DETECTION OF MAGNETIC FIELDS USING FIBRE OPTIC INTERFEROMETRIC SENSORS

by

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A thesis submitted to the Faculty of Mathematical and Physical Sciences at the University of Surrey for the Degree of Doctor of Philosophy

September 1985
For my Father and late Mother,
George and Renne Pratt
whose love made this possible.
The principle aim of the work described in this thesis is to determine a suitable optical detection system for d.c. and low frequency magnetic fields which are likely to be encountered in practical magnetometer applications. To construct a sensitive magnetometer one arm of an optical fibre Mach-Zehnder interferometer has been magnetically sensitised using a magnetostrictive material. Since the signal frequency range of interest was in the region of 0.01 to 10Hz, clearly the signal was in the same frequency band as the environmental noise associated with ambient temperature and pressure variations. Initially, a technique was developed to measure the magnetic field from the shift of the total fringe pattern generated by a modified Mach-Zehnder interferometer and a minimum detectable magnetic field of $10^{-7}$ tesla.m. was obtained. This minimum detectable magnetic field has been improved by a number of modifications. A technique has been developed which utilises an a.c. bias field to put the magnetic signal on a carrier so that it can be measured at a frequency where the amplitude of the interferometer 1/f noise is much reduced. To maintain maximum interferometric sensitivity to this signal active homodyne demodulation techniques have been developed to maintain the interferometer at quadrature by compensating for the environmental noise. A minimum detectable magnetic field of $5 \times 10^{-10}$ tesla.m. has been achieved with this system.

As an alternative to the Mach-Zehnder interferometer a Fabry-Perot interferometer, which utilises multiple-beam interference, has been considered. This type of interferometer consists of a single fibre with high reflectivity coatings on its ends. Such an interferometer has been used as a sensor and as an external cavity in laser frequency stabilisation scheme.
ACKNOWLEDGEMENTS

I wish to express my thanks to:

Dr. K. W. H. Foulds for his supervision and encouragement.
Professor G. D. Pitt for his enthusiasm, guidance and continued support.
Dr. R. E. Jones and Dr. P. Extance for their assistance, colleagueship and friendship.
Dr. D. N. Batchelder and Dr. J. P. Willson for many helpful discussions and comments.
T. Truelove and B. J. Scott for technical assistance.
F. Bristow and his colleagues in the Physics Department Mechanical Workshop for constructing the optical table and associated components.
Mrs. M. Gunney for typing the thesis.
STL drawing office and B. J. Denton-MacLennon of BAe drawing office for preparing the figures.
S.E.R.C. and STL for financial support.
B. Lucas for direction many years ago, and a special thank you goes to Janet.
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CHAPTER 1
INTRODUCTION TO OPTICAL FIBRE SENSORS

1.1. Introduction

The ideas that led to the development of optical fibre communications in the late 1960's and then optical fibre sensors in the middle of the 1970's probably started with the invention of the solid state laser by Maiman\(^1\) in 1960. This made available for the first time a source of coherent light that had characteristics similar to microwave radiation and thus led to the concept of using light for high bandwidth communications. The next advance was the development of the semiconductor laser by Hall et al\(^2\) which in its later forms\(^3\), is the light source that is used almost exclusively for fibre communications. Following this, it was realised in the middle of the 1960's that because of the attenuation suffered by light transmitted through the atmosphere, a guided wave system was likely to be the best choice. This led to the proposal by Kao and Hockham\(^4\) in 1966 to use circular glass fibres as dielectric waveguides for transmitting light over long distances. These fibres are constructed from a high refractive index core which is surrounded by a lower refractive index cladding. Light is guided by the fibre because of total internal reflection at the core cladding interface. Guides of this type are used in almost all optical communication systems today. A major advance in the development of optical fibres occurred in 1970 when a loss of less than 20dB/km was reported by Kapron et al\(^5\). The significance of this figure is that it had been identified as the highest loss fibre that could be used for economical communication systems. Silicon avalanche photodetectors had also been developed in parallel with these activities to detect the light transmitted by the fibre. Consequently, by the 1970's all the critical components were available to start operating optical fibre communications systems. The development of these systems and in particular the availability of the associated components
has encouraged interest in optical fibre sensors. Since 1976 magnetic, acoustic, temperature, acceleration, rotation, and current are among the fibre optic sensor types being investigated. This new technology, which has emerged from the optical communications industry, offers a number of advantages; increased sensitivity over existing techniques, geometric versatility in that fibre sensors can be configured in arbitrary shapes and dielectric construction so that it can be used in high voltage, electrically noisy, high temperature or corrosive environments.

A fibre optic sensor in its basic form consists of a light source, an optical fibre, a mechanism which modulates the light in the fibre, and a photodetector. Other components such as optical couplers, connectors and electronic signal processors may be necessary depending on the sensor type. The sensor types can be readily classified by the parameter of the optical beam which the measurand modulates. This parameter may be amplitude, phase, frequency, or state of polarisation. This discussion is confined to those signal modulation techniques which can be accomplished in what is referred to as the 'all-fibre' approach, that is, the modulation is performed while the light remains guided within the fibre. This removes the difficult problem of achieving a stable low loss interface between a bulk optic or an integrated optic component and the optical fibre in the sensor system. Amplitude, phase and polarisation modulation can be accomplished using all-fibre schemes. However, efficient frequency modulation is limited to accomplishment in bulk Bragg cell media external to the fibre. Although it is interesting to note that Risk et al. have recently demonstrated frequency shifting using an all-fibre approach.

Generally, amplitude modulating sensors use multimode fibre components and broad band light emitting diodes as optical sources. One such
technique makes use of microbend modulation in which mechanical deformation of the optical fibre perpendicular to its axis of propagation causes higher order modes in the core to be transmitted through the core-cladding interface in an amount proportional to the deformation. This technique has been demonstrated by Fields and Cole, who sandwiched the fibre between a pair of toothed plates. These introduce the microbends into the fibre. If the plates are perturbed by a measurand such as an applied pressure the fluctuation of either the core or the cladding light will be directly proportional to the signal for small deformations. The advantages of these amplitude sensors are the simplicity of construction and compatibility with multimode fibre technology.

The most sensitive types of optical fibre sensors are those based on optical phase modulation. This phase modulation can be measured by converting it into amplitude modulation using an optical interferometer. In this way these devices are able to resolve phase shifts of $10^{-6}$ radians which corresponds to an optical length change of approximately $10^{-12}$ metres for a light source wavelength of 1000nm. The work presented here is mainly concerned with constructing interferometers using single mode optical fibre and components in order to detect magnetic fields. Therefore, before reviewing the types of fibre optic interferometers which have been used as sensors the basic principles of optical interference are discussed.

1.2. Optical interference

Consider the overlap of two monochromatic electromagnetic (e.m.) waves of the same frequency at a point, P, in space. If these waves are linearly polarised and propagating in the z direction the electric and magnetic components of the waves are described by the well known solutions to Maxwell's wave equation, and are given by:
\( E_1(x,y,z,t) = E_{o1}(x,y) \exp[j(2\pi vt-k_0n_z\theta_1)] \)
\( H_1(x,y,z,t) = H_{o1}(x,y) \exp[j(2\pi vt-k_0n_z\theta_1)] \)
\( E_2(x,y,z,t) = E_{o2}(x,y) \exp[j(2\pi vt-k_0n_z\theta_2)] \)
\( H_2(x,y,z,t) = H_{o2}(x,y) \exp[j(2\pi vt-k_0n_z\theta_2)] \)

where \( E_0 \) and \( H_0 \) represent the electric and magnetic amplitudes respectively, \( x, y \) and \( z \) are the rectangular cartesian coordinates, \( v \) is the frequency of the e.m. wave, \( n \) is the refractive index of the medium, \( \theta \) is the phase of the wave at its source and \( k_0 = \frac{2\pi}{\lambda} \) is the free space wave number where \( \lambda \) is the wavelength of light in free space. Since the wave equation satisfies the principle of superposition the resultant electromagnetic disturbance at point \( P \) is equal to the vector sum of the separate waves \([E_1, H_1]\) and \([E_2, H_2]\) and is given by:

\[
\begin{align*}
\textbf{E} &= E_1 + E_2 \\
\textbf{H} &= H_1 + H_2
\end{align*}
\]

The \( \textbf{E} \) and \( \textbf{H} \) fields for optical waves vary in time at approximately \( 10^{14} \text{Hz} \) making the electric field an impractical quantity to detect. Therefore, the energy of the optical wave in space is measured over some finite interval of time. This quantity is known as the intensity and is given by the time average of the real part of the Poynting vector

\[
\langle S \rangle = \frac{1}{2} \Re \left[ \textbf{E} \times \textbf{H}^* \right]
\]

where the asterisk indicates the complex conjugate. The factor \( 1/2 \) is to account for the time average taken over a large interval compared with the period of the optical wave \( T = \frac{1}{v} \). It can be shown using Maxwell's equations that \( \frac{E}{H} = \sqrt{\frac{\varepsilon_0}{\mu_0}} \) where \( E \) and \( H \) are mutually perpendicular therefore the intensity of an e.m. wave is proportional to the square of the amplitude of the electric field;

\[
I = \frac{1}{2} \sqrt{\frac{\varepsilon_0}{\mu_0}} \textbf{E}_0^2
\]
or alternatively the square of the amplitude of the magnetic field:

$$I = \frac{1}{2} \sqrt{\frac{\mu_0}{\varepsilon_0} H_0^2}$$  \hspace{1cm} \text{(1.5)}$$

where $\mu_0$ is the permeability and $\varepsilon_0$ is the permittivity of free space.

In the work described here the constants are neglected and the optical wave will be represented exclusively by the electric field. The resultant optical disturbance at P arising from the interaction of the fields given in equation 1.1 can be measured in terms of intensity. Accordingly:

$$|E|^2 = (E_1 + E_2) \cdot (E_1 + E_2)$$

$$|E|^2 = |E_1|^2 + |E_2|^2 + 2E_1 \cdot E_2$$  \hspace{1cm} \text{(1.6)}$$

The last term is known as the interference term and can be written as:

$$E_1 \cdot E_2 = E_{o1}(x,y)E_{o2}(x,y) [\cos(k_0 n z_1 + \theta_1) \cos 2\pi vt + \sin(k_0 n z_1 + \theta_1) \sin 2\pi vt]$$

$$\times [\cos(k_0 n z_2 + \theta_2) \cos 2\pi vt + \sin(k_0 n z_2 + \theta_2) \sin 2\pi vt]$$  \hspace{1cm} \text{(1.7)}$$

where the real parts of the electric fields given in equation 1.1 have been used. The intensity at point P is given by taking the time average of both sides of equation 1.6 and can be expressed by:

$$I = I_1 + I_2 + E_{o1} \cdot E_{o2} \cos \phi$$  \hspace{1cm} \text{(1.8)}$$

where $\phi$ is the phase difference arising from the optical path length difference and the initial phase difference between the waves and is given by:

$$\phi = k_0 n (z_1 - z_2) + \theta_1 - \theta_2$$  \hspace{1cm} \text{(1.9)}$$

If waves are in phase at the emitter $\theta_1 - \theta_2 = 0$, then equation 1.9 simplifies to:
\[ \phi = k_o n (z_1 - z_2) \]  \hspace{1cm} 1.10

It can be seen from equation 1.8 that the magnitude of the interference term depends strongly on the polarisation states of the two waves because of the vector relationship \( \mathbf{E}_{o1} \cdot \mathbf{E}_{o2} \). When \( \mathbf{E}_1 \) is parallel to \( \mathbf{E}_2 \) (polarisations aligned) and the amplitudes of both waves are equal, that is, \( E_{o1} = E_{o2} \) equation 1.8 simplifies to:

\[ I = 2I_0 (1 + \cos \phi) \]  \hspace{1cm} 1.11

where \( I_1 = I_2 = I_0 \). These conditions can easily be achieved in an amplitude-splitting interferometer such as the Mach-Zehnder. The optical fibre version of this interferometer configuration is shown in Figure 1.1. Linearly polarised light from a coherent source is divided into two equal intensity components by a beam splitter. After propagating over different optical paths the two separate waves are recombined by another beam splitter. Providing the path lengths differ by a distance which is less than the coherence length of the source, interference between the beams can be observed on a suitable detector. The coherence properties of interferometer light sources will be discussed in 2.4.1. Small perturbations of the optical path length difference can be measured by the interferometer if the instantaneous intensity given by equation 1.11 is monitored by a suitable photodector. The resulting photocurrent is turned into a voltage, filtered and integrated. Clearly from inspection of equation 1.11 the resulting electrical
Fig. 11. Optical fiber interferometers
signal is dependent upon the starting value of $\phi$, and linear small signal operation is obtained only at $\phi = \pm \pi/2 + n\pi$ where $n$ is an integer. For accurate measurements the value of $\Delta\phi$ must be maintained at $\pi/2$ with a high degree of stability. This is an important fundamental constraint on sensor interferometers and its implications on system design will be discussed in Chapter 3.

1.3. Optical fibre interferometers

The principles of phase detection described in 1.2 can be implemented with an interferometer constructed from single mode optical fibre. Single mode fibre is chosen so that the guided waves which interfere each propagate at a single phase velocity just as in conventional interferometry. The interferometer phase difference in this case is given by $\phi = nk_o z$ where $n$ is the effective refractive index of the mode guided by the fibre and $z$ is the physical length difference between the optical fibres forming the interferometer arms. In this thesis the symbol $n$ always represents the effective index of a mode guided by the fibre. Variations of either $n$ or $z$ cause phase changes.

The four different types of amplitude splitting interferometer which have been used to construct fibre optic sensors are shown in Figure 1.1. These are the Mach-Zehnder, Michelson, Fabry-Perot and the Sagnac. The first three interferometers can be used to detect changes in the refractive index and the physical length of an optical fibre which can be produced by variables such as magnetic fields, temperature, pressure and acoustic fields. The Sagnac interferometer has been used exclusively as a gyroscope, and rotation of this interferometer effectively shortens the optical path length taken by one beam in comparison to the other. All such interferometers have some components in common. Generally, the
output from a single frequency laser (coherent) is coupled into an optical fibre (light emitting diodes can be used with Sagnac interferometers because they generally operate with an optical path length difference of less than one wavelength). The light coupled into the input fibre is split by a 3dB-coupler into the two arms of the interferometer. The 3dB-coupler serves the same role in fibre optic interferometers as half-silvered mirrors in conventional interferometers. The light in one arm of the interferometer serves as a reference for light in the other arm of the interferometer. In the case of either the Sagnac or the Fabry-Perot interferometers, the light is confined to a single fibre. In the Sagnac interferometer, the optical output of the 3dB coupler is inserted into opposite ends of the fibre optic coil and propagates in both clockwise and counterclockwise directions. In all except the Fabry-Perot interferometer, after passing through the interferometer, the light in the two arms is recombined by either the same or a second 3dB coupler, and interference occurs. In the Fabry-Perot interferometer, the interference takes place between successively transmitted waves (there is also a complementary interference function generated by the reflected waves). This device will be discussed in detail in Chapter 4. In all cases, the interference between the signal arm and the reference arm light causes phase modulation to become amplitude modulation. This output is transformed into an electrical signal proportional to the amplitude of the phase shift induced by the measurand by a suitable demodulation scheme.

An alternative form of interferometer which utilises the polarisation properties of single mode fibre, can be used to detect the presence of a measurand. A change in state of the polarisation of the light propagating in a single mode fibre can be observed due to a change in the fibre birefringence. This change in birefringence is a result of the phase velocities of the two polarisation modes of the single mode fibre being
altered unequally by the action of the measurand. The change in the polarisation of the light at the output of the fibre can be measured using a polariser and a photodiode. Consequently, the sensor can be viewed as a differential interferometer; the two light beams travel within the same fibre but with orthogonal polarisations. As a result this sensor configuration is simpler than the dual arm interferometers just described; but because it operates in a differential mode a loss of sensitivity usually results. Rashleigh has used this principle to measure acoustic and magnetic fields. In each case the fibre is attached to a cylinder made from a material which is sensitive to the field to be detected in such a manner that the response of the material asymmetrically stresses the fibre cross-section and thus unequally changes the phase velocities of the two orthogonally polarised modes. Alternatively magnetic fields have been detected by Smith utilising the direct interaction between the fibre and the field via the Faraday effect.

Multimode fibres can also be used as an interferometer by producing phase changes between the many different modes in the same fibre. The sensitivity is again less than the single mode fibre interferometer because of the differential effect. The detection of the modulated signal requires the detector to distinguish between the different modes in the fibre. One technique reported by Culshaw et al. is to sample the speckle (modal) pattern at the fibre output by restricting the field of view of the detector to a small number of speckles.

1.4. Objectives of the present work

The present work is principally aimed at developing a high sensitivity all-fibre Mach-Zehnder interferometer to detect d.c. and low frequency magnetic fields. In addition all-fibre Fabry-Perot interferometers are constructed and used as sensors and as external cavities in a laser
The content of the thesis is arranged as follows:

In Chapter 2 the basic components which are used in single mode optical fibre interferometers are reviewed. They include the optical fibre, directional coupler, phase modulator and the light source.

In Chapter 3 the general principles of the all-fibre Mach-Zehnder interferometer are described and results on its ability to detect small phase shifts ($<<\pi$) using two homodyne demodulation schemes are reported. The first scheme utilises electrical feedback from the interferometer output to a phase modulator in one of the interferometer arms. This maintains a stable phase difference between the two arms by compensating for environmentally induced phase shifts and operates the interferometer at its most sensitive point. The second demodulation scheme performs the same functions, but uses a feedback signal to tune the frequency of the laser.

In Chapter 4 all-fibre Fabry-Perot interferometers are discussed and results are reported on their construction and performance as sensors. Their ability to detect small phase shifts is assessed using homodyne modulation. A Fabry-Perot temperature sensor is reported and a new method for determining the sign of the temperature change proposed. Finally, a short optical fibre Fabry-Perot cavity is used as a frequency sensitive device in a feedback loop to improve the frequency stability of a laser diode.

In Chapter 5 the high sensitivity of a Mach-Zehnder interferometer to optical path length changes is exploited to detect small d.c. magnetic fields using the magnetostrictive effect. Results on two different interferometer configurations are reported. In the first configuration a spatial fringe pattern is generated and the displacement of the fringes is measured on a photodiode array to determine the magnetically induced
phase shift. This avoids the requirement to maintain a fixed phase
difference between the interferometer arms and can allow for polarisation
fluctuations in the fibres and source amplitude fluctuations. The second
configuration is based on the all-fibre Mach-Zehnder interferometer
described in Chapter 3. To overcome the interferometer 1/f noise which
normally obscures small d.c. signals a technique is developed for putting
the d.c. magnetic field signal on a carrier frequency. This allows the d.c.
signal to be measured at a frequency where the amplitude of the 1/f noise
is small using phase sensitive detection.

In Chapter 6 an assessment of optical fibre magnetometer development
is presented.

Finally, in Chapter 7 some concluding remarks are presented on the
work described in this thesis.
CHAPTER 2
SINGLE MODE OPTICAL FIBRE INTERFEROMETER COMPONENTS

In this Chapter components which have been used in the fibre optic interferometers reported in this work are reviewed. They include the optical fibre, directional coupler, phase modulator and the laser light source.

2.1. The optical fibre

A step index optical fibre consists of a core of refractive index, $n_1$, which is enclosed in a cladding of lower refractive index, $n_2$. This fibre is surrounded by various coatings for mechanical protection. The basic principles of the optical fibre waveguide are shown in Figure 2.1. The guiding of light by the fibre can superficially be explained by ray optics and Snell's law of total internal reflection at the core-cladding boundary. Only rays with an angle of incidence $\sin \theta > \frac{n_2}{n_1}$ are totally internally refracted at the boundary. This leads to the maximum coupling angle, $\alpha$, at the entrance to the fibre being given by:

$$\sin \alpha = n_1 \sin (90 - \theta) \quad 2.1$$

The quantity $\sin \alpha$ is called the numerical aperture (N.A.) of the fibre and can also be written as:

$$\text{N.A.} = (n_1^2 - n_2^2)^{\frac{1}{2}} \quad 2.2$$

The fibre radius, $a$, the N.A. and the free space operating wavelength $\lambda$, are generally combined into a normalised frequency $V$, which is expressed as:

$$V = \frac{2\pi}{\lambda} a \text{ (N.A.)} \quad 2.3$$

The precise conditions for light propagation must be determined by
Fig. 2.1. Numerical aperture ($\sin \alpha$) of a step index fibre.

Fig. 2.2. Bessel functions versus fibre V value showing the allowed $L_{P_{lm}}$ modes.
wave optics. Electromagnetic waves propagating in a fibre can be subdivided into low loss core modes and lossy cladding modes. The term mode refers to a single e.m. wave satisfying Maxwell's equations and the boundary conditions given by the fibre. Each mode propagates in the fibre with a different phase velocity. The exact solutions of Maxwell's equations for the fibre lead to a complicated set of HE, EH, TE and TM modes which have been discussed by Marcuse. However, a convenient approximation is to use the linearly polarised (LP) mode solutions which have been analysed by Gloge. Each mode is specified by two mode numbers, a radial number, \( m \), and an azimuthal number, \( \ell \). These modes exist in fibres with a refractive index difference, \( \Delta n \), such that:

\[
\Delta n = \frac{n_1 - n_2}{n_1} \ll 1
\]

In general, optical fibres are designed with a small index difference in order to reach high bandwidth in data transmission systems. This is achieved because the resulting small N.A. minimises modal dispersion. A summary of the derivation of the LP\(_{\ell m}\) mode solutions is given in Appendix A. The electric field of an e.m. wave propagating in the \( z \) direction which is confined to the fibre core can be represented by:

\[
E_z^{\text{core}} = E_1 J_\ell (K_1 r) e^{i \ell \phi} e^{i(\omega t - \beta z)}
\]

The cut off condition for a mode guided by the fibre corresponds to the condition that the parameter \( Y_2 \) (see Appendix A) representing the evanescent field in the cladding becomes imaginary. In this case evanescent field turns into a radiation field, and the wave is no longer guided by the fibre. Therefore, as shown in Appendix A, mode cut-off corresponds to the condition \( J_{\ell - 1}(K_1 a) = 0 \) where \( J \) is a Bessel function. Thus for a given value of \( K_1 a \) a specific group of modes will propagate in the fibre. The value of \( K_1 a \) for the optical fibre is
equivalent to the \( V \) value defined in equation 2.3 and therefore depends on the N.A. The lowest order mode (the LP\(_{01}^\text{L} \)) has a field distribution described by \( \xi=0 \) and cuts off at the first zero of the \( J_1 \) Bessel function which occurs when the guide ceases to exist (\( K_\text{r}=0 \)). The next order mode has a field distribution described by \( \xi=1 \) and cuts off at the first zero of the \( J_0 \) Bessel function. This mode is labelled the LP\(_{11}\) and cuts off when \( K_\text{r}=2.405 \). Thus for single mode operation at a particular wavelength \( V \) must be less than 2.405. This is easily achieved by selecting the appropriate core diameter. For example, a fibre whose N.A. is 0.1, operated at a wavelength of 0.9\( \mu \)m, must have a core diameter \( (2a) \) of less than 7\( \mu \)m for single mode operation. Figure 2.2 shows how the Bessel functions vary with fibre \( V \) value and indicates the regions in which a given LP mode is the highest one allowed of given \( \xi \) value group. Also shown in the Figure are the mode labels, HE, TE and TM which arise from the solutions of Maxwell's equations when the assumption that the refractive index difference between the core and cladding is small is not made. The relationship between the propagation constant and the mode type, as a function of the \( V \)-value of the fibre is shown in Figure 2.3. The power carried by the guided modes is not completely contained inside the fibre core. There is an electromagnetic field outside the core and this field decays exponentially with distance from the core-cladding interface (evanescent field).

The three principle types of optical fibre which have been developed for data transmission systems are shown in Figure 2.4. A step index multimode fibre supports many LP\(_{lm}\) modes each of which has a different phase velocity (propagation constant). The consequent differential mode
Fig. 2-3. Modes allowed to propagate in the fibre core
Multimode step index

Multimode graded index

Excellent bandwidth, Good for sensors

Single mode

Fig. 2.4. Comparison of types of optical fibres
delay causes pulse broadening and so restricts the bandwidth of the transmission system. To improve the bandwidth, and still retain a large core diameter, graded index fibres are used. In this case refractive index of the core $n(r)$ is a function of radial position, $r$, relative to the fibre axis. The phase velocity of a mode is proportional to $c/n(r)$ and therefore the phase velocity of each mode is made similar. For the highest performance in bandwidth single mode fibres are used, that is, a fibre which supports only the $LP_{01}$ mode. This type of fibre has also found widespread applications in fibre optic interferometric sensors. This is because the field distribution of the single spatial mode propagates through the fibre at a well defined speed enabling precise information about the phase of the wave to be determined by conventional interferometry.

2.1.1. Polarisation properties

It is important to note that for a single $LP_{\ell m}$ fibre mode four field distributions are possible because the orthogonal polarisations can each be coupled with either $\cos \phi$ or $\sin \phi$ in the azimuthal direction. Thus the $LP$ mode is formed from a combination of $HE_{\ell-1,m}$ mode and the $EH_{\ell+1,m}$ mode each of which has the possibility of $\cos \phi$ and $\sin \phi$ dependence. The field distribution in terms of the electric field is shown for the lowest order mode, namely, the $LP_{01}$ (or $HE_{11}$) mode in Figure 2.5. This mode which propagates in a single mode fibre consists of two orthogonal polarisation modes. Consider the case in which these modes are linearly polarised and have their respective electric fields polarised along the $x$ and $y$ axes of the fibre. Then the resultant electric field vector of any monochromatic e.m. wave propagating along the $z$ direction can be described by a linear superposition of these two modes, and is given by:

$$E(x,y,z,t) = [R_x E_x(x,y) + R_y E_y(x,y)] e^{j\omega t}$$

2.6
Fig. 2.5. The two polarisations of the LP$_{01}$ (or HE$_{11}$) mode in a single mode fibre.
$R_1 = r_1 e^{-j\beta_1 z}$ are the complex coefficients describing the amplitudes and phases of the modes, $E_i(x,y)$ describe the spatial variation of the electric fields and $\beta_i = k_0 n_i$ are the propagation constants of the polarisation modes.

The resultant state of polarisation of the wave propagating in the fibre is described by the resultant electric field vector which is formed by the two polarisation modes. In ideal fibres with perfect circular symmetry, the two modes are degenerate with $\beta_1 = \beta_2$ and any polarisation state injected into the fibre would propagate unchanged. In practical fibres imperfections break the circular symmetry of the core producing an anisotropic refractive index distribution which lifts the degeneracy of the two modes. These imperfections can result from either a geometrical deformation of the core or a material anisotropy through various strain-optic, magneto-optic and electro-optic refractive index changes. Thus the two modes have different propagation constants and this difference between their effective refractive indexes is known as the fibre birefringence and is expressed as:

$$\Delta \beta = k_0 (n_y - n_x)$$

where the $x$-mode is taken to be the fast mode. Hence, when the two modes are propagating in the fibre, one will slip in phase relative to the other as they propagate. When the phase difference is an integral number of $2\pi$ the input polarisation state will be reproduced and the modes are said to beat. Thus the effect of a uniform birefringence is to cause a general polarisation state to evolve through a periodic sequence of states as it propagates. The length over which this beating occurs is known as the fibre beat length $L_B = 2\pi/\Delta \beta$. Conventional
telecommunication single mode fibres are designed to be low birefringent fibres and have beat lengths of several metres (not necessarily uniform) corresponding to effective index differences of $10^{-7} < n_y - n_x < 10^{-5}$. Consequently, the polarisation state of the light launched into the fibre is, in general, not the same as the state of polarisation of the light at the fibre output, and varies with factors such as temperature, pressure and the mechanical configuration of the fibre. This is of little consequence in data transmission systems where the receiver simply detects intensity variations of the light propagating through the fibre. Although the differential mode delay, which arises because of the different propagation constants of the two polarisation modes, limits the bandwidth of the system. This bandwidth is extremely large compared to a multimode fibre system.

However, optimum operation of any device that depends upon the interference of two coherent light beams, as in homodyne\textsuperscript{23,24,25} or heterodyne\textsuperscript{25,26,27} detection, requires that the interacting beams have identical polarisations. This is particularly relevant to fibre optic interferometric sensors such as those described in 1.3. Ultimately, these applications will require that fibres maintain polarisation over several hundreds of metres. Therefore, current research effort is devoted towards maximising the internal birefringence of the fibre so as to achieve a high degree of polarisation preservation. This has given rise to a type of optical fibre which has elliptical symmetry in the core induced either by index variations\textsuperscript{28} or through the physical shape of the core\textsuperscript{29}. This means that the propagation constants of the two polarisation modes are very different and light injected into the fibre in a linear polarisation along one of the mode axes emerges along this axis, still linearly polarised. To date, most fibre optic interferometers have used the more commonly available low birefringent fibres and achieved good sensitivity in shielded environments\textsuperscript{7,8,9,11}. 
2.2. Directional couplers

Fibre couplers are important optical components for dividing, recombining and monitoring optical signals in both single mode and multimode data transmission systems. Single mode couplers are also finding application in fibre optic interferometric sensors. Two of the advantages of these sensors are compact all fibre construction and ruggedness. These advantages are achieved when the optical beams are guided by dielectric waveguides throughout the interferometer. This also removes the problem of achieving a low loss and a low reflection interface between the bulk-optic components and the fibre. However, in most interferometers constructed prior to 1982, conventional beam splitters have been used to divide and recombine the optical beams. Single mode fibre directional couplers can be used to replace the bulk beam splitters. Their operation depends on the transfer of light from one fibre to another by evanescent wave coupling, which requires that the fibre cores be brought close enough to enable the interaction of the evanescent fields which propagate in the cladding. To gain access to the evanescent field much of the cladding is usually removed by etching or mechanical lapping. This is because the spatial distribution of the intensity in the guided HE_{11} (or LP_{01}) mode is approximately gaussian with a full width at half maximum of about one core diameter. The distribution is determined by the numerical aperture and the core diameter.

The theory of the directional coupler has been treated in detail by several authors. The discussion here is concerned only with basic operating principles. The coupling mechanism can be understood by considering two single mode fibres with identical propagation constants which are parallel and in close proximity. Each fibre supports two orthogonal polarisation modes. Therefore, in the coupler
case four modes are possible and they may be divided by polarisation and field distributions. For a given state of polarisation the electric field magnitude of the two normal modes are equal in each of the fibres, but for one the electric fields are aligned and for the other they are opposed. The resultant electric field direction (polarisation state) in each fibre is formed from a linear superposition of the two normal modes and is shown in Figure 2.6 (top). For two fibres in close proximity the field distributions deform differently in each fibre and the propagation constants as a result are perturbed differently. This is shown in Figure 2.6 (bottom). It is this difference between propagation constants that determines the coupling ratio. The intensity of the light in each fibre is proportional to the time average of the vector product of the electric fields representing the normal modes. Therefore the light intensity in each fibre is determined by the relative phase difference between the normal modes. Since the two modes propagate at different phase velocities, their relative phase changes with distance and the light intensity is periodically transferred between the fibres as a function of distance. This process has been described analytically by Marcuse\textsuperscript{21} who has shown that for coupling between two modes the complex amplitudes of the electric fields of the waves propagating through the coupler are governed by two linear differential equations:

\[
\frac{3a_1(z)}{3z} = -j\beta_1 a_1(z) + jC a_2(z)
\]
\[
\frac{3a_2(z)}{3z} = -j\beta_2 a_2(z) + jC a_1(z)
\]

where \( a_\nu = e^{-j\beta_\nu z} \), \( \nu = 1,2 \) are the complex amplitudes, \( \beta_1 \) and \( \beta_2 \) are the propagation constants, \( C \) is the coupling coefficient and \( z \) is the interaction length. The solution for equation 2.8 may be expressed by:
Fig. 2.6. The symmetric and antisymmetric modes (normal modes) of two identical interacting fibres for different core separations.

Fig. 2.7. The optical power exchange between two parallel single mode fibres in close proximity as a function of interaction length.
Substitution of the input condition \( (E_0, 0) \) into equation 2.9 leads to a light intensity distribution at \( z = L \) inside a lossless coupler which is given by

\[
I_1 = I_0 \cos^2 CL
\]

\[
I_2 = I_0 \sin^2 CL
\]

where \( I_0 \) is the light intensity launched into fibre 1 at \( z = 0 \). This solution shows clearly that light intensity is continuously exchanged between the two fibres. The coupling length over which complete intensity transfer occurs between the fibres is known as the coupler beat length and is given by:

\[
L_c = \frac{\pi}{2C}
\]

This is shown clearly in Figure 2.7.

The coupling coefficient for the two identical fibres is given by the spatial overlap of the two interacting fibre modes:

\[
C = \frac{-2\pi v}{4I_0} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} (n^2 - n_2^2) E_1^* E_2 \, dx \, dy
\]

where \( n(x,y) \) is the refractive index profile of the fibre cores, \( n_2 \) is the refractive index of the cladding, and \( E_1 \) and \( E_2 \) are the electric field distributions of the interacting modes. The result of this integral for the coupling between two \( HE_{11} \) modes has been reported by Snyder and McIntyre. Their result indicates that the amount of coupled light intensity between the fibres depends on the fibre N.A., core radius, the distance between the cores, the interaction length, \( L \), and the free space wavelength of the light. This latter feature makes it
possible to construct a wavelength multiplexer or demultiplexer. For example, in a wavelength multiplexer the coupling coefficient is adjusted so that if light of wavelength $\lambda_1$ is launched into one input port and light of wavelength $\lambda_2$ is launched into the other input port, both wavelengths are coupled into the same output port.

For single mode fibre interferometers the guided light intensity must be split into two equal parts (3dB) by the coupler. For a fixed coupling coefficient the coupled intensity ratio can be adjusted by 3dB by choosing the interaction length, $L$, to satisfy the equation:

$$L = \frac{\pi n}{4C}, \quad n = 1, 2, \ldots$$  \hspace{1cm} 2.13

Three different techniques have been reported for fabricating single mode couplers. Bergh et al have developed a tunable low loss fibre coupler using mechanical lapping techniques. An unjacketed fibre is bonded into a slot in a fused quartz block. The block and the fibre are then ground and polished to within a few micrometres of the fibre core. Placing two such blocks in contact permits optical coupling between the mated polished faces of the fibres. The coupler configuration is shown in Figure 2.8. The light intensity in each fibre is continuously tunable by sliding one block and fibre with respect to the other. This adjusts the distance between the fibre cores, and therefore the strength of coupling. In an alternative method Sheem and Giallorenzi have twisted unjacketed fibres together. The fibres are then etched almost to the core-cladding interface until the evanescent field of the optical beam being guided in the input fibre couples light into the adjacent fibre. This type of coupler provides a fixed coupling ratio. A fixed coupling ratio device has also been reported by Villarruel and Moeller in which two fibres are twisted together then fused and tapered in an oxy-butane flame. This causes the spatial field distribution of the guided mode allowed in each of the cores to spread out so that evanescent field coupling can take place. A fused coupler which is manufactured by ITT Electro-Optical
Fig. 2.8. Crossectional view of a polished single mode fibre directional coupler.

Fig. 2.9. Photograph of a fused single mode fibre 3dB directional coupler which is manufactured by ITT.
Products Division is shown in Figure 2.9. This type of coupler was used in the interferometers described in the present work.

2.3. Phase modulators

The purpose of an optical fibre phase modulator is to effect a change in the index of refraction or the length or both of a fibre in order to control the phase of the propagating light.

The total phase, \( \phi \), of the light propagating through an optical fibre is given by

\[
\phi = k_0 n \xi
\]

(or \( \phi = \beta \xi \)) where \( \xi \) is the fibre length. This phase can be modulated by variations in the fibre length, refractive index and guide dimensions which can be achieved by mechanically stressing the fibre. The resulting optical phase shift is written as:

\[
d\phi = \beta d\xi + \xi d\beta
\]

The first term represents the effect of the change in length of the fibre and is given in terms of the axial strain \( \varepsilon_3 \) (the subscript 3 refers to the direction of light propagation in the fibre). The second term, the change in \( \phi \) due to a change in \( \beta \) is a result of two effects: the strain-optic effect whereby the strain changes the refractive index of the fibre, and a waveguide mode dispersion effect due to a change in fibre diameter, \( D \), produced by the strain. Thus:

\[
\varepsilon d\beta = \varepsilon [k_0 n \Delta + \left( \frac{\beta}{\partial D} \right) dD]
\]

It has been shown by Hocker\(^8\) that the waveguide dispersion term is negligible such that:

\[
\Delta d\beta \gg \left( \frac{\beta}{\partial D} \right) dD
\]

The strain-optic effect appears as a change in the optical indicatrix which can be expressed using the form of the photo-elastic tensor \( p_{ik} \):
\[ \Delta \left[ \frac{1}{n^2} \right] = p_{ik} \varepsilon_k \quad (i = 1, 2 \ldots 6) \quad 2.17 \]

where the standard subscript contraction has been used, \( p_{ik} \) are the tensor coefficients and \( \varepsilon_k \) are applied strains. With no shear strain inside the fibre the strain components \( \varepsilon_4 = \varepsilon_5 = \varepsilon_6 = 0 \). Only the \( i,k = 1,2,3 \) elements of the tensor are used for a homogeneous isotropic material and thus it can be written as:

\[
p_{ik} = \begin{vmatrix}
p_{11} & p_{12} & p_{12} \\
p_{12} & p_{11} & p_{12} \\
p_{12} & p_{12} & p_{11}
\end{vmatrix}
\quad 2.18
\]

Light propagating along the fibre in the 3-direction and either polarised along the 1 or the 2-directions thus sees a change in the indicatrix of:

\[ \Delta \left[ \frac{1}{n^2} \right]_{1,2} = (p_{11} + p_{12}) \varepsilon_r + p_{12} \varepsilon_3 \quad 2.19 \]

where a radially symmetric strain field \( \varepsilon_1 = \varepsilon_2 = \varepsilon_r \) has been assumed. Therefore the change in refractive index is given by:

\[ \Delta n = -\frac{1}{2n^3} \Delta \left[ \frac{1}{n^2} \right]_{1,2} = -\frac{n^3}{2} [(p_{11} + p_{12}) \varepsilon_r + p_{12} \varepsilon_3] \quad 2.20 \]

Hence, the effect of the orthogonal strains \( \varepsilon_1, \varepsilon_2 \) and \( \varepsilon_3 \) (\( \varepsilon_3 \) along the axis of propagation of the optical fibre) is to change the magnitude of the dielectric constants along the principal axes. This is shown in Figure 2.10.

