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Title of paper:

Robustness and Resilience of Structures under Extreme Loads

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Abstract

Many of modern life activities involve the risk of fire, explosions and impacts. In addition, natural extreme events are becoming more and more common. Thus, robustness, the ability to avoid disproportionate collapse due to an initial damage, and resilience, the ability to adapt to and recover from the effects of changing external conditions, represent two important characteristics of current structures and infrastructures. Their definitions are reviewed in this paper with the aim of sorting and describing the different approaches proposed in the literature and in the international standards. A simple example is also analysed in order to compare different methods

1. Introduction

Despite advances never experienced before in technological development, catastrophic failures of structures and infrastructure systems happen from time to time as a consequence of natural or man-made extreme events. This is an effect of both a changing climate and general changes in our society with an increasing pressure in optimizing the design and management of infrastructure including a more sustainable use of materials, structures and land use. We are building taller and larger structures than

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ever under increasing construction pace and also within extreme environments which would not be considered possible in the past.

Absolute safety can never be achieved; therefore it is important to consider what would happen should one or several elements of a structure fail:

- Would element failure in a system lead to the collapse of the entire system or a significant part of it?
- Would the system’s functionality be limited after such a failure?
- What is an acceptable and tolerable performance under such circumstances?

To answer these questions the robustness of the structure needs to be assessed and evaluated. In short, robustness is often described as the structure’s ability to avoid disproportionate collapse due to an initial damage.

Besides limiting damage due to extreme events, it is important to consider how the built environment can be refurbished or rebuilt after a disaster in an efficient and timely manner. Therefore, the topic of infrastructure resilience has gained an increasing attention in the recent years. Resilience, roughly refers to the ability of the infrastructure to adapt to and recover from a disturbance or damage during a disaster.

The present paper is organised as follows: in Section 2 the qualitative definitions of robustness and resilience are presented while their main quantitative measures are described in Section 3 with a representative example. Structural design considerations are stated in Section 4 and finally, in Section 5, some conclusive remarks are drawn.

2. Definitions of fundamental concepts

Robustness and Resilience are two terms referring to two similar, yet different properties of general systems. In order to avoid confusion and to outline their respective characteristics, starting from the etymological origins of the two words, this section discusses their qualitative definition and the main historical events that led to their delineation and provides some clarification regarding structural systems.

2.1. Robustness

Most living organisms are able to survive under significantly varying conditions. Internal failures might influence overall performance; however, the most fundamental functions are maintained even under serious internal failures. This differs significantly from human designed systems, where the failure of a single element can paralyse the entire system. This natural ability to withstand failures and errors is often referred to as robustness. The word comes from the Latin word “robus”, which means oak and symbolises strength and long life [1].
Robustness of structures received wide attention after the 1968 Ronan Point gas explosion ([2], [3]) and became an even more important research topic after the 2001 World Trade Center attacks ([4]). The insensitivity of a structural systems to local failure has been an important and widely discussed topic since then [5-9]. During the past two decades, it has become obvious that even modern structural design codes do not sufficiently address system behaviour and focus too much on the verification of individual members and explicit consideration of system performance is required to ensure overall structural safety i.e. to avoid consequences disproportionate to the originating cause.

Several approaches have been proposed to deal with the issue of disproportionate collapse of tall buildings [10], large span structures [11] and bridges [12-13]. However, these papers and documents do not always use the same terminologies to describe the same phenomena or system characteristics. Therefore, various attempts had been made to define a common framework robustness assessment, such as e.g. the European COST Action TU 1406 “Structural Robustness” [14-16].

The issue of structural robustness has been recognised in structural design codes, e.g. in and ISO 2394:2015 [17] and EN 1991-1-7 [18], where it is defined as “the ability of a structure to withstand events like fire, explosions, impact or the consequences of human error, without being damaged to an extent disproportionate to the original cause”. It is however, not clearly defined what is considered as disproportionate.

According to ISO 2394:2015 [17], for structures “where failure and damage can imply very serious consequences”, the assessment of structural robustness should be based on a systematic risk-based approach. A methodology for such assessments and a categorization of structures and consequences is suggested to help decide if such a risk-based robustness assessment is needed. If a risk-based approach cannot be justified, the system’s robust behaviour should be ensured through robustness provisions, such as critical member design, structural ties, and structural segmentation and whose effectiveness will depend on both the structural system itself and the consequences of system failure.

Starossek [6-7], Haberland [19] and Lind [20], suggest that the general requirements for a useful definition of robustness should be: expressiveness, objectivity, simplicity, calculability and generality. It is also clear that these characteristics can be in conflict with each other. Haberland [19] proves that expressiveness cannot be developed together with calculability: often a quantitative approach tends to be very complex and its physical meaning is easily lost. At the same time each structure is characterized by different collapse mechanisms, so it is not easy to have a general approach that is objective and simple at the same time.

