HEAT TRANSFER AND FLUID FLOW

IN LEPOL GRATE CALCINERS.

R.K. MEERABUX.

Presented for the Degree of
Doctor of Philosophy at
University of Surrey, Chemical Engineering
Stag Hill, Guildford, Surrey

March 1977

CE - TD - 9
SUMMARY

The semi-dry Lepol Process of cement manufacture consists of drying and calcining a shale and limestone nodule mix on a travelling "Lepol" grate. The resulting semi-calcined mixture then flows down a shute into a shortened version of a conventional rotary kiln. A better understanding of the fluid flow and heat transfer taking place around the nodules on such a Lepol Grate Calciner will unquestionably lead to the more efficient operation and design of such installations. The work carried out for this thesis involved first establishing the fundamental aerodynamics of a Lepol Grate - Rotary Kiln system and then using this knowledge to develop a realistic mathematical model to predict the bed temperature along the calciner section of the furnace; neither of these two experiments having been attempted before.

The aerodynamics of the system was established using a 1:24 scale, perspex, water model with the flow suitably adjusted to represent plant flow conditions and obtaining typical flow diagrams to present the results. The salient features observed in these studies, using polystyrene beads as tracers, were:-

(a) The flow patterns obtained in the Lepol grate model were essentially independent of both the Reynolds numbers range and the bed voidages investigated.

(b) The initial assumption of plug flow, through the drier and calciner was erroneous. Indeed, for the
calcining section, the flow was found to be restricted to 4/5ths of the grate area, due to the geometrical constraints on the system.

(c) The fluid flowing above the calcining section formed a double vortex.

(d) The fluid issuing into the drier chamber behaved as a confined jet.

A numerical mathematical model to describe the processes occurring on the Lepol grate calciner at steady state was derived, and tested against experimental results of Weber (1). This analysis involved dividing the bed of the calciner into 4 equal strata and then subdividing each stratum into increments. This model yielded the following equation for the top stratum of the bed:

\[
T_1 = \frac{\alpha - \gamma + \delta + \xi - \eta + m \cdot \bar{c} \cdot T_0}{m \cdot \bar{c}}
\]

where

- \( T_0 \) = Temperature at beginning of increment.
- \( T_1 \) = Temperature at end of increment.
- \( m \) = Mass flowrate of solids.
- \( \bar{c} \) = Specific heat of solids.
- \( \alpha \) = Heat transfer by convection.
- \( \gamma \) = Energy used for calcining.
- \( \delta \) = Energy radiated from the furnace walls.
- \( \xi \) = Energy radiated from the furnace gases.
- \( \eta \) = Energy re-radiated from the increment.
For the other 3 strata,

\[ T_1 = \alpha + \frac{m.c.}{m.c.} T_0 - \gamma + V \]

where \( V \) = Energy transferred by radiation between
the increment being considered and the
increment above.

In this way, the heating curves of the bed for the four strata at
steady state were obtained using an iterative procedure and compared
with the results of Weber.

Several correlations to characterise the heat transfer from
the gas to the bed of particles in the calciner were tested and
finally that of Lof and Hawley (2) was found to give the best fit
to Weber's results.
Acknowledgements

To my supervisor, Mr. F. D. Moles for all the help and encouragement shown by him over the last three years.

To the technicians of the Department of Chemical Engineering namely, Pat O’Sullivan, John Webb and Gerald Shurlock for their help in the manufacturing and assembling of the apparatus and their useful suggestions as to improvements.

To my fellow research students for their suggestions.

To the Associated Portland Cement Manufacturers Limited for sponsoring the work over the last three years.
CONTENTS

Acknowledgements.

1. Introduction.
   1.1. The Lepol grate-Rotary Kiln system.
   1.2. The physical model.
   1.3. The mathematical model.

2. Literature survey.
   2.1. Drying on the grate.
       2.1.1. Rate of drying.
   2.2. The calcination of limestone.
       2.2.1. Dissociation temperature.
       2.2.2. Heat consumed.
       2.2.3. Calcination rate.
   2.3. Review of physical modelling.
       2.3.1. Modelling in general.
       2.3.2. Pioneer Work in the Steel Industry.
       2.3.3. University of Surrey Work related to the Cement Industry.
       2.3.4. Modelling of moving grates.
   2.4. Measurement of solid temperatures on a grate.
   2.5. Pressure drop in fixed beds.
   2.6. Heat transfer in packed beds.
       2.6.1. Important factors.
       2.6.2. Bed packing.
       2.6.3. Saunders and Ford's work.
       2.6.4. Lof and Hawley's work.
       2.6.5. General correlation of coefficients.
   2.7. Mathematical modelling of moving grate systems.
3. Theoretical model to predict bed temperature at steady state in the calciner.

3.1. Introduction.

3.1.1. Assumptions.
3.1.2. Conduction in the bed.
3.1.3. Radiation between adjacent strata.
3.1.4. Emissivity of the bed.

3.2. Heat balance for stratum 1 at steady state.

3.2.1. Heat transfer by convection. \((\alpha)\)
3.2.2. Surface area of solid. \((\AA)\)
3.2.3. Rise in enthalpy of the solids. \((\beta)\)
3.2.4. Energy used for calcining. \((\gamma)\)
3.2.5. Energy radiated from the furnace walls. \((\delta)\)
3.2.6. Energy radiated from the gas. \((\xi)\)
3.2.7. Energy re-radiated from the increment. \((\eta)\)
3.2.8. Design equation.

3.3. Heat balance for strata 2,3 and 4 in the bed.

3.4. Analytical approach.

3.4.1. Disadvantages of the analytical method.

3.5. Isothermal scale modelling.

3.5.1. Similarity criteria.
3.5.2. Satisfying these criteria.

4. Description of the Hydraulic System.

4.1. Operating fluid.

4.1.1. Polystyrene bead-tracing.

4.2. Design and construction of the apparatus.

4.2.1. Pumps.
4.2.2. Piping.
4.2.3. Valves.
4.2.4. Flow measurement.
4.2.5. Sump tanks.
4.2.6. Bed resistance.
4.2.7. Lighting and photography.

4.3. Operation of the model.
4.3.1. Reynolds number in the model.
4.3.2. Operational difficulties.

5. Results.
5.1. Results and model operating conditions.
5.1.1. Reynolds numbers around the model.
5.1.2. Voidage of gauze used to represent the bed.
5.1.3. Re. for prototype calciner bed assuming plug flow.
5.1.4. Test 1.
5.1.5. Test 2.
5.1.6. Test 3.

5.2. Mathematical model.
5.2.1. Results of the mathematical model. (Equations 3.9 & 3.11)
5.2.2. Modified flow through the bed.
5.2.3. Modified heat transfer coefficient.

6. Discussion of results.
6.1. Flow patterns from the Water Model.
6.1.1. Flow patterns with no bed resistance.
6.1.2. Flow patterns in the calciner chamber. Voidage=0.25
6.1.3. Region of random flow and recirculation.
6.1.4. Separation of the boundary layer.
6.1.5. Consequences of flow separation.
6.1.6. Flow patterns in the above calciner (ε=0.25).
6.1.7. Flow patterns in the drier chamber (ε=0.25).
6.1.8. Flow patterns with a voidage of 0.36.
6.1.9. Implications of the water model results.

6.2. Discussion of the mathematical model.
6.2.1. Comparison between measured and predicted results.
6.2.2. Estimation of the mean gas temperature.
6.2.3. Change of grate speed.
6.2.4. The overall enthalpy balance.
6.2.5. Calcination heat requirement.
6.2.6. Radiation contribution to enthalpy gain of solids.

6.3. Conclusions.
6.3.1. Water model results.
6.3.2. Mathematical model results.

Nomenclature.

References.

Appendices.
1. Introduction

The research programme can be conveniently divided into two sections:

(a) To establish the aerodynamic flow patterns around the Lepol grate.

(b) To construct a mathematical model for the calciner in the light of the established flow patterns.

Except for simple furnace configurations, the prediction of the flow patterns for the furnace gases is impractical or impossible by ordinary analytical methods, and the use of scale models is an excellent method for solving flow problems.

For dynamic similarity, the Reynolds number \(= \text{velocity} \times \text{length}/\text{kinematic viscosity}\) should be the same in model and prototype. The kinematic viscosity of hot furnace gases is about 12 times that of cold air and about 120 times that of cold water. A twelfth size air model would therefore require a cold air velocity equal to the hot gas velocity in the prototype, whereas in a twelfth size water model, the water velocity would be only one tenth of it. For this reason water models are more suitable for flow visualization. Moreover, to investigate the aerodynamics of systems hitherto unknown, such as the Lepol grate - Rotary Kiln system, a water model is particularly suitable as it provides rapid, qualitative results. These results act as pointers for future quantitative velocity measurements by pitot tube on air models.
1.1. The Lepol grate - Rotary Kiln system

Lepol Grate Calciners have proved to be very attractive plants for the production of cement because of their lower fuel consumption as compared with the older Long Wet Process Kilns. Although Suspension Preheater Kilns show even better fuel economies, in the foreseeable future wet process kilns are much more likely to be replaced by semi-dry process Lepol kilns rather than the completely dry feed kilns.

The Lepol system is surprisingly compact considering its multiple functions. A plant consists of a two-pass Lepol preheating travelling grate, downdraft, and totally enclosed, followed by a short large diameter rotary kiln, plus a high efficiency, counterflow, cooler.

The raw material, pellets of limestone and shale from the granulators, passes on to the preheating grate through which hot gases pass and in so doing, the pellets are dried and heated to calcining temperatures before entering the kiln. The grate is arranged for double-pass operation so that hot gases which have passed through the calcining end of the grate are cleaned by a bank of cyclones and are then passed through the grate again to dry the incoming pellets. In the kiln proper the material is fully calcined and sintered before passing to the cooler. While cooling the clinker, the cooler provides high temperature secondary air for the kiln burner.

1.2. The physical model

The dimensions of the Lepol grate modelled were 20.9 metres in
length by 4.1 metres in width, and the kiln was 48 metres long with an outside diameter of 3.8 metres. A typical Lepol grate is shown in Figure 1.1 together with the temperatures encountered in the above and below sections of the drier and calciner chambers. A 1:24 scale representation was fabricated in perspex, water being chosen as the operating fluid. As the chief area of interest was the grate itself, only one third of the kiln length was reproduced, and the relatively thin bed of charge on the shute was not modelled.

In addition, the moving charge on the grate itself was modelled for static conditions using fine stainless steel gauze. This was justifiable since the gas velocities through the bed are much greater than the speed of the grate, and would not be expected to affect the aerodynamic flow patterns. In this way, the fundamental flow patterns, hitherto unknown, were established around the Lepol grate.

1.3. The Mathematical Model

During the last two decades, investigations of industrial furnaces have been aided by mathematical models which steadily increase in realism and refinement. At first a furnace had to be treated as a uniform temperature enclosure or "stirred tank" (3). Then one dimensional analyses were employed for long furnaces of modest width (4). Recent developments in computers and numerical methods have made it possible to make two dimensional analyses and procedures are now being developed for solving the equations of 3-dimensional mathematical models (5).
Mean gas temperatures:

Above calciner = 1000°C
Below calciner = 450°C
Above drier = 400°C
Below drier = 100°C

Bed height = 18 cms.
Pressure drop = 2.5 ins. w.g.

Figure 1.1  THE LEPOL GRATE-KILN SYSTEM.
One of the objects of this work was to develop a mathematical model which would adequately describe the calciner chamber and allow prediction of performance for a desired change. The operator of the furnace may also find, by the use of the model, an improved manner of working which the demands of continuous production would prevent his searching for by trial and error. Mathematical models are being used increasingly by furnace designers and operators; the reasons being that furnaces and related equipment are becoming more and more expensive and their standards of efficiency more competitive. By a series of careful and convincing tests, the physical inputs to the model have to be demonstrated to be realistic so that their mathematical implications are to be trusted beyond the range of existing experimental information.
2. Literature survey.

2.1. Drying on the grate

Although the analysis of heat transfer on the Lepol grate was confined to the calciner section, the drier section plays an important role in removing moisture from the nodules before they enter the calciner chamber and a brief treatment of the possible drying processes involved is given here. The analysis of drying in beds is more complicated than heat transfer or isothermal mass transfer alone. Different mechanisms can control and different drying regimes commonly called the constant rate and the falling rate regimes may be observed. The possible drying processes involved is given here and is based on the treatment of Treybal (6).

2.1.1. Rate of drying

Data for the exposure of a batch sample of nodules to air at constant temperature, humidity, and velocity constitutes drying under constant drying conditions. This yields a curve of moisture content as a function of time as illustrated in Figure 2.1. Much information can be obtained, however, if the data are converted to rates of drying and plotted against moisture content as in Figure 2.2. This may be done by measuring the slopes of tangents drawn to the curve of Figure 2.1 or by determining from the curve small changes in moisture content $\Delta X$ for corresponding small changes in time $\Delta \theta$ and calculating the rate as:

$$ N = -L_s \cdot \Delta X / A \cdot \Delta \theta \tag{2.1} $$

Here $L_s$ is the weight of dry solid and $A$ is the cross-section of the bed measured at right angles to the direction of gas flow. While different solids and different conditions of drying will frequently give
Figure 2.1 Batch drying, constant drying conditions.

Figure 2.2 Typical rate of drying curve, constant drying conditions.
rise to curves of very different shape in the falling-rate period, the curve shown in Figure 2.2 occurs frequently.

The salient features of Figure 2.2 are:

A-B Initial adjustment. In the beginning, the solid and the liquid surface are usually colder than the ultimate surface temperature and the evaporation rate will increase while the surface temperature rises to its ultimate value during the period AB on these curves.

B-C Constant Rate Drying. During the constant rate period, it is assumed that drying takes place from a saturated surface of the material by diffusion of the water vapour through a stationary air into the air stream. When the average moisture content of the solid has reached a value $X_c$, the critical moisture content, the surface film of moisture has been reduced by evaporation.

C-D First falling-rate period. At point C the surface is no longer capable of supplying free moisture to saturate the air in contact with it. Under these conditions the real factor influencing the rate of drying is the mechanism by which the moisture from inside the material is transferred to the surface.

D-E Second falling-rate period. At the conclusion of the first falling rate period we may assume that the surface is dry and that the plane of separation is moving into the solid. In this case, evaporation will be taking place from within the solid, the vapour reaching the surface by molecular diffusion through the material. The forces controlling the vapour diffusion will determine the final rate of drying, and these will be largely independent of the conditions outside the material. At point E, the moisture content of the solid has fallen to the equilibrium moisture content $X^*$ for
the prevailing air humidity, and drying stops.

2.1.2. Time of drying

By definition, the rate of drying is

\[ N = -\frac{Ls \cdot dX}{A \cdot d\Theta} \quad (2.1) \]

which can be rearranged and integrated to obtain the drying time while the moisture content changes from its initial value \(X_1\) to its final value \(X_2\).

\[ \Theta = \int_{\Theta_0}^{\Theta_1} d\Theta = \frac{Ls}{A} \int_{X_2}^{X_1} \frac{dX}{X} \quad (2.2) \]

The constant-rate period

For drying within the constant-rate period so that \(X_1\) and \(X_2 > Xc\) and \(N\) will be constant at \(Nc\) the equation 2.2 becomes

\[ \Theta = \frac{Ls \cdot (X_1 - X_2)}{A \cdot Nc} \quad (2.3) \]

The falling rate period

Frequently the entire falling rate curve may be taken as a straight line between points C and E in figure 2.2. During this period the rate of drying is directly proportional to the free moisture content \(X - X^*\). Hence

\[ N = \frac{m \cdot (X - X^*)}{Xc - X^*} \quad (2.4) \]

Substitution into equation 2.2 gives
\[ \theta = \frac{L_s}{A} \int_{X_2}^{X_1} \frac{dX}{m(X - X^*)} \]  
\[ = \frac{L_s}{\dot{m}A} \ln \left( \frac{X_1 - X^*}{X_2 - X^*} \right) \]  
\[ = \frac{L_s (X_c - X^*)}{N_c A} \ln \left( \frac{X_1 - X^*}{X_2 - X^*} \right) \]

Hence the total drying time is equal to the sum of the equations 2.3 and 2.7.

2.2. The calcination of limestone

The calcination of limestone (7) is an example where a decomposition reactions proceed according to the unreacted core model. Figure 2.3 illustrates this process. Here the reaction proceeds at a narrow front which moves into the solid particle, and the reactant is completely converted as that front passes by. The driving force for this highly endothermic reaction is the conduction of heat through the product layer.

The chemical reactions are as follows:

\[ \text{CaCO}_3 \text{ (high calcium limestone) + heat} \overset{\neq}{\rightleftharpoons} \text{CaO (h.c. quicklime) + CO}_2 \]
\[ 100 \quad 56 \quad 44 \]

\[ \text{CaCO}_3 \text{ MgCO}_3 \text{ (dolomitic limestone) + heat} \overset{\neq}{\rightleftharpoons} \text{CaO MgO (dol. quicklime) + } +2\text{CO}_2 \]
\[ 100 \quad 84 \quad 56 \quad 40 \quad +2\text{CO}_2 \quad 88 \]
Figure 2.3: Unreacted core model. Here reaction proceeds at a narrow front which moves into the solid particle. Reactant is completely converted as the front passes by.
Boynton (8) states three essential factors in the kinetics of the decomposition of limestone:

1. The stone must be heated to the dissociation temperature of the carbonates.
2. This minimum temperature (but practically a higher temperature) must be maintained for a certain duration.
3. The CO\textsubscript{2} evolved must be removed.

2.2.1. Dissociation temperature

Values of the dissociation temperature of CaCO\textsubscript{3} which were developed by Johnston (9) are still generally recognised. For calcite it is 898°C for 760 mm pressure for a 100% CO\textsubscript{2} atmosphere. Boynton states that magnesium carbonate dissociates at a much lower temperature of 402°C to 480°C. The dissociation temperature of dolomite, however, is not nearly so explicit. Since the proportion of MgCO\textsubscript{3} to CaCO\textsubscript{3} differs in the many species of dolomitic and magnesian limestone, the dissociation temperature naturally also varies and is therefore much more difficult to calculate. Boynton states the commencement of dissociation for a highly crystalline dolomite occurs near 750°C. Hence, for the Lepol system considered, calcination will be assumed to commence at 800°C.

2.2.2. Heat consumed

Gygi (10) states the amount of heat required for the dissociation of CaCO\textsubscript{3} as 396 kcal/kg. Weber mentions a heat requirement of 422 kcal/kg CaCO\textsubscript{3} and 310 kcal/kg MgCO\textsubscript{3} for the dissociation of these
materials. The calorific requirement for dolomitic stone would be less because of its lower dissociation point (725°C) for a pure equimolecular double carbonate. Boynton states the heat consumed for this case to be 346 kcal/kg. Clearly as impurities increase in the limestone, this thermal requirement is reduced correspondingly since there is less carbonate to decompose. The author has chosen the calorific requirement for calcination to be 422 kcal/kg CaCO₃.

2.2.3. Calcination rate

Calcination of limestone takes place in a very narrow zone which is the phase boundary between CaCO₃ and CaO. The data reported in the paper of Furnas (11) were given as rates of advance of this line of calcination from the outside to the inside pieces of limestone. The purely empirical equation given was

\[ \log_{10} R = 0.003145t - 3.3085 \quad (2.8) \]

where \( R \) = rate of advance of the line of calcination in centimetres per hour.

\( t = \) temperature °C.

Satterfield and Feakes (12) illustrate a typical time-centre temperature history for the studies made with agglomerates formed from 10 to 15 micron calcium carbonate.

Boynton (8) has presented a curve of the loss in weight against time for the calcination of a fine-grained, compact limestone possessing organic impurities, and the data can be approximated by the equation \( y = 0.0183X \). \( (2.9) \)
\[ y = \text{fraction of mass calcined.} \]
\[ X = \text{time in minutes.} \]

From the foregoing discussion, the starting point for calcination on the grate was chosen to be 800°C and Boynton's curve was used to obtain the % calcination as the iteration procedure used to calculate solids temperature (See Appendix L) progressed, since the time for the material to travel through an increment was known.

2.3. Review of physical modelling

Despite the original Lepol patent dating back to 1928 and the widespread use of the process in the cement industry, the published data relating to fluid flows within the system is negligible. The classic work of Gygi (10) and Anselm (13) have improved the available knowledge of the thermodynamics of the cement kiln process but they do not comment upon fluid flow characteristics of the system studied. The most recent treatment of rotary cement kiln systems due to Weber (1) provides useful gas conditions and velocity data for these kilns, but except for the measurement of the module temperature in the drier and calciner sections of the grate, little else is available for the preheater system.

2.3.1. Modelling in general

Johnstone and Thring (14) present similarity criteria and scale equations for use in pilot plant and model studies. Evans and Patrick (15) give an excellent review of modelling. This latter review considered boiler furnace modelling specifically, but there existed a background of other modelling work and general fluid dynamic studies. Philbrook (16)
summarizes similarity criteria for scale model and pilot plant experiments. He states that practically there are so many different natural processes involved each with its own set of similarity criteria that it is impossible to accomplish complete similarity.

Putman and Ungar (17) discuss the basic assumptions upon which cold-flow modelling of combustion systems is carried out, the application of modelling to various combustion problems and the type of results that should be expected from the application of modelling techniques. Rydderch (18) gives an excellent review of the question of simulation of buoyancy and discusses the work of several other researchers on the subject.

Gray and Robertson (19) have described the equipment and methods employed in the solution of industrial fluid flow problems by model techniques using air as the modelling fluid. The techniques used were illustrated by examples of typical problems investigated, viz: flow distribution in pipes and furnaces, mixing and entrainment, and flow of particles in gas streams. In order to obtain a set of results it is necessary to study photographic techniques.

The work of Winter and Deterding (20) describe the equipment and techniques required for continuous illumination, time exposures, flash photography and ultra violet photography with water models. For the observation of flow by these methods it is necessary for the fluid to carry with it a visible tracing material and the authors recommend the use of polystyrene beads with a mean S.G. of 1.00 for use with water models. In addition the authors recommend that water models be immersed in a rectangular tank to prevent photographic distortion when models of complicated shape are used.
2.3.2. Pioneer work in the Steel Industry

Chesters (21) using a 1/24 scale water model investigated the complex flow pattern obtained in an 80 ton open-hearth furnace and showed both that an improved roof life could be attained, and general wear in the furnace was largely explainable in terms of the deposition of harmful oxides expected from such flow patterns. Hulse (22) also determined flow distributions of open hearth furnace regenerators by means of cold trials. He commented that one of the main attractions was the ease and rapidity with which a mock-up of a device for improving flow distribution such as a baffle wall could be introduced and its effect determined.

2.3.3. University of Surrey work related to the Cement Industry

The flow patterns at the hot end of a rotary cement kiln were established by Lain (23) who used water and air isothermal modelling techniques to elucidate the flow conditions. For flow visualization in the water model, Lain used polystyrene tracer particles recommended by Winter and Deterding (20), coupled with high-speed cine and still photography. The conclusions reached from this model work were verified by Moles, Watson and Lain (24) who used both "hot" and "cold" experimental trials on full size kilns. Recently (25), the effect of buoyancy on a confined horizontal flame has been carried out using a water model to investigate the effect of density differences on the jet path. The technique used was developed by Thring and Horn (26) who used a jet of magnetite slurry to represent the flame, and water to simulate the combustion air. The jet of slurry being heavier than water, sank, and so provided an upside down picture of the rising flame jet.
2.3.4. Modelling of moving grates

Rolfe (27), with the aid of a combustion pot and an air flow model, evolved the design of undergrate baffles for chain grate stokers and showed that they result in a substantially improved primary air distribution. This uniform air distribution not only increased the permissible combustion rate by approximately 18% but also improved the efficiency by about 1 - 1½%. Ball (28) modelled combustion chamber conditions in a 1/3 size Lancashire boiler furnace tube. Full size grate links were used so that the fuel and the fuel bed thickness were the same as in an actual boiler. This made combustion conditions very difficult to model with the result that predictions for full scale work were only qualitatively correct and the model investigation was abandoned in favour of experiments in full scale equipment.

2.4. Measurement of solid temperatures on a grate

Grumell and Dunningham (29) considered the various factors affecting the combustion of a fuel bed of coal on a travelling grate with uniform air flow throughout the length of the grate. Measurements of bed resistance and grate temperature were recorded, and the authors established the ignition plane of the fuel bed. Hayward (30) on the other hand discussed the advantages of the chain grate stoker on shell boilers and also the limiting factors of the fuels which could be burnt on the grate, and Wright (31) measured the temperature distribution in the combustion chamber and investigated the heat transfer rates attainable on these grates.
In order to measure the temperature of the nodules on a Lepol grate, Weber (1) used travelling Iron-constantan thermo-couples which were embedded and sealed into a number of granules. These granules were positioned in the layer of nodules on the grate travelling along with the latter towards the kiln inlet. The leads of the thermocouple trailed along the grate and the temperature was recorded at intervals of one minute. Weber found this measuring procedure extremely troublesome, consequently the results suffered from errors of about ± 10%. He states therefore that the temperatures determined at different levels in the layer of granules were only approximate. He did not measure gas temperature in the bed at all.

Levenspiel et al. (32) determined directly film coefficients of heat transfer between gas and solid particles in beds of fluidized solids. They found that the determination of the temperature difference between gas and solid presented the principle problem in their study. The gas temperature was measured by means of a suction thermocouple, but there was no direct way of measuring the true solid temperature. The authors state that a bare thermocouple in the bed will measure a temperature between that of gas and solid. As the solid particle size decreases, a bare thermocouple should read closer to the solid temperature because there will be more points of contact of solid and couple per unit weight of solid present.

