Equalization of BFWA Channels: Theory and Analysis

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Abstract— **Broadband Fixed Wireless Access (BFWA) is quickly emerging as a strong network access alternative for the delivery of voice, data, Internet, video and multimedia type applications to business and residential customers. However, the physical limitations of the wireless channel present a fundamental technical challenge to system capacity and reliable communications. Previous studies have shown that BFWA channels are dispersive, they introduce intersymbol interference (ISI) to the transmitted signals, which greatly deteriorates the system performance. An equalization algorithm based on the algebra matrix is introduced and theoretically analyzed in this paper. The results show that this algorithm exhibits a good potential to combat ISI under certain conditions, which suggests the solutions for the future BFWA systems.**

I. Introduction

The interest in the BFWA systems as an alternative to digital subscriber line (DSL) or cable has recently increased. BFWA systems offer a very cost-effective way of building an access network. Easy maintainability, incremental costs and portability are key benefits of the wireless alternative [1]. Some work concerning standards for the BFWA systems is currently taking place under the auspices of IEEE 802.16 standard [2] and the ETSI HIPERMAN group [3]. In order to be competitive, the BFWA systems must offer similar data rates to their wire-line counterparts. At high data rate, the ISI induced by dispersive channels becomes a severe problem. The key building block in combating ISI is the equalizer. The higher the data rate, the more complex the equalizer. Different equalization algorithms for the BFWA channels were examined, e.g., in [4, 5]. Here, we introduce a new approach to equalization which has good asymptotic AI performance. The theory and performance analysis of this algorithm are provided in details in this paper.

A multipath channel can be modeled by an equivalent baseband system where the transmit filter, the channel and the receive filter, are represented by a discrete-time L-tap transversal filter with finite-length impulse response $h_n = \sum_{l=0}^{L-1} h_l \delta_{n-l}$ where h_l denotes the complex channel coefficients. Tailored for different terrain conditions, a set of 6 typical channel models called Standford University Interim (SUI) Channel Models were proposed in [6] for simulation, design, development and testing of technologies suitable for fixed broadband wireless applications. All of them are simulated using 3 taps, having either Ricean or Rayleigh amplitude distributions. For the purpose of this study, we select SUI-3 channel with tap spacing of 500ns, and maximum tap delay at 1000ns. Under the assumption that the transmitted data rate is 4Mbps, the multipath fading can be modelled as a tapped-delay line with adjacent taps equally spaced at symbol rate. The received signal is formed as

$$r_n = h_0 s_n + h_1 s_{n-1} + h_2 s_{n-2} + v_n \tag{1}$$

where the channel coefficients h_0, h_1, h_2 are complex Gaussian random variables and assumed to remain constant during the transmission of one block of data. They, however, vary from block to block. The transmitted PSK/QAM symbol at time instant n is denoted as s_n, and v_n is the complex additive white Gaussian noise with zero mean and variance N_0. To simplify the performance analysis, we use QPSK modulation for the purpose of this study. However, the proposed algorithm also applies to and the analytical results can be easily extended to other modulation schemes.

The task of the receiver is to detect the transmitted symbols $\{s_n\}$ given the received observation $\{r_n\}$. From (1), we see that the desired symbol is corrupted with ISI and AWGN noise. An equalizer is needed to combat ISI. Several equalization algorithms have been introduced in the literature, like minimum mean square error (MMSE) linear equalizer, decision feedback equalizer (DFE), and adaptive algorithms like least mean square (LMS), recursive least square (RLS) [7], square root Kalman (SRK) algorithm [8], etc.. Here, we derive a new equalization algorithm that exhibits a good potential for removing the detrimental effect of ISI.

II. A New Equalization Scheme

Based on (1), the received signal can be written in vector form as

$$\underbrace{\begin{bmatrix} r_n \\ r_{n+1} \\ r_{n+2} \end{bmatrix}}_{\mathbf{r}_n} = \underbrace{\begin{bmatrix} h_2 & h_1 & h_0 & 0 & 0 \\ 0 & h_2 & h_1 & h_0 & 0 \\ 0 & 0 & h_2 & h_1 & h_0 \end{bmatrix}}_{\mathbf{H}} \underbrace{\begin{bmatrix} s_{n-2} \\ s_{n-1} \\ s_n \\ s_{n+1} \\ s_{n+2} \end{bmatrix}}_{\mathbf{s}_n} + \underbrace{\begin{bmatrix} v_n \\ v_{n+1} \\ v_{n+2} \end{bmatrix}}_{\mathbf{v}_n} \tag{2}$$

