Health Monitoring in Proactive Reliability Management of Deteriorating Concrete Bridges

A thesis submitted for the degree of Doctor of Philosophy

by

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Abstract

Predicting future condition and performance of the infrastructure facilities for a foreseeable part of the remaining service life is vital for their effective management. Research in this area has led to the development of a number of predictive models (scientific, semi-empirical and empirical) for a wide range of materials and exposure conditions. These models are now reaching a state of maturity even though they contain numerous assumptions and simplifications. Due to the considerable amount of uncertainties in the input parameters, these models are of limited practical use and are useful only for short range predictions. Confidence in the prediction models can be increased by introducing additional information, through a range of inspection and monitoring procedures carried out at different stages of the structure’s life. Bayesian updating can be used to rigorously incorporate engineering judgements and prior beliefs about the condition and performance, into the more quantitative data obtained through inspections and health monitoring.

During this study, it has been demonstrated (in a specific area, i.e. chloride induced deterioration of reinforced concrete structures) that the uncertainty associated with performance prediction can be reduced considerably through the use of Bayesian techniques. Monitoring of the propensity to corrosion has been treated using Bayesian methods, leading to the prediction of structural performance with increased confidence. Sensitivity studies on the prior and posterior performance prediction (i.e. before and after incorporating data obtained through health monitoring systems) have been carried out to quantify the effects of various input parameters. It has been concluded that these parameters have a strong influence on the prior performance predictions but are relatively less sensitive to the posterior predictions. The cost-benefit of obtaining the additional information has been quantified using life-cycle cost analysis of various maintenance strategies (with and without the use of health monitoring systems). The strategy in which decisions regarding maintenance are supported through health monitoring systems has been found to be more economical and effective.

The benefit from the application of the techniques developed in this thesis lies in that we can prolong the service life (or extend the utility) of these structures by gaining additional information. Alternatively we can show and quantify the potential benefit of obtaining such information. Given the amount of public funds directed towards infrastructure maintenance and repair, the cost implication from adopting these techniques is significant.
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1.1. Preamble

Reinforced concrete has been used widely as a construction material for civil infrastructure facilities in the UK and many other countries in the last century. The main motivation in its use is the ability to be moulded into virtually any shape and size, and its perceived high durability in early years. Understanding of the latter was very limited until the 70’s; as a result, the structures comprised of concrete are deteriorating at rates higher than envisaged in the original design. A system of regular inspections and, wherever necessary, maintenance is in place to keep them safe and functional for their intended users. The number of structures (i.e. bridges, buildings, retaining walls, buried pipes etc) has grown very rapidly in the last few decades. Consequently a comprehensive inspection, maintenance, repair, rehabilitation and replacement program is not economically feasible. Hence a system of management is essential to keep these structures in a safe and serviceable condition whilst optimising the resources required in doing so and keeping the disruptions to their users at the minimum.

Due to the deterioration, and ever increasing demands with time, assessment of existing structures at various stages of their service life is vital for their effective management. Regular inspections are carried out to assess the condition of these structures e.g. for bridge stock in the UK, general inspections and principal inspections are carried out at every two years and six years interval respectively for the assessment purposes. Both of these approaches are practically visual except that the principal inspections are carried out at an arm’s distance from the structure and are characterised by a very limited testing. Maintenance and repair actions are carried out if a defect is indicated by the visual inspections (reactive management approach).

A major drawback of the existing management systems is that they rely on the change of condition of the structures rather than their safety levels to plan management activities. Any sound management strategy depends on the correct evaluation of the safety levels.
inherent in the structures at any point in time (Das, 1994). Hence the management systems should be based on safety characteristics of the structures in addition to their condition assessment (Das, 1998a). Uncertainties associated with the nature and rate of deterioration, the demand (past, present and future) and the actual performance of the structures are considerable, and subject to change during their service life. Because of these uncertainties, it is not possible to guarantee absolute safety of the structures hence management systems based on reliability are gaining the attention of the engineering community. In this approach, the performance of a structure is represented by its reliability i.e. the probability that the structure will not attain any of the defined limit states during its intended service life. Maintenance and repair is required when the reliability of the structure approaches a predefined minimum acceptable reliability level.

Various preventative measures against the deterioration at different stages in the service life of the structures can lead to reduced future maintenance needs (proactive management approach). These may cause an increase in the initial costs but are expected to yield cost benefits by reducing the future maintenance requirements. In an attempt to optimise available resources, the UK’s Highways Agency drafted guidelines for the life-cycle assessment of bridges (Draft BA81/00). These guidelines lay down the basis for comparison of various repair and maintenance strategies (essential as well as preventative) during the entire service life of a bridge and the strategy with minimum cost and maximum safety levels is generally selected.

Monitoring is used occasionally in conjunction with the inspections and testing etc. to improve the confidence in structural performance by reducing associated uncertainties e.g. BA 79/98 provides the guidelines for the use of monitoring of bridges if they are found to be substandard during the assessment but are not showing an urgent safety concern. Monitoring is also used to check and verify the effectiveness of the repair or rehabilitation methods.

1.2. Scale of the Deterioration in Concrete Structures

There are about 81,000 bridges in the UK administered by various managing authorities. 38% of which involves concrete (based on year 2000 bridge census). The Highways Agency alone is managing about 15,600 structures valued at £22 billion, of which 9,700 are bridges (many of these structures were built in the last 50 years). About 80% of these contain concrete (either reinforced or pre-stressed) as the main structural material (Flaig &
Lark, 2000). The UK Department of Transport estimates a total repair cost of £616.5 million due to corrosion damage to motorway bridges (Wallbank, 1989). These bridges are about 12% of the total bridge inventory in the UK hence the total cost would be approximately 8 times that of estimated by the Department of Transport.

In the US, there are about 600,000 highway bridges. About 29% of these have been classified as functionally deficient and will require $10.6 billion per year over the next 10 years to remedy the situation (ASCE, 2001). The cost of damage due to the de-icing salts alone is between $325 to $1000 million per year to bridges and car parks (Broomfield et al., 2002).

Similar problems are encountered in many other countries with aging infrastructure, typically built in the 1950’s and 1960’s throughout developing countries.

1.3. Problem Description

Predicting the future condition and performance of infrastructure facilities at element or system level for a foreseeable part of the remaining service life is vital for the intelligent management and effective use of resources. Research has taken place in this area related to different materials e.g. concrete, steel and masonry that has led to the development of predictive models for a range of conditions.

The input parameters of these models are based on the past experience and field observations from the similar categories of bridges. Due to the lack of available information, proper understanding of mechanisms involved, assumptions regarding deterioration phenomenon, and necessary idealisations to allow modelling, these parameters exhibit considerable degree of both aleatory and epistemic uncertainties. These factors hinder the effective use of these models in the efficient management of these structures.

Recent innovations in sensing and measurement technology have lead to the development of state-of-the-art sensors that can be used for permanent monitoring of structures. These health-monitoring systems have a great deal of potential in the management of infrastructure facilities. These can be used to obtain structure specific information on actual loading, response, mechanisms and rate of deterioration etc. Confidence in the output of predictive models can be increased by reducing the associated areas of uncertainties using additional information obtained through these methods. Moreover it is
possible to quantify the potential of obtaining the additional information, which is also important in terms of helping with the management decisions.

Estimating the extent of deterioration using predictive models, and the development of health monitoring methods to assess the condition and performance of structures have so far been treated separately in the contemporary management systems. A powerful tool can be developed by integrating the two areas that can estimate and predict the extent of deterioration with greater confidence (by reducing the associated epistemic uncertainties) throughout the service life of structures. Hence, it can be used to aid management decisions such as optimisation of inspection and maintenance activities.

1.4. Project Objectives

The primary objective of this project is to establish a methodology to effectively integrate the data obtained by permanent health monitoring of deterioration prone structures into the assessment procedures and management systems. The effectiveness of the methodology is illustrated with particular reference to the concrete bridges subjected to chloride induced deterioration by quantifying the reduction in epistemic uncertainties for a variety of scenarios and exposure conditions, and by comparing life-cycle costs (cost of operation, maintenance and inspection for its entire service life) for different inspection and maintenance strategies.

The key activities of the project are

- Study of predictive models for the governing deterioration mechanisms in concrete structures.
- Study of available instruments to monitor the deterioration processes and the type of data thus obtained.
- Highlighting different aspects of the management system where the monitoring data can be utilised effectively.
- Development of methodologies to effectively incorporate the data obtained by health monitoring into the management of structures for different possible scenarios.
- Quantifying the effectiveness of the proposed methodology for a range of sensor outputs.
• Quantifying the effectiveness of the proposed updating methodology for a range of conditions (different exposure conditions, concrete and workmanship qualities and instrument uncertainties) using sensitivity analysis of different parameters involved in the prediction of performance.

• Comparison of the case when the input parameters of the deterioration models are improved individually by gaining respective information through health monitoring methods with the overall performance updating.

• Life cycle cost comparison for different strategies with and without the use of health monitoring methods.

It is envisaged that the results of the project will help ascertain the effectiveness (quantitatively) of using permanent health monitoring systems in the management of deterioration prone structures especially at the earlier stages of its service life by increasing the quality of performance prediction at any time throughout its service life. This will provide effective and economical tools that could be used by the manager / owner of the structures to rationalise decisions regarding optimal repair, maintenance and inspection strategies and hence minimise its life-cycle costs.

1.5. Thesis Organisation

The thesis is divided into eight chapters. The description of the project and its key objectives are highlighted in chapter 1. Chapter 2 provides an introduction to the current management process, the main deterioration phenomena associated with concrete, their mechanisms and modelling, and highlights monitoring instruments that can be used for permanent health monitoring at various phases of the service life of reinforced concrete structures. A review is made of various methodologies available in literature to incorporate the information obtained through health monitoring into the management systems.

Chapter 3 describes the fundamentals of probability and statistical analysis associated with applications in structural engineering. Bayesian methods are elaborated together with the areas in which these methods have been used successfully. Simulation methods are briefly discussed together with variance reduction methods used in the current study.

Chapter 4 propounds different areas of a reliability based management system where health monitoring can be used effectively. The development of an updating methodology to integrate the data obtained through health monitoring systems with the existing
information regarding systems performance is also described in this chapter. In addition, a systems approach for the performance updating of an entire system with multiple sensors installed at various locations of the system is also developed.

Chapter 5 elaborates applications of the updating methodology in concrete bridges subjected to chloride induced deterioration. Simulated results of the prior and posterior performance are also presented in this chapter.

Chapter 6 discusses the results of sensitivity analysis of various input parameters on the prior and posterior predicted performance to establish robustness of the developed updating methodology.

Chapter 7 focuses on the comparison of life cycle costs for various management strategies with and without the use of health monitoring methods. Finally, Chapter 8 contains some general conclusions and summarises the main achievements of the current research. It also includes recommendations for further research.
2.1. Introduction

An overview of the project along with the research objectives are summarised in the previous chapter. A review of the related areas is presented in this chapter.

The general description of a typical bridge management system (BMS), its main modules, inspection categories and assessment levels are summarised in the first section. This also includes the development of a management system based on reliability and the applications of life-cycle cost analysis in optimising inspections and maintenance activities.

Estimation and prediction of the rate of deterioration in structures is an essential part of a management system that enables the manager / owner of the structures to plan the inspection, maintenance and repair activities. Understanding of the nature of deterioration processes involved and the physical phenomena responsible for these processes is vital in this respect. Various deterioration mechanisms for concrete structures are highlighted. The corrosion of steel embedded into the concrete is elaborated and a review of research efforts related to its modelling for the governing deterioration processes (chloride and carbonation induced deterioration) is presented.

Data obtained through health monitoring can be used to reduce uncertainties in the structural performance assessed at any point in time or predicted through the use of deterioration models. Some key instruments related to the health monitoring of structures for various stages in the service life of concrete structures are highlighted in Section 2.5. Finally, various attempts to integrate the information obtained from these instruments into the management systems are summarised followed by some conclusions from the literature review that emphasise the relevance of the project in the on-going research related to the management of deterioration prone structures.
2.2. Bridge Management System

With the implementation of BE 4, regular inspection of bridges was materialised in the UK in 1977 (Ryall, 2001). A decision for an increase in the vehicle weight from 38 to 40 tonnes was made in response to an EU directive in 1984 in which the international road vehicles were to be allowed into the UK roads from January 1999. This lead to a comprehensive rehabilitation programme for the local and trunk road bridges in the UK in 1988. A countrywide survey of concrete highway bridges in the UK for the Transport and Road Research Laboratory (Brown, 1987) and for the Department of Transport (Wallbank, 1989) suggested that the corrosion rates are higher than originally anticipated in design and hence the design life of 120 years will not be realised in a significant number of concrete bridges. The importance of formulating systematic procedures for planning management activities was realised and a flurry of research followed in this area.

In the UK, a bridge management system (BMS) was developed by High-Point Randle (HPR) called HiSMIS (Highways Structures Management Information System) and was implemented in 1990. Other management systems include BridgeMan developed by TRL in association with Oxfordshire County Council, NATS that is the Department of Transport own in-house system, and COSMOS developed by Surrey County Council that is part paper-based and part electronic. In the USA, two main BMS in use now-a-days are PONTIS developed through a collaborative effort between the Federal Highway Administration and State Department of Transportation, and BRIDGIT developed by the National Cooperative Highways research program (Ryall, 2001).

Most bridge management systems divide the structures into their elements to collect, store, and analyse the relevant information assigned to them individually and use this to optimise management activities at project and/or network level. Figure 2.1 highlights the main modules of a typical bridge management system.

General information regarding the bridges is stored in the inventory module. The inspection module stores information from the inspection proforma and reports including information about the general condition of different parts of the bridges, any specific treatments, and past remedial works etc (Ryall, 2001). The condition module is responsible for assessing the condition of elements based on the information stored in the inspection module and rates them using a predefined condition rating system.
In the UK, the maintenance module is used to keep record of any maintenance or repair works carried out during the lifetime of the bridges whereas in the USA, it is also responsible for making decisions on the maintenance and repairs. The budget & cost module contains all the information related to costs from the past and present projects and the budgetary constraints etc.

The estimation and prediction of deterioration rates for the bridges is vital for the planning of future maintenance activities. These rates are provided by the deterioration module based on the history and current condition of the bridge components. The optimization module combines information from the other modules to schedule management activities such that the required resources are minimised whilst maximising the benefits.

Due to the budgetary constraints, optimal strategies are not always pursued but the maintenance activities have to be prioritised. This objective is achieved through the prioritization module. A maintenance priority factor (MPF) is assigned to each element based on the condition of the element, its location, and importance to the safety of the overall bridge e.g. the highest priority factor used in the COSMOS is 3.7 and a lowest priority factor is 100. The priority is given to all structural elements with MPF less than 20, while all other works are considered as preventative maintenance (Maraki, 2003).

The bridge management decisions are made at both ‘project level’ and ‘network level’. The management actions at project level are related to specific bridge requirements e.g. the timing and type of maintenance, etc, on any particular bridge. These decisions do not depend entirely on factors associated with the particular bridge; they also depend on factors associated with other bridges in the stock (Vassie, 2000) e.g. budgetary constraints, traffic disruption and long-term benefits, etc. These strategic goals are achieved through
the network level management. Many bridge management systems utilise the concept of Markov Chain to predict the condition of bridges / elements in future, but Ng and Moses (1996) and Das (1998a) highlight its limitations. These include its inability to differentiate between bridges of different ages, and the assumption that the future states depend only on the current condition state. The use of reliability analysis has been proposed to remove the limitations of the Markov Chain model (Frangopol, 1997; Das, 1998b. Frangopol et al. 2001; Ryall, 2001).

2.2.1. Inspection Categories

At present, inspections are carried out at a regular interval to assess the condition of bridges and to report any defects. The inspections are divided into four main categories (BD63/94, BA63/94) namely,

- Superficial Inspection;
- General Inspection;
- Principal Inspection;
- Special Inspection.

The superficial inspection comprises a cursory check for obvious deficiencies which might lead to accidents, or potentially high maintenance costs, e.g. impact damage to superstructures and bridge supports, flood damage etc. These are usually carried out on an ad hoc basis.

The general inspection comprises a visual examination of representative parts of the structure for its condition assessment. The observed defects, if any, are reported using a preformatted checklist, which in turn may initiate a more detailed inspection. It is usually carried out every two years and is from the ground or deck level and may require binoculars.

The principal inspection requires close examination (within touching distance) of all inspectable parts of the structure against a preformatted checklist. Suitable access and traffic management may need to be provided to serve the purpose. This type of inspection is usually carried out every six years. Limited field testing e.g. half cell potential survey, cover and carbonation etc. may also be required (BA35/90).

The special inspection consists of close examination of a particular area, or defect, causing concern. It is usually carried out to investigate a specific problem, either found during
general or principal inspection or already discovered on other similar structures that would be detrimental for the structural safety or serviceability, or to assess the effectiveness of repair method used (BD63/94).

2.2.2. Bridge Assessment Levels

Prior to 1978, structures were designed using BS153 that was initially developed in two parts to account for material and workmanship, and stresses and construction in 1922 and 1923 respectively. These were revised several times in the next few decades, the latest being BS 153 (1972). In addition to this code, various other technical memoranda (BE1,77, and BE2/73) and codes of practice (e.g. CP114, CP115, CP116) were available. These codes were amalgamated into BS5400 in 1978 that was later revised in 1988. Several modifications and revisions have been carried out in this code to date to account for its limitations and to incorporate the updated knowledge regarding these structures. In conjunction with the design codes, the existing structures were initially assessed using BE4 compiled by the Department of Transport in the 1970’s. As stated earlier, a comprehensive rehabilitation programme for local and trunk road bridges was launched in the UK in 1988. The preliminary assessment and tests on the bridges (Brown, 1987 and Wallbank, 1989) revealed that a large number of bridges were failing the assessment criteria defined by the above codes. With the availability of numerous design and assessment guidelines at different points in time, the need to have a good quality and consistent assessment code was realised that lead to the development of assessment code BD 21. The initial versions of these codes were considering the same requirements for bridges of different material types, spans and structural forms etc. This was later refined and a number of different guidelines were prepared and implemented such as BD44 for the assessment of existing concrete structures, BD 56 for the steel, and BD 61 for the composite materials etc. These documents are constantly updated using the findings from extensive research programs commissioned by the Highways Agency to improve the accuracy of assessment methods. Most of these improvements were aimed at incorporating the conservative nature of structural systems in the assessment process by improving the understanding of structural behaviour of various bridge types and their components.

In the current practice, the assessment of RC highway bridges is covered by BD 44/95 that should be used in conjunction with BD21/01 along with their advice notes BA16-97 and BA44 96 respectively. Other advice notes, i.e. BA 51/95 and BA 52/94 have been provided
for the concrete bridges prone to reinforcement corrosion and alkali silica reaction respectively.

The process of assessment for the bridges is vital for their economic and effective maintenance. If assessments are unduly conservative, the bridges will be unnecessarily strengthened, or needless load restrictions will be imposed. Conversely if the method is not rigorous enough, some bridges could actually fail during service (Das, 1997). An advice note on the management of substandard structures was drafted in 1998 (BA 79/98). According to this guideline, the assessment of bridges should be carried out in stages of increasing sophistication, aiming at the greater precision at each higher level. The following are such possible levels of assessment. A flow chart outlining the categorical use of each level of assessment is presented in BA 79/98.

Level 1 is the simplest form of assessment, giving a conservative estimate of load capacity. Only the simplest methods are used and partial factors for load and resistance are obtained from the assessment standards, i.e. BD21/01 and BD44/95 respectively.

Level 2 assessment is characterised by more refined analysis and better structural idealisation. Analysis techniques include grillage analysis, finite element analysis or yield line analysis. Characteristic strength is also determined based on the available data e.g. mill tests on steel reinforcement etc.

Level 3 assessment includes bridge specific live loading. In addition material testing is carried out to determine characteristic or yield strength as well as the worst credible strengths of materials. Assessment is based on code implicit safety levels.

Level 4 assessment also takes into account the safety characteristic of bridge and uses bridge specific minimum safety / reliability levels to assess safety of the bridge.

Level 5 assessment involves full reliability analysis of particular structures or type of structures. This analysis requires probability data for all variables involved in the limit states.

Detailed guidance for carrying out the Level 1 to 3 assessments is available in the design manual for roads and bridges e.g. BD 21/01 and BD 44/95 etc. while guidance for the Levels 4 and 5 assessments that are based on reliability based procedures are still in the development phase.
2.2.3. Reliability based Bridge Management

During the assessment of existing highway structures (under the comprehensive rehabilitation program in the UK), a large number of highway bridges and other structures were found to be sub-standard with a little sign of deterioration at the surface. This large backlog of substandard bridges faced by many highway authorities in the UK indicated that the past management procedures (i.e. condition based inspections and reactive management approaches) have not been entirely successful (Das, 1994). A similar situation exists in the United States (ASCE, 2001) and in many other developed countries that have a legacy of aging infrastructure. These management systems were based on the subjective information obtained through the visual inspections of the condition of bridges which does not necessarily detect the deterioration phenomenon e.g. for pitting corrosion, it is possible for a bar to be completely corroded throughout without any visual indication (Vassie & Arya, 2003). These systems were primarily based on reactive management procedures i.e. despite the availability of non-destructive methods; these are applied only when the damage is identified by the visual inspections and are used to locate the damaged areas and to quantify the required repair works (Vassie & Arya, 2003).

Another drawback of these systems was that they relied on the change of condition of the structures rather than their safety levels to plan the management activities. It has been shown that the performance of bridges does not change consistently with the condition states depending upon the nature and location of defects and exposure conditions etc (Thoft-Christensen, 1999). Hence, the management system should also be based on safety criteria rather than just condition states (Das, 1998 & 1999 and Frangopol, 2000a). Uncertainties associated with the nature and rate of deterioration, the demand (past, present and future) and the actual performance of the structures are considerable, and subject to change during their service life. Because of these uncertainties, it is not possible to guarantee absolute safety and hence management systems based on reliability are gaining the attention of the engineering community.

The concept of reliability based procedures with particular reference to the civil infrastructure systems was initiated in the 70's e.g. Benjamin & Cornell (1970), Ang & Tang (1975) and Thoft-Christensen & Baker (1982) etc. The concept was initially used for the assessment of offshore structures. The first major research project on the reliability based management of bridges was supported by the European Union (Thoft-Christensen.
Since then this has been under a continuous investigation throughout the world and has now reached a level of maturity. Among the extensive list of contributors in this area some widely cited references include Cesare et al. (1993); Das (1994); Micic et al. (1995); Corotis (1996); Das (1996); Thoft-Christensen et al. (1996); Frangopol et al. (1997b); Val et al. (1997 & 1998); Das (1998b); Thoft-Christensen (1998); Enright & Frangopol (1999); Frangopol & Das (1999); Frangopol et al. (2000); Val et al. (2000); Rostam (2001); Sterrit et al. (2001); Chryssanthopoulos & Sterrit (2002); Kong (2003); Neves & Frangopol (2003) etc.

The use of reliability states in addition to the condition states was proposed by Frangopol & Das (1999) and Thoft-Christensen (1999) that was used in the Markov model to predict the deterioration rate and time to rehabilitation at the network level bridge management. This was a step forward towards the development of reliability based bridge management system (RBBM).

In the RBBM, safety levels, or performance, of bridges is expressed in terms of the probability of failure, $P_f$, proportional to the shaded areas in Figure 2.2 that also reflects change in the reliability of the structures due to various management activities.

The information from various modules of the existing BMS (as discussed in the Sec. 2.2) is used as the input for load and deterioration models and the uncertain input parameters are considered as random variables. Limit states functions (ultimate, serviceability, and durability limit state etc) can be obtained by defining the objective functions (safety, serviceability, aesthetics etc) (Das, 2000). Reliability analysis is then used to estimate the
structural performance and deterioration rate of the bridge at any point in time (that in turn leads to the so-called reliability profile), which can then be used to rationalise the decisions regarding management activities. Fig. 2.3 shows a schematic diagram of a reliability based BMS indicating principal information flow between and within the various modules.

![Figure 2.3: Key activities of Reliability based Bridge Management](image)

### 2.2.4. Optimisation and Whole Life Cost Analysis

Measures that improve the durability of a structure usually increase its initial cost but could reduce future costs by delaying maintenance activities, and hence the concept of optimisation was introduced in the BMS. Various tools available for optimisation include neural networks (Flood & Kartam, 1994), genetic algorithms (Liu et al., 1997; Busacca et al., 2001 and Malioka & Onoufriou 2002) and whole life cost analysis. The neural network method does not explicitly account for the deterioration phenomenon and load model etc. hence its use has been limited in the BMS. The genetic algorithms provide an effective tool for the optimisation of management activities throughout the life time of the structures. These are also being used in conjunction with the whole life cost analysis to optimise preventative maintenance actions (Tentelle, 2005).

The whole life cost analysis has been used in the management of bridges often for the optimization of management activities as it involves a direct comparison of various maintenance strategies for a foreseeable part of the service life of a structure on economic
grounds. Thoft-Christensen (1997), Frangopol et al. (1997 a-d) and Enright & Frangopol (1999 & 2000) are among those whose work is widely cited on the life cycle cost (LCC) analysis and its application in reliability-based bridge management. This concept is increasingly being used to plan and optimise management activities such as inspection and maintenance (e.g. Liu & Frangopol, 2004). The life cycle cost analysis enables bridge managers / owners to consider the consequences of future actions in the present day monetary terms. Thus, the total cost (i.e. for the entire service life) is estimated in present monetary terms, generally referred to as the ‘net present value’. The costs considered in this analysis are related to the design, construction, repair and maintenance, and failure of structures. Comparing such costs for all possible maintenance and repair schemes, the option with minimum cost can then be selected assuming cost is the only criterion, or Pareto optimal solutions can be found in a multi-objective optimisation situation. The model for LCC analysis of bridges is given by (Frangopol et al., 1997b)

\[ C_T = C_D + C_C + C_I + C_R + C_F \]  

.........Eq. 2.1

The optimization problem consists of minimising the total expected cost under a reliability constraint, i.e.

\[ \text{Min } C_T \text{ subjected to } \beta(t) \geq \beta_L \]

Where \( C_T \), \( C_D \), \( C_C \), \( C_I \), \( C_R \), and \( C_F \) are the expected net present total costs, design costs, construction costs, inspection costs, repair and maintenance costs, and failure costs respectively, whereas \( \beta(t) \) and \( \beta_L \) are reliability index at any point in time and lifetime target reliability index respectively. As \( \beta(t) \) is a function of time, it should be ensured that this does not fall below a prescribed target reliability index, \( \beta_L \), at any time during the service life.

All the costs in Eq. 2.1 can be presented as ‘net present values’ (NPV) using the following relation.

\[ NPV = \frac{C}{(1 + r)^n} \]  

.........Eq. 2.2

Where \( C \) is the cost associated with the activity at a certain time \( t_a \), \( r \) is the discount rate per annum and is currently specified as 3% for infrastructure projects by the UK’s Treasury (Treasury, 2005), and \( n \) is the time difference between \( t_a \) and present time in years.
The examples for the life-cycle reliability based assessment are presented in Frangopol (1997) and Faber (1997) for reinforced concrete bridges and Frangopol & Estes (1997) for steel bridge. The optimal balance between reliability and costs for different inspection strategies of a bridge deck is illustrated in Estes & Frangopol (1997). Frangopol et al. (1997a) showed the application of LCC analysis in the design where it was emphasized that modifying the dimensions of a bridge element changes the time between repairs. Hence using the life-cycle cost design, optimal dimensions can be chosen to minimise the total life-cycle costs. The use of life cycle reliability analysis to optimise management activities based on system reliability approach is proposed in Frangopol & Estes (1997). A procedure to estimate the rehabilitation rate for the elements of a bridge is presented in Frangopol et al. (2000a) and its use for the management of bridge stock (network level bridge management system) is discussed in Frangopol & Das (1999) & Frangopol et al. (2000b).

The timing of maintenance activities in the future is an uncertain quantity and so are the associated costs in terms of their net present values hence these should be represented using random variables. A methodology to estimate the distributions of various costs in Eq. 2.1 has been developed by Rubakantha (2001). Val & Stewart (2003a) derive the probability distribution of the cost of failure of a single structure and a group of identical structures when single or multiple failures are encountered / assumed during their life-time.

2.2.4.1. Maintenance Planning

A typical structural performance curve obtained using reliability analysis is shown in Figure 2.4. It is clear from the figure that the ideal performance curve, shown by thick line, indicates the assumption regarding the durability at the design stage. Actual reliability profiles (assumed) for two bridges are shown by separate curves having different deterioration rates (Bridge1 & Bridge2). Possible course of actions in this case could be

- to update bridge reliability to its original position (major repair or rehabilitation);
- to maintain/repair the bridge at intermittent intervals (minimum required maintenance etc.);
- do nothing now and replace the bridge when its performance approaches minimum acceptable limit.
The selection of best possible strategy is based on the optimisation procedures as described in previous section. Based on the above concept, maintenance works have been categorised into following groups (Das, 1999 & Draft BA 81/00):

- Routine Maintenance;
- Essential Maintenance;
- Preventative Maintenance.

The routine maintenance consists of work of minor nature which should be carried out at regular intervals to ensure safety of the structure stock, keep the stock in good order and minimise deterioration e.g. cleaning drains and channels and removal of debris from the bearing shelves etc.

The essential maintenance includes tasks that are required to maintain safety of the bridges e.g. where structures are assessed to be inadequate for carrying specified load or other safety concerns. In terms of reliability, essential maintenance improves the performance of bridge by strengthening or replacing elements of the bridges as shown in Figure 2.5.

The preventative maintenance includes work undertaken to prevent or slow down the rate of deterioration. It may also enhance the condition of the bridge (not usually the case). In case of concrete bridges, preventative works can be carried out to stop or delay the initiation of deterioration (proactive approach), or to reduce the rate of deterioration once it has initiated (reactive approach). Examples of preventative maintenance include repainting of steel, reapplication of sealers for concrete, use of cathodic protection system and washing of concrete elements to reduce the surface concentration of de-icing salts etc. A model for maintenance work, presented by Frangopol (1997), is shown in Figure 2.6.
2.2.4.2. Inspection Planning

In order to elaborate the importance of optimum inspection planning, a schematic variation of the bridge performance with both regular inspection intervals (current practice) and optimised inspections intervals have been plotted in the following figure.

As can be seen from Fig. 2.7a that the fixed inspection intervals might not yield much useful information at the early age of structures (i.e. for new bridges). As the deterioration progresses, the same fixed interval for inspections might not be enough to ensure safety or serviceability requirements and the structural performance may fall below the minimum specified performance levels(Fig. 2.7a). In the case of optimally planned inspections (Fig. 2.7b), the interval is optimised depending upon the multi-objective functions (Das, 2000) i.e. type and rate of deterioration, minimum performance requirements, and minimising the costs involved etc. This not only ensures the minimum safety and serviceability requirements throughout their service life but may also result in fewer numbers of inspections thus achieving effectiveness and economy in the management activities. Another benefit of this type of inspection scheme is that the inspections can be concentrated on critical areas of the bridges that would be identified by the limit states having a high probability of being exceeded e.g. vertical tension cracks in case of bending
and diagonal shear cracks in case of shear as the critical limit state and the visual signs of rust stains at the surface of concrete or the cracks in parallel to the reinforcement for structures prone to reinforcement corrosion etc.

2.3. Deterioration of Concrete Structures

Since the discovery of deterioration in concrete bridges in the late 70’s, detailed and extensive study of the concrete’s durability revealed several mechanisms responsible for its deterioration. These mechanisms can be divided into two main groups namely

- Chemical deterioration mechanisms &
- Physical deterioration mechanisms.

Chemical deterioration includes chloride attack, carbonation, acid attack, sulphate attack, and alkali-aggregate reaction while physical deterioration involves freeze-thaw, leaching, erosion and cracking etc (Brown, 1987; Wallbank, 1989; Takewaka, 1998; CEB, 1989; Basheer et al., 1996).

Reinforced concrete bridges, in practice, have been reported to be deteriorating primarily due to the corrosion of reinforcement embedded into concrete rather than the concrete itself. Primary mechanisms responsible for this are carbonation and/or chloride attack (BA 51/95). These two processes are unusual in that the aggressive agents (i.e. CO₂ or Cl⁻) do not attack the integrity of the concrete directly (Broomfield, 1997) but penetrate through the surface till these reach the rebar level and help initiate corrosion of reinforcement, which in turn causes the deterioration of concrete (i.e. cracking and spalling etc) due to the expansive rust products.

2.3.1. Transport Mechanism in concrete

Pore structure and crack configuration, and the filling of pores and cracks with water are determining factors for the transport of water and dissolved substances. The rate of transport primarily depends on the mechanism involved as well as on the binding ability of concrete for the substance being transported (Fig. 2.8).
2.3.2. Corrosion of Steel in Concrete

Corrosion is a thermodynamically spontaneous & unavoidable reaction of metals which is adverse to the metallurgical process of the production of metals from raw ores (Song & Shayan, 1998). If an electric potential difference is developed along the steel in concrete, it sets up an electrochemical cell (Neville, 1995). This forms anodic and cathodic regions connected by electrolyte in the form of pore water in the hardened cement paste. The positively charged Fe^{++} ions at the anode pass into the solution (pore water), if not utilised, attains a state of equilibrium. Negatively charged electrons pass through the steel towards the cathode and are absorbed into the electrolyte and in most cases combined with the water and oxygen to form OH⁻. The possible anodic and cathodic reactions are presented below. These depend on the pH of the pore solution, presence of aggressive anions, the existence of electrochemical potential on the steel surface, and availability of the oxygen in the vicinity of steel surface (Ahmad, 2003).
### ANODIC REACTIONS

<table>
<thead>
<tr>
<th>Reaction</th>
<th>CATHODIC REACTION</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\text{Fe} \rightarrow \text{Fe}^{++} + 2e^-$</td>
<td>$4e^- + \text{O}_2 + 2 \text{H}_2\text{O} \rightarrow 4(\text{OH})^-$</td>
</tr>
<tr>
<td>$\text{Fe} \rightarrow \text{Fe}^{+++} + 3e^-$</td>
<td>$2\text{H}^+ + 2e^- \rightarrow \text{H}_2$</td>
</tr>
<tr>
<td>$\text{Fe} + 2\text{H}_2\text{O} \rightarrow \text{HFeO}_2^- + 3\text{H}^+ + 2e^-$</td>
<td>$4\text{H}^+ + \text{O}_2 + 4e^- \rightarrow \text{H}_2\text{O}$</td>
</tr>
<tr>
<td>$2\text{Fe} + 3\text{H}_2\text{O} \rightarrow \text{Fe}_2\text{O}_3 + 6\text{H}^+ + 6e^-$</td>
<td>$2\text{H}^+ + 2e^- \rightarrow \text{H}_2$</td>
</tr>
<tr>
<td>$3\text{Fe} + 4\text{H}_2\text{O} \rightarrow \text{Fe}_3\text{O}_4 + 8\text{H}^+ + 8e^-$</td>
<td>$4\text{H}^+ + \text{O}_2 + 4e^- \rightarrow \text{H}_2\text{O}$</td>
</tr>
</tbody>
</table>

The net reaction of the processes (at the cathode and anode) is the formation of iron hydroxides.

$$\text{Fe}^{++} + 2(\text{OH})^- \rightarrow \text{Fe}($$OH)$_2$  

The iron hydroxide has low solubility and will precipitate. In the presence of oxygen at the reinforcement level, it is transformed into iron (hydro) oxides (Hematite and Goethite).

$$2\text{Fe}($$OH)$_2 + \frac{1}{2}\text{O}_2 \rightarrow \text{Fe}_2\text{O}_3 \text{ (Hematite)} + 2\text{H}_2\text{O}$$

$$2\text{Fe}($$OH)$_2 + \frac{1}{2}\text{O}_2 \rightarrow 2\text{Fe}_2\text{O}($$OH) \text{ (Goethite)} + 2\text{H}_2\text{O}$$

Another case of corrosion exists when the water is also available at the reinforcement level.

$$4\text{Fe}($$OH)$_2 + 2\text{H}_2\text{O} + \text{O}_2 \rightarrow 4\text{Fe}($$OH)_3$$

$$2\text{Fe}($$OH)$_3 \rightarrow \text{Fe}_2\text{O}_3\text{.H}_2\text{O} + 2\text{H}_2\text{O}$$

Unhydrated ferric oxide has a volume of about twice that of the steel it replaces, when fully dense. When it becomes hydrated, it swells even more and becomes porous. This leads to the volumetric increase at the steel / concrete interface from about 2 to 10 times (Broomfield, 1997), which leads to the cracking and spalling of concrete. The formation of rust disrupts the equilibrium of Fe$^{++}$ ions and hence more ions are passed into the solution and this causes the loss of steel section.

The difference in electrical potential that sets up the electrochemical cell can arise from differences in the environment (Neville, 1995) and the heterogeneous nature of the concrete (Song & Shayan, 1998) i.e.

- Microstructure and surface state in the steel surface.
- One part of the structure is permanently submerged and other exposed to periodic wetting and drying.
Substantial difference in cover thickness generates similar conditions. Due to the variation in salt concentration in pore water. Due to the non-uniform access of oxygen.

Under alkaline condition (pH > 10.7), the hematite, goethite or ferric oxide (γ-Fe₂O₃) formed by the corrosion process is insoluble and forms a thin passive layer on the surface of the steel (Montemor et al., 2003). The pore water in recently cast concrete has a pH-value of about 13 (Schiessl, 1988). Hence the reinforcement is generally in the passive state under normal conditions. This passive layer acts as a physical barrier for the oxygen and water to be in contact with the steel surface thus stopping, or at least reducing, further corrosion. The presence of this oxide layer also interferes with the transfer of electrons at the metal water interface (i.e. at cathode) hence polarising the cathode reaction and limiting the anodic dissolution. This passive layer forms primary defence mechanism against corrosion, in addition to the protection provided by cover concrete by limiting the availability of oxygen and water (Neville, 1995).

In order for the corrosion of reinforcement to initiate and progress in concrete, this passive layer has to be broken. The passive state of reinforcement depends on the pH-value and the chloride contents. If the pH drops below 10, more iron atoms dissolve and corrosion propagates. If the chloride contents near reinforcement exceed a threshold value, passivity is lost even though pH is still well above 10 (Neville, 1995; Ahmad, 2003). The chloride increases the solubility of the iron from the oxides that form a passive layer. Hence the two major phenomena that cause the loss of passive layer are carbonation (that causes the pH-value to fall below 10) and chloride attack (that helps dissolving the passive layer of iron (hydro) oxides etc.

Two types of corrosion are commonly observed on steel in concrete, general corrosion and localised corrosion. Unlike general corrosion, in which the rusting is almost uniform along the length of steel bar, localised corrosion is characterised by the small corrosion patches along the length of steel bar (a few centimetres of corrosion and then up to a meter of clean passive bar). This indicates the separation of anode and cathode (macro-cell formation) due to high levels of moisture availability that can easily transport ions to far off places from the cathode. Localised corrosion is usually observed in case of chloride induced corrosion. Local corrosion tends to be very variable along the length of member. A random
distribution of even quite severe local corrosion may have a little effect on the overall strength (BA 51/95), but of course this also depends on the actual stresses.

### 2.3.3. Chloride Induced Deterioration

In the UK and many other countries, chloride attack is found to be the governing mechanism for corrosion initiation as indicated by Figure 2.9 (Browne, 1987; Wallbank, 1989; Mutsuyoshi, 2001 & Gaal et al., 2002). Bridges are usually exposed to chlorides from three major sources.

- De-icing salt spray
- Sea-water spray (for structures in marine environment)
- Chloride ions cast into the concrete.

