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Multiple scattering effects in proton nucleus elastic scattering at intermediate energies

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The second-order corrections to the Kerman-McManus-Thaler optical potential for the elastic scattering of protons from ^{16}O are calculated at 135, 200, and 300 MeV incident energies, paying particular attention to the nonlocalities inherent in the potential. It is shown that these effects result in a significant reduction in the proton-target absorption.

Recent detailed investigations of nucleon-nucleon (NN) transition amplitudes calculated from realistic interactions [1] have shown their on- and off-shell behavior to be remarkably stable to the choice of interaction [2]. This, combined with more detailed information on nuclear wave functions now available from electron-scattering data, has stimulated a renewed interest in finite nucleus calculations of the nucleon-nucleus interaction based on the free NN transition amplitude and multiple-scattering expansions as formulated, for example, by Kerman, McManus, and Thaler (KMT) [3]. For nucleon-nucleus (NA) scattering at intermediate energies, calculations based on the KMT formalism are thus expected to be relatively insensitive to the assumed NN interaction. This is certainly not the case in the evaluation of medium effects, both at low and intermediate energies, using the nuclear matter G-matrix approach [4]. Such approaches also use the local-density approximation in applying the nuclear matter results to finite nuclei, an approximation which is suspect [5].

The first-order term of the KMT nucleon-nucleus potential, known as the single-scattering approximation (SSA), is given by the expectation value of the free NN transition amplitude in the target nucleus ground state. The effects of nonlocality on the evaluation of this first-order term, have recently been the subject of a number of very detailed investigations [6–8]. The significance of any agreement or disagreement of such first-order calculations with the experimental data are however impossible to assess without realistic calculations of the importance of higher order and, in particular, second-order contributions to the interaction. In this Rapid Communication we address this problem. We present the most complete calculations to date of the second-order term of the KMT nucleon-nucleus potential. We pay particular attention to the effects of the nonlocalities, inherent in the potential, on the calculated differential and integrated cross sections. We neglect, in the first instance, the spin-orbit component of the second-order potential. An estimate of the importance of this term for polarization data was made recently by Feshbach [9].

The second-order term of the multiple-scattering expansion of the NA potential involves a double scattering between the incident and target nucleons (DSA). This term, bilinear in the NN amplitude, involves two-particle correlations in a crucial way. Nonlocalities appear in this term from two basic sources. First, as in the first-order term, nonlocality arises through the NN amplitude. An additional nonlocality arises from the propagation of the nucleons in the intermediate state between scattering events. This complicates enormously the evaluation of the DSA terms. For this reason previous calculations have attempted to localize the second-order potential using the eikonal approximation to the propagator, simple models for the correlation function and approximate prescriptions for the NN transition amplitude [10]. Two attempts, by Feshbach and co-workers [11] and Johnson and Martin [12], have been made to include the nonlocality associated with the intermediate states propagator. However neither approach included fully the folding of the nonlocal NN transition amplitude with the target wave function. These effects are included explicitly in the present work. An analysis of the importance of such effects, within a relativistic context, was also made recently by Kaki [13].

According to the KMT multiple-scattering formalism, the first-order optical potential for proton-nucleus elastic scattering can be written

$$\hat{U}^{(1)}=\frac{A-1}{A}\sum_{\alpha}\langle\alpha|t_{01}(\omega)|\alpha\rangle\,, \tag{1}$$

where we have assumed that the target ground state is described by a single determinant of occupied single-particle wave functions $|\alpha\rangle$, with single-particle energies ϵ_α, and $t_{01}(\omega)$ is the antisymmetrized free NN transition operator for energy parameter ω. We do not consider those effects arising from the struck nucleon-core potential in the NN propagator. If, for the determination of ω, we also neglect the momentum of the struck nucleon and take the momentum of the incident nucleon to be the on-shell value, then the appropriate NN energy is half the beam energy, i.e., $\omega=\frac{1}{2}E$ [8].