It is straightforward to see that $\mathbf{H}^{-1}\mathbf{r}_n = \mathbf{H}^{-1}\mathbf{H}\mathbf{s}_n + \mathbf{H}^{-1}\mathbf{v}_n = \mathbf{s}_n + \mathbf{H}^{-1}\mathbf{v}_n$. In this way, an estimate of symbol vector $\mathbf{s}_n$ can be easily obtained by multiplying the received vector $\mathbf{r}_n$ with the inverse of the channel matrix

$\mathbf{H}$. However, $\mathbf{H}$ is not a square matrix as shown in (2), and is therefore not invertible. A solution which works around this problem is to replace $\mathbf{r}_n$ with

$$\mathbf{y}_n = \mathbf{H}^*\mathbf{r}_n = \mathbf{H}^*\mathbf{H}\mathbf{s}_n + \mathbf{H}^*\mathbf{v}_n = \mathbf{R}\mathbf{s}_n + \mathbf{z}_n$$

where $\mathbf{z}_n = \mathbf{H}^*\mathbf{v}_n$, and

$$\mathbf{R} = \mathbf{H}^*\mathbf{H} = \begin{bmatrix} h_2^* & 0 & 0 \\ h_1^* & h_2^* & 0 \\ h_0^* & h_1^* & h_2^* \\ 0 & h_0^* & h_1^* \\ 0 & 0 & h_0^* \end{bmatrix} \begin{bmatrix} h_2 & h_1 & h_0 & 0 & 0 \\ 0 & h_2 & h_1 & h_0 & 0 \\ 0 & 0 & h_2 & h_1 & h_0 \end{bmatrix} =$$

$$\begin{bmatrix} |h_2|^2 & h_2^*h_1 & h_2^*h_0 & 0 & 0 \\ h_1^*h_2 & |h_1|^2+|h_2|^2 & h_1^*h_0+h_2^*h_1 & h_2^*h_0 & 0 \\ h_0^*h_2 & h_0^*h_1+h_1^*h_2 & |h_0|^2+|h_1|^2+|h_2|^2 & h_1^*h_0+h_2^*h_1 & h_2^*h_0 \\ 0 & h_0^*h_2 & h_0^*h_1+h_1^*h_2 & |h_0|^2+|h_1|^2 & h_1^*h_0 \\ 0 & 0 & h_0^*h_2 & h_0^*h_1 & |h_0|^2 \end{bmatrix} \tag{3}$$

The superscript operator $()^*$ is the conjugate transpose operation when applied to matrices, and simply the conjugate when applied to scalars. Apparently, $\mathbf{R}$ is a hermitian matrix satisfying the condition $\mathbf{R} = \mathbf{R}^*$. We can now use the inverse of $\mathbf{R}$ to obtain an estimate of the symbol vector $\mathbf{s}_n$, which is denoted as $\hat{\mathbf{s}}_n$, i.e.,

$$\begin{aligned} \hat{\mathbf{s}}_n &= \mathbf{R}^{-1}\mathbf{y}_n = \mathbf{R}^{-1}(\mathbf{R}\mathbf{s}_n + \mathbf{z}_n) \\ &= \mathbf{s}_n + \mathbf{R}^{-1}\mathbf{z}_n = \mathbf{s}_n + (\mathbf{H}^*\mathbf{H})^{-1}\mathbf{H}^*\mathbf{v}_n \end{aligned} \tag{4}$$

which is an unbiased estimate of $\mathbf{s}_n$ since $\mathrm{E}[\hat{\mathbf{s}}_n] = \mathbf{s}_n$. However, this procedure is computationally complex due to the matrix inverse operation for each symbol vector. To simplify the computation, let us decomposite the matrix $\mathbf{R}$ into 3 matrices $\mathbf{R} = \mathbf{D} + \mathbf{L} + \mathbf{U}$ where $\mathbf{D}$ is a diagonal matrix, $\mathbf{L}$ is a strictly lower left triangle matrix, and $\mathbf{U}$ is a strictly upper right triangular matrix. For the matrix $\mathbf{R}$ expressed in (3), $\mathbf{L}$ and $\mathbf{U}$ are

$$\mathbf{L} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ h_1^*h_2 & 0 & 0 & 0 & 0 \\ h_0^*h_2 & h_0^*h_1+h_1^*h_2 & 0 & 0 & 0 \\ 0 & h_0^*h_2 & h_0^*h_1+h_1^*h_2 & 0 & 0 \\ 0 & 0 & h_0^*h_2 & h_0^*h_1 & 0 \end{bmatrix},$$

$$\mathbf{U} = \begin{bmatrix} 0 & h_2^*h_1 & h_2^*h_0 & 0 & 0 \\ 0 & 0 & h_1^*h_0+h_2^*h_1 & h_2^*h_0 & 0 \\ 0 & 0 & 0 & h_1^*h_0+h_2^*h_1 & h_2^*h_0 \\ 0 & 0 & 0 & 0 & h_1^*h_0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{5}$$

