

# Scaling properties of the perturbative Wilson loop in two-dimensional non-commutative Yang-Mills theory

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## Abstract

Commutative Yang-Mills theories in 1+1 dimensions exhibit an interesting interplay between geometrical properties and  $U(N)$  gauge structures: in the exact expression of a Wilson loop with  $n$  windings a non trivial scaling intertwines  $n$  and  $N$ . In the non-commutative case the interplay becomes tighter owing to the merging of space-time and “internal” symmetries in a larger gauge group  $U(\infty)$ . We perform an explicit perturbative calculation of such a loop up to  $\mathcal{O}(g^6)$ ; rather surprisingly, we find that in the contribution from the crossed graphs (the genuine non-commutative terms) the scaling we mentioned occurs for large  $n$  and  $N$  in the limit of maximal non-commutativity  $\theta = \infty$ . We present arguments in favour of the persistence of such a scaling at any perturbative order and succeed in summing the related perturbative series.

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## I. INTRODUCTION

One of the most interesting and intriguing features of noncommutative field theories is the merging of space-time and “internal” symmetries in a larger gauge group  $U(\infty)$  [1,2]. Peculiar topological properties can find their place there and be conveniently described under the general frame provided by the K-theory [3].

On the other hand some interplay occurs also when theories are defined on commutative spaces; in [4] it has been shown that in two space-time dimensions a non trivial holonomy concerning the base manifold and the fiber  $U(N)$  appears when considering a Wilson loop winding  $n$  times around a closed contour, leading to a peculiar scaling law intertwining the two integers  $n$  and  $N$

$$\mathcal{W}_n(\mathcal{A}; N) = \mathcal{W}_N\left(\frac{n}{N}\mathcal{A}; n\right), \quad (1)$$

$\mathcal{W}$  being the exact expression of the Wilson loop and  $\mathcal{A}$  the enclosed area. When going around the loop the non-Abelian character of the gauge group is felt.

One may wonder whether similar relations are present in the noncommutative case and, in the affirmative, what they can teach concerning the tighter merging occurring in such a situation.

Noncommutative field theories have been widely explored in recent years. Although their basic motivation relies, in our opinion, by their relation with string theories [5–7], they often exhibit curious new features and are therefore fascinating on their own [8,9].

The simplest way of turning ordinary theories into non-commutative ones is to replace the usual multiplication of fields in the Lagrangian with the Moyal  $\star$ -product. This product is constructed by means of a real antisymmetric matrix  $\theta^{\mu\nu}$  which parameterizes non-commutativity of Minkowski space-time:

$$[x^\mu, x^\nu] = i\theta^{\mu\nu} \quad \mu, \nu = 0, \dots, D - 1. \quad (2)$$

The  $\star$ -product of two fields  $\phi_1(x)$  and  $\phi_2(x)$  can be defined by means of Weyl symbols

$$\phi_1 \star \phi_2(x) = \int \frac{d^D p d^D q}{(2\pi)^{2D}} \exp \left[ -\frac{i}{2} p_\mu \theta^{\mu\nu} q_\nu \right] \exp(ipx) \tilde{\phi}_1(p-q) \tilde{\phi}_2(q). \quad (3)$$

The resulting action makes obviously the theory non-local.

A particularly interesting situation occurs in  $U(N)$  gauge theories defined in one-space, one-time dimensions ( $YM_{1+1}$ ).

The classical Minkowski action reads

$$S = -\frac{1}{4} \int d^2x \text{Tr} \left( F_{\mu\nu} \star F^{\mu\nu} \right) \quad (4)$$

where the field strength  $F_{\mu\nu}$  is given by

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ig(A_\mu \star A_\nu - A_\nu \star A_\mu) \quad (5)$$

and  $A_\mu$  is a  $N \times N$  hermitian matrix.

The action in Eq. (4) is invariant under  $U(N)$  non-commutative gauge transformations

$$\delta_\lambda A_\mu = \partial_\mu \lambda - ig(A_\mu \star \lambda - \lambda \star A_\mu). \quad (6)$$

We quantize the theory in the light-cone gauge  $n^\mu A_\mu \equiv A_- = 0$ , the vector  $n_\mu$  being light-like,  $n^\mu \equiv \frac{1}{\sqrt{2}}(1, -1)$ . This gauge is particularly convenient since Faddeev-Popov ghosts decouple even in a non-commutative context [10], while the field tensor is linear in the field with only one non vanishing component  $F_{-+} = \partial_- A_+$ .

In this gauge two *different* prescriptions are obtained for the vector propagator in momentum space, namely

$$D_{++} = i [k_-^{-2}]_{PV} \quad (7)$$

and

$$D_{++} = i [k_- + i\epsilon k_+]^{-2}, \quad (8)$$

$PV$  denoting the Cauchy principal value. The two expressions above are usually referred in the literature as 't Hooft [11] and Wu-Mandelstam-Leibbrandt ( $WML$ ) [12,13] propagators.

They correspond to two different ways of quantizing the theory, namely by means of a light-front or of an equal-time algebra [14,15] respectively and, obviously, coincide with the ones in the commutative case.

The *WML* propagator can be Wick-rotated, thereby allowing for an Euclidean treatment. A smooth continuation of the propagator to the Euclidean region is instead impossible when using the *PV* prescription.

In the commutative case, a perturbative calculation for a closed Wilson loop, computed with the 't Hooft's propagator, coincides with the exact expression obtained on the basis of a purely geometrical procedure [16,17]

$$\mathcal{W} = \exp\left(-\frac{1}{2}g^2 N \mathcal{A}\right). \quad (9)$$

The use instead of the *WML* propagator leads to a different, genuinely perturbative expression in which topological effects are disregarded [19,20]

$$\mathcal{W}_{WML} = \frac{1}{N} \exp\left(-\frac{1}{2}g^2 \mathcal{A}\right) L_{N-1}^{(1)}(g^2 \mathcal{A}), \quad (10)$$

$L_{N-1}^{(1)}$  being a Laguerre polynomial.

One can inquire to what extent these considerations can be generalized to a non-commutative  $U(N)$  gauge theory, always remaining in 1+1 dimensions. This was explored in ref. [21] by performing a fourth order perturbative calculation of a closed Wilson loop.

