Estimation of Carrier Frequency Offset for Generalized MC-CDMA Systems by Exploiting Hidden Pilots

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***Abstract*—This letter proposes a novel carrier frequency offset (CFO) estimation method for generalized multicarrier code-division multiple access systems in unknown frequency-selective channels utilizing hidden pilots. It is established that CFO is identifiable in the frequency domain by employing cyclic statistics (CS) and linear regression (LR) algorithms. We show that the CS-based estimator is capable of mitigating the normalized CFO (NCFO) to a small error value. Then, the LR-based estimator can be employed to offer more accurate estimation by removing the residual quantization error after the CS-based estimator. Simulation results are presented together with the theoretical analysis, and a good match between them is observed.**

***Index Terms*—Carrier frequency offset (CFO), cyclic statistics (CS), hidden pilot, linear regression (LR), multicarrier code-division multiple access (MC-CDMA).**

I. Introduction

GENERALIZED multicarrier code-division multiple access (GMC-CDMA) has recently received increasing interests for high data rate transmissions [1], [2]. Relying on the precoding redundancy, [1] has shown that signal recovery for GMC-CDMA systems is independent of channel null (deep fade) locations. Hence, for uncoded systems, GMC-CDMA outperforms the conventional MC-CDMA in frequency-selective fading channels. As an orthogonal frequency-division multiplexing (OFDM)-based technique, GMC-CDMA is very sensitive to carrier frequency offset (CFO) [3]. A good CFO estimator is acquired to improve the overall system performance. In literature, many data-aided and nondata-aided CFO estimators have been reported for multicarrier systems (e.g., [6]–[10]). However, very few publications so far have addressed exploiting hidden pilots for the CFO estimation. This motivated us to develop a novel CFO estimator by using hidden pilots.

Originally, techniques using hidden pilots were proposed for low-complexity estimation of static channels (or slowly time-varying channels) in single-carrier systems (see [4] and [5]). In this letter, we deploy the hidden pilots in the GMC-CDMA system for the accurate CFO estimation. The proposed CFO estimator consists of two subestimators operating in the frequency domain. Using the cyclic-statistics (CS) algorithm, the first subestimator is able to mitigate the normalized CFO (NCFO) to a small error value. This error is mainly from the quantization noise in the CS-based subestimator. The second subestimator is based on the linear-regression (LR) algorithm. It is shown that the LR-based estimator performs well for the case of small NCFO and is a good compensation to the CS-based estimator. Then, we use computer simulation to compare the proposed estimators with the pulse-shaping [8], CP-based [7], and null-subcarriers-based [9] approaches. Simulation results show that the proposed approach outperforms the others.

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II. GMC-CDMA With Hidden Pilots

The discrete-time equivalent model for the downlink of GMC-CDMA system has been presented in [1] and [2]. Prior to transmission, the information-bearing symbols are grouped into blocks $\mathbf{s}_M(n)$ with the size of $M \times 1$, where n denotes the block index. Then, these blocks are fed to a $K \times M$ block precoder $\mathbf{\Theta}$ to produce blocks $\mathbf{s}_K(n) = \mathbf{\Theta}\mathbf{s}_M(n)$. In the GMC-CDMA system, $\mathbf{\Theta}$ is used for the user signatures and also introduces the precoding redundancy $(K > M)$ [1]. Letting $L_t = K - M$, we construct another block precoder $\mathbf{\Theta}_t$ with the size of $K \times L_t$. This precoder is used to precode the hidden-pilot blocks $\mathbf{p}(n)$, with the input–output relationship of $\tilde{\mathbf{p}}(n) = \mathbf{\Theta}_t\mathbf{p}(n)$. Then, we feed the sum of $\mathbf{s}_K(n)$ and $\tilde{\mathbf{p}}(n)$ to the CP-OFDM modulator $\mathbf{F}$. The matrix $\mathbf{F} = [\mathbf{F}_{\rm cp}^T, \mathbf{F}_K^T]^T$ has the size of $J \times K$ $(J = K + L_{\rm cp})$, where $L_{\rm cp}$ denotes the length of cyclic prefix (CP), $(\cdot)^T$ denotes the matrix transpose, $\mathbf{F}_K$ denotes the inverse discrete Fourier transform (IDFT) matrix with the size of $K \times K$, and $\mathbf{F}_{\rm cp}$ is formed by collecting the last $L_{\rm cp}$ rows of $\mathbf{F}_K$. The output of CP-OFDM modulator (or transmitted block) is given by

$$\mathbf{x}(n) = \mathbf{F}\mathbf{W}\left[\mathbf{s}_M^T(n), \mathbf{p}^T(n)\right]^T \tag{1}$$

where $\mathbf{W} = [\mathbf{\Theta}, \mathbf{\Theta}_t]$ is usually formed by a Walsh matrix in GMC-CDMA systems. The detail description on how to obtain the output (1) can be found in [1].

