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An Axial- Azimuthal Hybrid Simulation of Coaxial Hall Thrusters

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We report on progress towards the development of a Hall thruster simulation in the axial-azimuthal (z - θ) computational space. Unlike most computational studies of closed-drift Hall accelerators which have been in one dimension (1D) along the axial direction or in two dimensions (2D) in the axial and radial dimensions, and which require some specification of the axial transport mechanism, this z - θ numerical simulation developed here self-consistently evolves the azimuthal electron drift velocity. The simulation is, in principal, capable of capturing correlated azimuthal disturbances in plasma properties which may give rise to cross-field transport, and makes no use of *ad-hoc* transport models. Preliminary analysis of the results indicates that azimuthal plasma instabilities may contribute to the axial electron transport process.

Nomenclature

$\mathbf{B}$ = magnetic induction vector
$\bar{c}_e$ = mean electron speed
$D_\perp$ = classical cross-field electron diffusion coefficient
e = electron charge
$\mathbf{E}$ = electric field vector
E_θ = azimuthal electric field component
E_z = axial electric field component
I_{sp} = specific impulse
k = Boltzmann constant
K_B = mobility scaling constant
m_e = electron mass
r = radial coordinate
T = electron temperature
$u_{e\theta}$ = azimuthal electron velocity
θ = azimuthal coordinate
σ_{en} = electron-neutral elastic collision cross section
$\mu_\perp$ = cross-field electron mobility
ν = electron-neutral collision frequency
ω_c = electron cyclotron frequency
z = axial coordinate

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I. Introduction

HALL thrusters are an electric propulsion technology that have attracted considerable interest concomitant with the increased number of government and commercial satellites in orbit.[1] The main use of these relatively low power, high I_{sp} rockets lies in low thrust applications such as orbit transfer and orbit station keeping. In Hall thruster plasmas, electrons emitted from a cathode, migrate towards the anode at the base of the discharge channel, and ionize xenon neutrals upon collision. The resulting ions are accelerated out of the device by the electric potential drop between the cathode and the anode, typically a few hundred volts. Such a potential drop is localized near the exit of the channel by an external radial magnetic field. While the overall operation of these thrusters is reasonably understood, key physics issues remain open for investigation. Of particular interest is the study of electron transport across the applied magnetic field. It is well known that the electron conductivity in the Hall thruster is much larger than can be inferred from classical collisions alone.[2] Fluctuations in plasma properties and electron scattering from the channel wall are believed to have a major effect on the local cross-field transport, which, in turn, affects the local electric field, ionization and acceleration of the ions, thrust, and overall discharge behavior. A good understanding of the processes involved in determining this anomalous transport remains the subject of important research.

With the exception of the work by Hirakawa,[4] and Adams et al.,[5] prior computational work on Hall thrusters has been one dimensional (1D) in the axial (z) direction or two dimensional (2D) in the axial and radial (r) directions. These prior studies have had reasonable success in describing the overall behavior of the plasma discharge. Most notably, they capture the dominant instability in these devices, the so-called "breathing mode" instability. However missing in these descriptions is the azimuthal electron dynamics. Besides drifting axially along z, opposite to the electric field, electrons in the thruster drift azimuthally as a result of the imposed crossed ($\boldsymbol{E} \times \boldsymbol{B}$) electric and magnetic fields. Azimuthal perturbations arise from the established equilibrium and, if properly correlated, result in a net axial transport of electrons.[6] This effect is not captured in z - r descriptions and the anomalous axial cross-field transport needs to be modeled. Most 2D studies have used a Bohm-type mobility ($1/B$ scaling) given by