2.5. Pressure drop in fixed beds

Pressure drop in packed beds are highly sensitive to voidage as well as packing arrangements. Ergun (33) investigated the pressure drop $\Delta P$ in a non-fluidized packed bed of height $L$ consisting of
spherical particles with diameter $d_p$ and arrived at the following relation:

$$\frac{\Delta P}{L} \cdot \frac{d_p}{\rho_f U_o^2} \cdot \frac{\varepsilon^3}{1-\varepsilon} = 150 \cdot \frac{1-\varepsilon}{Re_p} + 1.75 \quad (2.10)$$

These experiments covered a range of the porosity $\varepsilon$ between 0.40 and 0.65. In the above equation $\rho_f$ is the density of the fluid and $U_o$ is the velocity which the fluid would have if no particles would be present (the approach velocity). The Reynolds number is based on this approach velocity $U_o$ and the particle diameter $d_p$

$$Re_p = \frac{U_o \cdot d_p \cdot \rho_f}{\mu_f}$$

Ergun's equation can be transformed into the form:

$$\Delta P = a \cdot \nu_f U_o + b \cdot \rho_f U_o^2 \quad (2.11)$$

in which the parameters $a$ and $b$ depend only on the geometry of the bed. This equation is often interpreted as indicating that the total resistance consists of a viscous part (first term) and a form resistance (second term). At low Reynolds numbers the viscous losses predominate and at high Reynolds numbers only the kinetic energy losses need be considered. In the intermediate regions both terms must be used.

Kirov and Szpindler (34) obtained results for the relationship between pressure drop and air mass velocity for fuel beds of different voidage. The particle size markedly influenced the air flow through the packed fuel bed. When the values of the pressure drop through the bed were plotted against the air mass velocity on logarithmic co-ordinates, a series of straight lines of varying slopes were obtained with the mean particle diameter as a parameter. The
slope of these lines varied from 1 for laminar flow to 2 for fully developed turbulent flow.

2.6. Heat transfer in packed beds

When a stream of gas is passed through a stationary granular bed, the problem of heat transfer becomes mathematically very complex because of the variation of temperature in both the gas and solid phases with respect to both time and position. In a bed of broken solids there is a great variation in the shape, arrangement, and surface conditions of particles. In addition, the gas follows an undetermined path at an unknown velocity. For the case where the particle diameters are small and the thermal conductivity high, so that temperature gradients within any one particle and the resistance to heat transfer by conduction in either the fluid or the solid particles is negligible, Schumann (35) formulated and solved these complicated equations.

2.6.1. Important factors

A very significant piece of experimental work was carried out by Furnas (36), and in these studies, the area exposed to the gas stream depended upon the size, shape and degree of packing of the particles in the bed.

Furnas found that gas velocity, bed voidage and particle size had considerably more effect than the temperature. He argued that if most of the resistance to heat transfer to the solid piece resided in the surface film, the gas velocity may be expected to have a major effect because of the decreasing thickness of the film with increasing
velocity. On the other hand, if most of the resistance to transfer resided in the body of the particle, then the gas velocity will have less effect. In Furnas's work, the coefficient was found to vary with the 0.7 power of the velocity, indicating that the internal resistance of the particles did not have any great effect, even in large sizes. Thus, increasing the gas velocity increased the coefficient of heat transfer probably because the stagnant gas film surrounding the particles became thinner as the gas velocity increased.

2.6.2. Bed packing

The degree of packing in a bed has a very large influence on the resistance to fluid flow. Most factors which increase resistance to flow may be expected to increase the coefficient of heat transfer. A decrease in voids increases the coefficient considerably, probably due to the fact that as the bed becomes more compact the gas stream has to follow a more tortuous path, which causes more head-on-collisions with the solid particles, thus aiding the transfer. Decreasing the voids also increases the linear velocity of the gas according to Furnas, which increases the coefficient. Furnas interpreted his experimental data as showing that the coefficient of heat transfer from a gas stream to a bed of broken solids can best be expressed by the empirical equation:

\[
h = k \cdot c_o^{0.7} \cdot d^{0.3} \cdot 10^{1.68 \epsilon - 3.56 \epsilon^2} / \left( \frac{d^{0.9}}{p} \right)
\]

(2.12)

where \( h \) was the volumetric heat transfer coefficient, \( k \) was a constant
dependent on the bed material, \( G_0 \) was the mass velocity of the fluid, \( T \) was an average of the entering air temperature and the initial bed temperature, \( \varepsilon \) was the fraction voids in the bed, and \( d_p \) was the average particle diameter.

2.6.3. Saunders and Ford's work

Saunders and Ford (37) approached the problem of heat transfer from a gas to a bed of solid particles from first principles in order to relate the heat transfer to the shape and dimensions of the bed and particles, the physical characteristics of the material, the velocity and temperature of the gas. They demonstrated theoretically that the heat transfer was governed by the dimensionless groups

\[ u_0 T' c' / d_p c_s \] and \[ u_0 d_p c' / k_s \] and by the shape of both bed and particles.

\( u_0 \) = mean linear velocity of the gas entering the bed.
\( T' \) = time.
\( c' \) = specific heat of unit volume of gas at constant pressure.
\( d_p \) = particle diameter.
\( c_s \) = specific heat of unit volume of material.
\( k_s \) = thermal conductivity of the particles.

Furthermore, if the size \( d_p \) of the particles is small enough or their conductivity \( k_s \) large enough for the effects of temperature differences in their interiors to be negligible, \( u_0 d_p c' / k_s \) may be neglected. The highest value of \( u_0 d_p c' / k_s \) in their experiments was 4. In the discussion of this paper, Rosin expressed the opinion that even at values of 30 or 40 for the term \( u_0 d_p c' / k_s \), this term would not appreciably influence the result, the only limit being the
capacity of the bed to store heat in relation to the amount of heat carried into it by the flow.

Saunders and Ford criticized the experiments of Furnas because the heat capacity of his insulated container was several times that of the particles inside and his results may therefore not be free from its effect which would be different for different beds and might make comparison between results unreliable. They concluded that it was doubtful whether Furnas's results were applicable except in the particular circumstances of his experiments.

2.6.4. Lof and Hawley's work

Lof and Hawley (2) investigated the heat transfer from a flowing fluid to a bed of loose solids and it was found that the results could be correlated by the equation:

\[ h = 0.79 \left( \frac{G_o}{d_p} \right)^{0.7} \]  

(2.13)

where \( h \) has units of Btu per hour per cubic foot of bed volume per °F difference between air and solid, \( G_o \) is the air flow rate lbs. per hour per square foot of bed cross-section, and \( d_p \) is the equivalent spherical diameter of the particles in feet. Change in temperature of the entering air had no appreciable effect on the coefficient.

These authors concluded that the results of Saunders and Ford were not valid for a bed of broken solids. The sharper temperature rises and the higher coefficient obtained by the latter were largely due to the fact that the smooth uniform spheres used in their work permitted much more effective and complete gas solid contact than was
possible with the rough irregular shapes used in their own experiments. The close packing possible with spherical particles caused a decreased voids fraction and results in higher values of linear gas velocity, surface area per unit volume of bed and heat transfer coefficients.

The fraction voids employed by Saunders and Ford covered the range 37.5% to 38% whereas Lof & Hawley's work involved voids of 42.6% to 45.4%.

2.6.5. General correlation of coefficients

A resume of the correlations used in the present studies is now given.

The definition of particle diameter $d_p$ varies somewhat among the investigators. Saunders and Ford used the actual diameter of the uniform spheres, Furnas used a weighted mean based on screen analysis and Lof & Hawley used a value based on the weighted mean volume of the particles. Lof & Hawley presented the correlation of Saunders and Ford and Furnas in the following way:

\[
\text{Saunders and Ford : } h = 0.152 \left( \frac{G}{d_p} \right)^{3/4} \text{kcal/hr m}^3 \text{°c.} \tag{2.14}
\]

\[
\text{Furnas : } h = (a \text{ constant}) \left( \frac{G}{d_p} \right)^{0.7} \left( \frac{1}{d_p^{0.2}} \right) \tag{2.15}
\]

The constant is a function of the nature of the material, fraction voids and the average temperature. For limestone Furnas evaluated this constant to be 0.452. Lof & Hawley with a type of gravel presented the correlation:

\[
h = 0.79 \left( \frac{G}{d_p} \right)^{0.7} \text{Btu/hr ft}^3 \text{°F} \tag{2.16}
\]
Gamson, Thodos and Hougen (38) experimented with the transfer properties of water vapour from a bed of spheres to an air stream passing through the bed and analysed their results using the j-transfer factors developed by Colburn (39). For turbulent flow of gases the heat transfer coefficient could be correlated by the equation:

\[ h = 1.064 \left( \frac{d_p G_c}{\mu_f} \right)^{-.41} \frac{k_f}{\mu_f} \left( \frac{C_f \mu_f}{k_f} \right)^{2/3} \text{ kcal/hr m}^2 \text{ oC.} \]  

(2.17)

For the flow of cooling air through a bed of heated copper spheres, Denton et al (40) suggested the average heat transfer coefficient for a given sphere in a bed as a function of the local bulk temperature of the fluid and the local mass flowrate per unit area of bed by the relationship:

\[ h = 0.72 C_f G_c (Re)^{-.3} \text{ for Pr = 0.73 } \left( \frac{\text{kcal/hr m}^2 \text{ oC.}}{\text{ft}} \right) \]  

(2.18)

Ranz (41) characterized the heat transfer rate in packed beds based on the properties of a single particle and the configuration in which it lay. According to Ranz the heat transfer coefficient between solids and flowing fluid in packed beds were given by:

\[ \frac{h d_p}{k_f} = 2.0 + 0.6 \left( \frac{C_f \mu_f}{k_f} \right)^{1/3} \left( \frac{d_p G_c}{\mu_f} \right)^{1/2} \]  

(2.19)

Ranz argued that although the superficial velocity could be obtained easily, this velocity would be considerably smaller than the actual velocity of the fluid past the particle. Kunii and Suzuki (42) suggested that this equation explained well the experimental data in the turbulent flow range, since turbulence of fluid in void spaces may
degrade effects of adjacent particles and boundary layer should develop on surfaces of particles in a similar manner to the case on an isolated particle. In the low Reynolds number, however, boundary conditions describing heat transfer in flowing media were quite different.

2.7. Mathematical modelling of moving grate systems

Madejski (43) derived equations of radiative heat transfer between surfaces in relative motion and as an example he studied the case of parallel plates. He neglected however the simultaneous heat transfer by convection and conduction and the influence of radiation at the reverse side of the plate. Van Nood and Beek (44) developed a general theory for the heating of moving surfaces and considered the total heat transfer by convection, the radiation to and from the surface, and the longitudinal heat transfer by conduction in thin metal plates. The present work differed from these two authors in several ways:

(a) Complete heat penetration in the bed cannot be assumed as Weber's results indicated. (Figure 3.1)

(b) The gas transferred heat within the bed by convection.

(c) Within the bed the hotter particles above radiated to the colder particles below.

(d) At solids temperature above 800°C the endothermic reaction of CaCO₃ decomposition proceeded.

Recently, a powerful zone method developed by Hottel and Sarofim (5) has been evolved to accomodate space variation of gas and surface temperatures with local variations in gas composition provided
sufficient knowledge of the flow pattern and combustion process was available.

However, the rigours of the zone method were not necessary since the gas above the grate was isothermal and also radiation was not the key mode of heat transfer for the system studied.
3. Theoretical model to predict bed temperature at steady state in the calciner.

3.1. Introduction.

Unlike a fluidized bed where movement of particles is rapid and gas-solid contact is enhanced, the temperature of the solids cannot reasonably be taken to be independent of location in the bed as the results of Weber shown in Figure 3.1 clearly indicate. Thus, a model which assumed complete heat penetration of the bed could not be accurate. Hence for this analysis, the bed was divided into 4 equal strata as in Figure 3.2.

At steady state on the slowly moving grate, the temperature of the nodules will not vary with time at specific points in the bed, and using Weber's data the material temperature at the beginning of the calciner at the centre of the chosen depth was used as the starting point in the computation. The temperature of the solid computed at varying distances along the grate was then compared to the measured values of Weber's data of Figure 3.1.

3.1.1. Assumptions.

1. The bed was divided into 4 strata.
2. Stratum 1 receives heat by radiation and convection.
3. Strata 2, 3 and 4 receive heat by convection and radiation from the adjacent layer above.
4. Conduction in both the horizontal and vertical directions is negligible.
5. Resistance to heat transfer within the particle is negligible. (See Appendix G)
Fig. 3-1 Temperatures on the Lepol grate.
Figure 3.2 Section on bed showing division into 4 equal strata for analysis.
6. Each stratum will be divided into 15 increments and a mean temperature for both solid and gas phase in each increment will be evaluated.

7. Plug flow of gas through the bed will be assumed and modified in the light of experimental results.

3.1.2. Conduction in the bed.

The voidage of the bed of spherical particles was 0.4 and the point to point contact area among adjacent particles for conduction will be small compared to the total surface area exposed for forced convection heat transfer to occur. Also Van Nood and Beek (44) found longitudinal conduction to be negligible in their studies.

3.1.3. Radiation between adjacent strata.

The radiation between adjacent strata was estimated by considering radiation exchange between two parallel planes, which were located at the centre point of each stratum. The standard equation for this situation is:

\[ Q = \frac{\sigma \cdot A}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2}} \left( T_1^4 - T_2^4 \right) \]  

(3.1)

where \( \varepsilon_1 \), \( T_1^M \) refer to surface 1

\( \varepsilon_2 \), \( T_2^M \) refer to surface 2

\( A \) is the area of emitting surface

3.1.4 Emissivity of the bed.

The bed of nodules consists primarily of CaCO\(_3\) and McAdams (45) quotes the emissivity of CaCO\(_3\) as 0.93 at 20°C. However, the emissivity of non-conductors generally decrease with increase in temperature
and Mc. Adams (45) quotes values of emissivity of these materials in the range 0.3 to 0.8 at furnace temperatures. Folliott (46) measured the emissivity of CaCO$_3$ when decarbonation was occurring and obtained a value of 0.3. Lack of data necessitates the use of a single value of emissivity in the bed i.e. $\varepsilon_1 = \varepsilon_2$ and equation (3.1) becomes

$$Q = \frac{\sigma \cdot A \left( \frac{2}{\varepsilon_1} - 1 \right)}{1 - \varepsilon_1} \left( T_{1M}^4 - T_{1M}^4 \right)$$

(3.2)

3.2 Heat balance for stratum 1 at steady state.

Figure 3.3 illustrates the heat balance for an increment in stratum 1.

The considered system consists of a bed of solids which moves with constant velocity $\overline{V}$ parallel to a radiating surface $T_0$ and gases of a known composition are drawn through the bed.

Assuming complete heat penetration in the stratum we obtain the following heat balance for an element $dx$, thickness $L/4$ and width $B$

Total heat transfer by convection = Rise in enthalpy of the solids.

+ Energy used for calcining.

- Energy radiated from the furnace walls.

- Energy radiated from the furnace gases.

+ Energy reradiated from the increment.
Figure 3.3 Heat balance for stratum 1 at steady state.
3.2.1. Heat transfer by convection. (α)

\[ \alpha = h \cdot \bar{A} \cdot (T_{gl} - T) = \text{kcal/hr}, \]  \hspace{1cm} (3.3)

where \( h \) is the heat transfer coefficient (kcal/hr m² °C) (See Appendix D)
\( \bar{A} \) is the surface area of solid in contact with gas (m²).
\( T_{gl} - T \) is the temperature difference between the gas and the solid in the increment. (°C). (See Appendix H)

3.2.2. Surface area of solid. \( \bar{A} \)

The evaluation of the surface area of solid in contact with gas is performed thus:

Let \( V' = \text{Volume of increment.} \)
\( V = \text{Volume of particle.} \)
\( n = \text{Number of particles in the increment.} \)
\( \varepsilon = \text{Voidage in the bed.} \)
\( S = \text{Fraction of } V' \text{ occupied by particles} = 1 - \varepsilon \)
\( r = \text{Radius of particle (assumed spherical).} \)

Total surface area = \( 4\pi r^2 n. \)

Also \( n.V = S.V' \)

\[ n = \frac{S.V'}{\frac{4}{3}\pi.r^3}. \]

Hence \( 4\pi r^2 n = \frac{3.4\pi.r^2. S.V'}{4\pi.r^3} \)

\[ \bar{A} = \frac{3.(1 - \varepsilon). V'}{r} \]  \hspace{1cm} (3.4)

Hence the surface area of solid in the increment is easily evaluated.
3.2.3. Rise in enthalpy of the solids. \( (\beta) \)

\[ \beta = m \bar{c} dT = \text{kcal./hr.} \]  (3.5)

where  \( m \) = mass flowrate of solids through the increment. (kg/hr).

(See Appendix L)

\( \bar{c} \) = weighted mean specific heat of solids. (kcal/kg°C).

\( dT \) = rise in temperature of the solids (°C).

3.2.4. Energy used for calcining. \( (\gamma) \) (See Section 2.2)

The calorific requirement for calcining was taken as 422 kcal/kg \( \text{CaCO}_3 \).

\[ \gamma = \frac{\text{Fraction of mass flowrate through increment which is calcined} \times \text{Calorific requirement.}}{\text{kg} \times \text{kcal/hr} \times \text{kg}} \]

= kcal/hr.

3.2.5. Energy radiated from the furnace walls. \( (\delta) \)

\[ \delta = A_1 F_{12} \varepsilon_w \sigma T_w^4 = \text{kcal./hr.} \]  (3.6)

\( A_1 \) = Surface area of increment. (m².)

\( F_{12} \) = View factor from wall to increment (See Appendix K)

\( \varepsilon_w \) = Emissivity of the wall. = 0.5 (Hottel and Sarofim(5))

\( T_w \) = Absolute temperature of the wall (°K). (See Appendix J)

3.2.6. Energy radiated from the gas. \( (\xi) \)

\[ \xi = A_1 F_{13} \varepsilon_g \sigma T_g^4 = \text{kcal/hr.} \]  (3.7)

\( A_1 \) = Surface area of the increment. (m²).

\( F_{13} \) = View factor from gas to bed. (See Appendix K)

\( \varepsilon_g \) = Emissivity of the gas. (See Appendix F)

\( T_g \) = Absolute temperature of the radiating gas (°K).
3.2.7. Energy re-radiated from the increment. \( (\eta) \)

\[
\eta = A_1 \cdot \varepsilon_1 \cdot \sigma \cdot T^h \quad (3.8)
\]

- \( A_1 \) = Surface area of the increment. \( (m^2) \)
- \( \varepsilon_1 \) = Emissivity of the bed.
- \( T \) = Mean of the increment. \( (^\circ K) \).

3.2.8. Design equation.

Equating these relations yield:

Enthalpy rise of solids = \( m \cdot c \cdot \left( T_1 - T_0 \right) \)

\[
= \alpha - \gamma + \delta + \xi - \eta
\]

Thus in an increment the exit temperature of the solids \( T_1 \), is expressed as follows:

\[
T_1 = \frac{\alpha - \gamma + \delta + \xi - \eta + m \cdot c \cdot T_0}{m \cdot c} \quad (3.9)
\]

The iterative procedure used for evaluating the temperature of the solids for the stratum against distance along the calciner at steady state is shown in Figure 3.4.
For \( I = 1 \) to \( 15 \)

ENTER

PHYSICAL PROPERTIES OF
FURNACE WALLS, BED AND GAS.

\[ T_0 \]

\[ T = T_0 \]

\[ T_{1}, T_{\text{gout}} \]

\[ T_{gm} = \frac{T_{\text{gin}} - T_{\text{gout}}}{\ln(T_{\text{gin}}/T_{\text{gout}})} \]

\[ T = \frac{T_{1} - T_{0}}{\ln(T_{1}/T_{0})} \]

\[ T_{1}(n+1) - T_{1}(n) > 1 \]

PRINT \( T_{1}, T_{\text{gout}} \)

\[ T_{0} = T_{1} \]

NEXT \( I \)

Figure 3.4
3.3. Heat balance for strata 2, 3 and 4 in the bed.

Consider an increment of width \(dx\), thickness \(L/4\) and width \(B\): Within the bed, 

Energy transferred by convection = Enthalpy rise of solids 
+ Energy used for calcining 
- Net energy exchange by radiation between increments.

Using the notation as before:

\[
T_{2} - T_{1} = m\cdot c\cdot dT + m\cdot R - \alpha A
\]

(3.10)

where 

\(T\) = mean temperature of solids in increment considered.

\(T_{A}\) = mean solid temperature of the increment above.

\(R\) = calcination rate heat requirement. kcal/kg.hr.

For an increment where \(T_{o}\) is the solids entering temperature and \(T_{1}\) is the solids exit temperature:

\[
dT = T_{1} - T_{o}
\]

Hence

\[
m\cdot c\cdot T_{1} = h\cdot A\cdot \left( T_{g1} - T \right) + m\cdot c\cdot T_{o} - m\cdot R + \frac{\sigma A}{\varepsilon - 1} \left( T_{A}^{\gamma} - T^{\gamma} \right)
\]

(3.11)

Thus in an increment the exit temperature of the solids \(T_{1}\) is expressed as follows:

\[
T_{1} = \alpha + m\cdot c\cdot T_{o} - \gamma + V
\]

\[
\frac{m\cdot c.}{m\cdot c.}
\]

(3.11)

where 

\(\alpha\) = Energy transferred by convection.

\(\gamma\) = Energy used for calcining.

\(V\) = Energy transferred by radiation between the increment being considered and the increment above.
In this way, the exit temperature of the increment was evaluated and the iterative procedure was repeated for 15 increments identical to the loop used in Figure 3.4 for stratum 1. Hence, the heating curves of the bed for strata 2, 3 and 4 at steady state were obtained and compared to the measured results of Weber.
3.4. Analytical approach.

An alternative analytical approach was given in Appendix P where it was shown that the heating of the moving top stratum on the grate could be described by a first order non-linear differential equation as follows:

\[ \frac{d\theta}{d\zeta} = K_1 - K_2 \cdot \theta - K_3 \cdot \theta^4 \]

where \( \theta \) = temperature of increment.
\( \zeta \) = distance along the grate.
\( K_1, K_2 \) and \( K_3 \) were physical inputs to the system.

The heating of strata 2, 3 and 4 were also shown to be described by a first order non-linear differential equation as follows:

\[ \frac{dT}{dx} = K_4 - K_2 + K_3 \cdot T^4 \]

where \( T \) = temperature of increment.
\( x \) = distance along the grate.
\( K_4, K_2 \) and \( K_3 \) were physical inputs to the system.

These equations were solved using the Runge-Kutta numerical method as follows for stratum 1:

\[ \theta_1 = \theta_0 + 1/6 \ (P_0 + 2P_1 + 2P_2 + P_3) \]
where

\[ P_0 = Z \times f(\theta_0) \]

\[ P_1 = Z \times f(\theta_0 + P_0/2) \]

\[ P_2 = Z \times f(\theta_0 + P_1/2) \]

\[ P_3 = Z \times f(\theta_0 + P_2) \]

\[ Z = \text{step length.} \]

These results were compared with those obtained from the numerical method outlined in Sections 3.1 to 3.3 and were plotted in Figures P1 to P8 in Appendix P.

3.4.1. Disadvantages of the analytical method

(a) The analytical method assumed that the whole increment could be assigned a single temperature, whereas the numerical method evaluated a mean temperature for the increment. A single temperature for the increment is correct if small steps are made in the integration.

(b) Inspection of Figures P1 and P5 showed that at higher temperature gradients experienced as in stratum 1, the assumption of a single solid temperature for the whole increment was too coarse, and the log mean temperature of the solids evaluated in the numerical model was a better approximation to the realistic situation.

(c) The numerical solution was simpler and furthermore the analytical equations ultimately required the Runge-Kutta numerical solution.
3.5. Isothermal scale modelling

The use of isothermal scale models for the investigation of the aerodynamics of furnaces and other industrial plant is well established. Models have been used extensively for evaluating flow patterns and velocity distributions in a variety of conventional combustors and have yielded much information useful to designers of furnaces in the steel, cement and power industries.

3.5.1 Similarity criteria

(i) Geometric.

Geometric similarity between the model and furnace was obtained by directly scaling the linear dimensions of the furnace chamber. The scale used was 1 : 24.

(ii) Kinematic

Kinematic similarity required that velocity ratios in the furnace be reproduced in the model. Velocity distribution in the furnace chamber is controlled by the momentum of the fluid leaving the kiln tube. However, measurements of velocity ratios in the furnace do not exist, and hence kinematic similarity could not be reproduced.

(iii) Dynamic.

Dynamic similarity requires equal Reynolds numbers in both the prototype furnace and in the model (47). Thus
It is not generally possible to specify a precise Re for a furnace because of uncertainty as to the appropriate length and velocity terms. The Reynolds numbers were calculated using a hydraulic mean diameter of the calciner and drier chamber grate area thus:

\[
\frac{(\rho \cdot D_m \cdot V)}{\mu}_{\text{model}} = \frac{(\rho \cdot D_m \cdot V)}{\mu}_{\text{furnace}}
\]

In practice, Evans and Patrick (15) state, that with forced convection, provided Re \(> 10,000\) in both model and prototype, it need not be the same in both. Philbrook (16), Curtis and Johnson (48) also suggest that once fully developed turbulent flow has been attained, flow patterns, mixing and dynamic pressure losses are essentially independent of Re. Therefore, as long as the flow was well up into the turbulent region, the Re. model can be considerably less than in the prototype without seriously affecting the validity of the results, unless the model is to be used to determine frictional pressure drop. In this case, it may be necessary to duplicate Reynolds numbers to assure accuracy.

