Robust hypercardioid synthesis for spatial audio capture: microphone geometry, directivity and regularization

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ABSTRACT

Frequency-invariant beamformers are useful for spatial audio capture since their attenuation of sources outside the look direction is consistent across frequency. In particular, the least-squares beamformer (LSB) approximates arbitrary frequency-invariant beampatterns with generic microphone configurations. This paper investigates the effects of array geometry, directivity order and regularization for robust hypercardioid synthesis up to 15th order with the LSB, using three 2D 32-microphone array designs (rectangular grid, open circular, and circular with cylindrical baffle). While the directivity increases with order, the frequency range is inversely proportional to the order and is widest for the cylindrical array. Regularization results in broadening of the mainlobe and reduced on-axis response at low frequencies. The PEASS toolkit was used to evaluate perceptually beamformed speech signals.

1 Introduction

Spatial filtering with microphone arrays can be used for many applications, including speech enhancement, source separation, dereverberation, tracking and localization. In object-based audio, there exist opportunities to apply beamforming with the aim of isolating [1] or enhancing [2] certain audio objects in the sound scene when recording with a microphone array.

There are many classical techniques for beamforming, including delay-and-sum, superdirective beamforming, and beamforming with linear-constraints (e.g. LCMV, MVDR) [3, 4]. These methods are optimized over a narrow bandwidth, meaning that the beamwidth and sidelobe positions change significantly with frequency. Thus, the frequency response of off-axis sources can be far from flat. In contexts where audio quality is of concern, methods to approximate a frequency-independent beampattern are desirable, providing more consistent attenuation with frequency than narrowband beamforming and shotgun microphones [5, 6].

In the literature, there exist families of methods to design such frequency-independent beampatterns. Differential microphones achieve equivalent high-order directivities to their first-order counterparts by combining the output of more than two microphones. However, their inherent sensitivity to self-noise and positioning errors often limits their practical directivity to second or third order [7]. While recent work [8, 9, 10, 11] has considered robust design of differential microphones for linear and circular arrays, these do not overcome the very low robustness at low frequencies unless a constraint on white noise gain (WNG) is also imposed. Modal beamforming (see e.g., [12, 13]) performs a decomposition in terms of harmonic basis functions whose coefficients are linearly combined to synthesize arbitrary beampatterns. Alternatively, least-squares methods to directly approximate target directivity patterns have been proposed [5, 14]. The spatial PCM sampling (SPS) [15] is a particular case which uses a cardioid target directivity, and has been used for spatial audio recording by processing multiple virtual microphones in parallel, steered uniformly in 3D. The advantage of the least-squares beamformers (LSBs) is that they are in principle generalizable to any given arrangement of microphones in the array. However, design choices in terms of directivity order and array design impact the directivity and frequency range of the LSBs in practice.

The aim of this work is to identify the optimal array geometry and directivity order for robust spatial audio capture with LSBs in horizontal sound fields. To that end, three microphone array designs (rectangular grid, open circular, and circular with cylindrical baffle) with a fixed number of microphones and array
aperture are used to synthesize varying orders of hypercardioid beampatterns, which maximize the directivity index (DI) for a given order [7] and are frequency-independent. The effects of array geometry, directivity order and regularization on the array performance are evaluated in terms of: the WNG of the resultant microphone weights and the error in DI between the target and synthesized beampatterns; the frequency range within which LSB is truly frequency invariant; and the directional and on-axis responses. Furthermore, the relationship between target quality and interference suppression is investigated using the perceptual evaluation methods for audio source separation (PEASS) toolbox [16]. The frequency-independent LSB is introduced in Sec. 2, and Sec. 3 describes the simulation setup and evaluation metrics. Objective results in terms of the effects of regularization, array design and directivity order are presented in Sec. 4, and the results from the PEASS toolbox are discussed in Sec. 5. Finally, main conclusions are drawn in Sec. 6.

