A STUDY OF POLARIZATION EFFECTS IN
SUB-COULOMB (d,p) REACTIONS

by

Jeffrey Allan Tostevin

A thesis submitted to the Faculty of
Mathematical and Physical Sciences
of the University of Surrey in requirement
for the degree of Doctor of Philosophy

April 1979
ABSTRACT

The accuracy of sub-coulomb (d,p) reaction tensor analysing powers, calculated using the conventional DWBA theory, is investigated. Two outstanding uncertainties in the DWBA method, relating to a) break-up of the deuteron in the coulomb field of the target nucleus, and b) the presence of $T^*$ tensor interactions in the deuteron target system, are studied quantitatively. Particular emphasis is placed upon the reaction $^{208}\text{Pb}(d,p)^{209}\text{Pb}$, at $E_d = 9$ MeV and $E_d = 7$ MeV, and its applicability to the measurement of the parameter $D_2$ of Johnson and Santos.

Coulomb break up is treated in dipole approximation using perturbation theory and an adiabatic prescription. Special attention is paid to the use of a realistic state dependent interaction in the P-wave n-p continuum. Coulomb and strong tensor $T^*$ terms, obtained from both elastic scattering and break up models, are discussed and are included through a generalization of the conventional, Local Energy Approximation, DWBA stripping formalism. The resulting coupled differential equations are solved numerically.

Results of calculations show that both effects studied must be accounted for in any analysis of precision experimental data at 9 MeV and/or 7 MeV. Theoretical uncertainties in the strong interaction contribution to the $T^*$ potential, to which calculations at 9 MeV are sensitive, are found to be unimportant at an energy of 7 MeV. The effects of singlet ($^1S_0$) break-up of the deuteron are also shown to be completely negligible.
ACKNOWLEDGEMENTS

The work presented in this thesis was suggested by and performed under the supervision of Dr. Ron Johnson. It is with pleasure that I acknowledge his helpful advice, encouragement and seemingly limitless enthusiasm throughout the progress of this work.

My thanks go also to all members of staff and research students of the Nuclear Physics Group and Physics Department for their help in ways too numerous to mention.

I would like to thank the operators and advisory staff of the University of Surrey Computing Unit for their assistance.

I gratefully acknowledge financial support, from the States of Guernsey for a research studentship, and from the Science Research Council in the latter stages of this work.

Thanks also, to Mrs. Mickey Fortuna for her typing of this thesis.
## CONTENTS

<table>
<thead>
<tr>
<th>Chapter</th>
<th>Title</th>
<th>Page No.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>1.1 Information from the (d,p) Reaction</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>1.2 The Parameter D&lt;sub&gt;2&lt;/sub&gt;</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>1.3 Reaction Observables and Formal Theory</td>
<td>11</td>
</tr>
<tr>
<td></td>
<td>1.4 Distorted Waves Theory</td>
<td>18</td>
</tr>
<tr>
<td>2</td>
<td>THE STRIPPINGAMPLITUDE</td>
<td>25</td>
</tr>
<tr>
<td></td>
<td>2.1 The DWBA at Sub-Coulomb Energies</td>
<td>26</td>
</tr>
<tr>
<td></td>
<td>2.2 The Distorted Waves</td>
<td>29</td>
</tr>
<tr>
<td></td>
<td>2.3 The Reaction Formfactor</td>
<td>31</td>
</tr>
<tr>
<td></td>
<td>2.4 The LEA Method</td>
<td>34</td>
</tr>
<tr>
<td></td>
<td>2.5 The LEA at Sub-Coulomb Energies</td>
<td>45</td>
</tr>
<tr>
<td>3</td>
<td>COULOMB POLARIZABILITY IN STRIPPING REACTIONS</td>
<td>48</td>
</tr>
<tr>
<td></td>
<td>3.1 Three Body Aspects</td>
<td>49</td>
</tr>
<tr>
<td></td>
<td>3.2 Sub-Coulomb Considerations</td>
<td>50</td>
</tr>
<tr>
<td></td>
<td>3.3 The Adiabatic Approximation</td>
<td>54</td>
</tr>
<tr>
<td></td>
<td>3.4 The Deuteron-Target Interaction</td>
<td>60</td>
</tr>
<tr>
<td></td>
<td>3.5 The Modified n-p Internal Wavefunction</td>
<td>66</td>
</tr>
<tr>
<td></td>
<td>3.6 Relative Importance of the Multipoles</td>
<td>68</td>
</tr>
<tr>
<td>4</td>
<td>POLARIZABILITY CORRECTIONS</td>
<td>73</td>
</tr>
<tr>
<td></td>
<td>4.1 General Observations</td>
<td>74</td>
</tr>
<tr>
<td></td>
<td>4.2 The Break-up Amplitude</td>
<td>76</td>
</tr>
<tr>
<td></td>
<td>4.3 The Dipole Approximation</td>
<td>86</td>
</tr>
<tr>
<td></td>
<td>4.4 P-wave Finite Range Effects</td>
<td>88</td>
</tr>
<tr>
<td></td>
<td>4.5 Calculations for the Reaction (^{208}\text{Pb}(d,p))</td>
<td>93</td>
</tr>
<tr>
<td></td>
<td>(^{209}\text{Pb}) at (E_d = 9) MeV and 7 MeV</td>
<td></td>
</tr>
<tr>
<td>Chapter</td>
<td>Title</td>
<td>Page</td>
</tr>
<tr>
<td>----------</td>
<td>----------------------------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>5</td>
<td>Tensor Force Effects in the DWBA</td>
<td>97</td>
</tr>
<tr>
<td></td>
<td>5.1 Spin Dependence in the Distorted Waves</td>
<td>98</td>
</tr>
<tr>
<td></td>
<td>5.2 The LEA Reformulated</td>
<td>105</td>
</tr>
<tr>
<td></td>
<td>5.3 Computational Detail</td>
<td>115</td>
</tr>
<tr>
<td></td>
<td>5.4 Program Checks</td>
<td>118</td>
</tr>
<tr>
<td>6</td>
<td>Tensor Force Corrections</td>
<td>120</td>
</tr>
<tr>
<td></td>
<td>6.1 Tensor Forces in Sub-Coulomb Stripping</td>
<td>121</td>
</tr>
<tr>
<td></td>
<td>6.2 Radial Formfactors</td>
<td>125</td>
</tr>
<tr>
<td></td>
<td>6.3 Calculations for the Reaction</td>
<td>133</td>
</tr>
<tr>
<td></td>
<td>$^90\text{Zr}(d,p)^{91}\text{Zr}$ at $E_d = 5.5$ MeV</td>
<td></td>
</tr>
<tr>
<td></td>
<td>6.4 Calculations for the Reaction</td>
<td>137</td>
</tr>
<tr>
<td></td>
<td>$^{208}\text{Pb}(d,p)^{209}\text{Pb}$ at $E_d = 9$ MeV</td>
<td></td>
</tr>
<tr>
<td></td>
<td>6.5 Calculations for the Reaction</td>
<td>148</td>
</tr>
<tr>
<td></td>
<td>$^{208}\text{Pb}(d,p)^{209}\text{Pb}$ at $E_d = 7$ MeV</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>Summary and Conclusions</td>
<td>149</td>
</tr>
<tr>
<td>A</td>
<td>Mongan Separable Interactions</td>
<td>153</td>
</tr>
<tr>
<td>B</td>
<td>Evaluation of the Continuum P-Wave Eigenfunctions</td>
<td>154</td>
</tr>
<tr>
<td></td>
<td>Corresponding to Mongan's Separable Interaction</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>The Function $H_{4k}^{ijL}(r)$</td>
<td>160</td>
</tr>
<tr>
<td>D</td>
<td>Numerical Solution of the Coupled Equations</td>
<td>161</td>
</tr>
<tr>
<td>E</td>
<td>Physical Solutions of the Coupled Equations</td>
<td>163</td>
</tr>
<tr>
<td>F</td>
<td>Optical Model Potential Formfactors</td>
<td>165</td>
</tr>
</tbody>
</table>
CHAPTER 1

INTRODUCTION

1.1 Information from the (d,p) reaction

The detailed study of nuclear reaction processes is responsible for our present knowledge of nuclear structure and nuclear interactions. The deuteron, the simplest composite nucleus, is an obvious probe for experimental reaction studies and a wealth of accumulated reaction data has resulted in a continued active theoretical interest in deuteron initiated reactions.

In this thesis we shall deal exclusively with the deuteron stripping reaction \( A(d,p)B \), and in particular reactions in which deuterons are incident at energies below the Coulomb barrier of the target nucleus \( A \). At such an energy the process is amenable to semiclassical interpretation and such simple models \( (Le 62, Vi 73, Kn 74) \) have enjoyed much success. In the simplest model the deuteron, treated as a rigid elementary particle, follows an assumed classical trajectory, attaining its minimum velocity at the classical distance of closest approach of the \( d-A \) system. There results a finite probability of the neutron being captured, from the deuteron, about target \( A \) (assumed unexcited) with a particular value of orbital angular momentum \( \ell_n \) (and associated total angular momentum \( j_n = \ell_n \pm \frac{1}{2} \)).

While such models are instructive, extracting nuclear structure information from experimental reaction data requires a careful comparison of data with the predictions of a realistic reaction theory. The Distorted Waves Born Approximation, or DWBA (see for e.g. To 61, Sa 66, Au 70, Le 73, Gl 75), is now used almost universally in the study of the (d,p) reaction,
treat the process as a direct one-step transfer between states
describing the elastic scattering of the incident and outgoing projectiles.
Agreement between experiment and the DWBA is sufficiently good qualita-
tively, that the (d,p) reaction proves a most useful tool for nuclear
spectroscopy, as systematic $\lambda_n$ and $j_n$ dependences of the reaction data
(Bu 51, Bu 59, Le 64a, Le 64b, Jo 71) are understood.

With the advent of high intensity polarized deuteron beams (Ha 67) in the
last ten years, the spectroscopic usefulness of the reaction has increased
(e.g. Yu 68, Br 71, Sa 76, Ab 77). In addition, considerable interest has
been stimulated (see for e.g. Ba 71, Gr 76) as to the spin dependence of
the reaction mechanism. With a reaction initiated by a polarized deuteron
beam (Ha 74a), not only is the outgoing proton angular distribution measurable
but also its sensitivity to the incident beam polarization. This results
in new and independent reaction information in the form of the vector ($iT_{11}$)
and tensor ($T_{20}, T_{21}, T_{22}$) analyzing powers (Mc 71). The result is that the
reaction theory must reproduce not one but five angular distributions and
has resulted in a more sophisticated treatment of the DWBA stripping amplitude.

In particular, as shown in the theoretical investigation of Johnson
(Jo 67), and as has since been confirmed by explicit calculation (e.g.
Br 71, Kn 73, Jo 73, Ro 73, Kn 75, Kn 75a), in order to get even qualitative agreement with experimental (d,p) tensor analyzing powers, $T_{2q}$, then
the $^3D_1$ or D-state component of the deuteron wavefunction must be taken
into account. In fact the $T_{2q}$ in sub-coulomb reactions are completely
(Kn 74, Kn 77) dominated by the D-state and thus provide a rather direct
experimental observation of the $^3D_1$ component which remains quantitatively
poorly understood (Sp 75, Th 77). This prompts the question: can we learn
anything about the deuteron from a study of (d,p) tensor analysing powers?

Predictions of the DWBA are, in general, not accurate in a quantitative sense. However, much work has previously been done in numerical investigations of the accuracy of a DWBA at sub-coulomb energies (e.g. Go 65, Sm 65, Go 67) with a view to obtaining accurate spectroscopic factors. We conclude that the theory is most accurate when applied at incident deuteron energies well below the Coulomb barrier, to (d,p) transitions with Q values near zero. It is precisely in this energy regime that Knutson and Haeberli (Kn 75) have suggested that measured $T_{2q}$ be used to determine the parameter $D_2$ of Johnson and Santos (Jo 71). In the Johnson/Santos finite range treatment of D-state effects, the single parameter $D_2$ carries all information concerning the D-state. Thus, given that the $T_{2q}$ at sub-coulomb energies (and $Q \approx 0$) scale nearly linearly with $D_2$, the $D_2$ 'measurement' requires fitting the absolute magnitude of the experimental tensor analysing powers.

The present investigation is prompted by the observation of Knutson and Haeberli that the presently available $T_{2q}$ data is 'best fitted' by a $D_2$ value which is significantly smaller than that calculated from existing theoretical, and phenomenological, nucleon-nucleon potentials. This conclusion is however based upon the results of a conventional DWBA

+ Calculations in (Kn 75, Kn 77) were performed using the program DWCODE (Ha 70) of Harvey, Santos and Johnson.
stripping calculation and is therefore subject to the uncertainties inherent in the DWBA theory, and those due to the approximate evaluation of the DWBA amplitude. In this thesis we shall approach two such uncertainties, present in these previous calculations, which might be expected to alter considerably the calculated $T_{2q}$ and consequently the 'measured' $D_2$ value. We shall

i) relax the DWBA constraint that the deuteron internal wave-function is unchanged in passing through the interaction field of nucleus A, allowing the target Coulomb field to stretch the deuteron, and

ii) include in the deuteron target interaction a rank-2 tensor interaction of the $T_R$ type (Sa 60), known (De 69, De 70) to be of importance in $T_{2q}$ calculations. We shall also comment briefly upon the effects of allowing the deuteron to break up into a relative $^1S_0$ state (Ha 70a, Ha 74) previously studied by Wales (Wa 77).

It has been suggested that a low energy experiment might eventually yield $D_2$ to an accuracy of 3-4% (Kn 75), thus, the present investigation looks not only at the presently available data, but also at the suitability of such a proposed experiment to an improved determination of $D_2$.

In the following section we shall summarize the presently available knowledge of $D_2$, and, in the remainder of the chapter introduce the formal theoretical aspects of the (d,p) reaction and its DWBA description.

* $^{208}$Pb(d,p)$^{209}$Pb at incident laboratory energy of 7 MeV, rather than the presently available 9 MeV data.
1.2 The Parameter \( D_2 \)

As is well known (see for e.g. Bl 52), the deuteron is the only
bound state of the two-nucleon system and exists, not in a pure \( 3S_1 \) state,
but an admixture of \( 3S_1 \) and \( 3D_1 \) relative n-p configurations. While the
deuteron is rather well understood qualitatively, as pointed out in two
recent review articles (Sp 75, Th 77) the D-state component remains poorly
determined quantitatively. No model independent description of the \( 3D_1 \)
deuteron component exists to date.

In the notation of Johnson (Jo 67) we define the deuteron wavefunction

\[
\phi_{S_1}^{s_1}(r,p,n) = \sum_{L=0,2} i^L u_L(r) Y_{Ls_1}^{s_1}(\hat{r},p,n),
\]

where \( u_0(r) \), \( u_2(r) \) are respectively the S and D-wave radial components of
the wavefunction and \( Y \) is the spin-angle function

\[
Y_{Ls_1}^{s_1}(\hat{r},p,n) = \sum_{\Lambda \sigma'_1} (\Lambda s_1' \sigma'_1 | s_1 \sigma_1) i^L Y_{L}^{\Lambda}(\hat{e}) \chi_{s_1'}^{\sigma'_1}(p,n).
\]

In equation (1.2), \( \chi_{s_1'}^{\sigma'_1} \) is an eigenfunction of the total intrinsic n-p spin
and the Spherical Harmonic functions \( Y_{L}^{\Lambda} \) and angular momentum coupling
coefficients are defined according to the convention of Brink and Satchler
(Br 62). It is understood that \( s_1 = s_1' = 1 \) in the \( 3S_1 - 3D_1 \) deuteron ground
state.

With the adopted phase convention of equation (1.1), outside the range
of the n-p interaction

\[
u_0(r) = N e^{-ar}/r
\]

\[
u_2(r) = N_2 e^{-ar}/r(1 + 3/2a + 3/(4a^2))
\]
where $\frac{n^2 a^2}{2\mu}$ ($\mu$ the n-p reduced mass) is the deuteron binding energy, $\varepsilon_d$, and $\eta$ the asymptotic D-state to S-state ratio. The overall normalization constant $N$ is determined by the deuteron effective range $\rho(-\varepsilon_d, -\varepsilon_d) = 1.764 \pm 0.006$ fm (No 72) through the relation,

$$N^2 = \frac{2a}{1 - a\rho(-\varepsilon_d, -\varepsilon_d)}.$$ (1.4)

Conventionally the D-state component is quantified through a single parameter $P_D$, the D-state probability

$$P_D = \int_0^\infty dr r^2 |u_2(r)|^2.$$ (1.5)

Recently however, Amado et al. (Am 78) have advocated that one should use not $P_D$ but $\eta$ as a model independent measure of the D-state. They suggest that $\eta$, not usually considered an observable property of the deuteron, is capable of being measured directly from high precision elastic p-d tensor polarization data. In deuteron stripping reactions, then according to the finite range treatment of Johnson and Santos (Jo 71), the natural measure of D-state effects is the parameter $D_2$, defined in configuration space through

$$D_2 = \frac{1/15}{\int_0^\infty dr r^2 u_0(r) / \int_0^\infty dr r^2 u_2(r)}.$$ (1.6)

We shall introduce $D_2$ formally in the following chapter; important for the present discussion however is that $D_2$ is closely related to $\eta$ (Jo 71) and, typically to an accuracy of 1% (Kn 75) for presently available models of the n-p interaction, satisfies the relation

$$D_2 \sim \eta/a^2.$$ (1.7)
It is therefore possible, to a very good approximation $^+$, to consider a $D_2$ measurement as an $\eta$ determination.

In table 1.1 we summarize the presently available knowledge of $D_2$ and $\eta$ from existing models of the neutron-proton interaction, $V_{np}(^3S_1 - ^3D_1)$. Interactions which tend at large $r$ to the one pion exchange potential (Sa 67)

$$V_{np}(\text{OPEP}) = -\frac{f^2}{4\pi} \mu c^2 \frac{e^{-x}}{x} \left\{ 1 + \frac{\hat{S}_{np}}{3} \left( 1 + \frac{3}{x} + \frac{3}{x^2} \right) \right\}$$

where $\mu(x = \mu cr/\hbar)$ is the pion mass, $\hat{S}_{np}$ the usual tensor operator

$$\hat{S}_{np} = 3(\hat{s}_n \cdot \hat{p})(\hat{s}_p \cdot \hat{p}) - (\hat{s}_n \cdot \hat{s}_p) \quad (1.8)$$

and $f^2/4\pi$ the effective one pion coupling constant (conventionally taken (G1 62) as 0.08), are labelled with an asterisk. The other model values presented are for information only and we shall assume that the correct meson theoretic OPEP tail behaviour is a necessary constraint upon a realistic physical interaction. $D_2$ values for this group of potentials span the range of values $0.456 < D_2 < 0.540$, however, several presented entries require qualification.

Of note are the investigations of Glendenning and Kramer (G1 62) and of Iwadare et al. (Iw 56). In the latter analysis the correct OPEP tail was assumed in the region $r >$ pion Compton wavelength $\chi(\approx 1.4 \text{ fm})$, and in the absence of a detailed knowledge of $V_{np}$ for $r < \chi$ the deuteron wavefunction given a simple polynomial form. In this model Iwadare finds that

$^+$ Assuming no, presently unknown, long range components in $V_{np}$ which would alter the validity of equation (1.3) at large but finite $r$. 
<table>
<thead>
<tr>
<th>Model</th>
<th>$P_D$(%)</th>
<th>$\eta$</th>
<th>$D_2$(fm$^2$)</th>
<th>Notes/D$_2$ source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hamada-Johnston (Ha 62)</td>
<td>6.90</td>
<td>0.0264</td>
<td>0.487</td>
<td>* (Kn 75, Sp 75)</td>
</tr>
<tr>
<td>Holinde et al. (Ho 72)</td>
<td>5.70</td>
<td>0.0245 e</td>
<td>0.456</td>
<td>* (Kn 75)</td>
</tr>
<tr>
<td>Knutson 'Measured' (Kn 75)</td>
<td>-</td>
<td>0.0232 e</td>
<td>0.432</td>
<td></td>
</tr>
<tr>
<td>Kermode Local (Ke 78)</td>
<td>6.40</td>
<td>-</td>
<td>0.530</td>
<td></td>
</tr>
<tr>
<td>Reid Soft Core (Re 68)</td>
<td>6.47</td>
<td>0.0262</td>
<td>0.484</td>
<td>* (Kn 75)</td>
</tr>
<tr>
<td>Hulthen-Sugawara (Hu 57)</td>
<td>3.00</td>
<td>0.0294</td>
<td>0.532</td>
<td>(Kn 75)</td>
</tr>
<tr>
<td>Hulthen-Sugawara (Hu 57)</td>
<td>5.00</td>
<td>0.0252</td>
<td>0.466</td>
<td>(Kn 75)</td>
</tr>
<tr>
<td>Glendenning-Kramer (Gl 62)</td>
<td>5.6</td>
<td>0.0265</td>
<td>0.494 e</td>
<td>*</td>
</tr>
<tr>
<td>Yamaguchi (Ya 54)</td>
<td>4.0</td>
<td>0.0280</td>
<td>0.524</td>
<td>(Jo 71)</td>
</tr>
<tr>
<td>Wong (Wo 59)</td>
<td>-</td>
<td>0.0290</td>
<td>0.540 e</td>
<td>*</td>
</tr>
<tr>
<td>de Tourreil SSC (To 73)</td>
<td>5.45</td>
<td>0.0255</td>
<td>0.475 e</td>
<td>* (Sp 75)</td>
</tr>
<tr>
<td>Yale Potential (La 62)</td>
<td>6.96</td>
<td>0.0252</td>
<td>0.476 e</td>
<td>* (Sp 75)</td>
</tr>
<tr>
<td>Iwadare et al. (Iw 56)</td>
<td>6.7</td>
<td>0.0255</td>
<td>0.476 e</td>
<td>*</td>
</tr>
</tbody>
</table>

An * beside a presented $\eta$ or $D_2$ value indicates that the result is estimated using equation (1.7).

$\eta$ is almost completely determined by the deuteron quadrupole moment $Q_d$ and effective range $\rho(-\varepsilon_d, -\varepsilon_d)$, as shown in Figure 1.1. Using the presently accepted experimental values, $Q_d = 0.2860$ fm$^2$ (Re 75) and $\rho(-\varepsilon_d, -\varepsilon_d) = 1.764$ fm (No 72), we estimate the value $\eta = 0.0255$, of Table 1.1. Glendenning and Kramer also assume that for $r \lesssim \chi$ the form of $V_{np}$ is fixed as OPEP, but search upon the interaction for $r \gtrsim \chi$ to fit the deuteron and low energy n-p data.
Eight acceptable local interactions are found with $\eta$ values in the range 
$0.0256 \leq \eta \leq 0.0271$, or using equation (1.7), $0.478 \leq D_2 \leq 0.506$. The values in table 1.1 correspond to potential $\delta$ of G1 62. Thus, a local interaction which fits $Q_d$ determines $\eta$ rather accurately.

Kermode, Mines and Mustafa (Ke 76) suggest that for a non-local

$V_{np}$ there is no significant relation between $Q_d$ and $\eta$ and obtain considerable variation of $Q_d(\Delta Q_d \approx 40\%)$ with different phase equivalent OPEP tailed n-p interactions. Allen et al. (Al 78) however, have attributed the large effects found by Kermode et al. to their having effected relatively large changes in the deuteron wavefunction in the region $r \geq 2$ fermis. This is regarded as undesirable in a region where the interaction is dominantly OPE. Allen et al. find that only relatively small changes in $Q_d(\Delta Q_d \approx 4\%$ with an upper bound of $\approx 10\%)$ may be obtained by variation of the wavefunction for $r < 1.5$ fm.

The smallest $D_2$ value of table 1.1, that of Holinde et al. is obtained from a wavefunction which predicts $Q_d = 0.27$ fm$^2$ and $\rho(-\varepsilon_d, -\varepsilon_d) = 1.82$ fm; these values are plotted as H in Figure 1.1. The $\eta$ value obtained from Figure 1.1 and that estimated from $D_2$ via equation (1.7) are in general agreement and thus tend to indicate that the rather small $D_2$ value is a result of the rather poor fit to $Q_d$ and $\rho$. The model independent but approximate $\eta$ value of Wong (Wo 59) places a convenient upper limit upon the presented values, however, its accuracy has not been assessed.

In conclusion therefore, we feel that the limits placed upon $D_2$ (in table 1.1) by an OPEP tailed $V_{np}$, namely $0.456 \leq D_2 \leq 0.540$ are generous, in particular at the smaller value. Most interactions tend to a value in the region of 0.48. This is of significance as Knutson and Haeberli (Kn 75)
Figure 1.1 Loci of $Q_d, \phi(-\varepsilon_d, -\varepsilon_d)$ and $\eta$ values predicted by the deuteron wavefunction when the OPE Potential is assumed for $r > \bar{x}$, in the presence of hard core radii $r_c = 0.57$ fm and $r = 0$ fm. $\eta$ is obtained from the parameter $\beta$ on the curves by $\eta = 0.01986 \beta$. The wavefunction for $r < \bar{x}$ is assumed quadratic in $r$. 
analysing presently available (d,p) T_{2q} data, obtain a value
\[ D_2 = 0.432 \pm 0.032 \text{ fm}^2 \] or \( \eta \approx 0.0232 \). This value (see Figure 1.1) is
difficult to reconcile with the picture of a local OPEP tailed interaction.
Clearly knowledge of \( D_2 \) or \( \eta \) to an accuracy of 3-4\% (Kn 75) would place
a powerful, and at present absent, constraint upon an acceptable model of
\[ V_{np}(^3S_1 - ^3D_1). \] Indeed an independent determination of \( \eta \), together with
equation (1.4), would determine the deuteron wavefunction uniquely beyond
\( r > \xi \), allowing for a more detailed probing of the wavefunction and the
non-locality of the interaction at small n-p separations.

It has been reported (Gr 78) that the originally proposed method
of \( \eta \) measurement of Amado et al. is not applicable to the evaluation of
a reliable value of \( \eta \). A more recent analysis, however, determines \( \eta \) to
within 20\% (Am 78a), and is consistent with all presented values of Table
1.1. A \( D_2 \) measurement would provide equally useful and potentially more
accurate (Kn 75) information.

1.3 Reaction Observables and Formal Theory

A deuteron, incident upon a target nucleus A, comprising A nucleons
and of mass \( M_A \), may initiate any one of a number of reaction processes
allowed by energy conservation. In the present section, reaction observables for the stripping process
\[ A + \vec{d} \rightarrow B(= A + n) + p, \]
initiated by a polarized deuteron beam are related to the formal quantum
mechanical transition matrix. It is assumed that the residual nucleus B
is a bound state of the n + A system.
We shall make use of the co-ordinate system of Figure 1.2, in which \( R \) and \( r_1 \) shall be used as equivalent labels for the deuteron-target centre of mass separation. The remaining co-ordinates, as used by Johnson (Jo 67), are self evident and explicitly

\[
\begin{align*}
\Gamma_2 &= \Gamma + \gamma \Gamma_n, \\
\Gamma_1 &= (\Gamma_2 + (2 - \gamma) \Gamma_n)/2.
\end{align*}
\]

(1.9)

Here \( \gamma = M_A/(M_A + M_n) \approx A/(A + 1) \), where \( M_n \) is the neutron rest mass.
The incident deuteron beam momentum is defined as direction \( k_1 \), and the deuteron and target \((A)\) binding energies are respectively \( \varepsilon_d \), \( \varepsilon_A \), defined positive. The deuteron centre of mass energy is thus

\[
E_1 = \gamma^2 k_1^2/2\mu_1,
\]

where the \( \mu_1 \) are the channel reduced masses. If the neutron is bound to \( A \) in the final state with energy \( \varepsilon_n > 0 \) then \( \varepsilon_B = \varepsilon_A + \varepsilon_n \) and the proton final state energy is

\[
E_2 = \gamma^2 k_2^2/2\mu_2 = E_1 + Q.
\]

(1.10)

To be definite we shall fix the outgoing proton momentum in direction \( k_2 \), and the reaction \( Q \) value of equation (1.10) is

\[
Q = E_2 - E_1 = \varepsilon_n - \varepsilon_d.
\]

(1.11)

\[\text{‡} \text{ As far as possible we adhere to the convention that subscript 1 labels the incident deuteron channel and 2 the outgoing proton channel.}\]
Figure 1.2 Definition of the (d,p) reaction channel variables.
The total hamiltonian, $H$, for the $A + 2$ body problem may be conveniently decomposed as follows. In the incident channel

$$H = (H_A + H_{np}) + K_1 + V_{dA},$$

and in the outgoing channel

$$H = H_B + K_2 + V_{np} + V_p.$$ 

Here, $H_A$, $H_B$, $H_{np}$ are the internal hamiltonians of the target, residual nucleus and deuteron, and $K_i$ the appropriate channel kinetic energy operators. $V_{np}$ is the free neutron-proton interaction and $V_{dA} = V_n + V_p$, the sum of the many body neutron and proton-target interactions $V_n$, $V_p$. If we define the total energy of the initial d-A system, $E = E_1 - \varepsilon_A - \varepsilon_d$, then solution of the many $(A + 2)$-body Schrodinger equation

$$H\psi_{k_1}^{(+)}(\xi, p, n) = E\psi_{k_1}^{(+)}(\xi, p, n)$$

for state $\psi_{k_1}^{(+)}$, would yield all information available on all open reaction channels at energy $E$. In equation (1.14), $\xi$ represents all the internal degrees of freedom of the target nucleus $A$, and the physical asymptotic boundary conditions of a deuteron beam incident along $k_1$ and radially outgoing waves are shown explicitly. In fact, outside of the range of interaction $V_{dA}$ of equation (1.12), the incident waves part of the solution of equation (1.14) is

---

* We assume for simplicity free particle boundary conditions. If $V_{dA}$ contains a conventional d-A coulomb interaction, equation (1.15) should strictly have the appropriate coulomb conditions (see for e.g. (Ja 70)).
\[ \psi_{k_1}^{(+)}(\xi,p,n)_{\text{inc}} = e^{ik_1 \cdot r_1} \phi_s(\xi,p,n)_{\sigma_1} \phi_{aa_1}^A(\xi) \]  

(1.15)

where \( \phi_{s_1}^{\sigma_1} \) is the deuteron wavefunction and \( \phi_{aa_1}^A \) the initial state (spin \( a \), projection \( \alpha \)) of the target. Strictly, \( \psi_{k_1}^{(+)} \) should also carry subscripts \( a \) and \( \sigma_1 \) which are understood implicitly.

Provided that one or more stripping channels are open, then asymptotically (Me 61), \( \psi_{k_1}^{(+)} \) must contain outgoing spherical proton waves. With an implied summation on the right hand side over all directions of \( k_2 \), all open stripping channels, \( \beta \) and \( \sigma_2 \), then

\[ \psi_{k_1}^{(+)}(\xi,p,n) = \frac{\mu_2}{2\pi n^2} <b_\beta s_2 \sigma_2 | T| a_{aa_2} \sigma_1, k_1 > \]

[1.16]

where \( r_2 \) goes to infinity in direction \( k_2 \). State \( \phi_{b\beta}^B \) of the residual nucleus (spin \( b \), projection \( \beta \)) is an eigenstate of \( H_B \) and \( \chi_{s_2}^2(p) \) is the intrinsic spin wavefunction of the proton. The amplitude for each outgoing wave component is given by the matrix element of the \( T \) or transition operator which connects the incident channel \( (aa_{\sigma_1} \sigma_1, k_1) \) to the final state \( (b_\beta s_2 \sigma_2, k_2) \). Thus, all \((d,p)\) reaction information for a transition to a particular final state of \( B \) is contained within only a small number of matrix elements of the complete transition operator \( T \). We shall rename as \( T(d,p) \), this \((2b + 1)(2s_2 + 1)\) by \((2s_1 + 1)(2a + 1)\) matrix of amplitudes \( ^\dagger \) in the spin space of the particles involved, introduced in

\( ^\dagger \) Clearly, with \( k_1 \) fixed, \( T(d,p) \) is a function of the direction of \( k_2 \).
equation (1.16). The evaluation of these amplitudes shall comprise
a large part of the present thesis. We shall however, prior to this
investigation, define the reaction observables in terms of the formal
stripping matrix $T(d,p)$.

In a stripping reaction initiated by a polarized deuteron beam, it
is possible to measure simultaneously (Ro 73), not only the outgoing
proton cross section but its sensitivity to the incident beam polarization;
namely the reaction analysing powers. To make this definition precise we
introduce the set of irreducible tensor operators (Br 62, La 55) $\hat{T}_{kq}(s)$,
complete in the space of spin $s$, and for which *(Ro 74),

$$<s_s'|\hat{T}_{kq}(s)|s_s> = \hat{k}(s_s k_q |s_s')$$  \hspace{1cm} (1.17)

In addition, the $\hat{T}$ are orthogonal in the sense that

$$\text{Tr}[\hat{T}_{kq}(s)\hat{T}_{k',q'}(s)] = s^2 \delta_{kk'} \delta_{qq'}$$  \hspace{1cm} (1.18)

where the Hermitian conjugate satisfies (Ba 71)

$$\hat{T}_{kq}^+(s) = (-)^q \hat{T}_{k,-q}(s)$$  \hspace{1cm} (1.19)

The polarization of a beam of particles is conveniently described
using the density matrix (Fa 57) formalism. Using the completeness of
the $\hat{T}_{kq}(s)$ then we may write the initial state density matrix $\rho_1(s_1)$ as

$$\rho_1(s_1) = \frac{1}{s^2_1} \int \hat{T}_{kq}(s_1)\hat{T}_{kq}^+(s_1) \times \text{Tr}\{\rho_1(s_1)\}$$  \hspace{1cm} (1.20)

* We shall use stat $k = \hat{k} = (2k + 1)^{\frac{1}{2}}$.
or, upon inversion, the beam moments \( t_{kq}(s_1) \) are

\[
t_{kq}(s_1) = \frac{\text{Tr}\{\rho_1(s_1)\hat{t}_{kq}(s_1)\}}{\text{Tr}\{\rho_1(s_1)\}}
\]  

(1.21)

which describe completely the polarization of the deuteron beam. The final state density matrix for the outgoing protons (in a fixed direction \( \hat{k}_2 \)) is given (La 55) by

\[
\rho_2(s_2) = T(d,p)\rho_1(s_1)T(d,p)^\dagger,
\]  

(1.22)

where \( T(d,p) \) has been introduced previously. The proton differential cross section due to the polarized beam \( \rho_1(s_1) \) is then given, with the adopted normalization of \( T(d,p) \) (equation (1.16)) as

\[
\frac{d\sigma}{dQ}_{\text{pol}} = \frac{k_2}{k_1} \frac{u_1u_2}{(2\pi\hbar^2)^2} \frac{\text{Tr}\{\rho_2(s_2)\}}{\text{Tr}\{\rho_1(s_1)\}}.
\]  

(1.23)

where the analysing powers \( T_{kq} \) are defined by

\[
T_{kq} = \frac{\text{Tr}\{T(d,p)^\dagger \hat{t}_{kq}(s_1)T(d,p)^\dagger\}}{\text{Tr}\{T(d,p)\} \text{Tr}\{T(d,p)^\dagger\}}
\]  

(1.25)

and the \( t_{kq}(s_1) \) are as in equation (1.21). In equation (1.24) the unpol. cross section, that due to an unpolarized incident deuteron beam, is

\[
\frac{d\sigma}{dQ}_{\text{unpol}} = \frac{k_2}{k_1} \frac{u_1u_2}{(2\pi\hbar^2)^2} \frac{\text{Tr}\{T(d,p)\} \text{Tr}\{T(d,p)^\dagger\}}{a_a^2\delta_1^2},
\]

At first sight it would appear that there are eight independent \( (d,p) \) analysing powers. However, if one adopts the reaction co-ordinate system of the Madison convention (Ba 71) in which the positive z axis of a right
handed cartesian set is directed along \( k_1 \) and the \( y \) axis along \( k_1 \wedge k_2 \), then, provided the reaction conserves parity, we have identically (Da 71)

\[
T_{kq} = (-)^{k+q} T_{k-q} . \tag{1.26}
\]

Thus, together with the identity (Ba 71)

\[
T_{kq} = (-)^q T_{k-q}^* ,
\]

only four independent analysing powers remain, three tensor analysing powers \( T_{20}, T_{21}, T_{22} \) which are purely real and the vector analysing power \( T_{11} \) which is purely imaginary. All consist (equation (1.25)) of ratios of weighted bilinear combinations of the matrix elements of \( T(d,p) \), which shall be reduced in the following section to a numerically calculable form via the DWBA.

1.4 The Distorted Waves Theory

The formal \((d,p)\) stripping or transition amplitude \( T(d,p) \) of the preceding section may be written in the post form (Au 70), as

\[
T(d,p) = \langle \phi_{k_2} |(V_{np} + V_p)\psi_{k_1}^{(+)}\rangle , \tag{1.27}
\]

where \( \psi_{k_1}^{(+)} \) is the exact many body wavefunction of equation (1.14). This function, with the incident boundary condition \( |\psi_{k_1}^{(+)}\rangle^{\text{inc}} \) of equation (1.15), is solved subject to the physical boundary condition of radially outgoing waves in the asymptotic region. Thus in the notation of section 1.3,

\[
|\psi_{k_1}^{(+)}\rangle = \Omega^{(+)}(E) |\psi_{k_1}^{(+)}\rangle^{\text{inc}}
\]

where the Møller operator \( \Omega^{(+)}(E) \) is
\( \Omega^{(\pm)}(E) = 1 + (E - H + i\epsilon)^{-1}(V_n + V_p) \).

