Monte Carlo Simulation of Gas-Filled Radiation Detectors

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Abstract

A new simulation code has been developed that allows the response of gas-filled proportional counters to be calculated. The code is an electron transport code that simulates the elastic and inelastic scattering processes that occur as a result of electron-impact collisions with the gas atoms. The simulation concentrates on the avalanche development after the primary ionising particle has freed electrons in the gas volume, by tracking electrons until they reach the anode of the counter. The dynamics of the ions that accumulate in the gas volume are also considered. A major motivation for this work is the general renewed interest in proportional counters over the last decade, since the advent of micro-pattern detectors such as the micro-strip and the micro-gap detector. It is argued that the low relative cost, intrinsic amplification and environmental stability of these detectors gives them considerable advantages over other types of radiation detectors. The code has been benchmarked against experimental data. The manner in which the variation in the avalanche statistics affects the energy resolution properties of the detector is examined for single wire counters, micro-strip and micro-gap counters. The stability of micro-gap detectors when subjected to high rates of irradiation is also examined. It is envisaged that these detectors will be used in the future as part of a multiphase flow tomography device for imaging the flow of oil/water/natural gas mixtures that have been pumped through pipes from the seabed.
Acknowledgements

I would like to thank my main supervisor, Dr. Ed Morton, for giving me the opportunity to do this work in the first place, and for his helpful advice and suggestions over the period of this endeavour.

I must also thank my other supervisor, Dr. Walter Gilboy, for lending me the considerable weight of his experience on a number of occasions.

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Chapter 1

Introduction

1.1 Thesis Overview

The act of constructing computer-based simulations to model a system has undergone an explosion of interest in recent years. Such work has been performed across a wide range of disciplines. Actual cases include the use in economics to predict stock market trends [Phy 99], and the application in the field of artificial intelligence to model neural networks [Hay 99].

It is no coincidence that this rise in interest has happened at the same time as the recent phenomenal rise in computing power, since the usefulness of a simulation is directly related to the time taken to get an accurate answer. It is now possible to do calculations that are more numerically intensive than at any time in the past. The widespread availability of powerful computers has spurred the development of new fields of research into algorithms. One such example of this is the new area of Genetic Algorithms (GAs) [Dav 91].

It has also become possible to obtain higher orders of accuracy from well-known techniques. This includes the method of Monte Carlo [Pre 96]. As well as being used to solve deterministic problems, like calculating the value of an integral, one of its applications has been to model the transport of radiation through matter [Egs 00].

In this thesis the Monte Carlo technique is used to simulate the behaviour of gas-filled radiation counters. The aim of the work is to construct a computer code that can be used to further understand the drift, diffusion and eventual avalanching of electrons that occurs as part of the detection process. The investigation was motivated in part by renewed interest in the whole field of gas detectors over the last few years, since the arrival on the scene of “micro-pattern” detectors. Before discussing the simulations themselves, the
reasons why there is a sustained research effort at Surrey to develop such detectors will be outlined.

1.2 Overview of the Work at Surrey

1.2.1 Background

Since the discovery of x-rays by Roentgen in 1895, they have been put to use to a huge extent for practical purposes. Famously they are used for medical imaging and as a security tool (for example, to scan luggage in airports).

The x-ray imaging group at Surrey University is looking to exploit this effect in order to solve a problem that certain oil companies involved in sub-sea extraction are interested in. When a well is being pumped a mixture of water, oil and natural gas is obtained. It is important to know the composition of this mixture for two reasons. The first reason is to obtain the measurement of a quantity known as the "water cut". This is the volume of water expressed as a % of the total liquid in the mixture that is being pumped out of the well. Knowledge of this quantity enables the lifetime of an oil field to be assessed. The second reason is to calculate the flow rates of the various components. In order to make quantitative assessments of the amount of each quantity for tax purposes (known as fiscal metering), the rate of flow of the individual constituents must be obtained. The problem is that these different components can flow at different rates through the pipe. Currently a measurement of the water cut can be obtained by diverting a portion of the mixture into a side tank and using capacitance measurements or γ-ray attenuation measurements to obtain an estimate of the water cut. The ideal method would not perturb the flow at all.

Taking an image of a slice through the pipe with a tomographic method provides a means of measuring the latter. A system can be envisaged where two such devices are placed a fixed distance apart and some characteristic of the produced image is tracked. There are a number of tomographic procedures which have been suggested, all of which have drawbacks. For example one such suggestion is capacitance tomography, which relies on electronic sensors being placed at various locations on the wall of the pipe. The
image produced is actually a map of the electrical permittivity of the pipe cross section. The major problem with this method is that if the amount of water flowing is too high (>50%) then its inherent salinity can cause short circuits between the sensors. This is a problem because oil companies can consider wells to be economically viable until the water cut reaches 90%. Key has provided an account of this and other potential methods of measurement of this quantity [Key 99].

X-ray Computed Tomography has not really been considered to be a practical method of solving the problem until now. This is because there has been no means of mechanically rotating an x-ray tube around the pipe quickly enough to obtain an image. The time available is fundamentally set by the flow rate of the mixture. Luggar and Morton have designed an x-ray tube that consists of a series of multi-emitting sources that are wrapped around the pipe [Lug 99a]. These can be switched on and off in rapid order to achieve a complete scan. The proposed system is shown in Figure 1.1. A detailed overview of the general system has also been published [Mor 99].

It can be seen that the multi-emitter x-ray source completely encapsulates the pipe in

![Figure 1.1 The proposed multiphase flow tomography rig.](image_url)
an annular ring. The segmented detector plane is offset with respect to the source plane so that after passing through the pipe the x-rays will bypass the other side of the source. The interaction point can be determined because the outer annulus is built up out of a series of segmented detectors.

1.2.2 Factors influencing the choice of detector

There are four clearly identifiable areas where the ideal detectors for the segmented system should perform well. The first two described below are properties that are, for the most part, intrinsic to the detector. It is possible that engineering solutions can be found if deficiencies occur in the other areas. This might have an impact on the choice of the solution from an economic or practical point of view.

Detector stability at high fluence rates

This is of paramount importance. The region being imaged is altered dynamically as the multiphase mixture moves through the pipe. The oil industry has suggested that acceptable images can be produced if the mixture moves a maximum of 10 mm during a scan. For a typical flow velocity of 1 ms\(^{-1}\) this means that 10 ms is available to scan all the way around the pipe completely. Each scan is made up of a number of sub-scans as the elements of the x-ray source are switched on and off. By counting the number of photons that are detected, a photon attenuation map of the pipe can be produced.

After each burst of radiation the detector segment must reset itself back to a state so that exactly the same burst of x-rays applied again would achieve exactly the same result. The total time of the irradiation plus the time for the detector parameters to return to their pre-irradiated state is called the reset time.

Signal Amplification

The greater the intrinsic signal produced by the detector the less complicated the associated readout electronics can be. This is a bonus in a commercial system.
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Compactness

Oil rigs do suffer from a shortage of available space. Therefore a compact design would have a distinct advantage over a bulky one. This means that a large coefficient for photoionisation is desirable. In order to achieve maximal spatial resolution, the individual elements of the detector also need to be placed as close together as possible.

Sensitivity to the Environment

This includes the effects of temperature and humidity, which can affect the detector-readout assembly in various ways. Engineering solutions might well be found to maintain a constant temperature or to prevent water vapour from seeping into the apparatus, but these would have to be considered in terms of the overall design. As well as economic considerations, the overall bulkiness of the system needs to be considered (see Compactness).

1.3 Possible choices for the detector

There are three categories of detector that might be used. Gas-based detectors have already been mentioned. The other two categories of detector are scintillator-based and semiconductor-based.

1.3.1 Scintillator Devices

The basic mechanism for scintillation relies on the fact that the energy of the incoming radiation particle is ultimately converted into light. The electrons of the scintillation medium are promoted to various excited states. They de-excite by emitting light photons. There are various wavelengths of light that can be emitted. These correspond to states with varying lifetimes. Depending on the material, these can range from a few nanoseconds up to tenths of a second.

The light output can be converted to an electronic signal, if the scintillator is coupled to a photomultiplier tube. These are quite bulky by nature. For imaging applications, where spatial resolution is usually important, photodiodes can be used. Low bandgap
photodiodes such as silicon or germanium have a high “dark current” at room
temperature. This is highly temperature sensitive. This can complicate the collection of
the signal.

A scintillator material that is in widespread use as the detector array in X-ray
Computed Tomography systems is the BGO scintillator [Bic 00]. It has a chemical
formula of Bi$_4$Ge$_3$O$_{12}$. The bismuth component has an atomic number, Z, of 83. This
means that it has extremely high detection efficiency per unit volume. It is also
mechanically robust and immune to the effects of water vapour, unlike other scintillation
candidates such as NaI(Tl) or CsI(Tl). Its particular advantage is that its long-lived
excited states are practically non-existent, and so the detector returns to its non-irradiated
state in around 200 ns. This can be contrasted with NaI(Tl) scintillator material. Although
the latter material has a much higher light yield, it has long lived excitation states with
lifetimes that last up to 0.15 s [Koi 73].

However its light output is relatively low compared to that of other scintillators, so
care must be taken to reduce the possibility of a noisy signal. In addition the light output
will fall with increasing temperature. According to Melcher, there is a factor of 3
reduction in the output in moving from 0 to 100 °C [Mel 85].

1.3.2 Semiconductor Devices

A semiconductor detector can be created by diffusing a small quantity of electron
donor atoms into one side of a semiconductor material such as germanium (to produce n-
type doping) and a small quantity of electron acceptor atoms into the other side of it (to
produce p-type doping). A typical doping concentration is 2 parts per million [Kno 89].

The n-type region has an excess amount of electrons and the p-type region has an
excess number of holes in the lattice, where the acceptor atoms have been substituted into
the basic lattice structure. At the boundary between the n- and p-type regions there occurs
some diffusion of the electrons into the p-type region and holes into the n-type region.
The interface becomes depleted of electrons and holes and is therefore known as the
depletion region. As a direct result of the charge migration an electric field is set up
across the depletion region. This electric field can be enhanced, by applying an appropriate potential across the detector, using a technique that is known as reverse biasing. When radiation is incident on the depletion region new electron–hole pairs are created which then both drift under the influence of the applied field. A current pulse is initiated which, depending on the bias voltage, can last the order of 10 ns [Kno 89]. This gives semiconductor devices an extremely fast response.

Since the depletion region is effectively the active volume of the detector, commercial detectors usually have some intrinsic material (no electron-hole imbalance) placed between very thin heavily doped n- and p- type regions in order to expand it. The n- and p- types function as 'blocking contacts' in that they oppose the free flow of current through the bulk semiconductor material. This is emphasised in Figure 1.2.

![Figure 1.2 Schematic of a semiconductor detector.](image)

Semiconductors can be modelled as a dual energy level system consisting of the valence band (low energy) and the conduction band (high energy). Electrons must be promoted into the conduction band in order to migrate as a current. If the temperature of the environment in which the detector is placed is sufficiently high then there will be a significant current, even in the absence of radiation. This leakage current can end up swamping the signal induced by incident radiation if it is too large. The energy difference between the conduction band and the valence band (known as the bandgap) is plotted in Table 1.1.
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<table>
<thead>
<tr>
<th>Material</th>
<th>Bandgap energy / eV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Germanium</td>
<td>0.67</td>
</tr>
<tr>
<td>Silicon</td>
<td>1.12</td>
</tr>
<tr>
<td>Cadmium Telluride</td>
<td>1.50</td>
</tr>
</tbody>
</table>

Table 1.1 Bandgap energies of typical semiconductor materials [Sch 95].

Detectors based around the first two materials in Table 1.1 are cooled with liquid nitrogen before being used. This means that if they were to be used as a practical environmental sensor, then a temperature-controlled cryogenic environment needs to be set up around the detector segments.

There has been an explosion of interest in cadmium telluride (and its zinc-based derivatives) over the last few years, because of its potential for near-room-temperature operation. De Antonis has given an account of its potential for x-ray detection [Dea 98].

There are unfortunately problems associated with the use of cadmium telluride. For instance, a cadmium telluride crystal can catch electrons and holes in trapping centres (impurities and defects in the crystal). Over many incident x-ray pulses the charge can build up to such an extent that the crystal becomes polarised. The internal electric field is changed from its equilibrium value and the detector response will behave non-linearly with time. The greater in magnitude the incident x-ray fluence rate is, the more severe this effect is likely to be.

In addition to this, because the signal is itself quite small (~200 nA for a 30 keV x-ray photon that has been completely absorbed) and there is no intrinsic gain in the number of charge carriers created, some signal amplification must be incorporated into the detector readout instrumentation.

1.3.3 Comparison with gas detectors

Table 1.2 shows the absolute value of the linear photon attenuation coefficient for an example of a scintillator material, a semiconductor and a material used as a fill gas in,
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proportional counters. The underlying cross section data is taken from the XCOM calculation program written by Berger and his co-workers [Ber 87].

<table>
<thead>
<tr>
<th>detector</th>
<th>linear photon attenuation coefficient / cm$^{-1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>bismuth germanate</td>
<td>3.40</td>
</tr>
<tr>
<td>germanium</td>
<td>1.49</td>
</tr>
<tr>
<td>xenon gas (1 atmosphere)</td>
<td>9.63 x 10$^{-4}$</td>
</tr>
</tbody>
</table>

Table 1.2 A comparison of the relative efficiencies of three popular detector materials for the detection of 30 keV x-rays.

Xenon is a popular choice as the fill gas for proportional counters, because its relatively high atomic number of 54 means that it has a high photoelectric cross section.

It appears that both the scintillator and the semiconductor material would make a vastly more compact detector. However both of these detectors are temperature sensitive. A stable temperature environment would have to be engineered. In the final analysis, the amount of space actually saved is going to be much less than is predicted by Table 1.2. Neither do these detectors have the intrinsic signal amplification that is present in gas counters.

If a segmented gas detector could be found that is stable enough to cope with the high fluence rate capability that is required in order to perform multiphase flow imaging successfully, then it would make a reasonable candidate for use in the instrument pictured in Figure 1.1.

1.4 The Gas Detector

This section outlines the evolution of gas detectors. A description of each type of detector is interwoven with a general explanation of how they work.

Figure 1.3 represents the characteristic response curve of a gas-filled detector as the potential difference between the anode and the cathode becomes more positively biased.
1.4.1 The ionisation chamber

In its simplest form this consists of a pair of parallel plates with a potential difference applied across them, contained in a gas envelope (see Figure 1.4). Away from the edges of the chamber the electric field is uniform. For the x-ray to be detected it must interact with the gas particles to produce electrons and ions, in a process known as primary ionisation. This can be either in the form of a Compton scatter event, or a photoelectric interaction. In the case of the former, part of the x-ray energy goes towards the freeing of a bound electron and the remainder goes into producing a new photon with a lower energy. In the case of the latter all of the x-ray energy goes into the freeing of the bound electron. The ideal imaging system would incorporate a material that made the ratio of the probability of photoelectric interaction to that of scattering as high as possible. The freed electron then goes on to produce further electrons and ions in electron-impact ionising collisions in secondary ionisation processes.
When a very small potential difference is applied across the gas volume, the electric field is too low to permit any electrons that are freed in the initial ionisation process to escape from the region where they are created. The reason for this is that this region contains a relatively high density of ions that originated from the original x-ray interaction. Electrons in the vicinity of these ions have an extremely high probability of recombination.

As the electric field strength increases, more and more electrons escape from the original interaction region. An electron cloud is formed which drifts towards the anode of the counter. A cloud of positive ions that drift towards the cathode complements this. This constitutes an electric current and it initially increases in magnitude as the electric field strength is increased. Eventually the current ceases to increase when virtually all of the electrons escape from the original interaction volume. A steady-state situation is created, where the charge carriers being produced due to further x-ray interactions balance those collected at the anode. The value of this saturation current is virtually proportional to the radiation fluence rate, for a given incident energy. The ion saturation region of Figure 1.3 has been reached.

The ionisation chamber describes a picture where, in the ideal case, all of the electrons created in the original ionisation cloud are collected at the anode of the counter. Typically the electrons will drift through a potential equivalent to many times the ionisation energy.
of the gas before reaching the anode. In weak electric fields they never attain enough energy to produce further ionisation. This is because the electrons are constantly undergoing collisions with the gas molecules. This dissipates any energy gained from the electric field.

1.4.2 The Cylindrical Single Wire Proportional Counter

This consists of a wire of uniform circular cross that forms the anode of the counter. This is enveloped by a gas, which in turn is contained by a cylindrical metallic enclosure that forms the cathode. The resulting electric field lines radiate out from the centre of the anode. The value of the electric field $E$ at a distance $r$ from the centre of the counter is given by

$$E = \frac{V_a}{\log_e \left( \frac{r_c}{r_a} \right) r}.$$  

(1.1)

Here $V_a$ is the potential of the anode relative to the cathode, $r_c$ is the radius of the cathode and $r_a$ is the radius of the anode. Typical values of $r_a$ and $r_c$ are 50 μm and 2.5 cm respectively.

If $V_a$ is raised then eventually the electrons will, on average, gain enough energy between collisions to produce further ionisation. This will ideally occur in a small volume of the detector very close to the anode, for it is only here that the electric field is large.

Figure 1.5 The cylindrical single wire proportional counter.
enough to promote further ionisation. This is called the \textit{avalanche region} of the detector. The \textit{drift region} of the detector, where the magnitude of the electric field is too small to effect electron multiplication, is where the cloud of ion-electron pairs from the original ionising particle is formed.

As the magnitude of $V_a$ is increased, further electrons are produced. Each electron in the cloud can now initiate an avalanche. This avalanche is terminated when all of the electrons are collected at the anode of the counter. The mean number of electrons created per avalanche is called the \textit{single-electron-induced mean avalanche gain} $M$. It is defined exactly by

$$M = \frac{G_r W}{E_x}. \quad (1.2)$$

In Equation 1.2 $G_r$ represents the total number of electrons that are collected at the anode as a result of a single x-ray interaction and $E_x$ is the energy of the incident x-ray. $W$ is the $W$-value of the gas. This is defined as the total energy of the primary incident photon divided by the mean total number of ion-electron pairs created in the drift region by secondary ionisation processes.

It is this intrinsic signal amplification that is an attractive feature of the gas detector. It is possible for $M$ to reach values of the order of $10^5$ or higher whilst continuing to maintain stable operation [Hen 72]. In this region of operation the detector is said to be in \textit{proportional} mode. At small incident fluence rates the number of electrons collected at the anode is directly proportional to the number of ion pairs produced in the original ionisation cloud.

\subsection*{1.4.3 The multiwire proportional counter}

Georges Charpak invented the multiwire counter in 1967 (and was subsequently awarded the Nobel Prize for Physics in 1992) in order to track ionising particles at CERN [Cha 79]. Essentially a large number of anode wires are arranged parallel to each other. Reading out the signal from each anode individually facilitates a measurement of the position of the original ionisation. The planar cathode can be cut into strips perpendicular...
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Figure 1.6 The multiwire proportional counter.

to the anode so that 2-dimensional localisation of the original interaction position is possible.

1.4.4 Positive ion gas detector physics

Even when the strength of the electric field is large enough in magnitude to cause electron avalanching, the ions do not themselves gain enough energy between collisions to part electrons from neutral atoms (unless the counter is operated at pressures that are very much lower than one atmosphere). Their much higher mass compared to electrons means that the value of their acceleration in the direction of the electric field is much less. In fact, given a typical ion velocity of 1 m/s, and bearing in mind that a typical single wire proportional counter has cathode radius of 1 cm, the transit time of an ion is about 10 ms [Kno 89]. This means that the reset time of gas-filled proportional counters is much greater than that of the other detector types mentioned above. The drift velocity of electrons is typically 10^3 times greater than that of ions. The ions will tend to drift along the electric field lines. There will be a random element in their motion induced by collisions with neutral gas particles.

The presence of ions in the gas volume can have an impact on the detector response in two ways. If the value of \( M \) is large enough, then enough ions are generated in a single avalanche to eventually quench it. The electric field is reduced below the minimum value required to cause amplification and new electrons fail to be generated. This is known as self-induced space charge. The counter is no longer proportional because, when this ion
density is reached in the avalanche region, no more electron-ion pairs are formed. This is
irrespective of the energy of the incoming radiation. In fact this is what happens in a
Geiger-Müller counter. The latter is useful for measuring counts, but cannot be used for
the purposes of making an energy measurement. The remedy to the above problem is
simply to adjust the potentials on the electrodes to give a lower value for \( M \).

The second problem stems from the fact that the ion drift velocity is very much less
than its equivalent for electrons. If the x-ray fluence rate is sufficiently high then a
detection event can occur whilst the positive charge from the preceding event has not yet
reached the detector cathodes. This effect will be particularly severe in the avalanche
region of the detector, where there is naturally a high density of ions. As more and more
avalanches overlap, the gas gradually becomes more and more positively charged. This
alters the value of the electric field inside the detector over time. This is called general
space charge and it is more severe at higher incident fluence rates. It is this problem
which might conceivably be encountered in the imaging system of Figure 1.1.

The non-linear behaviour described here might be avoided if the anode and cathode
could somehow be placed closer together. Charpak suggested a modification to the
multiwire chamber that offered up this possibility [Cha 94]. If every alternate wire of
Figure 1.6 were to be negatively biased to form a cathode, then ions created in avalanches
at the anode wires would be swept towards their neighbours. If these wires were placed a
few hundred \( \mu m \) apart then a significant decrease in the reset time might be observed.
Charpak's modified detector is called the asymmetric wire chamber.

There are problems with using wires as electrodes in proportional counters. The wires
must have a uniform cross section, so that the electric field does not change in an
unpredictable fashion. The wires must also be kept under high tension in order to ensure
that the anodes and the cathodes remain at a fixed distance from each other. There is then
the problem of wire sagging occurring over a period of time.
1.5 Micro-pattern detectors

The advent of advanced photolithographic techniques enabled electrode strips to be patterned onto large areas of supporting insulator material. Oed was the first person to realise the potential of this methodology as a means of revolutionising the whole field of gas-filled radiation detectors. In 1988 he invented the micro-strip detector [Oed 88].

1.5.1 The micro-strip detector

This design consists of alternating anode and cathode strips laid down on an insulator surface such as glass. A typical anode width is 10 μm. A typical cathode width is 90 μm. A typical anode-anode pitch is 500 μm. The distance between the insulator plane and the drift cathode is usually around 5 mm. Figure 1.8 illustrates the electric field pattern and equipotential lines inside the detector.

Having the anode in close proximity to the cathode enables the ions to be swept out of the path of incoming electrons. There is a complication due to the fact that the gas is in direct contact with the surface of the insulator. Electric field lines do end up on the surface of the insulator. This means that the positive ions will tend to drift to the exposed surface. If the material is a good insulator, then the ion is going to be there a relatively long time before it picks up an electron from the insulator. The charging of the substrate material causes the non-linearity of the detector to reappear [Bou 92].
There are two possible remedies for this. One that was first proposed by Oed in his original paper relies upon coating the bottom of the substrate with a metallic substance in order to form a "back plane" electrode. This electrode is given a positive bias that acts to lessen the number of electric field lines in the gas that end up on the insulator surface.

Another solution to the problem is to have a thin layer of material on the top surface of the insulator with a resistivity whose value is below that of the bulk material, thereby allowing a positive ion to be neutralised. A thin coating of semiconductor material can be used to achieve this effect. ZnSe is one such material that has been used as a coating [Sud 97]. This trades off the possibility of an increased leakage current between the anode and the cathode of the detector with a reduced gain dependency on surface charging.

A separate problem with these detectors is the fact that the measured value of \( M \) can change over a matter of hours, when the detector is pulsed with very short bursts of radiation [Bou 92]. The reason for this could vary according to the type of material that is used for the insulator. In certain glasses the actual resistivity can change with time, as ions embedded in the bulk material migrate to the electrodes under the influence of the electric field, depleting it of charge carriers.
1.5.2 The micro-gap detector

This design, first proposed by Angelini [Ang 93], almost removes the insulator from the detector, except for strips used to raise the anode above the cathode plane. This means that there will be no charging up of the surface of the detector, except on the thin insulator strips that are exposed to the gas itself. Figure 1.9 shows a schematic of a micro-gap detector. A typical anode-anode pitch is 500 µm. A typical distance between the drift cathode and the cathode is 5000 µm. The separation between the anode and the cathode is typically 1 µm (this is, in effect, the thickness of the insulator).

It can be seen that the insulator strip overhangs the anode strip. This is necessary due to the extreme proximity of the anode to the cathode. If there was no overhang then the electric field between the anode and the cathode of the detector could be so large as to promote breakdown of the gas. If the operational value of $M$ is kept low enough, then it is hoped that this possibility can be minimised. The advantage, in terms of improved fluence rate capability, might then be realised. It is this possibility that is investigated in Chapter 7.

![Figure 1.9 The micro-gap detector.](image-url)
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1.6 Stability of Gas Detectors

There are generic problems with gas detectors that are related to breakdown. A feedback situation is possible where an avalanche initiated from a single electron becomes self-perpetuating, thereby paralysing the detector. The possible routes to a breakdown situation are discussed in this section [Rae 64].

1.6.1 Field Emission

In the absence of ionising particles in the gas volume it is possible for breakdown to be initiated through a process of field emission. Even though there is a potential barrier that locks the electron into the electrode, wave mechanics suggests that an electron has a finite probability of tunnelling through this barrier to reach the surface and be swept away by the electric field. This effect is expected to be more prevalent when the electric field is higher. In practice, surface contamination of the electrodes can reduce the magnitude of the electric field necessary to promote this effect [Lle 66].

1.6.2 Photon Feedback

There are two principal ways in which photon feedback can be initiated. The first arises when a photon is emitted during an avalanche. This photon might escape from the avalanche region and cause a neutral gas particle to be ionised. The avalanche can then start all over again when the newly created free electron drifts into the avalanche region.

Avalanche photons can also reach the cathode and release electrons, if their energy has a value that lies above the energy of the work function of the electrode. The work function, $\phi$, of various metals that might be used as electrodes is shown in Table 1.3. The data is taken from Kaye & Laby [Kay 86]. Ultraviolet photons produced in the avalanche have sufficient energy to free additional electrons.
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<table>
<thead>
<tr>
<th>elemental symbol</th>
<th>$\phi / eV$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ag</td>
<td>4.26</td>
</tr>
<tr>
<td>Al</td>
<td>4.28</td>
</tr>
<tr>
<td>Au</td>
<td>5.10</td>
</tr>
<tr>
<td>Cr</td>
<td>4.44</td>
</tr>
</tbody>
</table>

Table 1.3 The work functions ($\phi$) of possible candidates for the electrodes in micro-pattern detectors.

1.6.3 Ion feedback

If an argon ion, whose first ionisation potential is 15.76 eV, reaches the cathode of a counter made of silver (Ag), then it can be seen from Table 1.3 that there is enough residual energy to free an additional electron from the cathode. This can then start a new avalanche in the gas.

1.6.4 Streamer Development

Streamers are narrow luminous tracks that connect the anode to the cathode and have the devastating effect of causing permanent damage to the detector by melting the electrodes.

When a high density of charge is present in a highly localised area of the detector, the field can be raised to such an extent that additional avalanching can be promoted within this region. Additional electrons that are generated by photoionisation reinforce this effect. The source of photons for this interaction is the primary avalanche. These electrons are accelerated towards the high charge density areas of the developing avalanche. This high field region eventually spreads all of the way from the anode to the cathode. The outcome of the process is the creation of a densely ionised, low-resistivity channel between anode and cathode, inevitably leading to discharge.

The streamer channel is effectively a high density plasma where the internal field is so large that there is a distinct possibility that any charged particle travelling through it smashes into the electrodes with enough energy to severely damage them. In cylindrical
Chapter 1: Introduction

single wire counters this is not such a huge problem, because the electric field in the presence of zero irradiation falls away along a line extending radially out from the anode to the cathode (see Equation 1.1, for example). Therefore a streamer is not sustained. However, in a micro-pattern detector, even though there might be a high field gradient between the anode and the cathode, the streamer can bridge the gap more easily. This is because the two electrodes are so close to each other.

The shortest path from anode to cathode lies along the insulator surface. In fact the situation becomes even worse when the resistivity of the insulator is reduced in a microstrip in order to prevent surface charging. Lower ionisation densities are then able to sustain a streamer. Fonte gives a review of this effect [Fon 97]. The Raether criterion states that about $10^7$ or $10^8$ electrons need to be within the proximity of a given avalanche to initiate streamers [Bre 99]. Experimentally, streamers are characterised by a sudden surge in gain. Another characteristic is an intense ($>10 \, \mu\text{A}$) and short ($<50 \, \text{ns}$) current pulse.

