Abstract—Operational indices optimization is crucial for the global optimization in beneficiation processes. This paper presents a multi-tasking multi-objective evolutionary method to solve operational indices optimization, which involves a formulated multi-objective multifactorial operational indices optimization problem (MO-MFO) and a proposed multi-objective multifactorial optimization algorithm for solving the established MO-MFO problem. The MO-MFO problem includes multiple levels of accuracy models of operational indices optimization, which are generated on the basis of a dataset collected from production. Among the formulated models, the most accurate one is considered to be the original functions of the solved problem, while the remained models are the helpers tasks to accelerate the optimization of the most accurate model. For the multifactorial optimization algorithm, the assistant models are alternatively in multi-tasking environment with the accurate model to transfer their knowledge to the accurate model during optimization in order to enhance the convergence of the accurate model. Meanwhile, the recently proposed two-stage assortative optimization in order to improve the performance of the model during optimization is used to transfer knowledge among multi-tasking tasks. The proposed multi-tasking framework for operational indices optimization has conducted on 10 different production conditions of beneficiation. Simulation results demonstrate its effectiveness in addressing the operational indices optimization of beneficiation problem.

Note to Practitioners—Operational indices optimization is a typical method to achieve global production optimization by efficiently coordinating all the indices to improve the production indices. In this paper, a multi-objective multi-tasking framework is developed to address the operational indices optimization, which includes a multi-taking multi-objective operational indices optimization problem formulation and a multi-taking multi-objective evolutionary optimization to solve the above formulated optimization problem. The proposed approach can achieve a solution set for the decision making. The simulation results on a real beneficiation process in China with 10 operational conditions show that the proposed approach is able to obtain a superior solution set, which is associate with a higher grade and yield of the product.
The concept of multi-tasking optimization proposed in [30], [31] is able to simultaneously tackle multiple optimization tasks, which are defined as multifactorial optimization (MFO) problems. Meanwhile, multifactorial evolutionary algorithms (MFEAs) [30], [31] have been developed for addressing MFO problems. MFEAs allow implicit knowledge transfers across different optimization tasks via two approaches, i.e., assortative mating and vertical cultural transmission. By transferring positive knowledge across tasks, MFEAs are effective in exploring superior solutions of MFO problems due to problems are seldom isolated and implicitly related to each other. MFEAs have been successfully applied to many real-world problems, e.g., knapsack problems [30], rigid-tool liquid composite molding processes [31], capacitive vehicle routing problem [32], [33], bi-level optimization problems [34], expensive computational problems [35].

Motivated by the effectiveness of multi-tasking optimization, an ideal methodology to solve operational indices optimization of beneficitation process (OIOB) problem by using the multi-tasking optimization framework is proposed in this paper. The multi-tasking optimization framework for the operational indices optimization of beneficitation process involves a developed multifactorial operational indices optimization problem and a proposed multi-objective multifactorial algorithm for solving this aforementioned problem. In other words, the proposed multifactorial operational indices optimization problem that contains different models for the single operational indices optimization problem is also a kind of multiform optimization [36]. As suggested in [36], each of the formulation in this paradigm is likely to possess a unique search behaviors in a multi-task environment, thereby benefiting the exploration of optimal solution by leveraging positive knowledge of each formulation. In particular in this work, the formulated multi-objective multifactorial operational indices optimization problem involves different level of accurate models for the operational indices optimization formulated on the basis of the process dataset. Among them, the most accurate model is considered to be the original functions of the operational indices optimization, which usually be more difficult to be solved due to the complicity of the operational indices optimization problem. The remaining lower accurate models are the assistant tasks, they equipped with simpler structures are likely to be easier solving. By combining all the models into multi-task paradigm, the superior knowledge quickly explored by assistant models can accelerate the convergence of the accurate model via knowledge transfer.

The recently proposed two-stage assortative mating for multi-objective multifactorial optimization algorithm(TMO-MFEA) [37] is expected to solve the above MFO problem. TMO-MFEA first clusters decision variables into diversity-related variables(DV) and convergence-related variables(CV) by using decision variable clustering method proposed in [38], where DV helps to distribute individuals on the whole Pareto front (PF) widely and CV helps to push individuals to the true PF. Thereafter, these two types of variables undergo assortative mating independently by using different random mating probability to generate the integral offspring to enhance both the convergence and diversity in solving multi-objective MFO problems. In the literature of multi-tasking optimization, all tasks in MFO problems are often treated equally [30]–[34]. However, the developed multifactorial operational indices optimization problem involves three optimization tasks, and only the accurate complex model receives the most attention among the models. To this end, TMO-MFEA alternates multi-tasking environment for all the assistant tasks with the accurate task to enhance the accurate model obtaining much useful information, termed as ATMOMFEA. Specifically, at each generation in ATMOMFEA, only one assistant task is selected to transfer knowledge to the accurate task by the multi-tasking optimization algorithm, and all the assistant tasks are alternately under multi-tasking environment with the accurate task during the entire optimization.

