Application of WLF to OFDMA MU-MIMO Systems II: Frequency-Domain Packet Scheduling

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***Abstract*— We propose a Mutual Information (MI) based spatial frequency domain packet scheduling algorithm for uplink Orthogonal Frequency Division Multiple Access (OFDMA) multiuser multiple-input, multiple-output (MIMO) systems in this paper. The proposed scheduler is designed particularly for signal constellation constrained MIMO systems. The superiority of the investigated scheduler coupled with the innovative iterative receiver scheme over conventional solutions is verified by both simulation and analytical results.**

I. Introduction

MIMO techniques in combination with Orthogonal Frequency Division Multiple Access (OFDMA) have been commonly used by most 4G air-interfaces, e.g., WiMAX, Long Term Evolution, 802.20, Wireless broadband, etc. In the IEEE 802.16e mobile WiMAX standard, OFDMA has been adopted for both downlink and uplink transmissions [1], [2]. In 3GPP LTE, Single Carrier (SC) Frequency Division Multiple Access (FDMA) [3] is used for uplink transmission, whereas the OFDMA signaling format is used for the downlink transmission [4]–[6]. There are also some proposals on using OFDMA for uplink transmission in LTE advanced (LTE-A) standard, in which both SC-FDMA and OFDMA can be considered as two options for uplink transmission.

Frequency domain packet scheduling is used to allocate radio resources to individual users based on the channel information, incoming data status and QoS of traffic sessions. A power efficient frequency domain packet scheduling algorithm for OFDMA systems was proposed in [7], where the authors proposed a sub-optimal resource allocation scheme based on a greedy algorithm. In [8], a modified Proportional Fair (PF) was proposed for OFDMA systems to extend the conventional PF scheduling algorithms. In [9], a low complexity frequency domain packet scheduling algorithm was proposed. In the literature, the existing scheduling algorithms are based on maximum achievable rate using Gaussian signalling as channel input. However, in LTE systems, discrete time finite size signal constellations, e.g., M-QAM, QPSK are employed. Therefore, the achievable rate based on Gaussian signalling is inappropriate to model the LTE systems. In this paper, we investigate space frequency domain scheduling for OFDMA based multi-user MIMO system. The novelties of this paper are the derivation of the received Signal to Interference plus Noise Ratio (SINR) and the proposal of a scheduling algorithm for finite size signal constellations that can work in conjunction with the iterative receiver we proposed in [10].

Throughout this paper, $(\cdot)^{\mathcal{T}}$ denotes matrix transpose, $(\cdot)^{\mathcal{H}}$ matrix conjugate transpose, $(\cdot)^*$ matrix conjugate, $\mathrm{E}[\cdot]$ expectation, $\|\cdot\|$ Euclidean norm, $\|\cdot\|_F$ Frobenius norm, $\mathrm{Tr}(\cdot)$ trace operation, and $\mathbf{I}_N$ an $N \times N$ identity matrix.

II. System Model

Refer to [10] for a detailed description of the system model.

III. SINR expression and the mutual information for SDM MIMO-OFDMA schemes

In this section, we will derive the SINR expression and the Mutual Information (MI), for the MIMO-OFDMA system under question, which are needed for the spatial frequency multiuser scheduling algorithm introduced in the following section.

A. SINR expression for conventional MMSE FDE

For the conventional MMSE frequency domain equalizer (FDE), the received signal vector in the time domain can be expressed as

$$\mathbf{z} = \mathbf{G}^{\mathcal{H}}(\mathbf{HPs} + \mathbf{w}). \qquad (1)$$

Let $\mathbf{B} = \mathbf{G}^{\mathcal{H}}\mathbf{H}\mathbf{P}$, $\mathbf{G}$ and $\mathbf{B}$ can be expressed as

$$\mathbf{G} = \begin{bmatrix} \mathbf{G}_{11} & \mathbf{G}_{12} & \cdots & \mathbf{G}_{1K} \\ \mathbf{G}_{21} & \mathbf{G}_{22} & \cdots & \mathbf{G}_{2K} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{G}_{K1} & \mathbf{G}_{K2} & \cdots & \mathbf{G}_{KK} \end{bmatrix};$$