In the non-commutative case the Wilson loop can be defined by means of the Moyal product as [22,23,1]

$$\mathcal{W}[C] = \frac{1}{N} \int \mathcal{D}A e^{iS[A]} \int d^2x \operatorname{Tr} P_{\star} \exp\left(ig \int_C A_+(x + \xi(s)) d\xi^+(s)\right), \quad (11)$$

where  $C$  is a closed contour in non-commutative space-time parameterized by  $\xi(s)$ , with  $0 \leq s \leq 1$ ,  $\xi(0) = \xi(1)$  and  $P_{\star}$  denotes non-commutative path ordering along  $x(s)$  from left to right with respect to increasing  $s$  of  $\star$ -products of functions. Gauge invariance requires integration over coordinates, which is trivially realized when considering vacuum averages [24].

The perturbative expansion of  $\mathcal{W}[C]$ , expressed by Eq. (11), reads

$$\mathcal{W}[C] = \frac{1}{N} \sum_{n=0}^{\infty} (ig)^n \int_0^1 ds_1 \dots \int_{s_{n-1}}^1 ds_n \dot{x}_-(s_1) \dots \dot{x}_-(s_n) \langle 0 | \text{Tr } \mathcal{T} [A_+(x(s_1)) \star \dots \star A_+(x(s_n))] | 0 \rangle, \quad (12)$$

and it is shown to be an even power series in  $g$ , so that we can write

$$\mathcal{W}[C] = 1 + g^2 \mathcal{W}_2 + g^4 \mathcal{W}_4 + g^6 \mathcal{W}_6 + \dots \quad (13)$$

If we consider  $n$  windings around the loop, the result can be easily obtained by extending the interval  $0 \leq s \leq n$ ,  $\xi(s)$  becoming a periodic function of  $s$ .

The main conclusion of [21] was that a perturbative Euclidean calculation with the WML prescription is feasible and leads to a regular result. We found indeed pure area dependence (we recall that invariance under area preserving diffeomorphisms holds also in a non-commutative context) and continuity in the limit of vanishing non-commutative parameter. The limiting case of a large non-commutative parameter (maximal non-commutativity) is far from trivial: as a matter of fact the contribution from the non-planar graph does not vanish in the large- $\theta$  limit at odds with the result in higher dimensions [8].

More dramatic is the situation when considering the 't Hooft's form of the free propagator. In the non-commutative case the presence of the Moyal phase produces singularities which cannot be cured [21]. As a consequence 't Hooft's context will not be further considered.

Another remarkable difference between 't Hooft's and WML formulations in commutative Yang-Mills theories was noticed in [4]. When considering  $n$  windings around the closed loop, a non trivial holonomy concerning the base manifold and the fiber ( $U(N)$ ) (Eq.(1)) took place in the exact solution. The behaviour of the WML solution was instead fairly trivial ( $\mathcal{A} \rightarrow n^2 \mathcal{A}$ ), as expected in a genuinely perturbative treatment. However it is amazing to notice that the expression (10) with  $n$  windings, when restricted to *planar* diagrams becomes

$$\mathcal{W}_{WML}^{(pl)} = \sum_{m=0}^{\infty} \frac{(-g^2 \mathcal{A} n^2 N)^m}{m!(m+1)!} = \frac{1}{\sqrt{g^2 \mathcal{A} n^2 N}} J_1(2\sqrt{g^2 \mathcal{A} n^2 N}). \quad (14)$$

Scaling (1) is recovered.

In the non-commutative case this issue acquires a much deeper interest thanks to the merging of space-time and “internal” symmetries in a large gauge group  $U(\infty)$ , or, better, in its largest completion  $U_{cpt}(\mathcal{H})$  [2]. Also for the WML formulation we expect a non trivial intertwining between  $n$  and  $N$ , which might help in clarifying some features of this merging. Actually this is the main motivation of the present research.

Lacking a complete solution, we limit ourselves to a perturbative context. A little thought is enough to be convinced that the function  $\mathcal{W}_2$  in Eq. (13) is reproduced by the single-exchange diagram, which is exactly the same as in the commutative  $U(N)$  theory. Actually all planar graphs contributions coincide with the corresponding ones of the commutative case [19], being independent of  $\theta$  (see Eq.(14)). Although they dominate for large  $N$  and  $n$ , they are a kind of “constant” background, which is uninteresting in this context. Therefore in the following we will concentrate ourselves in calculating and discussing the properties of non-planar graphs  $\mathcal{W}^{(cr)}$  in the *WML* (Euclidean) formulation.

The contributions  $\mathcal{W}_4^{(cr)}$  and  $\mathcal{W}_6^{(cr)}$  with  $n$  windings will be presented in detail. At  $\theta = 0$  the commutative result is recovered, together with its trivial perturbative scaling, the result being continuous (but probably not analytic there).

Surprisingly, at  $\theta = \infty$  and at  $\mathcal{O}(g^4)$ , we recover the non trivial scaling law (1) of the exact solution in the commutative case; however, for the sake of clarity, we stress that such a scaling is here realized in a quite different mathematical expression. At  $\mathcal{O}(g^6)$  the scaling receives corrections, decreasing at large  $n$ ; as a consequence we can say that it holds only at large  $\theta$  and large  $n$ . We also realize that diagrams with a single crossing of propagators dominate, making possible the extension to higher perturbative orders. This evidence is partly based on a numerical evaluation of an integral occurring in the calculation of diagrams with a double crossing (see Appendix B).

We present arguments in favour of the persistence of such a scaling in the limits  $(n, N, \theta) \rightarrow \infty$  at any perturbative order and eventually succeed in summing the related perturbative series.

As soon as we move away from the extreme values  $\theta = 0, \theta = \infty$ , corrections appear which are likely to interpolate smoothly between small- $\theta$  and large- $\theta$  behaviours.

In Sect.2 we present the  $\mathcal{O}(g^4)$  calculation; the  $\mathcal{O}(g^6)$  results are reported in Sect.3 together with our conjecture concerning the leading terms at large  $n, N$  and  $\theta$  at any perturbative order. The details of the calculations are deferred to the Appendices. Final considerations are discussed in the Conclusions.