Field emission presents a fundamental limit to the magnitude of the electric field inside the detector, and by extension, indicates that $M$ cannot continue to rise indefinitely. The effects of ion and electron feedback can be greatly reduced by the addition to the gas volume of a quenching agent. The choice of gas is examined in more detail in Chapter 3.

The existence of streamers becomes a distinct possibility when a certain threshold charge density is established in the gas volume. This can occur at high values of $M$. More important from an imaging point of view is the fact that if the incident fluence rate is of a large magnitude, then avalanches that overlap in time can create the necessary charge density to sustain a streamer. Issues surrounding the high fluence rate operation of micro-gap detectors are considered further in Chapter 7.

1.7 Summary

This chapter has given an overview of the work at Surrey University. The aim is to develop a tomographic rig for multiphase flow imaging to serve the oil industry, as shown in Figure 1.1. Gas detectors appear to be a reasonable choice, because of their
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intrinsic signal multiplication when operated in proportional mode, as well as their stability with respect to temperature variations. The aim is to try to develop micro-strip and micro-gap detectors that will have the additional benefit of a high fluence rate capability.

There is another reason for trying to develop these detectors – it is just for the plain and simple reason that they are interesting devices! The rest of this thesis will concentrate on the development of gas detector simulations. As the physics of these detectors is examined more closely in the following chapters via simulation, any information that might be relevant to their effectiveness for being used in the above imaging system will be emphasised.

However part of the motivation for the work is to try to further understand the fundamental physics behind what is going on inside these detectors, in terms of signal development. The rest of this thesis concentrates on the development and use of the simulations.
Chapter 2

Developing a Simulation

2.0 Introduction

This chapter outlines the construction of the simulation code. Firstly the tomography problem described in Chapter 1 is analysed in more detail, to establish what would be useful for a simulation to measure. Then similar simulations that have been constructed in the gas detector area are reviewed. The reason for the decision to construct a simulation from scratch is defended in this context. Finally the algorithms actually used are described in detail.

2.1 Analysis of the Problem

After an x-ray photon has interacted inside the detector an electron cloud is quickly generated (for example, see Section 1.4.1). If a gas detector is going to be used for x-ray imaging then there are two possible properties that might be considered to be important. One consideration is the ability of the detector to remain stable at high fluence rates. This has been mentioned in Chapter 1 (and will be dealt with in Chapter 7).

The other important consideration is that of spatial resolution. This depends on two factors. The first is the size of the electron cloud that originally forms inside the drift region of the detector after the x-ray photon is absorbed. At this point the energies of all of the electrons in the cloud lie below the ionisation threshold of the gas. The second factor is the extent to which the cloud spreads out as it drifts towards the anode of the counter.

As well as electron-impact ionisation collisions, there are other phenomena that make a contribution to the electron numbers in the cloud. If an electron is removed from an inner shell of the gas in an ionisation event, the energy released when an electron falling into it from a shell further out fills the gap can take two forms. It can either go directly
Chapter 2: Developing a Simulation

into the freeing of an Auger electron, or a fluorescence photon can be emitted. In the case of the latter the photon can then initiate the formation of a new electron cloud. The degree to which this is detached from the old cloud depends on the mean free path of the photon through the gas.

For more than a decade a research group at the University of Coimbra have been developing a simulation code that enables the size of the electron cloud in xenon to be assessed, starting from the original x-ray ionisation event [Dia 97, Rac 99]. The development of the cloud can be followed until it is fully formed (when all electrons have reached an energy that lies below the ionisation potential of the gas). Electron and photon cross sections for all of the relevant elastic and inelastic processes must be provided. These must span an energy range from zero right up to the energy of the x-ray photon (tens of keV). Appendix A goes into this subject in more detail, assessing the impact of the finite size of the electron cloud on the spatial resolution.

In this work the decision was made to concentrate on what happens after the electron cloud was formed, purely because this was deemed to be a more manageable problem in the time available. It will be seen that the key electron transport physics happens at energies that lie below a few hundred eV. As the electron cloud drifts towards the anodes of the counter it will spread out. This has some impact on the spatial resolution. Electron diffusion is an effect whose magnitude can be estimated by the code described here.

When the electron nears an anode of the counter avalanching will occur. It will be seen in Chapter 6 that statistical variation in the single-electron-induced avalanche gain has a significant influence on the energy resolution of the counter. It must be said that this is not directly relevant to the construction of the tomography device itself (since the gas detector array would not be used to make spectroscopic measurements). However it is interesting to study how the variation of the electric field along the path of the avalanche affects the single-electron-induced gain distribution.

Finally, in Chapter 7, detector stability at high fluence rates is assessed for the micro-gap detector. The importance of being able to work at high fluence rates in imaging applications has already been discussed in Chapter 1.
2.2 Electron Transport Simulation

The idea behind the simulation is that each electron will be followed individually. As it moves through the electric field it will be scattered by the gas atoms. The scattering is stochastic in nature. This means that a Monte Carlo technique is well suited to this problem.

There are a number of electron transport simulation codes that are publicly available. These include MCNP [Mcn 00] and EGS [Egs 00]. EGS stands for Electron Gamma Shower. Due to the fact that this work is primarily interested in electrons, the following discussion relates to electrons only. EGS is considered to be the gold standard for accurate electron transport, not least because of the work done by Bielajew on improving the way that electron scattering has been implemented [Bie 96].

The strength of EGS lies in the fact that electrons (and photons) with energies ranging from a few keV to many MeV can be transported. However there is a catch. Even with the remarkable speed of today’s desktop computers it would be computationally prohibitive to try to model each and every interaction discretely, following each electron in the shower down to below the ionisation threshold of the material. It would also, in many cases, be unnecessary. One of the main uses of EGS is in radiotherapy treatment planning [Jen 88]. The tissue that would be irradiated is divided into a series of voxels, in which the energy deposited will be scored. Consider the case when an electron finds itself inside a certain voxel and its energy is sufficiently low that it will not escape from that voxel. In this instance there is no point in wasting CPU clock cycles whilst continuing to calculate its trajectory. The electron’s remaining energy should merely be assigned to that voxel.

In order to get around this problem a technique called condensed history can be used. This is illustrated in Figure 2.1. First of all a parameter called the energy cut is defined. If an electron collides with an atom in the transport medium and if the energy transferred is above the energy cut then the collision is modelled discretely. In practice, because the minimum energy cut is a few keV, this means that the collision will be an ionisation. Only in a discrete ionisation is a new electron introduced into the shower.
Chapter 2: Developing a Simulation

![Figure 2.1 An electron trajectory (as might be generated by EGS). The black lines represent the electron trajectory. The green circles represent points where a discrete collision event actually occurs. In the case of the circle on the right this is an ionisation event.](image)

It can be seen from Figure 2.1 that between discrete collisions the electron undergoes a series of changes in direction. These direction changes can be calculated from the scattering theory of Molière [Bet 53]. In effect, Molière was able to relate the angular deflection of an electron to the number of atoms that participate in this multiple scattering (which is determined by the length of the path segments in Figure 2.1). In other words the theory is being used to estimate what would happen over a large number of collisions, instead of doing the transport on a collision-by-collision basis. The mean energy loss from energy transfer along a path segment is calculated using stopping power formulae that are due to Berger [Ber 64], which are based on the Bethe-Bloch expression.

Unfortunately, a condensed history technique is not really feasible for use in the avalanche region of a gas detector. This is because almost all of the electrons that are created in the avalanche will escape from the region where they are generated and will be collected at the anode of the counter. The crucial difference between this case and the voxelated radiotherapy treatment planning simulation described earlier is the absence in the latter of the electric field. This means that an upper limit can be placed on the range of the electron. This vastly reduces the number of discrete entities that need to be simulated.

The solution to this problem appears to be to set the energy cut to 0 so that every collision is modelled discretely. Regrettably this won't work either. The $W$ values (the mean energy required to produce an electron-ion pair) for three fill gases used in proportional counters are shown in Table 2.1.
Chapter 2: Developing a Simulation

<table>
<thead>
<tr>
<th>gas</th>
<th>W-value / eV per ion pair</th>
</tr>
</thead>
<tbody>
<tr>
<td>neon</td>
<td>36.2</td>
</tr>
<tr>
<td>argon</td>
<td>26.2</td>
</tr>
<tr>
<td>xenon</td>
<td>21.5</td>
</tr>
</tbody>
</table>

Table 2.1 The W values for three fill gases used in proportional counters [Kno 89].

Clearly a lot of the energy transfer happens in collisions that involve electrons with energies of a few tens of eV.

The cross sections that describe the discrete collision processes in EGS are not accurate at these energies. The theory on which the calculation of the ionisation cross section is based, known as Møller scattering [Mol 32], assumes that the atomic electrons are essentially free. This works if the ionising electron has an energy that is very much greater than the binding energy of the atomic electron.

Above energies of a few keV the theory is fine. It has the considerable advantage that it applies to any material quite generally, provided the energy is high enough. This means that EGS does not have to be distributed with a large database of cross sections for all of the media that it might be desired to simulate electron transport in. The ionisation cross sections for a material can be calculated directly from the analytical formulation due to Møller.

When the energy is down to a few tens of eV, however, then numerical techniques need to be applied to calculate the appropriate cross section. Electron-impact collisions in this energy region have been the subject of much research. This subject is dealt with in Chapter 3. Here it will merely be noted that it is the numerical results of such calculations that are used as the input data to the simulations described here.

2.3 Previous Gas Detector Simulations

From the discussion in Section 2.2 it can be seen that a condensed history technique is not really suitable for this problem. Instead, each collision will be simulated discretely.
Chapter 2: Developing a Simulation

This is known as an analogue transport technique. In this section various analogue gas detector simulations that have been constructed in the past will be reviewed.

2.3.1 Literature Review

Groh [Gro 89,90] performed simulations in argon-methane single-wire counters. Single-electron-induced avalanches were examined in detail. Recombination effects between electrons and ions, photon emission and photon absorption were ignored. Due to the nature of the simulations (large gains with a magnitude of around $10^5$ were considered) the field was solved for every time interval that corresponded to a change in the total number of electrons of more than 3%. The time for an individual avalanche could be anything up to 75 minutes on a machine with an IBM-3084 CPU.

The large time required for each avalanche to be simulated meant that the relevant collision cross section was not calculated for each electron on an individual collision-by-collision basis. Instead an ensemble average was taken over all electron energies for all electrons that existed in the simulations after a given transport time interval had been completed. This mean energy was used as a starting point for calculating the mean transport time for electrons for the next transport time interval. This was held fixed for all electrons during the next time segment of the simulation.

The problem with this approach in non-uniform electric fields is that there is a correlation between the mean energy of the electron in an avalanche and its distance from the centre of the anode. This mean energy is expected to rise as the anode is approached, because the electric field is increasing. Taking the average over the entire electron ensemble means that the time used in the simulations to transport the electrons is somewhat removed from the true mean time at a given distance from the anode.

This work was particularly interested in the use of the simulations to calculate the dimensions of an avalanche (the distribution along the wire length of the position where the electron hits the anode). Also of interest was the correlation between the radial position of the original electron in the avalanche and the eventual single-electron-induced avalanche gain. It was found that for a fill gas that was a 90-10 argon-methane mixture
Chapter 2: Developing a Simulation

held at 1 atmosphere and at 300 K with a value of \( M \) (the single-electron-induced mean avalanche gain) of about \( 10^4 \), the avalanche extended about 150 \( \mu \text{m} \) along the central wire.

Pruchova looked at the problem of conducting simulations in micro-dosimetry counters filled with methane [Pru 95]. This gas was chosen because of the relative availability of cross sections for the relevant interaction processes. Pruchova analysed the spatial characteristics of the avalanches and found that the overall value of \( M \) is related to the number of electrons that are produced in what is called the seeding region of the avalanche.

Two separate “partial” avalanches are seen to develop in Figure 2.2 in the high electric field region close to the anode. If the ionisation has not occurred in the seeding region characterised by the electric field then the avalanche numbers would be approximately halved. This obviously has an impact on the energy resolution of the counter (see Chapter 6 for a detailed examination of this effect).

Rachinhas developed a model for single wire gas counters using xenon as the fill gas [Rac 94,97]. The simulated values of \( M \) for various anode voltages and anode wire radii were compared with their equivalent experimental values. Reasonable agreement was obtained between the two cases.

Segur analysed proportional counters that use tissue equivalent gas mixtures, as well as traditional argon-methane mixtures [Seg 89, 95a, 95b]. Tissue-equivalent mixtures
Chapter 2: Developing a Simulation

typically contain C₃H₈, CO₂ and N₂. In particular the influence of the changing electric field on the development of the avalanche in single wire counters was emphasised.

Without constructing a wire counter simulation, the value of $M$ can be estimated if the value of the Townsend first ionisation coefficient $\alpha_T$ is known as a function of electric field. In an infinite homogeneous medium subject to a uniform electric field $\alpha_T$ can be estimated from Equation 2.1.

$$G_c = \exp(\alpha_T x)$$

In Equation 2.1 $G_c$ is the mean number of electrons in the avalanche after it has progressed a distance $x$ parallel to the direction of the electric field from its point of origin. The value of $M$ in a single wire counter is then given by

$$\ln M = \int_{E_c}^{E_a} \frac{K_w \alpha_T(E)}{E^2} dE.$$  

Here $E$ is the electric field strength, $E_a$ is the electric field strength at the anode, $E_c$ is the electric field strength at the cathode and $K_w$ is a constant whose value depends on the counter dimensions and the voltages applied to the electrodes. Amongst others, Zastawny has advocated the use of this formula to estimate the value of $M$ [Zas 97].

There are two reasons why Equation 2.2 does not necessarily give a good estimate of $M$. The presence of an electric field gradient in the case of the single wire means that the application of $\alpha_T$ (which relates to the constant field case) is not really valid. The mean energy of the electron at a given radius from the anode centre tends to lag behind its value in the equivalent constant field. This means that the ionising power of an electron is altered. This is an example that highlights one of the strengths of Monte Carlo simulation. Such a simulation does not rely on any assumption about the electron’s energy at any point in the field. It merely calculates it from first principles.

The other reason why the use of Equation 2.2 is not quite so accurate when it is applied to the counters is related to the boundary conditions of the problem. Due to the fact that the boundary is treated as an absorbing surface there is a dip in the ionisation
Chapter 2: Developing a Simulation

coefficient close to the anode, as shown in Figure 2.3a. The reason for this can be explained with the help of Figure 2.3b.

In track B of Figure 2.3b an electron passes through an equi-field line towards the anode. It then scatters back out through the equi-field line and undergoes an ionising collision. Finally both electrons pass through the equi-field line as they are drawn towards the anode. The gain measured along the dotted line of Figure 2.3b is enhanced, in a relative manner.

![Diagram showing track A and track B near an anode with dotted line indicating gain enhancement.]

Figure 2.3a There is a characteristic dip in the ionisation coefficient ($\alpha$) as the anode is approached in proportional counters [Kun 99a].

![Diagram illustrating the boundary effect near an anode with an equi-field line highlighted.]

Figure 2.3b Illustration of the boundary effect that occurs close to the anode. The anode is treated as an absorbing surface.

This can be contrasted with track A. An anode in a proportional counter acts like a highly absorbing surface. If an electron passes through the equi-field line that represents the anode surface then it is absorbed. The possibility of an electron being released back
into the gas (the equivalent of a backscatter event) must be quite low, otherwise the avalanche runs the risk of being self-perpetuating.

Recently CERN have made their GARFIELD code freely available for download from their web server. This can be downloaded and run locally, or it can be run on one of their servers [Gar 00]. It is possible to drift electrons through two- or three-dimensional geometries that have been computed by finite element analysis programs such as Maxwell [Max 00]. Also the drifting of particles, including diffusion, avalanches and current induction can be considered.

Biagi has produced a Monte Carlo simulation code that is able to calculate the drift and diffusion parameters of gases in combined electric and magnetic fields [Bia 99].

Bellazzini has performed some simulations in argon/methane-filled micro-strip and micro-gap counters [Bel 94], particularly in the area of signal development. The time required for the signal to form was calculated to be less than a nanosecond.

2.3.2 Assessment of the ‘State of the Art’

It can be seen from the above section that a number of simulation codes have been developed for the analysis of the behaviour of proportional counters. However the decision to produce another can be justified on the basis that the ultimate aim in this case is to try and gain some information about the dynamically changing nature of the electric fields. This is due to the charging up of the gas volume after many avalanches. The dynamic motion of the positive ions has not really been considered for micro-pattern detectors.

The electric field has to be recalculated many times during a simulation. This means that an algorithm that rapidly recalculates the electric field must eventually be tightly coupled to the actual electron transport simulation. This facilitates an insight into how the size and shape of the residual positive ion cloud affects the development of further avalanches. It was decided that it would be much easier to start again than to try to ‘graft’ this feature on top of some existing code.
Chapter 2: Developing a Simulation

All of the codes that have been written so far, with the exception of the work associated with GARFIELD, have been written for one specific type of detector. It will be seen in Chapter 5 that the code that is developed in this work can be used across a wide range of counters.

There is another reason for starting afresh. It is unlikely that the designer of any code would have anticipated all of the quantities that might need to be measured by an outsider. Anybody who attempted to modify the source code would first have to expend considerable effort to understand the way it is structured. At some point it becomes reasonable to consider a complete rewrite.

Although the study of these detectors was conceived for imaging purposes, as described in Chapter 1, there are other interesting problems that exist in the gas detector area. The way in which the electric field influences the ultimate energy resolution of the detector is one such problem. This will be discussed in Chapter 6.

2.4 Construction of the Simulation

The electron transport physics that it is necessary to model is fundamentally stochastic in nature. A Monte Carlo technique is well-suited to this. The remaining parts of this chapter detail the manner in which the code was set up to produce a Monte Carlo simulation.

2.4.1 The Pseudo-Random Number Generator

A good Pseudo-Random Number Generator (PRNG) lies at the heart of a Monte Carlo simulation. Its job is to produce a uniformly distributed source of random numbers, usually spanning the range between 0 and 1. Whilst there are processes in nature that are intrinsically random, such as radioactive decay, it is a distinctly non-trivial task to get a computer to produce a pseudo-random sequence of numbers. Knuth gives a good introduction to the subject of PRNGs [Knu 69]. On a fundamental level computers receive their instructions from deterministic algorithms. This means that, for example, if the number $x$ is used as input in an algorithm then the number $y$ that results from
application of that algorithm is always the same if \( x \) is not varied. Therefore any random sequence produced by computational means must be prefixed by the word *pseudo*.

There is always the worry that the generator is not producing a uniformly distributed source of random numbers. The simulations described here were run on a mixture of Linux-based PCs and Windows NT-based PCs. NT-based simulations used the pseudo-random number generator from the Digital Visual Fortran 6.0 Compiler [Dig 00]. On Linux the Linear Congruential Generator taken from the EGS simulation code package was used [Egs 00], which was distinct from the Digital Fortran PRNG.

The code development environment was Windows NT. Each time the code was modified its compiled version was run at least once on both NT and Linux platforms with the same input parameter set (including the same random number seed). In no cases were the differences more than a few percent, which was consistent with the expected experimental uncertainty. As will be seen in Chapter 3, the uncertainties in the cross section interaction data that the simulation used as input could be of the order of 10% or more.

As a final check, a third PRNG was used to test the validity of the 16 individual results obtained in Figure 5.9. The PRNG in question was due to Marsaglia [Jam 90]. The ratio of the results obtained using the PRNGs described above to those obtained with the Marsaglia PRNG is plotted on the y axis of Figure 2.4 (the x axis numbers are just indices to differentiate the individual results).

![Figure 2.4 An estimate of the ratio of the results obtained with the PRNGs used for the bulk of the work in this thesis to those obtained with the Marsaglia PRNG.](image)

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On the basis of this evidence, it is argued that the PRNGs used in the simulations gave a reasonable approximation to a random number generator.

### 2.4.2 The Monte Carlo Transport Method

Each electron transport cycle in the simulation starts with an electron that has been allocated with a specified speed, position and direction (its *phase space* parameters). The total cross section for electron-impact interactions at the speed $v$ is $\sigma_i$.

If $\xi$ is a pseudo-random number uniformly distributed between 0 and 1 then it will be shown that the time for the electron to travel to the next interaction point $t_c$ is given by

$$t_c = -\frac{1}{N\sigma_i v} \ln(\xi).$$  \hspace{1cm} (2.3)

Here $N$ is the number density of the gas.

Consider an electron incident upon an elemental block of scattering material of length $\delta x$ and cross sectional area $A_c$, as shown in Figure 2.5.

![Figure 2.5](image)

*Figure 2.5 An electron is incident on the surface of a block of scattering material. Each of the scattering centres is depicted as green circles.*

The cross sectional area of each of the green scattering centres is $\sigma_a$. The total cross-sectional area for scatter presented to the electron is $\sigma_a N A_c \delta x$. The probability of having a scatter in the element is the ratio of the total scatter cross section to the total cross section $A_c$. This probability is also the fraction of electrons that will be absorbed in the element.
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If the reduction in the electron flux is \( \Delta I \) over the scattering element, then letting \( \Delta \lambda \rightarrow 0 \) and \( \Delta \lambda \rightarrow 0 \) causes the change in the flux \( I, dI \), to be given by

\[
\frac{dI}{I} = -N\sigma_a d\lambda.
\]  

(2.4)

In order to bring in explicit time dependence \( dx \) can be written as \( v dt \). \( dt \) is the time it takes to cross the scattering element. The integral that needs to be solved for \( I \) is

\[
\log_e \left( \frac{I}{I_o} \right) = -N \int_0^{t_c} \sigma_a v dt,
\]  

(2.5)

where \( I_o \) is the initial electron flux. \( v \) may vary with \( t \) as the electron moves through the electric field. However the quantity \( I/I_o \) is the fraction of electrons that last up until the time \( t_c \) without interacting. In other words it is the cumulative probability function of electrons that are left at the time \( t_c \). The cumulative probability function \( P \) that electrons will have been absorbed after the time \( t_c \) is given by

\[
P(t_c) = 1 - \exp \left( -N \int_0^{t_c} \sigma_a v dt \right).
\]  

(2.6)

In order to find the value of \( t_c \) that corresponds to the pseudo-random number \( \xi P \) must be inverted.

\[
t_c = P^{-1}(\xi).
\]

(2.7)

This technique is known as the direct method of sampling and is the crucial element of the Monte Carlo technique. There are many references on probability theory and Monte Carlo sampling methods [Ham 64, Kin 66, Spa 69]. For the purposes of this work it will merely be noted that if Equation 2.7 can be solved then a link can be made between \( t_c \) (which is designated the random variable) and \( \xi \).

Using the information in Equation 2.7 means that drawing the random number \( \xi \) causes the time for transport between collisions \( t_c \) to be given by
Chapter 2: Developing a Simulation

\[- \log_x (1 - \xi) = N \int_0^{t_c} \sigma_v v dt. \quad (2.8)\]

Drawing the random number \( \xi \) is functionally equivalent to drawing \( 1 - \xi \). Equation 2.8 can be rewritten as

\[- \log_x (\xi) = \int_0^{t_c} C_t dt. \quad (2.9)\]

In Equation 2.9 the \( N\sigma_v\nu \) product from Equation 2.8 has been rewritten as the collision frequency \( C_t \).

2.4.3 The fictional collision frequency

Solving an integral equation like Equation 2.9 can be extremely numerically intensive. This is particularly true given that this procedure would be repeated millions of times during a simulation, each time an electron’s position is to be adjusted. The problem is that as the electron moves through the electric field its energy is changing. Figure 2.6 shows \( C_t \) as a function of electron energy for argon gas at 1 atmosphere pressure and 298 K. If \( C_t \) were to remain constant throughout the proposed transport step (from \( t = 0 \) to \( t = t_c \)) then solving Equation 2.9 for \( t_c \) becomes trivial.

The solution relies upon creating a fictional collision frequency \( C_f \). \( C_f \) is guaranteed to remain constant throughout the transport step. After each transport step the random number \( \xi \) is drawn. If \( \xi < C/C_f \) then the interaction is designated to be a “true” interaction. The appropriate elastic or inelastic scattering process can then be modelled. Otherwise, no change in the electron’s phase space parameters is made.
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2.4.4 The null collision method

One way of resolving the problem of the changing collision frequency along a transport step is to use the maximum possible collision frequency that occurs across the relevant energy range for $C_f$. Another method, first proposed by Skullerud, is called the null collision method \[\text{[Sku 68]}\]. The algorithm is laid out in flow chart form in Figure 2.7.

In the box with the red outline in Figure 2.7 it is suggested that $C_f$ should be "augmented". A possible way in which this could be done is described with the help of Figure 2.8.

A trial step is performed (covering an energy range that lies between the two orange dots of Figure 2.8) using as a value for $C_f$ the value of $C_t$ at the beginning of the transport step. This step is rejected due to the increasing true collision frequency between these two points. Then $C_f$ is chosen to be the maximum collision frequency between the two points, $t_c$ is re-sampled and the step is repeated. This time the step may or may not be accepted. This depends on the new random number that is drawn. This procedure continues until an electron transport step is finally accepted. Only at this point are the

Figure 2.6 The collision frequency $C_t$ is plotted as a function of electron energy for 1 atmosphere of argon gas at 298 K.
Chapter 2: Developing a Simulation

Figure 2.7 An example of the null collision method [Sku 68].

Accept the transport step and update the electron’s phase space parameters.

Sample $t_c$ (from Equation 2.9) using the collision frequency $C_f$.
Move electron to new position

Does the actual collision frequency $C_t$ exceed the fictional one $C_f$ at any point along the transport path?

Augment $C_f$ and move electron back to original position

YES

NO

Brennan [Bre 91] optimised this technique by calculating a value for $C_f$ before the simulation was actually run. For a given electric field strength $E$ and starting energy $T_0$, the problem becomes one of finding the minimum value of $C_f$ that will cause all of the electrons at that energy to be absorbed before they move for a long enough $t_c$ such that $C_f$ is exceeded.
Figure 2.8 At any point along the electron transport step, if the collision frequency rises above the value used to calculate $t_c$, then the transport step is invalid. For the step represented by the arrow, any value for $C_f$ below the maximum value of $C_t$ is going to be unacceptable (if Equation 2.9 is going to be integrated on the basis of a constant collision frequency).

It can be seen from Equation 2.9 that there is a small but finite probability of the electron travelling for a time that corresponds to an extremely large value of $t_c$ before colliding. Therefore the only truly correct answer is that $C_f$ must be equal to $C_m$, the maximum possible collision frequency. However this criterion might be relaxed slightly. A case might be considered where a fraction of electrons $f$ do not quite obey the criterion. $f$ must be very much less than 1.

The case of argon is considered in Figure 2.9. The factor by which $C_t$ needs to increase in order for $C_f$ to be large enough to satisfy the constraint placed upon it by the value of $f$ in each case is plotted for an electric field strength value of 250 kV cm$^{-1}$. This is actually a strong avalanching field. For values of the single-electron-induced mean gain that are less than $10^4$ (corresponding to almost all of the simulations described in this thesis) electric field strengths greater than this are only likely to be encountered very close to the anode of the counter. Smaller values of $f$ correspond to a much larger maximum value of $t_c$ and so, for a given energy, the value of $C_f$ needs to be correspondingly larger.
Figure 2.9 Ratio of $C_f/C_i$ for various values of $f$, using the optimised null collision method of Brennan [Bre 91]. The gas medium is argon at 1 atmosphere and 298 K. The electric field strength is 250 kV cm$^{-1}$.

Figure 2.10 shows the effect of changing the electric field strength on the value of $C_f$. A clear trend can be observed. Increasing the electric field strength means that the value of $C_f$ is either the same or greater than its corresponding value at the lower field strength. This is because for a given value of $C_f$ (and therefore mean value for $t_c$) a greater change in energy is possible in the case of the stronger field. This means that a valid value for $C_f$ must be greater in magnitude for the stronger electric field.