According to the Eurocode 1 [18], robustness of a structural system can be defined as the attitude of the system to survive to a given set of exposures and characterises the entire system rather than its individual components. This definition is, however, rather broad and general. A formal, more restrictive definition of robustness has been recently suggested e.g. by CEN/TC250/WG6 [21] referring to the ability of the
system to avoid disproportionate collapse: “Structural robustness is an attribute of a structural concept, which characterizes its ability to limit the follow-up indirect consequences caused by the direct damages (component damages and failures) associated with identifiable or unspecified hazard events (which include deviations from original design assumptions and human errors), to a level that is not disproportionate when compared to the direct consequences these events cause in isolation.” According to this definition robustness is seen as an indicator of the ratio between direct and indirect consequences due to certain hazards. This can be quantified in several ways as described in Section 3.1.

2.2. Resilience

Besides being robust, another important feature that natural systems possess is the ability of to restore their original functionality after stressors and stresses. Sometimes the restored system even has an improved performance compared to that prior to the stressor. This ability of systems to recover and adapt is often characterized by the term resilience. The word comes from Latin as well, in which the verb “resilire” means to rebound or recoil [22]. In his seminal paper, Holling [23] introduced the concept of resilience to the analysis of ecological systems, which later became popular in other fields of natural and social sciences. This was then followed by technological research areas and engineering, see for example [24].

Various definitions of resilience exist depending on the discipline, research field or industry sector. Resilience representing the ability of a system to recover from an extreme event has gained a wider significance in recent years. The concept is often used in earthquake engineering, economic and social studies apply the resilience concept to communities, markets, socio-political and financial systems and also to natural environments. For example, Bhamra [25] presents an interesting classification of the resilience definitions in physical, ecological, social, engineering, organisational systems. Rose [26] discusses an innovative economic analysis on the disaster resilience from a conceptual and operational point of view. Yumarni [27] reports on economic resilience after an earthquake.

A generic, high level definition of disaster resilience is given by UNISDR [28]: “The ability of a system, community or society exposed to hazards to resist, absorb, accommodate to and recover from the effects of a hazard in a timely and efficient manner, including through the preservation and restoration of its essential basic structures and functions.”

The definition suggests that resilience has a certain temporal dimension, which needs to be considered when developing resilience measures. A convenient and simple visualization of this temporal dimension is possible through the so called “resilience triangle”, see Figure 1, typically applied for technological systems, such as the built infrastructure [29]. The triangle illustrates the abrupt performance loss and the gradual recovery over time, typical for earthquakes impacting a larger area and a portfolio of structures.
The resilience triangle is a useful representation especially for demonstrating the resilience of technological systems. Complex systems, however, are dependent on a) the managing organisations and b) on other interconnected systems. The overall resilience might be influenced by attributes beyond the actual technological system, as also schematically illustrated in Figure 2.

Consider two engineering systems with the exact same performance loss and recovery characteristics for a given hazard. The two systems might use resources quite differently. In the presented case, for example, system A is more efficient during normal conditions; however, it uses resources more extensively during emergency response and recovery. System B, on the other hand, is less resource efficient during normal operation, because for example it has more operating personnel, stores more supplies, or has a monitoring system implemented. However, during crisis these resources are easier to
mobilise, whereas for system A external resources need to be involved leading to additional costs. One could argue that system B is more resilient, however, it might not be straightforward to decide. A possible solution is to define a weight between e.g. costs and performance (or several weights if more performance indicators are used), see [30].

2.3. Robustness versus resilience

The traditional way to mitigate the risks that structures are exposed to has been to protect them, i.e. to increase the resistance of the structural elements and enhance the robustness of the system. However, protection against all types of hazard is impossible and improving structural robustness might not be economical after a certain level of tolerable risk. Recent research activities and incentives therefore have been focusing on ensuring resilient design concepts [31]. By doing so, Bruneau et al. [29] define four attributes of resilience:

- robustness,
- redundancy,
- resourcefulness, and
- rapidity

In this view, robustness is seen as part of resilience and can be associated with the drop of the performance in the resilience triangle of Figure 1.

Marjanishvili et al. [32] argues that the difference between expected and observed structural performance originates from the assumption that member-based design methods will adequately influence the global resistance from which structural robustness is derived. The authors proposed to consider robustness as a fixed property of the system as a function of topology and geometry. Topology here refers to the structure’s configuration relative to the site and characterises the expected exposure toward extreme loads. Geometry describes the layout of the structural load-bearing elements. Both attributes are fixed, i.e. cannot be changed without modifying the overall configuration of the structure, thus by defining the system's geometry the structural robustness is defined as well.