![Figure 2.9: Causes of Spalling (Gaal et al. 2002)](image)

The amount of chloride cast into the concrete is limited by various codes e.g. BS8110: Part 1, ACI 318 etc. thus the problem of chloride attack normally arises when the chloride ions ingress from the outside. In the USA, approximately 10 million tonnes of salt are applied on the roads every year whereas it is about 1 to 2 million tonnes per year in the UK, which is in proportion to the smaller road network (Broomfield, 1997). Similarly, structures on or near sea are in contact with chloride ions that are either in direct contact with water, due to air borne chloride ions or due to splash of water containing these ions.

#### 2.3.3.1. Ingress Mechanism(s)

Dry concrete imbibes salt water by absorption (Fig. 2.8) and, under some conditions, may continue to do so until it become saturated (Neville, 1995). If the external conditions then change to dry, the direct movement of water becomes reversed and the water evaporates from the ends of capillary pores open to the ambient air and the salts being left behind thus
concentration of the salts in the water retained in concrete increases near the surface. The concentration gradient thus established drives the salts near the surface of concrete towards the zones of lower concentration i.e. inwards; thus is transported by diffusion.

Thus in general the water moves outwards and the salt inwards. The next cycle of wetting with salt water will bring more salts into the capillary pores. The concentration gradient now decreases outwards from a peak value at certain depth from the surface, and some salt may diffuse towards the surface of concrete. If however the wetting period is short & drying restarts quickly, the ingress of salt water will carry the salts well into the interior of the concrete; subsequently drying will remove pure water, leaving the salts behind.

The ingress of chloride ions into concrete is strongly influenced by the exact sequence of wetting and drying. This sequence varies from location to location i.e. movement of the wind, location of the element etc. This is why different parts of the same structure may undergo a different pattern of wetting and drying, leading to considerable variation in the extent of corrosion damage in a single structure (Neville, 1995).

Once the concrete is cracked, its ingress mechanism is somewhat different. Based on the experiments carried out on reinforced concrete beams, Li (2002) observed that the chloride ingress rate in cracked concrete is significantly higher than the un-cracked concrete. Win et al. (2003) studied the effects of various parameters (i.e. w/c ratio, single & multiple cracks, exposure directions, crack width, chloride concentration and cover thickness) on the penetration profile of chloride in the cracked concrete sections and suggest that the increase in w/c ratio increases the chloride ingress rate in both cracked and un-cracked concrete. It was also observed that the penetration depth from crack surface (on each side of the crack) was equal to or slightly higher than that from the exposed surface for moderate to high w/c ratios. The movement of chloride ions along the steel was also observed. This is attributed to the high porous zone at the steel-concrete interface. The above findings are also verified by the experimental work of Ismail et al. (2004) i.e. for crack less than 53µm, crack does not have any significant effect on the diffusion process but for cracks equal to or greater than 60µm, the diffusion process propagating from the surface of crack is same as that from the surface of concrete.
2.3.3.2. Chloride Induced Corrosion

Regardless of the source and its ingress mechanism, when there is sufficient concentration of chloride ions at the rebar level to break-down the passive layer, corrosion may start depending on the availability of other agents like water and oxygen. Chloride ions attack the passive layer and unlike carbonation, there is no overall drop in the pH. Chlorides act as a catalyst i.e. they are not consumed in the reaction but they help breakdown the passive layer.

\[
\begin{align*}
Fe^{++} + 2Cl^{-} & \rightarrow FeCl_2 \\
FeCl_2 + 2H_2O & \rightarrow Fe(OH)_2 + 2HCl
\end{align*}
\]

HCl causes a further drop of the pH value, accelerating the dissolution of iron. Once the passivity layer is diminished and corrosion initiated, the rate of corrosion depends on a number of factors e.g. availability of oxygen and water, chloride concentration at the reinforcement level (Bamforth et al., 1997), temperature, resistance of concrete etc. The rate of corrosion is not a linear function of time as it tends to slow down due to the oxide layer formation on the steel reducing the penetration of oxygen and water. On the other hand, it increases once the cracks of certain width and depth are developed in the concrete providing easy access to the aggressive agents.

2.3.4. Carbonation Induced Deterioration

Carbonation is the result of interaction of carbon dioxide gas in the atmosphere with the alkaline hydroxides in the concrete (Broomfield 1997). The carbon dioxide diffuses through the concrete by dissolving in the pore water. This results in the formation of carbonic acid, which is capable of reacting with most of the calcium bearing compounds in the hydrated cement paste particularly with calcium hydroxide, which reacts to form calcium carbonate.

\[
\begin{align*}
CO_2 (\text{Gas}) + H_2O (\text{Pore Solution}) & \rightarrow H_2CO_3 (\text{Carbonic acid}) \\
H_2CO_3 + Ca(OH)_2 (\text{Calcium hydroxide}) & \rightarrow CaCO_3 (\text{Calcium carbonate}) + 2H_2O
\end{align*}
\]

This neutralises the alkalinity of the concrete, hence allowing the pH to fall below the threshold value. The passive layer breaks down leaving the reinforcement vulnerable to corrosion attack. The corrosion reaction of steel in concrete is dramatically enhanced when the pH of the pore solution falls below 9 (Song & Shayan, 1998). In addition, calcium
silicates and calcium aluminates become de-stabilised due to the transformation of calcium hydroxides into calcium carbonates. This leads to the formation of hydrated silicates and aluminates, which have little or no strength (Song & Shayan, 1998).

The rate of carbonation depends on the concentration of CO₂, the permeability of cement paste, the moisture content of the concrete, the relative humidity of the environment and the temperature. The rate of carbonation is maximum at an intermediate humidity. At high humidity (> 90%), the pores in concrete are almost completely blocked by water hence does not allow CO₂ to diffuse into concrete. At lower humidity (< 25%) the quantity of moisture is insufficient to form carbonic acid.

Carbonation affects the diffusion of chloride in the concrete by changing its pore structure (Patel et al., 1985). Ngala & Page (1997) found a reduction in the total porosity and a redistribution of the pore sizes as a result of the carbonation (the proportion of large pores increased). The chloride binding capacity is also decreased with carbonation.

2.3.5. Corrosion Damage

The corrosion of steel embedded in concrete is detrimental for structures in the following ways.

- It causes loss of steel cross-section, thus load carrying capacity of the structure is reduced.
- The product of corrosion process is several times larger in volume than the steel itself, thus causing cracks in the concrete. This also provides path for aggressive elements to penetrate and cause further deterioration.
- The expansion of steel after rusting causes delamination (which might lead to spalling) resulting in the significant reduction of bond strength. Val et al. (1998) have proposed a model for the reduction in bond strength due to reinforcement corrosion. The effect of bond strength reduction on the safety of the structure may be more severe in some cases than the loss of steel section (Sterritt & Chryssanthopoulos, 1999).
- It may cause reduction of ductility of the steel (e.g. in the case of pitting corrosion).
2.4. Deterioration Modelling

The corrosion induced deterioration process i.e. the corrosion of steel embedded into concrete is generally modelled using the two distinct phases; ‘initiation’ and ‘propagation’ as shown in Figure 2.10. Most of the corrosion models in the literature have adopted the same general form e.g. Tuutti (1980) & (1982); Browne (1980); Mangat & Gurusamy (1987); Thomas (1991); Cady & Weyers (1992); Mangat & Molloy (1994); Thoft-Christensen et al. (1996); Maage et al. (1996); Sarja & Vesikari (1996); Frangopol et al. (1997b); Engelund & Sorensen (1998); Stewart & Rosowsky (1998); Duracrete (1998); Quillin (2001) etc. Various models available in the literature for the initiation and propagation phase are described in the following sections.

![Figure 2.10: Corrosion induced deterioration model.](image)

2.4.1. Corrosion Initiation Models

The available models for the corrosion initiation can be divided into three groups (DuraCrete, 1998 & Thoft-Christensen, 2002a).

- Empirical models
- Semi-empirical models
- Scientific models

The empirical models are derived entirely by regression of the experimental / field data without taking any theoretical considerations into account. The semi-empirical models are those that are developed by simplifying the underlined physical phenomenon. Their parameters are estimated by regression analysis on experimental / field data. The scientific
models are those that are developed giving full consideration to associated phenomena involved in the deterioration process. These models are very complex in nature involving such parameters that cannot be obtained easily through the field measurements alone hence are of limited practical use. Hence the use of semi-empirical models is recommended (DuraCrete, 1998).

2.4.1.1. Chloride Ingress Model

Regardless of the external source, chloride penetration is a complex phenomenon. Typically, it involves initial absorption of salt laden water (particularly for dry concrete), possibly followed by capillary movement and diffusion, which transports chloride ions deep into the concrete (Fig. 2.8). There are other opposing mechanisms that slow down the chloride penetration e.g. its reaction with constituents of concrete and adsorption etc. Chloride ions can exist in concrete in one of the following states

- Free Chloride ions
- Physically Bound Chloride ions
- Chemically Bound Chloride ions

Free chloride ions are those that are dissolved in the capillary pore solution and are available for the reaction to break the passive layer hence are mainly responsible for the corrosion initiation (Bamforth et al., 1997; Broomfield, 1997; Song & Shayan, 1998). The amount of free chloride ranges from 40 to 45% of the total chloride contents in concrete (Tritthart & Cavlek 2000; Henry et al. 2000).

Physically bound chloride ions are those that are attached to the surface of the C-H-S gel through adsorption onto the free surface. These move towards the lower concentration on the gel at a much lower rate than the free chloride in the pore water and are not completely immobile and can, under certain circumstances, become available for reaction. Chemically bound chloride ions are said to be effectively immobile as a result of previous chemical reaction with some constituent of the cement paste e.g. C₃A & C₄AF, hence increasing the resistance of concrete to corrosion (Ahmad, 2003). These are also argued to become available if the pH drops below 11 for any reason and/or due to carbonation or when exposed to sulphates (Gaal et al., 2002). Hence the corrosion risk presented by the bound chlorides may be very similar to that presented by the free chlorides (Reddy et al. 2002).
Models based on the theory of diffusion have been developed to best represent the chloride ingress in concrete and are widely used in practice to predict the initiation of reinforcement corrosion in concrete (Andrade et al. 1996). Win et al. (2003) suggested that the effect of capillary suction is considerable and should also be considered while predicting the chloride ingress into the concrete.

Diffusion is mathematically represented by the partial differential equation using Fick’s 2nd law of diffusion as follows (Crank, 1975)

\[
\frac{\partial C}{\partial t} = \frac{\partial}{\partial x} \left( D \frac{\partial C}{\partial x} \right) + \frac{\partial}{\partial y} \left( D \frac{\partial C}{\partial y} \right) + \frac{\partial}{\partial z} \left( D \frac{\partial C}{\partial z} \right)
\]

.......... Eq. 2.3

Where \( C \) represents the concentration of diffusing substance at time \( t \) at a location defined by the coordinates \( x, y \) and \( z \), and \( D \) is the diffusion coefficient. For a one dimensional diffusion process with constant diffusion coefficient, the Eq. 2.3 would be reduced to

\[
\frac{\partial C}{\partial t} = D \frac{\partial^2 C}{\partial x^2}
\]

.......... Eq. 2.4

The solution for the above equation has been derived in Crank (1975) for a variety of scenarios (i.e. time dependent surface chloride concentration, and time dependent diffusion coefficient etc). Collepardi et al. (1970) appears to be the first to apply Fick’s second law to mimic chloride diffusion in concrete due to the de-icing salt whereas Takewaka & Matsumoto (1988) have presented a Fickian chloride ingress model for the marine environment. Work by Browne (1980) & Tuutti (1982) is also widely cited in this context. The model used for chloride ingress due to de-icing salts (based on the solution of Eq. 2.4) is as follows

\[
C(x, t) = C_o \left\{ 1 - \text{erf} \left( \frac{x}{2\sqrt{Dt}} \right) \right\}
\]

..........Eq. 2.5

Similarly the model for marine environment (with a time dependent surface chloride concentration) is

\[
C(x, t) = 2M \left\{ \sqrt{\frac{t}{\pi D}} \exp \left( - \frac{x^2}{4Dt} \right) - \frac{x}{2D} \left[ 1 - \text{erf} \left( \frac{x}{2\sqrt{Dc}\cdot t} \right) \right] \right\}
\]

..........Eq. 2.6
Where $C_0$ is the surface chloride concentration; $D$ is the effective diffusion coefficient; $x$ is the depth at which chloride concentration is required; $t$ is the time of exposure; $C(x,t)$ is the chloride concentration at depth $x$ and time $t$, and $M$ is the quantity of accumulated chloride on concrete surface per unit time (assumed to be a constant).

The following assumptions are inherent in the derivation of the above models (Crank, 1975)

- Semi-infinite medium of chloride ions.
- Chloride concentration is uniform throughout the surface.
- The initial concentration throughout the medium is zero.
- Diffusion takes place only in one direction (along the depth).
- Concrete is isotropic and homogenous material and the diffusion coefficient is constant in space and time.
- Load induced cracks are not considered in the model.

Several improvements and modifications have been proposed by various researchers since then, e.g. Hoffman & Wayers (1996) ignored the diffusion process for the first 1.27cm of cover concrete to account for adsorption within that depth. Suryavanshi et al. (1998) based on laboratory tests proposed a two dimensional diffusion model to account for lateral movement of chloride ions in the concrete cover and proposed the following model.

$$C(x, y, t) = C_s \left\{ 1 - \text{erf} \left( \frac{x}{2\sqrt{Dt}} \right) \text{erf} \left( \frac{y}{2\sqrt{Dt}} \right) \right\} \quad \text{...............Eq. 2.7}$$

Where $y = \text{movement of Cl^- ions parallel to the main reinforcement}$.

An alternate solution of the diffusion process (Crank, 1975), assuming a finite quantity of available substance, $M$, is given by Frangopol (1997) as

$$C(x, t) = \frac{M}{\sqrt{\pi Dt}} \exp \left( -\frac{x^2}{4Dt} \right) \quad \text{...............Eq. 2.8}$$

Mejbro (1996) proposed a complete solution to Fick’s second law with time-dependent diffusion coefficient and surface concentration. The model is very complex and is considered unrepresentative of the field conditions etc. Further it is argued that not all the parameters of its model can be evaluated based on site observations (Sterrit, 2000).
In view of random and systematic uncertainties present in the deterioration variables, the use of probabilistic models has found an increasing appeal. Thoft-Christensen et al. (1996) and Sorensen & Engluend (1996) appear to be the first to use a probabilistic framework for the corrosion initiation and propagation at rebar level. Frangopol et al. (1997b), Stewart & Rowosky (1998b), and Vu & Stewart (2000) are amongst those who further developed and used probabilistic models to predict time-varying bridge performance under chloride attack.

Other modifications include improvements in the input parameters of the model (distribution type and values of its parameters etc) e.g. Stewart & Rosowsky (1998a) similarly Vu & Stewart (2000) considered diffusion coefficient, not as a physical quantity but as an ‘effective’ parameter, dependent on w/c ratio, aggregate to cement ratio, mass densities of cement and aggregates and presented a model for it, originally developed by Papadakis in 1996. Another analytical model for the diffusion coefficient has been proposed by Oh & Jang (2004) considering the relationship between the diffusivity and the microstructure of concrete between interface of cement and aggregate, and of cement paste itself. Kong et al. (2002) considered w/c ratio and curing time as random variables in the prediction of chloride ingress into concrete.

In situations where the structure is subjected to wetting and drying cycles, the concrete can be considered to have ‘inner’ and ‘outer’ zones. The inner zone is assumed to be close to full saturation and the predominant transport mechanism is likely to be diffusion whereas the outer zone is likely to be affected by other transport mechanisms (Quillin, 2001). Sterritt (2000) proposed a model that combines the effects of initial absorption along with the diffusion to predict the corrosion initiation time.

Thoft-Christensen (2002a) incorporated chloride binding ability of concrete into deterioration models, based on experimental observations. A few researchers considered diffusion coefficient as time dependent and proposed different modifications e.g. Mangat & Molloy (1994) and Duracrete (1998) etc but Vu & Stewart (2000) argued that the time dependency of diffusion coefficient is primarily due to the hydration that changes the pore structure and reduces significantly during the first few years of construction and then approaches a constant value.

Due to the development of cracks under service loads in a flexural member, mechanism of diffusion becomes less dominant. Li (2002), based on his experimental work, proposed a
model for the corrosion initiation time in concrete including the effects of cracks, which is as follows.

\[ \mu_c(t) = C_0 \exp^{at} \]  ..................Eq. 2.9

\[ V_c(t) = bt + 0.1433 \]  ..................Eq. 2.10

where \( \mu_c(t) \) is the mean chloride content at reinforcement surface as a function of time; \( V_c(t) \) is the coefficient of variation of the chloride contents; \( C_0 \) is the mean initial chloride content at reinforcement surface; \( a \) is the rate of chloride ingress. ‘a’ and ‘b’ are obtained through regression of the observed data. The cover depth is not considered here as it is assumed to be irrelevant if crack width exceed 0.1mm.

Li et al. (2003) presented an alternate solution to Fick’s 2\textsuperscript{nd} law for diffusion to eliminate the assumptions regarding standard solution presented by Crank (1975) that has been the basis of most of the chloride ingress models and are argued to be impractical (Li et al. 2002 & 2003). It was suggested based on the work of Cunningham & Williams in 1980 that the mechanism of diffusion is a combination of Kundson flow and Viscous flow. Following models were developed for both of these categories.

When \( C(x, t) < C_r \)  (Kundson flow)

\[ C(x, t) = C_s + (C_i - C_s) \sum_{n=1}^{\infty} U_n(x, t, D_{c1}) \]  ..................Eq. 2.11

When \( C(x, t) > C_r \)  (Viscous flow)

\[ C(x, t) = C_s + (C_i - C_s) \sum_{n=1}^{\infty} \frac{U_n(x, T_r, D_{c2}) U_n(x, t, D_{c2})}{U_n(x, T_r, D_{c2})} \]  ..................Eq. 2.12

The transition between Kundson flow and Viscous flow, \( C_r \), was estimated to be 0.044% by weight of concrete.

\[ U_n(x, t, D_r) = \frac{4}{(2n-1)\pi} e^{\frac{D_r(2n-1)^2\pi^2}{4t^2}} \sin \left( \frac{(2n-1)\pi x}{2l} \right) \]  ..................Eq. 2.13

Where \( l \) is the crack length; \( C_s \) is the amount of chloride content at the surface of concrete; \( C_i \) is the initial chloride concentration in concrete; \( T_r \) is the time at which critical chloride
concentration is attained, and \( D_{c1} \) and \( D_{c2} \) are the diffusion coefficients for Kundson and Viscous flow respectively and have been calibrated using experimental data.

A summary of various other models for the initiation and propagation models is presented in Duracrete (1998) and Liang et al. (2002). Alternate methods to model concrete deterioration are also available in literature e.g. deterioration modelling of bridges using markov chain models (e.g. Cesare et al., 1992 & Destefano & Grivas, 1998) and neural network models (e.g. Elkordy et al., 1993; Szewezyk & Hajela, 1994; Glass et al., 1997 & Zhao, et al., 1998), etc.

### 2.4.1.2. Carbonation Models

According to Browne (1987) the penetration rate for carbonation can be determined from the simple diffusion law.

\[
X = k \sqrt{t} 
\]

Where \( X \) is the distance penetrated after time \( t \); \( k \) is the diffusion coefficient obtained from the calibration procedure.

CEB-FIP Model Code (1993) recommended the following relationship based on Fick’s first law for the estimation of carbonation depth at a given time as

\[
d_c^2 = 2D_{CO_2} \frac{C_u}{C_v} t 
\]

where \( d_c \) is the depth of carbonation at time \( t \) (m); \( D_{CO_2} \) is the diffusion coefficient of CO\(_2\) through carbonated concrete (m\(^2\)/sec); \( C_u \) is the concentration of carbon dioxide in the air (g/m\(^3\)), and \( C_v \) is the amount of CO\(_2\) required for complete carbonation of unit volume of concrete (g/m\(^3\)).

Lindvall (1998) used a model presented by CEB Task Group V and established the models for its environmental parameters. The model is mathematically represented as follows.

\[
x_c = \sqrt{\frac{2k_1 k_2 D_{eff} C_s}{a}} \left( \frac{t_0}{t} \right)^a 
\]

Where \( D_{eff} \) is the effective diffusion coefficient at a defined execution and environmental conditions; \( a \) is the binding capacity for CO\(_2\); \( t \) is the time in service; \( t_0 \) is the reference
period, e.g. 1 year; \( k_1 \) is a factor which considers the influence of execution on \( D_{\text{eff}} \) (e.g. influence of curing) and \( k_2 \) is a factor which considers the influence of environment on \( D_{\text{eff}} \) (e.g. influence of moisture and temperature at the surface of concrete structure), and \( n \) is the factor which considers the influence from environment on the time-evolution of \( D_{\text{eff}} \) (e.g. shelter against rain and moisture conditions at the surface of structure).

On the other hand a mathematical model for the carbonation process is presented by Saetta et al. (1995) based on the moisture, heat, and pollutant flow equations and a sensitivity studies of various parameters involved in the model is summarised in Saetta & Vitaliani (2004). The model involves a series of parameters that are difficult to estimate in the field hence makes it difficult to be utilised in practice.

Duracrete (1998) has summarised various available models for the carbonation in concrete and recommended the model presented by CEB Task Group V (Eq. 2.16) based on the evaluation by comparison of predictions with data produced from 16-year exposure tests on 27 concrete mixes and for the different exposure conditions.

### 2.4.2. Corrosion Propagation Models

Various models have been proposed by different researchers for the propagation of corrosion. Almost all of them have used corrosion current as the measure of rate of deterioration.

Broomfield (1997) has given following guidelines for the corrosion rates (using linear polarisation measurement with ring guard).

- **Passive condition**: \( I_{\text{corr}} < 0.1 \ \mu\text{A cm}^{-2} \)
- **Low to moderate corrosion**: \( 0.1 \text{ to } 0.5 \ \mu\text{A cm}^{-2} \)
- **Moderate to high corrosion**: \( 0.5 \text{ to } 1 \ \mu\text{A cm}^{-2} \)
- **High corrosion rate**: \( I_{\text{corr}} > 1 \ \mu\text{A cm}^{-2} \)

These measurements are sensitive to temperature and relative humidity (RH) which should be accounted for while interpreting field or laboratory results. Using Faraday's law of electrochemical equivalence, the corrosion current can be related to the section loss (Broomfield, 1997)

\[
1 \ \mu\text{A cm}^{-2} = 11.6 \ \mu\text{m section loss / year}
\]

\[\text{Eq. 2.17}\]
In the case of analytical modelling for the corrosion propagation, simple models have been used i.e. general corrosion on the whole bar circumference (Thoft-Christensen et al. 1996; Frangopol et al. 1997; Enright & Frangopol, 1998; Stewart & Rosowsky 1998 etc.) resulting in the following equation.

$$D(t) = D - 2r_{cor} (t - T_{i})$$ ................................Eq. 2.18

Where $D$ represents original bar diameter; $D(t)$ is the diameter of bar at time $t$; $r_{cor}$ represents the rate of corrosion (based on the relation between corrosion current and sectional loss (Eq. 2.17) and the assumption of uniform corrosion, $2r_{cor}$ has been estimated in literature as 0.023 $i_{cor}$; $t$ is the time since construction of the bridge, and $T_{i}$ is the time to corrosion initiation.

Corrosion of the reinforcing bar over the entire circumference is rare. Therefore by introducing a measured corrosion rate (based on field measurement) to the above equation, it is probable that overly conservative prediction will be made of a component’s service life (Tuutti 1982, Vassie 1984).

Sarveswaren & Roberts (1999) presented effects of corrosion propagation in terms of the area loss as

$$A_{loss} = i . t_{cor}$$ ................................Eq. 2.19

Where $i$ is the rate of corrosion and is given by ‘(-mE - c) + C_{cor}’; $t_{cor}$ is the time of the corrosion i.e. ‘$t_{now} - t_{ini}$’; $m$ and $c$ are the empirical coefficients derived from the site test data; $E$ is the measured half-cell potential value in mV; $C_{cor}$ is the measure of uncertainty in the prediction of corrosion rate; $t_{now}$ is the time of assessment, and $t_{ini}$ is the time taken for chlorides to reach a critical threshold value sufficient to initiate corrosion at the level of the reinforcement.

In the above models, corrosion rate is obtained from the field observations while Vu & Stewart (2000), based on accelerated laboratory testing, proposed that the corrosion rate is a function of concrete quality and cover depth (in addition to the availability of oxygen and water) as these affect the transport of oxygen into the concrete. The empirical model presented by them is

$$i_{cor} (t_p) = i_{cor} (1) . 0.85 t_p^{0.29}$$ ................................Eq. 2.20
\[ i_{\text{corr}}(1) = \frac{37.8(1-w/c)^{-1.64}}{X_c} \text{ (\(\mu A/cm^2\))} \]

Where \(i_{\text{corr}}(1)\) is the corrosion rate at the start of corrosion propagation; \(X_c\) is the concrete cover in cm, and \(t_p\) is the time since corrosion initiation.

It is clear from the above equations that the corrosion rate is not assumed constant as is the case with the previous models. Another model available in literature for the \(i_{\text{corr}}\) is that of Lawanwisut et al. (2003).

\[ i_{\text{corr}} = 0.3683 \ln(t) + 1.1305 \]

Cairns et al. (2003) presented an empirical model for the residual yield strength of corroding steel embedded into concrete to estimate the effects of corrosion propagation as follows.

\[ T_{\text{res}} = A_{SO} (1-0.02Q_{\text{cor}}) f_{y0} \text{ (salt contamination)} \]

\[ T_{\text{res}} = A_{SO} (1-0.01Q_{\text{cor}}) f_{y0} \text{ (carbonation)} \]

Where \(T_{\text{res}}\) represents the residual yield tensile strength of steel; \(A_{SO}\) and \(F_{y0}\) are the original x-section area and yield strength respectively, and \(Q_{\text{cor}}\) is the average loss of cross-section expressed as percentage of the original section.

A resistance degradation function (also known as ‘deterioration function’) was suggested by Mori & Ellingwood (1993) to model the time variant resistance.

\[ R(t) = R_o g(t) \]

Where ‘\(R_o\)’ is the initial resistance and ‘\(g(t)\)’ represents the resistance degradation function. This can be evaluated using the initiation and propagation models (Frangopol et al. 1997). Li (2003) used experimental data to develop a model for the deterioration function for the chloride induced corrosion, which is as follows.

\[ \mu(t) = \phi_0 e^{-\gamma t} \]

\[ V_\phi(t) = \delta t + V_o \]

Where \(\mu(t)\) is the mean deterioration function; \(V_\phi(t)\) is the coefficient of variation of the deterioration function; \(\phi_0\) is the initial deterioration function (i.e. at \(t=0\)); \(\gamma\) is the coefficient representing the rate of structural deterioration; \(V_o\) represents the initial
variation of concrete structural properties, and $\delta$ represents the increase in uncertainty during the deterioration process. The values of $\gamma$ and $\delta$ are experimentally determined as 0.096 and 0.014 respectively whereas $V_0$ and $\phi_0$ are taken as 0.1 and 1.0 respectively (Li, 2003);

### 2.4.3. Cover Cracking, Delamination and Spalling Models

The expansive products of the corrosion reaction, once it is initiated, causes internal tensile stresses into the concrete. These would result in cracking when increases the tensile strength of concrete. The formation and propagation of these cracks in concrete due to the corrosion of reinforcement is predominantly in the radial direction initiating from the steel until it reaches the surface of concrete. (Andrade et al., 1993 and Uddin et al. 2004).

A steel section loss of about 0.1 mm may initiate cracking (Broomfield 1997) in cover concrete. Clear developed an empirical model in 1976 (reported by Broomfield, 1997) to find the time to first cracking.

$$T = \left( \frac{0.052d^{1.22}t^{0.21}}{Z^{0.24}P} \right)^{0.83} \quad \text{Eq. 2.28}$$

Where $T$ is the time to first cracking (years), $d$ is the depth of cover in mm, $Z$ is the surface chloride concentration, $t$ is the age at which $Z$ was measured (years) and $P$ is the water / cement ratio.

A mechanistic cover cracking model has been presented by Bazant (1979). He assumed an increase in the diameter of rebar due to the formation of rust from $D$ to $D+\Delta D$ and hence incorporated the pressure applied to cover concrete. The time to cracking was then estimated by comparing it with maximum permissible concrete stresses. This model has been improved and modified by Liu & Weyers (1998) by dividing the amount of corrosion product into two categories; the amount of product required to fill the volume of pores at the steel concrete interface ($W_{\text{porous}}$) and the corrosion product required to generate the critical tensile stresses. The later corrosion products (required to generate critical tensile stresses) is further subdivided into two categories by Thoft-christensen (2002a) as the volume of rust products needed to fill in the space due to the expansion of concrete around the reinforcement ($W_{\text{expan}}$) and the amount of corrosion products that generates the cracking ($W_{\text{steel}}$).
The models for these stages are presented in detail in Thoft-Christensen (2002a) and Lawanwisut et al. (2003).

\[ W_{\text{crit}} = W_{\text{porous}} + W_{\text{exp ne}} + W_{\text{steel}} \] .......................... Eq. 2.29

\[ W_{\text{crit}} = \rho_{\text{rust}} \left( \pi (d_s + d_o) D + \frac{W_{\text{steel}}}{\rho_{\text{steel}}} \right) \] .......................... Eq. 2.30

Where \( D \) is the diameter of steel, \( \rho_{\text{rust}} \) and \( \rho_{\text{steel}} \) is the density of rust and steel respectively, \( d_o \) is the thickness of pore band around the steel / concrete interface and \( d_s \) is the thickness of corrosion products needed to generate tensile stresses. Idealising the concrete around the reinforcement by a think walled cylinder, with inner radius ‘a’ \( (a = (D+2d_o)/2) \) and outer radius of ‘b’ \( (b=C+(D+2d_o)/2) \), Liu and Weyers (1998) approximated the value of \( d_s \) as

\[ d_s = \frac{Cf_t'}{E_{\text{ef}}} \left( \frac{a^2 + b^2}{b^2 - a^2 + \nu_c} \right) \] .......................... Eq. 2.31

Where \( \nu_c \) is the poisons ratio and \( E_{\text{ef}} \) is the effective elastic modulus for concrete. For the constant corrosion rate, the time to cracking, \( T_{\text{cr}} \), can be estimated as

\[ T_{\text{cr}} = \frac{W_{\text{crit}}^2}{2K_p} \] .......................... Eq. 2.32

where \( K_p \) is the rate of rust production and is related to the corrosion current as follows

\[ K_p = 0.098 \sqrt[\alpha]{\frac{1}{\alpha} \pi D i_{\text{corr}}} \] .......................... Eq. 2.33

Where \( D \) is the diameter of bar; \( i_{\text{corr}} \) is the annual mean corrosion rate and \( \alpha \) accounts for the type of rust product. The limitation of the above model is the assumption of uniform corrosion of the entire bar which is not often observed in practice (Vassie, 1984; Broomfield, 1997). Cairns et al. (2003) empirically established the corrosion penetration thresholds required for the spalling of cover concrete for different top and bottom reinforcement levels assuming the non-uniform corrosion propagation (Fig 2.11b).
The empirical models for the time to delamination presented by Sarja & Vesikari (1996) and Cady & Wayers (1984) are presented by Equation

$$t_{DEL} = 80 \frac{C}{D \cdot R} \quad \text{..................Eq. 2.34}$$

$$t_{DEL} = 0.4918 (C-2.0)^2 \quad \text{..................Eq. 2.35}$$

Where $t_{DEL}$ is the time from the corrosion initiation to delamination; $C$ represents the cover depth (mm); $D$ is the rebar diameter (mm), and $R$ is the corrosion rate ($\mu$m/yr). Various other empirical models has been summarised in Ahmad (2003).

A model for the time required to spalling after the initiation of first crack is derived empirically by Vu & Stewart 2002 as

$$T_{sp} = \frac{A \times 10^{-3} \times \left(\frac{w/c}{C}\right)^{-B}}{i_{corr}(1)} \times 100 \quad \text{crack width > 0.3 mm only} \quad \text{...........Eq. 2.36}$$

Where $w/c$ is the water to cement ratio, $C$ is the concrete cover, $i_{corr}(1)$ is the corrosion rate at the start of corrosion propagation (Eq. 2.21), $A$ & $B$ are 6.5 & 0.573 for 0.5mm crack (as derived by Vu & Stewart (2002) using linear regression). It is assumed here that the first crack would be of the order of 0.05mm while the spalling would occur when cracks are joined together to form a crack width of about 0.4 to 0.5mm. The above equation is valid only for the constant corrosion rate. For the case of time variant corrosion rate, the expression can be found in Vu & Stewart (2002).

Thoft-Christensen (2000b & 2002a) has assumed a linear relation between the increase in corrosion product and rebar diameter loss due to corrosion based on the experimental work of Andrade et al. (1993). Hence the time to spalling can be evaluated as
\[ T_{\text{ser}} = T_{1\text{st}} + \frac{[W_{\text{crit}} - W_o]}{2} r_{\text{corr}} \gamma \] ..........................Eq. 2.37

Where \( W_{\text{crit}} \) is the crack width assumed to initiate spalling (\( = 0.5\text{mm} \) by Vu & Stewart 2002), \( W_o \) is the crack width for the first crack (taken as \( 0.05\text{mm} \)), \( r_{\text{corr}} \) is the corrosion coefficient (see Eq. 2.18 for details) and \( \gamma \) is the proportionality constant between the increase in corrosion product and rebar diameter loss, which is approximated as 1.4 to 4.2 by Thoft-Christensen (2002a & b).

### 2.4.4. Spatial Variability Model

Some of the input variables of the deterioration models may also be varying spatially. These parameters are not completely independent at different points in space but some degree of correlation exists among them. Further, correlation may also exist between different parameters (e.g. cover and diffusivity etc) thus spatial variability should be accounted for in such cases. Stewart & Faber (2003) highlighted the effects of spatial and temporal variability and its effects on the decisions regarding management.

Engelund & Sorensen (1998), Sterritt & Chryssanthopoulos (1999), Karimi & Ramachandran (2000) and Vu & Stewart (2002) introduced random field modelling for some of the model parameters in order to determine the effect of spatial variations on the results of a probabilistic analysis e.g. Sterritt & Chryssanthopoulos (1999) used cover depth and diffusion coefficient as random fields while Vu and Stewart (2002) used concrete compressive strength, concrete cover and surface chloride concentration, and Karimi and Ramachandran (2000) assumed diffusion coefficient and surface chloride concentration as random fields. Ying (2004) evaluated the effects of spatial variability on the maintenance and repair decisions for concrete structures. However, lack of field / experimental data led to different assumptions being made about the choice of spatially random variables and their auto-correlation functions. It has been suggested that the spatial variability does have a great impact on the deterioration process of concrete structures but it may not influence the final choice of the optimal maintenance strategy and one may find the same optimal maintenance strategy with the models that do not consider spatial variability (Ying, 2004).

### 2.5. Monitoring Instruments

The monitoring of reinforcement corrosion in concrete is of significant practical importance if premature failure of the reinforced concrete structures is to be prevented (Broomfield 1997). There is a growing tendency to use non-destructive techniques for
testing the durability of reinforced concrete structures. The predictive performance models can benefit from having such additional information available (Gulikers & Polder, 2003).

Non-destructive evaluation can play vital role for the continuous safety, capacity and serviceability assessment of bridges. Measurement of various parameters and thus different monitoring instruments are required to develop the true structural behaviour. The instruments can be categorized on the basis of different types of measurements required for a typical bridge (Bergmeister & Santa, 2000). Commonly used instruments to quantify the behaviour of a bridge are outlined in Figure 2.12. Details about the instruments, their working principle and possible applications can be found in Rafiq (2001).

Figure 2.12: Classification of common Structural Monitoring Instruments

Bridge monitoring systems based on various deterioration mechanisms are shown in Figure 2.13. These are broadly categorized into two groups.

- Global monitoring methods;
- Local monitoring methods.

Global monitoring is required when sufficient information is not available for the general structural integrity or a specific area of damage or where visual inspection is not possible due to the fact that the flaws may be internal to the structure. Local monitoring describes the situation where monitoring is applied to investigate more detailed characteristics of a known defect. Usually, local monitoring follows as a result of some form of the global monitoring and relies on prior knowledge of the area to be monitored. The global monitoring can be accomplished by calculating dynamic characteristics of the structures.
i.e. monitoring change in frequency, mode shape or comparing dynamic frequency of the structures or by monitoring acoustic emissions or structural displacements etc.

As this study focuses on the deterioration of concrete structures, with particular reference to the corrosion of reinforcement, instruments useful only for this particular deterioration mechanism will be discussed in some detail. This deterioration mechanism has two distinct phases (Section 2.4), i.e. ‘initiation phase’ & ‘propagation phase’, thus available instruments may be divided accordingly into two groups.

### 2.5.1. Instruments for Initiation Phase

Lifetime calculations, and prediction of the residual service life of structures, require quantitative information on cover-zone properties and threshold values for corrosion initiation (McCarter et al. 2001). Initiation of corrosion is a function of surface chloride concentration, temperature & humidity (environmental aspects), diffusion coefficient, permeability & adsorption (concrete durability aspects), threshold chloride contents to initiate corrosion and electrical properties of concrete in the vicinity of rebars (Section 2.3.2). Available instruments / methods that can be used for the permanent monitoring to estimate / predict corrosion initiation are as follows.

#### 2.5.1.1. Chloride Ions Measurement

Chloride profiles can be used to estimate & predict corrosion initiation at various levels in cover concrete. They can also be used to estimate diffusion coefficient and surface chloride concentration (by calibrating the deterioration models), thus predicting the time to corrosion initiation (at rebar level) at any point in time.
In practice, coring or drilling method is used to obtain samples at various depths of concrete cover. These samples are then tested in laboratory to establish chloride profiles (Broomfield 1997). Details for these methods can be found in literature e.g. Duracrete (1998); Henry et al. (2000); Long et al. (2001), & Inoue et al. (2002) etc. These methods are not completely non-destructive hence are not suitable for permanent monitoring of bridges. Further, the accuracy and repeatability of these methods is questionable (Gulikers, 2000).

Zimmermann et al. (1997) developed an electrochemical sensor and used it as multi-probe system to determine free chloride concentration and humidity at various depths of concrete cover but needs to be embedded into the concrete (which can only be done for either new structures or for existing structures during repair etc).

Under a European funded project named ‘Smart Structures’, in an attempt to develop permanent monitoring system for the concrete structures, many new sensors were developed (Klinghoffer et al. 2002). Chloride content sensor, developed in this project, is based on the electrochemical potential measurements between two half-cells. One of these half cells consists of silver chloride, plated on pure silver rod. The other half-cell is the commercial MnO2 reference electrode. The electrochemical potential of silver chloride is dependent on concentration of chloride but MnO2 is independent of it so the difference of potential is used to determine changes of chloride concentration (Klinghoffer et al. 2002).

2.5.1.2. Corrosion Risk Sensors

Schießl & Raupach (1993) devised a method to determine penetration of threshold chloride contents using reinforced steel electrodes acting as anodes embedded at various depths of cover concrete and a stainless steel or platinum coated titanium bar as cathode (Fig. 2.14). These sensors can only be installed in either new structures or during repair in old structures to verify effectiveness of the repair works. This method was extended in the ‘smart structures’ project by Raupach (2002) who developed an expansion ring system to determine corrosion risk at various depths for new as well as existing structures (Fig. 2.15). It consists of several rings of steel at varying depths and can be drilled into the existing structures.
Half-cell measurements are required for the corrosion risk sensors to determine the initiation of corrosion at various depths. As the value of potential corresponding to corrosion initiation is not known with certainty, a probabilistic model is required for this purpose. Lentz et al. (2002) developed a probabilistic model for half-cell potential measurements based on study carried out on large number of measurements performed on Danish and Swiss highways bridges.

Details for the other corrosion risk sensors e.g. metallic nail system can be found in Klinghofer et al. (2002).