The matrix inversion in (4) can be solved iteratively by the Jacobi algorithm [9, 10]

$$\mathbf{s}_n^i = \mathbf{y}_n - (\mathbf{L} + \mathbf{U})\mathbf{s}_n^{i-1} \tag{6}$$

where i is the iteration index. Substituting (5) into (6) yields

$$\begin{bmatrix} s_{n-2}^i \\ s_{n-1}^i \\ s_n^i \\ s_{n+1}^i \\ s_{n+2}^i \end{bmatrix} = \begin{bmatrix} h_2^*r_n \\ h_1^*r_n + h_2^*r_{n+1} \\ h_0^*r_n + h_1^*r_{n+1} + h_2^*r_{n+2} \\ h_0^*r_{n+1} + h_1^*r_{n+2} \\ h_0^*r_{n+2} \end{bmatrix} -$$

$$\begin{bmatrix} h_2^*h_1 s_{n-1}^{i-1} + h_2^*h_0 s_n^{i-1} \\ h_1^*h_2 s_{n-2}^{i-1} + (h_1^*h_0 + h_2^*h_1) s_n^{i-1} + h_2^*h_0 s_{n+1}^{i-1} \\ h_0^*h_2 s_{n-2}^{i-1} + (h_0^*h_1 + h_1^*h_2) s_{n-1}^{i-1} + (h_1^*h_0 + h_2^*h_1) s_{n+1}^{i-1} + h_2^*h_0 s_{n+2}^{i-1} \\ h_0^*h_2 s_{n-1}^{i-1} + (h_0^*h_1 + h_1^*h_2) s_n^{i-1} + h_1^*h_0 s_{n+2}^{i-1} \\ h_0^*h_2 s_n^{i-1} + h_0^*h_1 s_{n+1}^{i-1} \end{bmatrix}$$

Finally, we can derive

$$\begin{aligned} s_n^i &= h_0^*r_n + h_1^*r_{n+1} + h_2^*r_{n+2} - h_0^*h_2 s_{n-2}^{i-1} \\ &\quad - (h_0^*h_1 + h_1^*h_2) s_{n-1}^{i-1} - (h_1^*h_0 + h_2^*h_1) s_{n+1}^{i-1} - h_2^*h_0 s_{n+2}^{i-1} \\ &= h_0^*(r_n - h_2 s_{n-2}^{i-1} - h_1 s_{n-1}^{i-1}) + h_1^*(r_{n+1} - h_2 s_{n-1}^{i-1} - h_0 s_{n+1}^{i-1}) \\ &\quad + h_2^*(r_{n+2} - h_1 s_{n+1}^{i-1} - h_0 s_{n+2}^{i-1}) \\ &= \underbrace{(|h_0|^2 + |h_1|^2 + |h_2|^2) s_n}_{\text{MRC combined signal}} + \underbrace{h_0^*h_2(s_{n-2} - s_{n-2}^{i-1})}_{\text{cancellation residual}} \\ &\quad + \underbrace{(h_0^*h_1 + h_1^*h_2)(s_{n-1} - s_{n-1}^{i-1}) + (h_1^*h_0 + h_2^*h_1)(s_{n+1} - s_{n+1}^{i-1})}_{\text{cancellation residual}} \\ &\quad + \underbrace{h_2^*h_0(s_{n+2} - s_{n+2}^{i-1})}_{\text{cancellation residual}} + \underbrace{h_0^*v_n + h_1^*v_{n+1} + h_2^*v_{n+2}}_{\text{noise}} \end{aligned} \tag{7}$$

One can see from (7) that the decision statistic for the symbol s_n at the i^{th} iteration are obtained by cancelling the interference using the symbol estimates at the $(i-1)^{th}$ iteration, the inference cancelled signals from different paths are combined using maximum ration combining. This algebraic matrix approach leads to the interference cancellation (IC) and maximum ratio combining (MRC) based equalization. With the decision statistic $s_n^{(i)}$, the symbol estimate can be obtained using the maximum likelihood decision rule $\hat{s}_n = \arg\min_{s_m} |s_n^{(i)} - s_m|^2$. In the case of QPSK modulation, $s_m \in \{s_0, s_1, s_2, s_3\}$. Note that in the beginning of the iterative process, there is no estimate of symbols available. We can use coherent non-cancellation detection to obtain an initial estimate of the transmitted symbols so that the interference cancellation can be carried out in the subsequent stage. Denote $\hat{h}_0$ as an estimate of h_0. To detect the transmitted symbols coherently, we correct the phase shift by multiplying the received signal with the conjugate of $\hat{h}_0$ before making a symbol decision, i.e.,