In contrast to robustness seen as absolute system property, resilience represents a variable property which can be changed with specific design decisions. If resilience is seen as the structure’s ability of balancing between resisting, adapting to, and recovering from extreme events, then resistance represents the engineer’s effort to withstand a prescribed hazard. However, structures may encounter some level of damage due to the design level of an extreme load. Even if damage is limited and members do not fail, remedial actions might be required leading to a reduced functionality of the structure for a certain period of time. Adaptation can be understood as the availability of plans for emergency situations to
restore functionality after an extreme event. Recovery describes the time-varying process of restoration through remedial actions.

Then, Marjanishvili [32] propose a revised formulation of resilience and exclude adaptation and recovery, as they cannot be easily influenced and quantified by structural design. Hence, structural resilience focuses on the resistance component of generic resilience expression and is broken down into two main attributes: robustness and hazard. Structural resilience is thus associated with a specific hazard magnitude mitigated by a structural design with an assigned robustness. This definition allows the structural designer to quantify resilience and robustness and provides a basis for post-event structural assessment.

3. Measures of robustness and resilience

3.1. Robustness measures

In a very general conceptual approach the robustness $R$ can be expressed as:

$$ R = 1/(1 + S), \quad (1) $$

where $S$ represents the variation of system properties with respect to the variation of a generic system variable. In this way an extremely robust structure has $R=1$ whereas the opposite end is given by $R=0$.

Following the approach presented in [6] and [9] it is possible to divide the robustness assessment methods into five main categories: risk-oriented models, reliability-based models, static stiffness based, energy based and accumulative damage based.

3.1.1. Risk-oriented models

In case of risk-oriented strategies the robustness definition is linked to a risk assessment. An important contribution to this approach has been produced by [33]. In his work, the consequences associated with element damage are divided into direct and indirect, or, respectively proportional or disproportionate to the damage. Janssens [34] clearly distinguishes direct consequences, normally associated with initial damage or partial collapse of some constituent elements of the structure and indirect consequences that would extend beyond initial damage and be associated with any progressive collapse as well as loss of functionality or other negative impacts. On this basis it is possible to introduce an index of robustness $I_{ROB}$.
\[ I_{ROB} = \frac{R_{Dir}}{R_{Dir} + R_{Ind}} \quad (2) \]

Where \( R_{Dir} \) is the direct risk and \( R_{Ind} \) is the indirect one. \( I_{ROB} \) can also be expressed in a more general way introducing \( R_{Ind} = R_{Ind}/R_{Dir} \) and transforming Eq.(2) into:

\[ I_{ROB} = \frac{1}{1 + R_{Ind}} \quad (3) \]

The main advantage of this formulation is to calculate \( I_{ROB} \) even if there is not direct risk measure as in the case of a total loss of a structural member [9].

Faber [35] noted that Eq.(2) can only be used as a rough approximation, since the hazards, direct and indirect consequences are “decoupled” from each other. In fact, a more precise formulation would be:

\[ I_{ROB} = E \left( \frac{c_D(DS,H)}{c_D(DS,H)+c_{ID}(SS,DS,H)} \right), \quad (4) \]

where \( E[\_] \) is the expected value operator, \( c_D \) and \( c_{ID} \) are the direct and indirect consequences respectively, originating from various scenarios of hazards \( H \), constituent damage states \( DS \) and system states \( SS \).

### 3.1.2. Reliability-based models

A reliability-based measure of robustness \( \beta_R \), focusing on the redundancy of the structural system, is defined by Frangopol and Curley [36]:

\[ \beta_R = \frac{\beta_{intact}}{\beta_{intact} - \beta_{damaged}}, \quad (5) \]

where \( \beta_{intact} \) is the reliability index of the intact system and \( \beta_{damaged} \) is the reliability index of the damaged system. Higher values of \( \beta_R \) represent larger robustness.
3.1.3. Static stiffness based models

Robustness can be linked to the variation of the determinant of the stiffness matrix and the ratio between the determinant corresponding to the intact and to the damaged structure. Indeed, a structure that tends to instability has an almost singular stiffness matrix. Figure 3 shows two examples of variations in the structural system after an extreme event in a building and a bridge during World War II, as reported by Baker [37] and Thomas [38].

Figure 3. Examples of changes in structural system after local damage during extreme event (bombings during WWII): (a) seven-storey steel-framed office building reported by [37] after the main plate girder was blown down by a direct hit, (b) damage over the Oissel Bridge over the Seine reported in [38] - Photographs used with the permission of the Institution of Civil Engineers, ICE.
Nafday [39] proposes an interesting discussion about the skeletal structures safety. In particular the link between robustness and stiffness matrix properties is investigated. The ratio between the normalized determinant of the intact structure $|K_n|$ and the normalized determinant of the one $|K_{n}^*|$ corresponding to a damaged state is proposed as the importance factor $I$. More critical members will have a higher importance factor [9].

$$ I = \frac{|K_n|}{|K_{n}^*|}. $$

(6)

In [7] the static stiffness properties are used to define another synthetic robustness index:

$$ R_s = \min_j \frac{|K_j|}{|K_0|}. $$

(7)

Where $|K_0|$ is the intact structure stiffness matrix determinant while $|K_j|$ the stiffness matrix determinant after removing the $j$-structural element. Also in this case, robust structures present higher $R_s$ values. Starrosek [7] points out that this approach appears to be effective for structures susceptible to zipper-type collapse and less accurate for the ones susceptible to pancake-type or domino-type collapse.

