Revisiting the Relationship between Host Attitudes and Tourism Development:
A Utility Maximization Approach
Abstract

Host attitudes towards tourists are critical to the sustainable development of the tourism industry. Although numerous studies have focused on investigating host attitudes towards tourists and tourism development, the theoretical support from an economic perspective in this field is still underdeveloped. By following the social exchange theory and applying a utility maximization model, the current study not only explains Doxey’s Irridex model from an economic perspective but also complements the findings of the tourism area life cycle model proposed by Butler. Results show that the public resources at the destination, along with the ability of local community in channeling (foreign) tourism income into productivity advancement, influence the optimal level of tourism development in a destination.

*Keywords:* host attitudes; sustainable tourism; Doxey’s irrigidex model; Tourism Area Life Cycle model; social exchange theory
1 Introduction

Tourism has become an increasingly important factor in the economic and social development of a destination. Tourism development can boost the economic growth, expand employment opportunities, and improve income for both the government and the residents (Gursoy & Rutherford, 2004; Seetanah, 2011; Tao & Wall, 2009). The development of tourism may also preserve peace and order (Pratt & Liu, 2016) and minimize the perceived cultural distance between the tourists and the hosts (Fan, Zhang, Jenkins, & Lin, 2017). However, such benefits are not achieved without any costs. Tourism development in an area or region may also lead to various social and environmental issues, such as traffic congestion and pollution (Choi & Murray, 2010; Zhang, Fan, Tse, & King, 2017). Such problems often lead to a negative host attitude towards tourism and restrain the destination to establish a sustainable tourism development. As argued by Sharpley (2014), the success of tourism development depends on the balance of the relationship between the tourists and the hosts.

Positive host attitudes towards tourism development can improve the service quality and intangible infrastructure provided at the destination, which, in turn, enhance destination image and attract more tourists (Sharpley, 2014). Conversely, negative host attitudes may damage the destination image or even cause conflicts between hosts and tourists—a phenomenon that deviates from the initial motivation of tourism development. Although all destinations aim to benefit the hosts through the development of the tourism industry, the irritated voice has already been heard from a few destinations, including Hong Kong (Zhang et al., 2017), Barcelona (Gordon, 2014), and Amsterdam (Haines, 2016). Without the support of the hosts, achieving sustainable development in the tourism industry could be a challenging task. Hence, the academe and the industry must investigate the relationship between host attitudes and tourism development.

Indeed, numerous studies have already investigated the relationship between host attitudes and tourism development; however, the majority of these are empirical studies based
on one-off surveys (Sharpley, 2014). The findings obtained from such research are more likely to answer their specific research questions and have limitations in generalization. Sharpley (2014) and Bimonte and Punzo (2016) all argued that a conceptual framework is hence needed to comprehensively understand host attitudes towards tourism development. However, the theoretical support in this research field is still underdeveloped. To fill this research gap, a utility maximization model is employed in the current study to theoretically explain how and why host attitudes change along with the development of tourism from an economic perspective.

The remainder of the study is presented as follows. The frequently used theories regarding host attitudes towards tourism development are reviewed briefly in Section 2. Section 3 introduces the utility maximization model in detail and outlines the key characteristics of the model. This is followed by the findings and discussions of the model in Section 4. Finally, Section 5 concludes the study and discusses the limitations and future research directions.

2 Literature Review

In this section, the existing literature relating to host attitudes towards tourism development is reviewed. Particularly, the tourism area life cycle (TALC) model (Butler, 1980) is introduced to explain the tourism development process in a destination. This is followed by introducing of various factors which influence host attitudes towards tourism development from a sustainable viewpoint. Then, two theories related to the change of host attitudes, (Doxey, 1975)’s Irridex model and social exchange theory, are reviewed. The research gaps are identified and feasible solutions are suggested to bridge those gaps.

2.1 Tourism area life cycle model

The TALC model (Butler, 1980), one of the most renowned and widely used models to explain the tourism development in an area, uses a series of stages that characterize the
life cycle of a destination. Since its introduction in the 1980s, hundreds of articles have assessed and justified the TALC model from different perspectives (Lagiewski, 2006).

According to the model, the development of a tourism area typically experiences six stages, namely, the exploration, involvement, development, consolidation, stagnation, and post-stagnation stages. Host attitudes towards tourism development may vary at different stages. Many studies have applied the TALC model to evaluate the tourism development of different destinations, which range from a single tourism resource (e.g. Zhong, Deng, & Xiang, 2008) to a varied-feature destination (e.g. Gu, Ryan, & Yu, 2012), from tangible attractions (e.g. Getz, 1992; Kapeczyński & Szromek, 2008) to intangible events (e.g. Yang, Ryan, & Zhang, 2014), and from the Western perspective (e.g. Agarwal, 2002; Caldicott & Scherrer, 2013) to an Asian point of view (e.g. Bao & Zhang, 2006; Lee & Weaver, 2014). The TALC model is generally consistent with the abovementioned empirical observations.

However, the debates regarding TALC theory focus on the post-stagnation stage. Baum (1998) argued that the natural or human-induced changes in an area may result in the abandonment of the traditional tourism products and the rise of other emerging industries. By taking coastal resorts as the study context, Agarwal (1997, 2002) proposed a series of restructuring efforts implemented before the decline. One stage, “reorientation”, has been added between the stagnation and the post-stagnation stages of the TALC model to represent the dynamic process of restructuring. The restructuring can be induced by internal (e.g., uniqueness of resources and attractions, local residents and their attitudes, management, service practices, and qualities) and external factors (e.g., producers, consumers, and regulating authorities) in the tourism development process (Agarwal, 2002; Keller, 1987; Zhong et al., 2008). Meanwhile, Zimmermann (1997) suggested the simultaneous existence of multiple cycles of different forms of tourism. An aggregated pattern of development based on the European tourism products from 1860 to 2000 has also been established to illustrate patterns of activity popularity or life cycles (Butler, 2011).
One of the disadvantages of TALC is that the stages are difficult to identify and cannot be easily measured (Getz, 1992), which makes the prediction of the post-stagnation stage even more difficult. With the limited support of empirical facts concerning the post-stagnation stage, no consensus has been achieved in the academe regarding the reasons that lead to the decline, stabilization, or rejuvenation of tourism in an area. More efforts should thus be made to strengthen the theoretical framework for analyzing the post-stagnation stage in the TALC model.

2.2 The impact of tourism development on a destination

The impact of tourism development on a destination can be categorized into economic, social, and environmental dimensions (Ap & Crompton, 1998; Brougham & Butler, 1981), which correspond to the different dimensions defined in tourism sustainability. Previous studies have concluded that a perceived sustainable level significantly affects host attitudes towards tourism development (Gursoy & Rutherford, 2004; Tao & Wall, 2009) and the factors from the three dimensions can influence a destination from both positive and negative ways (Boley, McGehee, & Hammett, 2017; Choi & Sirakaya, 2005; King, Pizam, & Milman, 1993; Milman & Pizam, 1988).