The final state function \( \phi_{k_2} \), like \( \psi_{k_1}^{(+)} \), carries implied target and projectile spin projection labels and

\[
\phi_{k_2}(\xi, p, n) = \langle r_2, p | k_2 \sigma_2 \rangle \phi_B^{(\xi, n)} \quad (1.28)
\]

where

\[
\langle r_2, p | k_2 \sigma_2 \rangle = e^{-i k_2 \cdot r_2 \sigma_2(p)} .
\]

As yet no mention has been made of the physical requirement that the collection of \( n + p + A(= N + Z) \) fermions be described, in the absence of isospin labels, by wavefunctions antisymmetrized individually in the neutron and proton co-ordinates (Go 64). In fact \( T(d,p) \), defined by equation (1.16) and to this point referred to as the exact transition matrix has been formulated assuming the incident n-p pair are distinguishable from their counterparts in target A, and does not have the required symmetry. The correctly symmetrized stripping matrix \( T_{sym}(d,p) \) is (Au 70)

\[
T_{sym}(d,p) = \sqrt{N + 1}(T(d,p) + Z T_{ex}(d,p)) \quad (1.29)
\]

where \( T(d,p) \) is given by equation (1.27) in which it is now assumed that target states \( \phi_B \), \( \phi_A \) are correctly symmetrized. The 'exchange amplitude', \( T_{ex} \), corresponds to processes in which the outgoing proton originates from the target and not the incident deuteron whereas the 'direct amplitude' \( T(d,p) \) describes the more usual picture of the d,p process in which the proton of the deuteron remains in the continuum.
In the present work we make the usual approximation \[ T_{\text{sym}}(d,p) = \sqrt{N + 1} T(d,p), \] which results from antisymmetrizing in the neutron co-ordinates alone.

Equation (1.27) for \( T(d,p) \) may be written in many alternative, but exact, forms using the Gell-Mann, Goldberger transformation (Ge 53).

For an arbitrary proton-core interaction \( \tilde{V}_p \), then

\[ T(d,p) = \langle X_k^{(-)}(\tilde{V}_p)|V_{np} + (V_p - \tilde{V}_p)|\psi^{(+)} \rangle \]  

(1.30)

where \( \langle X_k^{(-)}(\tilde{V}_p)| \) is generated according to

\[ \langle X_k^{(-)}(\tilde{V}_p)| = \hat{\Omega}^{(-)}(E)|\phi_k \rangle, \]  

(1.31)

\[ \hat{\Omega}^{(-)}(E) = 1 + (E - i\epsilon - H_B - K - \tilde{V}_p)^{-1}\tilde{V}_p. \]  

(1.32)

Clearly, if we replace \( \tilde{V}_p \) by a one body p-B interaction \( U_p \), then we obtain the factorization

\[ \langle X_k^{(-)}(U_p)| = \langle X_s^{(-)}(k_2)|\phi^B_{bb}(\xi,n). \]  

(1.33)

Here the p-B relative wave function or distorted wave

\[ \langle X_s^{(-)}(k_2)| = \{(1 + (E_i - \epsilon - K - U_p)^{-1}U_p^+)\} \langle k_2 \sigma_2 | \]  

(1.34)

---

\(^{\dagger}\) We continue to use the amplitude \( T(d,p) \). When antisymmetrization is included, the conventional procedure (Au 70) is to multiply the reaction cross section of section 1.3 by \( N + 1 \) and then symmetrize the states \( \psi^A, \psi^B \), within \( T(d,p) \).
and therefore (exactly),

$$T(d,p) = \langle \chi_{k_2}^{(-)}(U_p) | V_{np} + (V_p - U_p) | \psi_{k_1}^{(+)} \rangle . \quad (1.35)$$

This expression is the conventional starting point for the discussion of the DWBA, which proceeds through three approximations.

1) It is assumed that in the proton final state, $V_p$ generates only weak coupling to neighbouring excited states of target B. The dominant process is therefore the $p-B$ (in state b6) elastic channel, in which case if $U_p$ is chosen to fit the observed elastic scattering data, it is argued that considerable cancellation between $V_p$ and $U_p$ in equation (1.35) occurs. In the DWBA, $U_p$ is chosen as a phenomenological optical model potential, which through its imaginary part (Ho 63) describes the overall loss of proton flux to open inelastic channels.

2) A similar weak coupling situation is assumed in the incident deuteron channel. In this case the situation is more complex as a result of the internal structure of the deuteron. If it is assumed that target A remains in its initial state throughout the reaction process then we may factorize,

$$\psi_{k_1}^{(+)}(\xi, p, n) = \psi_{k_1}^{(+)}(\xi, \xi_1) a^A_{\alpha\alpha}(\xi) . \quad (1.36)$$

Here, $\psi_{k_1}^{(+)}(\xi, \xi_1)$ is the projection of the many body wavefunction $\psi(\xi, p, n)$ onto the ground state $(\alpha, \alpha)$ of the target nucleus, and describes exactly all open reaction channels in which the target nucleus remains in its ground state. The three body $(n + p + A)$ function therefore contains, asymptotically, outgoing waves describing deuteron-elastic scattering, deuteron breakup and certain stripping channels. Approximations 1 and 2 give the simplified amplitude
\[ \Gamma_{\text{DWBA}}(d,p) = \left< \chi^{(-)}_{s_2\sigma_2} (k_2, p) \phi^B_{b\beta} \right| v_{\text{np}} \left| \phi^A_{\alpha\alpha} \psi^{(+)}_{k_1} (r, \xi_1) \right> , \quad (1.37) \]

in which we allow for the fact that, at the time at which the neutron is captured, the n-p pair, as a result of the weak deuteron binding energy, may no longer look like a deuteron: in fact the deuteron may be internally distorted or broken up prior to stripping.

From the nuclear structure point of view, the importance of equation (1.37) is that \( V_{\text{np}} \) is independent of the internal co-ordinates, \( \xi \), of \( A \). The reaction is assumed therefore to take place from initial to final state by simple capture of a neutron about \( A \), and can proceed only via that component of \( \phi^B \) in which \( A \) nucleons are in the same state of motion as \( \phi^A \). This neutron form factor (integrating over \( \xi \))

\[ <\phi^B_{b\beta}(\xi, n) | \phi^A_{\alpha\alpha}(\xi) > = F^{AB}_{\alpha\beta}(n)^* , \quad (1.38) \]

or structure matrix element, can therefore provide direct spectroscopic information on the structure of nucleus \( B \). The exact evaluation of \( F^{AB}(n) \) is in practice an impossible many body problem (Pi 65, Be 65).

The final approximation of the DWBA is:

3) It is assumed that the three-body nature of the deuteron target system is unimportant, arguing that the elastic component of \( \psi^{(+)}_{k_1} (r, \xi_1) \) is the dominant contribution to amplitude (1.37). The three-body function is thus replaced by the product form

\[ \psi^{(+)}_{k_1} (r, \xi_1) = \sum_{\sigma_1} \chi^{(+)}_{\sigma_1 \sigma_1} (k_1, r_1) \phi^A_{\sigma_1} (r, p, n) = \chi^{(+)}_{\sigma_1 \sigma_1} (k_1, p, n) , \quad (1.39) \]

of the deuteron internal wavefunction, and a phenomenological deuteron optical model wavefunction, \( \chi^{(+)}_{k_1, \xi_1} \), which reproduces the observed d-A elastic scattering. The deuteron is thereby treated as a rigid elementary
particle, throughout its passage of the target field, all loss of flux from the elastic channel being indiscriminantly taken into account through the absorptive optical model term. We obtain the conventional DWBA amplitude, via the indicated approximations, namely

\[ T_{\text{DWBA}}^{(d,p)} = \langle X^{(+)}(k_{1},p,n) | V_{np} | X^{(+)}(k_{2},p,n) \rangle, \quad (1.40) \]

the remaining incalculable element of which is \( F^{AB} \).

\( F^{AB} \), a function of the captured neutron co-ordinates only, may be expanded (Pi 65), exactly, in states of definite angular momentum transfer \( j \), as

\[ F_{\alpha \beta}^{AB}(n) = \sum_{j m} (a_{\alpha jm}|b_{\beta}) \phi_{j}^{m}(n) \quad (1.41) \]

which comprise the wavefunction of the captured neutron. It is standard practice in most DWBA calculations to assume that

\[ \phi_{j}^{m}(n) = \{S(n \lambda j;AB)/N + \frac{1}{2}\} \psi_{n \lambda j}^{m}(n) \quad (1.42) \]

where \( \psi_{n \lambda j}^{m} \) is a normalized single-particle\(^{\dagger}\) wave function with, for a fixed \( j, \lambda = j \pm \frac{1}{2} \) fixed by the parity of nucleus \( B \): or parity change \( \Pi_{AB} = (-)^{\lambda} \) in the reaction. In equation (1.42), \( N \) is the number of target neutrons and \( S(n \lambda j;AB) \), the spectroscopic factor for the transition to state \( n \lambda j \), measures the probability that state \( \phi^{R} \) has a parentage based on the ground state of target \( A \) with a single neutron in the shell model state \( \psi_{n \lambda j} \).

\(^{\dagger}\) Subscript \( n \) on the single particle function refers to the principal quantum number of the state.
Theoretically this procedure is really only justified for a closed shell target $A$ where the interaction of the neutron with the complete shells gives rise to a Hartree-Fock type one-body interaction (Le 73).

In practice, equation (1.42) is used in most calculations. The $\psi_{n\lambda j}$ are calculated here in a single particle Wood-Saxon shaped potential, with shape parameters obtained from the neutron optical potential in the limit (Ma 77) of zero bombarding energy. The well depth however, is varied to produce an eigenfunction with the correct experimental binding $\varepsilon_n$. In general only one value of $j$, and hence $\lambda$, enter the form-factor $F_{AB}(n)$, and we shall assume that

$$
F_{AB}^{A\beta}(n) = \sum_{m} (a\alpha j|m\beta) \alpha_{\lambda j}^{AB} \psi_{\lambda j}^{m}(r_n),
$$

where the principal quantum number, $n$, is no longer shown explicitly and the factor $(S(\lambda j, AB)/(N + 1))^\frac{3}{2}$ has been rewritten for convenience as $\alpha_{\lambda j}^{AB}$.

In the present section the approximations inherent in the conventional DWBA have been introduced without discussion of their probable accuracy. In the following chapter we shall assess these approximations in the sub-coulomb stripping regime and obtain an approximate expression for the amplitude $T_{DWBA}(d,p)$ when the deuteron D-state component is accounted for in the approximate finite range treatment of Johnson and Santos (Jo 71).
CHAPTER 2

THE STRIPPING AMPLITUDE

In the previous chapter the many body (d,p) transition matrix has been simplified, through the DWBA. The resulting amplitudes comprise matrix elements of the free n-p interaction, $V_{np}$, between two body scattering states which reproduce the observed elastic scattering in the initial and final states. Nevertheless, the nuclear rearrangement of the particles involved means that the resultant amplitude is a six dimensional integral. Traditionally this problem has been overcome either by using Butler theory (Bu 51), in which the deuteron and proton motions are replaced by plane wave states, or by assuming that $V_{np}$ has zero range (To 61). More recently however, methods have been developed which take account of the finite range of $V_{np}$, and will be referred to as finite range (F.R.) stripping treatments.

Both exact and approximate F.R. calculations have been performed, for a $^3S_1$ deuteron, using the proposed methods of Austern et al. (Au 64) and of Buttle and Goldfarb (Bu 64, Be 64), respectively. The latter approximate procedure which has been shown (Di 65, Mi 66) to reproduce rather accurately, in most cases, the exact results has been used extensively as a result of its simplicity and computational efficiency. However, since Johnson's observation (Jo 67) that (d,p) reaction tensor analysing powers should receive a sizable contribution from the $^3D_1$ deuteron component, the two procedures outlined above have been extended to include the deuteron D-state. Exact F.R. ($S + D$) state calculations have been reported by Delic and Robson (De 70, De 74) and the approximate
procedure of Buttle and Goldfarb, commonly known as the local energy approximation (LEA), has been generalized by Johnson and Santos (Jo 71). Although the capability of performing exact F.R. calculations is of great importance, their complexity and large computer time requirements have resulted in the use of the LEA method (Ha 70) for the routine analysis of experimental (d,p) and (p,d) data (Ro 73, Kn 73, Br 71).

In one such analysis, Knutson and Haeberli (Kn 75) suggest that it may be possible to obtain quantitative information about the deuteron from a study of sub-coulomb (d,p) reaction tensor analysing power data; in particular the number \( D^2 \). However, \( D^2 \) enters as a parameter in stripping theory only through the Johnson-Santos LEA treatment of finite range effects. Thus any measurement of \( D^2 \) necessarily assumes that the approximate LEA amplitude accurately represents the exact DWBA calculation, and in turn that the DWBA provides a good description of the reaction process.

In this chapter, we shall discuss briefly the accuracy of the DWBA approximations in the sub-coulomb regime. The LEA treatment of the stripping amplitude shall then be introduced, and finally we shall comment upon its accuracy in sub-coulomb reactions.

2.1 The DWBA at Sub-coulomb Energies

Consider a (d,p) transition, with Q value near zero, initiated by a deuteron with bombarding energy less than the target coulomb barrier height. In this limit, the emergent proton is also sub-coulomb, and the reaction takes place predominantly (Go 66, Le 62), far from the nuclear surface. In fact, the dominant contribution to the stripping amplitude is from the
region of the mean classical distance of closest approach \((Go \ 67, \ Bu \ 71)\) of the projectile-target systems. Within this radius, both deuteron and proton distorted waves decay exponentially and are extremely small in magnitude within the nuclear volume, there being little probability of the projectiles penetrating the coulomb barrier.

Thus, in the amplitude \(T_{\text{sym}}\) (equation (1.29)), contributions from the nuclear interior are almost totally suppressed, and many uncertainties of the DWBA, present at higher energies and which relate to the treatment of the nuclear interior, \((Gl \ 75, \ Go \ 66)\) disappear. We revisit briefly the approximations of section 1.4, namely the DWBA.

a) Neglect of the exchange amplitude \(T_{\text{ex}}\)

If exchange processes are to take place, good overlap is required \((Au \ 70)\) between both the proton and deuteron distorted waves and the nuclear states \(\phi^B, \phi^A\) respectively. This term is therefore small in the sub-coulomb limit.

b) The weak coupling approximation

The small overlap of distorted and target nuclear wavefunctions, means the probability of target excitation in both initial and final states is small. In the final state, the incomplete cancellation of the interaction \(V_p - U_p\), responsible for the excitation of the residual nucleus, is non zero only within the nuclear volume and thus will not contribute to the stripping amplitude. Therefore \(U_p\), which generates the correct elastic phase shifts, accurately represents the proton motion everywhere outside of the nuclear surface.
c) The neutron form factor

Beyond the nuclear surface, $F_{AB}^{AB}(n)$ is determined, but for an overall normalization, by the spin, parity and binding energy of the residual nucleus $\psi^B$. Thus, apart from a multiplicative constant, the single particle state $\psi_{\lambda j}$ of section 1.4, has the correct functional form in the important tail region.

d) Ignoring three body effects

The replacement of the three body function $\psi_{\zeta 1}^{(+)}(r, r_1)$ by a deuteron elastic scattering function is the most difficult step of the DWBA to justify. The loosely bound deuteron, even if it does not penetrate the region of the nuclear interactions $V_n$, $V_p$, may yet be broken up by the long ranged p-A coulomb force. We shall discuss this problem in the following chapters, however, assuming that elastic scattering is the dominant process, the deuteron distorted wave is again fixed beyond the nuclear surface by the elastic phase shifts.

In conclusion, the approximations of the DWBA are at their most accurate in sub-coulomb, $Q \lesssim 0$, transitions. In this limit, each of the functions entering the stripping amplitude is required only outside of the nucleus. Here, all are insensitive to the geometry of the potential from which they were generated, their form being rather precisely\footnote{Here we are particularly interested in tensor analysing powers. These are (see equation (1.25)) ratios of bilinear combinations of amplitudes, and the asymptotic normalization uncertainty in $\psi_{\lambda j}$ is unimportant.}
determined by the observed elastic scattering, and properties of the residual nucleus \( B \), formed in the reaction. The process is therefore insensitive (Kn 77, Go 65) to the short ranged nuclear optical potentials, and in particular small spin-dependent terms. In the following sections the LEA will be introduced without the added complication of spin dependence; the inclusion of which shall be the subject of chapter 5.

2.2 The Distorted Waves

We take as starting point for the LEA discussion,*

\[
T_{	ext{DWBA}}^{\text{D}}(d,p) = \langle \chi^{(-)}_{s_2\sigma_2,-s_2,p}\phi^B_{b\beta}|V_{np}|\phi^A_{a\alpha}\chi^{(+)}_{s_1\sigma_1,-s_1,p,n} \rangle . \quad (2.1)
\]

If the deuteron and proton optical potentials are designated \( U_1, U_2 = U_p \) respectively, then the corresponding distorted waves in the above, satisfy the Schrödinger equations

\[
[H + U_1(r) + U_{np}(r) - E + \epsilon_d] \chi^{(+)}_{s_1\sigma_1}(k_1,p,n) = 0 \quad (2.2)
\]

\[
[H_2 + U_2^+(r_2) - E_2] \chi^{(-)}_{s_2\sigma_2}(k_2,p) = 0 \quad , \quad (2.3)
\]

where \( H_{np} \) is the deuteron internal hamiltonian, and the \( U_i \) are supposed to contain the usual projectile-target centre of mass coulomb interaction. So, in general we have

\[
\chi^{(+)}_{s_1\sigma_1}(k_1,p,n) = \sum_{\sigma_1'} \chi^{(+)}_{\sigma_1'\sigma_1}(k_1,p,n) \phi^I_{s_1}(r_1,p,n) \quad , \quad (2.4)
\]

* The left hand side carries the indices \( \sigma_2\beta, \sigma_1\alpha \), implicitly throughout.
and where time reversal invariance requires (Sa 64) that

\[ \chi^{(-)*}_{\sigma' \sigma} (\kappa, \tau) = (-)^{\sigma - \sigma'} \chi^{(+)}_{-\sigma' -\sigma} (\kappa, \tau) \],

(2.6)

We shall assume at present that the U. are central \( (U_i = U_{ci}) \), thus \( \chi^{(+)}_1 \) and \( \chi^{(-)}_1 \) are diagonal in their respective spin spaces, and

\[ \chi^{(+)}_{\sigma' \sigma} (k_1, \tau_1) = \delta_{\sigma' \sigma} \chi^{(+)}_1 (k_1, \tau_1) \],

(2.7)

\[ \chi^{(-)}_{\sigma' \sigma} (k_2, \tau_2) = \delta_{\sigma' \sigma} \chi^{(-)}_2 (k_2, \tau_2) \].

(2.8)

Performing a partial wave expansion of these functions in eigenstates of the projectile-target relative orbital angular momentum \( \lambda_i (i = 1, 2) \), then

\[ \chi^{(+)} (k_i, \tau_i) = \frac{4\pi}{r_i} \sum \frac{\lambda_i \lambda'_{i-1} (k_i) \chi^{(+)}_{\lambda_i-1, \lambda_i} (k_i, \tau_i)}{r_i^2} Y_{\lambda_i}^\lambda (\kappa_i)^* Y_{\lambda_i}^\lambda (\kappa_i) \chi^{(+)} (k_i, \tau_i) \].

(2.9)

Here the radial functions \( \chi_{\lambda_i}^\lambda \), are solutions of the differential equations

\[ \left\{ - \frac{\hbar^2}{2u_i} \frac{d^2}{dr_i^2} - \frac{\lambda_i (\lambda_i + 1)}{r_i^2} \right\} + U_{ci} (r_i) - E_i \chi_{\lambda_i}^\lambda (k_i, \tau_i) = 0 \],

(2.10)

and, defining

\[ \chi_{\lambda_i}^\lambda (\kappa_i) = \frac{\lambda_i \lambda'_{i-1} (k_i, \tau_i) Y_{\lambda_i}^\lambda (\kappa_i) / r_i}{\lambda'_{i-1} \lambda_i (k_i, \tau_i)} \],

(2.11)

equation (2.6) gives, using \( Y_{\lambda_i}^\lambda (- \kappa_i) = (-)^{\lambda_i} Y_{\lambda_i}^\lambda (\kappa_i) \);

\[ \chi^{(-)} (k_2', \tau_2) = \frac{4\pi}{r_2} \sum \frac{\lambda_2 \lambda'_{2-1} (k_2', \tau_2) \chi^{(-)}_{\lambda_2} (k_2', \tau_2)}{r_2^2} \].

(2.12)
With the adopted normalization of equation (2.9), asymptotically the radial functions go as

\[ \chi_{\ell}(k, r) \bigg|_{r \to \infty} = \frac{i \sigma_{\ell}}{k} \left\{ F_{\ell}(kr) + C_{\ell}(G_{\ell}(kr) + iF_{\ell}(kr)) \right\}. \]  

(2.13)

Here \( F_{\ell}, G_{\ell} \) are respectively, the regular and irregular coulomb functions (Ab 65), \( \sigma_{\ell} \) the usual coulomb phase shift and \( C_{\ell} \), the elastic amplitudes, are related to the conventional S matrix partial wave amplitude \( S_{\ell}(Ro 67) \), according to

\[ C_{\ell} = 1/2i(\sigma_{\ell} - 1) \]  

(2.14)

The amplitudes \( C_{\ell} \), are experimental inputs to the stripping calculation via the observed elastic scattering and optical potentials. The distorted waves describing the relative motion of the projectile-target systems carry, in the DWBA, all information relating to the dynamics of the reaction process.

2.3 The Reaction Form Factor

The transition amplitude of equation (2.1) may be rewritten using the results of equations (2.4-8), corresponding to no spin-dependent distortion, as

\[ T_{\text{DWBA}}(d, p) = \int \int d\tau \chi_{(-)}(k_2, \tau) \chi_{(+)}(k, \tau) (s_{s_1} \sigma_{a_1} b_{b_1}) |_{np} |_{s_{s_1} \sigma_{a_1}} \]  

(2.15)

The closed bra-ket in the above is known as the reaction form factor, and in the notation of the previous chapter.
$$\langle s_2 s_2 b | V_{np} | s_1 s_1 \alpha \alpha \rangle = \sum_{np} \int d\xi \chi_{s_2}^g (p) \phi_{b}^*(\xi, n)$$

$$\times V_{np} (\tau) \phi_{s_1}^1 (\tau, p, n) \phi_{\alpha \alpha}^A (\xi),$$

(2.16)

where $\sum_{np}$ denotes summation over the neutron and proton spin variables and $\int d\xi$ all co-ordinates internal to target A.

This form factor contains all information on the nuclear structure of the target nuclei and projectiles in the amplitude (2.15). In section 1.4 we have already discussed the target nuclei overlap

$$F_{\alpha \beta}^A (n) = \int d\xi \phi_{\alpha \beta}^B (\xi, n) \phi_{\alpha \alpha}^A (\xi),$$

and, from equation (1.43), we approximate

$$F_{\alpha \beta}^A (n) = (\alpha \beta | n_n | b \beta) \alpha \beta \phi_{\alpha \beta}^B (\tau, n) \psi_{\alpha \alpha}^n (\tau, n).$$

(2.17)

Here $i^i_n$ are the orbital and total angular momenta of the captured neutron, spin $s_3,$ and

$$\psi_{i^i_n j^i_n} (\tau, n) = R_{i^i_n j^i_n} (\tau, n) \mathcal{Y}_{i^i_n s_3 j^i_n}^{m_n} (\hat{\tau}, n),$$

(2.18)

where, if $\chi_{s_3}^3 (n)$ is the intrinsic spin function of the neutron, then

$$\mathcal{Y}_{i^i_n s_3 j^i_n}^{m_n} (\hat{\tau}, n) = \sum_{s_3^3} \langle i^i_n s_3 j^i_n | s_3 \rangle | j^i_n m_n \rangle \chi_{i^i_n}^3 \chi_{s_3}^3 (n).$$

(2.19)

The remaining terms in equation (2.16) contain the structure of the projectiles, and in particular the deuteron wavefunction. With the deuteron wave function, $\phi_{s_1}^s,$ defined in equations (1.1-2), the explicit
appearance of $V_{np}(r)$ can be removed from equation (2.16) using the Schrödinger equation for the deuteron. Therefore,

$$
V_{np}(r)\phi_{s1}(\xi,p,n) = \left(\frac{\hbar^2}{2\mu} \frac{\nabla^2}{\xi} - \varepsilon_d\right)\phi_{s1}(\xi,p,n)
$$

$$
= \sum_{L=0,2} v_L(r) \mathcal{Y}_{Ls_1s_1}(\hat{r},p,n) , \quad (2.20)
$$

where $\mu$ is the reduced mass of the $n-p$ system, $\nabla_{\xi}$ is the gradient operator in the relative $n-p$ co-ordinate, $\xi$, and the radial functions $v_L(r)$ are (Jo 67)

$$
v_L(r) = \frac{1}{L} \left[ \frac{d^2}{dr^2} + \frac{2}{r} \frac{d}{dr} - \frac{L(L+1)}{r^2} \right] u_L(r) . \quad (2.21)
$$

Collecting the results of equations (2.16-21), together with equation (1.2) for $\mathcal{Y}_{Ls_1s_1}^{s_1}$, we obtain, using the relation (Br 62)

$$
\mathcal{Y}_{L}^{\Lambda} (\hat{r})^* = (-)^{\Lambda} \mathcal{Y}_{L}^{-\Lambda} (\hat{r}) ,
$$

and the triplet $n-p$ spin eigenfunction

$$
\chi^{s_1}_{s_1}(p,n) = \sum_{\sigma_2,\sigma_3} (s_2\sigma_2 s_3\sigma_3 | s_1\sigma_1) \chi^{s_2}_{s_2}(p) \chi^{s_3}_{s_3}(n) , \quad (2.22)
$$

the result that:

$$
<s_2\sigma_2 b\beta | V_{np} | s_1\sigma_1 a\alpha> = \sum_{\lambda,\lambda'} \mathcal{B}^{\lambda\lambda'}_{n,n}(\sigma_2\beta;\sigma_1\alpha,\lambda,\lambda' - \Lambda) i^{L - \xi} (-)^{\lambda} \quad (2.23)
$$

$$
R_{j\beta n}(r_n) v_{L}(r) \left( \mathcal{Y}_{\lambda\lambda'}^{\Lambda}(\hat{r}_n) \mathcal{Y}_{L}^{\Lambda}(\hat{r}) \right)^* . \quad (2.23)
$$
where
\[ B^{\lambda E}_{n j n}(\sigma_2^\beta;\sigma_1^\alpha,\lambda_n,\Lambda) = a^{AB}_{n j n}(a a_j m_n | b \beta) \]

(2.24)

\[ \times \sum_{\sigma_3^\sigma_1^1} (\ell_n^\lambda n s_3^\sigma_3 | j_{n n}^m) (L a s_1^* \sigma_1^1 | s_1^1 \sigma_1) (s_2^2 s_3^2 | s_1^1 \sigma_1^1) . \]

It should be pointed out that the present formalism (i.e. no spin dependent distortion in either channel), will predict identically zero reaction tensor analysing powers \( \dagger \) (Go 60, Jo 67), \( T_{2q} \), unless the deuteron D-state \( (L = 2) \) component is included in equation (2.23). Thus if \( V_{np}(\xi) \) is assumed central, then

\[ V_{np}(r) \phi_{s_1}(\xi, p, n) = (4\pi)^{-1} v_0(r) \chi_{s_1}(p, n) \]

and (Go 60) \( T_{kq} = 0 \), for \( k > 2s_3 \). This demonstrates rather clearly the importance of the deuteron D-state in sub-coulomb \( T_{2q} \) calculations.

2.4 The LEA Method

Using the explicit expressions for the projectile distorted waves (equations (2.9-12)) and reaction form factor (equation (2.23)), the partial wave decomposition of equation (2.15), for \( T^{DWBA}(d,p) \), is

\[ T^{DWBA}_{kq} = (3)^{-1/2} \epsilon_{kq}^* . \]

\[ \dagger \] The efficiency tensors \( \epsilon_{kq} \) of Goldfarb and Johnson (Go 60), are related to the \( T_{kq} \) of the Madison Convention (Mc 71) according to

\[ T_{kq} = (3)^{-1/2} \epsilon_{kq}^* . \]
\[ I_{\text{DWBA}}(d,p) = (4\pi)^2 \sum \lambda_{1-}^{l_1-} \lambda_{2-}^{l_2-} Y_{l_1-2}^{*}(\hat{r}_-2) Y_{l_2-1}^{*}(\hat{r}_-1) \]

\[ B_{n^j n}^{n b n} = (\sigma_2 \beta ; q_2 \alpha, \lambda n_{1-A}) L(-) L^{A}_{F L A} \]

(2.25)

with summation over \( \lambda_n, L, A, \lambda^{'}, \lambda_1, \lambda_2 \). The Jacobian for the transformation from co-ordinates \((r_1, r_2)\) to \((\tau_n, \tau)\) is unity and therefore

\[ F_{12}^{LA} = \int d\tau_n d\tau \gamma_{2n}^{l_2} (\tau) \gamma_{1n}^{l_1} (\tau) R_{j_n}^{l_1} (\tau) Y_{l_n}^{n} (\hat{r}_n) \]

\[ \chi_{2}^{l_2} (\tau_n + \frac{1}{2} \tau) \]

\[ \chi_{1}^{l_1} (\tau_n + \frac{1}{2} \tau) \]

(2.26)

where only the LA labels are shown explicitly upon \( F_{12}^{LA} \), and \( \chi_{1}^{l_1} \) are given by equation (2.11).

As outlined earlier, the approximate LEA treatment, by Buttle and Goldfarb (Bu 64), of finite range (F.R.) effects for a \( ^3S_1 \) deuteron, is known to be a good approximation (Di 65) to exact F.R. calculations. This led Johnson and Santos (Jo 71, Sa 68), to generalize the method to include the deuteron D-state components \( F_{12}^{2A} \). Following the Buttle-Goldfarb procedure we make a Taylor expansion of the functions \( \chi_{l_1}^{l_1} \), thus

\[ \chi_{l_1}^{l_1} (\tau_n + \frac{1}{2} \tau) = \exp(\frac{1}{2} \tau \cdot \nabla_{1}) \chi_{l_1}^{l_1} (\tau_n) \]

\[ \chi_{l_2}^{l_2} (\gamma \tau_n + \tau) = \exp(\gamma^{-1} \tau \cdot \nabla_{2}) \chi_{l_2}^{l_2} (\gamma \tau_n) \]

(2.27)

where \( \nabla_{1}, \nabla_{2} \) are gradient operators with respect to \( \tau_n \), with the proviso that they operate on \( \chi_{l_1}^{l_1} \) and \( \chi_{l_2}^{l_2} \), respectively.
Formally we may rewrite, exactly (Jo 71)

\[ F_{12}^{jA} = (2\pi)^{3/2} \int dr \, (\tau_1)^{jA}_{-1/2} (K_1/2 + K_2) \chi_{F_2} (\gamma r_n) \]

\[ \times R_{j1} (r_n) Y_{L}^{\pm} (\gamma r_n) \chi_{L_1}^{\pm} (\gamma r_n) , \]  

(2.28)

where

\[ \phi_{jA} (K) = (2\pi)^{-3/2} \int dr \, e^{-iK \cdot r} v_L (r) Y_L^{A} (\gamma r_n) \]  

(2.29)

and

\[ iK_1 = \nabla_1 , \quad iK_2 = \nabla_2 / \gamma . \]  

(2.30)

The angular integration in equation (2.29) can be performed analytically, and, using the definition

\[ (2\pi)^{3/2} \phi_{jA} (K) = 4\pi \tilde{v}_L (K) Y_L^{A} (\gamma r_n) \]  

(2.31)

for the functions \( \tilde{v}_L (K) \), then clearly

\[ \tilde{v}_L (K) = iL \int_0^\infty dr \, r^2 j_L (Kr) v_L (r) . \]  

(2.32)

However, using the definition of the \( v_L (r) \) of equation (2.21), then (Jo 71)

\[ \tilde{v}_L (K) = - \eta^2 / 2\mu (K^2 + \alpha^2) u_L (K) \]  

(2.33)

where the \( u_L (K) \) are the deuteron S(L = 0) and D(L = 2) state radial functions in momentum space, i.e.
\[ u_L(K) = \int_0^\infty \, dr \, r^2 j_L(Kr) u_L(r) \]  

(2.34)

and \( \alpha = \eta^{-1}(2\mu e_d)^{\frac{1}{2}} = .2316 \text{ fm}^{-1} \).

The basic idea of the LEA to replace the function \( \phi_{LA}(K) \) by a simple functional form, and in particular by a low order polynomial in \( K \). The complex differential operator \( \phi_{LA}(\frac{1}{2} K_1 + K_2) \) in equation (2.28), is then replaced by an approximate form, involving only low order derivatives of the functions \( \chi_{\nu}^{i} \); reducing the amplitude \( F_{12}^{LA} \), to a calculable form. For a Hulthén wavefunction (Hu 57)

\[ u_0(r) = N_H \left( e^{-\alpha r} - e^{-\beta_0 r} \right) / r \]  

(2.35)

where \( \beta_0 \approx 6\alpha \), and \( \beta_0^{-1} \) is a measure of the range of \( V_{np} \), then (Bu 64)

\[ \phi_{00}(K) = \beta_0^2 / (\beta_0^2 + K^2) \]  

which falls off rapidly for \( K \approx \beta_0 \). Thus, if equation (2.28) were rewritten in momentum space, the contributions to \( \phi_{00}(K) \) from \( K \ll \beta_0 \), would be expected to dominate the amplitude.

The Buttle-Goldfarb prescription for the deuteron S-state, is to replace \( \phi_{00} \) by a polynomial which is an accurate representation of the function for small \( K \). Consider now the small \( K \) expansion of the Bessel function \( j_L(Kr) \) appearing in equation (2.32), namely (Ab 65)

\[ j_L(Kr) = (Kr)^L \sum_{m=0}^{\infty} C_{mL} (-)^m(Kr)^{2m} \]  

(2.36)
where
\[ C_{mL} = \frac{(m + L)!2^{L}/(m!(2m + 2L + 1)!)}{m!} \quad (2.37) \]

Substituting in equation (2.32) then gives (Sa 73, Sa 74)
\[ \tilde{V}_L(K) = \sum_{m=0}^{\infty} (-)^m C_{mL} K^{L+2m} d_{mL} \quad (2.38) \]

where
\[ d_{mL} = \int_0^\infty dr r^{L+2m+2} u_L(r) I^L_1 \quad (2.39) \]
or using equation (2.33)
\[ \tilde{V}_L(K) = - \frac{\kappa^2/2\mu K^2 + \alpha^2}{K^2 + \alpha^2} \sum_{m} (-)^m C_{mL} K^{L+2m} d_{mL} \quad (2.40) \]

where
\[ d_{mL} = \int_0^\infty dr r^{L+2m+2} u_L(r) \quad (2.41) \]

In fact the Buttle-Goldfarb S-state treatment retains only the first two terms of equation (2.40), and following the notation of Johnson and Santos, the approximation is written
\[ (2\pi)^{3/2} \tilde{\phi}^{00}(K) = D_0(1 - K^2/\beta^2) \quad (2.42) \]

and therefore the LEA approximation to \( \tilde{V}_0 \) is
\[ \tilde{V}_0^{\text{LEA}}(K) = (4\pi)^{-1/2} D_0(1 - K^2/\beta^2) \quad (2.43) \]

Comparison with equation (2.40) shows that
\[ D_0 = -\frac{\pi^2}{2\mu(4\pi)i\{(K^2 + \alpha^2)u_0(K)\}_{K=0} = (4\pi)^{\frac{1}{2}}V_0(0) \]

\[ = -\epsilon_d(4\pi)^{\frac{1}{2}}d_{00} = (4\pi)^{\frac{1}{2}}d_{00} \quad (2.44) \]

and from equation (2.38), we obtain (Sa 73)

\[ \beta^{-1} = \{d_{10}/6d_{00}\}^{\frac{1}{2}} \]

\[ = \{d_{10}/6d_{00} - \alpha^{-2}\}^{\frac{1}{2}}, \quad (2.44b) \]

thus \( \beta^{-1} \) is a measure of the range of \( V_{np} \) and if \( u_0(r) \) is the Hulthén wavefunction, equation (2.25), then \( \beta = \beta_0 \). In all calculations to be presented in this thesis the values of the parameters \( D_0 \) and \( \beta \) are taken to be (Kn 77):

\[ D_0 = -1.251(\pi^2/2\mu)\{8\pi\alpha\}^{\frac{1}{2}}, \quad (2.45) \]

\[ \beta = 1.341 \text{ fm}^{-1}, \]

the values predicted by the Reid Soft-Core deuteron wavefunction (Re 68).

On the basis of the success of the S-state approximation, Johnson and Santos suggest a corresponding approximation to \( \phi_{2A}(K) \), accurate for \( K \ll \beta \). In this case (Jo 71), retaining the first term of \( \tilde{V}_2 \), then

\[ (2\pi)^{\frac{3}{2}}\phi_{2A}(K) = (4\pi)^{\frac{1}{2}}D_0D_2K^2\tilde{V}_2(K) \quad (2.46) \]

to the same order in \( K \) as the S-state component of equation (2.42), therefore

\[ \tilde{V}_{2LEA}(K) = D_0D_2(4\pi)^{-\frac{1}{2}}K^2 \quad (2.47) \]
where $D_2$, the parameter introduced in section 1.2, is

$$D_2 = \left\{ \frac{(1/15)d_{02}}{d_{00}} \right\}$$

$$= \text{Lim}_{K \to 0} \{ \tilde{V}_2(K)/(K^2 \tilde{V}_0(K)) \} , \quad (2.48)$$

and contains all the quantitative and model dependent deuteron D-state information in the approximate $\phi_{2\hbar}$. We shall investigate the accuracy of the LEA treatment to sub-coulomb reactions in the following section.