$$\begin{aligned} r'_n &= \hat{h}_0^* r_n = \hat{h}_0^*(h_0 s_n + h_1 s_{n-1} + h_2 s_{n-2} + v_n) \\ &= \hat{h}_0^* h_0 s_n + \underbrace{\hat{h}_0^*(h_1 s_{n-1} + h_2 s_{n-2} + v_n)}_{\text{combined ISI and noise}} \end{aligned}$$

$$s_n^{(1)} = \arg\min_{s_m \in \{s_0, s_1, s_2, s_3\}} |r'_n - s_m|^2 = \arg\max_{s_m} \mathrm{Re}\{s_m^* r'_n\} \tag{8}$$

Denote the channel vector $\mathbf{h} = \begin{bmatrix} h_0 & h_1 & h_2 \end{bmatrix}^T$, and $\hat{\mathbf{h}}$ as an estimate of $\mathbf{h}$. The estimate of the symbol s_n at the i^{th} stage is denoted as $s_n^{(i)}$ and derived as

$$\begin{aligned} &\mathbf{r}_n^{(i)} = \mathbf{r}_n - \mathbf{H}\tilde{\mathbf{s}}_n^{(i-1)} = \mathbf{H}[\mathbf{s}_n - \tilde{\mathbf{s}}_n^{(i-1)}] + \mathbf{v}_n, \quad z_n^{(i)} = \hat{\mathbf{h}}^* \mathbf{r}_n^{(i)} \\ &s_n^{(i)} = \arg\min_{s_m} |z_n^{(i)} - s_m|^2 = \arg\max_{s_m} \mathrm{Re}\{(\hat{\mathbf{h}} s_m)^* \mathbf{r}_n^{(i)}\} \end{aligned} \tag{9}$$

where $z_n^{(i)}$ is the decision statistic at the equalizer output, and serves as input for the decision device. The vector $\mathbf{r}_n^{(i)}$ is the interference cancelled version of the received vector at the i^{th} cancellation stage. The vector $\tilde{\mathbf{s}}_n^{(i-1)}$ is defined as $\begin{bmatrix} s_{n-2}^{(i-1)} & s_{n-1}^{(i-1)} & 0 & s_{n+1}^{(i-1)} & s_{n+2}^{(i-1)} \end{bmatrix}^T$, and initialized with the coherent non-cancellation detection expressed by equation (8).

Now we analyze the performance of this multistage equalization scheme. To simplify the notations, the iteration index is omitted sometimes whenever no ambiguity

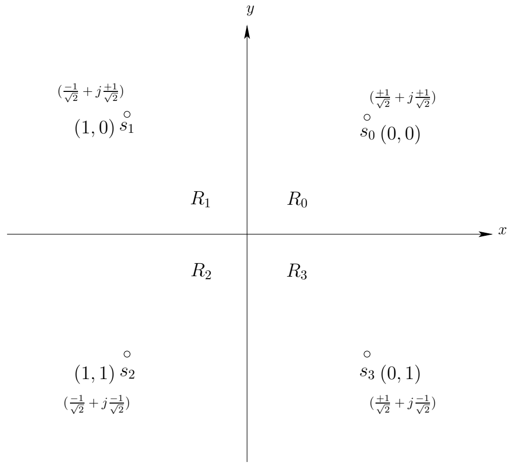

Fig. 1. QPSK constellation, bit-symbol mapping and the ML decision regions.

arises. ISI cancelled version of the received signals can be written as

$$\begin{aligned} r'_n &= h_0 s_n + (h_1 s_{n-1} - \hat{h}_1 \hat{s}_{n-1}) + (h_2 s_{n-2} - \hat{h}_2 \hat{s}_{n-2}) + v_n \\ r'_{n+1} &= h_1 s_n + (h_0 s_{n+1} - \hat{h}_0 \hat{s}_{n+1}) + (h_2 s_{n-1} - \hat{h}_2 \hat{s}_{n-1}) + v_{n+1} \\ r'_{n+2} &= h_2 s_n + (h_0 s_{n+2} - \hat{h}_0 \hat{s}_{n+2}) + (h_1 s_{n+1} - \hat{h}_1 \hat{s}_{n+1}) + v_{n+2} \end{aligned} \tag{10}$$