$$\mathbf{B} = \begin{bmatrix} \mathbf{B}_{11} & \mathbf{B}_{12} & \cdots & \mathbf{B}_{1K} \\ \mathbf{B}_{21} & \mathbf{B}_{22} & \cdots & \mathbf{B}_{2K} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{B}_{K1} & \mathbf{B}_{K2} & \cdots & \mathbf{B}_{KK} \end{bmatrix}, \quad (2)$$

where $\mathbf{G}_{ij} \in \mathbb{C}^{N\times N}$ is the equalization matrix between the jth transmitter and the ith receiver antenna. $\mathbf{B}_{ij}$ is defined similarly. The received signal vector for the ith user, $i \in \{1, 2, \cdots, K\}$, in the time domain can be expressed as

$$\mathbf{z}_i = \sum_{j=1, j\neq i}^{K} \mathbf{B}_{ij}\mathbf{s}_j + \mathbf{B}_{ii}\mathbf{s}_i + \sum_{j=1}^{K} \mathbf{G}_{ij}^{\mathcal{H}}\mathbf{w}_j. \quad (3)$$

The kth element, $k \in \{1, 2, \cdots, N\}$, of $\tilde{\mathbf{z}}_i$ can be expressed as

$$\mathbf{z}_i(k) = \mathbf{B}_{ii}(k,k)\mathbf{s}_i(k) + \sum_{j=1, j\neq i}^{K} \mathbf{B}_{ij}(k,k)\mathbf{s}_j(k) + \sum_{j=1}^{K} \mathbf{G}_{ij}^{\mathcal{H}}(k,k)\mathbf{w}_j(k). \quad (4)$$

The first term on the right hand side of (4) represents the desired signal, the second term is the interference from the other substreams, and the third one is the noise. The received SINR for the kth symbol of the ith user is thus

$$\gamma_{con}^{i}(k) = \frac{\mathbf{B}_{ii}(k,k)\mathbf{B}_{ii}^{\mathcal{H}}(k,k)}{\sum_{j=1,j\neq i}^{K} \mathbf{B}_{ij}(k,k)\mathbf{B}_{ij}^{\mathcal{H}}(k,k) + N_0 \sum_{j=1}^{K} \mathbf{G}_{ij}^{\mathcal{H}}(k,k)\mathbf{G}_{ij}(k,k)}. \quad (5)$$

B. SINR for conventional iterative FDE

For the conventional iterative FDE, the filter output can be expressed as

$$z_n = \mathbf{w}_n^{\mathcal{H}}\mathbf{r}_n = u_n s_n + \xi_n, \quad (6)$$

where the combined noise and residual interference ξ_n can be approximated as a Guassian random variable [11], i.e., $\xi_n \sim \mathcal{CN}(0, N_\xi)$. The parameters u_n and N_ξ can be determined as

$$u_n = \mathrm{E}\{z_n s_n^*\} = \mathrm{E}\{\mathbf{w}_n^{\mathcal{H}}\mathbf{r}_n s_n^*\} = \mathbf{w}_n^{\mathcal{H}}\mathbf{h}_n$$
$$N_\xi = \mathrm{E}\{|\xi_n|\} = \mathrm{E}\{|z_n - u_n s_n|\} = u_n - u_n^2. \quad (7)$$

Based on Eqs. (6) and (7), we can derive the SINR at the filter output as

$$\gamma_{ConvIter}^{i}(k) = \frac{u_n^2}{N_\xi} = \frac{u_n^2}{u_n - u_n^2} = \frac{u_n}{1-u_n} = \frac{\mathbf{w}_n^{\mathcal{H}}\mathbf{h}_n}{1-\mathbf{w}_n^{\mathcal{H}}\mathbf{h}_n}. \quad (8)$$

C. SINR expression for improved iterative FDE

For the improved iterative FDE, the frequency domain signal is given by [10] $z_n = \mu_n s_n + \nu_n$, where $\mu_n = \mathbf{g}_n^{\mathcal{H}}\mathbf{h}_n$, $N_\nu = \mu_n - \mu_n^2$. We can thus derive the SINR at the filter output as

$$\gamma_{ImprIter}^{i}(k) = \frac{\mu_n^2}{N_\nu} = \frac{\mu_n^2}{\mu_n - \mu_n^2} = \frac{\mu_n}{1-\mu_n} = \frac{\mathbf{g}_n^{\mathcal{H}}\mathbf{h}_n}{1-\mathbf{g}_n^{\mathcal{H}}\mathbf{h}_n}. \quad (9)$$

