A CTOD equation based on the rigid rotational factor with the consideration of crack tip blunting due to strain hardening for SEN(B)

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Abstract
Crack tip opening displacement (CTOD) from national and international standards was shown to give different values. This paper investigates the feasibility of CTOD determined based on the concept of rigid rotational factor in single-edge notched bend (SENB) specimens. Based on validated modelling methods, finite element (FE) models were simulated for crack ratios $0.3 \leq a_0/W \leq 0.7$ and yield-to-tensile ratio $0.44 \leq \sigma_{ys}/\sigma_{uts} \leq 0.98$. This covers cases of shallow to deeply cracked specimens and a wide range of strain hardening properties. CTOD obtained from the FE models was used as the basis of a newly implemented strain hardening corrected rotational factor, which considers the effects of crack tip blunting due to strain hardening, $r_{ps\ h}$. An improved equation considering strain hardening was implemented based on the $r_{ps\ h}$. The equation gives accurate estimation of CTOD from the FE models compared with the equation from BS 7448-1, ASTM E1820, and WES 1180.

KEYWORDS
CTOD, rotational factor, strain hardening, varied crack length

1 | INTRODUCTION

The crack tip opening displacement (CTOD) is one of the best known strain-based elastic-plastic fracture toughness parameter in fracture mechanics. The concept was originally developed in TWI\textsuperscript{1} in the 1960s, commonly used in the oil and gas industry. CTOD is advantageous in cases where the concept of linear elastic fracture...
mechanics is insufficient to account for ductile deformation, such as in pressure vessels, offshore platforms, and pipelines. Other well-known fracture toughness parameters includes (but not limited to) the $J$-integral, stress intensity factor, and crack tip opening angle. CTOD is often preferred in cases where the component is loaded primarily in strain, ie, offshore pipe reeling.

There are several different definition for CTOD, but all describes CTOD as a material parameter relative to the physical opening of the crack at the crack tip region. In general, CTOD is described as the displacement-based fracture toughness of a material at the point of maximum load, initiation of stable crack extension, and unstable crack extension. In lab conditions, specimens with notches representing cracks are loaded, in which the load-displacement data and specimen set-ups details are recorded for the calculation of CTOD. In addition to the typical specimen set-up, ie, C(T) and SEN(B), practical full-sized specimen set-up, ie, PRN(B), had been developed in the recent years, which reduces machining and the effects because of subsized specimens.

Fracture toughness testing (often described simply as “CTOD testing”) became standardised in the 1970s and is currently represented in a number of standards including BS 7448, ISO 12135, ASTM E1820, and WES1108. However, different assumptions about the determination of CTOD are used in each, which can give different values of CTOD.

It is important to define the value of CTOD with accuracy, particularly when CTOD is being used to determine tolerable flaw sizes for structural integrity assessments. Underestimates of CTOD would lead to either rejection of the material or unnecessary repairs, increasing the cost of operation and/or fabrication. On the other hand, overestimates of CTOD might lead to potentially unsafe structures being assessed as fit for service.

2 | THE CONCEPT OF RIGID ROTATIONAL FACTOR FOR THE DETERMINATION OF CTOD

When a SEN(B) specimen is loaded under 3-point bending, the crack tip would experience tensile stress, whereas there would be a region in the uncracked ligament that experiences compression. In the calculation of the plastic component of CTOD, BS 7448-1, ISO 12135, WES1108, early versions of ISO 12135, ASTM E1290-93, and E1820-01 assumed that the specimen flanges rotate about a stationary point within the uncracked ligament ahead of the crack tip, a distance equal to $r_p \times B_0$, where $r_p$ is the rotational factor and $B_0$ is the remaining ligament ahead of the crack tip. This concept was introduced by Dawes in 1979, using a 2-D plastic hinge model with assumptions of plane strain condition at the crack tip. Based on the argument that there is a tensile and compressive region ahead of the crack tip, the idea of a rotational hinge point had also been applied in the calculation of the crack tip opening angle (CTOA).

According to the assumption of slip line theory, the flanges of the specimen would deform as rigid arms rotating about a rotational point. Green’s observation on photo-elastic images of notched bend specimens showed that the yielding pattern at the crack tip region is similar to that predicted in a slip-line field. Digital image correlation observation on a SEN(B) specimen showed that the strain distribution near the crack tip is similar to that described in the slip line field theory by Green.