Regarding the impacts of economic dimension, studies (Gursoy & Rutherford, 2004; Seetanah, 2011; Tao & Wall, 2009) have argued that the economic benefits gained from tourism, such as greater employment, investment, and business opportunities; poverty alleviation; and increased government revenues, positively influence the level of support shown by residents to ongoing tourism development efforts. Past studies also supported the empirical findings that residents have a positive perception of the economic impacts of tourism (Choi & Sirakaya, 2005; King et al., 1993; Sinclair & Stabler, 2002). However, other studies have argued that certain aspects of economic impacts, such as seasonal operations, trade-off between the tourism related and other public investments, increased prices, economic leakage, and distributional inequality, may negatively affect host attitudes towards tourism development (Altinay, 2000; Giannoni & Maupertuis, 2007; Sandbrook,
In terms of social impact, several components (e.g., individual crimes, drug addiction, alcoholism, traffic conditions, and influence on the traditional way of life and values) have been identified to negatively affect host attitudes towards tourism development (King et al., 1993; Milman & Pizam, 1988; Wearing, 2001). The social benefits of tourism include enhanced quality of life, the efficient provision of hospitality to strangers, and enhanced confidence among residents (King et al., 1993; Milman & Pizam, 1988). In the Hong Kong context, Zhang et al. (2017) explored three dimensions of social sustainability in tourism, namely, host-guest conflict, social tolerance, and social acceptance. Concerns about the environmental impacts induced by tourism have been related to air, land, and water pollution; the loss of biological diversity and natural heritage; and changes in wildlife and natural habitats (Choi & Murray, 2010; Weaver & Lawton, 2001).

2.3 Irridex model

Host attitudes towards tourism development have been extensively studied from diverse perspectives due to its essential role in tourism planning and development. The hosts hold varying attitudes towards tourism in different development stages of a destination. As the most frequently applied model in studies of host attitudes (Kwon & Vogt, 2010; Nunkoo, Smith, & Ramkissoon, 2013; Şanlıöz-Özgen & Günülü, 2016), Doxey (1975) proposed the Irridex model, which assumes that the responses of a community to tourism development depend on the social interrelations within the host community. Host attitudes can be classified into four stages, namely, euphoria, apathy, annoyance, and antagonism. At the beginning stage, the hosts are euphoric at the potential economic and social benefits brought by tourism development. However, as the development of the destination occurs and the number of tourists increases, host attitudes gradually become apathetic, annoyed and, eventually, antagonistic. Ap and Crompton (1993) argued that the reaction of the hosts towards tourism development can be classified into four stages, namely, embracement, tolerance, adjustment, and withdrawal, which correspond to their attitude changes during
2.4 Social exchange theory

Rooted in economic theory, social exchange theory was first modified and applied by Thibaut and Kelley (1959), who studied the social psychology of groups. Social exchange theory emphasizes the perceived costs and benefits of a certain relationship as well as the implication on the relationship satisfaction of the parties involved. Sutton (1967) claimed that exchange is a social characteristic that identifies the traveling encounter between the hosts and the tourists. Long, Perdue, and Allen (1990) described social exchange theory as an appropriate framework that can explain the hosts’ perceptions of the impact of tourism.

The desire of the hosts in stimulating the economic and social development of the destination is the initiation of the exchange, which is the first stage of social exchange theory, followed by the formation stage where the exchange actually takes place. At the third stage, the evaluation stage, the hosts evaluate the benefits and costs brought about by tourism development. At the fourth and the last stage, two possible consequences of the exchange may emerge. If the benefits overwhelm the costs, the hosts are likely to support the tourism development; otherwise, they may show a negative attitude towards tourism development (Ap, 1992).

Three essential elements are considered in social exchange theory: comparison, power, and trust. Ward and Berno (2011) highlighted the importance of comparison, which provides the standard for all relationship judgments. Moreover, in their host-tourist perception study, Andereck, Valentine, Knopf, and Vogt (2005) found that host attitudes towards tourism and their support level for tourism development are heavily influenced by their subjective evaluation of the tourism industry’s impacts upon themselves and their communities. Similarly, another study on the community support level for tourism (Nunkoo & Ramkissoon, 2012) reported that the hosts’ trust in the government actors
and their power in influencing tourism are important determinants of the hosts’ support for the tourism industry.

2.5 Research gap

Several gaps in the literature have been identified based on existing studies on theories and practices related to host attitudes. First, numerous studies have investigated the factors influencing host attitudes towards tourism development; however, the theoretical support identifying the importance and effects of those factors remains lacking. Second, although social exchange theory illustrates that host attitudes towards tourism development are determined by their evaluation of the benefits and costs brought about by tourism (Ap, 1992), existing research has failed to use this theory to explain how and why host attitudes change throughout the development process in Doxey (1975)’s Irridex model. Therefore, an integrated model is necessary. Third, in contrast to numerous empirical studies, the reasons that cause the changes of the benefits and costs throughout the process of tourism development remain a mystery.

To fill the aforementioned research gap, a utility maximization model is employed in the current study to demonstrate the change of host attitudes throughout the evolution of tourism development. The contributions are four-fold. First, the model provides theoretical support for host attitude changes from the economic perspective. Second, it uses social exchange theory to explain the Irridex model of Doxey (1975). Third, the generated model can demonstrate the change of host attitudes towards tourism development using economic theories. Finally, the model can provide some insights into the conditions that facilitate decline or rejuvenation during the post-stagnation stage of tourism development in TALC model. Such findings are not only valuable for the academe but also helpful for governments in planning the sustainable development of tourist destinations.
3 Utility Maximization Model

Host attitudes towards tourism are highly related to their own welfare (Andereck et al., 2005; Pérez & Nadal, 2005). Therefore, in the current study, the changes in welfare, which can be measured by changes in utility in economic models, are used to approximate changes in host attitudes towards tourism development. Welfare improvement can lead to more supportive host attitudes towards tourism development, whereas welfare decline can lead to less support from the hosts.

The microeconomic models have been proven to be effective in investigating host attitudes towards tourism development. Hazari (1993) and Hazari and Kaur (1995) used conceptual models to illustrate the notion that the consumption of non-tradable goods by tourists may lead to a decline in the welfare of the hosts, which can then lead to a change in host attitudes towards tourists and tourism development. Bimonte and Punzo (2007), applying game theory, revealed different objectives between the hosts and the tourists; they found that the tourist carrying capacity can, therefore, be defined from the perspectives of the hosts, tourists, and the environment. As an extension of their previous work, Bimonte and Punzo (2016) employed the Edgeworth box to illustrate the exchange between the hosts and the tourists in terms of tourism resource and money. The different endowments stimulate the “exchange” between the two parties. Considering that the resources provided by the hosts are non-reproducible, the overdevelopment of tourism may decrease the welfare of hosts. Thus, tourism development must reach an equilibrium.

Bimonte and Punzo (2007, 2016) demonstrated the first step of social exchange theory and explained why “exchange” happens in tourism. Meanwhile, Hazari (1993) and Hazari and Kaur (1995) focused on the third step of social exchange theory, which evaluates the outcome of the “exchange”. However, as argued by Sharpley (2014) and Bimonte and Punzo (2016), a more comprehensive model is still needed to illustrate the entire process explaining the attitude change of the hosts following tourism development.