$$\mu_{\perp} = K_B \frac{1}{16B} \tag{1}$$

as a model for the electron mobility across the magnetic field,[3,7] where K_B is an adjustable constant. Many plasma transport experiments have demonstrated that electron transport can vary greatly and does not always conform to a Bohm-type scaling. A recent measurement of electron mobility in Hall discharge channel has revealed that such transport is a complicated function of axial location as well as operating parameters.[8] In particular, at high voltage, Bohm transport seems to fare well in the middle of the channel while near the channel exit the transport is reduced to near its classical value, and beyond the channel exit, it appears to be anomalous again. It is apparent from this study that a single model does not properly describe the full plasma region from the anode to the near plume. The use of this experimental spatially-varying mobility in 2D simulations reproduces the measured plasma properties reasonably well,[9] and such a model has been used to predict the erosion history of the channel walls.[10] However, this mobility applies only to the discharge geometry and conditions for which it was measured, and it is not expected to apply to other thrusters. The lack of a transport model for simulating changing geometries and changing operating conditions severely limits the flexibility and usefulness of these 2D simulations. Reliable Hall thruster simulations are a vital step toward developing and implementing Hall thruster technology, both to support laboratory experiments and to predict the thruster performance in space applications.

This paper describes describes our continued progress towards the development of a numerical simulation of a Hall thruster carried out in the axial-azimuthal (z - θ) coordinate plane. This model is based on our previous experience in formulating a z - r hybrid particle-in-cell (PIC) simulation, as described in Fernandez, Cappelli, and Mahesh[3]. The z - θ model self-consistently evolves the azimuthal drifts and makes no use of transport parameters. This is to be contrasted with z - r models, many of which are based on the original model of Fife.[11] A schematic of the computational space for the z - r and z - θ model is provided in the illustration in Fig. 1 and Fig. 2 respectively. It is hoped that the z - θ simulations can capture high frequency azimuthally-propagating instabilities, and in doing so might account for the anomalous axial transport of electrons. Like our past z - r simulation, this z - θ simulation uses a hybrid fluid/particle-in-cell (PIC) description with the electrons treated as a fluid, and the ions and neutrals treated as discrete particles.

As described in more detail below and previously[12] the simulation was run for different operating conditions tested in the laboratory. The preliminary results indicate that the simulation was able to capture high frequency plasma oscillations that resemble experimental observations of emission fluctuations seen using high speed streak photography. Computed contributions to the axial current due to azimuthal fluctuations were comparable in magnitude to those attributed to classical electron scattering, however, they appeared to be much lower than needed

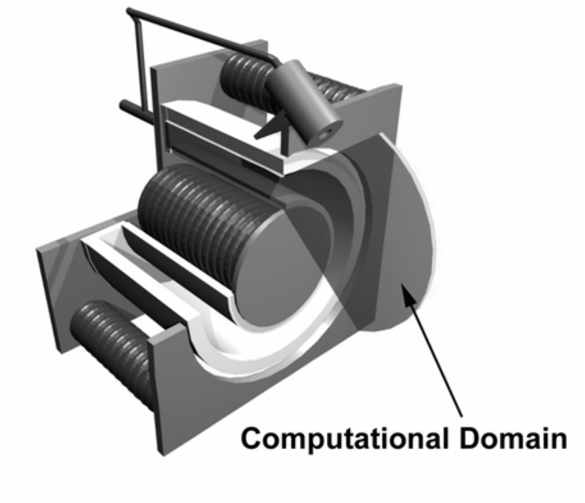

Figure 1. Traditional *z* - *r* computational plane

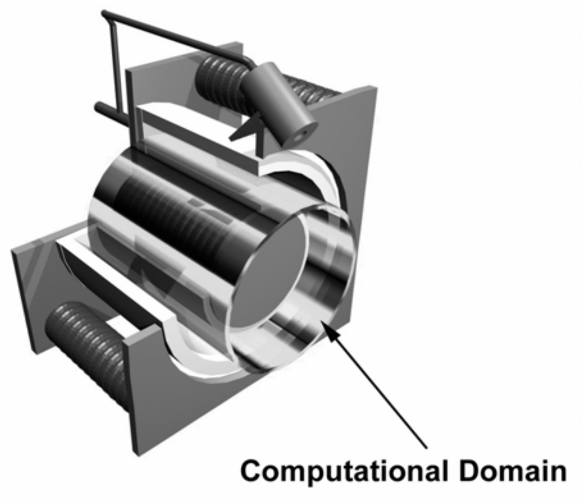

Figure 2. Alternative *z* - θ computational plane

to account for the measured discharge current. Since these earlier computational studies, which exhibited some numerical instabilities resulting in unphysical plasma potential spikes, we have implemented new numerical routines to reduce computational time and increase numerical accuracy. These previous computational results, along with the preliminary results from the computations are briefly described below.