D. Mutual Information and maximum achievable rate

Based on Gaussian signaling and Shannon's capacity theorem, the maximum achievable spectrum efficiency in bits/second/Hz for the kth user can be expressed as $r_k = \sum_j \log_2(1+\gamma_j)$, where γ_j is the received SINR for the jth substream of the kth user, it is given by (5) for the conventional linear MMSE equalizer, by (8) for the conventional iterative equalizer and by (9) for the improved iterative equalizer.

For broadband wireless communication systems, e.g., 3GPP LTE downlink, the total bandwidth B is usually divided into a number of M subcarriers. Among M subcarriers, N subcarriers ($N < M$) are allocated for data transmission. L contiguous subcarriers form a scheduling RB. Let $I_{sub,i}$ and $|I_{sub,i}|$ be the index set of subcarriers assigned to user i and the length of the set $I_{sub,i}$, respectively. Denote by P_t^i the total transmitted power of user i. Assuming that the power is equally allocated over $I_{sub,i}$, then $p_{n,i} = P_t^i/|I_{sub,i}|$. The maximum achievable rate in bits per second for the kth user can then be written as

$$C_k = \sum_j \frac{B|I_{sub,k}|}{M} \log_2(1+\gamma_j). \quad (10)$$

So far, we discussed the maximum achievable rate by considering the channel input using Gaussian signaling. In real LTE systems, discrete time finite size signal constellations, e.g., MQAM, are employed. The maximum achievable rate approach based on Gaussian signaling, e.g., (10), are therefore likely to be too optimistic for estimating the achievable rate. In this work, we consider to employ the mutual information between the discrete channel input $\mathbf{u}$ and the channel output $\mathbf{v}$. For a MIMO channel $\boldsymbol{\Lambda}$ with n_T transmit antennas and n_R receive antennas, we have $\mathbf{v} = \boldsymbol{\Lambda}\mathbf{u} + \boldsymbol{\upsilon}$. Here $\boldsymbol{\upsilon} \in \mathbb{C}^{n_R\times 1}$ is the white Gaussian noise, with $\mathbf{E}[\boldsymbol{\upsilon}\boldsymbol{\upsilon}^H] = \mathbf{I}_{n_R}\sigma_\upsilon^2$. The mutual information can be calculated by

$$\Psi(\mathbf{u};\mathbf{v}) = H(\mathbf{v}) - H(\mathbf{v}|\mathbf{u}), \quad (11)$$

where $H(\cdot) = -\mathbf{E}[\log_2(p(\cdot))]$ is the entropy function, and $p(\cdot)$ represents the Probability Density Function (PDF). The mutual information $\Psi(\mathbf{u};\mathbf{v})$ can be calculated accordig to [12] as

$$\Psi(\mathbf{u};\mathbf{v}) = -\mathbf{E}\left\{\log_2\left(\frac{1}{2^{M_c n_T}}\frac{1}{(2\pi\sigma_\upsilon^2)^{n_R}}\sum_{\mathbf{u}\in\mathbb{S}}\exp\left[-\frac{||\mathbf{v}-\boldsymbol{\Lambda}\mathbf{u}||^2}{2\sigma_\upsilon^2}\right]\right)\right\} - n_R\log_2(2\pi e\sigma_\upsilon^2), \quad (12)$$

In general Eq. (12) cannot be expressed in a closed form. Nevertheless, it can be evaluated by Monte-Carlo simulations. The mutual information is a function of the received SNR at the receiver antennas. The mutual information for uncoded 1×1 and 2×2 MIMO system with QPSK signaling and the system with 4-ASK signaling are shown in Fig. 1. The x-axis is the received SINR in dB. Both the simulation results and the curve fitting results are given.