The total phase shift of the light propagating through the fibre is the sum of the strain-optic index change as well as that due to the physical elongation. Neglecting modal dispersion, the phase change in a length of fibre \( \lambda \), for light polarised along either the 1-direction or the 2-direction is given by:
Fig. 2.10. The effect of applying a radially symmetrical strain field $\varepsilon_1 = \varepsilon_2 = \varepsilon_r$ on the indicatrix of an isotropic material.

\[
n_3 = n - \frac{n^3}{2} \left( 2p_{12} \varepsilon_r + p_{11} \varepsilon_3 \right)
\]

\[
n_1 = n_2 = n - \frac{n^3}{2} \left[ (p_{11} + p_{12}) \varepsilon_r + p_{12} \varepsilon_3 \right]
\]
So-called 'single mode' fibres with nominal circular symmetry about the fibre axis are in fact bimodal as discussed in 2.1.1. They can propagate two nearly degenerate orthogonal polarisations of the HE_{11} (or \text{LP}_{01}) mode. Birefringence is introduced in a fibre whenever the circular symmetry of the ideal fibre is broken thus producing an anisotropic refractive index distribution in the core region. For example, an asymmetrical transverse strain introduces a linear birefringence via strain-optic refractive index changes. In this case the strain field is not radially symmetric and the phase change in the fibre for the orthogonal polarisations of the HE_{11} mode can be written as:

\[
d\phi_{1,2} = k_0 n^2 \left[ \varepsilon_3 - \frac{n^2}{2} \left( p_{11} \varepsilon_1 + p_{12} \varepsilon_2 + p_{12} \varepsilon_3 \right) \right]
\]

\[
d\phi_1 = k_0 n^2 \left[ \varepsilon_3 - \frac{n^2}{2} \left( p_{11} \varepsilon_1 + p_{12} \varepsilon_2 + p_{12} \varepsilon_3 \right) \right]
\]

\[
d\phi_2 = k_0 n^2 \left[ \varepsilon_3 - \frac{n^2}{2} \left( p_{12} \varepsilon_1 + p_{11} \varepsilon_2 + p_{12} \varepsilon_3 \right) \right]
\]

using equations 2.17 and 2.18. The induced linear birefringence causes a phase difference between the modes which can be written as:

\[
d\phi_{1} - d\phi_{2} = k_0 n^2 \frac{3}{2} (p_{11} - p_{12}) (\varepsilon_1 - \varepsilon_2)
\]

This birefringent phase delay is independent of \varepsilon_3 and therefore does not have any axial dependence. It is important to notice that for a radially symmetric strain the induced birefringence is zero (\varepsilon_1 = \varepsilon_2 = \varepsilon_3).

In order to be able to use equations 2.21, 2.22 and 2.23 to characterise fibre optic phase modulators it is useful to consider two operating regimes.

(1) the fibre is axially unconstrained

(2) the fibre is axially constrained.

In regime (1) an induced axial strain, \varepsilon_3, corresponds to a change in the optical fibre length. The remaining principal strains \varepsilon_1 and \varepsilon_2 are
determined by the strain distribution in the optical fibre. If the induced radial strain distribution is not symmetric then the optical phase shift is given as in equation 2.22 and the induced birefringence as in equation 2.23. However, if the strain distribution has radial symmetry the optical phase shift is given by equation 2.21. This equation describes the phase shift induced in a fibre optic magnetometer by a linear magnetostrictive stretcher. These devices will be described in detail in Chapter 5. Equation 2.21 also describes the phase shift produced by an applied pressure wave which has a wavelength much greater than the axial dimension of the optical fibre subjected to the strain. For the axially constrained regime (\(\varepsilon_3 = 0\)) with a non-uniform radial strain distribution the optical phase shifts for the orthogonal polarisation modes become:

\[
d\phi_1 = -k_0 n^3 k \frac{1}{2} (p_{11} \varepsilon_1 + p_{12} \varepsilon_2) \\
d\phi_2 = -k_0 n^3 k \frac{1}{2} (p_{12} \varepsilon_1 + p_{11} \varepsilon_2)
\]

Equation 2.24

For a radially symmetrical strain \(\varepsilon_1 = \varepsilon_2 = \varepsilon_r\) the phase shift becomes:

\[
d\phi_{1,2} = -k_0 n^3 k \frac{1}{2} (p_{11} + p_{12}) \varepsilon_r
\]

Equation 2.25

This axially constrained situation is encountered in hydrophone applications where the acoustic wavelength is much less than the fibre interaction length.

The most popular mechanism for modulating the phase of the light propagating through an optical fibre is dynamic mechanical stressing of the fibre. This can be accomplished in response to an electrical signal by attaching the fibre to a piezoelectric material. Such a fibre optic phase modulator for multimode and single mode fibres was first proposed by Davies and Kingsley\(^{26}\) in 1974. In this modulator the fibre is wrapped around a hollow cylindrical PZT (lead-zirconate-titanate) transducer as shown in Figure 2.11, with its protective jacket intact. A voltage across the
Fig. 2.11. Piezoelectric cylinder phase modulator

Driving voltage

Optical fibre

PZT cylinder

Circumferential expanding mode
wall of the cylinder varies the radius of the tube and therefore strains the length of the fibre attached to it. This causes an optical phase shift in the light propagating through the optical fibre via a length change and a refractive index change as given in equation 2.21. The change in circumference of the PZT is given by:

$$\Delta L = \frac{d_{31}}{2\pi R} t V$$

2.26

where $V$ is the applied voltage, $t$ is the PZT wall thickness, $R$ is the PZT radius, and $d_{31}$ is the PZT strain-voltage coefficient normal to the direction of the applied voltage. Hence accounting for the PZT characteristics the optical phase shift given in equation 2.21 becomes:

$$d\phi = k_o n \left( \frac{d_{31}}{t} V N \right) \left( 1 - \frac{n^2}{2} [p_{12}-(p_{11}+p_{12})\mu] \right)$$

2.27

where $N$ is the number of fibre turns on the PZT, $k = 2\pi R$ and the radial strain is given by $\varepsilon_R = -\mu\varepsilon_3$ where $\mu$ is the Poisson ratio for the optical fibre. Using the parameter values for silica fibre, that is, $n=1.46$, $p_{11}=0.12$, $p_{12}=0.27$ and $\mu=0.16$ it is found that the contribution due to the change in refractive index $\frac{n^2}{2} [p_{12}-(p_{11}+p_{12})\mu] = 0.22$ is smaller than, but not insignificant in comparison to the contribution due to the direct length change. The material PZT-5H ceramic has the highest strain voltage coefficient and has been used to obtain a phase shift of 0.09 radians V$^{-1}$ turn$^{-1}$ by Davies and Kingsley. When operated in its resonance modes phase shifts that are one order of magnitude greater can be obtained. The disadvantage of this type of modulator is that it is bulky and also causes birefringence modulation in single mode fibres by coupling energy between the orthogonal polarisation modes. This polarisation modulation can be quite detrimental in many applications. For example, it could manifest itself as an intensity error signal in fibre optic interferometers (see 3.1.1). Although some sensors, called Polarimeters, actually use this modulation of the birefringence (see 1.3).
The asymmetric strain causing the birefringence in fibres which are wound on cylindrical formers is produced by the bend radius and the tension in the fibre coil. Ulrich et al has shown that the birefringence due to bending is given by:

$$\Delta \beta_b = \frac{2\pi \lambda}{4} \left( p_{11} - p_{12} \right) (1+\mu) \frac{r^2}{R^2} \tag{2.28}$$

and that the tension coiled birefringence is given by:

$$\Delta \beta_{tc} = \frac{2\pi \lambda}{2} \left( p_{11} - p_{12} \right) (1+\mu) \left( \frac{2-3\mu}{1-\mu} \right) \frac{r}{R} \epsilon_3 \tag{2.29}$$

where \(r\) is the radius of the fibre and \(R\) is the radius of curvature of the bend. The magnitude of the birefringence modulation caused by changes in \(R\) will now be compared to the phase modulation given in equation 2.21. The phase difference between the two orthogonal polarisations which is introduced by these birefringences is given by:

$$d\phi_1 - d\phi_2 = (\Delta \beta_b + \Delta \beta_{tc}) \frac{2\pi NR}{R} \tag{2.30}$$

where \(\lambda = 2\pi NR\) the length of the fibre. The modulation of this phase difference with change in \(R\) is dominated by the change in \(\epsilon_3\) in \(\Delta \beta_{tc}\) and therefore \(\Delta \beta_b\) can be neglected and the modulation of the phase difference can be written as

$$\left( d\phi_1 - d\phi_2 \right) = \frac{4\pi^2 N}{\lambda} \left( \frac{n^3}{2} \right) \left( p_{12} - p_{21} \right) (1+\mu) \left( \frac{2-3\mu}{1-\mu} \right) \frac{r}{R} \Delta R \tag{2.31}$$

where \(\Delta R\) is the change in \(R\). The ratio of this modulation of the phase difference between the two orthogonal polarisations to the phase modulation given in equation 2.21 (with \(\Delta \lambda = 2\pi NAR\) and \(\epsilon_\lambda = -\mu \epsilon_3\)) is:

$$\frac{\left( d\phi_1 - d\phi_2 \right)}{d\phi} = \frac{1}{2} \frac{r}{R} \tag{2.32}$$

where the constants for the silica fibre have been used. For a fibre
radius of 60\mu m and a PZT radius of 20mm the ratio is approximately $10^{-3}$. Thus the modulation of the polarisation achieved with such a phase modulator is very small in comparison to the modulation of the phase.

A PZT phase modulator, such as the one described here, with 100 turns of optical fibre and a supply voltage of approximately ± 15V can give up to ± 135 radians of optical phase shift. This large dynamic range of controllable phase shift has been used in many interferometric sensor systems in which this type of modulator is used to compensate for environmentally induced phase variations\(^{15,24}\).

Fibre optic phase modulators have taken on a variety of configurations. Carome and Adamovsky\(^{41}\) have developed a transducer which acoustically induces a cylindrically symmetric strain in the optical fibre. The modulator consists of a PZT cylinder filled with epoxy. The optical fibre is mounted with its axis along the axis of the cylinder as shown in Figure 2.12. The optical fibre is axially constrained and the optical phase shift is directly proportional to the induced radial strain through the strain-optic effect as given in equation 2.25. Thus the modulator does not produce any polarisation modulation. This is a resonance device, and it has been operated at frequencies from 300kHz to 7MHz. Nosu et al\(^{42}\) have obtained phase shifts of $5.8 \times 10^{-3}$ radians $\text{V}^{-1}\text{mm}^{-1}$ with this type of device. In 1980 Koo and Carome\(^{43}\) proposed bonding a piezoelectric plastic to an optical fibre as a mechanism for phase modulation of light propagating through the fibre. One of the plastics available for this purpose is polyvinylidene fluoride (PVF\(_2\)). The PVF\(_2\) film stretches the fibre when a voltage is applied across the thickness of the film. This produces an optical phase shift in the light propagating through the fibre via a length change and the strain-optic effect as given in equation 2.21. Koo and Carome\(^{43}\) have obtained phase shifts of $4.1 \times 10^{-3}$ radians $\text{V}^{-1}\text{mm}^{-1}$ up to a few kilohertz. The response decreases when the length
Fig. 2.12. A piezoelectric resonator phase modulator developed by Carome et al.\textsuperscript{41}

Fig. 2.13. A radially poled piezoelectric polymer (PVF\textsubscript{2}) jacket phase modulator developed by Jarzynski.\textsuperscript{44}
of fibre becomes comparable to the wavelength of the extensional elastic waves in the fibre. Jarzynski\textsuperscript{44} has extended this idea such that the optical fibre is jacketed by the PVF\textsubscript{2} plastic as shown in Figure 2.13. This method has several advantages over bonding the plastic to the fibre. The technology of melt extrusion plastic coating is well developed in the wire and cable industry and the same techniques can be used to produce long lengths of optical fibre with a PVF\textsubscript{2} jacket. Also the jacket thickness of 25 to 100\(\mu\)m will enhance the radial strain contribution to the phase shift (strain-optic effect). This means that the modulator can operate over a wide frequency range extending up to megahertz\textsuperscript{44} because the radial strain produces phase shifts in the axially constrained regime (\(\varepsilon_3=0\)). For a radially symmetric strain field the phase shift is given by equation 2.25.

2.4. Laser light sources

Optical fibre Mach-Zehnder interferometers generally operate with an optical path length difference of several millimetres and therefore require a coherent light source. Most early experimental sensors used Helium Neon (HeNe) lasers the main benefits being direct visibility to the eye, high power, single frequency (high coherence) and a centre frequency stability of better than 1 part in 10\textsuperscript{6}. However, there are several compelling reasons for replacing the HeNe laser with a semiconductor diode laser in optical fibre interferometers. These reasons are: the small size, robust package, low cost and low electrical power consumption of the diode laser. Before discussing these two light sources further a summary of the spectral characteristics of laser light is given.

2.4.1. Modal structure and temporal coherence

The electromagnetic wave within a laser cavity takes on a standing wave configuration determined by the optical length between the mirrors. The
cavity resonates when there is an integer number, \( m \), of half wavelengths between the mirrors. Thus:

\[
\frac{m \lambda}{2} = L n_k \quad \text{for resonance (m)}
\]

and

\[
\frac{(m+1) \lambda}{2} = L n_k \quad \text{for resonance (m+1)} \tag{2.33}
\]

where \( L \) is the length of the cavity and \( n_k \) is the refractive index of the medium inside the cavity. From these equations, the mode spacing between two adjacent modes is given by:

\[
\Delta \lambda = \frac{\lambda^2}{2 L n_k} \tag{2.34}
\]

and corresponding frequency difference is given by:

\[
\Delta \nu = \frac{c \Delta \lambda}{\lambda^2} = \frac{c}{2 L n_k} \tag{2.35}
\]

The resonant modes of the cavity are considerably narrower in frequency than the bandwidth of the laser radiation transition. Since sustained laser action can only occur at those frequencies within the lasing transition for which the cavity is resonant, the output of the laser contains a number of narrow frequency bands. These frequencies correspond to the longitudinal modes of the laser. These concepts are illustrated in Figure 2.14. Each longitudinal mode is associated with a set of transverse field distributions. In conventional gas lasers (no transverse waveguide) these field distributions are perpendicular to the propagation axis and they are known as TEM\(_{mm}\) modes (transverse electric and magnetic). These modes are determined by the transverse dimensions of the particular cavity and they determine the spatial distribution of energy in the emerging laser beam. The fundamental TEM mode is the most widely used because the optical intensity is ideally gaussian over the beam cross-section, the beam is spatially coherent and it can be focussed down to a small spot size.
Fig. 2-14. The combination of the lasing transition lineshape with the resonant cavity modes gives the resulting output of a laser.
The electron transitions responsible for the generation of light have a duration of approximately $10^{-8}$ s. Hence the emitted wave trains are finite in time and there is a spread in the emitted frequencies known as the natural linewidth. This linewidth is generally increased by other effects such as Doppler broadening, collision broadening and crystal field interactions depending on the type of optical source. The configuration of each wave train is assumed such that the square of its Fourier transform resembles an intensity versus frequency distribution often observed for spectral lines. A detailed account of this analysis can be found in Hecht and Zajac. The main result is that as the wave train becomes infinitely long in time because of the stimulated emission process, its frequency spectrum decreases to a single line in the frequency domain. This is the limiting case for an idealised monochromatic wave. In fact the frequency bandwidth is of the order of magnitude of the reciprocal of the temporal extent of the wave train:

$$\Delta v = \frac{1}{\Delta t}$$ (2.36)

where $\Delta v$ is taken to be the width of the spectrum where the intensity has dropped to $\frac{1}{2}$ of its maximum value. The time, $\Delta t$, which satisfies this equation defines the time for which the wave emitted by the source remains predictable in phase and is referred to as the coherence time. The coherence length is the distance that the light travels in the coherence time and is given by:

$$L_c = c \Delta t$$ (2.37)

The amplitude splitting interferometers described in this work measure the phase difference between two beams which is introduced by a time delay and therefore the measurement requires that the beams are temporally coherent. Suppose a light wave is divided into two identical disturbances
of the form:

$$E(t) = E_0 e^{i\phi(t)}$$  \hspace{1cm} 2.38

and then recombines to generate an interference signal. The interference term given in equation 1.7 can be written as:

$$Y(\tau) = \langle e^{i\phi(t+\tau)} e^{-i\phi(t)} \rangle$$  \hspace{1cm} 2.39

where \( \tau \) is time delay between the beams.

Hence

$$Y(\tau) = \lim_{T \to \infty} \frac{1}{T} \int_0^T (\cos \Delta \phi + i \sin \Delta \phi) dt$$  \hspace{1cm} 2.40

where \( \Delta \phi = \phi(t+\tau) - \phi(t) \). For a monochromatic plane wave of infinite coherence length \( \phi(t) = k.r - \omega t \) and \( \Delta \phi = -\omega \tau \) the interference term becomes: \( e^{-i\omega \tau} \). Hence \( |Y(\tau)| = 1 \) (fringe visibility is unity) and there is perfect correlation between the optical fields. For a quasimonochromatic source where \( \tau \) is greater than the coherence time \( \Delta t \), \( \Delta \phi \) will be random by varying between 0 and \( 2\pi \) such that the integral averages to zero and \( |Y(\tau)| = 0 \). This corresponds to complete incoherence. The condition \( 0 < |Y(\tau)| < 1 \) corresponds to partial coherence.

Amplitude splitting interferometers with adjustable optical path lengths can be used to measure the temporal coherence of optical sources. The output of such a device as a function of optical path length difference is shown in Figure 2.15 for an idealised zero line width source and a finite line width source. An optical path difference of 1m corresponds to a time delay between the recombining beams of 3ns in air. This is roughly the coherence length of a single frequency semiconductor laser. For comparison white light has a bandwidth of about \( 0.3 \times 10^{15} \) Hz and the coherence length is about \( 10^{-6} \) m. Therefore the interferometer optical path length difference will have to be less than \( 10^{-6} \) of a metre if interference is to be observed.
It is important to realise that \( Y \) is a measure of the coherence time and therefore bandwidth of the source. In fact the Fourier transform of \( Y \) gives the spectral energy distribution of the source\(^{46}\). This concept is illustrated in Figure 2.15 where the interferometer output is shown for an idealised zero linewidth source and a finite linewidth source.

2.4.2. Lasers used in optical fibre interferometers

Javan et al\(^{47}\) reported the operation of a continuous wave (c.w.) Helium Neon gas laser at a wavelength of 1152nm. Since then radiative transitions in Neon have been used to obtain laser oscillation at 633nm and 3390nm. These laser transitions are shown on the energy level diagram in Figure 2.16. A schematic diagram of a HeNe laser which operates at 633nm is shown in Figure 2.17. The gas discharge tube windows are at Brewster's angle \( \theta_B \) so that light with its electric field polarised vertically as shown in Figure 2.17 suffers no reflection losses at the windows. This causes the light output to be polarised in this direction, since the orthogonal polarisation experiences reflection losses at the windows and consequently has a higher threshold.

The linewidth of a HeNe laser is determined by Doppler broadening of the very narrow emission line between the atomic states and is approximately 1.5GHz. Since the cavity length determines the number of oscillating modes (see equation 2.35) a single mode can be obtained by reducing the cavity length so that only one resonance occurs in the lasing linewidth. The linewidth of the single mode is determined by the quality factor of the resonator and can be as small as a few kilohertz. This corresponds to a coherence length in air of many 10's of kilometres. Hence the laser is an ideal light source for optical fibre interferometers where there is often a large optical path length difference. A Tropel 200 single
Fig. 2.15. Interferometer output as a function of optical path length difference for an idealised zero linewidth source and a finite linewidth source.
Fig. 2.17 Helium Neon laser system

Fig. 2.16 Helium Neon laser energy levels
longitudinal mode HeNe laser emitting 1mW of optical power has been used in this work.

The first semiconductor lasers which were developed in 1962,²,⁴⁸ consisted of chips of GaAs of rectangular shape into which a planar p-n junction was diffused. The external appearance of such a laser is shown in Figure 2.18. The Figure shows the GaAs chip bonded to a metal base with a wire contact applied to the top. The chip has smooth cleaved end-facets out of which the light is emitted. The position of the p-n junction and the direction of light emission is shown. Passage of a current through the contact causes electrons and holes to be injected across the p-n junction where they recombine. Stimulated emission starts to occur in this region when the current density is raised to a sufficient level. Because of the high refractive index of the GaAs the end faces of the chip act as reflectors (R = 30%) and form a Fabry-Perot cavity (gain is high enough for R = 30% to give lasing). This cavity is excited into laser oscillation, approximately over the thickness of the region where the injected carriers recombine, when the current is raised above the threshold. The high rate of stimulated emission possible in semiconductors means that the length of the Fabry-Perot cavity may be much smaller than in most other lasers, a length of between 0.2 and 1mm being suitable.

In order to obtain sustained stimulated emission at the lowest possible current present day diode lasers consist of multilayer heterostructures³,⁴⁹ which confine both the carriers and the light in the plane perpendicular to the junction. These structures are formed by cladding the GaAs active layer with n- and p- type GaAlAs passive layers which have a larger band gap than the active layer. The injected carriers are confined by the potential barriers that exist at the interface with the layers of larger band gap and the light is confined by the waveguide that is created by the lower refractive index of these layers.
Fig. 2.18. Basic structure of a GaAs laser diode.

Fig. 2.19. 20μm stripe heterostructure GaAlAs/GaAs laser

\[ \lambda = 850 \text{ nm} \]
\[ I_{th} = 150 \text{ mA} \]
It is also important to provide optical and carrier confinement in the plane parallel to the junction. This has been achieved in several different ways. In an oxide insulated stripe laser lateral confinement is produced simply by restricting the area of the electrical contact to typically 10 to 20μm wide. In this structure there is no deliberate confinement of the light in the lateral direction. A typical room temperature operating current is in the range from 100 to 200mA and they require a drive voltage of about 1.8V. An STL 20μm stripe laser is shown in Figure 2.19. Lasers in which there is a lateral waveguide have a very narrow lasing filament less than 3μm wide. Two examples of such lasers are the channel substrate and the buried heterostructure laser. The threshold current if the best of these devices is in the 10 to 30mA range.

In all the laser structures described the waveguide dimensions are carefully controlled so that only the zero order transverse mode (TEM\textsubscript{00}) propagates and a single elliptical beam is emitted from the output face. The light output from the laser leaves an approximately rectangular aperture at the end of the laser cavity. The aperture is typically 0.2μm thick and 5 to 20 micrometres wide. The light is diffracted at the aperture and so for the fundamental transverse mode the beam divergence is typically 30 degrees vertically and 10 degrees horizontally.

A large number of semiconductor materials, in addition to GaAs, have been used to make injection lasers, the aim being to extend the operating wavelength into the 1100nm to 1600nm range where silica fibres have extremely low losses. The semiconductor material used in the active layer of a laser must have a direct gap to produce satisfactory stimulated emission. It is possible to produce direct-gap semiconductors which cover a continuous variation of band gap by employing solid solutions between different binaries. The most important set of binary compounds
are the III/V's. These comprise of compounds between one of the group III elements Al, Ga or In and one of the group V elements P, As or Sb.

2.4.3. Spectrum of a semiconductor laser

The need for a single frequency laser source in fibre optic interferometers has been highlighted in 2.4.1. The spectrum of a semiconductor laser is a function of output power, temperature and modulation conditions. Many lasers fluctuate from one longitudinal mode to the next as drive current and temperature change. It is important to know the spectral width, number of modes and the peak wavelength over the range of likely operating conditions. For an 850nm laser the absolute operating frequency is $3.5 \times 10^{14}$ Hz and the mode spacing corresponds to a wavelength separation of 0.35nm for a 200μm laser cavity (see equation 2.34). The linewidth of the laser transition is approximately $10^{13}$ Hz, this is considerably broader than the HeNe laser owing to the fact that photon emission takes place as a result of electron motion between two bands of energy levels. The energy level bands are approximately 3kT wide and are a result of the crystal structure. This is the basic reason why semiconductor lasers are prone to multilongitudinal mode operation. This behaviour is illustrated in Figure 2.14.

In order to achieve a stable single longitudinal mode output much work has been carried out to produce a more sophisticated laser resonator than the simple Fabry-Perot cavity. This work has involved external cavities, and distributed feedback gratings. The distributed feedback grating is an ideal solution because it can be incorporated into the multi-layer laser structure to perform the function of the normal end reflectors. The gratings are formed by ultrafine (approximately 0.2μm
period) corrugation of the active layer of the laser. This forms a Bragg reflection at a single wavelength determined by the period of the grating and thus the laser oscillation is sustained at this wavelength instead of the multiple wavelengths for the basic Fabry-Perot cavity. These type of devices are known as DFB lasers and the technology for incorporating the grating into the laser structure is still in its infancy. In view of this many laboratories are trying to find a simple and robust external cavity method for controlling the laser output spectrum. This technique makes use of optical feedback through reflection from an external grating or mirror and has the effect of modifying the reflectivity of the mirror facet of the laser. For an external cavity with a length of 1/20 the effective length of main laser cavity a periodic modification of the reflectivity is produced with peaks every 20 longitudinal modes. Thus laser oscillation can be sustained at a single longitudinal mode. In 4.5 of this thesis a technique for electronically locking a longitudinal mode of a diode laser to an external cavity resonance in order to improve the centre frequency stability is described.

As the temperature of the laser increases the peak of the laser gain spectrum moves towards longer wavelengths because of the reducing band gap of the semiconductor. This is at 0.2nm K⁻¹ which corresponds to 80GHz K⁻¹. The effective refractive index also slowly increases at about one quarter of the rate of the peak wavelength. The effect of this is for the peak wavelength to move from one longitudinal mode to the next. However, for certain drive currents and temperature ranges the radiative linewidth can be less than the longitudinal mode spacing so that the laser operates predominantly in a single longitudinal mode. The main difficulty then lies in preventing the laser hopping from mode to mode as drive current and temperature conditions vary. If the temperature drifts by as little as
0.5K it can hop to an adjacent mode. In order to reduce these effects in the present work the laser heat sink was temperature stabilised. An N.T.C. thermister, mounted next to the laser, has a resistance which is sensitively dependent upon temperature. This resistance is monitored in a bridge circuit and the resultant output used to drive a Peltier cooler which is in good thermal contact with the laser on one side and a heat-sink on the other. The temperature control circuit is shown in Appendix B(i). This technique provided a device temperature stability of better than ±0.1K at approximately 25°C.

2.4.4. Performance characteristics of two GaAlAs lasers

The light versus current curve for an STL GaAlAs laser diode (device No. 2920) is shown in Appendix C(i) and the laser drive circuit is shown in Appendix B(ii). A photograph of this laser is shown in Figure 2.20. The STL laser is an oxide insulated stripe device with a stripe width of approximately 20 micrometres and has a threshold current of 150mA at a temperature of 25°C. This threshold current rises by about 1% K⁻¹ heat sink temperature and the optical output is lowered by 0.8% K⁻¹ according to the manufacturers. At a drive current of 176mA it can supply 7mW of optical power into a numerical aperture of 0.5 (60° cone). The output spectrum of this laser has a centre frequency of approximately 350THz (λ = 856nm) and is dependent upon drive current and temperature. The dependence on drive current was measured using a diffraction grating spectrometer. This is shown in Figure 2.21 for increasing drive currents. The longitudinal modes correspond to the cavity resonances. The spacing between the modes is 0.18nm, corresponding to 74GHz. This can be verified by substituting the effective refractive index of the active region of 4.5 and the cavity length of 400μm into equation 2.35. At a drive current of 168mA the linewidth of the laser transition is less than
Fig. 220. STL GaAlAs diode laser.
Fig. 221. Output spectrum of STL laser diode no. 2920 at 25°C as a function of output power.
the longitudinal mode spacing and the laser operates at a wavelength of 866nm in a single longitudinal mode as illustrated in Figure 2.21. The linewidth of an individual longitudinal mode is well below the resolution of the spectrometer. Interferometric measurements by Epworth et al show that this type of laser typically has a coherence length of about 1m corresponding to a linewidth of 300MHz.

Data on the temperature dependence of the wavelength for a stripe laser was supplied by STL. The temperature coefficient of the peak wavelength is 0.22nm K\(^{-1}\) and the temperature coefficient of an individual mode wavelength is 0.06nm K\(^{-1}\).

The Laser Diode Laboratories device is a channel substrate planar (CSP) laser. This CSP structure provides a built in effective index guide that varies across the lateral junction plane confining the propagating e.m. wave as well as stripe insulation carrier confinement. These confinement techniques minimise the active volume of the laser which results in laser emission at extremely low drive currents. The light versus current characteristic for device No. 27 is shown in Appendix C(ii). The laser has a threshold current which is strongly temperature dependent with a positive coefficient of approximately 0.5mA.K\(^{-1}\). At a device temperature of 25°C the threshold current is 32mA. The optical power output is lowered by about 1% K\(^{-1}\). At a drive current of 46mA it can supply 7mW of optical power into a numerical aperture of 0.5. The output spectrum of the laser has a centre frequency of 363THz (\(\lambda = 825nm\)). This spectrum is plotted at increasing drive currents in Figure 2.22. The spectrum is multi-longitudinal mode at low optical powers with a mode spacing of 0.4nm corresponding to 176GHz. This can be confirmed by substituting the effective refractive index of the active region of 4.5 and the cavity length of 200\(\mu m\) into equation 2.35. At optical powers above 3mW the laser operates in a single longitudinal mode as shown in Figure 2.23. The linewidth of an individual mode is typically
less than 300MHz according to the manufacturers which corresponds to a coherence length of about 1m.

The temperature dependence of the peak wavelength for a typical device is 0.27nm.K^{-1} according to Laser Diode Laboratories.
Fig. 22. Output spectrum of CSP laser diode no. 27 at 25°C as a function of output power.
CHAPTER 3

USING AN OPTICAL FIBRE MACH-ZEHNDER INTERFEROMETER AS A SENSOR

The use of the optical fibre Mach-Zehnder interferometer as a sensor was mentioned in Chapter 1. In this interferometer one fibre is subjected to a signal which changes its optical path length, while the other fibre is isolated from the signal. In this Chapter the general principles of the optical fibre Mach-Zehnder interferometer are described and the experimental results obtained using two homodyne demodulation schemes are presented.

3.1. Single mode optical fibre Mach-Zehnder interferometer

In a single mode optical fibre Mach-Zehnder interferometer, as shown in Figure 3.1, beam splitting and combining are carried out using optical fibre directional couplers (see 2.2). The laser light is first divided into two equal intensity beams by a directional coupler. One beam propagates along a signal fibre and the other along a reference fibre, after which the beams are recombined at a second directional coupler. The output electric field amplitudes of a fibre directional coupler are given by field transfer matrix defined in equation 2.9.

In a Mach-Zehnder interferometer with optical inputs \( a_1(o) = E_0 \) and \( a_2(o) = 0 \), the outputs of the coupler at ports 3 and 4 in the absence of any coupler loss are given by:

\[
\begin{align*}
  a_1(L) &= E_0 \cos (CL) \\
  a_2(L) &= -jE_0 \sin (CL)
\end{align*}
\]

As the two beams propagate along the two interferometer arms, they experience, in general, different phase shifts. Thus the electric field inputs for the second directional coupler become:
Fig. 3-1 Fibre optic Mach-Zehnder interferometer
\[ E_{0s} = E_{01} e^{i\phi_s} \cos (CL) \]
\[ E_{0r} = -jE_{02} e^{i\phi_r} \sin (CL) \]  \hspace{1cm} (3.2)

where \( \phi_r \) and \( \phi_s \) are the total phase changes in the reference and signal arms respectively. \( E_{01} \) and \( E_{02} \) are the amplitude coefficients in the reference and signal fibres respectively. They may differ from \( E_0 \) due to different losses in the two arms of the interferometer. It should be noted that, in addition to the phase difference \( \phi_s - \phi_r \), the beam in the reference arm experiences a \(-\pi/2\) phase shift with respect to that in the signal fibre while propagating through the 3dB coupler. The outputs of the second directional coupler can be found by substituting the input electric fields (3.2) into equation 2.9. These outputs are generally expressed in terms of light intensity, \( I \). Hence the final outputs of the Mach-Zehnder interferometer, which are 180° out of phase because of the phase shift accumulated in the 3dB couplers, are given by:

\[ I_1 = \frac{1}{4} \left[ 4E_{01}^2 \cos^4(CL) + 4E_{02}^2 \sin^4(CL) - 2E_{01}E_{02}\sin^2(2CL) \cos \phi \right] \]
\[ I_2 = \frac{\sin^2(2CL)}{4} \left[ E_{01}^2 + E_{02}^2 + 2E_{01}E_{02} \cos \phi \right] \]  \hspace{1cm} (3.3)

The light intensity in the signal fibre, \( I_1 \), is proportional to \( \langle E_1 \rangle = \frac{E_{01}^2}{2} \) and the light intensity \( I_2 \) in the reference fibre is proportional to \( \langle E_2 \rangle = \frac{E_{02}^2}{2} \). Also for 3dB couplers it was shown in 2.2 that \( \alpha = \frac{2\phi}{\pi} \) and therefore equation 3.3 can be written in the simplified form:

\[ I_1 = \frac{I_T}{2} (1 - \alpha \cos \phi) \]
\[ I_2 = \frac{I_T}{2} (1 + \alpha \cos \phi) \]  \hspace{1cm} (3.4)

where the total light intensity, \( I_T \), in each coupler output port is equal to \( I_1 + I_2 \). The fringe visibility is defined as \( \alpha = \frac{E_{01}E_{02}}{I_1 + I_2} \) and depends
on the differential power loss and polarisation effects in the reference and signal arms. In practice the differential power loss is small, because of the small interferometer optical path length difference and low fibre loss, and \( \alpha \) depends mainly on the state of polarisation in the two interferometer arms.

3.1.1. Fringe visibility

In general the state of polarisation of the light propagating in single mode optical fibre drifts due to environmentally induced strain birefringence. This effect has been discussed in 2.1.1. In a Mach-Zehnder interferometer this results in a decrease in fringe visibility, \( \alpha \), and in the case where the polarisations of the interfering beams differ by 90° the visibility is zero and the interferometer output intensity becomes insensitive to optical path length changes. This problem can be solved by using polarisation maintaining optical fibre\(^{28,29}\) (see 2.1.1) or by deliberately introducing a compensating stress birefringence to control the polarisation\(^{57}\). In the work described here the Mach-Zehnder interferometer was operated in a shielded laboratory environment which minimised the effect of polarisation induced signal fading. In order to justify this it is important to note that it was shown by equation 2.32 in 2.3 that the modulation of the polarisation achieved with a PZT phase modulator can be 1000 times smaller than the modulation of the phase. In addition equation 2.23 shows that for radially symmetric strains and for purely axial strains, the induced birefringence is zero. Also Rashleigh\(^{58}\) has reported the sensitivity of an optical fibre Mach-Zehnder interferometer in which the signal fibre is strained axially as \( \frac{d\phi}{d\xi} = 11.3 \text{ radians m}^{-1}\mu\text{m}^{-1} \) where \( \xi \) is the physical length of the sensitised fibre and \( d\xi \) is the change in \( \xi \). He compares this sensitivity with a sensitivity of 0.22 radians m\(^{-1}\mu\text{m}^{-1} \) (a factor of
51 smaller) for a fibre polarimeter in which the intrinsic fibre birefringence is enhanced by wrapping it tightly around a small diameter cylinder. Hence in an ideal interferometer, the interfering beams have equal intensities and identical polarisations, and the interferometer outputs given in equation 3.4 reduce to:

\[ I_1 = I_0 (1 - \cos \phi) \]
\[ I_2 = I_0 (1 + \cos \phi) \]

where \( I_0 \) is the interferometer output at half maximum intensity. These outputs are shown in Figure 3.2.

3.1.2. Sensitivity to phase changes

The phase difference between the signal fibre and the reference fibre in a Mach-Zehnder interferometer is \( \phi = k_o L (L=nz) \) where \( L \) is the optical path length difference between the interfering beams. This optical path length difference is principally perturbed by small changes in the refractive index, \( n \), and the total sensitive length, \( L \), of each interferometer arm and thus produces a phase change, \( d\phi \), in the interferometer output. This phase change produces a change in the interferometer output intensity such that:

\[ \left| \frac{dI_1}{d\phi} \right| = I_0 \sin \phi \]

Thus, for a given signal \( d\phi \) the output change \( |dI_1| = |dI_2| \) is proportional to \( \sin \phi \). The phase \( \phi \) can drift over many radians due to low frequency ambient pressure and temperature variations. Thus it is possible to detect a small signal only when the signal frequency of \( d\phi \) is outside the frequency band of this interferometer phase drift. Even in this case the signal output \( dI_1 \) suffers from fading when \( \sin \phi \) changes...
Phase $\phi = \frac{\text{path difference} \times 2\pi}{\text{wavelength}}$

**Fig. 3-2** The two outputs from a Mach–Zehnder interferometer

$\Delta \phi$ is the modulation in $\phi$ due to external signal.

**Fig. 3-3** Variation in intensity change for different path differences
due to the drift as shown in Figure 3.3. There is no output signal, \( dI \), when \( \sin \phi \) vanishes at \( \phi = m\pi \), where \( m = 0, 1, 2, \ldots \).

### 3.1.3. The quadrature condition

The signal fading problem can be eliminated in a two beam interferometer by operating it at quadrature\(^{15,24,25} \) where the phase difference between the optical beams is given by:

\[
\phi = \frac{\pi}{2} \pm m\pi
\]

where \( m \) is an integer.

This corresponds to an optical length difference between the interfering beams of \( \lambda/4 \). At quadrature the sinusoidal interferometer output function is at its maximum gradient where the output intensity is linearly proportional to phase, and at its most sensitive to small phase changes. The sensitivity \( \left| \frac{dI}{d\phi} \right| \), given in equation 3.6, takes on its maximum value because \( \sin \phi \) is equal to unity. If the interferometer is maintained at quadrature over a restricted frequency range a phase signal which is in a different frequency band from the phase drift can still be determined from the output. Two techniques for achieving the quadrature condition will be described in section 3.2. Figures 3.2 and 3.3 illustrate the advantages of operating an interferometer in phase quadrature.

### 3.1.4. Phase modulation of the signal fibre

If the optical path length, \( l_n \), of the signal fibre is sinusoidally modulated at frequency \( \omega \), the total phase difference between the beams is given by:

\[
\phi = \phi_r - \phi_s + k_o f_s \sin \omega t
\]
where \( \phi_s \) and \( \phi_r \) represent the phase drift in the signal and reference fibres respectively. This section deals with magnitude of the harmonic component of the phase difference, \( \phi \), at frequency \( \omega \), as a function of the optical path length variation, \( \ell_n \). For the phase difference defined in equation 3.8, the interferometer output signal, \( I_1 \), can be written as:

\[
I_1 = I_o (1 - \cos (k_0 \ell_n \sin \omega t + \phi_s - \phi_r))
\]

3.9.

This signal may be represented as a Fourier series which can be obtained using the appropriate Bessel expansions and has the following form:

\[
I_1 = I_o \left\{ 1 - \sum_{n=1}^{\infty} J_0 (k_0 \ell_n) + 2 \sum_{n=1}^{\infty} J_2 (k_0 \ell_n) \cos (2n \omega t) \right\} \cos (\phi_s - \phi_r)
\]

\[
+ \left[ 2 \sum_{n=0}^{\infty} J_{2n+1} (k_0 \ell_n) \sin (2n+1) \omega t \right] \sin (\phi_s - \phi_r)
\]

3.10

Spectral analysis of this signal leads directly to the amplitude of the sinusoidal phase modulation.