The determination of the rotational factor is based on the geometrical analysis of a specimen under 3-point bend loading. Consider a deformed SEN(B) specimen (Figure 1), the distance of the rotational point from the crack mouth opening displacement (CMOD), $H$ is defined in terms of rotational factor, $r_p(W - a) + a$.

![Diagram for the evaluation of the geometrical based crack tip opening displacement](image)
From Figure 1, it is shown that

\[ r_p = \frac{H - a}{W - a} \]

\[ V_p = 2[r_p(W - a) + a + \delta] \sin \theta_p \]

\[ \delta_p = 2r_p(W - a) \sin \theta_p. \]

Relating \( V_p \) into \( \delta_p \) gives

\[ \delta_{pl} = \frac{r_p(W - a)V_p}{[r_p(W - a) + a + \delta]}, \]

where \( \delta_{pl} \) is the plastic component of CTOD, where \( \delta = \delta_{el} + \delta_{pl} \). Anderson et al found that \( r_p \) is insensitive to the specimen geometry for the same material. Wells shown that in the case of the SEN(B) specimen geometry, initially, the rotational point would be close to the crack tip \( (r_p < 0.1) \), which extends and converge to a point within the unbroken ligament ahead of the crack tip \( (r_p \approx 0.45) \) with increasing load and after general yielding. BS 7448 assumed a constant value of \( r_p = 0.4 \) in the calculation of CTOD. The basis for the constant \( r_p \) for SEN(B) is not in the public domain, but it was understood that this value is determined through extensive experiments and should underestimate the actual CTOD.

3 | METHODOLOGY

3.1 | Finite element modelling techniques

A total of 50 quarter Bx2B single-edge notched bend (SENB) specimen models were generated to investigate CTOD for a range of \( a_0/W \) and \( \sigma_{ys}/\sigma_{ult} \) using ABAQUS v6.14. A blunted crack tip of 0.03-mm radius was used in the models, which allows better plastic deformation at the crack tip region. The modelling technique used in this paper was shown to give good prediction of CTOD, further described in previous published works. The crack tip was designed to blunt continuously as the crack open without stable ductile tearing and thus leads to lower values of CTOD.

Ten different strain hardening properties were generated using the modified Ramberg-Osgood power law for \( 0.44 \leq \sigma_{ys}/\sigma_{ult} \leq 0.98 \), given below:

\[ \varepsilon = \frac{\sigma}{E} + \alpha \left( \frac{\sigma}{\sigma_{ys}} \right)^{n-1}, \]  

where \( \alpha = 0.002, E = 207 \text{GPa}, \) and \( \sigma_{ys} = 400 \text{MPa} \). The decrease of \( n \) in Equation 1 gives increasing strain hardening properties (decreasing \( \sigma_{ys}/\sigma_{ult} \)). The true stress strain curves obtained using Equation 1 were used to describe the material tensile properties of the models (Figure 2).

In addition to the different strain hardening properties, five different crack length models were generated to cover \( 0.3 \leq a_0/W \leq 0.7 \). These cover the cases of shallow crack up to deeply cracked specimen set-up. Figure 3 shows the symmetry boundary conditions specified on a typical model and technique used to extract the CTOD values.

3.2 | The determination of \( r_p \) based on the similar triangles method

The concept of rigid rotational point has been successfully implemented for the determination of CTOD with various degree of accuracy. To investigate the rotational factor, \( r_p \) for SEN(B) specimens, an opened crack was investigated geometrically based on the similar
triangles concept (Figure 4). This method extrapolates the crack face angles into the unbroken ligament ahead of the crack tip, where the intersection of the angles is described as the rotational point. Similar concept with minor modifications had been employed by BS 8571, ExxonMobil, and DNV in their calculation of CTOD. The following terms were used to simplify the derivation, \( r_pB_0 = Y \), \( z_1 + a_0 = C \), and \( a_0 + z_2 = D \). To relate the lower and upper clip gauge opening, \( V_{g1} \) and \( V_{g2} \) respectively to the point of rotation,

\[
\sin \theta = \frac{V_{g1}}{C + Y} = \frac{V_{g2}}{D + Y}
\]

leading to

\[
\frac{V_{g2}}{V_{g1}} = \frac{D + Y}{C + Y}.
\]

Expanding \( D \) and factoring \( C + Y \) gives

\[
\frac{V_{g2}}{V_{g1}} = \frac{C + Y + (z_2 - z_1)}{C + Y} = 1 + \frac{(z_2 - z_1)}{C + Y}.
\]

