ANALYSIS, DESIGN AND EVALUATION OF A FIBRE OPTIC
PHYSIOLOGICAL PRESSURE TRANSDUCER

A THESIS SUBMITTED FOR THE DEGREE OF DOCTOR OF PHILOSOPHY
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BY
PETER JOHN BRAND

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A pressure transducing system utilising light for the modulation of the deflection of a catheter tip pressure sensing diaphragm is investigated. Optical fibres are used for transporting the input light signal to, and the modulated signal output from, the probe tip. The emergent modulated light signal falls on to a photo detector which gives an analogue electrical output. The analysis of the total system is achieved by first considering the effect which the parameters of the optical fibres and their arrangement in the distal tip of the probe have on the fraction flux received to flux emitted as a function of the separation of a 'flat' reflecting diaphragm. This relationship is derived from first principles and considers the emitting fibre ends to be Lambertian sources. This implies that the numerical aperture (N.A.) of the fibre(s) is unity and that the polar distribution is uniform. The resulting expression is termed 'reflection factor', from which a simple method for accounting for fibre systems of other N.A.s is proposed. The conditions for which it is valid are discussed, at which time a critical appraisal is made of the one other original author in this field. The derived expression for reflection factor is modified by introducing a normalised concept for the separation 'd' of the reflecting surface to the distal tip face, and the outside radii 'R' of the receiving system, these being the separation ratio 'B' and the 'A' value respectively. When 'B = d/r' & 'A = R/r', noting that 'A' signifies the fibre arrangement which, geometrically is that of a central emitter surrounded by an annular receiver. The introduction of these normalised terms allows for a general description to be defined for the gradient of the reflection factor response curve (slope factor) which is dimensionless. It also allows for the description of what is termed the 'reflection factor total ratio'. This is the ratio of the reflection factor at a separation ratio of 'B' to that when the separation ratio has changed by 'delta B' this being the normalised deflection ratio. Before applying the above mentioned conceptual approach
which defines the optical fibre response for different positions for a 'flat' diaphragm, in order to predict the response when a real diaphragm is used, an assessment is made of the errors incurred by considering the real diaphragm to deflect with a flat profile. A separate section has been devoted to considerations of a diaphragm, its deflection characteristics and natural frequency. A graphical presentation by which a choice of diaphragm can be exercised and a factor of safety can be determined has been developed. The light source and photodetector are considered from the point of view of what type and mode of operation are most suitable for application in this system and a relationship for the minimum acceptable signal to noise ratio and the set level of system output is derived for different values of reflection factor total ratio. All the discrete system parameters are combined to form a graphical aid for the specification of system parameters (G.A.S.S.P.), which is used to provide a theoretically proposed system and its response characteristics. It is later applied to a system where system noise and probe reflection factor characteristics have been determined experimentally, to aid the choice as to which diaphragm should be used and at what separation it should be set. The fibre optic probe constructed thereby is experimentally evaluated, the diaphragm being 25 microns thick Berrylium Copper of 1.5 mm radius, for which when set at an initial separation of 35 microns a pressure resolution of 0.5 mm Hg. (sensitivity 0.186 mV / mm Hg.) is achieved.
ACKNOWLEDGEMENTS

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VALUES OF CENTRAL DEFLECTION \( (y_{\text{max}}) \) FOR \( R_{\text{dm}} = 1.5\text{mm} \)

DEFLECTION RELATIONSHIP Be/Cu

DEFLECTION RELATIONSHIP STAINLESS STEEL

DEFLECTION RELATIONSHIP GLASS

DEFLECTION AGAINST THICKNESS - \( R_{\text{dm}} = 1.0\text{mm} \)

DEFLECTION AGAINST THICKNESS - \( R_{\text{dm}} = 1.5\text{mm} \)

DEFLECTION PROFILES \( R_{\text{dm}} = 1.0\text{mm} \) P = 300mm Hg

DEFLECTION PROFILES \( R_{\text{dm}} = 1.5\text{mm} \) P = 300mm Hg

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APPROXIMATE RELATIONSHIP BETWEEN THE RATIO OF A RUPTURE OR YIELD PRESSURE OF A DIAPHRAGM WHEN USING THE "APPROXIMATE ENERGY METHOD" TO THAT WHEN USING THE "LINEAR METHOD"

UNDAMPED NATURAL FREQUENCY Be/Cu

UNDAMPED NATURAL FREQUENCY - STAINLESS STEEL

UNDAMPED NATURAL FREQUENCY - GLASS

COMPOSITE RAY PLOT OPTICAL FIBRE/DIAPHRAGM

COMPOSITE RAY PLOT OPTICAL FIBRE/"FLAT" DIAPHRAGM

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SECTION ONE

INTRODUCTION
1.0.0. INTRODUCTION

"All men, the leading dictator of philosophy, naturally wish to know... The sign of this is the pleasure which they get from using their senses, among which I choose as the outstanding one vision, because this is the one most conducive to our knowledge of anything and makes plain differences.

The research method in use today is very unsuitable and misleading, since very many enquire not what things are, but what is said about them by others... I therefore whisper in your ear, friend Reader, that you weigh in the exact balance of experience whatever I treat of in these essays... that indeed you use them only insofar as you find them most firmly corroborated by the direct evidence of your own senses."

ARISTOTLE

The concept of blood circulation was established in 1616 by WILLIAM HARVEY (1627), a very lucid and logical thinker. However, although it was observed that blood spurted from punctured vessels, over a hundred years passed before STEPHEN HALEs (1733) quantitively measured the mean blood pressure in an unanesthetized horse, and noticed the synchronous nature of heartbeat and oscillations of mean pressure readings. It is worth noting that Pascal's first law of fluid pressure became generally accepted late in the 17th century. From Stephen Hales' epic experiment the determination of blood pressure has been of interest to clinicians and physiologists. Many methods enabling the determination
of physiological vascular pressures (cardiovascular, by far the most common) have been evolved. However, many of these have had severe limitations, either because they are too large, difficult to operate or most important of all not accurate (for which there are many causes).

The measurement of intravascular pressures can be categorised into two sections; direct pressure measurement, where the pressure within a vessel is measured directly by the insertion of a pressure detector; and secondly, indirect pressure measurement where an estimate of the pressure within a vessel is obtained by external means, an example being the use of an inflatable cuff as used for the determination of Brachial artery pressure.

It is the direct pressure measurement system which will be studied as it is a far more accurate method, specifically the use of a new catheter tip transducer employing fibreoptics as the pressure transducing element. The pressure sensing diaphragm is located at the catheter tip and obviates the need for any fluid filled catheter coupling, which would otherwise preclude the accurate representation of a pressure wave form.

It should be noted that any intravascular blood transducer will, unless specifically stated (on the grounds of size, pressure range, construction), be capable of measuring other physiological pressures, notably those in the cerebro-spinal system, ocular system, urinary system, in fact any fluid filled physiological system.

1.1.0. INTRAVASCULAR PRESSURES & THEIR SIGNIFICANCE

Physiological fluid pressures whether cardio-vascular, cerebro-spinal, ocular or urinary do have certain qualitative and quantitative similarities. The pressure levels are all relatively low when compared with the "real world", ranging from -5 to 300 mm Hg., and the
and the pressure/time relationship will usually vary in a cyclic fashion about a mean level above atmospheric pressure. This is most predominant in the cardio-vascular system (blood pressure).

The term intravascular pressure, is taken to mean the pressure of blood within the cardio-vascular system, although in its literal context it would mean any vessel; nevertheless the problems to be discussed in the determination of blood pressure are mostly applicable to other physiological fluid pressures.

The essential features of the cardio-vascular system of mammals had been known for some time, as is shown by the illustration taken from VESALIUS (1543) FIG 1, where the most important omission is that of the capillaries. The heart is composed of four chambers arranged into two sections. The thin walled atria are connected via a valve and an orifice to the muscular thick walled ventricles. Each ventricle in turn connects to a major distributive artery, the exit paths are valve controlled. The left ventricle leads to the Aorta and carries blood which is oxygenated for distribution on its outward journey to the body organs and tissues. The oxygen transport takes place in the capillaries from which point on the blood is in the venous system. It returns to the heart, not under the influence of the left ventricle ejectate pressure which propels the arterial blood, but by a combination of the effects of the negative pressure created in the thorax due to respiration and the muscle pump effect of muscles squeezing the vein. On return to the heart the venous blood enters the right atria and is drawn down into the right ventricle during diastole. On the following systole it is ejected into the Pulmonary artery (at a lower pressure -20 mm Hg - than the simultaneous ejection of oxygenated blood from the left ventricle into the Aorta -100 mm Hg mean-) for
FIG. 1. The systemic arterial system. (From Vesalius, 1543.)
circulation in the lungs where oxygenation occurs. After which the re-oxygenated blood passes back into the left Atria. FIG 2 tabulates approximate values of intravascular pressures at different locations for an adult male.

Thus the prime mover which generates blood pressure is the heart. The level of arterial pressure is not constant but varies synchronously with heart beat. If the pressure wave form was recorded faithfully in the Aorta close to its exit from the heart, the wave form resulting FIG 3 would show a cyclically varying pressure having a peak value (systolic pressure) and a minimum value (diastolic pressure) corresponding to the heart in systole (pumping stroke) and diastole (resting stroke). The time of the cycle is the inverse of the pulse rate, thus the basic frequency of the wave form is 1-2Hz. Now superimposed can be seen irregularities having a much higher frequency but of small amplitude. These are effects caused by heart valves causing turbulence, and the filling of the ventricles.

The finite values of systolic and diastolic pressures alone give a poor assessment of patient wellbeing as the range of normal values is very large. However, it is the combined information from several parameter sensors (flow, oxygen saturation, temperature, etc. plus clinical observations) which enable very accurate diagnosis to be made, in specific cases in an operative situation where the surgeons modus operandi are finalised in theatre after consultation with the instrumentation.

The objectives underlying the routine clinical determination of physiological parameters is two-fold.

1) To allow an objective assessment of the present condition of a patient to be made. By implication this necessitates that the
<table>
<thead>
<tr>
<th>VESSEL</th>
<th>PRESSURE mm./Hg.</th>
<th>DIA. mm.</th>
</tr>
</thead>
<tbody>
<tr>
<td>AORTA &amp; LARGE ARTERIES</td>
<td>120/80</td>
<td>30-10</td>
</tr>
<tr>
<td></td>
<td>mean 100</td>
<td></td>
</tr>
<tr>
<td>SMALL ARTERIES</td>
<td>120/70</td>
<td>10-1</td>
</tr>
<tr>
<td></td>
<td>mean 90</td>
<td></td>
</tr>
<tr>
<td>CAPILLARIES</td>
<td>20</td>
<td>0.008</td>
</tr>
<tr>
<td>SMALL VEINS</td>
<td>10</td>
<td>5</td>
</tr>
<tr>
<td>MAIN VEINS</td>
<td>-5 to 5</td>
<td>40</td>
</tr>
<tr>
<td>PULMONARY ARTERY</td>
<td>25/10</td>
<td>40</td>
</tr>
<tr>
<td></td>
<td>mean 15</td>
<td></td>
</tr>
<tr>
<td>PULMONARY CAPILLARIES</td>
<td>5</td>
<td>0.01</td>
</tr>
<tr>
<td>PULMONARY VEINS</td>
<td>0</td>
<td>10</td>
</tr>
</tbody>
</table>

FIG. 2 INTRAVASCULAR PRESSURES: APPROXIMATE VALUES AND LOCATIONS.
FIG. 3  HIGH FIDELITY RECORDING OF PRESSURE WAVEFORM

SYSTOLIC PRESSURE

MEAN PRESSURE

DIASTOLIC PRESSURE

WAVELENGTH OF FUNDAMENTAL PRESSURE SIGNAL

HIGH FREQUENCY LOW AMPLITUDE SIGNALS CONSTITUTING "SOUNDS AND MURMOURS"
parameter determination system has a fast and accurate response, if action is to be effectively taken.

2) By using serial or continuous monitoring of important parameters, the tracing, establishing and predicting of trends which enables the observer (man or machine) to take the relevant action at the earliest opportunity for the patients welfare.

1.2.0. MEASUREMENT OF INTRAVASCULAR PRESSURE

Before describing modern methods for the direct measurement of intravascular pressures it is of more than academic interest to consider briefly the developments leading to the first steps taken in the development of pressure measurement in the cardio-vascular field.

As already mentioned it was in about 1616 that William Harvey began to demonstrate the circulation of blood. He and others had observed the effects of blood pressure, namely the pulsation of an artery and the spurting of blood from a severed artery. The concept of pressure eluded them.

There are two distinct trails of discovery and thought which led to the measurement of blood pressure. These can be described as the Anatomical and Physical trails.

Describing these developments together and in the reverse order by a process of logic we can deduce that before a system can measure a variable, the system itself must be understood. Then the concept of the variable must be fully understood. In the case of measuring blood pressure it is the concept of fluid pressure. These are what will be classed physical developments. If the concept of fluid pressure is known and applied to the human vascular system then this presupposes the anatomical layout of circulation is understood and the existence of an intravascular pressure. As will be mentioned
capillaries cannot be seen by the naked eye and although their existence can be deduced, confirmation of this requires the development of a microscope. This infers techniques for lens production have been developed and certain design criteria have been formulated.

A factor of prime importance which must be considered is that of interdisciplinary communication and what now is readily accepted communication between workers in the same field. In the period of interest communication relied on published papers and books with the emphasis on personal communication by letter and meeting. Time scales between published work and initial conception of an idea were long as were the times for the general acceptance of a new idea.

The aim of this section is to help one clarify the factors upon which scientific thought lay, with a view to further understanding of problems underlying present day achievements.

Through history man has been interested in himself both spiritually and physically. It is CLAUDIUS GALEN, an Hellenistic physician born in Paryanum in Asia Minor AD 129 to whom many of the middle century physicians referred when dealing with physiological and anatomical matters. Galen produced many written books dealing with the fact that arteries carry blood not air, the function of glands; he was one of the first to consider the brain as the site of knowledge and he used practical dissection as a basis for conclusions.

Up to 1555 Galen terminology was used widely in Europe when describing physiological or anatomical details. In that year Andrea Vesalius, a Flemish Anatomist and Physiologist born 1514, published his works on Human Anatomy. He used Galenic techniques (of dissecting and exploring) but overthrew Galenic anatomy which had been based on non-human materials. His description of the vascular system however
omitted capillaries, which he could not see with the naked eye.

At this time measurement had not progressed much from ancient times, observations were not adequately quantified. And for some years progress in the physical fields predominated, which as will be seen, enabled further discoveries concerning the human form to be made.

In 1586 GALLIEL GALILEO born in 1564 was the first to consider practical relationships between weight and volume and produced a hydrostatic balance to determine the specific gravity of solids. In 1590 he demonstrated that objects fall at the same rate independant of their weight.

Along different lines in 1590 ZACHARIUS JANSEN, a Dutch optician, whilst working with his father, trying to construct a telescope, accidentally produced the first compound microscope which was 6 feet long by 1" dia.; Galileo himself in 1610 produced a similar microscope, his innovation was the inclusion into the system of a screw threaded barrel and mount, allowing focussing to be achieved. Around this time knowledge of basic units of measurement was growing and the compound microscope became more prevalent.

In 16oo HILDANUS FABRICUS described the structure of venous valves, this led directly to WILLIAM HARVEY'S concept of the circulation of blood in 1628, for as he described to ROBERT BOYLE on how he came to consider blood circulation he answered "The valves in the veins of so many parts of the body were so placed that they give free passage of blood towards the heart but opposed the passage of venal blood in the contrary way."

The circulation of blood was described in detail by Harvey and he deduced the presence of capillaries but although it was noticed
that blood would spurt from a pierced artery the concept of fluid pressure alluded physicians.

The fluid pressure story took a major step forward in 1648 when HORIUS PERIEN performed experiments to determine Barometric Pressures in various localities and altitudes near Clermont in France. They were directed by BLAISE PASCAL who in 1649 proposed the critical dictum that a fluid will transmit a pressure, the same in all directions.

When in 1660 ROBERT HOOKE was investigating capillary action of fluids in tubes, MARCELLO MALPHIGI discovered the capillary bed in the lungs where oxygen uptake occurred. Later, the systemic capillary systems were described. For such an observation a microscope would have been available, one derived from the early type, probably Galillean in descent as Malphigi also worked in Pisa. Thus the circulation was fully described. Measurement required NEWTON in 1688 to make public his three Principia, the laws of motion, which gave an understanding between force, mass and gravitational acceleration. Manometer systems were developed, which led Stephen Hales 1733 to use an inverted U-tube inserted into a horse's femoral artery to view the height to which the blood rose in it. He had observed the use of U-tube manometry in the measurement of pressure of sap in plants, and was also aware of the use of mercury as a fluid in a manometer, but a century elapsed before POISELLE (1828) used a mercury filled U-tube to measure the pressure of dogs femoral artery.

Thus 1733 marked the true beginnings of the measurement of blood pressure. The steps leading to this event are numerous and their interrelationships complex. If a conclusion can be drawn from the described developments it is that three factors help to advance knowledge of ourselves and the universe; the existence of a few men of vision, many men of stolid perseverance and finally a good communication system.
1.2.1. FLUID FILLED MANOMETRIC SYSTEMS

By far the most commonly used systems for displaying intravascular pressures are those which employ a fluid filled catheter which links the vessel (by cannulation) to a form of fluid manometer. The height of the fluid column supported thus being a measure of the mean intravascular pressure.

1.2.2. FLUID COUPLED SYSTEMS

The manometric method just outlined uses a fluid coupling that is a fluid filled catheter which by cannulation connects the pressure system of interest to the pressure sensing element. The use of fluid filled catheters as a means of transmitting the pressure of interest to other forms of the pressure transducers, is now widely used. The fluid coupling does though modify the performance of the pressure transducer, different lengths and diameters of catheter affecting the performance of the systems. (Chapter 2.2.3.)

1.3.0. PRESSURE TRANSDUCING SYSTEMS

The term transducer is applied to systems which transform the physical variable of interest into another type, usually accomplished through the application of more than one physical process, which is then used to quantify the original. The manometric system already mentioned is one of the simplest examples of a transducing system. A sensor is the generic term for a system or system element which can detect a physical variable. Thus a transducer is a sub-group of the main group sensor. And it should be noted that a fluid manometric system is one of the few sensors that is also a transducer.

All the pressure transducers to be briefly described here use a diaphragm as the pressure sensing element. The fluid pressure to be applied to one side of the diaphragm (the other usually subject to atmospheric pressure). The diaphragm will deflect away from the greatest
pressure. For a circular diaphragm which is edge clamped, the maximum deflection will occur at its centre. It is only for small deflections that there is a linear relationship between deflection and pressure. Non-linearity becoming significant when the deflection approaches half the diaphragm thickness.

It is the method whereby the deflection of the pressure sensing diaphragm is measured that gives its name to the type of overall pressure transducer. With one exception namely "Optical Transducers". Systems whereby the pressure signal is transduced into an electrical analogue will be discussed.

1.3.1. RESISTIVE

The most commonly used pressure transducer after manometric systems are of the strain gauge type. They rely upon the principle that a small change in the length of an electrical conductor produces a small change in its electrical resistance. The magnitude of the deflection of the pressure sensing diaphragm is achieved by fixing several strain gauges in the form of a Wheatstone bridge to the diaphragm and determining the change of resistance of the gauge elements. Piezo-resistive gauges are a recent advance of strain gauges. The material used is a semi-conductor, which is far more sensitive than metal strain gauges.

1.3.2. INDUCTIVE

Inductive systems are seldom used. The mode in which they function is for the diaphragm to change the inductance of a coil, or the inductive coupling between two coils. This necessitates the mechanical fixing of a coil or a core, to the diaphragm with the detrimental effect of increasing the effective mass of the diaphragm.

1.3.3. PIEZO-ELECTRIC

If crystals such as Rochelle salt or quartz are mechanically
stressed a charge appears across the crystal which can be recorded. The magnitude of the charge is low and its D.C. capabilities are poor. However its high frequency response is excellent.

1.3.4. CAPACITIVE

This system is the first mentioned to involve the use of a non-contact method for measuring the deflection of the diaphragm. The diaphragm is allowed to form the moving "plate" of a parallel plate capacitor; deflections of the diaphragm thus changing the capacitance of the system.

1.3.5. OPTICAL

Optical methods themselves are not usually governed as the previous methods discussed by physical phenomena, but use light in a quantitative manner or spacial manner to gauge the position of the diaphragm, a photoelectric detector carrying out the final phase of the transducing, in the quantitative mode detector, and optical levers and scale for spacial mode detector.

1.3.6. CATHETER TIP SYSTEMS

All the mentioned types of pressure transducers can be used clinically by connecting to the transducer a fluid filled catheter and then introducing its other end into the vessel of interest (fluid coupling). The limitations of such a system are discussed later, suffice to say that a pressure transducer with its pressure sensing element in the actual vessel is an extremely desirable feature. Those that do are termed "catheter tip". This precludes a large diaphragm and transducing element which make the design of the system more complex.

1.4.0. BASIC DESIGN CRITERIA FOR AN INTRAVASCULAR PRESSURE TRANSDUCER

A full discussion of the physiological and instrumentation requirements for an intravascular pressure transducer follows in the next
section (2.0.0.). This critique upon which the design is based are concerned with the full understanding of the factors which effect and specify the operation of and limitations of an intravascular pressure transducer or indeed any unit of instrumentation.

The first and most important is the concept of accuracy, for if measurements are to be made and used confidently a measure of the system's accuracy must be known. In very general terms the accuracy of a system is dependant on two features of the system. These are the "directness" by which the variable of interest is related to the measured variable and the system's "uniqueness" which means how likely is the measured variable effected by an external variable. As a sub-group of the latter is the system's non-reactance, that is, does the act of measurement itself change the variable?

Having generally explained the features of a transducing system which effect its accuracy, a quantitive specification of accuracy will enable the performance of the system to be evaluated on paper; these are:-

1. The system's stability. How much does the measured variable change with respect to time, compared to the variable of interest?

2. The system's sensitivity. How well does the system detect small changes in the value of the variable of interest?

3. The system's range. What will be the maximum and minimum values of the variable of interest which can be measured?

4. The system's frequency response. This is really a measure of the sampling rate possible which results in no distortion of the variable interest.

The other factors which specify the system's limitations such as utility, cost, etc., will be discussed later (2.0.1.).
1.4.1. UNITS

The size of a roll of cloth was, over 800 years ago, measured, albeit indirectly, against the length of King Henry I's arm. The year 1791 witnessed the height of a French horse to be stated as a function of the distance from Dunkirk to Barcelona. The historical development of different methods of measurement has resulted in a wide variety of units. The measurement of physiological pressure producing the unit the mm Hg (millimetre of mercury), was the result of manometric systems extensively employing mercury.

In recent years agreement has been internationally reached on a set of favoured units (S.I.). In this system the pressure unit is the Pascal, however throughout this treatise physiological pressures will be quoted in mm Hg. All other units being C.G.S. However it is worth noting that 1 mm Hg approximately equals 133 Pascals.
SECTION TWO

INTRAVASCULAR PRESSURE TRANSDUCING SYSTEMS
2.0.0. REQUIREMENTS FOR AN INTRAVASCULAR PRESSURE TRANSDUCING SYSTEM

In the measurement of physiological pressures, ideally a record is required which conforms identically to the value of the pressure to be determined. In practice this is impossible to achieve because of errors arising from the measurement. There are two types of errors, static and dynamic, and it is the combined effect of these errors which result in the overall "accuracy" of the system.

Static errors involve the ability of the system to record fixed or very slowly oscillating pressures. The system's stability being a measure of its static error features.

The static accuracy of the system can be considered to be described by the term stability. When recording very slowly varying pressures, the base line, that is the transducer output for zero pressure must not change, or if it does, by how much must be known. Stability is also effected by the stem gain which will also effect the base line value. The major cause of base line in-stability is the effect of temperature. Stability or drift is quoted in mm Hg per hour while temperature sensitivity (or drift) in mm Hg/degree °C. The dynamic error in the distortion of the real time pressure wave-form is quantified by the system noise level and frequency response. The noise inherent in a system are the signals that although time varying are not related to the signal of interest. If they are of the same frequency then it is extremely difficult to remove them from the output signal - FRY, D.L. NOBLE, F.W. and MALLOS, A J (1957). Noise of significance arises from electronic components within or external to the system, but mechanically induced noise and thermally caused noise will also be present, the relative importance of each being dependant on the system in question.
The effect of noise is to cause a distortion in the amplitude of the output waveform, consequently the system noise must be reduced to its minimum level.

The frequency response relates the system's accuracy in reproducing signals of pressure which are time dependant and its determination can be achieved both by theoretical means and practical. Both methods investigate the transfer function between the input and output waveform amplitudes and phase relationships. Phase distortion being the delay between in and out signals which if of a constant shift for all frequencies is of no consequence.

The frequency response of a pressure transducer can be ascertained theoretically, if all the component specification is known, although it is most conveniently determined experimentally in the following manner. Sinusoidal varying frequencies of fixed amplitude simulate the input signal to the system. The resulting values of signal amplitude and corresponding frequency, form the frequency response curve from which a range of frequencies over which the signal amplitude remains constant is chosen. LATIMER (1969) gives an excellent account of a practical system investigation.

The physiological features which specify the system requirements must now be discussed. In the vascular system the pressures range from -5 to 300 mm Hg; in the arterial side the mean pressure is 100 mm Hg, with an oscillatory component of say 30 mm Hg. The pressure in the superficial veins in the supine position ranges from 5 to 15 mm Hg. The peak pressure of heart sounds in the ascending Aorta is about 10 mm Hg. The required frequency response of the system is probably the most difficult to confidently state. If only mean pressures are required then the frequency response can
be low. However for analysis of the derivative of the pressure waveform the noise at high frequencies will become large on differentiation and thus the system response should be high. Its value is then dependant on the number of harmonics of the fundamental thought to be significant.

The number of harmonics necessary to truthfully record the pressure waveform has been quoted by many workers - MCDONALD, D A (1960) shows that 95% or better of the variance of a number of pulse waves can be represented by 5 harmonics whilst VAN BRUMMELEN (1961) suggests 60. The rationale really is, that the better the response, the more likely it is that one can determine from the information received in the way of pressure waveforms, whether additional significant information can be obtained.

In specifying the requirements of a physiological pressure transducer, the specific purpose to which the pressure measurements are made ought to be specified as the prerequisite factor in the decision. The following specification is based on the resolve that if a system specification is such that it meets its most arduous requirements then it will also be suitable for a less demanding task, even though some form of system modification (external) might be needed.

The system requirements are:

- **Range**: -5 to 300 mm Hg.
- **Frequency response**: 0 to 1 KHZ
- **Temp. sens**: 1.0 mm/°C - ARTERIAL  0.1 mm/°C - VENOUS
- **Hysterisis**: ± 1% F.S.
- **Sensitivity**: .2–30 mV/100 mm Hg
- **Vol displacement**: .001 mm³/100 mm Hg see chapter (2.3.0.)
- **Non-traumatic**
- **Sterilizable**: Gas or Fluid Dip
2.0.1. CRITERIA FOR THE SPECIFICATION OF AN INTRAVASCULAR PRESSURE TRANSDUCER

There is no best method for the measurement of physiological pressures, for each pressure determination and its requirements might lead to a different method being chosen and this would infer a compromise needs to be made between methods. The reasons or criteria upon which selection depends are:

- **Accuracy** (this has been discussed earlier)
- **Utility** - an all embracing term for the practical suitability of the method. Utility is concerned with
  - **Ease of use** - ease of moving apparatus,
  - **Noise it produces**;
  - **Cost**;
  - **Availability** and finally the safety aspects of the method.

It is the aspect of safety that should be of primary concern for in recent years concern has been directed towards instrument design for safety in use in a hospital environment HEIB G E and GREEN (1971), SOVIE M D and FRUERMAN C T (1972) and also an editorial leader in Journal of Health Devices (1973).

2.1.0. TYPES OF PRESSURE TRANSUDING SYSTEMS

Pressure transducing systems employing diaphragms as the pressure sensor have been outlined. The purpose of the following is to expand on the theoretical basis and practical limitations of these systems, to allow an overall appraisal of the present expertise in this field to be made.
2.1.1. RESISTIVE

The effect of strain on the electrical properties of a metal not reported until TOMLINSON (1876) showed that when a wire was stretched its resistance changed. The relationship between \( R, L, A \) and \( P \) being:

\[
R = \frac{P \cdot L}{A}
\]

where \( R \) = resistance of wire
\( L \) = length of wire
\( P \) = resistivity
\( A \) = cross sectional area

then

\[
\frac{\Delta R}{R} = \frac{\Delta L}{L} + 2Z \frac{\Delta L}{L} + \frac{\Delta P}{P}
\]

where \( Z \) = poissons ratio

which has been shewn for a number of alloys (nickel and copper) is linear. therefore

\[
\frac{\Delta R}{R} = K \frac{\Delta L}{L}
\]

the value of \( K \) most commonly being, for a metal, 2. This is called the strain gauge factor.

A strain gauge, that is a gauge for sensing a change in dimension was first introduced in America in 1939 by RUGE and SIMMONDS then patented by Simmonds in 1942. Since that time they have been used widely and have evolved into 4 classes:- bonded wire; bonded foil; unbonded wire and semi-conductor devices. It is the last mentioned group that have been only recently available. They employ a semi-conductor (usually silicon) which has a gauge factor of 140, a factor of 70 times better than metal gauges. Temperature changes do however effect the gauge factor which is not the case for metal gauges. The use of balanced bridge systems will allow the compensation of temperature to be effected.

The construction and mounting of the gauge onto the pressure sensing diaphragm is difficult FORSE (1965). Recent methods of integrated
circuit techniques have provided an alternative method of construction. The pressure sensing diaphragm can itself be made from silicon in the form of a circular disc. Silicon dioxide crystals are then grown onto the disc prior to then being etched off in the areas where the final gauge is to be located. After the etching process the disc is exposed to the vapour of a suitable dopant. The "unit diaphragm" is then complete. A good account of semi-conductor gauges is given by MASON W P (1957).

To summarise, strain gauges which are not of the catheter tip type have been used widely in physiological pressure measurement - LAMBERT & WOOD (1947), BIERMANN & JENKINS (1951). The more recent types of construction perform well and can be miniaturised in the form of catheter tip pressure transducers - ANGELAKOS E T (1964), WARNICK (1958), MILLAR (1972). The following TABLE 1 shows the specification for those that are commercially available. The only severe criticism that can be made about them is that they are inherently an electrical hazard and rather fragile.

2.1.2. INDUCTIVE

The inductance of a coil of wire is given by the following relationship

$$L = \frac{n^2 G \mu_0}{\mu}$$

where \(n\) is the number of turns, \(G\) is a geometric form factor, and \(\mu_0\) is the permeability of the media. All these terms can be modified mechanically, thus the deflection of the diaphragm can be utilised to give, in several ways a change in the inductance of a coil. Change in inductance can be measured relatively accurately and simply, but will be effected by external magnetic fields.

There are three types of systems - self inductance (single coil), mutual inductance (twin coil), differential transformer (twin coil). The first and last types use a ferrite coil for inductive coupling, which is connected to the diaphragm.
### "BELL & HOWELL" (EXTERNAL TYPE)

<table>
<thead>
<tr>
<th>EXTERNAL DIAMETER</th>
<th>PRESSURE RANGE</th>
<th>OVER PRESSURE</th>
<th>SENSITIVITY</th>
<th>HYSTERESIS &amp; LINEARITY</th>
<th>THERMAL STABILITY</th>
<th>NATURAL FREQUENCY</th>
<th>VOLUME DISPLACEMENT</th>
</tr>
</thead>
<tbody>
<tr>
<td>mm.</td>
<td>mm.Hg.</td>
<td>mm.Hg.</td>
<td>mV/100 mm.Hg.</td>
<td>% F.S.</td>
<td>mm.Hg/°C</td>
<td>Hz.</td>
<td>mm³/100 mm.Hg</td>
</tr>
<tr>
<td>36</td>
<td>± 300</td>
<td>1500</td>
<td>2</td>
<td>± 1 *</td>
<td>0.1</td>
<td>100 **</td>
<td>0.0125</td>
</tr>
</tbody>
</table>

### "MILLAR" (CATHETER TIP TYPE)

<table>
<thead>
<tr>
<th>EXTERNAL DIAMETER</th>
<th>PRESSURE RANGE</th>
<th>OVER PRESSURE</th>
<th>SENSITIVITY</th>
<th>HYSTERESIS &amp; LINEARITY</th>
<th>THERMAL STABILITY</th>
<th>NATURAL FREQUENCY</th>
<th>VOLUME DISPLACEMENT</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.65</td>
<td>± 300</td>
<td>5000</td>
<td>30</td>
<td>± 0.5</td>
<td>0.1</td>
<td>35 . 10³</td>
<td>0.001</td>
</tr>
</tbody>
</table>

*Over a pressure range of -50 to 300 mm.Hg.

** Connected to a stainless steel catheter , .5 mm bore, 150 mm long

---

**TABLE 1**

**SPECIFICATIONS OF TWO COMMERCIALLY AVAILABLE SEMI-CONDUCTOR PHYSIOLOGICAL PRESSURE TRANSUDCERS**
Differential transformer: probably the most successfully applied variable induction system. The construction features are described NEUBERT (1963). They can be miniaturised and consequently may be used as a catheter tip device GAUER & GIENAPP (1950). More recently PIEPER & PAUL (1968) described a flow and pressure transducer in one 3 mm diameter catheter, frequency response being of the order of 1 kHz.

Mutual Inductance: VAN CITTERS (1966) gives a good description of this type and states that the main limitations are the moving coil must only move in the mutual axis otherwise significant error will be experienced and the total movement must be small for the system to operate in a fairly linear range. Practical systems have been documented by MOTLEY (1947).

Self Inductance: although this method has been used a number of times LAURENS (1959) and ALLARD (1962), it does suffer from the problems of non-linearity.

2.1.3. PIEZO-ELECTRIC

The charge produced across a crystal when stressed can be measured by using a charge amplifier which converts the charge into a voltage. Problems still arise due to charge leaking from the crystal, however the system's frequency response is excellent. This system differs from the previous systems in as much as it is an active system, that is, when a force is applied to such a crystal a charge is created across the crystal, consequently the deformations resulting need only be of the order of a few thousandths of a millimetre for full load, in other words it acts as a very stiff membrane detector yet gives a high sensitivity.
2.1.4. CAPACITIVE

By using a stiff metal diaphragm as the pressure sensor and incorporating within the unit and close to the diaphragm a fixed disc (metallic), a variable capacitance system is formed, where for relatively small changes in plate separation a large change in capacitance is achieved because the plate separation (initial) is set to be small. With small separations though, capacitance is of the order of pico Farads, consequently high excitation frequencies must be used (100 KHz).

A satisfactory circuit for quantifying the capacitance changes is to make it part of the resonant circuit of an oscillator, changes in its value causing a change in oscillator frequency (i.e. based on frequency mod. of carrier wave). The frequency modulated signal is then converted into a proportional d.c. signal, HAINES J (1960).

The relationship between the separation of two plates and its capacitance is of an hyperbolic nature

i.e. \( C = \frac{K}{d} \)

where \( K \) is composed of dielectric constants, and the area of the plates. However if \( d \) is small and its change is small, linearity is approximated.

i.e. \( C + \Delta C = \frac{K}{d + \Delta d} \)

\[ \Delta C = \frac{K}{d + \Delta d} - \frac{K}{d} = K \left( \frac{1 - \Delta d}{d + \Delta d} \right) = K \left( \frac{1 - \Delta d}{d} \right) \left( \frac{1}{1 + \frac{\Delta d}{d}} \right) \]
if \( \frac{\Delta d}{d} \) is small

\[
\Delta C = \frac{K}{d}
\]

The only criticism of this technique is the effect of stray capacitance on the system. In its favour are its inherent high frequency (low mass, high stiffness diaphragm) and small volume displacement.

In the manufacturing stage, by changing the initial separation between diaphragm and plate the sensitivity of the system can be changed. The constructional details of a capacitive system as outlined have been described by HANSEN (1949), PRESSEY (1953) also discussed temperature compensation methods; applications and modifications have been many and varied - LILLY (1942); LILLY, LEGALLIS & CHERRY (1947); BUCHTAL & WARBURG (1943); BAXTER (1951).

Recently FROBENIUS (1973), described a catheter tip transducer and shows that it has an equivalent gauge factor 10 times greater than the best piezo-resistive type, with a linear response from 0 - 300 mm Hg.

2.1.5. OPTICAL

The term optical when applied to pressure transducers is very ambiguous. In its widest sense all transducers may be termed optical as the final interpretation of the system output is usually accomplished visually i.e. meter readout. A more rigorous explanation of the term is to consider that by an optical means the pressure signal is modified allowing an optical signal to then be related to the pressure signal. The method by which the optical signal can be used and how it is modified by the pressure signal can be achieved in different ways.

There are two methods whereby the optical signal can be used; spacially or quantitively. The spacial use of light is an opto-mechanical
system where the light is used as a light lever to magnify a rotational or positional change of a mirror fixed to the pressure sensing diaphragm - FRANK (1924), HAMILTON (1934). In the quantitative method the energy content of the light is converted into an electrical analogue by using a photo-electric device, in other words a photo-electric system. For such a photo-electric system the manner in which the pressure signal is made to modulate the light is a critical feature. Both systems, that is spatial or quantitative employ a pressure sensing diaphragm which in deflecting causes the following mode to be either spatial or quantitative. It is the quantitative systems that have found more current acceptance due to the inherent difficulties of opto-mechanical systems such as optical levers needing to be of the order of metres in length, and precluding the construction of catheter tip transducers. Photo-electrical method (quantitative system) also enables the pressure sensing diaphragm to be isolated from the rest of the system, that is, not be in physical contact with any other part of the system and so allowing the diaphragm to be of a minimum mass. The ways in which the diaphragm deflection can be made to modulate a light signal are numerous - REIN H (1940), MUELLER (1954), GREER (1964), but are all by nature of their design fluid coupled systems. All catheter tip systems reported - DE LA CROIX (1966), LINDSTROEM (1969/70), BRAND P J (1972/3), MORIKAWA (1972), utilise optical fibres to transmit light (see chapter 3.0.2.) and return light from a catheter tip diaphragm. The modulation mode and efficiency being a very complex function of the optical system parameters. However the overall frequency response of the system will be high. Also from the electrical safety standpoint they are excellent.
2.2.0. FLUID COUPLED SYSTEMS

Because of their size all the forementioned systems of pressure transduction may, in a clinical situation, have to be used externally. This is achieved by using a fluid filled tube to transmit the vessel pressure to the pressure transducer. The total system then incorporates a column of fluid which modifies the performance capabilities of the transducer. The theoretical and practical explanations of the degradation fluid coupling causes will be discussed.

2.2.1. UNDAMPED NATURAL FREQUENCY

A simplified catheter/diaphragm system is shown FIG 4. The diaphragm is shown to be a piston moving under the action of the fluid pressure $P$ in a friction free fashion and it is weightless. The stiffness of the diaphragm is represented by a spring of stiffness $K$. $M$ represents the mass of fluid in movement and $A$ is the cross-sectional area of the diaphragm.

For this initial appraisal it is assumed that the system is free of any viscous force, frictional resistance force, the fluid is incompressible, and the tube is rigid.

Then the dynamic response of the system can be written as:

$$MD^2 \dot{x} + Kx = F(t) \ldots \ldots \ldots \ldots \ldots \ldots \ldots 1$$

where $D$ is a time operator and $F$ is the disturbing force which may or may not be time dependant. For this case with no viscous forces the response of the system to a step input will be discussed:

The differential equation describing movement becomes

$$MD^2 \dot{x} + Kx = F$$

which on taking Laplace transforms and substituting initial conditions leads to

$$\ddot{x} = \frac{F}{3(Ms^2 + K)}$$
on taking reverse transforms gives

\[ x = \frac{Kp}{K} \left( 1 - \cos \sqrt{\frac{K}{M}} t \right) \]

This shows that the diaphragm would oscillate with a natural undamped frequency of \( w_n \).

\[ w_n = \sqrt{\frac{K}{M}} \]

and will do so continuously. In any practical system frictional forces will cause the output value to stabilise at the static value of 'x' to be expected.

Let a more realistic model of the system include a viscous resistance term:- FIG 5

The differential equation describing motion becomes:-

\[ MD^2 x + C D x + K x = F(t) \]

Two cases will be investigated; when the forcing function is a step and when it is of a sinusoidal nature.

**STEP INPUT**

The input signal is \( F \)

\[ \therefore \quad M D^2 + C D x + K x = F \]

the transient response gives

\[ x = \frac{F}{K} \left[ 1 - \frac{\xi w_n t}{1 - \xi^2} \right] \sin \left[ w_n \left( 1 - \xi^2 \right) t - \phi \right] \quad \ldots \ldots . \ 2 \]

where \( \xi = \frac{C}{2w_n M} \)

and is termed the damping ratio and the damping frequency of the system is \( w_d = w_n \sqrt{1 - \xi^2} \)

The following FIGS 6 & 7 show the manner in which by changing the value of the damping ratio, the response to a step input is varied. The critical features of the response curves are the value of the peak overshoot, and the time taken to reach a certain fraction of the final value.
FIG. 5 SECOND EQUIVALENT PISTON
FIG. 6
RESPONSE TO STEP INPUT
SECOND ORDER SYSTEM
FIG 7  RESPONSE TO STEP INPUT
SECOND ORDER SYSTEM
1/2  FRACTION UNDAMPED PERIOD TIME  1/2
The overshoot can be found upon differentiation of equation 2. Then equating it to zero. The first overshoot is the maximum value and occurs at a time

\[ t_{\text{max}} = \frac{\pi}{w_d} \]

\[ = \frac{\pi}{w_n \sqrt{1 - 3^2}} \]

and has a value as a % of the step input value of

\[ \frac{\xi}{100} \left( 1 - 3^2 \right)^{\pi} \]

The following TABLE 2 shows the various values of \( \xi \) and the corresponding overshoot resulting.

Thus by varying \( K, M & C \) the response of the system can be altered so that a different damping line is followed. Values for a damping ratio of .3 - .7 give tolerable overshoot with fast response, the choice of \( \xi \) does depend on the finite value of the undamped frequency. For our purposes the limiting value of \( \xi \) is unity for at this value there is no overshoot and there is no oscillatory motion, however the time to reach its acceptable level, is long.

With a damping factor of about .7 the first overshoot is about 0.5% for a step input, and a maximum range of uniform response to a sinewave input is achieved (up to .5 of the undamped natural frequency of the system). Also the phase delay resulting is linear with increasing frequency. Now if the undamped natural frequency is large compared to the maximum value of the frequency to be investigated then it is possible that a system with a low value for damping could be used. This would produce at low frequencies a small and linear phase delay with a small amplitude error and for steady state response, although the value of the first overshoot will be large, the time to reach the steady state condition will be small.
<table>
<thead>
<tr>
<th>DAMPING FACTOR</th>
<th>0.1</th>
<th>0.2</th>
<th>0.3</th>
<th>0.4</th>
<th>0.5</th>
<th>0.6</th>
<th>0.7</th>
<th>0.8</th>
<th>0.9</th>
<th>1.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>% OVERSHOOT</td>
<td>73.0</td>
<td>52.6</td>
<td>37.2</td>
<td>25.4</td>
<td>16.3</td>
<td>9.5</td>
<td>4.6</td>
<td>1.5</td>
<td>0.15</td>
<td>0.00</td>
</tr>
</tbody>
</table>

**TABLE 2**

VALUES OF DAMPING FACTOR AND RESULTING OVERSHOOT FOR A SECOND ORDER SYSTEM
The response to a sinusoidal input will now be investigated.

The characteristic equation is

\[ M \ddot{x} + c \dot{x} + kx = F \sin \omega t \]

where for steady state conditions

\[
x = \frac{F}{K} \frac{\sin(\omega t - \phi)}{(1 - \left(\frac{\omega}{\omega_n}\right)^2 + 4 \left(\frac{\omega}{\omega_n}\right)^2)^{-1}}
\]

\[
t \omega_n^{-1} \phi = -2 \frac{\omega}{\omega_n} \mathcal{E}
\]

the magnification factor is

\[
X = \frac{0/P}{I/P} = \frac{1}{(1 - \left(\frac{\omega}{\omega_n}\right)^2 + 4 \left(\frac{\omega}{\omega_n}\right)^2)}
\]

that is, 'X' is the value of a factor by which the zero frequency displacement must be multiplied to give the output magnitude.

It should be noted that 'w' is the forcing function frequency and \( \omega_n \) the undamped natural frequency. The following graphs of (FIGS. 8 & 9) amplitude magnification and phase angle relationships for different damping ratios at varying \( w/\omega_n \) ratios show that a resonance condition occurs at \( w/\omega_n = 1 \) with a 90 degree phase shift between input and output. The aim would be to have a '\( \omega_n \)' as high as possible and an '\( \mathcal{E} \)' value which both satisfied the transient response requirements and also provides for an acceptable frequency range having minimal magnification factor.

Before using these results to predict the effect of using a fluid filled catheter, the effect of second order errors in the simplified analysis will be discussed.
FIG. 9
PHASE ANGLE RELATIONSHIP / SECOND ORDER SYSTEM
2.2.2. ASSUMPTIONS IN THE MODEL

The investigations performed assumed that the fluid was compressible, and that the catheter was rigid, and of the same diameter as the diaphragm. These assumptions are obviously not consistent with most practical systems, and their effects will be discussed. Within the realm of the model specified there is one effect not so far mentioned, namely the increase in actual system fluid mass caused by the diaphragm deflecting. It will be seen, however, that the increase in mass caused, is negligible, compared to the effect of the mass within the catheter SHITER (1962). This is because the fluid within the catheter is in motion and assumed to have a parabolic velocity distribution across the catheter diameter, as predicted by the Poiselle-Hagenbach equation. From which the fluids dynamic inertance can be found (so called equivalent mass) (see 2.2.3) and turns out to be for practical systems of catheter/transducer systems much greater than its actual mass. Similarly the viscous resistance force 'C' is deduced from the parabolic velocity distribution assumption, and it also becomes higher than would at first be thought. The assumption of a poiselle flow is made in the further analysis performed here. However, WOMERSLEY (1957) predicts that for a rigid catheter and incompressible fluid that the values of fluid inertance and viscous resistance themselves are also frequency dependent, this is because at high frequency oscillations of the fluid will tend it to move within the catheter in an en bloc manner (slug-like), with shearing forces at the periphery of the catheter bore.