If the quadrature condition can be maintained then \( \sin (\phi_s - \phi_r) = 1 \) and it is only necessary to measure the amplitude of the \( J_1 (k_0 \ell_n) \) component in equation 3.10 to determine the amplitude of the signal fibre phase modulation \( \phi_\omega \) at frequency \( \omega \). The interferometer output resulting from this phase modulation takes its maximum value when \( J_1 (k_0 \ell_n) \) is a maximum, which for \( \lambda = 633 \text{nm} \) is equivalent to an optical length modulation of about 180nm or a phase modulation of 1.8 radians; this corresponds to almost the full modulation of the interferometer output.
3.1.5. Noise in the interferometer

In a Mach-Zehnder interferometer, noise produces a signal which is indistinguishable from the phase signal. The minimum detectable phase shift (noise floor) is determined by a number of noise sources, such as intensity noise and phase noise, which are a result of the fundamental properties of the laser source together with the detector quantum shot noise. Thermal noise in the detector should also be considered as well as contributions from the interferometer itself such as the environmental drift and local acoustic signals. The minimum detectable phase shift depends on the resultant amplitude of these noise sources. It was shown in 3.1.4 that a phase shift in the signal arm of frequency $\omega$ appears as a signal on the output of the interferometer at $\omega$ (the first order Bessel function $J_1$), the interferometer being kept in quadrature. In the phase shift range below 0.1 radian the interferometer output at $\omega$ is directly proportional to the phase shift. This maximum phase shift is determined by the allowed non-linearity of the sensor response (chosen to be approximately 0.1 radian here) and limits the dynamic range with some demodulation schemes. The amplitude $J_1$ of the signal at $\omega$ reaches a maximum $J_1(\text{max})$ at a phase shift of 1.8 radians. Consequently, to measure phase shifts of approximately $10^{-6}$ radians, it is necessary to measure variations at the signal frequency of approximately $10^{-6}$ of the interferometer's d.c. output. At the quadrature condition the d.c. output is the mean of the maximum and minimum outputs of the interferometer. Thus, to detect $10^{-6}$ radians, the relative amplitude noise $\frac{\Delta I}{I_0}$ of the interferometer must be less than $10^{-6}$ (i.e. -120dB of the d.c. level to detect these small phase shifts). The limits imposed on the minimum detectable phase of a Mach-Zehnder interferometer by the predominant noise sources are now treated quantitatively.
For an interferometer with 100 percent fringe visibility the maximum sensitivity, which occurs at quadrature, is given by equation 3.6 and leads to a signal at a photodetector given by:

\[
\left| \frac{dV_1}{d\phi} \right| = V_0 \sin\phi = V_0
\]

where the electrical signal, \( V \), is proportional to the light intensity, \( I \), and \( V_0 \) is the voltage produced by the d.c. output of the interferometer at quadrature.

(a) Shot noise

The shot noise, which results from a statistical fluctuation in the number of electrons and holes created at the photodiode junction, produces an r.m.s. noise voltage \( dV_s \) across the load resistor, \( R \), of the photodiode amplifier and is given by:

\[
dV_s = (2eV_o \Delta F R)^{\frac{1}{2}}
\]

where \( e \) is the electronic charge, \( \Delta F \) the detection bandwidth, and \( V_o \) the d.c. voltage across the load resistance. Thus the minimum detectable r.m.s. phase shift, \( \phi_s \), due to the shot noise is given by:

\[
\phi_s = \frac{dV_s}{V_o}
\]

For silicon photodiodes used in the present work \( R \) was 100k\( \Omega \) and with \( V_o = 2V \), the shot noise voltage is \( 2.5 \times 10^{-7} \) V Hz\(^{-\frac{1}{2}}\) corresponds to a minimum detectable phase shift of \( 1.3 \times 10^{-7} \) radians r.m.s. Hz\(^{-\frac{1}{2}}\).

(b) Intensity noise

The intensity noise, \( dI \), of the laser produces a noise voltage, \( dV_I \),
which is related to the minimum detectable phase shift, $\phi_I$, by

$$\phi_I = \frac{dV_I}{V_0}$$  \hspace{1cm} 3.14

For the HeNe laser, $dV_I = 2\mu V \text{ r.m.s. Hz}^{-\frac{1}{2}}$ at 1kHz therefore, $\phi_I = 1 \times 10^{-6}$ radians r.m.s.Hz$^{-\frac{1}{2}}$.

(c) Phase noise

When the interferometer is at quadrature small frequency excursions of the laser produce a noise voltage $dV_F$ which is given by:

$$dV_F = \frac{V_0 2\pi L}{c} dv$$  \hspace{1cm} 3.15

where $dv$ is the source frequency fluctuation (jitter) and $L$ is the interferometer optical path length difference ($L=\lambda z$). Thus phase noise limited minimum detectable phase shift, $\phi_F$, is given by:

$$\phi_F = \frac{2\pi L}{c} dv$$  \hspace{1cm} 3.16

In general for a diode laser, $dv = 50kHz \text{ r.m.s. Hz}^{-\frac{1}{2}}$ at 1kHz and therefore an interferometer with a path difference of 0.01m can detect $1 \times 10^{-5}$ radians r.m.s.Hz$^{-\frac{1}{2}}$.

(d) Johnson noise

The thermal noise of the detector load resistance is given by the Johnson noise equation which is:

$$dV_J = (4KTR\Delta F)^\frac{1}{2}$$  \hspace{1cm} 3.17

where $K$ is Boltzmann's constant and $T$ is the resistor temperature. This noise produces a minimum detectable phase of approximately $10^{-8}$ radians r.m.s. Hz$^{-\frac{1}{2}}$. 
Hence the minimum detectable phase shift (noise floor) is given by:

\[ \phi^2 = \phi_S^2 + \phi_I^2 + \phi_F^2 + \phi_J^2 \]  

3.18

It should be noted that the intensity and phase noise have a 1/f frequency dependence while detector shot noise and Johnson noise are independent of frequency. From the results above the chief noise sources in an interferometer powered by a laser diode will be the phase noise and the intensity noise. All the other noise sources are at least 1 order of magnitude smaller. Hence, in the design of an interferometric sensor it is essential that the laser source has low amplitude noise as well as good frequency stability and that the interferometer has a small path difference.

The actual measured minimum detectable phase shift, \( \phi_m \), varies from the calculated value, \( \phi_c \), as a function of the normalised fringe visibility, \( V \), such that:

\[ \phi_m = V\phi_c \]  

3.19

where \( V = \frac{V_{\text{max}} - V_{\text{min}}}{V_{\text{max}} + V_{\text{min}}} \) and \( V_{\text{max}} \) is the maximum detector voltage and \( V_{\text{min}} \) is the minimum.

3.2. Signal recovery from a Mach-Zehnder interferometer

There are essentially two approaches to the retrieval of the signal from the interferometer, namely, homodyne\(^{23,24,25}\) and heterodyne\(^{25,26,27}\) demodulation. Both require that the signal be in a different frequency band to the low frequency environmental noise which produces the differential drifts between the arms. In homodyne demodulation, in which the two interferometer beams are at the same frequency, the drift causes
signal fading as well as distortion of the signal. In fact, the detected signal can be zero under certain conditions. A phase modulator can be included in one arm to maintain the interferometer at the maximum sensitivity (quadrature) by compensating for the low frequency drifts while allowing the system to respond to the higher frequency signal. In the heterodyne approach one or both of the interferometer optical beams are frequency shifted, the signal appears as frequency modulation on a carrier. This carrier is equal to the difference frequency between the two beams. The fading problem is eliminated and the drift produces frequency deviations of the carrier which can be separated from the frequency deviations produced by the signal provided they occur at different frequencies. Although heterodyne demodulation has the advantage of eliminating the drift problem the necessity to incorporate acousto-optic frequency shifters into the interferometer arms make the approach less attractive. These frequency shifters are expensive and it is difficult to achieve a stable low loss and low reflection interface between the frequency shifter and the optical fibre. Alternatively, if an all-fibre frequency shifter is developed the heterodyne approach will undoubtedly become more popular. Another possibility consists of applying a high frequency phase modulation through a PZT phase modulator placed in one arm of the interferometer, thus producing a phase modulated signal \( \cos(\phi_s - \phi_r + \phi_{\text{sig}}^+ \phi_m \cos(\phi_m t)) \). This technique is called synthetic heterodyne and has the disadvantage that the interferometer output signal can only yield the desired output of \( \phi_s - \phi_r + \phi_{\text{sig}} \) after complex electronic demodulation. For these reasons a homodyne demodulation approach was adopted in the work described here.

3.2.1. Homodyne demodulation using a fibre phase modulator

In this scheme, a phase modulator is used to maintain the quadrature...
condition by supplying an error signal, $\phi_e$, to compensate for any differential phase drift between $\phi_s$ and $\phi_r$. Hence, the quadrature condition becomes:

$$\phi_s - \phi_r - \phi_e = \frac{\pi}{2} \text{ mm} \quad 3.20$$

The phase signal $\phi_e$ is obtained by applying a voltage to the phase modulator which changes the optical path length of the interferometer reference arm. A signal, $I_3$, which is proportional to the cosine of the phase difference between the interferometer arms can be obtained by subtracting the interferometer outputs given in equation 3.5. $I_3$ is defined by:

$$I_3 = 2I_0 \cos (\phi_s - \phi_r - \phi_e + \phi_\omega) \quad 3.21$$

where $\phi_\omega$ is the phase modulation produced by the signal. $I_3$ can also be written as:

$$|I_3| = 2I_0 \sin [\phi_\omega + \phi_s - \phi_r - \phi_e + \frac{\pi}{2}] \quad 3.22$$

Near the quadrature condition this becomes:

$$|I_3| = 2I_0 [\delta - \phi_e] \quad 3.23$$

where $\delta = \phi_\omega + \phi_s - \phi_r - \frac{\pi}{2}$.

If an appropriate feedback signal can be produced from $I_3$ and applied to the phase modulator, the error phase, $\phi_e$, can be made to exactly cancel $\delta$, thus driving $I_3$ to zero. Such a feedback signal is the integral of $I_3$:

$$\int I_4 = \frac{1}{RC} \int |I_3(t)| \, dt \quad 3.24$$
This signal is amplified and used to drive the phase modulator. A schematic diagram of the feedback system is shown in Figure 3.4. If the interferometer phase modulation corresponds to frequencies much smaller than \( \frac{1}{RC} \) (where \( RC \) is the integrator time constant) the system ensures that \( \delta = \phi_e \) which is the quadrature condition. A low pass filter can be incorporated in the system so that the feedback only compensates for the low frequency environmental noise and therefore the phase signal \( \phi_\omega \) at a frequency \( \omega \) which is beyond the bandwidth of the compensator will modulate the amplitude of the signal detected at the interferometer output.

Dandridge and Tveten\(^6\) have demonstrated that the sensitivity and dynamic range of this type of demodulation scheme can be improved by a factor of 10 by measuring the signal after the interferometer outputs have been subtracted. In this way laser intensity fluctuations which are common mode to both interferometer arms cancel out and the first order term of the laser intensity noise is eliminated from the interferometer output signal.

3.2.2. The phase modulator

A cylindrical PZT phase modulator like the one reported by Davies and Kingsley\(^2\) which is described in 2.3, was chosen to provide the error phase signal, \( \phi_e \). A photograph of the phase modulator is shown in Figure 3.5.

A hundred turns of a single mode fibre (16 metres) were wrapped helically onto a Vernitron PZT cylinder made from 5H ceramic. The 5H material was chosen because it has the highest strain per unit electric field
Fig 3-4 Feedback System

Interferometer Outputs
\[ \begin{align*}
I_0(1+\cos \theta) \\
I_0(1-\cos \theta)
\end{align*} \]

Output Signal \( KX_0 \sin wt \)

Subtractor \( I_2 \)

Amplifier \( I_4 \)

Spectrum Analyser

Phase Sensitive Detector

\[ I_4A \]

Used to drive the phase modulator.
Fig. 3.5. The Vernitron PZT phase modulator.

Fig. 3.6. Phase modulator calibration
a) 0 to 1 V ramp at 200 Hz applied to the modulator
b) Interferometer output driven through $2\pi$ radians by the ramp
coefficients of the commercially available cylinders. The cylinder dimensions were: length, L, 76mm; diameter, D, 51mm; and wall thickness, T, 5.1mm. The optical phase shift produced by the modulator can be calculated from equation 2.27 using the strain per unit electric field coefficient \( (d_{31}) \) of 2.74\( \text{A}^0\text{V}^{-1} \) for 5H ceramic and parameter values for silica fibre given in 2.3. Hence the calculated optical phase shift is 0.096 radians V\(^{-1}\) turn\(^{-1}\) which for a 100 turn modulator is 9.6 radians V\(^{-1}\). The change in amplitude of an optical fibre Mach-Zehnder interferometer output signal produced when the cylinder was driven with a 0 to 1V ramp function at 200Hz is shown in Figure 3.6. The induced phase change per volt can be measured directly from the figure and is 2\( \pi \) radians V\(^{-1}\). This is in good agreement with the calculated value and demonstrates that the strain coupling between the cylinder and the fibre is extremely efficient. The breakdown voltage of the ceramic cylinder is greater than 1000V so that the total phase shift could be greater than 6000 radians. However, the range of the modulator is usually limited by the instrumentation supplying the driving voltages to the cylinder.

3.2.3. The compensator system

The interferometer light source was a single longitudinal mode laser. Results have been obtained for both a Tropel 200 HeNe laser and a semiconductor laser diode. In the case of the HeNe laser the light was coupled into the optical fibre using x10 microscope objective lens, and the light from the semiconductor laser was butt coupled into the fibre. It is important to obtain a high light intensity coupling efficiency between the small fibre core (<10\( \mu \text{m} \)) and the laser source. For this purpose the fibre terminations were bared to the cladding over a length of 100mm and the fibre was then clamped in mechanical mount,
incorporating a neoprene pad, which was attached to an Aerotech XYZ manipulator. This was found to give an exceptionally rigid structure, coupled with very fine sensitivity of movement (approximately 0.1µm) allowing the fibre core to be aligned accurately with the light beam. The neoprene acted as an efficient cladding mode stripper.

The Mach-Zehnder interferometer and its homodyne demodulation scheme is shown in Figure 3.7. The laser light was split by an optical fibre 3dB directional coupler (ITT fused coupler - see 2.2 and Figure 2.9) into two components. One component was transmitted in a reference fibre which was coupled to the optical fibre PZT phase modulator. The other component was transmitted in a signal fibre whose phase can be made sensitive to the chosen measurand. For example, in Chapter 5 the details of a technique for making the signal fibre sensitive to magnetic fields are described. The two optical components are then recombined in another 3dB coupler. The couplers, phase modulator and single mode optical fibres were joined using a Power Technology fusion splicer for robustness. The two optical outputs from the final 3dB coupler carried the interference fringes given by equation 3.5. The outputs were converted to electrical signals by silicon photodiodes A and B which are shown in Figure 3.7.

In the normal laboratory environment, the outputs were constantly varying due to the interferometer's inherent sensitivity to the environmental noise. In order to reduce this low frequency coupling into the interferometer the complete system was mounted on a vibration isolation table and enclosed in a perspex box. In addition to these precautions it was necessary to maintain the homodyne interferometer at quadrature (maximum sensitivity) by using the low frequency part of the interferometer output signal, \( I_1 - I_2 \), to drive the phase modulator in order to maintain the low frequency noise
Fig. 3.7. Fibre optic Mach-Zehnder interferometer with active phase compensation
signal equal to 0 as explained in 3.2.1. This was achieved by subtracting the photodiode outputs, which were 180° out of phase, in a differential amplifier to produce a signal proportional to the cosine of the phase difference between the interferometer arms. Consequently, the output from the differential amplifier is a suitable feedback signal for the phase modulator since it will vary in order to maintain the condition $I_1 = I_2$, that is, quadrature. The feedback signal was integrated with an op-Amp integrator which had a time constant, $RC$, of $10^{-3}$ s. The output of the integrator was then amplified by a factor of 20 and applied to the phase modulator as the correction signal $\phi_e$. The basic principles of the feedback system have been illustrated in Figure 3.4 and a circuit diagram is shown in Appendix B(iii). The electronic components used limited the range of the compensator to ±100V which corresponded to a phase compensation (tracking) range of ±200π radians. A disadvantage of the compensator is that once this voltage range is exceeded the feedback signal must be reset to zero. These resets cause a temporary loss of signal and an increase in the system noise.

The dotted line in Figure 3.7 shows how the feedback signal may also be used in an alternative homodyne demodulation scheme which will be described in 3.2.6. Essentially, the laser operating frequency is modulated by the feedback signal to provide phase compensation instead of the optical path length of the reference fibre.

3.2.4. Performance of the compensator

The photograph in Figure 3.8 shows the interferometer outputs detected at photodiodes A and B when a 0 to 2V ramp signal at 70Hz was applied to the phase modulator. The characteristic 180° phase difference between the two interferometer outputs is clearly demonstrated. At the
Fig. 3-8 Modulation of the Mach-Zehnder interferometer phase difference showing the $180^\circ$ phase relationship between the interferometer outputs
quadrature condition (half the peak to peak amplitude of the photodiode outputs) the silicon photodiodes each registered a value of 2V which corresponded to an optical power of 36 μW.

To demonstrate the near unity fringe visibility of the interferometer powered by the single frequency HeNe laser the phase modulator was driven with a sinusoidal signal at 100Hz such that the interferometer output was swept through two fringes. A photograph of the output detected at photodiode A is shown in Figure 3.9. The laser source supplying the interferometer was then switched off and the resulting d.c. level recorded. It can be seen in the photograph by the coincidence of the minimum of the transmitted fringes with the remaining d.c. level, that the fringe visibility is unity. Therefore, the interfering beams have equal intensities and parallel polarisations. Very little change in the fringe visibility was observed during the experiments, therefore it was assumed that the phase signals encountered by the fibres did not introduce any significant birefringence as predicted in 3.1.1. It has been shown in 3.1.5 that the minimum detectable phase shift is proportional to the fringe visibility, v, and takes its smallest value at v = 1.

The effect of the compensation system on the interferometer output over a time interval of several minutes is shown in Figure 3.10. The measurement was taken by monitoring the output of photodiode A on an X-Y plotter (X axis represents time at a rate of 20 seconds per division). The steady trace records the output with the compensation system holding the interferometer continuously at quadrature. The irregular oscillatory trace, which begins after approximately 4 minutes, is the output without the compensator. The departure from quadrature caused by the phase drift is clearly demonstrated by the oscillations. This phase drift is caused by low frequency temperature and pressure variations.
Fig. 3.9 Modulation of the Mach-Zehnder interferometer phase difference demonstrating the near unity fringe visibility
Fig. 3.10 Phase compensation in a Mach-Zehnder interferometer using a phase modulator.
In order to investigate the frequency response of the compensator, a sinusoidal voltage was applied to the phase modulator in addition to the error signal from the compensation system. The signal amplitude was held constant and the interferometer output measured on photodiode A at different frequencies. The interferometer response, $V_R$, to this phase modulation as a function of frequency is shown in Figure 3.11. The results are normalised with respect to the amplitude of the modulation, $V_1$, at a frequency which is well beyond the bandwidth of the compensation circuit. The graph shows that the demodulation system compensates for phase variations effectively in a frequency band ranging from 0 to 20Hz.

A spectrum of the noise on the interferometer output from d.c. to 5kHz with the compensator circuit in operation is shown in Figure 3.12. The result was obtained by using a spectrum analyser to measure the output of photodiode A. The amplitude of the noise measurement is normalised with respect to 1V r.m.s. by the spectrum analyser and is -100dBV at 1kHz in a bandwidth of 4Hz. The corresponding minimum detectable phase has been calculated from equation 3.11 and takes the value of $2.5 \times 10^{-6}$ radians r.m.s. Hz$^{-\frac{1}{2}}$. The additional noise in the present system is probably due to the source intensity noise (see 3.1.5(b)). This performance could certainly be improved upon by using an instrumentation type differential amplifier with a high common mode rejection ratio to subtract the interferometer output signals and measuring the interferometer output after this differential amplifier.

To investigate the response of the interferometer to small amplitude phase modulations, while the compensator maintains the quadrature condition, an additional drive voltage at a frequency of $\omega_m$ was applied to the phase modulator. The frequency $\omega_m$ was beyond the bandwidth of the compensator and the amplitude of the resulting phase modulation was determined by monitoring the output of photodiode A at $\omega_m$ with a lock-in amplifier.

The phase modulator drive voltage was increased from zero until the
Fig. 3-11 Frequency Response of the Compensation System
Fig. 3.12. Noise Spectrum of the Output of a Mach-Zehnder Interferometer in quadrature
lock-in amplifier output reached a first maximum at 260mV. This corresponds to the maximum value of the \( J_1(k_o s n) \) Bessel function which occurs at 1.8 radians. Smaller values of phase were then determined by measuring the lock-in amplifier voltage for other values of phase modulator drive voltage below that corresponding to \( J_1(\text{max}) \) and comparing the results with the tabulated values of Bessel functions.

Figure 3.13 shows the interferometer output \( J_1(k_o s n) \) in radians at 1kHz\( (\omega_m) \) as a function of \( V_m \) which is the amplitude of phase modulator drive voltage at 1kHz. A linear relationship between \( J_1(k_o s n) \) and the phase modulator drive voltage was observed until \( k_o s n \) was approximately 0.1 radian after which the relationship between the interferometer phase difference and the output intensity becomes non-linear (see 3.1.5). The sensitivity of the interferometer to phase modulator drive voltage is given by the gradient of the graph in Figure 3.13 and is equal to 6 radians V\(^{-1}\). This value agrees with the PZT calibration of 2\( \pi \) radians V\(^{-1}\) which was given in 3.2.2. These results are a consequence of the linear voltage-expansion coefficient of the phase modulator at 1kHz, this linearity has been reported by other authors. Jackson et al. have shown that a linear phase shift over 5 orders of magnitude from 1 to 10\(^{-5}\) radians for frequencies ranging from d.c. to 5kHz can be induced in the light transmitted by an optical fibre PZT phase modulator. Also it is apparent from Figure 3.13 that the optical fibre Mach-Zehnder interferometer is ideally suited to measuring these extremely small phase changes (usually much less than 1 radian) caused by small linear perturbations of the optical path length difference.

It should be noted that there is no increase in the noise on the interferometer output at a detection frequency of 1kHz as the optical path length difference \( L(nz) \) is increased from 10mm to 1m when the HeNe laser is used as the light source owing to the laser's good frequency stability.
Fig. 3-13 Variation of interferometer phase shift as a function of phase modulator drive voltage at 1kHz
However, if a diode laser is used as the light source the noise on the interferometer output is observed to increase linearly with optical path length difference. This noise (phase noise) is due to the laser's frequency jitter which has been discussed in 2.4. The interferometer noise resulting from this jitter has a $1/f$ dependence and has been reported by Dandridge et al. Results for the two diode lasers described in 2.4.4. have been obtained, the STL diode and the Laser Diode Labs diode, each device operated at a wavelength of approximately 850nm in single longitudinal mode with an optical output power of 5mW.

In Figure 3.14 the interferometer noise output at quadrature for each laser is given in dBV, as a function of optical path length difference. The measurement bandwidth is 4Hz and is centered at 1kHz. The corresponding minimum detectable phase shifts can be calculated using equation 3.11 ($V_o = 2V$) and the results are given in Figure 3.14. The minimum detectable phase shift for the STL laser is $8 \times 10^{-5}$ radians r.m.s. Hz$^{-1/2}$ at 1kHz for a path difference of 0.1m. The amplitude of the frequency jitter causing this phase noise is 38kHz r.m.s. Hz$^{-1/2}$ (calculated using equation 3.16). For the same optical path length difference the minimum detectable phase shift in the interferometer powered by the C.S.P. laser is $5 \times 10^{-5}$ radians r.m.s. Hz$^{-1/2}$ at 1kHz and is due to a diode frequency jitter of 24kHz r.m.s. Hz$^{-1/2}$. To achieve sensitivities in the $10^{-6}$ radian range using these laser diodes it is necessary to match the optical path lengths of the interferometer arms to within millimetres which may prove difficult in sensors using long lengths of fibre. These results are in agreement with those of Kersey et al. who have reported a noise floor of $5 \times 10^{-5}$ radians at 1kHz which they attribute to a frequency jitter of 50kHz r.m.s. at 1kHz in a 1Hz bandwidth. Their interferometer optical path length difference was 0.05m.
Fig. 3.14. Interferometer phase noise as a function of optical path length difference at a detection frequency of 1kHz (4 Hz B.W.)
The phase noise can be reduced by stabilising the laser's emission frequency (see 4.5), or by locking the interferometer at quadrature on the zero order fringe \((m=0\) in equation 3.7) using a phase modulator, or by adopting the polarimetric sensor approach in which the path lengths are effectively balanced (the small difference is given by the product of the fibre length and the normalised fibre birefringence, that is, about \(10^{-4}\) relative path length difference).

The photograph in Figure 3.15 shows the interferometer output detected at photodiode A for a 0.04m optical path length difference when the interferometer phase difference is modulated by \(4\pi\) radians. The STL laser diode was then switched off and the resulting d.c. level recorded. This result shows that the fringe visibility is effectively unity for the diode laser source at an interferometer path difference of 0.04m. An identical result was obtained for the Laser Diode Labs laser. Since the coherence length of these lasers has been reported by Epworth\(^{45}\) to be approximately 1m, good fringe visibility at a path length difference of 0.04m is expected. Hence, the sensitivity of the interferometer to small phase shifts is maximised as indicated by the relationship between sensitivity and visibility in equation 3.19.

3.2.5. Conclusions

An homodyne optical fibre interferometer has been held at maximum sensitivity (quadrature) by using a phase compensation scheme that applies a correction signal to a PZT phase modulator. The correction voltage range applied to the cylinder is \(\pm 100V\) and the resulting phase tracking range is \(\pm 200\pi\) radians. The minimum detectable phase shift is limited to \(2.5 \times 10^{-6}\) radians r.m.s. Hz\(^{-\frac{1}{2}}\) at 1kHz by laser amplitude noise in the interferometer powered by the HeNe laser. In the system using the laser diode sources operating at 850nm laser frequency jitter
Fig. 3-15 Fringe visibility of a Mach-Zehnder interferometer powered by an STL laser diode. (Path difference 0.04 m)
is the dominant noise contribution limiting the minimum detectable phase shift to the $10^{-5}$ radian range for optical path length differences of a few tens of millimetres.

The bandwidth of the compensation system depends on the time constant of the integrator and normally ranges from zero to 20Hz. Therefore, only low frequency, temperature and pressure induced phase variations are compensated for and the higher frequency phase signals are observed on the output photodiodes as an intensity modulation. This intensity modulation is linear for phase shifts with amplitudes up to 0.1 radian, hence the measured dynamic range of the demodulation scheme is from $10^{-6}$ to 0.1 radian. The dynamic range can be extended by reducing the integrator time constant in order to increase the compensation circuit bandwidth. In this case both the low frequency drifts and the relatively high frequency signal are compensated for and therefore the signal may be measured by monitoring the voltage fed back to the phase modulator at the signal frequency. The dynamic range in this system is limited by electrical feedback loop gain.

The main disadvantages of active homodyne demodulation are as follows: the amplitude of the phase drift causing the compensator to reach the end of its tracking range, the upper frequency response of the compensator is limited by the resonant frequency of the PZT cylinder (10kHz), the interferometer is not electrically passive because of the correction signals applied to the phase modulator and the modulator is a bulky device limiting the compactness of the sensor.

3.2.6. Homodyne demodulation using laser frequency tuning

In some sensor applications it is desirable that the demodulation scheme be electrically passive which means that no electrically powered elements can be used to control the phase, frequency or the polarisation of the light propagating through the interferometer arms. Shajenko and
Green\textsuperscript{66} have reported a method of keeping a Michelson interferometer in quadrature by shifting the wavelength of a HeNe laser light source, by adjusting its cavity length, to compensate for the environmental phase drift. Although the technique eliminates the electrically controlled phase modulator in the reference arm of the interferometer it still requires an active feedback signal, derived from the interferometer output, to tune the laser frequency and is therefore not a truly passive system.

The tuning of the emission frequency of a semiconductor laser diode by adjusting its drive current has been reported by Kobayashi et al\textsuperscript{67}. The following sections describe a compensation scheme based on the method of Shajenko and Green\textsuperscript{66} but using a laser diode source and a Mach-Zehnder interferometer. A similar approach has also been adopted by Dandridge and Tveten\textsuperscript{68}.

The modulation of the injection current in a laser diode produces both intensity and frequency modulation of the laser output. The intensity modulation results from a change in the operating point of the laser, and the frequency modulation from a change in the optical path length of the laser cavity, so that a change in the injection current, $i$, produces a change in the output frequency, $v$, given by:

$$dv = \left(\frac{dv}{di}\right)_v \ di$$

where $\left(\frac{dv}{di}\right)_v$ is the frequency current characteristic of the laser.

In an unbalanced interferometer the output phase difference is dependent on the optical path length difference, $L$, and the frequency of the light source, $v$, such that increments $dL$, and $dv$ result in a phase shift which can be written as:

$$d\phi = \frac{2\pi}{c} [vdL + Ldv]$$

where $v = c/\lambda$. 

Therefore, the amplitude of the current induced phase shifts, $d\phi_1$, is given by:

$$d\phi_1 = \frac{2\pi L}{c} \frac{dv}{dI}$$

From equations 3.26 and 3.27, it is apparent that the current to a laser diode powering an interferometer may be changed so that a phase shift attributable to an optical path length difference increment, $dL$, can be compensated by a shift in the frequency of the laser, $dv$.

To maintain the quadrature condition continuously, it is necessary to be able to tune the laser over a minimum wavelength interval $(\Delta \lambda)_2\pi$ corresponding to a full-cycle, $2\pi$, variation in $\phi$. In this way it will be possible to compensate for variations in path difference between the two interferometer arms that may amount to many wavelengths of light. The laser frequency tuning range is determined by the interval of current over which the laser remains in a single longitudinal mode. This frequency range is typically several gigahertz for the semiconductor lasers used here as shown in 2.4.4. From equation 3.27, the minimum tuning range is:

$$d(\nu)_{2\pi} = \frac{c}{L}$$

Thus the frequency tuning range required from the laser can be reduced by increasing interferometer optical path length difference, $L$.

However, the optical path length difference is limited by the phase noise produced by the laser frequency jitter as shown in 3.2.4. The phase noise produced by the frequency jitter of the STL diode is $3.2 \times 10^{-5}$ radians r.m.s. Hz$^{-\frac{1}{2}}$ at a detection frequency of 1kHz for an optical path length difference of 0.04m (see 3.2.4). As the tracking range and the phase noise in the interferometer are both proportional to path difference, a compromise must be made. Thus for a 0.04m optical path difference
and a laser tuning range of $10\text{GHz}$ the tracking range is approximately $2.6\pi$ radians, conversely, if a reduced tracking range can be tolerated the minimum detectable phase shift can be decreased by decreasing the optical path length difference.

3.2.7. The compensator system

The system is based on the optical fibre Mach-Zehnder interferometer shown in Figure 3.7. The interferometer was operated with the STL laser diode and a $0.04\text{m}$ optical path length difference. The compensation scheme is analogous to a phase modulator system except that the error signal, $\phi_e$, is provided by tuning the laser frequency instead of changing the optical path length of the reference fibre with a phase modulator. This is illustrated schematically in Figure 3.7 where the feedback signal follows the dotted line in the Figure. The two outputs of the interferometer were electronically subtracted to produce $I_3$ which was then integrated ($I_4$) and used to impose a current upon the laser d.c. drive current. Hence, the schematic diagram of the compensator system is identical to that shown in Figure 3.4 except that the output voltage from the integrator is used to produce an error current. This was achieved by applying the integrator output voltage to the feedback input of the laser drive circuit (See Appendix B(ii)). The integrator time constant was $10^{-3}\text{s}$.

3.2.8. Performance of the compensator

For the successful design of this form of compensation system it is essential that the magnitude of the current induced frequency shift $\frac{dv}{dt}\nu$ is known, within the chosen operating bandwidth for the laser diode.
To measure $\frac{dv}{dt}$, a signal generator was used to superimpose a periodic sawtooth signal on the d.c. laser drive current. The d.c. drive current for the STL diode was 168mA which produces a single frequency optical output of 5mW as shown in Figure 2.21. During the ramp of the sawtooth modulation $\frac{di}{dt}$ is constant and consequently the induced phase shift is linear, that is, $\frac{d\phi}{dt}$ is constant. Figure 3.16 shows the laser drive current and the interferometer output measured at photodiode A for a sawtooth period of 2 milliseconds. The optical frequency modulation of the laser produces the sinusoidal variation in the photodiode output corresponding to the frequency induced phase shift and the intrinsic intensity modulation accounts for the amplitude difference between the fringe maxima during the sawtooth period. For the result in Figure 3.16 the sawtooth amplitude was 5mA and this produced a phase shift of two fringes during the period of the ramp, that is, $4\pi$ radians. The source frequency shift resulting from this current ramp can be calculated from equation 3.27 and is 15GHz. A value for $\frac{di}{dt}$ for the ramp can be found directly from Figure 3.16. Hence, $\frac{dv}{dt}$ and $\frac{di}{dt}$ are known and the diode frequency-current characteristic $\left(\frac{dv}{dt}\right)$ can be calculated from equation 3.27 and is equal to 3GHz mA⁻¹. The frequency current characteristic was determined for a set of frequencies ranging from 0 to 5kHz and was found to be constant up to 4kHz. The results of these measurements are shown in Figure 3.17, the decrease in $\left(\frac{dv}{dt}\right)$ at about 4kHz is probably due to the bandwidth of our detectors, since Jones et al. have reported that the maximum frequency response of the frequency-current characteristic begins to decrease at 100kHz.

The effect of the compensation system on the interferometer output over a time interval of several minutes is shown in Figure 3.18. The steady trace records the output with the compensation circuit locking the
Fig. 3-16 Mach-Zehnder interferometer output as a result of source frequency modulation.
(a) Interferometer output driven through $4\pi$ radians by the modulation
(b) 0 to 5 mA ramp with a period of 2 milliseconds modulating the laser drive current
Fig. 3.17 Variation of $\frac{d\nu}{dI}$ with modulation frequency for an STL laser diode.
Fig. 3-18 Phase Compensation in a Mach-Zehnder interferometer using laser frequency tuning
interferometer at quadrature. The tracking range can be calculated from equation 3.27 using the measured value of \( \frac{\text{dv}}{\text{dt}} \_v \), the interferometer optical path length difference and current interval over which the laser remains in a single longitudinal mode, which for the STL diode is approximately 8mA. The resulting phase tracking range is approximately 6π radians. The discontinuities are due to phase drifts which are sufficiently large to take the system beyond this phase tracking range. The unstable trace, which begins after 330 seconds, is the output without the compensation circuit. The oscillations are due to the phase drift caused by low frequency temperature and pressure variations.

A spectrum of the noise on the interferometer output from d.c. to 1kHz with the compensator circuit in operation is shown in Figure 3.19. The amplitude of the noise measurement is normalised with respect to 1V r.m.s. by the spectrum analyser and is -80dBV at 1kHz (4HZ\(^{-1/2}\)). The corresponding minimum detectable phase has been calculated from this measurement using equation 3.11 and is \( 2.5 \times 10^{-5} \) radians r.m.s. Hz\(^{-1/2}\) at 1kHz. The measured interferometer noise floor agrees well with the value of \( 3.2 \times 10^{-5} \) radians r.m.s. calculated directly from equation 3.16 which is an expression for the phase noise. The STL laser frequency jitter of 38kHz r.m.s. Hz\(^{-1/2}\) and the optical path length difference of 0.04m were used in this calculation.

3.2.9. Conclusions

It has been shown that an unbalanced interferometric sensor may be kept in quadrature by tuning the emission frequency of a laser diode source by changing the drive current of the laser, thus eliminating the need for an electrically controlled phase modulator in the reference arm. The maximum laser frequency shift over which the STL diode laser
Fig. 3-19 Noise spectrum of the output of a Mach-Zehnder interferometer in quadrature
remains in a single longitudinal mode is 24GHz. This frequency shift corresponds to a phase tracking range of $6\pi$ radians for an interferometer with an optical path length difference of 0.04m. The response of the interferometer to phase shifts was found to be linear over 5 orders of magnitude with a minimum detectable phase shift of $2.5\times10^{-5}$ radians r.m.s Hz$^{-1}$ at 1kHz. However, this homodyne demodulation scheme requires a non-zero optical path length difference to operate and therefore the minimum detectable phase shift is limited by phase noise. As the tracking range and the phase noise in the interferometer are both proportional to the path difference a compromise is necessary. With laser frequency stabilisation techniques like those described in 4.5 it may be possible to extend the tracking range by increasing the optical path length difference while still retaining a minimum detectable phase shift in the $10^{-5}$ radian regime.

The demodulation circuit can be operated with a large or a small compensation bandwidth by selecting the integrator time constant accordingly. The main disadvantage of the demodulation scheme is that in order to detect small phase shifts the tracking range must be restricted to a few $\pi$ radians because of the laser frequency jitter.

3.3. Review of homodyne demodulation techniques

Two techniques to solve the signal fading problem caused by differential phase drifts between the arms of an optical fibre Mach-Zehnder interferometer have been demonstrated. The first approach maintained the interferometric output signal in quadrature by feeding back an error signal to a phase modulator (piezoelectrically stretched fibre coil) in the reference arm of the interferometer. However, the piezoelectric phase modulator is bulky and electrical control must be applied within the interferometer for the modulator to operate. In the
second approach a Mach-Zehnder interferometer has been maintained at quadrature by deriving an error signal from the interferometer output and using it to tune the emission frequency of a laser diode. Hence, the phase difference in an unbalanced interferometer is controlled without a piezoelectric phase modulator in the interferometer reference arm. Unfortunately, the phase tracking range is limited by the phase noise caused by the laser diode frequency jitter, for example, to achieve a minimum detectable phase shift of $10^{-5}$ radians the tracking range is only a few $\pi$ radians and this is often exceeded by the environmentally induced phase drifts.

