THE DEPENDENCE OF X-BAND GUNN OSCILLATOR

FREQUENCY ON TEMPERATURE

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Summary

The reasons for frequency variation with temperature in X-band Gunn diodes have been examined theoretically and experimentally. Frequency changes arise from the sensitivity of the diode admittance to r.f. voltage, to changes in the velocity field characteristic, the dielectric constant and the expansion coefficient of GaAs and to changes in the nature of the contacts to the diode. The diode cannot be considered in isolation from the circuit because of the heavy coupling between the two. A theory is presented which enables the $\frac{df}{dT}$ to be calculated in terms of the injection locking Q and the loading on the diode from any model of diode operation. The way in which the typical X-band diode operates is largely unknown. The various possibilities are discussed and the values of the parameter $Q \frac{df}{f} = \frac{dG_d}{dT} G_{od}$ appropriate to each mode are calculated. It is shown that a model of the hybrid mode, where the degree of domain build-up varies from slice to slice, can explain the experimentally observed $\frac{df}{dT}$ and in particular the zero values which have been found.

Investigations on the temperature variation of the sub-threshold properties of AgSn contacted and n+ GaAs contacted diodes have revealed departures from the ideal which are attributed to the contact regions, but no identifiable effect from this source has been found on the room temperature measurements of $\frac{df}{dT}$.

From the circuit work a model for some S4 outline packages in reduced height waveguide 16 has been obtained and the existence of a second cavity resonance at low frequencies, between the diode capacitance and the evanescent inductance of the circuit, has been demonstrated. Information on the harmonic impedances over which the Gunn diode can work has been obtained from measurements on a scaled model of circuit and package and the temperature variation of the dielectric constant of GaAs has been measured.
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Electric field components \( E_z, E_\phi \)
Magnetic field components \( H_z, H_\rho, H_\phi \)

\( \omega \) — frequency
\( \mu \) — permeability of medium
\( \mu_0 \) — "" free space permeability
\( g \) — conductivity of medium
\( \varepsilon \) — dielectric constant of medium
\( \varepsilon_r \) — relative dielectric constant
\( \varepsilon_0 \) — dielectric constant of free space
\( i = \sqrt{-1} \)
\( k \) — wave vector
\( k' \) — wave vector of perturbed frequency in free space
\( k_0 \) — free space wave vector of zero order cavity resonance frequency
\( k_4 \) — free space wave vector of fourth order cavity resonance frequency
\( E_0, E'_0 \) — scalar field amplitudes unperturbed and perturbed
\( J_m(\ ) \) — Bessel function of order \( m \)
\( N_m(\ ) \) — Neumann function of order \( m \)
\( Y_m(\ ) = J_m(\ ) + N_m(\ ) \)
\( h \) — height of cavity
\( a \) — radius of cavity
\( \pi b^2 \) — cross sectional area of dielectric rod
\( d\Delta \) — element of area
\( r \) — relative amplitude of fourth order cavity mode.
Chapter 10

\( f \) \quad frequency

\( T \) \quad temperature

\( C_d \) \quad diode capacitance

\( \xi \) \quad length of transmission line

\( Z_0 \) \quad impedance of transmission line

\( c_o \) \quad velocity of light

\( \omega \) \quad angular frequency

\( G_d \) \quad diode conductance

\( G_c \) \quad circuit conductance

\( B_d \) \quad diode susceptance

\( B_c \) \quad circuit susceptance

\( V \) \quad r.f. voltage amplitude

\( R_d \) \quad diode resistance

\( R_c \) \quad circuit resistance

\( X_d \) \quad diode reactance

\( X_c \) \quad circuit reactance

\( V_o, i_o \) \quad output voltage and current at diode terminals

\( V_i, i_i \) \quad injected voltage and current at diode terminals

\( \phi, \psi \) \quad phase between \( V_o \) and \( V_i \)

\( Y_o \) \quad load admittance at diode terminals

\( j \) \quad \(-1\)

\( \phi_o, \psi_o \) \quad phase between \( V_o \) and \( V_i \) at edge of locking band

\( Q_L \) \quad injection locking Q

\( V_B \) \quad bias voltage

Chapter 11

\( G_{od} \) \quad low field conductance of diode

\( R_{od} \) \quad low field resistance of diode

\( f_o \) \quad centre frequency of oscillator

\( PV \) \quad \((\text{peak to valley ratio})^{-1}\)
Appendix 1

$V_o$  dc bias voltage
$V_1$  rf voltage amplitude
$t_1$  time of domain nucleation
$t_2$  time of domain annihilation
$K$  (peak to valley ratio)$^{-1}$
$V_t$  threshold voltage
$i_t$  threshold current
$C_o$  diode low field conductance
tt  domain transit time from cathode to anode
$i_o$  dc current
$G_d$  fundamental diode conductance
$B_d$  fundamental diode susceptance

$$A = \frac{V - KV_o}{V - V_o t}$$

n  fractional change in $v(E)$ characteristic with temperature

$\delta$  fractional change in $t_1$ in delayed mode
$\rho$  fractional change in $t_1$ in quenched mode
m  fractional change in $t_2$ in quenched mode

Appendix 2

$i_o$  dc bias current
$i_1$  rf current amplitude
$V_D$  domain voltage
$C$  domain capacitance
$K$  (peak to valley ratio)$^{-1}$
$i_t$  threshold current
$R_o$  diode low field resistance
tt  time of domain extinction
tt  transit time of domain from cathode to anode
$a$  fractional change in $V(E)$ characteristic with temperature
$\delta$  fractional change in $i_o$ with temperature
$\gamma$  fractional change in $t_1$ with temperature
$R_d$  fundamental diode resistance
fundamental diode reactance

Appendix 3

$X_d$ electric field amplitude at position $x$, time $t$

$n_0$ doping density

$n(x,t)$ instantaneous electron concentration

e electronic charge

$\varepsilon$ dielectric constant

$J(t)$ total current density

$v(E)$ electron velocity as function of field

$D$ diffusion constant

$x$ x coordinate of the cathode

$L$ total length of diode

$\mu_o$ low field mobility

$E_{TH}$ threshold field

$E_V$ valley field

$\mu_l$ negative differential mobility

$\phi_D$ domain voltage

$\phi_{D1}$ domain voltage at fields between $E(k)$ and $E_{TH}$

$\phi_{D2}$ Domain voltage at fields between $E_{TH}$ and $E_V$

$E_D$ peak domain field
1. **Introduction**

Since its discovery in 1963 the Gunn Effect has been utilised in oscillator diodes. Today these diodes are beginning to find systems applications and for this reason, the second order parameters such as noise, temperature stability and injection locking are being investigated. So far the largest use of Gunn Diodes is in doppler intruder alarm systems where the transmitted frequency must only be kept within quite broad limits specified by the GPO. However, for the more demanding applications which are now being examined, such as radar local oscillators and phased array transmitters, frequency stability to a few MHz at X-band is imperative.

This stability can be achieved by compensating the cavity by mechanisms based on thermal expansion, by referring the frequency to that of a high Q cavity in a closed loop electrical system or by an open loop system in which the temperature is sensed by thermistors. The first of these methods cannot be used with pulsed diodes because of the need to compensate for in-pulse heating of the diode. The second and third methods are quite complex (see, for example, the system devised by the author for a microstrip module\(^{(1)}\)), and there is therefore considerable interest in obtaining diodes with zero \(\frac{df}{dT}\). If this is impossible, the problem should at least be understood so that steps may be taken to achieve greater uniformity among diodes.

Three main causes of frequency variation with temperature suggest themselves. The velocity field characteristic which is basic to Gunn operation is very sensitive to temperature change. In addition, displacement current arises both from the cold capacitance of the diode and from space charge build up and decay and will be temperature sensitive because the dielectric constant is a function of temperature. Finally, though a small effect compared with the first two, there is thermal expansion. It has been the intention of the work reported in this thesis to consider these various causes of \(\frac{df}{dT}\), to predict the effect of them on practical oscillators and to compare the theory with experimental measurements.
The problem is a complicated one in as much as the detailed operation of the 10 μm thick Gunn diode with practical contacts has yet to be realistically modelled. Those theories which have so far emerged have contained simplifications, many of them drastic, which make them extrapolations, some in one direction, some in another, of the actual behaviour. It is only recently that some of the more elementary features of transferred electron operation, such as the need for electrons entering the diode to fall through a sufficient voltage, (i.e. to travel a sufficient distance) to become hot, or once hot for a sufficient time to elapse for them to cool down again, have been taken into account and this can only be done accurately by the use of complicated simulations on the computer. Similarly, the difference between various types of contacts and, more especially, the effect which they will have on the way in which space charge builds up and moves through the diode are only now beginning to be appreciated.

The present work therefore contains, besides the necessary background which traces the origins, both historical and physical, of the Gunn effect, quite detailed accounts of the processes by which GaAs slices were grown, contacted and made up into the diodes which have been used in the experimental work. These diodes themselves have been extensively characterised below the Gunn effect threshold voltage in order to see how closely they compare with the theoretical ideal. From these experiments conclusions are drawn as to the probable nature of both metal and n+ GaAs contacts.

It is likewise important that the detailed interaction between diode and circuit should be fully understood. A full discussion of the various simple limiting modes of operation which have been proposed is given and comparisons are made with published computer simulations and experimental data. The magnitude and sign of $\frac{df}{dT}$ which can be expected is discussed in general admittance terms in a theory which is device and mode independent and which also removes the frequency sensitivity of the circuit from the problem. This enables the circuit engineer to trade off parameters of interest such as $\frac{df}{dt}$, injection locking Q and pushing figure one against another.
Returning specifically to the Gunn diode, the results of a series of calculations which derive the $\frac{df}{dT}$'s expected from the models of the simple modes are presented. In view of the neglect of the space charge build-up and decay processes in these simple models a more detailed theory which follows the time evolution of the domain has been produced. From this it is deduced that the standard X-band c.w. device operates in a hybrid mode. $\frac{df}{dT}$ has been calculated for this type of operation.

Experimentally the circuits which have been used have been extensively characterised at the fundamental and modelled on the computer. From experiments with a scale model of the circuit and package the harmonic admittances presented to the diode have been derived in an attempt to discover further details of operation of the devices.

Finally, using the derived circuit model, the observed $\frac{df}{dT}$'s of devices from slices are compared with those predicted from the various theories and it is shown that while the hybrid mode model is capable of explaining most of the observations, it is necessary in one case to consider a greater degree of domain formation.
2. The Historical Development of the Gunn Effect

2.1 Initial Observations

In 1963-4 J.B. Gunn was studying the hole and electron velocity saturations which occur in semiconductors at electric fields of about $10^3 \text{ Vcm}^{-1}$. The saturation arises when electrons are heated sufficiently by the field to interact with the optical lattice waves in the semiconductor, a scattering process which can lead to very large reductions in electron energy. In the particular cases of n-type GaAs and n-type InP Gunn found\(^{(2,3)}\), rather than a saturation, an instability in the velocity. The onset of the instability occurred at average threshold electric fields of around $2.5 \times 10^3 \text{ Vcm}^{-1}$, in GaAs and $6 \times 10^3 \text{ Vcm}^{-1}$ in InP, being slightly dependent on the length of specimen used. He was able to show that for short samples of GaAs the instability took the form of a periodic decrease in the carrier velocity with a basic frequency closely corresponding to the transit time of electrons across the specimen. In longer pieces of GaAs and in InP the periodicity was absent but it is now evident that this and the dependence of threshold field on length are the result of a combination of inhomogeneous material and of the higher domain fields in longer samples. It was demonstrated that the effect did not vary with the cross-sectional area of the diodes used and that it was contact independent in GaAs and almost so in InP.

In subsequent work, using capacitively coupled probes on the surface of the semiconductors, Gunn\(^{(4)}\) was able to show that the instability took the form of a voltage step which periodically traversed the specimen from cathode to anode. The field in the region of the voltage step was higher than the threshold field whilst that in the remainder of the material fell below threshold. A new voltage step was launched from the cathode immediately the previous one was discharged at the anode.

In InP the arrival at the anode of the first of the high field regions usually resulted in avalanche hole injection from the contact. Because of this and because of the much better state of GaAs as a semiconductor material interest in the InP instabilities
waned and has only recently revived\(^5\). All that follows here will refer to GaAs unless the converse is specifically stated.

2.2 The Explanation of the Gunn Effect

Although Gunn considered several possible mechanisms which might account for his observations he was unable to accept completely any of these. He was inclined to favour a travelling wave interaction with optical phonons, using as a parallel the well-understood mechanism of the acoustic amplifier. (In a piezoelectric crystal electrons drifting at velocities near those of acoustic phonons interact with the travelling field pattern produced by coherent phonons in much the same way that drifting electrons interact with slow waves in a travelling wave tube.) However, Gunn recognised that it would be difficult to reconcile satisfactorily the observed frequencies with the rather lengthy cycle time of such a model.

It was not until several months later that Kroemer\(^6\) suggested that the origin of the Gunn Effect should be sought by bringing together the discussions of negative resistance which had been given in 1961 by Ridley and Watkins\(^7\) and 1962 by Hilsum\(^8\).

Ridley and Watkins were concerned with mechanisms which would lead to the achievement of a negative resistance medium. Kroemer himself had previously suggested that if the charge carriers in a crystal could be made to occupy states in the \(E(k)\) diagram for which \(\left(\frac{d^2E}{dk^2}\right)^{-1}\) (the effective mass) was negative then the crystal would exhibit a negative resistance and the material could be used in the construction of amplifiers and oscillators. The difficulty with such an approach in practice is that it would be almost impossible to maintain an adequate population of carriers in the required states.

The line of attack which Ridley and Watkins took was a rather different one. They argued that carriers heated by an electric field could be made to populate additional minima lying a little higher in energy in the \(E(k)\) diagram than the levels normally occupied. By stipulating that carriers in the higher lying states had a smaller mobility than when in the lower ones and that the rate of population of the higher levels with increasing electric field was sufficiently rapid a differential negative resistance could be
obtained. The effect of different scattering mechanisms was discussed in as far as they affected the variation of mobility with field and it was shown, for example, that ionized impurity scattering, where the energy lost in a collision reduces with increasing approach velocity, worked against the negative resistance while increased Joule heating of the lattice at higher fields was advantageous as it tended to reduce mobilities.

The case of two spherical bands at slightly different energies was treated analytically in some detail for the case where the intra-band scattering dominated over the inter-band scattering and the electron distributions in the two valleys were, therefore, at different temperatures. It was shown that for two valleys centred on \( k = 0 \), collisions with longitudinal optical phonons could quite easily give rise to a negative resistance for spherical minima but with much more difficulty in the case of ellipsoidal valleys. It was suggested that p-type Ge and Si under uniaxial strain (which splits the degenerate valence band at \( k = 0 \)) would be suitable materials but that III-V compounds with naturally occurring sub-bands were also worthy of attention. The significant fact was pointed out that a bulk negative resistance was inherently unstable and would result in a break-up of the material into domains of high and low field.

Domain formation was taken a stage further by Ridley\(^9\) who showed by Prigogine's principle\(^{10}\) ('In a system in which irreversible processes are occurring, the steady state is that of minimum entropy production subject to any external constraints') and from an examination of electrical stability, that domain formation was both thermodynamically and electrically favoured in those materials which otherwise would show a bulk negative resistance. Formation of a domain resulted in a state of lower current density than existed beforehand giving rise to a lower rate of entropy production. Domain movement due to the drifting of the space charge walls supporting the high field region under the influence of the applied field was introduced by Ridley in the above paper with special reference to domains originating from the impurity barrier type of negative resistance.
Hilsum's contributions were to point out that certain III-V compounds possessed a band structure which could enable an intervalley transfer negative resistance to occur and he further recognised that scattering between the low lying central valley and the higher valleys lying towards the edge of the Brillouin zone would be by optical phonons. He was then able to use the relationship between the temperature of the electron population and the applied field derived by Stratton (11) (assuming the population was thermalised by inter-electron collisions). Making reasonable assumptions concerning the parameters of GaSb at 70°K and GaAs at 273°K, he was able to calculate velocity field plots for the two materials. These showed negative resistance regions at fields of a few thousand volts per cm. Hilsum was foreshadowing many later calculations of velocity as a function of field when he pointed out that many of the assumptions used were of doubtful validity, particularly those concerning the scattering mechanisms and electron temperatures. Nevertheless, his calculations for GaAs proved qualitatively correct while GaSb has yet to be grown in a sufficiently pure state to be checked.

Kroemer argued that the intervalley transfer negative resistance predicted generally by Ridley and Watkins and specifically for GaAs by Hilsum, together with high field domain formation as described by Ridley, provided the explanation of the periodic current variations and high field transits observed by Gunn. Intervalley transfer had, in fact, been one of Gunn's possible explanations but had been rejected by him because the electron temperature necessary for transfer was worked out to be unattainably high at around 4000°C. This calculation was, however, in error through an underestimation of the density of states available in the upper valleys; Hilsum's calculation giving a more reasonable figure of about 670°K.

2.3 Confirmatory Evidence for the Mechanism

As the explanation suggested by Kroemer fell naturally into two parts, electron transfer negative resistance followed by domain formation and travel, so can the confirmatory experiments be similarly divided. On the one hand there are a series of experiments
which show that the Gunn Effect is obtained only in semiconductors with suitable band structures and that modification of these can suppress the effect, and on the other there are observations on the properties of the high field domains in GaAs showing that in their behaviour they are consistent with the proposed explanation.

2.3.1 Suitability of Band Structure

The requirements that must be met in order that a semiconductor should show an inter-valley transfer differential negative resistance are very complex as they depend both on the band structure and on the details of the inter- and intra-band scattering processes. As these latter are to a large extent unknown in many of the less common semiconductors the discussion here will be kept to the bare essentials.

The first requirement is that the band structure should possess an upper valley or valleys in which the carriers will have a reduced mobility. To some extent this will depend on the scattering mechanisms but a large effective mass and high density of states are a definite advantage. A multiplicity of upper valleys not only increases the density of states available but also reduces the mobility through equivalent valley scattering. The sub-band gap between the normal and the higher energy states must be small enough that carriers heated by the electric field can be scattered across but large enough so that competing thermal population is small. In terms of kT this means an upper limit of say ten or twenty kT depending on the density of states ratio between upper and lower valleys and a lower limit of about two to five kT, i.e. at room temperature the sub-band gap should lie between 0.05 and 0.5eV.

The second requirement is that hot electrons should collide with the lattice and be scattered to the higher energy states and should not lose their energy by the creation of electron hole pairs. This in turn implies that the conduction-valence band gap should be rather larger than the energy at which electrons transfer to the upper states.
The two most important experiments in confirming the supposed explanation of the Gunn effect consisted of observing the disappearance of the effect when GaAs was modified so that the conditions on band structure described above no longer applied.

The band structure of GaAs is shown in Fig.1. It can be seen that it has a band gap of about 1.53eV and a sub-band gap of about 0.38eV. The triply degenerate or X valleys are relatively shallow compared to the central valley indicating favourable mobility and density of states ratios. Under hydrostatic pressure it is well established, that the energy bands of zinc blende structure materials (a class which includes silicon, germanium, diamond and the III-V compounds) behave similarly with the approximate pressure coefficients which are shown in Fig.1.

Thus under hydrostatic pressure the sub-band gap falls at a rate of about \(1 \times 10^{-6} \text{ eV kg}^{-1} \text{ cm}^2\) and if intervalley transfer were the origin of the Gunn effect the application of about thirty atmospheres should eliminate the effect. The experiment was first performed by Hutson, Jayaraman, Chynoweth, Corriell and Feldman who found that the Gunn Effect ceased upon application of 25k bars. The author with Lees and King used demonstrably purer material and was able to make some comparisons with the theoretical model of intervalley transfer being developed at that time by Butcher and Fawcett. An interesting comparison between the measured threshold field as a function of pressure and the displaced Maxwellian theory of Butcher and Fawcett is shown in Fig.2. Apart from a small uncertainty in the length of the sample used the agreement is good showing the increase in electron temperature required to dominate thermal transfer when the sub-band gap becomes small. Also shown are three curves for different scattering parameters derived using a simplified theory of the type used in their pioneering papers by Ridley and Watkins and by Hilsum. It can be seen that relatively crude models of Gunn Effect phenomena are capable of approaching quite closely the computer simulations which are required for exact modelling.
A similar effect to that obtained with pressure results from the alloying of GaP with GaAs. GaP is an indirect band gap material with the [100] minima having the lowest energy. It forms a complete range of solid solutions with GaAs in which the alloys with more than about 55% of GaAs have a direct gap, as in GaAs itself, but with a reduced sub-band gap (19,20,21) dependent on the composition. Allen, Shyam, Chen and Pearson (22) investigated the Gunn Effect in the alloy system and found a reduction in threshold field with composition which was in close agreement with the observation on GaAs under pressure. The Gunn Effect was not present in alloys with more than 48% GaP. Similar results have been obtained by the author in which long samples of material with 15% of GaP exhibited the characteristic spiky current waveform in a resistive circuit.

Both the pressure and alloying experiments demonstrate that altering GaAs so that intervalley transfer can no longer occur inhibits the Gunn Effect. The reverse is demonstrated by experiments in which materials incapable of field induced intervalley transfer are modified and then show the Gunn Effect.

InAs has a structure which in many ways is suitable for intervalley transfer. The sub-band gap between the high mobility Γ minimum and the lowest low mobility sub-band (L in the [III] direction) is 0.86 eV but the valence-conduction band gap is only 0.36 eV. Thus impact ionization occurs at fields lower than those necessary for intervalley transfer and the latter cannot be observed. From Fig.1 it can be seen that the application of pressure to InAs samples results in an increase in the valence-conduction band gap and a decrease in the Γ-L sub-band gap. Allen, Shyam and Pearson reported that 14 k bar of uniaxial [III] pressure enabled intervalley transfer to occur before impact ionization and that they obtained Gunn Effect oscillations. Their detailed interpretation has been questioned by Smith and Camphausen (24) who found that under 25 k bar of hydrostatic pressure they could observe domain transits in InAs. It is likely that any discrepancies arise from the assumption by Allen, Shyam and Pearson of pure uniaxial stress which is very difficult to achieve in practice.

A negative differential resistance arising from transfer between the [III] and [100] valleys in germanium at temperatures below
150°K has been predicted by Fawcett and Paige\(^{(25)}\) and observed by Chang and Ruch\(^{(26)}\). Elliott and McGroddy\(^{(27)}\) have observed the transit of domains in this material at 27°K and Smith\(^{(28)}\) has observed low temperature current oscillations which are believed to be due to a bulk negative resistance arising when the four degenerate [III] valleys are split by uniaxial stress.

Fawcett and Ruch\(^{(29)}\) have shown theoretically that the conduction band of InSb should show a negative differential mobility at fields in excess of 600V cm\(^{-1}\). With very short pulses Smith et al\(^{(30)}\) avoided the build-up of impact ionization across the narrow valence-conduction band gap and were able to see the Gunn Effect.

Of other materials in which the Gunn Effect should occur it has been found, as has already been discussed, in InP, in CdTe\(^{(31)}\) with a threshold of 13kV cm\(^{-1}\) and in ZnSe\(^{(32)}\) with a threshold of 38kV cm\(^{-1}\). It has not yet been found in GaSb, ZnS, ZnTe or CdSe presumably because these materials have yet to be obtained in sufficiently pure forms.

2.3.2 Detailed Examination of Domain Behaviour

By careful measurement of domain velocity and voltage as a function of the voltage applied to very long samples Heeks\(^{(33,34)}\), using a probe technique devised by Gunn\(^{(9)}\), was able to reconstruct the velocity-field characteristic of GaAs on the assumption of Ridley Watkins domain formation. This was in good agreement with calculated characteristics and gave very solid evidence for the mechanism. In addition Heeks' measurements had sufficient resolution to show that the high field domain was basically triangular and as a consequence the peak field was very much higher than had been previously supposed. Indeed impact ionization within the domain was observed at sufficiently high domain voltages showing the field was then of the order of 200-300 kV cm\(^{-1}\).

The simple theory of Ridley and Watkins assumes that material with a bulk differential negative resistance forms a high field domain in a low field background. The field magnitudes are obtained by applying current (or velocity) continuity through the two regions on
the velocity-field characteristic and choosing the integral of the field over the device length to be equal to the applied voltage. Butcher (35) pointed out that this is not correct when applied to a moving domain because it makes no allowance for displacement current. The simple picture can be applied however to a somewhat modified velocity field characteristic which is given by an equal areas rule. The major change is that the negative resistance takes on a greater value and provides an even better fit with Heeks' observations.

There is then considerable experimental evidence to show that the Gunn Effect occurs only in those materials which have suitable band structures for an electron transfer differential negative resistance to occur. The detailed behaviour of samples exhibiting the Gunn Effect is consistent with domain formation resulting from a bulk differential negative resistance as given by the Ridley and Watkins theory but modified by Heeks' recognition of the triangular domain and Butcher's allowance for domain dynamics.
3. GaAs Growth

3.1 Introduction

Over the past few years GaAs has proved to be an extremely useful semiconducting material. In addition to Gunn Effect microwave generators it has been used to make lamps and lasers (emitting naturally in the i-r and also in the visible when alloyed with GaP), field effect transistors, Impatt diodes, detector and mixer diodes and i-r detectors. With so many applications it is not surprising that it should have received attention second only to silicon, and possibly germanium. In contrast to these elemental semiconductors GaAs is a compound and its production presents, besides the usual problems of attaining crystalline perfection and control of impurities, the added difficulty of maintaining stochometry at the high temperatures necessary for growth.

In the preparation of the STL Gunn device, three different growth processes are used. These are the gradient freeze method for producing ingots and the parallel techniques of chloride vapour transport and liquid phase epitaxy for the growth of thin films on substrates. These techniques are described in detail below.

3.2 Gradient Freeze Method

Most of the early Gunn Effect work used relatively large devices which were cut from ingots of bulk GaAs. The techniques for producing these ingots were already well established in 1963 and had been successfully applied to many other compound semiconductors such as indium antimonide. The gradient freeze method is now used to obtain substrates for use in the epitaxial processes which produce today's device material.

The equipment used is shown in Fig. (3a). The quantities of gallium and arsenic required to obtain stochiometric GaAs are weighed out from 99.999% purity starting materials. They are placed in a silica boat which has been sandblasted to reduce wetting by the mix and the consequent incorporation of too much silicon in the final material. Depending upon the requirement, the material may be grown undoped to yield source material for the liquid epitaxy process or elemental chromium may be added to a proportion of 0.05%. This is incorporated in the material at a final concentration of 1 in $10^6$ and together with residual
oxygen in the reaction tube normally gives rise to semi-insulating material. The latter is used as substrates for the growth of check slices on which Hall measurements of epitaxial layers can be made. Alternatively $n^+$ material, to be used as substrates for the production of high frequency device slices by epitaxy, is obtained by adding an $n$ type dopant such as selenium or tellurium to a final concentration of $10^{18}$ cm$^{-3}$.