### 2.5.1.3. Conductance Measurement Method

Chloride contents can also be determined indirectly by the use of electrical conductivity measurements (e.g. Streicher & Alexander 1995, McCarter et al. 2001. Long et al. 2001). Electrical conductivity of concrete varies with the presence of chloride ions that can be calibrated to estimate chloride ions concentration. Mackechine & Alexander (2000)
questions the suitability of chloride conductivity test for high performance concrete due to uneven level of drying damage, and inadequate saturation of low water / binder ratio.

2.5.2. Instruments for Propagation Phase

Electrochemical techniques are virtually the sole methods of assessing the condition of the steel reinforcement with respect to corrosion without removal of the concrete cover (Gulikers & Polder 2003). The rate of corrosion, once initiated, depends primarily on the quantity of water and oxygen available for the corrosion process (Section 2.3). It also depends on the amount of chloride ions available at the reinforcement level (Bamforth et al., 1997; Quillin, 2001; Ahmad, 2003). The parameters that are of importance in this phase are half-cell potential, corrosion current, resistivity, humidity and temperature of concrete etc. An array of non-destructive methods is available for this phase, ranging from the very mundane e.g. the chain drag method, to the highly sophisticated e.g. electrochemical impedance spectroscopy but only the methods suitable for permanent monitoring are elaborated in the following sub-sections.

2.5.2.1. Half Cell Potential Measurement

The half cell consists of a piece of metal in a solution of its own ions (i.e. copper in copper sulphate etc). By moving a standard half-cell at the concrete surface, condition of embedded steel below the half-cell can be estimated (Fig. 2.16).

![Figure 2.16: Half cell potential measurement method (Broomfield, 2002)](image)

If the steel is passive, the potential measured is small (Zero to -200mV for Cu/CuSO₄, or even positive). If the passive layer is failing or increasing amount of steel are dissolving, the potential becomes more negative thus condition of the steel can be estimated. Different potential values and their respective corrosion conditions are summarised in Table 2.1 for different half cells. Li (2002) developed a probability distribution model for the Cu/CuSO₄...
half cell potentials corresponding to the corrosion initiation based on 78 test beams. It is reproduced in the Fig. 2.17.

Table 2.1: ASTM criteria for corrosion of steel in concrete (Broomfield 1997).

<table>
<thead>
<tr>
<th>Copper / Copper Sulphate</th>
<th>Silver / Silver chloride, 4M KCl</th>
<th>Standard hydrogen electrode</th>
<th>Calomel electrode</th>
<th>Corrosion Condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>&gt; -200 mV</td>
<td>&gt; -106 mV</td>
<td>&gt; +116 mV</td>
<td>&gt; -126 mV</td>
<td>Low (10% risk of corrosion)</td>
</tr>
<tr>
<td>-200 to -350 mV</td>
<td>-106 to -256 mV</td>
<td>+116 to -34 mV</td>
<td>-126 to -276 mV</td>
<td>Intermediate Corrosion risk</td>
</tr>
<tr>
<td>&lt; 350 mV</td>
<td>&lt; -256 mV</td>
<td>&lt; -34 mV</td>
<td>&lt; -276 mV</td>
<td>High (&lt;90% risk of corrosion)</td>
</tr>
<tr>
<td>&lt; -500 mV</td>
<td>&lt; -406 mV</td>
<td>&lt; -184 mV</td>
<td>&lt; -426 mV</td>
<td>Severe corrosion</td>
</tr>
</tbody>
</table>

Figure 2.17: Probability of corrosion initiation as measured in Half cell potential (Li, 2000)

An advantage of half cell measurements is that these are not affected by the presence of surface treatments (Cairns & Melville 2003). It is important to mention here that the half cell estimates the condition and not the rate of corrosion (Gulikers & Podler, 2003). The half cell is a mixed potential representing an anodic and cathodic areas on the rebars. It is not the driving potential in the corrosion cell. Very negative potentials can be found in standard conditions where there is plenty of dissolved iron and no oxygen to form passive layer or to form rust thus should be combined with other measurements e.g. humidity measurements or resistivity measurements etc. It also does not give reliable values in defective concrete, especially concrete with voids (Leelalerkiet et al. 2004).
2.5.2.2. Corrosion Current Measurement

Corrosion is an electro-chemical process involving a continuous exchange of electrical charge between the anodic & cathodic locations on the surface of a steel reinforcing bar i.e. it involves the flows of current between the anode and the cathode. The rate of corrosion can be estimated by monitoring this corrosion current. The overall electrochemical behaviour of the corrosion interface between the steel electrode and the chloride electrolyte can be represented by a simplified electronic diagram, known as Randles circuit (Millard et al. 2001), shown in Figure 2.18.

![Figure 2.18: Equivalent circuit representing the steel-concrete interface (Gulikers and Polder 2003).](image)

Different instruments are used to measure the corrosion current e.g. linear polarisation method with or without ring guard and galvanostatic pulse method. The working principal and the pros and cons of both methods can be found elsewhere e.g. Broomfield (1997), Andrade et al. (2002), and Gulikers and Polder (2003) etc.

In the linear polarization technique, polarization of the steel is carried out using electric current and its effect on the half cell potential is monitored. It is carried out with a half cell incorporating an auxiliary electrode and a variable low voltage DC power supply. Sometimes, another ring shape electrode is used to confine the current flow in a particular rebar area, known as ring guard.

The half cell potential is measured and then a small current is passed from the auxiliary electrode to the reinforcement. The change in the half cell potential is simply related to the corrosion current by the Stern-Geary’s equation.

\[
I_{\text{corr}} = \frac{B}{R_p}
\]

\[\text{Eq. 2.38}\]
Where B is a constant (in concrete 26 to 52 mV, 26mV for actively corroding steel and 52mV for passive steel) and $R_p$ is the polarization resistance (in ohms).

$$R_p = \frac{\text{change in potential}}{\text{applied current}} \quad \text{Eq. 2.39}$$

By applying a second AC perturbation at a high frequency the double layer capacitance, $C_{dl}$, provides a short circuit bypass to $R_p$ and hence a measurement of $R_\Omega$ alone can be made (Fig. 2.18). This is subtracted from the overall potential resistance measurement to give $R_p$.

$$R_p = R_p, \text{measured} - R_\Omega \quad \text{Eq. 2.40}$$

In the galvanostatic pulse transient analysis, a short current pulse is applied to the steel from a counter electrode placed on the concrete surface. The pulse produces a transient change in the potential of the reinforcement (Fig. 2.20), which is continuously monitored using a reference electrode. A detailed numerical analysis of the measurement data allows the interfacial resistance, and hence the corrosion current, to be calculated (Gulikers & Polder 2003).

![Figure 2.19: Linear Polarization measurement method (Broomfield, 2002).](image)

![Figure 2.20: Excitation and response of a galvanostatic pulse (Gulikers & Polder, 2003).](image)
\[ \Delta E = I_{\text{app}} R_t = I_{\text{app}} (R_\Omega + R_i) \]  
\text{.................. Eq. 2.41}\]

Where \( I_{\text{app}} \) is the imposed electrical current; \( R_t \) represents the overall electrical resistance; \( R_\Omega \) is the effective electrolyte resistance between reference and working electrode; \( R_i \) is the effective electrical resistance of the electrochemical interface and is given by

\[ R_i = R_p \left( 1 - \exp \left( -\frac{t}{R_p C_{\text{dl}}} \right) \right) \]  
\text{.................. Eq. 2.42}\]

Where \( R_p \) representing the polarisation resistance of the reinforcement steel and \( C_{\text{dl}} \) as the double layer capacitance of the steel-concrete interface (Fig. 2.18). The \( R_p C_{\text{dl}} \) is the time constant for the corrosion process and can be determined using the galvanostatically induced potential gradients.

The corrosion current is very sensitive to the atmospheric changes e.g. temperature and humidity in pores etc. and actually measures the instantaneous corrosion rate, thus should be averaged over various readings taken at different conditions. It can be used to measure the amount of steel section lost during corrosion at the time of measurement (rate of corrosion) using Faraday’s law (Sec. 2.4.2).

The uncertainty concerning the actual value of \( C_{\text{dl}} \) is assumed to be the principal cause of uncertainty on the on-site determination of \( I_{\text{corr}} \) (Gulikers & Polder 2003). The corrosion current measurements are significantly influence due to surface coating and hence cannot be used to measure the corrosion rate for coated surfaces (Cairns & Melville 2003). It was suggested that both reference electrode and working electrode (steel reinforcement) should be on the same side of the coating to get sensible results.

### 2.5.2.3. Resistivity & Humidity Measurement

As the corrosion is an electrochemical process, the electrical resistance of concrete has a predominant effect on the corrosion rate as ions from cathode have to travel to anode for the reaction to progress. Four-Probe resistivity meter is used to measure the resistance of concrete that can be used to indicate the possible corrosion activity if the steel has been depassivated. Current is applied at the two outer probes and potential difference is measured at the two inner probes. This approach eliminates possible effects due to surface contact resistance. The resistivity, \( \rho \), for homogenous materials can be given by
where $a$ is the electrode spacing; $I$ is the applied current, and $V$ is the potential measured across inner probes. The resistivity measurements can also be used to locate the corrosion cracks in reinforced concrete structures (Lataste et al. 2003).

Humidity is another factor affecting the rate of corrosion as it indicates the amount of water available in concrete pores for reaction. Chloride induced corrosion is believed to be at a maximum at a relevant humidity of around 90-95% (Tuutti, 1982). Increasing the moisture content will limit the oxygen availability thus reduces the rate of corrosion. Similarly reducing water content also reduces corrosion rate due to limited availability of water. A method of humidity monitoring based on frequency measurements using capacitance probe is presented by Klinghoffer et al (2002). The frequency is proportional to the capacitance of the concrete and thereby to the humidity. The laboratory results by Klinghoffer et al. (2002) have shown that the frequency is inversely proportional to the water content. The sensor consists of two rubber electrodes between which frequency is measured. The measured frequency is converted to a volt signal and corresponds to the average water content in concrete expressed in percentage.

2.6. Integration of Monitoring Data into BMS

Monitoring systems reveal defects but do not usually indicate the condition / performance of the material or elements of the structure under consideration. The existence of deterioration detected by the monitoring systems does not by itself indicate whether a structure is threatened. Interpretation of the data obtained through monitoring of structures is equally important to aid decisions regarding management activities. Despite a considerable amount of research in the field of non-destructive testing and structural monitoring etc, very little have been done to effectively integrate the data obtained through these methods into the management system.

Various attempts to use NDE results within the framework of reliability based management systems to assess the performance and to illustrate their effects on the inspection and maintenance activities are discussed in some detail in Rafiq (2001). A brief summary of these methods are presented here.
2.6.1. Qualitative Approach

Present management systems use integer-valued condition ratings to establish the conditions of bridge elements, to predict the deterioration rate, and to determine the needs for maintenance and repair. Hearn & Shim (1998) redefined these condition states as stages in the service life of a bridge element. These condition states are distinguished by attributes of the stages (Table 2.2) that can be detected or measured by NDE methods. The applications of these condition states for different materials i.e. steel and concrete, and various NDT instruments for different states are elaborated further in Hearn & Shim (1998).

Table 2.2: Integration condition states (Hearn & Shim, 1998).

<table>
<thead>
<tr>
<th>Stage</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Protected 1</td>
<td>Element is protected against agents that can cause deterioration.</td>
</tr>
<tr>
<td>Exposed 2</td>
<td>Elements does not have protection. Aggressive agents have not reached a concentration that may initiate deterioration.</td>
</tr>
<tr>
<td>Vulnerable 3</td>
<td>No deterioration process active. Aggressive agent is present, and a deterioration process may become active at any time.</td>
</tr>
<tr>
<td>Attacked 4</td>
<td>A deterioration process is activated.</td>
</tr>
<tr>
<td>Damaged 5</td>
<td>Element is measurable or visibly damaged</td>
</tr>
</tbody>
</table>

2.6.2. Statistical Distribution Approach

Data obtained by monitoring can be used to establish statistical properties of the measured condition. These can be used along with different limit states (i.e. Ultimate, Serviceability, and Durability limit state etc) to establish failure probabilities as shown by Equation 2.44 (Bergmeister & Santa, 2000) which can then be used in the bridge management procedures (section 2.2.3).

\[
P_f = P \left[ \text{Monitored Data} \geq \begin{cases} \text{Ultimate limit state} & \text{ULS} \\ \text{Serviceability limit state} & \text{SLS} \\ \text{Durability limit state} & \text{DLS} \end{cases} \right] \quad \text{Eq. 2.44}
\]
2.6.3. Whole Life Optimisation Approach

Frangopol et al. (1997a) presented a methodology for the integration of non-destructive methods into the reliability based management system through the life-cycle cost analysis of bridges. The quality of the non-destructive instruments is related to the costs of inspections conducted using these instruments as shown by Equation 2.45.

\[ C_{\text{ins}} = \alpha_{\text{ins}} (1 - \eta_{\text{min}})^{20} \] .......................... Eq. 2.45

Where \( C_{\text{ins}} \) represents the cost of real inspection; \( \eta_{\text{min}} \) is the minimum detectable damage intensity using instruments used for the inspection; \( \alpha_{\text{ins}} \) is the cost for the ideal inspection (i.e. \( d(\eta) = 1 \) for \( \eta > 0 \)); \( d(\eta) \) is the probability of damage detection. Combining these with the cost of repairs etc for a variety of strategies using the life-cycle cost analysis, the optimum maintenance plan is selected based on the minimum cost criteria (Eq. 2.1) as explained in the Section 2.2.4.

2.6.4. Reliability updating Approach

The data from non-destructive testing and monitoring is never perfect, and depends on many uncertain factors e.g. condition of the structure under inspection and its service environment, sensitivity of inspection equipment, material imperfections and operator training skills etc (Zheng & Ellingwood, 1998). These uncertainties must also be taken into account while estimating the reliability of the structures.

A methodology was proposed by Tang (1973) and Madsen (1987) to update the probability of failure assuming the outcome of inspection as an event. The methodology has been successfully developed and used in the management of offshore structures. Zheng & Ellingwood (1998) and Zhang & Mahadevan (2000) have used this concept to update the fatigue reliability of steel bridges using the non-destructive testing results. The uncertainties associated with the testing instruments are incorporated using the probability of detection (POD) curve that relates the probability of detection (instrument uncertainty) to the size of defect.

Faber & Sorensen (2001 & 2002) proposed a generic framework to integrate the non-destructive inspection results to update the ‘defect rate’ of the elements of a structure. Where the ‘defect rate’ is assumed to be the rate of occurrence of any state of the considered components e.g. visual indication of corrosion in the case of concrete
structures. An indicator for the condition state of the structure is inspected which is then used to update the knowledge regarding the ‘defect rate’ using Bayesian methodology. The instrument uncertainty is expressed in terms of the probability that the instrument indicates degradation given that the degradation has initiated together with the probability that the instrument indicates degradation given that the degradation has not initiated.

For the case where the inspection indicates a defect, the updated defect rate would be

\[
f'_{θ|D}(θ|F) = c(f'_{θ|F}(θ|F)p_i + f'_{θ|F}(θ|F')1-p_i) \]  \hspace{2cm} \text{Eq. 2.46}

and the posterior defect rate for the case where the inspection indicates that there is no defect would be

\[
f'_{θ|D}(θ|F) = c(f'_{θ|F}(θ|F)q_i + f'_{θ|F}(θ|F')1-q_i) \]  \hspace{2cm} \text{Eq. 2.47}

Where \(f'_{θ|D}(θ|D)\) and \(f'_{θ|D}(θ|D)\) are the posterior defect rates where the inspection does and does not indicates a defect respectively; \(f'_{θ|F}(θ|F)\) and \(f'_{θ|F}(θ|F')\) are the defect rates given the inspected component is defective and not defective respectively; \(p_i\) and \(q_i\) are the probabilities (\(P(F|D)\) & \(P(F|D')\) respectively) that the inspected component is defective (F) given the observation that the indicator does and does not indicate a defect respectively and \(c\) is the normalizing constant. Application of the methodology with respect to the concrete structures subjected to corrosion is illustrated in Malioka & Faber (2003) where the cover depth measurements and the ladder arrangement are used as inspection indicators in order to update the probability of visual corrosion indication at a given time.

### 2.6.5. Critique on the Available Methods

The methodology presented by Hearn & Shim (1998) is a good step towards the integration of NDE method into the management systems but is a qualitative approach. This methodology cannot take into account the extend of damage within the defined limit states in quantitative manner that are obtainable with the state-of-art health monitoring methods. The framework for the integration of monitoring into the reliability analysis by Bergmeister & Santa (2000) uses health monitoring data to establish the statistical properties (i.e. mean and standard deviation etc) of the parameter under consideration. These are then related to the defined limit states to evaluate reliability of the structures.
being monitored. This methodology cannot be readily applied to the deterioration prone concrete structures as the monitoring instruments are not yet available to provide direct assessment of the corrosion damage (particularly in the initiation phase). Hence, the evaluated statistical properties of the monitored parameters cannot be related directly to the defined limit states (i.e. durability, serviceability and ultimate limit states) as proposed in the methodology. This method also fails to take into account the reliability of various monitoring methods used to detect the defects.

The framework for the integration of non-destructive data into the reliability based management system through life-cycle cost analysis seems very effective, but its application requires extensive field data regarding the instruments before the approach can be applied to the real structures with confidence. The instruments assumed during the development of methodology were unrealistic with hypothetical probability of detection curves. Furthermore, the relation used to relate the cost of inspection to the quality of instruments for the inspection is also imaginary hence the proposed methodology is of limited use with the scarce available field data. The damage intensity used to relate cost of inspection to the quality of instrument relates to the loss of steel section, hence the approach is not suitable for proactive management systems. The method also involves substantial amount of work to calculate costs related to the options selected for comparison and yet the selected option may not be the true optimized solution as some cases may be accidentally ignored.

The methodology presented by Faber & Sorensen (2001 & 2002) uses Bayesian approach to update the information regarding the attainment of defined condition states at a given time. This methodology is also capable of incorporating formally the uncertainty associated with instruments / measurements within the updating framework. A limitation of this methodology is that the outcome of the inspections is assumed to be either ‘defective’ or ‘not defective’ that does not fully take into account the potential of available state-of-art monitoring instruments. Another limitation of this method is that it updates the probability of attaining certain condition state at a particular point in time e.g. the probability of getting a visual corrosion indication at 50 years, hence requiring a significant computational effort to establish the time variation of the reliability curve for structure under consideration. It can not account for the defect detected at different point in time for various components (zones of the structure) under consideration, and hence does not fully utilise the potential of permanent health monitoring of structures.
2.7. Discussions and Conclusions

In order to manage the expanding structural stock economically, effectively and with efficiency, a management system is vital and because of the inherent uncertainties associated with load and deterioration models and geometric properties (e.g. concrete cover) etc, systems based on reliability is gaining more popularity.

Inspections are mainly carried out only at the fixed intervals and no information is available regarding the performance of structures in-between the inspections. Hence these cannot be used solely for the prediction of future condition and reliability of structures, which is the key ingredient for the reliability based management system. Principal inspections that carry somewhat detailed investigation of the condition of structures is usually expensive and also requires traffic disruption or management and is carried out at fixed interval (every six years) thus an effective inspection strategy (in addition to the optimum repair and maintenance plans) is also required that can target specific areas and problems (based on certain limit state being approached) predicted by the reliability analysis.

Key mechanisms for corrosion of steel embedded in concrete include chloride attack and carbonation. Chloride induced corrosion is the governing phenomenon for the deterioration in the UK and many other countries. Of all the predictive models available, Fick’s law has been most widely used in practice to model the initiation of corrosion in concrete whereas its propagation is based on simplistic corrosion penetration/section loss relationships. Crack initiation is modelled either empirically or by comparing stresses generated by expansive rust products with the tensile strength of concrete.

The input parameters of these predictive models are fraught with uncertainties that resulted in the development of probabilistic deterioration modelling. The output of these models contains significant uncertainties hence limits the effective use of these models for practical applications. These uncertainties can be reduced considerably by the effective use of structure specific data obtained through health monitoring methods.

Different methodologies used in the context of integration of the non-destructive testing and monitoring methods into the management systems have been presented and their limitations are discussed in the previous section. The ideas from these methods are extended in this project and a methodology is developed that can effectively incorporate
the information obtained through in-service health monitoring of structures to update the quality of performance prediction by reducing associated uncertainties, and hence improve the confidence in decisions regarding maintenance and repair activities.
Chapter 3

Structural Safety and Reliability

3.1. Introduction

Uncertainties are unavoidable in the design and planning of engineering systems (Ang & Tang, 1975). The need to rationally treat these uncertainties has induced a lot of concern among the scientists and engineers of today. The best way to deal with these uncertainties is considered to be the use of reliability analysis for the design and assessment of structures (Benjamin & Cornell, 1970; Ang & Tang, 1975; Thoft-Christensen & Baker, 1982; Melchers, 1999; Nowak & Collins, 2000). The concept of structural safety and reliability is introduced in this chapter. Various sources of random uncertainty are highlighted and the method of their quantification is illustrated. The reliability analysis methods used in the present research and the simulation methods used for its evaluation are elaborated. Bayesian theorem, its applications for the event updating case, and its use in the decision theory is also presented in this chapter.

3.2. Classification of Random Uncertainty

Uncertainty may be interpreted as the lack of precise information / knowledge about some quantity. Broadly speaking, uncertainty is of two main types; ‘systematic’ and ‘random’. Depending upon its nature, the random uncertainty is subdivided into two main categories, namely, ‘aleatoric’ and ‘epistemic’ (Melchers, 1999 and O’Hagen & Oakley, 2004).

3.2.1. Aleatoric Uncertainty

Aleatoric uncertainty (also known as ‘objective’ or ‘physical’ uncertainty) is described as the one arising from the inherent variability or randomness in the physical quantity e.g. loads, material properties and dimensions etc. These uncertainties cannot be reduced or eliminated due to greater availability of data regarding the phenomenon.
3.2.2. **Epistemic Uncertainty**

Epistemic uncertainty is due to the imperfect knowledge or incomplete information. In theory, these uncertainties can be completely eliminated or at least reduced when additional data regarding the phenomenon becomes available. Depending upon the nature, these uncertainties are divided broadly into two main groups, namely, ‘statistical uncertainty’ and ‘modelling uncertainty’ (Melchers, 1999; Duracrete 1999).

In addition to these categories, many other types of random uncertainties have been defined e.g. decision uncertainties, phenomenological uncertainties and human error etc. The details of these can be found in Ang & Tang (1975), Schneider (1997) & Melchers (1999) etc.

### 3.2.2.1. Statistical Uncertainty

A typical way of describing random physical uncertainty is through the use of a probability distribution. Its statistical estimators i.e. mean and higher moments are generally estimated from the available data. This induces additional uncertainty as the estimators of the sample does not perfectly represent the moments of the whole population and may vary from sample to sample. This type of uncertainty is termed as ‘statistical uncertainty’. It can be incorporated into the reliability analysis by describing the moments of the distribution as random variables or the reliability analysis might be repeated using different values of the parameters to indicate sensitivity (Melchers, 1999).

### 3.2.2.2. Modelling Uncertainty

Scientific, semi-empirical or empirical models are generally used to mimic an actual phenomenon or a physical process. Assumptions and simplifications have to be made to allow the modelling of such phenomena. Modelling uncertainty is referred to as the uncertainty associated with the difference between true and estimated / predicted behaviour obtained through such modelling. Modelling uncertainty is often associated with limited knowledge and hence can be reduced with research (leading to better models) or increased availability of data (Melchers, 1999). It is usually expressed in terms of the ratio of true response to the one predicted through modelling.

\[ X_m = \frac{\text{actual response}}{\text{predicted response}} \]
3.3. Uncertainty Modelling

Modelling or, in other words, quantification of uncertainty for the load and deterioration variables is important for the evaluation of safety characteristics of structures. The most widely accepted method for the representation of a random uncertain quantity is by considering it as a random variable and assigning a probability distribution function (PDF) for it. Each value of the distribution function represents outcome of the physical quantity along with the probability of its occurrence. The shape of the distribution function and its parameter values are generally established through physical reasoning and available laboratory, or field data, e.g. the concrete cover depth can be modelled using a normal distribution.

Various distributions used in the course of this research have been elaborated in some detail in Appendix A.

3.4. Moments of a Distribution

The characteristics of a random variable are completely described by its probability distribution function. However, in certain cases, the distribution function may not be known or it may not be suitable to fit any specific function to any particular sample. Hence an approximate description of a random variable is often necessary (Ang & Tang, 1975) such as in the case of empirical distributions. More concise descriptors (moments of the distribution), summarising only the dominant features of the random variable, are sometimes sufficient for engineering purposes (Benjamin & Cornell, 1970). These can be used more conveniently within the reliability analysis framework and can be readily obtained from observed data.

3.4.1. First Moment (Mean)

The first moment, also referred to as ‘mean’ or ‘expected value’, is the most commonly used and best understood measure of central tendency. It is simply the weighted average of all the possible outcomes of a random variable. The expression for the first moment for a continuous random variables, X, is given in Equation 3.1.

\[ E(X) = \mu_X = \int_{-\infty}^{\infty} x f_X(x) \, dx \]

....................Eq. 3.1

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3.4.2. Second Moment (Variance)

The second moment, or variance, is the measure of dispersion or variability around the central value i.e. how closely the values of the variable are clustered around the expected value and is given by Eq. 3.2.

\[
Var(X) = \int_{-\infty}^{\infty} (x - \mu_x)^2 f_X(x) dx \\
\text{Eq. 3.2}
\]

\[
Var(X) = E((x - \mu_x)^2) = E(X)^2 - (\mu_x)^2 \\
\text{Eq. 3.3}
\]

Another way of representing the dispersion of data is through the standard deviation, \( \sigma \), which is defined as

\[
\sigma_x = \sqrt{Var(X)} \\
\text{Eq. 3.4}
\]

A more widely used method to represent the dispersion of data about its mean value is by normalising the standard deviation by the mean value termed as the coefficient of variation (COV). It is a unitless quantity and gives the relative dispersion of data around the mean and is sometimes more useful than the variance or standard deviation itself (Ang & Tang, 1975)

\[
COV(X) = \frac{\sigma_x}{\mu_x} \\
\text{Eq. 3.5}
\]

3.4.3. Third Moment (Skewness)

Another useful property of a random variable is the symmetry or lack of symmetry of its probability distribution, which is measured by the third moment.

\[
E((X - \mu_x)^3) = \int_{-\infty}^{\infty} (x - \mu_x)^3 f_X(x) dx \\
\text{Eq. 3.6}
\]

A convenient non-dimensional measure of skewness is the skewness coefficient,

\[
\gamma_1 = \frac{E((X - \mu_x)^3)}{\sigma_x^3} \\
\text{Eq. 3.7}
\]

This would be ‘0’ if the distribution is symmetric about its mean. A positive value of the \( \gamma_1 \) corresponds to the PDF with dominant tails on the right and a negative value refers to the distribution with long tails on the left.
3.4.4. Fourth Moment (Kurtosis)

The fourth moment about the mean is a measure of the distributions flatness, and is given by

\[ E(X - \mu_X)^4 = \int_{-\infty}^{\infty} (x - \mu_X)^4 f_X(x)dx \]          \text{Eq. 3.8}

A non-dimensional measure of the flatness of a distribution is termed as ‘coefficient of kurtosis’ and is given by

\[ \gamma_2 = \frac{E(X - \mu_X)^4}{\sigma_X^4} \]                     \text{Eq. 3.9}

The coefficient of kurtosis is usually compared to a standard value i.e. the coefficient of kurtosis for the normal distribution is ‘3’. A value of \( \gamma_2 \) higher than ‘3’ indicates a more peaked distribution than the normal distribution and vice versa. Other higher moments can also be defined similarly, but for engineering purposes the first four moments are considered sufficient (Melchers, 1999). These four moments can be estimated easily through the available data and are sufficient to enable fitting of a probability density function to be undertaken using the Pearson family curves (Elderton & Johnson, 1969).

3.4.5. Covariance and Correlation

The uncertain physical quantities, represented by random variables, may be linked through functional relationships. In this case, they are not completely represented by their marginal distributions. A joint probability distribution is required in order to capture fully their characteristics.

The joint or bivariate cumulative distribution function of two random variables, X and Y, is given by

\[ F_{X,Y}(x,y) = P(X \leq x, Y \leq y) = \int_{-\infty}^{x} \int_{-\infty}^{y} f_{X,Y}(u,v)dvdu \]          \text{Eq. 3.10}

The joint second moment about the means \( E(X) \) and \( E(Y) \) is termed as covariance of X and Y. This provides a summary of the degree to which two random variables are associated. It is given by

\[ Cov(X,Y) = E[(X - \mu_X)(Y - \mu_Y)] = E(XY) - E(X)E(Y) \]          \text{Eq. 3.11}

Where \( E(X) \) and \( E(Y) \) are the expected values of marginal probability density functions, \( f_X(x) \) and \( f_Y(y) \), of the random variables X and Y respectively.
and $E(XY)$ is the expectation of the product of two random variables $X$ and $Y$ and is given by

$$E(XY) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} xy f_{x,y}(x,y) \, dx \, dy$$

..................Eq. 3.13

Where $f_{x,y}(x,y)$ is the joint probability density function for the random variables. If $X$ and $Y$ are statistically independent, Eq. 3.13 would become

$$E(XY) = \int_{-\infty}^{\infty} x f_x(x) \, dx \int_{-\infty}^{\infty} y f_y(y) \, dy = E(X)E(Y)$$

..................Eq. 3.14

The Cov $(X,Y)$ will be ‘0’ if $X$ and $Y$ are statistically independent (from Eq. 3.11 & Eq. 3.14). The covariance is basically a measure of the degree of linear interrelationship between the variates $X$ and $Y$. Normalised covariance is generally used to show this relationship, known as correlation coefficient, which expresses the relative strength of the association between $X$ and $Y$. Its value lies between -1 and +1

$$\rho = \frac{\text{Cov}(X,Y)}{\sigma_x \sigma_y}$$

..................Eq. 3.15

### 3.5. Bayesian Statistics

The most common probability measure expresses the long-run frequency for an event occurring in many repeated experiments (Lapin, 1990). Such probability is generally termed as ‘objective probability’ as there can be no disagreement about how these are obtained.

Decisions in engineering and business worlds must often be made under uncontrolled conditions where formalism of traditional statistics is at best impractical – and sometimes even impossible to implement i.e. there is often no population from which the samples can be drawn (Lapin, 1990). Because the usual sample data are not available in so many decisions that must be made under uncertainty, the approach taken must be different. This approach involves so called ‘Bayesian Statistics’. Thomas Bayes originally proposed the use of probabilities to quantify a person’s judgement regarding uncertain event for the
cases where repeatable experiments cannot be performed. Such probabilities are generally termed as ‘subjective probabilities’ or ‘personal probabilities’.

In 1763 Bayes presented an approach to update probabilities interpreted as the ‘degree of belief’ by rigorously combining the earlier understanding, or judgement, of the scientist or engineer regarding the phenomenon with data obtained through additional experiments. The earlier understanding of the phenomenon is termed as the ‘prior belief’ (belief or understanding held prior to observing the current set of data), and the new belief resulted after updating the prior belief is termed as the ‘posterior belief’. This approach is very useful in the cases where experimental data is scarce and/or contains uncertainties, and hence cannot be used solely for the determination of distribution parameters using the objective approach provided by classical statistical theory (Ang & Tang, 1975).

A major application of Bayesian approach is in the decision making subjected to uncertainty. A generic framework for the use of Bayesian approach in such a case is shown in the following figure.

Figure 3.1: Bayesian analysis (French & Smith, 1997)
It begins by structuring the problem to separate issues of uncertainty, belief, and knowledge (modelled by probabilities) from issues of preference or value judgement (modelled by utilities). They are recombined at the end when the expected utilities of the possible actions are formed. Next, the prior beliefs are modelled using probability distributions (Prior probability distribution). The Bayesian analysis is then used repeatedly in a cyclic manner to update the prior beliefs (Posterior distribution) in the light of merging information using the output of previous analysis as prior for the next, until the next decision has to be taken. At this stage, the posterior probabilities are combined with the expected utilities to rank possible strategies.

Bayesian theorem in its basic form is given in Eq. 3.16, derivation of which can be found in Appendix B. Other forms of Bayesian theorem starting from the simple cases, i.e. discrete data and discrete parameter, to the most general form i.e. continuous data and continuous parameter can be found in Press (2003).

\[
P(E_i | A) = \frac{P(A \cap E_i)}{P(A)} = \frac{P(A | E_i)P(E_i)}{\sum_{i=1}^{n} P(A | E_i)P(E_i)}
\]

...............Eq. 3.16

Where \( P(A) \) = the probability of occurrence of event \( A \);
\( P(E_i) \) = the probability of occurrence of event \( E_i \);
\( P(A|E_i) \) = the probability occurrence of event \( A \) given the event \( E_i \) has taken place;
\( P(E_i|A) \) = the probability of occurrence of event \( E_i \) given the event \( A \) has taken place.

In its simplest form Bayesian theorem can be represented as

Posterior distribution = Constant x Likelihood x Prior distribution

i.e. the posterior distribution is proportional to the product of likelihood times the prior information. The constant in the above expression represents the normalizing factor that would ensure that the area under the PDF is equal to 1. It can be seen from the above expression that quality of both the prior as well as the likelihood function (representing the quality of experimental output) plays an important role in the development of posterior distribution. This is illustrated graphically in Figure 3.2. It can be seen from the figure that good quality data will shift the posterior distribution towards the likelihood function if the prior belief about the parameter is not very strong. Similarly greater confidence in the prior belief will have more weight in establishing posterior distribution if the quality of data is
poor. A good quality prior coupled with high quality experimental data could result in a considerably tighter distribution (with reduced uncertainty) of the posterior performance prediction (Fig. 3.2).

![Figure 3.2: Effects of prior and likelihood on posterior distribution](image)

In addition to the quality of experimental outcome, the quantity of information is also important. As discussed previously (Fig. 3.1), the Bayesian analysis is repeated in a cyclic manner as more data becomes available hence the uncertainty can be reduced continuously with the merging information as illustrated in Figure 3.3.

![Figure 3.3: Effect of data quality and quantity in reducing uncertainty](image)
The primary goal for the engineer is to establish consistent decision basis for the planning, design, construction, and management of engineering systems such that the overall life cycle benefits of the system are maximised whilst maintaining the safety above minimum prescribed levels. In many practical cases, the consequences of a decision depend on some factors that are not known with certainty i.e. the decisions have to be based on uncertain information. Bayesian statistical decision theory provides a mathematical model for making engineering decisions in the presence of uncertainty (Benjamin & Cornell, 1970). A decision tree comprising of action, state, and consequence (Fig. 3.4) is generally used to represent the decision problem in which the engineer has to choose an action from several possible alternatives that, depending on the uncertain state of nature, would lead to associated consequences (e.g. costs).

Figure 3.4: Decision tree for prior and posterior analysis (Benjamin & Cornell, 1970)

For decisions regarding management activities of deterioration prone systems the possible actions could be the time to first intervention of the system for either repair or a detailed investigation to determine the extend of deterioration etc. The uncertain state of action in this case would be the actual time at which the performance will approach minimum prescribed safety level while the consequence would be the costs related to the actions and the state of nature. The idea in this type of decision analysis is to delay the intervention time whilst keeping the safety levels above prescribed limits so as to minimise the life cycle costs.
It is often feasible, but not necessarily economical, to obtain more information concerning the state before choosing an action from various alternatives. The question in this case would be (Benjamin & Cornell, 1970)

- How to combine this new information with the previous probabilities before making the decision analysis
- Whether (and how) one should obtain more such information before making a final decision.

Based on the above objectives, the decision analysis is divided into three main categories.

- Decision analysis with given information – Prior analysis
- Decision analysis given new information – Posterior analysis
- Decision analysis given new unknown information – Pre-Posterior analysis

The decision tree for the prior and posterior analysis is same (Fig. 3.4) except that the posterior probabilities for the state of action are used after updating using Eq. 3.16.

![Decision Tree](image.png)

**Figure 3.5: Decision tree for pre-posterior analysis (Benjamin & Cornell, 1970)**
Primary objective in the pre-posterior analysis is to establish benefits and potential of getting additional information and establishing its feasibility so that the decision regarding the nature of experiment can be made (i.e. whether or which experiment to be performed to minimise the costs). A typical decision tree for the pre-posterior analysis is shown in Figure 3.5.

The posterior analysis has been used in this study for the comparison of various management strategies based on (see Ch. 8 for details)

- Decisions supported by regular inspections only
- Decisions supported by predictive models updated by regular inspections
- Decisions supported by predictive models updated by optimised inspections
- Decisions supported by permanent health monitoring systems

3.6. Structural Reliability Analysis

Because of the random nature of load, material and deterioration variables, the performance of structural systems (that is dependent on these variables) is also a random variable. The performance of such systems is expressed in terms of its reliability or safety index. In general, the reliability of a system (or an element of a system) means its likelihood or probability of satisfying particular design or operational objectives (Das, 2000). A graphical illustration of the reliability analysis as an input/output problem is presented in Figure 3.6.

![Figure 3.6: Illustration of the procedure for reliability analysis (Shetty & Chubb, 2000)](image_url)
The limit state function or safety margin, $M$, can be evaluated using

$$M(t) = R(t) - S(t) \quad \text{.................Eq. 3.17}$$

Where $M(t) > 0$ indicates that the system is safe at any given time $t$, $M(t) < 0$ indicates its failure and $M(t) = 0$ is the limit state between the safety and failure; $R(t)$ and $S(t)$ represents the time dependent function (model) describing the strength or resistance and the load effects respectively. It is important to mention here that the failure of a limit state does not mean structural failure. In most cases it refers to a situation when the performance of the structure exceeds a predefined limit state, e.g. initiation of rebar corrosion would be referred as a ‘failure’ if the chloride concentration at the rebar level exceeds the threshold chloride concentration.

The durability design can be presented in two different, but theoretically equivalent, formats for the limit state functions. These are the ‘intended service period design’ and the ‘life time design’ (Duracrete, 1999). In the intended service period design the condition is that the limit state may, with certain reliability, not reach within the intended service period.

$$P_{f,T} = P\{R(t) - S(t) < 0\} \leq P_{acc} \quad \text{.................Eq. 3.18}$$

Where $P_{f,T}$ is the probability of failure of the structure within intended service period, $T$, and $P_{acc}$ is the accepted maximum value of the probability of failure. In the lifetime design, the explicit function of life time of the system, $L$, is defined as (See Fig. 3.7)

$$L = t\{R, S\} \quad \text{.................Eq. 3.19}$$

The reliability of the structure can be introduced by limiting the probability of exceeding the accepted value.

$$P_f = P\{L < T\} \leq P_{acc} \quad \text{.................Eq. 3.20}$$
Another related quantity is the failure rate or hazard function, $h(t)$, expressing the conditional probability of failure at a given time ‘$t + dt$’ given that the structure has not failed at time $t$.

$$h(t) \, dt = P[\text{failure in } t, t + dt / \text{no failure in } (0,t)] \quad \text{.........Eq. 3.21}$$

To find $h(t)$, one should divide $f_T(t)$ by the probability that no failure occurs (also called survival probability) (Ying, 2004).

$$h(t) = \frac{f_T(t)}{1 - P_F(t)}$$

A special case for the limit state function occurs if either $R$, or $S$, or both, are independent of time. Even if the load and resistance are time dependent, the limit state functions for designing structures are rarely formulated in this way and are simplified to time independent quantities (Duracrete, 1999). In this case, reliability of the system can be evaluated using Eq. 3.22.
Reliability = 1 - P_f  
...............Eq. 3.22

P_f = P (M 0) = P (R - S 0)  
...............Eq. 3.23

In this case, the probability of failure or the life time function is constant in the time scale as shown.