where $\hat{s}_m$ denotes the estimated s_m (a hard decision) at the previous iteration. Assume accurate channel estimation, i.e., $\hat{\mathbf{h}} \approx \mathbf{h}$, the signals expressed in equation (10) can be reformed as

$$\begin{aligned} r'_n &= h_0 s_n + h_1(s_{n-1} - \hat{s}_{n-1}) + h_2(s_{n-2} - \hat{s}_{n-2}) + v_n \\ r'_{n+1} &= h_1 s_n + h_0(s_{n+1} - \hat{s}_{n+1}) + h_2(s_{n-1} - \hat{s}_{n-1}) + v_{n+1} \\ r'_{n+2} &= h_2 s_n + h_0(s_{n+2} - \hat{s}_{n+2}) + h_1(s_{n+1} - \hat{s}_{n+1}) + v_{n+2} \end{aligned}$$

At the i^{th} cancellation stage, the combined signal after maximum ratio combining becomes

$$\begin{aligned} z_n^{(i)} &= h_0^* r'_n + h_1^* r'_{n+1} + h_2^* r'_{n+2} \\ &= (|h_0|^2 + |h_1|^2 + |h_2|^2) s_n + w_n^i \end{aligned}$$

where

$$\begin{aligned} w_n^i &= h_0^* h_2 (s_{n-2} - s_{n-2}^{(i-1)}) + (h_0^* h_1 + h_1^* h_2)(s_{n-1} - s_{n-1}^{(i-1)}) \\ &\quad + (h_1^* h_0 + h_2^* h_1)(s_{n+1} - s_{n+1}^{(i-1)}) + h_2^* h_0 (s_{n+2} - s_{n+2}^{(i-1)}) \end{aligned}$$

To compute the variance of w_n^i (the combined residual ISI and noise), we need to know the energy of cancellation residual, i.e, $\mathrm{E}[|s_m - \hat{s}_m|^2]$. Without loss of generality, we assume $s_m = s_0$, then

$$\begin{aligned} &\mathrm{E}[|s_0 - \hat{s}_0|^2] = P(\hat{s}_0 = s_0)|s_0 - s_0|^2 + P(\hat{s}_0 = s_1)|s_0 - s_1|^2 \\ &\quad + P(\hat{s}_0 = s_2)|s_0 - s_2|^2 + P(\hat{s}_0 = s_3)|s_0 - s_3|^2 \\ &= P(\hat{s}_0 = s_0) \cdot 0 + P(\hat{s}_0 = s_1) \cdot |2/\sqrt{2}|^2 \\ &\quad + P(\hat{s}_0 = s_2) \cdot |2/\sqrt{2} + 2j/\sqrt{2}|^2 + P(\hat{s}_0 = s_3) \cdot |2j/\sqrt{2}|^2 \end{aligned}$$

From Fig. 1, we see that the signals s_1 and s_3 are at the same distance to s_0. It can be easily shown that $P(\hat{s}_0 = s_1) = P(\hat{s}_0 = s_2)$. The above equation can be reformed as

$$\begin{aligned} \mathrm{E}[|s_0 - \hat{s}_0|^2] &= 2P(\hat{s}_0 = s_1) + 4P(\hat{s}_0 = s_2) + 2P(\hat{s}_0 = s_3) \\ &= 4P(\hat{s}_0 = s_1) + 4P(\hat{s}_0 = s_2) = 4(P_{e1} + P_{e2}) \end{aligned}$$

where $P_{e1} = P(\hat{s}_0 = s_1) = P(\hat{s}_0 = s_3)$, and $P_{e2} = P(\hat{s}_0 = s_2)$. The same results hold for other symbols. The energy of the interference cancellation residual at the i^{th} cancellation stage depends on the symbol error probability at the $(i-1)^{th}$ stage, i.e.,

$$\begin{aligned} &\mathrm{E}[|s_0 - \hat{s}_0|^2] = \mathrm{E}[|s_1 - \hat{s}_1|^2] = \mathrm{E}[|s_2 - \hat{s}_2|^2] = \mathrm{E}[|s_3 - \hat{s}_3|^2] \\ &= 4(P_{e1}^{i-1} + P_{e2}^{i-1}) \end{aligned}$$

The variance of w_n^i at the i^{th} cancellation stage can now be formed as

$$\begin{aligned} N_w^i &= |h_0|^2 \{4(P_{e1}^{i-1} + P_{e2}^{i-1})(|h_1|^2 + |h_2|^2) + N_0\} \\ &\quad + |h_1|^2 \{4(P_{e1}^{i-1} + P_{e2}^{i-1})(|h_0|^2 + |h_2|^2) + N_0\} \\ &\quad + |h_2|^2 \{4(P_{e1}^{i-1} + P_{e2}^{i-1})(|h_0|^2 + |h_1|^2) + N_0\} \end{aligned}$$