II. Numerical Model

The 2D (z - θ) cylindrical coordinate system is used in the simulation. First, we construct the simplest description able to treat electron transport self-consistently. In particular, the new description must resolve the inhomogeneous Hall current and the associated azimuthal fluctuations. As mentioned above, azimuthal waves have been seen experimentally in a number of Hall thrusters. Their amplitude is large and their power spectrum is complex, with coherent structures among random, broadband turbulence. Linear stability analyses have predicted azimuthal unstable waves at low frequency (a few hundred kilohertz) and at high frequency (megahertz), driven unstable by resistivity and equilibrium gradients in plasma density and magnetic field.[13] A nonlinear theory describing this cross-field transport mechanism is presently absent, except under simplifying assumptions. An example is the theory of Yoshikawa and Rose[14] which predicts a Bohm scaling for the electron transport with a coefficient proportional to the relative electron density fluctuation power. An objective of the model presented in this paper is to gain some understanding about the azimuthal fluctuations and their associated transport.

The z - θ model draws from our experience with the z – r model, which treats the neutral and singly-ionized xenon constituents as discrete particles, and the electrons as a fluid. The main differences come from the electron fluid description, which includes the steady-state electron momentum equations for migration perpendicular and parallel to the magnetic field lines. The system is closed with an equation for quasi-neutrality. For the results presented here, it is assumed that the magnetic field is purely radial (obtained from experiments), and does not vary with azimuthal position. The electron temperature is assumed to be constant in the azimuth direction and its axial variation is specified in accordance with that measured experimentally. This later assumption is particularly severe. We make it at this point in order to simplify the system. A time-dependent electron energy equation that includes ionization, joule heating, wall damping (including sheath saturation[15]), and conductive and convective fluxes will be implemented in future simulations. In time, the former assumption about the azimuthal symmetry of the magnetic field will also be relaxed, as it is found that axial asymmetry in the magnetic field appears to mode-lock lower frequency oscillations to the configuration of the magnetic poles.[16]

With these assumptions the electron momentum equation for the axial component to the electron drift velocity, u_{ez}, becomes:

$$u_{ez} = -\mu_\perp E_z - \frac{D_\perp}{n_e}\frac{\partial n_e}{\partial z} - \frac{1}{1+(\nu/\omega_c)^2}\frac{E_\theta}{B} - \frac{1}{1+(\nu/\omega_c)^2}\frac{kT_e}{en_eBr}\frac{\partial n_e}{\partial \theta} \tag{2}$$

Here, $u_{e\theta}$ is the azimuthal electron velocity, $\mu_\perp$ is the classical perpendicular mobility, ν is the electron-neutral collision frequency, ω_c is the electron cyclotron frequency, and $D_\perp$ is the classical diffusion coefficient arising from electron-neutral collisions. The last four quantities are given by:

$$\mu_\perp = \frac{e/m_e\nu}{1+(\nu/\omega_c)^2} \tag{3}$$

$$\nu = n_n \sigma_{en} \bar{c}_e \tag{4}$$

$$\omega_c = \frac{eB}{m_e} \tag{5}$$

$$D_\perp = \mu_\perp \frac{kT}{e} \tag{6}$$

In Eq. (4), $\sigma_{en} = 27 \times 10^{-20}$ m^2 is the electron-neutral elastic collision cross section, $\bar{c}_e$ is the mean electron speed (assuming a Maxwellian distribution), and m_e is the electron mass. We note that the axial electron momentum equation (Eq. (2)) now has components involving E_θ and $\partial n_e/\partial\theta$. These terms are important at low neutral densities and high magnetic fields. We expect these terms to dominate near the exit of the channel, where ionization depletes the neutral concentration and the magnetic field is strongest. In the opposite limit, at high neutral densities and low magnetic fields characteristic of the anode region, these terms are expected to be small. A full description of the equations solved for the electron fluid component is given in Ref. 12.

