What kind of preference maximization does the weak axiom of revealed non-inferiority characterize?*

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Abstract

A choice correspondence is weak justified if a non-chosen alternative is dominated by any other obtainable alternative, and for each discarded alternative there is *some* chosen alternative which dominates it. This definition allows us to build a connection between the behavioral property expressed by the weak axiom of revealed non-inferiority and a weak form of maximality. It is weaker than the form of maximality characterized by the weak axiom of revealed preference.

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1 Introduction

Eliaz and Ok (2006) accommodate preference incompleteness in revealed preference theory by studying the implications of weakening the fundamental choice-consistency condition of the weak axiom of revealed preference (WARP) in the weak axiom of revealed non-inferiority (WARNI).[1] They show that WARNI implies that the revealed preference relation (not necessarily complete) is regular, reflexive, and transitive whenever the domain of choice $\mathfrak{D}$ includes all unit sets of its underlying universal domain X and is closed under finite union; furthermore, the converse holds under the technical assumption that $\mathfrak{D}$ includes all countable subsets of X. Eliaz and Ok (2006)'s specification of $\mathfrak{D}$ is a weak specialization of the abstract choice domain exploited by Richter (1966), Kim (1987), and Bandyopadhyay and Sengupta (2003). Nevertheless, this specification is not the appropriate domain of choice for many important economic problems (Herzberger, 1973; Suzumura, 1983). For example, a choice domain $\mathfrak{D}$ which includes all unit sets of its underlying universal domain X and is closed under finite union violates the classical postulate of Walras' law (as observed by Herzberger 1973, p. 233), and this violation reduces enormously the empirical attraction of a revealed preference theory that allows for the empirical validity of the integrability conditions without the need to assume a particular functional form of preferences (Blundell, 2005). Moreover, most interesting problems in consumer economics must often rely only on a finite number of observations, i.e, 'consumer budget surveys' (Blundell, 2005). Last but not least, the observed behaviour is often inconsistent with the WARP and it opens the problem of assessing the nature of consumer (ir)rationality by using observational data alone (Blundell, Browning, and Crawford, 2003). This motivates the question in the title.

In this paper we follows the guidelines of revealed preference theory; we define a choice correspondence C on an arbitrary choice domain $\mathfrak{D}$; and we introduce a weaker notion of maximality, as follows. We say that a choice correspondence C over $\mathfrak{D}$ is weak justified if there exists a binary relation J on X (dubbed weak justification) such that, for any feasible set, no available alternative is J-related to any chosen alternative, and for each rejected alternative there is *some* chosen alternative which is J-related to it. Therefore, the binary relation J is a strict (not necessarily complete) preference relation; and a decision-maker makes weak justified choices if she can assert that non-chosen alternative is dominated by any other obtainable one, and for each discarded alternative there is some chosen alternative which dominates it. Our form of maximality is weaker than that introduced by Mariotti (2008), but stronger than that of asymmetric motivation introduced by Kim and Richter (1986) - and obviously stronger than the von Neumann-Morgenstern stable set.

Our main result is that a choice correspondence C on $\mathfrak{D}$ is asymmetric weak justified if and only if it satisfies WARNI.

[1] On a finite universal set WARNI is identical to one of the behavioral properties suggested by Bandyopadhay and Sengupta (1993).

2 Analysis

Let X be a non-empty set of alternatives. Let $\mathfrak{D}$ be the collection of non-empty subsets of X. A choice correspondence C is a map defined on $\mathfrak{D}$ such that $\emptyset \neq C(S) \subseteqq S$ for every $S \in \mathfrak{D}$.
A binary relation $J \subseteqq X \times X$ is said to be asymmetric if, for all $x, y \in X$, $(x, y) \in J$ implies $(y, x) \notin J$.
If there exists a binary relation J on X such that, for all $S \in \mathfrak{D}$:

