THE DESIGN OF
COMPOSITE WORK ROLLS

by

D.J. WESTGARTH

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University of Surrey
Guidford, Surrey
Great Britain
SUMMARY

The service life of work rolls can be substantially increased by substituting special wear-resistant sleeves for the conventional roll surface and a tough steel arbor for the roll core. In practice, the feasibility of such a structure is dependent on the method of securing the sleeves to the arbor.

This thesis describes the design of a novel composite roll system which utilises the elastic properties of a pre-tensioned arbor to provide controlled axial and radial sleeve clamping. The elastic stresses and overall deformation behaviour of a typical design have been determined using \( \frac{1}{3} \) scale three-dimensional photoelastic models.

This work was undertaken in the light of a preliminary study which revealed:

(a) The possibility of component overstressing under static and dynamic loading.

(b) A degree of uncertainty associated with clamping load distribution and interface contact conditions in the assembled roll.

The photoelastic model tests identified arbor pre-tensioning, for sleeve assembly, as the most critical static loading condition. By introducing an undercut fillet radius at the arbor bore termination, the
associated SCF was reduced from 14 to about 4.5. The critical stresses in the modified design are below the levels which would cause low cycle fatigue failure.

The effect of superimposing rolling load is not critical when compared with the mean stresses developed in the assembled roll. The roll assembly is shown to behave as a monolithic structure under transverse bending. This feature, which has considerable significance when related to mill stiffness and product tolerance control, is not available in alternative composite roll systems. An original method for analysing SCF's in a composite assembly is described.

Sleeve geometry is found to have a considerable effect on the flexural rigidity of the roll assembly. Modifications to sleeve and arbor design are recommended, and have been included in a proposed composite roll prototype.
ACKNOWLEDGEMENTS

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The author would like to acknowledge the expertise of Mr. K. Bedford, Senior Technician at the University of Surrey, in the manufacture of the photoelastic models described in this thesis.

Finally, the author wishes to thank his fiancée, Philippa, for her unfailing encouragement and good humour.
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**NOMENCLATURE**

**Composite Rolls:**

- \( a \) = Arbor bore diameter
  = Thrust Rod diameter

- \( b \) = Nominal arbor shank diameter
  = Nominal sleeve bore diameter

- \( c \) = Nominal sleeve external diameter

- \( DN \) = Sleeve new diameter

- \( DD \) = Sleeve discard diameter

- \( r_1 \) = Internal radius of Collet-Shell

- \( r_2 \) = External radius of Collet-Shell

- \( r_3 \) = External radius of Retaining sleeve

- \( L \) = Simply-supported roll length

- \( LB \) = Roll barrel length

- \( LL \) = Distance from L.H. support to position of rolling load application

- \( L2 \) = Distance from R.H. support to position of rolling load application

- \( IN \) = Retaining nut length

- \( LS \) = Arbor shoulder length

- \( W \) = Sleeve width

- \( WB \) = Arbor pre-tension

- \( LA(WB) \) = Effective arbor length under pre-tension

- \( WR \) = Residual load (Axial clamping force)

- \( LA(WR) \) = Effective arbor length under residual loading

- \( LA(WR) \) = Effective sleeve length under residual loading

- \( \Delta \) = Total diametral shrink-fit allowance between sleeves and arbor

- \( WR/WB \) = Axial clamping efficiency

- \( P \) = Transverse bending component of rolling load
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<td>Torsional component of rolling load</td>
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<td>( SF )</td>
<td>Rolling load shock factor at bar entry</td>
</tr>
<tr>
<td>( M )</td>
<td>Bending moment</td>
</tr>
<tr>
<td>( R )</td>
<td>Radius of curvature</td>
</tr>
<tr>
<td>( I )</td>
<td>Moment of inertia</td>
</tr>
<tr>
<td>( y )</td>
<td>Radial distance from neutral axis</td>
</tr>
<tr>
<td>( \sigma )</td>
<td>Stress</td>
</tr>
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<td>Poisson's ratio</td>
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**Subscripts:**

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<tr>
<td>( RS )</td>
<td>Retaining sleeves</td>
</tr>
<tr>
<td>( zz )</td>
<td>Axial co-ordinate</td>
</tr>
<tr>
<td>( \phi \phi )</td>
<td>Hoop co-ordinate</td>
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<tr>
<td>( rr )</td>
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Photoelastic Models:

\[ N = \text{Isochromatic fringe number} \]
\[ q_p = \text{Maximum principal stress} \]
\[ q_q = \text{Minimum principal stress} \]
\[ f = \text{Model material fringe value} \]
\[ t = \text{Model slice thickness} \]
\[ \theta = \text{Isoclinic parameter} \]
\[ \alpha = \text{Isoclinic angle} \]
INTRODUCTION

The rolling process is widely used in the steel industry where high yield and low operating costs on bulk tonnages are essential\(^{(1)}\). The desired shape of the workpiece is obtained by plastic deformation between two work rolls, having parallel axes and revolving in opposite directions. The metal is drawn into the rolls by friction. Rolling equipment consists of one or several stands of rolls, the whole being termed a rolling mill. The mill is normally sub-divided into roughing, intermediate and finishing stands.

During rolling the work roll surface suffers a gradual deterioration, which can lead to loss of control over product shape and quality. Consequently the rolls require periodic removal from the mill for redressing. This procedure is repeated until the roll diameter is reduced to some pre-determined minimum, based on its peripheral speed, when the roll is scrapped.

It follows that, the basic criterion for all types of work roll is that they should possess a tough core to withstand rolling loads and a wear resistant surface, to ensure a long life in terms of tonnage rolled. For conventional rolls these conflicting metallurgical requirements invariably result in a compromise due to the limitations of available heat treatment and casting techniques.
The mechanism of roll wear in hot rolling is complex, but can generally be attributed to either abrasion, thermal fatigue or a combination of both. Thermal fatigue of the roll surface is due to the temperature cycles undergone by the outer layers as they are alternately heated and cooled by the hot metal and water coolant. This is most prevalent in slow speed rolling applications such as the roughing stands in slab mills. Abrasion of the roll surface is caused by contact with the hot metal. It is, therefore, most prevalent in high speed rolling applications such as the finishing stands in rod, bar and light section mills.

Roll changing, due to wear, often constitutes an undesirable interruption in an otherwise continuous production process. The economic aspects are such that, in modern high production mills, there is a growing demand for substantial improvements in roll life. This demand is reflected in the considerable R and D effort which, both in this country and abroad, has been directed towards this end over the last 15 years.

Recent, or on-going, research work in the British Steel Corporation is fairly representative of the international effort, and can be sub-divided into:

(i) Improvements in roll cooling.
(ii) Introduction of lubricants at the roll surface.
(iii) Development of new types of work rolls and work roll surfaces.

Various methods of roll reclamation with welded or spray coated deposits and different types of sleeved, or composite, rolls can be placed in the latter category.
Since work is continuing on all of these developments it would be
difficult, and possibly misleading, to attempt an assessment of their
relative merits at this time. However, both methods of roll reclamation
with welded and coated deposits have suffered from problems of poor
surface finish, due to porosity, and relatively small improvements in wear
resistance. For composite rolls the main objection in the past has been
one of size limitation. This is mainly due to the method of sleeve
clamping which, for the majority of existing systems, has relied on a
radial interference (shrink fit) technique. The sleeves are, therefore,
required to be fairly thick in order to sustain the large hoop stresses.
This is especially critical if expensive materials such as tungsten carbide
are used since high wear resistance (up to an order of magnitude greater
than that associated with conventional work roll materials) is invariably
accompanied with tensile weakness.

An equally important, although less publicised, objection to
composite rolls is related to the penalties associated with possible sleeve
separation during rolling.

Under the action of rolling load (transverse bending and torsion)
there is a tendency for the sleeves to move, either relative to each other,
or with respect to the arbor shank\(^{(18)}\). If this were to occur, it would
almost certainly cause fretting between the component interfaces, possibly
leading to high cycle fatigue failure\(^{(19)}\). Moreover, the action of trans-
verse bending is to induce axial separation of the sleeves, which effectively
reduces the roll's external diameter. In extreme cases this could result
in a 500\% reduction in flexural rigidity, coupled with a proportionate loss
in mill stand stiffness. Product tolerance control would obviously suffer
as a consequence.
For the composite roll system described in this thesis, the sleeves are held in situ by a predominantly axial compressive force. This has three advantages:—

(i) Very thin sleeves can be economically exploited without the risk of developing critical hoop stresses.

(ii) Axial clamping can ensure monolithic behaviour of the composite assembly when subjected to transverse bending. Consequently, there is no risk of a reduction in roll stiffness due to axial sleeve separation. Moreover, if sleeves with a high elastic modulus are used (E tungsten carbide = $2\frac{1}{2}$ × E steel) a considerable increase in roll stiffness is possible.

(iii) Since there is no inherent restriction on sleeve size, the composite roll is no longer limited to low-torque applications.

The sleeve clamping method is based on the elastically deformed arbor principle illustrated in Figure 1, which has been applied to a typical prototype in the manner shown in Figure 2.

Referring to Figure 1(a), it can be seen that the application of an axial tensile load (WB) to a cylindrical arbor results in an elastic extension on length which is accompanied by a proportional elastic contraction on diameter. On releasing the load, the deformed arbor will revert to its original dimensions. If this recovery is prevented by the introduction of a number of concentric sleeves, then these will be subjected to a reactive loading (WR) as shown in Figure 1(b).
Fig. 1  Schematic representation of Elastically deformed arbor principle

(a) Pre-Tensioned condition

(b) Residual clamping condition

Clamped sleeves under retaining nut
residual compression = WR = residual tension in arbor
This reactive, or residual, loading will be opposite in sign, and proportional to, the final strain in the arbor just prior to release of external loading, the relative dimensions of the arbor and sleeves, and their respective elastic modulii.

Thus, for a simple idealisation, a relationship between arbor pre-tension and residual sleeve clamping can be derived by consideration of deformation compatibility. In this respect the sleeves are treated as plain cylinders, whilst the roll arbor is considered as a simple axisymmetric body.

Figure 2(a) illustrates the method of providing pre-tension in the prototype by means of a hydraulically loaded thrust rod positioned inside the 'blind-bore' of the arbor. The reactive forces are absorbed by the split collet-shell assembly which contains the pressure cap and piston. Equilibrium of the collet assembly requires retaining sleeves to be located at each end.

Figure 2(b) shows how the under-sized sleeves can be assembled and then radially clamped by partially relaxing the initial arbor pre-tension. This procedure, which can be accurately controlled to give any desired radial clamping, ensures the concentricity of the assembly and the absence of any undesirable clearance between the arbor shank and sleeve bores.

The retaining nut is then hand tightened and the oil pressure released to atmosphere, Figure 2(c). The arbor's axial contraction is arrested only when the compressive load in the sleeves is equal to the residual tension in the arbor.
Fig. 2. Composite work rolls, loading and assembly sequence
Figure 2(d) shows the forces to which a pair of simply-supported composite work rolls are subjected during the rolling process.

To ensure monolithic behaviour under the action of transverse bending, an expression of force equilibrium can be developed in terms of the maximum bending moment, or separating force, and the minimum required sleeve clamping force (WR).

To ensure no-slip transmission of the rolling torque a similar force equilibrium expression can be developed in terms of the applied torque and the frictional resistance between each sleeve interface.

A simple mathematical model has been developed along these lines, which relates the rolling load specification to the sleeve clamping and arbor pre-tensioning requirements. To assess the validity of this theory it was necessary to conduct an experimental feasibility study\textsuperscript{(20)}. This work was done with the aid of a simple $\frac{1}{3}$ scale model arbor and sleeve assembly. Theoretical estimations of clamping potential and roll stiffness characteristics were found to be in good agreement with experimental results. The amount of axial clamping required to prevent sleeve separation under transverse bending was found to be at least double that required to prevent rotational slippage due to applied torque.

However, the above tests also served to highlight a number of critical design features which, if related to a full sized prototype, would certainly give some cause for concern.
If a typical 305 mm diameter work roll (for rolling rod and bar up to 20 mm diameter) is used as an example, it can be shown that the minimum required arbor pre-tension is in the order of 7000 KN. This implies a nominal tensile stress in the arbor shank of some 0.23 KN/mm². The arbor's longitudinal extension and diametral contraction would be about 0.6 mm and 0.07 mm respectively. Although appreciable, a stress of this magnitude would not, in itself, be considered excessive for high strength alloy steel. Unfortunately, to accommodate pre-tensioning and sleeve support requirements, the arbor design is inherently non-uniform at its extreme ends. The geometric stress concentrations arising from such non-uniformity would undoubtedly result in the development of unacceptably high local stresses. This is especially critical in the vicinity of the arbor/thrust rod interface where contact stresses are extremely localised, and most probably, triaxial. A more detailed knowledge of these critical stress conditions was therefore considered advisable if future prototype arbors are to be designed with a high fatigue resistance.

The model arbor also displayed considerable non-uniformity of radial deformation along its length. When fully assembled with the sleeves and retaining nut, this non-uniform deformation behaviour resulted in each sleeve being subjected to different amounts of radial clamping. This is not acceptable for a full-sized composite roll.

Finally, the model tests were obviously limited in their relevance to the stress and deformation behaviour inside the assembled roll. During its service life the assembled roll is subjected to many cycles of fatigue loading, which makes it particularly sensitive to fretting damage in the vicinity of the sleeve/arbor interfaces. Consequently, detailed information
regarding the distribution of interface pressures, and the influence of sleeve and arbor design on the overall stiffness of the assembly is considered essential. The position and magnitude of localised stress concentrations are also required for a complete fatigue strength appraisal.

This present work is, then, concerned with the detailed analysis of a composite roll assembly subjected to three modes of loading, namely:

(a) Arbor pre-tensioning.
(b) Residual loading.
(c) Combined residual loading and transverse bending.

For reasons of compatibility it was considered necessary to use the same method of analysis throughout this investigation. It was therefore decided to utilise the photoelastic stress freezing technique. This method is equally suitable for analysing arbor pre-tensioning, a simple axi-symmetric loading case, as well as the assembled roll under combined loading which is a complex asymmetric loading case. The modeling technique permits direct measurement and observation of overall deformation behaviour which can be conveniently related to a full-sized prototype. In addition, complex tri-axial stress separations can be made at any desired location, together with direct measurement of local stress concentrations. The facility to exactly reproduce the way in which load is applied and distributed is unique to photoelasticity. It is also an essential facility for this particular project since, although basically simple in concept, the composite roll assembly is a complex structure to analyse.
In all, some '6' model idealisations were examined. Three models were required for the analysis of arbor pre-tensioning, whilst residual loading and combined loading were studied with one and two models respectively. The main objectives of these tests was to provide a general quantitative indication of the overall elastic deformation behaviour, to establish the way in which the applied load is shared between the separate components and to identify positions of local stress concentration which might constitute a source of weakness in the prototype assembly. If these objectives are satisfied it should be possible to establish a comprehensive mathematical model of composite roll behaviour which will serve as a design procedure for prototype equipment.

Finally, results from the photoelastic analysis of arbor pre-tensioning are also compared with those obtained from a Finite-Element idealisation of a similar arbor. The method of restraint and load input was endorsed from observations made in the photoelastic model. It is envisaged that the finite element technique could be used for the rapid assessment of various alternative arbor designs in the future.
CHAPTER 2

DESIGN ANALYSIS AND PROTOTYPE LOADING

The criteria for composite roll design, outlined in Chapter 1, required the sleeves to be held in situ with a clamping force capable of ensuring monolithic behaviour of the assembled roll during rolling.

The design analysis approach is, therefore, to initially convert the rolling load specification into sleeve clamping requirements which satisfy the equilibrium conditions necessary for monolithic behaviour. The minimum arbor pre-tension required to develop this clamping, or residual loading, is then determined. Finally, an estimate is made of the maximum arbor pre-tension necessary to facilitate sleeve assembly.

(i) Sleeve Clamping Requirements

2.1 Axial Compressive Clamping, WR

This is provided by the longitudinal contraction of the pre-tensioned arbor when all external loading is removed. The axial compressive force thus imparted to the sleeves is equal and opposite to the residual tension in the arbor. Axial compression is the primary mode of clamping, having two essential functions:

(a) To prevent axial separation of the sleeves due to transverse bending.

(b) To ensure no-slip transmission of rolling torque.
2.1.1 Roll Under Bending

For the purposes of evaluating nominal stresses and roll deflection it is convenient to treat the composite roll as an equivalent homogeneous beam, so that, for the idealised configurations shown in Figure 3(a).

The moment at any cross section is given by:

\[ M = \frac{EA.I_a}{R} + \frac{Es.I_s}{R} \]

or \[ M = \frac{EI}{R} \] ........................ (1)

where \( R \) = Radius of curvature of deformed roll
and \( EI = \) Combined flexural rigidity of the arbor and sleeves.

The above relationship assumes that no sliding occurs between the sleeves and arbor.

Substituting for \( R = \frac{E.y}{\delta} \) in equation (1) gives:

\[ M = \frac{EI.\delta}{E.y} \]

where \( \delta = \) Bending stress
\( y = \) Distance from neutral axis.

In the sleeved assembly, it is the tensile component of bending stress which tend to axially separate the sleeves.
Fig. 3 Comparisons between the idealised and actual composite roll assemblies

(a) Idealised case

(b) Actual case
Thus, the maximum separating stress in the sleeves $\delta_s$ is given by:

$$\delta_s = \frac{M_{\text{max}} \cdot E_s \cdot Y_s}{E I} \quad \text{......... (2)}$$

where $M_{\text{max}}$ = Maximum bending moment
and $Y_s = C/2$

It follows that, the axial compressive clamping force $W_{R_B}$ required to prevent sleeve separation due to transverse bending is given by:

$$W_{R_B} = \frac{M_{\text{max}} \cdot E_s \cdot C \cdot A_s \cdot S_F}{2. E I} \quad \text{......... (3)}$$

where $A_s$ = Nominal cross sectional area of sleeves
and $S_F$ = Bending moment magnification, or shock factor, at bar entry.
(Usually in the order of 2 ~ 3 times the nominal rolling load).

The above relationship between sleeve clamping and roll bending is based on the idealised structure shown in Figure 3(a). A number of important assumptions are therefore implied:

1. The sleeves are treated as plain cylinders.
2. The sleeves occupy the entire length of the arbor between the roll supports.
3. All sleeves have identical geometry.
4. All sleeves have identical material properties.

Figure 3(b) serves to illustrate the actual structure's deviation from this idealised representation.
An effective sleeve modulus $E_{se}$, based on the aggregate effect of individual sleeves and sleeve supports, is derived in Appendix 2, equation (a.5).

2.1.2 Roll Under Torsion

The torque transmitting capability of the composite roll assembly is analogous to that associated with a conventional friction plate clutch having two active friction faces compressed together under an axial load, $WR_T$.

Thus, assuming the normal laws of friction and a uniform pressure distribution between each sleeve interface, it can be shown that,

$$T = 2\pi \mu p \int_{r_i}^{r_o} r^2 dr$$

where $T =$ Friction torque

$\mu =$ Coefficient of dry friction at sleeve interface

$p =$ Uniform pressure $= \frac{WR_T}{\pi (r_o^2 - r_i^2)}$

and $r_i$ & $r_o =$ The internal and external radii of sleeve respectively.

It follows that, the sleeve clamping force $WR_T$ necessary for no-slip torque transmission in a composite roll assembly is given by:

$$WR_T = \frac{3. T. SF}{2\mu \left( \frac{c^3 - b^3}{c^2 - b^2} \right)}$$

\[ \text{....... (4)} \]
where \( SF \times T = \text{Rolling torque at bar entry} \)
\[c = \text{Sleeve external diameter}\]
\[b = \text{Sleeve bore}\]

2.1.3 Axial Clamping and Roll Temperature

The distribution of temperature in work rolls is a subject which has to date received relatively little attention, and for which, very little experimental data is available. The problem of differential thermal expansion between the roll surface and core is known to be a complex transient thermal condition, which is probably more complicated in the context of composite rolls.