The treatment of the ions and the neutrals as particles follows that of the z - r model. Particle injection is done by inverting a Maxwellian flux distribution function.[17] The nonlinear ionization rate is given by a fit to experiment according to the formula proposed by Ahedo et al.[18] The geometry used in the simulation corresponds to that of the Stanford Hall Thruster (SHT), with a channel length of approximately 8 cm, and a circumference of approximately 34 cm. The computational domain extends from z = 0 (anode) to z = 12 cm (4 cm past the channel exit).

The boundary conditions for the electric potential in the axial coordinate are Dirichlet at the anode, where the anode potential is set at the discharge voltage ($\phi_{anode} = \phi_{discharge}$). A Dirichlet condition is also applied at the downstream boundary (downstream of the exit plane at a presumed location of the cathode plane), where the potential is set to zero ($\phi_{zmax} = 0$). In the azimuthal direction the boundary condition is periodic. As mentioned above, our earlier results were obtained with a direct-solve method. The solution is found via a direct-solve method. The time step size for the iteration on the electron fluid equations is computed using the stability requirement[5] that it be less than the electron plasma frequency. Details of the discretization of the electron fluid equations and second-order finite difference technique used in our original formulation can be found in the paper by Fernandez et al.[19] Reference 19 also presents details on the advancement of the particles using the PIC method. The overall flow process is shown schematically in Fig. 3. Note that the electron fluid equations are solved in steady state, and only the ions and neutrals are advanced each time-step. The determination of ion particle positions gives, by quasi-neutrality, the plasma density. The axial electron current $J_{ez} = en_e u_{ez}$, is then obtained along with an effective mobility. This mobility can be compared with the "classical" value and an assessment of fluctuation-induced transport can be made.

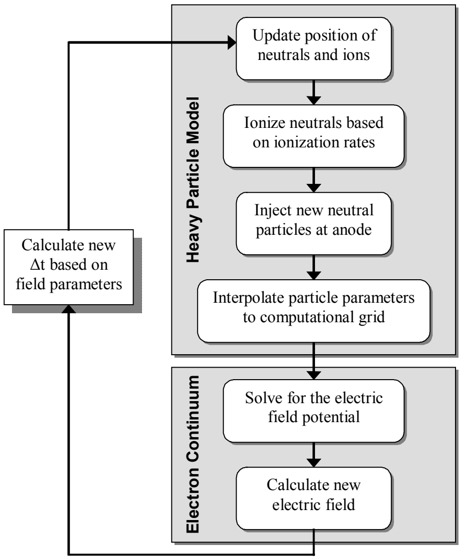

Figure 3: Flow chart for the numerical simulations

III. Results and Discussion

A typical simulation is initialized with a background of ions and neutrals placed uniformly in the domain. Their velocities are obtained by inverting a Maxwellian distribution function. A key parameter in the simulation is the value of the Hall parameter, ω_c/ν. If this parameter has a small initial value, due for instance to a large value of n_n, the electron transport is found to be classical and azimuthal disturbances negligible. The electric field is localized axially to the region where the magnetic field is strongest and the azimuthal electric field is very small. Simulations that use uniform profiles for the magnetic field and electron temperature yield a uniform profile for the axial electric field, as expected. However, a simulation started with a small value of the Hall parameter does not remain with that small value except near the anode where the magnetic field is low and the neutral density is high. This is due to ionization, which eventually depletes the large initial neutral density and raises the value of the Hall parameter. Azimuthal fluctuations then emerge, along with their associated transport. An azimuthal electron velocity develops as shown in the $u_{e\theta}$ field rendering in Fig. 4, which is at a time of 1 μs into a calculation, where the peak magnetic field is 50G, and the voltage is 150V. It is apparent in the figure that the strongest fluctuations in the azimuthal velocity are near the region of maximum magnetic field. It also seems as though there is a coherent structure to these fluctuations, although a quantitative assessment of these oscillations is not yet complete. The classical and anomalous contributions to electron transport (where here anomalous is taken to be the transport associated with the last two azimuthal terms of Eq. 2), are shown in Fig. 5. It seems that the azimuthal contributions are indeed important, although numerical anomalies prevented us from making definitive statements about cross-field transport due to correlated fluctuations at this time.