Womersley's solution of the linearized Navier Stokes equation leads to the conclusion that for rigid catheters that increasing frequency causes an increase in viscous resistance and decrease in fluid inertance - NOORDERGRAAF (1970).
The effects mentioned and also of catheter compliance and fluid compressibility are discussed by Latimer K E (1968); the analysis performed are complex and aim to enable a method of impedance matching to be achieved, in order to increase the frequency range of the catheter/transducer system. The analysis is thorough but finally because it is difficult to obtain values for tube compliance and take account of the variability of fluid compressibility it is advised that a practical evaluation is made. Other workers FRY (1960), SHIRER (1962) and YANOF (1963) have approached the problems of degradation caused by fluid coupling, Shirer for example omits the \( t^{4/3} \) term in the expression for fluid inertance and makes no comment on the omission. However it must be pointed out that all authors mentioned do show that, in the most favourable conditions fluid coupling is only satisfactory, if a low frequency response is required.

Using the assumptions of rigid catheter, incompressible fluid and independence of frequency on the skin effect parameters (inertance and viscous resistance), the length/diameter relationship for catheters connected to different volume displacement transducers will be shown, assuming a damping factor of 0.7.

2.2.3. PREDICTED PERFORMANCE OF CATHETER SYSTEMS

For the purpose of further analysis here of the dynamic response of a fluid filled catheter system, it is assumed that the catheter is rigid and the fluid is incompressible. The term \( M \) as introduced was simply the mass of the fluid in the system, actually it is a lumped term for the mass of the diaphragm \( M_D \), the mass equivalent of the fluid within the transducer (inertance) \( M_{INT} \), and the mass equivalent of the fluid within the catheter 'inertance' \( M_{INC} \), see FIG 10.
The inertance of a fluid is its dynamic impedance and is best assessed by kinetic energy considerations, the following theoretical determination of fluid inertance makes the assumption that the velocity distribution in the system is parabolic, that is Poiselles equation holds (sometimes termed skin effect).

Let \( V_o \) be the maximum velocity within the system then the velocity at of a particle at any radius \( r \) is

\[ V_r = V_o \left( 1 - \left( \frac{r}{R} \right)^2 \right) \]

the Kinetic energy in a catheter \( L_c \) becomes

\[ KE_{LT} = \frac{1}{2} L_c \int_0^R 2\pi r V_o^2 \left( 1 - \left( \frac{r}{R} \right)^2 \right)^2 dr \]

\[ = \frac{A_L C_v^2}{3} \left( L_c \right) \quad \text{.................. 2a} \]

now for continuity the volume flow rates of fluid and diaphragm displacements will be equal

i.e. with 'x' referred to diaphragm displacement

\[ A_D \frac{dx}{dt} = 2\pi \int_0^R r V_o \left( 1 - \left( \frac{r}{R} \right)^2 \right) dr \]

\[ = \frac{1}{2} V_o A_C \]

\[ \therefore V_o = \frac{A_D}{A_C} \frac{dx}{dt} \quad 2 \]

substitute into 2a for \( V_o \)

\[ KE = \frac{1}{2} \frac{A_L C_v^2}{3} \quad 4 \left( \frac{A_D}{A_C} \right)^2 \left( \frac{dx}{dt} \right)^2 \]

i.e. \( M_{INC} V_o^2 = \frac{1}{2} \frac{4}{3} \left( \frac{A_D}{A_C} \right)^2 M_c V^2 \)

or \( M_{INC} = \frac{4}{3} \left( \frac{A_D}{A_C} \right)^2 M_c \)

Thus \( M_{INC \ \text{CATHER}} = \frac{4}{3} \left( \frac{A_D}{A_C} \right)^2 M_c \)

similarly for the fluid within the transducer noting that \( \frac{dx}{dt} \) is
already referred to diaphragm displacement:

\[ M_{\text{IN TRANS}} = \frac{4}{3} M_T \]

where 'A' is area of piston

and 'a' is area of catheter

and 'M_c' is actual mass in catheter

and 'M_T' is actual mass in transducer

It can be seen at this stage that an increase in diameter of the catheter (perhaps caused by catheter compliance) causes an increase in mass, but the effect on inertance, is to reduce it!

Thus \[ M = M_D + \frac{4}{3} (\frac{A}{a})^2 M_c + \frac{4}{3} M_T \]

the term \( M_D \left( \frac{4}{3} (\frac{A}{a})^2 M_c \right) \) and will be neglected

As is also the term \( \frac{4}{3} M_T \) or the inertance of fluid in the transducer, as this is small compared to the \( M_{\text{INC}} \) term as \( (\frac{A}{a})^2 M_c \) for practical systems

Finally then

\[ M = \frac{4}{3} (\frac{A}{a})^2 M_c \]

this value for \( M \) will be used the descriptions for 'w' & 'E' as formulated earlier.

Now for a circular diaphragm which has been replaced by an equivalent piston and with ref to FIG 10

\[ \Delta V = A \Delta x \] where 'x' is displacement of diaphragm

Now volume displacement is defined as change in volume per unit pressure

\[ V_d = \frac{\Delta V}{\Delta P} \text{ and } \Delta P = \frac{Kx}{A} \]

\[ K = \frac{A^2}{V_d} \]

Thus the original differential equation becomes

\[ M D^2 x + CD + Kx = F(t) \]

\[ \therefore f_{UN} = \frac{1}{2} \sqrt{\frac{K}{M}} \]
\[ Z = \frac{2\sqrt{K}}{\pi} \]  

where \( C \) is specified in terms of \( A \) & \( L_c \) & \( d \); and is also a second
skin effect dependant - WOMERSLEY (1955)

from \( C \frac{dx}{dt} = \Delta P \cdot A \)  

and \( \phi \) from poiselle

\[ \frac{d}{8\mu L_C} \Delta P \]  

\( \mu \) is viscosity  

also \( \phi = a \cdot \bar{V} \) mean volume

remembering 'x' is referred to piston

\[ \bar{V} = \frac{A}{a} \frac{dx}{dt} \]  

\[ \phi = A \frac{dx}{dt} \]  

equating \( \phi \) from 6 & 7

\[ A \frac{dx}{dt} = \frac{\mu \cdot \bar{V}^4}{8\mu L_C} \Delta P \]  

and substitute\( \Delta P \) from 5

\[ C = \frac{A^2 \mu L_C^2 \bar{V}^4}{d^4 \pi} \]  

then

\[ Z = \frac{16u}{d^3} \sqrt{\frac{3L_C v_d}{\pi 1.33 \times 10^8}} \]  

Now:

generally if \( M = Y M C \left( \frac{A}{a} \right)^2 \)

\[ K = X \frac{A^2}{V_d} \]  

\[ f_r = \frac{1}{2\pi} \sqrt{\frac{X^2 \frac{A}{V_d} a^2}{YM A^2}} \]
If \( Z = 0.7 \), which is a usual compromise for overshoot and frequency range, with \( \mu = 0.01 \) poise

\[
P = 1 \text{ gm/cc}^2
\]

\[
L_c = 26.6 \times 10^8 d^6 \quad \frac{V_d}{V_d} \quad \text{......................... 9}
\]

Consequently the length of a catheter, knowing \( V_d \) and \( d \) can be determined to give a damping factor 0.7.

It will be seen that the low value of \( w_n \) resulting, is the limiting factor of the systems performance.

The following FIG 11 shows how for specific catheter inside diameter the length of the catheter required for \( Z = 0.7 \) varies with the type of transducer used i.e. its volume displacement, and also the resulting undamped natural frequency of the system. The graphs have been plotted using equations 3 & 9 and taking into account the effective mass of the fluid. SHIRER (1962) ignores the \( \frac{4}{3} \) factor for which reason the 'plot' is shown here.

From FIG 11 it follows noting \( Z = 0.7 \) throughout

1) if \( d \) is constant \( \frac{V_d}{V_d} \) remain constant when reducing the length of catheter if \( V_d \) is increased & vice versa.
FIG. 11
CATHETER/TRANSUDER PERFORMANCE CHARACTERISTICS

UNDAMPED NATURAL FREQUENCY $F = 174$ DIA = 0.04 cm

$F = 22$ $D = 0.035$

$F = 30$ $D = 0.030$

$F = 43$ $D = 0.025$

$F = 68$ $D = 0.020$

$F = 121$ $D = 0.015$
2) \( L \) is constant; increase of \( V_d \) requires an increase of \( d' \) to maintain a \( F_n \) value, vice versa.

The choice of catheter will probably be determined in order of priorities.

1) type of transducer i.e. \( V_d \), available
2) tube diameter & lengths available
3) minimum value of \( F_{UN} \) acceptable

2.3.0. DISCUSSION

The measurement of intravascular pressure can be achieved by using any of the mentioned transducer systems, if the transducing element is large, then a fluid filled catheter will have to be used as a pressure coupling device. The degradation has been discussed, using an approximate solution for defining the systems response in general terms, this does however indicate the performance limitations of using a fluid filled coupling. There are also problems in filling such a system with saline to ensure no air enters the system, the effects of minute air bubbles within the system have been shown by HENRY (1967), on filling a 80 cm catheter following stringent precautions lead to the inclusion of .2 -.3 mm air bubbles & caused a reduced natural frequency of 15% of the undamped value. In consequence then the problems in using a fluid filled system cause further degradation of the overall performance.

If the overall system frequency response is to have an upper working frequency of say 100 HZ, then it is almost certainly best achieved by the use of a catheter tip transducer which will have an undamped natural frequency well over 100 HZ. Although it is possible for a fluid filled catheter to have an undamped frequency of the order of hundreds of HZ, this results in a damping factor which is very small of the order of 0.01. Thus two factors limit its working frequency range; with low damping the ratio of \( w/w_n \) which is useable is very small;
and secondly increasing the damping value further reduces the useable frequency range. (See equation 2)

Of the transducer systems mentioned, all but two, have for a specific diaphragm, a fixed value for gain or sensitivity. The exceptions being the capacitive and fibreoptic systems. For these, a change in sensitivity can be achieved by arranging the diaphragm in different initial positions. This feature, as will be discussed later, provides an extra degree of freedom in a transducer design.

Finally then the conclusion to be drawn is the advisability of using a catheter tip system when determining accurately intravascular pressures, also that when considering factors of utility and safety then a fibre optic reflection transducer is indicated. Catheter tip systems possess by virtue of their design very low volume displacements, thus if they are for any reason used in a fluid coupled mode the degredation in response resulting will be of minimal effect.
SECTION THREE

FIBROPTIC PRESSURE TRANSDUCER

THEORETICAL ASSESSMENT
3.0.0. INTRODUCTION

The oldest and most commonly used method of determining intravascular pressure is to couple the system of interest via a fluid filled catheter to an external pressure transducer. The degradation of the transduced signal is severe and it would be far more useful to utilise a catheter tip transducer. It has been mentioned that various types of catheter tip transducers have been constructed. However it is only the strain gauge type that has found any degree of acceptance in the medical field, the reasons why this is so are not immediately apparent, but is probably due to a mixture of factors apart from the performance of the transducer... producibility, profitability and perseverance of individuals who favour the system in question. However the author has found that strain gauge catheter tip systems main fault is their fragility.

The fibre optic system to be investigated is most similar to the capacitive system. A capacitive catheter tip system is described by FROBENIUS W D (1973) the similarities being its robustness and its inherent sensitivity which is variable at the construction phase, dependent on the original position of the pressure sensing diaphragm.

A fibre optic system for transducing intravascular pressure relies on the diaphragm as it deflects, modulating the intensity of a light signal. The pressure dependent signal then being externally converted to an electrical analogue. Before discussing in more detail the operational mode of such a transducing system a brief historical review and theory and terminology of fibre optics follows.
3.0.1. HISTORICAL REVIEW, THEORY & TERMINOLOGY OF FIBRE OPTICS

Fibre optics is the technique of transmitting light through long thin fibres of glass or any other transparent material. During the last decade the technology of fibre optics has been successfully developed and applied to overcoming diagnostic problems encountered in medicine and surgery.

Before discussing some new work the history of fibre optics and its principles will be described.

An "optical fibre" is a fibre which consists of a transparent core (glass or plastic) surrounded by a transparent sheath, again glass or plastic.

A coherent bundle is composed of many optical fibres, each of which is flexible. The ends of the fibres are grouped and bonded together in such a way that the position of any fibre at one end is identical to its position at the other. Then, if an image is projected onto the end-face of a coherent bundle it will be transmitted without distortion to the other end. The resulting image is composed of many discrete points of information.

An incoherent bundle usually termed 'light guide' is a group of optical fibres used to transfer luminous energy from one place to another. The individual fibres in a light guide are of greater diameter than those in a coherent bundle, but are similarly flexible.

The Egyptians were the first to use glass rods to transmit images, which were in the form of mosaics. The mosaics were produced by the union of glass rods of various colours in such a way as to form a pattern. The rods, so formed were reheated and drawn out until reduced to a very small size, then cut transversely into small tablets each transmitting the same pattern. The remarkable point is that their method of producing groups of large-diameter rods to form a multifibre rod of small calibre is one of the most recent of present fibre optic
production techniques. The Egyptians established this between 1600 and 1000 B.C.

In A.D. 140 PTOMELY, a famous astrologer and geographer of Alexandria, attempted to establish laws to explain the phenomena of refraction. He failed but concluded that there was a relationship between the input light ray and the refracted ray that one day would be established. It was only in 1620 that SNELL, a Dutch professor succeeded.

The itemised laws of reflection and refraction led to the concept of the transmission of light by glass fibres. The combined effect of using many fibres to build up an image was first patented by JOHN LOGIE BAIRD (1927). There then followed many similar patents underlining the observation that the idea, though admittedly ingenious, is a somewhat obvious application of what had been known about transparent solids for sometime.

The techniques needed to produce optical fibres and align them required time to develop, and HOPKINS (1954) and KAPANY (1952), at Imperial College, London and VAN HEEL (1954) in Holland independently began to make significant progress in this respect, they also managed to produce sheathed fibre, that is a fibre whose core glass is surrounded by a protective sheath of glass of a lower refractive index, which prior to 1952 had not been attempted.

It was the production of such a true optical fibre that enabled applications for fibre optics to become not just pipe-dreams but reality.

Optical fibres, at present, are manufactured using a reduction process, i.e. one starts with large masses of the core and sheath materials and by using heat and a drawing process one obtains optical fibres of the correct size. To utilise this process, materials must be used which exhibit a gradual decrease in viscosity with temperature. This restricts materials to two types: a) glasses b) plastics.
Most manufacturers use glass materials at the present time and the technique for manufacture is as follows:

A rod of core glass (usually 30 mm diameter) is suspended in a vertical muffled furnace. Around the rod is a tube of sheath glass whose bore is slightly larger than the rod diameter. The combination is heated to softening point usually about $800^\circ C$ and the two drawn down fusing together. The fused fibre is drawn out of the furnace at a predetermined speed. To replace the withdrawn glass, the rod plus tube is fed into the furnace slowly. This feed rate and the withdrawal rate determine the fibre diameter. The wall thickness of the sheath tube and the reduction ratio determines the sheathing thickness of the fibre.

Two methods of drawing the fibre can be used:

1) for large fibres, the drawing can be done with a pair of spring loaded rollers.

2) for small fibres, which are flexible, the fibre can be drawn onto a drum which imparts the drawing rate and acts as a store for fibre already drawn.

The choice of method is dependent on the diameter of fibre required, the former is used for fibre over 250u in diameter and the latter for smaller diameter fibres.

Work is presently being carried out to improve the performance of fibre optic components and is centered on the following areas:

a) Producing optical quality glass sheath material, as present sheaths are fairly crude with regard to the inclusion of unhomogeneous materials - BRAND (1968).

b) Producing a super optical quality core glass.

c) Perfecting bonding techniques used to hold the ordered fibres to each other at the ends of a coherent bundle.

d) Improve the layering techniques used to produce a coherent bundle.

e) Improving pre-production preparation treatments. BRAND (1967)
f) Improving end-facing polishing techniques.

The most recent development of significance is the production of an optical fibre "Selfoc" UCHIDA (1970), whose core glass refractive index varies quadratically with the distance from the axis of the fibre. The significant feature of such a fibre is that it acts like a lens with a short focal length and thus appears to have great potential in the field of endoscopy. (see following analysis)

Light is propagated through an optical fibre by a series of total internal reflections from wall to wall of the fibre. A light ray incident on the fibre end-face will only be propagated through the fibre if, after refraction onto the core glass, its angle of incidence at the core/sheath interface is greater than the critical angle for that interface. Thus the ray will then suffer total internal reflections as it progresses on its zig-zag path within the fibres core. FIG 12 shows an axial ray passing through an optical fibre. Now the angle of incidence up to which light is accepted and propagated by the fibre is a function of the refractive index of the core glass \(n_1\), that of the sheath glass \(n_2\), and the external media refractive index \(n\). With respect of FIG 12

\[
\sin \theta_{\text{max}} = \left( n_1^2 - n_2^2 \right)^{\frac{1}{2}}
\]

As in conventional optics the term 'numerical aperture' is used as a means of describing the light gathering capability of an optical system, it is equal to \(n \sin \theta_{\text{max}}\). Values of N.A. for most commercially available fibres range from 0.54 \((2\theta = 66^\circ)\) to 0.60 \((2\theta = 72^\circ)\), the larger the N.A. becomes the more is the light that can be collected from a point source which is usually desirable for a light guide.
\[
\text{Optical Fibre - Ray Propagation}
\]
Inspection of equation 10 will show that if \( n_2 = 0 \), that as there is no sheath glass present one would greatly increase the fibres N.A. which is usually desirable. However, in practice the surface of an unclad fibre deteriorates, causing severe leakage of light. For this reason a glass sheath is fused around the core glass, thus protecting the interface at which total internal reflection occurs.

Unfortunately the quantity of luminous energy entering an optical fibre is greater than the emergent energy. There are three factors which affect this loss of light transmission.

a) Absorption of light by the glass core
b) Losses due to total internal reflections.
c) Losses due to reflection at each of the end faces of the fibre.

a) Losses due to absorption by glass core.

When light passes through any transparent medium a certain quantity is absorbed. In the case of glass this absorption partly results from the inclusion of impurity ions and is also partly due to scattering caused by included particles and local density fluctuations.

A useful term for describing the transparency of a medium is its internal transmittance factor \( \tau \), which is defined as the ratio of the radiant flux at the end, to the radiant flux at the beginning of the optical path within the glass (the path length must be stated.).

For example SCHOTT GLASS TYPE F2 at a wavelength of 500 nm has a \( \tau = 0.996 \) for 2.5 cms path length. Computing (see over) the percentage of light transmitted through 50 cms we see that 92% transmission occurs.
Consequently one must determine the path length of a ray in the optical fibre in order to compute the transmission of light through the fibre assuming absorption is the only loss factor. This path length is a function of the length of the fibre (L), the diameter of the fibre (d), and the internal angle incidence (α) which is related to the angle Φ of the entrance ray. From FIG. 12 it can be shown that \( L_p \) the path length in the core glass.

\[
L_p = L \left( \sin \alpha \right)^{-1}
\]

and

\[
t_{core} = t_{t} L \left( \sin \alpha \right)^{-1}
\]

b) Losses due to total internal reflection

For each internal reflection it can be shown both theoretically - JOOS G (1943) and practically - QUINCKE G (1866) that the light penetrates the sheath glass by a small yet finite amount. Consequently the sheath material will absorb a finite amount of the ray suffering the total internal reflection.

The factors affecting this loss are: the transmittance of the sheath material, the internal angle of incidence (α) of the light ray and the number of reflections the ray suffers. POTTER (1961) gives experimental values of 0.9995 for glass and .99 for plastic fibres as this reflection coefficient.

\[
t_r = r_{cs} L \left( \text{d tan} \alpha \right)^{-1}
\]

where \( r_{cs} \) is the reflection coefficient

where \( L \left( \text{d tan} \alpha \right)^{-1} \) is the number of internal reflections the ray suffers.
c) Losses due to reflection occurring at the fibre end-faces

There is a loss of light at each end-face, both when light enters the fibre and when it emerges from it. This is due to a specific amount of light being reflected back by the end-faces and is known as a Fresnel reflection loss.

A Fresnel reflection occurs when the light passes from one medium to another, the amount reflected being dependent on the refractive indices of the two media, which in our case are the core glass and external medium.

This loss, when expressed as a transmission is shown below

\[ t_{fr} = \left( \frac{n - n_1}{n + n_1} \right)^2 \text{ per interface} \] \[ ............. 15 \]

If, as in most cases, the internal medium is air \((n = 1)\) and we consider a typical core refractive index \(n_1 = 1.62\)

\[ t_{fr} = 0.947 \text{ per interface.} \]

as there are two interfaces the end losses come to approximately 10%.

Summarising 13, 14 & 15 the transmission percentage of an optical fibre may be described by:

\[ t_c = t_{core} \frac{L(d \tan \alpha)^{-1}}{L} \frac{L(d \tan \alpha)^{-1}}{L} (1 - 2((n - n_1)(n + n_1)^{-1})^2) \] \[ ............. 16 \]

FIG 13 shows the transmission percentage of white light through a bundle of 50 \(µ\) diameter fibre plotted against fibre length.

The loss of light at near zero length is partly due to the Fresnel reflection loss as described, but is also due to the fact that the total 'cross sectional' area of a bundle of fibres is greater than the cross sectional area of the core glass which accepts the light for propagation. These are two causes of end-face area losses; the ratio of sheath area to core area and the closeness of packing fibres in the bundle.

When an image is formed upon an end of an aligned bundle of optical fibres, each fibre relays to its other end a discrete point of information from the projected image see FIG 14.
FIG 13

TRANSMISSION OF WHITE LIGHT
THROUGH FIBRES OF 50MICRON DIAMETER
The resolution, that is the degree of detail that can be conveyed, of a coherent bundle is dependent on the diameters of the individual fibres composing the bundle. The method of assessing resolution is to project a fine line test grid onto one end of the bundle and assess its image at the other end. A fine line test grid is a mask composed of alternate parallel black and white lines, the thickness of the lines being equal. These grids are classed by how many of the pairs (a black and a white line) there are per millimetre.

If a fine-line test grid is projected onto the end of a coherent bundle such that the lines lie along the minimum fibre pitch (Fig 15) then the condition for the smallest size lines which can be resolved is that the lines composing the grid must not be narrower than the optical fibre diameter. If they were, a fibre at one end could find half its area white and the other half black, which when relayed to its other end will simply appear grey. Thus the maximum resolution of a coherent bundle will occur when the lines of the test grid are laid along the minimum fibre pitch and then the maximum resolution is approximately the reciprocal of twice the fibre diameter.

\[ R_{\text{max}} = \frac{1}{2d} \text{ line pairs/mm} \] 

where \( d \) = fibre diameter mm.

A coherent bundle having an individual fibre diameter of 10 \( \mu \)m will consequently have a maximum theoretical resolution of 50 line pairs/mm. In practice it is difficult to produce accurate layering, i.e. fibre pitch, and there are high scrap rates encountered in the production of coherent bundles.

Picture contrast is an important aspect of image quality. Poor contrast is caused by leakage of light from one fibre into another ('cross-talk').
Image Transmission by a Coherent Bundle
One cause of 'cross-talk' between fibres is the sheath thickness, which if too small (approaching the wavelength of the incident light), allows a significant percentage of light to 'leak' from the fibre through the sheath. Another cause may be found if the interface between core and sheath is of poor quality, the result of contamination of the raw surface during production of the fibre. If this is the case total internal reflection may not occur and the light leakage would be apparent. More detailed analysis, especially for skew rays has been detailed - KAPANY (1958) and TIEDERKEN (1967).

**New generation optical fibre**

'Selfoc'

For the sake of completeness a brief discussion of the physical parameters and performance of a new type of fibre generically termed 'Selfoc' is given below.

Selfoc differs from conventional optical fibres in two ways - it has no sheath glass and its refractive index varies with radial position within the fibre.

By various means the relationship between the refractive index \( n_y \) at any radius \( y \) is given by:

\[
 n_y = n_o (1 - \frac{1}{4} ay^2) 
\]

where \( a = \text{constant} \)

\( n_o \) = refractive index on fibre axis

with respect to FIG 16 which shows a ray passing through the fibre.

Using Snells law.

\[
 n \cos \phi = (n - dn) \cos(\phi - d\phi) 
\]

\[
 = (n - dn) (\cos \phi \cos d\phi + \sin \phi \sin d\phi) 
\]

\[
 = (n - dn) (\cos \phi + d\phi \sin \phi) 
\]

\[
 dn \cos \phi = nd\phi \sin \phi 
\]

................. 18

................. 19
FIG 16
RAY PATH THROUGH ELEMENT OF SELFOC TYPE FIBRE

\[ n \cdot \text{dn} \]

\[ d\theta \]

\[ dy \]

\[ dx \]
For PARAXIAL RAYS 19 becomes
\[
\frac{dn}{n} = \phi \frac{d\phi}{dy}
\]
which can be written
\[
\frac{dn}{dy} \frac{1}{n} = \phi \frac{d\phi}{dy}
\]
\[
\frac{d\phi}{dy} = \frac{d\phi}{dx} \frac{dx}{dy}
\]
from FIG 16
\[
\frac{dy}{dx} = \phi
\]
differentially 23 wrt x
\[
\frac{d\phi}{dx} = \frac{d^2 y}{dx^2} \frac{1}{\phi}
\]
using 23 and 24 22 becomes
\[
\frac{d\phi}{dy} = \frac{n \frac{d^2 y}{dx^2}}{\phi}
\]
combining 25 and 21
\[
\frac{dn}{dy} = n \frac{d^2 y}{dx^2}
\]
now we have for the ray two conditions
\[
\frac{d^2 y}{dx^2} = \frac{1}{n} \frac{dn}{dy}
\]
\[
y_y = n_o (1 - \frac{1}{2} ay^2)
\]
differentially 18 wrt y
\[
\frac{dn}{dy} = -n_o ay
\]
using 27 and 26
\[
\frac{d^2 y}{dy^2} = \frac{1}{n} n_o ay
\]
but
\[
(y) \quad \frac{1}{2} a_y g^2 \left(1 - \frac{1}{2} a_y g^2 \right)
\]
18 may be written
\[
\frac{d^2 y}{dx^2} = -ay
\]
a differential equation which solves to

\[ y = c \sin (ax + b) \]  

The above description of the path of a ray within the fibre is that of a sinewave. Consequently, by suitable adjustment of the constants \(a\) & \(b\), a fibre of specified length will act like a conventional lens and relay an image - KITANO I (1969), UCMIDA T (1970), GATAK A K (1972) - see FIG 17 for deduced form of ray path through selfoc.

3.0.2. FUNDAMENTAL PRINCIPLE OF OPERATION OF A FIBRE OPTIC REFLECTION PRESSURE TRANSDUCER

Two bundles of optical fibres are grouped together and ensheathed within a catheter at their distal ends (FIG 18). A light reflecting diaphragm is located close to and enclosing this end and is clamped to the catheter at its periphery.

Light from a constant source is projected onto the efferent bundle of fibres 'E' and consequently emerges from the distal ends of these fibres. A specific quantity of this emergent light energy, after being reflected by the diaphragm falls onto the afferent fibres 'A' in the distal tip group (FIG 19). It is the position of the diaphragm that alters the light propagated by the afferent fibres and as the effective position of the diaphragm (its deflection) is proportional to the applied differential pressure on the diaphragm we have a light output from the afferent fibres directly related to the pressure to which the diaphragm is subjected.

A photo-sensor located at the proximal end of the afferent fibres converts the optical signal to an electrical analogue. It is appropriate now to predict, using a very simple two fibre model, the form of the relationship between the position of a piston like diaphragm and the resulting afferent output.
FIG. 17
DEDUCED RAY PATH THROUGH "SELFOC"
FIG. 18

SYSTEM SCHEMATIC  Fibre Optic Pressure Transducer
A two fibre model is shown in FIG 20a with one edge of the efferent cone of light shown. Positions A B C and D are different positions for the diaphragm. With the diaphragm at position 'A' very little light reaches the afferent fibre. At position 'B' light is shown to cover about half of the afferent fibre giving rise to output 'B' as shown in the static calibration curve graph (FIG 20b). When the position 'C' is reached the whole of the afferent fibre is covered and consequently the maximum output position has been reached. At position 'D' no more light can reach the afferent fibre and the response is shown to decrease in an inverse square law manner.

The curve shown in FIG 20b is what will be termed the static calibration curve for a fibre optic pressure transducing probe and it is convenient here to introduce two more terms relating to this curve, viz. sensitivity and range. The sensitivity is the slope of the curve (output/d distance) and the range being described as the distance from contact with the face of the fibres to that distance where a maximum output from the afferent fibres is achieved.
FIG 20a

EDGE OF REFLECTED CONE

A B C D

EDGE OF EXIT CONE OF LIGHT

AFFECTENT

EFFERENT

TWO FIBRE MODEL

SUCCESSIVE POSITIONS OF DIAPHRAGM

FIG 20b

STATIC CALIBRATION CURVE

AFFECTENT OUTPUT

DIAPHRAGM POSITION
3.1.0. DISCUSSION

Thus there are four discrete components comprising the pressure transducing system. These are the light source, the fibre optic probe, the diaphragm and finally the photo-detector (which may or may not include an amplifier). Also there are three distinct phases in the transduction process - pressure causing the diaphragm to deflect; the position of the diaphragm leading to a specific amount of light to be deflected onto the afferent fibres and finally the photo electric conversion of the output from the afferent fibres. Each of these phases will be the subject of an in depth appraisal in subsequent chapters.

However, at this stage from the two fibre model it can be deduced that the following will be expected to affect the static calibration curve:

1. the separation of the fibres, one from another
2. the diameter of the fibres
3. the numerical aperture of the fibres

If one considers the system's response with a diaphragm in situ, a 'static pressure calibration curve' could be drawn (FIG 21), the horizontal abscissa now being the differential pressure on the diaphragm. Then the form of this curve will first be dependent on the static calibration curve and where the diaphragm was initially placed, viz. its initial separation from the optical fibres and second the relative stiffness of the diaphragm, (a measure of how much the diaphragm deflects).

A further factor found from the two fibre model is that inspection of the static calibration curve shows that there are two positions of the diaphragm which result in the same afferent output. However it should be noted that the output value for the smaller separation position will usually possess a greater sensitivity value.

Thus so far it has been hypothesised that the relative amount of light falling onto the afferent fibres (reflection factor) is dependent on the separation of the surface reflector, the separation of the fibres
FIG. 21  STATIC PRESSURE CALIBRATION CURVE, SHOWING HOW FOR ONE TYPE OF DIAPHRAGM, ITS INITIAL POSITION EFFECTS THE FORM OF THE STATIC CALIBRATION CURVE OBTAINED.
in the distal end, the diameter and numerical aperture of the fibres. When the static pressure calibration curve is plotted its form will also be dependent on the diaphragm's stiffness and its initial separation from the face of the fibres in the distal tip.

The two fibre model thus serves as a good guide to the understanding of the relationship between fibre probe, parameters and diaphragm, but no comments have been introduced to describe the photodetector or light source for this model except by inference that the light source output is constant.

The concept of using optical fibres to transduce the deflections of a pressure sensing diaphragm was conceived independently by the author. However during the initial literature survey and during the work on this thesis other workers in this field were encountered.

3.2.0. PREVIOUS WORKS

After conception of the idea for a fibre optic pressure transducer some preliminary work was performed (see sec. 5.2.0.) to confirm some of the predictions resulting from the developments of the two fibre model described. Upon confirmation of the predictions arrived at as described earlier, a literature survey was performed. At that time two other authors were found to have attempted to describe such a system for measuring pressure, DE LA CROIX (1966) and LINSTROEM (1969). Two further original descriptions were published during work on this thesis, LINSTROEM (1970) and MORIKAWA (1972). Detailed comments on their analysis of a fibre optic pressure transducer and its performance will be made later (Ch 9 & Ch 19) as to introduce certain concepts at this time is likely to confuse rather than illuminate!

De La Croix, may be said not to have realised the complexity of his system, as he was unaware that rays leaving an optical fibre do not do so in a parallel manner. He does, however, show predicted static
pressure calibration curves for different diaphragm parameters and initial separations, although his computing model is incorrect because as mentioned no account of the fibre's N.A. is made. He was also unaware of the 'duality' effect of the initial position of the diaphragm and throughout his experiments only computed system performance with the diaphragm located closer than the maximum range position, i.e. on the higher sensitivity section of the static calibration curve. (11.2.0.) and (14.2.1.)

De La Croix is however the only author to describe the problems in performing a dynamic calibration of a pressure transducer and the requirements for such a pressure generator.

Lindstroem and Morikawa both produced functioning systems, each performing the analysis of the system from different viewpoints, omitting to give a comprehensive or correct analysis of the total system. Lindstroem does not fully approach the mechanical parameter relationships for the total system performance of the diaphragm. Morikawa partially describes for diaphragm parameters the deflections and yield relationships. He does not attempt to relate the optical fibre parameters' effect on the static calibration response curve, whilst Lindstroem does so for two conditions of the diaphragm separation, but does not develop the significance of the relationship, i.e. why not use a single fibre? (see sec. 9) (ch19). Also the two conditions for which his relationship is said to hold are not in the author's view relevant for a practical intravascular pressure transducer system, for reasons which will be discussed later. (Ch. 19).

Problems in the construction of the probe are not mentioned by either Lindstroem or Morikawa. De La Croix and Morikawa had the American Optical Company manufacture their probes which resulted in non-specified fibre arrangements in the probe's distal tip.

The critical effect of the photo-detector specification in relation to the parameter specification of the probe/diaphragm
combination are omitted by all the authors.

The following TABLE 3 shows some of the specifications of their systems (Ch.19). Morikawa is the only author to mention stability of the light source (note all used incandescent sources). He achieved this requirement by an unmentioned method of comparison with a reference. One such system is described in 13.5.1.

3.2.1. DISCUSSION

At this juncture it can be seen that although the basic principle of the manner in which a fibre optic transducer functions is easy to follow, the relationships, both individual and composite of the system components will be complex. The following analysis of the system, initially in component form, then as an inter-related description, will enable an optimisation procedure to be accomplished in the overall system design. This will then be applied to the needs of an intra-vascular pressure transducer. The next two chapters (4 & 5) will discuss the components of the system in some detail especially the optical fibre system (probe).
<table>
<thead>
<tr>
<th>PROBE LENGTH cms.</th>
<th>PROBE DIA. mm.</th>
<th>LIGHT SOURCE</th>
<th>DETECTOR</th>
<th>BUNDLE O.D. mm.</th>
<th>FIBRE DIA. mm.</th>
<th>FIBRE N.A. ARRGMT.</th>
<th>DIAPM. MATL.</th>
<th>DIAPM. DIA. mm.</th>
<th>DIAPM. THICNS. mm.</th>
<th>SEP. mm.</th>
<th>RANGE mm/Hg.</th>
<th>FREQUENCY RESPONSE Hz.</th>
</tr>
</thead>
<tbody>
<tr>
<td>91</td>
<td>1.5</td>
<td>INCAND-ESCENT</td>
<td>SILICON VOLTG. SOURCE</td>
<td>2.5</td>
<td>.075</td>
<td>&quot;De La Croix&quot;</td>
<td>MACRO</td>
<td>MYLAR 2.5</td>
<td>.075</td>
<td>1.5</td>
<td>-5 to 300</td>
<td>2 K</td>
</tr>
<tr>
<td>60</td>
<td>1.5</td>
<td>INCAND-ESCENT</td>
<td>PHOTO-TRANSTR</td>
<td>&quot;Lindstroem&quot;</td>
<td>MICRO .56</td>
<td>BE/CU 1.0</td>
<td>.006</td>
<td>-50 to 200</td>
<td>15 K</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>INCAND-ESCENT.</td>
<td>1.47</td>
<td>&quot;MORIKAWA&quot;</td>
<td>GLASS 1.3</td>
<td>.02</td>
<td>.06</td>
<td>-300 to 300</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**TABLE 3**

DETAILS OF FIBRE OPTIC CATHETER TIP PRESSURE TRANSDUCERS DOCUMENTED
CHAPTER FOUR

SYSTEM DESCRIPTION
CHAPTER 4 SYSTEM DESCRIPTION

4.0.0. INTRODUCTION

The system utilising a fibreoptic reflection method for the transduction of intravascular pressure thus comprises four discrete elements viz; optical fibre system (probe); diaphragm; light source and finally the photo-detector (including amplifier). This chapter outlines their function and critical features.

4.1.0. OPTICAL FIBRE SYSTEM

The function of the optical fibre system (probe) is to convey light from a light source, from its proximal end to its distal end where the diaphragm is located, and then receive the reflected light from the diaphragm, which is transmitted back to its proximal end where a photo-detector is located. The probe should be flexible, of an acceptable outside diameter, and be non-traumatic. Such a probe can be constructed by producing it in the form of a 'Y' guide, each limb composed of optical fibres ensheathed within flexible catheters. The arms of the 'Y' guide contain the efferent and afferent fibres. The efferent arm has light focussed on its end face, whilst the afferent arm has a photo-detector facing its end. The vertical leg of the 'Y' guide is composed of both afferent and efferent fibres. At each end of the 'Y' guide the fibres are encapsulated within a metallic ferrule which is fixed into position onto the ensheathing catheter using a suitable adhesive (16.1.0.). The resulting optical fibre system is thus totally insulated from the external environment (i.e. loose fibres prevent no safety hazard). Each end of the end faces is ground then polished so that its surface is perpendicular to the axis of the fibre (16.1.0.). The bifurcation is accomplished by a potting technique.
The design constraints to be considered thus far are - the length of probe needed, i.e. the length of the vertical limb; the diameter of the probe, i.e. the outside diameter of the ensheathing catheter; and finally that the probe should be flexible, which precluded the inclusion of fibres greater than 15 thousandths of an inch in diameter. Critical features such as spectral transmission; diameter; spatial arrangement in the distal common end and the numerical aperture of the fibres used, will be discussed in the next chapter.

4.1.1. DIAPHRAGM

The function of the catheter tip diaphragm is to reflect light which emerges from the efferent fibres in the distal tip group onto the afferent fibres in that group, consequently its reflective properties in the region of the wavelength of the incident light falling on it should be high. Also the deflection produced for a given pressure should be repeatable, in other words the diaphragm should be elastic in the Hookean sense and follow deformation in its elastic range. This is because, if it should, for one system, undergo a permanent deformation then a new static calibration determination would be required and the resulting performance will not be reliable.

The undamped natural frequency of the pressure transducing probe is for all practical considerations solely a function of the undamped natural frequency of the diaphragm. This is for an idealised piston like system FIG 5; consequently the stiffer and lighter the diaphragm becomes the higher the undamped natural frequency becomes. The diaphragm is held in place at a predetermined initial separation from the distal tip fibre group (c 14), and is effectively edgeclamped, thus creating an enclosed air gap (c 11.3.0). (Ch 19)

The theoretical analysis of undamped natural frequency for diaphragms having different parameters and the description of its deflection performance as an independant system function is to be found in Chapter 10.
The factors affecting the choice of the diaphragm are thus:

1. It should be an elastic material, and not be subject to fatigue problems.
2. Its parameters must allow it to be used in its elastic range.
3. Its parameters give it an acceptable undamped natural frequency.
4. A certain overpressure should be considered as a safety factor.
5. As it is in contact with the intravascular fluid it should be of an untraumatic material.
6. It should be specularly reflective.

As will be shown later (c 11 and 12) these factors can only be finalised when considering the system as a whole.

4.1.2. DETECTOR AMPLIFIER AND DISPLAY

The detector is located close to and facing the proximal end of the afferent fibres. Its function is to convert the pressure modulated light signal intensity falling on it into an electrical analogue, this is then displayed in a suitable manner using perhaps a digital meter of some form, or say a C.R.T. An auxiliary amplifier to boost the output from the photo-detector may be needed. This will be dependent on the finite changes in output from the photo-detector caused by pressure changes of the required resolution of the system i.e. 1, 2, 3, .. 10.or 20 mm Hg, and noting that the drift and the stability of the amplifier then needs to be considered as another factor in the determination of the system's resolution. There are various types of photo-detectors and modes of operation and amplification, which are discussed in chapter 12.

At this time the critical features of a photo-detector amplifier and display can be stated as:-
1. The detector's response should be maximal at the wavelength to be detected.
2. It should be of an acceptable size.
3. It should be rugged.
4. It should be small.
5. It should preferably be battery powered.
6. It should be stable.
7. It should allow for efficient coupling to the afferent fibres.

4.1.3. LIGHT SOURCE

The light source generates electromagnetic radiation, most suitably at a wavelength that afford good transmission by the optical fibres, allows good reflection from the diaphragm, and efficient photo-electric conversion by the photo-detector. As will be seen in chapter 14 the stability and noise of the light source is a critical factor in the system overall pressure resolution capability. There are two forms of light source available - an incandescent source which relies on the black body radiation principle and a solid state device (semiconductor source) which has a narrow spectral band-width. The relative merits of these types of sources is given in chapter 13 noting that it should be small, portable. The problems in the light source specification will be in the choice and definition of:

1. a stable source, of low noise
2. source wavelength, or filters required
3. source output
4. source size
5. source afferent fibre coupling
6. source efficiency
The inclusion of the source in the overall system specification and analysis is given in chapter 14.

4.2.0. DISCUSSION

The reasons for giving the system components more detailed requirements at this stage in the work is to give an idea of the initial conditions for the system, which will later be used in chapter 14 to provide a practical solution to the theoretical analysis of the system as a whole. It also provides a better understanding of the system prior to the analysis section.

To illustrate the parameters upon which the output from the system, for a given pressure are dependent, the following general functional dependance equation can be written:

\[ Y_{op} = f(r, d_0, n, sc, p, R_{dm}, t_d, C, E, z, w, l, C_f, e_i, P_d, C_d, G, t_c) \]

where

- \( r \) is the fibre radius
- \( d_0 \) .... diaphragm initial separation from the distal end
- \( n \) .... numerical aperture of the fibres
- \( sc \) .... spacial configuration in the distal tip (at this stage infers separation of fibres)
- \( p \) .... differential pressure to which the diaphragm is subjected
- \( R_{dm} \) .... radius of diaphragm
- \( t_d \) .... thickness of the diaphragm
- \( C_r \) .... reflectivity of the diaphragm at a wavelength of \( w \)
- \( E \) .... Young's modulus for the diaphragm
- \( z \) .... Poisson's ratio
- \( w \) .... wavelength of the source radiation
- \( l \) .... length of the optical fibres
- \( C_f \) .... coupling factor between the light source and the efferent fibres
\( e_i \) \hspace{1cm} \text{power emitted from the source}

\( P_d \) \hspace{1cm} \text{photo-detector sensitivity at } \omega

\( C_d \) \hspace{1cm} \text{coupling factor between the efferent fibres and the photo-detector}

\( G \) \hspace{1cm} \text{gain of the auxiliary amplifier}

\( t_c \) \hspace{1cm} \text{transmission coefficient of fibres}

18 parameters have been introduced as having an effect on the system output; such a number of parameters are too cumbersome to handle in one group. For subsequent analysis it will be convenient to separate these into functions which describe the critical transduction phase dependencies.

\[
i.e. \quad Y_{op} = f(F_d, d, y_p) = f(e_i, c_f, c_d, t_c, G)
\]

where \( F_d \) is a relationship between the reflected light on the afferent fibres and the emergent light from the efferent fibres, for different separations of the mirror-like reflector (\( d \)). The form of spatial configuration in the distal tip group is defined by S.C. fibre diameter, and numerical aperture (chapter 5 the static calibration effect) and \( y_p \), is a relationship of the deflection produced by a general pressure on a diaphragm of specific parameters, chapter 10.

\( f(d, y_p, F_d) \) then is the functional relationship for one fibre configuration and parameters and one diaphragm type and initial separation (\( d \)) or ratio (\( d/r \)) (Ch. 8) for the ratio of the reflected light to the emergent light for different pressures on the diaphragm – static pressure calibration effect Chapter 11.

This function (I) will purely result in a dimensionless ratio. In order to ascertain the quantitative outputs, function (II) must be introduced in a general form which is described later, (Ch. 14) this will have the units describing the output specified. The above functional dependence of the system output has been introduced here to further prepare an understanding of the rationale of the subsequent analysis of the system. It has been mentioned that the system's resolution is dependent on certain factors – these 'boundary conditions' are investigated in Chapter 12.
CHAPTER FIVE

OPTICAL FIBRE SYSTEM
CHAPTER 5  OPTICAL FIBRE SYSTEM

5.0.0.  INTRODUCTION

It has been deduced from the simple two fibre model introduced in chapter 3.1.0. that the range and sensitivity of the static calibration curve will be effected if changes are made in fibre diameter, numerical aperture and the spatial configuration of the fibres in the distal tip of the probe (fibre separation). This chapter discussed these parameters in more detail and describes an initial practical investigation into the actual effects of changing two of these parameters; namely fibre diameter and spatial configuration in the distal tip. The investigation was performed prior to commencing a thorough analysis of the optical fibre system. An additional factor discussed is the effect of the spectral transmission characteristics of the fibre on the system performance. However in practice one has to accept the spectral characteristics of the fibre which satisfied the other parameter's requirements. In fact, as will be seen, the final system specification is a compromise of components that are available, and limitations due to construction problems.