During the initial stages of rolling the sleeve in contact with the hot metal will expand more than the arbor or surrounding sleeves. This will cause a temporary increase in total axial compressive clamping, irrespective of the sleeve material. It is not clear how this simple interpretation is modified by the effect of water coolant.

However, for the purposes of this present analysis, it is assumed that the thermal conditions potentially most detrimental to sleeve clamping exist when the entire roll assembly has experienced an overall uniform increase in temperature \((AT)\). This is a steady-state thermal condition which is attained after prolonged and continuous rolling.

It follows that, for sleeve materials having similar thermal properties to the arbor, no significant change in axial clamping will be apparent at elevated temperatures. This will not be the case for the majority of composite roll applications since materials having a high wear resistance
usually possess a low coefficient of thermal expansion. Consequently, at elevated temperatures the arbor will experience a longitudinal expansion greater than the cumulative sleeve expansion. This will cause a loss in axial clamping $\delta \text{WR}$ which, if the differential thermal expansion is of sufficient magnitude, could invalidate the force balance implied in equations (3) and (4).

An expression for the maximum possible, temperature related, loss in axial clamping is derived in Appendix 2, equation (a.6).

2.1.4 Minimum Required Axial Clamping

A preliminary study of potential composite roll applications, based on equations (3) and (4), showed that for simply supported rolls with a slenderness ratio ($L/c$) of 2 or more it is necessary to consider the effects of both transverse bending and torque. For rolls having a slenderness ratio of less than 2, the possibility of sleeve separation due to bending can usually be ignored.

Nevertheless, as a general rule, the minimum required axial clamping $\text{WR}_{\text{min}}$ is taken to be the greater of $(\text{WR}_B + \delta \text{WR})$ or $(\text{WR}_T + \delta \text{WR})$.

2.2 Radial Interference Clamping

The main reason for radial interference clamping is to provide an accurate and concentric location for the sleeves when assembled on the arbor. This precludes the possibility of a clearance existing between the sleeve bores and arbor shank.
In this context, radial clamping is not expected to act as a means of transmitting rolling torque. It is usually sufficient, therefore, to assign an arbitrary value for initially developed interference $\Delta'_{i\text{min}}$; where $\Delta'_{i\text{min}}$ is, for convenience, defined in terms of the total diametral shrink-fit allowance, based on minimum material conditions for the sleeve bores and arbor shank (Figure 4). The significance of machining tolerances ($S_T$ and $A_T$) at the sleeve/arbor common diameter ($b$) is more fully discussed with respect to sleeve assembly later in this Chapter (Section 2.5, p.23).

2.2.1 Thermal Considerations

As with axial clamping, for sleeves having a coefficient of thermal expansion equal to that of the steel arbor, no change in radial clamping will be apparent after prolonged and continuous rolling.

However, if wear-resistant sleeve materials are used, radial clamping at elevated temperatures ($= \Delta f_{\text{min}}$) will be greater than that developed during assembly (equation (a.7), Appendix 2).

(ii) Arbor Pre-Tensioning Requirements

To facilitate sleeve assembly and develop the previously discussed axial and radial clamping, the arbor must first be axially pre-tensioned.

It is convenient to express arbor pre-tensioning requirements in terms of the implied equilibrium conditions for axial and radial deformation compatibility.
2.3 For Axial Clamping

Referring to Figure 1, axial deformation compatibility implies the following equilibrium condition:

\[
\begin{bmatrix}
\text{Axial extension of arbors due to pre-tension} \\
\text{WB}
\end{bmatrix} - \begin{bmatrix}
\text{Axial contraction of sleeves due to residual loading} \\
\text{WR}
\end{bmatrix} + \begin{bmatrix}
\text{Axial extension of arbors due to residual loading} \\
\text{WR}
\end{bmatrix} = 0
\]

or

\[
\frac{W_B}{A_A \cdot E_A} = \frac{W_R \cdot L_s}{A_s \cdot E_s} + \frac{W_R \cdot L_A}{A_A \cdot E_A}
\]

From which

\[
W_B = W_R \left( \frac{A_A \cdot E_A}{A_s \cdot E_s} \cdot \frac{L_s}{L_A} + 1 \right)
\]

The above relationship between arbor pre-tension and axial clamping assumes:

1. The sleeves are treated as plain cylinders.
2. All sleeves have identical geometry.
3. All sleeves have identical material properties.
4. Effective arbor length under pre-tension is equal to effective arbor length under residual clamping.
5. Perfectly rigid sleeve supports at each end of the arbor.

Assumptions 1-3 are the same as those referred to in equation (3). Therefore, a similar expression for an equivalent sleeve modulus \(E_{s_e}^{'}) can be derived (equation (a.5), Appendix 2).
Figure 3(b) shows that the effective arbor length under pre-tension, \( LA(WB) \), is less than the effective arbor length (= effective sleeve length) under residual loading, \( LA(WR) \). The choice of data for these effective lengths is somewhat arbitrary, but previous experimental work has shown them to be sufficiently accurate for the purposes of analysis.

For assumption 5, it is known that a certain amount of flexing at the sleeve supports and thread bending at the connection between the arbor and retaining nut is inevitable in practice. Sleeve flatness errors and surface asperities are also expected to contribute to the overall stiffness loss in a full size roll assembly\(^{(21)}\).

In composite roll terminology, stiffness losses are equivalent to redundant arbor contraction (axial) and thus represent a loss in clamping efficiency \((WR/WB)\). An expression for the potential reduction in clamping efficiency is outlined in Appendix 2 (equations a.8 and a.9) and is given the factor designation \( F3 \); where \( F3 \) is the amount by which the original axial clamping requirements have to be increased to compensate for estimated stiffness losses.

It follows that, substituting for \( E_{s_e'} \), \( LA(WB) \), \( LA(WR) \) and \( F3 \) in equation (5) gives:

\[
WB = \frac{AA \cdot EA \cdot LA(WR)}{As \cdot E_{s_e'} \cdot LA(WB)} + \frac{LA(WR)}{LA(WB)} \times F3
\]

or

\[
\frac{WB_{\text{min}}}{\text{min}} = \frac{WR_{\text{min}} \cdot K \cdot F3}{\text{min}} \quad \text{........... (6)}
\]
where \( W_{R\min} = \) Minimum required axial compressive clamping

\( W_{Bf\min} = \) Final, minimum required, pre-tension in arbor just prior to release of external loading

and \( K = \frac{A.A. E.A. L.A(WR)}{A.s. E.s'. L.A(WB)} + \frac{L.A(WR)}{L.A(WB)} \)

2.4 For Radial Clamping

Consideration of deformation compatibility between the sleeves and arbor also indicates the necessity for developing radial clamping prior to axial clamping. It is argued that, if the final pre-load condition \( (W_{Bf\min}) \) is considered a strain datum then, on removal of external loading, both sleeves and arbor contract an equal amount on length and, therefore, expand an equal amount at their common diameter \( (b) \).

Thus, the initially developed radial clamping will remain virtually unchanged during the development of axial clamping.

It follows that, the initial arbor pre-tension \( W_{Bi\min} \) prior to partial relaxation to develop radial clamping and, eventually, axial clamping is given by:

\[
W_{Bi\min} = W_{Bf\min} + Z \cdot \Delta't_{i\min} \quad \text{(7)}
\]

where \( Z = \frac{A.A. E.A}{b \cdot \nu A} \)

The stiffness losses referred to in the previous section have the effect of slightly modifying the assumed deformation compatibility implied in equation (7). This is because any redundant arbor axial contraction will
also be accompanied by an excess arbor radial expansion, which is in addition to the initially developed radial interference.

The additional radial clamping so accrued is derived in Appendix 2, (equations a.10 and a.11).

2.5 For Sleeve Assembly

Referring to Figure 4, it is apparent that, in addition to satisfying sleeve clamping requirements, arbor pre-tension must also be capable of:

1. Providing sufficient diametral clearance $C_{L_{\text{min}}}$ to facilitate sleeve assembly, where $C_{L_{\text{min}}}$ is estimated to be $1.10^{-4}$ mm/mm of diameter.

2. Accommodating a maximum material condition (largest arbor diameter with smallest sleeve bore). The minimum required arbor pre-tension ($W_{B_{\text{min}}}^i$ or $W_{B_{\text{min}}}^f$) is, by definition, based on an assumed minimum material condition ($S_T = A_T = 0$).

It follows that, arbor pre-tension required to facilitate assembly of the smallest sleeves on to the largest arbor is given by:

$$W_{B_{\text{Cmin}}} = W_{B_{\text{min}}}^i + z (S_T + A_T + C_{L_{\text{min}}}) \quad \text{........ (8)}$$

Finally, since $W_{R_{\text{min}}}$ is a function of $W_{B_{\text{min}}}^f$, any additional axial clamping associated with a maximum material condition represents an excessively high mean stress in the arbor. This situation, which can be alleviated by increasing the initially developed radial clamping, is discussed in Appendix 2.
Fig. 4 Composite roll loading sequence, with maximum and minimum sleeve/arbor tolerance conditions

WB_{Cl,\min} = \dagger \text{Max. pre-tension in arbor to a sleeve assembly}

WB_{i,\max} = \dagger \text{Pre-tension in arbor, prior to relaxation to achieve radial interference sleeve clamping,}

WB_{f,\max} = \dagger \text{Pre-tension in arbor, prior to relaxation to achieve axial compressive sleeve clamping}

WB_{i,\min} = \ddagger \text{As for } WB_{i,\max}, \text{ for } \Delta_{\min}

WB_{f,\min} = \ddagger \text{As for } WB_{f,\max}, \text{ for } WR_{\min}

\text{where, } \dagger = \text{Max. material conditions}
\ddagger = \text{Min. " " " "}
(iii) Case Study for a Typical Prototype

The composite roll prototype described in the following case study is a 305 mm diameter, 10 sleeved assembly, for rolling rod and bar up to 20 mm diameter. All sleeves are manufactured in Tungsten carbide.

The rolling load and dimensional data (data sheet 1) is based on a survey of many existing conventional rolls and is, therefore, a fairly typical example.

The composite roll loading specification, together with nominal stresses and deformation, is shown in Table 1. A closer study of these results reveals the following points of interest:

1. Arbor pre-tensioning required to facilitate sleeve assembly and accommodate machining tolerances is about 3 times that required to develop sleeve clamping.

2. Sleeve clamping required to prevent axial separation of the sleeves (due to transverse bending) is almost double that required for no-slip transmission of rolling torque.

3. Maximum axial clamping is about 1.5 times the minimum required clamping.

The latter observation is an indication of the margin of error associated with the design analysis. This has to be tolerated due to the uncertainty associated with the actual size of arbor and sleeves after machining.
DATA SHEET 1

Typical prototype, all carbide sleeve system

\[ P = 100 \text{ KN} \]
\[ T = 6000 \text{ KN mm} \]
\[ SF = 2.0 \]
\[ a = 76 \text{ mm} \]
\[ b = 210 \text{ mm} \]
\[ c = 305 \text{ mm} \]
\[ W = 50 \text{ mm} \]
\[ L = 1268 \text{ mm} \]
\[ L_1 = 634 \text{ mm} \]
\[ L_2 = 634 \text{ mm} \]
\[ L_B = 760 \text{ mm} \]
\[ L_4 = 254 \text{ mm} \]
\[ L_N = 130 \text{ mm} \]
\[ L_S = 130 \text{ mm} \]
\[ L_{A(WB)} = 565 \text{ mm} \]
\[ L_{A(WR)} = 519.5 \text{ mm} \]
\[ r_1 = 146.2 \text{ mm} \]
\[ r_2 = 186 \text{ mm} \]
\[ r_3 = 221 \text{ mm} \]

\[ v_A = v_s = 0.3 \]
\[ \alpha_A = 11 \times 10^{-6} \text{ mm/mm/}^\circ\text{C} \]
\[ EA = 207 \text{ KN/mm}^2 \]
\[ \alpha_S = 5.5 \times 10^{-6} \text{ mm/mm/}^\circ\text{C} \]
\[ E_S = 517.5 \text{ KN/mm}^2 \]
\[ \mu = 0.1 \]
\[ \Delta T = 20^\circ\text{C} \]
\[ \Delta T = 0.013 \text{ mm} \]
\[ ST = 0.013 \text{ mm} \]
\[ CI_{\text{min}} = 0.02 \text{ mm} \]
### Table 1: Typical prototype—all carbide sleeve system

<table>
<thead>
<tr>
<th>Loading specification</th>
<th>Nominal stresses and deformations</th>
<th>1. Sleeve assembly</th>
<th>2. Sleeve clamping</th>
<th>3. Roll bending</th>
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</thead>
<tbody>
<tr>
<td></td>
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<td>(based on (W_B))</td>
<td>(based on (W_R))</td>
<td>(At mid-span)</td>
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<tr>
<td>(E_S)</td>
<td>163.5 kN/mm²</td>
<td>Parameter</td>
<td>Magnitude</td>
<td>Parameter</td>
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<td>(W_B)</td>
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<td>(\sigma_{zz,nom})</td>
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<td>(w_{A,nom})</td>
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<td>(\Delta_i min)</td>
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</table>
CHAPTER 3

PHOTOELASTIC MODELS

The model tests were designed to examine the proposed composite roll arrangement with respect to three principal conditions of loading.

The first loading condition is arbor pre-tensioning for sleeve assembly. In all, some three arbor designs were investigated, since the original arbor was found to be severely overstressed. Also included in this series of tests is an analysis of the novel collet-shell assembly. In the future development of composite rolls it is proposed to use this assembly for the accommodation of a large range of smaller diameter arbors by introducing spacer sleeves between the collet and arbor shoulder (not illustrated).

Results from the photoelastic analysis of arbor pre-tensioning are compared with those obtained from a finite-element idealisation of a similar arbor, described in Appendix 3.

The second model test is related to the residual loading conditions which exist when the sleeves are finally clamped onto the arbor. This constitutes the mean stress condition, to which all subsequent fatigue-life predictions are related.

The third model test is concerned with the effect of superimposing rolling load on the residually clamped assembly. For test purposes, rolling load was simulated by simply-supporting the model roll and subjecting it to transverse bending at mid-span. Unlike the previous tests, transverse
bending constitutes an asymmetric loading condition, for which two identically loaded models were required. The case for torsional loading is less severe and was, therefore, not examined.

Each model is analysed with respect to three criteria:

1. **Stress Concentrations**
   The position and magnitude of local stress concentration are identified, and related to some convenient nominal stress as stress concentration factors (SCF's).

2. **Overall deformation behaviour**
   Wherever possible, deformation is measured directly from the stress frozen model. Component interface and localised contact conditions are studied, especially in the context of assumed monolithic behaviour of the assembled roll.

3. **Stress distribution**
   Tri-axial stress separations are executed in the vicinity of local stress concentration to obtain curves of stress distribution.

The procedure adopted for tri-axial stress separation in a photoelastic model is fully discussed in Appendix 1. The effort required to obtain curves of stress distribution is considerable, but essential, for a complete understanding of the proposed design. In some cases, a knowledge of stress distribution can indicate the desirability of design modifications which would not otherwise be apparent.
The photoelastic method also allows the reproduction of Isoclinic lines, from which patterns of principal stress trajectories can be derived. This information is complementary to the above stress and deformation analysis and often serves to confirm, or dispute, apparent anomalies in measurement.

It is intended to use the results from this work to establish a generalised design procedure, complementary to the analysis outlined in Chapter 2, for future composite roll arrangements.

**Model Manufacture**

All components were cast oversize in Araldite CT200, a hot setting epoxy resin, using simple metal moulds. The unpolymerised resin was melted at $110^\circ C$, and 25 parts per 100 of Phthalic Anhydride which had been melted separately at about $140^\circ C$ were added. The hot mix was stirred thoroughly, filtered and poured into the moulds at $130^\circ C$, when the mix is sufficiently mobile to allow entrained air to escape. The moulds were retained outside the oven until the temperature fell to $115-120^\circ C$. The resin was then allowed to set and part cure by returning the moulds to an oven set at $110^\circ C$ for 16 hours.

After this time the temperature was reduced to ambient at a rate of $2\frac{1}{2}^\circ C$ per hour and the moulds were dismantled. The resin curing cycle was completed by subjecting the castings to an annealing cycle in which the oven temperature was raised slowly to $140^\circ C$ and then reduced to ambient at $2\frac{1}{2}^\circ C$ per hour. The castings were rough machined and annealed for a
second time. All the castings required for this investigation were manufactured from the same batch of resin and subjected to the same curing process. This procedure ensured that similar mechanical and optical properties were obtained throughout the various mating components.

The parts for each model were finish machined using tipped tools. Special form tools were used to obtain the fillet radii, which are estimated to be accurate to 0.05 mm. The cylindrical outer surface and shoulder of the arbor shank, together with the bores and end faces of the sleeves were ground using a copious supply of coolant. Subsequent examination showed that the flatness and concentricity of these components was sufficient to reproduce the ideal elastic deformation behaviour of the mating surfaces.

Figure 5 shows the overall model dimensions. The scale is approximately ¼ of the full size prototype described in Chapter 2, but is only a 3-sleeved assembly. This was considered sufficient to allow both the overall behaviour and critical features of the composite roll design to be fully assessed.

Model calibration

In carrying out a photoelastic analysis it is necessary to establish a relationship between the Isochromatic fringe number (N) and the principal stress difference (σp−σq) in appropriate units.

Initially the mean axial stress in the pre-tensioned arbor was determined in terms of the optical sensitivity of the photoelastic material.
A longitudinal slice was cut from the stress frozen model and the Isochromatic fringe number measured at a series of stations in a region of uniform stress, coinciding with a $0^\circ$ Isoclinic mid-way along the arbor shank.

Thus, the axial stress at any radius is given by:

$$\sigma_{zz} = \frac{Nr \cdot f}{t}$$

where $Nr =$ Isochromatic fringe number at radius $r$

$f =$ Material fringe value

$t =$ Thickness of longitudinal slice.

It follows that, the total axial load on the section is given by:

$$WB = 2\pi \int_{r_i}^{r_o} \sigma_{zz} \cdot r \cdot dr = \frac{2\pi f}{t} \int_{r_i}^{r_o} Nr \cdot r \cdot dr \quad ............ (9)$$

where $r_i$ and $r_o$ are the inner and outer radii of the arbor shank respectively.

Defining the nominal axial stress as:

$$\sigma_{zz, nom} = \frac{WB}{\pi (r_o^2 - r_i^2)} = \frac{N_{nom} \cdot f}{t}$$

which on substitution of equation (9) gives the nominal fringe order as:

$$\frac{N_{nom}}{t} = \frac{2}{\pi (r_o^2 - r_i^2)} \int_{r_i}^{r_o} Nr \cdot r \cdot dr \quad ............ (10)$$
This method of calibration has the advantage of being independent of the actual value of the applied load.

Expressing the value of the tangential stress $\sigma_{zz}$, measured at any chosen location on the surface of a slice of thickness $t_b$, in terms of the nominal stress $\sigma_{zz}^{\text{nom}}$ it follows that:

$$\frac{\sigma_{zz}}{\sigma_{zz}^{\text{nom}}} = \frac{N_b}{t_b} \times \frac{t}{N_{\text{nom}}}$$  \hspace{1cm} (11)

where $N_b$ is the isochromatic fringe number observed at the chosen location.

The maintenance of strict geometric similarity between the prototype and model leads to the conclusion that the corresponding stress in the prototype arbor will be given by:

$$\sigma_{zz}^p = \frac{N_b}{t_b} \times \frac{t}{N_{\text{nom}}} \times \sigma_{zz}^{\text{nom}p}$$

where $\sigma_{zz}^{\text{nom}p}$ is the nominal axial stress in the prototype arbor shank.

**Material fringe value, $f$:**

From equation (9)

$$f = \frac{WB}{2\pi} \int_{r_i}^{r_o} \frac{N_r t}{r} \, dr$$

for the model arbor, \hspace{1cm} $WB = 0.4 \text{ KN}$ \hspace{1cm} $r_i = 9.4 \text{ mm}$

$t = 1.27 \text{ mm} \hspace{1cm} r_o = 26.2 \text{ mm}$
and \( \int_{r_i}^{r_o} \frac{N_r}{r} dr \) is found to be 276.8 mm from graphical integration.

\[ f = \frac{0.4}{2\pi} \times 276.8 = 2.3 \times 10^{-4} \text{ KN mm}^{-1} \text{ f}^{-1} \]

where \( f \) is a constant for all models, which were manufactured from the same batch of araldite.

