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NUMERICAL OPTIMIZATION OF LOUDSPEAKER CONFIGURATION FOR SOUND ZONE REPRODUCTION

Philip Coleman, Philip J. B. Jackson, Marek Olik

Centre for Vision, Speech and Signal Processing, University of Surrey, Guildford, Surrey, GU2 7XH, United Kingdom,
e-mail: p.d.coleman@surrey.ac.uk

Jan Abildgaard Pedersen

Bang & Olufsen A/S, Peter Bangs Vej 15, DK7600, Struer, Denmark

The topic of sound zone reproduction, whereby listeners sharing an acoustic space can receive personalized audio content, has been researched for a number of years. Recently, a number of sound zone systems have been realized, moving the concept towards becoming a practical reality. Current implementations of sound zone systems have relied upon conventional loudspeaker geometries such as linear and circular arrays. Line arrays may be compact, but do not necessarily give the system the opportunity to compensate for room reflections in real-world environments. Circular arrays give this opportunity, and also give greater flexibility for spatial audio reproduction, but typically require large numbers of loudspeakers in order to reproduce sound zones over an acceptable bandwidth. Therefore, one key area of research standing between the ideal capability and the performance of a physical system is that of establishing the number and location of the loudspeakers comprising the reproduction array. In this study, the topic of loudspeaker configurations was considered for two-zone reproduction, using a circular array of 60 loudspeakers as the candidate set for selection. A numerical search procedure was used to select a number of loudspeakers from the candidate set. The novel objective function driving the search comprised terms relating to the acoustic contrast between the zones, array effort, matrix condition number, and target zone planarity. The performance of the selected sets using acoustic contrast control was measured in an acoustically treated studio. Results demonstrate that the loudspeaker selection process has potential for maximising the contrast over frequency by increasing the minimum contrast over the frequency range 100–4000 Hz. The array effort and target planarity can also be optimised, depending on the formulation of the objective function. Future work should consider greater diversity of candidate locations.

1. Introduction

Many techniques exist by which sound zones can be reproduced over loudspeakers (e.g. [1, 2, 3, 4]). Although methods based on sound field synthesis may require specific loudspeaker and microphone geometries to represent the sound field in terms of convenient basis functions, many methods can theoretically be applied to arbitrary loudspeaker arrays. Nevertheless, classical circular and line array loudspeaker arrays have generally been adopted in the sound zone literature. Where relatively few loudspeakers have been available, line array geometries have usually been used, facilitating closer driver spacing and raising the upper frequency of cancellation performance. However, at low frequencies, a wider array aperture is desirable. Additionally, when line arrays are placed in reflective environments, the reflected energy may need compensation [5]. This may be partially achieved by steering the energy peaks and nulls appropriately to the reflecting surfaces [6], but the ability to use loudspeakers surrounding the zones (including near the reflecting surfaces) may also aid the room compensation. Therefore, when considering placement of a few loudspeakers, there are competing demands on array aperture, inter-element spacing and the compensation for reflections.