Rearranging the equation leads to

\[
Y = r_pB_0 = \left( \frac{z_2 - z_1}{\frac{V_{g2}}{V_{g1}} - 1} \right) - (z_1 + a_0),
\]

where the rotation factor, \( r_p \), based on \( V_{g1} \) and \( V_{g2} \) is given as

\[
r_p = \left[ \left( \frac{z_2 - z_1}{\frac{V_{g2}}{V_{g1}} - 1} \right) - (z_1 + a_0) \right] \times \frac{1}{B_0}.
\]

Equation 2 allows the rotational factor to be calculated based on two clip gauges positioned at different heights above the crack mouth in standard laboratory tests. Similarly, the rotational factor based on the plastic
displacement can be obtained by simply replacing the lower and upper clip gauge displacement, \( V_{g1} \) and \( V_{g2} \) with the plastic lower and upper clip gauge displacement, \( V_{p1} \) and \( V_{p2} \).

### 3.3 | Strain hardening corrected \( r_p \) based on finite element modelling, \( r_{p\ sh} \)

The SEN(B) models were modelled under 3-point bend loading and the apex of the crack face opening was identified as the hinge location. Findings from experimental work and finite element (FE) modelling shows that as the crack open due to loading, the effects of strain hardening due to plasticity cause the crack tip to deform away from the crack face plane.\(^7\)\(^{28-30}\) To account for the effects of crack tip blunting due to strain hardening, \( r_p \) was extracted based on the intersection line extrapolated from the CMOD and CTOD to the symmetry line (Figure 5). This \( r_p \) accounts for crack tip blunting and should give lower \( r_p \) than that obtained from crack face angles, thereafter described as \( r_{p\ sh} \).\(^7\)\(^{21}\)

To extract \( r_{p\ sh} \) from the finite element (FE) models, Equation 2 was modified based on the CTOD and CMOD, described as

\[
r_{p\ sh} = R_1 \left( \frac{\sigma_{ys}}{\sigma_{uts}} \right) + R_2
\]

\( R_1 = -0.62(a_0/W) + 0.76 \)
\( R_2 = 0.61(a_0/W) - 0.22 \).

### 4 | RESULTS

#### 4.1 | Strain hardening \( r_p, r_{p\ sh} \) extracted from the models

To obtain a better resolution of the \( r_{p\ sh} \) distribution due to the effects of strain hardening, data were extracted from the FE models and processed using Equation 3. CTOD was extracted for \( 0.2 \text{ mm} < \delta_{FE} < 1.0 \text{ mm} \) to minimise the influence of the elastic CTOD and large deformation.\(^7\)\(^{21}\) Figure 6 shows \( r_{p\ sh} \) for the increasing FE CTOD, \( \delta_{FE} \).

Figure 6 shows the dependency of \( r_{p\ sh} \) on tensile ratio, where in general, \( r_{p\ sh} \) decreases with the increase of strain hardening. With the increase of CTOD, \( r_{p\ sh} \) would increase gradually with decreasing gradient. To exhibit the distribution of \( r_{p\ sh} \) due to strain hardening and crack length-specimen width ratio (\( a_0/W \)), \( r_{p\ sh} \) was plotted to \( \sigma_{ys}/\sigma_{uts} \) (Figure 7). In general, the decrease of \( a_0/W \) from 0.7 to 0.3 shows an overall decrease of \( r_{p\ sh} \). As \( \sigma_{ys}/\sigma_{uts} \) move towards 1, \( r_{p\ sh} \) for all \( a_0/W \) tends to converge and give a similar value. Models from \( a_0/W = 0.5 \) were validated experimentally\(^7\)\(^{21}\) and thus were used as the baseline for the fitting. A linear line was fitted to the \( r_{p\ sh} \) data from their respective \( a_0/W \), while passing through point \( \sigma_{ys}/\sigma_{uts} = 0.98, r_{p\ sh} = 0.51 \). This is based in the assumption where \( r_{p\ sh} \) converges at a fixed constant value for materials behaving in an elastic–perfect plastic behaviour. Data from \( \sigma_{ys}/\sigma_{uts} = 0.98 \) would be exhibiting elastic dominant properties where a blunted crack tip could give a significant larger CTOD and therefore was not used in the fitting.\(^{31,32}\)

Based on the fitting of the data, the \( r_{p\ sh} \) equation can be described using the following equation:

\[
r_{p\ sh} = R_1 \left( \frac{\sigma_{ys}}{\sigma_{uts}} \right) + R_2
\]

\( R_1 = -0.62(a_0/W) + 0.76 \)
\( R_2 = 0.61(a_0/W) - 0.22 \).