3.5.2. Satisfying these criteria

High exit kiln velocities and the spiralling nature of the flow due to the double vortex nature of the flow above the calciner chamber (see Plate 5.2) promote uniform gas temperatures above the
bed. In the present case of high speed flow within a fully running system, it is clear that gravitational and surface tension effects can be considered negligible. Hence buoyancy effects can be discarded. Chigier (49) states that in combustion systems it is not possible to satisfy all the similarity criteria that are pertinent to the particular system, and that experience has shown that certain of the similarity criteria are more important than others. If one is prepared to accept a certain degree of approximation the following guide rules may be applied:

(a) **High Reynolds number**

When the Reynolds number exceeds 4000 in ducted flow systems, transfer processes of momentum, heat and mass are controlled by turbulent forces and molecular transfer processes can be neglected. There is therefore no need to maintain equal Prandtl or Schmidt numbers in model and prototype. The Prandtl represents the ratio of kinematic viscosity to thermal diffusivity and the Schmidt number is the ratio of the kinematic viscosity to the diffusivity.

\[
Pr = \left( \frac{C_p \mu}{k} \right)_{\text{fluid}}
\]

\[
Sc = \left( \frac{\mu}{\rho \cdot D} \right)_{\text{fluid}}
\]

(b) **High Froude number**

\[
Fr = \frac{U^2}{gD} = \frac{\text{inertia force}}{\text{gravitational force}}
\]  

where \( U \) = fluid velocity
When the ratio of inertial to gravitational forces is high, buoyancy effects are negligible and there is no need to maintain the same Froude number in model and prototype. In furnaces with large diameter burners and low air velocities, buoyancy effects cannot be neglected.

(c). Non-isothermal conditions

When non-isothermal conditions in the prototype are simulated with isothermal flow in the model, the geometrical similarity ought to be abandoned in order to maintain the mass flow and velocity ratios while distorting the nozzle diameter in the model by the ratio of the square root of the densities of the cold fluid and of the hot flame gases according to the Thring-Newby criterion(50).
4. Description of the Hydraulic System.

Plate 4.1 shows an overall photograph of the apparatus, and Fig. 4.1 represents a line flow diagram. The apparatus was a closed loop design using water as the operating fluid and the perspex model was mounted on a steel frame which was bolted to the floor. Essentially, the water was first pumped through the kiln tube and into the calciner part of the model Plate 4.2 using pump A, then pump B directed the water from the calciner chamber to the drier chamber Plate 4.3. The water flow from the drier was connected to the suction of pump A and in this way, the loop was completed.

4.1. Operating fluid.

Water was chosen as the operating fluid since this will yield qualitatively results about this relatively unknown system, and allowed simultaneous observation of the whole field of flow. In addition observations were made at the time that the flow pattern existed, and flow similarity was obtained using water at much lower velocities than in a gas, due to viscosity characteristics thus permitting results to be obtained by:—

(a) Visual observation and drawing.
(b) Photography both still and cine.

4.1.1. Polystyrene bead-tracing.

For the observation of flow, polystyrene beads were chosen since they had the following advantages as stated by Winter and Deterding(20).
Fig 4.1

Line diagram of flow circuit

FLOW LOOP
--- ORIFICE PLATE
--- ROTAMETER

VALVES
1- MAINS WATER SUPPLY
2- SUMP ISOLATOR
3- RECYCLE
4- SHUTE INLEAKAGE
5- CHoke
6- CALCINER RIDDINGS INLEAKAGE
7- DRIER RIDDINGS INLEAKAGE
8- EX-CALCINER CHAMBER 3 OFF
9- EX-DRIER CHAMBER 3 OFF
10-12- AIR BLEED

PUMPS
A- CALCINER DELIVERY - DRIER SUCTION
B- DRIER DELIVERY - CALCINER SUCTION
Plate 4.2. Side view of calciner chamber and shute.
(a) They remained as discrete bodies in the flow.
(b) They possessed a specific gravity close to water, the maximum range of relative density being 0.93-1.05, the majority being between 0.98 and 1.02.
(c) The beads were spherical in shape.
(d) They possessed the optical property of high proportional reflection or re-radiation of light normal to the incident beam.

4.2. Design and construction of the apparatus.

The geometric design of the model was based on a 20.8 m. long and 4.1m. wide Lepol Grate and scaled at 1/24th the full size (¼'' = 1'). Since the chief area of interest was around the grate, only 1/3 of the prototype kiln length was modelled. Figures 4.2 to 4.7 were the key drawings used in the fabrication of the perspex model and its ancillaries. Construction of the model was carried out using perspex of 1" thickness with glued and screwed joints where necessary. The calciner and drier suction tubes (Plate 4.4) were fabricated in 24 guage stainless steel (Fig. 4.7). Pump B transported the fluid from calciner chamber to drier chamber and the square duct manifold through which the water flows into the drier chamber is shown on Plate 4.5.

The kiln tube and the curved part of the top of the model were surrounded in a rectangular tank made in perspex. The tank was essential with these curved surfaces to prevent optical distortion when being viewed and also eliminated the problem of condensation on these outside surfaces.
Figure 4.2. Key elevations of perspex Lepol grate.
Figure 4.3. Side view of below grate.
Figure 4.4. Grate to kiln flange arrangement.
Figure 4.5. Development of kiln shute.
Figure 4.6. Kiln end reducer.
Figure 4.7. Calciner and drier suction tubes.
Plate 4.4. Calciner and drier suction tubes.
Plate 4.5. Drier duct in-situ.
The various parts of the rig are detailed below.

4.2.1. Pumps.

A general view of the pumping bay is shown in Plate 4.6. The system consisted of two centrifugal pumps labelled A and B on the line diagram in Fig. 4.1. Pump A transported the fluid into the calciner chamber and out of the drier chamber, and Pump B circulated the water from the calciner chamber into the drier chamber, thus forming a closed loop. The characteristics of the pumps were as follows:

A : 4DM4/35 5h.p. ~ 300 gpm 30 ft. head.
B : 2DM12/39A 5h.p. ~ 100 gpm 90 ft. head.

Both pumps manufactured by the Worthington Simpson Company had 3-phase, drip-proof motors. Pump A was fitted with a recycle (valve 3 in Fig. 4.1). Since only part of its output was required and by allowing the pump to work at full output, the possibility of cavitation of the impeller was reduced. A recycle was not necessary for Pump B since it was operating at full output for most of the experiment.

4.2.2. Piping.

All piping used was the Le Bas u P.V.C. 2" or 4" diameter to reduce corrosion. For assembly all 2" joints were threaded and 4" diameter pipes glued. Ducting of 4" diameter u P.V.C. conveyed the water from the sumps via pump A and the choke valve (valve 5 in Fig. 4.1) to the model kiln tube. Provision was made for the shute inleakage and the drier and calciner riddlings inleakage by tapping off at three convenient positions between Pump A and the choke valve with 2" piping. Finally, both ex-drier and ex-calciner flows were conveyed in 2" piping.
Plate A.6. Pumping bay.
4.2.3. Valves.

The sump isolator, recycle and choke valves were 4" rubber-lined butterfly valves to resist corrosion, and, all other flows were controlled with 2" u P.V.C. angle seat valves.

4.2.4. Flow Measurement.

All flow measurements were achieved using rotameters supplied and calibrated by G.E.C. Elliott Ltd., and orifice plates constructed according to BS1042 PT.2. There were three similarly constructed orifice plates of 1" orifice situated in 2" pipe to monitor the smaller inleak flows, and the pressure differentials were measured using mercury manometers. Flow through the calciner and drier chamber were each controlled by means of a bank of 3 rotameters in parallel. Each rotameter had a range 16-200 l/min. water at 20°C. Flow regulation was obtained by adjusting the angle seat valves situated before the orifice plates and rotameters.

4.2.5. Sump tanks.

Two identical sumps, fabricated from polythene to reduce corrosion were used and the dimensions of each tank was:

61" x 37\frac{1}{2}" x 36" deep capacity = 295 gallons.

They were linked by a 2" pipe, situated 6" above the base of each tank, and high-low level indicators were fitted into the sump connected to mains water supply to facilitate the filling of the tanks. Furthermore, these tank capacities were adequate to prevent vortexing due to pump suction and in addition the tanks were placed 10 feet above the pump to provide positive suction head at maximum flowrate.
4.2.6. Bed resistance.

The bed of nodules in the prototype was simulated by square brass mesh in the model. The mean voidage in the prototype was 0.40. For experimental work, two types of mesh were used to create two values of voidage based on the surface area of the brass. The dimensions are stated below:

<table>
<thead>
<tr>
<th>Diameter of Wire</th>
<th>Length of enclosed air space</th>
<th>Voidage based on area</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.010</td>
<td>0.020</td>
<td>0.25</td>
</tr>
<tr>
<td>0.020</td>
<td>0.060</td>
<td>0.36</td>
</tr>
</tbody>
</table>

The mesh was stretched over a rigid rectangular framework of stainless steel strips (\(\frac{1}{16}\)" wide) which was spot welded along the edge. The excess mesh was then trimmed and the framework was then located in the model on \(\frac{1}{4}\)" square perspex runners. Whereas the coarser mesh allowed the polystyrene beads tracer to pass through, the finer mesh did not permit this. In this case, the framework was given a slack fit on the runners which enabled the beads to trickle down the side to the lower half of the model.

4.2.7. Lighting and Photography.

The entire centre plane of the model was illuminated by 4 x 1000 watt quartz halogen lights evenly spaced and situated above
1" wide slit in the shape of a tunnel. The tunnel painted in matt black reduced excess internal reflection in the model. Both oblique and back lighting were tried but found to be unsatisfactory. The background of the area to be studied was made black with hard board painted with matt black. This provided good contrast against the white polystyrene beads for photography.

There were many problems photographing the lower half of the model. This was due to three reasons:

(a) The brass gauze reduced the light intensity of the sources of light above.
(b) The suction tubes obstructed light from above and cast shadows in the model.
(c) The sloped nature of the lower half of the model resulted in excessive internal reflection of light incident on these surfaces.

Fortunately, the fluid was moving relatively slowly in these regions, which enabled flow patterns to be drawn by hand.

Various still and cine trials were conducted to find the best photographic conditions, and suitable results were obtained filming at a speed of 200 f.p.s. with a Hycam K20 S4E camera fitted with a 10 mm, f/1.8 Schnieder lens. Ilford Mk.V film rated 400 A.S.A. nominal was used. Stills were taken using a Pentax SP 1000 single lens reflex with shutter speeds varying from 1/250 sec to 1/30 sec depending on the region under investigation. The cine film produced was projected onto a 45° silvered plane, then onto graph paper where the observed results were analysed and in this way the flow patterns were derived and reduced to static conditions.
4.3. Operation of the Model.

Prior to filling the model, the air bleed taps were opened and 500 cc of polystyrene beads were emptied into the main tanks. After filling with water, the mixture was stirred vigorously for several minutes. The sump isolator valve and the remaining valves were then opened, and the head of water from the tanks filled the model expelling the air trapped in the system. The bleed valves fitted on the centrifugal pumps were opened and the air allowed to escape. After several minutes, the pumps were completely primed and free of air. Both pumps were then started simultaneously and remaining air trapped in the system was expelled via the air bleed taps. After expulsion of the air the bleeds were then closed. By this time, the polystyrene bead tracers were well distributed in the system, and the calciner and drier flowrates were then set at the appropriate flows and the model allowed to run for 20 minutes to attain steady state. After such time, filming was commenced.

4.3.1. Reynolds Number in the Model.

The Reynolds number in the real furnace based on the calciner grate area was $4.13 \times 10^4$. However, to maintain this value in the water model would require excessive flow velocities particularly at the kiln exit. Consequently, the flow was set up using as the limiting condition the flow obtainable by the calciner suction pump A. This resulted in a Reynolds number based on the model calciner area of $2.2 \times 10^4$ which was approximately half that calculated for the real furnace for conditions of plug flow.

4.3.2. Operational difficulties.

During circulation, the water underwent pressure changes
which resulted in the release of gas bubbles, for example at the valves bends, orifice plates, and changes of section. The water also picked up any air which was not displaced from the system and carried it into the model as a dispersion of bubbles. The presence of such bubbles can render the water opaque and prevent the projection of a straight beam of light through the model. Twenty minutes was found to be an adequate period for the substantial elimination of troublesome gas bubbles after the system was newly filled. A few ounces of detergent in the circulating water assisted the dispersion of bubbles from the walls of the system. The inside surfaces of the model were also normally wetted with detergent before the system was filled with water.

The water was found to exhibit a corrosive action on the impellers of the pumps, resulting in the dispersion of the corrosion products in the water and on the perspex surfaces and consequent increase of the opacity. Slight deterioration of the water occurred, but the increasing cloudiness presented no problem over a period of several days. Changing the water twice a week and flushing the model through at the same time were ample precautions to maintain adequate clarity at the model.
5. RESULTS

5.1. Results and model operating conditions.

The operation and construction of the model was given in Section 4, and was based on the considerations of Section 3.5 which describes the modelling criteria for flow scaling. Representation of the grate was made using two grades of wire gauze. Three series of runs are presented for the model and in each case two distinct flowrates were used and these conditions are summarized in Table 5.1.

The experiments were divided into 3 groups:

Test 1: No bed resistance. Figures 5.1 to 5.3
Test 2: Bed resistance with a voidage of 0.25. Figures 5.4 to 5.11.
Test 3: Bed resistance with a voidage of 0.36. Figures 5.12 to 5.18.

5.1.1. Reynolds numbers around the model.

Reynolds number in calciner chamber (Plug flow)

Total flowrate = 450 1/min.

= 27000 1/hr.

= 27 m³/hr.

Area of calciner grate = 6½" x 20½"

= 0.168m. x 0.521m.

Hydraulic mean diameter = \( \frac{2 \times 0.521 \times 0.168}{0.168 + 0.521} \) = 0.254 m.

Velocity based on plug flow through calciner

= 27/ (0.168 x 0.521)

= 308.5 m/hr.

Hence Re. calciner = \( \frac{1000 \times 0.254 \times 308.5}{3.6} \)

= 2.18 \times 10^4
Reynolds number at kiln exit.

Flowrate = 450 l/min.
Diameter of kiln exit = 3.7 ins = 0.0939 m.
Fluid velocity = \( \frac{2.7 \times 4}{\pi \times 0.0939^2} \) m/hr
= 4288 m/hr.
Hence Re. at kiln exit = \( \frac{1000 \times 0.0939 \times 4288}{3.6} \)
= 1.1 \times 10^5

Reynolds numbers at the lower flowrate.

Similarly with a flowrate through the model = 300 l/min.
= 18 m³/hr.
Re. in calciner chamber based on plug flow = 1.45 \times 10^4
Re. at kiln exit = 6.8 \times 10^4

Calculation of Re. in drier section.

Drier area = 13\(\frac{1}{4}\)" x 6\(\frac{1}{2}\)"
= 0.343 m. x 0.168 m.
= 0.0576 m²
Hydraulic mean diameter = 0.225 m.
Flowrate = 450 l/min. = 27 m³/hr.
Velocity based on plug flow through drier bed
= \frac{27}{0.0576}
= 469 m/hr
Re. = \( \frac{1000 \times 0.225 \times 469}{3.6} \)
= 2.93 \times 10^4

Similarly for a flowrate of 300 l/min = 18 m³/hr.
Re. in drier chamber = 1.95 \times 10^4
Reynolds number down duct into drier.

The entry duct into the drier consisted of a 3" x 3" square cross-section

\[ = 6.889 \times 10^{-3} \text{ m}^2. \]

Hydraulic mean diameter = 0.083 m.

For a flowrate \( = 450 \text{ l/min} = 27 \text{ m}^3/\text{hr}. \)

\[ \text{Re. in duct} = 9 \times 10^4 \]

Similarly for a flowrate \( = 300 \text{ l/min} = 18 \text{ m}^3/\text{hr}. \)

\[ \text{Re. in duct} = 6 \times 10^4 \]

5.1.2. Voidage of gauze used to represent the bed.

Two grades of mesh were used:

Fine grade.

Looking at one square:

\[ \text{Diameter of wire} = 0.010 \text{ ins.} \]

\[ \text{Air space area} = 0.020 \times 0.020 = 0.0004 \text{ sq.ins.} \]

\[ \text{Total area of space + wire} = 0.04 \times 0.04 = 0.0016 \text{ sq.ins.} \]

\[ \text{Voidage } \varepsilon = \text{fraction of area occupied by space}. \]

\[ \frac{0.0004}{0.0016} = 0.25 \]
Coarse grade.

\[ \text{Air space area} = 0.06^2 = 0.0036 \text{ sq.ins.} \]
\[ \text{Total area of space + wire} = 0.1 \times 0.1 = 0.01 \text{ sq.ins.} \]
\[ \varepsilon = \frac{0.0036}{0.01} = 0.36 \]

<table>
<thead>
<tr>
<th></th>
<th>Flowrate</th>
<th>1/min</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>450</td>
<td>300</td>
</tr>
<tr>
<td>Re. ex-kiln</td>
<td>$1.1 \times 10^5$</td>
<td>$6.8 \times 10^4$</td>
</tr>
<tr>
<td>Re. calciner area</td>
<td>$2.2 \times 10^4$</td>
<td>$1.5 \times 10^4$</td>
</tr>
<tr>
<td>Re. drier area</td>
<td>$2.9 \times 10^4$</td>
<td>$2.0 \times 10^4$</td>
</tr>
<tr>
<td>Re. duct</td>
<td>$9.0 \times 10^3$</td>
<td>$6.0 \times 10^3$</td>
</tr>
</tbody>
</table>

Table 5.1. Reynolds numbers around the model under plug flow conditions
5.1.3. Re. for prototype calciner area assuming plug flow.

The Reynolds number for the prototype based on the calciner area which represents the largest area available for flow is as follows:

\[ \text{Gas flow ex-kiln at } 1000^\circ\text{C} = 1.88 \times 10^5 \text{ m}^3/\text{hr at 1000} \]
\[ \rho \text{ gases at } 1000^\circ\text{C} = 3.281 \times 10^{-1} \text{ kg/m}^3 \]
\[ \mu \text{ gases at } 1000^\circ\text{C} = 1.836 \times 10^{-1} \text{ kg/hr.m} \]
\[ \text{Length of calciner} = 12.2\text{m} \]
\[ \text{Width of calciner} = 4.1\text{m} \]
\[ \text{Hydraulic mean diameter} = 6.14\text{m} \]
\[ \text{Velocity based on calciner X-section} = 3.76 \times 10^3 \text{m/hr} \]

Hence Re. prototype

\[ = \frac{3.281 \times 10^{-1} \times 6.14 \times 3.76 \times 10^3}{1.836 \times 10^{-1}} \]
\[ = 4.13 \times 10^4 \]

Hence the Re. no. in the model based also on a plug flow assumption is 0.53 times the Re. no. in the prototype.

\[ \text{Re. model} = 0.53 \text{ Re. prototype}. \]
5.1.4. Test 1.

Figures 5.1 to 5.3 represent the flow patterns of the calciner and drier chambers with no bed resistance. These results are of no practical importance but represent base line conditions against which the flow patterns could be compared when a resistance was inserted in the model. Essentially, these results verified that the gauze used to simulate the bed did alter the flow.

5.1.5. Test 2.

Figures 5.4 to 5.11 represent the flow patterns of the calciner and drier chambers with the voidage as calculated in 5.1.6 equal to 0.25. The effect of riddlings air inleakage into the calciner chamber was also investigated and Figure 5.7 shows this result.

The flow patterns around the drier chamber in Figures 5.8 to 5.11 were obtained under the two main flowrates through the model of 450 and 300 l/min. In addition, the effect of shute and riddlings in-leakages were investigated and these are plotted in Figures 5.10 and 5.11 for the drier.

5.1.6. Test 3.

Figures 5.12 to 5.18 represent the flow patterns of the calciner and drier chamber with the voidage of the bed increased from 0.25 to 0.36 as calculated in 5.1.6.
Figure 5.1. Derived overall flow pattern of calciner chamber with no bed resistance.

Two flowrates used:
450 litres/min.
300 litres/min.
Figure 5.2. Derived overall flow pattern of drier chamber with no bed resistance.
Figure 5.3. Derived overall flow pattern of drier chamber with no bed resistance.
Figure 5.4: Derived overall flow pattern of calciner chamber. (Higher flowrate)

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Voidage based on area of bed</td>
<td>0.25</td>
</tr>
<tr>
<td>Flowrate through model</td>
<td>450 litres/min.</td>
</tr>
<tr>
<td>Riddlings inleakage</td>
<td>nil.</td>
</tr>
<tr>
<td>Re. ex-kiln</td>
<td>$1.1 \times 10^8$</td>
</tr>
<tr>
<td>Re. based on area of bed</td>
<td>$2.2 \times 10^4$</td>
</tr>
</tbody>
</table>
Figure 5.5: Derived overall flow pattern of calciner chamber. (Lower flowrate)
Figure 5.6: Derived overall flow pattern of calciner chamber. (Higher flowrate + inleakage)
Distance from kiln exit.

Figure 5.7: Flow pattern of above calciner cross-section at three positions along the bed. Flowrate = 450 litres/mi
Flowrate through model = 450 litres/min.
Voidage based on area of bed = 0.25
Shute inleakage = nil.
Re. through duct = 9.0 x 10^4
Riddles inleakage = nil.
Re. based on area of bed = 2.9 x 10^4

**Figure 5.8:** Derived flow pattern of drier chamber. (Higher flowrate)
Flowrate through model = 350 litres/min. Voidage based on area of bed = 0.25
Shute inleakage = nil. Re. through duct = $6 \times 10^4$
Riddlings inleakage = nil. Re. based on area of bed = $1.9 \times 10^4$

Figure 5.9. : Derived flow pattern of drier chamber. (Lower flowrate)
Flowrate through model = 450 litres/min. Voidage based on area of bed = 0.25
Shute inleakage = 50 litres/min. Re. through duct = 9 x 10^4
Riddlings inleakage = nil. Re. based on area of bed = 2.9 x 10^4

Figure: 5.10. Derived flow pattern of drier. (Higher flowrate + shute inleak...
Flowrate through model = 450 litres/min. Voidage based on area of bed = 0.25
Shute inleakage = 50 litres/min. Re. through duct = $9 \times 10^4$
Riddlings inleakage = 50 litres/min. Re. based on area of bed = $2.9 \times 10^4$

Figure 5.11. : Derived flow pattern of drier chamber. (Higher flowrate + shute + riddlings inleak)
Figure 5.12: Derived overall flow pattern of calciner chamber (higher flowrate)

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Voidage based on area of bed</td>
<td>0.36</td>
</tr>
<tr>
<td>Flowrate through model</td>
<td>450 litres/min.</td>
</tr>
<tr>
<td>Riddlings inleakage</td>
<td>nil.</td>
</tr>
<tr>
<td>Re. ex-kiln</td>
<td>$1.1 \times 10^5$</td>
</tr>
<tr>
<td>Re. based on area of bed</td>
<td>$2.2 \times 10^6$</td>
</tr>
<tr>
<td>Description</td>
<td>Value</td>
</tr>
<tr>
<td>------------------------------------------</td>
<td>------------------------------------</td>
</tr>
<tr>
<td>Voidage based on area of bed</td>
<td>0.36</td>
</tr>
<tr>
<td>Flowrate through model</td>
<td>450 litres/min.</td>
</tr>
<tr>
<td>Riddlings inleakage</td>
<td>50 litres/min.</td>
</tr>
<tr>
<td>Re. ex-kiln</td>
<td>$1.1 \times 10^5$</td>
</tr>
<tr>
<td>Re. based on area of bed</td>
<td>$2.2 \times 10^4$</td>
</tr>
</tbody>
</table>

**Figure 5.13.** Derived overall flow pattern of calciner chamber (higher flowrate + inleakage)

- **Recirculation**
- **Bed resistance**
- **Suction tubes**
- **Random flow**
- **Kiln tube**
- **Kiln Shute**
- **Region of upward flow**
Figure 5.14.: Derived overall flow pattern of calciner chamber (lower flowrate)

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Voidage based on area of bed</td>
<td>0.36</td>
</tr>
<tr>
<td>Flowrate through model</td>
<td>300 litres/min.</td>
</tr>
<tr>
<td>Riddlings inleakage</td>
<td>nil.</td>
</tr>
<tr>
<td>Re. ex-kiln</td>
<td>$6.7 \times 10^4$</td>
</tr>
<tr>
<td>Re. based on area of bed</td>
<td>$1.5 \times 10^4$</td>
</tr>
</tbody>
</table>
Vortex Flowrate = 50 litres/min
Voidage = 0.36

Figure 5.15. Flow pattern of above calciner cross-section at 10cms. from kiln exit for two flowrates.
Figure 5.16: Flow pattern of above calciner cross-section at 20 cms. from kiln exit for two flowrates.

Flowrate = 450 litres/min
Voidage = 0.36

Flowrate = 300 litres/min
Voidage = 0.36
Flowrate = 450 litres/min.
Voidage = 0.36

Flowrate = 300 litres/min.
Voidage = 0.36

Figure 5.17. : Flow pattern of above calciner cross-section at 37 cms. from kiln exit for two flowrates.
Voidage based on area of drier bed = 0.36
Flowrate through model = 450 litres/min.
Riddlings inleakage = Nil
Shute inleakage = Nil
Re. through duct = $9 \times 10^4$
Re. based on area of drier bed = $2.93 \times 10^4$

Figure 5.18.: Flow pattern around the drier bed
TITLE OF PLATES.

PLATE 5.1 : Typical pattern at the kiln shute obtained at f = 8 and a shutter speed of 1/125 secs using a central slit of 1000 watts intensity.

PLATE 5.2 : Typical double vortex on the above calciner cross-section, at a distance 8 ins. from the kiln shute at f = 8 and a shutter speed of 1/125 secs with a slit of 1000 watts intensity.
5.2. Mathematical model.

5.2.1. Results of the mathematical model. (Equations 3.9 and 3.11)

The temperature (VS) distance relationships developed for the calciner in Section 3 were evaluated using a digital PDP 8 computer. The results were plotted for each stratum in the bed, and are illustrated in Figures 5.19 to 5.24. Appendix M gives the results of the computations for each stratum.