If R and S are statistically independent then

\[ P_f = \int_{-\infty}^{\infty} F_R(x)f_s(x)dx \]  
...............Eq. 3.24

In many cases, the load and resistance variables are not independent, but are functions of other random variables i.e. material properties and dimensions etc. In this case the limit state is considered as a function of all the basic variables, X_i.

\[ G(X_i; i=1,2,...N) = 0 \]  
...............Eq. 3.25

and the failure probability expression will become
In many cases, the time-variant problem (Eq. 3.18) can be simplified into a time invariant problem. Following are the examples of such cases.

**Case 1: When only load or actions are time dependent**

The safety margin in this case would be

\[ M(t) = R - S(t) \]  
\[ \text{Eq. 3.27} \]

Typical realisations for the load and resistance variables are presented in Fig. 3.10.

Considering the maximum value of load or action for any given time period, \( T \), the time variant reliability problem can be reduced down to time invariant problem. In this case, the probability of failure will be defined for the time interval ‘\( T \)’, for which the maximum loads have been established.

\[ P_f(T) = P\left[ \min_{0<t<T} M(t) \leq 0 \right] = P\left[ R \leq \max_{0<t<T} S(t) \right] \]  
\[ \text{Eq. 3.28} \]

**Case 2: When only resistance is time dependent**

Similar to the previous case, the time variant reliability problem can be reduced to time invariant problem by considering the minimum of resistance for a specified time interval, \( T \).

The safety margin in this case would be

\[ M(t) = R(t) - S(t) \]  
\[ \text{Eq. 3.29} \]

And the probability of failure for the time interval ‘\( T \)’ would become
\[ P_f(T) = P\left[ \min_{0 < t < T} M(t) \leq 0 \right] = \left[ \min_{0 < t < T} R(t) \leq S \right] \]

.........Eq. 3.30

**Case 3: When both load and resistance are time dependent**

The load and resistance are both time dependent but they are independent of each other. There is no simple procedure in this case to calculate the reliability accurately but sometimes an upper bound approximation is applied to reduce the problem to time-invariant reliability analysis as follows (Ying, 2004)

\[ M(T) = \min_{0 < t < T} R(t) - \max_{0 < t < T} S(t) \]

.........Eq. 3.31

hence the probability of failure within the time interval, \( T \), would become

\[ P_f(T) = P\left[ M(T) \leq 0 \right] \]

.........Eq. 3.32

The integral in Eq. 3.26 can be solved in several ways i.e.

- Analytical Integration
- Numerical Integration
- Numerical Approximation (FORM & SORM)
- Monte Carlo Simulation Method

Close form solution (i.e. analytical and numerical integration) of the integral in Eq. 3.26 is possible only in very simple cases. For most engineering applications, numerical approximation and Monte Carlo methods have been utilised. The involvement of complex mathematical functions (i.e. inverse error function in the predictive models) and the conditional probabilities evaluation involving intersection function in the expressions (i.e. Eq. 3.16) makes it extremely difficult to apply numerical approximation methods i.e. FORM & SORM. Hence Monte Carlo simulation methods have been used in this study to solve the above integral (Eq. 3.26).

**3.7. Monte Carlo Simulation**

Monte Carlo simulation involves ‘sampling’ at ‘random’ to simulate artificially a large number of experiments and to observe the results (Melchers, 1999). In structural reliability analysis problem, samples of each random variable, \( X_i \), are generated randomly and the
limit state function, $G(x)$ (Eq. 3.25), is evaluated for each set of the generated random variables. The probability of failure for ‘N’ conducted trials can be given approximately by (Melchers, 1999)

$$P_f \approx \frac{n(G(x_i) \leq 0)}{N}$$  

\[\text{................Eq. 3.33}\]

Where $n(G(x_i) \leq 0)$ refers to the number of trials n for which ‘$G(x_i)$ $\leq 0$’. The direct sampling method (described above) is the simplest Monte Carlo approach for reliability problems but not the most efficient (Melchers, 1999). The efficiency of the simulation process depends on the magnitude of the probability of failure i.e. total number of the samples required, N, depends on the accuracy required for the $P_f$. The smaller the value of $P_f$, the larger will be the number of samples required to get reasonable values of the $P_f$.

Different variance reduction methods are available in literature that can be used to increase the efficiency (required number of samples and hence the computation time and computer resources etc) of the simulation process, these include

- Latin hypercube sampling
- Importance sampling
- Stratified sampling
- Adaptive sampling
- Directional simulation method
- Antithetic variates method
- Conditional & Generalised conditional expectation method

Among these variance reduction techniques, Latin hypercube sampling method and conditional & generalised conditional expectation methods offer an advantage over other methods in that they do not require information regarding the important regions or variables in advance and are equally applicable to time invariant and time variant problems (Sundararajan, 1995). In this study the failure probability is required to be evaluated at different points in time throughout the service life of the systems and the design point would be varying at each time step, hence it was envisaged that the methods utilising the location of design point e.g. importance sampling and stratified sampling etc would be uneconomical to use. Hence Latin hypercube method and generalised conditional expectation method have been selected for the simulation process. An outline of these methods is provided in the following.


3.7.1. Latin Hypercube Sampling Method, LHS

The LHS method provides a constraint sampling scheme instead of random sampling as in the case of direct simulation (Sundararajan, 1995). In this method, the uniform random variable ranging from 0 to 1 is divided into N equal non-overlapping regions and a sampling point is generated from each region as the probability of occurrence within each region is same (Ding et al. 1998 & Olsson et al. 2003).

\[ u_i = \frac{u}{N} + \frac{i+1}{N} \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \text{Eq. 3.34} \]

Where \( u \) is a random number in the range \([0, 1]\) and \( u_i \) \( (i=1,2,\ldots,N) \) is the random value for the \( i_{th} \) interval. Inverse transformation can then be used to generate values for the input random variables. One value from each input variable is then picked at random and substituted into the limit state equation. This procedure is repeated N times to calculate the failure probability using Eq. 3.33.

Distributions of threshold chloride concentration generated through direct sampling method and LHS method are shown in Figure 3.11. It comprises of a uniform distribution ranging from 0.6 to 1.2 Kg/m³. The figure clearly indicates that the Latin Hypercube sampling results in much smoother distribution as compared to the direct sampling method hence would have a significant effect on the accuracy of computed failure probabilities.

![Figure 3.11: Distribution obtained through a) Direct sampling b) Latin hypercube sampling](image)

3.7.2. Generalised Conditional Expectation Method, GCEM

In the conditional expectation (CE) method all the basic variables in the limit state function (Eq. 3.25) are generated randomly except one called ‘control variable’, \( X_k \). The other random variables termed as ‘conditional variables’ should be those with least variability and the resulting conditional expectation (Eq. 3.35) must be computable by some known...
expression (Sundararajan, 1995). This method can be used for any performance function with any probability distribution of random variables. The only limitation is that the control variable, $X_k$, must be statistically uncorrelated to the other random variables. The limit state function in Eq. 3.25 can be re-written in the following form.

$$X_k = G_k(X_i; 1, 2, ..., n; i \neq k) \quad \text{.................Eq. 3.35}$$

The failure state according to the Eq. 3.35 is given by $X_k < G_k$ and the survival state by $X_k \geq G_k$. For the $i$th simulation cycle, the probability of failure can be computed as (Sundararajan, 1995)

$$P_{fi} = F_{X_k}[G_k(X_i; i=1, 2, ..., n; i \neq k)] \quad \text{.................Eq. 3.36}$$

The mean value for the failure probability can be computed by

$$\bar{P}_f = \frac{\sum_{i=1}^{N} P_{fi}}{N} \quad \text{.................Eq. 3.37}$$

and the variance and the coefficient of variation (COV) of the estimated probability of failure can be computed by

$$Var(P_f) = \frac{\sum_{i=1}^{N} (P_{fi} - \bar{P}_f)^2}{N(N - 1)} \quad \text{.................Eq. 3.38}$$

$$COV(P_f) = \sqrt{Var(P_f)} \quad \text{.................Eq. 3.39}$$

The conditional expectation method can be generalised by allowing the number of the control variables to be larger than one. The selected computational steps according to this generalised approach are (Ayyub and Chia, 1992; Sundararajan, 1995).

- The performance function should be defined using Eq. 3.25.
- The control random variables, $X_k = (X_{k1}, X_{k2}, ..., X_{km})$, are selected on the basis of criteria described earlier for the conditional expectation method. All other random variables are considered as the conditional random variables.
- In the $j$th simulation cycle, the conditional random variables are generated randomly. The probability of failure in the $j$th simulation cycle is given by
Here only the $X_{k1}, X_{k2}, \ldots, X_{km}$ are random variables and the remaining $(n-m)$ variables are deterministic (generated) values of the conditional variables for the $i_{th}$ cycle. The function can be solved to compute failure probability for each trial.

The failure probability can then be determined by Eq. 3.37 while the statistical parameters of failure probability can be computed using Eq. 3.38 & 3.39.

### 3.7.3. Generation of Correlated Samples

In practice, the random variables in some cases might not be completely independent but some degree of correlation exists among them e.g. the probability of corrosion initiation at some point in time, $t + dt$, would have a strong correlation with the probability of corrosion initiation at time $t$ provided that the ‘$dt$’ is small but would have a very weak or no correlation if ‘$dt$’ is very large. In this case (see chapter 8 for details) the samples for various time steps must be generated simultaneously. The procedure for this is as follows.

Let $X_i$ ($i=1,2,\ldots,N$) be $N$ correlated random variables and their correlation defined through the correlation coefficient matrix, $\rho_{X_i}$. The covariance matrix, Cov$(X_i; i =1,2,\ldots,N)$, for these variables can be formulated using Eq. 3.15. The $i^{th}$ set of sample for the correlated variables can then be generated using the following relationship (Sundarajan, 1995 and Nowak & Collins, 2000).

$$ x_i = l' u_i + a $$

Where the vector $x_i$ represents the $i_{th}$ set of generated sample from the correlated random variables, $X_i$; $l'$ is the lower triangular matrix for the Cholesky’s decomposition of Cov$(X_i)$, where $l \times l' = \text{Cov}(X_i)$; $a$ is the vector with mean values of the correlated random variables, and $u_i$ is the vector with values generated randomly from independent standard normal distribution, $N(0,1)$.

### 3.8. Summary and Conclusions

The probabilistic and statistical methods used in the course of this research have been presented. These procedures have been used to model uncertainties associated with input parameters of the deterioration models described in the previous chapter. The highlighted
methods for reliability analysis have been used for the estimation and prediction of system performance.

The Bayesian approach for event updating is also presented in this chapter. This has been used for the integration of data obtained through health monitoring methods with the available prior information from other similar structures elsewhere or from the past. Application of Bayesian statistics in decision theory is also summarised. This has been used for the comparison of various management strategies to establish the feasibility and potential of using health monitoring methods in the reliability based management of deterioration prone systems.
4.1. Introduction

The process of assessment is of fundamental importance for maintaining the infrastructure systems in a safe and serviceable condition. In order to ensure that the systems are not unnecessarily repaired or needless restrictions are imposed on them, it is important that assessment procedures are not unduly conservative and explore all aspects of safety to determine the structures fitness for the purpose (Das, 1997). Scarce data obtained through regular inspections at intermittent interval makes it difficult to establish the extent of deterioration. This allows deterioration to propagate unnoticed in between inspections, thus making efficient preventative maintenance and repair strategies practically impossible (Aktan & Grimmelsman, 1999). In recent years, health monitoring systems have been developed that provide structure specific information on the actual loading and its effect on structural systems, the nature, rate, and extent of deterioration on a continuous basis and hence can overcome the above limitations. This chapter propound different areas of a reliability based management system that can be benefited through use of such health monitoring systems. An updating methodology based on the Bayesian approach is also presented that can be used for the integration of data obtained through health monitoring systems with engineering judgement and prior information from similar structures elsewhere or from the past. This approach is aimed at identifying and reducing areas of uncertainty, and hence improves the quality of performance prediction.

4.2. Health Monitoring in Management of Systems

Health monitoring of systems is mainly used now-a-days to aid inspections or to check the effectiveness of repair works. In this section, emphasis is made on various facets of a reliability based management system where the introduction of health monitoring systems can provide useful information in reducing the epistemic uncertainties and hence improve...
the confidence regarding systems performance. The key activities of a reliability based management system (RBMS) were outlined in Fig. 2.3. The role of monitoring in the reliability based management of deterioration prone systems is presented graphically in Fig. 4.1.

![Diagram of Health Monitoring System](image)

**Figure 4.1: Role of monitoring in management of deterioration prone bridges.**

It can be seen from the figure that the data obtained by monitoring certain element(s) of a system or the system as a whole (local or global monitoring) can be used in different areas of a RBMS with varying degree of importance. This depends on what parameters are being monitored and how the output is being combined with previous information at hand. As outlined in the figure, health monitoring has its applications in the modelling of loads and deterioration variables, and to expand and update the existing database being used to plan and manage infrastructure systems. Another major application of the health monitoring is in performance estimation and prediction section of the RBMS where it can be used to update overall performance profiles of the systems by reducing associated uncertainties. It also has some applications in the management module e.g. it may be used in planning the location and type of detailed inspections on the system.

Possible applications of health monitoring systems and the benefits that can be achieved by integrating monitoring data into each of the areas (Fig. 4.1) are elaborated in some details the following subsections.
4.2.1. Deterioration Modelling

The three key activities in the deterioration modelling section of a RBMS are shown in Fig. 4.1. Decisions regarding the selection of leading mechanism responsible for deterioration of the systems are dealt with in the ‘deterioration mechanism’ section. In this section, among the list of possible deterioration processes, the governing mechanism is identified for further consideration (e.g. their modelling) so that the management activities can be planned based on their predicted extent of damage in future. Once the governing mechanism is identified and modelled using scientific, semi-empirical or empirical models, the information regarding input variables of these models is required. These comprise values of the input variables (if deterministic), and their distribution type and characteristics if they are of random nature. This process is carried out in the ‘variable modelling’ section. Finally if correlations among input parameters of the deterioration models and their variation in space have a dominant effect on the predicted performance, spatial aspects of these input variables are considered for the evaluation of overall systems performance.

4.2.1.1. Deterioration Mechanism

The applications of health monitoring systems in the selection of governing deterioration mechanism and its associated predictive models are illustrated in Fig. 4.2.

![Deterioration Mechanism Diagram](image)

**Figure 4.2 : Role of monitoring in deterioration mechanism and model selection**

**Identification of Mechanism**

The deterioration of systems may be caused by one, or more, mechanisms depending on environmental conditions and the quality of its materials, e.g. chloride induced deterioration, carbonation, freezing and thawing, etc, in the case of concrete, fatigue and corrosion in the case of steel. The data obtained by health monitoring methods can be used
to establish (for new structures) or confirm (for existing structures) a pre-selected governing mechanism. For existing structures, reference may be made to past inspection results for the selection of governing mechanism(s). In the case of new structures, the selection of mechanism(s) can be made based on the conditions of predefined location of the structure, and the experience of similar structures and conditions elsewhere.

Important parameters (indices) for different deterioration mechanisms can be defined that can then be used to develop a plan of which instruments should be installed to detect the mechanisms causing deterioration. As an example, carbonation generally causes a drop of pH in concrete, which can be detected by the help of pH measuring probe and hence the deterioration caused by carbonation can be quantified. Similarly penetration of chloride can be monitored either by measuring the chloride concentration at various depths, or by monitoring the penetration of threshold chloride concentration at various depths. The instruments that can be used to monitor such mechanisms were explained in Section 2.5. The use of monitoring in this context might not be very cost effective as monitoring all parameters for a variety of possible deterioration processes is not economically viable. Hence inspection results, engineering judgement, and past experience on similar systems should be used in advance to shortlist possible mechanisms. The governing mechanism(s) can then be identified by the use of health monitoring systems.

Selection of Predictive Model

Once the governing mechanism is identified (established or confirmed through health monitoring systems), the next step is to choose a mathematical model (physical or empirical) to represent the actual mechanism so as to enable / facilitate prediction of the deterioration rate at any point in time.

Occasionally, different physical phenomena may be used to model a deterioration process. Even for the same physical phenomenon, a set of different assumptions are being made by different researchers to allow modelling. As a result, the outputs obtained through these models fluctuate significantly e.g. Fick’s 2nd law of diffusion may be used to predict chloride ingress into the concrete but different boundary conditions have been adopted leading to different results. In addition, different processes have been proposed to mimic chloride induced deterioration (e.g. absorption - see section 2.4).
Health monitoring results can be used in this respect to select and fine tune the best possible model for the deterioration mechanism under consideration e.g. by comparing the chloride profiles in concrete obtained by health monitoring with the profiles predicted by different models. Similarly monitoring data can also be used to estimate the initial & boundary conditions and other input parameters for the selected model so that the quality of performance prediction can be improved to give more confidence in decisions regarding management activities.

**Updating Applications**

Health monitoring data can also be used to update the previously identified mechanism and/or associated mathematical model if discrepancies between the actual and predicted output are observed when additional data becomes available. They can also be used to update the variable representing modelling uncertainty thus increasing confidence regarding the output of the predictive models.

4.2.1.2. **Basic Variables and their Modelling**

The applications of monitoring in the selection and modelling of basic variables is presented in Fig. 4.3. The nature of basic input variables (deterministic or random) depends on the type of model selected to represent the deterioration and on the variability involved (usually both aleatoric and epistemic uncertainty), which can be determined using either monitoring or available field / laboratory testing. The data obtained through health monitoring along with the field / laboratory tests can be used to estimate the moments (mean and COV etc) for the random variables.

The selection of distribution types (shape of the distributions) depends on the nature of random variables. These are generally decided through the physical reasoning (nature of the physical process involved) and keeping in view the estimated distribution moments. These can also be aided by field / laboratory testing and the data obtained through health monitoring methods.
Once the distribution type and its moments are established for the basic random variables, these can be updated further to reduce the epistemic uncertainties when the additional data becomes available i.e. data obtained through health monitoring methods.

4.2.1.3. Spatial and Temporal Variability

The uncertainties associated with some of the deterioration variables are subjected to changes in time and space (Stewart et al., 2003). These variables at different points in time and space are not completely independent but some degree of correlation exists among them. Further, correlation may also exist among different variables of a deterioration model.

In most reliability analysis problems, material properties and exposure conditions are considered spatially homogenous but the appearance of damage (e.g. rust stains, cracking, and spalling in concrete structures) is rarely homogenous across the entire surface of a structure. In addition, the environmental conditions and the properties of some materials may also vary with time hence temporal variability should be accounted for in such cases.

The role of monitoring in different sections of a spatial variability model is similar to that of the model without spatial variability. In addition to its use in the deterioration modelling without spatial variability (See fig. 4.1) i.e. finalizing governing deterioration phenomenon, selection of spatial variables, their distribution shape and values of its parameters, monitoring data can be used to update prior estimates of correlation lengths and auto-correlation functions, which is currently a major limitation in the use of such models in practical situations (Chryssanthopoulos & Sterrit, 2002). The updating
procedures may include updating of the parameters of the distributions or updating the results of theoretical deterioration model as a whole (depends on the parameters being monitored and procedures used for integration).

The computed probability of failure using reliability analysis is used mainly for comparison purposes i.e. safety characteristics and decisions regarding management activities are based on its comparison with some values. Hence in quantifying advantages of using health monitoring instruments (by comparing updated and prior structural performance), the use of models with or without spatial variability would not have any significant difference as long as the analysis with prior and posterior information is consistent. Spatial variability models are not considered explicitly in this study.

4.2.2. Systems Performance

Systems performance may be determined in terms of reliability profiles as explained earlier (Fig. 4.1). There is a considerable uncertainty associated with the deterioration processes and future load predictions hence long-term predictions of the structural performance of deteriorating structures are inherently inaccurate and the results will hardly be useful for practical purposes (Stewart et al., 2003). The long-term predictive capability of these models needs to be improved. The integration of data obtained through the health monitoring systems can be used to continuously improve the quality of overall structural performance and hence could improve the predictive capabilities of the deterioration models in the face of these uncertainties.

The reduction of uncertainties in this case is a function of dependence of the parameters being monitored on the limit state function used for systems performance assessment i.e. a higher degree of dependence between the limit state function and the parameter being monitored will produce a tighter updated distribution.

4.2.3. Management Module

For systems prone to deterioration, variation of their performance with time (the so called ‘performance profiles’) is a basic ingredient to optimise management activities (Fig. 4.1) that can be improved by the methods explained in the previous sections. Health monitoring also has some direct applications in the management module. Once monitoring of the system reveals the attainment of certain limit state e.g. initiation of corrosion, initiation of cracks, etc, the management strategy has to be changed altogether, e.g. in the case where
initiation of corrosion is confirmed, the inspections would have to be concentrated on the amount of corrosion products, corrosion rate and resistance of concrete etc rather than on the amount of chloride present in cover concrete. Thus the health monitoring systems are expected to have a profound effect on maintenance and repair strategies.

4.3. Proactive Health Monitoring

Health monitoring systems can be divided broadly into two categories

- Direct monitoring methods;
- Indirect monitoring methods.

The direct monitoring methods are those in which a parameter under consideration is monitored directly to quantify its condition and/or performance state, e.g. monitoring the corrosion rate of steel embedded into the concrete. Indirect monitoring methods involve monitoring of some parameter other than that of direct interest but related through physical, or semi-empirical models, to the latter, e.g. monitoring the penetration of threshold chlorides into the concrete cover to estimate the corrosion initiation time at the rebar levels.

The term 'proactive health monitoring' is used herein in the sense that the data obtained by indirect monitoring methods, as explained above, is used to develop a framework for an early warning to failure, e.g. in the Fig. 4.4, the primary limit state under consideration is $Z_1$ (which is either difficult, or is costly to monitor, or its monitoring would not yield any significant benefit in terms of the proactive maintenance actions) whereas the limit state being monitored is $Z_2$. If the relation between the limit states $Z_1$ and $Z_2$ can be established using scientific, semi-empirical or empirical relations then any additional information obtained through health monitoring methods regarding the limit state $Z_2$ can be used to improve the prediction of attainment of the limit state $Z_1$.

It is worth mentioning here that the statistical correlation between the limit state $Z_1$ and $Z_2$ is of utmost importance in this context. This correlation could be in terms of a functional relationship between these limit states, or because some (or all) of the parameters defining these limit states are the same. Strength of this correlation among these limit states will define the improvement in quality of prediction for the attainment of limit state $Z_1$ given $Z_2$ is being monitored.
4.4. Health Monitoring Vs Inspection

Consider the space and time frame illustrated in Fig 4.5. The major difference between the inspection and health monitoring is that the inspection results provide information regarding the entire element/system at a particular point in time whereas the health monitoring systems can provide information at a particular location for the entire service life. Combining the information from these two sources can lead to a powerful tool that can enhance our ability to predict the performance of systems with increased confidence for longer time periods and hence can be used to optimise management activities.

4.5. Updating Methodology

The need for updating methodologies has been emphasized in Section 4.2, where the uncertainty associated with the mechanism of deterioration, selection of appropriate predictive model, and the modelling of basic random variables involved in the
deterioration process can be reduced by introducing the data obtained through the health monitoring methods and hence the quality of performance prediction can be improved. The effects of spatial and temporal variation of deterioration parameters (as explained in the previous section) cannot be evaluated through the scarce data obtained via visual inspections alone and these should be supplemented with the data obtained through testing and the use of health monitoring systems.

A powerful and versatile approach dealing with performance evaluation and prediction of systems in the face of uncertainty is the Bayesian approach. These techniques have had a significant impact in nuclear plants assessment and in the health care systems. More recently, these have been used successfully in offshore structures and steel bridges etc for the planning and optimization of inspection and maintenance schedules (Madsen 1987; Madsen & Sorensen, 1990; Onoufriou et al. 1994; Zheng & Ellingwood, 1998; Onoufriou, 1999). However, these applications have focused on very specific deterioration mechanisms and inspection methods delivering ‘hard’ data, e.g. crack sizes in fatigue analysis of steel structures.

The Bayesian updating approach (as described in the Sec. 3.5) can be used to incorporate information obtained from different sources at different points-in-time during long service lives, e.g. either from detailed inspections and monitoring or even from the qualitative assessment methods i.e. visual inspections or service records etc. As an example, it has been used for crack size evaluation at various point in time for steel structures subjected to fatigue (Zheng & Ellingwood, 1998), updating the reliability of steel gates on dams using visual inspection results (Estes et al. 2003 & Estes et al. 2004) and in condition prediction of deteriorating concrete bridges (Estes and Frangopol 1999), where the effects of inspection updating is illustrated for existing bridges. Faber & Sorensen (2001 & 2002) have presented a framework for the integration of inspection results obtained through instrumentation to evaluate the condition states of bridges at a given time.

The Bayesian framework is adopted in this research for the integration of data from health monitoring methods with the existing information and engineering judgement from the similar structures elsewhere and from the past.
4.5.1. Continuous and Discrete Output

The frequency of measurements required on a system depends primarily on the phenomenon being monitored, e.g. the frequency of information obtained through sensors should be very high for live load measurements on a structure to avoid any important reading being missed out. On the other hand, if the phenomenon being monitored is the corrosion of a concrete structure, the sensors output need not to be very frequent because of the slow nature of the process.

Even though the process of health monitoring may be continuous by nature, the output from the sensors could either be of a continuous or a discrete form depending on the parameters being monitored and the type of sensors being used. As an example, the fatigue crack growth in a steel element would be a case of the continuous output if health monitoring methods yield crack size as the output. But it would be a case of the discrete output if the sensor can only give a signal when the crack has reached a predefined size. Similarly, for concrete structures subjected to chloride induced deterioration, an example of the continuous output case could be the monitoring of chloride concentration at certain depth of concrete cover. Whereas the discrete output for this example could be the monitoring of threshold chlorides penetration in the cover concrete in which case the sensor may yield one of the two outputs. The first scenario is that it would retain its previous state (i.e. passivity confirmation at the sensor location or the conclusion that the threshold chloride concentration has not yet reached the sensor location). In other words, the limit state has not been attained at the sensor location at the time of monitoring). While the second scenario is that the sensor would change to a new state (i.e. initiation confirmation, or the confirmation that the threshold chloride concentration has reached the sensor location, i.e. the limit state has been attained at the sensor location).

For the continuous output case, the updating process can be split into the two cases (similar to the discrete output case) by relating the measured value to a limiting value (see Sec. 3.6), e.g. for the case where sensor measures the chloride concentration at any particular depth, the limiting value could, for instance, be the attainment of threshold chloride concentration at that depth and hence the safety margin can be developed for the 'failure' or 'safety' cases.
For the cases where the health monitoring information can be transformed in the form of either ‘fail’ or ‘safe’ mode, the two possible updating scenarios are explained in the following sub-sections.

4.5.1.1. Confirmation of ‘Safety’ at the Sensor Location

Assuming that the prior probability of failure that can be evaluated using Eq. 3.23 as follows;

\[ P_f^*(T) = P[M(T) \leq 0] \]

The first updating scenario is the case when the information from the health monitoring system confirms that the predefined limit state has not been attained at a particular point in time (at the time of monitoring) at the sensor location.

The ‘exact time to failure’ at the sensor location is not known in this case and the only information obtained is that ‘the time to failure at the sensor location (for a predefined limit state) is greater than the time of monitoring’ i.e. ‘\( M_s(t_m) > 0 \)’. Hence the posterior probability of failure would become

\[ P_f^*(T) = P[M(T) \leq 0 | M_s(t_m) > 0] \quad \text{...... Eq. 4.1} \]

Where \( P_f^*(T) \) and \( P_f^*(T) \) are the prior and posterior failure probabilities respectively; \( T \) is the time span for which the decisions regarding management activities are required, it can be selected arbitrarily as 10, 20 or 30 years etc; \( M(T) \) is the prior safety margin at the desired location (e.g. rebar level) for the predefined limit state at time \( T \), and \( M_s(t_m) \) is the safety margin for the same limit state at the sensor location at the time of monitoring, ‘\( t_m \)’.

Using the Bayesian updating principle, Eq. 4.1 would become

\[ P_f^*(T) = \frac{P[M(T) \leq 0 \cap M_s(t_m) > 0]}{P[M_s(t_m) > 0]} \quad \text{......... Eq. 4.2} \]

For the discrete output case, the updating would reduce associated uncertainties as the sensor confirms ‘safety’ for successive time steps. It is important to note that the sensor would be unable to make any distinction for the outputs in consecutive time steps until the limit state at the sensor location changes from ‘safety’ to ‘failure’. For example, for reinforced concrete structure subjected to chloride induced deterioration, the sensor would
continue to indicate that the threshold chloride has not reached the sensor level for successive time steps until it is actually attained.

In the case of continuous output, the sensor results could be distinct for different time step (e.g. chloride concentration at the sensor level) and hence the quality of information obtained in this case is better than the discrete output and would be reflected in the posterior distribution (see Fig. 3.3 for details).

Similarly for the given prior ‘time to failure’ i.e. $F_T(t) = P(T \leq t)$, the cumulative distribution function for the posterior ‘time to failure’ can be computed as follows

$$F_T(t) = P(T \leq t | T_S > t_m)$$  \hspace{1cm} \text{Eq. 4.3}$$

Where ‘$T_S$’ is the ‘time to failure’ at the sensor location and ‘$t_m$’ is the time at which information regarding the ‘$T_S$’ is obtained (time of monitoring).

4.5.1.2. Confirmation of ‘Failure’ at the Sensor Location

When the health monitoring system confirms the attainment of a limit state at the sensor location at a given time, the time to failure (or attainment of the limit state) would become available and hence the updating is expected to yield a much tighter distributions with significant reduction in uncertainty.

In this case, the time of failure at the sensor depth would be equal to the time of attainment of limit state as indicated by the health monitoring systems i.e. $M_S(t_m) = 0$. The posterior failure probability would be

$$P_f(T) = P[M(T) \leq 0 | M_S(t_m) = 0]$$  \hspace{1cm} \text{Eq. 4.4}$$

Applying Bayesian principle, the Eq. 4.4 would become

$$P_f(T) = \frac{P[M(T) \leq 0 \cap M_S(t_m) = 0]}{P[M_S(t_m) = 0]}$$  \hspace{1cm} \text{Eq. 4.5}$$

The quality of information in this case would be same for the both discrete and continuous outputs and would yield the same posterior distribution. Similar to the case with ‘safety confirmation’, the posterior ‘time to failure’ can be evaluated as follows

$$F_T(t) = P(T \leq t | T_S = t_m)$$  \hspace{1cm} \text{Eq. 4.6}$$
and its expansion using the Bayesian law would lead to

\[ F_T(t) = \frac{P(T \leq t \cap T_s = t_m)}{P(T_s = t_m)} \] \[ \text{Eq. 4.7} \]

### 4.5.2. Instrument / Measurement uncertainty

The statistical characteristics of the posterior distribution would also depend on the accuracy and precision of the instrument being used as more uncertainty in the instrument or measurement method would reduce the confidence of the posterior prediction. This instrument / measurement uncertainty must also be incorporated within the updating framework. This can be carried out by considering a full distribution for it or making some simplifications depending on the sensor output and parameter being monitored.

A possible simplification would be to replace the ‘\( M_s(t_m) = 0 \)’ in the Eq. 4.5 by two constraints ‘\( M_s(t_m) \leq 0 \)’ and ‘\( M_s(t_m-t_{ins}) > 0 \)’, i.e. a sensor confirms the attainment of limit state at the time ‘\( t \)’ whereas it would not have been attained the limit state at the time ‘\( t_m-t_{ins} \)’.

Here the time interval ‘\( t_{ins} \)’ would reflect the uncertainty in instrument and measurement method used. Higher value of the time, \( t_{ins} \), would reflect higher instrument / measurement uncertainty and would reduce the confidence in the posterior predicted performance and vice versa. The updated failure probability expression would become

\[ P_f^* (T) = P[M(T) \leq 0 | M_s(t_m) \leq 0 \cap M_s(t_m-t_{ins}) > 0] \] \[ \text{Eq. 4.8} \]

using Bayesian theorem to convert the conditional probability would yield

\[ P_f^* (T) = \frac{P[M(T) \leq 0 \cap M_s(t_m) \leq 0 \cap M_s(t_m-t_{ins}) > 0]}{P[M_s(t_m) \leq 0 \cap M_s(t_m-t_{ins}) > 0]} \] \[ \text{Eq. 4.9} \]

In terms of the ‘time to failure’ the posterior distribution would become

\[ F_T(t) = P(T \leq t | T_s \leq t_m \cap T_s > (t_m-t_{ins})) \]

\[ F_T(t) = \frac{P[T \leq t \cap T_s \leq t_m \cap T_s > (t_m-t_{ins})]}{P[T_s \leq t_m \cap T_s > (t_m-t_{ins})]} \] \[ \text{Eq. 4.10} \]
4.5.3. Systems Updating Approach

An inherent assumption in the above methodology is that the deterioration is uniform over the monitored domain. Due to the temporal and spatial effects of the exposure conditions and concrete quality within the member and/or for different members of a system or a network etc, the actual performance could be different for different members of a system and even at different locations of the same member. In order to explore the effects of the above mentioned assumption, the monitored domain can be divided into a number of smaller zones with the possibility of installing sensors within each zone. Now considering the same prior performance for each zone, the predicted performance can be updated based on the sensor information from each zone. The distance between the sensors and hence the physical size of zone should be large enough to avoid any spatial correlation on sensor outputs. On the other hand, the zone should be small enough to justify the assumption of uniform performance over its entire physical size.

Another scenario where multiple sensors may be required is, when more confidence in the performance prediction is required at some critical location or more robust / redundant monitoring system is required because of the critical nature of the zone.

There would also be cases where a combination of both scenarios would be required, e.g. a bridge deck element. In this case for each lane of traffic, multiple sensors can be located at critical shear and moment locations in addition to the sensors at other locations. Structurally critical areas of a bridge can be determined from the design calculations or from a load assessment of the bridge and locations for the health monitoring instruments can be identified. Vassie & Arya (2003) proposed a risk based assessment of corrosion and its use in identifying health monitoring systems for the high risk areas.

The updating procedures based on the data from multiple sensors have been developed for each of the two cases bearing in mind the nature of the decision that needs to be considered. In the former case (multiple sensors in different zones), the interest is to determine updated predicted performance for the entire member under consideration. In the later case (multiple sensors in the same zone), the interest lies in improving the confidence in prediction of performance within the individual zone. Of course, the two cases could also exist in combination, as shown schematically in Figure 4.6.
4.5.3.1. Observations from Different Zones

Consider a member divided into a number of small zones and a sensor located in each zone. The outcome of the health monitoring system would be the time of attainment of the limit state at the sensor location of each zone, \( T_i \). The difference between sensor initiation times at each of these zones would reflect the spatial variation of deterioration phenomenon. Hence the time of attainment of the limit state at sensor location for the entire monitored domain would become a random variable, which can be represented by either a fitted distribution e.g. normal distribution, or an empirical distribution using the data obtained from multiple sensors located along the plan. The updated time of attainment for the primary limit state (see Sec. 4.3) for the entire member (composed of different zones) can be obtained by integrating over the entire monitored domain.

\[
F_T^{\text{sys}}(t) = \int F_T^{\text{zone}}(t | x = T_i) f_{T_i}(x) dx
\]

...............Eq. 4.11

Where \( F_T^{\text{sys}}(t) \) and \( F_T^{\text{zone}}(t | x = T_i) \) is the posterior distribution for the time of attainment of the primary limit state (distribution for the service life of system) for the entire system and for the zone ‘i’ given the sensor initiation time ‘\( T_i \)’ respectively and the \( f_{T_i}(x) \) is the distribution for the time of attainment of limit state at the sensor locations. It is clear from the above equation that the output using this procedure is the same as that of the weighted average of the predicted performances for each zone separately.

The failure probability can then be obtained by defining associated safety margin functions as follows.

\[
P_f^{\text{sys}}(T) = P[M^*(T) \leq 0]
\]

...............Eq. 4.12
Where $M''(T) = T^{sys} - T$; $T^{sys}$ is the random variable representing the time of attainment of primary limit state for the entire member/system (evaluated through Eq. 4.10) and $T$ is the time span within which the failure probability is required.

### 4.5.3.2. Observations from the Same Zone

In this case, as explained in the previous section, the objective is to either increase robustness of health monitoring instruments or to increase the confidence in prediction depending upon the critical nature of the grid element under consideration. Hence, the sensors would be located relatively close to each other (within the same zone) and the assumption is that the deterioration would be uniform within that zone. Bayesian updating can thus be applied for the multiple sensors to improve confidence regarding prediction of performance of the zone under consideration. Let 'i' represent the sensor number along the first dimension and ‘j’ represent the sensor number along the 2nd dimension within a zone then the expression for posterior time of primary limit state attainment (Sec. 4.3) using the multiple sensor data would become

$$F''_T(t) = P\left\{ \cap_{i=1, j=1}^{n_i, n_j} M_{i,j} \leq 0 \cap_{i=1, j=1}^{n_i, n_j} M(X_{i,j}) > 0 \right\}$$

...............Eq. 4.13

Where $M_{i,j}$ is the safety margin for the sensor identified through ‘i’ and ‘j’, and ‘ni’ and ‘nj’ represent the total number of sensors along the 1st dimension and 2nd dimension respectively.

### 4.6. Conclusions

In the first part of this chapter, different areas within the context of reliability based management systems are highlighted where the output from health monitoring methods can be used with the view of increasing confidence in the long term performance predictions. These include selection and modelling of deterioration phenomenon and their updating along with the updating of overall performance of the systems.
The development of an updating methodology is presented in the later part of the chapter. Different scenarios based on the sensor output are identified and formulated to obtain a general updating methodology that is applicable for a variety of deterioration phenomena.

Finally the methodology is extended to incorporate information from multiple sensors located at different points along the space and their two possible scenarios (i.e. a sensor located at the different zones of a member, and multiple sensors located within the same zone) are also elaborated and formulated that can be used to estimate the 'time to failure' of the member or a system as a whole.
Chapter 5

Applications of Health Monitoring to Reinforced Concrete Structures

5.1. Introduction

The applications of health monitoring in the civil infrastructure systems and the formulation of a general updating methodology are outlined in the previous chapter that effectively integrates the data obtained through health monitoring with the existing information regarding the structures performance to improve the confidence in its long term predictions. In recent years, significant attention has been given to the reinforced concrete bridges that forms a major part of the transport network across the world. A more detailed formulation of the methodology for this specific case, i.e. reinforced concrete structures has been elaborated in this chapter (see Sec 1.2. for the scale of the deterioration in concrete bridges).

Chloride induced deterioration has been identified as a primary source for the degradation of reinforced concrete structures in the UK (Wallbank, 1989) and in many other countries. Gaal et al. (2002) highlighted the relative importance of chloride attack on concrete spalling (Fig. 5.1). Hence the chloride induced deterioration has been selected to show the potential of health monitoring systems in reducing the prediction uncertainty regarding performance of deterioration prone systems. The simulation method used, input data validation and verification, and the results for a number of cases of exposure conditions and material quality have been presented and discussed in this chapter.
5.2. Performance Based Deterioration Model

In the existing model for chloride induced deterioration (Fig. 2.10, reproduced in Fig. 5.2a), the performance degradation is related to the loss of steel section due to corrosion. Degradation of a bridge, or its elements, is not considered until the corrosion has actually initiated at the rebar level, as indicated by horizontal line on the curve in Figure 5.2a. This model is very useful to predict the remaining life of a bridge, or its elements, by targeting ultimate limit states in the reliability analysis, but has its limitations in serviceability limit states analysis. An example of such case would be the management of existing bridges where it undermines the importance of, and potential benefits from, proactive measures (such as preventative maintenance to avoid corrosion initiation etc) to optimize management and repair costs.