The silica boat is placed at one end of a long sealed silica reaction tube and inserted into the furnace arrangement shown in the figure. The boat is positioned close to the interface between two high temperature furnaces while the other end of the reaction tube contains elemental arsenic and is in a low temperature furnace. The function of this arsenic is two-fold. In the first place excess arsenic vapour presents a loss of arsenic from the mix which would give rise to non-stochiometry. Secondly, because the reaction tube is held at about 1270°C which is above the softening point of silica, the vapour pressure in the tube must be kept at precisely one atmosphere. This is done by maintaining solid arsenic in the coolest part of the tube and keeping the temperature of it constant at 610.0°C ± 0.1°C.

The two high temperature furnaces are allowed to stabilise so that both are at 1270°C and then, while the power to the one containing the boat is gradually lowered, the power to the other is maintained constant. This results in the temperature profiles which are shown in Fig. 3b, where it can be seen that the freezing point of 1240°C gradually moves through the boat. It is obviously important to prevent rapid temperature fluctuations during this period and a control of ±0.25°C is placed on the lower temperature furnace. After about 60 hours the freezing zone will have passed completely through the boat leaving single crystal material behind it.

Because of the high segregation coefficient for most impurities between liquid and solid GaAs most of the impurities will be concentrated in the last grown part of the ingot which can be discarded. There remains however a small amount of silicon ($10^{16}$ cm$^{-3}$) which appears to be neutral together with any intentional dopants. About 250 gms of GaAs can be grown at a single charge and since the material tends to grow along the $[111]$ axis it is a simple matter to orient the ingot and cut off (100) slices of a suitable size for epitaxy.
A similar technique which has been used is Czochralski pulling using molten boron oxide to prevent arsenic loss.\(^{(36)}\) There have been reports however of considerable quantities of boron being incorporated in such material.

### 3.3 Chloride Vapour Transport Epitaxy

The standard technique for the growth of thin layers of high
goodness device material involves the vapour transport of GaAs and its
or subsequent deposition on a substrate which may be of semi-insulating
or n\(^+\) doped GaAs depending on the requirement. The apparatus which is
used for this process is shown in Fig. 4. Hydrogen is purified by
filtering through the walls of red hot palladium tubes and is then
bubbled through very pure AsCl\(_3\) (0.999999). The resulting concentration
of the trichloride in the vapour is closely controlled by either placing
the bubbler in a water bath at a chosen temperature or by condensing out
excess trichloride with a similar water bath.

As the arsenic trichloride enters the hot furnace most of it
reacts with the hydrogen to give arsenic vapour and hydrogen chloride.
The hydrogen chloride reacts with GaAs contained in a silica boat at
825\(^{\circ}\)C according to the reversible reaction.

\[
4\text{HCl} + 4\text{GaAs} \rightleftharpoons 4\text{GaCl} + \text{As}_4 + 2\text{H}_2
\]

This reaction is reversed further down the furnace at 760\(^{\circ}\)C where an
epitaxial film is formed on the surface of the GaAs substrates.

The process is a specific version of a general method of
growing III-V films in which the material is transported by halogens but
owes its success to the purity which is maintained throughout the process.
Palladium filtering removes almost all impurities from hydrogen whilst
repeated distillation is used to remove impurities from the arsenic
trichloride. The resulting hydrogen chloride, prepared in the reaction
tube, contains consistently fewer impurities than gas which is obtained
in cylinders. The selection procedures which Tietjen and Amick\(^{(37)}\) find
necessary when HCl gas is used as one of the starting materials are evidence
of this. The high level of purity in the technique is further maintained
by growing the source GaAs within the reactor as a subsidiary process rather
than the using the alternative of powdered, and probably rather impure,
material.
To grow the source material gallium which is 0.999999 pure is washed and etched, placed in the boat in the reactor and baked overnight in a stream of purified hydrogen at a high temperature. Arsenic trichloride is passed over and dissociates. The gallium reacts with the arsenic produced and with the hydrogen chloride as follows.

\[
\text{As}_4 + 4\text{Ga} = 4 \text{GaAs}
\]

\[
6\text{HCl} + 6\text{Ga} \rightarrow 6\text{GaCl} + 3\text{H}_2 \rightarrow 3\text{H}_2 + 2\text{GaCl}_3 + 4\text{Ga}
\]

at cold reactor exhaust

A thin skin of GaAs grows over the gallium in the boat and when complete the reactor is ready to grow by the transport process already discussed. Besides being grown from very pure starting materials the source GaAs also benefits from the advantageous segregation coefficients for impurities between the crystalline skin and the saturated solution of arsenic in gallium.

Taking several precautions this process can be used to grow epitaxial layers with thicknesses between 2 and 100 μm and carrier concentrations between \(10^{14}\) and \(10^{17}\) cm\(^{-3}\). It is found advisable to prepare the substrates very carefully. They are cut 3° off (100) to avoid facet and pyramid formation and etch-lapped and etched to a smooth finish with NaOCl and 3:1:1 H₂SO₄, H₂O₂, H₂O. Pyramid formation is thought to be due to the deposition on the surface of the substrate of metallic gallium\(^{38}\) and for this reason the crust of source GaAs must remain intact during growth. As the transport of GaAs uses up the gallium a stage is reached where the metal in the boat balls up because of surface tension forces and the crust breaks. This places an upper limit on the amount of epitaxial material which can be grown from a single charge of Ga.

During growth crystalline GaAs is also deposited on the reactor walls in the zone around the substrates and competes with them as reaction nucleation centres. To avoid a serious drop in growth rate and the build-up from growth to growth of impurities a purge run at high temperature is carried out between growths.

With these precautions Barry\(^{39}\) has shown that the resultant carrier concentration in the grown material is in practice a function of the temperature of the water bath controlling the arsenic trichloride concentration and that there are slight but consistent variations from growth to growth during the useful life of the gallium source. The
mechanism of the doping process is not yet firmly established but the relative concentrations of $\text{As}_4$ and $\text{GaCl}$ in the gas stream seem to be important and the etching of the reactor tube walls by the hydrogen chloride to form chlorosilanes with the subsequent incorporation of silicon into GaAs as substitutional donor or acceptor or in complexes with vacancies cannot be ignored.

3.4 Liquid Phase Epitaxy

This process, while altogether simpler in concept than the vapour phase epitaxy, can yield very pure GaAs with total carrier concentrations of around $10^{13}$ cm$^{-3}$. However in the present context it has been used, because of its directness, to grow heavily doped contact layers of good quality GaAs.

The technique consists of dissolving GaAs in 0.999999 Ga at about 800°C to form a saturated solution. A substrate is brought into contact with this solution and the temperature is lowered, with the result that the gallium arsenide in solution crystallises out on the surface of the substrate. As in the case of the vapour phase source material the segregation coefficients for impurities between the melt and the solid reduce the impurities in the latter to a very low level.

Fig. 5 shows the apparatus. The melt is contained in a long silica boat within a tube which is continually flushed with palladium filtered hydrogen. The whole furnace is tilted so that the melt falls to one end of the boat while the substrate is clipped into the other end. When the temperature of the furnace has stabilised the melt is tipped onto the substrate and the temperature is slowly lowered.

The thickness of material to be grown depends on the temperature drop employed and at the end of the cooling period the furnace is again tilted so that the melt runs off the substrate. The doping level of the resultant material depends on a number of factors. Dopants such as tin, selenium or tellurium can be intentionally added to the melt to produce $n$-type material but because the segregation coefficients are functions of temperature slight doping gradients through the material are usually found.

There is always a certain amount of compensation in liquid epitaxial material due to silicon being leached out of the boat by the melt according to the reaction
2H₂ + SiO₂ → Si + 2H₂O

This can be controlled by growing at low temperatures or by controlling the water or oxygen level in the gas stream\(^{(41)}\). However, the problem which causes most trouble in growing thin heavily doped contacts is uneven growth due to constitutional supercooling. To reduce this the temperature profile through the furnace is kept very flat by controlling three zones separately, and the cooling rate is kept suitably low. For n⁺ contacts about 5μm of n-type material, selenium doped to 10^{18} \text{cm}^{-3}, is usually grown.
4. Gallium Arsenide Properties Relevant to the Gunn Device

4.1 Band Structure

The band structure of GaAs has already been discussed in some detail in Chapter 2.

4.2 Velocity Field Characteristic

The displaced Maxwellian calculations of Butcher and Fawcett mentioned in Chapter 2 have been supplemented by similar calculations using a Monte Carlo method in which the various scattering processes undergone by any given electron are simulated on the computer. The results of Boardman, Fawcett and Rees[42] are compared with the displaced Maxwellian in Fig. 6 where it is seen that the principle difference is one of velocity magnitude. These computer calculations, although very sophisticated, are only as good as the input data describing band structure and scattering mechanisms, much of which is very uncertain. As an example a Monte Carlo calculation with a change of one scattering parameter[43] is also shown in Fig. 6. For this reason there have been many attempts to measure the velocity field characteristic of gallium arsenide directly.

The experiments fall into three groups. Very fast pulse techniques and the use of high frequency r.f. enable the measurements to be made within a fraction of the dielectric relaxation time so that there is no appreciable build-up of space charge. Timing the flight of a very few carriers injected into semi-insulating material again prevents field distortion while at the other extreme information can be obtained from the current passed by fully formed domains in long Gunn oscillators.

Gunn and Elliott[44] used a novel bouncing ball pulse generator to produce pulses 250 ps long. These were applied to material of 100 cm estimated to have a dielectric relaxation time of about 200 ps. Their general conclusion was that the negative differential mobility was lower than had been previously supposed, but Copeland[45] pointed out that the addition of a field sensitive hole generation rate would remove this discrepancy. This explanation does not now seem to be reasonable and it is likely that Gunn and Elliott's material was non-uniform, the consequence of which will be discussed later in this chapter.
Acket and de Groot (46) and T'Lam and Acket (47) applied an r.f. voltage at 35.5 GHz and 33 GHz respectively to their samples of GaAs which were between 0.2 and 3Ω cm. At such a frequency intervalley scattering times are small enough so that it is still meaningful to define a velocity field characteristic but at the same time measurement is made in a shorter time than the minimum negative dielectric relaxation time so that space charge build up is minimised. By analysis of the behaviour of a small d.c. current, the power absorbed by the sample and the attenuation produced by the sample as functions of the applied microwave field a consistent picture of the velocity field characteristic was obtained.

By timing the flight of an electron pulse across a piece of biassed semi-insulating gallium arsenide Ruch and Kino (48) were able to measure the velocity field characteristic directly, provided that the injected current density was kept sufficiently small that the field was not perturbed. This produced results in quite good agreement with the theoretical characteristics.

Lastly it is possible to draw conclusions on the form of the velocity field characteristic from experiments on long Gunn devices where mature domains can form. An example of this is the work of Heeks (34) where sophisticated probe measurements were used to measure the velocity as a function of applied bias. By applying Butcher's equal areas rule a velocity field characteristic can be derived.

All these different measurements are compared with the two theories in Fig.6. It will be seen that there are differences. The microwave heating techniques of ref.46 tend to show rather broader velocity peaks as a consequence of the curve fitting which is used in the evaluation of the experiments and there is in general a considerable variation in the amount of negative resistance which is found.

Glover (49) has shown that microwave heating experiments tend to overestimate the threshold field and current due to residual energy relaxation effects. He has also demonstrated that the effect of inhomogeneities and doping gradients is to reduce the negative conductance calculated on the basis of homogeneous material and it is notable that where Glover has ensured that his specimen of GaAs has no doping gradients or inhomogeneities the form of the velocity field characteristic is very close to that of Ruch and Kino and the displaced Maxwellian and modified
Monte Carlo calculations. From the available evidence it seems that the Monte Carlo calculation with modified parameters is quite a good approximation to the actual characteristic and for that reason has been used in the work reported here.

4.3 Variation of the Velocity Field Characteristic with Temperature

Ruch and Fawcett\(^{(43)}\) have extended the Monte Carlo calculations to other temperatures. The most notable feature to emerge is the sharp knee in the low field characteristic at low temperatures which is due to the interaction of electrons with polar phonons causing increased scattering over and above that due to the ionized impurity scattering which tends to dominate at low field strengths. The knee is more pronounced at low impurity concentrations since the scattering rates for the two processes differ most in this region. Other results of the calculation are that the threshold field is virtually independent of temperature, rising by about 5% between 77°K and 300°K, the peak to valley ratio is practically constant over the same temperature range falling from a maximum of 2.15 at 200°K to 1.5 at 700°K while above threshold the velocity at all fields falls nearly linearly with temperature.

Experimental measurements as a function of temperature have been made by Ruch and Kino\(^{(50)}\) using the time of flight method discussed earlier, by Acket, 'T Lam, and Heinle\(^{(51)}\) at low temperatures, Bostock and Walsh\(^{(52)}\) at high temperatures and by Inoue et al\(^{(53)}\) by microwave heating techniques; and by Higashisaka\(^{(54)}\) with long Gunn diodes. The results are compared with the theory of ref.\(^{(43)}\) in Fig. (7). It is seen that the agreement is quite reasonable over the range 77 to 500°K lending considerable confidence to the choice of parameters for the Monte Carlo calculation.

An interesting feature which is illustrated by Fig. 7 is that specimens may have very different velocity field characteristics at room temperature depending on the total impurity concentration. However this tends to disappear at low temperatures due to polar phonon emission becoming the limiting scattering process in the [000] valley and optical phonon scattering being dominant in the [100] valleys at electron concentrations below 10\(^{17}\) cm\(^{-3}\). Thus all qualities of
material tend to show the same value of peak velocity of around $3.2 \times 10^7$ cm s$^{-1}$ at 77°K. In the mode calculations of this thesis the approximation has been made that the velocity field characteristic varies linearly with temperature about room temperature at a rate of 0.0029°K$^{-1}$.

4.4 Hall and Resistivity Measurements

The primary method of assessing the quality of epitaxial material grown for Gunn Devices is by measurement of the resistivity and Hall mobility. For these purposes check substrates of semi-insulating GaAs are included in the reactor in each growth run. From the thin conducting layers grown on these substrates clover-leaf shaped samples are cut ultrasonically. This particular form of specimen is chosen to reduce errors resulting from contact non-uniformity to a minimum. From a comparison of the measured carrier concentration and Hall mobility the general quality of the material can be assessed since the absolute mobility to be expected from a given impurity concentration is well known both from theory and experiment. Thus material with a measured carrier concentration of $10^{15}$ cm$^{-3}$ would be rated good if the Hall mobility was 7800 cm$^2$V$^{-1}$s$^{-1}$ but poor if the mobility was found to be 6500cm$^2$V$^{-1}$s$^{-1}$. This point will be returned to later.

4.5 Doping Profiles

Whilst Hall effect and resistivity measurements can give some idea of the overall quality of the GaAs, the work of Glover clearly shows that the terminal negative resistance properties of the finished device are functions of the doping uniformity through the thickness of the active region since this governs the way in which space charge builds up. It is therefore very important that the flatness of the carrier concentration across the sample should be measured.

For thick layers of about 100 μm it is possible to use a technique developed at Cornell University in which a laser beam focussed across the sample creates a strip of very low resistance material by causing impact ionization across the band gap. As the strip is moved along the sample the variation in resistance change is proportional to the original resistivity of the material. Because
of the difficulties of obtaining small light spots this method is not really suitable for X-band material although there seems no reason why a focused beam of electrons should not be used to give better resolution.

For thin layers an alternative method is to produce a Schottky barrier on the surface of the material and to open up a depletion layer by reverse biasing. It is then found that the depth of penetration is simply given by the reciprocal of the capacitance. The doping at that depth is derived from the capacitance cubed divided by the rate of change of capacitance with voltage.

In practice there are difficulties in the use of the technique in the analysis of material. The diameter of the diode enters as the fourth power and unless great care is taken the uncertainty in the result is large. Similarly unless mesa construction is employed the area of the depletion layer will be a function of depth since it will spread out sideways as well as through the depth of the crystal. Lastly the analysis of the results is rather tedious.

At STL the need to deposit metal films to form the Schottky barrier is avoided by pressing a meniscus of mercury against the clean GaAs surface as suggested by Carroll. The second contact is formed by the large capacitance between a metal ground plane and the n⁺ substrate. The area of the contact is defined by the diameter of the capillary tube in which the mercury is held and the 'diode' can be moved at will over the surface of the material. The non-mesa construction is compensated for by a correction applied later. The lengthy measurement and calculation processes are removed by the use of an automatic profile plotter designed at RRE. This instrument applies a combined d.c. and a.c. voltage to the Schottky barrier and from the current flow the doping density and depth of penetration are calculated by simple logic circuits. Although the estimated errors in the determination of carrier concentration as a function of depth by this method can amount to a factor of two the results in practice point to rather better consistency and particularly within any one measurement it is a good measure of the flatness of the doping.
4.6 **Dielectric Constant**

This has long been taken as 12.5\(^{(57)}\). Following the report of a resonance in the value in X-band by Larrabee and Hicinbothem\(^{(58)}\) a number of determinations were made\(^{(59-62)}\) by different methods. These failed to show any resonance and produced values ranging from 10 to 13.3 with a preponderance between 12.5 and 13.

The most accurate determinations seem to be those of Champlin, Erlandson, Glover, Hauge and Lu\(^{(59)}\) with a mean value of 12.9±.5 between 8.5 and 70 GHz.

4.7 **Variation of the Dielectric Constant with Temperature**

Measurements have been made at 70 GHz by Tong Lu, Glover and Champlin\(^{(63)}\), who obtained a value of 1.0 \(\times\) 10\(^{-4}\) °C\(^{-1}\) for \(\varepsilon\), while at the same frequency Champlin and Glover\(^{(64)}\) obtained 1.2 \(\times\) 10\(^{-4}\) °C\(^{-1}\). There is no data available on the value in X-band.

4.8 **The Elastic Constants of GaAs**

The elastic constants are important in the calculation of the strain under a metal contact, and a number of determinations have been made\(^{(65-68)}\). The following are values taken from these for which the agreement is best.

\[
\begin{align*}
\varepsilon_{11} &= 1.188 \times 10^{+12} \text{ dynes cm}^{-2} \\
\varepsilon_{12} &= 0.5367 \times 10^{+12} \text{ dynes cm}^{-2} \\
\varepsilon_{44} &= 0.594 \times 10^{+12} \text{ dynes cm}^{-2}
\end{align*}
\]

4.9 **Expansion Coefficient**

This is given by Feder and Light\(^{(69)}\) as 6.0 \(\times\) 10\(^{-6}\) at room temperature.
5. Measurement of the Variation of the Dielectric Constant of GaAs with Temperature at 9.5 GHz

5.1 Technique

Measurements were made using a cavity perturbation technique. The radial cavity shown in Fig. 8 is constructed from gold plated invar with a diameter of 2.42 cm so that the resonant frequency of the $H_{01}$ mode lies at 9.5 GHz, while the height of 1.97 cm ensures that other resonances are at suitably higher frequencies. Power is fed into and out of the cavity by loosely coupled loops in the side walls while the top face is removable and forms a radial choke. A 5 $\mu$m film of Mylar ensures correct action of the choke. Samples are centred in the cavity and held in position with a little grease or Araldite. The temperature is measured by a copper-constantan thermocouple inserted deep into the cavity wall. The $Q$ of the empty cavity as constructed was approximately 3000.

5.2 Theory of the Experiment

5.2.1 Description of Fields in the Cavity

The $E_{01}$ mode is one in which there is no variation in the electric and magnetic field strengths along the axis of the resonator. Maxwell's equations in cylindrical coordinates with

\[
\begin{align*}
\frac{\partial}{\partial z} &= 0 \text{ are:} \\
1) \quad \frac{\partial E_z}{\partial \phi} &= -i \omega \mu \rho H \rho \\
4) \quad \frac{\partial H_z}{\partial \phi} &= (g + i\omega) \rho E \rho \\
2) \quad -\frac{\partial E_z}{\partial \rho} &= -i \omega \mu H \phi \\
5) \quad -\frac{\partial H_z}{\partial \rho} &= (g + i\omega) E \phi \\
3) \quad \frac{\partial}{\partial \rho} (\rho E \phi) - \frac{\partial E}{\partial \phi} = i \omega \mu H z \\
6) \quad \frac{\partial}{\partial \rho} (\rho H \phi) - \frac{\partial H}{\partial \phi} = (g + i\omega) \rho E z
\end{align*}
\]

From these equations wave equations can be set up of which:

\[
\rho \frac{\partial^2 E_z}{\partial \rho^2} + \rho \frac{\partial E_z}{\partial \rho} + \omega \mu \rho \frac{\partial^2 E_z}{\partial \phi^2} = -\frac{\partial^2 E_z}{\partial \phi^2} \text{ for the case } g = 0
\]

...(7)
is of most interest here.

Writing \( k^2 = \omega^2 \mu \epsilon \) we have:

\[
k^2 \rho^2 \frac{\partial^2 E_z}{\partial \rho^2} + k\rho \frac{\partial E_z}{\partial \rho} + k^2 \rho^2 E_z = -\frac{\partial^2 E_z}{\partial \phi^2} \quad \ldots \ldots (8)
\]

Adding \(-m^2 E_z\) to both sides we have on the LHS terms dependent only on \( \rho \) and on the RHS terms only dependent on \( \phi \). The equation is therefore separable with both sides equal to the same constant which we can choose to be zero.

The LHS is Bessel's equation with a solution

\[
E_z = E_0 J_m(k\rho) + E'_0 J'_m(k\rho)
\]

which because \( J_\gamma(z) = (-1)^\gamma J_\gamma(z) \) for positive integral \( \gamma \) reduces to \( E_z = E_0 J_m(k\rho) \)

The RHS gives a solution \( E_z = E_0 \cos m\phi + E'_0 \sin m\phi \) which by a suitable choice of origin for \( \phi \) reduces to

\[
E_z = E_0 \cos m\phi.
\]

A general solution for the wave equation is

\[
E_z = \sum_{m=0}^{\infty} E_m J_m(k\rho) \cos (m\phi) \quad \ldots \ldots (9)
\]

There is another class of solution which will concern us, namely:

\[
E_z = \sum_{m=0}^{\infty} E_m N_m(k\rho) \cos (m\phi) \quad \ldots \ldots (10)
\]

The major difference lies in the behaviour as \( k\rho \to 0 \);
Solution (9) \( \to 1 \) whereas solution (10) \( \to -\infty \)
5.2.2 The Perturbation Solution

It is easily shown that the introduction of a piece of dielectric into an empty cavity results in a change in resonant frequency which is given by

$$\Delta \varepsilon \cdot \omega \varepsilon_0 \int_E E_O^* d\tau = \int (\omega_o^2 - \omega^2) (\varepsilon_o E_O^* + \mu_o H_H^*) d\tau$$

where $\Delta \varepsilon$ is the change in dielectric constant of the material introduced, i.e. $\Delta \varepsilon = \varepsilon_r - 1$, and the terms with and without subscript zero refer to conditions before and after insertion of the dielectric.

We make two assumptions: (1) that the field patterns in the cavity are basically unaltered by the insertion of the dielectric and (2) that the field is essentially constant over the volume of the dielectric (both these requirements are satisfied for a cylindrical rod of radius less than $\frac{1}{5}$ of the radius of an $E_{01}$ mode cavity divided by $\sqrt{\varepsilon_r}$).

We can therefore write $E = F_{00} J_0 (k_p)$ and since at resonance the stored electrical and magnetic energies are equal

$$\Delta \varepsilon \cdot \omega \varepsilon_0 \int J_0 (k_p) \int_{a}^{b} H^2 d\tau = \int (\omega_o^2 - \omega^2) \varepsilon_o J_0^2 (k_p) d\tau$$

... (11)

$$\Delta \varepsilon \cdot \omega \varepsilon_0 \int_{a}^{b} \int \pi b^2 h = 2 (\omega_o^2 - \omega^2) \varepsilon_o h \int J_0^2 (k_p) 2\pi \rho dp$$

... (12)

where $\pi b^2$ is the area of the rod inserted, $a$ is the radius of the cavity and $h$ its height.

Now the integral on the RHS of eqn. (12) runs from $k_p = 0$ to $k_p = 2.40$ (the first zero of $J_0 (k_p)$) and is equal to $\pi a^2 J_1 (2.40)$

$$\Delta \varepsilon = \frac{\pi a^2}{\pi b^2} \frac{\omega_o^2 - \omega^2}{\omega^2} \cdot 2 \times 0.270$$

... (13)

or

$$\varepsilon_r - 1 = \frac{\pi a^2}{\pi b^2} \frac{\omega_o^2}{\omega^2} - \frac{\Delta \omega}{\omega} \times 0.54$$

... (14)
and
\[ \frac{\partial \varepsilon_r}{\partial T} = -\frac{\pi a^2}{2\mu b^2} \frac{\omega_0}{\omega^2} \times 0.54 \times \frac{\partial \omega}{\partial T} \] \quad \ldots (15)

which is the perturbation solution for \( \varepsilon_r \) in terms of the frequency shift on introducing a dielectric sample along the axis of an H_{01} radial mode cavity. The solution is independent of the shape of the dielectric rod and depends only on its area.

5.2.3 Comparison with Exact Solution for Cylindrical Rod

As a check on the validity of the perturbation solution when the dielectric constant of the sample is very high (\( \varepsilon_r \) for GaAs is \( \approx 13 \)) it can be compared with the exact solution for a cylindrical sample. This has been calculated by Gevers \(^{73} \) by expressing the electric fields within the sample in terms of the zeroth term of eqn. (9) and outside the sample in terms of a combination of the zeroth terms of eqns. (9) and (10). (Eqn. (10) cannot be a solution within the dielectric because of the singularity at \( \rho = 0 \)). The electric and magnetic fields are matched at the boundary of the sample and the electric field is set equal to zero at the boundary of the cavity. This results in an expression for \( \varepsilon_r \) of

\[ \varepsilon_r = \left[ 1 - \frac{k_1'b}{b} (\varepsilon_r - 1) \right] \frac{Y_0(k_1'a)}{J_0(k_1'a)} - \frac{Y_0(k_1'b)}{J_0(k_1'b)} \frac{Y_0(k_1'a)}{J_0(k_1'a)} - \frac{Y_0(k_1'b)}{J_0(k_1'b)} \] \quad \ldots (16)

where it has been assumed that \( b \sqrt{\varepsilon_r} \) is small compared with \( a \).

In Fig. 9 \( \varepsilon_r \) derived from eqn. (16) is compared with the same quantity derived from eqn. (14) as a function of the reduction in frequency caused by the sample. It is seen that there is reasonable agreement for frequency reductions of around 5% even for \( \sqrt{\varepsilon_r} b/a \) as high as 0.2 but that thereafter the degree of agreement depends upon the exact ratio of \( b/a \) that is chosen.
5.2.4 Variational Solution

Although the perturbation and exact solutions show reasonable agreement for a cylindrical rod, in the case of a square sectioned rod there is always the possibility that other modes may be excited in the cavity and that the full description of the fields in the cavity would be a combination of all the terms of eqns. (9) and (10).