Recently Sheem et al\textsuperscript{70} have demonstrated that the signal fading problem in a homodyne Mach-Zehnder interferometer can be solved using a truly passive demodulation scheme. The optical demodulator has the advantage that unlike the homodyne demodulation schemes described here signal recovery may be performed without active feedback from the interferometer output to a phase modulator or the laser source, thus avoiding problems associated with the reset of a phase tracker. The demodulation scheme has a large dynamic range, is electrically passive and does not require a non-zero interferometric optical path difference. The basic principle of Sheem et al's\textsuperscript{70} optical demodulator is to derive two optical signals from the interferometer which are proportional to $\sin \phi$ and $\cos \phi$ (i.e. $\frac{\pi}{2}$ out of phase), where $\phi$ is composed of a low frequency drift term and a higher frequency signal term. These optical outputs have the following form:

$$I_1 = I_0 \sin \phi$$
$$I_2 = I_0 \cos \phi$$

where the d.c. component from each photodetector has been ignored (a.c. coupling). As $\sin \phi$ and $\cos \phi$ can never be zero simultaneously at the same value of $\phi$, the value of $\phi$ may be recovered by performing the
operations of differentiation and multiplication such that:

\[ I_1 \frac{dI_2}{dt} - I_2 \frac{dI_1}{dt} = \frac{d\phi}{dt} \quad 3.30 \]

this output may then be integrated to give a signal which is directly proportional to the phase shift of the interferometer. It follows from the functional operations in equation 3.30, which can be implemented with analog circuits, that both the differential phase drift and the signal can be detected continuously so that the system has a large dynamic range. Finally, the high frequency signal is filtered from the low frequency drift. The main difficulty of the optical demodulator is that two output signals which are \( \frac{\pi}{2} \) out of phase must be produced. One technique proposed by Sheem et al.\(^70\) is the use of two source frequencies to obtain the \( \frac{\pi}{2} \) phase shift. It has been shown in 3.2.6 that the output of an interferometer is frequency dependent. Thus if the interferometer input light is composed of or equivalently switched between two frequencies \( \nu_1 \) and \( \nu_2 \), such that:

\[ |\phi(\nu_1) - \phi(\nu_2)| = \frac{\pi}{2} \quad 3.31 \]

then from equations 3.5 and 3.31 the interferometer outputs are:

\[ I_1 = I_0(1+\cos\phi) \]
\[ I_2 = I_0(1+\sin\phi) \quad 3.32 \]

resulting in equation 3.29 when the d.c. light levels are ignored.

Another method for producing the \( \frac{\pi}{2} \) phase shift using a \( 3 \times 3 \) optical fibre directional coupler instead of a standard \( 2 \times 2 \) as the interferometer arm combiner has been demonstrated by Koo et al.\(^71\). A schematic of the interferometer output configuration is shown in Figure 3.20. The three interferometer output signals are given by:
Fig. 3-20 Schematic of single mode fibre interferometer using a 3x3 directional coupler as the second beam splitter.
\[ I_1 = -2B_2(1+\cos\phi) \]
\[ I_2 = B_1+B_2\cos\phi+B_3\sin\phi \]
\[ I_3 = B_1+B_2\cos\phi-B_3\sin\phi \]

where \( B_i (i=1,2,3) \) are constants to be determined by coupling coefficients between the fibre waveguides within the coupler interaction length \( L \).

An output signal proportional to \( \sin\phi \) may be obtained by subtracting \( I_2 \) and \( I_3 \) and is given by:

\[ I_2-I_3 = 2B_3\sin\phi \]

Koo et al. have reported a minimum detectable field of \( 10^{-6} \) radians at 1kHz in a 1Hz bandwidth using a 3 x 3 coupler to perform the optical demodulation. It is important to note that the electronic processing is complex compared with the feedback systems described in this Chapter, requiring an analog circuit realisation of \( \sin^2\phi+\cos^2\phi=1 \) which leads to distortion of the detected signal limiting the minimum detectable phase shift. Yet each demodulation scheme has advantages and drawbacks which must be evaluated with reference to the particular application.
CHAPTER 4

OPTICAL FIBRE FABRY-PEROT INTERFEROMETERS

The most popular interferometer for the detection of phase changes in optical fibre sensors is the two arm Mach-Zehnder interferometer, as discussed in Chapter 3. Jackson et al\textsuperscript{24} have employed this interferometer to measure phase changes as small as $10^{-6}$ radians. These interferometers are relatively complicated instruments that require beam splitting and recombining elements. Recently, all-fibre Fabry-Perot interferometers have been demonstrated which utilise multiple beam interference in a single fibre by several authors\textsuperscript{16,72,73,74}. In a Fabry-Perot interferometer the transmission as a function of phase is a series of sharp peaks whereas the Mach-Zehnder transmission varies as a cosine function. A small phase change in the interferometer will therefore provide a larger change in transmitted light intensity in the Fabry-Perot than in the Mach-Zehnder interferometer. Apart from its potential improved phase sensitivity, a single fibre interferometer is also an attractive proposition because it eliminates the need for the beam splitting and recombining elements. An investigation into the operation of the optical fibre Fabry-Perot interferometer as a sensor and as an external cavity in a laser frequency stabilisation scheme is presented in this Chapter.

4.1. The Fabry-Perot interferometer

A conventional Fabry-Perot interferometer in its most simple form consists of two plane, parallel, highly reflecting surfaces separated by a distance, $z$, with a medium of refractive index, $n$. The general principles of the interferometer are shown in Figure 4.1. Light entering the interferometer is repeatedly reflected from the end mirrors, with some of the light being transmitted through the mirrors at every pass. The light propagating within the cavity, which is coherent over at least
Refractive index $n$

Reflected signal

Input light, $\lambda$

Mirror

Length $L$

Transmitted signal

Fabry-Perot cavity

Transmitted intensity

$T_t(n+1)$

Phase change (radians)

$\Delta \theta$

Fig. 4-1. Fabry-Perot interferometer
twice the cavity length, builds up a standing wave pattern determined by the optical path length, \( n z \), between the mirrors. The cavity resonates when there is an integral number of half wavelengths between the mirrors and at each resonance the light intensity transmitted through the interferometer is a maximum. These transmission maxima are referred to as Fabry-Perot interference fringes. The optical fibre Fabry-Perot interferometer developed here, which will be discussed in detail in 4.2, consists simply of a single monomode fibre with silver coated end faces. The Fabry-Perot interferometer transmission equations are well known for plane waves in non-guided resonators\(^{14}\) and they can be used to interpret the optical fibre interferometer results. Although it is important to realise that this theory is not rigorously applicable to a guided wave resonator. In particular, losses produced at every reflection by coupling to non-guided modes are not taken into account. The main results for the Fabry-Perot theory for plane waves and monochromatic light are summarised here.

The light intensity transmitted through the interferometer is expressed as a function of the phase difference, \( \phi \), between two successively transmitted waves. This phase difference is determined by the optical path length between the mirrors. Hence the Fabry-Perot transmission equation is given by:

\[
I = I_1 \left( \frac{T}{1 - R} \right)^2 \left[ 1 + \frac{4R}{(1 - R)^2} \sin^2 \frac{\phi}{2} \right]^{-1}
\]

where \( I \) is the transmitted light intensity, \( I_1 \) is the incident light intensity, \( T \) is the transmittance of the mirrors, and \( R \) is the reflectivity of the mirrors. This equation assumes that the cavity loss is zero. The maximum light intensity, \( I_{\text{max}} \), which can be transmitted through the interferometer is given by:
In this equation the incident light intensity is normalised to unity. The resonator finesse, $F$, is defined as the ratio of the separation of adjacent fringes to their width at half maximum intensity and depends on the reflectivity of the mirrors, and is given by:

$$F = \frac{\pi \sqrt{R}}{1 - R}$$

The substitutions of equations 4.2 and 4.3 into equation 4.1 means that the Fabry-Perot transmission equation becomes:

$$I = I_{\text{max}} \left[ 1 + \left( \frac{2F}{\pi} \right)^2 \sin^2 \frac{\phi}{2} \right]^{-1}$$

In a practical interferometer a cavity loss, $A$, must also be taken into account and is related to the reflectivity and the transmittance by:

$$T + A + R = 1$$

The metal film mirrors are mainly responsible for the loss because the cavity transmission medium can be chosen to be optically transparent at the wavelength of the interferometer light source. In a Fabry-Perot cavity with loss $A$, the maximum transmission through the cavity is given by:

$$I_{\text{max}} = \left[ 1 - \frac{A}{1 - R} \right]^2$$

where $I_{\text{i}} = 1$.

The phase difference, $\phi$, between successively transmitted waves is given by:

$$\phi = \frac{4\pi n z}{\lambda}$$

Peaks in Fabry-Perot transmission occur at specific values of phase...
difference \( \phi = 2\pi m \), where \( m \) is an integer. The spacing between the peaks in wavelength is given by:

\[
\Delta \lambda = \frac{\lambda^2}{2nz}
\]

or in terms of frequency:

\[
\Delta \nu = \frac{c}{2nz}
\]

and is called the free-spectral of the Fabry-Perot cavity.

4.1.1. Sensitivity of the interferometer to phase changes

Small increments in \( \lambda \), \( n \) and \( z \) result in a phase change \( d\phi \) given by:

\[
d\phi = \frac{4\pi}{\lambda} (ndz + zdn) - \frac{4\pi nz}{\lambda^2} d\lambda
\]

This equation is the derivative of equation 4.7. The sensitivity of a Fabry-Perot interferometer to such phase changes is proportional to the derivative of equation 4.4, and is given by:

\[
\frac{dI}{d\phi} = -I_{\text{max}} \frac{2F^2}{\pi^2} \sin \phi F(\phi)^{-2}
\]

where \( F(\phi) = [1 + \left(\frac{2F}{\pi}\right)^2 \sin^2 \frac{\phi}{2}] \).

The maximum sensitivity to phase changes is achieved by operating in the transmission region where \( \frac{dI}{d\phi} \) is a maximum. The phase corresponding to this transmission value is derived in Appendix D(i) and is given by:

\[
\phi_o = \cos^{-1} \left[ \frac{1}{2} \left( 9 + \frac{\pi^2}{F^2} + \frac{4}{4F^4} \right) - \frac{\pi^2}{4F^2} - \frac{1}{2} \right]
\]

If \( F > 1 \), \( \cos \phi_o \) is approximately given by:

\[
\cos \phi_o = 1 - \frac{\pi^2}{6F^2}
\]

In this case the sensitivity takes on its maximum value of:
\[
\frac{dI}{d\phi}\bigg|_{\phi_o} \sim I_{\text{max}} 0.2F \quad 4.14
\]

at \( \phi_o \sim \pm \frac{\pi}{\sqrt{3}F} \).

It is also useful to define the sensitivity in terms of phase delay per pass, i.e. \( \phi = \frac{2\pi n z}{\lambda} \). In this case the interferometer transmission is given by:

\[
I = I_{\text{max}} \left[ 1 + \frac{4R}{(1 - R)^2} \sin^2 \phi \right]^{-1} \quad 4.15
\]

and the sensitivity becomes:

\[
\frac{dI}{d\phi}\bigg|_{\phi_o} \sim I_{\text{max}} 0.4F \quad 4.16
\]

The majority of the results are interpreted using equation 4.14, but some results are discussed using this alternative sensitivity definition. The phase \( \phi_o \) corresponds to a cavity transmission value, according to equation 4.4, of \( 0.75 I_{\text{max}} \). Equation 4.14 demonstrates that the sensitivity of the interferometer to phase changes is directly proportional to the cavity finesse. This sensitivity can be much larger than that of a dual arm Mach-Zehnder interferometer which takes its maximum value of \( 0.5 I_{\text{max}} \) at quadrature as explained in 3.1.3.

The sensitivity of the phase to temperature in an optical fibre Fabry-Perot interferometer is given by:

\[
\frac{d\phi}{dT} = \frac{4\pi z}{\lambda} \left[ n \frac{1}{z} \frac{dz}{dT} + \frac{dn}{dT} \right]
\]

\[
\frac{d\phi}{dT} = \frac{4\pi z}{\lambda} \left[ n\alpha + \beta \right] \quad 4.17
\]

where \( \frac{1}{z} \frac{dz}{dT} = \alpha \) is the thermal expansion coefficient of the fibre used for the Fabry-Perot interferometer and \( \frac{dn}{dT} = \beta \) is the temperature coefficient of the refractive index.
4.2. The optical fibre Fabry-Perot interferometer

The fibre Fabry-Perot interferometers consisted of lengths of optical fibre with semi-silvered end faces, which acted as mirrors forming a cavity. The fibre Fabry-Perot cavities were made, in lengths ranging from 0.1m to 10m, from ITT fibre (80 micrometre cladding diameter, 4.5 micrometre core diameter, 0.12 numerical aperture) and from STL fibre (125 micrometre, 7 micrometre core, 0.12 numerical aperture). The ITT fibre supported a single mode at a wavelength of 633 nm guiding the $LP_{01}$ mode only. The STL fibre guided two modes, the $LP_{01}$ and $LP_{11}$ at a wavelength of 633 nm. The fibre end faces were cleaved and silver was deposited on the ends by means of electron beam evaporation. The system was calibrated for silver thickness versus evaporation time. A glass slide was coated together with the fibre end faces. The reflectivity and transmission of the glass slides were measured as a function of wavelength using a spectrometer. Figure 4.2 shows a slide transmission spectrum. A transmission of 14% was obtained at 633 nm for a silver coating thickness of 30 nm. The coating thickness on the slides were measured independently with a Tally step and were in good agreement with the vacuum system calibration. The result of a silver coated slide thickness measurement is shown in Figure 4.3. The thickness of 30 nm corresponded to a reflectivity of 85% at 633 nm. These techniques were used to obtain reflectivities in the range 65 to 85% on a number of fibre Fabry-Perot interferometers.

4.2.1. Performance of the fibre interferometer

The experimental arrangement used to investigate the properties of the fibre Fabry-Perot interferometers is shown in Figure 4.4. The light source was a Tropel 200 single frequency HeNe laser operating at 633 nm.
Fig. 4.2. Silver film transmission spectrum

Coating thickness \( \sim 300 \text{Å} \)

Fig. 4.3. Glass slide silver film thickness

Reflectivity \( \sim 85 \) percent at 633nm
Fig. 4.4. Optical fibre Fabry Perot interferometer
The output power was nominally 1mW. The laser light was launched into the Fabry-Perot device via a Melles Griot lens of focal length 7 mm. In order to avoid optical coupling between the Fabry-Perot back reflected light and the laser cavity, the incident light can be passed through a polariser and a quarterwave-plate in the conventional manner. In the present work the fibre ends were cleaved at an oblique angle as often happens, and therefore back reflections into the laser cavity were avoided. This method provided adequate launching efficiency but prevented the reflected light from interfering with the light in the laser cavity and producing mode hopping. Part of the fibre Fabry-Perot was wound onto PZT phase modulator in an identical manner to that described in 3.2.2 so that the cavity length could be modulated.

Figures 4.5 and 4.6 show the light intensity transmitted through four different fibre Fabry-Perots. The horizontal axis represents the phase change $\phi$ defined in equation 4.10. In the present work the third term was negligible since the linewidth of the HeNe laser was approximately 1KHz and the centre frequency stable. The transmission functions were therefore considered as those for perfectly monochromatic light. From these functions the finesse can be determined. The zero offset shown in Figures 4.5 and 4.6 shows that there was a d.c. light level present. This light level represents the minimum Fabry-Perot transmission ($I_{\min}$) and was due to the incomplete stripping of the cladding modes. In order to allow for this d.c. light in Fabry-Perot transmission analysis an equivalent experimental finesse, $F_e$, was defined:

$$F_e = \frac{2\pi}{\Delta \phi}$$

where $\Delta \phi$ is the full width of a transmission fringe, in radians when the transmission $I = 0.5 \left[ I_{\max} - I_{\min} \right]$ as shown in Figure 4.7. Note in the conventional definition of finesse $\Delta \phi$ is the fringe width at
(i) Temperature induced phase shift
Length = 1m. Finesse = 15

(ii) Approximately 2π p-p phase modulation at 50Hz
Length = 0.5m. Finesse = 6

Fig.4-5. Observed Fabry-Perot transmission functions
(i) Approximately $4\pi$ p-p phase modulation at 25Hz
Length = 0.5 m, Finesse = 2

(ii) $2\pi$ p-p phase modulation at 35Hz
Length = 10 m, Finesse = 6

Fig. 4-6 Observed Fabry-Perot transmission functions
Fig. 47. Definition of experimental finesse
half maximum transmission \( (0.5 I_{\text{max}}) \). The function \( F(\phi) \), defined in equation 4.11 varies between unity and \( \frac{1}{1 + \left( \frac{2F}{\pi} \right)^2} \) and leads to a relationship for the experimental finesse (see Appendix D(ii)) given by:

\[
F_e = \frac{\pi}{2} \left[ \sin^{-1} \left( \frac{1}{\sqrt{(\frac{2F}{\pi})^2 + 2}} \right) \right]^{-1}
\]

4.19

The calculated fineses were obtained from equation 4.19 using the measured glass slide reflectivity and are listed in Table 4.1 together with the corresponding observed fineses of the Fabry-Perot devices. The calculated values were in general higher than the observed ones. The likely reasons for the discrepancy were that the end faces of the cavity mirrors were tilted to the fibre axis and thus the internally reflected light on the end face of the fibre could not perfectly couple with the fibre mode. Yoshino et al have shown that the presence of tilt on the reflection surfaces reduces the effective reflectivity and consequently the Fabry-Perot interferometer finesse. In addition cavity losses arise due to absorption in the evaporated silver films and coupling to non guided modes. Figure 4.5(i) shows a transmission function for a Fabry-Perot interferometer of length 1m and mirror reflectivity 85%. The maximum transmitted intensity was about 27μW (1.5V on detector) which corresponded to a measured value of \( I_{\text{max}} \approx 30\% \). Hence the cavity losses were estimated, using equation 4.6, to be \( \Delta \nu \approx 7\% \).

<table>
<thead>
<tr>
<th>FP LENGTH</th>
<th>MIRROR REFLECTIVITY</th>
<th>REFLECTIVE FILM THICKNESS (nm)</th>
<th>FINESSE</th>
<th>FREE SPECTRAL RANGE ( \Delta \nu ) (MHz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>z(m)</td>
<td>R(%)</td>
<td></td>
<td>( F_e ) observed</td>
<td>( F_e ) calculated</td>
</tr>
<tr>
<td>0.5</td>
<td>70</td>
<td>12</td>
<td>2</td>
<td>9</td>
</tr>
<tr>
<td>1</td>
<td>85</td>
<td>30</td>
<td>15</td>
<td>19</td>
</tr>
<tr>
<td>5</td>
<td>82</td>
<td>23</td>
<td>6</td>
<td>16</td>
</tr>
<tr>
<td>10</td>
<td>80</td>
<td>20</td>
<td>6</td>
<td>14</td>
</tr>
</tbody>
</table>

**TABLE 4.1.** FIBRE FABRY-PEROT INTERFEROMETER CHARACTERISTICS
4.2.2. Measured sensitivity of the interferometer

The highest measured finesse value of $F = 15$, corresponded to a maximum transmission of $I_{\text{max}} \approx 30\%$. At $\phi_0$ where the Fabry-Perot was most sensitive to small phase changes (i.e. much less than 1 fringe) the predicted sensitivity, according to equation 4.16 is $\left| \frac{dI}{d\phi} \right|_{\phi_0} = 6 I_{\text{max}}$. Petuchowski et al. have obtained similar results, they obtained a value of $\left| \frac{dI}{d\phi} \right|_{\phi_0} = 7.2 I_{\text{max}}$ with a fibre Fabry-Perot of finesse 18.

For comparison a two arm Mach-Zehnder interferometer at quadrature has a maximum sensitivity $\left| \frac{dI}{d\phi} \right| = 0.5 I_{\text{max}}$. The recent development of low loss fibre 3dB couplers has brought $I_{\text{max}}$ in the Mach-Zehnder close to unity. In an ideal lossless Mach-Zehnder interferometer the maximum sensitivity is 0.5. Fabry-Perot interferometers in the present work and those described by Petuchowski et al. have a reduced sensitivity improvement over the ideal Mach-Zehnder interferometer due to losses. For example, the Fabry-Perot of finesse 15 with $I_{\text{max}} = 0.3$ had a sensitivity $\left| \frac{dI}{d\phi} \right|_{\phi_0} = 1.8$ which is about a factor of 3 better than an ideal Mach-Zehnder. Petuchowski et al. reported a sensitivity of $\left| \frac{dI}{d\phi} \right|_{\phi_0} = 0.54$ which is marginally superior to the ideal Mach-Zehnder interferometer for a finesse of 18 and $I_{\text{max}} \approx 0.075$. They have used external silver mirrors butt-coupled to the fibre ends to form the Fabry-Perot interferometer and attributed their poor value of $I_{\text{max}}$ to coupling losses at the cavity ends. It is evident that the silver coated fibre ends in the present work have produced an improvement in sensitivity by reducing the Fabry-Perot losses. A further improvement in $I_{\text{max}}$ should be possible by using low loss multilayer dielectric coatings for the Fabry-Perot mirrors. Yoshino et al. have coated their fibre ends with multilayer dielectric films by vacuum evaporation and obtained a sensitivity $\left| \frac{dI}{d\phi} \right|_{\phi_0} = I_{\text{max}} 5.2$. 

\[ -121 - \]
for a Fabry-Perot of finesse 13, but they do not give a measured value of $I_{\text{max}}$.

4.2.3. The dual mode interferometer

The small peaks observed in Figures 4.5(i), 4.5(ii) and 4.6(ii) in the minimum transmission region of the Fabry-Perot output function were caused by the excitation of the $LP_{11}$ mode in addition to the fundamental. These particular fibre Fabry-Perot cavities were fabricated from the STL fibre which supported the $LP_{01}$ and $LP_{11}$ modes at a wavelength of 633nm. Each mode has its appropriate phase velocity and hence the condition for constructive interference is different for the two modes. Hence the Fabry-Perot transmission function was a superposition of the transmission functions of the two different fibre modes. The ratio of the intensities of the light in the two modes were controlled by adjusting the coupling of the light into the fibre FP. Figure 4.6(ii) shows the output transmission of a two mode fibre Fabry-Perot when the optical path was modulated by applying a sinusoidal voltage to the PZT cylinder producing a $2\pi$ radian phase shift. The modulation clearly demonstrates that the order of the peaks depends on the sign of the optical path length change.

4.2.4. Summary of the interferometer characteristics

Fibre Fabry-Perot interferometers utilising the completely guiding dielectric optical fibre structure with integral mirrors have been demonstrated. The results have been interpreted using plane wave theory applied to conventional Fabry-Perot cavities. The coupling of some light into non-guided modes was not taken into account by this theory. These additional losses may contribute to the discrepancies between the measured and the calculated finesses.
Fibre Fabry-Perot interferometers with finesses ranging from 2 to 15 have been obtained. A finesse of 15 corresponds to a fibre end reflectivity \( R \), of 85% which is achieved with a 50nm silver coating. The transmitted light intensity as a function of cavity length is easily scanned using a PZT phase modulator. The almost unlimited geometric versatility of the fibre Fabry-Perot cavity has excellent prospects for sensor coils. The Fabry-Perots have been constructed in lengths ranging from 0.01\( \text{m} \) to 10m. Fibre supporting the \( \text{LP}_{01} \) mode only and fibre supporting both the \( \text{LP}_{01} \) and \( \text{LP}_{11} \) modes has been used to fabricate the Fabry-Perots.

In the Fabry-Perot interferometer described here the sensitivity to phase changes has been discussed as a function of the transmitted light intensity. The reflected light intensity from the near and far fibre ends of the interferometer produces a complementary intensity variation as a function of phase. Therefore, the reflected light intensity can also be used to determine changes in phase. This configuration has the advantage that the light source and detector electronics are used near the same fibre end and is consequently suitable for remote sensing applications. Kersey et al\(^74\) have reported using the reflected light from the fibre Fabry-Perot to measure phase changes. Their interferometer has a low finesse because the reflection coefficients of the bare fibre ends (0.04%) formed the resonator mirrors.

4.3. A high sensitivity fibre Fabry-Perot sensor

For maximum sensitivity, the fibre Fabry-Perot interferometer was operated at \( \phi_0 \) as defined in equation 4.12 using active homodyne demodulation. The interferometer and the demodulation system are shown in Figure 4.8. The length of the interferometer was 10m and the mirror reflectivity was 80%. Eight metres of the cavity were wrapped helically onto a PZT cylinder (50 turns of fibre) to form a phase modulator (the
Fig. 4.8. Fibre Fabry-Perot sensing interferometer
PZT details are given in 3.2.2). This modulator was used to compensate for changes in the optical length of the cavity in a similar way to the system described in 3.2.1. The Fabry-Perot output signal was fed via a photodetector into one input of a differential amplifier. The other input was set at a reference voltage equivalent to 0.75 I_{max} transmission which was obtained by deviating 5% of the Fabry-Perot input light onto a photodetector using a glass slide. The output from the differential amplifier was proportional to the difference between the Fabry-Perot transmission and the reference signal. This output was amplified and used to provide an error voltage V_e which was used to control the phase modulator.

The sensitivity of the phase difference between the transmitted waves of the Fabry-Perot interferometer to the modulator was 0.19 radians V^{-1} turn^{-1}. This sensitivity was a factor of two greater than when the modulator was used in one arm of a Mach-Zehnder interferometer in 3.2 because of the double pass made through the modulator by one of the interfering waves with respect to the other in the Fabry-Perot configuration. For an error voltage range of ± 100V the fifty fibre-turn modulator provided a phase tracking range of ±302 radians. The frequency response of the feedback loop was designed (i.e. amplified and integrated) so that the system would compensate for ambient pressure and temperature changes below 20Hz but not interfere with the signal frequencies which were greater than a few 100Hz. The details of the electronics were identical to those for the Mach-Zehnder compensation circuit shown in Appendix B(iii).

The higher frequency signals were simulated by applying a sinusoidal voltage to the PZT phase modulator. This modulated the optical path length and hence the light intensity transmitted by the interferometer.
This output signal was detected on a silicon PIN photodiode and displayed on an oscilloscope and a spectrum analyser in order to evaluate the interferometer performance.

4.3.1. Performance of the sensor

In order to determine the finesse of the interferometer a sinewave was applied to the phase modulator. This caused the Fabry-Perot interferometer output to go through four fringes per cycle of the modulation waveform, as shown in Figure 4.6(ii). It can be seen from the figure that the experimental finesse is approximately 6.

Figure 4.9 demonstrates the effect of closing the control loop around the interferometer with no phase modulation applied. The interferometer output was maintained at 0.75 of its maximum transmission for 255 seconds after which the feedback was switched off. Figure 4.10 shows the spectrum of the stabilised Fabry-Perot output when the phase is modulated with a sinewave of $12 \times 10^{-3}$ radian p-p at 480Hz. The resulting intensity modulation of the output light produced a -40dBv signal at 480Hz on the spectrum analyser. If this signal is substituted into equation 4.14 a value for the phase perturbation $d\phi$ at 480Hz of $8.3 \times 10^{-3}$ radians r.m.s. is obtained where the interferometer finesse is 6 and the output light intensity at $I_{\text{max}}$ produced a detector signal of 1v.

The minimum detectable phase for the interferometer was determined by making measurements at a signal to noise ratio of unity for the $12 \times 10^{-3}$ radians p-p signal at frequencies from 100 to 1 kHz at 100 Hz intervals. Figure 4.11 shows the results of these measurements, where the spectrum analyser output (dBv) has been converted into phase shift (radians) using equation 4.14. The interferometer minimum detectable phase has a 1/f dependence and has a value of $5 \times 10^{-6}$ radians r.m.s. Hz$^{-1}$ at 1kHz for the 10m Fabry-Perot. The shot noise limit of the detector, $dV_s$, has been
Fig. 4.9. Fabry-Perot interferometer output as a function of time.
(i) Represents a phase modulation of $12 \times 10^{-3}$ radians at 480 Hz

(ii) Represents the 2nd harmonic component of the 480 Hz phase modulation

Fig. 4-10. Fabry-Perot interferometer output as a function of frequency
Fig. 4.11. Fabry-Perot interferometer minimum detectable phase shift as a function of frequency.
defined in equation 3.12 and is $1.6 \times 10^{-7} \text{V.Hz}^{-\frac{1}{2}}$. The noise limits the minimum detectable phase, $d\phi_{\text{g}}$, to $1.3 \times 10^{-7}$ radians r.m.s. Hz$^{-\frac{1}{2}}$ (calculated by substituting the shot noise voltage into the Fabry-Perot sensitivity equation 4.14 where $F = 6$ and $I_{\text{max}} = 1\text{v}$). These results show that the Fabry-Perot minimum detectable phase was a factor of 38 above the shot noise limit at a detection frequency of 1kHz. The discrepancy between the measured and calculated minimum detectable phase was attributed to the sensitivity of the interferometer to laser intensity noise and ambient acoustic noise.

4.3.2. Conclusions

In conclusion the sharp transmission characteristics of the Fabry-Perot interferometer, i.e. its high finesse, have been used to recover small periodic phase signals. Active homodyne demodulation has been used to maintain the interferometer at its maximum sensitivity point $\phi_0$. A minimum detectable phase of $5 \times 10^{-6}$ radians.Hz$^{-\frac{1}{2}}$ has been obtained at a detection frequency of 1kHz using a 10m Fabry-Perot with a finesse of 6.

4.4. A fibre Fabry-Perot temperature sensor

In some sensor applications where the environment is very hostile there is a requirement for an electrically passive sensor, and extreme sensitivity is not necessary. Measurements have been taken with the Fabry-Perot interferometers described here when the signals perturbing the fibre cavity produce length changes of several tens or even hundreds of wavelengths. The detection was simply performed by counting the number of fringes produced at the Fabry-Perot output photodetector. This counting technique was used to detect temperature over
a range of 25°C to 60°C. The interferometer was made sensitive to
temperature by attaching 0.03m of the cavity, stripped to the silica
cladling, to a small electric heater with heat sink compound. The
fibre temperature was monitored with a copper/constantan thermocouple
which was in contact with the fibre.

4.4.1. Performance of the sensor

The temperature induced phase change was measured with a Fabry-Perot of length 1m and mirror reflectivity of
70% which propagated the fundamental mode only (LP_{01}). The output
transmission from the Fabry-Perot as a function of temperature change
is shown in Figure 4.12(i). The interval between two adjacent interference
fringes corresponded to a phase change of 2\pi and a temperature change
of 1.3K. Figure 4.12(ii) shows the Fabry-Perot fringe count, N, as
a function of temperature. From the gradient of the curve the sensitivity
of phase to temperature \( \frac{d\phi}{dT} \) was deduced. This was approximately 5 radians
K\(^{-1}\) for a sensitive length of 0.03m. This gives a projected sensitivity
for 1m of temperature sensitive fibre of 167 radians K\(^{-1}\)m\(^{-1}\). For silica
glass, the temperature coefficient of the refractive index, \( \beta \), and the
thermal expansion, \( \alpha \), are known to be \(-10^{-5} K^{-1}\) and 5 x 10\(^{-7}\) K\(^{-1}\)
respectively. This leads to a calculated temperature sensitivity (See Eq.4.17)
of 184 radians K\(^{-1}\)m\(^{-1}\) (29 fringes K\(^{-1}\)m\(^{-1}\)) which was comparable to the
experimental value. Yoshimo et al\(^{16}\) have reported a similar result of
36 fringes K\(^{-1}\)m\(^{-1}\) for bared silica fibre with an all-fibre Fabry-Perot.
In a Michelson interferometer, Corke et al\(^{75}\) have improved the sensitivity
of silica fibre to temperature by bonding it to a metal with a large
coefficient of thermal expansion. The sensitivity achieved here could
be improved by using a similar technique.
(i) Fabry-Perot output as a function of temperature interferometer
Length = 1m. Finesse = 4

(ii) Fabry-Perot fringe count N, as a function of temperature

Fig. 4.12 Fabry-Perot interferometer temperature sensor
In the fibre Fabry-Perot interferometer temperature sensor described here, only the magnitude of the temperature change was retained. The Fabry-Perot transmission consisted of a series of identical fringes spaced evenly in phase for both decreasing and increasing optical path length. Yoshimo et al have demonstrated a technique for identifying the direction of optical path length change. Their HeNe laser output contained two stable longitudinal cavity modes with different intensities. The constructive interference condition is slightly different for the two modes in the Fabry-Perot interferometer, and the Fabry-Perot interferometer therefore has two 'paired' peaks of different intensity in one period of its transmission function. The order of appearance of these peaks in the transmission function depends on the sign of the optical length change. They illustrate the effect by showing chart recordings of Fabry-Perot interferometer output as a function of temperature (heating followed by cooling). For the temperature sensor described here a new technique for determining the direction of the temperature change is proposed. It is shown in Figure 4.6(ii) that the sign of the optical path length change can be discriminated by a fibre Fabry-Perot interferometer propagating in the $L_{p0}$ and $L_{p1}$ modes. This technique is explained in detail in Section 4.2.3. Such a Fabry-Perot interferometer can be used to determine the sign of the temperature change.

4.4.2. Conclusions

In conclusion the number of fringes seen by the detector has been counted as a function of cavity temperature. The results have given a sensitivity of 27 fringes K$^{-1}$m$^{-1}$ which can be improved upon by using a metal clad optical fibre. Only the magnitude of the temperature change is determined from the basic Fabry-Perot interferometer because its transmission function is symmetrical. A technique has been proposed and
demonstrated to determine the sign of the temperature change by producing an asymmetric transmission function using a fibre which supports the \( \text{LP}_{01} \) and \( \text{LP}_{11} \) modes and is the subject of a patent application by Jones and Pratt. A copy of this application can be found at the end of this thesis.

4.5. The use of optical fibre Fabry-Perot cavities for the reduction of diode laser frequency jitter

It has been shown by Dandridge et al.\(^{62,63,64}\) and in 3.2.4 of the present work that the phase noise in interferometers caused by source frequency jitter increases in direct proportion to the interferometer optical path difference. The large path differences intrinsic in Fabry-Perot sensors make them inherently sensitive to source frequency jitter. For example, in a Fabry-Perot interferometer with a sensing length of 10m, a source stability of 2Hz would be required for a sensitivity of \( 10^{-6} \) radians (see equation 4.10). Consequently a stable HeNe laser has been used to make the measurements reported in this Chapter.

Frequency jitter is particularly relevant with diode lasers as discussed in 2.4. With the single mode STL diode laser used in this work (frequency jitter 38kHz at 1kHz - see 3.2.4) it would be necessary to have an optical path length difference of less than a few millimetres to obtain a minimum detectable phase shift of \( 10^{-6} \) radians. Clearly, the diode laser emission frequency needs to be stabilised in order to reduce the effect of the phase noise contribution to the Fabry-Perot and the unbalanced Mach-Zehnder interferometer outputs. Several authors have reported the use of external cavities to ensure the single longitudinal mode operation of a diode laser as discussed in 2.4.3. Goldberg et al.\(^{77}\) have shown that optical feedback from a mirror also improves the frequency stability by a factor of 10. Another method to improve the frequency stability of the diode laser is to lock the laser output to a reference frequency provided by an external
Fabry-Perot interferometer\(^{78,79}\). Cobb and Culshaw\(^{78}\) have reduced the amplitude of the frequency jitter by an order of magnitude using this technique. A similar approach has been adopted here, but an optical fibre Fabry-Perot interferometer has been substituted for the bulk optics device used conventionally.

4.5.1. The Fabry-Perot interferometer output dependence on source frequency

The relationship between a change in source emission frequency and the Fabry-Perot output phase \(d\phi\) is given by:

\[
\frac{dI}{d\nu} \bigg|_{\nu_{\text{ref}}} = 0.8 \frac{\pi n c}{z} \frac{F}{I_{\text{max}}} \quad 4.20
\]

Hence for a given cavity length and refractive index, the transmitted output is dependent on source frequency. The frequency of the source can be tuned to the transmission value of 0.75 \(I_{\text{max}}\) for maximum frequency sensitivity, with cavity parameters \(n\) and \(z\) constant. The Fabry-Perot output intensity is directly proportional to the source frequency in this operating region. Hence the phase noise on the Fabry-Perot output gives a direct measure of the source frequency jitter.

An expression for the Fabry-Perot frequency sensitivity can be obtained by substituting for \(d\phi\) from equation 4.20 into equation 4.14 and is given by:

\[
\frac{dI}{d\nu} \bigg|_{\nu_{\text{ref}}} = \frac{0.4\pi}{\Delta \nu_{\text{FSR}}} \frac{F}{I_{\text{max}}} \quad 4.21
\]

or

\[
\frac{\Delta \nu}{\Delta \nu_{\text{FSR}}} = 2\sqrt{3F} \quad 4.22
\]

at

\[
\nu_{\text{ref}} = \pm \frac{c}{4nz\sqrt{3F}} = \frac{\Delta \nu_{\text{FSR}}}{2\sqrt{3F}}
\]

where \(\nu_{\text{ref}}\) is the source frequency which corresponds to an interferometer
These equations show that the interferometer sensitivity to source frequency jitter is directly proportional to the cavity finesse and the cavity length.

4.5.2. The frequency stabilisation system

In order to stabilise the frequency of diode laser the output was fed into an all-fibre Fabry-Perot interferometer via a beam splitter. The interferometer was a 0.2m length of single mode optical fibre with semi-silvered end faces and was installed in an aluminium box to reduce its sensitivity to the environmental noise. It had a free spectral range of 500MHz and a finesse of 10, and was used in a feedback loop to control the laser frequency by adjusting its drive current. A schematic diagram of the stabilisation technique is shown in Figure 4.13. The Fabry-Perot output signal, via a photodetector, went to one input of a differential amplifier. The other input was a reference signal equivalent to 0.75 $I_{\text{max}}$ transmission, which corresponded to a reference optical frequency $v_R$, and was produced by deviating 20% of the Fabry-Perot input light onto another photodetector. The output from the differential amplifier was proportional to the difference between the laser diode frequency and the reference frequency $v_R$. This output was used to produce an error current, $I_e$, which was imposed on the laser drive current. Hence the feedback loop maintains the frequency $v_R$ by adjusting the laser drive current. The feedback loop was designed so that the amplitude of the laser frequency variations occurring in the frequency range from d.c. to approximately 5kHz were reduced.

4.5.3. The frequency stabilisation of an STL diode laser

The single mode operation of the STL diode laser at a drive current of 168mA ($1.12 I_{\text{th}}$) and a temperature of $25 \pm 0.1^\circ\text{C}$ has been reported in 2.4.4. The laser produced an output of 5mW at 866nm and was used in the stabilisation system described here. The magnitude of the phase noise...
Fig. 4.13. Fibre Fabry-Perot laser frequency stabilisation system

- Control current
- Spectrum analyser
- Temperature stable package
- Single mode fibre Fabry-Perot
- Frequency stabilised laser output

Feds (I) Mach-Zehnder magnetometer
(II) Fabry-Perot temperature sensor

M.O. = Microscope objective
P.D. = Photodetector
C. = Control operational amplifier
A. = Current amplifier
B.S. = Beam splitter
on the Fabry-Perot output was measured on a spectrum analyser. Figure 4.14 shows the phase noise from d.c. to 1kHz for both the free running and stabilised laser (for this spectrum analyser 0dBV' corresponds to an input voltage of 0.225V r.m.s.). The noise has a 1/f dependence, and its magnitude at 1kHz (50Hz B.W.) for the free running laser was $\sim -30\text{dBV}'$ and for the stabilised laser was $\sim -42\text{dBV}'$. The frequency jitter causing this phase noise was calculated using equation 4.22 and the Fabry-Perot constants ($F=10$, $v_{\text{FSR}}=500\text{MHz}$, $I_{\text{MAX}}=1\text{V}$). The frequency jitter at 1kHz (normalised to 1Hz B.W.) for the free running laser was 40kHz and for the stabilised laser 10kHz. Hence the current feedback scheme reduces the laser frequency jitter by about a factor of 4 at 1kHz.