![Figure 5.2: Chloride induced deterioration model a) Existing b) Modified](image)

A refinement in this deterioration model is suggested as shown in Figure 5.2b. In addition to the limit states affected by the active corrosion phase (i.e. serviceability and ultimate...
limit states), durability limit states have been accounted for in this case. As shown in the Fig 5.2b, the chloride ingress phase has also been incorporated in quantifying performance degradation. This, in other words, would mean that the indication of performance degradation would start as soon as the chloride ions start to penetrate into the concrete (corrosion initiation phase), i.e. well before the actual corrosion of rebars would take place. This can be achieved by defining the performance of a bridge or any of its elements as a function of the propensity to corrosion. This refinement promotes the introduction, and facilitates the use, of proactive monitoring measures and of preventative maintenance techniques, in this case maintenance before the corrosion of rebars has actually initiated and hence, can be used to optimise the required resources.

5.3. Classification of Limit States

The definition of limit states for a structure describing the end of their function and effective service lives is a key ingredient in the probabilistic design and assessment approaches. It can be seen from the Figure 5.2 that for the structure subjected to chloride induced deterioration, the limit states can be distinctively separated into three broad categories.

5.3.1. Durability Limit State

The first category of limit states are related to the chloride ingress phase i.e. initiation of corrosion (e.g. Thoft-Christensen et al., 1996; Engelund & Sorensen, 1998; Gaal et al., 2001; Rostam, 2001; Stewart & Faber, 2003 etc). This would not have any significant effect on the capacity of the structures but would strongly impair their potential long-term durability, hence is termed as the “durability limit state”.

Rearranging the chloride ingress model (Eq. 2.5) and replacing the C(x,t) with the threshold chloride concentration for the initiation of corrosion, $C_{th}$, a model for the time to corrosion initiation can be obtained as follows.

$$T_I = \frac{E_{mod}X^2}{4D\text{erfc}^{-1}\left(\frac{C_{th}}{C_o}\right)^2} \quad \text{Eq. 5.1}$$

Where $E_{mod}$ is the modelling uncertainty variable that accounts for the discrepancies between the selected physical process and its associated mathematical model. It also
accounts for other processes not explicitly considered while modelling degradation. The distribution for the corrosion initiation time for a typical set of published data (Table 5.4) is shown in the following figure.

![Cumulative Distribution Function](image)

**Figure 5.3:** A typical distribution for the corrosion initiation time.

The durability limit state in this case can be defined as a maximum probability for the initiation of corrosion, e.g. 5%, 10% or 20% etc. The time corresponding to the attainment of the above limit state can be evaluated using the distribution shown in Fig. 5.3.

### 5.3.2. Serviceability Limit State

The second category of limit states is related to the active corrosion phase that will affect serviceability of the structures. This may, or may not, lead ultimately to the structural failure but hinders with the intended purpose of these structures. Some examples of these serviceability limit states are the time to first cracking, critical crack width (usually 0.3 to 0.5mm) at the concrete surface, the initiation of delamination and / or specific area of member being delaminated, and spalling etc (Liu & Weyers, 1998; Stewart & Rosowsky, 1998; Sterritt & Chryssanthopoulos, 1999; Thoft-Christensen, 2000; Rostam, 2001; Vu & Stewart, 2000 & 2002; Li et al., 2003).
Table 5.1: Typical input parameters for corrosion propagation models (Vu & Stewart, 2000 & Thoft-Christensen, 2002a)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho_{\text{rust}}$</td>
<td>Density of rust products</td>
<td>N(3600,360) Kg/m$^3$</td>
</tr>
<tr>
<td>$\rho_{\text{steel}}$</td>
<td>Density of steel</td>
<td>N(8000, 800) Kg/m$^3$</td>
</tr>
<tr>
<td>$d_0$</td>
<td>Thickness of porous zone at steel concrete interface</td>
<td>LN(12.5,2.54) µm</td>
</tr>
<tr>
<td>D</td>
<td>Diameter of reinforcement</td>
<td>N(25,2.5) mm</td>
</tr>
<tr>
<td>C</td>
<td>Cover depth</td>
<td>N(40,5) mm</td>
</tr>
<tr>
<td>$f'_c$</td>
<td>Characteristic compressive strength of concrete</td>
<td>40 MPa (deterministic)</td>
</tr>
<tr>
<td>$E_c$</td>
<td>Young’s modulus</td>
<td>4600 ($f'_c$)$^{0.5}$</td>
</tr>
<tr>
<td>$E_{\text{eff}}$</td>
<td>Effective Young’s modulus</td>
<td>$0.5 \times E_c$</td>
</tr>
<tr>
<td>$i_{\text{corr}}$</td>
<td>Corrosion rate</td>
<td>N(3,0.3)</td>
</tr>
<tr>
<td>$\nu$</td>
<td>Poissons ratio</td>
<td>0.25</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Proportionality constant for increase in corrosion product and rebar diameter loss</td>
<td>Uniform (1.4 – 4.2)</td>
</tr>
</tbody>
</table>

The Liu & Weyers (1998) model has been used to estimate the time to first cracking (Eq. 2.29 to 2.33) whereas two separate models for the time to spalling (time required for the crack, once initiated, to reach 0.5mm in width) has been evaluated using Vu & Stewart (2000) and Thoft-Christensen (2000) shown by Eq. 2.36 and 2.37 respectively. The typical input values for these models are shown in Table 5.1 and their distributions are plotted in Fig. 5.4.

![Figure 5.4: Serviceability limit states](image-url)
5.3.3. Ultimate Limit State

The third and final category of limit states would be those that are related directly to the failure of the structure, i.e. local or global instability, failure of shear and moment capacity, and bonding failure, etc (Frangopol et al., 1997b; Thoft-Christensen, 1998; Rostam 2001; Thoft-Christensen, 2002). Once corrosion is initiated, the rebar diameter and cross-sectional area reduces that results in reduced strength of the member. This reduction in rebar diameter and cross-sectional area has been plotted in the following figure based on the corrosion propagation models (Sec 2.4.2) that can be used to evaluate the ultimate limit states.

![Figure 5.5: Reduction in area and diameter due to corrosion.](image)

5.3.4. Discussion on Limit States

Many researchers have used initiation of corrosion as the limit state for performance evaluation because once initiated, repairs of corroded structures are quite cost intensive. The time between the event of corrosion initiation and cracking is very small compared to the structure’s life as can be seen from the Fig. 5.4 where the mean time to first cracking after corrosion initiation has been estimated as 2.30 years while the time from first cracking to the critical crack width (initiation of spalling) is 2.69 (Vu & Stewart model) to 4.55 years (Thoft-Christensen model). This has been reported in literature to vary between 2 to 5 years as referred to by Li & Weyers (1998) and 1 to 7 years as stated by Kirkpartick et al. (2002). The relative importance of durability limit state can be concluded from the above discussion where the time period from the corrosion initiation to serviceability limit states is very small and the repairing of the structure for the serviceability limit states is considerably higher than the proactive management, i.e. either preventative measures or...
the replacement of only concrete cover etc. In the remaining part of the thesis the emphasis will be made primarily on the durability limit state but the concept can easily be extended to the serviceability and ultimate limit states where necessary.

5.4. Probabilistic Modelling of Deterioration Variables

In laboratory, field tests or on actual bridges, the chloride profiles are generally measured to establish deterioration characteristics for the bridges prone to chloride induced deterioration. These profiles are used to establish the surface chloride concentration and the effective diffusion coefficient by fitting the profiles to the diffusion based deterioration model (e.g. Eq. 2.5) using non-linear regression analysis. The threshold chloride contents have been established from the laboratory experiments and field trials. This section outlines the characteristics of variables used in the deterioration modelling. The details of their modelling aspects are treated separately in the later chapter.

5.4.1. Surface Chloride Concentration

One of the inherent assumptions of the corrosion initiation model (Eq. 5.1) is that the surface chloride concentration and diffusion coefficient are constant. It is suggested in literature that the concentration of chloride at the concrete surface may be greater than that in the surrounding environment (Quillin, 2001). This concentration increases with time (Nilson et al., 1997) but tends to stabilise relatively early in the lifetime of concrete at some distance below the concrete surface (Bamforth 1999) hence the assumption of constant surface chloride is prevailed. However there are significant variations in the steady state value over the surface of concrete (Quillin, 2001).

The variability of the surface chloride concentration comes from the fact that the exposure condition would be different for different bridges in the network (The amount of salt sprayed would be different depending upon the environmental conditions etc). It would also be different for different elements of the same bridge depending upon their location, e.g. for a cross beam under a faulty expansion joint would be much more vulnerable to chloride ingress than a beam under a continuous deck. Some other factors influencing surface chloride concentration are environmental factors, exposure time, mix design and curing, cement types and admixtures used, concentration of salt solution and binding ability of the concrete to chloride ions etc (Bamforth et al., 1997).
5.4.2. Diffusion Coefficient

The effects of material quality on the chloride ingress model are represented through the diffusion coefficient. A good quality concrete would have a lower less diffusion coefficient and hence higher corrosion initiation period. The diffusion coefficient may vary in time and space (along the depth and / or along the plan). In literature, its value has been reported to vary between 70 to 1350 mm²/yr. Not enough evidence is available in literature to confirm that the diffusion coefficient is constant (or variable) along the depth. It is a function of water to cement ratio, cement type, temperature and humidity in addition to the workmanship quality, i.e. placing, compacting and curing etc (Page et al., 1981; Tuutti, 1982; Thomas, 1991; Liam et al., 1992, Maruya et al. 1994; Zhang & Gjrov, 1996 etc). Some researchers have argued that the diffusion coefficient decreases with time, e.g. HETEK (1996). This variation is attributed to the hydration of cement but tends to become small after the first few years from construction (Bamforth et al., 1997).

5.4.3. Threshold Chloride Concentration

When the chloride concentration at any given depth of concrete cover (e.g. at rebar level) exceeds a certain amount, generally known as ‘threshold chloride concentration’, corrosion is said to be initiated. This threshold value is a function of w/c ratio, cement type, micro-cracks at steel-concrete interface, temperature and humidity etc. A value of 0.4% by weight of cement is often used but it is clear that it cannot be represented by a single value (Quillin, 2001) hence is represented by a random variable.

5.4.4. Concrete Cover

Cover depth on highway bridges may vary significantly from the specified depths (Wallbank, 1989). Investigations have shown this variability is related to construction quality, i.e. steel fixing, formwork erection, concrete casting and on site quality checks (Mirza & MacGregor, 1979; Morgan et al., 1982; Marosszeky & Chew, 1990). It was also suggested that the cover depth is significantly affected by contractor’s practice but no systematic variation was found between the horizontal and vertical faces (Sterritt, 2000).

5.4.5. Modelling Uncertainty

The modelling uncertainty variable, $E_{mod}$, represents the uncertainty associated with the mathematical representation of the selected governing physical phenomenon. It also accounts for the lack of knowledge regarding other physical phenomena involved.
5.5. Updating Model for Chloride Induced Deterioration

The level of uncertainty associated with the corrosion initiation time distribution can be observed from the Fig. 5.3 where the coefficient of variation for the distribution is 2.09 with the mean value of 26 years. The philosophy behind the updating here is to try to minimise the uncertainty (i.e. coefficient of variation) in the predicted performance (which is the corrosion initiation time in this particular case). Once the model for the time to corrosion initiation is known (e.g. Eq. 5.1, but other more complicated models i.e. including absorption etc (Sterritt (2000) can also be used), the probability of failure (which is the probability of corrosion initiation in this case) can be evaluated using the following relation.

\[ M(t_s) = T_l - t_s \]

\[ P_f(t_s) = P(M \leq 0) \] ........................Eq. 5.2

Where \( t_s \) is the time period for which the probability of corrosion initiation is required for decision purposes (this can be set arbitrarily, e.g. 10, 20, 30 years etc.). For the formulation purposes, assume a sensor located at depth ‘\( X_s \)’ from the surface of concrete. It is assumed here that the health monitoring of the member is carried out using either the ladder arrangement or using the expansion ring system (sec. 2.5). The sensor would yield the confirmation of either passivity or initiation at the sensor location at the time of monitoring, ‘\( t_a \)’. Hence the updating procedure can be divided into two parts as follows.

5.5.1. Passivity Confirmation at Sensor Location

In this case assuming that at time ‘\( t_a \)’ the passivity is confirmed at the sensor location. The time for corrosion initiation at the sensor location is not known yet but the information for passivity confirmation (that the corrosion initiation time at the sensor location is greater than the time of monitoring ‘\( t_a \)’) can be used to update the corrosion initiation time at the rebar level as follows.

\[ T_{IS} > t_a \] ........................Eq. 5.3

or \[ T_{IS} - t_a > 0 \]

i.e. \[ M_{X_s} > 0 \]

Where \( M_{X_s} = T_{IS} - t_a \)
\( t_s \) = The time at which the probability of corrosion initiation is being updated.

\( T_{IS} \) = The predicted corrosion initiation time at the sensor location, can be calculated as

\[
T_{IS} = \frac{E_{mod} X_s^2}{4D \left[ \text{erf}^{-1} \left( \frac{C_{th}}{C_0} \right) \right]^2}
\] .......................... Eq. 5.4

\( X_s \) = The depth of sensor from the concrete surface, taken as random variable to account for uncertainty associated with sensor location.

The updated probability for corrosion initiation at the rebar level can then be computed using

\[
P_{f, upd}(t_s) = P\left[ (M(t_s) \leq 0) \mid M_{X_s} > 0 \right] .......................... Eq. 5.5
\]

Using Bayesian principle the conditional probability can be transformed to

\[
P_{f, upd}(t_s) = P\left( \frac{(M(t_s) \leq 0) \cap (M_{X_s} > 0)}{M_{X_s} > 0} \right) .......................... Eq. 5.6
\]

5.5.2. Initiation Confirmation at Sensor Location

Assuming in this case that at time, \( t \), the corrosion is initiated at the sensor location. The time for the initiation of corrosion at sensor location would be known in this case. This information can be used to update the corrosion initiation time at the rebar levels as follows

\[
T_{IS} = T_{as} .......................... Eq. 5.7
\]

or \( T_{IS} - T_{as} = 0 \)

or \( M_s = 0 \)

Where \( M_s = T_{IS} - T_{as} \) and

\( T_{as} \) = The actual corrosion initiation time at the sensor location.

The updated probability for the corrosion initiation at rebar level can be computed using

\[
P_{f, upd}(t_s) = P(M(t_s) \leq 0 \mid M_s = 0) .......................... Eq. 5.8
\]
and using the Bayesian law

\[ P_{t_s}^{upd}(t_s) = P\left( \frac{(M(t_s) \leq 0) \cap (M_s = 0)}{M_s = 0} \right) \]  

\[ \text{Eq. 5.9} \]

### 5.5.3. Instrument / Measurement Uncertainty

The output from the monitoring instruments is in the form of potential values measured between the sensor and the reference electrode installed in the concrete cover. The working principal for these instruments is similar to that of the half-cell. Increase in the chloride concentration at the sensor location will cause an increase in the negative potential that can be monitored continuously to get the required information (i.e. passivity or initiation confirmation). The potential value corresponding to the initiation of corrosion has been used as a deterministic value in literature e.g. Glass & Buenfeld (1997), Alonso et al. (2000), Castellote et al. (2002), Gaal et al. (2003b) etc. In practice, it is not a single value but is dependent on various other parameters including the quality of concrete, the properties of steel–concrete interface and the availability of other agents such as water and oxygen etc.

During the corrosion process, the negative potential starts to increase when the passive layer is attacked by chloride ions. This is usually repaired by the passivity of surrounding concrete and the instantaneous potential value shifts up and down during the process. On average the potential value increases with the increase in chloride ions until the passive layer is completely dissolved and the potential value reaches the upper limit as shown by the Fig. 5.6.

![Figure 5.6: Potential value Vs Probability of corrosion initiation](image-url)
The actual values for mV1 & mV2 are dependent on the type of sensors used (see Table 2.1 for different available half cells), e.g. these values are for the cupper/cupper sulphate sensor is -200mV and -350mV respectively. The above results have also been verified by Lentz et al. (2002) in which the authors have established probability density functions for half cell potential values corresponding to ‘corrosion’ and ‘no corrosion’ cases using observations from a number of field observations where after half cell observations, the reinforcement was exposed to determine the condition of steel.

It can be seen from Fig. 5.6 that the probability of corrosion initiation below mV1 is very small hence the sensor is assumed to be passive below this potential value. Similarly the probability of corrosion initiation above the mV2 is very high and the sensor is assumed to be initiated. Between these values the event of corrosion initiation or passivity confirmation is indecisive that reflects the instrument uncertainty. The time required for the sensor to attain the upper potential limit (mV2) from the lower potential limit (mV1) is termed as ‘tins’ and represents the instrument / measurement uncertainty. This time has been observed to be relatively small (Raupach & Schießl, 2001 and Raupach, 2002). Instead of using a single potential value to represent initiation of corrosion, the two limiting values have been proposed to incorporate this uncertainty within the updating methodology. The limit state for the corrosion initiation confirmation case (Eq. 5.7) would become

\[
T_{ls} \leq T_{as} & T_{ls} > (T_{as} - t_{ins}) \quad \text{.................Eq. 5.10}
\]

or

\[
T_{ls} - T_{as} \leq 0 \cap T_{ls} - (T_{as} - t_{ins}) > 0
\]

or

\[
M_s \leq 0 \cap M_{xs} > 0
\]

Where \(T_{as}\) is the time corresponding to the upper potential limit and \((T_{as} - t_{ins})\) is the time corresponding to the lower potential limit. Hence the updated failure probability (similar to Equation 4.9 and 4.10) would be

\[
P_f^*(t_s) = P[M(t_s) \leq 0 \mid M_s \leq 0 \cap M_{xs} > 0] \quad \text{.................Eq. 5.11}
\]

\[
P_f^*(t_s) = \frac{P[M(t_s) \leq 0 \cap M_s \leq 0 \cap M_{xs} > 0]}{P[M_s \leq 0 \cap M_{xs} > 0]} \quad \text{.................Eq. 5.12}
\]
Another source of uncertainty in the health monitoring system is due to the location of the sensors in the concrete cover. The exact location of the sensor is not known hence the variable $X_s$ should also be taken as random variable in the deterioration model (Eq. 5.4).

### 5.5.4. General Case

The above methodology can be extended easily for more sensors installed at various depths along the concrete cover as follows. The following symbols will be used.

- $T_i(X = X_i)$ = a priori predicted initiation time at depth $X_i$.
- $X_i$ = depth of sensor $i$ from concrete surface.
  - $= X_C$ for $i = n+1$.
- $X_C$ = Cover depth.
- $n$ = Total number of sensors.
- $i = 1, 2, ..., n$ Representing sensor number at depth $X_i$.
- $M(t_s)$ = Safety margin for prior corrosion initiation time at rebar level at time $t_s$.
- $M(x_i)$ = Safety margin for corrosion initiation time at depth $X_i$ from the surface of concrete at any time $t_a$.
  - $= T_i(X = X_i) - t_a$, when passivity is confirmed at depth $X_i$.
  - $= T_i(X = X_i) - (T_{ii} - t_{ma})$ when corrosion has initiated at depth $X_i$ & the time to corrosion initiation of sensor, $T_{ii}$, becomes known.
- $M_i$ = Safety margin between predicted and actual initiation time for corrosion.
  - $= T_i(X = X_i) - T_{ii}$, when the time to corrosion initiation of sensor $i$ becomes known.
  - $= 0$ for passivity confirmation case.
- $T_{ii}$ = time at which initiation is detected by the sensor $i$.
- $t_s$ = The time at which probability of corrosion initiation at rebar level is required for decision purposes (This can be set arbitrarily e.g. 10, 20, 30 years etc)
time interval between the two events i.e. 'corrosion initiation confirmation' and ‘passivity confirmation’ that reflects the inaccuracy of monitoring instruments in determining the time of corrosion initiation.

\( t \) = Any instantaneous time between 0 to \( t_s \).

\( P_f^{\text{upd}}(t_s) = \) Posterior / updated probability of corrosion initiation at rebar level at time \( t_s \).

Once the additional information is available, updated \( P_f \) at any time \( t = t_a \) can be computed as

\[
P_f^{\text{upd}}(t_s) = P\left[ M(t_s) \leq 0 \bigg| \bigcap_{i=1}^{n} M_i \leq 0 \bigcap_{i=1}^{n+1} M(X_i) > 0 \right]
\]

\[
P_f^{\text{upd}}(t_s) = P\left[ \frac{M(t_s) \leq 0 \bigcap_{i=1}^{n} M_i \leq 0 \bigcap_{i=1}^{n+1} M(X_i) > 0}{\bigcap_{i=1}^{n} M_i \leq 0 \bigcap_{i=1}^{n+1} M(X_i) > 0} \right]
\]

\textbf{Eq. 5.13}

Similar to the above procedure for the evaluation of posterior failure probability, Bayesian updating can be used to derive the updated / posterior distribution of time to corrosion initiation. Specifically, let the prior distribution for corrosion initiation time be given by

\[
F_T(t) = P\left[ T(X = X_c) \leq t \right]
\]

then the updated distribution can be obtained using

\[
F_T^{\text{upd}}(t) = P\left[ T(X = X_c) \leq t \bigg| \bigcap_{i=1}^{n} M_i \leq 0 \bigcap_{i=1}^{n+1} M(X_i) > 0 \right]
\]

\[
F_T^{\text{upd}}(t) = P\left[ \frac{T(X = X_c) \leq t \bigcap_{i=1}^{n} M_i \leq 0 \bigcap_{i=1}^{n+1} M(X_i) > 0}{\bigcap_{i=1}^{n} M_i \leq 0 \bigcap_{i=1}^{n+1} M(X_i) > 0} \right]
\]

\textbf{Eq. 5.14}

In the proposed scheme, the updated distribution is obtained numerically by considering number of steps along the time axis. The results of prior and posterior failure probabilities and corrosion initiation time at the rebar level for various sensor initiation times are shown in Sec. 5.7.
5.5.5. Limitation and Modification in Updating Methodology

It was observed during the updating process that if the assumed sensor initiation times of second and subsequent sensors (e.g. at 20 & 30mm etc) were somewhat less, or more, than their expected corrosion initiation times, the updating methodology failed to give any results and the output was a number divided by ‘zero’. In order to understand and remedy this limitation, the following procedure was adopted.

From the model of time to corrosion initiation at any given depth X (Eq. 5.1), we can write

\[ T_i = K_i X^2 \] .......................... Eq. 5.15

Where \( K_i = \frac{E_{\text{mod}}}{4D \left[ \text{erfc}^{-1} \left( \frac{C_{ik}}{C_0} \right) \right]^2} \) .......................... Eq. 5.16

and the initiation time at the sensor depth, \( X_s \), would be

\[ T_{is} = K_i X^2_s \] .......................... Eq. 5.17

Assume three sensors are placed along the cover depth at 10, 20 and 30mm depth. From the above equation, we can write

\[ \frac{T_{20}}{T_{10}} = \frac{K_{1}X_{20}^2}{K_{1}X_{10}^2} \text{ or } \frac{T_{20}}{T_{10}} = \frac{X_{20}^2}{X_{10}^2} \] .......................... Eq. 5.18

\[ \text{or } T_{20} = \frac{X_{20}^2}{X_{10}^2}T_{10} \] .......................... Eq. 5.19

Similarly we can also write

\[ \frac{T_{30}}{X_{30}^2} = \frac{T_{10}}{X_{10}^2} \text{ and } T_{c} = \frac{X_{c}^2}{X_{10}^2}T_{10} \] .......................... Eq. 5.20

Where \( T_{10}, T_{20} \) and \( T_{30} \) are the predicted corrosion initiation times at depths 10, 20 & 30mm respectively.

From the above equations, it is clear that the ratio between the corrosion initiation times at various depths is independent of all the deterioration variables involved in the variable
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This conclusion conflicts with the observations from the in-service structures. In order to understand this aspect more closely, following cases were considered.

5.5.5.1. No Instrument / Measurement Uncertainty

Assume that there is no instrument / measurement uncertainty, (i.e. the location of the first sensor, $X_{10}$, and its corrosion initiation time, $T_{10}$, are obtained deterministically through the health monitoring system). In this case, the corrosion initiation time at any other depth (i.e. 20mm, 30mm & cover depth etc) can be evaluated deterministically (Eq. 5.19 & 5.20) once the initiation time at the first sensor becomes available.

\[ T_{20} = \frac{0.02^2}{0.01^2} T_{10} = 4 T_{10} \quad \text{similarly} \quad T_{30} = 9 T_{10} \quad \text{...............Eq. 5.21} \]

Hence the realizations of $T_{10}$ and $T_{20}$ are fully correlated in this case. If the corrosion initiation time for the other sensors (e.g. at 20mm depth) comes out to be different than the values stated in Eq. 5.21 (i.e. 4 $T_{10}$ in this case), the numerator and denominator in Eq. 5.13 and 5.14 would be ‘0’ and the updating methodology fails to yield any result.

5.5.5.2. Uncertainty only in the Location of Sensors

Now consider the case when there is some level of uncertainty associated with the placement of the instrument though the sensor locations are fixed with respect to each other. We can write

\[ T_{20} = \frac{X_{20}^2}{X_{10}^2} T_{10} = \frac{(X_{10} + 0.01)^2}{X_{10}^2} T_{10} = K_2 T_{10} \quad \text{...............Eq. 5.22} \]

Where \[ K_2 = \frac{(X_{10} + 0.01)^2}{X_{10}^2} \]

Using simulation to evaluate the value for $K_2$, we get

\[ \mu_{K2} = 4.0522 \quad \text{COV} = 0.1053 \]

The mean value of $K_2$ approaches to 4.0, as it was in the previous case (when $X_{10}$ & $T_{10}$ are deterministically known). Now if the initiation time of first two sensors becomes available (i.e. $T_{10} = T_{a10}$ and $T_{20} = T_{a20}$), the value of $K_2$ can be evaluated deterministically using Eq. 5.22 as there is only one unknown variable in the equation (i.e. $X_{10}$). Hence the initiation
time for the remaining of the sensors and the reinforcement can again be evaluated
deterministically.

5.5.5.3. Uncertainty only in the Sensor Initiation Time

Assuming a case where the sensor location is deterministic but the sensor initiation time
contains a certain level of uncertainty. Assuming that the sensor indicated initiation at the
time ‘T_{a10}’ whereas confirms passivity at time ‘T_{a10-t_{ins}}’ i.e.

\[(T_{a10} - t_{ins}) < T_{10} < T_{a10}\]  \hspace{1cm} \text{.........Eq. 5.23}

Using Eq. 5.21, we can write

\[T_{20}^{\text{min}} = 4(T_{a10} - T_{ins}) \quad \& \quad T_{20}^{\text{max}} = 4(T_{e_{10}})\]  \hspace{1cm} \text{.........Eq. 5.24}

i.e. \(T_{20}\) should lie between \(T_{20}^{\text{min}}\) and \(T_{20}^{\text{max}}\) (which might not be the case in practice) and
the updating methodology would fail once again to yield any result outside this range.

5.5.5.4. Uncertain Sensor Location and its Corrosion Initiation Time.

The variable \(K_1\) (defined by the Eq. 5.15) is

\[K_1 = \frac{T_{1}}{X^2}\]

With the most general case, where both the sensor location and its corrosion initiation time
would contain some degree of uncertainty, the value of \(K_1\) would lie in the following range
(Similar to the procedure adopted in Sec. 5.5.5.3)

\[\frac{T_{a10} - t_{ins}}{X_{10}^2} < K_1 < \frac{T_{a10}}{X_{10}^2}\]  \hspace{1cm} \text{.........Eq. 5.25}

Here \(X_{10}\) is a random variable hence the minimum and maximum value for \(K_1\) would also
be a random variable. The corrosion initiation time at the 20mm depth should lie in the
following range.

\[K_1^{\text{min}} X_{20}^2 < T_{20} < K_1^{\text{max}} X_{20}^2\]  \hspace{1cm} \text{.........Eq. 5.26}
i.e. the time to corrosion initiation has to be within the range defined by the above equation. This again limits the updating methodology as in practice the sensor initiation time at 20mm depth can vary well above or below the specified range.

An inherent assumption in the above methodology is that ‘$K_1$’ is spatially uniform and is fully correlated along the depth (Eq. 5.18). In other words the deterioration variables i.e. $E_{mod}$, $D$, $C_{th}$ and $C_o$ are uniform along the depth and can be represented as random variables that are fully correlated along the depth. This is not a realistic assumption because the ratio of actual corrosion initiation times for various depths is not completely independent of these deterioration variables. Hence, the corrosion initiation time for a given depth may not lie in the range computed by Eq. 5.19 to 5.26, limiting the applicability of the updating methodology. This limitation can be overcome if ‘$K_1$’ is considered to be statistically independent along the depth. The Eq. 5.18 would become

$$\frac{T_{20}}{T_{10}} = \frac{(K_1)_{10}}{(K_1)_{20}} \frac{X_{20}^2}{X_{10}^2}$$

\[\text{Eq. 5.27}\]

Where $(K_1)_{10}$ and $(K_1)_{20}$ are the realizations of $K_1$ at 10mm and 20mm depth respectively. Re-arranging the above Eq. we get;

$$T_{20} = \left(\frac{(K_1)_{10}}{(K_1)_{20}} \frac{X_{20}^2}{X_{10}^2}\right) T_{10}$$

\[\text{Eq. 5.28}\]

This means that the ratio of corrosion initiation times at various depths will not be a deterministic value even if there is no instrument / measurement uncertainty. From Eq. 5.28 it is clear that there is theoretically an infinite range of possible corrosion initiation time at 20mm depth given the initiation time of 10mm depth becomes available. This increases the scope of the updating methodology to update predicted performance for a wide range of deteriorating systems.

The surface chloride concentration, $C_o$, and threshold chloride concentration, $C_{th}$, are likely to be uniform throughout the depth. Non-linear regression of the deterioration model (Eq. 2.5) on the data obtained through testing of various samples (obtained from HETEK, 1996) indicated the diffusion coefficient is also uniform along the depth (see appendix D for details). Hence the variation of corrosion initiation times at various depths can be attributed
to the modelling uncertainty, $E_{mod}$. In other words, the random variable $E_{mod}$ should be treated as independent along the depth (e.g. at 10mm, 20mm & 30mm etc). This modification not just rectifies the limitation of the updating methodology but is also a more realistic representation of the practical cases.

5.6. Simulation of Probabilistic Performance Prediction

Monte Carlo simulation has been used to estimate the prior and posterior performance predictions. The output of the simulation is in the form of a histogram of corrosion initiation time at the rebar level (Eq. 5.1 and Eq. 5.14) or the probability of corrosion initiation (Eq. 5.2 and Eq. 5.13) for a given time interval (e.g. 20 years in this case). A typical distribution for the corrosion initiation time has been plotted in the following figure.

![Figure 5.7: A typical distribution for the corrosion initiation time.](image)

The results from the above distribution can be interpreted in two different ways. The probability of corrosion initiation at a given time interval can be evaluated that can then be compared to a predefined maximum probability of corrosion initiation to trigger management actions. On the other hand the time required by the member in question to reach a predefined probability of corrosion initiation (e.g. 10%, 20% etc) can be estimated. The second way of interpreting the results is to consider the cumulative distribution function of the corrosion initiation time as the area of member showing corrosion activity normalised by the total area. The limit state in this case would be the maximum tolerable percentage area of member showing corrosion activity.
5.6.1. Input Data Verification

The random numbers for the deterioration variables are generated using Matlab™ 6.0 random number generator. These set of generated random numbers must be verified and validated before they can be used to evaluate the distribution for the corrosion initiation time.

The input file used to generate these input variables and to perform the updating based on sensors information is shown in Appendix C. Figure 5.8 below shows the cumulative distribution functions and probability density functions of the input variables generated through the program.

![Figure 5.8: Distributions for various deterioration variables](image)

The figure clearly shows that the moments of random variables generated through computer program (MATLAB™) are in good agreement with the assumed distribution characteristics for the input random variables. The distributions for assumed cover depth and sensor locations are plotted in Fig. 5.9. The sensors are assumed to be located at a...
nominal depth of 10, 20 and 30mm depth and are fully correlated (because positions of the sensors are fixed with respect to each other). The correlation among these variables are plotted in Fig. 5.10.

![Figure 5.9: Distributions for cover and sensor depths](image)

![Figure 5.10: Correlation among various sensors and cover depths](image)

From the Equation 2.5, it can be seen that the amount of chloride ions diffused at a depth \( x \) and time \( t \), i.e. \( C(x,t) = C_{th} \), cannot exceed surface chloride concentration. Hence the set of randomly generated values having the ratio of \( C_{th}/C_0 \) greater than 1.0 must be discarded.

### 5.6.2. Upper Bound for probabilistic performance prediction

The mean values for the time to corrosion initiation at rebar level, for different sample sizes and various upper limits are shown in Table 5.2.

It is clear from the table that even for same number of samples used, the mean value for time to corrosion is different for various upper limits of the distribution. The reason for this can be explained by Figure 5.11.
Table 5.2: Time to Corrosion Initiation for different Sample sizes and Various Upper Limits

<table>
<thead>
<tr>
<th>Sampling Points</th>
<th>Mean (Limited to 1000 Years)</th>
<th>Mean (Limited to 500 Years)</th>
<th>Mean (Limited to 250 Years)</th>
</tr>
</thead>
<tbody>
<tr>
<td>100,000</td>
<td>26.22</td>
<td>23.79</td>
<td>21.80</td>
</tr>
<tr>
<td></td>
<td>26.07</td>
<td>23.93</td>
<td>21.81</td>
</tr>
<tr>
<td></td>
<td>26.37</td>
<td>24.11</td>
<td>21.77</td>
</tr>
<tr>
<td>1000,000</td>
<td>26.07</td>
<td>23.84</td>
<td>21.70</td>
</tr>
<tr>
<td></td>
<td>26.05</td>
<td>23.81</td>
<td>21.75</td>
</tr>
<tr>
<td></td>
<td>26.14</td>
<td>23.93</td>
<td>21.76</td>
</tr>
<tr>
<td>5000,000</td>
<td>26.17</td>
<td>23.87</td>
<td>21.74</td>
</tr>
<tr>
<td></td>
<td>26.06</td>
<td>23.83</td>
<td>21.73</td>
</tr>
<tr>
<td></td>
<td>26.06</td>
<td>23.85</td>
<td>21.72</td>
</tr>
</tbody>
</table>

Figure 5.11: Chloride Profiles for different $C(x,t)/Co$ values

As can be seen from the figure and the diffusion model (Eq. 2.5), the time required for chloride concentration at a depth $x$ to reach $C_o$ approaches to infinity i.e. for the randomly generated input for the deterioration model, the values having $C_{th}/Co$ ratio close to 1 (which is practically impossible to attain) will generate very high time to corrosion initiation thus the ratio of $C_{th}/Co$ has to be limited to some practical value. This also explains the reason for why increasing number of sampling points increases the mean value of time to corrosion initiation (Table 5.2).
A simple method of limiting the ratio, \( C_{th}/C_0 \), to a practical value would be to define an upper limit for the time to corrosion initiation (Table 5.2). In literature, different methods have been used to serve the purpose e.g. fitting standard distribution types (Lognormal or Weibull) to the data obtained (Ciampoli et al. 2002, Gaal et al. 2002, Thoft-Christensen 2002b) will automatically discard the unnecessary values. The percentage of sample lost and area under PDF for time to corrosion due to various limits are summarised in Table 5.3.

<table>
<thead>
<tr>
<th>Truncated at (Years)</th>
<th>% Sample Lost</th>
<th>Area under PDF Curve</th>
</tr>
</thead>
<tbody>
<tr>
<td>2000</td>
<td>0.34</td>
<td>0.9997</td>
</tr>
<tr>
<td>1000</td>
<td>0.53</td>
<td>0.9948</td>
</tr>
<tr>
<td>500</td>
<td>0.88</td>
<td>0.9918</td>
</tr>
<tr>
<td>250</td>
<td>1.49</td>
<td>0.9851</td>
</tr>
<tr>
<td>125</td>
<td>3.05</td>
<td>0.9693</td>
</tr>
</tbody>
</table>

A value of 1000 years has been selected as the upper limiting value to compute statistical parameters (mean and standard deviation) for prior time to corrosion initiation. The percentage of sample loss for this case is about 0.5%. This error induced due to this assumption is very small and can be safely ignored.

5.6.3. Variance Reduction methods

Two variance reduction methods (i.e. Latin hypercube sampling method, LHS, and Generalised conditional expectation method, GCE) have been used in this project to increase the efficiency of simulation process. A comparison of results obtained through the LHS and GCE (combined with LHS) has also been carried out to establish their effectiveness in terms of their convergence and consistency characteristics. The results for these two cases have been presented and discussed in the following subsections. A typical failure probability curve for 20 years time, i.e. the probability of corrosion initiation at the rebar level (40mm cover depth) in 20 years time assuming three sensors located at 10, 20 and 30mm cover depths is shown in the following figure.
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Figure 5.12 : A typical prior and posterior failure probability curve.

The trend of the coefficient of variation (COV) for the probability of failure at various times is shown in the following figure.

Figure 5.13: A typical trend of the COV for the probability of failure.

As can be seen from the figure, the COV for the probability of failure increases with time. It also increases swiftly with the increase in complexity of the problem (i.e. the increase in number of sensors along the cover depth). Based on the above curves, following three points in time have been selected to characterize the effectiveness of the two variance reduction methods.
The prior case
Just before the information from the 3rd sensor is incorporated
After the information from the 3rd sensor is incorporated

5.6.4. Latin Hypercube Sampling Method, LHS

The estimation of prior and posterior predicted performance through simulation can be split up into four main sections as can be seen from the flow chart (Fig. 5.14) for the evaluation of time to corrosion initiation and failure probabilities. After applying the latin hypercube sampling in accordance with the Sec. 3.7.1, the inverse transformation functions are used to generate all the deterioration variables. The validation rules (described in the previous sections) are used to discard the inconsistent data and the posterior performance i.e. time to corrosion initiation after the updating is evaluated using the information obtained through the sensors. The loop in the flow chart is used to evaluate the posterior failure probabilities for the specified time span (i.e. 20 years in our case) at a given time interval (0.1 years in this case). The curves of failure probabilities against time are plotted in the output section. The histograms for prior and posterior time to corrosion initiation at any given time are generated from the simulated results and their distribution characteristics are also evaluated in the output section (see Fig. 5.14 for details).
The mean and COV for the probability of failure at the three comparison points (Sec. 5.6.3) are plotted in the following figure.
Figure 5.15: Mean and COV for the probability of failure using LHS

It is clear from the Figure 5.15 that the mean probability of failure becomes constant after 15 x 10^6 cycles and the sampling effect is almost completely eliminated. Whereas the coefficient of variation for the probability of failure is considerably high at 15x10^6 cycles which is reduced continuously with the further increase in number of simulation cycles. The figure also shows that the coefficient of variation for the two posterior performance cases i.e. before and after the 3rd sensor initiation becomes equal at about 100x10^6 cycles.

In order to check the consistency characteristics of the LHS, five different seed values have been used for the random number generation of the deterioration variables. The output for these cases is plotted in Figure 5.16.

Figure 5.16: Consistency characteristics for LHS

As expected, the scatter of results is reduced about its mean value with the increase in simulation cycles. For the prior case, the result tends to converge practically to a single
value at 42\times10^6 cycles, whereas the failure probability results in the posterior case has converged considerably at about 100 \times10^6 cycles.

In terms of the time required to run a simulation, approx. 60 minutes are required to run the 50\times10^6 simulation cycles. It increases linearly to about 120 minutes for 100 \times10^6 simulation cycles.

5.6.5. Generalized Conditional Expectation Method, GCE

The input and output section for the flow chart of GCE method is similar to that of the LHS case. The performance prediction section (that is different in this case) is shown in the following figure.

![Flowchart of GCE method](image)

---

Figure 5.17: Performance prediction section for flowchart of GCE method
In the generalised conditional expectation method, the effect of varying control variable as well as conditional variables has been analysed in the following subsections.

5.6.5.1. Effects of Control Variables

The probability of failure and its corresponding COV as a function of the size of control variable, keeping the conditional variable as constant (equals to 10,000 in this case) is shown in the following figure.