5.1.0.  FIBRE DIAMETER

The diameter of the fibres used in the probe have an effect on the static calibration curve response, the smaller the diameter the steeper the response (increase in sensitivity) of the ascending portion of the static calibration curve. However in chapter 3.0.1., mention is made of one limitation of fibre diameter (crosstalk), which is due to light leakage caused by the sheath thickness having reached the wave-length of the incident light. For commercially available fibres this means the diameter of the fibre would be of the order of microns. With such small fibres the actual problems are in handling them during construction. Consequently it is this factor that limits the use of
the minimum size of optical fibre used. Also the smaller the fibre
diameter, the lower the light transmission through the fibre, because
the light travels further through the core glass of the fibre, and it
suffers more internal reflections. So these factors must be considered
in the quantitative analysis of the system. The primary interest in
fibre diameter is its effect on sensitivity. It is worth noting here
that the term sensitivity is the slope of the static calibration curve,
i.e. \( \text{delta o/p} \div \text{delta separation} \), or, if put into the terms
introduced in chapter 4.2.0. \( \text{dy/dd} \), noting that "\( y \)" is a quantitive
parameter in the manner stated here.

The fibres have been assumed to be of circular cross-section.
Apart from simplifying the subsequent analysis through the use of this
assumption, it is also the only cross-section of optical fibres
commercially available in loose form.

The number of fibres used in the probe will depend on the
calibre of the protective catheter used for ensheathing the probe and
also the diameter of the fibres. With respect to the numbers of fibres
the only problem to be considered is the ratio of the number of efferent
to afferent fibres in the probe, but this cannot be resolved until the
total system elements are considered. 14.2.0. (app# 2)

5.1.1. NUMERICAL APERTURE OF FIBRE

The cone angle (N.A.) at which light emerges from the efferent
fibres will also affect the sensitivity and range of the optical fibre
probe (deductions from twin fibre model), all other factors remaining
constant. A very small N.A. affording a low sensitivity and long range
to the probe, a large N.A. giving the reverse characteristics – these
two statements are meant to be interpreted as relative effects of one to
another. From chapter 3.0.1. equation it can be seen that the N.A. of a fibre could fall into a wide range of values. In fact by suitable choice of core and sheath glass the N.A. can be greater than 1. So far excluded in the description of the significance of N.A. is that within this cone the illumination may not be uniform and consequently the polar distribution of the intensity of illumination within the N.A. should be known (see FIG. 22). This distribution is best assessed practically for the system in question, as the input signal to the fibre will also modify the actual distribution within the N.A.. If a fibre is illuminated with a high numerical aperture source and its other end is viewed at an angle greater than its corresponding aperture, then instead of the end appearing dark, bands of light appear. This is a skew ray effect which POTTER (1961) described and RICHTER (1966) theoretically confirms. The distribution should be known or included in any analysis of the effects of N.A. on the performance of the probe.

5.1.2. ARRANGEMENT OF FIBRES IN DISTAL END

The two fibre model has shown how a basic static calibration curve will appear. The term spatial configuration has been introduced somewhat prematurely and for the two fibre model, the term "fibre separation" is more appropriate to describe another factor affecting the sensitivity and range of the probe. But it can be deduced that increasing the fibre separation slightly causes a reduction in sensitivity and increase in range.

In a practical situation system there will be more than two fibres present and they will be tightly packed together. In other words, fibre separation is not the actual parameter variable that concerns us, but rather the spatial relationship between the efferent and afferent fibres in the distal tip group (fibre arrangement) which is a legitimate extension to the two fibre model. FIG 23 shows two proposed extremes of fibre spatial configuration (arrangement).
FIG 22
POLAR DISTRIBUTION WITHIN NUMERICAL APERTURE

APERTURE DEGREES

φ + φ₀

0.8  0.6  0.4

-60  -40  -20  20  40
in the distal tip group. That shown above is termed "micro". Each efferent fibre is surrounded by six afferent fibres. Such an orientation can be thought of as many small transmitter and receiver systems and would give a high sensitivity, low range probe, when compared with the arrangement shown underneath termed "Hemi" for reasons which are apparent.

These two systems can be considered as only differing in the scale of equivalent fibre sizes, that is the "Hemi" system really behaves as if it were made up from two much larger fibres ('TWO FIBRE MODEL'). This rationale for the effects of fibre orientation will become apparent later (8.1.0.) and the terms micro and macro system of fibres will be used. In other words the two systems are in fact micro and macro systems respectively where, shall we say, the effective fibre diameter as a function of the sender/receiver size is the variable (R/r ratio. Ch 8).

5.1.3. SPECTRAL TRANSMISSION

For a quantitative analysis of the output from the optical fibre system to be performed, the spectral transmission of the fibre type used must be known. The significance of spectral transmission has been introduced earlier in chapter 4, but as has been mentioned earlier the fibre's diameter and length also affects the quantitative output from the system.

The spectral transmission characteristics of the fibre only affect the quantitative output of the system and have no effect on the probe's range. The factors critical to the choice of the spectral transmission of the fibres used in the pressure transducing probe are that it should be high in the region of the wavelength of the peak sensitivity of the photo-detector.
FIG. 23  TWO PROPOSED EXTREMES FOR FIBRE ORIENTATION IN THE DISTAL TIP GROUP
5.2.0. INITIAL PRACTICAL INVESTIGATION INTO THE EFFECTS OF FIBRE DIAMETER AND SPATIAL CONFIGURATION ON OPTICAL FIBRE SYSTEM RESPONSE

Four optical fibre pressure transducing probes were constructed (for construction details see chapter 16.1.0.), that is special 'Y' guides without a distal tip diaphragm in situ. They were all of the same length and comprised of fibres of the same N.A. (equivalent to 56 degrees total cone angle). Of the four probes two were composed of one fibre diameter, the other two of a different diameter. Also, one from each pair had the fibres in its distal tip arranged in the "micro" manner, the other in the "Hemi" manner.

The objective was to compare the static calibration curves of the four probes to note the effects of the use of different fibre diameters and distal tip orientation. The static calibration curves were obtained for each probe by fixing the distal tip ferrule of the probe into a holder on an optical bench, initially just in contact with a gold plated mirror. The mirror was held in place on the end of a micrometer screw, the body of which was clamped to a holder on the optical bench. It was arranged that rotation of the micrometer thimble did not rotate the mirror but only altered the separation of the mirror from the distal tip of the probe. FIG 24 shows the static calibration set up used. (Later termed 'in-vitro calibration set up!')

For this initial investigation a powerful light source was used, it is shown in FIGURE 25, and can be seen to be constructed on the double skinning principle. This obviates the need for a cooling fan, as the outside of the box remains cool whilst the inside box housing the lamp allows the lamp to run at its optimum temperature. The lamp used was a 150 watt quartz halogen lamp with an integral elliptical mirror. A simple bayonet light guide connector coupling was made between light source and the proximal end of the probe,
FIG. 25

INITIAL LIGHT SOURCE USED
(note later a direct coupling is used between light source and efferent fibres). A similar type of connection was made between the afferent fibres and the photo-detector. The photo-detector used was a Plessey SCI silicon photocell and was used in its photo-voltaic mode in open circuit conditions. No amplifier was needed as the light source provided adequate efferent illumination. Display was shown on a digital voltameter.

Such crude systems for photo-detection and light source design were used as the problems of reducing the power of the light source would have led to more time being spent on developing more sensitive detector systems, and as stated then the objectives were simply to corroborate the deductions arising from the TWO FIBRE MODEL and also to obtain a "feel" for the system (see following discussion).

The results of this preliminary investigation are shown in table 4 from which it can be seen that:

1. For the same fibre diameter, the effect of the "micro" orientation compared to the "hemi" is to reduce the range and thus increase the sensitivity of the probe (values of sensitivity are not shown in the table).
2. For the same distal tip fibre orientation, if one say increases the fibre diameter, the result will be to increase the probe's range and consequently reduce its sensitivity.

The conclusion from these investigations confirms the deductions made from the two fibre model, which will serve as a foundation upon which the results of a theoretical analysis can be compared.
The practical investigations performed enabled the system performance as predicted by the TWO FIBRE MODEL to be confirmed. The problems of light stabilisation were highlighted. In order to overcome light source drift which would occur between the calibration of the two probes, the light source was set prior to each calibration to give the same reference value on a reference detector. The actual value of drift after two hours warm up for the light source was $\pm 0.5\%/\text{min.}$ Each calibration lasted 5 min. inferring a $\pm 2.5\%$ error in baseline drift. As the only requirement at this time was to demonstrate the effects of fibre diameter and orientation, the range figure alone has been stated in Table 4. This is unaffected by drift, the sensitivity value though is, at best, a direct function of the range, i.e. $1/\text{range}$. Furthermore these range values agree with those arrived at from later predictions (c.f. transition 'point' 8.1.1.).

To compensate for the light source drift, a compensatory system was later devised, the principle of which is discussed in chapter 13.5.1.

Another problem noted, although in the investigation described it was taken into account, is the effect on repeatability, of the coupling orientation between light source, light guide and proximal end of the efferent fibres of the pressure transducer. There are two effects here. First, considering the coupling of the light guide to the efferent fibres, a change in the coupling orientation effectively changes the fibre arrangement in the distal tip (particularly for the "micro" system) as some fibres may not line up with the illuminating light guide fibres causing less sender systems than was first expected. Secondly, the coupling of the light guide to the light source which, if the illumination of the source itself is not uniform, will cause a change in the quantitative transmission of the system.
Thus coupling orientation can affect the repeatability in the system's performance in a quantitative manner. The effect of coupling orientation has been symbolised earlier by $C_f$ for the light source and $C_d$ for the afferent fibres/photo-detector. In practice in order to ensure that these effects are eliminated, that is, that there are no errors in repeatability due to coupling differences, positive locations for the connections between the respective components must be provided.

In these initial tests such locations were provided by way of bayonet end fittings.
<table>
<thead>
<tr>
<th></th>
<th>PROBE 1</th>
<th>PROBE 2</th>
<th>PROBE 3</th>
<th>PROBE 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>RANGE OF PROBE</td>
<td>0.0635</td>
<td>0.5</td>
<td>0.254</td>
<td>0.762</td>
</tr>
<tr>
<td>DIA METER OF CATHETER</td>
<td>2.7</td>
<td>2.7</td>
<td>6.35</td>
<td>6.35</td>
</tr>
<tr>
<td>FIBRE DIAMETER</td>
<td>0.077</td>
<td>0.077</td>
<td>0.203</td>
<td>0.203</td>
</tr>
<tr>
<td>FIBRE ORIENTATION</td>
<td>MICRO</td>
<td>HEMI</td>
<td>MICRO</td>
<td>HEMI</td>
</tr>
<tr>
<td>NUMERICAL APERTURE</td>
<td>0.56</td>
<td>0.56</td>
<td>0.56</td>
<td>0.56</td>
</tr>
</tbody>
</table>

**TABLE 4** RESULTS OF INITIAL PRACTICAL INVESTIGATION INTO THE EFFECTS OF FIBRE DIAMETER AND ORIENTATION ON PROBE RANGE AND SENSITIVITY
CHAPTER SIX

THEORETICAL ANALYSIS

OPTICAL FIBRE SYSTEM
6.0.0. INTRODUCTION AND OBJECTIVES

The fundamental principle of operation of the pressure transducer is based on the change in afferent light output caused by a change in the position of the mirror-like diaphragm. Consequently, the basic objective of this theoretical analysis is to obtain a relationship between the diaphragm position from the distal tip of the probe and the afferent fibre output. It has been deduced earlier that the probe's characteristics will modify its response. Thus the following analysis will attempt to account for all the variables' characteristics of the fibre optic pressure sensing probe.

The basic theory (6.1.0.) considers the amount of light received by a disc of radius 'R', from another disc, coaxial to the first and of radius 'r' separated by a distance 'a' (FIG. 30). The relationship derived is shown from first principles of radiometry whereby the conditions for which it holds can be clearly seen. Section 6.1.1. applies this relationship to the optical fibre system of the pressure transducer. This enables the effect of diaphragm separation from the distal tip of the probe to be related to the afferent fibre output (static calibration curve).

The application of the basic theory to the optical fibre system of the pressure transducer is achieved by considering the afferent fibres in the distal tip group of the probe to form the central radiating 'disc', and the afferent fibres to form an annulus around the emitting disc. Then two forms of fibre arrangements in the distal tip group can be specified, (FIG. 26). If the emitting disc is a single fibre of small diameter and is surrounded by a receiving annulus of six fibres, such that there are many such emitter/receiver systems in the distal tip group, then we have the 'micro' system shown, which is the simplest micro system. However, if more than one fibre forms the central emitting disc then as long as there are more than one receiver/emitter system within the distal tip group the arrangement is still to be termed 'micro' (also irrespective of the
number of fibres composing the annular receiver. The 'macro' system as shown is such that only one emitter/receiver system in the distal tip group of the probe is formed. The annular receiver and emitting 'disc' can be composed of any number of fibres respectively. The limiting case though for a 'macro' system is when there is only one fibre or group of fibres as the emitter/receiver, that is when the emitting fibres also are utilised as the receivers. This special case is described in section 6.1.2.
DENOTES AFFERENT FIBRES (receivers)

"MICRO"

"MACRO"

"2R"
6.1.0. BASIC THEORY

With respect to FIG. 27, consider \( dA \), an element of a perfectly diffusing surface, that is a surface which radiates according to the cosine law. Let \( \phi \) be the flux radiating from this surface, normally per unit area.

Thus the flux radiated at an angle \( \theta \) to the normal is \( \phi \cos \theta \) per unit area.

Consider now element \( dB \) of a surface parallel to, and a normal distance "a" from \( dA \), and also at \( \theta \) to \( dA \).

Then the flux intensity on element \( dB \) received from \( dA \) is

\[
\frac{dA(\phi \cos \theta)}{a \sec^2 \theta}
\]

Thus the flux received from \( dA \) by \( dB \) is

\[
\frac{dA \phi \cos \theta}{a \sec^2 \theta} \cdot B \cos \theta
\]

\[
= \frac{dA \cdot dB \phi \cos^4 \theta}{a^2}
\]

Thus the flux received from a radiating disc by an elementary area parallel to and coaxial with it is, if 'R' is the radius of the disc (FIG. 28).

\[
\frac{dB \phi}{d^2} \int_0^R \cos^4 \theta 2\pi b \, db \quad \text{where} \quad b = a \tan \theta
\]

\[
= 2\pi R \int_0^{\tan^{-1} \frac{R}{a}} \cos \theta \sin \theta \, d\theta
\]

\[
= 2\pi R \int_0^{\tan^{-1} \frac{R}{a}} \sin \theta \, d(\sin \theta)
\]

\[
= 2\pi R \left[ \frac{\sin^2 \theta}{2} \right]_0^{\tan^{-1} \frac{R}{a}}
\]

\[
= \frac{\pi R^2}{2} \left( \frac{1}{2} - \frac{1}{2} \right)
\]

\[
= \frac{\pi R^2}{4} - \frac{\pi R^2}{4}
\]

\[
= 0
\]

\[
= 2\pi d\phi \int_0^{\tan^{-1} \frac{R}{a}} \cos \theta \sin \theta \, d\theta
\]

\[
= \frac{\pi R^2}{2} - \frac{\pi R^2}{2}
\]

\[
= 0
\]
FIG. 27
FLUX FROM RADIATING ELEMENT "dA" RECEIVED BY A PARALLEL ELEMENT "dB".

FIG. 28
FLUX FROM RADIATING DISC OF RADIUS "R" RECEIVED BY AN ELEMENT PARALLEL AND COAXIAL TO IT.
Now the flux received from a radiating disc, as before, of radius 'R', by an elementary area parallel to it, but this time displaced at a distance 'p' must be found.

With respect to FIG 29, let the centre of a radiating disc of radius 'R' be 'C'. Also let 'O' represent the projection in its plane of the elementary receiving area. Let OC be "p" and OPQ allow COQ to be 'α'.

Geometrically from FIG. 29

\[ PT = TQ \]
and \[ OT = p \cos \alpha \]

\[ OP = p \cos \alpha - PT \] ........................................... 33
\[ OQ = p \cos \alpha + PT \] ........................................... 34

now using cosine law on \( \triangle OPC \) & \( \triangle OQC \)

\[ R^2 = OP^2 + p^2 - 2OP \cdot p \cos \alpha \] ........................................... 35
and \[ R^2 = OQ^2 = p^2 - 2OQ \cdot p \cos \alpha \] ........................................... 36

from 35

\[ \cos \alpha = \frac{OP^2 + p^2 - R^2}{2 OP \cdot p} \]

and from 36

\[ \cos \alpha = \frac{OQ^2 + p^2 - R^2}{2 OQ \cdot p} \]

\[ \frac{OP^2 + p^2 - R^2}{2 OP \cdot p} = \frac{OQ^2 + p^2 - R^2}{2 OQ \cdot p} \]

\[ OQ \cdot (OP^2 + p^2 - R^2) = OP \cdot (OQ^2 + p^2 - R^2) \]
\[ OQ \cdot OP^2 + OQp^2 - OQR^2 = OP \cdot OQ^2 + OOp^2 - OPR^2 \]
\[ OQ \cdot p^2 - OQ \cdot r^2 - OOp^2 = OP \cdot p^2 - OP \cdot R^2 - OQ \cdot OOp^2 \]
\[ OQ \cdot (p^2 - R^2 - OP \cdot OQ) = OP \cdot (p^2 - R^2 - OP \cdot OQ) \] ........................................... 37

obviously one condition from 37 is

\[ OQ = OP \]
FIG. 29  FLUX FROM RADIATING DISC OF RADIUS "R", RECEIVED BY A PARALLEL BUT NON-COAXIAL ELEMENT (where "O" is its projection in the plane of the disc).
otherwise

\[(p^2 - R^2 - 10P,0Q) = 0 \] ...........................................38

but from 33 and 34

\[OP,OQ = (p \cos \alpha - PT)(p \cos \alpha + PT)\]

\[= (p \cos \alpha)^2 - PT^2 \]

\[\therefore 38 \text{ becomes} \]

\[p^2 - R^2 - (p \cos \alpha)^2 - PT^2 = 0 \]

\[PT^2 = R^2 - p^2 + p^2 \cos^2 \alpha \]

\[PT^2 = R^2 + p^2 (\cos^2 \alpha - 1) \]

\[PT^2 = R^2 - p^2 \sin^2 \alpha \] ...........................................39

thus

\[PT = TQ = \sqrt{R^2 - p^2 \sin^2 \alpha} \] ...........................................40

substituting 39 and 40 into 33 and 34

\[OP = p \cos \alpha - \sqrt{R^2 - p^2 \sin^2 \alpha} \] .......................41

\[OQ = p \cos \alpha + \sqrt{R^2 - p^2 \sin^2 \alpha} \] .......................42

Now the flux reaching the elementary area from area POQP' of the disc

(where QOQ' = d\alpha) is the difference between that reaching it from O0Q' and OPP', and will take the form of equation 32 if 'dB' is area of element.

\[d(FR) = \frac{d\alpha}{2\pi} (TdB \phi) \frac{OQ^2}{OQ^2 + a^2} - \frac{OP^2}{OP^2 + a^2} \]

thus the total flux received from the whole disc is

\[FR = \frac{\phi}{2} \int \frac{\sin^{-1} \frac{R}{P}}{\sin^{-1} \frac{-R}{P}} \frac{OQ^2}{OQ^2 + a^2} - \frac{OP^2}{OP^2 + a^2} \ d\alpha \] .....................43

rationalizing integrand

let OQ = A where A = p \cos \alpha + \sqrt{R^2 - p^2 \sin^2 \alpha}

let OP = B where B = p \cos \alpha - \sqrt{R^2 - p^2 \sin^2 \alpha}
considering the integrand of 43

\[ \frac{A^2}{A^2 + a^2} - \frac{B^2}{B^2 + a^2} \] ........................................ 44

\[ \frac{a^2 (A^2 - B^2)}{A^2 B^2 + a^2 (A^2 + B^2) + a^4} \] ........................................ 45

and let \( x = R^2 - p^2 \sin^2 \alpha \) ........................................ 46

for clarity; the numerator above (45) expands on its own

\[ a^2 (p^2 \cos^2 \alpha + 2p \cos \alpha \cdot x + x^2 - p^2 \cos \alpha \cdot x + 2p \cos \alpha \cdot x - x^2) = 4a^2 p \cos \alpha \cdot x \] ........................................ 47

and substituting for 'x'; \( = 4a^2 p \cos \alpha \cdot (R^2 - p^2 \sin^2 \alpha) \) ............. 48

The denominator for 44 on expansion becomes

\[ (p^2 \cos^2 \alpha + 2p \cos \alpha \cdot x + x^2)(p^2 \cos^2 \alpha - 2p \cos \alpha \cdot x + x^2) + a^4 + a^2 \]

\[ (2p^2 \cos^2 \alpha + 2x^2) \]

on further expansion

\[ p^4 \cos^4 \alpha - 2p^3 \cos^3 \alpha + x^2 p^2 \cos^2 \alpha + 2p^3 \cos^3 \alpha - 4p^2 \cos \alpha \cdot x^2 + \]

\[ 2p \cos \alpha \cdot x^3 + p^2 \cos \alpha \cdot x^2 - 2p \cos \alpha \cdot x^3 + x^4 + a^4 + 2a^2 p^2 \cos^2 \alpha + 2a^2 x^2 \]

collecting terms

\[ p^4 \cos^4 \alpha + 2x^2 p^2 \cos^2 \alpha + x^4 + a^4 + 2a^2 p^2 \cos^2 \alpha + 2a^2 x^2 - 4p^2 \cos^2 \alpha \cdot x^2 \]

substituting for x

\[ p^4 \cos^4 \alpha + 2(R^2 - p^2 \sin^2 \alpha) p^2 \cos^2 \alpha + (R^2 - p^2 \sin^2 \alpha)^2 + a^4 \]

\[ + 2a^2 p^2 \cos^2 \alpha + 2a^2 (R^2 - p^2 \sin^2 \alpha) - 4p^2 \cos \alpha \cdot (R^2 - p^2 \sin^2 \alpha) \]

and expanding

\[ p^4 \cos^4 \alpha + 2R^2 p^2 \cos^2 \alpha - 2p^4 \sin^2 \alpha \cos^2 \alpha + R^4 - 2R^2 p^2 \sin^2 \alpha + p^4 \sin^4 \alpha \]

\[ + a^4 + 2a^2 p^2 \cos^2 \alpha + 2a^2 R^2 - 2a^2 p^2 \sin^2 \alpha - 4p^2 \cos^2 \alpha \cdot (R^2 - p^2 \sin^2 \alpha) \]

collecting terms

\[ a^4 + 2a^2 p^2 \cdot (\cos^2 \alpha - \sin^2 \alpha) + 2a^2 R^2 + p^4 (\cos^4 \alpha + \sin^4 \alpha) + \]

\[ 2p^2 R^2 \cdot (\cos^2 \alpha - \sin^2 \alpha) - 2p^4 \sin^2 \alpha \cos^2 \alpha + R^4 - 4p^2 \cos^2 \alpha \cdot (R^2 - p^2 \sin^2 \alpha) \]

.......................... 49
using identity for \( \cos 2\alpha \sin \alpha \) in 49, i.e. \( \cos^2 \alpha - \sin^2 \alpha \).

\[
a^4 + 2a^2 p^2 \cos 2\alpha + 2a^4 R^2 + 2p^2 R^2 \cos 2\alpha + R^4 + p^4
- 4p^2 \cos^2 \alpha (R^2 - p^2 \sin^2 \alpha)
\]

now \( \cos^2 2\alpha = (\cos^2 \alpha - \sin^2 \alpha)^2 \)

\[
= \cos^4 \alpha - 2\sin^2 \alpha \cos^2 \alpha + \sin^4 \alpha
\]

using \( \cos^2 \alpha \) identity (51) in 50

\[
a^4 + 2a^2 p^2 \cos 2\alpha + 2p^2 R^2 \cos 2\alpha + R^4 + p^4 \cos^2 2\alpha
- 4p^2 \cos 2\alpha (R^2 - p^2 \sin^2 \alpha) + 2a^2 R^2
\]

which finally becomes

\[
(a^2 + R^2 + p^2 \cos 2\alpha)^2 - 4p^2 \cos^2 \alpha (R^2 - p^2 \sin^2 \alpha)
\]

thus integrand of equation 43, is equation 48, divided by equation 52

\[
\frac{4a^2 p \cos \alpha \sqrt{R^2 - p^2 \sin^2 \alpha}}{(a^2 + R^2 + p^2 \cos 2\alpha)^2 - 4p^2 \cos^2 \alpha (R^2 - p^2 \sin^2 \alpha)}\]

and integral of equation 43 becomes

\[
FR = \frac{\phi}{2} \int \frac{\sin^{-1} \frac{R}{p}}{-\sin^{-1} \frac{R}{p}} \frac{4a^2 p \cos \alpha \sqrt{R^2 - p^2 \sin^2 \alpha}}{(a^2 + R^2 + p^2 \cos 2\alpha)^2 - 4p^2 \cos^2 \alpha (R^2 - p^2 \sin^2 \alpha)} \, d\alpha
\]

using the substitution introduced equation 46; \( x^2 = R^2 - p^2 \sin^2 \alpha \) we have

\[
d\alpha = \frac{xdx}{p^2 \sin \alpha \cos \alpha}
\]

and \( \sin^2 \alpha = \frac{R^2 - x^2}{p^2} \)

\( \cos^2 \alpha = \frac{p^2 - R^2}{p^2} \)

\( \cos 2\alpha = \frac{p^2 - 2R^2 - 2x^2}{p^2} \)

the integrand of equation 54 on substituting for \( d\alpha \), \( \sin \alpha \), \( \cos^2 \alpha \) and \( \cos 2\alpha \) is

\[
\frac{4a^2 x^2 \, dx}{\sqrt{R^2 - x^2} \left( (a^2 + R^2 + p^2 - 2R^2 - 2x^2)^2 - 4x^2 (p^2 - R^2 + x^2) \right)}
\]
multiplying brackets
\[\frac{4a^2 x^2 dx}{\sqrt{R^2 - x^2}} \left( (a^2 - R^2 + p^2)^2 + 2(a^2 - R^2 + p^2) 2x^2 + 4x^4 - 4x^2 p^2 + 4x^2 R^2 - 4x^4 \right)\]

expanding
\[\frac{4a^2 x^2 dx}{\sqrt{R^2 - x^2}} \left( (a^2 - R^2 + p^2)^2 + 4a^2 x^2 - 4x^2 R^2 + 4x^2 p^2 + 4x^4 - 4x^2 p^2 + 4x^2 R^2 - 4x^4 \right)\]

which reduces to
\[\frac{4a^2 x^2 dx}{\left( (a^2 - R^2 + p^2)^2 + 4a^2 x^2 \right) \sqrt{R^2 - x^2}}\]

which further reduces to
\[\frac{x^2 dx}{\left( (a^2 - R^2 + p^2)^2 + x^2 \right) \sqrt{R^2 - x^2}}\]

writing 't' for \(a^2 - R^2 + p^2\) in 55 the integral 54 becomes
\[2a\int_{0}^{R} \frac{x^2 dx}{(t^2 + x^2) \sqrt{R^2 - x^2}}\]

noting the significance of x, i.e. the distance P.T.; see substitute equation 46.

If \(x = R \sin \theta\)

then \(x^2 = R^2 \sin^2 \theta\) and \(\sqrt{R^2 - x^2} = R \cos \theta\)

also \(dx = R \cos \theta d\theta\)

substituting for \(x^2, \sqrt{R^2 - x^2}\) and \(dx\) in 56
\[FR = \phi dB \int_{0}^{\Theta_1} \frac{R^2 \sin^2 \theta d\theta}{R^2 \sin^2 \theta + t^2}\]

on long division of the integrand
\[FR = \phi dB \int_{\Theta_2}^{\Theta_1} d\theta - \phi dB \int_{\Theta_2}^{\Theta_1} \frac{t^2}{R^2 \sin^2 \theta + t^2} d\theta\]
where \( I_1 = \phi \frac{dB}{d\phi} \int_{\theta_2}^{\theta_1} d\phi \) ................................. 60

\[ I_2 = \phi \frac{dB}{d\phi} \int_{\theta_2}^{\theta_1} \frac{t^2}{R^2 \sin^2 \theta + t^2} \frac{d\phi}{R^2 \sin^2 \phi + t^2} \] ................................. 61

operating on equation 60

\[ I_1 = \phi \frac{dB}{d\phi} \left[ \begin{matrix} 0 \\ \theta \end{matrix} \right]_{\theta_2}^{\theta_1} \]

resubstituting using 57 for \( \phi \)

\[ I_1 = \phi \frac{dB}{d\phi} \left[ \begin{matrix} -1 \times \\ \sin \frac{R}{\phi} \end{matrix} \right]_{R}^{0} \] ................................. 62

Now operating on equation 61 and rewriting

\[ I_2 = \phi \frac{dB}{d\phi} \int_{\theta_2}^{\theta_1} \frac{t^2}{R^2 \sin^2 \phi + t^2} \frac{d\phi}{R^2 \sin^2 \phi + t^2} \] ................................. 63

and introducing the substitution

\[ y = \tan \phi \] ................................. 64

the following can be shown

\[ d\phi = dy/1 + y^2; \quad \sin^2 \phi = y^2 / 1 + y^2; \quad \sec^2 \phi = y^2 + 1 \]

and substituting for \( d\phi \) and \( \sin^2 \phi \) in 63

\[ I_2 = \phi \frac{dB}{d\phi} \int_{Y_1}^{Y_2} \frac{t^2}{(R^2 \frac{y^2}{1+y^2} + t^2)} \frac{dy}{(1 + y^2)} \]

\[ \therefore I_2 = \phi \frac{dB}{d\phi} \left( R^2 + t^2 \right) \int_{Y_1}^{Y_2} \frac{t^2}{y^2 + \frac{t^2}{R^2 + t^2}} \frac{dy}{R^2 + t^2} \]

which is in standard form and on integration becomes

\[ I_2 = \phi \frac{dB}{d\phi} \frac{t}{\sqrt{R^2 + t^2}} \tan^{-1} \left[ \frac{y \sqrt{R^2 + t^2}}{t} \right]_{Y_1}^{Y_2} \] ................................. 65
using 64 and 57,

\[ y = \frac{x}{\sqrt{R^2 - x^2}} \]

which on substitution into 65

\[ I_2 = \phi \frac{dB}{\sqrt{R^2 + t^2}} \tan^{-1} \left[ \frac{x}{t} \frac{\sqrt{R^2 + t^2}}{\sqrt{R^2 - x^2}} \right] \quad 0 \rightarrow R \]

combining 66 and 62

\[ FR = I + I_2 \]

\[ = \phi \frac{dB}{\sqrt{R^2 + t^2}} \left[ \sin^{-1} \left( \frac{x}{R} \right) - \frac{t}{\sqrt{R^2 + t^2}} \tan^{-1} \left( \frac{x}{t} \frac{\sqrt{R^2 + t^2}}{\sqrt{R^2 - x^2}} \right) \right] \quad 0 \rightarrow R \]

on inserting limits

\[ FR = \phi \frac{dB}{\sqrt{R^2 + t^2}} \left[ \pi - \frac{\pi}{2} \frac{t}{\sqrt{R^2 + t^2}} \right] \]

\[ = \frac{\pi}{2} \phi \frac{dB}{\sqrt{R^2 + t^2}} \left[ 1 - \frac{t}{\sqrt{R^2 + t^2}} \right] \]

thus the flux received by element

\[ \frac{\pi}{2} \phi dB \left[ 1 - \frac{t}{\sqrt{R^2 + t^2}} \right] \]

noting \( t = \frac{a^2 - R^2 + p^2}{2a} \)

Thus using equation 67, the flux received by a disc of radius 'r' can be found by integration over a radius and of new variable 'p'.

Noting that \( dB = 2\pi p dp \)

Let the flux received by a disc of radius 'r' from a disc of radius 'R' be 'FR' then from 67

\[ FR = \frac{\pi}{2} \phi \int_0^R \left( 1 - \frac{t}{\sqrt{R^2 + t^2}} \right) 2\pi p dp \]

\[ \quad \left[ 0 \rightarrow R \right] \]

\[ \quad \text{.................. 68} \]
substituting for \( t \) in 68

\[
FR = \frac{\pi \phi}{2} \int_0^r \left( 1 - \frac{a^2 - R^2 + p^2}{2a \sqrt{R^2 + (a^2 - R^2 + p^2)^2}} \right) \cdot \frac{2 \pi \rho dp}{d \phi
}
\]

\[
= \frac{\phi}{2} \int_0^r \left( 1 - \frac{a^2 - R^2 + p^2}{\sqrt{a^2 R^2 + (a^2 - R^2 + p^2)^2}} \right) 2 \pi \rho dp
\] 

The term within the square root sign on expansion becomes:

\[
a^4 + 2a^2 R^2 + 2a^2 p^2 + p^4 + R^4 - 2R^2 p^2
\]

can be written

\[
a^4 + 2a^2 R^2 + 2a^2 p^2 + p^4 + R^4 + 2R^2 p^2 - 4R^2 p^2
\]

which reduces to

\[(a^2 + R^2 + p^2)^2 - 4R^2 p^2 .... A\]

therefore 69 becomes

\[
FR = \frac{\pi \phi}{2} \int_0^r \left( 1 - \frac{a^2 - R^2 + p^2}{\sqrt{a^2 R^2 + (a^2 - R^2 + p^2)^2}} \right) \cdot \frac{2 \pi \rho dp}{d \phi
}
\]

if 'v' is used for \( a^2 - R^2 + p^2 \)

\[
dv = 2 \pi \rho dp
\]

and

\[
p^2 = v - a^2 + R^2
\]

substituting for \( dv \) and \( p^2 \) in equation 70

\[
FR = \frac{\pi \phi}{2} \int_{v_1}^{v_2} \left( 1 - \frac{v}{\sqrt{(a^2 + 2R^2)^2 - 4R^2 (v^2 - a^2 + R^2)}} \right) dv
\]

\[
= \frac{\pi \phi}{2} \int_{v_1}^{v_2} \left( 1 - \frac{v}{\sqrt{v^2 + 4R^2 a^2}} \right) dv
\]

\[
= \frac{(\pi \phi)^2}{2} \int_{v_1}^{v_2} \left( \frac{v}{\sqrt{v^2 + 4R^2 a^2}} \right) dv
\]

let

\[
\int_{v_1}^{v_2} \frac{v}{\sqrt{v^2 + 4R^2 a^2}} dv = I_3
\]
then using substitution

\[ v = 2Ra \tan \theta \] .............................. 71

thus \[ dv = 2Ra \sec^2 \theta \, d\theta \]

\[ I_3 \] becomes

\[ I_3 = \int_{\theta_2}^{\theta_1} 2Ra \tan \theta \sec \theta \, d\theta \]

\[ I_3 = \int_{\theta_2}^{\theta_1} \frac{2Ra \sin \theta \, d\theta}{\cos^2 \theta} \]

\[ = \left[ + \frac{2Ra}{\cos \theta} \right]_{\theta_2}^{\theta_1} \]

but from equation 71

\[ \cos \theta = \frac{2Ra}{\sqrt{4Ra^2 + v^2}} \]

thus \[ I_3 = \left[ \frac{1}{\sqrt{4Ra^2 + v^2}} \right]_{v_2}^{v_1} \] .............................. 72

thus \[ FR = \frac{\pi}{2} \phi \left[ v \right]_{v_2}^{v_1} - \frac{\pi}{2} \phi \left[ \sqrt{4Ra^2 + v^2} \right]_{v_2}^{v_1} \]

but \[ v = a^2 - R^2 + p^2 \]

Therefore \[ FR = \frac{\pi}{2} \phi \left[ a^2 - R^2 + p^2 \right]_{0}^{r} - \frac{\pi}{2} \phi \left[ \sqrt{4Ra^2 + (a^2 - R^2 + p^2)^2} \right]_{0}^{r} \]

from identity developed on page 134 (A)
Thus the flux received by a disc of radius 'r' from a disc of radius 'R'
(FIG. 30) which is parallel to, and a distance 'a' from it is

\[ FR = \frac{\pi}{2} \phi \left[ r^2 - \frac{1}{\pi} \phi \left[ \frac{(a^2 - R^2 + r^2 - (a^2 - R^2 + o))^2 - 4R^2r^2}{(a^2 + R^2 + o)^2 - 4R^2o} \right] \right] \]

73

It can be seen from the symmetry of equation 73 that 'r' and 'R'
are interchangeable, that is 'FR' is the flux received from either a disc
of Radius 'R' or 'r' by a disc of radius 'r' or 'R' respectively. From
this juncture 'R' will be considered to be the receiving disc and 'r'
the emitting disc, i.e. R \rightharpoonup r.

The expression 'FR' e.g. equation 73 derived from first
principles makes the assumption that the emitting disc behaves in a
Lambertian manner, that is its luminance is uniform when viewed from any
position. This assumption is of direct importance when the derived
relationship is applied to the optical fibre system and will be discussed
later.
6.1.1. REFLECTION FACTOR AND APPLICATION TO OPTICAL FIBRE/MIRROR SYSTEM

Consider the situation shown in FIG. 31 where an emitting disc of radius 'r' faces an annulus which is both coaxial and parallel to it. The outside radius of the annulus is 'R' and its inner radius 'r'. Also the separation between the disc and annulus is 'a'. One can consider this system to be analogous to an emitter/receiver system as mentioned earlier with the emitter axially displaced from the receiver.

Thus using equation 73, the flux received by the annulus becomes noting \( R > r \)

\[
FR = \frac{1}{2} \phi \pi \left[ \frac{r^2 + R^2 + a^2 - \sqrt{(r^2 + R^2 + a^2)^2 - 4R^2r^2}}{r} \right] - \frac{1}{2} \phi \pi \left[ \frac{r^2 + R^2 + a^2 - \sqrt{(r^2 + R^2 + a^2)^2 - 4R^2r^2}}{r} \right]
\]

in other words the annulus receives the difference between the amount of flux a total disc of radius 'R' would receive less the amount of flux a disc of radius 'r' would receive.

Collecting terms in \( R > r \)

\[
FR = \frac{1}{2} \phi \pi \left[ \frac{r^2 + R^2 + a^2 - \sqrt{(r^2 + R^2 + a^2)^2 - 4R^2r^2}}{r} \right] - \frac{1}{2} \phi \pi \left[ \frac{r^2 + R^2 + a^2 - \sqrt{(r^2 + R^2 + a^2)^2 - 4R^2r^2}}{r} \right]
\]
FIG. 30  VIEW OF PARALLEL COAXIAL DISCS FOR WHICH FLUX RECEIVED BY ONE FROM THE OTHER IS GIVEN BY

\[ FR = \frac{1}{2} \phi T^2 \left[ r^2 + R^2 + a^2 - \sqrt{(r^2 + R^2 + a^2)^2 - 4R^2r^2} \right] \]

FIG. 31  PRELIMINARY SYSTEM, PRIOR TO APPLICATION TO LIGHT COLLECTING ABILITY OF AFFERENT FIBRES IN DISTAL TIP OF FIBRE OPTIC PRESSURE TRANSDUCER.
is the reflection coefficient of the mirror, the amount of flux an annulus receives from a co-planar central disc is on substituting from 'a' in equation 75 '2d'

\[ FR = C | \phi | \pi^2 \left[ \frac{R^2 - r^2}{2 \pi^2} + \frac{4 \sqrt{r^2 d^2 + d^2}}{(r^2 + R^2 + 4d^2)^2 - 4R^2 r^2} \right] \]

This result would give a quantitative value of the flux received by the afferent fibres forming the receiving annulus; if 'φ' the emergent flux from the afferent fibres is known. However, it was mentioned earlier (sec. 4.2.0.) that the analysis would best be performed by introducing 'Fd' (equation 31) as the relative amount of light received by the annulus from the central disc.

Thus again considering FIG. 32, the total flux emitted from the disc 'r' is given by

\[ \pi r^2 \phi \] where φ is the flux emitted in the normal direction per unit area,
i.e. Total flux emitted = \( \pi r^2 \phi \) .................. 77

The 'reflection factor' i.e. Fd is obtained by division of equation 76 by equation 77.

\[ Fd \ (R \ Factor) = \frac{1C}{2 \pi^2} \left[ \frac{R^2 - r^2}{2 \pi^2} + \frac{4 \sqrt{r^2 d^2 + d^2}}{(r^2 + R^2 + 4d^2)^2 - 4R^2 r^2} \right] \] .... 78

Thus 'Fd' is the 'reflection factor' and gives the fraction of flux which the receiving annulus obtains from the central emitting disc. This relationship will allow the effect of fibre orientation and fibre diameter on the reflection factor for different diaphragm separations to be determined. In other words, a relative static calibration curve can be obtained for various system parameters. The above expression assumed the fibre's N.A. is unity and its polar distribution uniform. A simplified method for accounting for a Numerical Aperture of less than one follows in 6.1.3.
Fig. 32  Fundamental Emitter/Receiver System and Specularly Reflective Diaphragm: For which flux received by annulus is

\[ FR = \frac{1}{2} \pi \left[ \frac{R^2 - r^2 + 4}{r^2 \tilde{d} + d^4 - (r^2 + R^2 + 4d^2 - 4R^2) r^2} \right] \]
6.1.2. SPECIAL CASE - SINGLE FIBRE SYSTEM

If a single fibre system is used as an emitter and receiver, as is practically possible by incorporating into the proximal end of the system a beam splitter (FIG. 33), then using a definition introduced earlier we have the limiting case for a 'Macro' system where one emitter/receiver group comprises the distal face of the pressure transducing probe.

With respect to equation 73, 'r' then becomes 'R' and as a reflective diaphragm is located a distance 'd' from the face of the fibres, i.e. \( a = 2d \), we have from equation 73,

\[
FR = C \phi \pi^2 \left[ \frac{2R^2 + 4d^2}{\sqrt{(2R^2 + 4d^2)^2 - 4R^4}} \right]
\]

thus the reflection factor becomes

\[
Fd = \frac{1C}{2R^2} \left[ \frac{2R^2 + 4d^2 - 4 \sqrt{R^2 d^2 + d^4}}{R^2 d^2 + d^4} \right] \quad \text{79}
\]

Noting \( R_{dm} \approx \frac{R}{2} \)

At this juncture it can be mentioned that such a system utilizing one fibre would have as one limitation the minimum and maximum radius of a fibre which will afford mechanical suitability. The other limitations will be discussed later.
REFLECTING DIAPHRAGM

"R"

OPTICAL FIBRE

"Q"

BEAM SPLITTER

PHOTO-DETECTOR

LIGHT SOURCE

SINGLE FIBRE SYSTEM—SIMPLIFIED LAYOUT

FIG. 33
6.1.3. A METHOD OF ACCOUNTING FOR NUMERICAL APERTURE OF OPTICAL FIBRE

Initially, a simplified method of accounting for the numerical aperture of the optical fibres comprising the pressure transducing probe is given. (See ch. 9). Now if an axial ray (FIG. 34) is considered to emerge from the central emitter at its periphery having its maximum possible exit angle $\theta$ ($\sin \theta =$ Numerical Aperture), then after reflection on the mirror-like diaphragm, its radial position upon falling on the annular receiver is $'Z'$. This value is dependent on the value of the separation $'d'$ of the diaphragm from the distal tip face/emitter/receiver plane, and is the radius of the effective receiving annulus.

Consequently the value of $'R'$ (radius of collecting fibre) as used in equation 78, the expression for the reflection factor, is dependent on the value of $'d'$ in that expression. If equation 78 is re-written with $'Z'$ as $'R'$

$$F_d = \frac{1}{2r^2} \left[ \frac{Z^2 - r^2 + 4}{r^2d^2 + d^4} \left( r^2 + Z^2 + 4d^2 \right)^2 - 4Z^2r^2 \right] \ldots \ldots 80$$

where $'Z'$ is the conditional effective outside radius of the annulus receiver, dependent on the numerical aperture of the fibre and the separation distance $'d'$ such that from (FIG. 34)

$$2d \tan \theta - r \geq R \quad Z = R \quad (i)$$

$$R \geq 2d \tan \theta - r \geq r \quad Z = 2d \tan \theta - r \quad (ii)$$

$$2d \tan \theta - r \leq r \quad Z = r \quad (iii)$$

This is the worst condition possible for the system's response; for considering conditions (iii) and (ii), light will in practise not conform to the direct cut off implied.

However, the expression for reflection factor now allows the effect of fibre orientation, fibre diameter, numerical aperture and diaphragm separation on probe characteristics to be assessed and it should be noted that if $\theta = 90$ (Numerical Aperture = 1) equation 80 reverts to equation 78 upon substituting for the above conditions, and also that when $'d'$ and $'\theta'$ satisfy (i) above the modified response returns to standard form.
The limitations of this simplified method for the accounting
for a system having an N.A. of less than unity is discussed in chapter 9,
and comparisons between a theoretical static calibration curve and that
obtained practically, for a system composed of optical fibres with an
N.A. of less than unity is made in appendix 1.
FIG. 34  SIMPLIFIED METHOD FOR ACCOUNTING FOR FIBRE NUMERICAL APERTURE (N.A.)
6.2.0. THE EFFECT OF CHANGING THE VARIABLES R, r, d & N.A.

The effect of the fibre orientation can be considered to be simply a matter of the finite sizes of 'R' and 'r', for if a macro system is compared to a micro system then the only differences in performance will be due to finite values of 'R' and 'r'. This will become apparent when the concept of [R/r] and [d/r] dimensionless ratios are introduced (8.1.0.) and also it will be possible to show that a specific Macro system may have an identical performance to a certain Micro system. Thus fibre orientation can be considered to be defined by 'R' and 'r'. The other two variables comprising the Reflection factor relationship (d & \( \theta \)) allow many combinations of fixed and variable parameters for analysis.

The most convenient manner in which the information obtained by changing variables in equation 80 can be communicated is graphically. All the graphs which appear subsequently, unless otherwise mentioned, were obtained from their respective mathematical model, using a Hewlett Packard 9820 programmable calculator and the same manufacturer's X-Y plotter (9862A).

The graphs obtained from the expression for reflection factor \( F_d \) (equation 80), are in effect theoretical static calibration curves, noting that the 'y' abscissa is a fractional ratio indicating the proportion of the efferent flux received by the afferent fibres in the distal tip group. By virtue of the form of the expression (80) it will be necessary at this time to use finite values for 'R', 'r' and 'd'. The range of values which 'R' and 'r' can take are dependent on practical system considerations, irrespective of Macro or Micro formats except that 'R' maximally, for the Macro system is nearly the outside radius of the actual optical probe and thus will have a maximum value of say 1.5 mm.