The model starts with an economy with one private good produced according to the
technology described as

$$Y = AL,$$

(1)

where $Y$, $L$, and $A$ represent output level, labor supply, and productivity of each unit of labor, respectively. The output, $Y$, is shared between the hosts and the tourists. The ratio of the tourists’ consumption level, $Q_T$, to the hosts’ consumption level, $Q_R$, is denoted as $\rho$. Hence, the consumption level of the tourists can be represented as $\rho Q_R$. The total output is expressed as

$$Y = Q_R + Q_T = (1 + \rho)Q_R.$$  

(2)

In tourism demand analysis, the consumer price index is usually employed as a proxy for tourism price due to the high correlation of the two indices (Lin, Liu, & Song, 2015; Song & Lin, 2009). This means the consumption structure of residents and tourists should also be highly correlated (Martin & Witt, 1987). Thus, without loss of generality, normalizing the hosts’ population to 1, $\rho$ becomes the number of tourists in the economy.

As the development of inbound tourism, the increased tourism receipts could be utilized in purchasing foreign capital and foreign technology, which can then stimulate the economic growth of a tourist destination (Song, Dwyer, Li, & Cao, 2012). The tourism receipts, $Q_T$, along with a transit ratio, $\phi$, can be channeled into productivity following

$$A = \phi Q_T.$$  

(3)

The technology transit ratio, $\phi$, describes how efficient the local community can transform the foreign capital into productivity advancement. The advancement may come from the technological improvement brought about by new machinery or human capital enhancement achieved by education or training.
Substituting (2) and (3) into (1), Equation (1) can then be rearranged into

\[ L = \frac{(1 + \rho)}{\phi \rho}. \]  

Therefore, the labor supply of hosts is determined by the ratio of tourists’ consumption to the hosts’ own consumption, \( \rho \), and the transit ratio of local community in channeling tourism income into productivity improvement, \( \phi \).

As reviewed in Section 2.2, the impact of tourism development on a destination can be categorized into economic, social, and environmental aspects. The economic factors, such as income level, living standard, and unemployment rate (Deery, Jago, & Fredline, 2012; Lindberg, Andersson, & Dellaert, 2001; Milman & Pizam, 1988) can be generalized into the private goods consumption level and labor supply, whereas social and environmental factors (although they cover traffic congestion, environmental pollution, the occupation of public resources, and many other aspects) (Deery et al., 2012; King et al., 1993; Zhang et al., 2017) can be generalized into the enjoyment of tangible and intangible public resources. Here, the factor “public resources” should be distinguished from the term “public goods” in economics. The public resources in the current model refer to all the goods and services that have the feature of “sharing”. That is, any newly introduced consumer would decrease the quantity or quality of the goods or services enjoyed by the existing consumers. Public resources can include the public goods (e.g. the traffic congestion problem can be considered as the share of the goods “room” and the noise problem can be considered as the share of the goods “peace”), as well as the rival goods (e.g. the shortage of particular consumption goods can be considered as the share of “supply”). The social and environmental factors, rival or non-rival, are categorized into public resources. Households are assumed to decide how to allocate resources among the three factors in each period in order to maximize the utility of the current period. As a result, a constant elasticity of substitution (CES) type of utility for hosts can be specified.
as

\[ U_R = \left[ Q_R^\gamma + k \left( \bar{L} - L \right) \gamma + m \left( \frac{Q_P}{1 + \rho} \right)^\gamma \right]^{\frac{1}{\gamma}}, \tag{5} \]

where \( U_R \) is the overall utility of the hosts, and \( \bar{L} \) represents the maximum level of time endowed by the hosts; thus \( \bar{L} - L \) represents the leisure enjoyed by the hosts. \( Q_P \) denotes the total amount of public resource provision. As the tourist arrival increases, the public resources shared by the hosts decrease, which then leads to a decline in welfare. The model setting is consistent with the argument in literature. As reviewed by Uysal, Woo, and Singal (2012), residents perceived positive economic impacts and negative or neutral social and environmental impacts as the development of tourism. The private goods consumption, \( Q_R \), and leisure, \( \bar{L} - L \), measure the economic factors that influence the welfare of the hosts, whereas the public resource provision, \( Q_P \), measures the social and environmental factors. Here, \( \gamma \) is a parameter that defines the elasticity of substitution among private goods consumption, leisure, and public resource consumption by \( \frac{1}{1 - \gamma} \) and \( k \) and \( m \) are the share parameters of leisure and public resource provision, respectively. The CES utility function is widely used in economic studies to aggregate various types of goods in utility functions (Dixit & Stiglitz, 2012). Some other forms of functions are the special cases for CES functions. When \( \gamma \) approaches zero, the goods are perfect substitutes and the function reduces to Cobb-Douglas form; when \( \gamma \) approaches negative infinite, the goods are perfect complement and the function transforms to Leontief form which is used frequently in the computable general equilibrium studies.

Given that public resource consumption has no influence on labor market equilibrium, the hosts’ labor supply decision remains the same as described in Equation (4). By substituting Equation (4) into Equation (5) and by rearranging terms, we obtain

\[ U_R = \left[ Q_R^\gamma + k \left( \frac{1 + \rho}{\phi \rho} \right) \gamma + m \left( \frac{Q_P}{1 + \rho} \right)^\gamma \right]^{\frac{1}{\gamma}}. \tag{6} \]
With Equation (6), the partial derivative of $U_R$ with respect to $\rho$ can be derived as

$$
\frac{\partial U_R}{\partial \rho} = X \cdot [\bar{L} - \frac{1}{\phi} \left( \frac{1}{\rho} + 1 \right)]^{\gamma-1} \frac{1}{\phi \rho^2} - X \cdot m Q_R^\gamma \left( \frac{1}{1 + \rho} \right)^{\gamma+1},
$$

(7)

where

$$
X (\rho; Q_R, Q_P, \bar{L}, m, k, \phi, \gamma) = \left\{ Q_R^\gamma + k \left[ \bar{L} - \frac{1}{\phi} \left( \frac{1}{\rho} + 1 \right) \right]^\gamma + m \left( \frac{Q_P}{1 + \rho} \right)^\gamma \right\}^{\frac{1}{\gamma+1}}.
$$

The partial derivative of $U_R$ with respect to $\rho$ describes the changes of the welfare of the hosts with the changes in the number of tourists. Therefore, a positive partial derivative can be described as an increase in the welfare of the hosts, which then leads to positive host attitudes towards tourism development. In comparison, a negative partial derivative represents negative host attitudes towards tourism development.

Appendices A.1 and A.2 prove that $\frac{\partial U_R}{\partial \rho}$ starts with a positive value and gradually decreases to zero as $\rho$ increases. $\frac{\partial U_R}{\partial \rho}$ reaches zero when $\rho$ increases to a critical level and becomes negative afterwards, it converges back to zero as $\rho$ approaches infinite.