A vulnerability index has been proposed by Lu [40] which is based on the form of the structure. The main concept is that poor form and connectivity yields to disproportionate consequences in damaged structure. The interested reader can see also [41].

### 3.1.4. Energy based models

The principles of energy absorption and energy balance have been often applied to the assessment of robustness. Pinto [41] and Agarwal [42] present a general discussion of the main characteristics of these approaches.

A method to evaluate the collapse resistance of a structure is described in [43]. In this case critical sequences of damage events that produce the structural collapse are analysed and the corresponding strain energy is calculated. The most critical sequences are those with the lowest energy requirement.

Starossek [7] proposed a simple approach based on the comparison of the energy released during the initial failure and the energy necessary for the failure development:
\[ R_e = 1 - \max_j \frac{E_{r,j}}{E_{f,k}}. \] (8)

Where \( E_{r,j} \) denotes the energy released during the initial failure of a structural element \( j \) and contributing to the damage of the subsequently affected structural element \( k \). \( E_{f,k} \) is the energy required for the collapse of the subsequently affected structural element \( k \). Actually, Eq. (8) is useful for structure that are susceptible to pancake-type or domino-type collapse. In other case the assessment of \( R_e \) is quite complex and requires a complete structural analysis.

Izzudin [44] proposed a ductility centre robustness assessment framework based on energy balance principles for the dynamic considerations using a sudden column removal approach. In their work they proposed the use of the pseudostatic response in combination with some ductility criteria for assessing structural robustness and acknowledge that on its own, energy absorption is not a property to quantify structural robustness.

### 3.1.5. Accumulative damage models

Accumulative damage models are based on the quantification of the damage progression. Starossek [7] proposes a robustness index based on damage measure:

\[ R_d = 1 - p/p_{\text{lim}}. \] (9)

where \( p \) is the maximum total damage resulting from a certain initial damage. \( p_{\text{lim}} \) is the corresponding acceptable total damage. \( R_d \) equal to one represents a perfect robustness condition, it means that no additional damage occurs. When \( p > p_{\text{lim}} \), Eq. (9) yields to negative values that highlight a not safe condition. This formulation is mainly focused on the assessment on progressive collapse that is characterised by a huge disproportion between the magnitude of the initial damage and the resulting collapse of large part of the structure. Another version of this \( R_d \) can be defined with an integral formulation:

\[ R_{d,\text{int},\text{lim}} = 1 - \frac{2 \int_{i_{\text{lim}}(2-i_{\text{lim}})}^{i_{\text{lim}}} (d(i) - i) \, di}{i_{\text{lim}}(2-i_{\text{lim}}) \int_0^{i_{\text{lim}}} (d(i) - i) \, di}. \] (10)
Where $i_{\text{lim}}$ represents the assumable maximum extent of the initial local damage, $d(i)$ is the maximum total damage resulting from the initial damage characterised by an extent $i$. These damages measures can be expressed as mass, volume, area variation of the structural element or even by their costs. Both the damage measures $d(i)$ and $i$ are dimensionless values obtained by dividing the damaged value by the corresponding undamaged value. As reported in [7] the pre-factor of Eq.(10) is necessary to modulate the effect of disproportionate local failure. The $i_{\text{lim}}$ value can be tuned for each case. This formulation is effective in assessing design objective in terms of robustness, more information can be found in [19].

Another robustness assessment method has been presented in [45] and [46]. It is based on load-capacity evaluation and damage condition limit states. Load factors are defined as the multiplier for the load corresponding to a certain damage condition (e.g. failure of a structural element, cross section damage etc.).

Given the load factor to reach the functionality or ultimate limit state $LF$ and the one corresponding to critical member strength capacity $LF_i$ it is possible to define a system reserve factor:

$$ R_f = \frac{LF}{LF_i} \quad (11) $$

It is important to point out that $R_f$ is dependent upon the system properties regardless of the design load level.

### 3.2. Resilience measures

System functionality has been considered to be a key parameter for resilience measurements in [47-48]. In particular, Henry [48] gives an interesting review of resilience metrics in different field (psychology, infrastructure, economy etc.) and proposes an innovative method to characterize a time dependent resilience measure using figure-or-merit.