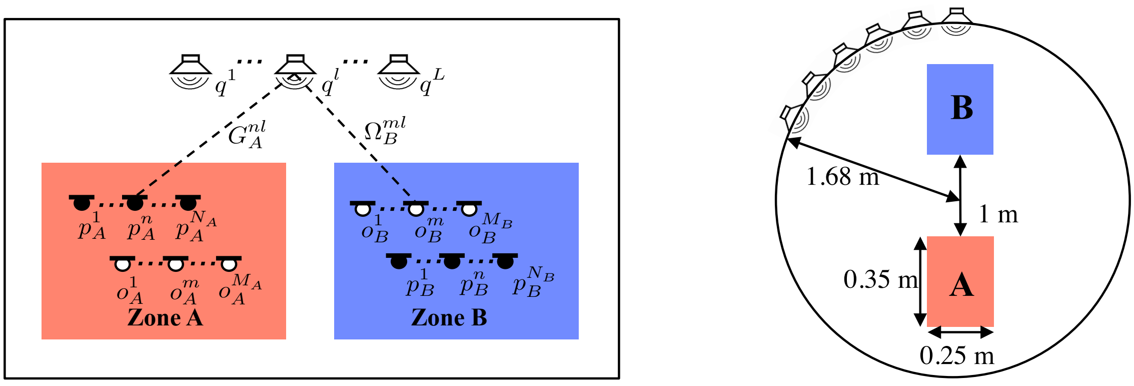
Loudspeaker placement for sound field reproduction has occasionally been investigated. In [7] an optimal combination of loudspeakers was selected for automotive sound zones using physical metrics, and in [8], a perceptual model of 'distraction' was used in the objective function. The robustness of crosstalk cancellation systems has also been addressed. In [9] the effect of loudspeaker spacing on matrix condition number and frequency was considered. This was extended in [10] by using several pairs of loudspeakers corresponding to different frequency ranges. A numerical search approach was used in [11], with the benefit of making the entire array available for crosstalk cancellation. For plane wave synthesis, a number of loudspeakers were selected from a spherical array, considering matrix condition number and desired reproduction accuracy as constraints [12], and in optimal locations have also been chosen for least-squares reproduction [13]. In [14], an ideal singular value matrix was defined based on a given number of loudspeakers, whose positions were then modified based on a candidate set.

The work presented in this paper is novel in that it focuses on a number of important properties specific to sound zone reproduction, uses measured transfer functions in a reflective environment, and optimizes over a large area (covering two fairly large zones) with up to 30 loudspeakers. We consider reproduction using acoustic contrast control (ACC) [1], choosing loudspeakers from a candidate set comprising a 60 channel circular array. Experimental results measured in a reflective room are presented for optimizations considering several objective function elements pertinent to sound zones.

2. Background

Figure 1 shows the sound zone system notation and geometry. Two audio programs A and B are to be reproduced in zones A and B, respectively. The rest of the room is uncontrolled. The zones (defined by the control microphone positions) and loudspeakers may be placed arbitrarily in the room. For each frequency, the source weights can be written in vector notation as $\mathbf{q} = [q^1, q^2, \ldots, q^L]^T$, where there are L loudspeakers and q^l is the complex source weight of the lth loudspeaker. Similarly, the complex pressures at the control microphone positions in zones A and B are written as $\mathbf{p}_A = [p_A^1, p_A^2, \ldots, p_A^{N_A}]^T$ and $\mathbf{p}_B = [p_B^1, p_B^2, \ldots, p_B^{N_B}]^T$ respectively, where there are N_A control microphones in zone A and N_B in zone B, and the complex pressures at the nth microphones in each zone are p_A^n and p_B^n. The observed pressures at the monitor microphones in each zone are denoted as $\mathbf{o}_A = [o_A^1, o_A^2, \ldots, o_A^{M_A}]^T$ and $\mathbf{o}_B = [o_B^1, o_B^2, \ldots, o_B^{M_B}]^T$ respectively, where there are M_A monitor microphones in zone A and M_B in zone B, and the complex pressures at the mth microphones in each zone are o_A^m and o_B^m. Spatially distinct microphones are used in order to reduce possible bias due to measurement of performance at the exact control positions. The pressures at the microphones may be written as $\mathbf{p}_A = \mathbf{G}_A\mathbf{q}$, $\mathbf{o}_A = \mathbf{\Omega}_A\mathbf{q}$, $\mathbf{p}_B = \mathbf{G}_B\mathbf{q}$ and $\mathbf{o}_B = \mathbf{\Omega}_B\mathbf{q}$, where $\mathbf{G}_A$ and $\mathbf{\Omega}_A$ are the control and monitor microphone plant matrices, respectively, with respect to zone A, and similarly $\mathbf{G}_B$ and $\mathbf{\Omega}_B$ are the plant matrices with respect to zone B. The notation is illustrated in Figure 1a.

