A NEW METHOD OF GENERATING COMPLEX MACHINED SURFACES

- by -

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The manufacture of complex contoured components, such as turbine blades, presents great difficulties for the production engineer, especially when these parts are required as single items or in small quantities.

In this work the requirements for prototype turbine blades are examined and available methods of manufacture discussed. It is shown that most existing manufacturing techniques require a pattern or die that completely defines the shape of the required contour and that this prerequisite is not economically acceptable for small quantity batches.

Two machining systems namely, numerically controlled machines and those using interpolating tracer control, which meet the small quantity specification, are discussed in detail and shown to have disadvantages with regard to cost and flexibility in operation.

This work proposes a new method to provide a means of producing accurately machined parts having complex surfaces, with emphasis on low cost together with simple setting procedures.

The complex contour is considered as the sum of a number of simple contours which may be defined by a single parameter. During the cutting operation these parameter values will either be constant or appear as a rate of change.
Analogue signals describing these values are generated and the work and cutter movements guided by the sum of these signals.

The generation of signals to provide for parameters such as taper or angular motion is relatively simple. However, the derivation of input data for the generation of section form requires far more sophisticated techniques and it is in this section of the work that completely new ideas are developed.

The input methods investigated relate to devices utilising conformal mapping techniques, a powerful mathematical tool when working in two dimensions, which to the author's knowledge has not been considered in the field of machine tool control.

The transformation from one plane represented by a regular figure, the input device, to a second plane which transforms to a predicted complex shape, the work/tool relationship, presents an ideal machine tool control situation.

A particular transformation known as Zhukovsky’s transformation is discussed in detail and used in the design of an input device. This transformation produces a family of aerofoil shaped curves, by applying a certain mapping technique to the flow about a circular cylinder.
The Author wishes to acknowledge the interest shown in this study and the encouragement given by Professor J.M. Zarek of the University of Surrey.

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<table>
<thead>
<tr>
<th>CHAPTER</th>
<th>TITLE</th>
<th>PAGE NO</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>CUTTING TOOL AND WORK TABLE CONTROL</td>
<td>12</td>
</tr>
<tr>
<td>3</td>
<td>PARAMETER INPUT - SECTION FORM</td>
<td>27</td>
</tr>
<tr>
<td>4</td>
<td>PARAMETER INPUT - TAPER</td>
<td>36</td>
</tr>
<tr>
<td>5</td>
<td>PARAMETER INPUT - TWIST</td>
<td>42</td>
</tr>
<tr>
<td>6</td>
<td>SETTING THE MACHINE</td>
<td>44</td>
</tr>
<tr>
<td>7</td>
<td>THE OPERATING SEQUENCE</td>
<td>47</td>
</tr>
<tr>
<td>8</td>
<td>CONCLUSION</td>
<td>50</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>APPENDIX</th>
<th>TITLES</th>
<th>PAGE NO</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>A SURVEY OF TWO DIMENSIONAL FLUID FLOW LEADING TO CONFORMAL MAPPING TECHNIQUES</td>
<td>54</td>
</tr>
<tr>
<td>2</td>
<td>CONFORMAL MAPPING AND POSSIBLE APPLICATIONS FOR MACHINE TOOL CONTROL</td>
<td>58</td>
</tr>
<tr>
<td>3</td>
<td>THE ZHUKOVSKY TRANSFORMATION</td>
<td>62</td>
</tr>
<tr>
<td>4</td>
<td>NUMERICAL CONTROL AND INTERPOLATING TRACER</td>
<td>69</td>
</tr>
<tr>
<td>5</td>
<td>SPECIFICATION OF ORIGINAL GEAR SHAPER</td>
<td>75</td>
</tr>
<tr>
<td>6</td>
<td>SPECIFICATION OF BRISTOL SIDDELEY MODIFICATIONS</td>
<td>77</td>
</tr>
<tr>
<td>7</td>
<td>SPECIFICATION OF MODIFICATIONS FOR THIS PROJECT</td>
<td>79</td>
</tr>
</tbody>
</table>

REFERENCES

FIGURES
Fig 1. DIAGRAM SHOWING METHOD OF DIMENSIONING TURBINE BLADES
Fig 2. BLADE PARAMETERS
Fig 3. BASIC MACHINE TOOL EQUIPPED WITH TEMPLATE COPYING MECHANISM
Fig 4. METHOD OF RELATING MACHINE MOTIONS TO SPECIFIED PARAMETERS
Fig 5. SCHEMATIC OF DUAL CUTTER BOX AND DRIVE
Fig 6. PHOTOGRAPH OF DUAL CUTTER BOX AND DRIVE
Fig 7. SCHEMATIC OF ELECTRO-HYDRAULIC SERVO VALVE
Fig 8. BLOCK DIAGRAMS OF TOOL ACTUATION AND TABLE ROTATION CIRCUITS
Fig 9. PHOTOGRAPH SHOWING TOOL ACTUATION AND TABLE DRIVES
Fig 10. SCHEMATIC OF COPYING AND TAPER INPUT DEVICE
Fig 11. PHOTOGRAPH OF COPYING AND TAPER INPUT DEVICE
Fig 12. SCHEMATIC OF ASSEMBLY BLOCK 'E' OF FIGS 10 AND 11
Fig 13. GEOMETRIC CONSTRUCTION TO OBTAIN TRANSFORMED POINT P (DESIGN STUDY I)
Fig 14. GEOMETRIC CONSTRUCTION DESIGN STUDY I
Fig 15. COMPUTER PLOT OF APPROXIMATE ZHUKOVSKY TRANSFORMATION SOLUTION
Fig 16. COMPUTER PROGRAM TO OBTAIN FIG 15
Fig 17. GEOMETRIC CONSTRUCTION DESIGN STUDY 2
Fig 18. GEOMETRIC CONSTRUCTION FOR BLADE TAPER (1)
Fig 19. GEOMETRIC CONSTRUCTION FOR BLADE TAPER (2)
Fig 20. CIRCUIT FOR TAPER CONTROL
Fig 21. ARRANGEMENT OF WORKPIECE TENSIONING DEVICE
Fig 22. DIAGRAM ILLUSTRATING THEORY OF STREAMLINES AND VELOCITY POTENTIAL
Fig 23. DEFINITION OF STREAM FUNCTIONS AND EQUIPOTENTIALS
Fig 24. BASIC PRINCIPLE OF CONFORMAL TRANSFORMATION
Fig 25. DERIVATION OF ZHUKOVSKY TRANSFORMATION FUNCTION $\omega = z + \frac{b^2}{z}$
Fig 26. TRANSFORMATION OF A CIRCLE TO A STRAIGHT LINE USING FUNCTION $\omega = z + \frac{b^2}{z}$
Fig 27.) TRANSFORMATION OF A CIRCLE TO A CAMBERED AEROFOIL

Fig 28.) GEOMETRIC CONSTRUCTION OF TRANSFORMATION $\omega = z + \frac{b^2}{r}$

Fig 29.) GEOMETRIC CONSTRUCTION OF TRANSFORMATION $\Omega = z + \frac{b^2}{z}$

Fig 30.) GEOMETRIC CONSTRUCTION OF TRANSFORMATION $\Omega = z + \frac{b^2}{z}$

Fig 31.) SIMPLE EXAMPLE OF DATA PREPARATION FOR CONTINUOUS CONTOURING NUMERICAL CONTROLLED MACHINE TOOLS

Fig 32.) DIAGRAM OF INTERPOLATING TRACER CONTROL SHOWING BOTH SHAPE AND TWIST CONTROL UNITS

Fig 33.) PHOTOGRAPH SHOWING THE ARMATAGE PROFILING UNIT

Fig 34.) FRONT VIEW OF THE MAXICUT 3a GEAR SHAPER

Fig 35.) MODIFIED MAXICUT GEAR SHAPER SET UP FOR CUTTING EXTERNAL BLADES

Fig 36.) MODIFIED MAXICUT GEAR SHAPER SET UP FOR CUTTING INTERNAL BLADES
NOTATION

\( A_v \): Area of rotary actuator vane

\( a \): Radius of transformed circle

\( b \): Zhukovsky's transformation factor \( \omega = z + \frac{h^2}{z} \)

\( e \): Eccentricity factor

\( h \): Vertical eccentricity factor

\( I \): Moment of inertia

\( i \): Increment of mechanical feed

\( K \): Stiffness of spring

\( K_t \): Torsional spring constant

\( K_i \): Magnification factor of section

\( L \): Length of blade

\( l \): Width of actuator vane

\( M \): Mass

\( N \): Bulk modulus of fluid

\( O \): Origin of co-ordinates

\( P \): Transformed point

\( \Delta P \): Pressure

\( Q \): Total velocity of a uniform stream

\( Q_r \): Local resultant velocity

\( R \): Fixed radius of tool point about ram centre

\( r \): Radius vector

\( r_e \): Average radius of actuator vane

\( r_{m} \): Maximum radius of vane

\( r_{m} \): Minimum radius of vane

\( T \): Torque

\( t \): Aerofoil section thickness
\( u \) Abscissa in \( \omega \) plane

\( v \) Ordinate in \( \omega \) plane

\( V_R \) Volume per radian of actuator

\( V_S \) Swept volume of actuator

\( \mathcal{W} \) Complex potential function \((\phi + i\psi)\)

\( z \) The complex variable \((z + i\psi)\)

\( \beta \) An angle defining vertical shift in Zhukovsky's Transformation

\( \Theta \) Angle of twist

\( \phi \) Angle of mechanical feed (i.e. multiple of \( i \))

\( \phi \) Velocity potential

\( \psi \) The stream function

\( \omega \) Complex variable in transformed plane \((\omega + i\psi)\)
1. **INTRODUCTION**

With an increasing demand for turbine type machinery there arises a need for extensive research to obtain design data for the optimum configuration of turbine components, working under known environmental conditions.

The research can involve consideration of fatigue phenomena, dynamic effects, temperature, material selection, corrosion and other physical conditions.

Turbine components include such diverse items as a centrifugal compressor rotor for a diesel turbocharger, a radiator fan for a vehicle, a cooling blower for the traction motors of a diesel locomotive, a diffuser for the centrifugal compressor of a gas turbine, blading for a torque convertor transmission and a water pump for a liquid cooled engine.

All these components share one characteristic, namely, a surface having a shape so complex that it cannot be generated by anything as simple as a lathe.

Investigators in this field of research require specimen components in very small quantities, and very often as single items. The prototypes must be manufactured to prescribed dimensions and finished to a high degree of accuracy.
The object of this project, is to determine methods of producing these specimen components, which offer flexibility in operation, together with relatively low manufacturing and equipment costs.

2. PARAMETERS OF PROTOTYPE BLADES

This investigation is concerned with the generation of turbine blades of specified dimensions. There is no reason, however, why items such as rotors with integral blades could not be produced by the process, where design of these items permits.

Due to the complexity of turbine blade profiles used in practice, the methods of dimensioning them vary, depending on the company concerned and the application.

The general method of dimensioning is illustrated in Fig.1. Two centre lines at 90° to each other are arbitrarily chosen about the form at the tip of the blade. Points on the form are defined by co-ordinates measured from the centre lines. The shape of the profile at several sections taken along the length of the blade is also defined by co-ordinates. Thus by considering the loci of points at sections along the blade a progressive thickening or tapering of the section may be apparent. Further, the points at successive sections may impart helix or twist to the form. For this project the configuration of the blade will be defined by parameters rather than the locus of single points. These parameters are as follows:
(a) **Basic Form and section area**
This may be of any section i.e. aerofoil, rectangular, circular, etc. The given form will be specified as applying to a section of the blade, which will normally be the tip section.