The rest of this paper is organized as follows. Section II briefly describes multi-tasking optimization, including the evolutionary multi-tasking optimization problem and algorithm. The description of solved OIOB problem and its optimization function are presented in Section III. Section IV describes the multi-tasking framework for OIOB, including the formulation of the multifactorial operational indices optimization problem, as well as the proposed multifactorial multi-objective optimization algorithm. The computational tests on different operating Condition s and the results are analyzed in Section V. Finally, Section VI concludes the paper.

II. EVOLUTIONARY MULTI-TASKING OPTIMIZATION

This section presents the evolutionary multi-tasking optimization. The multi-tasking optimization problem, also name multifactorial optimization (MFO) problem, is first introduced. Then the multifactorial optimization algorithm (MFEA) to solve MFO problems is presented.

A. Multifactorial Optimization Problem

An MFO problem involves multiple optimization tasks to be tackled simultaneously by a single solver. Suppose $K$ optimization tasks are in an MFO and all tasks are assumed to be minimization problems. The MFO can then be defined as follows:

\[
\{x_1, x_2, \ldots, x_K\} = \text{argmin}\{F_1(x), F_2(x), \ldots, F_K(x)\}
\]

s.t. $x_i \in X_i, i = 1, 2, \ldots, K$

where $F_i(x), i = 1, 2, \ldots, K$ represents a single optimization task and, $x_i$ is a set of feasible solutions in the search space of the $i$-th task $X_i$.

The multi-tasking optimization is based on the evolutionary optimization algorithm. Therefore, to facilitate the evolutionary multi-tasking optimization algorithm, each individual $p_i$ in MFOs has the following new definitions [30]:

**Definition 1.** Factorial Rank: The factorial rank $r_i^j$ of individual $p_i$ for task $T_j$ is the index of $p_i$ in the list of population members, which is sorted in decreasing order of preference with respect to $T_j$.  

**Definition 2.** Skill Factor: The skill factor $\tau_i$ of individual $p_i$ represents which task of $p_i$ is associated with.

**Definition 3.** Scalar Fitness: Scalar Fitness of individual $p_i$ in a multi-tasking environment is defined as $\phi_i = 1/r_i^t$.

The Factorial Rank (Definition 1) of an individual is obtained by comparing all individuals in the population in terms of one task. If the tasks are MOPs, then the individuals are compared on the basis of non-dominated fronts and crowding distances. Suppose two given individuals $p_1$ and $p_2$, whose non-dominated sorting fronts are NF1 and NF2 and crowding distances are CD1 and CD2, respectively, are in Task 1. $p_1$ is considered to be preferred over $p_2$, which means factorial rank $r_1^1 < r_2^1$, if one of the following two conditions is met:

1) NF1 < NF2
2) NF1 = NF2 and CD1 > CD2

Once the scalar fitness (Definition 3) of each individual is calculated in each generation, individuals from different tasks can be compared directly. For example, individual $p_1$ is considered dominant over $p_2$ during evolutionary multi-tasking if $\phi_1 > \phi_2$.

B. Multifactorial Optimization Algorithm

The multi-tasking optimization algorithm preserves the general procedures of traditional EAs, e.g., population initiation, evaluation, offspring generation, and environment selection. The basic structure of an MFEA is presented in Algorithm 1. MFEA incorporates individuals of all tasks into one population to enable all tasks to be optimized simultaneously. MFEA proposes a unified search space $Y$ as a shared solution space to facilitate all tasks to share the same knowledge. All individuals are distributed in $Y$, and each individual is assigned a skill factor in the population initiation of MFEA, which are steps 1 and 2 in Algorithm 1. During evaluation, all
Algorithm 1 Framework of MFEA

**Input:** $NP$, number of individuals in parent population; $K$, number of tasks.

**Output:** The best solution of each task.

1) Randomly generate $NP$ individuals in the unified search space $Y$ as an initial population $P$.
2) Assign skill factor for every individual, for case the $j$th individual, its skill factor $\tau_j = \text{mod}(j,K) + 1$.
3) Population evaluation
4) While termination criterion not fulfilled do
5) Assortative mating
6) Vertical cultural transmission
7) Offspring evaluation
8) Environment selection based on scalar fitness
9) End while

C. Unified Search Space $Y$

Implicit knowledge transfer among tasks is a unique feature of MFEAs, in which the knowledge indicates the solutions. To allow knowledge to be shared across different tasks, individuals in MFEAs are distributed across a unified search space $Y$. The boundaries of $Y$ are $0$ and $1$, and the dimension of $Y$ is $D_{\text{max}}$, which is the maximum dimension of all tasks in MFO. An individual is decoded into the solution space of a specific task before it is evaluated by that same task. For instance, suppose a solution $y_i$ is decoded into the solution space of $T_k$. The decoded solution $x_k$ is $x_k = y_i(1:D_k) \times (U_k - L_k) + L_k$, where $D_k$ is the dimension of $T_k$, and $U_k$ and $L_k$ are the upper and lower boundaries of $T_k$ separately.