The minimum value of 'r' is dependent on the smallest manageable fibre diameter which is about 10\( \mu \)m in diameter. The value for 'd', for static calibration purposes, is when it is considered simply as a
a position of a mirror surface. However, it will be necessary to relate a diaphragm's parameters to its deflection $\gamma_D$ (Chapter 10.1.0.) and then form a composite relationship for reflection factor, diaphragm initial position and parameters (Chapter 11) and introduced earlier by function (1) equation 31.
6.2.1. DISCUSSION

By considering fibres to be grouped in the distal tip of the pressure transducing probe, such that the afferent fibres form a receiving annulus around a centrally emitting efferent fibre(s); a relationship for the system's Reflection Factor (Fd) or RF, has been developed from the first principles of Radiometry. The expression for Reflection Factor (RF) allows the ratio of flux received to flux emitted to be calculated for given values of 'R', 'r', 'd' and 'N.A.'.

This analysis has assumed that the efferent fibres behave like Lambertian sources; in fibre optic terms this means their Numerical Aperture would be unity and their polar distribution within that aperture is uniform. It is possible to obtain such fibres, however, using a simplified approximation, account has been taken for N.A. but still assumed a uniform polar distribution within that aperture.

Another assumption, that of fibres grouped together to form discs or annuli is not without error. This is because the area of annulus or disc will not be perfectly packed by fibres. Indeed, the radii so formed will not be true radii. The effect will be minimal as the fibre diameter reduces and the radii in question increases, the error due to packing though, will still be of a similar order. Mathematical explanation of errors introduced is given in Chapter 9. The diameter of the diaphragm must be greater or equal to R-r, or R/2 for the single fibre case, due to equivalence with the conditions for which the co-planar system was derived.

Due to the complex nature of the expression for RF (Fd) (equation 80 or 78) interpretation of the resulting conditions for different values of the parameters is performed graphically.
Finally, the reflecting surface has been assumed flat; for an actual deflecting diaphragm this surface will have curvature. Errors in assuming a flat diaphragm deflection profile are discussed in Chapter 11.

The experimental verification of the reflection factor expression can be found in appendix 1. Logically this verification could be performed in Chapter 17. i.e. subsequent to discussions and descriptions of constructional details of the light source and detector units. Although pre-empting the continuity of this thesis, it is valid now to explain that it was thought that the 'pressure transducing probe' constructed would enable the verification of the reflection factor expression in Chapter 17. This was not so, as the fibre arrangement within the pressure probe was not in the form prescribed and could not be related to the reflection factor expression requirements, i.e. central emitter/annular receiver. Consequently, subsequent to this failure in verification, a new probe was constructed which although not suitable as a pressure transducing probe does have a fibre arrangement of the form necessary for use in the reflection factor expression. The results of this work is then presented in appendix 1.
CHAPTER SEVEN

GRAPHICAL INTERPRETATION

OPTICAL FIBRE SYSTEM
7.0.0. INTRODUCTION AND OBJECTIVES

Using a programmable calculator, the effect of utilizing different fibre diameters and numerical apertures was determined and the results plotted. The aim was to clarify the response due to changing the system parameters and determine further methods by which the resulting static calibration curves could be classified. These curves have 'y' ordinates of Reflection Factor (Fd) or RF, and should be called 'static response curves'.

7.1.0. THE EFFECT OF CHANGE OF FIBRE DIA. & EMITTER/RECEIVER DIAMETERS.

If one considers the micro arrangement system, then a change in fibre diameter, considering the 6:1 system (all of the same diameter) will simply cause a proportional change in 'R', the effective outside diameter of each receiving annulus formed with 'r' as equivalent to the fibre radius and equal to 10, 20, 30 and 50μ respectively. FIG. 35 shows the static response curves obtained from equation 80 with a fibre of N.A. = 1. It can be seen that for the simplest of Micro system R = 3r, and follows from the definition of this simplest form.

From the figure it can be seen that the greater the individual fibre diameter the longer the 'range' becomes, at the expense of the forward going and negative sensitivities. Also, as shown, when the aspect ratio R:r is constant, the maximum value of reflection factor is constant. This is demonstrated mathematically 8.1.0. and has from the graph a value of approximately 0.525. This figure has assumed a diaphragm mirror to have a reflection coefficient of unity and unless stated is considered unity throughout.

From the graph, the range for the different fibre diameters is approximately: 5μ for 10μ radius fibre; 13μ for 20μ fibre, 22μ for 30μ fibre and 39μ for 50μ fibre. It will be shown later that the separation for maximum reflection factor is proportional to the ratio of 'd/r' (8.1.1.)
FIG. 35. THE EFFECT OF FIBRE SIZE ON PROBE STATIC RESPONSE (THEORETICAL)
Thus fibre diameter is shown to effect the probe's static response characteristics, but only when considered in the context of the basic micro system where individual fibres comprise the emitter/receiver system. If more than one fibre, say 7, comprise the emitter and say 31 the annular receiver, then 'r' is the equivalent diameter of the central 7 fibres and 'R' the equivalent outside diameter of the receiving annulus so formed. Thus if 'r' and 'R' are specified, the actual fibre sizes within these bands are irrelevant, except from considerations of packing errors (mentioned earlier).

This is illustrated by FIG. 36 which shows two theoretical static calibration curves, one where the central radius is 20μ (probably a single fibre) with an annulus of three times that radius, i.e. on basic micro format; the other depicts a central emitter of 666μ also with an annulus of three times that radius. Now the manner in which it is comprised i.e. of 20, 30, etc. μ fibres is of no consequence in the formation of its response curve, which will be noticed to have an identical maximum value of reflection factor as per the smaller system, yet with a longer range and lower sensitivities.

The use of finite values for 'R' and 'r' and 'd' do give an appreciation of absolute performance, but the symmetry of various parameter combinations is masked.

If one now considers what effect fibre diameter has when using a 'single fibre' for emitting/receiving, then one can initially state in a similar manner to above, that it is of no consequence whether the 'single' fibre is comprised of one or many smaller fibres, for the effect of many smaller fibres will be to simulate the condition for one larger. Using equation 74 and again assuming the mirror/diaphragm has a reflection coefficient of unity, the response for 'single' fibres of 10, 20, 30, 50 and 500μ radius were plotted FIG. 37 and FIG. 38 (same a absciae as FIG. 35). Comparison of FIG. 38 and 35 shows that the
FIG. 36  FIBRE SIZE & EQUIVALENT FIBRE SIZE  EQUIVALENCE EFFECT
FIG. 37 THE EFFECT OF "SINGLE FIBRE" SIZE ON PROBE STATIC RESPONSE (THEORETICAL)
FIG. 38

"SINGLE FIBRE" RESPONSE - abscissae as in fig. 35.

"SINGLE FIBRE" - N.A. = 1

R = 50
R = 30
R = 20
R = 10

SEPARATION D MICRONS
20
30
40
50

R = 500 MICRONS

REFL. FACTOR
sensitivities are all negative going which is to be expected, for as 
'd' increases the reflection factor can only decrease. The comparison 
of sensitivities for various separations cannot be made accurately at 
this time, but it is a salient question to bear in mind. In addition, 
true comparisons between the single fibre case and Micro or Macro systems 
will be better accomplished later. One simple fact from FIG. 38 is that 
at near zero separation we have maximum value of reflection factor.

Thus considering the annular systems; it appears that by 
altering 'r' and 'R' but keeping R:r constant, one attains a family of 
responses with the same peak response but changing sensitivities and 
ranges. An increase of 'r' causes a decrease in sensitivity and an 
increase in range. By increasing annular radius only the peak response 
is increased with a probable reduction in sensitivity and increase in 
range.
7.1.1. EFFECT OF CHANGING NUMERICAL APERTURE OF OPTICAL FIBRES WITHIN SYSTEM

There are several values for the numerical aperture of optical fibre which are commercially available. Most commonly .866 (60° = θ); .56 (34° = θ) and also 1 (θ = 90). As the theory has been based on a Numerical Aperture of unity the next most suitable fibre would be one having a Numerical Aperture = .866. Consequently, values of N.A. of unity and .866 were used in equation 80 to determine the comparative effects on probe response for different numerical aperture fibres.

FIG. 39 shows, for the simple micro-systems shown; having central emitters (or fibres) of 20μ and 35μ radii the effect of changing from a N.A. of unity to an N.A. of .866.

From the two systems having a Numerical Aperture of unity, one can come to the same conclusions as arrived at in 7.1.0. But when the effect of changing the N.A. for one system is considered, several points can be deduced (which are implied in the simple method used for accounting for N.A. effects).

These are, that the response is delayed by a finite separation 'd'; for the 20μ case approximately 12μ, and for the 35μ case approximately 21μ. The exact value can be deduced from condition (3)(page 142) in section 6.1.3. i.e. when $d = \frac{r}{\tan θ}$ the response commences.

The resulting curve then, is formed by (with reference to 6.1.3.) the substitution for 'R' of '2d tan θ - r', until a separation is reached when the condition (1) is reached, i.e. $2d \tan θ - r = R$ when, as can be seen from FIG. 39 the curve then follows the unmodified form where N.A. = 1. This transition occurs as shown graphically, when for the 20μ = r case at approximately 23μ separation and for the 35μ = r case when the separation is approximately 40μ. The exact values of the separation for this transition being numerically from condition (1) $d = \frac{R + r}{2 \tan θ}$, which infers
that for larger values of 'R' one would further displace this transition point; where the basic (NA = 1) response curve commences.

Thus the response curve commences at a separation such that 'd' is a function of 'r', and its transition to the standard curve occurs such that 'd' is a function of 'R', both being dependent on θ.

FIG. 40 illustrates the dependence on the transition point of the radius 'R' when compared to the situation shown in FIG. 39. The former fig. shows the condition for R = 10r, and it can be seen that the transition points now lie off the scale.

It should be noted that the curves produced using the method for accounting for Numerical Aperture are in both figures (for the same 'r') identical up to the transition point (for R = 3r). This will become apparent in Chapter 8, and also that the sensitivities of a system for Numerical Aperture = 1 and Numerical Aperture = .866, differ (pre-transition point). Quantitive values are not yet shown, however, the salient feature which can be seen is that it is possible to obtain, with the lower numerical aperture system, a sensitivity equal to the high numerical aperture case but at a greater separation, although the corresponding value of reflection factor will be lower.

These two factors are quite relevant to the subsequent analysis, for a diaphragm could be placed further away, yet give a good sensitivity value, which perhaps may be a more practicable system. This feature of the effect of Numerical Aperture is further discussed in 8.1.2.

However, as has been mentioned the validity of the expression prior to the point where transition to the standard case occurs is suspect.
FIG. 39  EFFECT OF NUMERICAL APERTURE ON PROBE STATIC RESPONSE (THEORETICAL)
7.1.2. DISCUSSION

The interpretation of the effects of fibre diameter, emitter/receiver diameters and numerical aperture, as described from the results of the theoretical analysis, has been accomplished graphically. The generalised effects of changing these variables has been discussed. However, because of the many finite combinations of these parameters it is not possible (or even desirable) to plot response curves for each condition.

The main deduction which has been drawn is the significance of 'R:r' ratios for which similarities appear in response curves for similar ratios. It has also been shown that sensitivity theoretically depends on 'r'.

At this time, although finite values of sensitivity and reflection factors for various mirror-like diaphragm separations could be ascertained, no use could be made of them for a practical composite system, as the diaphragm investigation had not yet been performed, and the composite relationship between 'Fd' and diaphragm displacements for specific applied pressures has not been shown. These two relationships are 'y_D' in equation 31, and the composite function 'I' in equation 31, respectively.

The objectives of this initial interpretation of the mathematical model (Equation 80) have been accomplished. In fact the results have shown that the use of finite values for the parameters is to cloud the real appreciation of the effects of changing the parameters.

The simplified method for accounting for a system possessing an N.A. of less than unity is not very rigorous, however, due to other analytical limitations it will be seen that theoretically a system is best specified past the 'transition point'. (Pages 172 & 276)
CHAPTER 8

FURTHER DEVELOPMENTS OF THE THEORY
8.0.0. INTRODUCTION AND OBJECTIVES

A normalised expression from the reflection factor 'RF' which incorporates the dimensionless parameters 'R/r' and 'd/r' is used to clarify graphically the basic features of the optical probe which alter the response curve. The curves so produced are now termed normalised response curves. In order to determine the relationship between different combinations of 'R/r' and N.A. at different 'd/r' values for specific sensitivity values; the 'slope factor' for a system is developed and then used to show graphically the trade off in the gain in separation 'd' and loss in 'Fd' when a Numerical Aperture = 1 system is compared to an N.A. = .866.

Now because the pressure sensing diaphragm can be introduced at any initial separation 'd' then the initial reflection factor for this initial condition (i.e. differential pressure on the diaphragm on the diaphragm of zero), will be dependent on this separation 'd' and the resulting reflection factor will be termed 'F_o'. For one type of diaphragm and maximum differential pressure, the resulting maximum central deflection will be constant, thus the final reflection factor corresponding to 'F_o' will be termed 'F_p_m', the reflection factor resulting at maximum pressure. These terms will be used to show how their ratio depends on the initial separation 'd_o' and the effective deflection or movement of the diaphragm. The method used to specify the displacement of the diaphragm at any value of 'd' is to introduce a normalised displacement term \( \Delta (d/r) \) where the displacement is shown as a fraction of the normalised separation ratio 'd/r'.

The objectives are to obtain a relationship for 'F_p_m' and the initial separations at which different normalised deflections occur. These will later be related to finite diaphragm parameters. The significance of the ratio 'F_p_m/F_o' will be discussed.
8.1.0 NORMALISED REPRESENTATION OF EFFECTS OF R, r & d.

Considering the expression for \( RF \) or \( F_d \) derived in Chapter 6, equation 78 where Numerical Aperture is unity

\[
F_d = \frac{C}{2r^2} \left[ \sqrt{\frac{R^2-r^2+4}{r^2d+d^2}} - \sqrt{\frac{(R^2+r^2+4d^2)^2 - 4R^2r^2}{16d^4+4R^2d^2+8r^2d^2+8r^2d^2}} \right]
\]

then expanding

\[
F_d = \frac{C}{2r^2} \left[ \sqrt{\frac{R^2-r^2+4}{r^2d+d^2}} - \sqrt{\frac{R^4+r^4+16d^4-2R^2r^2+8r^2d^2+8r^2d^2}{R^2-r^2+4}} \right]
\]

therefore

\[
F_d = \frac{C}{2r^2} \left[ \sqrt{\frac{R^2-r^2+4}{r^2d+d^2}} - \sqrt{\frac{R^4+r^4+16d^4-2R^2r^2+8r^2d^2+8r^2d^2}{R^2-r^2+4}} \right] 
\]

now if \( 'A' = \frac{R}{r} \) (O/R by R) and \( 'B' = \frac{d}{r} \) N.B. \( A \gg 1 \)

equation 81 becomes

\[
F_d = \frac{C}{2} \left[ A^2 - 1 + 4 \sqrt{B^2+B^4} - \sqrt{A^4+1+16B^4-2A^2+8B^2(A^2+1)} \right] \quad \text{(82)}
\]

noting that \( RF \) or \( (F_d) \) is still the same function, i.e. the fraction of flux received. However, from the expression it can be seen that it is dependent on the ratio of outside to inside radii \( (R & r) \) and the separation ratio \( '(d)' \).

Taking into account the effect of N.A. as before, the conditions for this normalised case are that if \( Z_n \) is the normalised effective outside radius of collecting annulus (c.f. 6.1.3.), i.e. \( (Z) \) this replaces \( 'A' \) in equation 82.

The conditional requirements for \( Z_n \) are

\[
2B \tan \theta - 1 > A \quad Z_n = A \quad \text{(i)}
\]

\[
A > 2B \tan \theta - 1 \quad Z_n = 2B \tan \theta - 1 \quad \text{(ii)}
\]

\[
2B \tan \theta - 1 < 1 \quad Z_n = 1 \quad \text{(iii)}
\]

when

\[
F_d = \frac{C}{2} \left[ Z_n^2 - 1 + 4 \sqrt{B^2+B^4} - \sqrt{Z_n^4+1+16B^4-2Z_n^2+8B^2(Z_n^2+1)} \right] \quad \text{.......... (83)}
\]

it can be seen that if \( Z_n = 1 \)

\[
F_d = 0
\]
and from condition (iii) above, the response curve will commence when
\[ B = \frac{1}{\tan \theta} \]
which is to be expected from the graphical representation of standard response curves shown earlier. Similarly, the transition point to the standard curve occurs when condition (i) is reached
i.e. \[ B = A + 1 \frac{1}{2 \tan \theta} \]

It is in these normalised conditions that the significance of the ratio 'A' and 'B' can be shown, and that the finite value of 'r' is of prime importance to sensitivity. With \( \theta = 90^\circ \) the equation 83 reduces to the standard form equation 82, for which FIG. 41 shows the normalised response curves for 'A' values of \( \sqrt{2}, 3, 5, 10, 20 \) and FIG. 42 using a different scale for \( A = 3, 10, 50 \) and 100. Two salient features can be deduced; these are

1) increasing the value of 'A':- the maximum reflection factor possibly increases, the range apparently maintains approximately a good positive going sensitivity (section 8.1.2.). This assumes 'r' is constant.

2) the effect of changing 'r' for any 'A' value is to alter the separation at which peak response is achieved (its value remaining the same) with the consequence of altering the sensitivity. Thus the true separation for a specific reflection factor is obtained using the 'd/r' value and the finite value of 'r'.

The normalised response expression for the 'single fibre' system can be developed similarly from equation 79 which is
\[ F_d = \frac{C}{2R^2} \left[ \frac{2R^2 + 4d^2 - 4 \sqrt{R^2d^2 + d^4}}{R^2d^2 + d^4} \right] \]
when for this case, 'ratio 'A' is of no consequence, i.e. always unity and 'B' is introduced as before equal to 'd/R' where 'R' is the common radii
\[ F_d = \frac{C}{2} \left[ 2 + 4B^2 - 4 \sqrt{B^2 + B^4} \right] \]
FIG. 41  NORMALISED STATIC RESPONSE CURVE 1 (THEORETICAL)
A \left( \frac{B}{r} \right) = 100

A = 50

A = 10

A = 3

FIG. 42
NORMALISED STATIC RESPONSE CURVE 2 (THEORETICAL)
This expression is shown graphically in FIG. 43 where it can be seen that

1) peak reflection factor occurs at contact position and if \( C = 1 \),
   \( F_d = 1 \)

2) only the finite size of \( 'R' \) affects the response curve, and only changes the sensitivity of the response.

Later, the specific value for sensitivity (slope factor) at different 'd/r' values will be compared to that of the annular condition response (Ch. 19).

If account is taken now for the effect of numerical aperture for the annular situation (micro or macro), then by using equation 83, FIG. 44 shows for two values of \( 'A' \) (\( R/r \)) 10 & 3 the normalised response curves when the Numerical Aperture of the fibres are 1\( (90^0) \); .9397\( (70^0) \); .866\( (60^0) \); .7071\( (45^0) \) and .5\( (30^0) \).

This shows that by increasing Numerical Aperture the positive going sensitivity increases, with a transition point of \( 'd/r' = A + 1 \), and is thus dependent on \( 'A' \) and \( 'r' \). As for the previous cases described, the sensitivity is dependant on \( 'r' \), all other things being constant.

The interesting feature is that from FIG. 44 it appears that one can increase the 'd/r' separation ratio for a specific \( 'A' \) value when decreasing Numerical Aperture, and still obtain a similar sensitivity (precise values given in 8.1.2.) assuming \( 'r' \) is constant. FIG. 45 shows further cases where \( 'A' \) is 3, 10, 50 and 100 and the x axis scale extends to 10.

All the normalised response curves plotted confirm the significance of \( 'R/r' \) and \( 'r' \) on the characteristics of the probe, and lend themselves to convenient application to a composite relationship between diaphragm/optical probe.

A method for quantifying, in a normalised manner, the sensitivity at any separation ratio 'd/r' is discussed in the next section. This is in order to investigate whether there are advantages in using lower Numerical...
FIG. 43 NORMALISED STATIC RESPONSE CURVE-SINGLE FIBRE (THEORETICAL)
Aperture fibres; and also to obtain a more accurate idea of sensitivity values. Although it should be noted from Figures 44 and 45 that up to 'A' of say 30 with θ = 60° a serious discontinuity exists at the transition point and consequently the simplified method for accounting for the effects of Numerical Aperture must be questioned. However, in Chapter 11 another system factor will show that the taking into account of Numerical Aperture prior to transition point is not theoretically possible and consequently Numerical Aperture considerations, up to the condition when the afferent fibres are completely covered by efferent light, are not required.
8.1.1. SLOPE FACTOR

Let the 'slope factor' be the normalised sensitivity value at any separation ratio \( d/r \), \( B \). Then using the basic normalised equation for \( F_d \) (equation 82) which is

\[
F_d = \frac{C}{2} \left[ \frac{A^2 - 1 + 4 \sqrt{B^2 + B^4}}{4} - \frac{A^4 + 1 + 16B^4 - 2A^2 + 8B^2(A^2 + 1)}{2} \right]
\]

the expansion for slope factor can be determined as slope factor

\[
\frac{d(F_d)}{dB} = \frac{d(F_d)}{d(d)}
\]

noting \( B = \frac{d}{r} \)

Thus

\[
\frac{1}{r} \frac{d(F_d)}{dB} = \frac{d(F_d)}{d(d)}
\]

in other words the actual value of sensitivity at a separation 'd' is '1/r' times the slope factor.

Now \( \frac{d(F_d)}{dB} = \frac{C}{2} \left[ \frac{4}{2} \left( \frac{(B^2 + B^4)^{-\frac{1}{2}}}{2} - \frac{(A^4 + 1 + 16B^4 + 8B^2(A^2 + 1))^{-\frac{1}{2}}}{2} \right) \right] \]

\[
\times (64B^3 + 16B(A^2 + 1)) \]

To obtain an expression for the slope factor which accounts for a Numerical Aperture other than unity equation 83 must be considered

\[
F_d = \frac{C}{2} \left[ \frac{Z_n^2 - 1 + 4 \sqrt{B^2 + B^4}}{4} - \frac{Z_n^4 + 1 + 16B^4 - 2Z_n^2 + 8B^2(1 + Z_n^2)}{2} \right]
\]

from which the reflection factor \( F_{NAd} \) for the modified response occurs

when \( Z_n = 2B \tan \theta - 1 \)

therefore \( F_{NAd} = \frac{C}{2} \left[ \frac{(2B\tan \theta - 1)^2 - 1 + 4 \sqrt{B^2 + B^4}}{4} - \frac{(2B \tan \theta - 1)^4}{1 + 16B^4 - 2(2B \tan \theta - 1)^2 + 8B^2 + 8B^2(2B \tan \theta - 1)^2} \right] \]

thus the slope factor \( \frac{d(F_{NAd})}{dB} \)

\[
= \frac{C}{2} \left[ \frac{4 \tan \theta (2B\tan \theta - 1) + 2(B^2 + B^4)^{-\frac{1}{2}} (2B + 4B^3)}{2B + 4B^3} \right]^{-\frac{1}{2}} \left[ (2B\tan \theta - 1)^4 + 1 + 16B^4 - 2(2B\tan \theta - 1)^2 + 8B^2 + 8B^2(2B\tan \theta - 1)^2 \right]^{-\frac{1}{2}} \left[ 8 \tan \theta (2B\tan \theta - 1)^3 + 64B^3 - 8 \tan \theta (2B\tan \theta - 1)^3 + 16B + 16B (2B \tan \theta - 1)^2 + 32B^2 (2B \tan \theta - 1)^2 \tan \theta \right]
\]
again noting that
\[
\frac{d(F_{\text{NA}})}{d(B)} = \frac{d(F_{\text{NA}})}{d(d)}
\]
and that 'slope factor' is the rate of change of reflection factor with the initial separation ratio and is dimensionless, and also that equation 86 is now specific and is related only to Numerical Aperture of less than unity; equation 85 being the 'normal' case only for NA = 1.

If the 'single fibre' system is now considered for the normal case, the slope factor can be found using equation 84.

\[
F_d = \frac{C}{2} \left[ 2 + 4B^2 - 4 \sqrt{B^2 + B^4} \right]
\]

therefore
\[
\frac{d(F_d)}{d(B)} = \left[ \frac{C}{2} \left( 8B - 2(B^2 + B^4)^{-\frac{1}{2}} (2B + 4B^3) \right) \right]
\]

Using the expression derived for slope factor equation 85 and 86, a graph of slope factor against separation ratio B was plotted, FIG. 46; for three conditions of 'A' (3, 10, & 50) and also three conditions for the N.A. (1; .866 & .707). Explanation of this figure will enable the following more complex plots to be appreciated more easily. Considering first the situation shown with A = 3 and NA = 1, the x intercept (slope factor = 0) signifies the separation for peak response and by definition the range. Consequently, the range is readily determined from this form of representation. If one now compares the standard situations depicted i.e. NA = 1, when A = 3, 10 & 50, it can be seen that as 'A' increases it is possible to achieve an identical slope factor yet possess a greater separation ratio. This is perhaps an advantage when considering practical construction of a pressure transducer. However, reference to the reflection factors achieved is advocated in order to assess fully what the actual reflection factor is for these conditions. One can assess this with reference to FIG. 44, where one considers for example B = .4 and A = 3 at which \( F_d = \text{approx.}.45 \), then referring to FIG. 46 one can determine what the slope factor is for this condition (B = .4 A = 3) which is shown to be about .5. Reading across to the A = 10 condition this slope factor...
FIG. 46 SLOPE FACTOR AGAINST SEPARATION RATIO "B"
(c.f. fig. 44)
is achieved with $B = .60$. Using this value for $B$ the corresponding value for reflector factor from FIG. 44 is .66. In other words, the same slope factor can be achieved at a greater separation ratio by increasing the $'A'(R)$ ratio and also results in a better reflection factor. To simplify this comparative procedure relating slope factor and reflection factor, composite graphs will later be shown.

FIG. 46 also shows the effect of accounting for the fibre's N.A. Two conditions are shown: NA = .866 and .707. As described earlier the transition points where these curves re-join (or follow) the standard curve (NA = 1) can be computed as $B = \frac{A + 1}{2\tan\theta}$, and it can be seen that when $A = 3$ there is a large discontinuity of slope factor; but as $'A'$ increases, so the discontinuity decreases. Also that as the N.A. decreases so it appears that the same slope factor can be attained at a much greater separation ratio.

The next section discusses further cases in more detail. However, from FIG. 46 it can be seen that the positive values of slope factor have a maximum value of 2 and a minimum value of about -.2. These values will be of use when the diaphragm parameters are considered together with the probe response characteristics.
However, let us now consider the conditional value for the separation ratio 'B', which gives a peak value for the reflection factor when using the standard form, i.e. N.A. = 1. Its value can be assessed from say FIG. 45 and can be seen to be a function of the 'A' value used. Its precise value is to be found now, using the differentiation technique for maximum and minima determination.

Now the first derivative of 'reflection Factor' is 'slope factor' and is shown by equation 85, which can be written

$$\frac{dF_d}{dB} = \frac{c}{2} \left[ 2(B^2 + B^4)^{-\frac{1}{2}} (2B + 4B^3) - \frac{1}{2} (A^2 - 1)^2 + 16B^4 + 8B^2(A^2 + 1) \right] ... \quad 86a$$

Using the substitution $X = A^2 - 1$ and $Y = A^2 + 1$ and equating the above to zero for determination of the turning point(s), noting that the conditions for 'A' are such that $A > 1$ and $B$ is always positive

$$0 = 2(B^2 + B^4)^{-\frac{1}{2}} (2B + 4B^3) - \frac{1}{2} (X^2 + 16B^4 + 8B^2Y)^{-\frac{1}{2}} (64B^3 + 16BY)$$

$$\frac{2(4B^3 + BY)}{(X^2 + 16B^4 + 8B^2Y)^{\frac{1}{2}}} = \frac{1 + 2B^2}{(1 + B^2)^{\frac{1}{2}}}$$

$$4(4B^3 + BY)^2(1 + B^2) = (1 + 2B^2)^2 (X^2 + 16B^4 + 8B^2Y)$$


$$64B^6 + 32B^4Y + 4B^2Y^2 + 64B^8 + 32B^6Y + 4B^4Y^2 = X^2 + 16B^4 + 8B^2Y + 4B^2X^2 + 64B^6$$

$$+ 32B^4Y + 4B^4X^2 + 64B^8 + 32B^6Y$$

$$B^4(4Y^2 - 4X^2 - 16) + B^2(4Y^2 - 4X^2 - 8Y) - X^2 = 0$$

but $Y - X^2 = (A^2 + 1)^2 - (A^2 - 1)^2 = 4A^2$

Therefore considering the terms within the brackets of the equation to be solved

$$4Y^2 - 4X^2 - 16 = 16(A^2 - 1)$$

$$& 4Y^2 - 4X^2 - 8Y = 8(A^2 - 1)$$
Therefore
\[ 16(A^2 - 1)B^4 + 8(A^2 - 1)B^2 - (A^2 - 1)^2 = 0 \]
\[ (A^2 - 1)^2 \left[ 16B^4 + 8B^2 - (A^2 - 1) \right] = 0 \]

now as \( A > 1 \) \( \rightarrow \) \( A^2 - 1 \neq 0 \)

then \( A^2 - 1 \neq 0 \)

thus \( 16B^4 + 8B^2 - (A^2 - 1) = 0 \)

which may be written
\[ (4B^2)^2 + 2(4B^2) - (A^2 - 1) = 0 \]
\[ 4B^2 = \frac{-2 \pm \sqrt{4 + 4(A^2 - 1)}}{2} \]
\[ 4B^2 = -1 \pm A \]

now as \( A > 1 \) and \( B \) is positive.
\[ 4B^2 = A - 1 \]
\[ B^2 = \frac{A - 1}{4} \]
\[ B = \sqrt{\frac{A - 1}{2}} \]

Thus for any value of \( 'A' \) i.e. \( \frac{R}{r} \), the separation ratio at which peak reflection factor occurs can be found from equation 87. The following list displays the values for this separation ratio \( B_{\text{peak F}} \) for several values of \( 'A' \).

<table>
<thead>
<tr>
<th>'A'</th>
<th>( B_{\text{peak F}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sqrt{3} )</td>
<td>0.427</td>
</tr>
<tr>
<td>3</td>
<td>0.707</td>
</tr>
<tr>
<td>5</td>
<td>1.0</td>
</tr>
<tr>
<td>10</td>
<td>1.5</td>
</tr>
<tr>
<td>20</td>
<td>2.18</td>
</tr>
<tr>
<td>50</td>
<td>3.5</td>
</tr>
<tr>
<td>100</td>
<td>4.97</td>
</tr>
</tbody>
</table>

Let us now consider the conditions which determine whether the transition point lies before, at, or after the position where peak reflection factor is obtained for the standard form i.e. (N.A. = 1).
Now the separation ratio for transition is

\[ B_{trans} = \frac{A + 1}{2 \tan \theta} \]

and for peak reflection factor

\[ B_{peak F} = \sqrt{\frac{A - 1}{2}} \]

with one proviso \( A > 1 \)

1) For coincidence of \( B_{trans} \) & \( B_{peak F} \)

\[ \frac{A + 1}{2 \tan \theta} = \sqrt{\frac{A - 1}{2}} \]

\[ \tan^2 \theta (1 - A) + (A + 1)^2 = 0 \]

Therefore

\[ \tan^2 \theta = \frac{(A + 1)^2}{A - 1} \]

\[ \tan \theta = \frac{A + 1}{\sqrt{A - 1}} \]

This conditional relationship for \( \theta \) (the numerical aperture angle) and \( A \) (the \( R \) ratio) is shown plotted graphically by FIG. 46a, from which any pair of co-ordinates of the resulting curve give rise to coincidence of the separation ratio at which transition occurs, and the separation at which a peak value of reflection factor occurs (when N.A. = 1).

From this curve it can be seen that if \( \theta < 70^\circ \) then irrespective of the \( 'A' \) value, coincidence cannot occur, mathematically this condition can be found by solving the initial equation for \( 'A' \) in terms of \( \theta \).

2) For \( B_{trans} \) to be greater than \( B_{peak F} \) or less than \( B_{peak F} \).

If the separation ratio for transition is greater than that where peak reflection factor occurs (when N.A. = 1) then

\[ \frac{A + 1}{2 \tan \theta} = \sqrt{\frac{A - 1}{2}} \]

will have a real positive value. If the reverse is true the above value will be negative.
FIG. 46a

GRAPHICAL REPRESENTATION OF THE CONDITIONS EFFECTING THE TRANSITION AND PEAK REFLECTION FACTOR SEPARATION RATIO.

"B_t < B_{peak F}"

"B_t = B_{peak F}"

"B_t > B_{peak F}"

FIBRE NUMERICAL APERTURE (DEGREES) θ

"A" ratio (R/r)
Now the graph shown as FIG. 46a depicts what is, in effect, a 'mapping' of a boundary line surface between when the difference, described as \( \frac{A + 1}{2 \tan \theta} - \frac{\sqrt{A - 1}}{2} \), is positive and negative, i.e. equal to zero. Consequently, logically any pair of '\( \theta \)' & 'A' values forming a co-ordinate of the total 'surface' shown, will either result in a negative or positive value for the difference \( B_{\text{peak F}} - B \) dependent on whether the point lies in the surface, above or below the boundary.

With respect to FIG. 46a, consider the two pairs of co-ordinate points, \( P_1 & P_2 \), \( P_3 & P_4 \); where \( P_1 \) & \( P_3 \) lie below the boundary and \( P_2 \) & \( P_4 \) lie above the boundary.

The co-ordinate values being

<table>
<thead>
<tr>
<th>( \theta )</th>
<th>( A )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_1 )</td>
<td>72.5</td>
</tr>
<tr>
<td>( P_2 )</td>
<td>85.0</td>
</tr>
<tr>
<td>( P_3 )</td>
<td>72.5</td>
</tr>
<tr>
<td>( P_4 )</td>
<td>85.0</td>
</tr>
</tbody>
</table>

Let us now determine whether the difference \( B_{\text{peak F}} - B \) is negative or positive for the 'points' chosen (any such points could have been picked).

Again the difference is

\[
B_{\text{peak F}} - B = \frac{A + 1}{2 \tan \theta} - \frac{\sqrt{A - 1}}{2}
\]

Now for \( P_1 \):

\[
\begin{align*}
B_{\text{peak F}} - B & = \frac{17}{2 \tan 72.5} - \frac{\sqrt{1.5}}{2} \\
& = + 0.743
\end{align*}
\]

For \( P_3 \):

\[
\begin{align*}
B_{\text{peak F}} - B & = \frac{2.5}{2 \tan 72.5} - \frac{\sqrt{1.5}}{2} \\
& = + 0.040
\end{align*}
\]

For \( P_2 \):

\[
\begin{align*}
B_{\text{peak F}} - B & = \frac{17}{2 \tan 85} - \frac{\sqrt{1.5}}{2} \\
& = - 1.192
\end{align*}
\]

For \( P_4 \):

\[
\begin{align*}
B_{\text{peak F}} - B & = \frac{2.5}{2 \tan 85} - \frac{\sqrt{1.5}}{2} \\
& = - 0.244
\end{align*}
\]
When it can be seen that 'points' lying below the boundary line within the resulting surface give values of $B_t$ to be greater than $B_{\text{peak} F}$, and if the point lies above the boundary line for $B_t$ to occur prior to $B_{\text{peak} F}$.

Thus for any values of $\theta$ and $A$ it is possible to rapidly determine the relative values of $B_t$ and $B_{\text{peak} F}$. The significance of such a determination becomes clear much later in Chapter 14 where a total system theoretical appraisal is given. However, it can be seen now that if $B_t$ occurs before $B_{\text{peak} F}$ then the standard form of the reflection factor expression encompasses both a positive and negative slope portion of the static response curve.
8.1.2. GRAPHICAL REPRESENTATION OF SLOPE FACTOR, SEPARATION RATIO AND REFLECTION FACTOR

In order to simplify the comparative procedure outlined for ascertaining the characteristics for different types of probes having the same slope factor, the following graphs have been plotted. Each is composed of two relationships, namely slope factor against separation ratio; and slope factor against reflection factor. The latter relationship was formed by a programming procedure using equations for reflection factor and separation ratios and slope factor and separation ratios. The 'y' abscissa being slope factor for each of these curves.

Following the two FIGS. 47 and 48 which depict slope factor as the dependent variable, there follows several figures where finite sensitivity is depicted against finite separation and reflection factor, which enable a precise comparison for different system features to be ascertained.

Consider now FIG. 47 which shows for two conditions of $R/r(A)$ i.e. 3 and 10, the relationship between slope factor, separation ratio and reflection factor when N.A. = 1, and .866. The curves have been labelled alphabetically, and let 'A' be curve as marked A, nor 'A' as R. Then 'A' depicts $R = 3$, N.A. = 1; 'B' $R = 10$, N.A. = 1; 'C' R = 3, N.A. = .866; 'D' $R = 10$, N.A. = .866.

Consider a comparison between 'A' and 'B'. It can be seen that for any positive value of slope factor this value can be achieved at a greater separation ratio for curve 'B' with a correspondingly greater value of reflection factor. Consider such a comparison when the slope factor is .38. With curve 'A' the separation ratio needed is about .49 and gives a corresponding reflection factor of about .5. This same slope factor considering curve 'B' gives a required separation ratio of .75 and a corresponding reflection factor of about .75. Therefore, in this case by using an $R/r$ of 10 instead of 3 one obtains an increase of about 50% in separation ratio and reflection factor. It can be seen
also that if greater slope factors are compared the increase mentioned reduces. Similarly as lower slope factors are compared there is an increase in parameters 'B' and 'F_d'.

The negative slope comparison is more confusing as there will be a duality of slope factors with their corresponding values of separation ratio and reflection factor. The most convenient method to resolve this duality graphically, is to realise that as slope factor decreases an anti-clockwise route is taken; this will be compared to the value of reflection factor, where a clockwise route is followed. Again with respect to FIG. 47, if a desired slope factor is -.01, using curve 'A' the separation ratio required is .8 leading to a reflection factor of .53, compared with for 'B' a separation ratio .24 and a reflection factor of .74.

It can also be seen by reversing this comparative procedure that the duality of reflection factors possible always gives the positive going slope factor a higher value than the negative going slope factor value; a feature which has been discussed earlier.

Considering now curves 'C' and 'D' which depict the effect of N.A. These curves, as mentioned earlier form part of the resulting description for a system with a fibre having an N.A. of a value less than unity, for, 'conditionally' they revert back to the standard form. These transition points have been marked 't_1' and 't_2'. Where 't_1' is the transition point for N.A. = .866; A = 3 and 't_2' when A = 10. Thus it can be seen that 'C D' is a continuum dependent on the value of 'A'. The transition point 't_1' is followed by 't_1' (on the slope factor/separation ratio graph), and similarly for the reflection factor graph.

Let the effects of using an N.A. = .866 be compared to the A = 3 and 10 case, (N.A. = 1), which themselves have been compared. Then, as before, if the slope factor considered is .38 then the condition prior to 'transition' for A = 10 (curve 'D') is met and also this condition is just met for curve 'C', i.e. A = 3. Consequently, the effect of 'A' for this value of slope factor is of no consequence. Now the separation ratio
necessary with $\theta = 60^\circ$ becomes 1.1 and gives rise to a reflection factor of .46. Thus the effect is to increase the maximum separation ratio required when compared to $A = 10$ with N.A. = 1 but reduce the minimum reflection factor when $A = 3$. In the case where the slope factor considered lies before the transition point, the effect of different $A(R)$ values is of no consequence, and generally the separation ratio necessary is larger than the largest 'A' value compared, and also the reflection factor smaller than for that value when the lowest 'A' value is considered.

It is worthwhile comparing the case of the single fibre case in standard form FIG. 48 which shows a normalised static response curve together with a 'slope factor curve', with those for the annular condition (FIG. 47). The curves FIG. 48 can be seen to be unique, that is, only dependent on the implicit value of 'r'. For separation ratios of less 1, comparison of the slope factors obtained, shows that they are very similar except that for the single fibre case they are negative. At this stage there would not appear to be any reason for preferring one system to another; critiques by which choice can be exercised will be discussed later. Also from FIG. 47, it can be seen that for both values of 'A', the slope factor is nearly a linear function of separation ratio, up to say a value of .5 for 'B'.
FIG. 47  COMBINED GRAPH OF SLOPE FACTOR AGAINST SEPARATION RATIO AND REFLECTION FACTOR
FIG. 48  COMBINED GRAPH OF SLOPE FACTOR AGAINST SEPARATION RATIO AND REFLECTION FACTOR
FOR THE SINGLE FIBRE CASE
8.1.2.1. FINITE VALUES OF SLOPE FACTOR (SENSITIVITY)

The manner in which ratios \( \frac{d}{r} \) and \( \frac{R}{r} \) effect system performance has been discussed. It will prove to be useful to specify finite values for 'r' to obtain an idea of actual sensitivities and separations which will be used later.

Consider FIG. 49 which shows actual sensitivity against separation and reflection factor for the simplest \( \mu \) system, i.e. \( A = 3 \). Where 'r' has values of 20\( \mu \) and 35\( \mu \) and N.A. of 1 and .866. These fibre sizes and Numerical Apertures are commercially available and manageable. It can be shown that considering the standard form (N.A. = 1) for both fibres there is a dilemma as to which would be more suitable for these curves exhibit a cross over point (x) on the sensitivity separation curves; which means that above the cross over point an identical sensitivity can be attained using a smaller fibre and allow a much greater value in reflection factor to result. If the sensitivity chosen falls below (x), the separation feature is reversed and the difference in reflection factor attained diminishes.

To illustrate this condition consider a high sensitivity value of \( 40 \left[ \frac{d(F)}{d(F)} \right] \) dF/mm, the separation in \( \mu \) that this achieves is 6 with 'r' = 20 and 4.5 with 'r' = 35; the corresponding values for reflection factor being .4 and .22 respectively. Now let the value of sensitivity chosen be .13 as shown, then for the 20\( \mu \) radius fibre the required separation is 10.5 and 15 for the 35\( \mu \) radius fibre. The corresponding reflection factor being .51 and .48 respectively.

The features which will determine the choice of optical system, are required sensitivity, limitations on diaphragm location and the fibres' diameter. The scales shown for sensitivity are more usefully stated as \( \left[ \frac{dF \times 10^{-3}}{\text{micron}} \right] \), and dependent on the initial value of RF, one can arrive at a value of \( \%RF \) change/micron.
FIG. 49

GRAPH OF SENSITIVITY AGAINST SEPARATION AND REFLECTION FACTOR
(MICRO SYSTEM-THEORETICAL)
An important feature to be later discussed is the linearity of the sensitivity/separation curve, or its curvature, which can be seen to be linear up to a separation of about 10μ and 15μ for the 20 and 35μ cases. FIG. 49 depicts what are in effect 'μ' arrangements. It is very informative to consider FIG. 50 which shows a μ system's characteristics compared with a Macro system's. In both cases A = 3 but for one, \( r' = 20μ \) and, the other has \( r' = 330μ \), inferring that the outside diameter of the probe is approximately 2 mm. Considering the situation when N.A. = 1, i.e. curves 'A' and 'B' it can be seen that a much higher value of sensitivity can be achieved with the μ system (curve A) than the macro system (curve B), and that the rate of change of sensitivity with separation is very large when compared to that of ('B'). It will be shown that for a practical pressure transducing system the sensitivity required necessitates a form of μ system.
FIG. 50 GRAPH OF SENSITIVITY AGAINST SEPARATION AND REFLECTION FACTOR, SHOWING COMPARISON BETWEEN MICRO AND MACRO SYSTEMS—THEORETICAL
8.1.3. DISCUSSION

Using a normalised equation for reflection factor, the effect of different \( R/r \) (A) values on probe response has been shown. The greater the \('A'\) value the greater the peak reflection factor obtained and the larger the range becomes the response still maintains a good forward going sensitivity (and imparting certain advantages – see below). Changing \('r'\) for any \('A'\) value alters the separation at which peak response is attained, consequently altering the range and sensitivity.

A measure of the sensitivity at any separation has been introduced for normalised response curves, namely slope factor. The combined slope factor against separation ratio and reflection factor graphs have been shown, from which it can be seen that by increasing \('A'\), the separation ratio \('B'\) can be increased for an identical slope factor as that obtained for the original \('A'\) value. The resulting reflection factor would also be increased.

Using a simple method for accounting for the Numerical Aperture can be seen to be questionable for low \( R/r \) values, as the slope factor at transition point changes markedly. However, from FIG. 47 the effect of a lower N.A. is to allow a greater separation \( \frac{d}{r} \) ratio for a given slope factor, but impart a lower reflection factor. This effect is the same as that obtained when comparing a micro to macro system on a finite basis, i.e. an \('A'\) ratio which is constant where \('r'\) is in one case small and in the other case large. In such a situation the \( \mu \) system has a high sensitivity value at a small separation, whereas the macro system gives a low sensitivity throughout its range, although peak value of reflection factor will be identical.

The single fibre case is rather interesting, for as shown, it gives values of slope factor similar to the other systems described. It is of the Macro type and consequently is independent of the actual fibres composing it. Now for reasons to be discussed later it would not be
practicable to produce a probe of less than 1.5 mm diameter across the fibres of the distal group (primarily due to coupling of a conventional light source to the efferent fibres, however, if a true laser source is used the following comments are invalidated (Ch. 19). Consequently the minimum 'R' value will be of the order of .75 mm. Then using FIG. 48 with the value of slope factor at .5 = B is .75 with $R = \frac{750}{\mu}$ the sensitivity becomes .75, approximately .001 which is at least 1000 times less than a $\mu$ system's possible sensitivity (also see FIG. 35).

