

# Robust reproduction of sound zones with local sound orientation

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**Abstract:** Pressure matching (PM) and planarity control (PC) methods can be used to reproduce local sound with a certain orientation at the listening zone, while suppressing the sound energy at the quiet zone. In this letter, regularized PM and PC, incorporating coarse error estimation, are introduced to increase the robustness in non-ideal reproduction scenarios. Facilitated by this, the interaction between regularization, robustness, (tuned) personal audio optimization, and local directional performance is explored. Simulations show that under certain conditions, PC and weighted PM achieve comparable performance, while PC is more robust to a poorly selected regularization parameter.

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## 1. Introduction

Sound zone systems render audio content to a listening zone through a set of loudspeakers, with minimal interference to other region(s) within a shared space. To increase the richness, quality, and functionality of sound content delivered to the listening zone, it is also desirable to generate a directional sound field at the listening zone. Two numerical optimization approaches are investigated for this purpose—pressure matching (PM) (Betlehem *et al.*, 2015), minimizing error between the reproduced and the desired sound pressures at the controlled points, and planarity control (PC) (Coleman *et al.*, 2014b), maximizing correlation between the reproduced sound field and the preferred directional components of the local sound field.

In practice, the reproduction system suffers performance loss caused by perturbations in reproduction, e.g., inconsistencies among loudspeakers' sensitivities. To increase robustness, regularized PM and PC are usually adopted (Coleman *et al.*, 2014a, 2014b). Several strategies can be used to determine the diagonal load, such as applying a constraint on array effort (AE) (Elliott *et al.*, 2012) or the largest singular-value (Shin *et al.*, 2014). However, it can be difficult to set an appropriate threshold. Monte-Carlo simulation (Bai and Chen, 2014) could be used to find the optimal solution, but at a high computational cost.

This letter introduces an alternative strategy to determine the diagonal loads for PM and PC. Probability-Model Optimization (PMO) (Doclo and Moonen, 2003) with an assumed coarse error model is applied to formulate regularized PM and PC (denoted as PM-AEQ and PC-AEQ, where AEQ represents using simple Additive Error model for “Quick” implementation). Their diagonal loads are determined by estimation of the maximal error amplitude. This strategy obtains a proper regularization parameter with relatively low complexity and computational cost, compared to the state-of-the-art methods mentioned above. Our previous work (Zhu *et al.*, 2017) showed that ACC-AEQ [i.e., regularized acoustic contrast control (ACC) (Choi and Kim, 2002) using the same approach] is capable of generating robust sound zones with acceptable contrast under non-ideal reproduction conditions. However, a well-known limitation of ACC (Coleman *et al.*, 2014a) is that it cannot generate directional sound at the listening zone.

In this letter, we extend our work in Zhu *et al.* (2017) and Coleman *et al.* (2014a, 2014b) with two novel contributions. First, we apply PMO to PC and PM, leading to novel variants of these methods (i.e., PM-AEQ and PC-AEQ), which directly include modulation matrices for regularization based on an estimate of the

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errors in the system. Second, we expose novel insights into the design of directional sound zone systems. Specifically, we show that PC-AEQ and (weighted) PM-AEQ can give equivalent performance, and that, for given error conditions, PC-AEQ is more robust than PM-AEQ to a wrongly estimated error bound.

## 2. Methods

The cost functions ( $J$ ) and the corresponding loudspeaker weights ( $\mathbf{w}$ ), for ACC, PM, and PC are listed in Table 1.  $\mathbf{G}_L$  and  $\mathbf{G}_Q$  are  $M \times L$  transfer function (TF) matrices defining the listening zone and the quiet zone, respectively, where  $L$  is the number of loudspeakers for reproduction and  $M$  is the number of control points in each zone. The  $m$ th row and  $l$ th column element  $\mathbf{G}_{(m,l)}$ ,  $m = 1, 2, \dots, M$ ,  $l = 1, 2, \dots, L$ , represents the sound propagation gain and delay between the  $l$ th loudspeaker and  $m$ th control point.  $(*)^H$  denotes the complex conjugate matrix transpose. The three sound zone optimizations pursue minimized sound energy ( $\|\mathbf{G}_Q \mathbf{w}\|^2$ ) at the quiet zone and have different objectives at the listening zone. At the listening zone, ACC maximizes the sound energy, PM aims to reproduce pre-defined sound amplitude and phase ( $\mathbf{p}_{\text{des}}$ ) at each control point, and PC maximizes the sound energy toward a certain spatial range. In ACC and PC,  $\Phi(\ast)$  denotes the eigenvector corresponding to the maximum eigenvalue of a matrix. As introduced in Coleman *et al.* (2014b),  $\Lambda = \mathbf{Y}^H \mathbf{\Gamma} \mathbf{Y}$  in PC, where  $\mathbf{\Gamma}$  is a diagonal matrix for selecting the range of acceptable angles, and the rows of  $\mathbf{Y}$  are populated by regularized fixed-beam width super-directive beamforming, which gives a high-resolution estimate of the plane wave components in the listening zone, following Coleman *et al.* (2014b). The non-negative parameters  $\lambda_{\text{ACC}}$ ,  $\lambda_{\text{PM}}$ , and  $\lambda_{\text{PC}}$  weight the relative importance between the listening zone and quiet zone objectives in the optimization.