D. Assortative Mating

MFEAs diversify the solutions of a task by applying assortative mating to generate new individuals, thus avoiding falling into local optima. The notion of assortative mating in the natural world means that individuals prefer to mate with those belonging to the same background. On the basis of this principle, MFEA not only enables to keep task-specific knowledge by encouraging crossovers between individuals from the same task and but also diversify task-specific knowledge by allowing mating between individuals from different tasks. The diversified knowledge of a task obtained by assortative mating helps MFEA escape local minimaums. The assortative mating process is described in Algorithm 2, where $rmp$ is the random mating probability and rand is a random number between [0, 1].

E. Vertical Cultural Transmission

After the offspring are generated, a skill factor that represents one task is assigned to each new individual by vertical cultural transmission, which indicates in MFEAs that an offspring randomly imitates one task that its parents are associated with. For instance, suppose an offspring $o$ is generated by the mutation of $p$. The skill factor of $o$ will be same with $p$. However, if the offspring is generated by two parents $p_1, p_2$ with factors are $\tau_1, \tau_2$, the skill factor $\tau$ of offspring $o$ will be $\tau_1$ or $\tau_2$. Furthermore, in case the two parents $p_1, p_2$ are from different tasks, then the offspring randomly imitates the skill factor mode in vertical cultural transmission implies, thus indicating knowledge transfer into a certain degree. Algorithm 3 lists the pseudo code of the vertical cultural transmission.

III. Operational Optimization of Beneficiation Process

This section first reviews the beneficiation process that to be solved in this paper. After that, the modeling of multi-objective operational indices optimization function is described.

A. Description of Beneficiation Process

The beneficiation process generates concentrate ore via multiple components, including raw ore processing, shaft furnace roasting, grinding, high- and low-intensity magnetic dressing, weak-intensity magnetic dressing, and concentrated ore and tailing ore processing, as shown in Fig. 1. The raw ore is first classified into two types, i.e., particle (015 mm) and lump ore (larger than 15 mm), in a screening unit and processed by different process lines. The separated particle ore is conveyed to a high-intensity magnetic production line (HMPL), whereas the lump ore is delivered to a low-intensity magnetic production line (LMPL). The lump ore, which has low intensity magnetic, is roasted in a shaft furnace before being transferred for grinding, which aims to improve its intensity magnetic and remove the contained waste rock. Meanwhile, the particle ore is delivered directly to the grinding process, as shown in HMPL. In the grinding units of the two lines, the grinding process breaks the feeding ore into pulp slurry with a suitable particle size. The pulp slurry enters the high- or low-intensity magnetic separation units, in which the concentrated ore and tailing are separated and sent to dewatering process units. After being dewatered, the mixed concentrated ore obtained from the two lines is considered the final product and sent into a storeroom. Meanwhile, the mixed tailing ore is sent to a tailing dam.
TABLE I

<table>
<thead>
<tr>
<th>Name</th>
<th>$\eta$</th>
<th>$\beta_1$</th>
<th>$\xi_1$</th>
<th>$\rho_1$</th>
<th>$\beta_2$</th>
<th>$\xi_2$</th>
<th>$\rho_2$</th>
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<tr>
<td>Lower bound</td>
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<td>16</td>
<td>46</td>
<td>18</td>
<td>68</td>
<td>60</td>
</tr>
<tr>
<td>Upper bound</td>
<td>85</td>
<td>58</td>
<td>20</td>
<td>52</td>
<td>23</td>
<td>87</td>
<td>85</td>
</tr>
<tr>
<td>Unit</td>
<td>%</td>
<td>%</td>
<td>%</td>
<td>mesh</td>
<td>%</td>
<td>%</td>
<td>mesh</td>
</tr>
</tbody>
</table>

B. Multi-objective Operational Indices Optimization Modeling

Operational indices optimization aims to improve the global production indices by coordinating the operational indices of the process line. Therefore, the production indices are considered the optimization objectives of the operational indices optimization function, and the operational indices are the decision variables. In this study, the two important production indices, mixed concentrate grade ($G$) and mixed concentrate yield ($Q$), are taken as the objectives of operational indices optimization functions. In the beneficiation process, mixed concentrate grade represents the percentage of valuable mineral composition in the mixed concentrate ore. Meanwhile, mixed concentrate yield measures the production efficiency and equipment utilization rate in the production process, which influence the production cost. In the beneficiation process, both of these production indices are expected to be maximized.

