Unified Power Allocation for Receive Spatial Modulation Based on Approximate Optimization

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This work was supported in part by the National Science Foundation of China under Grant 61501095, the Foundation Project of National Key Lab. of Sci. and Tech. on Comm. under Grant 6142102010702 and the International Science and Technology Cooperation Project of Sichuan Province (No.2017HH0009).

ABSTRACT In this paper, a novel unified power allocation (PA) framework is proposed for receive (pre-coding aided) spatial modulation (RSM). We find that the PA matrix design can be formulated as a non-convex quadratically constrained quadratic program (QCQP) problem, whose solution is generally intractable. To tackle this problem, we propose a pair of solvers having different trade-offs in terms of bit-error-rate (BER) and complexity. Specifically, we first propose a successive convex approximation (SCA) method, to convert the non-convex QCQP problem under consideration into a series of linear convex subproblems, where the latter can be easily solved by the classic polynomial-time based optimization method, i.e., the interior point method. To further reduce the computational complexity, we propose an augmented Lagrangian multiplier (ALM) method, which transforms the challenging non-convex constrained PA optimization problem into its unconstrained counterpart, which can be efficiently solved by an iterative manner. Our simulation results show that both the proposed SCA and ALM methods are capable of substantially improving the system error performance compared with conventional RSM system without PA as well as conventional PA-aided RSM schemes.

INDEX TERMS Approximate approximation, index modulation, power allocation (PA), receive spatial modulation (RSM), pre-coding aided spatial modulation (PSM).

I. INTRODUCTION

RECEIVE spatial modulation (RSM), also dubbed pre-coding aided spatial modulation (PSM), has been proposed as a new low-cost high-efficiency multiple-input multiple-output (MIMO) transmission scheme [1]–[4]. RSM exploits the multiplicity of antennas available at the receiver as a spatial constellation diagram to improve spectral efficiency with low complexity. The operating principle of RSM is that the transmitter activates all of its antennas and beamforms the transmit signals using a pre-designed linear precoder to a single receive antenna in a multi-antenna receiver [5], [6]. In RSM, the information bits are carried by two units: the conventional PSK/QAM constellations and the indices (or the index combination) of the activated receive antennas [7].

Due to the benefits of low-cost and low-complexity at the receiver in the RSM, there are some research activities on RSM/PSM, especially for unbalanced MIMO-aided downlink transmission scenarios. Specifically, the concept of RSM was first presented in [1], which is motivated by space-domain based index modulation. A natural extension of RSM, namely generalized RSM (GRSM), was proposed in [8] for increasing the multiplexing gain through activating multiple receive antennas so as to carry a great number of information symbols during each time slot. In [9] and [10], the authors presented closed-form bit error rate (BER) upper bounds of conventional RSM and GRSM systems, which also quantified the transmit diversity order of RSM. Recently, as a further advance, the principle of RSM was extended to multiple-antenna equipped broadcast fading channels in
[11]–[13], where a novel mathematical framework for computing the average bit error probability (ABEP), the diversity order, and the coding gain of RSM-aided MIMO broadcasting systems was proposed. More recently, the paradigms of RSM and mmWave-MIMO were combined in [14]–[16] with the objective of reducing the costs of multiple radio frequency (RF) chains. In [17], a variety of low-complexity detectors, such as the transmitter zero-forcing and transmitter minimum-mean-square-error detectors, were investigated for RSM, with the objective of ascertaining the varying requirements in complexity and reliability in practical implementation. In the aforementioned closed-loop RSM designs, apart from the conventional modulation function (precoder design for information conveying), the channel state information can be further exploited by link adaptation techniques for achieving improved performance.

One of the promising link adaptation techniques for closed-loop RSM-MIMO is power allocation (PA), which has been intensively researched in the context of spatial modulation systems. Specifically, a low-complexity Euclidean distance-based PA algorithm was proposed in [18], [19]; proposed a “worse-case-first”-based PA (WCF-PA) algorithm, whereas [20] developed two generalized precoder designs to enhance the BER in SM-MIMO systems. Optimal PA in SM-MIMO systems was considered from an information-theoretical perspective in [21]. Furthermore, in [22], the optimization criteria were discussed and compared for the precoder design in conventional SM. As a new variant of SM technique, RSM may also benefit from being combined with PA for accommodating time-varying channels. However, to the best of our knowledge, very few existing studies in the literature have attempted the integration of PSM and PA techniques. Compared to the original SM scheme, its RSM counterpart exploits linear precoding for modulation in the transmitter, thus the above-mentioned PA algorithms designed for SM in [18]–[21] may not be applicable to RSM in a straightforward manner. On this research topic, only the authors of [23] proposed a PA algorithm for the RSM scheme, which is designed through minimizing the upper bound of the instantaneous bit error probability. However, it is shown in [12] that the proposed PA only provides a marginal BER gain.