Meanwhile, equations (2.42) and (2.28) give the S-state amplitude

$$p_{12}^{00} = D_0 \int d\tau_n \left\{ 1 - K^2/\beta^2 \right\} \lambda_l^2(\gamma\tau_n) R_{j_n} \lambda^2(\tau_n) Y_{j_n}^{\lambda}(\tau_n)^*$$

$$\times \lambda_l^1(\tau_n), \quad (2.49)$$

where $K = \frac{1}{2} K_1 + K_2$; with $K_1, K_2$ given by equation (2.30). As $R_{j_n}$ vanishes for $r_n \to \infty$, Green's Theorem can be used to show that in equation (2.49), we may make the replacement (Sa 68)

$$K^2 = a_1 K_1^2 + a_2 K_2^2 + a_3 K_3^2 , \quad (2.50)$$

(by analogy with equation (2.30), $iK_3 = \nabla_3$, is the gradient operator with respect to $r_n$, operating only upon $R_{j_n}$) and

$$a_1 = -(2 - \gamma)/4\gamma , \quad a_2 = (2 - \gamma)/2 , \quad a_3 = 1/2\gamma . \quad (2.51)$$

Therefore, using the Schrödinger equations for the functions appearing in equation (2.49), for example
then equations (2.49-50) give

$$ F_{12}^{00} = D \int d\tau \Lambda_{12}(r) X_{n}^{\lambda}(\hat{r}) Y_{n}^{\lambda}(\hat{r}) X_{1}^{\lambda}(r) X_{n}^{\lambda}(r) = 0 \quad (2.52) $$

where the multiplicative factor $\Lambda_{12}$ is (Bu 64)

$$ \Lambda_{12}(r) = 1 - (\alpha^2/\beta^2 \epsilon_d) \{ U_c^{(1)}(r) - U_c^{(2)}(r) - U_{j_{12}}(r) - \epsilon_d \} \quad (2.53) $$

Here $U_{j_{12}}$ is the potential used to generate the neutron bound state function $R_{j_{12}}$ (Appendix F).

Use of the addition theorem for the Spherical Harmonics (Br 62), namely

$$ Y_{\ell_1}^{m_1}(\hat{r}) Y_{\ell_2}^{m_2}(\hat{r}) = (4\pi)^{-1/2} \sum_{\ell_3 m_3} \hat{\lambda}_{\ell_1 \ell_2 \ell_3}^{\ell_3 m_3} (\ell_1 \ell_2 \ell_3 | m_1 m_2 m_3) $$

$$ \times (\ell_0 \ell_2 \ell_0 | \ell_3 \ell_0) Y_{\ell_3}^{m_3}(\hat{r}) \quad (2.54) $$

gives finally, the S-state amplitude

$$ F_{12}^{00} = D \int d\tau \Lambda_{12}(r) X_{n}^{\lambda}(\hat{r}) Y_{n}^{\lambda}(\hat{r}) X_{1}^{\lambda}(r) X_{n}^{\lambda}(r) $$

$$ \times F(\ell_1 \ell_2 \ell_3) \quad (2.55) $$

where $F$ is the radial integral

$$ F(\ell_1 \ell_2 \ell_3) = \int_0^\infty dr \Lambda_{12}(r) X_{n}^{\lambda}(k_2, \gamma r) R_{j_{12}}(r) X_{1}^{\lambda}(k_1, r) \quad (2.56) $$

$$ \times F(\ell_1 \ell_2 \ell_3) \quad (2.57) $$
The evaluation of the D-state terms $F_{12}^{2A}$ of equation (2.28), because of the directional dependence of $\phi_{2A}(K)$, as given by equations (2.31) and (2.46), is somewhat more complex. It should be noted however, that the approximation introduced, namely

$$(2\pi)^{3/2} \phi_{2A}(K) = (4\pi)^{1/2} D_0 D_2 K^2 Y_2^A(\hat{r})$$

(2.58)

treats the angular dependence of $\phi_{2A}$ exactly. The operator identity of equation (2.50), can be generalized (Sa 68), to any homogeneous quadratic (HQ) function, $\phi_{HQ}(K)$, of the components of vector $K(=K_1/2 + K_2)$ appearing in the integral of equation (2.28), therefore

$$\phi_{HQ}(K) = \sum_{i=1}^{3} a_i \phi_{HQ}(K_i) \quad (i = 1, 2, 3)$$

(2.59)

with the $a_i$ given by equation (2.51). This is true in particular for the approximation to $\phi_{2A}$ of equation (2.58): therefore (dropping now the $n$ subscript upon $r_n$),

$$F_{12}^{2A} = (4\pi)^{1/2} D_0 D_2 \int d\tau(-)^{3/2} \sum_{i=1}^{3} a_i K^2 Y_{2i}^A(\hat{r}_i) Y_{2j}(\gamma, \eta)$$

$$\times R_{i}^{n}(r) Y_{k}^{n}(\hat{r}_i) Y_{l}^{1}(\gamma, \eta) \quad (2.60)$$

For an arbitrary operator $K$, such that $iK = \nabla$,

$$K Y_{1}^{n}(\hat{K}_{\lambda}) = (3/4\pi)^{1/2} K_{\mu} = -i(3/4\pi)^{1/2} \nabla_{\mu}$$

(2.61)

where $\nabla_{\mu}(K_{\mu})$ are the spherical components (Br 62) of the gradient ($K$)
operator. Repeated use of the gradient formula (Br 62)

\[ \nabla \phi(r) Y^m_l(\hat{r}) = \sum_{\ell', m'} (\ell m \mu | \ell' m')(2010 \ell' 0) \hat{g}^{\ell - 1}_{\ell', \ell} \phi(r) Y_{\ell'}^m(\hat{r}) , \]

where the operators \( \hat{g}_{\ell', \ell} \), acting upon \( \phi(r) \), are

\[ \hat{g}_{\ell + 1 \ell} = \frac{d}{dr} - \frac{\ell}{r} , \quad \hat{g}_{\ell - 1 \ell} = \frac{d}{dr} + \frac{\ell + 1}{r} , \] (2.63)

can be used to show that (Sa 73)

\[ K^2 Y^\Lambda_2 (K \nabla) \phi(r) Y^m_l(\hat{r}) = - \frac{5}{4\pi} \sum_{\ell, m'} (\ell m 2\Lambda | \ell' m') \times (20\ell' 0 \ell 0) Y_{\ell'}^m(\hat{r}) \hat{\Omega}_{\ell', \ell} \phi(r) , \] (2.64)

where the second derivative radial operators, \( \hat{\Omega}_{\ell', \ell} \), are

\[ \hat{\Omega}_{\ell', \ell} = \hat{g}_{\ell', \ell} \hat{g}_{\ell' \ell} \] (2.65)

and \( \ell' = \ell - 2(\ell'' = \ell - 1) \), \( \ell (\ell'' = \ell + 1) \) or \( \ell + 2(\ell'' = \ell + 1) \).

Equation (2.60) and (2.64) give therefore for \( F^{2\Lambda}_{12} \),

\[ F^{2\Lambda}_{12} = - \gamma^{-1} \frac{5}{4\pi} \frac{1}{2} \frac{1}{2} \frac{3}{2} \sum a_i F^{(i)}_{2\Lambda} , \] (2.66)

and, using the addition theorem of equation (2.55)

\[ F^{(i)}_{2\Lambda} = \sum_{L_1 L_2 M_1 M_2} \Omega^{(i)}_{\Lambda}(L_1 L_2 M_1 M_2) F_{12}^{(i)} , \]

where the \( \Omega^{(i)}_{\Lambda} \) are defined by
\[ \Phi_A^{(1)}(L_1', M_1') = (-)^L_A (t_1' t_2' - L_1' M_1') \langle 20L_1'0 | 2_2'0 \rangle \]

\[ \times \langle L_1'0 | \ell_2'0 \rangle \langle L_1' M_1' | \ell_2' \rangle \hat{L}_1' \hat{\ell}_2' \hat{L}_2'^{-1} , \quad (2.68) \]

\[ \Phi_A^{(2)}(L_2', M_2') = (-)^L_A (t_2' t_2' - L_2' M_2') \langle 20L_2'0 | 2_2'0 \rangle \]

\[ \times \langle L_2'0 | \ell_2'0 \rangle \langle L_2' M_2' | \ell_2' \rangle \hat{L}_2' \hat{\ell}_2' \hat{L}_2'^{-1} , \quad (2.69) \]

\[ \Phi_A^{(3)}(L_3', M_3') = (t_n^{-2} A | L_3' M_3') \langle 20L_3'0 | \ell_2'0 \rangle \]

\[ \times \langle L_3'0 | \ell_2'0 \rangle \langle L_3' M_3' | \ell_2' \rangle \hat{L}_3' \hat{\ell}_2' \hat{L}_2'^{-1} , \quad (2.70) \]

and the radial integrals, \( F_{12L_1}^{(i)} \), are

\[ F_{12L_1}^{(1)} = \int_0^\infty dr \ r \ \chi_{L_2'}^2(k_2, \gamma r) \hat{R}_{L_1L_1'} (r) \{ \hat{\delta}_L_{L_1} (\gamma \chi_{L_1'} (k_1, r) / r) \} , \quad (2.71) \]

\[ F_{12L_2}^{(2)} = \gamma \int_0^\infty dr \ r [ \hat{\delta}_L_{L_1} \{ \chi_{L_1'}^2(k_2, \gamma r) / \gamma r \} ] \hat{R}_{L_1L_1'} (r) \chi_{L_1'} (k_1, r) \quad (2.72) \]

and

\[ F_{12L_3}^{(3)} = \int_0^\infty dr \ \chi_{L_2'}^2(k_2, \gamma r) \{ \hat{\delta}_L_{L_1} \hat{R}_{j_n} (r) \} \chi_{L_1'} (k_1, r) \quad (2.73) \]

(in equation (2.72), \( \hat{\delta}_L_{L_1} \) operates upon the argument \( \gamma r \)).

The overlap integrals (2.71-73) remain numerically clumsy, as the \( \hat{\delta}_L_{L_1} \) contain second order derivatives. As outlined for the S-state component, these can be removed using the radial Shrödinger equations satisfied by \( \chi_{L_1} \), \( \chi_{L_1'} \) and \( R_{j_n} \). This gives (cf. equation (2.66))
\[ F_{2A} = - \gamma^{-1}(5/4\pi)^{1/2} P_0 P_2 \{ a_1 F_{2A}^{(1)} + a_2 F_{2A}^{(2)} + \bar{G}_{2A} \} , \quad (2.74) \]

where \( F_{2A}^{(i)} \) (i = 1, 2) are obtained from the \( F_{2A}^{(i)} \), upon making the replacement of \( \hat{\delta}_{1i}^{(i)} \) by \( (\hat{\delta}_{1i}^{(1)} - \hat{\delta}_{1i}^{(2)}) \) in the radial integrals, \( F_{12L_{1i}}^{(i)} \) of equations (2.71-72). \( \bar{G}_{2A} \) is given by

\[ \bar{G}_{2A} = \sum_{L_{13}^3 M_3^1} \Omega_{2A}^{(3)} (L_{33}^1, M_3^1) G_{12L_{13}^3}, \quad (2.75) \]

where \( \Omega_{2A}^{(3)} \) is as per equation (2.70), and

\[ G_{12L_{13}^3} = \int_0^\infty \frac{d \chi_{2l_2}^m(k_2, \gamma r) \chi_{1l_1}^{n}(k_1, r)a_3(\hat{\delta}_{1i}^{(1)} - \hat{\delta}_{1i}^{(2)} \hat{\delta}_{1i}^{(2)})}{(\Lambda_{12}(r) - 1)^{1/2} j_{\ell n}^{\mp}(r)} \quad (2.76) \]

\( \Lambda_{12} \) is the S-state finite range correction factor of equation (2.54).

2.5 The LEA at Sub-coulomb Energies

The accuracy of the LEA treatment of the finite range of \( V_{np} \) has recently been studied by Knutson (Kn 77), using the method due to Goldfarb and Parry (Go 68). The argument is that in a sub-coulomb reaction with \( Q \neq 0 \) the product of the distorted waves, \( \chi^{(+)}(k_1, r_1) \) and \( \chi^{(-)}(k_2, r_2) \) of equations (2.7-8), is slowly varying in the region of the classical distance of closest approach (see for e.g. Figures 4 and 6 of (Bu 71)), from which the dominant contributions to the stripping amplitude arise.

On the other hand, the short range of \( V_{np} \) favours contributions to the amplitude from small r and it is supposed that the product of the projectile distorted waves is accurately represented by
\[ \chi^{(-)}(k_2, \xi_2) \chi^{(+)}(k_1, \xi_1) \approx \chi^{(-)}(k_2, \xi) \chi^{(+)}(k_1, \xi) \quad (2.77) \]

where the true arguments have been replaced by their mean value (see equation (1.9))

\[ \xi = \frac{1}{2} (\xi_1 + \xi_2) = \xi_2 - \frac{1}{4} \xi_1 \quad (2.78) \]

and we have assumed that \( \gamma = 1 \). It is supposed therefore, that the product (2.77) varies little over the range of \( V_{np} \). Clearly this picture is consistent with the LEA procedure of replacing the differential operator \( \phi_L(\xi/2 + k_2) \) of equation (2.29), by one involving only low order derivatives of the distorted waves. These approximate expressions with low derivatives are expected to determine rather well the \( \chi^{(\pm)} \), within the range of \( V_{np} \), if the functions are slowly varying.

If now we assume that in the important large \( \rho \) stripping region the neutron bound state function, \( \psi_{n, m}^{m, n} \) takes on its asymptotic form, that is

\[ R_{j_n, m} (r_n) \approx N(\xi, j_n) h^{(1)}_{n, m} (i \mathcal{K}_n r_n) \quad (2.79) \]

where \( h^{(1)}_{n, m} \) is the spherical Hankel function (Sc 68a), \( N \) an overall normalization, \( \varepsilon_d + Q = \pi^2 \mathcal{K}_n^2(2\mu) = \varepsilon_n \), the neutron separation energy and \( \mu_n \) the n-A reduced mass (\( \mu_n \approx M \), the nucleon mass). Under these two physically reasonable assumptions Knutson has shown (Kn 77) that the transition amplitude (of equation (2.25)) takes the form

\[ T_{DWBA}^{(d,p)}(d,p) \propto \sum_{L=0,2} \tilde{V}_L(3/4i \mathcal{K}_n) F_L(\sigma_2 m_n; \sigma_1) \quad (2.80) \]

with

\[ \mathcal{K}_n = \pi^{-1} \{ 2M(\varepsilon_d + Q) \}^{\frac{1}{2}} \quad (2.81) \]
where $\tilde{V}_L(K)$ is the radial part of $\tilde{\phi}_{LA}(K)$, defined by equations (2.32-33).

So the stripping amplitude in the sub-coulomb limit depends on the $\phi_{LA}$ through the numbers $\tilde{V}_L(3/4iK_n)$ which multiply the $S$ and $D$-state parts of the amplitude. In this approximation therefore, the accuracy of the LEA depends only upon the accuracy with which the numbers $\tilde{V}^{\text{LEA}}_L(3/4iK_n)$ obtained via the LEA approximation agree with the exact values $\tilde{V}_L(3/4iK_n)$; $\tilde{V}^{\text{LEA}}_0, \tilde{V}^{\text{LEA}}_2$ are given by equations (2.43) and (2.47), respectively. For the momentum space wavefunction of Yamaguchi (Ya 54), then (Sa 68)

$$\tilde{V}_0(K) = N_y/\left(\beta^2 + K^2\right)$$

$$\tilde{V}_2(K) = N_yK^2t/(\gamma^2 + K^2)^2 ,$$

where $N_y$ is an overall normalization constant and, given that

$$\beta = 5.759 \ a, \quad \gamma = 6.771 \ a, \quad t = 1.784 ,$$

equation (2.48) gives $D_2 = 0.5248 \ \text{fm}^2 = t\beta^2/\gamma^4$. So for the ratio's

$$f_L(K) = \{\tilde{V}^{\text{LEA}}_L(K)/\tilde{V}_L(K)\}$$

corresponding to a reaction of zero $Q$ value ( $K_n = .3277 \ \text{fm}^{-1}$), then

$$f_0(3/4iK_n) = .999, \quad f_2(3/4iK_n) = .952 .$$

Thus, as $(d,p)$ tensor analysing powers are predominantly linear in the deuteron $D$-state amplitude (Jo 67), (a point seen particularly well in available sub-couloomb (Kn 75) calculations in which the $T_{2q}$ scale very nearly linearly with the assumed value of $D_2$), Knutson estimates that the conventional LEA is expected to underestimate by about 5% the calculated $T_{2q}$ when compared with the results of an exact finite range calculation.
CHAPTER 3

COULOMB POLARIZABILITY IN STRIPPING REACTIONS

Stripping reactions in which the energy of the incident deuteron is below the target coulomb barrier are of special interest. For transitions with Q value near zero, quantitative nuclear structure information may be extracted (Go 65) from the reaction through the detailed testing of experimental data against the predictions of a reaction theory. However, if the extracted number is to be considered a 'measurement', the testing must investigate small corrections neglected in the usual theoretical treatment.

In the previous chapter formalism has been presented for the calculation of the (d,p) stripping amplitude within the DWBA framework. A fundamental assumption of the DWBA is that the integrity of the deuteron is maintained, throughout its traverse of the interaction field of the target A. In the present chapter this constraint is relaxed to the extent that the target coulomb field, acting only upon the proton within the deuteron, may stretch or distort the n-p internal wavefunction from that of an isolated deuteron.

In conventional DWBA calculations, (d,p) tensor analyzing powers arise almost exclusively from the deuteron D-state (Jo 67), or non spherical component of the wavefunction (Kn 74). The dominant effect of coulomb polarizability, or stretching, is to introduce additional non-spherical terms into the n-p relative wavefunction; in particular relative P-states (Dr 66, Cl 65). Further, the effect of polarizability upon the spin
dependence of the n-p centre of mass motion, in light (De 69, De 70) of
the sensitivity of calculated tensor analysing powers to the presence of
rank 2 tensor interactions, is of great importance.

The quality of fits to experimental data in the sub-coulomb regime,
obtained in the conventional DWBA (Kn 77, Go 65), strongly suggests the
polarizability corrections may be handled in a perturbative manner. Clearly
however, an accurate quantitative estimate of these effects is essential
to the interpretation of a fit to tensor analysing power data as a D_2
measurement.

3.1 Three Body Aspects

As was pointed out in Chapter 1, in the limit in which coupling
between the target initial and excited states and the residual nucleus
bound and excited states is weak, the many body d-A system reduces to an
equivalent (e.g. Ju 67, Au 68, Ju 69) three body model. In this model the
neutron and proton, comprising the deuteron, interact mutually and with the
target core A, assumed inert. The neutron may be captured about A, into a
bound state F^{AB}(n) leaving the proton in the continuum to reach the detector
in direction $k^2$ say. Within this model, in which the passive co-ordinates of
target A have been integrated out, the stripping amplitude may be written
(see e.g. equation (1.37)) in the post form, and using the co-ordinate system
of Figure 1.2 as

$$T(d,p) = \langle k_2^{AB}(n)|V_p + V_{np}|\psi_{k_1}^{(+)}\rangle$$

$$= \langle \chi(-)(k_2,p)F^{AB}(n)|V_{np}|\psi_{k_1}^{(+)}(\xi,\xi)\rangle .$$

(3.1)
In line with previously stated assumptions, the proton distorted wave $\chi^{(-)}(k_2, p)$ is generated by an optical potential which reproduces the observed elastic scattering of the proton from the residual nucleus B (A + n).

Formally $\psi_3^{(+)}(r, R)$ is the three body $n + p + A$ wavefunction, the exact solution for which describes all reaction processes in which target A remains in its ground state. The philosophy of the present approach however, like many previous 3-body stripping models (see e.g. Gi 66, Jo 70, Fa 76), is to calculate in an approximate way the function $\psi_3^{(+)}$, which is, on physical grounds, deemed a good approximation to the exact 3-body state. This approximate solution need accurately represent the exact function, only within the limited important region of configuration space contributing to the stripping amplitude; indeed asymptotically ($R \to \infty$), the approximate solution need not in fact support stripping channels (Jo 70, Gi 66, Fa 76).

Calculation of the stripping amplitude thus proceeds via equation (3.1), replacing the usual DWBA product form of equation (1.39), i.e.

$$\psi_3^{(+)}(r, R) \approx \chi^{(+)}(k_1, R) \phi_{s_1}^{(p,n)}$$

by an improved but manageable approximation to the exact three body function.

### 3.2 Sub-Coulomb Considerations

For sub-coulomb incident deuteron energies the proposed 3-body stripping model is most appropriate. In a low Q value transition upon a heavy target nucleus both deuteron and proton have values for the coulomb parameter
\[ \eta_i = \frac{Z e^2 \mu_i / h^2 k_i}{(i = 1, 2)} \]

in significant excess of unity \(^\dagger\), enabling a rather clear semiclassical description of the process. In this case the incident deuteron follows a nearly classical hyperbolic coulomb trajectory. It is stripped of its neutron in the region (Go 68, Bu 71) of the classical distance of closest approach, \( q_c \), of the deuteron-target system. This point, some distance \(*\) from the nuclear surface region, is determined (Vi 73) by the reaction Q value and scattered angle of the emergent proton. However, for all reaction angles, stripping takes place predominantly in the tail region of the bound state function \( F^{AB}(n) \).

The amplitude for exciting the target nucleus in this limit becomes negligible. Further, the incident channel is coupled strongly only to relatively weakly bound states of the n-A system, reducing nuclear rearrangement in the residual nucleus. While the preceding ideas rely upon the DWBA picture of a structureless deuteron, the success in fitting sub-coulomb data, even with simple semiclassical models (Le 62, Vi 73, Kn 74),

\[^\dagger\] We have in mind the reaction \(^{208}\text{Pb}(d,p)^{209}\text{Pb}\) at incident deuteron energy \( E_d = 9 \text{ MeV} \). In this case, for \( Q \gtrsim 0 \), typically

\[ \eta_1 \approx 6.1 \text{ and } \eta_2 \approx 4.4. \]

\(^*\) For deuterons incident upon \(^{208}\text{Pb}\) at \( E_d = 9 \text{ MeV} \),

\[ q_c \geq \frac{Z e^2}{E_1} \approx 13.4 \text{ fm, the value for a head on collision. This compares with a nuclear radius } \approx 7 \text{ fm}. \]
reinforces the perturbative nature of DWBA corrections in this energy regime.

Emphasis is to be placed therefore upon obtaining an accurate approximate form for the 3-body function \( \psi_{k_1}^{(+)} \), in the \( R \gtrsim q_c \), large separation region of \( R \) space. At these separations the deuteron-target interaction

\[
V_{dA}(\xi, R) = V_n(R - \frac{1}{2} \xi) + V_p(R + \frac{1}{2} \xi) + \frac{Ze^2}{|R + \frac{1}{2} \xi|},
\]  

(3.2)

is dominated by the coulomb field of the target nucleus, assumed that of a point charge, and the relatively short ranged neutron and proton-core interactions \( V_n, V_p \), are expected to be of lesser importance. Austern, Vincent and Farrell (Fa 76, Au 78) have shown that by far the dominant effect of including break up contributions from \( V_n \) and \( V_p \) into the 3-body wavefunction, is to modify the n-p centre of mass motion from that predicted by the DWBA, within the nuclear interior. As already pointed out, such interior contributions are strongly supressed in the sub-coulomb limit and will not be considered further in the present discussion.

We take as the appropriate approximate 3-body hamiltonian,

\[
H = K_R + H_{np}(\xi) + \frac{Ze^2}{|R + \frac{1}{2} \xi|}
\]  

(3.3)

where the n-p hamiltonian \( H_{np} \) is the sum

\[
H_{np}(\xi) = K_\xi + V_{np}(\xi)
\]  

(3.4)

of the relative kinetic energy and interaction operators and \( K_R (= K_1) \) is the n-p centre of mass kinetic energy operator. We decompose \( H \) in the form

\[
H = H(R) + H_{np}(\xi) + \Delta V(\xi, R)
\]  

(3.5)
where
\[ H(R) = K_R + Z e^2 / R \; ; \]  
(a purely coulomb distorting \( ^\dagger \) hamiltonian dominating the motion of the n-p centre of mass. The remaining term in equation (3.5), \( \Delta V(r, R) \), absent in the usual DWBA treatment, couples the relative and centre of mass motion of the n-p pair and is known as the 'polarizing potential'; explicitly
\[ \Delta V(\xi, R) = Z e^2 \{ 1 / |R + \frac{1}{2} \xi| - 1 / R \} \; . \]  
The significant region of overlap in the 3-body stripping amplitude, \( T(d,p) \) of equation (3.1), is given by the radial values:
\[ r < \text{[Range of } V_{np}] \]  
\[ R > \text{[Distance of closest approach } q_c] \), for which \( 2R >> r \). In this region of space we may expand \( \Delta V \) in the multipole series (Sc.68a)
\[ \Delta V(\xi, R) = \sum_{k=1}^{\infty} \Delta V^k(\xi, R) \; , \]  
\( ^\dagger \) Clearly any central deuteron-core optical potential term may be added to \( H(R) \) without complication.
\[ \Delta V^k(r, R) = 4\pi(Ze^2/R) \sum_q k^{-2}(- r/2R)^k Y^q_k(\hat{R})^* Y^q_k(\hat{r}) \]

\[ = \sum_q \beta_k(R) Y^q_k(\hat{R})^* Y^q_k(\hat{r}) r^k \]  

(3.10)

where \(- k \leq q \leq k\). Conventionally, elastic scattering calculations (Ba 77, Cl 62) at this point consider only the leading \((k = 1)\), or dipole term of the expansion (3.9), assuming that the series is rapidly convergent, i.e.

\[ \Delta V(r, R) \approx \Delta V^1(r, R) \]

\[ = - (Ze^2/2R^3) \cdot \cdot \cdot \]  

(3.11)

this approximation has also been used by Gibson and Kerman (Gi 66) in stripping calculations.

We shall return to the accuracy of this approximation, to stripping, later in this and the following chapter. Prior to this however, we shall investigate rather generally the effects of \(\Delta V(r, R)\) without this imposed limitation. The effect of polarizability upon the n-p centre of mass motion is now studied in the framework of the Born-Oppenheimer (Ba 77) or Adiabatic (Te 66) approximation.

3.3 The Adiabatic Approximation

The polarizing potential \(\Delta V(r, R)\) of equation (3.7) can be seen by inspection of the multipole expansion, equations (3.9-10), together with the corresponding series for \(2R < r\) (Sc 68a) to satisfy the commutator

\[ [J \cdot \hat{r}, \Delta V(r, R)] = 0 \]  

(3.12)

Here \(J\) is the total angular momentum operator, relative orbital + neutron spin + proton spin, of the n-p pair, and in the notation of chapter 2,
\[ J = - i \hat{\mathbf{L}} \mathbf{r} + \hat{z}_2 + \hat{z}_3. \] (3.13)

In addition, it is of course assumed that

\[ [J^2, H_{\text{np}}(\mathbf{r})] = [J_{\alpha}, H_{\text{np}}(\mathbf{r})] = 0, \quad (\alpha = x, y, z). \]

Consider the Schrödinger equation for the relative n-p motion (at a fixed centre of mass position \( \mathbf{R} \)) in the presence of the interaction \( \Delta V(\mathbf{r}, \mathbf{R}) \); or

\[ \{ \varepsilon f, - H_{\text{np}}(\mathbf{r}) - \Delta V(\mathbf{r}, \mathbf{R}) \} \left| \phi_{f\sigma F}^\prime (\mathbf{R}), \hat{\mathbf{R}} > \right. = 0. \] (3.14)

In equation (3.14) we have assumed, as a result of equation (3.12) that we may find solutions, \( |\phi_{f\sigma F}^\prime (\mathbf{R}), \hat{\mathbf{R}} > \), which are simultaneous eigenstates of both energy and \( J \cdot \hat{\mathbf{R}} \), although not of \( J^2 \). So, the inclusion of the qualification \( \hat{\mathbf{R}} \) within the ket implies an eigenstate of \( J_z \) referred to a \( z \) axis directed along \( \hat{\mathbf{R}} \), and explicitly

\[ J \cdot \hat{\mathbf{R}} |\phi_{f\sigma F}^\prime (\mathbf{R}), \hat{\mathbf{R}} > = \sigma F |\phi_{f\sigma F}^\prime (\mathbf{R}), \hat{\mathbf{R}} >. \] (3.15)

Asymptotically, as \( R \to \infty \), then clearly

\[ \Delta V(\mathbf{r}, \mathbf{R}) \to 0, \text{ for all finite } r, \]

and therefore from equation (3.14)

\[ \left. \varepsilon_{fii}^\prime (\mathbf{R}) \right|_{\mathbf{R} \to \infty} = \varepsilon_f, \] (3.16)

\[ \left. |\phi_{f\sigma F}^\prime (\mathbf{R}), \hat{\mathbf{R}} > \right|_{\mathbf{R} \to \infty} = |\phi_{f\sigma F}^\prime \left( \mathbf{R}, \hat{\mathbf{R}} > \right) \] (3.17)
where
\[ J^2 |\phi_{f s_f, \hat{R}}\rangle = s_f (s_f + 1) |\phi_{f s_f, \hat{R}}\rangle \]

\[ H_{np}(r) |\phi_{f s_f, \hat{R}}\rangle = \varepsilon_f |\phi_{f s_f, \hat{R}}\rangle ; \]

the \( \hat{R} \) in the ket also indicating an eigenstate of \( J \cdot \hat{R} \).

For the present we shall represent states of the n-p system with a discrete index \( f \), where in line with chapter 2, \( f = 1 \) labels the deuteron ground state; thus

\[ |\phi_{1s_1, \hat{R}}\rangle = |\phi_{s_1, \hat{R}}\rangle \]

when referred to the intrinsic \( \hat{R} \) co-ordinates. The equation we actually must solve in the present three body model is not equation (3.14), but

\[ \{E - H_{np}(r) - \Delta V(r, R) - H(R)\} |\psi^{(+)}_{s_1, 1s_1}\rangle = 0 , \]

where the asymptotic boundary conditions * satisfied by the function \( |\psi^{(+)}\rangle \) are, as \( \Delta V \rightarrow 0 \) for \( |R| \rightarrow \infty \);

\[ ^\dagger \text{States } |\phi_{f s_f, \hat{R}}\rangle \text{ should strictly also carry a label } s_f, \text{ where from equation (3.17), } s_f \text{ is the eigenvalue of total angular momentum } J \text{ of the state as } R \rightarrow \infty. \text{ For simplicity we do not show } s_f \text{ explicitly.} \]

* For convenience we continue to use free particle boundary conditions in the following discussion.
where energy conservation requires that the final state energy

\[ E_f = E - \varepsilon_f = \hbar^2 k_f^2 / 2 \mu_f , \]

and \( \mu_f \) is the reduced mass in the final state. In equation (3.21), the kets \( |\phi^f_{s} > \), without label \( \hat{R} \), are eigenstates of \( H_{np}(x) \), \( J_z^2 \) and \( J_z \) referred to a fixed reaction co-ordinate system. Throughout this thesis we have adopted the co-ordinate set recommended in the Madison Convention (Mc 71) for the description of reaction \( T_{2q} \). We shall designate this set \( \{Oxyz\} \), and so the two sets of basis states, \( |\phi^f_{s} > \) and \( |\phi^f_{s} , \hat{R} > \), can be related through the introduction of a rotation operator, \( \mathcal{R}(\hat{R}) \), which rotates co-ordinate set \( \{\hat{R}\} \), with \( z \) axis along \( \hat{R} \), into \( \{Oxyz\} \). Using the rotation matrices \( \mathcal{D}^{sf} \), as defined by Brink and Satchler (Br 62), then

\[
\mathcal{R}(\hat{R}) |\phi^f_{s} , \hat{R} > = \sum_{\sigma_f} \mathcal{D}^{sf}_{\sigma_f' \sigma_f | \phi^f_{s} } \sigma_f' , \hat{R} > = |\phi^f_{s} > . \tag{3.22}
\]

Using the fact that for each \( \hat{R} \), the solutions of equation (3.14) constitute a complete set in \( r \)-space, we may expand the three body function

\[
<\hat{R} | \psi^{(+)}_{k_1, \sigma_1} > = \sum_{fsf} (|\phi^f_{s} > , \hat{R} > \mathcal{D}^{sf}_{\sigma_f' \sigma_f | \phi^f_{s} } \sigma_f' , \hat{R} > )\chi^{sf}_{\sigma_f' \sigma_f | \chi^{sf}_{\sigma_f' \sigma_f | k_1 , \hat{R} } > . \tag{3.23}
\]
Some care must be taken in interpreting this equation; at first sight the bracketed term (summed over $\sigma_i$) looks like a simple rotation of state $|\phi'\rangle$, however, inspection of equations (3.17-18), show that $|\phi'\rangle$ is an eigenstate of $J^2$ only at infinity and the expansion (3.23) is purely formal. Using equations (3.17) and (3.22) however, then asymptotically, comparison with equation (3.21) gives

$$
\chi_{\sigma_f \sigma_1}^{f^*}(k_1, R) \bigg|_{R \to \infty} = \delta_{f^* f} \delta_{\sigma_1 \sigma_i} \delta_{s_f s_i} e^{\frac{i k_f R}{R}} + \frac{e^{\frac{i k_f R}{R}}}{R} \int_{\sigma_f \sigma_1}^{s_f s_i} (\hat{R}) (3.24)
$$

where $\int_{\sigma_f \sigma_1}^{s_f s_i} (\hat{R})$, of equation (3.21), is the amplitude for exciting state $|\phi_f^{s_f}\rangle$ of the n-p system when a deuteron is incident in state $\sigma_1$.

The differential equations satisfied by the $\chi_{f^* f}^{\sigma_f \sigma_1}$ are obtained upon substituting equation (3.23) into the Schrödinger equation (3.20), or

$$
\sum_{f^* f} \{ E - H(R) - c_{f^* f}^{\sigma_f \sigma_1}(R) \} |\phi_f^{s_f} (R), \hat{R}\rangle \cdot \oint_{\sigma_f \sigma_1}^{s_f s_i} (\hat{R}) (3.25)
$$

$$
\times \chi_{\sigma_f \sigma_1}^{f^*}(k_1, R) = 0
$$

solution of which would yield the exact three-body function $\psi^{(+)}$, corresponding to the model hamiltonian of equation (3.3). Clearly operator $K_R$, within $H(R)$, operating to the right upon the kets $|\phi'\rangle$, introduces coupling between the $\chi_{f^* f}^{\sigma_f \sigma_1}$ of different $f$. The result is that an exact solution of equation (3.25) would require solving an infinite set of coupled equations. Although such continuum coupling has been handled in an approximate manner using the technique of 'momentum bins' (Ra 75, Fa 76, Au 78), the enormous numerical
commitment required in such a calculation, together with the long range nature of the coupling terms in equation (3.25), favour an alternative approach.

Rather, at this point we make the Adiabatic approximation (Cl 62, Gi 66, Da 63), the mathematical consequence of which is to neglect all terms in equation (3.25) which involve derivatives of the kets $|\phi'_{\sigma_f}(R),\hat{R}\rangle$, with respect to $R$. Physically, in assuming such terms are negligible when compared with other terms in the equation, we have implied that the characteristic time associated with the deuteron internal motion is small compared with the time taken for its centre of mass to traverse a distance in which the break up potential $AV(z,R)$ has changed appreciably (Cl 62, Da 63). Inspection of equations (3.9-10) for $AV(z,R)$ shows that the interaction varies only slowly with $R$ (as $1/R^2$) about the important stripping region near the turning point. In addition, the local velocity of the deuteron centre of mass at this point is small. The deuteron internal motion, in which each nucleon has an average kinetic energy $<K> \approx 26$ MeV (Ke 70), is therefore fast compared to its centre of mass translation and the n-p relative motion can be considered as being in a fixed field (cf. equation (3.14)). Clearly such a treatment would be more doubtful in treating nuclear break up effects, as performed by Testoni and Gomes (Te 66), in which the break up potentials are more rapidly varying; particularly in the important nuclear surface region.

Upon ignoring the derivatives mentioned, and using the orthogonality of the kets $|\phi'_{\sigma_f}(R),\hat{R}\rangle$,

$$\{E - \epsilon_{\sigma_f}(R) - H(R)\} \sum_{\sigma_f} \int d^3 R \, s_{\sigma_f}^{s_f}(\hat{R}) \chi_{\sigma_f}^{s_f}(k_1,R) = 0 \quad , \quad (3.26)$$
the $\chi^s_f$ having become uncoupled. The Adiabatic approximation has therefore replaced the physical real deuteron break up problem by one involving a virtual excitation of the n-p system, the incident channel ($f = 1$) being unable to couple to broken-up states ($f > 1$) of the n-p pair. So, just as an exact solution of our approximate three body equation (3.20) would yield no stripping channels asymptotically, there being no interaction present to bind the neutron to the core, in addition our approximate solution, obtained via the Adiabatic approximation, will contain no asymptotic n-p break up channels. Nevertheless the approximate solution, from equation (3.23):

$$\langle R | \Psi (R) \rangle \approx \sum_{\sigma_1' \sigma_1} \{ | \phi^\prime_{1 \sigma_1} (R) \rangle \otimes \sum_{f=1}^{s_1} D^{s_1}_{\sigma_1 \sigma_1'} (R) \} \chi^s_{\sigma_1' \sigma_1'} (E_{1 \sigma_1}, R) \tag{3.27}$$

is expected, within the region of configuration space contributing to the stripping amplitude, to reproduce rather accurately the details of the break up contributions to $\psi^{(+)}$.

In the above (equation (3.27)) we have set all $\chi^{s_f}_{f}$ with $f$ other than unity to zero, all coupling to the incident channel having been removed. We now see that the energy eigenvalue $\epsilon_{1 \sigma_1} (R)$, appears in the Schrödinger equation (3.26), for the n-p centre of mass function $\chi^s_{1 \sigma_1}$; thus modifying its motion from that of a purely coulomb distorted deuteron.