It can be seen from the figure that the increase in control variables does not have any significant effect on the mean failure probability and its COV. Neither the mean failure probability is stabilised in this case nor is there any significant reduction in the coefficient of variation of the probability of failure. The same conclusion can also be drawn from the following figures where the results (failure probabilities) for five set of independently generated inputs using different seed values for the deterioration variables have been plotted.

---

Figure 5.18: Effect of control variables on probability of failure

Figure 5.19: Consistency characteristics for GCE (effects of control variables)
The figure clearly shows that the scatter of data about its mean value is almost constant for the selected range of the size of control variables.

### 5.6.5.2. Effects of Conditional Variables

The probability of failure and its corresponding COV as a function of the size of conditional variable, keeping a constant control variable size (10000 in this case) is shown in the following figure.

![Figure 5.20: Mean and COV for the failure probability (effects of conditional variables)](image)

The stabilisation of mean failure probability and a reducing trend in the COV for the failure probability is apparent from the figure. The mean failure probability becomes almost constant at about $70 \times 10^6$ cycles. The scatter of data about the mean probability of failure for the prior and posterior case is plotted in the following figure. As can be seen from the figure that for five different seeds for input variables, the probability of failure is converging with the increase in size of conditional variables, but this convergence is not very significant as compared to the latin hypercube method.

![Figure 5.21: Consistence characteristics for GCE (effects of conditional variable)](image)
5.6.5.3. Combined effects of control and conditional variables

The probability of failure and its corresponding COV as a function of the size of control and conditional variable is shown in the following figure.

The coefficient of variation in this case is slightly less than the two individual cases but still is considerably higher than the LHS method. The same is true regarding the consistency characteristics as shown in the following figure.

In terms of the time required for simulation using GCE, approx 218 minutes are required to run a simulation with the control and conditional variable size of 20,000. This time increases non-linearly to 906 minutes for 40,000 simulation cycles, which is almost 9 times that of LHS with $50 \times 10^6$ cycles.
It is clear from the above results that the latin hypercube sampling method is more effective and efficient in the case of Bayesian updating than the generalised conditional expectation method. The convergence characteristics of the failure probability using LHS is considerably better than the GCE method. The number of simulation cycles required by the LHS is very high compared to the GCE but the time required to complete a simulation with suitable simulation cycles in the case of LHS is considerably less than the GCE hence the latin hypercube method has been adopted for the simulation of prior and posterior predicted performance.

5.6.6. Optimum Simulation Cycles

The output from the simulation is either in the form of probability of failure (probability of corrosion initiation) at a given time interval or the distribution for the time to corrosion initiation. The optimum number of simulation cycles for both of these cases is different and are treated separately in the following sub-sections.

5.6.6.1. Probability of Failure

Having established that the maximum number of simulation cycles would be required for the case when the deepest sensor in the concrete cover shows corrosion initiation, the case with third sensor showing initiation has been considered in this case to establish optimum simulation cycles. The mean and COV for the probability of failure with the third sensor showing corrosion initiation (for various sensor initiation times) has been plotted in Figure 5.24.

![Figure 5.24: Mean and COV for the probability of failure](image)

It is clear from the Fig. 5.24a that the delayed sensor initiation time would lead to the lower probability of failure (corrosion initiation at rebar level) which is in accordance with
the general conception regarding chloride penetration. The Fig. 5.24b indicates that the COV for the probability of failure is considerably less in the range between 5 to 8 years, i.e. the number of simulation cycles required in this range would be considerably less to get the same level of confidence in predicted failure probability as it would be for other initiation times. The reason for this lies in the fact that the peak of the distribution for corrosion initiation time at 30mm depth (3rd sensor location) is located in this region hence more samples will be concentrated into this region thus increasing the accuracy of the results. The effect of the number of simulation cycles on the posterior probability of failure is shown in Figure 5.25.

The Fig. 5.25 presents the mean and coefficient of variation for the probability of failure as a function of number of simulation cycles for various sensor initiation times. It is clear from the figure that the mean failure probability is practically constant with the increase in number of simulation cycles from 40E6 to 100E6 for all the considered sensor initiation times. The COV for the failure probability also shows similar trend except for the higher sensor initiation times e.g. at 10.0 years where the COV is reduced abruptly till 50E6 simulation cycles after which this reduction becomes less significant. Based on the above discussion the optimum simulation cycles required for this case is 50E6.

5.6.6.2. Corrosion Initiation Time Distribution

Similar to the probability of failure, being the worse case scenario, the optimum number of simulation cycles required to obtain a reasonable prediction of the time to corrosion initiation is established for the case where third sensor shows corrosion activity at 30mm depth. The mean time to corrosion initiation for the posterior distribution after updating for

Figure 5.25: Effects of number of simulation cycles on the probability of failure.
the third sensor is plotted in Fig. 5.26a whereas the coefficient of variation for the first four moments of the distribution is plotted in Fig. 5.26b.

![Figure 5.26: Posterior corrosion initiation time  A) Mean  B) COV for the first four moments](image)

The mean time to corrosion initiation at the rebar level reduces continuously with the increase in simulation cycles but tends to be stabilised at about 200E6 cycles. The COV for the first two moments for the corrosion initiation time becomes practically constant at 150E6 cycles whereas the variation in third and forth moment becomes less significant at about 250E6 cycles as shown in Fig. 5.26b. In the literature, only the first two moments have been used to analyse and solve engineering problems but being on the safer side, 250E6 simulation cycles have been used for the estimation of corrosion initiation time distribution.

### 5.7. Example 1

A typical structural element of a bridge (e.g. slab, beam, or a cross beam etc) subjected to de-icing salts is considered. The probabilistic modelling of various parameters involved in the corrosion model are shown in Table 5.4. The sensors are assumed to be located at nominal depths of 10, 20, & 30mm while the reinforcement is located at a nominal depth of 40mm.
5.7.1. Prior Corrosion Initiation Time

The distribution for the corrosion initiation time (PDF and CDF) at depths of 10, 20, 30 and 40mm based on the prior information (Table 5.4), the last value corresponding to the assumed level of rebar, are shown in the following figure.

![Figure 5.27: Prior PDF of time to corrosion initiation at various depths.](image)

The expected corrosion initiation time at rebar level is estimated to be 26 years (with a COV of 2.09). In addition to the mean values, 95% fractile for all the cases have been presented in the figure to indicate the spread of the distribution. As can be seen, the
simulated distributions for various depths of concrete cover appears to form a family of
distribution with the mean value ratio increasing non-linearly with depth (i.e. 1.95 / 7.08 =
0.28, 7.08 / 15.64 = 0.45, 15.64 / 26.00 = 0.60). This (along with the shape of distributions)
suggests that when chloride ingress is modelled through Fick’s 2\textsuperscript{nd} law of diffusion, the
resulting probability distributions could be fitted to a Weibull (or lognormal) family.

From the inspection point of view, the final distribution (i.e. for $T$ where $E(T) = 26.00$ yrs)
could be used to estimate an appropriate time for an initial half cell potential survey; this
could vary between 6 and 8 years, with the probability of corrosion initiation at rebar level
being 10% and 20% respectively.

\subsection{5.7.2. Probability of Failure}

If we now consider a case where, e.g. an expansion ring system, or a ladder arrangement
system has been installed to the structure to monitoring corrosion risk at various depths
below the concrete surface, the results for the prior and posterior failure probabilities (at $t_s$
= 20 years) versus time (where corrosion initiation at rebar level represents ‘failure’),
assuming different times for corrosion initiation at sensor level, are shown in Fig. 5.28 for
one sensor at 10mm depth. Similarly Fig. 5.29 and Fig. 5.30 shows the results for the case
where two sensors (at 10 & 20mm depths) and three sensors (at 10, 20 & 30mm depth) are
fitted in the concrete cover.
It can be seen from these figures that:

- Increase or decrease in the failure probability (from the prior value) strongly depends on the time at which the sensors indicate that corrosion initiation is reached.

- If the first sensor (here assumed at 10mm depth) does not detect corrosion initiation in the first 3 years after the surface is exposed to chloride ions, the probability of corrosion initiation at rebar level in 20 years time is negligible and the bridge can be considered as robust; of course, this assumes that the sensor is functioning properly and that exposure conditions and material properties will remain the same throughout this period. This result would suggest that a more detailed half-cell potential survey of the structure should be delayed beyond the value indicated...
above, as it is unlikely to yield any useful information about the condition of this particular structure.

- Conversely if the first sensor detects corrosion initiation at 0.5 years, then the corrosion initiation at rebar level by year 20 is practically certain. This can be used to bring forward the time for a half-cell potential survey and would also emphasise the need for preventative actions to be taken (e.g. cathodic protection).

- The evolution of posterior probability profiles for the case of two and three sensors, assuming different scenarios is presented in Fig. 5.29 & 5.30 respectively. In these cases, it is the combined information from the sensors that becomes relevant for drawing the appropriate conclusions regarding the inspection and maintenance regime for the structure.

- As can be seen from the Fig. 5.30 that the slope of the posterior failure probability curve after incorporating subsequent sensor initiation times is considerably reduced. This reduction in the rate of change of posterior failure probability refers to the reduction in the effects of additional information being incorporated on the posterior failure probability hence is an indication of the increase in confidence level in the posterior performance prediction.

### 5.7.3. Posterior Corrosion Initiation Time

Fig. 5.31 & 5.32 illustrates the effectiveness of introducing a proactive monitoring system in the structure. The reduction in uncertainty can be quantified by comparing prior and posterior distributions for the time to corrosion initiation assuming sensor initiation (Fig. 5.31) or simply confirmation of passivity at sensor locations (Fig. 5.32).

![Figure 5.31: Posterior corrosion initiation time for initiation confirmation case. a) PDF b) CDF](image)
It can be seen from the figures that:

- Uncertainty is reduced continuously as more information becomes available, be it in the form of confirmation of passivity or in detecting initiation at sensor locations. The reduction in uncertainty (in terms of the COV) is more pronounced when the actual time to initiation at sensor location becomes available rather than when only passivity is confirmed at any specific point in time.

- The percentage reduction in COV, with one sensor in position, is around 76 % and is practically constant regardless of the time to corrosion initiation at the sensor level (see Fig. 5.31). In the case of confirmation of passivity, the COV reduces continuously with time and approaches 50% after about 4 years (Fig. 5.32).

- The additional percentage reduction of COV in the presence of a second sensor (at 20mm) in place is around 7% of the previous updated value, and for the third sensor in place (at 30mm depth) the COV is reduced further by another 5% of the previous updated value (Fig. 5.33b).

- The change in updated corrosion initiation time at the rebar level (from its prior value) depends upon the early or delayed sensor initiation time from its prior expected value e.g. with the prior mean value for the sensor initiation time (at 10mm depth) of 1.95 years (Fig. 5.27), the mean value of the updated time to corrosion initiation at rebar level reduces (from the prior value of 26.0 years) to 15.8 years if sensor detects initiation at 1 year time, or increases to 29.94 years for sensor initiation time of 2.0 years (Fig. 5.31).
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Based on the prior information, the time of first intervention on the bridge is 4.9, 6.0 and 8.0 years for the 5%, 10% and 20% distribution fractile respectively. These time of interventions for different cases of passivity confirmation and sensor initiation times is summarised in Fig. 5.34. For example it can be seen that the time to corrosion initiation at rebar level (using the 10% distribution fractile) changes from 6.0 years (prior information) to about 8 years (if the corrosion initiation is detected at the sensor location, at 10mm cover depth, after 1 year) or 12 years (if passivity is confirmed by the 10mm sensor after 1 year). The results are clearly different for different distribution fractile (i.e. 10%, 20% etc), and for different scenarios. As a result, the first intervention on the bridge (e.g. detailed inspection using half cell survey etc) can be brought forward or postponed accordingly.

5.8. Example 2

The potential of utilising health monitoring systems to update the performance predictions has been illustrated in the previous section using an example of the reinforced concrete bridges subjected to chloride induced deterioration. The effectiveness of the methodology
(in reducing uncertainties and hence improving the quality of performance prediction) has been demonstrated for a variety of cases assuming different sensor initiation times.

This example deals with the systems updating approach (formulated in Sec. 4.5.3) for the same deterioration phenomenon i.e. the chloride induced deterioration in reinforced concrete structures. Two separate approaches for the different scenarios, as identified in Sec. 4.5.3, to incorporate information obtained from multiple sensors have been considered and discussed. These include updating of performance prediction based on the observations from different zones (if a sensor is located within each zone) and the updating based on the observations from the same zone (if multiple sensors are located within the same zone at a critical location).

In practice both procedures might be required, first to obtain the updated distribution within each zone, and subsequently to obtain the posterior predicted performance of the entire member under consideration. It is important to highlight here that in addition to the performance prediction of the entire member, performance of individual zones would also be required to optimise maintenance, repair and rehabilitation strategies. In this section, the results of the two procedures are shown separately to explore their characteristics.

5.8.1. **Multiple Sensors in different Zones**

The posterior predicted performance (corrosion initiation times) assuming five sensors (at 10mm cover depth) distributed along the plan (i.e. member divided into five zones) are shown in Figure 5.35. The scenario is that the number of sensors indicating corrosion initiation at 10mm depth varies from zero to five.

![Figure 5.35 : Corrosion initiation time at rebar level for different initiation times of five sensors (at 10mm depth).](image)
The posterior predicted performance of the member assuming 2nd and 3rd subsequent sensor information at 20 & 30mm cover depths are shown in Figure 5.36. The assumed initiation time for 10mm, 20 mm and 30mm depth are 1.0 years, 4.0 years and 9.0 years respectively for all zones.

**Figure 5.36**: Rebar corrosion initiation time for different sensor initiation times at 20 & 30mm depth

It can be observed from the figures that:

- if all the sensors yield the same output at a given point in time, i.e. either passive or initiated (i.e. no spatial effects), the distribution for the time to corrosion initiation is the same as that for the case where only one sensor is installed in the entire member (See also Fig. 5.31 & 5.32). This validates the systems updating methodology for the multiple sensors installed at different zones.

- the mean value for the corrosion initiation time is shifting consistently from the state when all sensors show passivity confirmation to the state when all sensors show initiation confirmation at various sensor locations (Figure 5.35). A similar pattern can be observed for subsequent sensors at 20 & 30mm depth (Figure 5.36).

- if all the sensors yield the same output, i.e. either corrosion initiation or passivity confirmation at a given point in time, the uncertainty associated with the predicted performance is considerably less than the case where the sensors show diverse results, for example the COV of the posterior rebar corrosion initiation time distribution for the former cases are 0.48 and 1.71 respectively (i.e. for the same initiation time and passivity confirmation case) whereas its value is 1.87 for the later case where 3 out of 5 sensors indicate initiation at year 1.0. The reason for this lies in the fact that the spatial effects are negligible in the two border line cases (i.e. when all sensors are either...
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passive or have the same initiation time) but not in the case where mixed sensor outputs are obtained.

- the coefficient of variation for the rebar corrosion initiation time is continuously reduced with the increase in number of sensors along the depth i.e. the quality of performance prediction is improved with the increase in number of sensors. e.g. the COV for the case with one sensor at 10mm depth (Fig. 5.35) ranges from 0.5 to 1.9 compare to the prior case of 2.09. This COV is reduced to about 0.45 for the case with 2 sensors (Fig. 5.36a) and further down to 0.41 with three sensors at 10, 20 & 30mm cover depth (Fig. 5.36b).

The probability of failure within 20 years time with one sensor is installed at 10mm cover depth are plotted in the following figure.

![Figure 5.37: Failure probability curves with one sensor at 10mm cover depth.](image)

It can be seen from the figure that the probability of failure increases continuously with the increase in number of sensors indicating initiation of corrosion and hence the time to intervention on the bridge needs to be modified accordingly.

5.8.2. Multiple Sensors in the Same Zone

The results (posterior corrosion initiation time) assuming one, two or three sensors in the same zone are shown in Figure 5.38. The updating is carried out at 1.0 year, assuming all the three sensors are confirming passivity at that point in time.
Figure 5.38: Bayesian updating for multiple sensors along the plan showing Passivity Confirmation.

It is clear from the figure that increasing the number of sensors in the same zone would increase the confidence regarding the prediction of performance (as COV for the corrosion initiation time is reducing continuously). A similar trend can be seen for the case where the sensors show initiation of corrosion at year 1.0 (Figure 5.39).

Figure 5.39: Bayesian Updating for multiple sensors along the plan for Initiation confirmation.

The effects of Bayesian updating for multiple sensors in the same zone showing different results are shown in Figure 5.40 and 5.41. Figure 5.40 summarises the posterior corrosion initiation time for the case where the sensors are either passive or are initiated at year 1.0, whereas Figure 5.41 summarises similar results for initiation detected at year 1.0 or 1.2 years by different sensors.
The reduction in mean value in Fig. 5.40 is due to the fact that the number of sensors indicating initiation are increasing whereas the increasing trend in the mean value in Fig. 5.41 is due to the increase in number of sensors indicating initiation at a delayed time (i.e. 1.2 years from 1.0 years).

### 5.9. Summary and Conclusions

A performance based deterioration model is proposed in this chapter for the chloride induced deterioration of reinforced concrete structure after highlighting the limitations of the existing deterioration model. The proposed modification in the existing model indicates the degradation of performance much earlier than the conventional model by combining the durability limit states with the serviceability and ultimate limit states. This model enables us to explore the potential benefits of, and justifications for the proactive management activities.
A comparison of various limit states concluded that the durability limit state (defined by the time to corrosion initiation in our case) is the most important among others because once initiated the repairs and maintenance of the structure gets quite cost intensive and the time required for the structure to attain other limit states (i.e. serviceability and ultimate limit state) is very small compared to the effective life of structure.

After the formulation of updating methodology for the specific case of chloride induced deterioration in reinforced concrete structures, a comparison of two variance reduction methods has been carried out and it was concluded that even though the use of generalised conditional expectation (GCE) method considerably reduces the required number of simulation cycles but latin hypercube method is more economic and efficient as it requires significantly less time to run a simulation compared to the generalised conditional expectation method and also its convergence characteristics are better in the case of Bayesian updating than the GCE method.

The result from a case study for different sensor initiation times clearly shows the effectiveness of the updating methodology in reducing the prediction uncertainties. It has been shown that the information from the health monitoring can be used effectively along with the existing information and previous experience to increase the confidence in predicted performance and hence this can be used as a decision support tool to optimise the management activities and ultimately the available resources.

An example of a systems approach for combining data from multiple sensors at various locations of the bridge is also presented in this chapter. Two main cases of updating have been identified and results for both of these cases have been summarised and discussed. It has been shown that the performance of a monitored domain representing a member or a structure can be updated using the proposed procedures. The application of the proposed methodologies for chloride induced deterioration bridge element has shown their effectiveness in reducing the associated uncertainties or in obtaining overall performance prediction of the member/structure by rationally combining similar data obtained through sensors at different locations of the bridge element.
6.1. Introduction

A number of deterioration models are available in literature for each deterioration phenomenon. These are the result of variations in the assumptions and simplifications carried out to allow for the modelling, or due to the consideration of different physical processes altogether. Furthermore, the modelling of input parameters of these models is subject to controversy that has a strong influence on the output of these models and hence on the predicted performance of bridges (Enright & Frangopol, 1998a). An extensive sensitivity study on the prior and posterior performance prediction (i.e. before and after incorporating data obtained through health monitoring systems) of bridges prone to chloride induced deterioration is presented in this chapter. The effect of different values for the probabilistic distribution parameters of random input variables, such as diffusion coefficient and threshold chloride concentration, is quantified. The uncertainty related to model variable and to the health monitoring system (instrument and measurement uncertainty) is also investigated.

6.2. Probabilistic Modelling of Deterioration Variables

6.2.1. Modelling Uncertainty for Deterioration Model, $E_{mod}$

The modelling uncertainty variable, $E_{mod}$, represents the uncertainty associated with the mathematical representation of selected governing physical phenomenon. It could also account for the absence of explicit models to represent other physical phenomena involved. As is well known, the ingress of chloride is a complex process. Different processes have been proposed to model chloride penetration into concrete and a variety of scientific and semi-empirical models have been put forward. Although chloride ingress model represented by Eq. 2.5 is widely used by those advocating diffusion is the dominant process, the distribution type and parameters for the model uncertainty are not well established in literature. Lentz et al. (2002) have used lognormal distribution with mean
and COV of 1.0 & 0.1 respectively whereas Faber & Sorensen (2001) and (2002) have used lognormal distribution with mean and COV of 1.0 and 0.05 respectively. Comparison of actual time to corrosion initiation (published in literature) with predicted values using Eq. 5.1 point towards higher modelling uncertainty levels. The distributions of modelling uncertainty considered in the present sensitivity analysis are as follows and are presented graphically in Figure 6.1. These figures are truncated along x-axis for presentation purposes.

**MU1.** Lognormal; Mean=1.0, COV=0.1

**MU2.** Lognormal; Mean=1.0, COV=0.25

**MU3.** Lognormal; Mean=1.0, COV=0.5

![Figure 6.1: Distributions for modelling uncertainty variables](image)

It is clear from the figure that the increase in uncertainty level (COV) for modelling uncertainty also affects higher distribution moments, e.g. skewness of the distribution increases with the increase in COV. The distributions for prior corrosion initiation times for the three modelling uncertainty models are plotted in Figure 6.2.
It can be seen from the figure that the modelling uncertainty has a negligible effect on the prior time to corrosion initiation (represented by the COV of the distribution), e.g. increase in the COV for the prior time to corrosion initiation at rebar level is only 0.48% due to an increase in the COV for modelling uncertainty of 60% (i.e. from 0.1 to 0.25). Similarly increase in the COV for the prior rebar corrosion initiation time is only 2.4% (i.e. from 2.08 to 2.13) for an increase in the modelling uncertainty of 100% (i.e. from 0.25 to 0.5). The reason for this small effect of $E_{\text{mod}}$ on the total COV is due to the dominating influence of uncertainties in the estimation of other basic random variables, i.e. $X$, $D$, $C_{th}$ and $C_{o}$.

The uncertainty associated with the posterior corrosion initiation time however, is proportional to the modelling uncertainty as can be seen from Figure 6.3 for various assumed sensor initiation times (at 10mm cover depth) e.g. for the sensor initiation time of say 1.0 years, an increase in the modelling uncertainty of 60% (i.e. from MU1 to MU2 case) causes an increase in the COV of rebar corrosion initiation time of over 37%. Similarly further increase in modelling uncertainty of 100% (i.e. from case MU2 to MU3) causes an increase in the COV of rebar corrosion initiation time of over 58%.

It can also be seen from the figure that the uncertainty associated with the rebar corrosion initiation time has been reduced considerably from its prior prediction, e.g. this reduction in COV for the case of MU2 is in the range of 76 to 78% depending on the sensor initiation times (i.e. from 2.09 for the prior case to about 0.48 for the posterior case).
Figure 6.3: Posterior corrosion initiation times for various uncertainty levels and sensor initiation times

The increase in mean value for the rebar corrosion initiation time is a linear function of the sensor initiation time as can be seen from Figure 6.4. Another important conclusion from the same figure is that the coefficient of variation of the posterior rebar corrosion initiation time is practically independent of the sensor initiation time.

Figure 6.4: Mean and COV for posterior rebar initiation time vs. sensor initiation time.

In the event that multiple sensors are installed along the cover depth, variation in the mean and COV for the posterior times to corrosion initiation at the rebar level can be seen in Figure 6.5. The ‘zero’ sensor case corresponds to the prior time to corrosion initiation. Increase or decrease in the mean value for the time to corrosion initiation is strongly influenced by the actual initiation time at the sensor locations (Initiation times assumed for the three sensors at 10, 20 & 30mm depth are 1.0, 4.0 and 9.0 years respectively) but are converging with the increase in no. of sensors. This shows the effectiveness of updating in
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reducing epistemic modelling uncertainty. The coefficient of variation is also reducing with the increase in number of sensors (as is clear from Figure 6.5). This reduction is considerable for the first updating but less so for subsequent updating. The difference in COV for different modelling uncertainty distributions is almost constant regardless of the number of sensors after the first updating. This provides a rationale for always striving for better predictive models, with modelling uncertainties as low as possible.

![Figure 6.5](image)

**Figure 6.5**: Effect of number of sensors in reducing uncertainty for time to corrosion initiation.

6.2.2. Exposure Conditions, $C_o$

Three main sources of chloride in concrete include chloride cast in concrete (e.g. plasticizers etc), the influence of a marine environment and de-icing salts. Modern material specifications limit the amount of chloride cast into concrete. Stewart & Rosowsky (1998) have reported that structures within 3km range of coast are affected by marine environment to a varying degree depending on the distance from the coast.

Surface chloride concentration in case of de-icing salts being a dominant source is a function of the amount of salts sprayed which is dependent on weather conditions, amount of rain, traffic flow etc. Hence a probabilistic model is used to describe the surface chloride concentration e.g. Table 6.1 summarises some recently used surface chloride concentration models. A graph presented by Vu & Stewart (2000) shows that the mean value is, as might be expected, location dependent and hence would be different for different networks of bridges.
Table 6.1: Summary of recently used models for surface chloride concentration

<table>
<thead>
<tr>
<th>Reference</th>
<th>Type</th>
<th>Mean (Kg/m³)</th>
<th>St. Dev. (Kg/m³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thoft-Christensen et al. (1996)</td>
<td>Normal</td>
<td>3.24</td>
<td>0.22</td>
</tr>
<tr>
<td>Stewart &amp; Rosowsky (1998) &amp;</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Vu &amp; Stewart (2000 &amp; 2002)</td>
<td>Lognormal</td>
<td>3.5</td>
<td>1.75</td>
</tr>
<tr>
<td>Faber &amp; Sorensen (2001 &amp; 2002)</td>
<td>Lognormal</td>
<td>µ, s =Normal</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(µ= 9.2, σ=0.92)</td>
<td></td>
</tr>
<tr>
<td>Lentz et al. (2002)</td>
<td>Lognormal</td>
<td>µ, s =Normal</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(µ=5.52, σ=0.46)</td>
<td></td>
</tr>
<tr>
<td>Lounis &amp; Amleh (2003)</td>
<td>Lognormal</td>
<td>3.81</td>
<td>1.53</td>
</tr>
</tbody>
</table>

Based on the above discussion, the following three cases have been considered in the sensitivity studies. All three cases would be typical of a bridge in an environment of average severity as a result of de-icing salt application and are presented graphically in Figure 6.6.

EC1. Lognormal; Mean = 3.5 Kg/m³, COV=0.5

EC2. Lognormal; Mean = 3.5 Kg/m³, COV=0.25

EC3. Lognormal; Mean = 4.5 Kg/m³, COV=0.5

![Figure 6.6: Models for different exposure conditions.](image)

The prior distributions for corrosion initiation time at rebar level are shown in Figure 6.7. The figure shows a significant difference in the distribution characteristics of prior
corrosion initiation times for different exposure conditions. A reduction in the uncertainty associated with surface chloride concentration (i.e. Case EC2, where the COV is reduced by 50% from the Case EC1) not only has resulted in a reduction in the uncertainty of the corrosion initiation time, i.e. COV from 2.09 to 0.73 (about 65%) but also has significantly reduced its mean value i.e. from 26.0 to 15.3 years (about 41%).

![Graph showing cumulative distribution function and probability density function for different cases.](image)

**Figure 6.7: Prior corrosion initiation times at rebar level for different exposure conditions.**

Similarly an increase in the mean value of surface chloride concentration (more severe environment, Case EC3 compared to Case EC1) by about 29% causes a reduction in mean value of corrosion initiation time from 26.0 to 16.70 years (i.e. 36%). Only a slight reduction in uncertainty of corrosion initiation time (about 9%) is observed.

Figure 6.8 shows the posterior time to corrosion initiation at rebar level for various exposure conditions, and different sensor initiation times at 10mm cover depth. The reduction in uncertainty associated with the posterior time to corrosion initiation is evident for all assumed sensor initiation times e.g. for the case EC1, this reduction is in the range of 76% to 78% depending on the sensor initiation time. Similarly for the cases EC2 and EC3, this reduction is in the range of 33% to 40% and 74% to 76% respectively.
Figure 6.8: Posterior rebar corrosion initiation times for various sensor initiation times and different exposure conditions

Figure 6.9 summarises the relation between the mean rebar corrosion initiation time and the sensor initiation times at 10mm cover depth. Similar to the modelling uncertainty case, the mean rebar corrosion initiation time is a linear function of the sensor initiation time. Furthermore, the COV for posterior rebar corrosion initiation time is also a linear function of sensor initiation time (Fig. 6.9).

The uncertainty associated with posterior corrosion initiation time at rebar level is further reduced by the use of additional sensors along various depths of concrete cover as shown in Figure 6.10. The assumed corrosion initiation times at the three sensors located at 10, 20 and 30mm cover depth are 1.0, 4.0 and 9.0 years respectively. The figure clearly shows that the posterior distributions for time to corrosion initiation at rebar level for various exposure conditions have converged considerably (the mean value as well as the
uncertainty). In other words, the posterior time to corrosion initiation is not very sensitive to uncertainties associated with exposure conditions. It can also be concluded from the above results that performance updating is likely to be more beneficial than monitoring and updating of exposure condition models.

Assuming 10% probability of corrosion initiation as the durability limit state, the time of first intervention on the bridge (or element under consideration) would be 6.1 years for the EC1 case (Figure 6.7). This time is increased to 6.4 years for the case where uncertainty is reduced by 50% i.e. case EC2. Similarly, time of first intervention would be reduced to 5.05 years if the mean surface chloride concentration is increased by 28.6%, i.e. case EC3.

The time of first intervention based on posterior corrosion initiation time would be 7.95, 7.88 and 7.48 for the cases Co1, Co2 and Co3 respectively for one sensor at 10mm cover depth showing initiation at 1.0 year, which is sufficiently accurate for all practical purposes. Naturally, the range becomes even smaller in the presence of more sensors (Figure 6.10).

6.2.3. **Threshold Chloride Concentration, C_{th}**

Threshold chloride concentration is a function of material factors i.e. w/c ratio, pH of the pore solution, capacity of the cement paste to bind the chloride ions, micro-cracks at steel-concrete interface, capillary structure, temperature and humidity (Quillin, 2001 & Montemor et al. 2003). The laboratory vs. field conditions also have a strong influence on the threshold chloride values, e.g. preparation of reinforcement (clear vs. rusted bars), compacting, curing, temperature and moisture (Gaal et al. (2003b).
Different threshold chloride concentration models have been reported in literature. A comprehensive summary of these models is available in Li (2002) & Glass et al. (2003a). Glass & Buenfeld (1997) reported the minimum and maximum values for threshold chloride as 0.2% and 2.0% by mass of binder. These can be approximated as 0.6 Kg/m³ and 6 Kg/m³ respectively. Lentz et al. (2002) have used normal distribution with mean and COV of 2.3 Kg/m & 0.3 respectively. Similarly Thoft-Christensen (2000a) have used normal distribution with mean and COV of 0.90 Kg/m³ and 0.15 respectively. Stewart and Rosowsky (1998) modelled it as a uniform distribution between 0.6 and 1.2 Kg/m³. The same distribution has been used by Vu & Stewart (2000 & 2002). Faber and Sorensen (2001) have used lognormal distribution with mean and COV of 0.45 and 0.33 respectively.

Kirkpatrick et al. (2002) performed sensitivity analysis of threshold chloride concentration on the predictive model using a triangular distribution with lower limit of 0.6 Kg/m³ and upper limit of 1.2, 2.0, 3.0, 4.0 and 5.0 Kg/m³ respectively. Considerable variation in the output was observed with different threshold chloride models. In this study, three different models are examined. The distribution characteristics of these models are as follows and are presented graphically in Figure 6.11.

TC1. Uniform (0.6 -1.2) Kg/m³; Mean=0.9 Kg/m³, COV= 0.19

TC2. Uniform (0.6 - 2.0) Kg/m³; Mean = 1.3 Kg/m³, COV=0.31

TC3. Normal ; Mean=0.9 Kg/m³, COV=0.17

The prior corrosion initiation times at rebar level for the three threshold chloride models are shown in Figure 6.12. It is clear from the figure that, for the same moments, the tail
characteristics of the two distributions (Uniform and Normal) have no effect on the
distribution of corrosion initiation time.

![Graph showing cumulative distribution functions and probability density functions for different threshold chloride concentration models.](image)

**Figure 6.12:** Prior rebar corrosion initiation times for various threshold chloride concentration models.

It can also be seen that increasing the upper limit for $C_{th}$ from 1.2 Kg/m$^3$ (case TC1) to 2.0 Kg/m$^3$ (Case TC2), i.e. an increase in mean and COV of about 44% and 63% respectively causes an increase in the mean for corrosion initiation time of about 98% while its COV remains practically unaffected. Similar results have been obtained by Enright & Frangopol (1998a) and Kirkpatrick et al. (2002).

![Graph showing posterior initiation times for various threshold chloride models and sensor Initiation times.](image)

**Figure 6.13:** Posterior initiation time for various threshold chloride models and sensor Initiation times

The posterior predicted time to corrosion initiation at rebar level for various corrosion initiation times at 10mm cover depth and for different threshold chloride models are shown in Figure 6.13. Similar to the other cases, the uncertainty in corrosion initiation time is reduced considerably for different hypothesized initiation times at sensor location (10mm cover depth) e.g. the COV is reduced from 2.09 (prior case) to about 0.5 (posterior case).
It can also be seen from the figure that the effect of uncertainty in modelling threshold chloride concentration is considerably reduced in the posterior predicted time to corrosion initiation and is reduced further by the installation of additional sensors along various depths within the concrete cover as shown in Figure 6.14.

![Figure 6.14: Effect of sensor numbers in reducing uncertainties for various threshold chloride models.](image)

The figure clearly shows that the mean and COV for posterior predicted corrosion initiation time is converging asymptotically to a single value, i.e. the uncertainty associated with modelling of threshold chloride is reduced by the effective use of data obtained through monitoring. From the above discussion it can be concluded that updating of the overall performance is likely to be more beneficial than obtaining additional information and hence improving the threshold chloride model prior to performance evaluation.

Similar to the previous cases, mean and COV for the posterior corrosion initiation times have a linear relation to the sensor initiation times as shown in Figure 6.15. Hence, these curves can be used to establish posterior rebar corrosion initiation times for concrete structures in practice for any measured sensor initiation times at 10mm cover depth.

![Figure 6.15: Posterior rebar initiation time vs sensor initiation time. a) Mean b) COV](image)
Based on the prior predictive model, the ‘time to first intervention’ on bridges, assuming 10% probability of corrosion initiation as the durability limit state, are 6.1, 7.5 and 6.13 years i.e. a difference of 1.4 years between TC1 and TC2 case, which is reduced to 0.53 years (7.92 and 8.45 years for the first sensor initiated at year 1.0). This difference is further reduced to 0.36 years and 0.33 years by updating using the subsequent second and third sensors information respectively.

6.2.4. Workmanship and Material Quality

The uncertainty associated with the quality of material and workmanship has a considerable effect on the expected performance of a concrete structure. Workmanship quality is reflected in the present study through the concrete cover uncertainty whereas the uncertainty in material quality is associated with the probabilistic model adopted for diffusion coefficient. These effects are correlated since good level of workmanship generally would be associated with good concrete quality, and thus less variation in concrete cover. Hence, the probabilistic modelling for concrete cover and effective diffusion coefficient are considered in tandem.

6.2.4.1. Cover Depth, $X_c$

Cover depth on highway bridges may vary significantly from the specified depths (Wallbank, 1989). Investigations have shown that this variability is related to construction quality i.e. steel fixing, formwork erection, concrete casting and a number of quality checks performed on site (Mirza & MacGregor, 1979; Morgan et al., 1982; Marosszeky & Chew, 1990; Clark et al., 1997). It has been suggested that cover depth is significantly affected by contractor’s practice. The average cover depth provided by some contractors is frequently greater than the design specification while that of others is frequently below the specified value. No systemic variation is found between horizontal and vertical faces, but complex steel fixing can lead to low cover (Sterritt, 2000).

6.2.4.2. Diffusion Coefficient, $D$

Diffusion coefficient values have been reported in the literature to vary from 70 to 1350 mm²/year. Diffusion is a function of water to cement ratio, cement type, temperature and humidity in addition to workmanship quality i.e. placing, compacting and curing etc (Page et al., 1981; Tuutti, 1982; Thomas, 1991; Liam et al., 1992; Maruya et al., 1994; Zhang & Gjorv, 1996; Glass & Buenfeld, 1997). Some researchers have argued that diffusion
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coefficient decreases with time e.g. HETEK (1996). This variation is attributed to the hydration of cement but tends to become small after the first few years from construction (Bamforth et al., 1997). Thoft-Christensen (2002a) and Vu & Stewart (2000) suggested that diffusion coefficient is dependent on w/c and temperature and is not a real physical constant. A model error (random variable) was assigned to the diffusion coefficient to indicate its variability rather than using a random variable for it directly. Vu & Stewart (2000) presented a model for diffusion coefficient, originally developed by Papadakis in 1996 whereas the model error based on field observations was used to estimate its dispersion (COV). Sensitivity analysis by Enright and Frangopol (1998a) observed that the COV of diffusion coefficient has a small effect on the corrosion initiation time.

Sterritt (2000) analysed field samples obtained from different published sources and came up with three different broad categories related to concrete and workmanship quality named ‘Good Quality’, ‘Average Quality’ and ‘Poor Quality’. The diffusion coefficients and concrete cover models for these cases are given below and are also adopted in the current study:

WQ1. $D_{\text{Good}} = \text{Lognormal}; \text{Mean}=5 \times 10^{-5}, \text{St. Dev.}=1 \times 10^{-5} \text{ m}^2/\text{year}$ ;

$X_{\text{Good}} = \text{Normal}; \text{Mean}= 40 \times 10^{-3}, \text{St. Dev.}=5 \times 10^{-3} \text{ m}$

WQ2. $D_{\text{Avg}} = \text{Lognormal}; \text{Mean}=10 \times 10^{-5}, \text{St. Dev.}=2 \times 10^{-5} \text{ m}^2/\text{year}$ ;

$X_{\text{Avg}} = \text{Normal}; \text{Mean}= 40 \times 10^{-3}, \text{St. Dev.}=10 \times 10^{-3} \text{ m}$

WQ3. $D_{\text{Poor}} = \text{Lognormal}; \text{Mean}=15 \times 10^{-5}, \text{St. Dev.}=3 \times 10^{-5} \text{ m}^2/\text{year}$ ;

$X_{\text{Poor}} = \text{Normal}; \text{Mean}= 40 \times 10^{-3}, \text{St. Dev.}=15 \times 10^{-3} \text{ m}$

These models for diffusion coefficient and concrete cover are plotted in Figure 6.16 and the associated prior corrosion initiation times are shown in Figure 6.17.
As can be seen, the reduction in quality (i.e. 100% increase in mean diffusion coefficient and 100% increase in COV of concrete cover) results in an early corrosion initiation (i.e. reduction of mean value (by about 44%) and an increase in uncertainty of corrosion initiation time (i.e. COV increase of approx. 35%) at reinforcement level.

The posterior time to corrosion initiation at reinforcement level for different models of concrete and workmanship quality and for various sensor initiation times is shown in Figure 6.18. The increase or decrease in mean value of time to corrosion initiation depends strongly on the initiation time at sensor location as can be seen from Figure 6.18, but as can be seen, there is a considerable reduction in uncertainty of posterior performance for all hypothesised sensor initiation times. The reduction in COV is about 70% regardless of concrete quality and sensor initiation time.
Figure 6.18: Posterior corrosion initiation time for various models of concrete and workmanship quality and for various sensor initiation times.

Figure 6.19 summarises the effects of the number of sensors in reducing the uncertainties associated with predicted performance. The reduction in uncertainty is evident (albeit mild after first updating) showing continual effectiveness of updating methodology in reducing associated uncertainties. In contrast to the previous cases, the figure shows that the relative difference in COV of corrosion initiation times among different concrete qualities is practically constant after the first updating.

