Martian Coaxial Tiltrotor Aerobot: Aerodynamic Shape Design of Coaxial Tiltrotor and Robust Flight Control

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Summary

Mars has been an active planet for space exploration since the 1960s. The future Mars missions, such as searching for life, could be accomplished by the technique up to date, but the Martian aerobots allow a longer range and a lower altitude compared with rovers and orbiters, respectively. Hyperion is an autonomous fixed-wing solar-electrical vertical take-off and landing aerobot proposed based on the previous work to investigate the Isidis Planitia region. The proposed aerobot is a coaxial tiltrotor design with two auxiliary rotors at the wing-tips. This thesis focuses on two topics: aerodynamic design of the coaxial tiltrotor system and robust flight control for transition phases.

Due to the limited solar flux on Mars, it is necessary to improve the efficiency of the coaxial tiltrotor system. The vortex based theory has the highest computational efficiency (high computational precision and low computational cost). Two models, the Prescribed Wake Model (PWM) and the Free Wake Model (FWM), are used in this work. The PWM is used in the optimization process; while the FWM is used to provide data to determine the empirical equations and validate the PWM results. The optimal coaxial rotors for both hover and cruise have significant improvement (approximately 10% less power) compared with the baseline design. The final coaxial tiltrotor is obtained by a weighted average of the two optima in hover and cruise conditions. The performance of the proposed coaxial tiltrotor is very close to that of the optima for both hovering and cruise conditions.

The flight control system is another important aspect for such a special Martian aerobot, especially for the transition and the conversion between hover and cruise. It is known that the aerodynamic property for an aerobot is difficult to predict, so the aerodynamic coefficients used in this work follow the usual practice that an uncertainty (±20%) is imposed for the aerodynamic terms. In this thesis, a robust controller based on $\mu$ synthesis and Divide and Conquer gain scheduling method is proposed to design the controller for transition flight. A 6-degree-of-freedom simulation with aerodynamic uncertainty validates the feasibility and the robustness of the proposed controller. The simulation shows that the transition can be accomplished within 150s. Although the altitude in the simulation shows a acceptable steady state error (±1m). The other state variables are robustly stabilized during the transition flight.

Key words: Martian VTOL aerobot, Coaxial Tiltrotor Design, Robust Transition Control.

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Glossary of Terms

Variables for rotor design:

\( A \) \hspace{1cm} \text{rotor disk area, } A = \pi R_{lp}^2 \text{ (m}^2\text{)}

\( A_c \) \hspace{1cm} \text{wake contraction ratio}

\( c \) \hspace{1cm} \text{rotor blade chord (m)}

\( c_{la} \) \hspace{1cm} 2D lift curve slope of airfoil

\( c_p \) \hspace{1cm} \text{power coefficient, } c_p = \frac{P}{\rho A (\omega R_{lp})^3}

\( c_q \) \hspace{1cm} \text{torque coefficient, } c_q = \frac{Q}{\rho A R_{lp} (\omega R_{lp})^2}

\( c_{ql} \) \hspace{1cm} \text{torque coefficient of lower coaxial rotor}

\( c_{qu} \) \hspace{1cm} \text{torque coefficient of upper coaxial rotor}

\( c_r \) \hspace{1cm} \text{thrust coefficient, } c_r = \frac{T}{\rho A (\omega R_{lp})^2}

\( c_{r0} \) \hspace{1cm} \text{design point of thrust coefficient}

\( d \) \hspace{1cm} \text{distance between two rotor disks of the coaxial tiltrotor system}

\( d_{\text{aux}} \) \hspace{1cm} \text{longitudinal distance between the nose of Hyperion and the auxiliary rotors}

\( d_c \) \hspace{1cm} \text{longitudinal distance between the CG and the nose of the Martian aerobot}

\( dD \) \hspace{1cm} \text{longitudinal distance between the AC and the nose of the Martian aerobot}

\( dT \) \hspace{1cm} \text{longitudinal distance of the coaxial tiltrotor and the nose of the Martian aerobot}

\( k_1, k_2, k_3 \) \hspace{1cm} \text{parameters determining the position of collocation point}

\( M_b \) \hspace{1cm} \text{number of bound vortex sections on the blade}

\( N_b \) \hspace{1cm} \text{number of blades}

\( N_s \) \hspace{1cm} \text{number of collocation points on one tip vortex}

\( N_{\nu} \) \hspace{1cm} \text{number of azimuth positions}
**Glossary of Terms**

- $Q_l$: torque of lower coaxial rotor (N·m)
- $Q_{tol}$: tolerance of torque balance
- $Q_u$: torque of upper coaxial rotor (N·m)
- $R_{tip}$: rotor blade radius (m)
- $r$: radial position of rotor blade (m)
- $\overline{r}$: nondimensional radial position of the tip vortex collocation point
- $r_c$: radius of rolled-up tip vortex core (m)
- $\overline{r}_{i,k}^n$: nondimensional coordinates of collocation points for the $i^{th}$ azimuth location at the $k^{th}$ tip vortex segment at the $n^{th}$ iteration
- $\overline{r}_{RMS}$: RMS of the collocation points
- $\overline{r}_{tol}$: predefined tolerance of the RMS of the collocation points
- $T$: thrust of the coaxial tiltrotor
- $T_{aux}$: total thrust of the auxiliary rotors
- $T_{aux1}$: total auxiliary rotors thrust required for taking off
- $T_{aux1}$: total auxiliary rotors thrust required for landing
- $T_{min}$: lower limit of the midpoint of the total thrust interval of auxiliary rotors
- $u$: induced velocity at rotor disk (m·s⁻¹)
- $V_{\infty}$: free stream inflow rate (m·s⁻¹)
- $v_i$: induced rate, $v_i = u/(\omega R_{tip})$
- $\overline{z_T}$: nondimensional axial position of the tip vortex collocation point
- $\Gamma$: strength of a vortex (m²·s⁻¹)
- $\Gamma_b$: blade bound circulation (m²·s⁻¹)
- $\Gamma_{b(RMS)}$: RMS of blade bound circulation
- $\Gamma_{b(tol)}$: RMS tolerance of the blade bound circulation
- $\Gamma_{b(obj)}$: objective bound circulation for the lower coaxial rotor
Glossary of Terms

\( \Gamma_{b(o)}^{\text{obj}} \) \hspace{1em} \text{objective bound circulation for the upper coaxial rotor}

\( \Gamma_{\text{flag}(l)} \) \hspace{1em} \text{flag sign representing the relationship between the objective bound circulation and the real value for lower coaxial rotor}

\( \Gamma_{\text{flag}(u)} \) \hspace{1em} \text{flag sign representing the relationship between the objective bound circulation and the real value for upper coaxial rotor}

\( \Gamma_{v} \) \hspace{1em} \text{blade tip vortex strength (m}^2\text{·s}^{-1})

\( \zeta \) \hspace{1em} \text{vortex age in the wake (rad)}

\( \delta \Gamma_{b} \) \hspace{1em} \text{bound circulation increment (m}^2\text{·s}^{-1})

\( \delta \theta \) \hspace{1em} \text{blade pitch change (rad)}

\( \theta_{75} \) \hspace{1em} \text{blade pitch angle at 0.75} \text{\( R_{ip} \) (rad)}

\( \theta_{l} \) \hspace{1em} \text{blade pitch of lower coaxial rotor (rad)}

\( \theta_{u} \) \hspace{1em} \text{blade pitch of upper coaxial rotor (rad)}

\( \lambda_{w} \) \hspace{1em} \text{tip speed ratio,} \, \lambda_{w} = \frac{V_{w}}{\omega R_{ip}}

\( \rho \) \hspace{1em} \text{atmosphere density (kg·m}^{-3})

\( \sigma \) \hspace{1em} \text{rotor solidity,} \, \sigma = \frac{N_{b}c}{\pi R_{ip}}

\( \psi_{b} \) \hspace{1em} \text{rotor blade azimuth distance,} \, \psi_{b} = \frac{2\pi}{N_{b}} \text{ (rad)}

\( \omega \) \hspace{1em} \text{rotational speed of rotor (rad·s}^{-1})

Variables for controller design:

\( AR \) \hspace{1em} \text{aspect ratio of the wing}

\( \bar{b} \) \hspace{1em} \text{wing span (m)}

\( C_{D} \) \hspace{1em} \text{aerodynamic drag coefficient}

\( C_{D0} \) \hspace{1em} \text{aerodynamic drag at minimum lift}

\( C_{D\delta e} \) \hspace{1em} \text{variation of the drag coefficient due to the effective elevator deflection}

\( C_{D\delta a} \) \hspace{1em} \text{variation of the drag coefficient due to the effective aileron deflection}

\( C_{L} \) \hspace{1em} \text{aerodynamic lift coefficient}
Glossary of Terms

\( C_{l_q} \) variation of the lift coefficient due to the pitch rate

\( C_{l_A} \) variation of the lift coefficient due to the AoA

\( C_{l_A} \) variation of the lift coefficient due to the AoA rate

\( C_{l_{se}} \) variation of the lift coefficient due to the effective elevator deflection

\( C_i \) aerodynamic roll moment coefficient

\( C_{rp} \) variation of the roll moment coefficient due to the roll angular rate

\( C_{l_{y}} \) variation of the roll moment coefficient due to the yaw angular rate

\( C_{l_{p\beta}} \) variation of the roll moment coefficient due to the AoS

\( C_{l_{sa}} \) variation of the roll moment coefficient due to the effective aileron deflection

\( C_m \) aerodynamic pitch moment coefficient

\( C_{m_{00}} \) pitch moment coefficient at the aero AoA

\( C_{m_{pq}} \) variation of the pitch moment coefficient due to the pitch angular rate

\( C_{m_{oa}} \) variation of the pitch moment coefficient due to the AoA

\( C_{m_{oa}} \) variation of the pitch moment coefficient due to the AoA rate

\( C_{m_{se}} \) variation of the pitch moment coefficient due to the effective elevator deflection

\( C_{o} \) aerodynamic yaw moment coefficient

\( C_{og} \) variation of the yaw moment coefficient due to the roll angular rate

\( C_{or} \) variation of the yaw moment coefficient due to the yaw angular rate

\( C_{og} \) variation of the yaw moment coefficient due to the AoS

\( C_{osa} \) variation of the yaw moment coefficient due to the effective aileron deflection

\( C_y \) aerodynamic side force coefficient

\( C_{yp} \) variation of the side force coefficient due to the roll angular rate

\( C_{yr} \) variation of the side force coefficient due to the yaw angular rate

\( C_{y_{p\beta}} \) variation of the side force coefficient due to the AoS
<table>
<thead>
<tr>
<th>Term</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_{\alpha} \delta a$</td>
<td>variation of the side force coefficient due to the effective aileron deflection</td>
</tr>
<tr>
<td>$\bar{c}$</td>
<td>wing average chord length (m)</td>
</tr>
<tr>
<td>$D$</td>
<td>aerodynamic drag of the airframe (N)</td>
</tr>
<tr>
<td>$D_{aero}$</td>
<td>aerodynamic drag of the airframe in VTOL phase (N)</td>
</tr>
<tr>
<td>$e$</td>
<td>wing span efficiency</td>
</tr>
<tr>
<td>$\bar{F}_{aero}$</td>
<td>$3 \times 1$ aerodynamic force vector (N)</td>
</tr>
<tr>
<td>$\bar{F}_{prop}$</td>
<td>$3 \times 1$ force vector generated by the propulsion system (N)</td>
</tr>
<tr>
<td>$\bar{g}$</td>
<td>$3 \times 1$ gravity vector (m·s$^{-2}$)</td>
</tr>
<tr>
<td>$\bar{I}$</td>
<td>$3 \times 3$ moment of inertia matrix (kg·m$^2$)</td>
</tr>
<tr>
<td>$I_{xx}$</td>
<td>total moment of inertia in $X_B$ axis (kg·m$^2$)</td>
</tr>
<tr>
<td>$I_{yy}$</td>
<td>total moment of inertia in $Y_B$ axis (kg·m$^2$)</td>
</tr>
<tr>
<td>$I_{zz}$</td>
<td>total moment of inertia in $Z_B$ axis (kg·m$^2$)</td>
</tr>
<tr>
<td>$I_0$</td>
<td>moment of inertia of the coaxial tiltrotor along the rotation shaft (kg·m$^2$)</td>
</tr>
<tr>
<td>$I_i$</td>
<td>moment of inertia of the coaxial tiltrotor perpendicular to the rotor shaft (kg·m$^2$)</td>
</tr>
<tr>
<td>$i_p$</td>
<td>nacelle angle (rad)</td>
</tr>
<tr>
<td>$k_{aux}$</td>
<td>thrust curve slope of the auxiliary rotor (N·rad$^{-1}$)</td>
</tr>
<tr>
<td>$k_{diff_{coax}}$</td>
<td>torque curve slope of the coaxial tiltrotor (N·m·rad$^{-1}$)</td>
</tr>
<tr>
<td>$k_{tot_{coax}}$</td>
<td>total thrust curve slope of the coaxial tiltrotor (N·rad$^{-1}$)</td>
</tr>
<tr>
<td>$L$</td>
<td>aerodynamic lift of the airframe (N)</td>
</tr>
<tr>
<td>$\bar{L}$</td>
<td>aerodynamic roll moment of the airframe (N·m)</td>
</tr>
<tr>
<td>$M$</td>
<td>aerodynamic pitch moment of the airframe (N·m)</td>
</tr>
<tr>
<td>$\bar{M}_{aero}$</td>
<td>$3 \times 1$ aerodynamic moment vector (N·m)</td>
</tr>
<tr>
<td>$M_{B1}$</td>
<td>$3 \times 3$ conversion matrix from the inertia frame to the body frame</td>
</tr>
<tr>
<td>$M_{BW}$</td>
<td>$3 \times 3$ conversion matrix from the wind frame to the body frame</td>
</tr>
<tr>
<td>$M_{coax}$</td>
<td>torque generated by differential collective pitch of the coaxial tiltrotor (N·m)</td>
</tr>
</tbody>
</table>
Glossary of Terms

\( M_D \) aerodynamic drag induced pitch moment in VTOL phase (N·m)

\( M_l \) torque generated by the left auxiliary rotor (N·m)

\( M_r \) torque generated by the right auxiliary rotor (N·m)

\( \tilde{M}_{\text{prop}} \) 3×1 moment vector generated by the propulsion system (N·m)

\( m \) total mass of Hyperion (kg)

\( N \) aerodynamic yaw moment of the airframe (N·m)

\( p \) roll angular rate (rad·s\(^{-1}\))

\( q \) pitch angular rate (rad·s\(^{-1}\))

\( r \) yaw angular rate (rad·s\(^{-1}\))

\( S \) wing area (m\(^2\))

\( T_{\text{coax}} \) thrust generated by the coaxial tiltrotor (N)

\( T_l \) thrust generated by the left auxiliary rotor (N)

\( T_r \) thrust generated by the right auxiliary rotor (N)

\( u \) velocity component in the \( X_B \) axis (m·s\(^{-1}\))

\( u_{\text{cmd}} \) command value of the velocity component in the \( X_B \) axis (m·s\(^{-1}\))

\( \bar{V} \) 3×1 velocity vector (m·s\(^{-1}\))

\( V_{cr} \) cruise speed (m·s\(^{-1}\))

\( V_{\text{cmd}}^{cr} \) command value of the cruise speed (m·s\(^{-1}\))

\( V_{cr}^{\text{tol}} \) tolerance of the cruise speed to generate the switching signal (m·s\(^{-1}\))

\( V_r \) relative inflow rate (m·s\(^{-1}\))

\( v \) velocity component on \( Y_B \) axis (m·s\(^{-1}\))

\( v_{\text{cmd}} \) command value of the velocity on \( Y_B \) axis (m·s\(^{-1}\))

\( w \) velocity component on \( Z_B \) axis (m·s\(^{-1}\))

\( w_0 \) nominal vertical speed in VTOL mode (m·s\(^{-1}\))

\( w_e \) velocity component on \( Z_O \) axis (m·s\(^{-1}\))
$x_{aux}$  longitudinal distance between the CG and the auxiliary rotors (m)

$x_c$  position of the Martian aerobot on $X_G$ axis (m)

$Y$  aerodynamic side force of the airframe (N)

$y_{aux}$  lateral distance between the auxiliary rotors (m)

$y_c$  position of the Martian aerobot on $Y_G$ axis (m)

$z_e$  position of the Martian aerobot on $Z_G$ axis, i.e. negative value of altitude (m)

$z^\text{cmd}_e$  command value of negative altitude (m)

$\alpha$  AoA (rad)

$\beta$  AoS (rad)

$\delta_l$  deflection of the “left” elevon (rad)

$\delta_r$  deflection of the “right” elevon (rad)

$\delta_a$  effective aileron deflection, $\delta_a = (\delta_r - \delta_l)/2$ (rad)

$\delta_e$  effective elevator deflection, $\delta_e = (\delta_r + \delta_l)/2$ (rad)

$\theta$  pitch attitude angle (rad)

$\theta^\text{cmd}$  command value of pitch attitude angle (rad)

$\theta^\text{diff}_{aux}$  differential blade pitch of the auxiliary rotors (rad)

$\theta^\text{tot}_{aux}$  total blade pitch of the auxiliary rotors (rad)

$\theta^\text{diff}_{coax}$  differential blade pitch angles of the coaxial tiltrotor (rad)

$\phi$  roll attitude angle (rad)

$\phi^\text{cmd}$  command value of roll attitude angle (rad)

$\xi$  damping of the actuators

$\psi$  yaw attitude angle (rad)

$\psi^\text{cmd}$  command value of the yaw attitude angle (rad)

$\ddot{\omega}$  $3\times1$ angular rate vector (rad·s$^{-1}$)

$\omega_n$  natural frequency of the actuators (rad·s$^{-1}$)
# List of Acronyms

<table>
<thead>
<tr>
<th>Acronym</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>AC</td>
<td>Aerodynamic Centre</td>
</tr>
<tr>
<td>AIAA</td>
<td>American Institute of Aeronautics and Astronautics</td>
</tr>
<tr>
<td>AoA</td>
<td>Angle of Attack</td>
</tr>
<tr>
<td>AoS</td>
<td>Angle of Sideslip</td>
</tr>
<tr>
<td>ARES</td>
<td>Aerial Regional-scale Environmental Survey</td>
</tr>
<tr>
<td>AVL</td>
<td>Athena Vortex Lattice</td>
</tr>
<tr>
<td>BEMT</td>
<td>Blade Element and Momentum Theory</td>
</tr>
<tr>
<td>CFD</td>
<td>Computational Fluid Dynamics</td>
</tr>
<tr>
<td>CG</td>
<td>Centre of Gravity</td>
</tr>
<tr>
<td>CNES</td>
<td>French Space Agency</td>
</tr>
<tr>
<td>D&amp;C</td>
<td>Divide and Conquer</td>
</tr>
<tr>
<td>DoF</td>
<td>Degree of Freedom</td>
</tr>
<tr>
<td>EoM</td>
<td>Equations of Motion</td>
</tr>
<tr>
<td>EPFL</td>
<td>Ecole Polytechnique Federale de Lausanne</td>
</tr>
<tr>
<td>EPS</td>
<td>Electric Power System</td>
</tr>
<tr>
<td>ESA</td>
<td>European Space Agency</td>
</tr>
<tr>
<td>FWM</td>
<td>Free Wake Model</td>
</tr>
<tr>
<td>GGI</td>
<td>General Grid Interface</td>
</tr>
<tr>
<td>HALE</td>
<td>High Altitude Long Endurance</td>
</tr>
<tr>
<td>IEEE</td>
<td>Institute of Electrical and Electronics Engineers</td>
</tr>
<tr>
<td>JPL</td>
<td>Jet Propulsion Laboratory</td>
</tr>
<tr>
<td>KARI</td>
<td>Korea Aerospace Research Institute</td>
</tr>
<tr>
<td>L/D</td>
<td>Lift to Drag ratio</td>
</tr>
<tr>
<td>LDI</td>
<td>Linear Differential Inclusion</td>
</tr>
<tr>
<td>LPV</td>
<td>Linear Parameter Varying</td>
</tr>
<tr>
<td>Acronym</td>
<td>Description</td>
</tr>
<tr>
<td>---------</td>
<td>-------------</td>
</tr>
<tr>
<td>LQ</td>
<td>Linear Quadratic</td>
</tr>
<tr>
<td>LTI</td>
<td>Linear Time Invariant (system)</td>
</tr>
<tr>
<td>MABVAP</td>
<td>Mars Aerobot Validation Program</td>
</tr>
<tr>
<td>MAP</td>
<td>Mars Aerial Platform</td>
</tr>
<tr>
<td>MARV</td>
<td>Martian Autonomous Rotary-wing Vehicle</td>
</tr>
<tr>
<td>MASSIVA</td>
<td>Mars Surface Sample Imaging VTOL Aircraft</td>
</tr>
<tr>
<td>MATADOR</td>
<td>Mars Advanced Technology Aeroplane for Deployment, Operations, and Recovery</td>
</tr>
<tr>
<td>MIT</td>
<td>Massachusetts Institute of Technology</td>
</tr>
<tr>
<td>MTR</td>
<td>Mono Tilt-Rotor</td>
</tr>
<tr>
<td>MRF</td>
<td>Multi-Reference Frame</td>
</tr>
<tr>
<td>NASA</td>
<td>National Aeronautics and Space Administration</td>
</tr>
<tr>
<td>NN</td>
<td>Neural Network</td>
</tr>
<tr>
<td>NRL</td>
<td>(US) Naval Research Laboratory</td>
</tr>
<tr>
<td>PC2B</td>
<td>Predictor-Corrector 2\textsuperscript{nd}-Backward</td>
</tr>
<tr>
<td>PCC</td>
<td>Predictor-Corrector Central</td>
</tr>
<tr>
<td>PD</td>
<td>Proportional Derivative</td>
</tr>
<tr>
<td>PID</td>
<td>Proportional Integral Derivative</td>
</tr>
<tr>
<td>PIPC</td>
<td>Pseudo-Implicit Predictor Corrector</td>
</tr>
<tr>
<td>PSO</td>
<td>Particle Swarm Optimization</td>
</tr>
<tr>
<td>PWM</td>
<td>Prescribed Wake Model</td>
</tr>
<tr>
<td>R/C</td>
<td>Remote Control</td>
</tr>
<tr>
<td>RMS</td>
<td>Root Mean Square</td>
</tr>
<tr>
<td>SMC</td>
<td>Sliding Mode Control</td>
</tr>
<tr>
<td>SSC</td>
<td>Surrey Space Centre</td>
</tr>
<tr>
<td>STARMAC</td>
<td>Stanford/Berkeley Testbed of Autonomous Rotorcraft for Multi-Agent Control</td>
</tr>
<tr>
<td>TsAGI</td>
<td>Kamov Design Bureau and Central Aero-hydrodynamics Institute</td>
</tr>
<tr>
<td>UAV</td>
<td>Unmanned Aerial Vehicle</td>
</tr>
<tr>
<td>USA</td>
<td>United States of America</td>
</tr>
<tr>
<td>Acronym</td>
<td>Description</td>
</tr>
<tr>
<td>---------</td>
<td>----------------------------------</td>
</tr>
<tr>
<td>USSR</td>
<td>Union of Soviet Socialist Republics</td>
</tr>
<tr>
<td>VLM</td>
<td>Vortex Lattice Method</td>
</tr>
<tr>
<td>VRS</td>
<td>Vortex Ring State</td>
</tr>
<tr>
<td>VTOL</td>
<td>Vertical Take-Off and Landing</td>
</tr>
<tr>
<td>WFF</td>
<td>Wallops Flight Facility</td>
</tr>
</tbody>
</table>
Chapter 1

1 Introduction

This thesis aims to use the technology of the state of art to discuss the aerodynamic shape design of the coaxial tiltrotor and transition/conversion control of Hyperion, an all solar-electric-powered Vertical Take-Off and Landing (VTOL) Martian coaxial tiltrotor aerobot proposed by the Surrey Space Centre (SSC). The objective of this project at the current stage is to build a demonstrator on Earth rather than a real life Martian aerobot. Some terms used in this thesis are given as follows:

The term “rotor” denotes the helicopter rotor with low inflow rate to counteract the weight of the vehicle; while the term “propeller” denotes the fixed propeller with high inflow rate to counteract the aerodynamic drag. The term “transition” in this thesis denotes the transient phase from hover to cruise; while “conversion” denotes the reverse process from cruise to hover.

1.1 Why Mars

The motivation for Mars exploration originates from the hypothesis that there were life on Mars [Perminov, 2011]. This was firstly proposed by Giovanni Virginio Schiaparelli, the Italian astronomer and director of the observatory in Milan. He discovered that there are “canals” linked with “lakes” and “oceans” on Mars by the powerful telescopes. Strughold, the American scientist, proposed that there might be primitive vegetation on Mars in 1953. Schklovskiy, the Soviet astrophysicist, proposed that there was intelligent life on Mars, and he also suggested sending probes to Mars.

These scientific hypotheses are proven false based on the recent Mars exploration, but the research shows that there might be life on Mars before. There is a hypothesis in astronomy that Venus and Mars might be the past and future of Earth [Strobel, 2011]. Exploration of the two neighbours of the Earth will help to validate if the hypothesis is correct, and also to understand the evolution of solar system.

Table 1-1 gives some key facts for the three planets [NASA, 2012a, NASA, 2012c, NASA, 2012d]. The general characteristics of the three planets are similar. Comparing with the Earth, the Venus is a “hot” planet with a “high” pressure “acid” atmosphere; while the Mars is “cold” with a “low” pressure “inert” atmosphere. The thunders are very common on Venus. Therefore, the
Chapter 1. Introduction

Venus is not suitable for close exploration based on the technology up to date, compared with the Mars. There was still many probes sent to the Venus since 1961 [Curtis, 2012]. The former USSR focused on the landers, while the USA focused on the flyby and orbiters. Unfortunately, none of the landers were successful due to the high pressure and thick acid atmosphere. No further mission for the Venus is proposed after 1997.

Table 1-1: Facts and figures for the planets Venus, Earth and Mars

<table>
<thead>
<tr>
<th></th>
<th>Venus</th>
<th>Earth</th>
<th>Mars</th>
</tr>
</thead>
<tbody>
<tr>
<td>Orbit size around the sun (km)</td>
<td>$1.0820948\times10^8$</td>
<td>$1.4959826\times10^8$</td>
<td>$2.2794382\times10^8$</td>
</tr>
<tr>
<td>Mean radius (km)</td>
<td>6,051.8</td>
<td>6,371.0</td>
<td>3,389.5</td>
</tr>
<tr>
<td>Mass (kg)</td>
<td>$4.8673\times10^{24}$</td>
<td>$5.9722\times10^{24}$</td>
<td>$6.4169\times10^{23}$</td>
</tr>
<tr>
<td>Density (kg-m$^{-3}$)</td>
<td>$5.243\times10^3$</td>
<td>$5.513\times10^3$</td>
<td>$3.934\times10^3$</td>
</tr>
<tr>
<td>Surface gravity (m s$^{-2}$)</td>
<td>8.87</td>
<td>9.81</td>
<td>3.71</td>
</tr>
<tr>
<td>Length of “day” (Earth day)</td>
<td>-243.018 (retrograde)</td>
<td>1</td>
<td>1.026</td>
</tr>
<tr>
<td>Length of “year” (Earth day)</td>
<td>224.70</td>
<td>365.26</td>
<td>686.98</td>
</tr>
<tr>
<td>Mean orbit velocity (m s$^{-1}$)</td>
<td>$3.5020\times10^4$</td>
<td>$2.9783\times10^4$</td>
<td>$2.4077\times10^4$</td>
</tr>
<tr>
<td>Orbit eccentricity</td>
<td>0.00677672</td>
<td>0.01671123</td>
<td>0.0933941</td>
</tr>
<tr>
<td>Min/Max surface temperature (°C)</td>
<td>462/462</td>
<td>-88/58</td>
<td>-87/-5</td>
</tr>
<tr>
<td>Atmospheric constituents</td>
<td>CO$_2$, N$_2$</td>
<td>O$_2$, N$_2$</td>
<td>CO$_2$, N$_2$, Ar</td>
</tr>
<tr>
<td>Surface atmosphere pressure (atm)</td>
<td>~100</td>
<td>1</td>
<td>0.07</td>
</tr>
</tbody>
</table>

1.2 Mars Mission Overview

During the period of 1960 to 1989, the USSR and the USA were very active in Mars exploration projects, and today the USA has a number of on-going active missions at the “red planet” [Perminov, 2011]. Mars exploration helps us to gain a better understanding of the evolution of Earth and the solar system. Before 1989, the USSR proposed a series of projects such as 1M, 2MV, M-69, M-71, 5NM, and 5M, and launched Marsnik 1&2, Sputnik 22&24, Mars 1, 2, 3, 4, 5, 6, 7, Zond 2, Phobos 1&2, etc; but unfortunately, none were successful. The USA proposed the Mariner Program and Viking Program during the same period, and launched Mariner 3, 4, 6, 7, 8, 9, and Viking 1&2 with the success rate of more than 50%. From 1990 to the present, more countries have joined in the exploration of Mars, and the missions have become more ambitious. Two Mars rovers, Spirit and Opportunity, were launched by the US in 2003 and were successfully deployed onto the Martian surface. Russia launched Mars 96, but failed again unfortunately. Japan launched Nozomi but this failed too. Mars Express, a European Space Agency (ESA) mission, was a success. More recently, in 2011, Russia launched Phobos-Grunt (which also carried a Chinese Martian satellite Yinghuo-1) to Phobos, a moon of Mars. Unfortunately, the rocket failed to complete the burn to set the correct course for Mars. A list of the Mars missions is listed in Appendix A.

Rovers, landers, and satellites have helped us to understand the Mars much better, but many questions still remain. Proposals for the future Mars missions are as follows:
• Searching for signs of life, such as methane or other organic molecules
• Gaining a better understanding of the Martian magnetic field
• Finding and locating landing sites for future missions
• Analyzing the detailed constituents of the Martian atmosphere
• Getting very high resolution pictures of the Martian surface

These missions could be accomplished by the techniques used up to date, however, the use of aerobot technology would allow a much longer exploration range than a rover, and yet would still give detailed scientific data due to its low-altitude compared with the orbiters.

1.3 Why Aerobot

The concept of a Martian aerobot in itself is not new. Three types of aerobots have already been considered for Mars exploration; they are lighter-than-air vehicles (balloons and airships), fixed wing aircraft, and rotorcraft. NASA considered two kinds of balloons for Mars: helium super-pressure balloons and solar Montgolfiere balloons. However, the results of their precursor projects, such as the Mars 96 Aerosat by the French Space Agency (CNES) [Vargas et al., 1997]; MAP (Mars Aerial Platform) [Goebel, 2002] by NASA Wallops Flight Facility (WFF), and MABVAP (Mars Aerobot Validation Program) [Goebel, 2002] by the Jet Propulsion Laboratory (JPL) are not encouraging; therefore, the practical use of lighter-than-air aerobots for Mars needs further development. There are also many fixed-wing proposals, such as the long-endurance Mars aircraft [Colozza, 1990] proposed by NASA Lewis Research Center (now Glenn Research Centre); the Kitty Hawk [McKay, 1998] by JPL; ARES (Aerial Regional-scale Environmental Survey) [Levine, 2009] by NASA Langley Research Center; Sky-Sailor [Noth, 2006] by the Ecole Polytechnique Federale de Lausanne (EPFL), and MATADOR (Mars Advanced Technology Aeroplane for Deployment, Operations, and Recovery) [Franchi, 2004] by the US Naval Research Laboratory (NRL). Proposals for rotorcraft include coaxial helicopters, tiltrotors, and tilt-nacelle Vertical Take-Off and Landing (VTOL) rotorcraft [Young, 2001, Young et al., 2005, Young et al., 2002] such as that by NASA Ames Research Center; MARV (Martian Autonomous Rotary-wing Vehicle) [Datta et al., 2000] by the University of Maryland, and Mesicopter [Kroo et al., 2001] by the University of Stanford. Other design concepts include the flapping wing Mars Entomopter [David, 2002] by the NASA Institute for Advanced Concepts, and the Gas Hopper [SpaceRenaissance, 2010] by NASA. A recent research on Martian aerobot is proposed by [Forshaw and Lappas, 2012] based on the twin rotor tailsitter concept design.
## Table 1-2: Martian Aerobots

<table>
<thead>
<tr>
<th>Aerobot</th>
<th>Mass (kg)</th>
<th>Wing span (m)</th>
<th>Wing area (m²)</th>
<th>Aspect ratio</th>
<th>Propulsion system</th>
<th>Endurance &amp; range</th>
<th>Cruise Speed (m·s⁻¹)</th>
<th>Cruise altitude</th>
<th>Control devices</th>
<th>Navigation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mars Solar Aircraft</td>
<td>567.16</td>
<td>51.6</td>
<td>166.33</td>
<td>16</td>
<td>Propeller, 25% GaAs Solar array, fuel cell, electrical motor</td>
<td>Long endurance, N/A</td>
<td>34</td>
<td>N/A</td>
<td>Elevator, aileron, rudder</td>
<td>N/A</td>
</tr>
<tr>
<td>AME</td>
<td>203.8</td>
<td>12.4</td>
<td>12.24</td>
<td>12.65</td>
<td>Rocket, Lithium/hydrogen peroxide fuel cell</td>
<td>2 flights (8.8 hrs in total), 3400km</td>
<td>110.6</td>
<td>Elevation changes</td>
<td>Elevator, aileron, rudder</td>
<td>N/A</td>
</tr>
<tr>
<td>Kitty Hawk by JPL</td>
<td>N/A</td>
<td>2</td>
<td>N/A</td>
<td>N/A</td>
<td>Glider, no propulsion</td>
<td>20 min, 100km</td>
<td>~80</td>
<td>2km</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>Kitty Hawk by AMES</td>
<td>135</td>
<td>9.75</td>
<td>N/A</td>
<td>N/A</td>
<td>Rear-mounted propeller, Hydrazine</td>
<td>3 hrs, 1800km</td>
<td>160</td>
<td>1-9km</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>ARES</td>
<td>150(wet)</td>
<td>6.25</td>
<td>7</td>
<td>5.6</td>
<td>Bi-propellant Liquid Rocket (MMH/MON-3)</td>
<td>1 hrs, &gt;500km</td>
<td>145</td>
<td>1-2km</td>
<td>Ruddervators/Vtail</td>
<td>IMU, radar, air data</td>
</tr>
<tr>
<td></td>
<td>101(dry)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Canon-Flyer</td>
<td>20</td>
<td>2.2</td>
<td>0.77</td>
<td>6.3</td>
<td>Propeller driven, battery powered electric motor or hydrazine-powered motor</td>
<td>15 min, 130km</td>
<td>144</td>
<td>500m</td>
<td>Elevator, aileron, rudder</td>
<td>IMU, radar, air data, Sun sensor</td>
</tr>
<tr>
<td>Sky-Sailor</td>
<td>2.5</td>
<td>3.2</td>
<td>0.96</td>
<td>10.7</td>
<td>Propeller, solar powered, Li-Po battery, electrical motor</td>
<td>12 hrs, 1700km</td>
<td>30-40</td>
<td>1.5km</td>
<td>N/A</td>
<td>Vision</td>
</tr>
<tr>
<td>NRL MATADOR</td>
<td>N/A</td>
<td>4</td>
<td>N/A</td>
<td>N/A</td>
<td>Rocket</td>
<td>45 min to 1 hr, 300-400km</td>
<td>143</td>
<td>4km</td>
<td>Cold-gas rejection, thrust vector</td>
<td>N/A</td>
</tr>
<tr>
<td>Project</td>
<td>Mass (kg)</td>
<td>s</td>
<td>Isp (s)</td>
<td>Thruster Type</td>
<td>Duration</td>
<td>Range</td>
<td>Control System</td>
<td>Notes</td>
<td></td>
<td></td>
</tr>
<tr>
<td>------------------</td>
<td>-----------</td>
<td>---</td>
<td>---------</td>
<td>---------------</td>
<td>----------</td>
<td>--------</td>
<td>-------------------------------------------------</td>
<td>-------------------------------------</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Minerva by Cranfield</td>
<td>141.5(wet) 93(dry)</td>
<td>6.18</td>
<td>6.675</td>
<td>5.72</td>
<td>Aerojet reaction-control thrusters, bipropellant</td>
<td>75 min, 620km</td>
<td>155</td>
<td>6km</td>
<td>Flaprons, ruddervators/ Vtail</td>
<td>N/A</td>
</tr>
<tr>
<td>MACE by Delft</td>
<td>120.7</td>
<td>8</td>
<td>3.07</td>
<td>16</td>
<td>Electric engine, propeller, Lithium semi-fuel cells</td>
<td>6 hrs, 2680km</td>
<td>123.6</td>
<td>0-10km</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>MASSIVA by Surrey</td>
<td>15</td>
<td>8.5</td>
<td>6</td>
<td>12</td>
<td>Propeller driven, electric motor, solar powered, NiMH battery</td>
<td>10 days, 1000km</td>
<td>30</td>
<td>70km</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>Halcyon by Surrey</td>
<td>25</td>
<td>8.558</td>
<td>8.23</td>
<td>6.6</td>
<td>Propeller, electric motor, solar powered, Li-Po battery</td>
<td>10 days, 1000km</td>
<td>50</td>
<td>1km</td>
<td>Elevator, aileron, rudder</td>
<td>Terrain contour matching, Sun sensor</td>
</tr>
<tr>
<td>Tailsitter by Surrey</td>
<td>15</td>
<td>7.5</td>
<td>7</td>
<td>8.5</td>
<td>Propeller, electric motor, solar powered, Lithium based cells</td>
<td>100km(reusable) 450km(single-use)</td>
<td>44</td>
<td>1km</td>
<td>Elevon, twin rotors</td>
<td>N/A</td>
</tr>
</tbody>
</table>
The design parameters of some proposed Martian aerobots are listed in Table 1-2. When considering the cruise speed, there are two design concepts, i.e. low speed and high speed. Sky-sailor, Mars Solar aircraft, and the three Martian aerobots proposed by Surrey are low subsonic aeroplanes with the Mach number below 0.2; while AME, Kitty Hawk, ARES, Canon-flyer, MATADOR, Minerva, and MACE are high subsonic speed Martian aircraft. The propulsion system of the two kinds of aeroplanes is not the same. The high speed aeroplanes usually use the rocket engines, while the low speed counterparts usually use the propeller driven by the electric engine. The wing span of the low speed Martian aerobot is usually large.

The main advantages of the three aerobots proposed by Surrey are the VTOL capability and the use of solar power. Due to the three properties, the Martian aerobots proposed by Surrey are reusable. Therefore, the mission of these Martian aerobots can be extended if necessary. However, these Martian aerobots also has disadvantages. One main disadvantage is that the solar insolation on Martian surface is limited, see Section 2.1.2. In this case, the time cost for charging the batteries is quite long for each flight.

Table 1-3: Comparison of balloons/airships, fixed wing aircraft, and rotorcraft

<table>
<thead>
<tr>
<th></th>
<th>Advantage</th>
<th>Disadvantage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Balloons/Airships</td>
<td>Low cost</td>
<td>Deployment, inflation, leakage</td>
</tr>
<tr>
<td></td>
<td>Hover property</td>
<td>Slow speed</td>
</tr>
<tr>
<td></td>
<td>Low power</td>
<td>Poor wind resistant</td>
</tr>
<tr>
<td></td>
<td>Using Mars atmosphere circulation</td>
<td></td>
</tr>
<tr>
<td>Fixed wing aircraft</td>
<td>Flexible control</td>
<td>Poor hover property</td>
</tr>
<tr>
<td></td>
<td>Directional control</td>
<td>“One time” flight only</td>
</tr>
<tr>
<td>Rotorcraft</td>
<td>Flexible control</td>
<td>Very high power consumption</td>
</tr>
<tr>
<td></td>
<td>Directional control</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Hover property</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Multiple flight</td>
<td></td>
</tr>
</tbody>
</table>

Table 1-3 gives a comparison of the three types of aerobots. Each kind of the aerobot has advantages and disadvantages. Balloons and Airships are suitable for preliminary exploration, since the cost and power requirements are relatively low; however, they not suitable for long range missions due to their inherent slow speed and poor wind resistance. The balloons/airships can make use of the Mars atmosphere circulation, but the deployment, inflation, leakage are the main technical difficulties until now. Rotorcraft makes good candidates for Martian scouts as they offer excellent hovering capability and flexible control; however, their significantly high power requirements pose a great challenge for power system design. As such, long range rotorcraft exploration will perhaps become practical only when combined with robotic and human exploration [Young et al., 2005]. Fixed wing aircraft have relatively low power requirements, good wind resistance, and flexible control; however, they can be used “one time only” because the take-off and landing for fixed wing aircraft needs a relatively smooth runway. To sum up, none of
these three kinds of aerobots are without some form of compromise for Mars exploration. Thus, we propose a new hybrid design combining the lower power advantages of a fixed wing aircraft, with the VTOL and hover capability of a rotorcraft – a unique coaxial tiltrotor embedded in high aspect ratio wing design – *Hyperion*.

In order to celebrate the 100 anniversary of the Wrights’ flight on Earth, NASA held a competition for Mars flight. Most of the Martian aerobots presetend above were proposed during this period, at the beginning of the 21st century. The *ARES* proposed by the NASA Langley Research Center won the competition and was planning to be launched in a launch window of 2011 or 2013. However, the *MAVEN* (Mars Atmosphere and Volatile EvolutioN), an orbiter, was chosen instead by NASA [Hautaluoma, 2007]. Most Martian aerobots proposed in this competition were of fixed-wing and rotorcraft configurations. According to the previous studies, the main challenges for Martian aerobot design are briefly introduced in Section 1.3.1 to 1.3.3. Details can be found in [Song, 2008].

### 1.3.1 Propulsion System

The environment on Mars is quite different from that on Earth, refer Section 2.1 for details. The main factors have influence on the propulsion system are the constituents and density of the atmosphere. The Martian atmosphere is composed of inert gases (95% of CO$_2$, 4% of N$_2$, and 1% of Ar). Under such conditions, the possible propulsion technologies can be used for Martian aerobots are the internal combustion system, electrical motors, and rocket system. [Colozza, 2003] gave a good comparison on the three solutions of the propulsion systems. The mass and performance characteristics for these possible systems are evaluated. The conclusion of this research is that the 4 cycle internal combustion engine produces the lowest mass system for the flight duration up to 4 hours; the electrical system with fuel cells is a better choice for longer time flight; the rocket engine has the highest TRL (Technology Readiness Level), but its mass will grow rapidly with the flight duration.

Because the majority (95%) of the Martian atmosphere is CO$_2$ rather than O$_2$, the conventional air-breathing propulsion system used on Earth can not be used on Mars. Some CO$_2$ engines for small aircraft have been developed [Gasparin, 1999], but they are far from being used in this study. The air-breathing system is possible to be used in the future.

The rocket engine can be used as the propulsion system for Martian aerobots. The flight duration is very limited for such aerobot, since this is totally determined by the fuel carried. The propulsion system of *ARES* is rocket engine. The flight duration of *ARES* is only one hour. So the rocket engine is possible to be used on Martian aerobots, but the duration is too short.
The current possible solution for the propulsion system is the electrical motor, which is the choice of most proposed Martian aerobots. However, the power supply for the electrical motor is another serious challenge.

### 1.3.2 Power System

The fuel cell and the solar panel with rechargeable cell are two possible solutions as the power system for the electrical motors.

The fuel cell is new technology which has not been used for aerobots. If the fuel cell is used for the Martian aerobot, the aerobot has to carry the tanks of propellant and oxidant during the whole mission. The tanks will be additional dead weight for the aerobot. Another disadvantage is that the flight duration is determined by the fuel in the tanks. Moreover, the propellants may contaminate the Martian atmosphere, which is undesirable for Mars exploration.

The electrical propulsion system with solar panel and rechargeable cell is used in some High Altitude Long Endurance (HALE) UAV on Earth, such as the Helios [Dryden, 2002] and Pathfinder [Galante, 2002]. Helios is operating with an altitude of 30km on Earth. The environment is quite similar to that on Martian surface. Although Helios project terminated with a crash, it has proven the feasibility of such propulsion system.

Therefore, the fuel cell has potential for the future. The solar panel with rechargeable cell is the power system of the current technology. The solar panel with rechargeable cell is the technology can be used currently for Martian aerobot.

### 1.3.3 Martian Atmosphere Property

Because of the maximum diameter of the present launch vehicle is approximately 3.5m, the size of the Martian aerobot is impossible to be very large. The Martian aerobot wings have to be folded in order that it can be embedded into the aeroshell, so the wing spans of the proposed Martian aerobots are usually no larger than 10m.

The atmosphere density on Martian surface is 0.0135kg·m⁻³ (comparing with 1.29kg·m⁻³ on Earth). The sound speed at Martian surface is about 250m·s⁻¹ (comparing with 340m·s⁻¹ on Earth with aero altitude at sea level). The design of HALE aircraft on Earth can be used as a reference.

The daily temperature on Martian surface is from -87°C to -5°C. The atmosphere density is low. This poses a challenge for thermal control subsystem. It is difficult for the traditional air-cooling technology to dissipate the heat.
1.4 Summary of Previous Work

*Hyperion* is the third generation of Mars VTOL aerobots proposed by the Surrey Space Centre. The previous two being *Massiva* [Fielding, 2004] and *Halycon* [Song, 2008]. Fielding proved the validity of the hybrid design of Martian aerobot and proposed the *Massiva* ( Martian Surface Sampling and Imaging VTOL Aircraft). *Halycon*, proposed by Song, is more practical with larger mass budget and more detailed design consideration. *Hyperion* is a revised version based on *Halycon*. Therefore, a short introduction of *Halycon* is given. Refer [Song, 2008] for the details.