Against the above background, we propose a novel unified PA framework based on the maximum minimum Euclidean distance (MMD) criterion for RSM-MIMO communication systems. The main contributions of this paper are as follows.

- We investigate the benefits of adaptive PA based on the MMD and derive the optimal PA precoder for BPSK-modulated (4 × 2)-element RSM, which is used a benchmark in order to gain the insight into the benefits of our proposed PA algorithms. Then, it is found that the formulated MMD problem is a non-convex quadratically constrained quadratic program (QCQP) problem. To tackle this problem, we propose a successive convex approximation (SCA) method, which converts this complex problem into a series of linear convex problems. It is found that the proposed SCA method is able to offer a considerable BER performance gain.
- To reduce the computational complexity, we shrink the transmit signal constellations and only exploit the Euclidean distances of a few dominant error vectors for optimizing the PA matrix, where only spatial constellation points are considered. Then, we propose a novel method, namely leveraging augmented Lagrangian multiplier (ALM) method, to transform the challenging PA optimization problem into a sequence of unconstrained subproblems. The proposed ALM method exhibits an attractive trade-off between the BER performance and computational complexity.

The organization of the paper is as follows. Section II presents the system model of PA-RSM. Section III presents the closed-form solution for the MMD-based PA algorithm. This is followed by the proposed PA algorithms in Section IV, where both the SCA and ALM methods are detailed. Our simulation and performance comparison results are presented in Section V. Finally, Section VI concludes this paper.

Notations: Boldface capital and lowercase symbols represent matrices and column vectors, respectively. The superscript (·)T, (·)H and (·)−1 represent transpose, Hermitian transpose and inverse, respectively. ∥·∥ represents the Frobenius norm of A, while Re{·} denotes the real part of a complex variable. diag{·} refers to the diagonal operation. |·| represents the cardinality of a given set, or the magnitude of a complex quantity. The order of complexity for the PA algorithms is denoted by O(.). tr(·) is taken to mean the trace operator (sum of the diagonal elements).

II. SYSTEM MODEL

A. RSM SYSTEM MODEL WITH POWER ALLOCATION

Consider a MIMO communication system equipped with $N_t$ transmit and $N_r (N_r \leq N_t)$ receive antennas shown in Fig. 1. The essence behind RSM is to use both the index of the receive antennas and the index of the amplitude and phase modulation (APM) constellations to convey information. At each time slot, the information bits conveyed by RSM are divided into two parts, namely the antenna bits with a length of $b_1$ and the QAM/PSK bits with a length of $b_2$. More specifically, the total symbol rate of RSM is $b_1 + b_2$ and the first $b_1 = \log_2(N_r)$ bits (i.e., the antenna bits) are mapped to a spatial constellation point, which is an element of the set

$$S_{\text{spatial}} = \{e_0, e_1, \ldots, e_i, \ldots, e_{N_r-1}\},$$

where $e_i \in \mathbb{R}^{N_r}, i = 0, \ldots, N_r - 1$, is the $i$-th column of the identity matrix $\mathbf{I}_{N_r}$, which denotes the activation of the $i$-th receive antenna. Then, the last $b_2 = \log_2(M)$ (i.e., the QAM/PSK bits) are mapped to the QAM/PSK symbol $s$ selected from the signal codebook

$$S_{\text{signal}} = \{s_0, s_1, \ldots, s_i, \ldots, s_{M-1}\}.$$
where $s_m \in \mathbb{C}, m = 0, \ldots, M - 1$, is a power normalized $M$-level modulated symbol. The resulting RSM symbol $x = s_m e_i \in \mathcal{S}$ can be represented as

$$
\mathcal{S} = \{ s_0 e_0, s_1 e_0, \ldots, s_M e_0, \ldots, s_{M-1} e_{N_r-1} \},
$$

where $\mathcal{S}$ is the Cartesian product of the sets $\mathcal{S}_{\text{signal}}$ and $\mathcal{S}_{\text{spatial}}$.

At the receiver, the corresponding received signal vector of the conventional RSM system is given by

$$
y = \mathbf{H} \mathbf{P} \mathbf{B} \mathbf{Q} \mathbf{x} + \mathbf{n},
$$

where $\mathbf{y}$ is the $N_r \times 1$ received signal vector and $\mathbf{H} \in \mathbb{C}^{N_r \times N_t}$ is the MIMO channel matrix. Furthermore, $\mathbf{P}$ is an $N_t \times N_r$ matrix, which denotes the linear precoder for bit modulation in RSM. $\mathbf{B}$ is the diagonal matrix used for normalizing the power of each column of the linear precoding matrix. Finally, $\mathbf{n} \in \mathbb{C}^{N_r \times 1}$ is the additive complex Gaussian noise vector with mean zero, variance $\sigma^2$ and i.i.d. elements.