The problem can be tackled by the variational technique which gives the resonant frequency in terms of the electric field distribution in the cavity.

\[
\omega^2 \varepsilon_0 \mu_0 \left\{ \varepsilon_r E^2 d\tau - \int E V^2 E d\tau \right\} = \ldots (17)
\]

If the electric field distribution is not known exactly it can easily be shown that the distribution which gives the best value of \( \omega \) is that which minimises \( \bar{V} \). Thus it is possible to observe the convergence of \( \omega \) towards its actual value as more and more of the possible cavity modes are included.

For the case of a square cross section dielectric rod only those modes for which \( m \) is a multiple of four will be excited and the field in the cavity is given by \( E_z = \sum m J_m (k_r \rho) \cos m\phi \). \( m = 0, 4, 8, 16, \ldots \)

To determine the influence of these higher order modes the resonant frequency of a 2.416 cm diameter cavity with a square rod of side 0.1 cm and dielectric constant 13 has been calculated.

As a first order approximation it was assumed that the field in the cavity could be represented by \( J_0 (k_r \rho) \) where this is the zeroth mode of the empty cavity.

Thus eqn (17) reduces to

\[
\omega^2 \varepsilon_0 \mu_0 \left[ \int \varepsilon_r J_0^2 (k_r \rho) dA + \int J_0^2 (k_r \rho) dA \right] = k_0^2 \int J_0^2 (k_r \rho) dA
\]

area of sample remainder of cavity whole cavity
where because \( \frac{\partial \mathbf{E}}{\partial z} = 0 \) the integrals have been reduced to two dimensions and the wave equation \( \nabla^2 \mathbf{E} = k^2 \mathbf{E} \) has been used. The integrals were evaluated on a digital computer, (the approximation \( J_0(k_0 \rho) = 1 \) over the sample area was found to make a negligible difference to the integral over the sample area) and \( \omega \) was calculated to be 9.0686 GHz.

As a second order approximation the field in the cavity was assumed to have a contribution from the fourth empty cavity mode.

i.e. \( E_z = J_0(k_0 \rho) + r J_4(k_4 \rho) \cos 4\phi \)

In this case eqn (16) becomes

\[
\omega^2 \varepsilon_\circ \mu_0 \varepsilon_r \left[ J_0^2(k_0 \rho) + 2r J_0(k_0 \rho) J_4(k_4 \rho) \cos 4\phi + r^2 J_4^2(k_4 \rho) \cos^2 4\phi \right] dA
\]

Area sample

\[
+ \left[ J_0^2(k_0 \rho) + 2r J_0(k_0 \rho) J_4(k_4 \rho) \cos 4\phi + r^2 J_4^2(k_4 \rho) \cos^2 4\phi \right] dA
\]

remainder of cavity

\[
= \left[ k_0^2 J_0^2(k_0 \rho) + r(k_0^2 + k_4^2) J_0(k_0 \rho) J_4(k_4 \rho) \cos 4\phi + r^2(k_4^2) J_4^2(k_4 \rho) \cos^2 4\phi \right] dA
\]

whole cavity

Once again the integrals were evaluated by computer and by differentiation the value of \( r \) corresponding to minimum \( \omega \) was found to be \( 3.16 \times 10^{-6} \), leading to a resonant frequency differing from that obtained by assuming the fourth order mode is not excited by roughly 1 part in \( 10^{12} \).

It was judged that the convergence was sufficiently rapid that any of the higher order modes would not be appreciably excited, and it can be concluded that the perturbation calculation is sufficiently accurate when only the \( H_{01} \) mode is considered.

For comparison purposes an observed frequency of 9.06 GHz would lead to an \( \varepsilon_r \) of 13.00, calculated by the variational principle, and to a value of 13.02 from perturbation. \( \partial \varepsilon_r / \partial \mathbf{T} \) is of course evaluated...
from an observation of \( \frac{3\omega}{\delta T} \) by way of \( \frac{\delta \varepsilon_r}{\delta \omega} \). This latter quantity evaluated under the same conditions takes the values \(-4.77 \times 10^{-9}\) and \(-4.56 \times 10^{-9}\) respectively.

The values of the integrals used in this work are:

<table>
<thead>
<tr>
<th>Function</th>
<th>Over sample</th>
<th>Over remainder of cavity</th>
<th>Over whole cavity</th>
</tr>
</thead>
<tbody>
<tr>
<td>( J_0^2(k_o) )</td>
<td>0.0025</td>
<td>0.306391</td>
<td>0.308891</td>
</tr>
<tr>
<td>( J_0(k_o \rho) J_4(k_4 \rho) \cos 4\phi )</td>
<td>1.18 \times 10^{-7}</td>
<td>-1.18 \times 10^{-7}</td>
<td>0</td>
</tr>
<tr>
<td>( J_4^2(k_4 \rho) \cos^2 4\phi )</td>
<td>7.34 \times 10^{-13}</td>
<td>0.0412695</td>
<td>0.0412695</td>
</tr>
</tbody>
</table>

5.3 Experimental Results

The cavity was set up in the transmission mode and the resonance was observed on a spectrum analyser. A direct reading cavity wavemeter was used to measure frequency.

As checks a rod of PTFE and a tube of alumina were cut to length, placed in the cavity and the reduction in frequency measured. For the PTFE a resonance of 9.150 GHz together with a cross sectional area of 0.0807 cm\(^2\) yields a value for \( \varepsilon_r \) of 2.102. In the case of the tube a resonant frequency of 9.035 GHz and an area of 0.01447 cm\(^2\) gives \( \varepsilon_r = 9.24 \). Both these values are in good agreement with published data. Checks were also made to ensure that the results were not influenced by the details of the sample mounting, the top face fastening, or by too tight a coupling of the input and output loops.

The first GaAs to be examined was cut from the end of the single crystal material of a semi-insulating ingot (502) grown as described previously. It was reputed to be of greater than \( 10^8 \, \Omega \text{cm} \) and was cut and etched (3.3.1) to a cross sectional area of 0.0792 cm\(^2\). No resonance could be found so the sample was lapped thinner to an area of 0.0291 cm\(^2\). A resonance was found at 8.162 GHz with a 3 dB width of about 250 MHz. This would give a value of \( \varepsilon_r \) of 14.85 and a resistivity, using the calculation for \( \tan \delta \) given in ref. 73, of 15 \( \Omega \text{cm} \). As a check a similar sample was prepared with a series of seven alloyed tin contacts along its
length. A current of 1 mA was passed through the outermost and from the potential drops the resistivity was calculated to be 21.5 Ωcm, in good agreement with the cavity value.

Henceforth material was taken from ingot 532 and was found to have a suitably high resistivity. A series of measurements were made of ε_r and ∂ε_r/∂T as a function of sample cross sectional area. The latter were made by observing the drift of the resonance as the cavity was alternately heated and cooled over about 15°C in an oven.

It was found that consistent results were obtained after the first temperature cycle during which the cavity was apparently settling down. The shift in the resonance was initially monitored on the spectrum analyser set to the stabilised mode, but as the size of the samples was reduced it became obvious that residual drift in the analyser was a problem. An alternative method was therefore used in which the resonance of the test cavity and a high Q wavemeter were displayed together on an oscilloscope as the input frequency was swept over a few MHz (measured initially on the spectrum analyser). Thus, provided the sweep width did not drift, the deviation of the test resonance from the standard was easily measured to within 0.1 MHz.

A HP 5245L frequency counter and transfer oscillator later became available which enabled very low rates of frequency drift to be measured with good stability. The coupling loops were removed from the cavity and one of the faces was milled flat leaving a thin circular iris. The cavity was clamped to a length of gold plated invar WG 16 and a Pound discriminator was set up as shown in Fig. 10. The outputs from a matched pair of crystal detectors were equalised by shunting one of them and a low drift high gain differential amplifier (using op amps) provided a voltage proportional to the excursion of the cavity resonance over approximately 3 MHz. This was fed as a correction signal into the sweeper to form a phase locked loop, the frequency of which was monitored on the counter. Two samples from ingot 532, with cross sectional areas of about 0.002 cm^2, were measured with this system.

The collected results are shown as a function of area in Fig. 11. It is found that the apparent values of both ε_r and ∂ε_r/∂T fall with decreasing sample area to limiting values of 14.4 and 2.45 x 10^-3 °C^-1.
The value of 14.36 appears rather high for GaAs but it is of interest that the measurement on the sample from the other ingot examined (502) also gave a high value. The material was examined with an electron probe microanalyser to see whether pockets of the chromium which is used to dope the gradient freeze ingots had been incorporated into the material, but nothing was found. However the resolution of the instrument, at 1 part in 1000 maximum, is such that only very large aggregations would be visible.

The value of \( \frac{1}{\varepsilon} \frac{\partial \varepsilon}{\partial T} \) of \( 1.708 \times 10^{-4} \) is rather higher than that measured by Tong Lu, Glover and Champlin of \( 1.0 \times 10^{-4} \text{ oC}^{-1} \) and Champlin and Glover of \( 1.2 \times 10^{-4} \text{ oC}^{-1} \) at 70 GHz but not enormously so.

As a last check that the square cross section of the sample was not influencing the experiment a radial cavity with a much reduced height of 0.180 cm was made out of brass. Coupling was to the E field by way of an OSM connector mounted flush in the top face of the cavity. A cylindrical GaAs sample 0.130 cm diameter was cut from a thick slice of ingot 532 with an ultrasonic drill. This produced a frequency shift of -0.728 GHz \( (f_o = 9.760 \text{ GHz}) \) leading to a value for \( \varepsilon_r \) of 15.2.

This is plotted in Fig. 11 and is in good agreement with the measurements on square sectioned samples of the same cross sectional area.
6. The Fabrication of Gunn Diodes

6.1 Geometry

The earliest Gunn diodes were made from pieces of GaAs cut from bulk ingots. "Ohmic" contacts were produced by alloying large pieces of tin to opposite faces in an atmosphere of hydrogen and wet hydrogen chloride. In cases where the GaAs extended beyond the confines of the contact the excess material was often lapped or air abraded away. (See Fig 12a) This method of construction was limited at that time by the difficulties of cutting GaAs to less than 0.004 in thickness and so produced oscillators with transit times of greater than one nanosecond. It was on this type of device that much of the early diagnostic work on the Gunn Effect was done. Such diodes were, however, limited in the frequency at which they could operate, they could not be operated C.W. because of the large thermal resistance of the thick GaAs and could not be produced by a batch process.

For the latter two separate solutions have been forthcoming. In the first Sandbank, King and Wasse (75) suggested that the device should be formed in epitaxial material grown on a semi-insulating substrate. As can be seen from Fig 12b such a transverse diode is easily produced by photo-lithography and has the advantage that heat can be effectively removed since the active material is spread over a large area. However, the geometry puts severe limits on the uniformity of the material which can be used and the experimental results at high frequencies have not been spectacular.

The design has, however, two interesting features. If a diode is made with a divergent geometry from cathode to anode it can be shown that the distance which the domain travels before quenching is dependent of the applied bias. Jeppsson, Marklund and Olsson (76) have made diodes with the electrodes arranged concentricly and have produced diodes tunable from 3.8 GHz to 8.5 GHz.

At much lower frequencies it is possible to use diodes fabricated in this planar form as vehicles in which the travelling domain can be put to use as a generator or store of information.
This class of device was first described by Sandbank (77) and given the title of DOFIC (Domain Originated Functional Integrated Circuit). As an example, an analogue to digital convertor can be made by again using a divergent geometry but also varying the cross section area of the device in a sawtooth fashion along its length. Variation of the applied bias causes quenching on a different tooth while at the same time, the passage of the domain over previous teeth gives rise to a number of pulses in the current passed by the device. Alternatively, applying a voltage pulse to the bias when a domain is in transit across a uniform diode can drive the domain field into avalanche temporarily, and artificially alter the local resistivity. This local resistivity change can be read out again on the diode current during subsequent domain transits. Thus this type of DOFIC acts as a store.

Ladbrooke and Carroll (78) have suggested that the efficiency of Gunn diodes would be improved by a special shaping of the cross-sectional area along the length of the device so that the voltage takes on a particularly favourable waveform. The planar construction on a semi-insulating substrate would be most suitable for this.

The second type of Gunn diode construction to use photolithography was first suggested by Hilsum, Smith and Taylor (79) and is illustrated in Fig 12c. Here a diode of mesa construction is cut from an epitaxial layer on an n⁺ substrate. This is now the standard form of device for operation at frequencies above 1Ghz. In its original form with the substrate soldered to the heat sink the thermal resistance is quite poor but this can be improved considerably by inverting the device so that heat is extracted directly from the epitaxial layer. For the same reason devices are often split into a number of smaller diodes wired in parallel in the same cartridge. Using these techniques it is possible to dissipate about 12 to 15 W in a single device to yield C.W. output powers of up to $\frac{3}{4}$ W in X-band.

6.2 Contacts

A variety of metal ohmic contacts have been used on n-type GaAs. Pure tin has already been mentioned and pure indium is widely used in Hall Effect measurements. Both these metals, however, migrate
rapidly through GaAs under the influence of high fields and so cannot be used in the fabrication of commercial Gunn devices where long life is a desirable property.

Eutectic alloys of germanium with silver or gold (80) with the occasional addition of nickel have been widely used in the USA. The tendency of the contact material to ball up during the alloying cycle is prevented by the hard skin of nickel.

In Britain more attention has been focussed on contacts which are basically tin but in which this metal is prevented from migration into the GaAs by alloying the excess with silver to form a much harder material. A similar contact can be made with indium and gold (81).

In the process in use at STL (82) 1,000\(^\text{\AA}\) of tin and 10,000\(^\text{\AA}\) of silver are evaporated in that order on to the GaAs slice which has been thoroughly cleaned and lightly etched in 3:1:1 \(\text{H}_2\text{SO}_4 : \text{H}_2\text{O}_2 : \text{H}_2\text{O}\). The slice is then placed in a furnace which is similar to that used for the growth of vapour epitaxial GaAs and is heated to 610° C. This results in a certain amount of GaAs being taken into solution in the tin and then recrystallising out as a tin doped epitaxial layer as the temperature is lowered. The process takes place in an atmosphere of arsenic and hydrogen which minimises the loss of arsenic from the surface of the slice (83), avoiding the formation of high resistance or p-type regions under the contact. As a variation a glassy layer of SiO\(_2\) is often deposited over the metal film before alloying. Not only does this help to minimise arsenic loss, it also makes for a much smoother surface and finer structure of the alloyed contact (84).

The final stage is either to evaporate or chemically to deposit a thin gold layer to prevent tarnishing.

An alternative contacting technique is to grow a thin layer of \(n^+\)Se doped material on the surface by liquid epitaxy, as has already been described. A standard metallic contact is then applied to the \(n^+\) layer.
6.3 Photo-Etching

Normal photo-resist techniques are followed and surplus GaAs is removed using a 10% Br$_2$ in Methanol etch. Both square and round mesas have been used with equal success with the diameters of the individual contacts for C.W. devices lying between 0.003 and 0.006 in. After etching the slices are diamond scribed and broken as for silicon.

6.4 Encapsulation

To prevent damage to the diodes they are mounted in ceramic diode cartridges of the S4 outline. The substrate material is soldered to the header with AuGe eutectic alloy in an inert atmosphere and a gold tape or wire is thermocompression bonded to the metal contact on top of the mesa. The free ends of this wire are trapped under the cap of the cartridge when this is welded on. Alternatively, for improved heat sinking the chip may be thermocompression bonded, epitaxial layer down, to the header of the encapsulation and the wire applied as before. Thermal resistances for a 0.004 in diameter device in these two configurations are approximately 130° CW$^{-1}$ and 35° CW$^{-1}$, respectively. The latter thermal resistance can also be achieved but much more repeatably by the use of a plated heat sink intermediate between the epitaxial layer and the header.
7. Variation of the Sub-Threshold Properties of the Practical Diode with Temperature

7.1 The Ideal Diode

The temperature variation of properties of bulk GaAs such as the velocity-field characteristic, dimensions and dielectric constant has been discussed. What must now be considered is their effect when brought together with contact regions in the form of a practical diode of limited thickness.

For the purposes of comparison with theory the ideal diode would consist simply of a thin layer of bulk GaAs sandwiched between two "ohmic" contacts; the latter being defined as boundaries at which the field falls abruptly as possible to zero and which drop no voltage irrespective of the current density or flow direction through them. Since all practical contacts will be variants on the metal semiconductor or high-low carrier density themes this ideal is not achievable.

The degree to which perfection is approached in the practical diode can be assessed through a comparison of the temperature behaviour of the below threshold current-voltage characteristic with that expected from the bulk properties. For a diode in which the field rises to threshold over a distance of 0.1 μm at each contact the extra space charge density necessary at the contact amounts to $2 \times 10^{14}$ cm$^{-3}$ which for a 10 μm thick diode with a doping level of $1 \rightarrow 2 \times 10^{15}$ cm$^{-3}$ means that the properties of the perfect diode are essentially those of the bulk material. The relevant current voltage plot and its temperature variation is therefore to be obtained from the velocity-field characteristics of Ruch and Fawcett (43). In brief it is expected that on cooling from room temperature to 77°K the threshold current will increase by a factor of 1.55, the threshold voltage will decrease by about 5% and the low field resistance will decrease by a factor of between six and eleven times depending on the total ionized impurity concentration, (donors and acceptors). Furthermore the device will be electrically symmetrical.

Table 1 lists the GaAs crystals used in this work together with their cleave and stain thicknesses, Hall effect mobility and carrier concentration, details of Schottky barrier measurements where applicable, and the results of an analysis of the temperature induced changes in the
below threshold current-voltage characteristics. The devices have been grouped into three types, metal contacted, vestigially contacted (with a limited doping increase directly under a metal contact) and full n+ contacted, for ease of comparison.

7.2 Compensation

The first point to note is that the Hall measurements at 300°K and 77°K indicate that, within the experimental accuracy, the vapour epitaxial material has a carrier concentration which is independent of temperature and that the material is also moderately compensated. The compensation is illustrated in Fig. 13 where the ratio of $\mu_{77}/\mu_{300}$ determined from the Hall measurements is compared with the value of $N_d - N_a$ obtained from the same source. Assuming that the mobility ratio is approximately equal to the ratio of the drift mobilities, $N_d + N_a$ can be obtained from Ruch and Fawcett, and is also plotted. It can be seen that for heavily doped slices ($N_d - N_a > 3 \times 10^{15} \text{ cm}^{-3}$) there is little compensation, while growth at higher resistivity is obtained in large measure by the addition of acceptors.

7.3 Contact Effects

7.3.1 AgSn Contacted Devices

The perfect diode discussed above would show a threshold current increase of 1.55 on cooling to 77°K and this is found to be the case when n+ GaAs contacts are used. However, values of about 1.2 are typically found on AgSn contacted diodes which are in addition electrically very asymmetric for currents approaching threshold.

From the work of Colliver et al.\(^{(86)}\) on InP it would appear that the reason for this departure from the expected behaviour lies in the nature of the contact formed between the metal and the semiconductor. In all cases this will be a Schottky barrier but its detailed behaviour will depend on the height of the barrier, which will be mainly controlled by GaAs surface states at the interface, and the width of the barrier which will depend on the doping density in the material immediately under the contact. While surface states may be expected to be a fairly repeatable feature of GaAs the doping under the contact will be critically dependent on a balance between arsenic loss which gives rise to acceptor sites and diffusion or solution regrowth from tin which gives rise to donor sites. Thus it is to be expected that the nature of the alloyed
cathode contact will be greatly dependent on such parameters as the thickness of metal films, the alloying temperature and time and the control of arsenic pressure during alloying.

Evidence that this is so comes from the alteration in the V-I characteristic and the $\frac{df}{dT}$ and locking Q values measured in the full height waveguide circuit when diodes were retested after air baking cycles at 400°C. Fig. 14a shows the current voltage plot degrades when the diode is baked in air at 400°C, and partially recovers when the baking temperature is increased to 500°C. In Fig. 14b the changes in microwave operation for a series of 3 one minute bakes at 400°C are recorded.

When narrow barriers result, either by good control or by making the contact to a heavily doped layer, electrons can tunnel through the barrier with little voltage drop and in the limit of a very thin barrier an ohmic contact results. With wider barriers tunnelling is no longer dominant and the current which can be drawn is controlled by thermal emission over the barrier. The latter is very temperature sensitive, a fact which has been used by Tantraporn (87) to measure the barrier height. Colliver et al have shown that the detailed build up of space charge, and hence the efficiency, of a thin Gunn device is controlled by the field in the region of the cathode and that a potential barrier which raises this field and removes the "acceleration distance" present in an n$^+$ contacted diode can be advantageous. However in agreement with Gurney (88) they conclude that the presence of this high field region serves to depress the threshold current below that expected from the bulk properties of the material and that the diode will also be highly asymmetrical in the threshold region.

Thus it appears that the failure of the AgSn contacted diodes to increase their threshold current to the value expected at 77°C is due to a Schottky barrier contact. If we assume that the AgSn forms a wide barrier, we can postulate that at room temperature it can provide a thermally generated current which is near to or above the Gunn threshold current. At 77°C however it may be unable to do this so that on cooling there is a reduction in the current density at which threshold occurs. This implies that the barrier height lies within the range 0.04 eV to 0.2 eV.
Measurements of the pulsed threshold current as a function of temperature in the range 20°C to 150°C show that for n⁺ devices the fall is as predicted by Ruch and Fawcett. For Ag-Sn contacted devices this is also observed in some cases. In others there is a transition from an almost constant current at low temperatures to one falling off correctly and in yet other devices the threshold current does not fall at the bulk rate within the above temperature limits and may even rise. If the transition in the slope of the threshold current is interpreted as being the temperature beyond which the threshold current flow is not limited by the barrier we can use the method of Tantraporn to obtain the barrier height. Values of up to .36 eV have been found in practice and in addition a correlation has been found between the barrier height and the efficiency of diodes.

For comparison Bolton and Jones\(^{(89)}\) found that their metal contacted diodes exhibited a large series resistance which could be characterised by activation energies between 0.04 and 0.15 eV. They also found that the room temperature low field resistance of the devices was some six times greater than it should be and that the threshold voltage decreased with temperature. Such behaviour is consistent with a very wide potential barrier contact since in this case the voltage dropped by the contact rises quite steeply with increasing current once the thermal generation current in the barrier is exceeded.

A final complication is that the strain which is built into the epitaxial layer-metal interface during alloying and cooling might be sufficient to alter the band structure and so reduce both the threshold voltage and current density. However, even assuming that the metal and the GaAs show no plastic deformation on cooling from 300°C to 77°C, the stress built in will be only 3.1 kbars. This would not cause any significant change in the v(E) characteristic and in any case is probably an overestimate since a measurement of the radius of curvature of an alloyed slice after cooling from 900°C to 300°C showed a strain equivalent to a stress of only 0.76 kbars.

The vestigially contacted devices shown in Table 1 were grown with a doping density of around 2 \times 10^{16} cm\(^{-3}\) immediately under the metal contact in an endeavour to remove or mitigate some of the effects described above. It was found that these diodes were indistinguishable
from conventional AgSn contacted diodes. However, a suspicion exists that the thin highly doped layer was being largely removed in subsequent processing.

7.3.2 $n^+$ GaAs Contacted Diodes

The $n^+$ contacted slices show the expected behaviour of the threshold current on cooling. This is shown in Fig. 15 where the current voltage characteristic of slice R360C/VILE158 is compared with the theory of Ruch and Fawcett for the same carrier concentration. (The curves are normalised at the 300°K threshold). Many of the anomalous contact effects have been removed and the devices have been air baked at 400-500°C without degradation. There is though still a slight electrical asymmetry with the diode passing more current when the substrate is biased negative and the diodes generally operate at low efficiency in this polarity. The splitting of the characteristic is exaggerated at low temperatures with, in extreme cases, avalanche breakdown for positive bias on the epitaxial layer. At the same time the change in low field resistance on cooling both AgSn and $n^+$ contacted diodes is found to be typically a factor of 2 less than that expected from the mobility change of the starting material.

The following experiments were made to determine the origin of this additional resistance which has a negative temperature coefficient relative to the bulk of the GaAs.

To check whether it was due to trapping out when the heavily doped $n^+$ substrate and contact layers were cooled, Hall measurements were made on typical slices of substrate and on a contact layer on semi-insulating substrate. The results, shown in Table 2, demonstrate that there is no trapping and that the mobility does not change appreciably between 300°K and 77°K. This is as predicted for $10^{18}$ cm$^{-3}$ material by Ruch and Fawcett. Taking a worst possible current distribution and assuming that the $n^+$ contact, the active layer and the substrate all have the same cross sectional area the substrate will have at room temperature a resistance of, at the most, one twenty second of the active layer resistance. This resistance ratio is not sufficiently high to explain the observed behaviour on cooling.

The resistance of the encapsulation and the wire inside the package is very low and in any case has a positive temperature coefficient.
The gold germanium solder used to bond the diodes to the header was eliminated by inverting a diode from slice R360C 6LE158 (A30) on to the header with a thermo-compression bond. This diode gave a resistance decrease on cooling by a factor of 4.46 and 4.37 for the two polarities, which may be compared to 4.79 measured on a device bonded with the solder. Within the limits of the experiment this represents identical behaviour.

It was considered possible that the interface between the AgSn contact and the n⁺ layers might be contributing resistance and so a dummy slice with no epitaxial layer was processed. Small diodes (approximately 0.005 inch cubes) were cleaved from this and showed the expected resistance at 300°K. On cooling to 77°K the resistance decreased by a factor of 1.16 on one diode and 1.18 on another, in broad agreement with the results of Table 2 thus showing that the resistance of the metal-semiconductor interface was not having any measurable effect.

The extra resistance then must lie within the interface between the n⁺ regions and the active layer. Since AgSn contacted diodes, liquid epitaxy n⁺ contact diodes and the vapour epitaxy n⁺ contacted slices supplied by Plessey all show similar behaviour the substrate interface must be the chief suspect. The fact that the threshold voltage decrease on dropping the temperature to 77°K is always rather more than expected for a perfect diode (see Table 1) seems to indicate a certain amount of field concentration within the diode.

7.4 Conclusions

Neither the AgSn contacted diodes nor those with n⁺ GaAs contacts behave as a bulk GaAs diode should. Both show the effects of a resistive region which probably lies at the interface between substrate and active layer, and this presumably arises from the finite depth required before the growth settles down. It is tempting to consider that such a region may be largely to blame for the poor current cut-back observed on 10 µm vapour grown layers as compared with the near theoretical values of 2:1 which are often observed with 100 µm layers.

In addition the AgSn contacted devices show a marked contact effect on cooling which is due to the presence of a Schottky barrier which arises during alloying as a result of a balance between Sn doping and As loss. In the worst cases, which STL process seems to avoid, the barrier
dominates the behaviour of the device and causes great temperature sensitivity. At room temperature ambient almost all STL devices run at a temperature high enough that the barrier is of no consequence and the contact appears ohmic.