We write the second-order potential $\hat{U}^{(2)}$ as the sum of two components [14]

$$\hat{U}_I^{(2)}=-\frac{A-1}{A}\sum_{\alpha,\beta}\langle\alpha|t_{01}(\omega)|\beta\rangle \times\frac{1}{E^+-K_0+\epsilon_\alpha-\epsilon_\beta}\langle\beta|t_{01}(\omega)|\alpha\rangle\,, \tag{2}$$

$$\hat{U}_{II}^{(2)}=\frac{1}{A-1}\left[\hat{U}^{(1)}\frac{1}{E^+-K_0}\hat{U}^{(1)}\right], \tag{3}$$

where K_0 is the kinetic-energy operator of the incident nucleon and the sums run over all occupied single-particle states. The term $U_{II}^{(2)}$ arises naturally in the KMT formalism [3] and must be included to account correctly for all contributions from ground-state matrix elements of the *NN* transition amplitude.

In the present work all matrix elements of $t_{01}(\omega)$, in Eqs. (1)–(3), were calculated assuming the optimal factorization approach [4–6].

At intermediate energies, it is reasonable to introduce the closure approximation in the intermediate states propagator in Eq. (2). This replaces the single-particle energy differences $\epsilon_\alpha - \epsilon_\beta$ by some average energy. With this energy taken to be zero, this can be shown to provide an excellent description of the elastic-scattering observables [14]. In this approximation, the momentum space matrix elements of $U_I^{(2)}$ can be rewritten in terms of the target correlation function [15]

$$D(\mathbf{r},\mathbf{r}') = A^2\rho(r)\rho(r') - A(A-1)\rho(\mathbf{r},\mathbf{r}') , \quad (4)$$

where $\rho(\mathbf{r},\mathbf{r}')$ is the probability of finding a nucleon at position $\mathbf{r}$ and another at position $\mathbf{r}'$, and $\rho(r)$ is the nuclear density normalized to unity.

Using the KMT decomposition of the *NN* transition amplitude [3], the central component of $\hat{U}_I^{(2)}$ is

$$\langle \mathbf{k}|\hat{U}_I^{(2)}|\mathbf{k}'\rangle = -\frac{A-1}{A}\int d\mathbf{k}''\beta(\omega,\mathbf{k},\mathbf{k}'',\mathbf{k}')g(k'')\int d\mathbf{r}\int d\mathbf{r}' e^{-i(\mathbf{q}\cdot\mathbf{r}+\mathbf{q}'\cdot\mathbf{r}')}D(\mathbf{r},\mathbf{r}') , \quad (5)$$

where $g(k'')$ is the intermediate states propagator (in momentum space representation) and $\mathbf{q}=\mathbf{k}-\mathbf{k}''$ and $\mathbf{q}'=\mathbf{k}''-\mathbf{k}'$ are the momentum transfers at the two *NN* vertices.

For a target of zero total spin and isospin

$$\beta(\omega,\mathbf{k},\mathbf{k}'',\mathbf{k}') = \left[\frac{\hbar^2}{\mu_{NN}(2\pi)^2}\right]^2 \{\mathcal{A}^2+\mathcal{B}^2+2\mathcal{C}^2+\mathcal{D}^2+\mathcal{E}^2+2\mathcal{F}^2\} , \quad (6)$$

with μ_{NN} the *NN* reduced mass and $\mathcal{A}^2$–$\mathcal{F}^2$ [14] are given in terms of the isoscalar and isovector components of the KMT *NN* amplitudes according to

$$\mathcal{A}^2 = \mathcal{A}_0^2 + 3\mathcal{A}_\tau^2 . \quad (7)$$

The momentum space matrix elements of $\hat{U}_{II}^{(2)}$ can similarly be written

$$\langle \mathbf{k}|\hat{U}_{II}^{(2)}|\mathbf{k}'\rangle = (A-1)\left[\frac{\hbar^2}{\mu_{NN}(2\pi)^2}\right]^2 \times \int d\mathbf{k}''[\mathcal{A}_0^2+\mathcal{C}_0^2]g(k'')\rho(q)\rho(q') . \quad (8)$$