![Figure 1-1: Layout of Halcyon [Song and Underwood, 2007]](image)

**Table 1-4: Important parameters of Halcyon**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total mass (kg)</td>
<td>25</td>
</tr>
<tr>
<td>Wing span (m)</td>
<td>8.558</td>
</tr>
<tr>
<td>Wing airfoil</td>
<td>zagi10</td>
</tr>
<tr>
<td>Wing area (m²)</td>
<td>8.23</td>
</tr>
<tr>
<td>Average wing chord length (m)</td>
<td>1.3046</td>
</tr>
<tr>
<td>Coaxial rotor diameter (m)</td>
<td>2.0</td>
</tr>
<tr>
<td>Propeller diameter (m)</td>
<td>1.0</td>
</tr>
<tr>
<td>Nominal cruise speed (m·s⁻¹)</td>
<td>50</td>
</tr>
<tr>
<td>Coaxial rotor rate (rpm)</td>
<td>1819</td>
</tr>
<tr>
<td>Propeller rate (rpm)</td>
<td>3000</td>
</tr>
<tr>
<td>Power for coaxial rotor (W)</td>
<td>~2000×2</td>
</tr>
<tr>
<td>Power for propellers (W)</td>
<td>~310×2</td>
</tr>
<tr>
<td>Number of blades for coaxial rotor</td>
<td>2×2</td>
</tr>
<tr>
<td>Number of blades for propellers</td>
<td>2</td>
</tr>
</tbody>
</table>

Figure 1-1 shows the 3D structural layout of *Halcyon*. It has a flying wing embedded with a pair of 2×2-bladed coaxial helicopter rotor for Vertical Take-Off and Landing (VTOL) and two 2-bladed highly efficient propellers for cruise. The wing can be folded to an aeroshell within the
radius of 2.65m. The surface of the wing is covered by the solar panels to supply energy for the electric power system. The coaxial rotor and propellers are driven by the electric motor. Two pairs of elevons and one pair of rudders are designed to control the attitude of the vehicle in cruise. Table 1-4 shows some important parameters of Halcyon. The power consumed by the coaxial rotor and propellers are only for reference since they are estimated by the simple Momentum Theory.

1.5 Motivation and Novelty

1.5.1 Motivation

Halcyon is a good design as Martian scout in general, but there are still some serious problems given as follows. The objective of this work is to try to do some research on some problems in the previous work, listed as follows. It is impossible to build a real life Martian aerobot directly, so we hope to build a scaled demonstrator on Earth first in the future.

- The performance of the coaxial helicopter rotor in the design process is estimated by the simple Momentum Theory, which is a rough estimation of the rotor performance without considering the structure of the rotor. The Halcyon design is close to the design limit of technology up to date, so it is necessary to improve the efficiency of the coaxial rotor to be practical.

- The coaxial helicopter rotor is used in hover only, whilst the propellers are working in cruise only. In other words, the coaxial helicopter rotor and propellers are useless in cruise and VTOL phases, respectively. It is possible to reduce the mass budget of the propulsion system by introducing the concept of coaxial tiltrotor.

- The Aerodynamic Centre (AC) and Centre of Gravity (CG) for VTOL and cruise modes should be different to guarantee inherent stability (Section 4.2 and Chapter 5 give a discussion on the dynamic stability of the Martian aerobo). In the Halcyon design, a sliding mass is required to be shifted backward during the transition phase, while be shifted forward in conversion. This design is very difficult to implement in reality.

- The previous work only includes the controller design for cruise. The control strategy in VTOL, especially in transition and conversion, needs to be considered.
1.5.2 Contributions

According to the existing problems described above, the contributions of this thesis are listed as follows:

- Novel coaxial tiltrotor aircraft design for Mars
- Application of the Prescribed Wake Model (PWM) based coaxial tiltrotor design method to the Martian aerobot
- Novel transition and conversion control strategy based on \( \mu \) synthesis and Divide and Conquer (D&C) gain scheduling method for the Martian aerobot

1.6 Structure of the Thesis

This thesis is organized as follows:

Chapter 2 is a review chapter, and firstly provides a survey on the Martian surface environment. The survey on the Martian environment is already given in the previous work, so we just list some important conclusions for this part. Secondly, the unique structure of Hyperion is proposed based on previous Halcyon design with consideration of the problems given in Section 0. Then, a brief overview of the existing coaxial tiltrotor aircraft is given. After that, a comprehensive review on the aerodynamic model for rotor and the control strategy for VTOL, cruise, and transition/conversion are given. At last, the mission profile and aerodynamic property of the proposed Hyperion is discussed to provide further information for the following chapters.

Chapter 3 gives an introduction on the proposed Martian aerobot. Since this work is a continuing project, most subsystems introduced in this chapter are based on previous design. Firstly, the mechanical structure is proposed. Then, a brief introduction on the mission profile, the power budget, power system, thermal control system, scientific payloads, airframe physical structure, flight control system, and mass budget are given according to previous design. At last, the aerodynamic performance of the Martian aerobot is discussed. This analysis shows that the cruise speed selected in previous design is corresponding to the maximum Lift to Drag (L/D) ratio of the airframe.

Chapter 4 focuses on the aerodynamic design of the coaxial tiltrotor to improve its efficiency and the auxiliary rotor design. Although the rotor system should be considered as low Reynolds number condition, the Reynolds number of the rotor system is not too low. The influence of the Reynolds number is not significant, especially before the stall point. Therefore, the design method and the models for high Reynolds number rotor system can be used for this work. The PWM with high computational efficiency is selected as the design process, while the Free Wake Model
(FWM) with high precision is selected for validation. These two models are validated by the measurements of Harrington coaxial rotors and Hamilton coaxial propellers in the literature. The coaxial tiltrotor is a coaxial rotor working for hover (low inflow rate) and cruise (high inflow rate); therefore, it is a compromise of the optima in both conditions. The global optima for coaxial helicopter rotors and coaxial propellers are still open problems, so some suboptimal solutions are available based on the aerodynamic theory. The coaxial rotors with uniform bound circulation are known to have high efficiency close to the global optima, so this criterion is used to find the optima in both conditions. The resulting coaxial tiltrotor is obtained by a weighted average of the optima in hover and cruise. Its performance is very close to the optima in each condition. The design of auxiliary rotors used for pitch and roll control is considered from the control perspective. Two thrust changing methods, rotational speed and blade pitch, are considered. The detailed discussion shows that both of these two methods are suitable for the auxiliary rotors, and the blade pitch mechanism is selected because of its lower power cost.

Chapter 5 concentrates on the robust flight control of the proposed Martian aerobot. The rigid-body dynamic and aerodynamic models are derived first. Because of the inherent uncertainty of the aerodynamic model, the proposed controller must be robustly stable under all the conditions. The robust controllers for VTOL and cruise are proposed based on the control of quadrotors and fixed wing aircraft. The transition corridor and transition trajectory are presented based on the nominal aerodynamic model. The nonlinear dynamic model is linearized based on the given transition trajectory. A $\mu$ synthesis D&C gain scheduling method is proposed to control the linearized model. The robustness of the transition controller is guaranteed by the $\mu$ synthesis theory for Linear Time Invariant (LTI) systems. The simulation results show the validity of the proposed controllers.

Chapter 6 summarizes the work of this thesis and indicates the areas for further study.
Chapter 2

2 Literature Review and Mission Profile

A comprehensive review is necessary before detailed discussion on the proposed Hyperion. The Martian environment, especially the property of the Martian atmosphere, is briefly introduced first. Then the general structure of Hyperion is proposed based on the previous design. After that, the existing coaxial tiltrotor aircraft, rotor aerodynamic theories, and the flight control strategy for tiltrotor aircraft are reviewed. At last, the mission profile and the general character of Hyperion are presented.

2.1 Martian Environment

Mars is the planet most similar to Earth in the solar system. Recent research has found that there is ice locked in the polar caps and gaseous water in its atmosphere. If the sign of life on Mars can be proved, the exploration of Mars can help us to understand the evolution and the future of Earth. Some general features of the Mars are listed in Table 1-1. The Mars is hostile to the human being due to the low temperature and lack of water and oxygen.

2.1.1 Martian Surface Atmosphere

We are more interested in the Martian atmosphere, in which the Martian aerobot is operating. Detailed information of the Mars can be found in [Tillman, 1997] based on the measurements in the Viking 1 & 2 landers (Viking 1 landing at 22°41'49"N 48°13'19"W, Viking 2 Landing at 48°16'08"N 225°59'24"W) and the pathfinder.

The Martian atmosphere is quite different from that on Earth. The Martian atmosphere contains 95% of carbon dioxide and 5% of Nitrogen and other inert gas. The density and pressure of the Martian atmosphere vary with different places and altitude, but its average density and pressure are of approximately 1% and 0.69% of the sea level atmosphere on Earth, respectively.

2.1.1.1 Martian Surface Atmosphere Pressure

The average atmosphere pressure on Martian surface is approximately 700Pa, only 0.69% for that on Earth. Figure 2-1 and Figure 2-2 present the daily average pressure variation measured by the Viking landers. Figure 2-1 shows the annual CO$_2$ condensation – sublimation cycle for both
landing sites. $L_s$ in the figure denote the time line for measurements. As shown in the figure, $90^\circ$ denotes the local summer; $180^\circ$ denotes the local autumnal equinox; $270^\circ$ denotes the winter; and $0^\circ$ denotes the spring. The subsequent years are plotted on the first year. The atmosphere pressure variation is quite similar for each year. The annual pressure variation is from approximately 7,000 Pa to 1,000 Pa. The pressure reaches highest in winter, especially the Viking 2 in northern part. This is caused by the local weather change, which is the same as that on Earth.

Figure 2-1: Martian surface pressure variation measured by the Viking landers each Martian year

[Tillman, 1997]

Figure 2-2: Martian surface pressure variation for 3.3 Martian years measured by the Viking landers

[Tillman, 1997]
2.1.1.2 Martian Surface Atmosphere Density

Figure 2-3 shows a typical Martian atmosphere density distribution at noon for a local summer Martian day. The Martian surface geography is given by Figure 2-4. Generally speaking, the surface atmosphere density follows the variation of local altitude. The density of the atmosphere ranges from $2\times10^3\text{kg}\cdot\text{m}^{-3}$ to $3.4\times10^3\text{kg}\cdot\text{m}^{-3}$. The minimum density located at the mountains annotated, such as the Olypus Mons, Ascræus Mons, Pavonis Mons, Arsia Mons, and Elysium Mons. The Isidis Planitia region is also annotated in Figure 2-3.

![Figure 2-3: Martian atmosphere density distribution [MCD, 2012]](image)

![Figure 2-4: Topographic map of Mars [JPL, 1999]](image)

The average atmosphere density on Martian surface is approximately 0.0135kg·m⁻³, approximately only 1% of the atmosphere on Earth at sea level. The Martian surface gravity is
3.71 m s$^{-2}$, which is about 1/3 of that on Earth at sea level. Therefore, it is more difficult to fly an aerobot on Mars due to such a low atmosphere density. In this case, the size Martian aerobot should be very larger or the cruise speed should be faster, compared with the same amount of mass budget. However, due to the size of the launch vehicle, the wing span can not be too large. Therefore, the cruise speed for Martian aerobot is not low. The cruise speed of the ARES airplane is 145 m s$^{-1}$ with its total mass of 185 kg [Kuhl, 2009]. However, the high cruise speed will pose a challenge for the propulsion system.

2.1.1.3 Martian Surface Atmosphere Temperature

The average atmosphere temperature at Martian surface is about -63°C [Tillman, 1997]. The typical maximum and minimum temperature for a Martian day is -25°C and -89°C with the maximum of 27°C and the minimum of -143°C in record. Figure 2-5 shows the daily temperature cycle measured by the Viking I & 2 from the 95th to the 97th Martian day (local summer). The temperature on the Martian surface varies periodically. Because the Martian atmosphere is very thin, the temperature on Martian surface is mainly influenced by the solar heating and the infrared cooling. The heat exchange within the atmosphere is very little.

![Figure 2-5: Martian surface atmosphere temperature measured by the Viking I & 2 [Tillman, 1997]](image)

![Figure 2-6 Martian surface atmosphere temperature measured by the Mars Pathfinder and Viking landers [Tillman, 1997]](image)
A comparison of the temperature for the same period measured by the Mars Pathfinder is given by Figure 2-6. The temperature variation is quite similar to the measurements by the Viking landers. It is worth noting that the temperature sensor of the Viking landers is located at 1.5m above the surface. The Pathfinder has three temperature sensors located at 1.0m, 0.5m, and 0.25m above the petal. Generally, the Pathfinder temperatures are about 10°C hotter than the Viking lander.

The thermal control is necessary for the Martian aerobot to keep the important payloads operating at a particular temperature. The temperature variation poses a challenge for the thermal control system.

2.1.1.4 Martian Surface Wind and Dust

The wind during the local summer on Martian surface is measured by the Viking landers [Tillman, 1997]. The wind speed ranges from 1m·s⁻¹ to 10m·s⁻¹. However, the wind will become stronger in winter up to 23m·s⁻¹ for Viking 2. The wind speed can not be calculated at Viking 1 since the wind sensors partially failed. The estimation of the wind speed is about 28m·s⁻¹. Such a wind speed variation in winter poses a great challenge for the Martian aerobot system.

The wind variation will cause the daily variation of the “atmospheric dust”. A heavy wind will cause a dust storm, which is a disaster for close exploration. During a dust storm, the atmosphere temperature will increase. The sunlight reaching the Martian surface will be only 5% of that without dust storm. In addition, the dust storm will damage the mechanical components of the aerobot. Therefore, it is almost impossible for Martian aerobot operating in a dust storm.

Figure 2-7: West-East component of Martian surface wind speed at noon of a Martian summer day [MCD, 2012]
Chapter 2. Literature Review and Mission Profile

2.1.2 Martian Surface Solar Insolation

Figure 2-9: Martian surface solar insolation [MCD, 2012]

Figure 2-8: South-north component of Martian surface wind speed at noon of a Martian summer day [MCD, 2012]

It should be mentioned that the wind will also cause unstable when the Martian aerobot is landing on the ground. As shown in Figure 2-7 and Figure 2-8, the maximum wind speed in the *Isidis Planitia* region in summer is no more than 10 m·s⁻¹. The lift caused by the wing at wind speed of 10 m·s⁻¹ is 3.72 N ($L = \frac{1}{2} \rho V^2 SC_l = 3.72$). Therefore, the wind on the Martian surface will not have significant influence on the stability when landing.
The highest solar insolation is about 589W·m⁻², but this is only a theoretical value at the point where the sun can best illuminate the surface, see Figure 2-9. The insolation at the equator is the highest, which is the same as the solar insolation on Earth. The Isidis Planitia region is close to the equator, so we can make use of this advantage. However, this solar insolation is approximately half of that on Earth at a clear day. This also poses a great challenge for the power system.

2.1.3 Mars Magnetic Field

According to recent research [Connerney et al., 2005], Mars has no global magnetic field. However, it must have one in the past, just like the Earth. For the present, Mars has a very strong crustal magnetic field, which is as strong as 30 times of that on Earth. The reason for the formation of such a strange magnetic field is not known. This is a good research area for future Mars investigation missions.

2.1.4 Isidis Planitia Region

The picture of the Isidis Planitia by the Mars Global Surveyor [NASA, 2012a] is shown in Figure 2-11. The name, Isidis, denotes Isis, the name of Egyptian god of heaven and fertility. Isidis Planitia is the third largest impact giant basin on Mars, whose centre is at 12.9°N, 87.0°E. The basin is covered by the dust and has a diameter of 1500km. The mineral magnesium carbonate,
which indicates the presence of water before, is found around the basin. Therefore, detailed exploration is required at this region to find signs of life.

![Figure 2-11: Isidis Planitia by Mars Global Surveyor (NASA, 2012a)](image)

### 2.2 Overview of the Coaxial Tiltrotor Aircraft

Since there was no aerobot on Mars by now, a short introduction on the coaxial tiltrotor aircraft is presented in this section. The first tiltrotor design was patented [Lehberger, 1930] in 1930; and it is a coaxial tiltrotor design. After that, more tiltrotor aircraft came out, but these tiltrotor aircraft focus on the twin rotor configuration. The largest tiltrotor manufacturer in the world is the Bell Helicopter. Their famous models include Bell XV-3, Bell XV-15, Bell “Eagle Eye”, V-22 Osprey, BA609, etc. The history of tiltrotor aircraft is about 80 years, but the technical publications are rare and most focus on the history and introduction of this technology.

Although proposed early, the use of the coaxial tiltrotor is a recent event. There are many projects on the coaxial tiltrotor aircraft, and two outstanding designs are the MTR and Verticopter®. Although both are still in design process, but their achievements are distinguished.

#### 2.2.1 MTR (Mono Tilt-Rotor)

The MTR [Baldwin, 2007, Baldwin, 2008, Baldwin, 2010, Baldwin, 2011] might be the first aerobot use the coaxial tiltrotor, which is proposed by the Baldwin Technology Company, LLC, designed by the Glenn L. Martin Institute of Technology, and funded by the US Office of Naval Research [Leishman et al., 2004]. The object of this project is to design a heavy-lift rotorcraft for Navy Sea Basing with Ship to Objective Maneuvre, and Army Future Combat System with mounted manoeuvre and air mobility. The most innovative features of MTR were aerodynamically deployed wing panels, pitch axis suspended load, and the coaxial tiltrotor design. Comparing to the traditional single and coaxial rotor helicopter for the same mission, the MTR
Chapter 2. Literature Review and Mission Profile

[Baldwin, 2010] is of half size, 1/3 weighted, and 1/3 fuel consumption. A scaled demonstrator is built and the main features are given by Table 2-1. The demonstrator in three different flight phases is shown in Figure 2-12.

![MTR demonstrator in hover, transition, and cruise modes](theworacle, 2009)

Table 2-1: Technical Data for the Scaled MTR Demonstrator [Baldwin, 2010]

<table>
<thead>
<tr>
<th>Feature</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gross weight (kg)</td>
<td>4,264</td>
</tr>
<tr>
<td>Rotor diameter (m)</td>
<td>7.62</td>
</tr>
<tr>
<td>Cruise speed (m·s⁻¹)</td>
<td>103</td>
</tr>
<tr>
<td>Cruise altitude (m)</td>
<td>6,096</td>
</tr>
<tr>
<td>Number of blade</td>
<td>2×4</td>
</tr>
</tbody>
</table>

2.2.2 Verticopter®

The embedded coaxial tiltrotor concept is used in the Verticopter® design [Garrow, 2011]. The Verticopter® is designed by the Garrow Aircraft LLC to supply the service for aerial surveillance, reconnaissance, border patrol, news coverage, VIP transportation, and medical evacuation. The Verticopter® has a series of scalable VTOL models ranging from 1/10 to 2/1 for different missions. The numbers of blades for coaxial tiltrotor also vary with the scale number.

![An R/C Verticopter® model](Garrow, 2011)

Figure 2-13 shows an R/C Verticopter® model with a 2×3-bladed coaxial tiltrotor. The model has a canard wing airframe with a pair of embedded coaxial tiltrotor. Six elevons plus rudders on the
wings are designed for flight control during the cruise mode. The coaxial tiltrotor used in the Verticopter® design is actually a pair of coaxial propeller, which can not perform collective or cyclic pitch change. The actuators for pitch and roll manoeuvring in VTOL mode are embedded micro-thrusters at the nose and wing tips for the present design. The coaxial tiltrotor just coincide with the CG of the airframe. This design will help to reduce the complexity in controlling the model, especially in VTOL mode.

The technical data for a series of the Verticopter® models are shown in Table 2-2. The coaxial tiltrotors of the mini R/C and the reg. R/C models are electric powered 2×3-bladed rotors. Because of the power limit of electric power system, their weight and power are much smaller compared with the other three models. The Drone/UAV model uses two gas engines to drive a 2×7-bladed coaxial tiltrotor. The 2-seater and 7-seater models use the turbine and jet engines, respectively. The main contribution of thrust in cruise for these two large models is from the turbine and jet engines rather than the coaxial tiltrotor.

<table>
<thead>
<tr>
<th>Model</th>
<th>Scale</th>
<th>Max speed (m·s⁻¹)</th>
<th>Wingspan (m)</th>
<th>Weight (kg)</th>
<th>Motor Power (kW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>mini R/C</td>
<td>1/10</td>
<td>59.0</td>
<td>0.9144</td>
<td>1.225</td>
<td>N/A</td>
</tr>
<tr>
<td>reg. R/C</td>
<td>1/5</td>
<td>75.1</td>
<td>1.829</td>
<td>5.443</td>
<td>2×0.895</td>
</tr>
<tr>
<td>Drone/UAV</td>
<td>1/2</td>
<td>120.0</td>
<td>4.572</td>
<td>86.20</td>
<td>2×15.66</td>
</tr>
<tr>
<td>2-seater</td>
<td>1/1</td>
<td>187.0</td>
<td>9.144</td>
<td>1134</td>
<td>2×335.6</td>
</tr>
<tr>
<td>7-seater</td>
<td>2/1</td>
<td>214.6</td>
<td>18.29</td>
<td>3583</td>
<td>2×1254</td>
</tr>
</tbody>
</table>

2.3 Review of the Rotor or Propeller Aerodynamic Theory

According to the aerodynamic theory [Anderson, 2001], the Reynolds number is one of the three important variables (AoA, Mach number, and Reynolds number), which determine the performance of the lifting body in the flow. The Reynolds number is a variable that has more influence on the formation of the boundary layer (flow very close to the lift body, or the sectional airfoil performance); while the Mach number and the AoA has more influence on the flow far from the lifting body (the wake structure and the influence). The Reynolds number gives a measure of the ratio of inertial forces to viscous forces. It quantifies the relative importance of the two types of forces for a given flow condition. In the traditional analysis method for fluid motion, the fluid is usually divided into the boundary layer and the flow field far from the lifting body are usually separated, since we need completely different models to describe their performance. This thesis focuses on general performance of the lifting bodies. A brief review on the low Reynolds number for lifting body performance is also given, but it will have influence on the performance of the sectional airfoils for rotor blades or wings only. The Reynolds number belongs to the further detailed design regime, which is even related with the roughness of the surface. Moreover, the Reynolds number of the rotor system is not too low. It has no significant influence on the
performance of sectional airfoil. Therefore, this factor is left as a potential research area in the future. In order to complete the design loop for sectional airfoil performance, the thin airfoil theory is used instead for simplicity. This sectional airfoil model is independent of the wake model, so it can be replaced by a more precise sectional airfoil model in future work.

The terms "rotor" and "propeller" stand for a hub with a number of radial airfoils rotating around the hub. The rotors or propellers can be used to provide thrust for aircraft. Although "rotor" and "propeller" are used to represent the rotating system working at zero and high inflow rates respectively; the fundamental theories of their operation are the same. The term "rotor" is often used in the context of helicopters, where it is used to counteract the weight. However, it is worth noting that a helicopter rotor is also called a lifting propeller or lifting airscrew in some old literature [Bsdkin et al., 1973]. The term "propeller" often stands for the thrust generator of fixed wing aircraft. In this thesis, the terminology will follow this rule to avoid ambiguity.

2.3.1 Aerodynamic Theories for Single Rotors and Propellers

Because of the different working conditions for the rotor and propeller, the literature of both helicopter and fixed wing aircraft has been studied. Conlisk [Conlisk, 1997, Conlisk, 2001] and Johnson [Johnson, 1986] give very good reviews on the general aerodynamic theory of helicopter rotors. Some specific reviews on Computational Fluid Dynamics (CFD) application [Caradpna, 1992], rotor wake behaviour [Gessow, 1986], rotor vortex wake [Gray, 1992, McCroskey, 1995], are also available. Wald [Wald, 2006] reviewed the aerodynamic theories for propellers. The coaxial tiltrotor will act as hovering rotor and cruising propeller under the different flight phases, so both cases need to be considered in this thesis. It is very difficult to give a complete review of the rotor and propeller aerodynamic theories, but generally, these theories can be divided into three categories: the classical Blade Element and Momentum Theory (BEMT), the vortex wake based theory, and the grid based CFD methods. The rotor or propeller theories are reviewed by a short introduction of these three categories.

2.3.1.1 Blade Element and Momentum Theory (BEMT)

The Wright brothers might have been the first to use BEMT to design efficient propellers for their aircraft. BEMT was formally proposed by Gustafson and Gessow [Gessow, 1948] in 1948 to analyze the aerodynamic properties of helicopter rotors. In BEMT, the rotor disk is divided into small annular elements. Both Blade Element Theory (also called blade strip theory) and Momentum Theory are applied to these elements. Assuming the annular elements are independent of each other, the annular elements can be treated independently. The spanwise induced airflow rate is solved by Momentum Theory. The blade loadings obtained by BEMT are reasonable but not very precise. However, it is still widely used, due to its simplicity and computational
efficiency. BEMT usually gives lower blade loadings at the blade root and higher loadings at the blade tip than would be obtained in practice. This is caused by the independence assumption of annular elements not being true in reality. The classical BEMT is briefly introduced in Appendix B-2.

2.3.1.2 Vortex Based Theory

The famous Kutta-Joukowski theorem forms the basis of the vortex wake based theory for rotors and propellers. The general concept is the same as the vortex lattice model for finite wings. The vortex based theory is a combination of vortex evolution theory and strip theory. In essence, the difference between vortex based theory and classical BEMT is the method of getting the induced velocity. The vortex system of finite wing or rotor blade is composed of the lifting line or lifting surface and the trailed vortex sheet (containing the trailed and shed vortices) representing the wake. According to the conservation of vorticity, the trail vortex strength is equal to the spatial rate of change of the bound circulation along the blade; and the shed vortex strength is equal to the temporal rate of change of the bound circulation. The induced velocity field can be calculated by the Biot-Savart law. The thrust and power are determined by the bound circulation. The main and most difficult problem for vortex based theory is to calculate the influence factor of the vortex system.

The vortex theory for a propeller is relatively simple. The superimposed high inflow rate will shed the helix trail vortex sheet away quickly, and the influences of the “old” vortex filaments become small very fast. Moreover, the interaction of the trail vortices and the bound vortex are also very small. The concentration and roll-up process of vortex sheet can usually be ignored. The helix vortex sheet generated is usually regarded as rigid and undeformed; that is, the radius and the screw pitch of vortices remain constant in the wake. Because the shed vortices strength is usually very small, they are often neglected in the model. The trail vortex filaments in the wake are helical rather than straight, a lot of effort is made to calculate the influence factor with different screw pitches. [Goldstein, 1929] gave a series of approximation equations to estimate the influence factor of the helical vortex for a lightly loaded propeller. Theodorsen [Theodorsen, 1948] pointed out that the undeformed trail vortex model could be used for moderate loaded propellers by shifting the focus to the wake far behind the propeller.

For the helicopter rotors, the inflow rate is much lower. The “old” vortex filaments also have significant influence on the induced rate at blade. Due to the absence of high inflow rate, the distances between vortices in the wake are much smaller. The interaction among trail vortex filaments and the bound vortex will cause the helical vortices to contract and roll-up. These contraction and roll-up effects will have significant influence on the induced velocity at the rotor disk. In other words, the structure of the vortex filaments in the wake cannot be simply
determined as in the propeller case. Therefore, two approaches are proposed to predict the structure of the vortex wake. One is the Prescribed Wake Method (PWM), based on empirical equations derived from measurements and analytical results; the other is the Free Wake Method (FWM), in which the vortex filaments are allowed to freely distort in the velocity field.

The widely used PWM is usually proposed for single rotor configurations only, because it is not easy to get accurate data on the coaxial helicopter rotor wake. The most commonly used PWM for helicopter rotor wakes is proposed by Landgrebe [Landgrebe, 1972, Landgrebe, 1969]. The PWM can give higher precision of the circulation distribution with acceptable computation cost compared with BEMT; however, it is worthwhile to note that the wake of a steady state operating helicopter rotor is inherently unstable, so the shape of the prescribed wake, especially in the far wake, is debatable. The shape usually used in PWM is usually an average position value.

In FWM, the shape of the vortex structure is determined by the spatial collocation points along the vortices in the wake. These collocation points are allowed to distort freely in the flow field to achieve equilibrium. Because the tip vortices have much more influence on the flow field, usually only the tip vortex is modelled to largely reduce the computational cost. Finite revolutions of the vortex filaments are modelled since it is impossible to calculate with infinite length; moreover, the influence of the very far wake can be neglected. At first, a time marching method was widely used to solve the FWM, in which the collocation points were initialized and allowed to freely distort with time. The vortex filaments are represented by a series of finite length line segments connected with the collocation points in the wake. The first time marching free wake calculation is given by Scully [Scully, 1967]. A vortex with a finite core radius equal to the blade length is enforced in the calculation to represent the shed vortex sheet. Clark & Leiper [Clark and Leiper, 1969] and Sadler [Sadler, 1971] gave the first calculation based on a relaxation technique. Both time marching and relaxation methods were the subject of intense study later on. It was found that the convergence rate of the time marching method is very slow, and usually results in an oscillating solution. This is not caused by the numerical error, but by the physical nature of the helicopter rotor wake. The relaxation methods have better convergence property. The converged solution obtained by the relaxation method is usually an average position of the oscillating wake. Miller and Bliss [Miller and Bliss, 1993] use a linearized Biot-Savart equation, and a fully implicit method, to get a result for the hovering and forward flight cases, but their results for forward flight do not match experimental data. Bagai and Leishman [Bagai, 1995, Bagai and Leishman, 1995] proposed a Pseudo-Implicit Predictor Corrector (PIPC) method to calculate the steady state wake of a helicopter rotor, which shows a robust convergence rate and conformity with experimental data. This method is also applicable for multi-rotor configurations [Bagai and Leishman, 1996], but the convergence property is not as robust as that for single rotor configuration. In order to study the transient dynamics of helicopter rotor wakes, Bhagwat and
Leishman [Bhagwat and Leishman, 2000, Bhagwat and Leishman, 2001] proposed Predictor-Corrector 2\textsuperscript{nd}-Backward (PC2B) and Predictor-Corrector Central (PCC) difference algorithms. These two algorithms are based on the previous PIPC algorithm and the time marching starts after the PIPC convergence.

### 2.3.1.3 Grid Based CFD Method

The grid based CFD method generally refer to the grid based numerical methods of the Navier-Stokes equation or equivalent governing equations of fluid dynamics. For the CFD method, the difference between the helicopter rotor and propeller lies in the different setup of boundary conditions. The application of CFD to rotors and propellers is a broad topic. Most grid generation and numerical methods are taken directly from the subsonic fixed wing cases. A special point for rotor and propeller simulation is the treatment of the rotation of the blades in simulation. Additional rotating domains are required besides the stationary domain. These domains are connected by a common interface, such as the General Grid Interface (GGI), sliding interface, stitch interface, etc. These different names imply different algorithms are used to pass information through the interface. The solver often used is the Multi-Reference Frame (MRF) solver. The CFD method is usually used for validation and final improvement design due to its large computational cost and yet high accuracy. CFD methods will not be further discussed in this thesis. Further information can be found in References [Conlisk, 2001] and [Caradpnnna, 1992].

### 2.3.2 Research on the Coaxial Rotor Design

Aerodynamic theories and design methods for coaxial rotors or tiltrotors are derived from the theories for single rotors. Public literature on coaxial rotor theories and design are relatively few. Coleman [Coleman, 1997] gives a very good review on the aerodynamic theories for coaxial rotors, although some conclusions are contradictory. NACA Langley Research Center [Harrington, 1951, Taylor, 1950] and De Lackner Helicopter Inc. [Sweet, 1960] use a single rotor with the same solidity to predict coaxial rotor performance. The Kamov Design Bureau and Central Aero-hydrodynamics Institute (TsAGI) in Russia made great efforts to develop the theory for coaxial rotors [Bsdkin et al., 1973, Tischenko, 1989]. Their research methods included modelling blades by lifting line and lifting surfaces theory and using various wake types and vortex theory. They also conducted a lot of experiments to reduce vibration. Their research enabled Russia to become the largest user of coaxial helicopters in the world. Nagashima [Nagashima and Nakanishi, 1981] considered the coaxial rotor as a type of variable geometry rotor. The wake of the coaxial rotor could be optimized by adjusting rotor parameters. His theoretical models used both actuator disk and free-wake analysis. He also proposed a set of equations to describe the interference between two rotors with different rotor spacing. Saito and Azuma [Saito and Azuma, 1981] conducted
local momentum theory with a modified Landgrebe wake, their results show good experiments correlation. Westland Helicopter Ltd. conducted a series of experiments for coaxial rotor systems. Andrew [Andrew, 1981a, Andrew, 1981b] proposed the vortex/momentum/blade element approach (BEMT with a tip vortex correction). Based on experimental results, Landgrebe [Landgrebe, 1971, Landgrebe and Bellinger, 1974] assumed the wake for a single rotor showed stronger and weaker contraction compared with the upper and lower rotor respectively, and this assumption was validated by the results of experiments in Japan and Russia. Zimmer [Zimmer, 1985] developed a method based on a curved lifting-line/vortex wake/blade element/momentum concept. The blades were regarded as adjacent 2D airfoil elements with a curved lifting-line. Shed vortices were accounted for over a short distance behind each station, and the trailed vortices were carried on downstream.

Most of the public literature on coaxial rotor design is based on the classical BEMT. The interference between two rotors is largely simplified such that the influence of the lower rotor on upper rotor is often ignored, while the influence on the lower rotor is assumed to be due to the upper rotor’s uniformly distributed fully developed slip stream. The research group in the University of Maryland proposed an optimization method for a coaxial rotor based on BEMT (such as the work by Leishman [Leishman and Ananthan, 2006] and Bohorquez [Bohorquez, 2007]). Leishman [Leishman and Ananthan, 2006] pointed out that the best L/D ratio for a specific airfoil is valid for a small range, so the blade twist is optimized while the taper is fixed. The proposed coaxial tiltrotor is obtained by a linear combination of the twist distributions. Rand and Khromov [Rand and Khromov, 2010] also use the BEMT as a basis. The optimum model is solved analytically using a new calculus of variations based on optimal conditions. The objective is to minimize the power required for a given thrust requirement for different axial inflow rates. The spanwise effective Angle of Attack (AoA) is adjusted to be uniform by changing the chord distribution. As expected, the optimal upper rotor has exponential twist and exponential chord along the radius. The optimal lower blades have two parts due to the influence of the upper rotor, and both have an exponential twist and exponential chord distribution.

2.3.3 Low Reynolds Number Aerodynamics

According to the previous design [Song, 2008], the Reynolds number for the wings is approximately 50,000 ~ 60,000; while the Reynolds number span for the rotor system is rather wide, since the speed at each section of the rotor is different. The Reynolds number of rotor system is a range rather than a specific value. The mid-point of the Reynolds number span is usually used to represent the Reynolds number of the rotor. For the present Martian aerobot, the Reynolds number for the rotor system is from 14,000 to 47,000 (Re= \frac{\rho v L}{\mu}, note the Reynolds
Chapter 2. Literature Review and Mission Profile

number of 478,000 used in the previous work is wrong), so the Reynolds number of the rotor system is 30,000 in this work. This Reynolds number regime is the same as that for the Micro Air Vehicle (MAV) on Earth. The corresponding study on MAV can be referred in considering the performance of the Martian aerobot.

![Graph of NACA 0012](image1)

**Figure 2-14:** Comparison of the aerodynamic properties of the NACA 0012 for a series of Reynolds numbers from 30,000 (low Reynolds number) to 1,230,000 (high Reynolds number)

![Graph of Eppler 387](image2)

**Figure 2-15:** Comparison of the aerodynamic properties of the Eppler 387 for a series of Reynolds numbers from 30,000 (low Reynolds number) to 1,230,000 (high Reynolds number)

Both the wing and rotor system of the Martian aerobot are operating at the low Reynolds number regime (Re<10^6). This thesis focuses on the aerodynamic property of the rotor system. The wing performance will just follow the conclusions in previous work. The Reynolds number of the rotor system is much lower than the low Reynolds number limit (10^6), the influence due to the Reynolds
number factor is significant, while the variation trend of the sectional lift or drag coefficients before separation is quite similar (i.e. the lift coefficient is an approximately linear function of AoA, while the variation of the drag coefficient is not significant before separation). The NACA 0012 and Eppler 387 are used to show this claim. Figure 2-14 and Figure 2-15 show the polar views of the NACA 0012 and Eppler 387 at the Reynolds numbers from 30,000 (low Reynolds number) to 1,230,000 (high Reynolds number). The data in these two figures are computed by JavaFoil [Hepperle, 2007], which is an open source code used to analyze the 2D performance of the airfoil. It includes the potential flow analysis and the boundary layer analysis. NACA 0012 is an airfoil used for traditional helicopter rotor. Eppler 387 is an efficient airfoil designed to be operating at low Reynolds number, so it has higher maximum lift coefficient before separation. The Eppler 387 has been used in some Martian aerobot rotor system proposals, such as [Young et al., 2002] and [Forshaw and Lappas, 2012]. The two figures show that the lift coefficients at these Reynolds numbers are almost the same before stall. However, the stall will occur for lower AoA with respect to lower Reynolds number. While the drag coefficient increase significantly for low Reynolds number. The exact reason for this physical phenomenon is still under research. Usually, the performance difference of a specific airfoil at different Reynolds numbers is determined by the size of the wake, which is determined by the structure of the boundary layer.

The Reynolds number has significant influence on the structure and formation of the boundary layer, then the boundary layer will influence the performance of the rotor system. However, the Reynolds number is not the only factor of the structure of the boundary layer. Even the surface smoothness will have great influence on the boundary layer. Moreover, considering the Reynolds number will require more powerful tools, such as the grid based CFD solver, wind tunnel experiments, etc. In this preliminary research, the Reynolds number factor is not considered for the following reasons: the lack of funding; our focus to the overall flow field of the rotor wake; the variation trend of sectional airfoil performance is similar before separation. However, considering the Reynolds number factor in the rotor system is very important will be a good research topic in the future to build a real life Martian aerobot. A brief introduction on this concept is given in the remaining part of this section. Extensive reviews of airfoil performance at low Reynolds numbers can be found in [Carmichael, 1981], [Gad-el-Hak, 2007], [Lissaman, 1983], and [Shyy et al., 2008].

Equation 2-1 gives the definition of Reynolds number. It is worth noting that the Reynolds numbers vary at different wing sections or blade sections. An average Reynolds number is often used for specific operating conditions.

\[
Re = \frac{\rho UL}{\mu} \quad \text{Equation 2-1}
\]
Where, $\rho$ denotes the density of the fluid (0.0135 kg m$^{-3}$ for Martian surface atmosphere); $U$ denotes the total flow velocity at the point of interested; $L$ denotes the reference length (a dimensional value of the rotor system); and $\mu$ denotes the fluid viscosity ($1.1 \times 10^{-5}$ kg m$^{-1}$ s$^{-1}$ for Martian surface atmosphere).

[Schmitz, 1942] might be the first study of low Reynolds number aerodynamics in this range. He measured the forces generated by airfoils in a wind tunnel with the Reynolds number range of approximately $2 \times 10^4 \sim 2 \times 10^5$. The performance of three airfoils, a thin flat plate, a thin cambered plate, and a thick cambered airfoil (N60), was measured in his study. His conclusion was that the thick cambered airfoil has a critical Reynolds where the performance changes drastically. Schmitz’s results were verified and expanded by [Abbott et al., 1945], [Riegels, 1961]. The critical Reynolds number for most airfoils is $10^4 \sim 10^5$ [McMasters and Henderson, 1979].

Figure 2-16 is one of the figures in McMasters’ study. In general, the smooth airfoils have a higher L/D ratio than rough airfoils at high Reynolds numbers. However, below the critical Reynolds number of about $10^5$, the rough airfoil has better performance than the smooth counterpart.

The reason for this phenomenon might be the formation and separation of the boundary layer near the airfoil surfaces. The flow for low Reynolds numbers is laminar; while for high Reynolds numbers is turbulent. For the flow with high Reynolds numbers, the flow will be attached for both smooth and rough airfoils. While the roughness of the airfoil will increase the drag; therefore, the smooth airfoil has higher L/D ratio. However, the laminar boundary layer attached to the smooth airfoil is unable to resist the strong pressure gradients. The separation will happen for low Reynolds number condition. This boundary layer separation will largely degrade the performance of the airfoil. While the boundary layer of the rough airfoil is still turbulent for low Reynolds number condition, so the separation for rough airfoil condition is much smaller.

Another explanation of such phenomenon is the “laminar separation bubble” [Shyy et al., 2008]. For the boundary layer in low Reynolds number regime, the separated laminar boundary layer
rapidly transit to a turbulent flow, while the turbulent flow will reattach the boundary layer back to the body surface. In this case, a bubble is formed. The size of the bubble will dramatically increase the drag of the airfoil, since a larger wake is formed after the airfoil. The separation bubble is undesired in practical use. The laminar separation bubble will be formed only while the Reynolds number is higher than 70,000. The physical chord for lower Reynolds numbers is too short to be reattached [Lissaman, 1983].

As described above, the low Reynolds number has more influence on the boundary layers rather than the entire flow field. Since it is very difficult to predict the separation of boundary layer without more powerful tool for analysis, this problem is left for future research. In this preliminary work, we propose to use “rough” airfoil for the blade section, which will increase the turbulence in the boundary layer. Although it will increase the drag, it will prevent the separation. In this case, the variation trends of the lift and drag coefficients are similar to the high Reynolds number counterpart with an increase in drag. Therefore, a linear thin airfoil theory for high Reynolds number condition before stall and separation is used in this work to complete the design loop. The corresponding “geometry” can be regarded as the results of symmetric airfoils. For asymmetric airfoils, we just need to add the zero-lift AoA to the blade pitch at each section. Considering the Reynolds number factor, there is no symmetric airfoil available to be efficiently operating at high lift coefficient. The special designed low Reynolds number airfoils are recommended for this work, such as the Eppler 387.

To sum up, two compromise methods are used to simplify the Reynolds number influence in this work. First, the rough blade surface can be used to suppress the separation for each blade section. This method will increase viscous drag, and then decrease the efficiency; however, studying this topic will require more powerful tools, such as the CFD solver. Since the CFD solver is not available in this research, this compromise has to be made. Second, careful placement of the sectional AoA to make sure the separation will not happen. This method is more practical, but the low Reynolds number will place a constraint on the thrust coefficient. With the two approaches, the model for this low Reynolds number rotor system can be largely simplified.

2.4 Review of Flight Control Strategy

The flight control strategy is a critical problem for the Martian aerobot with such a unique design. Figure 3-1 shows the three flight modes (VTOL, cruise and transition/conversion) of Hyperion. The control strategies for three flight modes are different. The Martian aerobot in VTOL mode is quite similar to the quadrotor, since all the three control channels of the attitude angles are fully decoupled. While the cruise mode is the same as the flying wing aircraft, whose control strategy is similar to the conventional fixed wing aircraft. Therefore, the control technology of Hyperion in
these two modes is well developed. The control of the tiltrotor aircraft on Earth can be used as reference to solve the control problem in transition and conversion, since the control concepts are similar.

Mathematical modelling is the first step for a controller design in every flight mode. A precise mathematical model can usually guarantee the validity of a controller with higher performance. In this thesis, the focus is the preliminary controller design, so the rigid body based dynamic model is used. However, the aerodynamic property of the airframe and the thrust and torque generated by the rotors are difficult to predict. Therefore, we will focus on the robust control strategy of the Martian aerobot with the aerodynamic uncertainties.

The Martian aerobot in VTOL (hover), transition/conversion, and cruise modes can refer to the control of the quadrotors, tiltrotor aircraft, and fixed wing aircraft. Because the control of the fixed wing aircraft is a well-developed technique and has already been discussed in previous design, the control strategy for fixed wing aircraft is not reviewed in this thesis. A review on the modelling and control strategy of the quadrotors and tiltrotor aircraft are presented as follows.

### 2.4.1 Modelling and Control in VTOL (Hover) Mode

For the Hyperion in VTOL (hover) mode, the pitch control is implemented by adjusting the thrust of the auxiliary rotor together; the roll angle is controlled by differentiate the thrust of auxiliary rotors; and the differentiation of coaxial tiltrotor will cause yaw moment to adjust the yaw angle. The altitude is controlled by increasing or decreasing the thrust of the coaxial tiltrotor. The horizontal flight direction is controlled by tilting the vehicle for specific direction. The three attitude angles and three velocity channels are fully decoupled, as the quadrotors.

The robotic community shows great interest to the quadrotors since 1990s. A series of quadrotor projects for different application are proposed. Some good work is the Mesicopter [Kroo and Prinz, 1999, Kroo et al., 2001], STARMAC (Stanford/Berkeley Testbed of Autonomous Rotorcraft for Multi-Agent Control) [Hoffmann et al., 2004, Hoffmann et al., 2009], quadrotor projects from the University of Pennsylvania [Kumar, 2012] and Massachusetts Institute of Technology (MIT) [How, 2008], OS4 quadrotor [Bouabdallah, 2007, Bouabdallah et al., 2004a, Bouabdallah et al., 2004b, Bouabdallah and Siegwart, 2007], microdrones GmbH UAV platform [microdrones, 2012], etc.