## II. THE FOURTH ORDER CALCULATION

We concentrate our attention on  $\mathcal{W}_4^{(cr)}$  and resort to an Euclidean formulation, generalizing to  $n$  windings the results reported in [21].

By exploiting the invariance of  $\mathcal{W}$  under area-preserving diffeomorphisms, which holds also in this non-commutative context, we consider the simple choice of a circular contour

$$x(s) \equiv (x_1(s), x_2(s)) = r(\cos(2\pi s), \sin(2\pi s)). \quad (15)$$

Were it not for the presence of the Moyal phase, a tremendous simplification would occur between the factor in the measure  $\dot{x}_-(s)\dot{x}_-(s')$  and the basic correlator  $\langle A_+(s)A_+(s') \rangle$  [19]. The Moyal phase can be handled in an easier way if we perform a Fourier transform, namely if we work in the momentum space. The momenta are chosen to be Euclidean and the non-commutative parameter imaginary  $\theta \rightarrow i\theta$ . In this way all the phase factors do not change their character.

We use WML propagators in the Euclidean form  $(k_1 - ik_2)^{-2}$  and parameterize the vectors introducing polar variables in order to perform symmetric integrations [12,19]. Then we are led to the expression

$$\begin{aligned} \mathcal{W}_4^{(cr)} = & r^4 \int_0^n ds_1 \int_{s_1}^n ds_2 \int_{s_2}^n ds_3 \int_{s_3}^n ds_4 \times \\ & \int_0^\infty \frac{dp}{p} \frac{dq}{q} \int_0^{2\pi} d\psi d\chi \exp(-2i(\psi + \chi)) \exp(2ip \sin \psi \sin \pi(s_1 - s_3)) \times \\ & \exp(2iq \sin \chi \sin \pi(s_2 - s_4)) \exp(i \frac{\theta}{r^2} p q \sin[\psi - \chi + \pi(s_2 + s_4 - s_1 - s_3)]) \end{aligned}$$

$$= \mathcal{A}^2 F\left(\frac{\theta}{\mathcal{A}}, n\right). \quad (16)$$

Integrating over  $\psi$  and  $p$ , we get, after a trivial rescaling

$$\mathcal{W}_4^{(cr)} = \pi r^4 n^4 \int [ds]_4 \int_0^\infty \frac{dq}{q} \oint_{|z|=1} \frac{dz}{iz^3} e^{-q \sin[n\pi(s_4-s_2)](z-\frac{1}{z})} \frac{1 - \frac{\gamma}{z} e^{-in\pi\sigma}}{1 - \gamma z e^{in\pi\sigma}}, \quad (17)$$

where  $\sigma = s_1 + s_3 - s_2 - s_4$  and

$$\gamma = \frac{\theta q}{2r^2 \sin[n\pi(s_3 - s_1)]}, \quad \int [ds]_4 = \int_0^1 ds_1 \int_{s_1}^1 ds_2 \int_{s_2}^1 ds_3 \int_{s_3}^1 ds_4.$$

We can further integrate over  $z$ , obtaining series of Bessel functions. Integration over  $q$  and resummation of the series [25] lead to

$$\begin{aligned} \mathcal{W}_4^{(cr)} &= 2n^4 \mathcal{A}^2 \int [ds]_4 \left[ \frac{1}{2} + \frac{2}{\beta^2} (\exp[i\beta \sin \alpha] - 1 - i\beta \sin \alpha) \right] \\ &= 2n^4 \mathcal{A}^2 \int [ds]_4 \left[ \frac{1}{2} + \frac{2}{\beta^2} \sum_{m=2}^{\infty} \frac{(i\beta \sin \alpha)^m}{m!} \right], \end{aligned} \quad (18)$$

where

$$\alpha = n\pi(s_1 + s_3 - s_2 - s_4), \quad \beta = \frac{4\mathcal{A}}{\pi\theta} \sin[n\pi(s_4 - s_2)] \sin[n\pi(s_3 - s_1)]. \quad (19)$$

It is an easy calculation to check that the function  $F$  is continuous (but probably not analytic) at  $\theta = 0$  with  $F(0) = \frac{n^4}{24}$ , exactly corresponding to the value of the commutative case obtained with the WML propagator [19].

The first order correction in  $\theta$  can also be singled out

$$\mathcal{W}_4^{(cr)} \simeq 2n^4 \mathcal{A}^2 \int [ds]_4 \left[ \frac{1}{2} - \frac{2i}{\beta} \sin \alpha \right]. \quad (20)$$

The calculation is sketched in Appendix A and the result is

$$\mathcal{W}_4^{(cr)} \simeq \frac{n^4 \mathcal{A}^2}{24} + i\theta \frac{n^3 \mathcal{A}}{4}. \quad (21)$$

One might recover the trivial scaling  $\mathcal{A} \rightarrow \mathcal{A}n^2$  provided  $\theta \rightarrow \theta n$ ; however this is ruled out by the large- $\theta$  behaviour we are going to explore.



The large- $\theta$  behaviour can be obtained starting from Eq.(18); the first terms in the expansion turn out to be

$$\mathcal{W}_4^{(cr)} = -\frac{n^2 \mathcal{A}^2}{8\pi^2} + i \frac{n^3 \mathcal{A}^3}{8\pi^2 \theta} + \frac{8n^4 \mathcal{A}^4}{3\pi^2 \theta^2} \left( \frac{1}{256} + \frac{175}{3072} \frac{1}{n^2 \pi^2} \right) + \mathcal{O}(\theta^{-3}). \quad (22)$$

We notice that the large- $\theta$  limit (first term in Eq.(22)) obeys the scaling (1), which, in the commutative case, was present in the exact solution for the gauge group  $U(N)$ . This scaling is different from the trivial one at  $\theta = 0$ .

### III. THE SIXTH ORDER CALCULATION AND BEYOND

The motivation for exploring the sixth order is to see whether the scaling law we have found in the fourth order result at  $\theta = \infty$  still persists in higher orders. In the affirmative case one would be strongly encouraged to resum the series in order to inquire about the persistence of such a scaling beyond a perturbation expansion. This, in turn, might have far-reaching consequences on the interpretation of the theory in the extreme non-commutative limit.