For stratum 1, the packed bed heat transfer correlations were tested, namely those of Saunders and Ford (37), Gamson et al. (38) and Lof and Hawley (2). All three were used to calculate the solids temperature against distance along the calciner for plug flow of gas through the grate. The results are plotted in Figure 5.19 and are compared to the measured results of Weber. Using the Lof and Hawley correlation, a negligible deviation was obtained, but the Saunders and Ford relationship gave overestimated results with a 27% maximum deviation, and similarly the Gamson et al. results showed a 39% maximum deviation from Weber's results. Hence, the results for strata 2, 3 and 4 were then calculated using the Lof and Hawley correlation since there was good agreement between predicted and measured results for stratum 1.

5.2.2. Modified flow through the bed.

The results in Section 5.1 for the water model indicate that the plug flow assumption was not correct since flow of gas through the bed was confined to 80%
of the grate area. In the light of this information, the
Lof and Hawley heat transfer coefficient was modified
using a gas velocity based on this reduced grate area,
and for the region of the grate where gas flow through
the bed did not exist, the heat transfer coefficient
was equated to zero.
FIGURE 5.19 Predicted temperature of solids along calciner under plug flow conditions.
Figure 5.20 Predicted temperatures along calciner under plug flow and modified flow conditions using the Lof and Hawley correlation.
Figure 5.21 Predicted temperatures under plug flow and modified flow using the Lof and Hawley correlation.
Figure 5.22 Predicted temperatures under plug flow and modified flow using the Lof and Hawley correlation.
Figure 5.23 Predicted temperatures under plug flow and modified flow using the Lof and Hawley correlation.
STRATUM I

Region of no gas flow.

Fractional distance along calciner.

Figure 5.24 Solids temperature attained at various grate speeds.
5.2.3. Modified heat-transfer coefficient.

Total calciner area = 48.4 m²
Active area of gas flow = 4/5 x 48.4 = 38.72 m²

Gas flow ex-kiln = 1.88 x 10⁵ m³ at 1000°C.
Density gas mixture at 1000°C = 3.281 x 10⁻¹ kg/m³
Hence gas mass flowrate modified =

\[ \frac{1.88 \times 10^5 \times 3.281 \times 10^{-1}}{38.72} = 1.59 \times 10^3 \text{ kg/hr.m}^2 \]

\( Go \text{ mod.} = 1.59 \times 10^3 \text{ kg/hr.m}^2 = 3.26 \times 10^2 \text{ lbs/hr ft}^2 \)

\( d = 0.03125 \text{ ft} \)

\( h = 0.79 \cdot (Go/d)^{0.7} \text{ modified. (Lof and Hawley).} \)

\( = 513 \text{ Btu/hr ft}^3 \text{ °F} \)

The surface area in 1 ft³ of bed = 115.2 ft²

\( h = 4.453 \text{ Btu/hr ft}^2 \text{ °F} \)

\( = 21.73 \text{ kcal/hr m}^2 \text{ °C}. \)

The heat transfer coefficient based on a plug flow assumption using the Lof and Hawley correlation was 18.68 kcal/hr m² °C. Hence, the modified heat transfer coefficient represents a 16.3% increase.

Using this value of heat transfer coefficient the modified flow prediction for both the mean gas temperature in an increment and the exit solid temperature from an increment were evaluated and compared to the results with a plug flow assumption. These results were plotted in Figures 5.20 to 5.23 for stratum 1 to 4.

Finally the effect of varying the speed of the grate was investigated for modified flow conditions and these results are plotted in Figure 5.24.
6. Discussion of results.

6.1. Flow patterns from the Water Model.

The water model results shown provide a qualitative representation of the flow taking place in the Lepol model system. The flow diagrams were produced by making detailed cine and still photography, using polystyrene beads as tracers, as described in Section 4.1.1. As Winter and Deterding(20) observed, in the turbulent flow regime, with flow patterns which were substantially two dimensional, a sufficient degree of orderliness frequently occurred for the flow to be plotted qualitatively by direct inspection of a single set of tracers.

6.1.1. Flow patterns with no bed resistance.

A short series of experiments were conducted to show that the insertion of the stainless gauze to represent the bed did in fact alter the flow. These results, Figures 5.1 to 5.3, were baseline conditions and bear no relevance to actual plant operation, but they do show many of the features that were inherent in the other model results.

These were a region of random flow above the kiln shute, and a small region of recirculation at the beginning of the calciner chamber for both flowrates used. However, in the absence of a bed resistance, the flow swirl around the suction tubes extended up into the calciner chamber above the "would-be" grate level.

In the drier chamber, Figures 5.2 and 5.3 show that at the higher flowrate, a region of recirculation exists in the upper part of the chamber. The upward flow of fluid in the outer regions of the drier for both flowrates was due to the fluid impinging on the bottom of the chamber and then deflecting.
6.1.2. Flow patterns in the calciner chamber. Voidage = 0.25

With this bed resistance and a flowrate of 450 l/m., the following observations were made from Figure 5.4 for the central plane:

(a) As the fluid exited the kiln tube, it was accelerated due to a reduced cross-section.
(b) A region of random or non directional flow was created above the shute.
(c) Fluid leaving the shute impinged on the calciner roof and was reflected.
(d) A region of recirculation was observed at the beginning of the calciner.
(e) The fluid drawn through the grate followed a sweeping curve.
(f) Flow of fluid drawn through the grate was confined to 4/5 ths. of the available bed area.

When the flowrate through the model was reduced to 300 l/m. identical trends were observed as shown in Figure 5.5 but the region of recirculation was marginally reduced. This showed that the flow patterns were independent of the flows used. This observation was in agreement with Philbrook(16) who suggested that once fully developed turbulent flow has been attained, flow patterns, mixing and dynamic pressure losses are essentially independent of Reynolds number.

Finally, it was observed in Figure 5.6 that when inleakage of fluid at the riddlings outlet was simulated, the pattern below the grate was altered. The previous upflow nature of the flow at the end of the calciner was transformed to an overall downward flow of fluid.
In effect, the riddlings inleakage was acting as a jet issuing into a confined space and the intrinsic property of such a jet is to entrain slower moving surrounding fluid. This property manifests itself by drawing fluid down the latter part of the grate, but does not affect flow through the bed itself.

6.1.3. Region of random flow and recirculation.

The region of random flow or "dead fluid" which occurred above the shute and the area of recirculation in the above calciner originated from the geometry of these regions and can be explained by boundary layer theory. For fluid flow across a surface there is a zone of retarded fluid called the boundary layer and the mathematical treatment of this layer is well quoted in the literature (51,52).

According to Rouse(51) two characteristics of the boundary layer are:

(1) Within a very small distance normal to the boundary the velocity increased from zero to practically that of the corresponding potential motion.

(2) The pressure intensity throughout the boundary layer is governed by the surrounding flow.

So long as the pattern of the corresponding potential motion indicates either a constant or a steadily increasing velocity at the outer border of the boundary layer, the boundary pressure gradient in the direction of flow will be equal to or less than zero. If on the other hand, the potential pattern requires deceleration along the boundary, the decrease in velocity must be accompanied by a corresponding
increase in pressure intensity according to Bernoulli's equation. The kinetic energy lost in the growth of the boundary layer is no longer available for restoration of pressure, and such a restoration is essential if the potential motion is to continue. From this conclusion it would seem clear that flow under given geometrical boundary conditions is physically possible only if the potential pattern so modifies itself that no deceleration is required. This seemingly paradoxical statement merely signifies that the potential flow net must detach itself from the boundary at the point where deceleration becomes appreciable, and beyond that point enclose a region of discontinuity instead of a solid boundary.

6.1.4. Separation of the boundary layer.

Separation of the flow from a boundary necessarily involves a complete change in the velocity distribution within the layer. This is indicated schematically in Figure 6.1, in which conditions within the boundary layer are clarified by enlarging the scale of normal distance y.

If there is a pressure gradient force outside the boundary layer decelerating the fluid it is possible that, since the fluid in the boundary layer is moving more slowly, it may be brought to rest by the same pressure field. Beyond this point the flow close to the surface is in the opposite direction, that is towards low pressure (See Figure 6.2). As the boundary layer thickens along the direction of motion, the viscous stress which is urging the fluid close to the boundary forward decreases, because the shear decreases. Ultimately it becomes small enough for the pressure gradient force to overcome it
Figure 6.1 Separation of flow from a flat boundary; vertical scale enlarged.
Figure 6.2  Fluid flow up the pressure gradient causing separation.

Figure 6.3  Addition of momentum to the boundary layer to prevent separation.
and reverse the flow. Separation occurs when the gradient of velocity at the boundary is zero for just beyond this point the only force on the stagnant fluid is the pressure gradient force. Once separation has occurred the rigid boundary no longer exerts a controlling influence over the flow. The point of separation may often be determined by boundary layer theory (52), as the point at which \( \frac{\partial v}{\partial y} = 0 \), \( y \) being measured in the direction away from the boundary.

6.1.5. Consequences of flow separation.

Separation occurs in regions where the fluid is flowing up the pressure gradient and separation will occur on a convex surface (Figure 6.2) where the flow channel widens out and this is similar to the geometry of the shute-kiln arrangement. According to Scorer (53) separation can be prevented in two ways:

(a) Addition of momentum to the boundary layer. This increases the shear at the boundary and can be done by:

(1) Motion of the boundary itself as in Figure 6.3.
    If the shoulder in the case illustrated is a circular cylinder which is made to rotate the flow adheres to the boundary in the fashion illustrated.

(2) By injecting fast moving fluid into the boundary layer.

(b) Removal of the boundary layer by suction. If the fluid composing the boundary layer is removed by suction, either through slots or holes, it is replaced by fluid from the main stream and so since the velocity gradient at the surface is maintained, separation is prevented.
It must be pointed out here that boundary layer theory predicts that the dead region above the shute should have a direction, but this was not easily observed possibly for the reason that flow in this region was not substantially two dimensional along the vertical centre plane.

When the flow is completely enclosed in a tunnel or tube it may be difficult to cause the fluid to flow round a bend without separation. In wind tunnels, flow round right angle bends is divided up by a large number of vanes in the corner (Figure 6.4) which shear the flow as it passes through them. It then emerges as a uniform stream again.

Even in a bend with curved walls (Figure 6.5) which approximates to the region at the beginning of the calciner the pressure is greater and the velocity smaller on the outside of the curve. Separation is therefore likely to take place on the outside of the bend where the boundary layer grows towards the high pressure region.

6.1.6. Flow pattern in the above calciner ($e = 0.25$).

Figure 5.7 demonstrated the double vortex nature of the flow that existed along the above calciner chamber along its entire length. On leaving the shute, the fluid impinged on the curved calciner roof and followed the contour of the confined shape, spiralling its way along the chamber and exited through the greater part of the width of the grate. Clearly, as the fluid travelled along the grate, momentum was lost and the patterns became less vigorous. A similar situation arose when the model was operated at a lower flowrate.
Figure 6.4  The shearing of flow at right angle bends to produce a uniform stream.

Figure 6.5  Separation on the outside of a curved wall.
6.1.7. Flow patterns in the drier chamber ($e = 0.25$).

The flow patterns in the drier chamber were given in Figures 5.8 to 5.11. The fundamental observations that arose from these experiments were:

(a) The square duct which conveyed fluid from the calciner chamber to the drier chamber essentially acted as a square jet issuing into a confined space and the vortex regions above the grate were due to jet entrainment.

(b) Below the grate the fluid followed the shape of the containing chamber and a semi-circular pattern was produced. For the higher flowrate in Figures 5.8 and 5.10 the driving force for the preferred direction of flow was determined by higher momentum fluid below the shute which did not impinge on the suction tubes. For lower flowrate, however, in Figure 5.9, lower momentum fluid gave rise to flow in both directions below the grate.

(c) Fluid flow did not take place throughout the length of the grate in the drier, and indeed the attempted passage of fluid through the grate and the upward nature of flow below the grate gave rise to a region of random flow at the partition between the drier and calciner sections below the grate.

(d) The shute inleakage (Figures 5.10 and 5.11) transformed the previous vortex motion in that area to one of random flow due to the net zero balance of fluid motion from the drier and inleaking fluid entering the shute section as shown in Figure 5.10.
(e) The introduction of a riddlings inleakage did not alter the flow pattern below the grate significantly, but the entrainment effect of a jet entering a confined space was observed near the riddlings inleakage entry (Figure 5.11).

6.1.8. Flow patterns with a voidage of 0.36.

(i) Calciner chamber.

The flow patterns in Figures 5.12 to 5.17 were observed when the grate resistance was reduced. These were similar to those with the higher resistance as discussed in Section 6.1.2. However, the essential differences were:

(a) The area of random flow above the shute increased slightly.

(b) The area of recirculation at the beginning of the grate was reduced to less than half that observed at the lower voidage.

(c) The introduction of a riddlings inleakage did not enhance the flow of fluid at the end of the grate as was first observed in Figure 5.6, since with a reduced grate resistance the fluid below the grate possesses a higher momentum.

(ii) Above calciner.

These results were given in Figures 5.15 to 5.17. Identical flow features were produced to those discussed in Section 6.1.6., viz the double vortex nature of the flow at the position nearest the kiln shute which became less defined further away from the shute when the fluid lost momentum.
(iii) Around the drier.

For the voidage of 0.36 in Figure 5.18, the confined jet characteristics of the fluid issuing into the drier chamber were again a feature and the symmetrical regions of reverse flow were again observed.

6.1.9. Implications of the water model results.

The results of the water model indicated that an assumption of plug flow of fluid through both the calciner and drier chambers was not correct. For the calciner the flow was confined to 4/5ths of the area, and implied that the heating of the material by the passage of hot gases through the moving bed on the prototype was confined to 4/5ths of the available grate area. Hence, the actual Reynolds numbers through the model grate were greater than those indicated in Table 5.1 based on a plug flow assumption. The revised table recalculated on the reduced area of flow for the calciner section is given in Table 6.1 for the higher flowrate of 450 l/min.

<table>
<thead>
<tr>
<th>Re. through calciner grate area.</th>
<th>Plug flow</th>
<th>Actual flow</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$2.2 \times 10^4$</td>
<td>$2.6 \times 10^4$</td>
</tr>
</tbody>
</table>

Table 6.1

It was found for the voidage of 0.25 that the introduction of a jet below the calciner reversed the trend for the upward flow region, but the region of no gas flow through the bed would still persist due to the geometric constraints of the flow path of fluid exiting the kiln.
The region of random flow above the shute did not contribute to the required flow of fluid through the bed and this part of the furnace was redundant. A sloped roof in this region would suffice but this complicates the problem of refractory installation on hanging surfaces. Hence, from the practical point of view the present case of the semi-circular arch with a vertical wall may present less problems in construction.

The square jet issuing into the drier chamber presented an uneven mass flow of fluid through the drier grate. Although the flow patterns provided only a qualitative indication, it was observed visually that the greater part of the mass flowrate was confined to the area below the square duct. In practice this indicated that the raw material was heated unevenly and the sudden introduction of wet nodules into this region of high mass-flowrate of hot gases would result in "thermal shatter" due to the rapid evolution of water vapour from within the particles.

6.2. Discussion of the mathematical model.

The results from equations 3.9 and 3.11 developed in Section 3 were illustrated in Figures 5.19 to 5.24 for the bed which had been conveniently divided into 4 strata for analysis. The Lof and Hawley correlation for the heat transfer from the gas stream to the solid bed was found to give excellent agreement between the measured results of Weber and the mathematically predicted results for stratum 1. The maximum error observed for the bed temperature using the Saunders and Ford, and Gamson et al. correlations for this region was 27% and 39% respectively for the plug flow and modified flow situations.
A discussion of these correlations was given in Section 2, where it was established that the higher coefficients obtained by Saunders and Ford were largely due to the fact that the smooth uniform spheres used in their work permitted much more effective and complete gas solid contact than was possible with the rough irregular shapes used in Lof and Hawley's experiments. In addition, the fraction voids employed by Saunders and Ford covered the range 37.5% to 38% whereas Lof and Hawley's work involved voids of 42.6% to 45.4%. For the bed under consideration the mean voidage measured experimentally was 40%. Thus, the Lof and Hawley conditions were nearer to those investigated.

The results of the water model indicated that plug flow of gas through the bed did not exist and that flow was confined to 4/5 ths. of the grate. This was incorporated into the mathematical model where both a plug flow and a modified flow heat transfer coefficient as stated in Section 5.2.3 were used, and the analysis for the 4 strata plotted in Figures 5.20 to 5.23. Using the modified flow conditions, the Lof and Hawley heat transfer coefficient increased by 16.3% compared to plug flow conditions and manifested itself by sharper rises in the heating curves and showed better agreement with Weber's measured temperatures.

6.2.1. Comparison between measured and predicted results.

A general inspection of the results showed that the mathematical model developed, compared favourably with measured results of Weber, and the trend observed in the water model encouraged a better fit to measured values. The greatest error between measured and theoretical results occurred in stratum 3 where the maximum deviation was 18.75% below using the modified flow prediction and 28.9% below using the plug flow assumption. The latter was untrue, however, hence the maximum error for the mathematical model was an 18.75% underestimation.
Nevertheless, for stratum 1, 2 and 4 the predicted results under modified flow conditions were all within 10.7% of Weber's measured values. For stratum 3 the maximum deviation observed of 18.75% was due to the method of evaluating gas convective temperature in the bed as discussed in Section 6.2.2.

For both strata 1 and 2 the temperature of the solid decreased sharply in the region of no flow of gas. This was due to the endothermic reaction of calcination as discussed in Section 2.2 which started at 800°C and proceeded to utilize the enthalpy of the solids. There was a discrepancy between Webers and the modified flow results in these no flow regions. However, the maximum deviation observed was 10.7% which was acceptable allowing for the 10% experimental accuracy of Webers measurements. For Strata 3 and 4 the temperature of the solid remained constant in these regions since the calcination temperature of 800°C was not achieved.

6.2.2. Estimation of the mean gas temperature.

One of the key relationships in the mathematical model was that developed in Appendix H for calculating the gas temperature as it exits an increment and hence the log mean gas temperature within an increment. The relationship was based on a straightforward equation which said that for each increment the following balance existed:

\[
\text{Heat lost by gas} = \text{Heat gained by solids}
\]

The final form of the equation was

\[
tg_2 = (tg_1 - T) \cdot \exp \left( \frac{-h_A L}{Go \cdot Cf} \right) + T
\]

The mean temperature of the gas in contact with the bed at \( T \) was given by:
More correctly, however, the balance should read for stratum 1 which was unique:

Heat transferred by convection
+ Heat radiated by furnace walls
+ Heat radiated by gases
+ Heat used for calcination
= Enthalpy rise of solids
+ Heat re-radiated by solids.

To estimate the order of magnitude of the radiation terms, specimen calculations were performed for each stratum choosing the initial and final increments under conditions of plug flow, and these results are in Appendix N.

For stratum 1, the radiation contribution to the total heat transferred to the solids was 19.44% and 22.66% at the beginning and end of the calciner respectively.

For strata 2, 3 and 4 within the bed, however, this equation would be converging to the truth since the only radiation term was that between the stratum being considered and the stratum above, both of which were at lower temperatures. The correct balance now reads for these regions:

Heat transferred by convection
+ Net radiative transfer between increment above and below.
= Enthalpy rise of solids.

The radiation contribution to the total heat transferred were
calculated in Appendix N and the results summarized in Table 6.2 for all strata.

<table>
<thead>
<tr>
<th>Stratum</th>
<th>Radiation contribution</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Initial increment</td>
<td>Final increment</td>
</tr>
<tr>
<td>Stratum 1</td>
<td>19.44%</td>
<td>22.66%</td>
</tr>
<tr>
<td>Stratum 2</td>
<td>1.75%</td>
<td>9.6%</td>
</tr>
<tr>
<td>Stratum 3</td>
<td>0.188%</td>
<td>9.5%</td>
</tr>
<tr>
<td>Stratum 4</td>
<td>0.097%</td>
<td>7.6%</td>
</tr>
</tbody>
</table>

Table 6.2. Radiation contribution for each stratum.

The greatest deviation between predicted and measured results occurred in stratum 3 and it was due to the use of the gas equation. Since heating of the nodules was not entirely due to the passage of gases through the bed, the exit gas temperatures for an increment should be higher than those calculated. Hence the predicted solids temperatures will tend to be lower than the measured values. Nevertheless, the equation was shown to give acceptable results for the remaining strata.

6.2.3. Change of grate speed.

The design equation was then used to evaluate the solids temperature attained for considered changes in grate speed for a constant bed depth of 18 cms. The normal operating grate speed was 80.5 m/hr. and speeds of half and twice this value were investigated and the results were plotted in Figure 5.24 for modified flow conditions in stratum 1. However, it must be pointed out that although the trend in the curves
were correct, clearly the solids temperature leaving the drier section and entering the calciner section would also be dependent on the grate speed. Nevertheless, if inspection was confined to the calciner alone the results were justified, and the graph demonstrated the usefulness of the design equation to investigate a salient design parameter.

6.2.4. The overall enthalpy balance.

The necessary check to the design equation was an overall enthalpy balance. Although plug flow conditions did not exist through the grate, the gas temperatures below the grate for the modified flow regime in the region of no flow was difficult to predict due to the aerodynamics of the fluid flow below the calciner. Nevertheless, an overall enthalpy balance for the plug flow situation should yield a reasonable result. The computed values for the plug flow condition are given in Tables 5.3 to 5.6 Appendix M and the relevant quantities for the enthalpy balance are illustrated in Figure 6.6. An arithmetic mean was chosen to represent solids temperature in and out and also the gas temperature out.

Mass flowrate of solids for the four strata

\[
\text{Mass flowrate of solids for the four strata} = B \cdot dx \cdot L \cdot (1-d) \cdot \rho \quad \text{kg/hr} \quad \text{(See Appendix L)}
\]

\[
B = 4.1 \, \text{m}
\]

\[
dx = 12.2/15 \, \text{m}
\]

\[
L = 0.18 \, \text{m}
\]

\[
d = 0.4
\]

\[
\rho = 2480 \, \text{kg/m}^3
\]

Solids flowrate = 38429 kg/hr.
Gas flowrate $= 61637 \text{ kg/hr.}$
Specific heat gas mixture (mean) $= 0.296 \text{ kcal/kg°C}$
Mean gas temperature in (measured) $= 1000°C$
Arithmetic mean gas temperature out $= 430°C$

Solids flowrate $= 88429 \text{ kg/hr.}$
Arithmetic mean solids temperature in $= 177.5°C$
Arithmetic mean solids temperature out $= 752°C$
Specific heat of solids: Arithmetic mean of limestone (0.217) and lime (0.26) $= 0.238 \text{ kcal/kg°C}$

Figure 6.6 Overall enthalpy balance for calciner.
Mass flowrate of gas = 3131 m³/min at 1050°C (See Appendix B)

\[ \rho_g = 3.281 \times 10^{-1} \text{ kg/m}^3 \] (See Appendix C)

Hence gas flowrate = 61637 kg/hr.

6.2.5. Calcination heat requirement.

As discussed in Section 2.2, the percent calcination can be expressed by

\[ y\% = 1.83 \times X \]

where \( X \) = time in minutes

Time spent in each increment = 0.0101 hr (See Appendix L)

Calcination was assumed to commence at 800°C, and referring to Figures 5.20 and 5.21 it is observed that calcination temperatures are attained only in strata 1 and 2.

For stratum 1, time spent on grate after attaining 800°C = 5.4 mins.
For stratum 2, time spent on grate after attaining 800°C = 1.2 mins.

Hence for stratum 1, % calcination = 9.9%
stratum 2, % calcination = 2.2%

Mass flowrate/stratum = 88429/4 kg/hr (See Appendix L)
= 22107 kg/hr.

Heat required = 422 kcal/kg CaCO₃

Mass calcined stratum 1 = 22107 x 0.099 = 2188.6 kg/hr
Mass calcined stratum 2 = 22107 x 0.022 = 486.4 kg/hr

2675.0
Heat consumed = 2675 x 422
                 = 1,128,850 kcal/hr.

6.2.6. Radiation contribution to enthalpy gain of solids.

From Table 6.2, the net radiation contribution to the heating of stratum 1 was 21.05%. Also,

Arithmetic mean solids temperature in stratum 4 = 337°C
Arithmetic mean gas temperature out = 430°C

Hence the driving force for radiation transfer between the exiting gas and stratum 4 will be small and was neglected. The radiation contribution to the heating of the whole calciner bed was taken to be the arithmetic mean for the four strata = 5.26%.


Enthalpy gain of solids

= 88429 x 0.238 x (752 - 177.5)
= 12,090,986 kcal/hr

Heat loss from furnace shell

Total surface area of semicircular calciner furnace of 2.44 m outside radius and 12.2 m in length = 93.52 m²

Heat loss from shell = 413 kcal/hr m²                (See Appendix J)

Total heat loss from semicircular shell = 38624 kcal/hr.

Enthalpy loss of gas

= 61637 x 0.296 x (1000 - 430)
= 10,399,395 kcal/hr.
Enthalpy gain of solids = 12,090,986 kcal/hr
Shell losses = 38,624 kcal/hr

12,129,610

Enthalpy loss of gas = 10,399,395 kcal/hr
Calcination requirement = 1,128,850 kcal/hr
Radiation contribution (5.26%) = 635,986 kcal/hr

12,164,231

This enthalpy imbalance in the system indicated that 0.28% of the heat input into the system was unaccounted for. In practice the shell losses are greater than the figure calculated since the heat losses for the bottom half of the calciner chamber was not considered due to lack of data. Nevertheless, using a straightforward arithmetic mean to define the solids and gas temperatures yielded a good approximation to an overall enthalpy balance.