A simple blade of this form is shown in Fig. 2 as type 1.

(b) **Non-uniform section or taper**
This is shown as type 2 in Fig. 2, and describes how the tip section progressively changes towards the root of the blade. In the first instant only constant rate of taper change will be considered, although variable taper forms will be included in later stages.

(c) **Lead angle or angle of twist**
This type is illustrated as type 3 in Fig. 2.
Initially only uniform twist will be considered, but the problems associated with non-uniform twist will be investigated.

(d) **Length**
At this stage no consideration is given to blade roots as this will depend on the user's requirements and also methods of supporting the work piece.
3. **SURVEY OF POSSIBLE METHODS OF MANUFACTURE**

In considering the possible methods of manufacture of any item three general lines of approach may be followed, namely, casting, processes of mechanical deformation, and machining.

Casting, whether by sand moulds, shell moulding or investment casting, requires a master or pattern to form a mould cavity. Due to the complexity of the shape of a blade pattern, casting only becomes an economic method where quantities of similar items are required. Further, unless the casting process is carried out under strict control, casting defects can so alter the physical properties of the specimens to make them unsuitable for experimental work.

Of the processes of mechanical deformation, rolling or continuous stamping may be used for simple forms as type 1 in Fig.2. For the more complex forms forging is a possible solution, but specially prepared dies would be required again implying quantity manufacture.

The inherent surface finish produced by the above two methods implies a need for subsequent hand finishing which tends to result in dimensional inaccuracies.

The machining of turbine blades requires the continuous positioning of the workpiece and cutter to a degree of accuracy that makes manual control impossible.
The process of copy milling is used to produce these complex forms. In this method the position of the cutter, with respect to the work, is varied in relationship with a stylus or probe moving over a facsimile of the complex form, at the same feed-rate as the workpiece.

In all the cases so far cited a pattern or die that completely defines the shape of the desired contour is a prerequisite and therefore makes them unsuitable methods for very small quantities of parts.

There are, however, two available machining methods where the input to the machine tool is not derived from a complete complex master, namely,

(a) Numerical control systems
(b) Three-dimensional tracer control

The use of continuous path, three-dimensional, numerically controlled milling machines for the manufacture of turbine blades is well established. (Ref 1).

For programming the machine input data for a complex shape such as turbine blade profile, a digital computer is required due to vast numbers of change points on the form.

Whilst the method is extremely accurate and results in fast machining the first cost and maintenance costs of the control equipment and ancillary units is very high.

Further, a small change in blade dimension requires the reprogramming of part of the machine tape input.
The second method of three-dimensional tracer control has advantages in producing the type of product being considered. In this case a conventional machine tool is equipped with mechanical or hydraulic controls. The basic form of the part is derived from a probe acting on a plate profile of the shape to be produced. Taper and to a limited extent twist, are applied by interpolating secondary probes acting on inclined planes (Ref 2), (Ref 3).

The author considers that in order to cover the complete range of existing processes connected with this work, this particular method of tracer control and a survey of numerical control techniques warrants an appendix in this report. (See Appendix 4, page 69).

4. SPECIFICATION TO BE COVERED BY THIS PROJECT

(a) That the process developed should be capable of producing the complex profiles previously described.

(b) The operation should be automatic and continue to the completion of the part.

(c) Tooling and setting time must be kept to a minimum.

(d) The process must be suitable for small quantity manufacture making it impracticable to derive data from exact complete copies of the profile.

(e) Adjustment of parameter values must be simple and quickly obtained. For instance, it may be required to produce sets of specimens where each item in the set is identical, except for small changes in one parameter.
Accuracy is of prime importance if subsequent research is to be of value.

(g) The overall cost of the development to be low.

(h) The range of parameter values obtainable must be suitable to cover the investigator's requirement.

These values to be as follows:

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<thead>
<tr>
<th>Parameter</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length variable</td>
<td>0 - 9&quot;</td>
</tr>
<tr>
<td>Angle of twist</td>
<td>0 - 100°</td>
</tr>
<tr>
<td>Taper section change</td>
<td>1/5 - 1</td>
</tr>
</tbody>
</table>

5. **OUTLINE OF THE METHOD TO BE DEVELOPED**

A general description of the method to be adopted is as follows:

To consider the complex contour as the sum of a number of simple contours which may be defined by a single parameter. During the operation, or part of the operation, these parameter values will either be constant or appear as a rate of change.

Analogue signals describing these values will be generated and the work and cutter movements guided by the sum of these separately derived signals.

6. **APPLICATION OF PROPOSED METHOD**

From the discussion concerning possible methods of manufacture, it was shown that a machining operation is the only possible solution to produce small quantities economically.
Thus a system must be developed which offers continuous control to the cutter and workpiece, together with a means of providing input signals to the system to obtain the desired cutter workpiece relationship.

At this stage several possibilities must be explored and many decisions made. These can be listed as follows:

(a) Type of cutting tool best suited to the operation.
(b) Type of cutting action to be used.
(c) Methods of controlling the tool-workpiece relationship.
(d) Methods of generating the analogue input signals defining the physical parameters.

Considering the general form of the blades in Fig. 2, it can be seen that progressive changes in twist angles and section occur in the longitudinal direction. Therefore, blades can more easily be generated by tool paths in the longitudinal direction. This directional requirement limits the choice of cutter to two forms; either a single point cutter as used on a lathe or a multi-edge rotary cutter as used on a milling machine. The multi-edge cutter has the advantage of an increased tool life, but the use of such a tool for this work may lead to a situation wherein the tool centre path that is satisfactory when working in one plane may permit removal of stock, in an adjacent plane, which belongs to the finished part. This interference is not evident with a single point tool which can easily be shaped to suit particular requirements.
For these reasons a single-point tool was selected for use in this work. This decision was made with the knowledge that the surface finish obtained would be adequate for the prototype blades required but that some hand finishing could be required on components for practical usage.

Taking into consideration the need to cut along the length of the blade together with the preference for using a single point tool, a machine tool with a reciprocating action was the obvious choice.

7. THE BASIC MACHINE TOOL

The machine tool to be equipped for the manufacture of turbine blades is a modified Drummond Maxicut Gear Shaper which was presented to the author by Bristol Siddeley Engines Ltd. (see appendix 6).

The machine had been used for manufacturing turbine blades by a tool control mechanism utilising a magnified copy of the blade profile (see appendix 6).

Fig. 3 shows an illustration of the machine. The cutter spindle is carried in a ram and is operated by two independent motions. The reciprocating motion is obtained from a rocking shaft (a) carrying a pinion which engages in a rack cut in the spindle ram. The amount of reciprocation (or stroke of the cutter) is varied by adjusting the throw of the operating crank at the side of the machine. The rotary motion is obtained from a feed shaft (b) which passes through the head of the saddle and carries a worm which engages a worm wheel on top of the cutter spindle.
Thus a tool attached to the cutter spindle is given reciprocating motion for cutting and a rotary motion at the end of the stroke to align the tool to the next path.

The connecting rod operating the reciprocating motion can be placed on either side of the centre of the area which enables the machine to be used for up or down cutting.

There are available four cutter speeds and a choice of six rates of feed, together with facility for stroke adjustment.

8. WORKPIECE AND CUTTER MOTION

Fig. 4 shows schematically the proposed method of relating mechanical motions to the required parameters.

From the previous section it was shown that two mechanical motions were available on the machine tool. A reciprocating ram to carry the cutting tool and a rotary motion about the ram centre to move the tool from one cutter path to the next. These rotary increments are made in small angular steps, each step being made after a complete cycle of the ram.

To describe the principle of operation of the process, reference will be made to the types of component referred to in Fig. 2.

The only parameter concerned in type 1 is that of form, the blade having constant section. In this case it is required to position the cutting edges for each cutter-path at the commencement of the stroke, such that the correct form is generated. This requirement implies accurate control of the horizontal position of the cutting tools and also a means of defining the tool position consistent with the profile to be formed.
At the start of the cutting stroke the type 2 component will require the same control as for type 1 but, during the stroke the tools will move horizontally at a velocity proportional to that of the cutter ram, to produce the taper. It will be shown in a later chapter that the rate of change of the tool position during the cutting stroke is the sum of two components of taper, namely front and side taper.

A type 3 component will require the cutting tool control as for types 1 and 2, in order to effect form and taper. To provide for twist the work is mounted on a rotary table which is controlled to rotate a preset angle during the ram stroke. The angular velocity of the table must be proportional to the ram velocity. The control system required for this rotation is discussed in a later chapter.

From the foregoing it can be seen that two major tasks must be undertaken.

1. To equip the machine tool with control systems that will govern the horizontal position of the tools at the commencement and during the stroke and also provide rotary position to the work. The controls used must have high response rates and be stable over the frequency range encountered.

2. To provide a means of deriving inputs to the control systems consistent with the component parameters.
CHAPTER 2

CUTTING TOOL AND WORK TABLE CONTROL

To provide for the generation of the forms, it is necessary to develop a means of supporting and controlling the position of a cutting tool together with a rotating table to support the work.

Actuation and Control of the Cutting Tool

In considering the method of tool actuation the following points must be appreciated:

1. Due to vibration problems input mechanisms must be remote from the machine tool. Also since the system requires addition of signals from more than one source, inputs must be electrical.

2. Changes in tool position may either occur during the idle stroke of the machine ram or during the cutting stroke. The change may either result from a step input or, in the case of taper, as a rate of change.

3. The cutting tool and mounting must be designed to provide sufficient stiffness to withstand the high cutting forces such that the resulting deflection is minimal and accuracy of cutting maintained.

4. The response time for effecting a change of position must be very small.
5. The design must be accommodated in the existing framework of the machine.

Two possible methods of actuation were considered, namely, an electrical solenoid with a mechanical linkage, and an electro-hydraulic system.

After consideration of both methods it was felt that the advantage lay with the hydraulic system. The following points indicate the advantage:

1. Ease of remote control - the input being derived as a voltage, or sum of several voltages, and used to drive the hydraulic power to support the cutting forces.

2. Ease of providing damping to prevent overshoot.

3. Fast response time possible with modern electro-hydraulic valves. (.001 sec.)

4. Standard hydraulic actuators available for both linear and rotary actuation.

The cutting tool mounting had to be carefully designed and the following points considered:

(a) Attachment of the cutting tool mounting.

(b) Location of the cutting tools in the work mounting to ensure cutting takes place radially, with the ram centre.
(c) Weight of the mounting to be as low as possible. The necessary offset position of the tools could lead to high bending moments in the structure and unwanted deflection.

(d) The actuating hydraulic cylinders must be part of the assembly and to reduce the effects of compressibility, the volume of oil between valve and motor to be kept to a minimum.

(e) Provision for feedback information.

The initial outline design proposed direct actuation of the cutting tool by constructing the tool box with the cylinders in the horizontal position. It was subsequently considered that the direct application of the cutting force to the servo-system could lead to tool chatter, Ref 4. Further, the length of the hydraulic ram in this position would lead to obstruction of the machine frame under certain conditions.

A schematic and photograph of the final design is shown in Figs. 5 - 6. Two hydraulic cylinders are mounted vertically and form structural members of the tool box. This arrangement reduces the hydraulic force required and with direct actuation the mechanical diminution of input movement provides a greater control of movement and hence improved accuracy.
The tool box is designed to give simultaneous cutting on both sides of the blade so that the workpiece will be subjected to opposite and fairly equal tangential forces during the cutting stroke, with the attendant advantage of balancing bending loads.