3.4 The Deuteron-Target Interaction

Using the unitarity property of the rotation matrices $D$, namely (Br 62)

$$\sum_{\sigma} D^{s}_{\sigma \sigma_1'} (R) D^{s}_{\sigma_1' \sigma_1} (R) = \delta_{\sigma_1, \sigma_1'} \tag{3.28}$$

and dropping the now redundant reference to channel 1 (with the understanding
that $s = s_1 = 1$), we may rewrite the Schrödinger equation of the n-p centre mass (equation (3.26))

$$(E_1 - H(R)) \chi^{(+)}_{\sigma_1 \sigma_1}(k, R)$$

$$= \sum_{\sigma_i} \tilde{V}_{\sigma_i \sigma}^\dagger(R) \chi^{(+)}_{\sigma_1 \sigma}(k, R) .$$

(3.29)

We have used the relation $E_1 = E - \varepsilon_1$ where $\varepsilon_1 = -\varepsilon_d$, and $\varepsilon_d$ is the deuteron binding energy, therefore the potential matrix $\tilde{V}$, is defined by

$$\tilde{V}_{\sigma \sigma'}(R) = \sum_{\sigma} D_{\sigma \sigma}^S(R) (\varepsilon_{\sigma}(R) + \varepsilon_d) D_{\sigma \sigma'}^S(R) ,$$

(3.30)

and the outgoing wave boundary conditions upon the centre of mass are now shown explicitly.

Thus, polarizability has introduced a spin dependent interaction $\tilde{V}$ into the d-target system. To further simplify equation (3.30) we use the symmetry of the $D$ matrices

$$D_{\sigma \sigma}^S(R) = (-)^{-\sigma - \sigma'} D_{-\sigma - \sigma'}^S(R)$$

(3.31)

together with the additional identity (Br 62)

$$D_{-\sigma - \sigma'}^S(R) D_{\sigma \sigma'}^S(R) = \sum_{KQ} (s, -\sigma, s, \sigma | KQ) D_{0Q}^K(R)$$

(3.32)

$$\sum_{KQ} (-)^{-\sigma - \sigma} (s, -\sigma, s, \sigma | KQ) \{ \varepsilon_{\sigma}(R) + \varepsilon_d \}$$

then

$$\tilde{V}_{\sigma \sigma'}(R) = \sum_{KQ} (-)^{-\sigma - \sigma} (s, -\sigma, s, \sigma | KQ) \varepsilon_{\sigma}(R) + \varepsilon_d$$

$$D_{0Q}^K(R)$$
and, using the symmetry of the Clebsh-Gordan coefficients and equation (3.31):

\[
\bar{V}_{\sigma''\sigma'},(R) = \sum_{KQ\sigma} (-)^Q K^2 \sigma^2 - 2 (\sigma\sigma' K\sigma) \{ \epsilon_\sigma(R) + \epsilon_d \}
\]

\[
\times (\sigma\sigma' K - Q|\sigma''\sigma') \mathcal{D}_Q^K (\hat{R}^{-1}) .
\]  

(3.33)

The rotation \( \hat{R}^{-1} \), the inverse of \( \hat{R} \), takes the fixed reaction \( z \) axis of \( \{Oxyz\} \) into the direction \( \hat{R} \), and the Euler angles corresponding to this rotation are \((\phi_R, \theta_R, \gamma)\), where \( \gamma \) is arbitrary. Angles \((\theta_R, \phi_R)\) are the usual polar angles of vector \( \hat{R} \) in the fixed co-ordinates \( \{Oxyz\} \), and using the identity, for arbitrary Euler angles \((\alpha\beta\gamma)\)

\[
\mathcal{D}_Q^K (\hat{R}^{-1}) = (4\pi)^{1/2} \hat{R}^{-1} Y_K^Q (\hat{R}) \ .
\]

(3.34)

then

\[
\mathcal{D}_Q^K (\hat{R}^{-1}) = (4\pi)^{1/2} \hat{R}^{-1} Y_K^Q (\hat{R}) \ .
\]

(3.35)

Making use of the properties

\[
\hat{\tau}_{KQ}^\dagger (s) = (-)^Q \hat{\tau}_{K-Q}^\dagger (s)
\]

\[
<s\sigma'|\hat{\tau}_{KQ} (s)|s\sigma''> = \hat{K}(s\sigma' KQ|s\sigma'')
\]

of the irreducible tensor operators \( \hat{\tau}_{KQ}(s) \) introduced in chapter 1 (equations (1.17-1.19)), together with equation (3.35) enables us to rewrite for \( \bar{V} \):

\[
\bar{V}_{\sigma''\sigma'},(R) = \sum_{KQ} (4\pi)^{1/2} Y_K^Q (\hat{R}) <s\sigma'|\hat{\tau}_{KQ}^\dagger (s)|s\sigma''> \bar{V}_K (R)
\]

(3.36)
where

\[ \bar{V}_K(R) = \sum_\sigma \hat{s}^2 \langle \sigma \sigma | K | \sigma \rangle \{ \epsilon_\sigma(R) + \epsilon_d \}. \tag{3.37} \]

So, making use of the identity (Lo 77):

\[ \hat{T}_R = (\hat{s} \cdot \hat{R})^2 - 2/3 = (2/3) (2\pi/5) \sum_{Q} Y_{2Q}^0(\hat{R}) \tau_{2Q}^+(s), \tag{3.38} \]

where \( \hat{T}_R \) is Satchler's (Sa 60) rank-2 tensor operator, the \( K = 2 \) term of equation (3.36) may be written

\[ \bar{V}^{(2)}(R) = 3(5/2) \bar{V}_2(R) \hat{T}_R. \tag{3.39} \]

In addition there is a central potential term

\[ \bar{V}^{(0)}(R) = \hat{s}^2 \sum_\sigma \{ \epsilon_\sigma(R) + \epsilon_d \}, \tag{3.40} \]

while time reversal invariance of equation (3.14) dictates that \( \epsilon_{+1} = \epsilon_{-1} \) and hence, \( K = 1 \) contributions to \( \bar{V} \) vanish identically.

Dependent therefore upon the exact functional form of \( \epsilon_\sigma(R) \) upon \( \sigma \), which in turn is entirely dependent upon what is assumed for Hamiltonian \( H_{np}(r) \) in equation (3.14), polarizability allows for the possibility of long ranged tensor \( \hat{T}_R \) terms in the deuteron-target interaction. In view of the known sensitivity (De 70, De 69) of calculated \( T_{2q} \) to the presence of such interactions, and in particular the importance of long ranged interactions in the sub-coulomb regime, a knowledge of under what conditions these terms arise and which terms dominate, is of considerable importance.

Before looking at this problem from a perturbation theory standpoint, an exact result can be stated. If \( H_{np}(r) \) is spin independent then clearly \( \epsilon_\sigma(R) \) is independent of \( \sigma \) and a simple product solution
\[<\xi,p,n|\phi^\pi_\sigma(R),\hat{R}> = \phi(\xi,R)\chi^\sigma_s(p,n,\hat{R}) \]  

(3.41)

exists, where if \( s = s^2 + s^3 \), then

\[ s \cdot \hat{R} \chi^\sigma_s(p,n,\hat{R}) = \sigma \chi^\sigma_s(p,n,\hat{R}) \]  

(3.42)

In this case no tensor terms are present and

\[ \tilde{V}^{(0)}(R) = \varepsilon(R) + \varepsilon_d \]  

From a perturbative approach, the change in the deuteron binding energy, \( \varepsilon_\sigma(R) + \varepsilon_d \), as a result of the polarizing potential, \( \Delta V(R,R) \), may be evaluated in non-degenerate perturbation theory using the set of basis states \( |\phi^\pi_\sigma_f,\hat{R}> \) which are unconnected by \( \Delta V \). To first order in the complete interaction \( \Delta V \), we have

\[ \{\varepsilon^{(1)}_\sigma(R) + \varepsilon_d\} = \langle \phi^\pi_\sigma,\hat{R}|\Delta V(R,R)|\phi^\pi_\sigma,\hat{R}\rangle \]  

(3.43)

which for \( \Delta V \) given by equations (3.9-10), gives the exact result (Ha 72)

\[ \{\varepsilon^{(1)}_\sigma(R) + \varepsilon_d\} = (3/2)Q_d Ze^2/R^3(\sigma^2 - 2/3) \]  

(3.44)

where \( Q_d \) is the deuteron quadrupole moment. Substitution into equations (3.39) and (3.37) gives

\[ \tilde{V}^{(2)}_{f_\sigma,0}(R) = (3/2)Q_d Ze^2/R^3 T^R_{\sigma} \]  

(3.45)

the first order \( T_R \) contribution, which is just the usual coulomb \( T_R \) potential. This was first pointed out by Raynal (Ra 64) and obtained using the Watanabe (Wa 58) folding procedure in which the deuteron D-state is included. This interaction is obtained even in the absence of deuteron break up (Ke 73).
Closed expressions of the form of equation (3.45) cannot be obtained for higher order energy corrections. Consider however the dominant dipole 
(k = 1) term in the break up potential, $\Delta V^1(r, R)$ of equation (3.10). To second order in $\Delta V^1$, Ramsey et al. (Ra 53) obtain the result that

$$\{\epsilon^{(2)}_\sigma(R) + \epsilon_d\} = -1/2(Ze/R^2)^2\alpha_d^o$$  \hspace{1cm} (3.46)

where $\alpha_d^o$ is the deuteron static polarizability (Ba 77) and is (Ra 53) explicitly:

$$\alpha_d^o = \alpha_{SS} + 3\alpha_{SD}\{\sigma^2 - 2/3\} + 9\alpha_{DD}\{\sigma^2 - 2/3\}^2.$$  \hspace{1cm} (3.47)

The terms $\alpha_{SD}$, $\alpha_{DD}$ are linear and quadratic in the deuteron D-state, respectively while $\alpha_{DD}$ << $\alpha_{SD}$ and is neglected. Ramsey et al. calculate that

$$\alpha_{SD} = 0.027 \text{ fm}^3,$$  \hspace{1cm} (3.48)

$$\alpha_{SS} = 0.56 \text{ fm}^3$$

giving rise to the tensor and central second order contributions to $\tilde{V}$: namely

$$\tilde{V}^{(2)}_{s\cdot 0}(R) = -3/2\alpha_{SD}(Ze/R^2)^2 \hat{T}_R,$$  \hspace{1cm} (3.49)

$$\tilde{V}^{(0)}_{s\cdot 0}(R) = -(1/2)\alpha_{SS}(Ze/R^2)^2.$$  \hspace{1cm} (3.50)

Clearly equations (3.45) and (3.49) comprise the largest $\hat{T}_R$ contributions to the potential $\tilde{V}(R)$, any higher order contributions going at least as $1/R^6$, and arising from second order corrections due to the quadrupole (k = 2) term of $\Delta V$, or third order dipole corrections. It should be noted that unless the deuteron D-state, or equivalently a tensor term in $V_{np}$, is present, there are no first or second order small contribut-
ions to the tensor potential \( V^{(2)} \). We return now to the effects of polarizability upon the relative n-p motion, also from a perturbative approach.

3.5 The Modified n-p Internal Wavefunction

Within the developed Adiabatic approximation the approximate solution of the three body problem is

\[
\langle \tilde{R} | \psi_{k_1, \sigma_2, \sigma_3} \rangle = \sum \langle \phi^{(+)}_{\sigma_1' \sigma_1''} (R), \tilde{R} | \phi^{(+)}_{\sigma_1'' \sigma_1, 1} (\tilde{R}) \rangle \chi_{\sigma_1' \sigma_1}^{(+)} (k_1, R), \tilde{R} \rangle , \tag{3.51}
\]

where \( \chi^{(+)} \) satisfies equation (3.29). We shall define the modified deuteron wavefunction at point \( R \) according to

\[
| \phi^{(+)}_{\sigma_1'} (R) \rangle = \sum \mathcal{D} \langle \sigma_1'' \sigma_1' | \phi_{\sigma_1'' \sigma_1, 1} (R), \tilde{R} \rangle | \phi^{(+)}_{\sigma_1'} (R), \tilde{R} \rangle . \tag{3.52}
\]

From equations (3.17) and (3.22), then

\[
| \phi^{(+)}_{\sigma_1'} (R) \rangle \bigg|_{R \to \infty} = | \phi_{s_1} \rangle
\]

however \( | \phi^{(+)}_{\sigma_1'} (R) \rangle \) as given by equation (3.52) is not an eigenstate of either \( J^z, \tilde{R} \) or \( J_z, \tilde{R} \), and the label \( \sigma_1' \) denotes only the boundary condition of the state into which \( | \phi^{(+)}_{\sigma_1'} (R) \rangle \) evolves as \( R \to \infty \).

So equation (3.51) may be rewritten to resemble more closely the usual DWBA expression for \( \psi^{(+)} \) (cf. equation (1.39)), i.e.

\[
\langle \tilde{R} | \psi_{k_1, \sigma_2, \sigma_3} \rangle = \sum \langle \phi^{(+)}_{\sigma_1'} (R), \tilde{R} | \chi_{\sigma_1' \sigma_1}^{(+)} (k_1, R) \rangle . \tag{3.53}
\]
Exact solution of equation (3.14) for the functions |φ^1 (R), R⟩, is not possible, however treating the break up potential \( ΔV \) to first order, we find

\[
|φ^1 (R), R⟩ = |φ^1 s_1, R⟩ + \sum_{s_f \neq 1} |φ^1 s_f, R⟩(−ε_d − ε_f)
\]

\[
× <φ^1 s_f, R | ΔV (r, R) | φ^1 s_1, R⟩,
\]

(3.54)

where once again we have used the fact that \( ΔV \) does not couple \( J \cdot \hat{R} \) basis states. So, using the definition of \( |φ^1 s_1 (R)⟩ \) of equation (3.52)

\[
|φ^1 (R), R⟩ = |φ^1 s_1⟩ + \sum_{s_f \neq 1} |φ^1 s_f, R⟩(−ε_d − ε_f)
\]

\[
× <φ^1 s_f, R | ΔV (r, R) | φ^1 s_1, R⟩
\]

(3.55)

where a redundant \( s_f \) sum has been introduced; or

\[
|φ^1 (R)⟩ = |φ^1 s_1⟩ + \sum_{s_f \neq 1} |φ^1 s_f, R⟩(−ε_d − ε_f)
\]

\[
× <φ^1 s_f, R | ΔV (r, R) | φ^1 s_1⟩.
\]

(3.56)

Therefore using the closure relationship

\[
\sum_σ |φ^1 σ, \hat{R}⟩⟨φ^1 σ, \hat{R}| = \sum_σ |φ^1 σ⟩⟨φ^1 σ|
\]

we obtain finally:
where $|\delta \phi_{1}^{k}(R)>$, the perturbation to the deuteron internal wavefunction due to the $k$th multipole $\Delta V^{k}(r,R)$ (equation (3.10)) of the polarizing potential, is

$$|\delta \phi_{1}^{k}(R)> = \sum_{k} |\phi_{1}^{k}(R)>$$

Note that this is exactly the result which would have been obtained upon ignoring the threefold degeneracy of the deuteron ground state, the effects of which have been taken into account in potential $\bar{\nu}$. So, substituting for $\Delta V^{k}$ from equation (3.10) into (3.58) and using the Wigner-Eckart theorem (Me 61), we define $\Delta V^{k}_{fsf}(R)$.

$$\langle \phi_{fsf} |\{r^{k}Y_{k}(\hat{r})\}|\phi_{1}> = (s_{1}s_{f}|s_{f}s_{f})\Delta V^{k}_{fsf}(R)$$

and hence

$$|\delta \phi_{1}^{k}(R)> = \sum_{s_{f}s_{f}|s_{f}s_{f}} (s_{1}s_{1}|s_{f}s_{f})Y_{k}(\hat{r})^{*} |\phi_{fsf}>\Delta V^{k}_{fsf}(R)/(-\epsilon_{d} - \epsilon_{f}).$$

3.6 Relative Importance of the Multipoles

In section 3.3 we commented that it is conventional in both stripping and elastic scattering calculations, which include polarizability, to retain only the leading dipole term $\Delta V^{1}$ of the multipole series, equation (3.9). That is, the break up wavefunctions $|\delta \phi^{k}>$, of equation (3.58), for $k > 1$ are assumed small in some sense, when compared with $|\delta \phi^{1}>$. In the stripping amplitude, $T(d,p)$, the $|\delta \phi^{k}(R)>$ are operated upon from the left by $V_{np}$ and,
as stated previously, results, at sub-coulomb energies, in the important contributions to the amplitude coming from the r and R values indicated in equation (3.8). Therefore, ignoring for the present any state dependence of the interaction \( V_{np} \), the relative importance, to stripping, of the multipole functions \( |\delta \phi^k(R)> \), for \( k > 1 \), is of order

\[
R_k \approx \left| \langle \tau > 0|\delta \phi^k(q_c R)> \right|/\left| \langle \tau > 0|\delta \phi^1(q_c R)> \right|. \tag{3.61}
\]

We shall assume that in equation (3.58) we may introduce, for the \( n-p \) continuum, plane wave states, i.e.

\[
\sum_{\sigma_f} \int \frac{d^k f}{(2\pi)^{-3}} \langle \phi_f |s_\sigma > \langle f |s_\sigma |k \rangle |s_\sigma |k \rangle = \langle k |s_\sigma |k \rangle \tag{3.62}
\]

where

\[
\langle \xi,p,n |k \sigma s_1 \rangle = \exp(ik \cdot \xi) \chi_{s_1}^1(p,n) . \tag{3.63}
\]

So, using the identity (Ro 67)

\[
(2\pi)^{-3} \int \frac{d^k f}{-\xi_d - \xi_k} = -\frac{2\mu l}{\pi^2} \frac{1}{4\pi} \exp(-\alpha |\xi - \xi'|) \tag{3.64}
\]

for the free particle Greens Function, where \( \xi_d = \pi^2 \alpha^2/2\mu \), and neglecting the deuteron D-state component for the present magnitude estimate, then

\[
\langle \xi,p,n |\delta \phi^k_{s_1} (R) > = \left\{ \int dr' \frac{\exp(-\alpha |r - r'|)}{|r - r'|} \Delta V^k(\xi,R) \nu_0(r') \right\} \times \chi_{s_1}^1(p,n) . \tag{3.65}
\]
We shall insert, in the above, the expansion (Br 62)

\[
\exp\left(-\frac{\alpha |r-r'|}{\alpha |r-r'|}\right) = -4\pi \sum_{\ell,\lambda} j_{\ell}(i\alpha r)h_{\ell}^{(1)}(i\alpha r')\times y_{\ell}^{\lambda}(r')y_{\ell}^{\lambda}(r)^* \tag{3.66}
\]

where \(h_{\ell}^{(1)}, j_{\ell}\) are the spherical Hankel and Bessel functions (Sc 68a), respectively and \(r_>, r_<\) are the lesser and greater of \(r\) and \(r'\). Then,

\[
\langle \Sigma, p, n | \delta \phi_{\sigma_1}^k (R) \rangle \propto (2R)^{-k-1} f_k(r) \left\{ 4\pi^{-2} \sum q y_{q}^{q}(r) y_{q}^{q}(r') \right\} \times \chi_{\sigma_1}^\sigma(p,n) \tag{3.67}
\]

where the radial function \(f_k\) is

\[
f_k(r) = j_k(i\alpha r) \int_r^{\infty} h_k^{(1)}(i\alpha r') r'^{k+2} u_0(r') \, dr' + h_k^{(1)}(i\alpha r) \int_0^r j_k(i\alpha r') r'^{k+2} u_0(r') \, dr'. \tag{3.68}
\]

The angular dependent bracketed term in equation (3.67) is of order unity, for all \(k\), and the importance of each multipole, \(k\), is essentially determined by the radial part, \(f_k(r)\), of \(|\delta \phi^k\rangle\). The use of the plane wave continuum in the calculation of \(|\delta \phi^k\rangle\) is expected to be a good first approximation.

The energy denominator in the Green's Function of equation (3.58), highlights the relative importance of low energy \(n-p\) break up configurations whereas, for a spin-independent \(V_{np}\), equation (3.67) shows that the \(k = 1\) and \(k = 2\) wavefunctions correspond to \(n-p\) relative \(P\) and \(D\) waves, respectively.

Scattering in \(P\) and \(D\) states, however, becomes important only at relatively high energies (e.g. Pr 75).
Using a Hulthén form for $u_0(r)$ (Hu 57), numerical evaluation of equation (3.68) shows that, within the range of $V_{np}(r \leq 7 \text{ fm say})$, $f_2(r) < 5f_1(r)$. So, for the ratio $R_2$ (equation (3.61)), the relative importance of quadrupole to dipole distortion, we obtain $R_2 \leq 5/2q_c$. For sub-coulomb deuterons incident upon $^{208}\text{Pb}$, at 9 and 7 MeV for example, $q_c$ takes the values $q_c \geq 13.5 \text{ fm}$ and $q_c \geq 16.9 \text{ fm}$, respectively, for which $R_2 \leq \frac{1}{5}$ and $R_2 \leq \frac{1}{6}$.

To conclude therefore, on the basis of the present rather simple plane wave estimate, we find that quadrupole electric break up of the deuteron is less important than the corresponding dipole term. This estimate fails however to take account of the state dependence of the n-p interaction, which multiplies the $|\Theta_{cf}^k>$ when inserted in the stripping amplitude. We shall return briefly to this problem in section 4.3 the following chapter.

In this chapter formalism has been developed for the treatment of coulomb break up of the deuteron in an Adiabatic approximation. We have seen that, from the point of view of tensor analysing power calculations, the problem divides itself rather naturally into two important parts, namely:

i) Polarizability introduces long ranged, coulombic tailed $T_R$ tensor interactions into the deuteron target interaction, and

ii) Polarizability distorts the deuteron wavefunction, from that of an isolated deuteron, in the important stripping region of configuration space.
Treatment of each of these aspects requires considerable effort and in order to investigate both their individual importance and structure are studied separately. In the following chapter the problem of the modified relative n-p wavefunction is studied quantitatively while a discussion of the tensor interaction problem is postponed until chapters 5 and 6.
POLARIZABILITY CORRECTIONS

Since the calculation of Oppenheimer and Phillips (Op 35) in 1935, several authors have tackled the problem (Cl 65, Go 65a, Da 63, Gi 66) of deuteron polarizability in stripping reactions at differing levels of sophistication. Of these, the present work follows most closely the approach of Gibson and Kerman (Gi 66).

It is recognized that while polarizability corrections are important, and hence in theory more amenable to observation when treating the (d,p) amplitude in the prior formalism (Cl 65, Go 65a), the difficulty then in treating the initial state dictates that an accurate quantitative calculation should proceed via the post formulation. The post form transition amplitude, comprising matrix elements of $V_{np}$, is dominated by the relative n-p wavefunction at small separations. Polarizability however, primarily introduces long ranged components into the relative n-p wavefunction corresponding to broken up states of the two nucleon system. It is argued therefore (Go 65a, Cl 65) that polarizability corrections are small in the post treatment of the stripping amplitude, a view confirmed by existing (Gi 66, Da 63) (d,p) reaction cross section calculations. The non-spherical nature of the break up contributions to the n-p relative wavefunction however suggests (Kn 74) that stretching effects may yet be of considerable importance in the analysis of tensor analysing power data.

The present chapter is concerned with the numerical evaluation of the (d,p) stripping matrix and corresponding reaction $T_{2q} s$, when the n-p relative function contains explicit coulomb break up components. In view
of the suggestion (Kn 75) that it may be possible to measure $D_2$ to an accuracy of 3-4%, particular attention is paid to obtaining an accurate quantitative estimate of the break up effects. Essential to this end is to treat correctly the strong state dependence (Pr 75) of the $n-p$ interaction $V_{np}$, whose strength multiplies the stripping amplitude.

4.1 General Observations

In the present chapter the formalism of chapters 2 and 3 is used, and generalized, to study the effect upon calculated $(d,p)$ tensor analysing powers of deformation or stretching of the deuteron internal wavefunction. As was argued previously, spin dependent distortion in both deuteron and proton channels is ignored at present, a study of which shall comprise the following chapter.

In this case, the deuteron centre of mass function in the presence of polarizability, as given by equation (3.29), is assumed diagonal in deuteron spin space and can be equated to the deuteron distorted wave $\chi^+(k_1, R)$ of chapter 2. Hence the approximate three body function of equation (3.53) is now

$$<R|\psi_{g_1}(k_1, \sigma_1)^+|> = |\phi_{g_1}(R)^+\chi^+(k_1, R)$$

(4.1)

where the deuteron ground state function $|\phi_{g_1}>$ of the usual DWBA formalism (e.g. equation (1.39)) has been replaced by the modified deuteron state

\* This is not strictly true as equation (3.28) may yet contain the central components $\overline{V}^{(0)}(R)$ of potential $\overline{V}(R)$, of section 3.4.
of equation (3.52). In chapter 2 it was pointed out that in the absence of the deuteron D-state (i.e. a spin independent $V_{np}$) and spin dependent channel distortions then the DWBA predicts that reaction tensor analysing powers are identically zero. An analogous exact result can be stated even when including polarizability of the deuteron wavefunction.

If $V_{np}$ is spin independent then quite generally, using equations (3.41) and (3.52) we obtain

$$<\xi, p, n|\phi_{1}(R)> = \phi(\xi, R)\chi_{s_{1}}(p, n)$$

resulting once again in identically zero $T_{2q}$ (Go 60). Hence, although $\phi(\xi, R)$ contains non S wave relative n-p states and is deformed from spherical, the induced deformation and deuteron spin direction are not coupled and will not contribute to the calculated $T_{2q}$. This result is also in agreement with the semiclassical model of Knutson (Kn 74).

Clearly therefore, unlike differential cross section corrections, previous calculations for which (Gi 66, Da 63) have been performed with a function of the form of equation (4.2), a tensor analysing power calculation requires that some care be taken over the choice of $V_{np}$. A non-zero $T_{2q}$ relies upon the presence of coupling of space and spin degrees of freedom, dictating that a realistic spin dependent interaction be used. The interaction $V_{np}$, appears in the stripping amplitude through the product $V_{np}|\phi_{1}(R)>$, in which $V_{np}$ appears essentially twice. Firstly, a knowledge of $V_{np}$ is required to calculate the function $|\phi_{1}(R)>$, be it exactly through equations (3.14) and (3.52), or using the perturbation formulae of section 3.5. Secondly, one must evaluate the product $V_{np}|\phi_{1}(R)>$, and clearly should use the interaction of the first step if unambiguous and quantitatively good results are to be obtained.
This has not in fact been the practice in previous stripping calculations. For instance, both Gibson et al. (Gi 66) and Dar et al. (Da 63) make the assumption that \(|\phi_{s_1}(R)\rangle\) may be calculated using a \(V_{np}\) which vanishes in all but the assumed \(3S_1\) deuteron ground state; in which it is \(V_{np}(3S_1)\). Therefore, retaining only the dipole term \(\Delta V^1\) of the polarizing potential, \(|\delta\phi_{s_1}^1(R)\rangle\) is calculated by inserting in equation (3.58), plane wave continuum n-p states, in the manner of equation (3.65). Calculation of \(V_{np}|\phi_{s_1}(R)\rangle\) then proceeds by multiplying the entire function \(|\phi_{s_1}(R)\rangle\) by \(V_{np}(3S_1)\), although the correction to the deuteron ground state, \(|\delta\phi^1\rangle\), comprises a \(3P\) relative n-p configuration (see for example equation (3.67)) in which it is known that \(V_{np}\) acts less strongly (Pr 75).

In the following treatment †, the aforementioned ambiguities will be removed by the consistent use of a realistic spin-dependent separable model for \(V_{np}\) in the calculation of the break up component \(|\delta\phi_{s_1}^1(R)\rangle\), of the modified deuteron wavefunction, \(|\phi_{s_1}(R)\rangle\). Finite range effects in the break up part of the stripping amplitude are included through a generalization of the conventional S and D state treatments of chapter 2.

4.2 The Break up Amplitude

In the previous chapter an approximate expression for \(|\delta\phi_{s_1}^k(R)\rangle\), the change in the deuteron wavefunction to first order in \(\Delta V^k(x,R)\), was obtained; namely

† A preliminary account of this work has been presented elsewhere (To 77).
where we have introduced a continuum index $p$ into equation (3.58), and
the $|\phi_{p_{s_{f}}\sigma_{f}}\rangle$ are normalized to

$$|\phi_{p_{s_{f}}\sigma_{f}}\rangle = \frac{\delta(p - p')/p^2}{\delta(p - p')} ;$$

(4.4)

here $\delta(a - b)$ is the Dirac delta function (Me 61). No renormalization of
the deuteron wavefunction

$$|\phi_{p_{s_{f}}\sigma_{f}}\rangle = |\phi_{s_{1}}\rangle + |\delta_{k_{1}}^{\sigma_{1}}(R)\rangle ,$$

which includes multipole $k$ break up, is performed.

So, inclusion of the internal polarizability effects of multipole $k$
requires that a break up amplitude $T_{kseed}^{B,U}(d,p)$ be added to the conventional
amplitude $T_{kseed}^{D,WBA}(d,p)$ of equation (2.15); $T_{kseed}^{B,U}$ is obtained by replacing, in
the reaction form factor (equation (2.16)), $V_{np}|\phi_{s_{1}}\rangle$ by $V_{np}|\delta_{k_{1}}^{\sigma_{1}}(R)\rangle$. Using
the Schrödinger equation for the continuum states $|\phi_{p_{s_{f}}\sigma_{f}}\rangle$, we remove the
explicit appearance of $V_{np}$, therefore

$$V_{np}|\delta_{k_{1}}^{\sigma_{1}}(R)\rangle = \sum_{\epsilon_{p}} \int dp d\epsilon_{p} (\epsilon_{p} - K_{f}) |\phi_{p_{s_{f}}\sigma_{f}}\rangle \langle\phi_{p_{s_{f}}\sigma_{f}}|/(-\epsilon_{p} - \epsilon_{d}),$$

(4.5)

in which form, provided the $|\phi_{p_{s_{f}}\sigma_{f}}\rangle$ are calculable, there is no possibility
of inconsistent use of $V_{np}$. As $V_{np}$ is assumed to conserve parity and total
n-p spin $s(= s_{2} + s_{3})$, then if $\ell_{f}$ labels the relative n-p orbital angular
momentum

\[
\sum_{\ell' f} \phi_{\ell' f} \langle \phi_{\ell' f} | = \sum_{\ell' f} \phi_{\ell' f} (\ell'_f \ell''_f s) > \times < \phi_{\ell' f} (\ell''_f \ell''_f s) | , \quad (4.6)
\]

where \((-) = (-) = (-) \), and

\[
<_{\ell, p, n} \phi_{\ell' f} (\ell''_f \ell''_f s) > = \psi_{\ell' f} (\ell''_f \ell''_f s) \mathcal{U}_{\ell''_f s_{\ell f}} (\hat{r}, p, n) . \quad (4.7)
\]

Here \(\mathcal{U} \) is the spin angle function of equation (1.2) and the physical outgoing wave boundary conditions on the break up states, which are normalized to (Go 64)

\[
\sum_{\ell' f} \int dr r^2 \psi_{\ell' f} (\ell''_f \ell''_f s) \psi_{\ell' f} (\ell''_f \ell''_f s) = \frac{1}{p^2} \delta(p-p') \delta_{\ell''_f \ell''_f} , \quad (4.7b)
\]

are shown explicitly.

Rewriting equation (3.59), using the functions of equation (4.7) gives

\[
\sum_{\ell' f} \langle \phi_{\ell' f} (\ell''_f \ell''_f s) | r^k \beta_k(R) \delta_{k(R)} \psi_{\ell' f} \phi_{\ell''_f s''_f} > \quad (4.7)
\]

\[
= (s_1^1 s_1^1 k q \delta_{s''_f s''_f} \delta_{s''_f s''_f} ) \quad (4.8)
\]
where using equations (4.7), (1.1), (2.5), together with the angular momentum addition identity (Br 62)

\[ \hat{c}^2 c \hat{W}(abcd;ef) = \sum (aab\beta|e\epsilon)(eed\delta|c\gamma) \]

\[ \times (b\delta\epsilon|f\phi)(a\alpha\phi|c\gamma), \quad (4.9) \]

the sum on the right over all \( z \) components, then

\[ \Delta_{s_f}^k \Psi_{l_p}^l(R) = \beta_k(R) \hat{K} \Psi_{l_p}^l(4\pi)^{-\frac{1}{2}} \]

\[ \times \sum_{l'_f, l''_f} L'(l'0k0|l''_f0)W(s_1 s'_1 k l' f ; l''_f s_f l' f)^{-\frac{1}{2}} \]

\[ \times \int_0^\infty dr r^{k+2} \psi_{l''_f p}^s (r) u_{l''_f} (r) \]

\[ (4.10) \]

where \( L' (= 0, 2) \) and \( s'_1 (= 1) \) are the deuteron internal orbital and spin angular momenta of equation (1.2).

So,

\[ \left< \psi_{np} \right| \Delta_{s_1}^k \Psi_{l_p}^l(R) = \sum_{q s_f} \int_{l''_f} dp p^2 (\epsilon_p - K_{l_f}) \phi_{s_f}^l (l''_f p^{s''_f} s'_1) / (-\epsilon_d - \epsilon_p) \]

\[ \times (s_1 s_f k q|s_f s_f) \gamma_{k q}^l (R) \Delta_{s_f}^k \Psi_{l_p}^l(R) \]

\[ (4.11) \]

\[ ^\dagger \quad \text{Equation (4.8) shows that } s = s'_1: \text{ For simplicity we now drop the } s \text{ label upon the continuum states } \psi_{l''_f p}^s s_f^{s'_1}, \text{ with the understanding that all functions correspond to } ^3(l_f)_s \text{ break-up states of the } n-p \text{ system.} \]
where

\[ K_{\ell_f} = \left( -\frac{\ell^2/2\mu}{} \right) \left\{ \frac{d^2}{dr^2} + \frac{2}{r} \frac{d}{dr} - \frac{\ell_f(\ell_f+1)}{r^2} \right\}. \] (4.12)

If we define

\[ Z_{s_f \ell_f}^k(r,R) = \sum_{\ell''_f} \int dp \, p^2 (\varepsilon_p - K_{\ell_f}) \psi_{s_f \ell_f}^{(+)}(r) \langle \ell''_f \psi_{\ell''_f}^{(+)}(r) \left( -\frac{\varepsilon_p}{\varepsilon_{\ell''_f}} \right) \rangle \] (4.13)

then equation (4.11) may be rewritten

\[ \langle r,p,n|V_{np}|\Delta^{k}\phi_0(\bar{R})\rangle = \sum_{q_{s_f \sigma_f \ell_f}} (s_{1 \sigma_1 kq} | s_{f \sigma_f} \rangle \chi_{kq}^{(R)} \Delta_{s_f \ell_f}^{k} \chi_{s_f \ell_f}^{(R)}(r,R) \] (4.14)

and, using the angular momentum identity (Br 62)

\[ \sum_{\varepsilon} (a\alpha \beta | e\epsilon \delta | c\gamma) = \sum_{\ell_f} \varepsilon_{\ell_f} W(abcd;ef) \] (4.15)

together with the symmetry of the Clebsh-Gordan coefficients gives

\[ \sum_{\sigma_f} (s_{1 \sigma_1 kq} | s_{f \sigma_f} \rangle \langle \lambda_f \sigma_f s_{1 \sigma_1} \rangle | s_{f \sigma_f} \rangle = \sum_{\ell A} S_{1 \ell A}^{\ell_f \ell_f} W(\ell_f s_{1 \sigma_1} k; s_{f \sigma_f} \rangle (-)^{L-\ell_f} \] (4.16)

Finally, substituting (4.16) into equation (4.14),
\begin{equation}
\langle r, p, n | V_{np} | \Phi_{s_1}^{\sigma_1} \rangle = \sum_{L \Lambda \sigma_1} (L \Lambda s_1 \sigma_1 | s_1 \sigma_1) \\
\times W_{L \Lambda}^{k}(r, R) \chi_{s_1}^{\sigma_1}(p, n) ,
\end{equation}

(4.17)

where we have defined

\begin{equation}
W_{L \Lambda}^{k}(r, R) = \frac{L}{s_1} \sum_{s_f} \hat{s}_f (-) ^ {L-k_f} W(\ell_f s_1, s_f R) \gamma_{s_f}^{k_f} (r, R) \\
\times T_{\ell_f k_f, L \Lambda}^{L \Lambda}(r, R) ;
\end{equation}

(4.18)

here \( T_{L_1 L_2, L_3 \Lambda_3}^{R_1, R_2} \) is the bipolar harmonic (Br 62)

\begin{equation}
T_{L_1 L_2, L_3 \Lambda_3}^{R_1, R_2} = \sum_{\Lambda_1 \Lambda_2} (L_1 \Lambda_1 L_2 \Lambda_2 | L_3 \Lambda_3) Y_{L_1}^{\Lambda_1}(R_1) Y_{L_2}^{\Lambda_2}(R_2)
\end{equation}

(4.19)

and the Racah coefficient in equation (4.18) restricts \( L \) to the values 0, 1 or 2.

