THE STIFFNESS OF SOILS AND WEAK ROCKS
AT VERY SMALL STRAINS

by

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To my parents
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SUMMARY

This thesis addresses the stiffness of natural geomaterials at very small strains. Three natural geomaterials with wide ranging stiffnesses and strengths were investigated. These were a weak rock (Chalk), a very stiff clay (London clay) and a soft clay (Bothkennar clay). The experimental programme was laboratory-based and all tests were performed in the triaxial apparatus.

A major aim of the project was to develop sufficiently accurate triaxial local-strain instrumentation to observe the linear stress-strain response of the three geomaterials. This was achieved by the development of a Fabry-Perot interferometer. The interferometer was used in two ways. Firstly, it was used as an accurate reference instrument against which commercial LVDTs were calibrated. These LVDTs where then used as local-strain instruments. Secondly, the interferometer was used directly as a local-strain instrument.

The strain limit of linear behaviour was remarkably similar for all three geomaterials and was less than 0.004% axial strain in all tests. In addition, the stiffness degradation with strain was found to be broadly similar for all three geomaterials.

After a rest period, the stiffness at very small strains of a geomaterial reverts back to the maximum stiffness, regardless of the direction of the incoming or outgoing stress path. It was suggested that during such rest periods healing of the material occurs. At intermediate strain levels, stiffness is affected by the direction of the current stress path. Softer response occurs when the stress path is directed towards the yield surface as opposed to away from the yield surface. In addition, it was shown that once the influence of creep due to prior loading had been properly accounted for, recent stress history had no effect on geomaterial stiffness.

Comparison of the triaxial stiffness with the field seismic stiffness showed that the triaxial stiffness, at very small strains, are closer to the field seismic stiffness than previously believed.
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SYMBOLS

A = constant
B = plate width, magnetic field component of electromagnetic waves
$B_0$ = amplitude of magnetic component of electromagnetic waves
$c$ = velocity of light in a vacuum
$c_u$ = undrained shear strength
$D_r$ = relative density
$D_{60}$ = largest particle size in the smallest 60% of particles
d = distance
$E$ = Young's modulus, electric field component of electromagnetic waves
$E_{\text{max}}$ = Young's modulus at very small strain levels from triaxial test
$E_0$ = Young's modulus measure in-situ by seismic techniques
$E_i$ = amplitude of electric component of electromagnetic waves
$E_i$ = amplitude of incident light
$E_r$ = amplitude of reflected light
$E_t$ = amplitude of transmitted light
e = void ratio
$e_0$ = in-situ void ratio
$e_0, e_1$ etc. = atomic energy levels
$F$ = coefficient of finesse
$\mathcal{F}$ = finesse
$f(e)$ = void ratio function
$G_0$ = shear stiffness measure in-situ by seismic techniques
$G_{\text{max}}$ = shear stiffness at very small strain levels from laboratory tests
$G_p$ = the shear stiffness at the end of primary consolidation
$h$ = cavity length, Planck's constant
$I$ = light intensity
$I_L$ = liquidity index
$LL$ = liquid limit
$n$ = index of refraction
OCR = over consolidation ratio
$PL = plastic limit$
$p = pressure$
$p' = mean effective stress$
$p'_{i} = initial mean effective stress$
$p'_{o} = in-situ mean effective stress$
$p'_{st} = mean effective stress at the start of a shear stage$
$p'_{y} = isotropic yield stress$
$m = constant
$n = constant, porosity, index of refraction$
$R = reflectivity$
$r = ratio of reflected and incident light amplitudes$
$T = transmissivity$
$t = ratio of transmitted and incident light amplitudes$
$UCS = unconfined compressive strength$
$V_s = shear wave velocity$
$Y_1, Y_2, Y_3 = yield surfaces (see Jardine et al. (1991) and Smith et al. (1992))$
$x = optical path length
$q = deviator stress$

$\epsilon_a = axial strain$
$\epsilon_{linear} = strain limit of linear behaviour$
$\Delta G = increase in shear stiffness per log cycle of time$
$\Delta t = time increment$
$\Delta h = change in cavity length$
$\Delta \delta = change in phase difference$
$\delta = phase difference$
$\phi = phase angle$
$\lambda = wave length$
$\lambda_0 = light wavelength in a vacuum$
$\rho = bulk density$
\( \sigma_1, \sigma_2, \sigma_3 = \text{principal total stress} \)

\( \sigma'_1, \sigma'_2, \sigma'_3 = \text{principal effective stress} \)

\( \sigma'_{\gamma} = \text{vertical effective yield stress} \)

\( \theta, \phi = \text{angle} \)

\( \nu = \text{Poisson's ratio, frequency} \)

\( \omega = \text{angular velocity} \)

\( \upsilon = \text{velocity of light in a medium other than a vacuum} \)
1. INTRODUCTION

Early geotechnical design methods focused on the failure state of soil. Excessive movement of the soil or structure was prevented by applying a sufficiently large factor of safety to the most critical failure scenario. Recently, the emphasis during geotechnical design has focused on calculating the movements of structures. Such design relies on the ability to model the interaction between the structure and the adjacent soil. The complexity of such calculations largely depends on the type of structure, ground conditions and the soil model. Complex problems may be analyzed by means of numerical techniques including finite element, finite difference and boundary element techniques. The recent trend towards powerful desk top computers has made the required computational capability available to practising engineers.

An important geotechnical parameter required in order to solve soil-structure interaction problems is the stiffness of the soil. Considerable effort has been applied to develop methods for the measurement of soil stiffness. In recent years advances in stiffness measurement have been made both in the field and in the laboratory.

The measurement of stiffness by means of field seismic geophysics has shown considerable promise as a method to measure the small strain stiffness of geomaterials (see for example Matthews et al. (1996)). Seismic techniques are based on the measurement of the seismic shear wave or Rayleigh wave velocities of a geomaterial. From the wave velocity and a knowledge of the bulk unit weight of the material, the shear stiffness can be calculated. The advantage of seismic geophysics is the fact that the material is tested undisturbed and at the in-situ stress. This avoids problems with regards to sampling disturbance and changes caused by modification of the stress state. As small strains are imposed by the propagated waves during seismic geophysics, the stiffnesses measured by this technique corresponds to the stiffness at very small strains. However, the stiffness of most geomaterials is strongly dependent on strain level (see for example Jardine et al. (1984)). Therefore the stiffness measured
by means of seismic geophysics may require modification if it is to be applied to
geotechnical design.

The most notable recent advance with regards to stiffness measurement in the
laboratory has been the development of triaxial local strain instrumentation (see for
example Burland and Symes (1982), Clayton and Khatrush (1986), Tatsuoka
(1988)). Local strain instrumentation measures the strains on a triaxial specimen
remote from the ends. This avoids inaccuracy caused by effects such as bedding errors
and apparatus compliance. Measurements made using local instrumentation have
shown that the stress-strain behaviour of geomaterials are strongly non-linear with
high stiffnesses at very small strain levels (see for example Jardine et al. (1984)).

To date, linear stress-strain behaviour has rarely been observed in the triaxial
apparatus for natural soils with low levels of bonding. In contrast, linear stress-strain
behaviour has been observed for well bonded geomaterials such as weak rock (see for
example Matthews and Clayton (1993), Cuccovillo and Coop (1997b)). This raises the
philosophical question as to whether linear behaviour has not been observed for soils
because the available instrumentation has been inadequate or indeed whether soils do
not exhibit linear behaviour.

The question whether natural soils behave as a linear-elastic material at very small
strains is not only important from the viewpoint of scientific curiosity; it has some
fundamental implications. Firstly, for linear-elastic soil the stiffness will be a constant
for all strain levels up to the limit of linear behaviour ($\varepsilon_{linear}$). For such conditions,
elasticity theory may be used to model soil behaviour. Furthermore, linear-elastic
behaviour implies that the stiffness of the material will be independent of the mode of
shear and the shear rate. Stiffnesses measured by different techniques should therefore
yield the same value. In particular, the comparison of stiffness from field tests such as
seismic geophysics and that from laboratory tests such as the triaxial test would be
valuable. At very small strains the stiffnesses should be the same. Seismic stiffness
could therefore be used as a bench mark against which the triaxial stiffness at small
strains may be compared. Mair (1993) pointed out that practitioners find it extremely valuable when different tests yield the same parameter value as it increase the level of confidence in the parameter. In addition, it may be argued that any discrepancies between the stiffness from the two methods will be indicative of either sampling disturbance or differences introduced by discontinuities, layering and non-homogeneity (assuming additional effects such as stress level, strain level and creep has been accounted for). Indeed, comparison of field and laboratory stiffnesses may present a effective tool to evaluate effects such as sampling disturbance.

As a long term goal, it might be speculated that if knowledge of the stiffness degradation behaviour of geomaterials was well established then seismic geophysics, carried out in isolation, might give sufficient data to the practitioner to allow design. This would significantly reduce the time and cost needed to obtain stiffness parameters.

This research project has been laboratory based. All tests were conducted in the triaxial apparatus. The principal aim was to develop local strain instrumentation capable of detecting the linear stress-strain range of a number of natural geomaterials. The aim was achieved by developing a Fabry-Perot interferometer which was used in two ways. Firstly, commercial LVDTs were calibrated against the interferometer to levels of accuracy higher than those achieved by other workers. These LVDTs were then used as local instrumentation for triaxial testing. Secondly the interferometer was used directly as a local strain instrument.

Three natural geomaterials were investigated. These were the Bothkennar clay, the London clay and the Chalk. These materials were chosen on the basis of their wide ranging strengths and stiffnesses. A number of questions were addressed. These included the strain range of linear behaviour, the effect of current stress path direction and the effect of recent stress history on the stiffness response of geomaterials. In addition, seismic geophysical data was available from sites close to where the triaxial samples were taken. The opportunity was therefore used to compare the triaxial
stiffnesses at very small strains with stiffnesses measured by seismic geophysical techniques.

The arrangement of the thesis is as follows:

A review of the current literature is made in Chapter 2. This includes a brief introduction to the stress-strain behaviour of geomaterials and the factors that affect stiffness. Furthermore, current measurement and modelling techniques are reviewed.

The local instrumentation developed as part of this project is shown in Chapter 3. The optical principles required to understand the operation of the Fabry-Perot interferometer is briefly reviewed. In addition, the use of the interferometer as a stand-alone local-strain device and as a calibration reference is presented in some detail.

Three triaxial systems were used during this project. These were a low-pressure, a medium-pressure and a high-pressure apparatus. These apparatuses are discussed in Chapter 4. The measurement instrumentation is described in the context of calibration and application.

Chapter 5 describes the testing of the three geomaterials. Emphasis is placed on sampling, specimen preparation and testing procedures. Typical test results are shown for each geomaterial. In addition some general issues are discussed. These include creep rates prior to shear and a discussion on the techniques by which to present stiffness data at very small strains.

The stiffness behaviour observed for the three geomaterials is discussed in Chapter 6. The influence of a number of factors on geomaterial stiffness is discussed with reference to the observed behaviour. Topics such as the limit of linear stress-strain behaviour and the degradation of stiffness with increasing strain are addressed. In addition, comparisons are made between stiffness data from the triaxial tests to field seismic stiffnesses for all three geomaterials.

Chapter 7 presents the main conclusions of this thesis.
2. LITERATURE REVIEW

This chapter presents a review of current knowledge of geomaterial stiffness behaviour. Emphasis is placed on the factors which influence geomaterial stiffness. Some field and laboratory methods used to measure stiffness are discussed. In addition, some of the well known mathematical models used to predict soil behaviour are briefly presented.

2.1. Introduction to stress-strain behaviour of geomaterials

In theory, an assembly of particles in contact will not exhibit a linear stress-strain response (see for example Bowden and Tabor (1964) and Johnson (1985)). However, all natural materials will invariably have been subjected to processes such as ageing, creep, cementation and over-consolidation. Work conducted by means of the resonant column apparatus suggest that soils subjected to such processes tend to exhibit a region of linear elastic behaviour (see for example Anderson and Richart (1976), Georgiannou et al. (1991)).

Even though the resonant column test is a valuable tool for the investigation of the stiffness of geomaterials at very small strains it is strictly only suitable for measuring stiffness in the linear elastic region. Attempts have been made to extend the measurement range of the resonant column measurements to the non-linear region (see for example Anderson and Richart (1976)), but difficulties remain. These will be discussed in Section 2.3.1.

The triaxial test allows the stiffness response of geomaterial to be investigated over a wide range of strains. Table 2-1 shows some of the currently available data on the strain limit of linear behaviour (\(\varepsilon_{\text{linear}}\)) for a wide range of geomaterials. All tests were monotonic triaxial loading tests with local instrumentation used to measure axial strains.
<table>
<thead>
<tr>
<th>Material</th>
<th>$\varepsilon_{\text{linear}}$ (%)</th>
<th>Description and comments</th>
<th>Ref</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dogs Bay sand</td>
<td>$&lt; 1 \times 10^{-3}$*</td>
<td>Reconstituted, uniform, angular, biogenetic, carbonate sand. $D_{50} = 0.3$ mm.</td>
<td>3</td>
</tr>
<tr>
<td>Leighton Buzzard sand</td>
<td>$2 \times 10^{-3}$</td>
<td>Reconstituted, uniform, sub-rounded, quartz sand. $D_{50} = 0.62$ mm.</td>
<td>7</td>
</tr>
<tr>
<td>Kaolinite</td>
<td>$&lt; 2 \times 10^{-3}$+</td>
<td>Reconstituted clay. $PL = 44%$, $LL = 82%$.</td>
<td>5</td>
</tr>
<tr>
<td>Berthierville clay</td>
<td>$&lt; 2 \times 10^{-3}$*</td>
<td>Soft silty clay. $c_u = 15$kPa, vertical yield stress ($\sigma'_y$) $\approx 50$kPa, $I_L = 1.67$.</td>
<td>8</td>
</tr>
<tr>
<td>Bothkennar clay</td>
<td>$&lt; 2 \times 10^{-3}$*</td>
<td>Soft marine clay. $c_u = 25$kPa, vertical yield stress ($\sigma'_y$) $\approx 70$kPa, $I_L = 0.9$.</td>
<td>8</td>
</tr>
<tr>
<td>Queen’s borough clay</td>
<td>$&lt; 2 \times 10^{-3}$*</td>
<td>Soft silty clay. $c_u = 15$kPa, vertical yield stress ($\sigma'_y$) $\approx 80$kPa, $I_L = 0.7$.</td>
<td>8</td>
</tr>
<tr>
<td>Osaka Bay clay</td>
<td>$1 \times 10^{-3}$</td>
<td>Stiff, overconsolidated clay (OCR = 1.2). $LL = 65%$, $PI = 37%$, $c_u = 330$kPa.</td>
<td>6</td>
</tr>
<tr>
<td>Vallericca clay</td>
<td>$&lt; 10^{-2}$#</td>
<td>Weakly cemented, overconsolidated clay. ($\sigma'_y$) = 1.9MPa, $p'_c = 430$kPa, CaCO$_3$ content 30%.</td>
<td>2</td>
</tr>
<tr>
<td>Calcarenite</td>
<td>$1 \times 10^{-2}$</td>
<td>Weak rock, carbonate sand grains cemented by calcite bonds. Isotropic yield stress ($p'_y$) = 3.4MPa.</td>
<td>1</td>
</tr>
<tr>
<td>Sandstone</td>
<td>$2 \times 10^{-2}$</td>
<td>Weak rock, quartz grains weakly bonded by iron oxide. UCS = 1MPa, $p'_y = 11$MPa.</td>
<td>1</td>
</tr>
<tr>
<td>Chalk</td>
<td>$3 \times 10^{-2}$</td>
<td>Weak rock. UCS = 9MPa. $\varepsilon_{\text{linear}}$ ranged between 5x$10^{-3}$ % for $n = 29%$ and 4x$10^{-2}$% for $n = 45%$</td>
<td>4</td>
</tr>
</tbody>
</table>

* Linear stress-strain range not observed at small-strain capability of local instrumentation.
+ Stress reversal at $\varepsilon = 2 \times 10^{-3}$% showed plastic strains.
# $\varepsilon_{\text{linear}} = 10^{-2}$ from resonant column tests, however the triaxial tests show this limit to be inconclusive (see Figure 2-17).

1. Cuccovillo and Coop (1997b)
2. Georgiannou et al. (1991)
5. Mukabi et al. (1991)
6. Mukabi et al. (1994)
7. Park (1993)
8. Smith (1992)

Table 2-1. Strain limit of elastic behaviour for a number of geomaterials.
The literature shows that for soils the limit of linear behaviour has only been identified successfully in very few cases. These include work on Leighton Buzzard sand (Park (1993)) and work on Osaka Bay clay (Mukabi et al. (1994)). Indeed, Table 2-1 confirms for soils the authors failed to measure the strain limit of linear behaviour in most cases. In contrast, linear stress-strain behaviour has been observed more often for well bonded geomaterials such as weak rock (see for example Matthews (1993), Cuccovillo and Coop (1997b)). This raises the question as to whether linear behaviour has not been observed widely for soils because the available instrumentation has been inadequate or indeed whether soils do not exhibit linear behaviour.

The linear-elastic idealisation of soil behaviour is attractive because of its conceptual simplicity. However, it is now widely recognised that some yielding occurs well inside the state boundary surface of a geomaterial (Leroueil and Vaughan (1990), Jardine (1992), Hight and Higgins (1995)). This leads to non-linear stress-strain behaviour of soil at intermediate strain levels. Non-linear stress-strain response, with high stiffness at small strains, was postulated by Simpson et al. (1979). They based their postulation on the results of numerical back analysis of a large excavation in London clay. In particular they found it impossible to match the calculated displacement patterns with observed patterns when using a ground model with a linear relationship between stress and strain. On the other hand, when using a non-linear stress-strain model, they found good agreement between calculated and observed displacement magnitudes and patterns. Their postulation has since been confirmed by numerous researchers (see for example Jardine et al. (1984), Clayton and Khatriush (1986), Tatsuoka (1988)).

Non-linear stress strain behaviour is shown schematically in Figure 2-1. The typical strain levels for a number of geotechnical structures are included in Figure 2-1 (Mair (1993)). It shows that the strain levels where stiffness degradation is most rapid coincides with the strain levels found for geotechnical structures in practice. This implies that designers have to take due consideration of the non-linear stress-strain response of geomaterials.
2.2. Factors that affect soil stiffness

Hardin and Richart (1963) used the results from numerous resonant column tests on clays and sands to propose an empirical equation for soil stiffness at very small strains \( G_{\text{max}} \):

\[
G_{\text{max}} = A f(e) (p')^n OCR^m
\]  

(2-1)

Equation (2-1) emphasises the importance of void ratio, mean effective stress and stress history on soil stiffness. All other factors are collectively grouped together as a constant \( A \). Recent research has confirmed the particular importance of void ratio and mean effective stress on soil stiffness, but in addition a number of other factors have been identified. Table 2-2 is a summary of the most important factors that influence soil stiffness and is taken largely from Hardin and Drnevich (1972), Clayton et al. (1984) and Jardine (1995).

<table>
<thead>
<tr>
<th>Void ratio ( (e) ):</th>
<th>Loose soils deform by interparticle sliding and dense soils deform by particle deformation. Therefore, stiffness decreases with increasing void ratio. An empirical expression to link ( G_{\text{max}} ) to ( e ) for clean granular material was suggested by Hardin and Richart (1963) as ( f(e) = (2.17-e)^2/(1+e) ). Jamiolkowski et al. (1991) proposed a relationship applicable to a wider range of geomaterials as ( f(e) = 1/(e^{1.3}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average particle size ( (D_{50}) )</td>
<td>More particles per unit volume leads to more contacts per unit volume, allowing greater interparticle sliding. Stiffness decreases with decreasing average particle size.</td>
</tr>
<tr>
<td>Coefficient of uniformity ( (D_{60}/D_{10}) )</td>
<td>Uniform soils have higher void ratio than well-graded soils. Stiffness decreases with decreasing coefficient of uniformity.</td>
</tr>
<tr>
<td>Parameter</td>
<td>Description</td>
</tr>
<tr>
<td>-------------------------</td>
<td>----------------------------------------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>Particle angularity</td>
<td>Park and Tatsuoka (1994) showed that sub-rounded sand is distinctly stiffer than sub-angular sand at similar void ratio and effective stress. This is explained by local yielding at particle contacts in angular particles. Therefore stiffness decreases with increasing particle angularity.</td>
</tr>
<tr>
<td>Particle roughness</td>
<td>Rough particles inhibit sliding. Therefore stiffness decreases with decreasing particle roughness.</td>
</tr>
<tr>
<td>Mineralogy</td>
<td>Stiffness increases with increasing mineral strength.</td>
</tr>
<tr>
<td>Plasticity</td>
<td>Jardine (1995) demonstrated that the stiffness of low plasticity clay-silts and clay-sands is comparable to the stiffness of clean sand and showed a trend of decreasing stiffness with increasing plasticity.</td>
</tr>
<tr>
<td>Cementation</td>
<td>Interparticle bonds must be broken before sliding can occur between particles. Stiffness increases with increasing cementation.</td>
</tr>
<tr>
<td>Ageing</td>
<td>Stiffness increases as ageing increases (see section 2.2.4).</td>
</tr>
<tr>
<td>Depositional (inherent)</td>
<td>Sediment particles settle into their most stable configuration, so stiffness decreases as load and bedding direction diverge (see section 2.2.3).</td>
</tr>
<tr>
<td>anisotropy</td>
<td></td>
</tr>
<tr>
<td>Degree of saturation</td>
<td>Hardin and Drnevich (1972) found the degree of saturation to be relatively unimportant for sands but very important for clays. Stiffness decreases for an increase in degree of saturation.</td>
</tr>
<tr>
<td>Mean effective stress</td>
<td>Stiffness increases as mean effective stress increases. But this effect is reduced as levels of bonding increase (section 2.2.1).</td>
</tr>
<tr>
<td>Shear stress</td>
<td>Shear stress increases particle sliding and therefore deformation accelerates as $\sigma_{1}/\sigma_{3}$ approaches $(\sigma'<em>{1}/\sigma'</em>{3})<em>{r}$. Yu and Richart (1984) proposed the following relationship to account for the effect of shear stress level of dry sands: $G</em>{max} = c f(e) \sigma^{0.5}_n (1-0.3k^{1.5})$</td>
</tr>
<tr>
<td><strong>Over-consolidation ratio (OCR)</strong></td>
<td>OCR has very little influence on $G_{max}$ for clean silica sand (see for example Hardin and Drnevich (1972), Shibuya et al. (1992) and Porovic and Jardine (1994)). However Fioravante et al. (1994) showed a significant increase in $G_{max}$ with increasing OCR for a crushable sand (Quiou sand) consisting of 74% shell fragments and 15% calcium carbonate aggregates. For clay the effect of OCR on $G_{max}$ is still controversial, with Vigiani (1991) reporting $G_{max} = (G_{max})<em>{nc} OCR^{0.2}$ for reconstituted clays and Jamiołkowski et al. (1995) arguing that OCR has little effect on $G</em>{max}$ for clays. Stiffness at intermediate strain levels is influenced by OCR. Jardine (1995) showed that OCR has the strongest effect on silty sandy clays, whilst reconstituted sand and silt experience the least effect.</td>
</tr>
<tr>
<td><strong>Recent stress history</strong></td>
<td>Atkinson et al. (1990) found that recent stress history influences the stiffness at small strain levels. The stiffness increases as the effective stress path direction changes. Limited evidence from Hird and Pierpoint (1997) suggest that the effect of recent stress history is not as important as claimed by Atkinson et al. (1990) (see section 2.2.5).</td>
</tr>
<tr>
<td><strong>Stress path direction</strong></td>
<td>When soil is sheared from close to the yield surface, the stiffness response is softer when the stress path direction is towards the yield surface (see section 2.2.6).</td>
</tr>
</tbody>
</table>

Table 2-2. Factors that affect geomaterial stiffness.

Some of the factors that have an influence on the stiffness of geomaterial and have been considered to be of particular importance in the context of this thesis are discussed in more detail in the following sections.
2.2.1. Mean effective stress

Equation (2-1) emphasises the fact that shear stiffness is a function of the mean effective stress \((p')\) raised to the power \(n\). If the mechanical behaviour of soil is considered to be purely frictional, its strength and stiffness should vary linearly with mean effective stress (Viggiani and Atkinson (1995)). This implies that \(n = 1\). On the other hand if soil is regarded as an assembly of elastic spheres, contact mechanics theory predicts that the elastic normal and shear stiffnesses will vary with the mean effective stress, raised to a fractional power (Duffy and Mindlin (1957), Deresiewicz (1974) and Johnson (1985)). Duffy and Mindlin (1957) suggested on the basis of contact mechanics that for an assembly of elastic spheres, \(n = 1/3\).

However, more recent numerical work on an assembly of elastic spheres of different sizes have shown that overall deformations consist of elastic deformations, as well as particle slippage and rearrangement (Dobry, Ng and Petrakis (1989)). This indicates that \(n\) will be slightly larger than \(1/3\) even at very small strain levels.

The values of \(n = 1\) and \(n = 1/3\) for the two theories noted above give valuable benchmarks against which the behaviour of geomaterials may be judged. Experimental data show that for uncedmented materials both the theories are valid, but at different strain levels (Porovic and Jardine (1994)). At very small strains, soil stiffnesses vary as a function of mean effective stress raised to a fractional power and at large strain levels they vary linearly with mean effective stress. When the value of \(n\) is found to be outside the range \(1/3 < n < 1\), it is an indication that neither of the two theories are valid. One example of such a case is when interparticle bonding occurs. This will be discussed below.

The resonant column test allows the stiffness of soil to be investigated at very small strains. Table 2-3 show the exponent \(n\) (equation (2-1)) for a number of geomaterials as measured in the resonant column apparatus.
<table>
<thead>
<tr>
<th>Material</th>
<th>( n )</th>
<th>Description</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kaolinite</td>
<td>0.65*</td>
<td>Reconstituted clay of medium plasticity.</td>
<td>3</td>
</tr>
<tr>
<td>Carbonatic sand</td>
<td>0.63</td>
<td>Reconstituted crushable sand, consisting of 74% shell fragments and 15% calcium carbonate aggregates.</td>
<td>2</td>
</tr>
<tr>
<td>Ham River sand</td>
<td>0.51</td>
<td>Reconstituted, uniform, quartz sand with rounded particles. ( D_{60} = 0.315 \text{mm} ). Test conducted at relative density = 76%.</td>
<td>5</td>
</tr>
<tr>
<td>Ticino sand</td>
<td>0.43</td>
<td>Reconstituted, uniform, sub rounded silica sand. ( D_{50} = 0.5 \text{mm} ).</td>
<td>4</td>
</tr>
<tr>
<td>Artificially cemented sand</td>
<td>0.43</td>
<td>Reconstituted, subangular to subrounded, medium to fine silica sand. Artificially cemented by 0, 1, 2 and 4% Portland cement with ( D_r = 50% ) and UCS ranging between 19 and 59kPa for increasing cementation. ( n ) was found to be 0.43 for all levels of cementation (see Figure 2-2).</td>
<td>1</td>
</tr>
<tr>
<td>Weakly cemented silty sand</td>
<td>0.13</td>
<td>Intact, weakly cemented silty sand. Specimen cut from block sample. The cementing agent was CaCO(_3) (15% by weight), ( e_0 = 0.58 ).</td>
<td>6</td>
</tr>
</tbody>
</table>

*Changes in void ratio were neglected

1. Acar and El-Tahir (1986)  
2. Fioravante et al. (1994)  
3. Humphries and Wahls (1968)  
4. Lo Presti et al. (1993)  
5. Porovic (1994)  

Table 2-3. Dependence of \( G_{\text{max}} \) on mean effective stress.

From Table 2-3 it may be concluded that at low levels of bonding, stiffness increases with increased mean effective stress as demonstrated by values of \( n \) between 0.43 and 0.65.

It is interesting to note from the work by Acar and El-Tahir (1986) on artificially
cemented sand, that increasing the fraction of Portland cement up to 4% by weight increased the stiffness, but did not reduce the exponent $n$ (Figure 2-2). They speculate that the insensitivity of $n$ to cementation indicates that the relative contribution to resistance from cement-to-cement or sand-to-cement bonds is insignificant. From this they conclude that the cementation-induced increase in shear stiffness is due to the confining effect from the cement at sand-to-sand interfaces. These conclusions may be debatable, but at the very least, the study shows that sands must have significant levels of bonding before the dependence on mean effective stress reduces.

Results similar to those in Table 2-3 were shown by Kohata et al. (1997) from triaxial tests. At very small strains, $n$ varied between 0.49 and 0.57 for a total of eight uncemented sands and gravels. In contrast they found $0 < n < 0.11$ for sedimentary soft mudstone from four different sites and $0.15 < n < 0.22$ for sedimentary soft siltstone from two sites.

Further evidence of the effect of bonding in reducing the dependence of stiffness on mean effective stress has been given by Stokoe et al. (1995). They used a resonant column apparatus to investigate intact weakly to moderately cemented silty sand at various confining pressures. The specimen was cut from a block sample retrieved by excavating a test pit. The cementing agent was CaCO$_3$ (15% by weight) and the results are shown in Figure 2-3. Two further tests were conducted on remoulded specimens. In one, material from the intact specimen was remoulded, and in the second the specimen was remoulded after first removing the CaCO$_3$ by soaking it in an acidic solution for two days. Both remoulded specimens were re-compacted to void ratios similar to that of the intact specimen ($e = 0.58$). The limited effect of the mean effective stress on stiffness for the intact material is reflected by an exponent $n$ of 0.13. This is in contrast to $n = 0.65$ and $n = 0.63$ for the two remoulded specimens. It is interesting to notice that the mere presence of the cementing agent made very little difference to the stiffness of the remoulded geomaterial. This suggests that once the bonds are broken, the bonding agent had no further influence
on the material stiffness, at least at very small strains. It would have been interesting to investigate whether the cementing agent had an influence on the stiffness at larger strain levels where the frictional nature of the material dominates.

The mean effective stresses used during tests on the well-bonded material discussed in the previous paragraphs were in all cases significantly lower than the isotropic yield stress. It may be more interesting to investigate the change of stiffness at stress levels close to the isotropic yield stress. First consider the behaviour of reconstituted kaolin, a clay with low levels of bonding (Figure 2-4). Humphries and Wahls (1968) conducted this test in a resonant column apparatus. The sample was taken through a number of isotropic consolidation and swelling cycles, resulting in the specimen being normally consolidated and overconsolidated at various stages of the test. This allowed the stiffness change to be evaluated as the mean effective stress level was taken up to, and beyond the yield stress, thereby allowing the investigation of the effect of yielding on the stiffness. Figure 2-4 shows that the stiffness of the reconstituted kaolin consistently increased as the clay was taken through the isotropic yield point (A). This indicates that yielding *per se* does not drastically alter the effect of mean effective stress on material stiffness.

Different behaviour was observed for well-bonded calcarenite by Cuccovillo and Coop (1997b). They investigated the effect of mean effective stress on the stiffness of intact and reconstituted calcarenite in a high pressure triaxial apparatus (for the material description see Table 2-1). The results are shown in Figure 2-5. The isotropic yield stress of the intact material ranged between 2.2 and 3.4 MPa. From Figure 2-5 it may be seen that the reconstituted material followed the fractional power relationship with $n = 0.6$. In contrast, the stiffness of the intact material was independent of the mean effective stress for stresses lower than the yield stress. Furthermore, a marked decrease in stiffness was evident as the mean effective stress approached the yield stress and destructuring of the material occurred. This clearly indicates the effect of bonding in reducing the effect of mean effective stress on stiffness. It is interesting to note from Figure 2-5 that at very high mean effective
stress (8.4MPa), a convergence of the stiffness of the reconstituted and intact samples is evident, indicating that the intact sample has been largely destructured.

When samples are taken from different depths, the material will have been subjected to different in-situ stress levels. During laboratory testing, these samples are often reconsolidated back to the in-situ stress. Comparison of stiffness behaviour from different samples becomes difficult because of these effects. Normalisation techniques are used to overcome this problem. It is common practice to normalise stiffnesses from triaxial tests by \((p')^l\) (Clayton et al. (1994)) and stiffnesses from resonant column tests by \((p')^{0.5}\) (Hardin and Drnevich (1972), Hicher (1996)). From the discussion in the previous paragraphs, it is clear that these normalisation procedures are only applicable in special cases; namely \(n = 1\) at relatively large strains (i.e. after bonds have been broken) and \(n = 0.5\) at very small strains for reconstituted unbonded material. Porovic and Jardine (1994) used results from resonant column and torsional shear tests on reconstituted Ham River sand to determine the variation of \(n\) with strain level. Their results are shown in Figure 2-6. They indicate that the stiffness of uncememented sand can be successfully normalised with regards to mean effective stress if the strain level is accounted for. However, the discussion in the previous paragraphs has shown that stiffness becomes insensitive to stress level as bonding increases. This suggests that for cases where \(n = 0\), such as hard rock, no normalisation with regards to stress level is necessary. However, it also shows that for materials such as hard soils and weak rocks, normalisation is less than straightforward.

2.2.2. Bonding

Leroueil and Vaughan (1990) noted a number of causes of geomaterial bonding. These included:
- solution and deposition of silica at particle contacts in sands,
- cold welding at interparticle contacts under high pressure,
- deposition of carbonates, hydroxides and organic matter from solution,
• re-crystallisation of minerals during weathering,
• modification of the adsorbed water layer,
• interparticle attractive forces in clayey soils.

Bonding increases stiffness by inhibiting interparticle sliding (Clayton et al. (1984)). Increased stiffness for increased levels of bonding has been shown experimentally by numerous investigators (see for example Acar and El-Tahir (1986) (Figure 2-2), Cuccovillo and Coop (1997b) (Figure 2-5) and Bressani (1990)). In addition to increasing stiffness, bonding reduces the influence of mean effective stress on stiffness (see section 2.2.1).

Evidence of bonding may be found by a number of techniques. These include the comparison of the mechanical behaviour of bonded material to the same material when reconstituted, as well as visual and chemical methods to detect the presence of bonding agents. A number of techniques for identifying bonding are discussed below.

a) Bonding is evident in soils which exhibit higher void ratios at a certain stress level, than possible for the same material when reconstituted. The bonding therefore permits states which are outside the states possible for material exhibiting no structure. Such behaviour has been observed by numerous investigators (see for example Leroueil and Vaughan (1990) and Aversa et al. (1993)). Structure-permitted space as a means to identify bonding is particularly sensitive to porosity. Aversa et al. (1993) showed examples of bonded soils that yielded on and inside the reconstituted normal compression line. They showed that materials of lower porosity tended to exhibit less structure-permitted space, regardless of the level of bonding.

b) Reduction of stiffness as destructuring occurs. When mean effective stress approaches the isotropic yield stress a reduction in stiffness may occur (see section 2.2.1). Once again this method is only suitable for high porosity
material, as low porosity masks the effect.

c) Increased geomaterial stiffness when compared to equivalent uncemented material (see for example Acar and El-Tahir (1986) (Figure 2-2), Cuccovillo and Coop (1997b)). The problem with judging geomaterial bonding by increased stiffness is the multitude of other factors that also influence geomaterial stiffness (see Table 2-2). These factors could mask the effect of bonding. This technique was used by Hight et al. (1997) to show the structured nature of very dense marine sand. Even though cementing was not obvious from inspection, the stiffness measured from seismic velocities were higher than typical laboratory stiffnesses of uncedmented dense sand.

d) Chemical agents such as carbonates and oxides indicate that the material may be bonded. However, the mere presence of a possible cementing agent does not guarantee bonding. Cementing agents has to be located at interparticle contacts to ensure geomaterial bonding.

e) Mitchell et al. (1968) proposed a method based on Rate Process Theory to demonstrate the presence of bonding. The method relates the effects of bonding and temperature to the rate at which atoms in a solid continuum flow relative to each other. This method is potentially very powerful as it allows the quantification of bonding. However to be used for soils, a number of important assumptions have to be made, not least the fact that continuum behaviour is applicable to a particulate medium such as soil. For this reason, Rate Process Theory has not found wide acceptance as a means to identify bonding.

The above discussion show that a number of methods exist to identify geomaterial bonding. It also shows that no one technique is suited to identify bonding in all types of geomaterials. This makes it difficult (and in many cases impossible) to judge whether one material is more bonded than another.
2.2.3. Fabric

The effects of fabric on geomaterial stiffness have been observed by a number of investigators. Some examples as discussed below.

Figure 2-7 shows that the stiffness of Bothkennar clay is strongly influenced by the material fabric (Clayton et al. (1992)). Triaxial tests were conducted on Laval and Sherbrooke samples which were reconstituted to their in-situ stresses. The mottled clay was stiffer than the bedded clay which in turn was stiffer than the laminated clay. This indicated that the stiffness increased as the bedding features became more disrupted and the material became more homogeneous. However (Hight et al. (1992)) pointed out that laminated samples invariably suffer more from sample disturbance than homogeneous samples. In addition, for the mottled Bothkennar samples, bioturbation may have led to additional cementation.

Well-bonded geomaterials suffer less from sample disturbance and are therefore more suitable for the investigation of fabric effects on stiffness. Clayton et al. (1994) showed that discontinuities had a marked effect on the stiffness of fractured Chalk. Locally instrumented intact samples tested in the triaxial apparatus had a much stiffer stress-strain response than the stiffnesses measured in-situ using surface wave geophysics.

Both from field (King et al. (1994)) and laboratory tests (Jamiolkowski et al. (1995)) investigators have used anisotropy of shear wave velocity to draw conclusions on the effect of fabric on stiffness. The difficulty in using this technique is the inability to separate the effects of fabric and stress anisotropy on the shear wave velocities in different directions. Jamiolkowski et al. (1995) attempted to overcome this problem by comparing shear-wave velocities in different directions for a number of intact Italian clays under isotropic stress conditions. They found \( G_{bh}/G_{vh} \) ranged between 1.4 and 1.5 and concluded that this was evidence of the influence of the soil fabric on the stiffness.
The above examples show that even though it is difficult to isolate the effect of fabric on stiffness, evidence does exist to suggest that fabric influences stiffness behaviour.

2.2.4. Ageing

Ageing increases stiffness. Two examples of mechanisms by which ageing increases stiffness are increased bonding and creep. Bonding increase stiffness by inhibiting interparticle sliding and creep reduces the void ratio over time without change in the stress state of the geomaterial. These mechanisms are discussed below.

Bjerrum (1973) recognised the fact that ageing in the form of creep had a significant influence on the compressibility of normally consolidated clays. He observed that if a normally consolidated reconstituted clay was held at constant mean effective stress (after primary consolidation), the clay exhibited a yield stress on subsequent loading which was higher than the current stress. Such behaviour is in conflict with classical consolidation theory where the yield stress of a normally consolidated clay is equal to the current stress. Bjerrum (1973) used time contours to model the effect of different creep periods on the yield stress (see Figure 2-8). Note that when the material is loaded after creep, it yields on the normal compression line of the reconstituted material. Bjerrum’s model therefore only accounts for the effect of creep and ignores any bonding that may have developed during the rest period.

Leonards and Ramiah (1959) investigated the one-dimensional compression of reconstituted clay in an oedometer. They conducted two types of tests. In one creep was allowed to occur, and in the other creep was prevented during the rest period. The results are shown in Figure 2-9. They illustrate that the yield stress increases after the rest period regardless of whether creep is prevented or allowed. They also showed that after the rest period the clay exhibited structure-permitted space. Both these factors suggests that some form of interparticle bonding developed during the rest period.
The work of Leonards and Ramiah (1959) and Bjerrum (1973) indicates that both the ageing processes of creep and increased bonding will increase the stiffness of clay. In practice however, it is often difficult to separate these two mechanisms.

The stiffness of freshly deposited clean sand increases significantly when held at constant stress over periods as short as a few days. This phenomenon has been observed both in the field (Mitchell 1986) and in the laboratory (Daramola (1978), Mitchell and Solymar (1984), Stokoe et al. (1995)). Daramola (1978) investigated the stiffness of Ham River sand in the triaxial apparatus (also see Daramola (1980)). Figure 2-10 show the results of four tests where the samples were held at constant mean effective stress of 400kPa prior to shear for periods ranging between 0 and 152 days. The results showed a significant increase in the stiffness of the sand, even during engineering time scales.

Mesri et al. (1990) have reported numerous cases where increased stiffness was observed as a result of ageing. They used equation (2-2) as a basis to quantify the increase in stiffness as a function of time:

\[ G_{0(\text{aged})} = [1 + (N_G) \log(\Delta t)] \ G_{0(\text{unaged})} \]  (2-2)

and:

\[ N_G = \frac{\Delta G}{G_p} \]  (2-3)

Where \( \Delta G \) = is the increase in shear stiffness per log cycle of time, \( G_p \) is the shear stiffness at the end of primary consolidation and \( \Delta t \) is time increment.

Mesri et al. (1990) examined data from 22 cases on a wide range of soils. The data showed that stiffness increased faster as the particle size decreased. They reported
the stiffness increase ratio per log cycle of time ($N_C$) to be:

- $N_C = 1$ to 3% for sand (6 cases)
- $N_C = 3$ to 6% for silt (5 cases)
- $N_C = 6$ to 19% for clay (11 cases)

The comparison of the stiffness increase for sand and silt is particularly interesting. For these materials, effects such as electrostatic forces, effects from absorbed water and pore water chemistry are much less important than for clay. As the number of contact points increases with decreasing particle size, the greater stiffness increase of silt compared to sand suggests that bonding at interparticle contacts is the mechanism by which the stiffness is increased.

Schmertmann (1991) brought together a considerable amount of evidence in support of the fact that ageing increases stiffness. He noted that almost all investigators have suggested that bonding is the mechanism by which the stiffness is increased. He believes that these investigators were mostly speculating, as they offered very little evidence to support this conclusion. On the basis of results from a technique to separate the cohesive and frictional components of a geomaterial undergoing shear (Schmertmann and Osterberg (1961)), he concluded that the stiffness increase was entirely due to frictional effects. The technique he used was based on the interpretation of the results from triaxial tests on two different samples at roughly the same void ratio but different mean effective stresses. Mohr-circles for the two tests were constructed at equal strain levels and the mobilised cohesion ($c''$) and mobilised friction angle ($\phi''$) determined by graphically fitting a straight line tangential to both circles. A typical example of the mobilised components for reconstituted Kaolinite is shown in Figure 2-11. He concluded that the mechanism by which geomaterial stiffness increases during ageing was entirely due to increased interparticle friction.

Schmertmann's conclusion should be regarded with caution for the following reason. Figure 2-11 shows that the technique can only be used reliably at axial
strains in excess of 0.5%. In the context of triaxial testing this is a relatively large strain. However, as discussed in Section 2.2.1 the effect of bonding is particularly important at small strains and once destructuring occurs as the strain level increases, frictional effects become more important. It therefore seems that Schmertmann (1991) came to the conclusion that bonding is of little importance as a mechanism to increase stiffness by using a technique that was unable to detect the influence of bonding before destructuring occurred.

The examples discussed in this section indicate that ageing increases stiffness, often through the mechanism of increased bonding.

2.2.5. Recent stress history

The effect of recent stress history on the stiffness of soils at very small strains ($G_{\text{max}}$), may be investigated in a resonant column apparatus. However, stress excursions applied during resonant column torsional shear are complex. They involve rotation of the principal stress directions and can not therefore be represented in triaxial stress space. If it is assumed that at very small strain levels the mode of shear does not influence stiffness measurements (see for example Shibuya et al. (1992)), the effect of recent stress history may be investigated by changing the stress path prior to vibration. Such tests were conducted by Hardin and Black (1968) on reconstituted kaolin. Figure 2-12 shows the stress paths followed for two tests, where a stress state $p' = 237\,\text{kPa}$ and $q \approx 23\,\text{kPa}$ were approached from different directions. One stress path approached the stress state in drained compression and the other in undrained extension. The void ratio was 0.86 in both cases and both samples were normally consolidated insofar as the current mean effective stress was the highest in its history. $G_{\text{max}}$ was determined as 165MPa for the drained approach path and 172MPa for the undrained approach path. This constitutes only 4% difference in stiffness which indicates that recent stress history has an insignificant effect on stiffness at very small strains in the resonant column apparatus. Hardin and Black (1968) concluded from a number of similar tests that
$G_{\text{max}}$ is strongly dependant on mean effective stress, but virtually independent of the deviatoric stress.

The stiffness of soils at small and intermediate strain levels is influenced by recent stress history (Atkinson et al. (1990), Jardine et al. (1991), Smith (1992)). Atkinson et al. (1990) investigated the effect of recent stress history by conducting four shear stages on a single sample of reconstituted London clay using external strain measurement. Before each shear stage the required stress state was approached from a different direction in triaxial stress space, resulting in an angle ($\theta$) between incoming and outgoing stress paths of $0^\circ$, $\pm 90^\circ$ and $180^\circ$. The results are shown in Figure 2-13 and indicate that the stiffness at intermediate strain levels are higher for large angles between the incoming and outgoing stress path. In fact the stiffness at $\varepsilon_s = 0.01\%$ was of the order of five times higher when a stress path reversal occurred ($\theta = 180^\circ$) compared to when no stress path reversal was applied ($\theta = 0^\circ$).

When interpreting the results from tests which involve changes in stress path direction, the effect of creep needs to be considered as it is difficult to separate strains from continuing creep under a previous load and strains induced by the current loading. Atkinson et al. (1990) allowed three hours between reaching the required stress state and commencing shear. This was twice the period required for primary consolidation to occur and in addition they state that no creep could be detected by the external volume gauge prior to shear. Some judgement on the magnitude of creep that may have occurred prior to shear can be made from the work of Bishop and Lovenbury (1969). They investigated the creep behaviour of undisturbed London clay over extended periods of more than three years. One of their conclusions was that creep occurred at all deviatoric stress levels and that no threshold was apparent below which creep did not occur. Depending on the deviatoric stress state, they found creep rates of London clay after three hours to range between 0.2 and 2%/day. Atkinson et al. (1990) did not specify the shear rate at which they conducted the tests to investigate the effect of recent stress history, but on the basis of the results of Bishop and Lovenbury (1969), it is fair to assume
that a significant proportion of the strains they measured could be attributed to creep. Because of this, they could be expected to significantly overestimate stiffness after a stress path reversal and underestimate it for no stress path reversal.

Jardine et al. (1991) recognised the possibility that creep might lead to erroneous stiffness measurements and used a restriction by which enough time was allowed until the creep rate prior to shear was less that 1% of the shear rate. They found more modest increases in stiffness for natural London clay at intermediate strain levels. Ratios of undrained compression and extension stiffness at 0.01% strain varied between 0.75 and 2.52 for a number of consolidation paths.

During the course of research work evidence became available of the effect of recent stress history on the stiffness of natural Oxford clay (Hird and Pierpoint (1997)). Their test strategy included numerous constant \( p' \) and constant \( q \) stress paths on single triaxial specimens. The effect of creep was minimised by typically allowing 2 to 3 days rest between stress excursions. Even though the data on recent stress history effects were limited, they suggested that recent stress history does not have as much effect on the stiffness of clay as found by previous workers. They tentatively proposed that recent strain history may have a more important influence on geomaterial stiffness.