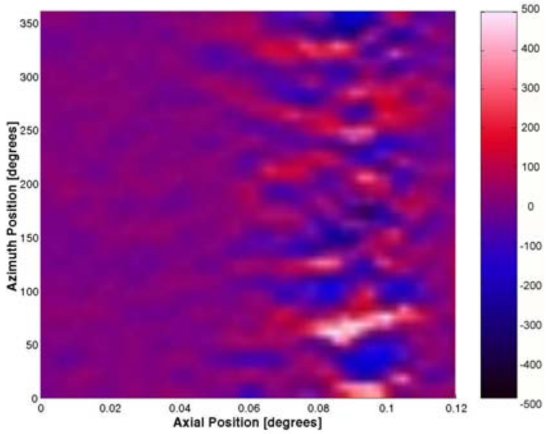

Figure 4. Field rendering of the computed azimuthal electron velocity.

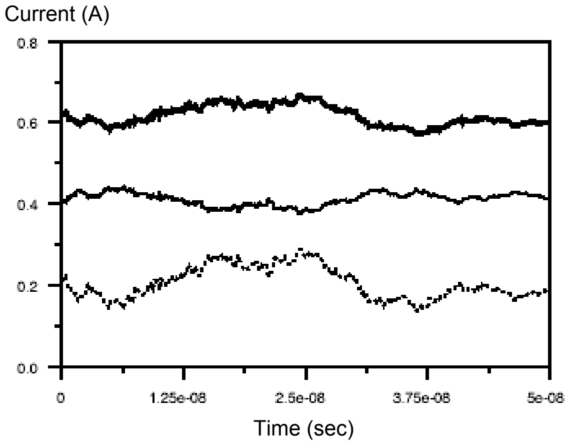

Figure 5. Electron current as a function of time. The bold (upper) trace, solid trace (middle) and dashed trace (lower) represent the total current, the classical contribution, and the azimuthal fluctuation-induced contribution respectively.

Calculations were performed for cases corresponding to discharge conditions for which we have considerable experimental data.[8] The majority of the results presented here are for a 100 G peak magnetic field, a mass flow rate of 2 mg/s through the 9 cm (inner diameter) channel approximately 1.2 cm in width. The simulated discharge time for each case was 1 - 2 μs. While this time may be too small to reach the quasi steady state operating conditions of the thruster, it does provide partial insight into how well the simulation captures the structure of the discharge. It is noteworthy that a typical run takes an average of two weeks to compute on a single processor (Pentium 4) 3.0 GHz machine with 1 Gigabyte of RAM running in a Windows XP environment.

A simple time averaging of the potential field over the duration of the simulation was carried out for comparison to the time-averaged potential measured within the discharge channel. Unphysical spikes in the potential (and hence in the electric field), most notably in the region of the cathode plane, were observed in the earlier simulations. We attribute these anomalies to the numerical method used for the analysis. A filtering was implemented on the results to exclude the effect of these fluctuations for comparison to experiments. The resulting comparison between measured and simulated (but filtered) plasma potential is depicted in Fig. 7. It can be observed from this figure that the simulation predicts the plasma potential reasonably well throughout the length of the acceleration channel. While the agreement between measurements and predictions is encouraging, this agreement may be somewhat fortuitous, in light of the required filtering and the appearance of these high frequency anomalies in the data.

