On the interplay between topology and controllability of complex networks

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Complexity theory has been used to study of a wide range of systems in biology, nature but also in business and socio-technical systems, e.g., see [1]. The ultimate objective is to develop the capability of steering a complex system towards a desired outcome. Recent developments in network controllability [2] concerning the reworking of the problem of finding minimal control configurations allow the use of the polynomial time *Hopcroft-Karp algorithm* instead of exponential time solutions. Subsequent approaches built on this result to determine the precise control nodes, or *drivers*, in each minimal control configuration [3], [4]. A browser-based analytical tool has been developed in [5] which can be used by stakeholders in collaborative decision-making, and has been applied to policy-making in industrial networks.

One key characteristic of a complex system is that it continuously evolves, e.g., due to dynamic changes in the roles, states and behaviours of the entities involved. This means that in addition to determining driver nodes it is appropriate to consider an evolving topology of the underlying complex network, and investigate the effect of removing nodes (and edges) on the corresponding minimal control configurations. The study of the ability of each node to control the network in [6] showed that control centrality is determined by the degree distribution and in some cases (absence of loops in directed weighted networks) by the node's layer index.

In our work we take this one step further and investigate the effect of removing nodes (and edges) on the corresponding minimal control configurations. We propose a classification scheme that combines existing work on topological features [7, 8], with principles of controllability in complex networks. In particular, we consider three categories of nodes based on the effect their removal has on controllability, in terms of the cardinality of the maximum matching, C_{MM}, in the network: a node is *delete-redundant*, iff C_{MM} is unchanged; *delete-ordinary,* iff C_{MM} is reduced by one; and, *delete-critical* iff C_{MM} is reduced by more than one. We experimented with randomly generated directed networks of varying size, and studied pertinent characteristics such as in- and out-degree, average degree distribution as well as connectivity and isolated nodes.

Firstly, nodes from each category were removed and we examined the effect on the control configurations of the network. As the edge probability approaches 1, delete-critical nodes decrease rapidly while beyond 1 all nodes tend to become delete-ordinary. Secondly, some nodes (5%, 10%) were randomly removed and we examined the effect this has on the different categories of nodes in the network (for N=500, N=1000). It transpires that the delete-redundant category is the less stable category while delete-ordinary is the most stable as the probability of edge distribution increases.

The results of our analysis confirm our hypothesis – structural control theory provides information which is orthogonal to network analysis. The combined information provides a solid basis for the behavioural modeling of a complex system and can provide an important dimension when it comes to scenario appraisal for collaborative decision-making.

References

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