The cylinder rod ends carry wedge shape pieces. One side of the wedge is tapered to provide for a 1/4" forward motion, of the tool, for the full cylinder stroke of 1.1/2". The wedge face butts against a roller which is an integral part of the tool holder. The tool holder is mounted as a slider on two widely spaced rods and contact is maintained between roller and wedge by suitable springs.

The reverse side of the wedge is tapered such that a full cylinder stroke will drive an ac pick off, used for feedback position measurement, within its limited linear range of ± 0.030".

The dimensions of the tool box were selected to allow for the maximum stroke of the machine ram and the cutting path of the tool edges to be at a 4" radius about the ram centre. This value of radius 'R' was chosen to suit the existing machine design and the size range of the required blades. Since the height of the skeleton of the aerofoil at mid chord position is 2b it is shown in appendix 3 that the approximate radius of the skeleton is b. Thus to ensure minimum horizontal movement of the tool between adjacent cuts R should be approximately equal to b.
The development of the equations for vector lengths required that $\beta$ was small (i.e. $\beta \approx \cos \beta$) and in this respect $\beta$ should not be greater than .12 radians.

Hence $R = 4 = \frac{b}{\beta} = \frac{b}{.12}$

$\therefore b = .48''$ say $\frac{1}{2}''$ max.

Maximum chord $= 4b = 2''$ which is satisfactory for the blades in question.

$\beta = .12$ then percentage camber $= \frac{.12}{2} \times 100\%$

$= 6\%$

Thus the maximum width of blade cut at this $4''$ radius is $2''$ with a camber of $6\%$. Obviously blades of less width and reduced camber can be produced and further it should be emphasised that matching the skeleton to a close radius is convenient but not absolutely essential. However, the radius could easily be made adjustable to suit all ranges of blade sizes.

**COMPONENT SELECTION**

Factors affecting the selection of the hydraulic equipment are the forces acting during cutting and the maximum response time allowed for the tool to take up a position corresponding to an input signal.

At this stage only estimates could be made of frictional and inertia values to provide an equation of the motion.
Complete analysis of the cutting forces on a single point cutting tool is extremely complex, even when considering the simple case of orthogonal cutting. In this project forces related to the relative work-tool motion must be considered together with the actual cutting motion of the machine ram.

In view of this complexity it is very difficult to predict the forces to which the work and tool will be subjected, and assumptions have to be made in the original design of the tool box structure and the work holding methods.

Due to the small width to length ratio of the blades to be produced, it is a fair assumption that the limiting horizontal force that can be tolerated will be that which causes buckling of the work piece.

As the workpiece can be firmly clamped to the table, it was decided to use up stroke cutting to provide extra tension to the workpiece. Further fixed tension was provided by the top attachment of the workpiece which allows rotation through a thrust bearing.

Initial tests showed that with small tool rake angles, the depth of cut was the critical factor in selecting the allowable horizontal movement of the tool for any particular cutting stroke.

This fact was as would be expected in such a cutting situation and has a bearing on the setting for taper and helical work.
From the foregoing it can be assumed that the load inertia is negligible and a value of about 300 lbf, due to the cutting action constitutes the main load on the cutting tool. The value of 300 lbf is comparable to that obtained on a conventional shaping machine.

There are two possible cases to consider with regard to the response time for the horizontal cutting tool motion. Firstly when cutting parallel sections the variation in position from cut to cut will be a maximum of .006". This change can take place during the idle return stroke of the machine which taken at the maximum cutting speed of 47 cycles per minute allows 0.65 seconds. Secondly in the case of taper cutting the maximum rate of displacement of the tool during the cutting stroke will be 0.25" in 0.65 seconds (i.e. .4" per second).

A survey was made of all the available standard electro-hydraulic servo-valves and from the information obtained the Elliott type 610 valve was chosen. As stated earlier it was important to fit the valve close to the actuating cylinder to reduce compressibility effects and therefore the valve needed to be attached to the tool frame with the attendant need to keep weight and size to a minimum. The 610 valve at 1.6 lb proved to be the lightest of the valves surveyed and also the simplest to fit, which had characteristics suitable for this application.
control of hydraulic power is a single stage type, electrically actuated, proportional flow, three way valve. A feature of the valve which is important for this project is that it has facility for zero lap adjustment. (Ref. 5)

An electro-hydraulic servo valve shown schematically in Fig. 7 provides an oil flow proportional to an applied voltage.

The voltage signal is applied to a torque motor which actuates a spool valve thus allowing oil to flow through either port A or B. The applied voltage is proportional to the rotation of the torque motor which is in turn proportional to the movement of the spool valve and hence the opening of the port. In a closed loop system the torque motor is fed with an error signal and the spool will be driven in the direction to reduce the error.

Due to manufacturing problems it is difficult to arrange for the annulus of the spool valve to exactly coincide with the ports. Three conditions can be obtained.

1. Underlap - In this case the annulus is smaller than the port such that there is always a leakage through the valve. This has the effect of power wastage. Some authorities consider that underlapping has the advantage of providing for a "state of readiness".
The Elliott valve has the facility for adjusting the lap position and is usually set for zero-lap conditions.

The control unit provides a dither output such that the spool is continually vibrating at 50 c.p.s. at a very small amplitude. This motion prevents a slip-stick situation which can lead to non-linear operation.

As the valve was of the three way type with only one control port, a compounded cylinder was used, the annulus area being half the full bore area. In this way system pressure acting on the annulus side effectively biased the controlled pressure on the full area; the raising or lowering of the control pressure adjusting the piston position either side of its mid-position.

Dimensionally, a 1.1/2" cylinder stroke was considered suitable and the drive wedge angle arranged to give 1/4" forward tool movement for this stroke.

Taking into consideration the mechanical advantage of the tool drive and estimating viscous friction resistance, a maximum working load of 500lbf. was assumed.

Using a supply pressure of 1000 lbsf/in², the effective control pressure is 500 lbf/in² and as the maximum efficiency of the valve controlled system is 66% a pressure of 333 lbf/in² is used in calculations.
A cylinder of 1 1/2" dia. was chosen giving an effective area of 1.77 inches$^2$. With a piston rod diameter of 1 1/16" the annulus area was .88 inches$^2$.

At the maximum working stroke of the machine the cutting stroke period is .65 seconds; thus the maximum velocity of the ram for a full actuation of 1 1/2" = \frac{1.5}{.65} = 2.3$ inches per sec.

The required flow = 2.3 x 1.77 = 4.06 inches$^3$ per sec. = .87 gallons per minute

The 610 valve is designed for a 2 g.p.m. flow at 3000 p.s.i., or 1 g.p.m. at 1000 p.s.i.

Analysis of the electro-hydraulic system was attempted but lack of information relating to frictional and inertial values indicated that results obtained would be of little use in design. However, the natural frequency of the actuator was estimated at 300 c.p.s., which represents a much higher loop gain, compatible with adequate stability than is required for operation.

It was considered that the main problem to be encountered would concern overdamping due to friction in the tool holder guides. Great attention was given to the design of the phosphor-bronze guide bearings and the lubrication of these.

Position feedback is obtained by Sperry ac Pick-offs fitted in the tool holder assembly such that the probe is actuated by the slope of the tool drive wedge. The body of the pick-off is adjustable in its housing to enable it to operate within its linear range of $^\pm .030"$. 

transisterised control unit manufactured by Elliott. The unit contains an ac error amplifier, a modulator/demodulator, a dc torque motor amplifier, a dither oscillator and stabilized power supplies. The unit has provision for 3 ac input signals and one dc signal. A separate 400 c.p.s. generator is used as a reference supply.

ACTUATION AND CONTROL OF THE WORK TABLE

To provide for helix it is necessary to develop a drive to a rotary work table such that the table rotates through a predetermined angle during the cutting stroke of the machine tool. The proposed setting angles were between 0° - 90° in steps of 5°.

The original machine had a fixed work station but was still equipped with a bearing tube used to provide indexing of gear wheels.

This tube, which closely fitted in the machine bed, was an ideal housing for the work table bearings.

Again an electro-hydraulic drive was selected to power the table drive. Besides providing a uniform system, the advantages are the ease of obtaining the variable velocity and the high torque features of the drive.

As with the linear actuators, it was difficult to estimate the power output required for the rotary actuator. Taking into consideration the forces acting at the tool point, combined with the variable depth of cut in helical cutting, a torque of about 1500 lbf-in was considered adequate.
TORQUE OF ROTARY ACTUATORS

\[ \Delta P = \text{pressure drop across the actuator} \]

\[ r_D = \text{outside vane radius} \]

\[ r_S = \text{inside vane radius} \]

\[ \ell = \text{vane width} \]

\[ \theta = \text{vane angle of rotation} \]

The pressure is applied on the area of the vane at the average radius.

\[ T = \Delta P \left[ (r_D - r_S) \ell \right] \frac{r_D + r_S}{2} \]

\[ = \frac{\Delta P \ell}{2} \left( r_D^2 - r_S^2 \right) \]

Volume of Unit = \[ \frac{r_D^2 - r_S^2 \ell}{2} \theta \] \[ \theta \text{ in radians} \]

Then Volume per radian = \[ V_R = \left( \frac{r_D^2 - r_S^2}{2} \right) \ell \]

\[ \therefore T = \left[ \left( \frac{r_D^2 - r_S^2}{2} \right) \ell \right] \Delta P = V_R \Delta P \]

Torque is equivalent to volume per radian x pressure drop.
The natural frequency of a spring mass system is
\[ f_m = \frac{1}{2\pi} \sqrt{\frac{2K}{M}} \]

- \( K \) = stiffness of spring
- \( M \) = mass
- \( I \) = moment of inertia

For a vane type rotary actuator, the hydraulic natural frequency, when an inertia load is driven, will be found by substituting a torsional spring constant for linear \( K \) and substituting \( I \) for \( M \). Where \( I = \)

Swept Volume = \( V_s = \frac{(r_D^2 - r_S^2)l\theta}{2} \)

Area of vane = \( A_v = (r_D - r_S)l \)

Average radius of vane = \( \frac{r_D + r_S}{2} = r_c \)

Torque = \( N = \frac{\ell}{2} (r_D^2 - r_S^2) \)

- \( N \) = bulk modulus

Angular Deflection = \( \frac{V_s}{A_v r_c} = \frac{\theta l}{4 (r_D - r_S)(r_D + r_S)} = \theta \)

Then \( K_T = \frac{\text{torque}}{\text{angular deflection}} \)

\[ K_T = \frac{N\ell}{6} (r_D^2 - r_S^2) \]

substituting in \( f_m \) equation

\[ f = \frac{1}{2\pi} \sqrt{\frac{2N\ell}{\theta I} (r_D^2 - r_S^2)} \]

\[ V_R = \left( \frac{r_D^2 - r_S^2}{2} \right) \ell \]

\[ f = \frac{1}{2\pi} \sqrt{\frac{4NV_R}{\theta I}} = \frac{1}{2\pi} \sqrt{\frac{NV_R}{\theta I}} \]
Of the various vane type rotary actuators, a Keelavite Rotax HN 32 double vane actuator was chosen which had suitable shaft and body dimensions.

The Rotac has a displacement of 6.6 cub. ins. for a maximum sweep angle of 100°.

\[
\text{Torque} = Vr \Delta P = \frac{6.6 \times \pi}{1.8} \times \frac{2}{3} \times 1000 = 3500 \text{ lb-in.}
\]

Natural frequency = \( \frac{1}{\pi} \sqrt{\frac{NVR}{\theta I}} \)

If \( N = 2 \times 10^5 \text{ lb/inch}^2 \)

and \( I = 40 \text{ lb-inch}^2 \) then

\[
= \frac{1}{\pi} \sqrt{\frac{2 \times 10^5 \times 6 \times 3 \pi}{1.8 \times 1.8 \times 40}}
\]

= 58 \text{ cps}

The estimated moment of inertia I of the rotary table and shafting is very small and again, as with the linear actuators, the natural frequency of the rotary actuator is high, indicating stability would not be a problem.