The transmitted blocks experience the propagation channel $\mathbf{h} = [h(0), h(1), \ldots, h(L_u)]^T$, where L_u denotes the upper bound of the channel order. Considering the CFO, the received blocks can be formulated as (see [1] and [6])

$$\mathbf{y}_J(n) = \mathbf{\Omega}_J\mathbf{H}_l\mathbf{x}(n) + \underbrace{\mathbf{\Omega}_J\mathbf{H}_u\mathbf{x}(n-1)}_{\rm IBI} + \mathbf{v}_J(n) \tag{2}$$

where $\mathbf{H}_l$ and $\mathbf{H}_u$ are $J \times J$ lower and upper triangular matrices with entries $[\mathbf{H}_l]_{n,k} = h(n-k)$ and $[\mathbf{H}_u]_{n,k} = h(J+n-k)$. $\mathbf{v}_J(n)$ is the zero-mean Gaussian noise with the size of $J \times 1$. The diagonal matrix $\mathbf{\Omega}_J = \text{diag}\{\Phi^{nJ}, \Phi^{nJ+1}, \ldots, \Phi^{(n+1)J-1}\}$ comprises the CFO information with $\Phi = \exp(j(2\pi/K)\phi)$, where ϕ denotes the CFO normalized by the subcarrier spacing Δf. Normally, for the CP length $L_{\rm cp} \geq L_u$, the interblock interference (IBI) part can be removed by discarding CP at the receiver, and the remaining part becomes

$$\mathbf{y}_K(n) = \mathbf{\Omega}_K \mathbf{C}_K \mathbf{F}_K \mathbf{W} \left[\mathbf{s}_M^T(n), \mathbf{p}^T(n)\right]^T + \mathbf{v}_K(n) \quad (3)$$

where $\mathbf{\Omega}_K = \text{diag}\{\Phi^{nJ+L_{\rm cp}}, \Phi^{nJ+L_{\rm cp}+1}, \ldots, \Phi^{(n+1)J-1}\}$, $\mathbf{C}_K$ is the circulant channel matrix addressed in [1], and $\mathbf{v}_K(n)$ is the corresponding noise vector. We perform the DFT on $\mathbf{y}_K(n)$ and obtain the frequency-domain blocks as

$$\tilde{\mathbf{y}}_K(n) = \Phi^{nJ+L_{\rm cp}} \Big(\boldsymbol{\mathcal{D}}_K \mathbf{W} \left[\mathbf{s}_M^T(n), \mathbf{p}^T(n)\right]^T + \mathbf{i}_{\mathbf{s}}(n) + \mathbf{i}_{\mathbf{p}}(n)\Big) + \tilde{\mathbf{v}}_K(n) \quad (4)$$

where $\tilde{\mathbf{v}}_K(n) = \mathbf{F}_K^{-1}\mathbf{v}_K(n), \mathbf{i}_{\mathbf{s}}(n)$, and $\mathbf{i}_{\mathbf{p}}(n)$ are the intercarrier interferences (ICIs) contributed by the information symbols and pilots, respectively; $\boldsymbol{\mathcal{D}}_K = \mathbf{D}(\tilde{\mathbf{h}})$, where $\tilde{\mathbf{h}}$ denotes the channel frequency-response (CFR) with the size of $K \times 1$, and $\mathbf{D}(\mathbf{a})$ denotes the diagonal matrix with $\mathbf{a}$ in its diagonal. Then, we have the following three comments on (4).