In practice, a value of $f$ extremely close to 0 (0.001) was used throughout the simulations. This means that only 1 in 1000 transport steps is computed erroneously. The effect of this was assessed for the set of simulations in Chapter 5 for the case where the drift velocity and diffusion coefficients were measured in pure argon for constant electric field strengths. The difference in the results between using a value of $f$ equal to 0 and a value equal to 0.001 was less than 1%, which was within the bounds of experimental uncertainty. A speed-up factor of an order of magnitude was observed between the two cases. It would be interesting to compare these results with results from the null collision method to see if Brennan’s method is indeed the optimal method for this situation.
Even though it may have been possible to find a higher value for $f$, it was decided to be cautious when running the simulations. Therefore a value for $f$ of 0.001 was consistently used. This option was chosen in part because the actual running times of the simulations never became unmanageably large. It will be seen that even when dealing with the non-linear effects of space charge in Chapter 7 that it took no more than about 3 days to run a simulation of 2500 avalanches on a 300 MHz Pentium-powered PC. Therefore the time demands made by the simulations never motivated an increase in the value of $f$ (which would have increased the systematic error). The Monte Carlo radiation transport community traditionally adopts this cautious approach. Alex Bielajew, who has done much to enhance the EGS code over the last two decades, has famously quipped “Don’t be in a hurry to get the wrong answer”.

It is known from Figure 2.10 that the value of $f$ is dependent on the value of electric field strength. In a proportional counter there is a significant electric field gradient. $C_f$ might be calculated for a given value of $f$ using the strongest value of the electric field within the detector. It can be seen in Figure 2.10 that this would lead to a much higher value of $C_f$ than would be necessary in regions where the electric field strength is much
lower. The original plan for the simulation was to define two regions inside the detector, broadly corresponding to the drift region and the avalanche region. Two different values of $C_f$ (for a given electron energy) would be used, depending on the region the particle was actually in. In the end the value of $C_f$ was calculated on the basis of the strongest electric field inside the detector. It is suggested that there is some scope for further optimisation of the simulation along these lines in the future.

2.4.5 Electric field transport algorithms

Once the transport time $t_c$ has been fixed the electron's correct phase-space parameters must be calculated. An issue arises as to whether a relativistic treatment of the electron's change in momentum through the electric field is needed. It will be seen that the maximum figure for $M$, the single-electron-induced mean avalanche gain, obtained in the simulations throughout this thesis was $\sim 10^4$. Figure 2.10b plots the energy distribution of electrons when they reach the anode of a single wire counter filled with argon in two separate cases.

![Figure 2.10b](image)

Figure 2.10b A plot of the energy spectrum (obtained by simulation) of electrons collected at the anode of a cylindrical single wire counter for two different anode wire thickness values. In both cases the gas mixture was 90% argon and 10% methane, held at 1 atmosphere and 298 K. In both cases the value of $M$ was around 8000.
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It can be seen that the electron energies can be considered to be non-relativistic. For example the relativistic factor $\gamma$ for 50 eV electrons is 1.0001.

This implies that the coupled equations of motion that need to be solved are

$$\frac{dr}{dt} = v, \quad \text{and} \quad \frac{dv}{dt} = -\frac{eE}{m}. \quad (2.10)$$

Here $r, v$ and $E$ represent the position, velocity and electric field vectors respectively. $e$ is the magnitude of the electron charge and $m$ is the mass of the electron.

For all of the counters described in this work, it is only in the uniform field that an analytical solution is possible. Three algorithms are considered here. For the purposes of this discussion only the $x$ direction will be considered. However the discussion applies equally to the $y$ and the $z$ dimensions.

The first algorithm used is due to Euler [Pre 96].

$$x^{n+1} = x^n + v_x^n h$$

$$v_x^{n+1} = v_x^n - \frac{eE_x^n}{m} h \quad (2.11)$$

Here $x^n$ is the position of the electron at the $n$-th transport step. $v_x^n$ is its equivalent velocity. $h$ is the time over which the electron will be transported and $E_x$ is the electric field in the $x$ direction.

The second algorithm is due to Verlet [All 97] and is called the Verlet Velocity Algorithm (VVA).

$$x^{n+1} = x^n + v_x^{n+1} h - \frac{eE_x^n}{2m} h^2$$

$$v_x^{n+1} = v_x^n - \frac{1}{2} h \left( \frac{eE_x^n}{m} + \frac{eE_x^{n+1}}{m} \right) \quad (2.12)$$

The third algorithm is due to Runge-Kutta [Lap 71].
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\[ x^{n+1} = x^n + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4) \]
\[ v_x^{n+1} = v_x^n + \frac{1}{6}(l_1 + 2l_2 + 2l_3 + l_4) \]  

(2.13a)

where

\[ k_1 = h v_x^n \]
\[ l_1 = -\frac{eE_x[x_n]h}{m} \]
\[ k_2 = h \left(v_x^n + \frac{l_1}{2}\right) \]
\[ l_2 = -\frac{eE_x[x_n + \frac{k_1}{2}]h}{m} \]  

(2.13b)

\[ k_3 = h \left(v_x^n + \frac{l_2}{2}\right) \]
\[ l_3 = -\frac{eE_x[x_n + \frac{k_2}{2}]h}{m} \]
\[ k_4 = h (v_x^n + l_3) \]
\[ l_4 = -\frac{eE_x[x_n + k_3]h}{m} \]

Here \( n \) corresponds to the \( n \)th time step, and \( h \) is the time step. The square brackets \[ \] in Equation 2.13b indicate that \( E_x \) is to be evaluated at the position described by the quantity inside them. There are two factors to consider when evaluating a candidate procedure for solving the coupled equations of Equation 2.10. These are accuracy and efficiency.

i) Accuracy

There are two sources of error associated with a numerical method. Round-off error results from the finite number of significant figures that a given computer uses to represent a number. This is not inherent in the numerical method itself because if it was possible to gain access to an ‘infinite precision’ machine then this would vanish, irrespective of the algorithm used. Truncation error on the other hand results from the fact that the continuous differential equations are being represented by discrete sets of values. For instance, consider three consecutive steps of the VVA. Eliminating the dependence on velocity means that

\[ \frac{x^{n+1} + x^{n-1} - 2x^n}{h^2} = -\frac{eE[x^n]}{m} \]

(2.14)
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How closely does this approximate the underlying differential equation? Taylor expanding about the point \( x^n \) reveals that

\[
\left( \frac{d^2 x}{dt^2} \right)_{x=x^n} = -\frac{eE_z[x_n]}{m} + \frac{h^2}{12} \left( \frac{d^4 x}{dt^4} \right)_{x=x^n} + \ldots \quad (2.15)
\]

It follows from this that the VVA is not actually solving the differential equation at hand, but a version that differs from it by an amount given by all but the first term on the right hand side of Equation 2.15. Since all of the unwanted terms of the Taylor expansion have \( h \) coefficients which have an order that is greater than or equal to two the VVA is known as an \textit{order-two} method. In a loose sense the truncation error can be defined as the difference between the solution obtained with a finite time step and the one calculated with an infinitesimal one. The Euler algorithm is an \textit{order-one} method and the Runge-Kutta method described here is \textit{order-four}.

In order to illustrate this point a simple harmonic oscillator equation was solved. This was given by

\[
\frac{d^2 y}{dx^2} = -y \quad y(0) = 0 \quad \frac{dy}{dx}(0) = 1 \quad (2.16)
\]

The analytic solution of Equation 2.16 is \( y = \sin (x) \). This is plotted in Figure 2.11 for

![Figure 2.11 Solving the simple harmonic oscillator using the 3 candidate methods.](image)

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the three methods using the same step size $dx$ of 0.4 in each.

This illustrates the alarming variations that can occur when using a method that does not display an accuracy of a sufficient order. The Euler method was rejected for this reason.

Clearly the larger the time step used the greater the discrepancy will be for a given method. However it is also true that for a given time step a higher order method will give greater accuracy. It can also be seen from Equation 2.15 that a larger electric field gradient implies the derivative associated with each power of $h$ will be larger in magnitude.

Bearing this in mind, a simulation parameter was created that was dubbed *the maximum fractional change in the electric field along a time step, $s_f$*. If, during a simulation, the fractional change in the electric field exceeded $s_f$, then the time step was reduced and the step was taken again. This procedure was repeated until a step size was found that met the required condition. It must be noted at this point that adopting this approach did not change the total time between collisions $t_c$. It merely meant that Equation 2.10 was integrated over smaller time steps than $t_c$.

The best test of a given method is to actually insert it into a simulation and then see how it affects any of the quantities it is actually desired to measure. It would be useful to know how the remaining algorithms impact upon the characteristics of an avalanche. In order to illustrate this, the gain and spatial extent of an avalanche was measured for an avalanche in a single wire counter.

Some explanation is perhaps needed of the quantities in the two columns to the right of Table 2.2. The quantity $\Theta$ is measured by logging the angular position of each electron's arrival at the anode (the anode of a single wire counter has a circular cross section). After each avalanche the standard deviation of this quantity is taken. Over a large number of avalanches (5000) the mean of the standard deviation is then taken. The quantity $z$ is measured in a similar fashion. The only difference is that in this case the position along the length of the wire is logged.
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<table>
<thead>
<tr>
<th>algorithm</th>
<th>relative gain</th>
<th>relative Θ spread</th>
<th>relative z spread</th>
</tr>
</thead>
<tbody>
<tr>
<td>Runge Kutta</td>
<td>1.000 ± 0.002</td>
<td>1.000 ± 0.010</td>
<td>1.000 ± 0.017</td>
</tr>
<tr>
<td>VVA</td>
<td>1.004 ± 0.007</td>
<td>1.003 ± 0.013</td>
<td>0.994 ± 0.013</td>
</tr>
</tbody>
</table>

Table 2.2 Avalanche characteristics produced with the two candidate algorithms in a single wire counter. The counter used in the simulation was filled with a 90-10 argon-methane mixture at 1 atmosphere pressure and 298 K. The anode radius was 5 µm, the cathode radius was 2.5 cm and the anode potential was 1400 V. This configuration led to a single-electron-induced mean gain of ~8000. The value of $s_f$ used in both cases was 0.01.

The reason why the two methods give similar results is that the time step that is chosen is small enough so that all of the unwanted terms of Equation 2.15 are too small to affect the result significantly. The reason for this lies in a combination of the total value of the time between collisions and the value of $s_f$ used being sufficiently small.

Table 2.3 shows the effect on the mean single-electron-induced avalanche gain of using different values of $s_f$, for a simulation that was performed inside a single wire counter using the VVA.

<table>
<thead>
<tr>
<th>$s_f$</th>
<th>mean gain</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.001</td>
<td>11927 ± 122</td>
</tr>
<tr>
<td>0.01</td>
<td>11889 ± 82</td>
</tr>
<tr>
<td>0.05</td>
<td>11730 ± 90</td>
</tr>
</tbody>
</table>

Table 2.3 The mean single-electron-induced avalanche gain is plotted for various values of $s_f$ for simulations run in a single wire counter. The counter had a 5 µm anode and a 2.85 cm cathode. The anode was held at 1070 Volts relative to the cathode. The gas mixture was a 90% argon, 10% methane ratio which was held at 1 atmosphere and 298 K.

It was concluded from this that a value for $s_f$ of 0.01 was satisfactory for use throughout the simulations.

**ii) Efficiency**

This refers to the relative speed of the algorithm. Broadly speaking, a lower order method will do fewer calculations to achieve its end than its higher order equivalent,
although this is not always the case [Hoc 88]. In a simulation the bottleneck portion of a
transport calculation tends to arise in the bit of the code where the force is evaluated. In a
single wire counter things are not too bad, because it is possible to write down the electric
field as a function of position (see Equation 1.1). Even so, the Runge-Kutta method was
found to run about 20% slower than the VVA in this case.

The decision was therefore taken to use the VVA as the motion integrator for the
particle transport simulations. This is in contrast to the approach used by Rachinhas [Rac
97]. A different second order method is used, known as the Verlet Leapfrog Algorithm
(VLA).

\[
\frac{x^{n+1} - x^n}{h} = v_x^{n+1/2}
\]

\[
m \left( v_x^{n+1/2} - v_x^{n-1/2} \right) = -E(x^n)e
\]  (2.17)

Over three position steps it renders exactly the same expression as Equation 2.14, so it
has the same accuracy as the VVA. At first glance it appears to only require one force
evaluation per time step and therefore appears to be more efficient. However it pulls the
positions and the velocities half a time step out of synchronisation. Since the initial
values of the velocity and position are known together, the velocity needs to be moved
back half a time step, by some other integrator, or possibly by a high order Taylor
expansion. Just before a collision is to occur, the position and velocity need to be brought
back together so that they are known at the same time.

For the simulations the VVA was the algorithm chosen. This was because it maintains
the same degree of accuracy as the VLA, whilst the position and the velocity of the
electron remain in synchronisation. It can be seen from Table 2.2 that the method is of
sufficient accuracy that it yields very similar results to fourth-order Runge Kutta, yet the
algorithm is less time-intensive.
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2.4.6 Interpolation of the cross sections and collision frequencies

The cross sections are calculated at a user-specified number of points spanning the energy range. This is done in a pre-processor code prior to running the simulations. The values are calculated on the basis of a cubic spline interpolation across the energy range [Pre 96].

Between each point the cross section/collision frequency is assumed to show no variation. This means that the energy region of interest is divided up into a series of bins, with a particular value of the cross section allocated to each bin. There are two aspects to consider when choosing a suitable scheme for the allocation of the cross section bins with respect to energy. The first is that the binned data must adequately represent all of the features of the cross section across the energy range. The second is that the bin corresponding to a given electron energy must be locatable with a high efficiency. Since this operation is performed many times during a simulation there is a potential bottleneck here. A binary search [Pre 96] over the array containing the data is going to be fairly inefficient.

The solution chosen was to divide the energy region up in such a way that in $\log_e$ space the bins were equally spaced. In other words the bin number corresponding to a given energy is given by

$$\frac{\log_e (T) - \log_e (T_{min})}{\log_e (T_{max}) - \log_e (T_{min})} N_b = I_b$$  \hspace{1cm} (2.18)

Here $T$ is the energy of the electron whose corresponding cross section/collision frequency it is desired to find. $T_{min}$ and $T_{max}$ are the bounding energies of the range chosen. $N_b$ is the total number of bins and $I_b$ is the index of the bin corresponding to $T$. $I$ is actually an integer so any fraction left over from the left hand side of Equation 2.18 is rounded up. As the energy is increased the bin size increases. Looking at the cross sections graphs in Chapter 3 it will be seen that there is a very rapid variation with respect to energy in the low energy region (sub-20 eV). Arranging the bin sizes in this
Chapter 2: Developing a Simulation

way captures this variation quite well – particularly the characteristic minimum in the cross section below 1 eV.

For all of the results published here, 500 bins were used to hold the cross section data between an energy of 0.01 eV and 1000 eV. It was found that doubling the bin size had very little effect on the measured value of the drift velocity and diffusion coefficients measured in pure argon (the difference was less than 1% in the worst case) in constant uniform electric fields.

2.4.7 Simulation of the electron-impact collision

Once the electron has moved to the interaction point the next decision that needs to be made is what sort of interaction will happen. Another pseudo-random number \( \xi \) is drawn. If \( \sigma_i \) represents the partial cross section for the \( i \)-th process (normalised such that the total cross section is equal to 1) then the interaction type that is chosen is given by

\[
\sum_{i=1}^{j-1} \sigma_i < \xi < \sum_{i=1}^{j} \sigma_i.
\] (2.19)

In this case the type of interaction chosen would be that corresponding to the partial cross section \( \sigma_{j+1} \). An exception is made in the case when \( \xi < \sigma_j \). Here \( \sigma_j \) is the cross section that must correspond to the chosen interaction.

Now that the type of interaction has been chosen the probability distributions for the angle of scatter, and, if is an ionising collision, the energy of the outgoing particles can be sampled. For the case of an isotropic scatter, if \( (u,v,w) \) is a normalised direction vector that is to be sampled, and \( \xi_1 \) and \( \xi_2 \) are two pseudo-random number generated numbers, then the new direction of the incident electron is calculated using the sequence described in Equation 2.20.

\[
\theta = \cos^{-1}(1 - 2\xi_1),
\]
\[
\phi = 2\pi\xi_2,
\]
\[
u = \sin(\theta)\sin(\phi), \quad w = \cos(\theta)
\] (2.20)

\[u = \sin(\theta)\cos(\phi)\]
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At this point the new position, direction and velocity of the electron has been set. The electron can then embark on a new step. This cycle is continued until an electron reaches the boundary of the problem. This is usually the anode of the counter, but in a micro-pattern detector it could be the insulator surface exposed to the gas (as explained in Chapter 1).

2.5 Summary

The simulation code that forms the main body of this work is a Monte Carlo code. Eventually this code will be used to examine the energy resolution properties of gas-filled counters and detector stability at high incident fluence rates. However before these problems are examined, suitable cross sections for all of the relevant electron-impact interactions need to be found. The issue of solving the electric field in the micro-pattern detectors needs to be addressed. Then the simulation code must be benchmarked against experimental results. These topics will form the subject of the next three chapters.
Chapter 3

The Simulation Input Data

3.1 Introduction

A simulation of avalanching in gas chambers relies upon electron-impact cross section data for all of the relevant types of interaction. These cross sections are made available in the literature, either as a result of scattering experiments, or from applications of theory to the scattering issue. In this chapter a detailed examination of the methods used to obtain the actual data used in the simulations is undertaken, for each of the relevant processes. In addition the physics that underpins the ion transport is considered.

3.2 Choice of gas mixture

Before studying the cross sections themselves, an overview is given of some of the gases that might be used inside the detectors.

3.2.1 The fill gas

If the counter is going to be used for x-ray imaging then it must contain a gas that has a reasonable ability to stop x-rays. The attenuation coefficient for photoelectric absorption for three suitable gases is plotted in Figure 3.1. It follows from the fact that xenon has the largest photoelectric absorption coefficient that this gas is therefore an extremely good fill gas to use for the detector. The plan is to use it in the final multiphase flow tomography system shown in Figure 1.1.

The downside of using xenon stems from reasons of pure economy. Its relatively small concentration in the atmosphere relative to argon makes it extremely expensive to isolate. This means that for practical laboratory purposes the gas has to be recycled after use.
Figure 3.1 The photoelectric absorption coefficients for 3 noble gases that might be used as the fill gas in proportional counters. The data are taken from the XCOM database of Berger [Ber 87]. The coefficients are calculated at 1 atmosphere pressure and 298 K.

Moving up the periodic table, the next member of the noble gas family is krypton. It is not as expensive as xenon. However there are still problems associated with its use in proportional counters. Krypton gas produced by atmospheric liquefaction is a radioactive emitter. This would not be helpful if the detector was going to be used for a low-level counting experiment. Key has pointed out that high fluence rates are more likely to be the problem in the proposed tomography system [Key 99]. This raises the intriguing possibility of using krypton for the detectors in the final assembly.

It can be seen from Figure 3.1 that argon has a lower attenuation coefficient for x-rays than the other two gases. Therefore it will not be used for the final system. However it has the advantage that it is the cheapest gas of the three. When it is finished with it can just be leaked to the surrounding atmosphere. It was considered to be an ideal gas with which to develop the prototype detectors. It will be seen in Chapter 5 that some micro-strip detectors filled with argon were manufactured by Key at Surrey University. Using argon in the simulations provided an opportunity with which to benchmark the code against that data. It will also be seen that there is other data in the literature that can be used to facilitate a comparison with the simulations. Therefore argon is the fill gas chosen for use in the simulations.
Chapter 3: The Simulation Input Data

The reason why noble elements are very popular choices for proportional counter gases is because of their extremely low electron affinity. Electrons have an extremely low probability of attaching to neutral noble gas atoms. If they attach to atoms to form negative ions, then the much larger mass that is now associated with this negative signal carrier will no longer promote avalanching. Multiplication of the original number of signal carriers will not ensue.

One of the inherent difficulties that needs to be overcome in order to obtain the successful operation of a proportional counter lies in the fact that air contains material components that will grab electrons willingly. Oxygen and water vapour are two gases that are particularly prone to this. If they leak into the detector then they can pick electrons off as they drift down from the point where they are created towards the avalanche region of the detector.

3.2.2 The quench gas

Argon is very rarely used in proportional counters in pure form. This is because counters find it extremely difficult to operate in a stable fashion. A small proportion of quench gas is usually added to remedy this.

The problem is partly one of photon feedback (see Section 1.6.2). A quench gas can be added that will intercept the photons before they escape from the avalanche region of the detector or release electrons from the cathode. A quench gas is molecular by nature. This means that it can absorb a photon's energy and dissociate, rather than produce an additional electron that can go on to reinitiate an avalanche. There is also another possibility. If an argon atom collides with a quench gas molecule whilst it is still in an excited state, then its energy can be transferred to the quench gas molecule. The quencher then dissociates. In this case no photon is produced.

Methane is widely used as a quench gas in cylindrical single wire proportional counters. If it is added in small quantities to an argon-filled counter (5-10%) then it is a good promoter of detector stability. It has been found that it is difficult to obtain values of the single electron induced mean avalanche gain $M$ of more than $\sim 10^2$ in a single wire.
counter [Kno 89]. By adding 10% methane to the counter, values of $M$ of $10^5$ or greater can be obtained.

Methane has been used in micro-pattern detectors in the past [Alu 94]. However Bouchier has suggested that it is not suitable over the long term for use in such detectors [Bou 92]. The problems arise as a direct result of the fragmentation process that can occur. Unfortunately the fragments can polymerise inside the detector. This leads to their solidification and the lower surface of the detector becomes coated. This can lead to a change in the gain characteristics, after the detectors have been irradiated for a few weeks. This process is known as ageing of the detectors.

On the other hand, as for argon, there is a range of actual experimental data available in the literature (see Chapter 5) that can be used to benchmark the simulation code. The extensive use of methane in the simulations described here is not meant to be taken as a suggestion that this is the best gas to use in micro-pattern detectors. The argument is that the code can be benchmarked for methane. This means there will be more confidence in results obtained with gas mixtures containing methane, than for other gases that might be used. The behaviour of methane-filled detectors can then be used to give a qualitative explanation of the behaviour of the detectors.

Another gas that might be used for quenching is carbon dioxide. Key developed micro-strip detectors using argon/carbon dioxide mixtures. This experimental data is also used for benchmarking purposes in Chapter 5. Detectors filled with carbon dioxide do not suffer so heavily from the problem of ageing. Carbon dioxide and methane are the two quench gases used in the simulations.

3.3 Cross Sections: Initial Concepts

Before discussing the electron-impact physics in detail the concept of a differential cross section (DCS) is clarified. The idealised scattering experiment is depicted in Figure 3.2.

Consider a pencil beam of electrons of initial energy $T_0$, directed along the z-axis, of
Chapter 3: The Simulation Input Data

Figure 3.2 Schematic diagram of a scattering experiment [McC 95].

$N_e$ electrons per second. The target consists of $N_t$ particles located at the origin. The number of particles, $N_j$, scattered per second into the solid angle $d\Omega$ making an angle $(\theta, \phi)$ defined by the direction $k_j$ relative to the incident direction, with energies between $T_j$ and $T_j + dT_j$, is given by

$$N_j dT_j d\Omega = d^2\sigma_j (T_0, T_j, \Omega) N_e N_t$$

where the subscript $j$ denotes the particular excitation process leading to the final state $j$.

The formulation of the double differential cross section (DDCS), $d^2\sigma / dT_j d\Omega$, implies that $T_j$ is a continuous variable. This statement is true for ionisation processes. There is actually an additional degree of freedom which occurs in ionisation that relates to the direction of the newly freed electron, $\Omega_2$. This leads to the triple differential cross section (TDCS), $d^3\sigma / dT_j d\Omega_1 d\Omega_2$. The single differential cross section (SDCS), $d\sigma / d\Omega$, is obtained by integrating over $T_j$. This cross section is a measure of the probability of scattering the primary electron into a solid angle $d\Omega$ in the direction $(\theta, \phi)$. It is relevant to excitation and elastic scattering as well as ionisation.

It should be pointed out that the collision problem is formulated in the centre of mass (COM) frame of reference. There is therefore the need to relate the scattering angle
obtained in the COM frame to the laboratory frame. For this situation the electron has a much smaller mass than the scattering centre and so it is safe to assume that the centre of mass frame of reference moves at the same speed as the gas particle. In particular, argon atoms at 300 K move at speeds of \( \sim 400 \text{ m s}^{-1} \). A typical avalanche electron energy is of the order of eV. A 0.1 eV electron therefore moves at a speed of \( \sim 10^5 \text{ m s}^{-1} \). The gas atom is effectively at rest relative to the electron, even at the lowest energies, and so negligible error occurs in assuming that the scattering angle refers to the 'rest' reference frame. The latter is, of course, the laboratory reference frame.

3.4 Establishing the electron-impact scattering cross sections

It can be seen in Figure 2.10b that the vast majority of the electron-impact collisions with gas atoms will occur at energies that are below 100 eV. The tools used to obtain cross sections within this energy range will now be discussed. The way the scattering is actually performed in the simulations will also be discussed.

3.4.1 Elastic scattering

It can be shown [Dav 69] that the SDCS for elastic scattering \( \frac{d\sigma_{el}}{d\Omega} \) can be written as

\[
\frac{d\sigma_{el}}{d\Omega} = \frac{1}{4k^2} \sum_{l=0}^{\infty} (2l + 1)[S_l(k) - 1]P_l(\cos(\theta))^2.
\]

Here \( k \) is the wavenumber associated with the electron and \( \theta \) is the angle of scatter relative to the original direction. The remaining quantities in Equation 3.2 will be revealed in the following discussion.

\( \frac{d\sigma_{el}}{d\Omega} \) is clearly going to depend on the form of the scattering potential \( V \). There are a number of possible contributions to \( V \).

\[
V = V_{st} + V_{ex} + V_p.
\]

\( V_{st} \) accounts for the net force the free electron would experience if it encountered the scattering particle with electrons in their ground state conformation. \( V_{ex} \), the exchange
potential, is a correction term which accounts for the Pauli exclusion principle, as applied to the free electron and its bound counterparts. Effectively this is a requirement that electrons of like spins cannot occupy the same location in space - each electron is surrounded by a zero-probability hole called a Fermi hole. \( V_p \) takes account of the polarisation of the target that results from the presence of a free electron in its vicinity.

\( l \) appears in Equation 3.2 as the angular momentum quantum number. The scattered electron wavefunction \( \Psi \) can be written as a sum of spherical waves, each characterised by a different \( l \), so that

\[
\Psi = \frac{1}{2i k r} \sum_{l} i^l (2l + 1) P_l(\cos(\theta)) \left[ e^{-i(kr-l\pi/2)} + S_l(k) e^{i(kr-l\pi/2)} \right].
\] (3.4)

The first exponential term in Equation 3.4 represents the incoming wave and the second one represents the outgoing wave. \( r \) represents the distance from the scattering centre and \( P_l \) are the Legendre polynomials.

\( S_l(k) \) is a function of the phase shift \( \delta \) that occurs for each angular momentum wave component as it is stretched or shrunk in the scattering potential \( V \).

\[
S_l(k) = e^{2i(\delta + l\beta_l)}
\] (3.5)

If there is negligible change in phase then the \( S_l(k) \) tend to 1, and from Equation 3.2 it can be seen that there can be no contribution to the scattering cross section.

It turns out that a maximum value of \( l \), \( l_{max} \) can always be defined such that for all \( l \) greater than \( l_{max} \), \( S_l \) is virtually equal to 1. In other words large-valued \( l \) waves don’t contribute much to the scattering process. This can be explained classically by considering the distance of closest approach of a particle with angular momentum \( L_T \) to a centre of force which is Coulombic in nature and whose potential \( V_{cen}(r) \) therefore has a \( 1/r \) shape. In \((r, \theta, z)\) coordinates the \((\theta, z)\) part can be collected up into the total angular momentum of the system \( L_T \). The total energy of the particle \( E_T \) can then be written as

\[
E_T = \left[ \frac{L_T^2}{2mr^2} - V_{cen}(r) \right] + \frac{1}{2} m v_r^2.
\] (3.6)
In Equation 3.6 $v_r$ is the radial velocity of the free electron. The term involving $L_T$ on the right hand side of Equation 3.6 is effectively a repulsive potential. For a given value of $E_T$, if the particle has a large value of $L_T$, then it is not going to get very close to the scattering centre. This is good news for those who are adopting the wave mechanics approach to calculate a cross section because it means that the sum of Equation 3.2 does not extend to infinity. The sum is terminated at $l_{\text{max}}$, where $S_{l_{\text{max}}}(k)$ is practically equal to 1.