2.2.6. **Stress path direction**

Lade and Duncan (1976) investigated the stress-strain response of sand by conducting triaxial tests. The test series included numerous test with varying stress path directions. They showed that when shearing the sand from close to the yield surface, a significantly softer response was observed when the stress path direction was towards the yield surface compared to when the stress path direction was away from the yield surface.

Similar results were found by Pierpoint (1996). He sheared intact Oxford clay from
the estimated in-situ stress. This stress was close to passive failure. When conducting constant $p'$ shear excursions on a single sample, he found that the stiffness response was significantly softer when the direction of the stress path was toward the yield surface (decreasing $q$) as opposed to when the stress path was away from the yield surface (increasing $q$).

The above evidence shows that when soil is sheared from close to the yield surface, the direction of the stress path has a strong influence on the stiffness response. The stiffness response is softer when the stress path direction is towards the yield surface. This behaviour is not altogether surprising. It is known that plastic strains start to dominate as the stress path approaches the yield surface (see for example Jardine et al. (1991)). This implies that for some materials, yielding is a gradual process which commences before large scale yielding is evident.

2.3. Measurement of geomaterial stiffness

Numerous methods have been developed to measure geomaterial stiffness. These methods may be classified in a number of ways. One simplistic system to classify stiffness tests is to categorise tests according to whether a test is conducted on in-situ material (in the field), or in the laboratory on material removed by sampling. This classification system conveniently groups all stiffness tests into one of two possible categories. However, it does not give any detail on the theoretical basis for the test or the method of execution. A more rational classification may be made according to the measurement principle. This basis may be used to classify stiffness measurement techniques as either direct, indirect or empirical.

Direct techniques typically consist of a test where the strain increment is measured for a particular stress increment. The ratio of the stress and strain increments constitutes the material stiffness. Examples of direct laboratory tests for measuring soil stiffness are the oedometer test (Terzaghi (1923), Casagrande (1936)), the triaxial test (Bishop and Henkel (1962)) and the hydraulic consolidation test (Rowe
and Barden (1966)). Direct measurement of soil stiffness in the field may be made by pressuremeter test (Ménard (1957)) and dilatometer tests (Marchetti (1980)).

Indirect techniques to measure soil stiffness require a theoretical basis which establishes a relationship between soil stiffness and an additional material parameter. Seismic geophysics is an example of an indirect stiffness test. It is based on the relationship between shear wave velocity \( V_S \) and shear modulus \( G_0 \) of an elastic material as shown in equation (2-4).

\[
G_0 = \rho V_S^2
\]  

(2-4)

where: \( \rho \) is the bulk density.

The shear wave velocity may be measured both in the laboratory and in the field. Examples of laboratory stiffness measurements which are based on shear wave velocity measurements include resonant column tests (Hardin and Richart (1963)) and bender element tests (Shirley and Hampton (1977)). Numerous techniques have been developed to measure shear wave velocity in the field (see for example Matthews et al. (1996)). These include techniques where shear waves are propagated either as surface waves or body waves. Examples of surface wave techniques are the continuous surface wave method and spectral analysis of surface waves. Body wave techniques include cross-hole and down-hole techniques.

Empirical techniques are based on past experience where correlations have been made between stiffness and an additional parameter measured by the specific test. In the case of in-situ penetration tests, the resistance to penetration is often correlated to stiffness. Such relationships have been established for the Standard Penetration Test (see for example Stroud (1989)) and the cone penetration test (see for example Meigh (1987)). Empirical techniques are not particularly accurate. They often rely on correlations made from other sites, and conditions may change from one site to the next. A further limitation is the fact that the empirical parameters are generally
material-specific and some knowledge of the material type is therefore required. This information may not always be available.

The following sections will discuss methods to measure geomaterial stiffness in the context of field and laboratory techniques. With regards to laboratory tests, triaxial testing and in particular recent advances in local-strain measurements are discussed. In addition, the advantages and limitations of laboratory and field tests are briefly considered.

2.3.1. Stiffness from laboratory tests

Laboratory tests requires a soil specimen to be sampled and brought to the laboratory. This has the disadvantage of possible sampling disturbance. However, sampling allows the opportunity to visually study the material. From this, the nature of the soil may be identified and physical features may be noted. A further advantage of laboratory tests is the well-defined boundary conditions applied to the specimen. The stress state and drainage conditions of the soil element are known accurately during laboratory tests. Also, control over the imposed boundary conditions are available during testing. Freedom to manipulate the boundary conditions differ from one type of test to another, but in general the sophistication of a test apparatus increases with increased boundary conditions control. A test such as the triaxial test permits a pre-determined and well defined stress path to be followed. This allows, for example, the behaviour of the material to be studied when simulating stresses imposed by construction.

Laboratory tests have some disadvantages. As mentioned above, sampling disturbance may result in a significant discrepancy between in-situ and laboratory behaviour. In addition, laboratory tests are conducted on small samples. The behaviour of a small volume of material may differ significantly from the behaviour of a large volume. This is particularly true when the material is non-uniform. Features such as fissures and joints may give rise to significant differences between
the intact and mass stiffness of a geomaterial. Furthermore, non-uniform fabric in
the form of layering is often found in natural geomaterials. Again this will lead to
differences between the intact and mass behaviour of the material.

When using laboratory tests directly to measure geomaterial stiffness, particular
attention is required when measuring strain increments. The accuracy of external
strain measurements may be reduced significantly by a number of errors. These
errors were discussed in the context of triaxial testing by Baldi et al. (1988) and
include:

- bedding errors caused by lack of fit between specimen ends and apparatus,
- seating errors from closure of gaps between apparatus components,
- apparatus compliance,
- non-uniform strains resulting from specimen restraint.

These errors can not easily be overcome for tests such as the oedometer and
hydraulic consolidation test. In contrast, the triaxial tests allows strain increments to
be measured remote from the specimen ends. This measurement technique is known
as local-strain measurement. The advantage of local-strain measurement is that it
overcomes the errors listed above. For this reason, the triaxial test combined with
local strain instrumentation provides the opportunity to measure soil stiffness more
accurately than is possible by other direct laboratory tests.

Since its introduction the triaxial test has been widely used to measure soil strength.
However it was not popular, until the late 1970s, as a test to measure geomaterial
stiffness. This development should be seen in its historical context. During the
1960s and 1970s it was found that triaxial tests gave consistently lower stiffnesses
than those measured by other means. Such evidence was shown by Marsland (1971)
and Marsland (1975) for large diameter plate tests and triaxial tests on stiff clays.
Figure 2-14 shows the results from 38mm and 98mm triaxial samples and 865mm
diameter plate tests on London clay. Further examples of the relatively low
stiffnesses measured by triaxial tests were reported by St John (1975) (Figure 2-15).
Stiffnesses back calculated from a number of full scale engineering projects in London clay were shown to be significantly higher than those measured by triaxial tests. These discrepancies between stiffnesses from field and triaxial tests led to a critical re-evaluation of the techniques used during triaxial testing.

External strain measurement, where the displacement of the loading ram is taken as equal to the sample displacement, was common practice up to the mid 1970s. Brown and Snaith (1974), Daramola (1978) and Costa-Filho (1980) were amongst the first to use local strain instrumentation, where displacement is measured directly on the specimen and remote from the ends.

An early example of local displacement instrumentation was the commercial LVDT used by Brown and Snaith (1974). These LVDTs (linear variable differential transformer) were bulky and required special care to be fixed to the specimen. For example, Brown et al. (1980) used a specially designed structure next to the specimen to support the weight of the LVDTs. Subsequently, a number of purpose made local displacement instruments have been developed. These include:

- Electrolytic level (Burland and Symes (1982), Jardine et al. (1984)).
- Hall effect gauges (Clayton and Khatrush (1986), Clayton et al. (1989)).
- Local deformation transducer (LDT) (Tatsuoka (1988), Goto et al. (1991)).

Typical characteristics of local displacement instruments as reported in the literature are shown in Table 2-4.
<table>
<thead>
<tr>
<th>Instrument</th>
<th>Range (mm)</th>
<th>Accuracy (µm)</th>
<th>Resolution (µm)</th>
<th>Calibration instrument</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Electro level gauge</td>
<td>15</td>
<td>±2</td>
<td>&lt; 1</td>
<td>LVDT with accuracy of 0.2µm</td>
<td>1, 8</td>
</tr>
<tr>
<td>Hall effect gauge</td>
<td>2.5</td>
<td>±6</td>
<td>&lt; 1</td>
<td>Micrometer with 2.54µm resolution</td>
<td>4, 5</td>
</tr>
<tr>
<td></td>
<td>7</td>
<td>±30</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LDT</td>
<td>0.2</td>
<td>0.09</td>
<td>0.12</td>
<td>Micrometer with 2.54µm resolution</td>
<td>6</td>
</tr>
<tr>
<td>Proximity transducer</td>
<td>5</td>
<td>±2</td>
<td>1</td>
<td>Slip gauges with accuracy of ±2µm</td>
<td>7</td>
</tr>
<tr>
<td>LVDT</td>
<td>5</td>
<td>-</td>
<td>1</td>
<td>Micrometer</td>
<td>2, 3</td>
</tr>
<tr>
<td></td>
<td>0.02</td>
<td>0.05</td>
<td>0.005</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

3. Cuccovillo and Coop (1997a)
5. Clayton et al. (1989)

Table 2-4. Characteristics of local displacement instrumentation.

On first inspection, the LDT reported by (Goto et al. (1991)) and the LVDT reported by (Cuccovillo (1995) and Cuccovillo and Coop (1997a)) have significantly higher accuracies than the other instruments. This observation is discussed in more detail below.

Goto et al. (1991) reported the accuracy of the LDT as $10^{-6}$ axial strain. This level of accuracy for LDTs has subsequently been reported by a number of researchers (see for example Tatsuoka and Shibuya (1992), Shibuya et al. (1992)). The strain accuracy of $10^{-6}$ was for a gauge length of 90mm, which reduces to a displacement accuracy of 0.09µm. However, Goto et al. (1991) based this accuracy on calibration against a micrometer with 1µm resolution. Claiming an accuracy better than the
accuracy (or resolution) of the calibration device constitutes poor calibration practice. In fact, a number of specialist publications on instrumentation have recommended that the accuracy of the calibration device should be 3 to 10 times better than the accuracy of the instrument under calibration (Doebelin (1990), Sydenham, Hancock and Thorn (1989), Collett and Hope (1983), Sydenham (1982)).

Cuccovillo (1995) used LVDTs as local strain instruments. She stated that overall accuracy can be determined by summing up errors from non-linearity, hysteresis and drift. In addition, she noted that at small values of the output voltage, these errors were usually negligible when compared to electrical noise. The noise level from her system corresponded to a displacement of 0.05μm and from this she concluded the accuracy to be 0.05μm. Again this is poor calibration practice because in the context of the stated accuracy, the LVDTs were not calibrated against a suitably accurate reference device.

On the basis of the above discussion, the claimed accuracy of 0.09μm for the LDT and 0.05μm for the LVDT must be treated with some caution. From Table 2-4 it may be seen that currently the most accurate local instrumentation for which the accuracy has been clearly established is ±2μm.

Careful comparison of Table 2-1 and Table 2-4 bring an important point to light. Even though the limit of linear stress-strain behaviour for geomaterials with low levels of bonding has been identified for very few cases, Table 2-1 suggests the limit to be 0.002% or less. On a typical triaxial specimen with a length of 200mm and local strains measured over the middle third, Table 2-4 shows that (disregarding the LDTs and LVDTs) the best available local strain instrumentation offers a measurement accuracy of approximately ±0.002% axial strain. This is inadequate to detect the linear stress-strain region of weakly cemented sands and clays.
It was stated earlier in this section that the discrepancy between triaxial and field stiffnesses reported by (Marsland (1971), Marsland (1975) and (St John (1975)) prompted a fundamental re-evaluation of triaxial strain measurement techniques. Jardine, Fourie et al. (1985) showed that when local instrumentation were used, good agreement was found between back calculated field and triaxial stiffnesses (see Figure 2-16). Similar evidence was shown by Jardine et al. (1991) for a number of geotechnical structures and soil conditions. This suggested that errors from external strain measurements were responsible for the lower triaxial stiffnesses and dispelled a common belief, held before the introduction of local strain measurements, that sample disturbance was the cause of the discrepancy.

As stated above, the resonant column test is an indirect test. It relies on the relationship between shear wave velocity and shear stiffness as shown in equation (2-4). This equation is based on elasticity theory and is therefore only valid if the material behaves in a linear elastic fashion. The resonant column apparatus is therefore well suited to measure stiffness in the linear stress-strain region. Stiffness measurements extended beyond the linear region suffer from a number of difficulties related to the testing method, the soil response and interpretation. Testing difficulties include the fact that when the soil behaviour is non-linear, the natural frequency depends on the amplitude and whether the natural frequency was approached by increasing or decreasing the excitation frequency. Also, corrections have to be made for the contribution to damping by the apparatus itself. Difficulties with regards to the soil response include the fact that energy is dissipated within the specimen. This must be accounted for. Furthermore, the soil behaviour will be affected by factors such as strain rate, cyclic pre-straining, stress non-uniformity and pore pressure generation. From the above it is clear that the measurement of soil stiffness in the non-linear region using the resonant column apparatus requires a number of assumptions and corrections. Invariably, data outside the linear region becomes progressively less reliable.
A typical resonant column result of stiffness vs. shear strain amplitude is shown in Figure 2-17 for Vallericca clay (Georgiannou et al. (1991)). A plateau of stiffness vs. strain is clearly observed for very small strain levels and the stiffness response becomes non-linear at larger strains. Figure 2-17 also shows the results of triaxial tests on the same material. Local strain instrumentation was used for the triaxial tests and the data is plotted down to shear strains of 0.01%, giving an overlap with the resonant column data. First inspection of the data suggest a good fit at the strain levels for which the data overlaps. However, on close examination it is clear that the data from the two tests diverge at shear strains of 0.01%. Shibuya et al. (1992) suggested that the cyclic pre-straining which occurs during resonant column testing may result in an increase of the linear stress-strain region. Even so, Figure 2-17 shows that the trend of the triaxial stiffness is to rise above that of the resonant column stiffness. This poses the question as to whether effects such as strain rate and mode of shear influences the stiffness measured by the two apparatuses. Theoretically this should not be the case for a homogeneous isotropic material exhibiting linear elastic behaviour. Furthermore, a number of investigators including Shibuya et al. (1992) and Lo Presti et al. (1993) have shown that for sands effects such as strain rate and mode of shear become less important in the linear stress-strain region. If these effects are not the cause of the divergence observed in Figure 2-17, the question remains as to how to account for the different behaviour. Clearly it would be of significant benefit if the full stress-strain behaviour, from the linear range up to failure could be measured in a single apparatus. One of the aims adopted for this research project was to develop triaxial local displacement instrumentation, accurate enough to measure the full stress-strain behaviour of natural clays with low levels of bonding.

2.3.2. Field stiffness measurements

When geomaterial stiffness is measured in the field, the material is tested in situ. The implication is that the material does not need to be removed and it is therefore subjected to little disturbance. This is particularly advantageous for materials which
are sensitive to sampling disturbance such as sands and gravels. The fact that no sampling is required does not necessarily imply that no disturbance occurs. In the case of pressuremeter tests, some disturbance will occur during preparation of the test cavity. This disturbance is in the form of stress relief and smearing of the cavity walls. For plate load tests, disturbance is less than for pressuremeter tests, as access is available to the test area. The surface may therefore be prepared by removing all disturbed material. However, disturbance in the form of stress relief will still occur. In addition, both the pressuremeter and plate load test suffer from bedding errors. In the case of plate tests, this may be eliminated by installing a probe to measure the displacement at a number of positions below the plate (Marsland and Eason (1973)). The major advantages of seismic field tests are that the material is tested at the in-situ stress levels and in an undisturbed state (except perhaps for effects of borehole installation, which should be minor).

In general, a larger volume of soil is tested by field tests than laboratory tests. Field tests may therefore be more suitable for assessing the mass behaviour of a geomaterial. Judgement on whether a test is indicative of the intact or mass behaviour can be made by comparing the volume of soil tested to the fabric features of the material. In particular spacing of joints and fissures and laminations should be taken into consideration.

Field tests have a number of disadvantages when compared to laboratory tests. The boundary conditions in terms of stresses and strains are often not known accurately. In addition, only limited control may be available over these boundary conditions. These factors complicate the interpretation of the test data. Drainage conditions may not be known during field tests. For direct tests such as plate load tests and pressuremeter tests, some assumption has to be made with regards to the drainage condition. The rate at which the test is conducted in conjunction to the permeability is often used to judge whether a test is drained or undrained. In reality partial drainage will occur.
The stress and strain fields are often not uniform during field tests. Again this complicates the rational interpretation of the data and additional assumption may be required to account for this difficulty. Such an assumption further reduces the confidence with which stiffness measurements can be made when using field techniques.

Field tests are limited with regard to the loading condition and mode of deformation that may be imposed to the soil. For example, most plate load test are conducted by applying a vertical stress increments, whereas, pressuremeter tests apply an axisymmetrical horizontal load. These loading conditions may be different to the loads imposed by structures.

Seismic tests do allow some freedom with regards to orientation of the measured stiffness by varying the direction of polarisation of the propagated waves. This may be used to assess the anisotropy of the material. Seismic methods have some limitations however. The stress level at which tests are conducted cannot be varied significantly. In addition no control is available over the strain level. In general it is assumed that the measured stiffness is the in-situ stiffness at very small strain ($G_0$). It is widely believed that the stiffness required for geotechnical design (at strain levels between 0.01% and 1%, see Figure 2-1) will be much lower (perhaps by an order of magnitude) than $G_0$ measured by seismic techniques (see for example Hardin and Drnevich (1972)).

**2.3.3. Comparison of field and laboratory stiffnesses**

The discussion in the previous sections has shown that various field and laboratory techniques exist to measure geomaterial stiffness. It was also shown that all tests have certain limitations. Nevertheless, valuable insights into the behaviour of geomaterials may be gained by comparing field and laboratory stiffnesses. When such comparisons are made a number of factors have to be taken into consideration.
These factors include:

a) geomaterial stiffness is strongly dependant on strain level (see Figure 2-1).

b) differences in disturbance, stress level, strain level, shear rate, mode of shear and drainage conditions need to be accounted for.

c) field and laboratory tests which measures soil stiffness directly by measuring the strain increment for a particular stress increment suffer from varying degrees of bedding errors.

d) effects from discontinuities must be accounted for. In particular, the spacing of discontinuities relative to the volume of material tested will dictate whether the mass or intact behaviour of a material is being measured by a specific technique.

In recent years, the comparison of field seismic stiffness with laboratory stiffnesses has become more frequent. This stems from the fact that seismic techniques measure the soil in an undisturbed state and at the in-situ stress level. Seismic techniques therefore give a benchmark against which laboratory stiffnesses may be evaluated.

In theory, stiffness measured by the resonant column apparatus may be compared directly to the seismic stiffness measured in the field. Both tests are indirect tests which are based on the measurement of shear wave velocity. In addition both techniques include only very small strains. Comparison of field seismic stiffness to resonant column stiffness have now been reported by many investigators (Burghignoli et al. (1991), Tatsuoka and Shibuya (1992), Stokoe et al. (1995), Jacobs and Butcher (1996), Hight et al. (1997)). Agreement between seismic and resonant column stiffnesses have been varied. Burghignoli et al. (1991) found good agreement between resonant column and seismic stiffnesses for Fucino clay. They attributed this to the lack of macrostructural features and the fact that the clay had high levels of bonding (CaCO₃ varied between 10% and 90%).

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Agreement between resonant column and seismic techniques have been less successful for other materials. A number of reasons have been suggested:

a) In general it has been observed that stiffness comparisons for clays are better than that for sands (see for example Shibuya et al. (1996) and Hight et al. (1997)). Sample disturbance was suggested by the authors as the reason for this observation. In contrast to clays, sands are more susceptible to sample disturbance on account of their inability to sustain pore fluid suctions.

b) Solid resonant column specimens give better comparisons than hollow cylinder specimens. Preparation of hollow cylinder specimens require more time, effort and skill than solid cylinder samples. This introduces higher levels of mechanical disturbance as well as disturbance from moisture changes (Hight et al. (1997)).

c) Stokoe et al. (1995) noted sampling bias as the reason for poor correlation between cross-hole and resonant column results on weakly bonded layered sand. Successful sampling and trimming was only possible from layers with relatively high levels of cementation. This resulted in higher measured stiffness in the laboratory than in the field.

Comparisons of field seismic stiffness with triaxial stiffness suffer from all the problems discussed above for resonant column tests. However, some additional factors have to be considered in the case of triaxial stiffness. These include differences in strain levels, as well as differences in mode of shear and rate of shear. A number of cases have been reported which compare stiffnesses from the two techniques (see for example Powell and Butcher (1991), Tatsuoka and Shibuya (1992), Clayton et al. (1994), Miyazaki et al. (1994)).

In general, good agreement has been found for weak rocks with no or few fabric features (Tatsuoka and Shibuya (1992), Shibuya et al. (1992)). The agreement was particularly good for an artificial cement treated sandy soil and became
progressively worse for mudrocks with more pronounced fabric. Clayton et al. (1994) compared stiffnesses measured by surface wave techniques with stiffnesses from unconfined compression tests for natural Chalks. The Chalks were from three different sites and were highly fractured in all cases. The intact dry densities varied significantly between sites. The mass stiffness from surface wave measurements were broadly similar for all three sites. In contrast however, the intact laboratory stiffnesses varied according to the intact dry density with increased stiffness for increased density. $E_{\text{max}}$ measured in the laboratory were up to 25 times higher than $E_0$ measured in the field. Clayton et al. (1994) concluded that the relatively low stiffness of the discontinuities dominated the behaviour of the Chalk. The differences between laboratory and field seismic stiffness reduced as the intact dry density reduced.

Comparison of field seismic stiffness with triaxial stiffness for sand has been made by Tokimatsu and Oh-hara (1990) (as reported by Tatsuoka and Shibuya (1992)). They found that triaxial stiffnesses were higher than seismic stiffness for loose sand and lower for dense sand. Stiffnesses measured by the two methods differed by up to a factor of 10. In addition they found close agreement between triaxial and seismic stiffnesses when the sand was frozen in situ before sampling. On the basis of these results they concluded that the discrepancy between triaxial and seismic stiffnesses was entirely due to sampling disturbance.

Differences between field seismic stiffnesses and triaxial stiffnesses for clay have been investigated by Powell and Butcher (1991), Smith (1992) and Clayton et al. (1994). A number of clays ranging from stiff heavily overconsolidated clay to soft recent clay deposits have been investigated. The laboratory stiffnesses were generally lower than the seismic stiffnesses. All triaxial strains were measured locally, but the linear plateau was not detected in any of the tests. This made it difficult to judge whether the discrepancy between seismic and triaxial stiffness was due to differences in strain level or due to another cause such as sampling disturbance. For London clay, Clayton et al. (1994) found $G_0$ (from seismic cross-
hole) to be approximately twice that of $E_{0.01}$ measured in the triaxial apparatus and approximately 8 times more than $E_{0.1}$.

In summary it may be said that significant differences have been found between the field seismic and laboratory small strain stiffnesses of geomaterials. This does not necessarily indicate that either technique is in error. Discrepancies may typically be attributed to differences in strain level, differences in mass and intact behaviour or sampling disturbance. In fact, comparison of laboratory stiffnesses at very small strains with field seismic stiffness might be very useful. Uses include:

a) judgement on the degree of sampling disturbance (Tatsuoka and Shibuya (1992), Hight (1993)),

b) evaluation of the effect of fabric by comparing mass and intact behaviour (Tatsuoka and Shibuya (1992), Clayton et al. (1994)),

c) assessment of normalisation techniques (Clayton et al. (1994)),

d) in cases where triaxial tests can not detect the linear stress-strain region, seismic stiffness is useful to give and upper bound which may be used during extrapolation of triaxial stiffness to small strain levels (Clayton et al. (1994)).

From the evidence in the previous paragraphs it might be argued that if the effects of sampling disturbance and fabric were small, the stiffness at very small strains as measured in the field and laboratory might be expected to be the same.

2.4. Modelling of soil behaviour

The behaviour of soils are complex. For this reason a conceptual framework is required within which the experimentally observed behaviour of soils can be interpreted. Mathematical soil models provide such a framework. In addition, models which describe the behaviour of soils are essential during geotechnical design in order to predict behaviour.

A range of soil models have been developed to date. The models which have found
favour amongst geotechnical researchers and practitioner are models based on elasticity and plasticity theory.

2.4.1. Soil models based on elasticity theory

Classical continuum mechanics provides a number of models based on elasticity theory. Under certain conditions these models may be used to describe soil behaviour.

The simplest relationship between stress and strain is based on elasticity theory with a linear relationship between stress and strain:

\[ \{\sigma\} = |C| \{\varepsilon\} \]  \hspace{1cm} (2-5)

where: \{\sigma\} and \{\varepsilon\} are stress and strain vectors and \(|C|\) is the stiffness matrix.

In general, there are 6 independent stress and 6 independent strain components. \(|C|\) is therefore a 6x6 matrix. This implies that for the most general case the stiffness matrix will have 36 independent terms. However the matrix will be symmetrical (see for example Love 1927) and therefore the number of independent terms reduces to 21. A model requiring 21 parameters is not practical. Indeed for most geotechnical problems a model of such generality is unnecessary. For most soils an additional assumption of cross-anisotropy may be made. For cross-anisotropic behaviour, the stress-strain and shear stiffness is isotropic within a plane. This plane is normally the horizontal plane. The elastic coefficients are different for stresses and strains outside this plane. For a cross-anisotropic model, the number of independent parameters reduces to 5. If the soil is furthermore assumed to be isotropic, the stiffness response is independent of direction and only 2 parameters are required to describe the behaviour of the continuum. These two parameter are either the Young's modulus \((E)\) and the Poisson's ratio \((\nu)\) or the bulk modulus \((K)\) and shear modulus \((G)\). For soil mechanics problems it is often more convenient to
use bulk and shear modulus as it readily allows volumetric and shear strains to be separated.

The bulk modulus and shear modulus of an isotropic linear-elastic material can be determined in the triaxial apparatus. This may conveniently be done by conducting a test along a constant p' stress path to determine the shear stiffness. In addition a test along a constant q stress path is required to determine the bulk stiffness.

The 5 parameters required for a cross-anisotropic linear elastic model cannot all be determined in the triaxial apparatus. This makes the model unattractive for use in geotechnical practice. Graham and Houlsby (1983) developed a 3 parameter model which takes anisotropy into consideration by means of a parameter which accounts for the coupling between shear and volumetric strains.

Linear elastic models are attractive on the basis of conceptual and theoretical simplicity. However such models are only suitable for use under limited conditions. In particular, such models are only suitable to be used during monotonic loading at small strains. A footing on very stiff material is one example.

It was shown in Section 2.1 that some yielding occurs well inside the state boundary surface (Leroueil and Vaughan (1990), Jardine (1992), Hight and Higgins (1995)). The result is a non-linear stress-strain response of soil at intermediate strain levels. The simplest model which accounts for non-linear stress-strain behaviour is the non-linear elastic model. Duncan and Chang (1970) adopted a hyperbolic stress-strain curve to describe the stiffness response of soil. The initial slope of the curve is taken to be a function of the mean effective stress by using a relationship similar to equation (2-1). Furthermore, the asymptotic value is taken as equal to the deviator stress at failure. In its most elementary form the model therefore requires 4 parameters namely; A (constant), n (exponent), c' (effective cohesion), φ' (effective angle of friction).
Duncan and Chang (1970) used a non-linear elastic model to simulate a footing on sand. Comparison of the calculated settlements with test data showed good agreement for settlements up to 10% of the footing width. Gunn (1993) demonstrated the marked improvement when calculating tunnel settlement profiles using a non-linear elastic model compared to a linear-elastic model. Non-linear elastic models may be used at higher strain levels than linear-elastic models. However non-linear models are not well suited to cases where unload-reload excursions occur.

Soil behaviour is characterised by the development of elastic and plastic strains during stress excursions. Such behaviour can not be described by theories based on elasticity. However, plasticity theory allow this behaviour to be modelled.

2.4.2. Soil models based on plasticity theory

The minimum requirement for a plasticity model is a yield criterion and a flow rule. Such models are known as perfectly-plastic models. Hight and Higgins (1995) found that for a footing on soft clay an elastic perfectly-plastic model gave good agreement between predicted and measured settlements. They concluded that the importance of stress-strain non-linearity did not dominate this type of problem.

More advanced plasticity models which allows the size of the yield surface to change during yielding require a hardening law in addition to the yield surface and flow rule. Early plasticity models which were specifically developed to model soil behaviour include the Cam Clay Model (Schofield and Wroth (1968)) and the Modified Cam Clay Model (Roscoe and Burland (1968)). These models assume elastic behaviour for all stress excursions inside the yield surface. Plastic strains only occur when the yield surface is engaged.

Jardine et al. (1986) developed a non-linear elasto-plastic model where the stress-strain behaviour inside the yield surface is non-linear. The non-linear response of
the soil is quantified by fitting a periodic logarithmic function to data from triaxial tests. In particular, it allows for high stiffnesses at small strains. Jardine et al. (1991) reported results from numerical analysis with this model for six geotechnical case histories. The yield criterion used included a Mohr-Coulomb and a Modified Cam Clay yield criterion. The six projects which were evaluated consisted of a deep excavation, a cut and cover tunnel, tension piles, a gravity platform, a bored tunnel and a pipe jacked tunnel. The ground conditions included London clay, Glacial till and sands. In general good agreement was found between the predicted and measured movements. The average ratio between prediction and measurement was 1.16.

Models such as the Cam Clay Model and the Modified Cam Clay Model assume isotropic hardening. This implies that during work hardening, the yield surface expands uniformly about the hydrostatic axis. Experimental evidence of a number of materials which exhibit plastic behaviour (including metals) has shown that the yield surface does not expand uniformly during hardening. This behaviour is shown schematically in Figure 2-18. When the material is loaded in compression along ABC past its yield stress at B, work hardening accompanied by plastic strains occurs. If the material is now unloaded to E, it will move along slope CE. Reloading in compression from E will cause the material to move along ECD where it will yield at C. However, if the material was loaded in extension from E it would move along the curve EF and will therefore yield before it reaches point G (EGH is a mirror image of ABD). From point E, the yield stress in tension therefore is less than the yield stress in compression. This behaviour is known as the Bauschinger effect. From Figure 2-18 it may be seen that the yield surface does not expand isotropically during work hardening along BC.

The Bauschinger effect may be modelled by permitting the yield surface to translate instead of a yield surface expanding uniformly (Mróz (1967)). A number of frameworks have been developed to take account of non-linear behaviour of geomaterials and the effect of stress reversals (see for example Al-Tabbaa and
Wood (1989), Simpson (1992), Stallebrass and Taylor (1997)).

Stallebrass and Taylor (1997) compared the results from a three surface kinematic hardening model to a centrifuge test of a circular footing. The various stages of the test included consolidation, swelling, pore pressure equalisation as well as one unload-reload loop during loading. The predicted settlement curve showed similar characteristics to the measured curve. In particular, the hysteretic nature of the unload-reload loop was similar in both cases. In addition, good agreement was found between predicted and measured settlement profiles. However, the magnitude of the predicted and measured settlements were significantly different. The predicted settlement was less than the measured settlement by a factor of almost three.

The above discussion shows that no single model can fully describe the behaviour of soil. Hence, for a given geotechnical structure and ground conditions it is necessary to recognise the modelling aspects that are important. The chosen model should therefore be a balance between complexity of the model and accuracy of the prediction.

5. Conclusions from literature review

The following is a summary of the most important conclusions made in this chapter.

a) In the triaxial apparatus, the linear stress-strain response of sands and clays with low levels of bonding have only been identified successfully in very few cases. From the available evidence it is estimated that the linear stress-strain range for clays and sands are smaller than 0.002% axial strain (Table 2-1). Examination of local instrumentation characteristics indicate that these strain limits are outside the capabilities of most current state of the art local strain instrumentation (Table 2-4).

b) Geomaterials exhibit non-linear stress-strain at intermediate strain levels.
c) Mean effective stress has a strong influence on stiffness for weakly bonded soils, but this influence is reduced as levels of bonding increase.

d) Ageing increases stiffness. The increase in stiffness is largely a result of interparticle bonding which develops during ageing.

e) It is generally believed that recent stress history has an influence on the stiffness of soils at small and intermediate strains.

f) When soil is sheared from close to the yield surface, the direction of the stress path has a strong influence on the stiffness response (Lade and Duncan (1976), Pierpoint (1997)). The stiffness response is softer when the stress path direction is towards the yield surface.

g) The major advantage of field techniques are the fact that no sampling is required. This reduces possible sampling disturbance and indeed allows testing of materials such as uncedmented sands and gravelly soils that can not be sampled. However it was argued that field tests suffer from limited control of boundary conditions including drainage condition, stress path, strain level and strain rate.

h) The major advantages of laboratory testing are the well-defined boundary conditions. In addition, control may be available for some of the boundary conditions.

i) It has been shown that comparison of field and laboratory stiffnesses may give valuable insights into the behaviour of geomaterials. A number of factors were identified which need to be addressed when comparing laboratory and field stiffnesses. These included differences in disturbance, stress and strain level, mode of shear, rate of shear, drainage condition and differences in the volume of material tested.

j) No single model can fully describe the behaviour of soil. In practice it is important to balance the complexity of the model with the required accuracy of the prediction.
On the basis of the above discussion, a number of shortcomings in scientific knowledge on geomaterial stiffness behaviour can be identified:

a) Dynamic techniques have given valuable insights into the behaviour of geomaterials at small strains. However, the question remains whether static stiffness will be similar. In particular the linear stress-strain range has not been observed widely for these materials in triaxial loading. This may be a result of the fact that current state of the art local strain instrumentation is not able to measure accurately to sufficiently small strain levels.

b) Stiffness degradation of weakly bonded geomaterials has not yet been fully established during triaxial shear over the entire range from linear behaviour to failure. As the operational stiffnesses at the strain levels found in practice are believed to be considerably lower than the seismic stiffnesses, the rate of stiffness degradation is important when employing seismic techniques to obtain stiffnesses for design.

c) The effect of factors such as recent stress history and stress path direction on geomaterial stiffness is not yet fully understood.

d) No unified method of quantifying geomaterial bonding exists. This implies that in many cases it is difficult, or even impossible, to judge whether one material is more bonded than another.

From these shortcomings in scientific knowledge, the aims for this thesis were identified as follows:

a) To develop triaxial local instrumentation capable of measuring the stress-strain behaviour of a range of geomaterials, from inside the linear stress-strain region, up to failure.

b) To study the stiffness degradation of geomaterials from the linear plateau up to failure.

c) To investigate the effect of current stress path direction on the stiffness of geomaterials at small and intermediate strains.
d) To investigate the effect of recent stress history on the stiffness of geomaterials at small and intermediate strains.

e) To compare triaxial stiffness in the linear stress-strain region with seismic field stiffness for a range of geomaterials.
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3. DEVELOPMENT OF LOCAL INSTRUMENTATION

A major goal of this research project was the development of high accuracy triaxial local displacement transducers, capable of identifying the linear stress-strain behaviour of a number of natural geomaterials ranging from soft clays to weak rock. It was foreseen that local displacements would have to be measured accurately at levels smaller than 0.1μm. This was accomplished by developing a Fabry-Perot interferometer for use both as a local displacement instrument, and as an accurate reference system to calibrate commercial LVDTs. Calibrating the LVDTs against the interferometer allowed the LVDTs to be used at significantly higher levels of accuracy than those specified by the manufacturer. In addition, the interferometer was used as a stand alone local strain device. The interferometer therefore played a pivotal role in achieving accurate local displacement measurements. The development and application of the interferometer will be discussed in this chapter. The theoretical background required to understand its operation is summarised in Sections 3.1 and 3.2. The remainder of the chapter focuses on the applications and calibration of the interferometer.

3.1. Optical principles

3.1.1. Light as electromagnetic waves

Light is part of the electromagnetic spectrum. The electromagnetic spectrum ranges from wavelengths of $10^6\text{m}$ for radio waves to $10^{-14}\text{m}$ for gamma rays. The visible range of the spectrum has wavelengths between approximately 0.4μm and 0.8μm. Electromagnetic waves consists of an electric field ($E$) and a magnetic field ($B$) that are perpendicular to one another and to the direction of propagation as shown in Figure 3-1. The oscillating behaviour may be expressed in terms of either the electric or magnetic field, which is a function of both time and displacement:
\[ E(x,t) = E_0 \cos(\omega t - \theta) \]  \hspace{1cm} (3-1)

Where \( \theta = \frac{2\pi x}{\lambda} \), and \( (\omega t - \theta) \) is known as the phase. It is often convenient to write wave equations in complex format as the exponent correspond to the phase of the wave. Equation (3-1) is equal to the real part of the complex equation:

\[ E(x,t) = E_0 e^{-i(\omega t - \theta)} \]  \hspace{1cm} (3-2)

The intensity of light \( (I) \) is defined as the average rate at which the electromagnetic waves carry energy across a unit area. It may be shown from electromagnetic theory (see for example Wolfson and Pasachoff (1995)), that:

\[ I \propto E_0^2 \]  \hspace{1cm} (3-3)

This is analogous to the work rate per unit cross sectional area of mechanical waves travelling in a three dimensional medium, where the average power of the waves is directly proportional to the square of the wave amplitude. If a wave of arbitrary amplitude is considered (as will be the case in the following sections), equation (3-3) may be rewritten as:

\[ I = E_0^2 \]  \hspace{1cm} (3-4)

The complex conjugate of any complex number \( z = re^{i\theta} \) is \( z^* = re^{-i\theta} \). This may be used to calculate the modulus of a complex number as \( |z| = \sqrt{zz^*} = \sqrt{r^2} = r \). Accordingly, if a light wave is expressed in terms of a complex number the intensity \( (I) \) is calculated as:

\[ I = EE^* \]  \hspace{1cm} (3-5)
3.1.2. Lasers

Natural light is a combination of electromagnetic waves of various wavelengths and phases. A laser (light amplification by stimulated emission of radiation) produces light of high temporal and spatial coherence. The following description of the operation of lasers is based on a number of references (Alonso and Finn (1969), Burns and Macdonald (1970), Wolfson and Pasachoff, (1995)).

A photon can interact with an atom in a number of ways. Figure (3-2) shows three types of interaction that are particularly important for laser operation. Firstly, an atom in the ground energy state \((e_0)\) may absorb an incident photon (Figure 3-2(a)) thereby raising the energy of the atom to an excited state \((e_1)\). This is called stimulated absorption. The difference in energy levels before and after absorption will be equal to the energy of the photon, which is a function of its frequency \((\nu)\) and Planck's constant \((h)\):

\[
e_1 - e_0 = \nu h
\]  

(3-6)

Secondly, an atom in an excited state \((e_1)\) may spontaneously fall back to the ground state \((e_0)\) and in the process emit a photon in a random direction (Figure 3-2(b)). Alternatively, the atom may cascade to the ground state by falling to one or a number of intermediate states and emitting a photon each time it falls to a lower state. This fall back is known as spontaneous emission. The frequencies (and wavelengths) of the emitted photons may be calculated from equation (3-6).

Thirdly, an atom in an excited state may be stimulated to fall back to the ground state by an incident photon (stimulated emission). This will only occur if the incident photon has the same energy as the energy of the excited atom. The photon emitted upon falling back to the ground state will have the same energy, direction, phase and polarisation as the incident photon. In this case amplification of the light occurs, as the condition starts with one photon (and an excited atom) and ends with two identical photons.
A simplified illustration of laser energy levels is shown in Figure 3-3. Under normal circumstances, a substance will have most of its atoms in the lowest possible energy state known as the ground state \((e_0)\). Subsequently stimulated emission will seldom occur. This condition may be changed by “pumping”, where energy from an external source is used to raise the majority of the atoms to a number of excited levels collectively called \(e_2\). Different sources can be used for pumping, including electric discharge, flash lamps, sunlight, and chemical reactions. The atoms of most elements will stay in the excited state for only a very short period of time, before they spontaneously cascade through a number of energy levels back to the ground level. They remain at each of these levels for approximately \(10^{-8}\) seconds. However, some energy levels are special because atoms remain at these levels for much longer (approximately \(10^{-3}\) sec), before they decay further. Such an energy level is called a meta-stable level and is fundamental to the operation of lasers. As the average lifetime at the meta-stable level \((e_2)\) is of the order of a hundred thousand times longer than the lifetimes at other levels, atoms begin to collect at this level and soon a large number of the atoms may be found at \(e_2\) instead of at the ground level. This condition is known as population inversion. The first atoms at level \(e_2\) that spontaneously fall back to level \(e_1\) will emit a photon of energy \(e_2-e_1\) in the process. This photon is called the seed photon as it then induces other atoms at level \(e_2\) to decay to level \(e_1\) through stimulated emission. In the process each atom will emit an identical photon. Level \(e_1\) is know as the lower lasing level. Once at the lower lasing level, atoms will quickly fall back to the ground state and if pumping is still occurring, the atoms are ready to repeat the cycle.

Stimulated emission of the radiating medium is significantly enhanced by placing the medium between two parallel mirrors of which one reflects (nearly) all the photons that impinge on it (reflectance > 99.9%) and the other mirror reflects approximately 99% and transmits 1%. Therefore, the intensity of light emitted by a laser is only a fraction of the intensity of the light between the mirrors. The mirrors reflect the stimulated photons backwards and forwards, thereby stimulating more
excited atoms at level \( e_2 \) to emit photons of similar wavelength and direction. In the case of a gas lasing medium the gas is contained in a glass tube with the mirrors at the ends. The mirrors are placed a whole number of wavelengths apart resulting in a standing wave with high temporal and spatial coherence inside the laser cavity.

Apart from absorbing a photon (Figure 3-2), an atom may be raised to an excited state by collision with another atom or with a free electron. These mechanisms are important in the helium neon (HeNe) gas laser. The lasing medium consists of 85% helium and 15% neon contained in a laser cavity at low pressure. An electric current is passed through the gas medium exciting the helium atoms by collision with free electrons and raising them to a meta-stable level (Figure 3-4). This level is close to the meta-stable level of neon and collisions between neon atoms at the ground state and excited helium atoms transfers energy to the neon atoms, raising them to the meta-stable level at 20.66eV. These neon atoms drop to a lower level at 18.70eV during stimulated emission and from equation (3-6) it may be calculated that the wavelength of the emitted light is 0.6328\( \mu \)m. The atoms eventually drop back to the ground level. A high number of neon atoms are maintained at the meta-stable level by continuous collisional pumping with excited helium atoms.

HeNe lasers have a number of advantages including high stability, high power and low cost. In addition, they produce light in the visible spectrum. These attributes make them popular light sources for laboratory applications.

3.1.3. Interference of two light beams

Young (1802) performed the classic interference experiment to demonstrate the wave nature of light. He passed coherent light through two narrow slits to produce two wave fronts as shown in Figure 3-5. When these two wave fronts were projected onto a screen, bright and dark fringes were visible indicating constructive and destructive interference. Constructive interference occurred when two wave crests coincided resulting in a bright fringe. Bright fringes were observed at
positions on the screen where the difference in path lengths of the two waves were a multiple of the wavelength of the light. Consequently, the fringe spacing was only a function of the light wavelength and experiment geometry (slit spacing and screen distance). It follows therefore, that for a known experiment geometry, the wavelength may be determined and vice versa.

3.1.4. Refraction of light

The velocity (c) of light in a vacuum is constant. It is approximately $2.8 \times 10^8$ m/s and becomes slower in a denser medium. This phenomenon is used to define the index of refraction ($n$) of a medium in terms of the velocity of light in the medium ($v$):

$$n = \frac{c}{v}$$  \hspace{1cm} (3-7)

In addition, when a light wave passes between two mediums with different refractive indices (see Figure 3-6) the direction of propagation changes. According to Snell’s law:

$$\frac{\sin \phi_1}{\sin \phi_2} = \frac{n_2}{n_1}$$  \hspace{1cm} (3-8)

The refractive index of air is close to unity and shows slight sensitivity to changes in pressure and temperature. Eden (1966) showed that the refractive index of air changes by one part in a million for a change in temperature of 1°C and a change in pressure of 0.4kPa.
3.1.5. Division of light amplitude

Sir George Stokes (1883) investigated the amplitude of reflected and refracted light rays. He used the principle of ray reciprocity to derived relationships between the reflected and transmitted amplitudes of light incident on a refracting surface.

Consider two media in contact with refractive indices $n_1$ and $n_2$ ($n_1 < n_2$) (Figure 3-7). An incident light ray with amplitude ($E_i$) is propagated in medium $n_1$ and impinges on the interface. According to Snell's law a portion of the light is reflected back into medium $n_1$ and the rest of the light is transmitted into medium $n_2$ (assuming no absorption). The reflection and transmission coefficients are defined in terms of the amplitude of the reflected light ($E_r$) and amplitude of the transmitted light ($E_t$) as:

$$r = \frac{E_r}{E_i}$$ (3-9)

$$t = \frac{E_t}{E_i}$$ (3-10)

(The reflection and transmission coefficients $r'$ and $t'$, for a light ray refracted into medium $n_1$ are defined in a similar way). For the case shown in Figure 3-7(a) the amplitude of the reflected light is $rE_i$ and that of the transmitted component is $tE_i$. Figure 3-7(b) shows the reverse of the situation in (a) where rays $rE_i$ and $tE_i$ are incident on the surface. According to the principle of ray reciprocity, these two (virtual) rays will combine to produce $E_i$. In reality, these two rays will produce two reflected waves and two refracted waves (Figure 3-7(c)). The cases shown in Figure 3-7(b) and (c) are equivalent and therefore:

$$E_i = r^2E_i + t'tE_i$$ (3-11)
and

\[ 0 = r'tE_i + trE_i \]  \quad (3-12)

These equations may be rewritten as:

\[ tt' = 1 - r^2 \]  \quad (3-13)

and

\[ r = -r' \]  \quad (3-14)

Equations (3-13) and (3-14) are known as the Stokes relations.

Fresnel (1832) showed that when a light wave is reflected from either side of a refracting surface, the amplitude of the reflected wave will be the same in both cases. However, a \( \pi \) phase change will occur if the light is incident from the side of higher velocity (lower index), whereas no phase change will occur if the light is incident from the side of lower velocity. This is in accordance with Equation (3-14).