### 3.1 Model Subjected to Pre-Tension

#### 3.1.1 Testing of original model

For test purposes the axial pre-tension (WB) was provided by dead weight loading applied through a fine stranded steel wire and an araldite CT200 distribution pad machined to represent the termination of the prototype thrust rod. This arrangement, which is illustrated in Figure (6), avoided any difficulties associated with the stability of the thrust rod or friction in the bore of the arbor.

The symmetry of both the geometry and applied loading made it possible to determine the location of suitable slices by inspection, as shown in Figure (7). Slices were cut using a bandsaw and milling machine, with the surfaces being finished by means of a diamond tipped fly-cutter.

For accuracy of fringe measurement, it is necessary to ensure that the applied loading is sufficient to produce an adequate number of fringes per mm in the stressed model. However, the applied loading must not cause the model to be over strained since this would result in deformations which could not be related to those experienced by the prototype.
FIG. 6 MODEL UNDER PRE-TENSION
FIG. 7  ISOCHROMATIC FRINGE PATTERN OF ORIGINAL MODEL UNDER PRE-TENSION
Therefore, assuming the optical properties of the model material to be such that,

\[
1 \text{ fringe/mm} = 266.10^{-6} \text{ KN/mm}^2
\]

and that an adequate number of fringes per mm in the arbor shank is 0.8, the desired value of model pre-tension can be determined in the following manner:

The nominal axial tensile stress in the arbor is given by,

\[
\sigma_{zz}^{\text{nom}} = \frac{WB \cdot 4}{\pi (b^2 - a^2)}
\]

where \( WB = \text{Arbor pre-tension} \)
and \( b \& a = \text{Arbor shank external and internal diameters respectively.} \)

But \( \sigma_{zz}^{\text{nom}} \) is also equivalent to 0.8 fringes/mm.

From which, \( WB = 0.4 \text{ KN} \)

This value of pre-tension is equivalent to an acceptable 1% axial strain in the model arbor. The prototype strain would be about \( \frac{1}{5} \) of this value.

As a check on the total applied load, the fringe data is analysed in accordance with the procedure implied in equation (9).
Thus, for the pre-tensioned model arbor, the axial load mid-way along the arbor shank is given by:

\[
WB = \frac{2\pi f}{t} \int_{r_i}^{r_o} Nr. r. dr
\]

By graphically integrating the measured fringe values (Nr) the total applied load is calculated to be 0.38 KN. This result is within 5% of the specified dead weight loading.

3.1.2 Model displacements

Following stress freezing the radial deformation of the arbor shank was measured at a series of transverse sections.

These results have been expressed in terms of the prototype deformation using the approximate relationship

\[
U_p = \frac{WB_p}{WB_m} \times \frac{Em}{Ep} \times \frac{Em}{Ep} \times \frac{vm}{vp} \times Um
\]

\[
\text{where } \quad \begin{align*}
WB_p &= \text{Prototype pre-tension} = 7000 \text{ KN} \\
WB_m &= \text{Model pre-tension} = 0.4 \text{ KN} \\
lm/lp &= \text{Geometric scale factor} = \frac{1}{4} \\
Em &= \text{Elastic modulus of model arbor} = 0.0207 \text{ KN/mm}^2 \\
Ep &= \text{Elastic modulus of prototype arbor} = 207 \text{ KN/mm}^2 \\
vm &= \text{Poisson's ratio of model arbor} = 0.48 \\
vp &= \text{Poisson's ratio of prototype arbor} = 0.3
\end{align*}
\]

From which \( U_p = 0.27 \times Um \)
The measured results from the photoelastic model were adjusted in accordance with equation (12) and are shown in Figure (8). Also included in Figure (8) are the results from a finite element idealisation of a similar pre-tensioned arbor which is described in Appendix 2. A comparison of these two methods will be discussed later.

From Figure (8) it can be seen that, for approximately 80% of the shank length, the deformation profile exhibits an almost parallel contraction (within 0.02 mm for a full sized arbor). Resistance to contraction is evident in the vicinity of the sleeve support shoulder, and is probably due to the 'bulk' effect offered by this shoulder. Nevertheless, this profile characteristic is a considerable improvement (more parallel) over that associated with a previous arbor design (20), which did not incorporate an under-cut fillet radius at the shoulder. It seems reasonable to assume, therefore, that the under-cut fillet radius has the effect of reducing the shoulder's resistance to radial contraction. Deviation from parallel contraction is also evident in the vicinity adjacent to the bore termination. This is probably due to the axial position, and not the geometry, of the bore termination. Supporting evidence of this assertion will be discussed in the context of results from further test work described later in this chapter (Section 3.1.6).

3.1.3 Isoclinic and Stress Trajectories

Figure (9a) shows isoclinic lines of all possible parameters superimposed on the pre-tensioned arbour. These lines are the loci of points of constant principal stress direction, from which curves of principal stress, or stress trajectories Figure (9b) can be directly obtained using the procedure described in Appendix 1.
Fig. 8 Diameter profile of pre-tensioned arbor for typical prototype.
Fig. 9(a) Isoclines for pre-tensioned arbor (original model)

Fig. 9(b) Stress trajectories for pre-tensioned arbor (original model)
The stress trajectories along the arbor's uniform section (shank) are essentially parallel to the longitudinal axis. This indicates that any transverse shear stress is negligible. Nevertheless, this section can be subjected to combined bending and tension, so that the deformation profile shown in Figure (8) is consistent with these results. The overall pattern of the stress trajectories is smooth which, although not to be confused with load flow or distribution, does suggest that the arbor design is not unsuitable for its purpose.

3.1.4 Stress Concentrations

The axial symmetry of both the external load and arbor geometry implies that the principal stresses at critical locations can be determined directly by making normal observations of the optical patterns at the boundaries of longitudinal and transverse slices cut from the stress frozen model. The isochromatic fringe pattern obtained when these slices are observed in a light field circular polariscope is shown in Figure (7). Positions of critical stress are located by the concentration of Isochromatic fringes.

Thus, for the arbor subjected to axial pre-tension, attention is focussed primarily on five specific locations, as shown in Figure (10).

In accordance with equation (11), stress concentration factors in the pre-tensioned arbor can be conveniently expressed in terms of the nominal axial tensile stress $\sigma_{nom}$ mid-way along the arbor shank where,

$$\sigma_{nom} = \frac{WB}{\pi} \frac{4}{b^2 - a^2}$$
Fig. 10 Critical stresses in pre-tensioned arbor (original design)

All SCF's are multiples of nominal axial tensile stress, $\sigma_{\text{nom}}$, in arbor at section 2
With the exception of the fillet radius at the bore termination (Section 3), the stress concentration factors in the arbor are all less than 3. The SCF value of 2.7 for the shoulder fillet radius (Section 1) is within 8% of a result obtained for a similar configuration studied by Lee and Ades (22).

The SCF measured in the fillet radius at the arbor bore termination was about 14, although the Isochromatic fringes were so concentrated in this vicinity that this measurement can only be an approximation. The extremely high value for this SCF can be attributed to the severe contact conditions which exist between the arbor and simulated thrust rod. It is appreciated that, with a nominal stress in the order of 0.2 kN/mm², an elastic SCF of 14 could never exist in a steel prototype.

In subsequent tests with alternative bore termination geometries the maximum fillet stress was reduced to about 4.5 × Nominal arbor tension.

3.1.5 Stress Distributions

The procedure used to obtain the distribution of the three separate stresses, \( \sigma_{zz} \), \( \sigma_{rr} \) and \( \sigma_{\phi\phi} \) through the arbor section is fully discussed in Appendix 1.

Figure (11) shows the distribution of these stresses, expressed as multiples of the nominal axial stress, \( \sigma_{Ax}^{\text{nom}} \) at Section 2, for Sections 1 and 3. At Section 3 the application of load produces a large bending moment with the maximum tensile stress being further increased by the geometrical stress concentration. The effect of the bending moment is to cause 'belling'.
Fig. 11 Stress distributions in pre-tensioned arbor (original photoelastic model)
of the arbor which tends to neutralise the diametral contraction due to axial pre-tensioning. The maximum tangential stress of \(6 \times \sigma_{\text{nom}}\) does not correspond with the directly measured SCF of 14 since it was not convenient to separate the individual stresses at the point of maximum stress intensity. Section 1 is also subjected to a bending moment due to the off set of reaction developed at the collet-shell/Arbor pre-loading shoulder interface. However, the resultant moment is largely diffused by the stiffness of the arbor shoulder so that the axial stress distribution is essentially perturbed only by the geometrical stress concentration associated with the sleeve support should fillet.

As would be expected, the radial and hoop stress are significantly lower than the corresponding axial stresses, and largely reflect the rate of change of the latter. It should be noted that there is no direct correlation between the hoop stress and the axial stress on any particular section since the magnitude of the former is really determined by the radial deformation and hence, the local structural stiffness of the component.

3.1.6 Modifications to Bore Termination Geometry

The geometry of bore termination in the proposed arbor design (original model) was considered most unsatisfactory due to the very large concentration of stress in the fillet radius. This was mainly attributed to the relative geometry of the mating surfaces which apparently caused the SCF associated with the arbor fillet radius to be superimposed on the SCF due to contact between the arbor and thrust rod, Figure (12a).
FIG. 12 ISOCHROMATIC FRINGE PATTERNS FOR ORIGINAL AND MODIFIED ARBOR DESIGNS
It was considered essential, therefore, to investigate alternative bore terminations in an attempt to reduce the associated stress concentration.

For the purposes of investigation, a dummy arbor with two bore terminations geometries ($\beta = 40^\circ$ and $50^\circ$) was subjected to the same axial loading as the original arbor. Figure (12b) shows the Isochromatic fringe patterns for the two modified arbor designs. The under-cut fillet radius is a feature common to both modified designs.

The tangential SCF's measured at the bore termination fillet radius for $\beta = 40^\circ$ and $\beta = 50^\circ$ were approximately 4.3 and 4.5 respectively. Figure (13) shows these results compared with the original SCF value of approximately 14. This comparison indicates a two-thirds reduction in maximum fillet stress for the modified arbor design. The hoop SCF's are approximately 60% less than their respective tangential SCF's.

Since the difference in SCF value for $\beta = 40^\circ$ and $\beta = 50^\circ$ is small, it appears that under-cutting the fillet radius, and not any particular cone angle, is mainly responsible for the above reduction in fillet stress. This seems reasonable, since the under-cut fillet radius effectively prevents the SCF associated with arbor geometry being superimposed on the SCF associated with contact between the thrust rod and bore termination.

The profile of diametral contraction for the modified arbor designs is not significantly different from that measured in the original model Figure (8). This would tend to confirm the assertion that the profile characteristics of a pre-tensioned arbor are dependent on the axial position, and not the geometry, of any particular bore termination design.
Fig. 13 Critical stresses in pre-tensioned arbor and collet-shell assembly
Load/unit length of circumference,
\[
\frac{2WB}{\pi(b+a)}
\]

Fig. 14 Resolution of forces at arbor bore termination
Analysis of bore termination fillet stress

Observations from the photoelastic model indicated that the maximum fillet stress could be sensibly expressed in terms of the axial tension in the arbor and the local bending effect of the thrust rod acting on the bore termination.

Figure (14) shows this proposed force idealisation, from which the following expression for maximum fillet stress \( \sigma_{3\text{max}} \) is obtained,

\[
\sigma_{3\text{max}} = \sigma_{3\text{nom}} \times Q_1 + \sigma_3 \times Q_2
\]

or

\[
\frac{\sigma_{3\text{max}}}{\sigma_{3\text{nom}}} = Q_1 + \frac{\sigma_3}{\sigma_{3\text{nom}}} \times Q_2
\]  \hspace{1cm} (13)

where \( Q_1 \) and \( Q_2 \) are SCF's which express the individual contributions of axial tension and local bending, respectively.

\[
\sigma_{3\text{nom}} = \text{Local axial tension at Section 3}
\]
\[
= \frac{4. \, \text{WB}}{\pi (b^2 - a'^2)}
\]

and

\[
\sigma_3 = \text{Local bending stress at Section 3}
\]
\[
= \frac{6M}{(b - a')^2} - \frac{6. \, \text{WB}}{\pi (b + a')^2} \left[ \frac{b + a'}{2} - a'' + (a' - a'') \cot^2 \beta \right]
\]

\[
= \frac{(b - a')^2}{(b - a')^2}
\]

The maximum fillet stress \( \sigma_{3\text{max}} \) is obtained directly from fringe measurements made in the model arbor, so that the constants \( Q_1 \) and \( Q_2 \) are found from the simultaneous solution of equation (13) for \( \beta = 40^\circ \) and \( \beta = 50^\circ \).
Table 2 compares the critical stress relationships derived for $\beta = 40^\circ$ and $\beta = 50^\circ$, together with an extrapolation for the original half-cone angle of $60^\circ$ with an under-cut fillet radius. The extrapolated SCF for $\beta = 60^\circ$ is also included in Figure (13).

<table>
<thead>
<tr>
<th>$\beta$</th>
<th>$\frac{\sigma_3}{\sigma_{3\text{nom}}}$</th>
<th>Tangential SCF</th>
<th>Hoop SCF</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\sigma_3^\text{max}/\sigma_{3\text{nom}}$</td>
<td>$Q1$</td>
<td>$Q2$</td>
</tr>
<tr>
<td>$40^\circ$</td>
<td>2.040</td>
<td>4.27</td>
<td>5.3</td>
</tr>
<tr>
<td>$50^\circ$</td>
<td>1.638</td>
<td>4.46</td>
<td>&quot;</td>
</tr>
<tr>
<td>$60^\circ$</td>
<td>1.461</td>
<td>4.57</td>
<td>&quot;</td>
</tr>
</tbody>
</table>

The modified bore termination geometry was incorporated in subsequent model tests concerned with residual loading and transverse bending.

3.1.7 Collet-Shell Assembly

The notation used in analysis is shown in Figure (15). The collet-shell assembly shown in Figure (13) is an axisymmetric structure consisting of three $118^\circ$ segments and two retaining sleeves. The assembly transmits the full reactive loading during arbor pre-tensioning, and would normally contain the pressure cap and piston arrangements (Figure 2).

A preliminary examination of the stress frozen model suggested that the collet-shell could be considered as a two-dimensional beam, subjected to a uniform axial tensile load which is offset from the point of application. This offset, or eccentricity $e$, is caused by the inclination of the collet-shell/Arbor shoulder interface, as shown in Figure (15a). Observations from
(a) Actual structure ~ 3 dimensional shell

- Collet-shell Mid-section
- Retaining sleeve
- Interface pressure distribution between arbor and collet-shell

(b) Idealised structure ~ 2 dimensional beam

- Deformed profile
- Elastic beam
- Rigid body

Fig. 15 Resolution of forces in collet-shell assembly
the model showed the load eccentricity to be equal at each end of the assembly, thus implying a uniform bending moment along the axis of symmetry.

A similar examination of the retaining sleeves revealed that, due to bending and radial expansion of the collet-shell, the sleeves were subjected to non-uniform internal pressure, both in the axial and circumferential directions.

Figures (7) and (16) show respectively the isochromatic fringes patterns in a longitudinal slice cut from the collet-shell assembly and a circumferential slice from one of the retaining sleeves (Section F). Locations of maximum stress, for the collet-shell are seen to be in the vicinity of the fillet radius. Maximum stress in the retaining sleeve occurs at the extreme edge of the sleeve Figure (7) and in the vicinity of the inter-collet clearance, Figure (16).

Model measurements

(a) Collet-Shell

Fringe measurements were made at the shoulder fillet radius and the nominal mid-section (Sections 8, 9 and 7 in Figure 13) for six longitudinal slices cut from one 118° segment of the collet-shell. Confirmatory measurements were made on one longitudinal slice from a diametrically opposite segment. Circumferential variations in fringe values were found to be negligible.
Total axial load = WB

Circumferential pressure distribution

FIG. 16  ISOCROMATIC FRINGE PATTERN OF RETAINING SLEEVE ~ HOOP SLICE, SECTION F
Critical stress values

Average tangential SCF's calculated for the fillet radius at each end of the collet-shell are 2.7 and 3.0 for Sections 9 and 8 respectively. Referring to Figure (13), it seems reasonable to suggest that the above critical stress difference can be attributed to the different sleeve positions with respect to the collet fillet radius. It follows that, the sleeve position associated with Section 9 is superior to that associated with Section 8.

Stress distributions

Figure (17a) shows the tangential stress distribution around the inner and outer radii at the mid-section for one 118° segment of the collet-shell.

Although the variation in stress is not excessive, it is apparent that bending about the neutral axis causes the extreme ends of the collet-shell (positions A1 and A6) to be subjected to additional tensile stress. Conversely, the area above the neutral axis is subjected to a compressive stress superimposed on the axial tension.

Also included in Figure (17a) is a theoretical distribution of axial stress. This is based on the assumption that the total stress in the collet-shell \( \sigma_c \) can be expressed in the following manner:

\[
\sigma_c = \frac{W}{A_c} + \frac{M_v}{I}
\]

\[\text{........... (14)}\]
(a) Tangential stress distributions in collet shell (mid plane)

(b) Deformed profile along collet shell

Fig. 17  Collet-shell analysis
where \( W = \text{Total axial load} = \frac{W_B}{3} \)

\( Ac = \text{Cross-sectional area of collet-shell} \)

\[ = \theta (r_2^2 - r_1^2) \]

\( M = \text{Applied moment at mid-plane} = W e_1 \) (\( e_1 = \text{load eccentricity at mid-plane} \))

\( y = \text{Distance from NA} = (r \cos \theta - y) \)

\[ = (r \cos \theta - \frac{2(r_2^3 - r_1^3) \sin \theta}{3(r_2^2 - r_1^2) \theta}) \]

and \( I = \text{Moment of Inertia of collet-shell about NA} \)

\[ = \frac{(r_2^4 - r_1^4)}{4} \times (\theta + \sin \theta \cdot \cos \theta) - \frac{4(r_2^3 - r_1^3)^2 \sin^2 \theta}{9(r_2^2 - r_1^2) \theta} \]

The above expression for total stress at any location around the collet-shell circumference assumes simple beam behaviour.

To determine \( e \)

For the model assembly, the total force acting on the collet-shell is given by:

\[ W = \frac{W_B}{3} \int_{-\theta}^{\theta} (A + Br) r \, dr \, d\theta \]

where \( A = \frac{\sigma_1 r_2 - \sigma_2 r_1}{r_2 - r_1} \)

and \( B = \frac{\sigma_2 - \sigma_1}{r_2 - r_1} \)

Values for \( \sigma_2 \) and \( \sigma_1 \), corresponding to radii \( r_2 \) and \( r_1 \), are measured directly from the model collet-shell at mid-plane.
Similarly, the total moment acting on the collet-shell is given by,

\[ M = W \times e_1 = \int_{-\theta}^{\theta} \int_{r_1}^{r_2} (A + Br)(r \cos \theta - \bar{y}) r \, dr \, d\theta \]

Graphical integration of the above expression gives,

\[ W = 0.114 \, \text{KN}, \quad \text{or} \quad W_B = 0.341 \, \text{KN} \]

and

\[ M = -0.082 \, \text{KN:mm} \]

The derived value for \( W_B \) (0.341 KN) is some 15% less than the specified dead weight loading of 0.4 KN. This is the expected margin of error associated with graphical integration.

Since \( M = W \times e_1 \)

It follows that,

\[ e_1 = -\frac{0.082}{0.114} = -0.73 \, \text{mm} \]

The negative sign for \( e_1 \) indicates that the effective offset of applied load is below the collet-shell neutral axis.