The main conclusion which can be drawn is that there are several systems which give similar characteristics, i.e. slope factor and reflection factor at certain separation ratios. The intention of this section has been to enable the 'rules' for specification of alternative systems to be understood. The forementioned graphs will be used to help specify probe characteristics when plotting the results of a mathematical model for the diaphragm/probe system (11.2.0.).
8.2.0. REFLECTION FACTOR TOTAL RATIO \( \frac{F_{pm}}{F_0} \) AND NORMALISED DEFLECTION RATIO \( \Delta B \)

As a prelude to actually considering the diaphragm and diaphragm/optical probe system, and also the overall system resolution, two further concepts are to be introduced and are both closely concerned with the final practical system.

Let us consider a stylised diaphragm to be placed in position (1) such that \( B = 0.5 \) as shown in FIG. 41 and assume that this is its initial position when it has a zero differential pressure applied to it. If \( A = 3 \) the resulting reflection factor is seen to be 0.505, let this be termed \( F_0 \) i.e. the reflection factor achieved at zero applied differential pressure.

Now consider the diaphragm to be placed in position (2) such that \( B \) is now 0.25 and is thus a value of \( \Delta B = 0.25 \) less than before, the resulting reflection factor is seen to be 0.37. This value is termed the \( F_{pm} \) reflection factor and is defined as the resulting value when the diaphragm has been deflected due to the maximum applied differential pressure causing a normalised deflector ratio of \( \Delta B \).

Thus, from situations shown 1 & 2

\[
\frac{F_{pm}}{F_0} = \frac{0.37}{0.505} \approx 0.73 \quad A = 3 \\
\Delta B = -0.25 \\
B = 0.5
\]

The above ratio is less than unity; if situations 3 & 4 are now considered

\[
\frac{F_{pm}}{F_0} = \frac{0.34}{0.29} \approx 1.17 \quad A = 3 \\
\Delta B = -0.25 \\
B = 2
\]

(also see FIG. 53, curve C)

Thus dependent on which side of the normalised response curve the 'action' takes place, so the ratio of \( \frac{F_{pm}}{F_0} \) may be less (L.U.) or greater than unity (G.U.)
It will be of importance later to define what the conditions are for equivalence of \( \frac{F}{F_0} \) for less the unity (L.U.) and greater than unity (G.U.) and their significance.

The obvious geometrical conditions as can be seen from conditions 1 & 2 and 5 & 6 show that a geometric equivalence occurs when

\[
\frac{1}{\frac{F}{F_0} \text{ (L.U.)}} = \frac{F_{\text{pm}}}{F_0} \text{ (G.U.)}
\]

which, incidentally for this situation, necessitates a greater \( 'B' \) value, for the G.U. case ( \( 1.7 = B \) for the case mentioned).

Looking further into the significance of the ratio \( \frac{F}{F_0} \) it can be seen that in terms of the change in reflection factor to initial value.

For less than unity case

\[
\frac{F_0 - F_{\text{pm}}}{F_0} = 1 - \frac{F_{\text{pm}}}{F_0}
\]

and for greater than unity case

\[
\frac{F_{\text{pm}} - F_0}{F_0} = \frac{F_{\text{pm}} - 1}{F_0}
\]

Thus, when considering features of the system where the change in reflection factor to original value are relevant, equivalence effects occur when

\[
\left[ \frac{F_{\text{pm}}}{F_0} \right] \quad \left( \ll 1 \right) = \left[ 2 - \frac{F_{\text{pm}}}{F_0} \right] \quad \left( \gg 1 \right)
\]

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Whilst when considering features of the system where the ratio is the dependent feature, equivalence occurs when

\[
\left[ \frac{F_{\text{pm}}}{F_0} \right] \quad \left( \ll 1 \right) = \left[ \frac{1}{\frac{F_{\text{pm}}}{F_0}} \right] \quad \left( \gg 1 \right)
\]

Please note Chapter 12 applies these equivalence conditions to the total system.
It is possible to use the results from the previous section dealing with slope factor, to arrive at \( \frac{F_{pm}}{F_o} \) ratio, etc. However, it is of more practical interest to directly consider this ratio from equation 89 which is yet to be introduced.

The significance of \( \frac{F_{pm}}{F_o} \) for different normalised deflection ratios \( \triangle(d) \) is that as \( F_{pm} \) is the reflection factor resulting from the maximum differential pressure to which the diaphragm is subjected. The total change in \( \frac{F_{pm}}{F_o} \) must then be resolved into smaller fractions dependent on the pressure resolution of the system required. The utilisation of different 'A' values will also be investigated. Later the use of finite 'r' values and then actual deflection values as \( \triangle d \) values will be shown.
8.2.1. THEORETICAL DESCRIPTION OF RELATIONSHIP BETWEEN $\frac{F_{pm}}{F_0}$ & $\triangle(\beta)$

Considering the standard form expression for reflection factor

(82) we have

$$F_d = \frac{C}{2} \left[ A^2 - 1 + 4 \sqrt{B^2 + B^4} - \sqrt{A^4 + 1 + 16B^4 - 2A^2 + 8B^2(A^2 + 1)} \right]$$

if $B_o$ is the separation ratio, let us call it 'initial separation ratio' where differential pressure across the diaphragm is zero and if $\triangle B$ is the normalised deflection ratio

$$\frac{F_{pm}}{F_0} = A^2 - 1 + 4 \sqrt{B_o^2 + B_o^4} - \sqrt{A^4 + 1 + 16B_o^4 - 2A^2 + 8B_o^2(A^2 + 1)}$$

For convenience let $B_{pm}$ be $B_o - \triangle B$

thus

$$\frac{F_{pm}}{F_0} = \frac{A^2 - 1 + 4 \sqrt{B_{pm}^2 + B_{pm}^4} - \sqrt{A^4 + 1 + 16B_{pm}^4 - 2A^2 + 8B_{pm}^2(A^2 + 1)}}{A^2 - 1 + 4 \sqrt{B_o^2 + B_o^4} - \sqrt{A^4 + 1 + 16B_o^4 - 2A^2 + 8B_o^2(A^2 + 1)}}$$

The above description for reflection factor total ratio is readily programmable and enables graphs depicting the effect of different normalised deflection ratios and initial positions, on $\frac{F_{pm}}{F_0}$ for specific values of $A$, to be formulated.
8.2.2 GRAPHICAL REPRESENTATION OF THE EFFECTS OF NORMALISED DEFLECTION RATIO, INITIAL SEPARATION RATIO AND 'A' (R/r) RATIO ON REFLECTION FACTOR TOTAL RATIO \( \frac{F_{pm}}{F_0} \)

The following figures referred to in this section were obtained from equation 89 (considering fibres of N.A. of unity), using an incremental programming procedure. First, for each separation ratio, the \( F_0 \) value was calculated, then the \( F_{pm} \) value when the quoted normalised deflection ratio was subtracted from the initial separation ratio. This procedure was then repeated with an incrementally small decrease in initial separation, such that the curves depict what the \( \frac{F_{pm}}{F_0} \) ratio would be at any initial separation ratio for given normalised deflection ratio.

As an introduction to this section refer to FIG. 51 and 52 which both depict for two 'A' (R/r) values when the Numerical Aperture of the fibres is unity, the effect of different normalised deflection ratio on \( \frac{F_{pm}}{F_0} \).

From these can be deduced, the effect of changing 'A', \( \Delta(d/r) \) being constant and the effect of changing \( \Delta(d/r) \) 'A' being constant, for any separation 'd/r'.

The significance of the ratio \( \frac{F_{pm}}{F_0} \) is that for ratios less than one, the lower the ratio the greater the change in reflection factor due to the effect of the normalised deflection ratio, and vice versa for \( \frac{F_{pm}}{F_0} \) values greater than unity.

Now considering FIG. 51 where \( \Delta B \) at each initial separation ratio B is .75, it can be seen that up to the boundary condition \( \frac{F_{pm}}{F_0} = 1 \) the separation ratio 'B' can be greater for A = 10 compared to A = 3 for the same \( \frac{F_{pm}}{F_0} \) ratio, also that at the same separation ratio 'B' a lower \( \frac{F_{pm}}{F_0} \) is attained with the larger 'A' ratio. This can also be seen from
FIG. 51: FPM/FO AGAINST SEPARATION RATIO "B"
(THEORETICAL)
FIG. 52 where $\triangle B = .4$. Now if the two figures are compared, it can be seen that, as is to be expected, for one 'A' ratio the effect of decreasing $\triangle B$ is to increase the $F_{pm}$ ratio at any separation ratio up to where the boundary condition $F_{pm} = 1$, thence to decrease it. The 's' symbol on each of the curves denotes the transition point where the curve (not shown) for accounting for Numerical Aperture rejoins the standard.

FIG. 53 shows how for one 'A' ratio the reflection factor total ratio is effected by assumed different normalised deflection ratios. The value of the $F_{pm}$ ratio at a separation ratio of 2 and .5, for $\triangle B$ of .25 is shown to illustrate the conclusions discussed earlier on the formation of $F_{pm, L.U.}$ and $F_{pm, G.U.}$.

An interesting feature of the case when $A = 3$ can be shown; that it is possible to obtain a change equivalence (change divided by original) of $F_{pm}$ ratios for a given value of $\triangle d/r$, that is have a $F_{pm, L.U.} = 2F_{pm, G.U.}$ with different initial separation ratios; yet maintaining the value of normalised deflection ratio $\triangle B$ (using $F_{pm, G.U.}$).

When relating to a practical system. This means a greater separation can be achieved. However, the finite value of $F_{o}$ will be shown to diminish. The criteria for this form of equivalence has been stated (equation 88) and in FIG. 53 two pairs of these equivalent reflection factor total ratios are shown, conditions 7 & 8, and 9 & 10.

It can be seen that the 'G.U.' equivalent value, for each of those pairs of equivalence, is the limiting feature upon which the $\triangle B$ value for the condition must be met.
FIG. 53 FPM/FO AGAINST SEPARATION RATIO "B" WITH "A"=3 FOR DIFFERENT VALUES OF NORMALLISED DEFLECTION RATIO, DELTA d/r (THEORETICAL)
Thus considering situations 9 & 10 where \( \frac{F_{pm}}{F_0} \) \( \text{L.U.'} \) = .8 and the G.U. value is 1.2, then using the 1.2 value it can be seen that this can only be met with a \( \Delta B \) value of say greater than 0.35, or considering the \( \Delta B \) values shown only with \( \Delta B \) of .4 and this occurs at \( B = 1.35 \) (10'). Consequently, for direct equivalence (\( \Delta B \) included) the L.U. separation ratio becomes .75 (9').

Situations 7 & 8 depict a smaller change in reflection factor i.e. \( \frac{F_{pm}}{F_0} \) = .95 and the 'G.U.' value becomes 1.05. Considering again the G.U. value (8) it can be seen that for all the conditions for \( \Delta B \) shown, one can attain the value of G.U. Thus there will be five cases of equivalence (in each case for the same \( \Delta B \)). These are not shown on the figure, but are arrived at in the manner described for situation (7 & 8).

It is also possible to obtain the equivalence consideration by using one value of \( \Delta B \) to obtain the 'L.U.' value of \( \frac{F_{pm}}{F_0} \) and a different value of \( \Delta B \) to obtain the equivalent G.U. value.

One example of such a manoeuvre is depicted in FIG. 54 which shows in addition to the state with \( 'A' = 3 \) the comparison when \( 'A' = 10 \) of the normalised deflection ratio on the ratio \( \frac{F_{pm}}{F_0} \). Now the conditions of equivalence shown (9 & 10) have, for this example, been chosen such that there is only one possible equivalence condition for the G.U. condition (10'\( D \)) i.e. when \( B = \Delta 0.4 \) at \( B = 1.35 \) (\( A = 3 \)). However, for the L.U. value of \( \frac{F_{pm}}{F_0} \) there can be seen to be four situations where equivalence can be satisfied. These are shown as 9'\( A' \), 9'\( B' \), 9'\( C' \), 9'\( D' \), when \( \Delta B = .1 \) at \( B = .3 \); \( \Delta B = .2 \) at \( B = .5 \); \( \Delta B = .3 \) at \( B = .65 \) and \( \Delta B = .4 \) at \( B = .7 \) respectively. Thus it can be seen that each of the forementioned situations are themselves equivalent. That is, the true value of \( \frac{F_{pm}}{F_0} \) L.U. can, by increasing \( \Delta B \) & \( B \), be achieved with the different conditions mentioned. All these conditions are equivalent to the \( \frac{F_{pm}}{F_0} \) G.U. state, described by condition 10'\( D' \).
different equivalent values to \( \frac{F_{pm}}{F_o} \) been chosen it would be possible to
have had further identities other than \((10')_D \), i.e. perhaps a \((10')_C \).

Also depicted on this figure is the family of curves when
A = 10 and, as has been mentioned earlier, there will be identities
and equivalents as for A = 3 condition between \( \frac{F_{pm}}{F_o} \) L.U. and G.U.
respectively, for these two 'A' values.

The following FIG. 55 extends the separation ratio scale up to
10 and enables the features for the G.U. condition to be further explored.
Considering the A = 10 condition, then even with a\( \triangle \) of .7 the biggest
change in reflection factor possible is when \( \frac{F_{pm}}{F_o} \) is 1.15 (12), the
equivalent ratio shown (11) being .85. Thus although for L.U. condition
\( \frac{F_{pm}}{F_o} \), values signifying large changes in reflection factor are possible,
the best G.U. value is in an equivalent form 0.85 (L.U.), and approximately
from the separation ratio 'B' = 5.0 shown by (12') remains fairly constant
for several orders of 'B'. A similar logic can be used to describe the
'A' = 3 situation except that a larger 'B' value is possible.

As yet no description of the effects for geometrical equivalence
i.e. \( \frac{F_{pm}}{F_o} \) G.U. has been discussed, but it can be deduced that
a larger value of \( \frac{F_{pm}}{F_o} \) G.U. is required for this equivalence condition,

than the change equivalence value for \( \frac{F_{pm}}{F_o} \) G.U. The following table 5
shows for specific values of \( \frac{F_{pm}}{F_o} \) L.U. the \( \frac{F_{pm}}{F_o} \) G.U. values for the two
equivalence conditions.
EQUIV. CONDITION

\[
\frac{F_{pm}}{F_o} < 1
\]

\[
\frac{F_{pm}}{F_o} > 1
\]

<table>
<thead>
<tr>
<th>(F_{pm}/F_o)</th>
<th>Change from original</th>
<th>Total Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>.7</td>
<td>1.3</td>
<td>1.43</td>
</tr>
<tr>
<td>.75</td>
<td>1.25</td>
<td>1.33</td>
</tr>
<tr>
<td>.8</td>
<td>1.2</td>
<td>1.25</td>
</tr>
<tr>
<td>.85</td>
<td>1.15</td>
<td>1.18</td>
</tr>
<tr>
<td>.9</td>
<td>1.1</td>
<td>1.11</td>
</tr>
<tr>
<td>.95</td>
<td>1.05</td>
<td>1.05</td>
</tr>
</tbody>
</table>

TABLE 5

from which it can be seen that at near unity values of \(\frac{F_{pm}}{F_o}\), the error in the G.U. values for the two conditions is minimal.

At greater changes of \(\frac{F_{pm}}{F_o}\) i.e. \(\frac{F_{pm}}{F_o} > 1\); there is a significant difference between the G.U. values. The significance can be seen from FIG. 54, where condition 9 shows \(\frac{F_{pm}}{F_o}\) of .8 and the corresponding equivalence condition for change/original situation is condition 10 i.e. \(1.2 = \frac{F_{pm}}{F_o}\). Now, as shown with a normalised deflection of .4, the required separation ratio is 1.35. If the equivalence condition for geometric situation is now considered, the condition shown as 10 (Geom. Eqn.) is required, i.e. \(\frac{F_{pm}}{F_o} = 1.25\); This requires a separation ratio of 1.52; significantly different to the other equivalence requirement.

One point should be borne in mind when considering these figures of \(\frac{F_{pm}}{F_o}\) against initial separation ratio where normalised deflection occurs, and is the following.

Reference to the actual static response curve or normalised static response curve should be made to ensure that, for the initial
separation considered, the normalised deflection ratio causes the $F^m_{pm}$ position to occur on the same side of the response curve as the initial condition.

The significance of the conditions for equivalence will become apparent when system resolution is discussed (Chapter 12). It appears that an optimum (maximum) value for $\frac{F^m_{pm}}{F_c}$ G.U. can be extracted from these figures, especially noticeable with $A$ equal to $3$. The significance of this optimum ratio for reflection factor total ratio is mentioned in 11.2.0, and discussed in 14.1.2.
The introduction of the concepts of reflection factor total ratio \( \frac{F_{pm}}{F_o} \) and normalised deflection ratio \( \triangle B \), has enabled the effective change in reflection factor resulting from having a stylised diaphragm in different initial positions, when subjected to various values of normalised deflections.

It has been shown that there are two possible conditions for \( \frac{F_{pm}}{F_o} \) depending on whether the initial separation and final separation both lie on the negative or positive slope factor side of the normalised response curve.

A value of \( \frac{F_{pm}}{F_o} \) of greater than unity (G.U.) results in the former condition and less than unity in the latter condition. That there are two equivalence states for \( \frac{F_{pm}}{F_o} \) of L.U. and G.U. has been shown, and one has its basis in geometric considerations. The other (equation 88) is based on the ratio change in \( F/\text{original} \), being constant.

From FIGS. 51 - 55 inclusive, the following deductions can be drawn:

1) For any separation value and normalised deflection ratio the effect of increasing \( 'A' \) is to decrease the reflection factor total ratio \( \frac{F_{pm}}{F_o} \);

2) For any separation value and \( 'A' \) ratio up to that at which \( F_{pm} = 1 \); the effect of increasing the normalised deflection ratio \( \frac{F_{pm}}{F_o} \) is to decrease the reflection factor. An increase results above 1.

These two factors are compatible for they both mean that an increase in the change in the reflection factor results.

By altering either \( 'A' \), \( 'B' \) or \( \triangle B \) the conditions for equivalence can be met, for \( \frac{F_{pm}}{F_o} \) (L.U.) and \( \frac{F_{pm}}{F_o} \) (G.U.)

Thus if a specific \( \frac{F_{pm}}{F_o} \) value is known to be required for the pressure transducing probe, then using FIGS. 51 - 55 various combinations of...
'B', $\triangle B$ and 'A' can be selected to fulfil this requirement. This will be laid down by the detector/amplifier characteristics.
CHAPTER 9.

SUMMARY OF THEORETICAL ANALYSIS
OF OPTICAL FIBRE SYSTEM.
The ratio of the amount of light received by the afferent fibres, to the emergent light from the efferent fibres was calculated from first principles and shown to be dependent on

1) separation of the mirror-like diaphragm
2) central emitter size
3) annulus of receiver size

Two fundamental assumptions were made

1) That the fibres emitting do so in a Lambertian manner, which, in fibre optic terms, infers that their Numerical Aperture is unity and the polar distribution is uniform.
2) That the arrangement of the fibres in the distal tip group is such that emitters simulate a central disc with receivers grouped in an annulus around that group.

Considering the latter assumption and with reference to FIG. 56, it can be seen that, because of imperfect packing of the fibres, an error is involved in using the effective outside diameter of the annulus for 'R' and 'r' and its inside diameter. The figure depicts the simplest case where all the fibres are of the same diameter and also for the simplest micro system, i.e. where R = 3r (A = 3), which is the most practical system.

Now the area of the assumed annulus is $\pi (R^2 - r^2)$ and the actual area of the receivers forming the annulus is $6\pi r^2$.

Therefore, if the value of reflection factor calculated using $R = 3r$ is used is ($F_3$) then the error corrected value will be

$$\frac{6 \pi r^2}{\pi (R^2 - r^2)} F_3$$

assuming uniform intensity of illumination conditions prevail.

Now let $K_E$ be error factor i.e.

$$K_E = \frac{6 \pi r^2}{\pi (R^2 - r^2)}$$
Figure 5.6 "Receiver Radii" - Errors Due to Packing Factor
but $R = 3r$

$$K_{E3} = \frac{6 \cdot r^2}{8r^2} = .75$$

i.e. 75%

thus if $F_{E3}$ is error corrected reflection factor

the $F_{E3} = K_{E3}F_3$

when $K_{E3} = \frac{6}{8}$

$F_{E3} = .75 F_3$

So for $R/r = 3$ the error in neglecting the packing effect is independent of the fibre size if they are all the same within the system i.e. simplest micro or macro. A generalised evaluation for accounting for different $R/r$ values and mixed fibre diameters is not given here.

Three methods for interpreting the effects of changing the system parameters have been shown, all derived from the initial reflection factor relationship

1) (a) By plotting static response curve
   (b) By plotting normalised static response curves where the x axis represents a separation ratio $B (d/r)$

2) (a) By plotting slope factor against separation ratio
   (b) By plotting sensitivity against separation.

3) By plotting reflection factor $\frac{F_{pm}}{F_o}$ total ratio against an initial separation ratio $B$ for a normalised deflection ratio delta $B$.

It is these representations 1, 2 & 3 from which different equivalent parameter conditions have been specified which will enable various systems to be modelled and their performance theoretically evaluated. The ratio $\frac{F_{pm}}{F_o}$ will prove to be the critical factor by which a specific system(s) will emerge as satisfactory for application as a pressure transducing probe. Values of slope factor required can be specified from this, although being dependent on the value of $\triangle B$ which is practicably possible.
Also for (3) the resulting $\frac{F_{pm}}{F_0}$ values assume that the
stylised diaphragm exhibits zero curvature when suffering the theoretical
normalised deflection ratios. How this assumption effects the previous
analysis is discussed in Chapter 11.
It is prudent here to restate the fundamental analysis approach that is necessary to determine the separation response of an optical fibre system. The aim is to define the level of illumination intensity falling on the afferent fibre(s) and consequently to be able to determine the flux received. This requires considerations of the system's N.A. and its polar distribution within the N.A. This author's analysis and that of Lindstroem's both commence with a derivation of a mathematical expression which gives the level of illumination intensity as a function of the separation of the receiving surface at radially offset positions, from the efferent fibre. Our analysis utilises the fundamental expression for describing the illumination intensity on a surface, when the source size is of the same order as the separation (Lambert's Cosine Law). In our case, we evaluate the integration steps leading to the final expression for flux intensity for a system of an N.A. of unity, assuming that the polar distribution is uniform. (A Lambertian source). Lindstroem too, assumes a uniform polar distribution. However, he only integrates his expression for flux intensity received for conditions when the separation distance is small compared to the fibre radius, and, when using a value of numerical aperture of 0.53. He then plots this intensity distribution relationship and further notes that at separations which are (with an N.A. = 0.53) less than 0.1 of the fibre radius i.e. \(d/r < 0.1\), the flux intensity that falls onto the efferent fibre is uniform. Now for these conditions at separation ratios of less than 0.1 and of an N.A. of 0.53, he computes the ratio of flux received to flux emitted, using a geometric coupling factor which is complex due to the fact that his fibre system considers only two adjacent fibres, noting the fact that the illumination level on the efferent fibre has been set to be uniform which simplifies this calculation.

One point to be noted is that by using a system with a larger N.A., Lindstroem's critique, that the separation should be small enough to give a uniform flux intensity on the efferent fibre, up to which the resulting expression for reflection factor is valid, reduces. It cannot, therefore, be applied to a system which has an N.A. of unity.
In order to consider the response at larger separations than 
$0.1 > d/r$, rather than pursuing his original method for the determination of received flux intensity which is based on Lambert's Cosine Law, he sets a further boundary for separation ratio, such that it is large so can then assume that the fibre source is a 'point source'. This enables him to redescribe his relationship for flux intensity and separation, for different offsets, to be dependent only in an inverse square law manner. This description incorporates a cut off to account for the N.A.

Our analysis is basically similar to that of Lindstroem's when he considers the fibre source not to be a 'point source', but we have considered a fibre system with an N.A. of unity and all our integration limits are based on this, furthermore, in his analysis section, which is comparable to ours, he only considers intensity ratio. Because our fibre arrangement has been considered to be that of a central emitter and annular receiver, calculation of the ratio of flux received to that emitted, is simple for which no approximations have been necessary and furthermore our relationship for an N.A. of unity covers the whole of the response curve. What must now be discussed is the validity of, and conditions for which the use of our simple method for accounting for a system with an N.A. of less than unity is limited. The fundamental error will be that, especially for small separation ratios, the radiant flux levels with a system of N.A. of less than unity, will be greater than if the N.A. were unity. In other words, the actual value of reflection factor will be greater than initially predicted. Consequently the prime condition necessary to allow our simple method for accounting for system N.A. to overcome this discrepancy, is when the fibre source tends to a 'point source' and this implies that the transition point has been passed i.e. that the total receiving annulus is covered by the emergent, reflected illumination cone. Thus the larger the N.A. considered becomes, the more useful this simple method is, because 'smaller separation ratios' satisfy the requirements. This has the opposite effect when compared to Lindstroem's results. Although it has been seen that it appears that for
large 'A' values there is less of a discontinuity between 'standard' and 'N.A.' plots, from what has been now discussed it can be seen that the smaller the 'A' value the better. Due to the many imperfections of a practical system; oblique fibres, poorly polished fibres, poor transmitting fibres, any theoretical predictions must be tested for practical validity. This is discussed in appendix one, which shows that for a fibre system of 'macro' format with 'A' = 2.5 and of an N.A. 0.53, our expression for reflection factor is experimentally verified for separations which are subsequent to the transition point. Furthermore, that it is shown in the conclusions to this thesis that the results of Lindstrom's analysis for large separation ratios compare well with those derived here, for a system with an N.A. of 0.53 and 'A' = 3.
CHAPTER 10

THE PRESSURE SENSING DIAPHRAGM

THEORETICAL ANALYSIS OF RESPONSE CHARACTERISTICS
10.0.0. INTRODUCTION

The objective of this chapter is to first obtain a relationship between diaphragm parameters, applied differential pressure and the resulting central deflection a diaphragm will suffer, and then to consider for different diaphragm materials the central deflection for specific diaphragm thicknesses and applied pressure. In order to enable finite values to be used in later computations, a maximum applied pressure of 300 mm Hg. and a maximum diameter for the diaphragm of 3mm are used (cf Chapter 2). This chapter will consider the diaphragm as an entity in itself. However, because up to now a 'stylised' flat diaphragm has been inferred, and this is not the real situation an expression for the deflection and slope of the diaphragm at any position on its diameter is shown.

The frequency response of the diaphragm and the detector/amplifier system are the two parts of the system upon which the overall frequency response of the transducer depend. Consequently, the undamped natural frequency of a diaphragm is given and computed for different diaphragm materials and thickness.
10.1.0. THEORETICAL ANALYSIS - DIAPHRAGM DEFLECTION

A circular diaphragm clamped at its edge may be analysed by considering it to be a plate whose stiffness is not negligible compared to the restoring tensile force where the load is symmetrically distributed about an axis perpendicular to the plate through its centre. All the deflections are also symmetrical to the axis (TIMOSHENKO, S. 1940) (ROARK, R.J. 1943).

Let us assume initially that:

i) the plate is flat with uniform thickness 't' and is composed of an isotropic, homogeneous material of density 'ρ';

ii) the thickness is less than 1/8 unsupported diameter 'Ddm' and greater than twice the maximum deflection;

iii) all loads are applied perpendicularly

iv) the plate is never strained beyond its elastic limits.

Then with respect to FIG. 57 the equation for the static deflection of such a plate (TIMOSHENKO, S. 1940) (ROARKS, R.J. 1943) is for any point on its radius \( r \).

\[
y_D = \frac{3P (1-Z^2)R_{dm}}{16Et^3} \left[ \frac{1 - \left( \frac{r}{R_{dm}} \right)^2}{2} \right] \]

Now the first part of assumption (ii) allows for the omission of the effect of accounting for shearing stresses, whilst the second part i.e. \( y \leq \frac{t}{2} \) justifies the omission of the effects of middle plane stress in the deflection analysis. This chapter also includes a discussion of more accurate descriptions for large diaphragm deflections.

Noting that the flexural rigidity term used in equation 90 is

\[
\frac{Et^3}{12(1-Z^2)}
\]
where

\[ y(r_{dm}) = \frac{3P (1-z^2) R_{dm}^4}{16Et^3} \left[ 1 - \frac{\xi^2}{R_{dm}^2} \right]^2 \]

and the central deflection is

\[ y_{(\text{max})} = \frac{3P (1-z^2) R_{dm}^4}{16Et^3} \]

where  
P = differential pressure on diaphragm  
z = Poisson's ratio  
E = Young's modulus of elasticity

**FIG. 57**  EDGE CLAMPED DIAPHRAGM - RELATIONSHIPS
From equation 90, the maximum deflection can be seen to occur when
\[ r_{dm}^4 = 0 \]
Therefore
\[ y_{\text{max}}(y_D) = \frac{3P(1-Z^2)^4 R_{dm}^4}{16Et^3} \]
\[ \text{cf } y_D - \text{Page 105} \]

Now the boundary conditions which have been assumed are:

1) \( y_{rdm} = 0 \) zero deflection at edge
   when \( r_{dm} = R_{dm} \) edge condition

2) \( \frac{d(y)}{d(r_{dm})} = 0 \) when \( r_{dm} = R_{dm} \)
   zero slope at edge

3) \( \frac{dy}{d(r_{dm})} = 0 \) at \( r_{dm} = 0 \) zero slope at centre

4) \[
\frac{d}{d(r_{dm})} \left[ \frac{1}{r_{dm}} \frac{d}{d(r_{dm})} \left[ r_{dm} \frac{dy_{rdm}}{d(r_{dm})} \right] - \frac{P}{2Et^3} \right] (12-Z^2) = 0
\]
   moment sum

It is convenient here to obtain a relationship for the slope of the diaphragm at any radius \( r_{dm} \). Later this will be used to account for a 'curved' rather than flat diaphragm as has been assumed so far when considering the optical fibre response relationship.

Using equation 90
\[
\frac{dy_d}{d(r_{dm})} = \frac{-3P}{4 \frac{1-Z^2}{Et^3}} R_{dm}^4 \left[ 1 - \frac{r^2}{R_{dm}^2} \right] \frac{2r_{dm}}{R_{dm}^2} \]
\[ = \frac{-3}{4} \frac{P(1-Z^2)}{Et^3} R_{dm}^2 \left[ 1 - \frac{r^2}{R_{dm}^2} \right] (r_{dm}) \]
\[ = \frac{-3}{4} \frac{P(1-Z^2)}{Et^3} r_{dm} \left[ \frac{2r_{dm}^2 - r_{dm}^2}{R_{dm}^2} \right] \]
\[ \text{……….92………..} \]
and let \( \frac{d(y_d)}{d(r_{dm})} \) be termed "GRAD" of diaphragm
Before continuing with graphical representation, another condition for the diaphragm can be made to supplement the above mentioned ones: the minimum diameter of the diaphragm is 2R-2r (only relevant for macro systems) which was mentioned in 6.2.1. Therefore, $3mm > 2R_{dm} > 2(R-r)$.

In order to ascertain the safety factor, i.e. the ratio of the rupture pressure to the design maximum pressure (300 mm Hg), that a specific diaphragm would possess it is necessary to consider the conditions for which rupture occur.

Theoretically it can be shown (TIMOSHENKO 1940) that the maximum stress for the diaphragm occurs at its edge and has a value (using the same initial assumptions as before).

$$f_{max} = \frac{3}{4} P \frac{R_{dm}^2}{t^2}$$  

whereby $P = 4 \frac{t^2}{R_{dm}^2} f_{max}$

Now for brittle materials the rupture pressure can be found from 94 by using the material's ultimate tensile stress in place of $f_{max}$ (SNYDER, R.D. 1973). However, for ductile materials an empirical value for the rupture pressure has been found to give results obtained from observation, (HODGE, P.G. 1959) and he uses the value of the material's yield point stress $\sigma_{yp}$ and an empirical constant;

Thus $P_{rd} = \frac{\sigma_{yp} t^2}{R_{dm}^2}$

$P_{rd}$ is rupture pressure for ductile material.

Thus for brittle materials

$$P_{rb} = \frac{t^2}{R_{dm}^2} \frac{4}{3} \sigma_{UTS}$$  

and for ductile materials

$$P_{rd} = 2.814 \frac{t^2}{R_{dm}^2} \sigma_{yp}$$
Later it will be shown that as these values for rupture or yield pressure are based on the linear small deflection theory, the values obtained will always be on the safe side.

Considering now the central deflection 'y_{max}' corresponding to the pressure at which rupture occurs and using the value of 'P_{rb}' and 'P_{rd}' above in equation y_{max} (91).

We have

1) \[ y_{max} \text{ (rupture)} = \frac{3}{16} (1-Z^2) \frac{R_{dm}^2}{E} \frac{4}{3} \frac{f_{UTS}}{t} \text{ [brittle materials]} \]

2) \[ y_{max} \text{ (rupture)} = \frac{3}{16} \frac{R_{dm}^2}{E} \frac{2.814 f_{yp}}{t} \text{ [ductile materials]} \]

Now also from 91

\[ \frac{y_{rupt}}{y_P} = \frac{P_{rupt}}{P} \]

if P = the design pressure (maximum), thus the safety factor \( \frac{P_{rupt}}{P} \) is equal to the ratio of the deflection at rupture, divided by the deflection at the design pressure.
Three materials have been considered for the pressure sensing diaphragm: Berylium copper, stainless steel and glass. The basic requirements for these materials as a pressure sensing diaphragm are; non-traumatic and springy (not plastic or very ductile).

To meet these requirements the metallic materials were chosen in specific hardened conditions. Berylium Copper (Be/Cu), an alloy of 2% Be, 98% Cu, which is softened then precipitation hardened, and for stainless steel (S.S.) a martensitic alloy EN56D which is tempered at 300 - 350°C.

The glass considered was that of a typical microscope slide.

When considering different materials for diaphragms it is possible to have diaphragms of different materials and different thicknesses showing the same displacement and deflection profiles. If the diaphragm radius is Rdm and the pressure on it is considered constant; then from equations 90 and 91

\[
y \left( \frac{dy}{d_{\text{rdm}}} \right) \lesssim \frac{(1-Z_2^2)}{Et^3}
\]

thus with the suffixes 1, 2 and 3 for Be/Cu, glass and stainless steel respectively we have

\[
\frac{(1-Z_1^2)}{E_1 t_1^3} = \frac{(1-Z_2^2)}{E_2 t_2^3} = \frac{(1-Z_3^2)}{E_3 t_3^3}
\]

expressing in terms of Berylium Copper (1) we have

\[
t_1^3 = \frac{E_2 t_2^3}{E_1} \left( \frac{1-Z_2^2}{E_1} \right) = \frac{E_3 t_3^3}{E_1} \left( \frac{1-Z_3^2}{E_1} \right)
\]

on substitution of values for materials given, see Page 228

\[
t_1 = .75 \ t_2 = 1.146t_3
\]

\[\text{99a}\]
which describes the equivalent thickness for these different material
diaphragms such that they, for the same diaphragm radius, give the
same deflection characteristics.

Now considering the deflection at which rupture would occur
for these different material diaphragms we have, using equation 98
for the Be/Cu and stainless steel diaphragm and 97 for the glass
diaphragm

\[
y_{1 \text{ rupt}} \propto \frac{1-Z_{1}^{2}}{E_{1}t_{1}} 2.814 \times 10^{5} \quad \text{(Be/Cu)}
\]

\[
y_{2 \text{ rupt}} \propto (1-Z_{2}^{2}) \frac{4}{3} 0.7 \times 10^{4} \quad \text{(glass)}
\]

\[
y_{3 \text{ rupt}} \propto 1-Z_{3}^{2} \frac{2.814 \times 1.34 \times 10^{5}}{E_{3}t_{3}} \quad \text{(stainless steel)}
\]

On substitution of the equivalent thicknesses of \( t_{2} \) and \( t_{3} \) relative to \( t_{1} \),

\[
y_{1 \text{ rupt}} \propto \frac{(1-Z_{1}^{2})}{E_{1}t_{1}} \frac{2.814 \times 10^{5}}{3}
\]

\[
y_{2 \text{ rupt}} \propto (1-Z_{2}^{2}) \frac{0.756}{3} 0.7 \times 10^{4}
\]

\[
y_{3 \text{ rupt}} \propto (1-Z_{3}^{2}) \frac{1.146 \times 2.814 \times 1.34 \times 10^{5}}{E_{3}t_{3}}
\]

Substituting the values for \( E \) & \( Z \)

\[
y_{1 \text{ rupt}} \propto \frac{1.9253}{t_{1}} 10^{-2}
\]

\[
y_{2 \text{ rupt}} \propto \frac{1.113}{t_{1}} 10^{-3}
\]

\[
y_{3 \text{ rupt}} \propto \frac{1.9666}{t_{1}} 10^{-2}
\]

Thus

\[
\frac{y_{2 \text{ rupt}}}{y_{1 \text{ rupt}}} = 5.78 \quad 10^{-2}
\]

\[
\frac{y_{3 \text{ rupt}}}{y_{1 \text{ rupt}}} = 1.0253
\]
thus as all the diaphragms have the same deflection characteristics
the safety factor $\frac{y_{\text{rupt}}}{y_p}$ for the different diaphragms will be with

Be/Cu as reference

<table>
<thead>
<tr>
<th>Material</th>
<th>$\frac{y_{\text{rupt}}}{y_p}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Be/Cu</td>
<td>1</td>
</tr>
<tr>
<td>Glass</td>
<td>0.0578</td>
</tr>
<tr>
<td>Stainless Steel</td>
<td>1.0253</td>
</tr>
</tbody>
</table>

The significance of the above mentioned relative safety factors
is that glass shows itself clearly to possess poor safety features when
compared to Be/Cu and Stainless Steel.

The values of Young's modulus $E$, the ultimate tensile strength
$f_{\text{uts}}$, the yield point $f_y$, and Poisson's ratio are shown below.

<table>
<thead>
<tr>
<th>Material</th>
<th>$E \times 10^6$ (gm/mm$^2$)</th>
<th>$f_y \times 10^5$ (gm/m$^2$)</th>
<th>$f_{\text{uts}} \times 10^4$ (gm/mm$^2$)</th>
<th>$\rho$ (gm/m$^3 \times 10^{-3}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Be/Cu</td>
<td>13.3</td>
<td>1</td>
<td>-</td>
<td>8.25</td>
</tr>
<tr>
<td>S.S.</td>
<td>20.0</td>
<td>1.34</td>
<td>-</td>
<td>7.8</td>
</tr>
<tr>
<td>GLASS</td>
<td>6</td>
<td>-</td>
<td>0.7</td>
<td>0.23</td>
</tr>
</tbody>
</table>

NOTE for glass the $f_{\text{uts}}$ is shown, as it is considered a brittle material
in the context earlier described.

The above values were used in equation 91 to compute the values
of diaphragm central deflection (tables 6 & 7) for two conditions of
radius $R_{dm}$: 1mm & 1.5mm. The differential applied pressure considered
being 300mm Hg. The values of deflection greater than half the diaphragm
thickness denoted (*)

Now with respect to tables 6 and 7 it can be seen that even for
the glass diaphragm the maximum central deflection in only 15 $\mu$ when
($R_{dm} = 1.5\text{mm}$) when 35 $\mu$ thick. The larger deflections must be discounted
as they are greater than half the diaphragm's thickness (see 10.1.2.)
Thus it is worth noting that, considering the normalised deflection ratio
introduced earlier, i.e. $\frac{d}{r}$, we note that $\triangle d'$ (the deflection)
<table>
<thead>
<tr>
<th>THICKNESS MM</th>
<th>GLASS</th>
<th>BE/CU</th>
<th>ST. STEEL</th>
</tr>
</thead>
<tbody>
<tr>
<td>.020</td>
<td>.015094</td>
<td>.006543</td>
<td>.004351</td>
</tr>
<tr>
<td>.025</td>
<td>.007728</td>
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<tr>
<td>.030</td>
<td>.004472</td>
<td>.001939</td>
<td>.001289</td>
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<td>.035</td>
<td>.002816</td>
<td>.001221</td>
<td>.000812</td>
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<td>.000818</td>
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<td>.045</td>
<td>.001325</td>
<td>.000574</td>
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<tr>
<td>.055</td>
<td>.000726</td>
<td>.000315</td>
<td>.000209</td>
</tr>
</tbody>
</table>

**TABLE 6  VALUES OF CENTRAL DEFLECTION**

For P 300 mmHg

R 1
<table>
<thead>
<tr>
<th>THICKNESS MM</th>
<th>DEFLECTION MAX MM</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>GLASS</td>
</tr>
<tr>
<td>.020</td>
<td>.076415</td>
</tr>
<tr>
<td>.025</td>
<td>.039125</td>
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<td>.030</td>
<td>.022642</td>
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<td>.035</td>
<td>.014258</td>
</tr>
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<td>.040</td>
<td>.009552</td>
</tr>
<tr>
<td>.045</td>
<td>.006709</td>
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<tr>
<td>.050</td>
<td>.004891</td>
</tr>
<tr>
<td>.055</td>
<td>.003674</td>
</tr>
</tbody>
</table>

**TABLE 7 VALUES OF CENTRAL DEFLECTION**

For P 300 mmHg

R 1.5
is say 15 \( \mu \), thus 'r' must be of that order to obtain values shown earlier of \( B = 0.2 - 0.7 \); greater values of 'r', as one would have for macro systems, would result in exceedingly small values of \( \frac{\Delta d}{r} \) and hence values of \( \frac{F_{pm}}{F_0} \) approaching unity. This will be discussed later.

To enable the deflection/thickness relationship for different '\( R_{dm} \)' values to be assessed for the different materials all subjected to a 300 mm Hg differential pressure FIG. 58, 59 and 60 were plotted (using equation 91) for Be/Cu, Stainless Steel and Glass respectively. Also, in order to allow two conditions for the diaphragm to be met; namely, the deflection should be less than half the thickness and that a safety factor regarding over-pressure rupture can also be assessed, these graphs also show

1) \( \frac{a \gamma_{max}}{2} = t \) (limit line)

2) curves showing '\( \gamma_{rupt} \)' against thickness (using equation 97 & 98 where applicable) for different diaphragm radii (\( R_{dm} \))

These figures allow much information about diaphragm performance as a pressure sensing element to be assessed. The 'y' axis is on a logarithmic scale to show the 'linear theoretical' rupture deflections for a full range of thickness of diaphragm.

Consider now FIG. 58 which depicts for the Be/Cu diaphragm with different radii \( R_{dm} \), the relationship between the central deflection (\( \gamma_{max} \)) and diaphragm thickness for a pressure of 300 mm Hg (lower curves) and the upper curves showing the 'linear theoretical' deflection at which rupture would occur in each case.

**N.B.** \( \gamma_{max} \) is the central deflection with maximum applied differential pressure \( P_{max} \) applied to the diaphragm.
FIG. 58 DEFLECTION RELATIONSHIP Be/Cu
FIG. 59  DEFLECTION RELATIONSHIP STAINLESS STEEL
DEFLECTION
MICRONS

Y = t/2

R = 0.5
R = 0.8
R = 1.0
R = 1.2
R = 1.5
R = 1.8

"Ymax"
(P = 300 mm. Hg.)

"Yrupt"

R = 1.2

THICKNESS MICRONS

FIG. 60  DEFLECTION RELATIONSHIP GLASS
From these 'pairs' of curves it is possible to determine for any thickness 't' and radius 'R_{dm}' the safety factor and hence the rupture pressure. This is accomplished using the relationship described (equation 99) in the following manner.

Let us choose a diaphragm thickness of 30 μ shown by condition (13) then if \( R_{dm} = 1.5 \text{mm} \)

\[
y_{\text{max}} \text{ for } P = 300 = 10 \mu
\]
and \( y_{\text{max}} \text{ for rupture } = 260 \mu \)

therefore safety factor \( \frac{y_{\text{rupt}}}{y_{p}} = 26 \)

and thus rupture pressure = \( 26 \times 300 \)

\( = 7,800 \text{ mm Hg} \).

Similarly if \( R_{dm} = 1.0 \text{ mm} \) is chosen for the diaphragm radius

safety factor \( \frac{y_{\text{rupt}}}{y_{p}} = \frac{125}{2} \)

\( \simeq 60 \)

Also from this form of graph, equivalent diaphragm parameters can be found which give the same central deflection. The manner in which this is accomplished is shown in FIG. 59 which now considers a stainless steel diaphragm. If a desired deflection of 4 μ is chosen for a differential pressure of 300 mm Hg (condition\( \Delta \)), then as shown there would be five pairs of thickness/radii diaphragm that would be possible; one example with \( R_{dm} = 1.5 \text{ mm} \) and 35 μ diaphragm.

At this time a value of required \( y_{\text{max}} \) for 300 mm Hg (\( y_{\text{max}} \)) has not been specified. However, as has been mentioned, the \( \frac{F_{pm}}{F_{o}} \) values shown earlier (Chapter 8) were obtained by considering normalised deflection ratios \( \Delta B \) occurring at a separation ratio of \( B \) with values of .2 - .7 for \( \Delta dB \). The required ratio \( \frac{F_{pm}}{F_{o}} \) has not been specified yet.
However, if a value of $\triangle B = .5$ is considered, then as $B = \frac{d}{r}$
$\triangle B = .5 = \triangle (d)$, now as $\triangle (d)$ the deflections shown have a maximum
value of 15 $\mu$ then for $\triangle B = .5$, 'r' must be 30 $\mu$. This indicates
a micro type system is necessary. For if $r$ were 500.0 $\mu$ the value
of $\triangle B$ would be very small, i.e. 0.03 and consequently the $\frac{F_{pm}}{F_0}$
would be very small. The use of these FIGS. 58, 59 and 60 depicting
diaphragm deflection characteristics will be used as discussed later
when the composite optical fibre/diaphragm system is described
(Chapter 11.2.0. & ch. 14).

The equivalent condition for diaphragm thicknesses of different
materials giving the same deflection characteristics has been described
in equation 99a.

$$t_1 = .756t_2 = 1.146t_3$$

and the relative safety factors to a Be/Cu have been shown to be

Be/Cu : 1
GLASS : .0578
Stainless Steel : 1.0253

Thus if a Be/Cu diaphragm is such that it exhibits a safety
factor of 60 then a glass diaphragm will exhibit a safety factor of
60 x .0578 = 3.47 and a stainless steel diaphragm 61.5; this can be
seen graphically to be true. Considering FIG. 58 with a (Be/Cu) diaphragm
radius of 1.0 mm and a thickness $t_1 = 30 \mu$ as described earlier, the safety
factor is 60.