In any device modelling contact effects should be taken into account but most theories neglect them. We therefore expect to find better correlation with experiments made on $n^+$ GaAs contacted diodes and on AgSn diodes above room temperature than we would do on AgSn contacted devices cooled below room temperature. It will be shown later that no significant difference was found between $n^+$ GaAs and AgSn contacts in measurements of room temperature $\frac{df}{dT}$. 
8. The Interaction of the Gunn Diode with Resonant Circuits

8.1 Introduction

It is first useful to discuss the operation of a Gunn device in a resistive circuit. The current waveform of a Gunn device to which a steady d.c. bias is applied is shown in Fig. 16. Once the bias field exceeds the threshold value $E_{TH}$, a domain begins to grow at the cathode. Provided that the product of the doping density ($cm^{-3}$) and length (cm) is much greater than $10^{12}$ then the domain grows rapidly to an equilibrium size as shown in Fig 16b, with a maximum field $E_H$, which is very high and is on the flat part of the equal-areas-rule velocity field characteristic for all bias voltages. The domain moves across the sample with a velocity $V_v$ corresponding to this valley velocity and on reaching the anode is discharged. During the transit the current remains at a level corresponding to the valley ($neV_v$) level and a current pulse is generated by the domain discharge. A new domain quickly forms at the cathode to repeat the process. The regular series of current pulses shown in Fig. 16c could be used to excite a high Q resonant circuit at the same frequency, but because of the low fundamental content of the waveform the efficiency of this resonant transit time mode would be very small.

More efficiency is obtainable if the voltages set up in the resonant circuit are allowed to react back on the diode to control the time of nucleation of a domain, its growth and decay in response to the time varying voltage and its extinction due to the applied voltage becoming insufficient to support the voltage drops demanded across the domain and the remainder of the material by current continuity. This latter limit is known as the domain sustaining voltage and was first noted by Heeks, Woode and Sandbank (33), in experiments where the voltage across a long GaAs sample containing a domain in transit was progressively reduced. Domain extinction occurs at roughly half the threshold voltage.

Depending on the precise circuit conditions several alternative modes of operation are possible and it is these which form the subject of the remainder of this chapter.
8.2 Delayed Domain Mode

Provided that the doping density is relatively high the current passed when a high field domain is present in a piece of GaAs is virtually independent of the bias voltage and corresponds to the valley velocity. Therefore, as the threshold field is exceeded in a GaAs device driven by a sinusoidal voltage, a domain will form at the cathode and the current drawn from the circuit will fall to the valley level. While the domain is in transit this current will remain fixed and will not change until the domain is discharged at the anode. If the sinusoidal voltage has then fallen below the threshold level a new domain is prevented from forming at the cathode and the diode appears ohmic. The current then follows the form of the voltage until threshold is once again exceeded in the next cycle. The process is illustrated in Fig. 17. With a suitable choice of r.f. amplitude and a ratio of frequency to transit frequency in the range 0.7 to 1 a negative resistance is obtained and r.f. power can be extracted.

This form of operation of the Gunn diode is known as the delayed domain mode and shows its best efficiency with a delay of about one quarter period when values of around 6% peaking at biases of just over twice threshold can be obtained.

8.3 Quenched Domain Mode

If the r.f. voltage swing on the device is increased it is possible to reduce the voltage on the diode below the sustaining voltage while the domain is in transit. The domain is discharged in mid-flight and another cannot be formed until the bias reaches threshold later in the cycle. Again the current is near the valley value while the domain is in existence and the device appears ohmic after the quenching. With a suitable choice of parameters a negative resistance can be obtained (Fig. 18). The peak efficiency of the quenched mode is less than that of the delayed mode, being around 2½%, and again occurring near twice the threshold voltage. The competing processes of domain quench whilst in flight and delay before the next domain is formed mean that the frequency can range from about 0.8 times the transit frequency upwards. The model as formulated shows no upper frequency limit. However a practical limit is set by the power lost by passage of the disregarded domain.
charging and discharging current through the part of the device which is not 'used' and which appears ohmic while the domain is changing amplitude.

These simple models of the delayed and quenched domain modes make certain assumptions. They assume that irrespective of the applied voltage the presence of the domain determines the current as that corresponding to the valley value. This is a reasonable approximation for highly doped long samples where the domain voltage, even when but a fraction of the applied voltage, is high in absolute terms and where the heavy doping pushes up the peak domain field. It is also assumed that domain formation and annihilation are instantaneous events. This is not so since the domain can only build up by accumulation of space charge in a negative resistance region and must grow from some nucleating centre. The exact size and nature of this centre will control the domain formation time. However, as an order of magnitude it can be calculated that in the negative resistance region of the v(E) characteristic the domain will grow in 10cm material to 10 times its original size in about $10^{-10}$ seconds or one cycle of X-band. Similarly domain annihilation takes a finite time. The effect can only be fully dealt with by computer simulations and even with this type of modelling a choice of incorrect boundary conditions often makes it necessary to include a domain nucleating notch or inhomogeneity in the resistivity profile assumed for the device.

Finally, the simple explanations neglect entirely the current flow arising from the charging and discharging of the domain as the voltage across the device varies. This again demands knowledge of domain growth and decay rates in order to be properly taken into account. However, if a constant domain capacitance is assumed some idea of its effect can be obtained.

8.4 Current Driven Modes

Carroll (92) has shown that the domain capacitance can be used to increase efficiency by virtue of storing and releasing charge at appropriate times in the r.f. cycle. By assuming sinusoidal current, such as would arise in a circuit with high impedance at harmonic frequencies, he found that the efficiency of the quenched domain mode could be increased to about 15% and that of the delayed mode to about 14%. The waveforms for these modes are shown in Figs 19 and 20.
8.5 High Efficiency Waveforms

There is considerable uncertainty as to whether present day Gunn diodes are operated with sinusoidal voltage, sinusoidal current or either in the typical circuit. A case in which neither the current nor the voltage are sinusoidal occurs when both are square waves\(^{(93)}\). As is well known such switching characteristics can lead to high efficiency and 27% has been predicted at the fundamental in the case of the Gunn diode. However, these waveforms require a resistive loading at all harmonics. This is negative in the case of the second harmonic of ref 93 and would therefore be difficult to produce in practical circuits at high frequencies. Kino and Kurü\(^{(93)}\) have also discussed a particular case which leads to even higher efficiency, yet is realisable with simple circuit elements and avoids harmonic power output. If the voltage on the diode takes the form of a half sinusoid while the current is a square wave, then the fundamental is resistively loaded, odd harmonics are open circuited and even harmonics are short circuited. The efficiency can rise to 33% for high bias levels. The analyses presented by Kino and Kuru are only applicable to long Gunn diodes at low frequencies and once again domain formation and annihilation times are neglected together with domain charging currents and the cold capacitance of the device. Nevertheless they serve to show that proper loading at the harmonics is essential if efficiencies of over 10% are to be achieved.
9. Computer simulation of Gunn effect devices

9.1 Domain Modes

From an examination of the interaction of the Gunn device with a resonant circuit it has become apparent that the detailed build-up of space charge in the diode and its response to the applied voltage is of considerable importance and yet the total problem is beyond the reach of simple analyses.

Techniques have been developed for use on the digital computer in which the operation of the device is divided into a mesh of time and space points within which the electric field intensity and carrier density are calculated from current and particle continuity and Poisson's equation, using as boundary conditions the time behaviour of the circuit voltage and current.

The earliest examples of this type of analysis applied to the Gunn Effect were by McCumber and Chynoweth and by Kroemer. Although the starting assumptions of the exact form of the velocity field plot were different the conclusions reached were very similar. With uniformly doped material an accumulation layer of electrons detached itself from the cathode and propagated across the device growing as it went. In front of the layer the field rapidly increased so that all the electrons in this part of the crystal became heavy while behind the accumulation layer the field dropped to a low value and the electrons remained light. The accumulation layer could travel very much faster than the saturated velocity of the heavy electrons and this was the mechanism of its growth.

By the inclusion of a small region of slightly lighter doping near the cathode McCumber and Chynoweth were able to convert the accumulation layer to a dipole domain. Kroemer showed that a random doping profile had much the same effect; many small domains launched initially decayed at the expense of a dominant one. Both obtained the familiar spiked current response observed in resistive circuits from periodic dipole domain transits.

Copeland performed essentially similar calculations but used a parallel LCR circuit to provide the boundary conditions. In a 10 μm device at about 11 GHz with 5 x 10^{14} carriers he found a single resonant "Gunn mode" which was tunable over about an octave at frequencies
above the "transit" frequency (corresponding to electrons travelling at the valley velocity), thus showing that domain growth was occupying a considerable portion of the transit. He predicted an optimum efficiency of about 4.1% at a bias of around $2.6V_{TH}$ with a load conductance of 0.033 times the cold conductance. At this load conductance the efficiency could be further increased to 5.4% if the carrier concentration were increased to about $1.2 \times 10^{15}$ cm$^{-3}$. As will be discussed later these figures are fairly typical of present day X-band Gunn diodes and can be compared with the predictions of the simple delayed domain mode where the maximum efficiency of 7% occurs at $1.8V_{TH}$. In this case, however, the frequency is 0.82 times the transit frequency since instantaneous domain formation is assumed and the space charge would travel only slowly. The loading in the simple model is $0.166G_o$ representing the more complete current modulation obtained when domain growth times are neglected.

9.2 LSA Mode

A further result of Copeland's computer simulations$^{(97)}$ was the identification of another mode of operation at frequencies very much higher than the transit frequency. This he termed a "limited space charge accumulation mode" because the circumstances under which it occurred led to the transit of a small accumulation layer rather than a dipole domain. By a very rapid change of voltage across the sample resulting from high frequency operation the time spent in the negative resistance region of the $v(E)$ characteristic was small. With the additional help of a low doping level the space charge growth could be kept low so that the departure of the whole material from the fundamental $v(E)$ characteristic was small and the device behaved as a true bulk negative resistance. It is obvious from the above that setting a limit on the space charge growth puts a limitation on the doping and frequency and in fact Copeland, somewhat arbitrarily, suggested an upper limit for the ratio $\frac{n}{f}$ of $2 \times 10^5$ s cm$^{-3}$. Because the small amount of space charge which does accumulate while the material is above threshold must be eliminated in the part of the cycle in which the material is taken below threshold, it is similarly possible to put a lower limit on $n/f$. This Copeland chose to be $2 \times 10^4$ s cm$^{-3}$, allowing a complete order of magnitude difference between the growth and decay factors per cycle to prevent cumulative build-up of charge from cycle to cycle.
The LSA mode makes special demands not only on the ratio of \( \frac{N}{f} \) but also on material homogeneity, for the work of Kroemer\(^{(95)}\) showed how doping fluctuations can cause accumulation layers to change into voltage robbing dipole domains. In addition special precautions are necessary to prevent a slow traverse of the voltage through the negative resistance region of the \( v(E) \) characteristic before the r.f. oscillation has built up to its full magnitude. For this reason pulses with very fast rise times are used to excite a ringing of the cavity and this is aided by designing into the circuit structures which reflect almost 100% of the initial voltage step across the diode one potential LSA period later.

Copeland calculated the maximum efficiency for the LSA mode to be 18.5% and because it is transit time independent high frequencies can be obtained from relatively long diodes. Useful powers have been produced up to 94GHz.

### 9.3 Relaxation LSA

A related mode is the relaxation LSA discussed by Jeppesen and Jeppsson\(^{(98)}\). In this case higher carrier concentrations are used so that the conduction current makes a significant contribution to the total current passed by the device, and the condition of a high Q circuit which leads to the sinusoidal r.f. voltage of Copeland's mode is removed. If the diode is presented with a load which is conductive at the fundamental and inductive both at the fundamental and harmonics it is found that, again assuming minimal space charge build-up, the voltage wave-form is markedly non-sinusoidal. It is composed of two main parts. First an exponential growth from low voltage up to the Gunn threshold representing the charging of a resistive (and slightly cold-capacitive) device through the circuit inductance. Secondly, when the device becomes negative resistive and then purely capacitive above threshold, a damped oscillation takes place, driving the voltage up to a high value and back down towards zero again. Applying an analysis similar to Copeland's a range of \( \frac{N}{f} \) between 1 and \( 5 \times 10^5 \) s cm\(^{-3}\) is found to give effective space charge control.

The efficiency predicted when the same inductive load is used at all harmonics is about 13% but clearly this can be improved by varying the harmonic loading and values nearer 30% have been reported from devices\(^{(99)}\) thought to be operating in this mode. The attractiveness of this
method of operation lies in the simplicity of the circuit requirements, the use of overlength samples to increase both device impedance and potential power output, the more readily attainable higher carrier concentrations which can be used and, arising from the low Q nature of the oscillation, its freedom from starting problems.

9.4 Hybrid Mode

This mode was independently reported by Huang and MacKenzie (100,101) as a result of an analytic theory and by Bott and Fawcett (102) who generated it during computer simulations of a Gunn diode. In this case a dipole domain forms and due allowance is made for the time taken for it to grow to its full amplitude. If the rate of voltage increase applied across the diode by the circuit is greater than the rate of growth of the domain voltage, the field in the material external to the domain will be pushed above threshold thus giving the mode some of the characteristics of LSA.

In the simulation of Bott and Fawcett the domain never obtained complete equilibrium with the applied voltage with the result that the r.f. field in the whole device was above threshold for most of the cycle. Huang and MacKenzie (100), however, looked at a wider range of conditions and included also the case where the domain reached maturity during the cycle. With a sinusoidal voltage drive efficiencies of up to 22% were predicted and apparent experimental confirmation was provided by several authors (103,104), including Huang and MacKenzie themselves. The range of applicability of the mode was calculated by Huang to lie between $n/F$ values $2 \times 10^4$ and $4 \times 10^5$ thus covering a larger range than Copeland's LSA. Its disadvantage when compared with the latter mode is that higher peak fields and therefore easier avalanche breakdown result from the use of dipole rather than accumulation layers, but conversely the extreme doping uniformity demanded by LSA is no longer necessary.

9.5 Conclusions

This summary of the interactions of the Gunn device with resonant circuits has by no means been an exhaustive one. It has omitted, for example, the various amplifier modes which of course can appear, with a suitable circuit, as oscillator modes. However, it has attempted to show that the problem is exceedingly complex, that simple
models are of some limited use and that the assignment of a given device-circuit combination to any particular mode is extremely difficult. For example consider the various mode charts of Fig. 21 and take a typical c.w. diode with a doping of $10^{15}$ cm$^{-3}$ and a length of 10 μm operating in X-band. Without further information we have the choice of voltage or current driven delayed or quenched modes, hybrid, LSA and relaxation LSA. If we knew that the efficiency were more than 10% we could perhaps rule out the voltage driven delayed and quenched modes but without a detailed knowledge of voltage drive levels and preferably voltage waveforms it would be very difficult to decide amongst the remainder.

In any case the boundaries between the modes must be expected to be very indistinct in practice because of the effect of space charge growth current and material inhomogeneities. Jones and Rees have shown how dipole domains nucleated at high cathode field regions can, under suitable circuit conditions, be largely converted to accumulation layers, but similarly the effect of inhomogeneities on accumulation layers may well tip the balance in the other direction.

The idealised modes which have been discussed do, at least, present reasonably simple, intelligible pictures of circumstances which can only be fully examined by complex computer simulations.
10. The Influence of the Microwave Circuit on $df/dt$

So far we have dealt almost exclusively with the Gunn diode and any mention of the circuit has been rather general, i.e., we have supposed that a sine or square or other waveform has been developed in a suitable circuit. We must now consider how the presence of this circuit will affect the measured value of $df/dT$ and derive relationships which will enable a comparison between experiment and theory to be made. It is shown in this chapter that the final $df/dT$ is not only a function of the variation of the diode susceptance with temperature but that the diode conductance is also coupled into the equation because both these terms are functions of the r.f. voltage in the circuit. Similarly it is shown that the variation of both the real and the imaginary parts of the circuit admittance with frequency also enters into the calculation of $df/dT$. Because these latter quantities are not easy to evaluate in practical circuits, the solution to the problem is recast in terms of the more readily available injection locking $Q$ and load admittance.

The influence of different circuit environments is readily demonstrated by the two simple case illustrated in Figs. 22a and 22b. Here a diode which is purely capacitive is resonated in the one case by a lumped inductance and in the other by a length of transmission line. It is easily shown that the rate of frequency change with temperature resulting from a capacitance change is given in the lumped case by:

$$\frac{df}{dT} = -\frac{1}{2} \frac{f}{C_d} \cdot \frac{dC_d}{dT}$$

and in the distributed case by:

$$\frac{df}{dT} = -\left[\frac{C_d}{f} + \frac{\ell}{f z_0 c_0} \left(1 + (w_c z_0)^2\right)^{-1}\right] \frac{dC_d}{dT}$$

where $\ell$ and $z_0$ are the length and impedance of the transmission line and $c_0$ is the velocity of light. For small $\ell$ the two cases become equivalent as expected.

Now the usual microwave circuit for Gunn devices is far more complicated than either of the two simple cases considered here and it is apparent that some completely general description of the circuit is required before changes in the diode can be adequately related to changes in the circuit as a whole. A model in which only fundamental voltages occur will be considered.
When the temperature of a Gunn diode in a circuit is changed both the frequency and amplitude of oscillation are in general altered but in such a way that the oscillation condition of diode and load admittances being conjugate is preserved. For the case of a general parallel circuit such as in Fig. 22c we may write:

\[
\frac{\partial}{\partial \omega} \left( G_d + G_c \right) \cdot \Delta \omega + \frac{\partial}{\partial T} \left( G_d + G_c \right) \cdot \Delta T + \frac{\partial (G_d + G_c)}{\partial V} \cdot \Delta V = 0
\]  
\[\ldots(3)\]

\[
\frac{\partial}{\partial \omega} \left( B_d + B_c \right) \cdot \Delta \omega + \frac{\partial}{\partial T} \left( B_d + B_c \right) \cdot \Delta T + \frac{\partial (B_d + B_c)}{\partial V} \cdot \Delta V = 0
\]  
\[\ldots(4)\]

where subscripts d and c refer to diode and circuit respectively and \( V \) is the a.c. voltage amplitude which is taken as a measure of oscillation amplitude. If we make the assumptions of an invar circuit and a linear circuit then these equations reduce to:

\[
\frac{\partial}{\partial \omega} \left( G_d + G_c \right) \cdot \Delta \omega + \frac{\partial G_d}{\partial T} \cdot \Delta T + \frac{\partial G_d}{\partial V} \cdot \Delta V = 0
\]  
\[\ldots(5)\]

\[
\frac{\partial}{\partial \omega} \left( B_d + B_c \right) \cdot \Delta \omega + \frac{\partial B_d}{\partial T} \cdot \Delta T + \frac{\partial B_d}{\partial V} \cdot \Delta V = 0
\]  
\[\ldots(6)\]

Since we are interested in frequency change with temperature we can eliminate the r.f. amplitude change \( \Delta V \) to give

\[
\frac{\Delta \omega}{\Delta T} = \frac{-\left[ \frac{\partial B_d}{\partial T} - \frac{\partial G_d}{\partial T} \cdot \frac{\partial B_d}{\partial G_d} \right]}{\frac{\partial (B_d + B_c)}{\partial \omega} - \frac{\partial (G_d + G_c)}{\partial \omega} \cdot \frac{\partial B_d}{\partial G_d}}
\]  
\[\ldots(7)\]
where the term \( \frac{\partial B_d}{\partial G_d} \) should be understood to mean \( \frac{\partial V}{\partial G_d} \).

Thus \( \frac{df}{dt} \) has been split into two factors, one of which depends only on the diode and reflects the change, not only of the susceptance with temperature, but also of the conductance through the coupling term \( \frac{\partial B_d}{\partial G_d} \). The second depends on both the diode and circuit and reflects the admittance changes of these with frequency. All the information which we require of the circuit is contained in the terms:

\[
\frac{\partial (B_d + B_c)}{\partial \omega} - \frac{\partial (G_d + G_c)}{\partial \omega} \cdot \frac{\partial B_d}{\partial G_d}
\]

Similarly in the case of a series circuit as shown in Fig. 22d it can be shown that

\[
\frac{\Delta \omega}{\Delta T} = \frac{\frac{\partial X_d}{\partial T} - \frac{\partial R_d}{\partial T} \cdot \frac{\partial X_d}{\partial R_d}}{\frac{\partial (X_d + X_c)}{\partial \omega} - \frac{(R_d + R_c)}{\partial \omega} \cdot \frac{\partial X_d}{\partial R_d}} \quad \ldots (8)
\]

In order to proceed further it is necessary to find some other phenomenon which also connects the variables \( \omega, V, B_d, B_c, G_d \) and \( G_c \). Injection locking measures the response of the frequency to a change in r.f. voltage and is therefore suitable for our purposes.

Consider the general equivalent circuit of Fig. 22e in which an oscillator feeds a transmission line load through a frequency dependent network. Let the output voltage and current across and through the load be \( V_o, i_o \). Let an injected signal of the same frequency but with transformed amplitude \( V_i, i_i \) be also applied to this load and let the phase difference between the two voltages be \( \phi \).

Then the total voltage and current at this plane, \( V, i \), are given by:
\[ V = V_0 + V_i \cos \phi + j V_i \sin \phi \]
\[ i = i_o - i_i \cos \phi - j i_i \sin \phi \]
\[ Y = Y_o + \Delta Y_o = \frac{i}{V} = \frac{i}{V_0} \left[ 1 - \left( \frac{i_i}{i_o} + \frac{V_i}{V_o} \right) \cos \phi - j \left( \frac{i_i}{i_o} + \frac{V_i}{V_o} \right) \sin \phi \right] \]

where a binomial expansion assumes \( i_i \ll i_o, V_i \ll V_o \). Now since the signals appear across the same load \( \frac{i_i}{i_0} = \frac{V_i}{V_o} \) and the input signal appears to modify the admittance at the oscillator terminals by an amount
\[ \Delta Y_o = - Y_o \cdot \left( 2 \frac{V_i}{V_o} \cos \phi + 2j \frac{V_i}{V_o} \sin \phi \right) \quad \ldots(9) \]

We now apply the oscillation condition \( \Delta G = \Delta B = 0 \) and obtain from a linear circuit:
\[ \frac{3}{\partial \omega} \left( G_d + G_c \right) \Delta \omega + \frac{3G_d}{\partial V} \Delta V - 2 \frac{V_i}{V_o} \left[ G_c \cos \phi - B_c \sin \phi \right] = 0 \]
\[ \ldots(10) \]
\[ \frac{3}{\partial \omega} \left( B_d + B_c \right) \Delta \omega + \frac{3B_d}{\partial V} \Delta V - 2 \frac{V_i}{V_o} \left[ B_c \cos \phi + G_c \sin \phi \right] = 0 \]
\[ \ldots(11) \]

We can, as before, eliminate the unknown \( \Delta V \) to obtain an expression relating the amount of frequency pulling to the amplitude of the injected signal.
\[ \Delta \omega \cdot \left[ \frac{3(B_d + B_c)}{\partial \omega} - \frac{3B_d}{\partial G_d} \cdot \frac{3(G_d + G_c)}{\partial \omega} \right] = 2 \frac{V_i}{V_o} \left\{ \left( G_c + \frac{3B_d}{\partial G_d} B_c \right) \sin \phi + \left( B_c - \frac{3B_d}{\partial G_d} G_c \right) \cos \phi \right\} \]
The maximum value of the RHS occurs for \( \phi = \phi_0 \) where \( \tan \phi_0 = \frac{\partial B}{\partial G} \cdot \frac{B}{G} \),

\[
G + \frac{\partial B}{\partial G} \cdot B
\]

We may also write \( V_i \) and \( V_o \) in terms of the input and output powers to obtain

\[
\Delta \omega_{\text{max}} = 2 \sqrt{\frac{P_i}{P_o}} \cdot (G + B \frac{\partial B}{\partial G}) \frac{|\sin \phi_0|}{\left\{ \frac{\partial (B + B)}{\partial \omega} - \frac{\partial B}{\partial G} \cdot \frac{\partial (G + G)}{\partial \omega} \right\}}.
\]

...(12)

This may be compared with the phenomenological description of injection locking:

\[
\Delta \omega_{\text{max}} = \frac{\omega_0}{2Q_L} \sqrt{\frac{P_i}{P_o}}
\]

...(13)

so that we can equate the locking \( Q(Q_L) \) with:

\[
Q_L = \frac{\omega_0 \cdot |\sin \phi_0|}{4 G_c \cdot (1 + \frac{c}{G_c} \cdot \frac{\partial B}{\partial G})} \left\{ \frac{\partial (B + B)}{\partial \omega} - \frac{\partial B}{\partial G} \cdot \frac{\partial (G + G)}{\partial \omega} \right\}
\]

...(14)

In a series circuit a similar result is obtained:

\[
Q_L = \frac{\omega_0 \cdot |\sin \psi_o|}{4 R_c \cdot (1 + \frac{c}{R_c} \cdot \frac{\partial X}{\partial R})} \left\{ \frac{\partial (X + X)}{\partial \omega} - \frac{\partial (R + R)}{\partial \omega} \cdot \frac{\partial X}{\partial R} \right\}
\]

...(15)

where

\[
\tan \psi_o = \frac{R_c + \frac{\partial X}{\partial R} \cdot X_c}{X_c - \frac{\partial X}{\partial R} \cdot R_c}
\]
Combining equations 14 and 15 with 7 and 8 and recognising that $G_c = -G_d$, $B_c = -B_d$ etc we have eliminated the frequency dependence of the circuit and obtain

$$\frac{\Delta \omega}{\Delta T} = - \left[ \frac{3B_d}{\partial T} - \frac{3B_d}{\partial G_d} \right] \cdot w_0 \cdot |\sin \phi|$$

$$\frac{4 Q_L}{G_d} (1 + \frac{B_d}{G_d} \cdot \frac{3B_d}{\partial G_d})$$

...(15)

and for the series case

$$\frac{\Delta \omega}{\Delta T} = - \left[ \frac{3X_d}{\partial T} - \frac{3X_d}{\partial R_d} \right] \cdot w_0 \cdot |\sin \psi|$$

$$\frac{4 Q_L}{R_d} (1 + \frac{X_d}{R_d} \cdot \frac{3X_d}{\partial R_d})$$

...(17)

These two equations 16 and 17 therefore imply that, provided all the diode operating conditions remain constant (i.e. $w, G_d, B_d$) then $\frac{df}{dT}$ will vary as $\frac{1}{Q_L}$.

It is difficult, in practice, to make controlled experiments in which $Q_L$ is varied while the other parameters remain fixed. However Hobson quotes a result in which the addition of an extra half wavelength to a waveguide cavity doubled the $Q$ (as measured by a frequency pulling technique) and halved $\frac{df}{dT}$.

An alternative method of eliminating the frequency sensitivity of the circuit is to use the frequency pushing figure instead of the locking $Q$.