To evaluate the system based on the monitor microphone pressures, the metrics of acoustic contrast, control effort and planarity were used, as in [15]. Acoustic contrast C describes the attenuation achieved between the zones, control effort E is the energy that the loudspeaker array requires, relative to a reference source q_r producing the same pressure in the bright zone, and planarity η is the extent



(a) Notation (b) Geometry

Figure 1: Experiment (a) notation and (b) reproduction system geometry (not to scale).

to which the sound field in the bright zone resembles a plane wave [16]. The metrics are defined as:

$$C = 10\log_{10}\left(\frac{M_B \mathbf{o}_A^H \mathbf{o}_A}{M_A \mathbf{o}_B^H \mathbf{o}_B}\right);\; E = 10\log_{10}\left(\frac{\mathbf{q}^H\mathbf{q}}{|q_r|^2}\right);\; \eta = \frac{\sum_i w_i \mathbf{u}_i \cdot \mathbf{u}_\alpha}{\sum_i w_i}, \tag{1}$$

where $\mathbf{u}_i$ is the unit vector associated with the ith component's direction, $\mathbf{u}_\alpha$ is the unit vector in the principal direction $\alpha = \arg\max_i w_i$, and $\cdot$ denotes the inner product. The energy components $w_i = \frac{1}{2}|\psi_i|^2$ at the ith angle correspond to the plane wave components ψ_i, and $\mathbf{w} = [w_1, w_2, \ldots, w_I]^T$. The steering matrix $\mathbf{H}_A$ $(I \times M_A)$, populated by a regularized max-SNR beamformer with fixed beamwidth [2, 15, 16], maps between the observed pressures at the microphones and the plane wave components, $\mathbf{w} = \frac{1}{2}|\mathbf{H}_A\mathbf{o}_A|^2$.

3. Acoustic contrast control

The ACC cost function is written as a minimization of the pressure in the zone B, with constraints on the zone A sound pressure level and array effort [1, 17]:

$$J = \mathbf{p}_B^H\mathbf{p}_B + \mu(\mathbf{p}_A^H\mathbf{p}_A - A) + \lambda(\mathbf{q}^H\mathbf{q} - Q), \tag{2}$$

where $A = N_A|p_r|^2 \times 10^{T/10}$, with T as the target spatially averaged level in decibels relative to the threshold of hearing $p_r = 20\ \mu$Pa, and $Q = |q_r|^2 \times 10^{E/10}$ (cf. Eq. (1)). The cost function may be minimized by taking the derivatives with respect to $\mathbf{q}$ and the Lagrange multipliers μ and λ, and setting to zero:

$$-(\mathbf{G}_A^H\mathbf{G}_A)^{-1}(\mathbf{G}_B^H\mathbf{G}_B + \lambda\mathbf{I})\mathbf{q} = \mu\mathbf{q};\;\; \mathbf{p}_A^H\mathbf{p}_A = A;\;\; \mathbf{q}^H\mathbf{q} = Q, \tag{3}$$

The eigenvector $\hat{\mathbf{q}}$ corresponding to the maximum eigenvalue of $(\mathbf{G}_B^H\mathbf{G}_B + \lambda\mathbf{I})^{-1}(\mathbf{G}_A^H\mathbf{G}_A)$ [17] is proportional to the solution $\mathbf{q}$. The constraints are met, as in [15], by appropriate scaling.