Royce [49] defines three resilience capacity: absorptive capacity, restorative capacity, adaptive capacity. Absorptive capacity can be expressed as the degree to which a system can absorb the system perturbations and minimize consequences with little effort [50]. Adaptive capacity expresses the ability of a system to change in response to adverse impacts. Restorative capacity of a resilient system is the attitude to return to normal or improved performance and reliability. A new resilience factor $\rho_i$ is proposed in [49] based on the resilience capability mentioned above and the recovery time after disaster, so that:
\[ p_i = S p \frac{F_i F_d}{F_0^2}, \]  

(12)

\[ Sp = \begin{cases} \left( \frac{t_0}{t_i} \right) \exp[-\alpha(t_r - t_r^*)] & \text{for } t_r \geq t_r^* \\ \left( \frac{t_0}{t_i} \right) & \text{otherwise} \end{cases} \]  

(13)

In Eqs. (12)-(13), \( Sp \) is the speed recovery factor, \( F_0 \) is the original stable system performance level, \( F_d \) denotes the performance level immediately after disruption, \( F_r \) represents the performance level at a new stable level after recovery. In addition, \( t_r \) is the time to final recovery, \( t_r^* \) is the time to complete initial recovery actions, \( t_0 \) denotes the slack time. It is the maximum amount of time post-disaster that is acceptable before recovery ensues, where \( \alpha \) is a numerical parameter. The absorptive capacity is represented by the ratio \( F_d / F_0 \) that is a measure of the system performance after the disruption compared to the intact system performance. Therefore, the adaptive capacity can be expressed by the ratio \( F_r / F_0 \) that assesses the degree of the system performance change at the new stable condition compared to the initial system performance. In [49], this method is enhanced in a probabilistic environment and several interesting applications are presented.

A very complete review on resilience measures is reported in [51]. The authors discuss qualitative and quantitative assessment approaches. It is interesting how the latter approaches are divided into structural based and general. Structural-based approaches examine how the structure of a system impacts its resilience, in this category it is possible to distinguish deterministic [52] and probabilistic approaches [53]. General resilience measures evaluate system performance, regardless of the structure of system. The main idea in this approach is to quantify the system performance before and after disruption. Into this framework it is possible to include optimization [54], fuzzy logic [55] and simulations model approaches [56].

In the literature significant amount of research is focused on the definition of infrastructure resilience. [57] presents a resilience index as the ratio of the probability of failure and recovery of the system. [58] enhances this method using belief functions framework, its main applications are highway networks. Instead, a network topology approach has been proposed in [59]. In this work the resilience factor is the ratio of the value delivery of a network after a disruption to the delivery value of the undamaged system.

Reed [60] evaluates the resilience of a networked infrastructure introducing a quality function \( Q(t) \). Its value is 1 when the system is fully operable and 0 when is failed. An interesting contribution is given in [61], where the most advanced resilience metrics, cost – and non –cost-based are described for air traffic.
management research. Interesting research in the field of transport network vulnerability (i.e. resilience) can be found in [62-63] and recent publication on “The Future of National Infrastructure” [64].

For water supply systems, Todini [65] presents an interesting optimization problem of water distribution performance in which cost and resilience are the two objective functions. Surplus water is used to characterize resilience of the looped network. Indeed, it can be seen as an intrinsic capability of overcoming system collapses.

The study of direct consequences in terms of structural damage is relevant and interesting research has been carried out on how resilient structures can reduce the damage produced by impact and explosion [66-68].

Recently, the integration of sustainability and resilience has been addressed [69-70], but quantitative measures of resilience are not generally available for specific events such as fire and blast in concrete buildings.

Instead, in specific earthquake engineering area, the resilience metrics play an important role [67-74]. Takewaki [75] discusses the development of critical excitation methods as worst scenario analysis to upgrade the buildings earthquake resilience.

Platt [76] reports various approach to assess recovery after seismic event. Satellite images analysis, volunteered geographic information, ground survey and observation, social audit, household surveys, insurance data and official reports are compared and tested. The interesting conclusion of the authors is that currently it would appear to be challenging to directly measure resilience and that it is easier to analyse the recovery after disruption.

According to the community seismic resilience framework [29], resilience with respect to a specific earthquake can be calculated as the integral defined by the resilience triangle (see Figure 1):

\[ R = \int_{t_0}^{t_1} [100 - Q(t)] dt \] (14)

where \( t_0 \) is to the time of the disruptive event; \( t_1 \) is time at full recovery; and \( Q \), the quality of infrastructure, is expressed in percentage as a function of time \( t \).

Using Eq.(14) for measuring resilience might be difficult, since an increased duration of interruption could lead to an increased resilience, by integrating over a longer time period. To address this aspect, several authors proposed a fixed period of time. For example, [77] defines resilience as the normalized area under the functionality curve:
where $T_{LC}$ stands for control time.

Lange and Honfi [78] argue that it is important to account for anticipation and adaptation, i.e. that the performance is not 100% at hazard onset and at the end of recovery. They suggest that instead of a single resilience measure a set of indicators is needed which provide more insights about the shape of the performance loss function and can be compared with criteria developed based on public expectations.