#### 2.4.1.1 Modelling of VTOL (Hover) Mode

In the public literature on the quadrotor control problems, the rotor aerodynamic theories used to estimate the thrust are the simple Momentum Theory [Orsag and Bogdan, 2009], BET [Orsag and Bogdan, 2009], BEMT [Bouabdallah et al., 2004a, Bouabdallah et al., 2004b, Bouabdallah and
Siegwart, 2007]. Among all the three methods, only the BEMT can give a reasonable estimation of rotor performance. On the prediction of the rotor torque, some paper just use the general assumption that the rotor torque are proportional to the square rotor speed [Berbra et al., 2009, Coza and Macnab, 2006, Dierks and Jagannathan, 2010, Dikmen et al., 2009, Erginer and Altug, 2007, Mian and Wang, 2008]. The accuracy of this estimation is very low. While in some publications, the authors just skipped the aerodynamic problem. The control variables selected are the thrusts and moments as control variables [Czyba, 2009, Das et al., 2008], or some variables relevant with the thrusts [Xu and Ozguner, 2006], or even the attitude angles [Al-Hiddabi, 2009]. In the Mesicopter project proposed by Stanford University, A comprehensive analysis of the rotor aerodynamics is given by the 3D CFD software OVERFLOW-D [Kroo et al., 2001].

2.4.1.2 Control Methods of VTOL (Hover) Mode

The objective of the VTOL controller for Hyperion is to adjust the attitude angles and the altitude. In the public literature, almost all the control methods have been used to quadrotor, including Proportional Derivative (PD) control [Dikmen et al., 2009], Proportional Integral Derivative (PID) control [Bouabdallah, 2007], Linear Quadratic (LQ) control [Bouabdallah, 2007], dynamic inversion control [Dikmen et al., 2009], feedback linearization control [Al-Hiddabi, 2009, Voos, 2009], Sliding Mode Control (SMC) [Dikmen et al., 2009, Efe, 2007, Zhou et al., 2008], high order SMC [Benallegue et al., 2006], back-stepping control [Dikmen et al., 2009, Madani and Benallegue, 2006, Madani and Benallegue, 2007], Neural Network (NN) control [Dierks and Jagannathan, 2009]; and other hybrid control methods such as adaptive-fuzzy control [Coza and Macnab, 2006], model independent PD control [Tayebi and McGilvray, 2006], NN output feedback control [Dierks and Jagannathan, 2008], robust NN control [Nicol et al., 2008], robust control [Coza and Macnab, 2006, Lee et al., 2009, Mokhtari et al., 2005], and etc. These controller structures are also different. The quadrotor is a good testbed for control methods. All these methods can be used for the control of Hyperion for VTOL mode, since its dynamics is quite similar to the quadrotors. Because the objective of the control system is to stabilize the dynamics, we will use the traditional control loops and robust controller design method in this work.

2.4.2 Control in Transition and Conversion Modes

The general transition/conversion control concept of Hyperion is similar to the tiltrotor aircraft on Earth, although the aerodynamic configuration is not the same and the coaxial tiltrotor of Hyperion are designed not to perform cyclic control. The first two experimental tiltrotor aircraft [Markman and Holder, 2000] are the Transcendental Model 1G & 2 manufactured by the Transcendental Aircraft Corporation in the 1954. The Model 1G experimental aircraft was crashed
in 1955. The first complete transition from hover to cruise [Markman and Holder, 2000] was completed by the Curtiss-Wright X-100 experimental tiltrotor aircraft, designed by the Curtiss-Wright Corporation. However, public literature on the tiltrotor aircraft is limited. Most publications focus on the history [Apostolo, 1988, Harding, 2000, Markman and Holder, 2000] and the mathematical model for engineering applications [Harendra et al., 1973, Marr et al., 1973]. Most of the technical publications use very simple transition dynamic model or even skip this part. Since the general concept of the conversion control is the same as that for transition, most authors did not mention the conversion control in their publications. A summary of the available publications on the transition control of tiltrotor aircraft is reviewed as follows.

Calise and Rysdyk [Calise and Rysdyk, 1996] proposed the Neural Network augmented model inversion method for tiltrotor aircraft; and they [Rysdyk and Calise, 1999] continued and implemented with an internal mode of a feedback linearization and an outer loop with PD controller. The focuses of both articles are to design the inner loop controller to improve the dynamic property. Under the support of Smart UAV Development Program by Korea Aerospace Research Institute (KARI), Lee [Lee et al., 2007] proposed the same inner-outer loop control structure for attitude control. The inner loop is a PD compensator to improve the dynamic property; and the outer loop is a PI controller to stabilize the attitude dynamics. The Particle Swarm Optimization (PSO) is used to find the optimal parameters for controllers. But the optimal controller found is the optimum under the proposed controller structure only. The reason of using PD compensator and PI controller for inner and outer loops is not given. Kang [Kang et al., 2008] continued and conducted the flight tests for this design. The main objective in the flight test is to verify the attitude controller and speed controller during transition by manually tilting and automatic tilting. Song [Song and Wang, 2009] pointed out that the transition trajectory should be selected within the transition corridor; and the trajectory is selected to ensure the transition as stable as possible. The controller structure in his work also has two loops. The inner loop was a state feedback compensation to decouple the system variables and to improve the dynamic property. The outer loop was a PI output feedback controller for attitude stabilization. In the last of the paper, flight experiment results validated the proposed design. Kendoul [Kendoul et al., 2006] proposed a back stepping based tracking controller for small tiltrotor aircraft model. The back stepping control method can provide robustness and improve the dynamic property. Yu [Yu et al., 2005] proposed a nonlinear adaptive neural networks internal model controller for a tiltrotor experimental demonstrator. The neural network controller was trained first offline and then online. Lyapunov Theory and simulation results validated its feasibility. Baldwin [Baldwin, 2011] proposed to use apprenticeship learning algorithm to train controller of the Mono Tilt-Rotor (MTR). The controller can learn from flight demonstrations and find the flight trajectory.
In these publications, the transition controllers for tiltrotor aircraft have two functions: improve the dynamic property and stabilize the system. Although the proposed control methods are not the same, most controllers have an inner-outer loop structure. The inner loop controller will improve the dynamic property, and the outer loop controller will guarantee the stabilization. Because the error dynamics of the trajectory linearized model is a Linear Parameter Varying (LPV) system, it is difficult to use the linear compensator to improve the dynamic property; therefore, the compensator selected in the literature for the inner loop is usually very complex. Other methods, such as those derived from machine learning, are used also because of the LPV property.

The transition corridor is a critical concept for transition control for two different steady operating states with large variation. It is a virtual corridor that connects two important working conditions. In the aircraft control area, it is very important for maneuver flight. The transition corridor is composed of all the steady states during transition. Taking the tiltrotor aircraft as an example, the transition corridor is usually represented by the figure of the cruise speed v.s. the nacelle angle (Figure 5-65 shows the transition corridor for Hyperion). For a tiltrotor aircraft, both the lift and the rotor thrust can be used to counteract the weight during flight. The lift and thrust can be adjusted by changing specific control variables, so the number of combinations is infinite for specific cruise speed. In this case, the transition corridor has two boundaries. One is the stall boundary, for which the wing of the aerobot will generate the maximum lift; the other is the power boundary, for which rotor will generate the maximum thrust. The area between the two boundaries are the transition corridor.

If the proposed controller can guarantee the flight states fall within the transition corridor during the whole transition, then the proposed controller can stabilize the aerobot for transition. This is the main concept for transition control used in this thesis. Refer Section 5.10.2 for details.
Chapter 3

3 Overview of Hyperion

The objective of this chapter is to give an introduction on the subsystems of the proposed Martian UAV, Hyperion. Most subsystems are based on the previous work [Song, 2008], such as the system design, mission profile, mass budget, power budget, thermal control system, scientific payloads, etc. We also give some general calculation on the performance of the Martian aerobot in the end. This chapter will help the readers not need to go back to read the previous work first. The design key requirements or constraints will just follow the previous work [Song, 2008].

- Mass budget: 25kg
- Aeroshell diameter: 2.65m
- VTOL capability (reusable design)
- Payload mass: 3kg

3.1 General Structure of Hyperion

The following modifications are made to solve the problems shown in Section 1.5.1.

- The blade number of the coaxial rotor is increased from 2×2 to 2×4. This is due to the fact that the blade sectional AoA of coaxial rotor in previous design is approximately 17° [Song, 2008]. This is impractical for an aerobot design. The performance of the coaxial rotor in Halcyon is estimated by the simple Momentum Theory, which usually overestimates the performance. This modification will increase the rotor solidity and make the coaxial rotor more practical for detailed preliminary design, which is the content of Chapter 4.

- The coaxial tiltrotor is used instead of the coaxial rotor and propellers for Hyperion. One reason is the coaxial tiltrotor design will improve the efficiency of the propulsion system, that is, there is no useless payload in all the flight phases. Another reason is the mass of the coaxial rotor will increase due to the increase of blade number; the coaxial tiltrotor design will help to keep the total mass and power budget of the propulsion system.
• The coaxial tiltrotor is designed not to perform the cyclic pitch operation to reduce the mechanical and control complexity of the coaxial tiltrotor and to avoid possible blade collision during transition and conversion. Although it is possible to use the full helicopter coaxial tiltrotor mechanics, such as the one used in the MTR (Mono Tilt-Rotor) [Baldwin, 2008], the mechanical and control complexity is still a great challenge for the embedded design. The blades of the coaxial tiltrotor may collide with each other especially when tilting during transition and conversion.

• Two small auxiliary rotors are embedded at the wing tips. There are two reasons for such design. Firstly, the two auxiliary rotors are the actuators to control the pitch and roll angles since the coaxial tiltrotor is designed without cyclic inputs. Secondly, the relative positions of the AC and CG for VTOL and cruise flights are not the same. In the Hyperion design, the sliding mass mechanism is not needed and the relative position of the AC and CG is designed for stable VTOL requirements. It is worthwhile to note that the auxiliary rotors will operate at VTOL and low speed transition/conversion phases. The covers for the auxiliary rotors will be closed to increase the wing area during the transition.

• A 2-elevon flaps design is used instead of the previous 4-elevon plus 2-rudder design. As we can see that the aerobot will be folded to an aeroshell when launching (Figure 1-1), more flaps will increase the complexity in the folding wing design. But such design will largely reduce the manoeuvrability; therefore, such a modification is debatable. This modification is used in the thesis just to validate the possibility.

![Hyperion in VTOL (hover), transition/conversion and cruise phases](image-url)

Figure 3-1: Hyperion in VTOL (hover), transition/conversion and cruise phases

Figure 3-1 shows the different configurations of Hyperion in VTOL (hover), transition/conversion and cruise phases. The cover mechanism of the auxiliary rotors is not shown. The following coaxial tiltrotor design and robust transition/conversion control problems in this thesis are based on this design.
Chapter 3. Overview of Hyperion

Figure 3-2: Coaxial tiltrotor concept for Hyperion

Figure 3-3: Dimensions of the airframe (mm) [Song, 2008]

The detailed coaxial tiltrotor concept is given by Figure 3-2. The distance between two rotors is $d = 0.3\text{m}$. The rotor radius is $R_{\text{rp}} = 1.0\text{m}$ based on the previous design. Since the coaxial tiltrotor will be tilted forward to about $90^\circ$, the radius of the duct is at least $R_{\text{duct}} > \sqrt{R_{\text{rp}}^2 + (d/2)^2} = 1.012\text{m}$. Because the radius of the rotor is not small, the aeroelastic deformation requires the radius of the duct to be larger than the limit of $1.012\text{m}$ for safety. Since the aeroelastic model for this project is not available, so we can only give a brief estimation of the duct radius ($R_{\text{rp}}$). In this work, we propose the radius of the duct to be about $1.10\text{m}$ for safety. That is, the radius of the duct is larger than the radius of the rotor by $0.1\text{m}$. In this case, the duct can not provide significant additional
thrust. Therefore, the ducted fan effect is not considered in this thesis. The duct can be redesigned in the future if the high precision aeroelastic model is build.

The airframe used in this work is the same as the previous design. The exact dimensions are given by Figure 3-3.

3.2 Mission Profile

The scientific investigation for this Martian aerobot includes high resolution image of the landscape, near surface atmosphere investigation, surface/sub-surface chemical experiments, surface sampling, etc. All of these missions should be accomplished in exploration region. The landing site must be safe and be valuable for investigation. Since the Mars mission aims to find the signs of life, we will focus on the regions there have traces of water.

The *Isidis Planitia* region is selected as the exploration site due to the following reasons [Song, 2008]:

- The *Isidis Planitia* region is a basin around which the magnesium carbonate is found (see Section 2.1). The existence of magnesium carbonate shows the presence of water before. Therefore, it is possible to find the signs of life in this region.

- The *Isidis Planitia* region (12.9°N) is close to the equator, so the solar radiation is not far from its maximum. The solar-electric power system can get enough energy during the mission.

- The *Isidis Planitia* region is an ideal landing site because it is vast and the density of surface is suitable for landing. There are no mountains or valleys in this region.

The Martian aerobot should operate at an altitude less than 2.5km, since the atmosphere density will not decrease too much. This design could fly for one hour per Martian day, so it would take at least 10 Martian days (i.e. 10 flights) to complete the mission.

The Mars Express Beagle-2 was planned to land in the Eastern part of the *Isidis Planitia* region, but it was lost when landing. The *Isidis Planitia* region might be a bay of the northern sea [Song, 2008], so this is a good region for searching for signs of life.

3.2.1 Flight Season and Path

Because of the heavy wind, it is not safe for the Martian aerobot to fly in local winter. Therefore, *Hyperion* is proposed to be flying in spring or summer, when the wind speed is relatively low. Since the *Isidis Planitia* region is close to the equator, the solar insolation is high for any season.
Figure 2-7 and Figure 2-8 show that the wind direction in local summer is from East to West; therefore, the proposed flying direction is from West to East, as the red arrow shown in Figure 3-4.

![Figure 3-4: Flight path on Mars [Song, 2008]](image)

The flight path is about 100km. We need 10 flights (Martian days) to complete this flight. A more possible flight path is given by the yellow line in Figure 3-4 in previous work by following the “bay” of the northern “sea”. Since the image of Mars is available in NASA, it is possible to use the terrain contour matching strategy for navigation during flight.

### Table 3-1: Mission profile for each flight

<table>
<thead>
<tr>
<th>Phase</th>
<th>Flying Mode</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Take-off</td>
<td>VTOL</td>
<td>Climb vertically to the transition altitude with a constant speed</td>
</tr>
<tr>
<td>Transition</td>
<td>Transition</td>
<td>Transit to cruise and close the covers of the auxiliary rotors</td>
</tr>
<tr>
<td>Cruise</td>
<td>Cruise</td>
<td>Climb to the cruise altitude</td>
</tr>
<tr>
<td>Cruise</td>
<td>Cruise</td>
<td>Cruise at a constant speed and explore the <em>Isidis Planitia</em> region</td>
</tr>
<tr>
<td>Cruise</td>
<td>Cruise</td>
<td>Descend to the transition altitude</td>
</tr>
<tr>
<td>Conversion</td>
<td>Conversion</td>
<td>Convert to hover and open the covers of the auxiliary rotors</td>
</tr>
<tr>
<td>Landing</td>
<td>VTOL</td>
<td>Land vertically with a constant speed</td>
</tr>
</tbody>
</table>

![Figure 3-5: Mission profile for each flight](image)

The mission profile for each flight is shown in Figure 3-5 and Table 3-1. In each flight, the aerobot will take off vertically and climb to a transition altitude. Then it will transit from hover to cruise. After that, the vehicle will climb to the cruise altitude and cruise for about an hour. Finally, *Hyperion* will descend to the transition altitude, transit to hover and land vertically. The transition
altitude in the mission profile is to guarantee the safety during the transition, because there might be some altitude loss during transition.

### 3.2.2 Landing Site

The landing site for *Hyperion* has two meanings. One is the landing site for the aeroshell; the other is the landing site for each flight during the exploration. The exact landing site for this project is not confirmed yet. Deciding the exact landing sites for each flight requires detailed mission design. Deciding the landing sites include many aspects, such as the position of interest, properties of the Martian surface, etc. In this thesis, some possible requirements are proposed for the landing site selection.

The requirements for aeroshell landing are the same as that for the landers and rovers projects. Some possible requirements are listed as follows:

- The landing site for aeroshell should be not far from the exploration region. In this thesis, the landing site should be selected at somewhere at the western part of the *Isidis Planitia* region due to the flying direction is from West to East (Figure 3-4).
- The surface of the landing site for aeroshell should not be too hard to prevent possible crashing.
- The landing site for aeroshell should be flat. The rocks on the surface can cause damage to the aeroshell.

The possible landing site for each flight has to be smooth. Since the *Isidis Planitia* is a flat region, satisfying this requirement is not difficult.

### 3.3 Power Budget and Power System

#### 3.3.1 Power Budget

Table 3-2 shows the possible operations of the Martian aerobot for each Martian day. The figures in the table are based on a brief calculation of the power (or energy) budget in previous work [Song, 2008]. The Martian aerobot, *Hyperion*, is a revised version based on previous work. The revision is the propulsion system, so the operational concept and the power budget of other subsystems will just follow the figures in [Song, 2008]. Since this thesis is not on the power system, a brief description on this topic is given in this section.

Based on [Song, 2008], assume the efficiencies of the electronic controls, the drive motor, and the gearbox are $\eta_e = 0.98$, $\eta_m = 0.9$, and $\eta_g = 0.85$, respectively. Therefore, the efficiency from the
power system to the rotor is $\eta = \eta_c \eta_m \eta_g = 0.75$. Therefore, the power computed in Chapter 4 should be converted to the power budget of the power system by dividing the proposed efficiency.

According to the conclusion of Chapter 4, the aerodynamic power required by the rotor system is given as follows:

- Coaxial rotor in VTOL mode (hover): 3399W (power coefficient of $C_p = 0.0117$) in total
- Coaxial rotor in cruise mode: 542W (power coefficient of $C_p = 0.0074$) in total
- Auxiliary rotors: 40W for each

Therefore, the power budget for propulsion system is given as follows:

- Coaxial rotor in VTOL mode (hover): 4532W
- Coaxial rotor in cruise mode: 723W
- Auxiliary rotors: 53W for each

Table 3-2: Energy budget for each Martian day (some data from [Song, 2008])

<table>
<thead>
<tr>
<th>State</th>
<th>Operations</th>
<th>Power (W)</th>
<th>Duration</th>
<th>Total Power (W)</th>
<th>Energy (kJ)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Morning</td>
<td>Contact experiment</td>
<td>15</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Imaging</td>
<td>5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Orbiter communication</td>
<td>50</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>On-board computer</td>
<td>10</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>3.5 hrs</td>
<td>80</td>
<td>1008</td>
</tr>
<tr>
<td>Take-off</td>
<td>Coaxial rotor propulsion</td>
<td>4532</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Auxiliary rotor</td>
<td>106</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Flight control</td>
<td>5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Imaging</td>
<td>5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Orbiter communication</td>
<td>50</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>On-board computer</td>
<td>10</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1 min hover + 1 min transition</td>
<td>4708</td>
<td>565</td>
</tr>
<tr>
<td>Cruise</td>
<td>Forward propulsion</td>
<td>723</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Imaging</td>
<td>5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Flight control</td>
<td>5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Orbiter Communication</td>
<td>50</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>On-board computer</td>
<td>10</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1 hr</td>
<td>793</td>
<td>2854</td>
</tr>
<tr>
<td>Landing</td>
<td>VTOL propulsion</td>
<td>4532</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Auxiliary rotor</td>
<td>106</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Flight control</td>
<td>5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Imaging</td>
<td>5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1 min hover + 1 min transition</td>
<td>4708</td>
<td>565</td>
</tr>
</tbody>
</table>
Chapter 3. Overview of Hyperion

<table>
<thead>
<tr>
<th>Time</th>
<th>Activity</th>
<th>Orbiters</th>
<th>On-board computer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Afternoon</td>
<td>Contact experiments</td>
<td>15</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>Imaging</td>
<td>5</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Orbiter communication</td>
<td>50</td>
<td>10</td>
</tr>
<tr>
<td>Night</td>
<td>On-board computer</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td>Thermal Control</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Total: 7.15 MJ

### 3.3.2 Solar Energy and Solar Cell

The solar insolation flux on Mars is 589 W·m⁻² in theory (as presented in Section 2.1.2). According to the previous work [Song, 2008], a more reliable estimation of \( E_p = 400 \text{W·m}^{-2} \) is used.

The total energy for a Martian day is briefly estimated by

\[
E = \int_0^{12} E_p \sin \alpha \, dt = \int_0^{12} E_p \sin \frac{\pi}{12} \, dt = 3056 \text{kW·h·m}^{-2}
\]

Equation 3-1

Where, \( E_p \) is the solar insolation flux on Mars; \( \alpha \) is the zenith angle of the sun. The total energy available for the Martian aerobot is approximately 10.8 MJ·m⁻².

Assume the efficiency of the solar cell is 15\%, the total solar energy for the Martian aerobot is 12.96 MJ for each Martian day (the wing area covered by the solar panels is about 8 m²).

Table 3-3: Solar Cell Comparison [Song, 2008]

<table>
<thead>
<tr>
<th>Characteristics</th>
<th>Emcore</th>
<th>DayStar</th>
<th>Uni-Solar</th>
<th>NREL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Specific Power/Mass (W·kg⁻¹)</td>
<td>164</td>
<td>519</td>
<td>453</td>
<td>750</td>
</tr>
<tr>
<td>Power/Area (W·m⁻²)</td>
<td>138</td>
<td>76</td>
<td>45</td>
<td>96</td>
</tr>
<tr>
<td>Mass/Area (kg·m⁻²)</td>
<td>0.84</td>
<td>0.1465</td>
<td>0.0992</td>
<td>0.128</td>
</tr>
<tr>
<td>Efficiency</td>
<td>27.5%</td>
<td>15.2%</td>
<td>9%</td>
<td>19.2%</td>
</tr>
<tr>
<td>Area Required (m²)</td>
<td>5.61</td>
<td>10.13</td>
<td>17.12</td>
<td>8.02</td>
</tr>
<tr>
<td>Mass (kg)</td>
<td>4.70</td>
<td>1.49</td>
<td>1.69</td>
<td>1.026</td>
</tr>
</tbody>
</table>

The preferred solar cell in previous work is NREL thin-film cell by comparing four candidate solar cells with practical application: Emcore high efficiency triple junction cells, Daystar thin film CuInGaSe₂ solar cells, NREL ZnO/Cds/CuInGaSe₂ thin-film solar cells, and the UNISOLAR. The comparison is given by Table 3-3.
### 3.3.3 Battery for Energy Storage

In the previous work [Song, 2008], three kinds of rechargeable batteries (lithium ion, lithium polymer, and lithium-sulphur) in use are considered for energy storage for the Martian aerobat. A comparison is given in Table 3-4. The conclusion is to use the Kokam SLPB8043128H 3.2Ah (lithium polymer) for this Martian aerobat.

### 3.4 Thermal Control System

The temperature range of a Martian day is from -90°C to -20°C. The thermal control system will have two functions. One is to keep the temperature of the components (heating); the other is to cool down the electric motors. The operating temperature for each component is listed in Table 3-5.
According to the previous design [Song, 2008], both the passive and active thermal control strategies are required for this Martian aerobot. An aerogel-insulated warm box is proposed for the central electronic devices and the rechargeable batteries.

Based on the calculations in previous work, approximately 296Wh energy is sufficient to support the central warm boxes. The servos for elevons are far from the central warm box, so extra heating is required. The separate Radioisotope Heater Units (RHU) was proposed to be used for the servos.

During the taking off phases, the temperature of the motor will increase by over 45°C. Since the initial temperature of the motor is the same as the environment (very low) and the duration of taking off is only 1 minute, the cooling device is not working during this time. However, the cooling device is working all the time during cruise.

### 3.5 Scientific Payloads

The selection of scientific payloads depends on the mission objective and the mass budget. A possible payload package proposed in previous work is listed in Table 3-6. This is only a possible choice for 3kg mass budget for payload.

<table>
<thead>
<tr>
<th>Instruments</th>
<th>Power (W)</th>
<th>Mass (g)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Contact experiment</td>
<td></td>
<td>1970</td>
</tr>
<tr>
<td>Alpha particle X-ray spectrometer</td>
<td>1.5</td>
<td>570</td>
</tr>
<tr>
<td>Mossbauer spectrometer</td>
<td>1.6</td>
<td>500</td>
</tr>
<tr>
<td>Microscope imager</td>
<td>0.5</td>
<td>300</td>
</tr>
<tr>
<td>Abrasive tool</td>
<td></td>
<td>300</td>
</tr>
<tr>
<td>Imaging system</td>
<td></td>
<td>1030</td>
</tr>
<tr>
<td>Wide angle imager (downward)</td>
<td>1.5</td>
<td>115</td>
</tr>
<tr>
<td>Wide angle imager (forward)</td>
<td>1.5</td>
<td>115</td>
</tr>
<tr>
<td>4 Narrow angle imager (downward)</td>
<td>2.0</td>
<td>200×4</td>
</tr>
<tr>
<td>Overall</td>
<td></td>
<td>3000</td>
</tr>
</tbody>
</table>

### 3.6 Communication Scheme

The communication between the aerobot on Mars and the ground station on Earth is a critical problem. Since there is no Martian aerobot before, we have to refer some other projects. The communication scheme for the Martian landers and rovers can be used as a reference. Since the author knows very little about this area, the content in this section will just show the main idea.
For the Mars missions launching landers or rovers to Mars, an orbiter (i.e. satellite) will also be launched. Taking the Viking lander 1 [NASA, 2012d] as an example. The orbiter can be used independently for Mars observation by the payloads installed. This orbiter can also be used as a relay satellite for the lander.

Such a communication scheme can be used for the Martian aerobot. Figure 3-6 gives a possible communication scheme between the Martian aerobot and the ground station on Earth. The Martian aerobot is required to transmit measurement data to ground station on Earth; the ground station will also transmit the command to the Martian aerobot. Therefore, the bidirectional communication between the Martian aerobot and the ground station is required. It is worth noting that we have two possible methods to transmit the information between the ground station and the orbiter. One is direct transmission (blue arrow); the other is via a relay satellite for Earth (black arrows). The exact solution for the transmission scheme between ground station on Earth and the orbiter belongs to the orbiter design regime, so this is not discussed in detail.

Since the Martian aerobot is fully autonomous, the communication between the Martian aerobot is not required to be real-time. The flight command to be transmitted from the orbiter to the Martian aerobot is the possible flight path. For the measurement data obtained by the Martian aerobot, we propose that the measurement data are stored in the memory of the Martian aerobot. The data is transmitted only when the orbiter passes the zenith of the Martian aerobot. Since the Martian aerobot will fly for only one hour per Martian day, we can even propose that the data are not transmitted during flight. This will largely simplify the control scheme in flight.

### 3.7 Structure of Airframe and Rotor

The cruise speed of the Martian aerobot is proposed to be 50m·s⁻¹ in previous work. In this case, the Reynolds number for the wings is about 60,000 [Song, 2008]. Based on this property, six
possible airfoils (E387, E326, zagi10, MH60, MH45, and sipki11) were studied in previous work. At last, the zagi10 is selected as the wing airfoil because of its preferable aerodynamic characteristics. The airfoil of the rotor is not confirmed, since its working environment is more complex. This problem is left in the future work.

Material is another important factor for the wing of the Martian aerobot. The material used must be very light weight with high strength. The primary materials selected for previous design are the same as those used for the HALE (High Altitude Long Endurance) aircraft on Earth, the "Helios". The main tubular wing spar is made of carbon fibre. The wing ribs are made of epoxy and carbonfiber. The shaped Styrofoam is filled into the leading edge. Mylar is used to cover the wing. The rotors are also made in the same way.

The Martian aerobot has a wing span of approximately 8m. This aerobot must be folded and be stowed into an aeroshell for launch. In this project, a pathfinder like aeroshell with a diameter of 2.65m is used. The dimension of the aeroshell is shown in Figure 3-7.

![Aeroshell configuration](image)

Figure 3-7: Aeroshell configuration [Song, 2008]

The structure of the Hyperion is shown in Figure 3-1. The wing platform is a zagi wing configuration, which is the same as the previous work. The wing is folded in order to be stowed into the aeroshell. The coaxial tiltrotor is embedded in the middle of the airframe.

### 3.8 Flight Control System

#### 3.8.1 Control Surfaces and Control Schemes

Because the VTOL and cruise flight modes are different, the corresponding control surfaces and control schemes used are not the same. This problem will be discussed in detail in Chapter 5. A brief discussion will be given in this section.
The cruise mode of the Martian aerobot is the same as the conventional fixed-wing aircraft on Earth, so the control schemes used are quite similar. As shown in Figure 3-1, only a pair of elevons is introduced in the current Martian aerobot. This is debatable since it will largely degrade the performance of the lateral dynamics. The advantage is that it will simplify mechanism of the actuator system. The current work on this problem is only to validate the possibility of such a strategy. The pitch and roll angles are controlled by deflecting the elevons. The yaw angle is controlled by the roll attitude angle. The cruise speed is adjusted by changing the thrust generated by the coaxial tiltrotor. The altitude is controlled by the pitch angle.

The VTOL mode is the same as the conventional quadrotor, so the control strategy for the quadrotors can be used as a reference. The pitch and roll angles are controlled by changing the thrust generated by the auxiliary rotors. The yaw angle is controlled by differentiating the coaxial tiltrotor. The altitude is controlled by changing the thrust of the coaxial tiltrotor. The horizontal velocity is controlled by tilting the wing planform with pitch and roll angles.

The transition mode is a little complex. We divide the transition mode into two phases, low speed transition and high speed transition. During the low speed transition, the control strategy used is the same as that in VTOL mode. During the high speed transition, the control strategy is similar to that used in cruise mode. Details can be found in Section 5.10.

3.8.2 Servos

The servo motor is an important component of the control system. A comprehensive discussion on selecting appropriate servo for the Martian aerobot is given in previous work [Song, 2008]. A brief introduction on the main achievement is listed in this section.

The proposed electric servo for the Martian aerobot is the Futaba S3155 high speed slim digital servo. The advantage of this servo motor is high torque, low weight, fast response, high precision, and small dimension.

<table>
<thead>
<tr>
<th>Futaba S3155 high speed slim servo [Song, 2008]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
<tr>
<td>volts</td>
</tr>
<tr>
<td>4.8V</td>
</tr>
<tr>
<td>6.0V</td>
</tr>
<tr>
<td>Dimensions</td>
</tr>
<tr>
<td>1.2 x 0.4 x 1/14 in.</td>
</tr>
</tbody>
</table>
3.8.3 Sensors

Sensor is another critical component of the whole control system. The digital electronic sensors can be selected based on the mission requirement. The sensor suite proposed in previous work [Song, 2008] can be used in this research. The sensors and relevant strategies selected are listed as follows:

- Analog Devices Accelerometers ADXL202 for 3-axis accelerations
- MuRata Enc-03J Analog Gyros for 3-axis angular rates
- Dual ported MPXV50045 4kPa dynamic pressure sensor plus MPX4115A Barometric pressure sensor for airspeed and altitude
- The attitude angle is estimated by integrating the angular rates measured by the gyros. The drift is corrected by the Sun sensor
- Since the GPS system is not available on Mars, landmark and terrain contour matching strategy can be used for navigation

All these sensors are based on the MEMS technology, so all these sensors are less than 20g.

3.9 Mass Budget

The mass budget of the Martian aerobot is 25kg in previous work [Song, 2008], as given in Table 3-7. The mass budget is very important for this project. Although we will make some modification in this work, Table 3-7 is also valuable as a reference. A comparison of the mass budget of the propulsion system is given in Section 4.3. The other subsystems are the same as the previous design, as shown in Table 3-7.

Table 3-7: Mass Budget of previous design, Halcyon [Song, 2008]

<table>
<thead>
<tr>
<th>Subsystem</th>
<th>Mass Budget (kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Propulsion System</strong></td>
<td>6.5</td>
</tr>
<tr>
<td>Motors/Gear boxes/Controllers</td>
<td>2.0</td>
</tr>
<tr>
<td>Rotors/Props/Swashplate</td>
<td>4.1</td>
</tr>
<tr>
<td>Wiring/Controller/Mounting</td>
<td>0.4</td>
</tr>
<tr>
<td><strong>Power system</strong></td>
<td></td>
</tr>
<tr>
<td>Batteries</td>
<td>2.1</td>
</tr>
<tr>
<td>Container</td>
<td>0.2</td>
</tr>
<tr>
<td>Solar Array</td>
<td>1.02</td>
</tr>
<tr>
<td>Wiring/Regulator</td>
<td>0.2</td>
</tr>
</tbody>
</table>
3.10 Aerodynamic Properties of Airframe

The airframe of Hyperion is the same as previous Halcyon design in cruise. The aerodynamic property of the wing will decide the cruise speed corresponding to the best L/D. There are other choices of the cruise speed, such as the minimum power consumption. Considering the mission of the Martian aerobot, the cruise speed is required not too low. Therefore, we will still use this cruise speed in this work. The aerodynamic coefficients of the wing are based on the previous work, which are obtained by AVL [Drela, 2011], an open source code for predicting the aerodynamic property of fixed wing aircraft based on the Vortex Lattice Method (VLM). The aerodynamic coefficients of the airframe in cruise are listed in Appendix B-1.

Figure 3-9 gives the L/D of the airframe for different cruise speed conditions with lift just counteracting the weight. The best L/D corresponds to the cruise speed of about 44m·s⁻¹. The cruise speed selected in previous design is 50m·s⁻¹, which is very close to the best value in this analysis. In this work, the cruise speed used in this thesis will follow the previous used value. This cruise speed will result in a large AoA of 5.4° (see Figure 3-10).

Figure 3-11 shows that the required thrust reaches the minimum at the same cruise speed corresponding to the best L/D. The lift of the vehicle remains the same for different cruise speed conditions, since it is equal to the weight. The point of the maximum L/D should also be the point with minimum thrust.
Chapter 3. Overview of Hyperion

Figure 3-9: The L/D of the airframe with different cruise speed (Calculated based on the aerodynamic property in previous study)

Figure 3-10: The AoA of the working point with different cruise speed (Calculated based on the aerodynamic property in previous study)

Figure 3-11: The thrust required to counteract the drag with different cruise speed (Calculated based on the aerodynamic property in previous study)
For incompressible flow, the VLM can accurately predict the lift and induced drag, while the prediction of the profile drag of the airframe is very difficult. Therefore, the drag coefficient predicted by AVL is questionable. In this thesis, a safe estimation value of 10N used, as in the Halcyon design.

3.11 Summary

Before the end of this chapter, a brief summary is presented. This will help to clarify the design requirement in the following chapters.

Hyperion is the third generation Martian aerobot proposed by the Surrey Space Centre to investigate the Isidis Planitia region on Mars. It is a large solar-electric aerobot based on a flying wing design (to maximize lift and minimize drag) with a pair of coaxial tiltrotor to provide a VTOL capability and forward propulsion for level flight. The coaxial tiltrotor is designed not to perform cyclic control to simplify the mechanical structure. The auxiliary rotors at the wing tips are used to control the roll and pitch attitude angles of the Martian aerobot in VTOL and low speed transition/conversion phases, whilst conventional elevons are used for attitude control in high speed transition/conversion phases. The covers of auxiliary rotors will be closed during transition to increase the wing area and to improve the aerodynamic property (see Figure 3-1). The thrusts of the coaxial tiltrotor and auxiliary rotors can be adjusted by changing the collective blade pitch angles. All the rotors are driven by the Electric Power System (EPS). The aerobot would carry a small (approximately 3kg) science payload to investigate the Martian surface and atmosphere. Hyperion will act as a long-range Mars scout, flying over the Isidis Planitia region, flying for an hour or so each Martian day (sol) around local noon when the Sun can best illuminate the solar cell covered wings. Each flight would cover at least 100km (cruise speed of 50m·s⁻¹), and it would take off and land vertically autonomously, and carry out scientific studies both whilst on the surface and when flying. In this way, it will take about 15 flights to across the region.

Table 3-8: General design parameters of Hyperion

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total mass (kg)</td>
<td>25</td>
</tr>
<tr>
<td>Wing span (m)</td>
<td>8.558</td>
</tr>
<tr>
<td>Wing airfoil</td>
<td>zag10</td>
</tr>
<tr>
<td>Wing area (m²)</td>
<td>8.23</td>
</tr>
<tr>
<td>Average wing chord length (m)</td>
<td>1.3046</td>
</tr>
<tr>
<td>Nominal cruise speed (m·s⁻¹)</td>
<td>50</td>
</tr>
<tr>
<td>Coaxial tiltrotor diameter (m)</td>
<td>2.0</td>
</tr>
<tr>
<td>Number of blades for coaxial tiltrotor</td>
<td>2×4</td>
</tr>
<tr>
<td>Coaxial tiltrotor rate in hover (rpm)</td>
<td>1819</td>
</tr>
<tr>
<td>Coaxial tiltrotor rate in cruise (rpm)</td>
<td>1150</td>
</tr>
</tbody>
</table>
Chapter 3. Overview of Hyperion

Some general design parameters of *Hyperion* are shown in Table 3-8. Most parameters are from the previous design [Song, 2008]. The mission profile during each flight is presented in Figure 3-5 and Table 3-1. This thesis will focus on the aerodynamic design of coaxial tiltrotor and robust transition/conversion control, which are the main topics in the following two chapters.
Chapter 4

4 Aerodynamic Design of Coaxial Tiltrotor and Auxiliary Rotor

The focus of this chapter is the aerodynamic design of the propulsion system for Hyperion, including the coaxial tiltrotor and the auxiliary rotors. The design method of coaxial tiltrotor is the second novelty of this thesis shown in Section 1.5.2.

This chapter focuses on the geometric structure design of the coaxial tiltrotor and the auxiliary rotors. The rotor blades are assumed to be rigid bodies. The blade root/tip design and the airfoil selection are left for future study in this thesis. Because the radius of the coaxial rotor is large, it is impractical to build the rotor duct very close to the rotor tip. Moreover, the duct is not deep due to the geometry of the airframe. Therefore, the duct influence of the rotor is not significant. So in this design procedure, the duct effect is not considered. The coaxial rotor and auxiliary rotors regarded as independently operating rotor system.

4.1 Coaxial Tiltrotor Design

The coaxial rotor is not a new concept. It was patented by Henry Bright in the British Office [Coleman, 1997]. The coaxial tiltrotor is actually a pair of coaxial rotor to work under two completely different conditions, hover (with zero inflow rate) and cruise (with high inflow rate). Therefore, the coaxial tiltrotor design is a multi-objective optimization problem. The coaxial tiltrotor is a compromise of the optimum coaxial rotors in each condition. The coaxial tiltrotor of Hyperion is required to generate thrust of about 93N (the total mass of the Martian aerobot is
25kg; the Martian surface gravity is 3.71 m/s\(^2\) for VTOL mode and about 10 N (an estimation of aerodynamic drag in nominal cruise [Song, 2008]) for cruise mode.

### 4.1.1 Coaxial Rotor Aerodynamic Model Selection

There are three main aerodynamic theories for the rotors or propellers, as presented in Section 2.3. Their relationship is shown in Figure 4-1. The theory with higher accuracy needs higher computational cost.

Classical BEMT is the simplest with least computational cost. However, due to the independence assumption between the annuli and the way of handling interference between the upper and lower rotor, its accuracy is questionable. CFD (i.e. Navier-Stokes equation based theory in this thesis) is very accurate, but the computational cost is very high, therefore, the CFD method is not best suited for preliminary design calculations. Since the Reynolds number of the rotor system does not have significant influence on the performance, the design method and model for high Reynolds number can be used without causing large error. The vortex based theory has two models: prescribed wake and free wake. The accuracy of the FWM is known to be of the same order as the CFD method for subsonic rotors or propellers; therefore, FWM method is also sometimes called the grid free CFD method. The FWM can be used for preliminary design, but the computation cost is still high. For the vortex based models, the time cost is mainly on computing the Biot-Savart induced factors. The computation burden for PWM is much lower compared with FWM, because it is not necessary to calculate the induced factor at each of the collocation points in the wake. A lot of experimental and empirical data are required to decide the empirical parameters in PWM. In this thesis, FWM is used to determine those empirical parameters used in PWM, and then the optimization procedure makes use of the computationally efficient PWM. The final result is then checked by the FWM to validate the results. The coaxial tiltrotor configuration is a compromise between hover and cruise optima. In this thesis, only the hover and cruise states are considered in the design process.

The FWM and PWM for a single rotor configuration are also briefly addressed in this thesis, because the optimum single rotor can provide a good initial value for the optimization. Moreover, the computational cost for single rotor is much less than that for coaxial rotors, which will speed up the optimization process.

### 4.1.2 Free Wake Model

In FWM, the flow is assumed to be incompressible. The blades are represented by a lifting-line with rigid near-wake trailer vortices extending to a certain azimuth angle (typically 30°). The far wake vortices are assumed to roll up into tip and root vortices. Usually, only the tip vortex is
modeled, since the influence of the root vortex can be neglected. The far wake tip vortices are represented by straight vortex line segments connected by Lagrangian collocation points, which are allowed to freely distort in the wake under the influence of local velocity field. The governing equation is a first order partial differential equation, which is solved by a finite difference method. The FWM used in this thesis is based on the PIPC method proposed by Bagai and Leishman [Bagai, 1995, Bagai and Leishman, 1995, Bagai and Leishman, 1996]. The blades are modeled by the Weissinger-L lifting surface model. Rotor blades are assumed to be rigid. The strength of the far wake tip vortex is equal to the maximum of the bound circulation. The “vortex strength - moment” approach is used to determine the release point of rolled-up tip vortices. The tip vortex is assumed to be Lamb-Ossen vortex [Saffman et al., 1992] with a finite core.

![Vortex model with finite core](image)

The original reason for this assumption is to prevent the singularity at the evaluation point on the influencing vortex when applying the Biot-Savart law. However, some experiments show that the tip vortex generated does have a viscous core with finite radius [Widnall, 1972, Muller, 1990, Tung et al., 1983]. A series of vortex with visous core models used in aerodynamic modelling are given in [Vatistas et al., 1991]. The tangential induced velocity \( v_t \) of a 2D vortex with core radius of \( r_c \) is given by Equation 4-1.

\[
v_t = \frac{\Gamma r}{2\pi \left(r_c^2 + r^2\right)^{\frac{3}{2}}}
\]

Equation 4-1

Where, \( n \) is an integer, \( \Gamma \) is the vortex strength; \( r \) is the distance between the induced point and the vortex line; \( r_c \) is the radius of rolled-up tip vortex core. It is worth noting that the maximum tangential velocity occurs at \( r = r_c \). With appropriate selection of \( n \), some commonly used vortex model can be obtained.
With $n \to \infty$, we have the Rankine vortex model, which is commonly used in practice. The corresponding profile velocity is given by Equation 4-2.

$$v_i = \begin{cases} \frac{\Gamma r}{2\pi r^2}, & 0 \leq r \leq r_c \\ \frac{\Gamma}{2\pi r}, & r \leq r_c \end{cases}$$

Equation 4-2

With $n = 1$, the model is called Scully vortex. The corresponding profile velocity is given by Equation 4-3.

$$v_i = \frac{\Gamma r}{2\pi \left( \frac{r^2 + r_c^2}{r_c^2} \right)}$$

Equation 4-3

With $n = 2$, the model is called Lamb-Ossen vortex. The corresponding profile velocity is given by Equation 4-4. This model is used in this thesis, because it can describe the real profile velocity of an actual vortex profile [Bagai and Leishman, 1993]. This model is also widely used in the free wake analysis for rotary-wing.

$$v_i = \frac{\Gamma r}{2\pi \sqrt{r^4 + r_c^4}}$$

Equation 4-4

The core radius ($r_c$) is assumed to decay by following the second order Lamb-Ossen rule (Equation 4-5) [Saffman et al., 1992].

$$r_c = 1.12\sqrt{4\nu\delta t}$$

Equation 4-5

Where, $\delta$ is the decay coefficient (with the value from $10^3$ to $10^5$), which is determined by experiments; $\nu$ is the kinematic viscosity of the fluid (Martian atmosphere); $t$ is the age of the vortex segment.

The effect of shed vortices is much smaller than the trail vortices, so this is estimated by the empirical indicial factors [Beddoes, 1984] to reduce the computational cost. These empirical indicial factors are independent of the airfoil and Mach number. The iteration can start from any initial wake condition, and the convergence is reached when the Root Mean Square (RMS) of collocation points ($\overline{\theta_{RMS}}$) in two consecutive iterations is smaller than the predefined tolerance ($\overline{\theta_{tol}}$). Figure 4-3 gives the general procedure of the PIPC algorithm (see Appendices B-3 and B-4 for details). This algorithm can be used for single and coaxial configurations. The coordinates of the collocation points in Equation 4-6 are nondimensionalized by the rotor radius ($R_{op}$).
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**Initialization**

- Calculate the predicted collocation points
- Calculate the corrected collocation points
- Solve Weissinger-L model for prediction step
- Solve Weissinger-L model for correction step
- Calculate induced velocity at collocation points for prediction step
- Calculate induced velocity at collocation points for correction step
- Calculate the predicted collocation points

**Figure 4-3: PIPC algorithm flow chart**

\[
\overline{r}_{\text{RMS}} = \frac{1}{N_r N_s} \sqrt{\sum_{i=1}^{N_r} \sum_{k=1}^{N_s} (\overline{r}_{i,k}^n - \overline{r}_{i,k}^{n-1})^2}
\]

**Equation 4-6**

where, \( \overline{r}_{i,k}^n \) denotes the nondimensional coordinates of collocation points for the \( i \)th azimuth location at the \( k \)th tip vortex segment at the \( n \)th iteration; \( \overline{r}_{\text{RMS}} \) is the RMS of \( \overline{r}_{i,k}^n \); \( N_r \) denotes the number of azimuth positions; \( N_s \) denotes the number of collocation points on one tip vortex.