A Sperry 4 way electro-hydraulic valve was chosen to drive the rotary hydraulic actuator.

Feedback signals are obtained by an ac rotary pick-off driven by the rotary table drive. A reduction gearbox between the drive and the pick-off ensures the unit is working in its linear range.
To obtain signals corresponding to velocity the output of the ac pick-off is passed through a demodulator and subsequently through an R.C. circuit to differentiate the position output to obtain a voltage proportional to velocity. This dc signal is passed directly to the error amplifier of the valve control unit.

A similar circuit is used as a controlled input to the system. The input ac pick-off being driven by the cutter ram shaft via a reduction gearbox.

Hydraulic supplies are obtained from a Keelavite hydraulic pump equipped with a 5 micron filter.

A block diagram of the electrical and hydraulic circuits are shown in Fig.8.

Photograph Fig. 9 shows both the tool box and table actuation drive. The Electro-hydraulic drive units and hydraulic power supply unit can be seen at the right of the machine.
PARAMETER INPUTS – SECTION FORM

In the first chapter it was shown that the shape of a turbine blade could be described by three parameters—form, taper and twist. Each of these parameters will be discussed in depth in this and the next two chapters.

It was stated earlier that turbine blade sections are most often developed by experimentation and that these cannot usually be defined mathematically. In these cases a copying technique or numerically controlled machine are invariably used.

The first attempts to provide input information for this machine system utilised a copying mechanism which is illustrated in Figs. 10 and 11.

In describing the machine tool it was shown that the nominal path of the cutting tool was at a fixed radius about the centre of the cutting ram and that increments of position were obtained by small angular increments on this radius.

Therefore, in providing a copying system it is necessary to move a probe along a radius proportional to the cutter path radius.

The block A, manufactured in aluminium, has two milled slots into which twice full size facsimiles of the required form of either side of the blade are fitted.
The block slides freely on linear hearings across the two slide bars.

The stationary assembly E (Fig.12) is designed to provide a voltage proportional to the local mechanical position on the template.

The main block carries two spring-loaded pointed spindles on which are fitted ground conical pieces. The spindles protrude from the block and touch the template face. Two ac pick-offs are fitted in the block at right angles to the spindles, their probes touching the cones. Facility is provided to adjust the positions of the pick-offs to ensure probe movement is in the linear range of the device.

Thus a movement of the spindle resulting from a change of position of the template will effect a proportional change in voltage from the pick-offs.

A second function of assembly E is to provide a signal at the end of the cutting stroke to relieve the tools away from the work during the return motion of the cutting ram.

The solenoid M drives a wedge between two extensions fitted to the cones which rotates them to a position on their surface ground with a .010" flat. The .010" movement of the pick-off probes produces a voltage which applied to the servo system moves the cutters away from the work.
The indexing of block A, to match the machine tool feed, is produced by rod B which is in turn driven by lead screw D. The lead screw is actuated by a second solenoid operating a pawl on a free wheel mechanism.

Fig. 10 also illustrates the method of providing side taper. This will be described in the next chapter.

The amount of angular motion for each index position is adjusted by moving the lead screw assembly block along longitudinal slide bars.

The essential aim of the work is to develop a method of providing a means of generating the section form without the need of manufacturing a template to define the form.

The Zhukousky transformation (see appendix 3) is used in investigating the study of the flow of air around an aeroplane wing. When one considers the use of the transformation for turbine blade sections, the size difference is such that it becomes easier to fit the curves obtained from the transformation to the specified section of a turbine blade. Thus a method of generating specified sections becomes available. This factor is important when single or very small quantities of components are required.

Fig. 28 and the associated equations, of appendix 3, show that by neglecting small quantities the vectors \( r \) and \( b^2 \) can be given by
\[ r = b + be + be \cos \theta + b \beta \sin \theta \]
\[ b^2 = b - be - be \cos \theta - b \beta \sin \theta \]

where \( b + be = a \) the radius of the circle in the z plane

\( be \) = horizontal shift from origin

\( \theta \) = angular position of vectors

\( \beta \) = angle of vertical shift from origin

\( be \) and \( \beta \) are constants for a particular thickness ratio and 'a' relates to length of the section required.

A computer program was written for the vector summation of \( r \) and \( \frac{b^2}{r} \) assuming values for \( a, be \) and \( \beta \). The programme and the plot read out is shown in Figs 15 and 16.

The computer, a Honeywell H120 machine, was programmed using Fortran IV language by card input. The output was produced by Calcomp plotter.

Design Study I

The original intention was to develop a mechanism which directly provided the summation of the two vectors

\[ r \quad \text{and} \quad \frac{b^2}{r} \]

Consider a disc A of radius \( a = b + be \) as one circle of Fig (13). The centre of the disc is fixed with respect to a datum such that 'be' is the horizontal offset and \( \beta \) is the angle subtended by the vertical offset. The length of vector \( r \) for all angles of \( \theta \) can be provided by a link, equipped with a slider, joining the origin to the circle circumference such that the length \( Ou = r \) for all values of \( \theta \). A second disc C of radius \( b - be \) with centre offset from \( O \) may provide in a similar manner the vector length \( ov = \frac{b^2}{r} \) for all values of \( - \theta \). Thus by controlling the disposition of the links such that \( \theta \) and \( - \theta \) are numerically equal the vectors \( r \) and \( \frac{b^2}{r} \) are obtained directly.
It is now required to summate these two vectors by completing the parallelogram $OuPv$. Mechanical summation involves ensuring that for all angles $\theta$, $Ov$ is parallel to $uP$ and $Ou$ to $vP$ and at the same instant $Ov = uP$ and $Ou = vP$. Various designs were considered to achieve this objective based on pantograph type mechanisms.

The method of vector summation was dealt with by firstly considering the direction of the line on which the point is located and secondly the position of the point along that line. Referring again to Fig (13) by producing the line $uP$ to meet the Ox line in $w$, an isosceles triangle $Ouw$ is formed. The vector summation point is located on the side $uw$ at a distance $uP$ from the apex.

The length $Ou$ can be obtained for any angle $\theta$ by the method previously detailed and the length $uw$ obtained by a second disc $B$, a mirror image of disc $A$ with its origin at $w$. The second problem of locating the point $P$ along $uw$ can be solved by displacing the origin of the smaller disc $C$ to the point $u$, whilst maintaining the attitude relationship to its centre.

Fig. (14) illustrates these two operations. Disc $A$ is considered a datum and the centre of disc $B$ moves along Ox with respect to disc $A$. For each position of disc $B$ there is an associated isosceles triangle given by each discrete value of $\theta$. Two such triangles are illustrated.
The apex of the isosceles triangle, the intersection of the two vector links, is the origin of the disc C. Thus the vector summation point P is given by the point of intersection of the vector link r of disc B and the circumference of disc C, shown at P, P', P'' and P'''.

The problem of maintaining the space relationship of disc C relative to discs A and B was solved by joining the centre of disc C to that of another disc, C', positioned at the opposite apex point formed by discs A and B. The centres of the two 'C' discs were connected by a vertical slotted link with a tension spring to draw the disc centres to touch the circumferences of discs A and B. The locus of points at the intersection of the circumference of C, and the link joining disc origin B generated the lower profile of the aerofoil section.

In a practical design of this device it was envisaged that the pin touching the circumference of disc C (i.e. a locus point on the aerofoil) would contact the probe of an ac pick-off carried on an arm about the cutter radius centre. Thus for each feed increment an analogue voltage would be obtained proportional to the radial distance from the cutter path to the generated point. This voltage could then be used so the input signal to the cutting tool position servomechanism.
The constructed model produced a satisfactory pencil outline of a section which when compared with the computer printout previously described, showed a consistancy within a tolerance zone of ± .01 "

A recent publication (Ref 13) states "The limitation on aerofoil shape must be maintained within ± .003 in on each surface", a tolerance which could be easily satisfied by this technique.

**Design Study 2**

Consider Fig. 17, the two circles for rotating vectors \( r \) and \( b^2 \) are drawn twice full size and the points at the circumference of vector \( 2r \) at \( \theta \) shown as A and vector \( 2b^2 \) at \( -\theta \) shown as B. The geometry of the figure shows that the centre point of the line AB will be the summation point of vector \( r \) and \( b^2 \) (see appendix 3). Two constructed summation points are shown in the diagram.

This construction offers a simple method of obtaining the transformed shape. Again a model was constructed which gave a satisfactory outline of the transformed shape.

It is necessary to control two variables in the mechanisation of this method. Firstly it is required to maintain the vector links at equal and opposite angles and secondly to effect a bisection of the link joining the ends of the vector links. The maintenance of vector link position is obtained by moving a rod along the axis, thus moving the lightly loaded links around the circumference of the disc at equal angles to the origin.
At the top dead centre position a second rod is required to maintain angular disposition in the second and third quadrants. Bisection of the joining link is obtained by two further equal links pivoted at the vector link ends and joined by a further pivot at the other end. The two equal links thus form an isosceles triangle with the base formed by the vector joining link, which is varying in length with the instantaneous vector position. Bisection of the joining link can be achieved by bisection of the apex angle of the isosceles triangle. Two spring loaded rollers hold a third pivoted link such that it bisects the angle and hence the vector joining link. It is envisaged that every point position relative to radial centre line would be obtained from an electrical pick-off in a similar manner to that described in the previous design study.

As stated previously models of both these devices described as Design Studies I and 2 were constructed to give pencil outlines of the transformed shapes.

The first design is obviously the most complicated since it requires three discs two of which are moving along constrained paths. The greatest difficulty is to ensure disc C maintains its correct position during the operation since it has components of motion parallel to both the $\phi$ and $\gamma$ axis.

The second design is simpler and could be adapted to cater for a range of values of $a$, $b$, $be$ and $\beta$. 
This variability could be achieved using different sized rings, instead of discs, which could be positioned relative to an origin on a base plate, the bisecting linkages could be standard for a wide range of values. The greatest problem with this design is that although the rollers controlling vector link positions can be of small diameter the in-line position at $\theta = 0$ and $\theta = \pi$ cannot be obtained and further consideration needs to be given to this point.
PARAMETER INPUTS - TAPER

It is important to define the geometry of magnification of complex shaped solids before proceeding to discuss methods of obtaining taper.

Fig 18 shows a typical aerofoil shape to be machined. The intersection of the diagonals of the circumscribed rectangle may be considered the apex point of the tapered solid. Radial lines, extending beyond the form, are drawn through the apex point 0.

Lengths are measured along the lines such that in each position the measured length from 0 is a multiple of the distance from 0 to the intersection of the radial line with the form i.e. \( \frac{ob}{oa} = \frac{od}{oc} = K_i \) where \( K_i \) is a constant specifying the magnification of the shape.

The inputs for form will normally be related to the top section of the blade and the shape will progressively increase in area to the lower end of the blade to provide for taper.

Figs. 18 and 19 illustrate the relationship between the defined taper of the blade and the variables of the process. The cutters move about a radius \( R \) from the cutter ram centre and can be adjusted horizontally in proportion to the signal applied to the servo controller. Discrete cutter paths are obtained by radial increments of the cutter about the ram centre.
Considering one side of the form, the cutter enters
the work at point a and moves progressively during
to the stroke to point b at the larger section. The
rate of change of position of the tool is proportional
to the velocity of the cutter ram.