The loudspeaker weights ( $\mathbf{w}'$ ) optimized by PMO are also listed in Table 1. The proposed modulation matrices  $\mathbf{E}_Q$ ,  $\mathbf{E}_L$ , and  $\mathbf{E}'_L$  [Eq. (2)], are introduced to the solutions. Assuming the errors ( $A_i$ ) in the acoustic TFs have a predictable distribution ( $f_{A_i}$ ), PMO aims to optimize the average cost function (Doclo and Moonen, 2003)

$$\bar{J} = \int_{A_1} \dots \int_{A_N} J(A_1, \dots, A_N) f_{A_1} \dots f_{A_N} dA_1 \dots dA_N. \tag{1}$$

Here,  $J$  can be  $J_{\text{ACC}}$ ,  $J_{\text{PM}}$ , or  $J_{\text{PC}}$ , listed in Table 1. Consider the case where additive error exists in the reproduction system and errors in different TFs are independent and identically distributed. Then,  $\tilde{\mathbf{G}}_{(m,l)} = \mathbf{G}_{(m,l)} + ae^{j\phi}$ , where  $j = \sqrt{-1}$ , and  $a$  and  $\phi$  are the amplitude and phase of the additive error. The statistical properties of the error are defined as  $\mu_a = \int_a a f_a da$ ,  $\mu_\phi = \int_\phi e^{-j\phi} f_\phi d\phi$ ,  $\sigma_a = \int_a a^2 f_a da$  and  $\sigma_\phi = (\int_\phi \cos \phi f_\phi d\phi)^2 + (\int_\phi \sin \phi f_\phi d\phi)^2$ . Defining  $\mathbf{O}_{X \times Y}$  as an  $X \times Y$  matrix with each element equal to one, and  $\mathbf{I}$  as an  $L \times L$  identity matrix, then

$$\begin{cases} \mathbf{E}_Q = \mu_a \mu_\phi^* \mathbf{O}_{L \times M} \mathbf{G}_Q + \mu_a \mu_\phi \mathbf{G}_Q^H \mathbf{O}_{M \times L} + M(\sigma_a - \mu_a^2 \sigma_\phi) \mathbf{I} + M \mu_a^2 \sigma_\phi \mathbf{O}_{L \times L}, \\ \mathbf{E}_L = \mu_a \mu_\phi^* \mathbf{O}_{L \times M} \mathbf{G}_L + \mu_a \mu_\phi \mathbf{G}_L^H \mathbf{O}_{M \times L} + M(\sigma_a - \mu_a^2 \sigma_\phi) \mathbf{I} + M \mu_a^2 \sigma_\phi \mathbf{O}_{L \times L}, \\ \mathbf{E}'_L = \mu_a \mu_\phi^* \mathbf{O}_{L \times M} \Lambda \mathbf{G}_L + \mu_a \mu_\phi \mathbf{G}_L^H \Lambda \mathbf{O}_{M \times L} + s_1(\sigma_a - \mu_a^2 \sigma_\phi) \mathbf{I} + s_2 \mu_a^2 \sigma_\phi \mathbf{O}_{L \times L}, \end{cases} \tag{2}$$

where  $\mu_\phi^*$  is the complex conjugate of  $\mu_\phi$ ,  $s_1$  is the sum of diagonal elements in  $\Lambda$ , and  $s_2$  is the sum of all the elements in  $\Lambda$ . Compared to Zhu *et al.* (2017), the expressions

Table 1. Personal audio optimization cost functions and solutions.