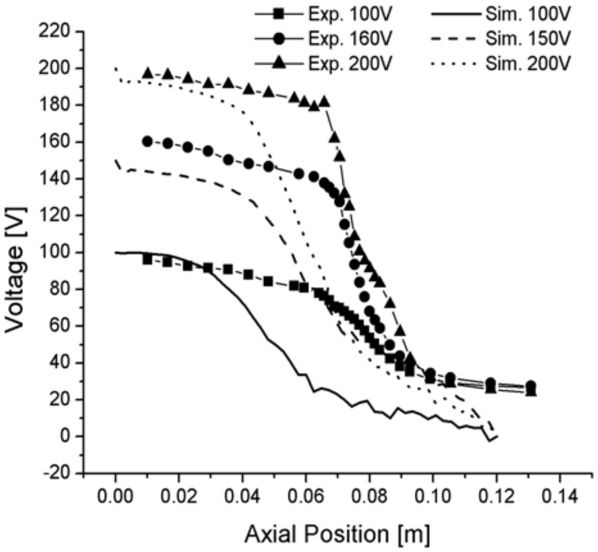

Figure 6. Comparison between measured and predicted (filtered) plasma potential.

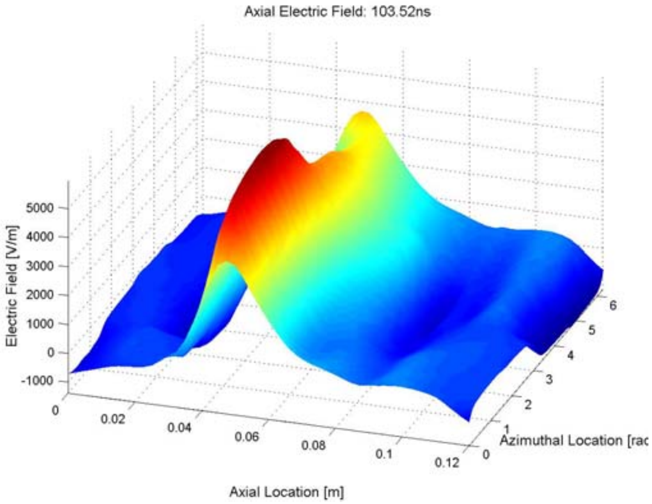

Figure 7. Simulated electric field variation with modified numerical analysis.

As mentioned earlier, the numerical scheme used for the integration of the electron fluid momentum equation was modified in more recent analyses. The new discretization scheme is computationally more expensive, and to date, preliminary results have been obtained for a limited set of conditions, and for a limited computational time (~ 0.1 μs). A snapshot of the spatial variation in the electric field predicted from these more recent calculations is shown in Fig. 7. It is apparent that the simulations tend to capture, even for this short computational duration, a reasonable field distribution, with peak fields at a 6 cm location from the anode, reaching 5 x 10^3 V/m, in good agreement with experiment[8]. It is noted that this distribution has evolved from an initial field that was uniform in its properties. However, for these short simulations, ions (and neutrals) have not yet reached a quasi-steady state needed for a quantitative comparison to measured ion velocities.

IV. Summary

We have investigated electron transport in Hall thrusters via numerical simulation using a hybrid fluid/PIC model in the z - θ coordinate space. At present, it represents a simplified description of Hall thruster plasmas but is able nonetheless to capture azimuthal flows, fluctuations, and their associated transport. Preliminary results indicate that for the large experimental values of the Hall parameters found in these discharges, the electron transport associated with azimuthal disturbances is significant. However, a question in the numerical stability of our first calculations, and the limited extent to which a revised (more accurate) simulation has evolved has prevented us from drawing any definitive conclusions at this time.

Future work with the z - θ model lies in improving the numerical stability of the method of solution, and then subsequently examining the azimuthal fluctuations and transport in detail. The theory will be expanded by adding the electron energy equation to determine the spatial variation in the electron temperature. This equation should include a wall damping model that accounts for the temperature-dependent secondary electron emission as well as sheath saturation effects. The method of solution has been refined and but the added computational expense has precluded the carrying out of sufficiently long computations that evolve the ion motion to quasi-steady state, and that capture the "breathing mode", at this time. An examination must also be made of the sensitivity of the solution to the overall grid structure.

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