4. Search procedure

The objective function selection is formulated similarly to [11] (although here we consider robustness in terms of errors rather than the 'sweet spot' size), and is given as:

$$Y = \upsilon_c C - \upsilon_e E + \upsilon_m M + \upsilon_\eta \eta, \tag{4}$$

where C, E and η are defined in Eq. (1), υ indicates a real weighting value pertaining to the term indicated by the underscore, and $M = -10\log_{10}\left(\|\mathbf{G}_B^H\mathbf{G}_B\|_1\|(\mathbf{G}_B^H\mathbf{G}_B)^{-1}\|_1\right)$. If each coefficient were

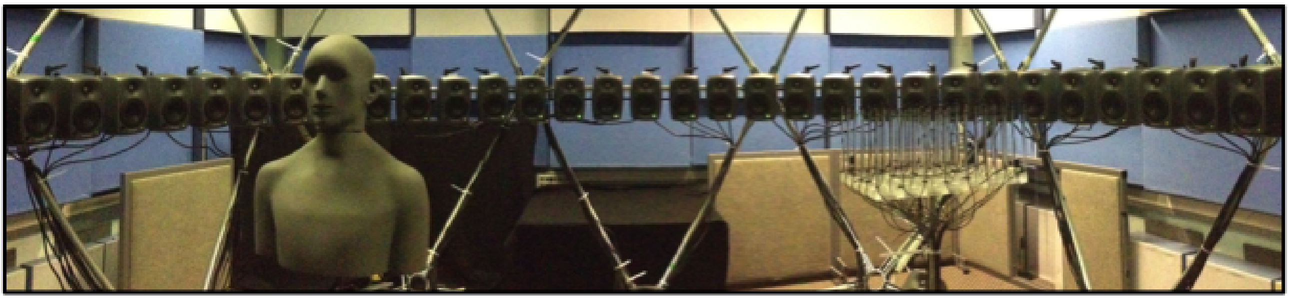


Figure 2: Wide-angle photograph of the experimental system showing the circular loudspeaker array

set at 1, then 10 dB of acoustic contrast would trade off against 10 dB of effort, a matrix condition number reduction by a factor of 10, and 10% of planarity. The matrix condition number penalty M is similar to [12] but uses the logarithm of the reciprocal matrix condition number as this allows the penalty to tend towards minus infinity. A sequential forward-backward search (SFBS) [18] was used to select a number of loudspeakers from the candidate positions. This is fast and simple, yet allows for a backward search step to help avoid nesting of a solution. The SFBS comprised two iterations of a sequential forward search algorithm, followed by one iteration of a sequential backward search algorithm. In order to maximise the performance in both zones, the ranking was based on the minimum of the zone A and zone B scores.

5. Reproduction system realization

A reproduction and measurement system was designed and mounted on a bespoke spherical structure, the "Surrey Sound Sphere", placed in an acoustically treated room of dimensions 6.55 $\times$ 8.78 $\times$ 4.02 m (RT60 235 ms averaged over 0.5 kHz, 1 kHz and 2 kHz octave bands). The loudspeakers (Genelec 8020b) were clamped to the equator of the sphere to form a 60 channel circular array (as Figure 1b) used as the candidate set, and 48 microphones (Countryman B3 omni) were attached to a grid mounted on a microphone stand, with 8 positions of the microphone stand measured per zone. A photograph is shown in Fig. 2. A Mac Pro computer running Matlab was used to play the audio and also to record the signals from the microphones, via the 'playrec' utility. A 72 channel MOTU PCIe 424 sound card was used, with the microphone inputs first passed through a pre-amplifier (PreSonus Digimax D8). Level differences between the input and output signal channels were compensated through calibration. Room impulse responses (RIRs) between each microphone position and each loudspeaker were measured at 48 kHz using the maximum length sequence (MLS) approach (15th order) and cropped at 150 ms. Finite impulse response (FIR) filters were populated and measured by considering a bin-by-bin approach. The RIRs were first down-sampled to 20 kHz, and a 8192 point fast Fourier transform (FFT) was taken. The source weights were collated, the negative frequency bins populated by complex conjugation, and the inverse FFT taken to obtain a time-domain filter. Regularization was applied by initializing λ (Eq. (3)) such that the condition number of the matrix to be inverted did not exceed 10^{10}, before enforcing a control effort limit of 0 dB relative to a single loudspeaker equidistant from both zones reproducing the same sound pressure level (76 dB) in the bright zone [15]. A 4096 sample modelling delay was applied to ensure causality. Performance was measured by convolving an MLS sequence with each of the FIR filters, replaying it through the loudspeakers, and sampling the reproduced sound pressures with the microphone array. Finally, the FFT was taken of the recorded system responses, and the evaluation metrics were calculated in the frequency domain.