From the measurement of $\frac{dv}{dt} \sim 3\text{GHz/mA}$ for an STL laser in Chapter 3, the feedback current was estimated to be $3.3 \times 10^{-6}\text{mA}$ at 1kHz. The Fabry-Perot input light gives a direct measure of the laser intensity noise. The intensity noise from zero to 1kHz is shown in Figure 4.15 and remained the same for both the free running and the frequency stabilised laser.

The variation of laser output intensity with current was determined for the operating point ($I = 168\text{mA}$, $P = 5\text{mW}$) from laser characteristic shown in Appendix C(i) such that: $\frac{1}{I} \frac{dI}{dt} = 0.03\text{mA}$. Consequently, the feedback current at 1kHz had little effect on the laser amplitude noise i.e. $\frac{dI}{I} = 10^{-7}$. The frequency stabilised STL laser (10kHz at 1kHz) gave a calculated minimum detectable phase of $8 \times 10^{-6}$ radians r.m.s. Hz$^{-\frac{1}{2}}$ for a Mach-Zehnder interferometer with a path difference of 0.04m (see equation 3.16). A noise floor of about $10^{-5}$ radians is reported in Chapter 5 for a Mach-Zehnder interferometer with a 0.04m optical path difference, which is in agreement with the calculated value.

The long term laser frequency jitter was measured by recording the output of the Fabry-Perot interferometer over several minutes as shown
Fig. 4.14. STL laser phase noise
(i) Free running laser
(ii) Frequency stabilised laser

Fig. 4.15. STL laser intensity noise

Measurement bandwidth = 50Hz
$0\text{dBV'} = 0.225\text{V r.m.s.}$
in Figure 4.16. When the feedback loop was closed the frequency jitter was reduced from approximately 30MHz p-p to less than 2MHz p-p.

4.5.4. Conclusions

A feedback loop controlling the injection current has been used for locking an STL diode laser longitudinal mode frequency to a reference frequency provided by an external optical fibre Fabry-Perot cavity. When the feedback loop was closed a substantial reduction in laser frequency jitter was achieved and at a detection frequency of 1kHz a stability of 10kHz was achieved. This would allow an optical fibre interferometer to detect $10^{-5}$ radians at 1kHz while operating with a 0.04m optical path difference. This stabilised STL diode laser was used to power a Mach-Zehnder interferometer configured as a magnetometer. These magnetic measurements will be reported in Chapter 5. The stabilisation system is the subject of a patent application by Jones and Pratt. A copy of this application can be found at the end of this thesis.

Kist and Wolfelschneider have reported using a short optical fibre Fabry-Perot to stabilise the frequency of a diode laser to within a few 100kHz. In their system an electronic feedback loop was used to lock the laser frequency to a reflection peak of the Fabry-Perot interferometer.

4.6. A comparison of the Fabry-Perot sensor and the Mach-Zehnder sensor

A comparison of the optical fibre Mach-Zehnder sensor and the optical fibre Fabry-Perot sensor is shown in Table 4.2. The fibre Fabry-Perot is an attractive sensor proposition because it is particularly simple, with only a single fibre, has no requirement for 3dB couplers and is free from false signal due to a reference arm. For the recovery of small periodic signals, active homodyne demodulation can be used to maintain the Fabry-Perot interferometer at its maximum sensitivity point $\phi_0$. In this
Fig. 4.16. Laser frequency jitter over several minutes
### TABLE 4.2. A COMPARISON OF THE MACH-ZEHNDER SENSOR AND THE FABRY-PEROT SENSOR

<table>
<thead>
<tr>
<th>Fibre components</th>
<th>Mach Zehnder sensor</th>
<th>Fabry-Perot sensor</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Two 3dB couplers</td>
<td>Sensing fibre</td>
</tr>
<tr>
<td></td>
<td>Sensing fibre</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Reference fibre</td>
<td></td>
</tr>
<tr>
<td>Laser phase noise sensitivity</td>
<td>$d\phi = 2\pi L \frac{dv}{c}$</td>
<td>$d\phi = 4\pi L \frac{dv}{c}$</td>
</tr>
<tr>
<td>Sensor path length difference</td>
<td>A few millimetres</td>
<td>A few metres</td>
</tr>
<tr>
<td>Laser phase noise (gives minimum detectable phase)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(i) diode Laser at 850nm (dv=10KHz)</td>
<td>$d\phi \approx 10^{-6}$ radians</td>
<td>$d\phi \approx 10^{-3}$ radians</td>
</tr>
<tr>
<td>(ii) Helium Neon Laser at 633nm (dv=1Hz)</td>
<td>$d\phi \approx 10^{-10}$ radians (other noise sources will limit this figure)</td>
<td>$d\phi \approx 10^{-7}$ radians</td>
</tr>
<tr>
<td>Reported sensitivity to a.c. signals</td>
<td>$10^{-6}$ radian for a HeNe and a diode laser</td>
<td>$10^{-5}$ radian for a HeNe laser</td>
</tr>
</tbody>
</table>


interferometer the sharp transmission fringes at first sight appear
to be highly advantageous giving a potentially larger sensitivity than
a two beam interferometer, but the large path differences necessary in
sensor Fabry-Perot's make them inherently sensitive to laser phase
noise. While with the Mach-Zehnder interferometer it is possible to
have a long length of signal fibre and keep the path difference small
thereby reducing the phase noise. Hence, to exploit the high sensitivity
of an optical fibre Fabry-Perot interferometer it is necessary to have an
extremely stable source frequency, typically of the order of a few Hz.
This is a demanding criterion, even for HeNe gas lasers and it is unlikely
that stabilised laser diode modules can provide stability to this degree
in the near future. Therefore, sensors such as the magnetometer and the
hydrophone which require high sensitivity (10^{-6} radian resolution), small
size, a robust package and long lengths of sensitised fibre are likely
to be based on Mach-Zehnder interferometers powered by diode lasers.
However, a simple Fabry-Perot device may find low sensitivity applications
such as temperature and pressure sensing where fringe counting techniques
can be used, particularly in environments where conventional techniques
are unsatisfactory.
CHAPTER 5
THE DETECTION OF D.C. MAGNETIC FIELDS USING
OPTICAL FIBRE MACH-ZEHNDER INTERFEROMETERS.

Considerable interest has recently been shown in magnetostrictive optical fibre sensors, particularly because a high sensitivity, room temperature, magnetic field sensor would represent a significant advance in magnetometer technology. The most sensitive magnetometers at present are superconducting quantum interference devices (SQUID's) which are capable of detecting magnetic fields as small as $10^{-16}$ tesla in the laboratory. SQUID's require cryogenic instrumentation for their operation which limits the conditions under which they can be utilised. Fibre optic sensors measure the magnetic induction (flux density), $B$, in free space resulting from an external magnetic field, $H$, and therefore the sensitivity is given in the units of magnetic induction which are tesla (T). It has become the convention in the published literature on these optical sensors to refer to this magnetic induction as the magnetic field and this convention has been adopted here. An interferometric sensor, utilising a magnetostrictive stretcher, capable of detecting $10^{-12}$ T for one metre of sensitive fibre was first proposed by Yariv and Windsor in 1980. Such a sensor would approach the SQUID's performance for sensitised lengths of 100 to 1000m.

5.1. Optical fibre magnetometers

The principle of sensor operation is associated with the measurement of the longitudinal strain produced in a single mode optical fibre bonded to or jacketed by a magnetostrictive material. Magnetostriction is a change in dimension of a ferromagnetic material when it is placed along the axis of an applied magnetic field. Hence the ferromagnetic material produces a strain in the jacketed optical fibre in the presence of an applied magnetic field which perturbs the optical path length, hence
changing the propagation time of the light guided by the fibre. A key problem which is examined in the present work is the identification of high magnetostriction materials and the incorporation of such materials into a fibre sensor by appropriate bondings or coating. Two sensor configurations have previously been explored for measuring the change in optical path length caused by the magnetostriction, the Polarimeter and the Mach-Zehnder interferometer. The operating principles of the Polarimeter are described in 1.3. In the magnetometer application the fibre is wound around, and bonded to a nickel cylinder. Rashleigh has claimed a minimum detectable magnetic field of $10^{-10}$ tesla for 1 metre of magnetically sensitive fibre (T.m.)

A more sensitive approach, which has been adopted by a number of authors has used the magnetostrictive strain to induce an optical path length change in one arm of a single mode optical fibre Mach-Zehnder interferometer. In this configuration, the sensing fibre's optical path length is compared to that of a non-magnetically sensitive reference fibre. Dandridge et al. reported the first experimental results on fibre magnetic field sensors and measured minimum detectable a.c. magnetic fields as low as $8 \times 10^{-12}$ T.m. for bulk nickel devices operating at 1 to 10 kHz. In this paper and many subsequent publications the minimum detectable magnetic field has been defined in the units tesla per metre, that is, the minimum detectable field increases with the length of sensitised fibre. Clearly, the minimum detectable field should decrease as the sensitised length increases and the correct units are given by the product of the field and the length (T.m). Somewhat higher minimum detectable fields were found by Dandridge et al. for thin film nickel jacketed fibres. Jarzynski et al. calculated the respective magnetic responses anticipated for various metallic alloys and calculated the reduced elongation of thin metallic coatings versus thickness which arises from restraining forces of the rigid glass fibre.
To obtain a magnetostriction approaching that of the bulk material (restraining forces negligible), coatings of approximately 10 micrometres on 100 micrometre diameter fibre silica fibres are required. The use of a bulk stretcher such as the nickel cylinder recently reported by Cole et al provides an alternative structure which is not affected by fibre restraining forces.

In both the Polarimeter and the Mach-Zehnder interferometer the detection of small alternating magnetic fields at known frequencies in the range from 100Hz to 10kHz at optimal d.c. bias magnetic fields has been achieved by the authors. The alternating magnetic signal enables the separation of the magnetic effect (at relatively higher frequency) from the effects of the environmental phase drift. Thus the signal can be recovered from the sensor (whether it is a Polarimeter or an interferometer) using a homodyne demodulation scheme (as described in 3.2). In addition narrow band phase sensitive detection can be used to reduce noise at the chosen signal frequency. These techniques have enabled Mach-Zehnder interferometers to detect a.c. magnetic fields as small as $10^{-12}$ T. However, in practical magnetic field sensing applications such as magnetic anomaly detection which can be used to identify rocks beneath the earth's surface (geomagnetic surveying) and magnetic heading sensors for navigation, the measurement of small d.c. and low frequency magnetic fields is required. D.C. magnetic field detection is a more difficult problem because the signal is in the same frequency band as the environmental phase drift and also one has no prior knowledge of the signal waveform. Hence the advantages of measuring the signal where the amplitude of the sensor 1/f noise is low using phase sensitive detection cannot be realised.
This Chapter reviews the principles of magnetic field detection using optical fibre Mach-Zehnder interferometers and reports results on two d.c. magnetometers which are based on the Mach-Zehnder interferometer.

5.1.1. Sensitivity of an optical fibre to magnetic fields

The basic configuration of a Mach-Zehnder interferometric sensor has been shown in Figure 3.7. In the present application the signal fibre is magnetically sensitised by coupling it to a magnetostrictive material. The basic geometry of a single mode optical fibre jacketed by a magnetostrictive material is shown in Figure 5.1. The total phase change experienced by a light beam propagating in such a fibre can be calculated using a model based on a bulk stretcher which assumes the fibre restraining forces are negligible. Consider the magnetostrictive jacket to be applied directly to the fibre cladding and the cross-sectional area of the jacket to be much larger than that of the silica fibre. Therefore the strains in the fibre core and cladding are assumed to be equal to the strains in the jacket material. The magnetic field is applied along the axis of the jacketed fibre (light propagation direction) and the magnetic field terms perpendicular to the axis are neglected. The consequent magnetostriction is taken to be associated with zero net volume charge, so:

\[ 2\varepsilon_1 + \varepsilon_3 = 0 \]

where \( \varepsilon_1 = \varepsilon_2 = \varepsilon_r \) are the strain components perpendicular to the axis and to each other, while \( \varepsilon_3 \) is the strain along the axis. Therefore in addition to the strain \( \varepsilon_3 \) created by the axial magnetic field, the fibre is also subjected to a radially symmetric strain such that:
Fig. 5.1. Basic geometry of a silica fibre jacketed by a magnetostrictive material.
and consequently the induced birefringence is zero as shown by equation 2.23. The effect of such strains on the phase of a light beam propagating in the fibre has been discussed in 2.3. An expression for the total phase change experienced by the light in a fibre of length $L$ is given in equation 2.21 and is the sum of a strain induced refractive index change and a physical elongation. Using the values of the photoelastic constants for silica (see 2.3), $P_{11} = 0.12$, $P_{12} = 0.27$ as well as the refractive index, $n = 1.46$ and the fact that $\varepsilon_1 = \frac{-\varepsilon_3}{2}$ in the model, thus the phase change in a length of fibre $L$ becomes:

$$\Delta \phi = \frac{2\pi n L 0.92 \varepsilon_3}{\lambda}$$

where it is assumed that the orthogonal polarisation modes which the fibre supports are degenerate. An identical result has been derived by Yariv and Windsor. The main contribution to the phase change in equation 5.3 is due to the physical elongation of the fibre since the changes in refractive index due to $\varepsilon_1$ and $\varepsilon_3$ almost cancel each other.

A general expression for the length change $\Delta L$ of a fibre jacketed by a magnetostrictive material has been given by Lenz and Mitchell and is:

$$\Delta L = \int_{0}^{L} \frac{dc}{dB} B \cdot d\ell$$

where $c$ is the magnetostrictive strain ($c = \frac{\Delta L}{L}$), $B$ is the applied magnetic field in free space, and $d\ell$ is the component of the jacketed fibre which aligns with the magnetic field. The gradient of the magnetostriction versus applied field curve, $\frac{dc}{dB}$, is referred to as a material (piezomagnetic) constant and its value depends on the permeability of the magnetostrictive material. For the case when the magnetic field
is applied along the fibre, \( \Delta \ell \) is equal to an increment on the fibre and when the magnetic field is perpendicular to the fibre, \( B \Delta \ell \) is zero.

The axial strain \( \epsilon_3 \) versus axial magnetic field strength curves for some magnetostrictive materials \( ^6,25,88,89 \), are shown in Figure 5.2. This strain can be either positive (expansion) or negative (contraction) depending on the material. These curves clearly demonstrate that the functional dependence of the axial magnetostriction varies according to the field regime. Hence for magnetic fields applied along the fibre the following scalar relationships are valid:

\[
\begin{align*}
\epsilon_3 & \propto B^2 \quad \text{for } B \ll \frac{B_{\text{sat}}}{2} \\
\epsilon_3 & \propto B \quad \text{for } B = \frac{B_{\text{sat}}}{2} \\
\epsilon_3 & \propto C \quad \text{for } B \gg \frac{B_{\text{sat}}}{2}
\end{align*}
\]

where \( B_{\text{sat}} \) is the field at which the magnetisation, and hence the magnetostriction, saturates and \( C \) is a constant. Hence it is possible to identify regions of steep gradient where a small change in magnetic field, \( \Delta B \), produces a maximum change in the longitudinal strain. This variation of the magnetostrictive response with known applied magnetic fields has led to use of a bias field, \( B_0 \), in most sensors to place the device in a region of maximum sensitivity. Therefore, assuming that the total applied field \( B \) is the sum of a bias field \( B_0 \) and a time varying field \( B \sin \omega t (B \ll B_0) \), the resulting magnetostrictive strain is proportional to \( B_0 + B \sin \omega t \). The sensitivity of the magnetostrictive fibre sensor to the time varying field \( B \sin \omega t \) can be expressed as:

\[
\Delta \epsilon_3 = \left. \frac{d \epsilon_3}{dB} \right|_{B_0} \Delta B \sin \omega t
\]

where in this case the material constant defined equation 5.4 is

\[
\left. \frac{d \epsilon}{dB} \right|_{B_0} = K_.
\]

Hence the sensitivity depends on the gradient of the magneto-
Fig. 5.2 Magnetostrictive extension of various materials as a function of field.
strictive response at the bias field $B_0$. The amplitude of the optical phase shift produced by the time varying magnetic field can be obtained by substituting the amplitude of equation 5.6 into equation 5.3 and is given by:

$$\Delta \phi = \frac{2 \pi \alpha 0.92 K_1 \Delta B(T)}{\lambda}$$

For the metallic glass Fe$_{80}$B$_{20}$ the material constant, $K_1$, at maximum sensitivity ($B_0 = 10^{-3}T$) can be deduced approximately from the curve in Figure 5.2 and takes on a value of $12 \times 10^{-3}T^{-1}$. Hence, the phase shift produced at maximum sensitivity to magnetostrictive strain is:

$$\Delta \phi = \frac{10^{-1} \varepsilon(m) \Delta B(T)}{\lambda}$$

where $n=1.46$.

This magnetically induced phase shift is inversely proportional to the light source wavelength and at a nominal wavelength of 1000nm, which is close to the magnetometer operating wavelengths of 633nm and 850nm used in this work, the phase shift is given by:

$$\Delta \phi = 10^5 \varepsilon(m) \Delta B(T)$$

To compare sensor performance, a characteristic sensitivity, $S$, is defined as the optical phase shift induced by the magnetostriction, at a given wavelength, normalised by the length of the fibre strained and the value of the applied magnetic field:

$$S = \frac{\Delta \phi}{\Delta B(T) \varepsilon(m)} \text{ (radians } T^{-1} \text{ m}^{-1})$$

5.10
5.1.2. Minimum detectable magnetic field

It is well documented \(^{24,25}\) that optical path length changes as small as \(10^{-7}\) of a wavelength i.e. \(10^{-13}\) m may be detected with a Mach-Zehnder interferometer. This corresponds to a minimum detectable phase shift of \(10^{-6}\) radians, which will give rise, according to equation 5.9, to a minimum detectable magnetic field of approximately \(10^{-11}\) T for 1 metre of magnetically sensitive fibre. In order to approach this level of performance it is necessary to operate the interferometer at its shot noise limit. This can be achieved when the optical powers on photodetectors are high and other noise sources such as laser phase noise and Johnson noise can be neglected. The derivation of the minimum detectable magnetic field for a fibre optic Mach-Zehnder interferometer operating in the shot noise limited situation follows. If a Mach-Zehnder interferometer is at quadrature then the signal output current from the mixing detector will be:

\[
i_s = \frac{P e n}{h \nu} \Delta \phi
\]

5.11

where

\(P\) = total power in the optical beam
\(e\) = electronic charge
\(n\) = quantum efficiency of the detector
\(\nu\) = the laser frequency
\(h\) = Planck's constant \((6.6 \times 10^{-34}\) Js\)
\(\Delta \phi\) = \(10^5 \times \ell(m) \Delta B(T)\)

The bulk stretcher model described in 5.1.1 for the sensitivity of the optical phase to magnetic fields has been adopted and d.c. magnetic field of \(4 \times 10^{-4}\) T(\(B_0\)) is assumed to bias the magnetostrictive material to the point of maximum sensitivity. Since \(\Delta B(T)\) is time varying, this time dependence will be imparted to the signal current, \(i_s\), and the time average of this current is given by:
where \( \sin^2 \omega t = \frac{1}{2} \). The shot noise has been defined in 3.12 and the shot noise current associated with the average current output from the mixing detector is simply:

\[
<i_s^2> = \frac{1}{2} \left( \frac{Pn\Delta\phi}{h\nu} \right)^2
\]

The signal to noise electrical power ratio as obtained from equations 5.12 and 5.13 is:

\[
\frac{S}{N} = \frac{<i_s^2>}{<i_n^2>} = \frac{Pn(\Delta\phi)^2}{4 \, h\nu \Delta F}
\]

where \( \Delta F \) is the detection bandwidth. Equation 5.14 can be expressed in terms of magnetic field by substituting for \( \Delta\phi \) using equation 5.9 and thus becomes:

\[
\frac{S}{N} = \frac{10^{10} Pn(\Delta B(T))^2 \ell (m)^2}{4 \, h\nu \Delta F}
\]

The minimum detectable magnetic field, \( \Delta B \), is defined as that value of \( \Delta B \) for which \( S/N = 1 \). From equation 5.15:

\[
\Delta B = \sqrt{\frac{2h\nu\Delta F}{10^{10} Pn\ell^2 (m)}}
\]

Using a detection bandwidth \( \Delta F = 1\text{Hz}, P = 30 \times 10^{-6} \text{W}, \ell = 1\text{m}, n = 0.5, \) and \( \nu = 3 \times 10^{14} \text{Hz} (\lambda = 1000\text{nm}) \), the shot-noise-limited minimum detectable field is \( 2 \times 10^{-12} \text{T.m} \). This corresponds to a minimum detectable phase shift of about \( 2 \times 10^{-7} \) radians. Hence an optical fibre Mach-Zehnder interferometer, such as the one described in Chapter 3, utilising a one metre length of magnetostrictively coupled fibre is theoretically capable of detecting \( 2 \times 10^{-12} \text{T} \). In practice, a finite jacket thickness will reduce sensitivity, and the magnetostriction characteristics at high
frequencies may differ from those observed under d.c. fields so the theoretical minimum detectable magnetic field may be considered a lower limit.

When assessing the minimum detectable field claimed for a fibre optic magnetometer, it is necessary that the length of fibre used is stated, since the sensitivity is proportional to the length of magnetostrictively coupled fibre as shown in equation 5.9. Figures are conventionally quoted normalised to a 1m length of fibre, although at present most experiments use a 0.1m length and assume a proportional improvement to 1m. A further factor to be considered is the bandwidth of the measurement. This is now conventionally taken as 1Hz, although practical low field sensing may be concerned with smaller bandwidths of say, 0.1Hz.

5.1.3. Magnetostrictive materials

In order to optimise the sensor response careful selection of the magnetostrictive material is necessary. Magnetostrictive materials are soft ferromagnets whose dimensions change along a given axis accompanying magnetisation of the material in that direction. A ferromagnetic material is described in terms of a domain structure, each domain contains some atoms whose magnetic moments are aligned in the same direction. In an unmagnetized volume of a material, the domains are arranged in a random fashion with their magnetic axes pointing in all directions so the material has no resultant magnetic moment. Under the influence of a magnetic field the domains tend to align in the direction of the applied field giving a resultant magnetic moment and magnetization. Eventually the field strength causes complete domain alignment with the direction of the applied magnetic field, and this results in magnetic saturation in the applied field direction. The magnetostriction which
results in a change in dimension of the material along a given axis accompanies magnetization of the material in that direction and is a result of the domain wall movement. Some examples of magnetostrictive materials are Fe, Co, Ni and various alloys and compounds of these elements. Data presented by Giallorenzi et al, which is listed in Table 5.1 gives typical values of the saturation magnetostriction materials in their crystalline form, that is, the strain at magnetic saturation. Nickel which is the most commonly used magnetostrictive transducer exhibits a large negative magnetostriction and has a large saturation magnetostriction of $-30 \times 10^{-6}$.

The magnetostrictive strain of alloys composed of the elements mentioned can be enhanced when they are produced as amorphous metals, commonly called metallic glasses since they solidify in the glassy state. Unlike the crystalline solid, the amorphous solid has no discernable long range order or grain boundaries. Hence the domain wall motion responsible for magnetostriction is not impeded and magnetostriction occurs in much smaller applied magnetic fields. A listing of amorphous alloy magnetostrictions reproduced from Luborsky is given in Table 5.2. Amorphous metals exhibit magnetostrictive strains per unit applied field of about two orders of magnitude greater than that of Nickel. This means that although the saturation magnetostriction is approximately the same, the applied field required to reach saturation is much less in the case of the amorphous alloy. Forester et al have shown that the magnetostriction of Fe$_{80}$B$_{20}$ saturates at $30 \times 10^{-6}$ in an applied field of $3 \times 10^{-3} \text{T}$ whereas $-30 \times 10^{-6}$ for thin film nickel requires a much larger field of about $10^{-1} \text{T}$. This is illustrated in Figure 5.2.

The magnetostrictive materials used in this work were all amorphous metals. They were either in the form of a jacket sputtered onto the cladding of the fibre for about 50mm of its length, or metallic glass
### TABLE 5.1.

**MAGNETOSTRICTION OF SOME CUBIC CRYSTALS AT MAGNETIC SATURATION**

(<100> and <111> directions)

<table>
<thead>
<tr>
<th>Material</th>
<th>S &lt;100&gt; (x 10^-6)</th>
<th>S &lt;111&gt; (x 10^-6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fe</td>
<td>20</td>
<td>-20</td>
</tr>
<tr>
<td>40% Ni-Fe</td>
<td>-7</td>
<td>30</td>
</tr>
<tr>
<td>60% Ni-Fe</td>
<td>-27</td>
<td>22</td>
</tr>
<tr>
<td>73% Ni-Fe (annealed)</td>
<td>14</td>
<td>7</td>
</tr>
<tr>
<td>73% Ni-Fe (quenched)</td>
<td>15</td>
<td>14</td>
</tr>
<tr>
<td>80% Ni-Fe</td>
<td>9</td>
<td>0</td>
</tr>
<tr>
<td>Ni</td>
<td>-46</td>
<td>-30</td>
</tr>
<tr>
<td>3% Si-Fe</td>
<td>27</td>
<td>-5</td>
</tr>
<tr>
<td>7% Si-Fe</td>
<td>-5</td>
<td>3</td>
</tr>
<tr>
<td>Ni$<em>{0.8}$Fe$</em>{2.2}$O$_4$</td>
<td>-36</td>
<td>-4</td>
</tr>
<tr>
<td>Co$<em>{0.8}$Fe$</em>{2.2}$O$_4$</td>
<td>-590</td>
<td>120</td>
</tr>
<tr>
<td>Alloy</td>
<td>(x 10⁻⁶)</td>
<td></td>
</tr>
<tr>
<td>-----------------------</td>
<td>----------</td>
<td></td>
</tr>
<tr>
<td>Co B</td>
<td>-4.0</td>
<td></td>
</tr>
<tr>
<td>Fe₈₀B₂₀</td>
<td>31.0</td>
<td></td>
</tr>
<tr>
<td>*2605</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fe₈₆P₁₀B₁C₃</td>
<td>30.0</td>
<td></td>
</tr>
<tr>
<td>*2615</td>
<td>29.0</td>
<td></td>
</tr>
<tr>
<td>Fe₄₀Ni₄₀P₁₄B₆</td>
<td>11.0</td>
<td></td>
</tr>
<tr>
<td>*2826</td>
<td>11.0</td>
<td></td>
</tr>
<tr>
<td>Fe₂₉Ni₄₉P₁₄B₆Si₂</td>
<td>3.0</td>
<td></td>
</tr>
<tr>
<td>*2826B</td>
<td>5.0</td>
<td></td>
</tr>
<tr>
<td>Fe₈₀P₁₅C</td>
<td>19.0</td>
<td></td>
</tr>
<tr>
<td></td>
<td>31.0</td>
<td></td>
</tr>
<tr>
<td></td>
<td>31.0</td>
<td></td>
</tr>
<tr>
<td>Co₄₆.₆Ni₃₁.₁P₂₂.₃</td>
<td>-9.0</td>
<td></td>
</tr>
<tr>
<td>Fe₀.₆Co₃₅.₂Ni₃₉.₈P₂₄.₄</td>
<td>-11.5</td>
<td></td>
</tr>
<tr>
<td>Fe₀.₇Co₄₆.₉Ni₃₅P¹₇.₄</td>
<td>-12.0</td>
<td></td>
</tr>
<tr>
<td>Fe₃.₉Co₃₆.₅Ni₄₂.₈P₁₆.₈</td>
<td>-8.5</td>
<td></td>
</tr>
<tr>
<td>Fe₄.₅Co₃₉.₉Ni₃₇.₃P¹₈.₅</td>
<td>-4.0</td>
<td></td>
</tr>
<tr>
<td>Fe₆Co₇₄B₂₀</td>
<td>&lt;0.4</td>
<td></td>
</tr>
<tr>
<td>Fe₅Co₇₀.₃P₁₆B₁₆Al₃</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>Fe₄.₇Co₇₀.₃Si₁₅B₁₀</td>
<td>-0.1</td>
<td></td>
</tr>
<tr>
<td>Co₇₅Si₁₅B₁₀</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>Co₉₁P₉</td>
<td>-4.₃</td>
<td></td>
</tr>
</tbody>
</table>

* Allied Chemical Co. Metglas designation
ribbons to which the fibre was bonded.

Metallic glasses are available commercially from the Allied Chemical Corporation in the U.S.A. and Vacuumschmelze in West Germany. They are prepared by a common glass forming technique known as 'drum quenching' which involves cooling the molten alloy on a spinning flywheel at a rate of one million degrees per second. This produces very thin ribbons of approximately 25 micrometre thickness (necessary to achieve rapid cooling rates) and widths of a few centimetres. The Vacuumschmelze metallic glasses 'Vitrovac' (trade name) 4040 (Fe_{39}Ni_{39}Mo_{4}Si_{6}B_{12}) and 6025 (Co_{66}Mo_{2}Fe_{4}Si_{16}B_{12}) were used in the present work. They both exhibit positive magnetostriction, their respective quoted maximum strain being $10 \times 10^{-6}$ (see Figure 5.2) and $0.5 \times 10^{-6}$ corresponding to saturation in applied magnetic fields of approximately $10^{-3} \text{T}$. Clearly (see Tables 5.1 and 5.2) these ribbons are not optimised specifically for magnetostrictive transducer applications. They were chosen because of their availability and similarity in composition to amorphous materials known to have large values of magnetostriction.

5.1.4. Magnetic sensor heads

The sensor head is the length of magnetically sensitised fibre which enables the applied magnetic field to be converted into an optical path length change. The incorporation of magnetostrictive sensor heads into optical fibre interferometers has been the focus of recent work by a number of authors. The three main categories of interest in magnetic sensor head fabrication have been:

1. A magnetostrictive metal jacket applied to the surface of the fibre by either electroplating$^{25,86}$ or R.F. sputtering$^{92}$.

2. Bulk crystalline metal cylinders with fibre bonded to the circumference$^{18,83}$.
3. Fibre bonded to amorphous metallic glass strips of high magnetostriction in either a straight line or a cylindrical configuration.

The optical fibre magnetometers which will be described in 5.2 and 5.3 mainly use sensor heads from category 3, although results will be reported on a fibre sensor head which has been sputter coated with a 5 micron layer of amorphous \( \text{Fe}_{80}\text{B}_{20} \) alloy.

A general difficulty in fabrication of the sensor heads is that of stripping the protective coatings from the single mode ITT step index fibre and replacing them with a magnetostrictive material without breaking the fibre. This was achieved by mechanically stripping the plastic secondary coating and dissolving the silicone rubber primary coating in sulphuric acid. The fibres were then washed in electronic grade methanol to give a good clean surface to which adhesion of a suitable coating or bonding can be effected.

Initially, a linear sensor head was constructed by coating a 50mm length of bared fibre with an amorphous film of \( \text{Fe}_{80}\text{B}_{20} \) alloy. The film was deposited by R. E. Jones at the Cavendish Laboratory, Cambridge using a co-sputtering process. A cross-sectional view of this linear sensor head is shown in diagram (a) in Figure 5.3. This material was chosen because it had been found by Forester et al\(^{88}\) to have a large magnetostriction and a small saturation field as shown in Figure 5.2. The coating process involved mounting a fibre stripped to its cladding in the vacuum chamber of an R.F. sputtering system such that a 50mm length would be in the plasma. The co-sputtering target consisted of lumps of boron on a 50mm thick square piece of iron foil. A sputtering time of 10 hours was required to produce a 5 micrometre coating. Subsequent analysis of such a coated fibre by the STL materials evaluation centre revealed that the
(a) Sputter coated fibre

(b) Fibre bonded to metallic glass strip

(c) Fibre bonded to metallic glass on circular former

Fig. 5·3
coating thickness was non-uniform. This is demonstrated by their photographs taken with an electron microscope shown in Figure 5.4. On one side of the fibre (photo (b)) there is clearly a coating which is approaching 5μm thick and on the other side of the fibre (photo (a)) there is no coating. This non-uniformity was expected since no attempt was made to alter the orientation of the fibre during the coating operation. Figure 5.5 shows an end view of the coated fibre with the visible trace representing a spectrometer scan establishing the presence of iron in the coating.

Sensor heads from category 3 were constructed by bonding the fibre directly to a strip of metallic glass ribbon, this offers relative simplicity of preparation and potential low manufacturing cost while retaining sensitivity. Interest is also encouraged because an on-line jacketing process for modified fibres has not yet been developed. Conventional 2-part Araldite epoxy resin provides a rigid bond between the fibre and the metallic glass. An adhesive with little mechanical compliance is essential to ensure that the strain induced in the metallic glass is transmitted efficiently to the fibre (high fibre-metallic glass strain coupling efficiency). The two sensor head configurations investigated that use metallic glass ribbon are shown in diagrams (b) and (c) in Figure 5.3. The sensor head in (b) has a linear configuration and the one in (c) has a circular form.

The linear sensor heads were constructed by bonding strips of Vacuum-schmelze metallic glass ribbon (25 micrometres thick) to lengths of fibre which had been bared to the silica cladding. Structures with the fibre bonded to one side and both sides were fabricated. One cylindrical sensor head consisted of a strip of metallic glass ribbon, 0.16m long, wrapped tightly on a circular glass former of 25mm diameter and bonded
(a) One side of the fibre shows minimal traces of the metal coating.

(b) After the fibre has been rotated through 180° the 5 micrometer thick metal coating is clearly seen.

Fig. 54. Electron microscope photographs of a fibre which has been cleaved at the centre of its coated length.
Fig. 5.5. Electron microscope photograph of the coated fibre, the superimposed spectrometer line scan responds to the iron component in the coating.
in place. Around the circumference of the metallic glass ring one turn of single-mode optical fibre was attached with the Araldite. Perpendicular to the fibre and the metallic glass ring, 200 evenly distributed turns of varnish insulated copper wire (diameter 0.5mm) were wrapped in order to produce a magnetic field along the longitudinal axis of the fibre turn. A second cylindrical sensing former was constructed in a similar fashion, except that the metallic glass was bonded to a plastic former.

5.2. The use of an interference pattern to detect d.c. magnetic fields.

Magnetic field measurement is usually required at d.c. or low frequencies, the frequency range of interest being in the region 0.01 to 10Hz., as explained in 5.1. Clearly, the magnetic signal is in the same frequency band as the noise normally associated with optical fibre Mach-Zehnder interferometers, which includes phase drift and polarisation fluctuations of the light propagating the fibres. Therefore an alternative signal recovery scheme to the active homodyne demodulation scheme described in 3.2 is considered. A modified Mach-Zehnder interferometer has been developed to reduce some of the low frequency noise sources which become the limiting factors encountered in d.c. magnetic field measurement.

A review of this work was published at the first international conference on optical fibre sensors in London in 1983. A copy of the paper can be found at the end of this thesis.

5.2.1. The modified Mach-Zehnder interferometer

The interferometer was built using bulk optic components and is shown schematically in Figure 5.6. A photograph of the interferometer is shown in Figure 5.7. The light source was a Tropel 200 single frequency
Fig. 5.6 Magnetostrictive fibre optic sensor
Fig. 57. Modified optical fibre Mach-Zehnder interferometer.
HeNe laser which emitted 1mW of optical power at a wavelength of 633nm. The use of a single frequency source relaxes the constraints placed on matching the lengths of the interferometer fibre arms because it has a coherence length of tens of metres (see 2.4). This enabled several sensor head configurations involving different lengths of fibre to be substituted directly into the interferometer without adjusting the reference arm length of 1m.

At the time of construction of the interferometer (1981), an all-fibre 3dB coupler operating at 633nm was not available. A dielectric coated beam splitter was used to split the light into two equal intensity components that were launched into a reference fibre and signal fibre. The fibre used was single mode at 633nm, with a core diameter of 4 micrometres and cladding diameter of 125 micrometres. About 100mm of the signal fibre was used to construct a magnetic sensor head as described in 5.1.4. The sensor head was then placed in a calibrated Helmoltz coil (approximately 100mm in length), along with the reference fibre to compensate for thermal effects arising from the coil. The recombination of the light beams was achieved by clamping the two fibre ends together in neoprene pad so that they are parallel and in close contact. The neoprene acted as a cladding mode stripper. The light from the two fibres interfered to produce 2-slit interference fringes which were imaged onto a photodiode array. A photograph of the fringes is shown in Figure 5.8(a).

5.2.2. The measurement of phase using an interference pattern

The separation between successive fringes, $\Delta x$, is given by the usual relationship:

$$\Delta x = \frac{D \lambda}{a}$$ 5.17
(a) Cosine squared fringes of output interference pattern

(b) Output of 128 element photodiode array showing 4 fringes

Fig. 5-8. Modified Mach-Zehnder interferometer output.
where, $D$, is the distance from the photodiode array to the fibre ends and, $a$, is the distance between the fibre cores. Both fibres were clamped together so that they were touching, the minimum separation of the fibre cores being limited by the cladding diameter to 125 micrometres. Hence, the fringe separation is approximately 0.8 mm for a distance of 0.15 m between the diode array and the fibre ends. The photodiode array consists of a row of 128 silicon detectors. The separation between each detector in the array is 24 micrometres which corresponds to about 33 diodes per fringe. Figure 5.8(b) shows the output when 4 fringes are imaged onto the 128 diodes where each video pixel represents a discrete photodiode output.

The variation of the light intensity across the photodiode array resulting from the interference pattern can be represented by:

$$I = I_T(1 + \alpha \cos \left( \frac{2\pi x a}{\lambda D} + A \right)$$

where $I_T = I_1 + I_2$ is the total light intensity, $\alpha$ is the fringe visibility (see 3.1.1 and 3.1.2), $\frac{2\pi x a}{\lambda D}$ determines the spatial distribution of the fringes on the array, $x$ is a measure of the fringe position relative to the zero order spatial fringe (central fringe) and $A$ is the phase difference between the reference fibre and the signal fibre. This equation is slightly different from the conventional Mach-Zehnder interferometer output equation 3.5 which is given in 3.1.1. In a conventional detection system the amplitude of the zero order spatial fringe ($x = 0$) is used as a measure of the interferometer phase difference. Also because a second beam splitter is not used to recombine the light equation 5.18 is a factor of two greater than equation 3.5. If the fringe visibility, $\alpha$, and the intensity of the light forming the fringe pattern are normalised to unity then equation 5.18 becomes:
When the phase difference, $A$, is zero this equation is analogous to the result produced by two slit sources in Young's experiment. The difference between two fringe patterns with a phase difference, $\phi$, between them is given by:

$$I_A - I_{A+\phi} = 2 \sin \frac{\phi}{2} \sin \left( \frac{\frac{2\pi x a}{\lambda D} + A}{2} \right)$$

The derivation of this equation is given in Appendix E. It can be seen that for small values of $\phi$ the difference term is a sine wave whose maximum amplitude is directly proportional to the phase difference between the fringe patterns. The phase difference is then determined by finding the r.m.s. value of the difference term (equation 5.20) and multiplying by $\sqrt{2}$ to give the absolute phase shift. The amplitude of equation 5.20 and therefore the measurement of the phase, is made independent of the variations in intensity and fringe visibility between the two interference patterns by normalising them to unity before the subtraction. Effectively, the interferometer phase is determined from the relative position of the fringes whereas noise effects such as polarisation fluctuations and intensity variations only affect the fringe visibility.