In this case

$$\frac{3}{2 \omega} (G_d + G_c). \Delta \omega + \frac{3G_d}{\partial V_B} \cdot \Delta V_B + \frac{3G_d}{\partial V} \cdot \Delta V = 0$$

$$\frac{3}{2 \omega} (B_d + B_c). \Delta \omega + \frac{3B_d}{\partial V_B} \cdot \Delta V_B + \frac{3B_d}{\partial V} \cdot \Delta V = 0$$

Eliminating $\Delta V$ we have
\[
\frac{\Delta \omega}{\Delta V_B} = -\frac{\frac{\partial B_d}{\partial V_B} - \frac{\partial G_d}{\partial V_B} \cdot \frac{\partial B_d}{\partial G_d}}{\frac{\partial (B_d + B_c)}{\partial \omega} - \frac{\partial (G_d + G_c)}{\partial \omega} \cdot \frac{\partial B_d}{\partial G_d}} 
\]

which can be combined with equation 7 to give

\[
\frac{\Delta \omega}{\Delta T} \cdot \frac{\Delta V_B}{\partial \omega} = \frac{\frac{\partial B_d}{\partial T} - \frac{\partial G_d}{\partial T} \cdot \frac{\partial B_d}{\partial G_d}}{\frac{\partial (B_d + B_c)}{\partial \omega} - \frac{\partial (G_d + G_c)}{\partial \omega} \cdot \frac{\partial B_d}{\partial G_d}} 
\]

Therefore in order to make valid comparisons between experiment and theory it is necessary to measure \(\frac{df}{dT}\) and either the locking Q or the frequency sensitivity to bias. The appropriate combination of measurements is then to be compared with the theoretical value of the relevant expression 16 or 19 which contains only terms which can be derived from models of diode operation.

We have therefore arrived at a reasonable method of dealing with cases in which only fundamental voltages exist in the circuit.

Where harmonic voltages are present the situation is much more complicated. It is no longer generally true that the admittances of device and circuit are conjugate and the determination of an oscillation frequency must proceed by way of the formulation given by Van der Pol\(^{107}\) and by Tan and Foulds\(^{108}\). It is beyond the scope of this thesis.
11. \textit{df/dT} Calculations on Models of the Simple Modes

11.1 Introduction

In chapter 8 the simplest pictures of the interaction of a resonant circuit with a Gunn diode were discussed. These types of interaction are now examined more closely to determine the effect of temperature change.

As was shown in the last chapter the $df/dT$ of an active diode in any circuit depends on the form of the circuit through the injection locking $Q_L$ and the loading of the diode $G_d$, $B_d$. It depends on the frequency $f_0$ and on the details of the operation of the diode through the terms

$$\left[ \frac{\partial B_d}{\partial T} - \frac{\partial G_d}{\partial T} \cdot \frac{\partial B_d}{\partial G_d} \right] |\sin \phi_0|$$

$$\left[ 1 + \frac{B_d}{G_d} \frac{\partial B_d}{\partial G_d} \right]$$

We can arrange equation 16, chapter 10, to the form

$$\frac{Q_L}{f_0} \frac{df}{dT} \frac{G_d}{G_{od}} = \frac{1}{4} \cdot \frac{\left[ \frac{\partial B_d}{\partial T} - \frac{\partial G_d}{\partial T} \cdot \frac{\partial B_d}{\partial G_d} \right] |\sin \phi_0|}{G_{od} \left[ 1 + \frac{B_d}{G_d} \frac{\partial B_d}{\partial G_d} \right]}$$

where $G_{od}$ ($R_{od}$ in the case of a series circuit) the low field conductance (or resistance) of the diode has been introduced as a normalising term. On the left hand side of the above equation are quantities which will be measured in practical circuits and on the right hand side are quantities which can be obtained by calculation for any given mode of operation.

The results of analytical calculations on the simple modes will be given in paragraphs 11.2 and 11.3. The details are to be found in Appendices 1 and 2.

In the models of delayed and quenched modes it is assumed that domain formation and extinction are instantaneous processes, that the velocity field characteristic is composed of two straight lines, ohmic.
below threshold and saturated at the valley level above threshold, and
that this characteristic scales linearly with temperature as discussed in
chapter 4. The diode is driven with a sinusoidal voltage or current and
from the calculated current or voltage the admittance, power output and
efficiency are obtained by Fourier Analysis. The variation of the
dielectric constant with temperature is not included.

It will be seen that one of the quantities to be evaluated in
the calculations is \( \frac{\partial B_d}{\partial T} \) subject to the r.f. voltage and the frequency
remaining constant. If the Gunn diode were to be mounted directly across
a simple parallel RC circuit as shown in Fig. 22a and discussed at the
beginning of Chapter 10 then \( \frac{df}{dT} \) would depend directly on \( \frac{\partial B_d}{\partial T} \) subject to
the load conductance remaining constant. Calculations of \( \frac{\partial B_d}{\partial T} \) subject to
constant load and to constant frequency have provided a useful check on
the main calculation in two respects.

Firstly, if the diode is virtually decoupled from the load, i.e.
the circuit has a high locking Q, then conductance changes in the diode
become unimportant and the two calculations should give identical results.
Secondly, even when heavier loading occurs, both calculations must show
the same sign of \( \frac{df}{dT} \) and, in particular, conditions for which zero \( \frac{df}{dT} \) is
obtained must coincide. It was found that these checks were satisfied
for the present calculations and the results of these are discussed below
in terms of the rather more general Q. \( \frac{df}{dT} \) parameter.

To conclude the chapter the effect of temperature on the simple
models of Copeland's LSA and relaxation LSA where negligible space change
build-up is assumed is discussed.

11.2 Voltage Driven Modes

11.2.1 Delayed Mode

The results of this mode are shown in Figs. 23 a, b and c. In
the frequency conductance plane the delayed mode occupies a lune at
frequencies between about 0.7 and 1.0 times the transit frequency. The
efficiency is almost proportional to the load conductance and reaches a
maximum value of 6% at about 0.84 of the transit frequency for a bias of
2 \( V_{TH} \) and a peak to valley ratio of 2:1. It is found that \( \frac{df}{dT} \) is negative
at low frequencies and positive at high frequencies with the zero
condition lying around 0.84 of the transit frequency. Thus this model
predicts that it should be possible to achieve zero $\frac{df}{dT}$ with close to
optimum power output. At higher drive voltage or lower peak to valley
ratio a similar picture is obtained but with the maximum efficiency
reduced to 3.1% in the latter case. The results are summarised in
Table 3 for various combinations of drive voltage and peak to valley
ratio. For an X-band oscillator with a locking Q of approximately 50
$\frac{df}{dT}$ in this mode will be roughly between -1 and +2 MHz $^{o}C^{-1}$ falling to
about one half on increasing the bias to $3 V_{TH}$ or decreasing the peak to
valley ratio to 1.43:1.

11.2.2 Quenched Mode

The quenched mode occurs at higher r.f. voltage swings than
the delayed mode and in the conductance-frequency plane occupies a lower
conductance region than the delayed mode. It also has the capability of
operating at frequencies above the transit frequency being limited
ultimately only by the resistive loss in the unused portion of the device.
The range of the mode is shown in Fig. 23 (a). Figs. 24 a, b and c show
the efficiency and $\frac{Q}{f_{o}} \frac{df}{dT} \frac{G_{d}}{G_{od}}$ plotted against the loading for the same
biasses and peak to valley ratios as were used for the delayed mode.
$\frac{df}{dT}$ is always positive and increases from zero when completely decoupled
up to an approximately constant value as the loading becomes heavier.
Table 4 presents the summarised results.

For the same circuit as was considered in the delayed mode the
quenched mode would give a maximum $\frac{df}{dT}$ of about +80 kHz $^{o}C^{-1}$ with a
factor of about two reduction on going either to $3 V_{TH}$ drive or 1.43:1
peak to valley ratio.

The small positive value of $df/dT$ obtained in the quenched
mode reflects the response of the diode to changes in the conductivity
of the material. That is to say with an increase in temperature the
current passed by the diode is decreased which in turn leads to a
decrease in r.f. voltage amplitude to maintain the diode conductance
at the value allowed by the circuit. The susceptive current likewise
decreases but the decrease in r.f. voltage counters this leaving the
diode susceptance essentially unchanged.
In the delayed mode however a new factor is added in the form of the transit time. This increases with increasing temperature and shifts the phase of the current appreciably with respect to the voltage. It is this phase shift and its effect upon the conductive and susceptive currents and through them on the r.f. voltage which leads to the much larger positive and negative values of $\frac{df}{dT}$ which are obtained.

11.3 Current Driven Modes

In the current driven modes a fully formed domain with a finite capacitance is assumed. The quantity $\omega C_D R_{od}$ then becomes a parameter in the calculations. Reasonable values in the range 0.04 to 0.20 have been investigated. In the delayed domain mode a typical figure for $\omega C_D R_{od}$ of 0.08 has been assumed for the particular results discussed below.

11.3.1 Delayed Domain Mode

Figs. 25 a, b, c and d show that the efficiency in this mode peaks below the transit time as in the voltage driven mode but attains much higher values and increases with drive voltage. The value of

$$\frac{Q df}{f dt} R_D R_{od}$$

is generally negative at the optimum efficiency but can take on large positive or negative values at the extremes of the frequency range. Table 5 presents data for the optimum efficiency case.

The plots of efficiency show broad maxima at somewhat lower frequencies than those of the voltage driven mode whilst the value of

$$\frac{Q df}{R_{o dt}} R_{od}$$

generally shows a shallow minimum with the lowest values occurring at frequencies a little lower than that at which peak efficiency occurs. Higher drive voltage or a lowering of the peak to valley ratio leads to higher $\frac{df}{dt}$. Unlike the case of the voltage driven mode it is not possible to change the loading on the diode while retaining the bias voltage and $\omega C_D R_{od}$ fixed, unless the frequency relative to transit frequency is allowed to vary.

For our typical X-band diode in a circuit of $Q_L = 50$ this mode predicts a $\frac{df}{dT}$ of around -1.1 kHz °C⁻¹ at maximum efficiency but with
positive or negative values of up to two orders of magnitude higher at the frequency extrema of the mode.

11.3.2 Quenched Domain Mode

The results of calculations on this mode are given in Figs. 26 a, b, c and d, where curves are presented for the optimum efficiency case and for about one half of this efficiency (allowing \( \omega C_D R_{od} \) to vary). At optimum efficiency \( \frac{Q_{df}}{R_d} \) rises with bias from zero and passes through a maximum. At lower power output higher values are obtained, with a similar dependence on bias but at lower peak to valley ratios the results are shifted towards lower biases. Table 6 summarises the results of calculations on this mode.

It is seen that at optimum efficiency under any conditions \( \frac{df}{dT} \) is very low and positive and would correspond to 200 Hz/°C for our typical X-band circuit. Decoupling by 3 dB changes this value very little, but would of course involve changes in the parameter \( \omega C_D R_{od} \) as would bias voltage changes.

Thus in the current driven modes we find the same distinctions between delayed and quenched modes that occurred for the voltage driven modes, with both positive and negative values of \( \frac{df}{dT} \) in the delayed mode but with much smaller positive ones in the quenched mode.

11.4 The Copeland LSA Mode

The case of the Copeland LSA mode is trivial. Since no space charge is assumed to build up the susceptance is given simply by the cold capacitance of the diode and the conductance is given by the action of the imposed voltage or current waveform on the current-voltage, i.e. velocity field, characteristic. Although the conductance is a function of r.f. amplitude the susceptance is not and so there is no coupling between the temperature sensitivities of the two parts of the admittance. Frequency variations are therefore due entirely to the change in cold capacitance with temperature. From eq. 16 chapter 10, we have

\[
\frac{\Delta \omega}{\Delta T} = \frac{2B_d}{\omega_c} \left[ \frac{4 G_d Q_L}{\omega_c} \right] \frac{1}{G_d}
\]
which can be expressed in terms of the diode capacitance \( C_d \) as

\[
\frac{\Delta \omega}{\Delta T} \cdot \left[ \frac{4}{\omega_o} \frac{G_d Q_L}{\omega_o} + C_d \right] \cdot \frac{1}{\omega_o} = \frac{-3C_d}{\Delta T}
\]

From this it is very easy to express \( \frac{\Delta \omega}{\Delta T} \) in terms of the dielectric constant change with temperature.

For the case of a 0.004 in dia., 10 \( \mu \)m thick diode in a circuit with a Q of 50 at 9.5 GHz the frequency variation with temperature will be about -22.5 kHz \( ^\circ C^{-1} \). This is small simply because the highly temperature sensitive velocity field characteristic has no effect on the frequency.

11.5 The Relaxation LSA Mode

In the case of the relaxation LSA mode harmonic resonances become very important and a simple consideration of operation only at the fundamental, as was assumed for the case of Copeland's LSA, cannot be used. For the specific case of a lumped circuit Christensson, Woodward and Eastman give a very simplified equation for the period of this mode.

\[
\sim 2\pi \sqrt{LC_d} + \frac{L}{R_{od}} \cdot \frac{V_{TH}}{(2V_B - V_{TH})}
\]

If we assume that, for the case of a c.w. diode in an S4 capsule resonating with a WG16 evanescent inductance which we will be discussing later,

\[
L = 2.2 \text{ nH}, \quad R_{od} = 10\Omega, \quad C_d = 0.36 \text{ pF}, \quad V_{TH} = 4V, \quad V_B = 10V.
\]

Then the frequency will be 7.1 GHz. If we now consider the effect of a temperature change the parameter which will have the greatest influence will be the low field resistance \( R_{od} \). Taking our previous figure of +0.0029 for \( \frac{1}{R_{od}} \cdot \frac{dR_{od}}{dT} \) we arrive at a value of \( \frac{df}{dt} \) given by,

\[
\frac{df}{dT} = \frac{f^2}{R_{od}} \cdot 0.0029 \cdot \frac{L}{V_{TH}} \left( \frac{V_{TH}}{2V_B - V_{TH}} \right) \quad \text{or} \quad +8.04 \text{ MHz } ^\circ C^{-1}
\]
11.6 Conclusions

The chief conclusion to be drawn from the above work is that the $\frac{df}{dT}$ to be expected in practice will be extremely sensitive to the mode of operation. However it must be remembered that those modes which have been considered are in many ways extremes since assumptions have been made as to the shape of voltage and current drive waveforms, and to the extent of domain build up, which are to some degree unrealistic for the X-band c.w. device. In the following chapter we shall see the effect of better modelling where space charge build up and decay processes are taken into account.
12. Calculations on a Model of the Hybrid Mode

The simple models discussed above suffer from the grave disadvantage that they either assume instantaneous domain formation or, in the case of the LSA, ignore space charge build-up altogether. Since the measured susceptance of the c.w. Gunn diode at the fundamental is about twice that due to the cold capacitance the space charge cannot in practice be ignored but conversely consideration of the magnitude of the dielectric relaxation time shows that domain formation is far from instantaneous. Furthermore the domain growth time depends directly on the velocity field characteristic and so will be very sensitive to temperature change.

A model which reflects most of these features has been developed. In a device driven by a sinusoidal voltage a small fully depleted domain is assumed to exist at the cathode at the moment that the field in the diode rises above threshold. Such a domain might form as a result of a field concentration at the cathode or through the conversion, by inhomogeneities, of an accumulation layer launched from the high-low junction. Following Huang, it is shown in app.3 that the rate of growth of this domain can be derived from a geometrical analysis of the voltage under the domain. Strictly speaking, growth should alter the shape of the domain but it is assumed here that there is no diffusion and that the domain remains fully depleted. From the amplitude and rate of growth of the domain the current and space charge velocity can be calculated. On arrival at the anode the domain is discharged by neutralisation of the depletion layer. This process takes place at a rate dependent on the instantaneous velocity of the domain. The problem has been treated numerically, dividing the period into as many parts as were necessary to avoid hunting of the current waveform. (80 parts were normally taken). Typical parameters for the X-band diode were chosen, i.e. 10 μm thickness, 10^{15} cm^{-3} carrier concentration and 10 GHz frequency with the initial domain amplitude being left as a variable. The velocity field characteristic used consisted of three straight lines (V_{TH} = 3.5 kV cm^{-1}, V_v = 10 kV cm^{-1}) scaling linearly with temperature as previously discussed. The dielectric constant was taken to have a temperature variation of +1.5 \times 10^{-4} \degree C^{-1}.

The results proved to be of great interest. They showed that
once the domain reaches an appreciable size the rate at which it can grow is relatively slow. This is because only those parts of the domain where the field gives rise to a negative resistance can contribute to space charge growth. The field in the bulk of the domain is beyond the valley and gives rise to neither growth nor decay. The direct result of this is that, at X-band, voltage is applied by the circuit to the diode faster than the domain can absorb it, so that the field outside the domain is driven above threshold. The diode therefore operates in a hybrid mode. This is true even when the domain is assumed to have an initial amplitude of 0.4V and the carrier concentration is as high as \(2 \times 10^{15} \text{ cm}^{-3}\). With starting amplitudes of 0.04V the domain growth is much less and the oscillation assumes much of the character of Copeland's LSA with predicted efficiencies of over 20%. For the subsequent work an initial amplitude of 0.4V, i.e. \(\sim 10\%\) of the threshold voltage, was chosen as not being too large and yet still giving substantial domain behaviour.

The waveforms of the applied voltage, the field outside the domain, the domain voltage, the terminal current and domain leading edge position are shown for a typical case in Fig. 27. The hybrid character shows immediately in the near sinusoidal behaviour of the field outside the domain \(E_k\) while the terminal current is forced well below that appropriate to the valley velocity by the large displacement current. The cessation of space charge growth when the entire domain is beyond the valley is particularly clearly shown. These waveforms are similar to those given by Bott and Fawcett\(^{102}\) which result from a more complete analysis of the diode space charge (Fig. 28). Current waveforms of the same general type have been calculated by Ito et al.\(^{109}\), and in addition the latter authors predict values for diode conductance and susceptance as functions of r.f. amplitude, which are quantitatively similar to those calculated here.

The extent of the hybrid mode is limited by several features in these calculations. For large r.f. voltage swings the r.f. conductance becomes positive. At small r.f. voltages it is found that the field at the cathode is above threshold at the time that the domain is quenched by the anode, so that two domains per cycle would result. This case has not been calculated. At high temperatures, an additional limit occurs due to the space charge not being completely dissipated at the end of the cycle.
At these relatively high temperatures the domain velocity is reduced, the anode plays less part in discharging the domain and the limit is similar to that normally employed in the discussion of LSA in overlength diodes.

Plots of the admittance as a function of r.f. voltage and temperature are shown in Figs. 29 a, b, c and d. From this data the terms \( \frac{\partial G_d}{\partial T}, \frac{\partial B_d}{\partial T}, \frac{\partial G_d}{\partial T} \) have been evaluated. The resultant value of \( \frac{Q_L}{f_0} \frac{df}{dT} \frac{G_d}{G_{od}} \) is presented as a function of load conductance and temperature in Table 7. The corresponding efficiencies are also shown.

At 11 GHz the range of the mode is much more restricted, operation above 270°K falling outside the limits discussed above. The form of the conductance and susceptance variation is very similar to that at 10 GHz but with the curves shifted by -34.5°K. It is interesting to note that this is exactly the shift which is necessary to keep the frequency electron velocity product constant. The calculated values of \( \frac{Q_L}{f_0} \frac{df}{dT} \frac{G_d}{G_{od}} \) and efficiency at 11 GHz are shown in Table 8.

In the hybrid model there is no abrupt transition between "delayed" and "quenched" modes, but we may compare results at high conductance with those of the delayed mode and those at low conductance with the quenched mode. These latter are shown in Tables 3 and 4. It is seen that the hybrid model tends to predict negative \( \frac{df}{dT} \) in all but a very few cases. This is because the dominant \( \frac{\partial B}{\partial T} \) evaluated from plots such as Fig. 29d is generally positive. However, the overall magnitudes of \( \frac{df}{dT} \) at both high and low conductances are in fair agreement with those predicted by the simple models.

In a typical cavity with a locking Q of 50 at 9.5 GHz, a diode would show a typical \( \frac{df}{dT} \) of -760 kHz °C\(^{-1}\) which would fall to zero on decoupling. Positive values could occur at moderate loading but it is clear that any zero value of \( \frac{df}{dT} \) would occur only for a limited temperature range and examination of Fig. 29d shows that both convex and concave plots of frequency against temperature might be obtained depending on the loading, temperature and frequency employed.
13. Microwave Circuits

13.1 Full Height Waveguide Circuit

At the start of this work it was realised that the two contributions to $\frac{df}{dT}$ from the diode and from the circuit could not simply be added together but that the total frequency sensitivity resulted from a complex interplay between diode and circuit. For this reason it was decided to remove the contribution made by circuit expansion by using gold plated invar.

The first circuit to be made was a direct copy of a full height X-band waveguide oscillator which had been successfully developed for medium power Gunn diodes. This cavity is shown in Fig. 30. The only alteration from the original design was the replacement of the ceramic tuning rod by an invar screw. Coupling to the load could be varied by means of an iris mounted on the front of the cavity block.

It was found that, by a suitable adjustment of the iris and tuning screw, the circuit could be satisfactorily set to the chosen frequency of 9.5 GHz and the injection locking Q could be varied from 20 to 20,000. Measurements of $\frac{df}{dT}$ as a function of locking Q were made on a variety of diodes and will be discussed in detail later.

It is of interest to know the impedances presented to the diode chip as a function of the frequency and cavity setting. Although it was demonstrated that the individual components of the circuit such as the tuning screw and iris functioned in accordance with theory, when these are mounted in close proximity, together with the diode mounting post and a short circuit, the coupling between their evanescent modes will be considerable. Taken together with the problem of deriving an equivalent circuit for an encapsulated diode mounted at the bottom of a long post, the chances of successfully modelling this circuit appeared slim.

13.2 Reduced Height Waveguide Circuit

To avoid these uncertainties a reduced height waveguide cavity, as shown in Figs. 31 and 32, was constructed. The height of the guide was chosen to be 0.064 in so that the S4 capsule was mounted directly across the cavity, showing only a cylinder of uniform diameter in the
waveguide. The transition from reduced height into full height waveguide is by a six step Chebyshev transformer designed by D.S. Tennet. Measurements using the reduced height slotted line, to be described later, showed that the v.s.w.r. of this transformer was less than 1.17 over the band 7.5 - 11.2 GHz (Fig. 33).

The back of the cavity is closed by a sliding short circuit plunger. This is of the contacting type, fitted with short springy beryllium copper fingers. Measurements on this plunger in the reduced height slotted section indicated that the electrical short circuit was positioned approximately 0.015 cm in front of the face of the plunger. This is due to the reduction in guide height caused by the thickness of the fingers and was confirmed by measurements of the cavity length for zero output from a Gunn diode. This was again found to be about 0.015 cm greater than $\lambda_g$ or $\lambda_g/2$.

The body of the cavity and the lower heat sink chuck for the diode were made of gold plated invar, the choke of anodised aluminium and the short circuit plunger of brass. The latter is held in position at any setting by the pressure of the fingers. Following the philosophy of keeping the cavity circuit as simple as possible, an iris was not used to control the coupling to the load. This was allowed to vary with the short circuit plunger position and the frequency to give values of injection locking Q ranging from below 10 to several thousand.

13.3 Determination of the Waveguide Mode to Diode Chip Transformation for the Reduced Height Waveguide Circuit

It is essential to the interpretation of $df/dT$ results obtained in a given circuit that the detailed behaviour of the circuit should be well known. The reduced height waveguide circuit meets this requirement provided that the electrical transformation through the package between the waveguide and the diode chip can be characterized.

An extensive characterization of the S4 package from 4-24 GHz has been carried out at RMCS by Owens and Cawsey. Their work was carried out with the package mounted at the end of a 7 mm coaxial line and the results are therefore only applicable to this configuration. Owens has derived a configuration-independent equivalent circuit for the package using the scheme suggested by Getsinger to account for the transformation of the field from the coaxial mode in the line to the
radial mode which is assumed to exist at the outer surface of the package. The package is thus characterized in terms of lumped elements terminating the radial mode and, with the appropriate transformation included, can be used in any configuration.

There are, however, several uncertainties in the process. The extent to which the radial mode is developed at the end of the coaxial line is in doubt as is the validity of the model used by Getsinger to account for fringing capacitance in the mode transformation. The encapsulations used for the measurements at RMCS were supplied by RRE and had 0.002 in dia. gold wires and were 0.015 in deep from flange centre to pedestal top. The effects of changing the wire size and the package manufacturer were not known. For these reasons it was felt necessary to make a quick check of the package in the reduced height waveguide in which it was being used so that the behaviour of the diodes could be analysed with some degree of confidence.

The problem resolves itself into two parts. First a check that the transformation between the waveguide mode and the radial mode around the capsule agrees with the well established theory and secondly the measurement of the package parameters.

For the former experiment a series of radial shorts were made up. These brass pieces were machined to the external dimensions of the S4 package but had varying "alumina" diameters. It was found difficult to make measurements in the reduced height guide through the transformer because of the phase shifts introduced and so a reduced height slotted section capable of being mounted on a HP 809C probe carriage was constructed. The shorts were mounted in the waveguide shown in Fig. 34 which is a complete replica of the reduced height waveguide cavity, but without the transformer to full height guide. The contacting short circuit plunger used in the reduced height cavity was characterized and placed to present an open circuit at the plane of the diode.

The results of the measurements of the admittance of the radial shorts are shown in Fig. 35 where they are compared with the theoretical values given by Marcuvitz. There is good agreement but the measured values lie consistently at slightly higher impedances than the theory would predict. The discrepancy (in round terms, 0.04 nH) is thought to be due to the detailed method of mounting the diode in the waveguide. The
diode is recessed into the upper and lower faces of the guide. This results in an additional length of current path at the short position which will appear as a small inductance.

Ggetsinger shows that the transformation between waveguide and radial modes is as given in Fig. 36a. The capacitances $C_1$ are negligibly large and therefore if a short is placed in the radial line the transformation reduces purely to an inductance which is equal to the impedance of the shorting post as measured in the waveguide mode. (The negative capacitance, $C_2$ represents an element necessary to counter the possible addition of a positive geometrical capacitance $C_2$ which must represent unperturbed waveguide). Thus the transformation to a radial mode at the outer surface of the S4 package is obtained from the measured inductance of the 0.080 in radial short, which being an experimental value includes the extra inductance due to the mounting.

Two types of package are in use at STL (Fig. 36b). The one designated 'old' is dimensionally equivalent to that described in reference 110 but with a flange centre to pedestal top distance of 0.025 in while in the package termed 'new', this dimension is 0.033 in. Wires of 0.001 in and 0.002 in dia, together with the broad tape often used for high power diodes, were used in the assembly of open and short circuited capsules. Empty capsules were also measured.

The admittances of these dummy packages were obtained from the slotted line and by subtraction of the mode transformation described above, the lumped admittances presented to a radial wave at the outer surface of the package were found. These are compared in Fig. 37 with the values obtained by Owens from coaxial line measurements on his original 0.015 in capsules, and with later measurements using diodes provided by STL 0.035 in deep.