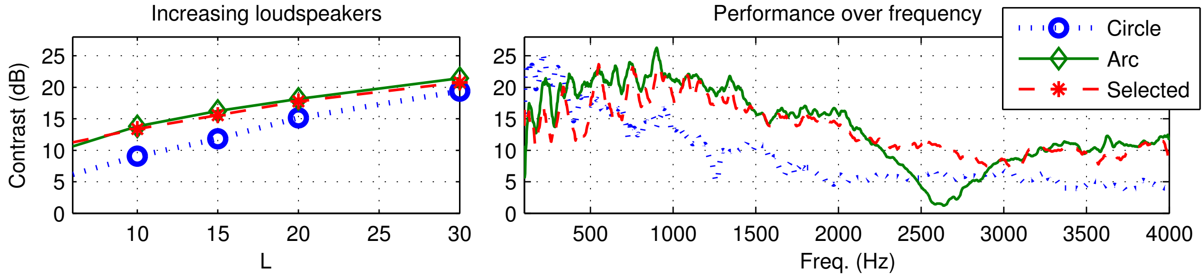


Figure 3: Acoustic contrast using the contrast-only cost function for selecting a number of loudspeakers from the candidate set (left) and the performance over frequency for the 10 loudspeaker case (right).

6. Performance

At each step of the SFBS algorithm, filter weights were calculated based on the loudspeakers populating each set. Equation (4) was evaluated based on predictions of sound pressure at the monitor microphone positions, obtained in the frequency domain by multiplying the source weights with the measured transfer functions between the microphone positions and the loudspeakers. The scores were calculated as the unweighted mean of the performance at the frequency bins nearest to 100 Hz intervals between 100–4000 Hz. When the loudspeaker set had been chosen, a final set of filters was calculated based on the chosen set, the performance of which was measured with the pressure microphone array. Thus, the recorded performance of the loudspeaker sets was independent from the predicted values, both in that the full bandwidth was considered for evaluation, and in that experimental measurement errors were present between setup and playback. Baseline circular and arc arrays were used for comparison with the selected sets. The trade-off between aperture and spacing is seen from Fig. 3, with the circular array superior below 500 Hz, and the arc array superior at higher frequencies. The objective function weightings used are shown in Table 1.

6.1 Optimal positioning of a fixed number of loudspeakers

First, selection of the optimal combination of a fixed number of sources was considered. Filters were calculated and the performance measured using 6, 10, 15, 20 and 30 loudspeakers, for the reference arrays and using the contrast-only cost function. The results are presented in Fig. 3. The mean scores plotted were calculated in the frequency domain over all frequency bins between 100–4000 Hz. For comparison, the mean measured performance over 100–4000 Hz using all 60 loudspeakers was 24.3 dB, 23.0 dB and 15.2 dB for ACC, PC and PM, respectively.

From Fig. 3 (left), it is clear that the circular array was suboptimal in terms of acoustic contrast for all control methods. The selected set can be noted, for each control method, marginally to outperform the reference arc with 6 loudspeakers. However, the optimal set of 6 loudspeakers also comprised an (off-centre) arc. The performance may therefore have been improved in the optimally selected set by accounting for the interaction with the room and zone geometry. Similarly, although the performance was only measured for target zone A, the selected arcs were designed to maximize performance across both zones. For greater numbers of loudspeakers, the reference arc array slightly outperformed the selected arrays. There are a number of possible reasons for this, including potential increased overall performance (i.e. to both zones), experimental errors leading to inaccurate predictions, and increased noise for a particular frequency bin influencing the scores. Moreover, there may not have been sufficient freedom in the selection procedure to reconfigure the array from 6 loudspeakers (where the selected set outperformed both references) to greater numbers.