We organize the sixth order loop calculation according to the possible topologically different diagrams one can draw. If we order the six vertices on the circle from 1 to 6, we denote by  $\mathcal{W}_{(ij)(kl)(mn)}$  the contribution of the graph corresponding to three propagators joining the vertices  $(ij), (kl), (mn)$ , respectively. Thus  $\mathcal{W}_{(14)(25)(36)}$  corresponds to the maximally crossed diagram (i.e. the one in which all propagators cross); then we have three diagrams with double crossing, namely  $\mathcal{W}_{(14)(26)(35)}, \mathcal{W}_{(13)(25)(46)}$  and  $\mathcal{W}_{(15)(24)(36)}$ . Finally we have six diagrams with a single crossing  $\mathcal{W}_{(12)(35)(46)}, \mathcal{W}_{(16)(24)(35)}, \mathcal{W}_{(15)(23)(46)}, \mathcal{W}_{(15)(26)(34)}, \mathcal{W}_{(13)(26)(45)}$  and  $\mathcal{W}_{(13)(24)(56)}$ . Diagrams without any crossing are not interesting since they are not affected by the Moyal phase; they indeed coincide with the corresponding ones in the commutative case.

The diagrams with a single crossing can be fairly easily evaluated; surprisingly the most difficult diagrams are the ones with double crossing. All integrations can be performed ana-

lytically, but a single one concerning the doubly crossed diagrams, which has been performed numerically.

The details of such a heavy calculation are described in Appendix B. Here we only report the starting point and the final results.

As an example of singly crossed diagram we consider  $\mathcal{W}_{(16)(24)(35)}$

$$\begin{aligned} \mathcal{W}_{(16)(24)(35)} = & -r^6 N n^6 \int [ds]_6 \int_0^\infty \frac{dp}{p} \frac{dq}{q} \frac{dk}{k} \int_0^{2\pi} d\phi d\chi d\psi \exp(-2i(\phi + \chi + \psi)) \times \\ & \exp(2ip \sin \phi \sin n\pi s_{16}^- + 2iq \sin \psi \sin n\pi s_{24}^- + 2ik \sin \chi \sin n\pi s_{35}^-) \times \\ & \exp\left(i \frac{\theta}{r^2} q k \sin[\psi - \chi + n\pi(s_{35}^+ - s_{24}^+)]\right), \end{aligned} \quad (23)$$

where  $s_{ij}^\pm = s_i \pm s_j$ .

The doubly crossed diagram  $\mathcal{W}_{(15)(24)(36)}$  leads to the expression

$$\begin{aligned} \mathcal{W}_{(15)(24)(36)} = & -r^6 N n^6 \int [ds]_6 \int_0^\infty \frac{dp}{p} \frac{dq}{q} \frac{dk}{k} \int_0^{2\pi} d\phi d\chi d\psi \exp(-2i(\phi + \chi + \psi)) \times \\ & \exp(2ip \sin \phi \sin n\pi s_{15}^-) \exp(2iq \sin \psi \sin n\pi s_{24}^-) \times \\ & \exp(2ik \sin \chi \sin n\pi s_{36}^-) \exp\left(i \frac{\theta}{r^2} \left(p k \sin[\phi - \chi + n\pi(s_{36}^+ - s_{15}^+)] + \right. \right. \\ & \left. \left. q k \sin[\psi - \chi + n\pi(s_{36}^+ - s_{24}^+)]\right)\right). \end{aligned} \quad (24)$$

Finally the maximally crossed diagram  $\mathcal{W}_{(14)(25)(36)}$  reads

$$\begin{aligned} \mathcal{W}_{(14)(25)(36)} = & -r^6 N n^6 \int [ds]_6 \int_0^\infty \frac{dp}{p} \frac{dq}{q} \frac{dk}{k} \int_0^{2\pi} d\phi d\chi d\psi \exp(-2i(\phi + \chi + \psi)) \times \\ & \exp(2ip \sin \phi \sin n\pi s_{14}^-) \exp(2iq \sin \psi \sin n\pi s_{25}^-) \times \\ & \exp(2ik \sin \chi \sin n\pi s_{36}^-) \exp\left(i \frac{\theta}{r^2} \left(p q \sin[\phi - \psi + n\pi(s_{25}^+ - s_{14}^+)] + \right. \right. \\ & \left. \left. p k \sin[\phi - \chi + n\pi(s_{36}^+ - s_{14}^+)] + q k \sin[\psi - \chi + n\pi(s_{36}^+ - s_{25}^+)]\right)\right). \end{aligned} \quad (25)$$

We notice that the  $U(N)$  factor is the same in all the three configurations.

The sum of the diagrams with a single crossing and  $n$  windings contribute at  $\theta = \infty$  with the following expression

$$\mathcal{W}^{(1)}(\theta = \infty) = \frac{\mathcal{A}^3 N n^4}{24\pi^2} \left(1 - \frac{6}{n^2\pi^2}\right). \quad (26)$$

The maximally crossed diagram in turn leads to

$$\mathcal{W}^{(3)}(\theta = \infty) = -\frac{\mathcal{A}^3 N n^2}{64\pi^4}. \quad (27)$$

Finally the diagrams with double crossing give

$$\mathcal{W}^{(2)}(\theta = \infty) = \frac{\mathcal{A}^3 N n^2}{12\pi^4}(1 + 0.2088). \quad (28)$$

As we have anticipated, the last term has been evaluated numerically. Its  $n$ -dependence has been checked up to  $n = 6$ , within the incertitude due to the numerical integration (see Appendix B).

Summing together all the contributions of diagrams with crossed propagators, we get

$$\mathcal{W}_6^{(cr)}(\theta = \infty) = \frac{\mathcal{A}^3 N n^4}{24\pi^2} \left( 1 - \frac{1}{n^2 \pi^2} \left( \frac{35}{8} - 0.4176 \right) \right). \quad (29)$$

We remark that the leading term at large  $n$

$$\mathcal{W}_6^{(cr)}(\theta = \infty) \simeq \frac{\mathcal{A}^3 N n^4}{24\pi^2}$$

exhibits the scaling (1). It comes only from diagrams with a single crossing. Diagrams with such a topological configuration can also be computed in higher orders; for instance, at  $\mathcal{O}(g^8)$  they lead to the result

$$\mathcal{W}_8^{(cr)}(\theta = \infty) \simeq -\frac{\mathcal{A}^4 N^2 n^6}{192\pi^2} + \mathcal{O}(n^4). \quad (30)$$

The integral over the loop variables provides a factor  $n^{-2}$ , turning the trivial  $n^8$ , due to the kinematical rescaling, into the factor  $n^6$ . Details are reported in Appendix C.