IV. Spatial Frequency Multiuser Scheduling

For localized OFDMA downlink multiuser MIMO transmission*, each OFDMA downlink transmission sub-frame can be partitioned into several RBs for the convenience of multiple

*In the localized OFDMA transmission scheme, each user's data is transmitted by consecutive subcarriers, while for the distributed transmission scheme, the user's data is transmitted by distributed subcarriers [6].

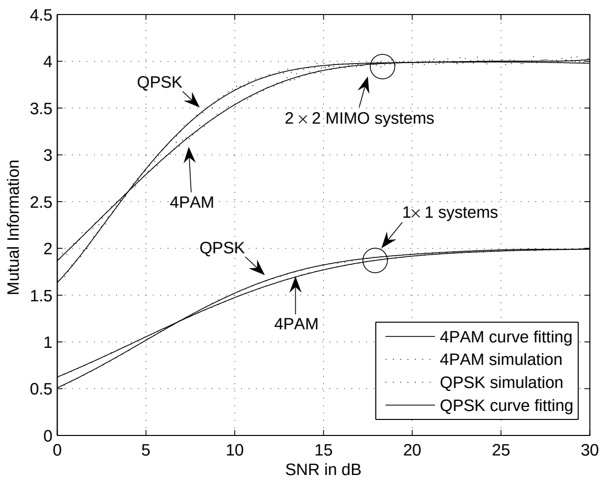

Fig. 1. Mutual Information versus received SINR for uncoded 1×1 and 2×2 MIMO systems with QPSK signaling and with 4ASK signaling.

user packet scheduling [6]. Let $I_{RB,i}$ be the index set of RBs assigned to user i within one sub-frame and $|I_{RB,i}|$ be the length, the number of total RBs in one sub-frame is $|I_{RB}|$. Then $|I_{RB,i}|L = |I_{sub,i}|$. Multiple contiguous RBs can be assigned to one user within one sub-frame.

Denote by $\boldsymbol{\phi}_j$ the jth set of K users which are selected from the total K_T users in the system and let $\boldsymbol{\Phi}$ be the whole set of K users chosen from total K_T users, $\boldsymbol{\phi}_j \in \boldsymbol{\Phi}, \forall j \in \{1, 2, \cdots, |\boldsymbol{\Phi}|\}$, where $|\boldsymbol{\Phi}|$ is the size of $\boldsymbol{\Phi}$, and $|\boldsymbol{\Phi}| = \binom{K_T}{K}$.

Let us define $U_j(\boldsymbol{\phi})$ as the utility function for the jth RB. As will show later, $U_j(\boldsymbol{\phi})$ is a function of the MIs of the scheduled users. The objective is to maximize the utility function by selecting the users group with appropriate channel condition and optimizing the power allocated for each user within one subframe. The optimization problem can be described as

$$\begin{aligned} \max_{\forall \boldsymbol{\phi} \in \boldsymbol{\Phi}; \boldsymbol{\phi}: I_{RB,i}, P_t^i, \forall i \in \boldsymbol{\phi}} \quad & U_j(\boldsymbol{\phi}), \\ s.t.: \quad & I_{sub,i}^{k+1} - I_{sub,i}^{k} = 1, \\ & \forall k \in \{1, 2, \cdots, |I_{sub,i}| - 1\}, \end{aligned} \tag{13}$$

where $I_{sub,i}^k$ is the kth element in the set $I_{sub,i}$. The sub-constraint corresponds to the localized downlink OFDMA transmission, i.e., the user data is transmitted by a group of consective subcarriers. The above optimization problem is to maximize the utility function for each RB subject to the user's power constraint.

We can define $U(\boldsymbol{\phi}) = \sum_{i \in \boldsymbol{\phi}} \Psi_i$, where Ψ_i is the MI for user i, which is defined by Eq. 12. Maximization of this utility function is equivalent to optimization of the maximum achievable rate for systems with a finite alphabet constrianed signal constellation. This may result in an unfair situation, i.e., only the users with good channel conditions get resources.