To gain greater insight into the loudspeaker sets selected by the optimization procedure, the measured contrast was studied across frequency. Figure 3 (right) shows this representation of the

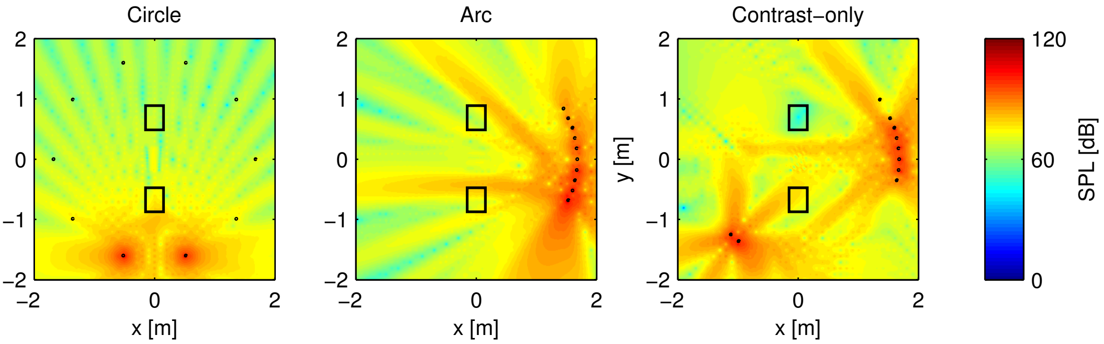


Figure 4: Sound pressure level distribution at 2650 Hz for ACC applied to 10 element loudspeaker arrays: contrast-only selected (left), arc (centre) and circle (right). The loudspeaker positions are marked with black circles. Source weights and sound pressures were based on anechoic responses for simulated sources and sensors at the same locations as the physical loudspeakers and microphones.

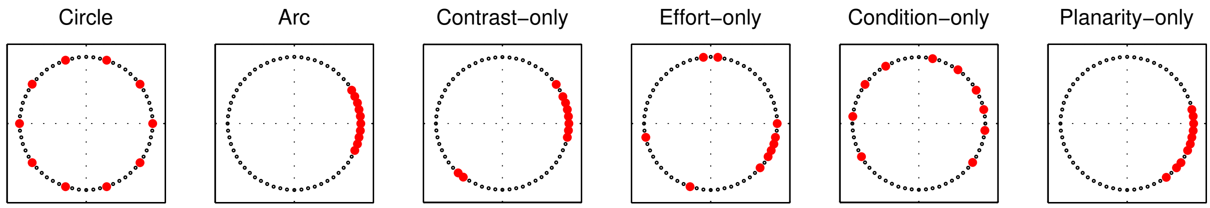


Figure 5: Arrays of 10 loudspeakers showing the reference circle and arc, and sets chosen using each objective function element in turn.

measured performance of each set, for 10 loudspeakers. An interesting trade-off between the minimum and maximum contrast can be noted. In particular, although the mean contrast scores were very similar for both the selected array and the reference arc (13.4 and 13.7 dB, respectively), the minimum (smoothed) contrast scores were 7.2 and 1.2 dB, respectively. So, although the selected set exhibited a lower contrast score than the arc below 2 kHz, it reduced the effect of the dip in contrast between 2–3 kHz. A visualization of the sound pressure level in a simulated anechoic room with equivalent geometry at 2650 Hz, corresponding to the frequency at which the selection procedure yielded the most benefit, is shown in Fig. 4, against the reference arrays. It is evident that combination of all 10 loudspeakers allows increased cancellation at this frequency, due to the ability of the array to create multiple beams focusing on the bright zone, where the arc array suffers from grating lobes.