Assuming the fringing capacitance $C_3$ from the wire to the post of Fig. 36b to be small, all the open circuit packages of a similar structure should show approximately equal admittance and this is found to be so. The short circuit capsules show more variation, as expected, but separate measurements on two similar 0.001 in dia wired 'new' capsules are in good agreement and the results for 'old' and 'new' capsules with 0.002 in dia wires are in the expected relationship one to another. Rather than to try to derive equivalent circuits by curve
fitting to the measurements it was considered better to take Owens' results, which were made over a much wider frequency range, and to change the values slightly to obtain a fit.

The present results are compared with those of Owens in Table 9.

The general levels are very similar and on increasing the depth of the capsule there is a reduction in capacitance and increase in inductance as is expected. The measurements made in waveguide appear to show more series inductance and shunt capacitance than those made in the coaxial line. It is possible that the discrepancy is due to the different height radial lines to which the measurements are referred. Whereas the waveguide measurements used a height of 0.065 in the coaxial line measurements were made in line of rather lower impedance and this would naturally tend to decrease the calculated series inductance in the coax. As to some extent series inductance and shunt capacitance are interchangeable over a narrow frequency band, this may also explain the lower value of the latter in coax.

It is more difficult to explain the greatly increased inductance when 0.002 in wires are used. It is possible that this should be interpreted as extra shunt capacitance due to the increased surface area of the 0.002 in dia wire and due to the finite curvature of the wire in the region of the chip leading to a value for \( C_3 \) which is not negligible.

For the present work the values obtained for the 0.033 in deep capsule with the 0.001 in dia wire are considered sufficiently accurate.

13.4 Theoretical Model for the Reduced Height Waveguide Circuit and its Application

Using the package and mode transformation obtained from the slotted line measurements a model of the circuit was set up as shown in Fig. 38a and a computer program used to calculate the admittance presented to the diode chip as a function of cavity length and frequency. This calculation assumes that the Chebyshev transformer is perfect. An estimate of the error involved in this assumption can be obtained from Table 10 where a complete model for the transformer including discontinuity susceptances as shown in Fig. 38b and Table 11, has been used to calculate the admittance presented to a particular diode, R360C 6LE158 A10, at the
frequencies of 9, 10 and 11 GHz and also at the second harmonic of these frequencies. A fuller discussion of the magnitude of these admittances will be given later. It is sufficient to note here that in agreement with the v.s.w.r. measurements reported earlier, there is very little error involved in the assumption of a perfect transformer.

With the aid of this model the loading presented to any diode can be found. As an example we consider a diode which shows the features observed during the measurements particularly clearly. In Fig. 39a are plotted the observed frequency and power variations with the length of the cavity. The frequency is sensibly constant at a high value for short cavity lengths and decreases steadily as the cavity length increases. Continued lengthening causes a jump from the λ/2 cavity mode to the λ mode. The power shows a medium level when the frequency is saturated, begins to fall at the commencement of frequency decrease, falls to zero as the cavity becomes exactly λ/2 or λ long and then rises again before the jump to the next cavity mode.

Taylor, Fray and Gibbs(115) have seen similar frequency saturation effects although these generally occurred between 16 and 22 GHz whilst those reported here were always between 10 and 13 GHz. A coaxial resonance of the diode with the package and its mounting structures was found to be the cause of the phenomenon reported by Taylor, Fray and Gibbs and to check whether the lower frequency saturation was caused by a similar resonance, a reduced height radial cavity with diameter equal to the waveguide width was made up. In this structure packaged Gunn diodes operated at around 6 GHz and this was also found to be the resonance frequency when an identically packaged varactor of similar capacitance was substituted for the Gunn diodes. It was therefore apparent that a coaxial resonance at 12 GHz was not limiting the frequency.

Fig. 39b shows the calculated conductance and susceptance corresponding to Fig. 39a. It is clear that when the frequency tunes smoothly with the length of the cavity the susceptance takes on a value of about 7 mmhos (about twice the cold susceptance) whilst the conductance varies from zero up to two or three mmhos, following closely the shape of the output power plot. The r.f. voltage swing at the fundamental, calculated from the power, is close to that required to take the total voltage below threshold. In the frequency saturated regions however, the conductance and susceptance take on very large values, the latter can
be of either sign, and the r.f. voltage is very small. This behaviour strongly suggests that the observed output frequency is not the fundamental and close examination with the spectrum analyser of leakage through the bias choke revealed a subharmonic frequency of about 6 GHz. The subharmonic was present only when the output frequency saturated and abruptly disappeared once the tunable region was reached.

The frequency cavity-length plot should therefore be considered as a composite of two different modes of operation. So long as the admittance presented to the diode chip by the circuit is less than a certain limit (typically about 2 mmhos), the diodes can operate conventionally with power output at the fundamental and be tuned by the varying circuit susceptance. If heavier X-band loading is applied the diode is forced into a mode with zero loading at the fundamental, in which it resonates with the evanescent inductance of the waveguide. This latter quantity transformed through the package is shown as a function of frequency in Fig. 40, and for reasonable diode dynamic susceptances, it is seen that resonance at around 6 GHz will take place. For very short cavity lengths the evanescent inductance is decreased which should lead to an increase in the saturated frequency. The opposite is often observed and it is likely that this is due to interference between the short circuit plunger and the radial fields local to the diode. Power is extracted at the second harmonic with a saturated frequency of around 12 GHz. (The similarity of a diode mounted in a radial cavity to that mounted across a cut off waveguide of the same dimension accounts for the observed resonance in that case of 6 GHz).

The two cavity modes do not occur for all devices. All show the evanescent resonance when heavily loaded at X-band and in some cases this mode persists even where the X-band resonance is lightly loaded. (See for example Fig. 41). These diodes can sometimes be forced into the X-band fundamental mode by setting the cavity to a position where the loading on this mode would be relatively heavy, i.e. the resonance would have a low loaded Q, hopefully lower than that of the evanescent mode, removing all capacitance on the bias input and applying the full drive voltage in a step. The diodes could then be tuned in the normal manner provided the limiting conductance for fundamental X-band operation was not exceeded. No striking correlation of the behaviour in this respect with the properties listed in Table 1 was observed, although it might be expected that the slice thickness and the carrier concentration profile through the slice would be among the determining factors.
13.5 Measurements with a Scale Model of Package and Circuit

One of the great unknowns of c.w. Gunn diode operation is the form that the r.f. voltage and current waveforms take. Largely this arises from the difficulty of making meaningful measurements at frequencies of up to 30 GHz, which would be necessary if the first three harmonics were to be included. Some idea of what might be expected can be obtained if the admittances presented to the diode chip at the various harmonics can be found. For instance, much higher load admittances at the harmonics compared with the fundamental would suggest that the voltage waveform would be likely to be sinusoidal whereas the current waveform would have a high harmonic content.

The theoretical model of the reduced height waveguide circuit can be used to calculate the admittance presented to the diode if the full representation of the Chebyshev transformer is used, if the dominant waveguide mode excited is the $TE_{01}$ and if the equivalent circuit of the package derived in X-band is still applicable. Table 10 presents second harmonic admittances calculated for a particular diode where these assumptions have been made. They do not differ greatly in order of magnitude from the fundamental admittances.

As a more valid check on the harmonic loading models of the circuit and package scaled by 3.5 times were made up. A probe of 0.141 inches semi rigid cable was terminated in a short length of air line which emerged through the post of the capsule and protruded by about 0.020 inches. The wire of the encapsulation was then soldered to this centre conductor.

By keeping the dimensions of the probe small compared with the wavelength, it was hoped that the transformation between the coaxial measuring mode and the modes existing in the capsule, whatever they might be, would be small enough to be neglected. As a correction factor we should perhaps add in some shunt capacitance to represent that removed by the presence of the probe.

The method of mounting the dummy package in the circuit followed exactly the procedure used in the X-band circuit. The waveguide beyond the Chebyshev transformer was terminated by a resistive vane attenuator with a measured return loss of about 78 dB.

Three typical frequency/cavity length plots given by different diodes were taken and the corresponding scaled circuit lengths and
measuring frequencies were set up. Measurements were made, on a slotted line, of the admittances seen by the chip at the fundamental, second and third harmonics. Fig. 42a shows the results for diode R360C/6LE 158/B2 where a comparison has been made at the fundamental with admittances derived from the theoretical circuit model.

The qualitative agreement between the measured and calculated curves is extremely good but quantitatively the measurements show an additional resistive loss of about 1.1 mmho which is not unreasonable, and lie lower in capacitance by about 0.095 pF at X-band. In part the capacitance difference is due to the presence of the probe but this cannot explain the total discrepancy. It is possible that the dielectric constant of the machinable alumina used to fabricate the scaled capsule is lower than that of the fired alumina used in the S4 construction. The measured susceptances should therefore have a correction of +12 mmhos applied at the second harmonic and +18 mmhos at the third.

From the corrected plots of Fig. 42b and 42c two features emerge. The conductances at the harmonics are largely controlled by the position of the short circuit plunger as is the case for the fundamental conductance. Peaks occur when the cavity length is an odd number of $TE_{01}$ wavelengths and low values when it is even. This implies that the dominant mode excited at the harmonics is $TE_{01}$, as might be largely expected from the symmetry of the arrangement. Secondly the conductance and susceptance at the second and third harmonics fluctuates wildly with a total admittance of roughly the same magnitude as that at the fundamental.

Small effects due to harmonic tuning can be seen in the power output of the device as a function of loading. When the fundamental r.f. voltage is calculated, it is rarely found to be sufficient to take the device from the bias condition to below the Gunn Effect threshold. As this is a condition for the existence of all modes other than those which rely on space charge dissipation by the anode it is necessary to postulate that the difference is made up by the additional loss found in the measurements and by harmonic voltages, and as these latter will be strong functions of cavity length we would expect the fundamental voltage to vary somewhat across the tuning band of the device. This is observed in practice. Similarly the effect of harmonic tuning is indirectly illustrated in Fig. 41 where the frequency of the evanescent cavity mode is perturbed by the tuning of the X-band second harmonic.
The lightly loaded conditions near second harmonic $\frac{\lambda_g}{2}$ and $\lambda_g$ cavity lengths, which are likely to favour the build up of high second harmonic voltage, increase the effective capacitance of the device at the fundamental thus giving a local frequency decrease.

In conclusion there is experimental evidence to show that significant harmonic voltages and currents can and do exist in the reduced height waveguide circuit. To a first order they do not appear to affect the fundamental frequency and power output very much but should if possible be taken into account when considering the detailed behaviour of the device.
14. Experimental Measurements of $\frac{df}{dT}$

14.1 Full Height Waveguide Cavity

Measurements were made in the full height invar waveguide cavity using the phase bridge of Fig. 43 to measure frequency drift from the phase shift produced an injection locked oscillator. (This relationship has been derived in Chapter 12). The temperature of the oscillator was varied slowly over a range of about 5°C by a small heater clamped to its outside face. A thermocouple inserted into a deep hole in the cavity block was used to measure temperature. The injection locking method was originally chosen because it was believed that, since the frequency of the locked oscillator remained constant throughout the measurement, the frequency sensitivity of the circuit would play no part and the diode would appear as if in a lumped circuit. However, by considering the possible paths to arrive at a given point in the phase frequency plane, it can be shown that phase changes of the oscillator, made under constant frequency, i.e. locked, conditions, reflect the distributed nature of the circuit. This is because pulling of the locked oscillator away from its natural frequency by the locking signal occurs prior to any changes which may take place at constant frequency. However, the bridge was useful in that small frequency changes of less than 100 kHz were easily measured and a fast display of frequency change within a pulse could be obtained by displaying the output on an oscilloscope.

The majority of the measurements were made on three silver tin contacted slices K258B, K217B and K167B with the properties listed in Table 1. These crystals cover a range of about five in n.1 product. This parameter was considered important in determining the degree to which domain mode (i.e. velocity field characteristic) or LSA (i.e. cold capacitance) behaviour would dominate. Results were taken at 8V bias with the iris being adjusted to give different locking Qs. The frequency was set to 9.5 GHz by means of the screw behind the diode.

As discussed previously the loading on the diode $G_d$ was not precisely known in this cavity and so Fig. 44 shows the measured values of $\frac{df}{dT}$ as a function of the injection locking Q for a total of eleven diodes from the three crystals. The following features are apparent,

1) $\frac{df}{dT}$ is negative for all the cases investigated.
2) There is an inverse relationship between $\frac{df}{dT}$ and locking Q which, since the loading on the diode decreases with increasing Q, shows a decrease in $|\frac{df}{dT}|$ with decoupling.

3) For a given Q diodes with the lowest resistivity and highest n,l product have the lowest $\frac{df}{dT}$.

4) There is a scatter of about 2:1 in the $\frac{df}{dT}$ values at a given Q for different diodes from the same slice.

The scatter in the individual resistivities of diodes from a given slice was small and the repeatability of measurement was checked by replacing a diode eight times in the cavity. This showed a spread in $\frac{df}{dT}$ of less than 10%. The effect of non-uniform packaging was investigated by use of varying numbers of wires to the chip and capsules of different depths. The results suggested that minor variations in the packaging would not account for the observed spread in $\frac{df}{dT}$. The thermal resistances of a batch of 27 diodes from slice R294D mainly clustered together around 125°C W⁻¹ with three being twice as large and one three times as large. The bonding technique was therefore quite uniform and the variability must lie with the diodes themselves. It was observed that, on keeping the tuning of the circuit fixed, the oscillation frequencies varied by about ±1%. However, on roughly transforming the circuit impedances back to the chip a spread of 30% in the effective diode capacitance was found. This is probably because the AgSn contact is likely to be affected by temperature changes such as might occur in the bonding cycle. Later measurements in the reduced height cavity showed that air baking of Fig. 14a reduced the calculated r.f. voltage at the chip from 3.07V to 2.54V and this will obviously affect the effective capacitance of the diode.

14.2 Reduced Height Waveguide Cavity

Measurements in this cavity were made using a Hewlett Packard frequency counter and transfer oscillator, the heating arrangements being similar to those used in the case of the full height circuit. The presence of two different modes in the reduced height cavity has been previously discussed. Roughly one third of the slices examined were not or could not be induced to operate in the fundamental X-band mode. These slices
tended to include those thicker than 10 μm and they often gave their best efficiencies in a coaxial test cavity at between 5 and 7 GHz. It was also notable that the only AgSn contacted slice included in their number, R305B, was particularly efficient at 7.6%. Thus resonance with the evanescent inductance at 6 GHz might be associated with a wider bandwidth of negative resistance in devices with better contacts. This view is supported by the case of R134B/5 LE7 which resonated with the evanescent inductance when biased normally (i.e. substrate + ve), but operated with a fundamental X-band resonance when reversed biased. Under the latter conditions the power output was very low. A similar low frequency oscillation has been recently found in full height waveguide cavities where the diode resonates with the evanescent inductance at about 3 GHz and appreciable power is coupled out at the third harmonic.

Of the diodes which did operate in the required TE\textsubscript{01} mode the AgSn contacted diodes all gave negative $\frac{df}{dT}$ values. For these slices the $\frac{df}{dT}$ varied from $-2.5$ MHz $\circ C^{-1}$ at a $Q_L$ of 20 to $-240$ kHz $\circ C^{-1}$ at a $Q_L$ of 500.

The devices with n\textsuperscript+ GaAs contacts were more variable. The general picture was of negative $\frac{df}{dT}$ regardless of the tuning of the cavity. The data for device B601BA/61 shown in Fig. 45 is typical although slice R368C/6LE192 produced much larger values of $\frac{df}{dT}$ than normal, particularly at high bias. This is shown in Fig. 46. However, the calculated values of $\frac{Q_L}{f_o} \cdot \frac{df}{dT} \cdot \frac{G_d}{G_{od}}$ are very much in accord with those of Fig. 45. Slices B539a and K172B/P631 both produced small isolated positive values at about 10.9 GHz with moderately heavy loading. These two slices have a similar thickness although slice B6018, which is also 8 μm, did not show any positive values, but as can be seen from Fig. 45 it did show a pronounced minimum in $\frac{Q_L}{f_o} \frac{df}{dT} \frac{G_d}{G_{od}}$ at 10.9 GHz. The exception to the general picture given above was slice R360C/6LE158 which, as is shown in Fig. 47 consistently produced positive $\frac{df}{dT}$ falling to zero as the frequency decreased and loading increased. This slice was somewhat thicker than most at around 12.5 μm. For all slices it was found that $\frac{df}{dT}$ did not decrease beyond a certain limit as the diode was more lightly loaded and the locking $Q$ became very large.

In the case of those diodes which resonated with the evanescent inductance positive $\frac{df}{dT}$ was normal although a variation in magnitude and excursions into negative values occurred as a function of cavity length.
The example given in Fig. 48 for slice R134/5LE7 demonstrates that the negative values occurred at cavity lengths where the second harmonic was lightly loaded and could have existed as a fundamental resonance in its own right. The positive values of $\frac{df}{dT}$ were sometimes quite large, $+6.4 \text{ MHz}^\circ\text{C}^{-1}$ being recorded on one diode from slice R134B/5LE7 and $+14 \text{ MHz}^\circ\text{C}^{-1}$ on a metal contacted diode from slice R305B. Analogous $\frac{df}{dT}$ behaviour has been found in full height cavities where the evanescent resonance is the third subharmonic of the detected X-band output.
Before drawing any conclusions from the experimental and theoretical work reported above it will be useful to assemble any further evidence which has been published on the subject.

Bird et al.\textsuperscript{(116)} have reported large negative $\frac{df}{dT}$, which increased at low temperatures, from AgSn contacted Gunn diodes and suggested that this was due to the temperature dependence of the contact resistance in such diodes. The use of $n^+$ GaAs contacts was found to improve matters considerably in that $\frac{df}{dT}$ was still negative but was much more constant with temperature. On decreasing the loading of the diode by 6 dB from its maximum value $\frac{df}{dT}$ assumed very low positive or negative values. In a later paper Bird, Bolton, Edridge and Geraghty\textsuperscript{(117)} stated that the amount of decoupling from the maximum load condition which was required to obtain low $\frac{df}{dT}$ could be less than 6 dB and in some cases was zero. It was suggested that operation in the low $\frac{df}{dT}$ state corresponded to a quenched domain mode. Hobson\textsuperscript{(106)} has given essentially similar experimental results to those of ref.\textsuperscript{(116)} and states that $\frac{df}{dT}$ is not a strong function of frequency. In addition he has performed a simple analysis for the voltage driven delayed and quenched domain modes which is similar to that discussed in Chapter 11. He obtained similar results with $\frac{df}{dT}$ being negative at low frequencies and positive at high frequencies in the delayed mode and positive in the quenched mode, but his analysis is specific to a parallel circuit in which the load resistance appearing across the Gunn diode is frequency independent. Clarke\textsuperscript{(118)} has correctly siezed on the velocity field relationship as being the dominant factor in determining $\frac{df}{dT}$, but his method of evaluating the characteristic by using the temperature variation of the phenomenological constants of the McCumber and Chynoweth\textsuperscript{(94)} analysis is much inferior to the Monte Carlo method of Ruch and Fawcett. His experimental results showing $\frac{df}{dT}$ of -1 to -2 MHz $^{-1}$ in a cavity with a frequency/temperature coefficient of -200 kHz $^{-1}$ are almost certainly taken on metal contacted diodes.

Davies, Gurney and Mircea\textsuperscript{(119)} report the results of an analysis based on computerised mobility or resistivity notch models. They find reasonable correlation between the value of $\frac{df}{dT}$ and the r.f. voltage swing across the diode. Positive $\frac{df}{dT}$ is predicted for large swings and negative for small swings. This is also found experimentally and they suggest
that devices with metal contacts naturally produce lower r.f. voltages and so must be decoupled much further from the optimum power output condition before zero $\frac{df}{dT}$ can be obtained.

Returning now to the results presented in Chapter 14 and dealing first with those diodes which resonated with the evanescent inductance at round 6 GHz. Both 6 GHz and 12 GHz oscillations coexist at all cavity settings and it appears that the $\frac{df}{dT}$ is largely determined by whichever is dominant. Away from the second harmonic $\frac{n\lambda}{2}$ settings the 12 GHz signal can exist only as a second harmonic of the 6 GHz and we have positive values of $\frac{df}{dT}$. Close to the $\frac{n\lambda}{2}$ settings the 12 GHz signal can exist in its own right but it appears also to sustain the 6 GHz oscillation. $\frac{df}{dT}$ is then negative, as for diodes operating with an X-band fundamental and no low frequency oscillation. If this picture is correct then it remains to explain the positive $\frac{df}{dT}$ associated with the low frequency operation.

It has been shown in Chapter 12 that a change of frequency in the hybrid mode is largely equivalent to a change in temperature and that we should therefore expect to obtain the variation of susceptance with temperature at 6 GHz by moving to lower temperatures on the plot of Fig. 29d. If this is done it is found that the dominant $\frac{dB}{dT}$ becomes negative for reasonable r.f. amplitudes and the corresponding $\frac{df}{dT}$ will be positive. Alternatively it can be argued that the 6 GHz operation is some form of relaxation oscillation. This will be poorly developed as only at the fundamental does the circuit approximate to a lumped inductance and the diodes are underlength. However on the basis of a relaxation mode it was calculated in section 11.5 that the second harmonic would show temperature variations of up to $+16 \text{ MHz} / \text{°C}$ which compares well with the $+14 \text{ MHz} / \text{°C}$ measured on one diode. On balance a hybrid mode with some relaxation character to account for high values seems to offer the best explanation of the observed positive $\frac{df}{dT}$.

The slices in which fundamental X-band operation was obtained appear to be in reasonable agreement with the hybrid mode theory of Chapter 12 in that $\frac{df}{dT}$ is generally negative with values of $\frac{O_L \frac{df}{dT} G_d}{f_0 \frac{df}{dT} G_{od}}$ which are about $-2 \times 10^{-4}$ when heavily loaded falling to around $-2 \times 10^{-5}$ on decoupling. Positive values of this parameter, when they occur, are low being $+2.4 \times 10^{-6}$ to $+1.2 \times 10^{-5}$ for B539a/12 and are at a moderately heavy loading. Both of these features are as
predicted by the theory. The only points of conflict occur at those cavity lengths where the loading on the diode, as measured by the power output of the cavity, becomes zero. As shown in Fig. 46, there is a slight discrepancy between these measured lengths and those at which the cavity would be $n \lambda / 2$ long, with the result that the calculated loading on the diode does not tend to zero at quite same length as the injection locking $Q_l$ tends to infinity. This produces artificially high values of $Q_l \frac{df}{dT} G_{od}$ at these cavity settings and zero values at the measured half wavelengths.

The only slice which does not fit this picture is R360C/6L6E158. This is a thick slice of 12.5 μm and it is therefore operating at relatively high frequencies. The hybrid mode model predicts low negative $\frac{df}{dT}$s for high frequencies (see high temperature regions of Fig. 29d). With the increased thickness and relatively high doping level this slice should perhaps be interpreted more in terms of the instantaneous domain formation models of Chapter 11 than the hybrid theory of Chapter 12 which has been evaluated for 10 μm and $10^{15}$ cm$^{-3}$ carriers. If this is done then the plots of Fig. 47 are to be interpreted as a parabola running from right to left across Fig. 23a Values of $Q_l \frac{df}{dT} G_{od}$ range from $1 \rightarrow 2 \times 10^{-4}$ at high frequencies down to zero at low frequencies in broad agreement with the values of Fig. 23a.

The other slices however are more difficult to explain in terms of the simple instantaneous domain formation models. It may be argued that these slices all work in the voltage driven delayed mode at frequencies between 0.7 and 0.85 of the transit frequency but, while they show appropriate $Q_l \frac{df}{dT} G_{od}$ the range of frequency and thickness makes this hypothesis unlikely. Operation in the current driven delayed mode is possible and the observed values of $Q_l \frac{df}{dT} R_{od}$ are again of the correct order but as with the voltage driven modes the isolated positive values of $\frac{df}{dT}$ at moderately heavy loading cannot be explained.

Therefore, to summarise, the present experimental results are explained quite well in terms of sign and magnitude on the basis of a hybrid mode theory except for those of the thicker crystal for which an instantaneous domain formation model appears more appropriate. More
generally the hybrid mode seems to explain well the observations of Bird et al. and Hobson that diodes with negative $\frac{df}{dT}$ can be decoupled into regions of low positive $\frac{df}{dT}$ and that such a change can also be brought about by temperature shifts. It is at variance with the results and theoretical work of Davies, Gurney and Mircea in that $\frac{df}{dT}$ is not calculated to be positive at the lowest conductances. However the precise details of the thicknesses and resistivity of the diodes used in the published accounts is unknown and it must be borne in mind that there will be a steady progression away from the hybrid model towards the simple models as the n.i product is increased.

From the present work no clear distinction emerges between n$^+$ GaAs and AgSn contacted diodes in terms of $\frac{df}{dT}$ or in terms of the r.f. voltage swings which can be calculated from the power output and admittance of the chip. This is possibly because the measurements have been limited to the region around room temperature where there is little difference between the AgSn and the n$^+$ GaAs contact. However it is worth noticing that positive $\frac{df}{dT}$ associated with fundamental X-band operation has not been found on metal contacted diodes and it is possible that contact effects which vary with temperature are responsible for this.
16. Conclusions

The values of $\frac{df}{dT}$ which have been observed have been shown to be largely consistent with a model of the hybrid mode in which the degree of domain character varies for different slices of GaAs. In particular slices of low thickness appear more consistent with the hybrid model of Chapter 12 while for greater thickness the voltage driven quenched and delayed modes with rapid domain formation appear to be a better description. By taking the calculations on these modes as useful limiting cases we can predict the values of the parameter $\frac{Q}{f_0} \cdot \frac{df}{dT} \cdot \frac{G_d}{C_{od}}$. In the hybrid mode the value falls from about $-3 \times 10^{-4}$ at heavy loading down to zero when completely decoupled. With a correct choice of operating frequency and temperature and with loading about 3 dB below maximum positive values of up to $+4 \times 10^{-5}$ can be obtained. In the voltage driven delayed and quenched modes values of $-1 \times 10^{-4}$ or $+5 \times 10^{-4}$, dependent on frequency, at heavy loading fall to zero on decoupling.

There will, of course, be a complete gradation of behaviour between these two limits and for any given slice the important parameters will be the doping level, length, doping distribution through the layer and the nature of the contact regions. In addition the circuit can be expected to play a significant role since the degree of space change build-up will depend on the time the diode spends with fields in the negative resistance region of the $v(E)$ characteristic. This will be controlled, via the current and voltage waveforms, by the harmonic impedances of the circuit.

The present $\frac{df}{dT}$ measurements have shown no great difference between AgSn contacted diodes and those with $n^+$ GaAs contacts, with the proviso that no AgSn slice has shown any regions of positive $\frac{df}{dT}$ in fundamental X-band operation. Differences are very apparent however in the temperature behaviour of the sub-threshold properties and it would not be surprising if there were greater effects on $\frac{df}{dT}$ at lower temperatures as reported by Bird et al. (116).