2 Background
Consider an $M$ element microphone array. The $M \times 1$ narrowband array manifold vector $a(k) = [a_1(k), a_2(k), \ldots, a_M(k)]^T$ describes the transfer function from a far-field sound source at the sth steering direction to each microphone, where $k = \omega / c = 2\pi f / c$ is the wavenumber. For the open arrays, this is simply the change in phase at the mth microphone due to the propagation distance,

$$a_m(k) = e^{jk \rho r_m} \frac{\sin (\theta_m)}{\sin \theta_m},$$

where $i = \sqrt{-1}$, $u$ is a unit vector pointing towards the look direction for a time harmonic dependence $e^{jk \rho r_m}$ [17], and $r_m$ is the position of the mth microphone. Eq. 1 can be rewritten in spherical coordinates as a Fourier Series as [18]:

$$a_m(k, \theta_0, \phi_0) = \sum_{n=-N_a}^{N_a} \hat{J}_n(k r_m \sin \theta_0) e^{j n (\theta_m - \phi_0)}$$

where $J_n$ is the Bessel function of order $n$. For a microphone on an infinitely long rigid cylinder, the manifold for the horizontal plane [19] can be extended for arbitrary plane wave propagation [20, 21]:

$$a_m(k, \theta_0, \phi_0) = \frac{\sin \theta_0}{\pi k r_m \sin \theta_0} \sum_{n=-N_a}^{N_a} \hat{H}_n^{(2)}(kr_m \sin \theta_0) e^{jn(\theta_m - \phi_0)}$$

where $H_n^{(2)}$ is the derivative of the Hankel function of the second kind, $\theta_0$ and $\phi_0$ are the inclination and azimuth angles of the source direction of arrival [17] and $r_m$, $\phi_m$ and $z_m$ are the microphone positions in cylindrical coordinates. The order $N_a$ is chosen in order to achieve an accurate representation in the spatial domain up to the maximum frequency of interest $f_{\text{max}} = 20$ kHz, following $N_a = [1.1 k_{\text{max}} r_m]$ [22]. The $M \times S$ matrix $A(k) = [a_1(k), a_2(k), \ldots, a_M(k)]$ can in turn be defined as the collection of the array manifold vectors for all $S$ steering directions. The signals captured by the microphones $x(\omega)$ can then be expressed as:

$$x(\omega) = A(\omega)s(\omega) + v(\omega)$$

where $s(\omega) = [s_1(\omega), s_2(\omega), \ldots, s_S(\omega)]^T$ is an (often sparse) vector of source signals at $S$ steering directions and $v(\omega)$ is the $M \times 1$ vector of additive noise signal with arbitrary spatial characteristics. The output of a filter-and-sum beamformer $y(\omega)$ is then expressed as:

$$y(\omega) = w^H(\omega)x(\omega).$$

To set up a least-squares problem, we define a (frequency-independent) target directivity pattern vector $d = [d_1, d_2, \ldots, d_M]$, where $d_\ell$ is the desired response at the $\ell$th steering direction. In this paper, we focus on the design of hypercardioid patterns for a certain order $N$:  

$$d_\ell = \frac{1}{2N+1} \sum_{n=0}^{N} b_n \cos[n(\phi_0-\phi_\ell)],$$  

where $\phi_\ell$ and $\phi_0$ are the azimuth angles for the look direction and $\ell$th steering direction, respectively and $b = [b_0, b_1, \ldots, b_N]$ are the real coefficients for natural $(n \geq 0)$ cylindrical harmonics [11], with $b_0 = 1$ and $b_n = 2 \forall n \neq 0 [19]$.

The objective of this beamformer is to minimize the least-squares error between the synthesized and desired directional responses:

$$\min J(\omega) = ||w^H(\omega)A(\omega) - d||^2 + \beta(\omega)||w(\omega)||^2_2.$$
3 Simulations

Simulations were performed to investigate the capability of each microphone array to reproduce increasing orders of virtual hypercardioid directivity pattern. This section describes the simulation conditions and evaluation metrics.

3.1 Setup

The array manifold matrices and least-squares weights described above were implemented in Matlab. The three microphone arrays (regular rectangular grid, open circular, and circular mounted on a rigid cylindrical baffle) each comprised \( M = 32 \) omnidirectional microphones, with a radius of 0.1 m. Their properties (aperture, inter-element spacing and spatial aliasing limit) are summarized in Table 1. Directivities up to the maximum order of the arrays \( N = \left\lfloor (M - 1)/2 \right\rfloor = 15 \) were synthesized [23], steering in \( S = 360 \) directions evenly distributed on the horizontal plane (i.e., one-degree steering resolution). A non-ideal scenario is included with microphone calibration error modeled as a normally distributed deviation from the nominal sound pressure level with a standard deviation of 0.2 dB. However, results are generalizable to other error types and magnitudes. The regularization parameter \( \beta \) to mitigate the beamformer’s sensitivity to array manifold errors was varied at 100 logarithmic intervals in the range \( 10^{-10} \) to \( 10^5 \).