To tackle this problem, we consider a resource fair allocation algorithm for each RB based utility function maximization. The key idea of the fair resource allocation algorithm is to limit the users with more RBs used in a past certain period T_{win}, and give priority to those users with less transmissions in the period T_{win}. The algorithm works as follow: Let $\alpha_{k,i}$ be the moving average of used RBs by the ith user in the past T_{win} at interval k and $\alpha_{k,i} = (1 - \frac{1}{T_{win}})\alpha_{k-1,i} + \frac{1}{T_{win}}\delta$, where δ is the MI or the capacity for the ith user if the user i gets scheduled, otherwise $\delta = 0$. We define the utility function at the kth interval as $U_k(\boldsymbol{\phi}) = \sum_{i \in \boldsymbol{\phi}} f(\alpha_{k,i}, \Psi_i, c)$, where $f(\alpha_{k,i}, \Psi_i, c)$ is a function of $\alpha_{k,i}$, c and Ψ_i, and is defined as $\Psi_i / \alpha_{k,i}^c$, where c is a constant. The per RB based scheduling problem then becomes

$$\boldsymbol{\phi}^* = \arg \max_{\forall \boldsymbol{\phi} \in \boldsymbol{\Phi}} \sum_{i \in \boldsymbol{\phi}} f(\alpha_{k,i}, \Psi_i, c). \tag{14}$$

Note that the above expression is in fact a generalized Proportional Fair (GPF) scheduling algorithm. When $c = 1$, it is a traditional Proportional Fair (PF) scheduling algorithm [13], [14] While for the case of $c = 0$, it becomes the maximum throughput scheduling algorithm. A value of c between 0 and 1 represents the tradeoff between the maximum throughput scheduling and traditional PF scheduling algorithm.

V. Numerical Results

In this section, we will give some analytical and numerical results to demonstrate the effectiveness of the proposed FDE and scheduling schemes. Here we consider the case with 2 antennas at the transmitter and single antenna at the MS. For the MU-MIMO case, two MSs are grouped together to form a virtual MIMO between the MSs and the BS. We consider a six-path fading channel which is normalized such that the average channel gain for each transmitted symbol is equal to unity. The fading coefficients for each path are modeled as independent identically distributed (i.i.d) complex Gaussian samples. The block size of the user data is 12, which is also the number of subcarriers in a resource block. The size of FFT is 256, and the length of Cyclic Prefix (CP) is 8. The power loss incurred by the insertion of the CP is taken into account in the SNR calculation. At each Monte-Carlo run, 500 sub-frames are used for data transmission and the power of each user is randomly generated to simulate the fact that users maybe in different locations.

Fig. 2 shows the simulation results for the maximum achievable rate in bits/second/Hz versus the number of available users for the downlink MIMO OFDMA systems with the maximum sum MI based spatial scheduling algorithm. The transmitted symbols are selected from the 4ASK signal constellation and the transmitted SNR is 20 dB. The transmitted SNR is defined as E_s/N_0, where E_s is the total transmitted power of the grouped users, and N_0 is the double-sided power spectrum density. GPF scheduling algorithms of Eq. 14 with $c = 0$, and $c = 1$ are investigated. $c = 0$ and $c = 1$ correspond to the traditional PF and the maximum achievable rate scheduling algorithms, respectively. Random user Pairing Scheduling (RPS) algorithm is also investigated for a baseline comparison. For random pairing scheduling, the first user is selected in a round robin fashion, while the second user is randomly selected from the rest of the users in the system. It can be seen that as the number of users increases, the multiuser diversity gain can be achieved for all the investigated systems except the one with the RPS algorithm. The reason is that those non-random pairing schedulers have more freedom to choose the MSs with good channel condition and multiuser diversity can thus be exploited.

The sum rate performance for both the conventional and improved iterative receivers for SDM-FDPS multiuser MU-MIMO schemes are investigated and compared in Fig. 2. It can

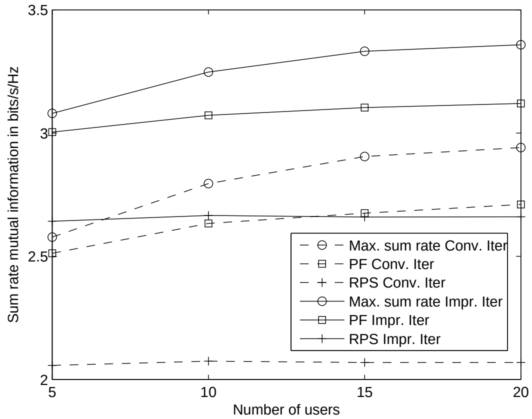

Fig. 2. Performance comparison of the improved and conventional iterative IC-MMSE schemes for the 4ASK system with 2×2 antenna setup. All the curves are plotted at the 3rd iteration when the systems reach convergence, SNR 20 dB.