The model described above can be used for any asymmetric flow conditions. In this thesis, we are more interested in the hover and axial flight conditions, which are axial symmetric. Because of the symmetry of the inflow and the rotor configuration, the computation of FWM can be largely reduced. The periodic property of rotor configurations was firstly used by Miller [Miller and Bliss, 1993]. With the symmetry of inflow, the model can be reduced further. For the single rotor case, the tip vortex collocation points and induced velocity should also be axially symmetric, that is, the calculation of the induced velocity is only needed for a specific azimuth location. The properties of flow and blade can be obtained directly by symmetry. For the coaxial rotor with the same rotational speed, the azimuth configuration for reasonable azimuth discretization is periodic. The flow field and vortex structure in the wake are also periodic theoretically.

Figure 4-4 shows the different configurations for a 2×4-bladed coaxial rotor with the azimuth discretization of 15°. The four configurations are given in accordance with their evolution with time. It is obvious that the relative position of configuration 4 is the same as configuration 1. Therefore, the flow field and blade loading of the two configurations can be transformed by a
simple rotation operation. That is, only 3 configurations are needed for free wake calculation. The flow field and blade loading at other azimuth locations can be obtained directly from the symmetric property.

Figure 4-4: 2x4-bladed coaxial rotor configurations (blue for upper rotor and red for lower rotor)

Torque balance is another very important problem for the coaxial rotor case. Many algorithms are proposed for torque trim. One method based on the bisection algorithm is used in this thesis (shown in Figure 4-5). The FWM and the PWM, shown later, will be used as a routine function to calculate rotor properties. The pitch change ($\delta \theta$) for torque trim should not be set too large,
otherwise the convergence speed is slow. The algorithm is slow in the first iteration, but the speed is very fast in the following iterations. The reason for this is that in subsequent iterations, the upper and lower rotor pitch configurations are known.

In the free wake calculation for the hover case, two or three additional revolutions of the far wake rigid vortex filaments are required to be attached to the end of the free tip vortices. This is to prevent the curling-up of the end part of the free wake due to the absence of the truncated far wake and the superimposed free stream. The radius and screw pitch are decided by the last revolution of free tip vortex.

4.1.3 Validation of Free Wake Model

The main objective of this chapter is to design a pair of coaxial tiltrotor for a Martian aerobot. The Reynolds number for the present coaxial tiltrotor is 30,000. This is a low Reynolds number design problem, so we should use some low Reynolds number measurements to validate this methodology. However, as we described in Section 2.3, we propose to use rough blade sectional airfoil and carefully place the sectional operating AoA before separation, Weissinger-L sectional model can still be used. Therefore, the model proposed in this chapter can be used for the Martian aerobot rotor system. The Reynolds number factor can be considered by replacing the airfoil performance model in the future, but it will require more powerful design tools, such as the grid based CFD solver.

4.1.3.1 Validation of FWM for a Coaxial Helicopter Rotor (Hover)

The wind tunnel measurements of Harrington 1 and 2 coaxial rotors [Harrington, 1951] are used to validate the FWM in hover in this thesis. Harrington 1 and 2 are both 2x2-bladed non-twisted helicopter coaxial rotors. The radius of the Harrington rotors is 3.81m (150 inches); the blade section airfoil is NACA0012; the rotor solidity for Harrington 1 is 0.054 and for Harrington 2 is 0.152; Harrington 1 rotors have a constant taper of 2.5 and Harrington 2 rotors have a constant chord. The distance between the upper and lower rotors is approximately 20% of the radius. In the calculation, the RMS tolerance of the collocation points ($T_{col}$) for the FWM is $10^{-4}$; the torque balanced tolerance ($\tilde{Q}_{bal}$) is $10^{-3}$; the initial pitch change ($\delta \theta$) used for torque trim in Figure 4-5 is 0.1°. We can select other values for these parameters, but there is little difference for different sets of reasonable parameters. The 2D airfoil property of NACA0012 airfoil can be found in the relevant reference [Gessow, 1948]. Three turns of the free tip vortex and two revolutions of the rigid far wake tip vortex are used for each blade.

Figure 4-6 and Figure 4-7 show that the FWM predicted performance of the Harrington 1 and 2 coaxial rotors fits well with the measurements. The free tip vortices of the Harrington 2 coaxial
rotor at the thrust coefficient of $c_T = 0.0058$ for zero azimuth difference between the upper and lower rotors are shown in Figure 4-8. The upper rotor tip vortices contract into the lower rotor. The tip vortices of the upper and lower rotor do not intertwine with each other but form outer and inner layers of vortices respectively. The screw pitch of the upper rotor is smaller than that of the lower rotor, because the upwash and downwash effects caused by the interference between two rotor tip vortices. These properties predicted by FWM follow the experiment results of coaxial rotor system, such as the smoke test given by [Akimov et al., 1994]. The ratio of the fully contracted radius of the upper rotor and the lower rotor is approximately equal to the inverse of the thrust loading ratio (approximately 1.1), which goes well with Nagashima’s work [Nagashima and Nakanishi, 1981]. The contraction ratio of upper rotor wake at the lower rotor is about 0.8, which is in good agreement with the simple momentum theory. The tip vortex in the z direction is approximately piecewise linear. The conversion point is the blade azimuth distance ($\psi_s$).

![Figure 4-6: Validation of FWM predicted performance of Harrington 1 coaxial rotors in hover](image1)

![Figure 4-7: Validation of FWM predicted performance of Harrington 2 coaxial rotors in hover](image2)
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4.1.3.2 Validation of FWM for a Coaxial Propeller (Cruise)

The wind tunnel measurements of a coaxial propeller (Hamilton Standard 3155-6 and 3156-6) conducted by [Biermann and Hartman, 1940] is used to validate the FWM in the axial flight condition. The Hamilton Standard 3155-6 and 3156-6 are two series rather than two propellers. These authors produced a set of data for scaled versions of their propellers. The blade twists of propellers in a particular series are not the same for different blade pitches as measured at \( \theta_{75} \). The experiments were conducted by changing the axial free stream rate for specific pairs of propellers with the same rotational speed. The diameter of the propellers used was approximately 3.048m (10 feet). The blade section airfoil is Clark Y. The distance between the propellers is 0.859m (33.8 inches). Their three-bladed coaxial propeller case (six blades in total) is used for validation in this thesis. In the calculation, the RMS tolerance of the collocation points \( \tau_{Rm} \) for FWM is \( 10^{-4} \); the 2D airfoil properties of Clark Y can be found in relevant reference [Silverstein, 1934]. Three revolutions of the free tip vortex are used in the calculation.

Figure 4-9 and Figure 4-10 show that the FWM generally can give a good estimation of the coaxial propeller performance. It is worth noting that the coaxial propellers used based on the Hamilton Standard 3155-6 and 3156-6 are two series rather than two propellers, see [Biermann and Hartman, 1940] for details. Figure 4-9 and Figure 4-10 show the validation of three coaxial propellers in the Hamilton Standard 3155-6 and 3156-6 series with the blade pitch at 0.75 radius of 20°, 30°, 40°. However, with the increase of the tip speed ratio \( \lambda_{w} \), the FWM tends to over predict the thrust and power coefficient. This is due to the small angle assumption used when solving the near wake Weissinger-L lifting surface model. For high inflow rate, the small angle assumption is questionable. Some modification, such as the method used in Reference [Naiman, 1943], can be used to improve the accuracy, but this will increase the computational cost.
Considering the tip speed ratio for Hyperion is not very high (approximately 0.4), the small angle assumption is used for axial flight in this thesis.

![Figure 4-9: Validation of thrust coefficient predicted by FWM for Hamilton 2x3-bladed coaxial propeller](image)

![Figure 4-10: Validation of power coefficient predicted by FWM for Hamilton 2x3-bladed coaxial propeller](image)

Figure 4-11 gives one case of the FWM predicted tip vortex for the Hamilton coaxial propeller. The contraction of the wake is very small – as expected. This result is very helpful in PWM modeling for coaxial propellers. The roll-up process of the tip vortex follows the “vortex-strength moment” rule. Because the maxima of the bound vortices are at the blade centre rather than near the tip in Harrington cases, the radius of the wake is approximately 0.9 $R_{tip}$. The screw pitch of the helical tip vortices is almost the same, due to the high inflow rate. The thrusts generated by fore- and aft-rotors are almost the same ($T_{f}/T_{a} \approx 1$) since the working conditions of both rotors are similar.
4.1.4 Prescribed Wake Model

4.1.4.1 Prescribed Wake Model for a Coaxial Helicopter Rotor (Hover)

There are many PWMs proposed for the single helicopter rotor configuration. The one most commonly used is proposed by Landgrebe [Landgrebe, 1969]. The algorithm proposed by Landgrebe is based on a time marching FWM, which includes calculating the induced velocity at the collocation points in the wake. The general fitting formulas for the tip vortex used in this thesis follow those used by Landgrebe. Because the influence of the inboard trailing vortices is smaller than the tip vortex, they are modeled to be a series of piecewise straight linear vortices in the blade rotating plane, as is the case for the near wake trailers in FWM. If the rotors are operating at the X-Y plane and the induced rate is downward (i.e. as in hover mode), the fitting formulas [Landgrebe, 1969] for one rotor can be written as Equation 4-7 and Equation 4-8.

\[
\begin{align*}
\bar{z}_T &= \begin{cases} 
  k_1 \zeta, & 0 \leq \zeta \leq \psi_b \\
  \bar{z}_T \mid_{\zeta=\psi_b} + k_2 \left( \zeta - \psi_b \right), & \zeta > \psi_b
\end{cases} \\
\bar{\bar{r}} &= \bar{A}_c + (1 - \bar{A}_c) e^{b \bar{z}}
\end{align*}
\]

Equation 4-7  
Equation 4-8

where, \( \bar{z}_T \) and \( \bar{\bar{r}} \) denote the nondimensional axial and radial position of the tip vortex collocation points in the wake; \( \zeta \) is the wake age of specific collocation points; \( \psi \) is the blade azimuth location; \( A_c \) denotes the contraction ratio; \( k_1, k_2, k_3 \) are the parameters determining the axial and radial position of collocation points. The relationship of \( k_1, k_2, k_3 \) with thrust coefficient (\( c_T \)) is assumed to have the form of Equation 4-9 to Equation 4-11. The general form of Landgrebe prescribed wake can be regarded as the rotor wake derived from momentum theory with a linear twist correction. It is worthwhile to note that a term of the linear twist influence should also be
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included in Landgrebe model. The blade twist for a real rotor does not have to be linear; but the twist effect of other forms is more complex. Fortunately, the blade twist has minor effects on the wake structure and is usually regarded as a correction for the momentum theory, so such a term is neglected in this thesis. An example can be found in the validation part in Section 4.1.5.

\[ k_1 = p_{11} \frac{c_T}{\sigma} + p_{12} \quad \text{Equation 4-9} \]
\[ k_2 = p_{21} \sqrt{c_T} + p_{22} \quad \text{Equation 4-10} \]
\[ k_3 = p_{31} c_T + p_{32} \quad \text{Equation 4-11} \]

For coaxial rotor configuration, Equation 4-9 to Equation 4-11 are solved for both rotors, and the total thrust coefficient is used. Since only the torque balanced case is considered in this thesis, the thrust share ratio \( (c_{T_1} / c_{T_2}) \) is a constant if the rotor distance is fixed; so there is little difference in using the total or separate thrust coefficients. For rotor solidity, we have to use a separate number for each rotor, since, in the general case, they could have different chords.

These parameters are estimated using a linear regression of FWM solutions. The collocation points on the tip vortex of the first revolution are used for linear regression.

Figure 4-12: PWM flow chart for helicopter rotor or propeller performance calculation

The procedure shown in Figure 4-12 can be used for single rotor/propeller or either one for coaxial rotor/propeller configurations. In the initialization process, the thrust coefficient \( (c_T) \), the tip vortex strength \( (\Gamma_T) \), and bound circulation \( (\Gamma_b) \) for the rotor are initialized by prior information or the classical BEMT. The tip vortex structure in the wake can be obtained by Equation 4-9 to Equation 4-11. The induced velocity created by the far wake can be calculated
using the Biot-Savart law for a finite core vortex (see Equation 4-1). The bound circulation distribution is solved by the Weissinger-L model with the influence of the rolled-up tip vortices in the wake. If the RMS error ($\overline{F}_{b(RMS)}$) of the bound vortex for two consecutive iterations is smaller than the predefined tolerance ($\overline{F}_{b(tol)}$), then convergence is reached; if not, the thrust coefficient is updated, and the loop is executed for the next iteration. The bound circulation in Equation 4-12 is nondimensionalized by its maximum value.

$$\overline{F}_{b(RMS)} = \frac{1}{N_{\varphi} M_{b}} \sqrt{\left(\overline{F}_{b(n)}^{(n)} - \overline{F}_{b(n-1)}^{(n-1)}\right)^{2}}$$  

Equation 4-12

Where, $\overline{F}_{b(n)}^{(n)}$ denotes the nondimensional bound circulation for $n^{th}$ azimuth location at $i^{th}$ bound segment at $n^{th}$ iteration; $\overline{F}_{b(RMS)}$ is the RMS of $\overline{F}_{b(n)}^{(n)}$; $N_{\varphi}$ denotes the number of azimuth position; $M_{b}$ denotes the number of bound vortex sections on one blade.

It is worth noting that the bound circulation ($\Gamma_{b}$), tip vortex structure, and tip vortex strength ($\Gamma_{t}$) for a single rotor are the same for different azimuth locations because of the axial symmetry; while it is more complex for the coaxial case. Firstly, the bound circulation distributions for different configurations are not the same. The reason is that the position of the control points on the blades and the azimuth location of tip vortices are different for different configurations. Secondly, the different bound vortices at different azimuth locations will lead to different strength distributions for single tip vortices at different vortex segments, and show a periodic property along the tip vortex. In this case, the tip vortex structure should not be the same for different azimuth locations, but the difference can be neglected since it is very small. Figure 4-13 and Figure 4-14 give an example on this claim. The two figures show the radius and depth of the collocation points in the wake of Harrington 1 coaxial rotor. It is obvious that the radius and depth of the collocation points in the wake are almost the same for different azimuth locations.
4.1.4.2 Prescribed Wake Model for a Coaxial Propeller (cruise)

The wake for moderate loaded single and coaxial propellers is usually modelled by a uniform helical vortex sheet [Wald, 2006]. Since the tip vortex is dominant in the wake and the FWM can also predict the performance of a coaxial propeller, the wake is modelled by a rolled-up tip vortex only in this thesis.

The wake model for a moderately loaded coaxial propeller is much simpler than that for coaxial helicopter rotors. Figure 4-11 shows that the rolled-up helical tip vortices in the wake are uniform. The radius and screw pitch are almost the same with the increase of wake age. The radius of the wake is mainly determined by the roll-up process of tip vortex, which is decided by the bound circulation distribution. The main contribution of the screw pitch for a coaxial propeller is the high inflow rate; while the induced velocity is comparatively smaller. The induced velocity is the
main problem in modelling a coaxial propeller. An estimation of the induced rate based on momentum theory is used in this section.

Assuming the induced velocity is uniformly distributed at the rotor disk surface (which is not true in reality), and further, assuming both rotors have the same thrust, according to the simple Momentum Theory [Leishman, 2006], we have

\[ T = \rho \left(V_\infty + u\right)2u \cdot 2\pi r dr \]  

Equation 4-13

where, \( T \) is the total thrust; \( \rho \) is the density of the incompressible flow; \( V_\infty \) is the free stream rate (i.e. inflow rate); \( u \) is the induced velocity at rotor disks; \( A \) is the rotor disk area. If we nondimensionalize Equation 4-13, we have

\[ c_T = 4(\lambda_\infty + v_i) v_i \]  

Equation 4-14

\[ v_i = \frac{\sqrt{\lambda_\infty^2 + 2c_T - \lambda_\infty}}{2} \]  

Equation 4-15

The axial position (z direction) of wake collocation points is given by Equation 4-16 and Equation 4-17.

\[ \bar{z} = \frac{\xi}{\omega}(\lambda_\infty + v_i) \]  

Equation 4-16

\[ \bar{r} = r_i/R_{ip} \]  

Equation 4-17

The general procedure is the same as the PWM shown in Figure 4-12.

### 4.1.5 Validation of Prescribed Wake Model

#### 4.1.5.1 Validation of PWM for a coaxial helicopter rotor (hover)

The proposed PWM is validated by the measurement of the Harrington coaxial helicopter rotors. Because the PWM in this thesis is used to design the optimum rotor, the blade spanwise load is also very important, not only calculating the total thrust but also for calculating the power consumption. There is little literature on rotor blade load measurements; therefore, the FWM results are used to verify the PWM predicted blade load. The parameters in Equation 4-9 to Equation 4-11 for the Harrington 2 coaxial rotors are solved using linear regression.

Figure 4-15 and Figure 4-16 show the PWM model predicted wake for the Harrington 2 coaxial rotor in the first revolution. It can be seen that the coincidence of wake geometry of upper rotor is better than that for lower rotor in the first revolution, but the PWM predicted wake generally goes well with the FWM predictions. In the PWM calculations, the RMS tolerance of the bound
circulation ($\Gamma_{b(0)}$) is $10^2$; the torque trim tolerance ($Q_{\text{tol}}$) is $10^3$; and the initial pitch change ($\delta\theta$) for torque trim is $0.1^\circ$. In order to quantitively analyze the difference between the wake shape predicted by the empirical equations and the wake shape predicted FWM, Equation 4-18 is proposed to give this answer. After some simple calculation, we have $\bar{r}_\text{RMS} \approx 3\%$ for upper rotor, $\bar{r}_\text{RMS} \approx 7\%$ for lower rotor. It can also be seen from the figures that the RMS of upper rotor is smaller than that for the lower rotor.

$$\bar{r}_\text{RMS} = \frac{1}{N_c N_C} \sqrt{\sum_{i=1}^{N_c} \sum_{k=1}^{N_C} \left( \bar{r}_{i,k}^{\text{PWM}} - \bar{r}_{i,k}^{\text{FWM}} \right)^2}$$  

Equation 4-18

Where, $\bar{r}_{i,k}^{\text{PWM}}$ denotes the positions of the collocation points predicted by the empirical equations for PWM calculation; $\bar{r}_{i,k}^{\text{FWM}}$ denotes the collocation points predicted by FWM simulation; $\bar{r}_\text{diff}$ denotes the prediction difference of the two models. The meanings of the other variables are the same as those in Equation 4-6.

Figure 4-17 and Figure 4-18 show that the PWM predicted performance goes very well with the measurements.

![Figure 4-15: PWM (black line) and FWM (circle) predicted wake radius for the Harrington 2 coaxial rotor](image)
Figure 4-16: PWM (black line) and FWM (circle) predicted wake in z direction for the Harrington 2 coaxial rotor.

Figure 4-17: Validation of PWM predicted performance of the Harrington 1 coaxial rotor in hover.

Figure 4-18: Validation of PWM predicted performance of Harrington 2 coaxial rotor in hover.
Figure 4-19: PWM and FWM predicted upper rotor tip vortices in the wake for the Harrington 2 coaxial rotor in hover at $C_t=0.0068$ (blue continuous line for PWM and red dashed line for FWM)

Figure 4-20: PWM and FWM predicted lower rotor tip vortices in the wake for the Harrington 2 coaxial rotor in hover at $C_t=0.0068$ (blue continuous line for PWM and red dashed line for FWM)

Figure 4-21: PWM and FWM predicted bound circulation and induced rate for the Harrington 2 coaxial rotor in hover at $C_t=0.0068$
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Upper PWM
Upper FWM
Lower PWM
Lower FWM

Figure 4-22: PWM and FWM predicted thrust and torque distribution for the Harrington 2 coaxial rotor in hover at $c_T=0.0068$

Figure 4-19 and Figure 4-20 give a comparison of the tip vortices in the wake by the PWM and FWM for the Harrington 2 coaxial rotor at a thrust coefficient of $c_T=0.0068$. The tip vortices of both rotors predicted by PWM and FWM almost coincide in the first two revolutions. It is known that the vortices closer to the blades have more influence on the performance. The coincidence of first two revolutions shows that the accuracy of the PWM is good enough for preliminary design modelling.

The bound circulation and the induced rate on the blades are shown in Figure 4-21. The spanwise thrust and torque coefficients are shown in Figure 4-22. It can be seen that the bound circulation, induced rate, thrust and torque distribution predicted by PWM go well with those predicted by FWM. The thrust share ratio ($c_{T_u}/c_{T_l}$) of the torque balanced state is 1.13 (1.1 for FWM). The upper rotor wake will cause a downwash effect at the inboard and an upwash effect at the outboard of the lower rotor, so the inflow rate at the inboard part is larger than the outer part; the inboard spanwise thrust of the lower rotor is smaller than that for the upper rotor but higher for the outboard. Due to the influence of the far wake rolled-up tip vortices, the induced rate near the tip is the highest.

Harrington coaxial rotors are both non-twisted, so the influence of twist can not be shown in the above validation cases. The optimum Harrington 1 coaxial rotor for a design point of $c_{T_D}=0.0058$ is used to examine the blade twist influence on the prescribed model. The twist of the Harrington 1 coaxial rotor is optimized by following the procedure given in Figure 4-31.
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Upper rotor
Lower rotor

Figure 4-23: Blade twist of the optimum Harrington 1 coaxial helicopter rotor for \( c_{\text{tw}} = 0.0058 \)

The spanwise pitch angles of both rotors are shown in Figure 4-23. Both rotors have exponential twist from root to tip. The twist range is about 25° for both rotors, the lower rotor having a slightly higher twist than the upper rotor. The PWM and FWM predicted bound circulation and induced rate are given by Figure 4-24. The bound circulation predicted by the PWM is a little higher than that predicted by the FWM; while the induced rate is a little lower. Comparing with Figure 4-21, we see the twist has a small effect. However, the PWM and FWM predicted results generally fit well with each other; therefore, the twist effect can be ignored.

4.1.5.2 Validation of PWM for a coaxial propeller (cruise)

The proposed PWM is validated using the wind tunnel measurements of the Hamilton Standard coaxial propellers conducted by Biermann and Hartman [Biermann and Hartman, 1940], as with the FWM for coaxial propellers.
Figure 4-25 and Figure 4-26 show that the proposed PWM generally can predict the performance of a coaxial propeller. The performance over prediction at high tip speed ratios ($\lambda_{\infty} > 0.5$) is caused by the small angle assumption, which is the same as in the FWM case. For low tip speed ratios ($\lambda_{\infty} < 0.1$), the thrust is under predicted. The reason is that the wake for a low tip speed ratio is more like the helicopter coaxial rotor wake with axial radius contraction.

![Figure 4-25: Validation of thrust coefficient predicted by PWM for the Hamilton 2×3-bladed coaxial propeller](image1)

![Figure 4-26: Validation of power coefficient predicted by PWM for the Hamilton 2×3-bladed coaxial propeller](image2)
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Figure 4-27: PWM and FWM predicted fore-propeller tip vortices of the Hamilton 2×3-bladed coaxial propeller at $\theta_{\tau}=30^\circ$, $\lambda_\alpha=0.3183$ (blue continuous lines for PWM and red dashed lines for FWM)

Figure 4-28: PWM and FWM predicted aft-propeller tip vortices of the Hamilton 2×3-bladed coaxial propeller at $\theta_{\tau}=30^\circ$, $\lambda_\alpha=0.3183$ (blue continuous lines for PWM and red dashed lines for FWM)

Figure 4-29: PWM and FWM predicted bound circulation and induced rate of the Hamilton 2×3-bladed coaxial propeller ($\theta_{\tau}=30^\circ$, $\lambda_\alpha=0.3183$)
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Figure 4-30: PWM and FWM predicted spanwise thrust and torque coefficient distribution of the Hamilton 2x3-bladed coaxial propeller ($\theta_{z5}^c=30^\circ$, $\lambda_c=0.3183$)

Figure 4-27 to Figure 4-30 give the comparison of the proposed PWM and FWM for the Hamilton 2x3-bladed coaxial propeller. The tip vortex systems generated by the PWM and FWM generally agree with each other. The screw pitch of the wake predicted by the PWM is a little smaller than that predicted by the FWM. However, this underestimation will not cause significant error since the free stream inflow rate is dominant. The aft-rotor bound vortex has a slight increase near the tip (see Figure 4-29). This is caused by the upwash effect of the fore-rotor wake boundary. The aft-rotor wake also has an influence on fore-rotor. A slight increase in the fore-rotor bound vortex can also be seen. These effects can also be seen in the spanwise distribution of thrust and torque coefficients in Figure 4-30. The loadings predicted by the PWM generally go well with those of the FWM.

It is worth noting that the coaxial rotor is embedded in the airframe. The existence of the airframe will have influence on the rotor system. This effect can be regarded as the combination effect of "pusher" configuration and "tractor" configuration. It is known [Catalano, 2004] that the mechanism for "pusher" interaction is that the wing in front of the rotor will increase the inflow rate of the rotor system, then it will have influence on the performance of the coaxial rotor; while the mechanism for "tractor" is that the wing will change the structure of the wake of the rotor system, then the performance will be influenced. Both of the heading parts in front and back of the coaxial coaxial rotor are very small (as shown in Figure 3-3). For the "pusher" effect, the lift generated by the front of the heading part is very small, so the speed variation can be neglected comparing with the cruise speed. For the "tractor" effect, the small rear part will not have significant influence on the wake structure. Therefore, this factor is not considered in this design work.

With the above validation work, we can confirm that the FWM and PWM used in this chapter can give a precise prediction on the performance and spanwise loading for a pair of coaxial rotor in
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hover (zero inflow rate) and cruise (high inflow rate) conditions. In this case, we can use such models as the fundamentals in designing the coaxial tiltrotor for the Martian aerobot.

4.1.6 Optimization Model and Optimization Criterion

Since the sectional airfoil is not determined yet, we will use the ideal 2D thin airfoil as an example to show the procedure. The corresponding results can be regarded as the common simple symmetric airfoils. The results for other asymmetric airfoils can be obtained just adding the zero lift AoA to the blade pitch angle each blade section.

Although the definitions of efficiency [Leishman and Ananthan, 2006] for the coaxial rotor in hover mode and the coaxial propeller in cruise mode are not the same, the essence in the definitions is that the coaxial rotor or propeller can generate a specific thrust with the lowest power consumed. The torque should also be balanced (see Equation 4-19 to Equation 4-21).

\[
\text{Min } c_p \quad \text{Equation 4-19}
\]

s.t.

\[
c_r = c_{r_0} \quad \text{Equation 4-20}
\]

\[
c_{Q_u} = c_{Q_l} \quad \text{Equation 4-21}
\]

Where, \( c_p \) denotes the power coefficient; \( c_r \) denotes the total thrust coefficient; \( c_{r_0} \) denotes the design point of the thrust coefficient; \( c_{Q_u} \) and \( c_{Q_l} \) are the torque coefficients of the upper and lower rotors respectively.

This problem can be solved analytically if the objective and constraint functions are differentiable [Coney, 1989]. Because the PWM used in this thesis is solved iteratively, it is not easy to get the derivative information. This optimization problem can be solved using the "greedy algorithm", if the model is discretized, because its form is quite similar to the knapsack problem with divisible items. This problem was first solved by [Dantzig, 1957]. However, using the greedy algorithm directly will need a lot computation, and this is not necessary. The global optima for coaxial rotors and propellers are still open problems until now, but the aerodynamic theory can give some suboptima whose performance is close to the global optima. The properties of these suboptima can largely simplify the optimization problem.

The optimum helicopter rotor is known to have uniform circulation and the same effective AoA for all the blade sections, so the blades of the optimum hover single helicopter rotor should have exponential blade twist and taper [Gessow, 1948]. The optimum coaxial rotor and propeller should have uniform loading along the blade. The exponential blade twist will ensure uniform
circulation and the exponential blade taper will ensure uniform effective AoA for each blade section. For the optimum coaxial helicopter rotor based on classical BEMT [Leishman and Ananthan, 2006], the upper rotor should be the same as the optimum single helicopter rotor because the influence of lower rotor is ignored. The lower rotor should have piecewise exponential distribution of both twist and chord due to the influence of the contracted upper rotor wake.

The circulation for the optimum propeller is not as simple as for the optimum helicopter rotor. The circulation of the optimum propeller is approximately elliptical [Wald, 2006], in which situation the swirl loss is the minimum. The efficiency of a coaxial propeller is known to be better than that of a single propeller because the swirl effect induced by fore-propeller can be recovered by the aft-propeller [Coney, 1989]. The coaxial propeller with uniform blade loading is known to have very good efficiency close to the optimum. Therefore, uniform loading is usually regarded as the optimum criterion for coaxial propellers.

The blade taper is fixed in the optimization process; because the blade section aerodynamic drag induced power consumption is of minor importance. Another reason is that the Weissinger-L model in the PWM is directly related to the rotor solidity. The relative position of the control points will change with different rotor chord. Moreover, the PWM model for a coaxial rotor in hover in this thesis is directly related to the rotor solidity. Therefore, the rotor chord is predefined and kept constant during the optimization in this thesis. The spanwise blade pitch (i.e. twist) at each blade section is selected as the optimization variable. The optimum in this thesis is not the global optimum but is not far away.

Figure 4-31 gives the general optimization procedure used in this thesis. This procedure is used to optimize the coaxial rotor in hover and the coaxial propeller in cruise since the optimization criteria in both cases are uniform bound circulation. This optimization procedure is described for the coaxial helicopter rotor without loss of generality. The optimization can start with a baseline coaxial rotor with any blade twist, but the computational cost can be largely reduced if the initial value is selected appropriately. The bound circulation objectives \( \Gamma_{b(u)}^{\text{obj}} \) and \( \Gamma_{b(l)}^{\text{obj}} \) are the estimation of the optimum coaxial rotor circulations at the torque balanced state. The simple BEMT can give good initial values for the two circulation objectives. A good initial twist for both rotors is the bound circulation of the optimum single rotor with a half thrust coefficient. The initial estimation is obtained from a torque balanced state. \( \Gamma_{\text{flag}(u)} \) and \( \Gamma_{\text{flag}(l)} \) are two sets of flags representing the relationship of the actual bound circulation (\( \Gamma_{b(u)} \) and \( \Gamma_{b(l)} \)) on each blade section and the objective bound circulation. Each set has \( M_b \) elements.
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Figure 4-31: Optimization procedure for the coaxial rotor and propeller

Equation 4-22 gives the definition of $\Gamma_{\text{flag}(u)}$ and $\Gamma_{\text{flag}(l)}$. This equation is applied to all the blade elements on both the upper and lower rotor.

$$\Gamma_{\text{flag}} = \begin{cases} 0, & \left| \Gamma_b - \Gamma_{b,\text{obj}} \right| / \Gamma_{b,\text{obj}} \leq \Gamma_{b,\text{(tol)}} \\ 1, & \left| \Gamma_b - \Gamma_{b,\text{obj}} \right| / \Gamma_{b,\text{obj}} > \Gamma_{b,\text{(tol)}} \text{ and } \Gamma_b < \Gamma_{b,\text{obj}} \\ -1, & \left| \Gamma_b - \Gamma_{b,\text{obj}} \right| / \Gamma_{b,\text{obj}} > \Gamma_{b,\text{(tol)}} \text{ and } \Gamma_b > \Gamma_{b,\text{obj}} \end{cases} \quad \text{Equation 4-22}$$

It is worth noting that the tolerance limit $\Gamma_{b,\text{(tol)}}$ used here is different from the tolerance $\bar{\Gamma}_{b,\text{(tol)}}$ used in the PWM calculation. $\Gamma_{b,\text{(tol)}}$ is used to ensure that the circulation along the blade is not too far from the objective circulation $\Gamma_{b,\text{obj}}$. The pitch at each blade element is adjusted by Equation 4-23.

$$\theta = \theta + \delta\theta \cdot \Gamma_{\text{flag}} \quad \text{Equation 4-23}$$

Where, $\delta\theta$ is the pitch change, a small angle for blade pitch adjustment at each blade section. Equation 4-23 shows that the optimum in this thesis is searched with small steps. This procedure is not necessary in the optimization procedure based on BEMT. In BEMT, the desired spanwise
pitch angle can be calculated directly because the blade sections are assumed to be independent of each other. But in the vortex based theory, any change of a specific blade element will cause all the induced velocities to change over the blade. This property can be seen from the near wake influence matrix of the Weissinger-L model. The near wake influence matrix of the Weissinger-L model is not diagonal. Although the diagonal elements are far larger than the interaction effects, the non-diagonal elements are not small enough to be neglected.

The objective circulation of the lower rotor \( \Gamma_{b(t)}^{\text{obj}} \) is updated in each iteration. This is because the predefined estimation of \( \Gamma_{b(w)}^{\text{obj}} \) and \( \Gamma_{b(t)}^{\text{obj}} \) may not be a torque balanced state for the optimum coaxial rotor. \( \Gamma_{b(w)}^{\text{obj}} \) is increased when uniform circulation is reached. \( \delta \Gamma_b \) should not be too large in order to ensure that the thrust coefficient is not too far from the design point. It should be noted that the selection of the blade pitch change \( \delta \theta \) is related to the bound circulation tolerance \( (\Gamma_{b(w)}^{\text{obj}}) \). The blade pitch should be selected to be small enough such that all the bound circulation can be adjusted to meet the uniform circulation criterion.

One thing that should be taken care of is that the circulation at the blade root and the tip should be zero, according to the boundary condition. That is, the circulation of the blade elements near the root and the tip should gradually tend to zero. These blade elements are called boundary elements in this thesis. The uniform circulation criterion is not applicable to these boundary elements. A simple method is to use extrapolation. However, the simple extrapolation sometimes can make the optimization unstable; especially extrapolation at the tip may result in chattering of the rolled-up tip vortex strength. Moreover, there is no strict rule on how many elements should be selected as the boundary elements. The method used in this thesis is as follows: The boundary elements near the blade tip are considered first. Since the circulation of the boundary elements should be close to zero, the bound circulations of the elements near blade tip are smaller than the bound circulation objectives. The bound circulation of the elements near the blade tip will cause the pitch to increase more than those at the blade centre (i.e. the half-radius), so there is a specific position beyond which the pitch of the element at the outer part is larger than that of the inner part. A rule is imposed on the optimization process such that the pitch of the outer elements should not be larger than those of the inner elements; i.e. \( \theta_i \leq \theta_{r-1}, \forall i = 2, \cdots, M_b \). The number of elements adjusted by this rule is recorded as the number of boundary elements. The pitch of the boundary elements at the blade root is calculated using extrapolation.

### 4.1.7 Optimum Hover Coaxial Rotor

In this section, the coaxial tiltrotor of \textit{Hyperion} is optimized to reach the maximum hover efficiency. The original coaxial tiltrotor is a 2×4-bladed coaxial rotor (8 blades in total) with zero
twist and constant chord. Rotational speeds for both rotors are 1819rpm (190rad·s⁻¹) and the tip Mach number is 0.76 (the speed of sound in the atmosphere at the Martian surface is approximately 250m·s⁻¹). The Reynolds number for the rotor system on Mars is 30,000. As explained at the beginning of this thesis, this operating condition belongs to a low Reynolds number regime. However, as explained in Section 2.3, we can still use the Weissinger-L model in this work, since the rough blade surface is used with carefully placement of the sectional AoA. If the more precise airfoil model is obtained, this part (Weissinger-L model) can be replaced easily since it is independent of the wake model.

Detailed parameters of the coaxial tiltrotor are shown in Table 3-8. The total mass of the vehicle is 25kg. The acceleration due to gravity on the surface of Mars is 3.71m·s⁻². The atmospheric density at the Martian surface is approximately 0.0135kg·m⁻³. Therefore, the design point for hover is $c_{r_0}=0.0606$. The blade section airfoil is not determined.

The coaxial rotor can be optimized by following the procedure shown in Figure 4-31. The bound circulation tolerance ($\Gamma_{\text{bd-rot}}$) in the optimization is $10^{-2}$; the pitch change ($\delta \theta$) is 0.1°; the bound circulation increment ($\delta \Gamma_h$) is 0.1m²·s⁻¹. The blade section airfoil used in the optimization process is the ideal thin airfoil (lift curve slope of 5.73, and drag is neglected). Three blade taper cases (1:1, 2:1, 3:1) for the same rotor solidity are calculated in this thesis. The optimum hover coaxial rotors with different blade tapers have the same power consumption, because power cost of the airfoil drag is neglected. However, the real airfoil drag is not zero and is approximately a second order effect of the AoA. One character of optimum rotors is that the effective AoA along the blade radius equals that which gives the best lift-to-drag ratio. The range of effective AoAs along the blade is helpful in selecting the best among the three taper cases and in choosing the blade section airfoil in future work.

![Figure 4-32](image)

**Figure 4-32:** Spanwise pitch and axial induced rate of the optimum hover coaxial rotor for Hyperion
The spanwise pitch and induced rate of the optimum hover coaxial rotors are shown in Figure 4-32. Both the upper and lower rotors have spanwise exponential twist. The optimum coaxial hover rotor with larger blade taper has lower blade twist. For a specific pair of coaxial rotors, the pitch of the upper rotor is higher than that of the lower rotor at the outboard of the blade ($r > 0.8R_{ip}$); and generally smaller at the inboard ($r > 0.8R_{ip}$). This is caused by the induced rate difference for the two areas.

The AoA and blade bound circulation of the optimum coaxial rotors is shown in Figure 4-33. Although the AoA distributions are different, the spanwise bound circulations are the same. The “uniform” bound circulation ensures the optimum. With the increase of blade taper, the range of spanwise AoA reduces. The AoA ranges of 2:1 and 3:1 cases are narrow and similar. The AoA range should include the highest lift-to-drag ratio AoA for the blade airfoil, which will ensure the torque caused by drag to be a minimum.

According to BEMT [Leishman and Ananthan, 2006], the thrust coefficients of both optimum hover rotors are linearly distributed; while the torque coefficient of the upper rotor is linearly distributed and is piecewise linearly distributed for the lower rotor. The piecewise linear distribution of the lower rotor is caused by the upper rotor wake influence. The result of the Weissinger-L model for the optimum coaxial rotor only (neglecting the far wake influence) will result in a linear distribution of the thrust and torque coefficients. When considering the influence of the far wake, there should be some change.

Figure 4-34 gives the thrust and torque distribution of the optimum hover coaxial rotors. Due to the influence of the upper, and especially the lower rotor far wake, the upper rotor torque coefficient at the inner part is no longer linearly distributed. The spanwise torque coefficient at the inner part is higher than for the outer part. This also affects the thrust distribution, but it is not significant. The spanwise torque coefficients for upper and lower rotor in Figure 4-34 still show

![Figure 4-33: Spanwise AoA and bound circulation of the optimum hover coaxial rotor for Hyperion](image-url)
some linearity. The comparison of the optimum cruise coaxial propeller for 2:1 taper case and the original design is shown in Figure 4-40.

![Figure 4-40: Spanwise thrust and torque load of the optimum hover coaxial rotor for Hyperion](image)

### 4.1.8 Optimum Cruise Coaxial Rotor

The coaxial tiltrotor is optimized to improve the efficiency for cruise (high inflow rate axial flight) by the procedure given in Figure 4-31. The axial inflow rate is equal to the cruise speed of 50 m/s\(^1\). The thrust required is equal to the aerodynamic drag of vehicle (10N). The rotational speed is reduced to 1150 rpm (120 rad/s\(^1\)). The design point of the total thrust coefficient required for cruise is \(c_T\) = 0.0164. The optimization is also conducted for three taper cases (1:1, 2:1, 3:1), as in the hover cases.

![Figure 4-35: Spanwise pitch and induced rate of the optimum cruise coaxial propeller for Hyperion](image)
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The spanwise pitch and induced rate of the optimum coaxial propellers are shown in Figure 4-35. Because of the high inflow rate, the interference between the propeller wakes is much less. The twists of fore- and aft-propellers for one pair of coaxial propellers are quite similar. The working conditions for both rotors are similar. Because the thrust loading in cruise is much lower than that in hover, the effective AoA is much lower than the inflow rate angle. The difference of blade twists for the three taper cases are very slight. The aerodynamic inflow angles are similar while the AoA is different for the different taper cases. The aerodynamic inflow angle for the blade section is much larger than the AoA, so the blade twist difference is not significant. The induced rates for both fore- and aft-propellers are uniform at the central part (i.e. centre radius). The induced rates are much smaller than the inflow rate.

Figure 4-36: Spanwise AoA and bound circulation of the optimum cruise coaxial propeller for Hyperion

The spanwise AoA and bound circulation of the optimum cruise coaxial propeller are given by Figure 4-36. The spanwise AoA ranges are much smaller than those of the hover cases (Figure 4-33). Comparatively, the effective AoA of the blade taper 2:1 is almost uniformly distributed.
The bound circulations of the three taper cases are the same, as expected. Bound circulations for one pair of fore- and aft-propellers are similar.

Figure 4-37 shows the spanwise thrust and torque coefficients of the optimum cruise coaxial propeller. The thrust and torque coefficients are linearly distributed along the radius, which is a standard result of the Weissinger-L model. Due to the high inflow rate, the contracted wake is blown far away very quickly. Wake influences are much smaller, so the linear distribution property is more significant. The linear spanwise thrust and torque distribution also confirm the uniform bound circulation. The comparison of the optimum cruise coaxial propeller for the 2:1 taper case and the original design is shown in Figure 4-41.

4.1.9 Proposed Coaxial Tiltrotor

For one pair of coaxial tiltrotors, the upper and lower rotors in hover will become the fore- and aft-propellers after tilting. For simplicity, the term “upper rotor” is used to indicate the upper rotor in hover and fore-propeller in cruise; and the “lower rotor” is used in the similar manner. The coaxial tiltrotor is a pair of coaxial rotors working at both hover (zero inflow rate) and cruise (high inflow rate) states; therefore, the resulting coaxial tiltrotor is a compromise of optima in hover and cruise. The definition of resulting coaxial tiltrotor depends mainly on the compromise method, so there will be infinite number of optima. The term “optimum coaxial tiltrotor” used in this thesis is one of these optima. Figure 4-33 and Figure 4-36 show that the effective AoA distributions of taper 2:1 are “uniform”. Therefore, the taper of the coaxial tiltrotor is selected to be 2:1 (3:1 would also have been suitable from a performance perspective, but would involve a larger chord at the root).

The compromise method used in this thesis is used in [Leishman and Ananthan, 2006], a weighted average of the blade twist since the optimum coaxial rotors in the hover and cruise modes in Sections 4.1.7 and 4.1.8 have different blade twists. The blade twist of the final coaxial tiltrotor at each section is the average of the corresponding blade twist on the optima for both hovering and cruising cases, see Equation 4-24. $\theta_{alt}$, $\theta_{hor}$, and $\theta_{cr}$ are the blade pitch angles for corresponding section for the coaxial tiltrotor, optimum hover coaxial rotor, and optimum cruise coaxial rotor; $\varepsilon$ is the weight for compromise ($\varepsilon=0.5$ in this work).

$$\theta_{alt} = \varepsilon \theta_{hor} + (1-\varepsilon) \theta_{cr}$$  \hspace{1cm} \text{Equation 4-24}

The proposed coaxial tiltrotor for the hover and cruise cases are shown in Figure 4-38 and Figure 4-39. The twists of both rotors are a weighted average of the optimum hover and cruise coaxial rotors. The blade collective pitch is adjusted to reach the thrust design points in hover and cruise modes. The weighted value of the twist used can be selected based on the power consumption in
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hover and cruise modes and the time that the aircraft will spend in these respective modes. The
final structure of the coaxial tiltrotor is directly related to the weights. In this thesis, hover and
cruise are considered to be equally important. In practice this will depend on the mission
operational scenario, which will be a big topic of mission design, but in this thesis we are just
establishing the methodology. The resulting coaxial tiltrotor for Hyperion is then a straight
average of the optima for the hover and cruise modes. The twists of the resulting “optimized”
coaxial tiltrotor are higher than for the optimum cruise case and are lower than for the optimum
hover case.

![Figure 4-38: Pitch distribution of the Hyperion resulting coaxial tiltrotor in hover](image1)

![Figure 4-39: Pitch distribution of the Hyperion resulting coaxial tiltrotor in cruise](image2)
The performance of the resulting coaxial tiltrotor in hover and cruise are shown in Figure 4-40 and Figure 4-41 respectively. The performance of the resulting coaxial tiltrotor in hover and cruise shows a slight decrease compared with the corresponding optima in hover and cruise. The efficiency of the resulting coaxial tiltrotor is improved significantly compared with the baseline design. As shown in Figure 4-40, the power consumed for hover flight by the final coaxial tiltrotor is approximately 7% less than that by the baseline coaxial rotor (thrust coefficient design...
point of 0.0606, see Figure 4-40); the power consumed for cruise flight is approximately 14.6% less (thrust coefficient design point of 0.0166, see Figure 4-41). A comparison is given by Table 4-1.

4.2 Auxiliary Rotor Design

The design of the auxiliary rotors is much easier than that for coaxial tiltrotor, since the auxiliary rotors are single rotors and the interactions between the coaxial tiltrotor and auxiliary rotors can be ignored. The auxiliary rotors are symmetric to each other because of the lateral symmetry of Hyperion.

4.2.1 Design Requirements for the Auxiliary Rotor

The design requirements for the auxiliary rotors are presented as follows:

- Control Requirement: The auxiliary rotors are designed to control the pitch and roll angles during VTOL (hover) and low speed transition/conversion phases, so the auxiliary rotors must be able to adjust the thrust continuously within a thrust interval.