Cost functions	$\begin{aligned} \max_{\mathbf{w}} J_{\text{ACC}}(\mathbf{w}) &= \ \mathbf{G}_L \mathbf{w}\ ^2 - \lambda_{\text{ACC}} \ \mathbf{G}_Q \mathbf{w}\ ^2 \\ \min_{\mathbf{w}} J_{\text{PM}}(\mathbf{w}) &= \ \mathbf{G}_L \mathbf{w} - \mathbf{p}_{\text{des}}\ ^2 + \lambda_{\text{PM}} \ \mathbf{G}_Q \mathbf{w}\ ^2 \\ \max_{\mathbf{w}} J_{\text{PC}}(\mathbf{w}) &= (\mathbf{G}_L \mathbf{w})^H \Lambda (\mathbf{G}_L \mathbf{w}) - \lambda_{\text{PC}} \ \mathbf{G}_Q \mathbf{w}\ ^2 \end{aligned}$
Solutions	$\begin{aligned} \mathbf{w}_{\text{ACC}} &= \Phi[(\mathbf{G}_Q^H \mathbf{G}_Q)^{-1} \mathbf{G}_L^H \mathbf{G}_L] \text{ (Choi and Kim, 2002)} \\ \mathbf{w}_{\text{PM}} &= (\mathbf{G}_L^H \mathbf{G}_L + \lambda_{\text{PM}} \mathbf{G}_Q^H \mathbf{G}_Q)^{-1} \mathbf{G}_L^H \mathbf{p}_{\text{des}} \text{ (Betlehem et al., 2015)} \\ \mathbf{w}_{\text{PC}} &= \Phi[(\mathbf{G}_Q^H \mathbf{G}_Q)^{-1} \mathbf{G}_L^H \Lambda \mathbf{G}_L] \text{ (Coleman et al., 2014b)} \end{aligned}$
Robust solutions	$\begin{aligned} \mathbf{w}'_{\text{ACC}} &= \Phi[(\mathbf{G}_Q^H \mathbf{G}_Q + \mathbf{E}_Q)^{-1} (\mathbf{G}_L^H \mathbf{G}_L + \mathbf{E}_L)] \text{ (Zhu et al., 2017)} \\ \mathbf{w}'_{\text{PM}} &= [(\mathbf{G}_L^H \mathbf{G}_L + \mathbf{E}_L) + \lambda_{\text{PM}} (\mathbf{G}_Q^H \mathbf{G}_Q + \mathbf{E}_Q)]^{-1} (\mathbf{G}_L^H + \mu_a \mu_\phi) \mathbf{p}_{\text{des}} \\ \mathbf{w}'_{\text{PC}} &= \Phi[(\mathbf{G}_Q^H \mathbf{G}_Q + \mathbf{E}_Q)^{-1} (\mathbf{G}_L^H \Lambda \mathbf{G}_L + \mathbf{E}'_L)] \end{aligned}$

for  $\mathbf{E}_Q$  and  $\mathbf{E}_L$  are formulated with a uniform, simplified, and compact matrix form to expose the similarities and differences between them. Specifically, the first two terms describe the interaction between the assumed errors and TFs, and the last two terms show the correlation of the assumed errors. If  $\Lambda = \mathbf{I}$ ,  $\mathbf{E}'_L$  reduces to  $\mathbf{E}_L$ , and  $\mathbf{w}'_{PC}$  becomes  $\mathbf{w}'_{ACC}$ , revealing the relationship between ACC and PC.

Due to the difficulty in estimating the error distribution, it is practical to use a simple error model to approximate the actual error, and thereby only the error bound estimate (i.e., the estimate of the maximal error amplitude  $a_{max}$ ) is required to calculate the loudspeaker weights. As an example, in the following simulations, the actual error is roughly estimated by a simple error model with amplitude  $a$  uniformly distributed at  $[0, a_{max}]$  and phase  $\phi$  uniformly distributed at  $[0, 2\pi]$ . In this case,  $\mu_a = a_{max}/2$ ,  $\sigma_a = a_{max}^2/3$ ,  $\mu_\phi = 0$ , and  $\sigma_\phi = 0$ , leading to  $\mathbf{E}_L = \mathbf{E}_Q = M\sigma_a\mathbf{I}$ ,  $\mathbf{E}'_L = s_1\sigma_a\mathbf{I}$ . Thus, the diagonal loads  $\delta_Q = \delta_L = M\sigma_a$  and  $\delta'_L = s_1\sigma_a$  act on  $\mathbf{G}_Q^H\mathbf{G}_Q$ ,  $\mathbf{G}_L^H\mathbf{G}_L$ , and  $\mathbf{G}_L^H\Lambda\mathbf{G}_L$ , separately. PMO, with this simple error model assumption, is denoted as AEQ. The sound zone optimizations are then denoted as ACC-AEQ, PM-AEQ, and PC-AEQ.