The seven operational indices, which represent the product quality of the unit processes in the production line, are taken as the decision
variables of the operational indices optimization function. They are the magnetic tube recovery rate ($\eta$), concentrate grades in the LMPL and HMPL ($\bar{\beta}_1, \bar{\beta}_2$), particle sizes of the low- and high-intensity magnetic ores ($p_1, p_2$), and low- and high-intensity magnetic tailing grades ($\xi_1, \xi_2$) as depicted in Fig. 1. These decision variables can be denoted as $x = (\eta, \bar{\beta}_1, \bar{\beta}_2, p_1, p_2, \xi_1, \xi_2)$.

In practice, the production indices are not only determined by operational indices but are also related to production conditions, including the grade of the waste rock ($a_2, a_3$), ball mill capability ($Q_1, Q_2$) and run time ($T_1, T_2$) in LMPL and HMPL, which are altogether denoted as $C = (a_2, a_3, Q_1, Q_2, T_1, T_2)$. Therefore, the input of the database models is $[x, C]$, whereas the output is $(G, Q)$.

The optimization functions of OIOB are developed by data-based method models because no exact fundamental model has been established yet for the formulation of the relationship between production and operational indices. The optimization problem model of the operational indices optimization of beneficiation can be described as follows:

$$\begin{align*}
\text{max} & \quad F_{f_1}(x) = \{G_{f_1}(x), Q_{f_1}(x)\} \\
\text{s.t.} & \quad x \in X
\end{align*}$$

(2)

where $G_{f_1}(x)$ and $Q_{f_1}(x)$ are the two objectives, concentrate grade and concentrate yield, that are trained by a machine learning method $f_1$ and $X$ is the search space of the decision variables. The boundaries of each decision variable are summarized in Table I.

As a standard formation of an MOP, Eq.2 can be formulated as follows:

$$\begin{align*}
\text{min} & \quad -\mathbf{F}_f(x) = \{-G_f(x), -Q_f(x)\} \\
\text{s.t.} & \quad x \in X
\end{align*}$$

(3)

Traditionally, existing researchers simply uses the multi-objective algorithm NSGA-II [18] to solve the established OIOB problem [15, 16]. However, the global optimal solutions of OIOB problem are difficult to obtain because unit processes in the beneficiation process are strongly coupling, thus leading to Eq.3 has many local optimums. To address this problem, a multi-tasking framework is proposed and presented in Section III.

IV. PROPOSED MULTI-TASKING MULTI-OBJECTIVE EVOLUTIONARY OPERATIONAL INDICES OPTIMIZATION

This section describes the multi-tasking optimization framework for solving OIOB. We first present the formulation of the multi-objective multifactorial operational indices optimization of the beneficiation problem. Then a brief description of TMO-MFEA [40]. Followed by is the proposed multi-tasking optimization algorithm ATMO-MFEA for solving this MO-MFO problem.

A. Multi-objective Multifactorial Operational Indices Optimization Problem Modeling

Theoretically, the any number of optimization tasks can be involved in multifactorial optimization problem. In this study, three models that are established on the basis of process data, are considered in the established multi-objective multifactorial operational indices optimization problem contains, an accurate model and two assistant models. In the multi-objective multifactorial operational indices optimization problem, the accurate model that represents the relationship between operational and production indices is the model from OIOB. Multilayer perception neural networks (MLP) is a powerful machine learning technique [41] and has been widely adopted in nonlinear regression and classification problems. In the current study, the MLP with two layers, in which the number of notes are 18 and 15, is applied to establish the accurate model of the operational indices optimization. The two assistant models, which assist the accurate model in finding optimal solutions by knowledge transfer, should be easily solved to generate useful knowledge. Thus, the two assistant tasks are modeled into simpler structures and generated by first-order polynomial regression model (PR1) and second-order polynomial regression model (PR2) separately. The constructed multi-objective multifactorial operational indices optimization problem can be described as follows,

$$\begin{align*}
\{x_{\text{MLP}}, x_{\text{PR1}}, x_{\text{PR2}}\} = \text{argmin} \{&-F_{\text{MLP}}(x), -F_{\text{PR1}}(x), -F_{\text{PR2}}(x)\} \\
\text{s.t.} & \quad x_{\text{MLP}}, x_{\text{PR1}}, x_{\text{PR2}} \in X
\end{align*}$$

(4)

where $-F_{\text{MLP}}(x), -F_{\text{PR1}}(x)$ and $-F_{\text{PR2}}(x)$ represent the MLP, PR1, and PR2, respectively, and $X$ is the search space of the decision variables.

Each optimization task in the multi-objective multifactorial operational indices optimization problem is trained using 400 groups of collected process datasets and validated with 50 groups of data. The validation results and the calculated root-mean-square error (RMSE) of these three models are shown in Fig. 2 and Table II separately.