To simplify the implementation, similar to [1], we consider the zero-forcing (ZF) precoder, given below

$$
\mathbf{P} = \mathbf{H}^H (\mathbf{H} \mathbf{H}^H)^{-1}.
$$

As can be seen from (5), the elements of the main diagonal of $\mathbf{B}$ take the following form

$$
\beta_i = \sqrt{\frac{1}{\| \mathbf{H} \mathbf{H}^H \|^{-1}_{ii}}}, \quad i = 1, 2, \ldots, N_r.
$$

To improve the BER performance, PA is adopted in our proposed PA-RSM system shown in Fig. 1. To be specific, the diagonal matrix $\mathbf{Q}$ allocates the total power $P_t$ to the different transmit antennas, yielding

$$
\mathbf{Q} = \begin{pmatrix}
q_0 & 0 \\
0 & \ddots \\
0 & 0 & q_{N_r-1}
\end{pmatrix},
$$

where the diagonal elements are limited by the total power constraint as per $\sum_{i=0}^{N_r-1} q_i = P_t$.

Thus, plugging (5) and (6) into (7), the received signal simplifies

$$
y = \mathbf{B} \mathbf{Q} \mathbf{x} + \mathbf{n}.
$$

### B. Maximum Likelihood Receiver

At the receiver side, the transmit bitstream is reconstructed by detecting the transmitted signal constellation point and the index of the receive antenna. Therefore, the maximum-likelihood (ML) decoder is formed as

$$
\hat{x} = \arg\min_{x \in \mathcal{S}} \| \mathbf{y} - \mathbf{B} \mathbf{Q} x \|^2.
$$

It is worth noting that the complexity of the ML detector for RSM is low, since $\mathbf{B}$ and $\mathbf{Q}$ are both diagonal matrices. To reduce the detection load, some low-complexity linear detectors were recently proposed in [17], which are capable of satisfying varying requirements for reliability and complexity. Based on the union bound theory for ML detection [7], the pairwise error probability (PEP) $P_e$ of RSM associated with the ZF detector is given by

$$
P_e(x_i \rightarrow x_j) \approx \lambda \cdot Q\left( \frac{1}{2N_0} d_{\min}(\mathbf{q}) \right),
$$

where $Q(\cdot)$ is the Q-function, and $\lambda$ is the number of neighboring constellation points associated with the minimum distance $d_{\min}(\mathbf{q})$. Here, $d_{\min}(\mathbf{q})$ is the minimum squared Euclidian distance between two PA-aided RSM symbols $x_i$ and $x_j$, which is defined as

$$
d_{\min}(\mathbf{q}) = \min_{i \neq j} \min_{\substack{x_i, x_j \in \mathcal{S}, \\mathbf{x}_i \neq \mathbf{x}_j}} \| \mathbf{B} \mathbf{Q} (x_i - x_j) \|^2.
$$

### C. Optimization Criterion

In this subsection, we will give the optimization criterion for the PA-RSM system. Our goal is to design the PA weights adaptively with the objective of minimizing the upper bound of $P_e$, i.e., the right-hand-side of (10). Since error events mainly occur in the nearest neighbors, maximizing $d_{\min}(\mathbf{q})$ in (11) is equivalent to reducing the error probability, espe-
We consider a (4 × 2)-element BPSK-modulated RSM with PA algorithms specifically at high signal-to-noise ratios (SNRs). The PA design can be formulated as the following MMD problem:

\[
\max_{\mathbf{q}} \quad d_{\min}(\mathbf{q}) \\
\text{s.t.} \quad \|\mathbf{q}\|^2 \leq P_t,
\]

where \(P_t\) is the total power constraint. Based on (8), the squared Euclidean distance \(d_{i,j}(\mathbf{q})\) in (11) can be calculated as

\[
d_{i,j}(\mathbf{q}) = \|\mathbf{BQ}(\mathbf{x}_i - \mathbf{x}_j)\|^2 \\
= (\mathbf{x}_i - \mathbf{x}_j)^H \mathbf{Q}^H \mathbf{B}^H \mathbf{BQ}(\mathbf{x}_i - \mathbf{x}_j) \\
\overset{(a)}{=} \text{Tr}(\mathbf{Q}^H \mathbf{B}^H \mathbf{BQ}\Delta \mathbf{X}_{i,j}) \\
\overset{(b)}{=} \mathbf{q}^H (\mathbf{B}^H \mathbf{B} \circ \Delta \mathbf{X}_{i,j}) \mathbf{q},
\]