The analysis of $\frac{df}{dT}$ in terms of the locking $Q$, the loading on the diode and the r.f. voltage and temperature dependence of the diode admittance can be used for any model of an active device in any mode and
enables the results of calculations to be expressed in a form useful to the circuit engineer who has to trade off one parameter of an oscillator against another.

It may be argued that the models which have been chosen in this work as limits are not particularly good representations of Gunn diode behaviour and this is no doubt true. The more sophisticated analysis of Jones and Rees (105) seems to be coming closer to the observed properties of X-band and thinner Gunn diodes and we may confidently expect that in the near future fairly complete explanations of all facets of the operation of such devices will be available. In addition GaAs material is now being grown much more repeatably with good control of carrier concentration and thickness, although it is only fair to add that the interface between the substrate and epitaxial layer is still a rather unknown quantity. The metal and n⁺ contacts to GaAs are beginning to be understood theoretically and are in practice under good control but the detailed quantitative links between experiment and theory are, at the moment, lacking. Taken together with the increased activity of the type reported here in the analysis of Gunn diode circuits it seems reasonable to expect that good control over second order parameters such as noise and frequency drift with temperature will be shortly achieved.

The present contribution has examined those features which will be important in the specification of \( \frac{df}{dT} \) and, although all the pieces of the jigsaw are not available at this time, has provided a good framework on which the total pattern may finally be built.
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### Table 1
Properties of the Crystals used in the Experiments
(a) Silver-Tin Contacted Diodes

<table>
<thead>
<tr>
<th>Slice</th>
<th>n-900 cm^{-3}</th>
<th>u-900 cm^{2}V^{-1}sec^{-1}</th>
<th>u-77 cm^{2}V^{-1}sec^{-1}</th>
<th>Thickness ( \mu )m</th>
<th>B300 E77 14th 77 1000 77 Vth 77 1000 77 300K splitting</th>
<th>77K splitting</th>
<th>( n ) 10^{15} cm^{-3}</th>
<th>Thickness ( \mu )m</th>
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(b) Vestigially Contacted Devices
(i.e. Doping raised to \( 2 \times 10^{15} \) cm^{-3} over 1-2 \( 10^{15} \) cm^{-3} followed by AgSn contact)

<table>
<thead>
<tr>
<th>Slice</th>
<th>n-900 cm^{-3}</th>
<th>u-900 cm^{2}V^{-1}sec^{-1}</th>
<th>u-77 cm^{2}V^{-1}sec^{-1}</th>
<th>Thickness ( \mu )m</th>
<th>B300 E77 14th 77 1000 77 Vth 77 1000 77 300K splitting</th>
<th>77K splitting</th>
<th>( n ) 10^{15} cm^{-3}</th>
<th>Thickness ( \mu )m</th>
<th>Comments</th>
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<td>+ve +ve avalanche +ve +ve avalanche</td>
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<td>1.24</td>
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<td>8.4</td>
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<td>E313 A</td>
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<td>7,700</td>
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<td>&lt;22</td>
<td>5.94</td>
<td>1.61</td>
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<td>E314E/515ET1</td>
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<td></td>
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<td>4.79</td>
<td>1.60</td>
<td>1.59</td>
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<td>0.43</td>
<td>6,600</td>
<td>68,000</td>
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<td>1.52</td>
<td>1.64</td>
<td>0.928</td>
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<tr>
<td>E355A</td>
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<td>7,700</td>
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<td>1.305</td>
<td>11.0</td>
<td>2.505</td>
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<td>5.21</td>
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<td>E355C</td>
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<tr>
<td>E355E</td>
<td>2.6</td>
<td>7,600</td>
<td>44,000</td>
<td>0.352</td>
<td>15.6</td>
<td>2.64</td>
<td>1.58</td>
<td>1.58</td>
<td>0.967</td>
</tr>
<tr>
<td>Flessey 5539a</td>
<td>~8</td>
<td>2.57</td>
<td>1.52</td>
<td>1.59</td>
<td>0.844</td>
<td>0.875</td>
<td>v. slight $+$ve to thresh</td>
<td>larger $+$ve up to thresh</td>
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<tr>
<td>Flessey 56018</td>
<td>~8</td>
<td>4.5</td>
<td>1.53</td>
<td>1.56</td>
<td>0.771</td>
<td>0.873</td>
<td>v. slight $+$ve at low fields</td>
<td>no splitting</td>
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### Table 2

Hall Parameters of Substrate and Contact Layer at 300°K and 77°K

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<th>300°K Measurements</th>
<th>77°K Measurements</th>
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<tr>
<td></td>
<td>n cm⁻³</td>
<td>μ cm⁻² V⁻¹ sec⁻¹</td>
</tr>
<tr>
<td>t 511</td>
<td></td>
<td></td>
</tr>
<tr>
<td>from front face</td>
<td>7.8 x10¹⁷</td>
<td>4250</td>
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<tr>
<td>from front face</td>
<td>1.0 x10¹⁸</td>
<td>3030</td>
</tr>
<tr>
<td>t 567</td>
<td></td>
<td></td>
</tr>
<tr>
<td>from front face</td>
<td>7.5 x10¹⁷</td>
<td>2780</td>
</tr>
<tr>
<td>from front face</td>
<td>2.46 x10¹⁸</td>
<td>1930</td>
</tr>
<tr>
<td>act Layer</td>
<td></td>
<td></td>
</tr>
<tr>
<td>201</td>
<td>9.8 x10¹⁷</td>
<td>3300</td>
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### TABLE 3

Voltage Driven Delayed Mode

<table>
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<tr>
<th></th>
<th>$2\ V_{TH}\ 2:1$</th>
<th>$3\ V_{TH}\ 2:1$</th>
<th>$2\ V_{TH}\ 1.43:1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Peak Efficiency %</td>
<td>6.0</td>
<td>6.54</td>
<td>3.12</td>
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<tr>
<td>Freq. for max eff. TT</td>
<td>0.83</td>
<td>0.86</td>
<td>0.85</td>
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<tr>
<td>$Q \frac{df}{dT} \frac{G_{d}}{G_{od}}$ max +</td>
<td>$0.644 \times 10^{-3}$</td>
<td>$0.292 \times 10^{-3}$</td>
<td>$0.363 \times 10^{-3}$</td>
</tr>
<tr>
<td></td>
<td>$-0.192 \times 10^{-3}$</td>
<td>$-0.722 \times 10^{-4}$</td>
<td>$-0.794 \times 10^{-4}$</td>
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</table>

### TABLE 4

Voltage Driven Quenched Mode

<table>
<thead>
<tr>
<th></th>
<th>$2\ V_{TH}\ 2:1$</th>
<th>$3\ V_{TH}\ 2:1$</th>
<th>$2\ V_{TH}\ 1.43:1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum efficiency %</td>
<td>2.6</td>
<td>2.45</td>
<td>1.16</td>
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<tr>
<td>Maximum value of $Q \frac{df}{dT} \frac{G_{d}}{G_{od}}$</td>
<td>$0.78 \times 10^{-5}$</td>
<td>$0.34 \times 10^{-5}$</td>
<td>$0.43 \times 10^{-5}$</td>
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</table>
### TABLE 5

**Current Driven Delayed Mode**

<table>
<thead>
<tr>
<th></th>
<th>$4V_{th} 2:1$</th>
<th>$5V_{th} 2:1$</th>
<th>$4V_{th} 1.43:1$</th>
<th>$3V_{th} 1.43:1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Optimum efficiency %</td>
<td>12.2</td>
<td>14.2</td>
<td>6.1</td>
<td>5.6</td>
</tr>
<tr>
<td>Freq. for opt. eff. TT</td>
<td>0.625</td>
<td>0.71</td>
<td>0.71</td>
<td>0.64</td>
</tr>
<tr>
<td>$\frac{Q \cdot df}{f_0 \cdot \frac{R_d}{dT \cdot R_{od}}}$ at opt. eff. $^\circ C^{-1}$</td>
<td>$-2.5 \times 10^{-4}$</td>
<td>$-4.8 \times 10^{-4}$</td>
<td>$-7.0 \times 10^{-4}$</td>
<td>$-4.0 \times 10^{-4}$</td>
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### TABLE 6

**Current Driven Quenched Mode**

<table>
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<tr>
<th></th>
<th>$2V_{th} 2:1$</th>
<th>$4V_{th} 2:1$</th>
<th>$2V_{th} 1.43:1$</th>
<th>$4V_{th} 1.43:1$</th>
<th>$3V_{th} 1.43:1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Optimum efficiency %</td>
<td>3.5</td>
<td>9.6</td>
<td>2.4</td>
<td>4.85</td>
<td></td>
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<tr>
<td>$\frac{Q \cdot df}{f_0 \cdot \frac{R_d}{dT \cdot R_{od}}}$ at opt. eff.</td>
<td>$-1.8 \times 10^{-6}$</td>
<td>$-3.4 \times 10^{-6}$</td>
<td>$-3.2 \times 10^{-6}$</td>
<td>$-3.2 \times 10^{-6}$</td>
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</tr>
<tr>
<td>$\frac{1}{2}$ Opt. eff. %</td>
<td>1.9</td>
<td>5.3</td>
<td>1.35</td>
<td>2.2</td>
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<tr>
<td>$\frac{Q \cdot df}{f_0 \cdot \frac{R_d}{dT \cdot R_{od}}}$ at $\frac{1}{2}$ opt. eff.</td>
<td>$-3.4 \times 10^{-6}$</td>
<td>$-3.4 \times 10^{-6}$</td>
<td>$-4.1 \times 10^{-6}$</td>
<td>$-3.2 \times 10^{-6}$</td>
<td></td>
</tr>
<tr>
<td>$\frac{G_d}{G_{od}}$</td>
<td>$\frac{Q}{G_{od}} \frac{df}{dT} \frac{G_d}{fo}$</td>
<td>eff %</td>
<td>$\frac{Q}{G_{od}} \frac{df}{dT} \frac{G_d}{fo}$</td>
<td>eff %</td>
<td>$\frac{Q}{G_{od}} \frac{df}{dT} \frac{G_d}{fo}$</td>
</tr>
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<td>.08</td>
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<tr>
<td>.06</td>
<td>$-1.12 \times 10^{-4}$ 9.75</td>
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<td>$-2.58 \times 10^{-4}$ 11.5</td>
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<td>$-4.35 \times 10^{-4}$ 11.32</td>
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<tr>
<td>.04</td>
<td>$-1.35 \times 10^{-4}$ 7.15</td>
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<td>$-6.85 \times 10^{-5}$ 6.7</td>
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<td>$-9.03 \times 10^{-5}$ 6.3</td>
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<td>.02</td>
<td>$-7.88 \times 10^{-5}$ 4.15</td>
<td></td>
<td>$-2.81 \times 10^{-5}$ 3.6</td>
<td></td>
<td>$-5.72 \times 10^{-5}$ 3.3</td>
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TABLE 7

Hybrid Mode, 10 GHz

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<tr>
<th>$\frac{G_d}{G_{od}}$</th>
<th>$\frac{Q}{G_{od}} \frac{df}{dT} \frac{G_d}{fo}$</th>
<th>eff %</th>
<th>$\frac{Q}{G_{od}} \frac{df}{dT} \frac{G_d}{fo}$</th>
<th>eff %</th>
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<td>.12</td>
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<td>.08</td>
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<tr>
<td>.06</td>
<td>$-9.50 \times 10^{-5}$ 11.75</td>
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<td>$-1.15 \times 10^{-4}$ 11.16</td>
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<td>.04</td>
<td>$-1.20 \times 10^{-3}$ 10.30</td>
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<td>$-1.79 \times 10^{-4}$ 10.05</td>
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<td>.02</td>
<td>$-1.17 \times 10^{-4}$ 8.18</td>
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TABLE 8

Hybrid Mode, 11 GHz
### Table 9

**Comparison of Equivalent Circuits for Different S4 Package Arrangements**

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<th>Package</th>
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<th>wire dia. inches</th>
<th>$C_A$ pF</th>
<th>$C_B$ pF</th>
<th>$L_d$ nH</th>
<th>$L_w$ nH</th>
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<tbody>
<tr>
<td>RRE S4²</td>
<td>.015</td>
<td>.002</td>
<td>.03</td>
<td>.195</td>
<td>.186</td>
<td>.167</td>
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<tr>
<td>STL S4³</td>
<td>.035</td>
<td>.002</td>
<td>.03</td>
<td>.105</td>
<td>.186</td>
<td>.405</td>
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<tr>
<td>STL S4</td>
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<td>.001</td>
<td>.02</td>
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<td>.195</td>
<td>-</td>
</tr>
<tr>
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<td>.001</td>
<td>.02</td>
<td>.160</td>
<td>.250</td>
<td>.350</td>
</tr>
<tr>
<td>STL S4</td>
<td>.025</td>
<td>.002</td>
<td>.02</td>
<td>.193</td>
<td>.195</td>
<td>.620</td>
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<td>.002</td>
<td>.02</td>
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<td>.250</td>
<td>.620</td>
</tr>
<tr>
<td>Frequency, GHz</td>
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<td>10</td>
<td>11</td>
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<td>cavity length, cm</td>
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<td>0.454</td>
<td>1.74</td>
<td>1.89</td>
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<tr>
<td>complete model</td>
<td>-j 10.8</td>
<td>-j 5.83</td>
<td>-j 16.8</td>
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<tr>
<td>Admittance, mmhos</td>
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<td>1.68</td>
<td>2.00</td>
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<tr>
<td>perfect transformer</td>
<td>-j 10.8</td>
<td>-j 5.78</td>
<td>-j 16.43</td>
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<tr>
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<td>8.81</td>
<td>8.44</td>
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<tr>
<td>second harmonic</td>
<td>-j 3.37</td>
<td>- 2.21</td>
<td>+j 24.5</td>
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</table>

**TABLE 10**

Admittances of Gunn Diode Load for R360C/6LF158/A16
from Theoretical Models
<table>
<thead>
<tr>
<th></th>
<th>9 GHz</th>
<th>10 GHz</th>
<th>11 GHz</th>
<th>18 GHz</th>
<th>20 GHz</th>
<th>22 GHz</th>
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<tr>
<td>$Z_1$</td>
<td>77.06</td>
<td>69.89</td>
<td>65.71</td>
<td>56.64</td>
<td>55.83</td>
<td>55.27</td>
</tr>
<tr>
<td>$Z_3$</td>
<td>81.87</td>
<td>74.26</td>
<td>69.82</td>
<td>60.18</td>
<td>59.32</td>
<td>58.72</td>
</tr>
<tr>
<td>$Z_4$</td>
<td>109.3</td>
<td>99.15</td>
<td>93.22</td>
<td>80.36</td>
<td>79.21</td>
<td>78.41</td>
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<tr>
<td>$Z_5$</td>
<td>194.1</td>
<td>176.0</td>
<td>165.5</td>
<td>142.7</td>
<td>14.6</td>
<td>139.2</td>
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<tr>
<td>$Z_6$</td>
<td>344.8</td>
<td>312.8</td>
<td>294.1</td>
<td>235.5</td>
<td>249.9</td>
<td>247.3</td>
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<tr>
<td>$Z_7$</td>
<td>459.9</td>
<td>417.1</td>
<td>392.2</td>
<td>338.1</td>
<td>333.3</td>
<td>329.9</td>
</tr>
<tr>
<td>$Z_8$</td>
<td>489.3</td>
<td>443.8</td>
<td>417.3</td>
<td>359.7</td>
<td>354.5</td>
<td>350.9</td>
</tr>
</tbody>
</table>

<p>| | | | | | | |</p>
<table>
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<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Omega$</td>
<td>1.738</td>
<td>1.660</td>
<td>1.260</td>
<td>2.365</td>
<td>2.078</td>
<td>1.499</td>
</tr>
<tr>
<td>$L_1$</td>
<td>0.738</td>
<td>0.813</td>
<td>0.865</td>
<td>1.004</td>
<td>1.018</td>
<td>1.029</td>
</tr>
<tr>
<td>$L_2$</td>
<td>0.789</td>
<td>0.870</td>
<td>0.927</td>
<td>1.073</td>
<td>1.089</td>
<td>1.100</td>
</tr>
<tr>
<td>$L_3$</td>
<td>0.767</td>
<td>0.834</td>
<td>0.838</td>
<td>1.030</td>
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**TABLE 11**
Parameters for full model of Chebyshev transformer.
Fig. 1 Band structure of GaAs

Fig. 2 Variation of gunn effect threshold field with pressure
Fig 3a Furnace construction for growing GaAs Ingots

Fig 3b Gradient Freeze Furnace: Temperature Distributions During Growth
Fig. 4 Vapour phase reactor

Fig. 5 Liquid epitaxial reactor
Fig. 6 Velocity field characteristic of GaAs at 300° K
Fig. 7 Velocity field characteristic of GaAs at various temperatures.

- Intrinsic
- 43 Modified Monte Carlo
- 60 Time of flight
- n = 8.8 x 10^16 cm^-3
- 51 Microwave heating, 36 GHz
- n = 3.8 x 10^17 cm^-3
- 52 Microwave heating, 36 GHz
- n = 1.0 x 10^18 cm^-3
- 53 Gunn diode
- \( \mu = 6500/33,000 \text{ cm}^2 \text{ V}^{-1} \text{ s}^{-1} \)
- 54 Microwave heating, 9.4 GHz

Velocity, 10^7 cm s^{-1}

Field kV cm^-1

0 5 10 15

0 1 2 3

110°K
160°K
313°K
453°K
600°K
500°K
300°K
300°K
77°K
77°K
96°K
160°K
300°K
300°K
545°K
300°K

Field kV cm^-1
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Fig. 14b Effect of air-baking silver-tin contacted diodes.
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Fig. 18 Voltage driven quenched domain mode

Fig. 19 Current driven quenched mode

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Fig. 21
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General representations of diode, circuit and transmission line load

(e) Injection locked oscillator

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Fig. 23b  Voltage driven delayed domain mode
Fig. 23c Voltage driven delayed domain mode
Fig. 24 Voltage driven quenched domain mode
Fig. 25 Current driven delayed domain mode
Fig. 26(a)  
Fig. 26(b)  
Fig. 26(c)  
Fig. 26(d)  

Fig. 26 Current driven quenched domain mode
$T = 300^\circ K$, $\ell = 10 \, \mu m$, $n = 10^{15} \, cm^{-3}$, $f = 10 \, GHz$, $V_B = 10V$, $V_{RF} = 7.275V$, Initial Domain Amplitude = 0.4V

Fig. 27 Hybrid mode waveforms
Fig. 28 Theoretical time dependence of current and electric field in hybrid mode
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Fig. 29 (c) and (d) show the susceptance, $B_0/G_0$, for the same conditions as above, with r.f. voltage levels of 6.48V, 7.27V, 8.05V, 8.3V, and 9.61V.

Fig. 29: Diode admittance from hybrid mode model.
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Symbols:
- \( \lambda_g \): Guide length
- \( \lambda_{g/2} \): Half guide length
Fig. 42  Admittance presented by scaled circuit
Fig. 43 Phase bridge for measuring $\frac{df}{dT}$
Fig. 44 $df/dT$ vs. $Q_L$ at 8V, 9.5 GHz in full height circuit
Fig. 47 Slice R380C/VILE 158 in the reduced height waveguide cavity

Fig. 48 $\frac{df}{dT}$ Measurements for diode R134B/5LE7/B (Evanescent resonance)
Appendix 1

Voltage Driven Delayed and Quenched Modes

Referring to figs 17 and 18 we assume that a voltage $V_q - V_1 \sin \omega t$ is applied to the diode from a bias supply $V_o$ and an r.f. circuit of resonant frequency $\omega$. The time $t_1$ is the time at which the waveform passes through threshold $V_t$ and a domain is formed at the cathode. The time $t_2$ is the time at which the domain is removed from the diode. In the case of the quenched mode this is effected by the voltage swinging below the sustaining value $K V_t$, where $K$ is the reciprocal of the peak to valley ratio of the material and in the case of the delayed mode by the space charge being annihilated at the anode.

On an instantaneous domain formation model we may therefore, for both modes, express the current waveform $i(t)$ shown in the lower parts of figs 17 and 18 as

$$
0 < t < t_2 \quad i(t) = K i_t \quad \text{where } i_t \text{ is the threshold current}
$$

$$
t_2 < t < t_1 \quad i(t) = C_o V \quad \text{where } C_o \text{ is the low field conductance}
$$

$$
t_1 < t < 2\pi/\omega \quad i(t) = K i_t
$$

From the nature of the modes we have the additional information that in the quenched mode $\sin \omega t_2 = \frac{V_o - KV_t}{V_1}$ and in the delayed mode

$$
t_1 - t_2 + t_t = \frac{2\pi}{\omega} \quad \text{where } t_t \text{ is the transit time of the domain.}
$$

Applying Fourier Analysis to these current waveforms leads to the following results:-

The d.c. current $i_o$ is given by:-

$$
i_o = \frac{\omega C_o}{2\pi} \left[ KV_t \left( \frac{2\pi}{\omega} + t_2 - t_1 \right) + V_o (t_1 - t_2) + \frac{V_1}{\omega} \left( \cos \omega t_1 - \cos \omega t_2 \right) \right]
$$

which for the delayed domain case can also be written as

$$
i_o = \frac{C_o}{2\pi} \left[ KV_t (\omega t_t) + V_o (2\pi - \omega t_t) + \frac{(V_o - V_t)}{\sin \omega t} \left( \cos \omega t \left(1 - \cos \omega t\right) + \sin \omega t \sin \omega t \right) \right]
$$
The a.c. current in phase with the voltage $i_{lin}$ is given by:

$$
i_{lin} = -\frac{C_0}{\pi} \left[ (V_0 - KV_t)(\cos \omega t_2 - \cos \omega t_1) - \frac{V_1}{2}(\omega t_1 - \omega t_2) + \frac{V_1}{4} \right] (\sin 2\omega t_1 - \sin 2\omega t_2) \right]$$

so that the a.c. conductance is given by $G_d$ where

$$G_d = \frac{-C_0}{\pi} \left[ \frac{(V_o - KV_t)}{(V_o - V_t)^2} \sin \omega t_1(\cos \omega t_2 - \cos \omega t_1) - \frac{1}{2} (\omega t_1 - \omega t_2) 
+ \frac{1}{4} (\sin 2\omega t_1 - \sin 2\omega t_2) \right]$$

In the case of the delayed mode this can be written as

$$G_d = \frac{-C_0}{\pi} \left[ \cos 2\omega t_1 (A \sin \omega t - \frac{1}{2} \sin 2\omega t) 
+ \sin 2\omega t_1 (A (\cos \omega t_1 - 1) + \frac{1}{2} (1 - \cos 2\omega t_1) 
- A \sin \omega t + \omega t - 2\pi) \right]$$

where $A = \frac{V_o - KV_t}{V_o - V_t}$

The a.c. current lagging the voltage by $\frac{\pi}{2}$ $i_{out}$ is given by:

$$i_{out} = \frac{-C_0}{\pi} \left[ KV_t - V_o (\sin \omega t_2 - \sin \omega t_1) + \frac{V_1}{4} (\cos 2\omega t_2) \right]$$

So that the ac susceptance $B_d$ is given by

$$B_d = \frac{-C_0}{\pi} \left[ KV_t - V_o (\sin \omega t) + \frac{1}{2} (\cos 2\omega t_1 - \cos 2\omega t_2) \right]$$
In the case of the delayed mode this can be written as

\[ B_d = -\frac{C_0}{2\pi} \int A \{ \cos 2\omega t_1 \cdot (\cos \omega t - 1) - \sin 2\omega t_1 \cdot \sin \omega t + 1 - \cos \omega t \} \]

\[ + \frac{1}{2} \{ \cos 2\omega t_1 \cdot (1 - \cos 2\omega t) + \sin 2\omega t_1 \cdot \sin 2\omega t \} \]

We may now calculate from the expressions for \( G_d \) and \( B_d \) the various parameters of interest in \( \frac{df}{dT} \) calculations. The delayed mode will be considered first. We allow a set of mutually consistent changes to take place in the parameters \( C_0 t_t \) and \( t_1 \) so that

\[ C_0 \rightarrow C_0 (1 + n) \]
\[ t_t \rightarrow t_t (1 - n) \]
\[ t_1 \rightarrow t_1 (1 + \delta) \]

So that a change with temperature of the velocity field characteristic of \( n\% \) causes the diode conductance to increase by \( n\% \) while the transit time decreases by \( n\% \). In accordance with the discussion of Ch 4.3 we shall set \( n = -0.0029 \degree C^{-1} \).

For any discussion of \( \frac{df}{dT} \) in a simple high Q parallel circuit placed across the diode terminals it has been shown in ch 10.2 that the quantity of interest is \( (\frac{3B}{dT})_d \omega G_d \). This is evaluated by the elimination of \( \delta \) from the following differential expressions for the changes in conductance and susceptance.
\[ \Delta G_d = -\frac{C_n}{2\pi} \left[ \cos 2\omega_t \left\{ \frac{1}{2} (2\omega_t \cos 2\omega_t - \sin 2\omega_t) + A(\sin \omega_t - \omega_t \cos \omega_t) \right\} \\
+ \sin 2\omega_t \left\{ \frac{1}{2} (1 - \cos 2\omega_t - 2 \omega_t \sin 2\omega_t) + A(\omega_t \sin \omega_t + \cos \omega_t - 1) \right\} \\
+ A \omega_t \cos \omega_t - A \sin \omega_t - 2\pi \right] \]

\[ \Delta B_d = -\frac{C_n}{2\pi} \left[ \cos 2\omega_t \left\{ A(\omega_t \sin \omega_t - 1 + \cos \omega_t) + \frac{1}{2} (1 - \cos 2\omega_t - 2 \omega_t \sin 2\omega_t) \right\} \\
+ \sin 2\omega_t \left\{ A(-\omega_t \cos \omega_t - \sin \omega_t) + \frac{1}{2} (\sin 2\omega_t - 2 \omega_t \cos 2\omega_t) \right\} + A \right] \]

\[ \left. -\frac{C_n}{2\pi} \left[ 2\omega_t \left( \sin 2\omega_t \left\{ \frac{1}{2} \sin 2\omega_t - A \sin \omega_t \right\} \\
+ \cos 2\omega_t \left\{ \frac{1}{2} (1 - \cos 2\omega_t) + A (\cos \omega_t - 1) \right\} \right) \right] \]

For the more general treatment of \( \frac{df}{dT} \) given in ch 10.2 eqn 16 we require the following expressions.