The production conditions are often subject to changes due to equipment maintenance. Eq. 4 shows that production conditions are related to the production indices. Thus, changes in production conditions will cause the production indices to deviate from their optimal values. To

<table>
<thead>
<tr>
<th>Name</th>
<th>MLP</th>
<th>PR2</th>
<th>PR1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yield</td>
<td>276.4462 (31.1268)</td>
<td>380.6579 (59.2188)</td>
<td>1.1108e+03 (198.3324)</td>
</tr>
<tr>
<td>Grade</td>
<td>(0.2913 (0.0513))</td>
<td>(0.5255 (0.07837))</td>
<td>(0.7149 (0.1254))</td>
</tr>
</tbody>
</table>
Algorithm 4 Framework of TMO-MFEA

Input: $NP$, number of individuals in parent population; $K$, number of tasks.

Output: The best solution of each task.

1) Randomly generate $NP$ individuals in the unified search space $Y$ as an initial population $P$.
2) Assign skill factor for every individual, for case the $j$-th individual, its skill factor $\tau_j = \text{mod}(j,K) + 1$.
3) Population evaluation.
4) $(DV, CV, DVind, CVind) = \text{Decision variables clustering method}$
5) While termination criterion not fulfilled do
   \hspace{1cm} Two stage assortative mating strategy
6) Off(DV) = Assortative mating($rmp_{CV}, CV$)
7) Assign skill factor for Off via Vertical cultural transmission
8) Off(CV) = Assortative mating($rmp_{DV}, DV$)
9) Offspring evaluation
10) Environment selection based on scalar fitness
11) End while

Algorithm 5 Framework of ATMO-MFEA

Input: $NP$, number of individuals in population; $K$, the number of tasks; $Maxgen$, maximum generations

Output: The best solutions of MLP task.

1) Randomly generate $NP$ individuals in the unified search space $Y$ as an initial population $P$.
2) Assign skill factor for every individual, for case the $j$-th individual, its skill factor $\tau_j = \text{mod}(j,K) + 1$.
3) Population evaluation.
4) $(DV, CV) = \text{Decision variables clustering method}$
5) For gen = 1 : $Maxgen$
   \hspace{1cm} If mod(gen,20) < 10
6) MLP and PR1 generate offspring by two-stage assortative mating.
7) PR2 generate offspring
8) Else
9) MLP and PR2 generate offspring by two-stage assortative mating.
10) PR1 generate offspring
11) End if
12) Environment selection based on scalar fitness.
13) End for
14) Non-dominate solutions of MLP

keep the production indices running on their optimal values and the global production optimization during the entire process, operational indices optimization should be re-optimized for any given production condition. In this paper, 10 typical production conditions during past production processes are considered in finding the optimal production indices, which are presented in Table III.

B. A Brief Summary of TMO-MFEA

The TMO-MFEA was developed to keep a good diversity as well as convergence for multi-objective MFO problems by inducing different $rmp$ (random mating probability) [40]. According to [30], a larger value of $rmp$ permits higher random mating, thereby may facilitating population diversity. In contrast, a smaller value of $rmp$ would benefit for population convergence. In the multi-objective optimization problems, it is desired to balance the diversity and convergence to ensure solutions well spread on the Pareto front (PF) as well as close to the PF. Therefore, appropriate of $rmp$ plays an important role in multi-objective multifactorial algorithm.

Fortunately, decision variables in multi-objective optimization problem can be generally separated into two types, i.e. diversity-related variables (DV) and convergence-related variables (CV), where DV takes charge of a wide distribution on the whole PF and CV contributes to pushing the true PF of the individuals [38] as shown in Fig. 3. According to the above findings, the proposed TMO-MFEA sets a smaller $rmp$ for DV and a larger one for CV to encourage to convergence of DV and diversity of DV.

The proposed TMO-MFEA follows a similar framework with MO-MFEA as presented in Algorithm 4, the main difference of TMO-MFEA during the offspring generating step. A general framework ATMO-MFEA in solving the MFO problem for OIOB, evolutionary algorithm (ATMO-MFEA) is proposed for addressing MFO problem for OIOB. In the ATMO-MFEA, MLP is under the multi-tasking environment with each assistant model alternatively formulated MFO problem for OIOB, the accurate task (MLP) needs more attention when be solved, whereas the assistant models are used to improve the convergence of the MLP. In this section, a two-stage assortative mating based alternative multi-objective multifactorial evolutionary algorithm (ATMO-MFEA) is proposed for addressing MFO problem for OIOB. In the ATMO-MFEA, MLP is under the multi-tasking environment with each assistant model alternatively to obtain the knowledge from the assistant models, leading to a fast convergence. The assistant models that equipped with simple structure would be easier to convergence, therefore, transferring the knowledge of assistant models to MLP enables a fast convergence of MLP due to the high similarity between assistant models and the accurate model. An illustration of how the assistant model can help the optimization of the target model (accurate model) is illustrated in figure 4. Note that, the TMO-MFEA is applied to realize the knowledge transfer among the multi-tasking optimization tasks in ATMO-MFEA during the offspring generating step. A general framework ATMO-MFEA in solving the MFO problem for OIOB,