The above value for \( e_1 \) is substituted in equation (14), and the resultant theoretical stress distribution is plotted for locations \( A_1 - A_6 \), at \( r_1 \) and \( r_2 \), as shown in Figure (17a). Although the bending stress contribution is relatively small, the reasonable correlation between the theoretical and experimental results would tend to suggest that the assumed beam behaviour and derived value for \( e_1 \) are valid.
Figure (17b) shows the deformed profile of the collet-shell along its axis of symmetry. The experimental curve was measured directly from the model. The theoretical curve is based on the beam assumption implied in equation (14) for collet-shell stress, so that:

\[ M'' = W \times e_1 = EI \frac{d^2u}{dz^2} \]

where \( W = \frac{WB}{3} \text{ KN} \)
\( e_1 = -0.73 \text{ mm} \)
\( u = \text{Radial deformation} \)
and \( z = \text{distance from mid-plane} \)

Observations from the model assembly suggested that the collet-shell could only be treated as an elastic beam up to the fillet radius location \((z=1)\). Beyond the fillet radius the collet-shell appeared to behave as a rigid body, being accompanied with relatively large radial deformation as shown in Figure (17b).

A comparison of the measured and theoretical profiles shows a reasonable correlation, although both the maximum deflection and slope for the theoretical result are less than the measured values (30% for deformation).
Isoclinics and Stress Trajectories

Figures (18a and b) show isoclinic lines and stress trajectories superimposed on a longitudinal section through the collet-shell.

The parallel trajectories indicate uni-directional stress with no appreciable transverse shear. This is compatible with the applied loading of combined tension and circumferential bending. This trajectory pattern is also consistent with the observed deformation behaviour and elastic beam assumption.

(b) Retaining sleeves

Three circumferential slices (A, B, C, D, E and F) were cut from each retaining sleeve as indicated in Figure (16). Fringe measurements were made every 15° around the circumference at the inner and outer radii, \( r_2 \) and \( r_3 \) respectively. These results were averaged over a 120° section, since the hoop stress was observed to be symmetrical about the 60° position, being maximum at 0° and 120°.

Sleeve Stress

From Lamé theory of thick cylinders under internal pressure, the sleeve hoop stress at any radial location is given by,

\[
\sigma_\phi = \frac{P}{r_3^2 - r_2^2} \left[ \frac{r_2^2}{r_3^2 - r_2^2} \left( 1 + \frac{r_3^2}{r_2^2} \right) \right]
\]
Fig. 18(a) Isoclinics for pre-tensioned collet-shell

Fig. 18(b) Stress trajectories for pre-tensioned collet-shell
From which, at \( r = r_2 \),
\[
\sigma_{\phi_2} = \rho \left( \frac{r_3^2 + r_2^2}{r_3^2 - r_2^2} \right)
\]

at \( r = r_3 \),
\[
\sigma_{\phi_3} = \frac{2\rho r_2^2}{r_3^2 - r_2^2}
\]

From the model, measured values of stress at \( r = r_2 \) are equivalent to \( \sigma_{\phi_2} + \rho \), whilst those at \( r = r_3 \) correspond to \( \sigma_{\phi_3} \).

It follows that, for the model sleeves \( r_2 = 44.5 \) mm and \( r_3 = 54 \) mm,

\[
\sigma_{\phi_2} = \rho \times 5.23
\]
and
\[
\sigma_{\phi_3} = \rho \times 4.23
\]
or
\[
\frac{\sigma_{\phi_2} + \rho}{\sigma_{\phi_3}} = 1.47
\]

Measured stress values (\( \sigma_{\phi_2} + \rho \) and \( \sigma_{\phi_3} \)) were substituted in equation (15) and compared with the nominal Lamé result of 1.47, as shown in Table 3.

**TABLE 3 Hoop Stress Results for Retaining Sleeves**

<table>
<thead>
<tr>
<th>Sleeve Section</th>
<th>( \frac{\sigma_{\phi_2} + \rho}{\sigma_{\phi_3}} )</th>
<th>Pav. ( \times 10^{-5} )</th>
<th>( \sigma_{\phi_3} )</th>
<th>( \sigma_{\phi_2} )</th>
<th>( \sigma_{\phi_3} ) max</th>
<th>( \sigma_{\phi_2} ) max</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1.46</td>
<td>2.14</td>
<td>9.04</td>
<td>1.11</td>
<td>10.36</td>
<td>18.97</td>
</tr>
<tr>
<td>B</td>
<td>1.55</td>
<td>1.55</td>
<td>6.57</td>
<td>8.12</td>
<td>7.47</td>
<td>18.25</td>
</tr>
<tr>
<td>C</td>
<td>1.54</td>
<td>0.47</td>
<td>2.00</td>
<td>2.47</td>
<td>2.20</td>
<td>3.57</td>
</tr>
<tr>
<td>D</td>
<td>1.07</td>
<td>0.61</td>
<td>2.60</td>
<td>3.21</td>
<td>5.88</td>
<td>4.46</td>
</tr>
<tr>
<td>E</td>
<td>1.46</td>
<td>1.45</td>
<td>6.12</td>
<td>7.57</td>
<td>6.75</td>
<td>15.63</td>
</tr>
<tr>
<td>F</td>
<td>1.43</td>
<td>3.69</td>
<td>15.61</td>
<td>19.30</td>
<td>18.97</td>
<td>37.24</td>
</tr>
</tbody>
</table>

64
A comparison of the measured ratio \( \left( \frac{\sigma_\phi f_2 + D}{\sigma_\phi f_3} \right) \) with the nominal Lamé result shows that, with the exception of Section D, the assumed Lamé behaviour for the retaining sleeves appears to be reasonably substantiated. Observations in the model indicated that the retaining sleeve in the vicinity of Section D was not in contact with the collet-shell. This would explain the low hoop stress measured at Section D.

Measured values of nominal and maximum hoop stress are also included in Table 3. These results are based on the average measured pressure Pav relevant to each section. Retaining sleeve SCF values (for \( r = r_2 \)) are shown in Figure (13), but are expressed in terms of a single equivalent force \( S \) for convenience. The derivation of equivalent force \( S \) is later described in the context of structural idealisation.

**Sleeve deformation**

From a consideration of Lamé, it can be shown that, the average sleeve deformation at \( r = r_2 \) is given by:

\[
\delta_2^{av} = (\sigma_\phi f_2 - v \cdot Pav) \frac{r_2}{B}
\]

where \( \sigma_\phi f_2 \) = Measured value of hoop stress at \( r_2 \)

and \( Pav = \) Average measured pressure.

Values of \( \delta_2^{av} \) were calculated for Sections A-F and, with the exception of sleeve D, found to be within 15% of the measured radial deformation. The error associated with Section D (35%) was again attributed to lack of contact with the collet-shell.
Structure Idealisation

It was apparent from the preceding photoelastic examination of a model collet-shell assembly that the actual structure would require considerable simplification to facilitate inclusion in the overall design procedure for composite rolls.

Observations from the model tests suggested that the three-dimensional collet-shell could be treated as a two-dimensional beam, separated into an elastic section and a rigid section, as shown in Figure (15b). The applied loading is represented by an offset axial tensile load $W$ on the collet-shell and an equivalent radial force $S$ at a location $l_2$ on the retaining sleeve. The axial load is offset from the neutral axis by $e_1$ at the collet-shell mid-section, and by $e$ at the point of load application. The collet-shell's second moment of area is the same for both the actual and idealised structures.

Deformation behaviour of the model assembly further indicated that it would not be unreasonable to assume matched rotation of the collet-shell and retaining sleeve $\phi$ in a full-sized prototype. It is also assumed that the load offset $e$, measured from the model, is comparable with that to be expected in any similarly proportioned configuration.

The problem is, therefore, to establish a relationship between the known parameters; $W$, $e$, $l$, $l_1$, $l_3$ and $l_4$ and the unknown variables; $S$, $l_2$, $\delta$ and $\phi$. 
Force on retaining sleeve

Referring to Figure (15b), let \( p_3 \) and \( p_4 \) be the radial pressure acting on the sleeve at positions \( \ell_3 \) and \( \ell_4 \) respectively.

If \( p \) is the pressure at a distance \( Z \) from the sleeve edge, then

\[
p = p_3 + \frac{p_4 - p_3}{\ell_4 - \ell_3} \times Z
\]

\[
\therefore S = \int_{\ell_3}^{\ell_4-\ell_3} p \times \sqrt{3} \times r_2 \times dz
\]

From which,

\[
S = \sqrt{3} \times r_2 \left[ \frac{p_3 + p_4}{2} \right] (\ell_4 - \ell_3)
\] ........................ (16)

Moment on retaining sleeve

Let \( z \) be distance along collet-shell, measured from mid-section.

Moments about position \( z = \ell_3 \), gives

\[
M = S \ (\ell_2 - \ell_3) = \int_{\ell_3}^{\ell_4-\ell_3} p \times \sqrt{3} \times r_2 \times z \times dz
\]

\[
= \sqrt{3} \times r_2 \frac{(\ell_4 - \ell_3)^2}{6} (p_3 + 2p_4)
\] ........................ (17)

Writing,

\[
\frac{6S(\ell_2 - \ell_3)}{\sqrt{3} \times r_2 \ (\ell_4 - \ell_3)^2} = p_3 + 2p_4
\]

and

\[
p_4 - p_3 = 2(p_3 + 2p_4) - 3(p_3 + p_4)
\]
Substituting from (16) and (17) gives,

\[
\frac{p_4 - p_3}{\sqrt{3} \times r_2 (\ell_4 - \ell_3)^2} = \frac{6S}{(2 \times \ell_2 - (\ell_4 + \ell_3))} \quad \ldots \ldots \quad (18)
\]

Radial displacement of retaining sleeve

\[
E \times \varepsilon = \frac{E \times \delta}{r_2} = \sigma \phi - \nu \times \sigma r
\]
where, \( \delta \) = Radial displacement

At \( r = r_2 \), \( \sigma r = - p \)

\[
\sigma \phi = p \times \frac{r_3^2 + r_2^2}{r_3^2 - r_2^2} \quad \text{from Lamé}
\]

\[
E \times \frac{\delta}{r_2} = p \left( \frac{r_3^2 + r_2^2}{r_3^2 - r_2^2} + \nu \right)
\]
Writing,

\[
\delta = Fl \times p
\]
where \( Fl = \frac{r_2}{E} \left( \frac{r_3^2 + r_2^2}{r_3^2 - r_2^2} + \nu \right) \)

it follows that,

\[
\delta_3 = Fl \times p_3
\]
and

\[
\delta_4 = Fl \times p_4
\]
where \( \delta_3 \), \( p_3 \) and \( \delta_4 \), \( p_4 \) are the radial displacements and pressures at positions \( \ell_3 \) and \( \ell_4 \) respectively.

The mean displacement of sleeve at \( \frac{\ell_4 + \ell_3}{2} \) is given by:

\[
\overline{\delta} = \frac{\delta_3 + \delta_4}{2} = Fl \times \frac{(p_3 + p_4)}{2}
\]
Substituting from (16)

\[ \bar{\delta} = F1 \left( \frac{S}{\sqrt{3} \times r_2 (\ell_4 - \ell_3)} \right) \]

\[ \therefore \quad \bar{\delta} = \frac{S}{\sqrt{3} \times (\ell_4 - \ell_3)E} \left( \frac{r_3^2 + r_2^2}{r_3^2 - r_2^2} + \nu \right) \] .............. (19)

Rotation of retaining sleeve

The rotation of sleeve is given by

\[ \phi = \frac{\delta_4 - \delta_3}{\ell_4 - \ell_3} = F1 \left( \frac{P_4 - P_3}{\ell_4 - \ell_3} \right) \]

Substituting from (18) for \( (p_4 - p_3) \)

\[ \phi = \frac{6S}{\sqrt{3} \times E} \frac{2l_2 - (l_3 + l_4)}{(l_4 - l_3)} \left( \frac{r_3^2 + r_2^2}{r_3^2 - r_2^2} + \nu \right) \] .............. (20)

Dividing (19) by (20),

\[ \frac{\bar{\delta}}{\phi} = \frac{(\ell_4 - \ell_3)^2}{6(2l_2 - (l_3 + l_4))} \] .............. (21)

Collet-Shell Deformation

Moment on collet section is given by,

\[ M = W e_1 = W e - S (l_2 - l_1) \]

but \[ M = E I \frac{d^2 u}{dz^2} \].
Integrating  \[ EI \frac{du}{dz} = Mz + A \]

Integrating  \[ EI \ u = \frac{Mz^2}{2} + Az + B \]

When \( z = 0; \ du/dz = 0; \ u = 0 \) \( \Rightarrow \) \( A = B = 0 \)

When \( z = l; \)

\[ \frac{du}{dz} = \phi = \frac{Mz}{EI} = \frac{\ell}{EI} (We - S(l_2 - l_1)) \]  \[ (22) \]

and  \[ u = \frac{Mz^2}{2EI} = \frac{\ell^2}{2EI} (We - S(l_2 - l_1)) \]  \[ (23) \]

From which,  \[ \delta = u + \left[ \frac{l_3 + l_4}{2} - l \right] \phi \]

\[ \Rightarrow \delta = \frac{l(l_3 + l_4 - l)}{2EI} (We - S(l_2 - l_1)) \]  \[ (24) \]

Dividing (24) by (23),

\[ \frac{\delta}{\phi} = \frac{(l_3 + l_4 - l)}{2} \]  \[ (25) \]

Equating (21) and (25), assuming matched rotation of collet-shell and retaining sleeve, gives

\[ l_2 = \frac{(l_4 - l_3)^2}{6(l_3 + l_4 - l)} + \frac{l_3 + l_4}{2} \]  \[ (26) \]

From which, it is apparent that the position of the equivalent radial force \( S \) is independent of the magnitude and offset of the applied load.
Finally, substituting for \( \ell_2 \) in (24) and equating with (19) to evaluate \( S \) in terms of \( e \), it can be shown that,

\[
S = \frac{We}{2I \left( \frac{r_3^2 + r_2^2}{r_3^2 - r_2^2} + v \right) + (\ell_2 - \ell_1)} \left( \sqrt{3} \times \ell (\ell_4 - \ell_3)(\ell_3 + \ell_4 - \ell) \right)
\] ............ (27)

\[S = \frac{0.036 \text{ KN}}{}\]

\[M = 0.66 \text{ KN.mm}\]

It follows that, the effective position of \( S \) is given by,

\[
(\ell_2 - \ell_3) = \frac{S}{M}
\]

\[\therefore \ell_2 = 39.3 \text{ mm}\]

Comparison with model measurements

Confirmation of simple beam behaviour for the collet-shell and Lamé analysis for the retaining sleeves, has already been demonstrated in the model analysis.

It is only required, therefore, to assess the validity of force idealisation on the retaining sleeves.

In the model, the total radial force \( S \) and moment \( M \) acting on the retaining sleeve are obtained by graphically integrating the measured average pressure \( P_{av.} \) along the sleeve length, from which,

\[S = 0.036 \text{ KN}\]

and \[M = 0.66 \text{ KN.mm}\]

\[\therefore \ell_2 = 39.3 \text{ mm}\]
If the expression for $l_2$ in equation (26) is evaluated for the model dimensions, then

$$l_2 = 36 \text{ mm}$$

Similarly, if $S$ is calculated from equation (27) then,

$$S = 0.022 \text{ KN}$$

A comparison of these results shows that, whilst the respective values for $l_2$ are in good agreement, the error between measured and theoretical values for $S$ is almost 40%. This error is equivalent to about 10% of the total axial load on each collet segment. Check measurements in the model analysis revealed that the maximum error associated with graphical integration would not affect the above result by more than 10-15%.

It was finally concluded that the above discrepancy between measured and theoretical values for $S$ must be attributed to the relatively large deformations experienced by the model assembly, which are particularly critical in the vicinity of the retaining-sleeve.
It is argued that the proposed theoretical treatment of equivalent force evaluation is more suited to a steel prototype since the corresponding deformations are considerably less than those associated with the photoelastic model.

3.2 Models Subjected to Residual Loading

3.2.1 Assembly and testing

The model arbor (with modified bore termination geometry) was pre-tensioned using the procedure adopted for the first-test. The radial displacement obtained was just sufficient to allow for assembly of the sleeves, and no special precautions were needed to ensure that the concentricity of the components was maintained during subsequent operations. The threaded connection between the retaining nut and arbor was replaced by a cemented joint since it was expected that the latter could be mechanically superior, be free from thread errors and ensure an even distribution of the residual clamping force (WR) between the interacting components. The quality of
the cemented joint was ensured by careful cleaning of the surface and injecting liquid cold setting epoxy into a circumferential groove machined at the forward end of the nut. Injection of the cement continued until the adhesive flowed freely from the opposite end of the nut. During this time the nut was maintained in the correct position by a jig which provided a small axial load on the nut and sleeve assembly. The adhesive was allowed to set and was then cured by raising the temperature to 60°C for two hours. The whole assembly was finally annealed to distribute the residual clamping force throughout the model assembly, which is shown in Figures (19) and (20).

This loading procedure is directly comparable with that employed for the prototype. Therefore, the relationship between arbor pre-tension \(W_B\) and sleeve clamping \(W_R\) in the model assembly is also in accordance with equation (6), so that:

\[
W_B = W_R \frac{AA \cdot EA \cdot LA(WR)}{AA \cdot Es ' \cdot LA(WB) + LA(WR)}
\]

ignoring stiffness losses

where \(EA = Es ' = 0.207 \text{ KN/mm}^2\)

\(AA = \frac{\pi}{4} (c^2 - b^2) = 2380 \text{ mm}^2\)

\(AA = \frac{\pi}{4} (b^2 - a^2) = 1879 \text{ mm}^2\)

\(LA(WB) = 76 \text{ mm}\)

\(LA(WR) = 89 \text{ mm}\)

From which, \(WR = 0.477 \cdot W_B\)

If \(W_B = 0.4 \text{ KN}\), then \(WR = 0.19 \text{ KN}\)
FIG. 19 ASSEMBLED ROLL UNDER RESIDUAL LOADING
However, using the direct measurement method of checking the total axial load retained in the model assembly indicated values of residual load considerably less than the theoretical prediction. The total tensile load in the arbor was found to be 0.13 KN, whilst the total compressive load in the sleeves was found to be 0.14 KN.

It was suspected that the difference between the measured and experimental values of residual load was due to the initial assumption of negligible stiffness losses in the model assembly. Evidence of sleeve flexing was obtained from fringe measurements made in the vicinity of the sleeve/sleeve interfaces, Figure (20). These observations indicated a rapid reduction in contact pressure at locations above the minimum groove diameter.

If the effective sleeve area is taken to be that which lies below the annular groove, then,

\[ A_{se} = \frac{\pi}{4} (c_e^2 - b^2) \]

where \( c_e = c - 2R = 67.6 \text{ mm} \) in Figure (5)

Thus, \[ A_{se} = 1428 \text{ mm}^2 \]

and, \[ WR = 0.37, \ \ \ \ WB = 0.147 \text{ KN} \]
FIG. 20  ISOCHROMATIC FRINGE PATTERN OF ASSEMBLED MODEL UNDER RESIDUAL LOADING

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This result is within 8% of the measured value of residual load and indicates that, in theoretical calculations, the effective external diameter of the sleeves \((c_e)\) coincides with the root diameter of the annular groove.

### 3.2.2 Isoclinics and Stress Trajectories

Figure (21a) shows isoclinic lines of all parameters superimposed on a longitudinal section through the residually clamped roll assembly. The stress trajectories shown in Figure (21b) are obtained directly from the isoclinics as previously described in Section 3.1.3.

Parallel trajectories along the arbor shank indicate the absence of transverse shear, whilst those through each sleeve are perturbed only by the annular groove. This tends to suggest that the critical sleeve stress could be expressed in terms of a geometric SCF related to the annular groove. The distribution of residual load through the roll assembly follows the trajectory route which changes from tension in the arbor to compression in the sleeves. Trajectories in the vicinity of load transfer from arbor to retaining nut are inclined at 45°. This indicates a condition of pure shear stress on transverse sections and serves to confirm the component stiffness idealisation for the model roll assembly.