### 5 | DISCUSSION

Equation 2 shown that based on the principles of similar triangles and assuming that the crack tip location would
be in line with the crack face plane, the rotational factor can be determined directly from the test specimen. Conceptually, this allows the \( r_p \) to be extracted at any instance of the test for the calculation of CTOD. This method requires two clip gauge measurements, positioned at different heights above the crack mouth.

An idealised case was investigated based on the typical double clip gauge set-up used in experiments, where
The lower and upper clip gauges, \( z_1 \) and \( z_2 \), were positioned 2 and 12 mm above the crack mouth, respectively. Equation 2 shows that the clip gauge ratio, \( V_{g2}/V_{g1} \), is the main determinant of the resultant \( r_p \). For the rotational point to fall ahead of the crack tip, \( V_{g2} \) must be larger than \( V_{g1} \) at all stages of loading. Figure 8 shows the effect of various \( V_{g2}/V_{g1} \) on the resulting \( r_p \). As \( V_{g2}/V_{g1} \) moves towards 1, \( r_p \) tends to move towards infinity (Figure 8, top). In real specimens, \( r_p \) lies in the unbroken ligament ahead of the crack tip, which falls between \( 0 < r_p < 1 \). Figure 8B shows that within the unbroken ligament, \( r_p \) decreases with the increase of the clip gauge ratio. Therefore, in laboratory tests, it should be noted that negative \( r_p \) and \( r_p \geq 1 \) are indications of false reading and do not reflect the actual \( r_p \) of the specimen.7

Generally, the increasing \( \sigma_{ys}/\sigma_{uts} \) (decreasing strain hardening) results in increasing \( r_{p,sh} \) in a linear relationship for each respective \( a_0/W \) set-up (Figure 7). As strain hardening decreases, the material tensile response exhibits increasing similarity to an elastic–perfect plastic tensile response (Figure 2). This implies that as strain hardening decreases (decreasing plastic behaviour), there will be decreased blunting at the crack tip because of the opening of the crack faces.7 In the ultimate case of elastic–perfect plastic situation (no strain hardening), the crack tip would fall almost in the plane of the crack face.30

The models suggest that as \( a_0/W \) increases, \( r_{p,sh} \) would move gradually towards the middle of the material ahead of the crack tip \( (r_{p,sh} \approx 0.5) \). Because of the different \( a_0/W \), the maximum difference of \( r_{p,sh} \) is seen in \( \sigma_{ys}/\sigma_{uts} = 0.44 \), the lowest difference in \( \sigma_{ys}/\sigma_{uts} = 0.89 \). The \( r_{p,sh} \) decreases sensitivity to \( a_0/W \) as strain hardening decreases (increasing \( \sigma_{ys}/\sigma_{uts} \)), and the \( r_{p,sh} \) trend from the different \( a_0/W \) seemingly intersects at a point between \( 0.89 \leq \sigma_{ys}/\sigma_{uts} \leq 1.00 \). This is portrayed by the \( r_{p,sh} \) at \( \sigma_{ys}/\sigma_{uts} = 0.98 \), where \( a_0/W = 0.7 \) gave the lowest \( r_{p,sh} \), followed by \( a_0/W = 0.6, 0.5, 0.4, \) and \( 0.3 \), which is the opposite of that observed in \( 0.44 \leq \sigma_{ys}/\sigma_{uts} \leq 0.89 \). It was suspected that this phenomenon is contributed by the way \( r_{p,sh} \) was extracted from the model, where the total CTOD was used instead of separating it into the elastic and plastic.

Given that Equation 4 is able to account for the effects crack tip blunting due to strain hardening in terms of \( r_{p,sh} \), the equation used in BS 7448-1 based on the similar triangles approach was modified to account for the effects of crack tip blunting due to strain hardening, described as

\[
\delta_{sh} = K^2 \left(1 - \nu^2\right) \frac{m_{wes} \sigma_{ys} E}{r_{p,sh} B a V_p} + \frac{r_{p,sh} B a V_p}{r_{p,sh} B a + a_0}, \tag{5}
\]

where \( m_{wes} = 4.9 - 3.5(\sigma_{ys}/\sigma_{uts}) \).