Suppose s_0 is transmitted, i.e., $s_n = s_0$, the probability of making correct decision is the probability of $z_n^{(i)}$ falling in the correct decision region R_0, i.e.,

$$\begin{aligned} &P^i(c|s_n = s_0) \\ &= P_r\{z_n^{(i)} = (|h_0|^2 + |h_1|^2 + |h_2|^2)s_0 + w_n^i \in R_0\} \\ &= P_r\left\{\frac{w_I}{\sqrt{N_w^i/2}} > -\frac{(|h_0|^2 + |h_1|^2 + |h_2|^2)}{\sqrt{2}\sqrt{N_w^i/2}}\right\} \\ &\quad \cdot P_r\left\{\frac{w_Q}{\sqrt{N_w^i/2}} > -\frac{(|h_0|^2 + |h_1|^2 + |h_2|^2)}{\sqrt{2}\sqrt{N_w^i/2}}\right\} \\ &= \left[1 - Q\left(\frac{|h_0|^2 + |h_1|^2 + |h_2|^2}{\sqrt{N_w^i}}\right)\right]^2 \end{aligned}$$

where w_I and w_Q are independent Gaussian random variable with variance $N_w^i/2$. Denote $x = |h_0|^2, y = |h_1|^2, z = |h_2|^2$. The symbol and bit error probabilities at the i^{th} cancellation stage can be expressed as

$$\begin{aligned} P_e^i(x,y,z) &= 1 - P^i(c|s_n = s_0) \\ &= 2Q\left(\frac{x+y+z}{\sqrt{N_w^i}}\right) - Q^2\left(\frac{x+y+z}{\sqrt{N_w^i}}\right) \\ P_b^i(x,y,z) &\approx P_e^i(x,y,z) \approx Q\left(\frac{x+y+z}{\sqrt{N_w^i}}\right) \\ &= Q\left(\frac{x+y+z}{\sqrt{a^i x + b^i y + c^i z}}\right) \end{aligned}$$

where

$$\begin{aligned} a^i &= 4(P_{e1}^{i-1} + P_{e2}^{i-1})(y+z) + N_0 \\ b^i &= 4(P_{e1}^{i-1} + P_{e2}^{i-1})(x+z) + N_0 \\ c^i &= 4(P_{e1}^{i-1} + P_{e2}^{i-1})(x+y) + N_0 \end{aligned} \tag{11}$$

Since $|h_0|$, the amplitude of the first tap is Ricean distributed due to the line of sight propagation, the random variable x is non-central chi-square distributed with 2 degrees of freedom and PDF

$$p(x) = \frac{1}{2\sigma^2} \exp\left(-\frac{x+s^2}{2\sigma^2}\right) I_0\left(\frac{\sqrt{x}s}{\sigma^2}\right), \qquad x \geq 0$$

The amplitudes of the other two taps ($|h_1|, |h_2|$) are characterized by a Rayleigh distribution. Therefore, each of the random variables y, z has a central chi-square distribution with 2 degrees of freedom and PDF

$$p(y) = \frac{1}{\gamma_1} \exp\left(-\frac{y}{\gamma_1}\right), \qquad y \geq 0$$
$$p(z) = \frac{1}{\gamma_2} \exp\left(-\frac{z}{\gamma_2}\right), \qquad z \geq 0$$

We must average $P_b^i(x, y, z)$ over distributions of x, y, z to obtain the average bit error probability at the i^{th} stage, i.e.,

$$\begin{aligned}
\bar{P}_b^i &= \int_0^\infty \int_0^\infty \int_0^\infty P_b(x, y, z) p(z) p(y) p(x) dx dy dz \\
&= \frac{1}{2\sigma^2 \gamma_1 \gamma_2} \int_0^\infty \int_0^\infty \int_0^\infty Q\left(\frac{x+y+z}{\sqrt{a^i x + b^i y + c^i z}}\right) \\
&\cdot \exp\left(-\frac{x+s^2}{2\sigma^2}\right) I_0\left(\frac{\sqrt{x}s}{\sigma^2}\right) \exp\left(-\frac{y}{\gamma_1}\right) \exp\left(-\frac{z}{\gamma_2}\right) dz dy dx
\end{aligned} \tag{12}$$