Table 1: Properties of tested microphone arrays.

<table>
<thead>
<tr>
<th>Aperture (m)</th>
<th>Rect.</th>
<th>Circ.</th>
<th>Cyl.</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.22</td>
<td>0.20</td>
<td>0.26</td>
<td></td>
</tr>
<tr>
<td>Element spacing (mm)</td>
<td>28.6</td>
<td>19.6</td>
<td>19.6</td>
</tr>
<tr>
<td>Spatial aliasing limit (kHz)</td>
<td>6.00</td>
<td>8.75</td>
<td>8.73</td>
</tr>
</tbody>
</table>

3.2 Evaluation

DI Error and WNG metrics were used to evaluate the performance of the beamformers under the tested conditions. The DI Error compares the achieved directivity \( \text{DI}_w \) against the directivity of the target beam \( \text{DI}_d \),

\[
\text{DI Error}(\omega) = \text{DI}_d - \text{DI}_w(\omega),
\]

where the directivity of the weights is defined as

\[
\text{DI}_w(\omega) = 10\log_{10} \frac{|w^H(\omega)a_i(\omega)|^2}{w^H(\omega)\Gamma_{\text{diff}}(\omega)w(\omega)}.
\]

and the diffuse field coherence matrix is given by

\[
\Gamma_{\text{diff}}(\omega) = \frac{1}{S} \sum_{s=1}^{S} a_s(\omega)a_s^H(\omega),
\]

where \( a_i(\omega) \) and \( a_s(\omega) \) are the array manifold transfer functions at the look direction and \( s \)th steering direction, respectively. The target DI for all odd orders up to 15 are given in Table 3. The WNG is related to the robustness of the weights, with low values indicating high sensitivity to errors. It is defined as

\[
\text{WNG}(\omega) = 10\log_{10} \frac{|w^H(\omega)a_i(\omega)|^2}{w^H(\omega)w(\omega)}.
\]

Additional objective metrics of the directional response for all orders under study are included in Table 3. \( BW_{3\text{dB}} \) is the beamwidth for a 3 dB attenuation with respect to the look direction response. The sidelobe suppression level (SSL) is the beampattern level difference at the target direction to that of the highest sidelobe. The Acoustic Contrast (AC) is the level difference at the target direction with respect to a predefined direction, in this case the interferer direction at \( \phi_i = 60^\circ \), which will be used for comparison in Sec. 5.

4 Results

This section presents the simulation results, analyzing the effects of the hypercardioid order, array design and regularization parameter on the tradeoff between directivity and robustness of the beampattern, the frequency range over which a frequency independent response is obtained and the on-axis response.
4.1 Directivity and Robustness

The relationships between frequency, regularization, and the metrics introduced above reveal the fundamental tradeoffs involved in the beampattern design. Fig. 2 (top) shows the effect of regularization on DI Error under ideal conditions, illustrated with a 7th-order target beam. It can be seen that, for each regularization value, there is a minimum frequency at which zero DI Error is achievable, and that the DI Error for a certain frequency increases with the regularization. This implies that, to achieve the most accurate directivity at lower frequencies, very little regularization must be applied. Fig. 2 (middle) shows the DI Error under non-ideal conditions. For small regularization parameters and low frequencies, the DI Error increases significantly, however, for higher regularization parameters there are minimal differences between the ideal and non-ideal performance. Thus, the performance is robust, but at a cost of reduced accuracy. This is a fundamental trade-off in regularized beamforming.

The effect of regularization on WNG is shown in Fig. 2 (bottom). Note the regions of high DI Error in Fig. 2 (middle) correspond to the lowest WNG values. The WNG required for robust performance depends on the array geometry, directivity order and manifold errors. Fig. 2 (middle) and (bottom) show very different performance for the three tested arrays. For all arrays, the WNG increases with regularization at low frequencies. Above a certain regularization, the WNG is positive. Similarly, above a given spatial onset frequency $f_o = 2.3$ kHz, the cylindrical array achieves a high WNG at all $\beta$, ensuring the same response as in ideal conditions. On the other hand, the circular array exhibits high DI Errors at specific high frequencies where the WNG dips. These correspond to Bessel function singularities in (2) when the sound field is sampled at the same radius. Although DI Error at these frequencies can be partially mitigated with higher regularization,
Fig. 4: Frequency range achieving the target response for all arrays and odd orders and $WNG_{\text{min}} = -10 \, \text{dB}$. Vertical lines indicate theoretical aliasing limits.