4 Findings and Discussions

4.1 Host attitudes towards tourism development without structural change

According to the proof shown in Appendix A.2, the partial derivative of the utility of a host, $U_R$, with respect to the number of tourist arrivals, $\rho$, $\frac{\partial U_R}{\partial \rho}$, is positive when $\rho$ is relatively small. The welfare of the hosts, which is approximated by $U_R$, increases from a low initial level to its peak value. The welfare of the hosts increases until $\rho$ increases to the critical level, $\rho^*$. In this “increasing phase”, host attitudes towards tourism development are positive as the hosts recognize an improvement in their welfare. After reaching $\rho^*$, $\frac{\partial U_R}{\partial \rho}$ becomes negative with further increase in $\rho$, which can be interpreted as a decrease in the welfare of the hosts. Such a decrease gradually slows
down, after which the level of hosts’ welfare stabilizes at a relatively low level. In this “decreasing phase”, host attitudes towards tourism development become negative as the hosts experience losses in their welfare. Appendix A.3 proves the existence and uniqueness of $\rho^*$, thereby indicating that without any structural changes such as the post-stagnation in TALC model, “increasing phase” and “decreasing phase” occur only once in the entire process of tourism development. The relationship between the welfare of the hosts, $U_R$, and the number of tourist arrivals, $\rho$, is demonstrated in Figure 1.

Figure 1 is about here.

Without any structural change in the economy, the welfare of the hosts will change according to the curve in Figure 1. The “increasing phase” from $\rho$ to $\rho^*$ replicates the “euphoric” stage in the Doxey (1975)’s Irridex model, wherein the hosts recognize the benefits brought about by tourism and positively perceive tourism development. Since the hosts make their decisions only to maximize the utility of the current period, they never know the optimal volume of tourist arrivals, which brings the maximal level of the welfare. After the volume of tourist arrivals surpasses $\rho^*$, the hosts will continue to develop the tourism industry and mistakenly think that their welfare would increase. This “overshooting phase” explains the “apathic” and “annoyed” stages in the Doxey (1975)’s Irridex model, where the hosts start to have a negative attitude towards tourism development, but do not withdraw from the market altogether. When $\rho$ exceeds a maximum volume, $\tilde{\rho}$, further development of tourism industry brings only losses in the welfare of the hosts. Some hosts may thus prefer to supply less labor and enjoy more leisure. The less labor input results in the less number of tourists the economy can host, and thus the volume of tourist arrivals decreases towards the optimal level, $\rho^*$. This “decreasing phase” describes the “antagonistic” stage in the Doxey (1975)’s Irridex model. However, given that there would always be hosts who will withdraw from the tourism market, the economy will not stay in this phase in the long run. Theoretically, if $\rho$ approaches an extremely large number (positive infinity in the current model), the harm
of overdeveloped tourism industry will stabilize which reflects the right tale of the curve in Figure 1.

In any given tourism development stage, the hosts can always produce private goods for tourists. Meanwhile, the hosts can also enjoy more private goods consumption and leisure due to the productivity improvement brought about by the money injection from the tourists. According to Bimonte and Punzo (2016), the different endowments of the hosts and the tourists are the initial conditions of the exchange. The existence of the difference is also the first step in social exchange theory (Ap, 1990). For the hosts to enjoy the benefits brought about by tourism development, the exchange must happen between them and the tourists. In evaluating the exchange, the hosts may discover that, despite the gains of private goods consumption and leisure, they also have to share public resources with the tourists. They will then realize that the more tourists they host, the less public resources they could enjoy. Based on their own judgment and past experiences, if the hosts evaluate the benefit in extra private goods consumption and leisure to be greater than the reduced public resource consumption, then the volume of tourist arrivals is between $\rho$ to $\rho^*$ in Figure 1; in this case, host attitudes towards tourism development become positive. Given that the gains and losses in private goods consumption, leisure, and public resource consumption are hardly quantifiable, when the benefits and costs are relatively close, the hosts may have an unclear evaluation of tourism development. Therefore, the hosts may hesitate supporting tourism development when the volume of tourist arrivals is between $\rho^*$ and $\bar{\rho}$ in Figure 1. This overshoot phase will end when the hosts clearly see that the losses in public resource consumption outweigh the gains in private goods consumption and leisure. Thus, after the overshoot, the volume of tourist arrivals must converge back towards $\rho^*$, after which the hosts can obtain maximal benefits. These findings explain why and how the benefits and costs change as the tourism industry develops, which also mimic the third and fourth steps in social exchange theory.
4.2 Influencing factors of the optimal tourism development level

As shown in Figure 1, the optimal volume of tourist arrivals, $\rho^*$, is crucial to host attitudes towards tourism development. Ideally, all the hosts would like to maintain the volume of tourist arrivals at $\rho^*$ to maximize their welfare. According to Equation (9) in Appendix A.3, the solution of $\rho^*$ is a function of the public resource provision, $Q_P$, and the transit ratio, $\phi$. Appendix B.1 proves that $\rho^* (Q_P, \phi)$ is a monotonically increasing function of $Q_P$ when the elasticity of substitution, $1/(1 - \gamma)$, is positive, but less than unity ($\gamma < 0$); further, they reported that it is a decreasing function when the elasticity of substitution is greater than unit ($0 < \gamma < 1$). Here, $Q_P$ has no impact on $\rho^*$ when the elasticity of substitution equals to zero or approaches to negative infinity.

In the case of $\gamma < 0$, the private goods consumption, leisure, and public resource consumption are perceived as complements instead of substitutes. In adopting an increase in public resource provision, the marginal utility from consuming public resource diminishes and falls below the marginal utility from consuming private goods and leisure. Rational hosts, in order to achieve the maximum level of utility, would sacrifice some consumption in public resource and increase the consumption in private goods and leisure. In the modeled economy, the only way to achieve such an increase would be to develop the tourism industry. The increased productivity brought about by the injection of foreign funds will not only increase the private goods production, but also decrease the labor required in the production process. In this scenario, the increment of public resources may be shared with an increasing number of tourists, but the benefit in private goods consumption and leisure can very well compensate the loss. However, when $0 < \gamma < 1$, the private goods consumption, leisure, and public resource consumption are perceived more as substitutes instead of complements. An increase in public resource provision increases the public resource enjoyment of each share obtained by the hosts. Therefore, the hosts are likely to withdraw from the tourism industry and decrease the volume of visitor arrivals to obtain more shares of public resources. Considering the substitute effect, the benefit in public resource consumption can very well compensate for the potential
loss in private goods consumption and leisure brought by the shrinkage of the tourism industry. With an increase in public resource provision, the welfare of the hosts shall expand consistently. Hence, the change of the optimal level of tourism development is determined by the elasticity of substitution among private goods consumption, leisure, and public resource consumption. If there are increased amount of public resources in the destination, the welfare of the hosts is expected to improve. However, whether the increase of public resource provision could stimulate the development of tourism industry depends on the characteristics of the hosts (γ).

Appendix B.2 proves that $\rho^* (Q_P, \phi)$ is a monotonically decreasing function of the technology transit ratio, $\phi$. A higher level of technology transit ratio ensures the hosts to more effectively channel foreign funds into productivity. With the same volume of tourist arrivals, the hosts would enjoy more private goods consumption and leisure without reducing their public resource consumption. Hence, it would be more beneficial for the hosts to sacrifice a fraction of the increment of private goods consumption and leisure to gain more public resource consumption. Therefore, monitoring the development of tourism industry is essential for the destination government in order to enjoy the sustainable development without the loss of hosts’ welfare. In contrast, destinations with lower level of technology transit ratio would embrace more tourists. According to Equation (2), inbound tourism could improve the productivity of the destination. Thus, tourism receipts generated by additional tourists need to achieve the appropriate productivity level, so that the consumption of private and public resources are balanced and the welfare of the hosts are maximized.