$$\begin{array}{ll} i) & \forall x \in C(S), \forall y \in S : (y, x) \notin J \\ ii) & \forall y \in S \backslash C(S) : (x, y) \in J \text{ for some } x \in C(S). \end{array} \tag{2.1}$$

then we say that J is a weak justification for C. If C has a weak justification, we say that C is *weak justified*. We will call J an asymmetric weak justification if J is asymmetric and satisfies $i)$ and $ii)$ for all $S \in \mathfrak{D}$. If C has an asymmetric weak justification, we say that C is *asymmetric weak justified*.
Our form of maximality is weaker than that provided by Mariotti (2008), according to which a choice correspondence C on $\mathfrak{D}$ is justified if there exists a binary relation J on X such that, for all $S \in \mathfrak{D}$:

$$\begin{array}{ll} i') & \forall x, y \in C(S) : (y, x) \notin J \\ ii') & \forall y \in S \backslash C(S), \forall x \in C(S) : (x, y) \in J \ . \end{array} \tag{2.2}$$

Therefore, J is not necessarily complete in both (2.1) and (2.2). However, while (2.2) requires that *all* chosen elements are J-related to all non-chosen elements, (2.1) requires that for each non-chosen element there exists *some* chosen element which is J-related to it. Furthermore, our form of maximality is stronger than the asymmetric motivation relation M introduced by Kim and Richter (1986). A choice correspondence C on $\mathfrak{D}$ is asymmetric motivated if there exists an asymmetric binary relation M on X such that, for all $S \in \mathfrak{D}$,

$$x \in C(S) \text{ iff } \nexists y \in S : (y, x) \in M.$$

Therefore, a weak asymmetric justified choice correspodence is an asymmetric motivated one as an asymmetric motivation coincedes with the part i) of an asymmetric weak justification. Finally, asymmetric weak justification also bears similarities to von Neumann-Morgenstern stable set. That is another structure that weakens maximization. Let R be an asymmetric relation on X. Then, the set V is a stable set solution of (X, R) if the following conditions are satisfied: $i'')$ *Internal Stability*: $\forall x, y \in V$, $(x, y) \notin R$; $ii'')$ *External Stability*: $\forall y \in X \backslash V$, $(x, y) \in R$ for some $x \in V$. For an asymmetric weak justification internal stability is required, whilst external stability is strengthened to: $\forall y \in X \backslash V : (x, y) \in R$ for some $x \in V$ and $(y, x) \notin R$ for all $x \in V$.
Eliaz and Ok (2006) suggest to read the statement "$x \in C(S)$" as "*x is revealed not to be inferior to any other obtainable alternative in S*" rather than to follow the classic interpretation of "*x is revealed to be at least as good as all other available alternatives in S*". Under this interpretation of

revealed preferences, they propose the weak axiom of revealed non-inferiority (WARNI). The idea behind this behavioral regularity is quite mild. It asserts that if an obtainable alternative from a set S is revealed not to be inferior to all of other chosen alternatives from S, then it must be chosen from S as well.

WARNI. $\forall S \in \mathfrak{D}, y \in S : [\forall x \in C(S) \, \exists T \in \mathfrak{D} : y \in C(T) \text{ and } x \in T] \Rightarrow y \in C(S)$.

This behavioral postulate is weaker than WARP which asserts that if $x \in C(S)$ and there exists a feasible set T such that $y \in C(T)$ and $x \in T$, then $y \in C(S)$. Furthermore, WARNI implies the canonical Property α (also known as Chernoff choice-consistency condition or basic contraction consistency), according to which an alternative that is deemed choosable from a feasible set S and belongs to a subset T of S must be deemed choosable from T (i.e., $x \in T \subsetneqq S$ and $x \in C(S) \Rightarrow x \in C(T)$).[2]

Theorem 1 shows that an asymmetric weak justified choice correspondence differs from an asymmetric justified choice correspondence, whilst Theorem 2 shows our main result that a choice correspondence C on D is asymmetric weak justified if and only if it satisfies WARNI.