The time to first intervention, assuming 10% probability of corrosion initiation as the durability limit state, based on prior performance model is 6.1, 2.4 and 1.1 years respectively i.e. difference of 5 years between the good and the poor quality case. This difference (i.e. 5 years between these two cases) is almost the same for the posterior predicted performance even though the uncertainty associated with posterior performance is considerably reduced. This demonstrates that workmanship and material quality have a strong influence on both the prior and the posterior performance prediction (corrosion
initiation time) unlike the first two parameters examined in this paper (e.g. exposure condition, threshold chloride concentration). This signifies the importance of obtaining additional information in this respect to improve / update the confidence in material and workmanship quality prior to performance evaluation.

6.2.5. Instrument / Measurement Uncertainty, $T_{ins}$

Monitoring the penetration of threshold chloride concentration at various depths of concrete cover is used to update the time to corrosion initiation at rebar level. Some common instruments used for this purpose are shown in Figure 2.14 & 2.15. Their working principle is the same as that of a half cell.

When the chloride concentration increases beyond its threshold level at certain cover depth, any steel at that depth depassivates and the negative potential and current value increases. By monitoring this variation in potential or current values, the time to corrosion initiation at the sensor location can be estimated. In general, a single potential/current value has been used to model the initiation of corrosion e.g. Raupach (2002) used 400mV and 10uA as the limiting values to model corrosion initiation for expansion ring system (Figure 2.15). These values depend on the type of instrument being used. Table 6.2 summarises the limiting values for different materials.

Table 6.2: ASTM criteria for corrosion of steel in concrete (Broomfield, 1997).

<table>
<thead>
<tr>
<th>Copper / Silver / Chloride</th>
<th>Standard Hydrogen Electrode</th>
<th>Calomel</th>
<th>Corrosion Condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>&gt;-200 mV</td>
<td>&gt;-106 mV</td>
<td>&gt;+116 mV</td>
<td>&gt;-126 mV</td>
</tr>
<tr>
<td>-200 to -350mV</td>
<td>-106 to -256mV</td>
<td>+116 to -34mV</td>
<td>-126 to -276mV</td>
</tr>
<tr>
<td>&lt; 350 mV</td>
<td>&lt;-256 mV</td>
<td>&lt;-34 mV</td>
<td>&lt;-276 mV</td>
</tr>
<tr>
<td>&lt;-500 mV</td>
<td>&lt;-406 mV</td>
<td>&lt;-184 mV</td>
<td>&lt;-426 mV</td>
</tr>
</tbody>
</table>

It is evident from the table that a limiting potential value corresponding to corrosion initiation cannot be modelled accurately as it involves a degree of uncertainty. This uncertainty in modelling the limiting potential value can also be observed from the data
published by Lentz et al. (2002) where, after half cell measurements, the steel bars were exposed to check the actual reinforcement condition. This additional uncertainty needs to be incorporated into the probabilistic methodology. As described in the previous chapters, a possible solution is to model the corrosion initiation using two limiting potential values. The first limiting potential would correspond to a very low probability of corrosion initiation while the second would correspond to a very high probability of corrosion initiation. The time required by the sensor to transit from first potential to the second, termed here as $t_{\text{ins}}$, represents instrument / measurement uncertainty. The value for $t_{\text{ins}}$ depends upon the chloride ingress rate, concrete quality and the difference between the upper and lower limiting potential values. From the curve of potential values vs time, presented by Raupach (2002), $t_{\text{ins}}$ can be approximated as 30 days. In addition, some degree of uncertainty would also exist due to the location of instrument / sensors in the concrete cover. This is incorporated using their location as random variable.

The following values of $t_{\text{ins}}$ have been considered in order to study the effects of this variable on the posterior corrosion initiation time.

- **IU1.** 0.05 year (18 days)
- **IU2.** 0.1 year (36 days)
- **IU3.** 0.15 year (55 days)

The instrument / measurement uncertainty affects only the posterior predicted corrosion initiation time. This effect is summarised in Figure 6.20 for different $t_{\text{ins}}$ values. It can be seen from the figure that for all the three cases of $t_{\text{ins}}$, the mean as well as the COV for the posterior predicted corrosion initiation time are practically the same i.e. the instrument / measurement uncertainty is insensitive to the predicted performance.
6.3. Summary and Conclusions

The results of the case study highlight the effects of various uncertainties on the prior as well as posterior predicted performance. It can be concluded from the above results that the uncertainties associated with input parameters affect significantly prior performance distributions whereas posterior performance (time to corrosion initiation in this case) distributions were found to be significantly less sensitive to these input uncertainties, i.e. exposure conditions, threshold chloride concentrations and instrument uncertainties. Hence the introduction of monitoring together with a methodology for performance updating would be highly beneficial in reducing uncertainties in the management of concrete structures.

A linear correlation between the sensor initiation time at certain depth and the mean posterior rebar corrosion initiation time is evident whereas the COVs for the posterior distributions are insensitive to sensor initiation times. Hence, in the practical applications, the posterior corrosion initiation time at rebar level can be estimated directly from these curves once the actual corrosion initiation time at the sensor location becomes available.

The study of effects of various threshold chloride concentration and exposure condition models on the predicted performance also concludes that updating of overall performance would be more beneficial than monitoring and updating of these parameters on an individual basis.
A comparison of various models for diffusion coefficient and concrete cover shows strong influence of workmanship and material quality on both the prior and posterior performance distributions, thus highlighting the importance of these parameters in predicting structural performance with better confidence. Similarly, the comparison of outcomes from various deterioration model uncertainty distributions clearly demonstrates the need for better predictive models (with lower modelling uncertainty) to improve the confidence in performance prediction and hence on decisions regarding management of the structures.
Management Strategies and Life-cycle Cost Comparison

7.1. Introduction

In modern management systems, decisions regarding management activities (i.e. maintenance and repair scheduling, and more recently, inspection planning etc) are based on predicted performance. The uncertainties (both epistemic and aleatory) associated with the probabilistic performance prediction being considerable, the decisions based on these predictions may result in unnecessary and costly repairs or in other extreme cases may prove fatal for the safety of structures.

Various management strategies are being implemented by different managing authorities depending upon the size of infrastructure, budgetary constraints and the availability of information within those budgetary constraints. A comparison of different management strategies is presented in this chapter that compares their safety characteristics and life cycle costs. The strategies are based (1) on regular inspections alone; (2) on predictive models updated through regular inspections; (3) on predictive models updated through optimized inspections; and (4) on predictive models updated using information from permanent health monitoring systems.

The updating methodology based on Bayesian framework (see chapter 5) is used to improve confidence in the predicted performance through data obtained by the inspections and/or health monitoring of the structures. In this chapter, the safety levels and life cycle costs for the management strategies are presented and discussed for various assumed inspections and health monitoring outputs (i.e. assuming different environmental severity and material properties). Furthermore, the relative effectiveness of these strategies from the point of view of safety and cost are also highlighted.
7.2. Bridge Management Strategies

A major input in LCC analysis is the actual timing of inspections, repairs and maintenance activities. The analysis would be straightforward provided the timings and the associated cost of actions became available for each strategy. In reality, uncertainties regarding bridge performance, availability of resources and financial constraints render the accurate determination of actual timings a difficult task. Hence, different strategies may be formulated, influenced also by the type of decision support tool available as well as the policy adopted by the management/owner of the bridge stock. In the following, four different management strategies are formulated starting from what is current practice and moving on to more advanced options. The consequences of adopting these strategies are evaluated through LCC analysis.

It is assumed here that the inspections carried out during the service life are principal inspections that include the use of half-cell potential measurements. The outcome of these inspections would be the extent (e.g. percentage area) of the members showing corrosion activity. The maximum tolerable area showing corrosion activity is assumed in this example as 10% of the total member area.

Following symbols are used in flowcharts for the management strategies.

- $T_S$ = Time framework for decision process (e.g. 20 or 30 years)
- $T_{Rp}^c$ = Predicted time to repair no. $c$ estimated through predictive models.
- $T_R^c$ = Time to repair no. $c$.
- $T_{1i}^c$ = Time for inspection number $i$ during repair cycle $c$.

$i = 1,2,3,\ldots,I_{\text{max}}$ (maximum number of inspections within a repair cycle).
$c = 1,2,3,\ldots,C_{\text{max}}$ (maximum number of repair cycles within decision period).

$\beta(T_{1i}^c) = \text{Performance measure at the time of inspection i, for repair cycle c.}$

$\beta_{\text{upd}}(T) = \text{Posterior performance measure at time T based on the inspection i, and repair cycle c.}$

$\beta_L = \text{Target value of performance measure.}$
7.2.1. Decisions based on Regular Inspections

In the UK and many other countries, principal inspections are carried out at 6 year intervals regardless of the age of bridges. Repairs would be carried out if the scheduled inspection (at every 6 years) revealed that the corroding area of the member is more than, or close to, the maximum specified limit. The inspection and repair decision process for this case is summarised in Figure 7.1. It is assumed that the concrete cover is replaced during repairs and the performance of the member is returned back to its original level.

\[ T^i_c(T) = \beta_L \] = Time at which the posterior performance measure for inspection i, and repair cycle c becomes equal to the target performance level.

**Figure 7.1: Management activities for regular inspections case.**

7.2.2. Decisions based on Predictive Models Updated through Regular Inspections

Probabilistic predictive models have been developed to predict the structural performance at any given time following the principles highlighted in section 2.4 (such as the one described in Eq. 2.5 & Eq. 5.1). These prior models can be updated every time a principal
inspection is carried out that reveals the actual performance of members at the time of inspection.

Let us assume that an inspection at time $T_{\text{insp}}$ of a member reveals that the actual percentage area indicating corrosion initiation is between $a_1$ and $a_2$. The difference between $a_1$ and $a_2$ represents the measurement uncertainty, i.e. the possible error in inspection results. The posterior distribution of the member’s condition given this inspection outcome can be obtained using the following expression.

$$F_{A^{\text{upd}}}(a) = P[A(t) \leq a \mid A(T_{\text{insp}}) > a_1 \cap A(T_{\text{insp}}) \leq a_2] \quad \text{.........Eq. 7.1}$$

Where $F_{A^{\text{upd}}}(a)$ is the posterior distribution of the percentage area indicating corrosion initiation; $A(T_{\text{insp}})$ represents the prior predicted percentage area indicating corrosion initiation at the time of inspection. The above equation can be rewritten using Bayes theorem as follows.

$$F_{A^{\text{upd}}}(a) = P\left[\frac{A(t) \leq a \cap A(T_{\text{insp}}) > a_1 \cap A(T_{\text{insp}}) \leq a_2}{A(T_{\text{insp}}) > a_1 \cap A(T_{\text{insp}}) \leq a_2}\right] \quad \text{.........Eq. 7.2}$$

In this case (i.e. where decisions regarding management activities are based on predictive models aided by regular inspections) the member in question would need repairing when the performance measure estimated through the predictive model and updated through the regular inspections (e.g. Eq. 7.2) falls below the target performance level before the next scheduled inspection. But if the allowable limit is expected to be reached at a time greater than the time of the next scheduled inspection, the performance is updated (after the scheduled inspection) and the time for repairs is modified based on the posterior performance with reduced uncertainties. The flowchart in Figure 7.2 shows the inspection and repair decision process for this case.
7.2.3. Decisions based on Predictive Models Updated through Optimal Inspections

It is generally accepted that inspection intervals for new bridges can be long whereas more frequent inspections may be required for aging structures. Hence, a strategy based on decisions made using predictive models and updated through optimised inspections has also been considered. The inspection intervals are not fixed in this case but are dependent on the performance of structures. An inspection is carried out every time the predicted reliability of the structure, before or after updating (based on previous inspection results), approaches allowable limit. The structure would need repairing if the updated reliability (following inspection results) falls below target reliability. The flowchart for this case is shown in Figure 7.3.

Figure 7.2: Management activities for predictive models updated through regular inspections.


**7.2.4. Decisions based on Predictive Models Updated through Health Monitoring Systems**

The posterior performance (time to corrosion initiation) based on additional information obtained through structural health monitoring is given by (Eq. 5.14) and is reproduced below.

\[
F^*_t(t) = P \left[ \left( T_i(X = X_i) \leq t \right) \bigcap_{i=1}^{n} M_i \leq 0 \bigcap_{i=1}^{n} M(X_i) > 0 \right]
\]

\[
\bigcap_{i=1}^{n} M_i \leq 0 \bigcap_{i=1}^{n} M(X_i) > 0
\]

----------Eq. 7.3

For the case where health monitoring system is installed, the prior predicted performance may be updated regularly at any given time interval (e.g. 0.1 years) and the time to repair would be given when the posterior performance distribution approaches target performance measure.

\[
T^c_R = T \left[ \beta^c_{\text{upd}}(T) \leq \beta_L \right] < T_S
\]

In this case, the additional costs associated with installation and operation of the health monitoring systems should also be incorporated into the life cycle cost analysis.
7.3. Results and Discussion

A structural system consisting of four members is considered as an example in order to compare life cycle costs for individual members and for the overall system. Examples of such a system could be for instance a crossbeam supported by three columns, a two lane slab supported by beams at the ends, or simply a beam or a slab element divided into four zones. The time for the attainment of limit state has been assumed to vary for the members to reflect the variability in their exposure conditions and material qualities etc. In order to illustrate the characteristics associated with the above strategies, the performance profiles are also plotted for each of the strategy for the decision period (30 years in this case).

Some general considerations are given below:

- Attainment of 10% probability of corrosion initiation has been taken as the limit state to trigger repair.
- The repair consists of replacement of cover concrete for the entire member (it is an upper bound assumption).
- A consistent time frame is required to enable comparison of the LCCs for the various management strategies. A decision time frame of 30 years has been considered for this purpose.
- Discount factor = 3% per annum
- The mean prior performance prediction (i.e. time to corrosion initiation) is obtained using the deterioration model presented in Eq. 5.1. The performance measure of interest is the normalised area indicating corrosion activity, $A_{ini}/A_{o}$, at a given point in time (Figure 7.4a).

![Figure 7.4 : Normalized member area indicating corrosion initiation a) Mean b) COV.](image)
The scatter associated with the performance measure is modelled through a coefficient of variation which is assumed to increase linearly with time as shown in Fig. 7.4b.

In practice, there would be some degree of correlation between the normalised corroding areas at various points in time; this correlation should be reducing with time. In the present study, this correlation is assumed to reduce linearly from perfect to zero in any time interval of 10 years.

The hypothesized outcomes of inspection at any given time are shown in Figure 7.5. The values of ‘a1’ and ‘a2’ (as described in section 7.2.2) are evaluated as ± 2.5% of the values obtained from this figure, i.e. the measurement error is assumed to be 5%, or, in other words, the inspection results are assumed to be 95% reliable. The different initiation characteristics for the members in Fig. 7.5 are assumed to result from the variation in exposure conditions and material and workmanship quality.

![Figure 7.5: Hypothesized inspection results at any given time.](image)

The cost data assumed for this study are as follows (Rubakantha, 2001)

**Half cell inspection** £30.0 / m²

**Patch Repair** £2240 / m² (i.e. cost of repair over cost of inspection \( C_R / C_I \) = 75)

Regarding the cost of health monitoring, a comparison of different monitoring to inspection cost ratios (\( C_M / C_I \)) ranging from 1 to 5 has been carried out.

### 7.3.1. Decisions based on Regular Inspections (Strategy A)

The hypothetical outcomes of the inspections (Fig. 7.5) at year 6, 12, and 18 for all the members are presented in Table 1. As can be seen the percentage area indicating corrosion activity at year 6 is well below 10% (limiting value) for all the members and none are
likely to attain the limit state in the near future. Hence, no maintenance is required for any member at year 6.

Table 7.1: Inspection results for the members

<table>
<thead>
<tr>
<th></th>
<th>M1</th>
<th>M2</th>
<th>M3</th>
<th>M4</th>
</tr>
</thead>
<tbody>
<tr>
<td>6 yr</td>
<td>2.95%</td>
<td>0.47%</td>
<td>0.09%</td>
<td>0.02%</td>
</tr>
<tr>
<td>12 yr</td>
<td>35%</td>
<td>12.5%</td>
<td>4.36%</td>
<td>1.6%</td>
</tr>
<tr>
<td>18 yr</td>
<td>2.95%</td>
<td>0.47%</td>
<td>19.5%</td>
<td>9.37%</td>
</tr>
</tbody>
</table>

The results at year 12 shows that members 1 and 2 have reached the limiting value and would be repaired, thus restoring them to the ‘as constructed’ condition whereas members 3 and 4 would not be repaired. Finally, inspection at year 18 reveals that member 3 has exceeded and member 4 is close to attaining the limit state, hence both members would be repaired at that point in time.

Based on the above information, the profile of \( A_{in}/A_o \) as a function of time is plotted for all members in Figure 7.6. Hence, life cycle costs for decision time frame of 30 years can be estimated. Since an existing structure is being assessed, the design and construction costs have already occurred. Hence, only the costs associated with inspections and repairs are considered for comparison (see Eq. 2.1 for details).

\[
LCC = \text{NPV for the cost of inspections and repairs} = \sum_{i=1}^{N} \frac{C_I}{(1 + r)^i} + \sum_{j=1}^{N} \frac{C_R}{(1 + r)^j}
\]

\[
LCC_1 = 1.837 C_I + 1.193 C_R = 1.217 C_R \quad \text{(given } C_R \text{ 75 C_I)}
\]

Similarly, LCC for the other members can be computed. These are summarised in the following table.

Table 7.2: Life-cycle costs for all members

<table>
<thead>
<tr>
<th></th>
<th>M1</th>
<th>M2</th>
<th>M3</th>
<th>M4</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1.837C_I+1.193C_R</td>
<td>1.837C_I+1.193C_R</td>
<td>2.313C_I+0.587C_R</td>
<td>2.313C_I+0.587C_R</td>
</tr>
<tr>
<td></td>
<td>1.217 C_R</td>
<td>1.217 C_R</td>
<td>0.618 C_R</td>
<td>0.618 C_R</td>
</tr>
</tbody>
</table>
The total LCC for the entire system would become

\[ \text{LCC}_{\text{sys}} = 3.67 \, C_R \]

The performance profiles for the members (Fig. 7.6) show that the durability limit state is violated for three of the members due to repairs being made after the pre-determined regular inspections. This implies that a required minimum performance is not guaranteed in this case. To account for this there is a need to use predictive models in conjunction with inspection results.

**7.3.2. Decisions based on Predictive Models Updated through Regular Inspections (Strategy B)**

At the outset, all members/areas are assumed to be identical and a common prior performance is considered as shown in Figure 7.4. It is clear that the prior performance will reach the allowable performance limit (10% probability of corrosion initiation) at year 6.1. This time is greater than 6 years (scheduled time for the first inspection), hence an inspection is carried out at year 6 for all the members and their hypothesised outcomes.

\[ \text{Durability Limit State} \]

\[ \text{Fig. 7.6: Variation of 'corrosion initiated area' with time.} \]
UniS

Management Strategies and Life-cycle Cost Comparison

(from Fig. 7.5) are presented in Table 7.3. These are the same as those used in the previous section. In this case, the instrument uncertainty should also be taken into account because this may have a strong influence on the posterior performance prediction. The inspection results incorporating measurement errors (i.e. ‘a1’ and ‘a2’) as explained in Sec. 7.2.2 are also shown in the table. A value of zero is assigned to the lower limit, a1, for members M2, M3 and M4 since an inspection cannot indicate ‘negative’ corrosion activity. Based on these probabilistic outcomes, the prior performance profile for all four members is updated and the predicted time to repairs is re-evaluated using the updated profiles.

Table 7.3: Inspection results for all members with their upper and lower limits.

<table>
<thead>
<tr>
<th></th>
<th>M1</th>
<th>M2</th>
<th>M3</th>
<th>M4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean (Fig. 7.5)</td>
<td>2.95 %</td>
<td>0.47 %</td>
<td>0.09 %</td>
<td>0.02 %</td>
</tr>
<tr>
<td>Lower Limit, a1</td>
<td>0.45 %</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Upper Limit, a2</td>
<td>5.45 %</td>
<td>2.97 %</td>
<td>2.59 %</td>
<td>2.52 %</td>
</tr>
</tbody>
</table>

The posterior performance profile for member 1 (representing normalised corroding area) is shown in Fig. 7.7. It is clear that the allowable limit for this member after updating is expected to be attained at 8.13 years, which is less than the time of next inspection (i.e. at 12 years). Hence the member will be repaired at 8.13 years. Similar to the previous case, it is assumed that the state of the member after repair will return to its original level and this cycle is repeated throughout the remaining decision time frame. The performance profile of the member for the entire 30 years period is also plotted in Fig. 7.7.

Fig. 7.7: Posterior A integral/Ao Vs time and updated performance profile for 30 years (member 1).
It can be seen from the figure that the member needs repairing at year 8.13, 16.26 & 24.39 whereas the principal inspections would be carried out every 6 years after the construction and major maintenance activity i.e. at 6, 14.13, and 22.26 years. Hence the LCC for member 1 for the 30 years period would be

\[ = 2.014 \ C_I + 1.891 \ C_R = 1.918 \ C_R \]  

(where \( C_R \) = 75 \( C_I \))

Similar to the procedure followed for member 1, the posterior predicted ‘area indicating corrosion’ and the life cycle performance profiles for member 2, 3, and 4 are shown in Figure 7.8. The main observations arising from these figures are:

---

**Fig. 7.8:** Posterior \( A_{in}/A_o \) Vs time and updated performance profile for 30 years.

Muhammad Imran Rafiq
The performance of the members does not violate the durability limit state.

The time to attainment of durability limit state for members 2, 3 and 4 is 10.45, 10.8 and 10.9 years respectively. These times are less than the next scheduled inspection (at 12 years), hence the members would be repaired at these times respectively.

The hypothesized times to corrosion initiation for the members 2, 3, and 4 are 11.30, 14.70 and 18.30 years respectively (Fig. 7.5). These are greater than the repair times suggested by this approach, hence it can be concluded that this approach is too conservative for optimal maintenance planning.

Let us assume that an additional inspection is carried out at year 12 for members 3 & 4. The posterior performance curves ($A_{	ext{ini}}/A_o$) for these members show that the time to repair for these members can be further delayed (as compared to the predicted values using this strategy). For instance, the predicted time for the attainment of allowable limit can be increased from 10.8 to 13.0 years for member 3, and 10.9 to 13.3 years for member 4. When the decisions regarding management activities are based on the ‘predictive models updated through regular inspections’, the procedure does not allow an additional inspection at year 12. The predicted performance in such cases would falls below the target performance level, e.g. an inspection at year 12 for member 3 would yield 4.35 % of the corroded area but the predicted ‘area indicating corrosion’ would have exceeded the maximum allowable value (i.e. 15.8 % instead of 10 %). In order to get an optimal management plan whilst maintaining the same target performance, the next inspection for member 3 must be carried out at 10.8 years. The same is also true for member 4. This provides a rationale for using optimised inspection schedule instead of a fixed interval.

Similar to the previous case, LCC for the members for 30 years period are summarised in Table 7.4.

**Table 7.4: Life cycle costs for all members.**

<table>
<thead>
<tr>
<th></th>
<th>M1</th>
<th>M2</th>
<th>M3</th>
<th>M4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost</td>
<td>2.014Ci + 1.891Cr</td>
<td>1.904Ci + 1.273Cr</td>
<td>1.887Ci + 1.254Cr</td>
<td>1.884Ci + 1.250Cr</td>
</tr>
<tr>
<td></td>
<td>1.918 Cr</td>
<td>1.298 Cr</td>
<td>1.279 Cr</td>
<td>1.275 Cr</td>
</tr>
</tbody>
</table>
Hence the total life cycle cost for the system would be

\[
LCC_{sys} = LCC_1 + LCC_2 + LCC_3 + LCC_4
\]

\[
= 1.918 \text{ CR} + 1.298 \text{ CR} + 1.279 \text{ CR} + 1.275 \text{ CR}
\]

\[
= 5.77 \text{ CR}
\]

The life cycle costs for the system is considerably higher than the previous case but the target performance levels are maintained, which is not possible for the case where decisions are based entirely on the regular inspections.

### 7.3.3. Decisions based on Predictive Models Updated through Optimal Inspections (Strategy C)

The previous case suggests that inspections at optimum intervals (based on the predicted structural performance) rather than the fixed intervals could be beneficial in reducing the LCC of the system whilst maintaining required performance levels.

As in the previous case, the prior predictive model (Fig. 7.4) reveals that the predicted time to the attainment of limit state for all members is 6.1 years. Hence the first principal inspection is required at 6.0 year for all the members. The inspection results at year 6 are the same as that of the previous case, (Table 7.3). The time for next inspection would be different from the previous case, and is estimated based on the posterior predicted performance (updated through the inspection results) as described in the flowchart (Fig. 7.3).

The prior and updated performance for member 1, based on the optimal inspection interval, is shown in Figure 7.9.
After the inspection and subsequent updating at year 6, the predicted time for the attainment of allowable limit is 8.13 years (Fig. 7.9). Hence, the next inspection would be carried out at 8 years, which would reveal the ‘actual corroding area’ of 10.0% (Fig. 7.5). Since the limiting value (10% corroding area) has been attained for member 1, the repairs would be carried out at 8 years.

The LCC for member 1 based on the performance profile in Figure 7.9 would be

\[ \text{LCC}_1 = 3.925 \ C_I + 1.904 \ C_R = 1.956 \ C_R \]

For member 2, the posterior predicted performance based on inspection at year 6 reveals the predicted time for limit state attainment as 10.45 years (Fig. 7.10). Hence, the next inspection is scheduled at 10 years, which reveals that the ‘normalised area indicating corrosion’ is 6.18% (Fig. 7.5). The actual performance of the member is higher than the allowable value (i.e. the actual ‘area indicating corrosion’ is less than the target value of 10%). Furthermore, the difference between predicted and actual performance at the time of inspection (i.e. 8.37% vs. 6.18%) is considerable. Hence, the predicted performance is further updated based on the inspection at 10 years.

The posterior performance (at 10 years) reveals that the predicted time for limit state attainment has been increased to 11.1 years. Hence, another inspection at 11 years is required to verify the results.
The mean value of ‘area indicating corrosion’ indicated by inspection at 11 years is 9.05% (Fig. 7.5), which is close to the predicted value at that year i.e. 9.42% (Fig. 7.10) hence the repair for member 2 is carried out at 11.1 years. The repair time proposed by this strategy is very close to the hypothesized time to reach allowable limit for this member (11.3 years). Assuming the same cycle of inspection and repairs for the 30 years period, the life cycle cost for the member would be

\[ 4.398C_I + 1.239C_R = 1.298C_R \]

Following similar procedure for the other two members, the results are shown in Fig. 7.11.
The thick green line in the figure shows the hypothesized initiation characteristics for the members (Fig.7.5). It can be seen that the predicted performance profiles updated through inspection results are converging gradually to the assumed performance for the members.

It can be seen from Figure 7.11 that the difference between the time for limit state attainment before and after updating through inspection 4 (at year 13 for both M3 and M4) is less than 1.0 year i.e. convergence becomes too slow. Hence the members M3 and M4 are assumed to be repaired at year 13.5 and 13.9 respectively. Their respective life cycle performance profiles for the 30 years period are also shown in the same figure.

The LCC for member 3 and 4 (based on the estimated time to inspections and repairs) are

\[
\begin{align*}
LCC_3 &= 4.953 C_I + 1.121 C_R = 1.187 C_R \\
LCC_4 &= 4.893 C_I + 1.103 C_R = 1.168 C_R
\end{align*}
\]

Hence the total life cycle cost for the system would be

\[
LCC_{sys} = LCC_1 + LCC_2 + LCC_3 + LCC_4
\]

\[
= 1.956 C_R + 1.298 C_R + 1.187 C_R + 1.168 C_R
\]

\[
= 5.609 C_R
\]

It can be seen that the LCC for this case (using optimised inspection intervals) is less than that of the previous case (regular inspection intervals) keeping the same target performance in both cases.

**7.3.4. Decisions based on Predictive Models Updated through Health Monitoring Systems (Strategy D)**

In this section, the effectiveness of an updating methodology (see Chapter 4 & 5) is highlighted, that uses data obtained through health monitoring systems to update the predicted structural performance. For simplicity, only one sensor is assumed in each member located at 10mm cover depth. The prior and posterior performance curves at rebar level are shown in Figure 7.12 for the various sensor initiation times. The sensor initiation times (located at 10mm cover depth) have been assumed based on the hypothesized
corrosion initiation characteristics for all the members at the rebar levels respectively (Fig. 7.5).

![Graph showing prior and posterior curves for 'Area showing corrosion' updated through monitoring.]

Figure 7.12: Prior and Posterior curves for 'Area showing corrosion' updated through monitoring.

Based on these posterior performance curves, the life cycle performance of the members for 30 years period are shown in Figure 7.13.

![Graph showing performance profiles for all members updated through health monitoring systems.]

Figure 7.13: Performance profiles for all members updated through health monitoring systems.
The life cycle cost for the members in this case would become

\[ LCC = \text{Cost of repairs} + \text{Cost of Instrumentation} \]

Assuming the cost of instrumentation to be twice the cost of inspection, the LCC for the members are shown in Table 7.5.

### Table 7.5: Life cycle costs for all members.

<table>
<thead>
<tr>
<th>M1</th>
<th>M2</th>
<th>M3</th>
<th>M4</th>
</tr>
</thead>
<tbody>
<tr>
<td>(2.0C_1 + 1.904C_R)</td>
<td>(2.0C_1 + 1.229C_R)</td>
<td>(2.0C_1 + 1.067C_R)</td>
<td>(2.0C_1 + 0.582C_R)</td>
</tr>
<tr>
<td>(1.931C_R)</td>
<td>(1.255C_R)</td>
<td>(1.094C_R)</td>
<td>(0.609C_R)</td>
</tr>
</tbody>
</table>

The total life cycle cost would become

\[ LCC_{sys} = LCC_1 + LCC_2 + LCC_3 + LCC_4 + LCC_5 \]

\[ = 4.889C_R \]

Hence LCC for the case where monitoring data is used to update prior prediction is most economical keeping the same target performance levels.

### 7.3.5. Comparison of Different Strategies

The LCC’s for the different strategies are shown in Figure 7.14 for all the members.
The figure clearly indicates strategy A in general has the least costs under the particular hypothesized inspection outcomes considered. However it is not possible to maintain a target performance level, and hence the probability of failure of the structure would be considerably higher compared to the other cases where a target performance level is maintained throughout the service life. This example highlights the inability of current practice of regular inspections to maintain consistent performance levels in the structure.

Among strategies B, C and D, where a target performance level is maintained throughout, strategy D is generally the most economic whereas strategy B is the most expensive as can be seen from Fig. 7.14. The only exception where strategy C is
expensive than strategy B is member 1. This is a special case where the updating after first inspection has yielded the actual initiation time (which would of course be very rare). The time to repair in this case is the same for strategy B and C, whereas for strategy C an additional inspection is required to verify the attainment of limit state before repairs hence slightly increases its LCC.

- Figure 7.14 also demonstrates that the cost of repair is the major factor contributing towards the life cycle costs and the inspection and monitoring cost is negligibly small compared to the repair costs. Hence the ratio of cost of monitoring to the cost of inspection would not have any significant effect on the total cost as is also confirmed by Fig. 7.15.

Figure 7.15: Life cycle cost comparison for various monitoring to inspection cost ratio.
As is clear from the figure that the ratio of monitoring to inspection cost does not have a significant effect on the total life cycle cost and its reduction is mainly due to the delay in repairing which is made possible due to the extra confidence in the performance prediction given by the health monitoring data.

The above figure also gives an estimation of the cost that can be spent on monitoring keeping the same LCC as for the other cases although the life cycle cost is also dependent on the initiation characteristics of the beam (Fig. 7.15).

7.4. Summary and Conclusions

A comparison of different management strategies has been presented in this chapter on the basis of safety levels and the life cycle costs. The strategies examined are based on regular inspection alone, on predictive models updated through regular inspections, on predictive models updated through optimised inspections, and on predictive models updated through the use of health monitoring systems.

The results from a number of cases indicated that the minimum performance levels for the deteriorating structural systems are impossible to maintain in the strategy where decisions are based entirely on regular inspections. The strategy including health monitoring system is found to be considerably better than the others in terms of optimisation of management activities as well as minimisation of the life cycle costs. It has also been concluded that the primary factor responsible for the life cycle costs of existing concrete structures is the repair activities, notwithstanding the indirect costs which are not examined herein. The life cycle cost can be reduced considerably by delaying the repair activities. This can be achieved by increasing confidence in the predicted performance, which was found to be significantly improved in the case where the performance is updated through health monitoring systems. Comparison of the results for different monitoring to inspection costs concludes that the overall life cycle cost is relatively insensitive to this factor hence the optimisation of management activities can be achieved considerably better with the health monitoring system installed into the systems.
Chapter 8

Conclusions and Recommendations

The overall objective of this thesis has been to improve the assessment and maintenance optimisation procedures of a management system for deterioration prone structures by establishing a methodology that effectively utilises the data obtained through health monitoring systems. In particular the thesis investigates the governing deterioration phenomena in concrete structures and available models for their prediction, an exploration of available instruments to monitor the deterioration processes, the development and appraisal of a methodology to integrate data obtained through structural health monitoring into the decision process of a management system, and a comparison of various management strategies by analysing their safety characteristics and life-cycle costs. As is evident, there is a wide diversity of research areas in this study.

8.1. Conclusions

A review of research areas pertinent to the study reported in this thesis is presented in Chapter 2. In particular, these included deterioration of concrete bridges, the governing deterioration mechanisms and their modelling, key activities of a management system used to administer the deteriorating stock of bridges, available instruments having the ability to monitor the extent of the deterioration mechanisms, and the methods to combine the information obtained through monitoring systems with the management procedures. The following conclusions can be drawn from this critical review.

- The prediction of future condition and reliability for the deteriorating bridge stock (on an individual and network basis) is vital for their effective management.

- The visual inspections carried out at regular intervals do not by themselves provide sufficient information regarding bridge condition and reliability. Hence, these are of limited use in predicting future performance of the structures.

- Detailed inspections (including testing on the bridges) require access to all parts of the structures and are very expensive (including direct costs, i.e. equipment, and
indirect costs, i.e. traffic management and delays), and hence are not practically feasible for application on a routine basis.

- The corrosion of steel reinforcement is a major factor causing degradation of concrete structures. The primary mechanisms responsible for this are chloride induced deterioration and carbonation.

- There is an increasing trend for the use of probabilistic methods in the area of deterioration prediction, and associated increase in the use of reliability based management systems to help manage deteriorating bridge stock.

- Extensive research has been carried out for the deterministic and probabilistic modelling of chloride induced deterioration in the last decade. This has resulted in a whole host of deterioration models for a variety of exposure conditions and material qualities. There is wide agreement regarding the need to standardise deterioration models and growing consensus trying to develop and calibrate physically-based models.

- There are considerable differences in the data collected on site and in the laboratories regarding these deterioration models which reduce the effectiveness of the predictive models. The data available for a particular set of exposure conditions and material quality is scarce and inconsistent, and hence methodologies to effectively utilise such data for the management purposes are very important but have been under researched.

- There is a need for the development of methodologies that can effectively integrate data obtained through inspections and in-service health monitoring systems into the decision support tools of a management system.

The probability and statistical methods relevant to this study, together with the Bayesian approach for event updating are introduced in Chapter 3. Bayesian approach has the ability to formally and rigorously incorporate the data (both qualitative and quantitative) obtained through a series of testing and health monitoring carried out at various stages of the service life of a structure. The uncertainties associated with instruments and measurement processes can also be incorporated within the updating methodology using this approach. Hence, Bayesian approach has been used for the development of a methodology to integrate data obtained through inspections and health monitoring methods with available
prior information from other similar structures elsewhere, or from the past. The application of Bayesian statistics to the decision process is summarised and has been used for the comparison of various bridge management strategies in Chapter 8.

In Chapter 4, different areas within the context of reliability based management systems are highlighted that can benefit from the use of health monitoring systems, from the point of view of increasing confidence in the long term performance predictions. These include selection of governing mechanisms, their modelling, their variation in time and space, loading and their effects on structural systems, estimation and updating of overall system's performance, and some direct applications of the health monitoring system in planning the nature and timing of inspections, maintenance, and repair activities. A generic updating methodology based on Bayesian framework has been developed that is applicable to a variety of deterioration prone structural systems. A key prerequisite for the application of this methodology is the availability of a prior predictive model.

Chapter 5 deals with applications of the updating methodology from the perspective of reinforced concrete bridges subjected to chloride induced deterioration. The main conclusions from this chapter are:

- A performance based deterioration model is proposed for structures prone to chloride induced deterioration that combines durability limit states with the serviceability and ultimate limit states. This model enabled exploration of the potential benefits of, and justification for, the proactive management activities.

- A comparison of various limit states for structures subjected to chloride induced deterioration concluded that the durability limit state (defined by the time to corrosion initiation in this case) is the most important for proactive management, hence the application of updating methodology was focused on this limit state.

- A comparison of Latin Hypercube Sampling (LHS) and Generalised Conditional Expectation (GCE) method suggests that LHS is more economical for the present case because the time required to run each simulation in the GCE method outweighs the benefits of reduced number of simulation cycles. Furthermore, the LHS method shows better convergence characteristics than the GCE method in the case of Bayesian updating.
Examples of the updating methodology for various sensor initiation times show the effectiveness of methodology in reducing prediction uncertainties. It has been shown that information from the health monitoring systems can be used effectively, along with existing information and previous experience, to increase the confidence in predicted performance.

Examples of a systems approach in order to combine information from multiple sensors at various locations of a bridge is also presented in Chapter 5. It has been concluded that the confidence in predicted performance of a monitored domain can be improved using the proposed procedures.

The effect of various uncertainties on the prior and posterior predicted performance is presented in Chapter 6. The main conclusions from this chapter are as follows.

- The uncertainties associated with input parameters significantly affect the prior performance distribution whereas posterior performance is relatively insensitive to the input uncertainties. Hence, the introduction of health monitoring together with a methodology for performance updating would be beneficial in reducing uncertainties in the management of concrete structures.

- A linear correlation between the sensor initiation time and the mean posterior rebar corrosion initiation time was observed and the COV for the posterior distribution was found to be insensitive to the sensor initiation time. Hence, in practical applications, the posterior corrosion initiation times at rebar level can be estimated directly from these curves once the actual corrosion initiation time at the sensor location becomes available.

- It is concluded from the comparison of various threshold chloride and surface chloride concentration models that updating of the overall performance would be more beneficial than monitoring and updating of input parameters on an individual basis. A strong influence of workmanship and material quality is observed on the prior and posterior performance distribution, highlighting the importance of these parameters in predicting structural performance with better confidence. Modelling uncertainty is also found to have a considerable effect on the confidence with which performance can be predicted, hence emphasizing the need for better predictive models (with lower modelling uncertainty).
Chapter 7 elaborates the comparison of different bridge management strategies on the basis of safety levels and life-cycle costs. It has been concluded that:

- Minimum performance levels for the deteriorating structural systems are impossible to maintain in the strategy where decisions are based entirely on regular inspections.

- The primary factors contributing to the life-cycle costs of reinforced concrete structures are the repair activities. Life-cycle costs can be reduced considerably by delaying these repairs. This can be achieved by an increase in confidence in the predicted performance. The latter was found to be significantly improved in the case where performance is monitored through suitable systems.

- The overall life-cycle cost was found to be relatively insensitive to the ratio of monitoring to inspection costs. Hence, the strategy including health monitoring systems was found to be considerably better than the others in terms of optimisation of management activities as well as minimisation of LCC.

- Health monitoring systems only provide information on the condition at the location at which instrumentation is situated, and hence a suitable combination of health monitoring along with the detailed inspections must be sought to produce a safe and sustainable structural stock.