### 3.2.3 Stress Concentrations in Arbor

Critical sections are again identified by the concentration of isochromatic fringes observed in the longitudinal and hoop slices cut from the model assembly, Figure (20).
Fig. 21(a) Isoclines for assembled roll under residual loading

Fig. 21(b) Stress trajectories for assembled roll under residual loading

---

- Stress trajectory of algebraically greater principal stress
- **q**, "smaller" ""
Figure (22) identifies the position of critical stresses, together with a tabulation of their associated SCF's measured directly from the model arbor. All results are expressed in terms of the nominal arbor tension $\sigma'_{A_{nom}}$. This approach allows a direct comparison with results from the arbor pre-tensioning test.

The critical stress values for arbor sections 1, 3, 5 and 6 are consistent with the observed redistribution of axial load from sleeve assembly to sleeve clamping conditions. Evidence of local bending in the vicinity of the arbor shoulder (Section 1) and the retaining nut (Section 6) is reflected in their respective SCF values, which are approximately double those measured in the pre-tensioned arbor. The clamping force in the sleeves is apparently reacted against the arbor sleeve supports, which causes an additional local bending stress to be superimposed on the nominal residual tension in the arbor. A more comprehensive analysis of these critical stress values is discussed in the context of subsequent tests with roll bending (Section 3.3.3).

The SCF associated with Section 4 is omitted in Figure (22), since, in the residually loaded model, it is no longer critical (less than 10% $\sigma'_{A_{nom}}$).

3.2.4 Stress Concentration in the Sleeves

The sleeves are subjected to axial compression, which results in local bending in the vicinity of the annular groove. This causes the sleeve to flex inwards, thus redistributing the interface pressure between
Fig. 22 Critical stresses in assembled roll under residual loading
each sleeve as previously described in Section 3.2.1. Consequently, a stress concentration is developed at the position of minimum groove diameter, where the effect of local bending is maximum.

It seems reasonable, therefore, to express the maximum sleeve stress \( \sigma'_{\text{max}} \) in terms of a nominal stress \( \sigma'_{\text{nom}} \) and a geometric SCF, so that:

\[
\sigma'_{\text{max}} = \sigma'_{\text{nom}} \times KR
\]  

(28)

where \( \sigma'_{\text{nom}} \) = Nominal compressive stress in residually loaded sleeves

\[
= -\frac{WR. 4}{\pi(c_e^2 - b^2)}
\]

and

\( KR \) = Geometric SCF associated with each sleeve.

In Figure (22), it can be seen that \( KR \) (Tangential SCF) varies from 1.5 for sleeves S2 and S3, to 1.8 for sleeve S1.

The higher stress in sleeve S1 is probably a function of the arbor fillet radius (Section 1) which undercuts the arbor shank and support shoulder. This has the effect of increasing the local bending stress on this particular sleeve.

3.2.5 Stress Distributions

Figure (23) shows the distribution of principal stresses \( \sigma_{zz}, \sigma_{rr} \) and \( \sigma_{\phi\phi} \) in the residually loaded roll assembly. All stresses are expressed in terms of the nominal stress in either the arbor or sleeves.
Axial stress, $\sigma_{zz}$  
Hoop stress, $\sigma_\phi$  
Radial stress, $\sigma_{rr}$  

$+$ve = Tensile stress  
$-$ve = Compressive stress

*Image 23: Stress distributions in assembled roll under residual loading*
The hoop and radial stresses, $\sigma_{\phi\phi}$ and $\sigma_{rr}$, are never critical, and largely reflect the rate of change of the more important axial stress, $\sigma_{zz}$. Zero radial stress indicates an unloaded boundary, whilst equal radial stress at each Sleeve/Arbor interface indicates uniform radial interference clamping along the arbor shank.

Residual load transference from sleeves to arbor causes local bending on Section 1, resulting in a peak stress of about $4 \times \sigma'_{\text{nom}}$. This is 67% greater than that associated with arbor pre-tensioning. Consequently, the SCF at Section 1 is probably a function of passing load in the arbor and local bending due to the sleeves acting on the arbor shoulder.

The slightly non-uniform distribution of axial stress at Section 2 can be attributed to the method of end restraint at each end of the assembly. The effect of transferring axial load through the retaining nut instead of the bore termination, reduces the peak stress at Section 2, from about 4.6 (modified arbor design) to $2.7 \times \sigma'_{\text{nom}}$.

The stress distributions observed in sleeves S2 and S3 can be considered typical for any comparable sleeve geometry, since neither are associated with local perturbations other than the annular groove. The SCF at the groove root is about 1.5 for both sleeves. This result is considerably less than that associated with comparable geometric discontinuities, such as a stepped shaft with a fillet radius, whose SCF is normally in the order of 1.9 - 2.1$^{(23)}$.

The axial stress distribution in sleeve S1 is consistent with the additional local bending stress acting on this sleeve, due to contact with the arbor support shoulder.
3.3 Models Subjected to Combined Residual Loading and Transverse Bending

3.3.1 Assembly and testing

Axisymmetric, residual, loading in the model assembly was accomplished using the procedures adopted in the first and second tests. The simulated rolling load (P) was applied through a cylindrical rod manufactured in Araldite CT200 as shown in Figure (24). The required bending load was relatively small when compared with the pre-tension and self-weight of the model. Consequently, to avoid the introduction of spurious loading effects during the stress freezing cycle, arrangements were made to immerse the whole model and loading device in oil having the same specific gravity as Araldite CT200.

The stress distribution due to combined bending and axial load is no longer axisymmetric and it was necessary to separate the individual stress components at the critical sections. This required the manufacture and testing of essentially two bending models from which longitudinal and transverse slices were cut at the precise position of load application.

If the ratio of pre-tension to bending load is to be the same for prototype and model, then,

\[
\frac{W_B}{P} = \frac{W_B}{P}\]

\[
\therefore \quad P_m = \frac{W_B}{P} \times P
\]
Fig. 24  Bending rig arrangement
where \( W_{B_m} = 0.4 \text{ KN} \)
\[ W_{B_p} = 7000 \text{ KN} \]

and \( P_p = 100 \times 3 = 300 \text{ KN} \) (for a pessimistic shock factor at bar entry, \( SF = 3 \))

From which, \[ P_m = \frac{0.4}{7000} \times 300 = 0.017 \text{ KN} \]

The resultant Residual stress/Bending stress ratio for the model arbor is estimated to be 3.5:1. This compares with an equivalent ratio of 11.5:1, for the 210 mm prototype arbor discussed in Chapter 2. The model loading conditions are, therefore, far more severe than those normally encountered in practice.

Apart from ensuring a pessimistic design appraisal, this approach is also of assistance to the photoelastic analysis. Exaggerating the bending stress component serves to emphasise the relative residual load and transverse bending contributions to the observed maximum stress.

Figure (25) shows the model assembly after slicing for analysis.

The total axial load retained in the roll assembly was verified in accordance with the procedure described for the two previous model tests. To avoid local load diffusion effects, due to component geometry or load contact conditions, fringe measurements were made through the assembly along a position corresponding to sleeve section S3 (Figure (23)) and not mid-way along the arbor shank.
FIG. 25  ISOCHROMATIC FRINGE PATTERN OF ASSEMBLED MODEL
UNDER COMBINED RESIDUAL LOADING AND TRANSVERSE BENDING

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On the 'compression' side of the assembly, sleeve clamping was found to have increased from 0.13 KN to 0.16 KN, whilst the arbor tension had decreased from 0.14 KN to 0.11 KN. An equivalent variation in distribution of axial load was also observed on the 'tension' side of the roll.

These results are within 10% of a theoretical prediction which assumes a linear distribution of bending stress. This would tend to suggest that, under the action of transverse bending, a composite roll assembly can be treated as a simple monolithic beam.

3.3.2 Isoclinics and Stress Trajectories

The stress trajectories shown in Figure (26b) were obtained directly from the Isoclinics (a) in the manner previously described.

When compared with the stress trajectories for the residually loaded model, Figure (21b), it is apparent that, for this particular arrangement, superimposing transverse bending has little effect on the roll's overall deformation behaviour. This observation is consistent with the above results from total load measurement. There is no evidence of appreciable transverse shear across the arbor shank although the effect of direct contact loading on the central sleeve S2 is clearly indicated.

The isoclinics and stress trajectories on the assemblies 'tension' side indicate limited axial sleeve separation, but only in the vicinity adjacent to the annular groove.
Fig 26(a) Isoclinics for assembled load

Fig 26(b) Stress trajectories for residual load
3.3.3 Stress Concentrations in Arbor

The isochromatic fringe pattern illustrated in Figure (25) shows that the location of critical sections, under combined loading conditions, remains essentially unchanged from those observed in the residual load model.

A preliminary analysis of the fringe measurements made at critical locations in the arbor was based on a similar procedure to that adopted for the residual load test. It was assumed that the maximum fillet stress \( \delta A_{\text{max}} \) at any particular location could be adequately described in terms of the nominal stress in the arbor due to the combination of residual loading and transverse bending. In which case, the maximum fillet stress is given by:

\[
\delta A_{\text{max}} = \sigma' A_{\text{nom}} \times Kr \pm \delta A_{\text{nom}} \times K_b \quad \text{(29)}
\]

where \( \sigma' A_{\text{nom}} \) = Nominal residual tension in arbor
\[= - \frac{4 \, \frac{WR}{\pi (b^2-a^2)^2}} \]
\( \delta A_{\text{nom}} \) = Nominal bending stress in arbor
\[= \pm \frac{M_y \cdot 64}{\pi (c^4-a^4)} \]

\( Kr \) = SCF associated with residual loading (measured in previous test)

and \( K_b \) = SCF associated with transverse bending.
However, this approach proved unsatisfactory since it was not possible to establish a consistent correlation between the measured values of maximum fillet stress and the SCF associated with transverse bending.

It was appreciated that an inconsistency of this nature would tend to suggest non-monolithic behaviour of the model roll assembly. This was considered most improbable in the light of results obtained from the total load checks and Isoclinic/Stress trajectory patterns which indicated simple beam behaviour.

A more detailed examination of the isochromatic fringe patterns from the residual load and transverse bending models suggested that critical fillet stresses in the arbor could be better expressed in terms of localised load diffusion between the mating components, rather than geometric SCF's specifically related to residual tension and overall bending. It was argued, for example, that the maximum stress at the arbor shoulder fillet (Section 1) is dependent partly upon the mean axial stress in the arbor and partly upon the local bending effect due to load transmitted through the sleeves into the shoulder. It follows that the maximum stress at a given location must be expressed in terms of the appropriate nominal stresses in both the arbor and the sleeves, so that

\[
\sigma_{\text{max}} = K_1 \times \sigma_1 - K_2 \times \sigma_2
\]

\[\text{…….. (30)}\]

where \( \sigma_{\text{max}} \) = Maximum arbor stress
\[\sigma_1 \] = Nominal arbor stress
\[\sigma_2 \] = Nominal sleeve stress

and \( K_1 \) & \( K_2 \) = SCF's which, respectively, express the effect of passing load in the arbor and local bending effect of sleeves acting on the arbor shoulder.
The general expression for $\sigma_{\text{max}}$ in equation (30) implies that it is equally applicable to the residual loading and combined loading cases. It is, therefore, assumed that SCF value $K_1$ and $K_2$ are independent of their respective nominal stresses.

In accordance with equation (30), for residual loading, the maximum arbor stress can now be written:

$$\sigma'_{A_{\text{max}}} = K_1 \times \sigma'_{A_{\text{nom}}} - K_2 \times \sigma'_{S_{\text{nom}}} \quad \ldots \quad (31)$$

Similarly, for the roll under combined residual loading and transverse bending, the maximum arbor stress is given by,

$$\dot{\sigma}_{A_{\text{max}}} = K_1 \left( \sigma'_{A_{\text{nom}}} \pm \dot{\sigma}_{A_{\text{nom}}} \right) - K_2 \left( \sigma'_{S_{\text{nom}}} \pm \dot{\sigma}_{S_{\text{nom}}} \right) \quad \ldots \quad (32)$$

From which, for the 'tension' side of the assembly,

$$\dot{\sigma}_{A_{\text{max}}} = K_1 \left( \sigma'_{A_{\text{nom}}} + \dot{\sigma}_{A_{\text{nom}}} \right) + K_2 \left( \sigma'_{S_{\text{nom}}} - \dot{\sigma}_{S_{\text{nom}}} \right)$$

or

$$\frac{\dot{\sigma}_{A_{\text{max}}}}{\sigma'_{A_{\text{nom}}}} = K_1 \left( 1 + \frac{\dot{\sigma}_{A_{\text{nom}}}}{\sigma'_{A_{\text{nom}}}} \right) + K_2 \left( \frac{\sigma'_{S_{\text{nom}}} - \dot{\sigma}_{S_{\text{nom}}}}{\sigma'_{A_{\text{nom}}}} \right)$$

In terms of the roll assembly's geometry,

$$\frac{\dot{\sigma}_{A_{\text{max}}}}{\sigma'_{A_{\text{nom}}}} = K_1 \left( 1 + \frac{8.M(b^2 - a^2)b}{\text{WR} \left( c_e^4 - a^4 \right)} \right) + K_2 \left( \frac{(b^2 - a^2)}{(c_e^2 - b^2)} - \frac{8.M(b^2 - a^2) c_e}{\text{WR} \left( c_e^4 - a^4 \right)} \right) \quad (33)$$
Similarly, for the 'compression' side of the assembly,

\[ \frac{\sigma_{AC_{\text{max}}}}{\sigma'_{A_{\text{nom}}}} = K_1 \left( 1 - \frac{8.M(b^2 - a^2)b}{W_R (c_e^4 - a^4)} \right) + K_2 \left( \frac{b^2 - a^2}{c_e^2 - b^2} + \frac{8.M(b^2 - a^2)c_e}{W_R (c_e^4 - a^4)} \right) \]  

(34)

The SCF's K1 and K2 can be obtained for any arbor location by substituting measured values of \( \frac{\sigma_{At_{\text{max}}}}{\sigma'_{A_{\text{nom}}}} \) and \( \frac{\sigma_{AC_{\text{max}}}}{\sigma'_{A_{\text{nom}}}} \), Figure (27), in equations (33 and 34) and solving simultaneously. Table 5 includes values of K1 and K2 for arbor Sections 1 and 3 together with a comparison of their respective maximum stresses in the residual load model, derived in accordance with equation (31) \( \frac{\sigma'_{A_{\text{Th}}}}{\sigma'_{A_{\text{nom}}}} \) and measured directly from the model \( \frac{\sigma'_{A_{\text{max}}}}{\sigma'_{A_{\text{nom}}}} \).

**TABLE 5 - SCF's Due to Passing Load and Local Bending**

<table>
<thead>
<tr>
<th>Arbor Section</th>
<th>( \frac{\sigma_{At_{\text{max}}}}{\sigma'<em>{A</em>{\text{nom}}}} )</th>
<th>( \frac{\sigma_{AC_{\text{max}}}}{\sigma'<em>{A</em>{\text{nom}}}} )</th>
<th>K1</th>
<th>K2</th>
<th>( \frac{\sigma'<em>{A</em>{\text{Th}}}}{\sigma'<em>{A</em>{\text{nom}}}} )</th>
<th>( \frac{\sigma'<em>{A</em>{\text{max}}}}{\sigma'<em>{A</em>{\text{nom}}}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.77</td>
<td>4.44</td>
<td>2.67</td>
<td>1.48</td>
<td>4.62</td>
<td>4.60</td>
</tr>
<tr>
<td>3</td>
<td>2.97</td>
<td>2.79</td>
<td>2.42</td>
<td>0.35</td>
<td>2.82</td>
<td>2.60</td>
</tr>
</tbody>
</table>

Similar measurements were made at arbor Sections 5 and 6, but the results were rather inconclusive due to the small bending moment associated with these locations. The error between derived and measured values of stress concentration for the residual load model at Section 1 is negligible, whilst that at Section 3 is about 8%.
Fig. 27 Critical stresses in assembled roll under combined residual loading and transverse bending.
It is apparent from the results tabulated in Figure (27) that the SCF for any arbor location is not linearly distributed about the axis of bending, as was initially anticipated in equation (29). This also serves to confirm the important influence of local bending effects, rather than overall bending moment, on the magnitude of critical fillet stresses in the arbor.

3.3.4 Stress Concentrations in Sleeves

In the residual load test it was suggested that the maximum sleeve stress could be conveniently expressed in terms of a nominal stress and a geometric SCF so that:

\[ \sigma_{s_{\text{max}}} = \sigma' s_{\text{nom}} \times KR \]

where \( \sigma' s_{\text{nom}} \) = Nominal compressive stress in sleeves

\[ = \frac{WR.4}{\pi(c_e^2 - b^2)} \]

and \( KR = \text{SCF associated with the geometry and location of each sleeve under residual loading.} \)

If, under combined loading conditions, the transverse bending contribution is treated in a similar manner to that initially adopted in the arbor analysis, then the maximum sleeve stress \( \dot{\sigma}_{s_{\text{max}}} \) may be written as:

\[ \dot{\sigma}_{s_{\text{max}}} = \sigma' s_{\text{nom}} \times KR \pm \dot{\sigma}_{\text{nom}} \times KB \]

\[ \text{............ (35)} \]
where \( \dot{\sigma}_{\text{nom}} \) = Nominal bending stress in sleeves due to \( P \)
\[
\dot{\sigma}_{\text{nom}} = \pm \frac{32 \cdot M \cdot c_\theta}{\pi(c_e^4 - a^4)}
\]

and \( KB = \) SCF associated with the geometry and location of each sleeve under transverse bending.

It follows that, the maximum sleeve stress on the 'tension' side of the assembly \( \dot{\sigma}_{\text{max}} \) is given by:

\[
\dot{\sigma}_{\text{max}} = \sigma'_{\text{nom}} \times KR + \dot{\sigma}_{\text{nom}} \times KB \quad \cdots \cdots \cdots (36)
\]

Similarly, for the 'compression' side,

\[
\dot{\sigma}_{\text{c max}} = \sigma'_{\text{nom}} \times KR - \dot{\sigma}_{\text{nom}} \times KB \quad \cdots \cdots \cdots (37)
\]

The maximum sleeve stresses, \( \dot{\sigma}_{\text{max}} \) and \( \dot{\sigma}_{\text{c max}} \) are obtained directly from fringe measurements in the model sleeves, Figure (27). The nominal sleeve stresses, \( \sigma'_{\text{nom}} \) and \( \dot{\sigma}_{\text{nom}} \), are conveniently calculated from simple theory.

Therefore, KR and KB can be determined for each sleeve from the simultaneous solution of equations (36 and 37). The results so obtained assume simple beam behaviour of the roll assembly. This can be verified by comparing the derived values of KR with those measured directly in the residual load test.
Referring to Figure (27), it is apparent that only sleeve S3 is unaffected by some localised perturbation in the loaded assembly. Sleeve S1 is affected by local load diffusion in the arbor shoulder vicinity, whilst the 'compression' side of sleeve S2 is affected by directed contact with the loading rod.

**Sleeve S3:**

From the substitution of measured fillet stresses, ($\sigma_{\text{max}}$ and $\sigma_{\text{sc max}}$) and calculated nominal stresses, ($\sigma'_{\text{nom}}$ and $\sigma_{\text{nom}}$) in equations (36 and 37), KR and KB were found to be 1.35 and 2.3 respectively. The result for KR is some 10% less than that obtained directly from the residual load test. This is considered an acceptable error for fringe measurement and data manipulation of the analysis procedure.

However, for design purposes, the directly measured value of KR = 1.5 is considered more appropriate.

It follows that, the maximum fillet stress in the sleeve groove is given by:

$$\sigma_{\text{max}} = 1.5 \times \sigma'_{\text{nom}} \pm 2.3 \times \sigma_{\text{nom}}$$

The constants, KR = 1.5 and KB = 2.3, are valid for all similar sleeves which are not affected by direct contact loading or local load diffusion. This assertion was verified by substituting for KR = 1.5 and KB = 2.3 in an equivalent expression for maximum sleeve stress on the
'tension' side of sleeve S2. (Apart from sleeve S3, this is the only sleeve groove location not affected by additional localised loading). The resultant value for maximum sleeve stress was found to be within 8% of the directly measured value included in Figure (27).

**Sleeve S2:**

The maximum sleeve stress on the 'tension' side of the assembly is given by equation (38).