### 3. Experimental conditions

PM-AEQ and PC-AEQ are tested on a realizable system for sound zone reproduction, with 11 loudspeakers directed toward the origin in an arc array and 48 control points in each zone. This system geometry is consistent with our previous work on regularized ACC (Zhu *et al.*, 2017, Fig. 2). The desired local sound orientation (LO) is first set as  $\theta_{des} = 180^\circ$ . Thus, the non-zero diagonal elements  $\gamma_\beta$  ( $\beta = 1, 2, \dots, 360$ ) of  $\Gamma$  in PC are  $\beta = 180$ , which is set to one, and  $\beta = 180 \pm 5$ , which are set with a raised-cosine weighting (Coleman *et al.*, 2014b). The elements of  $\mathbf{P}_{des}$  in PM correspond to those of a plane wave toward the desired orientation passing through the listening zone.

In the simulations, we suppose each loudspeaker acts as a monopole, defined by  $e^{jkr}/r$ , where  $k$  is the wave number and  $r$  is the distance between a loudspeaker and a control point. Perturbations are added to the spatial responses, assuming that the error has multiplicative form with Gaussian distribution between  $-3$  and  $+3$  dB in amplitude and uniform distribution between  $-10^\circ$  and  $+10^\circ$  in phase. Samples of  $\tilde{\mathbf{G}}_L$  and  $\tilde{\mathbf{G}}_Q$  are drawn from these distributions for Monte-Carlo trials.

The evaluation metrics of LO, acoustic contrast (AC), and AE are used to compare the performance of the optimization approaches. Similar to the definition of maximal (likelihood) direction of arrival, LO is defined as

$$LO(^{\circ}) = \max_{\beta} \mathbf{w}^H \tilde{\mathbf{G}}_L^H \mathbf{Y}_{\beta}^H \mathbf{Y}_{\beta} \tilde{\mathbf{G}}_L \mathbf{w}, \quad \beta = 1, 2, \dots, 360, \tag{3}$$

for each Monte-Carlo trial, where  $\mathbf{Y}_{\beta}$  is the  $\beta$ th row of  $\mathbf{Y}$ , introduced for PC. LO gives the direction of the sound wave at the listening zone. AC and AE are defined as

$$AC(dB) = 10 \log_{10} \left( \frac{\mathbf{w}^H \tilde{\mathbf{G}}_L^H \tilde{\mathbf{G}}_L \mathbf{w}}{\mathbf{w}^H \tilde{\mathbf{G}}_Q^H \tilde{\mathbf{G}}_Q \mathbf{w}} \right), \quad AE(dB) = 10 \log_{10} \left( \frac{\mathbf{w}^H \mathbf{w}}{|w_0|^2} \right), \tag{4}$$

where  $w_0$  is the input signal required to drive a single element at the center of the array so that the mean square pressure in the listening zone is the same as that when the array is driven by  $\mathbf{w}$  (Elliott *et al.*, 2012). AC describes the sound energy difference between the listening zone and the quiet zone, for each Monte-Carlo trial. The mean LO and mean AC averaged over 1000 Monte-Carlo trials describe the statistical performance of the system, and a low AE value refers to a reproduction with high energy efficiency.

### 4. Simulations

To make the assumed additive error set contain the real multiplicative error set, AEQ adopts  $a_{max} = \max\{\mathbf{G}\} \times a_{max,ME}$ , where  $\max\{\mathbf{G}\}$  is the maximum among all the TFs in  $\mathbf{G}$ , and  $a_{max,ME} = \sqrt{\mu^2 - 2\cos(\phi_{max})\mu + 1}$  with  $\mu = 10^{3/20}$ ,  $\phi_{max} = 10^\circ$ .

PM and PC applying AEQ are compared with two typical diagonal loading methods (SV and EL0). Compared with the solution formed by PMO, the traditional diagonal loading takes  $\mathbf{E}_Q = \delta_{DL}\mathbf{I}$ ,  $\mathbf{E}_L = \mathbf{E}'_L = \mathbf{0I}$  and  $\mu_a\mu_\phi = 0$ . SV applies  $\delta_{DL} = \sigma_1/10$ , where  $\sigma_1$  is the maximal singular value of  $\mathbf{G}_Q^H\mathbf{G}_Q$ . EL0 employs a  $\delta_{DL}$  leading to an AE equal to 0 dB. It can be derived that AEQ equals the traditional diagonal loading with  $\delta_{DL} = M\sigma_a(1 + 1/\lambda_{PM})$  in PM-AEQ and  $\delta_{DL} = M\sigma_a(1 - 1/\lambda_{PC} \cdot s_1/M)$  in PC-AEQ.