The reflectivity \( (R) \) of a surface is the ratio of the incident and reflected \emph{intensities} and may be calculated from Equation (3-4) and (3-9) as \( R = r^2 \). In a similar manner, the transmissivity \( (T) \) is defined as the ratio of transmitted to incident intensity. Assuming that no absorption takes place at the surface and that the principle of the conservation of energy holds:

\[ T + R = 1 \]  \quad (3-15)

\subsection*{3.1.6. Interference of multiple light beams}

In addition to the interference of two light beams, as discovered by Young, interference of multiple beams may occur. The theory of multiple beam interference is discussed in many physics and optics publications (see for example Born and Wolf (1964), and Pedrotti and Pedrotti (1993)).
Figure 3-8 shows a monochromatic light, incident on an optical plate with parallel sides, both covered by a refractive coating. Each time the light wave impinges on one of the refractive surfaces, a portion of the light will be reflected and the rest will be transmitted (assuming no absorption). Multiple reflections and transmissions of parallel waves will occur. These waves will interfere at an infinite distance from the plates. Alternatively, the waves may be projected through a lens in which case interference will take place at the focal point as shown in Figure 3-8. The amplitude of each wave may be calculated by observing the number of reflections and transmissions that have occurred. The phase of each of the waves depends on the path length travelled and the refractive index of the medium. It is important to note that the first reflected wave is the only wave that will undergo a $\pi$ phase change on reflection as it is the only wave that is reflected when incident from a medium of lower refractive index. The amplitude and phase of the first four reflected waves are shown below:

$$E_1 = (rE_0)e^{i(\alpha \gamma)}$$  \hspace{1cm} (3-16)

$$E_2 = (tt'r' E_0)e^{i(\alpha \gamma - \delta)}$$  \hspace{1cm} (3-17)

$$E_3 = (tt'r^{13} E_0)e^{i(\alpha \gamma - 2\delta)}$$  \hspace{1cm} (3-18)

$$E_4 = (tt'r^{15} E_0)e^{i(\alpha \gamma - 3\delta)}$$  \hspace{1cm} (3-19)

where $\delta$ is the phase difference between successive waves:

$$\delta = \left(\frac{2\pi}{\lambda}\right)(2n_h \cos \theta)$$  \hspace{1cm} (3-20)
By inspection of equations (3-17) to (3-19), the Nth reflection (for \( N > 1 \)) may be written as:

\[
E_N = (tt' r^{(2N-3)} E_0) e^{i\alpha - (N-1)\delta}
\]  

(3-21)

When all the reflected waves (including \( N = 1 \)) are superimposed, the resultant \( E_R \) is:

\[
E_R = rE_0 e^{i\alpha} + \sum_{N=2}^{\infty} tt' r^{(2N-3)} E_0 e^{i\alpha - (N-1)\delta}
\]

(3-22)

Which may be rewritten as:

\[
E_R = E_0 e^{i\alpha} \left[ r + tt' r' e^{-i\delta} \sum_{N=2}^{\infty} r^{(2N-4)} e^{-i(N-2)\delta} \right]
\]

(3-23)

This is a geometric series of the form:

\[
\sum_{N=2}^{\infty} x^{N-2} = 1 + x + x^2 + \ldots \ldots
\]

(3-24)

where:

\[
x = r^{12} e^{-i\delta}
\]

(3-25)

Since \(|x| < 1\), the series converges to the sum \( S = 1/(1-x) \). Therefore:

\[
E_R = E_0 e^{i\alpha} \left[ r + \frac{tt' r' e^{-i\delta}}{1 - r^{12} e^{-i\delta}} \right]
\]

(3-26)

Using Stokes relationships (equation (3-13) and (3-14)):
\[ E_R = E_0 e^{i \alpha} \left( r - \frac{(1 - r^2) re^{-i \delta}}{1 - r^2 e^{-i \delta}} \right) \]  

(3-27)

Which simplifies to:

\[ E_R = E_0 e^{i \alpha} \frac{r(1 - e^{-i \delta})}{1 - r^2 e^{-i \delta}} \]  

(3-28)

The intensity of the reflected beam \( I_R \) may be calculated from equations (3-5) and (3-28) as:

\[ I_R = |E_R|^2 = E_R E_R^* = E_0^2 r^2 \left[ \frac{e^{i \alpha}(1 - e^{-i \delta})}{1 - r^2 e^{-i \delta}} \right] \left[ \frac{e^{-i \alpha}(1 - e^{i \delta})}{1 - r^2 e^{i \delta}} \right] \]  

(3-29)

Using the identity:

\[ 2 \cos \delta = (e^{i \delta} + e^{-i \delta}) \]  

(3-30)

And the proportionality:

\[ \frac{I_R}{I_i} = \frac{|E_R|^2}{|E_0|^2} \]  

(3-31)

Equation (3-29) reduces to:

\[ \frac{I_R}{I_i} = \frac{2r^2(1 - \cos \delta)}{1 + r^4 - 2r^2 \cos \delta} \]  

(3-32)

But from \( R = r^2 \), equation (3-32) becomes:
\[ \frac{I_R}{I_i} = \frac{2R(1 - \cos \delta)}{1 + R^2 - 2R \cos \delta} \] (3-33)

It is interesting to note that at \( \delta = 0 \) the intensity of the reflected light is zero. This arises from the fact that the amplitude of the first reflected wave equals the sum of the amplitudes of all the subsequent waves, but the first reflected wave is \( \pi \) out of phase with all the subsequent waves, resulting in perfect cancellation.

Using the identity:

\[ \cos \delta = 1 - 2 \sin^2 \left( \frac{\delta}{2} \right) \] (3-34)

Equation (3-33) may be rewritten as:

\[ \frac{I_R}{I_i} = \frac{F \sin^2 \frac{\delta}{2}}{1 + F \sin^2 \frac{\delta}{2}} \] (3-35)

where:

\[ F = \frac{4R}{(1 - R)^2} \] (3-36)

\( F \) is known as the coefficient of finesse. From Equations (3-15) and (3-35) the ratio of transmitted to incident intensities are:

\[ \frac{I_T}{I_i} = \frac{1}{1 + F \sin^2 \left( \frac{\delta}{2} \right)} \] (3-37)
which is known as the Airy function. Note that at $\delta = 0$ the intensity of the transmitted light equals the intensity of the incident light.

3.2. The Fabry-Perot interferometer

Fabry and Perot (1897) developed an interferometer based on multiple reflections of light between two parallel optical plates. Figure 3-9 shows a typical configuration with the inner surfaces of the plates reflecting part of the light and transmitting the rest. In order to eliminate reflections from the outer surfaces of the plates, the outer surfaces should be covered with an anti-reflecting coating. The phase difference between successive beams may be calculated from equation (3-20) as:

$$\delta = \left(\frac{2\pi}{\lambda_{air}}\right)\left(2n_{air}h\cos\theta\right)$$  \hspace{1cm} (3-38)

The Fabry-Perot configuration differs from the configuration discussed in Section 3.1.6, on account of the air cavity. This has a number of important implications. Firstly, the index of refraction of air is close to unity ($n_{air} = 1.00029$) and $n = 1$ may be used without inducing a significant error. This simplifies equation (3-38). Secondly, for an air cavity, no phase shift occurs when the incident beam is reflected by the first surface. However a $\pi$ phase shift occur each time a beam is reflected inside the cavity as it is incident from a medium of lower refractive index.

Accordingly, each beam that is transmitted through the cavity after multiple reflections, will have been reflected $2m$ times (where $m$ is an integer), giving a total phase shift of $2m\pi$. Since the transmitted intensity function has a period of $2\pi$, it is not influenced by a $2m\pi$ phase shift for individual beams and consequently equation (3-37) is valid for a Fabry-Perot interferometer.

For the special case of light incident normal to the partial mirrors, it may be shown from equation (3-37) that the intensity of the transmitted light is a function of only
the mirror reflectance ($R$) and the phase difference $\delta$, where:

$$\delta = 4\pi \left( \frac{h}{\lambda} \right)$$  \hspace{1cm} (3-39)

Figure 3-10 show the normalised transmitted intensity (Airy function) for Fabry-Perot interferometers with different levels of partial mirror reflectance ($R$). Regardless of the mirror reflectance the peak to peak change in phase difference is $2\pi$. Figure 3-10 also show that the sharpness of the fringes is a function of the mirror reflectance only. For low mirror reflectance the Airy function approximates a sine wave, but at high mirror reflectance the peaks of the function are sharp. This characteristic of Fabry-Perot interferometers is quantified in terms of the finesse ($\mathcal{F}$) which is defined as the full width at half maximum intensity (FWHM) as a ratio of the fringe spacing. FWHM is the width between two points on either side of a maximum where the intensity has decreased to half of the maximum value. The finesse ($\mathcal{F}$) may be calculated as:

$$\mathcal{F} = \frac{\pi \sqrt{R}}{(1 - R)}$$  \hspace{1cm} (3-40)

The finesse ($\mathcal{F}$) should not to be confused with the coefficient of finesse ($F$). In fact:

$$\mathcal{F} = \frac{\pi \sqrt{F}}{2}$$  \hspace{1cm} (3-41)

Both high and low finesse interferometers have displacement measurement applications as will be seen in the following section.
3.3. Measurement techniques

Displacement measurements using a Fabry-Perot interferometer have to be tailored according to the magnitude of the displacement. Different methods are employed for large or small displacements. Recalling that the wavelength of visible light is between approximately 0.4 and 0.8 μm, a measurement of a few millimetres is regarded as a large displacement and displacements less than the light wavelength are considered as small. Large displacements can be measured accurately by the method of fringe counting. From equation (3-39), the change in phase difference ($\Delta \delta$) is related to the change in cavity length ($\Delta h$) as:

$$\Delta \delta = 4\pi \left( \frac{\Delta h}{\lambda} \right)$$  \hspace{1cm} (3-42)

Therefore, a $2\pi$ change in phase difference corresponds to a change in cavity length of $\lambda/2$.

Low finesse interferometers are particularly suited for the method of fringe counting where the number of peaks (or troughs) are counted during relative displacement of the two mirrors. This can be done by monitoring the analogue output signal and manually counting the number of fringes or by using electronic circuitry to convert the number of fringes directly to displacement.

Fringe counting has some difficulties. When used in conjunction with light from a gas laser where the incident light has the form of a sinusoidal wave, this technique can not determine the direction of relative displacement at point of zero sensitivity. This may result in measurement errors. For this reason, the user has to ensure that the relative displacement is always in one direction. Alternatively, some secondary displacement instrument can be used to indicate any displacement reversals. Another elegant way of overcoming this problem is to use a laser diode and to
modulate the intensity of the incident wave to some asymmetrical wave form such as a saw tooth wave. For an asymmetric incident wave, the output function will no longer be in the form of the Airy function, but will be asymmetric about an axis at \( \delta = 0 \). The output will therefore be different for decreasing and increasing relative displacements. Unfortunately the wavelength of laser diodes are more sensitive to changes in temperature compared gas lasers. In addition sophisticated electronics are required to modulate and demodulate the input and output signals.

When displacements of less than \( \lambda/2 \) are to be measured, the Fabry-Perot interferometer is suited to two measurement techniques in particular. The first is a zeroing technique, where the position of the mirrors are controlled by some mechanical means and the second is based on interpolation of the output function.

The zeroing technique requires one of the mirrors to have a translational degree of freedom parallel to the axis of the interferometer. Piezo electric actuators may be used for this purpose. The mirror is positioned to produce a maximum output \((\delta = 0)\), before inducing the displacement to be measured. The phase difference is then returned to zero by moving the mirror and observing the actuator displacement. The accuracy of the technique depends amongst other factors on the accuracy with which the zero phase shift can be identified and therefore high finesse interferometers are best suited. Essentially the interferometer is used only as a means to locate a reference position and the technique is limited by the performance of the actuator as well as the accuracy to which it can be calibrated. The calibration of piezo-actuators are difficult as their behaviour is typically non-linear and they exhibit hysteresis and creep. In addition noise in the control circuitry is converted to mechanical noise of the actuator. These problems may be addressed by using highly sophisticated actuators and electronic hardware, with the inevitable result of higher equipment cost. Even when these problems are adequately addressed, one requirement which still remains is the fact that the relative displacement of the two mirrors should be constant before and after the measurement, in order to locate the zero phase difference. This technique is therefore not particularly suitable for the
case where the interferometer is used directly as a local displacement instrument on
a triaxial sample, as the sample may creep after application of a load increment.

A second technique for measuring displacements smaller than \( \lambda/2 \) is based on the
interpolation of the output function. Figure 3-10 shows that between two peaks, a
change in transmitted intensity may be related to a change in cavity length. This
change in cavity length may be calculated using equations (3-37) and (3-42).
However, note from Figure 3-10 that the output function is highly non-linear. This
demands some care when using the technique of interpolation. The output function
has zero sensitivity at \( \delta = 0, \pi, 2\pi \) etc. and high sensitivity on the steep part of the
function. The position (and slope) of the most sensitive part of the function depends
on the interferometer finesse. For a finesse of 1, the position of highest sensitivity
will be at \( \delta = \pm0.5\pi \) about each peak and will be progressively closer to the peak as
the finesse increases. In order to start a measurement on the most sensitive part of
the output function, it is important to have some mechanical means by which to
manipulate the phase difference prior to displacement and a piezo-actuator is well
suited for this purpose.

Non-linearity and hysteresis of the piezo-actuator does not adversely affect the
interpolation technique, but the effect of creep and random noise needs to be
addressed. The most pragmatic approach to deal with the creep is to allow sufficient
time between positioning the actuator and commencing with the measurement in
order for the creep to decrease to an acceptable low level. In addition, random
mechanical noise may be minimised by using high quality control circuitry,
specially designed to produce a low noise input signal to the actuator. Other effects
which may further introduce errors during interpolation include fluctuation of laser
output power, noise of the electronic detection circuitry and changes in the
refractive index of the cavity medium. All these effects should be quantified during
calibration of the system.
Depending on the measurement technique employed, the importance of some interferometer characteristics change. Table 3-1 shows the importance of these factors for the different measurement techniques. Each factor must be judged in the context of a typical measurement range, with zeroing and interpolation normally used for displacements less that one wavelength and fringe counting for displacements of many wavelengths.

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<td>Piezo-actuator required</td>
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<td>Piezo-actuator calibration</td>
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<tr>
<td>Piezo-actuator creep</td>
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<tr>
<td>Piezo-actuator noise</td>
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<tr>
<td>Photo diode noise</td>
<td>•</td>
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<td>Change in cavity refraction index</td>
<td>•••</td>
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*••• very important, •• important, • relatively unimportant, _ unimportant*

Table 3-1. Comparison of different measurement techniques.

Both fringe counting and interpolation techniques were used during this work. Fringe counting was used to calibrate LVDTs (as described in Section 3.5) as well as for large local displacement measurements on a triaxial sample. In addition, the interpolation technique was used to measure very small local displacements on the same triaxial sample. This will be described in the following section.
3.4. The interferometer as a local displacement instrument

A pair of Fabry-Perot interferometers were developed to conduct local displacement measurements on triaxial specimens at ultra small strains. The following sections describe the instrument configuration and discuss the test procedure and results.

3.4.1. Description of Fabry-Perot local instrument

The layout of the interferometer is shown schematically in Figure 3-11. An 8mW Helium-Neon (HeNe) laser with a wavelength (\(\lambda\)) of 0.6328\(\mu\)m was used as a light source. The light was launched into a single mode optical fibre (core diameter 4\(\mu\)m, numerical aperture 0.12) via a laser-fibre pigtail assembly. The coupling efficiency of the pigtail assembly was of the order of 55%. A fibre coupler was used to split light from the laser from one fibre into two single mode fibres which carried the light to each of the two interferometers. As the light exited each fibre it was passed through a cylindrical graded index (GRIN) lens with a length of 4.3mm and diameter of 1.8mm, which collimated the light to a divergence angle of less than 0.3°. The collimated light subsequently impinged on a partial mirror with a diameter of 12.5mm and thickness of 1mm. The mirror was machined to a surface flatness of 1\(\mu\)m and a parallelism of better than 5 arc minutes. One side of the partial mirror had a dielectric coating which reflected 60% of the light (\(R = 0.60\)) whilst the other side had an anti-reflectance coating. The typical absorption of the dielectric coating was specified by the manufacturer as 0.5%. The light transmitted through the first mirror was projected onto another partial mirror with similar geometric and optical properties. This second partial mirror was fixed to a silicon photo diode with an active area of 15mm².

The interferometer is shown in Figure 3-12 and Figure 3-13. Both partial mirrors were fixed to optical stages, each with rotational degrees of freedom about two perpendicular axes at right angles to the axis of the interferometer. These stages were used to position the mirrors parallel to each other to create the Fabry-Perot
cavity. The orientation of the top stage was adjusted manually and therefore this could only be done when the cell top was removed. In contrast, the bottom actuator was adjusted by two miniature DC motors allowing the orientation of the bottom stage to be manipulated after closure of the cell top. This was necessary in order to realign the two mirrors should small movements occur whilst applying pressure to the triaxial cell. The top partial mirror was attached to the optical stage via an uni-axial piezoelectric actuator. The actuator had a range of 5μm and a resolution of 0.007μm. It was controlled by a purpose designed piezoelectric controller with low drift and low noise. The actuator provided a translational degree of freedom along the axis of the interferometer and allowed the phase difference to be controlled at any time during the test. This was particularly valuable in order to specify the phase difference at the start of the test. The outputs from the two photo diodes were monitored on an analogue oscilloscope as well as being captured in digital format after the output signal was amplified and converted by a dual slope analogue to digital converter.

3.4.2. Test procedure

As a result of time constraints, only one test was conducted with the interferometers as local displacement transducers. The material tested was high porosity Chalk (\(n = 0.47\)) from Needham in East Anglia. Of the three materials tested during this research project (London clay, Bothkennar clay and Chalk), the Chalk was chosen as it exhibited less creep than the two clays. Experience using LVDTs as local displacement transducers indicated that for either clay a rest period of months would be necessary before the creep rate would subside to acceptably small levels.

The 100mm diameter Chalk specimen (CH15) was cut from a block sample and shaped in a soil lathe. The ends were prepared by placing the specimen in a special cradle (see chapter 5) and the ends trimmed using a precision metal blade.
The test was conducted in two phases, a saturation phase and a shear phase. During the saturation phase the triaxial cell medium was water, whilst during the shear phase the cell medium was air. Air was used during shear, as some components of the interferometers were non-submersible. However, on safety grounds, the maximum cell pressure when using air was 200kPa.

The use of air as the cell medium required some attention to be paid to the specimen membrane. Latex rubber membranes are pervious to air and are therefore unsuitable for use in isolation (see for example Head (1986)). After some experimentation, the specimen was protected by three separate layers consisting of cling film, aluminium foil and a latex rubber membrane. One layer of cling film was placed against the Chalk and one layer of aluminium foil between the cling film and latex membrane. After placement of the latex membrane, it was sprayed with numerous layers of aerosol silicone sealant. Two LVDTs were fitted to the specimen prior to saturation.

The sample had an initial degree of saturation of 0.93. Full saturation could be achieved by using an elevated back pressure and according to Black and Lee (1973) a minimum back pressure of 350kPa would be required. This could readily be achieved when using water as the cell fluid, as the triaxial cell had a maximum working pressure of 1700kPa. It was recognised that even if all the air was driven into saturation at a high back pressure during the saturation phase, it would come out of saturation at a lower cell pressure during the shear phase. The problem was overcome by saturating the sample at 850kPa for 24 hours before flushing the sample with de-aired water over a period of several days by applying a small pressure differential of 15 kPa between the specimen top and bottom. A total of 285cm³ was flushed through the sample. Full saturation was confirmed by measuring constant B-values at increasingly higher back pressures.

Prior to the shear phase, the two interferometers were fitted in addition to two LVDTs (see Figure 3-14). Each pair was diametrically opposed and named according to the four compass directions as FP(N), FP(S), L₁(W), L₂(E), with south
facing towards the front of the compression machine. All brackets were glued to the specimen by silicone sealant and two pins per bracket used to provide further support. The gauge lengths were measured individually as the pin to pin distance between the top and bottom bracket and ranged between 71.0mm and 73.7mm for the four gauges.

The cell was pressurised by a compressor with a working capacity of 700kPa and a diaphragm type air regulator was used to control the cell pressure. The bottom of the cell was filled with water to cover the plugs in the cell base and the air pressure applied via one of the vent ports at the top of the cell. The test was conducted under undrained conditions at a cell pressure of 200kPa and a back pressure of 100kPa.

3.4.3. Test results

Before commencing to shear the specimen, the phase difference of each of the interferometers was positioned near the highly sensitive part of the output curve by means of the piezo-actuators. The resolution of the two interferometers, corresponding to one bit output, were 0.00057μm and 0.00083μm respectively. Sufficient time was allowed for the creep to subside and immediately before shear the creep rates were 0.010μm/min and -0.004μm/min for the two interferometers. This was approximately 2% of the locally measured shear rate at the start of the test. Figure 3-15 shows the output of the two interferometers and Figure 3-16 the displacements calculated from the change in phase difference. Displacements up to approximately 85 seconds (Figure 3-16) were calculated by interpolation of the output signal and subsequent displacements were determined by identifying the peaks and troughs for each interferometer. Figure 3-16 shows the smooth lift-off of the load at approximately 45 seconds and the immediate response of FP(S) with displacement at FP(N) delayed for approximately 15 seconds. The same behaviour was exhibited by the two LVDTs as shown in Figure 3-17. Even though the signal to noise ratios of the LVDTs were low at the start of the test, L2(E) clearly
responded soon after the start of load application, whereas L1(W) only responded approximately 15 seconds later.

FP(S), which responded first, showed an initial softer response compared to FP(N) but as shown in Figure 3-16 both were symmetrical about the load curve. This behaviour may be explained by one of two possible effects, namely discontinuity effects or specimen bending (local yielding was considered to be unlikely at these low stress levels). Of these two possibilities, it is suggested that discontinuity effects is the most likely mechanism that caused the observed behaviour. This view is supported by a number of factors. Firstly, the response from FP(S), which initially had a soft response, became markedly stiffer at higher strain levels (Figure 3-18 and Figure 3-19). This is consistent with closure of a discontinuity and contrary to the expected behaviour during bending. Secondly, the displacements of all four gauges at intermediate and high deviator stress, are not consistent with bending. FP(N) and L2(E) show approximately equal displacements, but displacements at FP(S) and L1(W) diverge throughout the test. Thirdly, the position of the contact point between the flat top cap and load ram was measured after the test as approximately 1mm from the top cap centre. On a specimen of 100mm diameter, an eccentricity of 1mm is unlikely to produce bending.

3.4.4. Interpolation technique

The shape of the output functions from the interferometers, when used as local displacement instruments for CH15 (Figure 3-15), was different to the expected shape of the Airy function for identical mirrors of 60% reflectivity (Figure 3-10). It is evident from Figure 3-15 that the output from the local instruments are of lower finesse than the expected function shown in Figure 3-10. This difference is explored below.

Recall the configuration of the interferometer, when used as a local displacement instrument (Figure 3-12). Light was transmitted through the cavity formed by two
partial mirrors \((R = 0.6)\) and then impinged onto a silicon photo diode glued onto the second mirror. A silicon interface with air has a reflectivity of 30%. If the surface of the silicon was to be parallel to the second mirror, a Fabry-Perot cavity would be formed between the two surfaces. This would result in two cavities in series. Some light would be reflected from the second cavity back into the first cavity. It was calculated from the beam diameter exiting the GRIN lens and the distance between the second mirror and the silicon surface, that for angles less than 10° between the two surfaces, a cavity would be formed. The angle was measured for both interferometers and found to be less than 0.5°. Both interferometers therefore consisted of two cavities in series.

Two cavities in series may be modelled as one cavity with the reflectivity of the second mirror equal to the ratio of the light reflected by the second cavity. This ratio will according to equation (3-35) be a function of the phase difference of the second cavity. If it is assumed that this phase difference remains constant, the effect of having a second cavity reduces to the equivalent of one cavity with two mirrors of unequal reflectivities. Equations have been developed to study the behaviour of such cavities (see for example Lee et al. 1992)). The typical response of a cavity with unequal mirror reflectivities are compared to a cavity with identical mirrors in Figure 3-20. It shows that the positions of peaks and troughs are not influenced by unequal reflectivities. In both cases peaks are at phase differences of \(2m\pi\) and troughs at \((2m+1)\pi\) where \(m\) is an integer. Also, the output function for unequal reflectivities remains symmetrical about \((\delta = 0)\). From Figure 3-20 and equation (3-37) it may be seen that for a cavity formed by identical mirrors, the peak transmitted intensity equals the incident intensity. Therefore normalised peak transmitted intensities equals unity. In contrast, cavities of unequal reflectivities produce normalised peak transmitted intensities less than unity. Furthermore, Figure 3-20 shows that when a cavity include one mirror of low reflectivity the finesse of the output function is significantly reduced.
Once the output from a cavity of unequal mirror reflectivities have been normalised with respect to maximum output, the Airy function (equation (3.37)) may again be used as the function to convert displacement to phase change. As an example, it may be shown that a cavity consisting of two mirrors of which $R_1 = 0.6$ and $R_2 = 0.2$ are approximated by the Airy function with an equivalent reflectivity ($R_\infty$) of 0.3464. The maximum error between the normalised output function and the Airy function was found to be less than $1 \times 10^{-4}$ for all values of $\delta$. When using this technique, some means is required to determine the equivalent reflectivity. One such technique is described below.

The shape of the Airy function for identical mirrors can be described uniquely in terms of one parameter. This parameter is the Finesse ($\mathcal{F}$). If the transmitted intensity is known at any value of $\delta$, the finesse may be calculated from equation (3.37). This implies that the function can be described uniquely by either the finesse or the normalised intensity at some value ($\delta = \pi$ may conveniently be used as it corresponds to a minimum value). The Finesse describes the shape of the Airy function and the normalised intensity at $\delta = \pi$ relates the position of the function relative to the origin.

For mirrors of unequal reflectivities two parameters are required to uniquely describe the shape and position of the output function. However, when using interpolation of the output function as the technique to measure displacement, only the shape of the output function is required. This is sufficient to establish a relationship between displacement and phase difference. The phase difference is therefore independent of the position of the function relative to the origin. The shape of the output function for unequal mirror reflectivities may be described in terms of the full width at mid intensity (FWMI). This is the width of the output function at an intensity midway between the minimum and maximum intensities. For a cavity formed by identical mirrors, a unique relationship exists between the finesse ($\mathcal{F}$) and the FWMI. This relationship is shown in Figure 3-21.
The output function shape of the two interferometers used as local gauges for CH15 were quantified in terms of the FWMI. As the displacement rate was not constant, each peak and trough were used as a reference point of known phase difference to convert the output function from the time to the phase domain. The width at mid intensity for the first 3 fringes (normalised by $2\pi$) was calculated as 0.4443 for FP(N) and 0.4373 for FP(S). This was used in conjunction with Figure 3-21 to determine the equivalent finesse as 0.416 for FP(N) and 0.482 for FP(S). The Airy function and equivalent finesse was used to interpolate values for the first 85 seconds of CH15.

5. Calibration of local LVDTs using the interferometer

In addition to using the interferometer as a local displacement instrument, it was used to calibrate commercial LVDTs. The LVDTs were much easier to use than the interferometers, and in addition functioned in water under high pressure. For these reasons, all triaxial tests with the exception of (CH15) were carried out using the LVDTs as local displacement instruments.

Each LVDT was calibrated over two ranges. The large range calibration was of the order of a few millimetres and the small range calibration was typically over $\pm 20\,\mu m$ to $\pm 50\,\mu m$. The large range calibration was carried out using a micrometer and will be discussed in the next chapter in the context of triaxial instrument calibration. The small range calibration of the LVDTs will be discussed below.

The aim of the small range calibration of the LVDTs was to maximise its accuracy. The output sensitivity of a measurement system may be improved by increasing the level of amplification. The level to which the signal can be amplified is limited by the high frequency electronic noise as both the signal and the noise is amplified. In addition, it is important to recognise that increasing the output sensitivity does not necessarily improve the accuracy of the system, and that judgement on the accuracy can only be made once the system is calibrated against a suitably accurate
calibration device. The accuracy of the reference system should be 3 to 10 times better than that of the instrument being calibrated (Doebelin (1990), Sydenham, Hancock and Thorn (1989), Collett and Hope (1983), Sydenham (1982)). Given the high accuracy of the interferometer it was ideally suited as a reference instrument for the calibration of the LVDTs.

The calibration arrangement is shown schematically in Figure 3-22 with both the LVDT and interferometer mounted on a linear stage. Fringe counting was used as the technique to calculate displacements during LVDT calibration. The optical components including the laser, partial mirrors, single mode fibre, GRIN lens, directional stage etc. were identical to those described in Section 3.4. Amplification of the demodulated LVDT output signal was maximised whilst keeping the amplification of the high frequency noise within acceptable levels. The micrometer handle was rotated by a geared motor at a rate of one revolution per hour. This was necessary given the relatively slow A/D converter. Hand rotation of the micrometer handle would have been satisfactory if a high speed converter was used. The digital outputs from the LVDT and photo detector were each logged at a rate of 11 readings per second, giving on average 27 data point between peaks. This corresponds to a resolution of 0.0117μm for the calibration device. Figure 3-23 shows a typical plot of the error between the measured (LVDT) displacements and the true (interferometer) displacements using a linear regression to convert the digital output to engineering units. The accuracy was calculated as ±2 times the standard deviation of the errors. Assuming a Gaussian distribution, this gave a confidence level of 95% that the difference between a measured value and the true value would be within the specified accuracy. For the LVDT shown in Figure 3-23 the accuracy was determined as ±0.0266μm. The calibration range was 60μm and the accuracy as a ratio of the range may be calculated as 0.044%.
3.6. Interferometer calibration

The interferometer was the key to accurate local strain measurements made during this project, both for use directly as a local gauge, and as a calibration device. It was therefore important to assess the accuracy of the interferometer itself. However, to measure the accuracy of the interferometer, it would have had to be calibrated against an instrument with a higher level of accuracy. The only instruments that are more accurate than a standard interferometer are interferometers for which particular measures have been taken to ensure very high accuracy. It was argued though, that even if the Fabry-Perot interferometer was calibrated against a more accurate interferometer, it was unlikely to be calibrated in the exact set-up used during this project. For these reasons, another course of action was taken in an attempt to estimate the accuracy of the interferometer. Firstly the performance of the interferometer was judged against its theoretical expected performance and secondly the external factors that could contribute to measurement inaccuracies were critically assessed.

The requirements for accurate measurements using a Fabry-Perot interferometer are different for different measurement techniques. For this reason the accuracy of the interferometer will be discussed separately in the context of fringe counting and interpolation techniques.

3.6.1. Accuracy of fringe counting technique

The Airy function shown in Figure 3-10 illustrates that the relative displacement required to change the phase difference from one peak to the next is only a function of the light wavelength. Therefore, the accuracy of fringe counting is dependent on the wavelength stability and the accuracy with which the light wavelength is known. A further factor is the level of random noise of the output signal which limits the exactness with which the peak can be identified.
The light wavelength produced by a HeNe laser should, according to quantum mechanics principles, be 0.6328μm (see Section 3.1.2). In order to confirm the proper operation of the laser used during this project and quantify any possible effects from the fibre and GRIN lens, the wavelength of the light exiting the GRIN lens was measured by an optical spectrum analyser with an accuracy of ±0.0001μm. The result is shown in Figure 3-24. The peak amplitude of the spectrum was at 0.6328μm and the full-width-at-half-maximum was 0.0014μm, affirming the near monochromatic light production of the laser at the expected wavelength.

Wavelength fluctuations of gas lasers occur if changes in temperature, barometric pressure or relative humidity take place. Collett and Hope (1983) reported that for Helium-Neon gas lasers, a change of one part per million of the wavelength will occur for a temperature change of 1°C, a pressure change of 0.3kPa or a change in relative humidity of 30%. When the interferometer was used to calibrate the LVDTs, the procedure took a few minutes only and changes in temperature, pressure or relative humidity were within the limits which would cause a change of wavelength of one part per million. For LVDT calibration therefore, it is estimated that the interferometer accuracy was better than the resolution of 0.0117μm (see Section 3.5).

External effects from the experimental set-up may introduce measurement errors that are not directly related to the interferometer. One example is tilt. Figure 3-22 shows the arrangement during LVDT calibration, with both the interferometer and LVDT fixed to the linear stage. An offset of 40mm existed between the axis of the LVDT and the axis of the interferometer and measurement inaccuracies would thus have been introduced if tilt rotation of the linear bearing of the stage occurred. Precision linear bearings were used as the sliding contact, but nonetheless this effect had to be evaluated.

Qualitative judgement could be made on the occurrence of tilt during the
measurements by examining the deterioration of the output signal. The operation of
the interferometer relied on the mirrors being parallel to create the Fabry-Perot
cavity and experience showed that the power of the output signal deteriorated
rapidly when small misalignments occurred. Typically, for a cavity length of about
10mm, the output signal would be completely lost at a relative angle of 1 to 2
degrees between the mirrors. During calibration, the power of the output signal at
the peaks typically deteriorated by less than 1% over the duration of the test. This
indicated that very little relative rotation of the mirrors took place.

Fringe counting was used to determine displacements larger than 0.25μm during test
CH15. Possible tilt rotation was assessed using the same qualitative method
discussed in the previous paragraphs. FP(S) showed no deterioration in the peak
output up to 1500 seconds (ΔL = 10μm), suggesting that no (or very little) relative
rotation of the mirrors occurred. Thereafter, up to the end of the test (ΔL = 23μm), a
10% reduction in the power output at peaks was observed, indicating a relative
mirror rotation of a few arc minutes. Similar behaviour was observed for FP(N)
with no reduction up to 1500 seconds and a 20% loss from 1500 seconds up to the
end of the test.

The index of refraction of air changes as changes in temperature and pressure
occurs. According to equation (3-38), this will introduce a change in phase
difference which is not related to mirror movement. Eden (1966) showed that the
refractive index of air changes by one part per million for a temperature change of
1°C and a pressure change of 0.4kPa. During test CH15, a maximum cell pressure
fluctuation of 1.1kPa was measured over the entire duration of the test. This would
have introduced a change of refractive index of 2.75 parts per million. For a cavity
length of 14mm and using equation (3-38) the apparent change in phase difference
may be calculated as 0.76 rad. This corresponds to an apparent change in cavity
length of 0.0385μm. This is an acceptable small error compared to the maximum
displacements of 23μm and 5μm measured by FP(N) and FP(S) respectively.
3.6.2. Accuracy of interpolation technique

Figure 3-10 shows the Airy function. Based on optical and mathematical principles, this is the expected output from a Fabry-Perot interferometer. Some judgement on the performance of a Fabry-Perot interferometer can be made by comparing the measured output to the expected output. This was done by mounting two identical partial mirrors onto an invar plate to form a cavity. Invar was chosen for its low thermal expansion coefficient (1 x 10⁻⁶°C) to minimise temperature effects. One mirror was kept stationary whilst the other was displaced using a piezo-actuator with a resolution of 0.004μm. This piezo-actuator was not of the same type used as part of the local instrumentation. The actuator had an open centre section which allowed the transmitted light from the interferometer to pass through the actuator. The intensity of the transmitted light could therefore be measured by an optical power meter. Figure 3-25 shows the measured output intensity of the interferometer as well as the Airy function for \( R = 0.6 \). A good correlation is observed indicating that the interferometer performed as expected.

As explained in section 3.4.4 the interferometers used as local displacement instruments consisted of two cavities in series. Interpolation of the output function relied on the accuracy with which the shape of the output function could be modelled mathematically. It was shown that if the equivalent finesse is known, the normalised output from two cavities in series could be approximated very closely with the Airy function. The accuracy of the technique therefore relied on the determination of the equivalent finesse by means of the full width at mid intensity (FWMI). The FWMI were calculated for the first three full fringes as 0.4443 ±0.00825 for FP(N) and 0.4373 ±0.00733 for FP(S). The range shown was taken as the maximum deviation from the mean value. From this, the maximum error of the output function slope could be calculated. The slope error varied between 12% at \( \delta = \pi \) and 17% at \( \delta = 2\pi \) for FP(N) and between 9% at \( \delta = \pi \) and 14% at \( \delta = 2\pi \) for FP(S). This was used to estimate the maximum measurement error up to the first peak as 0.011μm for FP(N) and 0.0091μm for FP(S).
The piezo-actuator used as one of the components of the local displacement instrument performed an important function by allowing phase difference manipulation prior to shear. Piezo-actuators have the tendency to drift even at constant voltage input. As it would be difficult to discriminate between actuator drift and specimen strains due to shear or creep, it was necessary to investigate the drift characteristics of the actuators. This was done by fixing the actuator and two mirrors to the invar plate, in a configuration similar to that used during test CH15. Temperature change was measured during the tests and in no case did it exceed 0.2°C. Each test was conducted over a period of approximately 15 minutes by positioning the output phase at the most sensitive part of the curve and calculating the displacement from the change in phase difference. A number of tests were conducted and a typical result is shown in Figure 3-26. Three different types of noise can be identified from Figure 3-26. These are:

a) High frequency noise which can be observed as a random scatter of data points.
b) Low frequency noise evident as a slow change in relative displacement over the entire test period.
c) Drift which occurred randomly over periods of 1 to 2 minutes, with stable periods in between. See for example the output in Figure 3-26 between 4.5 and 5.5 minutes.

High frequency noise (type (a)) which occurred as a random scatter of data points, was attributed to the inherent noise of the electronic components of the system. The magnitude of the noise should be judged in terms of the signal to noise ratio. The signal (in bits) may be defined as the difference between the output at a maximum and the output at a minimum. The noise was taken as the difference between a reading and the average of the previous five readings. For the test shown in Figure 3-26, 95% of the noise was within the range ±6 bits. From this, the signal to noise ratio was calculated as 46. This ratio is a sufficiently high ratio to indicate that noise should not pose a problem during measurements. The high frequency noise may also be evaluated in terms of apparent displacement. Figure 3-26 shows the
displacement as a function of time. The high frequency noise is evident throughout the test as a random scatter of data points. Again, the noise was taken as the difference between a displacement data point and the average of the previous five displacement readings. From this, 95% of the noise was calculated to be within the range ±0.0025μm.

Low frequency noise (type (b)) was evident in most of the tests as a slow change in relative displacement over the entire test period. The rate of this drift was typically of the order of 0.001μm/min, which was less than the thermal expansion of the invar for a temperature change of 0.2°C. This made it impossible to identify whether the source of the drift was the invar plate or the actuator. Nevertheless, the rate of drift was low relative to the locally measured shear rate during triaxial testing and therefore did not pose a problem.

The third type of noise (type (c)) occurred randomly over periods of 1 to 2 minutes, with stable periods in between. This noise was attributed to the piezo-actuator. In Figure 3-26 this type of drift is evident between 4.5 and 5.5 minutes at a rate of 0.013μm/min. This drift presented the most severe problem to measurement accuracy on account of the relatively high rate and random occurrence. It was observed in a number of tests, with a rate of 0.01μm/min being typical. The maximum rate detected was 0.02μm/min. For the test performed on Chalk (CH15), the locally measured shear rates at the start of the test were 0.42μm/min for FP(S) and 0.27μm/min for FP(N). This represented on average a ratio between possible actuator drift and locally measured shear rate of 3%. In the context of the very small strains measured, this was deemed acceptable.

Two further factors that could adversely affect the accuracy of the interpolation technique were the change in cavity refractive index and the output power stability of the laser. During the first 90 seconds of test CH(15) no change of cell pressure was detected and it was thought that any change in laboratory temperature over this
time period would have been considerably less than 1°C, which would have introduced a change of refraction index of less than one part per million.

Fluctuation of the laser output power would induce random noise of the light intensity transmitted through the cavity. The maximum fluctuation of the laser output power was specified as ±2.5% by the manufacturer. At the most sensitive part of the output function this is equivalent to an apparent change in phase difference of 0.0562 rad (equation (3-37)), which corresponds to an apparent change in cavity length of ±0.0028μm (equation(3-42)).

The above assessment of the measurement accuracy of the interpolation technique, showed that two effects in particular could have a significant influence on accuracy. These are errors from quantifying the output function shape and random actuator noise (type (c)). The maximum error on account of the shape of the output function was estimated as approximately 0.01μm. This should be compared to the maximum error as a result of random actuator noise of 0.013μm/min. This rate was shown to be 3% of the local shear rate measured during test CH15.
Figure 3-1. Light as an electromagnetic wave.
Figure 3-2. Interaction between photons and atoms.
Figure 3-3. Energy levels for lasers.
Figure 3-4. Energy levels for HeNe laser.
Figure 3-5. Interference of two wave fronts.
Figure 3-6. Refraction of light waves.
Figure 3-7. Figure to derive Stokes relations.
Figure 3-8. Multiple beam interference.
Figure 3-9. Reflections between two parallel plates.
Figure 3-10. Airy Function.
Figure 3-11. Schematic view of the Fabry-Perot interferometer.
Figure 3-12. Fabry-Perot local displacement instrument.
Figure 3-13. Fabry-Perot local displacement instrument.
Figure 3-14. Local instrumentation for test CH15.
Figure 3-15. Interferometer output at start of test CH15.
Figure 3-16. Interferometer displacements at start of test CH15.
Figure 3-17. Local displacements at the start of test CH15.
Figure 3-18. Local displacements at intermediate deviator stresses (CH15).
Figure 3-19. Local displacements at high deviator stresses (CH15).
Figure 3-20. Output function from cavities with equal and unequal reflectivities.
Figure 3-21. Relationship between finesse ($\mathcal{F}$) and FWMI for identical mirrors.
Figure 3-22. LVDT calibration arrangement.
Figure 3-23. Typical LVDT calibration result.
Figure 3-24. Wavelength spectrum of light from HeNe laser.
Figure 3-25. Output from Fabry-Perot interferometer.
Figure 3-26. Piezo-actuator drift.
4. **LABORATORY EQUIPMENT**

Three triaxial systems were used to conduct the tests for this research project. Table 4-1 compares the features of the three systems as well as the instrumentation used and material tested.

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<td>Max. cell pressure</td>
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<td>Top and bottom</td>
</tr>
<tr>
<td>Local gauges (axial)</td>
<td>2 x LVDT</td>
</tr>
<tr>
<td>Local gauges (radial)</td>
<td>LVDT (caliper)</td>
</tr>
<tr>
<td>External gauge (axial)</td>
<td>DC linear gauge</td>
</tr>
<tr>
<td>Volume gauge</td>
<td>None</td>
</tr>
<tr>
<td>Load cell</td>
<td>MIL Internal</td>
</tr>
<tr>
<td>Back pressure</td>
<td>Base and/or top</td>
</tr>
<tr>
<td>Mid-plane pressure</td>
<td>Flushable probe</td>
</tr>
</tbody>
</table>

Table 4-1. Comparison of triaxial systems.
4.1. Medium-pressure triaxial system

The medium-pressure triaxial system was used to conduct all tests on London clay, with the exception of LC5 which was carried out in the high-pressure system.

4.1.1. Description of medium-pressure triaxial system

The main components of the medium-pressure triaxial system were a 50kN load frame, a triaxial cell with 1700 kPa maximum working pressure, a double acting rolling Bellofram actuator and four GDS digital pressure controllers, each with a maximum supply pressure of 3MPa. The combination of a load frame and actuator enabled strain controlled and load controlled tests to be conducted. The system is shown in Figure 4-1 and a schematic view is shown in Figure 4-2.

The load frames used for all three triaxial systems were identical Wykeham Farrance “classic range” stepless load frames with 50kN load capacity. The loading rate could be varied from 0.0001mm/min to 0.0599mm/min in steps of 0.0001mm/min and from 0.01mm/min to 5.99mm/min in steps of 0.01mm/min. All load frames were placed on pneumatic mounts in order to reduce the effect of external vibrations.

The triaxial cell chamber was manufactured from perspex and had circumferential reinforcements to allow a working pressure of 1700kPa. It had an inside diameter of 226mm, giving ample clearance between the local instrumentation and the inside of the cell chamber. The cell was held in position by eight external tie rods and consequently the loading ram could only be connected to the top cap after the cell had been closed. The loading ram passed into the cell through a brass fitting at the top of the cell and was sealed by two o-rings. This had some implications for cyclic loading at small deviator stress excursions as discussed in Section 4.1.6. The cell was modified to include seven ports in the base through which instrumentation
cables, mid-plane pore pressure probe etc. were connected. Two vent holes in the top of the cell allowed air to be expelled from the cell during filling.

The base pedestal had a diameter of 100mm and was fitted with a high air entry ceramic (280 kPa). A recess was machined into the top of the pedestal with radial and circumferential grooves at the bottom of the recess to facilitate easy drainage. The ceramic disc (diameter 80mm and thickness 9mm) was glued into the recess using epoxy resin. Once the epoxy had cured, the face of the pedestal was machined in a lathe to ensure a flat surface parallel to the bottom of the pedestal. Another high air entry ceramic was fitted into the top cap using a similar procedure.

It is vital that uniform stresses are developed in the triaxial specimen when measuring stiffness. Baldi et. al (1988) pointed out that misalignment errors in triaxial testing can be separated into sources of misalignment associated with the specimen and sources of misalignment associated with the apparatus. Misalignment errors associated with the apparatus are:

- eccentricity of the ram,
- porous stones of non-uniform thickness,
- non-horizontal pedestal surface,
- non-verticallity of the loading ram.

The first three errors result in an off-centre contact point between the load ram and the top cap. Superficially this problem can be avoided by a top cap designed to force the contact point between the ram and the top cap to be centred as the load is applied. Two examples are top caps with a conical recess and a hemispherical ram end and top caps with a hemispherical protrusion and a conical recess in the ram. Such top caps often produce bending of the specimen, making it difficult to determine the strain level particularly at the start of triaxial shear (Baldi et. al (1988)). For this reason top caps that force alignment are undesirable when investigating small strain stiffness behaviour. The problem may be overcome by using a top cap that does not force alignment to the loading ram. One elegant
solution is to use a flat top cap and hemispherical loading ram end, as the load is applied to the top cap at the position of initial contact. However, any initial eccentricity will remain during the test and therefore care is required during specimen preparation and set-up to ensure concentric loading. A flat top cap is only suitable for applying positive deviator stresses. Hight et. al (1983) described a top cap which did not force the top cap to align and allowed negative deviator stress application. Three 37.5mm dia. by 41mm high pots were bolted to the top cap and filled with liquid polyester resin just before lowering the cell chamber and load cell. Threaded bolts protruding from the lower plate of the load cell mated with the resin pots and once the resin hardened, tensile loads could be applied.

The top cap used in the medium-pressure triaxial apparatus was modified from a commercial ball and socket extension top cap. It consisted of a flat metal disc sandwiched between two rubber discs, as shown in Figure 4-3 and Figure 4-4. The metal disc was connected to the loading ram via a standard M8 thread. This design was suitable for positive and negative deviator stress application and permitted lateral movement of the top cap relative to the loading ram, thereby avoiding the application of lateral loads. The rubber discs ensured gradual load transfer to the soil specimen, which proved to be an advantage while investigating small strain behaviour. The disadvantage of the rubber discs was the non uniform shear rate which developed during tests, even though the machine rate was held constant. Figure 4-5 shows strain rates measured by local instrumentation for compression and extension tests on London clay. It demonstrates that the shear rates in both extension and compression were relatively constant for strain levels up to 0.01%.