1) The pilot blocks are assumed to be invariant with respect to the block index n. So, we can omit the index n in pilot-related terms $\tilde{\mathbf{p}}(n), \mathbf{p}(n)$ and $\mathbf{i}_{\mathbf{p}}(n)$ for convenience.
2) The ICI term $\mathbf{i}_{\mathbf{s}}(n)$ is contributed by $\mathbf{s}(n)$ and, thus, has the zero mean and is uncorrelated for the block index n [3].
3) When ϕ and $\mathbf{h}$ are known at the receiver, the CFO and pilot blocks can be eliminated from (3). Then, the zero-forcing (ZF) equalization can be employed in the frequency domain. Reference [1] has shown that $\boldsymbol{\mathcal{D}}_K$ has at most L_u zeros on its diagonal, so the signal recovery is independent of channel null (deep fade) locations only when $L_t \geq L_u$ and any M rows of $\mathbf{\Theta}$ are linear independent.

Later on, we present how to use hidden pilots for the CFO estimation.

III. Estimation of CFO

A. CS-Based Estimator

Consider the autocorrelation of the received blocks (4) as

$$\mathcal{C}_{\tilde{\mathbf{y}}_K\tilde{\mathbf{y}}_K}(n, n_\tau) = \mathcal{E}\{\tilde{\mathbf{y}}_K(n, +n_\tau)\tilde{\mathbf{y}}_K^{\mathcal{H}}(n)\}. \quad (5)$$

Based on the comments 1) and 2) in Section II, it is easy to obtain

$$\boldsymbol{\mathcal{C}}_{\tilde{\mathbf{y}}_K\tilde{\mathbf{y}}_K}(n_\tau \neq 0) = \Phi^{n_\tau J}\mathbf{\Psi}_0 \quad (6)$$

where

$$\mathbf{\Psi}_0 = \boldsymbol{\mathcal{D}}_K \boldsymbol{\mathcal{C}}_{\tilde{\mathbf{p}}\tilde{\mathbf{p}}} \boldsymbol{\mathcal{D}}_K^{\mathcal{H}} + \boldsymbol{\mathcal{D}}_K \boldsymbol{\mathcal{C}}_{\tilde{\mathbf{p}}\mathbf{i}_{\mathbf{p}}} + \boldsymbol{\mathcal{C}}_{\tilde{\mathbf{p}}\mathbf{i}_{\mathbf{p}}}^{\mathcal{H}} \boldsymbol{\mathcal{D}}_K^{\mathcal{H}} + \boldsymbol{\mathcal{C}}_{\mathbf{i}_{\mathbf{p}}\mathbf{i}_{\mathbf{p}}}. \quad (7)$$

We can see that the matrix $\mathbf{\Psi}_0$ is constant with respect to the lag n_τ. Therefore, $\boldsymbol{\mathcal{C}}_{\tilde{\mathbf{y}}_K\tilde{\mathbf{y}}_K}(n_\tau > 0)$ is a periodic function of n_τ with the period of $(K/J\phi)$ by assuming that the matrix $\mathbf{\Psi}_0$ is not zero (justification of this assumption will be given later). For $n_\tau \in (0, N-1]$, we perform the DFT on (6) and obtain (see [11] for DFT property)

$$\tilde{\boldsymbol{\mathcal{C}}}_{\tilde{\mathbf{y}}_K\tilde{\mathbf{y}}_K}(m_\tau) = \frac{1}{N-1}\sum_{n_\tau=1}^{N-1} \boldsymbol{\mathcal{C}}_{\tilde{\mathbf{y}}_K\tilde{\mathbf{y}}_K}(n_\tau) e^{-j\frac{2\pi}{N-1}m_\tau n_\tau} = e^{-j\frac{2\pi}{N-1}(m_\tau - \frac{(N-1)J\phi}{K})}\mathbf{\Psi}_0 \mathcal{S}\left(m_\tau - \frac{(N-1)J\phi}{K}\right) \quad (8)$$

where

$$\mathcal{S}(x) = \frac{\sin(\pi x)}{(N-1)\sin\left(\frac{\pi}{N-1}x\right)} e^{-j\frac{N-2}{N-1}\pi x}.$$