If we compare equation (4.17) with that of the usual deuteron ground state expression, i.e.

\begin{equation}
\langle r, p, n | V_{np} | \Phi_{s_1}^{\sigma_1} \rangle = \sum_{L \Lambda \sigma_1} (L \Lambda s_1 \sigma_1 | s_1 \sigma_1) (i L_{L}^{\Lambda} \chi_{L}^{\Lambda}(r, R)) \chi_{s_1}^{\sigma_1}(p, n)
\end{equation}

obtained from equation (2.20), we see that the polarizability calculation has now the same structure as the conventional unperturbed problem; we obtain the break up amplitude, \( T_{k_u}^{B_u} \), upon replacing \( i L_{L}^{\Lambda} \chi_{L}^{\Lambda}(r) \) by \( W_{L \Lambda}^{k}(r, R) \) in equation (2.23) for the reaction form factor. So the counterpart of the reaction form factor of equation (2.16) for the break up component,
\[ |\delta \phi^k_{\sigma_1}(R)>, \]

of the modified deuteron wavefunction, which we shall designate \(<s_2\sigma_2b\beta|v_{np}|\delta \phi^k_{\sigma_1}a>|\]

is:

\[ <s_2\sigma_2b\beta|v_{np}|\delta \phi^k_{\sigma_1}a> = \sum_{L\Lambda \lambda n} B_{L\Lambda \lambda n}^{j_n j_n} (\sigma_2\beta;\sigma_1\alpha,\lambda,\Lambda,\lambda) \]

\[ \times R_j (r_n) i^{n\lambda_n^* (\hat{r}_n)} y_n^\lambda_n (\hat{r}_n) W_{L\Lambda}^* (r, R), \]  

(4.20)

and the break up amplitude, \(T^{B,u}_k (d,p)\), is given by

\[ T^{B,u}_k (d,p) = \sum_{L\Lambda \lambda n} B_{L\Lambda \lambda n}^{j_n j_n} (\sigma_2\beta;\sigma_1\alpha,\lambda,\Lambda,\lambda) B_{L\Lambda}^\lambda_n (k), \]

(4.21)

where \(B_{L\Lambda \lambda n}^{j_n j_n}\) is given by equation (2.24).

Here we have introduced

\[ B_{L\Lambda}^\lambda_n (k) = \chi^(-)(k_2, R) i^{n\lambda_n^* (\hat{r}_n)} y_n^\lambda_n (\hat{r}_n) W_{L\Lambda}^* (r, R) \chi^+(k, k_2) \]  

(4.22)

which may be used to generalize the results of Johnson (Appendix A of reference (Jo 67)) to include coulomb break up effects. If we define the transition amplitude \(T_c\), for stripping in the presence of coulomb break up, by

\[ T_c (d,p) = T^{DA} (d,p) + \sum_{k=1}^{\infty} T^{B,u}_k (d,p) \]

then equations (2.15), (2.23) and (4.21) give

\[ T_c (d,p) = \sum_{L\Lambda \lambda n} B_{L\Lambda \lambda n}^{j_n j_n} (\sigma_2\beta;\sigma_1\alpha,\lambda,\Lambda,\lambda) B_{L\Lambda}^\lambda_n (k_1, k_2); \]

here we have defined the generalization of equation (A.15) of (Jo 67)
and we have associated with $k = 0$, the unperturbed deuteron term of chapter 2, i.e.

$$
B_{n \Lambda}^{n \Lambda}(k_1, k_2) = \sum_{k=0}^{\infty} B_{n \Lambda}^{n \Lambda}(k),
$$

Expressions for the (d,p) cross section and polarizations are now given by equations (A.16-A.18) of (Jo 67).

For the purpose of calculation we rewrite the function $W_k^{\Lambda}(r, R)$ of equation (4.18) using equations (4.10), (4.13), as

$$
W_k^{\Lambda}(r, R) = \sum_{s_f^L, s_f^{L'}, q, L', \lambda_f, \lambda_f'} W_k^{s_f^L s_f^{L'} q, L', \lambda_f, \lambda_f'}(r, R) Y^{q}_0(\hat{R})
$$

the sum over $s_f^L, s_f^{L'}, \lambda_f, \lambda_f', q, L', \lambda_f, \lambda_f'$, where

$$
\sum_{s_f^L, s_f^{L'}, \lambda_f, \lambda_f'} W_k^{s_f^L s_f^{L'} q, L', \lambda_f, \lambda_f'}(r, R) Y^{q}_0(\hat{R}) = (4\pi)^{-1} \sum_{s_f^L, s_f^{L'}, \lambda_f, \lambda_f'} W^{s_f^L s_f^{L'} q, L', \lambda_f, \lambda_f'}(s_f^L, s_f^{L'}, s_f^L, s_f^{L'}) Y^{q}_0(\hat{R}) Y^{q}_0(\hat{R})
$$

and the radial function is
So, substituting in equations (4.21-22) for $T^B_{k,u}$ and using the partial
wave expansions of equations (2.9-12), we obtain

$$ T^B_{k,u}(d,p) = (4\pi)^2 \sum_{s_f^L, \lambda_f^L, \lambda_f^L} \sum_{q, L', \lambda_1, \lambda_2, \lambda, \lambda_n} \sum_{l_1, l_2} \sum_{\xi_2, \xi_1} \sum_{A, A'} \sum_{n, n'} \sum_{\sigma_2, \sigma_1, \alpha, \lambda_2, \lambda, \lambda_n}$$

$$ \times \int_{0}^{\infty} \int_{0}^{\infty} dp\, p^{k+2} \psi_f^{(+)}(\rho) u_L(\rho) . \quad (4.25)$$

summed over $s_f^L, \lambda_f^L, \lambda_f^L, q, L', \lambda_1, \lambda_2, \lambda, \lambda_n$, and where $F_{12}^{\lambda_f}(k)$, the counterpart in the P-wave break up states, of equation

$$(2.26)$$

is \{with $r_1 = 0\}$,

$$ F_{12}^{\lambda_f}(k) = \int \, d\hat{r}_1 \, d\hat{r}_2 \, (\hat{r}_2)^{2} R_{n, n'}^{(\lambda_2)}(\hat{r}_2) Y_{n, n'}^{(\lambda_1)}(\hat{r}_1) *$$

$$ \times \sum_{s_f^L, \lambda_f^L, \lambda_f^L} \sum_{q, L', \lambda_1, \lambda_2, \lambda, \lambda_n} \sum_{l_1, l_2} \sum_{\xi_2, \xi_1} \sum_{A, A'} \sum_{n, n'} \sum_{\sigma_2, \sigma_1, \alpha, \lambda_2, \lambda, \lambda_n}$$

$$ \times \{H_{f}^{\lambda_f^L}(r) Y_{f}^{\lambda_f^L}(\hat{r})\} \{\beta_2^{(R)} \gamma(R)^{(\lambda_2)} \chi_{f_1}^{(\lambda_2)}(\gamma R)\} , \quad (4.27)$$

with the $\chi_{f_1}^{\lambda_1}$ defined according to equation (2.11).

Finally therefore, performing the formal Taylor expansions,

$$ R_{j_2}^{(\lambda_2)}(r_2) Y_{j_2}^{(\lambda_2)}(\hat{r}_2)^{*} = e^{-ia_3 y K_3 \cdot r} R_{j_2}^{(\lambda_2)}(R) Y_{j_2}^{(\lambda_2)}(\hat{R})^{*} ,$$

$$ \chi_{f_2}^{(\lambda_2)}(r_2) = e^{ia_2 y K_2 \cdot r} \chi_{f_2}^{(\lambda_2)}(y R) ,$$
where \( a_2, a_3 \) are given by equation (2.41) then, by analogy with equation (2.30),

\[
iK_3 = \nabla^R_3, \quad iK_2 = \gamma^{-1} \nabla^R_2
\]  

(4.28)

where \( \nabla^R_2, \nabla^R_3 \) act only upon \( \{ x_{\ell_2} \} \) and \( \{ R_j^n \} \) respectively, and, as the notation suggests, operate upon the argument \( R \). Therefore exactly

\[
\Phi_f^{\ell_f \lambda_f} (k) = (2\pi)^{3/2} \int dR \Phi_f^{\ell_f \lambda_f} (a_2 K_2 - a_3 \gamma K_3) \chi_{\ell_2}^2 (\gamma R) R_j^n R \gamma^n \chi_{\ell_1}^1 (R) ^* \lambda_1
\]

\[
\times \{ \beta_k (R) Y^q_k (R) R \chi_{\ell_1}^1 (R) \}
\]

(4.29)

where we have defined, showing only the \( \ell_f \lambda_f \) dependence explicitly upon \( \Phi \), (cf. equation (2.29))

\[
\Phi_f^{\ell_f \lambda_f} (K) = (2\pi)^{-3/2} \int dR e^{-iK \cdot R} s_{\ell_f}^{\ell_f'} (K) Y^\ell_f (\hat{R})
\]

(4.30)

So, performing the angular integration

\[
(2\pi)^{3/2} \Phi_f^{\ell_f \lambda_f} (K) = 4\pi \tilde{H}_{\ell_f^*}^{\ell_f^*} (K) Y^\ell_f (\hat{R})
\]

(4.31)

where \( \tilde{H} \), the analogous radial momentum space wavefunction to \( \tilde{V}_L \) (equations (2.31-32)) of the unperturbed deuteron calculation, is

\[
\tilde{H}_{\ell_f^*}^{\ell_f^*} (K) = i \ell_f \int dr^2 j_{\ell_f} (Kr) H_{\ell_f^*}^{\ell_f^*} (r)
\]

(4.32)
In chapter 2 we have seen that the LEA treatment of the operator of the form \( \phi_{L}^{f} \) (\( a_{2}K_{2} - a_{3}K_{3} \)), appearing in equation (4.29), is to replace the radial function \( \tilde{H}(K) \), of equation (4.32), by a simple polynomial form, \( \tilde{H}_{\text{LEA}}(K) \), accurate for small \( K \). The chosen approximation of \( \tilde{H} \) to \( \tilde{H} \) depends of course upon the detailed form of the function \( H_{L}^{f} \) of equation (4.33), which shall be evaluated, in dipole approximation \((k = 1)\), in the following sections.

4.3 The Dipole Approximation

In section 3.6 we obtained, using a plane wave estimate and completely disregarding any state dependence of \( V_{np} \), the result that the quadrupole break up wavefunction \( |\delta\phi^{2}\rangle \) is a factor of order 5 smaller than the dipole term, \( |\delta\phi^{1}\rangle \), for the reaction \(^{208}\text{Pb}(d,p)^{209}\text{Pb} \), at \( E_{d} = 9 \text{ MeV} \). However, important in a stripping calculation is the relative strength of \( V_{np}|\delta\phi^{k}(R)\rangle \), requiring a knowledge of the relative n-p states within \( |\delta\phi^{k}\rangle \).

Using equations (4.10-11), then for \( k = 1 \), \( s_{f} \) may take the values \( s_{f} = 0(\ell_{f}'' = 1), 1(\ell_{f}'' = 1), 2(\ell_{f}'' = 1, 3) \). Schematically therefore, \( |\delta\phi^{1}\rangle \) comprises an admixture of the n-p states

\[
|\delta\phi^{1}\rangle = |^{3}p_{0} + ^{3}p_{1} + (^{3}p_{2} + ^{3}f_{2})\rangle . \tag{4.33}
\]

Similarly for \( k = 2 \) we have \( s_{f} = 1(\ell_{f}'' = 0, 2), 2(\ell_{f}'' = 2), 3(\ell_{f}'' = 2, 4) \), and therefore

\[
|\delta\phi^{2}\rangle = |(^{3}s_{1} + ^{3}d_{1}) + ^{3}d_{2} + (^{3}d_{3} + ^{3}f_{3})\rangle . \tag{4.34}
\]

It is well known that if \( V_{np} \) is assumed spin \((s_{f})\) independent but \( \ell \) depend-
ent, then the P-wave potential which multiplies equation (4.33) is very small ($P_r < 7.5$). In fact the early Serber (space exchange) interaction is identically zero in P-states. However, in section 4.1 we have already seen that a spin independent $V_{np}$ will in any case produce identically zero stripping $T_{2q}$. As noted there, it is precisely the spin dependence of the interaction which is responsible for a contribution to the $T_{2q}$, and the state dependent $V_{np}$ acting upon the terms $|^{3p_0}\rangle$, $|^{3p_1}\rangle$, $|^{3p_2 + ^3F_2}\rangle$ are certainly not individually weak ($Re 68$).

In the first instance we shall consider the leading dipole term, $|\delta \phi^1\rangle$, of the break up wavefunction. We have already noted the importance of D-state components of the deuteron wavefunction to calculated $T_{2q}$ and do not suggest that quadrupole break up effects are negligible, although, on the basis of the estimate of section 3.6, we expect that they are certainly of the same order as the dipole contributions. The importance of $k = 2$ contributions can however only be determined by detailed calculation. We shall not consider this term further at present.

Returning to the dipole function, $|\delta \phi^1\rangle$, we shall make the further approximation that we ignore the coupling between the $^{3p_2}$ and $^3F_2$ states. That this coupling is in any case weak is indicated by the small value of the mixing parameter $\epsilon_2$ (Ma 68), and, as argued in section 3.6, F wave scattering will be small in the low energy n-p break up configurations of particular importance in the integrand of equation (4.25). So, we shall make use of the uncoupled radial functions $\psi_{^3p_2}^{(+)}(r)$, where

$$s_f^{(+)} \psi_{^3p_2}^{(+)}(r) = s_f^{(+)} \psi_{^3p_2}^{(+)}(r) \delta_f^{(+)}$$

(4.35)
and similarly, define the function $H_{L^I}^{(r)}$ by

$$H_{L^I}^{(r)} = H_{L^I}^{(r)} \delta_{f^I}^{f^I},$$

obtained upon substituting equation (4.35) into equation (4.25).

### 4.4 P-wave Finite Range Effects

In the dipole approximation of the previous section and using the Mongan (Mo 68, Mo 69) Case I separable interaction model for $V_{np}$ in the continuum $^3P$ states, we show in Appendices B and C that

$$H_{L^I}^{(r)} = \mathcal{R}^{S^I} h_{L^I}^{(+)}(ia_r) - \mathcal{A}^{S^I} h_{L^I}^{(+)}(ia_r)$$

where $\mathcal{R}, \mathcal{A}$ are defined by equations (C.3-4) of Appendix C and $H_{L^I}^{(r)}$ by equation (4.36). $a_A, a_R$ are $s_f$ dependent and the values used are tabulated in Appendix A. The $\mathcal{A}, \mathcal{R}$, which contain integrals of the radial functions $u_L(r)$ of the deuteron ground state, in fact show little dependence upon the assumed deuteron interaction or wavefunction model, as we shall see shortly.

If we substitute equation (4.37) into equation (4.32) for $H_{L^I}^{(r)}$, then using the integral relation (Gr 65)

$$\int_0^\infty dr r^2 j_1(Kr)h_{L^I}^{(+)}(iar) = -\frac{K}{a^2(K^2+a^2)},$$

we obtain, exactly:

$$H_{L^I}^{(r)} = i\left(\frac{\mathcal{A}^{S^I}}{a_A^2} \frac{K}{(K^2+a_A^2)} - \frac{\mathcal{R}^{S^I}}{a_R^2} \frac{K}{(K^2+a_R^2)}\right).$$
If we now drop the redundant \( \ell_f, k \) labels, then to the same order in \( K \) as the LEA \( S \) and \( D \)-state treatments of chapter 2, we write for the \( P \)-states

\[
\hat{H}_{\mathrm{LEA}}(K) = i D_{L'}^L \left( \vec{\delta}_{\ell_f} \right),
\]

or, substituting in equation (4.31)

\[
(2\pi)^{3/2} \phi_{1\lambda}(K) = 4\pi i D^{s_{L'}^L} \left\{ K Y_{1}^{\lambda}(\hat{K}) \right\}.
\]

So, using equation (2.61) for the bracketed term above

\[
(2\pi)^{3/2} \phi_{1\lambda}(K) = (12\pi)^{1/2} D^{s_{L'}^L} i K_{\lambda} \delta_{\ell_f l}.
\]

where \( K_{\lambda} \) is the spherical component \( \lambda \) of vector \( K \), and the constants \( D^{s_{L'}^L} \) are therefore

\[
D^{s_{L'}^L} = \left\{ \hat{A}^{s_{L'}^L} / a_A^h - \hat{R}^{s_{L'}^L} / a_R^h \right\}.
\]

which completely parametrize that part of the \( 3^p \) n-p break up state which results from the \( L' \) component of the deuteron ground state. \( D^{s_{L'}^L} \) values obtained from several model deuteron wavefunctions are presented in table 4.1, and show little model dependence. It is seen that, ignoring any difference in angular momentum coupling coefficients, there is considerable cancellation between the values \( D_{0L'}^{s_f} \) and \( D_{1L'}^{s_f} \), i.e. the \( 3^p_0 \) and \( 3^p_1 \) break up components, as might have been expected from the discussion of section 4.3. For consistency we use in the following the values corresponding to the Reid soft core interaction.
Table 4.1

<table>
<thead>
<tr>
<th>Model</th>
<th>$D^{00}$</th>
<th>$D^{02}$</th>
<th>$D^{10}$</th>
<th>$D^{12}$</th>
<th>$D^{20}$</th>
<th>$D^{22}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>REID HARD CORE</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$P_D = 6.5%$</td>
<td>- 1.972</td>
<td>- .367</td>
<td>1.863</td>
<td>.327</td>
<td>-.575</td>
<td>-.122</td>
</tr>
<tr>
<td>(Re 68)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>REID SOFT CORE</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$P_D = 6.5%$</td>
<td>- 1.971</td>
<td>- .371</td>
<td>1.861</td>
<td>.330</td>
<td>-.575</td>
<td>-.123</td>
</tr>
<tr>
<td>(Re 68)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>HULTHEN (a)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$P_D = 3%$</td>
<td>- 1.987</td>
<td>- .325</td>
<td>1.875</td>
<td>.295</td>
<td>-.582</td>
<td>-.096</td>
</tr>
<tr>
<td>HULTHEN (b)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$P_D = 4%$</td>
<td>- 1.939</td>
<td>- .341</td>
<td>1.832</td>
<td>.307</td>
<td>-.570</td>
<td>-.106</td>
</tr>
<tr>
<td>HULTHEN (c)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$P_D = 5%$</td>
<td>- 1.987</td>
<td>- .361</td>
<td>1.876</td>
<td>.321</td>
<td>-.580</td>
<td>-.113</td>
</tr>
</tbody>
</table>

NOTES: 
(a) $\rho(-\varepsilon_d, -\varepsilon_d) = 1.734$ fm, $x_c = .1$ 
(b) $\rho(-\varepsilon_d, -\varepsilon_d) = 1.704$ fm, $x_c = 0$. 
(c) $\rho(-\varepsilon_d, -\varepsilon_d) = 1.734$ fm, $x_c = .13$ 
(Hu 57)
If we substitute equation (4.42) into (4.29), with \( k = 1 \), then using equation (4.28)

\[
F_{12}^{1\lambda}(1) = \delta_{\lambda_1}^{\lambda_2} \int (12n) \frac{1}{2} \sum_{i=2,3} a_{12}^{i} F^{(i)}_{12},
\]

(4.44)

where the remaining 3-d integrals are

\[
F_{12}^{(2)} = \int dR [\gamma^{-1} \lambda_{12} (\gamma R)] R_{\lambda_{12}}^{1} (R) Y_{n}^{\lambda_{12}} (R)^{*} \lambda_{1}^{1}(R) ,
\]

(4.45)

\[
F_{12}^{(3)} = - \gamma \int dR \lambda_{12} (\gamma R) (R) Y_{n}^{\lambda_{12}} (R)^{*} \lambda_{1}^{1}(R) ,
\]

(4.46)

Therefore, using the gradient formula of equation (2.62) and defining the angular integral

\[
\langle L_{1}^{M_{1}} | L_{2}^{M_{2}} L_{3}^{M_{3}} L_{4}^{M_{4}} \rangle = \int dR \frac{M_{1}^{1}}{L_{1}^{1}} \frac{M_{2}^{2}}{L_{2}^{2}} \frac{M_{3}^{3}}{L_{3}^{3}} \frac{M_{4}^{4}}{L_{4}^{4}}
\]

\[
= (-\frac{M_{2}}{L_{2}} L_{1}^{1} L_{2}^{2} L_{3}^{3} L_{4}^{4}) \frac{\hat{K}^{-2} (L_{1} M_{1} L_{2} - M_{2} | KQ)}{4\pi}
\]

\[
\times (L_{3}^{M_{3}} L_{4}^{M_{4}} | KQ) (L_{1}^{0} L_{2}^{0} | KQ) (L_{3}^{0} L_{4}^{0} | KQ) ,
\]

(4.47)

then equations (4.45-46) reduce to

\[
F_{12}^{(2)} = \sum_{L_{2}^{2}} \frac{L_{2}^{2} L_{1}^{1} (L_{2}^{2} \lambda_{2}^{1} 1\lambda_{1}^{1} | L_{2}^{2} L_{1}^{1}) (L_{2}^{2} 010 | L_{2}^{2})}{\langle L_{2}^{2} \lambda_{2}^{1} 1\lambda_{1}^{1} | L_{2}^{2} L_{1}^{1} \rangle} \frac{F^{P}(L_{2}^{2} \lambda_{2}^{1} 1\lambda_{1}^{1})}{2},
\]

(4.48)
\begin{equation}
F_{12\lambda}^{(3)} = (-)^{\lambda+1} \sum_{L_3, \Lambda_3} \hat{\lambda}_{n}^{L_3-1}(\ell_1, \lambda, 1 - \lambda|L_3, \Lambda_3)(\ell_2, 010|L_3, 0)
\end{equation}

\times << L_3, \Lambda_3 | \ell_2, \lambda, 1, q, l, \lambda | L_3 > \rangle F_{2}^{P}(\ell_1, q, \lambda, l, n, L_3), \quad (4.49)

where the radial (P-state) overlap integrals \( F_{2}^{P}, F_{3}^{P} \) are

\begin{equation}
F_{2}^{P}(\ell_1, \ell_2, j, n, L_2) = \int_{r_0}^{\infty} dR \left\{ \hat{g}_{L_2, \ell_2} (k_2, \gamma R) \right\} R_{j, n} (R) x^{(1)}_{\ell_1} (k_1, R)
\end{equation}

\times \beta_1(R), \quad (4.50)

and

\begin{equation}
F_{3}^{P}(\ell_1, \ell_2, j, n, L_3) = \int_{r_0}^{\infty} dR x^{(1)}_{\ell_1} (k_1, R) \left\{ \hat{g}_{L_3, \ell_2} \right\} R_{j, n} (R) x^{(1)}_{\ell_2} (k_2, R)
\end{equation}

\times \beta_1(R); \quad (4.51)

(in equation (4.50), \( \hat{g}_{L_2, \ell_2} \) acts with respect to \( \gamma R \)). In equations (4.50-51), a lower limit of \( r_0 \) has been placed on the integrals, as \( \beta_1(R) \rightarrow \infty \) as \( R^{-2} \) for \( R \rightarrow 0 \). In the subcoulomb regime this interior contribution is completely negligible and the spurious infinity in certain partial wave radial overlap integrals is removed by cutting off the radial integral at \( r_0 \); in the calculations of the following section \( r_0 \) is taken as 3 fm. The dipole break up amplitude \( T_{B, u}^{(d, p)} \) has been calculated using equations (4.26), (4.44) and (4.50-51), without further approximation.

The accuracy of the P-state LEA description of finite range effects can be estimated using the procedure of section 2.5. If we make the approximation that, with \( g \) given by equation (2.78),
\[ g(r_2, R) = X^{(-)}(k_2, r_2)X^{(+)}(k_1, R)\beta_1(R)Y_1^q(R) \cong G(\theta, \varphi), \quad (4.52) \]

then the accuracy of the LEA is dependent only upon the accuracy of replacing
\[ \tilde{H}_{LEA}^{sfL'}(3/4iK_n) \] by \[ H^{sfL'}(3/4iK_n) \]. For a reaction Q value \( \approx 0 \), we find that for all \( sf\equiv 0, 1, 2 \) and \( L'\equiv 0, 2 \), the important ratio
\[
\left[ \frac{H^{sfL'}_{LEA}(3/4iK_n)}{H^{sfL'}(3/4iK_n)} \right] > 0.90 ,
\]
and therefore the finite range effects are adequately represented in the proposed P-state LEA; with an accuracy comparable to that of the conventional D-state treatment (Jo 71).

In the following section we summarize the results of polarizability calculations for the reaction \( ^{208}\text{Pb}(d,p)^{209}\text{Pb} \); that proposed by Knutson and Haeberli (Kn 75) for the measurement of \( D_2 \).

4.5 Calculations for the reaction \( ^{208}\text{Pb}(d,p)^{209}\text{Pb} \) at \( E_d = 9 \text{ MeV} \) and \( 7 \text{ MeV} \)

The calculations of this section were performed by including the amplitude \( T_{k=1}^{B_u}(d,p) \) into the computer program DWCODE (Ha 70). In Figure 4.1 we present the results of these calculations for stripping to the \( j'' = 1/2^+(E_x = 2.03 \text{ MeV}, \text{ curve a}) \), \( j'' = 5/2^+(E_x = 1.57 \text{ MeV}, \text{ curve b}) \) and \( j'' = 9/2^+(E_x = 0.0 \text{ MeV}, \text{ curve c}) \) states of \( ^{209}\text{Pb} \) at \( E_d = 9 \text{ MeV} \). It is found that polarizability effects are small, but not completely negligible. The reaction \( T_{21}, T_{22} \) data fall to zero at large centre of mass angles (Kn 77) and changes to the \( T_{2q} \) are visible only in the observable \( T_{20} \), which is back angle peaked.
\[ \Delta T_{20} \equiv T_{20}[S+D+P] - T_{20}[S+D] \]

**Figure 4.1** Percentage changes in the calculated $T_{20}$ angular distributions when including dipole electric break up of the deuteron. Curves a to e are described in the text.
Results when including P-wave, or dipole, break up, $T_{20}[S + D + P]$, are presented as percentage deviations, $\Delta T_{20}\%$, from the calculated values, $T_{20}[S + D]$, of the conventional $(S + D)$ state calculation. Results for the $j^\pi = 1/2^+$ state, at $E_d = 7$ MeV are indicated by curve d. Calculations a to d indicate corrections due entirely to the stretching of the deuteron internal wavefunction, and, are calculated assuming purely coulomb distortion in the deuteron and proton channels. Inclusion of the central $R^{-4}$ polarization potential, $V^{(0)}_{so}(R)$ of equation (3.50), into the deuteron centre of mass motion, effected changes in the calculated values of less than 0.1%. Calculations were also found to be completely insensitive to the cut off radius, $r_0$, introduced in equations (4.50) and (4.51), provided its value was taken to be less than 6 fermis.

Curve e results when including dipole polarizability in the approximate sub-coulomb stripping model of Knutson et al. (Kn 73a, Kn 74); the calculation is for the $j^\pi = 1/2^+$ transition. The attractive feature of this model for the present investigation is, apart from its simplicity, that the internal structure of the deuteron and hence stretching effects are treated in an accurate manner, at the expense of any detailed consideration of the channel distorted waves. The product of the deuteron and proton distorted waves, which peak at the turning point, $q_c$, of the classical orbits $^+$, are replaced by the delta function form

$^+$ As $Q \leq 0$, $q_c$ is calculated at the mean energy of the incident and final projectiles.
\[ \chi^{(-)}(k_2,r_2) \chi^{(+)}(k_1,R) \propto \delta(p - q_c) , \]  \hspace{1cm} (4.53)

where \( p \) is given by equation (2.78), and which forces the stripping to take place at the point \( q_c \). In fact, when including polarizability we make the approximation (with \( R \equiv r_1 \))

\[ G(r_2,R) \propto \delta(p - q_c) \beta_1(q_c) Y_l^q(q_c') , \]  \hspace{1cm} (4.54)

with \( G \) given by equation (4.52). The evaluation of \( \gamma_{11}^{B,*}(d,p) \), equation (4.21), then follows obviously upon replacing the neutron bound state function by its asymptotic form ([Kn 73a], equation (2.79)).

This method therefore bypasses the numerical treatment of finite range effects, presented in the preceding section, and thus provides a valuable check of both the written computer program and formalism. In view of the simplicity of the model the results, curve e, agree remarkably well with the more exact calculation, curve a. For \( E_d = 7 \text{ MeV} \) also, the \( \delta \)-function result coincides with curve d, for \( \theta_{\text{cm}} \geq 130^\circ \), and indicates that the computed results are correct. More importantly however, the \( \delta \)-function model reproduces both the magnitude and energy dependence of dipole break up effects and may thus prove a very useful tool in the quantitative investigation of higher order multipole effects.

No study has been undertaken here of effects of dipole break up upon the observables \( \frac{d\sigma}{d\Omega} \) and \( iT_{11} \). The previous work of Gibson and Kerman ([Gi 66], while quantitatively in doubt (see section 4.1), remains qualitatively correct and places a useful upper bound upon P-wave break up contributions to \( \frac{d\sigma}{d\Omega} \).
CHAPTER 5

TENSOR FORCE EFFECTS IN THE DWBA

We have seen in the previous chapter that the effect, upon (d,p) tensor analysing powers, of distortion of the deuteron internal wave-function, are small. However, one aspect of the polarizability investigation not yet studied is the appearance, in chapter 3, of long ranged (coulombic tailed) tensor $T_R$ potential terms in the interaction describing the relative deuteron-target motion. In chapter 2, the Johnson-Santos LEA treatment of D-state effects was introduced, without spin dependent distortions, for clarity. In this chapter we shall generalize this treatment to include the usual spin-orbit distortion in both channels, and also the allowed rank-2 tensor interactions (Sa 60) in the deuteron channel.

Satcher has shown (Sa 60), that the deuteron target interaction may contain, assuming parity conservation and symmetry of the scattering matrix (reciprocity), at most $\dagger$ three rank-two tensor interactions of the form:

$$V_{R R} = \left( \frac{2}{3} \right) \hat{R}_{R} \hat{R}_{R} = \left( \frac{2}{3} \right) \left( \mathbf{S}_{1} \cdot \mathbf{S}_{1} \right) - 2 \mathbf{S}_{1} \cdot \mathbf{S}_{1}$$

and

$$V_{L L} = \left( \frac{2}{3} \right) \hat{L}_{L} \hat{L}_{L} = \left( \frac{2}{3} \right) \left( \mathbf{S}_{1} \cdot \mathbf{S}_{1} \right) - 2 \mathbf{S}_{1} \cdot \mathbf{S}_{1}$$

where

$$\hat{T}_{P} = \left( \frac{2}{3} \right) \left( \mathbf{S}_{1} \cdot \mathbf{P} \right) - 2 \mathbf{S}_{1} \cdot \mathbf{P}$$

$\dagger$ Neglecting any target spin effects.
Here, $r_1, \ell_1, \ell_2$ are the deuteron-target relative position, orbital angular momentum and linear momentum operators, $s_1$ the deuteron spin operator and the $V_i$ are the corresponding potentials (in general complex); assumed of known depth and radial dependence. The form of the interactions, obtained via nuclear models, will be discussed in chapter 6 but for the present discussion their detailed form is unimportant.

The principal modification to the calculation of the deuteron distorted waves, when including an interaction of the $T_R$ and/or $T_P$ types is that certain partial waves, differing by two units of orbital angular momentum, become coupled; the necessary generalization to the $(d,p)$ stripping formalism when including a $T_R$ interaction in an exact finite range calculation (De 70), has been presented elsewhere.

In the following sections we first outline the complications to the projectile distorted waves when including spin-dependent interactions, a purely elastic scattering problem; then, we shall generalize the Johnson-Santos LEA formalism to accept these improved elastic functions. However, in doing so we make no changes to the treatment of the deuteron $S$ and $D$ state functions, developed in chapter 2; to a large part now masked by the need for a more efficient treatment of the angular momentum algebra, for computation reasons.

5.1 Spin Dependence in the Distorted Waves

If in equations (2.2-3), the Schrödinger equations for the deuteron and proton distorted waves, $\chi_{s_1}\ell_1^{(+)}(k_1,p,n)$ and $\chi_{s_2}\ell_2^{(-)}(k_2,p)$, respectively, we allow the optical potentials $U_1, U_2$ to take on their most general form consistent with parity conservation and reciprocity, then
\[ U_1(r_1) = u_c^{(1)}(r_1) + u_{so}^{(1)}(r_1) \frac{\ell_1 \cdot s_1}{s_1} + u_R(r_1) \hat{T}_R + u_L(r_1) \hat{T}_L \]
\[ + \{ \hat{T}_p u_p(r_1) + u_p(r_1) \hat{T}_p \} , \] (5.1)

\[ U_2(r_2) = u_c^{(2)}(r_2) + u_{so}^{(2)}(r_2) \frac{\ell_2 \cdot s_2}{s_2} . \] (5.2)

Here \( \ell_i (i = 1, 2) \) are the projectile-target relative orbital angular momentum operators \( (\ell_i = -i r_i \times \nabla_i) \), \( s_i \) the projectile spin operators and \( u_c^{(i)}, u_{so}^{(i)} \) are the conventional central and spin-orbit components of the interaction.

We shall consider first the rather more complex deuteron distorted wave. The general solution of the Schrödinger equation (2.2), for \( \chi_1^{(+)} \), in the presence of interaction \( U_1 \) above, is given by equation (2.4), where, assuming rotational invariance (i.e. that \( J_1 = \ell_1 + s_1 \), the total angular momentum, is a good quantum number)

\[ \chi_1^{(+)}(k_1, r_1) = \frac{4\pi}{r_1} \sum (\ell_1 \chi_1 s_1 \sigma_1 | J_1 M_1 \rangle (\ell_1' \lambda_1' s_1' \sigma_1' | J_1' M_1 \rangle ) \]
\[ \times i \eta_{\ell_1, \ell_1'} (k_1) \eta_{\lambda_1, \lambda_1'} (r_1) \chi_{\lambda_1, \lambda_1'} (k_1, r_1) \] , (5.3)

with the summation over \( \ell_1, \lambda_1, \ell_1', \lambda_1', J_1, M_1 \). In equation (5.3) we have allowed for the fact that \( [\hat{T}_p^2, T_i] \neq 0 \) for \( i = R, P \).

Substituting equation (5.3) into the Schrödinger equation, shows (Sa 60) that the functions \( \chi_{\ell_1, \lambda_1} \), satisfy the set of coupled radial equations

\[ \frac{k^2}{2\mu_1} \chi_{\ell_1, \lambda_1} (k_1, r) \frac{J_1}{r} = \sum \langle J_1 \lambda_1 | U_1 | J_1 \lambda_1' \rangle \chi_{\ell_1', \lambda_1'} (k_1, r) \] , (5.4)
where $V^2_{1,1}$ is defined in Table 5.1. Also, we have defined

$$\langle J_{1,1}' | J_{1,1}'' \rangle = \sum \int \frac{d^3 \mathbf{r}}{2 \pi} Y^M_{J_{1,1}'} (\mathbf{r}, \sigma) \hat{U}_1 (r) Y^M_{J_{1,1}''} (\mathbf{r}, \sigma), \quad (5.5)$$

where

$$Y^M_{J_{1,1}'} (\mathbf{r}, \sigma) = \sum_{\lambda_1 \lambda_1' \sigma_1} (\lambda_1 \lambda_1' \sigma_1 | J_{1,1}'' \rangle Y^M_{J_{1,1}'} (\mathbf{r}) \chi_{s_1} (\sigma); \quad (5.6)$$

(N.B.: $Y^M_{J_{1,1}'}$ differs from $\mathbf{Y}^M_{J_{1,1}'}$ of equation (1.2) by a factor $i^J$).

Here spinor $\chi_{s_1}$ is taken to represent the triplet deuteron spin space and the matrix elements of equation (5.5) are independent of $M_1$, by rotational invariance. Therefore, removing the diagonal elements of the interaction, $U_1$, to the left hand side of equation (5.4), then

$$\left\{ \frac{\hbar^2}{2 \mu_1} \nabla^2_{J_{1,1}'} - U_{J_{1,1}'} (r) + E_{J_{1,1}'} \right\} \frac{\chi_{J_{1,1}'} (k_1', r)}{r} = \sum_{J_{1,1}''} \frac{\chi_{J_{1,1}''} (k_1', r)}{r} \cdot \frac{J_{1,1}'' (r)}{r}, \quad (5.7)$$

where

$$U_{J_{1,1}'} (r) = U^{(1)}_c (r) + U^{(1)}_{\text{so}} (r) K_{J_{1,1}'} (\xi, \xi_1') + U_L (r) K_{J_{1,1}'} (\hat{T}_L) + U_R (r) K_{J_{1,1}'} (\hat{T}_R) + U_P (r) K_{J_{1,1}'} (\hat{T}_P) + K_{J_{1,1}'} (\hat{T}_P) U_P (r) + K_{J_{1,1}'} (\hat{T}_R) U_R (r), \quad (5.8)$$

with

$$K_{J_{1,1}'} (\hat{\alpha}) = \langle \hat{\alpha} | J_{1,1}' \rangle, \quad (5.9)$$

and the coupling term

$$U^J_{\xi, \xi'} (r) = \{ \delta_{J, J_1'} + \delta_{J_1, J_1'} \} \delta_{\xi, \xi_1'} \delta_{\xi', \xi_1} \cdot \left\{ U_{J_1} (r) \langle J_{1,1}') | \hat{\xi}_R | J_{1,1}' \rangle | J_{1,1}'' \rangle + U_{J_1} (r) \langle J_{1,1}') | \hat{\xi}_P | J_{1,1}' \rangle | J_{1,1}'' \rangle \right\} \cdot \langle J_{1,1} | \hat{T}_P | J_{1,1}'' \rangle U_{J_1} (r) \}, \quad (5.10)$$
Table 5.1

Matrix Elements of the Spin-one Interaction Operators

<table>
<thead>
<tr>
<th>J=\xi+1</th>
<th>J=\xi</th>
<th>J=\xi-1</th>
</tr>
</thead>
<tbody>
<tr>
<td>\langle J</td>
<td>1\cdot S_1</td>
<td>J\rangle</td>
</tr>
<tr>
<td>6\langle J</td>
<td>T_L</td>
<td>J\rangle</td>
</tr>
<tr>
<td>3\langle J</td>
<td>T_R</td>
<td>J\rangle</td>
</tr>
<tr>
<td>3\langle J</td>
<td>T_p</td>
<td>J\rangle</td>
</tr>
</tbody>
</table>

\psi^2 = \frac{d^2}{dr^2} + \frac{2}{r} \frac{d}{dr} - \frac{\xi(\xi+1)}{r^2},

\langle J-1|T_R|J+1\rangle = \langle J+1|T_R|J-1\rangle = \{J(J+1)\}^{1/2}/(2J+1),

\langle J-1|T_p|J+1\rangle = -\frac{\{J(J+1)\}^{1/2}}{(2J+1)} \left[ \frac{d^2}{dr^2} + \frac{2J+3}{r} \frac{d}{dr} + \frac{J(J+2)}{r^2} \right],

\langle J+1|T_p|J-1\rangle = -\frac{\{J(J+1)\}^{1/2}}{(2J+1)} \left[ \frac{d^2}{dr^2} - \frac{2J-1}{r} \frac{d}{dr} + \frac{(J-1)(J+1)}{r^2} \right].
The matrix elements \( \langle \mathcal{J}_\ell | a | \mathcal{J}_\ell \rangle \), reported by Satchler (Sa 60), have been corrected according to the work of Johnson (Jo 62), and together with the off diagonal elements of operator \( \hat{T}_p \), given by Goddard (Go 77), are reproduced for completeness in table 5.1.