Consider the cutter at point c on the form at some
angle $\phi$ to the centre line of the form. The true
form magnification at point c is given by $d$ on the
radial line ocd such that $\frac{od}{oc} = K_r$. However due to the
straight line motion of the cutting tool the cutter
will move to point e on the larger form. The problem
is to generate the length ce to produce the correct
form. If the smaller form is of the same dimensions
as the copying template or generated form of the last
chapter, then f is the point on the smaller form
corresponding to e on the larger form. This implies
that as the cutter moves from point c to point e the
probe on the master form must move from c to f to
maintain section form. Thus the total horizontal
movement of the tool during a cutting stroke, at some
angle $\phi$ from the centre line, consists of a fixed front
taper movements suitably modified by a form correction
movement defined here as side taper.

Consider Fig. 18 it is assumed that local sections of the
curves are straight lined and that section thickness
is small compared with fixed radius $R$ then
\[ \frac{ob}{oa} = \frac{oc}{of} = \frac{od}{oc} = K_1 \text{ and from similar triangles} \]

\[ \frac{d'}{d} = K_1 \text{ where } d \text{ is the angle the probe must move across the master form to maintain section form.} \]

The angles \( \phi \) & \( \phi_1 \) are both specified in terms of \( \mathbf{N} \) where \( \mathbf{i} = \) the fixed incremental angular feed per stroke and \( \mathbf{N} \) the number of such increments from the form centre line.

The amount of change of position to correct for side taper will vary as \( \phi \) changes from the central position in either direction.

The problems which had to be resolved to generate signals to provide for taper were firstly to vary the initial form input signal to accommodate side taper and secondly the addition of a second signal to provide for front taper.

As front taper represented the greater rate of change, attempts were made to obtain a velocity input representing this component.

A long stroke rectilinear potentiometer was attached to the machine such that it was actuated by the ram. A d.c. voltage was applied to the outer terminals and output obtained from the slider terminal and the lower end terminal. A variable resistor was placed in series with this output to adjust for different values of taper. The output from the variable resistor was differentiated through R.C. network and then connected to the valve drive amplifier. (Fig.20).
During experiments it was found that, as the rate of taper was small and operation was in a relatively low frequency band, the changing ram position signal gave a reasonably smooth motion to the tool. The tool motion was further improved by providing a secondary feedback loop around the valve by connecting, in a negative sense, the a.c. pick-off directly controlled by the valve torque motor to the amplifier. This secondary loop is a velocity feedback direct from the torque motor driving the spool valve and hence gave the system anticipation of the positional change.

Workpieces cut with front taper using this arrangement showed no sign of the theoretical "staircasing" effect one would expect at high frequency operation.

The inputs derived for side taper must be provided from the form or profile since this component is required to maintain section homogeneity.

Consider Figs. 10 and 11, the block A carries the templates of each side of the blade to be produced, and moves along a pair of rods on linear bearings. The block A is actuated along its guide bars by arm B pivoted at point F. The cross plate carrying the fulcrum point F can be moved along the side bars in order to adjust the effective swing of arm B. Arm B is actuated by the lead screw assembly D, the initial position of which is adjusted such that the linear solenoid operating the ratchet wheel drive of the lead screw moves block A a proportional distance to the tool feed on the machine.
As stated earlier to provide for side taper it is necessary to effect a position change during the stroke of $\frac{\phi}{K_f}$. Thus as the tool moves further either side of the centre position the amount of section change increases.

To obtain this change mechanically the lead screw assembly D can be moved from its initial position along its guide bars by hydraulic cylinder G. The velocity of the actuator can be adjusted by a throttle valve H and is driven through a solenoid reversing valve J which effectively controls the stroke of the cylinder.

The operation of this unit can be described as follows:

1. Templates of both sides of the required form are fitted into the two slots in block A.

2. The pick-off block E is adjusted such that the probes which drive the taper cylinders touch the profile.

3. The angle subtended by arm B to the centre line $\phi$ is the same as the angle subtended to the tool to the centre line of the work.

4. During the cutting stroke of the machine, hydraulic cylinder G moves at the same rate as the cutting ram and actuates arm B and hence block A back towards the centre line on angle ($\phi - K_f\phi$)
The probes of assembly E and hence the a.c. pick-offs monitor this change and through the control system progressively adjust the tool position.

5. At the end of the stroke solenoid M is actuated by a micro switch, on the cutting ram motion, which rotates the tapered pieces to a .010 flat. The .010 movement of the pick-off probes changes the voltage input to the servo and hence relieves the tools away from the work.

6. The angle $\phi (I - K)$ is obtained by suitable adjustment of fulcrum point F, the initial position of lead screw assembly D and throttle valve H.

7. During the idle stroke of the machine the ratchet wheel drive moves block A to the next cutting position on the form.

The case cited implies a form copying situation. When the conformal mapping device is used to generate form positions the arrangement must be incorporated in the side taper unit described.
PARAMETER INPUTS - TWIST

For the experimental blades described in Chapter I, angles of twist from 0° - 90° are required.

The rotary table mechanism and drive methods are discussed in Chapter 2.

It is therefore necessary to provide inputs to the torque motor of the Sperry Electro Hydraulic valve such that the table rotation will have a velocity proportional to that of the cutting ram. Further it is required to provide a means of adjusting the voltages to give a discrete angular rotation for a particular setting.

The input from the cutting ram is obtained from the ram actuating shaft which turns through an angle of 180° during each stroke. An Elliot rotary a.c. pick-off is coupled to the shaft via a 1 : 6 reduction precision gearbox. The gearbox limits the rotation of the a.c. pick-off to within its linear range of ± 30°.

The output voltage from the pick-off is passed through a resistance capacitance differentiating network which produces a signal proportional to the velocity of the ram. This signal, applied to the valve amplifier unit, directly controls the spool valve via the valve torque motor. A variable resistance in the circuit is preset for a specified angular displacement per stroke.
A closed loop system is completed by connecting another a.c. pick-off via a 1:3 reduction gearbox to the rotary actuator shaft. The voltage generated is passed through an R.C. network to provide a velocity feedback to the power amplifier.

A block diagram of the hydraulic and electrical circuit is shown in Fig. 8.

Blades were cut with twist angles but it was appreciated that the calibrated input for a required angle demanded a more precise device. It would have been preferable to use a d.c. tachogenerator to provide signals for twist since the velocity component would be directly generated.
SETTING THE MACHINE

The setting procedure involves:

1. Supporting the work-piece on the rotary table
2. Setting of the ram stroke to suit the length of blade to be cut.
3. Setting of ram cutting speed to suit the material being cut.
4. Adjustment of the feed per cut (rotary motion per stroke about cutting ram)
5. Initial adjustment of tool position to coincide with probe position on input device.
6. Setting input device for taper.
7. Setting input device for twist.

The length to thickness ratio of specimens to be cut is large and therefore practical consideration has to be given to deflection of the workpiece due to the horizontal component of the cutting forces.

Fig. 21 shows the manner by which the workpiece is supported. The bar to be cut was prepared by turning a half inch diameter spigot on either end. One spigot is located in a plate clamped to the work table by means of a socket head screw. The top spigot is secured in a holder which is supported by a bracket on the machine frame.
The holder is of sufficient length to ensure clearance of the tool box when in the top of stroke position and made to be a running fit in the bracket. Workpiece tension adjustment is obtained by a nut on the threaded end of the holder. To ensure rotation a thrust bearing was assembled between the nut and the bracket.

Obviously turbine blades used in service require a variety of root forms and the spigot attachment would not be suitable for these. However, the main objective of this project is to provide sections for experimental purposes and spigot ends would satisfy this requirement. It would be a simple matter to manufacture the lower attachment to suit a particular root form and the top spigot could be removed on a second operation. The lower attachment at the table gives firm central support whereas the top bracket is subjected to bending forces and for this reason up cutting was used in this case.

Cutting in an upward direction requires that the smaller section of the taper is at the table end and the tools move progressively outwards as the ram rises.

The ram stroke was adjusted by turning the lead screw which set the offset position of the drive link to the fly wheel centre. The stroke length was set one inch greater than the blade length to allow for tool relief and reset motions.
The machine had been used by Bristol Siddeley Engines for cutting nimonic steels and therefore the range of speeds on this particular machine was low for cutting mild steel experimental blades. The top speed of 47 strokes per minute was invariably used in this work.

In all cases the tools were set initially .015" from the centre line of the workpiece with the tool drive hydraulic cylinders fully advanced. Thus the two tools were .030 apart at the initial setting with the tools in the forward position.

The tool head was rotated to a lead in position with respect to the work-piece and the probe on the form input device adjusted to a corresponding position.

The settings for taper and twist were made as described in Chapters 4 and 5 respectively.

High Speed Steel single point cutting tools were used with 10° rake angles. In most cases the depth of cut required to complete a blade was excessive for a single operation. For the first operation the tools were adjusted in the tool holders such that the maximum depth of cut was .040". The tools were then set to the original position for the final cut.
THE OPERATING SEQUENCE

A graphic description of the operating sequence of
the system requires consideration of both the machine
tool control system and the methods of deriving input data.

The machine tool control system is dependant on:

1. The velocity, initial position and displacement of the cutting ram - this information is obtained from:
   a) Rectilinear potentiometer
   b) ac pick-off on rocker arm
   c) Micro-switch indication of ram extreme position for relieving and repositioning cutting tools.

2. The position and, where necessary, the velocity of the cutting tools with respect to the fixed radial centre line obtained from:
   a) ac pick-off feedback elements actuated by tool drive wedges.
   b) ac pick-off velocity feedback elements actuated by servo valve torque motors.

3. The initial position and velocity of the work table obtained from:
   a) ac rotary pick-off actuated by rotary hydraulic actuator.
The inputs for position and rate of change of position for the tools are determined from -

a) The form input device either giving a static tool position for each feed increment or a rate of change modification for side taper.

b) A separate voltage signal to add to (a) above to provide for front taper.

Consider the tools set for up cutting with the ram at the lowest position, and the position input probe synchronised with the cutter box position. Any voltage error between the desired position given by the input device, for that line of cut, and the feedback pick-off on the tool box, drives the hydraulic cylinder through the electro-hydraulic valve to negate the error. If a tapered blade is required signals from the front and side taper input devices are combined and compared with the feedback voltage. The negation of this progressive error moves the tools outwards which thus cut the desired tapers. As the ram reaches the top position the relieving solenoid is switched to move the tools away from the work during the idle downward stroke. This sequence is repeated for each discrete increment position.
It was considered that the distance between the two tool points should not be less than .030" since a thinner section would tend to tear and cause tool damage. This implies that a .030" thickness of material will be left on the stock and that for taper work a thin web of material will require to be removed after machining.
CONCLUSION

It is considered that the most significant contribution this work has to offer is in the proposal to use conformal mapping techniques as a basic for a machine tool system.

This analogue presentation of data offers a truly continuous path system as compared with the digital systems, used in present day numerically controlled machine tools, where curvature is represented by co-ordinate points and the interval between them interpolated either by computer or machine tool controller action.

Taking the case of the Zhukovski transformation, were the equations of the resulting aerofoil expressed in cartesian or polar co-ordinates they would be very complicated. The Zhukovski transformation enables these unwieldy equations to be replaced, by the use of a very simple function $z + \frac{b^2}{z}$ and this is indeed a very elegant technique.