- Power Limit: The thrust generated by the auxiliary rotors should be large enough, while the power consumed should not be too large. This limit used in this work is 100W.

- Operating Condition: The auxiliary rotors are vented inside of the ducts at wing tips (see Figure 3-1), and they are working during VTOL (hover) and transition/conversion phases only. The inflow velocities of the two rotors are ±3m·s⁻¹ during VTOL (hover), about 0m·s⁻¹ for the low speed transition and conversion, so the velocity of inflow for auxiliary rotor design used in this thesis is zero.

- Geometry Limit: The auxiliary rotors are vented at the each tip of wing, so the maximum diameter of auxiliary rotor is 700mm. The diameter of the hub used in this work is 50mm.

- Aerodynamic Limit: The maximum rotational speed of the auxiliary rotors is 4775rpm (500rad·s⁻¹). The blade tip speed Mach number is 0.7.

4.2.2 Thrust Control Strategy

The thrust changing mechanism is considered first, as described in Section 4.2.1. The commonly used thrust control strategies are blade pitch control and rotational speed control. Both strategies can be used for the Hyperion auxiliary rotors. A comparison is given by Table 4-2. The rotational speed strategy can generate thrust for one direction only (either upward or downward), while it is possible to generate thrust for both sides using the blade pitch change strategy. The thrust
generated by a single rotor is given by Equation 4-25. The thrust generated by a rotor is approximately proportional to the second order of rotational speed and the blade pitch angle. This conclusion is an approximation result from the BEMT (see Appendix B-2).

\[ T = \rho A(\omega R)^2 c_r \]

Equation 4-25

<table>
<thead>
<tr>
<th>Thrust control strategy</th>
<th>Rotational speed</th>
<th>Blade pitch</th>
</tr>
</thead>
<tbody>
<tr>
<td>Structure</td>
<td>Simple</td>
<td>Complex</td>
</tr>
<tr>
<td>Relationship with thrust</td>
<td>Approximately square</td>
<td>Approximately linear</td>
</tr>
<tr>
<td>Response</td>
<td>Low</td>
<td>High</td>
</tr>
<tr>
<td>Gyroscopic effect</td>
<td>Yes</td>
<td>No</td>
</tr>
</tbody>
</table>

Table 4-2: A comparison of the thrust control strategies

Where, \( T \) denotes the thrust; \( \rho \) denotes the atmosphere density; \( A \) denotes the rotor disk area; \( \omega \) denotes the rotational speed; \( R \) denotes the radius of rotor disk; \( c_r \) denotes the thrust coefficient of rotor. Usually, the thrust coefficient is approximately a linear function of the rotor collective pitch.

According to the simulation by the ANSYS CFX® [ANSYS, 2010], the drag during vertical take-off and landing with the speed of 3m\( \cdot \)s\(^{-1} \) is \( D=1.22\text{N} \). The distance between the AC and the nose is \( d_p=2.285\text{m} \). Assume the thrust (\( T \)) of the coaxial tiltrotor in VTOL can balance the sum of gravity (\( G=92.75\text{N} \)) and the aerodynamic drag. The distance of the coaxial tiltrotor from the nose is \( d_r=1.5\text{m} \). The longitudinal distance between the nose and the auxiliary rotors is \( d_{aux}=3.05\text{m} \). The thrust of the coaxial tiltrotor in VTOL mode is a passive force, since it is used to control the altitude for force balance. In this section, the thrust of the coaxial rotor is \( T_{ coax } = G + D = 93.97\text{N} \) for take-off flight and \( T_{ coax } = G - D = 91.53\text{N} \) for landing flight.

Equation 4-25 holds only when the rotational speed is neither too high nor too low, so the rotational speed must be within a specific interval. Therefore, the thrusts of rotor using the rotational speed strategy cannot be in both directions (upward and downward). While the blade pitch control strategy can ensure the thrust in both directions. Differential of the auxiliary rotor thrust will cause rolling control moment. Suppose the range of thrust difference is from -0.5N to 0.5N, then the caused rolling moment interval will be from -3.5N\( \cdot \)m to 3.5N\( \cdot \)m. But the pitching limit is more critical because the arm in pitch control is much shorter than that in rolling control. The following discussion on the auxiliary rotor thrust control strategy will focus on the pitch angle control requirement. In this thesis, a 6N\( \cdot \)m pitch moment is assumed as an example to demonstrate the design concept. This value can be changed, but it has no influence on the conclusion.
4.2.2.1 Upward Auxiliary Rotor Thrust Control Strategy

As shown in Figure 4-42, when the thrusts of auxiliary rotors are upward, the CG (\(d_e\), yellow point in the figure) have to be allocated between the coaxial rotor thrust (\(T\), blue in the figure) and Drag (\(D\), red in the figure), as in Equation 4-26. Thrusts of two auxiliary rotors are at the tips of wing (\(T_{aux}\), black in the figure).

\[
d_r < d_e < d_D
\]

Equation 4-26

Equation 4-27 and Equation 4-28 show the relationship of the pitch moments for take-off and landing respectively. \(T_{aux1}\) and \(T_{aux2}\) denote the total thrusts of the auxiliary rotors during taking off and landing phases.

\[
T(d_e - d_r) + D(d_D - d_e) = T_{aux1}(d_{aux} - d_e)
\]

Equation 4-27

\[
T(d_e - d_r) - D(d_D - d_e) = T_{aux2}(d_{aux} - d_e)
\]

Equation 4-28

Then we have

\[
T_{min} \leq T_{aux2} \leq T_{aux1}
\]

Equation 4-29

\(T_{min}\) is the lower limit of the midpoint of the total thrust interval of auxiliary rotors for landing control. \(T_{min}\) must be positive. Since the propulsion system must be able to generate a pitch moment of 6N-m for both sides according to the design requirement, the pitch moment interval is [0, 12] N-m. Considering the rolling control requirement, a minimum of 1N difference should be maintained. Therefore, \(T_{min}\) is at least 4.5N. According to Equation 4-29, the total thrust of the auxiliary rotors is expected to be as small as possible, so the minimum value of \(T_{aux1}\) should be found.

\[
\frac{d(T_{aux1})}{d(d_e)} = \frac{T(d_{aux} - d_r) - D(d_{aux} - d_D)}{(d_{aux} - d_e)^3} > 0
\]

Equation 4-30
Equation 4-30 shows that $T_{aux}$ is a monotonically increasing function of $d_c$. The minimum value of $d_c$ corresponds to the minimum value of $T_{aux}$. From Equation 4-29 we have

$$d_c \geq \frac{T_d + D_d + T_{min}d_{aux}}{T + D + T_{min}}$$

Equation 4-31

The right half of Equation 4-31 is mono increase function of $d_c$, so the minimum value of $d_c$ is 1.581m. Then the midpoint of the total thrust interval of auxiliary rotors in take-off phase is 5.70N, based on Equation 4-27. Considering the pitching control and rolling control, the maximum thrust of upward thrust case should be at least 10.2N. In such case, the thrust interval for each auxiliary rotor is from 0N to 5.1N.

**4.2.2.2 Downward Auxiliary Rotor Thrust Control Strategy**

The force diagram for downward thrust control strategy is shown in Figure 4-43. The analysis is similar to that in the upward thrust control case. The CG, AC and the position of coaxial tiltrotor should follow Equation 4-32.

$$0 \leq d_c \leq d_f < d_D$$

Equation 4-32

The relationship pitch moments for take off and landing phases are given by Equation 4-33 and Equation 4-34, respectively.

$$T(d_f - d_c) - D(d_D - d_c) = T_{aux1}(d_{aux} - d_c)$$

Equation 4-33

$$T(d_f - d_c) + D(d_D - d_c) = T_{aux2}(d_{aux} - d_c)$$

Equation 4-34

Then the relationship in Equation 4-35 is obvious.

$$T_{min} \leq T_{aux1} \leq T_{aux2}$$

Equation 4-35

The minimum value of $T_{aux2}$ should be found as the design point of the auxiliary rotors. The derivative of $T_{aux2}$ is given by Equation 4-36, which shows that $T_{aux2}$ is a mono decreasing function of $d_c$. The minimum value of $T_{aux2}$ is obtained when $d_c$ reaches its maximum.
From Equation 4-35 we have

\[
\frac{d(T_{\text{aux}})}{d(d_c)} = \frac{T(d_i - d_{\text{aux}}) - D(d_D - d_{\text{aux}})}{(d_{\text{aux}} - d_i)^2} < 0
\]

Equation 4-36

The maximum value of \(d_c\) is 1.409m. The corresponding total thrust \(T_{\text{aux}}\) is 5.80N. Therefore, the thrust interval of this downward case is from 0N to 5.15N.

### 4.2.2.3 Blade Pitch Thrust Control Strategy

The blade pitch thrust control strategy is inspired from the collective pitch control of the helicopter rotors. The rotational speed is kept while the blade pitch angle will be adjusted by the actuators. Therefore, the mechanism of this strategy is more complex. But it can generate thrust for both sides and the thrust is approximately proportional to the blade pitch. This is a good property for control problem.

Figure 4-44 shows the force diagram for the blade pitch control strategy. The analysis of this strategy is much easier than the previous two cases. The CG coincides with the position of coaxial tiltrotor, since the take-off and landing are symmetric working conditions in this case, i.e. \(d_c = d_i\).

The auxiliary rotors will also counteract the pitch moment induced by the aerodynamic drag \(D\), so the pitch moment equations during taking off and landing are given by Equation 4-38 and Equation 4-39.

\[
D(d_D - d_c) = T_{\text{aux1}}(d_{\text{aux}} - d_c)
\]

Equation 4-38

\[
D(d_D - d_c) = T_{\text{aux2}}(d_{\text{aux}} - d_c)
\]

Equation 4-39

Then,

\[
T_{\text{aux1}} = T_{\text{aux2}} = \frac{D(d_D - d_c)}{d_{\text{aux}} - d_c}
\]

Equation 4-40
So, total auxiliary thrust required to counteract drag ($D=1.22\,\text{N}$) in take-off and landing phases are $0.6\,\text{N}$. This is a direct result from Equation 4-40. Considering the pitching and rolling control requirements, the maximum thrust required is $5.1\,\text{N}$, i.e. about $2.55\,\text{N}$ for each rotor in for both directions.

### 4.2.3 Proposed Auxiliary Rotors

The above discussion shows that all the three thrust control strategies can be used for Hyperion auxiliary rotor design. But the power consumption is not the same for the three strategies.

A comparison of the three thrust control strategies for auxiliary rotors is given by Table 4-3. The power consumed for the maximum thrust required is estimated by the simple Momentum Theory (Equation 4-41), which will usually underestimate the power consumption. However, the simple Momentum Theory can still give a reasonable estimation.

<table>
<thead>
<tr>
<th>Table 4-3: Comparison of three thrust control strategies</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum thrust (N)</td>
</tr>
<tr>
<td>---------------------</td>
</tr>
<tr>
<td>Upward thrust</td>
</tr>
<tr>
<td>Downward thrust</td>
</tr>
<tr>
<td>Blade pitch</td>
</tr>
</tbody>
</table>

$$P = \sqrt{\frac{T^3}{2\rho A}} \quad \text{Equation 4-41}$$

Where, $P$ denotes the power consumed; $T$ denotes the thrust; $\rho$ denotes the atmosphere density; $A$ denotes the rotor disk area. The simple Momentum Theory can only give a lower limit of the real rotors, but it can be seen that the power consumed by the upward and downward thrust control strategies is about 2.75 times of that by the blade pitch strategy. Therefore, the blade pitch thrust control strategy is proposed to be used for Hyperion.

Taking off and landing are two symmetric working conditions for the blade pitch thrust control strategy. The thrust required for both sides are the same; therefore, the blade for auxiliary rotors should be non-twisted. The maximum thrust coefficient of the auxiliary rotors is 0.016. We propose to use the 3-bladed non-twisted rotor as the design candidates.

Four constant blade taper cases ($1:1$, $2:1$, $2.5:1$, $3:1$) are considered in this section. The performance estimated by the BEMT is shown in Figure 4-45. The auxiliary rotors with larger taper are more efficient, but their performance generally is almost the same. Any of these design can be used for Hyperion, the taper case of $2.5:1$ is selected in this thesis to provide data for the control part in Chapter 5.
Chapter 4. Aerodynamic Design of Coaxial Tiltrotors

4.3 Summary

This chapter focuses on the preliminary aerodynamic design of the coaxial tiltrotor and the auxiliary rotors.

The power consumption for the coaxial tiltrotor is very high for the Martian aerobot, so the efficiency improvement is the design objective for the coaxial tiltrotor. As the coaxial tiltrotor will be operating under two conditions, hover (zero axial velocity) and cruise (high axial velocity), this is a multi-objective optimization problem. The medium-fidelity-low-cost models (PWM and FWM) and the uniform blade loading criterion are used as the fundamentals for the coaxial tiltrotor design problem. The final coaxial tiltrotor is obtained by a linear combination of the geometric pitch of the optima for both hover and cruise.

The power cost of the auxiliary rotor is much lower; moreover, the auxiliary rotors will be used in VTOL and low speed transition/conversion phases. Therefore, the design of the auxiliary rotors is analyzed from the control perspective. Three control strategies (upward thrust, downward thrust, and blade pitch control) are considered in this work. The final control strategy used for the auxiliary rotors is the blade pitch control strategy due to its fast response, lower power cost, and non-gyroscopic effect.

The aerodynamic design work for coaxial tiltrotor is only at the preliminary stage. More problems should be considered for a real-life coaxial tiltrotor; such as the blade section airfoil, rotor blade aeroelasticity, etc. It is worth noting that the forward flight performance of the coaxial tiltrotor should be considered before a real-life coaxial tiltrotor is built. Although it is only a transient state during the flight, it will have significant influence on the performance of the coaxial tiltrotor.

In this section, a brief discussion on the mass budget of the propulsion system is presented. A comparison of the mass budget of the Hyperion and the Halcyon is given by Table 4-4. Although

![Figure 4-45: Performance of the proposed candidate auxiliary rotors](image-url)
the total mass budget for the present *Hyperion* design is 1.1kg more than that for the *Halcyon* design, the present coaxial rotor design is more practical. Since the mass budget proposed in Table 3-7 has a margin of 3.5kg, the coaxial tiltrotor design is feasible. Therefore, the present design has a margin of 2.4kg. This is because in the *Halcyon* design, the performance of the coaxial rotor is estimated by the simple Momentum Theory for single rotor configuration. This estimation is usually much higher than the real-life situation (even much higher than the simple Momentum Theory for coaxial rotor system). The coaxial rotor system of *Halcyon* cannot generate such a high thrust under the Martian atmosphere. The detailed mass budget follows the estimation method used in *Halcyon* design [Song, 2008].

Table 4-4: Comparison of the mass budget of the propulsion system

<table>
<thead>
<tr>
<th></th>
<th>Hyperion</th>
<th>Halcyon [Song, 2008]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coaxial rotor</td>
<td>4.2kg</td>
<td>2.1kg</td>
</tr>
<tr>
<td>Propellers/Auxiliary rotors</td>
<td>1.0kg</td>
<td>1.0kg</td>
</tr>
<tr>
<td>Swashplate for coaxial rotor</td>
<td>0.0kg</td>
<td>1.0kg</td>
</tr>
<tr>
<td>Motors for coaxial rotor</td>
<td>1.6kg</td>
<td>1.6kg</td>
</tr>
<tr>
<td>Motors for propellers/auxiliary rotors</td>
<td>0.2kg</td>
<td>0.4kg</td>
</tr>
<tr>
<td>Wiring, controller and Mounting</td>
<td>0.4kg</td>
<td>0.4kg</td>
</tr>
<tr>
<td>Tilting mechanism</td>
<td>0.2kg</td>
<td>0.0kg</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>7.6kg</strong></td>
<td><strong>6.5kg</strong></td>
</tr>
</tbody>
</table>

- Coaxial Rotor: The mass budget of the 2-bladed coaxial rotors for previous *Halcyon* design is 2.1kg, so a mass estimation of the current 4-bladed coaxial rotor is approximately 4.2kg.
- Propeller/Auxiliary Rotor: The mass budget of the 2-bladed propeller (diameter of 1.0m) for the *Halcyon* design is 0.5kg each. Each auxiliary rotor in the *Hyperion* has 3 blades. The diameter is about 0.7m, so the mass budget estimation of the auxiliary rotor is 1.0kg.
- Swashplate for Coaxial Rotor: Since the *Hyperion* design will not perform the cyclic pitch operation, so the swashplate is not required.
- Motors for Coaxial Rotor: Because the maximum power cost of coaxial rotors for both *Halcyon* and *Hyperion* design are the same, so the same pair of electrical motors can be used.
- Motors for Propellers/Auxiliary Rotor: Figure 4-45 shows the maximum power consumption of the proposed auxiliary rotor is approximately 80W each. Because the weight of the electrical motor is approximately linear proportional to the power requirement, an estimation of 0.2kg for the motors is acceptable.
- Wiring, Controller, and Mounting: The wiring, controllers, and mounting system will follow the previous design and has a mass budget of 0.4kg.
• Tilting Mechanism: The tilting mechanism is implemented by two high torque servos. Most of the Futaba® high torque servos are less than 100g, so an estimation of 200g for the tilting mechanism is reasonable.
Chapter 5

5 Robust Flight Controller Design

In the previous work [Song, 2008], only the controller for cruise mode is proposed. This controller is based on the PID method. The control in cruise mode is quite similar to that for the conventional fixed wing aircraft. A lot of applications show that PID strategy can meet the stability requirement in cruise mode. However, the control for VTOL and transition/conversion modes is not considered in previous work. These two problems will be the two main topics in this chapter. Figure 5-1 shows the structure of the control system of a UAV system, which is also the control system used for the Martian aerobot. The work in this chapter focuses on the low level topics in the control system. That is the stabilization problem.

![Figure 5-1: Hierarchy of the control system of a typical UAV system [Kingston et al., 2005]](image)

In this chapter, the robust flight control strategy for Hyperion in VTOL, cruise and transition/conversion mode are presented in detail. The control method used for transition/conversion is the third novelty in this thesis. The transition/conversion is a flight mode that connecting the VTOL mode and cruise mode. A controller for cruise mode is also proposed in this chapter. However, this cruise controller is only used to stabilize the Martian aerobot after transition mode. This cruise controller is not proposed to be used for cruise flight at noon on each Martian day. The PID controller for cruise is proposed to be used during this cruise flight.

This chapter focuses on the lower level control problems. The controller will ensure the stability of the Martian aerobot when tracking specific trajectory. Higher level control problems, such as the trajectory planning, are beyond the scope of this work. The main topic in this chapter is the transition/conversion controller design. The controllers for VTOL (hover) and cruise are also briefly discussed.
5.1 Rigid Body Dynamic Model

The following assumptions are made to derive the dynamic model for the Martian aerobot.

- Assume the airframe and the rotor blades are rigid
- The CG of the airframe coincides with the centre of coaxial tiltrotor
- Assume the moment of inertia tensor of coaxial tiltrotor in body frame is diagonal
- Assume the gyroscopic effect of the auxiliary rotors are cancelled
- Ignore the rotation and the curvature of the planet (Mars)
- Ignore the gravity change with the altitude
- Assume the power system can meet the requirement of propulsion system
- Ignore the aerodynamic interference among the airframe, the coaxial tiltrotor, and the auxiliary rotors
- Ignore the aerodynamic damping when tilting the coaxial tiltrotor
- Ignore the wind gust and shear
- Ignore the ground effect for this preliminary study
- Ignore the atmospheric density variation with location
- Ignore the dusty effect on the aerobot

It is worth noting that the wind includes two kinds, the steady wind and the gust. The steady wind is predictable by the Martian atmosphere model; while the wind gust is unpredictable. The steady wind problem is considered in the trajectory planning level in the control system, which is beyond the content of this work. Too large wind gust is fatal disaster for all the aeroplanes. Therefore, only the small wind gust disturbance (10% of cruise speed) is considered in the simulation.

The ground effect also has influence on the performance of the Martian aerobot in VTOL model, i.e. the influence will be posed mainly on the rotors. The ground effect of the helicopter rotor is still not fully understood even now. A commonly used method is to introduce the image rotors. The operating conditions and the geometry of real rotors and the image rotors are symmetric about ground. This method is the similar to that for aerodynamic interference between the rotor and airframe. This problem is beyond the scope of this work, so it is left for future work. It is worth noting that the ground effect will increase the thrust and power of the rotor system. The ground effect can be neglected when the distance between the ground and the rotor disk is larger than \( 3R_{\text{ip}} \) [Leishman, 2006]. Therefore, neglecting this factor in this preliminary design
procedure is reasonable. But it is a good topic for future work. Figure 5-2 gives an example of the rotor thrust variation with the distance from the ground.

![Figure 5-2: Rotor thrust variation with distance from the ground [Leishman, 2006]](image)

It is known that the atmosphere density on Mars is different for different locations, just as that on Earth. However, the Isidis Planitia region is a flat land. The season for this mission is the local summer, in which the wind is steady. Therefore, the density variation on Mars is very small. This factor could be considered in the future if possible, but it can be neglected for the current preliminary stage.

It is known that the Martian surface is dusty. When the aerobot is operating at low altitude, it is possible to cause the dust flying into the rotor system. This will not only cause damage to the structure of the aerobot, it also has influence on the performance of the Martian aerobot. However, this effect will happen only when the flow is quite close to the surface. Considering this effect will require more powerful tools (such as the CFD software, wind tunnel), which are not available for the present study. This is a short period, so this will not be considered in this work.

The reference frames are shown in Figure 5-3. Their conversion follows the order of “Z-Y-X”. The Euler angles from the planetary frame to the body frame are yaw (ψ), pitch (θ), and roll (φ) angles; and the Euler angles from the wind frame to the body frame are Angle of Sideslip (AoS, β), and Angle of Attack (AoA, α). These definitions and the conversion matrices can be found in most literature on aircraft dynamics [Brockhaus, 2001]. A short introduction of the reference frames used in this thesis is shown below.
Chapter 5. Robust Flight Controller Design

Body frame \( (O_B - X_BY_BZ_B) \) is fixed to the airframe. The origin \( O_B \) coincides with the CG; the axis \( X_B \) points towards the nose; the axis \( Z_B \) is perpendicular to the axis \( X_B \) in symmetric plane of aircraft and points towards the bottom; the axis \( Y_B \) is perpendicular to the plane \( X_B - O_B - Z_B \) and points to the right wing.

Structure frame \( (O_S - X_SY_SZ_S) \) is fixed to the airframe. The origin \( O_S \) is at the nose of the vehicle; the axis \( X_S \) points towards the CG in the symmetric plane; the axis \( Z_S \) is perpendicular to \( X_S \) in the symmetric plane and points towards the bottom; the axis \( Y_S \) is perpendicular to plane \( X_S - O_S - Z_S \) and points to the left wing.

Inertial/Planetary frame \( (O_G - X_GY_GZ_G) \) is fixed to the ground surface. The origin \( O_G \) is fixed on the surface of the planet (Mars) where the aircraft take off; the axis \( X_G \) coincides with the initial axis \( X_B \) in body frame; the axis \( Z_G \) points downwards and is perpendicular to the surface of planet; the axis \( Y_G \) is perpendicular to the plane \( X_G - O_G - Z_G \) and all the three axes form a right-handed system.

Wind frame \( (O_W - X_WY_WZ_W) \) is fixed to the airframe. The origin \( O_W \) coincides with the CG; the axis \( X_W \) points opposite to the relative wind direction; the axis \( Z_W \) lies in the symmetry plane of the aircraft, and points downwards; the axis \( Y_W \) is perpendicular to the symmetry plane and points
towards the right wing. The wind frame is often used for wind tunnel measurements of the wings. This frame is not expected to be used for VTOL flight.

The transformation matrix from the inertial frame to the body frame is given by Equation 5-1.

\[
M_{bi} = M_1(\phi)M_2(\theta)M_3(\psi) = 
\begin{bmatrix}
\cos \theta \cos \psi & \cos \theta \sin \psi & -\sin \theta \\
\sin \phi \sin \theta \cos \psi - \cos \phi \sin \psi & \sin \phi \sin \theta \sin \psi + \cos \phi \cos \psi & \sin \phi \cos \theta \\
\cos \phi \sin \theta \cos \psi + \sin \phi \sin \psi & \cos \phi \sin \theta \sin \psi - \sin \phi \cos \psi & \cos \phi \cos \theta
\end{bmatrix}
\]

Equation 5-1

The transformation matrix from the wind frame to the body frame is given by Equation 5-2.

\[
M_{bw} = M_2(\alpha)M_3(-\beta) = 
\begin{bmatrix}
\cos \alpha \cos \beta & -\cos \alpha \sin \beta & -\sin \alpha \\
\sin \beta & \cos \beta & 0 \\
\sin \alpha \cos \beta & -\sin \alpha \sin \beta & \cos \alpha
\end{bmatrix}
\]

Equation 5-2

These transformation matrices can also be rewritten using the quaternion, which will not come to singularity while the pitch angle reaches 90°. Since Hyperion is not required to maneuver with large attitude angles, the Euler angle representation will not cause singularity.

Then the rigid body attitude dynamic equations and Equations of Motion (EoM) in the body frame are given by Equation 5-3 and Equation 5-4 [Brockhaus, 2001].

\[
\dot{\mathbf{\phi}} = \frac{1}{m} (\mathbf{\bar{F}}_{aero} + \mathbf{\bar{F}}_{prop}) + \mathbf{\bar{g}} - \mathbf{\bar{\omega}} \times \mathbf{\bar{v}} \quad \text{Equation 5-3}
\]

\[
\mathbf{\bar{I}} \mathbf{\ddot{\omega}} = \mathbf{\bar{M}}_{prop} + \mathbf{\bar{M}}_{aero} - \mathbf{\bar{\omega}} \times \mathbf{\bar{I}} \mathbf{\dot{\omega}} - \mathbf{\bar{j}} \mathbf{\dot{\mathbf{I}}} \quad \text{Equation 5-4}
\]

Where, \( \mathbf{\bar{v}} \) denotes the velocity vector; \( m \) denotes the total mass; \( \mathbf{\bar{F}}_{aero} \) denotes the aerodynamic force vector; \( \mathbf{\bar{F}}_{prop} \) denotes the propulsion force vector; \( \mathbf{\bar{g}} \) denotes the gravity vector; \( \mathbf{\bar{\omega}} \) denotes the angular velocity vector; \( \mathbf{\bar{I}} \) denotes the total moment of inertia tensor; \( \mathbf{\bar{M}}_{prop} \) denotes the moment generated by propulsion system (coaxial tiltrotor and auxiliary rotors); \( \mathbf{\bar{M}}_{aero} \) denotes the aerodynamic moment of the airframe; \( \mathbf{\bar{j}} \) denotes the unit vector in \( Y_g \) axis; \( I_t \) denote the moment of inertia of coaxial tiltrotor; and \( i_p \) denotes the nacelle angle. Note that the nacelle angle \( (i_p) \) is 90° for VTOL and -5.4° (nominal AoA during cruise) for cruise.

Figure 5-4 gives a brief side view of the coaxial tiltrotor system in hover mode. The blue rectangles in the figure denotes the two rotors; the red rectangle denotes the two engines used. This is only a preliminary proposal of the coaxial tiltrotor system, whose structure will be improved further.
two rotors are considered as two rotor disks when calculating the moment of inertia matrix about the CG of the aerobot (the same position as $O_b$).

According to Table 4-4 and the dimensions in Figure 5-4, we have

$$I_0 = I_{2b} = 2 \times \frac{1}{2} m_{rot} R_{tip}^2 = 2.1 \text{kg} \cdot \text{m}^2$$

Equation 5-5

$$I_t = I_{xb} = 2 \times \left( \frac{1}{4} m_{rot} R_{tip}^2 + m_{rot} \left( \frac{d}{2} \right)^2 + \frac{1}{3} m_{mot} \left( \frac{d}{2} \right)^2 \right) = 1.16 \text{kg} \cdot \text{m}^2$$

Equation 5-6

$$\overline{I}_{rot} = \begin{bmatrix} I_t & I_t & I_0 \\ I_t & I_t & I_t \\ I_t & I_t & I_0 \end{bmatrix} = \begin{bmatrix} 1.16 \\ 1.16 \\ 2.1 \end{bmatrix}$$

Equation 5-7

Where, $m_{rot}$ denotes the mass of the rotor (2.1 kg each); $m_{mot}$ denotes the mass of the electrical motor (0.8 kg each); $d$ denotes the "vertical" distance between the two rotors; $\overline{I}_{rot}$ denotes the moment of inertia matrix of the rotor system.

According to [Song, 2008], the total moment of inertia matrix of the whole Martian aerobot system is given by Equation 5-8.

$$\overline{I} = \begin{bmatrix} I_{xx} & I_{xy} & I_{xz} \\ I_{xy} & I_{yy} & I_{yz} \\ I_{xz} & I_{yz} & I_{zz} \end{bmatrix} = \begin{bmatrix} 123.18 & 0.0268 & 0.482 \\ 0.0268 & 8.587 & 8.6 \times 10^{-4} \\ 0.482 & 8.6 \times 10^{-4} & 131.69 \end{bmatrix}$$

Equation 5-8

The corresponding data given in the equation is the same as the present Martian aerobot in hover mode. Therefore, we have the moment of inertia of the airframe as follows:

$$\overline{I}_{ef} = \begin{bmatrix} I_{xx0} & I_{xy0} & I_{xz0} \\ I_{xy0} & I_{yy0} & I_{yz0} \\ I_{xz0} & I_{yz0} & I_{zz0} \end{bmatrix} = \begin{bmatrix} 122.02 & 0.0268 & 0.482 \\ 0.0268 & 7.427 & 8.6 \times 10^4 \\ 0.482 & 8.6 \times 10^4 & 129.59 \end{bmatrix}$$

Equation 5-9

The moment of inertia matrix for transition/conversion mode is given by by Equation 5-75.

The relationship between the attitude angles ($\phi$, $\theta$, $\psi$) and the three components of the angular rate of the vehicle is given by
Equation 5-10
\[
\begin{bmatrix}
  p \\
  q \\
  r
\end{bmatrix}
= \begin{bmatrix}
  \phi \\
  0 \\
  0
\end{bmatrix} + M_1(\phi) \begin{bmatrix}
  0 \\
  \dot{\theta} \\
  0
\end{bmatrix} + M_2(\theta) \begin{bmatrix}
  0 \\
  0 \\
  \dot{\psi}
\end{bmatrix}
\]

Equation 5-11
\[
\begin{bmatrix}
  \phi \\
  \dot{\theta} \\
  \dot{\psi}
\end{bmatrix}
= \begin{bmatrix}
  1 & \sin \phi \tan \theta & \cos \phi \tan \theta & p \\
  0 & \cos \phi & -\sin \phi & q \\
  0 & \sin \phi \cos \theta & \cos \phi \cos \theta & r
\end{bmatrix}
\]

The propulsion forces and moments in the equations can be written as:

\[
\vec{F}_{\text{prop}} = \begin{bmatrix}
  T_{\text{aux}} \cos i_F \\
  0 \\
  -T_{\text{aux}} \sin i_F + T_L + T_R
\end{bmatrix}
\]

Equation 5-12

\[
\vec{M}_{\text{prop}} = \begin{bmatrix}
  (T_L - T_R) y_{\text{aux}} + M_{\text{aux}} \cos i_F \\
  (T_L + T_R) x_{\text{aux}} \\
  M_{\text{aux}} \sin i_F + (M_L - M_R)
\end{bmatrix}
\]

Equation 5-13

Where, \( T_{\text{aux}} \) is the total thrust of the coaxial tiltrotor; \( M_{\text{aux}} \) is the torque generated by the coaxial tiltrotor; \( T_L \) and \( T_R \) are the thrust of the “left” and “right” auxiliary rotors in the body frame; and \( M_L \) and \( M_R \) are the corresponding torque generated by the auxiliary rotors.

The complementary equations are as follows:

\[
\alpha = \arctan \frac{v}{u}
\]

Equation 5-14

\[
\beta = \arcsin \frac{v}{V_r}
\]

Equation 5-15

\[
V_r = \sqrt{u^2 + v^2 + w^2}
\]

Equation 5-16

5.2 Mars Atmosphere Model

The aerodynamic forces and moments are directly relevant with the atmosphere density. Many Martian atmosphere models are available for the present stage. The early models include the model obtained from Earth based observations [Levine et al., 1965] before the first Mars missions, and the Martian Atmosphere Model Circa 1974 [NASA, 1974] based on the Earth observation and Mars Orbiter/fly-by missions with Global Circulation Models. Recent high precision Martian atmosphere model is based on the Mars mission measurements, such as Mars-GRAM 3.34 [Justus et al., 1996] based on the Viking measurements, Mars-GRAM 2001 [Justus and Johnson, 2001]...
Chapter 5. Robust Flight Controller Design

based on the Mars Global Surveyor measurements, Mars Climate Database (MCD) [MCD, 2012] based on the combination of all the measurements data, and etc.

The models listed above are too complex for the current design. The model [NASA, 2012b] used in the previous work is based on the measurement data of the Mars Global Surveyor in 1996. The corresponding equations are given by Equation 5-17 to Equation 5-19.

\[
T = \begin{cases} 
-31 - 0.000998h, & h \leq 7000 \text{ (°C)} \\
-23.4 - 0.00222h, & h > 7000 
\end{cases}
\]  

\[p = 0.699 \times e^{-0.00009h} \text{(kPa)} \]  

\[
\rho = \frac{p}{0.1921 \times (T + 273.1)} \text{(kg m}^{-3}\text{)}
\]

Where, \(T\) denotes the atmosphere temperature; \(h\) denotes the altitude; \(p\) denotes the atmospheric pressure; \(\rho\) denotes the atmosphere density.

Since the work in this thesis focuses on the transition flight, for which the altitude is not too high (approximately 50m), the atmosphere density change is not high. In this case, the atmosphere density in this thesis is assumed to be a constant of 0.0135 kg m\(^{-3}\).

5.3 Aerodynamic Model for the Airframe

The airframe will only generate aerodynamic drag \((D)\) and drag induced moment \((M_D)\) during the VTOL phases, as shown in Equation 5-20 and Equation 5-21. Equation 5-22 shows the relationship between \(M_D\) and \(D\).

\[
\vec{F}_{aero} = M_{BI} \begin{bmatrix} 0 \\ 0 \\ D \end{bmatrix}
\]

\[
\vec{M}_{aero} = M_{BI} \begin{bmatrix} 0 \\ M_D \\ 0 \end{bmatrix}
\]

\[M_D = D(d_D - d_e) \]

The aerodynamic forces and moments of the airframe in cruise is given by Equation 5-23 and Equation 5-24. It is worth noting that the aerodynamic forces in Equation 5-23 are expanded in the wind frame.
Equation 5-23
\[
\begin{bmatrix}
-D \\
Y \\
-L
\end{bmatrix} = \frac{1}{2} \rho V_r^2 S M_{\text{BF}} \begin{bmatrix}
-C_D \\
C_Y \\
-C_L
\end{bmatrix}
\]

Equation 5-24
\[
\begin{bmatrix}
\bar{L} \\
\bar{M} \\
N
\end{bmatrix} = \frac{1}{2} \rho V_r S \begin{bmatrix}
\bar{b} C_t \\
\bar{b} C_l \\
\bar{b} C_n
\end{bmatrix}
\]

Where, \(D, Y, L\) denote the aerodynamic drag, side force, lift, respectively; \(\bar{L}, M, N\) denote the aerodynamic moment in roll, pitch, yaw directions, respectively; \(\rho\) denotes the Martian atmosphere density; \(V_r\) denotes the relative flow velocity; \(S\) denotes the wing area; \(\bar{b}\) denotes the wing average chord length; \(\bar{b}\) denotes the wing span; \(C_D, C_Y, C_L, C_t, C_m, C_n\) are the aerodynamic coefficients of forces and moments. These aerodynamic coefficients are solved using the first order derivatives approximation, see Equation 5-25 to Equation 5-30. These aerodynamic derivatives of airframe are estimated by AVL [Drela, 2011], refer to Appendix B-1. AVL is an open source code for aircraft dynamic stability analysis written by Drela. AVL can give the estimation first order of aerodynamic derivatives using VLM:

Equation 5-25
\[
C_D = C_{D0} + \frac{(C_L - C_{L0})}{\pi eAR} + C_{D0e} \delta e + C_{D0a} \delta a
\]

Equation 5-26
\[
C_Y = C_{Y0} + C_{Y0e} \delta e + \frac{\bar{b}}{2V_r} (C_{Yp} p + C_{Yr} r)
\]

Equation 5-27
\[
C_L = C_{L0} + C_{L0e} \delta e + \frac{\bar{c}}{2V_r} (C_{L0a} \alpha + C_{L0q} q)
\]

Equation 5-28
\[
C_t = C_{t0} + C_{t0e} \delta e + \frac{\bar{b}}{2V_r} (C_{tp} p + C_{tr} r)
\]

Equation 5-29
\[
C_m = C_{m0} + C_{m0e} \delta e + \frac{\bar{c}}{2V_r} (C_{m0a} \alpha + C_{m0q} q)
\]

Equation 5-30
\[
C_n = C_{n0} + C_{n0e} \delta e + \frac{\bar{b}}{2V_r} (C_{np} p + C_{nr} r)
\]

A confidence interval of \(\pm 20\%\) is assigned to all the estimated aerodynamic coefficients \((C_D, C_Y, C_L, C_t, C_m, C_n)\). Then the actual values of aerodynamic coefficient should fall in the confidence interval.
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The aerodynamic model used during transition and conversion is the same as that during cruise. The covers of the auxiliary rotors will be closed at high cruise speed and the aerodynamic forces and moments for cruise low speed is very small, so this will not cause significant error.

5.4 Aerodynamic Model for Rotors

The performance of coaxial tiltrotor and auxiliary rotors is estimated by BEMT rather than FWM in this chapter. The reason is that BEMT can also give a good estimation of the rotor performance; and the high precision model of the coaxial tiltrotor is time dependent. Moreover, the objective of this chapter is to demonstrate the controller design concept, so the exact values are not important.

The performance of the proposed single auxiliary rotor predicted by BEMT is shown in Figure 5-5 and Figure 5-6. The auxiliary rotors will generate thrust lower than 2.5N in operation during VTOL and low speed transition/conversion; therefore the collective pitch will be within ±15°. The auxiliary rotors are embedded in the ducts at wing tips, so the variation of the axial inflow rate of
the auxiliary rotors is negligible in low speed. That is, the performance of auxiliary rotor remains for different cruise speed. The thrust of auxiliary rotors are approximately proportional to the collective pitch angle; while the torque is approximately proportional to the square of the blade pitch angle.

Figure 5-7: Thrust of a single auxiliary rotor with collective pitch variation range upto 40° (predicted by BEMT)

It is worth noting that the thrust of the auxiliary rotor is not exactly linear function of the pitch angle, especially at those states near zero pitch. In order to show its approximately linear property, we extend the pitch variation range to 40°, as shown in Figure 5-7. The thrust shows a linear relationship with the pitch angle while the blade pitch is greater than about 5°.

Figure 5-8: Total thrust of the coaxial tiltrotor in hover and transition/conversion with different axial inflow rate (predicted by BEMT)

The performance of the coaxial tiltrotor for hover and transition predicted by BEMT is given by Figure 5-8 and Figure 5-9. It is worth noting that only the inflow rate in axial direction is considered. The influence of the tangential inflow rate is of minor importance and is ignored in this work. Generally, the thrust of the coaxial tiltrotor is proportional to the blade pitch angle.
under different axial inflow rate. There is no net torque for the coaxial tiltrotor in operation. The torque generated by differentiate the collective pitch is proportional to the differential collective pitch.

![Figure 5-9: Torque of the coaxial tiltrotor in hover and transition/conversion with different inflow rate (predicted by BEMT)](image)

The thrust of the coaxial tiltrotor in cruise predicted by the BEMT is shown in Figure 5-10.

![Figure 5-10: Thrust of the coaxial tiltrotor in cruise with different inflow rate (predicted by BEMT)](image)

The same as the model for airframe aerodynamic performance, a variation interval of ±20% is imposed to the thrusts of the auxiliary rotors and the coaxial tiltrotor, and then the actual value should be within the confidential interval.

### 5.5 Actuator Model

The control actuators are also considered in the loops. For this Martian aerobot, the actuators are used to drive the flaps or rotor blades to a specific position. According to the engineering practice, this kind of actuator is driven by a motor with damper, and it is usually approximated by a second
order mass-spring-damper system. In the aerospace community, a second order spring-mass-damper system is usually used to approximate the dynamics of the actuator. There are high order models for such actuators, but this is not necessary since it is the focus of this thesis. In this case, the simple second order actuator is selected in this work. The delay is another factor should be considered for actuator model. Therefore, the corresponding transfer function is given by Equation 5-31.

\[ G_{act} = e^{-\tau T} \frac{\omega_n^2}{s^2 + 2\xi\omega_n + \omega_n^2} \]  

Equation 5-31

Where, \( \omega_n \) is the natural frequency; \( \xi \) is the damper; \( T \) is the delay of the actuator. Since Equation 5-31 represents the dynamic behaviour of an actuator, the performance, such as the adjusting time, overshooting, etc, should be well-posed. The commonly available actuator has a delay of no larger than 100ms, damping of 0.7, and natural frequency of 20Hz, which is the natural frequency for most actuators used in engineering. The motor rotational rate of the actuator should also be limited. The value used in this work is 60deg/s\(^{-1}\). This value is reasonable since the nominal value of the proposed servo is 0.15s/60° for 4.8V and 0.13s/60° for 6.0V (see Figure 3-8). The nonlinear factors, such as delay and rotational rate limits, are used in the simulation only. A reasonable actuator deflection limit is also applied for each actuator in the simulation.

### 5.6 Sensor Model

A sensor model can also be assumed in this work, but an ideal sensor model is used in this work for the following reasons.

- In engineering, the disturbance and uncertainties exist everywhere. For the aeroplane, the dominant uncertainty is the aerodynamic model, which can be about ±20%. Comparing with this, the disturbance in the sensors is much smaller. Therefore, the disturbance in the sensor can be ignored in this preliminary work.

- The sensor used in this model is not confirmed, so the model order for sensor assumed can not be high. In this case, what we can do only is to assume a possible small noise (or disturbance) under for the sensor. This factor is of minor importance comparing with the model uncertainties.

- The measurement data from the sensor is not the direct input of the controller. The output of the sensor data is the navigation system. Bandwith filters and state estimators will be applied on these data from the sensor. The outputs of the state estimators are the real "sensor" input data to the controller. It is impossible for the current design to give an estimation of the model of this navigation system.
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- The simulation results with noise will be very coarse. This will have influence on showing specific performance of the proposed controller.

5.7 Robust Control Method

The largest uncertainty for the aeroplane is caused by the uncertain aerodynamic estimation. Due to the existence of the uncertainty, it is necessary to select an appropriate control scheme that is tolerant to the aerodynamic uncertainty. This section aims to answer this question.

5.7.1 Robust Control Theory Selection

Because of the inherent large aerodynamic uncertainty of the aerobot control problem, the robustness of the controller with respect to the parameter uncertainty is critical. As shown in Figure 5-11, the transition control is an uncertain nonlinear control problem. Two methods are proposed to solve this problem.

- Robust nonlinear control method. The general concept of the robust nonlinear control method is to design a Lyapunov function for the system [Freeman and Kokotovic, 1996]; then a proper controller can be designed based on some procedure. Although some robust nonlinear control methods are proposed, most of these methods are proposed for the affine problem.

- Trajectory linearization plus the linear control theory. This method aims to use the trajectory linearization method to transform the nonlinear model into the linear
counterpart, and then the well developed linear control theory is used. This method is well
developed and widely used in aerospace area. More details on the trajectory linearization
method can refer to [Huang, 2007].

The trajectory linearization method is selected in this work due to the properties described. After
the trajectory linearization operation, the uncertain nonlinear control problem becomes a Linear
Parameter Varying (LPV) control problem. The varying parameter is the cruise speed during
transition. Three methods are proposed for LPV control problem. A comprehensive review on this
topic is given by [Leith and Leithead, 2000].

- Classical gain scheduling method (not listed in the figure). The classical gain scheduling
  method is widely used in the industry; however, the stability and robustness is
  questionable in theory [Turner and Bates, 2007].

- LPV control method. The LPV control approach is proposed for the LPV problem
  recently. This is an active area. The general concept of LPV control approach is to
  introduce a Linear Differential Inclusion (LDI) for the LPV control problem that the
  behaviour of the LPV model or even the nonlinear dynamic model is included in the LDI.
  It can be proved that if the controller can stabilize the proposed LDI, then the controller
  can stabilize the LPV model or the nonlinear dynamic model. Refer [Bruzelius, 2004] for
  more details.

- D&C gain scheduling method The D&C gain scheduling is the third method proposed for
  the control of LPV problem. The design concept is to introduce the D&C strategy into
  the controller design method. The LPV model is divided into several LPV subproblems
  with smaller parameter varying span. A candidate controller is designed for each
  subproblem using the simple linear control method. Then a proper method is used to
  combine these candidate controllers. The final controller is a switch system. The recent
developed supervisory control method is based on this strategy. Refer [Morse, 1996] and

Based on the discussion above, the D&C gain scheduling method is selected in this work. The
problem left is the design method for the LPV problem with small parameter variation. Due to the
existence of the aerodynamic uncertainties in the model, the linear robust control theory is used.
The linear robust control theory has three methods. The fundamental theory for the three methods
is the small gain theorem. Refer [Zhou et al., 1997] for a full definition, proof, and even examples
on the linear robust control theory.