4.3 Host attitudes towards tourism development with structural change

After the volume of tourist arrivals, $\rho$, reaches the optimal level, $\rho_t^*$, in a destination, the structural changes that increase the technology transit ratio, $\phi$, and public resource provision, $Q_P$, such as the improvement of human capital and the construction of infrastructures, can change the optimal volume of tourist arrivals in the next period, $\rho_{t+1}^*$, as
displayed in the upper quadrant of Figure 2. As discussed in Section 4.2, the impact of technology transit ratio on the optimal volume of tourist arrivals is negative, whereas the influence of public resource provision depends on the hosts’ rate of substitution among private goods consumption, leisure, and public resource consumption. In the cases wherein hosts’ perception of the three elements (i.e. private goods consumption, leisure, and public resource) inclines toward substitutes ($\gamma < 0$), a higher level of public resource provision would encourage tourism development. In comparison, in the case wherein the hosts perceive the three elements as complements ($0 < \gamma < 1$), a higher level of public resource provision would result in a lower optimal volume of tourist arrivals.

Consequently, in the current model, three possible scenarios emerge under the combined force of the increases in $\phi_t$ and $Q_{P,t}$. According to Equation (7), $U_{R,t}$ is a monotonic increasing function in $\phi_t$ and $Q_{P,t}$. An increase in technology transit ratio and public resource provision could lead to the improvement of the hosts’ welfare and, consequently, host attitudes towards tourism development. As shown in the first quadrant of Figure 2, with the same volume of tourist arrivals, an increase in transit ratio and public resource provision may result in the utility levels at $E'$, $E''$ or $E'''$, which are all higher than the original utility level at $E$. Two cases among the three, namely, Cases $A$ and $C$, however, are suboptimal.

When private goods consumption, leisure, and public resource consumption are considered substitutes ($\gamma < 0$), a higher level of technology transit ratio results in lower optimal volume of tourist arrivals, whereas a higher level of public goods provision leads to higher optimal volume of tourist arrivals. Therefore, the overall impact of an increase in technology transit ratio and public goods provision on the optimal volume of tourist arrivals depends on the respective magnitudes of individual impacts brought about by the improved technology transit ratio and increased public goods provision. If the impact of improved technology transit ratio is larger in magnitude, a drop in optimal volume of tourist arrivals can be observed, which is described as Case $A$ in Figure 2. If the impact of improved technology transit ratio is smaller in magnitude, an increase in optimal volume
of tourist arrivals emerges, which is represented by Case C in Figure 2. In the case wherein two offsetting impacts have the same magnitudes, the scenario is represented by Case B.

The situation is much simpler when private goods consumption, leisure, and public resource consumption are considered complements \((0 < \gamma < 1)\). Given that the individual impacts of improved technology transit ratio and increased public resource provision have the same direction, the overall impact of an increase in technology transit ratio and public resource provision on the optimal volume of tourist arrivals is always negative. In some special cases, wherein the utility of the hosts exhibits a Cobb-Douglas or Leontief form \((\gamma = 0\) and \(\gamma \to -\infty\), respectively), the changes in public goods provision have no impact on the optimal volume of tourist arrivals. In addition, the overall impact of an increase in technology transit ratio and public resource provision on the optimal volume of tourist arrivals is determined by the individual impact of the improved technology transit ratio, which is negative. These cases, which cover \(0 \leq \gamma < 1\) and \(\gamma \to -\infty\), are delineated as Case A in Figure 2.

In general, a decline in host attitudes towards tourism development and a contraction of the tourism industry are expected in scenarios described in Case A, whereas improvements in host attitudes towards tourism development and an expansion of tourism industry are expected in scenarios of Case C. Case B represents the individual impacts brought about by the improved technology transit ratio and increased public resource provision perfectly offset each other.

Two points can be made regarding the curves shown in Figure 2. First, if the initial structure change is negative, which leads to a decrease in technology transit ratio and public resource provision, all aforementioned changes in the optimal volume of tourist arrival and host attitudes towards tourism development are reversed. Second, the first quadrant of Figure 2 works only as an illustration. The positions of \(E', E'',\) and \(E'''\),
as well as the relative magnitudes of $U(\rho^*_{A,t+1})$, $U(\rho^*_{B,t+1})$, and $U(\rho^*_{C,t+1})$, are to be
determined with the specific values of the variables and parameters which may be different
in various destinations.

The evolution of host attitudes towards tourism development in accordance with a structure
change complements the explanation of the post-stagnation stage in the TALC model. As shown in the lower quadrant of Figure 2, before period $t$, the volume of tourist arrivals goes through the exploration, involvement, development, and consolidation stages. At the beginning of period $t$, the volume of tourist arrivals reaches the optimal level $\rho^*_t$, the economy then enters the stagnation stage in the TALC model. After period $t$, three possibilities emerge as the future development of the destination, namely, decline, stabilization, and rejuvenation. These three possibilities match Cases A, B, and C in the model of the current study, respectively. Butler (1980) argued that the construction of new attractions could attract more tourists, thus facilitating rejuvenation. In the current model, Butler’s conclusion only represents the hosts with insensitive (low) substitute elasticity among private goods consumption, leisure, and public resource consumption ($\gamma < 0$). As the attraction is also a type of public resources, insensitive hosts prefer to host more tourists when the new attractions are developed. The current model also proves that, if the hosts have sensitive substitute elasticity ($0 < \gamma < 1$), decline, stabilization, and rejuvenation are possible stages for the future development of the destination. The TALC model was developed based on the assumption that the hosts have insensitive substitute elasticity. The current study relaxed this assumption and proved that even if the hosts are elastic, the TALC model would be launched. This study also supports the findings of Baum (1998) and Agarwal (1997, 2002), who concluded that future development after the post-stagnation stage is determined endogenously by the characteristics of the destination.

5 Conclusions and Limitations

With the development of the tourism industry, its positive and negative impacts on the local community are observed. Host attitudes towards tourism development may fall into
the “development dilemma” (Telfer & Sharpley, 2015). In the present study, a static utility maximization model is developed to comprehensively investigate host attitudes towards tourism development. The economic impacts of tourism development are represented by changes in the private goods consumption and leisure. In general, the job opportunities and technological innovations brought about by tourism development can lead to higher wage and productivity rates. These changes in wage and productivity present the hosts with the greater possibility of enjoying more private goods and leisure as well as higher levels of welfare.

Meanwhile, the social and environmental impacts of tourism development are conceptualized to the changes in public resource consumption. The hosts must share public resources with the tourists; hence, the expansion of tourist arrivals leads to a reduction in public resource consumption by the hosts. The reduced public resource consumption leads to a loss in welfare, which is why impacts from social and environmental aspects are usually revealed as negative. The hosts maximize their welfare by trading-off among private goods consumption, leisure, and public resource consumption. The change in welfare leads to the change of host attitudes towards tourism development.

The current model shows that, from a relatively low initial status, the hosts become more supportive of tourism development due to the improvements in their personal welfare as the tourism industry develops. The gains in economic aspect outweigh the losses in social and environmental aspects in this stage. When the tourism industry develops to a critical level, the welfare of the hosts reaches the maximum level and starts to decline. By then, the losses in terms of social and environmental aspects surpass the gains in terms of economic aspects. As a result, the hosts generate negative attitudes towards tourism development and eventually become annoyed by it. Such findings can be used to explain the Doxey (1975)’s Irridex model, which describes the changes of host attitudes from euphoria to antagonism.