where \(\Delta \mathbf{X}_{i,j} = (\mathbf{x}_i - \mathbf{x}_j)(\mathbf{x}_i - \mathbf{x}_j)^H\), \(\mathbf{R}_{i,j} = \mathbf{B}^H \mathbf{B} \circ \Delta \mathbf{X}_{i,j}\), and \(\circ\) represents the Hadamard product. Step (a) in (13) uses the trace property of matrices \(\mathbf{A}\) and \(\mathbf{B}\) that \(\text{Tr}(\mathbf{AB}) = \text{Tr}(\mathbf{BA})\), while step (b) uses the matrix rule in [24, Th. 1.10.3]. Hence, (12) can be equivalently expressed as

\[
\max_{\mathbf{q}} \quad \min \mathbf{q}^H \mathbf{R}_{i,j} \mathbf{q} \quad \forall i,j, i \neq j \\
\text{s.t.} \quad \|\mathbf{q}\|^2 \leq P_t.
\]

III. CLOSED-FORM SOLUTION FOR THE MMD-BASED PA ALGORITHM

This section aims to derive a closed-form solution of the MMD-based PA algorithm (12). For ease of exposition, we consider a (4 × 2)-element RSM with BPSK. Hence, the signal alphabet set is \(\mathcal{S}_{\text{signal}} = \{1, -1\}\). As a result, all possible error vectors \(\mathbf{w}_{i,j} = \mathbf{x}_i - \mathbf{x}_j, i \neq j\) are listed in the following set: \(\mathbf{W} = \{(2, 0)^T, (-2, 0)^T, (0, 2)^T, (0, -2)^T, (1, 1)^T, (1, -1)^T, (-1, -1)^T, (-1, 1)^T\}\). Since some vectors in \(\mathbf{W}\) are collinear, the set to be studied can be simplified to \(\mathbf{W} = \{\mathbf{w}_1, \mathbf{w}_2, \mathbf{w}_3, \mathbf{w}_4\} = \{(2, 0)^T, (0, 2)^T, (1, -1)^T, (1, 1)^T\}\). Due to \(d_{i,j}(\mathbf{q}) = \|\mathbf{BQ}(\mathbf{x}_i - \mathbf{x}_j)\|^2 = \|\mathbf{BQw}_{i,j}\|^2\), the received signal distances are given as

\[
\begin{align*}
d_1 &= \|\mathbf{BQw}_1\|^2 = 4q_1^2\|\beta_1\|^2, \\
d_2 &= \|\mathbf{BQw}_2\|^2 = 4q_2^2\|\beta_2\|^2, \\
d_3 &= \|\mathbf{BQw}_3\|^2 = \|q_1\beta_1 - q_2\beta_2\|^2, \\
d_4 &= \|\mathbf{BQw}_4\|^2 = \|q_1\beta_1 + q_2\beta_2\|^2.
\end{align*}
\]

Based on (15), the optimization problem of (12) for (4 × 2)-element BPSK-modulated RSM can be simplified to

\[
\begin{align*}
\max_{\mathbf{q}} \quad & \min \{d_1, d_2, d_3, d_4\} \\
\text{s.t.} \quad & q_1^2 + q_2^2 = P_t.
\end{align*}
\]

As shown in Fig. 2, \(d_1\) and \(d_2\) are linear functions of parameter \(q_2^2\), while \(d_3\) and \(d_4\) are elliptic functions of \(q_1^2\). As can be observed from (16), the closed-form solution \(q_2^2\) is one of the intersections between distances \(d_1\), \(d_2\), \(d_3\) and \(d_4\). Let \(d_i^{(j)}\) denote the square of the PA weight for the \(j\)-th intersection point. As shown in Fig. 2, we have five possible intersection points and hence we need to calculate the values of \(q_1^{(j)}, j = 1, \ldots, 5\).

To be specific, the intersection point corresponding to \(q_1^{(1)}\) in Fig. 2 satisfies

\[
\begin{align*}
d_1 &= 4q_1^{(1)}\|\beta_1\|^2 = d_2 = 4q_2^{(1)}\|\beta_2\|^2 \\
q_1^{(1)} + q_2^{(1)} &= P_t.
\end{align*}
\]

Hence, we obtain\(^1\)

\[
\begin{align*}
q_1^{(1)} &= \beta_2^2/(\beta_1^2 + \beta_2^2)P_t, \\
q_2^{(1)} &= \beta_1^2/(\beta_1^2 + \beta_2^2)P_t.
\end{align*}
\]

Then, the power \(q_1^{(2)}\) associated with the second intersection of \(d_1 = d_3\) in Fig. 2 is given by

\[
\begin{align*}
d_1 &= 4q_1^{(2)}\|\beta_1\|^2 = d_3 = \|q_1^{(2)}\beta_1 - q_2^{(2)}\beta_2\|^2 \\
q_1^{(2)} + q_2^{(2)} &= P_t.
\end{align*}
\]