However, for the 'compression' side of sleeve S2 the measured fillet stress at the groove root (assumed position of rolling load application) was found to be about 90% less than that given by equation (38). Further examination of the contact conditions in this vicinity revealed the actual location of load application to be on either side of the groove, as shown by the concentration of isochromatic fringes in Figure (25). This can be attributed to lateral deformation of the loading rod, which effectively superimposed an additional axial tensile stress on the sleeve.

For the purposes of analysis, it is assumed that the above tensile stress is dependent on the magnitude of rolling load \( P \) and sleeve geometry, rather than the overall bending moment or residual clamping load.

It seems reasonable, therefore, to express the rolling load contribution \( \sigma_{R_{\text{max}}} \) to total sleeve stress in the following manner:

\[
\sigma_{R_{\text{max}}} = \sigma_{R_{\text{nom}}} \times K_P
\]

............... (39)
where $\sigma_{R_{\text{nom}}}$ = Nominal, transverse compressive stress on sleeve due to $P$
\[ \sigma_{R_{\text{nom}}} = \frac{P}{W(c_e - b)} \]

and

$K_p$ = SCF associated with the lateral deformation of the loading bar acting on the sleeve.

By substituting for $\sigma_{R_{\text{max}}}$, measured directly from the model, and $\sigma_{R_{\text{nom}}}, K_p$ was found to be about 3.1.

It follows that, the maximum fillet stress on the 'compression' side of sleeve S2 is given by:

\[ \dot{s}_{c_{\text{max}}} = 1.5 \times \sigma'_{s_{\text{nom}}} - 2.3 \times \dot{s}_{s_{\text{nom}}} - 3.1 \times \sigma_{R_{\text{nom}}} \quad \ldots \ldots \quad (40) \]

**Sleeve S1:**

Using the procedure described for sleeve S3, KR and KB were found to be 1.65 and 2.5 respectively. The increased maximum fillet stress associated with this particular sleeve is again attributed to the local load diffusion at the sleeve arbor interface (see comments in Section 3.2.4).

The above result for KR is some 11% less than that obtained from direct measurement in the residual load model, although the latter value (KR = 1.8) is considered the more appropriate for design purposes.

It follows that, for sleeves adjacent to the arbor shoulder, the maximum fillet stress in the sleeve groove is given by:

\[ \dot{s}_{s_{\text{max}}} = 1.8 \times \sigma'_{s_{\text{nom}}} \pm 2.5 \times \dot{s}_{s_{\text{nom}}} \quad \ldots \ldots \quad (41) \]
3.2.5 Stress Distributions

Figure (28) shows the distribution of principal stresses $\sigma_{zz}$, $\sigma_{\phi\phi}$ and $\sigma_{rr}$ on the 'compression' side of the assembled roll under combined residual loading and transverse bending. All stresses are expressed as multiples of the nominal residual stress in either the sleeve or arbor. This approach allows direct comparison with the previous model test.

The stress distributions for the 'tension' side of the arbor and sleeves S1 and S2 are not included since they are not significantly different from those shown in Figure (23) for the residual loading case.

As with the previous model, the hoop and radial stresses are never critical, and again largely reflect the rate of change of axial stress.

The effect of superimposing the compressive stress component of transverse bending on the tensioned arbor is most apparent at Section 1, where the critical stress is reduced by about 25%.

Lateral deformation of the loading bar causes a significant redistribution of stress in the central sleeve, S2. The distribution of axial stress reaches a maximum (about $1.8 \times \sigma'_{s_{nom}}$) just below the groove root, and then rapidly reduces to almost zero stress at the root. The distribution of hoop stress is similar, but for this particular case, is more critical than the axial stress, reaching a peak value of about $2.7 \sigma'_{s_{nom}}$. This condition of internal stress is not apparent in the tabulation of SCF's shown in Figure (27).
**Fig. 28** Stress distributions in assembled roll under combined residual loading and transverse bending — Compression side only
The increase in radial stress at the sleeve/arbor interface at Section 2, also indicates direct load transference from the central sleeve to the arbor section directly beneath it. This result is consistent with the Isoclinics and stress trajectories shown in Figure (26).

Also included in Figure (28) is the distribution of theoretical bending stress at the position of maximum bending moment $\frac{M}{Z}$ for the sleeve and arbor, $\sigma_s$ and $\sigma_A$ respectively. A comparison of these results shows that, whilst the overall behaviour of a composite roll is analogous to a simple elastic beam, individual stress distributions and maximum stress values are largely dependent on local contact and load diffusion effects. This is particularly apparent for the central sleeve, where the theoretical distribution of bending stress seriously misrepresents the actual stress conditions. Similar comparisons at locations nearer to the roll supports were found to be inconclusive due to the very small contribution of bending stress to total axial stress.
CHAPTER 4

DISCUSSION

The first part of this chapter (Section 4.1) contains a general discussion of the results from the photoelastic tests, and their implications to the design of prototype composite rolls and auxiliary apparatus.

In Section 4.2, the simple analytical model of an idealised composite roll is discussed in relation to results from the photelastic tests. The model accuracy and limitations are described, with recommendations for further experimental verification on a full size prototype.

In Section 4.3, results from a finite-element idealisation of a pre-tensioned arbor are compared with those obtained from the corresponding photelastic test. The relative merits of the two methods are also briefly discussed.

Critical stresses are particularly significant in the proposed composite roll design and have therefore been removed from the general discussion on photelastic tests. Moreover, some aspects of the analysis associated with stress concentration factors are shown to have a more generalised application in the presentation of fillet design data.

Finally, in Section 4.5, the relationship between present experimental work to the establishment of an appropriate design procedure is discussed.
The actual design procedure is complex and is therefore given in Appendix 4 (page 146). For completeness, an example is given which relates to the Lackenby composite roll prototype. The design of this prototype is not discussed in any detail.

4.1 Photoelastic Models

The photoelastic tests were designed to assess the overall validity of the simple analytical model and to provide general quantitative information regarding critical stresses in the roll arbor and sleeves.

It was appreciated that critical stresses in the roll assembly would be, to a large extent, dependent on the relative fit between the mating components. Moreover, the photoelastic tests were required to given an ideal solution for comparison with the simple analytical model. It was therefore necessary to ensure that the models were manufactured to at least the same order of accuracy and complexity as in the prototype.

Subsequent analysis of the model assembly has demonstrated that, in terms of the symmetry of fringe patterns and residual load distribution, this has been achieved.

First model test:-- Arbor pre-tensioning

Arbor: The simple analytical model has shown that the amount of arbor pre-tension required to facilitate sleeve assembly is in the order of 3 times that required to develop sleeve clamping (page 25). Therefore, to avoid overloading, the arbor shank should deform uniformly along its length.
Any localised resistance to radial contraction will necessitate a proportionate increase in arbor pre-tension to facilitate sleeve assembly over that region of the arbor. This is particularly critical in a prototype where, for example, 1000 KN of arbor pre-tension is required to obtain 0.01 mm of diametral contraction on a 210 mm diameter shank.

Uniform deformation of the arbor shank also implies equal radial clamping and optimum concentricity for the sleeves when finally assembled. This is considered expedient in a prototype since any excess radial clamping on one sleeve may result in insufficient radial clamping on the adjacent sleeve. It is realised that the condition for unqualified equal radial clamping is dependent on the maintenance of a very precise machining tolerance on the sleeve bore (expected to be no greater than ± 0.006 mm).

Previous experimental work on a simple model arbor\(^{(20)}\) showed that apart from a localised region (about 40% of shank length) mid-way along the arbor shank, the arbor deformation was extremely non-uniform. More importantly, radial contraction in the immediate vicinity of the arbor shoulder was found to be negligible.

Similar measurements of arbor deformation on the photoelastic model revealed that resistance to radial contraction is considerably reduced by the introduction of an undercut fillet radius between the shank and shoulder. Uniform radial contraction was obtained over 80% of the shank length and resistance to contraction was restricted to a localised region next to the shoulder which amounted to only 10% of the shank length. Minimum contraction was almost 70% of the nominal value measured on the uniform part of the shank.
It is apparent that the minimum contraction in shank diameter would not become worse with increasing shank length and, accordingly, a uniform contraction would be obtained along a greater proportion of the shank.

**Collet-shell assembly:** As an alternative to a one-piece threaded or bayonet-type connection, it was decided to examine a split collet-shell arrangement in the photoelastic tests. (The collet-shell segments are supported by retaining sleeves at each end of the assembly as shown in Figure (6), page 36). A particular advantage of this design is that a large range of smaller diameter arbors can be accommodated on a single pre-loading rig by introducing spacer sleeves between the collet and arbor shoulder.

The photoelastic tests showed that the total applied load $W_B$ was shared equally between the 3 collet segments. It was further demonstrated that each segment was subjected to a uniform bending moment caused by an effective offset at its extreme ends. In a simple idealisation the segments were treated as equivalent two-dimensional beams, and comparisons with fringe measurements made in the model showed that this would be sufficiently representative of the actual structure providing component deformation did not become too large.

The maximum stress in the collet-shell is about $3 \times$ nominal axial tension, and occurs in the reduced section at either end of the assembly. Radial expansion of the collet-shell at its extremities is shown to match the radial expansion of the retaining sleeves. Thus, the sleeve hoop stress can be expressed in terms of the total applied load, the effective eccentricity
of applied load, the deformation of the collet-shell and the assemblies overall geometry. Sleeve stress is shown to be in accordance with the predictions for Lamé-type thick cylinders, modified with a suitable stress concentration factor to reflect the non-uniform pressure distribution around the circumference and along the length of each sleeve. The total radial force $S$ on each retaining sleeve is shown to be no more than 10% of the total applied load.

Second model test:-- Residual loading

The method of sleeve clamping and, therefore, the distribution of residual load throughout the roll assembly, was directly analogous to that employed in a prototype. However to facilitate fringe examination and avoid errors associated with thread tolerance and drunkeness, the threaded connection between the retaining nut and arbor was replaced with a cemented joint. This procedure did not appear to alter the relative component stiffnesses.

An original prediction of residual load was found to be 40% greater than that measured in the model assembly. This theoretical derivation assumed that the sleeves could be treated as plain cylinders. However, fringe observations at the sleeve interfaces gave a clear indication that the presence of an annular groove gives rise to a substantial variation in axial clamping pressure. Thus, the actual sleeve stiffness is considerably less than that assumed in the simple analytical model.

The above result suggests that the effective external diameter of the sleeve is no greater than the minimum groove diameter. If this value is substituted into the expression for residual load calculation the theoretical result is found to be within 8% of the measured value.
The fringe patterns also indicated that the transfer of residual load through the sleeves was entirely due to axial and radial pressure, with insignificant shear stress at the sleeve/arbor interfaces. This confirms the assumption of negligible friction effects made in the analytical model. Equal fringe values at the bore of each sleeve shows that uniform radial clamping was obtained in the model assembly.

Third model test: Transverse bending of assembled roll

The roll assembly was simply-supported and subjected to transverse bending at mid-span. The bending load was applied through an araldite rod located in the central sleeve groove. The proportion of bending load to residual load was about $3 \times$ that estimated for a prototype. This was done to ensure a pessimistic design appraisal in the model test. The overload conditions also served to emphasise the relative bending and residual load contributions to the observed maximum stress. As an additional precaution, the roll assembly was immersed in oil during the stress freezing cycle, thus avoiding any spurious effects caused by the model's self weight.

Due to the asymmetry of loading, it was necessary to test two identically loaded models, from which longitudinal and hoop slices could be obtained at the precise position of bending load application.

Consideration of the stress distributions at selected locations in the sleeves and arbor confirm that the roll assembly behaved as a monolithic beam, with no visible sleeve separation. Theoretical calculation of total load redistribution based on the effective sleeve diameter, was within 10% of the measured values, again confirming that the simple idealisation is satisfactory.
Fringe observations on the 'tension' side of the assembly indicated that very little interface pressure existed between the sleeves. This would tend to suggest that, in the model tests, the applied loading was close to its limiting value for monolithic behaviour. According to simple theory, a 20% increase in bending load would have resulted in axial separation of the sleeves.

4.2 Scope of Simple Analytical Model

The analytical model describes the behaviour of composite rolls in terms of simple elastic theory for the purposes of estimating arbor pre-tension and sleeve clamping requirements. The roll arbor is treated as a uniform axi-symmetric body, and the sleeves as plain cylinders. This approach facilitates the derivation of a simple theoretical relationship between pre-tension and residual loading in terms of arbor and sleeve deformation compatibility. It also allows the roll assembly to be treated as an equivalent homogeneous beam when considering limiting values of rolling load for monolithic behaviour.

Particular attention was paid to the interdependence of sleeve and arbor deformation since it is this which determines the assembly sequence and how the residual load is finally distributed in the form of axial and radial clamping. For example, it is shown that once the retaining nut is tightened against the assembled sleeves, the interference condition between the sleeves and arbor will remain virtually unchanged during the development of axial clamping. Thus, it is necessary to clamp the sleeves radially prior to axial clamping. This is an important feature of the proposed
composite roll arrangement since it shows the way in which arbor deformation can be accurately controlled to give any desired combination of radial and axial clamping.

Most composite roll applications involve the use of sleeve materials having a high elastic modulus and a low coefficient of thermal expansion. Sleeves of differing geometry and material properties may also be combined on the same arbor. To accommodate this range of sleeve assemblies it was convenient to express the overall elastic modulus and thermal coefficient of the sleeves in terms of the aggregate effect of individual sleeves and sleeve supports.

Model accuracy:

The most significant factor associated with model accuracy is related to the accommodation of machining tolerances for the arbor shank and sleeve bores. This is because the proportion of arbor diametral contraction to accommodate machining tolerances is normally in the order of twice that required to provide minimum sleeve clamping. In practice, this can result in axial sleeve clamping being up to 60% greater than the minimum required value as given by the analytical model.

Model limitations:

The model does not include a transient thermal analysis, nor is it applicable to cantilevered composite rolls.
By assuming steady-state thermal conditions it is possible to estimate, and compensate for, potential losses in axial clamping due to differential expansion between the sleeves and arbor.

However, without experimental data on temperature distributions, it is not possible to evaluate the changes in radial clamping which will occur at various stages during rolling. This may be critical for extremes of temperature difference which could result in the development of large hoop stresses in the sleeve bores.

To facilitate a realistic transient thermal analysis it is recommended that on-line measurement of temperature distribution be made during the rolling trials of a prototype (see Section 4.5, page 119).

The case for cantilevered composite rolls is sufficiently different to warrant a separate investigation.

Finally, the simple analytical model takes no account of interface pressure distributions which have an important bearing on the arbor's resistance to fretting fatigue. Nor does it consider limiting factors such as critical stress concentrations or load contact/impact conditions.

However, photoelastic tests have shown that the model is sufficiently accurate for the purposes of predicting arbor pre-tension and sleeve clamping requirements for the criteria of monolithic behaviour. The effect of sleeve geometry can be accounted for by consideration of groove size and arbor dimensions. Moreover, the model is readily converted into a programmable format, thus making it a suitable vehicle for the more comprehensive design procedure discussed in Section 4.5 (page 119).
4.3 Finite-element Idealisation of a Pre-tensioned Arbor

During the initial stages of this project the possibility of using finite-element methods for the overall analysis of composite rolls was investigated\(^{(24)}\). Preliminary discussions with the program designers revealed that the finite-element approach would not be suitable for the asymmetric loading case of a roll under transverse bending. This was mainly attributed to the necessity for a three-dimensional mesh design which if adequately refined at the model boundaries, would require too many elements for the available core space (IBM 370/85 computer).

The case of axi-symmetric residual loading was attempted, but finally abandoned, due to the uncertainty associated with model boundary idealisation and pressure distributions at the component interfaces.

However, it was decided to analyse a pre-tensioned arbor, proportionately similar to that studied in the photoelastic test. A comparison of the two methods could show whether it was possible to use the numerical technique for the analysis of future arbor designs. Obviously, this would only be necessary if the proposed design was significantly different from that already studied in the photoelastic tests. Nevertheless, advocates of the finite-element technique strongly suggested that a design appraisal could be conducted with greater speed and economy.

The mesh design, load input idealisation, and results from the finite-element analysis are given in Appendix 3 (page 140).
Figure (8) (page 41) shows a comparison of diametral contraction along the pre-tensioned arbor shank. The maximum difference between the results is almost 20%, but average deviation from the nominal contraction, $U_{nom}$, is about the same. The error in $U_{nom}$ is approximately ± 5%.

Figure (A4) (page 145) shows separate stress distributions through the arbor shank and averaged values of tangential stress concentration at critical locations around the arbor boundaries. With the exception of Section 3, arbor bore termination, there appears to be no significant difference (3-10%) between these results and those shown in Figures (10 and 11) (pages 44 and 46) from the photoelastic test. No sensible comparison of results at the bore termination is possible since the geometry of fillet radius is different for both models. However, later photoelastic tests with arbors having comparable fillet designs to that incorporated in the finite-element model have given similar results for critical stress.

Therefore, on the basis of the above observations, there appears to be no reason why the finite-element method cannot be used for the analysis of future composite roll arbors. The order of accuracy associated with both methods is about the same, being related to user expertise.

The main advantage usually attributed to the finite element method is speed of analysis and, therefore, cost of analysis. It was the author's experience, however, that the time required for mesh design, data presentation and correction, is comparable to that required for model manufacture and stress freezing in photoelasticity. The time required for interpretation of results is also comparable, although the method of stress separation in photoelasticity was found to be rather tedious.
Nevertheless, although the two methods are undoubtedly complementary, the author is reluctant to recommend finite-element techniques for the evaluation of critical stress information on structures other than those with axi-symmetric loading or well-defined boundary conditions. For example, it is unlikely that the separate photoelastic design study of bore termination geometries (Chapter 3, page 47) could have been conducted with comparable ease. It is intended, however, to use the finite-element technique for the analysis of transient thermal conditions in a composite roll when sufficient temperature distribution data is available.

4.4 Critical stresses

Table 5 shows a summary of critical stress concentration factors measured in the pre-tensioned arbor and assembled roll.

<table>
<thead>
<tr>
<th>LOADING CASE</th>
<th>ARBOR Location</th>
<th>SCF</th>
<th>Location</th>
<th>SCF</th>
</tr>
</thead>
<tbody>
<tr>
<td>Arbor pre-tensioning</td>
<td>1</td>
<td>2.7</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$\frac{A_{\text{nom}}}{	ext{WR}} = \frac{WB}{AA}$</td>
<td>3 (original)</td>
<td>14.0</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>3 (modified)</td>
<td>4.5</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>1.4</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Residual loading*</td>
<td>1</td>
<td>4.6</td>
<td>S1</td>
<td>1.8</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>2.6</td>
<td>S2</td>
<td>1.5</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>2.9</td>
<td>S3</td>
<td>1.5</td>
</tr>
<tr>
<td>Combined Residual loading and</td>
<td>1</td>
<td>4.5</td>
<td>S1</td>
<td>2.1</td>
</tr>
<tr>
<td>Transverse bending*</td>
<td>3</td>
<td>4.8</td>
<td>S2</td>
<td>0.8</td>
</tr>
<tr>
<td></td>
<td>4.2</td>
<td>0.7</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>3.0</td>
<td>1.9</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>3.3</td>
<td>0.7</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* $\sigma'_{\text{nom}} = \frac{\text{WR}}{\text{AA}}$  $\sigma'_{\text{s nom}} = -\frac{\text{WR}}{\text{As}}$
Arbor pre-tensioning

This is the most critical static loading condition due to the high nominal stress and method of load application via a small diameter thrust rod. The severity of contact loading at the arbor bore termination (location 3) was particularly critical in the original design, where an elastic SCF of 14 was measured. For an equivalent steel prototype possessing this geometry and a nominal stress of 0.23 KN/mm², material in the vicinity of the bore termination would undoubtedly suffer localised plastic yielding.

Although the first load application would effectively reduce the fillet SCF, subsequent loading would also cause repeated, and cumulative, plastic deformation. In such cases, if the applied loading continues to be in excess of the material's shakedown limit (assumed = 2 x yield strength\(^{(25)}\)) the arbor would suffer incremental collapse, leading to low cycle fatigue failure.