In order for a simulation to model elastic scattering whilst taking into account the full single differential cross section that is described by Equation 3.2, the value of $S_l(k)$ needs to be known for all values of $l$ up to $l_{\text{max}}$, where $l_{\text{max}}$ is defined by the maximum energy of electrons in the simulation. Then the appropriate differential cross section can be calculated from Equation 3.2 for the appropriate value of $k$. Figure 3.3 shows $d\sigma_{el}/d\Omega$ for argon gas below 20 eV.

Figure 3.3 The differential cross section (dcs) for elastic scattering is plotted for argon gas [Das 85]. The dcs has been normalised so that when it is summed over all solid angle elements then its value is equal to 1.

In principle it is possible to work backwards from the values of $d\sigma_{el}/d\Omega$ to arrive at the values of $S_l(k)$. This would require $d\sigma_{el}/d\Omega$ to be known at each individual electron-impact energy for each value of $l$ up to and including $l_{\text{max}}$. This would allow the value of
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$d\sigma/d\Omega$ to be interpolated at any energy in the required range. The scattering angle could then be sampled using the rejection method of Monte Carlo sampling [Tav 98].

In the simulations isotropic scattering was assumed to occur when an elastic scattering collision happened. It was decided to proceed on the basis of this simpler approach to see if reasonably accurate results could be obtained, without having to resort to the computationally intensive procedure of interpolation and sampling described in the preceding paragraph.

A quantity related to the actual cross section for elastic scattering $\sigma_e$ is the momentum transfer cross section $\sigma_m$. This is defined in such a way that

$$\sigma_m = \int d\sigma_e (1 - \cos(\theta)) d\Omega.$$  

(3.7)

$\sigma_m$ is a weighted average over $d\sigma/d\Omega$. The weighting changes from 0 to 2 as $\theta$ increases and the scattering moves more in a backward direction. If the cross section is strongly forward scattering then the effective cross section is severely reduced and if scattering is strongly backward then the effective cross section is enlarged. Where it was available $\sigma_m$ was used in the simulations. In this way, even though scattering was modelled isotropically, some account was made of the shape of the differential cross section. Biagi has pointed out the fact that, in the case where the only contribution to the scattering comes from elastic mechanisms, the use of the momentum transfer cross section is formally exact [Bia 99].

The fractional energy loss of the electron in an elastic collision is proportional to the ratio of the mass of the electron to that of the scattering atom (see Equation 5.7).

3.4.2 Inelastic scattering

According to McCarthy [McC 95], the differential cross section for a discrete excitation process can be written as

$$\frac{d\sigma_{\text{ex}}}{d\Omega} = (2\pi)^4 \frac{k_i}{k_0} \sum |T_{i0}|^2.$$  

(3.8)
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The equivalent expression for ionisation is

$$\frac{d^3\sigma}{d\Omega_\mu d\Omega_\nu dE_i} = (2\pi)^4 \frac{k_\mu k_\nu}{k_0} \sum |T_{\mu0}(k_0, k_\mu, k_\nu)|^2.$$  (3.9)

Here $k_0$ denotes the initial wavevector of the free electron, $k_i$ denotes the final wavevector of the same electron and $k_\mu$ denotes the wavevector of the freed electron. $T_{\mu0}$, the term known as the $T$-matrix element, is the key to deriving the cross section. The sum is over degenerate states – there is possibly more than one transition that results in the given final state of the system.

It is interesting to note that whilst the excitation cross section for a discrete process is differential only in the angle of scatter of the free electron, the ionisation cross section has a dependency on the angle of emission of the freed electron. The differential cross section is also dependent on the amount of energy that is transferred in the collision. Hence the latter is known as the triple differential cross section (TDCS) for ionisation. It is to be noted that if the incoming electron energy is very much greater than the binding energy of the material then the TDCS becomes a SDCS (Single Differential cross section). Classically the second electron can then be treated as if it is practically at rest compared to the first. If the energy of the secondary electron is sampled then the angles of emission relative to the original direction are completely determined, as well as the secondary electron energy.

Of course from the Heisenberg Uncertainty Principle it is known that this is not true in a strict sense. When the bound electron’s momentum is comparable to that of the incoming electron then it is the uncertainty in the electron’s momentum that manifests itself in the probability distribution that gives the extra degrees of freedom. It is not possible to be sure in any individual case of the speeds and directions of the individual electrons that are involved in the collision. This implies the existence of a TDCS.

Near to the ionisation threshold it becomes difficult to find a suitable approximate potential to describe the physics of the process. McCarthy [McC 95] has suggested that this is because the motion of the free electron and the bound electron are highly
correlated. There have been a few attempts to calculate a TDCS at energies that lie way above the ionisation threshold [Ava 89]. However in simulations a lot of the electron-impact physics must happen at energies that lie around the ionisation threshold of the gas. This is because (by definition) this is where electron multiplication must start.

An attempt has been made to suggest empirical forms for the SDCS with respect to energy loss based on the scarce experimental data near to the ionisation threshold [Gre 72]. Unfortunately the theoretical basis of this expression is not verifiable.

In the simulations described in this work isotropic scattering is used to select the angle of the primary and the angle of any secondary electron generated. The SDCS for energy transfer after an ionising collision is also completely uniform. The validity of choosing such an approach will be seen in Chapter 5, where simulation is compared with experimental results.

3.4.3 The Boltzmann transport equation

Other than direct calculation, there are two other methods that can be used to obtain inelastic cross sections for electron-impact collisions. The first is by actually doing a scattering experiment. If the appropriate experiment is not available then there is another possibility. The Boltzmann transport equation can be used to infer cross sections from experimental swarm parameters.

Since the early pioneering work of Townsend, there has been much work done to measure the so-called swarm parameters that characterise the drift and diffusion of electrons through the gas. These include the drift velocity \(v_d\), the transverse diffusion coefficient \(D_T\) and the Townsend first ionisation coefficient \(\alpha_T\). In order to calculate the swarm parameters directly it is necessary to solve the Boltzmann transport equation. This can be written in the form

\[
\left\{ \frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla + \mathbf{a} \cdot \nabla \right\} n_f = -J(n_f),
\]

\[
a = \frac{e}{m} \mathbf{E}, \quad J(n_f) = -n \left( \frac{df}{dt} \right)_{\text{coll}}.
\]

(3.10 a,b,c)
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\( n(r,t) \) is the particle density function. It is defined by the fact that the number of electrons in the system to be found within a volume element \( dr \) is \( n(r,t)dr \). The function \( f(r,v,t) \) is the velocity distribution function, which is constrained to obey

\[
\int_v f(r,v,t)dv = 1. \tag{3.11}
\]

The \( v \cdot \nabla_r \) term of Equation 3.10a represents the net flow of particles out of a positional element of 6-dimensional phase space due to electron drift. The \( a \cdot \nabla_v \) term represents the net flow of particles out of a velocity element of phase space due to the presence of the electric field \( E \).

The term \( J \) refers to the collision operator. It represents the flow of particles out of an element of phase space due to collisions with gas particles. It is defined by the interaction cross sections. It can be seen that the solution of this equation is written as an \( nf \) product because then \( J \) only acts on the velocity distribution function \( f \). In other words, the value of the cross sections are only dependent on the energy of the free electron. Given a complete set of accurate cross sections, Equation 3.10 can be solved for \( f \) and the swarm parameters can be calculated.

In order to get unknown cross sections from known swarm parameters the problem must somehow be solved in reverse. The experimentally measurable swarm parameters are the drift velocity, the diffusion coefficients and the ionisation and attachment coefficients. Given such a limited set of experimental swarm parameters, it is not possible to determine a set of cross sections where many different types of interactions are possible. For polyatomic gases like methane or carbon dioxide there is the possibility of vibrational excitation, which can play an important role in determining the transport properties in the types of fields that exist in the drift region of a detector. It is possible that more than one set of cross sections would fit a given set of swarm parameters.

In order to start the process, all of the known experimental scattering data and theoretical scattering calculation data is collated. This is then used to constrain the cross section set. Any gaps in the known cross sections can then be filled in when the
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Boltzmann calculation is done. In addition, new cross sections can be added to describe processes that have not been characterised by experimental or theoretical means.

For example Stefanov has formulated a vibrational inelastic cross section for the CF₄ molecule [Ste 88], which is added to the known set of cross sections.

$$\sigma_i = C(u-1)u^{-\alpha} \ln(\beta u)$$  \hspace{1cm} (3.13)

Here $\sigma_i$ is the cross section, $\varepsilon$ is the energy, $u$ is equal to $\varepsilon_i/\varepsilon$, and the parameter set $(C, \alpha, \beta, \varepsilon_i)$ are all fitting parameters whose values can be adjusted.

Bordage argues that increasing the amount of experimental transport data makes the cross sections progressively more constrained [Bor 96]. Of course, part of the cross section space has been mapped out either from scattering experiments or quantum mechanics calculations. These are important constraints. Eventually a set of cross sections is established that is consistent with the swarm parameters for the gas in question. It is accepted that this might not be the only set of cross section data that is consistent with the experimental data.

3.5 The simulation electron-impact cross section data

The data actually used in the simulations will now be revealed for each of the gases used.

3.5.1 Argon

The cross sections used for excitation, ionisation and elastic scattering are shown in Figure 3.4

**Elastic scattering**

Above 10 eV the data is due to Jain [Jai 90]. Below 10 eV the data of Bordage are used [Bor 96]. It is noted that below about 11 eV only elastic scattering is possible. There is also a dip in the cross section that occurs at about 0.2 eV. This is known as the Ramsauer minimum.
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It occurs when the energy is such that there is a large amount of destructive interference between the individual partial waves in Equation 3.4. This feature will also be seen in methane and carbon dioxide. Indeed it is a characteristic of a wide range of atoms and molecules.

**Excitation**

Chutjian has measured the argon excitation cross section experimentally for excitation to 23 of the lowest lying excitation states above the ground state of argon [Chu 81]. Bretagne complemented this work by fitting a semi-empirical formula to Chutjian’s experimental data [Bre 86]. The expression used Chutjian’s work to influence its shape at low energies. At higher energies the trend matches what would be predicted if the Born Approximation was used to calculate the cross section. The results are shown in Figure 3.5 for the 4 lowest-lying excitation levels above the ground state energy.

It can be seen that there is a degree of uncertainty associated with Chutjian’s experimental work. There is also another problem. It is possible that there are many discrete transitions that can occur (ranging from the infra-red to the ultra-violet), each of
which are individually quite small. Furthermore, many of these are as yet unaccounted for in the literature.

![Graphs showing atomic cross sections for the lowest four excitation levels in argon gas.](image)

Figure 3.5 Atomic cross sections for the lowest four excitation levels in argon gas. The solid lines for each graph are calculated from the semi-empirical formula given by Bretagne [Bre 86]. The symbols are from the experimental measurements of Chutjian [Chu 81].

In this work the problem is reduced to something slightly less ambitious. The aim is to try to model a gas by choosing a cross section data set that causes the simulations to give a fair reproduction of experimentally observed behaviour in a wide range of circumstances. It is accepted that the cross section set used to generate these results might not be the only one that behaves in this way. There is therefore an argument against using the simulations to make accurate quantitative predictions.

On the other hand the match between experimental and simulation trends might be quite close (for example, the way the Townsend ionisation coefficient varies with energy for a given gas mixture). In this case it is argued that such a simulation cross section set is
plausible. It can therefore be used to give a plausible qualitative picture of the physics inside gas counters. Even though Chutjian's cross sections were the only ones used to model excitation in argon, it is acknowledged that there may be other sets that can give plausible results.

In the simulations, each of the levels of Chutjian is treated individually. The excitation cross section shown in Figure 3.5 is actually a composite of all of the excitation levels used in the simulation.

**Ionisation**

Straub has experimentally measured the cross section for single, double, triple and quadruple ionisation of the argon atom following electron impact [Str 95]. In the simulations each ionisation is considered to result in the freeing of only a single additional electron. The cross section used in the simulations is actually re-weighted to reflect this. This is achieved by writing the total ionisation cross section \( \sigma_i \) as

\[
\sigma_i = \sigma^{1+} + 2\sigma^{2+} + 3\sigma^{3+} + 4\sigma^{4+} + \ldots.
\]  

(3.14)

Here \( \sigma^{n+} \) represents \( n \)-fold ionisation after the process of electron-impact.

\( \sigma^{4+} \) is only non-zero above 300 eV. This is an energy that is very rarely encountered in the simulation regime. It was therefore deemed reasonable to terminate the series displayed in Equation 3.14 at the quadruple ionisation limit.

Straub suggests that the uncertainties in the measured cross sections are as much as 7.5%.

![Methane molecule](image.png)

**Figure 3.6** In methane 4 hydrogen atoms are arranged in tetrahedral fashion around a central carbon atom (see overleaf).
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3.5.2 Methane

Methane (or CH₄) is the simplest of the alkane family of saturated hydrocarbons. There are additional possibilities for energy absorption compared with monatomic argon. These include fragmentation (with or without the production of an additional electron), vibration of the molecule as a whole and rotation of the molecule as a whole.

The cross section set used for methane is shown in Figure 3.7.

Figure 3.7 The molecular electron-impact cross section set used for methane. It can be seen that there are additional possibilities for energy loss compared with argon.

Elastic scattering

Below an energy of 10 eV the cross section is due to the work of Bordage [Bor 96]. Above 10 eV the data is due to the work of Jain [Jai 86].

Excitation

Due to the molecular nature of methane, it tends to dissociate upon electron impact into a myriad of possible neutral fragments. Biagi has condensed the discrete excitation set into a single excitation cross section set that is consistent with experimental swarm parameters [Bia 89]. The onset of this discrete excitation occurs at 9.5 eV. Between 100 eV and 1000 eV the cross section is taken from the work of Gan [Gan 92].
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Vibration

Vibration characteristically gives rise to energy losses that coincide with the infrared ($\sim 10^{-1} \text{ eV}$) region of the electromagnetic spectrum. Rotational energy losses coincide with the microwave region. In fact, if infrared spectroscopy is used to interrogate the vibrational structure of the molecule, then a “banded” structure is obtained in the resulting spectrum [Atk 96]. This means that vibration transitions are accompanied by their rotational counterparts. In this work the possibility of rotational excitation is ignored. It is argued that the vibrational cross sections are themselves not sufficiently accurate to differentiate between the varying degrees of microwave rotational excitation that accompany the infrared vibrational transition.

If a vibrational event occurs then the free electron scatters isotropically. The energy deducted is the threshold energy for the vibration mode that is excited. Vibration in methane is modelled in the simulations as two composite energy levels, with thresholds at 0.367 \text{ eV} and 0.175 \text{ eV}. The data used for these cross sections is provided by Bordage [Bor 96].

Ionisation

Straub has published ionisation cross sections for methane [Str 97]. Straub actually measures the partial cross sections for the production of individual ionic species. For the simulations, unfortunately, this is of secondary interest. What is more important is whether or not an additional free electron is produced in an ionising collision. The problem can be explained with the help of Equation 3.15, by considering some of the possible fragmentations that can occur to produce ions.

\[
CH_4 + e^- \Rightarrow CH_3^+ + 2e^- \\
CH_4 + e^- \Rightarrow CH_3^+ + H^* + 2e^- \quad (3.15 \text{ a,b,c}) \\
CH_4 + e^- \Rightarrow CH_3^+ + H^- + e^-
\]

Here $H^*$ signifies the hydrogen free radical. Equations 3.15b and 3.15c both produce the $CH_3^+$ ion and therefore make a contribution to the cross section $\sigma_{CH_3^+}$ for the production
of this ion. However only Equation 3.15b produces an extra electron that actually contributes to the avalanche. In fact the only partial ionisation cross section for which it is certain that an extra electron is freed is for the production of $CH_4^+$. 

Biagi has published a single ionisation cross section that, in effect, collects together all of the possible ionisation channels that can lead to the production of a single electron [Bia 89]. This cross section was used in the simulations. Above 100 eV the cross section for the production of $CH_4^+$ measured by Straub is used to represent the ionisation cross section [Str 97].

### 3.5.3 Carbon Dioxide

![Carbon Dioxide Molecule](image)

Figure 3.8 Carbon dioxide is a linear molecule with two oxygen molecules flanking a central carbon atom

In the summer of 1999 Key developed some micro-strip detectors filled with mixtures of argon and carbon dioxide. In order to benchmark the simulations against this data, the literature was searched for a suitable set of carbon dioxide cross sections for electron-impact collisions. Sharma [Sha 98] has published a suggested set of cross sections for carbon dioxide. The excitation cross sections and vibration cross sections were not separately distinguished in this set. From the point of view of the simulations, it actually makes no difference. In each case the threshold energy associated with the collision is deducted from the free electron’s energy, and then an isotropic scatter is performed.
3.6 Ion transport physics

It has already been suggested that the accumulation of positive ions in the gas volume can have a profound effect on the single-electron-induced mean avalanche gain. This section explains how the ion transport is modelled and how the necessary data that is required to do the modelling is derived.

3.6.1 Modelling strategy

Three possible strategies were conceived. The first was to construct a full simulation based on tracking individual ions through the detector, on a collision by collision basis, in a similar manner to which electron transport was conceived.

The second was to construct a partial simulation based on tracking individual ions through the detector, updating each ion's position according to its macroscopic transport parameters, $D_i$ and $v_i$. $D_i$ is the diffusion tensor for the ion and $v_i$ is the drift velocity vector.

The third method does not treat each ion individually. Instead a mesh is placed over the whole detector volume. At equally spaced time intervals the charge that is generated through ionisation is allocated to the appropriate mesh point, depending on its position.
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The mesh charge is also redistributed as time passes, in order to account for the drift of the ions.

Given a complete set of atom-atom collision cross sections for the above process it is possible to use the first approach. If it were desired to know the exact position of each ion at all times this would be the only acceptable method. This method also has the biggest drain on both memory and CPU resources.

The second method is a sort of 'condensed history' positive ion transport (see Section 2.2 for an example of condensed history electron transport). It relies (from \(D_I\)) on knowing the appropriate probability distributions that are associated with an ion's position after a given time.

The final method is the most approximate of the three. The size of the array required to store the ion phase space parameters is reduced to the number of points in the grid. The average behaviour of the charge on each of these grid points is to drift along the field lines. This is the simplest approach of the three. It was decided to adopt this method in the simulations, because it was the fastest. If problems were encountered in trying to get it to run quickly enough, then the other methods would not be practical.

3.6.2 Ion transport

Figure 3.10 shows how the drift velocity varies with electric field strength for singly ionised positive argon ions (Ar\(^+\)) in argon gas.

The relationship between the ion drift speed \(v_I\) and the electric field \(E\) at low electric fields is

\[
v_I \propto E. \tag{3.16}\]

At high fields it is more like

\[
v_I \propto \sqrt{E}. \tag{3.17}\]

There are two forces that play a part in dictating the motion of positive ions. The first is due to polarisation. As a positive ion passes close to a gas particle, it polarises the
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Figure 3.10 The drift velocity is plotted as a function of electric field strength for singly ionised positive argon ions in argon gas. The gas pressure is 1 atmosphere and the temperature is 298K. The data are due to Hornbeck [Hor 51].

electron cloud surrounding the particle. This leads to an attraction between the two particles. The second force is the known as the hard sphere force. When particles get close to each other a strong repulsive force is felt that acts over a short distance as the electron clouds touch. The hard sphere force is responsible for the change in direction of the ion at a collision and is very short range. The polarisation force is an attractive force, the importance of which decreases as the field strength increases.

The reason for the relationships described above can be deduced from a simple argument. Imagine that each ion is accelerated from 0 to $2v_i$ in the direction of the electric field. Then $v_i$ is the average velocity in the direction of the electric field between collisions. $v_i$ can thus be connected to $E$ through Equation 3.18, where

$$v_i \propto \frac{eE}{m_i} \Delta t.$$  

(3.18)

Here $\Delta t$ is the inverse of the collision frequency and $m_i$ is the mass of the ion. At low electric fields the collision frequency is independent of $v_i$. This is because the electric field has only a small influence on the ion velocity.
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On the other hand, if the field is extremely high the overall velocity of the ion is strongly influenced by the electric field. In fact if $E$ is high enough then the mean ion velocity is the same as the drift velocity. Substituting for $\Delta t$ it can be seen that

$$v_i \propto \frac{qE}{m_i \Sigma v_i}.$$  \hspace{1cm} (3.19)

Here $\Sigma$ is the macroscopic collision cross section for the ion. It can be seen from Equation 3.19 that, upon rearrangement, $\sqrt{E}$ is proportional to $v_i$.

In these simulations it is assumed that the transport occurs in the low field region and so Equation 3.16 is valid. It may help to set this assumption in context by stating that the minimum value of the electric field that promotes avalanching in 1 atmosphere of argon is approximately 3.5 kV cm$^{-1}$. This is firmly within the proportional region of Figure 3.10. The high field region of a proportional counter, where Equation 3.16 may no longer apply, is at worst confined to a very small portion of the gas volume. Ions will therefore spend the vast majority of their time in the drift region of the detector, where Equation 3.16 is valid.

If the assumption made above is accepted to be true then the ion mobility $\mu_i$ can be defined, where

$$\mu_i = \frac{v_i}{E}. \hspace{1cm} (3.20)$$

It is pointed out that it is only in Chapter 7 that the influence of the ions generated in an avalanche is at all considered. The gas chosen for this is a mixture of 50% argon and 50% methane. So $\mu_i$ data needs to be obtained for this gas mixture.

Sharma has published data for $\mu_i$ for the gases used in the simulations [Sha 98]. This is shown in Table 3.1.
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<table>
<thead>
<tr>
<th>Host gas</th>
<th>Ion</th>
<th>Mobility / cm²V⁻¹s⁻¹</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ar</td>
<td>Ar⁺</td>
<td>1.00</td>
</tr>
<tr>
<td>Ar</td>
<td>CH₄⁺</td>
<td>2.07</td>
</tr>
<tr>
<td>CH₄</td>
<td>CH₄⁺</td>
<td>2.26</td>
</tr>
</tbody>
</table>

Table 3.1 Mobilities of ions in various host gases at 1 atmosphere pressure.

In order to correct the values of \( \mu_i \) that are displayed in Table 3.1 for ion transport in gas mixtures, Blanc's law can be used [Bat 62].

\[
\frac{1}{\mu_m} = \sum_i f_i \mu_i
\]  

Here \( \mu_m \) is the mobility in the gas mixture, \( \mu_i \) is the mobility of the ion in the \( i \)-th component of the gas and \( f_i \) is the gas fraction of that component.

The value of \( \mu_i \) for Ar⁺ moving in CH₄ needs to be found. The assumption is made that the speed of an ion is inversely proportional to its mass, as suggested by Equation 3.18. It can be seen that for CH₄⁺ ions the value of \( \mu_i \) is increased by a factor of 2.07 for transport in pure argon gas, relative to Ar⁺ ions in the same medium. If the suggested relationship held in a strict sense then it would be expected to go up by a factor of 2.43.

Part of this discrepancy must lie with the change in the effective collision cross section as the gas medium for the transport changes. However this approximation does appear to be a fairly reasonable one. The final values of \( \mu_i \) are shown in Table 3.2.

<table>
<thead>
<tr>
<th>gas</th>
<th>ion</th>
<th>mobility / cm²V⁻¹s⁻¹</th>
</tr>
</thead>
<tbody>
<tr>
<td>50% Ar, 50% CH₄</td>
<td>CH₄⁺</td>
<td>1.95</td>
</tr>
<tr>
<td>50% Ar, 50% CH₄</td>
<td>Ar⁺</td>
<td>0.96</td>
</tr>
</tbody>
</table>

Table 3.2 The mobilities of Ar⁺ and CH₄⁺ ions in an equal mixture of the two gases.
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It can be seen from Table 3.2 that only the $\text{CH}_4^+$ ion is considered in the simulations. In reality there is the possibility of more than one type of ionised species being produced on electron-impact ionisation of methane (see Equation 3.15, for example).

3.7 Summary

This chapter has detailed the important electron and ion transport physics that describes the signal carrier formation process in gas-filled proportional counters. The gas mixtures chosen were argon, carbon dioxide and methane. Once the electric field has been solved inside the micro-pattern detector, the simulation is ready to go. This forms the subject of the next chapter.
Chapter 4

The Electric Field Characterisation

4.1 Introduction

There are two particular situations where the value of the electric field needs to be calculated numerically. The initial value of the electric field inside a micro-strip or micro-gap detector needs to be evaluated. This then needs to be updated to reflect the charging up of the gas volume as the detector is irradiated. This chapter discusses how these calculations were performed.

4.2 The Poisson Equation

Poisson's equation is used to calculate the values of the potential inside the detector. This has the form

\[ \nabla^2 \phi = -\frac{\rho_c}{\varepsilon \varepsilon_0} \]  

Here \( \phi \) is the potential, the free charge density is \( \rho_c \), the relative permittivity is \( \varepsilon \) and the permittivity of free space is \( \varepsilon_0 \). If the detector is subject to a zero irradiation rate then \( \rho_c \) is 0. This is now the Laplace Equation.

4.3 Establishing the problem

The aim is to put a mesh over some subset of the detector space. \( \phi \) can then be evaluated at each mesh point. In the ideal case the mesh would extend over the whole active volume of the detector. This would mean that detector edge effects could be incorporated into the solution.

The problem with this is simply one of feasibility. The greater the number of mesh points that are used, the longer the calculation takes. Two choices present themselves. The mesh might be placed over a large volume, but then it would be fairly coarse. It will
be seen that this would negatively affect the accuracy of the solution. The other option is to concentrate on a small portion of the detector volume and then use a much finer mesh. This latter option was the one chosen. A micro-pattern detector consists of a structure that periodically repeats itself (see Figures 1.7, 1.9). It is argued that far away from the edges of the detector the electric field solution will itself be periodic (ignoring one-off detector blemishes such as anode imperfections). Therefore the decision was made to concentrate on one ‘period’ of the detector.

Furthermore, far away from the edges of the detector there will be no electric field along a line parallel to the strips. This removes a dimension from the problem. This assumption is similar to the one made in cylindrical single wire counters when Equation 1.1 is used to calculate the electric field. Therefore the electric field solution is calculated over two dimensions, even though the Monte Carlo electron transport is itself performed over three.

4.3.1 The micro-gap detector

![Figure 4.1 The area over which the mesh is placed in the micro-gap detector. Please note that the actual mesh spacing used in the calculations was not uniform, but was more finely grained close to the anode.](image)
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In Figure 4.1 one period of the detector would be obtained by reflecting the picture in the line AB. It is actually not necessary to solve the electric field over a whole period, because the electric field solution obtained over the mesh area can be reflected in either of the dotted lines to obtain the full solution.

On mesh points that correspond to electrodes, the potential is fixed throughout the calculation at the appropriate values (Dirichlet boundary conditions). On mesh points that lie on lines of symmetry of the potential (given by the dotted lines in Figure 4.1) the potentials at corresponding mesh points on either side of the symmetry line are set equal.

On mesh points that lie on the interface between the insulator surface and the gas, the boundary conditions of Equation 4.2 must hold.

\[
\begin{align*}
\varepsilon_2 E_{n2} - \varepsilon_1 E_{n1} &= \sigma_c / \varepsilon_0 \\
E_{p1} &= E_{p2}
\end{align*}
\]

Here the media either side of the boundary are labelled 1 and 2. The electric field is \(E\). The component of \(E\) normal to the interface is labelled \(n\) and the parallel component is labelled \(p\). The surface charge density is \(\sigma_c\). \(\varepsilon_1\) and \(\varepsilon_2\) are the relative permittivities of the respective media. Equation 4.2a follows directly from Gauss’ law and Equation 4.2b must be true if energy is to be conserved [Ble 89].
4.3.2 The micro-strip detector

The situation is similar to that for the micro-gap detector. Only half of a detector period is considered for the calculation of the potential.

4.4 Arranging the mesh

The dimensions and operating parameters of a typical micro-gap device are given in Table 4.1.

In a micro-pattern detector, the electric field gradient changes from near zero in the drift region of the detector to a very high value near the anode surface. In order to maintain a good degree of accuracy in the solution throughout the detector volume, the mesh must be finer-grained in the avalanche region of the detector than in the drift region.
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<table>
<thead>
<tr>
<th>detector property</th>
<th>dimension</th>
</tr>
</thead>
<tbody>
<tr>
<td>detector drift region</td>
<td>5 mm</td>
</tr>
<tr>
<td>insulator thickness</td>
<td>10 μm</td>
</tr>
<tr>
<td>anode thickness</td>
<td>0.2 μm</td>
</tr>
<tr>
<td>anode width</td>
<td>10 μm</td>
</tr>
<tr>
<td>insulator width</td>
<td>20 μm</td>
</tr>
<tr>
<td>anode-anode pitch</td>
<td>200 μm</td>
</tr>
<tr>
<td>drift cathode voltage</td>
<td>-2000 V</td>
</tr>
<tr>
<td>anode voltage</td>
<td>0 V</td>
</tr>
<tr>
<td>cathode voltage</td>
<td>-200 V</td>
</tr>
</tbody>
</table>

Table 4.1 Typical dimensions and operating parameters of a micro-gap detector.