And a quadratic equation with respect to \(q_1^{(2)}\) is obtained after the simplification

\[
3\beta_1^2q_1^{(2)} + 2\beta_1\beta_2q_2^{(2)}q_1^{(2)} - \beta_2^2q_2^{(2)} = 0,
\]

\[
q_1^{(2)} + q_2^{(2)} = P_t.
\]

Therefore, the solution to (20) is given by

\[
\begin{align*}
\beta_1^2q_1^{(2)} &= \beta_2^2q_2^{(2)}, \\
q_1^{(2)} + q_2^{(2)} &= P_t.
\end{align*}
\]

Comparing (17) and (21) leads to the revelation that these two problems are equivalent, and hence they have the same solution below which is also illustrated in Fig. 2.

\[
\begin{align*}
q_1^{(2)} &= \beta_2^2/(\beta_1^2 + \beta_2^2)P_t, \\
q_2^{(2)} &= \beta_1^2/(\beta_1^2 + \beta_2^2)P_t.
\end{align*}
\]

\(^1\)By the definition of \(\beta\) which is shown in Eq. (6), the value of \(\beta\) is real.
Similar to the evaluation process of $q_1^{2(2)}$, we can obtain the power $q_1^{2(3)}$ associated with $d_4 = d_1$, the power $q_1^{2(4)}$ associated with $d_3 = d_3$, and the power $q_1^{2(5)}$ associated with $d_2 = d_4$ step by step, which are given by:

$$
\begin{align*}
q_1^{2(3)} &= \beta_2^2/(\beta_1^2 + \beta_2^2) P_t \\
q_1^{2(4)} &= 9\beta_2^2/(\beta_1^2 + 9\beta_2^2) P_t \\
q_1^{2(5)} &= \beta_2^2/(\beta_1^2 + \beta_2^2) P_t.
\end{align*}
$$  

(23)

Then, the power $q_2^2$ corresponding to $q_2^2$ that satisfies the total power constraint can be obtained as follows:

$$
\begin{align*}
q_2^{2(3)} &= 9\beta_1^2/(9\beta_1^2 + \beta_2^2) P_t, \\
q_2^{2(4)} &= \beta_1^2/(\beta_1^2 + \beta_2^2) P_t, \\
q_2^{2(5)} &= \beta_1^2/(\beta_1^2 + \beta_2^2) P_t.
\end{align*}
$$  

(24)

In addition, the power $q_1^{2(6)}$ associated with $d_4 = d_4$ satisfies $q_1^{2(6)} q_2^{2(6)} = 0$. This implies that $q_1^{2(6)} = 0$ or $q_2^{2(6)} = 0$ which is not considered as a legitimate PA candidate. It is worth mentioning that although this method can obtain a closed-form solution to the MMD-based PA algorithm, the computational complexity becomes higher when the number of receive antennas and modulation order increase, since the solution is obtained by exhaustive search in a large search space encompassing by all legitimate candidate transmit symbols or error vectors. Hence, for a higher throughput, we propose two simple numerical approaches for this challenging non-convex MMD problem.

IV. PROPOSED POWER ALLOCATION ALGORITHM FOR RSM SYSTEMS

As can be seen from (14), the PA optimization problem is a non-convex QCQP one, since the objective function $q^H R_{i,j} q$ is a non-convex quadratic function. In this section, we present two methods to tackle this intractable problem. Firstly, by jointly considering both the signal and spatial constellations of RSM, we propose an SCA method, which decomposes the original high-complexity QCQP problem into a series of low-complexity convex problems. The SCA solver is capable of achieving a favorable BER performance. Then, to reduce computational complexity, we develop a new low-complexity but powerful solver, namely the ALM, which only considers dominant error vectors for optimizing the PA matrix, such as the error vectors generated by the spatial constellation points. Since the optimization space in ALM solver is shrink, the related complexity is considerably reduced.

A. SUCCESSIVE CONVEX APPROXIMATION (SCA) METHOD

The essence behind the SCA method is to approximate the original non-convex problem to a series of convex problems. And the solution of these approximate convex problems will gradually converge to a point (the solution).

Consider the following non-convex problem

$$
\begin{align*}
\min_{x} & \quad f_0(x) \\
\text{s.t.} & \quad f_i(x) \leq 0, i = 1, 2, ..., m,
\end{align*}
$$  

(25)

where both the objective function $f_0(x)$ and the constraints $f_i(x), \forall i$ are non-convex. In the initial step of the SCA algorithm, we first move the objective function $f_0(x)$ to the constraints by introducing an auxiliary scalar variable $t$ and re-write (25) as

$$
\begin{align*}
\min_{x} & \quad t \\
\text{s.t.} & \quad f_0(x) - t \leq 0, f_i(x) \leq 0, i = 1, 2, ..., m.
\end{align*}
$$  

(26)

Then, the constraint conditions are approximated as $f_i(x) \approx f_i(x), \forall x$. It turns out that if the approximations satisfy the following three conditions, the solutions to this series of approximations converge to a point satisfying the Karush-Kuhn-Tucker (KKT) conditions [25] of the original problem:

1. $f_i(x) \leq f_i(x)$ for all $x$;
2. $f_i(x_0) = f_i(x_0)$, where $x_0$ is the optimal solution to the approximate problem in the previous iteration;
3. $\nabla f_i(x_0) = \nabla f_i(x_0)$.