Now we have thickness equivalents of $t_2 = 40$ for glass and
$t_3 = 26$ for stainless steel (from equation 99a).

From FIGS. 59 & 60 conditions 19 & 20 ($R_{dm} = 1.0$) with $t_3 = 26$
and $t_2 = 40$ respectively, it can be seen that the safety factors as
graphically determined, are approximately 65 and 3.5 respectively,
confirming the validity of the relative safety factors' expression for
equivalent diaphragm (same $R_{dm}$, same pressure deflection characteristics).
To show more clearly the effect which diaphragm thickness has on central deflection, i.e. (proportional to $\frac{1}{t^3}$) and to enable more accurately the limiting boundary condition for which the 'linear theory' holds ($y < \frac{t}{2}$) to be determined, FIGS. 61 and 62 show deflection against thickness relationship for diaphragm radii of 1mm and 1.5mm respectively. It should be noted that for the glass diaphragm the safety factor is only a matter of units, consequently the rupture condition for glass is also shown on these graphs. The 'y' axis is this time plotted in a linear fashion showing the inverse $t^3$ dependence of the central deflection.

Again it can be seen that, for the diaphragm shown, with large radii 1.5mm (large for intra-vascular pressure transducer) that the deflections will be small.

Two further figs. are shown, and although they themselves are put to no practical use here, it is interesting to consider them. They are FIGS. 63 and 64 which show deflection profiles for different diaphragm parameters, all subjected to 300 mm Hg.

Their plotting was performed utilizing equation 90, and it should be noted that the 'y' axis scale is 50 times the x axis scale. The aim is to show that the gradient of the deflection at any radius $r_{dm}$ of the diaphragm is small and for thicker diaphragms becomes very small. Later the expression for the 'grad' equation 92 will be used to compare the effects of assuming the diaphragm to deflect with a flat profile (C11) when considering the optical fibre system/diaphragm system.
FIG. 63  DEFLECTION PROFILES  \( R_{dm} = 1.0 \text{ mm.} \)  \( P = 300 \text{ mm. Hg.} \)
10.1.2. LARGE DEFLECTION - CONSIDERATIONS

The maximum deflection for a circular diaphragm clamped (i.e. fixed and held) has been stated by equation 91

\[
y_{\text{max}} = \frac{3P(1-Z^2)R_{\text{dm}}^4}{(y_D) 16Et^3}
\]

and was stated to be valid up to a central deflection of half the thickness of the diaphragm; above this value the middle surface within the diaphragm becomes appreciably strained and the stresses in it can no longer be ignored.

Although we are only really interested in the value of \( y_{\text{max}} \), i.e. the central deflection assuming the small deflection equation 91, this expression was used earlier to obtain values of \( y_{\text{rupt}} \), the deflection at which rupture occurs and thence the safety factor. Its value (\( y_{\text{max}} \)) greatly exceeded one half the diaphragm's thickness and consequently the validity of the earlier determination of safety factor must be examined.

Let \( y_{\text{max}} \) continue to represent the central deflection obtained by using the 'linear' equation 91.

Now to determine the effect of using this linear expression for determination of \( y_{\text{rupt}} \) let us consider the next more accurate approximation for the central deflection (now termed \( y_a \)) which accounts for deflections greater than half the thickness of the diaphragm.

Such an expression was first described by TIMOSHENKO, S (1928) and is derived by strain energy methods. However, it only uses the first two terms for the description of the radial strain which the diaphragm suffers.

This relationship is

\[
\frac{3PR_{\text{dm}}^4}{16Et^4} = y_a \left( \frac{y_a}{t} \right)^3 \times .488
\]

\[
\text{.......................... 99b}
\]
The later work of WAY, S and PITTSBORGH, PA (1934) into the large deflections of diaphragms using an 'exact' power series strain energy analysis and the practical work of SANDWITH, C.J. and WIEDEMEEIER, D. (1972) show that the values of $y_a$ obtained from 99b are always larger than the actual values. Similarly for TIMOSHENKO's values for the maximum stress within the diaphragm

$$f_{max,a} = \frac{4.40E}{R^2} \frac{y_a t}{R_{dm}} + 0.476 \left(\frac{y_a}{R_{dm}}\right)^2$$

Now it is worth noting at this stage that the constant .488 in equation 99ab includes the value of Poisson's ratio for the material, and its value for this constant is .3. Similarly the constant 4.4 in equation 99c incorporates the value of $Z = .3$.

Let us consider the description for $y_a$ in equation 99a.

This may be rewritten

$$y_{max} = \frac{y_a}{t} + \left(\frac{y_a}{t}\right)^3$$

where it can be seen that if $\frac{y_{max}}{t} = \frac{1}{2}$

the ratio $\frac{y_a}{y_{max}} = .89; \text{ in other words the linear value is } 11\% \text{ in error.}$

FIG. 64a shows, using equation 99d, the ratio $\frac{y_a}{y_{max}}$ for different values of $y_{max}$ with various diaphragm thicknesses. It should be noted that the ratio $\frac{y_a}{y_{max}}$ is a function of a square of the value $\left(\frac{y_a}{t}\right)$.

Thus even at values of $y_{max}$ less than $\frac{t}{2}$; there will be an error in using this value from the linear description, but it will be small.

It is more important to compare the values of rupture or yield pressure using the linear method described earlier and the TIMOSHENKO approximation method. Let us first consider equation 99c

$$f_{a, max} = \frac{4.4 E t}{R^2} \frac{y_a}{R_{dm}} + \frac{0.476}{R_{dm}^2} y_a^2$$

by which, at given values for $f_{a, max}$, the central deflection can be
found at rupture. Then using this value of \( y_{a \text{ max}} \) in equation 99a the value of pressure \( P \) for the condition of rupture or yield can be found.

The following FIG. 64b shows the ratio of yield or rupture pressures when using the Timoshenko energy method to when using the linear method; from which it can be seen that for any condition the value found using the linear method is erring on the safer side. This is most marked as \( y_{a/t} \) increases, especially for very large values (or deflection) where this approximation does not hold. Furthermore, by using the exact power series analysis of Way and Pittsburgh it is possible to determine for larger deflections than is possible when using Timoshenko's two power series analysis the excess factor of safety possible when compared to the linear method result.

In conclusion, the linear method (i.e. assuming equation 91) for the description of \( y_{\text{max}} \) assumes a linear relationship between deflection and pressure. This becomes inaccurate when \( y/t \) is greater than one half, as the middle surface of the diaphragm becomes significantly stressed and must be accounted for. The effect is to no longer have the deflection proportional to the applied pressure.

Comparison between rupture or yield pressures obtained from linear, approximate series, and power series analysis, show that the linear method always produces a safer value (lower factor of safety) and consequently as one only requires for our purposes an approximate figure which is truly safe, the simple linear method is adequate.
FIG. 64a. APPROXIMATE RELATIONSHIP BETWEEN RATIO $\frac{Y_a}{Y_{\text{max}}}$ AND THE VALUE OF DEFLECTION OBTAINED FROM THE "LINEAR THEORY" $y_{\text{max}}$. 

- **A** ... Thickness $t = 40$ microns
- **B** ... Thickness $t = 35$ microns
- **C** ... Thickness $t = 30$ microns
- **D** ... Thickness $t = 25$ microns
- **E** ... Thickness $t = 20$ microns
Diaphragm Radius $R_{dm} = 1.5$ mm.

A .... Be/Cu Rupture
B .... St.St. Rupture
C .... Be/Cu Yield
D .... St.St Yield
E .... Glass Rupture

FIG. 64b. APPROXIMATE RELATIONSHIP FOR THE RATIO $P$ USING SMALL DEFLECTION EQUATION TO LARGE DEFLECTION EQUATION OF YIELD OR RUPTURE, PRESSURE FOUND FROM THE "ENERGY METHOD" TO THAT FROM THE LINEAR METHOD.
10.2.0. THEORETICAL ANALYSIS OF DIAPHRAGM NATURAL FREQUENCY

The expression for the undamped natural frequency of a plate or thin plate clamped at its edge is quoted widely. However, its form also varies widely according to the units used. For our purposes consider the following which is derived by Rocard, V. (1960)

\[(\text{Hz}) = \frac{0.47}{R_{dm}} \sqrt{\frac{E \cdot g}{\rho (1-z^2)}}\]

with \(t, R_{dm}\) in mm
\(E\) \quad gm/mm²
\(\rho\) \quad gm/mm³

this expression can be used with different consistent units throughout and reduces to the following by substituting the value for \(g (9,800 \text{ mm/s}^2)\)

\[(\text{Hz}) = 46.5 \frac{t}{R_{dm}^2} \sqrt{\frac{E}{(1-z^2)\rho}} \quad \text{................................. 100}\]

From the above relationship it can be seen that, for diaphragms of the same thickness and radius, that the undamped natural frequency is a function of \(\sqrt{\frac{E}{\rho (1-z^2)}}\) and let this value be called 'frequency factor'.

Thus the frequency factors for the three materials considered are, using the values mentioned earlier

<table>
<thead>
<tr>
<th>Material</th>
<th>Frequency factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Be/Cu</td>
<td>4.208 (10^4)</td>
</tr>
<tr>
<td>Stainless Steel</td>
<td>5.3 (10^4)</td>
</tr>
<tr>
<td>Glass</td>
<td>4.6 (10^4)</td>
</tr>
</tbody>
</table>

The relative values of each materials frequency factor shows that for a specific value of \(R_{dm}\) and \(t\), the Be/Cu diaphragm will exhibit the lowest natural frequency. The glass will have a 10% higher frequency and the stainless steel 26% higher than the Beryllium Copper diaphragm.
FIGS. 65, 66 and 67 show for the three materials how the undamped natural frequency varies with the thickness of the diaphragm values of diaphragm radii.

So even with Be/Cu diaphragm of 30 μ thick and \( R_{dm} = 1.5 \text{mm} \) the resulting undamped natural frequency will be 26KHz (condition 15 FIG. 65). Later reference will be made to those figures for the assessment of the undamped natural frequency of the diaphragm chosen. Although as can be seen, the frequency aspect does not appear to be a limiting factor in diaphragm selection.
FIG. 65  UNDAMPED NATURAL FREQUENCY Be/Cu
FIG. 66  UNDAMPED NATURAL FREQUENCY  STAINLESS STEEL
CHAPTER 11

OPTICAL FIBRE SYSTEM/DIAPHRAGM

AS COMPOSITE UNIT
11.0.0. INTRODUCTION

This chapter aims to form a mathematical model which represents the composite optical fibre/diaphragm system and gives a relationship for the reflection factor achieved in terms of diaphragm parameters and differential pressure applied across it. This expression will be the first part (1) of equation 31 discussed in Chapter 5.

In Chapter 8 a 'quasi model' was introduced which simulated the diaphragm by considering it to be subjected to parallel, specific normalised deflection ratios, at certain separation ratios. Although useful, as will be seen, as an initial design specification aid, it has three limitations:

1) The diaphragm (simulated) has been considered to deflect in a flat profile-
2) No account of incremental differential pressures were made i.e. (only initial and final reflection factors considered).
3) To relate to diaphragm parameters, the deflections of diaphragms had to be considered separately and then related to the ratio \( \triangle B \) as described. (Ch. 10.)

To assess the effect of assuming a stylised diaphragm (which deflects with a flat profile) to be equivalent to a 'real diaphragm', a mathematical ray plot is described, which enables a comparison between the position a 'ray' returns to on the distal tip face when there are 'flat conditions' or a 'true deflection' profile.

The effects of having an enclosed air gap between diaphragm and optical fibre distal face will also be investigated. There are two effects to be considered; the effect of temperature on the actual differential pressure to which the diaphragm is subjected, and the effect of initial internal pressure within gap on actual differential pressure.
11.1.0. THEORETICAL RELATIONSHIPS

There are two forms which the theoretical relationship for the optical fibre system/diaphragm can take; firstly an individual positional ray plot, secondly a quantitative reflection factor expression, where for both, the effects of diaphragm parameters, initial separations and differential pressure applied to the diaphragm are to be considered.

11.1.1. EFFECT OF CONSIDERING A 'FLAT' DIAPHRAGM

Considering the ray plot method first, which will allow for a comparison between considering the diaphragm to have a 'flat deflection' profile and a 'true deflection' profile.

With respect to FIG. 68, ABCD is an axial cross section of an emitting fibre(s) of radius 'r'. EF is the plane in which the receiving fibres lay. GH is a diaphragm of thickness 't' subjected to a differential pressure 'P' and has an initial separation from the fibres of 'd_0'.

One ray is shown leaving the emitter fibre(s), 'XY', at a radius 'r_e' and an angle 'α'; 'Y' being the point on the diaphragm where reflection takes place (and will vary according to the value of 'P').

The horizontal deflection at 'Y' is shown as y_{pr_{dm}} and after reflection on the diaphragm, is shown to return to the receiving fibres at a radius 'R_{pre}'. The suffix 'p,re' indicates pressure on the diaphragm and the radius from which the ray emerges from the emitting fibre(s).

Thus \[ R_{pre} = r_e + d_{pr_{dm}} \tan \alpha + d_{pr_{dm}} \tan (\gamma + \beta) \]

where \( d_{pr_{dm}} \) is the resulting separation of a point on the diaphragm of radius \( r_{dm} \) due to the applied pressure \( P \).

Now \( \alpha + \gamma = \beta \)

thus the above becomes
\[ R_{\text{pre}} = r_e + d_{\text{pr}_{\text{dm}}} \tan \alpha + d_{\text{pr}_{\text{dm}}} \tan (2\gamma + \alpha) \]

101

'\gamma' is shown as the angle the tangent to the diaphragm makes with the vertical axis at 'Y1', thus it is the 'grad' of the diaphragm at 'r_{dm}'. Now using equation 92, noting that with the same sign convention as used earlier '\gamma' is the complement of the grad angle.

\[ \gamma = \arctan \left( \frac{3}{4} \frac{P(1 - Z^2)}{E_t^3} \right) \]

102

The unknown factors are the value of 'd_{pr_{dm}}' in equation 101 and the value of 'r_{dm}' in 102. To evaluate these we must consider the two equations, one for the ray, the other for the diaphragm, which describe the point 'Y1' where ray meets diaphragm.

Now for the ray

1) \[ d_{pr_{dm}} = \frac{r_{dm} - r_e}{\tan \alpha} \]

103

and for the diaphragm

2) \[ d_{pr_{dm}} = d_o - y_{pr_{dm}} \]

104

where 'y_{pr_{dm}}' is the deflection of any point on the diaphragm and is described by equation 90. Noting that this is the approximate relationship which is valid up to about \( y_{pr_{dm}} (Y_{\text{max}}) \) of t/2; and when this condition is exceeded the value 'y_{pr_{dm}}' as calculated is greater than its true value and consequently it will be seen that the following analysis always errs on the pessimistic side.

\[ d_{pr_{dm}} = d_o - \frac{3}{16E_t^3} P(1 - Z^2)(r_{dm}^2 - r_e^2)^2 \]

105

thus we have from 103 and 105 by equating \( d_{pr_{dm}} \)

\[ \frac{r_{dm} - r_e}{\tan \alpha} = d_o - \frac{3}{16E_t^3} P(1 - Z^2)(r_{dm}^2 - r_e^2)^2 \]

106

so each side of the equation 106 is equal to 'd_{pr_{dm}}' the separation of the point on the diaphragm where the ray meets it.
By substituting incremental values of $r_{dm}$ into both sides of equation 106 until the equation holds, the value of $d_{pr_{dm}}$ and $r_{dm}$ can be determined, and then can be used first in 102 to determine $\gamma$; then $R_{pre}$ can be computed from 101.

Let us now consider the case if the diaphragm deflects with a flat profile (parallel to fibre face) of a magnitude of $y_{max}$ (from equation 91). This situation is shown in FIG. 69 which uses the same designatory symbols as the previous Fig. However, in this case, the diaphragm is assumed to move with a flat profile due to an applied pressure $P$, moving a distance $y_{max}$ (i.e. the value of the central deflection).

Thus $R = r + 2d \tan \alpha$ ........................................ 107

where $d_{P}$ is shown to be the resulting separation caused by the applied pressure (cf. $d_{pr_{dm}}$ which is dependent on $r_{dm}$) and

$$d_{P} = d_{o} - y_{max}$$

Using the expression for $y_{max}$ (equation 91) and substituting into the above equation, then substituting for $d_{P}$ in 107

$$R_{pre} = r_{e} + 2\left(d_{o} - \frac{3}{16} \frac{P r_{dm}^{4}}{E t^{3}} (1 - Z^{2})\right) \tan \alpha$$ ........................................ 108

Thus the radius $R_{pre}$ which is the radius on the exit plane at which an emergent ray XYZ leaving at a radius $r_{e}$ on this plane, can be found when considering the diaphragm to be 'real' or 'stylised' (FLAT).

The values thus are

$$R_{pre} (true) = r_{e} + d_{pr_{dm}} \tan \alpha + d_{pr_{dm}} (2 \gamma + \alpha)$$ ........................................ 101

where $\gamma = \arctan \left(\frac{3}{4} \frac{P}{E t^{3}} (1 - Z^{2}) \frac{r_{dm} (R_{dm}^{2} - r_{dm}^{2})}{r_{dm}^{2} - \frac{2}{r_{dm}}}ight)$ ........................................ 102

and $r_{dm}$ is obtained from

$$\frac{r_{dm} - r_{e}}{\tan \alpha} = d_{o} - \frac{3}{16Et^{3}} P (1 - Z^{2}) \left(R_{dm}^{2} - r_{dm}^{2}\right)^{2}.$$ ........................................ 106
FIG. 69  COMPOSITE RAY PLOT OPTICAL FIBRE /"FLAT" DIAPHRAGM
and the value of either side of equation 106, and for the stylised diaphragm

\[ R_{pre}^{(flat)} = r_e + 2 \left( d_0 - \frac{3}{16} \frac{P R_{dm}^4}{E t^3} \right) (1 - Z^2) \tan \alpha \]

where \( r_e \) is vertical exit position of the ray (a radial position)
\( d_0 \) is initial separation of diaphragm at zero applied differential pressure.
\( P \) is applied differential pressure on diaphragm
\( \alpha \) is the emergent angle of the ray
\( r_{dm} \) is the radius on the diaphragm where the ray meets it
\( R_{dm} \) is the radius of the diaphragm

Chapter 6 has discussed the optical fibre response when considering a flat reflection surface. If this relationship is to be applied to a real diaphragm which will take up different profiles for different applied pressures, then the effect on the reflected ray position of having a 'flat' diaphragm must be compared to the true diaphragm situation.

This can be accomplished by using the individual ray considerations for which equation 101 gives the true return radius \( R_{pre}^{(True)} \) of an emergent ray of angle \( \alpha \) and axial offset \( r_e \) when a pressure \( P \) is applied to a certain diaphragm and equation 108 gives the return ray position \( R_{pre}^{(FLAT)} \).

The FIGS. 70 - 74 inclusive, show graphically the true returned radius \( R_{pre}^{(TRUE)} \) plotted against the angle at which the ray emerges from the emitting fibre; also shown on each graph (the 'y' axis has two scales) is the % ratio of the returned radius for a flat diaphragm \( R_{pre}^{(FLAT)} \) to the return radius for a true diaphragm \( R_{pre}^{(TRUE)} \).

Because of the numerous values for the parameters involved in calculating \( R_{pre}^{(FLAT & TRUE)} \) the following real values have been used:
\( R_{dm} = 1.5 \text{ mm}; \) glass or Be/Cu diaphragms; initial separations \( (d_0) \) of 30 or 40 \( \mu \) (as from earlier work with a \( \mu \) system it is at such separations
that peak sensitivity is obtained).

The figures have been computed using an incremental procedure to obtain values of $R_{\text{pre}}$ (TRUE) as has been described earlier. Before describing these figures in detail, the conditions and assumptions by which the validity for which a 'flat diaphragm' can be considered rather than a true diaphragm must be stated.

1) If '$R$' is the radius of the receiving annulus as before, then if $R_{\text{pre}}$ (TRUE) $\geq R_{\text{pre}}$ (FLAT), then these values of $R_{\text{pre}}$ are outside the boundaries for the comparison as the derivation of the optical fibre response is only concerned with rays which return within one sender/receiver system.

2) Because a $\mu$ system is favoured due to its high sensitivity, the scales shown for $R_{\text{pre}}$ (TRUE)' are given in microns, and with condition 1 in mind, angles of only up to $60^\circ$ have been shown on the x abscissae.

3) Finally the critique must be that for a specific ray the $\frac{R_{\text{pre}}$ (FLAT)}{R_{\text{pre}}$ (TRUE)}$ must be such that for the value of $R_{\text{pre}}$ (TRUE), the finite value of $R_{\text{pre}}$ (FLAT) is within a certain amount; this amount will depend on the type of system considered, i.e. $R_\text{true}$ value and value of $r$.

The objectives will be to establish what the conditions are for the assumption of practical equality between $R_{\text{pre}}$ (FLAT & TRUE) and to show how other conditions alter the ray relationships between these two types of diaphragm.

FIGS. 70 and 71 show for different material diaphragms the effect of pressure on the '$R_{\text{pre}}$' against '$\alpha$' relationship. FIG. 72 shows for one diaphragm and pressure the effect of increasing the initial separation '$d_0$'.
The above mentioned figures all consider an axial central emergent ray \( r_e = 0 \); FIGS. 73 and 74 consider the effect of an offset axial ray \( r_e = 20 \mu \) and \(-20 \mu\) respectively. The effect when considering more axially offset rays is discussed later in Ch. 18.

Consider now FIG. 70 which shows the characteristics for a glass diaphragm when \( t = 40 \mu \) \( d_o = 30 \mu \) \( R_{dm} = 1.5\text{mm} \); it will be useful to refer to FIG. 64 which depicts its relative deflection profile when \( P = 300\text{mmHg}\). From which it can be seen that

1) For one pressure; increasing the ray emergent angle \( \alpha \), increases the value of \( R_{pre} \) (TRUE)' and decreases the \% ratio \( R_{pre} \) (FLAT)' which means the difference between \( R_{pre} \) (TRUE & FLAT) increases.

2) For one emergent ray angle \( \alpha \), increasing pressure causes a decrease in \( R_{pre} \) (TRUE)' and a decrease in ratio \( R_{pre} \) (FLAT)' .

The biggest finite difference between \( R_{pre} \) (FLAT)' and \( R_{pre} \) (TRUE)' will occur when \( P \) is a maximum and \( \alpha \) is a maximum and are shown by condition 16. Where \( P = 300 \text{ mm Hg} \) and \( \alpha = 60^{\circ} \) showing that \( R_{pre} \) (TRUE)' = 70 \( \mu \) and the 'ratio' .998 whereby \( R_{pre} \) (FLAT)' = 69.86; the difference resulting is infinitesimal.

Thus for the conditions shown in FIG. 70 the 'flat diaphragm' assumption is valid. This is to be expected when seeing the \% ratio scale value (99.5 to 100) and the small values of the 'true R' scale.

If a specific fibre system is considered, say a \( \mu \) system then if \( A = 3(\frac{R}{\mu}) \) and \( r = 26 \mu \) then the functioning boundaries for the system are shown by conditions 17 and 18 with \( R_{pre} \) as 26 and 78 thus \( \frac{d_o}{r} \)

becomes \( \frac{30}{26} = 1.15 \) and it is worth noting that for \( A = 3 \) and \( \frac{d_o}{r} = 1.15 \) that from FIG. 44 the response occurs on the negative slope of the static response curve.
FIG. 70 THE EFFECT OF CONSIDERING THE DIAPHRAGM TO DEFLECT IN A FLAT PROFILE - GLASS DIAPHRAGM
With the same fibre size \( r = 26 \mu m \) a separation \( \frac{d}{r} \) less than 1.15 would mean using a 'd' value of less than 30 \( \mu m \) as used in this Fig. Now as will be shown, reducing the initial separation reduces \( R_{pre}^{(TRUE)} \) and increases the ratio thus decreasing the error.

The main conclusions which can be drawn are that, for \( \mu \) systems

1) With separation of less than 30 \( \mu m \) and fibre radius of less than 26 \( \mu m \) the error in assuming a 'flat diaphragm' rather than a true diaphragm is negligible, (considering only near axial rays).

2) With \( A = 3 \); for this condition angles up to 60° need only be considered.

These are somewhat misleading conclusions, for although correct, the conditions for which they hold have been formed by considering practical conditions of separation and deflection but only for near axial rays.

FIG. 71 is included to show the equivalent ratio of two diaphragm materials having different thicknesses. The exact relationship for these thicknesses has been described earlier (equation 99a); With reference to FIG. 64 it can be seen that a 30 \( \mu m \) thick Be/Cu approximates to a 40 \( \mu m \) thick glass diaphragm. The figure shows the same situation as FIG. 70 except that a Be/Cu diaphragm of 30 \( \mu m \) thickness is shown.

FIG. 72 shows for the same diaphragm parameters as the previous fig, i.e. Be/Cu diaphragm \( R_{dm} = 1.5mm \) \( t = 30 \mu m \) \( p = 300 \text{ mm Hg} \), the effect on the \( R_{pre}^{(TRUE)} \) against \( \alpha \) relationship when the initial separation is 30 \( \mu m \) and 60 \( \mu m \).

As would be expected an increase in initial separation causes an increase in \( R_{pre}^{(TRUE)} \) and a decrease in ratio \( R_{pre}^{(FLAT)} \). The worst situation is shown by condition 21 where \( d_0 = 60 \mu m \) from which \( R_{pre}^{(TRUE)} = 175 \mu m \) and the ratio \( R_{pre}^{(FLAT)} = 99.52 \); thus the \( R_{pre}^{(FLAT)} = 174 \mu m \), the difference is still within 1/40 of a fibre
FIG. 71 EFFECT OF CONSIDERING THE DIAPHRAGM TO DEFLECT WITH A FLAT PROFILE - Be/Cu DIAPHRAGM

Axial Ray (r_e = 0)
Be/Cu diaphragm
t = 30 microns
d = 30 microns (initial sep.)
R_{dm} = 1.5 mm.

P=50 mm Hg.
P=150
P=300

"RATIO GROUP"

"TRUE R_{pre} GROUP"

TRUE RADIUS OF RETURNEO RAY "R_{pre} (true)" MIC

EXIT ANGLE \( \alpha \) DEGREES
FIG 72 THE EFFECT OF INITIAL SEPARATION ON THE ERRORS IN ASSUMING A "FLAT" DIAPHRAGM
diameter, but it should be noted that dependent on 'r' and 'R' the boundary condition may show that 'a' of less than 60° or greater than 60° is to be accounted for. The relationship between initial separation and 'R$_{pre}$ (TRUE)' and 'RATIO R$_{pre}$/FLAT' will be shown later, FIGS. 75 - 78, for the conditions of $P = 300 \, \mu m Hg$, a Be/Cu diaphragm with $R_{dm} = 1.5 mm$ and $t = 30 \mu m$. It will be seen that it is this relationship which is the crucial feature in defining the acceptability of the assumption (Page 260) for maximum error.

However, first let us consider the situation for an offset axial ray. FIG. 73 shows the effect when a positive offset is chosen i.e. $r_e = + 20 \mu m$ (deliberately chosen as about a normal fibre radius) using the maximum pressure condition ($p = 300 \, mm Hg$). Comparison with FIG. 71 shows that the ratio $R_{pre, FLAT}/TRUE$ has decreased and the value of $R_{pre, TRUE}$ increased. However, again the maximum errors are well under a fibre diameter.

An interesting case is when $r_e$ is negative ($-20 \mu m$) as shown in FIG. 74.

Where it can be seen that at an angle of about 45° the ratio of $R_{pre, (FLAT/TRUE)} = 1$.

Mathematically this condition can be found considering equation 101 for $R_{pre, TRUE}$ and 108 for $R_{pre, FLAT}$. It can be seen that they will be of the same value if

1) \( \gamma = 0 \)

thus $r_{dm} = 0$ from 102

In other words when 'a' allows the ray to meet the diaphragm at its centre, where 'grad' = 0 and $d_{pre, dm} = d_p$. Thus this will occur using equation 106 if

\[ \frac{-r_e}{\tan a} = d_0 - y_{max} \]

................................. 109
FIG. 73 THE EFFECT OF OFFSET AXIAL RAY IN ASSUMING DIAPHRAGM "FLAT"
Be/Cu DIAPHRAGM

- $t=30$ microns
- $d=30$ microns (initial separation)
- $P=300$ mm Hg.
- $r_e = -20$ microns
- $R_{dm} = 1.5$ mm.

FIG. 74 EFFECT OF "BELOW OFFSET" RAY WHEN ASSUMING FLAT DIAPHRAGM
For the curve shown in FIG. 74, using above equation 109 and TABLE 7
\[ \alpha = \tan^{-1} \frac{0.02}{0.03 - 0.009814} \]
\[ = 44.7 \]
This can be seen as shown by condition (22).

There can be seen to be two asymptotes for each of the ratio curves. Considering the top curve it can be seen that as the emergent angle decreases so the ratio increases, until 45° when the value of \( R_{\text{pre FLAT}} \) becomes larger than \( R_{\text{pre TRUE}} \) (although the finite values are very small). The asymptote occurs when the value \( R_{\text{pre TRUE}} \) becomes zero (i.e. central), at 26° for \( \alpha \) condition (23). The lower ratio curve then follows again from an asymptote as now \( R_{\text{pre FLAT}} \) is zero and as the angle \( \alpha \) decreases the ratio increases. Although rather complex this fig. shows that the errors incurred considering a flat diaphragm are still well within a nominal fibre diameter.

FIGS. 75 - 77 show how the radius of the returned ray is effected by the initial separation. \( \alpha \) is chosen as 60° with a pressure of 300 mm Hg. From these figures it can be seen that for the three cases shown \( r_e = 0 \) (FIG. 75), -20 (FIG. 76) and +20 (FIG. 77). microns that the error even for large displacements is small, especially when limits or boundary conditions for \( r \) and \( R \) are considered.

FIG. 78 shows clearly how for the same diaphragm the minimum separation is dependent on the maximum deflection of the diaphragm, which for the case shown is 0.009 \( \mu \)m (condition 24). Also note the change of scale of the vertical ratio axis which allows one to see the very small difference ensuing if a flat diaphragm is considered rather than a true diaphragm.

In conclusion then; for diaphragms considered which give deflections of the order of tens of microns, the error incurred considering a stylised diaphragm compared to a true diaphragm is negligible if rays of up to 60° are considered for near axial emergent rays.
FIG. 75  GRAPHICAL REPRESENTATION OF THE EFFECT OF THE INITIAL SEPARATION "d" ON THE ERRORS IN ASSUMING A FLAT DIAPHRAGM -2

Be/Cu Diaphragm

- 30 microns
- P = 300 mm Hg.
- a = 60 Degrees
- $R_m = 1.5 \text{mm}$
- $r_e = 0$
FIG. 7.6 GRAPHICAL REPRESENTATION OF THE EFFECT OF THE INITIAL SEPARATION \( d \) ON THE ERRORS IN ASSUMING A FLAT DIAPHRAGM.
FIG. 77 GRAPHICAL REPRESENTATION OF THE EFFECT OF THE INITIAL SEPARATION ON THE ERRORS IN ASSUMING FLAT DIAPHRAGM -4

Be/Cu Diaphragm

$t$=30 microns

$P$=300 mm.Hg.

$\alpha$=60 Degrees

$R_{dm}$=1.5 mm.

$r_e$=+20 microns
Fig. 78 Graphical representation of the effects of the initial step on the errors in assuming a flat diaphragm.

Initial separation diaphragm "d" microns

Be/Cu diaphragm

$P = 300$ mm Hg.

$\alpha = 60$ degrees.

$R_{dm} = 1.5$ mm.

$r_e = 0$.
In fact, as the values of $y_{\text{max}}$ as calculated from the 'linear theory' equation 91, are always greater than the true deflections (cf. 10.1.2.) the errors in ray position discussed are in fact pessimistic. But the effect of more axially offset rays is not given here and is to be found with reference to a specific system in Ch. 18.
11.1.2 REFLECTION FACTOR AS A FUNCTION OF THE OPTICAL FIBRE/'FLAT' DIAPHRAGM SYSTEM.

The previous section shows that for rays up to 60° the errors incurred in assuming a 'Flat' diaphragm rather than a true diaphragm are negligible for near axial rays. The 'Flat' diaphragm having a parallel deflection, for any pressure, equal to the central deflection of the true diaphragm.

Now using the notation used previously with

- $t =$ thickness of diaphragm
- $R_{dm} =$ radius of the diaphragm
- $E =$ Young's modulus
- $Z =$ Poisson's ratio
- $r =$ emitter radius
- $R =$ receiver outer radius
- $A = \frac{R}{r}$
- $d_o =$ initial separation of diaphragm at zero differential pressure
- $d_p =$ separation at specific pressure $'P'$
- $B = \frac{d}{r} =$ separation ratio
- $C =$ reflection coefficient for diaphragm

By using the earlier expression for reflection factor (78)

$$RF = \frac{C}{2r^2} \left( \frac{R^2 - r^2}{d_p} + \frac{4}{d_p} - \sqrt{(R^2 + r^2)^2 - 4R^2 r^2} \right)$$

noting that $d_p$ is the actual separation at any pressure.

Then with respect to FIG. 69

$$d_p = d_o - y_{max}$$

$y_{max}$ being the central deflection of the diaphragm for a pressure $'P'$

$$d_p = d_o - \frac{3P}{16Et^3} (1 - Z^2) R_{dm}^4$$

Now because the validity of the flat diaphragm assumption only holds for angles up to 60° for separations practicable then the above expression for reflection factor (110) should be modified for a 60° aperture
of fibre emergence rays i.e. \( \text{N.A.} = 0.866 \). However, it has been shown that the simplified method for accounting for Numerical Aperture equation 80, conditions 1, 2 and 3 (Page 142) is only satisfactory for separation past the transition point, then the minimum final separation \( d' \) when \( 'p' \) is its maximum value must be chosen such that condition (1) is fulfilled i.e. \[ 2d_p \tan \theta > R + r \] or \[ d_p > \frac{R + r}{2 \tan \theta} \] or \[ d_p > \frac{r(A + 1)}{2 \tan \theta} \]
so that the 'transition point' has been achieved.

Using the normalised notation

\[ B_{\text{p max}} = \frac{A + 1}{2 \tan \theta} \] \hspace{1cm} \text{112}

When this condition is used, the 'transition point' has been reached on the response curves (Chapter 8) and the standard form of equation 80 is used to represent this condition i.e. equation 110 where the value of \( d_p \) is found from 111.

Thus, by having the diaphragm located such that the condition in equation 112 is held, means that the derived expression for reflection factor (78) can be used even though the Numerical Aperture of the fibre considered is less than the unity condition assumed in its derivation.
It also means that the flat diaphragm assumption holds; that it is valid to apply the reflection factor expression to a curved (true) diaphragm by assuming it to be flat and deflect with a piston-like movement - a value of the central deflection that the true diaphragm would deflect (from considerations of near axial rays emergent).

Summarising then, the relationship between reflection factor of the optical fibre system/diaphragm is

\[ RF = \frac{C}{2r^2} \left( R^2 - r^2 + 4 \sqrt{r^2d_p^2 + d_p^4} - \sqrt{(r^2 + R^2 + 4d_p^2)^2 - 4R^2r^2} \right) \] \hspace{1cm} \text{110}

with \[ d_p = d_o - \frac{3P (1 - Z^2)R^4}{16Et^3} \] \hspace{1cm} \text{111}

with the condition that
1) \[ \frac{d_{max}}{r} \leq \frac{(A + 1) r}{2 \tan \theta} \]

2) \[ d_o \leq \frac{v_{max}}{\theta} \] (otherwise contact between diaphragm and fibres result)

3) The N.A. of the fibre is 0.866

4) The 'response' lies on only one side of static calibration curve.

Note any value of N.A. could be used less than 0.60.

It should be clarified that using the above relationship and conditions, a theoretical response curve for an optical fibre system and diaphragm unit for different pressures can be found if the separation is greater than the specified value.

This does not preclude a practical system being constructed of fibres of any N.A. with a diaphragm set at any initial separation for which a static calibration response curve can be determined practically. This will be of prime importance in the practical evaluation of the pressure transducer, as even if the Flat assumption does not hold, a practical system will allow for determination of its response. Now using equations 111 and 110 it is possible to plot for specific diaphragm parameters at a specific initial separation 'd_o' the reflection factor for any pressure; also the value of this reflection factor to the initial value when \( p = 0 \) (cf \( \frac{F_{pm}}{F_0} \))

Similarly using equation 89 such that

\[
\frac{F_p}{F_o} = \frac{A^2 - 1 + 4 \sqrt{B_p^2 + B_o^4}}{A^2 + 1 + 4 \sqrt{B_o^2 + B_o^4}} - \sqrt{A^4 + 1 + 16B_p^4 - 2A^2 + 8B_p^2} \frac{(A^2 + 1)}{p}
\]

where \( B_p \) is separation ratio \( \frac{d_p}{r} \); and \( B_o \) the initial separation ratio, the reflection factor ratio at any pressure \( p \) can be found ( \( \frac{d_p}{r} \) found from equation 111).
11.2.0. GRAPHICAL REPRESENTATION

It is now possible to plot a theoretical static pressure calibration response curve for a pressure transducer, given the optical fibre system parameters plus the diaphragm parameters and also the value of \( d_0 \) the initial separation. Then the ratio of the reflection factor at any pressure to its value at zero pressure can be determined.

In order to specify these parameters, let us consider some practical values. The diaphragm radius has a maximum permissible value of say 1.5 mm \( (R_{dm}) \); at this radius because of availability, a Be/Cu diaphragm say 30 \( \mu \) is a practical proposition. Its deflection being within the theoretical constraints for the diaphragm analysis.

From TABLE 6 its \( Y_{max} \) value is thus .009814 \( \mu \). (i.e. at \( P_{max} = 300 \text{mm Hg.} \))

The next problem is to decide at what initial separation to put this diaphragm.

Now, as has been shown earlier the finite value of \( d_0 \) (for a specific \( F \) and \( \frac{dF}{d(d)} \)) will be dependent on the finite size of \( r \) and \( R \); so let us consider values of \( r \) and \( A \). The dilemma as to which values can be chosen is best solved by first considering the relative merits of micro and macro systems functioning with a diaphragm which deflects .009814 \( \mu \). Earlier in Chapter 8 the value of the total reflection factor ratio was discussed \( \left( \frac{F}{F_0} \right) \), in relation to how this ratio was affected by what was termed the normalised deflection ratio \( \Delta B \). The normalised deflection ratio was defined as the ratio of the maximum deflection \( (Y_{max}) \) to the emitter radius. Then FIGS. 51 - 55 were shown to display how different values of 'A' and normalised deflection ratio '\( \Delta B \)' affected the ratio \( \frac{F_{pm}}{F_0} \). We now have a finite value to use; namely the deflection of the diaphragm. Thus in terms of normalised deflection ratio used we have it equal to \( \frac{0.009814}{r} \) as \( Y_{max} \) is defined, thus with any macro value for 'r' say 1mm.
We have very small values of the normalised deflection ratio, i.e. 0.009814, and with respect to the aforementioned figures it can be seen that the resulting value for $\frac{F_{pm}}{F_0}$ would be infinitesimal. Now for $r = 1$ we have the constraint that $A = 1.5$ as $R = 1.5$mm. So other values for $A$ must result in smaller values for $r$. As $r$ decreases so the normalised deflection ratio increases and the value $\frac{F_{pm}}{F_0}$ improves; however, for large $A$ values which result, it can be seen that a limitation on the initial separation becomes critical because as the response curve (FIG. 45) shows when $A = 50$ the useful part of this curve is only the positive side. Although this could be used it will be seen that the problem then becomes supplying enough light (Ch. 113) into a fibre of only say 50$\mu$m diameter ($dB$ then $\approx 4$, with $A = 60$). Thus macro systems will not prove suitable. (When using conventional light sources.)

Now for $\mu$ systems; any $\mu$ system apart from the simplest i.e. $A = 3$ (with $r =$ fibre radius) is not a practical system due to constructional problems. (Ch. 16)

So if true $\mu$ systems ($A = 3$) are considered, it has been shown that the smaller the value of $r$ the more sensitive the system. A commercially available fibre of good quality is produced by H.V. Skan and is 20$\mu$m radius and has a $60^\circ$ half angle with a very nearly uniform polar distribution. Consequently let us consider

$$r = 20\mu$$
$$R_{dm} = 1.5\text{ mm}$$
$$t = 30\mu$$
$$A = 3$$

The only consideration to be made now is at what separation the diaphragm should lie relative to the distal tip face of the optical fibres.

We do have a value for the minimum separation ratio for which the 'flat' diaphragm approximation holds, this as a separation ratio for the maximum applied differential pressure (equation 112).
\[
\frac{B_{\text{pmax}}}{2 \tan 60} \Rightarrow (A + 1) \text{ i.e. } B_{\text{pmax}} \gg B_t
\]

thus the minimum value of 'd', the initial separation is when accounting for the deflection of the diaphragm:

\[
d_{\text{pmax}} = d_o - Y_{\text{max}} \quad Y_{\text{max}} \text{ is central deflection of diaphragm with } p \text{ at max value}
\]

expressing in normalised terms

\[
\frac{A + 1}{2 \tan 60} = \left( \frac{d}{r_o} \right) - \left( \frac{Y_{\text{max}}}{r} \right) \quad (B_t = B_o - \Delta B)
\]

where \( \frac{Y_{\text{max}}}{r} \) is the normalised deflection ratio using the values given earlier (TABLE 6) equation 115 becomes

\[
\frac{2}{\tan 60} = \left( \frac{d}{r_o} \right) - 0.009814
\]

noting that the normalised deflection ratio becomes 0.49

\[
\frac{d}{r_o} = \frac{2}{\tan 60} + \frac{0.009814}{0.02000} = 1.645
\]

thus in finite terms the minimum initial separation becomes

1.645 x 0.020 \( \mu \) or 33 \( \mu \). (note condition 2 Page 277 is also satisfied)

This gives us a minimum value of 'd' to be used in equation 111 and 113 for the calculation of 'F' against 'd'. (and as it is past the B value the response \( p \) will lie in the same side) (Page 179)

The 'best' position for the diaphragm considering this ratio condition can be assessed graphically using FIG. 55 which shows reflection factor total ratio plotted against separation ratio and shows that a peak value for \( \frac{F_{\text{pm}}}{F_o} \) occurs at about 'd' = 2.1 (condition 25) with \( \frac{d_B}{B} = .5 \); and that for a separation ratio change of .1, very little change in \( \frac{F_{\text{pm}}}{F_o} \) results (condition 26). Inspection of graphs of slope factor (FIG. 47) against separation show that as the separation ratio increases so the slope factor decreases in this region i.e. \((2 - 2.2)\frac{d}{r}\).
but as the value of $\frac{F_{pm}}{F_o}$ is a ratio, the value of $F_{pm}$ will depend on the value of $F_o$. Now as can be seen on FIG. 55, there are two values for $A = 3$ curves when the ratio is the same; this is a ratioing effect, not a duality effect due to symmetric nature of response curve. Now as $\frac{\Delta d}{r}$ increases so $F_o$ decreases.

Now the closest initial separation is for analytical reasons as mentioned earlier on the previous page.

\[
d_o = 33 \mu
\]

with an optimum (this is discussed later) from FIG. 55 with respect to the ratio

\[
\frac{F_{pm}}{F_o} \text{ of } d_{opt} = 2.1 \times 0.02
\]

\[
= 42 \mu
\]

FIG. 79 shows both the value of $\frac{F_p}{F_o}$ and $\frac{F_m}{F_o}$ for different (ch. 14) applied differential pressures on the diaphragm when the initial separation is 33 and 42 microns (using equation 113). Condition (27) shows for the two initial separations the values of the ratio $\frac{F_{pm}}{F_o}$; these values could just as well have been obtained from FIG. 55 using the correct normalised deflection ratio (i.e. .5) curve at the correct separation ratio (1.64 & 2.1 respectively). However, FIG. 79 also shows how the ratio varies with pressure. It can be seen that for the diaphragm parameters chosen and optical fibre system that a very linear response is achieved.

Many such plots showing theoretical pressure static calibration response for different diaphragms and optical fibre systems could be performed using the criteria for definition of $d_o$ and $d_{opt}$ earlier discussed. However, it is best to consider the significance of the $\frac{F_{pm}}{F_o}$ ratio before looking at different diaphragms, etc. and this is discussed in context of detection amplifier system in the next chapter.
FIG. 79  STATIC PRESSURE CALIBRATION CURVE (THEORETICAL) AND REFLECTION FACTOR RATIO CURVE FOR A PRACTICAL SYSTEM

Be/Cu Diaphragm
- \( t = 30 \) microns
- \( R_{dm} = 1.5 \) mm.
- \( r = 20 \) microns (fibre rad)
- \( N.A. = 0.866 \)
- \( A = 3 \)
11.3.0. THE EFFECTS OF HAVING AN ENCLOSED AIR GAP BETWEEN THE DIAPHRAGM AND OPTICAL FIBRE SYSTEM

If there is an enclosed air gap between the diaphragm and end face of the fibres in the distal tip group, then the way in which the effective applied differential pressure on the diaphragm varies due to the following must be considered:

1) Changes in external pressure causing changes in enclosed air gap pressure
2) Changes in temperature causing changes in enclosed air gap pressure.

Both these effects are of interest as they can be seen to affect the actual differential pressure to which the diaphragm is subjected and hence the afferent fibre output.

11.3.1. THE EFFECT OF INCREASING EXTERNAL PRESSURE ON THE DIAPHRAGM

Let us first consider the volume displaced by a clamped circular diaphragm (FIG. 57) when subject to an applied differential pressure 'P'.