Returning to the radial functions, \( x^{(+) \ell_1 \ell_1}_\ell(k_1, r) \), clearly the index \( \ell_1 \) does not enter the radial equation (5.7). The significance of \( \ell_1 \) is evident only when we apply the correct asymptotic physical boundary condition that \( \chi^{(+) \sigma_1 \sigma_1}_\ell \) have an incident plane wave, provided \( \sigma'_1 = \sigma_1 \), else purely outgoing spherical waves. In this case (Ra 64), with the expansion adopted in equation (5.3), asymptotically

\[
\frac{J_1}{x^{(+) \ell_1 \ell_1}_\ell(k_1, r)} \left\{ \begin{array}{c}
\frac{e^{i \ell_1}}{k_1} \left( \begin{array}{c}
F_{\ell_1}^{(+) \ell_1}(k_1 r) \delta_{\ell_1 \ell_1}^2 + C_{\ell_1}^{(+) \ell_1}(k_1 r) \\
F_{\ell_1}^{(+) \ell_1}(k_1 r)
\end{array} \right) \\
+ i F_{\ell_1}^{(+) \ell_1}(k_1 r)
\end{array} \right\}
\]

(5.11)

where the elastic amplitudes \( C_{\ell_1}^{(+) \ell_1} \) are related to the more usual \( S \) matrix counterpart, \( S_{\ell_1 \ell_1}^{(+) \ell_1} \) (Jo 77), by

\[
C_{\ell_1}^{(+) \ell_1} = i^{\ell_1 - \ell_1'} \left( S_{\ell_1 \ell_1}^{(+) \ell_1} - \delta_{\ell_1 \ell_1} \right) / 2i
\]

(5.12)

being the generalization of equation (2.14).

Mathematically therefore, as a result of \( T_R \) or \( T_p \) interactions, partial waves, of fixed \( J_1 \), but which differ by two units of orbital angular momentum couple. Thus, an incoming deuteron with angular momenta \( J_1, \ell_1 = J_1 \pm 1 \), will have outgoing wave components with angular momenta \( J_1, \ell_1' = J_1 \pm 1 \) and \( J_1, \ell_1' = J_1 \mp 1 \), the amplitudes for which are

\[
C_{J_1 \ell_1 \pm 1, J_1 \pm 1} \text{ and } C_{J_1 \ell_1 \pm 1, J_1 \mp 1}
\]

respectively. \( \ell_1, \ell_1' \) are therefore referred
to as the incoming and outgoing partial waves. Clearly from equation (5.10), the function \( \chi_{\Omega_{11}}^{(-)} \) remains uncoupled, as does the function \( \chi_{11}^{0} \); this entire coupling scheme is represented schematically in Figure 5.1.

The case of the proton distorted wave, \( \chi_{\alpha^{'}, \sigma^{'}}^{(-)} \), is more simple. Both total angular momentum \( J_{2} (= \ell_{2} + s_{2}) \) and \( \ell_{2} \) are good quantum numbers, for the interaction \( u_{2} \) of equation (5.2). Using the time reversal relation (equation (2.6)) together with the general expansion of equation (5.3) (but with \( \ell_{2}^{'}, \ell_{2} = \ell_{2} \)), then

\[
\chi_{\alpha^{'}, \sigma^{'}}^{(-)} (k_{2}, r_{2})^{*} = (4\pi/r_{2}) \sum (\ell_{2}, \lambda_{2}, s_{2} - \sigma_{2}, J_{M_{2}}) (\ell_{2}, \lambda_{2}', s_{2}' - \sigma_{2}', J_{M_{2}})
\]

\[
\times (-)^{\sigma_{2} - \sigma_{2}'} \chi_{\ell_{2}}^{(-)} (k_{2} r_{2}) \chi_{\ell_{2}'}^{(-)} (k_{2} r_{2}) \quad \text{(5.13)}
\]

the sum over \( \ell_{2}, \lambda_{2}, \lambda_{2}', J_{2}, M_{2} \). The radial functions \( \chi_{\ell_{2}}^{J_{2}} \), satisfy the radial equation (Sa 64)

\[
\left\{ \frac{\hbar^{2}}{2\mu_{2}} \left[ \frac{d^2}{dr^2} - \frac{\ell_{2}(\ell_{2}+1)}{r^2} \right] \right\} - U_{J_{2} \ell_{2}} (r) + E_{2} \chi_{\ell_{2}}^{J_{2}} (k_{2}, r) = 0 \quad \text{(5.14)}
\]

where, as \( s_{2} = 1/2, J_{2} = \ell_{2} \pm 1/2 \), and therefore

\[
U_{J_{2} \ell_{2}} (r) = U_{c}^{(2)} (r) + U_{g_{o}}^{(2)} (r) \{ J_{2} (J_{2} + 1) - \ell_{2} (\ell_{2} + 1) - 3/4 \} \quad \text{(5.15)}
\]

while asymptotically, applying the physical boundary conditions,

\[
\chi_{\ell_{2}}^{J_{2}} (k_{2}, r) \bigg|_{r \to \infty} = \frac{i \sigma_{\ell_{2}}}{k_{2}} \left\{ F_{\ell_{2}} (k_{2} r) + C_{\ell_{2}} \left[ G_{\ell_{2}} (k_{2} r) + i F_{\ell_{2}} (k_{2} r) \right] \right\} \quad \text{(5.16)}
\]

These wavefunctions must now be combined within the DWBA formalism; the subject of the following section.
Figure 5.1 Coupling scheme in the deuteron partial waves when including a $T_R/T_P$ type interaction.
5.2 The LEA Reformulated

In this section we shall reformulate the LEA approach of chapter 2 to include the spin dependent interactions of the previous section. Using equations (2.4-5) the stripping amplitude (2.15) generalizes to

\[ T_{\text{DWA}}(d,p) = \sum_{\sigma_1'\sigma_2'} \int \frac{\partial^2 \chi_{\sigma_2'}(-)}{\partial x_2 \partial x_1} r_2 \chi_{\sigma_1'}(k_2, x_2)^* \]

\[ \times J<s_2 \sigma_2' b b | V_{\text{np}} | s_1 \sigma_1' a a> \chi_{\sigma_1}^{(+)}(k_1, x_1) , \quad (5.17) \]

where \( J \) is the Jacobian for the transformation from co-ordinates \((x, x_1)\) to \((x_2, x_1)\); all other symbols have been defined earlier. As noted by Satchler (Sa 64), the reaction form factor, \(<s_2 \sigma_2' b b | V_{\text{np}} | s_1 \sigma_1' a a>\), may be expanded, in a model independent way, in multipoles corresponding to the transfer of particular total (\(j\)), orbital (\(\ell\)) and spin (\(s\)) angular momentum

\[ j = b - a , \quad s = s_1 - s_2 , \quad j = \ell + s , \quad (5.18) \]

in the reaction. In fact, following Satchler, we write

\[ J<s_2 \sigma_2' b b | V_{\text{np}} | s_1 \sigma_1' a a> = \sum \imath^{\ell} G_{\lambda s j, \lambda}(x_2, x_1)(-)^{s_2-\sigma_2'} \]

\[ \times (a a m | b b) (s_1 \sigma_1' s_2 - \sigma_2' | s a) (\ell \lambda s | j m) , \quad (5.19) \]

where the sum is over \(\lambda, s, j, m, \sigma, \lambda\), or, alternatively

\[ G_{\lambda s j, \lambda}(x_2, x_1) = \imath^{\ell} S^{2\ell-2} \sum J<s_2 \sigma_2' b b | V_{\text{np}} | s_1 \sigma_1' a a> \]

\[ \times (-)^{s_2-\sigma_2'} (a a m | b b) (s_1 \sigma_1' s_2 - \sigma_2' | s a) (\ell \lambda s | j m) , \quad (5.20) \]
with summation over \( \sigma_1, \sigma_2, \sigma, m, \alpha, \beta \).

The stripping amplitude may therefore be expressed as (using equations (5.17), (5.19));

\[
T_{\text{DWBA}}(d,p) = \sum_{j} \hat{s}(a|\beta \beta) \beta_{s_j}^{\lambda \nu \sigma \lambda_1}(k_2, k_1),
\]

(5.21)

where \( \beta \), known as the 'reduced' amplitude, is

\[
\beta_{s_j}^{\lambda \nu \sigma \lambda_1}(k_2, k_1) = \int \sum_{s_1} (-)^{s_1} s_2^{\sigma_2} (s_1 s_2 - \sigma_2 | s) |
\]

\[
\times (\lambda \lambda \sigma | j m) \int d \varphi_2 x_2 \chi_{s_2}(k_2, r_2)^* G_{s_2 j, \lambda}(r_2, r_1) \chi_{s_2}(k_1, r_1),
\]

(5.22)

and the sum is over \( \sigma_2, \sigma_1, \lambda, \sigma \).

If we now use the expansions (equations (5.3) and (5.13)) for the deuteron and proton functions \( \chi_{s_2 \sigma_2}(-) \) and \( \chi_{s_1 \sigma_1}^{(+)} \), together with the angular momentum identities (Br 62),

\[
\sum_{\lambda_1 \lambda_2 \lambda' \lambda'} (\ell_1 \lambda_1 \lambda_2 | \ell' \lambda')(\ell_1 \lambda_1 \lambda_2 | \ell' \lambda') = 1,
\]

(5.23)

\[
\sum_{\lambda_1 \lambda' 2 \lambda_2 \lambda' \lambda} (-)^{s_2 \sigma_2} (s_1 s_2 - \sigma_2 | s) (\lambda \lambda \sigma | j m) (\ell_1 \lambda_1 \lambda_2 \lambda | \ell' \lambda')
\]

\[
\times (-)^{s_2 \sigma_2} (\ell_2 \lambda_2 \lambda_2 | \ell_2 \lambda_2 | \ell' \lambda')
\]

\[
= \hat{s}(s_2 \sigma_2) \hat{s}_1 \hat{s}_2 (J_1 M_1 J_2 M_2 | jm) \begin{pmatrix} \ell_1 & s_1 & J_1 \\ \ell_2 & s_2 & J_2 \\ \ell & s & j \end{pmatrix} \delta_{\ell \ell'} \delta_{\lambda \lambda'} \hat{c}^{-2},
\]

(5.24)
where the 9-J symbol (Br 62) has been introduced; we obtain the relation

\[
\sum_{s_1} \delta_{s_1 k_1} \delta_{s_2 k_2} \delta_{j_1 j_2} \delta_{\ell_1 \ell_2} \delta_{\sigma_1 \sigma_2} \delta_{s_1 s_2} = \left( \frac{\ell_1 - \ell_2}{s} \right) \left( \frac{\ell_1 + \ell_2 + \sigma_1 + \sigma_2}{s} \right)
\]

where we have introduced the function

\[
H_{\ell_1 \ell_2}^{\ell_1 \ell_2 \sigma_1 \sigma_2} (k_1, k_2) = \sum_{\lambda_1 \lambda_2 M_1 M_2} (J_{1 M_1} J_{2 M_2} | j_1 j_2 \sigma_1 \sigma_2)
\]

and

\[
I_{\ell_1 \ell_2}^{s_1 s_2} = 4 \pi \sum_{\lambda_1 \lambda_2} \delta_{\ell_1 \ell_2} Y_{\lambda_1} Y_{\lambda_2}^{*} (k_1, r_1) Y_{\lambda_2} (k_2, r_2)
\]

In equation (5.27), we have defined

\[
J_{1 \lambda_1}^{\ell_1} (r_1) = \frac{J_{1 \lambda_1}^{\ell_1} (k_1, r_1) Y_{\lambda_1} (r_1)}{r_1},
\]

\[
J_{2 \lambda_2}^{\ell_2} (r_2) = \frac{J_{2 \lambda_2}^{\ell_2} (k_2, r_2) Y_{\lambda_2} (r_2)}{r_2},
\]

the natural generalization of equation (2.11).
At this point we make the notation more specific; if we now assume that the reaction form factor is that of section 2.3, and given by equations (2.23) and (2.24), then substitution in equation (5.20), for \( G_{ksj,\lambda} \), gives (Jo 71)

\[
G_{ksj,\lambda}(r_2,r_1) = \delta_{jj} \sum_{L=0,2} A^L_{ksj_n} f^L_{ksj_n,\lambda}(r_2,r_1),
\]

(5.30)

where \( L, j_n \) are the deuteron internal orbital angular momentum and the total angular momentum of the captured neutron, respectively. Here, explicitly

\[
A^L_{ksj_n} = a^A_{n,j_n} s^2 \delta^L_{j_n L} \ U(Ls_s s_2 s_1, s_j),
\]

(5.31)

\[
\varphi^L_{ksj_n,\lambda}(r_2,r_1) = i \int R_{jn} (r_n) V_L(r) T^*_n L\lambda (r_n,r) ,
\]

(5.32)

in the notation of chapter 2, and where the bipolar harmonic, \( T^*_n L\lambda \), is given by equation (4.19).

In actual fact, Johnson and Santos (Jo 71) have shown that the \( L \) sum in equation (5.30) is redundant. There is a one-to-one correspondence between allowed \( L \) and \( s \) values (for a \((d,p)\) reaction), namely for \( L = 0(2), s = 1/2(3/2) \), and therefore

\[
G_{\lambda 1/2 j_n,\lambda}(r_2,r_1) = \delta_{\lambda,\lambda} A^0_{n,j_n} 1/2 J_{jn} (r_n) y^{(n)}(r_n) (4\pi)^{-1/2} y_0 (r) ,
\]

(5.34)
\[
G_{\ell 3/2j_n, \lambda} (\xi_2, \xi_1) = \Lambda^2_{\ell 3/2j_n} \sum^n_{l} R_{j_n l} (r_n) V_2 (r) \\
\times T^*_{2, \ell \lambda} (\xi_n, \xi) , \\
(5.35)
\]

with
\[
\Lambda^0_{\ell_n 1/2j_n} = (3/2)^{1/2} a_{\ell_n j_n} \\
(5.36)
\]
\[
\Lambda^2_{\ell 3/2j_n} = (3/10)^{1/2} a_{\ell_n j_n} \hat{\ell}_{n}^{\ell - 1} (-) j_n \frac{1}{2} - \ell_n \\
(5.37)
\]
\[
\times \{ 1 + (2 j_n^{2})^{-1} (-)^n j_n^{2} - \frac{1}{2} \left( \ell (\ell + 1) - \ell_n (\ell_n + 1) - 6 \right) \}^{1/2} .
\]

Returning now to equation (5.27), together with equations (5.30) and (5.32) for \( G_{s s j, \lambda} \), then (with \( s = \frac{1}{2}, \frac{3}{2}; L = 2s - 1 \))
\[
I_1 \frac{s}{2} \ J_1 \frac{s}{2} \ L_1 \ L_1' \ = \ 4\pi A^L_{s s j_n} \hat{r}^{L - 2} \sum \delta_{11'} \delta_{22'} \delta_{\lambda \lambda} \delta_{j j} \\
\times \frac{\ell - \ell_n - L}{F_{12}^{L \Lambda}} , \\
(5.38)
\]

the sum over \( \lambda, \lambda'_1, \lambda'_2, \lambda_n, L \), and where \( F^{L \Lambda}_{12} \) is the generalization of equation (2.26), namely
\[
F^{L \Lambda}_{12} = \int dr_2 dr_n \chi_{\ell_2} \ ^{J_2 \lambda_2} (r_2) R_{j_n l} (r_n) Y_{\lambda_n} (r_n) V_{L} (r) Y_{L \Lambda} (r_2) \chi_{L_1 L_1'} (r_1) . \\
(5.39)
\]

In the above we used the fact that the Jacobian transforming from set \( (r_1, r_2) \), to \( (r, r_n) \), is \( J^{-1} \). The label \( \ell'_1 \) upon \( \chi_{\ell_1 \ell'_1} \), is the orbital angular momentum eigenvalue of this function, as in chapter 2.
The analysis now follows that of chapter 2, precisely, upon making the necessary substitutions:

\[
\begin{align*}
\lambda^\prime_2 \chi_{\ell_2}^\prime (r_2) &= \chi_{\ell_2} (k_2, r_2) Y_{\ell_2}^\prime (r_2) / r_2 \\
+ J_2^\prime \lambda^\prime_2 \chi_{\ell_2}^\prime (r_2) &= J_2 (k_2, r_2) Y_{\ell_2}^\prime (r_2) / r_2 , \\
\lambda^\prime_1 \chi_{\ell_1}^\prime (r_1) &= \chi_{\ell_1} (k_1, r_1) Y_{\ell_1}^\prime (r_1) / r_1 \\
+ J_1^\prime \lambda^\prime_1 \chi_{\ell_1}^\prime (r_1) &= J_1 (k_1, r_1) Y_{\ell_1}^\prime (r_1) / r_1 .
\end{align*}
\]

The only further approximation required is in the use of the radial Schrödinger equation (2.52), to obtain the finite range correction factor of equation (2.54). Equation (2.52) is now no longer strictly true as the \( \chi_{\ell_1}^\prime (r_1) \) now satisfy the coupled equations of section 5.1. For the purpose of evaluating the correction term, \( \Lambda_{12} \), we ignore the small tensor force coupling term and assume that

\[
K_2^2 \chi_{\ell_1}^\prime (r_n) \approx (2\mu_1 / \hbar^2) [E_1 - U_{j n}^\ell_1 (r_n)] \chi_{\ell_1}^\prime (r_n) ;
\]

therefore, the generalization of equation (2.54), is

\[
\Lambda_{12} (r_n) = 1 - (\alpha^2 / \beta^2 \varepsilon_d) \{ U_{j_1 \ell_1} (r_n) - U_{j_2 \ell_2} (\gamma r) - U_{j_n \ell_n} (r_n) - \varepsilon_d \} ,
\]

where \( U_{j_1 \ell_1} \), \( U_{j_2 \ell_2} \) are given by equations (5.8) and (5.15), respectively, and \( U_{j_n \ell_n} \) by equation (F.5) of Appendix F.
Collecting the results obtained, then for the S-state

\[ F_{12}^{00} = D_0 \gamma^{-1}(4\pi)^{-\frac{1}{2}} \hat{\kappa}_{12}^{00} \hat{\chi}_{12}^{00} \left( \kappa_1 \lambda_1 \lambda_2 \lambda_2 \right) \left( \kappa_1 \lambda_2 \lambda_2 \lambda_1 \right) \left( \kappa_1 \lambda_0 \lambda_0 \epsilon_0 \right) \]

\[ \times F(J_{12}^{00}, J_{12}^{00}, J_{12}^{00}) , \quad (5.42) \]

(cf. equation (2.56)), where

\[ F(J_{12}^{00}, J_{12}^{00}, J_{12}^{00}) = \int_{-\infty}^{0} dr A_{12}(r) \sum_{n} \left( \kappa_1 \lambda_2 \lambda_2 \lambda_1 \right) \left( \kappa_1 \lambda_0 \lambda_0 \epsilon_0 \right) \delta_{jj_n} \]

and therefore, substituting in equation (5.38)

\[ I_{12}^{00} = (4\pi)^{\frac{1}{2}} D_0 \gamma^{-1} \hat{\kappa}_{12}^{00} A_1^{00} \hat{\chi}_{12}^{00} \left( \kappa_1 \lambda_2 \lambda_2 \lambda_1 \right) \left( \kappa_1 \lambda_0 \lambda_0 \epsilon_0 \right) \delta_{jj_n} \]

\[ \times F(J_{12}^{00}, J_{12}^{00}, J_{12}^{00}) . \quad (5.44) \]

For the deuteron D-state terms, then, from equation (2.74)

\[ F_{12}^{20} = -\gamma^{-1}(5/4\pi)^{\frac{1}{2}} D_0 D_2 \left\{ a_1 \overline{F}(1) + a_2 \overline{F}(2) + \overline{G}_2 \right\} , \quad (5.45) \]

where

\[ \overline{F}_{2\Lambda} (i) = \sum_{L_1 M_1} \Omega_A^{(i)} (L_1, M_1) F_{12}^{(i)} , \quad (i = 1, 2) \quad (5.46) \]

and

\[ \overline{G}_2 = \sum_{L_1 M_1} \Omega_A^{(3)} (L_3, M_3) G_{12}^{L_3} , \quad (5.47) \]

where the \( \Omega_A^{(i)} \) are given by equations (2.68-70), and
\[ F_{12L_1}^{(1)} = \int_0^\infty dr x_{l_2}^2(k_2, yr) R_{j_{11} \ell_{11}}(r) \{ (\hat{\alpha}_{11} \ell_{11} - \hat{\alpha}_{\ell_{11}}) (x_{l_1 \ell_{11}}^1(k_1, r)/r) \} , \]

(5.48)

\[ F_{12L_2}^{(2)} = \gamma \int_0^\infty dr x_{l_2}^2(k_2, yr) R_{j_{11} \ell_{11}}(r) \{ (\hat{\alpha}_{11} \ell_{11} - \hat{\alpha}_{\ell_{11}}) (x_{l_2 \ell_{11}}^2(k_2, yr)/yr) \} R_{j_{11} \ell_{11}}(r) x_{l_1 \ell_{11}}^1(k_1, r) \]

(5.49)

\[ G_{12L_3} = \int_0^\infty dr x_{l_2}^2(k_2, yr) x_{l_1 \ell_{11}}^1(k_1, r) (\hat{\alpha}_{11} \ell_{11} - \hat{\alpha}_{\ell_{11}}) + \beta^2(\Lambda_{12}(r) - 1) R_{j_{11} \ell_{11}}(r) . \]

(5.50)

Substituting then for \( F_{12}^{2\Lambda} \) from (5.45) into equation (5.33), and using the addition theorem for the Racah coefficient as given by equation (4.9) one obtains, very simply, the final result that

\[ I_{j_{12}}^{\ell_3/2j_{11}} \frac{1}{L_{11}} = 20n \frac{1}{2} \gamma^{-1} A^2 \delta_{j_{11}}^{l_{11}} D_{0j_{11}}^{l_{11}} \]

\[ \times \{ \sum_{L_{11}} (-1)^{\ell_{11} + L_{11}} \left| \Gamma(\ell_{11} \ell_{11}; L_{11})a_{11} F_{12L_1}^{(1)} \right| \}

+ \sum_{L_{12}} \left| \Gamma(\ell_{11} \ell_{12}; L_{12})a_{21} F_{12L_2}^{(2)} \right| \times (20l_{11}0|l_{11}0) G_{12L_3} \} ; \]

(5.51)

where

\[ \Gamma(\ell_{11} \ell_{12}; L_{11}) = \hat{\ell}_{11} \hat{\ell}_{12} H(2\ell_{11} \ell_{12}; L_{11}) (L_{11}0|L_{11}0) \]

\[ \times (20l_{11}0|l_{11}0) , \]

(5.52)
and \( I(\ell_2', \ell_1'; l_2', l_1) \) is obtained upon substituting in the above, \( l_2' \rightarrow l_2 \), \( l_1' \rightarrow l_1 \) and \( L_1' \rightarrow L_1 \).

The effect of including tensor distortion in the deuteron channel, upon the coupling introduced between the deuteron and proton channels is best seen by looking at the S-state contributions \( (s = 1/2) \) to the reduced amplitude \( \beta_{s_1 n} \). From equation (5.25), and suppressing all but the \( \ell_1 \) sums, \( \beta \) has the general structure

\[
\beta_{s_1 n} \propto \sum_{\ell_2} \sum_{\ell_1} \left\{ \begin{array}{ll} j_n & \ell_n \\ J_1 & \ell_1 \\ J_2 & \ell_2 \end{array} \right\} \left\{ \begin{array}{ll} j_1 & \ell_1 \\ s_1 & \ell_s \end{array} \right\} \ell_1/2 \beta_{s_1 n}, \quad (5.53)
\]

and tensor distortion has introduced an additional \( \ell_1' \) sum into the expression for \( \beta_{s_1 n} \), compared with that of Johnson and Santos (Jo 71); the present result reverts to theirs upon replacing, \( I_{J_2 J_1 J_2 J_1}^{s} = \delta_{J_2 J_1 J_2 J_1}^{s} \). So, for a fixed \( \ell_1 (> 1) \) in the outer sum (equation (5.53)), the parity \( ^{\dagger} \) coefficient in equation (5.44), for \( \ell_1' \equiv \ell_1+2 \), restricts \( \ell_1' \) to; \( \ell_1' = \ell_1, \ell_1 + 2(J_1 = \ell_1 + 1), \ell_1' = \ell_1(J_1 = \ell_1) \) and \( \ell_1' = \ell_1, \ell_1 - 2(J_1 = \ell_1 - 1) \). The result is that for a fixed \( \ell_1 \), and known \( j_n, \ell_n \), certain proton partial waves \( (J_2, \ell_2) \) now produce non zero S-state radial overlap integrals \( F(J_1 \ell_1, J_2 \ell_2, j_n, \ell_n) \) (equation (5.43)), by coupling to the deuteron incident partial wave, \( \ell_1 \), through the outgoing wave components, \( \ell_1' = \ell_1 \pm 2 \), introduced by the tensor distortion. In figure 5.2 we indicate the quantum numbers of these additional S-state radial integrals, zero in the limit of no tensor \( T_R/T_p \) forces, for the sets of values; \( j_n = 1/2, \ell_n = 0, j_n = 1/2, \ell_n = 1 \) and \( j_n = 3/2, \ell_n = 1 \).

\[ ^{\dagger} \quad (a0b0|c0) = 0, \text{ unless } a + b + c \text{ is even (Br 62)}. \]
Figure 5.2 Angular momentum quantum numbers $(J_2, l_2)$ of the additional non zero S-state radial overlap integrals, $F(J_1, l_1', J_2, l_2; n_1, n_2)$, arising from coupling to outgoing orbital angular momenta $l_1' = l_1 \pm 2$, introduced by a $T_R$ or $T_P$ interaction.
The effect of a tensor $T_R$ and/or $T_P$ force is therefore to allow each incident deuteron ($\ell_1$) state to overlap with a larger number of proton partial waves ($J_2\ell_2$) (a similar effect is found in the D-state amplitude $\beta_3/2j$) and thus altering the relative contributions from different terms in the $J_1\ell_1 J_2\ell_2$ sums, to the reduced amplitudes $\beta_{s_{j_n}}$.

5.3 Computational Detail

As seen in section 5.1, the inclusion of a $T_R$ and/or $T_P$ interaction complicates somewhat the distorted waves formalism. Computationally, in each deuteron partial wave ($J_1$ state), we need to solve coupled radial equations. For this reason, a number of early deuteron elastic scattering calculations were performed (Sc 68, Sc 69, Bu 75) which included a $T_R$ interaction but ignored explicit $\ell$-coupling; treating only the diagonal part of the interaction. This procedure has however been shown (Ba 75) not to be justified. In addition, the increased number of radial functions (2 per $J_1$ value), and non zero overlap integrals involving the off diagonal ($\ell_1' \neq \ell_1$) radial functions, requires increased computer time and storage requirements.

A computer program has been written to include, via the LEA formalism, the effects of $T_R$ and $T_L$ interactions upon the $(d,p)$, $(d,n)$ and inverse pick-up reactions; the $T_P$ interaction has been excluded from the present work. The coupled equations (5.7) are solved exactly and the radial functions, $X_{\ell_1} J_1$, obtained, used as in the formalism of section 5.2. Inclusion of a $T_R$ interaction required considerable modification to the computer program DWCODE (Ha 70), operating on the ICL 1905 Machines at the University of Surrey Computing Unit. The availability of the elastic scattering code DD, of Robson (Ro 67a), although most useful for checking purposes (see following section), was not used directly in the calculation of the radial
functions $\chi_{\ell_1^{\prime}}^{J_1^{\prime}}$; this would have resulted in an undesirable mixed language program.

The coupled equations (5.7) were solved using the Modified Numerov or Fox-Goodwin method, used in DD, and outlined in appendix D. For each total angular momentum quantum number $J_1 (> 0)$ in the deuteron channel we need to solve for five radial functions $\chi_{\ell_1^{\prime}}^{J_1^{\prime}}$, namely (see also Figure 5.1):

i) An uncoupled radial equation with $\ell_1 = \ell_1^{\prime} = J_1$, the solution of which has been described elsewhere (Ha 70), and is now routine; and

ii) two pairs of coupled equations, the coupled pairs being:

$$\chi_{J_1^{\prime}-1,J_1-1}^{J_1^{\prime}} \leftrightarrow \chi_{J_1^{\prime}-1,J_1+1}^{J_1^{\prime}}; \quad \chi_{J_1^{\prime}+1,J_1-1}^{J_1^{\prime}} \leftrightarrow \chi_{J_1^{\prime}+1,J_1+1}^{J_1^{\prime}}.$$

We have seen that for $J_1 = 0$, there is a single uncoupled function $\chi_0^{0,1}$.

A priori, all we know of the required physical solutions $\chi_{\ell_1^{\prime}}^{J_1^{\prime}}$ is that:

a) they are regular at the origin ($r = 0$), and

b) asymptotically, they are solutions of the homogeneous (Coulomb) equation corresponding to orbital angular momentum $\ell_1^{\prime}$. In particular, we require the asymptotic normalization of equation (5.11). In practice, a matching radius $r_m$ is chosen, outside which the tensor potential, coupling the equations, is deemed negligible when compared to the residual central

* DWCODE is an ALGOL program while DD is written in FORTRAN.
coulomb distortion. At radius $r_m$, we assume the coupled equations decouple, and reduce to radial coulomb equations with solutions, for $r > r_m$, given by equation (5.11). In all calculations to be presented, $r_m$ was chosen, due to coulomb function routine limitations (Gu 67), at 30 fermis. The accuracy of this prescription when long ranged, coulomb tailed, coupling is present is discussed in chapter 6.

The regularity of the $\chi_{j1}^{0,0}$ at $r = 0$, is insufficient information to obtain these physical functions directly in a single numerical solution of the coupled equations (5.7). Numerical integration from the origin with a set of regular, but arbitrary, starting values for the coupled functions, will generate a coupled solution of equation (5.7) which, in general, does not have the required physical asymptotic form. Clearly however this coupled solution, represented ($J_1$ fixed)

$$ X_{j1}^{(i)} \leftrightarrow X_{j1}^{(i)} \ , \quad (5.54) $$

where the $i$ serves to label the particular set of starting values used, must be some linear combination

$$ X_{j1}^{(i)} = \alpha_i X_{j1-1}^{J_1,J_1\pm1} + \beta_i X_{j1+1}^{J_1,J_1\pm1} \ , \quad (5.55) $$

of the regular physical functions.

† Note that $\ell_1$ does not enter the differential equations (5.7), only $\ell_1'$, which labels the functions in equation (5.54). $\ell_1$ appears upon the physical solutions $X_{\ell_1}^{\ell_1'}$, only through the imposed boundary condition (equation (5.11)) and has no significance upon mathematical solutions of equation (5.7).
In fact, we must generate two linearly independent coupled solutions, from two linearly distinct starting conditions, \( i = A, B \) say. A knowledge of these \( \chi^{(i)}_{\ell_1} \), and their first derivatives, at the matching radius \( r_m \), is then sufficient to obtain the required \( \chi^{\ell_1}_{\ell_1'} \), using the procedure of appendix E. Having obtained the functions \( J_1^{\ell_1} \chi^{\ell_1}_{\ell_1'} \), evaluation of the stripping matrix then proceeds as in the formalism of section 5.2.

### 5.4 Program Checks

The written computer program, based upon the DWBA reaction treatment, falls neatly into two rather distinct parts. Firstly, one calculates the required deuteron elastic scattering functions in the presence of a \( T_R \) and/or \( T_L \) interaction. Secondly, the functions are combined according to the stripping formalism of section 5.2.

The elastic amplitudes, \( C_{\ell_1' \ell_1} \) of equation (5.12), output by the present program were compared, for several calculations, with their counterpart from the much used Robson program DD (Ro 67a). The latter outputs amplitudes, \( \Delta_{J_1 \ell_1' \ell_1} \) (Ro 74), which are related to the present \( C_{\ell_1' \ell_1} \) through the formula

\[
\Delta_{J_1 \ell_1' \ell_1} = e^{-i(\ell + \sigma_{\ell_1'}) - 2\sigma_0} J_1 \left(2iC_{\ell_1' \ell_1} + \delta_{\ell_1' \ell_1}\right),
\]

or upon inversion

\[
C_{\ell_1' \ell_1} = (e^{i(\ell + \sigma_{\ell_1'}) + 2\sigma_0} \Delta_{J_1 \ell_1' \ell_1} - \delta_{\ell_1' \ell_1})/2i.
\]

The two programs are found to be in complete agreement with regard to these quantities.
The second element of the program, evaluation of the stripping matrix cannot be so thoroughly checked; particularly when one includes the deuteron D-state. There are no existing reliable D-state + $T_R$ calculations available, since the exact finite range calculations of Delic and Robson (De 70) have been found to be in error (De 74). However, for an S-state deuteron, correct zero range $T_R$ and $T_L$ calculations have been performed (De 69). The present program is in excellent quantitative agreement with the results of these available calculations.

This is at present as far as checking can proceed against established existing computer codes. All indications are however that the program is working correctly and as intended.

In the following chapter (S + D) state + $T_R$ calculations are presented, using the modified DWCODE, for the reactions $^{90}$Zr($^d$,p)$^{91}$Sr at 5.5 MeV, and $^{208}$Pb($^d$,p)$^{209}$Pb at 9 MeV and 7 MeV incident deuteron energies.
TENSOR FORCE CORRECTIONS

In the preceding chapter the formal apparatus has been derived for the inclusion of a tensor deuteron-target interaction into the DWBA and its LEA treatment. The DWBA dictates that the relative deuteron-target wavefunction should reproduce the observed elastic scattering (GI 75), thus treating deuteron break up effects only through the imaginary term of the optical model potential.

In chapter 3 however, the inclusion of explicit coulomb break up of the deuteron was found to generate, in addition to the usual $R^{-3}$ coulomb $T_R$ interaction (equation (3.45)), present in the absence of break up (Ha 72, Ra 64), an $R^{-4}$ $T_R$ term (equation (3.49)) resulting from break up of the deuteron into continuum $3p_{s_f}$ ($s_f = 0, 1, 2$) configurations of the n-p pair. However, as of yet no mention has been made as to nuclear break up contributions and their possible spin dependence. In this respect we shall, in this chapter, consider the predictions of the Adiabatic $^\dagger$ theory of Johnson and Soper, or J/S (Jo 70), and Johnson and Tandy (Jo 74), which treats explicitly contributions in which the deuteron is broken up into a relative continuum $3S_1$ state. Important for the following discussion however is that the J/S model wavefunction, unlike more sophisticated 3-body

$^\dagger$ It must be pointed out that the present, J/S, adiabatic approximation is entirely distinct from that of chapters 3 and 4: namely, that of Testoni and Gomes (Te 66).
stripping models for $\psi^{(+)}$ (Fa 76, Ra 75, Au 78), the three body wave-
function of equation (1.36), retains the simple DWBA form; i.e. a
simple product of deuteron internal and centre of mass functions for
which the formalism required in the evaluation of $T(d,p)$ has been
discussed.

In Appendix G we review briefly the theoretical and phenomenological
evidence for the presence of rank-2 interactions in the deuteron-target
system while in Appendix H we remark upon the effects of coupling, of
the deuteron elastic channel, to $^1S_0$ n-p break up configurations.

Calculations which include model $T_R$ interactions are presented for
$(d,p)$ transitions to $^{91}$Zr at incident deuteron energy $E_d = 5.5$ MeV and to
$^{209}$Pb at $E_d = 9$ MeV and 7 MeV.

6.1 Tensor Forces in Sub-Coulomb Stripping

Examples of $T_R$ forces have been met in Chapter 3, in particular, as
was stated there, the $R^{-3}$ interaction (equation (3.45)), a direct result
of the deuteron D-state, is a particular example of the Watanabe folding
procedure. According to the Watanabe prescription (Wa 58), if break up
and internal distortion of the deuteron is neglected, the deuteron target
relative motion should be described by the potential matrix $U_{11}^{\text{Wat}}(R)$ (in
deuteron spin space)

$$
[U_{11}^{\text{Wat}}(R)]_{\sigma_1' \sigma_1} = <\phi_{\sigma_1'}|U_{p}(R + 1/2\tau) + U_{n}(R - 1/2\tau)|\phi_{\sigma_1}> ,
$$

(6.1)

where $U_n$, $U_p$ ($\pm U_2$) are the neutron- and proton-nucleus optical potentials,
assumed local; this is not however a necessary constraint (e.g. Jo 72,
Ra 78). Conventionally (Jo 72), the U are evaluated at the incident deuteron energy, i.e., it is assumed that the n-p pair share equally the deuteron bombarding energy. Satchler has shown (Sa 60) that whenever the deuteron D-state component is included in the deuteron wavefunction \( \phi_{s_1} \), in equation (6.1), then \( U_{s_1}^{\text{Wat}} \) contains a \( T_\text{R} \) tensor term.