5.2.3. Magnetic field data collection system

For the detection of small phase shifts an accurate measurement of fringe position is required. In order to achieve this the signal from each photodiode in the array is digitised, in turn, and stored in an Apple II computer. The computer also performs the normalisation and the subtraction of the fringe patterns. A diagram of the data collection system is shown in Figure 5.9. The essential features of this system are summarised in Appendix F and a more detailed account is given by Willson
Fig. 5.9. Fibre optic magnetic sensor detection system.
and Jones. The phase shift resulting from the magnetostrictively coupled strain is measured therefore by subtracting two fringe patterns. A fringe pattern recorded in an applied magnetic field is normalised and subtracted from a previously recorded and normalized zero field fringe pattern. A typical difference curve (sine wave) corresponding to an applied d.c. magnetic field of \(3 \times 10^{-3}\) T is shown in Figure 5.10. The sensor head, in this case, consisted of a strip of Vacuumschmelze metallic glass (Vitrovac 4040) bonded to the signal fibre.

To minimise the effect of the phase drift in zero magnetic field, which is caused by the differential environmental perturbations on the two fibres, the Apple II is instructed to store a set of zero field fringes, and then \(20 \times 10^{-3}\) s later to take another set of fringes with the field applied. Effectively the system is recalibrated to zero for every measurement of magnetic field. Hence the sensitivity of this detection system which monitors more than one fringe is independent of the zero field fringe pattern, and no active compensation to retain quadrature at some fixed point in the fringe pattern is required. Therefore, the modified Mach-Zehnder interferometer can be electrically passive. The Apple II also controlled the Helmoltz coil current from within the program, thus allowing the automatic collection of data with and without an applied magnetic field.

The minimum detectable magnetic field is limited by the digitisation noise. For a fringe contrast ratio of unity and a 12 bit ADC the minimum detectable phase shift is \(8 \times 10^{-4}\) radians and is calculated from:

\[
\phi_{\text{noise}} = \pi \frac{1}{4096} \quad 5.21
\]

Therefore, assuming that the strain in magnetostrictive material is equal to the strain in the fibre core as explained in 5.1.1.
Fig. 5·10. Difference curve representing a phase shift of 1 radian produced by the application of a d.c. magnetic field of $3 \times 10^{-3} \text{T}$ to the 30mm 'Vitrovac' 4040 sensor head.
theoretical sensitivity of the optical phase to magnetic fields is given by equation 5.9 to be $10^5$ radians $T^{-1}m^{-1}$. Consequently, the digitisation noise limited minimum detectable magnetic field is $8 \times 10^{-9}$T for 1 metre of magnetically sensitised fibre.

5.2.4. Results on the various sensor head configurations

In order to measure the performance of the magnetic sensor heads each was placed, in turn, in one arm of the modified Mach-Zehnder interferometer and the sensitivity of the interferometer to small d.c. magnetic fields determined. Clearly the minimum detectable magnetic field for an interferometer system is limited by the random noise phase shifts present in the system. The phase drift in this interferometer caused by the differential environmental perturbations of the two fibres was reduced by ensuring that the fibres followed similar paths for an appreciable proportion of their total length and also by enclosing the interferometer in a perspex box. Even with these precautions the drift was still found to be a few radians per hour. The interferometer noise for the magnetic field detection system was determined by storing a set of zero field fringes, and then $20 \times 10^{-3}$s later storing another set of zero field fringes and subtracting the normalised patterns. A characteristic noise trace (difference curve) is shown in Figure 5.11 which corresponds to a minimum detectable phase shift of $2 \times 10^{-3}$ radians. This noise level is a factor of 2.5 above the digitisation noise limited phase shift of $8 \times 10^{-4}$ radians (see equation 5.21), indicating that its origins are from the environmental perturbations of the interferometer phase. This is confirmed by the periodic nature of the noise trace shown in Figure 5.11 which shows the fringe structure, indicating that the problem is a phase shift rather than detector noise. The position of the beam-splitter critically affects the optical path length in both
Fig. 5-11. Difference curve representing a phase shift of $2 \times 10^{-3}$ radians produced by the environmental noise.
arms of the interferometer. For example, a physical displacement of 0.1nm will produce a phase shift of about $2 \times 10^{-3}$ radians. Displacements of this kind, produced by acoustic and mechanical perturbations of the 30mm diameter plate beam-splitter, are thought to be responsible for the interferometric noise. Subsequently, this was confirmed by the replacement of the beam-splitter with an all-fibre 3dB single mode coupler (became available in 1983) which reduced the minimum detectable phase shift to the digitisation limit for the photodiode array system. The magnetic field results, which will be reported in this thesis, were taken using the plate beam-splitter.

The output of the interferometer as a function of applied magnetic field for the various sensor heads is shown in Figure 5.12. These results are summarised in Table 5.3. The sensitivity of the interferometer phase difference to magnetic fields has been defined for each sensor head using equation 5.10, and the minimum detectable magnetic fields determined for the system noise of $2 \times 10^{-3}$ radians.

The phase shift as a function of magnetic field for the sensor head which has been sputter coated with a 5 micrometre film of Fe$_{80}$B$_{20}$ is also plotted in Figure 5.13, together with the data published by Forester et al$^{88}$ on the magnetostriction of Fe$_{80}$B$_{20}$. It can be seen that the fit is very good and that magnetic saturation occurs in a similar field. This implies that the composition of the coating is close to the expected ratio. The sensitivity of $10^2$ radians T$^{-1}$m$^{-1}$ agrees well with the result obtained on the same interferometer by Willson and Jones.$^{92}$ However, the small quantity of amorphous alloy compared with the bulk of the optical fibre obviously reduces the coupling of the magnetostrictive strain into the core of the fibre and limits the sensitivity. The effect of the coating thickness on the sensitivity can be calculated by equating the force exerted on the metal jacket by the fibre to the force exerted by
Phase shift (x 10^3 radians)

Applied field (x 10^-4 Tesla)

Sensitised length

+ Linear metallic glass ribbon sensor (b) 30mm
☐ Linear sensor with fibre bonded to both sides of ribbon 60mm
● Ribbon and fibre bonded to circular glass former (d) 160mm
● Sputter coated fibre (a) 50mm

Fig. 5-12. Field dependences of phase shift
### TABLE 5.3

**COMPARISON OF THE SENSITIVITIES OF VARIOUS SENSOR HEADS**

<table>
<thead>
<tr>
<th>Sensitivity (rads/tesla.m)</th>
<th>Minimum Detectable Field (tesla.m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) 5μm Fe$<em>{80}$B$</em>{20}$ sputter coated on 125μm fibre. (fibre length = 50mm)</td>
<td>$1 \times 10^2$</td>
</tr>
<tr>
<td>b) 30mm Vitrovac 4040 bonded to 125μm fibre</td>
<td>$2.5 \times 10^4$</td>
</tr>
<tr>
<td>c) 65mm Vitrovac 6025 bonded to 125μm fibre</td>
<td>$3 \times 10^2$</td>
</tr>
<tr>
<td>d) 160mm Vitrovac 4040 wrapped on circular glass former. One turn of 125μm fibre bonded to the Vitrovac</td>
<td>$2 \times 10^2$</td>
</tr>
<tr>
<td>e) 160mm Vitrovac 4040 wrapped on circular plastic former. One turn of 125μm fibre bonded to the Vitrovac.</td>
<td>$1 \times 10^4$</td>
</tr>
<tr>
<td>f) 0.6μm Metallic Glass on 80μm fibre 3 $\times 10^2$ (fibre length = 100mm)</td>
<td>$3 \times 10^{-9}$</td>
</tr>
</tbody>
</table>

Item f) gives the results of Dandridge et al$^{82}$ for comparison. Their minimum detectable field is obtained for an a.c. field and phase sensitive detection.
Fig. 5:13 Fibre optic phase shift versus applied magnetic field.

Phase shift (x 10^-3 radians) vs. Magnetic field (x 10^-4 Tesla).

- Forester et al (1978)
by the metallic jacket. The strain created in the fibre-coating composite is given by:

\[
\frac{\Delta \ell}{\ell} = \frac{\Delta \ell}{\ell} \left( \frac{1}{E} \frac{A}{A + \frac{E}{E}} \right)
\]

5.22

where \( \Delta \ell \) is the magnetostrictive strain and the values of Young's modulus, \( E \), and the cross-sectional area, \( A \), for the materials are given in Table 5.4. In this calculation it has been assumed that there is no slippage at the metal-glass interface. Hence, in the case of a thin metal coating (<50μm) the strain transmitted to the fibre is largely dependent upon the ratio of Young's Moduli of the respective materials which in this case is approximately \( \frac{1}{2} \) (see Table 5.4). For the 5 micrometre Fe\(_{80}\)B\(_{20}\) coating the fibre-jacket composite extends by about 0.3 of the jacket material alone according to equation 5.22. For this coating the strain in the metal can be considered equal to strain in the fibre (bulk coating) for coating thicknesses in excess of about 50μm. Jarzynski et al. have reported that the restraining forces of the glass fibre reduce the sensitivity for Permalloy jackets which are less than 25μm on 80μm silica fibres. Conversely, another approach to improving the sensitivity would be to reduce the diameter of the cladding, either by etching or by pulling a complete reel of fibre specifically for this application. Unfortunately, the reduced cladding approach is likely to increase the vulnerability of the sensor fibre to breakage.

An additional increase in sensitivity can be obtained simply by increasing the length of the coated fibre. Thus, using a coated length of 50m will give an increase in sensitivity of three orders of magnitude.

The sensitivity of the interferometer was improved when the signal fibre was bonded directly to a 30mm length of Vitrovac 4040 metallic class [Table 5.3(b)]. Figure 5.12 shows this improvement
<table>
<thead>
<tr>
<th>Material</th>
<th>Diameter (µm)</th>
<th>Young's Modulus E($10^8$ N/m$^2$)</th>
<th>Cross-sectional Area A(m$^2$)</th>
<th>E.A. (N/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Core and cladding (silica)</td>
<td>125</td>
<td>730</td>
<td>$1.2 \times 10^{-8}$</td>
<td>$8.8 \times 10^{-6}$</td>
</tr>
<tr>
<td>Fibre with Fe$<em>{80}$B$</em>{20}$</td>
<td>135</td>
<td>1690</td>
<td>$2.3 \times 10^{-9}$</td>
<td>$3.9 \times 10^{-6}$</td>
</tr>
<tr>
<td>P.V.C.</td>
<td>30</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Glass</td>
<td>800</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
clearly, with magnetic saturation occurring in a magnetic field of about $2 \times 10^{-3}$ T. This agrees well with the value for the saturation field of the Vitrovac '4040' metallic glass which is shown in Figure 5.2. Kersey et al.\cite{Kersey95} have also reported observing magnetic saturation in an applied field of about $10^{-3}$ T for a Vitrovac '4040' linear sensor head. The difference curve shown in Figure 5.10 corresponds to a phase shift of 1 radian and is produced by the saturation field. The sensitivity is $2.5 \times 10^4$ radians T$^{-1}$ m$^{-1}$ and the minimum detectable magnetic field is $1 \times 10^{-7}$ T.m for the interferometer noise level of $2 \times 10^{-3}$ radians. The improvement achieved in the sensitivity (two orders of magnitude) is surprising and is considered to reflect the improved coupling of strain to the fibre core from the 10mm wide and 25 micrometre thick metallic glass ribbon which is large compared to the fibre. It was subsequently found that another effect was contributing to the sensitivity and this will be discussed later.

The directional response of a linear sensor head was investigated by measuring the sensitivity as a function of the angle, $\theta$, between the magnetic field and magnetostrictive axis of the fibre (axial direction). Figure 5.14 shows that the interferometer output approximately follows a $\cos \theta$ response predicted by equation 5.4. This is a result of the line integral response of the sensitised fibre which was defined in equation 5.4. (i.e. $\int B \, d\zeta$). The sensitivity was at a maximum when the fibre was aligned with the axial field and approached zero when the fibre was perpendicular to the field. Similar results on the directional sensitivity of nickel coated fibres have been reported by Lenz and Mitchell\cite{Lenz86}.

The use of the metallic glass 'Vitrovac' 6025, which has a higher permeability but lower magnetostriction (0.5 x 10^{-6} at $B_{sat}$),
Fig. 5.14. Sensitivity as a function of angle between the magnetic field and Vitrovac 4040 linear sensor head.
in a 65mm long sensor head resulted, predictably, in a lower sensitivity of $3 \times 10^2$ radians $T^{-1}m$. The phase shift as a function of magnetic field is shown in Figure 5.15. The minimum detectable magnetic field was $7 \times 10^{-6}T.m$ and magnetic saturation occurred at approximately $4 \times 10^{-3}T$.

In an effort to improve sensitivity while retaining small geometric size, a cylindrical sensor head was tried. The performance of the sensor head consisting of 160mm of 'Vitrovac' 4040 metallic glass bonded to a cylindrical glass former is shown in Figure 5.12. The response is representative of typical magnetostrictive behaviour for FeNi amorphous alloys with magnetic saturation occurring at about $2 \times 10^{-3}T$. The sensitivity of $2 \times 10^2$ radians $T^{-1}m^{-1}$ is of the same order of magnitude as the original Fe$_{80}$B$_{20}$ sputter coated fibre (see Table 5.3(d)) and the minimum detectable magnetic field is $1 \times 10^{-5}T.m$. This sensitivity is inferior to that of the fibre bonded to the free ribbon in the linear sensor head and may be attributed to the restraining forces of the glass former acting on the metallic glass. An improved sensitivity of $10^4$ radians $T^{-1}m^{-1}$ was obtained with an identical sensor head geometry, but constructed on a plastic former (Table 5.3(e)). This improvement was expected because the Young's modulus of the plastic former was almost two orders of magnitude less than the glass former modulus (Table 5.4). This sensor head had a minimum detectable field of $2 \times 10^{-7}T.m$ which was still inferior to that of the free ribbon linear sensor head.

In a practical magnetometer where the field to be detected is not toroidal the cylindrical sensor heads will have a reduced magnetostrictive response because of the $\int B \cdot d\ell$ directional dependence. Therefore a long 'barge' shaped mandrel with many turns of fibre is expected to produce the most satisfactory sensor head since the majority of
Sensing fibre characteristics: 65mm of "vitrovac" 6025 metallic glass bonded to single mode fibre

Note: "vitrovac" 6025:

$\text{Co}_{66}\text{Mo}_2\text{Fe}_4\text{Si}_{16}\text{B}_{12}$ composition

Data point error $\sim \pm 2$ milliradians

Fig. 5.15. Fibre optic phase shift versus applied magnetic field
sensitised fibre can then be aligned with the applied field.

The extremely high sensitivity of the linear sensor head (Table 5.3(b)) for just 30mm of fibre bonded to the metallic glass is clearly anomalous, and further experiments were carried out to determine the cause. The signal fibre was bonded to both sides of a 30mm length metallic glass ribbon. This would be expected to produce twice the sensitivity, but the actual phase shift observed in an applied field is shown in Figure 5.12 (curve represented by square data points). The linear behaviour, followed by saturation, seen in other sensors is not observed. This may be understood if the ribbon is assumed to bend in the applied field. The bonding of the fibre to the ribbon is then assumed to be more efficient on one side than on the other. As the ribbon bends, the fibre on the better bonded side will change its length. With a further increase in field, the less well bonded fibre begins to change its length, but in the opposite direction, and so the net phase shift along the total sensitised fibre length decreases. In the saturation limit, the phase shift approaches that of a purely magnetostrictive sensor.

The bending effect is a result of the motion of the domain walls through the material during magnetisation coupled with a tendency for a high permeability material to line up with an applied field. In a material with magnetisation anisotropy, such as the metallic glass ribbon, which is also very flexible, such bending is very likely to happen. In order to test this hypothesis, an optical lever type experiment was set up using a laser beam reflected from the ribbon which was subjected to a magnetic field. The deflection of the metallic glass was measured by monitoring the reflected beam on a photodetector. Considerable deflection was observed, and various mechanical resonances were apparent as the frequency of the applied
field was increased. The effect of these resonances on sensor head performance in a.c. magnetic fields has been investigated in an alternative Mach-Zehnder magnetometer detection system which will be discussed in 5.3.

An examination of the literature published on magnetic field interferometric sensors using linear magnetostrictive stretchers reveals that resonance peaks in the sensitivity versus magnetic field frequency curves have been reported. This work has involved detecting small a.c. magnetic fields, typically in the frequency range from 100Hz to 10kHz, superimposed on d.c. bias fields. Koo and Sigel have investigated the response of Allied Chemical Corporation metallic glass strips bonded to fibres in linear configurations. They have reported sharp resonances in the frequency range from 200Hz to 6kHz and obtained a minimum detectable magnetic field of $10^{-12}$ T.m at frequencies where the amplitude of the 1/f noise is small using phase-sensitive-detection. The origin of these resonance peaks was not identified. In magnetostrictive measurements on similar metallic glass sensor heads, Trowbridge et al have also observed resonance effects.

5.2.5. Conclusions

A modified Mach-Zehnder interferometer has been constructed and used to detect small d.c. magnetic fields. Particular emphasis has been placed on developing a detection system for measuring d.c. or slowly varying magnetic fields in the 0.01 to 10 Hz region which are likely to be encountered in practice.

The use of the Young's interference pattern generated by the interferometer to detect the magnetically induced phase shifts, as distinct from monitoring a single fringe has enabled the effects of polarisation
fluctuations in the fibres and laser intensity variations to be eliminated. To achieve this the contrast ratio and the intensity of each interference pattern examined were normalised to unity. The magnetically induced phase shift was then determined by subtracting a normalised non-zero field pattern from a previously recorded zero field pattern. This demodulation of the magnetic signal was performed by recording the position of the fringes on a photodiode array and digitising the resulting electrical signal. This signal is fed to an Apple II computer which performs the normalisation and then the subtraction. The detection system has the advantage of requiring no active compensation to retain quadrature because the sensitivity is independent of the zero-field fringe pattern, and therefore the interferometer can be electrically passive.

A number of different magnetic sensor heads have been constructed and used in the signal arm of the interferometer. The sensitivities have already been given in Table 5.3 but it is convenient to make a few comments here. The fibre with the 5 micrometre sputtered coating of Fe$_{80}$B$_{20}$ has a relatively low normalised sensitivity of $1 \times 10^2$ radians T.$^{-1}$m.$^{-1}$, but the actual sensitivity can be increased relatively easily by using longer lengths of fibre and possibly also by increasing the coating thickness. The most sensitive sensor heads used fibre bonded to magnetostrictive metallic glass, the highest sensitivity of $2 \times 10^4$ radians T.$^{-1}$m.$^{-1}$ being obtained with 'Vitrovac' 4040. It is thought that this enhanced response is brought about by mechanical distortion of the metallic glass in the magnetic field due to its magnetic anisotropy. In an effort to produce a more sensitive device in smaller volume and to eliminate the mechanical distortion a cylindrical sensor head was tried. This consists of a strip of metallic glass wound around a cylindrical glass former and a single turn of the fibre is in turn bonded
to the metallic glass. The sensitivity of $2 \times 10^2$ radians T.$^{-1}$m.$^{-1}$ is inferior to that of the fibre bonded to the free ribbon, which may be attributed to the lack of compliance of the glass former. When the glass is replaced with an identical plastic former an improved sensitivity of $10^4$ radians T.$^{-1}$m.$^{-1}$ is obtained.

It has been demonstrated that the magnetometer sensitivity depends on the cosine of the angle between the applied field and the fibre magnetostrictive axis. This may lead to applications in magnetic field line mapping where direct measurement of field line curvature is required. This could be accomplished by rotating the magnetometer and searching for maxima or minima in the output.

The interferometer minimum detectable phase shift is approximately $2 \times 10^{-3}$ radians, and the replacement of the beam splitter with an all-fibre 3dB coupler can reduce this to $8 \times 10^{-4}$ radians which is the digitisation noise limit for the photodiode array system. For this noise limit a projected minimum detectable d.c. magnetic field of $4 \times 10^{-8}$ T.m can be obtained with the 30mm 'Vitrovac' 4040 linear sensor head. Clearly, this magnetometer cannot compete with the very low minimum detectable d.c. magnetic fields obtained with devices such as the SQUID ($10^{-16}$T) or even the fluxgate magnetometer ($10^{-12}$T). Also because the detection system is independent of the zero field fringe position it cannot determine the sign of the magnetically induced phase shift and consequently the direction of the applied axial magnetic field. Therefore a new system, which is described in 5.3, has been developed in an attempt to lower the minimum detectable magnetic field and determine its direction.

5.3. The detection of d.c. magnetic fields using an a.c. bias field technique

In 5.1. it is pointed out that the most sensitive measurements
obtained with fibre optic magnetometers have been achieved for a.c. magnetic fields, a typical minimum detectable field being $10^{-11} \text{T.m.}$\textsuperscript{82} The signal recovery schemes without exception rely on the amplitude of the magnetic signal being greater than that of the interferometer 1/f noise at the chosen signal frequency. To improve the sensitivity of an optical fibre magnetometer to d.c. magnetic fields a technique has been developed for putting the signal on a carrier (frequency) and measuring its amplitude at a frequency where the amplitude of the 1/f noise is small. The approach relies upon the shape of the magnetostriction versus applied magnetic field curve which is shown in Figure 5.2. This is a non-linear, even function of magnetic field and by application of a suitable alternating bias field, the amplitude of the ambient d.c. field may be deduced by examining the response at various harmonics of the excitation frequency.

A review of this work was published in 1984 at the second international conference on optical fibre sensors in Stuttgart\textsuperscript{96}. A copy of the paper can be found at the end of this thesis.

5.3.1. D.C. detection scheme

Reference to Figure 5.2 shows that for magnetic fields well below half of the field for magnetic saturation, the magnetostriction versus axial field curve for the various materials is quadratic i.e. for $B \ll \frac{B_{\text{sat}}}{2}$, $\epsilon_3$, the axial strain is proportional to $B^2$ as defined in equation 5.5. Therefore, in a Mach-Zehnder interferometer magnetostrictive stretching of the signal arm fibre yields a differential path length change proportional to, $\Delta \ell$, and is given by

$$\Delta \ell = \ell k_2 B^2$$ \hspace{1cm} 5.23

where $\ell$ is the length of sensitised signal fibre and $k_2$ is the materials
constant for the quadratic magnetostrictive response. If \( B \) consists of a d.c. magnetic field \( B_d \) and an a.c. bias magnetic field \( B_a \sin \omega t \), then

\[
\Delta \ell = k_2 \ell [B_d + B_a \sin \omega t]^2 = k_2 \ell [B_d^2 + 2B_a B_d \sin \omega t + B_a^2 \sin^2 \omega t] \text{ for } B_d + B_a << -2 \frac{B_{\text{sat}}}{2}
\]

5.24

The output at \( \omega \) is \( 2k_a B_d \sin \omega t \) and phase sensitive detection is possible. Hence in the small field regime the magnetostrictive output at the bias field frequency is directly proportional to \( B_d \) for a fixed \( B_a \) or proportional to \( B_a \) for a fixed \( B_d \). The basic principles of this magnetic modulation scheme are illustrated in Figure 5.16. Diagram (a) in this Figure shows that in zero d.c. magnetic field an a.c. bias field at frequency \( \omega \) modulates the magnetostriction of the material at twice the bias field frequency. The effect of an additional small d.c. field on the modulation is shown in diagram (b). The magnetostrictive response now contains a component at the bias field frequency which is proportional to the d.c. field amplitude for a fixed a.c. field amplitude. A detailed analysis of the Mach-Zehnder interferometer output resulting from this magnetic modulation scheme follows.

The phase shift experienced by the light propagating in a magnetically sensitised signal fibre for a quadratic magnetostrictive response is calculated by substituting equation 5.23 into equation 5.3 (this equation represents the phase shift induced by a strain \( \varepsilon_3 \)) and is given by:

\[
\Delta \phi = \frac{2\pi \ell 0.92k_2B^2}{\lambda}
\]

5.25

This expression assumes that the strains in the fibre core are equal to the strains in the jacket material and neglects magnetic field terms perpendicular to the axial direction (see 5.1.1). If the total applied field comprises of an a.c. bias field of \( B_a \sin \omega t \) and a d.c. field
Fig. 5.16. Detection scheme for a d.c. magnetic field

(a) Bipolar bias field (zero d.c. field)

(b) Bipolar bias field (small d.c. field present)
B_d, the phase shift experienced by the light propagating in the signal
fibre becomes

$$\Delta \phi = Q\left[ B_d^2 + B_a^2 \sin^2 \omega t + 2B_aB_d \sin \omega t \right]$$  \hspace{1cm} (5.26)

where $Q = \frac{2\pi x \cdot 0.92k^2}{\lambda}$. In addition to the length change due to
magnetic effects, low frequency pressure and temperature variations
perturb the optical path length difference. Since the desired magnetic
signal at frequency $\omega$ is in a different frequency band to these noise
effects the active homodyne demodulation schemes described in 3.2 can be
used. Therefore, a PZT phase modulator is used to produce an error
phase signal, $\phi_e$, to maintain the quadrature condition for phase signals
below 20Hz. These include the d.c. magnetic terms as well as the
environmental phase drifts $\phi_s$ and $\phi_r$ of the signal arm and reference arm
respectively. A passive demodulator was not considered because electrical
connections were needed for the bias field and so the extra electronic
complexity of a passive scheme was not justified.

The interferometer output which is a direct measure of the phase
difference between the two fibre arms is given by equation 3.5 and for
the case of the magnetic signal given in equation 5.26 takes the form:

$$I = I_o \left[ 1 + \cos \left( 2QB_aB_d \sin \omega t - \frac{QB_a^2}{2} \cos 2\omega t \theta \right) \right]$$  \hspace{1cm} (5.27)

where $\theta = \frac{QB_a^2}{2} + QB_d^2 + \phi_s - \phi_r - \phi_e$ represents the low frequency terms.
This signal can be represented in terms of its Fourier components by
using the appropriate Bessel expansions as shown in Appendix G(i) and
these components are:

$$I = I_o \left\{ 1 + \sum_{n=1}^{\infty} J_0 \left( \frac{QB_aB_d}{2} \right) \cos 2n\omega t \right\} \hspace{1cm} (5.27)$$

$$* \left[ J_0 \left( \frac{QB_a}{2} \right)^2 \sum_{n=1}^{\infty} (-1)^n J_{2n} \left( \frac{QB_a}{2} \right) \cos 4n\omega t \right]$$

$$* \cos \theta$$
\[ + 2 \sum_{n=0}^{\infty} J_{2n+1} \left( \frac{2QB_a}{ad} \right) \sin (2n+1)\omega t \]

\[ \times 2 \sum_{n=0}^{\infty} (-1)^n J_{2n+1} \left( -\frac{a}{2} \right) \cos (2n+1) 2\omega t \]

* \cos \theta

\[ - 2 \sum_{n=0}^{\infty} J_{2n+1} \left( 2QB_aB_d \right) \sin (2n+1)\omega t \]

\[ \times \left[ J_0 \left( -\frac{a}{2} \right) + 2 \sum_{n=1}^{\infty} (-1)^n J_{2n} \left( -\frac{a}{2} \right) \cos 4n\omega t \right] \]

* \sin \theta

\[ + \left[ J_0 \left( 2QB_aB_d \right) + 2 \sum_{n=1}^{\infty} J_{2n} \left( 2QB_aB_d \right) \cos 2n\omega t \right] \]

\[ \times \sum_{n=0}^{\infty} (-1)^n J_{2n+1} \left( -\frac{a}{2} \right) \cos (2n+1) 2\omega t \]

* \sin \theta

5.28

When \( \theta \) is equal to \( \frac{\pi}{2} + m\pi \) the interferometer is at quadrature and the cos \( \theta \) terms are zero and the sin \( \theta \) terms are unity. Hence in this case the function of the active homodyne demodulator is to produce a phase error, \( \phi_e \), which compensates for the d.c. magnetic terms and the environmental drifts such that:

\[ \theta = \frac{QB_a}{2} + QB_d^2 + \phi_s - \phi_r - \phi_e = \frac{\pi}{2} + m\pi \]

5.29

From equation 5.28 it is clear that when \( B_d \) is zero only odd harmonics of \( 2\omega \) survive i.e. every other even harmonic of \( \omega \) since \( J_n(0) = 0 \) for all \( n \) except \( n = 0 \) in which case \( J_0(0) = 1 \). If a d.c. field, \( B_d \), is introduced
odd harmonics of $\omega$ in addition to the even ones are present in the output signal. These components, as can be seen from equation 5.28, are functions of $B_d$. Hence the a.c. bias field enables the d.c. signal to be extracted at $\omega$ using phase sensitive detection. The magnitude of this signal, $J_1(\phi)$, where $\phi$ is equal to $2 Q_b B_d$, is proportional to $B_d$ for $\phi < 0.1$ radians as explained in 3.1.5. The a.c. bias field can be at any frequency above the compensator bandwidth, a typical value being 1kHz. This is high enough to extract the signal of interest from the $1/f$ noise, whilst still giving good response from the magnetostrictive material. Metallic glasses operate at frequencies up to approximately 10kHz, above this frequency the permeability begins to fall off resulting in a reduced magnetostriction.

An important feature of this system is that it can provide information about the sign of the ambient d.c. field, whereas the straightforward magnetostrictive sensor described in 5.2., because of the square law dependence of the magnetostriction, can only give the magnitude of the field. From Figure 5.16 it can be seen that a reversal in d.c. field polarity results in a $180^\circ$ phase shift in the modulation of the magnetostriction. Hence the sign of the d.c. field is easily determined by measuring the interferometer output signal with a lock-in amplifier, a positive field results in a signal which is in phase with the a.c. bias field and a negative field results in a signal which is $180^\circ$ out of phase with the bias field.

If an a.c. magnetic field signal $B_s \sin \omega_s t$, is applied as well as the a.c. bias field and the d.c. field the resulting modulation of the magnetostriction has additional frequency components at $\omega_s, 2\omega_s$ and $\omega_s \omega_s$ as shown in Appendix G(ii). This modulation will generate additional Fourier components in the interferometer output. From Appendix G(ii) it can be seen that the amplitude of the sidebands on the bias field frequency ($\omega_s \omega_s$) are
proportional to the a.c. signal field strength. Hence, the magnetometer can recover both a.c. and d.c. magnetic fields.

5.3.2. Minimum detectable magnetic field for a quadratic magnetostrictive response

An expression for the minimum detectable a.c. magnetic field in a shot-noise-limited Mach-Zehnder interferometer has been given in equation 5.16. The equation is derived for the case where the magnetostrictive response to an applied magnetic field is linear. The total applied field consists of a d.c. field to bias the magnetostrictive material to its point of maximum sensitivity and an a.c. signal field. In the present d.c. magnetometer the quadratic region of the magnetostriction versus applied field curve is utilised as explained in 5.3.1. The total applied field consists now of an a.c. bias field and a d.c. signal field resulting in a magnetostriction of \( 2k_B B_a B_d \) at the bias field frequency, \( \omega \), as defined in equation 5.24. This magnetostriction, in turn, produces a phase sensitivity to magnetic fields at the bias field frequency (see equation 5.26) which is given by:

\[
\Delta \phi \mid_{\omega} = \frac{2\pi n}{\lambda} \xi \left( 2k_B B_a B_d \right) 0.92
\]

For \( n = 1.46, B_a = 4 \times 10^{-4} T, k_2 = 30(T^{-2}) \) and a nominal wavelength of 1000nm, which is near to experimental magnetometer operating wavelengths of 633nm and 850nm, the phase shift becomes:

\[
\Delta \phi \approx 2 \times 10^5 \Delta B_d (T) \xi (m)
\]

The minimum detectable magnetic field for a shot-noise-limited Mach-Zehnder interferometer can be obtained using the method described in 5.1.2. Hence, when the interferometer is at quadrature the signal to noise electrical power ratio is obtained by substituting the phase...
sensitivity of the quadratic system (equation 5.32) into equation 5.14

\[
\frac{s}{N} = \frac{10^{10} P_n(\Delta B_d(T))^2 k^2(m)}{h v \Delta F}
\]

Equation 5.35 is used to calculate the shot-noise-limited minimum detectable d.c. magnetic field (on its carrier frequency, \(\omega\)) which is \(1 \times 10^{-12}\) T.m.

It has been assumed that \(P = 30 \times 10^{-6}\) \(\omega\), \(\xi = 1m\), \(\eta = 0.5\) and \(v = 3 \times 10^{14}\) Hz \((\lambda = 1000\)nm\) and the detection bandwidth \(\Delta F\) is 1Hz. Hence this magnetometer is potentially capable of detecting a d.c. magnetic field of \(10^{-12}\) T for a one metre length of magnetostrictively coupled fibre at a 0-1Hz bandwidth.

5.3.3. The magnetometer system

Two fibre optic d.c. magnetometer systems were built, one operating at 633nm with a Tropel 200 HeNe laser light source and the second operating at 850nm using a frequency stabilised STL diode laser light source. Both of these systems consisted of a conventional all-fibre Mach-Zehnder interferometer with fibre directional couplers performing the beam splitting and recombining functions as described in Chapter 3. A schematic diagram of the magnetometer is shown in Figure 5.17, and a photograph is shown in Figure 5.18. The interference fringes were detected at the two output ports of the recombining coupler with a pair of silicon PIN photodiode detectors. The sensing fibre is bonded to Vacuumschmelze metallic glass forming the magnetic sensor head. Two sensor head configurations were employed, one using a single 0.1 m length of fibre bonded onto the metallic glass strip, the second with ten passes of the fibre along a similar strip of metallic glass. The interferometer used active homodyne demodulation which involved feedback to a PZT phase modulator as described in 3.2. The bandwidth of the compensator was from zero to 20Hz so the system retained sensitivity to signal fields above this frequency.
C.S. = Two coaxial solenoids
M.S. = Mode stripper
D.A. = Differential amplifier
I = D.C. current supply

Fig.5.17. Fibre optic Mach-Zehnder magnetometer.
Fig. 518 Optical Fibre Magnetometer.
A Helmoltz coil and a solenoid, mounted coaxially, were used to generate magnetic fields along the axis of the magnetically sensitive fibre. One supplied a d.c. field and the other an a.c. bias field. The resulting interferometer output signal can then be observed on either of the photodiode outputs. The magnitudes of the interferometer output frequency components were monitored using a spectrum analyser and a lock-in amplifier was also used to determine the magnitude of the signal in phase with the applied a.c. bias field. This latter feature was found particularly useful in determining the sign of the d.c. field, as a negative field results in a signal 180° out of phase with the bias field.

5.3.4. Performance of the magnetometer

The magnetometer response was examined by applying a 352Hz a.c. bias field and a small variable d.c. field. The applied d.c. field amplitude ranged from 0 to 10⁻⁴T. This ensured that the metallic glass remained within the quadratic region of its magnetostriction versus applied field curve. Consequently, the amplitude of the a.c. bias field was chosen to be greater than 10⁻⁴T (typically 4 x 10⁻⁴T) so that it remained bipolar for the range of applied d.c. fields. Hence, the desired magnetic response described in 5.3.1 and represented by equation 5.28 can be achieved. Figure 5.19 shows the electrical current supplied to the solenoid producing the a.c. bias magnetic field of 4 x 10⁻⁴T at 352Hz (upper trace) and the resulting interferometer output (lower trace) as measured on an oscilloscope for a d.c. field amplitude of 1 x 10⁻⁴T. This Figure demonstrates clearly that the interferometer output has a spectral component at the bias field frequency. The effect on the interferometer output when the polarity of the applied d.c. field is changed is shown in Figure 5.20. There is a 180° phase shift in the output at
Fig. 5-19. An oscillogram showing the magnetometer output (lower trace) produced by an a.c. bias magnetic field at 352 Hz in the presence of a d.c. field. The upper trace shows the electric current supplied to the solenoid to create the bias field.

Fig. 5-20. An oscillogram of the magnetometer output produced by an applied field of \((B_a + B_d)\) showing the effect of reversing the sign of the d.c. field. A positive d.c. field (upper trace) generates an output which is in phase with the a.c. bias field at 352 Hz. A negative field (lower trace) generates an output 180° out of phase with the bias field.
the bias field frequency. Inspection of Figure 5.16 shows that this phase shift should be expected and therefore the sign of the interferometer output on a phase sensitive detector (lock-in) varies with the direction of the d.c. field.

The spectral output at the bias field frequency \((f = 352)\) and its second harmonic \((2f = 704\text{Hz})\) were measured on a spectrum analyser as shown in Figure 5.21. When the d.c. field was set to zero there was still a spectral component at the bias field frequency in the interferometer output as shown in Figure 5.22. This corresponded to the amplitude of the resolved component of the earth's magnetic field, \(B_e\) \((10^{-5}\text{T})\) in the direction of the interferometer magnetostrictive axis. The interferometer output due to this component was removed by the application of an additional d.c. field of equivalent amplitude but in the opposite direction along the magnetostrictive material. When the earth’s field is perfectly nulled \((\text{i.e. } B_e - B_d = 0)\) there is no signal at the bias field frequency \((\text{in this case at 610Hz})\) as shown in Figure 5.23. The spectral components at frequencies higher than \(2f\) will be discussed later. The earth’s field has been nulled using this technique in the following measurements.

The amplitude of the output at the bias field frequency was directly proportional to d.c. magnetic fields up to \(10^{-4}\text{T}\). These measurements were obtained at an a.c. bias field frequency (carrier frequency) of 1700Hz using a lock-in amplifier operating at a 0-1Hz bandwidth. This data is shown in Figure 5.24 for the 0.1m and the 1m sensor head which have sensitivities of \(1 \times 10^3 \text{ radians T}^{-1} \ (0.5 \times 10^3 \text{V.T}^{-1})\) and \(4 \times 10^3 \text{ radians T}^{-1} \ (2 \times 10^3 \text{V.T}^{-1})\) respectively. The anticipated factor of 10 improvement was not achieved because of the difficulty in establishing uniform strain coupling between the fibre and metallic glass over long lengths of fibre. Once the d.c. field reached \(10^{-4}\text{T}\) the magnetostrictive response to the applied d.c. field of the metallic glass begins to depart from the quadratic form (as shown in Figure 5.2) and the amplitude of the output
Fig. 5.21. A spectrum of the magnetometer output produced by an applied a.c. bias magnetic field at 352 Hz (f) and a d.c. field. The signal component at f is a measure of the d.c. magnetic field.