(1) \( \frac{\partial \Delta G_d}{\partial C} \bigg|_{V_1 \omega} \)

Condition \( V_1 = \text{const} \) implies

that

\[ \delta = 0 \]

\[ \therefore \frac{\partial \Delta G_d}{\partial C} \bigg|_{V_1 \omega} \text{ is given by } \frac{\Delta G_d}{nC_0} \text{ where } \Delta G_d \text{ is calc with } \delta = 0 \]
\[ \left( \frac{\partial^2}{\partial C \partial t} \right)_{V,\omega} V_1 = -\frac{1}{2\pi} \cos 2\omega t \left\{ \frac{1}{2} (2\omega t \cos 2\omega t - \sin 2\omega t) + A(\sin \omega t - \omega t \cos \omega t) \right\} \\
+ \sin 2\omega t \left\{ \frac{1}{2} (1 - \cos 2\omega t - 2\omega t \sin 2\omega t) + A(\omega t \sin \omega t + \cos \omega t - 1) \right\} \\
+ A\omega t \cos \omega t - A\sin \omega t - 2\pi \right\} \]

(2). \[ \left( \frac{\partial^2}{\partial C \partial t} \right)_{V,\omega} V_1 \omega \]

This is evaluated similarly.

and is given by \( \frac{\Delta B_d}{nC_0} \) where \( \Delta B_d \) is calculated with \( \delta = 0 \)

\[ \left( \frac{\partial^2}{\partial C \partial t} \right)_{V,\omega} V_1 = -\frac{1}{2\pi} \cos 2\omega t \left\{ A(\omega t \sin \omega t - 1 + \cos \omega t) + \frac{1}{2} (1 - \cos 2\omega t - 2\omega t \sin 2\omega t) \right\} \\
+ \sin 2\omega t \left\{ A(\omega t \cos \omega t - \sin \omega t) + \frac{1}{2} (\sin 2\omega t - 2\omega t \cos 2\omega t) \right\} \\
+ A(1 - \cos \omega t - \omega t \sin \omega t) \right\} \]

(3). \[ \left( \frac{\partial^2}{\partial G \partial t} \right)_{C,\omega} V_1 \]

n=0 and \( \Delta V_1 = (V_o - V) \cdot \cos \omega t \cdot \omega t \cdot \delta \)

\( \left( \frac{\partial^2}{\partial G \partial t} \right)_{C,\omega} C_0 \omega \)

is given by \( \Delta B_d \) evaluated with n=0

\( \frac{\Delta B_d}{nC_0} \left( \frac{V_o - V}{\omega_t} \right) \cos \omega t \cdot \omega t \cdot \delta \)

\( \sin^2 \omega t \)

\( \left( \frac{\partial^2}{\partial G \partial t} \right)_{C,\omega} C_0 \omega \)

is given by \( \Delta G_d \) evaluated with n=0

\( \frac{\Delta G_d}{nC_0} \left( \frac{V_o - V}{\omega_t} \right) \cos \omega t \cdot \omega t \cdot \delta \)

\( \sin^2 \omega t \)
\[ \frac{\partial B_d}{\partial C_\omega} = \left\{ \cos 2\omega t_1 (-A \sin \omega t_1 + \frac{1}{2} \sin 2\omega t_1) \right\} \]

\[ + \sin 2\omega t_1 (-A(-1+\cos \omega t_1)-\frac{1}{2}(1-\cos 2\omega t_1)) \]

\[ \left\{ \cos 2\omega t_1 \left( \frac{1}{2}(1-\cos 2\omega t_1) + A(\cos \omega t_1 - 1 \right. \]

\[ + \sin 2\omega t_1 \left( \frac{1}{2} \sin 2\omega t_1 - A \sin \omega t_1 \right) \}

For the quenched mode the calculation follows a similar scheme with changes in parameters

\[ C_\omega \rightarrow C_\omega (1+n) \]

\[ t_1 \rightarrow t_1 (1+p) \]

\[ t_2 \rightarrow t_2 (1+m) \]

so that \( A \cos \omega t_1, \omega t_1 \rho = \cos \omega t_2, \omega t_2 m \)

Therefore to calculate \( \frac{\partial B_d}{\partial \omega} \) we evaluate \( G_d B_d \) as before

\[ \therefore \Delta G_d = - \frac{C_\omega n}{\pi} \left[ \sin \omega t_2 (\cos \omega t_2 - \cos \omega t_1) - \frac{1}{2} (\omega t_1 - \omega t_2) \right. \]

\[ + \frac{1}{4} (\sin 2\omega t_1 - \sin 2\omega t_2) \]

\[ - \frac{C_\omega \omega t_2 m}{\pi} \left[ \cos \omega t_2(\cos \omega t_2 - \cos \omega t_1) - \sin^2 \omega t_2 + \frac{1}{2} \cos 2\omega t_2 \right] \]

\[ - \frac{C_\omega \omega t_1 \rho}{\pi} \left[ \sin \omega t_2 \sin \omega t_1 - \frac{1}{2} + \frac{1}{2} \cos 2\omega t_1 \right] \]

The term in \( \rho \) is zero.
\[ \Delta B_d = -\frac{nC_0}{\pi} \left[ + \sin \omega_2 (\sin \omega_1 - \sin \omega_2) + \frac{4}{3} (\cos 2\omega_1 - \cos 2\omega_2) \right] \]
\[ - \frac{C_0}{\pi} \omega_2 m \left[ \cos \omega_2 (\sin \omega_1 - \sin \omega_2) - \sin \omega_2 \cos \omega_2 + \frac{1}{2} \sin 2\omega_2 \right] \]
\[ - \frac{C_0}{\pi} \omega_1 \rho \left[ + \sin \omega_2 \cos \omega_1 - \frac{1}{2} \sin 2\omega_1 \right] \]

By eliminating \( m \) and \( \rho \) and setting \( n = -0.0029^\circ C^{-1} \) we arrive at the desired result.

The calculation of the parameters required in Ch 10.2 Equ 16 proceeds as for the delayed mode.

\[ \frac{\partial G_d}{\partial c_0} \]

The condition \( V_1 = \text{const} \) implies \( \rho = 0, m = 0 \)

\[ \therefore \frac{\partial G_d}{\partial c_0} V_1 = \frac{\Delta G_d}{c_0} \] with \( \Delta G_d \) evaluated with \( \rho = 0, m = 0 \).

\[ \therefore \frac{\partial G_d}{\partial c_0} \frac{V_1}{\omega} = \frac{1}{\pi} \left[ - \sin \omega_2 \cos \omega_1 - \frac{1}{2} (\omega_1 - \omega_2) + \frac{1}{2} (\sin 2\omega_1 + \sin 2\omega_2) \right] \]

This is similarly given by \( \frac{\partial B_d}{\partial c_0} \) with \( \Delta G_d \) evaluated with \( \rho = 0, m = 0 \)

\[ \therefore \frac{\partial B_d}{\partial c_0} \frac{V_1}{\omega} = -\frac{1}{\pi} \left[ - \sin \omega_2 (\sin \omega_2 - \sin \omega_1) + \frac{1}{4} (\cos 2\omega_1 - \cos 2\omega_2) \right] \]

\[ n = 0 \text{ and } \Delta V_1 = \left( \frac{V_1}{V_0} \right) \cos \omega_1 \omega_1 \rho \]
\[ \sin^2 \omega_1 \]
\[ \frac{\partial B_{d}}{\partial V} \bigg|_{C_{0} \omega} \quad \text{is given by} \quad \Delta B_{d} \quad \text{evaluated with} \ n=0 \]
\[ \frac{\partial G_{d}}{\partial V} \bigg|_{C_{0} \omega} \quad \text{is given by} \quad \Delta G_{d} \quad \text{evaluated with} \ n=0 \]

\[ \Delta B_{d} = \frac{(V_{o}-V_{t}) \cos \omega \tau_{1} \omega_{1} \rho}{\sin^{2} \omega_{1}} \]

\[ \Delta G_{d} = \frac{(V_{o}-V_{t}) \cos \omega \tau_{1} \omega_{1} \rho}{\sin^{2} \omega_{1}} \]

\[ \therefore \frac{\partial B_{d}}{\partial G_{d}} \bigg|_{C_{0} \omega} = \left\{ A \cos \omega \tau_{1} (\sin \omega \tau_{1} - \sin \omega \tau_{2}) \right\} 
+ \left[ \sin \omega \tau_{2} \cos \omega \tau_{1} - \frac{1}{2} \sin 2 \omega \tau_{1} \right] \right\} \]

\[ \therefore \frac{\partial G_{d}}{\partial G_{d}} \bigg|_{C_{0} \omega} = \left\{ A \cos \omega \tau_{1} (\cos \omega \tau_{2} - \cos \omega \tau_{1}) \right\} 
+ \left[ \sin \omega \tau_{2} \sin \omega \tau_{1} - \frac{1}{2} + \frac{1}{2} \cos 2 \omega \tau_{1} \right] \right\} \]
Appendix 2

Current Driven Delayed and Quenched Modes

Consider a very simple model in which, when a domain forms, the conduction current passed by the domain is \( K_i \), while any difference in the total current from this value is displacement current which either charges or discharges the domain capacitance \( C \) which is assumed constant.

This model is illustrated in Figs. 19 and 20.

If the total current \( i = i_o + i_\perp \cos \omega t \).

And the domain voltage is \( V_D \).

Then \( \frac{\partial V_D}{\partial t} = \frac{1}{C} (i - K_i) \).

\[ V_D = \frac{1}{C} \left[ (i_o - K_i_t) t + \frac{i_\perp}{\omega} \sin \omega t \right] \]

and the total voltage dropped across the whole diode is given by

\[ V = (i_o + i_\perp \cos \omega t) R_o + \frac{1}{C} \left[ (i_o - K_i_t) t + \frac{i_\perp}{\omega} \sin \omega t \right] \]

since \( i_o + i_\perp = i_t \) we can replace \( i_\perp \) by \((i_t - i_o)\).

So that when a domain is present

\[ V = (i_o + (i_t - i_o) \cos \omega t) R_o + \frac{1}{C} \left[ (i_o - K_i_t) t + \frac{(i_t - i_o) \sin \omega t}{\omega} \right] \] \hspace{1cm} (1)

and when a domain is absent

\[ V = (i_o + (i_t - i_o) \cos \omega t) R_o \] \hspace{1cm} (2)

In the quenched mode the domain will cease to be present when all the charge is removed i.e. when \( V_D = 0 \). Let this time be \( t_1 \)

The condition is that \( \sin \omega t_1 = \frac{(i_o - K_i_t)}{(i_t - i_o) \omega t_1} \) \hspace{1cm} (3)
There is a limit on the values which $\omega t_1$ may take since we cannot have a situation where the domain is growing at its time of quench.

i.e. $t_1$ must be less than or equal to $t_2$ the time at which the current again begins to exceed $K_i t$.

The limit is given by $i_o + (i_t - i_o) \cos \omega t_2 = K_i t$

Combining with eqn (3) yields $\frac{\tan \omega t_2}{\omega t_2} = 1$ \hspace{1cm} \ldots (4)

with a solution $\omega t_2 = 1.43\pi$

In the delayed domain mode the domain ceases to exist when it reaches the anode at time $t = t_1$.

We require that the domain voltage at this time should be greater than zero.

i.e. $(i_o - K_i t) t_t + (i_t - i_o) \sin \omega t_t > 0$

or $\frac{\sin \omega t_t}{\omega t_t} > - (i_o - K_i t) \frac{(i_t - i_o)}{i_o}$

Both modes therefore lead to formally similar expressions but with different ranges of parameters and will be treated together.

The d.c. voltage and the diode resistance and reactance are obtained by fourier analysis of the voltage waveform.

The d.c. voltage $V_o$ is given by

$$V_o = i_o R_o + \frac{\omega}{2\pi C} \left( (i_o - K_i t) \frac{t_1^2}{2} - \frac{(i_t - i_o)}{\omega} \cos \omega t_1 - 1 \right)$$ \hspace{1cm} \ldots (5)
The in phase rf voltage $V_{\text{lin}}$ is given by

$$V_{\text{lin}} = R_0 \left( i_t - i_o \right) + \frac{R_0}{\omega R_0 C} \left[ (i_o - K_i t) \left( \omega t_1 \sin \omega t_1 + \cos \omega t_1 - 1 \right) \right]$$

and the out of phase rf voltage $V_{\text{out}}$ is given by

$$V_{\text{out}} = -\frac{R_0}{\omega R_0 C} \left[ (i_o - K_i t) \left( \omega t_1 \sin \omega t_1 + \cos \omega t_1 - 1 \right) - \frac{1}{4} (\cos 2\omega t_1 - 1) \right]$$

leading to expressions for the ac resistance and reactance

of $R_d = R_0 + \frac{R_0}{\omega R_0 C} \left( i_o - K_i t \right) \left( \omega t_1 \sin \omega t_1 + \cos \omega t_1 - 1 \right) - \frac{1}{4} (\cos 2\omega t_1 - 1)$ \hspace{1cm} (6)

and $X_d = -\frac{R_0}{\pi \omega R_0 C} \left( i_o - K_i t \right) \left( \sin \omega t_1 - \omega t_1 \cos \omega t_1 \right) + \frac{1}{4} \left( 2\omega t_1 - \sin 2\omega t_1 \right)$ \hspace{1cm} (7)

We assume that when the temperature is changed the various parameters change in magnitude according to

$$R_o \rightarrow R_o (1 - \alpha)$$

$$i_t \rightarrow i_t (1 + \alpha)$$

$$i_o \rightarrow i_o (1 + \beta)$$

$$\omega t_1 \rightarrow \omega t_1 (1 + \gamma)$$

As in the case of the voltage driven modes we shall set $\alpha = -0.0029^\circ \text{C}^{-1}$.

Also as in the case of the voltage driven modes we evaluate the incremental change in diode resistance $\Delta R_d$ and reactance $\Delta X_d$ so that in a simple series circuit we may calculate $\frac{\partial X_d}{\partial T}$.

$$\frac{\partial X_d}{\partial T} = \frac{1}{\omega R_d}$$
\[ \Delta R_d = \alpha \left[ -R_0 - \frac{\dot{i}_t(1-K)}{(i_t-i_0)^2} \cdot \frac{R_0}{\omega CR_0 \pi} \cdot \left[ \omega t_1 \sin \omega t_1 + \cos \omega t_1 - 1 \right] \right] \\
+ \delta \left( \frac{R_0}{\omega CR_0 \pi} \cdot \frac{\dot{i}_t(1-K)}{(i_t-i_0)^2} \cdot \left[ \omega t_1 \sin \omega t_1 + \cos \omega t_1 - 1 \right] \right) \\
+ \gamma \left( \frac{R_0}{\omega CR_0 \pi} \cdot \left[ \frac{(i_0-K_i_t)}{(i_t-i_0)} \cdot \omega t_1^2 \cos \omega t_1 + \frac{1}{2} \omega t_1 \sin 2 \omega t_1 \right] \right) \]...

and

\[ \Delta X_d = \frac{-R_0}{\pi \omega R_C} \cdot \left\{ \frac{\dot{i}_t(1-K)(\delta-\alpha)}{(i_t-i_0)^2} \cdot (\sin \omega t_1 - \omega t_1 \cos \omega t_1) \right\} \\
+ \gamma \left[ \frac{(i_0-K_i_t)}{(i_t-i_0)} \cdot \omega t_1^2 \sin \omega t_1 + 5 (\omega t_1 - \omega t_1 \cos 2 \omega t_1) \right] \]...

We may express equation (8) in the form

\[ \Delta R_d = \alpha \cdot \Delta + \delta \cdot D + \gamma \cdot G \]

in the quenched mode \( G = 0 \) and therefore \( \delta = -\frac{\alpha \Delta}{D} \)

in the delayed mode \( \gamma = -\alpha \) and therefore \( \delta = -\frac{\alpha \cdot (\Delta-G)}{D} \)

Substitution into equation (9) with the appropriate value of \( \alpha \) evaluates

\[ \frac{\partial X_d}{\partial T} \cdot \omega R_d \]

In a similar fashion to the voltage driven modes we wish to evaluate the various terms which make up equation 17 ch 10.2.
(1).  \[
\frac{\partial X_t}{\partial \omega R_o} \frac{\partial R_d}{\partial i} \omega R_o
\]

Since \( \omega, R_o \) are held constant \( \alpha = 0 \)
and \( i = i_t - i_o \) \( \therefore \Delta i = -i_o \delta \)

In the quenched mode the terms in \( \gamma \) in equations (8) and (9)
are identically = 0 which follows from equation (3).

In the delayed mode \( \gamma = -\alpha = 0 \)

Therefore in both cases.

\[
\frac{\partial X_t}{\partial \omega R_o} = \frac{R_o}{\pi \omega R_o C} \left\{ \frac{i_t(1-K)}{(i_t-i_o)^2} \right\} (\sin \omega t - \omega t \cos \omega t)
\]

\[
\frac{\partial R_d}{\partial i} \omega R_o = - \frac{R_o}{\pi \omega R_o C} \left\{ \frac{i_t(1-K)}{(i_t-i_o)^2} \right\} (\sin \omega t \omega t \cos \omega t - 1)
\]

and \( \frac{\partial X_t}{\partial \omega R_o} = - \frac{(\sin \omega t - \omega t \cos \omega t)}{(\omega t \sin \omega t + \cos \omega t - 1)} \)

(2).  \[
\frac{\partial X_t}{\partial R_o} \omega i
\]

\( \Delta R_o = -aR_o \)

and since \( i \) is held constant \( a_i_t = \delta i_o = 0 \)

\( \therefore \delta = \frac{a_i_t}{i_o} \)

In the quenched mode the term in \( \gamma = 0 \)
In the delayed mode \( \gamma = -\alpha \)
In the quenched mode

$$\frac{\partial \mathcal{X}_d}{\partial R_0 \omega_i} = \frac{1}{\pi R_0 C} \left\{ \frac{i_0 i_t (1-K) (\frac{i_t}{i_0} - 1)}{(i_t - i_0)^2} \right\} . (\sin \omega t_1 \omega t_1 \cos \omega t_1)$$

and in the delayed mode

$$\frac{\partial \mathcal{X}_d}{\partial R_0 \omega_i} = \frac{1}{\pi R_0 C} \left\{ \frac{i_0 i_t (1-K) (\frac{i_t}{i_0} - 1)}{(i_t - i_0)^2} \right\} . (\sin \omega t_1 \omega t_1 \cos \omega t_1)

- \frac{(i_0 - K i_t)}{(i_t - i_0)} \omega t_1^2 \sin \omega t_1 + 5 (\omega t_1 \omega t_1 \cos 2 \omega t_1))$$

This is evaluated with the parameters derived above to give.

in the quenched mode

$$\frac{\partial \mathcal{R}_d}{\partial R_0 \omega_i} = 1 + \frac{i_0 i_t (1-K) (1 - \frac{i_t}{i_0})}{(i_t - i_0)^2} \frac{1}{\omega R_0 \pi} \left[ \omega t_1 \sin \omega t_1 + \cos \omega t_1 - 1 \right]$$

in the delayed mode

$$\frac{\partial \mathcal{R}_d}{\partial R_0 \omega_i} = 1 + \frac{i_0 i_t (1-K) (1 - \frac{i_t}{i_0})}{(i_t - i_0)^2} \frac{1}{\omega R_0 \pi} \left[ \omega t_1 \sin \omega t_1 + \cos \omega t_1 - 1 \right]

+ \frac{1}{\omega R_0 \pi} \left[ \frac{i_0 - K i_t}{(i_t - i_0)} \omega t_1^2 \cos \omega t_1 + \frac{1}{2} \omega t_1 \sin 2 \omega t_1 \right]$$
Appendix 3

Hybrid Mode

1. Derivation of Domain Growth Equation

Poisson's eqn is:
\[ \frac{\partial E}{\partial x} (x,t) = (n(x,t) - n_o) \frac{e}{\varepsilon} \] ...

and current continuity implies that:
\[ J(t) = n(x,t) e v(E) - \frac{\partial}{\partial x} n(x,t) e D + \varepsilon \frac{\partial E}{\partial t} (x,t) \] ...

At some plane \( x=k \) near the cathode (or anode) we may postulate that \( n=n_o \) and therefore
\[ J(t) = n_o e v(E(k,t)) + \varepsilon \frac{\partial E}{\partial t} (kt) \] ...

Combining eqns (1) (2) and (3) gives
\[ \frac{\partial^2 E}{\partial x^2} (x,t) - v(E(x,t)) \frac{\partial E}{\partial x} (x,t) - \frac{n_o e}{\varepsilon} \{ v(E(x,t)) - v(E(k,t)) \} = \frac{3}{\varepsilon} [E(x,t) - E(k,t)] \] ...

Integrating w.r.t. \( x \) from \(-\frac{L}{2}\) to \(+\frac{L}{2}\) we obtain for a fully depleted domain
\[ \left| \frac{\partial E}{\partial x} (x,t) \right| \left|_{-\frac{L}{2}}^{\frac{L}{2}} \right. - \left. \left| \frac{\partial E(x,t)}{\partial x} \right| \left|_{-\frac{L}{2}}^{\frac{L}{2}} \right. \right. v(E(k,t)) \right. \] dx

\[ - \frac{n_o e}{\varepsilon} \left| \left|_{-\frac{L}{2}}^{\frac{L}{2}} \right. \left( v(E(x,t)) - v(E(k,t)) \right) \right. \] dx = \[ \frac{3}{\varepsilon} \left| \left|_{-\frac{L}{2}}^{\frac{L}{2}} \right. \left[ E(x,t) - E(k,t) \right] \right. \] dx

The first term is zero since \( \frac{\partial E}{\partial x} = 0 \) at \(-\frac{L}{2}, \frac{L}{2}\)

The second term can be shown to be zero for a three straight line characteristic as follows.
On a three straight line characteristic the domain of fig A1 gives

\[
\frac{L}{2} v(E) \frac{\partial E}{\partial x} dx
\]

\[
= \int_{-L/2}^{L/2} \mu_0 E \frac{\partial E}{\partial x} dx
\]

\[
+ \int_{x_1}^{x_4} \left[ \mu_0 E_{TH} - \mu_1 (E - E_{TH}) \right] \frac{\partial E}{\partial x} dx
\]

\[
+ \int_{x_1}^{x_3} \mu_0 E_{TH} - \mu_1 (E_v - E_{TH}) \frac{\partial E}{\partial x} dx
\]

\[
= \left[ \frac{1}{2} \mu_0 E^2 \right]_{-L/2}^{L/2} + \left[ \mu_0 E_{TH} + \mu_1 E_{TH} \right] \cdot E \left[ \int_{x_1}^{x_4} - \int_{x_1}^{x_3} \mu_1 E^2 - \int_{x_1}^{x_4} \right]
\]

\[
+ \left[ \mu_0 E_{TH} + \mu_1 (E_v - E_{TH}) \right] \cdot E \left[ \int_{x_1}^{x_4} \right]
\]

\[
= 0 + 0 + 0 + 0 + 0
\]

Since any \( v(E) \) characteristic may be split into an infinite number of straight line sections the above result is universally true.

\[
\therefore \frac{\partial}{\partial t} \left[ + \frac{L}{2} E(x,t) - E(k \xi) \right] dx = \frac{-ne}{\epsilon} \left[ \frac{L}{2} \left( v(E(x,t)) - v(E(k \xi)) \right) \right] dx \quad \ldots (5)
\]

the LHS represents the domain potential above the field \( E(k) \) while the RHS can be expressed in terms of the velocities derived from a three straight line characteristic.
\[ \frac{\partial}{\partial t} \phi_D = -\frac{ne}{\varepsilon} \int \{ u_o E(x,t) - u_o E(k,t) \} \, dx \]

over the part of the sample where \( E(x,t) < E_{TH} \)

\[ -\frac{ne}{\varepsilon} \int \{ u_o E_{TH} - u_1 (E(x,t) - E_{TH}) - u_o E(k,t) \} \, dx \]

over the part of the sample where \( E > E(x,t) > E_{TH} \)

\[ -\frac{ne}{\varepsilon} \int \{ u_o E_{TH} - u_1 (E_{V} - E_{TH}) - u_o E(k,t) \} \, dx \]

over the part of the sample where \( E(x,t) > E_{V} \)

Expressing the integrals of the fields as voltages we have

\[ \frac{\partial \phi_D}{\partial t} = -\frac{ne\mu_o}{\varepsilon} \phi_{D1} + \frac{ne\mu_1}{\varepsilon} \phi_{D2} \quad \ldots \quad (6) \]

where \( \phi_{D1} \) is that part of the domain voltage lying between fields of \( E(k) \) and \( E_{TH} \) and \( \phi_{D2} \) is that part of the domain voltage lying between fields of \( E_{TH} \) and \( E_{V} \), as shown in fig 11.

2. Application to a triangular domain

With the assumption that the positive wall of the domain is always fully depleted it is possible to calculate the rates of growth of a triangular domain at various stages in its cycle. These growth rates are given by eqn. (6) above with the added proviso that if the domain is colliding with the anode then it is being reduced in size by a different mechanism, namely discharge at an electrode. The various cases are trivial and only an example will be given.

if \( E(k) < E_{TH} \) and \( E_D < E_{TH} \), where \( E_D \) is the maximum field in the domain.

In this case the domain is decaying and

\[ \frac{\partial \phi_D}{\partial t} = -\frac{ne \mu_o \phi_D}{\varepsilon} \]
The current is given by $n e \mu_0 E(k) \frac{\partial E(k)}{\partial t}$

and we know that $E(k) = \frac{1}{L} (V - \phi_D)$

Therefore the current is $\frac{n e \mu_0}{L} (V - \phi_D) + \frac{\varepsilon}{L} \frac{\partial V}{\partial t} - \frac{\varepsilon}{L} \frac{\partial \phi_D}{\partial t}$

The domain velocity is given by $\mu_0 E(k)$

If the domain is colliding with the anode there are two contributions to domain growth. That due to the space charge as discussed above and that due to discharge by the anode. For the latter we assume that the depletion layer is discharged instantaneously by the anode. The domain is therefore progressively shortened as it runs into the anode. The rate of change of voltage on the domain from this source is given by

$$\frac{d\phi_D}{dt} \text{ discharge} = -\frac{\partial \phi_D}{\partial x} \cdot \frac{dx}{dt}$$

$$= -\frac{ne}{\varepsilon} \frac{dx}{dt}$$

where $x$ is the instantaneous domain width.

So that the total change in $\phi_D$ with time is given by

$$\frac{\partial \phi_D}{\partial t} = -\frac{ne}{\varepsilon} \frac{dx}{dt} + \frac{\partial \phi_D}{\partial t} \text{ space charge}$$

The corresponding values of $\frac{\partial E(k)}{\partial t}$ for use in the calculation of the current is given by

$$\frac{\partial E(k)}{\partial t} = \frac{1}{L} (\frac{\partial V}{\partial t} - \frac{\partial \phi_D}{\partial t})$$

It is a simple matter to calculate rate of domain growth, current and domain velocity for other situations. This being done the amplitude and position of the domain after a short time interval are easily found and with a succession of steps the variation of the current, domain voltage and domain position with time through the cycle may be arrived at.
Fourier analysis then yields values of the diode conductance and susceptance and by scanning values of the rf voltage and temperature (including the effect of dielectric constant change) a composite picture of the mode can be built up.
Fig. A1 Domain field profile