- $H_2$ method (also called LQG). This method is based on the 2-norm of the "signal" when
  applying the small gain theorem. The 2-norm is usually used as a measurement of the
strength of the "signal", so this method is usually used when the noise of the signal is the dominant disturbance in the system.

- $H_\infty$ method. This method is based on the $\infty$-norm of the "signal". The $\infty$-norm stands for the maximum value of the signal or system. When $\infty$-norm is used for control system, it stands for the maximum gain for all "frequency". When the maximum gain of the system and controller satisfy the small gain theorem, the controller is robust.

- $\mu$ synthesis. The general concept of $\mu$ synthesis is the same as the $H_\infty$ method. The difference is that the metric used is the $\mu$ value of the signal or system rather than $\infty$-norm. $\mu$ value is a structured singular metric for a system because the phase is also considered. However, the exact $\mu$ value of a system can not be solved. The usually used metric is its ceiling estimation. The usually used approach is the D-K iteration, see [Zhou et al., 1997] for details. Hankel order reduction method is usually used with $\mu$ synthesis, since the order of the controller designed by $\mu$ synthesis procedure is always very high. Hankel order reduction is based on the Hankel norm of the system. Hankel norm is a metric of the influence of the state in a system. The Hankel order reduction is processed by ignoring the less important states in the system, then the order of the system is reduced, see [Zhou et al., 1997] for details.

Therefore, the control method used for the candidate controller is the $\mu$ synthesis with Hankel order reduction.

In summary, the method used to solve the transition control problem is trajectory linearization, D&C gain scheduling, and the $\mu$ synthesis with Hankel order reduction. See Section 5.10 for details on the implementation on this approach.

For other flight modes (i.e. VTOL mode and cruise mode), the controller design method used is also $\mu$ synthesis, due to its robustness and good performance.

The general scheme of the $\mu$ synthesis method used in this thesis is shown in Figure 5-12. The "Plant" denotes the uncertain linearized model of each control channel. The "Actuator" denotes the model of corresponding actuator for the control problem. The "Controller" is the controller needs to be designed. $r$ denotes the reference signal, i.e. the command signal; $y$ denotes the output of the "Plant"; $dy$ denotes the noise of the sensor; $\hat{y}$ denotes the error between the output and the reference signal; $u^{cmd}$ denotes the desired command signal; and $u$ denotes the actual command signal produced by the actuator; and $du$ is the noise of the actuator. $Wy$, $Wu$, $Wr$, $Wdy$, and $Wdu$ are the weighting functions representing the performance of the controller.
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5.7.2 Robustness and Monte Carlo Style Simulation

The robustness of the proposed controller is guaranteed by the linear robust control theory developed approximately 30 years ago [Zhou et al., 1997], so the control method is used without proof. Detailed definitions, proofs, and even some examples can be found in [Zhou et al., 1997]. In order to show the uncertainty tolerance (robustness) of the proposed controller, a Monte Carlo style simulation is used to in the full states simulation. More discussion on the Divide and Conquer gain scheduling method is given by Section 5.10.3.

According to the Monte Carlo method, the simplest method used in control community is to select three values for each uncertainty (the minimum, the maximum, and the average value). The number of aerodynamic uncertainties for VTOL mode is 4 (coaxial rotor thrust, coaxial rotor torque, auxiliary rotor thrust, and aerodynamic drag), so the number of simulation required for VTOL simulation is \(3^4 = 81\). The number of uncertainty for cruise mode is 7 (aerodynamic lift, drag, side force, roll moment, pitch moment, yaw moment, and coaxial rotor thrust); therefore, the number of simulation required for cruise mode is \(3^7 = 2187\). The number of uncertainty for transition/conversion mode is 9 (aerodynamic lift, drag, side force, roll moment, pitch moment, yaw moment, coaxial tiltrotor thrust, coaxial tiltrotor torque, auxiliary rotor thrust). The number of simulation required for transition/conversion mode is up to \(3^9 = 19683\). It is worth noting that due to the time consuming 2-D look-up table and high order dynamic model used to represent the performance of rotors, the simulation time for one case is approximately half an hour. The simulation time cost will increase for future work if the N-D look-up tables for aerodynamic performance of the airframe and even the interference for airframe and rotors are introduced in the

![Figure 5-12: \(\mu\) synthesis scheme](image-url)
simulation. Therefore, the method of full Monte Carlo simulation to validate the robustness is not suitable for this work.

Based on the above analysis, it is very difficult to perform the full Monte Carlo simulation for the proposed controller. In this thesis, 10 simulations with uncertainty are performed to show the performance of the proposed controller. The two simulation cases of the maxima and minima of all the uncertain parameters are performed. The values of the aerodynamic uncertainties in the other 8 simulations are selected based on the Monte Carlo method (randomly selecting the values between the maximum, the minimum, and the average). The simulations are performed just to show the performance of the proposed controllers. It is not an evidence of the robustness of the proposed controller.

5.8 Robust Control Law for VTOL (Hover) Phases

The flight mode of the Martian vehicle in VTOL phases is similar to the quadrotors. The taking-off and landing are two symmetric flight phases with different direction of motion. Because the speed of taking off and landing cannot be too large due to the power and Vortex Ring State (VRS) constraints [Leishman and Ananthan, 2001], the difference of the flight conditions in taking off and landing are very small. Therefore, the flight modes for taking off and landing are presented together in this section.

The VTOL mode has two flight phases, taking off and landing. The free body diagrams of the two flight phases are shown in Figure 5-13 and Figure 5-14 respectively. It is worth noting that the two flight phases are quite similar. The only difference is the direction of the aerodynamic drag and the auxiliary rotor thrust. The thrust of the auxiliary rotor can be both upward and downward, see Section 4.2. Most of the gravity is counteracted by the thrust of the coaxial tiltrotor. The thrust of the auxiliary rotor is used to adjust the pitch attitude angle only.

![Figure 5-13: Free body diagram of the Martian aerobot in take-off flight](image-url)
5.8.1 Dynamic Model for VTOL Mode

The aerodynamic forces and moments of the airframe is given by Equation 5-23 and Equation 5-24. The nacelle angle \( \phi_U \) is 90°. Therefore, the thrust and moment generated by the propulsion system is given by Equation 5-32 and Equation 5-33.

\[
\begin{align*}
\vec{F}_{\text{aero}} & = \begin{bmatrix} 0 \\ 0 \\ -T_{\text{cox}} + T_L + T_R \end{bmatrix} \quad \text{Equation 5-32} \\
\vec{M}_{\text{prop}} & = \begin{bmatrix} (T_R - T_L) y_{\text{aux}} \\ (T_L + T_R) x_{\text{aux}} \\ -M_{\text{cox}} + M_L - M_R \end{bmatrix} \quad \text{Equation 5-33}
\end{align*}
\]

Where, \( T_{\text{cox}} \) denotes the total thrust generated by the coaxial tiltrotor; \( T_L \) and \( T_R \) are the thrust of the “left” and “right” auxiliary rotors; \( y_{\text{aux}} \) denotes the spanwise distance of the two auxiliary rotors; \( x_{\text{aux}} \) denotes the longitudinal distance between the CG and the auxiliary rotors; \( M_L \) and \( M_R \) are the torque of the “left” and “right” auxiliary rotors.

The simplified 6 Degree of Freedom (DoF) dynamic equations for controller design are given by Equation 5-34 to Equation 5-43. \( z_a \) denotes the negative value of altitude. The small angle assumption is used for these equations.

\[
\begin{align*}
\dot{\phi} & = p \\
\dot{\theta} & = q \\
\dot{\psi} & = r \\
\dot{p} & = \frac{1}{I_{xx}} (T_R - T_L) y_{\text{aux}} \quad \text{Equation 5-37}
\end{align*}
\]
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\[
\dot{q} = \frac{1}{I_{yy}} (T_i + T_r) x_{aux}
\]
Equation 5-38

\[
\dot{r} = \frac{1}{I_{zz}} (-M_{aux}^{diff} + M_i - M_r)
\]
Equation 5-39

\[
\dot{u} = rv - qw - \left( g + \frac{D_{aero}}{m} \right) \theta
\]
Equation 5-40

\[
\dot{v} = ru + pw + \left( g + \frac{D_{aero}}{m} \right) \phi
\]
Equation 5-41

\[
\dot{w} = qu - pv + g + \frac{D_{aero}}{m} - T_{aux} + T_i + T_r
\]
Equation 5-42

\[
\dot{\varepsilon} = -u \theta + v \phi + w
\]
Equation 5-43

5.8.2 Linearization Model for VTOL (Hover) Mode

The state space representation of the linearized attitude dynamic model (Equation 5-3) by the frozen coefficient method is given by Equation 5-44 to Equation 5-49. The frozen coefficient method is based on the calculus of variations and small deviation assumption. [Brockhaus, 2001] gives a good introduction on this method. The output equations are neglected. All the attitude angles and attitude rates are observable. The symbols with the prefix of \( \delta \) in the equations denote the corresponding variables in the linearized dynamic model.

\[
\begin{bmatrix}
\delta \phi \\
\delta \dot{p}
\end{bmatrix} =
\begin{bmatrix}
0 & 1 \\
0 & 0
\end{bmatrix}
\begin{bmatrix}
\delta \phi \\
\delta p
\end{bmatrix} +
\begin{bmatrix}
0 \\
\frac{k_{aux} x_{aux}}{I_{xx}}
\end{bmatrix}
\begin{bmatrix}
\delta \theta_{aux}^{diff}
\end{bmatrix}
\]
Equation 5-44

\[
\begin{bmatrix}
\delta \phi \\
\delta \dot{p}
\end{bmatrix} =
\begin{bmatrix}
1 & 0 \\
0 & 1
\end{bmatrix}
\begin{bmatrix}
\delta \phi \\
\delta p
\end{bmatrix} +
\begin{bmatrix}
0 \\
0
\end{bmatrix}
\begin{bmatrix}
\delta \theta_{aux}^{diff}
\end{bmatrix}
\]
Equation 5-45

\[
\begin{bmatrix}
\delta \dot{\phi} \\
\delta \dot{q}
\end{bmatrix} =
\begin{bmatrix}
0 & 1 \\
0 & 0
\end{bmatrix}
\begin{bmatrix}
\delta \theta \\
\delta q
\end{bmatrix} +
\begin{bmatrix}
0 \\
\frac{k_{aux} x_{aux}}{I_{yy}}
\end{bmatrix}
\begin{bmatrix}
\delta \theta_{aux}^{ext}
\end{bmatrix}
\]
Equation 5-46

\[
\begin{bmatrix}
\delta \theta \\
\delta q
\end{bmatrix} =
\begin{bmatrix}
1 & 0 \\
0 & 1
\end{bmatrix}
\begin{bmatrix}
\delta \theta \\
\delta q
\end{bmatrix} +
\begin{bmatrix}
0 \\
0
\end{bmatrix}
\begin{bmatrix}
\delta \theta_{aux}^{ext}
\end{bmatrix}
\]
Equation 5-47

\[
\begin{bmatrix}
\delta \psi \\
\delta \dot{r}
\end{bmatrix} =
\begin{bmatrix}
0 & 1 \\
0 & 0
\end{bmatrix}
\begin{bmatrix}
\delta \psi \\
\delta r
\end{bmatrix} +
\begin{bmatrix}
0 \\
\frac{k_{aux}^{diff}}{I_{zz}}
\end{bmatrix}
\begin{bmatrix}
\delta \theta_{aux}^{diff}
\end{bmatrix}
\]
Equation 5-48
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\[
\left[\frac{\delta \psi}{\delta r}\right] = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \left[\frac{\delta \psi}{\delta r}\right] + \begin{bmatrix} 0 \\ 0 \end{bmatrix} \left[\delta \theta_{\text{diff}}^{\text{coax}}\right]
\]

Equation 5-49

Where, \( k_{\text{aux}} \) denotes the thrust curve slope of the auxiliary rotor in Figure 5-5; \( k_{\text{coax}}^{\text{diff}} \) denotes the torque curve slope of the coaxial tiltrotor in Figure 5-9; \( \theta_{\text{tot}}^{\text{aux}} \) and \( \theta_{\text{aux}}^{\text{diff}} \) denote the total and differential values of the auxiliary rotor blade pitch angles; \( \theta_{\text{coax}}^{\text{diff}} \) denotes the differential collective pitch angle. The torque generated by the auxiliary rotors is much smaller than that by coaxial tiltrotor, so this term is neglected in the equation.

The linearized EoM (Equation 5-4) by the frozen coefficient method are shown by Equation 5-50 to Equation 5-52. They are first order differential equations.

\[
\delta \ddot{u} = -\left( g + \frac{D_{\text{aero}}}{m} \right) \delta \theta
\]

Equation 5-50

\[
\delta \dot{v} = \left( g + \frac{D_{\text{aero}}}{m} \right) \delta \phi
\]

Equation 5-51

\[
\delta \dot{w} = -\frac{k_{\text{coax}}^{\text{tot}}}{m} \delta \theta_{\text{coax}}^{\text{tot}}
\]

Equation 5-52

Where, \( D_{\text{aero}} \) is the aerodynamic drag induced by the airframe; \( k_{\text{coax}}^{\text{tot}} \) is the total thrust curve slope of the coaxial tiltrotor in hover, see Figure 5-8; \( \theta_{\text{coax}}^{\text{tot}} \) is the blade pitch angle of the torque balanced coaxial tiltrotor.

The \( \pm 20\% \) aerodynamic uncertainty is added to the variables \( k_{\text{aux}} \), \( k_{\text{coax}}^{\text{diff}} \), \( k_{\text{coax}}^{\text{tot}} \), and \( D_{\text{aero}} \), then Equation 5-44 to Equation 5-52 become the uncertain linear dynamic model for VTOL mode.

5.8.3 Control Loops for VTOL (Hover) Mode

The flight mode of Hyperion is quite similar to the quadrotors in VTOL (hover) phases. The control of the three attitude angles is fully decoupled. The horizontal speed \((u \text{ and } v)\) is coupled with the pitch and roll attitude angles, while others are decoupled; therefore, the structure of the control strategy for VTOL flight in this work is given by Figure 5-15 to Figure 5-18.

\[ \text{Figure 5-15: Control loop in the pitch channel for VTOL mode} \]
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Figure 5-16: Control loop in the roll channel for VTOL mode

The control loops in the pitch and roll channels are shown in Figure 5-15 and Figure 5-16. Both channels have the inner/outer loop control structure. The inner loops will stabilize the pitch and roll attitude angles; while the outer loop will be used to control the horizontal velocity. The pitch angle of the vehicle is controlled by changing the thrust of both auxiliary rotors together; while the roll angle is controlled by differentiating the auxiliary rotor thrusts. The horizontal velocity of the vehicle is controlled by tilting the vehicle towards specific attitude angle; that is, the command values of the pitch and roll angles ($\theta_{\text{cmd}}^\text{roll}$ and $\phi_{\text{cmd}}$) are generated by the horizontal velocity controller.

Figure 5-17 shows the yaw control loop of Hyperion in VTOL mode. The yaw angle is controlled by differentiating the collective pitch ($\theta_{\text{cmd}}^\text{coax}$) of the coaxial tiltrotor. Because the nacelle angle ($i_F$) in VTOL mode is 90°, the yaw channel is fully decoupled from the other attitude angle channels.

Figure 5-17: Control loop in the yaw channel for VTOL mode

As shown in Figure 5-18, the altitude channel is controlled by two steps. The inner loop will ensure the vehicle to climb or to descend with a constant speed not too large; and the outer loop will generate the motion speed command ($w_{\text{cmd}}$). The vertical velocity is controlled by the coaxial tiltrotor blade pitch ($\theta_{\text{cmd}}^\text{coax}$).

Figure 5-18: Control loop in altitude channel for VTOL mode

The controllers in Figure 5-15 to Figure 5-18, except the “altitude hold” in Figure 5-18, are solved by the general procedure of $\mu$ synthesis and Figure 5-12.
The “altitude hold” for take-off and landing are different, because the velocity when landing the ground has to be small enough. However, this is not necessary in taking off. The altitude holder for take-off flight is easier. In this work, the “altitude hold” controller in take-off flight is implemented by a saturation form in this thesis (Equation 5-53).

\[
\begin{align*}
\omega_{\text{cmd}} &= \begin{cases} 
\frac{w_0}{h_0}(z_{\text{cmd}} - z_e), & |z_{\text{cmd}} - z_e| \leq h_0 \\
 w_0, & \text{else}
\end{cases}
\end{align*}
\tag{5-53}
\]

Where, \(w_0\) is the proposed constant value of the vertical speed (3m·s\(^{-1}\) for taking off and 2m·s\(^{-1}\) for landing); \(h_0\) is a boundary of linear part of the function (3m used in this work).

In landing flight, the altitude holder used in this work is the altitude holder in the take-off flight (Equation 5-53) plus a safety condition. The general concept of the safety condition is to propose a “safe” altitude to adjust the horizontal velocity. That is, when the horizontal velocity is faster than a tolerance value, the command value of the altitude is kept at the “safe” altitude (Equation 5-54 and Equation 5-55).

\[
\begin{align*}
\omega_{\text{input}} &= \begin{cases} 
z_{\text{safe}}, & |u_e| + |v_e| < V_{\text{tol}} \\
z_{\text{cmd}}, & \text{else}
\end{cases}
\end{align*}
\tag{5-54}
\]

\[
\begin{align*}
\omega_{\text{cmd}} &= \begin{cases} 
\frac{w_0}{h_0}(z_{\text{input}} - z_e), & |z_{\text{input}} - z_e| \leq h_0 \\
w_0, & \text{else}
\end{cases}
\end{align*}
\tag{5-55}
\]

Where, \(z_{\text{safe}}\) is the safe altitude; \(z_{\text{cmd}}\) is the command value of the altitude; \(u_e\) and \(v_e\) are the horizontal velocity components; \(V_{\text{tol}}\) is the horizontal velocity tolerance (2m·s\(^{-1}\) in this work).

### 5.8.4 Simulation Result for VTOL (Hover) Mode

In this section, a 6 DoF simulation is used to validate the proposed control law for taking off and landing phases. Because the working conditions of taking off and landing phases are similar to each other, the simulation for taking off is discussed in detail; the simulation for landing is listed for comparison. The corresponding parameters used for the simulation are given by Table D-1.

#### 5.8.4.1 Simulation Results for Vertical Taking Off

According to the mission profile, the taking off controller will ensure the stability of vehicle when it climbs from the ground to a specific altitude. According to the mission profile, the initial condition for Hyperion is located somewhere on the Martian ground, so the initial velocity and angular rate should be zero. The initial conditions selected in this thesis are \([x_e, y_e, z_e]^T = [0, 0, 0]^T\),
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\[ [u, v, w]^T = [0, 0, 0]^T, [\phi, \theta, \psi]^T = [10^\circ, 10^\circ, -10^\circ]^T, [p, q, r]^T = [0, 0, 0]^T. \]

The control objective of the vehicle for taking off is hovering at specific position; therefore, we have the command values of

\[ z^{\text{cmd}}_e = -50 \text{m}, [u^{\text{cmd}}, v^{\text{cmd}}]^T = [0, 0]^T, \psi^{\text{cmd}} = 0^\circ. \]

The command climbing speed used in the simulation is \(-3 \text{m/s}\). A wind gust of \(-5 \text{m/s}\) is imposed in \(X_g\) direction between 10s and 20s to test the wind resistance. The uncertainty of the aerodynamic model is represented by scaling each aerodynamic parameter by a number within [0.8, 1.2]. A Monte Carlo style simulation is used to represent the results for the uncertain aerodynamic model (magenta dashed line). The simulation result for the state variables is given by Figure 5-19 to Figure 5-28; and the control inputs are given by Figure 5-29 to Figure 5-32. The black solid lines in the figures indicate the simulation results of the nominal model. Generally, the simulation results for state variables of the uncertain model are quite close to the results for nominal model (see Figure 5-19 to Figure 5-28), which also shows the robustness of the controller. The steady values of the state variables for uncertain model are quite close to those of the nominal model. The variation range of the states at convergence is within 1%. Since the performance of rotors is not the same, the control inputs are not the same.

Figure 5-19: 6 DoF take-off simulation result for the altitude position
Figure 5-20: 6 DoF take-off simulation result for the horizontal velocity in $X_G$ axis

Figure 5-21: 6 DoF take-off simulation result for the horizontal velocity in $Y_G$ axis

Figure 5-22: 6 DoF take-off simulation result for the vertical velocity in $Z_G$ axis
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Figure 5-23: 6 DoF take-off simulation result for the roll attitude angle

Figure 5-24: 6 DoF take-off simulation result for the pitch attitude angle

Figure 5-25: 6 DoF take-off simulation result for the yaw attitude angle
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Figure 5-26: 6 DoF take-off simulation result for the roll angular rate

Figure 5-27: 6 DoF take-off simulation result for the pitch angular rate

Figure 5-28: 6 DoF take-off simulation result for the yaw angular rate
Figure 5-29: 6 DoF take-off simulation result for the collective pitch angle of the coaxial tiltrotor

Figure 5-30: 6 DoF take-off simulation result for the differential collective pitch of the coaxial tiltrotor

Figure 5-31: 6 DoF take-off simulation result for the collective pitch angle of the left auxiliary rotor
The simulation result for the altitude of Hyperion in the take-off phase is shown in Figure 5-19. The vehicle reaches to the desired altitude with a prescribed climbing speed of $w_i = 3 \text{m/s}$. The altitude curves in the figure are almost linear before the altitude command value.

Figure 5-20 and Figure 5-21 give the horizontal velocity of Hyperion in take-off phase. The horizontal speed is caused when adjusting the pitch and roll attitude angles; and it slowly converges to zero during taking off. It is worth noting that due to the existence of wind gust between 20s and 30s, the aerobot tries to floating with the wind during this period.

The vertical climb speed during taking off is shown in Figure 5-22. The velocity is negative because the $Z_Q$ axis is defined to be downward. The value of the climb speed reaches the prescribed value ($w_i$) quickly at the beginning of the simulation. Then the speed keeps and decreases to zero while the altitude is close to the transition altitude.

Figure 5-23, Figure 5-24, and Figure 5-25 show the roll, pitch, and yaw attitude angles during taking off. The roll and yaw attitude angles are stabilized to be zero smoothly. The pitch angle generally converges to zero with a little fluctuation for the pitch channel. This is caused by the aerodynamic moment in this channel and the nonlinearity of the auxiliary rotors near zero thrust (see Figure 5-5). Although the roll angle is also controlled by the auxiliary rotors, its performance is much better; the main factor is the aerodynamic moment in this channel is zero.

The angular rates during taking off are given by Figure 5-26, Figure 5-27, and Figure 5-28. The pitch angular rate also shows a little fluctuation close to zero, which is indicated by the pitch angle in Figure 5-24. The performance of the take-off controller in roll and yaw channels is much better. The oscillation in Figure 5-27 is also caused by the nonlinearity of the auxiliary rotors near zero thrust. This is the same as the pitch angle explained.

Figure 5-29 shows the collective pitch of the coaxial tiltrotor during taking off. The coaxial tiltrotor with different aerodynamic performance is required to counteract the constant weight.
The steady state pitch angle varies due to the different aerodynamic performance. The acceleration at 0s and deceleration at about 18s of the vertical speed shown in Figure 5-22 are caused by the corresponding thrust change of the coaxial tiltrotor. The differential collective pitch of the coaxial tiltrotor to control the yaw angle is shown in Figure 5-30. The positive and negative yaw moment results in the acceleration and deceleration of the yaw angle.

The collective pitch angles of the two auxiliary rotors are given by Figure 5-31 and Figure 5-32. The initial (before 10s) differential collective pitch angle is used to stabilize the roll angle. The collective angles are the same after 10s, and fluctuating around zero at the end. This fluctuation results in the fluctuation of pitch angle and pitch rate.

### 5.8.4.2 Simulation Results for Vertical Landing

The landing phase is quite similar to the take-off, so a short introduction on the difference between them is given in this section. The initial condition for Hyperion is hovering at a specific altitude. The horizontal and vertical speeds are not strictly zero. Therefore, we select the initial conditions to be $[x_e, y_e]^T = [0, 0]^T$, $z_e = -50m$, $u = 5m \cdot s^{-1}$, $v = 5m \cdot s^{-1}$, $w = 1m \cdot s^{-1}$, $[\phi, \theta, \psi]^T = [0', 0', 0']^T$, $[p, q, r]^T = [0, 0, 0]^T$. The control objective of the vertical landing controller is to guarantee the altitude, velocity, attitude angles, and angular rates to be zero. This altitude and vertical speed criteria are very difficult to implement in real case, so an altitude tolerance ($z_{\text{tol}} = 0.05m$) is used to test if the Martian aerobot is safe by just falling freely in the simulation. A small prescribed descending speed of $-2m \cdot s^{-1}$ is used to prevent the VRS when landing. In order to test the wind resistance of the proposed controller, a wind gust of $5m \cdot s^{-1}$ is imposed in $X_e$ direction between 10s and 30s. The other setups are similar to the take-off phase.

The landing simulation results are given by Figure 5-33 to Figure 5-46.

![Figure 5-33: 6 DoF landing simulation result for the altitude position](image-url)
Figure 5-34: 6 DoF landing simulation result for the horizontal velocity in $X_G$ axis

Figure 5-35: 6 DoF landing simulation result for the horizontal velocity in $Y_G$ axis

Figure 5-36: 6 DoF landing simulation result for the vertical velocity in $Z_G$ axis
Figure 5-37: 6 DoF landing simulation result for the roll attitude angle

Figure 5-38: 6 DoF landing simulation result for the pitch attitude angle

Figure 5-39: 6 DoF landing simulation result for the yaw attitude angle
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Figure 5-40: 6 DoF landing simulation result for the roll angular rate

![Roll Angular Rate Graph](image)

Figure 5-41: 6 DoF landing simulation result for the pitch angular rate

![Pitch Angular Rate Graph](image)

Figure 5-42: 6 DoF landing simulation result for the yaw angular rate

![Yaw Angular Rate Graph](image)
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Figure 5-43: 6 DoF landing simulation result for the collective pitch angle of the coaxial tiltrotor

Figure 5-44: 6 DoF landing simulation result for the differential collective pitch of the coaxial tiltrotor

Figure 5-45: 6 DoF landing simulation result for the collective pitch angle of the left auxiliary rotor
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Figure 5-46: 6 DoF landing simulation result for the collective pitch angle of the right auxiliary rotor

The curves in Figure 5-33 to Figure 5-36 show that the proposed landing controller guarantees safely landing of the Martian aerobot. Both the velocity and the altitude at the end of the simulation are very small. Figure 5-37 to Figure 5-42 show that the attitude angles and angular rates also converge to zero except small fluctuations in the pitch channel, which is similar to the results in taking off simulation. The control inputs given by Figure 5-43 to Figure 5-46 show the corresponding variations of the state variables. This simulation also shows the robustness of the proposed landing controller. Figure 5-38 and Figure 5-41 show that the pitch angle and pitch rate also fluctuating around zero after 30s in the simulation.

The oscillation is as obvious as that in taking off simulation (Figure 5-24 and Figure 5-27). The reason for the oscillation is the same as that in the taking off simulation, i.e. the nonlinearity of the thrust curve near zero thrust.

Figure 5-33 shows that the altitude of the aerobot drops smoothly. However, due to the existence of wind gust between 10s and 30s, the aerobot will try to float with the wind. However, the horizontal velocity during this period and the later 10s are not small. The altitude of the Martian aerobot is kept at the safe altitude (5m in this simulation) until the horizontal velocity is smaller than the predefined tolerance, the altitude begins to drop again.

5.9 Robust Control Law for Cruise Phase

The flight mode of Hyperion in cruise mode is similar to the fixed wing aircraft. The control of the cruise mode has already been discussed in previous work [Song, 2008]. The control method used in previous work was root locus based PD controller. Since the flight control of fixed wing aircraft is a well developed technique, the conventional control loops are used in this section. Since this part has already been discussed in previous work, the contents in this section is only used to complete the simulation loop in transition flight in Section 5.10.
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Figure 5-47: Free body diagram for the Martian aerobot in cruise mode

The free body diagram of the Martian aerobot in cruise mode is shown in Figure 5-47. The lift generated by the airframe will counteract the weight; while the thrust of coaxial tiltrotor is used to counteract the aerodynamic drag; the pitch moment can be adjusted by changing the deflection of effective elevator. In this flight mode, the thrust required is much smaller than that in VTOL mode.

5.9.1 Dynamic Model for Cruise Phase

The aerodynamic forces and moments of the airframe are given by Equation 5-23 and Equation 5-24. The forces and moments generated by the propulsion system is given by Equation 5-56 and Equation 5-57.

\[
\begin{align*}
\vec{F}_{aero} &= \begin{bmatrix}
T_{\text{coax}} \cos i_F \\
0 \\
T_{\text{coax}} \sin i_F
\end{bmatrix} \\
\hat{M}_{\text{prop}} &= \begin{bmatrix}
M_{\text{coax}} \cos i_F \\
0 \\
-M_{\text{coax}} \sin i_F
\end{bmatrix}
\end{align*}
\]

Equation 5-56
Equation 5-57

The forces and moments by the propulsion system are very simple since only the coaxial tiltrotor is in operation. It is worthwhile to note that the nacelle angle \(i_F\) is fixed to be -5.4°, which is the minus value of the nominal AoA in cruise.

The moment of inertia matrix in cruise mode is given by Equation 5-58. The meanings of the symbols are the same as that of the VTOL mode in Section 5.8.1.

\[
\bar{I} = \begin{bmatrix}
I_{xx} & 0 & -I_{xz} \\
0 & I_{yy} & 0 \\
-I_{xz} & 0 & I_{zz}
\end{bmatrix} \approx \begin{bmatrix}
I_{xx0} + I_0 & 0 & -I_{xz0} \\
0 & I_{yy0} + I_t & 0 \\
-I_{xz0} & 0 & I_{zz0} + I_t
\end{bmatrix}
\]

Equation 5-58
Therefore, the simplified longitudinal dynamic equations are given by Equation 5-59 to Equation 5-63. The lateral dynamic equations are given by Equation 5-64 to Equation 5-68.

\[ \dot{\theta} = q \]  
\[ \dot{q} = \frac{I_{zz} - I_{xx}}{I_{yy}} \frac{pr}{I_{yy}} + \frac{M}{I_{yy}} \]  
\[ \dot{u} = rv - qw - g\theta + \frac{1}{m}(T_{\text{cont}} - D + L\alpha) \]  
\[ \dot{w} = qu - pv + g + \frac{1}{m}(T_{\text{cont}}l_F - L - D\alpha) \]  
\[ \dot{z_e} = -u\theta + w \]  
\[ \dot{\phi} = p \]  
\[ \dot{\psi} = r \]  
\[ \dot{\theta} = \frac{I_{yy}}{I_{xx}} qr + \frac{L + M_{\text{cont}}^\text{diff}}{I_{xx}} \]  
\[ \dot{\phi} = \frac{I_{xx}}{I_{zz}} pq + \frac{N - M_{\text{cont}}^\text{diff}l_F}{I_{zz}} \]  
\[ \dot{\psi} = -ru + pw + g\phi + \frac{Y}{m} \]

### 5.9.2 Linearization Model for Cruise Phase

The linearized longitudinal dynamic model is given by Equation 5-69 and Equation 5-70. \( \delta_e \) is the effective elevator deflection; \( A_{\text{long}}^\phi, B_{\text{long}}^\phi, C_{\text{long}}^\phi, \) and \( D_{\text{long}}^\phi \) are the matrices for the state space representation of the linearized longitudinal dynamic model in cruise mode.

\[
\begin{bmatrix}
\delta u \\
\delta z_e \\
\delta w \\
\delta \theta \\
\delta q
\end{bmatrix}
= \begin{bmatrix}
\delta u \\
\delta z_e \\
\delta w \\
\delta \theta \\
\delta q
\end{bmatrix}
+ \begin{bmatrix}
A_{\text{long}}^\phi \\
B_{\text{long}}^\phi \\
C_{\text{long}}^\phi \\
D_{\text{long}}^\phi
\end{bmatrix}
\begin{bmatrix}
\delta e \\
\delta \psi
\end{bmatrix}
\]  

Equation 5-69
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The linearized lateral dynamic model is given by Equation 5-71 and Equation 5-72. $\delta_{\alpha}$ denotes the effective aileron deflection; $A_{lat}^{cr}, B_{lat}^{cr}, C_{lat}^{cr},$ and $D_{lat}^{cr}$ are the state space matrices for the linearized lateral dynamics in cruise. The differential collective pitch of the coaxial tiltrotor is not used for roll control, because the elevens are more efficient in cruise.

\[
\begin{bmatrix}
\delta u \\
\delta z_e \\
\delta \theta \\
\delta q
\end{bmatrix} =
\begin{bmatrix}
\delta u \\
\delta z_e \\
\delta \theta \\
\delta q
\end{bmatrix} + D_{long}^{cr} \begin{bmatrix}
\delta \theta_{coax}
\end{bmatrix}
\]

Equation 5-70

The exact expressions of the state space matrices are given in Appendix C-1.

### 5.9.3 Control Loops for Cruise Phase

The cruise mode of Hyperion is quite similar to the control of the conventional fixed wing aircraft. The control of the cruise mode using PD controller has already been discussed in the previous work for Halcyon [Song, 2008], based on the root locus method. In this section, a robust flight controller for cruise mode is proposed based on $\mu$ synthesis to complete the design loop for transition/conversion mode in Section 5.10.

The control of the fixed wing aircraft is composed of the control of longitudinal and lateral dynamics. Many controller architectures have been proposed for the fixed wing aircraft. The longitudinal control loops in this thesis are given by Figure 5-48. The cruise speed is controlled by the coaxial tiltrotor. The altitude holder will generate the command value of the pitch angle; and the pitch angle is controlled by the effective elevator. The advantage of this kind of control loop is that it is suitable for trajectory tracking. There is coupling between the cruise speed and altitude channels, but the controller can be designed separately.
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5.9.4 Simulation Results for Cruise Phase

A 6 DoF simulation is used to validate the proposed control law for cruise in this section. The simulation parameters used are given by Table D-2. The cruise controller in this thesis is a low level controller, which will keep the states of the Martian aerobot not far from the expected working state. Therefore, the objective of the cruise simulation will focus on if the proposed controller can stabilize the Martian vehicle. The initial states of the vehicle is selected as $[x_e, y_e]^T = [0, 0]^T$, $z_e = -50m$, $u = 44.8 m/s^1$, $v = 0$, $w = 4.2 m/s^1$, $[\phi, \theta, \psi]^T = [0^\circ, 0^\circ, 2^\circ]^T$, $[p, q, r]^T = [0, 0, 0]^T$. The command value is selected as $V_{cr}^{cmd} = 50 m/s^1$, $z_e^{cmd} = -50m$, $\psi^{cmd} = 0^\circ$. In
order to test the wind resistance of the proposed controller, a short period wind gust of $5\text{m}\cdot\text{s}^{-1}$ is imposed in $X_s$ direction between 10s and 11s. The uncertainty of aerodynamic model is represented by scaling each aerodynamic parameter by a number within [0.8, 1.2]. A Monte Carlo style simulation is used to represent the results for the uncertain aerodynamic model (red dashed line). The state variables for the longitudinal dynamics is given by Figure 5-50 to Figure 5-54; states for the lateral dynamics is given by Figure 5-55 to Figure 5-59. The control inputs generated by the actuators are given by Figure 5-60 to Figure 5-62.

![Figure 5-50: 6 DoF cruise simulation result for the cruise speed](image)

![Figure 5-51: 6 DoF cruise simulation result for the performance of altitude holder](image)
Figure 5-52: 6 DoF cruise simulation result for the AoA

Figure 5-53: 6 DoF cruise simulation result for the pitch angle

Figure 5-54: 6 DoF cruise simulation result for the pitch rate
Figure 5-55: 6 DoF cruise simulation result for the roll angle

Figure 5-56: 6 DoF cruise simulation result for the roll rate

Figure 5-57: 6 DoF cruise simulation result for the yaw angle
Figure 5-58: 6 DoF cruise simulation result for the AoS

Figure 5-59: 6 DoF cruise simulation result for the yaw rate

Figure 5-60: 6 DoF cruise simulation result for the collective pitch of the coaxial tiltrotor
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The cruise speed is shown in Figure 5-50. The simulation result shows that the proposed controller ensures the cruise speed to reach the command value with little influence of the aerodynamic performance.

Figure 5-51 gives the altitude curves for the simulation. There is some altitude loss at the beginning of the simulation (before 10s). The reason is that the initial cruise speed and pitch angle is lower than those of the working condition. There is an altitude steady state error for the uncertain model (red dashed lines). The larger is aerodynamic uncertainty, the larger is steady state error. The maximum altitude error for the given range of uncertainty is ±1m, which is accurate enough for the altitude control.

The steady state AoA for different aerodynamic performance is not the same, see Figure 5-52. The weight of the Martian aerobot is a constant, so the different aerodynamic performance will result in different steady state AoA. The steady state AoA for the nominal model is approximately 5.4°, which is the proposed high L/D AoA in Section 3.10. The bias of the steady state AoA for
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the selected aerodynamic uncertain model is within ±1°. This variation has little influence on the L/D of the vehicle, see Figure 3-9 and Figure 3-10.

Figure 5-53 and Figure 5-54 show the pitch angle and angular rate for cruise. The command value of pitch angle is generated by the altitude controller. The different the pitch angles will usually cause different AoA's, but not always. Therefore, the steady state pitch angles for the model with different aerodynamic performance are also different. This trend goes with the AoA shown in Figure 5-52. The pitch increase at the beginning of the simulation (before 10s) is to increase the AoA quickly to generate enough lift to counteract the weight.

The roll angle and angular rate are shown in Figure 5-55 and Figure 5-56. The yaw angle, AoS, and yaw angular rate are shown in Figure 5-57 to Figure 5-59. The lateral dynamics is stabilized generally. These figures show that the performance of the controller in lateral dynamics is not good. A very low yaw angle error (2°) will cause a large roll angle (6°) and a large AoS (4°) to reach the stability. The settling time is long, but the lateral controller also shows its robustness and can ensure the stability. The other control flaps are required to improve the lateral controllability of the Martian aerobot.

Figure 5-60 shows the collective pitch of the coaxial tiltrotor in cruise. The figure shows that the coaxial tiltrotor accelerate the vehicle at the beginning of the simulation; and then keep the thrust to counteract the aerodynamic drag.

Figure 5-61 and Figure 5-62 show the deflections of the left and right elevons during cruise. The final deflections are approximately 9.2°, which is the effective elevator angle for the proposed working state. The differential of the elevons will generate the roll moment, which will change the roll angle, then will result in the changing of the yaw angle. This process is slow, so Hyperion is not suitable for large yaw angle manoeuvre.

5.10 Robust Control Law for Transition and Conversion Modes

The flight mode of Hyperion in transition and conversion phases is similar to those coaxial tiltrotor and conventional tiltrotor aircraft. The control methods used for tiltrotor aircraft are reviewed in Section 2.4.2. However, the robustness of those proposed controllers is not mentioned in those corresponding publications. This section aims to propose a robust transition and conversion control law for the Martian aerobot between hover and cruise. Because the general concepts for the control of transition and conversion are the same, they are discussed together in this section.
Figure 5-63: Free body diagram of the Martian aerobot in transition/conversion mode

Figure 5-63 shows the free body diagram of the Martian aerobot in transition/conversion mode. In this flight mode, the weight is counteracted by the aerodynamic lift and the vertical component of the coaxial tiltrotor thrust. The aerodynamic drag is counteracted by the horizontal component of the coaxial tiltrotor thrust. Both the aerodynamic pitch moment and the auxiliary rotor thrust can be used to adjust the pitch angle. The aerodynamic moment for higher cruise speed is more efficient and much larger than that caused by the auxiliary rotor thrust. Therefore, we propose to use the auxiliary rotor for low speed transition/conversion flight and the effective elevator for high speed flight.

5.10.1 Dynamic Model for Transition and Conversion Phases

The aerodynamic forces and moments of the airframe in transition/conversion are the same as those in cruise, i.e. Equation 5-23 and Equation 5-24. The forces and moments generated by the propulsion system are given by Equation 5-73 and Equation 5-74.

\[
\begin{align*}
\vec{F}_{\text{aero}} &= \begin{bmatrix} T_{\text{coax}} \cos i_F \\ 0 \\ T_{\text{coax}} \sin i_F + T_r + T_t \end{bmatrix} \\
M_{\text{prop}} &= \begin{bmatrix} M^\text{diff} \cos i_F + (T_r - T_t) y_{\text{aux}} \\ (T_r + T_t) x_{\text{aux}} \\ -M^\text{diff} \sin i_F + M_r - M_i \end{bmatrix}
\end{align*}
\]

Equation 5-73
Equation 5-74

It is worthwhile to note that the nacelle angle \( i_F \) is a control variable during the transition and conversion process. Both the auxiliary rotors and the elevons are used as the control variables for pitch angle.

The moment of inertia matrix for the transition and conversion process is given by Equation 5-75.

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\[
\mathbf{I} = \begin{bmatrix}
I_{xx} & -I_{xy} & -I_{xz} \\
-I_{xy} & I_{yy} & -I_{yz} \\
-I_{xz} & -I_{yz} & I_{zz}
\end{bmatrix}
\]

Equation 5-75

\[
\mathbf{I} = \begin{bmatrix}
I_{xx0} + I_1 \sin^2 i_P + I_6 \cos^2 i_P & -I_{xy0} & -I_{xz0} \\
-I_{xy0} & I_{yy0} + I_t & -I_{yz0} \\
-I_{xz0} & -I_{yz0} & I_{zz0} + I_t \cos^2 i_P + I_0 \sin^2 i_P
\end{bmatrix}
\]

Equation 5-75 shows that the moment of inertia matrix of the whole arobot system will change with the nacelle angle. This will cause great trouble in the control system design. However, we can use a constant inertia matrix instead due to the following reason. It is worth noting that only the elements in \(X_B\) and \(Z_B\) directions are changing with the nacelle angle. As presented in Equation 5-7 to Equation 5-9, the three main elements of the airframe moment of inertia are much larger than those of the rotor moment of inertia, especially the elements in \(X_B\) and \(Z_B\) direction. Therefore, we use constant moment of inertia in the controller design process. The constant moment of inertia is estimated by Equation 5-76 and Equation 5-77. These two numbers are only used for controller design. The original equation (Equation 5-75) is used in the final simulation work.

\[
I_{xx} = I_{xx0} + \frac{1}{2}(I_t + I_0) = 123.65 \text{kg} \cdot \text{m}^2
\]

Equation 5-76

\[
I_{zz} = I_{zz0} + \frac{1}{2}(I_t + I_0) = 131.22 \text{kg} \cdot \text{m}^2
\]

Equation 5-77

The simplified longitudinal dynamic equations are given by Equation 5-78 to Equation 5-82. The lateral dynamic equations are given by Equation 5-83 to Equation 5-87. Some items in the equations can be neglected, because the thrust and torque generated by the auxiliary rotors are much smaller than those by the coaxial tiltrotor.

\[
\dot{q} = q
\]

Equation 5-78

\[
\dot{q} = \frac{I_{zz} - I_{xx}}{I_{yy}} \frac{pr}{I_{yy}} + \frac{M + (T_0 + T_1) v_{max}}{I_{yy}} - \frac{I_t}{I_{yy}} \dot{i}_P
\]

Equation 5-79

\[
\dot{u} = rv - qw - g\theta + \frac{1}{m} (T_{ coax \ cos i_P} - D + L \alpha)
\]

Equation 5-80

\[
\dot{w} = qu - pv + g + \frac{1}{m} (T_{ coax \ sin i_P} + T_r + T_i - L - D \alpha)
\]

Equation 5-81

\[
\dot{\alpha} = -u\theta + w
\]

Equation 5-82

\[
\dot{\phi} = p
\]

Equation 5-83
\[
\dot{\psi} = r
\]

\[
\dot{\rho} = \frac{I_{yy} - I_{zz}}{I_{xx}} qr + \frac{L + M_{\text{diff}} \cos i_p + (T_c - T_i)y_{\text{ext}}}{I_{xx}}
\]

\[
\dot{\rho} = \frac{I_{xx} - I_{yy}}{I_{zz}} pq + \frac{N - M_{\text{diff}} \sin i_p}{I_{zz}} + M_i - M_l
\]

\[
\dot{v} = -ru + pw + g\phi + \frac{Y}{m}
\]

Equation 5-84
Equation 5-85
Equation 5-86
Equation 5-87

5.10.2 Transition Corridor and Transition Trajectory

The transition strategy in the previous work is not presented. If the vehicle in hovering state transits to cruise state directly, the disturbance and the altitude loss will be very large, so the Martian aerobot must climb to a very high altitude to guarantee the safety.