In Eqs. (6)–(8) we have adopted an abbreviated notation for the squares of the components of the *NN* amplitude, $\mathcal{A}_0^2 = \mathcal{A}_0(\omega,\mathbf{k},\mathbf{k}'')\mathcal{A}_0(\omega,\mathbf{k}'',\mathbf{k}')$, etc., and we make use of a small angle coplanar scattering approximation in treating the unit vectors entering the expansion of the transition amplitude in terms of the component KMT amplitudes $\mathcal{A}$, $\mathcal{B}$, etc. We note that the contributions from the isoscalar central and spin-orbit components $\mathcal{A}_0$ and $\mathcal{C}_0$ to $\hat{U}_I^{(2)}+\hat{U}_{II}^{(2)}$ is proportional to the correlation function $C(\mathbf{r},\mathbf{r}')$ of Ref. [11].

For the evaluation of the potential matrix elements the *NN* amplitudes were expressed in terms of the *NN* energy, the momentum transfer, and the total momentum. It was shown in Ref. [16] that the dependence of $\mathcal{A}$ and $\mathcal{C}$ on the angle between the latter two vectors is weak. We assume the same approximation is true for all the amplitudes $\mathcal{A}$–$\mathcal{F}$, and take the angle to be fixed at $\pi/2$. In this case the contribution from $\mathcal{F}$ vanishes. Further details will be given in Ref. [14]. The optical potential was then calculated, in momentum space, without additional approximation. We refer to such calculations as the nonlocal case.

In order to assess the importance of the nonlocalities arising from the *NN* transition amplitude and the intermediate state propagator on the second-order optical potential, we also consider three analogous but more approximate calculations. We first consider the situation where, in the evaluation of the second-order term, we assume the *NN* amplitude to take its on-shell values, leading to a local *NN* amplitude, a function of energy and momentum transfer only. The second-order potential obtained in this approximation treats correctly both the nonlocality of the intermediate states propagator and the folding of the finite range and angular dependence of the *NN* transition amplitude with the target wave function. The difference between these and the nonlocal calculations provide an estimate of the importance of the nonlocality of the *NN* amplitude. If we also consider the yet more approximate situation where, in the second-order term, the *NN* amplitude is fixed at its on-shell, zero momentum transfer value (the zero-range limit), we obtain what we will refer to as the zero-range potential. The comparison between this and the previous potential provides an estimate of the importance of carrying out correctly the folding of the finite ranged *NN* transition amplitude with the target wave function in the second-order term.

In order to remove all nonlocalities and arrive at an entirely local expression for the second-order optical potential, we fix the *NN* transition amplitude as in the zero-range case. In addition, the eikonal approximation [10, 17] is used for the intermediate state propagator. Finally, the correlation function $D(\mathbf{r},\mathbf{r}')$, taken from nuclear matter, is applied to the finite nucleus using the local-density approximation. This yields the local second-order potential

$$\langle \mathbf{k}|\hat{U}^{(2)}|\mathbf{k}'\rangle = iA(A-1)\frac{2\pi^3\mu_{NA}R_F}{\hbar^2 k_0}\beta_0(\omega)F(q) , \quad q=|\mathbf{k}-\mathbf{k}'| , \quad (9)$$

where μ_{NA} is the *NA* reduced mass, k_0 is the on-shell entrance channel momentum, R_F the Fermi correlation length, taken as $R_F = 1.38$ fm, and $F(q)$ is the Fourier transform of the square of the target density. The quantity $\beta_0(\omega)$ is given by Eq. (6) but with the NN amplitudes evaluated in the on-shell zero range limit.