A generic, time-dependent resilience index $I_{RES}(t)$ is proposed by [79], which aims to be consistent with previously mentioned risk-based measures:

$$I_{RES}(t) = E_X \left[ \frac{B_t(X,t)}{B_0(X)} \right]$$

(16)

where time $t$ denotes the time after the disrupting event; $B_0$ and $B_t$ are the benefits of the structure before and after the event respectively; the event. the expectation $E_X$ is taken over all relevant uncertainties $X$ influencing the benefits. The resilience index thus typically falls between 0 and 1. However, for if the recovered system is improved compared to the original, the resilience can be larger than 1.

### 3.3. Example

In order to test some of the above presented robustness and resilience measures an illustrative example is discussed as follows.

The steel frame, presented in Figure 4 with its geometrical characteristics, has been modelled in ANSYS® R18.1 [80]. The frame is fixed at the bottom of both columns and it is characterized by an IPE 200 cross section. The material elastic longitudinal modulus is $E=200$ GPa, its poisson ratio $\nu=0.3$.

As a first example the robustness evaluation according to Starrosek and Haberland [7], Eq.(7), has been developed. The ratios between the normalized stiffness matrix without the $j$th element and the normalized stiffness matrix of the intact system are reported in the rows of Table 1.

Thus, the minimum ratio represents a measure of structural robustness. In this case the deletion of element 4 or 8 yields to the lower value equal to $1.49 \cdot 10^{-7}$. As correctly reported by [7], this is more a measure of the structural connectivity and hardly can give an accurate measure of robustness. The authors agree with this consideration given that the elimination of one column (elements 2-15 or 10-11)
yields to a quite high stiffness matrix ratio even if the structural damages in this condition are more important that the ones obtained after the elimination of elements 4 or 8, see Figure 4.

**Figure 4.** Steel frame considered in the example.

<table>
<thead>
<tr>
<th>Element</th>
<th>( R_j )</th>
</tr>
</thead>
<tbody>
<tr>
<td>15-2</td>
<td>0.646588093187559</td>
</tr>
<tr>
<td>3</td>
<td>0.001579740685829</td>
</tr>
<tr>
<td>4</td>
<td>0.000000149150733</td>
</tr>
<tr>
<td>5</td>
<td>0.000000385626491</td>
</tr>
<tr>
<td>6</td>
<td>0.000231677343333</td>
</tr>
<tr>
<td>7</td>
<td>0.000000385626491</td>
</tr>
<tr>
<td>8</td>
<td>0.000000149150733</td>
</tr>
<tr>
<td>9</td>
<td>0.001579740685829</td>
</tr>
<tr>
<td>19</td>
<td>0.000002495241655</td>
</tr>
<tr>
<td>20</td>
<td>0.000231677343333</td>
</tr>
<tr>
<td>21</td>
<td>0.000002495241655</td>
</tr>
<tr>
<td>10-11</td>
<td>0.646588093187560</td>
</tr>
</tbody>
</table>

**Table 1** Stiffness matrix determinant ratios.
The energy-based measure of robustness presented in Eq.(8) was applied to the same frame structure. In this case a pancake-type collapse is considered. Thus, $E_{r,j}$ is the energy released during the failure of the second floor beam (finite elements 5-6-7). It has been approximated by its gravitational potential energy. Instead, $E_{f,k}$ is the energy required for the failure of the first floor beam. For the sake of simplicity only flexural failure has been considered and $E_{f,k}$ has been assumed equal to the ultimate strain energy absorbed by the beam in the collapse condition. The steel constitutive behaviour is modelled by a bilinear elasto-plastic curve whose yield stress is 275 MPa. The collapse mechanism considered here is characterized by 3 plastic-hinges: two at beam side and one at midspan.

The bending moment $M$ - curvature $\theta$ relationship has been represented by:

$$M = \bar{M} \tanh \left( \frac{K}{\bar{M}} \theta \right),$$

(17)

where $\bar{M}$ and $K$ are two parameters depending on the sectional and constitutive properties of the beam, see [67-68] for more details. Thus, the ultimate strain energy can be expressed as:

$$U = \int_0^1 \int_0^\theta \bar{M} \tanh \left( \frac{K}{\bar{M}} \theta \right) d\theta \, dx$$

(18)

In this way $R_v$, see Eq.(8), is equal to 0.95 which denotes a very robust structure, indeed a value equal to one denotes perfect robustness.