The two design studies considered indicated a method of obtaining output signals from a geometric construction, by Fig. 16 it can be seen that the equations are suitable for computer application and hence could be a quick means for providing input data.
for continuous path numerically controlled machine tools.

Although only one transformation formula has been considered in this work, there are in fact many others (Ref 9 section 8) some of which could be utilised in a similar way and provide a number of predicted curves to fit a designer's requirements.

In the particular case of aerofoil sections a designer has only to specify the width of section and chord/thickness ratio, from which the values b, e and $\beta$ could be established and the section machined.

Comparing this with the need to provide co-ordinate change points for each section the savings in design and drawing time are considerable.

Numerical control is well established in the manufacture of complex shaped parts but may be considered too expensive to supply the range of products this work aims to meet.

The tracer methods described suffer from the need to provide templates of the sections and the interpolating systems are suspect unless numerous templates are developed to define the total solid.

The proposed method of this work compares favourably with these two methods in that:

1. No templates are required when the form is generated and can be obtained by simple setting of arc radius and link lengths.
2. Taper and rotary functions are provided from the cutter motion.

3. The overall cost is lower.

This project grew from a requirement for experimental turbine blades for the dynamics section of the Department of Mechanical Engineering. In the long and sometimes difficult journey from those beginnings to this conclusion many avenues of approach have been explored and new ideas developed. Although the project itself is in a narrow field of work it has required the study of many facets of engineering and applied mathematics. These include:- Machine tools, Cutting tools, Two dimensional fluid flow, Complex numbers and their application, control theory and practice, Hydraulic operation of machines, Design and Manufacture.

The author cannot claim that the machine tool system described is, or has been, completely operational. However, sufficient work has been covered to indicate that the methods proposed can be used for machining the type of components concerned. Further the system is of relatively low cost and flexible in operation when compared with NC machines and those using interpolating tracer control.
Blades with form taper and helix were cut, the section contour being obtained from the copying device. The device based on conformal mapping was made as a model and produced pencil outlines of the aerofoil forms. By adjustment of disc size and linkage position the prescribed shape of various sections were obtained.

W W Sawyer, in his book Mathematicians Delight, states "It seems at first sight very strange that \( \sqrt{-1} \) something that no-one has ever seen, and which seems in its own nature to be impossible, should be useful for such material tasks as the design of dynamos, electric motors, electric lighting and radio." To this list can be added machine tool control systems.
To develop the shape of a blade the designer is largely concerned with the effects of air pressure on bodies, pressure distribution, and dynamic forces. However to convert physical behaviour into mathematical equations it is necessary to study the motion and behaviour of fluid elements.

It is convenient to consider flow in two dimensions only, such that in the subsequent mathematics there are only two variables, $x$ and $y$ in cartesian co-ordinates or $r$ and $\theta$ in polar co-ordinates. Two dimensional flow is fluid motion where the velocity at all points is parallel to a given plane.

For the purpose of this discussion two parameters of fluid flow must be outlined and defined. These are termed the STREAM FUNCTION and VELOCITY POTENTIAL.

The stream function, denoted by $\psi$, is the quantity of fluid passing a line across a stream per second. The shape and hence the length of the line crossing the stream is immaterial providing no fluid can enter or leave the stream.

Consider the line $op$ (Fig.22) set in a two dimensional stream of fluid moving from left to right.
Let the flow past the line at any point \( Q \), over a small length \( ds \) of the curve, be \( q \) where the direction of \( q \) makes an angle \( \beta \) to the tangent of the curve at \( Q \).

The component of the velocity \( q \) perpendicular to the element \( ds \) is \( q \cos \beta \) and assuming the depth of the stream to be unity the amount of fluid crossing the element \( ds \) is \( q \cos \beta \times ds \times 1 \) per second.

Adding all such quantities crossing similar elements along the line 0 to \( P \) the total flow past the line is

\[
\int_{0P} q \cos \beta \, ds
\]

This expression is called the stream function of \( P \) with respect to 0 and denoted \( \psi \).

A line connecting a series of points in the stream having constant \( \psi \) is known as a streamline and by definition no flow can cross a streamline, the velocity always being in the direction tangential to it (Fig. 23).

Thus the streamlines give one picture of the flow pattern. Another mathematical definition, giving a different plot of constants can be obtained by an expression giving the amount of flow along the line \( 0P \) (Fig. 22).

Then the component of velocity parallel to \( ds \) is \( q \cos \beta \) and the amount of fluid flowing along is \( q \cos \beta \times ds \times 1 \). The total amount of fluid flowing along the line towards \( P \) is given as

\[
\int_{0P} q \cos \beta \, ds
\]
This function is called the velocity potential of \( P \) with respect to \( 0 \) and is denoted \( \phi \).

A line in a flow connecting a series of points of constant \( \phi \) is called an equipotential and by definition there is no flow along such a line.

Fig. 22 illustrates a simple flow pattern comprising streamlines and equipotentials.

From the above statements it can be said that in a continuous fluid streamlines and equipotentials meet always at right angles.

The stream function gives a mathematical 'model' of a flow pattern in terms of the streamlines of the flow, and the velocity potential can give an alternative interpretation in terms of equipotentials of the same flow pattern.

Graphically these streamlines and equipotentials can be combined to give an orthogonal mapping of the stream.

Analytically the velocity potential and the stream function can be combined in a new function called the complex potential function by introducing the complex variable \( w = \phi + i\psi \).
This approach to two dimensional fluid flow is theoretical and furthermore assumes that air is incompressible, inviscid and the motion irrotational.

However, the topic does lead to the consideration of conformal transformation techniques as a solution to the problem of machine tool control.

It becomes a relatively simple to plot flow patterns from equations of $\psi$ and $\phi$ for simple shapes, for example a cylinder but far more difficult for complex shapes such as aerofoils.

It will be shown that a flow pattern can be constructed using conformal transformation techniques which can be achieved by geometrical means.

Although the correct transformation of the total flow pattern is irrelevant to this problem, the fact that the $\psi = 0$ line is the boundary of the aerofoil gives rise to the suggestion that a mechanical generation of the transformation would enable any aerofoil obtained by conformal transformation techniques to be manufactured. Further, it is possible that these methods are applicable to a wide variety of curve forming operations and provide a means of pure continuous path contouring which can compare with the established numerical control continuous path systems which interpolate between discrete points on the defined curve.
A real function of a real variable can easily be visualised by means of a graph.

If \( y = f(x) \) we interpret \( x \) and \( y \) as the absicissa and the ordinate of a point in a rectangular coordinate system and the set of points obtained in this way if \( x \) is made to take all values of an interval \( a > x > b \) represent the function \( y = f(x) \) in this interval.

Thus if it is required to machine a curve which can be defined by a relationship \( y = f(x) \) it is a simple matter to move either the tool or workpiece or both to obtain this relationship. This movement can be achieved by numerical control techniques interpolating between discrete values of \( x \) or by a gearing system between the \( x \) and \( y \) axes.

This concept can be extended to the case of complex valued functions of a complex variable with some modification. Since the values of both the function and the variable are complex numbers, a point representation of the function \( \omega = f(z) \) would require space representation of four real numbers.

A geometric representation of the function \( \omega = f(z) \) which is free of this disadvantage is obtained by regarding \( z \) and \( \omega \) as points in two different planes.
the z plane and the \( \omega \) plane, and by interpreting the function \( \omega = f(x) \) as a mapping of points in the z plane on to points in the \( \omega \) plane. Then corresponding to each point \((x, y)\) in the z plane defined by \( f(x+iy) \), there will be a point \((u, v)\) in the \( \omega \) plane where \( \omega = u+iv \). That is the function \( f(z) \) maps points in the z plane upon the \( \omega \) plane.

The correspondence between points in the two planes is called a mapping or transformation of points in the z plane into points in the \( \omega \) plane by function \( f(z) \). Corresponding points are called images of each other. The word image is also applied to a curve or region in one plane corresponding to a curve or region in the other.

Even though two separate planes are used to represent \( \omega \) and z it is often convenient to think of the mapping as effected in one plane thus permitting the use of geometric terms such as translation, inversion and rotation.

As with the case of real functions of real variables indicated earlier it is possible to apply these transformations to the work/tool relationships of a machine tool.

If the z plane is defined by a regular figure such as a circle, by applying a particular transform function it is possible to effect a transformation such that a
predicted complex shape is obtained in the $\omega$ plane.

If the transformation is reduced to a geometric operation the $z$ plane can provide the input to the system and the $\omega$ plane the output for work/tool positioning.

Consider the simple case shown in Fig. 24. $P$ and $Q$ are two points on a line given by $z_1, z_2$ in the $z$ plane. The two points are close together and separated by $dz$. The corresponding points in the transformed $\omega$ plane forming the line $P_1Q_1$ are given by $\omega_1, \omega_2$.

Now for all points
\[ \omega = f(z) \]

Differentiating with respect to $z$
\[ d\omega = f^1(z) \, dz \]

In the limit as $Q \to P$, $dz \to dz$, $Q \to P_1$, $d\omega \to d\omega$
\[ d\omega = (\text{vector } dz) \times (\text{Vector } f^1(z)) \]

Consider $f^1(z)$ rewritten in exponential form
\[ f^1(z) = re^{i\theta} \text{ where } r = \text{modulus of } f^1(z) \]

Then
\[ |d\omega| = |dz| \, r \]

and is in the direction of $dz$ after it has been rotated through $\theta$ the angular displacement of $f(z)$.

That is the transformed element equals the original element rotated through $\theta$ and multiplied by $r$.

If two elements intersect at some angle $\sim$ then the transformed elements are rotated through the same
angle, the angle of intersection \( \angle \) must remain unchanged during the transformation.

Re-writing equation 2

\[
\frac{d\omega}{dz} = f(z)
\]

Thus the length of the element in the \( w \) plane \( \frac{d\omega}{dz} \) is equal to the length of corresponding element in \( z \) plane \( dz \).

This ratio is known as the magnification of the transformation.

To summarize, it has been shown that conformal transformation is the study of methods whereby a geometric pattern in one plane comprised of certain shaped elements can be transformed into an entirely different, but predictable, pattern whilst the elements maintain their distinctive form and proportion.

The mode of the transformation depends on the transformation formula \( \omega = f(z) \) chosen.

The following transformation formulae illustrate the effect on relationships.

1. \( \omega = z + b \) A shift of origin
2. \( \omega = az \) A rotation and magnification
3. \( \omega = \frac{a^2}{z} \) An inversion in a circle

\( a \) and \( b \) are constants

The subject matter of this Appendix has been covered extensively by many authors and in particular Ref. 7 and Aerodynamics for Engineering Students, Houghton & Brock and Ref. 8 and 9. The next Appendix deals with an important transformation known as Zhukovsky's transformation.
THE ZHUKOVSKY TRANSFORMATION

The Zhukovsky transformation produces a family of aerofoil shaped curves, with their associated flow patterns by applying a certain transformation about a circular cylinder. Although the circle is only one streamline of the flow it will be shown that consideration of this simplified flow pattern is sufficient to give the required forms.

Consider the flow of an ideal fluid around a cylinder whose axis is at right angles to the direction of flow. (Fig. 25).

Let the trace of the cylinder in the z plane be a circle with radius a and centre at $z = 0, x = 0, y = 0$.

At very great distances from the circle, it will be supposed that the fluid has a velocity $V_0$ in the $x$ direction. The appropriate mapping function must map the line ABCDE into a line which is parallel to the real axis of the $\omega$ plane.