The deflection at any point on its radius 'rdm' is given by equation 90.

\[ y = \frac{3}{16Et} \frac{P}{3} (1 - z^2) \left( R_{dm}^2 - r_{dm}^2 \right)^2 \]

and let \( \frac{3}{16Et} \frac{P}{3} (1 - z^2) = H \); the volume displaced becomes

\[ V = \int_{0}^{R_{dm}} 2\pi r_{dm}^2 dy \]

\[ = 2\pi H \int_{0}^{R_{dm}} r_{dm}^2 \left( R_{dm}^2 - r_{dm}^2 \right) \left( R_{dm}^2 - r_{dm}^2 \right) \left( -2r_{dm} \right) d(r_{dm}) \]

\[ = -2\pi H \int_{0}^{R_{dm}} \frac{4r_{dm}^3}{4} \left( R_{dm}^2 - r_{dm}^2 \right) d(r_{dm}) \]

\[ = -2\pi H \int_{0}^{R_{dm}} \left( \frac{4r_{dm}^4}{4} \frac{R_{dm}^2}{r_{dm}} - \frac{4r_{dm}^6}{6} \right) d(r_{dm}) \]
\[ V = -2\pi H \left( R_{dm}^6 - \frac{2}{3} R_{dm}^6 \right) \]

\[ = -2\pi H \frac{1}{3} R_{dm}^6 \]

substituting for \( H \)

\[ V = -2\pi \frac{3P}{16Et^3} (1 - Z^2) \cdot \frac{1}{3} R_{dm}^6 \]

\[ = -\frac{3P(1 - Z^2)}{16Et^3} R_{dm}^4 \cdot \frac{2}{3} \pi R_{dm}^2 \]

\[ \text{which if } A_D \text{ is the normal cross section area of the diaphragm} \]

\[ V = -\frac{2}{3} A_D y_{\text{max}} \]

in other words the volume displaced is 2/3 of the value when considering the diaphragm as a 'flat' type which deflects an amount of 'y_{\text{max}}', the central deflection (piston-like).

Consider FIG. 80 which shows the initial condition for this analysis.

Now consider FIG. 81 which shows the external pressure to have increased by \( \Delta P_2 \) and consequently increasing the internal pressure an amount \( \Delta P_1 \), and hence causing a change in internal volume \( \Delta V_1 \).

At constant temperature using the gas laws,

\[ P_1 V_1 = (P_1 + \Delta P_1)(V_1 + \Delta V_1) \]

\[ \text{Now consider the diaphragm volume displacement noting that if } P_1 = P_2 \]

initially, \( P = \Delta P_2 - \Delta P_1 \).

Using equation 116

\[ \Delta V_1 = -\frac{3}{16Et^3} (1 - Z^2) R_{dm}^4 \cdot \frac{2}{3} A_D P \]

\[ \text{now let } : C = \frac{3}{16Et^3} (1 - Z^2) R_{dm}^4 \times \frac{2}{3} \]

\[ \therefore \Delta V_1 = -C A_D P \]

using value of 'P' mentioned above

\[ \Delta V_1 = -C A_D (\Delta P_2 - \Delta P_1) \]
FIG. 80 AIR GAP - INITIAL CONDITIONS

FIG. 81 AIR GAP CONDITIONS SUBSEQUENT TO INCREASE IN EXTERNAL PRESSURE ONLY
re-arranging equation 118
\[ 0 = \Delta V_1 \left( P_1 + \Delta P_1 \right) + \Delta P_1 \ V_1 \]
and substituting for \( \Delta V_1 \) from 119
\[ 0 = A_D C \left( \Delta P_2 - \Delta P_1 \right) \left( P_1 + \Delta P_1 \right) + \Delta P_1 \ V_1 \]
but \[ V_1 = A_D x \ d_o \]
\[ 0 = A_D C \left( \Delta P_2 - \Delta P_1 \right) \left( P_1 + \Delta P_1 \right) + \Delta P_1 \ A_D x \ d_o \]
which on simplification becomes
\[ 0 = \Delta P_1 \ ^2 + \Delta P_1 \ \left( P_1 - \Delta P_2 + \frac{d_o}{C} \right) - P_1 \Delta P_2 \] \[ \text{.................} \ 120 \]
noting that \( \Delta P_2 \) is the applied pressure on the diaphragm if the initial pressure is atmospheric \( (P_2) \), and the resulting differential pressure on the diaphragm is \( \Delta P_2 - \Delta P_1 \) if initially \( P_1 = P_2 \).

Thus from equation 120
\[ \Delta P_1 = - \left( P_1 + \frac{d_o}{C} - \Delta P_2 \right) + \sqrt{\left( P_1 + \frac{d_o}{C} - \Delta P_2 \right)^2 + 4P_1 \Delta P_2} \] \[ \frac{2}{2} \] \[ \text{.................} \ 121 \]

Therefore, for a specific initial separation \( d_o \) and diaphragm parameters, the value of the applied pressure and the resulting differential which the diaphragm suffers can be determined using initial conditions of atmospheric pressure within and external to the diaphragm.

The significance of this relationship is that the diaphragm will not deflect as much as would be expected because the actual differential pressure is lower than the applied pressure. Consequently the afferent fibre output will be related to the differential pressure which differs from the applied pressure. Consequently, a calibration curve of applied pressure and differential pressure is required.

This has been accomplished using equation 121 to determine \( \Delta P_1 \) (increase in internal pressure) for different values of \( \Delta P_2 \) (external pressure applied, above atmospheric) with specific values for \( d_o \) the diaphragm initial separation and \( C \) the parameters of the diaphragm.
FIG. 82 shows for a Be/Cu diaphragm of radius 1.5 mm and thickness 30 μ, the ratio of actual differential pressure on the diaphragm against the applied pressure; for different values of initial separation. It assumes the internal pressure initially is atmospheric and temperature remains constant.

Thus it can be seen, that as one would expect, increasing initial separation decreases the effect of the increase in internal pressure. For the practical situations depicted though, with d₀ = 50 μ, the differential pressure is only 65% of the applied pressure on the diaphragm. The real significance is that the sensitivity of the optical fibre system/diaphragm is reduced significantly because the deflection of the diaphragm for any value of applied pressure will also be reduced by this value (65%). Thus from the pressure standpoint(1) a vented system is indicated.

It is worth noting that a stiffer diaphragm would allow for a better sensitivity with regard to the ratio of actual differential pressure and applied pressure. However, the diaphragm as it is stiffer deflects less. (lower $\frac{F_{pm}}{F_0}$ results)
FIG. 82. GRAPH SHOWING THE EFFECTS ON THE DIFFERENTIAL PRESSURE ON A DIAPHRAGM DUE TO HAVING AN ENCLOSING AIR GAP (TEMP. CONST.)

Initial separation \( d = 200 \) microns

- \( d = 100 \)
- \( d = 50 \)
- \( d = 33 \)

FOR Be/Cu diaphragm

- \( t = 30 \) microns
- \( R_{dm} = 1.5 \) mm

\( y_{max 300'} = 0.009814 \) microns
11.3.2 THE EFFECTS OF TEMPERATURE ON THE VALUE OF ACTUAL DIFFERENTIAL PRESSURE TO WHICH THE DIAPHRAGM IS SUBJECTED

There are two causes of error here; one due to having an enclosed air gap where increases in temperature will cause increases in internal pressure. Secondly, although independent of the air gap factor, is the effect of temperature causing dimensional changes in the diaphragm system. Considering first the air gap temperature dependent feature and having initial conditions as shown in FIG. 80, then if the external pressure remains constant, i.e. \( P_2 \), and the temperature changes by \( \Delta T_1 \), there will be a change in internal pressure of \( \Delta P_1 \).

From gas laws

\[
\frac{P_1V_1}{T_1} = \frac{(P_1 + \Delta P_1)(V_1 + \Delta V_1)}{(T_1 + \Delta T_1)}
\]

\[
P_1V_1 + P_1V_1 \Delta T_1 = T_1(P_1V_1 + P_1 \Delta V_1 + \Delta P_1 V_1 + \Delta P_1 \Delta V_1)
\]

\[
P_1V_1 + P_1V_1 \Delta T_1 = P_1V_1 + P_1 \Delta V_1 + \Delta P_1 V_1 + \Delta P_1 \Delta V_1
\]

\[
o = P_1 \Delta V_1 + \Delta P_1 V_1 + \Delta P_1 \Delta V_1 - P_1V_1 \Delta T_1
\]

\[
o = \Delta V_1(P_1 + \Delta P_1) + V_1(\Delta P_1 - P_1 \Delta T_1)
\]

As before when considering the diaphragm

\[
\Delta V_1 = -CA_D P \quad \text{where} \quad P = (P_2 + \Delta P_2) - (P_1 + \Delta P_1)
\]

when if \( P_1 = P_2 \) initially and \( \Delta P_2 = 0 \) i.e. remains at atmosphere externally

\[
\Delta V_1 = -CA_D (\Delta P_2 - \Delta P_1)
\]

and also \( V_1 = A_D d_o \)

on substitution of \( \Delta V_1 \) & \( V_1 \) into

\[
\text{......................... 123}
\]
\[ 0 = -C_A d (\Delta P_2 - \Delta P_1) (P_1 + \Delta P_1) + d\Delta (\Delta P_1 - P_1 \Delta T_1) \]
\[ = - (\Delta P_2 - \Delta P_1) (P_1 + \Delta P_1) + \frac{d_o \Delta P_1}{C} - \frac{d_o P_1}{C} \frac{\Delta T_1}{T_1} \]

\[ \text{now if } \Delta P_2 = 0 \]
\[ 0 = + \Delta P_1 P_1 + \Delta P_1^2 + \frac{d_o}{C} \Delta P_1 - \frac{d_o P_1}{C} \frac{\Delta T_1}{T_1} \]
\[ 0 = \Delta P_1^2 + \Delta P_1 (P_1 + \frac{d_o}{C}) - \frac{d_o P_1}{C} \frac{\Delta T_1}{T_1} \]

\[ \text{thus } \Delta P_1 = - (P_1 + \frac{d_o}{C}) \pm \sqrt{(P_1 + \frac{d_o}{C})^2 + 4 \frac{d_o P_1}{C} \frac{\Delta T_1}{T_1}} \]

\[ \text{Thus if } P_1 = P_2 \text{ and the temperature changes by } \frac{\Delta T}{T} \text{ the increase in internal pressure } \Delta P_1 \text{ can be found. However, it is of more practical use to express this pressure increase for different values of the applied internal pressure when there is a change in internal temperature.} \]

So if \( P_2 \neq 0 \)

Then equation 124 must be expanded
\[ 0 = - \Delta P_2 P_1 - \Delta P_2 \Delta P_1 + \Delta P_1 P_1 + \Delta P_1^2 + \frac{d_o \Delta P_1}{C} - \frac{d_o P_1}{C} \frac{\Delta T_1}{T_1} \]
\[ 0 = \Delta P_1^2 + \Delta P_1 (P_1 - \Delta P_2 + \frac{d_o}{C}) - P_1 (\Delta P_2 + \frac{d_o}{C} \frac{\Delta T_1}{T_1}) \]

Now equation 127 describes the change in internal pressure \( \Delta P_1 \) due to a change in applied pressure \( \Delta P_2 \) and a change in internal temperature \( \Delta T_1 \).

Expressing \( \Delta P_1 \) from equation 127
\[ \Delta P_1 = - \left( P_1 - \Delta P_2 + \frac{d_o}{C} \right) \pm \sqrt{\left( P_1 - \Delta P_2 + \frac{d_o}{C} \right)^2 + 4 P_1 \left( \Delta P_2 + \frac{d_o}{C} \frac{\Delta T_1}{T_1} \right)} \]
As before $\Delta P_2 - \Delta P_1$ is the actual differential pressure on the diaphragm. It can be seen that this is a general equation which takes account of both temperature and pressure effects on the pressure within the diaphragm, for if $\Delta T_1 = 0$ equation 128 reverts to 121, the pressure relationship; and if $\Delta P_2 = 0$ the case for constant atmospheric applied pressure results, equation 126.

Our interest lies in how a change in environmental temperature effects the effective applied differential pressures for various values of applied pressure. Assuming that $T_1$ is the ambient temperature, i.e. $273 + 20 = 293^oK$; and body temperature is $(37^o + 273^o)K = 310^oK$, the effects of an increase in temperature of $17^oK$ will be shown together with the effects of a smaller change say $2^oK$, for this later case. Considering the initial temperature of the gap $T_2$ to be at blood heat $(310^oK)$. Thus for the case of a probe sealed at ambient temperature:

$$\frac{\Delta T_1}{T_1} = \frac{17}{273}$$

and for a probe sealed at blood temperature:

$$\frac{\Delta T_1}{T_1} = \frac{2}{310}$$

The following FIG. 83 shows how for these two different values of $\frac{\Delta T}{T}$, the actual differential pressure varies with applied pressure.

As is to be expected from inspection of equation 128, it can be seen that if $\frac{d_o}{C} \frac{\Delta T}{T}$ is small compared to the applied pressure $\Delta P_2$ (Group 1) then the resulting plot of the ratio of differential pressure to applied pressure against applied pressure, tends to the result obtained from equation 121 (accounting only for effects of $\Delta P_2$) and shown graphically in FIG. 82. However, if $\frac{d_o}{C} \frac{\Delta T}{T}$ is significant when compared to $\Delta P_2$ (Group 2) then a more severe effect on the ratio can be seen. Thus from the temperature change viewpoint, it is of prime importance that the air gap sealing temperature is as close to
its in-vivo temperature, as possible. As before, a larger initial separation and stiffer diaphragm will reduce the temperature effects on the air gap. However, the values shown in FIG. 83 are practical system values.

In conclusion it can be seen that for a fibre optic pressure transducer it is advisable to have a vented air gap. This aspect will be discussed in Chapter 15.

Let us briefly consider the effect of temperature on dimensional stability: if \( B_e \) = coefficient of linear expansion and suffixes 'o' and 't' are the initial and final temperatures respectively. The new value of say the initial separation 'd_o' becomes

\[
d_t = d_o (1 + B_e t)\]

Where \( B_e \) is the coefficient of linear expansion of the annulus which separates the diaphragm from the distal fibre group.

Thus \[
\frac{d_t}{d_o} = 1 + B_e t
\]

for Be/Cu \( B_e = 17 \times 10^{-6} \)

i.e. \[
\frac{d_t}{d_o} = 1 + 17 \times 10^{-6} t
\]

and as 't' will be of the order of only degrees in a maximum condition.

The change in position, with 'd_o' equal to tens of microns, will be seen to have a negligible effect on the separation of the diaphragm, especially with respect to the separation ratio \( \frac{d}{r} \) and normalised deflection ratio \( \frac{d'}{r} \) where \( \frac{d'}{d_o} \approx 1.002 \).
FIG. 83 EFFECT OF TEMPERATURE CHANGE FOR DIFFERENT APPLIED PRESSURES ON THE ACTUAL VALUE OF THE DIFFERENTIAL PRESSURE ON THE DIAPHRAGM
It has been shown how a theoretical static pressure calibration response curve can be formed for a fibre optic intravascular pressure transducer and that these curves are theoretically correct for a Numerical Aperture up to $60^\circ$ and an 'A' value of 3 or less, when the separations are past the transition point and only near axial rays are considered.

The relationship of against separation has also been included because if it is approximately linear then the ratio attained from these curves; or from the figs. discussed in Chapter 8, describing normalised deflection ratios (using the conditions for which they hold in equation 112), as will be discussed, enable the optical fibre/diaphragm specification to be chosen when considering the resolution of the detection/amplifier system as the limiting criteria factor in system design (Ch. 14)

The effect of an enclosed air gap between the diaphragm and distal fibre group face has been investigated. The investigation shows that because, for a estimate practical system, the volume of air entrapped is of the same order as the volume which the diaphragm would displace on its own, it would be far preferable to have a vented air gap (which is also useful as a means for checking pressure whilst measuring).
CHAPTER 12

INVESTIGATION OF

DETECTOR AND AMPLIFIER

RESPONSE CHARACTERISTICS.
There are various types of photo-detectors available, which are based on two principles; devices which measure the rate which radiant energy is absorbed (thermal detectors) and secondly those which measure the rate which 'quanta' are absorbed (photon detectors) (KRUSE, P.W. 1963). These will be discussed with a view to specifying which type could be most suitable for transducing the afferent fibre output into an electrical analogue.

A total system block diagram is shown, FIG. 86; This is based on the earlier system schematic FIG. 18, but incorporates several system parameters, including a general transducing factor 'f' for the photo-detection which is now to be considered.

The system output expression equation 129 shows how all the parameters contribute to the output signal, and in Chapter 14 are discussed. It is the aim of this Chapter to investigate the effects of different detector modes, i.e. different 'f' functions, on system output.

For the most suitable detector type and modes of operation, the resolution (a fraction of total range of pressure to be measured), capable by the detector in terms of the total change in reflection factor $F_{\text{pm}}$, is defined. By using a 'signal to noise ratio' criteria based on the detector resolution, a value for the overall system noise is shown which allows the effect of other system parameters to be assessed.

Certain problems associated with the amplifier system are discussed.
12.0.1 TYPES OF PHOTO-DETECTORS

There are two fundamental types of photo-detectors; Thermal Detectors and Quantum detectors. Thermal detectors measure the change in temperature of an absorber, caused by the incident radiation falling on them. The output obtained from them can be in the form of a thermal e.m.f. (thermo-couple detector); a change in resistance of a conductor (Bolometer). Such detectors can be made to respond to radiation over a wide frequency band. However, because of the 'thermal inertia' of the detector their response times are slow (.1 to 0.1 sec.). It is the quantum type of detector which will be seen to be more suitable.

Quantum detectors depend on the interaction of radiation with the electrons in a solid, causing the electrons to be excited to a higher energy state. These effects depend on the quantum nature of radiation, consequently they are termed quantum detectors.

The responsivity of a quantum detector is a measure of its sensitivity to light and can be defined as the ratio of the current produced to the amount of light falling on it in watts. An idealised graph of response/watt (responsivity) against incident wavelength is shown in FIG. 84, where it can be seen that, as the wavelength increases the responsivity also increases. This is because, as the quantum energy of an electromagnetic radiation is proportional to the inverse of its frequency, then for a unit of energy at a higher frequency, there will be a greater number of quanta (can be considered as photons) each giving rise to an electron hole pair (current) within the semi-conductor. However, because the absorption and reflection coefficient for the radiation falling on the detector also varies with its wavelength, then in practice, a peak responsivity value is obtained (usually around 800n M) which gives a value for light source specification, and because quanta or photon detectors require incident photons to have more than a certain minimum energy before they are detected, above a certain wavelength the
FIG. 84 IDEALISED RESPONSE CHARACTERISTICS FOR A QUANTUM DETECTOR
response curve will cut off, (thereafter decreasing in practice). This gives rise to a spectral responsivity relationship.

There are two types of detector which use this effect, a photo-emission vacuum system using a metal as electron source and solid state semi-conductor type.

The photo-emissive type is such that when electrons are given enough energy to escape from a solid, they flow through a vacuum to give a current, however, for this to occur very high voltages must be applied across the anode and cathode of the system; they are also bulky.

Solid state systems themselves fall into two groups using semi-conductor materials, the difference being that one system uses a single type of semi-conductor (n-type); the other uses a 'junction effect' between an n and p type semi-conductor.

The single type semi-conductor produces a 'Photo-Conductor Bulk effect cell' and are normally of CdS (Cadmium Sulphide) or CdSe (Cadmium Selenide). They behave like resistors whose resistance decreases non-linearly as the light level increases. However, their response time is, for low light levels, about 1 sec. and they also suffer from a feature termed 'light memory' which means that the resistance of a cell at a specific light level is a function of the cells previous exposure to light (both level and duration).

Considering now a junction effect detector, variously called photo-voltaic cell, photo-transistor and photo diode. Then according to its externally formed circuit connections it can fulfil different operational modes. These are best discussed with respect to FIG. 85 which shows the photo-current and bias voltage relationship for various levels of incident light.

When the cell is working in its photo-voltaic mode there are two distinct forms the output can take, dependent on the bias value and the load value.
1) If the cell has an initial zero bias but has a large load resistor value (shown by load line 1) then the cell produces a current (consequently voltage) which varies logarithmically with the level of incident light; and the response occurs in quadrant 4, the photovoltaic region.

2) Again if initially the cell has a zero bias, but has a very low or zero load resistor, i.e. short circuited, then the photo-current produced varies linearly with the level of incident light. Also, it again is effectively working in the photovoltaic region of the response characteristics (quadrant four). One important feature is that when zero biased the device has effectively a zero value of $i_d$ the dark current.

3) By arranging for a negative bias on the device, the operational mode falls into the photo-conductive region (quadrant 3), when with a near zero load a vertical load line (3) results and a similar response to light as in (2) occurs, except that the value $i_d$ of the dark current is not zero and makes it comparatively unsuitable (as temperature changes affect $i_d$) for use as a low level photo-detector; and if the load is not zero but say $R_L$ (loadline 4) is achieved with $V_a$ as reverse bias voltage it will also suffer from temperature affects on $i_d$.

By introducing another layer into the junction region, this time a separating layer of 'intrinsic' silicon the device becomes more stable, more sensitive, consequently can be smaller. These have been termed PIN type photo-detectors by manufacturers and may be of two types "Planer Diffused or Schottky Barrier".

Thus for reasons of sensitivity, size and durability it is the p/n or PIN type of photo-detector which are operating in their photovoltaic region which will be considered in more detail.
As has been outlined, the response can be linear (current output or logarithmic (voltage output), dependent on how the device is 'wired' as has been shown by loadline 2 and 1 respectively, (FIG. 85). Bearing in mind that for the former mode, the value of the dark current \( i_d \) is zero and consequently the effects of changes in ambient temperature have no effect on the dark current.
FIG. 85 "JUNCTION EFFECT" PHOTODETECTOR V/I CHARACTERISTICS

- Bias voltage V

-1

+ Bias voltage

0.5

1.0

15 10 \( V_a \) 5

"QUADRANT 4" 
"PHOTOVOLTAIC REGION"

"QUADRANT 3" 
"PHOTOCONDUCTIVE REGION"

V_a

\( \frac{V_a}{R_L} \)

+I (Photocurrent)
THE DEPENDANCE OF SYSTEM OUTPUT ON THE TYPE OF PHOTO-DETECTOR 
AND OPERATING MODE

A total system block diagram of a fibre optic pressure 
transducer is shown in FIG. 86. Where as earlier described (p.104) 
'q' is the radiant flux emitted by the light source; 'Cf' is a coupling 
factor between the light source and efferent fibres, 'tc' is the 
transmission of flux by the efferent fibres; 'RF' the flux reflection 
factor of the diaphragm/optical fibre system and 'Cd' the coupling factor 
between the afferent fibres and photo-detector. Two further parameters 
are included; 'f' the photo-detector transducing function and 'G' the 
amplifier gain thus the system output is

\[ y = Gf(RF t_c^2 C_d e_i) \]

The units of 'y' will depend on those of 'G' and 'f' which will 
 vary with the type and mode of operation of the photo-detector. For 
photo-voltaic cells, which fulfil in general terms the requirements for 
our detector there are two modes of operation to be investigated.

These have been mentioned earlier; (1) when the cell is 
operated with a load connected across it, such that with respect to FIG.85 
load line (1) is achieved. This will provide a voltage output device such 
that the output is a logarithmic function of the flux falling on the cell 
and, (2) when the cell is zero biased with a very low value of load or 
preferably short circuited, when the output current from the device 
varies linearly with the flux falling on the cell (load line 2).

At this time it is worth noting that the current device would be 
advantageous, as for a zero biased system the dark current is also zero.

Before discussing in detail these two 'f' functions, a critical 
factor with respect to the manufacturer's published response curve must be 
clarified.

Manufacturers usually give photo-detector responses for values 
of flux intensity rather than flux falling on the cell, i.e. mW/cm² instead
FIG. 86  TOTAL SYSTEM-BLOCK DIAGRAM

NOTE
"f" the photo-detector transducing function depends on the material in which the detector is operating,
and that the length etc. of the efferent and afferent fibres must be equal, such that the transmission through the optical fibres is consistent.
of mW because customers usually require a flux intensity/response relationship. The photo-detector itself purely responds to the level of flux falling on it; manufacturers know the area of their detector and its responsivity. Consequently, they divide the flux by the area of detector to give a value of flux intensity for a specific response. The analysis here describes the value of light falling onto the photo-detector as a flux level (mW), which can be seen from the coupling factor $C_f$ and $C_d$. Thus, specifically for the voltage device, an adjustment must be made to either the manufacturer's response characteristics or the described value of 'flux on detector $e_i$' to enable a practical system to be defined by modeling. This is best achieved by simply dividing the flux falling on the detector, i.e. $R_f t^2 C_f C_d e$; by the detector area ($A_d$) then the manufacturer's response characteristics can be used directly.
12.1.1 LOGARITHMIC MODE DETECTOR

For a logarithmic mode detector its transducing function is in open circuit conditions (i.e. large load value)

\[ y = M \log \frac{I}{Q} + Q \text{ volts} \]

where \( M \) and \( Q \) are constants in voltage units.

Now \( I \) is the intensity of flux on detector, which if it is of area \( a_d \) and omitting 'G' the amplifier gain

\[ I = RF t^2 \frac{C_d C_f}{a_d} e_i; \]

now \( t^2 \frac{C_d C_f}{a_d} \) is a constant for one system

\[ \therefore \quad I = RF K_s e_i; \quad K_s = t^2 \frac{C_d C_f}{a_d} \]

\[ \therefore \quad \text{equation 130 becomes} \]

\[ y = M \log \left( RF K_s e_i \right) + Q \text{ volts} \]

Now consider outputs \( y_1 \) and \( y_2 \) for values of reflection factor \( F_1 \) and \( F_2 \) respectively

\[ y_1 = M \log \left( F_1 K_s e_i \right) + Q \]

\[ y_2 = M \log \left( F_2 K_s e_i \right) + Q \]

so the difference between the two outputs is

\[ y_1 - y_2 = M \log \frac{F_1}{F_2} \]

It is worth noting that due to the logarithmic nature of the response the magnitude of the light source output is not included in the expression of the difference in these.

If the system is to resolve a pressure of \( \triangle P \)

\[ \triangle P = \frac{1}{N} \left( P_{\text{max}} - P_{\text{min}} \right) \]

where

\[ P_{\text{max}} = \text{max differential pressure on diaphragm.} \]

\[ P_{\text{min}} = \text{min differential pressure on diaphragm.} \]
However, a problem now arises, because although it has been shown earlier that for small deflections of the pressure sensing diaphragm the pressure reflection factor relationship is linear, thus

\[ \Delta P = \frac{P_{\text{max}} - P_{\text{min}}}{N} \equiv \Delta F = \frac{P_{\text{pm}} - P_{o}}{1} \]

there is a problem in defining the 'sign' of the change in reflection factor \( \Delta F \) because if the point of operation lies on the positive slope of the static calibration curve \( P_{\text{pm}} < P_{o} \); and if it lies on the negative slope \( P_{\text{pm}} > P_{o} \) consequently, for the sake of clarity these two conditions which define the system will be treated separately.

A) When \( \frac{P_{\text{pm}}}{P_{o}} < 1 \)

then \( \Delta F = \frac{1}{N} (P_{o} - P_{\text{pm}}) \) ............................................ 135

now to find the change in system output for a change in reflection factor \( \Delta F \), which is that resulting from the minimum pressure change to be resolved then with respect to equation 133

\[ F_{2} = F_{1} + \Delta F \] .................................. 136

where \( F_{2} \) is the reflection factor resulting after the change \( \Delta F \) occurs from its prior level \( F_{1} \); the + sign denotes whether an increase or decrease of \( \Delta F \) is considered.

Thus using 136 in 133

\[ \Delta y_{N} = M \log \frac{F_{1}}{F_{1} + \Delta F} \]

and substituting for \( \Delta F \) from equation 135

\[ \Delta y_{N} = M \log \frac{F_{1}}{F_{1} + \frac{1}{N} (P_{o} - P_{\text{pm}})} \]

which may be written

\[ \Delta y_{N} = - M \log \left( \frac{F_{1} + \frac{1}{N} (P_{o} - P_{\text{pm}})}{F_{1}} \right) \]

…………………………. 137
where $\Delta y_N$ is the change in system output resulting from a change $\Delta F$
in reflection factor from an initial level $F^1$ and $\Delta F$ is the reflection
factor change due to an increment of pressure change. Let this value
be some proportion of the reflection factor ($F^0_o$) for zero applied
differential pressure on the diaphragm

\[ i.e. F^1 = x F^0_o \]

so equation 137 can be written

\[
\Delta y_N = - M \log \left( \frac{xf + \frac{1}{xN} (F_o - \pm \Delta F)}{xf o} \right)
= - M \log \left[ 1 \pm \frac{1}{xN} (1 - \frac{F_pm - \pm \Delta F}{F_o}) \right]
\]

\[ ....................... 138 \]

B) WHEN $\frac{F_pm}{F_o} \gg 1$

then $\Delta F = \frac{1}{N} (F^p_m - F^o_o)$

\[ ....................... 139 \]

when as before $F^2 = F^1 + \Delta F$

\[ ....................... 140 \]

and

\[
\Delta y_N = M \log \left( \frac{F^1}{F^1 + \Delta F} \right)
\]

\[ ....................... 141 \]

\[
\Delta y_N = M \log \left( \frac{F^1 \pm \frac{1}{N} (F^p_m - F^o_o)}{F^1} \right)
\]

\[ ....................... 142 \]

\[
\Delta y_N = - M \log \left( \frac{F^1 \pm \frac{1}{N} (F^p_m - F^o_o)}{F^1} \right)
\]

\[ ....................... 143 \]

and

again if $F^1 = x F^o_o$

\[
\Delta y_N = - M \log \left( \frac{x F^o_o \pm \frac{1}{N} (F^p_m - F^o_o)}{x F^o_o} \right)
\]

\[ ....................... 144 \]

and finally

\[
y_N = - M \log \left[ 1 \pm \frac{1}{N} \left( \frac{F_pm}{F_o} - 1 \right) \right]
\]

\[ ....................... 145 \]
Thus summarising:

\[ \frac{F_{pm}}{F_o} \ll 1 \]

\[ \Delta y_N = -M \log \left[ \frac{1 \pm 1}{N_x} \left( 1 - \frac{F_{pm}}{F_o} \right) \right] \quad \text{.............. 138} \]

and if \( \frac{F_{pm}}{F_o} \gg 1 \)

\[ y_N = -M \log \left[ \frac{1 \pm 1}{N_x} \left( \frac{F_{pm}}{F_o} - 1 \right) \right] \quad \text{.............. 145} \]

thus for \( \frac{F_{pm}}{F_o} \ll 1 \); \( \frac{F_{pm}}{F_o} \ll x \ll 1 \)

and \( \frac{F_{pm}}{F_o} \gg 1 \); \( \frac{F_{pm}}{F_o} \gg x \gg 1 \)

noting that the '±' sign in equation 138 and 145 denote whether there is an increase or decrease in reflection factor at the specific value 'xF_o' where the change occurs, and if \( \Delta y_N \) is positive the prior level of 'y' is greater than its subsequent values.

To obtain quantitative values of \( \Delta y_N \) for different x values with \( \frac{F_{pm}}{F_o} \) and N as set values; the magnitude of the constant M was required.

Plessey Electronics supplied literature concerning their 'SCI' series of photo-voltaic cells. From this literature for an 'SCI' in open circuit conditions, both M & Q were determined for flux values of incident light flux intensity \( 1 - 20 \text{ mw/cm}^2 \)

\[ M = 77 \text{ mV} \]

\[ Q = 330 \text{ mV} \]

hence \( \Delta y_N \) described in 138 and 145 is in mV when the value of M above is used.

The two conditions for describing \( \Delta F \) lead to the difference in form of equations 138 and 145 if \( \Delta F \) is written such that

\[ \Delta F = \frac{1}{N_x} + \sqrt{\frac{F_o - F_{pm}}{2}} \quad \text{.............. 146} \]
then we have the modulus of $\Delta F$, which is required. Then both forms of descriptions for $\Delta y_N$ can be described by

$$y_N = - M \log \left[ 1 \pm \frac{1}{xN} \left( \sqrt{\left( \frac{F_{pm}}{F_0} \right)^2 - 2 \frac{F_{pm}}{F_0} + 1} \right) \right]$$

Using equation 147, $\Delta y_N$ was plotted for different values of $\frac{F_{pm}}{F_0}$, against $x'$, the proportion of $F_0$ at which the equivalent $\Delta F$ reflection factor change due to the pressure to be resolved, occurred.

Before discussing the graphs obtained, the effect on the finite system output of changing the required resolution of the system is discussed. 'N' has been introduced as the integer by which the resolution is defined. The larger it becomes, the smaller the required resolution (see equation 134).

Let us consider how its value affects the system output, given that $\frac{F_{pm}}{F_0}$ & $x$ is fixed. Now $\Delta y_N$ is from equation 147

$$\Delta y_N = - M \log \left( 1 \pm \frac{1}{xN} \left( \sqrt{\left( \frac{F_{pm}}{F_0} \right)^2 - 2 \frac{F_{pm}}{F_0} + 1} \right) \right)$$

and for the sake of clarity let

$$\frac{1}{x} \sqrt{\frac{F_{pm}}{F_0}^2 - 2 \frac{F_{pm}}{F_0} + 1} = a \text{ constant } K_N$$

Now considering two different values of 'N'; say $N_1$ and $N_2$, then the system output changes for the resolution requirements is defined by $N_1$ & $N_2$

$$\Delta y (N_1) = - M \log \left( 1 \pm \frac{1}{N_1} K_N \right) \quad \text{148}$$

$$\Delta y (N_2) = - M \log \left( 1 \pm \frac{1}{N_2} K_N \right) \quad \text{149}$$

Therefore $\Delta y (N_2) = \frac{\log \left( 1 \pm \frac{1}{N_2} K_N \right)}{\Delta y (N_1) = \log \left( 1 \pm \frac{1}{N_1} K_N \right)}$
Therefore \( (1 + \frac{1}{N_1} K_N) \Delta y (N_1) = 1 + \frac{1}{N_2} K_N \) .......................... 150

as the value of \( \frac{1}{K_N} \) is very small ( .001) then the L.H.S. of equation 150 becomes, from the binomial expansion

\[
\frac{\Delta y (N_2)}{\Delta y (N_1)} (1 + \frac{1}{N_1} K_N) = 1 + \frac{y (N_2)}{y (N_1)} \frac{1}{N_1} K_N
\]

the equation 150 becomes

\[
1 + \frac{\Delta y (N_2)}{\Delta y (N_1)} \frac{1}{N_1} K = 1 + \frac{1}{N_2} K
\]

Therefore \( \frac{\Delta y (N_2)}{\Delta y (N_1)} = \frac{N_1}{N_2} \) ........................................ 151

In other words, if the finite value of the system output change is known for a specific desired system resolution, as defined by say \( N_1 \), then the system output change varies inversely with the desired resolution as defined by \( N \).

So by halving the pressure resolution required, the system output is increased by a factor of two.

And let us also consider the relevance of an \( \pm \) change of \( \Delta F \) at each value of \( F \).

If the two values of \( \Delta y_N \) obtained from equation 147, when considering a -ve change and a +ve change are considered; again using the value of \( K_N \) as before

\[
\Delta y_{N+} = -M \log (1 + \frac{1}{N} K_N)
\]

and

\[
\Delta y_{N-} = -M \log (1 - \frac{1}{N} K_N)
\]

therefore

\[
\frac{\Delta y_{N+}}{\Delta y_{N-}} = \frac{\log (1 + \frac{1}{N} K_N)}{\log (1 - \frac{1}{N} K_N)}
\]
from which
\[
(1 - \frac{1}{N}) \left( \frac{\Delta y_{N+}}{\Delta y_N} \right) = 1 + \frac{1}{N} K_N
\]

again using the binomial approximations for second order terms
\[
1 - \frac{\Delta y_N + 1}{\Delta y_N} K = 1 + \frac{1}{N} K
\]

Therefore
\[
\frac{\Delta y_{N+}}{\Delta y_{N-}} = -1
\]

Thus the effect of an increase of reflection factor \( \Delta F \) is identical but opposite in magnitude to the system output change when a decreasing value for \( \Delta F \) is chosen. This being considered for one value of \( \frac{F_{pm}}{F_0} \) and \( x \).

Now let us consider the significance of either an increase or decrease of \( \Delta F \) occurring at \( F_1 \); in terms of whether it signifies an increase or decrease of the value of the pressure to be resolved. There has been shown to be two conditions relating \( \Delta P \) and \( \Delta F \), namely equations 135 and 139 for \( \frac{F_{pm}}{F_0} \ll 1 \) and \( \gg 1 \) respectively.

Thus if \( \frac{F_{pm}}{F_0} \ll 1 \); then an increase of \( \Delta F \) at \( F_1 \) is equivalent to a decrease of \( \Delta P \); and conversely a decrease of \( \Delta F \) at \( F_1 \) is equivalent to an increase of \( \Delta P \) and if \( \frac{F_{pm}}{F_0} \gg 1 \); then an increase of \( \Delta F \) at \( F_1 \) is equivalent to an increase of \( \Delta P \) and conversely a decrease of \( \Delta F \) at \( F_1 \) is equivalent to a decrease of \( \Delta P \).

In other words, on the positive slope of the static calibration curve \( \frac{F_{pm}}{F_0} \ll 1 \), an increase of \( \Delta P \) causes a decrease in \( \Delta F \) and vice versa; whilst on the negative slope of the static calibration curve \( \frac{F_{pm}}{F_0} \gg 1 \) an increase of \( \Delta P \) causes an increase of \( \Delta F \) and vice versa.

Two figures are shown which depict the system output change against the proportion of the value \( F_0 \) at which the value \( \Delta F \) occurs. \( \Delta F \) being the change in reflection factor resulting in a change of 1/300 of the pressure range of the system, i.e. \( N = 300 \). Each figure shows how different \( \frac{F_{pm}}{F_0} \) values alter the finite system output charge.
FIG. 87 considers the effects of an increase in pressure $\Delta P$ for both cases (where $\frac{F_{pm}}{F_0} \ll 1 \& \gg 1$) and FIG. 88 the effects of a decrease in pressure.

Each of the graphs can be seen to be composed of two groups of curves; the group on the left being for $\frac{F_{pm}}{F_0} \ll 1$, whilst the group on the right for $\frac{F_{pm}}{F_0} \gg 1$, each curve itself being for one value of $\frac{F_{pm}}{F_0}$.

From these graphs it can be seen that for $\frac{F_{pm}}{F_0} \ll 1$; as $F_0 \to F_{pm}$ i.e. minimum value of $x$, then the system output change $\Delta y_N$ for either a positive or negative $\Delta F$, increase; consequently the minimum value of $\Delta y_N$ occurs when $x = 1$, i.e. at $F_0$ when $P = 0$ and for $\frac{F_{pm}}{F_0} \gg 1$; as $F_0 \to F_{pm}$ i.e. the maximum value of $x$, then the system output change $\Delta y_N$, for either a positive or negative $\Delta F$; decreases; consequently the minimum change occurs at the maximum value of $x$ i.e. at $F_{pm}$ when $P$ is a maximum.

Now the minimum value $\Delta y_N$, the change in system output caused by the minimum change in pressure to be resolved, must be greater than the system noise level ($S$)

i.e. $\Delta y_N \gg S$ .............................................. 152

In order to define the maximum permissible system noise level one must consider the noise generated within the system. Now if $y'$ is the dc level at which the pressure $\Delta P$ occurs, then the minimum acceptable signal to noise ratio in dB is as shown below using 152 and 132

$$ S/N \text{ ratio} = 20 \log \frac{\Delta y_N}{y} $$

$$ S/N \text{ ratio} = 20 \log \frac{y_N}{77 \log \frac{\phi}{y} + 300} .............................................. 153 $$

where as before $\phi = \frac{F t}{C_d C_F} e_i$; is flux intensity falling on detector.

Consequently as $\Delta y_N$ is independent of the magnitude of the flux as discussed earlier; the greater the value of the flux, 'y' so the value of 'y' increases, consequently increasing the signal to noise ratio. In
FIG. 87 LOGARITHMIC DETECTOR-OUTPUT CHANGE FOR AN INCREASE OF THE PRESSURE TO BE RESOLVED, OCCURRING AT ANY INITIAL VALUE

for N=300
other words the greater the flux, the greater will be the problem to achieve the specified signal to noise ratio, i.e. of the electronics

(12.2.0)

Finite values for S/N ratio necessitate finite definitions for $C_d C_f t_c^2 e_i$ and RF.
12.1.2 LINEAR MODE DETECTOR

If a photodetector of the junction type is used such that it is effectively short circuited, then the load line (2) becomes the detectors response characteristic FIG. 85. This is such that the signal current varies in a linear fashion with light flux falling on the cell.

Because of this linear relationship the term responsivity, can be represented for one cell by a constant $K$ amp/watt, the response of the cell to incident flux.

i.e. $I_s$ (signal current) = $K RF t^2 C_d C_e$  

from detector

where $RFt^2 C_d C_\text{e}$ is flux falling on detector

NOTE: the value of flux intensity is not necessary hence

no 'a' term;

thus $y$ (amps) = $K RF t^2 C_d C_e$  

(or $I_s$)  .................................................. 153

describing system output as a 'current', and again at this stage omitting amplifier gain $G$ (using $G = 1$). As for the logarithmic mode detector, the relationship between the resulting output change for the smallest input change will be derived as before

$\Delta F = \frac{1}{N} (\sqrt{(F_o - F_{pm})^2})$  .................................................. 146

and from 153 $\Delta y = Kt^2 C_d C_e \Delta F$  .................................................. 154

but using 153, with condition that

when $F = F_o$, $y = y_o$

$y_o = Kt^2 C_d C_e F_o$

thus $\frac{y_o}{F_o} = Kt^2 C_d C_e$  .................................................. 155

using 155 in 154

$\Delta y = y_o \left(\frac{\Delta F}{F_o}\right)$  .................................................. 156
Therefore substituting for $\Delta F$ from 146

$$y_N = y_o \left( 1 - \frac{1}{N} + \sqrt{\left( \frac{F_o - F_{pm}}{N} \right)^2} \right)$$

which specifically becomes if

$$\frac{F_{pm}}{F_o} \ll 1$$

$$\Delta y_N = \pm \frac{y_o}{F_o} \frac{1}{N} \left( F_o - F_{pm} \right)$$

$$\Delta y_N = \pm \frac{y_o}{N} \left( 1 - \frac{F_{pm}}{F_o} \right)$$

or if

$$\frac{F_{pm}}{F_o} \gg 1$$

$$\Delta y_N = \pm y_o \frac{1}{N} \left( \frac{F_{pm}}{F_o} - F_o \right)$$

$$\Delta y_N = \pm \frac{y_o}{N} \left( \frac{F_{pm}}{F_o} - 1 \right)$$

From the above it can be seen

1) the output resolution change $\Delta y_N$ is directly dependent on the initial 'set' output 'y_o'

2) the output resolution change is inversely dependent on 'N'

3) $\Delta y_N$ is constant through the range of $F_{pm}$ to $F_o$

4) and equation 158 and 159 are seen to be equivalent as

$$y_N = \pm \frac{y_o}{N} \frac{\text{total change in } F}{\text{original } F}$$

when as described earlier, Page 195

$$\frac{F_{pm}}{F_o} \text{ G.U.} = 2 - \frac{F_{pm}}{F_o} \text{ L.U.}$$

Now as will be described by connecting the detector to an operational amplifier, such that it functions as a current to voltage converter, (12.2.0 & FIG. 91), with a feedback resistor $R_f$, the output signal becomes

$$y(\text{volts}) = R_f K R_f \frac{2}{c} C d F \epsilon \frac{1}{3}$$
which enables the value of \( y_0 \) the initial set value to be determined, if \( F_0 \) is known. i.e. the initial value of \( RF \). At this time, although values for 'K' from manufacturer's literature can be extracted (.1 - .4 amp/watt) as \( C_d C_f \) have not yet been investigated and so the true value of \( y_0 \) cannot be ascertained. However, it can be shown that \( y_0 \) can vary from 0 - 15 volts although there will be limitations to having high set voltage levels.

The following FIG. 89 shows how the resolution change \( \Delta y_N \) (volts) varies with \( y_0 \) for different values of \( N \) & \( F \). (using equations 158 & 159)

This figure shows that output changes of the order of mV are obtained when \( N = 300 \) i.e. \( \frac{1}{300} \) of pressure range is detected. Also that as expected, the greater 'Vo' the greater the output change.

If the ratio of the dc signal level, at a specific reflection factor, to the change in output caused by a change of the pressure to be resolved is found, we have a 'signal to noise ratio' where the 'noise' can be considered to be the minimum change in signal to be resolved, and is a boundary condition. Let this ratio be termed the minimum acceptable signal to noise ratio of the detector system (MA S/N).

Therefore M.A. S/N =

\[
\frac{y}{\Delta y_N}
\]

where

\[
y = K RF t^2 C_d C_f e_i\]

from equation 153

and

\[
\Delta y_N = \frac{y_0}{N} \left(1 - \frac{F_{pm}}{F_0}\right) \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ll 1

or

\[
= \frac{y_0}{N} \left(\frac{F_{pm}}{F_0}\right) - 1
\]

\ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ll 1

The finite value of 'y' in equation 153 has a multitude of values; because 'RF' ranges between 'F_{pm}' and 'F_0' and consequently the maximum value of 'RF' must be used in 153 to ensure that the most stringent value for the M.A. S/N ratio is found i.e. using the largest value of 'y' signal d.c. voltage. Consequently two cases for M.A. S/N must be investigated.
Fig. 89: Linear Detector—Output change for an increase of the pressure to be resolved, against the initial set output level $V_0$. 