Now, let us see how $P_{e1}^{i-1} + P_{e2}^{i-1}$ in (11) can be obtained. As mentioned earlier, in the beginning of the process, there is no estimate of symbols available. We can use coherent detection to get an initial estimates of transmitted symbol so that interference cancellation can be carried out in the following stages. For the coherent non-cancellation stage, recall (8)

$$\begin{aligned}
r_n' &= \hat{h}_0^* h_0 s_n + \underbrace{\hat{h}_0^*(h_1 s_{n-1} + h_2 s_{n-2} + v_n)}_{\text{combined ISI and noise}} \\
&= \hat{h}_0^* h_0 s_n + w_n \approx |h_0|^2 s_n + w_n
\end{aligned}$$

where $w_n = w_I + j w_Q \sim \mathcal{CN}(0, N_w)$ and $N_w = |h_0|^2(\mathrm{E}[|h_1|^2] + \mathrm{E}[|h_2|^2] + N_0) = |h_0|^2(P_1 + P_2 + N_0)$. The conditional symbol error probabilities are

$$\begin{aligned}
&P^1(\hat{s}_n^{(1)} = s_1 | s_n = s_0) = P_r\{r_n' = |h_0|^2 s_0 + w_n \in R_1\} \\
&= P_r\left\{\frac{|h_0|^2}{\sqrt{2}} + w_I + j\left(\frac{|h_0|^2}{\sqrt{2}} + w_Q\right) \in R_1\right\} \\
&= P_r\left\{\frac{|h_0|^2}{\sqrt{2}} + w_I < 0\right\} \cdot P_r\left\{\frac{|h_0|^2}{\sqrt{2}} + w_Q > 0\right\} \\
&= Q\left(\frac{|h_0|^2}{\sqrt{N_w}}\right)\left[1 - Q\left(\frac{|h_0|^2}{\sqrt{N_w}}\right)\right] \\
&= Q\left(\frac{|h_0|}{\sqrt{P_1 + P_2 + N_0}}\right)\left[1 - Q\left(\frac{|h_0|}{\sqrt{P_1 + P_2 + N_0}}\right)\right] \\
&P^1(\hat{s}_n^{(1)} = s_2 | s_n = s_0) = P_r\{r_n' = |h_0|^2 s_0 + w_n \in R_2\} \\
&= P_r\left\{\frac{|h_0|^2}{\sqrt{2}} + w_I + j(\frac{|h_0|^2}{\sqrt{2}} + w_Q) \in R_2\right\} \\
&= P_r\left\{\frac{|h_0|^2}{\sqrt{2}} + w_I < 0\right\} \cdot P_r\left\{\frac{|h_0|^2}{\sqrt{2}} + w_Q < 0\right\} \\
&= Q^2\left(\frac{|h_0|^2}{\sqrt{N_w}}\right) = Q^2\left(\frac{|h_0|}{\sqrt{P_1 + P_2 + N_0}}\right)
\end{aligned}$$

The sum of the symbol error probabilities at the first non-cancellation stage is thus

$$\begin{aligned}
&P_{e1}^1 + P_{e2}^1 = P^1(\hat{s}_n^{(1)} = s_1 | s_n = s_0) + P^1(\hat{s}_n^{(1)} = s_2 | s_n = s_0) \\
&= Q\left(\sqrt{\frac{x}{P_1 + P_2 + N_0}}\right)\left[1 - Q\left(\sqrt{\frac{x}{P_1 + P_2 + N_0}}\right)\right] \\
&+ Q^2\left(\sqrt{\frac{x}{P_1 + P_2 + N_0}}\right) = Q\left(\sqrt{\frac{x}{P_1 + P_2 + N_0}}\right)
\end{aligned} \tag{13}$$

Following the similar routine, we can calculate the sum of the symbol error probabilities in the subsequent cancellation stage as

$$\begin{aligned}
&P_{e1}^i + P_{e2}^i = P^i(\hat{s}_n^{(i)} = s_1 | s_n = s_0) + P^i(\hat{s}_n^{(i)} = s_2 | s_n = s_0) \\
&= Q\left(\frac{x+y+z}{\sqrt{a^i x + b^i y + c^i z}}\right)\left[1 - Q\left(\frac{x+y+z}{\sqrt{a^i x + b^i y + c^i z}}\right)\right] \\
&+ Q^2\left(\frac{x+y+z}{\sqrt{a^i x + b^i y + c^i z}}\right) = Q\left(\frac{x+y+z}{\sqrt{a^i x + b^i y + c^i z}}\right)
\end{aligned} \tag{14}$$

III. Analytical and Numerical Results

Computer simulations are carried out to demonstrate the performance of the proposed algorithm. During each Monte-Carlo run, the block size is set to 10000 bits, which correspond to 5000 QPSK symbols. The channel coefficients of the BFWA channels vary from one data block to another, however, they are assumed to remain constant during the transmission of one block of data. It is therefore a quasi-static channel. Channel coefficients are assumed to be known to the receiver ($\hat{\mathbf{h}} = \mathbf{h}$). The simulated results are averaged over 1000 channel realizations. For the bit error probability calculation in (12), the parameters settings for the SUI-3 channel coefficients are $s^2 = 1, \sigma^2 = 0.25$, $P_1 = \gamma_1 = \mathrm{E}[|h_1|^2] = 0.3162$, $P_2 = \gamma_2 = \mathrm{E}[|h_2|^2] = 0.1$.