Fig. 5.22. A spectrum of the magnetometer output produced by an applied a.c. bias magnetic field at 352 Hz (f) in zero applied d.c. field. The output at f is due to the resolved component of the earth's field along the magnetostrictive axis.
Fig. 5.23 A spectrum of the magnetometer output produced by an a.c. bias field at 510 Hz (f) when the response to the earth's field has been magnetically nulled showing no signal component at the bias field frequency f.
at the bias field frequency saturates. This linear dependence of the interferometer output at the carrier frequency upon applied d.c. field is predicted by equations 5.24 and 5.28 which are valid for a quadratic magnetostrictive response.

Figure 5.23 shows some interesting features of the magnetometer spectral response. A 610Hz a.c. bias field in zero d.c. field produces spectral components in the interferometer output at 1220Hz(2f), 2440(4f) and 3660Hz(6f). The components at 2f and 6f are expected since in zero d.c. field ($B_d = 0$) equation 5.28 predicts that odd harmonic components of $2\omega$ will be generated. The small spectral component at 4f is thought to be a consequence of mechanical resonances in the magnetic sensor head which occur at certain bias field frequencies. These resonances cause the strain versus magnetic field curve of the metallic glass to depart from its quadratic form thereby generating additional harmonics.

Figure 5.25 shows that at a selected bias field frequency of 233Hz (zero d.c. field) the magnetic response of the material generates the spectral components predicted by equation 5.28 at 2f, 6f and 10f. Therefore, the magnetically induced strain is quadratic at this frequency. The result of applying a d.c. field as well as the bias field generates spectral components at f, 3f, 4f, 5f, and 8f according to the spectrum shown in Figure 5.26. Equation 5.28 predicts that in the presence of two magnetic fields, components at each harmonic of the bias field frequency should be observed. Therefore, the components at 7f and 9f which are not visible in Figure 5.26 are probably obscured by the system noise floor.

The amplitude of the interferometer output at the bias field frequency as a function of bias field frequency is shown in Figure 5.27. The bias field amplitude was at its optimum of $4 \times 10^{-4}$T and the applied d.c.
Fig. 5:24. Interferometer output versus d.c. magnetic field
• 0.1m of magnetically sensitive fibre
° 1m of magnetically sensitive fibre
a.c. bias field: $4 \times 10^{-4}$ tesla at 700Hz
Fig. 5.25 A spectrum of the magnetometer output produced by an a.c. bias field at 233 Hz (f). Signal components at 2f, 6f and 10f are shown.

Fig. 5.26 A spectrum of the magnetometer output produced by an a.c. bias field at 235 Hz (f) and a d.c. field. Signal components at f, 2f, 3f, 4f and 5f are shown.
Fig 5.27 Interferometer output signal at the bias field frequency versus a.c. bias field frequency

- d.c. magnetic field = $10^{-5}$ tesla
- a.c. bias field = $4 \times 10^{-4}$ tesla
field was $10^{-5}$ T. The low frequency cut-off in sensitivity was due to the detection scheme, which compensated for drifts and phase signals below 20Hz and the high frequency cut-off reflected the decrease in permeability of the metallic glass at higher frequencies. Resonances were observed in the sensitivity from 100 Hz to 5 Hz which were associated with the mechanical geometry of the sensor head. This frequency dependent sensitivity in linear metallic glass sensor heads has been reported in 5.2.4, and has also been observed by other authors. The origin of these resonance peaks in linear sensor heads has not been clearly identified. In a material with magnetisation anisotropy, such as the metallic glass, which is also very flexible, mechanical resonances are likely during magnetisation. A smoother frequency response was obtained by bonding the linear sensor head to a glass slide. This result is shown in Figure 5.28. However, the sensitivity to magnetostrictive strain was reduced by the restraining forces of the glass acting on the metallic glass.

The sensitivity of the magnetometer output at the bias field frequency as a function of bias field amplitude was investigated. The d.c. magnetic field amplitude was kept constant at $10^{-6}$ T, and the bias field varied from 0 to $6 \times 10^{-4}$ T. The interferometer output was proportional to the bias field magnitude within the quadratic region of the magnetostriction curve, as shown in Figure 5.29. This is expected because the magnetostrictive response is proportional to the product of $B_a B$ (see equation 5.24). The output signal at the bias field frequency saturated at an approximate field amplitude of $4 \times 10^{-4}$ T. This saturation corresponded to the magnetostriction versus applied field curve evolving from a quadratic to a linear dependence. It is important to realise that when the a.c. (bipolar) bias field becomes unidirectional, which in the present case occurs for a.c.
Fig. 5.28. Interferometer output at the bias field frequency versus a.c. bias field frequency for a linear sensor head which is bonded to a glass slide.
Interferometer output (x10^4 radians)

d.c. magnetic field = 10^-6 T
a.c. bias field frequency = 1700 Hz

Fig. 5.29. Interferometer output versus a.c. bias field amplitude.
field amplitudes < $10^{-6}$ T, then the interferometer output at the bias field frequency becomes non-linear as a function of both a.c. and d.c. field amplitude. Kersey et al.\cite{97,98} have reported a detection scheme for d.c. magnetic fields which relies upon the non-linear shape of the magnetostriction curve and utilises this to amplitude modulate the magnetostriction produced by a unidirectional a.c. bias magnetic field. This technique will be discussed in more detail in Chapter 6.

Figure 5.30 shows the output spectrum of the magnetometer (amplitude of the spectral signal in dBV) when the magnetically sensitive fibre was exposed to d.c. field of $10^{-6}$ T and an a.c. signal magnetic field of $10^{-6}$ T at 200Hz. The a.c. bias field frequency, $f$, (carrier frequency) is 1700Hz. As predicted by equations 2 and 6 in Appendix G(ii) the d.c. and a.c. signal fields were identified by the presence of spectral signals at $f$ and $f \pm f_s$, respectively. The amplitude of the side bands at 1700 ± 200Hz was proportional to the a.c. signal field from 0 to $10^{-4}$ T. This data is shown in Figure 5.31 for each side band. The interferometer output began to saturate at $10^{-4}$ T because the magnetostriction of the metallic glass departed from its quadratic form at this field. The a.c. magnetic field sensitivity was $6 \times 10^2$ radians T$^{-1}$ at 1500Hz ($f - f_s$) and $1.9 \times 10^3$ radians T$^{-1}$ at 1900Hz ($f + f_s$). The different sensitivity to the a.c. signal field shown by each side band (carrier frequency) was probably caused by the non-uniform frequency response of the sensor head.

5.3.5. **Measured minimum detectable magnetic field**

The magnetometer minimum detectable d.c. magnetic field was measured on a lock-in amplifier at a 0 to 1Hz bandwidth. The results for two systems will be given, one using a HeNe laser as the light source and the other a frequency stabilised diode laser. These results are summarised in Table 5.5. The result for the HeNe system is taken at an a.c. bias
Fig. 5:30 A spectrum of the magnetometer output produced by an a.c. signal magnetic field of $10^{-6}T$ at 200Hz($f_s$) and a d.c. magnetic field of $10^{-6}T$ at a bias field frequency of 1700Hz($f$). The signal component at $f$ is a measure of the d.c. field and the signal component at the mixed frequencies of $f \pm f_s$ (side bands) are measures of the a.c. field.
Fig. 5.31 Interferometer output versus a.c. magnetic field.
TABLE 5.5

COMPARISON OF MAGNETOMETER PERFORMANCE FOR THE TWO LASER SOURCES

<table>
<thead>
<tr>
<th>Laser source</th>
<th>Length of sensitised fibre (m)</th>
<th>Measured minimum detectable field (tesla Hz$^{-1}$)</th>
<th>Normalised minimum detectable field (tesla m.Hz$^{-1}$)</th>
<th>Interferometer Resolution (radians)</th>
</tr>
</thead>
<tbody>
<tr>
<td>HeNe</td>
<td>(a) 0.1</td>
<td>$5 \times 10^{-9}$</td>
<td>$5 \times 10^{-10}$</td>
<td>$10^{-6}$</td>
</tr>
<tr>
<td>STL diode</td>
<td>(b) 0.1</td>
<td>$5 \times 10^{-8}$</td>
<td>$5 \times 10^{-9}$</td>
<td>$10^{-5}$</td>
</tr>
</tbody>
</table>
field frequency (carrier) of 1700Hz which is where the amplitude of the interferometer 1/f noise is much reduced as shown in Figure 5.30. For a signal to noise ratio of unity, 0.1m of magnetically sensitive fibre detects a d.c. field of $5 \times 10^{-9} \text{T Hz}^{-1}$ at the bias field frequency (Table 5.5(a)). Assuming a proportional improvement for 1 metre of magnetically sensitive fibre the minimum detectable magnetic field becomes $5 \times 10^{-10} \text{T m Hz}^{-1}$. This minimum detectable field is a factor of 500 greater than the shot-noise-limited performance predicted by equation 5.32 ($10^{-12} \text{T m Hz}^{-1}$). The noise floor at 633 nm of the Mach-Zehnder interferometer used here is given in 3.2.4 and is a factor of 10 greater than the shot-noise limit. Also, recent work by Cole et al suggests that the static magnetostrictive constant, $k_2$, which has been used to calculate the sensitivity is inappropriate for time varying magnetic fields. This is because the piezomagnetic strain constant of the material is dependent on the permeability which decreases with magnetic field frequency. Hartman et al have reported that for nickel coated fibres the sensitivity at 10kHz is one order of magnitude less than at 10Hz. Vacuumschmelze do not give a piezomagnetic strain constant for their metallic glass. Therefore, considering the doubt about the frequency response of the magnetostriction and the possible poor transfer of strain from the magnetic glass to the fibre by the epoxy, the poor agreement between the measured and calculated minimum detectable magnetic fields is understandable.

The minimum detectable d.c. magnetic field has also been measured
for a magnetometer powered by a frequency stabilised STL diode laser operating at a wavelength of 850nm. The laser diode was driven with a current of 168mA, while maintained at a temperature of 25 ± 0.1°C. These conditions corresponded to a single frequency optical output of 5mW as shown in Figure 2.21. In order to stabilise this frequency using the technique described in 4.5.2, 20% of the laser output power was fed into an all-fibre Fabry-Perot interferometer via a beam-splitter. The remaining 80% of frequency stabilised laser output was launched into an all-fibre Mach-Zehnder magnetometer. The amplitude of the STL diode laser frequency jitter was reduced from 38kHz to 10kHz at a detection frequency of 1kHz (see 4.5.3). This frequency stabilised output produced an interferometer noise floor at 1kHz of $10^{-5}$ radians r.m.s. for an optical path length difference ($n_z$) of 0.04m. For a signal to noise ratio of unity, a 0.1m sensor head detects a d.c. field of $5 \times 10^{-8}$ T.Hz$^{-\frac{1}{2}}$ at a bias field frequency of 1700Hz (Table 5.5(b)). Assuming a proportional improvement for 1 metre of magnetically sensitive fibre the minimum detectable magnetic field becomes $5 \times 10^{-9}$ T.m.Hz$^{-\frac{1}{2}}$. This normalised minimum detectable field is three orders of magnitude greater than the shot-noise limited performance predicted by equation 5.32 ($10^{-12}$ T.m.Hz$^{-\frac{1}{2}}$). The discrepancy is largely due to the phase noise produced by the 10kHz frequency jitter limiting the interferometer resolution to $10^{-5}$ radians r.m.s. Also the reduction of the magnetostrictive response with frequency and the poor coupling of strain from the metallic glass to the fibre by the epoxy probably contribute to the discrepancy.
5.3.6. Conclusions

A fibre optic magnetometer capable of measuring both d.c. and low frequency a.c. magnetic fields has been designed and demonstrated. The system is based on an all-fibre Mach-Zehnder interferometer and uses active homodyne demodulation. The magnetic field detection scheme utilises the quadratic dependence of the magnetostriction to impose the low frequency magnetic signals on a carrier (frequency). This prevents these signals from being obscured by the 1/f noise which is characteristic of the all-fibre interferometer.

A useful feature of the magnetometer is that its output can easily be nulled (set to zero) in the presence of environmental magnetic fields such as the earth's. This has been achieved by adjusting the amplitude and polarity of an additional d.c. bias field.

Measurements have been carried out on this magnetometer system using two interferometer light sources, a HeNe laser and a semiconductor laser. Initially, a HeNe laser was used to demonstrate the capabilities of the system. The interferometer noise floor was approximately $10^{-6}$ radians at a carrier frequency of 1700Hz resulting in a minimum detectable magnetic field of $5 \times 10^{-10} \text{T.m}$ at a 0-1Hz bandwidth for both d.c. and a.c. fields. A significant advance towards a practical fibre optic magnetometer has been achieved by using a diode laser to drive the interferometer. A noise floor of $10^{-5}$ radians r.m.s. at 1kHz was obtained for an interferometer path length difference of 0.04m using a frequency stabilised STL diode laser. This noise level resulted in a minimum detectable magnetic field at the carrier frequency of $5 \times 10^{-9} \text{T.m}$ at 0-1Hz bandwidth for both d.c. and a.c. fields.

The magnetometer powered by the diode laser has a dynamic range of five orders of magnitude which ranges from its minimum detectable magnetic field to $10^{-4} \text{T}$. At $10^{-4} \text{T}$, the magnetostriction of the Vitrovac
'4040' begins to depart from its quadratic form and the sensitivity falls off. Hence the dynamic range depends upon the magnetostrictive properties of the material and to some extent on the particular sensor head because the permeability depends on the shape of the material.

A sharp frequency dependence of sensitivity has been observed for the linear sensor heads. Any restraining forces acting on the sensor head affects the amplitude and frequency of these resonances. Future production of metallic glass with a thickness in excess of 50 micrometres may eliminate the resonances. A smooth frequency response has been obtained, at the expense of sensitivity, by bonding the linear sensor head to a glass slide. Clearly, it would be advantageous to arrange for the magnetic bias field frequency (carrier) to coincide with a sensor head resonance to exploit the enhanced sensitivity. This approach, with appropriate sensor head geometry and loading, may give a significant improvement in device performance. However, the sensor head resonance may have a temperature dependence. The present magnetometer will provide a very sensitive means for investigating the loading effect, frequency response and the magnetostrictive behaviour of the magnetic material.
CHAPTER 6

ASSESSMENT OF OPTICAL FIBRE MAGNETOMETER TECHNOLOGY

This Chapter reviews the state of the art in optical fibre magnetometer technology.

6.1. Other d.c. detection schemes

A few publications are now addressing the problems of d.c. and low frequency magnetic field detection. Kersey et al.\(^7,8\) have reported a detection technique for low frequency (d.c. to 20Hz) magnetic fields which has a sensitivity of \(10^{-10}\) T. for one metre of magnetically sensitive fibre. The system is based upon an all-fibre Michelson interferometer which is maintained at quadrature using active homodyne demodulation. This interferometer consists of an all-fibre 3dB coupler and two 3 metre lengths of single mode fibre coated with silver at their end facets. The basis of their magnetic signal recovery technique relies upon the non-linear response of the magnetostrictive material as a function of d.c. magnetic field and utilises this to amplitude modulate an alternating magnetostrictive response produced by a high frequency unidirectional magnetic field (carrier frequency). Small changes in the amplitude of the d.c. magnetic field can be detected from the resulting magnetostrictive response at the carrier frequency. With the application of alternating signal magnetic fields, side bands are observed on the carrier frequency as in a classical amplitude modulated signal. As the carrier can be at a frequency where the amplitude of the interferometer 1/f noise is small, the detection system gives good signal to noise ratio for low frequency magnetic fields. Since the quantity being detected in this scheme is the amplitude of the signal, it is not possible to determine the sign of the field causing the effect using phase-sensitive-detection. An improvement in this system has been reported by Kersey et al.\(^5\) which
involves active feedback to a solenoid producing a d.c. bias field. In this case the output of the interferometer is monitored on a lock-in amplifier and used to generate the current supply to the bias field solenoid. The feedback arrangement maintains the lock-in amplifier output at zero which corresponds to the null point in the a.c. magnetostrictive response. Changes in a d.c. signal magnetic field are, therefore, compensated for by variations in the bias field and the output of the system is taken from the error current supplied to the bias field solenoid. The closed loop operation of the system ensures a larger linear dynamic range, greatly reduces noise associated with the stability of the high frequency magnetic field and enables the sign of the signal field to be determined simply from the polarity of the error signal. A magnetic feedback loop of this type could be incorporated into the quadratic detection scheme described in 5.3. in order to extend the dynamic range. In such a scheme the null point would correspond to zero output at the bias field frequency.

It is now clear that in order to obtain the required sensitivity to small d.c. fields from optical fibre magnetometers, an alternating bias field must be applied to the magnetostrictive sensor head. This means that the system is no longer passive, as electrical connections are needed for the bias field. This, in turn, means that it is no longer necessary to achieve a passive demodulation scheme for the interferometer, although it is expected that passive demodulation will be preferred in a practical magnetometer simply to reduce the size of the device.

A fibre optic d.c. magnetometer, which uses a similar detection scheme to the one described in 5.3, has been disclosed in two recent publications. In the first publication Koo et al. have utilised the quadratic magnetostrictive response to achieve a minimum detectable d.c. magnetic
field of $10^{-10}$ T.m. at a 0-1Hz bandwidth. The system uses an all-fibre Mach-Zehnder interferometer with a diode laser source and a passive optical demodulation scheme with a 3 x 3 coupler (see 3.3). Helmoltz coils are used to apply a.c. and d.c. biasing magnetic fields to a linear sensor head consisting of a strip of Allied Chemical Corporation metallic glass bonded directly to the signal fibre. A spectrum analyser is used to measure signals, created by d.c. magnetic fields, at the output of the interferometer which are at the frequency of the alternating bias field. The output is shown to be linear up to an applied field of $2 \times 10^{-4}$ Tesla and by comparing the signal due to a known field with the noise floor of the system the minimum detectable field is determined.

The second publication by Koo and Sigel\textsuperscript{100} describes a d.c. gradient magnetometer in which both arms of the interferometer are sensitised to the same degree. This configuration responds only to differences in field between the two arms caused by small local magnetic perturbations in the presence of strong spatially uniform background magnetic fields, whereas a magnetometer measures the background field and perturbations in it. The optical demodulation and the magnetic field detection scheme of the interferometer remain identical but there are now two sets of Helmoltz coils providing the bias field both operating at the same frequency. Both the amplitudes (a.c. and d.c.) and relative phase (a.c. only) of the bias fields to the two sensor arms are carefully adjusted so that the magnetometer output at the a.c. bias field frequency is zero in the presence of the strong background magnetic field. The ability to carry out this tuning electrically rather than having to match the magnetostrictive properties and the lengths of the two sensor heads is an advantage of this system. As a result, magnetic variations in the background field which are spatially uniform across the two sensor heads
(common mode) are suppressed. The sensitivity is similar to that of the simple total field sensor, that is, $10^{-10}$ T.m. but with the base line spacing used, which in this case was 0.01m. A gradiometer's major advantage over a magnetometer is that the gradiometer's signal is only a function of asymmetries in the magnetic field and hence only a function of the magnetic anomaly which may be caused by a metallic object or a magnetic dipole at some distance $r$. In a uniform background field, the gradiometer output is zero whereas for a magnetometer the output is the background field.

6.2. Future trends

The major use of high sensitivity magnetometers such as the SQUID is in magnetic anomaly detection. Optical fibre magnetometers with d.c. sensitivities of $10^{-12}$ T.m. would appear to approach the SQUID's performance for magnetically sensitised lengths of 100 to 1000 metres. At present most optical fibre magnetometers utilise approximately 0.1m. of sensitised fibre and have their sensitivities extrapolated to much longer lengths of fibre in order to claim spectacular minimum detectable fields. Therefore it is expected that many workers will turn their attention to developing sensor heads with longer lengths of sensitised fibre so that these sensitivities can be realised. A publication by Lenz and Mitchell discloses a technique for coating long lengths of fibre by electroplating. This process involves pulling a fibre, which is bared to its cladding, through a copper solution to produce a conducting undercoating. The undercoat is needed so that electrodeposition can be started. The fibre then enters an electroplating bath in which the composition and plating current are controlled to achieve the desired film properties. They have applied coatings of Ni, NiFe and
Ni alloys over several metres of fibre, but the velocity with which the fibre is pulled through the electroplating solution and the coating thickness are not given. This process should be faster than vacuum deposition and should be capable of applying uniform thickness coatings over long lengths of fibre.

The magnetostrictive materials used in the present work for which minimum detectable fields of $10^{-10}$ T.m. have been obtained are by no means optimised in composition or preparation. It is expected that ultimate sensitivity can be further enhanced both by annealing of materials and by selection of specific alloys and metallic glasses with larger magnetostrictions. A suitable metallic glass would be the Allied Chemical Corporation 2605 alloy which has a saturation magnetostriction of $30 \times 10^{-6}$. In the non-annealed state the amorphous alloys possess random residual strains which affect the magnetic properties of the material, in particular the magnetisation and hence the magnetostriction. Hartman et al. have reported an improvement of two orders of magnitude in the magnetostrictive response for a nickel coating after a thermal anneal at $1000^\circ$C in hydrogen. This has also been confirmed by Giallorenzi et al. The optimisation of the magnetic materials by various annealing schedules should be examined in future work.

Other magnetometer system improvements may involve magnetic subtraction techniques such as those employed in the gradiometer to reduce the sensitivity to background magnetic noise. Alternatively, the magnetic response could be increased in a total field sensor by applying a magnetostrictive stretcher to the reference fibre which has an opposite coefficient of magnetostriction to that of the signal fibre. Jarzynski et al. have proposed the use of pressure insensitive coatings to reduce acoustic perturbations of the fibres. They have shown that for a nickel clad
fibre the pressure induced phase shift has two components, the change in refractive index and the change in length. These two components are opposite in sign and in principle the coating thickness can be selected to give zero acoustic sensitivity while retaining a good magnetostrictive response.

Even if the fibre optic magnetometer fails to match the SQUID performance, there are a large number of applications which fall into the $10^{-6}$ to $10^{-12}$ T. region. One novel application in this sensitivity region is the magneto-optic compass which will provide magnetic heading.

6.3. Faraday rotation in optical fibres

Finally, it is interesting to compare the magnetostrictive magnetometer technique with that of detecting magnetic fields by Faraday rotation in silica fibres. A magnetic field is applied axially to the fibre to produce a rotation in the direction of linear polarisation. This rotation is due to paramagnetic impurities and its magnitude is given by:

$$\theta = VB\ell$$

where, $V$ is the Verdet constant of the silica, $B$ is the axial magnetic field and $\ell$ is the length of fibre. A Verdet constant of approximately $\pi$ radians $T^{-1}m^{-1}$ at a wavelength of 633nm for silica fibre has been reported by Stolen and Turner. Hence, Faraday rotation is a considerably smaller effect than the magnetostrictive phase shift of $10^5$ rad $T^{-1}m^{-1}$ predicted by equation 5.9. Yariv and Windsor have considered the doping of optical fibres with paramagnetic ions to raise the Verdet constants to much higher levels than those observed in conventional silica fibres. Rare earth ions can be incorporated into the silica to
greatly enhance the Faraday effect, but constraints on the optical absorption introduced by the ions limit the enhancement that can be achieved. The Faraday approach is thus less promising than magnetostriction because it requires specially doped fibres which also need acceptable polarisation preserving properties to achieve high sensitivities.
The construction of optical fibre interferometers has allowed the extreme sensitivity of interferometric measurement to be utilised in sensors which can ultimately be deployed in practical environments. Mach-Zehnder interferometers have been built and used to detect small strain induced optical length changes. The signal frequency was arranged to be quite different from that of the environmental noise and two active homodyne demodulation systems were developed to minimise the effects of this noise by continuously maintaining the quadrature condition. In the first technique the interferometer phase difference was controlled by means of a piezoelectric phase modulator and in the second technique interferometer phase difference was controlled by tuning the laser emission frequency.

As an alternative interferometric configuration several sensors based on single mode fibre Fabry-Perot interferometers have been built and demonstrated. Such an interferometer has also been used to reduce the frequency jitter of a diode laser.

Finally, the feasibility of making optical fibres sensitive to magnetic fields by coating them with and bonding them to ferromagnetic materials has been investigated. Extremely good d.c. magnetic field sensitivities have been realised by using the magnetically sensitised fibre as the signal arm in a Mach-Zehnder interferometer. Techniques to prevent the low frequency environmental noise obscuring small d.c. magnetic signals have been developed. Initially, a technique was developed to measure magnetically induced phase shifts using the total fringe pattern of a Mach-Zehnder interferometer. This system was aimed at eliminating the effect of the polarisation fluctuations in the
fibres, but could not allow for optical path length effects which were minimised by comprehensive precautions. This approach was found to be inadequate for very high sensitivity devices to compete with conventional magnetometers such as flux gates and SQUID's. The sensitivity was improved at low frequencies by putting the magnetic signal on a carrier frequency in order to allow the measurement of signals which would otherwise be obscured by the 1/f noise characteristic of the all-fibre interferometer. This approach enabled active homodyne demodulation techniques to be used to maintain the interferometer at maximum sensitivity. In addition, the use of a diode laser to power the all-fibre interferometer configured as a magnetometer represents a significant advance towards a practical optical fibre magnetometer.

However, there are at least two remaining issues before the full potential of the optical fibre magnetometer can be realised. The first issue is to develop a suitable technique for sensitising long lengths of optical fibre to magnetic fields and incorporating this fibre into a compact sensor head configuration. The second is the miniaturisation of the interferometer into a reliable package which can be used in a practical environment. This package will contain the laser, fibre couplers, bias field coils and a demodulator for the interferometer readout. Once these packaging issues are overcome the fibre optic magnetometer offers the design flexibility and the sensitivity to meet the requirements for many applications.


45. Epworth, R.E., "The temporal coherence of various semiconudctor light sources used in optical fibre sensors", Conf. on fibre optic rotation sensors and related technologies, Massachusetts Institute of Technology, 1981.


APPENDIX A

Propagation of electromagnetic waves in weakly guiding optical fibre waveguides

The analysis summarised here describes the electromagnetic light field distributions (modes) which are allowed to propagate in a step index optical fibre with a small refractive index difference between the core and the cladding. Fibres with this property are called weakly guiding fibres and the guided modes can be described by linearly polarised (LP) modes. The LP mode solutions were first introduced by Gloge\textsuperscript{22} in 1971. A more exact description of the e.m. field involves a complicated set of HE, EH and TE modes.

The LP mode solutions can be found by solving Maxwell's equations for the boundary conditions imposed by the optical fibre. The wave equation for the optical fibre geometry is most conveniently written in co-ordinates \((r, \phi, z)\) as:

\[
\frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2}{\partial \phi^2} + \frac{1}{z^2} \frac{\partial^2}{\partial z^2} \left\{ \begin{array}{c} H_z \\ E_z \end{array} \right\} = \mu_0 \varepsilon_0 \frac{\partial^2}{\partial t^2} \left\{ \begin{array}{c} H_z \\ E_z \end{array} \right\}
\]

(1)

where \(\mu_0\) and \(\varepsilon_0\) respectively are the magnetic permeability and the dielectric permittivity of free space. For a guided mode propagating in the \(z\) direction a general solution to the wave equation is given by:

\[
\left| \begin{array}{c} H_z \\ E_z \end{array} \right| = \left| \begin{array}{c} H_{zo} \\ E_{zo} \end{array} \right| R(r) F(\phi) e^{i(\omega t - \beta z)}
\]

(2)

where \(\beta\) is the axial propagation constant, monochromatic propagation is assumed and \(R(r), F(\phi)\) represent the transverse distributions of the fields. If \(F(\phi)\) has azimuthal periodicity such that

\[
F(\phi) = \exp \pm (i\xi \phi)
\]

(3)
Then equation (1) yields a differential equation for $R(r)$ which is given by:

$$\frac{\partial^2 R(r)}{\partial r^2} + \frac{1}{r} \frac{\partial R(r)}{\partial r} + \left( T_{K^2} - \frac{\ell^2}{r^2} \right) R(r) = 0 \tag{4}$$

where $T_{K^2} = \mu_0 \varepsilon_0 \omega^2 - \beta^2$.

Within the core $T_{K^2} = T_{K_1^2} = n_1^2 k_0^2 - \beta^2$ and $T_{K_1^2} \geq 0$.

Equation (4) has Bessel function solutions and one of its solutions can be used to describe the e.m. waves which are confined to the core. The solution inside the core must remain finite at $r = 0$ and is therefore given by:

$$R(r) = E_1 J_{\ell} (T_{K_1} r), \quad r \leq a \tag{5}$$

where $E_1$ is a constant. Hence the electric field of a wave propagating in the $z$ direction which is confined to the core can be represented by:

$$E_z^\text{core} = E_1 J_{\ell} (T_{K_1} r) e^{i \phi} e^{i(\omega t - \beta z)} \tag{6}$$

To obtain solutions for the field in the cladding it is necessary to consider the situation in which $T_{K_2^2} < 0$ and then equation (4) becomes:

$$\frac{\partial^2 R(r)}{\partial r^2} + \frac{1}{r} \frac{\partial R(r)}{\partial r} - \left( Y_2^2 + \frac{\ell^2}{r^2} \right) R(r) = 0 \tag{7}$$

$Y_2 = i T_{K_2} = \sqrt{\beta^2 - n_2^2 k_0^2}$ is the propagation parameter for the cladding. This is known as Bessels modified equation and one of its solutions can be used to represent the field in cladding. For the case of a guided mode the field in the cladding must decay for $r \to \infty$ and can be represented by:

$$R(r) = E_2 K_{\ell} (Y_2 r), \quad r \geq a \tag{8}$$

where $E_2$ is a constant. This is because $K_{\ell}(Y_2 r)$ is proportional to
exp(-Y_2r) for large values of the argument. Hence the electric field in the cladding for a guided wave propagating in the z direction can be represented by:

\[ E_z^{\text{clad}} = E_z K_2 Y_2 r e^{i(\omega t - \beta z)} \]  

(9)

From equations 6 and 9 (which represent the electric field of a guided wave propagating in the core and the cladding respectively) and making use of the assumption that \( \frac{n_2 - n_1}{n_1} \ll 1 \); expressions for the amplitudes of the x, y and z field components in the core and the cladding can be derived. These fields are given in terms of the electric field at the interface, \( E_z^* \), by:

\[
E_y, x = \frac{\zeta_0}{n_1} H_y, x = E_z \cos \phi \begin{cases} 
\frac{J_\ell(T_{K_1} r)}{J_\ell(T_{K_1} a)}, & 0 \leq r \leq a \\
\frac{K_\ell(T_{K_2} r)}{K_\ell(T_{K_2} a)}, & r > a 
\end{cases}
\]

\[
E_z = -\frac{iE_z^* T_{K_1}}{2K_0 n_1} \left[ \frac{J_{\ell+1}(T_{K_1} r)}{J_\ell(T_{K_1} a)} \sin(\ell+1)\phi + \frac{J_{\ell-1}(T_{K_1} r)}{J_\ell(T_{K_1} a)} \sin(\ell-1)\phi \right], \ 0 \leq r \leq a
\]

\[
E_z = -\frac{iE_z^* Y_2}{2K_0 n_2} \left[ \frac{K_{\ell+1}(Y_2 r)}{K_\ell(Y_2 a)} \sin(\ell+1)\phi + \frac{K_{\ell-1}(Y_2 r)}{K_\ell(Y_2 a)} \sin(\ell-1)\phi \right], \ r > a
\]

(10)

The requirement that the field components tangential to the core-cladding interface at \( r = a \) be continuous leads to the equation:

\[
T_{K_1} \frac{J_{\ell-1}(T_{K_1} a)}{J_\ell(T_{K_1} a)} = -Y_2 \frac{K_{\ell-1}(Y_2 a)}{K_\ell(Y_2 a)}
\]

(11)
where \( n_1 > n_2 \). When the propagation constant of a mode guided by the fibre, \( \beta \), becomes smaller than \( n_2 k_0 \) the parameter \( Y_2 \) of equation 7 becomes imaginary. This means the evanescent field in the cladding turns into a radiation field, and the wave is no longer guided by the fibre. Hence mode cut off corresponds to the condition:

\[ Y_2 = 0 \tag{12} \]

and therefore

\[ \beta = n_2 k_0 \tag{13} \]

In this case the solution of equation (11) leads to the condition:

\[ J_{\ell-1}(T_{K_1 a}) = 0 \tag{14} \]

For \( \ell = 0 \), this includes the roots of the Bessel function \( J_{-1}(T_{K_1 a})= -J_1(T_{K_1 a}) \) and \( J_1(0) = 0 \) is taken as the first root. The cut off values for the \( LP_{0m} \) and the \( LP_{1m} \) modes are shown in Figure 2.2 and the condition for single mode propagation is discussed in 2.1. Every solution for \( T_{K_1 a} \) is associated with one set of modes designated \( LP_{\ell m} \). For \( \ell \geq 1 \) each set comprises four modes.
APPENDIX B

(i) Temperature control circuit
(ii) Laser diode power supply
(iii) Feedback system
(i) Temperature control circuit
(ii) Laser diode power supply
(iii) Feedback System
APPENDIX C

(i) STL diode laser (No.2920) output characteristic
(ii) Laser Diode Laboratories diode laser (No.27) output characteristic.
(i) Light output versus drive current curve of STL laser diode no. 2920.
(ii) Light output versus drive current curve of CSP laser diode no. 27
APPENDIX D

(i) Fabry-Perot interferometer maximum sensitivity condition

An expression for the sensitivity of the interferometer to phase changes (see equation 4.11) is given by:

\[
\frac{dI}{d\delta} = - I_{\text{max}} \frac{R \sin^2 \delta}{(1 + R \sin^2 \delta)^2}
\]  

(1)

where \( R = \left( \frac{2F}{\pi} \right)^2 \) and \( \delta = \phi/2 \)

Maximum sensitivity to phase changes occurs when \( \frac{dI}{d\delta} \) is a maximum i.e. \( \frac{d^2I}{d\delta^2} = 0 \).

Let \( U = 1 + R \sin^2 \delta \) and \( \frac{du}{d\delta} = 2R \sin \delta \cos \delta = R \sin 2\delta \).

Then:

\[
\frac{d^2I}{d\delta^2} = - I_{\text{max}} \left[ \frac{2R \cos \delta}{(1 + R \sin^2 \delta)^2} - \frac{2R^2 \sin^2 2\delta}{(1 + R \sin^2 \delta)^3} \right]
\]

= \[-I_{\text{max}} \left[ \frac{2R \cos \delta (1 + R \sin^2 \delta) - 2R^2 \sin^2 2\delta}{(1 + R \sin^2 \delta)^3} \right] \]

(2)

The numerator of equation 2 must be zero for \( \frac{d^2I}{d\delta^2} = 0 \)

\[ . \cdot 2R \left[ \cos 2\delta + R \cos \delta \sin^2 \delta - R \sin^2 2\delta \right] = 0 \]

(3)

Using the trigonometric identities \( \sin^2 \delta = \frac{1}{2}(1 - \cos 2\delta) \) and \( \sin^2 2\delta = 1 - \cos^2 2\delta \)

equation 3 becomes:

\[
R[2 \cos 2\delta + R \cos 2\delta] - 2R[1 - \cos^2 2\delta] = 0
\]

\[
R[2 \cos 2\delta + R \cos 2\delta - R \cos^2 2\delta - 2R + 2R \cos^2 2\delta] = 0
\]

\[
R[R \cos^2 2\delta + \cos 2\delta(2R) - 2R] = 0
\]

(4)
Equation 4 is a quadratic in $\cos 2\delta$ and has the solution:

$$\cos 2\delta = \frac{-(2+R) \pm \sqrt{4+4R+R^2+8R^2}}{2R}$$

$$= \frac{1}{2} \left( \frac{4}{R^2} + \frac{4}{R} + 9 \right)^{\frac{1}{2}} - \frac{1}{R} - \frac{1}{2}$$

(5)

Hence substituting $R = \left( \frac{2F}{\pi} \right)^2$ and $\delta = \phi/2$, the optimal operating point for the Fabry-Perot interferometer can be written as:

$$\phi_o = \cos^{-1} \left[ \frac{1}{2} \left( 9 + \frac{\pi^2}{F^2} + \frac{4\pi^4}{9F^4} \right)^{\frac{1}{2}} - \frac{\pi^2}{2F^2} - \frac{1}{2} \right]$$

(6)

For $F >> 1$ and therefore $R >> 1$, an approximate expression for $\phi_o$ can be deduced as follows:

$$\delta_o \approx \frac{1}{2} \cos^{-1} \left[ \frac{1}{2} \left( 9 + \frac{4}{R} \right)^{\frac{1}{2}} - \frac{1}{R} - \frac{1}{2} \right]$$

$$\delta_o \approx \frac{1}{2} \cos^{-1} \left[ \frac{3}{2} \left( 1 + \frac{2}{9R} \right)^{\frac{1}{2}} - \frac{1}{R} - \frac{1}{2} \right]$$

(7)

Using the binomial expansion $(1+x)^p = 1 + px + \frac{p(p-1)}{2!} x^2$ equation 7 can be written as:

$$\delta_o \approx \frac{1}{2} \cos^{-1} \left[ \frac{3}{2} \left( 1 + \frac{2}{9R} \right)^{\frac{1}{2}} - \frac{1}{R} - \frac{1}{2} \right]$$

$$\cos 2\delta_o \approx 1 - \frac{2}{3R}$$

(8)

Substituting for $\delta$ and $R$, equation 8 becomes

$$\cos \phi_o = 1 - \frac{\pi^2}{6F^2}$$

(9)

Using the cosine series, i.e. $\cos x = 1 - \frac{x^2}{2!} \ldots$, $\phi_o$ can be written as:

$$\phi_o = \frac{\pi}{\sqrt{3F}}$$

(10)

By substituting this value of $\phi_o$ into equation 1 an approximate expression for the interferometer sensitivity to phase changes (see equation 4.14) is obtained.
(ii) Fabry-Perot interferometer experimental finesse

The experimental finesse is defined in equation 4.18 to be:

\[ F_e = \frac{2\pi}{\Delta \phi} \]  

(11)

where \( \Delta \phi \) is the full width of a transmission fringe, when the transmission is given by:

\[ I = 0.5 \left[ I_{\text{max}} - I_{\text{min}} \right] \]  

(12)

where at \( I_{\text{max}} \):

\[ F(\delta)_{\text{max}} = \frac{1}{1 + R \sin^2 \delta} = 1 \]  

(13)

and at \( I_{\text{min}} \):

\[ F(\delta)_{\text{min}} = \frac{1}{R + 1} \]  

(14)

(\( F(\delta) \) is defined in equation 4.11)

Therefore the point on the Fabry-Perot transmission curve corresponding to the transmission value defined in equation 12 can be represented by:

\[ I_e = \frac{F(\delta)_{\text{max}} - F(\delta)_{\text{min}}}{2} + F(\delta)_{\text{min}} \]

\[ = \frac{R}{2(1+R)} + \frac{1}{1+R} \]

\[ = \frac{R + 2}{2(1+R)} \]  

(15)

The Fabry-Perot transmission will drop to the value given in equation 15 whenever \( \delta_1, \delta_2 = \delta_{\text{max}} \pm \delta_e \). Hence the fringe width at this transmission can be calculated as follows:
\[
\frac{R + 2}{2(1+R)} = \frac{1}{1 + R \sin^2 \delta_e}
\]

\[
1 = \frac{R + 2 + R \sin^2 \delta_e (R + 2)}{2(1+R)}
\]

\[
\delta_e = \sin^{-1} \sqrt{\frac{1}{R + 2}}
\]  \hspace{1cm} (16)

The total fringe width at the transmission value defined in equation 15 is given by:

\[
\delta_1 - \delta_2 = (\delta_{\max}^{+\delta_e}) - (\delta_{\max}^{-\delta_e})
\]

\[
= 2\delta_e
\]  \hspace{1cm} (17)

and the experimental finesse can be written as:

\[
F_e = \frac{\pi}{2} \left[ \sin^{-1} \left( \frac{1}{\sqrt{\left(\frac{2F}{\pi}\right)^2 + 2}} \right) \right]^{-1}
\]  \hspace{1cm} (18)

where \(\delta = \phi/2\) and \(R = \left(\frac{2F}{\pi}\right)^2\).
APPENDIX E

MODIFIED MACH-ZEHNDER INTERFEROMETER OUTPUT

\[
\frac{I(B)-I(B+\epsilon)}{2} = \cos^2 B - \cos^2(B+\epsilon), \text{ where } B = \frac{1}{2}(\frac{2\pi x a}{\lambda D} + A) \text{ and } \epsilon = \frac{\phi}{2}
\]

\[
= \cos^2 B - (\cos B \cos \epsilon - \sin B \sin \epsilon)^2
\]

\[
= \cos^2 B - (\cos^2 B \cos^2 \epsilon + \sin^2 B \sin^2 \epsilon - 2\sin B \cos B \sin \epsilon \cos \epsilon)
\]

\[
= \cos^2 B (1 - \cos^2 \epsilon) - \sin^2 B \sin^2 \epsilon + \sin 2B \sin \epsilon \cos \epsilon
\]

\[
= \cos^2 B (1 - \cos^2 \epsilon) - (1 - \cos^2 B) \sin^2 \epsilon + \sin 2B \sin \epsilon \cos \epsilon
\]

\[
= \cos^2 B (1 - \cos^2 \epsilon + \sin^2 \epsilon) - \sin^2 \epsilon + \sin 2B \sin \epsilon \cos \epsilon
\]

\[
= 2 \sin^2 \epsilon \cos^2 B - \sin^2 \epsilon + \sin 2B \sin \epsilon \cos \epsilon
\]

\[
= -(1 - 2\cos^2 B) \sin^2 \epsilon + \sin 2B \sin \epsilon \cos \epsilon
\]

but \[\cos 2B = \cos^2 B - \sin^2 B = -(1 - 2 \cos^2 B)\]

\[
\frac{I(B)-I(B+\epsilon)}{2} = \cos 2B \sin^2 \epsilon + \sin 2B \sin \epsilon \cos \epsilon
\]

\[
= \sin \epsilon (\cos 2B \sin \epsilon + \sin 2B \cos \epsilon)
\]

\[
\frac{I(B)-I(B+\epsilon)}{2} = \sin \epsilon \sin(2B+\epsilon)
\]

\[
\therefore I_A-I_{A+\phi} = 2\sin^\frac{\epsilon}{2} \sin\left(\frac{2\pi x a}{\lambda D} + A\right) + \frac{\phi}{2}
\]

and for small values of \[\phi\] the amplitude of this function is proportional to \[\phi\]. Hence the phase difference, \[\phi\], is given by:

\[
\phi = \sqrt{2} \text{ r.m.s. } (I_A-I_{A+\phi})
\]
APPENDIX F

1. Machine Code Subroutines

1.1. Initialising the interface (See CCS 7720 handbook)

Binary file: CRAP - see listing

The interface card is plugged into slot 2 in the Apple, The addresses allocated to this slot are COAO - SOAF (see page 80, Apple II Ref. Manual).