Fig. 3. The role of DV and CV in a minimization bi-objective MOP

C. Proposed Algorithm for Multi-objective Multifactorial Operational Indices Optimization Problem

It is known from above that among the optimization tasks of the formulated MFO problem for OIOB, the accurate task (MLP) needs more attention when be solved, whereas the assistant models are used to improve the convergence of the MLP. In this section, a two-stage assortative mating based alternative multi-objective multifactorial evolutionary algorithm (ATMO-MFEA) is proposed for addressing MFO problem for OIOB. In the ATMO-MFEA, MLP is under the multi-tasking environment with each assistant model alternatively to obtain the knowledge from the assistant models, leading to a fast convergence. The assistant models that equipped with simple structure would be easier to convergence, therefore, transferring the knowledge of assistant models to MLP enables a fast convergence of MLP due to the high similarity between assistant models and the accurate model. An illustration of how the assistant model can help the optimization of the target model (accurate model) is illustrated in figure 4. Note that, the TMO-MFEA is applied to realize the knowledge transfer among the multi-tasking optimization tasks in ATMO-MFEA during the offspring generating step. A general framework ATMO-MFEA in solving the MFO problem for OIOB,
TABLE III
Ten Typical Operational Conditions in Production Process

<table>
<thead>
<tr>
<th>Condition</th>
<th>(a_{g3}(%))</th>
<th>(a_{g2}(%))</th>
<th>(Q_1(T/h))</th>
<th>(T_1(h))</th>
<th>(a_{g2}(%))</th>
<th>(Q_2(T/h))</th>
<th>(T_2(h))</th>
</tr>
</thead>
<tbody>
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<td>Condition 1</td>
<td>19.55</td>
<td>42</td>
<td>57.79</td>
<td>96</td>
<td>32</td>
<td>69.14</td>
<td>96</td>
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<tr>
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<td>42</td>
<td>50.50</td>
<td>63</td>
<td>32</td>
<td>86.56</td>
<td>85</td>
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<tr>
<td>Condition 3</td>
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<td>87.04</td>
<td>96</td>
<td>32</td>
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<tr>
<td>Condition 4</td>
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<td>89.34</td>
<td>96</td>
<td>32</td>
<td>51.89</td>
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<tr>
<td>Condition 5</td>
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<td>Condition 6</td>
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<td>54.00</td>
<td>63</td>
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<td>Condition 7</td>
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<td>40.89</td>
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<td>Condition 10</td>
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<td>72</td>
<td>32</td>
<td>40.00</td>
<td>72</td>
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</tbody>
</table>

Fig. 4. Illustration of the process of assistant model helping the accurate model. In the figure, the assistant model can obtain optimal solution quickly, while the accurate model prone to local optimal. Then the solutions of the assistant model are transferred to the accurate model via assortative mating and vertical cultural transmission benefit to the accurate model escaping local optimal, thereby converging to global optimal.

which involves one accurate model and two assistant models, is summarized as Algorithm 5.

### V. Experimental Studies

To verify the proposed multi-tasking framework for solving OIOB problem, an empirical experiment on 10 different operational Conditions in the beneficiation process has been studied in this section. Particularly, we compare the ATMO-MFEA with a multi-task algorithm [31] and the traditional single-task method (ST) via simulation experiment to examine the performance of the multifactorial operational indices optimization problem as well as the ATMO-MFEA algorithm separately. All the compared algorithms in both multi-task and single-task methods are all based on NSGA-II [18].

The hypervolume (HV) [25] and the number of non-dominated solutions are applied as the performance indicator in this experiment to compare the results of multi-task and single-task algorithms. A large HV indicates an excellent diversity and convergence of the corresponding algorithm, whereas large number of non-dominated solutions indicates the good convergence of the algorithm. To calculate the HV, we first combine all solutions that obtained by the three algorithms in 20 runs and normalize them to [0, 1]. Then the reference point (1, 1) is used for all Conditions in the operational indices optimization problem.

#### A. Parameter settings

The parameters in the compared algorithms and the OIOB problem are outlined as follows.

1) Population size: The population size \(NP\) is set as 300 in ATMO-MFEA and NSGA-II, whereas the number of output solutions is 100.
2) Maximum generations: \(Maxgen = 200\).
3) Independent number of runs: \(runs = 20\).
4) Evolutionary operators in ATMO-MFEA and NSGA-II: S-BX [42] crossover probability \(pc = 1\), index \(\eta_c = 20\), polynomial mutation probability \(pm = 1/n\), where \(n\) is the number of variables, mutation index \(\eta_m = 20\).
5) Randomly Mating Probability in assortative mating in ATMO-MFEA: \(rmp_{CV} = 0.3\), \(rmp_{DV} = 1\).
6) Randomly Mating Probability in assortative mating in MO-MFEA: \(rmp = 0.3\).