The semi-circular path BCD can be made to map on the real axis of the $\omega$ plane by letting $\omega = K \cos \theta$ for values of $z$ on the circle.

\[
\omega = K \cos \theta = \frac{K}{2} \left( e^{i\theta} + e^{-i\theta} \right) = \frac{a}{e^{i\theta}}
\]

\[
= \frac{K}{2} \left( \frac{z + \frac{a}{z}}{z} \right) = \frac{K}{2a} \left( z^2 + \frac{a^2}{z} \right)
\]
A constant can be determined from initial velocity conditions and therefore there is no loss in generality by taking the mapping function to be:

\[ \omega = z + \frac{b^2}{z} \]

This transformation function is known as the Zhukovsky Transformation.

A similar equation is obtained from the Circle Theorem of Milne-Thomson (Ref. 10).

Thus by using the transformation function

\[ \omega = z + \frac{b^2}{z} \]

a point \((p)\) in the \(z\) plane given by

\[ z = x + iy \]

can be transformed to \((p)\) in the \(\omega\) plane given by

\[ \omega = u + iv. \]

\[ \omega = z + \frac{b^2}{z} \]

\[ = re^{i\theta} + \frac{b^2}{r} e^{-i\theta} \]

\[ = r (\cos \theta + i \sin \theta) + \frac{b^2}{r} (\cos \theta - i \sin \theta) \]

\[ u + iv = (r + \frac{b^2}{r}) \cos \theta + (r - \frac{b^2}{r}) i \sin \theta \]

\[ \therefore u = \frac{(r + \frac{b^2}{r}) \cos \theta}{r}; \quad v = \frac{(r - \frac{b^2}{r}) \sin \theta}{r} \]

Thus the \(u\) \(v\) co-ordinates in the \(\omega\) plane result from the transformation of the point on a circle in the \(z\) plane.
As an example consider $r = a = b$

(centre of circle at the origin) Fig. 26

Then $u = \left(\frac{r + \frac{b^2}{r}}{r}\right) \cos \theta = 2a \cos \theta$

$v = \left(\frac{r - \frac{b^2}{r}}{r}\right) \sin \theta = 0$

The circle of radius $a$ in the $z$ plane transforms to a line of length $4a$ in the $w$ plane.

If the original transformation function $w = z + \frac{b^2}{z}$ is again used in conjunction with a circle in the $z$ plane passing through $z = b$ (or $z = -b$) but with its centre shifted from the origin some interesting and useful shapes can be generated in the $w$ plane.

Consider the shift to be in both horizontal and vertical directions. Fig. 27. The horizontal shift is given by $'b'$, a fractional part of the constant $b$, and the vertical shift given by $h$ which is conveniently expressed in terms of $\beta$, the angle between the radius intersecting the $\infty$ axis on the circumference and the $\infty$ axis.
Consider the enlarged diagram: (Fig. 28)

By dropping perpendiculars onto \( av \) from \( t \) and \( u \)

\[
\begin{align*}
r &= l + m + n \\
l &= a \cos \alpha \\
m &= h \sin \theta \\
n &= b \eta \cos \theta \\
r &= a \cos \alpha + h \sin \theta + b \eta \cos \theta
\end{align*}
\]

But \( a = (b + b\eta), \cos \alpha = \frac{l}{a} \) (\( \alpha \) very small)

\[
\begin{align*}
h &= (b + b\eta) \sin \beta \\
h &= b (1 + \eta) \beta \Rightarrow b\beta \text{ (neglect small quantities)}
\end{align*}
\]

\[
\begin{align*}
r &= b + b\eta + b\eta \cos \theta + b\beta \sin \theta \\
r &= l + \eta + \eta \cos \theta + \beta \sin \theta \\
\frac{b}{r} &= 1 - \eta - \eta \cos \theta - \beta \sin \theta \text{ (neglect } \eta^2 \text{ & values)}
\end{align*}
\]

\[
\begin{align*}
\frac{b^2}{r} &= b - b\eta - b\eta \cos \theta - b\beta \sin \theta \\
u &= b \left( \frac{r}{b} + \frac{b}{r} \right) \cos \theta \\
&= 2b \cos \theta \\
v &= b \left( \frac{r}{b} - \frac{b}{r} \right) \sin \theta \\
&= b(2\eta + 2\eta \cos \theta + 2 \sin \theta) \sin \theta \\
&= 2b \eta (1 + \cos \theta) \sin \theta + 2b \sin^2 \theta
\end{align*}
\]
The Thickness Chord Ratio

 Thickness chord = \( t = t_1 - t_2 \)

where \( t_1 \) & \( t_2 \) are the vertical ordinates of the upper and lower surfaces.

\[
v_1 = t_1 = 2be \left( 1 + \cos \theta_1 \right) \sin \theta_1 + 2b \sin 2\theta_1
\]

\[
v_2 = t_2 = 2be \left( 1 + \cos \theta_2 \right) \sin \theta_2 + 2b \sin 2\theta_2
\]

as \( t_1 \) & \( t_2 \) are at the same ordinate \( u \)

\[
\theta_1 = \theta_2
\]

\[
t = t_1 - t_2 = 4be \left( 1 + \cos \theta_1 \right) \sin \theta_1
\]

The chord is given by substituting \( \theta = 0 \) and \( \pi \) in

\[
u = 2be \cos \theta ,\ \text{when the chord is} \ 4b.
\]

Thickness/Chord Ratio = \( \frac{4be \left( 1 + \cos \theta_1 \right) \sin \theta_1}{4b} \)

Thus the shape of the aerofoil produced is dependent on the following parameters in the \( z \) plane

\(1\) Diameter of Circle

\(2\) Shift of origin from centre of circle

In the above calculations assumptions have been made and small values neglected. In order to check the resulting equation for \( u \) and \( v \), a computer programme was developed for the conditions \( e = 0.14 \), \( b = 1.500 \) and \( \beta = 0.1 \).

The programme and resultant plot are shown in Figs.15 and 16. The plot shows the \( z \) plane circle superimposed on the transformed aerofoil shape in the \( \omega \) plane.
Graphical Generation of Zhukovsky Profile

Producing the profile streamline of a Zhukovsky transformation is a relatively simple process, although somewhat lengthy. This is because the original streamline is a simple geometric shape, the ordinates of which can easily be absorbed as the controlled variables in the numerical process.

In order to be of value in this project the transformation must be considered from a geometric point of view.

The function $\omega = z + \frac{b^2}{z}$

can be rewritten $\omega = re^{i\theta} + \frac{b^2e^{-i\theta}}{r}$

Thus a transformed point $P$ is given by the vector $\omega$ which is itself the vector sum of (a). The vector of length $r$ and argument $\theta$ and (b) The vector of length $\frac{b^2}{r}$ and argument $-\theta$. (Fig.29).

From a geometric or mechanical point of view it is a simple matter to construct a circular disc with a link pivoted about a point eccentric with the circle centre. In this way the length of the link at the circle circumference will give values of $r$ for values of $\theta$ from the mean position.

To complete the requirement, another disc and link assembly is needed to directly obtain $\frac{b^2}{r}$ at values of $-\theta$. 
Let $C$ be the centre of the given circle meeting $Ox$ in $A$ and $B$ (Fig. 30). $AB$ is a chord passing through the origin.

Since $AO = b$ and at $\theta = \pi$, $b^2 = b_r$, then point $A$ is its own inverse and a point on the locus of $S$. By drawing $AD$ parallel to $CB$ to meet $CO$ produced in $D$ the centre of the locus circle for $A$ & $S$, can be found at $E$, on reflection in $Ox$.

Fig. 30 shows the vectors $r$ and $b^2$ inclined to the Ox axis at $\theta$ and $-\theta$ respectively and demonstrates that it is possible for the two vectors to be obtained for all values of $\theta$ between 0 and $\pi$.

Vector summation by construction of a parallelogram will obtain the transformed point $P$ on the required shape.

Another possible solution is to construct the two discs twice their actual size but at the same time maintain their centre positions relative to the origin as shown in the diagram (Fig. 31).

$$\frac{aq}{qp} = \frac{q_r}{q_P} = \frac{2}{1}$$

\[ P \text{ is the mid-point of } qr. \]
Numerical machine tool control and interpolating tracer control were two manufacturing methods specified in Chapter 1 which met the requirement for small quantity production.

CUTTING COMPLEX CONTOURS USING NUMERICAL MACHINE TOOLS CONTROL:

Numerical machine tool control systems can be broadly classified under two headings -

a) Point to point systems

b) Continuous contouring systems

Point to point systems are, in general, related to hole making machine tools. The work table moves under the control of a punched paper tape in two directions, namely the x and y axis movements. Some systems provide facility for straight line or pocket milling and have the z axis under limited control by means of preset cams. This type of system is not suited to the manufacture of complex shapes where the x, y and z axes are required to be continuously controlled during the cutting action.

Continuous contouring numerically controlled machine tools were initially developed to satisfy the requirements for the Aircraft and Aerospace industries. The use of such machine tools for the production of many complex shapes such as special cams, waveguides, templates, dies, etc. can give vast savings in manpower and machining times.
These continuous control systems can be further subdivided into 2D, $2\frac{1}{2}$D, 3D and 3D+ classes of milling machines.

2D systems are machines whereby the x and y axes are under continuous control, but z axis (cutter depth) movement is by manual adjustment.

$2\frac{1}{2}$D systems — here the x and y axes are under continuous control but the z axis is under incremental control.

3D systems — here the x, y and z axes are under continuous control.

3D+ systems, sometimes called 4 or 5 axes control systems — the x, y and z axes are under continuous control, also the workpiece can be rotated about both a vertical and a horizontal axis.

It is essential that when a system is under continuous control the feed rate and table position is continuously checked during any machining operation and for this reason some form of interpolation must be achieved either in the control system of the machine or by a computer programme.

Although some continuous contouring systems are capable of being programmed entirely by hand the process, except for relatively simple work, requires a large amount of calculations. Further, in N.C. systems without interpolating facilities the controller must have the interpolation carried out by a computer.
To illustrate the problems of data preparation for continuous contouring systems consider the simple example shown in Fig. 32 (a more complex and relevant example is shown in Ref. 1)

The programmer must decide on how the component is to be machined, in what sequence the operations are to be carried out, the speeds and feeds required and the clamping necessary.

For programming purposes he must decide –

1) The actual cutter centre path necessary to produce the component.

2) The change points on the component and also on the cutter centre path, that is the point where the contour changes from one geometric form to another.

3) The division of each section between the change points into a number of spans which will enable the interpolator in the controller to carry out the interpolation functions necessary to machine the part within the specified tolerances.

4) To fix a datum point and calculate the co-ordinate dimensions, with respect to this datum, of all the points which have been determined on the cutter path.

5) The method of starting, bringing the cutter up to the work without overshoot, moving the cutter round corners at the correct feed rate without
overshoot. All these factors require additional decisions and appropriate programming.

The interpolating functions can either be linear, circular or parabolic.

If the complex shape of a turbine blade is considered for a continuous contouring system, calculations for the co-ordinate positions (usually y, z axes) for change points of the surface have to be calculated.

For the required accuracy the change point intervals can be as small as .0005 dependant on the type of interpolation used and it is impractical to use manual programming.

Computer programmes using a specialised language such as A.P.T.* are used to convert physical data into programming language. The tapes obtained from the computer have to be passed through a post-processor to provide suitable input information for the particular machine tool control.

An installation of this type of machine tool could cost up to £250,000 and tapes would normally be processed at a manufacturer's centre.

* Automatic programme tools.
MACHINING COMPLEX CONTOURED SECTIONS USING INTERPOLATING RACER CONTROL

These systems, of which there are a number, are not so well established as NC control. The devices to be described have been used to control shaping machines, milling machines, grinding wheel shaping heads and flame cutting machines.