For $m_\tau \geq 0$, the norm of $\tilde{\boldsymbol{\mathcal{C}}}_{\tilde{\mathbf{y}}_K\tilde{\mathbf{y}}_K}(m_\tau)$ is given by

$$\left\|\tilde{\boldsymbol{\mathcal{C}}}_{\tilde{\mathbf{y}}_K\tilde{\mathbf{y}}_K}(m_\tau)\right\| = \|\mathbf{\Psi}_0\| \left|\mathcal{S}\left(m_\tau - \frac{(N-1)J\phi}{K}\right)\right| \quad (9)$$

which achieves its maximum along the m_τ direction only when $|\mathcal{S}(m_\tau - ((N-1)J\phi/K))|$ reaches its maximum, i.e., $m_\tau - ((N-1)J\phi/K) = 0$. Since m_τ is an integer, the maximum of $|\mathcal{S}(m_\tau - ((N-1)J\phi/K))|$ should be located at

$$\hat{m}_\tau = \left\lfloor \frac{(N-1)J\phi}{K} + \frac{1}{2} \right\rfloor \quad (10)$$

where $\lfloor \cdot \rfloor$ denotes the integer floor. Hence, we can first determine such a $\hat{m}_\tau$ by searching the maximum of $\|\tilde{\boldsymbol{\mathcal{C}}}_{\tilde{\mathbf{y}}_K\tilde{\mathbf{y}}_K}(m_\tau)\|$ over $m_\tau \in [0, N-2]$, and then, estimate the NCFO via

$$\hat{\phi} = \frac{K\hat{m}_\tau}{(N-1)J}. \quad (11)$$

Then, we provide the following three comments.

1) Estimation Range: For $m_\tau \in [0, N-2]$, the maximum identifiable NCFO, denoted by $\hat{\phi}_{\max}$, can be derived from (11)

$$\hat{\phi}_{\max} = \frac{(N-2)K}{(N-1)J}. \quad (12)$$

Due to the effect of frequency fold, the estimate range of CFO is given by $|\phi| \leq \lfloor((N-2)K/2(N-1)J)\rfloor$.

2) Quantization Noise: Since m_τ is an integer, the relationship between ϕ and m_τ can be derived from (10) and (11) as

$$\phi = \frac{K(m_\tau + \delta_M)}{(N-1)J} \quad (13)$$

where $|\delta_M| \leq 0.5$ is the quantization noise. This noise may incur the CFO estimation error as

$$|\hat{\phi} - \phi| = \frac{K|\delta_M|}{(N-1)J} \leq \frac{K}{2(N-1)J}. \quad (14)$$

Certainly, this error can be further mitigated by increasing N.

3) Low-Complexity Approach: The computation complexity of the CS-based approach is mainly from the DFT operation and autocorrelation. Particularly, (8) needs the complexity of $\mathcal{O}(K^2(N-1)\log(N-1))$ for the fast Fourier transform (FFT) operation. The low-complexity approach is intended to reduce the computation cost of FFT by considering the trace of $\boldsymbol{\mathcal{C}}_{\tilde{\mathbf{y}}_K\tilde{\mathbf{y}}_K}(n_\tau \neq 0)$ as

$$\text{Tr}\left\{\boldsymbol{\mathcal{C}}_{\tilde{\mathbf{y}}_K\tilde{\mathbf{y}}_K}(n_\tau \neq 0)\right\} = \Phi^{n_\tau J}\text{Tr}\{\mathbf{\Psi}_0\} \quad (15)$$

where $\text{Tr}\{\mathbf{A}\}$ denotes the trace of $\mathbf{A}$. We find that $\mathbf{\Psi}_0$ is a Hermitian matrix, and thus, $\text{Tr}\{\mathbf{\Psi}_0\} > 0$. Then, the DFT of (15) is given by

$$\text{Tr}\left\{\tilde{\boldsymbol{\mathcal{C}}}_{\tilde{\mathbf{y}}_K\tilde{\mathbf{y}}_K}(m_\tau)\right\} = e^{-j\frac{2\pi}{N-1}(m_\tau - \frac{(N-1)J\phi}{K})}\text{Tr}\{\mathbf{\Psi}_0\} \cdot \mathcal{S}\left(m_\tau - \frac{(N-1)J\phi}{K}\right). \quad (16)$$

For $m_\tau \geq 0$, the absolute value of $\text{Tr}\{\tilde{\boldsymbol{\mathcal{C}}}_{\tilde{\mathbf{y}}_K\tilde{\mathbf{y}}_K}(m_\tau)\}$ is given by

$$\left|\text{Tr}\left\{\tilde{\boldsymbol{\mathcal{C}}}_{\tilde{\mathbf{y}}_K\tilde{\mathbf{y}}_K}(m_\tau)\right\}\right| = \text{Tr}\{\mathbf{\Psi}_0\}\left|\mathcal{S}\left(m_\tau - \frac{(N-1)J\phi}{K}\right)\right|. \quad (17)$$

Then, the NCFO can be estimated by searching the maximum of $|\text{Tr}\{\tilde{\boldsymbol{\mathcal{C}}}_{\tilde{\mathbf{y}}_K\tilde{\mathbf{y}}_K}(m_\tau)\}|$. The trace-based algorithm only needs one

FFT and, thus, reduces the computation cost of FFT to $\mathcal{O}((N-1)\log(N-1))$.