6.2.8. Implications of the mathematical model.

The mathematical model developed in Section 3 to describe the heating of the calciner bed gave good agreement with measured values quoted in the literature, and was then used to investigate the effect of changing grate speed. This realistic model incorporated flow trends observed in the water model viz,

(a) The vigorous axial mixing in the above calciner justified
the assumption of isothermal conditions in the above calciner.

(b) Downward flow of fluid was confined to 4/5 ths. of the available grate area.

Moreover, the 15 zones used in the computation were adequate for this realistic model and this model can be used by the furnace design operator to investigate the effects of a considered change which the demands of production would prevent his searching by trial and error.
6.3. Conclusions.

6.3.1. Water model results.

(i) The flow patterns obtained in the Lepol grate water model were essentially independent of:
(a) The turbulent Reynolds numbers used.
(b) The voidage used to represent the bed resistance.

(ii) The plug flow assumption of fluid flow through the calciner and drier sections appeared to be erroneous. The water model showed that the flow through the calcining section was restricted to 80% of the available grate area due to the geometrical constraints of the system.

(iii) The region of dead space above the kiln shute could be explained by boundary layer theory. Thus in practice this area was redundant.

(iv) The fluid flow in the above calciner cross-section was double vortex in nature.

(v) Fluid issuing into the drier chamber behaved as a confined jet.
6.3.2. Mathematical model results.

(i) Physical measurements of the temperature of the material on the calciner bed were successfully predicted from two theoretical approaches viz, a numerical and analytical method, both of which entailed dividing the bed into 4 stratum, and each stratum into 15 increments. The former, however, was found to be the more accurate method when the two approaches were compared to the measured values of Weber.

(ii) Incorporating the results of the water model into the numerical method yielded a better fit to Weber's measured values of bed temperature.

(iii) For stratum 1, 2 and 4 the predicted results of the numerical method under modified flow conditions were all within 10.7% of Weber’s measured values. For stratum 3, however, the maximum deviation observed was 18.75%, due to the method of evaluating gas convective temperature within the bed.

(iv) Heat transfer from the gas to the bed was best described using the Lof and Hawley correlation.

(v) The mean radiation contribution to the heating of the calciner bed was 5.26%.

(vi) The numerical mathematical model established would be a considerable help to both the designers and operators of the Lepol grates.
NOMENCLATURE.

- **a**: Parameter in Ergun's pressure drop equation.
- **a_i**: Weighting coefficients.
- **A**: Area of emitting surface. \((m^2)\)
- **A_i**: Surface area of increment. \((m^2)\)
- **A**: Cross-sectional area of calciner bed. \((m^2)\)
- **A**: Surface area of solid in contact with gas. \((m^2)\)
- **A**: Specific area of solid. \((m^2/m^3)\)
- **b**: Parameter in Ergun's pressure drop equation.
- **B**: Width of grate. \((m)\)
- **c**: Weighted specific heat of solids. \((kcal/kg^0C)\)
- **c**: Mean specific heat of gases. \((kcal/kg^0C)\)
- **C_d**: Mass concentration of dust particles. \((kg/m^3)\)
- **C_c**: Correction for total pressure and partial pressure.
- **C_ω**: Correction for total pressure and partial pressure.
- **d**: Diameter of spherical particle. \((m)\)
- **d**: Mean dust particle size. \((microns)\)
- **d**: Maximum dust particle size. \((microns)\)
- **Dm**: Hydraulic mean diameter. \((m)\)
- **Do**: Outside mean diameter of cylinder. \((m)\)
- **Fr**: Froude number. Ratio \((inertia\ force/gravitational\ force)\)
- **F_12**: View factor from wall to increment.
- **F_13**: View factor from gas to bed.
- **g**: Gas to bed exchange area. \((m^2)\)
- **g**: Gravitational constant. \((m/hr^2)\)
- **Go**: Mass velocity of the gases. \((kg/hr.m^2)\)
- **Go_{mod}**: Modified mass velocity. \((kg/hr.m^2)\)
h  Heat transfer coefficient from gas to particles. 
    Area basis kcal/hr.m².°C

  k_f  Mean thermal conductivity of gases. (kcal/hr.m.°C)
  k_i  Specific absorption coefficient. (ft⁻¹.atm⁻¹)
  k_1, k_2  Thermal conductivity of refractory 1 and 2. (kcal/hr.m.°C)
  K  Constant in Furnas equation.
  K_1, K_2, K_3, K_4, K_2, K_3  Parameters in the analytical model.
  L  Depth of bed. (m)
  L_1, L_2  Thickness of refractory 1 and 2. (m)
  L_m  Mean beam length. (m)
  L_s  Weight of dry solid. (kg)
  m  Mass flowrate through an increment. (kg/hr)
  m  Slope of the falling rate curve.
  n  Number of particles in increment.
  N  Rate of drying. (kg moisture evap'd/hr.m²)
  P  Partial pressure. (atm)
  P_c  Partial pressure of CO₂. (atm)
  P_ω  Partial pressure of H₂O vapour. (atm)
  Pr  Prandtl number. (c_f.µ_f/k_f)
  P_o, P_1, P_2, P_3  Parameters in the Runge-Kutta.
  r  Radius of the spherical particle. (m)
  R  Calcination rate heat requirement. (kcal/kg.hr)
  R  Rate of advance of the line of calcination according to Furnas. (cms/hr)
  R̅  Radius of the calciner chamber. (m)
  Re_p  Reynolds number based on approach velocity u_o and particle diameter d_p.
s  Fraction of incremental volume occupied by particles. \( (1-\varepsilon) \)
SS  The calciner bed to roof exchange area. \( (m^2) \)
Sc  Schmidt number. Ratio (kinematic viscosity/diffusivity)
t_g2  Gas temperature out. (Gas equation Appendix H)
t_g1  Gas temperature in. (Gas equation Appendix H)
t_s  Mean solid temperature. (Gas equation Appendix H)
t_g.mean  Log mean gas temperature of \( t_{g2} \) and \( t_{g1} \).
U  Parameter in Thring's equation. (Appendix F)
u_o  Approach velocity of gas. \( (m/hr) \)
v  Speed of grate. \( (m/hr) \)
V'  Volume of increment. \( (m^3) \)
V  Volume of particle. \( (m^3) \)
x  Distance along the grate. \( (m) \)
X  Time in minutes.
X*  Equilibrium moisture content. \( (kg.\text{moisture}/kg.\text{dry solid}) \)
X_c  Critical moisture content. \( (kg.\text{moisture}/kg.\text{dry solid}) \)
y  Fraction or % of mass calcined.
Z  Step length in the Runge-Kutta.

Greek symbols

\( \alpha \)  Heat transferred by convection. \( (k.\text{cal/hr}) \)
\( \beta \)  Rise in enthalpy of the solids. \( (k.\text{cal/hr}) \)
\( \beta' \)  Coefficient of volumetric expansion. \( 1/\degree \text{C} \)
\( \gamma \)  Energy used for calcining. \( (k.\text{cal/hr}) \)
\( \delta \)  Energy radiated from the furnace walls. \( (k.\text{cal/hr}) \)
\( \varepsilon \)  Voidage.
\( \varepsilon_1 \)  Emissivity of surface 1.
\( \varepsilon_2 \)  Emissivity of surface 2.
Emissivity of steel shell.
Emissivity of furnace walls.
Emissivity of the furnace gases and dust. (In models)
Emissivity of dust particles.
Emissivity of gases and dust mixture. (In evaluation)
Emissivity of $CO_2$.
Emissivity of $H_2O$ vapour.
Energy radiated from the furnace gases. (k.cal/hr)
Mean density of bed material. (kg/m$^3$)
Density of kiln dust. (kg/m$^3$)
Mean density of calciner gases. (kg/m$^3$)
Dimensionless temperature ratio in analytical model.
Initial dimensionless temperature of increment.
End dimensionless temperature of increment.
Dimensionless distance ratio in analytical model.
Stephan Boltzman constant. (k.cal/hr.m$^2$.°K$^4$)
Mean viscosity of furnace gases. (Kg/hr.m)
Energy re-radiated from the increment. (k.cal/hr)
Mean solid temperature in increment. °C
Initial temperature of solid in increment. °C
Exit solid temperature from increment. °C
Temperature of radiating gas in chamber. °K
Log mean gas temperature in increment. °C
Gas temperature into increment. °C
Gas temperature out of increment. °C
Mean solid temperature of the increment above. °K
Mean wall temperature. °K
Ambient air temperature. °C
\( T_1 \) Temperature of steel shell. °C

\( T_2 \) Intermediate wall temperature. °C

\( T_3 = T_w \) Mean wall temperature. °K

\( T_{1M} \) Mean temperature of surface 1. °K

\( T_{2M} \) Mean temperature of surface 2. °K

\( V \) Energy transferred by radiation between the increment being considered and the increment above. (k. cal/hr)

\( \Delta X \) Change in moisture content.

\( \Delta \theta \) Change in time.

\( \Delta P \) Pressure drop. (ins. w.g.)

\( \Delta t \) Temperature difference between surface and ambient air. (°C)

\( \Delta T_{\text{max}} \) Temperature between surface and centre of nodule. (°C)

\( \Delta \epsilon \) Correction for a mixture of CO\(_2\) and H\(_2\)O.

**Note.** Other symbols used in the evaluation of:

(a) The view factors from gas to bed and from wall to bed are defined separately in Appendix K.

(b) The analytical model for the prediction of bed temperature are defined in Appendix P.
References

1. Weber, P.

Heat Transfer in Rotary Kilns with due regard to Cyclic Processes and Phase Formation.
Wiesbaden. Bauverlog GmbH.

2. Lof, C.O.G. and Hawley, R.W.

Unsteady-state heat transfer between air and loose solids.

3. Van Heerden, C.

Autothermic processes.

4. Csaba, J. and Leggett, A.D.

Prediction of the temperature distribution along a pulverized coal flame.

5. Hottel, H.C. and Sarofim, A.F.

Radiative Transfer.

6. Treybal, R.E.

7. Kunii, D. and Levenspiel, O.
   Fluidization Engineering.

8. Boynton, R.S.
   Chemistry and Technology of Lime and Limestone.

9. Johnston, J.
   The thermal dissociation of calcium carbonate.

10. Gygi, H.
    Thermodynamics of the Cement Kiln.

11. Furnas, C.C.
    The Rate of Calcination of Limestone.

12. Satterfield, C.N. and Feakes, F.
    Kinetics of the Thermal Decomposition of Calcium Carbonate.

13. Anselm, W.
    Die Warmerechnung bei Brennofen für Zement,
    Kalk Magnesit und Dolomit.

Pilot Plants, Models and Scale-up Methods in Chemical Engineering.

15. Evans, D.G. and Patrick, M.A.

The Use of Modelling - Studies in Boilers.

16. Philbrook, W.O.

Fundamentals of Scale Model Experiments.

17. Putnam, A.A. and Ungar, E.W.

Basic Principles of Combustion Model Research.

18. Rhydderch, M.J.

Buoyancy and Furnace Model Studies.

19. Gray, F.A. and Robertson, A.D.

The deflection of hot jets due to buoyancy.

20. Winter, E.F. and Deterding, J.H.

21. Chesters, J.H.
   The Flow Pattern Factor in Fuel Research.

22. Hulse, C.
   The aerodynamics of open hearth furnace regenerators.

23. Lain, P.B.
   Flow patterns in rotary cement kiln models.

24. Moles, F.D., Watson, D., Lain, P.B.
   The aerodynamics of a rotary cement kiln.
   *Fourth Symposium on Flames and Industry*,

25. Smith, R.M.
   The effect of buoyancy on enclosed turbulent flames.

26. Horn, G. and Thring, M.W.
   Buoyancy Effects in Furnaces and Boilers.

27. Rolfe, T.J.K.
   Distribution of Primary Air in Chain-grate Stokers.
28. Ball, G.
   Heat Transfer, Combustion and Temperature in a Furnace Tube Model.

29. Dunningham, A.C., and Grumell, E.C.
   Combustion of Fuel on a Travelling Grate.
   *J. Inst. Fuel.*, 12, 87-95, 1939.

30. Hayward, C.H.G.
   Developments in the Firing of Shell Boilers by means of Chain-grate Stokers.

31. Wright, S.J.
   Temperature and Heat Transfer Patterns in Chain-grate Stoker-fired Economic Boilers.

32. Walton, J.S., Olson, R.L., Levenspiel, O.
   Gas-Solid Film Coefficients of heat transfer in fluidized Coal Beds.

33. Ergun, S.
   Mass Transfer Rate in packed columns. It's analogy to pressure loss.
34. Kirov, N.Y., and a Donau Szpindler, G.

35. Schumann, T.E.W.
   Heat Transfer: A liquid flowing through a porous prism.
   J. Franklin Inst., 208, 405-416, 1929.

36. Furnas, C.C.
   Heat Transfer from a gas stream to a bed of broken solids.

37. Saunders, O.A. and Ford, H.
   Heat Transfer in the flow of gas through a bed of solid particles.
   J. I. S. I., 1, 291, 1940.

38. Gamson, B.W., Thodos, G., and Hougen, O.A.
   Heat, Mass and Momentum Transfer in the flow of gases through granular solids.

39. Colburn, A.P.
   A method of correlating forced convection heat transfer data and a comparison with fluid friction.
   The Heat Transfer and Pressure loss in fluid flow through randomly packed spheres.

41. Ranz, W.E.
   Friction and transfer coefficients for single particles and packed beds.

42. Kunii, D. and Suzuki, M.
   Particle to fluid Heat and Mass transfer in packed beds of fine particles.

43. Madejski, J.
   Radiative heat transfer between moving surfaces.

44. Van Nood, J.H. and Beek, W.J.
   Radiative heat transfer to continuously moving thin metal plates.

45. McAdams, W.H.
46. Folliot, A.

La transmission de chaleur dans le four rotatif à ciment.
Centre d'Etudes et de Recherches de l'Industrie des Liants Hydrauliques.

47. Langhaar, H. L.

Dimensional analysis and theory of models.

48. Curtis, R. W. and Johnson, L. E.

Use of flow models for Boiler Furnace design.

49. Chigier, N. A.

Application of model results to design of industrial flames.
Fourth Symposium on Flames and Industry,

50. Thring, M. W., and Newby, M. P.

Combustion length of enclosed turbulent jet flames.
Fourth Int. Symp. on Combustion, 789-796.
Williams & Wilkins, Baltimore 1953.

51. Rouse, H.

Fluid mechanics for hydraulic engineers.
52. Kay, J.M.


53. Scorer, R.S.

Natural Aerodynamics.

54. Anon.

Lepol grates.
A.P.C.M. Note, Cauldon Works.

55. Spiers, H.M.

British National Committee.

56. Perry, J.H.


57. Eckert, E.R.G. and Drake, R.M.


58. Bowers, T.G. and Read, H.L.

Heat transfer in rotary kilns.
(Heat transfer - Boston)
59. Coulson, J.M. and Richardson, J.F.


60. Johnson, T.R., and Beer, J.M.

The zone method of analysis of radiant heat transfer:
A model for luminous radiation.

Fourth Symposium on Flames and Industry.
British Flame Res. Comm. and Inst. Fuel,
Imperial College, Sept. 1972.

61. Thring, M.W.

Luminous Radiation from Flames.


62. Hottel, H.C. and Egbert, R.B.

The radiation of furnace gases.

A.S.M.E. Transactions, 63, 297, 1941.

63. Carslaw, H.S. and Jaeger, J.C.


64. Gray, W.A. and Muller, R.

Engineering calculations in radiative heat transfer.

65. Siegel, R. and Howell, J.R.

Thermal radiation heat transfer.
Appendices.

A. Composition of the raw material on the grate.

B. Gas composition inside calciner chamber.

C. Physical properties of the gas mixture inside calciner.

D. Use of correlations for the heat transfer coefficient between gas and solid. (Plug flow assumed).

E. Calculation of the dust particle size \(d_p\) in calciner.

F. Representation of a real gas for engineering calculations.

G. Temperature variation within particles.

H. Evaluation of the mean gas temperature in the bed.

J. Evaluation of the inside temperature of the chamber walls. \(T_w\).

K. Evaluation of view factors from gas to bed and from wall to bed for stratum 1.

L. Evaluation of the mass flowrate through an increment and the calcination requirement for a moving increment.

M. Results computed from equation 3.9, section 3.

N. Evaluation of the radiation contribution to the total heat transfer for increment 1 and increment 15 using equation 3.9 in section 3.
P. Analytical solution for the temperature of solids at steady state on the grate.
Calcination on the Lepol grate.

The analysis for the raw meal used by Weber (1) in the Lepol kiln was as follows:

<table>
<thead>
<tr>
<th>Component</th>
<th>% by weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>CaCO$_3$ + MgCO$_3$ (by titration)</td>
<td>77.50</td>
</tr>
<tr>
<td>Quartz</td>
<td>4.00</td>
</tr>
<tr>
<td>Kaolin</td>
<td>1.50</td>
</tr>
<tr>
<td>Illite</td>
<td>17.00</td>
</tr>
<tr>
<td>Montmorillonite</td>
<td></td>
</tr>
</tbody>
</table>

100.00

The clinker produced had the following analysis:

<table>
<thead>
<tr>
<th>Component</th>
<th>% by weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>SiO$_2$</td>
<td>20.60</td>
</tr>
<tr>
<td>Al$_2$O$_3$</td>
<td>6.00</td>
</tr>
<tr>
<td>Fe$_2$O$_3$</td>
<td>3.90</td>
</tr>
<tr>
<td>CaO</td>
<td>66.60</td>
</tr>
<tr>
<td>MgO</td>
<td>0.80</td>
</tr>
<tr>
<td>SO$_3$</td>
<td>0.80</td>
</tr>
<tr>
<td>K$_2$O</td>
<td>0.65</td>
</tr>
<tr>
<td>Na$_2$O</td>
<td>0.16</td>
</tr>
</tbody>
</table>

100.00

This product approximated to the clinker analysis produced in the Lepol system modelled and Weber states a raw meal requirement of
1.546 kg/kg clinker.

The CaCO₃ content of the raw material may be obtained from the % CaO in the clinker.

\[
\text{CaCO}_3 \xrightarrow{\Delta} \text{CaO} + \text{CO}_2
\]

\[
\begin{array}{c c c}
100 & 56 & 44 \\
\end{array}
\]

\[
\text{CaCO}_3 \text{ content} = \frac{\%\text{CaO}}{56} \times \frac{100}{100} \text{ kg/kg clinker.}
\]

\[
\text{MgCO}_3 \text{ content} = \frac{\%\text{MgO}}{40.3} \times \frac{84.3}{100} \text{ kg/kg clinker.}
\]
Gas composition inside calciner chamber.

Measured (54) volumetric flow ex-kiln = 3131 m³/min at 1050°C (wet basis).

Orsat measurement
- 0.9% O₂
- 31.7% CO₂ ex-kiln.
- 67.4% N₂

Coal consumption = 3.9 tonnes per hour.
Clinker output = 30.1 tonnes per hour.
Raw moisture in coal = 12.39% by weight.
Ash = 11.06% by weight dry basis.
C.V. = 7212 kcal/kg.

The coal burnt approximates to an 802 coal as given in Spiers (55).

Composition of mineral-matter free 802 coal:

<table>
<thead>
<tr>
<th>Element</th>
<th>Weight</th>
<th>Mols Comp.</th>
<th>Mols O₂ req.</th>
<th>Mols CO₂</th>
<th>Mols H₂O</th>
<th>Mols SO₂</th>
<th>Mols N₂</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>0.82</td>
<td>0.0683</td>
<td>0.0683</td>
<td>0.0683</td>
<td>-</td>
<td>-</td>
<td>0.2287</td>
</tr>
<tr>
<td>H</td>
<td>0.053</td>
<td>0.0265</td>
<td>0.0132</td>
<td>-</td>
<td>0.0265</td>
<td>-</td>
<td>0.0442</td>
</tr>
<tr>
<td>N</td>
<td>0.017</td>
<td>0.0006</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.0006</td>
</tr>
<tr>
<td>S</td>
<td>0.01</td>
<td>0.0003</td>
<td>0.0003</td>
<td>-</td>
<td>-</td>
<td>0.0003</td>
<td>0.0010</td>
</tr>
<tr>
<td></td>
<td>0.9</td>
<td>0.0957</td>
<td>0.0818</td>
<td>0.0683</td>
<td>0.0265</td>
<td>0.0003</td>
<td>0.2745</td>
</tr>
</tbody>
</table>

Consider 1 kg. coal 802. 

Table B1
1 kg. coal produces 0.477 kg H₂O

Raw moisture/kg coal 0.1239 kg H₂O

<table>
<thead>
<tr>
<th>Total water</th>
<th>0.6009 kg/kg coal</th>
</tr>
</thead>
</table>

Fuel rate 59.02 kg/min coal.

This produces 59.02 x 0.6009 kg H₂O/min = 35.47 kg H₂O/min.

ρH₂O vapour at 1000°C = 1.720 x 10⁻¹ kg/m³.

Hence, water present in chamber = 2.063 x 10² m³ H₂O/min.

Total flowrate = 3131 m³/min.

H₂O = 206.3 m³

O₂, CO₂, N₂ = 2924.7 m³

0.9% x 2924.7 = 26.32 m³ O₂

31.7% x 2924.7 = 927.07 m³ CO₂

67.4% x 2924.7 = 1971.31 m³ N₂

Final analysis of gases in chamber.

<table>
<thead>
<tr>
<th></th>
<th>Vol. %</th>
</tr>
</thead>
<tbody>
<tr>
<td>H₂O</td>
<td>206.5 m³</td>
</tr>
<tr>
<td>O₂</td>
<td>26.32 m³</td>
</tr>
<tr>
<td>CO₂</td>
<td>927.07 m³</td>
</tr>
<tr>
<td>N₂</td>
<td>1971.31 m³</td>
</tr>
<tr>
<td>------------</td>
<td>---------</td>
</tr>
<tr>
<td>3131.00</td>
<td>100.00</td>
</tr>
</tbody>
</table>
Composition of gases in the calciner.

Table B2 shows the composition in the calciner.

<table>
<thead>
<tr>
<th>Component</th>
<th>Volumetric analysis %</th>
<th>Molecular weight</th>
<th>% Vol x M.W.</th>
<th>Weight analysis %</th>
</tr>
</thead>
<tbody>
<tr>
<td>H₂O</td>
<td>6.6</td>
<td>18</td>
<td>118.8</td>
<td>3.70</td>
</tr>
<tr>
<td>O₂</td>
<td>0.84</td>
<td>32</td>
<td>26.88</td>
<td>0.84</td>
</tr>
<tr>
<td>CO₂</td>
<td>29.61</td>
<td>44</td>
<td>1302.84</td>
<td>40.57</td>
</tr>
<tr>
<td>N₂</td>
<td>62.95</td>
<td>28</td>
<td>1762.6</td>
<td>54.89</td>
</tr>
</tbody>
</table>

| Total gas flow for Webers data ex-kiln | 4.409 x 10⁴ Nm³/hr |
|                                          | 2.06 x 10⁵ m³/hr at 1000 °C. |
| Gas flow ex-kiln system studied          | 1.88 x 10⁵ m³/hr at 1000 °C. |
| Length of grate studied in calciner      | 12.07 m. |
| Width of grate studied in calciner       | 4.1 m.   |
| Area of calciner grate                   | 49.5 m²  |
Physical properties of the gas mixture in calciner.

Perry(56) gives the viscosities at 1000°C and the specific heats at 760°C for the gases.

<table>
<thead>
<tr>
<th>Component</th>
<th>( \mu_F = \text{kg/hr m(1000°C)} )</th>
<th>( C_F = \text{kcal/kg°C (760°C)} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( O_2 )</td>
<td>( 1.98 \times 10^{-1} )</td>
<td>( 0.260 )</td>
</tr>
<tr>
<td>( CO_2 )</td>
<td>( 1.84 \times 10^{-1} )</td>
<td>( 0.295 )</td>
</tr>
<tr>
<td>( N_2 )</td>
<td>( 1.84 \times 10^{-1} )</td>
<td>( 0.280 )</td>
</tr>
<tr>
<td>( H_2O )</td>
<td>( 1.69 \times 10^{-1} )</td>
<td>( 0.540 )</td>
</tr>
</tbody>
</table>

Evaluation of the thermal conductivity.

Although the specific heat, viscosity and thermal conductivity of a gas increase with temperature, the Prandtl number \( c\mu/k \) shows little dependence upon temperature. The value of \( c\mu/k \) calculated at any single temperature serves sufficiently well for the solution of problems involving the same gas at another temperature. The values of \( c\mu/k \) for the four gases are stated:

**Prandtl numbers at 1 atm and 212°F.**

<table>
<thead>
<tr>
<th>Gas</th>
<th>( c\mu/k )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( O_2 )</td>
<td>0.74</td>
</tr>
<tr>
<td>( CO_2 )</td>
<td>0.80</td>
</tr>
<tr>
<td>( N_2 )</td>
<td>0.74</td>
</tr>
<tr>
<td>Steam</td>
<td>0.78</td>
</tr>
</tbody>
</table>

Thus \( k = c\mu/\text{constant} \).
The weighted mean viscosity and specific heat of the gas mixture was found to be $1.836 \times 10^{-1}$ kg/hr m and $2.96 \times 10^{-1}$ kcal/kg °C respectively.

The evaluated thermal conductivities at 1000°C were:

<table>
<thead>
<tr>
<th>Gas</th>
<th>k (kcal/hr m °C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$O_2$</td>
<td>$6.96 \times 10^{-2}$</td>
</tr>
<tr>
<td>$CO_2$</td>
<td>$6.79 \times 10^{-2}$</td>
</tr>
<tr>
<td>$N_2$</td>
<td>$6.96 \times 10^{-2}$</td>
</tr>
<tr>
<td>Steam</td>
<td>$1.17 \times 10^{-1}$</td>
</tr>
</tbody>
</table>

The weighted mean thermal conductivity of the gas mixture was $7.07 \times 10^{-2}$ kcal/hr m °C.