We are led to argue that the dominant term at the  $(2m + 4)$ -th perturbative order increases with  $n$  not faster than  $n^{2m+2}$ . In turn it exhibits the highest  $U(N)$  contribution, behaving like  $N^m$

$$g^{2m+4} \mathcal{W}_{2m+4}^{(cr)}(\theta = \infty) \simeq \mathcal{K}_m (nN)^m (g^2 \mathcal{A} n)^{m+2}, \quad (31)$$

which obeys the scaling (1).

Further we conjecture that diagrams with a single crossing dominate; then the weights  $\mathcal{K}_m$  can be evaluated (see Appendix D) and lead to

$$g^{2m+4}\mathcal{W}_{2m+4}^{(cr)}(\theta = \infty) \simeq -\frac{(g^2\mathcal{A}n)^2}{4\pi^2} \frac{1}{m!} \frac{(-g^2\mathcal{A}Nn^2)^m}{(m+2)!}; \quad (32)$$

the related perturbative series can be easily resummed

$$\mathcal{W}^{(cr)}(\theta = \infty) = -\frac{g^2\mathcal{A}}{4\pi^2 N} J_2(2\sqrt{g^2\mathcal{A}n^2N}). \quad (33)$$

If we compare Eq.(32) with the corresponding term due to planar diagrams, which are insensitive to  $\theta$  (see Eq.(14)), we notice that, in the 't Hooft's limit  $N \rightarrow \infty$  with fixed  $g^2N$ , the planar diagrams dominate by a factor  $n^2N^2$ , as expected.

Our conjecture is open to more thorough perturbative tests as well as to possible non-perturbative derivations which might throw further light on its ultimate meaning and related consequences. For recent papers on non-perturbative approaches, see [26–28].

#### IV. CONCLUSIONS

Summarizing our perturbative investigation, we can say that, when winding  $n$ -times around the Wilson loop the non-Abelian nature of the gauge group in the non-commutative case is felt, even in a perturbative calculation making use of the *WML* prescription for the vector propagator. This is due to the merging of space-time properties with “internal” symmetries in a large invariance group  $U_{cpt}(\mathcal{H})$  [2,28].

One gets the clear impression that in a non-commutative formulation what is really relevant are not separately the space-time properties of the “base” manifold and of the “fiber”  $U(N)$ , but rather the overall algebraic structure of the resulting invariance group  $U_{cpt}(\mathcal{H})$ . To properly understand its topological features is certainly beyond any perturbative approach. Rather one should possibly resort to suitable  $\mathcal{N}$ -truncations of the Hilbert space in the form of matrix models leading to the invariance groups  $U(\mathcal{N})$ .

It is not clear how many perturbative features might eventually be singled out in those contexts, especially in view of the difficulty in performing the inductive limit  $\mathcal{N} \rightarrow \infty$ .

For this reason we think that our perturbative results are challenging. They indicate that the intertwining between  $n$ , controlling the space-time geometry, and  $N$ , related to the gauge group, is far from trivial. The presence of corrections to the scaling laws occurring at  $\theta = 0$  and at  $\theta = \infty$ , while frustrating at a first sight in view of a generalization to all values of  $\theta$ , might be taken instead as a serious indication that  $n$  and  $N$  separately are not perhaps the best parameters to be chosen unless large values for both (and for  $\theta$ !) are considered. In such a situation, perhaps surprisingly, the relation (1) is recovered.

Eqs.(14,32) are concrete realizations of the more general structure

$$\mathcal{W}_{2m+4} = (\mathcal{A}n^2N)^{m+2} f_m(n, N), \quad (34)$$

$f_m$  being a symmetric function of its arguments. We stress that Eq.(14) concerns only *planar* diagrams; crossed graph contributions in the commutative case cannot be put in the form (34) and violate the relation (1) <sup>1</sup>. In the non-commutative case, for large  $n, N$  and maximal non-commutativity ( $\theta = \infty$ ), the structure (34) is instead restored for the leading contribution of *crossed* diagrams. The presence of the function  $f_m$  in the *WML* context might be thought as a sign of the merging of space-time and internal symmetries.

All these difficult, but intriguing questions are worthy in our opinion of thorough investigations and promise further exciting, unexpected developments.

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<sup>1</sup>The structure (34) is shared also by the exact geometrical solution of the commutative case [4].

## APPENDIX A: SMALL $\theta$ LIMIT

The first significant term in the small  $\theta$  expansion is the second one in eq.(20), that is the  $\mathcal{O}(\theta)$  term in  $\mathcal{W}_4^{(cr)}$ ,

$$\begin{aligned}\mathcal{W}_\theta &= -in^4 \mathcal{A}\pi\theta \int_0^1 [ds] \frac{\sin[n\pi(s_1 + s_3 - s_2 - s_4)]}{\sin[n\pi(s_4 - s_2)] \sin[n\pi(s_3 - s_1)]} \\ &\equiv -in^4 \mathcal{A}\pi\theta I \quad ,\end{aligned}\tag{A1}$$

with measure  $[ds] = ds_1 ds_2 ds_3 ds_4 \theta(s_4 - s_3) \theta(s_3 - s_2) \theta(s_2 - s_1)$ . Integrating in  $ds_1$  and  $ds_4$  leads to

$$\begin{aligned}I &= \frac{1}{n^2 \pi^2} \int_0^1 ds_2 ds_3 \theta(s_3 - s_2) \left\{ -n\pi \cos[2n\pi(s_3 - s_2)] \times \right. \\ &\quad \left[ s_2 \log \left| \frac{\sin n\pi s_2}{\sin n\pi(s_3 - s_2)} \right| + (1 - s_3) \log \left| \frac{\sin n\pi s_3}{\sin n\pi(s_3 - s_2)} \right| \right] \\ &\quad + \sin[2n\pi(s_3 - s_2)] \left[ -n^2 \pi^2 s_2 (1 - s_3) + \log |\sin n\pi(s_3 - s_2)| \log \left| \frac{\sin n\pi(s_3 - s_2)}{\sin n\pi s_2 \sin n\pi s_3} \right| \right. \\ &\quad \left. \left. + \log |\sin n\pi s_2| \log |\sin n\pi s_3| \right] \right\} \equiv I_1 + I_2 \quad ,\end{aligned}\tag{A2}$$