Johnson and Soper however (Jo 70, So 69) advocate that when including, explicitly, break up of the deuteron into the \( 3S_1 \) n-p continuum in a stripping calculation, then the n-p centre of mass function should be generated not by \( U_{s_1}^{\text{Wat}} \) but by an Adiabatic potential \( U_{s_1}^{\text{Ad}}(R) \), where

\[
[U_{s_1}^{\text{Ad}}(R)]_{s_1}^{\sigma_1} = \frac{\langle \phi_{s_1}^{(+)\sigma_1} | V_{np} | \phi_{s_1}^{(+)\sigma_1} \rangle}{D_{np}}
\]

and

\[
D_{np} = \langle \phi_{s_1}^{(+)\sigma_1} | V_{np} | \phi_{s_1}^{(+)\sigma_1} \rangle.
\]

The Johnson/Soper model, formulated under the assumption (Jo 70, Jo 74) that break up is to low energy break up configurations, generates an Adiabatic distorted wave, \( \chi_{s_1}^{(+)} \) say, which does not reproduce elastic scattering. Rather, in the spirit of section 3.1, \( \chi \) accurately reproduces deuteron break up effects in the exact three body wavefunction \( \psi_{s_1}^{(+)} \), but only within the range of \( V_{np} \) needed for the evaluation of the transition amplitude, \( T(d,p) \).

\[\text{-----------------------------------------}\]

\[\dagger\] This is in fact the reformulated Adiabatic prescription of Johnson and Tandy (Jo 74). This is however, numerically, negligibly different from that of J/S (Jo 70).

\[\text{-----------------------------------------}\]
Inspection of equation (6.2) for $U_{1}^{Ad}$, together with equation (2.20) for the product $V_{np}^{s_1} \phi_{s_1}$, shows that like the Watanabe formula (equation (6.1)) the folding agents in $U_{1}^{Ad}$ are linear combinations of S- and D-state terms and will generate an Adiabatic $T_{R}$ potential (Sa 60, Wa 77).

$T_{2q}$ data for deuteron elastic scattering from $^{90}$Zr at 5.5 MeV and $^{208}$Pb at 9 MeV, energies below their respective barrier heights of 5.7 and 10.4 MeV (Kn 77), are well reproduced in the so called 'hybrid model' of Knutson and Haeberli (Kn 75b, Appendix G); that is, if a Watanabe $T_{R}$ force is used. At these energies the incident deuteron has little probability of penetrating the coulomb barrier and so corrections to the Watanabe term at small deuteron-target separations, as proposed by Ioannides and Johnson (Io 78a, Appendix G) and resulting from the action of the Pauli Principle, are not expected to be felt. Further, Goddard (Go 76, Go 77) has shown that, for low incident deuteron energies, the effects of a $T_{p}$ force upon the entire elastic scattering $S$ matrix are very nearly indistinguishable from those of a suitably chosen $T_{R}$ term.

Therefore, although Goddard's analysis uses a $T_{p}$ force obtained from existing calculations at higher energies (Io 78) and there are large uncertainties in the radial shape and magnitude of the $T_{p}$ interaction at sub-coulomb energies (Io 78b), nevertheless, the elastic phase shifts may be reproduced by a $T_{R}$ force alone. So, while a $T_{p}$ force may be in part responsible for the observed experimental elastic scattering $T_{2q}$, in the absence of more specific information and in view of the increased complexity (Go 77a) of the calculation when including a $T_{p}$ force we shall, in the following, neglect explicit $T_{p}$ terms.
The question as to which $T_R$ prescription, the Watanabe or Adiabatic terms, should be used as input to a sub-coulomb $(d,p)$ reaction is difficult to answer. The Watanabe interaction predicts the correct elastic phase shifts and therefore, as stripping takes place predominantly beyond the nuclear surface, then provided break up channels are only weakly populated the phase shift is expected to determine the deuteron target wavefunction very accurately in this region. However, inspection of the measured ratios of elastic to Rutherford cross sections (Kn 75b) for $^{90}\text{Zr}$ ($E_d = 5.5$ MeV) and $^{208}\text{Pb}$ ($E_d = 9$ MeV), of the order 0.75 at backward centre of mass angles, shows that the $n$-$p$ pair do in fact interact appreciably with the target via the strong nucleon-nucleus interaction. A priori, there is no reason therefore to expect that nuclear break up effects are negligible.

On the other hand, the Johnson/Soper Adiabatic model is essentially a higher energy approximation and has been used very successfully in the region $E_d \gtrsim 20$ MeV (e.g. Sa 71, Ha 71) in reproducing $(d,p)$ and $(p,d)$ cross section data only poorly fitted in the conventional DWBA. The fact that the present Adiabatic approximation assumes the opposite of that of chapter 3, namely that the characteristic time associated with the $n$-$p$ relative motion is long compared to the time taken for their centre of mass to traverse a distance over which the break up potential changes appreciably, need not be of great concern. As noted earlier, unlike the coulomb break up potential, the nuclear break up term is rapidly varying particularly in the important nuclear surface region.

Recent support for this view comes from the work of Cadmus (Ca 76) in a study of the reaction $^{116}\text{Sn}(d,p)^{117}\text{Sn}$ at $E_d = 8.22$ MeV; an energy at
which the J/S Adiabatic model would not be expected to work well.
However, Cadmus finds that good agreement with cross section and vector
polarization data is obtained when using the Adiabatic potential and,
more importantly, that these data could not be satisfactorily reproduced
when phenomenological potentials which fit the observed elastic scattering
were used. While further study is obviously needed, these results hint
that the J/S model may have a wider energy range of applicability than
previously thought.

The spin dependence, and in particular the $T_R$ component of the
Adiabatic potential, has not been studied previously. Therefore, in the
light of the results of Cadmus and the simplicity of applying the J/S
method, it proves a convenient theoretical starting point for the study of
the spin dependence of nuclear break up.

6.2 Radial Formfactors

We shall assume that the optical potentials $U_n$, $U_p$, appearing in the
folding formulae (equations (6.1-2)), are isoscalar. For the present
discussion we take this local potential as the mean of the neutron and
proton terms obtained from the work of Becchetti and Greenlees (Be 69);
evaluated at $\frac{1}{2}$ the incident deuteron energy, $E_d$.

\[ U_p \] is no longer assumed to contain the proton coulomb interaction.
Effects of this term in the Watanabe folding were accounted for in
chapter 3, while the J/S folding scheme does not, by definition,
include the coulomb term.
Thus, in the energy region of interest here, $E_d < 10$ MeV, with $i = n, p$ and $r_{np} = R + 1/2 \xi$, then

$$U_i(r_{ij}) = U_c(r_{ij}) + U_{so}(r_{ij}) \frac{r_{ij}}{s_i}$$  \hspace{1cm} (6.3)

where, in the notation of Appendix F (Be 69),

$$U_c(r) = -V f(x_v) - iS g(x_s)$$  \hspace{1cm} (6.4)

Consider the generalized folding formula

$$\left[ U(R) \right]_{\sigma_1 \sigma_1} = \langle \sigma_1 | \sum_{i=n,p} U_i(r_{ij}) | \sigma_1 \rangle$$  \hspace{1cm} (6.5)

where, by analogy with the deuteron wavefunction, equation (1.1), we define

$$\langle \xi, \pi, n | \sigma_1 \rangle = \int_{L=0,2} \int_{Ls_{1}} \int_{\sigma_1} \phi_{\sigma_1} (\xi, \pi, n)$$  \hspace{1cm} (6.6)

Both central and spin-orbit terms of the nucleon potentials, $U_i$, contribute to the $T_R$ term of $U(R)$. In the special case $f_L = u_L$, the Watanabe model, formulae have been presented by Raynal (Ra 64) and Keaton (Ke 71) for the calculation of this potential; the latter however contains a number of errors with regard to the nucleon spin-orbit contributions. Wales (Wa 77) has shown that in the general case, the central terms of the $U_i$ generate, in $U(R)$, a $T_R$ interaction with the radial form

$$U_R(R) = \frac{3}{\sqrt{2}} \int_0^\infty dr r^2 \int_{-1}^1 U_c(\rho) \left[ \left( u_0 - \frac{u_2}{\sqrt{2}} \right) f_2 + u_z f_0 \right] P_2(\mu) d\mu$$  \hspace{1cm} (6.7)

where $\rho = |R + 1/2 \xi|$, $\mu = \frac{\xi}{2} \cdot \xi$, and $P_2$ is the Legendre Polynomial (Br 62) of order 2. Nucleon spin-orbit contributions to $U_R$ have also been written down by Johnson (Jo 78a), in the general case (equation (6.5)).
Clearly, inspection of equation (2.20) shows that in the Adiabatic model, equation (6.2),

\[ |f_{\sigma_1}^{Ad}\rangle = \eta_{np}^{-1} v_{np} |\phi_{s_1}^{1}\rangle \]  

(6.8)

and therefore

\[ D_{np} f_{L}^{Ad}(r) = \left[ \frac{n^2}{2u} \left( \frac{d^2}{dr^2} + 2 \frac{d}{r} \frac{d}{dr} - \frac{L(L+1)}{r^2} \right) - \varepsilon_d \right] u_L(r) \] 

(6.9)

Alternatively, if \( V_{np} \) has the form (Re 68)

\[ V_{np} = V_C(r) + V_T(r) S_{np} + V_{LS}(r) L \cdot S_{1} \]

where \( S_{np} \) is the usual tensor operator of equation (1.8), then the \( f_{L}^{Ad} \) satisfy (De 70)

\[ f_{o}^{Ad}(r) = \{ V_C u_o + \sqrt{3} V_T u_2 \}/D_{np} \] 

\[ f_{2}^{Ad}(r) = \{ (V_C - 2V_T - 3V_{LS})u_2 + \sqrt{3} V_T u_o \}/D_{np} \]  

(6.10)

A computer program was written to evaluate the 2-d folding, equation (6.7), exactly and has been checked against the Watanabe calculations of Keaton et al. (Ke 73). With \( U_C \) given by equation (6.4), the real and imaginary parts of the \( T_R \) potential \( U_R \) obtained from both Adiabatic \( (|f_{\sigma_1}^{Ad}\rangle = |f_{\sigma_1}^{Ad}\rangle, \) equation (6.8)) and Watanabe \( (|f_{\sigma_1}^{Ad}\rangle = |\phi_{s_1}^{1}\rangle) \) models, for deuterons incident with \( E_d = 9 \) MeV upon \( ^{208}\text{Pb} \), are presented in Figure 6.1.

The deuteron wavefunction used in obtaining these curves was the Reid soft core function (Re 68), and the \( f_{L}^{Ad} \) were calculated using equation (6.10). In both Watanabe (\( U_R^{Wat} \)) and Adiabatic (\( U_R^{Ad} \)) foldings, corrections due to the spin-orbit components of the nucleon-nucleus interaction \( V_{so} \), were small.
Figure 6.1 Real and imaginary parts of the $T_R$ potential $U_R(R)$ predicted by the Adiabatic (dashed) and Watanabe (solid) folding models for $^{208}$Pb at $E_d = 9$ MeV. Calculations were performed using the Reid Soft-Core wavefunction.
Calculations, not shown in Figure 6.1, were also performed, using equation (6.9), for the phenomenological deuteron wavefunction of Hulthen and Sugawara (Hu 57). While the Watanabe predictions agree with those of the Reid wavefunction to better than 5%, the Adiabatic calculation in the latter case produces a potential of the same geometry, but 1.4 times the magnitude of the corresponding Reid predictions. Quite clearly therefore, the Adiabatic folding, \( U^\text{Ad} \), contains considerably more deuteron wavefunction model dependence than the Watanabe prescription. While this should be borne in mind in what follows, we shall not consider this point in detail at present.

Rather more important than the difference in magnitudes of the Watanabe and Adiabatic potentials, is their difference in geometry. The real and imaginary parts of potentials \( U^\text{Wat} \), \( U^\text{Ad} \), have the familiar (Ke 73) second and third derivative Wood-Saxon forms, namely

\[
\text{Re.} U^\text{Wat}_R(R) \approx -V^2_R R \left\{ \frac{d}{dR} \frac{d}{dR} f(x^R_R) \right\}, \quad (6.11)
\]

\[
\text{Im.} U^\text{Wat}_R(R) \approx -V^2_I R \left\{ \frac{d}{dR} \frac{d}{dR} g(x^I_R) \right\}, \quad (6.12)
\]

respectively, for suitably chosen \( V_i, r_i, a_i \) \((i = R, I)\) and where we have used the notation of Appendix F. Keaton and Armstrong (Ke 73) have demonstrated that the above formulae provide a good description of the exact folding calculation, equation (6.7). This view is confirmed by the present work and the functional forms, equations (6.10-11), prove far more convenient at the computation stage. The strength, radius and diffuseness parameters of these terms were determined using the least squares fitting

\[\dagger\] Wavefunction corresponding to \( P_D = 4\%, \rho(-\varepsilon_d, -\varepsilon_d) = 1.704, x_c = 0.\]
routine of Smith (Sm 69) and the results, for $^{90}$Zr ($E_d = 5.5$ MeV) and $^{208}$Pb ($E_d = 9$ MeV), are presented in Table 6.1.

Rather clearly, apart from the relative magnitudes, which as we have seen are expected to be rather model dependent, $U^{Ad}_R$ and $U^{Wat}_R$ differ exclusively in their diffuseness parameters as a result of the short ranged agent, $V_{np} |\phi_{s_1}\rangle$, in the folding integral for the former. The latter therefore falls off significantly more slowly for large $R$. This point is made very clearly in Figure 6.2 in which the real parts of $U^i_R$ ($i = Ad, Wat$) for $^{208}$Pb, together with the $R^{-3}$ and $R^{-4}$ coulomb $T_R$ terms of equations (3.45) and (3.49) are plotted in the large $R$ region. Following the usual practice (Bu 78, Ka 76) we shall, in the absence of a precise knowledge of the coulomb $T_R$ terms for $R < R_c$, assume (see equation (3.45)) that

$$ \bar{V}^{(2)}_{e.o}(R) \approx U^3_R(R) T_R, \quad (6.13) $$

where

$$ U^3_R(R) = \frac{3}{2} Q_d Z e^2 / R^3 \quad R > R_c \quad \Rightarrow \quad U^3_R(R_c) \quad R < R_c, \quad (5.14) $$

and similarly for $U^4_R(R)$, where from equation (3.49)

$$ U^4_R(R) = - (3/2) \alpha_{SD}(Ze)^2 / R^4. \quad (6.15) $$

The error in this procedure for sub-coulomb incident deuterons is expected to be completely negligible. Using the value of $\alpha_{SD}$ from equation (3.48) we see from Figure 6.2 that for $R \leq R_c$ (\% 7.7 fm for $^{208}$Pb) the coulomb terms $U^3_R(R) = 1.108$ MeV and $U^4_R(R) = -1.115$ MeV cancel nearly identically. The total coulomb $T_R$ potential, $U^3_R + U^4_R$, is therefore small and is also shown in Figure 6.2.
Table 6.1. Adiabatic and Watanabe $T_R$ potential parameters

<table>
<thead>
<tr>
<th></th>
<th>$V_R$</th>
<th>$a_R$</th>
<th>$r_R$</th>
<th>$V_I$</th>
<th>$a_I$</th>
<th>$r_I$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>208_{\text{Pb}}</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Watanabe:</td>
<td>7.8</td>
<td>1.03</td>
<td>1.16</td>
<td>1.04</td>
<td>.920</td>
<td>1.28</td>
</tr>
<tr>
<td>Adiabatic:</td>
<td>3.3</td>
<td>.814</td>
<td>1.16</td>
<td>.606</td>
<td>.701</td>
<td>1.29</td>
</tr>
<tr>
<td><strong>90_{\text{Zr}}</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Watanabe:</td>
<td>8.43</td>
<td>.950</td>
<td>1.17</td>
<td>1.26</td>
<td>.930</td>
<td>1.285</td>
</tr>
<tr>
<td>Adiabatic:</td>
<td>3.25</td>
<td>.811</td>
<td>1.17</td>
<td>.590</td>
<td>.692</td>
<td>1.290</td>
</tr>
</tbody>
</table>

* The potentials are defined as per equations (6.11-12) with $x^2, f(x_i), g(x_i)$ given by Appendix F.
Figure 6.2 Large R behaviour of the Adiabatic (Reid and Hulthén predictions), Watanabe and Coulomb $T_R$ interactions for $^{208}$Pb. Also shown is the sum of the $R^{-3}$ and $R^{-4}$ coulomb $T_R$ terms.
In the following sections we present calculations which incorporate the various form factors of this section in the manner described in chapter 5.

6.3 Calculations for the Reaction $^{90}\text{Zr}(d,p)^{91}\text{Zr}$ at $E_d = 5.5$ MeV

In this section we present calculations for the transitions to the following spin/parity, $j^\pi$, states of the residual nucleus $^{91}\text{Zr}$, with excitation energy $E_x$, namely $E_x = 1.20$ MeV ($j^\pi = 1/2^+$), $E_x = 0.0$ MeV ($j^\pi = 5/2^+$), $E_x = 2.04$ MeV ($j^\pi = 3/2^+$), previously the subject of a study by Knutson (Kn 77). It is not suggested that this reaction be used to measure $D_2$ as the ground state Q value is 4.97 MeV and, with the target coulomb barrier of 5.7 MeV (Kn 77), the protons in the final state are above the barrier height; thus the advantages, detailed in chapter 2, of experiments in the sub-coulomb regime, with $Q \approx 0$, do not apply.

We present the following calculations for two reasons, namely:

i) Since the original analysis of Knutson (Kn 77), an error has been found (To 78) in the program DWCODE used in these earlier calculations, and

ii) to date, no quantitatively correct $(d,p)$ calculations which include both deuteron D-state and a $T_R$ force have been published (De 70, De 74).

The curves a to d in Figures 6.3-6.5 correspond to the theoretical DWBA calculations as follows:

a) (full line) is the corrected $(S + D)$ state calculation in the absence of $T_R$ forces and supersedes curve b,

b) (long dashed line) reproduces the earlier, incorrect, calculation of Knutson, also calculated without a $T_R$ term,
Figure 6.3 Angular distributions for the tensor analysing powers to the $3/2^+(E_x = 2.03 \text{ MeV})$ state of $^{91}\text{Zr}$. The theoretical DWBA curves a-d are described in the text. Curves correspond to the value $D_2 = 0.484 \text{ fm}^2$. 
Figure 6.4 Angular distributions for the tensor analysing powers to the $5/2^+$ ground state of $^{91}$Zr. The curves have the same meaning as in Figure 6.3, and are calculated using $D_2 = 0.484$ fm$^2$. 

$^{90}$Zr(d, p) $^{91}$Zr $5/2^+$ $E = 5.5$ MeV
Figure 6.5 Angular distributions for the tensor analysing powers to the $1/2^+$ ($E_x = 1.20$ MeV) state of $^{91}$Zr. The curves have the same meaning as in Figure 6.3, and are calculated using $D_2 = 0.484$ fm$^2$. 
c) (short dashed line) includes the Adiabatic $T_R$ potential of Table 6.1 and equations (6.11-12), and

d) (dot-dashed line) includes the Watanabe $T_R$ potential also of Table 6.1. The experimental points are due to Knutson (Kn 77a). Only the calculated $T_{2q}$ have been presented as we find, in agreement with the work of Delic and Robson (De 69, De 70), that the $T_R$ force effects negligible changes in the observables $d\sigma/d\Omega$ and $iT_{11}$. The computer program error mentioned concerns only the treatment of the deuteron $D$-state component, which in turn provides only minor contributions to $d\sigma/d\Omega$, $iT_{11}$, and the calculations of Knutson (Kn 77) with respect to these quantities remain quantitatively good.

For ease of comparison, the calculations in Figures 6.3-5 were performed using the optical model parameters used by Knutson and tabulated in Table 6.2. While agreement with experiment is in general only qualitative, no attempt has been made to improve the fit by arbitrary variation of parameters. It is apparent nevertheless, that the effect of the Adiabatic $T_R$ force, as a result of its rather short range, is considerably less than that of the Watanabe term, while the large angle behaviour of the $3/2^+$ transition would appear to favour the presence of a $T_R$ potential.

### 6.4 Calculations for the Reaction $^{208}\text{Pb}(d,p)^{209}\text{Pb}$ at $E_d = 9$ MeV

In this section we reconsider the sub-coulomb $(d,p)$ transitions to the $j^\pi = 1/2^+(E_x = 2.03 \text{ MeV})$ and $j^\pi = 5/2^+(E_x = 1.57 \text{ MeV})$ states of $^{209}\text{Pb}$, proposed for the measurement of $D_2$.

In Figure 6.6 the effects of the Adiabatic (dot-dashed curve) and Watanabe (long dashed curve) $T_R$ potentials of Table 6.1 upon the $T_{2q}$ for the $j^\pi = 1/2^+$ transition are compared with the conventional $S + D$ state DWBA
Table 6.2. Optical Model Potential Parameters

<table>
<thead>
<tr>
<th>Projectile</th>
<th>V</th>
<th>r\textsubscript{v}</th>
<th>a\textsubscript{v}</th>
<th>S</th>
<th>r\textsubscript{s}</th>
<th>a\textsubscript{s}</th>
<th>V\textsubscript{so}</th>
<th>r\textsubscript{so}</th>
<th>a\textsubscript{so}</th>
<th>r\textsubscript{c}</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>90\textsuperscript{Zr}</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>d</td>
<td>94.8</td>
<td>1.20</td>
<td>.58</td>
<td>8.0</td>
<td>1.30</td>
<td>1.00</td>
<td>5.63</td>
<td>0.98</td>
<td>1.00</td>
<td>1.3</td>
<td>(Kn 77)</td>
</tr>
<tr>
<td>p</td>
<td>57.5</td>
<td>1.17</td>
<td>.75</td>
<td>11.0</td>
<td>1.32</td>
<td>0.59</td>
<td>6.2</td>
<td>1.01</td>
<td>0.75</td>
<td>1.25</td>
<td>(Be 69)</td>
</tr>
<tr>
<td>208\textsuperscript{Pb}</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>d</td>
<td>119.2</td>
<td>1.05</td>
<td>.86</td>
<td>6.02</td>
<td>1.50</td>
<td>0.93</td>
<td>7.0</td>
<td>0.75</td>
<td>0.50</td>
<td>1.3</td>
<td>(Kn 77)</td>
</tr>
<tr>
<td>p</td>
<td>61.9</td>
<td>1.17</td>
<td>.75</td>
<td>12.2</td>
<td>1.32</td>
<td>.66</td>
<td>6.2</td>
<td>1.01</td>
<td>0.75</td>
<td>1.25</td>
<td>(Be 69)</td>
</tr>
<tr>
<td>Bound State</td>
<td>n</td>
<td>Adjusted</td>
<td>1.2</td>
<td>.7</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>6.0</td>
<td>1.2</td>
<td>0.7</td>
<td>(Kn 77)</td>
</tr>
</tbody>
</table>

* Parameters not mentioned here are set to zero.

† The notation is that of Appendix F.
Figure 6.6 Angular distributions for the tensor analysing powers to the $1/2^+(E_X = 2.03\text{ MeV})$ state of $^{209}\text{Pb}$. The theoretical curves compare the results of a conventional $S + D$ state DWBA calculation (Solid curve) with calculations which include an Adiabatic (dot-dashed) on Watanabe (dashed) $T_R$ force. Curves correspond to a value $D_2 = 484\text{ fm}^2$. 
calculation (solid curve) in the absence of $T_R$ terms. As for the $^{90}\text{Zr}$ calculations of the preceding section, we do not reproduce the $d\sigma/d\Omega$ and $iT_{11}$ angular distributions which are presented elsewhere (Kn 77). Results for the $5/2^+$ transition, are presented in Figure 6.7.

Immediately apparent is the sizable difference in the predictions of the Watanabe and Adiabatic $T_R$ interactions, more so than the corresponding differences found in the $^{90}\text{Zr}$ case. Calculations for $^{208}\text{Pb}$ in which a cut off radius, $r_0$, within which the $T_R$ force is neglected, is incremented, reveal that corrections introduced by the $T_R$ interactions are determined, to better than 10%, by the potential tail region beyond a radius of 9 fermis. Inspection of Figure 6.2 shows therefore that the model dependence of the Adiabatic $T_R$ potential will not effect appreciably the calculations of Figures 6.6-7; as in agreement with the observation of Delic and Robson (De 69), we find that, for a fixed potential geometry, $T_R$ contributions to the calculated $T_{2q}$ scale very nearly linearly with the strength of the interaction.

Although somewhat inconsistent, in order to delineate $T_R$ force effects and for comparison with previous work, all calculations presented in this section which include nuclear distortion use the phenomenological deuteron and proton optical potentials of Table 6.2. Strictly, we should use with the Adiabatic $T_R$ term a central and spin-orbit distorting potential generated via equation (6.2). However, in view of the insensitivity (Kn 77) of the sub-coulomb $T_{2q}$, and in particular the $j^\pi = 1/2^+$ data, to the presence of nuclear distortion, such considerations are deemed of secondary importance.

In Figure 6.8 the effects due to the inclusion, into an $(S + D)$ state DWBA calculation, of the coulomb $T_R$ potentials $U_R^3$ (long dashed curves) and $U_R^4$ (short dashed curves) of equations (6.14) and (6.15), respectively, for
Figure 6.7 Angular distributions for the tensor analyzing powers to the 5/2⁺(Eₓ = 1.57 MeV) state of ²⁰⁹Pb. Curves are as in Figure 6.6.
Figure 6.8 Angular distributions for the tensor analysing powers to the $1/2^+$ ($E_x = 2.03$ MeV) state of $^{209}$Pb. The theoretical curves compare the results of an (S + D) state DWBA calculation (solid curve) including only coulomb distortion with calculations which include the potentials $V_R^3$ (dashed curve) and $V_R^4$ (dot-dashed curve). Curves correspond to a value $D_0 = 0.484$ fm$^2$. 
the $j^m = 1/2^+$ level are shown. These calculations correspond to purely
coulomb distortion in both deuteron and proton channels, the results for
which, in the absence of $T_R$ terms, are given by the solid curves.
Calculations for the $5/2^+$ level are qualitatively similar and are not
presented graphically.

A comment upon the quantitative accuracy of the adopted treatment of
the long ranged tensor interactions is in order. As stated in section 5.3,
the terms $U_R^i$ ($i = 3, 4$), of infinite range, and which form the coupling
term in equation (5.7), the coupled equations for the deuteron radial
functions $\chi_{Jl,k}$, are assumed to vanish beyond the matching radius, $r_m = 30$ fm.
The treatment of long ranged (coulombic tailed) coupling is a familiar
problem in coulomb excitation (Al 56, Al 75), numerical calculations for
which typically (Ro 77) use a matching radius $r_m$ of the order of a few
hundred fermis; such a radius would require considerable modification of the
existing computer program DWCODE. The problem in elastic and inelastic
calculations is however complicated by the fact that, unlike a stripping
calculation in which the finite range of the neutron bound state function
limits the number of 'active' deuteron partial waves, a considerable number
of partial waves contribute. The elastic/inelastic partial wave amplitudes
fall off only slowly in $\ell$ and therefore an accurate treatment of each
amplitude is required for an accurate determination of the slowly convergent
(Ki 66, Cl 70) partial wave sums present. Knutson and Haeberli (Kn 75) over­
come this problem in sub-coulomb deuteron elastic scattering using the DWBA
theory of elastic scattering and thus treat the $R^{-3} T_R$ force to first order,
however, other analyses (Ka 76, Bu 78) which include the $R^{-3}$ term in the
Robson program DD (which has no facility for the special treatment of long
range coupling) make no mention of the number of partial waves or matching
radius used.
As an accuracy check of the present calculation, the off-diagonal elastic amplitudes \( C_{j=\ell+1}^{l+2} \) output by DWCODE (obtained by matching at 30 fm) are checked against their corresponding DWBA predictions. Collecting a number of results from the work of Johnson (Jo 62, Jo 77) we obtain in DWBA, for the potential \( U^2_{R}(R) \) (with \( R_c = 0 \))

\[
C_{j=\ell+1}^{l+2} = \frac{2\mu_1 k_1}{\pi^2} \frac{[(\ell+1)(\ell+2)]^{1/2}}{2\ell+3} \left\{ \frac{3}{2} Q_d Z e^2 \right\} M_{\ell+2}^{3} , \tag{6.16}
\]

where (A1 56)

\[
M_{\ell+2}^{3} = \int_{0}^{\infty} d\rho F_{\ell+2}(\rho) \rho^{-3} F_{\ell}(\rho)
\]

\[
\left. \left\{ 6 |\ell + 1 + i\eta_1 ||\ell + 2 + i\eta_1 | \right\}^{-1} . \tag{6.17}
\]

We find that for \( \ell \leq 8 \), agreement between the numerical values and the analytic expression, equation (6.16), is better than 6% while for \( 8 < \ell \leq 15 \) the values agree, typically, to order 2%; that is, the amplitudes agree to one part in \( 10^4 \). The somewhat poorer agreement for small \( \ell \) is attributed to the inaccuracy of the DWBA expression rather than the matching procedure; as confirmed by the improved agreement in the higher partial waves where the DWBA is more accurate.

The present work also shows that the modifications to the \( T_{2q} \) arising from a \( T_R \) force, are almost exclusively the result of the additional S-state radial overlap integrals, due to coupling to the off-diagonal deuteron partial waves, which were discussed in section 5.2. These integrals are linear in the off-diagonal amplitudes \( C_{j=\ell}^{l+2} \); off-diagonal D-state integrals, \( 1^{3/2}j \) (equation (5.51)), are typically two orders of magnitude smaller. However, in turn, the off-diagonal S-state integrals are smaller by two orders of magnitude than the diagonal elements. So, the dominant coulomb
modifications to the conventional \((S + D)\) state calculated \(T_{2q}\), are bilinear in the S-state amplitudes \(i\sqrt{1/2j}\) (equation (5.44)) and linear in the off diagonal amplitudes \(c^{j}_{\ell\ell'}\). If therefore, we assume a comparable error, of order 2%, for the diagonal elastic amplitudes, \(c^{j}_{\ell\ell'}\), then we estimate that coulomb \(T_R\) contributions are accounted for correctly quantitatively, with an accuracy of \(\approx 4\%\), in the present 30 fm matching procedure.

Hence, although a more accurate numerical study is perfectly possible, using for instance the asymptotic series expansion method of Rösel et al. (Ro 77), the increased numerical complexity, and corresponding increase in computation time, were thought unnecessary for the present preliminary investigation. Evident from Figure 6.8 is some degree of cancellation from the \(U^3_R\) and \(U^4_R\) terms, as expected from Figure 6.2; their combined effect relative to the purely central coulomb distorted calculation (solid curves) is very accurately represented by the sum of the deviations from these curves, but is not shown in Figure 6.8.

In Figures 6.9 and 6.10 we combine the results of this section. The solid and short dashed curves are the results of a conventional DWBA \((S + D)\) state calculation and the corresponding results when including a tensor potential \(\{U^{\text{Wat}}_R + U^3_R + U^4_R\}\), respectively. These calculations use the value \(D_2 = 0.484 \text{ fm}^2\) of the Reid soft core wavefunction. Also shown (long dashed curves), are the results, in the absence of \(T_R\) forces, of an \((S + D)\) state calculation using the 'best fit' \(D_2\) value, \(D_2 = 0.432 \text{ fm}^2\), of Knutson and Haeberli (Kn 75). This \(D_2\) value best reproduces the data in the conventional calculation. Calculations, not shown in Figures 6.9 and 6.10, including the \(T_R\) potential \(\{U^{\text{Ad}}_R + U^3_R + U^4_R\}\), are very nearly coincident with the dot-dashed curves of Figures 6.6 and 6.7.
Figure 6.9 Angular distributions for the tensor analysing powers to
the $1/2^+ (E_x = 2.03 \text{ MeV})$ state of $^{209}\text{Pb}$. The theoretical
curves are described in the key, and in the text.
Figure 6.10 Angular distributions for the tensor analysing powers to the $5/2^+$ ($E_x = 1.57$ MeV) state of $^{209}\text{Pb}$. Curves are as in Figure 6.9.
So, when including a Watanabe $T_R$ force, calculations would indicate that the experimental data are consistent with an OPEP tailed interaction $D_2$ value; this is particularly true for the $1/2^+$ transition. However, when including a $T_R$ force generated according to the Johnson/Soper Adiabatic prescription, the data indicate the need for a somewhat smaller $D_2$ value.

6.5 Calculations for the reaction $^{208}\text{Pb}(d,p)^{209}\text{Pb}$ at $E_d = 7$ MeV

The calculations of the previous section have been repeated at an incident deuteron energy of 7 MeV, for which there is no presently available experimental $T_{2q}$ data. Inspection of the preceding calculations shows that the most meaningful measure of $T_R$ force effects is the induced percentage change in $T_{20}$, at backward centre of mass angles; we take $\Delta T_{20}(\theta_b)$, with $\theta_b = 175^\circ$. Calculations for the $j^\pi = 1/2^+$ transition show that at 7 MeV, effects of the nuclear $T_R$ forces $U_R^{\text{Nat}}$, $U_R^{\text{Ad}}$ are completely negligible with $\Delta T_{20}(\theta_b) \ll 1\%$. For $j^\pi = 5/2^+$, we find also that $\Delta T_{20}(\theta_b) \approx -1\%$ for $U_R^{\text{Nat}}$, and, as a result of the slightly higher $Q$ value we see that the reaction still feels the strong $T_R$ force.

When including the interactions $U_R^3$, $U_R^h$, (with $j^\pi = 1/2^+$) then $\Delta T_{20}(\theta_b) = -6.7\%$ and $+3.1\%$, respectively; their combined effect giving $\Delta T_{20}(\theta_b) = -3.6\%$, highlighting the perturbative nature of the coulomb $T_R$ contributions. Also, for both $5/2^+$ and $1/2^+$ transitions, calculated $T_{2q}$ are completely insensitive to the presence of central nuclear distortion (cf. 9 MeV calculations of Kn 77).

We see therefore, that the rather large nuclear $T_R$ force contributions present at 9 MeV are small when the energy is reduced to 7 MeV. Coulomb $T_R$ contributions continue however to be of importance in a precision calculation.
CHAPTER 7

SUMMARY AND CONCLUSIONS

In this thesis we have been concerned with the quantitative accuracy of conventional DWBA calculations for sub-coulomb (d,p) reaction tensor analysing powers, $T_{2q}$. Two effects, neglected in the theory (Ha 70, Jo 71) most used for the routine analysis of experimental data, namely

a) dipole electric break up of the deuteron, and

b) the presence of a deuteron-target $T_R$ force, have been investigated numerically.

While much theoretical effort has been directed at coulomb break up effects upon the (d,p) cross section, no previous study has been made of tensor analysing power corrections. This generalization required an accurate treatment of the state dependence of the n-p interaction and was shown to be most easily accomplished using a separable interaction model. Dipole break up corrections are shown to be small but necessary for the analysis of a precision sub-coulomb stripping experiment. An important result is that Knutson's (Kn 73a, Kn 74) approximate distorted waves stripping model is able to predict quantitatively the magnitude and energy dependence of the dipole break up corrections. The examination of higher multipole effects, not considered here, may therefore be investigated without the need for lengthy numerical distorted wave and finite range treatments necessary here.

In chapter 5 the LEA formalism of Johnson and Santos (Jo 71) was generalized to allow the inclusion of deuteron-target rank-2 interactions. $T_R$ forces arising from the Watanabe and Johnson/Soper folding model theories
were found to differ primarily in their diffuseness; the latter being considerably smaller as a result of the folding over the short ranged interaction, $V_{np}$. An observed strong model dependence, of the magnitude of the Johnson/Soper term, upon $V_{np}$, was found to be of secondary importance in calculation. The diffuseness difference however, generated markedly different theoretical $T_{2q}$ predictions.

So, while the use of both Watanabe (which ignores deuteron break up effects) and Johnson/Soper (essentially a high energy approximation) models is arguable, the results obtained reflect the present theoretical uncertainty associated with the strong interaction part of the $T_R$ force. Coulomb $T_R$ forces, which are obtained from the Watanabe and dipole break up calculations are found, as a result of their long range, also to be of importance at all sub-coulomb energies; however, unlike the strong $T_R$ force, coulomb contributions are calculable without ambiguity.

Of the reactions considered, the $j^\pi = 1/2^+$ transition in $^{208}\text{Pb}(d,p)\ ^{209}\text{Pb}$ at $E_d = 9$ MeV is theoretically the best suited for the measurement of $D_2$ (for which experimental data is available (Kn 77a)). The $j^\pi = 5/2^+$ level calculations at $E_d = 9$ MeV, although less effected by $T_R$ forces (Figure 6.10) continue to show (Kn 77) some sensitivity to the details of central nuclear distortion, no study of which has been undertaken here. A study of the reaction $^{208}\text{Pb}(d,p)^{209}\text{Pb}$ at $E_d = 7$ MeV would however be free of both strong central and strong $T_R$ force uncertainties. Therefore we conclude that:

i) the presently available experimental data (Kn 77a), for $^{208}\text{Pb}$ at $E_d = 9$ MeV, cannot be used to determine $D_2$ with any degree of accuracy until a better theoretical understanding of the strong tensor $T_R$ force, required as input to a stripping calculation, is obtained;
ii) this theoretical uncertainty may be bypassed by considering an experiment at \( E_d = 7 \) MeV;

iii) the analysis of precision experimental data, even at \( E_d = 7 \) MeV, will require corrections due to coulomb break up and long range coulombic \( T_R \) forces if \( D_2 \) is to be determined to an accuracy of 3-4\%; and

iv) no correction need be made for \( ^1S_0 \) deuteron break up configurations at \( E_d = 9 \) MeV or 7 MeV.