It is also found that the optimal level of tourism development is influenced by the public
resource provision of the destination and the technology transit ratio of the hosts. When
the hosts are less sensitive to the substitution among private goods, leisure and public
resources, additional investments in public resources by the destination government could
further stimulate the tourism sector and achieve sustainable tourism development in
the long term. This resembles the rejuvenation stage of the TALC model. When the
hosts are sensitive to the substitution among the three elements, the decision-makers
should be cautious on investment decisions. In such a case, the expansion in public
resource cannot lead to the rejuvenation of the destination. In addition, destinations
with lower technology transit ratios embrace more tourists for the needs of additional
foreign resources. On the contrary, destinations with stronger ability in channeling foreign
resources into productivity advancement need to monitor the development of tourism
industry carefully as the hosts would be more satisfied with less tourists and more public
resource consumption.

Different from most studies that empirically investigated the relationship between host
attitudes and tourism development, the current study sheds light on the theoretical
foundation of this research field. A static utility maximization model is used to explain
the Doxey (1975)’s Irridex model from the economic perspective. Furthermore, the model
shows that Butler (1980) only focuses on the residents with insensitive substitute elasticity.
This research complements the study of Butler (1980) and demonstrates that even if the
substitute elasticity is elastic, the findings of Butler (1980) could still be held. Practical
implications can also be obtained from the current study. The study reveals that, hosts’
attitude towards tourism development is determined by the technology transit ratio of
hosts and the public resources allocated in the destination. The dynamic homeostasis
of different factors determines the hosts’ attitude at a particular time. The study also
provides strategic insights which can inform the destination to format effective policies
and regulations regarding how to develop the sustainable tourism. For instance, after
the stagnation period in TALC, simply enhancing the public resource provision of the
destination may not always lead to a rejuvenation of the destination, and the ability
of local community in channeling tourism income into productivity advancement also
plays a very important role. Thus, before the destination makes important decisions regarding tourism development, market survey is suggested to recognize the feedback of the hosts and understand their preferences clearly. The study also shows that, along with the development of a destination, even though hosts’ attitude towards tourism may change from euphoria to antagonism, the final emotional status (i.e. antagonism) is not permanent. In the post-stagnation stage, destinations can carry out different fiscal policies according to the characteristics of the hosts to cultivate favorable perceptions of the hosts towards tourism and to build up a sustainable tourism in the long run.

As a theoretical model, the current study is not without any limitations. The current model is a simplified model that generalizes many empirically tested factors into three aspects. The current study aims to explain the changes in host attitudes towards tourism development from a theoretical perspective; thus, a simplified model is adopted to discuss the properties of functions. In future studies, a more comprehensive model could be developed with additional destination-specific factors. Although analytical solutions may not be obtained, simulations on host attitudes can be used for the purpose of empirical investigation. The current model is a static model with exogenous tourism demand. An inter-temporal endogenous model can be adopted to incorporate Doxey (1975)’s Irridex model, social exchange theory, and the TALC model into one dynamic framework. The current model could serve as a baseline model for further studies in this field. Finally, more theoretical research should be conducted to enhance the foundations of empirical studies in the context of the relationship between host attitudes and tourism development.
Figure 1. Host attitudes with respect to \( \rho \)
Figure 2. Host attitudes and the TALC Model
Appendix A
Proofs related to the sign of $\partial U_R/\partial \rho$

By definition, all the variables and parameters are positive, $\gamma \in (-\infty, 1)$ and the term $\left\{ \bar{L} - 1/ [\phi (1/\rho + 1)] \right\}$ is also positive.

Let

$$f (\rho; \bar{L}, k, \phi, \gamma) = k \left[ \bar{L} - \frac{1}{\phi} \left( \frac{1}{\rho} + 1 \right) \right]^{\gamma - 1} \frac{1}{\phi \rho^2}$$

and

$$g (\rho; Q_R, m, \gamma) = m Q_R^\gamma \left( \frac{1}{1 + \rho} \right)^{\gamma + 1}.$$

Equation (7) then becomes

$$\frac{\partial U_R}{\partial \rho} = X \left( \rho; Q_R, Q_R, \bar{L}, m, k, \rho, \gamma \right) \cdot \left[ f (\rho; \bar{L}, k, \phi, \gamma) - g (\rho; Q_R, m, \gamma) \right]. \quad (8)$$

A.1 Initial Value of $\partial U_R/\partial \rho$

When $\rho$ is relatively small, in which case the home country’s tourism development is in its early stage, let $\rho$ be $1/ (\phi \bar{L} - 1 - \Delta)$ where $\Delta$ is an extremely small number.

$$X \left( \rho \rightarrow \frac{1}{\phi \bar{L} - 1}; Q_R, Q_R, \bar{L}, m, k, \phi, \gamma \right) =$$

$$\left\{ Q_R^\gamma + k \left( \frac{\Delta}{\phi} \right)^\gamma + m \left[ \frac{Q_R \left( \phi \bar{L} - 1 - \Delta \right)}{\phi \bar{L} - \Delta} \right]^\gamma \right\}^{\frac{1 - \gamma}{\gamma}},$$

$$f \left( \rho \rightarrow \frac{1}{\phi \bar{L} - 1}; \bar{L}, k, \phi, \gamma \right) = k \left( \frac{\Delta}{\phi} \right)^{\gamma - 1} \left( \frac{\phi \bar{L} - 1 - \Delta}{\phi} \right)^2,$$
and
\[
g \left( \rho \to \frac{1}{\phi L - 1}; Q_P, m, \gamma \right) = mQ_P^\gamma \left( \frac{\phi L - 1 - \triangle}{\phi L - \triangle} \right)^{\gamma + 1}.
\]

When \( \gamma \in (-\infty, 0] \), as \( \triangle \to 0 \), it can be show that
\[
\lim_{\rho \to \phi L \leftarrow 1} X \left( \rho; Q_R, Q_P, \bar{L}, m, k, \phi, \gamma \right) = 0^+,
\]
\[
\lim_{\rho \to \phi L \leftarrow 1} f \left( \rho; \bar{L}, k, \phi, \gamma \right) = +\infty,
\]
and
\[
\lim_{\rho \to \phi L \leftarrow 1} g \left( \rho; Q_P, m, \gamma \right) = mQ_P^\gamma \left( \frac{\phi L - 1}{\phi L} \right)^{\gamma + 1} > 0.
\]

When \( \gamma \in (0, 1) \), as \( \triangle \to 0 \), it can be show that
\[
\lim_{\rho \to \phi L \leftarrow 1} X \left( \rho; Q_R, Q_P, \bar{L}, m, k, \phi, \gamma \right) = \left\{ Q_R^\gamma + m \left[ \frac{Q_P \left( \phi L - 1 \right)}{\phi L} \right]^\gamma \right\}^{\frac{1}{1-\gamma}} > 0,
\]
\[
\lim_{\rho \to \phi L \leftarrow 1} f \left( \rho; \bar{L}, k, \phi, \gamma \right) = +\infty,
\]
and
\[
\lim_{\rho \to \phi L \leftarrow 1} g \left( \rho; Q_P, m, \gamma \right) = mQ_P^\gamma \left( \frac{\phi L - 1}{\phi L} \right)^{\gamma + 1} > 0.
\]