The top cap had a recessed high air entry ceramic. Two drainage lines facilitated flushing of the cavity behind the ceramic and allowed top drainage of the soil specimen.

A total of five valves were fitted to the cell base. Two each for drainage of the base pedestal and top cap, and one for the cell fluid. The pedestal and top cap required
two valves (and drainage lines) each, to allow flushing of the cavities behind the high air entry ceramics. De-airing blocks were used with each of the three pressure transducers. All blocks were orientated with the transducers vertical and the sensing membrane facing upwards. This allowed the air to be expelled through the air bleed plug at the top of the block during flushing. As shown in Figure 4-2 valves were placed on both sides of each of the de-airing blocks. Even though this was not necessary when conducting the tests, it was particularly useful to allow calibration of the transducers without removing them from the de-airing blocks and zero checking of the transducers during long tests.

The valve manifold shown in Figure 4-1 and Figure 4-2 allowed a number of useful tasks to be performed. It allowed water from the de-airing reservoir to the routed to any one of a number of destinations including, the triaxial cell, base pedestal, top cap, mid-plane probe and the two water-filled pressure controllers. It furthermore allowed any one of the three pressure measurement systems to be pressurised by any one of the controllers. This gave particular freedom in flushing any of the systems or to double check measurements from the pressure transducers.

Nylon tubing was used to connect the main components of the medium-pressure triaxial system. It had an outside diameter of 8mm, inside diameter of 5.5mm and a maximum operating pressure of 8400 kPa. Nylon tubing is impervious to air, but allows some migration of water when there is a difference in vapour pressure between the inside and outside (Head (1986)). This characteristic, as well as the fact that it expands and contracts during pressure changes, makes it unattractive for use in conjunction with an external volume gauge. However, for this triaxial system a radial caliper was used to measure radial specimen strains.

4.1.2. Instrumentation of the medium-pressure system

The instrumentation of the medium-pressure triaxial system included LVDTs, pressure transducers, an internal load cell and an external linear gauge.
Two miniature submersible LVDTs were used as local axial gauges for all tests in the medium-pressure triaxial apparatus. The LVDTs were part of the D5/200WRA series manufactured by RDP Electronics Ltd and the specifications are shown in Table 4-2.

<table>
<thead>
<tr>
<th>Working Range</th>
<th>± 5mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linearity</td>
<td>better than 0.5% of FS.</td>
</tr>
<tr>
<td>Typical sensitivity</td>
<td>80mV/V/mm</td>
</tr>
<tr>
<td>Precision</td>
<td>1 μm</td>
</tr>
<tr>
<td>Temperature sensitivity</td>
<td>0.01% of FS/°C</td>
</tr>
<tr>
<td>Temperature range</td>
<td>-20 to 125 °C</td>
</tr>
<tr>
<td>Maximum working pressure</td>
<td>3500kPa</td>
</tr>
</tbody>
</table>

Table 4-2. Manufacturers specifications for LVDTs (D5/200WRA).

At the electrical zero the armature extended 26mm from the body giving a total transducer length of 81mm. Both axial LVDTs had right-angle connections between the transducer body and electrical lead cable. In addition the armature guiding bore extended through the body, allowing the armature to move freely even at large displacements, and thereby prevented straining of the soil or damage to the LVDT. The hermetic seal between the LVDT and the lead cable consisted of an internal rubber gland and a polyolefin shrink tube which covered the cable and right angle extension of the transducer body.

Two brackets were used to attach each LVDT to the 100mm diameter specimens (Figure 4-6). Both the top and bottom brackets had a thickness of 12mm and each had two holes to accept the pins that assisted in holding the brackets to the specimen. Two retaining screws on either side of the pin holes were tightened once the pins were pushed into the specimen thus ensuring a ridged connection between the brackets and the pins. The LVDT body was held in position by the top bracket and the armature rested on the pad clamped to the bottom bracket. The brackets
were designed to hold the LVDT body close to the specimen (3mm) in order to minimise possible errors from bracket rotation. After the LVDT was in place, small adjustments could be made to the output signal by adjusting the bottom pad. The set-up procedure was designed to ensure correct alignment and position of the brackets. Both brackets were clamped to an alignment tool at the desired positions and this was only removed once the brackets had been pinned and glued to the specimen.

A radial caliper based on the design of Bishop and Henkel (1962) was used to measure radial strains of specimens tested in the medium-pressure triaxial apparatus (Figure 4-7). The caliper consisted of a hinged ring designed to fit over the two axial LVDTs with sufficient clearance not to touch either, even at large specimen strains. The caliper was fitted with an LVDT electrically identical to the two axial gauges. The LVDT body and armature were held in position by two attachments, one designed to clamp the body and the other to accept the threaded armature. Both attachments could swivel in a horizontal plane, thereby ensuring the alignment of the body and armature throughout a test. The radial LVDT was mechanically different to the axial LVDTs insofar as it did not allow the armature free movement all the way through the transducer body. This did not present any operational difficulties, as the magnitude of the radial strains were sufficiently small as not to reach the maximum travel of the instrument.

Each LVDT system consisted of a number of components, namely the displacement transducer, a signal conditioning unit, pre-amplifier, main amplifier and analogue to digital converter. The LVDTs were excited by a 5V rms alternating voltage at a frequency of 5kHz. This was supplied by a signal conditioning unit which combined the modulation and demodulation of the signal as well as amplification of the analogue output signal. In the context of the LVDT system, the terms used for amplification will be “pre-amplification” for amplification by the signal conditioning unit and “main amplification” for amplification by the CIL amplifier (see Section 4.1.3). The pre-amplifier had fine and coarse amplification
adjustments. Coarse adjustment was achieved via an eight position slide switch of which only one setting could be engaged at a time. Fine adjustment was by rotary potentiometer. This design allowed initial coarse adjustment of the amplification level using the slide switch and subsequent fine adjustment by the potentiometer. The pre-amplifier did not have a built in steady supply voltage designed to determine the exact amplification level and therefore, once the system has been calibrated and the amplification was altered via the potentiometer, it was difficult to return the system to the initial amplification level. This problem was overcome by permanently securing the potentiometer at its mid point and only using the coarse switch to change the amplification to the required level. An additional feature of the signal conditioning unit was a zero input switch, which allowed the LVDT to be set at its electrical zero. When the switch was pressed, a zero input was supplied to the pre-amplifier. Once the output of the pre-amplifier had been zeroed, the switch was released and the electrical zero of the LVDT was found by adjusting the armature for a zero output signal. The analogue signal from the pre-amplifier was further amplified by the main amplifier converted to a digital signal and logged by the computer. The main amplifier and A/D converter is described in Section 4.1.3.

The determination of effective stresses in an undrained test requires the measurement of pore water pressure. Bishop and Henkel (1962) noted that non-uniform pore pressures are likely to occur during undrained triaxial test as a result of end constraints. They suggested two possible solutions to the problem. Firstly, the test could be performed at a slow enough rate that the pore pressures were uniform throughout the sample, or alternatively a probe could be inserted into the specimen to allow local measurement of the pore pressure. The technique of using a pore pressure probe was first used by Taylor (1944). Taylor (1948b) described the probe as:

“A porous device inserted diagonally into the centre of the sample and connected by a small plastic tube containing air free water to a fine-bore tube”.

4-8
Hight (1982) described a local pore pressure probe based on a miniature silicon diaphragm pressure transducer which was mounted with its porous ceramic face flush with the cylindrical sample. The advantage of this probe was that it did not penetrate the sample, making it particularly useful for small diameter specimens. Hight (1982) noted two operational difficulties with the probe. Firstly, as the probe could not be flushed, care was required to ensure that it stayed saturated. This may prove particularly difficult when installing it against a sample with high suction, because cavitation of the water between the porous ceramic and the silicon diaphragm may occur. Secondly, as a consequence of the small gap between the porous ceramic and the diaphragm (50\(\mu\)m), these two components could come into contact at high effective stresses, preventing the correct operation of the sensor.

As it was foreseen that high suctions would occur in the London clay specimens, a flushable mid-plane probe was used in the medium-pressure triaxial apparatus. The probe was based on the design of Sodha (1974) and consisted of a cylindrical high air entry ceramic of 11.7mm diameter, glued into a stainless steel casing with 15mm of the ceramic extending from the casing. Two small bore stainless steel tubes were brazed to the back of the casing and guided from the cell through one of the plugs in the cell base. Two tubes were required in order to flush the probe. The loose end of the entry tube was connected to a de-airing block and pressure transducer and the loose end of the exit tube was connected to a valve to close the system as shown in Figure 4-2. The tubes and the cavity behind the porous ceramic could be flushed at any time during the test, but is was rarely necessary to flush the probe more than once after the initial cell pressure was applied.

Internal load cells have the advantage that they eliminate load ram friction from the load measurement. The Imperial College load cell (Bishop et. al (1975)) was based on a design that transformed axial load into bending of a strain gauged triangular plate which consisted of cantilevers radiating from a common boss and bearing on the edge of a groove in the cylindrical loading cap. The load cell was not affected by cell pressure and only marginally affected by lateral and eccentric loads. However, it
has a number of disadvantages including high compliance (because of the bending plate), a discontinuous load-compliance response when changing from compressive to tensile loads (Jardine, Symes et. al (1985)), as well as high hysteresis on account of friction between the bending plate and load cell body. The load cells used for all tests during this research project were shear-web type internal load cells manufactured by Maywood Instruments Limited. The sensing element of the cell consisted of a circular disc of 75mm diameter which transformed axial load to shear of four web elements. Foil shear strain gauges are bonded to these elements and the sensitivity of the load cell can be modified at the design stage by specifying the appropriate shear web thickness. The eight gauges are arranged in a full Wheatstone bridge configured so as to minimise the effect of eccentric loading. This design had all the advantages of the Imperial College load cell, but in addition it ensured high transducer stiffness and low hysteresis. The low hysteresis was of particular importance in the medium-pressure triaxial apparatus, as cyclic tests were to be conducted.

The three pressure transducer used to measure cell, mid-plane and back pressures were all PDCR 810 transducers manufactured by Druck Ltd, with a pressure range of 150psi (1035kPa). The sensing element was a silicon diaphragm with an integrated silicon strain gauge bridge, giving high performance as reflected in the manufacturers specification of a combined non-linearity and hysteresis of ±0.1% of full scale.

Relative displacement between the ram and the cell top was measured using a direct current linear gauge manufactured by MPE Ltd. The gauge was spring loaded and had a range of 25mm. It operated on the principle of two parallel metal plates forced apart by the armature, resulting in bending of the plates. Bonded strain gauges, configured in a full Wheatstone bridge arrangement, sensed the strains in the plates and by calibration a relationship between displacement and output was established. As contact occurs between the armature and the plates, these transducers can be expected to suffer from hysteresis and mechanical wear.
4.1.3. **Signal conditioning**

The outputs from all transducers were conditioned by a SGA 1100 series signal conditioning system manufactured by CIL Electronics Ltd. The system was modular and consisted of a number of strain gauge amplifiers and one analogue to digital (A/D) converter. The amplifier modules could be wired in quarter bridge, half bridge or full bridge configurations by using precision resistors to complete optional circuits on the circuit board. The full bridge option was used for all transducers. The features of the amplifier modules included:

- direct current bridge supply which could be varied between zero and 12 volts,
- 4 position gain switch and a ten turn potentiometer providing an overall gain range of 1 to 10000 times,
- zero dial,
- low pass filter with frequency cut off of 1Hz to 1kHz,
- a calibration switch on the front panel which supplied a stable voltage to the amplifier. The input could be specified as 1mV, 3mV, 10mV or 30mV. This feature allowed a number of useful operations including the determination of absolute amplification, the checking of amplification levels and the changing of the amplification from one level to another as required.

The analogue to digital converter was a dual slope converter compatible with an IEEE bus. A semiconductor multiplexer allowed all the amplifiers to be serviced by one A/D converter. The final format of the 12-bit data was a number in the range ±4095 corresponding to an input voltage of ±10V. Conversion time for the converter was 30ms which was adequate in the context of the slow testing carried out during this project.
4.1.4. Data acquisition

Windows™ based software was developed to capture the relevant data during tests. The data were plotted on the screen in a processed format, enabling the evaluation of progress as the test proceeded. The latest numerical values for all displacement gauges, pressure transducers and the load cell were displayed in engineering units. In addition, three graphs were displayed:

- Two local axial gauges (µm) and deviator stress (kPa) vs. time (sec),
- Deviator stress (kPa) and volumetric strain vs. axial strain,
- Deviator stress (kPa) vs. mean effective stress (kPa).

All data were written to the hard disc immediately after logging to ensure that no data were lost in the event of a power failure. These data were in an unprocessed bit format. The full listing of the programme is given in Appendix A.

4.1.5. Pressure control

Four digital pressure controllers manufactured by Geotechnical Digital Systems Ltd (GDS) were used as part of the medium-pressure triaxial system (Figure 4-1). Each controller had a fluid storage capacity of 200cm³ and could deliver a maximum pressure of 3MPa. Two were filled with de-aired water and two with silicone oil. The water filled controllers supplied the pressure for the cell and back pressure and the oil filled controllers were used to pressurise the top and bottom chambers of the rolling Bellofram actuator. The controllers operated on the principle of varying the pressure in the cylindrical vessel by the inward and outward movement of a piston connected to a geared stepper motor via a threaded shaft. A feedback loop between a pressure transducer which monitored the pressure in the vessel and the control circuit of the stepper motor ensured that the required pressure was supplied. Pressure changes could be entered as instructions using the key pad, or alternatively the input could be made digitally by a computer via an RS232 port. The required pressure could be entered by the TARGET command, in which case the controller
located the specified pressure and held it indefinitely. Further options were to RAMP the pressure by specifying a start pressure, end pressure and rate of pressure change and a CYCLE command which continuously cycled the pressure between two specified values at the required rate. In addition commands were available to FILL or EMPTY the vessel and to ZERO the pressure transducer.

4.1.6. Cyclic loading

The medium-pressure triaxial system was designed to conduct cyclic tests in order to probe the $Y_1$ and $Y_2$ yield surfaces of the geomaterials under investigation (Jardine et al. (1991) and Smith et al. (1992)). Smith et al. (1992) suggested that the $Y_1$ and $Y_2$ yield surfaces may be identified by carrying out cyclic tests and systematically increasing the magnitude of load-unload cycles. All probing tests were conducted under undrained conditions. This limited the possible stress directions in $p'$ $q$ stress space to vertical (or near vertical) stress excursions, but had the advantage of requiring only axial strain measurement to determine the geomaterial stiffness.

The deviator stress was controlled by an actuator positioned between the cross beam and load ram as shown in Figure 4-1. The actuator was of the double acting rolling Bellofram type, with a stroke of 60mm and capable of producing tensile and compressive loads up to 5kN. This type of actuator was chosen on the basis of low frictional resistance against axial movement, as it was foreseen that stress excursions of a few kPa would be required to probe the $Y_1$ yield surface. Nevertheless, difficulties with regards to the relatively high friction between the loading ram and the cell top o-rings still remained. The problem was especially acute when conducting cyclic loading at small deviator stress excursions, as pressure increase in the actuator only resulted in an increase in deviator stress once the system friction had been overcome. The problem was addressed by using a closed loop procedure for the control software. The first stress excursion was initiated by ramping the pressure in the relevant actuator chamber at the required
rate whilst continuously monitoring the load cell output. Once the target deviator stress was reached, the actuator pressure was held at the current value. After a specified rest period, the deviator stress was reversed to the start value. The process was fully automated with all instructions contained in a data file and requiring no input during the test. The flow chart and software code is contained in Appendix A. Typical stress excursions are shown in Figure 4-8 (LC1). The stress increment was doubled for each subsequent load-unload loop whilst keeping the loading rate constant. Rest periods between load, unload and reload cycles were also increased between cycles.

4.2. High-pressure Triaxial System

The high-pressure system was used to conduct sixteen tests. This included one test on London clay (LC5) and all tests on the Chalk with the exception of CH15.

4.2.1. High-pressure system description

The high-pressure triaxial apparatus was used to conduct tests on 38mm specimens at cell pressures of up to 5000kPa. All tests were strain controlled at positive deviator stresses. Figure 4-9 shows a schematic lay-out of the system which consisted of a load frame, a high pressure triaxial cell, two hydraulic pressure controllers and a volume change gauge. The load frame was identical to the one shown in Figure 4-1 and described in Section 4.1.1 and rested on three pneumatic mounts designed to isolate it from external vibrations.

The triaxial cell had an allowable working pressure of 14MPa. It was manufactured by Wykeham Farrance to the specifications of the University of Surrey. The cell bottom consisted of a base plate of outside diameter 200mm and a 38mm pedestal fixed onto the base plate. Two ports passed through the pedestal and base plate to allow back pressure application and drainage of the sample. Application of back pressure to both ports was necessary to avoid pressure differences between them.
when placing a specimen directly onto the pedestal. Moreover, both ports were connected to the de-aired water supply via the valve manifold to permit individual flushing of each port. The pedestal side was polished in a workshop lathe to ensure a good seal between the pedestal and specimen membrane. Metal was used to manufacture the cell in order to facilitate high pressure operation, but this prohibited the visual monitoring of specimens during tests.

On assembly, the cell top was placed over the base pedestal with the six bolts in the base plate aligned with the holes in the cell top flange. A 25kN shear web type internal load cell was fixed to the load ram which passed through the top of the cell. Cell fluid was stored in a reservoir attached to one of the tension rods and was gravitated into the cell through one of the ports in the cell base whilst expelling air through the vent plug in the cell top. De-airing blocks with two valves each were fitted to all fluid entry ports with the exception of the cell fill port. Two pressure transducers were fitted to these blocks in an upright position to allow de-airing when required as well as calibration without removing the transducers from the system. The cell and back pressure were each measured with a 20MPa transducer manufactured by Druck Ltd.

Stainless steel tubing (OD = 3.2mm, ID = 1.4mm) was used for all pressurised lines. This was necessary in view of the elevated pressures and the inclusion of a volume gauge in the system. The valve manifold, shown schematically in Figure 4-9, gave the required flexibility to flush all lines and ports, as well as the volume gauge. It also allowed the volume gauge to be excluded from the system when required.

The cell and back pressure were controlled by digital pressure controllers. The back pressure controller was identical to those described in Section 4.1.5, apart from the fact that it was modified to allow volume measurement. The cell pressure was controlled by an advanced controller which was similar in concept to those described in Section 4.1.5, but it was manufactured to higher tolerances and in addition had a maximum working pressure of 5000kPa.
Volume change of the 38mm specimens was measured externally by a volume gauge. The gauge was an Imperial College type rolling Bellofram volume gauge (Head (1986)) which consisted of a thick-walled cylindrical casing containing the “floating” piston attached to a rolling Bellofram seal at either end. The bottom chamber was connected to the back pressure supply and the top chamber to the pedestal base via the valve manifold. Any flow of pore water into or out of the specimen was converted to an axial movement of the piston relative to the casing. This movement was detected by a MPE displacement transducer similar to the one used to measure external displacements in the medium-pressure triaxial apparatus (Section 4.1.2). The arrangement of tubes and valves shown in Figure 4-9 allowed the volume gauge to the included or excluded from the system as required, and the convenient flushing of both chambers without disconnection of any lines.

External displacement was measured by a transducer identical to the one described in Section 4.1.2. All signal conditioning, A/D conversion of the output signals and data acquisition hardware and software were similar to those used for the medium-pressure triaxial system (Section 4.1.3.).

4.2.2. Local instrumentation of the high-pressure system

Two LVDTs were used as local instrumentation to measure axial displacements of specimens in the high-pressure triaxial apparatus. The LVDTs were electrically identical to the submersible transducers described in Section 4.1.2. However, as the cell had a pressurising capacity of 5000kPa, which was higher than the operating pressure for the submersible transducers, a number of modifications were made. Two venting holes were drilled in each transducer body admitting the cell fluid into the body and giving rise to equal pressure inside and outside the transducer, regardless of the cell pressure. As a result, the electrical conductors were in contact with the cell medium and to avoid any stray currents, silicon oil was used as cell fluid. This had the added advantage of not requiring sealing of the LVDT lead.
wires, allowing four thin (φ = 0.4mm) copper wires to be soldered directly onto the transducer. These cables exited the cell via plugs in the bottom of the base plate. Each LVDT was held in place by two brackets similar to those described in Section 4.1.2. However, some differences warrant reference. Firstly, the brackets were smaller, with a thickness of 8mm as the specimens tested in the high-pressure apparatus were 38mm in diameter, as opposed to 100mm in the medium-pressure apparatus. Secondly, the armature rested on a fine pitch screw, threaded into the bottom bracket which provided a means to fine tune the armature position prior to closing the cell top. Thirdly, these brackets were not pinned to the specimens. During initial trials pinning was found to be unsatisfactory, as it resulted in pieces of chalk being dislodged from the specimen. Therefore the brackets were glued to the membrane using quick setting cyanoacrylate adhesive (Super Glue) and in addition, supporting latex rubber bands were placed around the sample and brackets. The LVDT bore extended through the transducer body giving the armature unrestricted movement through the body even at large specimen strains.

4.2.3. **Load alignment**

In the high-pressure triaxial apparatus, a flat metal top cap and a ram with a hemispherical end was used. The problem of eccentric loading was addressed by seeking to have the contact point as close as possible to the centre of the top cap. When the cell top was placed onto the base plate, the tapered bottom of the cell top rested on a large o-ring (thickness 8.5mm) which fitted against the right-angled shoulder of the base plate. When the cell top rested on the o-ring, a small gap remained between the cell top flange and the base plate. This allowed the alignment of the cell top and base plate to be adjusted during assembly by controlling the tightening procedure of the six flange bolts. The nuts were first hand tightened in a criss-cross sequence, before the individual nuts were tightened by a spanner. When the nuts were moderately tight, a vernier caliper was used to measure the distance from the top of the flange to the bottom of the base plate (to 0.1mm) at the four compass positions along the circumference of the base plate. Small changes were
made by tightening individual nuts as required. In this way the position of contact between the load ram and top cap could be governed during set-up.

The success of the procedure was determined by inspecting the top cap after completion of each test. The contact point between the ram and cap was clearly visible as a small indentation which could be measured relative to the centre of the top cap. This measurement was carried out with a magnifying lens commonly used to measure rock joint characteristics. The lens had an internal linear scale with a resolution of 0.1mm. As experience was gained, the accuracy with which the contact could be established close to the top cap centre increased and after some time, distances of less than 0.3mm were routinely obtained. After each test, the surface of the top cap was machined in a lathe to remove the previous indentation.

4.3. **Low-pressure triaxial system**

The low-pressure triaxial system was used to carry out the three tests on Bothkennar clay.

**4.3.1. Description of low-pressure triaxial system**

The main components of the low-pressure triaxial system were a load frame, triaxial cell and two pressure controllers, as shown schematically in Figure 4-10. The load frame allowed strain controlled shear and was identical to the one shown in Figure 4-1 and described in Section 4.1.1. It was placed on three pneumatic mounts in order to reduce possible mechanical noise. The triaxial cell, manufactured by Soiltech Ltd, was held onto the base plate by three external tie bars. It had a perspex cell chamber with inside diameter 165mm and outside diameter 200mm. The base pedestal had a diameter of 101.6mm and included a recessed high air entry ceramic. Two top caps were used as part of this system. Top drainage were possible for BK1 and BK2 via a top cap with a recessed high air entry ceramic and two drainage lines. These two tests showed that drainage occurred sufficiently quickly by means of base
and radial drains. For BK3 a solid perspex top cap was used. Both top caps had flat surfaces to make contact with the hemispherical load ram end and consequently only positive deviator stress application was possible.

Five connections were made to the cell base, two each to the top cap and base pedestal and one to the cell pressure supply. De-airing blocks with valves on either side were used to connect all the pressure transducers to the system. Two types of tubing were used. Nylon for the cell pressure supply and stainless steel for the back pressure and mid-plane probe. Steel tubing were required for the back pressure line, as volume changes were measured externally by the pressure controller. A valve manifold similar to the one described in Section 4.1.1 was used to connect all the system components.

4.3.2. Low-pressure system instrumentation

Two submersible LVDTs were used to measure local axial displacements. They were similar to the ones described in Section 4.1.2, but were manufactured to a higher standard. The linearity of these LVDTs were specified by the manufacturer as 0.1% of full scale, as opposed to 0.5% for the ones described in Section 4.1.2. Two 12mm thick brackets were used to fix each LVDT to the specimen. The top bracket clamped the transducer body in position and the armature rested on a fine pitched screw, threaded into the bottom bracket. An alignment tool were used to hold the brackets in the required position before pinning and gluing them to the specimen.

Three pressure transducers were used to measure the cell, mid-plane and back pressure. All were of the PDCR type manufactured by Druck Ltd with a maximum pressure range of 150 psi (1035kPa). The mid-plane pore pressure probe was identical to the one described in Section 4.1.2 and consisted of a high air entry ceramic, stainless steel housing and two thin bore stainless steel drainage lines, which exited the cell via one of the plugs in the cell base. Load application was
measured by a 1kN shear web type submersible load cell manufactured by Maywood Instruments and external displacement by a direct current linear transducer. All signal conditioning, data acquisition software and hardware were similar to those used for the other two triaxial systems and are described in Sections 4.1.3 and 4.1.4.

4.4. Transducer calibration

The calibration of all transducers, with the exception of the high resolution calibration of the LVDTs, will be described in this section. The high resolution calibration of the LVDTs was discussed in Chapter 3.

4.4.1. Calibration Methodology

Calibration of the measurement transducers was conducted to evaluate the accuracy of the transducers. Accuracy has been defined by the British Standards Institution (1986) as the closeness between the result of a measurement and the true value. In order to quantify this, a suitably accurate reference system is required. If fact, the accuracy of the reference system should be 3 to 10 times better than that of the instrument being calibrated (Doebelin (1990), Sydenham, Hancock and Thorn (1989), Collett and Hope (1983), Sydenham (1982)). During this research project, the accuracy of all transducers were determined using a consistent methodology which consisted of the following steps:

a) Calibration of the transducer system consisting of the transducer, signal conditioning unit(s) and A/D converter against a suitably accurate reference.

b) A least squares linear transfer function was determined to describe the relationship between the digital output (in bits) and the true value (engineering units).
c) The error was calculated for each calibration point as the difference between the true value (engineering units) and the measured value (output converted to engineering units using the transfer function).

d) The errors were plotted against the true value to allow rapid visual assessment of the maximum error, non-linearity and hysteresis of the instrument (see for example Figure 4-11 and Figure 4-12).

e) The accuracy of the transducer system was calculated as ±2 times the standard deviation of the errors. Assuming a Gaussian distribution of the errors, this gave a confidence level of 95% that the difference between a measured value and the true value would be within the specified accuracy.

4.4.2. **Dead-weight calibration system**

A dead-weight calibration system manufactured by Budenberg Gauge Co. Ltd. was used to calibrate the load cells and pressure transducers used during this research project. The system operates on the principle of pressure balance between a piston loaded by dead weights and the pressure at the outlet port. The outlet port could be connected directly to a pressure transducer to be calibrated, or alternatively a load cell could be calibrated in a purpose made load frame. Friction in the system was minimised by machining the pistons to such high tolerances as not to require seals and the slow leakage of oil past the pistons provided lubrication. Both pistons were continuously rotated during calibration to further reduce friction. All test weights and piston areas were calibrated against weights and pistons traceable to the National Physical Laboratory. Furthermore, the accuracy of the tester takes into account the effect of the buoyancy of the weights in air and the buoyancy of the piston in the oil. The certificate of accuracy of the dead weight calibration system shows that at a temperature of 20°C and at the standard gravitational acceleration of 9.80665m/sec², the error of the system does not exceed 0.04%.
4.4.3. Pressure transducer calibration

Pressure transducers were calibrated without removal from the triaxial apparatus. This was made possible by the valve manifold and the two valve system used with each de-airing block. The pressure outlet from the dead weight calibration system was connected to the valve manifold and the required transducer pressurised by selecting the appropriate valves. Zero was taken as atmospheric pressure. The calibration procedure consisted of a load and an unload component with typically a minimum of ten equally spaced pressure increments for each component. A typical calibration result is shown in Figure 4-11 as the error for each calibration point plotted against the applied pressure. A detailed determination of the accuracy was made by evaluating the standard deviation as discussed in 4.4.1. The accuracy as a ratio of the calibration range was better than ±0.1% for most pressure transducers. Two notable exception were the 20MPa Druck transducers. The accuracy as a ratio of the calibration range was found to be ±0.22% and ±0.23% for these two transducers. This relatively poor performance may be explained by the small calibration range relative to the design range. All calibration results are summarised in Table 4.3 to 4.5.

4.4.4. Load cell calibration

Three different load cells were used in the three triaxial systems. The design range of each cell was chosen on the grounds of the material to be tested and was 1kN, 5kN and 25kN for the low, medium and high-pressure systems. Material stiffnesses were determined over axial strains from 0.0001% to more than 10% and in order to maintain compatibility between stress and strain measurements, all load cells were calibrated over more than one range. The cells were calibrated using compressive and/or tensile loads. Compressive loads were applied using the Budenberg dead-weight system and tensile forces by suspending weights from a hanger threaded into the transducer. As a result of the shear web design of the load cell (see Section
4.1.2), some difficulty was encountered in quantifying the precise load applied to the cell at the transition between compression and tensile loads. It resulted from the fact that when the cell was hanging under its own weight, a small amount of shear was induced in the webs by the weight of the centre section of the sensing element. This weight was unknown. Likewise when the load cell was placed inside the Budenberg load frame, the applied load was that of the loading ram, connectors and the load cell, minus the (unknown) weight of the centre segment of the sensing element. The problem was addressed by using the following procedure:

a) Zero was defined as the output when the load cell was placed on its side on the laboratory bench.

b) The Budenberg calibration system was used to conduct a load-unload calibration cycle under compressive loads to establish the slope of the load vs. output curve (sensitivity).

c) The load cell was then suspended under its own weight and the “effective” weight of the centre section of the sensing element calculated from the measured output and the sensitivity.

d) A tensile load-unload calibration cycle was conducted suspending weights from the cell.

The calibration result of the 5kN load cell shown in Figure 4-12 indicates a smooth transition between compressive and tensile load excursions, suggesting that the method describe above adequately accounts for the unknown weight of the sensing element. It furthermore shows that no significant difference exists for the slope of the output curve in extension and compression. This is to be expected given the cell design. Some non-linearity and hysteresis was evident, but the maximum error did not exceed 0.1% of the calibration range. Section 4.1.1 explained why the shear web design resulted in an instrument with less hysteresis compared to the Imperial College internal load cell. Calibration results on a 4.5kN Imperial College load cell by Matthews (1997) were compared to the 5kN shear web load cell and are shown in Figure 4-13. They clearly demonstrates the lower level of hysteresis in the shear web type cell.
The high resolution calibration of the 1kN load cell was conducted in tension even though it was only used for positive deviator stress application. This was necessary as the required calibration range was 0.14kN and the minimum load increment that could be applied by the dead load system was 0.1kN. This technique was deemed acceptable on the basis of the load cell design and the evidence shown in Figure 4-12.

The accuracy of all three load cells, as a ratio of the calibration range, was found to be better than ±0.07% for all calibration ranges.

4.4.5. LVDT calibration

A rigorous calibration programme was designed to determine the accuracy and resolution of the LVDTs for both high and low resolution measurement and in addition, to quantify factors such as initial armature position and regression range. All high resolution calibrations were conducted using the interferometer as reference system, as described in the previous chapter in the context of the theory, development and application of the Fabry-Perot interferometer. For the large range calibration, the LVDTs were calibrated over a typical range of a few millimetres using a micrometer. The resolution of the LVDTs ranged between 0.4µm and 1.5µm and the digital output signal was stable in all cases, indicating that the noise level was less than the resolution.

The output responses of the LVDTs were linear near to the electrical zero, becoming non-linear further away. As a linear calibration function was used, care was required when choosing the regression range. To investigate the effect of regression range, an LVDT was calibrated between ±10mm and progressively smaller ranges used for regression calculation. Regression over the full range yielded a plot of error vs. displacement resembling a third order polynomial, with a large negative error at one end of the range, a large positive error at the other end and a small error near the
electrical zero (Figure 4-14 (a)). When the regression range was systematically decreased for the same data set, the plot came to resemble a second order polynomial with the maximum negative error near the ends of the range and maximum positive error near the electrical zero (Figure 4-14 (e)). This indicated that the regression range was sufficiently far away from the non-linear part of the output response curve, but that the error could be improved for the same regression range by using a higher order polynomial instead of a straight line. When the regression range was decreased even further, the error became randomly spread about zero (Figure 4-14 (g)), implying that the data could be best represented by a linear function.

Figure 4-14 (h) shows the accuracy, normalised by the regression range, plotted against the regression range. This curve is useful to identify the optimum measurement range for the LVDT system at the particular amplification. It shows that the normalised error increases rapidly for ranges greater than ±5mm. Also for regression ranges smaller than ±3mm there was no significant decrease of the normalised error. This indicates that for regression ranges smaller than ±3mm, the accuracy was inversely proportional to the range. It is interesting to note in the light of the ±5mm design range that the characteristic third order polynomial shape of error vs. regression range was found for all ranges above ±5mm, and that the second order shape was found for ranges smaller than ±5mm. Figure 4-14(h) shows that for the LVDT and amplification under consideration, the optimum measurement range was slightly more than ±3mm. For this reason a calibration range of ±3.2mm was chosen.

During set-up of an LVDT the pre-amplifier zero switch was used to position the armature relative to the electrical zero. The aim was to have the armature as close as possible to the electrical zero when the shear stage commenced. This required the armature to be set positive or negative relative to the electrical zero depending on the stress path to be followed prior to shear. It was recognised that the offset relative to the electrical zero would be different for every test and therefore the effect of
initial armature positions on the performance of the LVDTs were investigated. Figure 4-15 shows the results for the calibration over successive stages. Each time the output went out of range at the positive end (+4095 bits), the zero settings were used to adjust the output to -4095 bits before the calibration recommenced. A regression was carried out for the linear range of each stage to determine the output sensitivity. The sensitivity for all the cycles was found to be 0.1018±0.0007µm/bit, indicating that the effect of the initial armature position was small.

4.4.6. Volume gauge calibration

For the low-pressure triaxial apparatus, the pressure controller, connected to the back pressure line, was used to measure volume change. The controller was calibrated by measuring the weight of de-aired water expelled from (or entered into) the pressure chamber via a stainless steel tube. The end of the tube was submersed in a beaker of water placed on an electronic balance with a resolution of 0.01g. One fill and empty loop was conducted for a volume of approximately 15000 mm³. The accuracy of the volume change measurements were calculated as ±34.6 mm³.

In the high-pressure apparatus volume changes were measured by means of an Imperial College volume change gauge. The gauge was calibrated against an advanced GDS pressure controller. Calibration against the weight of the expelled water, as described in the previous paragraph, was not possible as both chambers of the gauge has to be pressurised during operation. In addition, the option of placing the gauge directly on a balance was rejected on the basis of the stainless steel tubing used to connect the gauge to the valve manifold. The judgement was made that the stiffness of the tube could lead to erroneous weight measurements. Two separate controllers were used during calibration. One to pressurise each of the pressure chambers. A small pressure difference was applied to move the gauge piston, but when taking a measurement, equal pressure was applied to both chambers by closing a valve between the top chamber and the second pressure controller. All readings were taken at a pressure of 500kPa. One fill and empty loop was conducted.
during calibration to assess the effect of hysteresis. The effect of hysteresis was found to be small and the accuracy of the gauge was calculated as ±22.3mm³.

From the description of the laboratory equipment, it is clear that the three triaxial apparatuses were significantly different. In particular, the differences in allowable cell pressures and load application are notable. This was necessary on account of the wide range of strengths and stiffnesses of the three geomaterials tested. Nevertheless, two common features may be observed for all three systems. Firstly, all the systems were equipped with LVDTs as local strain instrumentation to allow accurate measurements of axial strains. Secondly, particular care was taken for each system to avoid bending of the specimen during loading. In part this was achieved by accurate alignment of the loading ram and specimen and avoiding the development of horizontal forces between the ram and the top cap. A further important requirement to avoid specimen bending is right cylindrical specimens. Considerable care was taken to ensure that right cylindrical specimens were obtained during specimen preparation. These procedures will be discussed in the following Chapter.
<table>
<thead>
<tr>
<th>Transducer</th>
<th>Measurement</th>
<th>Company</th>
<th>Serial no.</th>
<th>Design Range</th>
<th>Calibration Range</th>
<th>Resolution</th>
<th>Accuracy</th>
<th>Accuracy Calib. Range (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>LVDT</td>
<td>Axial displacement</td>
<td>RDP</td>
<td>5716</td>
<td>± 5mm</td>
<td>± 3.2mm</td>
<td>1.45μm</td>
<td>± 4.52μm</td>
<td>± 0.07</td>
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<tr>
<td></td>
<td>(local)</td>
<td></td>
<td></td>
<td></td>
<td>± 50μm</td>
<td>0.0126μm</td>
<td>± 0.129μm</td>
<td>± 0.13</td>
</tr>
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<td>Axial displacement</td>
<td>RDP</td>
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<td>± 3.2mm</td>
<td>1.14μm</td>
<td>± 6.85μm</td>
<td>± 0.11</td>
</tr>
<tr>
<td></td>
<td>(local)</td>
<td></td>
<td></td>
<td></td>
<td>± 40μm</td>
<td>0.00991μm</td>
<td>± 0.159μm</td>
<td>± 0.20</td>
</tr>
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<td>Radial displacement</td>
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<td>± 3.2mm</td>
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<td>± 0.13</td>
</tr>
<tr>
<td></td>
<td>(local)</td>
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<td></td>
<td></td>
<td>± 40μm</td>
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<td>± 0.137μm</td>
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<td>± 56.4μm</td>
<td>± 0.23</td>
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<td>Axial load</td>
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<td>-1kN to +2kN</td>
<td>0.545N</td>
<td>± 1.8N</td>
<td>± 0.06</td>
</tr>
<tr>
<td></td>
<td>(internal)</td>
<td></td>
<td></td>
<td></td>
<td>0 to 8kN</td>
<td>1.99N</td>
<td>± 5.5N</td>
<td>± 0.07</td>
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<td>Pore pressure (local)</td>
<td>Druck</td>
<td>373590</td>
<td>1035kPa</td>
<td>700kPa</td>
<td>0.176kPa</td>
<td>± 0.727kPa</td>
<td>± 0.10</td>
</tr>
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<td>Pressure transducer</td>
<td>Back pressure</td>
<td>Druck</td>
<td>373589</td>
<td>1035kPa</td>
<td>500kPa</td>
<td>0.125kPa</td>
<td>± 0.274kPa</td>
<td>± 0.05</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>700kPa</td>
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</tr>
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<td>500kPa</td>
<td>0.125kPa</td>
<td>± 0.282kPa</td>
<td>± 0.06</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1000kPa</td>
<td>0.247kPa</td>
<td>± 0.971kPa</td>
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<td>Cell pressure</td>
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<td>213-126</td>
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<td>2000kPa</td>
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Table 4.3. Transducers of the medium-pressure triaxial system.
<table>
<thead>
<tr>
<th>Transducer</th>
<th>Measurement</th>
<th>Company</th>
<th>Serial no.</th>
<th>Design Range</th>
<th>Calibration Range</th>
<th>Resolution</th>
<th>Accuracy</th>
<th>Accuracy Calib. Range (%)</th>
</tr>
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<tbody>
<tr>
<td>LVDT</td>
<td>Axial displacement (local)</td>
<td>RDP</td>
<td>5344</td>
<td>± 5mm</td>
<td>± 2.2mm</td>
<td>0.570μm</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>± 30μm</td>
<td>± 30μm</td>
<td>0.00734μm</td>
<td>± 0.0266μm</td>
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<td>Axial displacement (local)</td>
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<td>± 2.2mm</td>
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<td>± 0.12</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td>± 30μm</td>
<td>± 30μm</td>
<td>0.00758μm</td>
<td>± 0.0442μm</td>
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<td>Axial displacement</td>
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<td>-</td>
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<td>12mm</td>
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<td>± 5.39μm</td>
<td>± 0.04</td>
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<td></td>
<td></td>
<td></td>
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<tr>
<td>Volume gauge</td>
<td>Volume change</td>
<td>IC</td>
<td>-</td>
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Table 4.4. Transducers of the high-pressure triaxial system.
<table>
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<th>Company</th>
<th>Serial no.</th>
<th>Design Range</th>
<th>Calibration Range</th>
<th>Resolution</th>
<th>Accuracy</th>
<th>Accuracy</th>
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<td>Calib. Range (%)</td>
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<td>± 1.7mm</td>
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<td>± 4.49μm</td>
<td>± 0.13</td>
</tr>
<tr>
<td></td>
<td>(local)</td>
<td></td>
<td></td>
<td></td>
<td>± 20μm</td>
<td>0.00547μm</td>
<td>± 0.0340μm</td>
<td>± 0.09</td>
</tr>
<tr>
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<td>Axial displacement</td>
<td>RDP</td>
<td>5896</td>
<td>± 5mm</td>
<td>± 1.7mm</td>
<td>0.438μm</td>
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<td>± 0.11</td>
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<td>(local)</td>
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<td>± 20μm</td>
<td>0.00557μm</td>
<td>± 0.0272μm</td>
<td>± 0.07</td>
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<td>Axial displacement</td>
<td>MPE</td>
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<td>(external)</td>
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<td>Volume gauge</td>
<td>Volume change</td>
<td>GDS</td>
<td>02006</td>
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<td>Axial load</td>
<td>MIL</td>
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<td>(internal)</td>
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<td></td>
<td></td>
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<td>± 0.05</td>
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<td>Pore pressure</td>
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<td>± 0.08</td>
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<td>Back pressure</td>
<td>Druck</td>
<td>373590</td>
<td>1035kPa</td>
<td>400kPa</td>
<td>0.105kPa</td>
<td>± 0.411kPa</td>
<td>± 0.10</td>
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<td>Pressure transducer</td>
<td>Cell pressure</td>
<td>Druck</td>
<td>65863</td>
<td>1035kPa</td>
<td>400kPa</td>
<td>0.105kPa</td>
<td>± 0.420kPa</td>
<td>± 0.11</td>
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</tbody>
</table>

Table 4.5. Transducers of the low-pressure triaxial system.
Figure 4-1. Medium-pressure triaxial system.
Figure 4-2. Schematic view of medium-pressure triaxial system.
Figure 4-3. Schematic view of top cap for the medium triaxial pressure system.
Figure 4-4. Top cap for the medium-pressure triaxial system.
Figure 4-5. Strain rates for London clay specimens.
Figure 4-6. LVDT brackets.
Figure 4-7. Radial caliper.
Figure 4-8. Typical stress excursions for cyclic loading.
Figure 4-10: Schematic view of low-pressure triaxial system.
Figure 4-11. Typical pressure transducer calibration result.
Figure 4-12. Typical load cell calibration result.
Figure 4-13. Comparison of shear web load cell (MIL)

with Imperial College load cell.
Figure 4-14. Effect of regression range on LVDT error.
Figure 4-15. Effect of initial armature position on LVDT performance.
5. LABORATORY TESTING

Three geomaterials were tested during this project. These were Chalk, London clay and Bothkennar clay. These materials were selected as they represent natural geomaterials with a wide range of strengths and stiffnesses. All tests were conducted in the triaxial apparatus on specimens cut from block samples.

5.1. London clay

During the period of this research project, tunnels were under construction at Heathrow airport (Terminal 4). These tunnels formed part of the new Heathrow Express underground rail service between Heathrow airport and the city of London. The tunnels were constructed in London clay using the New Austrian Tunnelling Method (NATM). As the tunnel face was exposed during construction, the opportunity became available to obtain block samples of London clay.

5.1.1. Description of London clay

London clay is a stiff marine clay deposited during the Eocene period. The dominant clay minerals are illite and montmorillonite, with subsidiary kaolinite (Chandler and Apted (1988)). London clay is heavily overconsolidated. The estimated overburden pressure to which this material has been subjected during its geological history is approximately 1500kPa (Skempton (1961), Chandler and Apted (1988)). Fissures dominate the macro fabric of London clay. Ward et al. (1965) reported the fissure spacing to gradually increase with depth. At Ashford Common, approximately 15 miles to the west of central London, they found the fissure spacing to vary from 40mm at a depth of 10m to between 300 and 600mm at a depth of 45m.
A summary of the ground profile, as described during the tunnel site investigation, is presented below:

0 - 1.5m  GRAVEL and SAND. (MADE GROUND).
1.5 - 2.5m  Dense GRAVEL and coarse SAND. (TERRACE GRAVELS).
2.5 - 3.0m  Grey brown CLAY. (WEATHERED LONDON CLAY).
3.0 - 17.7m  Stiff grey brown CLAY. Extremely closely spaced randomly orientated fissures. Occasional partings of silty fine sand. (LONDON CLAY).
17.7 - 24.6m  Very stiff grey brown CLAY. Very closely spaced random orientated fissures. Rare impersistent partings of silty fine sand. (LONDON CLAY).

During tunnel construction, the clay was indeed found to be highly fissured as described above. The material exhibited a blocky nature and tended to fail on pre-existing fissures during excavation.

As part of the site investigation, the natural water content and Atterberg limits were determined for a large number of samples. Some scatter of the natural water content was observed, but it was generally found to be between 22% and 28%, decreasing to around 20% at depths greater than 50m below the ground surface. The natural water contents of the samples tested as part of this research project ranged between 24.7% and 25.7%. The liquid limit tested during the site investigation showed significant scatter and ranged between 50% and 80%, but in contrast the plastic limit data was less scattered and largely between 22% and 30%. This was in agreement with the liquid and plastic limits determined for a sample of clay taken in the concourse at rib 53, which were found to be 71% and 29% respectively.

A large number of unconsolidated undrained shear strength tests were performed on 38mm and 100mm diameter specimens as part of the site investigation. The data showed significant scatter, but the undrained shear strength was found to increase
with depth from approximately 100kPa at a depth of 10m to approximately 250kPa at 25m. SPT results were less scattered and increased from an N-values of approximately 15 at 5 meters depth to N-values of approximately 40 at 25 meters.

As part of the programme to monitor the ground behaviour during construction, numerous piezometers were installed in the vicinity of the tunnels. Figure 5-1 shows the measured pore pressures, before any influence from tunnel construction occurred. The scatter of the data is significant, but as shown in Figure 5-1 a hydrostatic pressure distribution fits the data reasonably well, suggesting minimal effect from under-drainage. Upper and lower bounds for the data at hydrostatic pressure are superimposed in Figure 5-1 and suggest that the phreatic surface is located between 1m and 5m below ground surface.