To compare PM and PC applying EL0, AEQ, and SV, their reproduced sound pressure level (SPL) distributions at 1 kHz, averaged over 1000 Monte-Carlo trials, are visualized in Fig. 1(a). It is shown that the AC between the zones and the LO at the

listening zone are both affected by the value of the diagonal load. The AEQ-optimized loudspeaker weights achieve the highest mean AC for both PM and PC. EL0 fails to deliver the main sound lobe toward the listening zone, leading to the worst mean AC among the methods. Due to the adopted compact arc array, the mean LO metric is relatively more stable than the mean AC metric. It can be observed that PM-AEQ and PM-EL0 have mean LO in accordance with the desired value (within  $1.3^\circ$ ). PM-SV, PC-AEQ, and PC-SV have small deviations (less than  $4.5^\circ$ ), while PC-EL0 deviates by  $15.4^\circ$ . Table 2 shows the AC, LO, and AE averaged over 100–3000 Hz, i.e., within the spatial aliasing limit for the studied loudspeaker array, sampled every 50 Hz. The results in Table 2 show the observations in Fig. 1(a) to hold over frequency. Interestingly, increasing the diagonal load tends to improve the LO accuracy for PC but reduces it for PM. Overall, applying AEQ regularization achieves a good balance between AC and LO under the error conditions tested.

From Table 2, PM-AEQ (with  $\lambda_{PM}=1$ ) has better mean LO at the cost of worse mean AC, compared to PC-AEQ. However,  $\lambda_{PM}$  can be tuned to control the trade-off between mean AC and mean LO (Table 1). Figure 1(b) shows the effect of varying  $\lambda_{PM}$  on AC and LO, alongside PC-AEQ and ACC-AEQ (Zhu *et al.*, 2017). Increasing  $\lambda_{PM}$  leads to better mean AC but worse mean LO. An interesting case is when  $\lambda_{PM}=10$ , marked with a magenta dashed line on Fig. 1(b). Here, the mean AC for PM-AEQ becomes very close to that of PC-AEQ. Moreover, PM-AEQ no longer exhibits an obvious advantage in terms of mean LO. Further increasing  $\lambda_{PM}$  leads to unchanged mean AC but much lower mean LO. The AE of PM-AEQ generally decreases with increasing  $\lambda_{PM}$ . The mean AC, mean LO, and AE for PM-AEQ with  $\lambda_{PM}=10^{-1}$ ,  $10^0$ , and  $10^1$ , averaged over frequency, can be found in Table 2. Overall, if mean AC performance has priority over mean LO in sound zone system design, PC-AEQ should be used, since judicious selection of the weighting parameter  $\lambda_{PM}$  is not required. Furthermore, the interaction between the weighted PM-AEQ and PC-AEQ indicates that further performance gains (in terms of balancing between AC and LO) are not achievable by the physical system under test.

A final consideration for PM-AEQ and PC-AEQ is the effect of the estimated error bound  $a_{max}$  on the system performance. To investigate the performance degradations caused by wrongly estimated error bounds, the mean AC performance of PM-