where  $I_1$  and  $I_2$  refer to the first and second square brackets in (A2), respectively. The two integrals in  $I_1$  coincide. They can be easily performed leading to

$$I_1 = - \left( \frac{1}{6n\pi} + \frac{1}{4n^3\pi^3} \right) .\tag{A3}$$

Concerning  $I_2$ , the first term is trivial, and provides us with a factor  $1/(8n^3\pi^3) - 1/(12n\pi)$ , whereas in the remaining integrals it is more convenient to integrate first on one variable and then to add the integrands together before performing the final integration, *i.e.*

$$\begin{aligned}I_2 &= \frac{1}{8n^3\pi^3} - \frac{1}{12n\pi} - \frac{1}{2n^3\pi^3} \int_0^1 ds \sin n\pi s \times \\ &\quad \left[ 2n\pi s \cos n\pi s \log |\sin n\pi s| + \sin n\pi s (\log |\sin n\pi s| - 1) \right] \\ &= \frac{1}{4n^3\pi^3} - \frac{1}{12n\pi} .\end{aligned}\tag{A4}$$

Adding (A3) and (A4) and taking into account Eq.(A1), Eq.(21) follows.

## APPENDIX B: SIXTH ORDER CALCULATION

### 1. The singly crossed diagram

We show in some detail the formulas for  $\mathcal{W}_{(16)(24)(35)}$ , the other five diagrams being simply obtainable by renaming the variables.

Integrating (23) over  $\phi$  and  $p$  we recover an expression analogous to (16). Therefore we can use the result obtained at  $\mathcal{O}(g^4)$  to get

$$\mathcal{W}_{(16)(24)(35)} = -2n^6 \mathcal{A}^3 N \int [ds]_6 \left[ \frac{1}{2} + \frac{2}{\beta'^2} (\exp[i\beta' \sin \alpha'] - 1 - i\beta' \sin \alpha') \right] \quad (\text{B1})$$

where now

$$\alpha' = n\pi(s_2 + s_4 - s_3 - s_5), \quad \beta' = \frac{4\mathcal{A}}{\pi\theta} \sin[n\pi(s_4 - s_2)] \sin[n\pi(s_5 - s_3)]. \quad (\text{B2})$$

The large- $\theta$  limit is easily derived from this formula; summing all the singly crossed diagrams we find

$$- \mathcal{A}^3 N n^6 \left[ \frac{1}{4\pi^4 n^4} - \frac{1}{24\pi^2 n^2} \right]. \quad (\text{B3})$$

### 2. The doubly crossed diagram

Integrating Eq.(24) over  $\phi$  and  $p$ , and then over  $\psi$  and  $q$ , we get

$$\begin{aligned} \mathcal{W}_{(15)(24)(36)} &= -r^6 N n^6 \pi^2 \int [ds]_6 \int_0^\infty \frac{dk}{k} \oint_{|z|=1} \frac{dz}{iz^3} e^{-k \sin[n\pi(s_6 - s_3)](z - \frac{1}{z})} \\ &\quad \frac{1 - \frac{\gamma'}{z} e^{-in\pi\sigma'}}{1 - \gamma' z e^{in\pi\sigma'}} \times \frac{1 - \frac{\gamma''}{z} e^{-in\pi\sigma''}}{1 - \gamma'' z e^{in\pi\sigma''}} \end{aligned} \quad (\text{B4})$$

where

$$\begin{aligned} \sigma' &= s_1 + s_5 - s_3 - s_6 & \gamma' &= \frac{\theta k}{2r^2 \sin[n\pi(s_5 - s_1)]} \\ \sigma'' &= s_2 + s_4 - s_3 - s_6 & \gamma'' &= \frac{\theta k}{2r^2 \sin[n\pi(s_4 - s_2)]} \end{aligned}$$

We consider the identity  $e^{-k \sin[n\pi(s_6-s_3)](z-\frac{1}{z})} \equiv [(e^{-k \sin[n\pi(s_6-s_3)](z-\frac{1}{z})} - 1) + 1]$  in (B4); in the first term it is possible to send  $\theta$  to infinity in the integrand, obtaining the result

$$-\frac{r^6 N n^6 \pi^3}{3} \int [ds]_6 e^{2\pi i(2s_3+2s_6-s_1-s_5-s_2-s_4)}. \quad (\text{B5})$$

The other contribution can be exactly integrated over  $z$  and  $k$ , leading to the sum of two expressions

$$-\frac{r^6 N n^6 2 \pi^3}{3} \int [ds]_6 \exp(-i(\lambda + \omega)) [\cos(\lambda - \omega)]^3 \quad (\text{B6})$$

and

$$\frac{i r^6 N n^6 2 \pi^3}{3} \int [ds]_6 \exp(-i(\lambda + \omega)) \frac{\left(1 + \left|\frac{d}{c}\right| \exp(i(\lambda - \omega))\right)}{\left(1 - \left|\frac{d}{c}\right| \exp(i(\lambda - \omega))\right)} [\sin(\lambda - \omega)]^3, \quad (\text{B7})$$

where

$$c = -\sin[n\pi(s_1 - s_5)] \exp(-in\pi\sigma') = |c| \exp(i\omega),$$

$$d = -\sin[n\pi(s_2 - s_4)] \exp(-in\pi\sigma'') = |d| \exp(i\lambda).$$

The integrals (B5) and (B6) can be easily computed; when summed with the corresponding ones from  $\mathcal{W}_{(14)(26)(35)}$  and  $\mathcal{W}_{(13)(25)(46)}$ , they give

$$\frac{\mathcal{A}^3 N n^2}{12 \pi^4}. \quad (\text{B8})$$

Expression (B7) instead, together with the corresponding ones from  $\mathcal{W}_{(14)(26)(35)}$  and  $\mathcal{W}_{(13)(25)(46)}$ , is difficult to deal with. We can prove their sum is real and have evaluated such a sum numerically, for  $n = 1, \dots, 6$ . We present the result in the form