4.4.1 Setting up the equations

The equations that need to be solved will be set up by a technique known as the finite difference method [Pre 96]. For a mesh with a uniform spacing in each dimension the standard 5-point difference equation can be set up.

In Figure 4.3, each of the dots represent mesh points that can be identified by the appropriate number. The potentials associated with each of the grid points are connected with the charge density \( p_i \) on point 1 by the relationship described in Equation 4.3.

\[
\frac{\phi_2 + \phi_3 - 2\phi_1}{\Delta_x^2} + \frac{\phi_4 + \phi_5 - 2\phi_1}{\Delta_y^2} = -\frac{p_1}{\varepsilon_0}. \tag{4.3}
\]

If the left-hand side of the above equation is Taylor expanded about point 1 then

\[
\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{1}{12} \left[ \left( \frac{\partial^4 \phi}{\partial x^4} \right) \Delta_x^4 + \left( \frac{\partial^4 \phi}{\partial y^4} \right) \Delta_y^4 \right] + \text{higher order terms} \ldots \tag{4.4}
\]

is obtained. Comparing this with Equation 4.1 it can be seen that the use of Equation 4.3
Figure 4.3 The 5-point finite difference formulation. Here the spacing between points 1 and 2 is the same as that between points 1 and 3. The spacing between 1 and 5 is equal to that between 1 and 4.

...to approximate Poisson's equation is going to be in error. The error will be by an amount that is determined by all but the first two terms of the expansion.

There are two points that might be noted about this error. The first is that the primary error term in Equation 4.4 is proportional to $\Delta^2$. This means that the finite difference scheme described by Equation 4.2 is of order 2. Clearly the smaller the grid spacing the smaller the error term will be. The second point relates to the gradient term associated with the error. In a detector, this means that when there is a rapidly changing electric field gradient the error will be greater, for a given value of $\Delta$. In order to counter this effect a smaller $\Delta$ needs to be used.

If the mesh has an unequal spacing between points then the situation becomes slightly more involved. In the following discussion the y co-ordinate will be omitted for it is assumed that it can be freely interchanged with the x co-ordinate.

The problem with trying to use a non-uniform grid with a 5-point difference formulation is that the term that is proportional to $d^3\phi/dx^3$ does not cancel out. This means that the first error term is proportional to $\Delta$. The method is therefore of order 1. In order to get back to order 2 an extra point needs to be added to the grid.
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Figure 4.4 In this instance the mesh is more finely spaced at decreasing x.

The correct formulation to restore order-2 accuracy now becomes

\[
\left( \frac{\partial^2 \phi}{\partial x^2} \right)_0 = \phi_0 \left( -1 - \beta - \gamma \right) + \phi_1 + \beta \phi_{-1} + \gamma \phi_{-2},
\]

\[
= \frac{1}{2} \left[ a^2 + \beta b^2 + \gamma c^2 \right],
\]

where

\[
\beta = \frac{a}{b} \left( c^2 - a^2 \right), \quad \gamma = \frac{a}{c} \left( a^2 - b^2 \right).
\]

This uses a 7-point formula in 2 dimensions. It applies to all points that do not lie on boundaries of the problem.

At the interface between the insulator surface and the gas a different formula is used. Referring to Equation 4.2a, the relationship that needs to be satisfied is given by

\[
\varepsilon_1 \left( \frac{\partial \phi}{\partial x} \right)_1 - \varepsilon_2 \left( \frac{\partial \phi}{\partial x} \right)_2 = -\sigma_c / \varepsilon_0.
\]

Figure 4.5 may help to explain the situation.
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Figure 4.5 Representation of the interface between the insulator surface and the gas.

The electric fields are evaluated just inside their respective boundaries. In order to retain second order accuracy the following evaluation is performed.

\[
\frac{\varepsilon_1}{\varepsilon_2} \left( \frac{1 + \lambda_1}{a + \lambda_1 b} + \frac{1 + \lambda_2}{c + \lambda_2 d} \right) \phi_0 + \left[ \frac{\varepsilon_1}{\varepsilon_2} - 1 \right] \phi_{-1} + \left[ \frac{\varepsilon_1}{\varepsilon_2} \right] \phi_{-2} + \left[ \frac{\varepsilon_1}{\varepsilon_2} \right] \phi_1 + \left[ \frac{\varepsilon_1}{\varepsilon_2} \right] \phi_2 = \frac{-\sigma_0}{\varepsilon_2 \varepsilon_0}
\]

\[
\lambda_1 = -\frac{a^2}{b^2}, \quad \lambda_2 = -\frac{c^2}{d^2}.
\]

Figure 4.6 illustrates what happens on boundaries that correspond to lines of symmetry of the problem (the dotted lines of Figures 4.1 and 4.2). Outside the boundary is placed an additional ‘ghost’ point 1, which lies the same distance from the boundary as the point -1, and is assigned the same value as the potential at point -1. Equation 4.5 can thus be recast into a form that only depends upon potentials for points that lie inside the mesh.
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4.4.2 The Mesh Spacing

In regions of the mesh where the spacing between consecutive points was non-uniform, it was decided to use a geometric progression to relate the distances between successive points.

\[ a + ar + ar^2 + ar^3 + \ldots + ar^{n-1} = \frac{a(r^n - 1)}{r - 1}. \]  \hspace{1cm} (4.10)

\( a \) is the first term in the series, \( r \) is the multiplier and \( n \) is the number of terms in the series. The sum of the series corresponds to the total distance that the particular portion of the mesh is desired to span.

It might be desired to arrange such a mesh to span the whole drift region of the detector. Applying this to data from Table 4.1, it is found that the total drift distance that must be spanned is 5000 µm. This means that if the value of \( a \) is 0.05 µm (1/4 of the total anode thickness) then 47 points are required (if the value of \( r \) chosen is ~1.23). This can be contrasted with the 10⁵ points that would be necessary if a uniform spacing of 0.05 µm was used.

For the simulations, the value of \( r \) was chosen to be fairly close to 1 (actually it was in the range \( 1.1 < r < 1.3 \)). If the value of \( r \) is too large then the intervals become too large too quickly. This would mean that the error term of Equation 4.4 would become more
significant. If the value of $r$ is too close to 1 then the number of points that are needed to span the grid increase dramatically.

In the simulation itself the gaseous region of the detector is divided into a number of quadrilateral regions. Each electron carries around with it a number known as its region number. The electron is located according to the algorithm used to slice up that particular region. There are just 2 choices: either a geometric progression or (in effect the special case of a geometric progression with the value of $r$ equal to 1.0) an equally spaced grid. The colours that mark the line boundaries of the quadrilaterals of Figure 4.7 indicate the algorithm used to section that portion of the mesh along that particular direction.

![Key](image.png)

**Micro-gap Detector**

**Micro-strip Detector**

Figure 4.7 Meshing scheme for the micro-strip and micro-gap detectors.
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4.5 Solving the Equations

Using the finite difference method turns the partial differential equation represented by Equation 4.1 into a set of simultaneous equations, where each of the unknown variables is the potential on a grid point. This might be written in matrix form.

\[ A \mathbf{x} = \mathbf{b} \]  

(4.11)

In Equation 4.11 $A$ represents the finite difference matrix, $\mathbf{x}$ represents the potential vector and $\mathbf{b}$ represents the charge density (surface or volume, depending on where the location associated with that potential is).

A lot of work has been done over the years to enable solutions to equations of the type given by Equation 4.11 to be solved numerically. Press [Pre 96] gives a review of a selection of possible candidate algorithms.

Two such methods described by Press were used at different times through the course of this work. The first was a relaxation method known as Successive Overrelaxation. It was found that it took over an hour to solve a typical matrix on a machine powered by a 200 MHz Pentium CPU. This was fine for obtaining the initial value of the electric field inside the micro-pattern detectors. However it was decided that this was going to be far too slow to meet the demands of the space charge simulations of Chapter 7, where the electric field needed to be recalculated thousands of times during a simulation.

A method known as LU Decomposition was found to be much more effective. The solution is performed in two stages. Firstly $A$ is 'decomposed' into two matrices that are upper triangular and lower triangular in form. The product of these two matrices will return $A$. This decomposition is quite computationally intensive. It was found to take a few minutes to apply the algorithm to a typical matrix of size 1000 x 1000 on a 300 MHz Pentium desktop machine. However the key point is that during an electron transport simulation $A$ does not change. $A$ is merely dependent on how the mesh is set up. For example, in Equation 4.5 its elements are just the coefficients associated with the potentials. Prior to running one of these simulation $A$ can be processed to produce its decomposed form. This can then be read into the Monte Carlo simulation.
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This effort is worthwhile. It is mathematically trivial to solve a set of simultaneous equations that can be represented in upper-diagonal or lower-diagonal form. In the simulation, once the right hand side of Equation 4.11 becomes available, it takes under a second to arrive at a solution.

4.5.1 Checking the solution

Fortunately it is possible to check whether a given solution is reasonable. The mesh potentials can be inserted back into the finite difference form of Equation 4.1. A vector called the residual \( r_s \) can be defined.

\[
A x_s - b = r_s. \quad (4.12)
\]

Here \( x_s \) is a proposed solution to Equation 4.11, which is the matrix representation of a finite difference form of Equation 4.1.

Each element \( r_i \) of the residual vector can be expressed in the following way.

\[
r_i = \sum_j a_{ij} \phi_j - b_i. \quad (4.13)
\]

Here \( a_{ij} \) are matrix coefficients, \( b_i \) are charge densities associated with each of the grid points and the \( \phi \) values represent the solution at these grid points.

If all of the terms of the same sign on the right hand side of Equation 4.13 are added together, the sum \( \Sigma_r \) can be used to define the normalised residual \( r_n \).

\[
r_n = \frac{r_i}{\Sigma_r}. \quad (4.14)
\]

A smaller value of \( r_n \) indicates a better quality of solution. The minimum value of \( r_n \) depends on the precision to which the numbers are held on the computer. Figure 4.8 shows a plot of the variation of \( r_n \) over the detector mesh for both a micro-gap and a micro-strip detector. In this particular case the computer used to generate the data guaranteed at most 15 significant figures of accuracy for the floating point numbers used in the calculations.
Chapter 4: The Electric Field Characterisation

**Micro-gap detector**

![Graph showing normalised residual for micro-gap detector]

**Micro-strip detector**

![Graph showing normalised residual for micro-strip detector]

Figure 4.8 The normalised residual $r_n$ for micro-gap and micro-strip detectors. The $x$ and $y$ directions correspond to those displayed in Figure 4.1 and Figure 4.2 respectively.

It must be emphasised that this merely proves that the finite difference form of Equation 4.1 has been solved correctly. It does not account for how well the finite difference form approximates the true partial differential equation.
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According to Equation 4.4, making the mesh smaller should result in a solution that more closely matches the true solution to Equation 4.1. In other words, the solutions obtained on successively smaller meshes should eventually converge on the true solution.

The smallest feature size in a micro-gap detector is the anode thickness. The standard mesh used to solve the detector in this work used four intervals in the y direction to cover the anode. In order to see whether this was satisfactory, this mesh size was halved in a one-off experiment. This effectively meant that the mesh spacing was half its usual size just above the anode. The value of the electric field was then measured along the dotted line of Figure 4.9.

![Figure 4.9 The electric field values shown in Figure 4.10 were measured along the dotted line.](image)

The electric field just above the anode was then measured for each of these two mesh stencils. The results are shown in Figure 4.10. The fractional difference at the anode surface is about 1 part in $10^4$ between the two cases. The coarser mesh was thus assumed to be suitable for use in the simulations.

4.6 Examining the electric field solutions

This section looks at actual solutions for the micro-gap and micro-strip solutions calculated with the code described here. A comparison is made with cylindrical single wire counters wherever it is deemed to be appropriate.
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Figure 4.10 A comparison of the electric field solution on a fine mesh and a coarser mesh for a micro-gap detector. Here the anode potential is 0 V, the cathode potential is -228 V, the drift cathode potential is -2000 V, the anode-anode pitch is 300 μm and the thickness of the insulator strip is 10 μm. The anode thickness is 0.2 μm.

4.6.1 The micro-gap detector

Figure 4.11 shows the effect of changing the thickness of the insulator surface on the electric field above the anode surface. 10 μm, 5 μm and 1 μm thick insulators were considered. The parameters used were the same as described in Figure 4.10, except that the cathode voltage was -250 V.
electric field above the anode (measured along the dotted line of Figure 4.9). This result is entirely expected, because it is the proximity of the anode to the cathode that generates such a high value of the electric field.

It is interesting to see how the electric field varies with distance from the anode, when compared to a cylindrical proportional counter. The counter parameters are chosen such that the electric field and the electric field gradient at the anode are equal in both cases. This result is shown for a counter with a 1 µm thick slab in Figure 4.12.

Figure 4.12 The 1 µm thick insulator micro-gap from Figure 4.11 is shown in Figure 4.12. This time it is compared to the electric field in a single wire counter where the electric field and the electric field gradient at the anode surface have been fixed to the same value as that at the micro-strip anode surface.

The wire radius that gives rise to the electric field shown in Figure 4.12 for the single wire counter is 38.8 µm. If that counter is taken to have a cathode with a 2.5 cm radius, then the potential of the anode relative to the cathode would need to be 6880 V. This compares with a potential difference of 250 V between the anode and cathode of the micro-gap counter. Here the potential difference between the anode and the drift cathode is 2000 V. The ability to generate electric fields of large magnitude whilst using smaller potentials is an advantage of the micro-gap detector.
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Figure 4.13 shows the effect of varying the anode-anode pitch of the micro-gap detector. This time the thickness of the insulator surface was held fixed at 10 µm. Otherwise all of the parameters except for the anode pitch were held at the values described in the caption associated with Figure 4.11. Again the electric field is measured along the dotted line of Figure 4.9.

![Graph showing electric field strength vs. distance above anode](image)

**Figure 4.13** An analysis of the effect of changing the anode-anode pitch on the electric field strength inside the micro-gap detector.

As Figure 4.13 shows there is a gradual decrease in the electric field near to the anode surface as the pitch is decreased.

Figure 4.14 is a diagram of the two-dimensional variation of the electric field of the detector close to the anode surface.

![Diagram of electric field around micro-gap detector](image)

**Figure 4.14** The electric field around the micro-gap detector. The detector characteristics are the same as Figure 4.10. The key to the left of the plot gives the electric field in V µm⁻¹.
The white spot in the left diagram of Figure 4.14 corresponds to the electric field at the anode edge.

### 4.6.2 The micro-strip detector

Figure 4.15 shows the electric field above the anode in a micro-strip detector. It also shows the electric field for a single wire counter. In common with Figure 4.12 the electric field and the electric field gradient have been fixed at the same values for the two counters.

![Graph showing electric field vs. distance above anode](image)

Figure 4.15 The micro-strip detector used for this analysis had a drift cathode voltage of $-2000$ V, an anode voltage of $0$ V and a cathode voltage of $-250$ V. The back plane voltage supporting the insulator was held at $0$ V. The anode width was $10 \, \mu$m, the cathode width was $90 \, \mu$m and the anode thickness was $0.3 \, \mu$m. The detector consisted of a $700 \, \mu$m thick insulator substrate. The drift cathode plane was separated from the cathode plane by $5000 \, \mu$m. The anode-anode pitch was $400 \, \mu$m.

The rapid fall in the value of the electric field above the anode surface is again evident for the micro-pattern detector.

This time, if the single wire counter was chosen to have a cathode radius of $2.5$ cm then the anode radius would have to be $75 \, \mu$m. The anode potential would need to be $3530$ V relative to the cathode.

Figure 4.16 shows the two-dimensional variation of the electric field inside the micro-strip detector for the detector set-up described in the caption associated with Figure 4.15.
Figure 4.16 Two dimensional variation of the electric field in the micro-strip detector. The electric field in the key is measured in V $\mu$m$^{-1}$.

4.7 Interpolating the electric field

It has been demonstrated that the electric field inside a micro-pattern detector can be solved on a mesh. The field is stored at each of the mesh points. However the electric field needs to be calculated at an arbitrary position within the mesh to facilitate proper electron transport.

There are two stages to this. The electron first needs to be located. An advantage of choosing a geometric progression for the mesh intervals described in Section 4.4.2 is that the electron is easily locatable. If $d_x$ is the total distance along a particular direction from the point that represents the start of the series, then the number of intervals $n$ that $d_x$ contains is given by

$$n = \frac{\log_e \left( \frac{d_x (r-1)+1}{a} \right)}{\log_e (r)}.$$  \hspace{1cm} (4.15)

Here $r$ is the multiplication factor of the geometric progression and $a$ is the size of the first interval in the series.
Chapter 4: The Electric Field Characterisation

The next task is to allocate the appropriate value of the electric field to the electron. The method of area weighting was used [Bir 91]. This can be illustrated with the help of Figure 4.17.

![Figure 4.17 The area weighting strategy. The area of each of the quadrilaterals bounded by the dotted lines is given by the lower case letter that is associated with it.](image)

The electron is represented by the red cross of Figure 4.17. The value of the electric field where the electron is will be determined by some combination of the values at each of the mesh points represented by capital letters. The weighting factor associated with the point A is given by

\[
\omega_A = \frac{d}{(a + b + c + d)}. \tag{4.16}
\]

The interpolated electric field \( E_{int} \) is then given by

\[
E_{int} = \sum_{i=1}^{4} w_i E_i. \tag{4.17}
\]

Here \( E_i \) is the electric field associated with the grid point \( i \) and \( w_i \) is the appropriate weighting factor.

The inverse of this procedure will occur when the charge generated in the avalanche is injected onto the mesh. The charge \( C_i \) associated with the grid point \( i \) due to a given ion will be given by

\[
C_i = q \ast w_i. \tag{4.18}
\]

In Equation 4.18 \( q \) is the charge that is to be inserted into the grid.
4.8 Summary

A Poisson solver program has been created to obtain the electric field inside a micro-strip and micro-gap detector. The solution is obtained on a series of points defined by a mesh that is spread over one period of the detector. Although a relaxation method was used to initially solve the electric field, the method of LU decomposition was used in the final analysis. This was because in order to track the effect of space charge, the field would have to be recalculated many times during a simulation. LU decomposition was able to solve the electric field in under a second on a 200 MHz Pentium machine.

It is the tight coupling between the Monte Carlo electron transport simulation and the electric field solver that was a major motivation for writing the code whose effects were described in this chapter. It is unlikely that an off-the-shelf commercial package would have integrated into the Monte Carlo code as smoothly. It is likely that this would have implied an additional time penalty each time the electric field was solved.
Chapter 5

Testing the Simulation Code

5.1 Introduction

Chapters 2, 3 and 4 have detailed the construction of the simulation. This chapter reveals how the simulations perform in a wide range of circumstances. Swarm parameters are measured, both for electric field strengths that correspond to the drift region and the avalanche region of a detector. Then the single-electron-induced mean avalanche gain $M$ measured in single wire counters by simulation is compared with literature values. Finally, data for $M$ that were generated in micro-pattern counters manufactured in the Physics Department at Surrey University are compared with calculated results.

5.2 The Swarm Parameters in the Drift Region

In Section 3.4.3 the Boltzmann transport equation was introduced (Equation 3.10). The idea of measuring swarm parameters that characterise the transport of the electrons through the gas was introduced. There are three swarm parameters that are of interest in this work. Two of them will be introduced in this section. These are of particular interest in the drift region of a counter. The third will be introduced in a later section, and is connected with electric field strengths that are large enough to promote avalanching. The swarm parameters are measured in a constant electric field after the hydrodynamic regime has been established.

5.2.1 The hydrodynamic regime

The velocity distribution function $f$ was introduced in Equation 3.11. The actual swarm parameters are measured when the function $f'$ loses its time dependence. $f'$ is given by

$$f'(v,t) = \frac{1}{n_0} \int n(r,t)f(r,v,t)dr,$$  \hspace{1cm} (5.1)
where \( n_0 \) is the total number of electrons in the group at time \( t \). \( f' \) is effectively the velocity distribution function averaged over the whole group of electrons. \( f' \) will lose its time dependence as \( t \to \infty \). When this happens it will become solely a function of the velocity \( v \). It is at this point that the hydrodynamic regime is established.

Now the energy losses in collisions with the gas atoms are balanced by energy gains from the electric field. The swarm becomes characterised by a time-independent value of \( f' \). Whatever conditions existed when the swarm was initiated become irrelevant at this stage. Once \( f' \) has been fixed for a given strength of the electric field then the swarm parameters can be measured.

### 5.2.2 Definitions of the swarm parameters for drift fields

In the following analysis it will be assumed that the electric field is aligned with the \( z \) axis.

**Drift Velocity: \( v_d \)**

\[
v_d = \frac{d\bar{z}}{dt}.
\] (5.2)

\( \bar{z} \) is the mean position of an electron at time \( t \).

**Transverse Diffusion Coefficient: \( D_T \)**

\[
D_T = \frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{2nt}.
\] (5.3)

Imagine that an experiment is conducted where \( n \) electrons, each with phase space parameters sampled from \( f' \), are released independently from the same point in space. After time \( t \) their position along the \( x \) axis is measured. This is the value \( x_i \). \( \bar{x} \) is the mean \( x \) co-ordinate of the ensemble at this time. In this analysis \( y \) could be substituted for \( x \). All that matters is the direction represented by the co-ordinate is *perpendicular* to the electric field.
5.2.3 Simulation Protocol

In the simulations an electron was released from a specified point in a random direction and with an energy that lay between 0 and 1 eV. The electron was then tracked for a specified time period. At various equally spaced time intervals its position and velocity were noted. This procedure was repeated until 1000 electrons had been drifted. \( v_d \) and \( D_T \) were then calculated using Equations 5.2 and 5.3. The simulation was repeated and the quoted measurement was taken to be the mean of these two measurements.

The quoted uncertainty was taken to be half of the difference between the two measurements. It was important to choose a total time that was long enough to ensure that all memory of the boundary conditions had been lost and the hydrodynamic regime was indeed being probed. If any of the measured quantities varied by more than 5\% over 3 equally spaced time intervals then the run was rejected and the measurement repeated over a longer time span. As an additional check, one simulation for each of the data sets presented here was conducted with the initial energy range of the electron expanded to between 0 and 5 eV. The differences were less than 1 \%, which was of the order of the observed experimental uncertainty. In this way it was ensured that the value of \( f^* \) being probed was its final, time-independent one.

5.2.4 The Reduced Field

Instead of quoting the swarm parameters as a function of electric field, they are typically quoted as a function of the reduced electric field, \( S \). \( S \) is defined by Equation 5.4.

\[
S = \frac{E}{N}.
\]  

(5.4)

\( E \) is the value of the electric field strength and \( N \) is the number density of the gas. If \( E \) is varied in proportion to \( N \), then the increase in the energy gain per unit distance is offset by the decrease in the average distance between collisions. This means that \( f^* \) remains the same. Therefore any variation in the swarm parameters when they are plotted as a function of the reduced electric field is independent of \( f^* \). A special unit has been defined
for $S$. It is called the Townsend (Td). A reduced electric field of 1 Td is equivalent to $10^{-17}$ V cm$^2$.

### 5.2.5 Drift parameters: argon

The literature data that is used for the comparison of the drift velocity and the diffusion coefficients is referenced in the work of Makabe [Mak 86]. The drift velocity can be written in terms of $f^*$, yielding

$$v_d = \int v_z f^*(v) \, dv.$$  \hspace{1cm} (5.5)

Figure 5.1 shows a plot of $v_d$ against $S$ for a range of $S$ values.

Bordage has noted that the drift velocity is particularly sensitive to the energy loss ascribed to various collisions [Bor 96]. Addition of 0.5% of methane to a pure argon mixture can increase the measured drift velocity by a factor of 10 at 1 Td.

Robson [Rob 84] has found that the drift velocity can be written as

$$v_z = \left( \frac{eE}{N \, m \, C_r(v)} \frac{1}{\Lambda^2} \right)^{\frac{1}{2}},$$  \hspace{1cm} (5.6)

Figure 5.1 The drift velocity is plotted as a function of the reduced electric field for argon gas for both the simulation and results obtained from the paper by Makabe [Mak 86].
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where

$$\Lambda = \frac{1}{C_T} \left( \frac{2m}{M_g} \bar{\varepsilon} C_{el} + \sum_k \varepsilon_k C_k(\bar{\varepsilon}) \right).$$

Here $\bar{\varepsilon}$ is the mean energy of the drifting electrons, $C_T$ is the total collision frequency and $C_{el}$ is the elastic scatter collision frequency. $M_g$ is the mass of the gas atom. The first term in Equation 5.7 is the mean energy loss in elastic collisions between electrons and gas molecules and the summation in Equation 5.7 is over all inelastic processes $k$. $\Lambda$ is therefore the mean energy loss in a collision.

The mean energy of drift electrons at 1 Td is 0.788 eV in argon (the ionisation threshold lies at about 15 Td). There is no inelastic channel through which an electron can lose energy in argon gas at these energies. If methane is added to the gas then there is the additional possibility of a relatively large energy loss of 0.175 eV or 0.367 eV due to an inelastic collision. For the case of a 0.5% addition of methane, even though $C_k$ is about 3 orders of magnitude lower than $C_{el}$, the energy loss in an inelastic encounter is 5 orders of magnitude greater than for an elastic collision. Given the strong dependence of $v_i$ on $\Lambda$ this explains why a much larger drift velocity is obtained.

The reason why the drift velocity tends to increase when the mean energy loss per collision increases might be explained in the following way. If the energy after the collision is very much less than the energy before the collision then it stops the electron near dead. The electron is then pushed in the direction of the force due to the electric field after such a collision, thereby enhancing the drift velocity, which is measured in the direction of the electric field. If the energy loss per collision is decreased, then after a collision an electron will retain most of its energy. Its velocity will be more randomly directed than before.

Figure 5.2 shows $ND_T$ as a function of $S$ for pure argon gas. Huxley has derived a value for the diffusion coefficient $D$ for a situation where elastic scattering makes the only contribution to the electron transport [Hux 74], which will help explain why the product of the diffusion coefficient and the gas number density is plotted against $S$. 

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Here $f'(E)$ has been recast as a function of energy and $\sigma_{el}(E)$ represents the elastic scattering cross section. It can be seen from Equation 5.8 that if $f'$ is fixed (by choosing a constant value of $S$) then $ND$ is a constant.

\[ ND = \left( \frac{2}{9m} \right)^{\frac{1}{2}} \int E f'(E) \frac{dE}{\sigma_{el}(E)}. \] (5.8)

It is noted that there is a tendency in the simulation to underestimate $D_T$ compared to the literature data, although the trends are reproduced.

One possible reason for the discrepancy is the simple fact that isotropic scattering was used when a collision was modelled. It would be interesting to redo the calculations, substituting more realistic probability distributions for the scattering. However, it is argued that Figures 5.1 and 5.2 indicate that the simulations produce reasonable data for the drift of electrons in argon.

5.2.6 Drift parameters: methane

Davies [Dav 89] collected the literature data for methane that was used in the comparison described here.

Figure 5.3 shows the drift velocity for methane and Figure 5.4 shows the transverse diffusion coefficient, both plotted as a function of $S$. 

\[ \text{Figure 5.2 The transverse diffusion coefficient is plotted as a function of reduced electric field for argon gas for both simulation and literature data [Mak 86].} \]
It will be noted that the drift velocity drops before it increases in Figure 5.3. If the collision frequency rises as the electron mean energy $\bar{\epsilon}$ is increased (as the electric field strength is increased), then it is possible for the drift velocity to fall. This is described in Equation 5.8. The avalanche electric field threshold of methane lies at about 80 Td.
5.3 The Swarm Parameters in the Avalanche Region

So far the ionisation cross section has not been introduced into the proceedings. This quantity is found to be of crucial importance when measuring the Townsend first ionisation coefficient $\alpha_T$.