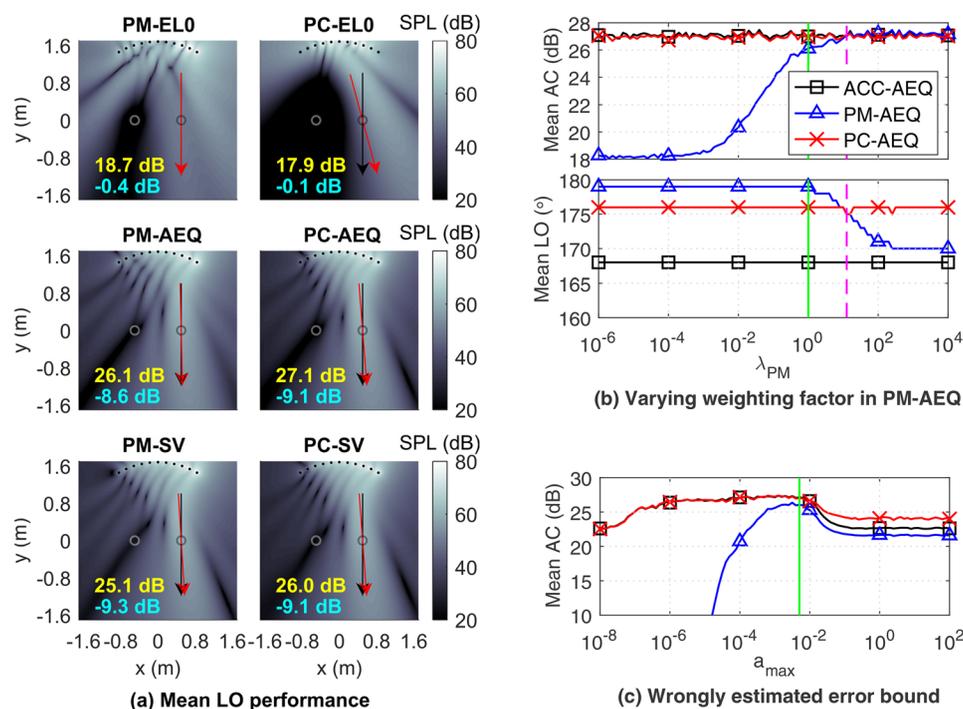


Fig. 1. (Color online) Simulated comparison at 1 kHz. (a) SPL of the reproduced sound field, averaged over 1000 Monte-Carlo trials, with EL0, AEQ, and SV applied to PM and PC. The black/white circles mark the listening/quiet zones. The black and red arrows mark the desired angle and the mean LO, respectively, in the listening zone. The mean AC (yellow) and AE (cyan) are also marked. (b) The mean AC and mean LO of PM-AEQ for varying  $\lambda_{PM}$ , alongside PC-AEQ and ACC-AEQ. (c) The mean AC of PM-AEQ, PC-AEQ, and ACC-AEQ applying different estimated error bound values into filter calculation. The green solid lines in (b) and (c) denote the initial values used. The magenta dashed lines in (b) denote  $\lambda_{PM}=10^1$ .

Table 2. Mean LO (Target 180°), mean AC and AE of different (tuned) personal audio optimization and regularization, averaged over 100–3000 Hz. Bold text illustrates the comparison between PC and tuned PM.

Personal Audio Optimization	PM							
	PC			$\lambda_{PM}$				
	EL0	AEQ	SV	$\lambda_{PM} = 1$		$\lambda_{PM} = 0.1$	$\lambda_{PM} = 10$	
Regularization	EL0	AEQ	SV	EL0	AEQ	SV	AEQ	AEQ
Mean LO (°)	168.6	<b>176.4</b>	176.7	179.8	177.6	176.5	178.0	<b>176.2</b>
Mean AC (dB)	19.3	<b>25.3</b>	24.0	17.4	23.9	23.1	22.0	<b>24.9</b>
AE (dB)	-1.5	<b>-8.2</b>	-8.3	0.0	-8.1	-8.5	-8.1	<b>-8.4</b>

AEQ, PC-AEQ, and ACC-AEQ using different  $a_{max}$  for filter calculation is presented in Fig. 1(c). The  $a_{max}$  value used previously [green line in Fig. 1(c)] leads to almost the optimal performance at 1 kHz. It can be observed that PC-AEQ has a broader range of  $a_{max}$  leading to a good mean AC (e.g., above 25 dB at 1 kHz), compared to PM-AEQ. We observed this pattern of results to hold for further simulations at 0.5 and 2 kHz. Overall, PC-AEQ is more robust than PM-AEQ to a wrongly estimated error bound of a certain actual error. This feature of PC-AEQ is very close to that of ACC-AEQ.

## 5. Conclusions

Robust reproduction of a certain sound orientation at the listening zone, along with a suppressed quiet zone, is desirable in real-world sound zone applications. In this letter, PM-AEQ and PC-AEQ were formulated to increase robustness against error in reproduction. With coarse error information incorporated, they obtained appropriate diagonal loads, giving better performance than other state-of-the-art approaches to determine the regularization parameter. Thereby, better performance on AC and the reproduced sound orientation at the listening zone was observed in simulations. The simulated result also showed that: PC-AEQ achieved almost the optimal AC with slight deviations from the desired LO; PM-AEQ can achieve close performance to PC-AEQ with a well-chosen weighting factor, but cannot offer better LO together with the same AC as PC-AEQ; PC-AEQ is more robust than PM-AEQ to the error bounds being wrongly estimated.

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