Condition (1) guarantees that the approximation $f_i(x)$ is tight for (16), and any solution to the approximate problem is a feasible point of the original problem. Condition (2) ensures that the approximate problem utilized in each iteration will further reduce the numerical value of the optimization function, i.e., $f_0(x(k)) \leq f_0(x(k-1))$, where $x(k)$ is the optimal solution to the approximate problem in the $k$-th iteration. Condition (3) ensures that the KKT conditions of the original problem are satisfied after solving a series of approximate problems. Based on the above observations, we now consider the optimization problem in (14). Since the proposed SCA method is formulated to consider both signal and spatial constellations, the dimension of calculation space of the PA weights $q$ and $R_{i,j}$ in SCA method are $MN_r$. Hence, the MMD optimization problem is re-written as

$$
\begin{align*}
\max_{q} & \quad \min_{R_{i,j}, \tilde{q}, \forall i, j, i \neq j} \tilde{q}^H \tilde{R}_{i,j} \tilde{q} \\
\text{s.t.} & \quad \|\tilde{q}\|^2 \leq P_t
\end{align*}
$$  

(27)

where $\tilde{q} \in \mathbb{C}^{MN_r}, \forall i, j, i \neq j$ are the PA weight vectors for all possible RSM symbol $x = s_m e_i \in S$, and $\tilde{R}_{i,j}$ is a $M N_r \times M N_r$ positive semi-definite matrix. Similar to problem (25), by introducing the auxiliary scalar variable $t$, problem (14) is converted to

$$
\begin{align*}
\max_{\tilde{q}} & \quad t \\
\text{s.t.} & \quad \tilde{q}^H \tilde{R}_{i,j} \tilde{q} \geq t, \forall i, j, i \neq j \\
& \quad \|\tilde{q}\|^2 \leq P_t.
\end{align*}
$$  

(28)

Based on the reformulated problem (28), the main processes of the proposed SCA method are as follows. Firstly, linearizing the constraints $\tilde{q}^H \tilde{R}_{i,j} \tilde{q}$ by writing $\tilde{q}^H \tilde{R}_{i,j} \tilde{q}$ as

$$
2\tilde{q}_k^H \tilde{R}_{i,j} \tilde{q}_k - \tilde{q}_k^H \tilde{R}_{i,j} \tilde{q}_k,
$$  

(29)

where $\tilde{q}_k$ is the solution in the last iteration. Obviously, linearizing approximation in (29) satisfies the aforementioned
Algorithm 1 Proposed SCA Algorithm

Input: Initialize $q_0 = 0$, $T = I_{M \times N_r}$, $\varepsilon = 0.01$, $k = 1$.

Repeat

Output: Output result

Step 1: $\tilde{q}_k = q_k$

Step 2: Solve the SCA problem (31) by the interior point method to get $\tilde{q}_{k+1}$

Step 3: $\tilde{q}_k = \tilde{q}_{k+1}$

Step 4: $k = k + 1$

Until $\|\tilde{q}_{k+1} - \tilde{q}_k\| \leq \varepsilon$

Return result

The augmented Lagrangian function for (32) is

$$L(a, s, \lambda, \mu) = -a^T G_{s, j} a + \lambda(a^T a + s - P_t) + \frac{\mu}{2}(a^T a + s - P_t)^2,$$  \hspace{1cm} (33)

where $\lambda$ is the Lagrangian multiplier, $\mu > 0$ is the penalty parameter, and $s \geq 0$ is a slack variable so that $a^T a + s - P_t = 0$.