Table 2: Slopes and coefficients of determination ($R^2$) of spatial onset and aliasing frequencies as a function of hypercardioid order $N$.

<table>
<thead>
<tr>
<th>$f_o$</th>
<th>Slope (Hz/N)</th>
<th>$R^2$</th>
<th>$f_a$</th>
<th>Slope (Hz/N)</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rect.</td>
<td>403</td>
<td>0.990</td>
<td>29</td>
<td>0.982</td>
<td></td>
</tr>
<tr>
<td>Circ.</td>
<td>387</td>
<td>0.992</td>
<td>-444</td>
<td>0.961</td>
<td></td>
</tr>
<tr>
<td>Cyl.</td>
<td>363</td>
<td>0.988</td>
<td>-455</td>
<td>0.972</td>
<td></td>
</tr>
</tbody>
</table>

the error will still be larger than for an equivalent cylindrical array. The rectangular array does not show such a distinct improvement above a given $f_o$ irrespective of $\beta$. For this order, there are no frequencies within its operating range that are robust with minimal regularization. Thus, at high frequencies the choice of $\beta$ becomes more sensitive with the rectangular and circular arrays, potentially increasing the DI Error compared to the cylindrical array.

Fig. 3 shows the effect of directivity order on the DI Error for the cylindrical array. In ideal conditions (top) DI Error increases with order for a given frequency and $\beta$, i.e. a smaller $\beta$ is required to approach the target directivity. However, this will result in very large DI Error for non-ideal conditions as mentioned above. The spatial onset frequency above which an error-free synthesis is achieved quickly rises with order as shown in Fig. 3 (bottom). In addition to higher onset frequencies, higher calibration errors would require larger regularization values to ensure a robust response.

4.2 Frequency Range

The effect of robust performance above $f_o$ was illustrated in Fig. 3 (bottom) for various orders. Fig. 4 shows the equivalent operative frequency range of the LSB with a WNG constraint ($WNG_{\text{min}}$) of $-10 \, \text{dB}$ for all odd hypercardioid orders and the three arrays tested. This was calculated as the frequency range within which the directional response error $d_{\text{error}} \leq -30 \, \text{dB}$, ensuring a minimum target response accuracy, where:

$$d_{\text{error}}(\omega) = 10\log_{10} \left( \frac{\sum_{s=1}^{S} |d(\omega, \phi_s) - d(\phi_s)|^2}{\sum_{s=1}^{S} |d(\phi_s)|^2} \right). \tag{14}$$

It can be seen clearly that as the order increases the frequency range over which the target frequency-independent directivity is synthesized narrows. At low frequencies the onset spatial frequency rises with order since the regularization is increased to meet the robustness constraint. For example, if we require performance down to 100 Hz, only a 1st order beam will give robust, frequency-independent performance.

In terms of the array geometry, the cylindrical array has consistently the widest bandwidth for all orders, followed by the circular array, with the rectangular being the narrowest. The onset frequency is lowest for the cylindrical array due to the diffraction around the baffle. The spatial aliasing frequency $f_a$ for each order of LSB can be compared with the theoretical aliasing frequency $f_a = c/(2d)$, illustrated with vertical lines in Fig. 4. The aliasing frequency of the rectangular array nearly matches its theoretical value of 6 kHz for all orders up to 7. Above that, this array cannot synthesize higher hypercardioid patterns without deviating from
the nominal response. Conversely, both circular arrays can do so up to 15th order. The aliasing frequency reduces with order for the circular arrays. Note this upper limit is extended beyond their theoretical value of 8.7 kHz for orders $N < 13$. Thus, when the directivity order is lowered from the limit imposed by the number of sensors ($N = \lfloor (M - 1)/2 \rfloor$), the aliasing frequency of circular arrays extends beyond its theoretical limit.

The slope of the onset and aliasing frequencies as a function of order is calculated using linear regression and given in Table 2. The rate of increase in $f_o$ is similar for the three arrays. $f_a$ decreases similarly for the two circular arrays but barely varies for the rectangular one. Note the magnitude of the slope for the aliasing frequency is larger than that of the onset frequency, as this is not obvious from Fig. 4 with a logarithmic frequency axis. The linear fit is confirmed by the coefficient of determination $R^2$ being above 0.96 for all cases except for $f_a$ of the rectangular array.

