# Abstract
A quantitative study of the astrophysically important sub-barrier fusion of $^{12}\text{C} + ^{13}\text{C}$ is reported. Low-energy collisions are described in the body-fixed reference frame using wave-packet dynamics within a nuclear molecular picture. In contrast to conventional methods, such as the potential model and the coupled-channels approach, these new calculations reveal three resonant structures in the S-factor, explaining some structures observed in the data. The structures in the data that are not explained are possibly due to cluster effects in the nuclear molecule, which need to be included in the new approach.

# Keywords
Nuclear molecule - Resonances - Quantum tunneling - Fusion
Chapter 1
Resonances in Stellar Carbon Fusion

Alexis Diaz-Torres and Michael Wiescher

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1.1 Introduction

The $^{12}\text{C}+^{12}\text{C}$ fusion cross sections at very low energies are critical for modelling
energy generation and nucleosynthesis during the carbon burning phase of stellar
evolution of massive stars ($M \geq 8M_\odot$) [1]. These cross sections also determine the
ignition conditions for type-Ia supernova explosions [2]. Variations of the fusion rate
in its traditional range of uncertainty moderately affect nucleosynthesis in the actual
type-Ia explosion event [3]. This situation would change if resonant structures in the
low-energy range of the fusion cross sections existed [4]. Such structures have been
observed at higher energies and are associated with molecular states. The possible
existence of such states at very low energies can significantly affect nucleosynthesis
in type-Ia supernovae [5] as well as superbursts on accreting neutron stars [6]. It
is therefore important to go beyond conventional approaches for averaged cross-
sections, to understand the nature of these molecular phenomena and their occurrence
at very low energies.

In [7], a new method based on wave-packet dynamics applied to the sub-barrier
fusion of $^{12}\text{C}+^{12}\text{C}$ is presented. In contrast to other traditional methods, such as

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the potential model [8] and the conventional coupled-channel approach [9], wave-
packet dynamics quantitatively explains some observed resonances in the fusion
cross sections. This is because the new method [7] allows one to include the effects
of molecular states on fusion. The present contribution highlights the main results
of the theoretical study reported in [7].

1.2 Wave Packet Dynamics

The method of wave packet dynamics has three steps:

(i) the definition of the initial wave function \( \Psi(t = 0) \),
(ii) the propagation \( \Psi(0) \rightarrow \Psi(t) \), dictated by the time evolution operator,
\[ \exp(-i \hat{H} t/\hbar) \], where \( \hat{H} \) is the total time-independent Hamiltonian,
(iii) after a long propagation time, the calculation of observables (cross sections,
spectra, etc.) from the time-dependent wave function, \( \Psi(t) \).

The wave function and the Hamiltonian are represented in a multi-dimensional
lattice. These are considered a function of a few collective coordinates such as the
internuclear distance and the spherical coordinate angles of the \(^{12}\text{C}\) symmetry axis
relative to the internuclear radius, thus reducing the complexity of the quantum
many-body reaction problem. The present method directly solves the time-dependent
Schrödinger equation, without the traditional expansion in a basis of energy eigen-
states, which is used in the conventional coupled-channels model. Despite this, the
numerically calculated total wave function accounts for all the coupled-channel
effects. The irreversible process of fusion at small internuclear distances is described
with an absorptive potential for fusion. The heavy-ion collision is described in the
rotating center-of-mass frame within a nuclear molecular picture [10]. Expressions
for the kinetic-energy operator, the collective potential-energy surface (PES) and the
time propagator are provided in Appendices of [7].

1.3 Model Calculations

Figure 1.1 shows real \(^{12}\text{C}+^{12}\text{C}\) total potentials for \( J = 0, 2, 4 \) partial waves. Figure
1.1a displays specific cuts in the PES for head-on collisions, while Fig. 1.1b
shows effective total potentials after folding the PES (including the centrifugal en-
ergy) of non-axial symmetric di-nuclear configurations with the probability density
of the ground-state wave-function of the two deformed, colliding \(^{12}\text{C}\) nuclei [7]. The
Equator-Equator alignment (thin solid line) facilitates the access by tunneling to the
potential pockets. All the alignments coexist and compete with each other, the kinetic
energy operator driving the system towards either re-separation or fusion in the po-
tential pocket of the Pole-Pole configuration (thick solid line). A strong, imaginary
Woods-Saxon potential centered at the minimum of the Pole-Pole potential pocket in
Fig. 1.1  a Some cuts in the real PES for $^{12}$C+$^{12}$C as a function of the internuclear distance and three alignments. The total angular momentum is $J = 0$. b Effective $^{12}$C+$^{12}$C total potentials for the $J = 0, 2, 4$ partial waves. The plot inserted shows the scattering phase shifts for specific partial waves. The occupation of these potential resonances (circles) causes the structures in the theoretical S-factor in Fig. 1.2.

Fig. 1.1a (thick solid line) provides a fusion absorption, which operates very weakly at the potential pockets of the non-axial symmetric configurations (dashed and thin solid lines). The strong repulsive core of the real potentials for non-axial symmetric di-nuclear configurations hinders the effect of the imaginary fusion potential on the potential resonances formed in the corresponding real potential pockets. These potential resonances are shown in the plot inserted in Fig. 1.1b. The occupation of these resonant states (circles in plot inserted) enhances the fusion cross sections, as reflected in the theoretical S-factor in Fig. 1.2.

Figure 1.2 displays model calculations of the astrophysical S-factor (lines) compared with various data sets (symbols) [4, 11–16]. It is observed that the theoretical S-factor curves, clearly affected by uncertainties of the PES of Fig. 1.1a, are rather flat at stellar energies ($<3$ MeV), without resonant structures. This is because the absence of potential resonances for these energies in Fig. 1.1b. However, cluster effects in the nuclear molecule (e.g., $^{20}$Ne + alpha and $^{23}$Na + p) can also be very important, possibly leading to additional resonances.
1.4 Summary

Molecular structure and fusion are closely connected. The present calculations indicate that the fusion excitation function monotonically declines towards stellar energies. In contrast to other conventional methods, the new approach reveals three resonant structures in the theoretical S-factor, which explain some observed structures. The structures in the data that are not explained are possibly caused by cluster effects in the nuclear molecule, which need to be included in the new approach.

References

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