In order to develop an example of resilience measure the approach proposed in [32] is applied in this case. Following this method, the resilience is seen as a function of hazard, topology and geometry of the structure. Actually, it is necessary to define the intensity measure $IM$ (representing the magnitude of the external event) and the $C$ function that describes the increase of the consequences as function of the pattern of $G(DM|IM)$. The latter is a deterministic function of the exceedance of the engineering response parameter limit of the structure. In this specific case, it measures the damage produced by the collapse of a given structural element. Thus, $C$ is a user-defined function capable of describing the increasing amount of structural failures associated with the location and extension of damage:

$$C(T) = \int \int CM(IM, DM)DMdIM \int \int G(DM|IM)DM(IM)dDMdIM$$

(19)

Where $CM$ is the overall consequence measure obtained as the product of $G(DM|IM)$ and $IM$. Now, if the rate of recovery after damage is assumed to be independent of the magnitude or type or functionality loss, the resilience can be approximated as the inverse of the consequences $C$ above defined [32]:

$$R(T) = 1/C(T)$$

(20)
This simplified approach assumes that the consequences measured as structural loss are governed by the order and location of element failure as the intensity of the blast threat increases. Thus, resilience can be assessed and it can influence the structural system configuration since early design.

In this case let’s assume that the threat is represented by a blast load located near elements 2-15 of the steel frame (see Figure 4). This load results in the failure of the first-floor left column (elements 2-15). Actually, the accurate sequence in which failure propagates from one column to the other parts of the structure can be assessed only by complex dynamic non-linear analysis. Here, for the sake of simplicity the damage propagation is assumed following the $IM$ graph presented in Figure 5 and the $G(DM|IM)$ one in Figure 6. With more details, while the former represents the engineering response parameter distribution on each structural element for the given load scenario (e.g. it can represent the Von Mises stress concentration in each element, or the maximum bending moment if the flexural failure is critical etc.), the latter presents the cumulative number of structural elements collapsed after the sequence of progressive failures presented in the x-axis. Thus, the collapse of the left column corresponds to the failure of 1 element, the consequent failure of the top beam or of the bottom beam corresponds to 2 elements failure (one column and one beam) and finally the collapse of the right column denotes the total collapse of the 4 elements (two columns and two beams). Clearly, the magnitude of this $G$ function distribution is arbitrary and there are many possible alternative values as there are many possible damage propagations depending on the considered scenarios.

![Figure 5. Topology plot of relative IM for all elements.](image-url)
The distribution of $CM$ is reported in Figure 7 and after the numerical calculation of the integral presented in Eq.(19), which represent the volume under the $CM$ surface, it is possible to calculate the resilience value $R=26.7\%$. Actually, this value becomes significant only when compared to other scenarios in order to find a design solution that maximize the resilience.

4. Design and structural considerations

Despite the large number of proposed measures for robustness and resilience discussed in previous sections, the implementation of such measures into practice is cumbersome for infrastructure developers/designers and asset owners as many of the proposed approaches are still under development.
and they have not been yet implemented in current design standards. Perhaps the most generally adopted
design philosophy, which is now being implemented in different codes, is the risk-based approaches in
which the type, probability and consequences of an event are compared against the cost of protection
and assumed potential loss [5]. Within such frameworks, systematic risk assessment methods are being
implemented in the field of structures, especially for cases of buildings with a high-risk of progressive
collapse [81].

In structural engineering, the use of risk or consequence classes for buildings have been widely used in
Europe and the US for some time; in this approach the probability of failure is not directly assessed but
risk is managed indirectly. For structures with a low risk of progressive collapse, robustness is not
directly quantified, and generally prescriptive rules are adopted to mitigate the potential loss of one or
some structural members. For higher consequence classes (i.e. Class 3 in the Eurocodes), systematic
risk assessments are needed such as the one presented in [81] which suggests the identification of
hazards, to eliminate (if possible) this hazard which give rise to the associated risk and for the hazard
that remain to develop risk mitigation measures so far as this is possible. Such approaches are
implemented with the idea that a structural design is conceived containing a level of structural robustness
suitable with the level of risk to which the structure is subjected.

Another relevant issue which is affecting infrastructure designers regarding robustness and resilience
considerations is the differentiation between existing and newly built infrastructure. The vulnerability
and mitigating measures that can be introduced in each case can be rather different and the use of
different measures for robustness and/or resilience might not be directly applicable to existing
infrastructure. In addition, the interface between new and existing building environment can be also be
problematic unless the problem is not approached as a system-of-systems.

Current structural design codes require the verification of strength and stability of structures based on
the limit state concept typically associated with the failure of individual members. It is also recognized
requirements to the overall performance of the entire structural system should be set to prevent
disproportionate collapse and mitigate the adverse possible effects of extraordinary situations which
cannot be fully covered by prescriptive design rules.

General aspects and approaches for structural design which enhances robustness have been widely
studied after World Trade Center attack in 2001, although the first principles of structural robustness
were introduced in the 1970s after the Ronan Point collapse [82]. Most recently in Europe, a major work
on this topic has been conducted within the COST Action TU0601 “Robustness of Structures”
http://www.cost-tu0601.ethz.ch/). Parallel reviews took place [16,83] raising similar limitations of
existing international codes to deal with robustness. A more recent review [84] has gathered research in
this field over the 21st century including the evolution of international codes [18], [85-93]. This work
concluded that recent refinements have been introduced in international codes regarding robustness although in many cases the changes in the general procedures adopted are not significant.