Fig. 2 shows that the theoretical analysis expressed by (11), (12), (13) is in close agreement with the simulated results for the first non-cancellation stage. The analysis expressed by (11), (12), (14) deviates from the simulation for the second stage when cancellation is performed. It is, however, accurate around 8 and 9 dB. Both simulation and analysis indicate that the improvement by applying the IC and MRC over coherent non-cancellation detection is significant and that equalization alone does not yield satisfactory performance in BFWA systems, the resulting bit error rate is well above 10^{-2}.

The convergence property of the proposed equalization scheme is illustrated in Fig. 3. It takes only 3 stages for the iterative scheme to converge. The dash-dot curve in the plot represents the theoretical lower bound of this equalization algorithm, which is derived by assuming perfect cancellation. The derivation and formula of this bound is given in [11], and omitted here to conserve space. Apparently, the actual performance of this algorithm is far from the performance bound. The rationale is that the error in the decision feedback significantly degrades the performance and prevents the algorithm from reaching its theoretical potential. There are different ways of tackling this problem, e.g., using channel coding to reduce the feedback error probability, and/or using soft cancellation rather than brutal force cancellation to prevent error propagation.

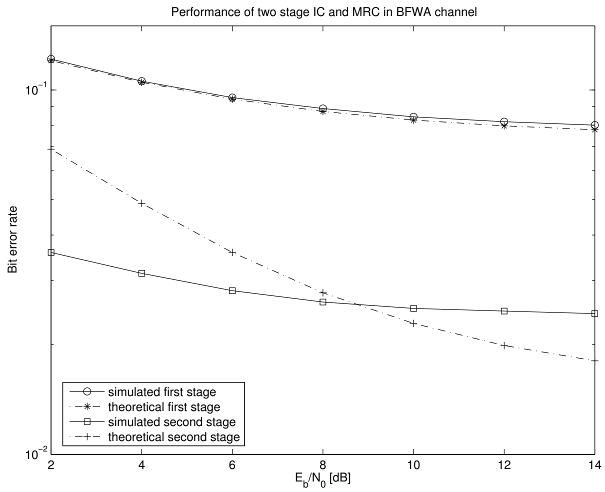

Fig. 2. Performance of the multistage equalization in uncoded QPSK: simulation vs. analysis.

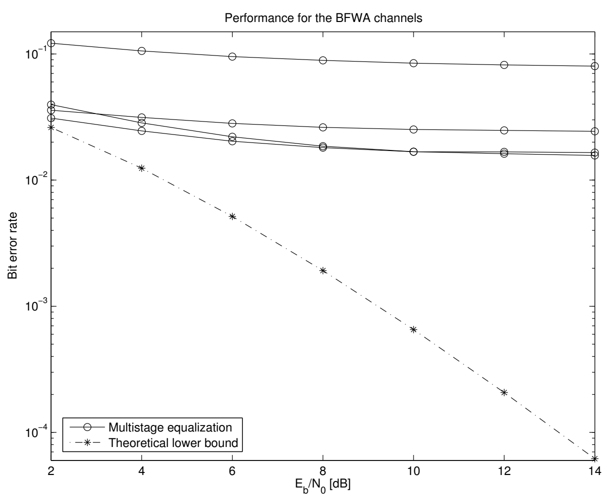

Fig. 3. Performance of the equalization scheme and comparison with its theoretical lower bound. For the multistage equalization, the topmost curve represents non-cancellation coherent detection stage and the second curve from the top represents the first stage equalization, the bottommost curve represents the 3rd stage equalization.

IV. Conclusions

In this paper, we introduced a linear algebraic approach to equalization. However, both simulation and analysis indicate that BFWA channels are very hostile. Equalization alone can not combat the detrimental effects of ISI. Results also show that this algorithm is far from its theoretical potential in an uncoded system, due to the fact that the errors in decision feedback will significantly degrade the performance. This suggests the use of channel coding and a soft interference cancellation scheme to reduce the feedback propagation errors. The combination of these ideas lead to joint equalization and decoding algorithm, which will be the future research topic for the authors.

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