Theorem 1 *There exists a choice correspondence C on $\mathfrak{D}$ that is asymmetric weak justified but not asymmetric justified.*

Proof. Let $X = \{x, y, z\}$. Suppose that $\mathfrak{D} = \{\{x, y\}, \{z, y\}, X\}$. Define C on $\mathfrak{D}$ by $C(X) = \{x, z\}$, $C(\{x, y\}) = \{x\}$, and $C(\{z, y\}) = \{z, y\}$. It is easy to see that C is asymmetric weak justified, but not asymmetric justified. For suppose that C is asymmetric justified. Then, since $C(X) = \{x, z\}$, we must have $(x, y), (z, y) \in J$. But $C(\{z, y\}) = \{z, y\}$ implies that $(z, y) \notin J$ yielding a contradiction. ■

Theorem 2 *A choice correspondence C on $\mathfrak{D}$ is asymmetric weak justified if and only if it satisfies WARNI.*

Proof. Assume that C is asymmetric weak justified. We show that C satisfies WARNI. Let $S \in \mathfrak{D}$, with $y \in S$, and suppose that for every $x \in C(S)$ there exists $T \in \mathfrak{D}$ such that $y \in C(T)$ and $x \in T$. As C is asymmetric weak justified it follows that $(x, y) \notin J$ for all $x \in C(S)$. Assume, to the contrary, that $y \notin C(S)$. Because C is asymmetric weak justified it follows that there exists $x \in C(S)$ such that $(x, y) \in J$ yielding a contradiction.

For the converse, let C on $\mathfrak{D}$ satisfy WARNI. We show that C is asymmetric weak justified. Define for all distinct $x, y \in X$:

$$(x, y) \in J \Leftrightarrow \exists S \in \mathfrak{D} : x \in C(S), \, y \in S \backslash C(S), \text{ and } \nexists T \in \mathfrak{D}, \; x \in T : y \in C(T).$$

[2]See Eliaz and Ok (2006, lemma 1, p. 81).

Then J is asymmetric. To show that C satisfies property i), let $x \in C(S)$ and $y \in S$ for some $S \in \mathfrak{D}$. By way of contradiction, let $(y, x) \in J$. Then there exists $S' \in \mathfrak{D}$ such that $y \in C(S')$, $x \in S' \backslash C(S')$, and for all $T \in \mathfrak{D}$, with $y \in T$, $x \notin C(T)$, which contradicts that $x \in C(S)$ and $y \in S$. Finally, we show that C meets property ii). Suppose that $y \in S \backslash C(S)$ for some $S \in \mathfrak{D}$. WARNI implies that there exists $x \in C(S)$ such that for all $T \in \mathfrak{D}$ it holds true $y \notin C(T)$ if $x \in T$. It follows that $(x, y) \in J$. ■

It is worth noting that Theorem 3 does not follow from Eliaz and Ok (2006) as our result relies on a more general domain of choice. Furthermore, Theorem 2 implies that our notion of maximality is not vacuous in the sense that not all choices have an asymmetric weak justification. Finally, we observe that a choice correspondence satisfying WARNI is rationalized by a unique asymmetric weak justification J whenever the domain of choice $\mathfrak{D}$ includes all pairs.

Theorem 3 *If J and J' are two weak asymmetric justifications on a nonempty set X such that* $\max\{S, J\} = \max\{S, J'\}$ *for all $S \subsetneqq X$ with $|S| = 2$. Then $J = J'$.*

Proof. Let $x, y \in X$, with $x \neq y$. Let

$$(x, y) \in J \Leftrightarrow \{x\} = \max\{\{x, y\}, J\} = \max\left\{\{x, y\}, J'\right\} \Leftrightarrow (x, y) \in J'.$$

The statement trivially follows. ■

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