Townsend observed that if an avalanche was induced via the photoelectric effect between two parallel plates separated by a distance $d_p$ where there is a constant electric field between them then the current $i_e$ induced by electrons on the plate is given by

$$i_e = \frac{i_0 \exp(\alpha_T d_p)}{1 - \gamma_T \left[\exp(\alpha_T d_p) - 1\right]}.$$  \hspace{1cm} (5.9)

Here $i_0$ is the current observed in the absence of an electric field and $\gamma_T$ is the second Townsend coefficient. When $\gamma_T$ is equal to 0 this expression reduces to a simple exponential form. This corresponds to a situation where the sole contribution to the avalanche is directly from electron-impact ionisation collisions. $\gamma_T$ gains significance as the magnitude of the reduced electric field increases in strength. $\gamma_T$ relates to secondary processes that can indirectly lead to an increase in electron avalanche number.

For example, suppose that the work function $\phi$ of the metal that acts as the cathode in the experimental apparatus has an energy value that is less than that of a particular excitation event in the gas. Photons that are produced in such excitation events can cause additional electrons to be produced at the cathode. Other secondary effects have already been discussed in Section 1.6.

Since the simulation does not take account of these secondary effects, any gain measurement will lead to a measurement of $\alpha_T$. If $\gamma_T$ is put equal to zero in the above equation, and it is assumed that current is directly proportional to the gain at the anode, then it can easily be seen that a plot of $\log_e(i_e)$ against $d_p$ will have a gradient equal to $\alpha_T$. $i_e$ will be proportional to the avalanche gain $G_c$ induced by a single electron. $G_c$ is the quantity that is actually measured in the simulations.
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The simulations were actually conducted in a slightly different manner to the Townsend experiment. Instead of setting up a number of simulations where the number of electrons traversing the gap $d_p$ are counted, the total distance covered by the simulation parallel to the direction of the electric field was split into 50 intervals. Each time an electron drifted past the planar boundary containing an interval, the value of $G_e$ was incremented by one. Figure 5.5 helps to illustrate this situation.

![Electron track](image)

**Figure 5.5** The gain $G_e$ is tallied at each of the intervals signified by the red dotted lines.

A single electron-induced avalanche was run 2000 times. The first time that an electron passed through an interval it was counted as part of the gain. If it then scattered in the opposite direction to the force due to the electric field, it was not counted twice.

The significance of $\alpha_T$ can be illustrated by considering how it relates to the electron ionisation cross section and electron velocity probability distribution. Consider an electron incident on an infinitesimally thin slice of gas along the field direction, as shown in Figure 5.6.

$dt$ is the time that an electron will take to drift across an infinitesimal slice of the slab of thickness $dz$. $dt$ is equal to $dz / v_d$. In the time $dt$ the electron undergoes $C_I dt$ ionising collisions, where $C_I$ is the mean ionisation frequency. Each of the newly generated electrons will eventually drift out of the slab. Therefore the net gain per electron incident on the slice $dG_e / G_e$ is equal to $C_I dt$. Eliminating $dt$ gives

$$\frac{dG_e}{G_e C_I} = \frac{dr}{v_d} \Rightarrow \frac{1}{G_e} \frac{dG_e}{dr} = \frac{C_I}{v_d} = \alpha_T.$$

(5.10)
In other words $\alpha_T$ is the ratio of the ionising collision frequency to the mean drift velocity. It can be calculated theoretically by noting that

$$\frac{C_I}{v_d} = \frac{N \int_{-\infty}^{\infty} f^*(v) \sigma_j(v) dv}{\int_{-\infty}^{\infty} f^*(v_j) v_z dv},$$

(5.11)

where $\sigma_j$ is the ionisation cross section of the gas, $v_z$ is the $z$-component of the velocity (the electric field direction is aligned with the $z$ axis), $v$ is the electron velocity and $N$ is the number density of particles in the gas.

### 5.3.1 Avalanche swarm parameters: argon

The reduced first Townsend ionisation coefficient $\alpha_T / N$ is plotted in Figure 5.7 as a function of the reduced electric field. The literature data is taken from the paper by Zastawny [Zas 97].
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Figure 5.7 Comparison between $\alpha_f$ measured in the simulation and the same parameter quoted in the literature (actually taken from the work of Zastawny [Zas 971]), for argon gas. The error bars are too small to show up on the plot.

Two sets of data are shown for the simulations. It was mentioned in Section 3.5 that the data of Chutjian was used as a basis for the cross sections [Chu 81]. It can be seen from Figure 3.5 that there is some uncertainty associated with the cross sections for the individual excitation levels.

Two potential schemes for the cross sections were analysed. One used the first 13 lowest lying excitation levels above the ground state of argon described by Chutjian. The second used all of the levels described by Chutjian. It was found that using 13 levels gave a better fit to the literature data (in a least squares sense). Therefore 13 levels were used in the cross section set for argon throughout this work (chronologically speaking, this was the first simulation that was performed out of all of the results described here).

It is argued that this is a legitimate approach for two reasons. The first is due to the uncertainty associated with the cross sections (as was graphically illustrated in Figure 3.5). The second reason is that Chutjian only published 23 excitation cross sections. There may be other cross sections that hold a similar importance to the ones described here, so even use of the full set would be approximate. Unfortunately other data that adequately describes the electron-impact physics in the low energy region (sub-50 eV) appears to be quite thin on the ground.
5.3.2 Avalanche swarm parameters: methane

Figure 5.8 shows $\alpha T / N$ plotted as a function of the reduced electric field. As before, the literature data is taken from the paper by Zastawny [Zas 97].

![Graph showing $\alpha T / N$ as a function of reduced electric field]

Figure 5.8 Comparison between simulation and literature for the quantity $\alpha T / N$ measured in methane gas.

5.4 The Cylindrical Single Wire Proportional Counter

Charles [Cha 72] made some measurements of the mean single-electron-induced avalanche gain $M$ in cylindrical single wire proportional counters filled with a 90:10 ratio of argon to methane and at 1 atmosphere pressure. An account was given of the various sources of error that might occur in an experimental measurement.

The first source stems from gas impurities. These can be minimised by constructing the chamber that holds the gas out of a material that does not lead to a lot of outgassing. The walls of the chamber can adsorb electronegative impurities like oxygen and water vapour when exposed to air. Even when the chamber has been pumped down these substances can remain. They are only released from the walls over a long period of time. Stainless steel is better than aluminium as a container in this respect. Impurities might also be introduced from the glue used to bond the anode wire to its guard ring. A second potential error is due to variations in the wire diameter along its length, which can lead to electric field variations in the gas volume.
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The discussion in the preceding paragraph helps to illustrate a valuable point about simulation work. This is that, even in the best-case scenario, it can only ever give the results that would be obtained in optimal conditions. It may be very difficult to engineer such conditions in practice.

The results are shown in Figure 5.9. Two avalanche sets, each consisting of 2500 electron histories were run in order to obtain each data point. The value of the gain was recorded after each history and two values for the mean were generated. The mean of the two values is plotted, and the uncertainty is quoted as the magnitude of half the difference between the means obtained in each run.

In the simulation the electrons were drifted in from a distance that corresponded to half the distance between the anode and the cathode, until they avalanched and were collected at the anode of the counter. This ensured that any ‘memory’ of their initial energy, which was randomly selected between 0 and 1 eV, was lost by the time that they reached the avalanche region of the detector. In order to double-check this, the 1400 V run was repeated for the 5 µm detector, this time randomly selecting the electron energy from between 0 and 5 eV. The difference was \(-1.5\%\), which was within the bounds of experimental uncertainty.

It can be seen that the simulation gain values slightly underestimate the measured gain. It can be seen from Figures 5.7 and 5.8 that the values of \(\alpha_T\) measured in the simulations show no trend of undercutting the experimental values. This might be due to the presence of additional ionisation channels in the gas mixture that are not present in the pure gas. The ionisation potentials of argon and methane are 15.76 eV and 12.6 eV respectively. It is possible that there are long-lived excited states of argon that lie within
Figure 5.9 Comparison of the gain measured in a cylindrical single wire counter experimentally (solid lines) and by simulation (symbols). The gas mixture is a 90% argon, 10% methane mixture, which was held at 1 atmosphere pressure at 298 K. The cathode radius was 2.845 cm. The anode radii were (from left to right) 5 µm, 12.5 µm, 25 µm, 37.5 µm and 50 µm respectively.

This range of energies. If an atom of argon in such an excited state collides with a methane molecule it can ionise it, contributing an extra electron to the measured gain.

It is worth pointing out the fact that a relatively small error in the effective ionisation cross section can lead to a large error in the measured gain. A rough estimate for $M$ (obtained by neglecting the electric field gradient) can be calculated along a path length from the cathode of the counter to the anode, by using the relation

$$\ln M = \int_{r_a}^{r_c} \alpha_f \, dr.$$  

(5.12)

Here $r_c$ is the radius of the cathode and $r_a$ is the radius of the anode. If there is a 10% error in the ionisation cross section then it can be seen from Equation 5.11 and 5.12 that the fractional error in $M$ will be $M^{0.1}$. Figure 5.10 plots this error as a function of $M$. It can be seen that it is greater than a factor of 2 if the value of $M$ is $10^4$. Therefore a relatively small intrinsic error in the cross section can be magnified, due to the exponential nature of the avalanche process.
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![Graph showing estimated fractional error in the gain as a function of the true gain.](image)

Figure 5.10 The estimated fractional error in the gain is plotted as a function of the true gain. This work assumes that mean error in the effective ionisation cross section is 10%.

5.5 The Micro-strip counter

Martyn Key manufactured some micro-strip detectors at the University of Surrey. These were then tested at one atmosphere in various mixtures of argon and carbon dioxide. A block diagram of the experimental set-up is shown in Figure 5.11.

![Block diagram of the experimental set-up.](image)

Figure 5.11 Schematic of the experimental set-up used to measure the gain of micro-strip counters.

In the experiment (which is detailed in Marytn Key’s doctoral thesis), an EG&G Ortec model 142PC pre-amplifier was used. This was known to have a conversion gain of 1 µV per ion pair collected. So the height of the voltage pulse coming out of the pre-amplifier can be related to the total charge that is collected at the anode of the counter, $C$. The gain
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$M$ is then just given by Equation 1.2. Unfortunately the $W$ values of the argon/carbon dioxide mixtures used were unknown (the $W$ value of pure argon is 26.2 eV [Kno 89]). The value of $W$ for gas mixtures should not be too far from this. The gains in Figures 5.12 are measured using an estimated value for $W$ of 25 eV per electron-ion pair.

The anode of the counter used was 10 $\mu$m wide, the cathode was 90 $\mu$m wide and the anode pitch was 400 $\mu$m. The electrodes were made of silver. The drift plane was arranged 5 mm above a borosilicate glass substrate, whose thickness was 700 $\mu$m. The back plane potential was 0 V and the anode potential was also held at 0. This meant that the cathode was held at a negative bias with respect to the anode. The drift cathode was at $-1000$ V.

![Figure 5.12](image)

Figure 5.12 The simulation gain is compared with the experimental gain for argon – carbon dioxide mixtures in a micro-strip counter. The experiment relates to 1 atmosphere of a mixture of argon and carbon dioxide gas at room temperature. The plot on the left is for a 90:10 argon:carbon dioxide ratio. The plot on the right is for an 80:20 argon:carbon dioxide ratio.

In the experimental arrangement detectors were irradiated with a low intensity x-ray source emitting fluorescence x-rays from a copper source with an energy of 8.05 keV.

The simulations tend to slightly overestimate the gain. It may be that the $W$ value used to calculate the experimental gain values was a bit too low. However it is encouraging that the general trend is accurately reproduced in both cases. The prototype detectors were quite small (2 inch x 2 inch). It is possible that there are slight variations in the electric field due to the finite size of the detector. These edge effects are not simulated.
5.6 Summary

The simulation code has been benchmarked in a variety of situations. These include measuring the drift velocity and diffusion coefficient in constant electric fields for argon and methane, measuring the Townsend first ionisation coefficient in argon and methane, measuring the single-electron-induced mean avalanche gain $M$ in single wire counters filled with argon and methane, and measuring $M$ in micro-strip counters filled with argon and carbon dioxide.

Figure 5.13 shows an actual electron track from a micro-gap simulation. The gain is 40 in this picture. In this instance the electron has drifted in from a distance above the anode plane of 2500 µm.
Chapter 6

Proportional Counter Energy Resolution

6.1 Introduction

It is readily acknowledged that proportional counters do not achieve the level of performance attributed to semiconductor detectors when it comes to making spectroscopic measurements. Table 6.1 shows a comparison between the energy resolution of a hyper-pure germanium detector and that of a micro-strip detector, measured at 8 keV.

<table>
<thead>
<tr>
<th>detector</th>
<th>energy resolution / %</th>
</tr>
</thead>
<tbody>
<tr>
<td>micro-strip</td>
<td>15</td>
</tr>
<tr>
<td>hyper-pure germanium</td>
<td>2</td>
</tr>
</tbody>
</table>

Table 6.1 The energy resolution of a micro-strip counter is compared to that of a hyper-pure germanium detector. The radiation energy corresponds to 8 keV Kα copper fluorescence x-rays. The micro-strip counter was filled with a gas mixture containing 90% argon and 10% carbon dioxide at 1 atmosphere pressure. It is described by Key [Key 99]. The germanium counter data was taken from Knoll [Kno 89].

In this chapter the factors that influence the energy resolution will first be discussed. Simulations that offer a new insight into the factors that affect the energy resolution will be described. These will apply in turn to single wire, micro-gap and micro-strip counters.

6.2 Signal Carrier Number Variation

The whole process of generating the signal inside the gas-filled detector is riddled with factors that can cause the detector output to vary, when a given input stimulus is applied. These will be discussed under separate headings within this section.
6.2.1 The number of electrons in the primary electron cloud

Consider a case where all of the energy of the photon is converted into electron/ion pairs in their ground state configurations, and where the gas molecules are in their ground state configurations. Since the total energy must be conserved, the variance in the number of electron/ion pairs must be equal to zero. This does not happen in the actual ionisation sequence that happens inside a gas counter.

Some of the energy will instead go towards the excitation of gas molecules. Since there are a large number of different ionisation and excitation channels, varying amounts of energy will be deposited in the gas each time a primary x-ray photon is detected. The possible energy dissipation channels determine the probability density function for the number of electron/ion pairs that are produced for a given primary ionising particle initial energy. This is not necessarily the Gaussian shape that characterises many counting experiments.

Gaussian statistics are expected when the experiment is set up in such a way that the quantity being measured is the number of successes from a series of independent identical trials, where there is a fixed probability of success or failure for each trial. When a detector is pointed at a gamma radiation source, each time a radiation particle is emitted there is a fixed probability of it interacting inside the detector. Furthermore, what happens to each particle is independent of all of the others. This is definitely not the case in the drift region of a proportional counter. The electron cloud is produced in an ionisation sequence that is initiated by the primary ionising particle.

The Fano factor \( F_s \) can be defined to take account of this. It is defined in Equation 6.1.

\[
F_s = \frac{s_e}{s_p}. \tag{6.1}
\]

Here \( s_e \) is the variance observed in the number of electrons in the original ionisation cloud and \( s_p \) is the variance that would be expected if Poissonian statistics were applicable to this situation.
6.2.2 Electron removal from the drift region

This is associated with the probability of an electron actually making it to the avalanche region, having been created in the drift region. An electron might drift out of the drift region, or it might attach to a particle in the gas, be it an ionic or neutral particle and thus fail to contribute to the signal. This is specific to the gas itself. The gas can be chosen to minimise the effects of electron attachment, by ensuring that it has a very small attachment coefficient. For argon, methane and carbon dioxide the attachment coefficient is very much lower than the typical electron-impact cross section of $10^{-16}$ cm$^2$. Datskos has produced data that suggests that it is definitely less than $10^{-20}$ cm$^2$ and it may be very much less than this [Dat 92]. There may be a significant contribution to attachment processes due to the presence of electronegative impurities from the atmosphere such as oxygen or water vapour.

6.2.3 Electronics-induced uncertainty

There are contributions here from two distinct sources. The first is from the potential applied by the voltage source, which can drift during operation. The second is from additive electronic noise in the pre-amplifiers and amplifiers that facilitate the readout process.

6.2.4 Avalanche number variation

When an electron enters into the avalanche region, it can induce a varying number of additional electrons to be collected at the anode of the counter. This is due to the fundamentally stochastic nature of the electron-impact physics. It is this phenomenon that will be examined by the simulation code.

6.3 Characteristics of the Avalanche Number Distribution

A form for the single-electron-induced avalanche number distribution $p_a$ has been suggested [Kno 89]. It is described in Equation 6.2 and is known as the Polya function.
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\[ p_a(z) = m_c^{m_c-1} z^{m_c-1} \exp(-m_c z). \]  
(6.2)

Here \( m_c \) is a constant for given counter parameters (namely gas composition, pressure, anode radius, cathode radius and applied voltage). \( z \) is given by

\[ z = \frac{A}{M}, \]  
(6.3)

where \( A \) is the number of electrons collected at the anode in a given avalanche and \( M \) is the mean avalanche number. The value for \( m_c \) usually lies between 1.3 and 1.6.

A quantity can be defined which is called the relative variance \( f_v \), where

\[ f_v = \left( \frac{(A - M)^2}{M^2} \right). \]  
(6.4)

It turns out that the value of \( f_v \) predicted by the Polya distribution obeys the relation

\[ f_v = \frac{1}{M} + \frac{1}{m_c}. \]  
(6.5)

Given a value for \( M \) that is greater than 100 (such as might be used in an operational proportional counter), it is known that the second term in Equation 6.5 will dominate. Therefore \( f_v \approx (1 / m_c) \).

It is noted here that the Fano process and the avalanche broadening process are absolutely intrinsic to the detector itself. If the other processes could somehow be eliminated, then this would give the ultimate resolution of the detector achievable with the given operating conditions. It can then be shown [Kno 89] that the fractional uncertainty in the charge collected at the electrodes is given by

\[ \frac{\sigma_Q}{Q} = \left( \frac{W(F_v + f_v)}{T_x} \right)^{1/2}, \]  
(6.6)

where \( W \) is the W-value of the gas (the mean energy expended to create an electron-ion pair) and \( T_x \) is the energy of the incoming radiation. \( Q \) is the charge collected at the anode of the counter and \( \sigma_Q \) is the standard deviation in this number. For a typical fill gas such
as argon, $F$ has a value of about 0.2, but $1/m_e$ is frequently more than twice this value (it actually depends, as will be discussed more fully later, on the counter geometry as well).

### 6.3.1 Experimental measurement

The pulse height distribution associated with a given avalanche number might be measured experimentally using the apparatus described in Figure 6.1.

![Figure 6.1 Schematic of a possible experimental arrangement for the measurement of the pulse height distribution due to a single-electron-induced avalanche (taken from Carver's experiment [Car 67]).](image)

An ultraviolet photon is only able to produce a single electron-ion pair in the gas. If multiple signal carriers are produced inside the drift region of the counter, then the signal pulse that is actually obtained is integrated over all of the pulses due to the individual avalanches.

### 6.3.2 Previous attempts to assess the avalanche number distribution

Alkhazov made a detailed attempt to characterise the avalanche number distribution from a theoretical perspective [Alk 70]. In this particular treatment, the assumptions made were that recombination, electron attachment to impurities, space charge and photoelectric effects were reasonably small. This is in common with the simulations described here.

Alkhazov approached the problem by suggesting an ad-hoc parameterisation for the distribution $\rho_f(l)$ of the distance $l$ that an electron travels from the point where it is created to the point where it can cause further ionisation, of the form

$$\rho_f(l) = a \exp(-al),$$

where $a$ is a parameter that can be related to $\alpha_T$, the Townsend first ionisation coefficient.
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Alkhazov derived a formula for \( f_v \), the relative variance measured in a single wire proportional counter.

\[
\begin{align*}
  f_v &= \int_{r}^{r_a} \frac{\alpha_1 f_o}{\exp \left[ \int_{r}^{r_a} \alpha_1 dr \right]} \, dr. \\
  f_o &= \frac{(4 \exp(-2\theta) - 4 \exp(-\theta) + 1)}{(4 \exp(-\theta) - 2 \exp(-2\theta) - 1)}.
\end{align*}
\]

Here \( r_c \) is the radius of the cathode, \( r_a \) is the radius of the anode and \( r \) is the distance from the anode. In Equation 6.8 \( f_o \) has the form

In Equation 6.9 \( \theta \) is equal to \( \alpha U_o / E \). \( U_o \) is approximately equal to the ionisation potential of the medium and \( E \) is the value of the electric field strength. It can be seen that \( U_o / E \) is the minimum distance required to cause an ionisation if an electron starts with 0 energy. \( 1 / \alpha_1 \) is the mean free path for an ionisation. \( \theta \) is a measure of the number of ionisation mean free paths that need to be traversed in order to gain enough energy to be able to cause further ionisation.

6.3.3 Relative benefit of using the Monte Carlo technique

There is a specific reason why the use of the simulation technique outlined in this thesis is considered to be of benefit, when compared with the use of Equation 6.8, to calculate \( f_v \). This is that Equation 6.8 is derived by assuming that the mean gain can be calculated using Equation 2.2. This latter expression is, in effect, an ‘extrapolation’ from the uniform field situation and therefore takes no account of the changing electric field gradient inside the counter. Alkhazov was able to arrive at the Polya distribution for the single wire counter through his analysis. It is interesting to see whether the simulation confirms this, whilst making no such assumption. It will also be seen that the avalanche statistics can be probed at intervals along the path of the development of the avalanche, in the simulations.
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6.4 Single Wire Counters

Firstly the degree to which the avalanche distribution function measured in a simulation matches the proposed Polya distribution is assessed. Secondly the code is used to help solve the problem of why counters with thinner wires give a better energy resolution for the same mean gain.

6.4.1 The Polya Distribution

Figure 6.2 illustrates the avalanche number distribution function for electrons collected at the anode of a single wire counter.

![Figure 6.2 The avalanche number distribution for a single wire counter is compared with a Polya distribution (see Equation 6.2) with the same mean avalanche number. The gas mixture was a 90:10 ratio of argon to methane held at 1 atmosphere pressure and at 298 K. The counter anode radius was 5 μm, the cathode radius was 2.5 cm and the anode potential was 1400 V. The Polya function plotted in Equation 6.2 has the same mean avalanche number as in the simulations.]

A Kolmogorov-Smirnov statistical test was run over the data represented in Figure 6.2 [Pre 96]. The Kolmogorov-Smirnov test is a statistical test that applies to continuous or quasi-continuous distributions, just as the Chi-Square test applies to discrete functions. The Kolmogorov-Smirnov statistic $D_K$ is defined as
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\[ D_K = \max \left| S_N(x) - P_N(x) \right|, \]

where \( S_N \) is the cumulative distribution function of the sampled variable \( x \), and \( P_N \) is the cumulative distribution function of the theoretical distribution \( x \) is supposed to be drawn from. From this, the probability \( p_D \) that the observed value of \( D_K \) should be greater than the value of \( D_K \) that is actually obtained can be established. If \( p_D \) is very much less than 0.5 then the distribution obtained and the theoretical distribution are not close. If \( p_D \) is very much greater than 0.5 then the uncertainties are very much smaller than should be expected from the performed experiment.

The results of this test, when it is applied to the Polya distribution, are as follows:

*The value of \( D_k \) is 5.513 x 10^{-3}.*

*The value of \( p_D \) is 0.86.*

This is not outside the range of values that statisticians deem to be an acceptable fit to the Polya model for the data distribution.

6.4.2 A comparison between counters with different anode wire thickness values.

Sakurai found that the energy resolution of xenon-filled proportional counters was somewhat improved for the same mean single-electron-induced gain \( M \) if thinner wires were used for the anode of the counter [Sak 92]. The following explanation is given for this:

Although the energy of electrons in the avalanche region goes into both ionisation and excitation of the gas atoms, at strong reduced electric fields the ionisation will be dominant, while at weak fields the excitation will be comparable to the ionisation. Now the variance in the avalanche size, and hence the energy resolution, will be controlled by fluctuations at the very beginning of the multiplication region.

The explanation appears to be similar to the reason for the existence of the Fano factor. If the excitation cross section competes successfully with the ionisation cross section in a given region of space, then there will be channels other than ionisation that enable the energy gained by electrons from the electric field to be dissipated. This means that there will be a greater level of fluctuation in the number of electrons generated, and therefore, by implication, \( f_e \). When the ionisation cross section becomes more dominant,
then when an electron exceeds the ionisation threshold it is more likely to produce an
additional electron than de-excite. This tends to cause the amount of fluctuation to be
lower than before. The ionisation cross section will generally become more competitive
with the excitation cross section as the energy is increased above the ionisation threshold
of the gas. This will occur at larger values of the electric field strength.

If this weakly ionising electric field region extends over a large distance, then this will
be reflected in a larger value of $f_v$. Consider the fluctuation in the avalanche number after
traversing the same potential along the avalanche direction at high electric field and at
low electric field. It is expected that a larger value of $f_v$ would be seen in the case of the
smaller electric field.

In order to minimise the value of $f_v$ it is desired that the electric field gradient where
the electric field is weakly ionising be fairly large, so that electrons pass through this
region quickly.

If the variable $K_C$ is defined as

$$K_C = \frac{V_o}{\ln \left( \frac{r_c}{r_a} \right)}, \quad (6.11)$$

where $V_o$ is the potential of the cathode relative to the anode of a single wire counter, $r_c$ is
the radius of the cathode and $r_a$ is the radius of the anode then the reduced field gradient $S$
is given by

$$S = \frac{K_C}{N r}. \quad (6.12)$$

In Equation 6.12 $N$ is the number density of the gas. The reduced field gradient in a
single wire cylindrical counter is then given by

$$\frac{1}{N} \left( \frac{dS}{dr} \right) = -\frac{K_C}{N^2 r^2} = -\frac{S^2}{K_C}. \quad (6.13)$$
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It can be seen that if $S$ is large or $K_C$ is small then the field gradient will also be large. In either of these cases it is expected from the above argument that a smaller value of $f_v$ will be obtained. Then, by implication, the energy resolution will be improved, as Equation 6.6 indicates.

Ideally it is required to calculate the values of $f_v$ as the avalanche develops. This concept can be further developed with the help of Figure 6.3. $f_v$ needs to be measured at various values of the reduced electric field, as marked out by the dotted lines of Figure 6.3.

![Developing avalanche with equi-field lines and anode](image)

Figure 6.3 The value of $f_v$ is measured as the avalanche develops at the equi-field lines. Each equi-field line corresponds to an independent measurement.

As far as the author is aware, this has not actually been done experimentally. On the other hand, it is a relatively easy matter in a simulation to count the number of electrons that cross the boundaries that are signified by the dotted lines of Figure 6.3. Then the variation of $f_v$ with reduced electric field can be plotted, as is done in Figure 6.4.

Two counters were set up with different wire radii. The operating parameters were set to give approximately the same mean single-electron-induced avalanche gain. The results are shown in Table 6.2.
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<table>
<thead>
<tr>
<th>$r_a / \mu m$</th>
<th>$K_C / V$</th>
<th>mean avalanche number</th>
<th>$f_r$</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>164.4</td>
<td>8209 ± 59</td>
<td>0.53 ± 0.01</td>
</tr>
<tr>
<td>40</td>
<td>360.4</td>
<td>8450 ± 72</td>
<td>0.71 ± 0.01</td>
</tr>
</tbody>
</table>

Table 6.2 A comparison between two detectors with broadly the same gain, but with different wire radii. The gas mixture is 90% argon, 10% methane, $r_e$ is equal to 2.5 cm and this was held at 1 atmosphere pressure and at 298 K.

This is consistent with what Sakurai observed experimentally [Sak 92]. Thinner wires result in a smaller value of $f_r$ and therefore better energy resolution. This simulation appears to back up the assertion that this is not simply due to some experimental artefact such as defects in the anode wires or impurities in the gas.

The value of $f_r$ is plotted against the reduced electric field strength for both counters in Figure 6.4.

Figure 6.4 The relative avalanche variance $f_r$ is plotted against the reduced electric field along the path of the avalanche development.

It can clearly be seen that $f_r$ increases rapidly at low $S$ in both cases. This does not continue through to the high $S$ limit. The difference is truly striking. The results published here back up the assertion that the value of $f_r$ is indeed controlled by what happens when the electric field is weakly ionising. For the 40 \( \mu m \) anode wire counter, the actual distance travelled by an electron, from a region where the electric field corresponds to the
onset of ionisation, to the point where \( f_v \) stops rising, is about 400 \( \mu m \). The equivalent distance is about 100 \( \mu m \) in the 5 \( \mu m \) anode wire counter. The weakly ionising field covers a much greater distance in the 40 \( \mu m \) wire counter than the 5 \( \mu m \) counter.