In order to obtain the optimal solution to (32), we first minimize the augmented Lagrangian function $L(a, s, \lambda, \mu)$ with respect to $a$ and $s$ in the $k$-th iteration, given by

$$\min_{a, s} L(a, s, \lambda, \mu)$$

$$s.t. \ s \geq 0.$$  \hspace{1cm} (34)

The partial derivative of the target function with respect to $s$ can be written as

$$\nabla_s L(a, s, \lambda, \mu) = \lambda_k + \frac{\mu_k}{2} (a^T a + s - P_t).$$  \hspace{1cm} (35)

With $\nabla_s L(a, s, \lambda, \mu) = 0$, we have

$$s = -a^T a + P_t - \frac{\lambda_k}{\mu_k}.$$  \hspace{1cm} (36)

Submitting (36) to (33), the augmented Lagrangian function can be simplified to

$$L(a, \lambda, \mu) = -a^T G_{s, j} a - \frac{1}{2\mu_k} \lambda_k^2.$$  \hspace{1cm} (37)

Based on the Lagrangian operation, the optimization problem (32) becomes equivalent to the following unconstrained optimization problem

$$\min_{a} L(a, \lambda, \mu).$$  \hspace{1cm} (38)

Eq. (38) can be solved by the conjugate gradient method. The gradient of $L(a, \lambda, \mu)$ with respect to $a$ is defined as

$$d^{(k)} = \nabla a L(a, \lambda, \mu).$$

Hence, the iterative process can be written as

$$a^{(k+1)} = a^{(k)} - \zeta_k d^{(k)},$$  \hspace{1cm} (39)

where $\zeta_k$ is the step size, which can be represented as

$$\zeta_k = \frac{(d^{(k)})^T d^{(k)}}{(d^{(k)})^T Dd^{(k)}},$$  \hspace{1cm} (40)

where $D = \nabla^2 L(a, \lambda, \mu)$ is the Hessian matrix.

When the approximate solution $a^{(k+1)}$ is obtained by the conjugate gradient method, we can update the Lagrangian multipliers and the penalty parameter with the following formula

$$\lambda_{k+1} = \lambda_k + \mu_k (a^{(k+1)}^T a^{(k+1)} - P_t),$$  \hspace{1cm} (41)

$$\mu_{k+1} = \rho \mu_k,$$  \hspace{1cm} (42)

where $\rho > 0$. The procedure for the proposed PA algorithm is detailed in Algorithm 2.
Algorithm 2 Proposed ALM Algorithm

Input: Initialize $a^{(0)} = I_{N_r}, \lambda_0 = 0.5, \mu_0 = 10, \rho = 2, k = 0, \varepsilon = 0.01$.

Output: Output result

Step 1: Update the primal vector $a^{(k+1)}$
$$a^{(k+1)} = \arg \min_a L(a, \lambda_k, \mu_k)$$

Step 2: Update the Lagrangian multiplier $\lambda_{k+1}$
$$\lambda_{k+1} = \lambda_k + \mu_k ((a^{(k+1)})^T a^{(k+1)} - P_t)$$

Step 3: Update the penalty parameter $\rho_{k+1}$
$$\rho_{k+1} = \rho \mu_k$$

Step 4: $k = k + 1$

Until $\|a^{(k+1)} - a^{(k)}\| \leq \varepsilon$

Return result

**TABLE 1. Complexity comparison of various PA schemes**

<table>
<thead>
<tr>
<th>PA schemes</th>
<th>$4 \times 2$-BPSK</th>
<th>$4 \times 2$-QPSK</th>
<th>$4 \times 2$-16QAM</th>
<th>$8 \times 4$-QPSK</th>
</tr>
</thead>
<tbody>
<tr>
<td>SCA method</td>
<td>2112</td>
<td>33024</td>
<td>8992704</td>
<td>526336</td>
</tr>
<tr>
<td>ALM method</td>
<td>896</td>
<td>12464</td>
<td>3146864</td>
<td>198528</td>
</tr>
</tbody>
</table>

**V. SIMULATION RESULTS AND DISCUSSION**

In this section, in order to validate the performance of the two proposed PA algorithms, simulation results of the proposed algorithms over the Rayleigh flat fading channel will be given. In addition, the number of realizations is $10^7$ at each SNR point.

**A. COMPLEXITY ANALYSIS**

In this subsection, we evaluate and compare the convergence behavior of the proposed SCA and the ALM methods for PA-aided RSM-MIMO systems with $N_t = 6$ dB and 9 dB. In Fig. 3, we consider the PA-RSM systems with $N_t = 4$, $N_r = 2$, and QPSK. As can be seen from Fig. 3, the ALM method exhibits a better convergence behavior than its SCA counterpart. Specifically, the ALM method only needs $C_{ALM} = 3$ iterations to converge, while the SCA method needs $C_{SCA} = 8$ iterations to achieve a satisfactory BER performance.

We then compare the computational complexity of the proposed SCA and ALM methods for PA-RSM systems. The main difference in complexity between these two methods lies in the complexity in solving for the PA matrix $Q$. The complexity order of the proposed SCA method is

$$O_{SCA} = N_t^2 M^2 N_t + C_{SCA} (M^4),$$

where $N_t^2 M^2 N_t$ is the computation required for constructing the convex optimization problem (28). The number of operations required for the convex solving process is $N_t^2 M^4$ due to computing $\text{Re} \{ 2\tilde{q}^H \tilde{R}_{ij} \tilde{q} - \tilde{q}^H \tilde{R}_{ij} \tilde{q} \}$. And $C_{SCA}$ is the number of iterations.