5.1.2. *In-situ stresses in the London clay*

During laboratory testing as part of the site investigation, the bulk unit weight of the London clay was found to be in the range 19kN/m³ to 20kN/m³ and for the purpose of calculating the vertical stresses in the soil, the bulk unit weight was taken as 19.5kN/m³. No testing was carried out on the fill material or Taplow Gravel and the bulk unit weight for both was assumed to be 17kN/m³. The pore pressure distribution was assumed to be hydrostatic with the phreatic surface at a depth of 3m. From the above, the pressure head at the sampling depth of 18m below the ground surface was calculated as 15m. This corresponds well with the data points in Figure 5-1. The vertical total stress was calculated as 344kPa and the vertical effective stress as 194kPa at the sampling depth of 18m below ground level.

A large number of self-boring pressuremeter tests were conducted during the site investigation and the results are shown in Figure 5-2. The scatter is significant but the horizontal total stress increase with depth are between one and three times the total vertical stress for most data points. Burland et al. (1979) calculated typical values for $K_o$ in London clay and showed the significant effect of surcharge
pressure. Their results are shown in Figure 5-3 for the two cases of no surcharge and 100kPa surcharge pressure, assuming hydrostatic pore pressure conditions. At the sampling depth of 15.5m below the top of the clay surface, $K_0$ ranges between 1.5 and 2.3 for the two cases. At Heathrow - Terminal 4, the surcharge pressure on the London clay is approximately 45kPa and on the basis of Figure 5-2 and Figure 5-3 a $K_0$ of 2.0 was used to calculate the horizontal effective stress as 388kPa.

5.1.3. Sampling and storage of London clay

The 7.9m diameter tunnel was constructed in a top-heading, bench and invert configuration. The sequence of excavation was to advance the top-heading 1.2m, then the bench by 1.2m and again the top heading and bench before advancing the invert by 2.4m. The advance lengths were decreased to 1.0m or 0.8 meters near cross passages or when overhead structures were particularly close to the crown. The closure of the invert lagged behind the front of the crown by approximately 5 meters.

All samples were taken from the bench at three different locations; the up-line tunnel at chainage 182 (19/3/96), the up-line tunnel at chainage 236 (15/5/96), and the concourse tunnel at rib 53 (30/10/96). The first step in the sampling process was to remove the 50mm temporary shotcrete lining from the tunnel face by excavator. The orientation of the in-situ clay was marked by painting horizontal lines on the exposed face. Bright spray paint was used for this purpose. Two trenches were then excavated into the bench approximately one meter apart. This isolated a primary block of material approximately 1 meter wide, 1.5 meters high to a depth of 1 meter. Horizontal lines were sprayed on the sides of the primary block as soon as the sides were exposed. Once the primary block had been sprayed, the excavator bucket was used to apply a lateral force to its side. This action had the effect of breaking the primary block into a number of secondary blocks. As a result of the blocky nature of the clay, it tended to break on pre-existing fissures. Suitable secondary blocks were placed to one side. A block was deemed to be suitable if sprayed lines were visible

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on at least two sides, allowing the *in situ* orientation to be determined. The whole operation took between ten and fifteen minutes.

Once the secondary blocks were transported to a quiet part of the tunnel, they were further evaluated. Preference was given to blocks which exhibited the least number of fissures. These blocks were cut into smaller sizes by using a machetti, as well as a hammer and builders chisel. Once again, the aim was to produce blocks with the least number of fissures, but as was discovered during the final sample preparation in the laboratory, no one sample was completely without fissures. During trimming of the blocks, care was taken to retain their orientation by spraying lines on the sides as they were reduced in size. The trimming operation took between two and three hours and once all the samples were of such a size as to fit into the wooden crates (400 x 400 x 400 mm), they were removed from the tunnel.

At the surface the samples were covered with coatings of paraffin wax and clingfilm. A minimum of three consecutive coatings of wax and clingfilm (3 layers of clingfilm per coating) was applied to each sample before it was placed into the wooden crates. The crates were filled with expanded foam to minimise relative movement between sample and crate. When samples were taken at the first location (19/3/96), the samples were brought to the surface at approximately midnight on a cold clear evening. Fissures started opening within a few minutes of reaching the surface and all samples were immediately covered in one layer of wax. Once this was complete, each sample was in turn coated with subsequent layers of wax and clingfilm. It was significant that there was no visible opening of fissures in the tunnel, even though the trimming process took a few hours, but fissures opened soon after reaching the outside. On the basis of this experience the sampling procedure was modified insofar as all trimmed samples were covered with clingfilm in the tunnel, before being transported to the surface. Limited working space in the tunnel prevented waxing of the samples under-ground. A total of eleven block samples were taken, all large enough to cut at least one 100mm diameter triaxial specimen to a length of 200mm.
As the samples taken from Heathrow Terminal 4 were stored for a considerable time, the decision was made to investigate the effectiveness of clingfilm and paraffin wax as protection against moisture loss. The opportunity was used to compare the effectiveness of different sealing techniques. During the sampling carried out on 30/10/96, additional samples were taken for this purpose. The samples were trimmed into six blocks and each protected against moisture loss by a different method. Sample A was covered only with a layer of paper towel. This sample was effectively unprotected against moisture loss and used as a reference. The purpose of the paper towel was only to prevent pieces of soil separating from the sample on drying. The protective treatments used for the other samples are shown in Table 5-1.

<table>
<thead>
<tr>
<th>Sealing Method</th>
<th>Sample size (mm)</th>
<th>Initial sample weight (g)</th>
<th>Sample sealing (no. of layers &amp; sealant weight(g))</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>110 x 100 x 90</td>
<td>3038.3</td>
<td>Paper towel only (1 layer, 4.1g)</td>
</tr>
<tr>
<td>B</td>
<td>120 x 110 x 80</td>
<td>2418.05</td>
<td>Cling film only (9 layers, 10.3g)</td>
</tr>
<tr>
<td>C</td>
<td>110 x 100 x 100</td>
<td>2574.80</td>
<td>Cling film (9 layers, 8.15g)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Aluminium foil (4 layers, 23.8g)</td>
</tr>
<tr>
<td>D</td>
<td>140 x 110 x 80</td>
<td>3125.8</td>
<td>Cling film (3 x 3 layers, 13.0g)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Paraffin wax (3 layers, 87.5g)</td>
</tr>
<tr>
<td>E</td>
<td>110 x 110 x 110</td>
<td>3083.0</td>
<td>Paraffin wax only (3 layers, 143.5g)</td>
</tr>
</tbody>
</table>

Table 5-1. Sealing of London clay.

The samples were placed in a cardboard box, large enough to ensure that there was no physical contact between the individual samples and stored in the same room as the samples used for triaxial testing. The room temperature was not controlled. Measurements of the temperature and relative humidity were made each time the samples were weighed. The temperature varied between 17°C and 24°C and the relative humidity between 27% and 65% over the test period of 15 months. The moisture losses are shown in Figure 5-4. Moisture loss was rapid for the unprotected
sample with a 6% loss after only one day and 50% moisture loss after 14 days. Similar results were found by Hvorslev (1949) on Boston blue clay. Figure 5-4 also shows how ineffective clingfilm is in protecting the clay, when used in isolation. This sample lost 7% of its moisture after 100 days. In contrast, the sample protected only by wax lost 1.5 % and the clingfilm - wax combination reduced the moisture loss to 0.6% over a period of 100 days. The clingfilm - wax combination produced better results than the wax in isolation, as the reinforcing effect of the clingfilm prevent small cracks from opening in the wax (Clayton et al. (1995)). Of all the methods investigated, the cling film - aluminium foil combination proved to be the most effective protection against moisture loss and was marginally better than the clingfilm - wax combination. This has some practical implications, as the time and effort required to use aluminium foil is much less than that required for wax.

All the London clay samples for this research project were protected against moisture loss by cling film and wax similar to Method D above. An assessment of moisture loss can be made from the data in Table 5-2. It shows the storage times and water contents during specimen preparation (w<sub>i</sub>).

<table>
<thead>
<tr>
<th>Test no.</th>
<th>Date Sampled</th>
<th>Location</th>
<th>Storage Time (Days)</th>
<th>w&lt;sub&gt;i&lt;/sub&gt; (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>LC1</td>
<td>19/3/96</td>
<td>Upline Ch182</td>
<td>155</td>
<td>25.5</td>
</tr>
<tr>
<td>LC2</td>
<td>15/5/96</td>
<td>Upline Ch236</td>
<td>162</td>
<td>25.6</td>
</tr>
<tr>
<td>LC3</td>
<td>19/3/96</td>
<td>Upline Ch182</td>
<td>269</td>
<td>25.2</td>
</tr>
<tr>
<td>LC4</td>
<td>30/10/96</td>
<td>Concourse Rib 53</td>
<td>128</td>
<td>24.7</td>
</tr>
<tr>
<td>LC5</td>
<td>15/5/96</td>
<td>Upline Ch236</td>
<td>512</td>
<td>25.7</td>
</tr>
</tbody>
</table>

Table 5-2. London clay.

The moisture contents of the samples showed relatively small variation. At the time of testing, the moisture contents of all the samples were between 25.2 ± 0.5%. The differences in moisture content between the samples were small even though the individual times between sampling and specimen preparation varied considerably.
Furthermore, there is no systematic variation in the moisture content for an increased storage time. This suggests that the combination of cling film and wax was sufficient protection to minimise moisture loss.

5.1.4. *London clay specimen preparation*

During the course of the research project, the method of specimen preparation for the London clay was continuously improved as experience was gained. The method described here was used for preparing 100mm diameter samples to be tested in the medium-pressure triaxial apparatus.

After removing a block of London clay from the wooden crate, the clingfilm and wax layers were removed and the sample inspected for any evidence of disturbance. Some samples showed mould growth, but in no case was any evidence of opened fissures found. The sizes of LC1 and LC2 were reduced by cutting pieces off the block by means of a hand held bow saw. This proved to be a slow and ineffective method. The clay cuttings tended to stick to the blade as it was remoulded by the forward and backward movement of the blade. A number of slices were taken off the sides of the block to produce an approximately cylindrical specimen with a flat base and a diameter of between 150 and 200mm. The specimen was then placed in a temporary shelter erected over the workbench using a large ground sheet. A humidifier which produced fine mist was also placed under the shelter in an attempt to minimise moisture loss from the specimen during preparation. The trimming process was continued by placing the specimen on its base and cutting small shavings off the sides by means of a knife. Once the specimen was small enough, it was placed in the soil lathe shown in Figure 5-5 and further trimmed down to the required diameter of 100mm using both the knife and a tensioned wire. Preparation time was between eight and ten hours per specimen. The long time for preparation proved to be a problem because fissures started to open on the clay surface.
The preparation method was significantly improved by replacing the bow saw with a band saw. As the band saw blade moved in one direction, it removed the clay cuttings from the sample, reducing the tendency of the cuttings to stick to the blade. This improved the cutting efficiency. The band saw was used from LC3 onwards. The process was started by making the first cut along a plane corresponding to the *in situ* horizontal plane. The block was then placed on this flat surface and subsequent slices taken off the sides of the sample. A circular paper template with a diameter of 104mm was held in place on the top of the sample with drawing pins. This was used as a guide to cut the specimen to a size suitable to be placed into the soil lathe. The band saw operation typically took 20 to 30 minutes. Once the sample was in the lathe, the knife and tensioned wire was used to trim the sample down to 100mm diameter. Total sample preparation time was reduced to less than 90 minutes.

Particular attention was given to trimming the ends of the specimen as it was recognised that the specimen had to be a right cylinder to ensure axis-symmetrical load application in the triaxial apparatus. This procedure was also continuously improved during the course of the research project as experience was gained. The ends of LC1 and LC2 were trimmed using the metal blocks shown in Figure 5-7 (Figure 5-7 shows the blocks used as a cradle when fixing the LVDT brackets). Two blocks were bolted together producing a relatively flat surface which was used as a guide when scraping the specimen end by means of a metal ruler. The result was not satisfactory, as the edges of the two metal blocks were not perfectly aligned. A new purpose-made end trimming cradle was manufactured (Figure 5-6) and used from sample LC3 onwards. The cradle was made from a hollow cylinder with an inside diameter of 103mm and a wall thickness of 18mm. Part of the cylinder wall was removed by machining, leaving the whole cylinder intact for a length of only 50mm. This allowed the specimen to be held stationary by hand, when placed inside the cradle. In this position, the end was trimmed flush with the face of the cradle. A thick metal scraper (250 x 50 x 10mm), machined with a sharp edge, was used as the trimming tool. The above method enabled specimen lengths to be trimmed
routinely to differ by less than 0.3mm when measured at four positions 90° apart along the circumference of the specimen.

On completion of the preparation, the specimen was weighed and the geometry measured using a pair of large digital vernier callipers. The diameter was recorded at six positions (top, middle and bottom, each at a 90° angle) and the length was measure at four positions.

5.1.5. London clay specimen set-up

A latex membrane was prepared by soaking it in de-aired water. After drying both the inside and outside using paper towels, the membrane was placed over the specimen and the specimen laid down horizontally as shown in Figure 5-7. In this orientation, the positions for the LVDT brackets were measured and marked on the membrane. Both brackets were temporarily clamped to an aligning tool (Figure 5-7) to ensure their accurate relative position. The brackets were first glued onto the membrane using silicone sealant and pinned using 25mm long pins (φ = 0.8mm) before tightening the pin retaining screws. The pin heads and retaining screws were subsequently covered with silicone sealant to prevent membrane leaks. The same procedure was repeated for the two brackets on the opposite side of the sample. All tests on 100mm specimens of London clay were carried out at a pin to pin gauge length of 72mm.

A small positive pressure was applied to the back pressure line until a thin layer of water was visible on the bottom high air entry ceramic before carefully sliding the specimen onto the base pedestal. The sliding action minimised the risk of trapping air between the sample and ceramic. Next the top cap was placed in position. (The rubber-metal interfaces of the top cap, where relative movement could occur, were lubricated before each test with grease to ensure unrestrained movement). A total of six O-rings were used to seal the top and bottom of the membrane against the base pedestal and the top cap. Next the radial caliper was fitted to the specimen.
Temporary supports held the radial caliper in position whilst the silicone sealant, used as adhesive between the caliper pads and membrane, was allowed to set. Once the supports were removed, preparations were made for the installation of the mid-plane probe. A small hole was cut into the membrane slightly below the position of the caliper and the flange of the latex grommet sandwiched between the membrane and the specimen. This was followed by drilling a slightly oversized hole to accept the probe of 11.7mm diameter. Some soft remoulded clay was replaced into the hole before the probe was inserted. This was done to avoid cavities between the high air entry ceramic and the clay. A pin held between the probe and grommet allowed air and excess remoulded clay to be expelled as the probe was pushed into the hole. The orientation of the probe was important. The first small-bore stainless steel tube entered the back of the probe casing at the centre and the second one entered the casing at the edge. The casing was orientated with the off-centre tube at the top. This was necessary to remove any air during the de-airing process, as de-aired water entered the probe through the centre tube and exited the probe through the off-centre tube. Two small O-rings were placed over the stretched membrane and grommet to seal the specimen from the cell water. Flushing of the probe was performed once a sufficiently high cell pressure was applied to ensure positive pore pressure. Saturation of the probe was confirmed by immediate response of the mid-plane pressure in subsequent B-value tests.

Each axial LVDT was clamped to the top bracket, taking care to position the armature to give an output positive relative to the electrical zero. The aim was to have the LVDTs as close as possible to the electrical zero once the sample reached the in-situ stress and therefore allowance had to be made for some axial extension during reconsolidation. This was necessary as the LVDT could only be zeroed, at high amplification, if the armature was sufficiently close to the electrical zero. Experience allowed judgement on the initial position of the LVDTs to be refined. It was possible to zero the LVDTs for all tests with the exception of LC1. A lower amplification level had to be used for LVDT1 during the shear stage of LC1. The
radial LVDT was clamped with its armature near to the electrical zero, as very little lateral strain occurred during reconsolidation to the in-situ stress.

Figure 5-8 shows a typical London clay specimen on completion of the set-up procedure.

One 38mm London clay sample (LC5) were tested in the high-pressure triaxial system. The purpose of this test was to investigate the isotropic consolidation behaviour of the clay and, if possible, to determine the isotropic yield stress. The set-up procedure was similar to that of the 38mm Chalk specimens and will be described in section 5.2.3. However, some differences must be mentioned. Only one membrane was used as opposed to the two membranes used for the tests on the Chalk. One membrane was deemed sufficient protection, even at the high cell pressures used, on account of the smooth texture of the clay surface. In the event, no membrane leak occurred during the test. Two vertical filter paper strips (5mm wide) were used to enhance drainage. The strips were diametrically opposed at positions avoiding the LVDT brackets. The drains reduced the consolidation period from several months to approximately three weeks. A saturated high air entry ceramic disc was placed between the clay sample and the base pedestal. A high air entry ceramic was used in order to minimise the migration of water from the ceramic into the specimen during set-up. Such migration would lead to disturbance in the form of a reduction in initial mean effective stress.

5.1.6. Disturbance of London clay prior to testing

The stiffness of geomaterials can be significantly altered if disturbance occurs prior to testing (see for example Hight (1986)). Some disturbance of material in the vicinity of a tunnel will inevitably occur as a result of the tunnelling process. Therefore, some judgement had to be made on the amount of disturbance that the material would have suffered prior to sampling.
The initial mean effective stress ($p_i'$) of the London clay specimens was measured by applying a sufficiently high cell pressure to produce a positive pore pressure. The results are shown in Table 5-3.

<table>
<thead>
<tr>
<th>Test no.</th>
<th>Storage Time (Days)</th>
<th>$w_i$ (%)</th>
<th>$p_i'$ (kPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>LC1</td>
<td>155</td>
<td>25.5</td>
<td>173</td>
</tr>
<tr>
<td>LC2</td>
<td>162</td>
<td>25.6</td>
<td>542</td>
</tr>
<tr>
<td>LC3</td>
<td>269</td>
<td>25.2</td>
<td>440</td>
</tr>
<tr>
<td>LC4</td>
<td>128</td>
<td>24.7</td>
<td>446</td>
</tr>
</tbody>
</table>

Table 5-3. London clay, initial mean effective stress.

Recall from Section 5.1.4 that in an attempt to counter moisture loss during sampling preparation, a humidifier was used. It should be recognised that the humidifier may indeed have introduced moisture to the specimens during preparation. However, the risk of moisture loss or introduction would have been considerably less for shorter specimen preparation times. The specimen preparation times were significantly reduced from LC3 onwards by the introduction of the band saw. It is noteworthy that $p_i'$ varied dramatically for the two samples with the long preparation times (LC1 and LC2), whereas $p_i'$ was almost identical for the two sample prepared by the improved technique (LC3 and LC4). The initial mean effective stress for these two samples were approximately 120 kPa higher than the estimated in-situ mean effective stress of 323 kPa. This evidence suggests that the material suffered some disturbance prior to sampling. An attempt was made to evaluate the level of disturbance.

One approach is to match the observed change in mean effective stress to experimentally determined changes in mean effective stress for which the strain path is known. This technique has been used to investigate the strains imposed during tube sampling (see for example Georgiannou and Hight (1994) and Siddique et al. (1998)). These workers have used the centre-line strains predicted by Baligh
(1985) as a basis to conduct triaxial strain path testing. Such strain paths include positive and negative strain cycles of known magnitude. For soft normally consolidated clays, a reduction in mean effective stress have been found from tube sampling (see for example Clayton et al. (1992)) as well as from the strain paths employed in the triaxial apparatus (see for example Siddique et al. (1998)). However, for stiff overconsolidated clay, agreement has not been good. This will be discussed below.

A stiff overconsolidated clay will exhibit an increase in initial mean effective stress during tube sampling. Chandler et al. (1992) found an increase in mean effective stress of approximately 220kPa when comparing the pore fluid suctions from U100 tube samples of London clay to those of block samples. It was argued that the higher suctions were as a result of the strains imposed on the specimens during sampling. Georgiannou and Hight (1994) were unable to reproduce this effect in the triaxial apparatus. They imposed triaxial strain paths of 1% positive strain, followed by 1% negative strain on two overconsolidated clays (London clay and Vallericca clay). This strain path resulted in a 10% reduction of the mean effective stress. They concluded that the increased mean effective stress observed for tube samples of stiff clays is dominated by the effect of the intense shearing which occurs at the periphery of the sample and not the centre-line strains.

The estimation of strains induced to the material in the tunnel face prior to sampling by matching observed change in mean effective stress to experimentally determined changes is an attractive method. However, the above example clearly indicates that in order to simulate the changes in mean effective stress accurately, it is important to apply the correct mode of shear. The mode of shear of the material in the tunnel face when full account is taken of the three dimensional effects are complex. The idealisation of these strains as either axis-symmetrical, plane-strain or simple-shear is probably an oversimplification. On the basis of the above, this method was rejected.
A research project is currently underway at the University of Surrey to investigate the performance of the station tunnels at Heathrow - Terminal 4. The project includes an extensive monitoring programme to measure the behaviour of the tunnel. In addition, three dimensional finite element analyses are being conducted to compare the predicted behaviour to the measured behaviour. Preliminary results from the analysis estimate the strain levels of the material on the centre line of the tunnel to be of the order of 1% (van der Berg (1998)). This may be compared to the typical strain level at failure of London clay. Bishop et al. (1965) used high quality block samples from Ashford Common to investigate the behaviour of London clay. He showed that in general failure in triaxial compression occur at strain levels of approximately 2%. Comparison of the estimated strain level prior to sampling and the typical strain level at failure leads to the conclusion that the London clay was subjected to significant levels of disturbance prior to sampling, but it was probably not failed.

5.1.7. London clay testing procedure

Immediately after filling the triaxial cell, a cell pressure of 700kPa was applied to the specimen under undrained conditions. The response of the mid-plane probe was monitored and compared to that of the pressure response at the base of the specimen. If the mid-plane response was sluggish, the probe was flushed with desired water. The de-airing procedure was such as to avoid a pressure in the cavity behind the ceramic in excess of the applied cell pressure. Once the probe was deaired, time was allowed for all the pressures to reach equilibrium. B-values were measured by applying a cell pressure increment and monitoring the response of the mid-plane and base pressures. Close correlation between the two pressures was typically observed. B-values ranged between 0.97 and 1.0 (see Table 5-7).

All specimens of London clay were reconsolidated to the estimated in-situ stress \( \rho_o' = 323\text{kPa}, q_o' = -194\text{kPa} \) prior to shear. Reconsolidation was carried out in two stages. During the first stage, specimens were swelled or consolidated under
isotropic conditions to a mean effective stress of 388kPa. The second stage comprised unloading to the in-situ stress along a drained stress path.

As a result of the top cap design, loading rates for the London clay were not constant. However it was shown in the previous chapter that the shear rate during the initial stages of the test was typically 0.2%/day for both compression and extension tests. The measured creep rate was typically less than 0.5% of the subsequent local shear rate (Table 5-8). LC1 was the only exception with a ratio of 2.17% Rest periods of between 6 and 12 days were required between reaching the in-situ stress and commencing with shear.

Undrained cyclic stress excursions were conducted during LC1, LC2 and LC3. The aim was to identify the $Y_1$ and $Y_2$ yield surfaces (Jardine et al. (1991)). As explained in the previous chapter, the cyclic loading was automated and the magnitude of the load excursions as well as the rest periods between excursions were increased between cycles. Nevertheless, significant problems were experienced with creep of the London clay. It was found that creep was re-activated by the load increments. By the time the next load cycle commenced, the creep from the previous cycle had not yet reduced sufficiently. A typical result is shown in Figure 5-9.

Hight and Higgins (1995) have pointed out a number of difficulties when conducting cyclic loading tests on material that exhibits significant creep. The behaviour after stress reversal depended on the rate of shear before reversal, the pause period and the overall strain level on reversal. They noted that if the pause period was sufficiently long to allow the creep rate to decrease to the same level as before shear commenced, the stiffness response appeared to be similar to that on first loading. However, they warned that if the pause period was not sufficiently long, strains could continue initially in the same direction as the direction of the applied stress immediately prior to reversal. This may lead to the erroneous interpretation of an infinite or negative stiffness. On the basis of the long rest period required for the London clay and the associated problems as outlined above, the
decision was made after completion of LC3 not to proceed with the cyclic test programme. A programme of monotonic loading from the in-situ stress state was adopted.

One test on London clay (LC5) was consolidated in the high-pressure triaxial apparatus. The aim of the test was to investigate the consolidation behaviour of the London clay under isotropic stress application and, if possible, to identify the isotropic yield stress. The test was conducted at a back pressure of 670kPa and the B-value before the start of consolidation was measured as 0.95. Consolidation was conducted in two stages. During the first stage the coefficient of consolidation was determined. This was done by rapidly increasing the cell pressure from 860kPa to 1300kPa and measuring the volume change (externally) over a period of 7 days. The time for 50% consolidation was determined as 6.1 hours by the Casagrande method (Casagrande (1938)) and the time to 90% consolidation as 27.6 hours by the method of Taylor (Taylor ((1948a)). The consolidation coefficient, as calculated by these two methods, differed by less than 5%, and the average value was 0.2m²/year. The second consolidation stage was conducted by continuously increasing the cell pressure at a low enough rate as to allow sufficient dissipation of excess pore pressures. The rate of cell pressure increase was calculated using the method suggested by Davison and Atkinson (1990) as 8 minutes/kPa. This rate was slow enough to ensure a maximum excess pore pressure of less than 20% of the mean effective stress. Davison and Atkinson (1990) found this criterion to be sufficiently stringent to give accurate measurements of the e-log p' curves of reconstituted London clay.

The second consolidation stage was conducted over a period of 17 days, up to a mean effective stress of 4100kPa. The result is shown in Figure 5-10 and includes local axial displacements as measured by the two LVDTs. The displacements measured by the two LVDTs were almost identical and good correlation exists between the shape of the void ratio curve, as measured by the external volume gauge and the two LVDTs. The isotropic yield stress can not be readily identified
from Figure 5-10. The compression data is re-plotted in Figure 5-11 to a log scale and includes the intrinsic compression line (ICL) as suggested by Burland (1990). It shows that the consolidation line of the intact natural clay progresses to the right of the intrinsic compression line. This behaviour is typical of materials exhibiting bonding (Burland (1990)). Burland et al. (1996) noted that for heavily overconsolidated clays it is typical for the consolidation line to bend down at high consolidation pressures as structural breakdown gradually occurs. Such response was not noticed for LC5 and a yield stress can not be identified from Figure 5-11.

A summary of the tests conducted on London clay is shown in Table 5-4.

<table>
<thead>
<tr>
<th>Tests conducted</th>
</tr>
</thead>
<tbody>
<tr>
<td>LC1 Cyclic loading. Monotonic undrained compression to failure.</td>
</tr>
<tr>
<td>LC2 Cyclic loading. (Specimen failed during cyclic loading on pre-existing fissure.)</td>
</tr>
<tr>
<td>LC3 Cyclic loading. Monotonic undrained extension to failure.</td>
</tr>
<tr>
<td>LC4 Monotonic undrained compression to ( q = -60 \text{ kPa} ). Monotonic undrained extension to failure.</td>
</tr>
<tr>
<td>LC5 Isotropic consolidation (conducted in high-pressure apparatus).</td>
</tr>
</tbody>
</table>

**Note:**
All first cyclic excursions and monotonic shear stages preceded by consolidation to the in-situ stress along a drained extension stress path.

Table 5-4. Tests conducted on London clay.

5.1.8. **Typical results of tests on London clay**

The stress paths of the monotonic undrained tests conducted on the London clay are shown in Figure 5-12. All the stress paths in \( p' - q \) stress space have negative slopes. The initial part of the slopes are remarkably similar with a slope of approximately 1:4. Non-vertical slopes of the stress path in \( p' - q \) stress space are typical of anisotropic material and stem from the cross-coupling between the volumetric and
shear components (Graham and Houlsby (1983)). Furthermore, the negative slope confirms that London clay has a higher horizontal stiffness than vertical stiffness. Too few tests were conducted to construct a yield surface. This difficulty is compounded by the fact that the isotropic yield stress can not be identified from test LC5 (Figure 5-10). Furthermore, two test failed prematurely (LC2 and LC4). During extension, LC3 and LC4 followed almost identical paths, but LC4 failed at a deviatoric stress of almost 100kPa less than that of LC3. Inspection of LC4 after removal from the triaxial cell showed that the specimen failed on an almost perfectly horizontal plane at the level of the top bracket pins. The marks where the pins were located in the soil were clearly visible. The failure plane was not a pre-existing fissure, but it is possible that the pins may have weakened the plane sufficiently to cause premature failure.

The stiffness behaviour of all the London clay specimens which underwent monotonic shear are shown in Figure 5-13. It may also be seen that the extent of the linear plateau was larger for both the compression tests (LC1 and LC4(Uco)) compared to the extension test (LC3 and LC4(Uex)). Some variation in the stiffness of the three specimens is evident at an axial strain of 0.001%. However, the stiffness of LC4 at 0.001% axial strain was very similar in compression and extension.

52. Chalk

Previous research carried out at the University of Surrey has been on the compressibility of weak rock (Matthews (1993)). One of the materials investigated at the time was high porosity Chalk from a quarry in Suffolk. The material was high porosity Chalk. The work by Matthews (1993) included field stiffness measurements in the form of seismic surveys. In Chapter 2 it was stated that one of the aims of this project was to compare field seismic stiffnesses with triaxial stiffnesses. Since the quarry is still in operation and samples could readily be obtained, it was decided to include Chalk from the same quarry in this study.
5.2.1. Description of Chalk

Chalk is a weak rock. It forms part of the Upper Cretaceous of Western Europe and outcrops over approximately 15% of the land surface of England. The following discussion on the origin of Chalk is taken largely from Clayton (1983) and Clayton (1990).

Chalk consists almost entirely of the skeletal remains of marine planktonic algae which drifted in the photic zone. Individual plates of the algae are arranged in disks known as coccoliths. Up to twenty coccoliths may have formed a spherical body known as a coccosphere. Soft white chalk is made up predominantly of the remains of coccoliths and coccospHERes. The result is a weakly cemented material with particle sizes ranging between 0.5μm and 10μm. Deposition of the algal debris occurred under gravity through water. This resulted in a high porosity carbonate mud on the sea floor. Diagenesis, in the form of cementation associated with burrowing benthos, started soon after the material reached the sea bed. Cementation imparted sufficient strength to the material to allow it to withstand subsequent overburden and resulted in the high void ratios of the material. Various additional diagenetical processes have significantly influenced the behaviour of Chalk. These include, consolidation, compaction, tectonism and late-stage solution.

The chemical composition of Chalk consists mainly of calcium carbonate (CaCO₃). Even though the chemical composition is uniform, the Chalk is physically highly variable, particularly with regards to density. Variations in dry density between 1.29 Mg/m³ and 2.46 have been reported by Clayton (1983). This represents a variation in porosity between 9% and 52%. At the one extreme, Chalk may have an in-situ moisture content far in excess of its liquid limit and to the other extreme it may have a porosity similar to that of limestone. This large variation in porosity results from differences in diagenetic and other post-depositional processes.
The strength of intact Chalk is strongly dependent on porosity. Matthews and Clayton (1993) showed that the strength of intact Chalk increases as the porosity decreases. They found this to be true for both dry and saturated Chalk specimens.

Discontinuities are common in Chalk and are often present in the form of sets of sub-horizontal and sub-vertical fractures. These fractures separate the rock mass into discrete intact blocks. As a result of weathering, fractures are more closely spaced near the surface. Clayton et al. (1994) showed that discontinuities have a significant effect on the mass stiffness of fractured Chalk. They compared the mass and intact stiffness of the Chalk from three sites with a wide range of porosities. Intact stiffness was determined from locally instrumented specimens tested in unconfined compression. The mass stiffness was measured using surface wave geophysics as well as large diameter plate tests (Matthews (1993)). They found that the mass stiffnesses from the three sites were broadly similar, but that the intact stiffness was strongly dependent on porosity. This led them to conclude that the mass stiffness of Chalk was dominated by the stiffness of the discontinuities.

5.2.2. Sampling of Chalk

Intact samples of the Chalk was taken at Needham Market Quarry in Suffolk. Matthews (1993) found the porosity of Chalk from the same site to be approximately 50% and the unconfined compressive strength of the intact material to range between 3.1MPa and 5.5MPa.

Intact Chalk samples were taken for this research project at the position shown as “1997 samples” in Figure 5-14. A mechanical shovel was used to excavate fresh Chalk from the quarry face. The action of the shovel produced material predominantly in the form of blocks up to a few hundred millimetres in size. These blocks were inspected for fissures, which were visible as brown stained lines. Preference was given to blocks with the least fissures. It was found during specimen preparation however, that no specimen was entirely without fissures.
The block samples selected for testing were covered in numerous layers of cling film and aluminium foil before being transported to the University of Surrey. Upon arrival at the university, all blocks were covered by additional layers of cling film and paraffin wax.

5.2.3. Chalk specimen preparation and set-up

The Chalk testing programme commenced once the London clay testing programme was at an advanced stage. At this time, specimen preparation procedures for the London clay had been refined to include the use of the end trimming cradle. This technique gave satisfactory results for the London clay and a similar procedure was therefore adopted for the preparation of the Chalk specimens. The block samples were trimmed down to the desired size by means of a bow saw. The remoulded material, produced by the cutting process, was easily removed from the blade by hand and clogging did not prove to be a problem as in the case of the London clay. A knife was used to reduce the size of the sample further before it was placed in a lathe and trimmed down to a diameter of 38mm. The ends of the sample were cut in a purpose made cradle similar to the one shown in Figure 5-6. The samples were checked to ensure that they conformed to a right cylinder by placing it upright on a flat surface and using a precision right angle to examine the verticality of the sample sides. If non-verticality was apparent, the sample ends were retrimmed. A vernier caliper was used to measure the sample diameter at six positions and the length at four positions in a configuration similar to that used for the London clay. Sample lengths were typically between 76 and 80mm. Because of the small sample size and ease with which the Chalk could be trimmed, preparation times were generally less than one hour.

Two latex membranes were used for each of the Chalk specimens tested in the high-pressure apparatus. Two membranes were adopted after a number of membrane leaks occurred during preliminary tests using a single membrane. For each test, both
membranes were tested before and after the test to check for leaks. This was done by submerging the air-inflated membrane in water and observing any air bubbles escaping. The inner membrane was cut to a length approximately 25mm longer than the length of the specimen and the outer membrane to a length approximately 55mm longer. Both membranes were placed over the specimen before laying it down horizontally in a purpose made cradle. As described in Chapter 4, the brass brackets used to hold the LVDTs in position were temporarily clamped to an alignment tool to facilitate alignment. The design of this tool was slightly different from the one used for the London clay. The top bracket was clamped to the tool in a similar fashion, but the bottom bracket was fixed to the tool via a thread extending from one end of the tool. The threaded hole in the bottom bracket was later used to accept a fine pitched screw which allowed accurate initial positioning of the LVDT armature. The brackets were glued to the membrane, at a centre-to-centre distance of 38mm by means of cyanoacrylate adhesive (Super Glue). Latex rubber bands were placed around the brackets to hold them in position during LVDT set-up. The use of pins was avoided because of the small specimen size and brittle nature of the Chalk.

The Chalk specimens were positioned directly on the base pedestal and the flat top cap was placed directly on top of the specimen. A total of eight o-rings were used to seal the two latex membranes. Two at the end of each membrane.

5.2.4. Chalk testing procedure

The testing procedure for the Chalk specimens was conducted in three stages, namely a saturation phase, consolidation phase and shear phase.

The average initial degree of saturation (S) of the Chalk was 95.7. The initial degree of saturation typically ranged between 93% and 98% (see Table 5-10). One notable exception was CH13 which had an initial degree of saturation of 84.3%.
It was attempted to fully saturate \( S = 1 \) all specimens in the triaxial apparatus. This was necessary in order to allow the measurement of pore pressure changes during undrained shear. In addition, fully saturated specimens were required as an external volume gauge was used to measure volume changes during consolidation.

Elevated back pressures were used during the saturation process. The theoretical minimum back pressure \( P_{100} \) necessary to fully saturate the Chalk samples was calculated from equation (5-2) (see section 5.4.1) as 212 kPa. According to Figure 5-24, 60 days would be the required time to fully saturate a Chalk sample with a degree of saturation of 95.7%, when applying the minimum back pressure. However, increasing the back pressure above \( P_{100} \) markedly reduces the required time to saturation. The minimum back pressure used during saturation was 600kPa. At this back pressure, the estimated time to full saturation is 20 hours for a sample with an initial degree of saturation of 95.7%.

The saturation procedure was started by first applying a cell pressure of 150kPa to the specimen at 50kPa back pressure. Both the back pressure and cell pressure were then simultaneously increased at a rate of 90 kPa per hour, maintaining a constant difference of 100kPa between the cell and back pressures. Once the target pressures were reached, a rest period was observed to allow the air to dissolve into the pore fluid.

Two factors govern the B-value of saturated \( S = 1 \) geomaterials for with the skeleton compressibility is equal to or more than that of water. These are the porosity and the ratio between the skeleton and water compressibility (Bishop (1973), see section 5.4.1). In particular, when the compressibility of the skeleton is low, B-values of less than unity will occur for a fully saturated \( S = 1 \) geomaterial. As an example, from the work of Bishop (1973) it may be shown that when the skeleton and water has the same compressibility, the B-value will be 0.5 for a porosity \( (n) \) of 1.
The level of saturation of the Chalk specimens was evaluated by measuring the pore pressure response to an increase in cell pressure. Typical B-values at full saturation ($S = 1$) was found to be between 0.88 and 0.92. Full saturation was confirmed by an unchanged B-value for a number of measurements at sufficiently long time intervals between each measurement (see section 5.4.1). If B-values were not in the range 0.88 to 0.92, the back pressure was raised further and more time allowed for saturation. These B-values of 0.88 to 0.92 may be compared to the values found by Wissa (1969). He investigated the B-values at full saturation of Portland cement stabilised soils. He found that the B-value reduced as the initial Youngs modulus of the material increased. At a modulus of 4 GPa, which is similar to the modulus measured for the Chalk, he found B-values of approximately 0.9.

Two specimens had markedly lower B-values than the rest. CH3 had a value of 0.65 and CH15 a value of 0.41. Inadequate saturation was initially suspected as the reason for these low values. As CH3 was tested in the high-pressure apparatus a high back pressure could be applied. For this test, the back pressure was incrementally raised to 1900kPa over a number of days. According to Black and Lee (1973) time to full saturation at 1900kPa was less than 2 hours. The specimen was conservatively held at 1900kPa for 36 hours before the B-value of 0.65 was measured. This was confirmation that the specimen was indeed saturated, despite the low B-value.

One Chalk specimen was tested in the medium pressure triaxial apparatus. This was test CH15. CH15 was the specimen for which the Fabry-Perot interferometer was used as local displacement instrument. This specimen was tested in the medium-pressure triaxial apparatus which did not allow high back pressure application. Water was used as the pressurising medium during the saturation phase and air during the shear phase. During saturation, the cell pressure was 1000kPa and the back pressure 850kPa. Even though full saturation was theoretically possible at these pressures, it was foreseen that the dissolved air would come out of solution at the lower pressure used during the shear phase. For this reason, de-aired water was
flushed through the sample under a pressure differential of a few kPa between base pedestal and top cap. This procedure was continued for several days. After flushing, constant B-values of 0.41 were measured over a period of four days at a back pressure of 850kPa. On the basis of the work done by Black and Lee (1973), this was taken to confirm full saturation.

Specimens were consolidated under isotropic conditions to the required mean effective stress prior to shear. CH1, CH3, CH4, CH5 and CH15 were sheared at initial mean effective stresses well below the isotropic yield stress of the Chalk. CH6, CH7, CH12 and CH13 were sheared after isotropic consolidation to stress levels close to the isotropic yield stress. In contrast CH8 and CH14 were sheared after consolidation to isotropic stresses well in excess of the isotropic yield stress.

After some experimentation, the (machine) shear rates were standardised at 0.6%/day. Creep rates prior to shear were monitored by the local instrumentation for all tests and are shown in Table 5-11. These creep rates were generally less than 2% of the local shear rates. Two notable exceptions were CH4 and CH7. Using a restriction on locally-measured creep rate as a ratio of externally measured shear rate may lead to errors on the unsafe side. This may be demonstrated by the fact that despite the care taken in preparing and setting up the specimens, the local shear rate was generally 2.5 to 4 times less than the external shear rate. However, in one case it was almost 7 times less (Table 5-11).

One practical problem encountered during the Chalk testing programme needs special mention. A small leak in the back pressure system was discovered at the contact between the base pedestal and the cell base plate. After noting the leak, tests were conducted to determine the rate of volume loss. It was found to be approximately 3mm$^3$/hour at a constant back pressure of 850 kPa. The problem was discovered and rectified after completion of CH2. Therefore, an estimation of the possible measurement errors introduced by the leak has been made for tests CH1 and CH2.
Test CH2 consisted only of saturation and consolidation phases (no shear phase was conducted). From the rate of volume loss and the test duration, the void ratio error up to yield is estimated as 0.004 and the void ratio error beyond yield as 0.002. These errors are judged to be relatively small.

Test CH1 consisted only of a saturation and an undrained shear phase. No consolidation was required, as the specimen was sheared undrained from \( p' = 100 \text{kPa} \), which was the mean effective stress after saturation. Inspection of the stress path for test CH1 clearly shows that the test did not follow an undrained path (Figure 5-16). In fact the pore pressure remained virtually constant throughout the test and it is believed that most of the pore pressure build-up during shear was lost as a result of the leak between the pedestal and base plate. The result may be compared with that of CH4 which was a drained test, also conducted from \( p' = 100 \text{kPa} \). The stress paths for CH1 and CH4 may be seen to conform closely. From this the judgement was made that CH1 was closer to a fully drained test than a fully undrained test.

5.2.5. Typical test results of Chalk

A summary of the test results for the Chalk testing programme is shown in Figure 5-15 (isotropic consolidation) and Figure 5-16 (shear behaviour). In addition, Table 5-10, Table 5-11 and Table 5-12 give information on sample characteristics, testing procedure and summarises some of the test results.

The initial void ratios of the Chalk samples varied between 0.91 and 0.95 (Table 5-10). As may be seen from Figure 5-15, the volumetric stiffness of all the Chalk specimens prior to yielding was high compared to the stiffness after yielding. Yielding could therefore be identified as a dramatic reduction in volumetric stiffness. The isotropic yield stress varied between specimens, but was approximately 2900kPa.
Figure 5-16 shows that the shear behaviour of the Chalk is clearly dependant on the mean effective stress at the start of shear. Different behaviour was observed for specimens sheared after isotropic yield, compared to those sheared without isotropic yield. CH1, CH3, CH4 and CH5 were sheared from mean effective stresses less than the isotropic yield stress and exhibited brittle failure without strain softening. Failure was observed as a sudden drop in deviatoric stress. Axial strain levels at failure was typically 0.08%. Upon removal from the apparatus, very little specimen damage was visible. The only signs of disturbance were small hairline cracks on the surface of the specimens.

CH6, CH7, CH12 and CH13 were sheared from close to the isotropic yield stress. The imminent onset of yield was determined by continuously monitoring the specimen during consolidation. Consolidation was terminated as soon as any evidence of yield was apparent. The first sign of yield was typically observed as a change in displacement rate, shown by one of the two local LVDTs. In practice, the procedure of terminating consolidation at the onset of yield was difficult as yield occurred suddenly and resulted in large volumetric and axial strains. On three occasions (CH9, CH10 and CH11) consolidation was not stopped in time. For these tests, the armature of the local LVDTs moved outside the range which allowed them to be zeroed at the highest amplification level. Subsequently no shear stages were conducted for these tests.

The four specimens that were successfully sheared (CH6, CH7, CH12 and CH13) all initially had near vertical stress paths (Figure 5-16). For two of the tests (CH12 and CH13), yielding was observed as a sharp turn to the left by the stress path at deviator stresses of 800kPa and 1000kPa respectively, before the onset of contracting behaviour. For the other two specimens (CH6 and CH7) yielding was initially more gradual with dramatic yielding evident at deviator stress levels of 1700kPa and 1800kPa.
All four specimens (CH6, CH7, CH12 and CH13) failed in a ductile fashion at axial strain levels between 0.06% and 0.11%. Some post peak strain softening was observed. Upon removal from the triaxial cell, severe strain localisation was evident in all cases. As specimens inside the high-pressure triaxial cell could not be observed visually during the test, it was difficult to judge whether the strain softening shown in the stress paths was accompanied by relative sliding of intact material. Even so, on the basis of the severe strain localisation apparent after removal of the specimen and the irregular appearance of the post yield stress paths, the judgement was made that post-peak strain softening was indeed accompanied (at least in part) by relative sliding of intact portions of the specimens. For these reasons, this behaviour can not be considered as the behaviour of the specimen as a whole.

Two specimens, CH8 and CH14, were consolidated to isotropic stress levels in excess of the isotropic yield stress, before being sheared in undrained compression. From Figure 5-16 it may be seen that both stress paths were near vertical at the initial stages of the tests and became progressively flatter accompanied by contractive behaviour. Strain softening occurred after yielding. It may also be observed that both stress paths were smooth at all stress levels. These observations suggest that progressive yielding occurred throughout the tests. Both specimens were examined after completion of the tests. CH8 showed a number of failure planes and some signs of barrelling. However, CH14 failed by strain localisation on a single failure plane. On first consideration, this may seem surprising, since the sample was consolidated to an isotropic stress approximately 2000kPa in excess of the yield stress. However, similar behaviour has been observed by Leddra and Jones (1990) and Petley et al. (1993). They subjected Chalk specimens with an intact porosity of 48% and an isotropic yield stress of 4.2MPa to high isotropic stresses before undrained shear. They found that after consolidation to 10MPa, specimens still exhibited strain localisation during shear. Strain localisation was in the form of numerous randomly orientated failure planes. In contrast, when they sheared the Chalk after isotropic consolidation to 30MPa, material deformation was uniform.
with barreling at large stain levels. For such specimens, no evidence of failure planes were visible. Their results indicate that isotropic stresses significantly in excess of the yield stress are required to cause extensive bond breakage. This is supported by Leddra and Jones (1990) who found that parts of the coccolith fabric remained intact even after being subjected to large volumetric strains.

When strain localisation occurs the behaviour of the material on the failure plane differs from that of the rest of the specimen. During this condition, further strains are concentrated on the failure plane and do not impact on the rest of the specimen. The result is that the specimen as a whole will not reach critical state during further shear. Strain localisation occurred in all tests performed on the Chalk which implies that critical state was not reached in any of the tests. However, it has to be pointed out that CH8 and CH14 followed a unique stress path during strain softening. This suggests that these stress paths may have been progressing towards the critical state. Previously Leddra and Jones (1990) have taken this line to be the critical state line.