There is an additional effect that needs to be pointed out, which causes the high reduced field region of each of the lines in Figure 6.4 to have a zero \( f_v \) gradient. As the reduced field \( S \) increases, the number of electrons that reach that value of the reduced field in any given avalanche is also increasing. This means that the avalanche distribution function that characterises the various \( S \) values is being sampled more extensively. The more times a distribution is sampled, the more likely its value is to converge upon the true mean multiplication factor. If there is a sufficient number of electrons already present in an avalanche, then all of the fluctuations will be ironed out over the large number of samplings that take place during each individual avalanche (for each electron in that avalanche).

To summarise, it appears that the best detector in terms of energy resolution is one in which the electric field changes from drift to strongly ionising over a very short distance. This minimises the change in \( f_v \) over the critical region where the number of electrons in the avalanche is relatively small. When the number of electrons becomes large enough the value of \( f_v \) becomes ‘locked in’. The avalanche distribution functions associated with the higher \( S \) values are more completely sampled, which is evidenced in the flat portion of each of the plots shown in Figure 6.4.

### 6.4.3 The development of the avalanche distribution function

The avalanche number distribution \( p_a \) has been plotted for the 5 \( \mu m \) counter described in Table 6.2. It is plotted at 3 different values of the electric field strength, as shown in Figures 6.5-6.7. The plots have been normalised so that the total area of each histogram is equal to 1.

At the start of the avalanche, the electric field is so weak that the chance of obtaining increasing numbers of electrons over a fixed distance is progressively smaller. The
distribution is not yet of the characteristic Polya form. This avalanche distribution function is plotted in Figure 6.5.

Figure 6.5 The avalanche number distribution is plotted for the 5 µm counter of Table 6.2 at 80 Td.

As the strength of the field increases, the chances of a single electron penetrating this far into the ionisation region of the counter without causing at least a single ionisation decreases. This is evidenced in Figure 6.6.

Figure 6.6 The avalanche number distribution is plotted for the 5 µm counter at 130 Td.

As the avalanche further progresses towards the anode the Polya distribution forms. It is characterised by the fact that the avalanche has now developed to the extent that there is a very small probability of this distance along the avalanche direction being reached without more than one electron being created. This means that there is now a clearly observed maximum in this function. This can be seen in Figure 6.7.
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Figure 6.7 The avalanche number distribution is plotted for the 5 μm counter at 190 Td.

Figure 6.8 shows a comparison of the avalanche distributions at the same value of the reduced electric field near the start of the avalanche for both the 5 μm and the 40 μm anode wire radius described in Table 6.2. It can clearly be seen that the avalanche distribution is more spread out in the case of the 40 μm wire.

Figure 6.8 The avalanche number distribution is compared for the 5 μm wire and the 40 μm wire counter of Table 6.2 at 80 Td.

It can clearly be seen that the 40 μm wire distribution leads to a larger value of $f_v$. 

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6.5 An Avalanche Statistics Comparison between Different Counters

A set of simulations were run in three different types of counter for approximately the same single-electron-induced mean gain $M$. The same gas mixture of 90% argon and 10% carbon dioxide, held at 1 atmosphere, was used in each. The feature sizes of the detectors actually used are shown in Tables 6.3 to 6.5.

<table>
<thead>
<tr>
<th>detector attribute</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>anode width</td>
<td>10 $\mu$m</td>
</tr>
<tr>
<td>cathode width</td>
<td>90 $\mu$m</td>
</tr>
<tr>
<td>anode / cathode thickness</td>
<td>0.3 $\mu$m</td>
</tr>
<tr>
<td>height of drift plane above substrate</td>
<td>5000 $\mu$m</td>
</tr>
<tr>
<td>substrate (insulator) thickness</td>
<td>700 $\mu$m</td>
</tr>
<tr>
<td>anode-anode pitch</td>
<td>400 $\mu$m</td>
</tr>
<tr>
<td>anode voltage</td>
<td>0 V</td>
</tr>
<tr>
<td>cathode voltage</td>
<td>-680 V</td>
</tr>
<tr>
<td>back plane voltage</td>
<td>0 V</td>
</tr>
<tr>
<td>drift cathode voltage</td>
<td>-1000 V</td>
</tr>
</tbody>
</table>

Table 6.3 Micro-strip detector attributes.

<table>
<thead>
<tr>
<th>detector attribute</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>anode radius</td>
<td>5 $\mu$m</td>
</tr>
<tr>
<td>cathode radius</td>
<td>25000 $\mu$m</td>
</tr>
<tr>
<td>anode potential</td>
<td>1075 V</td>
</tr>
<tr>
<td>cathode potential</td>
<td>0 V</td>
</tr>
</tbody>
</table>

Table 6.4 Cylindrical single wire detector attributes.
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<table>
<thead>
<tr>
<th>detector attribute</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>anode width</td>
<td>10 µm</td>
</tr>
<tr>
<td>anode thickness</td>
<td>0.2 µm</td>
</tr>
<tr>
<td>height of drift cathode above cathode</td>
<td>5000 µm</td>
</tr>
<tr>
<td>anode-anode pitch</td>
<td>300 µm</td>
</tr>
<tr>
<td>anode potential</td>
<td>0 V</td>
</tr>
<tr>
<td>cathode potential</td>
<td>-350 V</td>
</tr>
<tr>
<td>drift cathode potential</td>
<td>-2000 V</td>
</tr>
</tbody>
</table>

Table 6.5 Micro-gap detector attributes.

It can be seen from Tables 6.3-6.5 that the widths of the anode strips are set equal to each other for the micro-strip and micro-gap case. The anode diameter for the single wire case was also set to the same value.

The simulations were performed for 1000 avalanche histories. Each simulation was performed twice and the quoted uncertainty is half the difference between the two measurements. For the case of the micro-gap and micro-strip counter the electrons were drifted in from a distance of 2500 µm. For the case of the single wire counter a field of 10 Td was used for the value of the electric field at which the electron transport was initiated.

The value of $M$ for the three detector configurations is shown in Table 6.6.

<table>
<thead>
<tr>
<th>detector</th>
<th>$M$</th>
</tr>
</thead>
<tbody>
<tr>
<td>micro-gap</td>
<td>580.2 ± 4.7</td>
</tr>
<tr>
<td>micro-strip</td>
<td>582.8 ± 16.1</td>
</tr>
<tr>
<td>cylindrical single wire</td>
<td>576.6 ± 8.2</td>
</tr>
</tbody>
</table>

Table 6.6 The single-electron-induced mean avalanche gain is displayed for the three detectors.

It can be seen that the mean gain has been chosen to be nearly the same in all three cases.
Chapter 6: Proportional Counter Energy Resolution

In order to see how \( f_r \) develops along the path of an avalanche, this quantity was plotted for the three counters, in a similar fashion to that shown in Figure 6.4. Some clarification needs to be made about this, for the case of the micro-strip and micro-gap detectors. This can be established with the help of Figure 6.9, using a micro-gap detector as an example.

\[ \text{Figure 6.9 In each avalanche the number of electrons that are generated by the time an electron reaches the region defined by the horizontal dotted lines is recorded. } f_r \text{ is then calculated by applying Equation 6.4 over a series of avalanches.} \]

The electric field on the x axis of Figure 6.10 is plotted at the points where the horizontal dotted lines intersect with the vertical dashed line. It is acknowledged that the situation is slightly different in the case of the micro-pattern detectors compared to the single wire counters, because the regions defined by the horizontal dotted lines do not have the same field values all along their length.

In Figure 6.10 \( f_r \) is plotted against the reduced electric field \( S \) for three different types of detector. It can be seen for all three graphs that the general shape of Figure 6.4 is replicated. It would appear that the micro-gap detector gives the largest value of \( f_r \). The electric field close to the anode is plotted in Figure 6.11 for the three detectors. The electric field is broadly similar in all three cases. It does not really appear to explain why the micro-gap detector gives a substantially larger value of \( f_r \).

The electric field strength at the cathode that bounds the drift region of each detector is given in Table 6.7. It can be seen that the drift region electric field in the micro-gap counter is substantially greater than for the other two gas-filled detectors.
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Figure 6.10 A comparison of the variation in the relative variance of the avalanche distribution function $f_r$ with reduced electric field for three different types of counter.

Figure 6.11 The reduced electric field above the anode is plotted for the three detectors described in Tables 6.3 – 6.5.

In the case of the micro-gap detector this field must be very weakly ionising (or else $f_r$ would be 0 in this region). Since this weakly ionising field extends over a much greater region than for the micro-strip or single wire counter, the result is that $f_r$ is much greater.
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<table>
<thead>
<tr>
<th>counter type</th>
<th>reduced electric field at cathode / Td</th>
</tr>
</thead>
<tbody>
<tr>
<td>single wire</td>
<td>2.05</td>
</tr>
<tr>
<td>micro-gap</td>
<td>13.8</td>
</tr>
<tr>
<td>micro-strip</td>
<td>4.6</td>
</tr>
</tbody>
</table>

Table 6.7 The reduced electric field at the drift cathode for the three detectors considered in Table 6.3-6.5.

6.6 Summary

This chapter has analysed the role played by the avalanche statistics in the determination of the energy resolution of micro-strip, micro-gap and cylindrical single wire proportional counters. It was confirmed that the key to optimising the energy resolution is to create a counter where the electric field passes from being a drift field to being a strongly ionising electric field over a short distance. The advantage of a simulation is that the relative variance in the avalanche number can be calculated at various points within the counter – not just at the anode. This allows the dependency of the relative variance in the avalanche number (given by Equation 6.4, for example) to be plotted against the reduced electric field strength.
Chapter 7

High Fluence Rate Operation of the Micro-gap Detector

7.1 Introduction

The ultimate aim of the Radiation Imaging group at the University of Surrey is to be able to image the flow of multi-phase mixtures of oil, natural gas and water through pipes. The necessity of subjecting the detector to high x-ray fluence rates in order to acquire images in a dynamically changing environment was emphasised in Chapter 1. In fact the reduction of space charge is the principal reason for choosing micro-pattern devices to manufacture the detector elements of Figure 1.1 in the first place. The development of a simulation that corrects for space charge effects in micro-gap detectors forms the subject of this chapter.

7.2 Constructing the Simulation

This section introduces a number of issues that need to be considered in the context of the space charge simulations.

7.2.1 The Enhanced Simulation Model

In the simulations the detector was operated in a portion of parameter space where it was still very much in the proportional region in the low fluence rate limit. Mori indicates that this can extend up to values of $M$ (where $M$ is the single-electron-induced mean gain factor) in excess of $10^4$ [Mor 82]. The actual gains used were fixed at a value of around $10^2$, in order to comfortably satisfy this criterion. This meant that self-induced space charge did not greatly influence the avalanches being modelled here. It was the effect of general space charge that was explicitly being assessed. If the fluence rate is increased sufficiently then space charge will eventually become a problem.
Chapter 7: High fluence rate operation of the micro-gap detector

7.2.2 Previous Space Charge Simulations

Groh was the first person to attempt to include the space charge effect in a gas detector simulation. He was trying to obtain an understanding of self-induced space charge in a single avalanche in a cylindrical single wire proportional counter [Gro 90]. Due to the computationally intensive nature of the avalanches (each one took about 30 minutes) he was only able to perform an ensemble of 100 avalanches. Over this number of avalanches he did not observe very much change in the spatial distribution of the avalanche or the gain.

Bellazzini also performed some Monte Carlo simulation investigations of the space charge effect for micro-strip and micro-gap counters [Bel 94]. They chose one micro-gap detector and one micro-strip detector and tracked the variation in the gain as a function of the number of avalanches. The assumption was made that positive ions were fixed in space after each avalanche, so that the ion motion was not treated dynamically. As the number of avalanches increased the gain was observed to decrease. Again, because of the fact that each simulation took a large amount of time (of the order of weeks) to complete, they were not able to explore different detector configurations of the same counter.

In the calculations described here, the ion distribution was dynamically altered between avalanches, in order to take account of the time-evolving charge density. It was possible to take advantage of the availability of more powerful Pentium-based computers than were available when previous attempts to tackle the problem were made, in order to facilitate this.

7.2.3 Aim of the Calculations

The investigation can be separated into two distinct strands. It has already been pointed out in Section 1.5.2 that, for micro-gap detectors, the extreme proximity of the anode to the cathode makes breakdown a distinct possibility. By raising the level of the anode above the cathode plane, the electric field is reduced. This also means that ions will take a longer time to reach the cathode. This might have a negative effect on the stability of the detectors at higher fluence rates.
Secondly it is desired to know how varying the anode-anode pitch affects the fluence rate capability. An increased anode-anode pitch is expected to offer the promise of better stability because the individual avalanche region surrounding each anode is further away from its neighbours. This benefit comes at the price of a poorer spatial resolution. The five structures investigated all had the detector parameters described in Table 7.1. The gas mixture used was a 50:50 argon:methane mixture held at one atmosphere.

<table>
<thead>
<tr>
<th>detector attribute</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>anode width</td>
<td>10 µm</td>
</tr>
<tr>
<td>anode thickness</td>
<td>0.2 µm</td>
</tr>
<tr>
<td>insulator width</td>
<td>20 µm</td>
</tr>
<tr>
<td>separation of cathode from drift cathode</td>
<td>5000 µm</td>
</tr>
<tr>
<td>anode voltage</td>
<td>0 V</td>
</tr>
<tr>
<td>drift cathode voltage</td>
<td>-2000 V</td>
</tr>
</tbody>
</table>

Table 7.1 Micro-gap detector parameters that remained constant in this investigation.

7.2.4 Model Construction

It can be seen from Figure 1.1 that the irradiation of the tomography device will occur in a direction parallel to the strips. It is therefore expected that the charge density parallel to the strips at equilibrium will decrease as the x-ray path length through the gas increases, corresponding to exponential photon attenuation. Also, on neighbouring anode strips, at points corresponding to the same x-ray path length through the gas, it is expected that the charge density at equilibrium would be slightly different. This is because x-rays that have arrived at different anodes would have different paths through the material being imaged, and would have experienced different attenuation coefficients.

It would require a three-dimensional simulation of the electric field in order to model this situation. However the aim of this initial investigation is purely to determine the fluence rate capability of these detectors. If this can be modelled in a realistic fashion in
two dimensions, then the proposition of solving for the electric field multiple times during a simulation becomes more feasible from a computational point of view.

The Monte Carlo section of the code will generate a three-dimensional charge density function \( \rho_c \) that represents the position of the ions at any given time. If the average value of \( \rho_c \) is taken along an axis that lies parallel to the anode strips, then the dependence in this direction would be completely removed. This has inevitable consequences for the type of x-ray fluence rate that is being simulated. In particular the fluence rate is assumed to be uniform along the direction of the strips.

A schematic of the detector is displayed on the right hand side of Figure 7.1.

![Figure 7.1 A representation of a fictional partitioning of the detector into box-shaped segments. The dotted lines in the right-hand-side diagram lie halfway between anode centres. The blue quadrilaterals represent anodes.](image)

The "checkerboard" represented in the left-hand-side diagram of Figure 7.1 shows a plane that is parallel to the anode plane, and that lies somewhere between the anode plane and the drift cathode plane. Each electron cloud originating from a single x-ray will drift through this plane on its way down to the anodes. It is assumed that the electron fluence rate through each one of the squares of the checkerboard is identical. This means that the x-ray fluence rate in the region between this plane and the drift cathode must be uniform. The simulations focussed on one of these individual squares. When a fluence rate of \( F_p \) x-ray photons passes over this plane and generates electrons in the gas volume, the rate \( N_e \) at which electrons pass through a square is given by
Chapter 7: High fluence rate operation of the micro-gap detector

\[ N_e = \frac{F_p E_x A_f}{W}, \]  
(7.1)

where \( A_f \) is the area of the square, \( W \) is the \( W \)-value of the gas mixture and \( E_x \) is the x-ray photon energy. This electron fluence rate is then a measure of the fluence rate at which drift electrons are incident upon the detector avalanche region.

A uniform fluence rate means that, in the average case, \( \rho_c \) is a periodic function, with a period whose value is equal to the anode-anode pitch of the detector. It will be also be symmetric about the midpoint of each anode, just as the static electric field is. Therefore, to a first approximation, the electric field only needs to be solved over half the unit cell of the detector.

The other issue is what potential should be used on the electrodes. Strictly speaking the potential on the electrodes will change as the charged particles in the gas use the energy stored by the electric field to accelerate. The time it takes for the potential to return to its equilibrium value will depend on the external time constant of the circuit. In this case it is assumed that the time constant is extremely small so that the potentials applied to the electrodes remains constant. This is the same assumption as that made by Sipila in estimating the electric field change due to space charge build-up in cylindrical proportional counters [Sip 80].

Taking all of these arguments into consideration means that the boundary conditions of the problem can be set up in the same way as was described for the static solution, to a first approximation.

Given a uniform x-ray fluence rate applied for sufficient time, the periodicity described above would be asserted. The simulations themselves consisted of running 2500 avalanches, starting from a situation where there was 0 charge density in the gas volume. It took about 48 hours to complete each calculation of this type on a Pentium II 300 MHz machine running the Linux operating system. It might be argued that this was not sufficient time for equilibrium to be established in a real situation. However because the same model was used across all simulations, it is believed that there is a sound basis
Chapter 7: High fluence rate operation of the micro-gap detector

for using the results to make a qualitative argument for the differing degrees of non-linearity when the detectors are subjected to a high x-ray fluence rate.

The actual Monte Carlo electron transport is conducted over a single unit cell. If an electron leaves the unit cell on the left on its path to the anode it is re-injected on the right immediately, as shown in Figure 7.2.

![Figure 7.2 An electron is transposed when it reaches the edge of a unit cell. This procedure is justified because of the uniform irradiation of the detector.](image)

One single square of the checkerboard of Figure 7.1 was selected, and the whole simulation concentrated on a region that could be defined by extending this square out towards the anode plane in one direction and the drift cathode plane in the other. There was one constraint on the initial position of the electron within this region. This was that the electron originated halfway between the anode plane and the drift cathode plane of the detector. Its position in the two dimensions parallel to the anode plane was selected randomly within the square. The electron was given an initial energy of 1 eV. The large distance between the point of origin of the electron and the anode ensured that the electron attained hydrodynamic equilibrium in the drift region of the detector. This meant that the initial boundary condition on the electron’s energy would not affect the results of a given run (its energy was selected randomly from a uniform distribution spanning the energies of 0 and 1 eV).

The desired x-ray fluence rate was converted into an electron fluence rate according to Equation 7.1 and then an electron was originated every $I/N_e$ seconds. During each avalanche, any ions created were fixed in space for the duration of the avalanche. After each avalanche the ions were allocated to the grid using the weighting procedure.
Chapter 7: High fluence rate operation of the micro-gap detector

described in Section 4.7. The ions were drifted along the field lines for $t/N_e$ seconds and then fixed in space. The electric field was re-solved using the decomposition technique described in Section 4.5 and then the next avalanche was initiated.

It is argued that this method is reasonably effective until the electron fluence rate becomes so high that electrons are being fired into the avalanche region in time intervals that are smaller than the time it actually takes to create the ions in the avalanche itself. In order to get an estimate of the time taken for the avalanche to be completed, a set of simulations were set up where the time at which each ionisation happened was logged. Table 7.2 shows the mean times at which ions are produced after the first ionisation occurs in each avalanche, for three of the five detector variations analysed in this work. The selected parameters correspond to extremes in either the anode-anode pitch or in the thickness of the insulator used to support the anode.

<table>
<thead>
<tr>
<th>insulator thickness / μm</th>
<th>anode-anode pitch / μm</th>
<th>average signal formation time / ps</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>300</td>
<td>91.67 ± 5.2</td>
</tr>
<tr>
<td>5</td>
<td>300</td>
<td>77.21 ± 3.2</td>
</tr>
<tr>
<td>10</td>
<td>700</td>
<td>172.58 ± 9.1</td>
</tr>
</tbody>
</table>

Table 7.2 Mean time of ion formation after the first electron-impact ionising event in the avalanche.

It can be seen that the maximum average ion formation time is under 200 ps. It is expected that this quantity will be greater when the electric field that is large enough to sustain ionisation extends over a larger distance. The data in Table 7.2 can be converted into an estimate of the maximum fluence rate that can be considered using this simulation method.

7.3 Benchmarking the Simulation

There are a number of assumptions that are made in the above discussion. This section assesses their validity when applied to this problem.
Chapter 7: High fluence rate operation of the micro-gap detector

7.3.1 Electric field variation during a single avalanche

Throughout the whole of this work the assumption is always made that the electric field does not change significantly during an individual single-electron-induced avalanche. All of the simulations in this chapter were set up to have a value for \( M \), the mean single-electron-induced avalanche gain, of about 100.

Although the idea of solving the field many times during a single avalanche might be prohibitive from a computational point of view, indirect evidence as to its effect can be obtained from the following experiment. The electron fluence rate can be set to be extremely high, in order to ensure that the ions don’t move between avalanches. The effect of the distorted electric field on \( M \) can be analysed. Figure 7.3 shows a series of \( M \) values that were calculated when a given total charge was present in the gas (after a fixed number of avalanches).

![Figure 7.3](image)

Figure 7.3 The detector with the 10 \( \mu \)m insulator thickness and the 300 \( \mu \)m pitch was irradiated with an electron fluence rate of \( 2.5 \times 10^{18} \) \( \text{mm}^2 \text{s}^{-1} \) for a varying number of avalanche histories (ranging from 0 to 800). The mean gain calculated on the basis of the distorted value of the electric field (that is, with the ions present in the detector volume) is then measured. The purpose of using an abnormally high fluence rate is that the ions do not move significantly between avalanches and therefore accumulate in the gas volume. This allows an estimate to be made of the extent to which the number of ions around the detector anode influences the mean gain.

The shift in gain is about 4% for 10000 ions in the gas and is about 20% for 45000 ions. The electron fluence rate was chosen so that the ions were practically fixed in space at the point at which they were generated for the duration of the charge generation
process. Ions that reached the insulator surface were assumed to have zero lifetime on the insulator surface. This meant that any electric field distortion was solely due to the charge that was in the gas.

It can clearly be seen that the effect of increasing the charge near to the anode increases $M$, because it causes the electric field to be raised for points that lie above the charge cloud. The reduction in gain that can accompany space charge comes when the charge cloud has diffused outwards sufficiently that it pulls the electrons in the opposite direction to that due to the static electric field, in the zero-irradiation limit.

It was because the values of $M$ used in these simulations were so low that the electric field variation during a single avalanche was ignored.

**7.3.2 Non-linearity in the electric field variation during the positive ion motion.**

During the interval between avalanches, the positive ions were drifted along the electric field lines. As the ions move, the electric field dynamically changes. The choice of the time segment for successive refreshes of the electric field has the potential to have a significant influence on the detector simulation behaviour.

In order to gauge the effect of this, simulations were run where the electric field was solved a fixed number of times, $n_f$, between avalanches, at both high and low electron fluence rates. The electric field obtained after 2500 avalanches was then used to measure the new value of $M$. The results of this measurement are shown in Figure 7.4.

It is clear that there is not much difference in the mean gain in either case, for values of $n_f$ ranging from 1 to 100. An explanation for this lies in the fact that by the time the incident fluence rate is high enough to cause a serious gain distortion, the time between electric field refresh rates is very small. Consequently the distance positive ions can move in the time between each avalanche is also small and so therefore is the variation in the electric field as the ions are transported.
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Figure 7.4 The detector with the 10 μm insulator thickness and the 300 μm pitch was irradiated with a fluence rate of $2.5 \times 10^{12}$ mm$^{-2}$ s$^{-1}$, corresponding to a high electron fluence rate, and $0.05 \times 10^{12}$ mm$^{-2}$ s$^{-1}$, corresponding to a low electron fluence rate (within the context of the simulations described here). The electric field is re-calculated multiple times as the ions are drifted along the field lines.

7.3.3 Statistical variation in the mean gain

The experimental procedure relies upon solving the field after 2500 avalanches have been performed and calculating a mean gain based on the new electric field map obtained. Due to the stochastic nature of the problem, there is the possibility that the electric field will be sufficiently different after 2500 avalanches to cause the mean gain evaluated over two separate runs to differ significantly. In order to account for this, two separate electric field calculation runs were performed and the mean gain in each of these fields was calculated. Duplication also made possible a measurement of the uncertainty in the mean value, which is quoted as half of the difference between the two measurements. Over all the runs that were performed it was never more than a few percent.

7.3.4 The sensitivity of the results to a change in the simulated cross sectional area

It would be computationally quite daunting to try to simulate over the whole detector volume. Instead a 10 μm section along the length of the anode strips (which will be referred to here as the azimuthal length $a_z$) bounded the region within which the particle
transport occurred. There is the possibility that the choice of the size of this section could influence the experimental results. In order to gauge the effect of this, $a_z$ was varied. The fluence rate was kept constant which meant that the number of avalanches that needed to be fired into the simulation volume in order that the mean gain be measured at the same time across all simulations went up in proportion to $a_z$. The results of this are shown in Figure 7.5.

Figure 7.5 Changing the length of $a_z$ by a factor of 5 has very little effect on the mean gain in the electric field measured a fixed time after the simulation starts. The detector with the 10 $\mu$m insulator thickness and the 300 $\mu$m pitch was irradiated with a fluence rate of $2.5 \times 10^{12}$ mm$^{-2}$ s$^{-1}$. The time at which the measurement was taken corresponded to 2500 avalanches for the 10 $\mu$m result.

7.3.5 Contact between the ions and the insulator surface

The simulation aims to drift positive ions along electric field lines. The field lines eventually end up on one of the physical boundaries of the simulation. If that boundary is an electrode, then it is assumed that the ion will pick up an electron and remove itself from the problem instantaneously.

Some field lines will also end up on the insulator surface. This is not as significant a problem as it is with micro-strip detectors, where there is a large volume of insulator surface exposed to the gas. Depending on the relative conductivity of the surface of the insulator with respect to the bulk material, the ion will eventually accept an electron and be neutralised.
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This effect could be modelled quantitatively, if the appropriate probability density function for the lifetime of the ion on the insulator surface had been measured. In the absence of such a function it remains possible to gain an insight into the range of possible behavioural characteristics of the detector by considering the two extreme cases. One extreme is when the ions essentially stick to the surface (infinite lifetime) and the other is when the ions pick up an electron immediately after they hit the surface (0 lifetime), as suggested by Bellazzini [Bel 94].

The change in the relative gain, calculated by performing simulations for each of these two extreme cases, for a range of electron fluence rates, is plotted in Figure 7.6.

![Figure 7.6](image-url)

Figure 7.6 The relative gain is calculated by dividing the actual gain obtained for an insulator with a zero mean lifetime for ions by the equivalent measurement for an insulator with an infinite lifetime for an otherwise identical set of detector parameters. The detector used had a 10 µm thick insulator block and a 300 µm anode-anode pitch.

It can be seen that the effect of the charging up of the insulator strip seems to have very little effect on the value of $M$. This was the consideration that motivated the use of micro-gap detectors in the first place.
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7.4 Method

For a given detector configuration, the potentials on the electrodes were arranged so that the resultant electric field caused a value for $M$ in the zero irradiation limit of $94\pm1$. This applied to all of the detectors considered in the simulations. This would mean that detectors having two different geometries, but exposed to the same electron fluence rates, would initially have a similar amount of positive charge to deal with after the first avalanche in the sequence had been completed.

The simulations consisted of running 2500 avalanches for a given parameter set twice, whilst dynamically updating the ion positions. The altered electric fields that resulted from this were then used as the input to another 2 simulations of 2500 avalanches. This time the electric field was not updated from avalanche to avalanche. The aim was to assess the effect of the modified electric field on the value of $M$.

If 2500 avalanches are always used, then it is to be emphasised that the time at which the new mean gain is calculated is not the same in all cases. In fact the time will scale with the fluence rates. The purpose is to investigate how different detector configurations, subjected to higher and higher incident fluence rates, deal with what would have been similar amounts of charge deposited on them in the zero irradiation limit.

7.4.1 Varying the thickness of the insulator surface.

Figure 7.7 shows how altering the electron fluence rate changes the gain, for 3 different insulator thicknesses, corresponding to 1 µm, 5 µm and 10 µm respectively.