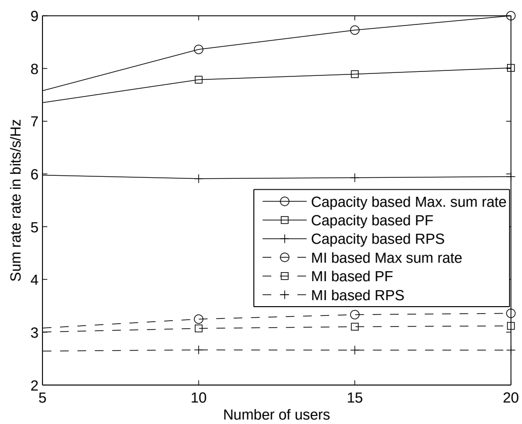

Fig. 3. Sum rate performance comparison for the improved iterative equalizer with MI and capacity based scheduling algorithms for SDM-FDPS MU-MIMO 4ASK systems with 2×2 antenna setup, SNR 20 dB.

be seen that the improved receiver scheme significantly outperforms its conventional counterpart for all the investigated scheduling algorithms. The improved iterative receiver scheme achieves better sum rate performance than its conventional counterpart by exploiting the complete second order statistics of the received signal.

Fig. 3 presents the performance comparison for the improved iterative receiver scheme using the MI based and the capacity based spatial frequency scheduling algorithms. It can be seen that with the capacity based scheduling algorithm, even with random user pairing scheduling algorithm, the sum rate already exceeds the 4 bits/sec/Hz, which is the limit of the 2×2 MIMO 4ASK system, therefore it is unrealistic.

Fig. 4 shows the sum rate performance comparison for the improved and conventional iterative equalizers using MI based scheduling algorithms for SDM-FDPS MU-MIMO QPSK system, the transmitted SNR is 30 dB. GPF scheduling algorithms with $c = 0$, $c = 1/2$, and $c = 1$ are investigated. $c = 1/2$ corresponds to a scheduling algorithm which has performance between the ones for the PF and the maximum achievable rate scheduling algorithm. The simulation results show that for all the investigated scheduling algorithms, the maximum achievable rate of the improved iterative receiver scheme is higher than that of the conventional scheme. For QPSK signaling, in comparison with conventional iterative receiver scheme, the improved iterative receiver scheme enhances the maximum sum rate, which is illustrated by Fig. 4.

Fig. 5 shows the maximum achievable rate performance versus the number of available users for the downlink MIMO systems with the maximum sum capacity based spatial scheduling algorithm. All the curves are plotted at the 3rd iteration when the systems reach convergence. The transmitted symbols are selected from the QPSK signal constellation and the transmitted SNR is 30 dB. Compared with Fig. 4, it can be seen that with the maximum sum capacity based scheduling, the maximum achievable rate is much higher than the one with the MI based scheduling algorithm. Even with the RPS scheduling algorithm, the maximum achievable rate already exceeds 4 bits/second/Hz, which is the capacity limit for the studied MIMO 2×2 channel under the QPSK signal constellation constraint. Therefore, the maximum achievable rate obtained under the maximum sum capacity scheduling algorithm is unrealistic.

Since MI is obtained under the signal constellation constraint, the performance of MI based scheduling algorithm is much closer to a real system. We can conclude that the commonly used sum capacity scheduling algorithm is too optimistic for practical applications, this is particular true for high SNR channels. Furthermore, maximum sum capacity based scheduling algorithm assumes that the input source is Gaussian distributed, which is unrealistic in a practical system. The complexity of the MI based scheduling is roughly the same as the sum capacity based scheduling.