#### B. Simulation Results and Discussions

The statistical mean and standard value of HV and the number of non-dominated solutions over 20 independent runs of each algorithm on 10 operational Conditions are shown in Table IV, where the best result of each test instance is highlighted. The Wilcoxon rank sum test is also performed at a significance level of 0.05, where the symbols “+” , “−” and “≈” denote that the result is significantly better, significantly worse, or comparable with that of ATMO-MFEA, respectively.

It can be observed in the Table IV that the MT algorithms, MO-MFEA and ATMO-MFEA achieve a large or competitive HV values against ST algorithm on all the Conditions. Meanwhile, the number of non-dominated solutions obtained by MT are substantially more than those obtained by ST on all operational Conditions, as shown in
Table IV. The results in the Table indicate that the MT works well with respect to both of convergence and diversity capability. The promising performance of MT may be attributed to the formulated multifactorical operational indices optimization problem. In multi-tasing environment, the implicit knowledge from the assistant models can accelerate the convergence of the accurate model and help MLP obtain superior solutions.

With respect to the two algorithms in MT, the proposed ATMO-MFEA loses none Condition in both of the HV and non-dominated solutions indicators, meaning that ATMO-MFEA achieves the best overall performance in comparison with MO-MFEA on the multi-factorical operational indices optimization problem. The superior performance of ATMO-MFEA may be because the accurate model gains much more attention than in MO-MFEA, which treat all models equally, thus facilitate the convergence of the accurate model.

To provide an overview of the performance of each algorithm during the optimization process, Fig. 5 depicts the mean and standard deviation of HV values over 20 runs across the optimization generation on Conditions 1, 2, and 9. This figure shows that ST achieves a good performance in the early optimization stage (before 20 generations), while generally stagnates later on. Meanwhile, the MT algorithms can maintain a relative convergence during all processes and thereby perform better than ST as the search progress with on the three Conditions. This result may be attributed to the fact that all individuals in the population belong to MLP in ST, whereas only one third of the individuals in the population belong to MLP in MT, thus making the MLP in ST obtain a larger number of superior solutions than that of MT at the early optimization process. However, the MLP in ST can easily be trapped in a local optimal and premature convergence. By contrast, with the help of knowledge from the assistant tasks, MLP in multi-task environment can skip the local optimal and achieve promising convergence.

In Fig. 5, we can also find that the MO-MFEA achieves a slow convergence speed in comparison with that of ATMO-MFEA. As mentioned above, this may be because MLP in ATMO-MFEA obtains more knowledge than in MO-MFEA, leading to a superior performance of ATMO-MFEA.

Fig. 6 plots the worst approximate Pareto fronts among the 20 runs of the 10 Conditions achieved by the two MT algorithms and ST algorithm in the objective space as listed in (a)-(j). This Figure shows that the solutions obtained by MT are distributed in a larger region on Conditions 6, 7, 8, 9 than those of ST. In other words, MT can find a high concentrate yield with an acceptable concentrate grade on these Conditions. While on the rest Conditions, e.g., Conditions 1, 2, 3, 4, 5, 10, approximate Pareto front of MT algorithms are closer to the Pareto front than ST algorithm, which means that both the higher grade and yield can be found by MT compared to ST. These observations confirm that MT algorithms can maintain better diversity than the original single-task framework, ST, thus benefit for the following decision-making.
Fig. 6. The approximate Pareto front obtained by multi-task method and single-task method on ten operational Conditions, where the dots are the values of multi-task method and circles are the single-task method.
VI. CONCLUSION AND FUTURE WORK

In this paper, a multi-tasking framework for addressing the OIOB problem was proposed. In the framework, the multi-tasking problem, including an accurate model and multiple assistant models, was first established. Afterward, ATMO-MFEA was developed for solving the formulated multi-tasking problem. The assistant models are alternatively in the multi-tasking environment with the accurate model in ATMO-MFEA to realize good knowledge transfer from the assistant models to the accurate model. The proposed multi-tasking optimization framework for operational indices optimization was tested through a numerical simulation experiment and compared with the traditional single-optimization method and a multi-task optimization algorithm.

The present study investigated the effectiveness of the multi-tasking optimization framework for operational indices optimization problem. For future work on the OIOB problem, we are interested in considering the uncertainties during production, such as change in resource grade. In addition, other optimization objectives can be considered to represent global production from more aspects. With respect to the multi-tasking algorithm, we will focus on improving the algorithm for solving problems with many objective optimization tasks. One possible way is to use fuzzy-dominated or indicator-dominated sorting instead of dominated sorting to calculate the scalar fitness of individuals. Although the knowledge transfer in the multi-tasking optimization algorithm has a positive influence on solving most MOPs, negative knowledge transfer, which degrades the solution quality of MOP, still exists. In the future, we would like to study how the negative transfer among optimization tasks can be reduced.