6.2 Positioning to achieve desired performance characteristics

The objective function introduced in Eq. (4) contains terms relating to four physical evaluation criteria. For comparison against the contrast-only case, the loudspeaker selection procedure was run using effort-only, condition-only and planarity-only weightings. The weightings and results of these experiments are shown in Table 1. Considering contrast, the loudspeaker sets chosen using the contrast-only and planarity-only cost functions performed the best. The planarity-only set outperformed the contrast-only set by 1.2 dB averaged over the frequency range 100–4000 Hz, although the contrast-only set still marginally achieved the highest minimum contrast (0.2 dB better than planarity-only). The effort-only and condition-only sets gave poorer contrast scores. These results suggest that the compact array geometries achieved by maximizing the target zone planarity are also beneficial in terms of the achieved contrast. Conversely, the effort-only and condition-only selected sets gave the best performance in terms of control effort. The condition-only sets gave performance close to the circular array results. The lowest effort (and also the lowest contrast) was achieved with this set.

Table 1: Objective function weightings and mean and minimum performance of the selected sets, over the range 100–4000 Hz.

	Weights				C (dB)		E (dB)		η (%)	
	υ_c	υ_e	υ_m	υ_η	Mean	Min.	Mean	Min.	Mean	Min.
contrast-only	1	0	0	0	13.4	**7.2**	-4.1	-0.1	60.3	4.0
effort-only	0	1	0	0	13.0	5.2	-4.9	-0.4	74.1	**23.9**
condition-only	0	0	1	0	6.2	0.7	**-9.3**	**-4.9**	36.4	-2.7
planarity-only	0	0	0	1	**14.6**	7.0	-4.4	-0.0	**83.5**	23.7

The planarity scores were highest with the planarity-only and effort-only sets. The effort-only and condition-only scores diverged under the planarity metric, with the effort-only metric giving arrays which reproduced relatively high planarity scores, suggesting that groups of sources combining as a beamformer use relatively little power for sound zone reproduction with few loudspeakers. Conversely, the condition-only set comprised an array with greater distance between the sources, which inevitably led to poor planarity scores for ACC, as for the circular arrays [15]. Although very basic weightings between the objective function elements were considered here, the individual components largely gave the expected performance.

7. Discussion

The loudspeaker selection investigation presented above may be considered as a preliminary study in to the kinds of irregular array geometries available for a limited number of loudspeakers, and the corresponding performance characteristics. Improvements may be made with a more extensive candidate set, and by improving the accuracy of performance predictions based on the measured RIRs in a reflective room. Furthermore, each objective function element was designed to correspond to a certain desirable feature of sound zones, and the weightings may be adjusted based on physical or perceptual criteria. Finally, the constraints on the ACC optimization were fixed throughout the above experiments, including a 0 dB control effort limit and a maximum matrix condition number of 10^{10}. The relationship between these constraints and the loudspeaker selection weighting coefficients may be explored. The concepts presented here may readily be applied to determining the best positions with fixed loudspeaker resources. Such a situation may occur in consumer living rooms, where using the proposed approach the design of the room, desired sound zone positions and desired source direction (e.g. a television) would all be considered. Similarly, loudspeakers installed with severe restrictions on placement, such as in cars, aeroplanes and offices, may be best combined to produce the desired sound field characteristics.

8. Summary

Motivated by the need to reduce the number of loudspeakers utilized in a practical sound zone system, a loudspeaker selection procedure was proposed. A novel objective function comprising weighted terms relating to contrast, effort, matrix condition number and planarity was applied to select subsets of loudspeakers based on various objective function weightings. The contrast-only set performed the best over 100-4000 Hz for 6 loudspeakers in terms of the mean contrast (measured in target zone A). Improvement in the minimum contrast over frequency was obtained with 10 loudspeakers. By altering the weight, the contrast-only and planarity-only sets gave the best contrast; effort-only and condition-only sets gave the least effort; and planarity-only and effort-only gave the best planarity. Further work may refine the search method, extend the candidate set, and explore objective function coefficient weightings.

9. Acknowledgements

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