For the ALM method, the complexity order of calculating the PA matrix $Q$ is given by

$$O_{ALM} = N_t^2 M^2 + N_t^2 N_t + C_{ALM} N_t^2 M^4 + C_{ALM} N_t^2 K_1,$$

where $N_t^2 M^2 + N_t^2 N_t$ is for constructing the optimization problem (32). Furthermore, the number of operations required for calculating the gradient in (35) is $N_t^2 M^4$. Moreover, $N_t^2 M^4$ indicates the complexity of the conjugate gradient method. $C_{ALM}$ is the number of iterations in the ALM method, and $K_1$ is the number of iterations of the conjugate gradient method. Due to the nature of conjugate direction, the method has quadratic termination, so $K_1 = 2$. As can be observed from (43) and (44), we find that the proposed ALM method is of lower complexity compared to the SCA method, since $C_{SCA} \gg C_{ALM}$, as also shown in Fig. 3.

More intuitively, the computational complexities of these two methods under various parameter configurations are shown in Table 1. The proposed ALM method can effectively reduce the complexity compared to the SCA method. For example, in the case of $N_t = 4$, $N_r = 2$ and $M = 4$, the complexity of the ALM method is 12464, which is reduced by 62.26% compared to its SCA counterpart.
the low-complexity PA-RSM (LCPA-RSM) scheme in [23] and the conventional RSM scheme under different parameter configurations.

In Fig. 4, the simulation parameters are $N_t=4, N_r = 2$, QPSK or 16 QAM. As shown in Fig. 4, the proposed ALM-aided PA-RSM, SCA-aided PA-RSM and LCPA-RSM schemes outperform the conventional RSM schemes. To be specific, the proposed ALM- and SCA-aided PA-RSM schemes and LCPA-RSM scheme provide an SNR gain about 2 dB, 2.5 dB and 1 dB over the conventional RSM scheme at BER $= 10^{-4}$, respectively. In Fig. 5, the $(4 \times 2)$-element and $(8 \times 4)$-element MIMO channels using BPSK modulation are considered. As shown in Fig. 5, the proposed ALM- and SCA-aided PA-RSM schemes and LCPA-RSM scheme still exhibit considerable BER performance improvements compared to the conventional RSM schemes. Therefore, it can be concluded that the proposed PA methods are capable of improving the BER performance considerably.

However, it is observed that the SCA method has a slightly better BER performance than the ALM method, as can be seen from Figs. 4 and 5. This is because that the SCA method jointly considers the signal and spatial constellations in the optimization process, while the ALM method considers only the spatial constellation. In other words, the search dimension of the PA solution in the SCA method is $MN_r$ while becomes $N_r$ for the ALM method. By contrast, the ALM method requires less iterations to converge compared to the SCA method. That is why the computational complexity of the ALM method is much lower than that of the SCA method. That is to say, the proposed ALM method offers an attractive trade-off between the BER performance and computational complexity compared to the SCA method.

Fig. 6 shows the BER performance of the conventional RSM, optimal PA-RSM, proposed two numerical PA-RSM and LCPA-aided PA-RSM of [23]. In Fig. 6, the BPSK-modulated $(4 \times 2)$-element MIMO channel is considered. It shows that the BERs of our proposed ALM- and SCA-aided PA-RSM are close to that of the optimal PA-RSM. Moreover, our proposed ALM- and SCA-aided PA-RSM are all superior to the PA-RSM system [23] in terms of BER.

Fig. 7 compares the BERs of the proposed ALM- and SCA-aided PA-RSM in the presence of Gaussian-distributed channel state information (CSI) errors obeying the distribution of $CN(0, \omega)$ with $w = 0.1$ and $1/\gamma$, where $\gamma$ is the average CSI estimation SNR at each receive antenna. As can be seen from Fig. 7, after introducing CSI estimation errors, the BER performances of our ALM- and SCA-aided PA-RSM systems are degraded. However, these two PA-RSM systems still provide considerable BER performance improvements compared to its conventional RSM counterpart under the same channel condition of $\omega = 1/\gamma$.

VI. CONCLUSION

In this paper, we have investigated the benefits of PA based on the MMD criterion, and have proposed a new unified PA framework for RSM-MIMO systems. Two new approximate
optimization methods, namely SCA and ALM, have been proposed to solve the formulated MMD problem. Simulation results demonstrated that the proposed SCA- and ALM-aided PA-RSM schemes are capable of providing considerably BER gains compared to the conventional designs. Our future work will apply the proposed schemes to transmit antenna selection [26], receive antenna selection [27] and other high-diversity index modulation techniques [28], [29].

REFERENCES


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