The effects of bonding on the shear behaviour of the Chalk may the examined by comparing the behaviour of bonded specimens, sheared at isotropic stresses significantly lower than the yield stress (CH1, CH3, CH4 and CH5), to specimens which suffered significant destructuring during isotropic yield (CH8 and CH14). A number of differences may be observed:

a) Progressive yielding occurred during shear of the destructured specimens. This was shown as a build-up of pore pressure as some of the load was shed from the soil skeleton onto the pore fluid. In contrast, for the bonded samples, yielding coincided with failure.

b) For the destructured specimens, the mode of failure was ductile with significant post-peak strain softening. However, for the bonded samples the mode of failure by fracturing. It occurred suddenly and no strain softening was evident.

c) The peak deviator stresses were surprisingly similar for bonded and destructured specimens. However the stress paths by which these peak stresses were attained were significantly different. When comparing CH8 and CH14, both of which
were destructured, it is clear that the peak deviator stress depends on the consolidation stress. This is to be expected of a material where frictional behaviour dominates. In contrast, the intact specimens show no clear increase in peak deviator stress for an increase in consolidation pressure. This suggests that the bonding dominated during shear of these materials.

Leddra and Jones (1990) also found that the behaviour of bonded and destructured Chalk differs greatly. They attempted to describe the behaviour of Chalk within the framework of Critical State Soil Mechanics. The behaviour of Chalk taken past the isotropic yield stress broadly conformed to critical state behaviour, with the exception of significant post yield softening. On the other hand, the behaviour of the intact material did not conform well to the expected critical state behaviour. In particular, the stress paths attained states outside the bounding surface. They do however suggest that the stress paths move back to the critical state line after failure.

An interesting question that arises from the different behaviours observed for the bonded and destructured materials is how much destructuring is required before the dominant mode of shear resistance changes from bonding to frictional. Aversa et al. (1993) investigated a number of bonded natural materials and commented that destructuring during shear is a long and continuous process which continue up to large strains. However, they showed that at axial strains in excess of 2% the behaviour of these materials can be described purely in terms of their frictional and dilational behaviour, which seems to suggest that any bonding that is still present no longer dominates the behaviour of the material. Similar behaviour can also be observed for the Chalk by examining the behaviour of the specimens sheared from close to the isotropic yield stress (CH6, CH7, CH12 and CH13). These specimens were consolidated up to the point where yield was first observed, but from Figure 5-15 it can be seen that consolidation was terminated before significant volumetric strains occurred. The shear behaviour of these specimens had a number of similarities to the destructured materials. Firstly, they contracted during shear.
Secondly, yielding was apparent at stress levels well below the peak stress. Also, these specimens showed ductile failure, but with only limited strain softening. Evidence of the fact that some bonding was still present is the relatively high peak deviator stresses that were reached by these specimens.

The stiffness response of the Chalk specimens varied markedly depending on whether they had been destructured prior to shear. This behaviour will be discussed in Chapter 6. However for completeness, a typical result of stiffness response is shown in Figure 5-17. The result is for CH5 and shows that the linear plateau could clearly be observed for axial strains up to 0.004%. For strain levels in excess of 0.004% the stiffness progressively decreased up to failure at 0.08%.

5.3. Bothkennar clay

5.3.1. Description of Bothkennar clay

The National Soft Clay Research Site is located at Bothkennar in Scotland. It is situated on the banks of the Forth Estuary approximately 35km west of Edinburgh. The clay from this site is a soft clay and forms part of the recent marine deposits which have been laid down within the last 5000 years. The description of the soil profile as reported by Nash, Powell et al. (1992) is given in Figure 5-18.

Paul et al. (1992) have discussed the engineering geology of Bothkennar clay and the following description is based largely on their work. The Clarets beds at Bothkennar extend to a depth of approximately 18m and four different facies can be identified. These are bedded, mottled, laminated and weathered facies. Features of the original sedimentary bedding is preserved in the bedded facies. The thickness of individual beds range between a few millimetres and 100mm. The beds are separated by primary sedimentary structures such as ripples, load casts, rip-up mudflakes and mudlumps features. The mottled facies have been reworked by bioturbation. The principle feature of the facies is the presence of mottling which
occupy 20 to 60% of the surface in vertical section. Primary sedimentary features are poorly defined or absent for the mottled facies. The laminated facies was found by Paul et al. (1992) at 7.75 to 9.40m depth. It consists of individual beds of silty clay separated by layers of silt typically 1 to 2mm thick. These silt laminae are about 30 to 50mm apart. The weathered facies was found in the top 4m metres of the profile. For this facies, primary sedimentary features are disrupted by fissures and other pedogenic features.

The average particle size decreases marginally with depth and for the profile as a whole, the clay fraction varies between 35 and 50%. The mineralogy of all three facies is remarkably similar. In the fine silt to clay fraction the material comprises of mainly illite and chlorite minerals with some traces of biotite and smectite. The coarse silt fraction consists mainly of quartz. The variation of water content and Atterberg limits with depth is shown in Figure 5-18.

A number of structural features have been identified by Paul et al. (1992). The bedded facies generally have a honeycomb structure with both edge-to-edge and edge-to-face contacts of the clay minerals visible. Local bonding between silt particles has been observed. The bonding agents were aluminosilicates, iron compounds and silica. The mottled facies show evidence of biogenic disturbance. For this facies, individual burrows with cemented linings have been observed.

5.3.2. Sampling of Bothkennar clay

A down-hole block sampling technique was used to sample the Bothkennar clay. The samples were taken by the Building Research Establishment (BRE) using a Sherbrooke sampler (Lefebvre and Poulin (1979)). Previous work carried out at the Bothkennar site, where various sampling techniques were compared, showed that the Sherbrooke sampler obtained the highest quality samples (Clayton et al. (1992)).
The sampler consisted of three cutting shoes at 120° intervals along its circumference (Figure 5-19). A vehicle mounted drilling rig was used to rotate the sampler. It was advanced by rotation and simultaneous flushing of water through the jets close to the cutting shoes. As a result an annular slot with an outside diameter of approximately 400mm diameter was cut. Once the sampler had been advanced by 350mm, a trigger was activated to release the three spring-loaded cutting blades at the bottom of the sampler. Rotation was continued and in the process the cutting blades separated the block sample from the bottom of the hole. The diameters of the blocks recovered varied between 200mm and 250mm. As soon as the block was at the surface, it was placed on a wooden board and taken into the laboratory to be sealed. Sealing was in the form of multiple layers of beeswax and clingfilm.

The hole was advanced to the next sampling depth by means of a flat bottomed auger. This advance was approximately 150mm, as block samples were taken at half meter intervals.

The borehole was filled with water throughout the sampling process. Lefebvre and Poulin (1979) suggested that either water or bentonite mud may be used as the borehole fluid. Water, as opposed to bentonite, may lead to sample disturbance by two possible mechanisms. Hopper (1992) showed that significant suctions develop at the bottom of a borehole on account of the total stress relief. The largest suctions develop to a depth of approximately one borehole diameter. When using water in the borehole, it will be drawn into the clay resulting in disturbance on account of swelling. This will result in a reduction in mean effective stress. The amount of disturbance will largely depend on the suctions developed, the permeability of the material and the time allowed for swelling to occur. It was attempted to minimise disturbance from swelling by removing samples from the hole as soon as possible after it had been separated from the bottom.

The second mechanism of disturbance is by base heave (see for example Bjerrum and Eide (1956)) of the material at the bottom of the hole as a result of a reduction
in the vertical total stress. This type of disturbance will further reduce the mean effective stress of a soft clay. Lefebvre and Poulin (1979) calculated that for a soft clay with an undrained shear strength of 15kPa, a trench can only be excavated to 4m depth if a factor of safety of 2 is to be maintained against bottom heave. Their results imply that disturbance from base heave will increase with depth. Notwithstanding the two possible disturbance mechanisms mentioned above, water was used as the drilling fluid. Upon enquiry, the BRE team explained that experience had shown that larger samples could be recovered when using water instead of bentonite.

Possible sampling disturbance may be assessed by comparing the estimated in-situ mean effective stresses and the initial mean effective stresses measured in the laboratory. These values are shown in Table 5-5. Comparison of the stress levels show good agreement between the in-situ and initial mean effective stresses for the two shallowest samples (BK1 and BK2) taken at 5.5 and 4.5m. On the other hand, the sample taken from 6.0m (BK3) showed a significant reduction in mean effective stress. This evidence suggest that BK1 and BK2 suffered minimal disturbance, whereas BK3 may have incurred some disturbance during sampling.

5.3.3. Bothkennar clay in-situ stress

The in-situ effective stresses were estimated on the basis of the work of Nash, Powell et al. (1992). From their data the relationship of vertical effective stress and depth (for depths between 1m and 16m) can be shown to conform closely to the following equation:

\[ \sigma'_{vo} = 6.17z + 11 \]  \hspace{1cm} (5-1)

Where \( \sigma'_{vo} \) is the vertical effective stress (kPa) and \( z \) is depth in meters. K\(_o\) was taken as 0.7. This value is in close agreement with values from Marchetti
dilatometer tests and marginally lower than values from self boring pressuremeter tests (Nash, Powell et al. (1992)).

The sampling depths and estimated in-situ effective stresses are shown in Table 5-5.

<table>
<thead>
<tr>
<th>Sample depth (m)</th>
<th>$\sigma'_{vo}$ (kPa)</th>
<th>$\sigma'_{ho}$ (kPa)</th>
<th>$p'_o$ (kPa)</th>
<th>$p'_i$ (kPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>BK1 5.5 - 5.85</td>
<td>46</td>
<td>32</td>
<td>37</td>
<td>33</td>
</tr>
<tr>
<td>BK2 4.5 - 4.85</td>
<td>40</td>
<td>28</td>
<td>32</td>
<td>34</td>
</tr>
<tr>
<td>BK3 6.0 - 6.35</td>
<td>49</td>
<td>34</td>
<td>39</td>
<td>23</td>
</tr>
</tbody>
</table>

Table 5-5. Bothkennar clay in-situ stresses.

5.3.4. Bothkennar clay specimen preparation and set-up

One 100mm diameter specimen was cut from the centre of each block sample. The block sample were first trimmed down by cutting the sides off by means of a tensioned wire. The exposed face allowed inspection of the sample fabric. Particular attention was given to the presence of sand or silt lenses. Sand or silt lenses can not sustain high suctions and as a consequence are much more susceptible to disturbance. For this reason such features were deemed as undesirable within the context of stiffness measurements. One block sample was rejected on the basis of a thin sand lens passing through the middle of the block, making it impossible to cut a sufficiently long specimen without including the lens. No sand or silt lenses were visible to the naked eye in the three samples that were tested. Final trimming was carried out in a soil lathe (Figure 5-5) and the ends were cut by placing the sample in the purpose made cradle used for all 100mm samples (Figure 5-6). After preparation, the sample dimensions were measured by means of a vernier caliper. Sample preparation times varied between 50 and 90 minutes for each of the three samples.
The set-up procedure for the Bothkennar clay specimens were slightly different to those for the London clay and Chalk. The Bothkennar clay samples were placed on the pedestal of the low-pressure triaxial apparatus, before placing the membrane and fitting the local LVDTs. Laying the specimen horizontally in order to fit the local gauges was avoided because of the soft consistency of the clay.

Four filter-paper strips were placed along the circumference of the specimen to enhance drainage. The membrane was placed by means of a membrane stretcher, before putting the bottom two o-rings in position. Once the filter paper strips, membrane and top cap were in place, the mid-plane pore pressure probe was installed. This was done according to the same procedure used for the London clay (see section 5.1.5). Next the LVDT brackets were fitted. An alignment tool was used to hold the two brackets in position as it was pinned and glued by means of silicone sealant. A pin-to-pin gauge length of 62.5mm was used for all tests on Bothkennar clay.

### 5.3.5. Bothkennar clay testing procedure

A cell pressure of 300kPa was applied to the specimen as soon as possible after set-up. Positive pore pressures were measured immediately after cell pressure application and flushing of the probe was not necessary in any of the tests. Time was allowed for the pressures to stabilise (typically overnight) before B-values were measured. B-values were 0.99 in all three tests. The initial mean effective stresses are shown in Table 5-5.

A number of consolidation and shear stages were conducted on the three Bothkennar clay specimens (see Figure 5-20 and Table 5-6). BK1 was sheared in undrained compression from close to the estimated *in situ* stress (path DE). BK2 was first sheared in undrained extension to zero deviator stress (path DF). The sample was then taken back along path FD before being sheared in undrained compression to failure (DE). BK3, was used to investigate the effect of recent stress...
history on the stiffness of Bothkennar clay. Three undrained compression shear stages were conducted from the same isotropic stress (point Q). However, prior to shear, this isotropic stress was each time approached from a different direction in triaxial stress space (RQ, SQ and TQ).

When reconsolidating BK1 and BK2 to the in-situ stress, the stress paths as suggested by Kirkpatrick and Khan (1984) were used. They suggested a stress path along the isotropic up to the in-situ mean effective stress (path AB) followed by undrained shear to the in-situ deviatoric stress (path BC). From a practical point of view, this stress path is easy to follow, particularly if only strain controlled loading is available, as was the case for the low-pressure triaxial apparatus. Hight et al. (1985) argued against this reconsolidation path on the basis of possibly damaging the specimen if it passes close to (or engages) the yield surface. One way to judge whether significant damage was done to the sample during reconsolidation is to observe the induced strains. Particularly in the case of a naturally bonded, high porosity clay such as Bothkennar clay, yield is accompanied by significant volumetric and shear strains. For BK1, the total axial strain during reconsolidation was 0.14%. For BK2 it was 0.15%. These low strain levels suggest that the specimens did not suffer significant damage during reconsolidation.

Reconsolidation to the in-situ effective stress along an undrained stress path was not entirely straight forward. In the case of BK1, isotropic consolidation to the in-situ mean effective stress (path AB) was followed by undrained shear at an axial strain rate of approximately 2%/day. The expected near-vertical stress path was observed during the deviatoric stress excursion (path BC). Once the in-situ deviatoric stress (point C) was reached a rest period followed to allow creep. As creep occurred, a small build-up of pore pressure was observed. The magnitude of the pore pressure build-up was approximately 5kPa and as a result the mean effective stress reduced to slightly less than the in-situ mean effective stress (path CD). BK1 was sheared from this stress level (point D) even though the initial mean effective stress was marginally lower than the in-situ mean effective stress.
The objective of test BK2 was to compare the stiffness of the clay during compression and extension when sheared from the same initial stress, approached along the same stress path. After reconsolidation to the in-situ effective stress the intention was to shear the specimen in undrained extension to an isotropic stress state, before re-applying the in-situ deviatoric stress and shearing it in undrained compression. During the rest period after reaching the in-situ stress (point C), pore pressure build-up was again observed (path CD). In this case it was unacceptable as it implied that the stress from which the sample was eventually to be sheared, was approached from a direction other that the intended undrained stress path direction. The specimen was therefore unloaded (path DF) and subsequently re-loaded (path FD) to the in-situ deviator stress. When held at the in-situ deviator stress after the second load cycle (point D), no pore pressure build-up was observed. Again the result was that the sample was sheared from a mean effective stress, slightly less than the in-situ mean effective stress. However, the main objective of the test was to evaluate the effect of stress path reversal on the stiffness of the clay and the fact that it was sheared from a stress slightly lower than the in-situ mean effective stress was regarded as being of secondary importance.

The stress paths followed for each test is summarised in Table 5-6 and shown schematically in Figure 5-20.

<table>
<thead>
<tr>
<th>Test</th>
<th>Shear stage</th>
<th>Excursion</th>
<th>Stress path</th>
</tr>
</thead>
<tbody>
<tr>
<td>BK1</td>
<td>BK1 (Uco1)</td>
<td>Consolidation</td>
<td>ABC</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Creep</td>
<td>CD</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Undrained compression</td>
<td>DE</td>
</tr>
<tr>
<td>BK2</td>
<td>BK2(Uex1)</td>
<td>Consolidation</td>
<td>ABC</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Creep</td>
<td>CD</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Unload - reload</td>
<td>DFD</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Undrained extension</td>
<td>DF</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Reload</td>
<td>FD</td>
</tr>
<tr>
<td></td>
<td>BK2(Uco1)</td>
<td>Undrained compression</td>
<td>DE</td>
</tr>
<tr>
<td>BK3</td>
<td>Consolidation</td>
<td>QRQ</td>
<td></td>
</tr>
<tr>
<td>---------</td>
<td>-----------------------</td>
<td>-----</td>
<td></td>
</tr>
<tr>
<td>BK3 (Uco1)</td>
<td>Undrained compression</td>
<td>QT</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Unload</td>
<td>TQ</td>
<td></td>
</tr>
<tr>
<td>BK3 (Uco2)</td>
<td>Consolidation</td>
<td>QSQ</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Undrained compression</td>
<td>QT</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Unload</td>
<td>TQ</td>
<td></td>
</tr>
<tr>
<td>BK3 (Uco3)</td>
<td>Undrained compression</td>
<td>QU</td>
<td></td>
</tr>
</tbody>
</table>

Table 5-6. Bothkennar stress paths.

5.3.6. *Typical test results of Bothkennar clay*

The shear excursions to failure of all the Bothkennar specimens are shown in Figure 5-21 and the stress-strain behaviour in Figure 5-22. The yield surface as determined by Smith et al. (1992), for Bothkennar clay from a depth of 6m is also shown in Figure 5-21. All three stress paths move to the right of vertical, as would be expected for an anisotropic material where the vertical stiffness is higher than the horizontal stiffness. Yield was observed as a dramatic turn to the left by the stress path as destructuring occurred and some of the load was shed onto the pore fluid. The yield points for all three specimens were close to the yield surface found by Smith (1992). Post-yield softening was evident in all three cases. Local axial strain at peak stress varied between 1.1% for BK2 and 2.2% for BK1 (see Figure 5-22). The mode of failure was different in all three specimen. BK1 failed by severe strain localisation on a single failure plane, BK2 failed by bulging and no strain localisation was evident. For BK3, both bulging and strain localisation was observed. The pore pressure response of the three specimen are shown in Figure 5-22 as a function of axial strain. These pressures were measured by the mid-plane probe. Inspection of Figure 5-22 shows that BK2 reached a critical state, with no further change in deviator stress or tendency of volume change. It also shows that BK3 was close to reaching critical state at the end of the test. In contrast BK1 did
not reach critical state, with a pore pressure increase still evident at 10% axial strain. These observations are consistent with the modes of failure noted above.

A typical plot of stiffness as a function of axial strain is shown in Figure 5-23 (BK3(Uco3)). It shows that the linear plateau of the material was clearly observed. Furthermore, from Figure 5-23 it may be seen that the stress-strain behaviour was linear up to an axial strain of approximately 0.003%.

5.4. General issues

In the previous sections, the testing techniques used during this project were discussed separately for each material type. A number of general issues which have implications for all three materials have been touched upon. These include saturation techniques, strategies to minimise the effect of creep and the presentation of stiffness data. Such issues are important, but were not discussed in detail as to avoid repetition and distraction from the topics at hand. These general issues will be discussed in more detail below.

5.4.1. Saturation

Triaxial specimens are saturated for a number of reasons. Firstly, fully saturated specimens (S = 1), used in conjunction with a positive back pressure, gives rapid response of the pressure measurement system to changes in pore pressure. Secondly, volume changes of saturated specimens may be measured externally by determining the volume of fluid to enter or exit the specimen. However, such measurements are subjected to inaccuracies caused by bedding errors and membrane penetration. Thirdly, to determine the effective stress of an unsaturated specimen it is necessary to measure both the pore air and pore fluid pressures. The effective stress can therefore not be determined unless additional equipment is used to measure these pressures. However, to determine the effective stress of a saturated specimen only the total stress and pore pressure has to be measured.
The time required to reach saturation \((S = 1)\) of triaxial samples using an elevated back pressure is governed by the diffusion time of air into the pore fluid. The required time is virtually independent of the permeability of the material (Black and Lee (1973)). The theoretical minimum back pressure for full saturation \((P_{100})\) for a specimen at an initial degree of saturation \((S_i)\) was shown by Lowe and Johnson (1960) to be:

\[
P_{100} = 49 P_i (1 - S_i)
\]  
(5-2)

where \(P_i\) is the absolute pressure (pressure relative to absolute zero). Black and Lee (1973) studied the time to saturation for specimens of coarse Ottawa sand at high density \((D_r = 100\%)\) from various initial degrees of saturation. They found that increasing the back pressure above the minimum back pressure \((P_{100})\) markedly reduced the time to saturation. Their results are replotted in Figure 5-24 and are normalised for sample size.

Bishop (1973) showed that the \(B\)-value of a saturated porous medium is governed by its porosity \((n)\), as well as the volumetric compressibility of the grains, the skeleton and the water. He also showed that if the compressibility of the soil skeleton \((C)\) is more than the compressibility of water \((C_w = 56 \times 10^{-6} \text{ m}^2/\text{kN})\), the \(B\)-value may be calculated as:

\[
B = \frac{\Delta u}{\Delta \sigma} = \frac{1}{1 + n \left(\frac{C_w}{C}\right)}
\]  
(5-3)

From equation (5-3) it is clear that for saturated geomaterials with low skeleton compressibility, \(B\)-values of less than unity will be measured. Bishop (1973) reported \(B\)-values of 0.53 and 0.77 for saturated sandstone. Wissa (1969) investigated the undrained pore pressure response of saturated samples of Portland
cement stabilised soil samples. He found that B-values decreased as the initial Youngs modulus of the material increased and reported B-values as low as 0.55.

Full saturation (S = 1) of triaxial samples may be confirmed in a number of ways:

a) If the compressibility of the soil skeleton is known, the theoretical B-value may be calculated (Bishop (1973)). This B-value may be compared to the measured B-value.

b) The B-value may be tested at a number of increasing back pressures (Wissa 1969). No increase in B-value for increased back pressure constitutes full saturation.

c) If back pressures higher than \( P_{100} \) are used, full saturation may be confirmed by no change in B-values, measured at appropriate time intervals.

d) Under less rigorous testing conditions, published guidelines of B-values at full saturation may be used. Chaney et al. (1979) suggested B-values at full saturation ranging from 0.999 for soft soil to 0.988 for stiff soil and 0.91 for very stiff soil.

Back pressures were used to facilitate full saturation in all samples tested during this project. Back pressures used for the Bothkennar clay were of the order of 300kPa, whereas the back pressures for the London clay ranged between 300kPa and 500kPa. Back pressures for the Chalk were generally of the order of 700kPa. However, as may be seen from Table 5-10, in some cases pressures significantly in excess of 700kPa were used. Saturation was confirmed by using the techniques (b) and (c) above. The B-values at full saturation were in agreement with those suggested by Chaney et al. (1979). The B-values for the Bothkennar clay were close to unity and those for the London clay ranged between 0.95 and 1.0. For the Chalk, B-values ranged between 0.87 and 0.92 apart from two case (CH3 and CH15). Details of these two samples are given in Section 5.2.4.
5.4.2. Creep rate prior to shear

In order to avoid measurement errors introduced by creep, the creep rate must be small compared to the subsequent shear rate. Hight and Higgins (1995) recommended a shear strain rate of 5%/day for undrained tests and suggested a creep rate prior to shear of less than 0.05%/day. Jardine (1995) also recommended a creep rate to shear strain rate ratio of less than 1%. None of these authors clearly state whether these shear strain rates should be measured locally or externally. It is assumed that they refer to the machine rate of strain.

The creep and shear strain rates for all tests conducted during this project are shown in Table 5-8, Table 5-11 and Table 5-14. During the testing programme, the ratio of the locally measured creep rate to the locally measured shear rate was generally less than 2%. The only two notable exceptions were tests CH4 and CH7. For the Bothkennar clay, the ratio was less than 1% in all cases and for the London clay the ratio was less than 0.5% (LC1 being the only exception).

Specifying a restriction based on the local creep and shear rates is a rational approach, but presents a difficulty. The operator only has control over the external shear rate and the local shear rate is often not known in advance. The local shear rate will always be less than the external shear rate and will vary according to the material type, bedding rate and apparatus compliance. As the majority of tests on each material type were conducted in one apparatus, the ratio between external and local shear rates were remarkably similar in each case. However, this ratio varied dramatically from one apparatus to the next. For the medium-pressure apparatus the external strain rate was between 4 and 9 times higher than the local strain rate. This high ratio may be accounted for on the basis of the top cap design. (Recall from Chapter 4 that the loading ram connection was sandwiched between two rubber pads). For both the high and low-pressure apparatuses a semi-hemispherical loading ram made contact directly on a flat metal or perspex top cap. In the case of the low-
pressure apparatus and the Bothkennar clay, the external rate was between 1 and 2 times faster than the local strain rate. For the chalk tested in the high-pressure apparatus, the ratio was generally between 2.5 and 4, but in one case was as high as 6.8. These values demonstrate the danger of using a criterion which restricts the ratio between the local creep and external strain rate.

Restricting the creep rate implies that a rest period has to be observed once the desired stress state has been reached and before commencing shear. Some interesting conclusions may be drawn by comparing the required rest periods for the different materials. From Table 5-11 it may be seen that for the Chalk the rest period differed dramatically for samples tested without isotropic yield and those tested after yield. Samples which were not subjected to yield required rest periods of less than one day. On the other hand the two specimens which underwent significant destructuring as a result of isotropic yielding (CH8 and CH14) required 6 and 13 days respectively. This clearly illustrates that materials with higher levels of bonding exhibit less creep.

When comparing the rest periods for the London clay to those for the Bothkennar clay, it is interesting to note that the London clay required much longer rest periods (Table 5-8 and Table 5-14). LC4, which was not subjected to cyclic shear and therefore did not undergo additional rest before cyclic loading, required a rest period of approximately 12 days. In contrast the Bothkennar specimens required only 2 to 3 days rest. This fact has to be judged in the light of the higher allowable creep rates for the Bothkennar clay on account of the higher shear rate. Furthermore, the London clay was subjected to a higher initial deviator stresses. Nonetheless, it was clear that the London clay had a higher tendency to creep. This may possibly be due to its fissured nature and the relatively large strains that it was subjected to prior to sampling. In contrast, all Bothkennar clay tests were carried out on material forming part of the mottled facies which has a uniform fabric.
5.4.3. Presentation of stiffness data

Presentation of stiffness data over a wide strain range introduces certain difficulties. In particular, it is difficult to give equal prominence to stiffnesses at all strain levels without resorting to numerous plots, over different strain levels, for each stress-strain curve. One way to overcome this problem is to plot stiffness against the logarithm of strain.

A further factor to consider is whether to plot secant or tangential stiffness. In theory, both methods should yield the same stiffness for a linear stress-strain relationship. However, as a result of random noise from the instrumentation, secant and tangential stiffnesses differ, even if the stress-strain relationship is essentially linear. When plotting non-linear stress-strain behaviour, using both methods to calculate stiffness has some advantages and limitations. These are explored below.

Tangential stiffness represents the stiffness at a particular strain level. It may be calculated numerically by performing a linear regression of a number of data points in the vicinity of the strain level under consideration. It is therefore independent of the choice of the origin. This is a major advantage. However, the sensitivity of tangent stiffness to random noise is the same at all strain levels. This is a disadvantage when plotting stiffness against the logarithm of strain. In addition tangential stiffness is sensitive to the number of data points used to perform the regression. This factor is particularly problematic if the data acquisition rate is varied throughout the test. During testing a higher acquisition rate was used at the start of a test as the stiffness at very small strains was of particular interest.

Secant stiffness has the advantage of being more sensitive at small than at higher strain levels. This makes it compatible with the approach of plotting stiffness against the logarithm of strain. However, a significant disadvantage when using secant stiffness is the fact that it is sensitive to the choice of origin.
After some trials, the decision was made to present all stiffness data as secant stiffness against the logarithm of strain. In order to minimise variation as a result of the choice of origin, a consistent method was used to choose the origin. Consider the typical result of deviator stress vs. axial strain of CH5 (Figure 5-25). Data acquisition started at point A and a rest period of 15 minutes was allowed before shear was initiated at point B. The rest period was used to monitor the creep rate of the specimen. A further time lag of a few seconds occurred before shear of the specimen commenced (point C). Rational points to use as an origin include (amongst others) points A, B and C. By way of illustration, the results of using points A and C as origin are shown in Figure 5-25. A scatter of data points are evident at strain levels below 0.001%. This is as a result of the random noise of the instrumentation. Furthermore, Figure 5-25 clearly shows that the calculated stiffness in the region between 0.001% and 0.0001% is significantly influenced by the choice of origin. It also shows that the calculated stiffness for strain levels above 0.002% are not significantly influenced by the choice of origin. In both cases the linear plateau can still be identified. If the assumption is made that the instrumentation noise is random, the appropriate choice of origin should result in a symmetrical spread of the data points about an axis that coincides with the linear plateau. Taking point D as the origin (Figure 5-25), resulted in an approximately symmetrical spread of the data point in the region 0.001% to 0.0001%.

On the basis of the results shown in Figure 5-25, some conclusions can be made concerning the influence of the choice of origin:

a) The calculated stiffnesses in the region 0.0001% and 0.001% are markedly influenced by the choice of origin.

b) The shape of the curve for strain levels in excess of 0.002% are not significantly influenced by the choice of the origin.
c) The value of the stiffness at 0.001% is marginally influenced by the choice of origin. In fact when the stiffnesses are compared at a strain level of 0.001% for the origins at points A, C, and D, the results are:

\[
\frac{E_{0.001}^C}{E_{0.001}^D} = 1.02 \quad \text{and} \quad \frac{E_{0.001}^A}{E_{0.001}^D} = 0.96
\]

On the basis of the above, a uniform method was adopted of taking the origin at a point that produced a symmetrical spread of the data points in the region 0.001% and 0.0001%. It should be noted that this method may not be appropriate if the linear plateau cannot be identified at strain levels which are unaffected by the choice of origin.
<table>
<thead>
<tr>
<th>Test</th>
<th>$w_1$ (%)</th>
<th>$e_i$</th>
<th>$e_f$</th>
<th>B-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>LC1 Uco</td>
<td>25.5</td>
<td>0.688</td>
<td>0.677</td>
<td>1.0</td>
</tr>
<tr>
<td>LC2 Uex</td>
<td>25.6</td>
<td>0.691</td>
<td>0.726</td>
<td>0.97</td>
</tr>
<tr>
<td>LC3 Uex</td>
<td>25.2</td>
<td>0.680</td>
<td>0.691</td>
<td>1.0</td>
</tr>
<tr>
<td>LC4 Uco &amp; Uex</td>
<td>24.7</td>
<td>0.667</td>
<td>0.686</td>
<td>0.98</td>
</tr>
<tr>
<td>LC5 Icon</td>
<td>25.7</td>
<td>0.694</td>
<td>0.645</td>
<td>0.95</td>
</tr>
</tbody>
</table>

Table 5-7. London clay sample characteristics.

<table>
<thead>
<tr>
<th>Test</th>
<th>Rest period (days)</th>
<th>Rest period (µm/hr)</th>
<th>$\dot{e}_\text{local}^\text{creep}$ (%/day)</th>
<th>$\dot{e}_\text{local}^\text{shear}$ (%/day)</th>
<th>$\dot{e}<em>\text{local}^\text{creep}$ / $\dot{e}</em>\text{local}^\text{shear}$ (%)</th>
<th>$\dot{e}<em>\text{external} / \dot{e}</em>\text{local}^\text{shear}$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>LC1</td>
<td>$6^{(2)}$</td>
<td>-0.13</td>
<td>-0.00433</td>
<td>+0.200</td>
<td>2.17</td>
<td>8.9</td>
</tr>
<tr>
<td>LC2$^{(3)}$</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>LC3</td>
<td>$7^{(2)}$</td>
<td>+0.025</td>
<td>+0.000833</td>
<td>-0.208</td>
<td>0.40</td>
<td>6.3</td>
</tr>
<tr>
<td>LC4(Uco)</td>
<td>12</td>
<td>-0.025</td>
<td>-0.000833</td>
<td>+0.204</td>
<td>0.41</td>
<td>5.2</td>
</tr>
<tr>
<td>LC4(Uex)</td>
<td>11</td>
<td>+0.020</td>
<td>+0.000667</td>
<td>-0.253</td>
<td>0.26</td>
<td>4.6</td>
</tr>
<tr>
<td>LC5</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

1. Local shear strain rate at start of test.
2. Rest period after last cyclic excursion.
3. Failed on pre-existing fissure during cyclic loading.

Table 5-8. London clay creep and shear rates.
<table>
<thead>
<tr>
<th></th>
<th>( p'_{si} ) (kPa)</th>
<th>( q_{si} ) (kPa)</th>
<th>( \varepsilon_{\text{linear}} ) (%)</th>
<th>( \varepsilon_{\text{peak}} ) (%)</th>
<th>( E_{0.001} ) (MPa)</th>
<th>( E_{0.01} ) (MPa)</th>
<th>( E_{0.1} ) (MPa)</th>
<th>( E_{1.0} ) (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>LC1</td>
<td>291</td>
<td>-206</td>
<td>0.004</td>
<td>2.5</td>
<td>260</td>
<td>245</td>
<td>151</td>
<td>52</td>
</tr>
<tr>
<td>LC2</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>LC3</td>
<td>315</td>
<td>-195</td>
<td>&lt; 0.001</td>
<td>1.8</td>
<td>274</td>
<td>233</td>
<td>98</td>
<td>30</td>
</tr>
<tr>
<td>LC4(Uco)</td>
<td>311</td>
<td>-200</td>
<td>0.004</td>
<td>-</td>
<td>228</td>
<td>229</td>
<td>129(1)</td>
<td>-</td>
</tr>
<tr>
<td>LC4(Uex)</td>
<td>319</td>
<td>-200</td>
<td>0.002</td>
<td>1.1</td>
<td>239</td>
<td>198</td>
<td>84</td>
<td>27</td>
</tr>
<tr>
<td>LC5</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

1. Projected value.

Table 5-9. Stiffness of London clay specimens.
<table>
<thead>
<tr>
<th></th>
<th>$S_i$ (%)</th>
<th>$w_i$ (%)</th>
<th>$e_i$</th>
<th>$e_f$</th>
<th>Saturation back pressure (kPa)</th>
<th>B-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>CH1</td>
<td>96.9</td>
<td>33.7</td>
<td>0.939</td>
<td>0.948</td>
<td>1093</td>
<td>0.90</td>
</tr>
<tr>
<td>CH2</td>
<td>97.2</td>
<td>33.8</td>
<td>0.939</td>
<td>0.813</td>
<td>1124</td>
<td>0.88</td>
</tr>
<tr>
<td>CH3</td>
<td>95.4</td>
<td>30.9</td>
<td>0.874</td>
<td>0.894</td>
<td>2768</td>
<td>0.65</td>
</tr>
<tr>
<td>CH4</td>
<td>96.9</td>
<td>33.7</td>
<td>0.939</td>
<td>0.940</td>
<td>1006</td>
<td>0.90</td>
</tr>
<tr>
<td>CH5</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.905</td>
<td>992</td>
<td>0.90</td>
</tr>
<tr>
<td>CH6</td>
<td>97.7</td>
<td>33.9</td>
<td>0.937</td>
<td>0.923</td>
<td>600</td>
<td>0.92</td>
</tr>
<tr>
<td>CH7</td>
<td>97.5</td>
<td>33.1</td>
<td>0.917</td>
<td>0.921</td>
<td>678</td>
<td>0.91</td>
</tr>
<tr>
<td>CH8</td>
<td>97.9</td>
<td>34.4</td>
<td>0.949</td>
<td>0.818</td>
<td>604</td>
<td>0.92</td>
</tr>
<tr>
<td>CH9</td>
<td>95.6</td>
<td>32.1</td>
<td>0.907</td>
<td>0.807</td>
<td>613</td>
<td>0.91</td>
</tr>
<tr>
<td>CH10</td>
<td>95.4</td>
<td>31.2</td>
<td>0.883</td>
<td>0.826</td>
<td>991</td>
<td>0.91</td>
</tr>
<tr>
<td>CH11</td>
<td>98.2</td>
<td>34.6</td>
<td>0.951</td>
<td>0.913</td>
<td>701</td>
<td>0.90</td>
</tr>
<tr>
<td>CH12</td>
<td>95.8</td>
<td>32.8</td>
<td>0.924</td>
<td>0.910</td>
<td>696</td>
<td>0.92</td>
</tr>
<tr>
<td>CH13</td>
<td>84.3</td>
<td>29.2</td>
<td>0.935</td>
<td>0.929</td>
<td>995</td>
<td>0.87</td>
</tr>
<tr>
<td>CH14</td>
<td>98.0</td>
<td>34.5</td>
<td>0.950</td>
<td>0.788</td>
<td>700</td>
<td>0.89</td>
</tr>
<tr>
<td>CH15</td>
<td>93.2</td>
<td>30.0</td>
<td>0.869</td>
<td>-</td>
<td>850</td>
<td>0.41</td>
</tr>
</tbody>
</table>

1. moisture content not measured for CH5 before test.

Table 5-10. Chalk sample characteristics.
<table>
<thead>
<tr>
<th></th>
<th>Rest period (days)</th>
<th>$\dot{\varepsilon}_{\text{creep}}$ (µm/hr)</th>
<th>$\dot{\varepsilon}_{\text{creep}}$ (%/day)</th>
<th>$\dot{\varepsilon}_{\text{shear}}$ (%/day)</th>
<th>$\frac{\dot{\varepsilon}<em>{\text{creep}}}{\dot{\varepsilon}</em>{\text{shear}}}$</th>
<th>$\frac{\dot{\varepsilon}<em>{\text{external}}}{\dot{\varepsilon}</em>{\text{shear}}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>CH1</td>
<td>0.4</td>
<td>-0.002</td>
<td>-0.00015</td>
<td>0.072</td>
<td>-0.2</td>
<td>3.0</td>
</tr>
<tr>
<td>CH2</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>CH3</td>
<td>0.9</td>
<td>0</td>
<td>0</td>
<td>0.890</td>
<td>0</td>
<td>3.1</td>
</tr>
<tr>
<td>CH4</td>
<td>0.8</td>
<td>-0.14</td>
<td>-0.0088</td>
<td>0.063</td>
<td>-14.0</td>
<td>3.7</td>
</tr>
<tr>
<td>CH5</td>
<td>0.8</td>
<td>+0.04</td>
<td>+0.0025</td>
<td>0.158</td>
<td>+1.6</td>
<td>3.9</td>
</tr>
<tr>
<td>CH6</td>
<td>0.2</td>
<td>+0.03</td>
<td>+0.0019</td>
<td>0.101</td>
<td>+1.9</td>
<td>6.8</td>
</tr>
<tr>
<td>CH7</td>
<td>2.7</td>
<td>-0.12</td>
<td>-0.0076</td>
<td>0.142</td>
<td>-5.4</td>
<td>3.9</td>
</tr>
<tr>
<td>CH8</td>
<td>6.0</td>
<td>+0.2</td>
<td>+0.013</td>
<td>1.2</td>
<td>+1.1</td>
<td>1.4</td>
</tr>
<tr>
<td>CH9</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>CH10</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>CH11</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>CH12</td>
<td>3.2</td>
<td>+0.04</td>
<td>+0.0025</td>
<td>0.22</td>
<td>+1.1</td>
<td>2.6</td>
</tr>
<tr>
<td>CH13</td>
<td>0.7</td>
<td>0</td>
<td>0</td>
<td>0.16</td>
<td>0</td>
<td>3.8</td>
</tr>
<tr>
<td>CH14</td>
<td>13.2</td>
<td>-0.28</td>
<td>-0.0017</td>
<td>0.24</td>
<td>-0.7</td>
<td>2.7</td>
</tr>
<tr>
<td>CH15</td>
<td>FP(N)</td>
<td>-0.24</td>
<td>-0.00782</td>
<td>0.45</td>
<td>-1.75</td>
<td>-3</td>
</tr>
<tr>
<td>CH15</td>
<td>FP(S)</td>
<td>+0.6</td>
<td>+0.0198</td>
<td>0.86</td>
<td>+2.3</td>
<td>-3</td>
</tr>
</tbody>
</table>

1. Local shear strain rate at start of test.
2. Not a fair reflection of creep rate as $L_1 = +1.4$µm/h and $L_2 = -1.4$µm/h.
3. Omitted on account of different top cap.

Table 5-11. Chalk creep and shear rates.
<table>
<thead>
<tr>
<th>Shear</th>
<th>$P'_{si}$ (kPa)</th>
<th>$\varepsilon_{\text{linear}}$ (%)</th>
<th>$\varepsilon_{\text{peak}}$ (%)</th>
<th>$E_{0.001}$ (GPa)</th>
<th>$E_{0.01}$ (GPa)</th>
<th>$E_{0.1}$ (GPa)</th>
<th>Status</th>
</tr>
</thead>
<tbody>
<tr>
<td>CH1</td>
<td>UC 108</td>
<td>0.04</td>
<td>0.07</td>
<td>3.2</td>
<td>3.2</td>
<td>-</td>
<td>failed</td>
</tr>
<tr>
<td>CH2</td>
<td>$^{(i)}$ -</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>CH3</td>
<td>UC 506</td>
<td>0.002</td>
<td>0.09</td>
<td>4.5</td>
<td>3.9</td>
<td>1.9</td>
<td>failed</td>
</tr>
<tr>
<td>CH4</td>
<td>DC 108</td>
<td>?</td>
<td>0.08</td>
<td>4.2</td>
<td>4.1</td>
<td>-</td>
<td>failed</td>
</tr>
<tr>
<td>CH5</td>
<td>UC 995</td>
<td>0.004</td>
<td>0.08</td>
<td>4.6</td>
<td>4.3</td>
<td>-</td>
<td>failed</td>
</tr>
<tr>
<td>CH6</td>
<td>UC 1976</td>
<td>0.003</td>
<td>0.06</td>
<td>4.8</td>
<td>4.1</td>
<td>-</td>
<td>failed</td>
</tr>
<tr>
<td>CH7</td>
<td>UC 2431</td>
<td>0.002</td>
<td>0.11</td>
<td>4.8</td>
<td>4.1</td>
<td>1.8</td>
<td></td>
</tr>
<tr>
<td>CH8</td>
<td>UC 3190</td>
<td>&lt;0.001</td>
<td>0.72</td>
<td>3.6</td>
<td>2.8</td>
<td>1.3</td>
<td></td>
</tr>
<tr>
<td>CH9</td>
<td>$^{(i)}$ -</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>CH10</td>
<td>$^{(i)}$ -</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>CH11</td>
<td>$^{(i)}$ -</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>CH12</td>
<td>UC 2220</td>
<td>0.001</td>
<td>0.08</td>
<td>4.6</td>
<td>3.9</td>
<td>-</td>
<td>failed</td>
</tr>
<tr>
<td>CH13</td>
<td>UC 2007</td>
<td>0.002</td>
<td>0.07</td>
<td>4.9</td>
<td>4.5</td>
<td>-</td>
<td>failed</td>
</tr>
<tr>
<td>CH14</td>
<td>UC 3798</td>
<td>&lt;0.001</td>
<td>0.75</td>
<td>3.5</td>
<td>2.9</td>
<td>1.5</td>
<td></td>
</tr>
<tr>
<td>CH15</td>
<td>UC 150</td>
<td>0.01</td>
<td>-</td>
<td>4.6</td>
<td>4.8</td>
<td>-</td>
<td></td>
</tr>
</tbody>
</table>

1. Not sheared. LVDTs out of highest amplification range at end of consolidation.

Table 5-12. Stiffness of Chalk specimens.
<table>
<thead>
<tr>
<th>Test</th>
<th>( w_i (%) )</th>
<th>( e_i )</th>
<th>( e_f )</th>
<th>B-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>BK1 Uco</td>
<td>69.4</td>
<td>1.874</td>
<td>1.860</td>
<td>0.99</td>
</tr>
<tr>
<td>BK2 Uco &amp; Uex</td>
<td>64.6</td>
<td>1.730</td>
<td>1.712</td>
<td>0.99</td>
</tr>
<tr>
<td>BK3 3 x Uco</td>
<td>69.1</td>
<td>1.856</td>
<td>1.860</td>
<td>0.99</td>
</tr>
</tbody>
</table>

Table 5-13. Bothkennar clay sample characteristics.

<table>
<thead>
<tr>
<th>Rest period (days)</th>
<th>( \varepsilon_{\text{local creep}} ) (( \mu \text{m/hr} ))</th>
<th>( \varepsilon_{\text{local creep}} ) (%/day)</th>
<th>( \varepsilon_{\text{local shear}} ) (%/day)</th>
<th>( \varepsilon_{\text{local shear}} ) (%)</th>
<th>( \varepsilon_{\text{external shear}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>BK1</td>
<td>11(^{(2)})</td>
<td>-0.059</td>
<td>-0.00228</td>
<td>2.070</td>
<td>0.11</td>
</tr>
<tr>
<td>BK2 (Uex)</td>
<td>2</td>
<td>+0.056</td>
<td>-0.00217</td>
<td>2.032</td>
<td>0.11</td>
</tr>
<tr>
<td>BK2 (Uco)</td>
<td>3</td>
<td>+0.042</td>
<td>-0.00163</td>
<td>2.071</td>
<td>0.079</td>
</tr>
<tr>
<td>BK3 (Uco1)</td>
<td>2</td>
<td>-0.30</td>
<td>-0.0115</td>
<td>1.402</td>
<td>0.82</td>
</tr>
<tr>
<td>BK3 (Uco2)</td>
<td>1</td>
<td>-0.23</td>
<td>-0.00883</td>
<td>1.133</td>
<td>0.78</td>
</tr>
<tr>
<td>BK3 (Uco3)</td>
<td>1.5</td>
<td>-0.13</td>
<td>-0.00499</td>
<td>1.382</td>
<td>0.36</td>
</tr>
</tbody>
</table>

1. Local shear strain rate at start of test.
2. Ready for shear after 4 days. Held longer to monitor.

Table 5-14. Bothkennar clay creep and shear rates.

| \( \sigma_{\text{si}} \) (kPa) | \( q_{\text{si}} \) (kPa) | \( \varepsilon_{\text{linear}} \) (%) | \( \varepsilon_{\text{peak}} \) (%) | \( E_{0.001} \) (MPa) | \( E_{0.01} \) (MPa) | \( E_{0.1} \) (MPa) | \( E_{1.0} \) (MPa) |
|-------------------------------|-----------------|-----------------|-----------------|----------------|----------------|----------------|----------------|----------------|
| BK1                           | 29.9            | 14.4            | < 0.002         | 2.2            | 30.0           | 24.3           | 10.9           | 3.2            |
| BK2 (Uex)                     | 25.9            | 11.7            | 0.002           | -              | 30.5           | 27.4           | -              | -              |
| BK2 (Uco)                     | 23.3            | 12.3            | < 0.001         | 1.1            | 29.1           | 21.8           | 10.9           | 3.2            |
| BK3 (Uco1)                    | 18.6            | 0               | 0.002           | -              | 26.5           | 21.7           | -              | -              |
| BK3 (Uco2)                    | 18.3            | 0               | 0.002           | -              | 24.8           | 21.7           | -              | -              |
| BK3 (Uco3)                    | 17.6            | 0               | 0.003           | 1.4            | 24.1           | 21.6           | 13.3           | 5.0            |

Table 5-15. Stiffness of Bothkennar clay specimens.
Figure 5-1. Pore pressures in the London clay.
Figure 5-2. Self-boring pressuremeter results in London clay.
Figure 5-3. Ko in London clay. (from Burland et al. (1979)).
Figure 5-4. London clay moisture loss for various sealing methods.
Figure 5-5. Soil lathe.
Figure 5-6. Cradle to trim triaxial sample ends.
Figure 5-7. LVDT brackets fixed to soil sample.
Figure 5-8. London clay specimen set-up.
Figure 5-9. Cyclic response of London clay (LC1).
Figure 5-10. Isotropic consolidation of London clay.
Figure 5-11. Isotropic consolidation of London clay.
Figure 5-12. Stress paths of London clay.
Figure 5-13. Stiffness behaviour of London clay.
Figure 5-14. Chalk sampling position at Needham Market.
(from Russell (1997)).
Figure 5-15. Consolidation results for Chalk.
Figure 5-16. Shear results for Chalk.
Figure 5-17. Typical stiffness response of Chalk (CH5).
Figure 5-18. Description and Atterberg limits of Bothkennar clay.
(from Nash, Powell et al. (1992)).
Figure 5-19. Sherbrooke sampler used to sample Bothkennar clay.
Figure 5-20. Schematic view Bothkennar clay stress paths.
Figure 5-21. Bothkennar clay stress paths to failure.
(includes yield surface from Smith et al. (1992)).
Figure 5-22. Bothkennar clay stress-strain behaviour.
Figure 5-23. Typical stiffness of Bothkennar clay (BK3, Uco3).
Figure 5-24. Time to saturation for coarse Ottawa sand (from Black and Lee (1973)).
Figure 5-25. Variation of stiffness with choice of origin.
DISCUSSION

This Chapter discusses the main findings of the thesis. The development of new local instrumentation formed an important part of the work carried out. This is discussed in section 6.1. The stiffness behaviour observed for a number of geomaterials is elaborated upon in Sections 6.2 to 6.4. Section 6.5 compares the stiffnesses measured during triaxial tests with the stiffnesses from a variety of field seismic stiffness measurements. In conclusion, the practical implications of the results of this thesis are discussed in section 6.6.