In the description of the target nucleus, we do not distinguish between protons and neutrons. We take the radial parts of the single-particle wave functions to be of harmonic oscillator (HO) form, with an oscillator parameter $a = (\hbar/M\omega)^{1/2} = 1.77$ fm [18] which produces a reasonable description of the charge form factors at small momentum transfers, typically $q \leq 2$ fm^{-1}. To estimate the uncertainty in the calculations due to nuclear correlations, we also calculate the single-particle wave functions using a Woods-Saxon (WS) potential. The potential parameters, obtained by fitting the electron scattering [14], provide an excellent description of the charge form factor for $q \leq 3$ fm^{-1}. The optical potential calculations were carried out in momentum space [19] for proton elastic scattering from ^{16}O at 135, 200, and 300 MeV incident energies using the free NN transition amplitude calculated from the Paris potential [20,21].

In order to obtain a first indication of the importance of the various nonlocalities present in the DSA terms on the absorptive nature of the optical potential we present, in Fig. 1, the calculated partial-wave reflection coefficients $\eta(+)$ corresponding to total angular momentum $J = L + \frac{1}{2}$. The calculations use the HO target wave functions. To isolate the effect of the treatment of the nonlocalities in the nuclear component of the second-order NA optical potential, these calculations were performed in the absence of the nucleon-nucleus Coulomb interaction. We see from Fig. 1 that, at all energies, the nonlocal second-order calculations (solid curves) reduce the absorption present in the lower partial waves in comparison with the first-order KMT calculations (dotted curves). The long-dashed curves, the results of calculations assuming the NN transition amplitude is on the energy shell, show that the contribution of the nonlocality of the NN amplitude to the second-order potential is negligible at these intermediate energies, at least within the framework the optimal factorization approximation. The zero-range (dashed curves) and local (dot-dashed curves) calculations, on the other hand, give considerably less absorption near the grazing partial waves and make clear the need for an accurate inclusion of the folding of the finite ranged NN amplitude with the target wave function, within the second-order term. This effect was taken into account in an approximate way in Refs. [10–12]. The effects associated with the nonlocality of the intermediate states propagator are, by comparison, small and tend to reduce the absorption compared with the local second-order potential calculation.

Figure 2 shows the elastic differential cross sections at the three energies under consideration. The nucleon-nucleus Coulomb potential is included in these calculations using the subtracted momentum space method [22] and assuming a uniform charge sphere density of radius $R_c = 1.3A^{1/3}$ fm, and a cutoff radius $R_{cut} = 10$ fm. The effect of the nonlocal second-order potential contributions on the elastic cross section is small, as seen by comparison with the first-order calculations (dotted curves). All