Figs. 6 and 7 show the BER performance comparison of the iterative algorithms for 4ASK and QPSK modulated OFDMA systems with the 2×2 and 4×4 MIMO configurations, respectively. The results are shown at the 3rd iteration. One can see that with the conventional iterative receiver algorithms, the QPSK modulated system outperforms the 4ASK system. However, the 4ASK system with the improved iterative scheme performs much better than the QPSK system with conventional iterative scheme. For both QPSK and 4ASK systems, the improved iterative receiver scheme has better performance especially at medium to high SNR region.

VI. CONCLUSIONS

In this paper and another submission [10], we propose a novel iterative receiver and a packet scheduler for OFDMA based MU-MIMO systems. The frequency domain iterative equalizer is derived by optimization of modified cost functions for the 4ASK system and by utilizing the complete second-order statistics for the QPSK system. The 4ASK and QPSK systems are used as an example for system design. However, the proposed iterative algorithm can be easily extended to arbitrary M-ASK, M-PSK and M-QAM systems. The performance for the presented MI-based scheduling algorithm is closer to a real LTE system compared with the one using the capacity based scheduling algorithm. It can be used to indicate the real system performance.

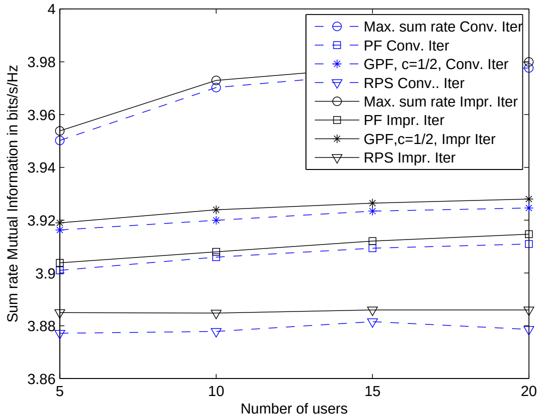

Fig. 4. Performance comparison of the MI based scheduling algorithms for the improved and conventional iterative IC-MMSE schemes for the QPSK system with 2×2 antenna setup. All the curves are plotted at the 3rd iteration when the systems reach convergence, SNR 30 dB.

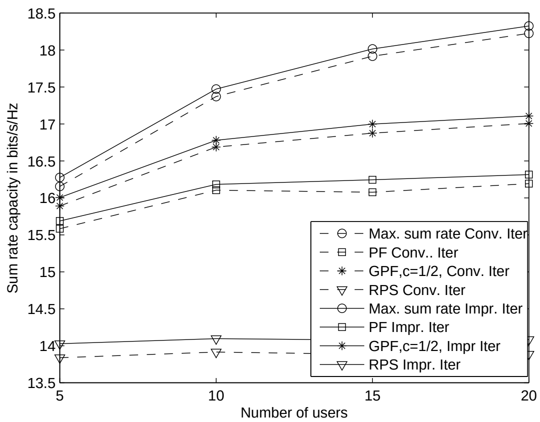

Fig. 5. Performance comparison of the Shannon capacity based scheduling algorithms for the improved iterative IC-MMSE schemes for the QPSK system with 2×2 antenna setup. All the curves are plotted at the 3rd iteration when the systems reach convergence, SNR 30 dB.

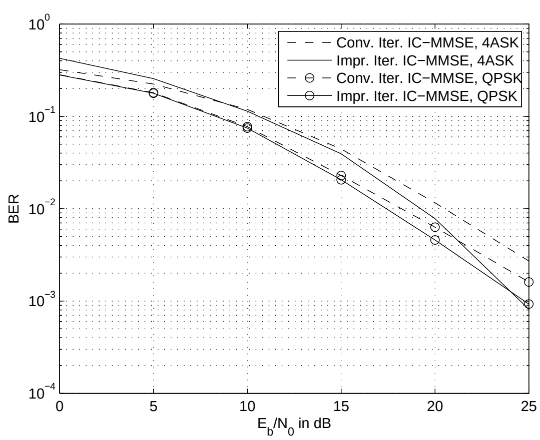

Fig. 6. BER performance of improved iterative receiver schemes with 2×2 antenna setup. All the curves are plotted at the 3rd iteration when the systems reach convergence.