1. Development of local strain instrumentation

In Chapter 2 it was stated that one of the major goals of this thesis was to develop accurate local instrumentation. The objective was to measure the stress-strain behaviour of a range of geomaterials, from inside the linear stress-strain region, up to failure. The development of the Fabry-Perot interferometer played a pivotal role in achieving this objective. The interferometer was used in two ways. Firstly, it was used directly as a local displacement instrument. Secondly, it was used as an accurate reference device to facilitate the high accuracy calibration of commercial LVDTs. These LVDTs were then used as local displacement instruments.

All LVDTs were electrically identical, but were different with regards to geometry, operational pressure and permissible cell fluid. In addition, the linearity of the LVDTs as specified by the manufacturer differed. The linearity of the standard LVDTs were specified as 0.5% of the full scale by the manufacturer. In contrast, the linearity of the high-performance LVDTs were 0.1% of full scale. In practice the performance of the high-performance LVDTs were found to be significantly superior to that of the standard LVDTs. The LVDTs were more resilient to harsh environmental conditions than the interferometer. For example they could operate at high pressures in various cell mediums including water and oil. This made them considerably easier to use as local instruments. Given these factors, and the success
of improving the accuracy of the LVDTs by calibration against the interferometer, the LVDTs were used for the majority of the tests conducted during this project.

6.1.1. LVDTs as local displacement instruments

The LVDTs used for local displacement measurements were miniature submersible instruments. All were electrically similar and had a working range of ±5mm. Some differences did exist between the LVDTs. These included differences in geometry, operating pressure, permissible cell fluid and differences in linearity specified by the manufacturer. The LVDTs were calibrated against the interferometer over ranges between ±20µm and ±50µm. The best accuracy achieved was for a high-performance LVDT and was found to be ±0.027µm. This represents an accuracy of strain measurement of ±0.43x10⁻⁶ for a gauge length of 62.5mm. The LVDT accuracy may be compared to the accuracy of local displacement instrumentation developed by previous workers (see Table 2-4). Comparison shows that a significant improvement has been made with regards to the accuracy of local displacement instrumentation. The accuracy of ±0.027µm constitutes an improvement of almost two orders of magnitude compared to the highest accuracy established by previous workers using good calibration practice (see for example Burland and Symes (1982), Clayton and Khatrush (1986), Hird and Yung (1989)).

The materials tested during this project were all natural geomaterials, but they varied significantly with respect to strength and stiffness. These materials included a soft clay (Bothkennar clay), a hard clay (London clay) and a weak rock (Chalk). The linear stress-strain range was observed for all three geomaterials during triaxial testing by using the LVDTs as local displacement instruments (see section 6.3). Linear stress-strain behaviour has not previously been observed for natural clays such as the Bothkennar clay and London clay.
6.1.2. Interferometers as local displacement instruments

One test was performed with two Fabry-Perot interferometers as local displacement instruments. The accuracy of the interferometers when used directly as a local displacement instrument could not be determined by calibration as a suitably accurate reference system was not available. An alternative approach was taken by investigating the factors which may introduce measurement errors. It was shown in Chapter 3 that two factors have a dominant influence on the measurement error of the interferometer when used in conjunction with the interpolation technique. These are piezo-actuator noise in the form of creep, and errors from quantifying the output function. The two types of errors can not be compared directly as one is a time-dependent error, and the other is an absolute error. However, the maximum possible error as a result of actuator noise was shown to be 0.013μm/min, which was 3% of the local shear rate measured during test CH15. In addition the maximum error on account of the shape of the output function was quantified as 0.01μm.

Two interferometers were fitted to a 100mm diameter Chalk specimen. In addition, two LVDTs were used as local instruments. For this test standard LVDTs were used. Each pair of instruments were diametrically opposed and named according to the four compass positions as FP(N), FP(S), L₁(W) and L₂(E). The load and interferometer displacements are shown in Figure 6-1 for the first 80 seconds of the test. The displacement was taken as the average of FP(N) and FP(S). The first part of the test was used to evaluate the creep prior to shear. Lift-off of the load occurred smoothly at 47 seconds. The load and displacement curves follow almost identical paths indicating that the ratio between load and displacement increments are constant. This constitutes linear behaviour. Figure 6-2 shows interferometer and LVDT displacements. The difference in high frequency noise levels of the two types of instruments is immediately apparent. The noise level of the interferometer was within 0.005μm, whereas for the LVDTs it was within 0.060μm. Random noise is particularly important when conducting stiffness measurements at very small
strains, as it limits the level to which meaningful measurements can be taken. In this respect, the performance of the interferometer was 12 times better than that of the LVDTs.

The load and displacements of CH15 for the first 180 seconds are shown in Figure 6-3. Up to approximately 90 seconds, displacements were calculated by the interpolation technique. From 90 seconds onwards, displacements were determined using the technique of fringe counting. Figure 6-3 shows that the displacement rates are similar for both techniques.

The initial stress-strain response of the Chalk is shown in Figure 6-4. Superimposed on the data points is a straight line of slope 4.6GPa. Figure 6-4 shows that the initial stress-strain response of the Chalk is essentially linear. In addition, the magnitude of the strain levels should be noted. Figure 6-4 shows that meaningful measurements of the stress-strain behaviour could be made down to a fraction of 0.0001% axial strain. Again, this constitutes a significant advance when these levels are compared to the small strain levels which have been measured by previous workers.

The stiffness of the chalk as a function of axial strain is shown in Figure 6-5. The strain was calculated as the average of the strains measured by the two interferometers. The data points show some scatter at strains smaller than 0.0001% and converge to a scatter of less than 10% of the stiffness at approximately 0.0001%. From Figure 6-5 a plateau is evident which seems to indicate linear stress-strain response of the Chalk. However, it was noted in Chapter 3 that the interferometer displacements curves diverged, particularly at large strain levels. For this reason, the extent of the linear plateau shown in Figure 6-5 should be treated with caution.

The stiffness of the Chalk as measured by the LVDTs is shown in Figure 6-6, taking the strain as the average of the two LVDTs. In contrast to the measurements made by the interferometer, the data points show large scatter at 0.0001% axial strain and
only converge at a strain level of approximately 0.001%. This is indicative of the higher noise level of the LVDTs compared with that of the interferometer.

At an axial strain level of 0.001%, the scatter in the stiffness data measured by means of the LVDTs reduced sufficiently to allow comparison of the stiffnesses measured by the two displacement measurement techniques. $E_{0.001}$ as measured by the two techniques was similar. In the case of the LVDTs, $E_{0.001}$ was approximately 4.5GPa, whereas $E_{0.001}$ as measure by the interferometers was approximately 4.6GPa. At strains between 0.001% and 0.01%, the stiffness measured by the interferometer remains practically constant, whereas the stiffness measured by the LVDTs reduced. It is suggested that this difference in behaviour is due to the more significant divergence shown by the interferometer measurements. In Chapter 3, it was shown that at intermediate strain levels the displacement measured by FP(S) diverged from the measurements made by the other instruments. It was argued that this was due to discontinuity effects.

2. Factors that influence geomaterial stiffness

6.2.1. Effect of bonding

Numerous methods have been suggested to identify the existence of geomaterial bonding (see for example Mitchell et al. (1968), Acar and El-Tahir (1986), Leroueil and Vaughan (1990), Aversa et al. (1993), Cuccovillo and Coop (1997b), Hight et al. (1997)). However, as noted in Chapter 2, no single method is suitable for identifying bonding for all types of geomaterials. In many cases, a number of tests (and supplementary evidence) are used to determine whether a geomaterial is bonded.

All three geomaterials used during this project were bonded. Given that all three materials were natural geomaterials and that care was taken during sampling, this was not unexpected. Strong evidence exists to show that both the Chalk and
Bothkennar clay are bonded. Bothkennar clay exhibits bond permitted space (see for example Smith 1992 and Nash, Sills et al. (1992)). Furthermore, both materials showed dramatic yielding during isotropic compression or shear. Evidence of bonding in the London clay is less clear. The geological age (Eocene) suggests that the material should be bonded. This is supported by the fact that during isotropic compression the state of the London clay proceeded to states above the intrinsic compression line.

The fact that no single method is suitable to identify bonding for all types of geomaterials makes it difficult, and sometimes impossible, to judge whether one geomaterial is more bonded than another. This difficulty is evident when attempting to rank the three geomaterials used, in order of bonding magnitude. Purely on the basis of stiffness and strength, a rational order for the three materials in decreasing order of bonding magnitude would be; Chalk, London clay and Bothkennar clay.

Some evidence is available to show that the Chalk is more bonded than Bothkennar clay. The geological age difference and material chemistry are two such factors. Furthermore, as the Chalk and Bothkennar clay are both high porosity materials, the significantly higher strength and stiffness of the Chalk suggests the existence of stronger bonds. However, classifying the London clay as more bonded than the Bothkennar clay may be contentious for two reasons. Firstly, the London clay had a low void ratio ($e \approx 0.68$) which may account for its high strength compared to Bothkennar clay. Secondly the London clay was highly fissured. Therefore, even if the intact material was bonded, low levels of bonding on the fissures may dominate the mass behaviour. This further complicates the question as to whether the mass behaviour of the London clay should be consistent with the expected behaviour of a bonded geomaterial.

As a result of the difficulty of ranking the three geomaterials in order of geomaterial bonding, no conclusions will be drawn concerning the effects of bonding on stiffness by comparing the behaviour of the three materials. Instead, the behaviour
of the Chalk before and after isotropic yielding will be used as a basis to investigate the effect of bonding. Isotropic yielding of the Chalk was accompanied by significant volumetric strains. This suggests that destructuring of the material occurred at it was taken past the isotropic yield stress. It may therefore be said that the Chalk was more bonded before yielding than after yielding.

Bonding increases stiffness. This may be observed by comparing the stiffness measured for the Chalk before and after yielding. See for example CH5 and CH14 shown in Figure 6-7. CH5 was isotropically consolidated to 995kPa. This was well below the yield stress of 2900kPa. In contrast, CH14 was destructured by isotropic consolidation to 3798kPa. Yield of the specimen was accompanied by significant volumetric strains of 8%. Both specimens were sheared undrained from the isotropic stress state. The specimen with the higher level of bonding (CH5) had a higher stiffness response, which was evident at all strain levels. The ratios of stiffness of the two specimen were 1.31 for $E_{0.001}$ and 1.48 for $E_{0.01}$.

The destructured specimen (CH14) was sheared at a higher mean effective stress than CH5. Even without accounting for this fact, the intact specimen (CH5) had a higher stiffness. A more realistic comparison can be made by taking the different stress levels into account. This may be done by normalisation. Figure 6-8 shows the stiffnesses of the two specimens normalised with respect to the initial mean effective stress at the start of shear ($p'_0$). After normalisation, the ratio of normalised stiffnesses of CH5 and CH14 was 5.1 for $E_{0.001}$ and 5.8 for $E_{0.01}$.

The above evidence clearly shows that bonding has the effect of increasing geomaterial stiffness. This is in agreement with the work of Acar and El-Tahir (1986), Bressani (1990) and Cuccovillo and Coop (1997b) (see Chapter 2).
6.2.2. *Influence of mean effective stress*

The testing programme carried out on the intact Chalk specimens allowed the influence of mean effective stress on the stiffness of bonded geomaterials to be investigated. Shear tests were conducted at mean effective stresses that ranged between approximately 100kPa and 3800kPa. As the isotropic yield stress of the Chalk was approximately 2900kPa, the stiffness before and after isotropic yield could be compared. Figure 6-9 shows the results of all the shear tests conducted on the Chalk. Some trends can be observed. For mean effective stresses up to the isotropic yield stress, \(E_{0.001}\) and \(E_{0.01}\) were found to be relatively insensitive to the mean effective stress. With the exception of one test (CH1) \(E_{0.001}\) ranged between 4100kPa and 4900kPa whereas \(E_{0.01}\) ranged between 3800kPa and 4500kPa. A reduction in \(E_{0.001}\) and \(E_{0.01}\) was observed for stress states in excess of the isotropic yield stress. \(E_{0.001}\) reduced to approximately 3500kPa and \(E_{0.01}\) to approximately 2800kPa. This again shows the effect of bonding on the stiffness behaviour of geomaterials. When geomaterials have significant levels of bonding, the stiffness is relatively insensitive to changes in mean effective stress. This is in contrast with geomaterials with low levels of bonding. As shown in Chapter 2, the stiffness of such materials is strongly influenced by the mean effective stress.

The above results are similar to those observed by Cuccovillo and Coop (1997b) for intact and remoulded Calcarenite.

6.2.3. *Effect of recent stress history*

One test was performed to investigate the effect of recent stress history. A number of shear stages were conducted on a single specimen of Bothkennar clay (BK3). The specimen was sheared in undrained compression from the same initial stress state on the isotropic line \((p'_{si} \approx 18\text{kPa})\). This point is shown schematically as point A in Figure 6-10. Three shear stages were conducted and each time \(p'_{si}\) was approached
from a different direction. Prior to the first shear stage (Uco1), the specimen was swelled isotropically to p'_si (path CA). Before the second shear stage (Uco2), the specimen was consolidated isotropically to p'_si (path DA). And before the third shear stage (Uco3), p'_si was approached along a constant p' stress path (path BA). The change in stress path direction was therefore 90° for Uco1, 90° for Uco2 and 180° for Uco3. Prior to each shear stage, a rest period was observed to allow creep levels to subside. The ratio between local creep and local shear rates were less than 1% for all three shear stages.

The above test strategy was similar to the one employed by Atkinson et al. (1990). Atkinson et al. (1990) included a fourth stress excursion where no change in stress path direction occurred. In order to conduct such a stress excursion, negative deviator stress application was required. This was not available in the low-pressure triaxial apparatus. The tests series of Atkinson et al. (1990) were on reconstituted London clay. The material was consolidated isotropically to 400kPa and swelled to 200kPa. It therefore had an over-consolidation ratio of 2 and an isotropic yield stress of 400kPa. The length of the approach stress paths were 90kPa prior to each shear stage. The authors commented that for approach paths of less than 90kPa, the subsequent behaviour was influenced by the length of the path. For the Bothkennar clay (BK2), the lengths of the approach paths were approximately 10kPa prior to each shear stage. The yield stress of the Bothkennar sample was approximately 40kPa and therefore the length of the approach path as a ratio of the isotropic yield stress was similar in both cases.

The results for the three shear stages are shown in Figure 6-10. It may be seen that the stiffness response of the three shear stages matched each other closely. The linear plateau was observed for all shear stages. E_max ranged between 24.1MPa and 26.5MPa. At a strain level of 0.01%, almost no difference in the stiffness of the three stages was observed. The evidence in Figure 6-10 clearly shows that recent stress history had a negligible effect on the material stiffness.
It was shown in Chapter 2 that the effect of recent stress history reduces as more time is allowed prior to shear to allow creep rates to subside. Atkinson et al. (1990) allowed only three hours between reaching the required stress state and commencing shear for reconstituted London clay. They found that depending on the change in stress path direction, a 5-fold variation in stiffness occurred. Jardine et al. (1991) used a restriction by which enough time was allowed until the (local) creep rate prior to shear was less that 1% of the (external) shear rate. They found a more modest recent stress history effect for intact London clay. $E_{0.01}$ ratios for undrained compression and extension varied between 0.75 and 2.52. Hird and Pierpoint (1997) typically allowed 2 to 3 days rest prior to shear of intact Oxford clay and found almost no recent stress history effects.

The above evidence and the results from BK3 shown in Figure 6-10 clearly shows that once creep due to the prior loading history is properly accounted for, recent stress history has a negligible effect on geomaterial stiffness.

6.2.4. Effect of current stress path direction

The previous section showed that recent stress history has no significant influence on the stiffness of a naturally bonded geomaterial such as Bothkennar clay. This section discusses the effect of current stress path direction on geomaterial stress-strain behaviour. Two tests were performed to investigate this effect. One test was on London clay (LC4) and the other on Bothkennar clay (BK2).

The London clay test (LC4) was conducted on a single specimen. The test strategy is shown in Figure 6-11. The specimen was brought to the in-situ stress ($p'_0 = 388kPa$, $q'_0 = -194kPa$) along a drained stress path (path AB). From the in-situ stress it was first sheared in undrained compression to $q = -60kPa$ (path BC). Subsequently the specimen was taken back to the in-situ stress, again along a drained stress path (Path CDB), before being sheared to failure in extension (path BE). Prior to each undrained shear excursion, a rest period was observed to allow
creep levels to subside to acceptable levels. Local creep to local shear rate ratios were less than 0.5% for both shear stages.

The aforementioned testing strategy was chosen for a number of reasons. Firstly, the test was conducted on one specimen to avoid effects that may arise from variation between samples. Secondly, the approach paths prior to the two shear stages were similar and therefore any possible effect that the recent stress history may have had on the stiffness behaviour was eliminated. Thirdly, the compression stage was conducted before the extension stage, because the compression path moves away from the yield surface, whereas the extension path moves towards the yield surface. This loading sequence was chosen in order to avoid damage to the specimen between the two shear stages.

The stiffness response for the two excursions are shown in Figure 6-11. The linear plateau was observed in both cases. The limit of linear behaviour was approximately 0.004% axial strain for the compression test as compared to 0.002% for the extension test. $E_{\text{max}}$ was found to be approximately the same for compression and extension (228MPa in compression and 239MPa in extension). However, the stiffness at intermediate strain levels were significantly different for compression and extension loading. At intermediate strain levels the stiffness was lower when the stress path was directed towards the yield surface ($U_{\text{ex}}$). In contrast, higher stiffnesses were observed when the stress path was moving away from the yield surface ($U_{\text{co}}$).

For the Bothkennar clay (BK2), the initial stress was approached along an undrained stress path (path AB in Figure 6-12). After a rest period at point B to allow creep to subside, the specimen was first sheared in undrained extension (path BA). The specimen was subsequently taken back to point B along path AB. After another rest period, the specimen was sheared in undrained compression (path BC). Creep as a ratio of the local shear rate was less than 0.2% for both shear stages.
The stiffness responses of the Bothkennar clay for the two shear stages are shown in Figure 6-12. The linear plateau was observed during undrained extension and the limit of linear behaviour was approximately 0.002%. In contrast, the linear plateau was not clearly observed for shear in undrained compression. At 0.001% axial strain, stiffness of the compression stage was marginally lower than the stiffness of the extension stage. However, for intermediate strain levels, the stiffness of the extension stage was significantly higher. In fact, \( E_{0.01} \) in extension was 26% higher than \( E_{0.01} \) in compression. Again it is notable that a softer response occurred when the current stress path moved towards the yield surface.

Comparison of LC4 (Figure 6-11) and BK2 (Figure 6-12) show that both materials behaved in a consistent manner. These two tests can be used to draw some conclusions on the effect of current stress path direction on the stiffness response of natural clays. In both tests the stiffnesses at very small strain levels were independent of the direction of the outgoing stress path. In contrast, it is clear that the stiffness responses at intermediate strain levels were strongly dependent on the stress path direction. The stiffness at intermediate strains was softer when the stress path moved towards the yield surface compared to when it moves away from the yield surface.

A number of factors other than the direction of the current stress path direction might possibly account for the behaviour observed above. These include:

a) variation of material structure,

b) effect of recent stress history,

c) differences in strain rate,

d) creep at start of the shear excursions,

e) differences in mean effective stress,

f) change in stress path direction.

These factors are investigated below.

Variation in specimen structure may account for differences in stiffness behaviour.
These include differences in fabric or differences in bonding levels. Variation of sample structure was minimised in two ways. Firstly, for each test, both shear stages were conducted on a single specimen. This eliminated possible differences in fabric and bonding levels that may occur between different specimens. Secondly, for both tests the first stress excursion was away from the yield surface and it was therefore unlikely that the specimen suffered addition damage between the first and second shear stages. The judgement is therefore made that the specimen structure was essentially similar for both shear stages.

For both tests, the incoming stress paths which preceded the shear stages were identical. Therefore any possible effects from recent stress history could have had no bearing on the results.

The local strain vs. time for the two tests are shown in Figure 6-13. The figure shows that strain rates were similar for the compression and extension stages of the two tests. This is particularly true for axial strains between 0.001% and 0.01%. The differences in stiffness observed at intermediate strain levels could therefore not be accounted for by differences in shear strain rate.

High creep rates prior to shear may lead to erroneous stiffness measurements, especially at very small strain levels. Effects from creep were minimised by permitting a sufficiently long rest period before each shear stage, in order to allow creep rates to subside. The local creep rates were less than 0.5% of the local shear rates for the two shear stages of the London clay and less than 0.2% for the two stages of the Bothkennar clay. These rates were sufficiently low to avoid any significant influence from creep.

Mean effective stress influences the stiffness of geomaterials. For the test on London clay (LC4), the undrained stress paths were non-vertical. This was probably as a result of the cross-coupling between shear and volumetric strains that occurs in anisotropic materials (see for example Graham and Houlsby (1983)). The result was
that the mean effective stress changed during the test. The influence of the changing mean effective stress can be reduced by normalising the stiffness with regards to the current mean effective stress. Figure 6-14 shows the normalised stiffness of the London clay. Normalisation did not change the main elements of the observed behaviour. The normalised stiffnesses at 0.001% axial strain were again approximately equal. Also, the normalised stiffness response at intermediate strains was again higher when the current stress path was moving away from the yield surface.

The stiffness of the Bothkennar clay, normalised with respect to the current mean effective stress is shown in Figure 6-15. Comparison of Figure 6-12 and Figure 6-15 show that the normalised stiffnesses for the extension and compression stages are matched even better at 0.001% axial strain. Again the main elements of the observed behaviour are the same as before normalisation. It is clear therefore that any differences in mean effective stress can not account for the observed difference in stiffness during compression and extension of LC4 and BK2.

Changes in stress path direction occurred for both LC4 and BK2. Therefore, the argument might be put forward that it was the change in stress path direction that resulted in the different stiffnesses at intermediate strains. In fact, it is true for both tests that at intermediate strain levels the stiffer response was measured for the stress path that had undergone a significant reversal in direction. This argument is dispelled by the fact that $E_{0.001}$ was approximately equal for the compression and extension stages of both tests. The reversal in stress path direction clearly had no influence on the stiffness at the start of the next loading stage (at very small strains). No mechanism can be postulated whereby a change in stress path direction will have an influence on the stiffness at intermediate strain level but no influence at small strains. The conclusion must be drawn therefore, that it was the direction of the current stress path that is the governing factor in the rate of stiffness degradation, and not the change in stress path direction.
The previous paragraphs have shown that none of the effects listed above can account for the behaviour observed for the London clay and Bothkennar clay. Therefore on the basis of the observed behaviour as shown in Figure 6-11 and Figure 6-12 the following conclusions are drawn.

a) When a rest period follows a shear excursion, the stiffness of a geomaterial reverts back to $E_{\text{max}}$. It may be said that during such rest periods the material “forgets” that it previously had a lower stiffness and perhaps that healing of any damaged material occurs.

b) After a rest period, the stiffness of the material is $E_{\text{max}}$, regardless of the direction of the incoming or outgoing stress path. $E_{\text{max}}$ is therefore independent of the previous and current stress path directions.

c) The stiffness at intermediate strain levels is dependent on the direction of the current stress path. When the stress path direction is away from the yield surface, the stiffness at intermediate strains is stiffer compared to when the stress path direction is towards the yield surface.

An additional comment needs to be made concerning the effect of the current stress path as outlined in (c) above. In the case of the London clay additional tests allowed the approximate positions of the yield surface to be established. From the in-situ stress, the path to the yield surface was significantly shorter in extension than in compression. For the Bothkennar clay, the yield surface established by Smith (1992) was used to judge the position of the in-situ stress relative to the yield stress. The in-situ stress was only marginally closer for shear in undrained compression than for undrained extension. Therefore, the criterion “moving towards the yield surface” was not as well defined for the Bothkennar clay as for the London clay.

The observations listed above may be compared to those found by previous workers. The recovery of stiffness that occurs when a rest period follows a shear excursion has as far as known, only been observed on one previous occasion. Hird and Pierpoint (1997) noted a recovery of stiffness of intact Oxford clay when applying a holding period. The instrumentation at their disposal was not sufficiently
accurate to measure the linear stress-strain response of the soil and their observations are therefore only valid for small, intermediate and large strains.

It was shown in Chapter 2 that both Lade and Duncan (1976) and Hird and Pierpoint (1997) found that when a soil was sheared from close to the yield surface, the stiffness response was significantly softer when the stress path was directed towards the yield surface as opposed to away from the yield surface. This is in agreement with the results shown in Figure 6-11 and Figure 6-12. Again, their observations were only at small and intermediate (other than very small) strain levels.

The observation that $E_{\text{max}}$ is independent of the incoming or outgoing stress path has not been observed before. This is probably due to the fact that sufficiently accurate local-strain instrumentation has not been widely available to date. The observed behaviour has some important implications for modelling of soil behaviour and will be discussed further in section 6.6.3.

6.3. Limit of linear stress-strain behaviour

It was shown in Chapter 2 that observations of linear stress-strain behaviour have only been made for materials with relatively high levels of bonding. For soils with low levels of bonding the linear plateau has not been observed to date. During this project the linear plateau was identified in 15 of the 20 shear stages conducted. For the London clay and Bothkennar clay, the linear plateau was observed in 7 out of the 10 shear excursions. This constitutes a significant advance.

From the behaviour observed during this project, some conclusions may be made regarding the limit of linear behaviour of natural geomaterials.

The limit of linear stress-strain behaviour ($\varepsilon_{\text{linear}}$) increases as the level of bonding increases. This conclusion may be drawn by comparing intact Chalk (CH5) with destructured Chalk (CH14). The results are shown in Figure 6-7. CH5, with the
higher level of bonding, showed almost no reduction in stiffness up to an axial strain of 0.004%. In contrast, no clear linear plateau was evident for CH14.

The three geomaterials tested during this research project had wide ranging strengths and stiffnesses. The stiffness of the London clay was about an order of magnitude higher than that of the Bothkennar clay. Similarly, the stiffness of the Chalk was about an order of magnitude more than that of the London clay. However, it was apparent that the strain limit of linear behaviour ($\varepsilon_{\text{linear}}$) did not vary significantly from one material to the next. In a number of tests, the linear plateau was not clearly observed at a strain level of 0.001%. However for these tests, the onset of the linear plateau was observed when plotted against the log of strain. This suggests that the linear plateau was only marginally outside the measurement capabilities of the LVDTs. This may be contrasted with the largest strain limit of linear behaviour observed (0.004% for LC4(Uco)). As an example, typical stiffness plots for all three materials are shown in Figure 6-16. For the three tests shown, the strain limits of linear behaviour were in a narrow range between 0.002% and 0.003%. However, when the stiffness data for the same three tests are plotted as a function of deviator stress, a different picture emerges. Figure 6-17 shows that the range of the stress limits for linear behaviour covers two orders of magnitude. The Bothkennar clay exhibited a linear stress-strain relationship up to a deviator stress of 1kPa. In contrast, for the Chalk it was up to 100kPa.

In summary, the above evidence has shown that bonding increases the strain limit of linear stress-strain behaviour ($\varepsilon_{\text{linear}}$). However, it is clear that bonding has a much stronger influence on stiffness and strength than on strain limit of linear stress-strain behaviour ($\varepsilon_{\text{linear}}$).
6.4. Non-linear stress-strain behaviour

In general, stiffness reduces with increasing axial strain. Of all the tests conducted during this project, only two tests showed an increased stiffness with increasing strain. These two tests were on intact Chalk (CH1 and CH4). Recall from Chapter 5 that both were drained shear tests and were performed at a mean effective stress of 100kPa. This stress level was small in comparison with the isotropic yield stress of 2900kPa. The results for the two tests are shown in Figure 6-18. In both cases, an increase in stiffness can clearly be observed for strain levels between 0.001% and 0.003%. Stiffness reached a peak at 0.003% before it decreased at larger strains. Peak stiffness for CH1 and CH4 were at deviator stress levels of 119kPa and 154kPa respectively. The behaviour of Chalk samples sheared at intermediate stress levels was significantly different. CH5 was sheared undrained from an initial mean effective stress of 995kPa. This was still well below the isotropic yield stress. The results for CH4 and CH5 are compared in Figure 6-19. In contrast to CH4, no stiffness increase was observed for the sample sheared at an intermediate stress level (CH5). This behaviour was consistent for all Chalk specimen sheared at mean effective stresses in excess of 500kPa.

The above behaviour may possibly be explained on the basis of fissure closure. All Chalk specimens contained discontinuities which were seen as thin brown lines on the surface of the specimen. Breaking a specimen along a discontinuity revealed an irregular surface with a rough texture. Brown staining was commonly observed. To the naked eye these fissures appeared closed, and failures along fissures seldom occurred during specimen preparation. It is significant that both specimen sheared at low mean effective stresses showed a stiffness increase at small strains. In contrast, none of the specimens sheared at mean effective stresses of 500kPa and above exhibited this behaviour. This is consistent with fissure closure, where stiffness increases as normal stress increases. At effective vertical stresses of approximately 220kPa and 250kPa CH1 and CH4 reached peak stiffness. This seems to suggest
that at these stress levels complete fissure closure occurred. At higher deviator stresses stiffness reduction again became apparent.

For all tests with the exception of CH1 and CH4, stiffness degraded with increased strain level. The rate of stiffness degradation is shown in Table 6-1.

<table>
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<th>Test</th>
<th>$\frac{E_{0.01}}{E_{0.001}}$</th>
<th>$\frac{E_{0.1}}{E_{0.001}}$</th>
<th>$\frac{E_{1.0}}{E_{0.001}}$</th>
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<td>CH3 (Uco)</td>
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<td>0.11</td>
</tr>
<tr>
<td>Bothkennar clay</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>BK1 (Uco)</td>
<td>0.81</td>
<td>0.36</td>
<td>0.11</td>
</tr>
<tr>
<td>BK2 (Uco)</td>
<td>0.75</td>
<td>0.37</td>
<td>0.11</td>
</tr>
<tr>
<td>BK3 (Uco)</td>
<td>0.90</td>
<td>0.55</td>
<td>0.21</td>
</tr>
</tbody>
</table>

Table 6-1. Geomaterial stiffness degradation.

Considering the wide ranging strengths and stiffnesses of the geomaterials tested, it is again surprising how little variation in stiffness degradation occurred. $E_{0.01}$ values were, with few exceptions, found to be between 0.80 and 0.95 of $E_{0.001}$. At 0.1% axial strain, the stiffness ($E_{0.1}$) had reduced further to ratios of between 0.35 and 0.55 of $E_{0.001}$. At 1% strain all intact Chalk samples had failed. However, the remainder of the specimens had secant stiffnesses of between approximately 0.10 and 0.20 of $E_{0.001}$ and were close to failure.

On close inspection, some differences in the rate of stiffness degradation are apparent. Specimens showed a lower rate of stiffness degradation when the stress
path was moving away from the yield surface. Compare, for example, LC1 (Uco) and LC4 (Uco) with LC3 (Uex) and LC4 (Uex). All specimens were initially at negative deviatoric stresses. The stress paths for the two extension tests were moving towards the closest part of the yield surface. On the other hand, the stress paths for the two compression tests were moving away from the yield surface. Similar behaviour was observed for the Bothkennar clay.

It is of considerable practical consequence that the rate of stiffness degradation is not significantly influenced by the strength and stiffness of the geomaterial. Field seismic stiffness can therefore be used in conjunction with the stiffness degradation rates shown in Table 6-1 to make a judgement on operational stiffness for design.

6.5. **Comparison of laboratory and field stiffness**

It was stated in Chapter 2 that one of the objects of this project was to compare triaxial stiffnesses at very small strain levels with seismic stiffnesses measured *in situ*. Geophysical field stiffness data was available for all three geomaterials tested. The triaxial stiffnesses are shown together with the geophysical stiffnesses for the three materials in Figure 6-20, Figure 6-21 and Figure 6-22.

The seismic stiffness of the Chalk was measured by means of the surface wave technique at the same quarry at Needham Market in Suffolk where the Chalk samples were taken. The results have been reported by Clayton et al. (1994). The surface wave stiffness ($E_0$) ranged between 750MPa and 1900MPa. As shown in Figure 6-20 these values are significantly lower than the small strain stiffness of the triaxial specimens. With the exception of the two destructured samples (CH8 and CH14) and CH1, the small strain stiffness ($E_{0.001}$) of the Chalk specimens were within a relatively narrow range. $E_{0.001}$ ranged between 4100kPa and 4900kPa. Therefore, the stiffness of the triaxial specimens at 0.001% strain was on average 3.4 times more than those measured by means of the surface wave technique. This discrepancy may be explained by considering the macro structure of the material.
Matthews (1993) has given a detailed description of the rock mass from the Needham Market Quarry. He found the rock mass to be highly fractured. It was characterised by vertical and sub-vertical rough joints together with wavy sub-horizontal bedding discontinuities. Four major sub-vertical joint sets were identified. The average spacing of the sub-vertical joint sets ranged between 8 and 120mm. In addition, the spacing of the horizontal bedding planes was investigated by excavating three test pits to a depth of approximately 5m. The bedding planes had apertures of up to 40mm and the average spacing was found to vary between 235mm and 364mm.

The above description of the discontinuities suggests that the rock mass stiffness will be strongly influenced by the behaviour of the discontinuities. It should be recognised that $E_0$ as measured by the surface wave technique is representative of a large volume of material at very small strains. In contrast, $E_{\text{max}}$ as measured in the triaxial apparatus only characterises a small volume of material at very small strains. The evidence in Figure 6-20 shows that the stiffness increases significantly when the number of joints and fissures reduce. This confirms that the stiffness of the rock mass is dominated by the discontinuities.

Triaxial and cross-hole stiffnesses of the London clay are compared in Figure 6-21. The seismic data was obtained from Butcher (1997). It consisted of a cross-hole survey conducted in London clay at a site approximately 1 mile to the North of Heathrow Terminal 4. The seismic waves were horizontally propagating and vertically polarised. At the sampling depth of 18m, the cross-hole stiffness ($E_0$) was approximately 350MPa. This may be compared to $E_{0.001}$ from the triaxial tests which ranged in a relatively narrow band between 235kPa and 274kPa. $E_{0.001}$ was therefore approximately between 20% and 30% lower than the cross-hole stiffness. In the previous Chapter it was suggested that the London clay samples were subjected to disturbance prior to shear as a result of the tunnelling process. It was estimated that the strain levels to which the samples were subjected were of the order of 1%. Strain levels of this magnitude may have resulted in some damage to
the material structure. The notion of damage to the material structure is consistent with the lower stiffnesses measured in the triaxial apparatus compared to the cross-hole stiffnesses.

Seismic cone and seismic cross-hole data was available from the Bothkennar site and are shown in Figure 6-22. Some scatter of the seismic stiffness is apparent. This is not surprising as Ricketts et al. (1996) argued that shear wave velocity can only be estimated to within 10% to 20%. A 10% error in the shear wave velocity will lead to a 20% error in the calculated stiffness.

Two of the triaxial tests were conducted on Bothkennar clay at initial mean effective stresses close to the in-situ mean effective stress (BK1 and BK2). However, the shear stages of BK3 was conducted at initial mean effective stresses significantly lower than the in-situ mean effective stress. In order to compare the triaxial and cross-hole data, all stiffnesses were normalised with respect to the mean effective stress. The stiffnesses are compared in Figure 6-22. The triaxial stiffnesses were taken as the stiffness at 0.001% axial strain, $E_{0.001}$ for the three shear stages conducted during BK3 were found to be approximately mid way between the upper and lower bounds of the seismic stiffness, whereas $E_{0.001}$ from BK1 and BK2 were close to the lower bound of the measured seismic stiffness. In fact, it may be argued that within the context of the measurement uncertainty of both data sets, no significant difference in the triaxial and field seismic stiffness can be observed for the Bothkennar clay.

Some important conclusions, which are fundamental to the theory and practice of soil mechanics, may be drawn from the above results.

a) Figure 6-22 shows that $E_{\text{max}}$ as measured in the triaxial apparatus is very similar to $E_0$ as measured by field seismic geophysics. This is true even though the two measurement techniques are fundamentally different. However, both are based on the principles of elasticity theory. This suggests that at very small strain levels, soil behaviour conforms to that of a linear-elastic material.
b) The strain rate at which seismic and triaxial tests are conducted are vastly different. Seismic field tests are essentially dynamic tests and the triaxial tests performed during this project were essentially static. The fact that similar stiffnesses were measured indicates that at very small strain, the stiffness of soils are rate independent.

c) If due care and attention is taken during sampling, storage and testing of soils, laboratory tests can provide results which gives a true reflection of the behaviour of the soil \textit{in situ}.

d) The level of disturbance as a result of sampling, storage and transportation may be evaluated by comparing field stiffness to laboratory stiffness.

e) Differences in mass and intact behaviour can be quantified by comparing stiffnesses measured in the laboratory on relatively small samples, to field seismic stiffness which measures the stiffness of a large volume of soil.

6.6. \textbf{Practical implications of the results from this thesis}

6.6.1. \textbf{Future local displacement instruments}

This project has increased the accuracy of measurements made with local displacement transducers. This was achieved by the development of a Fabry-Perot interferometer which has allowed local measurements to an accuracy of approximately $0.01\mu m$. In addition the interferometer has been used to calibrate commercial LVDTs to accuracies of up to $\pm 0.027\mu m$. Even though the accuracy of the interferometer was better than that of the LVDTs, it is foreseen that the interferometer will remain a research tool. This observation is based on a number of factors. Firstly, sophisticated equipment is required to develop an interferometer as a local displacement instrument. Secondly, considerable care is necessary during the use of the interferometer. Thirdly, analysis of the interferometer results was found to be time consuming. Finally, the present configuration of the interferometer does not allow it to be used in a cell medium other than air. Compressed air creates a
potential health hazard, and this restricts the allowable cell pressure that can be applied.

Even though the LVDTs were not as accurate as the interferometer, they were sufficiently accurate, after calibration against the interferometer, to allow the linear plateau to be observed for all three geomaterials investigated. The LVDTs were calibrated and used during testing without significant difficulty. In addition, it was relatively easy to analyse the data. Given the above factors, it is suggested that the LVDTs, combined with the interferometer as a calibration device, may find favour amongst other research and commercial laboratories.

6.6.2. Integrated approach to stiffness measurement

The development of the local strain instrumentation during this project has made it possible to directly compare $E_{\text{max}}$ measured in the triaxial apparatus with $E_0$ from seismic field stiffness measurements. $E_0$ from the field may be used to establish a benchmark against which $E_{\text{max}}$ from the triaxial apparatus can be compared. From this, effects such as the influence of the macro fabric (Clayton et al. (1994)) and sampling disturbance (Hight (1993)) can be evaluated.

Even though field seismic techniques can measure values of $E_0$ relatively quickly and economically, they can not measure the rate of stiffness degradation of stiffness with increasing strain. For this reason triaxial testing and local strain measurement will remain important to future geotechnical practice. This statement is based on the fact that field seismic techniques only provide information on the stiffness at very small strain levels. It was shown in Section 6.4 that the largest stiffness reduction occurs between 0.01% and 0.1% strain. For many soils, these are the strain levels which commonly occur in the vicinity of geotechnical structures (see for example Mair (1993)) and therefore, stiffness measurements are also required at higher strain levels.
6.6.3. Implications to modelling of geomaterial behaviour

Geotechnical constitutive models have typically been developed on the basis of experimentally observed behaviour. The highly-accurate local displacement transducers used during this research have allowed the stress-strain response of a number of geomaterials to be observed from the linear range up to failure. The behaviour observed for these materials can be compared with those predicted by current popular constitutive models.

One of the most advanced classes of constitutive models currently available to model geomaterial behaviour are kinematic hardening models (see for example Stallebrass and Taylor (1997)). These models were designed to take into consideration factors such as non-linear stress-strain behaviour and the effects of changes in stress path direction. In general these models do not take time effects into consideration.

The behaviour observed for both the London clay and Bothkennar clay is idealised in Figure 6-23. It shows a stress path excursion from A to B (Figure 6-23(a)). The length of the excursion (AB) is sufficiently long for the soil to display a non-linear stress-strain response. At B a rest period follows. From B, two stress paths are compared. The first is a stress path of increasing deviator stress (BC) and the second is a stress path of decreasing deviator stress (BD). The behaviour observed during this testing programme is idealised in Figure 6-23(b) and the behaviour as predicted by kinematic hardening models is idealised in Figure 6-23(c) (see for example Stallebrass and Taylor (1997)). The behaviours are clearly different. For the stress path reversal (BD), the modelled stiffness reverts back to the maximum stiffness. However, for no stress path reversal (BC), the model assumes the stiffness to be equal to the stiffness as point B was approached. This is significantly lower than the stiffness on reversal. As shown in Figure 6-23(b), the observed behaviour is significantly different. The initial stiffness after the rest period is the same.
regardless of the direction of the outgoing stress path. Furthermore, this stiffness is markedly higher than the stiffness response as point B is approached.

The above behaviour was observed for two tests on two different natural clays. Both were sheared from stress states close to the in-situ stress. These initial stress states were remote from the yield surface. The above behaviour might not be observed at initial stress states close to the yield surface, but it can be concluded that this behaviour will be important in most engineering situations.

A further consideration is the question of whether to model the effect of recent stress history. Models such as the Brick Model (Simpson 1992) and the Three Surface Kinematic Hardening Model (Stallebrass et al. (1994), Stallebrass and Taylor (1997)) have been developed to include history effects. This was based largely on the history effects observed experimentally in the triaxial apparatus (see for example Atkinson et al. (1990) and Stallebrass (1990)). This thesis has shown that once creep effects are accounted for, recent stress history has no effect on the stiffness of geomaterials. These results indicate that it is not necessary to model the recent stress history of geomaterials.
Figure 6-1. Interferometer displacements (CH15).
Figure 6-2. Interferometer and LVDT displacements (CH15)
Figure 6-3. Interferometer displacements (first 180 seconds) (CH15).
Figure 6-4. Initial stress-strain response of Chalk (CH15).
Figure 6-5. Stiffness of Chalk (CH15) (strains from interferometer).
Figure 6-6. Stiffness of Chalk (CH15) (strains from LVDTs).
Figure 6-7. Stiffness of Chalk before and after yielding.
Figure 6-8. Normalised stiffness of Chalk before and after yielding.
Figure 6-9. Stiffness of Chalk vs. initial mean effective stress ($p'_s_i$).
Figure 6-11. Effect of current stress path direction on London clay (LC4)
Figure 6-12. Effect of current stress path direction on Bothkennar clay (BK2)
Figure 6-13. Locally measured shear rates for LC4 and BK2.
Figure 6-14. Stiffness of London clay normalised with respect to the current mean effective stress (LC4)
Figure 6-15. Stiffness of Bothkennar clay normalised with respect to the current mean effective stress (BK2)
Figure 6-16. Stiffness of all three geomaterials as a function of $\varepsilon_a$. 
Figure 6-17. Stiffness of all three geomaterials as a function of deviatoric stress increment.
Figure 6-18. Stiffness of Chalk at low mean effective stress. (CH1 and CH4).
Figure 6-19. Stiffness of Chalk at low and intermediate mean effective stress.

(CH4 and CH5).
Figure 6-20. Triaxial and surface wave stiffness of Chalk.

\( E = 3G \) assumed for geophysical data
Figure 6-21. Triaxial and cross-hole stiffness of London clay.

(Triaxial stiffness at $e_a = 0.001\%$. Cross-hole data from Butcher (1997)).

($E = 3G$ assumed for geophysical data)
Figure 6-22. Triaxial and seismic stiffness of Bothkennar clay.

(Triaxial stiffness at $\varepsilon_a = 0.001\%$).

($E = 3G$ assumed for geophysical data)
Figure 6-23. Idealised geomaterial behaviour.
CONCLUSIONS

The following conclusions are drawn from this thesis:

- The linear stress-strain response of weakly bonded natural soils has to date not been observed widely. This is a consequence of the fact that in general, the local strain instrumentation developed prior to this project has not been sufficiently accurate to detect this response.

- A Fabry-Perot interferometer has been developed, and used both for calibration of LVDTs and as a local strain instrument in its own right. The best accuracy achieved for LVDTs used as local displacement instrumentation was ±0.027μm. This constitutes an improvement of almost two orders of magnitude compared to the highest accuracy established by previous workers using good calibration practice. The improved accuracy allowed linear stress-strain behaviour to be observed for geomaterials with widely varying strength and stiffness. Particularly for natural clay, these appear to be the first observations made of linear stress-strain behaviour.

- The maximum error of the interferometer, used directly as a local displacement instrument, was estimated as 0.01μm.