Other than the different insulator thicknesses, the detector dimensions were as described in Table 7.1. The potentials on the electrodes were chosen so that the gain measured in the zero electron fluence rate limit was the same. The dotted line of Figure 7.7 corresponds to a time interval between successive avalanches that marks the mean time of ion formation after the start of an avalanche. After this point the algorithm is not as valid, because a new avalanche is initiated before the old one has been fully completed.
Figure 7.7 It can clearly be seen that changing the thickness of the insulator has two effects on the gain. For the low insulator thickness the gain rises by the greatest amount before it falls away. All detectors had a 300 μm anode-anode pitch. The fluence rate measured is the electron fluence rate.

This means that not all of the ions that are actually present in the simulation when the next avalanche is initiated should be present. Nevertheless the measurement is useful because it puts an upper limit on the amount of electric field distortion that occurs. Naturally if a detector is subjected to such a high fluence rate, then it certainly cannot be considered to be stable.

In order to try to understand what is going on, it is instructive to be able to visualise the electric field and the ion density in a two dimensional fashion. Figure 7.8 shows a schematic of a micro-gap detector section, along with ion density and electric field distortion maps.
Figure 7.8 The ion density (plotted on the left) and the distortion in the electric field (plotted on the right) in the region enclosed by the dotted lines of the top figure, at an electron fluence rate of $2.5 \times 10^{12} \text{ mm}^2\text{s}^{-1}$, for the detector with the 10 $\mu$m thick insulator and the 300 $\mu$m anode-anode pitch.

For the ion density picture in Figure 7.8, the scale on the left represents the number of ions that are associated with that pixel in the diagram. The distortion of the electric field $\delta_e$ on a pixel is given by

$$\delta_e = E_f - E_0,$$

where $E_f$ is the electric field value of the pixel at a given fluence rate value and $E_0$ is the electric field at that pixel in the absence of radiation. The units of $\delta_e$ are in V/$\mu$m. The scale is linear in both cases.
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It can be seen that in the region between the anode and the area where the bulk of the volume space charge exists there is a reduction in the value of the electric field. The sharp structure at the left hand edge of the diagram of Figure 7.8 represents ions that end up at the drift cathode of the detector (as opposed to the cathode that is separated from the anode by the insulator surface).

Next the electron fluence rate was fixed at $2.5 \times 10^{12} \text{ mm}^{-2}\text{s}^{-1}$ and the electric field after 2500 avalanches was plotted along the dotted line specified in the left-hand diagram of Figure 7.9.

![Figure 7.9 The electric field is plotted at a high electron fluence rate for the 1 \( \mu \text{m} \) and the 10 \( \mu \text{m} \) detector along the dotted line of the left hand diagram after 2500 avalanches.](image)

The reduction in the electric field along this line explains why the value of $M$ is depressed in the 10 \( \mu \text{m} \) case. A map of the charge density in the two cases is shown in Figure 7.10.

Due to the increased distance between the anode and the cathode for the case of the 10 \( \mu \text{m} \) insulator, the space charge is found to exist in the gas for a longer time, before being quenched at the cathode surface. This explains the larger drop in the value of the electric field in this case. (The diagrams are plotted on the same scale in order to make the task of comparison easier).
Figure 7.10 The ion density is plotted after 2500 avalanches at an electron fluence rate of $2.5 \times 10^{12}$ mm$^{-2}$s$^{-1}$ for the 10 μm thick insulator detector and the 1 μm thick insulator detector (the units are the same as those of Figure 7.8).

There is an interesting structure that is apparent in Figure 7.7. The gain appears to rise before it falls for both the 1 μm and the 5 μm insulator thickness values. Initially it was thought that this might be due to charging up of the insulator surface. However when the simulations were run again, this time giving the electrons a mean lifetime of 0 seconds on the insulator surface, very similar results were obtained. In fact, at an incident electron fluence rate corresponding to $1 \times 10^{10}$ mm$^{-2}$s$^{-1}$ the value of $M$ changed from $113.6 \pm 0.20$ (for an effectively infinite lifetime of the positive ions on the insulator) to $108.7 \pm 2.1$ (for a zero lifetime). It is clear that there is another effect at work here.

In order to try to figure out what was happening here, the region described by the top diagram of Figure 7.11 was analysed. The electric field distortion $\delta_r$ is plotted on the bottom left and the ion density is plotted on the bottom right of Figure 7.11. The violet-coloured block on the left of this picture represents the insulator block plus the anode and can be taken as a zero reference for $\delta_r$. The large white-coloured area to the
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Figure 7.11 The value of $\delta_r$ (bottom left) and the ion density (bottom right) is plotted for the 1 $\mu$m detector at an incident fluence rate of $1 \times 10^{10}$ mm$^{-2}$s$^{-1}$ (the units are the same as those of Figure 7.8). The right of the anode-insulator structure indicates that there is an increase in the electric field in this region. There is a significant charge density above this area.

The suggested explanation for this is given in Figure 7.12. For the diagram on the left, the arrows represent lines of force for the electrons. The initial case corresponds to a situation prior to any irradiation. Moving to the diagram on the right, which represents a schematic of the situation after irradiation with 2500 avalanches, it is argued that the positive ion cloud above the green region will increase the vertical component of the force on the electrons. This will tend to encourage electrons that are in this region to move further towards the anode, enhancing the value of $M$. 
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Figure 7.12 The simulations show that there is a net increase in the gain at intermediate count rates. The charge cloud forms in such a way as to encourage more electrons that wander into the green region to move in the direction of the electric field arrows.

At intermediate fluence rate values, this more than offsets the effect on $M$ of the reduction in the value of the electric field in the region directly above the anode. It can be seen from Figure 7.11 that the space charge has a much greater component in the region that lies to the side of the anode rather than directly above it.

This also explains why this effect is seen for the micro-gap with the 1 μm thick block, but not the 10 μm thick anode. It is the close proximity of the anode to the cathode that forces electric field lines to change direction and end up on the cathode instead of the drift cathode. If more electric field lines end up on the cathode, then the $x$ component of the electric field vector, as defined in Figure 7.12, will on average be greater. This will tend to displace the ions along the $x$ direction by a greater amount. Finally, this leads to the effect illustrated in Figure 7.12, where there is a large ion density displaced laterally above the anode.

7.4.2 Changing the pitch of the detector

The effect of changing the pitch of the detector on the incident electron fluence rate is shown in Figure 7.13.
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Figure 7.13 The effect of changing the fluence rate for detectors with different anode-anode pitches is plotted. Each detector had an insulator thickness of 10 μm. Otherwise the detector parameters are the same as those listed in Table 7.1.

In the high fluence rate region of Figure 7.13 it can be seen that the smaller pitch detector gives a slightly bigger change in the value of $M$, although the effect is marginal. It is argued that this is reasonable because of the boundary conditions chosen on the edge of a detector unit cell. Since these are periodic this means that there are additional charge clouds either side of the charge cloud that is being simulated, corresponding to the charge around neighbouring anodes. Being positive, these would tend to cause a slight diffusion laterally away from the anode.

There is also the slight increase in the mean gain that occurs at intermediate count rates, in common with Figure 7.13. It is argued that this occurs for a similar reason. If the anodes are closer together then the field lines are such that there will be less of a tendency for ions to drift laterally, since this would lead them into the path of neighbouring anodes. The boost in the value of $M$ is therefore only seen for the detector with the largest anode-anode pitch.

7.5 Limitations of the simulations

An important issue that needs to be raised is attached to the fact that only the first 2500 avalanches are simulated on each anode. For an electron fluence rate of $1 \times 10^{10}$ mm$^{-2}$s$^{-1}$ this corresponds to the first 83 μs of the avalanche. Each simulation took
between 1 and 3 days to run. In the future, as computers become more powerful, it may be possible to simulate on a time scale that results in experimentally observable effects.

A second issue is the increased possibility of electron-ion recombination as the positive ion density increases. There are two sorts of recombination that are relevant to the transport of electrons in gases. The first results from electrons attaching themselves to gas atoms and then colliding with positive ions. The second results from electrons colliding with positive ions directly. The gases can be chosen so that their neutral constituents have a low electron affinity. This minimises the first possibility. The second is the process by which an electron recombines with an ion.

Given appropriate recombination coefficients, it can be envisaged how a simulation might incorporate these effects. The recombination coefficient $\alpha$ is defined in Equation 7.3.

\[
\frac{dn^-}{dt} = -\alpha_n n^+ n^-
\]  

(7.3)

Here, at time $t$, the positive ion number density is $n^+$ and the electron number density is $n^-$. This can be rewritten as

\[
\int \frac{dn^-}{n^-} = -\int \alpha_n n^+ dt.
\]  

(7.4)

In a situation where $n^+ >> n^-$ it can be assumed that $n^+$ is effectively constant, if measured over a small enough time segment. $\alpha_n n^+$ is, in effect, the collision frequency for recombination. The information a simulation would need to hold would include a map of the ion density so that $n^+$ could readily be obtained.

There are two major contributions to the value of $\alpha$.

**Radiative Recombination**

This can be summed up by Equation 7.5.

\[
A^+ + e^- \rightarrow A + h\omega.
\]  

(7.5)
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In order to calculate the recombination cross section from first principles, it is necessary to produce a weighted sum of the square of the dipole matrix from each electron’s bound state to the continuum, over the whole volume of the problem. The dipole matrix is proportional to

\[ |\psi_f^* r \psi_i| \ . \] (7.6)

where \( r \) is the separation of the electron from the centre of mass of the system, \( \psi_f^* \) is the complex conjugate of the final wavefunction of the electron (bound) and \( \psi_i \) is the wavefunction of the electron in the continuum.

**Three body Recombination**

\[ A^+ + e^- + X \rightarrow A + X \ . \] (7.7)

When electrons enter into the sphere of influence of an ion then there is a possibility that they will be captured. Clearly this possibility is greater if there is the prospect of a further energy loss collision during this time. If \( X \) corresponds to a neutral atom in the vicinity of the ion-electron pair then additional energy loss channels become available. It is also found that this quantity is dependent on the electric field strength. This has an important influence on the behaviour of ionisation chambers. The higher the value of the macroscopic electric field the more probable it is that electrons will be swept from the vicinity of an ion before undergoing an inelastic collision that allows itself to be recaptured.

The recombination cross section is a function of the electron energy, the gas number density and the reduced electric field. It would be instructive to repeat the experiment if a full parameter set for \( \alpha \) was obtained for the gases considered in the simulations. The introduction of ion-electron recombination is near the top of the list of potential enhancements to the simulation code in the future.
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7.5.1 Other detector instability possibilities

It has been observed that micro-gap detectors can show stability even up to measured x-ray fluence rates corresponding to $10^7 \text{mm}^2 \text{s}^{-1}$ experimentally, if the gain is kept low ($<500$) [Fon 97]. By multiplying by $E_e/W$, where $E_e$ is the x-ray energy and $W$ is the $W$-value of the gas, it can be seen that an electron fluence rate of the order of $10^9 \text{mm}^2 \text{s}^{-1}$ can be sustained. From Figure 7.7 it is observed that the gain starts to change at an electron fluence rate of around $10^{10} \text{Hz/mm}^2$.

In fact the problem of space charge has not been found to be the main barrier to the operation of the current generation of micro-gap detectors experimentally. It has recently been observed that the maximum value of $M$ that is achievable with various types of micro-pattern detectors falls with increasing fluence rate. In particular, Bressan and co-workers have gathered together a list of evidence that supports this [Bre 98]. Fonte also observed this.

It has been suggested that this may be due to a process known as defects activation [Fon 98]. In this case intense bombardment of the cathode by ions causes the work function of the cathode to be reduced [Kar 83]. This in turn increases the probability of emission of a further electron.

Photon bombardment of the cathode may also have a part to play. The reason why such high proportions of quench gas were used in the simulations was to “suppress” such a photon bombardment of the cathode. Figure 7.14 shows the effect on the maximum obtainable gain by varying the fluence rate for a real micro-gap detector.

Trying to relate this information to the simulation results is slightly difficult because a different anode pitch size was used in the Fonte experiment (200 μm). Also the gas mixture used to generate the data in Figure 7.14 was a 90:10 argon:methane ratio, whereas in the simulations the ratio was 50:50. However it can be seen that there is indeed a general decrease in the maximum value of the gain as the fluence rate is increased. Eventually, if the fluence rate is high enough, this maximum value will fall below the value of $\sim100$ that was used in the simulations.
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![Graph showing maximum achievable gain vs. x-ray fluence rate]

Figure 7.14 The maximum achievable gain is plotted as a function of x-ray fluence rate. The data is due to Fonte [Fon 98]. The incident x-ray fluence rate consisted of 6 keV photons interacting with a gas that was a mixture of 90% argon and 10% methane at 1 atmosphere.

Interestingly, it has been found that including a Gas Electron Multiplier (GEM) structure in the micro-gap detector drift region vastly increases the maximum stable achievable gain. This is because part of the amplification will now occur well away from the micro-patterned structure, inside the GEM. This structure was first proposed by Sauli. It typically consists of a thin foil of copper-coated kapton insulator. This is perforated by holes through which electrons can migrate. A schematic of the detector is shown in Figure 7.15.

Inside the GEM, the electric field is of sufficient magnitude to cause electron amplification. This means that extra gain amplification can be achieved in proportional counters. In particular, the maximum achievable gain can be greatly enhanced when a proportional counter is used in tandem with a GEM. This is now the preferred way to operate these detectors [Fra 98]. It is probably fair to say that the future of stable micro-gap operation lies in the coupling of the two technologies.
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Figure 7.15 The Gas Electron Multiplier. The solid lines represent electric field lines and the dotted lines represent potential contours.

7.6 Summary

The simulations described in this chapter looked at the issue of subjecting micro-gap detectors to successively higher electron fluence rates. This is a desirable characteristic for x-ray imaging. The effect of space charge in the gas was singled out for analysis. Due to limitations in the CPU power available at this fluence rate, the development of the avalanche was only tracked for about $1/10^{th}$ ms. It was found that if either the anode pitch is large enough (700 µm), or the thickness of the insulator is made sufficiently small (1 µm), then the electric field can change in such a way that the mean gain measured can actually increase from its value in the zero-irradiation rate limit.

It is acknowledged that the processes of electron-ion recombination, ion bombardment of the cathode and instability caused by the photons produced in the gas interacting with the cathode of the detector all have a part to play in determining the detector characteristics in the high fluence rate regime. It may be that trying out different metals
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for the cathode, or experimenting with different gas mixtures, can minimise the effects of these processes.

The work described here represents an initial step towards the understanding of this complex problem, by singling out space charge and analysing its effects.
Chapter 8

Conclusions

This work has detailed the development of a simulation from scratch to model electron transport within gas detectors. The simulation is able to calculate quantities that relate to the drift, diffusion and avalanching of electrons after they have been created by a primary ionising particle inside the detector.

A Monte Carlo electron transport technique is used. Each electron-impact collision is simulated discretely, on an interaction-by-interaction basis. This can be contrasted with other types of radiation transport code, such as the ‘Electron Gamma Shower’ code. In the latter, electron-impact collisions that occur with a low energy transfer are not treated individually. Instead, the accumulated energy loss and directional change over a given transport distance is calculated and then used to modify the electron’s phase space parameters.

The simulations were benchmarked in a variety of different situations against experimental data for argon, methane and carbon dioxide gas. These included the drift velocity and diffusion coefficients in argon and methane gas in a uniform field. The mean single-electron-induced avalanche gain calculated in a cylindrical single wire counter filled with a mixture of argon and methane gas was compared with experimental measurements. The gain measured in micro-strip detectors filled with mixtures of argon and carbon dioxide gas (which were manufactured in the Physics Department at Surrey University by Martyn Key) was also compared with simulation measurements.

An investigation was also performed into the energy resolution properties of gas detectors. In particular, the influence of the variation in the number of electrons collected at the anode of the counter on the energy resolution was analysed for single wire counters, micro-gap and micro-strip detectors. One of the advantages of doing simulation work is that the number of electrons that exist in the avalanche can be measured at any point along its path. This allows the relationship between the changing electric field
strength and the variance in the avalanche number to be characterised. It was confirmed that if the electric field gradient is low around the ionisation threshold of the gas, then the energy resolution is poorer.

One of the aims of this work was to tightly couple a ‘fast Poisson solver’ to the code in order to be able to dynamically solve the electric field inside the detector multiple times during a simulation. This was in order to correct for the influence of the positive ions that pile up in the detector volume over multiple avalanches.

It has been observed that experimentalists have found that high fluence rates might be difficult to achieve in practice, for reasons other than space charge build-up in the gas. It has been speculated that this might be due to intense ion bombardment of the cathode, causing the detector to be destabilised by positive feedback. GEM structures (Gas Electron Multipliers) are increasingly being used in micro-pattern detector systems to enhance their operating characteristics, both in terms of increasing the gain and also the stability of these detectors at high fluence rates. It may be that different detector materials can be used to overcome this effect. It is suggested that more work is needed in this area. Nevertheless, it is argued that the simulation methodology is itself sound in the case where space charge presents the limiting factor to the detector stability at high fluence rates. In this sense the simulations are useful for providing an upper limit on the detector stability in the high-rate limit.

The development of the code is an ongoing process. It is hoped that eventually the full development of the ionisation cloud in the detector will be simulated, stemming from the initial x-ray photoelectric event inside the detector. Due to the contribution of fluorescence and Auger transitions, this would require electron-impact cross sections for the sub-shells of the gases used to fill the active volume of the detector to be available. Tavora [Tav 98] has done some work to calculate the cross sections for atomic sub-shells that are accurate for electron impact energies right down to a few hundred electron Volts. Photon transport might also be added to widen the parameter space over which the code might be operated.
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It is suggested that the next step would be to use this information in the simulations. In this way a better estimate of the size of the electron cloud may be obtained. This is a factor that helps to determine the spatial resolution of the detector, along with the drift characteristics of the electrons as they drift towards the anode of the counter.

The motivation for this work is ultimately to come up with a simulation tool that will enable the detectors to be used for imaging. In particular, it is envisaged that they will be used in the multiphase flow imaging apparatus that was described in Chapter 1. The spatial resolution and fluence rate stability characteristics are particularly important for this purpose.

Argon-based gas mixtures were used in this work. This was principally because the detector development programme in the Physics Department at Surrey University involved the use of such gases. Therefore some validation of the simulation work was performed. Xenon gas would be a better candidate for use in the final imaging system because of its greater stopping power for x-ray photons. It might prove to be judicious to include xenon as a gas option in the simulations in the future.

Over the last couple of years or so a number of new detector types have been built, based around the technology that was used to introduce GEM structures in the first place. Advancements in kapton etching techniques have led to a number of new devices being introduced. In common with the GEM these incorporate thin kapton films, which are coated with a metallic substance (usually copper or gold). These include the WELL detector [Boz 99], the micro-groove detector [Bel 99] and the micro-slit detector [Lab 99]. The technology used to etch these is considered more reliable than that used to produce micro-gap or micro-strip detectors, in the sense that electrodes are produced with a lower defect density. In addition to this, detector manufacture is cheaper. It is a possibility that detector development will be concentrated in this area in the future. It would be interesting to use the interaction engine that was produced in this work with these new electric field structures.

Some publications are related to the work presented in this thesis. An account of the tomography system described in Chapter 1, including the Monte Carlo work, has been
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given [Mor 99]. Part of the energy resolution analysis, which pertains to single wire counters, and is described in Chapter 6, has also been published [Kun 99a]. Some of the space charge work was also presented at the SPIE symposium in Denver in 1999, including a proposed method of solution of the problem, which was published in the proceedings associated with that conference [Kun 99b].

As more and more features are added to the simulation the computational burden increases. As long as the remarkable increase in the CPU to cost ratio observed over the last thirty years continues on its current trend, the Monte Carlo technique (along with other numerical techniques) will become more and more useful in providing valuable insights into physical problems.
Appendix A

The Spatial Resolution of the Gas Detector Array

After an x-ray photon has interacted inside the detector an electron cloud is quickly generated (for example, see Section 1.4.2). If a gas detector is going to be used for x-ray imaging then there are two issues that need to be considered. The first issue is the robustness of the detector when it is subjected to a high fluence rate of x-rays. An analysis of the behaviour of these detectors at high rates is undertaken in Chapter 7.

Another issue is that of the spatial resolution of the detector. This can be explained more clearly with the aid of Figure A.1. A fan beam of x-rays is to be rotated around the source. A fan beam is desirable because it allows many different paths through the image to be interrogated at once, which reduces the scan time necessary to obtain each image. Key gives a detailed explanation of the principles of x-ray tomography [Key 991]. Each of the dotted lines represents a different path through the source and therefore a slightly different mean attenuation coefficient for the x-ray photons.

When a photon travels along one of these lines and is absorbed by the gas an electron cloud is produced around the original interaction point. The point at which the detection event occurred becomes less certain. This means that two lines that are separated by a distance that is less than the size of the electron cloud cannot be resolved. Therefore the spatial resolution is limited by the size of the electron cloud that results from the initial ionisation.

Apart from the straightforward electron-impact ionisation of gas atoms there are a couple of other effects that are important in determining the size of the electron cloud. If an electron or photon removes a bound electron from a gas atom’s inner shell then the residual ion is left in an excited state. It might de-excite by emitting an electron in an Auger process. This Auger electron then forms part of the electron cloud ensemble. An alternative de-excitation channel is via the production of an x-ray fluorescence photon. Due to the large distances that photons can travel relative to electrons, it is possible that they can extend the electron cloud substantially.
Appendix A: The Spatial Resolution of the Gas Detector Array

There are two thicknesses of oil pipeline whose cross sections need to be imaged. The first has a 2-inch diameter and the second has a 4-inch diameter. There are also two choices of target material for the x-ray tube. Using a tungsten target leads to the production of an x-ray spectrum with a main fluorescence peak at an energy of 60 keV. A silver target implies that the major fluorescence peak in the spectrum will occur at 22 keV. Table A.1 illustrates the range of electrons in 1 atmosphere of xenon gas at these two energies.

<table>
<thead>
<tr>
<th>Energy / keV</th>
<th>range / mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>22</td>
<td>0.4</td>
</tr>
<tr>
<td>60</td>
<td>2.4</td>
</tr>
</tbody>
</table>

Table A.1 The range of electrons in 1 atmosphere of xenon gas

The electron range has been calculated from an empirical formula that is due to Sauli [Key 99]. The range being measured is the distance that separates the point of origin of
Appendix A: The Spatial Resolution of the Gas Detector Array

the electron from its end point, and will be called the penetration. This can be contrasted with the range measured in the continuously slowing down approximation. The latter is the distance that would be obtained if the electron were considered to be travelling in a straight line as it was slowing down, instead of undergoing multiple scattering events that cause it to change direction.

Table A.2 shows the approximate mean free path of photons that are produced when an outer shell electron falls into an inner shell vacancy specified by the appropriate letter, following the ejection of an electron in an ionisation event.

<table>
<thead>
<tr>
<th>shell vacancy</th>
<th>photon energy / keV</th>
<th>photon mean free path / mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>K</td>
<td>34.5</td>
<td>302.3</td>
</tr>
<tr>
<td>L</td>
<td>5.1</td>
<td>2.3</td>
</tr>
<tr>
<td>M</td>
<td>1.4</td>
<td>0.3</td>
</tr>
</tbody>
</table>

Table A.2 The approximate mean free path of xenon fluorescence photons in 1 atmosphere of xenon

The data is taken from the XCOM database produced by Berger [Ber 87].

There are a number of ways that the spatial resolution can be enhanced. The first is to use 22 keV photons (instead of 60 keV). This cuts out the possibility of K-shell vacancies being created. Of course 22 keV photons are attenuated more than 60 keV photons, as Table A.3 shows.

<table>
<thead>
<tr>
<th>energy/ keV</th>
<th>2-inch pipe</th>
<th>4-inch pipe</th>
</tr>
</thead>
<tbody>
<tr>
<td>22</td>
<td>15.6</td>
<td>2.5</td>
</tr>
<tr>
<td>60</td>
<td>19.6</td>
<td>3.1</td>
</tr>
</tbody>
</table>

Table A.3 Percentage transmission through 2 diameters of oil pipelines filled with water

There is ~15% difference in the attenuation coefficient of oil with respect to water, so basing the calculation on pipes filled with water is reasonable. If 22 keV photons were used then it would mean irradiating for a longer time than with 60 keV photons in order to get the same statistical accuracy.
Appendix A: The Spatial Resolution of the Gas Detector Array

Another method is to increase the gas pressure. This would mean that the ranges of the electrons and photons are decreased. This would mean larger voltages having to be applied to the electrodes of the detector in order to get the same value of $M$, the mean single electron induced avalanche gain.

A third method would be to increase the radius of the detector array. A given size of electron cloud would then subtend a smaller solid angle at the x-ray point source elements. There are two downsides to this. The first is that the overall size of the system is increased. The second is that the x-ray photons have to pass through an increased amount of air on their way to the detector. They therefore have a greater possibility of scattering or being absorbed before they reach the detector.
Appendix B

Photon Transport

During an avalanche, some electron-impact collisions will leave the gas molecules in an excited state. One of the ways in which they can de-excite is by emitting a photon. This photon might then escape from the avalanche region and initiate a further avalanche. Alternatively it might undergo photoelectric absorption in the detector cathode, yielding an electron which can then restart the avalanche. This is the reason why proportional counter gases are usually a 2 component mixture, including a quench gas that absorbs photons and dissociates into neutral particles. This electron suppression leads to an enhancement in the stability of these counters. Examples of quench gases include methane and carbon dioxide. This section discusses the possible quench dynamics of argon – methane mixtures.

Figure B.1 illustrates a schematic of a small part of the energy level diagram of argon.

Figure B.1 A schematic of the electronic energy-level diagram for argon.

There are 2 possible situations where a photon that is produced can cause further ionisation. The first is where the photon produced has an energy that is above the 15.76 eV ionisation threshold for argon and can remove an electron from the atom in its ground state.
Appendix B: Photon Transport

Argon has 3 shells, K, L and M, corresponding to quantum numbers n of 1, 2 and 3 respectively. Their energies are 15.76 eV, 250 eV and 3 keV. Ejection of an electron from the L or the M shells by electron-impact ionisation can result in the production of photons that are energetic enough to cause further ionisation. There are quite possibly a multitude of sub-shells in argon within which electron transitions can occur, to produce a photon with an energy that lies above the ionisation potential of argon.

The situation is further complicated by the possibility of two-stage ionisation. Here many excitations occur in the original avalanche. According to Boffard there are actually two meta-stable states of argon, at energies of 11.6 and 11.7 eV (M1 and M2), which are important in plasma physics simulations [Bof 99]. These have a mean lifetime of 1 second and 37 seconds respectively as compared with a normal excited state, which has a lifetime that is on the order of nanoseconds.

As an electron avalanche rips through the gas these states can become populated from excitation events. Because only a further 4 eV are needed to ionise the gas, there will be a large number of photons produced in discrete excitations that can actually ionise an atom of the gas when it is in the M1 or M2 state.

This two step ionisation process might explain why pure argon in a single wire counter becomes unstable for multiplication factors of about 100 or more (Knoll p168), which is unlikely to involve electrons with enough energy to eject inner electrons from their shells. To get a feel for typical cross sections in this energy range the cross sections for CH4 photoionisation and photodissociation are plotted in Figure B.2 along with that for the argon photoionisation coefficient above the argon threshold.

If it was desired to test this assertion via simulation then an accurate cross section set that extends down to below 10 eV would be required. The argon data of Figure B.2 is due to Kennedy [Ken 72] and the methane data is due to Sampson [Sam 89].
Appendix B: Photon Transport

Figure B.2: The total photoabsorption cross section for argon and methane gas is plotted along with the partial cross section for producing electron-ion pairs. Notice the severe discrepancy between the latter two quantities at low energies. This is why methane is a good quencher – it absorbs ultraviolet photons without producing additional electrons.

A conventional explanation for the quenching effects of methane are that it, to quote Knoll [Kno 89], “preferentially absorbs photons in a mode that does not lead to further ionisation”. The energy threshold for the production of an extra new free electron is 12.6 eV. So this would tally with this explanation – if a photon is absorbed it must therefore fragment without producing an additional electron. However given that a small quantity of methane (5%) has a significant effect on the stability, it is suggested that as well as this ability to absorb photons and stop the metastable states of argon, there is also the possibility of collisional de-excitation occurring when in the metastable state. If the energy is transferred from the argon metastable atom to a methane molecule this molecule will dissociate without producing an additional electron. It is suggested that any simulation that aims to tackle this effect would need to include this possibility.
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