The transition corridor and the transition trajectory are two very important concepts for a smooth transition control problem. The working points of hover and cruise controllers for Hyperion are too far from each other. A series of temporary transition states and a transition trajectory are required to ensure the stability and small disturbance during the transition and conversion between hover and cruise. These transition states should be selected along the transition trajectory. There are infinite choices of transition trajectories; and there is infinite number of steady flight states on the trajectory. Usually a lot of experiments are required to decide the transition corridor and a suitable transition trajectory. The boundaries of the transition corridor are estimated by the aerodynamic property of the Martian aerobot in this thesis. Figure 5-64 shows the longitudinal force diagram during transition and conversion. The coaxial tiltrotor will counteract part of the gravity to keep the altitude and counteract the drag to keep the specific cruise speed.
The transition trajectory aims to accelerate the cruise speed from zero to the nominal value steadily and safely with little altitude loss. Assume the pitch angle and the AoA are fixed and there is no altitude change during the whole transition process, so the longitudinal dynamic equations in the inertia frame are given by Equation 5-88 and Equation 5-89.

\[
m \dot{V}_c = T_{\text{coax}} \cos(i_f + \alpha) - D \\
\dot{m} = -T_{\text{coax}} \sin(i_f + \alpha) - L + mg
\]

Consider those steady states in transition and conversion. The climb angle is zero, so the AoA \( \alpha \) is equal to the pitch angle \( \theta \) if there is no wind disturbance. The aerodynamic coefficients \( C_l \) and \( C_D \) are two constants. The aerodynamic forces \( L \) and \( D \) are proportional to \( V_c^2 \), according to Equation 5-23. The power of the coaxial tiltrotor is assumed to be able to reach the thrust requirement during the transition and conversion, then we have the relationship given by Equation 5-90.

\[
i_f = \arctan \left( \frac{mg - L}{D} \right) - \alpha
\]

In Figure 5-65, the wing stall boundary \( (\alpha=12^\circ) \) is represented by the green line. The boundary with very low AoA \( (\alpha=1^\circ) \), which is an estimation of the performance limit of coaxial tiltrotor, is represented by the red line. The enclosed area by these two boundaries is the transition corridor for Hyperion. Any point in the transition corridor represents a possible transition state. The transition trajectory should be selected within the transition corridor. The one used in this thesis is the trajectory corresponding to the AoA for cruise. The change of nacelle angle \( (i_f) \) and cruise speed \( (V_c) \) of the transition trajectory is shown by the blue dotted line in \( (\alpha=5.4^\circ) \) in the figure. All the transition states are selected from this trajectory.
5.10.3 Divide and Conquer Gain Scheduling Method

Trajectory tracking is not a new topic in aircraft control problems. Two approaches are usually used for such robust tracking problem. One is trajectory linearization control method based on the linear control theory [Mickle et al., 2004]; the other is the direct nonlinear control method based on the Lyapunov functions [Freeman and Kokotovic, 1996]. Because the linear control theory is well developed, the robustness of trajectory linearization control method is guaranteed theoretically. The robust nonlinear control theory up to date is proposed for affine systems with uncertainty independent of the control term only. Therefore, the trajectory linearization based method is selected in this thesis. The trajectory linearization controller is composed by trajectory pseudo-inverse dynamics and linear controller, see Figure 5-66. \( \eta \) denotes the system output; \( \mu \) denotes the system input; \( \bar{\eta} \) denotes the command value of \( \eta \); \( \bar{\mu} \) denotes the command value of \( \mu \); \( \tilde{\eta} \) denotes the output error; \( \tilde{\mu} \) denotes the output from the LPV controller.

![Figure 5-66: Trajectory linearization control scheme [Mickle et al., 2004]](image)

The linearized dynamics based on a nominal trajectory is usually a LPV model. Stabilization of the general LPV control problem is still an open problem. Some literature gives results on the robust control of LPV system. These results are much more complex and not easy to use than LTI problem. The gain scheduling method is one of the widely and successfully used control methods for LPV problems. The gain scheduling method originates from the 1960s and was regarded as an engineering trick until the early 1990s. The classical gain scheduling methods are based on the interpolation of controller parameters (including parameter coefficients, zeros, poles, state matrices, and etc.). Although classical gain scheduling is widely used [Fitzpatrick, 2003], these controllers even can not be proved to be stable in theory [Amato, 2006]. Considering the cost for launching a Martian aerobot, it is impractical to do the flight test for Mars enviroment. Therefore, the classical gain scheduling method is not appropriate.

Modern gain scheduling theory has two branches: one is the LPV gain scheduling method; the other is the D&C gain scheduling method. Leith and Leithead [Leith and Leithead, 2000] give a good survey on these two methods.
The general concept of LPV gain scheduling approach is to find a Linear Differential Inclusion (LDI) containing the dynamic behaviour of the nonlinear system. The controller is designed based on the LDI. It can be proved that if the controller can robustly stabilize the LDI, the controller can robustly stabilize the nonlinear system. A main shortcoming is there is no standard procedure to find a suitable LDI for a given nonlinear system. The resultant controller is also sensitive to the LDI. The LPV gain scheduling controller can be used for any nonlinear problems with an appropriate LDI. The performance of LPV controller is also sensitive to the selected LDI.

For D&C gain scheduling method, the original problem is broken into several subproblems. These subproblems are of the same kind with a much smaller parameter variation. Control of the subproblems is much easier. Appropriate combination of the controllers of the subproblems is the solution of the original control problem. D&C method can be used for time varying system when the dwell-time is small enough [Vu and Liberzon, 2011]. That is, the settling time after controller switching is faster than the variation of the parameters. For a given LPV control problem, the divided subproblems can be approximated by a LTI problem with appropriate parameter uncertainties, when the range of time varying parameters is small. Because the control theory for LTI is complete, the D&C gain scheduling method is the subject of considerable research activity.

The linearized tracking error dynamics of Hyperion in transition is a LPV model. The continuously varying parameters in the model are the increasing cruise speed ($V_c$). The variation span of cruise speed is from 0 m·s⁻¹ in hover to 50 m·s⁻¹ in cruise. It is very difficult to design a controller for such large variation of parameters. Therefore, the modern gain scheduling method is required. Considering the inherent uncertainty of aerodynamic model, and there is no prior information about how to select an appropriate LDI, The D&C gain scheduling is selected in this thesis.

![Figure 5-67: Transition states within the transition corridor](image-url)
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Theoretically, any points along the proposed transition trajectory shown in Figure 5-65 can be selected as the transition states. The figure shows that the nacelle angle change rate at low cruise speed is very small but very large for high cruise speed, with the increase of cruise speed. In such case, fewer transition states at low cruise speed are selected. For the states close to the nominal cruise speed, the change of nacelle angle is very large, especially from 45m·s⁻¹ to 50m·s⁻¹. The reason is as follows. The thrust of the coaxial tiltrotor is regarded as a passive force. The lift generated by the wing (or airframe) is proportional to the second roder of the cruise speed, so the lift variation rate for high speed flight is higher than that for low speed flight. The nancelle angle is related with the ratio between the thrusts required in vertical direction and in forward direction (Equation 5-90). Therefore, the variation rate for high speed is higher than that for low speed. If the variation rate is higher in low speed flight, the vertical component of thrust can not be counteracted, since the lift generated by the wing is negligible comparing with the gravity.

Many choices of the transition states are possible. The steady states used in this thesis are given by Table 5-1 and Figure 5-67.

Table 5-1: Transition states for transition/conversion controller design

<table>
<thead>
<tr>
<th>State</th>
<th>I (cruise)</th>
<th>II</th>
<th>III</th>
<th>IV</th>
<th>V</th>
<th>VI</th>
<th>VII</th>
<th>VIII (cruise)</th>
</tr>
</thead>
<tbody>
<tr>
<td>V₀ (m·s⁻¹)</td>
<td>0</td>
<td>10</td>
<td>20</td>
<td>30</td>
<td>35</td>
<td>40</td>
<td>45</td>
<td>50</td>
</tr>
<tr>
<td>i₀ (deg)</td>
<td>84.6</td>
<td>84.5</td>
<td>84.1</td>
<td>83.0</td>
<td>82.0</td>
<td>79.7</td>
<td>73.1</td>
<td>-5.4</td>
</tr>
<tr>
<td>Tₗ₀₀ₖ (N)</td>
<td>92.75</td>
<td>89.05</td>
<td>77.97</td>
<td>59.50</td>
<td>47.51</td>
<td>33.72</td>
<td>18.25</td>
<td>4.5 (10⁵)</td>
</tr>
<tr>
<td>L (N)</td>
<td>0</td>
<td>3.697</td>
<td>14.97</td>
<td>33.27</td>
<td>45.29</td>
<td>59.15</td>
<td>74.86</td>
<td>92.75</td>
</tr>
<tr>
<td>Vertical thrust (N)</td>
<td>92.75</td>
<td>89.05</td>
<td>77.78</td>
<td>59.48</td>
<td>47.46</td>
<td>33.60</td>
<td>17.89</td>
<td>4.5 (10⁵)</td>
</tr>
<tr>
<td>Horizontal thrust (N)</td>
<td>0</td>
<td>0.16</td>
<td>0.68</td>
<td>1.66</td>
<td>2.16</td>
<td>2.88</td>
<td>3.64</td>
<td>4.5 (10⁵)</td>
</tr>
</tbody>
</table>

The drag estimated by Equation 5-25 is too low. A more reasonable value (10N) is used for cruise mode

5.10.4 Linearization Model for Transition and Conversion Modes

The dynamic model given by Equation 5-78 to Equation 5-87 is linearized using the frozen coefficients method at the steady states in Table 5-1. Then, the linearized longitudinal and lateral dynamic models are given by Equation 5-91 to Equation 5-94. \( A_{\text{long}}^{\text{trans}}, B_{\text{long}}^{\text{trans}}, C_{\text{long}}^{\text{trans}}, D_{\text{long}}^{\text{trans}}, A_{\text{lat}}^{\text{trans}}, B_{\text{lat}}^{\text{trans}}, C_{\text{lat}}^{\text{trans}}, D_{\text{lat}}^{\text{trans}} \) denote the state-space representation matrices for linearized longitudinal and lateral error dynamic models. The state and control variables are the same as those for VTOL and cruise phases. It is worth noting that these equations are required to be solved for each state given by Table 5-1. The expressions of these state matrices are given in Appendix C-2.
5.10.5 Transition and Conversion Controller Structure

The transition/conversion controller in this thesis has two levels. The higher level is a switching system with a switching signal. The switch signal is generated by the switch logic to decide which lower level candidate controller is used for the present control task. The candidate controllers are in the lower level. These candidate controllers are designed using the \( \mu \) synthesis in Figure 5-12.

5.10.5.1 Divide and Conquer Controller Structure

In this thesis, eight candidate controllers (including the hover and cruise cases) for the steady states on the transition trajectory, corresponding to those in Table 5-1, are used. Figure 5-68 shows the structure of D&C gain scheduling controller used in this thesis. The candidate controllers for each state form a controller pool for selection. The switch logic will select the proper candidate controller based on the current state. Vu [Vu and Liberzon, 2011] pointed out that the dwelling time is very critical for the stability and performance of switching system. That is, the settling time of control system should be faster so as to the parameter will not change too much within the settling time. The transition is accomplished by reaching these steady states one
by one, and the switch logic is judging if the present steady state is reached. The criterion used in this thesis is the cruise speed and altitude errors are smaller than a predefined tolerance $V_{cr}^{\text{tol}}$.

![Diagram](image)

**Figure 5-68: Divide and Conquer gain scheduling control scheme for transition and conversion**

### 5.10.5.2 Candidate Controller Structure

After the Divide and Conquer operation, the subproblems are also LPV control problems, but the range of the variable parameter is much smaller. The subproblems can be treated as an LTI control problem using the objective steady state as nominal state, then the LTI control problem can be solved. The flight control of *Hyperion* in hover and cruise are solved in Sections 5.8 and 5.9. The focus in this section is the controller design of the transition states shown in Table 5-1.

![Diagram](image)

**Figure 5-69: Control loops of the candidate controllers for the longitudinal dynamics in transition and conversion**

The controller structure for the longitudinal dynamics is shown in Figure 5-69. The command pitch angle ($\theta^{\text{cmd}}$) and the command altitude ($z_e^{\text{cmd}}$) are constant during transition and conversion.
The command cruise speed \( V_{cr}^{cmd} \) will change step by step based on the switch logic in Figure 5-68. Both the total auxiliary rotor pitch and effective elevator both can be used to control the pitch angle. The auxiliary rotors are used to control the pitch angle for the low cruise speed, because the effect of elevator is negligible for low cruise speed. With the increase of cruise speed, the elevator is more effective and more efficient than the auxiliary rotors. Especially at high cruise speed, the pitch moment generated by the auxiliary rotors are much smaller than the deflection of the effective elevator. Hence, there must be a conversion point during the transition/conversion. There are infinite choices of the conversion point. The conversion point used in this thesis is the forward speed limit of VTOL flight mode \( V_{cr} = 10 \text{m/s}^2 \). It is worthwhile to note that the nacelle angle of all the transition states in Table 5-1 is larger than 70°, so the thrust of coaxial tiltrotor has more influence on the “vertical” direction (as shown in Figure 5-67). This “vertical” direction does not mean large vertical thrust requirement. It just shows that the thrust required in vertical direction is larger than that in horizontal direction. Therefore, the coaxial tiltrotor pitch \( \Theta_{rot} \) is used to control the altitude, and the nacelle angle \( i_r \) will be used to accelerate the cruise speed.

![Figure 5-70: Control loops of the candidate controllers in lateral dynamics in transition and conversion](image)

Figure 5-70 shows the control loop for lateral dynamics. The objective for lateral control is to stabilize the rolling and yawing attitude angles to be nominal values. Because the nacelle angles for the selected transition states in Table 5-1 are larger than 70°, the coupling for the roll and yaw channels are not large. The differential auxiliary rotor pitch \( \Theta_{aux}^{diff} \) and effective aileron \( \delta_a \) is used to control the roll, and the differential of coaxial tiltrotor \( \Theta_{coax}^{diff} \) is used to control the yaw angle. In this thesis, the conversion point selected for differential auxiliary rotor pitch and effective aileron is also the forward velocity limit in VTOL mode, which is the same as pitch control in longitudinal mode. It is worth noting that the AoS is not controlled explicitly, as in cruise control case. Because of the robust stability of the lateral controller, the AoS, which is an unobservable state variable, can be stabilized.
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It is worth noting that the nacelle angle should be changed slowly to prevent possible collisions of the rotor blades. Therefore, an actuator with low response is used. In this thesis, the natural frequency of the nacelle angle actuator is $\omega_n = 1\text{Hz}$; and the damping is $\xi = 0.7$.

5.10.6 Simulation Result for Transition and Conversion Phases

In this section, a 6 DoF simulation is used to validate the proposed control law transition and conversion phases. The corresponding parameters used for the simulation are given by Table D-3.

5.10.6.1 Simulation Results for Transition Phase

In this section, a 6 DoF simulation is used to validate the proposed transition control law. Hyperion is required to track the transition trajectory by passing the steady states from State I to State VIII in Table 5-1 during transition. The initial conditions are selected as $[x_v,y_v]^T = [0,0]^T$, $z_v = -50\text{m}$, $[\phi,\theta,\psi]^T = [3^\circ, 1.3^\circ, 2^\circ]^T$, $[p,q,r]^T = [0,0,0]^T$. According to the mission profile, the transition is the flight phase after taking off, so the initial conditions are the final states after taking off. The initial values of roll and yaw attitude angles are used to test the effect of lateral controller for transition. The command cruise speed ($V_{cr}^{cmd}$) during transition is governed by the switch logic in Figure 5-68. The nacelle angle change between the State VII and cruise (State VIII) in Table 5-1 is very large (approximately 70°); and this will cause the pitch angle to increase about 15°; then cause the altitude to increase significantly. Because the Martian aerobot is required to climb after transition (see Table 3-1), the command altitude for cruise can be set higher than that for transition, so the command altitude of 70m is used for cruise. The aerodynamic coefficients are scaled a number within $[0.8, 1.2]$. A proper saturation is assigned for every control actuators to prevent large control input. The simulation results are shown in Figure 5-71 to Figure 5-87. The red dashed lines in the figures represent a Monte Carlo style simulation results for uncertain model. The black solid lines denote the simulation results for nominal model. The state variables for longitudinal and lateral dynamics are shown in Figure 5-71 to Figure 5-75 and Figure 5-76 to Figure 5-80. The control inputs are given by Figure 5-81 to Figure 5-87. In general, the proposed controller can steadily ensure the vehicle to convert from hover to cruise.
Figure 5-71: 6 DoF transition simulation result for cruise speed

Figure 5-72: 6 DoF transition simulation result for altitude

Figure 5-73: 6 DoF transition simulation result for the AoA
Figure 5-74: 6 DoF transition simulation result for the pitch attitude angle

Figure 5-75: 6 DoF transition simulation result for the pitch angular rate

Figure 5-76: 6 DoF transition simulation result for the roll attitude angle
Figure 5-77: 6 DoF transition simulation result for the roll angular rate

Figure 5-78: 6 DoF transition simulation result for the yaw attitude angle

Figure 5-79: 6 DoF transition simulation result for the AoS
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Figure 5-80: 6 DoF transition simulation result for the yaw angular rate

Figure 5-81: 6 DoF transition simulation result for the nacelle angle

Figure 5-82: 6 DoF transition simulation result for the collective pitch of the coaxial tiltrotor
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Figure 5-83: 6 DoF transition simulation result for the collective pitch angle of the left auxiliary rotor

Figure 5-84: 6 DoF transition simulation result for the collective pitch angle of the right auxiliary rotor

Figure 5-85: 6 DoF transition simulation result for the deflection of the left elevon
The cruise speed and altitude during transition are given by Figure 5-71 and Figure 5-72. The cruise speed increase gradually by reaching the proposed steady states one by one in Table 5-1. Due to the existence of uncertainty in the model, the settling time for these same steady states is different; therefore, the time cost in the transition process is also different. For the worst case, the time required to increase to cruise speed is completed within 150s. Figure 5-72 shows that the altitude generally follows the command values. The simulation results show that there is some altitude loss for the cases with low aerodynamic performance. Especially for the uncertainty of 0.8, the altitude drops to 40m at about 85s. The drop results in an increase of cruise speed. So the time required for lower aerodynamic performance cases is less. This effect is not significant for uncertainty larger than 0.9.

The AoA and pitch attitude angle during transition are shown in Figure 5-73 and Figure 5-74. The AoA and the pitch angle generally are kept at the command value of 5.4°. The sharp changes
shown in the figures are caused by changing the nacelle angle. Since the nacelle angle changes between the first seven transition steady states in Table 5-1 are not large, the tilting induced pitch variations are also very small. The largest pitch and AoA changes are between the state VII and state VII (cruise), see Table 5-1. The final pitch and AoA are also different for the cases with different aerodynamic performance, which has already been discussed in the cruise case.

Figure 5-75 shows the pitch angular rate during transition. The proposed transition controller can stabilize the pitch rate to zero. The large variations in the figure are caused by the nacelle angle change when switching between the transition states.

The roll and yaw angles in the lateral dynamics are shown in Figure 5-76 to Figure 5-80. The roll and yaw dynamics are controlled separately for the first seven transition states, so the performance of the controller are very good. The roll and yaw angles converge very rapid. When Hyperion converts to cruise mode, the rolling will be an inner loop dynamics of the yaw angle. The convergence of both channels is not as fast as the other transition states. The AoS shown in Figure 5-79 converges very rapidly. The main contribution is the increase of cruise speed. The AoS is finally stabilized to zero only when converting to the cruise mode.

Figure 5-81 and Figure 5-82 show the nacelle angle and the collective pitch angle of coaxial tiltrotor during transition. The nacelle angle during transition generally shows a multi-step behaviour, which denotes that the nacelle angle for one control period is almost a constant. With the saturation of the control inputs, the nacelle angle will reach the maximum when accelerating, and will tilt back while the proposed cruise speed is reached. Because of the different aerodynamic performance, the switching time between transition states is different. While the Martian aerobot is passing through these transition states, the collective pitch angle of coaxial tiltrotor generally decreases with a bit increase at the beginning. The general trend is governed by the working points at each transition states. The thrust required from hover to cruise decrease rapidly (see Table 4). The increase of collective pitch is caused by the incremental of axial inflow rate and the requirement of acceleration. When Hyperion converts to the cruise mode, the rotational speed of coaxial tiltrotor will slow down to decrease the aerodynamic drag of blades. Therefore the pitch of coaxial tiltrotor jumps up quickly. The final collective pitch remains approximately 34°, which is the working point of the coaxial tiltrotor in cruise.

Figure 5-83 to Figure 5-86 show the collective pitch of two auxiliary rotors and the deflections of two elevons. As discussed in Section 5.10.5.2, the auxiliary rotors active only from the hover (State I) to State II, then the elevons will work starting from State II. The roll angle is stabilized to zero during this period (see Figure 5-76), so significant differential of auxiliary rotors while almost the same elevon deflections. Large deflections of elevons occur when the pitch angle has large variation from the command value. Figure 5-85 and Figure 5-86 show that the largest
deflection of the elevons occurs at the simulation time of about 35s, on which time the pitch has large error. Although the pitch variation at this time is smaller than that when converting to cruise, the elevons are not effective for the low cruise speed. Generally, such a control scheme can guarantee the stability during transition.

Figure 5-87 gives the differential collective pitch of the coaxial tiltrotor, which is used to control the yaw attitude angle. Because the allowable maximum torque generated by coaxial tiltrotor is large, yaw angle is controlled to almost zero from hover (State I) to State II. Due to the existence of AoS induced when controlling the roll and yaw angle, the yaw angle is not stabilized only approximately zero (see Figure 5-78). The reason is the AoS is not directly controlled during transition. Generally, the proposed yaw holder is effective and validated by the simulation.

5.10.6.2 Simulation Results for Conversion Phase

The 6 DoF simulation to validate the proposed conversion controller is presented in this section. In the conversion process, Hyperion is required to track the transition trajectory by passing the steady states in Table 5-1 from State VIII to State I. The nacelle angle change between the State VIII (cruise) and the State VII are almost 70°, so the tilting induced pitch angle is about 18°. The initial conditions are selected as \[ x, y, z \] = [0, 0, 0], \[ u, v, w \] = [48.7 m s\(^{-1}\), 0, -11.2 m s\(^{-1}\)], \[ \phi, \theta, \psi \] = [3°, -13°, 2°], \[ \rho, q, r \] = [0, 0, 0]. The corresponding initial cruise speed is 50 m s\(^{-1}\). According to the mission profile, the conversion is the flight phase after cruise, so the initial conditions can be regarded as the final states after cruise. The initial value of roll and yaw angles can be stabilized very close to zero. The initial values of roll and yaw attitude angles are used to test the effect of lateral controller for conversion. The command cruise speed (\( V_{\text{cmd}} \)) during transition is governed by the switch logic in Figure 5-68. The nacelle change between the State VII and cruise (State VIII) in Table 5-1 is very large (approximately 70°), so the pitch angle will decrease about 17°. This negative pitch angle will cause a significant altitude loss. Because Hyperion is required to land vertically after conversion (see Table 3-1), a larger initial altitude is selected for safety. The command altitude for conversion in this simulation is 50 m. The uncertainty of aerodynamic model is represented by scaling each aerodynamic parameter by a number within [0.8, 1.2]. The saturations assigned the control actuators are the same as those for transition process. The simulation results are shown in Figure 5-88 to Figure 5-103. The red dashed lines in the figures represent a Monte Carlo style simulation results for uncertain model. The black solid lines denote the simulation results for nominal model. The state variables for longitudinal dynamics are shown in Figure 5-88 to Figure 5-92. The state variables for lateral dynamics are given by Figure 5-93 to Figure 5-97. The control inputs are given by Figure 5-98 to Figure 5-103. In general, the proposed controller can steadily ensure the Martian aerobot to convert from cruise to hover.
Figure 5-88: 6 DoF conversion simulation result for the cruise speed

Figure 5-89: 6 DoF conversion simulation result for altitude

Figure 5-90: 6 DoF conversion simulation result for the AoA
Figure 5-91: 6 DoF conversion simulation result for the pitch attitude angle

Figure 5-92: 6 DoF conversion simulation result for the pitch angular rate

Figure 5-93: 6 DoF conversion simulation result for the roll angle
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Figure 5-94: 6 DoF conversion simulation result for the roll angular rate

Figure 5-95: 6 DoF conversion simulation result for the yaw attitude angle

Figure 5-96: 6 DoF conversion simulation result for the AoS
Figure 5-97: 6 DoF conversion simulation result for the yaw angular rate

Figure 5-98: 6 DoF conversion simulation result for the nacelle angle

Figure 5-99: 6 DoF conversion simulation result for the collective pitch of the coaxial tiltrotor
Figure 5-100: 6 DoF conversion simulation result for the collective pitch of the left auxiliary rotor

Figure 5-101: 6 DoF conversion simulation result for the collective pitch of the right auxiliary rotor

Figure 5-102: 6 DoF conversion simulation result for the left elevon deflection
The cruise speed and the altitude in conversion are shown in Figure 5-88 and Figure 5-89. It is worth noting that the cruise speed increases to approximately 55m·s⁻¹ at the beginning of conversion process. This increase is caused by the altitude loss during this period. After reaching the maximum, the cruise speed decreases step by step to approximately zero. The Martian aerobot dives from the altitude of 150m to about 45m when the cruise speed reaches its maximum. The altitude is kept at 50m during the whole conversion process. There is a small altitude jump when the Martian aerobot transformed into the VTOL mode for hover, but it falls back to the command value rapidly.

The AoA and pitch angle during the conversion mode are shown in Figure 5-90 and Figure 5-91. The AoA reaches almost 14° at the beginning of the conversion process. This is caused by the altitude loss. The AoA has little influence when Hyperion in VTOL mode, so the AoA for this period is represented by zero in Figure 5-90. The pitch angle is stabilized to the command value during the conversion. The pitch increased to about 20° to decelerate the cruise speed when transforming to the VTOL mode in Figure 5-91. The pitch at the final period also shows some little fluctuation, which is the same as in VTOL phases.

The pitch angular rate during the conversion process is shown by Figure 5-92. The disturbances are caused by tilting the nacelle. The pitch rate also shows the fluctuation caused by the nonlinearity of the auxiliary rotor thrust, as in the VTOL mode.

The lateral performance of Hyperion in conversion process is shown by Figure 5-93 to Figure 5-97. The roll angle is stabilized to zero very rapid; while the yaw angle is much slower. The AoS is caused when adjusting the yaw angle. There is a roll angle jump when transforming to the VTOL mode. This jump is used to control the lateral velocity (v). The AoS at the final part of Figure 5-93 is also represented by zero, which is similar to the AoA in Figure 5-90.
The nacelle angle during conversion mode shown in Figure 5-98 also has a multi-step property, which is the same as the transition mode in Figure 5-81. The values of the nacelle angle are sometimes more than 90°, so the coaxial tiltrotor is used to decelerate the cruise speed.

Figure 5-99 gives the collective pitch of the coaxial tiltrotor in conversion mode. The trend is quite similar to the transition mode.

The collective pitch angles and the elevon deflections are shown in Figure 5-100 to Figure 5-103. Because the elevons are very effective as "elevator" and "aileron", the large deflections only appear when changing the nacelle angle. The fluctuation of the auxiliary rotor in VTOL mode is caused by the nonlinearity of the thrust curve, as in the VTOL simulations.

5.11 Summary

This chapter presents a low level robust control law (attitude and velocity stabilization) for the proposed Martian aerobot in the whole flight envelope. The aerodynamic model during flight is based on the steady state vortex lattice method. The difficulty for this control problem is to guarantee the robustness for each flight phase, since the aerodynamic property of the Martian aerobot is very difficult to predict. A usual practice is to superimpose an uncertainty of ±20%.

The steady flight control problems, such as taking off, landing, and cruise, are much easier. The control objective can be reached by the linear robust control theory. The flight control loops for VTOL follow those of the quadrotors; the control loops for cruising follow those of the conventional flying-wing aircraft. The 6 DoF simulation shows the robustness and the performance of the proposed controller.

The flight control during transition and conversion between hover and cruise is the focus for the control problem. The transition and conversion flights are two transient flight phases. The control objective for transition and conversion flight is to guarantee that the flight states are within the transition corridor. Direct transition and conversion between hover and cruise are possible only in theory, since this will cause very large disturbance or even unstability. Conventional flight control method for transition and conversion is the classical gain scheduling method. However, the stability and robustness of such method is questionable. In this work, a combination of the trajectory linearization technique and the D&C gain scheduling method are used to simplify the control problem. The simplified problems can be solved by the linear robust control method. The robust stability of each subproblem is guaranteed by the linear robust control theory. The robust stability of the transition and conversion control law is guaranteed by the switching speed of the controller and the varying parameter (cruise speed), i.e. the switching criterion. Since the transition and conversion controller will guarantee the aerobot reach the transition states one by
one, the proposed transition and conversion control law is robustly stable. The 6 DoF simulation also shows the robustness and performance of the proposed transition and conversion controller.

The interaction between the rotors and the airframe is not considered in this thesis. For conventional helicopters, the existence of the airframe will increase the rotor thrust a little (no more than 5% according to [Leishman, 2006]). The reason for conventional helicopters is that the airframe under the rotor will change the structure of the wake generated by the rotor. However, the airframe of the present Martian aerobot design is not below the rotor, so this increase is expected to be smaller than 5%. In this case, the interaction can be neglected in this work for preliminary design.
Chapter 6

6 Summary and Conclusions

6.1 Summary

This thesis presents the design and control for Hyperion, which is the third generation of Martian aerobot proposed by the Surrey Space Centre to investigate the Isidis Planitia region on Mars. Hyperion is an improvement based on the previous design. Hyperion is a solar-electrical powered VTOL coaxial tiltrotor aerobot. It has a zagi flying wing structure with a pair of coaxial tiltrotor embedded at the centre and an auxiliary rotor at each wing tip. The coaxial tiltrotor will be used to counteract the weight in VTOL mode and aerodynamic drag in cruise mode. The auxiliary rotors are used to control the pitch and roll attitude angles during the VTOL and low speed transition phases. Two topics are selected as the main contribution of this thesis. One is the aerodynamic design of the coaxial tiltrotor due to the limited solar energy on Mars; the other is the robust flight control method, especially the control strategy for transition and conversion phases.

The coaxial tiltrotor is not a new concept, but it is only used in recent applications. The design of coaxial tiltrotor is a broad topic. In this thesis, the Reynolds number of the rotor system is 30,000. It falls into the low Reynolds number regime. However, the rough blade section and careful placement of sectional AoA will help to prevent the separation, so we can still use the Weissinger-L model for rotor blade. Since the rotor is not proposed to be operating at or beyond the stall point, the Reynolds number factor is ignored for this preliminary work. That is, the design method and the aerodynamic model for high Reynolds number will not cause significant error if used for the rotor system of the Martian aerobot. This thesis focuses on the efficiency improvement. In such a case, the coaxial tiltrotor design problem can be regarded as designing a coaxial rotor efficiently working under two different conditions (low and high inflow rate). This work emphasises on the preliminary aerodynamic shape design, so the other factors (such as the rotor distance, rotational speed, rotor diameter, and etc.) just follow the parameters from the previous design. The vortex based theory is selected as the fundamentals because of its lower computational cost comparing with the N-S equations based CFD method and very high precision of the blade load distribution comparing with the classical BEMT method. PWM and FWM are two main methods for the vortex based theory. Considering the spanwise loading precision and computational cost, the PWM is selected in the optimization process, while the FWM is selected for validation.
Chapter 6. Summary and Conclusion

The optima coaxial rotor for either hover or cruise is still an open problem based on the vortex based theory. The coaxial rotors with uniform blade loading for both rotors are know to be efficient close to the global optima, so the uniform circulation is used as the optimization criterion for both hover and cruise cases. According to the BEMT, the rotors with optimum efficiency are known to have exponential blade twist and exponential blade chord distributions along the blade. The blade chord distribution is kept not for optimization because the blade chord has less influence on the performance, and the Weissinger-L model is directly relevant with the chord distribution. The optimum blade twists of the coaxial rotor for both hover and cruise can be found using the method proposed, shown in Figure 4-31. Three blade taper cases (1:1, 2:1, 3:1) are considered. The blade pitch can be adjusted using the collective pitch mechanism, but impossible for the blade twist. Both the coaxial rotors with taper of 2:1 and 3:1 have relatively uniform spanwise AoA, so either case can be used to design the resulting coaxial tiltrotor. The one with taper case of 2:1 is selected in this work for demonstration and provide data for the control part in Chapter 5. The resulting coaxial tiltrotor is a weighted combination of the blade twists of two optima for hover and cruise, because the coaxial tiltrotor is a compromise of the optima in both conditions. An equal weighting factor is used in this thesis just to demonstrate the general concept. The performance of the resulting coaxial tiltrotor is very close to the optimum for either condition.

The design of the auxiliary rotor in this thesis emphasises more on the attitude control of Hyperion in VTOL mode. Due to the existence of the aerodynamic drag and its induced pitch moment, the auxiliary rotors must be able to stabilize the pitch angle. The commonly used mechanism to change the thrust of a rotor is changing the rotational speed and the blade pitch angle. The thrust direction can not be changed for the rotational speed mechanism, whilst the thrust direction can be adjusted by the blade pitch mechanism, so three cases, the upward and downward thrust for rotational speed mechanism and blade pitch mechanism, are considered. All the three methods are possible as the auxiliary rotor for Hyperion. The blade pitch mechanism is selected for the auxiliary rotors for its lower power budget. Because taking off and landing are two symmetric working conditions for the blade pitch mechanism case, non-twisted blades are used for the auxiliary rotors. Four blade taper cases (1:1, 2:1, 2.5:1, and 3:1) are considered in this thesis. Although the rotor with 3:1 taper performs best, the difference is very small due to the small power consumption. Therefore, it is possible to use anyone of the four taper cases as the auxiliary rotors. The taper case of 2.5:1 is used as the auxiliary rotors in the control part in Chapter 5.

The robust flight control is another important topic for such a special Martian aerobot. The control in this thesis focuses on the low level controller that guarantees the stability of Hyperion. Considering the inherent uncertainty of the aerodynamic coefficients in the model, it is necessary to consider the robustness of the controller. The $\mu$ synthesis method is used in this thesis as the
fundamental to guarantee the robustness of the proposed controller. The flight modes in VTOL and cruise are quite similar to the quadrotor and the fixed wing aircraft. Since the flight control of the quadrotor and fixed wing aircraft is a well developed technology and many publications can be found, the conventional control loops are selected for Hyperion in VTOL and cruise modes.

The flight control of Hyperion in transition mode is more complex and difficult. Although the general concept of transition control of Hyperion is quite similar to the tiltrotor aircraft, most publications on the transition and conversion control of the tiltrotor aircraft are on the history and introduction of the technology. Most of the public technical literature on tiltrotor aircraft use very simple mathematical model, so the robustness of these control methods is questionable. The boundaries of the transition corridor are estimated by the nominal model of the longitudinal dynamics. Any trajectory connecting the hover and cruise states within the transition corridor can be used as the transition trajectory. The transition trajectory used in this thesis is those states corresponding to the high L/D working conditions with AoA of 5.4°.

The trajectory linearization method is used to transform the longitudinal and lateral nonlinear dynamic models into the LPV models. The commonly used LPV control method is the gain scheduling. Commonly used gain scheduling methods are the classical gain scheduling, the LPV control, and the D&C gain scheduling. Considering the inherent uncertainty of the aerodynamic coefficients, the classical gain scheduling controller even can not guarantee the stability. The LPV controller is difficult to use since there is no prior information on how to design the LDI for the longitudinal and lateral LPV models. Therefore, the D&C gain scheduling method is used for the transition and conversion control problems. The transition and conversion controllers are both two-level controllers. The higher level will generate the switch signal to decide which lower level controller is used for the present control task. A series of transition states are selected to divide the LPV models for longitudinal and lateral dynamics into the corresponding LPV models with smaller variant range segments. The robust controller design for each segment is much easier, and the \( \mu \) synthesis method is used to guarantee the robustness of the controller for each segment. The transition and conversion process is accomplished by passing through all the transition states. The simulation results show that the proposed transition and conversion controllers can ensure the robust stability during the whole transition and conversion processes.

The work presented in this thesis emphasises mainly on the aerodynamic design of the coaxial tiltrotor for the Martian aerobot to improve its efficiency, and the robust flight control for all phases (VTOL, cruise, and transition/conversion). On the aerodynamic design on coaxial tiltrotor, efforts are made to give a comprehensive of the aerodynamic design of the coaxial tiltrotor to improve the efficiency. The performance of the final proposed coaxial tiltrotor is close to the optima in both hover and cruise, which shows much improvement comparing with the baseline design. On the robust flight control, a novel robust control strategy based on the D&C gain
scheduling method and \( \mu \) synthesis is proposed for the transition and conversion between hover and cruise. The simulation results show that the transition and conversion controllers can guarantee the robust stability of Hyperion in transition and conversion processes for the assumed uncertainty of the aerodynamic coefficients.

6.2 Conclusions

The work in this thesis contains two problems, the aerodynamic design of the propulsion system and the robust flight control. Some important conclusions drawn from the work are listed as follows.

6.2.1 Aerodynamic Design of Propulsion System

- The FWM based on PIPC algorithm and the PWM used in this thesis can accurately predict the performance of the coaxial rotor in hover and cruise with not too large tip speed ratio. The prediction error for large tip speed ratio \((\lambda_n > 0.5)\) is caused by the small angle assumption used when solving the Weissinger-L lifting surface model for the rotor blades.

- The Reynolds number of rotor system in this work is approximately 30,000. This has great influence on the rotor system. The rough low Reynolds number sectional airfoil in this work is recommended to be used in this work to simplify this problem, but it is worth noting that this will increase the viscous drag, and then decrease the efficiency. The proposed rotor system must be treated with caution. In order to further improve the efficiency, it is necessary to carefully study the boundary layer structure. However, the research on the boundary layer structure need more powerful CFD tools and wind tunnel experiments, so this is left for the future due to the lack of funding at present.

- The coaxial tiltrotor is a pair of coaxial rotor working for hover with low inflow rate and cruise with high axial speed. The "optimum" coaxial tiltrotor is essentially a compromise of the optima for both hover and cruise. A compromise method of the weighted average of the blade twist is used to get the final shape of the coaxial tiltrotor. The performance of the final coaxial tiltrotor is close to the optima in both hover and cruise, which shows an improvement of the efficiency comparing with the baseline design.

- Both the rotational speed and blade pitch mechanism can be used for the auxiliary rotors. Since the thrust direction can not be changed for the rotational speed mechanism, upward and downward thrust cases need to be considered. The blade pitch mechanism is selected in this work since its power consumption is the lowest.
6.2.2 Robust Flight Control Strategy

- The Martian aerobot can be robustly stabilized by the proposed control strategy in both VTOL and cruise modes for the given uncertainty range of the aerodynamic coefficients. For the VTOL control problem, the structure of the proposed controller is similar to that of the quadrotor that all the three attitude angles are fully decoupled. The direction of the horizontal velocity is controlled by changing the pitch and roll angles.

- For the cruise control problem, the structure of the longitudinal controller is similar to that of the conventional fixed wing aircraft. In the lateral control problem, the yaw angle is controlled by the roll angle in the lateral controller, since the lateral dynamic model is an underactuated control system (only one pair of elevons). The directional control using this design is much weaker, but it is still possible.

- The shape of the transition corridor shows that several transition states are required to ensure the stability during the transition and conversion processes. The simulation results show that the proposed transition/conversion controller can guarantee the robust stability during the transition/conversion phase for the given aerodynamic uncertainty of ±20%.

6.2.3 General Research Findings

- The Reynolds number of the rotor system is about 30,000, which belongs to the low Reynolds number regime. Since the sectional AoA (or lift coefficient) is operating before separation, moreover, we propose to use the rough sectional airfoil for this preliminary design; the trend of the sectional airfoil performance is similar to the high Reynolds number cases. However, this factor should be studied in detail before building a real life Martian aerobot in the figure.

- The rudders are very important in the lateral stability of the Martian aerobot. In this work, the rudders are abandoned to simplify the mechanical structure of the wing. However, the lateral dynamic property for cruise mode is not very good. Therefore, we should consider the Martian aerobot with rudders in the future.

6.3 Recommendation for Future Work

The work in this thesis has been in the development of a Martian aerobot to investigate the Isidis Planitia region on Mars. The aerobot in this thesis, Hyperion, is designed based on the previous work. The design of a Martian aerobot is a systems engineering problem contains design problems of many aspects. The work in this thesis focuses on two of these design problems, the preliminary aerodynamic design of the coaxial tiltrotor and the low level robust flight control.
6.3.1 Aerodynamic Design and Optimization

In this thesis, a preliminary aerodynamic shape design of the coaxial tiltrotor is presented. The focus in this work is the shape of the blades. However, other factors such as the rotor radius, rotational speed, rotor disk distance, and etc. also have large influence on the performance of the coaxial tiltrotor. These parameters should be determined prior to the work in this thesis, so these parameters used in this thesis are the same as those in the previous design. The performance of the coaxial tiltrotor should be validated using the commercial CFD software and the wind tunnel measurements if possible.

The proposed structure of the coaxial tiltrotor is still far from the ultimate production. The coaxial tiltrotor is designed under the assumption of inviscid incompressible flow, but this is not the fact for the Martian atmosphere. The Reynolds number is about 50,000 to 60,000 for the wing and 30,000 for the rotor system, so the conventional airfoils will not reach their design performance. One problem left for the future research is the airfoil selection for the rotor blades. The tip speed ratios of the rotor blades in hover and cruise are larger than 0.3, so the incompressible flow assumption is not strict for this problem. This should also be considered in future research.

The rotor blade tip is also an important factor which has significant influence on the formation of the rolled-up tip vortex and its structure, so this could also be a good topic to improve the efficiency. It is worthwhile to note that the coaxial tiltrotor is located at the duct of the airframe, so this factor can also be considered in future design process.

The airframe of the aerobot, which just follows the previous design, is a large zagi flying wing with a duct at the centre. The duct has large influence on the aerodynamic property of the zagi flying wing, so the aerodynamic optimization of the airframe is also a good topic for future research.

6.3.2 Structural Analysis

The airframe and the rotor blades in this thesis are all assumed to be rigid bodies, but this assumption is not appropriate considering their sizes and loading distributions. The deformation caused by the aerodynamic loads will have significant influence on the aerodynamic performance; therefore, some deformation modifications should be added for the structure.

As shown in Figure 3-1, the coaxial tiltrotor is located at the centre of the duct. The mechanical structure connecting the coaxial tiltrotor and the airframe is a very interesting topic. Two possible ways can be derived from the MTR and Verticopter® designs.
6.3.3 Dynamic and Aerodynamic Model in Transition and Conversion

The aerodynamic model used in this thesis is based on the steady derivatives. The aerodynamic property of *Hyperion* in transition and conversion is more complex since the dynamic changes of the AoA and AoS. The aerodynamic model for the coaxial tiltrotor in this thesis is also not precise. The modelling of the coaxial tiltrotor in transition and conversion is a good topic for further research. Especially the coupling between the airframe and the rotors should be considered.