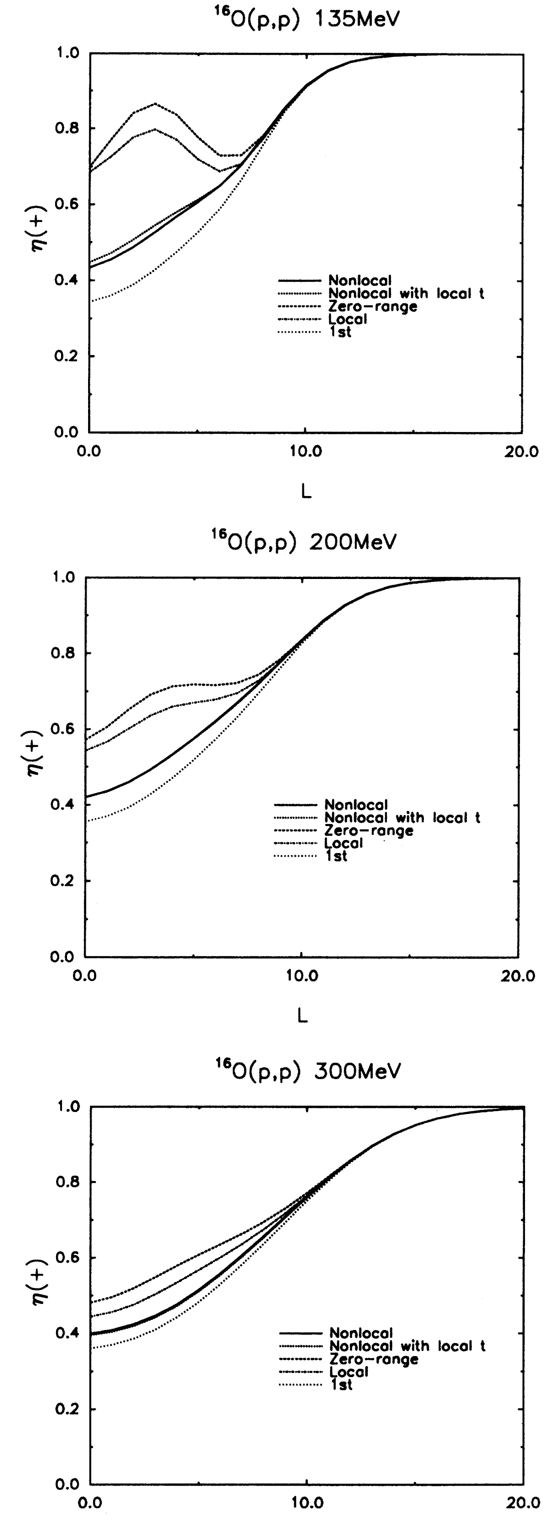

FIG. 1. Reflection coefficients in the $J = L + 1/2$ partial wave for ^{16}O(p,p) scattering at 135, 200, and 300 MeV, calculated using the first-order (dotted curve) and second-order nonlocal (solid curve), nonlocal with on-shell transition amplitude (long-dashed curve), and zero-range (dashed curve), and local (dot-dashed curve) optical potentials described in the text.