The modified arbor designs shown in Figure (12) (page 48) incorporated undercut fillet radii, which resulted in the critical fillet stress being reduced to about \(4.5 \times \sigma_{\text{nom}}\). This improvement is mainly attributed to the effective separation of the SCF associated with the bore fillet and the SCF due to contact loading. A critical stress of this magnitude is well below the shakedown limit for a prototype arbor material (Table 6, page 156).
The only disadvantage associated with the use of an undercut fillet radius at the bore termination is that, for very long arbors, it may present some difficult machining problems. For this reason, further model tests with simpler fillet geometries are at present being investigated.

**Assembled Roll**

**Arbor:** Under the action of residual loading the measured SCF values at the arbor shoulder (location 1) and thread start (location 6) are, respectively, 70% and 107% greater than the corresponding values measured in the pre-tensioned arbor. The actual stress value is, however, much less in the residually loaded arbor due to the large difference in nominal stress (about 1:10).

The increase in SCF values at arbor locations 1 and 6 reflects the effect of the sleeves acting on the arbor supports, causing a localised bending stress to be superimposed on the nominal fillet stress. Table 5 shows that the effect of superimposing transverse bending is marginal when compared with the mean stress levels developed during assembly. An alternating stress condition of comparable magnitude in a prototype roll is not considered a critical high cycle fatigue problem (see case study, Appendix 4, page 158).

The non-linear distribution of SCF values measured in the combined load model was initially interpreted as evidence of non-monolithic behaviour for the roll under bending. This apparent anomaly was eventually resolved by expressing the arbor fillet stress in terms of local load diffusion effects, rather than the overall contribution from transverse bending.
Thus, the critical stress at locations such as the arbor shoulder fillet was expressed in terms of the nominal stress in both the arbor and sleeves (equation 30, page 92). The corresponding SCF's were obtained from fringe measurements made on the 'tension' and 'compression' sides of the roll assembly. Substitution of these SCF values in a similar expression for residual loading (equation 31, page 93) verified their specific relevance to localised loading effects rather than particular values of arbor tension or transverse bending.

It is apparent that this type of stress concentration will occur in connection with a wide variety of loaded structures, such as loaded flanges and pre-strained bolts. It seems therefore that the procedure outlined here could find more general application in the presentation of fillet design data (26).

**Sleeves:** Sleeve stresses are not critical, and generally reflect the geometric stress concentration associated with the annular groove.

The expression for critical fillet stress in the sleeve subjected to direct contact with the loading bar is arbitrarily based on the sleeve width W and transverse bending load P (equation 40, page 100).

A more comprehensive procedure for sleeve analysis would require further work on tests specifically designed to isolate, and quantify, parameters affecting sleeve stress and sleeve support. For example, it is apparent from the photoelastic tests that sleeve stress and sleeve flexing could be reduced by reducing the height of sleeve support, (arbor shoulder and retaining nut diameter). However, except for very shallow
grooves, it is not clear what proportion of sleeve support can be sacrificed before the sleeve becomes unstable. This is particularly critical for sleeve designs required for the rolling of complex, sharp angled, sections.

4.5 **Prototype Design Procedure**

The photoelastic tests have shown the simple analytical model to be adequate for the purposes of estimating arbor pre-tension and sleeve clamping requirements to ensure monolithic behaviour of a composite roll during rolling. The model is readily modified to include limiting factors, related to permitted stress levels, determined from SCF and material strength data.

However, it is also desirable to assess each composite roll with respect to 3 optimum values for the arbor shank/sleeve bore diameter ($b$). The criteria for optimisation are:

(i) **Most Economic Sleeve Design** - $b \neq b_{\text{max}}$

Wear-resistant sleeve materials such as tungsten carbide are very expensive, and their cost is volume related.

(ii) **Maximum Sleeve Support** - $b = b_{\text{min}}$

For some applications, the existing roll and bearing dimensions may be such that a composite roll replacement is only feasible if the sleeves are given maximum support.

(iii) **Minimum Arbor Pre-tension** - $b_{\text{max}} \geq b \geq b_{\text{min}}$

Arbor pre-tension, for sleeve assembly, represents the most critical static stress condition in a composite roll. The
feasibility of some potential applications may, therefore
depend on a minimum arbor pre-tension requirement.

This optimisation procedure obviously requires a large number
of iterative calculations over the range, \( b = b_{\text{max}} - b = b_{\text{min}} \). Consequently,
a computer program has been written which determines the optimum 'b'
diameter together with related arbor pre-tension and sleeve clamping
requirements. A description of this program is given in Appendix 4 (page 196).
The program is sufficiently flexible to allow the user to decide which
of the available criteria is most suitable for any particular application.

The program includes an estimate of arbor fatigue strength, the
derivation of actual sleeve bore/arbor shank diameters (\( b_{\text{s,act}} \) and \( b_{A,\text{act}} \)
respectively) and is relevant to a large range of simply-supported composite
rolls involving single, twin, or four-strand rolling.

Example design - Lackenby composite roll prototype

The program was used to assess the feasibility and optimum dimensions
for a proposed composite roll prototype to be installed in the No.2 Rod
Mill, BSC, Lackenby works. The relatively small difference between roll
neck (ND) and barrel discard diameter (DD) necessitated maximum sleeve
support (\( b = b_{\text{min}} \)). This also coincided with a minimum arbor pre-tension
requirement.

Figure (A6), (page 154) shows the arrangement for the first trial.
It consists of two double-pass and six single-pass steel sleeves which
comply with the existing pitch requirement for twin-strand rolling. It is
intended to use this configuration as a vehicle for suitable instrumentation to obtain on-line quantitative information on shock loading and temperature distributions.

Subsequent trials in this mill will involve the use of sleeve materials such as tungsten carbide and Ni-hard.

A cost/benefit appraisal of such an arrangement has been conducted and shown to be appreciable\(^\text{(27)}\). The benefits are based on established wear-rates which, for carbide, are an order of magnitude less than that associated with conventional work roll materials.
CONCLUSIONS

The photoelastic model tests have shown that, whilst the proposed composite roll design is not unduly complicated, a considerable amount of information is required to fully describe its behaviour.

The technique of pre-tensioning an arbor to provide controlled axial and radial sleeve clamping is shown to be feasible and capable of preventing sleeve separation under simulated rolling load. This is an important feature of the design, which is not available in known alternative arrangements (8-11).

The simple analytical model is shown to be adequate for the purposes of estimating arbor pre-tension and sleeve clamping requirements. However, the original treatment of sleeves as plain cylinders was found to overestimate sleeve clamping efficiency by 40%. Critical stress information from the photoelastic tests is conveniently combined with the analytical model to give a basic design procedure for simply-supported composite rolls. This has been written into a computer program which enables the feasibility of a design proposal to be rapidly assessed in terms of mechanical stress levels and minimum sleeve dimensions.

The original pre-tensioned arbor was found to be severely overstressed in the vicinity of the bore termination where an SCF of 14 was measured. Further tests with alternative designs demonstrated that the critical stress could be reduced to a safe level with an SCF = 4.5.
by introducing an undercut fillet radius. The arbor deformation characteristics were also improved by using this type of fillet at the arbor shoulder.

Critical stress information for sleeve design is not as generalised as that obtained for the arbor, although the photoelastic tests did indicate a preferred manner of sleeve support. A comprehensive procedure for sleeve design would require a separate investigation, especially if composite rolls are to be used for the rolling of complicated or sharp angled sections.

Recommendations arising from the photoelastic tests have been included in the design of a composite roll prototype, to be installed in the No. 2 Rod Mill, BSC, Lackenby works. It is also recommended that, during the initial trials of this prototype, provision be made to measure on-line conditions of rolling load and temperature distributions through the assembly. This could be achieved by embedding suitable pressure transducers and thermocouples in the sleeves and arbor. The additional results from this work could enhance the reliability of the analytical model, and would be complimentary to the photoelastic design information described in this thesis.
REFERENCES


24. Hellen, T. 1972, Jan-May. Private communications to discuss the use of Finite-Element method for the design analysis of composite rolls.


32. ESDU No. 71010, 1973, Sept. "Design against fatigue, basic design calculations".

1.1 Isoclinics and Stress Trajectories

If the photoelastic model is examined in plane-polarised light, and if at any point the direction of one of the principal stresses coincides with the direction of polarisation of the incident light, that point will appear dark when viewed through the analyser. The locus of points at which principal stress direction coincides with the direction of polarisation appears as a dark line across the model. Such lines are known as Isoclinic lines. The angle $\theta$ which the direction of the principal stress makes with a reference axis is the parameter of the isoclinic. In a field of non-uniform stress, the directions of the principal stresses vary from point to point. If a curve is drawn in such a way that the direction of one of the principal stress is tangential to it at every point along its length, then such a curve is termed a Line of Principal Stress or Stress Trajectory. The stress trajectories are, therefore, drawn directly from the isoclinic lines which are projected from the polarising bench on to a screen. An inspection of the stress trajectories gives a valuable impression of the stress distribution in the model. Stress concentrations are clearly indicated by a crowding together of the trajectories, whilst a change in stress sign is recognised by an abrupt cross-over of the P and Q trajectories.

1.2 Stress Separations

In order to obtain complete stress distributions across a section in a slice cut from a photoelastic model it is necessary to measure both the isoclinic angle ($\alpha$) and the isochromatic fringe value ($N$) at various
positions across that section. The procedure for partial fringe determination was in accordance with the Sénarmont method, which is accurate to within ±0.01 fringes.

Referring to Figure (A1), for axial symmetry, the differential equation of equilibrium in polar co-ordinates reduces to:

\[ \frac{\partial \sigma_{rr}}{\partial r} + \frac{\sigma_{rr} - \sigma_{\phi\phi}}{r} + \frac{\partial \sigma_{\phi z}}{\partial z} = 0 \] ............ (a.1)

Neglecting body forces.

The following procedure is a graphical integration technique for the solution of equation (a.1) in accordance with the Tésar method of stress separation (28).

1. To determine \( \frac{\partial \sigma_{rz}}{\partial z} \):

from Mohr's circle, for a system of three-dimensional stress, it can be shown that:

\[ \sigma_{rz} = \frac{1}{2} (P - Q) \sin 2\alpha \]

\[ \because \quad \sigma_{rz} = \frac{N \rho}{t} \sin 2\alpha \] ............ (a.2)

where \( N \) and \( \alpha \) are measured in the \( r-z \) plane and \( \alpha \) is measured anti-clockwise from \( Z \) to \( r \).

Partially differentiating equation (a.2) with respect to \( Z \),

\[ \frac{\partial \sigma_{rz}}{\partial z} = \frac{f}{2t} \left[ \sin 2\alpha \frac{\partial N}{\partial Z} + 2 \cos 2\alpha, N \cdot \frac{\partial \alpha}{\partial Z} \right] \] ............ (a.3)
Fig. A1 Notation for photoelastic analysis
Photelastic observations give values for the slopes $\partial N/\partial Z$ and $\partial a/\partial Z$ at Mid-line which, together with the material fringe value $f$ and slice thickness $t$, enables equation (a.3) to be solved at various radii through the section.

2. To determine $\partial r - \sigma \phi$:

The stress difference $(\partial r - \sigma \phi)$ is measured directly from the circumferential slice for various radii, perpendicular to the Mid-line as shown in Figure (A1).

If the stress difference $(\partial r - \sigma \phi)$ is not constant around the circumference, as is the case for the asymmetric bending stress distribution, equation (a.1) can be solved by substituting

$$\frac{\partial r - \sigma \phi}{r} \quad \text{for} \quad \frac{\partial r - \sigma \phi}{\rho}$$

where $\rho = \frac{r}{\partial a/\partial \phi}$

3. To determine $\partial \sigma r / \partial r$:

From equation (a.1)

$$\partial \sigma r / \partial r = -\left(\frac{\partial a r}{\partial Z} + \frac{\partial r - \sigma \phi}{r}\right)$$

which, when integrated gives,

$$\sigma r_N = \sigma r_{r_1} \int_{r_0}^{N} \left(\frac{\partial a r}{Z} + \frac{\partial r - \sigma \phi}{r}\right) dv \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \l
By a process of graphical integration equation (a.4) can be solved for \( \sigma_{rr} \) at various radii.

Thus, from (3), \( \sigma_{zz} \) can be solved since,

\[
\sigma_{zz} - \sigma_{rr} = (P - Q) \cos 2\alpha
\]

\[
= \frac{N_f}{t} \cos 2\alpha
\]

.\text{.} \quad \sigma_{zz} = \frac{N_f}{t} \cos 2\alpha \pm \sigma_{rr}

and from (2), \( \sigma_{\phi \phi} \) can be solved since,

\[
(\sigma_{rr} - \sigma_{\phi \phi}) = \frac{N_f}{t_* t}
\]

.\text{.} \quad \sigma_{\phi \phi} = \pm \sigma_{rr} - \frac{N_f}{t_* t}

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2.1 Effective Sleeve Modulus, $E_{se}$

Roll under bending:

The composite roll is treated as an equivalent homogeneous beam by combining the flexural rigidity of the arbor and sleeves (equation 3, page 15).

This simple idealisation requires modification to account for:

1. Sleeves of differing material properties.
2. Sleeves of differing widths.

If all sleeves (NSr), and sleeve supports (NSE), are expressed in terms of material $EA$ and width $w$, the effective sleeve modulus can be approximated to:

$$E_{se} = EA \sum_{r=1}^{NST} \frac{Ns_r \cdot R_r + NSE}{\sum_{r=1}^{NST} NS_r \cdot R_r}$$

For the roll under residual loading (equation 5, page 20), a similar expression can be derived for $E_{se}'$ by substituting $NSE'$ for $NSE$ in equation (a.5).
NOMENCLATURE

\[ \begin{align*}
NST & = \text{Number of sleeve material types} \\
NSr & = \text{Number of sleeves of elastic modulus } E_{Sr} \text{ and width } W \\
R_s & = \frac{E_{Sr}}{EA} \\
NSE & = \text{Number of additional equivalent sleeves of elastic modulus } EA \text{ (Roll under bending)} \\
& = \frac{(LS + LN)}{W} \\
NSE' & = \text{Number of additional equivalent sleeves of elastic } \frac{EA}{EA} \text{ (Roll under residual loading)} \\
& = \frac{LS}{W} \\
NS_s & = \text{Number of sleeves of thermal coefficient } \alpha_{S_s} \text{ and width } W \\
R_s & = \frac{\alpha_{S_s}}{\alpha_A}
\end{align*} \]
2.2 **Sleeve Clamping and Roll Temperature** (steady state thermal conditions)

The use of sleeve materials with a lower coefficient of thermal expansion than the arbor will result in a loss in axial clamping and an increase in radial clamping at elevated temperatures.

**Axial clamping:**

If all sleeves \( (NS_s) \) are expressed in terms of thermal property \( \dot{\alpha}A \) and width \( w \), the loss in axial clamping due to an overall, uniform, increase in temperature is given by:

\[
\delta W_R = \sum NS_s \cdot R_s \cdot (\dot{\alpha}A \cdot w \cdot \Delta T \cdot EA \cdot AA) \quad .......... \quad (a.6)
\]

**Radial clamping:**

If the composite roll under consideration consists of sleeves with identical coefficients of thermal expansion (being less than the arbor material), the increase in radial clamping for any sleeve will be proportional to:

\[
\Delta f_{min} - \Delta f_{min}' = b \cdot \Delta T (\dot{\alpha}A - \dot{\alpha}s) \quad .......... \quad (a.7)
\]

The case for a composite roll with sleeves of differing thermal properties (e.g. steel and carbide) can be accommodated in the expression for \( \delta W_R \) without difficulty. However, to assess the effect of a similar mixed sleeve combination on radial clamping would require a detailed knowledge of the temperature distribution through, and along the roll assembly.
2.3 **Stiffness Losses**

The simple relationship between arbor pre-tension \( W_B \) and residual sleeve clamping \( W_R \) (equation 5, page 20) assumes that the sleeves are perfectly flat and perpendicular to each other. It further assumes absolute rigidity for the sleeve supports. The effect of this simple idealisation is to over estimate the clamping efficiency \((W_R/W_B)\) of the composite roll under consideration.

**Potential reduction in axial clamping:**

If stiffness losses are to be compensated for, then the original arbor pre-tension (to develop \( W_{R_{\text{min}}} \)) must be increased according to:

\[
W_{Bf_{\text{min}}} = W_{R_{\text{min}}} \times K + W_{e}
\]

where \( W_{e} \) = Potential reduction in effective arbor pre-tension due to stiffness losses.

Let \( W_{e}^* \) = Estimated component flexing under residual loading, \( W_{R_{\text{min}}} \)

\( W_{e}^* \) = Redundant arbor axial contraction

and \( W_{\text{nom}} \) = Nominal arbor axial extension under pre-tension, \( W_{Bf_{\text{min}}} \).

Then \( W_{e} = \frac{W_{e} \times AA \times EA}{LA(W_B)} \)

and \( W_{Bf_{\text{min}}} = \frac{W_{\text{nom}} \times AA \times EA}{LA(W_B)} \)

* From previous experimental work, values of \( W_{e} \) of up to 10\% \( W_{\text{nom}} \) have been measured.
Substituting for \( \text{WB}_e \) in equation (a.8) gives,

\[
\text{WB}_{f, \text{min}} = \frac{\text{WR}_{\text{min}} K + \frac{\text{W}_e}{\text{W}_{\text{nom}}} \times \text{WB}_{f, \text{min}}}{1 - \frac{\text{W}_e}{\text{W}_{\text{nom}}}}
\]

or

\[
\text{WB}_{f, \text{min}} = \frac{\text{WR}_{\text{min}} K \cdot \text{F}_3}{1 - \frac{\text{W}_e}{\text{W}_{\text{nom}}}}
\]

where \( \text{F}_3 = \frac{1}{1 - \frac{\text{W}_e}{\text{W}_{\text{nom}}}} \) ............... (a.9)

Potential increase in radial clamping:

The redundant arbor axial contraction will also be accompanied by an excess arbor radial expansion, which is in addition to the initially developed interference \( \Delta'_{\text{i}, \text{min}} \) referred to in Section 2.2 (page 19) and equation 7, page 22.

It follows that,

If \( \text{W}_e = \) Redundant arbor axial contraction

and \( \text{U}_e = \) Excess interference simultaneously developed

Then:

\[
\text{U}_e = \frac{\text{W}_e \cdot b \cdot vA}{LA(\text{WB})}
\]

............. (a.10)

Also, since

\[
\text{F}_3 = \frac{1}{1 - \frac{\text{W}_e}{\text{W}_{\text{nom}}}}
\]
\[ W_e = (1 - \frac{1}{F3}) W_{nom} \]

Substituting for \( W_e \) in equation (a.10) gives:

\[ U_e = \frac{(1 - \frac{1}{F3}) W_{Bf_{min}} \cdot b \cdot vA}{AA \cdot EA} \]

If the resultant interference is not to be greater than that which is required, then, providing \( U_e < \Delta'i_{min} \)

\[ \Delta'i_{min} = \Delta'i_{min} - U_e \]

where \( \Delta'i_{min} \) = Corrected, initially developed interference.

2.4 Maximum Sleeve Clamping

Minimum sleeve clamping requirements are, by definition, based on a minimum material condition between the arbor shank and sleeve boxes.

If a maximum material condition is found to exist in practice, the excess axial clamping can be dissipated by increasing the initially developed radial clamping. The amount by which the initial interference is increased will depend, to a large extent, on the tensile strength of the sleeve material.

Thus, the maximum interference \( \Delta'i_{max} \) is given some arbitrary value, say

\[ \Delta'i_{max} = \Delta'i_{min} + S_T \]
and the resultant hoop tension checked against permissible values.

It follows that maximum axial clamping \( WR_{\text{max}} \) is given by:

\[
WR_{\text{max}} = \frac{WB_{\text{f, max}}}{K \cdot F3}
\]

where

\[
WB_{\text{f, max}} = WB_{\text{i, min}} + Z \left( S_T + A_T - \Delta_i_{\text{max}} \right)
\]
APPENDIX 3 - FINITE ELEMENT IDEALISATION OF A PRE-TENSIONED ARBOR

The arbor design examined in the photoelastic tests is a fairly typical configuration, the analysis of which will be relevant to a wide range of composite roll applications.