6.3.4 Flight Control in Transition and Conversion

The flight control strategy for transition and conversion in this thesis just shows the possibility of being used for transition and conversion flights. The disturbance during transition and conversion is still not small, especially between the State VII and State VIII (cruise). Further research on the transition control can focus on the disturbance reduction and transition trajectory design.
### Appendix A  Mars Missions

#### Table A-1: Mars Missions [NASA, 2012c]

<table>
<thead>
<tr>
<th>Launch Time</th>
<th>Name</th>
<th>Type</th>
<th>Country</th>
<th>Result</th>
<th>Reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>1960</td>
<td>Korabl 4</td>
<td>Flyby</td>
<td>USSR</td>
<td>Failure</td>
<td>Did not reach Earth orbit</td>
</tr>
<tr>
<td>1960</td>
<td>Korabl 5</td>
<td>Flyby</td>
<td>USSR</td>
<td>Failure</td>
<td>Did not reach Earth orbit</td>
</tr>
<tr>
<td>1962</td>
<td>Korabl 11</td>
<td>Flyby</td>
<td>USSR</td>
<td>Failure</td>
<td>Reach Earth orbit, spacecraft broken</td>
</tr>
<tr>
<td>1962</td>
<td>Mars 1</td>
<td>Flyby</td>
<td>USSR</td>
<td>Failure</td>
<td>Radio failed</td>
</tr>
<tr>
<td>1962</td>
<td>Korabl 13</td>
<td>Flyby</td>
<td>USSR</td>
<td>Failure</td>
<td>Reach Earth orbit, spacecraft broken</td>
</tr>
<tr>
<td>1964</td>
<td>Mariner 3</td>
<td>Flyby</td>
<td>USA</td>
<td>Failure</td>
<td>Shroud failed jettison</td>
</tr>
<tr>
<td>1964</td>
<td>Mariner 4</td>
<td>Flyby</td>
<td>USA</td>
<td>Success</td>
<td>Returned 21 images</td>
</tr>
<tr>
<td>1964</td>
<td>Zond 2</td>
<td>Flyby</td>
<td>USSR</td>
<td>Failure</td>
<td>Radio failed</td>
</tr>
<tr>
<td>1969</td>
<td>Mars 69A</td>
<td>Orbiter</td>
<td>USSR</td>
<td>Failure</td>
<td>Launch vehicle failure</td>
</tr>
<tr>
<td>1969</td>
<td>Mars 69B</td>
<td>Orbiter</td>
<td>USSR</td>
<td>Failure</td>
<td>Launch vehicle failure</td>
</tr>
<tr>
<td>1969</td>
<td>Mariner 6</td>
<td>Flyby</td>
<td>USA</td>
<td>Success</td>
<td>Returned 75 images</td>
</tr>
<tr>
<td>1969</td>
<td>Mariner 7</td>
<td>Flyby</td>
<td>USA</td>
<td>Success</td>
<td>Returned 126 images</td>
</tr>
<tr>
<td>1971</td>
<td>Mariner 8</td>
<td>Orbiter</td>
<td>USA</td>
<td>Failure</td>
<td>Launch failure</td>
</tr>
<tr>
<td>1971</td>
<td>Kosmos 419</td>
<td>Orbiter</td>
<td>USSR</td>
<td>Failure</td>
<td>Stopped at Earth orbit</td>
</tr>
<tr>
<td>1971</td>
<td>Mars 2</td>
<td>Orbiter, Lander</td>
<td>USSR</td>
<td>Failure</td>
<td>Orbiter arrived, Lander destroyed</td>
</tr>
<tr>
<td>1971</td>
<td>Mars 3</td>
<td>Orbiter, Lander</td>
<td>USSR</td>
<td>Failure</td>
<td>Orbiter works for 8 months, lander works for 20 seconds</td>
</tr>
<tr>
<td>1971</td>
<td>Mariner 9</td>
<td>Orbiter</td>
<td>USA</td>
<td>Success</td>
<td>Returned 7,329 images</td>
</tr>
<tr>
<td>1973</td>
<td>Mars 4</td>
<td>Orbiter</td>
<td>USSR</td>
<td>Failure</td>
<td>Flew past Mars</td>
</tr>
<tr>
<td>1973</td>
<td>Mars 5</td>
<td>Orbiter</td>
<td>USSR</td>
<td>Success</td>
<td>Returned 60 images; last for 9 days</td>
</tr>
<tr>
<td>1973</td>
<td>Mars 6</td>
<td>Orbiter, Lander</td>
<td>USSR</td>
<td>Success, Failure</td>
<td>Occultation experiment produced data and lander failure</td>
</tr>
<tr>
<td>1973</td>
<td>Mars 7</td>
<td>Lander</td>
<td>USSR</td>
<td>Failure</td>
<td>Missed Mars; now in solar orbit</td>
</tr>
<tr>
<td>1975</td>
<td>Viking 1</td>
<td>Orbiter, Lander</td>
<td>USA</td>
<td>Success</td>
<td>Located landing site for Lander and first successful landing on Mars</td>
</tr>
<tr>
<td>1975</td>
<td>Viking 2</td>
<td>Orbiter, Lander</td>
<td>USA</td>
<td>Success</td>
<td>Returned 16,000 images and extensive atmospheric data and soil experiments</td>
</tr>
<tr>
<td>1988</td>
<td>Phobos 1</td>
<td>Orbiter</td>
<td>USSR</td>
<td>Failure</td>
<td>Lost route to Mars</td>
</tr>
<tr>
<td>1988</td>
<td>Phobos 2</td>
<td>Orbiter, Lander</td>
<td>USSR</td>
<td>Failure</td>
<td>Lost near Phobos</td>
</tr>
<tr>
<td>1992</td>
<td>Mars Observer</td>
<td>Orbiter</td>
<td>USA</td>
<td>Failure</td>
<td>Lost prior to Mars arrival</td>
</tr>
<tr>
<td>1996</td>
<td>Mars Global Surveyer</td>
<td>Orbiter</td>
<td>USA</td>
<td>Success</td>
<td>More images than all Mars Missions</td>
</tr>
<tr>
<td>1996</td>
<td>Mars 96</td>
<td>Orbiter</td>
<td>Russia</td>
<td>Failure</td>
<td>Launch vehicle failure</td>
</tr>
<tr>
<td>1996</td>
<td>Mars Pathfinder</td>
<td>Lander, Rover</td>
<td>USA</td>
<td>Success</td>
<td>Technology experiment lasting 5 times longer than</td>
</tr>
<tr>
<td>Year</td>
<td>Mission</td>
<td>Type</td>
<td>Country</td>
<td>Outcome</td>
<td>Notes</td>
</tr>
<tr>
<td>------</td>
<td>--------------------------</td>
<td>--------</td>
<td>---------</td>
<td>---------</td>
<td>-----------------------------------------------------</td>
</tr>
<tr>
<td>1998</td>
<td>Nozomi</td>
<td>Orbiter</td>
<td>Japan</td>
<td>Failure</td>
<td>No orbit insertion; fuel problems</td>
</tr>
<tr>
<td>1998</td>
<td>Mars Climate Orbiter</td>
<td>Orbiter</td>
<td>USA</td>
<td>Failure</td>
<td>Lost on arrival</td>
</tr>
<tr>
<td>1999</td>
<td>Mars Polar Lander</td>
<td>Lander</td>
<td>USA</td>
<td>Failure</td>
<td>Lost on arrival</td>
</tr>
<tr>
<td>1999</td>
<td>Deep Space 2 Probe</td>
<td>Probe</td>
<td>USA</td>
<td>Failure</td>
<td>Lost on arrival</td>
</tr>
<tr>
<td>2001</td>
<td>Mars Odyssey</td>
<td>Orbiter</td>
<td>USA</td>
<td>Success</td>
<td>High resolution images</td>
</tr>
<tr>
<td>2003</td>
<td>Mars Express and Beagle 2</td>
<td>Orbiter, Lander</td>
<td>ESA</td>
<td>Success, Failure</td>
<td>Orbiter imaging Mars, lander lost on arrival</td>
</tr>
<tr>
<td>2003</td>
<td>Spirit</td>
<td>Rover</td>
<td>USA</td>
<td>Success</td>
<td>Operating lifetime of more than 15 times original warranty</td>
</tr>
<tr>
<td>2003</td>
<td>Opportunity</td>
<td>Rover</td>
<td>USA</td>
<td>Success</td>
<td>Operating lifetime of more than 15 times original warranty</td>
</tr>
<tr>
<td>2005</td>
<td>Mars Reconnaissance</td>
<td>Orbiter</td>
<td>USA</td>
<td>Success</td>
<td>Returned more than 26TB data (more than all other Mars missions)</td>
</tr>
<tr>
<td>2007</td>
<td>Phoenix</td>
<td>Lander</td>
<td>USA</td>
<td>Success</td>
<td>Returned more than 25 GB data</td>
</tr>
<tr>
<td>2007</td>
<td>Dawn</td>
<td>Vesta Orbiter</td>
<td>USA</td>
<td>Success</td>
<td>Currently in operation</td>
</tr>
<tr>
<td>2011</td>
<td>Phobos-Grunt and Yinghuo-1</td>
<td>Phobos lander, sample, return and Mars orbiter</td>
<td>Russia</td>
<td>Failure</td>
<td>Failed to leave Earth orbit and rescue unsuccessful</td>
</tr>
<tr>
<td>2011</td>
<td>MSL Curiosity</td>
<td>Rover</td>
<td>USA</td>
<td>Success</td>
<td>Currently in operation</td>
</tr>
<tr>
<td>2013</td>
<td>MAVEN</td>
<td>Orbiter</td>
<td>USA</td>
<td>In plan</td>
<td></td>
</tr>
<tr>
<td>2013</td>
<td>Mars Orbiter</td>
<td>Orbiter</td>
<td>India</td>
<td>In study</td>
<td></td>
</tr>
<tr>
<td>2014</td>
<td>MetNet precursor</td>
<td>Impact lander</td>
<td>Finland</td>
<td>In plan</td>
<td></td>
</tr>
<tr>
<td>2016</td>
<td>ExoMars</td>
<td>Orbiter, Lander</td>
<td>ESA</td>
<td>In plan</td>
<td></td>
</tr>
<tr>
<td>2016</td>
<td>InSight</td>
<td>Lander</td>
<td>USA</td>
<td>In study</td>
<td></td>
</tr>
<tr>
<td>2016</td>
<td>Mars Geyser Hopper</td>
<td>Lander</td>
<td>USA</td>
<td>In study</td>
<td></td>
</tr>
<tr>
<td>2018</td>
<td>ExoMars</td>
<td>Lander, Rover</td>
<td>ESA</td>
<td>In plan</td>
<td></td>
</tr>
<tr>
<td>2018</td>
<td>Red Dragon</td>
<td>Lander</td>
<td>USA</td>
<td>In study</td>
<td></td>
</tr>
<tr>
<td>2022</td>
<td>Network</td>
<td>3 small impact landers</td>
<td>ESA</td>
<td>In study</td>
<td></td>
</tr>
<tr>
<td>2022</td>
<td>MMSR</td>
<td>Lander and ascent stage</td>
<td>ESA</td>
<td>In study</td>
<td></td>
</tr>
<tr>
<td>2022</td>
<td>Mars One</td>
<td>Manned mission</td>
<td></td>
<td>In study</td>
<td></td>
</tr>
<tr>
<td>2020s</td>
<td>MELOS-1</td>
<td>Orbiter, Lander</td>
<td>Japan</td>
<td>In study</td>
<td></td>
</tr>
<tr>
<td>2020s</td>
<td>Mars-Grunt</td>
<td>Orbiter, Lander, ascent stage</td>
<td>Russia</td>
<td>In study</td>
<td></td>
</tr>
</tbody>
</table>
Appendix B  Aerodynamic Property

B-1  Aerodynamic Coefficients of the Airframe

The first order derivatives of the aerodynamic coefficients of the airframe estimated by AVL in cruise are given by Table B-1. The mean aerodynamic chord of the airframe is \( \overline{c}=1.3046 \text{m}; \) the wing span is \( \overline{b}=8.558 \text{m}; \) the wing area is \( S=8.23 \text{m}^2. \)

Table B-1: Aerodynamic derivatives of the airframe in cruise estimated by AVL [Song, 2008]

<table>
<thead>
<tr>
<th>Longitudinal dynamics</th>
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<td>( C_{Le} )</td>
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<td>( C_{Dsa} )</td>
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<td>( C_{Lq} )</td>
<td>3.035447</td>
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<td>( e )</td>
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<td>( AR )</td>
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Lateral dynamics

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<td>( C_{lp} )</td>
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<td>-0.553168</td>
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<td>( C_{yr} )</td>
<td>0.042092</td>
<td>( C_{lp} )</td>
<td>0.064378</td>
</tr>
<tr>
<td>( C_{Ysa} )</td>
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<td>( C_{lp} )</td>
<td>-0.548309</td>
</tr>
</tbody>
</table>

B-2  Blade Element and Momentum Theory [McCormick, 1999]

Consider the annular element shown in Figure B-1, the thrust on the annular element according to the simple Momentum Theory is given by Equation 7-1.

\[
\begin{align*}
\frac{dT}{df} &= \rho dA(V_{\infty} + u \cos \phi)2u \cos \phi \cdot Ff \\
&= \rho (2\pi r dr)(V_{\infty} + u \cos \phi)2u \cos \phi \cdot Ff \\
\end{align*}
\]

Equation 7-1
\[ F = \frac{2}{\pi} \arccos \left( \frac{N_b f (R_{\text{tip}} - r)}{r \cos(\phi + \alpha_i)} \right) \]  
Equation 7-2

\[ f = \frac{2}{\pi} \arccos \left( \frac{N_b f (r - R_{\text{root}})}{r \cos(\phi + \alpha_i)} \right) \]  
Equation 7-3

Where, \( dT \) is the thrust increment on at the annular element; \( dA = 2\pi r \, dr \) denotes the area of the annular element; \( r \) denotes the radius of the annular element; \( V_\infty \) denotes the inflow rate; \( u \) denotes the induced velocity; \( \phi \) denotes the inflow rate angle (see Figure B-2); \( F \) denotes the Prandtl tip loss factor (Equation 7-2); \( f \) denotes the Prandtl root loss factor (Equation 7-3); \( N_b \) denotes the number of blades; \( R_{\text{tip}} \) and \( R_{\text{root}} \) denote the tip and root radius of the rotor; and \( \alpha_i \) denotes the induced AoA.

Figure B-2: Velocity and force diagram for blade element

Consider the blade element shown in Figure B-2, the thrust generated by the blade element is given by Equation 7-4.

\[ dT = dL \cos(\phi + \alpha_i) - dD \sin(\phi + \alpha_i) \]
\[ = \frac{N_b}{2} \rho V_r^2 \left[ c_{\alpha \alpha} \alpha_{\text{eff}} \cos(\phi + \alpha_i) - (c_{d0} + c_{d1} \alpha_{\text{eff}} + c_{d2} \alpha_{\text{eff}}) \sin(\phi + \alpha_i) \right] c \, dr \]  
Equation 7-4

\[ V_r = \sqrt{(V_\infty + u \cos \phi)^2 + (\omega r - u \sin \phi)^2} \]  
Equation 7-5

\[ \alpha_{\text{eff}} = \theta - \phi - \alpha_i \]  
Equation 7-6

Where, \( dL \) and \( dD \) are the lift and drag generated by the blade element; \( V_r \) is the total flow rate of the blade element (Equation 7-5); \( c_{\alpha \alpha} \) is the lift curve slope of the blade section airfoil; \( \alpha_{\text{eff}} \) is the effective AoA of the blade element (Equation 7-6); \( c_{d0}, c_{d1}, \) and \( c_{d2} \) are the coefficients.
representing the drag of blade section airfoil; \( c \) is the chord length of the blade element; and \( \omega \) is the rotational speed of rotor. For small angles of the induced AoA, we have Equation 7-7.

\[
\alpha_i = \arcsin \frac{u}{V_r} = \frac{u}{V_r} 
\]

Equation 7-7

Assume the thrust estimated by Equation 7-1 and Equation 7-4 are the same, and then the induced AoA can be solved iteratively by Equation 7-8 to Equation 7-13. The tip and root loss factors are calculated by Equation 7-2 and Equation 7-3 in each iteration.

\[
A\alpha_i^2 + B\alpha_i + C = 0 
\]

Equation 7-8

\[
A = 1 + K \left[ c_{d2} \sin \phi - (c_{d1} + 2c_{d2}) \cos \phi - c_{ia} \sin \phi \right] 
\]

Equation 7-9

\[
B = \frac{V_o}{\omega r} + K \left[ c_{ia} \cos \phi + c_{ia} \alpha_{eff} \sin \phi - (c_{d1} + 2c_{d2} \alpha_{eff}) \sin \phi + c_d \cos \phi \right] 
\]

Equation 7-10

\[
C = K \left[ c_d \sin \phi - c_{ia} \alpha_{eff} \cos \phi \right] 
\]

Equation 7-11

\[
c_d = c_{d0} + c_{d1} \alpha_{eff} + c_{d2} \alpha_{eff}^2 
\]

Equation 7-12

\[
K = \frac{N_\psi c}{8\pi \rho f \cos^2 \phi} 
\]

Equation 7-13

Then the thrust, torque and power of the rotor is given by Equation 7-14 to Equation 7-16.

\[
T = \int_{r_{\text{out}}}^{r_{\text{in}}} \rho (2\pi r)(V_\infty + u \cos \phi) 2u \cos \phi \cdot Ff \cdot dr 
\]

Equation 7-14

\[
Q = \frac{N_\psi}{2} \int_{r_{\text{out}}}^{r_{\text{in}}} V_\infty c \left[ c_{ia} \alpha_{eff} \sin(\phi + \alpha_i) + c_d \cos(\phi + \alpha_i) \right] r dr 
\]

Equation 7-15

\[
P = Q\omega 
\]

Equation 7-16

The equations above are used to estimate the performance of single rotor or propeller. The coaxial rotor or propeller uses the same equations for each rotor or propeller. The difference is the inflow rates caused by the intereference of rotors or propellers. The interference of coaxial rotor or propeller in this thesis follows the conclusions in reference [Leishman and Ananthan, 2006]. Only the influence of the upper rotor on the lower rotor is considered because it is more important. The lower rotor is usually assumed to be operating at the fully developed slip stream of the upper rotor.

The contracted area is \( A_c = \pi r_c^2 \). In ideal case, \( \frac{r_c}{r} = 0.707 \) for hover and \( \frac{r_c}{r} = 1 \) for cruise. The inflow rate of inner area of the lower rotor is \( V_\infty + 2u_\psi \) in hover and \( V_\infty + u_\psi \) in cruise for ideal case.
B-3 Weissinger-L Lifting Surface Model

Weissinger-L lifting surface model for the rotor blades is presented in this section. It is a finite difference method for rotor blades. The difference between the Weissinger-L model and the classical BEMT is the estimation method of the induced rate. The blade is divided into \( M_b \) blade sections. For each blade section, the effective AoA \( (\alpha_{\text{eff}}) \) is given by Equation 7-17.

\[
\alpha_{\text{eff}} = \theta + \frac{V_{\infty}}{V_r} + \frac{V_{z(\text{far})}}{V_r} + \frac{V_{z(\text{near})}}{V_r}
\]

Equation 7-17

\[
\Gamma = \frac{1}{2} V_r c c_m \alpha_{\text{eff}}
\]

Equation 7-18

Where, \( \theta \) denotes the geometric pitch of the blade section; \( V_{z(\text{far})} \) denotes velocity in z direction (axial direction) induced by the far wake (i.e. rolled-up tip vortices); and \( V_{z(\text{near})} \) denotes the velocity in z direction induced by the near wake (i.e. the bound vortex and near wake trailer vortices). The Weissinger-L model of the Harrington 1 rotor blade shown in Figure B-3 is used to explain the model. The bound vortex is located at the 1/4 blade chord line; the control points are located at the 3/4 blade chord line in the middle of each blade sections; the near wake trailer vortices are extended to a finite azimuth length (30°) in the rotor plane. As shown in Figure B-3, the trailers are represented by the broken lines.

![Figure B-3: Weissinger-L lifting surface model for the Harrington 1 rotor blade](image)

The induced velocity of straight vortex is calculated by Equation 4-1. The radius of the vortex core for near wake trailer vortices assumed to be zero. The strength of the two trailer vortices at blade root and tip is equal to that of the bound vortex at the root and tip; the strength of the trailer vortices at the central parts is equal to the strength difference of the bound vortex segments adjacent to it. Therefore, the near wake vortices can be regarded as \( M_b \) horseshoe vortices, which is the same as the finite wing case. The strength of each horseshoe vortex is equal to the
corresponding bound circulation of the segments, so the near wake function can be written as Equation 7-19.

$$
\begin{bmatrix}
1 & 0 & 0 & \cdots & 0 \\
0 & 1 & 0 & \cdots & 0 \\
0 & 0 & 1 & \cdots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & 0 & \cdots & 1 \\
\end{bmatrix}
\begin{bmatrix}
I_{11} & I_{12} & I_{13} & \cdots & I_{1M_0} \\
I_{21} & I_{22} & I_{23} & \cdots & I_{2M_0} \\
I_{31} & I_{32} & I_{33} & \cdots & I_{3M_0} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
I_{M_1} & I_{M_2} & I_{M_3} & \cdots & I_{M_M_0} \\
\end{bmatrix}
\begin{bmatrix}
\Gamma_1 \\
\Gamma_2 \\
\Gamma_3 \\
\vdots \\
\Gamma_{M_0} \\
\end{bmatrix}
= 
\begin{bmatrix}
A_1 \\
A_2 \\
A_3 \\
\vdots \\
A_{M_0} \\
\end{bmatrix}$$

Equation 7-19

Where,

$$A_i = \frac{1}{2} c_i c_{i0} V_p \left( \theta_i + \frac{V_{zi}}{V_p} + \frac{V_{zi(far)}}{V_p} \right)$$

Equation 7-20

Where, \( I_y \) denotes the induced velocity at the \( i \)th control points caused by the \( j \)th horseshoe vortex, which can be calculated directly by the Biot-Savart law; \( \Gamma_j \) denotes the strength of the \( j \)th horseshoe vortex, i.e. the bound circulation at the \( j \)th blade section; \( c_i \) denotes the chord length of the \( i \)th blade section.

Equation 7-19 can be solved directly with the information of the far wake, which is the objective of the FWM and PWM used in this thesis.

**B-4 Free Wake Model [Bagai, 1995]**

The FWM presented in this section is based on the PIPC algorithm shown in Figure 4-3. FWM is solved iteratively with the prescribed wake structure and the vortex strength. The rolled-up tip vortices of the rotor are represented by the straight segments connected by the collocation points. Figure B-4 shows the coordinate system and the Lagrangian discretization of the vortex filaments.

![Figure B-4: Coordinate system and Lagrangian discretization of the vortex filaments](Leishman et al., 2002)

The collocation points are governed by the Lagrangian description of vorticity (Equation 7-21). This equation can be solved by finite difference method, which is the original concept of FWM.
The induced velocity can be calculated by

\[ V_i(\psi', \zeta') = \frac{1}{4\pi} \sum_{j=1}^{N_b} \sum_{k=1}^{N_c} \Gamma_j(\psi', \zeta_k) \frac{h_{jk}}{\sqrt{h_{jk}^2 + r_c^2}} (\cos(\theta_{(jk)1}) - \cos(\theta_{(jk)2})) \hat{e} \]

\[ + \frac{1}{4\pi} \sum_{j=1}^{N_b} \sum_{k=1}^{N_c} \Gamma_j(\psi', i) h_{jk} \left( \frac{\cos(\theta_{(jk)1}) - \cos(\theta_{(jk)2})}{2} \right) \hat{e} \]

Where, \( V_i \) denotes the total induced velocity; \( N_b \) denotes the number of blades; \( N_c \) denotes the number of segments on one vortex; \( h_{jk} \) denotes the distance between the point of evaluation and the vortex; \( \Gamma_j(\psi', \zeta_k) \) denotes the strength of the rolled-up tip vortex; \( r_c \) denotes the core radius of the vortex; \( \Gamma_j(\psi', i) \) denotes the strength of the bound circulation; \( \theta_{(jk)1}, \theta_{(jk)2}, \theta_{(jk)1}, \theta_{(jk)2} \) are the angles to evaluate the Biot-Savart law.

The Lamb-Oseen vortex mode is used to describe the evolution of the rolled-up tip vortices, so the core radius is given by

\[ r_c = 0.00855 \sqrt{\frac{\delta \zeta}{\omega}} \cdot \frac{I_0}{l} \]

Where, \( \delta \) is the turbulent viscosity coefficient with the order of 10^5; \( \zeta \) is the wake age; \( l_0 \) is the original length of the vortex; and \( l \) is the length of the vortex.

Then the five points central PIPC scheme can be written as Equation 7-24 to Equation 7-29.

\[ \tilde{\tilde{\psi}}_{t, k}^n = \tilde{\psi}_{t-1,k-1}^n + (\tilde{\psi}_{t-1,k-1}^n - \tilde{\psi}_{t-1,k}^n) \frac{\Delta \psi - \Delta \zeta}{\Delta \psi + \Delta \zeta} \]

\[ + \frac{2}{\omega} \frac{\Delta \psi \Delta \zeta}{\Delta \psi + \Delta \zeta} \left( \tilde{\psi}_{t}^n + \frac{1}{4} \left( \tilde{\psi}_{t}(\tilde{\psi}_{t-1,k-1}^n + \tilde{\psi}_{t}(\tilde{\psi}_{t-1,k-1}^n) + \tilde{\psi}_{t}(\tilde{\psi}_{t-1,k-1}^n) + \tilde{\psi}_{t}(\tilde{\psi}_{t-1,k}^n) \right) \right) \]

\[ \tilde{\psi}_{t,k}^n = \tilde{\psi}_{t-1,k-1}^n + (\tilde{\psi}_{t-1,k-1}^n - \tilde{\psi}_{t-1,k}^n) \frac{\Delta \psi - \Delta \zeta}{\Delta \psi + \Delta \zeta} \]

\[ + \frac{2}{\omega} \frac{\Delta \psi \Delta \zeta}{\Delta \psi + \Delta \zeta} \left( \tilde{\psi}_{t}^n + \frac{1}{4} \left( \tilde{\psi}_{t}(\tilde{\psi}_{t-1,k-1}^n + \tilde{\psi}_{t}(\tilde{\psi}_{t-1,k-1}^n) + \tilde{\psi}_{t}(\tilde{\psi}_{t-1,k-1}^n) + \tilde{\psi}_{t}(\tilde{\psi}_{t-1,k}^n) \right) \right) \]

\[ \tilde{\psi}_{t}(\tilde{\psi}_{t-1,k-1}^n) = \epsilon \tilde{\psi}_{t}(\tilde{\psi}_{t-1,k-1}^n) + (1 - \epsilon) \tilde{\psi}_{t}(\tilde{\psi}_{t-1,k-1}^n) \]

\[ \tilde{\psi}_{t}(\tilde{\psi}_{t-1,k}^n) = \epsilon \tilde{\psi}_{t}(\tilde{\psi}_{t-1,k}^n) + (1 - \epsilon) \tilde{\psi}_{t}(\tilde{\psi}_{t-1,k}^n) \]
\[ \tilde{V}_i(\tilde{r}_{i,k}^n) = \varepsilon \tilde{V}_i(\tilde{r}_{i,k}^{n-1}) + (1-\varepsilon) \tilde{V}_i(\tilde{r}_{i,k}^{n-1}) \]  

Equation 7-28

\[ \tilde{V}_i(\tilde{r}_{i,k}^n) = \varepsilon \tilde{V}_i(\tilde{r}_{i,k}^{n-1}) + (1-\varepsilon) \tilde{V}_i(\tilde{r}_{i,k}^{n-1}) \]  

Equation 7-29

Where, \( \tilde{r}_{i,k}^n \) are the predicted positions of collocation points for \( n^{th} \) iteration; \( \Delta \psi \) and \( \Delta \zeta \) are the discretization in azimuth and wake development directions; \( \tilde{V}_i(\tilde{r}_{i,k}^{n-1}) \), \( \tilde{V}_i(\tilde{r}_{i,k}^{n-1}) \), and \( \tilde{V}_i(\tilde{r}_{i,k}^{n-1}) \) are the induced velocities at the positions of the previous iteration; \( \tilde{V}_i(\tilde{r}_{i,k}^{n-1}) \), \( \tilde{V}_i(\tilde{r}_{i,k}^{n-1}) \), \( \tilde{V}_i(\tilde{r}_{i,k}^{n-1}) \), and \( \tilde{V}_i(\tilde{r}_{i,k}^{n-1}) \) are the induced velocities at the predicted positions; \( \varepsilon \) is the relaxation parameter and is selected to be 0.5.

**Appendix C  Controller Design**

### C-1  Linearized Model for Cruise Mode

The linearized model for cruise mode can be written as Equation 5-69 to Equation 5-72 for the longitudinal and lateral dynamics. The expressions of the state matrices elements for longitudinal dynamics are given as follows. The elements without explicit expressions are zero. The method used to derive these equations can be found in [Brockhaus, 2001].

\[ A_{long}^{cr} (1,1) = \frac{1}{m} \rho V_c S (-C_D + C_L \alpha) + \frac{1}{m} K_y \]  

Equation 7-30

\[ A_{long}^{cr} (1,3) = \frac{1}{2m} \rho V_c S (C_L - C_D \alpha - 2C_{D\alpha} + C_{L\alpha} \alpha) \]  

Equation 7-31

\[ A_{long}^{cr} (1,4) = -g \]  

Equation 7-32

\[ A_{long}^{cr} (1,5) = -w + \frac{\alpha}{2m} \rho V_c SC_L \]  

Equation 7-33

\[ A_{long}^{cr} (2,1) = -\theta \]  

Equation 7-34

\[ A_{long}^{cr} (2,3) = 1 \]  

Equation 7-35

\[ A_{long}^{cr} (2,4) = -V_c \]  

Equation 7-36

\[ A_{long}^{cr} (3,1) = \frac{1}{2m} \rho V_c S (-C_D \alpha - C_L) - \frac{i_k}{m} K_y \]  

Equation 7-37

\[ A_{long}^{cr} (3,3) = \frac{1}{2m} \rho V_c S (-C_D - C_L \alpha - C_{L\alpha}) - \frac{i_k}{m} k_y \alpha \]  

Equation 7-38

\[ A_{long}^{cr} (3,4) = -g \theta \]  

Equation 7-39
Appendix

\[ A_{\text{long}}^e (3,5) = u - \frac{1}{2m} \rho V_e S \tilde{C}_{Lq} \]  
\[ A_{\text{long}}^e (4,5) = 1 \]  
\[ A_{\text{long}}^e (5,3) = \frac{1}{2I_{yy}} \rho V_e S \tilde{C}_{\text{swa}} \]  
\[ A_{\text{long}}^e (5,5) = \frac{1}{2I_{yy}} \rho V_e S \tilde{C}_{\text{swq}} \]  
\[ B_{\text{long}}^e (1,1) = \frac{K_T}{m} \]  
\[ B_{\text{long}}^e (1,2) = \frac{1}{2m} \rho V_e^2 S (-C_{D\alpha} + C_{L\alpha} \alpha) \]  
\[ B_{\text{long}}^e (3,1) = -\frac{1}{m} K_T \alpha \]  
\[ B_{\text{long}}^e (3,2) = \frac{1}{2m} \rho V_e^2 S (-C_{D\alpha} \alpha - C_{L\alpha}) \]  
\[ B_{\text{long}}^e (5,2) = \frac{1}{2I_{yy}} \rho V_e^2 S \tilde{C}_{\text{m\bar{e}}} \]  
\[ C_{\text{long}}^e (1,1) = 1 \]  
\[ C_{\text{long}}^e (2,2) = 1 \]  
\[ C_{\text{long}}^e (3,4) = 1 \]  

Where, \( K_T \) denotes the thrust curve slope with respect to the collective pitch of the coaxial tiltrotor for cruise; \( K_T \) denotes the thrust curve slope with respect to the axial inflow rate of the coaxial tiltrotor; \( C_{\text{Da}} = \frac{C_{L\alpha}^2}{\pi e AR} \) denotes the influence of the aerodynamic drag coefficient variation with the AoA derived from Equation 5-25.

The expressions for lateral dynamics are given by the following equations.

\[ A_{\text{lat}}^e (1,2) = 1 \]  
\[ A_{\text{lat}}^e (1,5) = \theta \]  
\[ A_{\text{lat}}^e (2,2) = \frac{1}{2I_{xx}} \rho V_e S \tilde{b}^2 C_{\bar{p}} \]  

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\[ A_{lat}^{cr} (2,4) = \frac{1}{2I_{xx}} \rho V_{cr}^2 SbC_{ip} \quad \text{Equation 7-55} \]

\[ A_{lat}^{cr} (2,5) = \frac{1}{2I_{xx}} \rho V_{cr} Sb^2 C_r \quad \text{Equation 7-56} \]

\[ A_{lat}^{cr} (3,5) = 1 \quad \text{Equation 7-57} \]

\[ A_{lat}^{cr} (4,1) = \frac{g}{V_{cr}} \quad \text{Equation 7-58} \]

\[ A_{lat}^{cr} (4,2) = \alpha + \frac{1}{2m} \rho SbC_{ip} \quad \text{Equation 7-59} \]

\[ A_{lat}^{cr} (4,4) = \frac{1}{2m} \rho V_{cr}SC_{\gamma \beta} \quad \text{Equation 7-60} \]

\[ A_{lat}^{cr} (4,5) = -1 + \frac{1}{2m} \rho SbC_{\gamma r} \quad \text{Equation 7-61} \]

\[ A_{lat}^{cr} (5,2) = \frac{1}{2I_{zz}} \rho V_{cr} Sb^2 C_{ip} \quad \text{Equation 7-62} \]

\[ A_{lat}^{cr} (5,4) = \frac{1}{2I_{zz}} \rho V_{cr} SbC_{\gamma p} \quad \text{Equation 7-63} \]

\[ A_{lat}^{cr} (5,5) = \frac{1}{2I_{zz}} \rho V_{cr} Sb^2 C_{\gamma r} \quad \text{Equation 7-64} \]

\[ C_{lat}^{cr} (1,1) = 1 \quad \text{Equation 7-65} \]

\[ C_{lat}^{cr} (2,3) = 1 \quad \text{Equation 7-66} \]

C-2  Linearized Model for Transition Mode

The linearized model for transition mode can be written as Equation 5-91 to Equation 5-94 for the longitudinal and lateral dynamics. The expressions of the state matrices elements for longitudinal dynamics are given as follows. Both the effective elevator and auxiliary rotor will be used as the pitch actuator in transition, so \( B_{long}^{trans} (5,1) \) has two expressions (Equation 7-85 and Equation 7-86).

\[ A_{long}^{trans} (1,1) = \frac{\cos i_f}{m} K_r \cos (i_f + \alpha) + \frac{1}{m} \rho V_{cr} S (C_D + C_{La}) \quad \text{Equation 7-67} \]

\[ A_{long}^{trans} (1,3) = \frac{\cos i_f}{m} K_r \cos (i_f + \alpha) \alpha - \frac{\cos i_f}{m} K_r \sin (i_f + \alpha) \]

\[ + \frac{1}{2m} \rho V_{cr} S (C_L - C_D \alpha - 2C_{La} \alpha + C_{La} \alpha) \quad \text{Equation 7-68} \]
Appendix

\[ A_{\text{long}}^{\text{trans}} (1,4) = -g \]  
Equation 7-69

\[ A_{\text{long}}^{\text{trans}} (1,5) = -w + \frac{2m}{W} \rho V_c SN C_{Lq} \]  
Equation 7-70

\[ A_{\text{long}}^{\text{trans}} (2,1) = -\theta \]  
Equation 7-71

\[ A_{\text{long}}^{\text{trans}} (2,3) = 1 \]  
Equation 7-72

\[ A_{\text{long}}^{\text{trans}} (2,4) = -V_c \]  
Equation 7-73

\[ A_{\text{long}}^{\text{trans}} (3,1) = \frac{m}{\sin(i_F)} \cos(i_F + \alpha) \frac{\alpha}{m} \rho V_c SC_{D} - \frac{1}{m} \rho V_c SC_{L} \]  
Equation 7-74

\[ A_{\text{long}}^{\text{trans}} (3,3) = \frac{\sin(i_F)}{m} K_{l} \cos(i_F + \alpha) + \frac{\sin(i_F)}{m} K_{f} \sin(i_F + \alpha) + \frac{1}{2m} \rho V_c S(-C_D - C_L \alpha - C_{Ls}) \]  
Equation 7-75

\[ A_{\text{long}}^{\text{trans}} (3,4) = -g \theta \]  
Equation 7-76

\[ A_{\text{long}}^{\text{trans}} (3,5) = u - \frac{1}{2m} \rho V_c SC_{Lq} \]  
Equation 7-77

\[ A_{\text{long}}^{\text{trans}} (4,5) = 1 \]  
Equation 7-78

\[ A_{\text{long}}^{\text{trans}} (5,3) = \frac{1}{m} \rho V_c SC_{m} \]  
Equation 7-79

\[ A_{\text{long}}^{\text{trans}} (5,5) = \frac{1}{2I_{yy}} \rho V_c SC^2 C_{au} \]  
Equation 7-80

\[ B_{\text{long}}^{\text{trans}} (1,1) = \frac{1}{2m} \rho V_c S(-C_{Dc} + C_{Lc} \alpha) \]  
Equation 7-81

\[ B_{\text{long}}^{\text{trans}} (3,1) = \frac{1}{2m} \rho V_c S(-C_{Dc} \alpha - C_{Lc}) \]  
Equation 7-82

\[ B_{\text{long}}^{\text{trans}} (3,2) = \frac{\sin(i_F)}{m} K_{l} V_c \sin(i_F + \alpha) - \frac{T}{m} \cos i_F \]  
Equation 7-83

\[ B_{\text{long}}^{\text{trans}} (3,3) = \frac{K_{f}}{m} \sin i_F \]  
Equation 7-84

\[ B_{\text{long}}^{\text{trans}} (5,1) = \frac{K_{m}}{m} x_{aux} \]  
Equation 7-85

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The expressions for lateral dynamics are given by the following equations. $B_{\text{lat}}^{\text{trans}} (2,1)$ has two expressions (Equation 7-103 and Equation 7-104).

\[ A_{\text{lat}}^{\text{trans}} (1,2) = 1 \]  
Equation 7-90
\[ A_{\text{lat}}^{\text{trans}} (1,5) = \theta \]  
Equation 7-91
\[ A_{\text{lat}}^{\text{trans}} (2,2) = \frac{1}{2I_{XX}} \rho V_{cr} S \beta C_{ip} \]  
Equation 7-92
\[ A_{\text{lat}}^{\text{trans}} (2,4) = \frac{1}{2I_{XX}} \rho V_{cr}^{2} S \beta C_{ig} \]  
Equation 7-93
\[ A_{\text{lat}}^{\text{trans}} (2,5) = \frac{1}{2I_{XX}} \rho V_{cr} S \beta^{2} C_{cr} \]  
Equation 7-94
\[ A_{\text{lat}}^{\text{trans}} (3,5) = 1 \]  
Equation 7-95
\[ A_{\text{lat}}^{\text{trans}} (4,1) = \frac{R}{V_{cr}} \]  
Equation 7-96
\[ A_{\text{lat}}^{\text{trans}} (4,2) = \alpha + \frac{1}{2m} \rho S \beta C_{yp} \]  
Equation 7-97
\[ A_{\text{lat}}^{\text{trans}} (4,4) = \frac{1}{2m} \rho V_{cr} S C_{y\beta} \]  
Equation 7-98
\[ A_{\text{lat}}^{\text{trans}} (4,5) = -1 + \frac{1}{2m} \rho S \beta C_{yr} \]  
Equation 7-99
\[ A_{\text{lat}}^{\text{trans}} (5,2) = \frac{1}{2I_{Z2}} \rho V_{cr} S \beta^{2} C_{np} \]  
Equation 7-100
\[ A_{\text{lat}}^{\text{trans}} (5,4) = \frac{1}{2I_{Z2}} \rho V_{cr}^{2} S \beta C_{n\beta} \]  
Equation 7-101
\[ A_{\text{lat}}^{\text{trans}} (5,5) = \frac{1}{2I_{Z2}} \rho V_{cr} S \beta^{2} C_{nr} \]  
Equation 7-102
The nominal values will be used to solve the expressions above, and then the state space matrices are obtained. Therefore, we can use the method proposed in Chapter 5 to design the controller. This frozen coefficient method is a standard method widely used in the aircraft control community [Brockhaus, 2001]. Since it is a linearized model, the variation range for the state deviations is limited. According to the community, the angular deviation is selected to be smaller than 10°. The velocity deviation selected is usually smaller than 10% of the nominal value. These are only the empirical values. The selection can be different for different situations.

**Appendix D  Simulation Parameters**

The corresponding parameters for simulations in VTOL, cruise, and transition/conversion modes are listed in Table D-1, Table D-2, and Table D-3, respectively. The aerodynamic coefficients of the airframe in cruise mode and transition mode are given by Table B-1.

<table>
<thead>
<tr>
<th>Aerodynamic drag</th>
<th>1.22N</th>
</tr>
</thead>
<tbody>
<tr>
<td>A.C. from C.G.</td>
<td>0.93m</td>
</tr>
<tr>
<td>Distance between C.G. and the auxiliary rotor in $X_B$ axis</td>
<td>1.7m</td>
</tr>
<tr>
<td>Distance between the two auxiliary rotor</td>
<td>7m</td>
</tr>
<tr>
<td>Rotational speed of auxiliary rotors</td>
<td>500rad\cdot s^{-1}</td>
</tr>
<tr>
<td>Rotational speed of coaxial rotors</td>
<td>190rad\cdot s^{-1}</td>
</tr>
<tr>
<td>Total mass</td>
<td>25kg</td>
</tr>
<tr>
<td>-----------</td>
<td>------</td>
</tr>
<tr>
<td>Moment of inertia $I_{xx}$</td>
<td>123.18kg·m²</td>
</tr>
<tr>
<td>Moment of inertia $I_{yy}$</td>
<td>8.587kg·m²</td>
</tr>
<tr>
<td>Moment of inertia $I_{zz}$</td>
<td>131.69kg·m²</td>
</tr>
<tr>
<td>Moment of inertia product $I_{xy}$</td>
<td>0.026825kg·m²</td>
</tr>
<tr>
<td>Moment of inertia product $I_{xz}$</td>
<td>0.48174kg·m²</td>
</tr>
<tr>
<td>Moment of inertia product $I_{xy}$</td>
<td>$8.5831\times10^4$kg·m²</td>
</tr>
<tr>
<td>Gravity</td>
<td>3.71m·s⁻¹</td>
</tr>
<tr>
<td>Frequency of actuators</td>
<td>20Hz</td>
</tr>
<tr>
<td>Damping of actuators</td>
<td>0.7N·s·m⁻¹</td>
</tr>
<tr>
<td>Delay of actuators</td>
<td>100ms</td>
</tr>
<tr>
<td>Pitch range of the auxiliary rotors</td>
<td>$-20°$~$20°$</td>
</tr>
<tr>
<td>Pitch variation limit of coaxial tiltrotor (corresponding to the working point)</td>
<td>$-3°$~$3°$</td>
</tr>
<tr>
<td>Pitch differential of the coaxial tiltrotor</td>
<td>$-2°$~$2°$</td>
</tr>
</tbody>
</table>

Table D-2: Simulation parameters for cruise mode

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Atmosphere density</td>
<td>0.0135kg·m⁻³</td>
</tr>
<tr>
<td>Wing surface</td>
<td>8.23m²</td>
</tr>
<tr>
<td>Mean aerodynamic chord of the wing</td>
<td>1.3046m</td>
</tr>
<tr>
<td>Wing span</td>
<td>8.558m</td>
</tr>
<tr>
<td>Nominal AoA</td>
<td>5.4°</td>
</tr>
<tr>
<td>Nancelle angle</td>
<td>-5.4°</td>
</tr>
<tr>
<td>Nominal elevon deflection</td>
<td>9.2°</td>
</tr>
<tr>
<td>Nominal cruise speed</td>
<td>50m·s⁻¹</td>
</tr>
<tr>
<td>Total mass</td>
<td>25kg</td>
</tr>
<tr>
<td>Moment of inertia $I_{xx}$</td>
<td>124.12kg·m²</td>
</tr>
<tr>
<td>Moment of inertia $I_{yy}$</td>
<td>8.587kg·m²</td>
</tr>
<tr>
<td>Moment of inertia $I_{zz}$</td>
<td>130.75kg·m²</td>
</tr>
<tr>
<td>Moment of inertia product $I_{xy}$</td>
<td>0.026825kg·m²</td>
</tr>
<tr>
<td>Moment of inertia product $I_{xz}$</td>
<td>0.48174kg·m²</td>
</tr>
</tbody>
</table>
Appendix

<table>
<thead>
<tr>
<th>Moment of inertia product $I_{XY}$</th>
<th>$8.5831 \times 10^{-4} \text{kg} \cdot \text{m}^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pitch command angle limit</td>
<td>$-5^\circ$ ~ $10^\circ$</td>
</tr>
<tr>
<td>Roll command angle limit</td>
<td>$-10^\circ$ ~ $10^\circ$</td>
</tr>
<tr>
<td>Coaxial tiltrotor pitch variation limit (comparing with the working pint)</td>
<td>$-5^\circ$ ~ $5^\circ$</td>
</tr>
<tr>
<td>Elevon limit</td>
<td>$30^\circ$ ~ $30^\circ$</td>
</tr>
</tbody>
</table>

Table D-3: Simulation parameters for transition/conversion mode

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Atmosphere density</td>
<td>$0.0135 \text{kg} \cdot \text{m}^{-3}$</td>
</tr>
<tr>
<td>Wing surface</td>
<td>$8.23 \text{m}^2$</td>
</tr>
<tr>
<td>Mean aerodynamic chord of the wing</td>
<td>$1.3046 \text{m}$</td>
</tr>
<tr>
<td>Wing span</td>
<td>$8.558 \text{m}$</td>
</tr>
<tr>
<td>Distance between C.G. and the auxiliary rotor in $X_\mu$ axis</td>
<td>$1.7 \text{m}$</td>
</tr>
<tr>
<td>Distance between the two auxiliary rotor</td>
<td>$7 \text{m}$</td>
</tr>
<tr>
<td>Nominal AoA</td>
<td>$5.4^\circ$</td>
</tr>
<tr>
<td>Nancelle angle</td>
<td>$-5.4^\circ$</td>
</tr>
<tr>
<td>Nominal elevon deflection</td>
<td>$9.2^\circ$</td>
</tr>
<tr>
<td>Nominal cruise speed</td>
<td>$50 \text{m} \cdot \text{s}^{-1}$</td>
</tr>
<tr>
<td>Total mass</td>
<td>$25 \text{kg}$</td>
</tr>
<tr>
<td>Moment of inertia $I_{xx}$</td>
<td>$123.65 \text{kg} \cdot \text{m}^2$</td>
</tr>
<tr>
<td>Moment of inertia $I_{yy}$</td>
<td>$8.587 \text{kg} \cdot \text{m}^2$</td>
</tr>
<tr>
<td>Moment of inertia $I_{zz}$</td>
<td>$131.22 \text{kg} \cdot \text{m}^2$</td>
</tr>
<tr>
<td>Moment of inertia product $I_{xy}$</td>
<td>$0.026825 \text{kg} \cdot \text{m}^2$</td>
</tr>
<tr>
<td>Moment of inertia product $I_{xz}$</td>
<td>$0.48174 \text{kg} \cdot \text{m}^2$</td>
</tr>
<tr>
<td>Moment of inertia product $I_{xy}$</td>
<td>$8.5831 \times 10^{-4} \text{kg} \cdot \text{m}^2$</td>
</tr>
<tr>
<td>Pitch command angle limit</td>
<td>$-5^\circ$ ~ $10^\circ$</td>
</tr>
<tr>
<td>Roll command angle limit</td>
<td>$-10^\circ$ ~ $10^\circ$</td>
</tr>
<tr>
<td>Coaxial tiltrotor pitch variation limit (comparing with the working pint)</td>
<td>$-5^\circ$ ~ $5^\circ$</td>
</tr>
<tr>
<td>Elevon limit</td>
<td>$30^\circ$ ~ $30^\circ$</td>
</tr>
<tr>
<td>Pitch range of the auxiliary rotors</td>
<td>$-20^\circ$ ~ $20^\circ$</td>
</tr>
<tr>
<td>Pitch differential of the coaxial tiltrotor</td>
<td>$-2^\circ$ ~ $2^\circ$</td>
</tr>
</tbody>
</table>
Bibliography


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