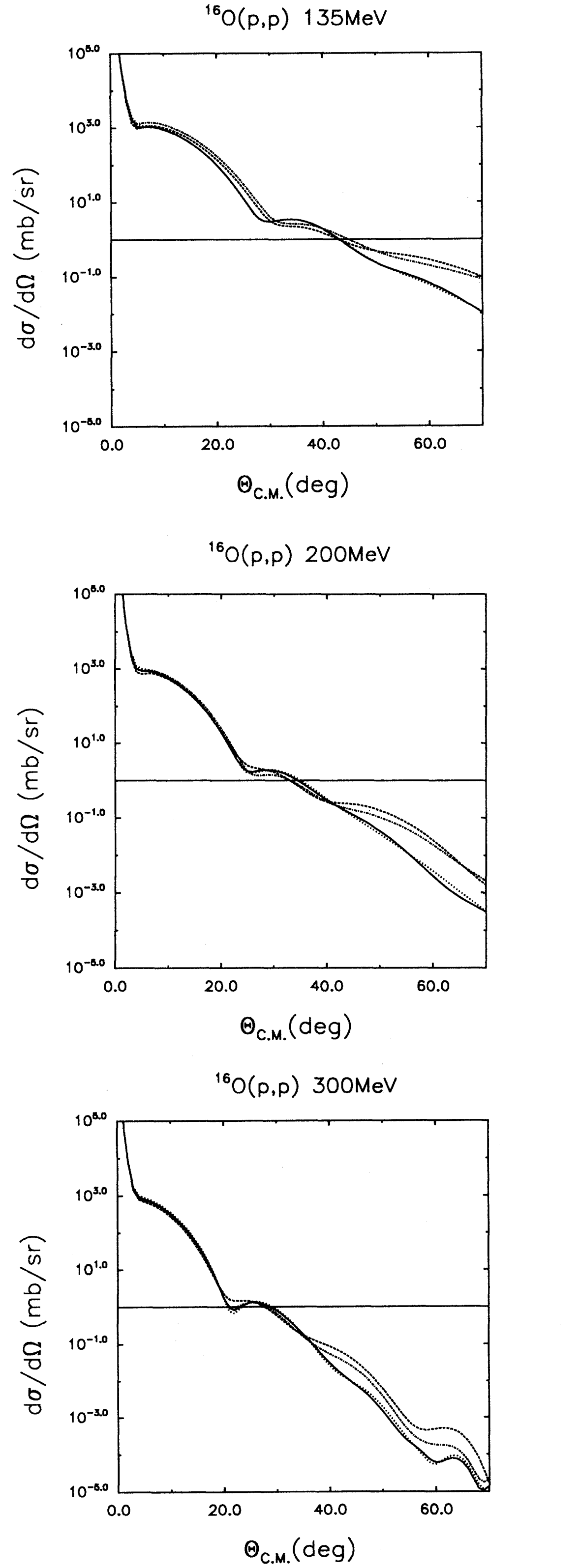

FIG. 2. Differential cross section for ^{16}O(p,p) at 135, 200, and 300 MeV, calculated using the first-order (dotted curve) and second-order nonlocal (solid curve), zero-range (dashed curve), and local (dot-dashed curve) optical potentials described in the text.

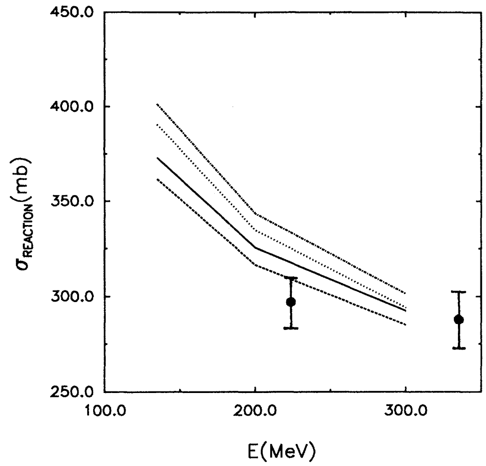

FIG. 3. Reaction cross sections for ^{16}O(p,p) as a function of the incident laboratory energy calculated using the first-order optical potential with HO (dotted curve) and Woods-Saxon (dot-dashed curve) target wave functions. The dashed and solid curves are the results of the second-order nonlocal calculations using the HO and WS wave functions, respectively. The data points are taken from Ref. [23].

curves have the same meanings as in Fig. 1. The zero-range and local second-order potentials by contrast, drastically overestimate the DSA terms at large angles, as might be anticipated from their use of the zero-range approximation to the NN transition amplitude.

Figure 3 shows the calculated and experimental reaction cross sections over the energy range of interest. Here the solid and dashed curves show the results obtained from the nonlocal calculations when using harmonic oscillator and Woods-Saxon wave functions, respectively. The dot-dashed and dotted curves show the corresponding calculations for the first-order potential only. The data points show the experimentally deduced values and are taken from Ref. [23]. It is evident that the nonlocal second-order potential calculations result in a significant reduction in the reaction cross sections as compared with those obtained from the first-order potential.

In conclusion, to obtain a realistic estimate of second-order corrections to the nucleon-nucleus optical potential for elastic scattering, we need to treat very carefully the folding of the finite range of the NN transition amplitude with the target wave function. In the optimal factorization form of the second-order potential, we find that the effects associated with the nonlocality of the NN amplitude can be neglected at intermediate energies. The multiple-scattering corrections result in a significant reduction in the proton-target absorption particularly in lower partial waves. Due to the surface dominance of the elastic scattering, the elastic observables are not very sensitive to such effects. Our results suggest, however, that the second-order terms in the KMT multiple-scattering expansion of the optical potential generate significant modifications to proton-nucleus wave functions in low partial waves. This may have implications for distorted wave calculations of nuclear reactions.

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