**Evaluation of the densities of the gas mixture.**

Using data obtained from Perry and Eckert and Drake (57) the densities of the gases were as follows at 1000°C.

<table>
<thead>
<tr>
<th>Component</th>
<th>Density kg/m³</th>
</tr>
</thead>
<tbody>
<tr>
<td>$O_2$</td>
<td>$3.061 \times 10^{-1}$</td>
</tr>
<tr>
<td>$CO_2$</td>
<td>$4.238 \times 10^{-1}$</td>
</tr>
<tr>
<td>$N_2$</td>
<td>$2.683 \times 10^{-1}$</td>
</tr>
<tr>
<td>$H_2O$(steam)</td>
<td>$1.720 \times 10^{-1}$</td>
</tr>
</tbody>
</table>

The weighted mean density of gas mixture was $3.281 \times 10^{-1}$ kg/m³.
The physical properties of the gaseous mixture at 1000°C were as follows:

\[ \mu_{mean} = 1.836 \times 10^{-1} \text{ kg/hr m.} \]
\[ C_{mean} = 2.96 \times 10^{-1} \text{ kcal/kg °C.} \]
\[ k_{mean} = 7.07 \times 10^{-2} \text{ kcal/hr m °C.} \]
\[ \rho_{mean} = 3.281 \times 10^{-1} \text{ kg/m}^3. \]
Use of correlations for the heat transfer coefficient.
Plug flow assumed.

Lof and Hawley. (2)

\[ h = 0.79 \left( \frac{G_o}{d_f} \right)^{0.7} \text{Btu/hr ft}^3 \text{°F} \]

\[ G_o = 1.280 \times 10^3 \text{ kg/hr m}^2 = 2.623 \times 10^2 \text{ lbs/hr ft}^2 \]

\[ d_f = 0.0094 \text{ m} = \frac{3}{8}'' = 0.03125 \text{ ft} \]

\[ h = 441 \text{ Btu/hr ft}^3 \text{°F} \]

The surface area in 1 ft\(^3\) of bed = \(\frac{3(1-\epsilon)}{r}\) where \(\epsilon = 0.4\)

\[ = 115.2 \text{ ft}^2 \]

\[ h = 3.828 \text{ Btu/hr ft}^2 \text{°F} \]

\[ = 18.68 \text{ kg cal/hr m}^2 \text{°C}. \]

Saunders and Ford. (37)

\[ h = 0.152 \frac{G_o}{d_p} \text{kcal/hr m}^3 \text{°C} \]

\[ = 54.05 \text{ kcal/hr m}^2 \text{°C}. \]

Gamson, Thodos and Hougen. (38)

\[ h = 1.064 C_f G_o \left( \frac{d_p G_o}{\mu_f} \right)^{-0.41} \left( \frac{C_f \mu_f}{k_f} \right)^{-2/3} \text{kcal/hr m}^2 \text{°C} \]

where \(C_f = 2.96 \times 10^{-1} \text{ kcal/kg °C}\)

\(k_f = 7.07 \times 10^{-2} \text{ kcal/hr m}^2 \text{°C/m}\)

\(\mu_f = 1.836 \times 10^{-1} \text{ kg/hr m}\)

\[ \left( \frac{d_p G_o}{\mu_f} \right)^{-0.41} = 0.1792 \]

\[ \left( \frac{C_f \mu_f}{k_f} \right)^{-2/3} = 1.191 \]

\[ h = 86 \text{ kcal/hr m}^2 \text{°C}. \]
Ranz. (41)

\[
\frac{h \cdot \Delta p}{k_f} = 2.0 + 0.6 \left[ \frac{C_f \cdot \mu_f}{k_f} \right]^{1/3} \left[ \frac{d_r \cdot \mu_o}{\mu_f} \right]^{1/2}
\]

which yields

\[ h = 48.51 \text{ kcal/hr m}^2 \text{ °C} \]

Denton, Robinson and Tibbs. (40)

\[ h = 0.72 C_f \cdot C_o \left( \frac{G_o \cdot d_p}{\mu_f} \right)^{-0.3} \text{ kcal/hr m}^2 \text{ °C.} \]

\[ = 77.8 \text{ kcal/hr m}^2 \text{ °C.} \]

Values for heat transfer coefficients.

Bowers and Read (58 ) have stated values for heat transfer coefficients for limestone kilns as 35.2 and 33.7 kcal/hr m² °C and for a dry cement rotary kiln 37.1 to 49.8 kcal/hr m² °C. However, the authors did not indicate their method or source of obtaining these results. Furnas computes the heat transfer coefficient for limestone to be 22.5 kcal/hr m² °C. The results for the correlations used were:

- Lof & Hawley : 18.68 kcal/hr m² °C *
- Saunders and Ford : 54.05 kcal/hr m² °C
- Gamson et al. : 86.0 kcal/hr m² °C
- Ranz : 48.51 kcal/hr m² °C
- Denton et al : 77.8 kcal/hr m² °C

* Modified in section 5.2.3. for non-plug flow conditions.
The heat transfer coefficients were conveniently grouped into three numerical categories as follows:

- Lof and Hawley = 18.68 kcal/hr m² °C
- Saunders and Ford = 54.05 kcal/hr m² °C
- Ranz = 48.51 kcal/hr m² °C
- Gamson et al. = 86.0 kcal/hr m² °C
- Denton et al. = 77.8 kcal/hr m² °C

For ease of analysis the Lof and Hawley, Saunders and Ford, and Gamson et al. were chosen to indicate the spread of values obtained from the use of the correlations (See Fig. 5.19) to represent heat transfer from gas to the bed of particles.

The value of the heat transfer coefficients obtained clearly show a trend viz. greater values where spherical particles were used, and this was expected for reasons stated in section 2.6.
Calculation of the dust particle size \((d_m)\) in calciner.

The maximum particle size \(d_m\) that can be supported in the calciner chamber gases can be obtained by a balance between the drag force on the particle (Stokes law) and the gravitational force on the particle. The terminal velocity will be taken as that of the gas as it enters the grate i.e. at kiln exit. Thus:

\[
3.\pi.\mu.f.d_m u = \frac{1}{6}\pi.d_m^3 \left( \rho_s - \rho_f \right) g
\]

where

- \(u = 3.06\) m/sec
- \(\rho_s = \) density of solid = 1713 kg/m\(^3\)
- \(\rho_f = \) density of the fluid = 3.281 \(\times\) 10\(^{-1}\) kg/m\(^3\)
- \(\mu_f = \) viscosity of the fluid = 1.836 \(\times\) 10\(^{-1}\) kg/hr m
- \(g = 9.81\) m/sec\(^2\).

Hence \(d_m = 409\) microns.

Calculation of the surface mean diameter.

Let the size analysis of the dust on a weight basis be represented by a straight line from 0% weight at 1 micron particle size to 100% weight at 409 microns particle size as shown in Figure E1.

The surface mean diameter \((59)\) is given by

\[
\frac{d' \rho'}{dp'} = \frac{1}{\Sigma \frac{x' \rho'}{d'}}
\]

Since the size analysis is represented by the continuous curve

\[
d' = 408 x' + 1.02
\]

\[
\frac{d' \rho'}{dp'} = \frac{1}{\int_0^1 \frac{dx'}{d'}}
\]
\[ = \frac{1}{\int_{0}^{1} \frac{dx'}{(408x' + 1.02)}} \]

\[ = 68 \text{ microns.} \]

**Figure E1** Size analysis of the dust in calciner.
Appendix F  Representation of a real gas for engineering calculations.

F1. Introduction.

F2. Interpretation of $a_i$ in terms of spectral energy distribution.

F3. Representation of the emissivity of dust in engineering calculations.

F4. Mean beam length in the calciner chamber. ($L_m$).

F5. Evaluation of gas emittance.

F6. Analysis of gases in chamber.

F7. The grey gas fit.

F8. Emissivity of dust + gas mixture in chamber.

F9. Data for the evaluation of the emissivity of the medium.
Introduction.

Real gases, such as CO$_2$ and water vapour are not grey but emit and absorb radiation only in discrete bands in the spectrum and therefore have mean absorption coefficients which vary with gas temperature and the temperature of the radiation source. The emissivity path length relationship for a grey gas is given by:

$$\varepsilon_g = 1 - e^{-k_iP_L m} \quad \ldots \quad (1)$$

where $k_i$ is the specific absorption coefficient.

$P$ is the partial pressure of the absorbing gas.

$L_m$ is the path length.

Hottel and Sarofim(5) have shown that the emissivity of a real gas may be visualized as the weighted sum of the emissivities of a number of grey gases:

$$\varepsilon_g = a_1\left(1-e^{-k'_1P_L m}\right) + a_2\left(1-e^{-k'_2P_L m}\right) + a_3\left(1-e^{-k'_3P_L m}\right) + \ldots$$

$$= \sum a_i - \sum a_i e^{-k_i P_L m} \quad \ldots \quad (2)$$

where $k'_1 < k'_2 < k'_3 < \ldots$

At large values of $P_L m e^{-k_i P_L m}$ is very small and

$$\sum a_i = \varepsilon_g \text{ at large } P_L m \quad \ldots \quad (3)$$

In order to determine the weighting coefficients ($a_i$) and the absorption coefficients ($k_i$), eq. (2) is rewritten in the form
\[ \ln(\Sigma a_i - \varepsilon g) = \ln(\Sigma a_i \cdot e^{-k_i \cdot P_L m}) \] .......(4)

Since \( k'_1 < k'_2 < k'_3 \) ......., then at large \( P_L m \)
\[ e^{-k'_1 \cdot P_L m} >> e^{-k'_2 \cdot P_L m} >> e^{-k'_3 \cdot P_L m} \] .......

and equation (4) becomes
\[ \ln(\Sigma a_i - \varepsilon g) = -k'_1 \cdot P_L m + \ln(a_1) \] .......(5)

If \( \ln(\Sigma a_i - \varepsilon g) \) is plotted against \( P_L m \), then \( \ln(a_1) \) and \( k'_1 \) can be determined by drawing the tangent to the curve at large \( P_L m \), and measuring its gradient \( (k'_1) \) and its intercept \( \left[ \ln(a_1) \right] \) on the ordinate.

F2. Interpretation of \( a_i \) in terms of spectral distribution.

If the spectral variation of \( k_\omega \) can be imagined to be approximated by a number of groups of constant absorption coefficient bands, each having an absorption coefficient \( k_n \), as shown in Figure F1 then the weighting factor \( a_i \) for each term in the exponential series (equation 2) represents the fraction of the black body energy in the wave number regions where the absorption coefficient is \( k_i \). This is illustrated in Figure F1 where the representation is shown for three terms in equation 2.

Since the distribution of black body energy in the spectrum varies with temperature, according to Planck's Law, and the size of the gas absorption bands may also depend on temperature, it follows that the weighting factors \( a_i \) will vary with temperature. Thus, by suitable selection of the values of \( k_i \) and \( a_i \) it is possible to use a fixed set of absorption coefficients, and let the full temperature dependence of emissivity be carried by the weighting factors \( a_i \).
Although in theory the emissivity of a gas approaches 1 at sufficiently high values of $P_L$, at all practically attainable values of $P_L$ the emissivity is considerably less than 1. Therefore, the fitted emissivity equation will usually have one term where $k_i = 0$, corresponding to the non-absorbing part of the spectrum between the strong bands. This is referred to as the clear gas component.

The number of terms used in the grey gas fit depends on the desired accuracy of the fit. The greater the number of terms the better will be the representation. However, Hottel suggests that for many purposes a one gray, one clear gas approximation is adequate. Hence equation 2 becomes.

$$\varepsilon_g = a_0 (1 - e^{-0 P_L}) + a_1 (1 - e^{-k_1 P_L}) \quad \ldots \ldots \quad (6)$$

and $a_0 = 1 - a_1$

---

**Figure F1.** Representation of real gas absorption bands. (After Johnson and Beer (60)).
F3. Representation of the emissivity of dust in engineering calculations.

Thring (61) states that the emissivity of a single particle can be described as follows

\[ \varepsilon_D = 1 - \exp \left( - \frac{3}{2} \frac{C_D}{\rho_S} \frac{L_m}{dp} \right) \]  \hspace{1cm} (7)

where \( C_D \) is the mass concentration of particles = kg/m³

\( \rho_S \) is the density of particles kg/m³

\( dp \) is the spherical particle diameter m

\( L_m \) is the path length.

In the calciner grate and in the kiln, a considerable amount of dust is generated. Rewriting equation (7) as follows

\[ \varepsilon_D = 1 - \exp \left( - U \cdot C_D \cdot L_m \right) \]  \hspace{1cm} (8)

where \( U = \frac{3}{2 \rho_S dp} \)

and incorporating this into equation (2) yields the emissivity of the medium:

\[ e_m = \sum a_i \left[ 1 - \exp \left( k_i \cdot P \cdot L_m + U \cdot C_D \cdot L_m \right) \right] \]  \hspace{1cm} (9)
Hottel (5) has tabulated the mean beam length for gas surface exchange for several situations. For the present case which will be treated as an infinite cylinder, of half circular section, radiating to a spot on the middle of the flat side, Hottel states that:

\[
\frac{L_m}{R} = 1.26 \quad \cdots \cdots (10)
\]

where \( R \) is the radius = 2.05 m

Hence the mean beam length = 2.58 m = 8.46 ft.

F5. Evaluation of gas emittance

Typically, emittance is a function of gas temperature, path length, gas total pressure and gas partial pressure and tabulations of experimental results must allow consideration of all these variables. The results of Hottel and Egbert (62) presented in Figures F2, F3, F4, F5 and F6 were designed to allow this consideration.

In figures F2 and F3, the reduced emittance \( \varepsilon_{gT} \) of \( \text{CO}_2 \) and \( \text{H}_2\text{O} \) are presented as a function of the absolute temperature of the gas and the product of the mean beam length and partial pressure. These values are called the reduced emittance because they are plotted for a reduced pressure; that is, in order to obtain smooth presentation of the data, all data is reduced to an equivalent partial pressure of zero.

The effect of total pressure is given in Figures F4 and F5. These figures give the ratio of the actual emittance at the state being considered to the reduced emittance values of Figures F2 and F3. The variables for this ratio are the total pressure and the product of
Figure F2. Emissivity of Carbon dioxide.

Figure F3. Emissivity of water vapour.
Figure F4. Correction factor for CO$_2$ emissivity.

Figure F5. Correction factor for H$_2$O emissivity.

Figure F6. Correction factor for band overlap when CO$_2$ and H$_2$O are present in an enclosure.
partial pressure times mean beam length.

When both water vapour and CO$_2$ are present in the gas, a correction for the mixture composition may be made from Figure F6. The values of $\Delta e$ in these figures are subtracted from the sum of the emittances of each component, as determined from Figures F2, F3, F4 and F5.

Thus with the mean beam length known, the value of the overall gas emissivity of the combustion products due to CO$_2$ and H$_2$O may be obtained from Figures F2 to F6 and following the notation of Hottel, we can write the emittance as:

$$\varepsilon_g = \varepsilon'_c \cdot C_c + \varepsilon'_w \cdot C_w - \Delta e \quad \ldots \ldots (11)$$

where $\varepsilon'_c$ is the emittance of CO$_2$ from Figure F2.

$C_c$ is the correction for total pressure and partial pressure from Figure F4.

$\varepsilon'_w$ is the emittance of H$_2$O vapour from Figure F3.

$C_w$ is the correction for total pressure and partial pressure from Figure F5.

$\Delta e$ is the correction for a mixture of CO$_2$ and H$_2$O from Figure F6.

**F6. Analysis of gases in chamber.**

<table>
<thead>
<tr>
<th></th>
<th>Vol %</th>
<th>wt %</th>
</tr>
</thead>
<tbody>
<tr>
<td>H$_2$O</td>
<td>206.5  m$^3$</td>
<td>6.60</td>
</tr>
<tr>
<td>O$_2$</td>
<td>26.32  m$^3$</td>
<td>0.84</td>
</tr>
<tr>
<td>CO$_2$</td>
<td>927.07 m$^3$</td>
<td>29.61</td>
</tr>
<tr>
<td>N$_2$</td>
<td>1917.31 m$^3$</td>
<td>62.95</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>3131.00</td>
<td>100.00</td>
</tr>
</tbody>
</table>
Hence:

Partial pressure of CO₂ (Pc) = 0.296 atm.
Partial pressure of H₂O vap. (Pω) = 0.066 atm.
Mean beam length (Lₘ) = 8.46 ft.
Total pressure = 1.0 atm.
Temperature of gases = 1000°C = 2292°F

Pc.Lₘ = 2.50 atm-ft.
Pω.Lₘ = 0.558 atm-ft.

Using Hottel's notation to evaluate the emissivity of this gas mixture:

c'c = 0.185
c_c = 1
ε'ω = 0.135
c_ω = 1.03
Δε = 0.045

εg = 0.185 x 1 + 0.135 x 1.03 - 0.045
= 0.279

The emissivity (Vs) PL curve was constructed with mixtures around that existing in the furnace, with the mean beam length held constant and the results are in table F1. Figure F7 shows the εg (Vs) PL plot.
F7. The grey gas fit.

Table F2 shows the choice of two values of weighting factors $E(a_i)$ in fitting $CO_2 - H_2O$ emissivity data at $2300^\circ R$, and Figure F8 shows the best fit of emissivity was given by $E(a_i) = 0.43$. This corresponds to intercept $a_1 = 0.39$ and slope $(-k')$ of $0.305 \text{ ft}^{-1} \text{ atm}^{-1}$.

Table F2

<table>
<thead>
<tr>
<th>$P_c \text{ (atm)}$</th>
<th>$P_w \text{ (atm)}$</th>
<th>$P_{Lm} \text{ atm ft}$</th>
<th>$\varepsilon_g$</th>
<th>$\Sigma a_i = 0.370$</th>
<th>$\Sigma a_i = 0.43$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>$\Sigma a_i - \varepsilon_g$</td>
<td>$\Sigma a_i - \varepsilon_g$</td>
<td></td>
</tr>
<tr>
<td>0.220</td>
<td>0.035</td>
<td>2.16</td>
<td>0.227</td>
<td>0.143</td>
<td>0.203</td>
</tr>
<tr>
<td>0.250</td>
<td>0.040</td>
<td>2.45</td>
<td>0.244</td>
<td>0.126</td>
<td>0.186</td>
</tr>
<tr>
<td>0.300</td>
<td>0.050</td>
<td>2.96</td>
<td>0.270</td>
<td>0.10</td>
<td>0.160</td>
</tr>
<tr>
<td>0.296</td>
<td>0.066</td>
<td>3.06</td>
<td>0.279</td>
<td>0.091</td>
<td>0.151</td>
</tr>
<tr>
<td>0.330</td>
<td>0.060</td>
<td>3.30</td>
<td>0.285</td>
<td>0.085</td>
<td>0.145</td>
</tr>
<tr>
<td>0.360</td>
<td>0.070</td>
<td>3.64</td>
<td>0.302</td>
<td>0.068</td>
<td>0.128</td>
</tr>
<tr>
<td>0.400</td>
<td>0.070</td>
<td>3.98</td>
<td>0.314</td>
<td>0.056</td>
<td>0.116</td>
</tr>
</tbody>
</table>
The emissivity of the gas mixture is given by

\[ \varepsilon_g = \sum a_i \left(1 - e^{-k_i P L_m}\right) \]  

Hottel. \hspace{1cm} \ldots \ldots (2)

The emissivity of the dust is given by

\[ \varepsilon_d = 1 - e^{-\frac{3C_D L_m}{2d_p \rho}} \]  

Thring \hspace{1cm} \ldots \ldots (7)

\[ = 1 - e^{-U C_D L_m} \]  

\hspace{1cm} \ldots \ldots (8)

Therefore combining these two equations for the emissivity of the medium yields

\[ \varepsilon_{medium} = \sum a_i \left(1 - e^{-k_i L_m}\right) \]  

\hspace{1cm} \ldots \ldots (12)

where \( k'_{i} = U C_D + k_i P \).

For one clear gas and one grey gas

\[ \varepsilon_m = a_o \left[1 - e^{-\left(U C_D L_m + k'_{o} P L_m\right)}\right] + \]

\[ + \]

\[ a_1 \left[1 - e^{-\left(U C_D L_m + k'_{1} P L_m\right)}\right] \]  

\hspace{1cm} \ldots \ldots (13)

where \( a_o = 1 - a_1 = 0.61 \)

\( k'_{o} = 0 \) for the clear component

Hence

\[ \varepsilon_m = a_o \left[1 - e^{-U C_D L_m}\right] + a_1 \left[1 - e^{-\left(U C_D L_m + k'_{1} P L_m\right)}\right] \]  

\hspace{1cm} \ldots \ldots (14)
Data for the evaluation of the emissivity of the medium.

\[ a_0 = 0.61 \]
\[ a_1 = 0.39 \]
\[ k.l' = 0.305 \text{ ft}^{-1} \text{ atm}^{-1} \]
\[ L_m = 8.46 \text{ ft} \]
\[ P = 0.362 \text{ atm} \]
\[ \rho_b = 106.94 \text{ lbs/ft}^3 \]
\[ d'_p = 2.015 \times 10^{-4} \text{ ft} \]
\[ U = 69.78 \text{ ft}^2/\text{lb} \]
\[ C_D = 2.9 \times 10^{-4} \text{ lbs/ft}^3 \]

This gives the emissivity of the medium = 0.384.
Figure F7. Emissivity of the CO$_2$-H$_2$O mixture at 2300°R, in dependence of $(P_c + P_\omega) \cdot L_m$ ft.atm.
Best fit is given by $\Sigma a_i = 0.43$

Figure F8  Fitting of CO$_2$-H$_2$O emissivity data at 2300°R.
Temperature variation within particles.

To estimate the temperature variation within the particle, the maximum temperature difference between the centre and the surface of a sphere whose outer surface temperature changed at a constant rate $\bar{\beta}$ ($^\circ$C/sec) was calculated. When dynamic equilibrium is reached, Fourier's conduction equation representing this sphere with a steadily rising temperature has been calculated by Carslaw and Jaeger (63) and their analysis yields:

$$\frac{dT}{dt} = \frac{k_s}{\rho \cdot \bar{C}} \left( \frac{2^2 T}{\partial r^2} + \frac{29T}{r \partial r} \right)$$

where $k_s$, $\rho$, and $\bar{C}$ are the thermal conductivity, density and specific heat of the material respectively,

$r$ = radial position within the sphere of diameter $d_p$.

The boundary conditions for this situation are:

$$\frac{dT}{dt} = \bar{\beta} \text{ for } r = 0 \text{ to } \frac{d_p}{2}.$$  
$$\frac{dT}{dt} = 0 \text{ for } r = 0 \text{ at centre}.$$

Kunii and Levenspiel (7) have computed this problem and state the solution to this equation as

$$|T_{\text{surface}} - T_{\text{centre}}| = \Delta T_{\text{max}} = \bar{\beta} \cdot \left( \frac{\rho \cdot \bar{C}}{k_s} \right) \cdot d_p^2.$$  

This equation was tested for the two cases of CaCO$_3$ and CaO on the grate.
<table>
<thead>
<tr>
<th></th>
<th>CaCO₃</th>
<th>CaO</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density (kg/m³)</td>
<td>2480 (Perry)</td>
<td>960 (Perry)</td>
</tr>
<tr>
<td>Thermal conductivity kg.cal hr m² °c/m</td>
<td>0.684 Satterfield and Feakes.</td>
<td>1.93 (McAdams)</td>
</tr>
<tr>
<td>Specific heat kg.cal kg °c</td>
<td>0.217 (Perry)</td>
<td>0.26 (Boynton)</td>
</tr>
</tbody>
</table>

Satterfield and Feakes (12) found that the CaCO₃ underwent a less than 8% shrinkage during calcination, and for these calculations a particle size of 0.0095 m = 3/8" was used and shrinkage considered negligible.

Let the sphere of CaCO₃ and CaO experience an abrupt temperature change = 100°C/sec = 6000°C/hr.

Then \( \Delta T \) max. CaCO₃ = 17.75°C
\( \Delta T \) max. CaO = 2.9°C

In practice, stratum 1 which experiences the most rapid gain in enthalpy, the solids temperature changes from 400°C to 870°C along the calciner.

Calciner length = 12.2 m.
Grate speed = 80.5 m/hr.
Temperature gain of solids = 470°C

Hence for stratum 1 = 0.86°C/sec.

The calculation thus shows that temperature variation within the particles will be unimportant.
Appendix H

Evaluation of the mean gas temperature in the bed.

For the theoretical model, a log mean solids temperature $T$ was evaluated for each increment. (See Figure 3.4). Thus the gas temperature $t_g$ varied simply with bed height.

For a differential height $dL$ of the increment,

\[
\text{Heat transferred} = \text{heat into section} - \text{heat out by solids by gas gas}
\]

\[hA\left(t_g - T\right)dL = -G_0 C_f dT_g\]

where

- $h$ = heat transfer coefficient. (kcal/hr m² °C)
- $A'$ = specific surface of solid. (m²/m³)
- $G_0$ = mass flowrate of gas through increment. (kg/hr m²)
- $C_f$ = mean specific heat of gas. (kcal/kg °C)

\[G_0 = U_0 \rho_f\]

where

- $U_0$ = velocity of gas through the bed. (m/hr)
- $\rho_f$ = mean density of gases. (kg/m³).

The convenient representative velocity $U_0$ chosen was that based on the total flowrate, and the total cross-sectional area of the bed, i.e. a plug flow assumption.