To initialise the CCS interface, the value 00 is loaded into addresses COA1 and COA2. The interface is required to be inputs on both data buses, so the next step is to load 00 into the DDR A and B. Alternatively, for outputs, the value loaded should be SFF.

Finally, 24 is loaded into COA1 and COA3. The initialisation is then complete.

1.2. Transferring data from interface to register

Binary file - TRANS2

The routine is designed to idle until the 'Start Scan' signal from the Reticon array arrives at the Apple, and then synchronises the loading of data from the first diode into the first allocated register. In addition, for each diode, the routine has to wait until a 'Data Valid' signal is sent by the ADC indicating that it has completed the conversion of the analogue signal.

The above required the use of two additional input lines, and the most convenient solution was to use the games i/o port. The corresponding memory addresses are C061 and C062. The 'Start Scan' signal from the Reticon is TTL high for 16 clock cycles. The Apple interprets a TTL high as 128 which means that the MSB is 1, and is interpreted as indicating a negative number. Thus to determine whether the 'Start Scan' has arrived, the Apple monitors the address C061 until it goes
negative, at which point the routine moves to the next stage.

The 'Data Valid' signal is a TTL low, so the procedure here is to monitor CO62 until it goes positive, at which point it loads the data from the two data bases into the two sets of induced registers, starting at 1800 and 1900.

The routine then loops back and idles until the next 'Data Valid' signal arrives, and then loads the next set of data.

2. 'BASIC' Programs

Program INSTANT 4 uses 'CRAP' and 'BINSTANT', the latter being a variant on 'TRANS2' which enables the Apple to switch on the magnetic field. This is accomplished by using an 'annunciator' softswitch (see Apple II Ref. Manual, page 79), and is a two-state TTL switch that can be toggled by addressing CO58 and CO59.

The routine loads one set of fringes with the field off, then switches on the field, idles for about 20 msecs, before loading the next set of fringes. INSTANT 4 then uses these two sets of fringes to calculate the phase.
0200- BE 00  LDA  #$00
0203- AA  TAX
0204- DW  STX  #$041
0207- SC  A5  D0  STY  #$043
020A- BE  A0  C0  STX  #$044
020D- BE  A2  C0  STX  #$042
0210- AB  24  LDR  #$24
0213- BE  A1  C0  STX  #$041
0216- BE  A3  C0  STX  #$043
0219- 60  RTS

0250- AC  58  C0  LDY  #$058
0253- AC  61  C0  LDY  #$061
0255- 16  FB  BPL  #$053
025A- A2  81  LDX  #$A1
025E- AC  52  C0  LDY  #$062
0262- 30  FB  BMI  #$05A
0266- AD  A0  C0  LDA  #$040
026B- AC  A2  C0  LDY  #$042
0270- 80  FF  17  STA  $17FF, X
0275- 80  FF  18  STA  $18FF, X
0275- CA  DEX
027F- 00  EB  BNE  #$035
0281- 00  FF  BNE  #$FF
0288- A0  82  LDY  #$62
028B- 80  FF  DEV
028F- 00  FD  BNE  #$0377
0294- CA  DEX

0287- 00  F8  BNE  #$0375
0290- EA  NOP
0293- EA  NOP
0296- EA  NOP
0299- EA  NOP
02A0- EA  NOP
02A3- EA  NOP
02A6- AC  61  C0  LDY  #$061
02A9- 16  FB  BPL  #$053
02AC- A2  81  LDX  #$A1
02AF- AC  52  C0  LDY  #$062
02B1- 30  FB  BMI  #$05A
02B9- AD  A0  C0  LDA  #$040
02CE- AC  A2  C0  LDY  #$042
02D2- 80  FF  17  STA  $15FF, X
02D7- 80  FF  18  STA  $16FF, X
02D7- CA  DEX
02DE- 00  EB  BNE  #$039A
02DF- AC  58  C0  LDY  #$058

02E8- A2  81  LDX  #$A1
02E9- AA  01  LDX  #$A1
02EC- 86  DEY
02ED- 00  FD  BNE  #$04A
02F0- CA  DEX
02F4- 00  F8  BNE  #$034A
02FA- 60  ATS
10 HOME
20 LET H=16400
30 DEF CH=4
40 N=125:TH=10000: SX=16
50 DIM V1(N),V2(N),NI(N),N2(N)
55 PRINT DEF: "LOAD CRAP": PRINT DEF: "LOAD BINSTANT"
100 UT=5
105 PRINT " YOUR OPTIONS ARE:"
106 PRINT
107 PRINT " 1. DATA ACQUISITION"
115 PRINT " 2. ESCAPE"
130 PRINT " WHICH OPTION DO YOU WANT?"
150 INPUT ANSWER
160 IF ANSWER = 1 OR ANSWER = 2 THEN GOTO 200
170 PRINT
180 PRINT " YOUR ANSWER WAS INVALID"
200 IF ANSWER = 1 THEN GOTO 600
300 GOTO 990
600 GOSUB 1000
700 GOSUB 2000
800 GOSUB 3000
900 GOSUB 4000
950 GOSUB 5000
990 END
1000 CALL 768
1095 PRINT
1100 PRINT " INTERFACE INITIALIZED"
1200 RETURN
2000 CALL 848
2095 PRINT
2100 PRINT " DATA LOADED"
2104 PRINT
2200 RETURN
3000 PRINT " NORMALISING DATA"
3010 MAX = 0:MIN = TH
3100 FOR I = 1 TO N
3120 AI = 6143 + 1:AR = 6793 + 1:AR = 5631 + 1:AR = 5987 + 1
3140 V1(I) = SX + Peek (AI) + ( Peek (AR)) / SX
3142 V2(I) = SX + Peek (AR) + ( Peek (AR)) / SX
3150 IF MAX > ABS ( V1(I) - V2(I) ) THEN GOTO 3180
3160 MAX = ABS ( V1(I) - V2(I) )
3170 GOTO 3200
3190 IF MIN < V1(I) - V2(I) THEN GOTO 3200
3190 MIN = V1(I) - V2(I)
3200 NEXT
3300 NX = 0:NN = TH
3310 FOR I = 1 TO N
3320 IF NX > V1(I) THEN GOTO 3580
3330 NX = V1(I)
3400 GOTO 3600
3500 IF NN < V1(I) THEN GOTO 3580
3550 NN = V1(I)
3600 NEXT: RETURN
4000 SUM = 0
4010 FOR I = 1 TO N
4020 SUM = (V1(I) - V2(I)) * 2 + SUM
4040 NEXT
4050 HSUM = SOR ( SUM / N )
4060 RETURN
5000 HDR : HCOLOR= 3: HPLT 260 / N, 80 - 80 - ( V1(I) - V2(I) ) / MAX
5020 FOR I = 1 TO N
5030 HCOLOR= 3: HPLT TO 1 = 260 / N, 80 - 80 - ( V1(I) - V2(I) ) / MAX
5040 NEXT
5045 HOME
5050 PH = 1, 414 = HSUM / ( NX - NN )
5060 PRINT " PHASE-SHIFT IS: INT (100000 * PH) / 100000" RADIANS"
5070 PRINT
5080 PRINT " TO CONTINUE PRESS RETURN".
5090 IF PEEK (= 16384) > 127 THEN RETURN
5095 GOTO 6000
APPENDIX G

(1) FOURIER COMPONENTS OF THE MAGNETOMETER OUTPUT

The interferometer output function given in equation 5.27 is:

\[ I = I_0 [1 + \cos(2QB_a B_d \sin \omega t - \frac{Q B_a}{2} \cos 2\omega t) + \theta] \]  

(1)

and can be represented as:

\[ I = I_0 [1 + \cos (X - Y + Z)] \]  

(2)

where \( X = A \sin \omega t; \ Y = D \cos 2\omega t, \) and \( C = 0, \ A = 2QB_a B_d, \) and \( D = \frac{Q B_a^2}{2} \)

Using a trigonometric identity equation 2 can be written as:

\[ I = I_0 [1 + (\cos X \cos Y + \sin X \sin Y) \cos Z \]

\[ - (\sin X \cos Y - \cos X \sin Y) \sin Z] \]  

(3)

The Fourier components can now be obtained by substituting the appropriate Bessel expansions into equation 3 and are as follows:

\[ I = I_0 \left[ 1 + \left[ J_0(A) + 2 \sum_{n=1}^{\infty} J_{2n}(A) \cos 2n\omega t \right] \right. \]

\[ \left. * \left[ J_0(D) + 2 \sum_{n=1}^{\infty} (-1)^n J_{2n}(D) \cos 4n\omega t \right] \right] \]

* \( \cos \theta \)

\[ + 2 \sum_{n=0}^{\infty} J_{2n+1}(A) \sin(2n+1)\omega t + 2 \sum_{n=0}^{\infty} (-1)^n J_{2n+1}(D) \cos(2n+1)2\omega t \]

* \( \cos \theta \)

\[ - 2 \sum_{n=0}^{\infty} J_{2n+1}(A) \sin(2n+1)\omega t \]

\[ * \left[ J_0(D) + 2 \sum_{n=1}^{\infty} (-1)^n J_{2n}(D) \cos 4n\omega t \right] \]

* \( \sin \theta \)
\[ + \left[ J_0(A) + 2 \sum_{n=1}^{\infty} J_{2n}(A) \cos 2n\omega t \right] \]
\[ \times 2 \sum_{n=0}^{\infty} (-1)^n J_{2n+1}(0) \cos(2n+1)2\omega t \]
\[ \times \sin \theta \]

(4)
(ii) MAGNETOMETER RESPONSE TO D.C. AND A.C. MAGNETIC FIELD SIGNALS

Analysis of the magnetostrictive response produced by a magnetic field comprised of an a.c. bias field, \( B_a \sin \omega t \), a d.c. signal field, \( B_d \), and an a.c. signal field, \( B_s \sin \omega_s t \). In this case the path length difference between the interferometer arms is proportional to:

\[
\Delta \ell = \ell k (B_d + B_a \sin \omega t + B_s \sin \omega_s t)^2
\]  

(1)

for \( B_d + B_a + B_s \ll \frac{B_{\text{sat}}}{2} \)

Expanding equation 1 and collecting terms according to their frequency components, one obtains:

1. d.c. components:

\[
\Delta \ell \bigg|_{\text{dc}} = \ell k \left[ B_d^2 + \frac{B_a^2}{2} + \frac{B_s^2}{2} \right]
\]

2. components at the a.c. bias magnetic field frequency \( \omega \)

\[
\Delta \ell \bigg|_{\omega} = \ell 2B_a B_d k_s \sin \omega t
\]

3. component at twice the a.c. bias magnetic field frequency \( 2\omega \)

\[
\Delta \ell \bigg|_{2\omega} = -\ell k B_a^2 \cos 2\omega t
\]

4. component at the a.c. signal magnetic field frequency \( \omega_s \)

\[
\Delta \ell \bigg|_{2\omega_s} = 2B_d B_s k_s^2 \sin \omega_s t
\]

5. component at twice the a.c. signal magnetic field frequency \( 2\omega_s \)

\[
\Delta \ell \bigg|_{2\omega_s} = -\ell k B_s^2 \cos 2\omega_s t
\]

6. components at the mixed frequencies between the bias and signal a.c. magnetic fields (\( \omega \pm \omega_s \))

\[
\Delta \ell \bigg|_{\omega \pm \omega_s} = \ell 2B_a B_s k_s [\cos(\omega - \omega_s) t - \cos(\omega + \omega_s)]
\]
MATERIAL REDACTED AT REQUEST OF UNIVERSITY
FIBRE OPTIC SENSOR

This invention relates to sensors for the sensing and measurement of a variety of parameters including temperature, pressure, etc.

In these arrangements the sensing element, which is subjected to the parameter to be sensed and measurement is an optical fibre Fabry-Perot interferometer. Such an interferometer consists of a length of an optical fibre with semi-silvered optically flat ends, which ends thus act as mirrors for the interferometer. The influence of the parameter being sensed, which can be exercised in various ways, e.g. by magneto-strictive means, piezo-electric means, thermal effects, acoustic effects, varies the length of the fibre. Thus detection depends on measuring the changes in the transmitted signals as functions of changes in length of the Fabry-Perot interferometer. Such a Fabry-Perot interferometer may use relatively long optical fibres, e.g. 40cm or longer, say 100cm.

An object of the invention is to provide an improved sensor arrangement of the type referred to above. According to the present invention, there is provided an optical fibre sensor arrangement, which includes an optical fibre Fabry-Perot etalon which uses single-mode fibre and which can support two or more transverse modes of radiation which are path-length distinct each of which modes produces its own sequence of peaks in the transmission characteristics of the sensor,
the peaks in the two sequences being of different
amplitudes so that the transmission function is
asymmetrical, wherein the parameter being monitored is
applied to the etalon in such a way as to vary the path
length of the fibre, thus varying the number of said
peaks which are produced in each of said sequences of
peaks, and wherein read-out means associated with the
sensor monitors the number of peaks produced from each
sequence, and thus the value and sign of the parameter to
which the sensor has been subjected.

The transmission function of a single-mode
all-fibre Fabry-Perot interferometer consists of a series
of identical and evenly-spaced peaks. Modulation of the
length of the fibre when used in a sensor causes
fluctuations in the light intensity seen at a detector at
one end of the fibre. This light comes from the other
end of the interferometer, e.g. from a laser controlled
in the manner set out in our Application No. 8401143
(R.E. Jones et al 5-1). Since the transmission function
of the sensor fibre is symmetrical, no information is
obtainable as to the sign of the perturbation of the
fibre due to the influence of the parameter being sensed.

In the present arrangement the fibre used is one
which supports more than one transverse mode of the
radiation used. Thus a single-mode fibre operating with
a 1.3 micrometre source wavelength supports four
transverse modes when illuminated by a 0.633 micrometre
source. Three of these modes are path length degenerate,
so that there are two distinct, in path length, modes
supported by the fibre. The energy in each of these
distinct modes is dependent on the light source (usually
a laser) - fibre launch condition. Hence two series of
peaks occur in the transmission functions, and these
peaks do not coincide. In general the peaks in the two
series are of different amplitude, with the result that
the transmission function becomes asymmetrical.
With such an arrangement the sign of the perturbation of the fibre due to the parameter being sensed is detected by observing the ordering and spacing of the peaks in the transmission circuit using a discriminating circuit fed from the interferometer's detector. The intensities of these peaks is registered using a photodetector, and logic circuits, which can be relatively simple, are used to determine the number of peaks and their order of appearance at the detector.

With two modes supported in the fibre, mode 1 is represented in such circuitry by logic 1 and mode 2 by logic 0.

An interferometer of the type referred to is made by coating the optically flat ends of an optical fibre with reflective material so that the fibre becomes a Fabry-Perot etalon. When such an etalon is used as a sensor incident light enters the fibre at one end and eventually leaves it at the other end, with its transmission function influenced by the parameter to be monitored. As already indicated, the transmission function of such a system consists of a series of sharp transmission peaks, and in a fibre which supports two transverse modes there are two series of these transmission peaks. Variations in the optical length of the fibre or the wavelength of the source of light cause variations in the transmitted intensity through the fibre. Such a fibre can be used for sensing, where changes in temperature or pressure radically alter the optical path length along the fibre. The detection techniques involve pulse counting of the two series of peaks, after discrimination circuitry has separated them, to determine the number of peaks. This gives an indication of the magnitude of the parameter being sensed, and due to the differing amplitudes of the peaks, the positions of the peaks of the two series with respect to each other indicating the sign of the parameter.
CLAIMS:

1. An optical fibre sensor arrangement, which includes an optical fibre Fabry-Perot etalon which uses single-mode fibre and which can support two or more transverse modes of radiation which are path-length distinct each of which modes produces its own sequence of peaks in the transmission characteristics of the sensor, the peaks in the two sequences being of different amplitudes so that the transmission function is asymmetrical, wherein the parameter being monitored is applied to the etalon in such a way as to vary the path length of the fibre, thus varying the number of said peaks which are produced in each of said sequences of peaks, and wherein read-out means associated with the sensor monitors the number of peaks produced from each sequence, and thus the value and sign of the parameter to which the sensor has been subjected.

2. An arrangement as claimed in claim 1, wherein the source of light applied to the etalon is a semiconductor laser which emits light at a wavelength of 0.85 micrometers or 1.3 micrometres.

3. An optical fibre sensor arrangement, substantially as described herein.
FIBRE OPTIC SENSOR

Abstract of the Disclosure

An optical fibre Fabry-Perot etalon can be used as a sensor if the parameter being monitored is allowed to influence the etalon's length, e.g. by pressure, temperature, magnetostrictive effects, piezoelectric effects, acoustically, etc. However, with single-mode fibre and only one set of peaks in the transmission function the device is direction insensitive so that the sign of the parameter being monitored cannot be detected.

In the present arrangement the etalon is driven in such a way as to support two different path-length distinct transmission modes, e.g. by the use as a light source of a laser emitting light at 1.3 micrometres wavelength. Two sequences of peaks are then produced in the transmission function which are peaks of different sizes, so that the transmission function is asymmetrical. The peaks are separated at the detection circuitry by discriminations followed by pulse counting means so that the arrangement becomes sign responsive.
MATERIAL REDACTED AT REQUEST OF UNIVERSITY
LASER STABILISATION CIRCUIT

This invention relates to an arrangement for the stabilisation of a semiconductor laser.

For some types of optical fibre sensor, light with a highly stable wavelength is needed. This is not as readily available from semiconductor diode lasers as it is from gas lasers. In view of the greater convenience of using the relatively small semiconductor lasers as compared with gas lasers this is a disadvantage. Hence an object of the invention is to produce an arrangement for stabilising a semiconductor laser's wavelength which is adequate for such purposes.

According to the invention, there is provided an arrangement for the stabilisation of a semiconductor laser, in which a proportion of the light output of the laser is applied to a Fabry Perot interferometer the output of which is applied to a light-responsive detector which generates an output appropriate to the characteristics of the light applied to the detector, in which the detector's output is compared with a reference parameter appropriate to what the light output should be, which comparator produces an error signal appropriate to the relation between the laser's light output and what it should be, and in which said error signal is fed back to the laser current supply circuit such as to adjust the laser current in a manner as to cause the laser's output to be altered towards what that output should be.
An embodiment of the invention will now be described with reference to the accompanying drawings, in which Fig. 1 represents the optical aspects while Fig. 2 represents the electronic aspects of an arrangement for stabilising a semiconductor laser.

In the arrangement shown in Figs. 1 and 2, the laser 1 is mounted on a support with a temperature sensor 2 and a Peltier cooler, which together co-operate to maintain the temperature of the laser at a suitable level. This is indicated by the block 4, Fig. 2, which provides the laser temperature control.

The light from the laser 1 passes via launching optics 5 and an optical fibre to a coupler 6, which may be fused or polished, and which splits the light beam into two outputs. Each of these outputs extends via a respective fibre-fibre coupler 7, 8 to a reference Fabry Perot interferometer and a sensing Fabry Perot interferometer 9, 10. These interferometers each has its own detector 11, 12. In the detector block, the output of the light-responsive device, e.g. a photo-diode, is compared with a reference quantity by a comparator. Hence the comparator output is an error signal appropriate to the relation between the light reaching the detector, and what the light should be. This error signal is applied to the laser current supply to influence the latter in the appropriate sense.

As can be seen from Fig. 2, the output of the reference interferometer 9 is fed back to the laser current supply circuit 13 to control the current supply for the laser in a sense appropriate to stabilisation of the wavelength of the light. The other Fabry Perot interferometer 10 is the sensing element, and its output is obtained via a discriminator 16 and a pulse counter 17, which counts distances between successive peaks in the transmission characteristics of the interferometer 10, which distances vary in accordance with the parameter being sensed.
At this point it is mentioned that each of these interferometers is a length of an optical fibre, such as a single mode fibre with semi-silvered optically flat ends, which ends thus act as mirrors for the interferometer. The influence of the parameter being sensed, which can be exercised in various ways, e.g. by magnetostriction, piezo-electric means, thermal effects, acoustic effects, varies the length of the fibre. Thus detection depends on measuring the changes in the transmitted signals as functions of changes in length due to the parameter being sensed. In one case the sensor interferometer was 40cm long, but the reference interferometer may be shorter.

In the arrangement described herein, a proportion, typically 5% of the laser's total drive current, is controlled by the output of the photo-detector 11 at the remote end of the reference Fabry Perot interferometer. Hence a closed loop is formed and the transmission of the interferometer is retained at a constant value. Isolation of the interferometer from environmental change by suitable protection means that the laser's output wavelength is stabilised. As can be seen, the light output of the laser is also applied to the sensing interferometer, so that the latter thus receives light at a stabilised wavelength.

The light is launched into each of the Fabry Perot interferometers via index-matching material provided by the couplers 7, 8. Each of these couplers may be in the form of a GTE elastomeric splice. Such splices are available from GTE Products Corporation, 2401 Reach Road, Williamsport, PA17701, USA.

In the arrangements described above, the ends of the interferometer are silvered, the reflective coatings can also be applied as multi-layer dielectric coatings.
CLAIMS:

1. An arrangement for the stabilisation of a semiconductor laser, in which a proportion of the light output of the laser is applied to a Fabry Perot interferometer, the output of which is applied to a light-responsive detector which generates an output appropriate to the characteristics of the light applied to the detector, in which the detector's output is compared with a reference parameter appropriate to what the light output should be, which comparator produces an error signal appropriate to the relation between the laser's light output and what it should be, and in which said error signal is fed back to the laser current supply circuit such as to adjust the laser current in a manner as to cause the laser's output to be altered towards what that output should be.

2. An arrangement as claimed in claim 1, and in which the Fabry Perot interferometer is a length of a single mode optical fibre with its ends half-silvered to produce two mirrors which are substantially parallel, light entering and leaving the fibre ends via the half-silvering.

3. An arrangement as claimed in claim 1, and in which the Fabry-Perot interferometer is a length of a single-mode optical fibre with its ends provided with multi-layer dielectric coatings.

4. An arrangement as claimed in claim 1, 2 or 3, in which the light from the laser is applied via an optical fibre to an optical fibre coupler where it is split into two beams one of which is applied to said Fabry Perot interferometer whilst the other forms the effective output of the laser.

5. An arrangement as claimed in claim 1, 2, 3 or 4, in which the laser mounting is provided with a temperature sensor and a Peltier cooler, which maintains the temperature of the laser at a desired level.
6. An arrangement as claimed in claim 4 or 5 as appendent to claim 3, in which the laser's effective output is applied to another Fabry Perot interferometer function as a sensor.

7. An arrangement for the stabilisation of a semiconductor laser, substantially as described with reference to the accompanying drawings.
LASER STABILISATION CIRCUIT

Abstract of the Disclosure

To stabilise the light output from a semiconductor laser (1), that output is split into beams in two optical fibres by a coupler (6). One output beam is the effective output of the laser while the other is a reference output. The reference output is applied via a Fabry Perot fibre optic interferometer (9) to a detector (11) where it is compared with a reference parameter. Any difference results in an error signal which is used to appropriately adjust the laser's current supply to give the light output the required characteristics.

As shown the effective output goes to another Fabry Perot optic interferometer (10) used as a sensor.
**Fig. 1.**

1. Sensor
2. Laser
3. Fiber
4. Temperature sensor
5. Laser current control
6. Reference FFP
7. Discriminant
8. Pulse counting
9. Output

**Fig. 2.**

1. Laser current supply
2. Reference FFP
3. Laser
4. Laser temp. control
5. Sensing FFP
6. Discriminant
7. Output
MATERIAL REDACTED AT REQUEST OF UNIVERSITY
Optical Fibre Magnetometer Using a Stabilised Semiconductor Laser Source

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INTRODUCTION

Fluctuations in the operating frequency of semiconductor lasers produce intensity noise in interferometric sensors with a path length difference. This noise has a 1/f frequency dependence and its magnitude increases in direct proportion to interferometer path length difference\(^1\)). An approximate value of the amplitude of these fluctuations, for the STL GaAlAs laser used to drive the magnetometer, is 10^5 Hz measured at a detection frequency of 1 kHz. To achieve a minimum detectable phase of 10^-6 radians in a fibre interferometer with such a laser it is necessary to balance the interferometer arms to better than 1 mm. In order to reduce this noise source some authors\(^2\) have reported locking the laser frequency to an external Fabry-Perot cavity by controlling the laser drive current. This is possible since the output frequency of a solid state laser is linearly dependent upon drive current. Compared with unstabilised lasers, improvements of 20 dB in laser output noise spectra have been reported using this technique. Similar results have been obtained in this work, but an all-fibre Fabry-Perot interferometer replaced the bulk optics device. This stabilised laser then acted as a source for a Mach-Zehnder sensing interferometer.

All-fibre Mach-Zehnder interferometers have been used extensively to measure small magnetic fields\(^3,4,5\). All of the interferometers use a magnetostrictive material to change the optical path length of one interferometer arm with respect to the other. The most sensitive measurements to date have been achieved with a.c. magnetic signals at frequencies ranging from 100 Hz to 10 kHz, a typical minimum detectable field being 10^-10 tesla (T) per metre of magnetically sensitive fibre. This detection is usually accomplished using active homodyne demodulation\(^6\) which maintains the interferometer at quadrature by compensating for low frequency phase variations such as those which arise from temperature. The system maintains maximum interferometric sensitivity to a.c. magnetic fields beyond the frequency range of the compensation. This signal recovery scheme relies on the amplitude of the magnetic signal being greater than that of the interferometer 1/f noise at the chosen signal frequency. D.C. magnetic field detection is a more difficult problem because the signal is in the same frequency band as the 1/f noise and also one has no prior knowledge of the signal waveform. A d.c. magnetometer developed previously at STL\(^7\) was capable of detecting a minimum field of 10^-7T. To improve this sensitivity a technique has been developed to measure the d.c. magnetic signal at a frequency where the amplitude of the 1/f noise is much smaller. This approach uses active homodyne demodulation with a piezoelectric phase modulator to maintain quadrature. A passive approach was not considered because active elements were present as part of the magnetic detection technique, and so the extra electronic complexity and higher noise level of a passive scheme was not justified.
THEORY

The transmission of a Fabry-Perot interferometer as a function of phase is given by:

\[ I = I_m \left[ 1 + (2 F/\nu) \sin^2 \frac{\phi}{2} \right]^{-1} \]  \hspace{1cm} (1)

where \( I_m \) is the maximum transmission through the cavity, \( F \) is the finesse and \( \phi \) the phase difference between successively transmitted waves. The relationship between a change in source emission frequency and the Fabry-Perot phase difference is given by:

\[ d\phi = 4\nu L n c^{-1} d\nu \] \hspace{1cm} (2)

Hence for a given cavity length and refractive index the transmitted output is dependent on source frequency.

In a Mach-Zehnder magnetometer, magnetostrictive expansion of the signal arm fibre yields an optical path length change given by:

\[ \lambda = KH^2 \] \hspace{1cm} (3)

where \( K \) is a constant dependent on the magnetostrictive material and the coupling of the strain to the optical core, and \( H \) is the applied magnetic field. The relationship is valid for small fields where the material magnetostrictive response is quadratic. In this small field regime a bipolar a.c. magnetic field (a.c. bias field) at frequency \( \omega_0 \) produces an interferometer output signal containing even harmonic components of \( \omega_0 \) only. An additional applied d.c. field produces components at the fundamental and odd harmonics, the magnitude of these components being proportional to the applied d.c. field for a.c. bias fields within the quadratic region of the magnetostrictive response. This is illustrated in Figure 1. Hence the bias field enables the d.c. signal to be extracted at \( \omega_0 \) using phase-sensitive detection. A similar d.c. magnetic signal recovery scheme has recently been published by the U.S. Naval Research Laboratories\(^4\).\(^5\).

EXPERIMENTAL

An STL GaAlAs laser diode was operated at 850 nm on a temperature-controlled mount. This provided a device temperature stability of better than \( \pm 0.1^\circ \text{C} \) at approximately 25\(^\circ\)C. The laser diode was driven with a current of 180 mA, which was a factor of 1.2 above threshold. These conditions corresponded to a single frequency optical output of 5 mW. In order to stabilise this frequency, 20% of the laser output was fed into an all-fibre Fabry-Perot interferometer via a beam splitter. The Fabry-Perot was used in a feedback loop to control the laser frequency by adjusting its drive current. This is shown schematically in Figure 2. The remaining 80% of frequency stabilised laser output was launched into an all-fibre Mach-Zehnder magnetometer. The signal arm was bonded to Vacuumschmelze magnetostrictive metallic glass for 0.1 m of its length. The trade name of the metallic glass used was 'Vitrovac' 4040. The interferometer used conventional active homodyne demodulation. A d.c. magnetic field and a 700 Hz a.c. bias field were applied to the signal arm. The magnitude of the odd harmonic components of the a.c. bias field, in the interferometer output, was found to be proportional to the d.c. field.
RESULTS

The operation of the Fabry-Perot stabilisation scheme is shown in Figure 3. The intensity fluctuations at the Fabry-Perot output resulting from laser frequency deviations were reduced by 20 dB from d.c. to 1 kHz when the feedback loop was controlling the laser drive current. The characteristic 1/f dependence was demonstrated. Consequently, the amplitude of the operating frequency fluctuations were reduced to $10^4$ Hz at a detection frequency of 1 kHz. This frequency stabilised output produced a magnetometer noise floor, at 1 kHz, of $10^{-5}$ radians for an interferometric path difference of 0.04 m. The magnetometer d.c. response was examined by applying the 700 Hz a.c. bias field and the d.c. magnetic field. The spectral components contained in the interferometer output are shown in Figure 4. The magnitude of the first harmonic response is linear with applied magnetic field from 0 to $10^{-4}$T as shown in Figure 5. This data was obtained using the lock-in amplifier at a 1 Hz bandwidth. Once the d.c. field reaches $10^{-4}$T the magnetostrictive response to the applied field begins to depart from the quadratic form and the first harmonic component magnitude saturates as a function of d.c. field. The magnitude of the a.c. bias field was $4 \times 10^{-4}$ p-p. The first harmonic component was measured in an applied d.c. field of $10^{-4}$T shown in Figure 5, giving a sensitivity of 0.05 mV/µT. For a signal to noise ratio of unity, 0.1 m of magnetically sensitive fibre detects $10^{-8}$T at 0 to 1 Hz bandwidth. A sensor head using 10 passes of fibre along a 0.1 m length of metallic glass gave a sensitivity of $4 \times 10^{-8}$T. The anticipated factor of 10 improvement was not achieved because of poor strain coupling between the fibre and metallic glass. At zero d.c. applied field there was a spectral component at the first harmonic of the a.c. bias in the interferometer output. This corresponded to the magnitude of the resolved component of the earths magnetic field ($10^{-4}$T) in the direction of the interferometer magnetostrictive axis. The interferometer output due to this component was removed by applying an additional d.c. field of equivalent magnitude but in the opposite direction along the magnetostrictive material.

The magnetometer sensitivity to d.c. magnetic fields as a function of a.c. bias field magnitude was investigated. The d.c. magnetic field magnitude was kept constant at $10^{-6}$T, and the bias field varied from 0 to $10^{-4}$T. The interferometer output was proportional to the bias field magnitude within the quadratic region of the magnetostrictive curve. The a.c. field was maintained at $4 \times 10^{-4}$ T p-p for maximum applied d.c. field sensitivity.

The interferometer first harmonic output as a function of a.c. bias field frequency is shown in Figure 6. The bias field magnitude was $4 \times 10^{-4}$T. The low frequency cut-off in sensitivity was due to the detection scheme, which compensated for drifts and phase signals below 100 Hz. Resonances were observed in the sensitivity from 100 Hz to 5 kHz which were associated with the mechanical geometry of the sensor head. A smoother frequency response, at the expense of phase sensitivity to magnetic fields, can be obtained by bonding the metallic glass to a rigid former. This frequency-dependent sensitivity was observed in our previous work on linear metallic glass sensor heads(7) and has also been reported by other workers(3). The origin of these resonance peaks in linear sensor heads has not been clearly identified. In a material with magnetisation anisotropy, such as the metallic glass, which is also very flexible, mechanical resonances are likely during magnetisation.

Figure 4 also shows the magnetometer spectral response to an a.c. signal field of $10^{-4}$T at a frequency of 160 Hz. The magnitude of the sidebands on the first harmonic of the bipolar a.c. field at $700 \pm 160$ Hz was proportional to the a.c. signal field magnitude from 0 to $10^{-4}$T. The a.c. sensitivity was $10^{-8}$T/m.
CONCLUSIONS

An unbalanced all-fibre Mach-Zehnder interferometer powered by a frequency stabilised GaAlAs laser diode has been operated with a noise floor at 1 kHz of $10^{-5}$ radians. This interferometer has been used as a magnetometer. A sensitivity of $10^{-9}$T/m at 0 to 1 Hz bandwidth has been achieved for both d.c. and a.c. applied fields. The output produced by an applied d.c. magnetic field output has been imposed on a carrier frequency of 700 Hz in order to allow measurement of signals which would otherwise be obscured by the 1/f noise characteristic of the all-fibre interferometer. The use of a frequency stabilised semiconductor diode laser to drive an all-fibre interferometer configured as a magnetometer, to measure low-frequency a.c. and d.c. fields, represents a significant advance towards a practical optical fibre magnetometer.

ACKNOWLEDGEMENTS

The authors thank STL Limited for permission to publish this work and are grateful to B.J. Scott for construction of the laser drive electronics and D.N. Batchelder for useful discussions. One of us (RHP) acknowledges support from the SERC through a C.A.S.E. award.

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Amplitude of oscillating bias field — 

No output at frequency of bias field at zero ambient field

Output at frequency of bias field at d.c. field

(a) Bipolar bias field (zero field)  
(b) Bipolar bias field (small d.c. field present)

Fig. 1 Detection scheme for a d.c. magnetic field

Peltier cooler  
Current supply  
Laser heat sink  
M.O.  
B.S.  
Temperature stable package  
Single mode fibre Fabry-Perot  
Frequency stabilised laser output  
Feeds (i) Mach-Zehnder magnetometer  
(ii) Fabry-Perot temperature sensor

M.O. — Microscope objective  
P.D. — Photodetector  
C — Control operational amplifier  
A — Current amplifier  
B.S. — Beam splitter

Fig. 2 Fibre Fabry-Perot laser frequency stabilisation system

Horizontal axis 100 Hz/div  
Vertical axis 10 dB/div

Fig. 3 Laser noise spectra demonstrating a 20 dB reduction in laser noise with the Fabry-Perot frequency stabilisation system
Fig. 4 Mach-Zehnder output spectrum showing:

(i) The fundamental component of a bipolar a.c. magnetic field at 700 Hz representing the presence of an additional $10^{-6}$ tesla d.c. field

(ii) Side bands $\omega_c \pm \omega_b$ corresponding to a $10^{-6}$ tesla r.m.s a.c. magnetic field at 160 Hz

Fig. 5 Interferometer output versus d.c. magnetic field
- 0.1m of magnetically sensitive fibre
- 0m of magnetically sensitive fibre
- a.c. bias field: $4 \times 10^{-4}$ tesla at 700Hz

Fig. 6 Interferometer first harmonic output signal versus a.c. bias field frequency