Fig. 5 shows the target DI and the frequency ranges of the three arrays in terms of number of octaves as a function of order. This exemplifies the tradeoff between directivity and bandwidth when choosing the order. The bandwidth reduction with order was also observed for modal beamforming with cylindrical [23] and spherical arrays [24].

### 4.3 Beampattern

Given the required tradeoff between robustness and DI performance, an important question remains about whether any performance degradation is acceptable for practical purposes. Following from the DI Error for the cylindrical array in non-ideal conditions shown in Fig. 2(b, right), the equivalent beampatterns corresponding to three (frequency-independent) regularization parameters are shown in Fig. 6. The left plot shows a completely saturated beampattern at low frequencies due to low robustness to simulated errors. This highlights the importance of regularization even for small calibration errors such as 0.2 dB. The center plot shows the beampattern when appropriate regularization is applied; here, the high-frequency performance (in the robust region of WNG) is equivalent to the leftmost plot, but the peak level and directivity decrease at lower frequencies compared to those for the target response. The right plot shows an extreme example of over-regularization. Here, the level is very low across all frequencies, and the normalized beampattern shows a complete collapse of directivity at low frequencies. Moreover, amplification at mid frequencies from the scattering of the baffle is observed since, for very large $\beta$, the directional response in this region is proportional to $d\mathbf{A}(\omega)\mathbf{A}(\omega)$, thus being dependent on the array manifold.

### 4.4 On-axis Response

In addition to the broadening of the main lobe, quantified by the DI Error, increased regularization impacts on the flatness of the on-axis response. The low-frequency roll-off effect can be seen in Fig. 7 for each
array and 1st, 5th and 11th order target beams with a fixed regularization parameter ($\beta = 2$). There are two broad trends; first, there is a minimum frequency of flat on-axis response, following the observations above, which increases with order, as also seen in Fig. 4; second, the array design influences the precise roll-off frequency while the roll-off rate is roughly constant across the arrays.

The cylindrical array consistently retains a flat frequency response to a lower frequency than the other arrays, and all arrays perform well up to their aliasing limits shown in Fig. 4, subject to notches in the response for the open circular array.

### 5 Evaluation using PEASS

The effects of modeling errors, regularization, directivity order and array geometry on the directional response have been evaluated objectively in Sec. 4. The aim of this section is to assess how the directivity order and array geometry influence the perceptual impression of LSB recordings. To that end, PEASS [16] was used as an objective metric derived from subjective listening data. PEASS was also used in the context of microphone array beamforming in [1, 25, 26], whereas only the signal-to-interferer ratio, rather than the perceptual scores, was used in [27].

#### 5.1 Stimuli creation

The sound scene comprised two speech signals: one target female English speaker and one interferer male Danish speaker, both being 10 s long.

To test the effect of microphone array geometry perceptually, microphone signals were simulated for the three arrays considered. Array transfer functions were modeled with a 1024-point FIR filter per sensor with a sampling frequency of 16 kHz. Microphone signals were synthesized with the stimuli using (4) with the target and interferer signals at $\phi_t = 0^\circ$ and $\phi_i = 60^\circ$, respectively. The beamformer coefficients were calculated with $W_{NG_{min}} = -10$ dB to ensure a robust response and the output signal for each order was obtained as per (5). The equivalent beampatterns for $N = 5$ and the three arrays in non-ideal conditions are shown in Fig. 8.

#### 5.2 Results

Perceptual scores from PEASS are calculated for the three microphone arrays and all odd directivity orders up to 15. Equivalent scores for an omnidirectional microphone at the center of the array and delay-and-sum beamformer (DSB) are also included for reference. All scores are shown in Fig. 9, ranging from 0 to 100 with larger values indicating better performance.

The overall perceptual score (OPS) increases from 9 for 1st order to 36-40 for 3rd order before flattening above that. This is caused by the same trend observed for the interferer perceptual score (IPS), rising sharply from 19 to above 80 for 1st to 3rd order before settling. Fig. 9 also shows that a LSB of order 3 or above outperforms the omni microphone and DSB, whereas both DSB and LSB of any order can reduce the influence of the interferer compared to the reference omni.