N.B. This velocity was modified in the light of experimental results.
Integration between the limits $L = 0$ and $L = L/4$ yields

$$\ln \left( \frac{t_{g1} - T}{t_{g2} - T} \right)_0 = \frac{h.A.L}{G_oC_f^4}$$

Thus the temperature of the gas at depth $L/4$ was given by

$$t_{g2} = \left( t_{g1} - T \right) \cdot \exp \left( -\frac{h.A.L}{G_oC_f^4} \right) + T$$

The mean temperature of the gas in contact with the bed at $T$ will be given by

$$t_{g\text{mean}} = \left( t_{g1} - t_{g2} \right) / \ln\left( t_{g1}/t_{g2} \right)$$

The above analysis is not strictly correct, since the bed of nodules is also affected by radiation from the gas and the walls and the radiative transfer between two adjacent strata. This has been quantified in appendix N and the results stated in Table 6.2.

The temperature of the gas calculated at depth $L/4(t_{g2})$ was used as the gas film entering temperature for the stratum below and the above calculation repeated until the gas exits the bed.

![Diagram](image)

**Situation in an increment.**
Appendix J  Evaluation of the inside temperature of the chamber walls. ($T_w$).

J1. Physical properties of the wall.


J3. Evaluation of the heat transfer coefficient from the outside wall to ambient air.

J4. Physical properties of air at 77°C.

J5. Temperature at the furnace walls.
J1. Physical properties of the wall.

The calciner chamber is a semicircular \( \frac{1}{4} \)" steel shell lined with two layers of refractory brick.

Figure J1 illustrates the cross section of the walls.

The thermal conductivity of refractory 1 and 2 is 0.1153 and 0.8680 kcal/hr m\(^2\) \(^{\circ}\)C/m respectively.

The thickness of refractory 1 and 2 is 0.127m and 0.1524m respectively.

The inside and outside radius of the furnace is 2.16m and 2.44m respectively.

The mean outside surface temperature of the steel shell measured was 132\(^{\circ}\)C and because the shell has a small thickness of 0.63 cms. and a high thermal conductivity the inside surface temperature of the steel shell will also be approximately 132\(^{\circ}\)C.


The inside temperature of the wall can be estimated by the following heat balance.

Heat lost by convection from steel shell

\[ + \text{heat lost by radiation to air} \]

= heat conducted through brickwork 1

= heat conducted through brickwork 2
Ambient $T_0 = 21^\circ C$

$T'_1 = 132^\circ C$

Thermal conductivity

- Steel shell: $38.74 \text{ kcal/hr m}^2 \text{ °C/m}$
- Refractory 1: $0.1153 \text{ kcal/hr m}^2 \text{ °C/m}$
- Refractory 2: $0.8680 \text{ kcal/hr m}^2 \text{ °C/m}$

Wall thickness

- Steel shell: 1/4ins. = 0.63cms.
- Refractory 1: 0.127m.
- Refractory 2: 0.1524m.

Calciner inside radius: 2.16m.
Calciner outside radius: 2.44m

Figure J1 Cross-section of the calciner walls.
Considering unit surface area:

\[ h_c \cdot \left( T_1 - T_0 \right) + \sigma \cdot e \left( T_1^h - T_0^h \right) = k_1 \frac{\left( T_2' - T_1' \right)}{L_1'} = k_2 \frac{\left( T_3' - T_2' \right)}{L_2'} \]

J3. Evaluation of the heat transfer coefficient \((h_c)\) from outside wall to air.

McAdams (45) has quoted a correlation for the natural convection coefficient of heat transfer from horizontal tubes to air in the laminar region and the data can be represented by the dimensionless equation:

\[ \text{Nu} = f (\text{Pr}, \text{Gr}) \]

or \[ \frac{h_c \cdot D_0}{k_f} = 0.53 \left[ \frac{D_0^3 \cdot \rho_f \cdot \beta_f \cdot \Delta t}{\mu_f^2} \left( \frac{\text{Cf} \cdot \mu_f}{k_f \cdot f} \right)^{0.25} \right] \]

where \(D_0\) = outside diameter of cylinder (m).
\(Cf\) = specific heat of air.
\(\mu_f\) = viscosity of air.
\(k_f\) = thermal conductivity of air.
\(\rho_f\) = density of air.
\(\Delta t\) = temperature difference between surface and ambient air.
\(g\) = acceleration due to gravity.
\(\beta_f\) = coefficient of volumetric expansion, \(1/°C\).

The physical properties \(Cf, k_f, \rho_f, \mu_f\) and \(\beta_f\) have been evaluated by McAdams at a film temperature between the temperature of the surface of the cylinder and the temperature of the ambient air. For the case considered the film temperature was \(77°C\).
Physical properties of air at 77°C.

The physical properties of air at 77°C have been evaluated by McAdams.

\[
\begin{align*}
C_f &= 0.2412 \text{ kcal/kg°C} \\
\mu_F &= 0.0776 \text{ kg/hr m} \\
k_F &= 0.0269 \text{ kcal/hr m}^2 \text{ °C/m} \\
\rho_f^2 \cdot \beta_{fg} \cdot C_f &= 3.623 \times 10^7 \\
\mu \cdot k_f &= \text{m}^3 \text{ °C} \\
\end{align*}
\]

These values substituted into equation (a) yield

\[hc = 2.44 \text{ kcal/hr m}^2 \text{ °C}.
\]

Temperature at the furnace walls.

McAdams quotes the emissivity of mild steel in the temperature region concerned at 0.15.

Substituting the data in the heat balance across the walls yields for unit surface area:

Heat loss by convection = \(2.44 \times (132-21) = 271 \text{ kcal/hr.}\)

Heat loss by radiation = \(4.88 \times 10^{-8} \times .15 \left[405^4-294^4\right]\)

\[= 142 \text{ kcal/hr.}\]

\[Q = \text{Total heat loss from 1m}^2 \text{ of shell surface} = 413 \text{ kcal/hr.}\]

Heat conducted through layer 1 = \(k_1 \left[\frac{T_2' - T_1}{L_1} \right]\)

\[\therefore \text{ The interface temperature } T_2' = \frac{Q \times L_1 + T_1'}{k_1} \]

\[= \frac{413 \times 0.127 + 132}{0.1153} = 587^\circ \text{C.}\]
The heat balance for the second layer of brick is

\[
\frac{k_1 (T'_2 - T'_1)}{L'_1} = \frac{k_2 (T'_3 - T'_2)}{L'_2}
\]

Hence \( T'_3 = 660^\circ \text{C} \) = Temperature at wall.
Appendix K  Evaluation of the view factors from
gas to bed and from wall to bed for
stratum 1.

K1. Definition of view factor and exchange area.

K2. Zoning.

K3. Evaluation of the view factors.
Evaluation of view factors from gas to bed and from wall to bed for stratum 1.

K1. Definition of view factor and exchange area.

The emission from black surface $A_1$ in all directions to one side of each element of it is $A_1 E_1$. Of this flux emitted in $2\pi$ steradians, let the fraction directed toward $A_2$ be $F_{12}$, the view factor. The one way flux $Q_{1 \rightarrow 2}$ will then be represented by $A_1 F_{12} E_1$. Similarly surface $A_2$ will send $A_2 F_{21} E_2$ toward $A_1$ and the net exchange will be

$$Q_{1 \rightarrow 2} = A_1 F_{12} E_1 - A_2 F_{21} E_2.$$ 

Since this net flux must become zero when the temperatures are the same, i.e. when $E_1 = E_2$, and since for black surfaces the factors $F$ are determined by geometry alone, it follows that $A_1 F_{12}$ must equal $A_2 F_{21}$. This product, having the same dimensions of area is called the direct-interchange area $S_1 S_2$. $F_{12}$ being dimensionless is reported in the literature for several situations; plainly it is $A_1 F_{12}/A_1$. By definition

$$F_{11} + F_{12} + F_{13} + \ldots = \sum_j F_{1j} = 1$$

or

$$\sum_j A_1 F_{1j} = A_1$$

The geometric situation in the calciner is shown in Figure K2.

The calciner bed is situated in a long semi-circular tunnel containing gases at approximately 1000°C. The furnace walls are assumed to be at a constant temperature and the walls radiate to the bed. Also the gases radiate to the bed and the bed re-radiates to the chamber.

K2. Zoning.

The semi-circular tunnel was divided into 15 zones each 0.8 m.
Figure K2 Geometric situation above the calciner bed.

Figure K3 Zoning of the calciner to evaluate the roof to bed exchange factor. (e.g. Increment 1 shown)
wide. For the evaluation of the calciner roof to bed exchange area $SS$, the results were calculated from the middle of the roof zone to the middle of the bed increment, and is illustrated in Figure K3. Hence, the bed in increment 1 has a view of the roof and gas from increment 1 to increment 15, and the view factor for the bed at this point is equal to the sum of the individual view factors evaluated.

$$\text{Since } \sum_{gs} \bar{g} \bar{s} + \sum_{ss} \bar{s} \bar{s} = A_1$$

$$\text{or } \frac{\sum_{gs} \bar{g} \bar{s}}{A_1} + \frac{\sum_{ss} \bar{s} \bar{s}}{A_1} = 1$$

where $A_1 = \text{Area of calciner roof increment} = 0.8 \times 4.1 \text{ m.}$

Then $\frac{\sum_{gs} \bar{g} \bar{s}}{A_1} = 1 - \frac{\sum_{ss} \bar{s} \bar{s}}{A_1}$

The calciner roof is arched, but as far as emission is concerned Gray and Muller (64) suggest that the concave surface may be replaced by a plane surface of area $A_2$, as in Figure K1.
Figure K4  Geometry for configuration factor between elemental area and finite long strip of differential width. The surfaces are in parallel planes.
K3. Evaluation of the view factors.

Siegel and Howell (65) have investigated the configuration factor between two elemental areas located on strips that have their surfaces parallel. The geometry for this system is shown in Figure K4.

The configuration factor between \( dA_1 \) and \( dA_2 \) was derived as follows:

The distance \( s \) can be expressed as

\[
s^2 = \frac{x^2}{z^2} + x^2
\]

and

\[
\cos \beta_1 = \frac{x \cos \phi}{s} = \frac{x \cos \phi}{\left(\frac{x^2}{z^2} + x^2\right)^{\frac{1}{2}}}
\]

The solid angle \( d\omega_1 \) subtended by \( dA_2 \) when viewed from \( dA_1 \) is related to the projected area of \( dA_2 \) and the distance between the differential elements by the relation

\[
d\omega_1 = \frac{dA_2 \cdot \cos \psi}{s^2}
\]

\[
= \frac{x \cdot d\phi \cdot dx \cdot \cos \psi}{s^2} = \frac{x \cdot d\phi \cdot dx}{s^2} \cdot \frac{x}{s}
\]

The fraction of energy leaving black surface element \( dA_1 \) that arrives at black element \( dA_2 \) is defined as the geometric configuration factor

\[
\frac{dF_{1-2}}{dA_1 \cdot dA_2} = \cos \beta_1 \cdot d\omega_1
\]

\[
= \frac{x \cos \phi}{\left(\frac{x^2}{z^2} + x^2\right)^{\frac{1}{2}}} \frac{1}{\pi} \cdot \frac{\frac{x^2 \cdot d\phi \cdot dx}{s^2}}{\left(\frac{x^2}{z^2} + x^2\right)^{\frac{3}{2}}}
\]
which is the desired configuration factor between \( dA_1 \) and \( dA_2 \).

When \( dA_2 \) becomes a strip of differential width

\[
dF_{d1 \text{-strip}} = \frac{x^3 \cos \phi \, d\phi}{\pi \left( x^2 + x^2 \right)^2} dx
\]

And since the integrand is an even function of \( x \) the final result is

\[
dF_{d1 \text{-strip}} = \frac{2x^3 \cos \phi \, d\phi}{\pi} \left[ \frac{x}{x^2 + x^2} + \frac{1}{2} \tan \left( \frac{x}{x^2 + x^2} \right) \right]_0^x
\]

The results computed from this equation are given in Table \( K_1 \).
<table>
<thead>
<tr>
<th>Zone</th>
<th>Roof to bed</th>
<th>Gas to bed</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.51</td>
<td>0.49</td>
</tr>
<tr>
<td>2</td>
<td>0.64</td>
<td>0.36</td>
</tr>
<tr>
<td>3</td>
<td>0.73</td>
<td>0.27</td>
</tr>
<tr>
<td>4</td>
<td>0.78</td>
<td>0.22</td>
</tr>
<tr>
<td>5</td>
<td>0.81</td>
<td>0.19</td>
</tr>
<tr>
<td>6</td>
<td>0.82</td>
<td>0.18</td>
</tr>
<tr>
<td>7</td>
<td>0.83</td>
<td>0.17</td>
</tr>
<tr>
<td>8</td>
<td>0.83</td>
<td>0.17</td>
</tr>
<tr>
<td>9</td>
<td>0.83</td>
<td>0.17</td>
</tr>
<tr>
<td>10</td>
<td>0.82</td>
<td>0.18</td>
</tr>
<tr>
<td>11</td>
<td>0.81</td>
<td>0.19</td>
</tr>
<tr>
<td>12</td>
<td>0.78</td>
<td>0.22</td>
</tr>
<tr>
<td>13</td>
<td>0.73</td>
<td>0.27</td>
</tr>
<tr>
<td>14</td>
<td>0.64</td>
<td>0.36</td>
</tr>
<tr>
<td>15</td>
<td>0.51</td>
<td>0.49</td>
</tr>
</tbody>
</table>

**Table K1.** Evaluated view factors.
Evaluation of the mass flowrate through an increment and the calcination requirement for a moving increment.

Length of calciner = 12.2 m
Number of increments = 15
Hence length of increment = 12.2/15 m

Speed of grate = 80.5 m/hr.
Hence time to travel increment = \( \frac{12.2}{15} \frac{m}{80.5} \) = 0.0101 hr.

Volume of increment = \( B \cdot dx \cdot L/4 \)
where \( B = \) width of grate = 4.1 m
\( dx = \) increment width = 12.2/15 m
\( L = \) depth of bed = 0.18 m

Volume of solids = \( B \cdot dx \cdot L \cdot (1-\varepsilon) / 4 \)
where \( \varepsilon = \) voidage = 0.4

Mass of solids = \( B \cdot dx \cdot L \cdot (1-\varepsilon) \cdot \rho \)
where \( \rho = \) weighted mean density of solids when both CaO and CaCO\(_3\) are present

\( \rho \text{ CaCO}_3 = 2480 \text{ kg/m}^3 \)
\( \rho \text{ CaO} = 960 \text{ kg/m}^3 \)

Hence mass flowrate through increment = \( B \cdot dx \cdot L \cdot (1-\varepsilon) \cdot \rho \text{ kg/hr.} \)
\( 0.0101 \frac{m}{4} \)
The calcination rate according to Boynton was:

\[ y = 0.0183 \times X \]

where \( y \) = fraction of mass calcined.

\( X \) = time in minutes.

Calcination requirement = 422 kcal/kg.

Considering a moving unit mass of increment the calcination heat requirement was evaluated thus when the bed attained 800°C. The maximum time experienced by an increment for calcination was 6.06 minutes.

<table>
<thead>
<tr>
<th>Time in increment (mins.)</th>
<th>Fractional mass of increment calcined</th>
<th>Heat required to calcine (kcal)</th>
<th>Actual heat required for moving unit mass (kcal)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0.606</td>
<td>0.011</td>
<td>4.642</td>
<td>4.642</td>
</tr>
<tr>
<td>1.212</td>
<td>0.022</td>
<td>9.284</td>
<td>4.642</td>
</tr>
<tr>
<td>1.818</td>
<td>0.033</td>
<td>13.926</td>
<td>4.642</td>
</tr>
<tr>
<td>2.424</td>
<td>0.044</td>
<td>18.568</td>
<td>4.642</td>
</tr>
<tr>
<td>3.03</td>
<td>0.055</td>
<td>23.210</td>
<td>4.642</td>
</tr>
<tr>
<td>3.636</td>
<td>0.066</td>
<td>27.852</td>
<td>4.642</td>
</tr>
<tr>
<td>4.242</td>
<td>0.077</td>
<td>32.494</td>
<td>4.642</td>
</tr>
<tr>
<td>4.848</td>
<td>0.088</td>
<td>37.136</td>
<td>4.642</td>
</tr>
<tr>
<td>5.454</td>
<td>0.099</td>
<td>41.778</td>
<td>4.642</td>
</tr>
<tr>
<td>6.06</td>
<td>0.110</td>
<td>46.42</td>
<td>4.642</td>
</tr>
</tbody>
</table>
Appendix M. Results computed from section 3, equation (3.9) and (3.11).

TABLES 5.2 to 5.10 were obtained with a grate speed of 80.5 m/hr, and a total bed depth of 18 cms. Hence the depth of each stratum was 18/4 cms.

The grate length was 12.2 m. and was divided into 15 increments for analysis.

Using the Lof and Hawley correlation,

\[ h_{\text{plug flow}} = 18.68 \text{ kcal/hr.m}^2.\text{°C} \]
\[ h_{\text{modified flow}} = 21.73 \text{ kcal/hr.m}^2.\text{°C} \]

The starting solid temperature for each stratum obtained from Weber was as follows:

<table>
<thead>
<tr>
<th>Strata</th>
<th>Temperature °C</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>400</td>
</tr>
<tr>
<td>2</td>
<td>125</td>
</tr>
<tr>
<td>3</td>
<td>95</td>
</tr>
<tr>
<td>4</td>
<td>90</td>
</tr>
<tr>
<td>Increment</td>
<td>Solid Exit (Lof &amp; Hawley)</td>
</tr>
<tr>
<td>-----------</td>
<td>---------------------------</td>
</tr>
<tr>
<td>1</td>
<td>505</td>
</tr>
<tr>
<td>2</td>
<td>592</td>
</tr>
<tr>
<td>3</td>
<td>662</td>
</tr>
<tr>
<td>4</td>
<td>719</td>
</tr>
<tr>
<td>5</td>
<td>766</td>
</tr>
<tr>
<td>6</td>
<td>804</td>
</tr>
<tr>
<td>7</td>
<td>815</td>
</tr>
<tr>
<td>8</td>
<td>824</td>
</tr>
<tr>
<td>9</td>
<td>832</td>
</tr>
<tr>
<td>10</td>
<td>838</td>
</tr>
<tr>
<td>11</td>
<td>844</td>
</tr>
<tr>
<td>12</td>
<td>849</td>
</tr>
<tr>
<td>13</td>
<td>855</td>
</tr>
<tr>
<td>14</td>
<td>861</td>
</tr>
<tr>
<td>15</td>
<td>869</td>
</tr>
</tbody>
</table>

These results have been plotted in Figure 5.19.
Temperatures evaluated for stratum 1 based on the Lof and Hawley correlation under plug flow conditions.

<table>
<thead>
<tr>
<th>Increment</th>
<th>Log mean gas.(^{\circ}\mathrm{C})</th>
<th>Gas* exit.(^{\circ}\mathrm{C})</th>
<th>Mean** solid.(^{\circ}\mathrm{C})</th>
<th>Exit solid.(^{\circ}\mathrm{C})</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>834</td>
<td>688</td>
<td>451</td>
<td>505</td>
</tr>
<tr>
<td>2</td>
<td>865</td>
<td>743</td>
<td>547</td>
<td>592</td>
</tr>
<tr>
<td>3</td>
<td>889</td>
<td>787</td>
<td>626</td>
<td>662</td>
</tr>
<tr>
<td>4</td>
<td>909</td>
<td>824</td>
<td>690</td>
<td>719</td>
</tr>
<tr>
<td>5</td>
<td>925</td>
<td>853</td>
<td>742</td>
<td>766</td>
</tr>
<tr>
<td>6</td>
<td>937</td>
<td>877</td>
<td>785</td>
<td>804</td>
</tr>
<tr>
<td>7</td>
<td>945</td>
<td>892</td>
<td>809</td>
<td>815</td>
</tr>
<tr>
<td>8</td>
<td>948</td>
<td>897</td>
<td>820</td>
<td>824</td>
</tr>
<tr>
<td>9</td>
<td>950</td>
<td>903</td>
<td>828</td>
<td>832</td>
</tr>
<tr>
<td>10</td>
<td>953</td>
<td>907</td>
<td>835</td>
<td>838</td>
</tr>
<tr>
<td>11</td>
<td>954</td>
<td>910</td>
<td>841</td>
<td>844</td>
</tr>
<tr>
<td>12</td>
<td>956</td>
<td>913</td>
<td>847</td>
<td>849</td>
</tr>
<tr>
<td>13</td>
<td>957</td>
<td>916</td>
<td>852</td>
<td>855</td>
</tr>
<tr>
<td>14</td>
<td>959</td>
<td>919</td>
<td>858</td>
<td>861</td>
</tr>
<tr>
<td>15</td>
<td>961</td>
<td>923</td>
<td>865</td>
<td>869</td>
</tr>
</tbody>
</table>

* Represents the gas film entrance temperature for Stratum 2.

** Represents the mean solid radiating temperature for the plane in Stratum 1 which radiates to the plane in Stratum 2.
Temperatures evaluated for stratum 2 based on the Lof and Hawley correlation under plug flow conditions.

<table>
<thead>
<tr>
<th>Increment</th>
<th>Log mean gas.(^{(°C)})</th>
<th>Gas* (\text{exit.}) (\text{(°C)})</th>
<th>Mean** (\text{solid.}) (\text{(°C)})</th>
<th>Exit solid.(\text{(°C)})</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>525</td>
<td>389</td>
<td>162</td>
<td>207</td>
</tr>
<tr>
<td>2</td>
<td>590</td>
<td>459</td>
<td>244</td>
<td>285</td>
</tr>
<tr>
<td>3</td>
<td>646</td>
<td>522</td>
<td>321</td>
<td>360</td>
</tr>
<tr>
<td>4</td>
<td>695</td>
<td>580</td>
<td>394</td>
<td>431</td>
</tr>
<tr>
<td>5</td>
<td>736</td>
<td>631</td>
<td>463</td>
<td>496</td>
</tr>
<tr>
<td>6</td>
<td>773</td>
<td>677</td>
<td>525</td>
<td>556</td>
</tr>
<tr>
<td>7</td>
<td>801</td>
<td>716</td>
<td>582</td>
<td>609</td>
</tr>
<tr>
<td>8</td>
<td>819</td>
<td>746</td>
<td>632</td>
<td>655</td>
</tr>
<tr>
<td>9</td>
<td>836</td>
<td>773</td>
<td>675</td>
<td>695</td>
</tr>
<tr>
<td>10</td>
<td>850</td>
<td>796</td>
<td>712</td>
<td>729</td>
</tr>
<tr>
<td>11</td>
<td>862</td>
<td>815</td>
<td>743</td>
<td>758</td>
</tr>
<tr>
<td>12</td>
<td>872</td>
<td>832</td>
<td>770</td>
<td>783</td>
</tr>
<tr>
<td>13</td>
<td>881</td>
<td>846</td>
<td>793</td>
<td>804</td>
</tr>
<tr>
<td>14</td>
<td>886</td>
<td>853</td>
<td>803</td>
<td>803</td>
</tr>
<tr>
<td>15</td>
<td>888</td>
<td>855</td>
<td>803</td>
<td>803</td>
</tr>
</tbody>
</table>

* Represents the gas film entrance temperature for Stratum 3.

** Represents the mean solid radiating temperature for the plane in Stratum 2 which radiates to the plane in Stratum 3.
Temperatures evaluated for stratum 3 based on the Lof and Hawley correlation under plug flow conditions.

<table>
<thead>
<tr>
<th>Increment</th>
<th>Log mean gas.(^{\circ})C</th>
<th>Gas(^{\ast}) exit.(^{\circ})C</th>
<th>Mean(^{\ast\ast}) solid.(^{\circ})C</th>
<th>Exit solid.(^{\circ})C</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>304</td>
<td>233</td>
<td>115</td>
<td>137</td>
</tr>
<tr>
<td>2</td>
<td>367</td>
<td>289</td>
<td>159</td>
<td>183</td>
</tr>
<tr>
<td>3</td>
<td>426</td>
<td>343</td>
<td>207</td>
<td>232</td>
</tr>
<tr>
<td>4</td>
<td>482</td>
<td>396</td>
<td>257</td>
<td>283</td>
</tr>
<tr>
<td>5</td>
<td>534</td>
<td>447</td>
<td>308</td>
<td>334</td>
</tr>
<tr>
<td>6</td>
<td>582</td>
<td>496</td>
<td>359</td>
<td>385</td>
</tr>
<tr>
<td>7</td>
<td>625</td>
<td>542</td>
<td>409</td>
<td>434</td>
</tr>
<tr>
<td>8</td>
<td>661</td>
<td>582</td>
<td>458</td>
<td>482</td>
</tr>
<tr>
<td>9</td>
<td>694</td>
<td>620</td>
<td>504</td>
<td>526</td>
</tr>
<tr>
<td>10</td>
<td>723</td>
<td>655</td>
<td>547</td>
<td>568</td>
</tr>
<tr>
<td>11</td>
<td>748</td>
<td>686</td>
<td>587</td>
<td>607</td>
</tr>
<tr>
<td>12</td>
<td>772</td>
<td>714</td>
<td>625</td>
<td>643</td>
</tr>
<tr>
<td>13</td>
<td>792</td>
<td>740</td>
<td>659</td>
<td>675</td>
</tr>
<tr>
<td>14</td>
<td>806</td>
<td>760</td>
<td>689</td>
<td>704</td>
</tr>
<tr>
<td>15</td>
<td>815</td>
<td>776</td>
<td>716</td>
<td>728</td>
</tr>
</tbody>
</table>

\(^{\ast}\) Represents the gas film entrance temperature for Stratum 4.

\(^{\ast\ast}\) Represents the mean solid radiating temperature for the plane in Stratum 3 which radiates to the plane in Stratum 4.
**Table 5.5: Stratum 4**

Temperatures evaluated for stratum 4 based on the Lof and Hawley correlation under plug flow conditions.