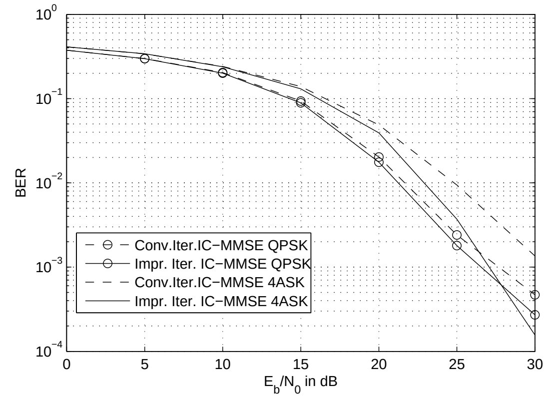

Fig. 7. BER performance of improved iterative receiver scheme with 4×4 antenna setup. All the curves are plotted at the 3rd iteration when the systems reach convergence.

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REFERENCES

[1] K. Etemad. "Overview of mobile WiMAX technology and evolution", *IEEE Commun. Mag.*, pp. 31-40, Oct. 2008.

[2] F. Wang, A. Ghosh, C. Sankaran, P. Fleming, F. Hsieh, S. Benes. "Mobile WiMAX systems: performance and evolution", *IEEE Commun. Mag.*, pp. 41-49, Oct. 2008.

[3] Z. Lin, P. Xiao and B. Vucetic. "Analysis of Receiver Algorithms for LTE SC-FDMA Based Uplink MIMO Systems", *IEEE Transactions on Wireless Communications*, Vol. 9, No. 1, Nov. 2010, pp. 60-65.

[4] Z. Lin, P. Xiao, T. B. Sorensen, B. Vucetic. "Spatial Frequency Scheduling for Uplink SC-FDMA based Linearly Precoded LTE Multi-user MIMO Systems", *Transactions on Emerging Telecommunications Technologies*, Vol. 21, No. 3, April. 2010, pp. 213-223.

[5] Z. Lin, T. B. Sorensen, P. E. Mogensen. "Downlink SINR distribution of linearly precoded multi- user MIMO systems", *IEEE Communications Letters*, Vol. 11, No. 11, Nov. 2007.

[6] 3GPP TR 25.814 V7.0.0, "Physical Layer Aspects for Evolved UTRA," Tech. Rep., June 2006.

[7] K. Lee, B. Ryu, C. Lee. "Power Efficient Frequency Domain Packet Scheduling for OFDMA Systems". *Proc. International Conference on Convergence Information Technology*, pp.1-6, 2007.

[8] C. Wengerter, J. Ohlhorst,A. Golitschek Edler von Elbwart. "Fairness and Throughput Analysis for Generalized Proportional Fair Frequency Scheduling in OFDMA". *Proc. VTC 2005-Spring*, pp.1903-1907, 2005.

[9] Z. Sun, C. Yin, G. Yue, "Reduced-Complexity Proportional Fair Scheduling for OFDMA Systems". *Proc. International Conference on Communications, Circuits and Systems Proceedings*, vol. 2, pp.1221-1225, 2006

[10] P. Xiao, Z. Lin, Y. Wu. "Application of WLF to OFDMA MU-MIMO Systems I: Frequency-Domain Equalization". *The Tenth International Symposium on Wireless Communication Systems, ISWCS 2013*. Invited paper to the Special Session on "Non-Circularity and Widely Linear Filtering in Radiocommunications".

[11] V. Poor, S. Verdu. "Probability of error in MMSE multiuser detection", *IEEE Trans. Commun.*, vol. 43, pp. 858–971, May 1997.

[12] B. M. Hochwald and S. T. Brink, "Achieving near-capacity on a multiple-antennas channel," *IEEE Trans. Commun.*, vol. 51, no. 3, pp. 389–399, Mar. 2003.

[13] T. Park, O.-S. Shin, and K. B. Lee, "Proportional Fair Scheduling for Wireless Communication with Multiple Transmit and Receive Antennas," in *IEEE VTC*, Florida, USA, October 2003, vol. 3, pp. 1573 – 1577.

[14] V. K. N. Lau, "Proportional fair space-time scheudling for wireless communications," *IEEE Trans. Commun.*, vol. 53, no. 8, pp. 1353–1360, Aug. 2005.