- Bonding has the effect of increasing geomaterial stiffness. This was demonstrated by comparing the stiffness of high porosity Chalk before destructuring and after destructuring by isotropic yielding. The destructured material had a lower stiffness.
• When geomaterials have significant levels of bonding, the stiffness is relatively insensitive to changes in mean effective stress. For Chalk with relatively high levels of bonding it was shown that the stiffness does not increase significantly for an increase in mean effective stresses, provided that the mean effective stress is less than the yield stress.

• Once creep due to the prior loading history is properly accounted for, recent stress history has no effect on geomaterial stiffness.

• When a rest period follows a shear excursion, the stiffness of a geomaterial reverts back to $E_{\text{max}}$. It was suggested that during such rest periods healing of any damaged material occurs and that the material “forgets” that it previously had a lower stiffness.

• After a rest period, the stiffness of the material is $E_{\text{max}}$ regardless of the direction of the incoming or outgoing stress path. $E_{\text{max}}$ is therefore independent of the previous and current stress path directions.

• The stiffness at intermediate strain levels is dependent on the direction of the current stress path. When the stress path direction is away from the yield surface, the stiffness at intermediate strains is stiffer compared to when the stress path direction is towards the yield surface.

• The limit of linear stress-strain behaviour ($\varepsilon_{\text{linear}}$) increases as the level of bonding increases. This conclusion is drawn by comparing Chalk before and after destructuring. However it was pointed out that the limit of linear behaviour is remarkably similar for natural bonded materials with a wide range of strengths and stiffnesses.
• For the three materials tested, little variation in stiffness degradation occurred. In general $E_{0.01}$ values were found to be between 0.80 and 0.95 of $E_{0.001}$. $E_{0.1}$ was approximately between 0.35 and 0.55 of $E_{0.001}$ and $E_{1.0}$ between approximately 0.10 and 0.20 of $E_{0.001}$.

• The stiffness of high porosity Chalk is dominated by its discontinuities. This conclusion is drawn by comparing the small strain stiffness of triaxial specimens containing relatively few discontinuities with the mass stiffness measured by means of surface wave geophysics. The stiffness increases significantly when the number of discontinuities reduce.

• Once due consideration is taken of effects from sampling disturbance, stress level, strain level and differences in mass and intact behaviour, $E_{\text{max}}$ as measured in the triaxial apparatus is similar to $E_0$ as measured by field seismic geophysics at very small strains. At the very least this shows that the stiffness of soils are rate independent and at most is shows that soil behaviour at very small strains are linear-elastic in nature.
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APPENDIX

Appendix A - Software development
Flowchart for triaxial cyclic loading software.
Program for Stress Path Triaxial Testing
(Version 1.1, 25 June 1996)

'THIS PROGRAM: (i) READS output from CIL System
(ii) PLOTS results on custom graphs
(iii) CONTROLS GDS pressure controllers

'REQUIRED FILES:
'SYSTEM FILES:
C:\WINDOWS\SYSTEMS\KRNLS386.EXE Used by GetModuleUsage statement (ReadSGAP procedure)
C:\WINDOWS\LYNX.DRV Quad card driver (see LYNX readme file for system settings)
C:\LAB\SGA\GH1.EXE Reads data from A/D converter
C:\LAB\SGA\GH1.PIF
C:\LAB\COMMS1.EXE VB4 Program to change GDS pressure at a specified rate
C:\LAB\COMMS1.PIF
C:\LAB\COMMS2.EXE VB4 Program to set current GDS pressure as target pressure
C:\LAB\COMMS2.PIF
C:\LAB\COMMS3.EXE VB4 Program to read GDS pressures
C:\LAB\COMMS3.PIF

'USER FILES:
C:\LAB\CONTROL.DAT (user input data)

'PROGRAM FILES:
C:\LAB\TEMP1.DAT Temporary file for SGA data.
C:\LAB\TEMP2.DAT Temporary file for GDS data.
C:\LAB\CONTROL.RAN Random access version of CONTROL.DAT.
C:\LAB\TESTFILE.DAT Output file of test data.
c:\LAB\PRESSURE.DAT Output file of GDS pressures

'FORMAT FOR CONTROL.DAT:

2
100, 200, Number of stress points
50,
60,
73,
-80,
350,
100, 200,
50,
60,
73,
-80,
-350,

Start time, End time in seconds
Target pressure for Controller 1
Target pressure for Controller 2
Target pressure for Controller 3
Target pressure for Controller 4
Change in loadcell force (N) for this cycle
Start time, End time in seconds
Target pressure for Controller 1
Target pressure for Controller 2
Target pressure for Controller 3
Target pressure for Controller 4
Change in loadcell force (N) for this cycle

e etc.

Notes on CONTROL.DAT:
Time in seconds since start of test.
Low value > -99 kPa.
High Value < 999 kPa.
Maximum rate of pressure increase is 1 sec/kPa.
Allow 100 minimum of 100 seconds between end of one stress point and start of next.
Declare Variables

Declare Function GetModuleUsage Lib "C:\WINDOWS\SYSTEMS\KRNL386.EXE" (ByVal hModu le As Integer) As Integer
'GetModuleUsage is a Windows DLL file for "ShellProcedure"

Public PlotInt As Integer 'Interval in seconds for dataset written to spreadsheet
Public Nth As Integer 'Nth dataset written to disk
Public LogNumber As Long 'Number of current dataset read from A/D Converter
Public RowNumber As Integer 'Row number in spreadsheet

Public OnceLoop1 As Integer 'Counter for Loop in "TimeProcedure"
Public OnceLoop2 As Integer 'Counter for Loop in "ChartProcedure"
Public Shift As Integer 'Shifts all data by 50 rows in WriteToSheetProcedure
Public CheckWrite As Integer 'Trigger in "MainProgram"

Public TimeSinceStart As Long 'Time in seconds since start of test
Public TimeTotal As Long 'Total seconds since January 1
Public TimeTotalStart As Long 'Total seconds since January 1 at start of tes t
Public NowMonth As Integer 'Number of days for whole months since January 1

Public Ch1, Ch2, Ch3, Ch4 As Integer 'Channel output in Bits
Public Ch5, Ch6, Ch7, Ch8 As Integer 'Channel output in Bits

Public LVDT1Factor, LVDT2Factor, LVDT3Factor As Single 'Calibration factors
Public ExLVDTFactor, LoadCellFactor, MidPlaneFactor As Single
Public BackPressFactor, CellPressFactor As Single

Public LVDT1Offset, LVDT2Offset, LVDT3Offset As Integer 'Offset values in bits
Public ExLVDTOffset, LoadCellOffset, MidPlaneOffset As Integer
Public BackPressOffset, CellPressOffset As Integer

Public LVDT1, LVDT2, LVDT3, ExLVDT As Single
Public LoadCell, MidPlane, BackPress, CellPress As Single

Public GaugeLength, SampleHeight As Single
Public Diameter, Area As Single

Public DeltaSigma1Tot, DeltaSigma1Eff As Single
Public DeltaSigma3Tot, DeltaSigma3Eff As Single
Public DeltaP, Deltaq As Single
Public AxialStrain, VolumeStrain As Double

Public NoOfStressPoints, StressPoint, RecordNumber As Integer
Public ControllerStartTime, ControllerEndTime As Integer
Public InitialLoad, CurrentLoadChange, TargetLoadChange As Integer
Public HaveRunComms1, HaveRunComms2 As Boolean
'Trigger to ensure controller is only addressed once for each 'dataset. Activated in ReadControllerDataProcedure and 'deactivated in CommandControllerProcedure

Page 2
'******************************************************************************
' Set User Variables
'******************************************************************************

Sub UserVariablesProcedure()
  PlotInt = 60  'Interval in seconds for dataset plotting to screen
  Nth = 1  'Each Nth dataset will be written to disk
  GaugeLength = 50  'LVDT Gauge length in mm
  Diameter = 100  'Sample diameter in mm
  LVDT1Factor = 1  'Calibration factor for LVDT1 (µm)
  LVDT2Factor = 1  'Calibration factor for LVDT2 (µm)
  LVDT3Factor = 1  'Calibration factor radial LVDT (µm)
  ExLVDTFactor = 1  'Calibration factor for external LVDT (µm)
  LoadCellFactor = 2.0423  'Calibration factor for Load cell (N)
  MidPlaneFactor = 0.17439  'Calibration factor for mid plane (kPa)
  BackPressFactor = 0.1743  'Calibration factor for back pressure (kPa)
  CellPressFactor = 0.17637  'Calibration factor for cell pressure (kPa)
End Sub

'******************************************************************************
' Set Program Variables
'******************************************************************************

Sub ProgramVariablesProcedure()
  OnceLoop1 = 0  'Counter for Loop in "TimeProcedure"
  OnceLoop2 = 0  'Counter for Loop in "PlotProcedure"
  LogNumber = 0  'Number of current dataset
  StressPoint = -1  'Stress point number
  RecordNumber = 1  'RecordNumbers in ReadControllerDataProcedure
  CheckWrite = -1
  CurrentLoadChange = 0  'Reset for each cycle in CommandControllersProcedure
End Sub

'******************************************************************************
' Set values at start of test
'******************************************************************************

Sub StartValuesProcedure()
  .Open "C:\LAB\TEMP1.DAT" For Input As #1
  ' Open file for input.
  Input #1, Ch1, Ch2, Ch3, Ch4, Ch5, Ch6, Ch7, Ch8  ' Read data.
  Close #1

  'Zero offset in Bits at start of test
  LVDT1Offset = -Ch1  'Offset for LVDT1 (Bits)
  LVDT2Offset = -Ch2  'Offset for LVDT2 (Bits)
  LVDT3Offset = -Ch3  'Offset for radial LVDT (Bits)
  ExLVDTOffset = -Ch4  'Offset for external LVDT (Bits)
  LoadCell1Offset = -Ch5  'Offset for Load cell (Bits)
  MidPlaneOffset = -Ch6  'Offset for mid plane transducer (Bits)
  BackPressOffset = -Ch7  'Offset for back pressure transducer (Bits)
  CellPressOffset = -Ch8  'Offset for cell pressure transducer (Bits)
End Sub

'******************************************************************************
' Set Controller Variables  (GDS Controllers)
'******************************************************************************

Sub ReadControllerDataProcedure()
Dim SeekVal(1 To 4), I As Integer
Dim ExecutionTime As Integer

StressPoint = StressPoint + 1 'Allows progress monitoring

Open "C:\LAB\CONTROL.RAN" For Random As #2

Get #2, 1, Value
NoOfStressPoints = Value
Debug.Print NoOfStressPoints

RecordNumber = RecordNumber + 1
Get #2, RecordNumber, Value
ControllerStartTime = Value
Debug.Print ControllerStartTime

RecordNumber = RecordNumber + 1
Get #2, RecordNumber, Value
ControllerEndTime = Value
Debug.Print ControllerEndTime

For I = 1 To 4

RecordNumber = RecordNumber + 1
Get #2, RecordNumber, Value
SeekVal(I) = Value
Debug.Print SeekVal(I)

Next I

RecordNumber = RecordNumber + 1
Get #2, RecordNumber, Value
TargetLoadChange = Value
Debug.Print TargetLoadChange

Close #2

ExecutionTime = ControllerEndTime - ControllerStartTime

Open "C:\LAB\TEMP2.DAT" For Output As #1
Print #1, ExecutionTime
Print #1, SeekVal(1) 'Notes:
Print #1, SeekVal(2) '(i)TargetLoadChange is not written to TEMP2,
Print #1, SeekVal(3) 'as it is not required by GDS controllers
Print #1, SeekVal(4) '(ii)LoadChange is Declared as Public
Close #1

HaveRunComms1 = False 'Enable trigger for "MainProgramProcedure"
HaveRunComms2 = False

End Sub

'Notes for this procedure:
'(i)"Value" variable is used as Get Statement returns "2" irrespective of correct
'value. This is particularly true for variables that are declared Public
'as opposed to declared Dim (however exceptions have been observed).
'(ii)Do not Dim "Value" it upsets Get Statement.
Procedure to initialize charts

Sub InitializeChartsProcedure()

    Application.DisplayFullScreen = True 'Create "Full Screen"
    'Initialize "time chart". Plots LVDT1, LVDT2 & q vs. time.

    Sheets("Sheet1").Select
    Range("A1").Select
    ActiveSheet.ChartObjects.Add(1, 27, 220, 295).Select
    Selection.Name = "TimeChart"
    ActiveChart.ChartWizard Source:=Range("A100:A199,B100:B199,C100:C199,F100:F199"), PlotBy:=xlColumns, CategoryLabels:=1, SeriesLabels:=0, HasLegend:=1, CategoryTitle:="Seconds", ValueTitle:="μm", ExtraTitle:="kPa"

    ActiveSheet.ChartObjects("TimeChart").Activate
    ActiveChart.SeriesCollection(1).Select
    ActiveChart.SeriesCollection(1).Name = "=""L1"

    ActiveChart.SeriesCollection(2).Select
    ActiveChart.SeriesCollection(2).Name = "=""L2"

    ActiveChart.SeriesCollection(3).Select
    ActiveChart.SeriesCollection(3).AxisGroup = 2
    ActiveChart.SeriesCollection(3).Name = "=""q"

    ActiveChart.Axes(xlValue, xlSecondary).HasTitle = True
    ActiveChart.Axes(xlValue, xlSecondary).AxisTitle.Select
    Selection.Characters.Text = "kPa"

    ActiveChart.Axes(xlValue, xlSecondary).Select
    With Selection.TickLabels.Font
        .Name = "Times New Roman"
        .FontStyle = "Bold"
    End With

    ActiveChart.Axes(xlValue, xlSecondary).AxisTitle.Select
    Selection.Font.Name = "Times New Roman"

    ActiveChart.PlotArea.Select
    .Selection.Left = 10
    .Selection.Top = 1
    .Selection.Width = 190
    .Selection.Height = 275

    ActiveChart.Legend.Select
    .Selection.Left = 120
    .Selection.Top = 280

    ActiveChart.Axes(xlCategory).AxisTitle.Select
    .Selection.Left = 60
    .Selection.Top = 280

    Windows("CYCLE1.XLS").Activate 'Deactivate "TimeChart"

    'Initialize "strain chart". Plots q vs. strain.

    ActiveSheet-chartObjects.Add(220, 27, 225, 147).Select
    Selection.Name = "StrainChart"
    ActiveChart.ChartWizard Source:=Range("D100:D199,F100:F199,G100:G199"), PlotBy:= _
ActiveSheet.ChartObjects("StrainChart").Activate
ActiveChart.SeriesCollection(1).Select
ActiveChart.SeriesCollection(1).AxisGroup = 1
ActiveChart.SeriesCollection(2).Select
ActiveChart.SeriesCollection(2).AxisGroup = 2
ActiveChart.Axes(xlCategory).Select
Selection.TickLabels.NumberFormat = "#E+0"
ActiveChart.Axes(xlValue, xlSecondary).HasTitle = True
ActiveChart.Axes(xlValue, xlSecondary).AxisTitle.Select
Selection.Characters.Text = "Vol Strain"
ActiveChart.Axes(xlValue, xlSecondary).Select
With Selection.TickLabels.Font
  .Name = "Times New Roman"
  .FontStyle = "Bold"
End With
ActiveChart.Axes(xlValue, xlSecondary).AxisTitle.Select
Selection.Font.Name = "Times New Roman"
ActiveChart.Axes(xlValue, xlSecondary).Select
Selection.TickLabels.NumberFormat = "#E+0"
ActiveChart.SeriesCollection(1).Select
ActiveChart.SeriesCollection(1).Name = "=""q"
ActiveChart.SeriesCollection(2).Select
ActiveChart.SeriesCollection(2).Name = "=""Vol Strain"
ActiveChart.PlotArea.Select
Selection.Left = 13
Selection.Top = 1
Selection.Width = 195
Selection.Height = 128
ActiveChart.Axes(xlCategory).AxisTitle.Select
Selection.Left = 50
Selection.Top = 230
ActiveChart.Legend.Select
Selection.Left = 200
Selection.Top = 150

Windows("CYCLE1.XLS").Activate 'Deactivate "pqChart"
'Initialize "p-q chart". Plots p' vs. q.
Selection.Name = "pqChart"
ActiveChart.ChartWizard Source:=Range("E100:E199,F100:F199"), PlotBy:= _
  xlColumns, CategoryLabels:=1, SeriesLabels:=0
ActiveSheet.ChartObjects("pqChart").Activate
ActiveChart.Axes(xlValue).AxisTitle.Select
Selection.Characters.Text = "q (kPa)"
ActiveChart.Axes(xlCategory).AxisTitle.Select
Selection.Characters.Text = "p' (kPa)"
ActiveChart.Legend.Select
Selection.Delete

ActiveChart.PlotArea.Select
Selection.Left = 13
Selection.Top = 1
Selection.Width = 205
Selection.Height = 130

ActiveChart.Axes(xlCategory).AxisTitle.Select
Selection.Left = 100
Selection.Top = 150

Windows("CYCLE1.XLS").Activate 'Deactivate "pgChart"

End Sub
Main Program Procedure

Sub MainProgram()

    Dim CheckPlot As Integer  'Trigger to initialize plot and write procedures
    Dim ProgramLoop As Integer  'Variable for endless loop

    Call UserVariablesProcedure  'Set user variables
    Call ProgramVariablesProcedure  'Set program variables
    DoEvents  'Yield to processor
    Call WriteSheetHeaderProcedure  'Write spreadsheet header
    Call InitializeChartsProcedure  'Initialize Charts on spreadsheet
    Call WriteGDSFile  'Converts CONTROL.DAT File to Random Access File

    Call ReadControllerDataProcedure  'Get GDS data for first stress point
    Call ReadSGAProcedure  'Place start values in temp file
    Call GetDataProcedure  'Get start values from temp file
    Call StartValuesProcedure  'Set start values to variables
    Call WriteToFileHeaderProcedure  'Write File Header

    Do
        Call TimeProcedure  'Get TimeSinceStart (time since start of test)
        CheckPlot = 0  'Trigger to initialize plot events
        If TimeSinceStart >= (LogNumber * PlotInt) Then CheckPlot = 1
        If CheckPlot = 1 Then
            Call ReadSGAProcedure  'Place data in temp file
            Call GetDataProcedure  'Get data from temp file
            Call CheckLoadProcedure  'Evaluate current load
            Call WriteToSheetProcedure
            CheckWrite = CheckWrite + 1
            If CheckWrite >= Nth Then
                Call WriteToFileProcedure
                CheckWrite = 0
            End If
        End If
    End If

    If CheckPlot = 1 Then LogNumber = LogNumber + 1
    If TimeSinceStart >= ControllerStartTime And _
        StressPoint < NoOfStressPoints And _
        'HaveRunComms1 = False Then Call CommandControllerProcedure
    If TimeSinceStart >= ControllerEndTime And _
        StressPoint < NoOfStressPoints Then Call ReadControllerDataProcedure

    'Read next data set
    ProgramLoop = 0
    Loop While ProgramLoop = 0  'Endless loop

End Sub

Procedure to write header for test data file

Sub WriteToFileHeaderProcedure()

    Open "C:\LAB\TESTFILE.DAT" For Output As "+  'Create file for data.
    Print "+, "Date: " + Date: Print "+,
    Print "+, "Start Time: " + Time: Print "+,
    Print "+, Tab(1); "Time"; Tab(10); "LVDT1"; _
        Tab(19); "LVDT2"; Tab(28); "LVDT3"; Tab(37); "ExLVDT"; _
        Tab(46); "Load"; Tab(56); "Mid";
        Tab(65); "Back"; Tab(73); "Cell" --
    Print "+,
    Print "+, Tab(1); " "; Tab(10); LVDT1Factor;
    Tab(19); LVDT2Factor; Tab(28); LVDT3Factor; Tab(37); ExLVDTFactor; _
        Tab(46); LoadCellFactor; Tab(56); MidPlaneFactor; _

Tab(65); BackPressFactor; Tab(73); CellPressFactor
Print #1,
Print #1,
Print #1, Tab(1); " 0"; Tab(10); Ch1; Tab(19); Ch2; Tab(28); Ch3; Tab(37); Ch4; Tab(46); Ch5; Tab(56); Ch6; Tab(65); Ch7; Tab(73); Ch8
Close #1

End Sub

*********************************************************************************************************************************************
' Procedure to write spreadsheet header
*********************************************************************************************************************************************

Sub WriteSheetHeaderProcedure()
  Sheets("Sheet1").Select
  Range("A1:J2").Select
  Selection.Clear 'Clear cells
  Selection.HorizontalAlignment = xlCenter
  Selection.Font.FontStyle = "Bold"
  Selection.Font.Name = "Times New Roman"

  Range("A1").Select
  ActiveCell.FormulaR1C1 = "Time"

  Range("B1").Select
  ActiveCell.FormulaR1C1 = "LVDT1"

  Range("C1").Select
  ActiveCell.FormulaR1C1 = "LVDT2"

  Range("D1").Select
  ActiveCell.FormulaR1C1 = "LVDT3"

  Range("E1").Select
  ActiveCell.FormulaR1C1 = "ExLVDT"

  Range("F1").Select
  ActiveCell.FormulaR1C1 = "LoadCell"

  Range("G1").Select
  ActiveCell.FormulaR1C1 = "MidPlane"

  Range("H1").Select
  ActiveCell.FormulaR1C1 = "BackPress"

  Range("I1").Select
  ActiveCell.FormulaR1C1 = "CellPress"

  Range("J1").Select
  ActiveCell.FormulaR1C1 = "Point"

  Range("A100:G199").Select
  Selection.Clear 'Clear cells

  Cells(100, 1).Value = 0 'Set initial values to plot
  Cells(100, 2).Value = 0
  Cells(100, 3).Value = 0
  Cells(100, 4).Value = 0
  Cells(100, 5).Value = 0
  Cells(100, 6).Value = 0
  Cells(100, 7).Value = 0

  Range("A1").Select 'Send screen to top

End Sub

*********************************************************************************************************************************************
' Procedure to retrieve data from "TEMP1.DAT"
*********************************************************************************************************************************************

Sub GetDataProcedure()
  Open "C:\LAB\TEMP1.DAT" For Input As #1 ' Open file for input.
  Input #1, Ch1, Ch2, Ch3, Ch4, Ch5, Ch6, Ch7, Ch8 ' Read data.
  Close #1
LVDT1 = (Ch1 + LVDT1Offset) * LVDT1Factor
LVDT2 = (Ch2 + LVDT2Offset) * LVDT2Factor
LVDT3 = (Ch3 + LVDT3Offset) * LVDT3Factor
ExLVDT = (Ch4 + ExLVDTOffset) * ExLVDTFactor
LoadCell = (Ch5 + LoadCellOffset) * LoadCellFactor
MidPlane = (Ch6 + MidPlaneOffset) * MidPlaneFactor
BackPress = (Ch7 + BackPressOffset) * BackPressFactor
CellPress = (Ch8 + CellPressOffset) * CellPressFactor

End Sub

Sub WriteToSheetProcedure()

Dim I, J, ColumnNumber As Integer

If OnceLoop2 = 0 Then Shift = 0 Else Shift = 50  'Shifts data
If LogNumber >= 49 Then OnceLoop2 = 1 Else OnceLoop2 = 0
RowNumber = (LogNumber - Int(LogNumber / 50) * 50) + Shift + 100

'Write data to be visable on screen
Cells(2, 1).Value = TimeSinceStart
Cells(2, 2).Value = LVDT1
Cells(2, 3).Value = LVDT2
Cells(2, 4).Value = LVDT3
Cells(2, 5).Value = ExLVDT
Cells(2, 6).Value = LoadCell
Cells(2, 7).Value = MidPlane
Cells(2, 8).Value = BackPress
Cells(2, 9).Value = CellPress
If StressPoint < NoOfStressPoints Then Cells(2, 10).Value = StressPoint - Else Cells(2, 10).Value = "Finish"  'Indicates status of test

Area = 0.7854 * (Diameter ^ 2)  'Initial sample area in mm^2
AxialStrain = (LVDT1 + LVDT2) / (0.002 * GaugeLength)  'LVDT in μm
VolumeStrain = (AxialStrain ^ 2) * ((0.5 + (LVDT1 + LVDT2)) / 10000)

DeltaSigma1Tot = (LoadCell / Area) * 1000 + CellPress  'Area not corrected
DeltaSigma3Tot = CellPress
DeltaSigma3Eff = CellPress - MidPlane
.Deltap = (DeltaSigma1Eff + 2 * DeltaSigma3Eff) / 3  'Change in Mean Effective Stress

Deltaq = DeltaSigma1Tot - DeltaSigma3Tot  'Change in Deviator Stress

'Write data to spreadsheet for plotting
Cells(RowNumber, 1).Value = TimeSinceStart
Cells(RowNumber, 2).Value = LVDT1
Cells(RowNumber, 3).Value = LVDT2
Cells(RowNumber, 4).Value = AxialStrain
Cells(RowNumber, 5).Value = Deltap
Cells(RowNumber, 6).Value = Deltaq
Cells(RowNumber, 7).Value = VolumeStrain

If RowNumber = 199 Then  'Reduce data routine
    For I = 102 To 200 Step 2
        J = (I - 100) / 2 + 100
        For ColumnNumber = 1 To 7
            Cells(J, ColumnNumber).Value = Cells(I, ColumnNumber)
        Next ColumnNumber
    Next I
    Range(Cells(151, 1), Cells(200, 7)).Clear
End If

Calculate

End Sub

'******************************************************************************
' Procedure to write dataset to file
'******************************************************************************

Sub WriteToFileProcedure()

Open "C:\LAB\TESTFILE.DAT" For Append As 
Print ", Tab(1); TimeSinceStart; Tab(10); Ch1;
   Tab(19); Ch2; Tab(28); Ch3; Tab(37); Ch4; Tab(46); Ch5;
   Tab(56); Ch6; Tab(65); Ch7; Tab(73); Ch8

Close 
' Data to file.
' Close File

End Sub

'******************************************************************************
' Procedure to calculate time
'******************************************************************************

Sub TimeProcedure()

'(Note: It takes 15msec to process this procedure on a 66MHz 486)

If Month(Date) = 1 Then NowMonth = 0
If Month(Date) = 2 Then NowMonth = 31
If Month(Date) = 3 Then NowMonth = 59
If Month(Date) = 4 Then NowMonth = 90
If Month(Date) = 5 Then NowMonth = 120
If Month(Date) = 6 Then NowMonth = 151
If Month(Date) = 7 Then NowMonth = 181
If Month(Date) = 8 Then NowMonth = 212
If Month(Date) = 9 Then NowMonth = 243
If Month(Date) = 10 Then NowMonth = 273
If Month(Date) = 11 Then NowMonth = 304
If Month(Date) = 12 Then NowMonth = 334

TimeTotal = Timer + Day(Date) * 86400 + NowMonth * 86400

Do While OnceLoop1 = 0
   'Do this loop only once
   TimeTotalStart = TimeTotal
   OnceLoop1 = 1
Loop
TimeSinceStart = TimeTotal - TimeTotalStart

End Sub

'******************************************************************************
' Procedure to read SGA output (in a DOS shell)
'******************************************************************************

Sub ReadSGAProcedure()

Dim X, Y As Integer

X = Shell("C:\LAB\SGA_GH1.EXE", 2)
Do
   Y = DoEvents 'Yield to processor, to execute DOS program
Loop While GetModuleUsage(X) <> 0 'X = 0 when no other applications
' demands time from processor
Open "C:\LAB\TEMP1.DAT" For Input As \\1 ' Open file for input.
Input \\1, Ch1, Ch2, Ch3, Ch4, Ch5, Ch6, Ch7, Ch8 ' Read data.
Close \\1

End Sub

********************************************************************************
' Procedure to Convert Sequential File to Random Access File
********************************************************************************

Sub WriteGDSFile()

    Dim Number, I As Integer
    If Dir("C:\LAB\CONTROL.RAN") = "CONTROL.RAN" Then Kill "C:\LAB\CONTROL.RAN"
    'Checks if file already exists and if so, deletes it.
    I = 1
    Open "C:\LAB\CONTROL.DAT" For Input As \\2
    Open "C:\LAB\CONTROL.RAN" For Random As \\3
    Do While Not EOF(2)
        Input \\2, Number
        Put \\3, I, Number
        I = I + 1
    Loop
    Close \\2
    Close \\3

End Sub

********************************************************************************
' Procedure to control GDS Controllers
********************************************************************************

Sub CommandControllerProcedure()

    Dim X, Y As Integer
    CurrentLoadChange = 0 'Set CurrentLoadChange to zero
    InitialLoad = LoadCell 'Set load at start of this cycle
    X = Shell("C:\LAB\COMMS1.EXE", 1)
    DoEvents 'Yield to processor
    DoEvents
    DoEvents
    SendKeys "%l" 'Send Alt+S to execute program (window already active)
    Do
        Y = DoEvents 'Yield to processor, to execute DOS program
        Loop While GetModuleUsage(X) <> 0 'X = 0 when no other applications 'demands time from processor
        DoEvents 'Yield to processor, to sent keys
        DoEvents
        DoEvents
        HaveRunComms1 = True 'used in "MainProgramProcedure" and 'ReadControllerDataProcedure

End Sub
Sub CheckLoadProcedure()

    Dim TestPositive, TestNegative, Test, Proceed As Boolean
    Dim X As Integer

    CurrentLoadChange = LoadCell - InitialLoad

    TestPositive = TargetLoadChange >= 0 And CurrentLoadChange >= TargetLoadChange
    TestNegative = TargetLoadChange <= 0 And CurrentLoadChange <= TargetLoadChange

    Proceed = TestPositive Or TestNegative  'Proceed if either is true

    If Proceed = True And HaveRunComms2 = False Then

        X = Shell("C:\LAB\COMMS2.EXE", 1)  
        "COMMS2.EXE" reads current GDS pressures of controllers on COM 7 & 8  
        'and sets the current pressures as the target pressures
        DoEvents  'Yield to processor
        DoEvents
        SendKeys "^2"  'Send Ctrl+S to make COMMS2 active window
        DoEvents
        SendKeys "%2"  'Send Alt+S to execute program
        DoEvents  'Yield to processor, to sent keys
        DoEvents
        HaveRunComms2 = True  'reset in "ReadControllerDataProcedure"

    End If

End Sub
Laser interferometry to evaluate the performance of local displacement transducers

G. HEYMANN,* C. R. I. CLAYTON* and G. T. REED*

The authors have developed a Fabry–Pérot laser interferometer system for displacement measurement, which is compact and can be used as a stand-alone local strain measuring system. Interferometry operates on the principle of interference of a coherent light source and therefore provides an absolute standard of measurement. The purpose of this development is to investigate the extent of the linear-elastic range of geomaterials in triaxial stress space, and to obtain values of stiffness which can subsequently be compared with those obtained in other ways, both in the laboratory (using resonant column or beaker elements) and in the field (using field geophysical techniques). The interferometer is also suitable as a high-accuracy instrument for calibration of other linear displacement transducers. This paper describes the instrument configuration and evaluates the performance of a commercial LVDT calibrated against the interferometer.

KEYWORDS: deformation; laboratory equipment; laboratory tests; stiffness.

INTRODUCTION

Local displacement transducers are now-commonly used to measure the small-strain behaviour of geomaterials. These transducers measure the relative displacements of positions on soil samples remote from the ends, thereby eliminating errors introduced by apparatus compliance, bedding and principal stress rotation near the sample ends. The ideal local displacement transducer should be accurate over a large measurement range and, furthermore, it should not be affected adversely by the cell fluid or cell pressure, or be damaged at large soil strains. Despite the large numbers of commercially available transducers such as LVDTs and proximity transducers, as well as a number of purpose-made local strain devices (Burland & Symes, 1982; Jardine et al., 1984; Clayton & Kharush, 1986; Clayton et al., 1989; Tatsuoka, 1988; Goto et al., 1991), no single device satisfies all these requirements. Different resolutions and accuracies are reported for these instruments, as shown in Table 1.

An instrument should be calibrated using a reference system having an accuracy 3–10 times better than the accuracy of the instrument being calibrated (Sydenham, et al., 1989; Doebelin, 1983; Collett & Hope, 1983; Sydenham, 1982). Table 1 shows that the performance of local displacement transducers has surpassed the calibration capabilities that are commonly available in research and commercial laboratories and clearly the need has developed for a cost-effective, high-accuracy calibration instrument. Previous workers have apparently used a Michelson interferometer to calibrate LVDTs and proximity transducers (e.g. Lo Presti et al., 1993). However, no details have been given with regard to the apparatus configuration, laser performance and signal interrogation

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Table 1. Characteristics of local displacement transducers

<table>
<thead>
<tr>
<th>Instrument</th>
<th>Reported range: mm</th>
<th>Reported accuracy: (\mu)m</th>
<th>Reported resolution: (\mu)m</th>
<th>Calibration instrument</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Electrolevel gauge</td>
<td>—</td>
<td>±2</td>
<td>—</td>
<td>—</td>
<td>Buriand &amp; Symes (1982)</td>
</tr>
<tr>
<td></td>
<td>15</td>
<td>—</td>
<td>&lt; 1</td>
<td>LVDT with accuracy of 0.2 (\mu)m</td>
<td>Jardine et al. (1984)</td>
</tr>
<tr>
<td>Hall effect gauge</td>
<td>2.5</td>
<td>±6</td>
<td>&lt; 1*</td>
<td>Micrometer with 2-54 (\mu)m resolution</td>
<td>Clayton et al. (1989)</td>
</tr>
<tr>
<td></td>
<td>7</td>
<td>±30</td>
<td>—</td>
<td>—</td>
<td></td>
</tr>
<tr>
<td>Local deformation transducer (LDT)</td>
<td>0.2†</td>
<td>0.09†</td>
<td>0.12</td>
<td>Micrometer with 1 (\mu)m resolution</td>
<td>Goto et al. (1991)</td>
</tr>
<tr>
<td>Proximity transducer</td>
<td>5</td>
<td>±2</td>
<td>1</td>
<td>Steel slip gauges with accuracy of ±0.2 (\mu)m</td>
<td>Hird &amp; Yung (1989)</td>
</tr>
<tr>
<td>LVDT</td>
<td>2</td>
<td>1</td>
<td>0.3</td>
<td>—</td>
<td>Scholey et al. (1995)</td>
</tr>
</tbody>
</table>

* Hall effect gauge resolution reported by Clayton & Kharush (1986).
† LDT range of 1.5 mm has been reported by Tatsuoka (1988).
‡ LDT accuracy calculated from reported strain accuracy of \(10^{-6}\) and gauge length of 90 mm.

**INTERFEROMETER CONFIGURATION AND PERFORMANCE**

A Fabry–Pérot interferometer (Fabry & Pérot, 1897) has been developed at the University of Surrey as a local displacement instrument for triaxial measurement of geomaterial behaviour at ultra-small strains. The interferometer is also suitable as a high-accuracy instrument for calibration of other linear displacement transducers. The layout is shown schematically in Fig. 1. An 8 mW helium–neon (HeNe) laser light source with a wavelength (\(\lambda\)) of 0.6328 \(\mu\)m is used. The laser has an output power stability of ±2.5% and a longitudinal mode spacing of 438 MHz. The light is launched into a single-mode optical fibre with a core diameter of 4 \(\mu\)m and numerical aperture of 0.12 by way of a laser-fibre pigtail assembly. The coupling efficiency of the pigtail assembly is of the order of 55%. As the light exits the fibre it is passed through a cylindrical graded index (GRIN) lens with a length of 4.3 mm and a diameter of 1.8 mm, which collimates the light (divergence angle less than 0.3°). The collimated light subsequently impinges on a partial mirror with a diameter of 12.5 mm and a thickness of 1 mm. It is machined with a surface flatness of 1 \(\mu\)m and a parallelism of better than 5 arc minutes. One side of the partial mirror has a dielectric coating which reflects 60% of the light and transmits 40%, while the other side has an anti-reflectance coating. The light transmitted through the first mirror is projected on to another partial mirror with similar geometric and optical properties. This second partial mirror is fixed to a silicon photodetector with an active area of 15 mm\(^2\). The two partial mirrors are positioned parallel to each other to form the Fabry–Pérot cavity.

Multiple reflections occur between the two partial mirrors, resulting in the well-known Fabry–Pérot transfer function shown in Fig. 2 (see, for example, Born & Wolf (1965)). The sharpness of the peaks is quantified as the full width at half maximum, which is the width between two points.

![Fig. 1. Schematic layout of Fabry–Pérot interferometer](image-url)
on either side of a maximum where the intensity has decreased to half its maximum value. The ratio of the separation of adjacent peaks and the full width at half maximum is called the finesse \((F)\) of the interferometer and is a function of the fraction of the incident light reflected by each mirror \((R)\):

\[
F = \frac{\sqrt{R}}{1 - R} \quad \cdots \quad (1)
\]

The normalized intensity may be calculated, for any relative movement of the two mirrors, in terms of the finesse \(F\) and the phase shift \(\phi\):

\[
\text{Normalized intensity} = \frac{\text{Output intensity}}{\text{Input intensity}} = \frac{1}{1 + F\sin^2\left(\frac{\phi}{2}\right)} \quad (2)
\]

Figure 2 illustrates the fact that the relative mirror displacement required to induce a \(2\pi\) phase shift (one peak to the next) is \(\lambda/2\) and therefore only a function of the light wavelength. Both high- and low-finesse interferometers have displacement measurement applications. High-finesse interferometers have the advantage that the peaks can be identified with high accuracy and are therefore particularly appropriate to be used in association with nulling techniques, where an induced displacement is countered by returning the mirror to zero phase shift using some mechanical means such as a piezo actuator. Low-finesse interferometers are particularly suited to the technique of fringe counting of a digital signal, and can be used effectively for large-range displacement measurements (several millimetres). Fig. 3 shows the measured normalized output intensity of the interferometer developed by the authors, as well as the theoretical transfer function. The two mirrors forming the cavity were mounted on to an invar plate to minimize any temperature effects. One mirror was kept stationary while the other was displaced using a piezo actuator with a resolution of 0·004 \(\mu\)m.

The accuracy of measurements using fringe counting relies on the phase stability of the peaks and is therefore dependent on the accuracy with which the wavelength is known, as well as on any fluctuation of the wavelength. The wavelength of the light exiting the GRIN lens was measured by an optical spectrum analyser with an accuracy of \(\pm 0\cdot0001\ \mu\)m. The peak amplitude of the spectrum was at 0·6328 \(\mu\)m and the full width at half maximum was 0·0014 \(\mu\)m. Wavelength fluctuations of gas lasers occur if changes in temperature, barometric pressure or relative humidity occur. Collett & Hope (1983) reported that for helium–neon gas lasers, a change of one part per million of the wavelength will occur for a temperature change of 1°C, a pressure change of 0·3 kPa or a
Fig. 3. Theoretical and measured interferometer output

change in relative humidity of 30%. When the interferometer was used to calibrate LVDTs, the procedure took only a few minutes and was carried out in a temperature-controlled laboratory. Changes in temperature, pressure and relative humidity were therefore within the limits which would cause a change of wavelength of one part per million.

Interpolation between peaks may introduce additional errors, as the Fabry–Perot transfer function has to be known accurately. Effects which may introduce errors into the function, such as uncertainties in the mirror reflectance and fluctuation of laser output power, must be assessed in order to quantify the accuracy of the interferometer when used with interpolation techniques.

LINEAR TRANSDUCER CALIBRATION

Ideally a local displacement transducer should have high accuracy and a large measurement range. These two characteristics are generally in conflict, as higher accuracy normally results in a smaller measurement range. A convenient way of improving the performance of electronic linear transducers is to modify the signal conditioning. This often takes the form of increasing the amplification of the output signal, thereby increasing the output sensitivity. The level to which the output signal can be amplified is limited by the high-frequency electronic noise, as both the signal and the noise are amplified. High-frequency noise is induced by electromagnetic sources such as the amplifier circuit components and external electronic devices often found in the laboratory. This is in addition to low-frequency noise, which may be caused by temperature changes or amplifier drift. The high-frequency noise may be reduced by shielding or by using filtering techniques. It is important to note, however, that increasing the output sensitivity or reducing the noise does not improve the accuracy. Judgement on the accuracy can only be made after the instrument has been calibrated against a suitably accurate calibration device.

The performance of a commercial LVDT with a range of ±5 mm, used for triaxial local strain measurements, was evaluated. Two calibration ranges were used. Firstly, the LVDT was calibrated over the full range using a micrometer with a resolution of 2.54 μm. Secondly, in order to evaluate the limit of the LVDT's performance, it was calibrated over a smaller range using the interferometer. A 12 bit analogue to digital (A/D) converter which integrated the signal over a period of 30 ms was used in both cases. This type of converter is particularly suitable for typical soil mechanics test applications. The result of the calibration using the micrometer is shown in Fig. 4 as the measurement error with respect to a linear regression. The resolution of the A/D converter was 2.7 μm and the digital output signal was stable, indicating that the noise level was less than the resolution. If the error is assumed to represent a Gaussian distribution and the accuracy is taken as the 95% confidence level
(±2 standard deviations), the accuracy may be calculated as ±16 μm.

The arrangement for the LVDT calibration using the interferometer is shown in Fig. 5. Both the LVDT and interferometer were mounted on a translational stage used for routine calibration of linear displacement devices. The stage slides on precision linear bearings, and possible roll or pitch errors have been neglected. The micrometer handle was rotated by a geared motor at a rate of one revolution per hour. This was necessary, given the slow A/D converter. Hand rotation of the micrometer handle would be satisfactory if a high-speed converter were used. The amplification of the demodulated output signal was maximized while keeping the amplification of the high-frequency noise within acceptable levels. At this high amplification, the range of the LVDT was ±50 μm and the resolution of the A/D converter 0.002 μm. Fig. 6 shows the error between the measured displacement and a linear regression. For a 95% confidence level, the accuracy may be calculated as ±0.13 μm.

Comparison of the results from the two calibrations shows that the ratio of the accuracy of the LVDT to the measurement range is 0.32% for the large calibration range and 0.26% for the smaller calibration range. Additional calibrations were carried out for a number of ranges smaller than ±5 mm and in all cases the accuracy was between 0.2% and 0.5% of the range; this compares favourably with the manufacturer's specification of 0.5%. This small variation indicates that the LVDT is the limiting component in the system.

The above results show that a compromise is required between accuracy and measurement range. This may be addressed by using high amplification for small-strain measurements and reducing the amplification at larger strains. This strategy is acceptable for most testing programmes as the accuracy requirements are normally less stringent at larger strain levels. It may, however, present a problem for cyclic testing programmes where small stress excursions are undertaken at various levels of strain.
DISCUSSION AND CONCLUSIONS

A Fabry–Pérot laser interferometer has been designed and constructed, and has been used with great effect to carry out calibrations of an LVDT intended for local strain measurement. This experience has demonstrated that the interferometer is sufficiently compact and robust to be used as a means of directly (i.e., without the use of LVDTs or Hall effect devices) measuring the ultra-small-strain stiffness behaviour of soil and weak rock specimens. Such testing is currently under way at the University of Surrey.

Many previously reported accuracies for local strain measurement devices have been better than the accuracy of the calibration system. This is unacceptable. The measurement of displacement with an interferometer depends only on the coherence of the light used, and therefore gas laser interferometers can provide a much higher accuracy than more conventional calibration systems.

Calibrations of the LVDT have shown that the system accuracy (including power supply stability, modulation/demodulation and analogue to digital conversion) is inversely proportional to the measurement range. Improved signal conditioning of electronic displacement transducers does not, of necessity, improve the accuracy of the system output. Accuracy can only be determined once the transducer system is calibrated against a suitably accurate reference system.

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REFERENCES


Appendix C

Definitions and typical geomaterial stress-strain response

<table>
<thead>
<tr>
<th>Definitions</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Brittle failure</td>
<td>Sudden failure of triaxial specimen.</td>
</tr>
<tr>
<td>Ductile failure</td>
<td>Development of large plastic strains after failure.</td>
</tr>
<tr>
<td>Post-peak strain softening</td>
<td>Gradual reduction of deviator stress after peak deviator stress.</td>
</tr>
<tr>
<td>Recent stress history</td>
<td>Most recent stress path experienced by a soil. Excludes stress paths which may have engaged the yield surface (Y_3) yield surface as defined by Jardine et al. (1991)).</td>
</tr>
<tr>
<td>Triaxial compression test</td>
<td>Triaxial tests during which the deviator stress increases.</td>
</tr>
<tr>
<td>Triaxial extension test</td>
<td>Triaxial tests during which the deviator stress decreases.</td>
</tr>
</tbody>
</table>

Axial strain at end of various stress excursions (\(\varepsilon_{\text{final}}\))

<table>
<thead>
<tr>
<th>Stress excursion</th>
<th>(\varepsilon_{\text{final}}) (%)</th>
<th>Stress excursion</th>
<th>(\varepsilon_{\text{final}}) (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>LC1(Uco)</td>
<td>2.4</td>
<td>CH1 (Uco)</td>
<td>0.070</td>
</tr>
<tr>
<td>LC2</td>
<td>-</td>
<td>CH2</td>
<td>-</td>
</tr>
<tr>
<td>LC3(Uex)</td>
<td>1.4</td>
<td>CH3 (Uco)</td>
<td>0.23</td>
</tr>
<tr>
<td>LC4(Uco)</td>
<td>0.080</td>
<td>CH4 (Dco)</td>
<td>0.10</td>
</tr>
<tr>
<td>LC3(Uex)</td>
<td>1.2</td>
<td>CH5 (Uco)</td>
<td>0.082</td>
</tr>
<tr>
<td>LC5</td>
<td>-</td>
<td>CH6 (Uco)</td>
<td>0.087</td>
</tr>
<tr>
<td></td>
<td></td>
<td>CH7 (Uco)</td>
<td>0.14</td>
</tr>
<tr>
<td></td>
<td></td>
<td>CH8 (Uco)</td>
<td>14.5</td>
</tr>
<tr>
<td></td>
<td></td>
<td>CH9</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td></td>
<td>CH10</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td></td>
<td>CH11</td>
<td>-</td>
</tr>
<tr>
<td>BK1(Uco)</td>
<td>10.2</td>
<td>CH12 (Uco)</td>
<td>12.8</td>
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<tr>
<td>BK2(Uex)</td>
<td>0.065</td>
<td>CH13 (Uco)</td>
<td>4.7</td>
</tr>
<tr>
<td>BK2(Uco)</td>
<td>14.6</td>
<td>CH14 (Uco)</td>
<td>26.0</td>
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<tr>
<td>BK3(Uco1)</td>
<td>0.063</td>
<td>CH15 (Uco)</td>
<td>0.023</td>
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<td>BK3(Uco2)</td>
<td>0.065</td>
<td></td>
<td></td>
</tr>
<tr>
<td>BK3(Uco3)</td>
<td>8.9</td>
<td></td>
<td></td>
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</tbody>
</table>
Typical stress-strain response of London clay (LC4, Uex1).
Typical stress-strain response of Chalk (CH5).
Typical stress-strain response of Bothkennar clay (BK3, Uco3).