<table>
<thead>
<tr>
<th>Increment</th>
<th>Log mean gas.(^{\circ}\text{C})</th>
<th>Gas* exit.(^{\circ}\text{C})</th>
<th>Mean solid.(^{\circ}\text{C})</th>
<th>Exit solid.(^{\circ}\text{C})</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>193</td>
<td>157</td>
<td>100</td>
<td>111</td>
</tr>
<tr>
<td>2</td>
<td>239</td>
<td>195</td>
<td>123</td>
<td>136</td>
</tr>
<tr>
<td>3</td>
<td>285</td>
<td>234</td>
<td>151</td>
<td>166</td>
</tr>
<tr>
<td>4</td>
<td>332</td>
<td>275</td>
<td>182</td>
<td>199</td>
</tr>
<tr>
<td>5</td>
<td>378</td>
<td>316</td>
<td>217</td>
<td>235</td>
</tr>
<tr>
<td>6</td>
<td>423</td>
<td>358</td>
<td>254</td>
<td>273</td>
</tr>
<tr>
<td>7</td>
<td>468</td>
<td>400</td>
<td>293</td>
<td>313</td>
</tr>
<tr>
<td>8</td>
<td>508</td>
<td>440</td>
<td>332</td>
<td>352</td>
</tr>
<tr>
<td>9</td>
<td>547</td>
<td>479</td>
<td>372</td>
<td>392</td>
</tr>
<tr>
<td>10</td>
<td>583</td>
<td>517</td>
<td>412</td>
<td>432</td>
</tr>
<tr>
<td>11</td>
<td>617</td>
<td>552</td>
<td>450</td>
<td>470</td>
</tr>
<tr>
<td>12</td>
<td>648</td>
<td>586</td>
<td>488</td>
<td>507</td>
</tr>
<tr>
<td>13</td>
<td>677</td>
<td>618</td>
<td>525</td>
<td>543</td>
</tr>
<tr>
<td>14</td>
<td>701</td>
<td>646</td>
<td>560</td>
<td>577</td>
</tr>
<tr>
<td>15</td>
<td>722</td>
<td>671</td>
<td>592</td>
<td>608</td>
</tr>
</tbody>
</table>

* Gas temperature leaving grate.
### Table 5.7: Stratum 1 Temperatures Evaluated Based on the LoF and Hawley Correlation

**Under Modified Flow Conditions**

<table>
<thead>
<tr>
<th>Increment</th>
<th>Log mean gas. (°C)</th>
<th>Gas* exit. (°C)</th>
<th>Mean** solid. (°C)</th>
<th>Exit solid. (°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>844</td>
<td>704</td>
<td>457</td>
<td>520</td>
</tr>
<tr>
<td>2</td>
<td>876</td>
<td>763</td>
<td>566</td>
<td>615</td>
</tr>
<tr>
<td>3</td>
<td>902</td>
<td>810</td>
<td>652</td>
<td>691</td>
</tr>
<tr>
<td>4</td>
<td>922</td>
<td>847</td>
<td>720</td>
<td>750</td>
</tr>
<tr>
<td>5</td>
<td>937</td>
<td>876</td>
<td>774</td>
<td>797</td>
</tr>
<tr>
<td>6</td>
<td>946</td>
<td>894</td>
<td>806</td>
<td>815</td>
</tr>
<tr>
<td>7</td>
<td>951</td>
<td>903</td>
<td>822</td>
<td>829</td>
</tr>
<tr>
<td>8</td>
<td>954</td>
<td>910</td>
<td>835</td>
<td>840</td>
</tr>
<tr>
<td>9</td>
<td>957</td>
<td>915</td>
<td>845</td>
<td>849</td>
</tr>
<tr>
<td>10</td>
<td>959</td>
<td>920</td>
<td>853</td>
<td>857</td>
</tr>
<tr>
<td>11</td>
<td>961</td>
<td>924</td>
<td>859</td>
<td>862</td>
</tr>
<tr>
<td>12</td>
<td>963</td>
<td>927</td>
<td>865</td>
<td>868</td>
</tr>
<tr>
<td>13</td>
<td>No flow</td>
<td>No flow</td>
<td>858</td>
<td>849</td>
</tr>
<tr>
<td>14</td>
<td>No flow</td>
<td>No flow</td>
<td>841</td>
<td>833</td>
</tr>
<tr>
<td>15</td>
<td>No flow</td>
<td>No flow</td>
<td>827</td>
<td>821</td>
</tr>
</tbody>
</table>

* Represents the gas film entrance temperature for Stratum 2.

** Represents the mean solid radiating temperature for the plane in Stratum 1 which radiates to the plane in Stratum 2.
TEMPERATURES EVALUATED BASED ON THE LOF AND HAWLEY CORRELATION
UNDER MODIFIED FLOW CONDITIONS

<table>
<thead>
<tr>
<th>Increment</th>
<th>Log mean gas.(°C)</th>
<th>Gas* exit.(°C)</th>
<th>Mean** solid.(°C)</th>
<th>Exit solid.(°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>545</td>
<td>413</td>
<td>169</td>
<td>223</td>
</tr>
<tr>
<td>2</td>
<td>618</td>
<td>493</td>
<td>267</td>
<td>316</td>
</tr>
<tr>
<td>3</td>
<td>679</td>
<td>563</td>
<td>358</td>
<td>403</td>
</tr>
<tr>
<td>4</td>
<td>731</td>
<td>626</td>
<td>441</td>
<td>482</td>
</tr>
<tr>
<td>5</td>
<td>774</td>
<td>680</td>
<td>516</td>
<td>553</td>
</tr>
<tr>
<td>6</td>
<td>806</td>
<td>725</td>
<td>583</td>
<td>615</td>
</tr>
<tr>
<td>7</td>
<td>830</td>
<td>760</td>
<td>641</td>
<td>668</td>
</tr>
<tr>
<td>8</td>
<td>849</td>
<td>790</td>
<td>690</td>
<td>713</td>
</tr>
<tr>
<td>9</td>
<td>864</td>
<td>815</td>
<td>731</td>
<td>750</td>
</tr>
<tr>
<td>10</td>
<td>877</td>
<td>836</td>
<td>766</td>
<td>782</td>
</tr>
<tr>
<td>11</td>
<td>888</td>
<td>853</td>
<td>795</td>
<td>808</td>
</tr>
<tr>
<td>12</td>
<td>895</td>
<td>863</td>
<td>809</td>
<td>810</td>
</tr>
<tr>
<td>13</td>
<td>No flow</td>
<td>No flow</td>
<td>801</td>
<td>791</td>
</tr>
<tr>
<td>14</td>
<td>No flow</td>
<td>No flow</td>
<td>781</td>
<td>772</td>
</tr>
<tr>
<td>15</td>
<td>No flow</td>
<td>No flow</td>
<td>762</td>
<td>752</td>
</tr>
</tbody>
</table>

* Represents the gas entrance temperature for Stratum 3.

** Represents the mean solid radiating temperature for the plane in Stratum 2 which radiates to the plane in Stratum 3.
## TABLE 5.9. STRATUM 3

TEMPERATURES EVALUATED BASED ON THE LOF & HAWLEY CORRELATION
UNDER MODIFIED FLOW CONDITIONS

<table>
<thead>
<tr>
<th>Increment</th>
<th>Log mean gas.(°C)</th>
<th>Gas* exit.(°C)</th>
<th>Mean** solid.(°C)</th>
<th>Exit solid.(°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>327</td>
<td>253</td>
<td>120</td>
<td>148</td>
</tr>
<tr>
<td>2</td>
<td>400</td>
<td>320</td>
<td>176</td>
<td>207</td>
</tr>
<tr>
<td>3</td>
<td>468</td>
<td>384</td>
<td>235</td>
<td>267</td>
</tr>
<tr>
<td>4</td>
<td>531</td>
<td>446</td>
<td>297</td>
<td>328</td>
</tr>
<tr>
<td>5</td>
<td>588</td>
<td>504</td>
<td>358</td>
<td>389</td>
</tr>
<tr>
<td>6</td>
<td>638</td>
<td>557</td>
<td>418</td>
<td>448</td>
</tr>
<tr>
<td>7</td>
<td>679</td>
<td>604</td>
<td>475</td>
<td>503</td>
</tr>
<tr>
<td>8</td>
<td>716</td>
<td>647</td>
<td>528</td>
<td>554</td>
</tr>
<tr>
<td>9</td>
<td>748</td>
<td>685</td>
<td>577</td>
<td>601</td>
</tr>
<tr>
<td>10</td>
<td>776</td>
<td>719</td>
<td>622</td>
<td>644</td>
</tr>
<tr>
<td>11</td>
<td>800</td>
<td>749</td>
<td>663</td>
<td>682</td>
</tr>
<tr>
<td>12</td>
<td>817</td>
<td>774</td>
<td>699</td>
<td>716</td>
</tr>
<tr>
<td>13</td>
<td>No flow</td>
<td>No flow</td>
<td>717</td>
<td>718</td>
</tr>
<tr>
<td>14</td>
<td>No flow</td>
<td>No flow</td>
<td>719</td>
<td>720</td>
</tr>
<tr>
<td>15</td>
<td>No flow</td>
<td>No flow</td>
<td>720</td>
<td>721</td>
</tr>
</tbody>
</table>

* Represents the gas film entrance temperature for Stratum 4.

** Represents the mean solid radiating temperature for the plane in Stratum 3 which radiates to the plane in Stratum 4.
TABLE 5.10. STRATUM 4

TEMPERATURES EVALUATED BASED ON THE LOF AND HAWLEY CORRELATION
UNDER MODIFIED FLOW CONDITIONS

<table>
<thead>
<tr>
<th>Increment</th>
<th>Log mean gas. (°C)</th>
<th>Gas* exit. (°C)</th>
<th>Mean solid. (°C)</th>
<th>Exit solid. (°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>209</td>
<td>171</td>
<td>103</td>
<td>117</td>
</tr>
<tr>
<td>2</td>
<td>266</td>
<td>218</td>
<td>134</td>
<td>152</td>
</tr>
<tr>
<td>3</td>
<td>322</td>
<td>268</td>
<td>171</td>
<td>191</td>
</tr>
<tr>
<td>4</td>
<td>379</td>
<td>318</td>
<td>212</td>
<td>234</td>
</tr>
<tr>
<td>5</td>
<td>433</td>
<td>369</td>
<td>257</td>
<td>280</td>
</tr>
<tr>
<td>6</td>
<td>485</td>
<td>419</td>
<td>303</td>
<td>328</td>
</tr>
<tr>
<td>7</td>
<td>532</td>
<td>466</td>
<td>351</td>
<td>375</td>
</tr>
<tr>
<td>8</td>
<td>577</td>
<td>511</td>
<td>398</td>
<td>422</td>
</tr>
<tr>
<td>9</td>
<td>617</td>
<td>554</td>
<td>445</td>
<td>468</td>
</tr>
<tr>
<td>10</td>
<td>655</td>
<td>594</td>
<td>490</td>
<td>513</td>
</tr>
<tr>
<td>11</td>
<td>689</td>
<td>631</td>
<td>533</td>
<td>555</td>
</tr>
<tr>
<td>12</td>
<td>718</td>
<td>665</td>
<td>574</td>
<td>594</td>
</tr>
<tr>
<td>13</td>
<td>No flow</td>
<td>No flow</td>
<td>595</td>
<td>596</td>
</tr>
<tr>
<td>14</td>
<td>No flow</td>
<td>No flow</td>
<td>597</td>
<td>599</td>
</tr>
<tr>
<td>15</td>
<td>No flow</td>
<td>No flow</td>
<td>600</td>
<td>601</td>
</tr>
</tbody>
</table>

* Gas temperature leaving grate.
### Solid Temperatures Evaluated Based on the LoF & Hawley Correlation

**For Modified Flow for Different Grate Speeds**

<table>
<thead>
<tr>
<th>Increment</th>
<th>( V = 40.25 \text{m/hr} )</th>
<th>( V = 80.5 \text{m/hr} )</th>
<th>( V = 170.0 \text{m/hr} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>622</td>
<td>520</td>
<td>463</td>
</tr>
<tr>
<td>2</td>
<td>761</td>
<td>615</td>
<td>518</td>
</tr>
<tr>
<td>3</td>
<td>827</td>
<td>691</td>
<td>567</td>
</tr>
<tr>
<td>4</td>
<td>867</td>
<td>750</td>
<td>611</td>
</tr>
<tr>
<td>5</td>
<td>889</td>
<td>797</td>
<td>650</td>
</tr>
<tr>
<td>6</td>
<td>903</td>
<td>815</td>
<td>685</td>
</tr>
<tr>
<td>7</td>
<td>911</td>
<td>829</td>
<td>717</td>
</tr>
<tr>
<td>8</td>
<td>916</td>
<td>840</td>
<td>745</td>
</tr>
<tr>
<td>9</td>
<td>918</td>
<td>849</td>
<td>770</td>
</tr>
<tr>
<td>10</td>
<td>920</td>
<td>857</td>
<td>792</td>
</tr>
<tr>
<td>11</td>
<td>922</td>
<td>862</td>
<td>812</td>
</tr>
<tr>
<td>12</td>
<td>924</td>
<td>868</td>
<td>810</td>
</tr>
<tr>
<td>13</td>
<td>900</td>
<td>849</td>
<td>792</td>
</tr>
<tr>
<td>14</td>
<td>883</td>
<td>833</td>
<td>774</td>
</tr>
<tr>
<td>15</td>
<td>874</td>
<td>821</td>
<td>759</td>
</tr>
</tbody>
</table>
Evaluation of the radiation contribution to the total heat transfer for Increment 1 and Increment 15, using equation 3.9, Section 3.

**Stratum 1.**

**Data for increment 1.**

Heat transferred by convection. = \(407547\) kcal/hr.
Radiation from furnace walls. = \(31400\) kcal/hr.
Radiation from furnace gases. = \(80298\) kcal/hr.
Re-radiation from solids. = \(13347\) kcal/hr.

Radiation contribution = 19.44%

**Data for increment 15.**

Heat transferred by convection. = \(101919\) kcal/hr.
Radiation from furnace walls. = \(31400\) kcal/hr.
Radiation from furnace gases. = \(80298\) kcal/hr.
Re-radiation from solids. = \(81830\) kcal/hr.

Radiation contribution = 22.66%

**Stratum 2.**

**Data for increment 1.**

Heat transferred by convection. = \(384324\) kcal/hr.
Net radiation between considered increment and increment above. = \(6850\) kcal/hr

Radiation contribution = 1.75%
Data for increment 15.

Heat transferred by convection. = 90912 kcal/hr.
Net radiation between considered increment and increment above. = 9703 kcal/hr.

Radiation contribution = 9.6%

Stratum 3.

Data for increment 1.

Heat transferred by convection. = 201317 kcal/hr.
Net radiation between considered increment and increment above. = 379 kcal/hr.

Radiation contribution = 0.188%

Data for increment 15.

Heat transferred by convection. = 105048 kcal/hr.
Net radiation between considered increment and increment above. = 11029 kcal/hr.

Radiation contribution = 9.5%

Stratum 4.

Data for increment 1.

Heat transferred by convection. = 98481 kcal/hr.
Net radiation between considered increment and increment above. = 96 kcal/hr.

Radiation contribution = 0.097%
Data for increment 15.

Heat transferred by convection  =  138472 kcal/hr.
Net radiation between considered increment and increment above  =  11390 kcal/hr.

Radiation contribution  =  7.6%
Appendix P

Analytical solution for the temperature of solids at steady state on the grate.

The approach is illustrated in Figures P1' and P2'.

The increment is of thickness \( dx \), width \( B \), and depth \( L/4 \). If the thickness \( dx \) is small compared with its length, and if the convection essentially controls the heat flow, transverse temperature gradients will be negligible, but there will be a large axial temperature distribution. The increment is assigned a temperature \( T \).

For analysis 15 increments were used.
Figure P2. Section on bed showing division into 4 equal strata for analysis.

Using the same notation as before in Section 3, the heat balance for stratum 1 reads as follows:

Heat stored in increment \( = B \cdot dx \cdot L \cdot (1-\varepsilon) \cdot \rho \cdot \bar{c} \cdot \frac{dT}{dt} \) \hspace{1cm} (1)

Heat utilized for calcination \( = B \cdot dx \cdot L \cdot (1-\varepsilon) \cdot \rho \cdot R \) \hspace{1cm} (2)

where \( R \) = calcination rate heat requirement.

Heat radiated by furnace wall \( = B \cdot dx \cdot F_{12} \cdot \varepsilon \cdot \sigma \cdot Tw^4 \) \hspace{1cm} (3)

Heat radiated by gas \( = B \cdot dx \cdot F_{13} \cdot \varepsilon g \cdot \sigma \cdot Tg^4 \) \hspace{1cm} (4)

Heat re-radiated by increment \( = B \cdot dx \cdot e_1 \cdot \sigma \cdot T^4 \) \hspace{1cm} (5)

Heat transferred by convection \( = B \cdot dx \cdot 3 \cdot h \cdot (1-\varepsilon) \cdot L \cdot \left( \frac{T_{gl} - T}{r} \right) \) \hspace{1cm} (6)

Equating and dividing through by \( B \cdot dx \cdot \varepsilon \cdot \sigma \cdot Tw^4 \) yields

\[
\frac{L \cdot (1-\varepsilon) \cdot \rho \cdot \bar{c} \cdot dT}{4 \cdot \varepsilon \cdot \sigma \cdot Tw^4} + \frac{L \cdot (1-\varepsilon) \cdot \rho \cdot R}{4 \cdot \varepsilon \cdot \sigma \cdot Tw^4} = \frac{F_{12}}{\varepsilon \omega} + \frac{F_{13} \cdot \varepsilon g \cdot Tg^4}{\varepsilon \omega \cdot Tw^4} - \frac{e_1 \cdot T^4}{\varepsilon \omega \cdot Tw^4} + \frac{3 \cdot h \cdot (1-\varepsilon) \cdot L \cdot \left( \frac{T_{gl} - T}{r} \right)}{4 \cdot r \cdot \varepsilon \omega \cdot Tw^4}
\]

\[
\tau = \frac{x}{\bar{V}} \hspace{1cm} dt = \frac{dx}{\bar{V}}
\]

Introduce the dimensionless quantities:

\[
\theta = \frac{T}{Tw} \hspace{1cm} \theta_G = \frac{T_G}{Tw} \hspace{1cm} \theta_{G1} = \frac{T_{gl}}{Tw}
\]

\[
\delta_1 = \frac{e_1}{\varepsilon \omega} \hspace{1cm} \delta_g = \frac{\varepsilon g}{\varepsilon \omega}
\]

Also \( \zeta = \frac{x}{L'} \) where \( L' \) = height of furnace above grate.

\[
\frac{dT}{dx} = \frac{d\theta}{d\zeta} \cdot \frac{Tw}{L'}
\]
Hence

\[
\frac{L \cdot (1-\varepsilon) \cdot \rho \cdot c \cdot V}{4 \cdot \varepsilon \cdot \sigma \cdot T_0^3 \cdot L'} \cdot \frac{d\theta}{d\zeta} + \frac{L \cdot (1-\varepsilon) \cdot \rho \cdot R}{4 \cdot \varepsilon \cdot \sigma \cdot T_0^3} = \frac{F_{12} + F_{13} \cdot \delta g \cdot \theta^* - \delta_1 \cdot \theta^* + 3 \cdot h \cdot (1-\varepsilon) \cdot L \cdot (\theta_{GL} - \theta)}{4 \cdot \varepsilon \cdot \sigma \cdot T_0^3}
\]  

(10)

This yields the dimensionless quantities

\[
\frac{L \cdot (1-\varepsilon) \cdot \rho \cdot c \cdot V}{4 \cdot \varepsilon \cdot \sigma \cdot T_0^3 \cdot L'} = \mu
\]

\[
\frac{L \cdot (1-\varepsilon) \cdot \rho \cdot R}{4 \cdot \varepsilon \cdot \sigma \cdot T_0^4} = C'
\]

\[
\frac{3 \cdot h \cdot (1-\varepsilon) \cdot L}{4 \cdot \varepsilon \cdot \sigma \cdot T_0^3} = H'
\]

Thus

\[
\mu \cdot \frac{d\theta}{d\zeta} + C' = F_{12} + F_{13} \cdot \delta g \cdot \theta^* - \delta_1 \cdot \theta^* + H \cdot \theta_{GL} - H \cdot \theta.
\]  

(11)

Let \( D' = F_{12} - C' + F_{13} \cdot \delta g \cdot \theta^* + H \cdot \theta_{GL} \).

\[
\mu \cdot \frac{d\theta}{d\zeta} = D' - H \cdot \theta - \delta_1 \cdot \theta^*
\]

\[
\frac{d\theta}{d\zeta} = K_1 - K_2 \cdot \theta - K_3 \cdot \theta^* \text{ where } K_1 = D'/\mu, K_2 = H'/\mu, K_3 = \delta_1/\mu
\]  

(12)

Hence the heating of the moving top stratum on the grate has been described by a first order non-linear differential equation.

This equation was solved by the Runge Kutta numerical method in the following manner.

\[
\theta_1 = \theta_0 + \frac{1}{6} \left[ P_0 + 2P_1 + 2P_2 + P_3 \right]
\]
where
\[ P_o = Z \times f(\theta_o) \]
\[ P_1 = Z \times f(\theta_o + P_o/2) \]
\[ P_2 = Z \times f(\theta_o + P_1/2) \]
\[ P_3 = Z \times f(\theta_o + P_2) \]
\[ Z = \text{step length}. \]

The results obtained were plotted in Figures P1 and P5 for the conditions of plug flow and modified flow and were compared to the results of the numerical solution. (Sect. 3)


Energy stored in increment
\[ = \frac{B \cdot dx \cdot L \cdot (1-\varepsilon) \cdot \rho \cdot c \cdot V \cdot dT}{4} \]  \hspace{1cm} (13)

Energy used for calcining
\[ = \frac{B \cdot dx \cdot L \cdot (1-\varepsilon) \cdot \rho \cdot R}{4} \]  \hspace{1cm} (14)

Energy transfer by convection
\[ = \frac{B \cdot dx \cdot L \cdot h \cdot 3 \cdot (1-\varepsilon) \cdot \left( T_{gl} - T \right)}{4} \]  \hspace{1cm} (15)

Energy exchange by radiation
\[ = \frac{\sigma \cdot B \cdot dx \cdot (T_a^b - T^b)}{4} \]  \hspace{1cm} (16)

\[ T_a = \text{Solid temperature of the increment above}. \]

Equating and multiplying through by
\[ = \frac{4}{L \cdot (1-\varepsilon) \cdot \rho \cdot c \cdot V} \]
yields
\[ \frac{dT}{dx} + \frac{R}{c \cdot V} = \frac{3 \cdot h \cdot \left( T_{gl} - T \right)}{\rho \cdot c \cdot V \cdot r} - \frac{4 \cdot \sigma \cdot \left( T_a^b - T^b \right)}{\left( \frac{2}{\varepsilon_1} - 1 \right) \cdot L \cdot (1-\varepsilon) \cdot \rho \cdot c \cdot V} \]  \hspace{1cm} (17)

Let
\[ K_1' = \frac{R}{c \cdot V}. \]
\[ K_2' = \frac{3h}{c \cdot V \cdot r} \]
\[ K_3' = \frac{4\sigma}{\left( \frac{2}{\varepsilon_1} - 1 \right) \cdot L \cdot (1-\varepsilon) \cdot c \cdot V} \]
\[ K_4' = K_2' \cdot T_{gl} - K_3' \cdot T_a - K_1' \]

Thus
\[ \frac{dT}{dx} = -K_4' T + K_3' T^b. \]  \hspace{1cm} (18)
This first order non-linear differential equation of equation (18) describes the heating of strata 2, 3 and 4 within the bed and was solved by the Runge-Kutta method.

The results of equations (12) and (18) were compared to those obtained from the numerical method in section 3 and were plotted in Figures P1 to P8 for the conditions of plug flow and modified flow through the calciner bed. The following observations were made:

(a) Plug flow conditions.
Stratum 1 gave the largest deviation between the analytical and numerical solutions of 18.6%, and for the remaining strata similar results were obtained with the results agreeing all within 3%.

(b) Modified flow conditions.
Stratum 1 gave the largest deviation of 24.7% between the analytical and numerical solutions and for stratum 2 and 3 the numerical solutions gave marginally improved results to fit Weber's results. For stratum 4, however, the analytical solution yielded a better approximation. This shows that at higher temperature gradients experienced as in stratum 1, the assumption of a single solid temperature for the whole increment was too coarse, and the log mean temperature of the solids evaluated in the numerical model was a better approximation to the realistic situation. Moreover, the numerical solution was a simpler model and was hence preferred to the analytical solution.
Figure P1. Comparison between predicted solids temperature under plug flow conditions using the numerical and analytical solutions. (h= Lof & Hawley)
Figure P2. Comparison between predicted solids temperature under plug flow conditions using the numerical and analytical solutions. (h= Lof & Hawley)
Figure P3. Comparison between predicted solids temperature under plug flow conditions using the numerical and analytical solutions. (h= Lof & Hawley)
Figure P4. Comparison between predicted solids temperature under plug flow conditions using the numerical and analytical solutions. (h= Lof & Hawley)
Figure P5. Comparison between predicted solids temperature under modified flow conditions using the numerical and analytical solutions. (h = Lof & Hawley)
Figure P6. Comparison between predicted solids temperature under modified flow conditions using the numerical and analytical solutions. (h = Lof & Hawley)
Figure P7. Comparison between predicted solids temperature under modified flow conditions using the numerical and analytical solutions. (h= Lof & Hawley)
Weber's measured values.

Predicted values.

Numerical solution.

Analytical solution.

Figure P8. Comparison between predicted solids temperature under modified flow conditions using the numerical and analytical solutions. (h= Lof & Hawley)