No attempt has been made here to assign a 'measured' \( D_2 \) value to the experimental data. In view of the remaining theoretical uncertainty such a value would be of little significance, however, there is no evidence, at present, for the rather small value, \( D_2 = 0.432 \) fm\(^2\), obtained from the analysis of Knutson and Haeberli (Kn 75).

Investigation of an experiment at \( E_d = 7 \) MeV, when performed, will ultimately require further consideration of

a) higher multipole coulomb break up contributions,

b) an upper bound upon nuclear break up effects, and

c) a rigorous quantitative assessment of the approximate LEA procedure.

Certain progress has been made here with regard to suggestion a.

Tremendous progress has been made, over the last 30 years, in our understanding of the nucleon-nucleon interaction and the deuteron. Nevertheless, the parameter \( D_2 \) (or \( \eta \)) remains only poorly determined. Sub-coulomb stripping is shown to provide a simple, and potentially very accurate,
method for the measurement of $D_2$. This new constraint would not only
differentiate between acceptable models of $V_{np}$ but could provide valuable
new information upon the short range behaviour of the deuteron wavefunction.
In addition, the input of a known $D_2$ value, into the sub-coulomb calcula-
tions presented here, would help answer the questions posed with regard to
the form of the strong deuteron-target $T_R$ force.
APPENDIX A

Mongan Separable Interactions

Mongan (Mo 68, Mo 69) has proposed four separable potential models of the nucleon-nucleon interaction (Cases I-IV of Mo 69). These potentials are rank two separable interactions of the form

\[ V_{jj}(k,k') = g_{jj}(k)g_{jj}^*(k') - h_{jj}(k)h_{jj}^*(k'), \quad (A.1) \]

parametrized in momentum space for each \( j, \ell, s \) \((2s+1\ell_j)\) state of the two nucleon system.

The case I parametrization (Mo 69) is particularly suited to treating \( \ell = 1 \) states; explicitly for case I the formfactors \( h_{\ell j}^j, g_{\ell j}^j \) are

\[ g_{\ell j}^j(k) = C_R \frac{k^{\ell+1}}{(k^2 + a_R^2)^2}, \quad (A.2) \]
\[ h_{\ell j}^j(k) = C_A \frac{k^{\ell+1}}{(k^2 + a_A^2)^2}, \quad (A.3) \]

where \( C_A, C_R, a_A, a_R \) carry an implied \( j, s \) dependence and subscripts \( A, R \) refer respectively to the attractive and repulsive parts of the interaction. These potential formfactors have, for odd \( \ell \), a particularly simple pole structure in the complex \( k \) plane and in addition provide the best overall fit to the Livermore \( 3P_j \) (Ma 68) phase shift data of the 4 cases presented by Mongan. Explicitly, we have used the parametrization for the \( 3P_j \) states (Mo 69);

<table>
<thead>
<tr>
<th>( j )</th>
<th>( a_R (\text{fm}^{-1}) )</th>
<th>( C_R (\text{MeV fm})^{1/2} )</th>
<th>( a_A (\text{fm}^{-1}) )</th>
<th>( C_A (\text{MeV fm})^{1/2} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2.258</td>
<td>118.2</td>
<td>1.326</td>
<td>16.48</td>
</tr>
<tr>
<td>1</td>
<td>0.697</td>
<td>3.498</td>
<td>2.322</td>
<td>18.89</td>
</tr>
<tr>
<td>2</td>
<td>0.0</td>
<td>0.0</td>
<td>1.509</td>
<td>5.349</td>
</tr>
</tbody>
</table>

where the last entry \( (3P_2) \) has been parametrized assuming no coupling to the \( 3F_2 \) state.
APPENDIX B

Evaluation of the Continuum P-wave Eigenfunctions Corresponding to Mongans Separable Interaction

The radial function \( \psi_{\ell,j}^{(+)} \), of equation (4.7), normalized according to equation (4.7b), satisfies the radial integral equation (Go 64), with \( s \) conserved,

\[
\psi_{\ell,j}^{(+)}(r) = (2/\pi)^{\frac{1}{2}} \int dp \ p^2 \ T_{\ell,j}^{is}(p,p';K^2) \left[ \frac{j_{\ell}(pr) + e^{i\delta_{\ell}(p')}}{\epsilon(p') + i\eta - \epsilon(p)} \right]
\]

(B.1)

where \( \epsilon(p) = \frac{\hbar^2 p^2}{2\mu} = \frac{e_p}{p} \) of section 4.2, and \( T_{\ell,s}^{is}(p,p';K^2) \) is the partial wave \( T \) matrix (Go 64). In the dipole approximation of section 4.3, then \( \ell = \ell' = 1 \) and \( s = 1 \) and therefore, with these values implied

\[
T_{\ell,s}^{is}(p,p';K^2) = T_{\ell}^{j}(p,p';K^2) \delta_{\ell,\ell'},
\]

(B.2)

and using equations (B.1-2), then the asymptotic form of the \( \psi_{\ell,j}^{(+)} \), which are complex, is

\[
\psi_{\ell,j}^{(+)}(r) \bigg|_{asy} = (2/\pi)^{\frac{1}{2}} \left[ j_{\ell}(p'r) + e^{i\delta_{\ell}(p')} \sin\delta_{\ell}(p')h_{\ell}^{(+)}(p'r) \right]
\]

(B.3)

where \( h_{\ell}^{(+)} \) are the outgoing wave Hankel functions of Messiah (Me 61) and \( \delta_{\ell}(p') \) is the usual nuclear phase shift and is related to the on-shell \( T \) matrix element \( T_{\ell}^{j}(p^2) \), by (Mo 69)
\[ T^j_{\ell}(p, p'; p^2) = T^j_{\ell}(p^2) = -(\pi \rho_{\varepsilon})^{-1} e^{i\delta^{\ell}_j(p)} \sin \delta^{\ell}_j(p) \], \quad (B.4) \\

where
\[
\rho_{\varepsilon} = p^2 \left( \frac{d\varepsilon(p)}{dp} \right)^{-1} = \rho \mu^- \hbar^- \ . \quad (B.5)
\]

For convenience we shall calculate the real functions \( \psi^j_\ell \) defined by
\[
\psi^j_\ell(r) = c^j_\ell(p) \psi^j_\ell(k) \ . \quad (B.6)
\]

In the present uncoupled P-states then \( T^j_{\ell} \) of equation \((B.2)\) is given by
\[
(Mo 68) \quad T^j_{\ell}(p, p'; K^2) = V^j_{\ell}(p, p') + \int_0^\infty dq \ q^2 \frac{V^j_{\ell}(p, q) T^j_{\ell}(q, p'; K^2)}{\varepsilon(K) - \varepsilon(q) + i\eta} \quad (B.7)
\]

and using the separable partial wave potential form of Mongan
\[
V^j_{\ell}(p, p') = g^j_{\ell}(p) g^j_{\ell}(p') - h^j_{\ell}(p) h^j_{\ell}(p') \quad (B.8)
\]

where \( g^j_{\ell} \) and \( h^j_{\ell} \) carry an implicit \( s \) label, then
\[
T^j_{\ell}(p, p', K^2) = N^j_{\ell}(p, p'; K^2)/D^j_{\ell}(K^2) \quad (B.9)
\]

where \((Mo 68)\)
\[
N^j_{\ell}(p, p'; K^2) = g^j_{\ell}(p) g^j_{\ell}(p') A^j_{\ell}(K) - h^j_{\ell}(p) h^j_{\ell}(p') B^j_{\ell}(K) \\
- (g^j_{\ell}(p) h^j_{\ell}(p') + h^j_{\ell}(p) g^j_{\ell}(p')) C^j_{\ell}(K) \quad , \quad (B.10)
\]

and we have defined
\[
A^j_{\ell}(K) = 1 + \int_k^j(h, \hbar; K) \quad , \quad (B.11)
\]
\[ B^j_\ell(K) = 1 - \mathcal{G}^j_\ell(g,g;K), \quad (B.12) \]

\[ C^j_\ell(K) = \mathcal{G}^j_\ell(g,h;K), \quad (B.13) \]

with
\[ \mathcal{G}^j_\ell(m,n;K) = \int_0^\infty dq \frac{m^j_\ell(q) n^j_\ell(q)}{\varepsilon(K) - \varepsilon(q) + i\eta}, \quad (B.14) \]

and
\[ d^j_\ell(K^2) = A^j_\ell(K) B^j_\ell(K) + (C^j_\ell(K))^2. \quad (B.15) \]

So, substituting for \( h^j_\ell, g^j_\ell \) the Mongan expressions of equations (A.2-3) of appendix A (with \( \ell = 1 \) understood), the integrands of the \( \mathcal{G} \) \((m,n;K)\) appearing in equation (B.11-13) are even in \( q \). Therefore, extending the range of integration to the entire real \( q \) axis and closing the contour in the upper \( \frac{1}{2} \) plane, then
\[ \mathcal{G}^j_1(h,h;K) = \Pi^R(A) - i\Pi^i(h,h), \quad (B.16) \]

\[ \mathcal{G}^j_1(g,g;K) = \Pi^R(R) - i\Pi^i(g,g), \quad (B.17) \]

\[ \mathcal{G}^j_1(g,h;K) = \frac{\mu \pi C_A C_R}{\eta^2} \left\{ \frac{a_3^3}{\nu^2 (a_2^2 - a_1^2)} \left( \frac{a_1^3}{(k^2 + a_1^2)} - \frac{a_3^3}{(k^2 + a_3^2)} \right) \right\} \]
\[ - i\Pi^i(g,h), \quad (B.18) \]

and with \( P, Q \) representing \( A \) or \( R \), then
\[ \Pi^R(Q) = \frac{\mu \pi C^2_Q a_Q}{\nu^2 (k^2 + a_Q^2)} \left( \frac{a_Q^2}{(k^2 + a_Q^2)} - \frac{3}{2} \right), \quad (B.19) \]

\[ \Pi^i(m,n) = \frac{\mu \pi C_A C_R}{\nu^2} m^j_\ell(K) n^j_\ell(K), \quad (B.20) \]

here \( C_A, C_R, a_A, a_R \) are those appropriate to \( j, \ell = 1, s = 1. \)
From equations (B.1-2), we need only the \( \frac{1}{2} \) off shell T matrix,
\[ T^j_\xi(p,p';K^2 = p'^2) \]
to calculate the \( \psi^{j(+)}_{p',\xi}(r) \), and
\[ T^j_\xi(p,p',p'^2) = g^j(p)F^j_g(p') - h^j(p)F^j_h(p') \] (B.21)
where \( \xi \) is no longer shown and where
\[ F^j_g(K) = [g^j(K)A^j(K) - h^j(K)C^j(K)]/D^j(K^2) \] , (B.22)
\[ F^j_h(K) = [h^j(K)B^j(K) + g^j(K)C^j(K)]/D^j(K^2) \] . (B.23)

Using equations (B.2) and (B.21) in (B.1) therefore gives
\[ \psi^{j(+)}_{p,\xi=1}(r) = \frac{2}{\pi^{\frac{1}{2}}} \{ j^1_1(pr) + F^j_g(p)\alpha^j_g(p) - F^j_h(p)\alpha^j_h(p) \} \] , (B.24)
where we have defined (with \( m = g \) or \( h \)),
\[ \alpha^j_m(p) = \int_0^\infty dq q^2 \frac{j^1_1(qr)m^j(q)}{\epsilon(p) - \epsilon(q) + i\eta} \] . (B.25)

The integrand in equation (B.25) is even in \( q \), so, using the identity (Me 61)
\[ j^1_1(z) = (2i)^{-1}[h^j_\xi^{(+)}(z) - h^j_\xi^{(-)}(z)] \] , (B.26)
and extending the \( q \) integration to the real \( q \) axis, then we may close the contours for the \( h^{(+)}(h^{(-)}) \) terms in the upper (lower) \( \frac{1}{2} \) plane, and we find
\[ \alpha^j_g(p) = -\mu\pi\eta^{-2}\{pg^j(p)h^{(+)}_1(pr) + GR(p)h^{(+)}_1(ia_xr)\} \] , (B.27)
\[ a_h^+(p) = -\mu \pi h^{-2} [ph_r^+(p)h_1^+(pr) + ^G_A(p)h_1^+(ia_A r)] \] (B.28)

where
\[ G^0(p) = C_q a_q^2/(p^2 + a_q^2) \] (B.29)

Therefore, from equations (B.27-28) and (B.24)

\[ \psi^{\dagger}(+)_{\text{asy}} = \psi^{\dagger}(+)_{\text{asy}} + \psi^{\dagger}(+)_{\text{asy}} \] (B.30)

where \( \psi^{\dagger}(+)_{\text{asy}} \) is the asymptotic wavefunction of equation (B.3) and we have used the fact that from equation (B.21),

\[ [g^j(p)F_g^j(p) - h^j(p)F_h^j(p)] = T^j_{\text{asy}} (p^2) \] (B.31)

where \( T^j_{\text{asy}} \) is the on shell matrix element of equation (B.4). \( \psi^{\dagger}(+) \) is a finite ranged function

\[ \psi^{\dagger}(+)_{p,1} (r) = (2/\pi)^{1/2} \mu \pi h^{-2} \{ ^G_A(p)F_h^j(p)h_1^+(ia_A r) \]

\[ - G^R(p)F_g^j(p)h_1^+(ia_R r) \} \] (B.32)

and combining terms for simplicity;

\[ \psi^{\dagger}(+)_{p,1} (r) = \tilde{F}_h^j(p)h_1^+(ia_A r) - \tilde{F}_g^j(p)h_1^+(ia_R r) \] (B.33)

where
\[ \tilde{F}_h^j(p) = (2/\pi)^{1/2} \mu \pi h^{-2} ^G_A(p)F_h^j(p) \] (B.34)

and similarly for \( \tilde{F}_g^j \) with \( A \rightarrow R \).
The phase shift \( \delta_j^1(p) \) is obtained from equations (B.31) and (B.4) through the relation (Mo 69)

\[
\tan\delta_j^1(p) = \frac{\text{Im} T_{\xi=1}^j(p^2)}{\text{Re} T_{\xi=1}^j(p^2)}
\]  

(B.35)

where \( \text{Re}, \text{Im} \) denote the real and imaginary parts, and therefore the real function, \( \tilde{\psi}_j^1(+) \) of equation (B.6), is

\[
\tilde{\psi}_j^1(+) = \frac{\text{Re}\psi_j^1(+)}{\cos \delta_j^1(p)}.
\]  

(B.36)

As checks of these formulae a small computer program was used to evaluate the \( \delta_j^1(p) \), and the ratio \( \frac{\text{Re}\psi_j^1(+)}{\text{Im}\psi_j^1(+)} \) as a function of \( r \). The calculated \( \delta_j^1(p) \) agreed at all energies with those of Mongan and the above ratio, evaluated through equation (B.24), gave the required value \( \tan\delta_j^1(p) \), for all \( r \).
The Function \( H_{11}^{1L}(r) \)

The function \( H_{11}^{1L} \), in the dipole approximation of section 4.3 (\( k = l = 1 \)), may be rewritten using the real functions \( \tilde{\psi}_j^{(+)}(r) \) of Appendix B (equation (B.6));

\[
H_{11}^{1L}(r) = \int_0^\infty dp \, p^2 \{ \varepsilon_p - K_{\ell=1} \} \tilde{\psi}_j^{(+)}(r) / \{ \varepsilon_p + \varepsilon_d \}
\]

\[
\times \int_0^\infty dp \, p^3 \tilde{\psi}_j^{(+)}(p) u_L(p) .
\]

From equations (B.30), (B.33) and (B.3) and using the reality of \( h_1^{(+)}(iz) \), for \( z \) real (Me 61), then

\[
H_{11}^{1L}(r) = R \, j_L^1(h_1^{(+)}(ia_r) - A \, j_L^1(h_1^{(+)}(ia_A r))
\]

where we have used the fact that \( \tilde{\psi}_j^{(+)}(r) \) is a solution of the free particle equation with energy \( \varepsilon_p \), i.e.

\[
\{ \varepsilon_p - K_{\ell=1} \} (\text{Re} \tilde{\psi}_j^{(+)}_{p=1}) / \cos \delta_j^{(p)} = 0
\]

and therefore in the notation of Appendix B

\[
R \, j_L^1 = \int_0^\infty dp \, p^2 \frac{(p^2 + a_1^2)}{(p^2 + a_2^2)} \frac{\text{Re} \tilde{\psi}_j^{(+)}(p)}{\cos \delta_j^{(p)}} \int_0^\infty dr \, r^3 \tilde{\psi}_j^{(+)}(r) u_L(r) ,
\]

\[
A \, j_L^1 = \int_0^\infty dp \, p^2 \frac{(p^2 + a_1^2)}{(p^2 + a_2^2)} \frac{\text{Re} \tilde{\psi}_j^{(+)}(p)}{\cos \delta_j^{(p)}} \int_0^\infty dr \, r^3 \tilde{\psi}_j^{(+)}(r) u_L(r) ,
\]

where \( \pi^2 a_2^2 / 2 \mu \) is the deuteron binding energy.
Numerical Solution of the Coupled Equations

We wish to solve, numerically, pairwise coupled second order differential equations of the form (see equations (5.7), (5.10) assuming a fixed $J_1 = J$)

\[
\frac{d^2 \chi_{J-1}}{dr^2} = k(r) \chi_{J-1} + \tilde{g}(r) \chi_{J+1}
\]

\[
\frac{d^2 \chi_{J+1}}{dr^2} = \tilde{k}(r) \chi_{J+1} + g(r) \chi_{J-1}
\]  

for the functions $\chi_{J-1}$ and $\chi_{J+1}$, where we have suppressed all but the single important outgoing orbital angular momentum label. The functional coefficients $k, \tilde{k}, g, \tilde{g}$ are assumed known for all $r$ and inspection of equation (5.10) shows that $g = \tilde{g}$ is the present, $T_R$ interaction, problem. According to the modified Numerov or modified Fox-Goodwin algorithm (Me 66, Ra 71, Ro 67a) solution proceeds as a step by step recurrence relation through a constant integration step length $h$.

Given the values of $\chi_{J-1}$, $\chi_{J+1}$ at $r$ and $r-h$, then

\[
\begin{bmatrix}
K(r + h) \tilde{G}(r + h) \\
G(r + h) \tilde{K}(r + h)
\end{bmatrix}
\begin{bmatrix}
\chi_{J-1}(r + h) \\
\chi_{J+1}(r + h)
\end{bmatrix}
= \begin{bmatrix}
\chi_{J-1}(r + h) \\
\chi_{J+1}(r + h)
\end{bmatrix}
\]

\[
\begin{bmatrix}
K(r - h) \tilde{G}(r - h) \\
G(r - h) \tilde{K}(r - h)
\end{bmatrix}
\begin{bmatrix}
\chi_{J-1}(r) \\
\chi_{J+1}(r)
\end{bmatrix}
- \begin{bmatrix}
K(r - h) \tilde{G}(r - h) \\
G(r - h) \tilde{K}(r - h)
\end{bmatrix}
\begin{bmatrix}
\chi_{J-1}(r - h) \\
\chi_{J+1}(r - h)
\end{bmatrix}
\]

\text{(D.2)}
where
\[ K(r \pm h) = 1 - \frac{h^2 k(r \pm h)}{12}, \]
\[ G(r \pm h) = -\frac{h^2 g(r \pm h)}{12}, \]
\[ \kappa = 2 + \frac{5h^2 k(r)}{12}, \]
\[ \gamma = \frac{5h^2 g(r)}{6} \] (D.3)
and similarly for \( \tilde{k}, \tilde{g}, \tilde{\kappa}, \tilde{\gamma} \) in terms of \( \kappa, g \). Solution thus requires a
starting condition comprising a set of values for \( \chi_{J-1}(r), \chi_{J-1}(r - h) \)
\( \chi_{J+1}(r), \chi_{J+1}(r - h) \) for \( r \) near the origin.

As outlined in the text, two linearly independent solutions of
equation (D.1) are needed to extract the physical solutions which have the
required asymptotic forms (equation (5.11)). The two mathematical solutions
(designated A and B in the text) are obtained from the regular linearly
independent starting conditions

\[
\begin{align*}
\chi_{J-1}(h) &= p_{J-1}(h) & \chi_{J-1}(2h) &= p_{J-1}(2h) \\
\chi_{J+1}(h) &= 3p_{J+1}(h) & \chi_{J+1}(2h) &= 3p_{J+1}(2h) \\
\chi_{J-1}(h) &= 3p_{J-1}(h) & \chi_{J-1}(2h) &= 3p_{J-1}(2h) \\
\chi_{J+1}(h) &= p_{J+1}(h) & \chi_{J+1}(2h) &= p_{J+1}(2h)
\end{align*}
\]
Set A,

\[
\begin{align*}
\chi_{J-1}(h) &= p_{J-1}(h) & \chi_{J-1}(2h) &= p_{J-1}(2h) \\
\chi_{J+1}(h) &= 3p_{J+1}(h) & \chi_{J+1}(2h) &= 3p_{J+1}(2h) \\
\chi_{J-1}(h) &= 3p_{J-1}(h) & \chi_{J-1}(2h) &= 3p_{J-1}(2h) \\
\chi_{J+1}(h) &= p_{J+1}(h) & \chi_{J+1}(2h) &= p_{J+1}(2h)
\end{align*}
\]
Set B

where \( p_{J-1} \) and \( p_{J+1} \) are regular solutions of equation (D.1) in the absence
of coupling \((g = \tilde{g} = 0)\), obtained by expansion about \( r = 0 \) (see e.g. Ha 69).

Extraction of the physical solutions \( \chi^J_{\lambda, \lambda'} (\lambda = J \pm 1, \lambda' = J \pm 1) \),
from the mathematical \( \chi^{(i)}_{J-1}, \chi^{(i)}_{J+1} (i = A, B) \), is the subject of the following
appendix.
APPENDIX E

Physical Solutions of the Coupled Equations

The numerical solutions \( \chi_{J-1}^{(i)}, \chi_{J+1}^{(i)} \) (\( i = A, B \)) of Appendix D, regular at the origin, are linear combinations (for \( J \) fixed > 0),

\[
\chi_{J-1}^{(i)} = a_i \chi_{J-1,J-1}^{J} + b_i \chi_{J+1,J-1}^{J},
\]

\[
\chi_{J+1}^{(i)} = a_i \chi_{J-1,J+1}^{J} + b_i \chi_{J+1,J+1}^{J},
\]

(E.1)

of the regular physical solutions \( \chi_{J}^{J} \), whose asymptotic form as given by equation (5.11), namely

\[
\chi_{J}^{J}(r) \left|_{r \to \infty} \right. = \frac{i \sigma}{k} \{ F_{J}^{(kr)} \delta_{J,J}^{J} + C_{J}^{J} [G_{J}^{(kr)} + i F_{J}^{(kr)}] \}.
\]

(E.2)

Use of the Wronskain relation (Ab 65),

\[
W(F_{J}^{(kr)},G_{J}^{(kr)}) = F_{J}'G_{J}^{(kr)} - F_{J}G_{J}'^{(kr)} = 1
\]

(where primes denote differentiation with respect to \( kr \)) together with the reciprocity condition (Ra 64),

\[
C_{J-1,J+1}^{J} = C_{J+1,J-1}^{J},
\]

(E.3)

therefore gives at the matching radius \( r_m \),
\( W'_1 \) = \[ W(F_{J-1} \chi^{(A)}_J r_m \r) = W_1 \quad W'_2 \r \begin{bmatrix} C^J_{J-1, J-1} \end{bmatrix}, \]

\( W'_3 \) = \[ W(F_{J-1} \chi^{(B)}_J r_m \r) = W_3 \quad W'_4 \r \begin{bmatrix} C^J_{J-1, J+1} \end{bmatrix}, \]

and

\( W'_2 \) = \[ W(F_{J+1} \chi^{(A)}_{J+1} r_m \r) = W_1 \quad W'_2 \r \begin{bmatrix} C^J_{J+1, J-1} \end{bmatrix}, \]

\( W'_4 \) = \[ W(F_{J+1} \chi^{(B)}_{J+1} r_m \r) = W_3 \quad W'_4 \r \begin{bmatrix} C^J_{J+1, J+1} \end{bmatrix}, \]

(E.4)

(E.5)

where the \( W_i \) are

\[
W_1 = - W(G_{J-1} + iF_{J-1} \chi^{(A)}_{J-1} r_m \r = \alpha_A e^{i \sigma_1 J_1 / k},
\]

\[
W_2 = - W(G_{J+1} + iF_{J+1} \chi^{(A)}_{J+1} r_m \r = \beta_A e^{i \sigma_1 J_1 / k},
\]

\[
W_3 = - W(G_{J+1} + iF_{J-1} \chi^{(B)}_{J-1} r_m \r = \alpha_B e^{i \sigma_1 J_1 / k},
\]

\[
W_4 = - W(G_{J+1} + iF_{J+1} \chi^{(B)}_{J+1} r_m \r = \beta_B e^{i \sigma_1 J_1 / k},
\]

(E.6)

and are evaluated numerically at the matching radius; as are the \( W'_i \). The amplitudes \( C^J_{J' \ell \ell'} \) are obtained by simple inversion of equations (E.4) and (E.5), and using equations (E.6) and (E.1), for \( J' = J-1 \) \( \ell' = J+1 \) we have

\[
\begin{bmatrix}
-e^{-i \sigma_1 J_1 / k} \chi^J_{\ell-1, \ell'} \\
\end{bmatrix}
\begin{bmatrix}
\begin{bmatrix}
W_4 - W_2 \\
W'_3 - W_1 \\
\end{bmatrix}
\end{bmatrix}
\begin{bmatrix}
\begin{bmatrix}
\chi^J_{\ell-1, \ell'} \\
\chi^{(A)}_{\ell} \\
\chi^{(B)}_{\ell} \\
\end{bmatrix}
\end{bmatrix} = \frac{1}{W_1 W_4 - W_2 W_3}
\]

(E.7)

Equation (E.7) holds at all radii, solving for the required radial functions, \( \chi^J_{\ell \ell'} \).
APPENDIX F

Optical Model Potential Formfactors

In the computer program DWCODE (Ha 70) the central and spin-orbit terms, $U_c, U_{so}$, of the deuteron and neutron/proton optical potentials $U_i(r)$ ($i = d(\equiv 1), n, p(\equiv 2); \text{e.g. equations (5.1-2)})$, defined by

$$ U_i(r) = U_c(r) + U_{so}(r) \frac{x_1}{s_1/s_i}, \quad \text{(F.1)} $$

are

$$ U_c(r) = V_c(r) - Vf(x_v) - i\{Wf(x_w) + S \ g(x_g)\}, \quad \text{(F.2)} $$

$$ U_{so}(r) = \{V_{so} + iW_{so}\} x^2 \frac{1}{r} \left\{ \frac{d}{dr} f(x_{so}) \right\}. \quad \text{(F.3)} $$

Here $x$ is the pion Compton wavelength with

$$ x^2 = \left(\frac{\hbar}{m_c}\right)^2 = 2.0 \text{ fm}^2, $$

$f(x_i)$ is the conventional Wood-Saxon formfactor

$$ f(x_i) = (1 + e^{x_i})^{-1} , \quad x_i = \frac{(r - r_i A^{1/3})}{a_i}, \quad \text{(F.4)} $$

where $A$ is the mass number of the target nucleus and

$$ g(x_i) = -4a_i \frac{d}{dr} f(x_i); $$

$r_i, a_i$ ($i = V, W, S, SO$) are the usual radius and diffuseness parameters.

\[\text{\footnotesize \begin{align*} &\text{\footnotesize } + \text{ Tensor formfactors are dealt with individually in the text of chapter 6.} \end{align*}}\]
$V_c(r)$, the coulomb potential between projectile and target, is taken as that due to a uniformly charged sphere of radius $R_c = r_c A^{1/3}$, and (Ja 70)

$$V_c(r) = \begin{cases} 
Z_p Z_t e^2 / 2 R \{3 - (r/R)^2\} & r < R_c \\
Z_p Z_t e^2 / r & r \geq R_c 
\end{cases} \quad (F.6)$$

where $Z_p, Z_t$ are the projectile and target charges, respectively.

The potential (Real) binding the neutron is similarly defined (with $s = \frac{1}{2}$),

$$U(r) = - V f(x_v) + \hbar^2 v_{so} \frac{1}{r} \left\{ \frac{d}{dr} f(x_{so}) \right\} \frac{\ell \cdot s}{s} \quad (F.7)$$

or for an angular momentum state $j, \ell$:

$$U_{j \ell}(r) = - V f(x_v) + \hbar^2 v_{so} \frac{1}{r} \left\{ \frac{d}{dr} f(x_{so}) \right\} [j(j+1) - \ell(\ell+1) - 3/4]. \quad (F.8)$$
Evidence for Tensor Interactions

While all calculations (e.g. Ir 74, Ke 73, Kn 75b, Ka 76, Ba 75) agree that the predicted Watanabe $T_R$ interaction makes important contributions to the calculated elastic scattering tensor analysing powers, phenomenological investigations are divided in their conclusions. A group of studies (e.g. Sc 68, Sc 69, Bu 75, Ka 76, Pe 77) indicate the need for a $T_R$ term of comparable strength and radial shape to the Watanabe form, while others (e.g. Ba 75, Bu 78) obtain interactions with little resemblance to the folding model predictions; all however obtain an optical model $T_R$ term. The observation of Raynal (Ra 63, Ra 64), that there is no evidence of a $T_R$ interaction in scattering from Ca$^{40}$, has been attributed (Ke 73) to the fact that the analysing power $T_{21}$, theoretically (Jo 71a, Ho 71) and in calculation (Ir 74, Co 68) the most sensitive parameter to the presence of rank-2 interactions, was not amongst the measured data. Thus, while suggesting the presence of a $T_R$ potential, results of existing elastic scattering analyses are inconsistent in their findings and in general †, fits to the polarization variables $i T_{11}$, $T_{2q}$ ($q = 0, 1, 2$) remain only qualitative.

Of great importance therefore, is the success of the 'hybrid' model of Knutson and Haeberli (Kn 73b, Kn 75b) in fitting sub-coulomb scattering data.

† A notable exception is the recent phenomenological study of Burgi et al. (Bu 78) upon $^{40}$Ar in which good fits, to all five observables at three deuteron energies, are obtained.
They combine the predicted folding model spin-dependent interactions with a phenomenological central interaction and in this way obtain a good description of all five reaction observables for deuteron energies below the target barrier height. Limited success has also been reported for this approach by Karban et al. (Ka 76), however, the model has been shown to be less reliable at higher energies (Ir 74, Kn 75b).

The Watanabe model described (equation (6.1)) fails to take account of two major perturbative effects in the scattered system, namely:

a) the Pauli Principle (see e.g. Po 75, So 69), and

b) deuteron break-up (Jo 70, Ha 71),

known to be of importance in the calculation of the central terms of $U_1(R)$.

Recently, Ioannides and Johnson (Io 76) and Austern (Au 76) have re-examined the spin dependence of the deuteron target interaction, resulting from the inclusion of the Pauli Principle (P/P). They find that the action of the P/P upon the D-state component of a deuteron propagating in nuclear matter, introduces an interaction of the $T_P$ type. An extended analysis by Ioannides and Johnson (Io 78) has shown that for finite nuclei (using a perturbation theory approach) there exists a $T_P$ interaction of comparable magnitude but of considerably smaller diffuseness than the Watanabe $T_R$ force. The P/P however, also has consequences for the $T_R$ force in the deuteron optical potential. The $T_P$ force is associated (Io 78) with a rapid drop in the deuteron D-state probability, with radial position $R$ within the nuclear volume, from that of its free space value $P_D$. However, in the Watanabe model the D-state is responsible (Ke 70) for the presence of the $T_R$ interaction. Calculations in which the $R$ dependent 'deuteron' wavefunction is folded in
equation (6.1), show (Io 78a, Jo 78) that the result is therefore to suppress the $T_R$ interaction within the nuclear volume, while beyond the nuclear surface, in which region the nuclear and deuteron momentum distributions are non-overlapping, the $T_R$ interaction coincides with the predictions of the conventional Watanabe folding.

Let us consider now break-up effects:

In chapter 3 the dominant spin dependence of coulomb break-up of the deuteron was the introduction of an $R^{-4} T_R$ interaction (equation (3.49)) in the $n-p$ centre of mass system. Stamp (St 70) has shown that in addition, break-up into a continuum $^3S_1$ $n-p$ state contributes only a very small $T_L$ interaction, second order in the nucleon-nucleus spin orbit interaction, assumed isoscalar. If however the nucleon optical potentials have an isovector spin orbit component then the elastic channel will couple to the $^1S_0$ $n-p$ break-up channel $^†$ (Ha 74), the effect upon the elastic channel being to introduce a non-local $T_L$ interaction which is second order (Ha 70a) in the isovector spin-orbit component. Phenomenological studies (Ir 74, Ka 76, Sc 69) agree with these theoretical results; namely, if present, a $T_L$ interaction must be small and recent analyses do not allow for such a term (Bu 78, Ka 76). Break-up effects within the framework of the Johnson-Soper (Jo 70) Adiabatic model are discussed further in chapter 6.

$^†$ We comment briefly upon this effect in Appendix H.
APPENDIX H

Singlet Break-up Effects

As was stated in Appendix G, if the neutron and proton, comprising the deuteron, interact with the target nucleus via nucleon optical potentials with an isovector spin-orbit component, then the deuteron elastic channel couples to singlet \((^1S_0)\) break-up states of the n-p system (Ha 70a, Wa 77). The effects of singlet coupling upon \((d,p)\) stripping observables, for an assumed \(^3S_1\) deuteron, have been studied previously by Harvey and Johnson (Ha 74), who conclude that corrections from singlet break-up are small, although not completely negligible at backward centre of mass angles. This study however concerned the reaction \(^{56}\text{Fe}(d,p)^{55}\text{Fe}\) at \(E_d = 23\) MeV and therefore, in view (Ha 70a) of the rank-2 nature of singlet break-up effects upon the deuteron elastic channel, an estimate of their importance upon the reaction \(^{208}\text{Pb}(d,p)^{209}\text{Pb}\) at \(E_d = 9\) MeV, under study here, is of considerable interest.

The computer program DWCODE allows the calculation of singlet break-up effects, including the \(^3D_1\) component of the deuteron wavefunction \(\dagger\), the method having been described fully elsewhere (Ha 70, Wa 77). Calculations have been performed using the optical model parameters of Table 6.2 and the isovector coupling potentials from the work of Wales (Wa 77). For the \((d,p)\) transitions to the \(j^\pi = 5/2^+(E_x = 1.57\) MeV) and \(j^\pi = 1/2^+(E_x = 2.03\) MeV)

\(\dagger\) In fact, when including D-state and singlet coupling effects in the approximate way described by Harvey and Santos (Ha 70), DWCODE requires certain corrections (To 78a). For this reason the previous calculations reported by Wales (Wa 77) are in error.
states of $^{209}$Pb at $E_d = 9 \text{ MeV}$, we find that singlet effects upon all five reaction observables, $d\sigma/d\Omega, iT_{11}, T_{2q} (q = 0, 1, 2)$, are completely negligible, changing, typically, the predicted values only in the fourth decimal place throughout the angular range.

Clearly, singlet channel coupling may be ignored in any further study of sub-coulomb stripping reactions.
REFERENCES


Be 65 : T. Berggren, Nucl. Phys. 72 (1965), 337.


Bu 78 : H.R. Bürgi, W. Grüebler, P.A. Schmelzbach, V. König, R. Risler and B. Jenny, to be published.


Da 71 : S.E. Darden, in ref. (Ba 71), p.39.

Fa 57 : U. Fano, Rev. Mod. Phys. 29 (1957), 74.


Ha 72 : W. Haeberli, Revista Brasileira De Fisica 2 (1972), 187.


Io 78a: A.A. Ioannides and R.C. Johnson, to be published.


Iw 56 : J. Iwadare, S. Otsuki, R. Tamagaki and W. Watari, 
Prog. Theor. Phys. (Kyoto), 16 (1956), 455; 


Jo 71a: R.C. Johnson, in ref. (Ba 71), p.143.


Jo 73 : R.C. Johnson, F.D. Santos, R.C. Brown, A.A. Debenham, G.W. Greenlees, 
A208 (1973), 221.


Jo 78 : R.C. Johnson, Paper presented at the International Symposium on 

Jo 78a: R.C. Johnson, Notes on Folding Model Spin-Orbit Contributions, 
(unpublished).


Ka 76 : O. Karban, A.K. Basak, J.A.R. Griffith, S. Roman and G. Tungate, 

Ke 70 : P.W. Keaton Jnr., E. Aufdembrink and L.R. Veeser, Los Alamos Report 


2 (1976), L113.

Ke 78 : M.W. Kermode, Private communication to R.C. Johnson (1978).


Kn 77a: L.D. Knutson, Tabulated data for the reactions $^{90}$Zr(d,p)$^{91}$Zr, $^{208}$Pb(d,p)$^{209}$Pb, Private communication, 1977.

La 55 : W. Lakin, Phys. Rev. 98 (1955), 139.


Me 71 : The Madison Convention, pp xxv-xxix, of (Ba 71).


Ra 78: J.A. Ramirez and W.J. Thompson, to be published.


To 77 : J.A. Tostevin and R.C. Johnson, Contribution to the Inst. of Physics Conf. on Nuclear Physics, University of Surrey, March 1977.

To 78 : J.A. Tostevin and R.C. Johnson, A correction to DWWCODE (Note to program users), University of Surrey, 1978, (unpublished).


Wa 58 : S. Watanabe, Nucl. Phys. 8 (1958), 484.


Ya 54 : Y. Yamaguchi, Phys. Rev. 95 (1954), 1628;
        Y. Yamaguchi and Y. Yamaguchi, Phys. Rev. 95 (1954), 1635.