In summary, it is concluded that there is a great deal of potential in the effective use of health monitoring systems within various aspects of the modern bridge management systems. The effective integration of data obtainable through such systems can provide a better understanding of the state of any ongoing deterioration mechanisms, help maintain minimum target reliability, and reduce the chances of missing important information.

8.2. Recommendations for Future Work

Many areas of research became apparent during the study that can promote and improve the reported research work. The results and conclusions reported here are by no means the end but the beginning of development in this area. Recommendations for appropriate future work in this area are as follows.

- During the literature review, it was observed that modelling uncertainty, which is associated with any predictive model, has not been explored sufficiently. A
sensitivity analysis on the prior and posterior predicted performance concluded that its effect is significant. Hence, reliable models for this variable should be developed for various available deterioration models through laboratory and field data.

- Application of the developed methodology has been presented for bridges subjected to chloride induced deterioration. It could be extended to other deterioration mechanisms e.g. carbonation and ASR in concrete, fatigue in steel and cracking in masonry structures. It can also be developed for other structures which would benefit from inspection and maintenance optimisation such as offshore structures, pipelines, transmission towers, overhead reservoirs and aerospace structures etc.

- An important issue in the implementation of health monitoring systems for the management of deterioration prone systems is the location of sensing equipment. Very limited research has been directed towards this issue, therefore there is a need to develop rational methodologies to address this problem as well as the issue of how much data should be collected to give sufficient information regarding structural performance.

- It is both important and desirable that the developed methodology should be tested using laboratory and field observations before it can be applied to practical applications.

- A generic framework for multi-stage updating and performance prediction should be developed that uses data from a wide range of sources (i.e. deterioration and loading data) and from various different monitoring instruments (e.g. half cell survey and chloride profiles etc) to increase the confidence in performance prediction of a deterioration prone system. This can lead to the development of an intelligent and robust infrastructure management system.
\[
F(x) = \Pr[X \leq x] = \begin{cases} 
0 & x < a \\
\frac{x - a}{b - a} & a \leq x \leq b \\
1 & x > b 
\end{cases} 
\]

Where \(a\) and \(b\) represents the upper and lower limits of the distribution respectively as shown in the following figure.

![Uniform Distribution Figure](image)

The first and second moments of the distribution can be obtained using following expressions.

\[
E[X] = \frac{a + b}{2} \\
Var[X] = \frac{(b - a)^2}{12}
\]

### A.3 Lognormal Distribution

The lognormal distribution is generally used if the random variable is expected to have a skewed shape (either positive or negative) or it cannot have a value below zero (i.e., negative values). The relation between normal and lognormal distribution can be expressed as

\[Y = \ln (X)\]

Where \(Y\) is a normally distributed random variable and \(X\) represents log normally distributed random variable.

The probability density function and cumulative distribution function for a log-normal distribution are
The first and second moment for the lognormally distributed random variable $X$ are as follows.

$$E(X) = e^{\mu + \frac{\sigma^2}{2}}$$

$$Var(X) = e^{2\mu + 2\sigma^2} - e^{2\mu + \sigma^2}$$

Figure A.3: Lognormal Distribution
Appendix B: Derivation of Bayesian Theorem

If the probability of an event depends on the occurrence or non-occurrence of another event, the associate probability becomes a conditional probability. In the sample space shown in Figure B.1 the conditional probability of $E_1$ assuming $E_2$ has occurred can be obtained using following relation.

\[
P(E_1 \mid E_2) = \frac{P(E_1 \cap E_2)}{P(E_2)}
\]  

\[\text{Figure B.1: Sample space}\]

Now consider a sample space, divided into $n$ mutually exclusive and collectively exhaustive events, $E_1$.....$E_n$. Let $A$ be an event in the sample space (Thoft-Christensen & Baker 1982).

\[
P(A) = P(A \ E_1) + P(A \ E_2) + \ldots + P(A \ E_n)
\]  

\[\text{Figure B.2: Sample space divided into n mutually exclusive events}\]

From Fig. B.2, it is clear that

\[
P(A) = P(A \ E_1) + P(A \ E_2) + \ldots + P(A \ E_n)
\]

\[\text{Figure B.2: Sample space divided into n mutually exclusive events}\]

Using Equation B.1, the above Eq. can be re-written as

\[
P(A) = P(A \mid E_1)P(E_1) + P(A \mid E_2)P(E_2) + \ldots + P(A \mid E_n)P(E_n)
\]  

\[\text{Appendix B: Derivation of Bayesian Theorem}\]
Hence,

\[ P(A) = \sum_{i=1}^{n} P(A \mid E_i)P(E_i) \] .............. B.4

From the definition of Eq. B.1, it follows that

\[ P(A \mid E_i)P(E_i) = P(E_i \mid A)P(A) \] .............. B.5

Re-arranging and substituting value of \( P(A) \) from Eq. B.4 gives Bayesian Theorem.

\[ P(E_i \mid A) = \frac{P(A \mid E_i)P(E_i)}{P(A)} = \frac{\sum_{i=1}^{n} P(A \mid E_i)P(E_i)}{P(A)} \] .............. B.6
Appendix C-1:

MatLab™ Input files for updating methodology using LHS methods

% Setting the seed values to ensure samples can be reproduced
rand('state',12598643)
randn('state',58749618)

% To is used to calculate time of run at the end of simulation
To=clock;

% Assumed Sensor Outputs at 10, 20 and 30mm depths
Ta10=1.0;
Ta20=4.0;
Ta30=9.0;

% Time framework for decision purposes
ts=30;

% Instrumentation / measurement uncertainty.
Tint=0.1;

% Initializing variables
clear Pfnum; clear Pfdenom; clear Pfupdnum; clear Pfupddenom;
for c=1:1:50
c*5000000
end

% Maximum number of simulation cycles
Iteration=500000;

% Latin hypercube Uniform random sample (increasing order)
i=(1:Iteration)';
U=(rand(Iteration,1) + (i-1))./Iteration ;

% Generation of Co
[a,b]=sort(rand(Iteration,1));
U=U(b);
Co=logninv(U,1.141191,0.472381);

% Generation of Cth
[a,b]=sort(rand(Iteration,1));
U=U(b);
Cth=0.6+(1.2-0.6)*U;

% Validation Rule for Cth and Co
Z2=(Cth./Co);
Z2=Z2(Z2<1.0);
Z3=erfcinv(Z2);
% Making number of simulation cycles equal to the validated sample size for Cth and Co
Iteration=length(Z3);
clear Co; clear Cth;

% Generation of standard normal random number
U=U(1:Iteration);
N=norminv(U,0,1);

% Sample for D
[a,b]=sort(rand(Iteration,1));
N=N(b);
D=5e-5+1e-5*N;

% Sample for X
[a,b]=sort(rand(Iteration,1));
N=N(b);
X=0.04+0.005*N;

% Model error for initiation time calculation.
[a,b]=sort(rand(Iteration,1));
U=U(b);
Emod = logninv(U,-0.03031231,0.2462207); %Emod(1,0.25)

% Sensor location uncertainties.
[a,b]=sort(rand(Iteration,1));
N=N(b);
X10=0.01+0.001*N;
clear Z*; clear X*; clear D; clear U2;
clear Emod; clear a; clear b; clear U; clear N; clear M;

% Evaluation of corrosion initiation time
Z5=4.*D.*(Z3).^2;
Z6=X.^2;
Z610=X10.^2;
Z620=(X10+0.01).^2;
Z630=(X10+0.02).^2;
T=Emod.*(Z6./Z5);clear Z6;
[a,b]=sort(rand(Iteration,1));
Emod=Emod(b);
T10=Emod.*(Z610./Z5);clear Z610;
[a,b]=sort(rand(Iteration,1));
Emod=Emod(b);
T20=Emod.*(Z620./Z5);clear Z620;
[a,b]=sort(rand(Iteration,1));
Emod=Emod(b);
T30=Emod.*(Z630./Z5);
clear Z*; clear X*; clear D; clear U2;
clear Emod; clear a; clear b; clear U; clear N;

j=0; t=0;
clear Time; clear Pf; clear Pfupd; clear Tupd;

% Prior failure probability evaluation
Pfnum(c)=length(T(T<=ts));
Pfdenom(c)=length(T);

% Evaluation of Posterior corrosion initiation time and probability of failure
i=1:1:Iteration;

% Case when Initiation is not detected at any sensor.
if (t<Ta10 & t<Ta20 & t<Ta30)
  Tupd =T(T10(i)>t & T20(i)>t & T30(i)>t);
  valid(c)=length(Tupd);
  %

% Initiation detected at 10mm sensor only
elseif (t>=Ta10 & t<Ta20 & t<Ta30)
  Tupd =T(T10(i)>(Ta10-Tint) & T20(i)>t & T30(i)>t & ... TiO(i)<=Ta10);
  valid(c)=length(Tupd);

% Initiation detected at 10mm and 20mm sensors only.
elseif (t>Ta10 & t=Ta20 & t<Ta30)
  Tupd =T(T10(i)>(Ta10-Tint) & T20(i)>(Ta20-Tint) & T30(i)>t ... & T10(i)<=Ta10 & T20(i)<=Ta20);
  valid(c)=length(Tupd);

% Initiation detected at 10mm, 20mm and 30mm sensors.
elseif (t>Ta10 & t>Ta20 & t>Ta30)
  Tupd =T(T10(i)>(Ta10-Tint) & T20(i)>(Ta20-Tint) ... & T30(i)>(Ta30-Tint) & T10(i)<=Ta10 & T20(i)<=Ta20 ... & T30(i)<=Ta30);
  valid(c)=length(Tupd);
  %
end

% Checking upper limit for corrosion initiation time distribution
T=T(T>O & T<ts);
clear T10; clear T20; clear T30; clear i; clear Tupd;
%
% Establishing moments for prior distribution
meanT=mean(T)
covT=std(T)/meanT
skewT=skewness(T)
KurT=kurtosis(T)
%
% Checking upper limit for posterior corrosion initiation time distribution
Tpost=Tpost(Tpost<=ts);
%
% Establishing moments for posterior distribution
meanTupd=mean(Tpost)
varTupd=var(Tpost)
covTupd=std(Tpost)/meanTupd
skewTupd=skewness(Tpost)
KurTupd=kurtosis(Tpost)
%
% Plotting prior and posterior distributions
plotcdf(Tprior)
figure
plotcdf(Tpost)
%
% Evaluating time for simulation
TimeElapsed=etime(clock,To)/60
%
% End of file

**MatLab™ Input files for updating methodology using GCE method**

% Setting the seed values to ensure samples can be reproduced
rand('state',12598643)
randn('state',58749618)
%
% To calculate time of run at the end of simulation
To=clock;
%
% No. of Iterations
Iteration=50000 % Conditional Variable
ControlVar=10000
%
% LHS sample (increasing order) for Control Variables
i=(1:ControlVar)';
U=(rand(ControlVar,1) + (i-1))./ ControlVar;
%
% To generate Standard Normal Random no.
[a,b]=sort(rand(ControlVar,1));
U=U(b);
N=norminv(U,0,1);
clear i;
\% Generation of Co
\[ [a,b]=\text{sort(rand(ControlVar,1))}; \]
\[ U=U(b); \]
\[ Co=\text{logninv}(U,1.141191,0.472381); \]

\% Generation of Cth
\[ [a,b]=\text{sort(rand(ControlVar,1))}; \]
\[ U=U(b); \]
\[ Cth=0.6+(1.2-0.6)*U; \]
\% clear a; clear b; clear U;

\% Validation rule
\[ Z2=Cth./Co; \]
\[ Z2=Z2(Z2<1.0); \]
\[ Z3=(\text{erfcinv}(Z2)).^2; \]
\[ \text{ControlVar}=\text{length}(Z3); \]
\% clear Z2; clear Co; clear Cth;
\% \% Assumed Sensors outputs
\[ \text{Ta10}=91.0; \]
\[ \text{Ta20}=93.8; \]
\[ \text{Ta30}=98.4; \]

\% Instrument / measurement uncertainty
\[ \text{Tint}=0.1; \]

\% Time frame for decision purposes
\[ ts=30; \]

\% LHS sample (increasing order) for Conditional Variables
\[ i=(1:1:Iteration)'; \]
\[ U=(\text{rand}(Iteration,1)+(i-1))./Iteration; \]

\% To generate Standard Normal Random no.
\[ [a,b]=\text{sort(rand(Iteration,1))}; \]
\[ U=U(b); \]
\[ N=\text{norminv}(U,0,1); \]
\% clear i;

\% Generation of sample for D
\[ [a,b]=\text{sort(rand(Iteration,1))}; \]
\[ N=N(b); \]
\[ D=5e-5+1e-5*N; \]

\% Generation of sample for X
\[ [a,b]=\text{sort(rand(Iteration,1))}; \]
\[ N=N(b); \]
X = 0.04 + 0.005*N;

% generation of sample for X10, X20 and X30
[a, b] = sort(rand(Iteration, 1));
N = N(b);
X10 = 0.01 + 0.001*N;
X20 = X10 + 0.01;
X30 = X10 + 0.02;

% Generation of sample for Emod
[a, b] = sort(rand(Iteration, 1));
U = U(b);
Emod = logninv(U, -0.004975, 0.099751);
[a, b] = sort(rand(Iteration, 1));
Emod2 = Emod(b);
[a, b] = sort(rand(Iteration, 1));
Emod3 = Emod(b);
[a, b] = sort(rand(Iteration, 1));
Emod4 = Emod(b);

% clear N; clear U; clear a; clear b;
clear PfPrior; clear T; clear T10; clear PfPost;
clear Pf; clear Pfupd

% Performance evaluation (Corrosion initiation time)
i = (1: ControlVar)';
for j = 1:1:Iteration

% Prior corrosion initiation time evaluation
T = (Emod(j) .* X(j) .^ 2) ./ (4 .* D(j) .* Z3);
T10 = (Emod2(j) .* X10(j) .^ 2) ./ (4 .* D(j) .* Z3);
T20 = (Emod3(j) .* X20(j) .^ 2) ./ (4 .* D(j) .* Z3);
T30 = (Emod4(j) .* X30(j) .^ 2) ./ (4 .* D(j) .* Z3);

% Prior failure probability evaluation
Pf(j) = length(T(T<=ts))/length(T);

a = 0; Time = 0;

% Posterior corrosion initiation time and failure probability evaluation
for t = 0:0.1:ts
   a = a + 1;
   Time(a) = t;

% Case when Initiation is not detected at any sensor.
if (t<Ta10 & t<Ta20 & t<Ta30)
   Tupd = T(T10(i)>t & T20(j)>t & T30(i)>t);
   Pfupd(j, a) = length(Tupd(Tupd<=ts))/length(Tupd);
valid(j,a)=length(T upd);

% Initiation detected at 10mm sensor only
elseif (t>=Ta10 & t<Ta20 & t<Ta30)
    T upd =T(T i0(i)>(Ta10-Tint) & T20(i)>t & T30(i)>t & ...
          T10(i)<=Ta10);
    Pfupd(j,a)=length(T upd(T upd<=ts))/length(T upd);
    valid(j,a)=length(T upd);

% Initiation detected at 10mm and 20mm sensors only.
elseif (t>Ta10 & t>=Ta20 & t<Ta30)
    T upd =T(T i0(i)>(Ta10-Tint) & T20(i)>(Ta20-Tint) & T30(i)>t
          & T10(i)<=Ta10 & T20(i)<=Ta20);
    Pfupd(j,a)=length(T upd(T upd<=ts))/length(T upd);
    valid(j,a)=length(T upd);

% Initiation detected at 10mm, 20mm and 30mm sensors.
elseif (t>Ta10 & t>Ta20 & t>=Ta30)
    T upd =T(T i0(i)>(Ta10-Tint) & T20(i)>(Ta20-Tint) & T30(i)>
          (Ta30-Tint) & T10(i)<=Ta10 & T20(i)<=Ta20 ...
          & T30(i)<=Ta30);
    Pfupd(j,a)=length(T upd(T upd<=ts))/length(T upd);
    valid(j,a)=length(T upd);
end
end
end
Maxlength=a;
a=1:1:a;

% Moments of distribution for the prior failure probability
PfPrior(a)=sum(Pf)/length(Pf);
i=1:1:Iteration;
temp=PfPrior(1);
VarPf = sum((Pf(i)-temp).^2)/(Iteration*(Iteration-1));
CovPf = sqrt(VarPf)/temp;
skewPf=skewness(Pf);
KurPf=kurtosis(Pf);
%

% Moments of distribution for the posterior failure probability
for a=1:1:Maxlength
    temp=Pfupd(:,a);
    temp=temp(finite(temp));
    Valid(a)=length(temp);
PfPost(a)=mean(temp);
i=1:1:Valid(a);
    VarPfPost(a)=var(temp)/(Valid(a)-1);
    VarPfPost(a) =sum((temp(i)-PfPost(a)).^2)/(Valid(a)*(Valid(a)-1));
    CovPfPost(a) = sqrt(VarPfPost(a))/PfPost(a);
clear i;
clear temp;
end
PfPrior
PfPost

% Plotting the prior and posterior probability of failure for the decision period
plot(Time,PfPrior,Time,PfPost)
legend('PfPrior','PfPost, Tservice=20 years, Tint=0.1 years')
xlabel('Time (Years)')
ylabel('Failure Probability (Pf)')
title('GCEM N=1E4,1E4, Ti=1.0,3.8,8.4 Pfupdgce.m')
clear t; clear j; clear a; clear X; clear X10; clear X20;
clear X30; clear T; clear T10; clear T20; clear T30; clear i;
clear Pfnum; clear Pfdenom;
clear Emod*; clear Z3; clear D;
TimeElapsed=etime(clock,To)/60
clear To; clear MaxT; clear Maxlength;

% End of file
Appendix C-2:

MatLab™ input file for Multiple Sensor Updating methodology

1. Updating procedure for sensors in different zones

warning off
clear

% Setting seed for random variables to ensure reproducible samples
rand('state',12598643)
randn('state',58749618)

% To calculate time of run at the end of simulation
To=clock;

% Assumed sensors outputs
Ta10all=[1.0 1.5 2.0];  % Max six sensors
Ta20all=[94.0 94 94];
Ta30all=[99.0 99.4 99];

% Time at which posterior corrosion initiation time is required
tall=[1.0 1.5 2.0];

% Instrument / measurement uncertainty
Tint=0.1;

% Limiting value for T and Tupd.
ts=1000;

Tpost=[]; Tupd1=[]; Tupd2=[]; Tupd3=[]; Tupd4=[]; Tupd5=[]; Tupd6=[];

% Maximum number of simulation cycles
for c=1:1:100
  % No. of Iterations = Iteration
  Iteration=5000000;
c*Iteration

  % Latin Hypercube Uniform random sample (increasing order)
i=(1:Iteration)';
  U=(rand(Iteration,1) + (i-1))./ Iteration ;

  % Generation of Co
  [a,b]=sort(rand(Iteration,1));
  U=U(b);
  Co=logninv(U,1.141191,0.472381);

  % Generation of Cth
  [a,b]=sort(rand(Iteration,1));
  U=U(b);
Cth=0.6+(1.2-0.6)*U;

% validation rule
Z2=(Cth./Co);
Z2=Z2(Z2<1.0);
Z3=erfcinv(Z2);

% Now Iteration will have to be redefined to get
% same matrix size.
Iteration=length(Z3);
clear Co; clear Cth;

% Generation of standard normal sample
U=U(1:Iteration);
N=norminv(U,0,1);

% Generation of sample for D
[a,b]=sort(rand(Iteration,1));
N=N(b);
D=5e-5+1e-5*N;

% generation of sample for X
[a,b]=sort(rand(Iteration,1));
N=N(b);
X=0.04+0.005*N;

% Model error for corrosion initiation time calculation.
[a,b]=sort(rand(Iteration,1));
U=U(b);
clear a; clear b; clear Z2;
Emod = logninv(U,-0.03031231,0.2462207); %Emod(1,0.25)

% Sensor location uncertainties.
[a,b]=sort(rand(Iteration,1));
N=N(b);
X10=0.01+0.001*N;
clear a; clear b; clear N; clear U;

% Time to corrosion initiation
Z5=4.*D.*(Z3).^2;
Z6=X.^2;
Z610=X10.^2;
Z620=(X10+0.01).^2;
Z630=(X10+0.02).^2;
%
T=Emod.*(Z6./Z5); clear Z6;
[a,b]=sort(rand(Iteration,1));
Emod=Emod(b);
T10=Emod.*(Z610./Z5); clear Z610;
[a,b]=sort(rand(Iteration,1));
Emod=Emod(b);
\[
T20 = Emod \cdot (Z620. / Z5); \text{clear } Z620;
\]
\[
[a, b] = \text{sort}(\text{rand}(\text{Iteration}, 1));\]
\[
Emod = Emod(b);
\]
\[
T30 = Emod \cdot (Z630. / Z5);
\]
\%
clear Z*; clear X*; clear D; clear U2; clear temp;
clear Emod; clear a; clear b; clear U; clear N;

\%

\textit{Posterior performance evaluation}

\%
i = 1 : 1 : \text{Iteration};

\%
TpostN = [];

for N = 1 : 1 : length(Tall0all)
\[
\begin{align*}
Ta10 &= Tall0all(N); \\
Ta20 &= Tall20all(N); \\
Ta30 &= Tall30all(N); \\
t &= tall(N);
\end{align*}
\]

\%

\textit{Procedure for Bayesian Updating}

clear Tupd;

\%

\textit{Initiation detected at 10mm, 20mm and 30mm sensors.}

if (t > Ta10 & t > Ta20 & t > Ta30)
\[
\text{Tupd} = T(T10(i) > (Ta10 - Tint) \land T20(i) > (Ta20 - Tint) \land T30(i) > (Ta30 - Tint) \land T10(i) \leq Ta10 \land T20(i) \leq Ta20 \land T30(i) \leq Ta30);
\]
valid(c) = length(T10upd);

\%

\textit{Initiation detected at 10mm and 20mm sensors only.}

elseif (t > Ta10 & t > Ta20 & t < Ta30)
\[
\text{Tupd} = T(T10(i) > (Ta10 - Tint) \land T20(i) > (Ta20 - Tint) \land T30(i) > t \land T10(i) \leq Ta10 \land T20(i) \leq Ta20);
\]
valid(c) = length(T10upd);

\%

\textit{Initiation detected at 10mm sensor only}

elseif (t > Ta10 & t < Ta20 & t < Ta30)
\[
\text{Tupd} = T(T10(i) > (Ta10 - Tint) \land T20(i) > t \land T30(i) > t \land T10(i) \leq Ta10);
\]
valid(c) = length(T10upd);

\%

\textit{Case when Initiation is not detected at any sensor.}

elseif (t < Ta10 & t < Ta20 & t < Ta30)
\[
\text{Tupd} = T(T10(i) > t \land T20(i) > t \land T30(i) > t);
\]
valid(c) = length(T10upd);

end

\%

\textit{Storing individual output for each sensor}

if N == 1
\[
\text{Tupd1} = \text{Tupd1} \cup \text{Tupd};
\]
validN(c, N) = length(Tupd);
else
N == 2
T upd2 = [T upd2; T upd];
validN(c, N) = length(T upd);
elseif N == 3
    T upd3 = [T upd3; T upd];
    validN(c, N) = length(T upd);
elseif N == 4
    T upd4 = [T upd4; T upd];
    validN(c, N) = length(T upd);
elseif N == 5
    T upd5 = [T upd5; T upd];
    validN(c, N) = length(T upd);
elseif N == 6
    T upd6 = [T upd6; T upd];
    validN(c, N) = length(T upd);
end
end

% Defining upper limit for corrosion initiation time
T = T(T > 0 & T < ts);
clear T10; clear T20; clear T30; clear i; clear T upd;
clear T postN;

% Defining moments for the prior corrosion initiation time distribution
meanT = mean(T)
covT = std(T)/meanT
skewT = skewness(T)
KurT = kurtosis(T)
clear T;

% Defining upper limit for posterior corrosion initiation time distribution
T post = T post(T post > 0 & T post <= ts);
meanT upd = mean(T post)
varT upd = var(T upd bound)
covT upd = std(T post)/meanT upd
skewT upd = skewness(T post)
KurT upd = kurtosis(T post)

% Plotting posterior distribution
V = axis;
axis([0 100 V(3) V(4)]);
figure
plotcdf(T post)

% Time of simulation
TimeElapsed = etime(clock, To)/60
2. Updating procedure for sensors in the same zones

warning off
clear
% Setting seed for random variables to ensure reproducible samples
rand('state',12598643)
randn('state',58749618)

% To calculate time of run at the end of simulation
To=clock;

% Assumed sensor initiation times
Ta10all=[1.0 1.6 1.8 2.0 2.5]; % Max. 5 sensors allowed in this file
Ta20all=[94 94 94 94 94 94];
Ta30all=[99 99 99 99 99 99];

t=2.5; % Time of Updating
%
% Instrument / measurement uncertainty
Tint=0.1;

% Limiting value for T and Tupd.
ts=1000;

Tpost=[];
Tpost1=[]; Tpost2=[]; Tpost3=[]; Tpost4=[]; Tpost5=[];
for c=1:1:50

% Maximum no. of simulation cycles
Iteration=5000000;
c*Iteration

% Latin Hypercube Uniform random sample (increasing order)
i=(1:Iteration)';
U=(rand(Iteration,1) + (i-1))./Iteration ;

% To generate Standard Normal Random no.
clear i;
[a,b]=sort(rand(Iteration,1));
U=U(b);
N=norminv(U,0,1);

% Generation of Co
[a,b]=sort(rand(Iteration,1));
U=U(b);
Co=logninv(U,1.141191,0.472381);

% Generation of Cth
[a,b]=sort(rand(Iteration,1));
U=U(b);
Cth = 0.6 + (1.2 - 0.6) * U;

% Concrete Quality Rating
[a, b] = sort(rand(Iteration, 1));
N = N(b);
D = 5e-5 + 1e-5 * N; % Good Quality
%D = 10e-5 + 2e-5 * N; % Average Quality
%D = 15e-5 + 3e-5 * N; % Poor Quality
D = D(D > 0);

% Data for X
[a, b] = sort(rand(Iteration, 1));
N = N(b);
X = 0.04 + 0.005 * N; % Good Quality
%X = 0.04 + 0.010 * N; % Average Quality
%X = 0.04 + 0.015 * N; % Poor Quality
X = X(X > 0);

% Validating rule
Z2 = (Cth / Co);
Z2 = Z2(Z2 < 1.0);
Z3 = erfcinv(Z2);
% Now Iteration will have to be redefined to get
% same matrix size.
Iteration = length(Z3);
clear Co; clear Cth;
U = U(1:Iteration);
N = N(1:Iteration);
D = D(1:Iteration);
X = X(1:Iteration);
%

% Emod = model error for initiation time calculation.
[a, b] = sort(rand(Iteration, 1));
U = U(b);
clear a; clear b; clear Z2;
Emod = logninv(U, -0.03031231, 0.2462207); % Emod(1, 0.25)

% Sensor location uncertainties.
[a, b] = sort(rand(Iteration, 1));
N = N(b);
X10 = 0.01 + 0.001 * N;
clear a; clear b; clear N; clear U;

% Time to corrosion initiation
Z5 = 4. * D .* (Z3).^2;
Z6 = X .^ 2;
Z610 = X10 .^ 2;
Z620 = (X10 + 0.01) .^ 2;
Z630=(X10+0.02).^2;  
%  
T=Emod.*(Z6./Z5); clear Z6;  
[a,b]=sort(rand(Iteration,1));  
Emod10=Emod(b);  
[a,b]=sort(rand(Iteration,1));  
Emod20=Emod(b);  
[a,b]=sort(rand(Iteration,1));  
Emod30=Emod(b);  
%  
clear Z*; clear D; clear U2; clear a; clear b;  
clear U; clear N;  

% prior and posterior performance evaluation  
for N=1:1:length(Ta10all)  
  Ta10=Ta10all(N);  
  Ta20=Ta20all(N);  
  Ta30=Ta30all(N);  
  if N>1  
    Iteration=length(Tupd);  
    X10=X10(1:Iteration);  
    X=X(1:Iteration);  
    Emod=Emod(1:Iteration);  
    Emod10=Emod10(1:Iteration);  
    Emod20=Emod20(1:Iteration);  
    Emod30=Emod30(1:Iteration);  
    T=Tupd;  
  end  
  T10=(T.*X10.^2.*Emod10)./(Emod.*X.^2);  
  T20=(T.*(X10+0.01).^2.*Emod20)./(Emod.*X.^2);  
  T30=(T.*(X10+0.02).^2.*Emod30)./(Emod.*X.^2);  
%  
i=1:1:Iteration;  

% Procedure for Bayesian Updating  
clear Tupd;  

% Initiation detected at 10mm, 20mm and 30mm sensors.  
if (t>Ta10 & t>Ta20 & t>=Ta30)  
  Tupd = T((T(i)>=(Ta10-Tint) & T20(i)>=(Ta20-Tint)) ...  
    & T30(i)>=(Ta30-Tint) & T10(i)<=Ta10 & T20(i)<=Ta20 ...  
    & T30(i)<=Ta30);  
  valid(c)=length(T10upd);  
% Initiation detected at 10mm and 20mm sensors only.  
elseif (t>Ta10 & t>=Ta20 & t<Ta30)  
  Tupd = T((T(i)>=(Ta10-Tint) & T20(i)>=(Ta20-Tint)) & T30(i)>t ...  
    & T10(i)<=Ta10 & T20(i)<=Ta20);  
  valid(c)=length(T10upd);  

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% Initiation detected at 10mm sensor only
elseif (t>=Ta10 & t<Ta20 & t<Ta30)
    Tupd = T(T10(i)>(Ta10-Tint) & T20(i)>t & T30(i)>t & ... 
    T10(i)<=Ta10); 
    valid(c)=length(T10upd);

% Case when Initiation is not detected at any sensor.
elseif (t<Ta10 & t<Ta20 & t<Ta30)
    Tupd = T(T10(i)>t & T20(i)>t & T30(i)>t); 
    valid(c)=length(T10upd);
end
validN(c,N)=length(Tupd);

% Storing individual outputs
if N==1
    Tpost1=[Tpost1,Tupd'];
elseif N==2
    Tpost2=[Tpost2,Tupd'];
elseif N==3
    Tpost3=[Tpost3,Tupd'];
elseif N==4
    Tpost4=[Tpost4,Tupd'];
elseif N==5
    Tpost5=[Tpost5,Tupd'];
end

% Storing cumulative output
Tpost=[Tpost,Tupd'];
validN(c)=length(Tpost);
end
clear T10; clear T20; clear T30; clear i; clear Tupd;
clear T; clear Emod*; clear X*;

% Establishing moments of the posterior performance distribution
Tpost=Tpost(Tpost<=ts);
meanTupd=mean(Tpost)
covTupd=std(Tpost)/meanTupd
skewTupd=skewness(Tpost)
KurTupd=kurtosis(Tpost)

% Plotting posterior performance distribution
plotpdf(Tpost)

% TimeElapsed=etime(clock,To)/60
Appendix C-3: Input file for Inspection Updating Procedure

```matlab
warning off
clear
% Fixing a seed value to ensure reproducible samples
seed = 931316785;
rand('seed', seed);
randn('seed', seed);

%To calculate time of run at the end of simulation
To = clock;

% Maximum number of simulation cycles
Iteration = 500000

% Loading Data for Covariance matrix to generate correlated data.
load Cov1yr.mat
%
% Detailed Inspection Results at year 6.
InspMin1 = 0.0045; InspMax1 = 0.0545;
%
% Generating correlated sample for 'area showing corrosion' at successive time steps
% using Cholesky's decomposition method.
for c = 1 : 1 : length(Cov)
    MeanA(c, 1 : 1 : Iteration) = Mean(c);
end
%
L = chol(Cov);
L = L';

% Initializing variables
A7post1 = []; A8post1 = []; A9post1 = []; A10post1 = []; A11post1 = []; A12post1 = [];
A13post1 = []; A14post1 = []; A15post1 = []; A16post1 = []; A17post1 = []; A18post1 = [];
A19post1 = []; A20post1 = [];
%
for c = 1 : 1 : 1000
    % No. of Iterations = Iteration
c*Iteration

    % Latin Hypercube Uniform random sample (increasing order)
i = (1 : Iteration)';
U = (rand(Iteration,1) + (i-1))./ Iteration ;

    % To generate Standard Normal Random no.
clear i;
%

    % Generation of standard normal distribution
Ncomb = [];```

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for a=1:1:length(Cov)
    [z,b]=sort(rand(Iteration,1));
    U=U(b);
    N=norminv(U,0,1);
    Ncomb=[Ncomb N];
end
clear z; clear b; clear U; clear a;
Ncomb=Ncomb';

% Correlated sample generation
X= L*Ncomb + MeanA;

% Correlated prior 'areas showing corrosion initiation'
A6=X(1,:); A7=X(2,:); A8=X(3,:); A9=X(4,:); A10=X(5,:);
A11=X(6,:); A12=X(7,:); A13=X(8,:); A14=X(9,:); A15=X(10,:);
A16=X(11,:); A17=X(12,:); A18=X(13,:); A19=X(14,:);
A20=X(15,:);

% clear X; clear N; clear Row; clear Sdev;
clear Mean; clear C; clear N;
i=1:1:Iteration;

% Posterior 'area showing corrosion' based on inspection results
A7 upd1=A7(A6(i)>InspMinl & A6(i)<=InspMaxl);
A8 upd1=A8(A6(i)>InspMinl & A6(i)<=InspMaxl);
A9 upd1=A9(A6(i)>InspMinl & A6(i)<=InspMaxl);
A10 upd1=A10(A6(i)>InspMinl & A6(i)<=InspMaxl);
A11 upd1=A11(A6(i)>InspMinl & A6(i)<=InspMaxl);
A12 upd1=A12(A6(i)>InspMinl & A6(i)<=InspMaxl);
A13 upd1=A13(A6(i)>InspMinl & A6(i)<=InspMaxl);
A14 upd1=A14(A6(i)>InspMinl & A6(i)<=InspMaxl);
A15 upd1=A15(A6(i)>InspMinl & A6(i)<=InspMaxl);
A16 upd1=A16(A6(i)>InspMinl & A6(i)<=InspMaxl);
A17 upd1=A17(A6(i)>InspMinl & A6(i)<=InspMaxl);
A18 upd1=A18(A6(i)>InspMinl & A6(i)<=InspMaxl);
A19 upd1=A19(A6(i)>InspMinl & A6(i)<=InspMaxl);
A20 upd1=A20(A6(i)>InspMinl & A6(i)<=InspMaxl);

valid(c,1)=length(A7 upd1);
valid(c,1)=length(A7 upd1);
A7 post1=[A7 post1 A7 upd1];
A8 post1=[A8 post1 A8 upd1];
A9 post1=[A9 post1 A9 upd1];
A10 post1=[A10 post1 A10 upd1];
A11 post1=[A11 post1 A11 upd1];
A12 post1=[A12 post1 A12 upd1];
A13 post1=[A13 post1 A13 upd1];
A14 post1=[A14 post1 A14 upd1];
A15 post1=[A15 post1 A15 upd1];
A16 post1=[A16 post1 A16 upd1];
A17 post1=[A17 post1 A17 upd1];
A18 post1=[A18 post1 A18 upd1];
A19 post1=[A19 post1 A19 upd1];
A20 post1=[A20 post1 A20 upd1];
end
clear MeanA; clear Ncomb;

% Evaluation of moments of the prior and posterior distribution
MeanA=[mean(A6),mean(A7),mean(A8),mean(A9),mean(A10),mean(A11),...
     mean(A12),mean(A13),mean(A14),mean(A15),mean(A16),...
     mean(A17),mean(A18),mean(A19),mean(A20)]
StdA=[std(A6),std(A7),std(A8),std(A9),std(A10),std(A11),std(A12),...
     std(A13),std(A14),std(A15),std(A16),std(A17),std(A18),...
     std(A19),std(A20)]

% MeanApostl=[mean(A7post1), mean(A8post1), mean(A9post1),...
     mean(A10post1),mean(A11post1),mean(A12post1),...
     mean(A13post1),mean(A14post1),mean(A15post1),...
     mean(A16post1),mean(A17post1),mean(A18post1),...
     mean(A19post1),mean(A20post1)]
StdApost1=[std(A7post1),std(A8post1),std(A9post1),...
     std(A10post1),std(A11post1),std(A12post1),...
     std(A13post1),std(A14post1),std(A15post1),...
     std(A16post1),std(A17post1),std(A18post1),...
     std(A19post1),std(A20post1)]

plotpdf(A7post5)
clear i; clear A*upd*
clear A6; clear A7; clear A8; clear A9; clear A10; clear A11; clear A12;
clear A13; clear A14; clear A15; clear A16; clear A17; clear A18; clear A19;
clear A20;
clear A*postl
TimeElapsed=etime(clock,To)/60
% End of file
Appendix D: Variation of diffusion coefficient along the depth

A number of samples (obtained from HETEK, 1996) have been used to estimate the diffusion coefficient and surface chloride concentration. These are obtained by non-linear regression of the chloride induced deterioration model (Eq. 2.5) onto the observed data. The details of three sample and the computed average diffusion coefficients for various depths are as follows.

Sample 1: (HETEK, 1996), $T = 2.074$ years

![Sample 1](image1)

<table>
<thead>
<tr>
<th>Depth (mm)</th>
<th>Avg. D ($\text{mm}^2/\text{yr}$)</th>
<th>$C_0$ (%mass)</th>
</tr>
</thead>
<tbody>
<tr>
<td>upt0 10</td>
<td>11.35</td>
<td>0.55</td>
</tr>
<tr>
<td>upt0 15</td>
<td>11.57</td>
<td>0.55</td>
</tr>
<tr>
<td>upt0 20</td>
<td>11.56</td>
<td>0.55</td>
</tr>
<tr>
<td>upt0 30</td>
<td>11.56</td>
<td>0.55</td>
</tr>
</tbody>
</table>

Sample 2: (HETEK, 1996), $T = 1.047$ years

![Sample 2](image2)

<table>
<thead>
<tr>
<th>Depth (mm)</th>
<th>Avg. D ($\text{mm}^2/\text{yr}$)</th>
<th>$C_0$ (%mass)</th>
</tr>
</thead>
<tbody>
<tr>
<td>upt0 10</td>
<td>10.57</td>
<td>0.29</td>
</tr>
<tr>
<td>upt0 15</td>
<td>10.59</td>
<td>0.29</td>
</tr>
<tr>
<td>upt0 20</td>
<td>10.59</td>
<td>0.29</td>
</tr>
<tr>
<td>upt0 30</td>
<td>10.59</td>
<td>0.29</td>
</tr>
</tbody>
</table>
Sample 3: (HETEK, 1996), $T = 2.088$ years

<table>
<thead>
<tr>
<th>Depth (mm)</th>
<th>Avg. D (mm$^2$/yr)</th>
<th>$C_0$ (%mass)</th>
</tr>
</thead>
<tbody>
<tr>
<td>upto 10</td>
<td>7.30</td>
<td>0.45</td>
</tr>
<tr>
<td>upto 15</td>
<td>7.49</td>
<td>0.45</td>
</tr>
<tr>
<td>upto 20</td>
<td>7.50</td>
<td>0.45</td>
</tr>
<tr>
<td>upto 30</td>
<td>7.50</td>
<td>0.45</td>
</tr>
<tr>
<td>upto 40</td>
<td>7.50</td>
<td>0.45</td>
</tr>
<tr>
<td>upto 50</td>
<td>7.50</td>
<td>0.45</td>
</tr>
</tbody>
</table>

It is clear from the results of all three samples that the average diffusion coefficient remains practically constant along the depth for all three samples. Hence, it is concluded that the diffusion coefficient is fully correlated along the depth and hence would have the same realization for the evaluation of corrosion initiation time at various depths (see section 5.5 for details).


CEB-FIP Model Code 1990 (1993), Lausanne, Switzerland.