However, it may be the case that a composite roll arrangement is required for some special application, where the overall proportions of the arbor differ significantly from those studied in the photoelastic model. In this instance, the existing procedure for arbor design may not be suitable, and may even be misleading.

For this reason it was decided to analyse a finite element idealisation of a similar arbor, with the intention of assessing its suitability for the rapid appraisal of alternative arbor designs.

All work was run on the CEBG computer (IBM 370/85) using their in-house suite of finite element programs\(^\text{(29)}\). The roll arbor was treated as a simple axisymmetric structure, being described with a mesh design (Figure A2) containing two element types, EX16 and EX12. These are rectangular and triangular elements respectively. Both have the advantage of a mid-side node facility.

Mesh refinement at the model boundaries and in the vicinity of the bore termination is about an order of magnitude greater than that midway through the arbor shank.

This degree of refinement was considered necessary if critical stresses near the arbor boundaries were to be quantified with sufficient accuracy.
The method of end restraint and idealised load input was based on qualitative observations made in the photoelastic model. Isochromatic fringes and stress trajectories in the vicinity of the arbor/thrust rod interface indicated that the intensity of pressure at the bore termination was maximum at the outer radius and linearly decreased towards the axis of symmetry.

The idealisation of this pressure distribution is shown in Figure A2.

Assuming a system of point loads, the magnitude of load at each node is determined in the following manner:

Let \( f \) be the force/unit length of circumference, then the total axial load acting on the arbor \( WB \) can be written as,

\[
WB = \sum_{n=1}^{m} 2\pi \cdot f_n \cdot r_n.
\]

where \( f_n = f \cdot r_n / r_m \)

\[
WB = \sum_{n=1}^{m} 2\pi \left( \frac{f}{r_m} \cdot r_n^2 \right)
\]

or

\[
f_n = \frac{WB \cdot r_n}{2\pi \sum_{n=1}^{m} r_n^2}
\]

From which,

\[
f_n = 0.292 \times r_n \text{ KN}
\]
Fig. A2 Mesh plot for finite-element model of pre-tensioned arbor

Nodes held against axial displacement

assumed contact load distribution

Total axial load = WB = 7000 kN

scale, 0.5 full size
Figure A3 shows the overall deformation plot for the pre-tensioned arbor. The radial contraction along the arbor shank is also included in Figure (8) (Chapter 3, page 41). Figure A4 shows the distribution of stress at Sections 1 and 3, together with tangential SCF values at all critical locations.

These results are discussed, in relation to the equivalent photoelastic model test, in Chapter 4.
Fig. A3  Deformation plot of pre-tensioned arbor  (finite element model)
Fig. A4: Stress distributions in pre-tensioned arbor (finite element model)
A computer program has been written which conforms to the general analysis of composite rolls discussed in Chapter 2. The program structure shown in Figure (A5), is arranged as a series of feasibility assessments related to the loading and sleeve clamping requirements for any proposed design. The controlling parameters are mainly associated with permissible stress levels which are based on the photoelastic tests described in Chapter 3.

Optimisation Procedure

Since the external dimensions of a composite work roll are largely dictated by those of the conventional roll it is replacing, the main objective is to select an optimum value for the sleeve/arbor common diameter, \( b \).

For most applications there will exist numerous satisfactory values of \( b \). Therefore, the program is designed to evaluate each application with respect to 3 design criteria, namely:

(i) **Most economic sleeve design** \( \sim b = b_{\text{max}} \).

Wear-resistant sleeve materials such as tungsten carbide are very expensive, and their cost is volume related.

(ii) **Maximum sleeve support** \( \sim b = b_{\text{min}} \).

For some applications, the existing roll and bearing dimensions may be such that a composite roll replacement is only feasible if the sleeves are given maximum support.
Most economic sleeve criteria

**Input**

Stage 1

- Reduce DN until \( \frac{H_1}{H_2} < F_2 \)
- Or until \( H_2 = H_{2\text{min}} = (DN - C) \)

Stage 2

- Increase \( b \) until \( \frac{H_1}{H_2} > F_2 \)
- \( b = b_{\text{max}} \)

Stage 3

- Determine all loads from \( \text{WR}_{\text{min}} \sim \text{WBc_{min}} \)

Stage 4

- Determine feasibility of:
  - Reduce \( b \) if permissible stress is exceeded
  - \( b = b_{\text{nom}}, b = b_{\text{max}}, b = b'_{\text{max}} \)

Stage 5

- Determine fatigue life prediction
  - \( b = b_{\text{max}} \) or \( b = b'_{\text{max}} \)

Stage 6

- Determine \( \text{BS}_{\text{act}} \) and \( \text{BA}_{\text{act}} \)

Stage 7

- Minimum arbor pre-tension criteria

Output answers

**END**

Fig. 15: Design procedure for composite work rolls ~ Optimisation case.
(iii) Minimum arbor pre-tensioning $b_{\text{max}} \geq b > b_{\text{min}}$.

Arbor pre-tensioning, for sleeve assembly, represents the most critical static stress condition in a composite roll. The feasibility of some potential applications may, therefore, depend on a minimum arbor pre-tensioning requirement.

Program Description

Referring to the flow-chart in Figure (A5), it can be seen that the program consists of seven separate stages.

STAGE 1 (Most economic sleeve design criteria)

Initially C is assigned a maximum value and b is assigned a minimum value, dictated by the existing roll and bearing dimensions.

The ratio $\frac{H_1}{H_2}$ (Supported sleeve height) is then compared with an empirical factor (F2) of acceptability. (Observations from the photoelastic tests have given some indication of minimum sleeve support requirements, although a design procedure for sleeve support would require a separate study).

If $\frac{H_1}{H_2} > F_2$, b is increased until $\frac{H_1}{H_2} = F_2$.

If $\frac{H_1}{H_2} < F_2$, the sleeve new size DN is reduced until $\frac{H_1}{H_2} = F_2$, or until DN is reduced to 50% of its original value ($H_2 = H_{2\text{min}} = (DN - C)/4$). This is an arbitrarily chosen limitation to new size reduction which can be altered if required.
The above procedure ensures that \( b \) is assigned a maximum value, and that the sleeves are of a minimum volume. Subsequent reductions in \( b \) are tolerated only if the related load and stress conditions exceed permitted values.

**STAGE 2**

The arbor bore diameter \( a \) is assigned a maximum value based on roll stiffness, wall thickness and thrust rod ruling section considerations. No subsequent reduction in \( a \) is permitted as this would only serve to worsen the contact stress condition at the thrust rod/arbor interface.

**STAGE 3**

Minimum sleeve axial clamping requirements \( W_{R_{\text{min}}} \) are determined from the rolling load input data. (This is normally supplied by the rolling mill personnel, but can be obtained from a separate program based on the Cook and McCrum theory of rolling load\(^{(30)}\)). Provision is made for the calculation of maximum bending moment for single, twin or four strand rolling.

Arbor pre-tensioning requirements for sleeve clamping and sleeve assembly are determined, assuming a maximum material condition for the sleeve bores and arbor shank.

As an approximation, the permissible value of maximum arbor pre-tension \( W_{\text{PERM}} \) is based on a piston diameter of about 60 mm less than \( C \) and a maximum oil pressure of 0.21 KN/m\(^2\). If necessary, these criteria for \( W_{\text{PERM}} \) can be confirmed from a separate analysis of the pressure cap and related apparatus\(^{(31)}\).
If \( W_{Cl\min} > W_{PERM} \), \( b \) is reduced until \( W_{Cl\min} = W_{PERM} \).

**STAGE 4**

Critical stresses in the thrust rod and arbor, during sleeve assembly (\( \sigma_{nom} \) and \( \sigma_{A_{max}} \) respectively), and for the arbor under residual loading (\( \sigma'_{A_{max}} \)), are compared with material strength specifications.

If any stress exceeds its permitted value, \( b \) is incrementally reduced until this condition is corrected.

The alternating stress component in the arbor is determined and compared with a conservative estimate of the arbor material fatigue strength in the following manner:

For the arbor subjected to mean axial loading \( W_R \) and alternating bending \( \pm \delta A \), it can be shown that (32), for an infinite service life the arbor's fatigue life ratio \( FL \left( \frac{\text{Material fatigue strength}}{\text{Factored alternating stress}} \right) \) must be greater than unity,

\[
FL = \frac{\text{Sam}(N) \times 2 \times KS_f}{(\delta At - \delta Ac)} \geq 1 \quad \text{.................. (a.12)}
\]

Assuming a constant stress amplitude

where \( \text{Sam}(N) \) = Fatigue strength of material laboratory specimen at a given mean stress for endurance \( N \); and

\[
\frac{(\delta At - \delta Ac)}{2 \times KS_f} = \text{Factored alternating stress (KS}_f\text{ is the surface finish factor).}
\]
For a conservative estimate of fatigue strength,

\[ \text{Sam}(N) = \text{Sao}(N) \left( 1 - \left( \frac{\sigma'_{\text{max}}}{\text{Ays}} \right)^m \right) \]

where \( \text{Sao}(N) = \text{Value of Sam}(N) \) at zero mean stress

\( m = \text{Index for the effect of mean stress} \)

\( \sigma'_{\text{max}} = \text{Maximum mean stress in arbor, at Section 1} \)

and \( \text{Ays} = \text{Arbor material yield strength} \)

Assuming \( \text{Sao}(N)/\text{Ays} = 0.4 \), \( m = 1 \) (Goodman line) and \( \text{KSf} = 0.92 \) then,

from equation (a.12).

\[ F_L = 0.4 \frac{(\text{Ays} - \sigma'_{\text{max}}) 2 \times 0.92}{(\delta A_t - \delta A_c)} \]

The above estimation of fatigue life implies the direct substitution of SCF's derived from the photoelastic investigation. This is a rather pessimistic approach since the effective SCF in a dynamically loaded steel prototype tends to be less than that measured in a photoelastic model\(^{(33)}\).

A minimum permissible value for the collet-shell's cross-sectional area \( AC_{\text{min}} \) is determined.

**STAGE 5**

Based on the previously calculated value of \( WB_{CL_{\text{min}}} \), the actual sleeve bore and arbor shank diameters \((b_{S\text{,act}} \text{ and } b_{A\text{,act}} \text{ respectively})\) are determined in the following manner.
Referring to Figure 4 (page 24), zero diametral clearnace will exist at values of arbor pre-tension = \( W_{B_{\text{imin}}} \) and \( W_{B_{\text{imax}}} \). Assuming maximum material conditions,

\[
b_{\text{S,act}} = b - \left[ \frac{W_{B_{\text{imax}}} \times V_{A \times b}}{E \times A} \right]^{+ST}_{-0}
\]

and

\[
b_{A,\text{act}} = b^{+O}_{-AT}
\]

**STAGES 6 AND 7 (Maximum sleeve support and minimum arbor pre-tension criteria, respectively)**

Stages 3-5 are repeated with reducing values of \( b \). Dimensional or permitted stress levels limit the value of \( b \) for maximum sleeve support criteria. A minimum value for arbor pre-tension often coincides with \( b = b_{\text{min}} \), but this is not always the case.

**Prototype Design**

The following is a brief summary of the design and stress analysis adopted for a proposed composite roll prototype to be installed in the No. 2 Rod Mill, BSC, Lackenby works. This is a 25 stand continuous mill consisting of 7 roughing, 10 intermediate and 8 finishing stands rolling two strands simultaneously. It is fed with 83 mm square by 10.7 m long billets from a single reheat furnace and normally produces between 3000 and 3400 tonnes per week of rod in the size range of 5.5 mm to 10 mm diameter. The rod is sold to be drawn into pins, staples, carding wire, etc. and is all high carbon steel.
The composite roll prototype, shown in Figure A6, is designed to replace the existing 340 mm diameter rolls in one of the intermediate stands. It consists of two double-pass and six single-pass sleeves which comply with the 311 mm pitch requirement for twin-strand rolling. In this particular example, the relatively small difference between the roll neck (ND) and barrel discard size (DD) dictates a maximum sleeve support criteria, where \( b = b_{\text{min}} \). (This also coincides with minimum arbor pre-tensioning requirements).

The overall size, sleeve clamping and arbor pre-tensioning requirements (Data Sheet 2 and Table 6 respectively) for the Lackenby prototype are not unlike those outlined for the example composite roll described in Chapter 2.

However, a number of design features have been considerably modified in the light of results from the photoelastic investigation. Sleeve design has been largely influenced by observations made in the second and third model tests. In this work it was shown that, for satisfactory clamping, the arbor shoulder and retaining unit diameter should not be significantly in excess of the sleeve diameter at the base of the annular groove (pass).

Other design improvements which have been incorporated in the Lackenby prototype include the stress relieving features in the vicinity of the arbor's sleeve support shoulder and bore termination.
Fig. A6 Composite work roll - Dimensional parameters for Lackenby prototype
DATA SHEET 2

Lackenby prototype, all steel sleeve system

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>NSR</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>PL</td>
<td>311 mm</td>
<td></td>
</tr>
<tr>
<td>P</td>
<td>60 KN</td>
<td></td>
</tr>
<tr>
<td>T</td>
<td>3072 KN.mm</td>
<td></td>
</tr>
<tr>
<td>SF</td>
<td>2.5</td>
<td></td>
</tr>
<tr>
<td>a</td>
<td>105.1 mm</td>
<td></td>
</tr>
<tr>
<td>b</td>
<td>243 mm</td>
<td></td>
</tr>
<tr>
<td>c</td>
<td>292.5 mm</td>
<td></td>
</tr>
<tr>
<td>W</td>
<td>55.58 mm</td>
<td></td>
</tr>
<tr>
<td>L</td>
<td>1079 mm</td>
<td></td>
</tr>
<tr>
<td>L1</td>
<td>388.75 mm</td>
<td></td>
</tr>
<tr>
<td>L2</td>
<td>379.7 mm</td>
<td></td>
</tr>
<tr>
<td>LB</td>
<td>800.1 mm</td>
<td></td>
</tr>
<tr>
<td>L4</td>
<td>139.7 mm</td>
<td></td>
</tr>
<tr>
<td>LN</td>
<td>101.0 mm</td>
<td></td>
</tr>
<tr>
<td>LS</td>
<td>134 mm</td>
<td></td>
</tr>
<tr>
<td>LA(MB)</td>
<td>633 mm</td>
<td></td>
</tr>
<tr>
<td>LA(WR)</td>
<td>661.7 mm</td>
<td></td>
</tr>
<tr>
<td>r1</td>
<td>150 mm</td>
<td></td>
</tr>
<tr>
<td>r2</td>
<td>183 mm</td>
<td></td>
</tr>
<tr>
<td>r3</td>
<td>221 mm</td>
<td></td>
</tr>
</tbody>
</table>

Additional values:

- \( v_A = v_s = 0.3 \)
- \( \alpha_A = 11 \times 10^{-6} \text{ mm/mm/}^\circ\text{C} \)
- \( E_A = 207 \text{ KN/mm}^2 \)
- \( \alpha_s = \alpha_A \)
- \( E_s = E_A \)
- \( \mu = 0.1 \)
- \( \Delta T = 30^\circ\text{C} \)
- \( N_{SI} = 1 \)
- \( N_{SI} = 10.165 \)
- \( A_T = 0.0127 \text{ mm} \)
- \( S_T = 0.0127 \text{ mm} \)
- \( C_L_{\text{min}} = 0.0243 \text{ mm} \)
Table 6  Lackenby prototype – All steel sleeve system

<table>
<thead>
<tr>
<th>Loading specification</th>
<th>Critical stresses and deformations</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1. Sleeve assembly (based on max. pre-tension – W8CLmin)</td>
</tr>
<tr>
<td></td>
<td>Parameter</td>
</tr>
<tr>
<td>E5e = 99.8 kn/m²</td>
<td>WR8 = 332 Kn</td>
</tr>
<tr>
<td>E5e' = 207 kn/m²</td>
<td>WR7 = 286 Kn</td>
</tr>
<tr>
<td></td>
<td>SWR = 0 Kn</td>
</tr>
<tr>
<td></td>
<td>WRmin. = 332 Kn</td>
</tr>
<tr>
<td></td>
<td>E5e' = 207 kn/m²</td>
</tr>
<tr>
<td>K = 2.98</td>
<td>&quot;</td>
</tr>
<tr>
<td>F3 = 1.18</td>
<td>OzzAnom</td>
</tr>
<tr>
<td>WBf min = 146.7 Kn</td>
<td>OzzCSmax</td>
</tr>
<tr>
<td>Δimin = 0.01 mm</td>
<td>S</td>
</tr>
<tr>
<td>uε = 0.0016 mm</td>
<td>OzzRes</td>
</tr>
<tr>
<td>Δimin = 0.0084 mm</td>
<td>OzzRnom</td>
</tr>
<tr>
<td>WBi min = 2045.8 Kn</td>
<td>Pmax</td>
</tr>
<tr>
<td>WBi max = 4764.8 Kn</td>
<td>UAnom</td>
</tr>
<tr>
<td>Δimin = 0.0211 mm</td>
<td>Anom</td>
</tr>
<tr>
<td>W8f max = 2506.2 Kn</td>
<td>UAnom</td>
</tr>
<tr>
<td>WR max = 725.4 Kn</td>
<td>UAnom</td>
</tr>
<tr>
<td>W8CL min = 7365.8 Kn</td>
<td>UAnom</td>
</tr>
</tbody>
</table>
SCF Determination

Due to small differences between the prototype and model's diametral ratio (less than 20%), the SCF's extrapolated for the prototype will inevitably be subject to some error. However, the larger bore size coupled with the smaller shoulder diameter on this particular prototype arbor will ensure that any errors associated with SCF evaluation will be conservative.

There are no critical stresses associated with the geometry of the Lackenby sleeves due to the shallow pass requirements for this particular mill stand position.

Strand Positioning

It can be seen from the figure included in Table 6, that the position of the first strand is aligned with the second sleeve from the left-hand bearing support RA. This particular strand positioning ensures that, in the following case study, the resultant bending moment is maximum.

Case Study - All steel sleeve system

This is the sleeve arrangement for the first proposed on-line trial in the rolling mill. The object of this trial will be to assess both the structural integrity of the composite roll design and the performance of various coated or surface-hardened steel sleeves. These sleeves are expected to have a service life of up to 5X that normally associated with conventional work roll materials. It is intended to recycle the coated sleeves indefinitely.
Referring to Table 6, arbor pre-tensioning for sleeve assembly

$W_{BCL\text{min}}$ is shown to be about 7400 KN if maximum material conditions are assumed. This is equivalent to a nominal arbor stress $\sigma_{\text{nom}}$ of 0.20 KN/mm$^2$ and a maximum fillet stress at the bore termination of about 0.84 KN/mm$^2$. However, it is probably over pessimistic to assume that both the sleeves and arbor shank will be finally machined to their respective maximum material tolerances. For instance, if the machining tolerance contribution to arbor pre-tensioning is halved then $W_{BCL\text{min}}$ is reduced to about 6200 KN. The related fillet stress at the bore termination is approximately 0.78 KN/mm$^2$.

The maximum axial compressive stress in the thrust rod is calculated to be about -0.85 KN/mm$^2$, whilst the maximum required oil pressure is found to be no greater than 0.18 KN/mm$^2$. The above stress levels are considered acceptable in relation to the material strength specifications given in Table 7. Maximum stress levels associated with sleeve clamping and alternating bending are insignificant when compared with sleeve assembly loading conditions. The arbor fatigue strength is estimated to be an order of magnitude greater than that required for an infinite service life. It is assumed that the arbor does not suffer fretting fatigue during rolling.

<table>
<thead>
<tr>
<th>TABLE 7 Material Specification (BS970, 1955)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Component</td>
</tr>
<tr>
<td>-----------</td>
</tr>
<tr>
<td>Arbor</td>
</tr>
<tr>
<td>Thrust Rod</td>
</tr>
<tr>
<td>Collet-Shell</td>
</tr>
<tr>
<td>Pressure Cap</td>
</tr>
<tr>
<td>Piston</td>
</tr>
</tbody>
</table>