$$\frac{4 \pi r^6 N n^4}{3} \times J_{NUM}, \quad (\text{B9})$$

where

$$J_{NUM}(n=1) = 1.32236(80 \pm 37) \times 10^{-3}$$

$$J_{NUM}(n=2) = 0.330(49 \pm 16) \times 10^{-3} \quad \frac{J_{NUM}(n=1)}{4} = 0.330592(01 \pm 93) \times 10^{-3}$$



$$\begin{aligned}
J_{NUM}(n=3) &= 0.146(97 \pm 35) \times 10^{-3} & \frac{J_{NUM}(n=1)}{9} &= 0.146929(80 \pm 41) \times 10^{-3} \\
J_{NUM}(n=4) &= 0.08(17 \pm 29) \times 10^{-3} & \frac{J_{NUM}(n=1)}{16} &= 0.082648(00 \pm 23) \times 10^{-3} \\
J_{NUM}(n=5) &= 0.05(10 \pm 40) \times 10^{-3} & \frac{J_{NUM}(n=1)}{25} &= 0.052894(72 \pm 15) \times 10^{-3} \\
J_{NUM}(n=6) &= 0.03(88 \pm 79) \times 10^{-3} & \frac{J_{NUM}(n=1)}{36} &= 0.036732(45 \pm 10) \times 10^{-3}
\end{aligned}$$

All the errors are three standard deviations. Within the numerical error,  $J_{NUM}$  scales as  $1/n^2$ .

### 3. The maximally crossed diagram

Integrating Eq.(25) over  $\phi$  and  $p$ , and then over  $\chi$  and  $k$ , we get, after a simple rescaling

$$\begin{aligned}
\mathcal{W}_{(14)(25)(36)} &= -r^6 N n^6 2\pi^2 \int [ds]_6 \int_0^\infty \frac{dq}{q} \int_0^{2\pi} d\psi \exp(-2i\psi) \times \\
&\exp\left(4i \frac{qr^2}{\theta} \sin \psi \sin[n\pi(s_2 - s_5)]\right) \left[ \frac{1}{2} \frac{\bar{\alpha}}{\alpha} \frac{\bar{\beta}}{\beta} + \right. \\
&\left. \frac{\theta^2}{8r^4 \alpha^2 \beta^2} \left( \exp\left(\frac{4ir^2}{\theta} \text{Im}(e^{in\pi\sigma'''} \bar{\alpha}\beta)\right) - \frac{4ir^2}{\theta} \text{Im}(e^{in\pi\sigma'''} \bar{\alpha}\beta) - 1 \right) \right], \quad (\text{B10})
\end{aligned}$$

where  $\sigma''' = s_1 + s_4 - s_3 - s_6$ , the bars denote complex conjugation and

$$\alpha = \sin[n\pi(s_1 - s_4)] + q \exp i(\psi + n\pi(s_1 + s_4 - s_2 - s_5)),$$

$$\beta = \sin[n\pi(s_3 - s_6)] - q \exp i(\psi + n\pi(s_3 + s_6 - s_2 - s_5)).$$

We can recognize in (B10) the same structure we have found in (18). We rewrite it as follows:

$$\begin{aligned}
\mathcal{W}_{(14)(25)(36)} &= -r^6 N n^6 2\pi^2 \int [ds]_6 \int_0^\infty \frac{dq}{q} \int_0^{2\pi} d\psi \exp(-2i\psi) \times \\
&\exp\left(4i \frac{qr^2}{\theta} \sin \psi \sin[n\pi(s_2 - s_5)]\right) \exp(-2i(\gamma_\alpha + \gamma_\beta)) \left[ \frac{1}{2} \cos(2n\pi\sigma''') - 2\gamma_\alpha + 2\gamma_\beta + \right. \\
&\left. \frac{1}{\pi i} \int_{\mu-i\infty}^{\mu+i\infty} ds \Gamma(-s) e^{-i\frac{\pi}{2}s} \left[ \frac{4|\alpha||\beta|r^2}{\theta} \right]^{s-2} [\sin(n\pi\sigma''' - \gamma_\alpha + \gamma_\beta)]^s \right], \quad 2 < \mu < 3, \quad (\text{B11})
\end{aligned}$$

where we have defined  $\alpha = |\alpha| \exp(i\gamma_\alpha)$  and  $\beta = |\beta| \exp(i\gamma_\beta)$ .

One can prove that, in the large- $\theta$  limit, the last integral goes to zero. Then, in this limit, equation (B11) can be easily evaluated:

$$\begin{aligned} \mathcal{W}_{(14)(25)(36)} \xrightarrow{\theta \rightarrow \infty} & -\frac{r^6 N n^6 \pi^3}{3} \int [ds]_6 \left( e^{2in\pi(2s_2+2s_5-s_1-s_4-s_3-s_6)} + \right. \\ & \left. e^{2in\pi(2s_1+2s_4-s_2-s_5-s_3-s_6)} + e^{2in\pi(2s_3+2s_6-s_1-s_4-s_2-s_5)} \right) \\ & = -\frac{\mathcal{A}^3 N n^2}{64\pi^4}. \end{aligned} \quad (\text{B12})$$

Here we notice that the integrand is completely symmetric in the three propagators (14)(25)(36), as it should.