The target perceptual score (TPS), rates the quality of the target signal extracted from the beamformer. TPS increases marginally for higher orders yet not monotonically. Higher TPS scores are also seen for the cylindrical array followed by the circular and lastly rectangular arrays, determining the same ranking of OPS. The artifacts perceptual score (APS), rating musical noise and other artifacts in the beamformed signal, is 87 for all arrays and LSB orders as well as DSB and omni.
Fig. 9: PEASS scores for speech with WNG_{min} = −10 dB in ideal conditions: omni microphone, DSB and 1st-15th order hypercardioid LSB.

5.3 Discussion

The steep rise of IPS from 1st to 3rd order and subsequent flattening suggests that an increased attenuation of the interferer is very noticeable from 1st to 3rd order but there is no obvious benefit of increasing the order further. The IPS correlates well with the AC (Table 3): it increases from 3.5 dB for first order to 16.9 dB for 3rd order, indicating that the objective improved attenuation is perceptually noticeable; for higher orders AC increases less rapidly and not monotonically since the interferer lies at different positions in the beampattern as the latter becomes more directional with order.

On the other hand, the increase of TPS with order, even if moderate, is opposite to ratings from informal listening, where the responses were perceived with reduced spectral content as order increased. This was caused by the narrowed frequency range of the LSB when increasing the order, resulting in on-axis roll-off at higher frequencies, as shown in Figs. 4 and 7. Therefore, it seems as though PEASS does not account for potential linear processing such as the bandpass filter introduced by LSB with higher f_o as order increases. This was confirmed by obtaining the same score (OPS=99, TPS=93) for all cases when no interferer was modeled at the microphone array input. Similarly, DSB achieved a TPS below 20, differing from informal listening ratings, where the target signal quality was maintained despite the interferer being low-pass filtered.

The higher OPS values obtained by the cylindrical array especially compared to the rectangular one are as a result of higher TPS, and IPS to a lesser extent. The higher IPS for the cylindrical array is likely to be due to the wider bandwidth compared to the other two arrays. This difference in bandwidth is accentuated as the order increases, which is also reflected in the IPS. However, as mentioned previously, the TPS does not consider the reduced bandwidth when increasing order, questioning the validity of the score rises of the circular arrays for high orders compared to the rectangular array. Informal subjective listening indicated that this difference of nearly 40 in OPS between the cylindrical and rectangular arrays may be overemphasized, albeit correctly ranked.

Finally, the APS is equally high for all cases considered, including the unprocessed omnidirectional microphone. This highlights the absence of artifacts from LSB compared to source separation and adaptive beamforming algorithms, which are signal dependent [24].

6 Conclusion

In this paper, we investigated the relationships between regularization, directivity order and array geometry for least-squares-synthesized high-order hypercardioid patterns and their effects on frequency range, robustness and directivity performance. The regularization parameter trades accuracy in directivity for robustness. When robust low-frequency performance is achieved the loss in accuracy manifests as a reduction of the frequency-independent range, broadening of the main lobe and a reduced on-axis response. The hypercardioid order balances directivity and frequency range of the LSB, with spatial onset and aliasing frequencies approaching one another as the order increases. For the 0.1 m radius array considered here, a compromise order of 3-5 may be appropriate, particularly when the target object is remixed back into the sound scene, where preservation of the target object frequency response with moderate attenuation of other objects may be preferable. In terms of the array geometry, the cylinder-baffled circular array provided the most robust response and widest frequency range. The circular array proved to be less robust at certain Bessel singular frequencies, requiring to increase regularization and hence reduce accuracy.
Both circular arrays extend their practical bandwidth from their theoretical aliasing limit when lowering the order from the maximum given by the number of microphones. The rectangular array achieved the narrowest bandwidth among the arrays tested and was unable to synthesize error-free beampatterns above 7th order.

The improved interferer attenuation with order and overall higher performance of the cylindrical array was validated by perceptually-derived PEASS scores. However, the latter neglected the bandpass filter of robust LSB, thus not penalizing the target signal as the order increased and the frequency range narrowed. This prevented us from deriving a perceptually-motivated optimum order, balancing directivity and frequency range, which may be explored in the future through listening tests alongside extending the work to 3D beams and other application-specific beampatterns.

**Acknowledgment**

This work was supported by the EPSRC Programme Grant S3A: Future Spatial Audio for an Immersive Listener Experience at Home (EP/L000539/1) and the BBC as part of the BBC Audio Research Partnership.

**References**