The elastic CTOD from WES 1108 was used as it was calibrated theoretically to account for the effects of strain hardening in the elastic loading region.6,7,16,20,21,29

The error in prediction of CTOD using Equation 5 was investigated by normalising it to the CTOD from the models (\( \delta_{FE} \)), described as CTOD ERR. For the range of \( 0.44 \leq \sigma_{ys}/\sigma_{uts} \leq 0.98 \) and \( 0.3 \leq a_0/W \leq 0.7 \), CTOD between 0.2 and 1.0 mm was considered for the investigation. CTOD below 0.2 mm is considered “critical” in standards, where the elastic-based stress intensity factor, \( K \), is more relevant13,5; CTOD beyond 1.0 mm is generally large, where some significant stable crack propagation would be expected in real specimens.7,20,33

The mean and standard deviation of CTOD ERR were calculated and plotted to \( a_0/W \) for all CTOD equations: CTOD from Equation 5 (\( \delta_{sh} \)), BS 7448-1,
WES 1108, and ASTM E1820 (Figure 9). The equations were compared relative with the respective $a_0/W$ and $\sigma_{ys}/\sigma_{uts}$ (Figure 9A,B).

For $0.3 \leq a_0/W \leq 0.7$, the CTODs considering the effects of strain hardening (CTOD from Equation 5, WES 1108 and ASTM E1820) gave lower overall error compared with BS 7448-1 (Figure 9A). The mean CTOD from BS 7448-1 overestimated CTOD for $a_0/W = 0.3$ and 0.4, as it is outside the valid range of $0.45 \leq a_0/W \leq 0.55$ specified in the standard. The three strain hardening CTODs gave comparable low values for the mean CTOD ERR, while CTOD from Equation 5 gave the most accurate estimation for the range of $0.3 \leq a_0/W \leq 0.7$. Likewise, the best consistency/dispersion of data was portrayed by CTOD from Equation 5 with the lowest standard deviation, followed by ASTM E1820 then both WES 1108 and BS 7448-1.

In terms of strain hardening, for the range of $0.44 \leq \sigma_{ys}/\sigma_{uts} \leq 0.98$, ASTM E1820 gave the best estimation for $0.44 \leq \sigma_{ys}/\sigma_{uts} \leq 0.54$, but the accuracy and consistency reduce as $\sigma_{ys}/\sigma_{uts}$ increases to 1 (Figure 9B). Similar to before, $\delta_{sh}$ gave best compromise of both accuracy and consistency/dispersion. BS 7448-1 did not consider strain hardening in the equation; it overestimated high strain hardening and underestimated low strain hardening, similar to that observed in previous works.\textsuperscript{7,16,20,21} In general, the lower estimation and accuracy shown by WES 1108 is contributed by the assumptions used in the calibration of the equation\textsuperscript{17}; ASTM E1820 converts $J$ into $\delta_{45,5,30}$ which gives lower CTOD compared with CTOD based on the opening of the original crack tip.\textsuperscript{7,20} which is employed in this work. The results shown that the rigid rotational concept is a robust and reliable technique for the estimation of CTOD, especially when strain hardening consideration is employed.

6 | CONCLUSIONS

The concept of rigid rotational factor is a robust concept implemented for the estimation of CTOD. A series of FE models generated within the range of $0.44 \leq \sigma_{ys}/\sigma_{uts} \leq 0.98$ and $0.3 \leq a_0/W \leq 0.7$ showed that SEN(B) tests do not exhibit a fixed value of $r_p$ but instead increase $r_p$ in a linear manner from 0.2 to 0.53 with the decrease of strain hardening (increasing $\sigma_{ys}/\sigma_{uts}$).

The models showed that for the same strain hardening property, $r_p$ would increase with increasing $a_0/W$ from 0.3 to 0.7. However, as the material strain hardening decreases, $r_p$ showed reduced sensitivity to $a_0/W$. Data suggest that that $r_p$ converges at a point independent of $a_0/W$ between 0.89 $\leq \sigma_{ys}/\sigma_{uts} \leq 1.0$, when the material exhibits significant elastic dominant properties. In the calculation of CTOD, $\delta_{sh}$ (Equation 5) showed the best compromise of both accuracy and dispersion compared with BS 7448-1, ASTM E1820, and WES 1108 for the range of $0.44 \leq \sigma_{ys}/\sigma_{uts} \leq 0.98$ and $0.3 \leq a_0/W \leq 0.7$.

In laboratory tests or modelling works, Equation 2 enables the extraction of $r_p$ for the calculation of CTOD based on the similar triangles approach. However, caution should be exercised to account for human and test equipment error, as the estimation is heavily dependent on the displacement input at the crack mouth region. It should be noted that this technique does not account for the effects of crack tip blunting due to strain hardening.

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