### APPENDIX C: HIGHER ORDERS

First we prove that singly crossed diagrams behave in the large- $\theta$  limit at least as  $\frac{1}{n^2}$  or subleading in the limit of a large number of windings  $n$ . We start by realizing that we can always express the integral of a generic diagram with  $m$  propagators and a single crossing generalizing Eq.(18)

$$\mathcal{I} \equiv \int_0^1 dt \int_0^t dz \int_0^z dy \int_0^y dx \int [ds]_{2m-4} \cos[2\pi n(x+z-y-t)], \quad (\text{C1})$$

$[ds]_{2m-4}$  being a measure depending on  $x, y, z, t$  only through the extremes of integration. As a matter of fact, it is always possible to single out the variables linked to the propagators which cross, suitably rearranging the other kinematical integrations. These integrations lead to polynomials

$$\mathcal{I} = \int_0^1 dt \int_0^t dz \int_0^z dy \int_0^y dx \sum_{k_1 k_2 k_3 k_4} c_{k_1 k_2 k_3 k_4} x^{k_1} y^{k_2} z^{k_3} t^{k_4} \cos[2\pi n(x+z-y-t)]. \quad (\text{C2})$$

Now we perform the change of variables  $\alpha = y+x$ ,  $\beta = y-x$ ,  $\gamma = t+z$ ,  $\delta = t-z$

$$\begin{aligned} \mathcal{I} = & \int_0^1 d\delta \int_\delta^{2-\delta} d\gamma \int_0^{\frac{\gamma-\delta}{2}} d\beta \int_\beta^{\gamma-\delta-\beta} d\alpha \\ & \sum_{q_1 q_2 q_3 q_4} c'_{q_1 q_2 q_3 q_4} \alpha^{q_1} \beta^{q_2} \gamma^{q_3} \delta^{q_4} \cos[2\pi n(\beta + \delta)] \end{aligned} \quad (\text{C3})$$

and then integrate over  $\alpha$ . Changing again variables to  $\psi = \beta + \delta$ ,  $\xi = \delta - \beta$ , we end up with

$$\mathcal{I} = \int_0^1 d\psi \int_{-\psi}^{\psi} d\xi \int_{\frac{3\psi-\xi}{2}}^{2-\frac{\psi+\xi}{2}} d\gamma \sum_{p_1 p_2 p_3} C_{p_1 p_2 p_3} \psi^{p_1} \xi^{p_2} \gamma^{p_3} \cos[2\pi n \psi]. \quad (\text{C4})$$

The integrals over  $\xi$  and  $\gamma$  can be easily performed giving, of course, a polynomial in  $\psi$

$$\mathcal{I} = \sum_r C'_r \int_0^1 \psi^r \cos[2\pi n \psi] d\psi. \quad (\text{C5})$$

Integrating by parts, we realize that only even inverse powers of  $n$  are produced, starting from  $n^{-2}$ .

Now we turn our attention to the  $U(N)$  factors. A direct computation of the traces involved in the diagrams with a single, a double or the triple crossing ( $\mathcal{O}(g^6)$ ), shows that they all share the common factor  $N^2$  (our normalization being  $t^0 = \mathbf{1}/\sqrt{N}$ ,  $Tr(t^a t^b) = \delta^{ab}$ ;  $a, b = 1, \dots, N^2 - 1$ ). As the Wilson loop is normalized with  $N^{-1}$ , at  $\mathcal{O}(g^6)$  the single factor  $N$  ensues.

It is now trivial to realize that any insertion of  $m - 3$  lines no matter where in such diagrams provided that further crossings are avoided, produces the factor  $N^{m-3}$ .

#### APPENDIX D: COMPUTATION OF THE WEIGHTS

In the previous appendix we have shown that the  $n$ -dependence of singly crossed diagrams in the large- $\theta$  limit takes the form  $\sum_{p=1}^P c_p n^{-2p}$ . To find the leading contribution at large  $n$  we have to evaluate  $c_1$ . This can be done as follows: at  $\mathcal{O}(g^{2m+4})$  we start drawing a cross and then add the remaining  $m$  propagators in such a way they do not further cross. From Eqs.(C2-C5) one can realize that  $c_1$  is different from zero only for a particular subset of these diagrams: if we label the four sectors in which the cross divides the circular loop as North (the sector containing the origin of the loop variables  $s_i$ ), West, South and East, then only diagrams with  $r$  propagator in the southern sector and  $m - r$  in the northern one contribute to  $c_1$ ; moreover these contributions are all equal. Therefore we can evaluate this integral once and then multiply it by the number of configurations in this subset.

We choose as representative the diagram with all the  $m$  non-intersecting propagators in the northern sector, starting from the origin and connecting  $s_1$  with  $s_2$ , ...,  $s_{2m-1}$  with  $s_{2m}$ . In this way the crossed variables are  $s_{2m+1}$ , ...,  $s_{2m+4}$ . We obtain the integral

$$\mathcal{I} = (-\pi)^{m+2} (gr)^{2m+4} N^m n^{2m+4} \int_0^1 dt \int_0^t dz \int_0^z dy \int_0^y dx \frac{x^{2m}}{(2m)!} \cos[2\pi n(x+z-y-t)]. \quad (\text{D1})$$

Following the procedure described in appendix C we get

$$\mathcal{I} = \frac{(-\pi)^{m+2} (gr)^{2m+4}}{(2m)!} N^m n^{2m+4} \int_0^1 d\psi \frac{1}{(2m+1)(2m+2)} \psi(1-\psi)^{2m+2} \cos[2\pi n\psi] \quad (\text{D2})$$

and finally

$$\mathcal{I} = -N^m \frac{(-g^2 \mathcal{A} n^2)^{m+2}}{(2m+2)!} \left( \frac{1}{4\pi^2 n^2} + \mathcal{O}\left(\frac{1}{n^4}\right) \right) \quad (\text{D3})$$

Now we have to count. We denote by  $S_{2r}$  the ways in which the  $r$  propagators in the southern sector can be arranged without crossing. A little thought provides the recursive relation

$$S_0 = 1, S_{2r} = \sum_{k=1}^r S_{2k-2} S_{2r-2k}, \quad (\text{D4})$$

which can easily be solved

$$S_{2r} = \frac{2^{2r} \Gamma(r + \frac{1}{2})}{\Gamma(\frac{1}{2}) \Gamma(r + 2)}. \quad (\text{D5})$$

The  $m-r$  propagators in the northern sector lead to the weight  $S_{2(m-r)}$  times the number of possible insertions of the origin, namely  $[2(m-r)+1]$ . The number of relevant diagrams is therefore

$$\mathcal{N}_m = \sum_{r=0}^m S_{2r} S_{2(m-r)} [2(m-r)+1] = \frac{2^{2m+2} (m+1) \Gamma(m + \frac{3}{2})}{\Gamma(\frac{1}{2}) \Gamma(m+3)}. \quad (\text{D6})$$

Multiplying Eqs.(D3) and (D6) we are led to Eq.(32).

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