General recommendations to achieve a robust design include various strategies such as event control, strengthening of critical elements, provision of alternative load path, segmentation etc [94]. One important strategy is to provide redundancy at various levels, i.e. at material level, member level and system level [95-96]. Besides improving internal redundancy, some authors argue that external redundancy could be seen as a measure to increase robustness, since if alternative means to provide the same functionality are available, the consequences of failure can be mitigated [97].

Another structural characteristic that is typically seen as beneficial for a robust behaviour is ductility especially at connections. Ductile materials and joint can accommodate larger deformations thus allow for redistribution of stresses and give a warning before collapse occur in contrast to brittle failure.

An important aspect raised by several researchers (i.e. [44]) influencing robustness is the energy absorption capacity of structures which can be considered as a useful additional property to consider in combination with redundancy and ductility.

According to EN1990 [93] “a structure shall be designed and executed in such a way that it will not be damaged by events such as: explosion, impact, and the consequences of human errors, to an extent disproportionate to the original cause.” In other word the structure should be sufficiently robust. It is, however, not obvious what is a sufficient level of robustness and how it can be measured.

Regarding Eurocodes, more details on robustness in EN1991-1-7 [18] concerning accidental design situations and related design strategies (see Figure 8).

![Figure 8. Strategies for accidental design situations according to EN1991-1-7 [18]](image.png)
According to EN1991-1-7 [18] in accidental design situations measures should be taken to mitigate the risk of accidental actions e.g. by ensuring that the structure has sufficient robustness.

This could be done in several ways, that is:

1) overdesigning and/or protecting key elements;

2) enhancing ductility to enable better energy absorption;

3) enhancing redundancy to provide alternative load paths.

The informative Annex A of EN1991-1-7 [18], “Design for consequences of localised failure in buildings from an unspecified cause”, provides design guidance to withstand local failure from an unspecified cause without disproportionate collapse. The annex contains prescriptive rules, based on the building’s consequence class, aiming to provide sufficient robustness and decrease the chance of collapse in case of unforeseen harmful events. However, these recommendations have very limited applicability and have little use apart from multi-storey RC buildings.

In general, it is widely accepted that even if a structure is extremely robust, it is impossible to resist against all kinds of hazards. Therefore, it needs to be considered what happens after failure. Performance based design initiatives take into consideration losses due to various system damage states [98].

Many interesting papers discussed at structural and substructure level the most important approach to improve resilience. For example, Xilin [99] presents a structural engineering approach to the development of earthquake resilient rocking or self-centering structures. The same self-centering approach is discussed in [100] for steel structures. Finally, also the specific bridge seismic resilience has been an interesting and wide research field [101-102].

Resilience-based approaches however, need to go even further [78]. A resilient design and operation of a structure should account for response, restoration of functions and recovery. To achieve a satisfactorily high resilience both the structure and the operating organization should have sufficient flexibility for reacting to the varying needs due to the changing circumstances. This could include adaptivity through automatized control mechanism but also well-established processes and sufficient resources, both human and materialized, in case of an emergency and in the aftermath of a disaster. Obviously what and how this needs to be done depends on the actual structure and incident considered. General requirements and guidelines, however, can be given especially with regards to expected response and recovery times and minimum levels of functioning during and after crisis times.
5. Conclusions

Catastrophic failures of structures and infrastructure systems happen from time to time as a consequence of natural or man-made extreme events. Therefore, it is important to consider what would happen if one or several elements of a system fail.

The quantitative and qualitative definitions of robustness and resilience have been reviewed in this paper. If the former is simply denoted as the ability to avoid disproportionate collapse due to an initial damage, the latter is the ability to adapt and recover from a disturbance or damage due to a disaster.

Quantitative measures of robustness can be obtained with risk oriented, energy based, static stiffness based, cumulative damage models. The effectiveness of each approach depends of the specific case because what is working well for a given structural system may became less accurate for another.

Resilience properties can be distinguished in absorptive capacity, restorative capacity, adaptive capacity. Quantitative measures of resilience can be divided into structural based and general. Structural-based approaches examine how the structure of a system impacts its resilience while general resilience measures evaluate system performance, regardless of the structure of system.

While most resilience definition can be applied to infrastructures very few are valid also for structures. The authors would like to underline that more research into resilient structural systems is needed, especially since adaptive and smart structures are becoming more important. In addition, current technological development requires the need of robust and resilient design even in sectors not traditionally linked to civil engineering. For example, “digital data management” affecting infrastructure development and operation of large assets can also be subject to similar principles of robustness and resilience. Data protection and security and the existence of “virtual infrastructure” will introduce new domains of research in civil engineering within the new context of Digital World and Digital Engineering.

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