Therefore, across the domain of \( \gamma \), we have
\[
\lim_{\rho \to \phi L \leftarrow 1} \frac{\partial U_R}{\partial \rho} > 0.
\]
A.2 Infinity approximation of $\partial U_R/\partial \rho$

With the development of home country’s tourism industry, the number of tourists, $\rho$, increases. As $\rho$ grow large and approaches infinity, when $\gamma \in (-\infty, -1]$,

\[
\lim_{\rho \to +\infty} X \left( \rho; Q_R, Q_P, \bar{L}, m, k, \phi, \gamma \right) = 0^+, \\
\lim_{\rho \to +\infty} f \left( \rho; \bar{L}, k, \phi, \gamma \right) = 0^+, \\
\]

and

\[
\lim_{\rho \to +\infty} g \left( \rho; Q_P, m, \gamma \right) = +\infty.
\]

In terms of convergence speed, we have

\[
X \left( \rho; Q_R, Q_P, \bar{L}, m, k, \phi, \gamma \right) = \mathcal{O} \left( \frac{1}{\rho^{1-\gamma}} \right), \\
f \left( \rho; \bar{L}, k, \phi, \gamma \right) = \mathcal{O} \left( \frac{1}{\rho^2} \right),
\]

and

\[
\frac{1}{g \left( \rho; Q_P, m, \gamma \right)} = \mathcal{O} \left( \frac{1}{\rho^{1+\gamma}} \right).
\]

Therefore, we have

\[
\lim_{\rho \to +\infty} X \cdot f = 0^+, \\
\]

and

\[
\lim_{\rho \to +\infty} (-X \cdot g) = 0^-, \\
\]
with

\[ X \cdot f = O \left( \frac{1}{\rho^{2-2\gamma}} \right), \]

and

\[ (-X \cdot g) = O \left( \frac{1}{\rho^{-2\gamma}} \right). \]

Since \( 2 - 2\gamma > -2\gamma > 0 \), \( X \cdot f \) converges to zero faster than \( (-X \cdot g) \). That is, \((X \cdot f - X \cdot g)\) becomes negative before it converges to zero.

When \( \gamma \in (-1, 0] \), we have

\[
\lim_{\rho \to +\infty} X \left( \rho; Q_R, Q_P, \bar{L}, m, k, \phi, \gamma \right) = 0^+,
\]
\[
\lim_{\rho \to +\infty} f \left( \rho; \bar{L}, k, \phi, \gamma \right) = 0^+,
\]

and

\[
\lim_{\rho \to +\infty} g \left( \rho; Q_P, m, \gamma \right) = 0^+,
\]

with

\[ X \left( \rho; Q_R, Q_P, \bar{L}, m, k, \phi, \gamma \right) = O \left( \frac{1}{\rho^{1-\gamma}} \right), \]
\[ f \left( \rho; \bar{L}, k, \phi, \gamma \right) = O \left( \frac{1}{\rho^2} \right), \]

and

\[ g \left( \rho; Q_P, m, \gamma \right) = O \left( \frac{1}{\rho^{1+\gamma}} \right). \]
We therefore have

$$\lim_{\rho \to +\infty} X \cdot f = 0^+, \quad \lim_{\rho \to +\infty} (-X \cdot g) = 0^-,$$

with

$$X \cdot f = \mathcal{O} \left( \frac{1}{\rho^{2(1-2\gamma)}} \right),$$

and

$$(-X \cdot g) = \mathcal{O} \left( \frac{1}{\rho^{(1-\gamma)(1+\gamma)}} \right).$$

Since $2(1-\gamma) > (1-\gamma)(1+\gamma) > 0$, $X \cdot f$ converges to zero faster than $(-X \cdot g)$. That is, once again, $(X \cdot f - X \cdot g)$ becomes negative before it converges to zero.

When $\gamma \in (0, 1)$,

$$\lim_{\rho \to +\infty} X \left( \rho; Q_R, Q_P, \bar{L}, m, k, \phi, \gamma \right) = \left\{ Q_R^2 + k \left( \bar{L} - \frac{1}{\phi} \right)^\gamma \right\}^{\frac{1-\gamma}{\gamma}} > 0,$$

$$\lim_{\rho \to +\infty} f \left( \rho; \bar{L}, k, \phi, \gamma \right) = 0^+,$$

and

$$\lim_{\rho \to +\infty} g \left( \rho; Q_P, m, \gamma \right) = 0^+,$$
with

\[ f(\rho; \bar{L}, k, \phi, \gamma) = \mathcal{O}\left(\frac{1}{\rho^{2}}\right), \]

and

\[ g(\rho; Q_{P}, m, \gamma) = \mathcal{O}\left(\frac{1}{\rho^{3+\gamma}}\right). \]

We therefore have

\[ \lim_{\rho \to +\infty} X \cdot f = 0^{+}, \]

and

\[ \lim_{\rho \to +\infty} (-X \cdot g) = 0^{-}, \]

with

\[ X \cdot f = \mathcal{O}\left(\frac{1}{\rho^{2}}\right), \]

and

\[ (-X \cdot g) = \mathcal{O}\left(\frac{1}{\rho^{(1+\gamma)}}\right). \]

Since \(2 > (1 + \gamma) > 0\), \(X \cdot f\) converges to zero faster than \((-X \cdot g)\). That is, similar to previous results, \((X \cdot f - X \cdot g)\) becomes negative before it converges to zero.

In general, across the domain of \(\gamma\),

\[ \lim_{\rho \to +\infty} \frac{\partial U_{R}}{\partial \rho} = 0^{-}. \]
A.3 Proof of the existence and uniqueness of $\rho^*$

The solution of setting Equation (8) to zero determines the value of $\rho^*$. Since $X(\rho; Q_R, Q_P, \bar{L}, m, k, \phi, \gamma)$ is strictly positive, the equation simplifies into

$$f(\rho^*; \bar{L}, k, \phi, \gamma) = g(\rho^*; Q_P, m, \gamma),$$

or

$$k \left[ \bar{L} - \frac{1}{\phi} \left( \frac{1}{\rho^*} + 1 \right) \right]^{\gamma-1} \frac{1}{\phi \rho^*} = m Q_P^\gamma \left( \frac{1}{1 + \rho^*} \right)^{\gamma+1}. \tag{9}$$

Since both sides of Equation (9) are positive, take natural logarithm on both sides, we have

$$\ln \left( \frac{k}{\phi} \right) + (\gamma - 1) \ln \left[ \bar{L} - \frac{1}{\phi} \left( \frac{1}{\rho^*} + 1 \right) \right] - 2 \ln \rho^* = \ln (m Q_P^\gamma) - (\gamma + 1) \ln (1 + \rho^*).$$

Rearranging terms, we have

$$(\gamma - 1) \ln \left[ \bar{L} - \frac{1}{\phi} \left( \frac{1}{\rho^*} + 1 \right) \right] - 2 \ln \rho^* + (\gamma + 1) \ln (1 + \rho^*) + \ln \left( \frac{k}{m \phi Q_P^\gamma} \right) = 0.$$