REFERENCES


Cuie Yang received the B.Sc. from Henan Polytechnic University in 2014 and the M.Sc. from Northeastern University in 2016. She is a Ph.D. degree candidate in control theory and control engineering from the State Key Laboratory of Synthetical Automation for Process Industry, Northeastern University. Her current research interest is multi-tasking evolutionary optimization, data-driven evolutionary optimization and their application.

Jinliang Ding (M’09-SM’14) received the Ph.D. degree in control theory and control engineering from Northeastern University, Shenyang, China, in 2012. He is a Professor with the State Key Laboratory of Synthetical Automation for Process Industry, Northeastern University. He has authored or co-authored over 100 refereed journal papers and refereed papers at international conferences. He is also the inventor or co-inventor of 17 patents. His current research interests include modeling, plant-wide control and optimization for the complex industrial systems, stochastic distribution control, and multiobjective evolutionary algorithms and its application. Dr. Ding was a recipient of the Young Scholars Science and Technology Award of China in 2016, the National Science Foundation for Distinguished Young Scholars in 2015, the National Technological Invention Award in 2013, two First-Grade Science and Technology Award of the Ministry of Education in 2006 and 2012, respectively, and the Best Paper Award of 2011-2013 for Control Engineering Practice.

Yaochu Jin (M’98-SM’02-F’16) received the B.Sc., M.Sc., and Ph.D. degrees from Zhejiang University, Hangzhou, China, in 1988, 1991, and 1996 respectively, and the Dr.-Ing. degree from Ruhr University Bochum, Germany, in 2001.

He is a Professor in Computational Intelligence, Department of Computer Science, University of Surrey, Guildford, U.K., where he heads the Nature Inspired Computing and Engineering Group. He is also a Finland Distinguished Professor funded by the Finnish Agency for Innovation (Tekes) and a Changjiang Distinguished Visiting Professor appointed by the Ministry of Education, China. His science-driven research interests lie in the interdisciplinary areas that bridge the gap between computational intelligence, computational neuroscience, and computational systems biology. He is also particularly interested in nature-inspired, real-world driven problem-solving. He has (co)authored over 200 peer-reviewed journal and conference papers and been granted eight patents on evolutionary optimization. He has delivered 20 invited keynote speeches at international conferences.

He is the Editor-in-Chief of the IEEE TRANSACTIONS ON COGNITIVE AND DEVELOPMENTAL SYSTEMS and Co-Editor-in-Chief of Complex & Intelligent Systems. He is also an Associate Editor or Editorial Board Member of the IEEE TRANSACTIONS ON EVOLUTIONARY COMPUTATION, IEEE TRANSACTIONS ON CYBERNETICS, IEEE TRANSACTIONS ON NANOBIOSCIENCE, Evolutionary Computation, BioSystems, Soft Computing, and Natural Computing. Dr Jin is an IEEE Distinguished Lecturer (2013-2015 and 2017-2019) and past Vice President for Technical Activities of the IEEE Computational Intelligence Society (2014-2015). He was the recipient of the Best Paper Award of the 2010 IEEE Symposium on Computational Intelligence in Bioinformatics and Computational Biology and the 2014 and 2017 IEEE Computational Intelligence Magazine Outstanding Paper Award. He is a Fellow of IEEE.

Tianyou Chai (M’90-SM’97-F’16) received the Ph.D. degree in control theory and engineering from Northeastern University, Shenyang, China, in 1985.

He has been with the Research Center of Automation, Northeastern University, Shenyang, China, since 1985, where he became a Professor in 1988 and a Chair Professor in 2004. He is the founder and Director of the Center of Automation, which became a National Engineering and Technology Research Center in 1997. He has made a number of important contributions in control technologies and applications. He has authored and coauthored two monographs, 84 peer reviewed international journal papers and around 219 international conference papers. He has been invited to deliver more than 20 plenary speeches in international conferences of IFAC and IEEE. His current research interests include adaptive control, intelligent decoupling control, integrated plant control and systems, and the development of control technologies with applications to various industrial processes.

Prof. Chai is a member of the Chinese Academy of Engineering, an academician of International Eurasian Academy of Sciences, and IFAC Fellow. He is a distinguished visiting fellow of The Royal Academy of Engineering (UK) and an Invitation Fellow of Japan Society for the Promotion of Science (JSPS). For his contributions, he has won three prestigious awards of National Science and Technology Progress, the 2002 Technological Science Progress Award from the Ho Leung Ho Lee Foundation, the 2007 Industry Award for Excellence in Transitional Control Research from the IEEE Control Systems Society, and the 2010 Yang Jia-Chi Science and Technology Award from the Chinese Association of Automation.