B. LR-Based Estimator

The linear regression estimator is based on the following equation:

$$\mathbf{C}(n_\tau) = \boldsymbol{\mathcal{C}}_{\tilde{\mathbf{y}}_K\tilde{\mathbf{y}}_K}(n_\tau)\boldsymbol{\mathcal{C}}^{\mathcal{H}}_{\tilde{\mathbf{y}}_K\tilde{\mathbf{y}}_K}(1) = \Phi^{(n_\tau-1)J}\mathbf{\Gamma} \tag{18}$$

where $\mathbf{\Gamma} = \mathbf{\Psi}_0\mathbf{\Psi}_0^{\mathcal{H}}$ is a Hermitian matrix. The trace of $\mathbf{C}(n_\tau)$ is given by

$$\text{Tr}\{\mathbf{C}(n_\tau)\} = \Phi^{(n_\tau-1)J}\text{Tr}\{\mathbf{\Gamma}\}. \tag{19}$$

Due to $\mathbf{\Psi}_0 \neq \mathbf{0}, \text{Tr}\{\mathbf{\Gamma}\}$ should be positive. So, we can obtain the phase of $\text{Tr}\{\mathbf{C}(n_\tau)\}$ using

$$\begin{aligned}\varphi(n_\tau) &= \arg\{\text{Tr}\{\mathbf{C}(n_\tau)\}\}\\ &= \frac{2\pi J\phi}{K}(n_\tau-1) - 2\pi\left\lfloor\frac{2n_\tau+T_\varphi}{2T_\varphi}\right\rfloor\end{aligned} \tag{20}$$

where $\arg\{x\}$ is a function to obtain the phase of x, for $\arg\{x\}\in[-\pi,\pi), T_\varphi = (K/J\phi)$ is the period of the function $\varphi(n_\tau)$. Assuming T_φ to be known, we can form a first-order polynomial $f(n_\tau) = an_\tau + b$ and determine the parameters (a,b) by solving the following equation:

$$(\hat{a},\hat{b}) = \arg\min_{(\hat{a},\hat{b})}\sum_{n_\tau=1}^{\lfloor T_\varphi/2\rfloor}|\varphi(n_\tau) - f(n_\tau)|^2. \tag{21}$$

It is easy to find that $\hat{a}$ is the estimated slope of $\varphi(n_\tau)$, which is identifiable by using the LR algorithm. Then, the NCFO ϕ can be determined by

$$\hat{\phi} = \frac{K}{2\pi J}\hat{a}. \tag{22}$$

The following points should be noted for the LR-based approach.

1) Estimation Performance: Equation (21) shows that the curve fitting performance improves with the increase of T_φ. However, T_φ is inverse proportional to the NCFO. Hence, the estimation performance becomes worse with the increase of NCFO.

2) About $\lfloor T_\varphi/2\rfloor$: Equation (21) shows that the knowledge of $\lfloor T_\varphi/2\rfloor$ is required for successful CFO estimation. To determine $\lfloor T_\varphi/2\rfloor$, we perform the differential operation on $\varphi(n_\tau)$ and have

$$\begin{aligned}\Delta\varphi(n_\tau) &= \varphi(n_\tau+1) - \varphi(n_\tau)\\ &= \begin{cases}\gamma, & \text{for } n_\tau = \lfloor(2i+1)T_\varphi/2\rfloor\\ \frac{2\pi J\phi}{K}, & \text{otherwise}\end{cases}\end{aligned} \tag{23}$$

where $i = 0, 1, \ldots$, and γ is very close or equal to -2π.[1] Due to $|(2\pi J\phi/K)| \ll |\gamma|$, we can find $\lfloor T_\varphi/2\rfloor$ by searching the first location of γ over $n_\tau \in (0, N-1)$. If they does not have such a γ, then we set $\lfloor T_\varphi/2\rfloor = N-1$.