Define

$$F(\rho) = (\gamma - 1) \ln \left[ \bar{L} - \frac{1}{\phi} \left( \frac{1}{\rho} + 1 \right) \right] - 2 \ln \rho + (\gamma + 1) \ln (1 + \rho) + \ln \left( \frac{k}{m \phi Q_P^\gamma} \right).$$

We can derive

$$\frac{\partial F(\rho)}{\partial \rho} = \frac{(\gamma - 1)(1 + \rho)}{\phi \rho} + [(\gamma - 1) \rho - 2] \left[ \bar{L} - \frac{1}{\phi} \left( \frac{1}{\rho} + 1 \right) \right] \rho (1 + \rho).$$
Since
\[
\left[ \bar{L} - \frac{1}{\phi} \left( \frac{1}{\rho} + 1 \right) \right] > 0,
\]
\[(\gamma - 1) < 0,
\]
and
\[
\rho (1 + \rho) > 0.
\]

We can conclude that
\[
\frac{\partial F(\rho)}{\partial \rho} < 0.
\]

When \( \rho \) is relatively small,
\[
\lim_{\rho \to \frac{1}{\phi L - 1}} F(\rho) = (\gamma - 1) \ln (0) - 2 \ln \left( \frac{1}{\phi L - 1} \right) + (\gamma + 1) \ln \left( \frac{\phi \bar{L}}{\phi L - 1} \right) + \ln \left( \frac{k}{m \phi Q P} \right).
\]

Since \( \lim_{\Delta \to 0} \ln (\Delta) = -\infty \) and \( (\gamma - 1) < 0 \), we have
\[
\lim_{\rho \to \frac{1}{\phi L - 1}} F(\rho) = +\infty.
\]

Rearranging \( F(\rho) \), we can have
\[
F(\rho) = (\gamma - 1) \left\{ \ln \left[ \bar{L} - \frac{1}{\phi} \left( \frac{1}{\rho} + 1 \right) \right] + \ln (1 + \rho) \right\} + 2 \ln \left( \frac{1}{\rho} + 1 \right) + \ln \left( \frac{k}{m \phi Q P} \right).
\]

When \( \rho \) approaches positive infinity,
\[
\lim_{\rho \to +\infty} F(\rho) = (\gamma - 1) \left\{ \ln \left( \bar{L} - \frac{1}{\phi} \right) + \ln (\infty) \right\} + 2 \ln (1) + \ln \left( \frac{k}{m \phi Q P} \right).
\]
Since $\ln(\infty) \rightarrow +\infty$ and $(\gamma - 1) < 0$, we have

$$\lim_{\rho \rightarrow +\infty} F(\rho) = -\infty.$$ 

According to intermediate value theorem, since $F(\rho)$ is continuous and monotonic decreasing, $\lim_{\rho \rightarrow \frac{1}{\delta t - 1}} F(\rho) = +\infty$, and $\lim_{\rho \rightarrow +\infty} F(\rho) = -\infty$, there must exist one unique solution to the equation $F(\rho) = 0$. That is, there exists one unique critical value of $\rho$, $\rho^*$.

With the derivative first being positive, equaling to zero at $\rho^*$, and approaching zero from negative side when $\rho$ is large, we can derive a utility representation as illustrated in Figure 1.
Appendix B
Proofs of the Influencing Factors of $\rho^*$

B.1 Change of $\rho^*$ with respect to $Q_P$

Assume Equation (8) equals to zero and take natural logarithm on both sides, rearranging terms, we can have

$$\ln Q_P = \frac{1}{\gamma} + \frac{\gamma - 1}{\gamma} \ln \left[ \bar{L} - \frac{1}{\phi} \left( \frac{1}{\rho^*} + 1 \right) \right] - \frac{2}{\gamma} \ln \rho^* + \frac{\gamma + 1}{\gamma} \ln (1 + \rho^*).$$

It can be shown that,

$$\frac{\partial Q_P}{\partial \rho^*} = e^{\ln Q_P} \cdot \frac{\partial \ln Q_P}{\partial \rho^*} = e^{\ln Q_P} \frac{1}{\gamma} \frac{\partial F(\rho^*)}{\partial \rho^*}.$$

Appendix A.3 shows that $(\partial F(\rho^*)/\partial \rho^*) < 0$. That is, when $\gamma < 0$, we have $(\partial Q_P/\partial \rho^*) > 0$, and when $\gamma \in (0, 1)$, we have $(\partial Q_P/\partial \rho^*) < 0$. Since $Q_P$ is a monotonic function of $\rho^*$, by chain rule, we can conclude that $(\partial \rho^*/\partial Q_P) > 0$ when $\gamma < 0$, and $(\partial \rho^*/\partial Q_P) < 0$ when $\gamma \in (0, 1)$.

In the special case of $\gamma = 0$, the CES utility function will become Cobb-Douglas type,

$$U_R = Q_R \left( \frac{L}{L} \right)^k \left( \frac{Q_P}{q + \rho} \right)^m.$$ 

The optimal level of $\rho$, $\rho^*$, is therefore,

$$\rho^* = \frac{1 + \sqrt{1 + 4\phi}}{2\phi},$$

which is not a function of $Q_P$. We therefore have $(\partial \rho^* / \partial Q_P) = 0$ when $\gamma = 0$.

In general, $(\partial \rho^* / \partial Q_P) > 0$ when $\gamma < 0$, $(\partial \rho^* / \partial Q_P) = 0$ when $\gamma = 0$, and
\( \partial \rho^*/\partial Q_P < 0 \) when \( \gamma \in (0, 1) \).

### B.2 Change of \( \rho^* \) with respect to \( \phi \)

Rearranging terms of \( F(\rho) \) from Appendix A.3, we can have

\[
F(\rho) = (\gamma - 1) \ln \left[ \frac{\bar{L}}{\phi} - \frac{1}{\phi}\left(\frac{1}{\rho} + 1\right) \right] - 2 \ln \rho + (\gamma + 1) \ln (1 + \rho) + \ln \left(\frac{k}{m}\right) - \ln \phi - \gamma \ln Q_P.
\]

As proved in Appendix A.3,

\[
\frac{\partial F(\rho^*, \phi, Q_P, \gamma)}{\partial \rho^*} < 0.
\]

It can also be shown that

\[
\frac{\partial F(\rho^*, \phi, Q_P, \gamma)}{\partial \phi} = -\frac{1}{\phi} \left[ \bar{L} - \gamma \frac{1}{\phi} \left(\frac{1}{\rho^*} + 1\right) \right] < 0.
\]

Therefore, by chain rule,

\[
\frac{\partial \rho^*}{\partial \phi} = - \frac{\partial F(\rho^*, \phi, Q_P, \gamma)}{\partial \phi} \frac{\partial \rho^*}{\partial \rho^*} < 0.
\]

In the special case of \( \gamma = 0 \),

\[
\frac{\partial \rho^*}{\partial \phi} = - \left( \frac{1 + 2\phi}{\sqrt{1 + 4\phi}} + 1 \right) \frac{1}{2\phi^2} < 0.
\]

We therefore have \( \partial \rho^*/\partial \phi < 0 \) for \( \gamma \in (-\infty, 1) \).
References