The essential features of these systems are firstly a tracer control which can be contacting or non-contacting and secondly a fair curve interpolating system.

In one particular tracer controlled milling system (Ref 3) templates are made to the shape of each cross section specified on the drawing. These are spread on an arbor and lined up so chordal tangents are all parallel (Fig. 33). Follower discs press against the template contour, these discs are directly attached to template follower bars which have knife edges which press against a strip at the same feed rate as the milling cutter. The arrangement so far described is an extension of a normal copying system. The most interesting part of the development is that the strip referred to is in fact a flexible member which deflects under the effect of the knife edge mounting. In this way a fair curve interpolation is obtained between template sections.

The desired twist is obtained through a differential gear train that rotates the stack of templates relative to the instantaneous position of the work.
The differential is under the control of a second interpolating system. In one type of control studied the tracer point is non contacting. The system operates to maintain a certain length spark gap between the flexible member and the tracer point. In this case similar controls are used to maintain the spark gap as those used in arc welding.

A shaping machine control system developed by Armytage (Tools) Ltd uses a mechanical-hydraulic system known as Hyprofile Tracer. The tracer unit is a hydraulic servo-valve, the output of which drives the tool slide of a conventional shaping machine.

Geometric and non-geometric shapes can be produced by the use of two or more templates (Fig.34). One template is always attached to the tracer and can be straight or contoured but always contacts the other template or templates irrespective of the difference in shape or depth of the stationary template.

This method enables two dimensional contours of profiles to be produced on standard machine tools.

Both methods of machine control described have been used to manufacture turbine blades (Ref 2 and 3).
SPECIFICATION OF THE MAXICUT NO 3A HEAVY DUTY GEAR SHAPER

The No 3A (Ref 11) is essentially a heavy duty machine for spur and helical gears up to 18" pitch diameter and 3/4 diametral pitch. Such gears can be produced singly or, where design permits, a number of blanks can be assembled on a mandrel to a maximum height of 5" for simultaneous cutting.

The cutter head is designed for rigidity under heavy cutting loads and the guides for spur or helical gears make full length contact with their mating halves which has the effect of reducing wear. The lubrication is by force feed pump to all working parts.

The work spindle slide is carried in large area ways on its bed which swings on a heavy bearing on the machine column. A tongue on the bed engages with a locating bearing when closed and pulling the locking lever renders the unit virtually solid with the machine body.

The relieving mechanism is by rack and quadrant, the latter being activated by cams in the gear box and push rods.

The required speed and feed are quickly obtained with direct reading hand wheel selectors. A door gives quick easy access to the crank disc for adjustment of stroke to the cutter spindle, the disc being graduated in inches and millimeters for both pull and push strokes.
The machine is fully self contained with electric motor housed in the base casting. The drive is by belt to the gear box.

Stepped pulleys enable two speeds to be obtained and these together with the speed changes in the gear box permit a choice of eight speeds. From the gear box drives are taken to the cutter spindle reciprocating crank, to the feed control gear box, while two cams on the main drive shaft operate the relieving mechanism.

The feed motions are direct to the cutter spindle and through pick-off gears to the work spindle enabling the latter to rotate with the former according to the gear being cut.

**PRINCIPLE DIMENSIONS**

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<thead>
<tr>
<th>Description</th>
<th>Value 1</th>
<th>Value 2</th>
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<tbody>
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<td>Capacity p.c. diameter</td>
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<td>457mm</td>
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<tr>
<td>Maximum face width</td>
<td>5&quot;</td>
<td>127mm</td>
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<tr>
<td>Maximum Dimetral Pitch</td>
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<td>6½ module</td>
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<tr>
<td>Speeds of cutter (Stroke per minute)</td>
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<td>121, 153,</td>
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<td></td>
<td>98, 162,</td>
<td>237, 300</td>
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<td>Feeds (Strokes per rev of cutter)</td>
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<td>2020, 2400</td>
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<td>Horse power of motor</td>
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<td></td>
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<td>Overall dimensions of machine</td>
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<td></td>
<td>Width 4'9&quot;</td>
<td>1450</td>
</tr>
<tr>
<td></td>
<td>Height 6'5&quot;</td>
<td>1956</td>
</tr>
<tr>
<td>Gross Weight</td>
<td>75 cwt.</td>
<td>3810 Kg.</td>
</tr>
</tbody>
</table>

A photograph is shown in Fig. 35 which indicates the essential features of the machine.
The following features of the gear cutting machine are retained for cutting turbine blades. (Ref. 12).

1. The reciprocating cutter ram.
2. The feed motion and gearing.
3. The relieving mechanism.
4. The hinged work tray.

The machine is modified to obtain the greater cutting length of 9" required for blades on the Ghost Engine. This single factor required an extensive change to the original machine design. A one foot thick spacing column was placed between the machine base and the upper casting to give the increase cutting distance. The cutting ram was also lengthened accordingly.

The principle of operation is a copying technique and therefore the drive and gears to the table motion were removed.

Inside the tool head 3 (Fig. 36) is a sliding member (4) which carries a roller (5) at one end and a tool holder with tool (6) at the other. There is a spring inside the tool head keeping the roller in contact with the forming cam (7) at all times. The forming cam, providing the horizontal motion to the cutting tool, together with the downward cutting and rotary feed motions, combine to produce the form on
the blade (8).

The set up described produced the near side of the form and is referred to as an internal machine. The same principles apply to an external machine, which cuts the second side. In this case the cutter box is constructed such that the tool has a greater offset from the ram centre. This feature is shown in Fig. 37.

A number of these machines were constructed to cater for production quantities. Some hand working was subsequently required to produce the specified surface finish.
SPECIFICATION OF DESIGNED PARTS AND PURCHASED ITEMS USED IN THIS PROJECT

Purchased Items:

2. Sperry Electro-Hydraulic valve 3150/1 mounted with and controlling rotary actuator.
3. Mounting block for Sperry E.H.S.V.
5. Sperry power oscillotor 23453.
7. Sperry Rotary Pick-off 21407 2 off.
8. Elliot 610 Electro Hydraulic valve 2 off.
9. Elliot Servo control unit 2 off.
10. Sperry linear pick-off 21681 4 off.
11. Keelovite Rotac hydraulic actuator Rn 32.
12. Penny & Giles 9" stroke Rectilinear Potentionmeter.
15. Keelovite restrictor valve.
17. Precision Gearbox 10.1 reduction.
18. Precision Gearbox 5.1 reduction.

Designed Items

1. Dual tool box fitting directly to cutting ram (Fig. 5) including hydraulic cylinders servo-valves and feedback pick-offs.

2. Work table and support (Fig. 21) including rotary actuator, servo valve and feedback pick-off.

3. Copying attachment (Fig. 10) including means of relieving cutting tools.

4. Rotary pick-off and gearbox connection to ram shaft.

5. Rectilinear potentiometer attachment to machine reciprocation.

6. Models of devices which generate the Zhukovsky transformation.
REFERENCES


4. Personal Communication to Author, Science Research Council 3.7.66.


11. Drummond-Asquith (Sales Ltd.) Maxicut Gear Shaper Technical Literature.

REFERENCES contd.

13 Production of Forged hollow turbine blades
Diagram showing method of dimensioning turbine blades by defining form at various sections by co-ordinates to chosen centre lines.

<table>
<thead>
<tr>
<th>SECTION</th>
<th>X</th>
<th>Y</th>
</tr>
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<tbody>
<tr>
<td>Tip</td>
<td>A</td>
<td>B</td>
</tr>
<tr>
<td>SECTION V.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Root</td>
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</table>
FIG. 2

BASIC FORM OF BLADE

TYPE 1: PARALLEL UNTWISTED FORM

TYPE 2: NON-UNIFORM SECTION (TAPER)
SECTION MAGNIFICATION GIVEN BY \( \frac{A}{B} = \frac{a}{b} \)

TYPE 3: TWISTED SECTION
\( \Theta = \text{LEAD ANGLE} \)

BLADE PARAMETERS
FIG. 3

SADDLE  RAM  a  b  GEAR CHANGE

FEED  CHANGE  STOP  CLUTCH

BASIC MACHINE EQUIPPED WITH TEMPLATE COPYING MECHANISM AS USED BY BRISTOL SIDDELEY LTD.
FIG. 4

Controlled Rotary Motion
of table to provide for blade helix

Feed increment per stroke
Rotary motion about ram centre φ = n°

Initial tool position at start of stroke provides for section form
Controlled motion of tool during stroke provides for taper

Diagram illustrating method of relating machine notions to specified parameters
Schematic of dual cutter box and drive

Feed back
Pick off

Tool drive
Cutting tools

Hydraulic ram

R''
PHOTOGRAPH OF DUAL CUTTER BOX AND DRIVE
Schematic of Electro-Hydraulic Servo Valve

Torque Motor
3 Way Pilot Valve
4 Way Main Valve
To Hydraulic Motor

DC Error Voltage
Photograph showing both tool actuation and table drive
SCHEMATIC OF COPYING 4 TAPER INPUT DEVICE
PHOTOGRAPH OF COPYING AND TAPER INPUT DEVICE
SCHEMATIC OF ASSEMBLY BLOCK 'E'

OF FIGS 10 & 11
CENTRE OF DISC 'C' MOVED TO POSITION RELATIVE TO 'u' TO CUT VECTOR vW AT P

GEOMETRIC CONSTRUCTION TO OBTAIN TRANSFORMED POINT P (DESIGN STUDY)
R
CUTTER RADIUS
ABOUT RAM.

GEOMETRIC CONSTRUCTION

DESIGN STUDY I
$E = 0.1400$
$B = 1.5000$
$\beta = 0.1000$

Computer plot for approximate Zhukovsky's solution
Computer program to obtain Fig. 15
C & C₁ ARE TWO POINTS ON AEROFOIL

GEOMETRIC CONSTRUCTION DESIGN STUDY 2
GEOMETRIC CONSTRUCTION FOR BLADE TAPER (I)
GEOMETRIC CONSTRUCTION
FOR BLADE TAPER (2)
CIRCUIT FOR TAPER CONTROL
ARRANGEMENT OF WORKPIECE TENSIONING DEVICE
Diagram illustrating theory of streamlines and velocity potential
Diagram illustrating stream function and equipotentials.
Basic Principle of Conformal Transformation
DERIVATION OF ZHUKOVSKY'S TRANSFORMATION

FUNCTION \( \Omega = z + \frac{b^2}{2z} \).
Transformation of a circle to a straight line using function \( w = z + \frac{b^2}{z} \).
Transformation of a Circle to a Cambered Aerofoil
GEOMETRIC CONSTRUCTION OF TRANSFORMATION \( w = z + \frac{b^2}{z} \)
Geometric constructions of transformation $\theta = z + \frac{b^2}{2}$.
CUTTER PATH (1" DIA CUTTER)

DATUM
A' (11.8)  B' (16.366, 8)

SIMPLE EXAMPLE OF DATA PREPARATION
FOR CONTINUOUS CONTORING N.C.M.T.

FINISHED DIMENSIONS

AB = 4"
AF = 4"
GE = 1.5"
ABC = 60°
FAB = 90°
AFE = 90°
DIAGRAM OF INTERPOLATING TRACER CONTROL SHOWING
BOTH SHAPE AND TWIST CONTROL UNITS
Illustrations show the production of Elliptical Dies by extrapolation of two templates to produce a tapered elliptical bore.
Front view of the No. 3A MAXICUT Gear Shaper.
SET UP FOR CUTTING INTERNAL BLADES