C. About $\mathbf{\Psi}_0$

In order to guarantee the CFO identifiability, we need the condition $\mathbf{\Psi}_0 \neq \mathbf{0}$. Because $\tilde{\mathbf{p}}$ and $\mathbf{i}_\mathbf{p}$ are constant, (7) can be rewritten into

$$\mathbf{\Psi}_0 = \boldsymbol{\mathcal{D}}_K\tilde{\mathbf{p}}\tilde{\mathbf{p}}^{\mathcal{H}}\boldsymbol{\mathcal{D}}_K^{\mathcal{H}} + \boldsymbol{\mathcal{D}}_K\tilde{\mathbf{p}}\mathbf{i}_\mathbf{p}^{\mathcal{H}} + \mathbf{i}_\mathbf{p}\tilde{\mathbf{p}}^{\mathcal{H}}\boldsymbol{\mathcal{D}}_K^{\mathcal{H}} + \mathbf{i}_\mathbf{p}\mathbf{i}_\mathbf{p}^{\mathcal{H}}. \tag{24}$$

In order to assure $\mathbf{\Psi}_0 \neq \mathbf{0}$, the pilot design should fulfill $\boldsymbol{\mathcal{D}}_K\tilde{\mathbf{p}} = \mathbf{0}$. Since $\boldsymbol{\mathcal{D}}_K$ contains at most L_u zeros on its diagonal, this condition can be achieved when $\tilde{\mathbf{p}}$ has at least L_u+1 nonzero elements.

[1] $\gamma = -2\pi$ holds only when T_φ is an integer.

IV. Simulation Results and Analysis

The root-mean-square error (RMSE) $\sqrt{1/\mathcal{I}\sum_{i=0}^{\mathcal{I}-1}|\hat{\phi}_i - \phi|^2}$ was used to benchmark the CFO estimation performance. Here, $\mathcal{I}$ is the number of Monte Carlo trials. The frequency-selective fading channel was modeled as a finite impulse response (FIR) filter with the maximum order of $L_u = 4$. Each tap was randomly generated according to the Rayleigh distribution with the variance of $1/(L_u+1)$. The block precoder $\mathbf{W}$ was formed by the $K\times K$ Walsh matrix. The parameters of GMC-CDMA systems were given by $K = 32, M = 28, L_{\text{cp}} = 4, L_t = 4$. The information-bearing symbols were drawn from the quadrature phase-shift keying (QPSK) constellation with the equal probability. The subcarrier spacing Δf of 312.5 kHz was the same as HIPERLAN/2 [13]. Elements in the pilot block $\mathbf{p}$ were identical. The pilot-to-information power ratio was given by L_t/M. The signal-to-noise ratio (SNR) is defined by the average received symbol energy to noise $E_s/\mathcal{N}_o$ [14]. Based on the above parameters, we find that the block $\tilde{\mathbf{p}}$ results in eight nonzero elements with equal amplitudes. As shown in Section III-C, the CFO identifiability can be guaranteed by employing this kind of pilot design.

Test Case 1: This experiment examines the estimation performance as a function of NCFO with SNR $=$ 5 and 15 dB, respectively. The RMSE results were obtained by taking the average of $\mathcal{I} = 500$ Monte Carlo trials. Each trial collected $N = 200$ GMC-CDMA blocks for the CFO estimation. Fig. 1(a) shows that the RMSE for the CS-based approach is very stable (around 2×10^{-3}). This result has a good match with the quantization error in (14). In the range of large NCFO (e.g., $|\phi| > 0.15$), we can observe that the LR-based approach cannot offer a good estimation due to the relatively short T_φ. The estimation performance improves significantly with the decrease of NCFO. When NCFO is small (e.g., $|\phi| < 0.1$), the LR-based approach outperforms the CS-based approach.

Test Case 2: In order to indicate the performance-complexity tradeoff, we examine the RMSE results as a function of data record length (DRL) N with $\phi = 0.0625$ and the typical SNR $=$ 12 dB. Fig. 1(b) shows that RMSE for CFO estimators decreases rapidly with the increase of N. They can achieve the best performance offered by the pulse-shaping-based approach [8] only using $N = 20$ blocks.

Test Case 3: To evaluate the subestimators for different SNR cases, we plot the RMSE results in Fig. 2 with the NCFO $\phi = 0.0625$ and the DRL $N = 100$ blocks.Fig. 2 shows that the CS-based subestimator is not sensitive to the SNR. The RMSE result for the LR-based estimator becomes stable and offers the best performance for SNR $>$ 10 dB. Interestingly, we observe that the low-complexity CS-based approach offers the same performance as the CS-based approach. In order to compare with state-of-the-art approaches, we also plot the RMSE results for the pulse-shaping, CP-based [7] and null-subcarriers (NS)-based [9] CFO estimators. Fig. 2 shows that the proposed estimators outperform the pulse-shaping-based and CP-based estimator. For the fair comparison with the NS-based approach, we investigate the OFDM system with the same setup as the GMC-CDMA system. The number of NSs is $N_z = 4$, which is

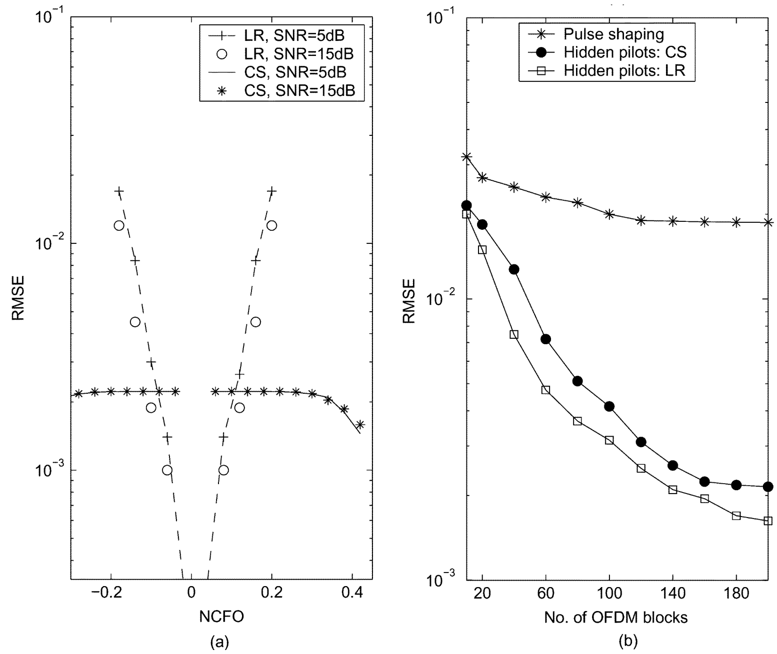

Fig. 1. RMSE results as a function of NCFO and DRL.

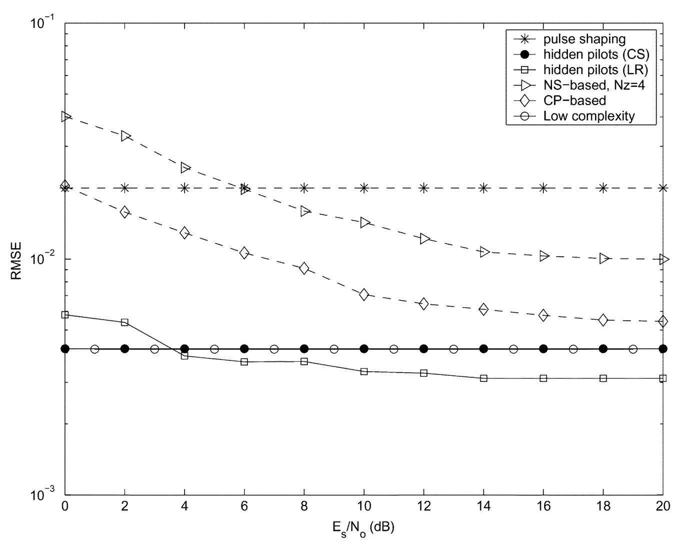

Fig. 2. RMSE results as a function of SNR.

the same as the number of hidden pilots in the GMC-CDMA system. Fig. 2 shows that the NS-based approach is sensitive to the SNR (particularly in the low SNR range). The proposed approaches also outperform the NS-based one.

V. Conclusion

Utilizing the hidden pilots, this letter has proposed a novel CFO estimators for GMC-CDMA in unknown frequency-selective channels. It has been established that the CFO is identifiable in the frequency domain by employing the CS-based and LR-based approaches. Both theoretical and simulation results have shown the excellent estimation performance of the proposed CFO estimator.

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