For $N=300$, the graph shows the output change (in volts) $\Delta V_N$ as a function of the initial set output voltage $V_0$ for different values of $\frac{F_{pm}}{F_0}$, which are $0.7$ or $1.3$ and $0.75$ or $1.25$. The graph scales from 0 to 0.10 volts on the vertical axis and from 2.0 to 14.0 on the horizontal axis.
When $\frac{F_{pm}}{F_o} < 1$, in this case the maximum value of the signal is found when 'RF' is 'F_o' as $F_o \gg F_{pm}$

then from 153

$$y_o = K F_o t^2 C_c C_e i$$

and using 158

Therefore $M.A. S/N = \frac{N y_o}{y_o (1 - \frac{F_{pm}}{F_o})}$

$$M.A. S/N = \frac{N}{1 - \frac{F_{pm}}{F_o}}$$

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When $\frac{F_{pm}}{F_o} \gg 1$

the maximum value of output signal occurs at

$F_{pm}$ as $F_{pm} \gg F_o$

thus from equation 153

$$y_{pm} = K F_{pm} t^2 C_c C_e i$$

but

$$y_o = K F_o t^2 C_c C_e i$$

Therefore $y_{pm} = y_o \frac{F_{pm}}{F_o}$

162

thus using equation 162 and 158

$$M.A. S/N = \frac{N y_o \frac{F_{pm}}{F_o}}{y_o (\frac{F_{pm}}{F_o} - 1)}$$

$$= \frac{N F_{pm}}{F_o} \frac{\left(\frac{F_{pm}}{F_o} - 1\right)}{\left(\frac{F_{pm}}{F_o} - 1\right)}$$

$$= \frac{N}{1 - \frac{F_o}{F_{pm}}}$$

163
Thus we have two values for the minimum acceptable detection signal to noise ratio; dependent upon whether $\frac{F_{pm}}{F_o} \gg 1$ noting that for equivalence of system M.A. S/N ratio we have $\frac{1}{\frac{F_{pm}}{F_o}} (L.U.) = \frac{F_{pm}}{F_o} (G.U.)$ 

These then are 

$$\text{M.A. S/N} \left( \frac{F_{pm}}{F_o} \leq 1 \right) = \frac{N}{1 - \frac{F_{pm}}{F_o}} \quad \cdots \quad 161$$

and 

$$\text{M.A. S/N} \left( \frac{F_{pm}}{F_o} \geq 1 \right) = \frac{N}{1 - \frac{F_o}{F_{pm}}} \quad \cdots \quad 163$$

FIG. 90 shows how the value of minimum acceptable detector signal to noise ratio (dB) varies with $\frac{F_{pm}}{F_o}$ ratio for different resolution capabilities, i.e. $N = 300; 150$ & $100$. Now it can be seen that to obtain the same minimum acceptable noise level of the actual system, the $\frac{F_{pm}}{F_o}$ value greater than unity is the inverse of the equivalent value less than unity. One example is shown as condition (28) for which the M.A. S/N is 63 dB, whereby when two values of $\frac{F_{pm}}{F_o}$ are boundary condition, i.e. .8 or 1.25 from graph (28A)
FIG. 90  LINEAR DETECTOR—"SIGNAL TO NOISE RATIO" AGAINST $F_{pm}/F_0$ FOR DIFFERENT "N" VALUES: MINIMUM ACCEPTABLE VALUE

RESOLUTION FACTOR "N"

"A"  $N=300$
"B"  $N=150$
"C"  $N=100$

MINIMUM VALUE ACCEPTABLE FOR SIGNAL TO NOISE RATIO (dB)

SYSTEM $F_{pm}/F_0$ VALUE
12.1.3 INTERNALLY GENERATED NOISE - LINEAR MODE DETECTOR

The causes of internally generated noise within the detecting system are:

1) Shot noise: resulting in a noise current

\[ i_{sn} = (2 \cdot e \cdot i_{dn} \cdot B)^{\frac{1}{2}} \]

2) Thermal Noise: resulting in a noise current (in detector and auxiliary resistor)

\[ i_{tn} = \left( \frac{4 \cdot K \cdot T \cdot B}{R_T} \right)^{\frac{1}{2}} \]

where

- \( e \) = electron charge
- \( i_{dn} \) = dark current
- \( K_B \) = Boltzmann's Constant
- \( B \) = Bandwidth Hz
- \( I_L \) = Photo-current
- \( T \) = absolute temperature
- \( K \) = Responsivity
- \( R_T \) = Effective load resistor across detection

i.e. total internal noise current is given by

\[ i_{in} = (2 \cdot e \cdot i_{dn} \cdot B)^{\frac{1}{2}} + \left( \frac{4 \cdot K \cdot T \cdot B}{R_T} \right)^{\frac{1}{2}} \]

Consider now a linear mode detector, i.e. with a zero bias voltage and effectively short circuited. Then as '\( i_{dn} \)' the dark current is zero, \( R_T \) becomes an unbiased equivalent impedance for the detector \( R_S \).

The total noise current \( i_{in} \) becomes from equation 164

\[ i_{in} = \left( \frac{4 \cdot K \cdot T \cdot B}{R_S} \right)^{\frac{1}{2}} \]

but the signal current is from equation 153

\[ 2 \cdot \text{amps} = K \cdot R_F \cdot \frac{T^2}{R_c} \cdot C_d \cdot C_f \cdot e_i \]

noting that as yet 'RF' has not been specified as either \( F_{pm} \) or \( F_0 \)

thus the true signal to noise ratio is
as before $RFt^2C_dC_fe^i$ is the flux falling on the detector and it can be
seen that improving the light flux falling on the detector improves the
signal to noise ratio.

Earlier, the minimum acceptable signal to noise ratio (M.A. S/N)
was defined in terms of $F_{pm}$, $F_o$ & $N$ by equations 161 and 163, for a linear
mode detector.

Thus as the actual signal to noise ratio is generally described
by equation 166; we have for the two cases

When $\frac{F_{pm}}{F_o} \ll 1$

$$\frac{K RF t^2C_dC_fe^i}{(4K_BT_B/R_s)^{\frac{1}{2}}} \gg \frac{N}{1 - \frac{F_{pm}}{F_o}}$$

Where the RF value must be chosen as the same value as for the condition
taken for the RHS i.e. $F_o$ as found from equations 162 and 161.

Therefore $K F_o t^2C_dC_fe^i$

$$\frac{(4K_BT_B/R_s)^{\frac{1}{2}}}{1 - \frac{F_{pm}}{F_o}}$$

Similarly when $\frac{F_{pm}}{F_o} \gg 1$

$$\frac{KF_{pm} t^2C_dC_fe^i}{(4K_BT_B/R_s)^{\frac{1}{2}}} \gg \frac{N}{1 - \frac{F_o}{F_{pm}}}$$

For these two inequality relationships the value of the RHS can
be found from FIG. 90 (in dB) and consequently the value of $F_o$ or $F_{pm}$
and $e_i t^2C_dC_f$ can be found to satisfy the actual detector noise level
requirement.

These original forms for the specification of actual detector
signal to noise ratio can be re-arranged to form a more immediately
apparent specification relationship.
On firstly re-arranging 167

$$\frac{F_{pm}}{F_o} \ll 1$$

$$\frac{1}{N} \left(1 - \frac{F_{pm}}{F_o}\right) F_o C_d C_f e_{i} \gg \frac{(4K_B T_B/R_S)^{\frac{1}{2}}}{K}$$

and for \( \frac{F_{pm}}{F_o} \gg 1 $$

$$\frac{1}{N} \left(F_o - F_{pm}\right) t_c^2 C_d C_f e_{i} \gg \frac{(4K_B T_B/R_S)^{\frac{1}{2}}}{K} \quad \ldots \quad 169$$

Consequently, we have two descriptions for the conditions for the minimum value of input light energy which is detectable by the photo-detector; because the RHS of these inequalities is what is termed the 'Noise Equivalent Power' of the detector (N.E.P.). With respect to the RHS then

$$\frac{(4K_B T_B/R_S)^{\frac{1}{2}}}{K} = N.E.P.$$  

This is obtained because as from equation 166 the actual detector signal to noise ratio

$$S/N = \frac{K R F t_c^2 C_d C_f e_{i}}{(4K_B T_B/R_S)^{\frac{1}{2}}}$$

then if $S/N = 1$ i.e. noise signal = input signal

$$\frac{(4K_B T_B/R_S)^{\frac{1}{2}}}{K}$$

is the actual value of the just detectable input light energy signal.

Consequently, the value of the LHS of 169 and 170 determines the minimum actual value of the light flux falling on detector and \( F_o, F_{pm} \)

\( N, t_c^2, C_d, C_f, e_{i} \) can be adjusted to satisfy the inequality. It should be noted that the L.H.S. is simply \( \frac{1}{N} \) of the change in light flux falling on the detector. Now because the choice of detectors is limited, values
for $R_s K$ can be given approximated values. However, thus far, no considerations for amplifier noise and the minimum value of signal which can be sensed by the amplifier has been made and this follows in the next section, but by choosing average values; these are

$$R_s \approx 5 \text{ M\text{\textOmega}}$$

$$K \approx 0.2 \text{ amp/watt}$$

and using a value for $B$ of $10^4$ Hz

the N.E.P. = \( \frac{1.28 \times 10^{-10}}{K} \left( \frac{B}{R_s} \right)^{1/2} \) watts

$$\approx 3 \times 10^{-11} \text{ watts}$$

thus for these particular detector equations 169 and 170 become

for

$$\frac{F_{pm}}{F_o} \ll 1$$

$$\frac{1}{N} \left( F_o - F_{pm} \right) t_c^2 C_d C_f e_i \gg 3 \times 10^{-11} \quad \text{................. 171}$$

and for

$$\frac{F_{pm}}{F_o} \gg 1$$

$$\frac{1}{N} \left( F_{pm} - F_o \right) t_c^2 C_d C_f e_i \gg 3 \times 10^{-11} \quad \text{................. 172}$$

in a similar manner the original signal to noise ratio relationship can be re-written then equation 167 and 168 become

for

$$\frac{F_{pm}}{F_o} \ll 1$$

$$\frac{10^{11}}{3} F_o t_c^2 C_d C_f e_i \gg \frac{N}{1 - \frac{F_{pm}}{F_o}} \quad \text{................. 173}$$

and for

$$\frac{F_{pm}}{F_o} \gg 1$$

$$\frac{10^{11}}{3} F_{pm} t_c^2 C_d C_f e_i \gg \frac{N}{1 - \frac{F_o}{F_{pm}}} \quad \text{................. 174}$$
For a logarithmic mode detector the internal noise comprises as before, of shot noise arising within the semi-conductor and thermal noise both due to detector and its load resistor when present. Writing the total noise in voltage form

\[ V_{\text{in}} = (2e_i dn B)^\frac{1}{2} R + (4K_B TBR)^\frac{1}{2} \]

where \( R \) is the effective resistance present.

Now to determine the value of \( i_{\text{dn}} \) relevant to the further development of an expression for actual signal to noise ratio, then the value of effective forward bias produced by a particular level of illumination must be known. This value also depends on the actual load resistance values. Previously it has been assumed that the device is to operate in open circuit conditions. If this is the case the problem then becomes defining the effective detector impedance, as this varies with level of illumination. Consequently, it is prudent to omit a theoretical analysis here of signal to noise ratio, and conclude that practical determination would be advised. Now as reasons mentioned earlier this form of detector mode is not favoured, and subject to the light source specification and analysis chapter's conclusion, this mode detector will not be pursued.
12.2.0 AMPLIFIER/DETECTOR COMPOSITE

The theory and characteristics of different amplifier types are beyond the scope of this thesis; however, it is necessary to be aware of the manner in which amplifier noise and drift affects the previous definition of detector/fibre optic parameters (equations 169-170 in general form).

Let us first consider the configuration of detector and amplifier to be utilised, FIG. 91. This represents a photo-detector operating in its linear mode coupled to a current to voltage converter (current amplifier). The detector effectively operating as a constant current source, thus having a very large equivalent impedance. Now the current amplifier presents almost zero load impedance to ground because the inverting input appears as a virtual ground itself. The input current to the amplifier from the detector is, $I_s$. However, it flows through the feedback resistor $R_f$ generating an output voltage

$$v_{volts} = I_s R_f$$

The actual input impedance of the current amplifier $Z_n$ is with the finite gain as 'A' and $Z_{id}$ the open loop differential input impedance.

$$Z_{in} = \frac{Z_{id}}{1 + (Z_{id})(1 + A)}$$

$$\approx \frac{R_f}{1 + A} \text{ as } Z_{id} \gg 1$$

thus normally the impedance presented to the detector is very small and consequently load line (2) on FIG. 85 (forward and reverse characteristics of a diode) will be the response form of the diode.

Also the voltage gain is

$$\frac{R_f + R_s}{R_s} = G_v$$

now as $R_s$ is of the same order as $R_f$, the gain will be in units;
12.2.1 NOISE

The noise of the amplifier will be of both current and voltage form; the manufacturers specify equivalent input noise figures for specific band widths. Voltage noise can be referred if required, almost directly to the output. For reasons just described, current noise must be considered to pass through 'R_f', the feedback resistor, to determine output voltage referred noise.

The objective is now to refer the values of 'I_{an}', input amplifier noise current, and 'V_{an}', input amplifier voltage noise, to a noise equivalent power which would fall on the detector.

Thus

\[
\text{N.E.P. (I_{an})} = \frac{I_{b}}{K} \quad \text{band width as stated}
\]

\[
\text{N.E.P. (V_{an})} = \frac{G V_{an}}{KR_f} \quad \text{by manufacturer}
\]

Thus the combined effects of noise for detector and amplifier result in a modification of the generalised form for minimum detectable change in flux sensed by detector

i.e. \[\frac{F_{pm}}{F_o} \ll 1\]

\[\frac{1}{N} \left( F_o - F_{pm} \right) t^{2C_f C_d e_i} \supseteq \text{N.E.P. (detector)} \quad \ldots \ldots \ldots \ldots 169\]

\[\frac{F_{pm}}{F_o} \ll 1\]

\[\frac{1}{N} \left( F_{pm} - F_o \right) t^{2C_f C_d e_i} \supseteq \text{N.E.P. (detector)} \quad \ldots \ldots \ldots \ldots 170\]

which now becomes generally

\[\frac{1}{N} \left( F_o - F_{pm} \right) t^{2C_f C_d e_i} \supseteq \text{N.E.P. total system} \quad \ldots \ldots \ldots \ldots 175\]

when N.E.P. (total system) = N.E.P. (detector) + N.E.P. (I_{an}) + N.E.P. (V_{an})

\[\ldots \ldots \ldots \ldots 175a\]
noting that $I_{an}$ and $V_{an}$ will be specified in RMS values.

Thus inequality (175) ensures that throughout the range of
radiant flux falling on the detector, that the change in detector
output, resulting in a change in flux caused by the maximum pressure
change to be resolved, is always greater than the system noise.

Thus far, no lower limit for the finite level of flux
falling on the detector has been specified for dc conditions of the
output. The following section discusses the prime criteria which
will enable the lowest value for $e_1$ to be specified.
12.2.2. MINIMUM SET OUTPUT CONDITIONS

This is practically governed by the value of the input bias current of the amplifier 'I_B', and is the input current needed to drive the amplifier. In other words, the signal current to the amplifier must be greater than the input bias current for the amplifier i.e. I_s > I_B.

Now the lowest set output occurs, depending whether F_p/F_o L.U. or G.U. at F_p or F_o respectively. Using equation 153 then

\[ I_s = y_m \text{ (lowest value of standing output)} = RF K t^{2c_f d_i} \]

\[ y_m \gg I_B \]

i.e. \( RF K t^{2c_f d_i} \gg I_B \) ............................ 176

thus knowing approximate values for RF, K, \( t^{2c_f d_i} \) and with a value of I_B from manufacturers specification, a value of minimum value of 'e_i' can be found.

Then this value of 'e_i' can be used

1) for selection of minimum power of light source -

(Chapter 13)

2) for selection of most stringent \( \frac{F_p}{F_o} \) value using condition(175)

Also it should be noted that the output voltage is not simply

\[ y_{volts} = I_s R_f \]

but

\[ y_{volts} = (I_s - I'_B) R_f \] ............................... 176a

where \( I_s = RF K t^{2c_f d_i} \)
12.2.3 DRIFT

Drift associated with the amplifier is due to changes in the values of input bias current with time and temperature, it is also due to changes in the value of input offset voltage and $R_f$. These are best assessed later.

Chapter 13 outlines drift compensation systems, whilst Chapter 17.1.0 investigates practically a system's drift characteristics.
12.3.0 DISCUSSION

The photo-detector to be used in the pressure transducer must be small and robust, consequently a semi-conductor type detector must be used. Because junction type semi-conductor photo-detectors have a zero value of 'dark' current when short circuited and with zero bias, they become a first choice for photo-detector system. Their response to light is linear and consequently knowing only the $\frac{F_{pm}}{F_0}$ value, a figure for the minimum permissible signal to noise ratio for the system can be found, FIG. 91 (or vice versa if the system signal to noise ratio is known).

A logarithmic mode detector utilised a load connected across the detector, a voltage created by the photo-current is produced across this load and varies in a logarithmic fashion with light intensity falling on the cell. Also, the dark current is not zero and consequently the device will be temperature dependent.

The exact nature of the external circuit connections (hookup) for detector types used follows later. Ch. 16.2.0.

Consideration is given prior to this, to light source performance and characteristics which enable finite values for $e_i$ to be defined.

For a logarithmic mode detector the minimum signal to noise ratio (boundary value) is a function of the flux falling on to the detector and then must be compared to the actual system noise.

A linear system, however, has a minimum signal to noise ratio (boundary condition) independent of the magnitude of the flux falling on to the cell, but also then must be compared to be signal to noise ratio of the functioning total system.

An interesting conclusion can be now made for the significance of the equivalence conditions introduced earlier for $\frac{F_{pm}}{F_0} \ L.U. \ & \ G.U.$

For a linear mode detector, equivalence defined by geometric considerations i.e. $\frac{F_{pm}}{F_0} \ L.U. = \left(\frac{F_{pm}}{F_0} \ G.U.\right)^{-1}$, signifies identical M.A. S/N ratios, whilst
FIG. 91 LINEAR MODE DETECTOR SYSTEM—SCHEMATIC
the equivalence of outputs occurs when \( \frac{F_{pm}}{F_o} \cdot \text{G.U.} = 2 - \frac{F_{pm}}{F_o} \cdot \text{L.U.} \). For a logarithmic mode detector it is only the latter equivalence condition which has relevance and applies both to noise and output conditions.

The noise and drift associated with an amplifier coupled to a photo-detector have been discussed, and selection of an amplifier is further discussed in the next chapter. The two relationships by which \( F_{pm}, F_o \) and \( e_i \) can be specified are

for minimum set output conditions to be met

\[
RF \left[ F_o \text{ or } F_{pm} \right] K \cdot t_c^2 \cdot C_f \cdot C_d \cdot e_i \geq I_B \quad \text{.................. 176}
\]

and for noise conditions

\[
\frac{1}{N} (F_o - F_{pm}) t_c^2 \cdot C_f \cdot C_d \cdot e_i \geq \text{N.E.P. (total system)} \quad \text{.................. 175}
\]

Use of equation 176; can, by approximately fixing a value for \( F_{pm} \) or \( F_o \) (whichever is to be lower), \( t_c^2 \), \( C_f \cdot C_d \) and using a realistic value for \( I_B \), give a minimum value for \( e_i \) which thus allows for the driving of the amplifier throughout the range of flux falling on to the detector (Chapter 13). This value of \( e_i \) then can be used in equation 175 to ascertain what the ratio of \( \frac{F_{pm}}{F_o} \) for this minimum source condition would be.
CHAPTER THIRTEEN

LIGHT SOURCE
The function of the 'light source' is to provide electromagnetic radiation which is then modulated by the applied pressure on the distal tip diaphragm; which after transmission by the afferent fibres is sensed by a photo-detector.

The term light source is misleading since it infers the necessity for a 'visual' light producing source; this is not the case, as any radiator which allows efficient fibre transmission, diaphragm reflection and photo-electric conversion will suffice.

The visual spectrum ranges from .38 to .76 μm wavelength from violet through to red; it should be borne in mind at this point that photo-detectors usually have a peak response at about .9 μm which is in the infra-red region of the spectrum and does not evoke a visual response.

Consequently, assuming efficient transmission at this wavelength by the optical fibres, and efficient reflection at the diaphragm, then ideally a light source emitting radiation at around .9 μm would be most favourable.

Two forms of light source will be considered; an incandescent filament type; and a light emitting diode. Both types require a source of electrical power input, which itself can be the cause of both noise and drift of the radiant flux from the source, and consequently methods for compensation of light source drift will be discussed.
13.0.1 UNITS AND DEFINITIONS

When discussing 'light flux' a distinction must be made as to whether the units should relate to one's subjective response to the 'light' or its actual electrical power or energy content. Because the human eyes' response varies at different wavelengths, various units have been evolved which account for the eyes response to different wavelengths; such units being lumens, foot candles, etc. However, we have no direct need to relate to these 'Photometric Luminosity' units because, as has been mentioned, only the actual energy or power of 'light' dissipated by a source is required. The need only arises if photo-detector manufacturers specify their devices response in visual response units.

The term 'light' will now be used to describe any source which emits electromagnetic radiation, visible or not.

Certain terms of importance are discussed:

**Radiant Energy** Q: this is the radiant energy of the electromagnetic radiation from a source and is found for all the radiation wavelengths present. It will have units of energy, i.e. ergs.

**Radiant Flux** \( \Phi \): this is the rate of flow of radiant energy, i.e. \( \frac{dQ}{dt} \). It will have units of watts.

Luminous energy and Luminous Flux are concerned, as those terms mentioned above with the energy and power of the radiation from a source. However, they are related to the visual response evoked by the radiation and consequently are relative to the human eyes' response to electromagnetic energy.
13.1.0 BLACK BODY RADIATION AND THE INCANDESCENT SOURCE

A 'Black body' is defined as a material which will absorb all radiation wavelengths; if such a body is held at a temperature $T^\circ K$ it will radiate energy according to the law first described by Planck in 1901 (equation 175). In other words, by heating a material to a high enough temperature it will emit electromagnetic radiation of wavelengths which are useful, i.e. detectable by photo-detectors. However, the materials used as filaments in these 'incandescent sources' are not true 'black bodies' and do not radiate fully as a black body does at all wavelengths. Tungsten, the most suitable filament material $MP 3680^\circ K$ (ALLEN, R.D.) is only about 35% as efficient as a black body and also this figure (emissivity $\varepsilon$) varies with wavelength.

Thus by supplying a filament with electrical energy its temperature can be raised (joule heating). The main loss in power efficiency is due to convection and conduction from the filament (approx. 10 - 20%). Consequently, about 80% of the electrical energy is radiated as radiant energy in the spectral distribution found from equation 175 and will be discussed.

One important factor to consider is the resistance of tungsten wire; this is 20 times greater at a running temperature of say 3200$^\circ K$ then at ambient. Consequently, on switching such a source on, a high current surge results initially. In order to ascertain approximate filament dimensions let us consider an incandescent source to operate at 3200$^\circ K$ and be nominally termed a 100W source (i.e., power supplied) with say a 12V supply.

Thus two conditions must be met

a) 100W raise temperature of filament to 3200$^\circ$

b) 100W supplied at 12V dissipates through resistance of filament.
Considering firstly (b) for filament, let its resistance at 3200°K be

\[ R_f \sim \]

Therefore

\[
\frac{100}{12} = I
\]

and

\[
100 = I^2 R_f
\]

Therefore

\[
R_f = \frac{12}{100}
\]

now the resistivity of tungsten at 3200°K is \(99.5 \times 10^{-6}\) \(\text{cm} \cdot \text{s}\). Thus

if \(A_f\) and \(L_f\) are filament cross sectional area and length respectively

then

\[
R_f = \frac{L_f}{A_f} \times 99.5 \times 10^{-6}
\]

therefore

\[
\frac{12}{100} = \frac{L_f}{A_f} \times 99.5 \times 10^{-6}
\]

\[
\frac{L_f}{A_f} \sim 12 \times 10^2
\]

with \(d_f\) as filament diameter

\[
\frac{L_f}{A_f} \sim \frac{\pi}{4} d_f^2 \cdot 12 \times 10^2 \text{ cm}^2
\]

Now as will be shown later, the total radiated flux \(\phi_{\text{Tot}}\) can be found for a filament at a specific temperature, this value being per unit surface area of the filament.

Thus remembering that there is about a 20% loss of power due to convection, etc.

\[
\phi_{\text{Tot}} \times L_f \pi d_f = .8E_t
\]

substituting for \(L_f\) from 176a

\[
\phi_{\text{Tot}} \pi d_f \frac{\pi}{4} d_f^2 \cdot 12 \times 10^2 = .8E_t
\]

\[
d_f^3 = \frac{0.8E_t \cdot 4}{\pi \cdot \phi_{\text{Tot}} \cdot 12 \times 10^2}
\]

Now for a 'T' of 3200 from Stefan's Law (Page 342) the value of \(\phi_{\text{tot}}\) is 594 Watts/cm².
Therefore \( \frac{d^3}{d_f} = \frac{0.8 \times 100 \times 4}{\pi^2 \times 594 \times 12 \times 10^2} \)

\( d_f = 0.0356 \text{ cms} \)

\( = 0.35 \text{ mm} \)

now \( \frac{L_f}{A_f} = 12 \times 10^2 \)

Therefore \( L_f = 12 \times 10^2 \times \frac{\pi}{4} \frac{d^2}{d_f} \)

\( = 115 \text{ mm} \)

thus summarizing:

For a 100W 12V lamp the filament will have dimensions of .35mm and 11.5 cms length. This in practice is effected by spiral winding. If a coil pitch of twice the diameter of the filament is assumed and a coil diameter of 4mm is chosen, then the length of coil is:

\( \frac{115}{4\pi} \times \frac{2d_f}{2} \)

\( = 6.5 \text{ mm} \)

Thus effectively the source would appear as a rectangular source .4 cms x .6 cms which as is discussed later, makes for poor coupling with the efferent fibres of the pressure transducer. If the coil diameter was to be 3mm the resulting overall length then becomes approximately 1.1 cm.

The fulfilment of condition a); namely that the rate of electrical energy supplied must enable the filament to reach and maintain the operating temperature desired, can usually be met. The main objective for performing such a calculation is to obtain the time for the filament to reach a steady state.

Now for a 'Black Body', Planck's equation gives the energy emitted by a source

\[ Q = \frac{8 \pi \chi}{\lambda^5} (e^{\frac{hc}{\lambda K_B T}} - 1) \]

where \( c \) is velocity of light \( 3 \times 10^{10} \text{ cm/s} \)

\( h \) is Planck's const \( 6.624 \times 10^{-27} \text{ erg sec} \)
$K_B$ is Boltzmann's const $1.38 \times 10^{-16}$ ergs/deg K

$\lambda$ is wavelength cms

$T$ is absolute temp °K

now $Q_\lambda$ from equation 177 has units of ergs/cm$^4$ and defines the energy density emitted/unit wavelength interval from a source.

The useful energy emitted by the source over a bandwidth $d\lambda$ becomes

$$Q_\lambda \cdot d\lambda = \left( \frac{8\pi c h}{4\lambda^5} \frac{hc}{e^{\frac{hc}{K_B AT}} - 1} \right) d\lambda$$

178

which has units of ergs/cm$^3$, the energy density emitted by the source over a bandwidth $d\lambda$, the power or radiant flux $\phi$ is obtained from equation 178 by multiplying by $c$

$$\phi = \frac{8\pi h}{4\lambda^5} \frac{10^{-7}}{\left( e^{\frac{hc}{K_B AT}} - 1 \right)} d\lambda$$

179

and has units of ergs/s/cm$^2$

Therefore $\phi = \frac{8\pi h}{4\lambda^5} \frac{10^{-7}}{\left( e^{\frac{hc}{K_B AT}} - 1 \right)} d\lambda$ watts/cm$^2$

$$\phi = \frac{c_1}{\lambda^5} \frac{c_2}{2} \frac{d\lambda}{(e^{\frac{hc}{K_B AT}} - 1)}$$

watts/cm$^2$

where in c.g.s. units

$c_1 = 3.7407 \times 10^{-12}$ watts cm$^2$

$c_2 = 1.438$ cm$^0$K with $\lambda$ & $d\lambda$ in cms

Thus using 180 the radiant flux emitted by a source at a wavelength $\lambda$ and temp $T^\circ K$ and a bandwidth $d\lambda$ can be found in watts/cm$^2$.

The radiant flux between $\lambda_1$ & $\lambda_2$ can best be found by numerical integration of equation 180 using a small bandwidth $d\lambda$. However, to obtain the total radiant flux one can use Stefan's Law.

$$M \text{ (watts/cm}^2\text{)} = \sigma (T^4 - T_o^4)$$

$$\sigma = 5.6712 \times 10^{-12}.$$
Where $T$ is temperature of black body, $T_o$ is ambient temperature.

The following FIG. 92 shows the radiant flux in mW/cm$^2$ over a 10$^8$ bandwidth for different wavelengths of a black body at several temperatures; where it can be seen that increasing the body's temperature shifts the peak power wavelength to a lower wavelength value, and also increases the value of this power.


\[ A = 3200 \text{ K} \]
\[ B = 3000 \text{ K} \]
\[ C = 2854 \text{ K} \]
\[ D = 2500 \text{ K} \]

**Fig. 92** Black Body Radiator—Radiant Flux Density of Source per 10A Waveband

Radiant Flux Density (watts/cm² per 10A Waveband) vs. Wavelength (\( \lambda \text{ um} \))
13.2.0 SEMI-CONDUCTOR SOURCE - LIGHT EMITTING DIODE

Luminescence is the term used to describe the emission of light by a material when some of the energy absorbed by the body is converted into photons.

Now certain materials, when subjected to an electric field, excite electrons within the body to higher energy levels. On decaying to ground level, 'quanta' in the form of photons may be emitted, their wavelength being dependent on the critical energy gap for the material.

Thus, the fact mentioned earlier that (p-n junction) materials can absorb light and produce an electrical current (photo-current) is in principle reversible, and the emitted light produced by electrical excitation is due to 'radiative recombination' and may be termed 'electroluminescence'.

If a semi-conductor of a suitable composition is forward biased, an electroluminescent device can result where the luminescence occurs at the junction in all directions, and the absorption and quantum efficiency of the device depend on its composition and physical dimensions (ARCHER, R.J. and KERPS, D. 1966).

The exact nature of the Quantum physics of semi-conductor junctions applied to describing electro-luminescence is very complex (ROOSBROECK, W. VAN & SHOCKLEY, W. 1954; DUKE, C.B. & HOZONYAK, N. 1973). The overall external efficiency of the diode is determined by its internal quantum efficiency and the absorption the photons emitted at the junction, suffer on their optical path to the surface of the diode. The internal quantum efficiency (electrons in to photons out) is generally high (50%) and is affected by material structure and purity, whilst the transmission of the photons to the external environment is dependent on path length, surface finish, etc. (also see sec. 3.0.1.) and is typically only of a few percent (NUÈSE, C.J.; KRESSEL, H. and LADANY, I. 1972).
Now for 'direct' radiative recombination', where in one step an electron of higher energy falls to its ground state, from Planck's law if \( E \) is the energy band gap for the junction then

\[
E_g = \frac{h c}{\lambda}
\]

where \( \lambda \) is the resulting wavelength of the photon released and \( h, c \) are Planck's constant and velocity of light respectively which becomes

\[
E_g (eV) = \frac{12400}{\lambda(\text{Å})}
\]

Thus by knowing the value of the forbidden band gap energy \( E_g \) an approximate value for the wavelength of the source can be found at one temperature. Increasing temperature causes increasing shift in peak output radiation in magnitude and wavelength, i.e., a detector peaking at peak wavelength initially will sense less O/P as temperature increases. The approximate value for quantum efficiency has been mentioned and is low (typically .5%). Unfortunately, another disadvantageous feature is exhibited by these devices, which is the degradation of output with time. Explanations for degradation are themselves complex and has been reported by BIARD, J.R.; PITTMAN, G.E. & LEEZR, J.F. (1966).

It is thus prudent to consider a commercially obtainable material used as a light emitting diode, namely Gallium Arsenide. This material has a \( E_g \) value of 1.4 eV at 300°k consequently the predominant emitted wavelength is from equation 180, 8857 Å (8857 μm). In practice there will be a finite bandwidth because of the inclusion of impurities within the material. Consider now a commercially available electroluminescent source Plessey Ltd. market their GaL2, a light emitting diode fabricated from Gallium Arsenide. This device is typical of the present state of the art of components commercially available. The following TABLE 8 shows the device's salient features.
<table>
<thead>
<tr>
<th><strong>Parameter</strong></th>
<th><strong>Value</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>Forward Current (amps)</td>
<td>1</td>
</tr>
<tr>
<td>Radiant Flux (mW)</td>
<td>10</td>
</tr>
<tr>
<td>Quantum Efficiency (%)</td>
<td>73</td>
</tr>
<tr>
<td>Power Efficiency (%)</td>
<td>67</td>
</tr>
<tr>
<td>Bandwidth (µm)</td>
<td>0.028</td>
</tr>
<tr>
<td>Peak Output Wavelength (µm)</td>
<td>0.925</td>
</tr>
<tr>
<td>Effective N.A.</td>
<td>0.866</td>
</tr>
<tr>
<td>Apparent Source Area (cm²)</td>
<td>0.01</td>
</tr>
</tbody>
</table>

**Table 8**
13.3.0. COMPARISON BETWEEN INCANDESCENT & SEMI-CONDUCTOR 'LIGHT' SOURCES

The prime requisite for a light source is that it emits adequate radiant flux which, on emerging from the afferent fibres within the optical probe system, is photo-detectable. To enable a definition of acceptable levels of emergent flux, reference must be made to detector response characteristics; fibre transmission characteristics and the optical fibre/diaphragm reflection factors achievable. Furthermore it is the change of flux resulting between two limiting pressure values which must be detectable (resolution). Prior to discussing finite minimum values for the flux entering a fibre, the following general comparisons are to be discussed.
13.3.1 POWER EFFICIENCY

Power efficiency is defined as the output radiant power from the source relative to the electrical power supplied to it. For an incandescent source, losses are due to conduction and convection from the filament and result in power efficiency of the order of 80%, whilst for semi-conductor sources the power efficiencies obtainable are very low, being of the order of 1% (NUESE et al). A specific value can be seen in TABLE 8 where for the commercially available device described, it is only .67%.

The relevance of power efficiency as a method for comparison however, is meaningless, if as is the case, there are no incandescent sources & L.EDS of the similar power consumption. If there were, an incandescent source would, on the basis of power efficiency, be far more satisfactory.
13.3.2 COUPLING FACTOR & DURABILITY

The fraction of radiant flux collected by the efferent fibres, to that emitted by a source, has been termed the 'coupling factor' \( C_f \). Now as the effective diameter of the efferent fibre group is of the order of a \( '\text{mm}' \), and the radiant source of an incandescent filament, (although depending on its power consumption), see Chapter 13.1.0., is many times greater than this, and in the form of a coiled helix is described earlier. Coupling factors of less than .1 arise. Problems also will be met with ensuring that the effective 'f' no. of the collection system is no larger then the N.A. of the fibres, as an over-estimate of the fibre collection results.

Semi-conductor sources, however, because of their small apparent active area (\( .01 \text{ cm}^2 \) TABLE 8) enable, a more efficient coupling to occur. In our situation of the order of \( C_f = .5 \) (private communication PLESSEY LTD.)

Briefly then, the coupling factor \( C_f \) is

- for an incandescent source \( .1 \) efferent fibre effective diameter of \( 1 \text{ mm} \),
- for a semi-conductor source \( .4 \) diameter of \( 1 \text{ mm} \).

Durability is included here because inherent in the principle of operation of these devices and the coupling factor achieved, is the fact that semi-conductor sources are rugged and safe (cannot implode or shatter as an incandescent source can) and are very small.

No mention has been made with respect to the coupling factor \( C_d \), the fraction of flux that the photo-detector receives from the afferent proximal fibres. This too is dependent on the geometrical aspects of these components and is now discussed. As mentioned earlier the coupling factors \( C_d \) & \( C_f \) are dependent on the geometry of the components concerned; the light source, the efferent and afferent fibres and the photo-detector. \( C_d \) is defined as the ratio of flux falling on to the detector to that emitted from the afferent proximal fibre group. If it is assumed that both the effective cross-sections of the afferent proximal group and the photo-detector are circular with \( 'D_d' \) as the detector diameter, and \( 'a' \)
as the separation between them, \( d_{\text{aff}} \) being the diameter of the afferent group; then ignoring the cosine law effect one can arrive at a value for the coupling factor of:

\[
C_d = \left( \frac{D_d}{d_{\text{aff}} + a \tan \theta} \right)^2, \quad \text{where} \quad \frac{\sin^{-1} \theta}{2} = \text{N.A.}
\]

This is in effect the ratio of the detector area to that of the base of the truncated cone of flux projected from the afferent fibre group on to the plane of the detector. \( C_d \) has an optimum value of unity, for if the denominator of the above expression is less than the numerator, then as the photo-detector responds to flux density, then the critique of all the flux from the afferent fibres reaching the detector is satisfied.

From the above expression one can say that, if there is no restriction for the value of the separation 'a' and if the condition \( d_{\text{aff}} \leq D_d \) prevails then a coupling factor of near unity is accomplished. However, let us at this time assume a more pessimistic state of affairs and consider that a value of 0.2 is realistic. The active area of an SCI is rectangular and consequently, modifications to the above description for \( C_d \) are necessary, the value of the sides of the rectangle being approximately 4 x 2 mm. Thus it is very likely that the above 'condition' will apply. The above description \( C_d \) also applies to the \( C_f \) value, but a correction is required to account for the source N.A. As mentioned Plessey Ltd. specified an approximate value for this coupling factor when using their Gal 2 source. Both the values for \( C_d \) & \( C_f \) are to be discussed later in this thesis.
13.3.3 SPECTRAL OUTPUT & POWER OUTPUT

An incandescent source output contains a spectrum of wavelengths, whilst a semi-conductor device emits within a narrow bandwidth (typically 0.028 µm). The output powers for incandescent sources are much greater than for semi-conductor sources, due partly to the non-availability of high power output L.E.D.'s and also, of course, the incandescent source is far more efficient in any case. However, at the present time, maximum value of output for an L.E.D. is of the order of 10 - 50 mW.

However, although an incandescent source may have a very large bandwidth, account must be made of the spectral absorption of the fibres, spectral reflectivity of the diaphragm and spectral response of the detector, to compare fairly its performance with a mono-chromatic source.

This could be achieved by an incremental numerical integration technique incorporating the various coefficients relevant for various wavelengths.

\[ \text{i.e. } P_d \quad \text{(detector response)} \]
\[ T_c \quad \text{(fibre transmission)} \]
\[ \sigma_E \quad \text{(emissivity)} \]

when the detector response could be found. A semi-conductor source, because it has a narrow bandwidth, significantly simplifies calculations of detector response.

In other words, the incandescent source will emit much of its radiation, in effect, inefficiently due to low detection sensitivity and transmission at large wavelengths.
13.3.4 STABILITY (DRIFT)

The stability of a light source is of prime importance if 'drift' is not to be compensated for.

An incandescent source, by virtue of its principle of radiation, i.e. thermal radiation from a hot body, is subject first to the effects of 'strong' cooling draughts. Also, even with quartz halogen lamps where, when the tungsten evaporates from the filament and forms a vaporised halogide which on cooling redeposits on to the filament, the output of the lamp will vary. Warm up is another factor to account for. This is the time taken for the lamp to reach an equilibrium state, and in order to achieve warm up, times of 15 mins. for a 12V 50W QI lamp need to elapse. This necessitates quite large size currents initially which load the filament severely.

Semi-conductor sources are also subject to drift (BIARD J.R. ET AL) but is long term drift compared to incandescent source drift.(Ch.18).
A first criteria for the selection of a light source is the minimum acceptable level of the flux from the source which will satisfy the requirements of the detector and amplifier as discussed in 12.2.2. For a linear mode detector coupled to a current amplifier we have, from equation 176, that the lowest standing input current to the amplifier can be determined.

\[
RF K t_c^2 C_f C_d e_i \gg I_B \quad \text{equation 176}
\]

noting that RF (the reflector factor) should be the lowest value of reflection factor for the system and is either \( F_{pm} \) or \( F_o \), dependent on whether \( F \) is L.U. or G.U. respectively.

Now taking values for \( K, C_d \) and \( C_f \), as previously thought to be practically achievable, i.e. 0.2 amp/watt, 0.2 and 0.4 respectively (i.e. \( C_f \) for on L.E.D.) with \( t_c^2 \) the transmission coefficient for optical fibres of 1 metre in length and 40 \( \mu \)m diameter at \( \lambda = 0.8 \mu \text{m} \) as being 0.25 and using a figure of 0.35 as the value for the minimum reflector factor, from equation 176.

\[
RF K t_c^2 C_f C_d e_i \gg I_B \\
0.35 \quad 0.2 \quad 0.25 \quad 0.4 \quad 0.2 \quad e_i \gg I_B \\
e_i \gg I_B/1.4 \times 10^{-3} \text{ watts} \quad \text{equation 183}
\]

Now for a good general purpose low drift operation amplifier \( I_B = 25 \text{nA} \)

(Analogue DEV 184L)

Therefore from equation 183

\[
e_i \gg \frac{25 \times 10^{-9}}{1.4 \times 10^{-3}} \\
e_i \gg 1.78 \times 10^{-5} \text{ W} \\
e_i \gg 1.7 \times 10^{-2}\text{mW}
\]
noting that the reflection factor considered must be the lowest value. Thus from only considerations of the current to drive the amplifier, it can be seen that a very low level of light source output radiant flux can be tolerated. In fact the output from a G.A.L.2 is virtually 6000% higher than the minimum required. Consequently, a solid state source is theoretically acceptable from the 'set output condition'.

However, as will be discussed later, 14.1.0. the specification of \( \frac{F_{pm}}{F_o} \) value which provides an adequate signal change for a change of pressure to be resolved, must also be performed. The manner in which this is accomplished is discussed here. The expression which conditionally specifies that the input change to the detector is greater than the N.E.P. of the system has been given by equation 175. For \( \frac{F_{pm}}{F_o} \leq 1 \)

\[
\text{i.e. } \frac{1}{N} (F_o - F_{pm}) t_c^2 C_f C_d e_i \geq \text{N.E.P. (system)} \]

then using values for parameters as before

\[
\text{i.e. } F_{pm} = 0.35 \quad \text{(as } \frac{F_{pm}}{F_o} \text{ is shown as L.U.)} \quad C_f = 0.4
\]

\[
C_d = 0.2, \quad t_c^2 = 0.25
\]

and with \( N = 300 \) and 'e_i' as calculated \( 1.7 \times 10^{-5} \). Equation 175 becomes

\[
\frac{1}{300} (F_o - 0.35) 0.25, 0.4, 0.2, 1.7 \times 10^{-5} \text{ N.E.P. total system}
\]

\[
1.33 \times 10^{-9} (F_o - 0.35) \text{ N.E.P. total system} \quad \cdots \cdots \cdots \cdots \cdots 184
\]

Considering an 'analogue device' operational amplifier 184L which has approximate values for noise voltage and current of \( 5 \mu \text{V (V}_{an} \) \), and \( 20 \mu \text{A (I}_{an} \) rms respectively over a bandwidth up to \( 10^4 \), then with the detector as assumed in 12.1.3 and using the relationship for N.E.P. of input current and voltage noise for amplifier 12.2.1

\[
\text{N.E.P. total system} = \text{NEP (det) + NEP (I}_{an} \) + NEP (V}_{an} \)
\]

\[
= (3 \times 10^{-11} + \frac{2 \times 10^{-11}}{2 (K)} + \frac{5 \times 10^{-6}}{2 \times 5 \times 10^5}) \text{ W}
\]

\[
= (3 + 10 + 5) 10^{-11} \text{ W}
\]

\[
18 \times 10^{-11} \text{ W} \quad \cdots \cdots \cdots \cdots \cdots 185
\]
it is worth noting that the amplifier current noise is of the same
significance level as that for the detector. Thus equation 184 becomes

\[ 1.33 \times 10^{-9} (F_o - 0.35) \gg 1.8 \times 10^{-11} \]

\[ F_o \gg 0.48 \]

Whereby for the condition of minimum radiated power from light source
the required ratio \( \frac{F_{pm}}{F_o} \) becomes

\[ \frac{F_{pm}}{F_o} = \frac{0.35}{0.48} = 0.729 \]

Summarising: the minimum level of dc flux falling on the detector is
set by the input current bias of amplifier and thus, the minimum value
for the radiant flux from the light source can be found from equation 176.
For an analogue device op/amp 184L this would be \( 1.7 \times 10^{-2} \text{mW} \) and an
L.E.D would suffice. To determine whether at this low level of flux
the required conditions can be met, use is made of equation 175 whereby
it is shown that a reflection factor total ratio 0.729 would be required
(a L.U. value). Because of the linear mode noise equivalence condition
(Ch. 12) this becomes 1.37 in G.U. conditions. See also FIG. 90
Condition 28A. Also from this figure it can be seen that the critical
signal to noise ratio is about 60 dB.

Now by increasing the value of \( \varepsilon \) above its minimum acceptable
value, as is possible using a GAL 2 which has a rated maximum output
of 10mW, equation 175 can be used to determine what the smallest \( \frac{F_{pm}}{F_o} \)
ratio could be, thus re-writing equation 175

\[ \frac{1}{N} (F_o - F_{pm}) t_c C_r C_d \varepsilon_i \gg \text{NEP (total system)} \]

which becomes, using values as previously
\[
\frac{1}{300} (F_o - 0.35) 0.25, 0.4, 0.2, 10 \times 10^{-3} \gg 18 \times 10^{-11}
\]
\[
6.6 \times 10^{-7} (F_o - 0.35) \gg 18 \times 10^{-11}
\]
\[
F_o - 0.35 \gg 2.73 \times 10^{-4}
\]
\[
F_o \gg 0.3503
\]
Therefore \( \frac{F_{pm}}{F_o} = 0.999 \) (note reference made later in Chapter 14)

Similarly the G.U. value is 1.001 and also from FIG. 90 the critical S/N ratio is about 90 dB.

With these conditions the effective amplifier current for minimum value of reflector factor

\[
I_{amp} = I_s - I_B
\]

where \( I_s = RF.K.t^2 C_{c}.C_{d}.C_{e}.i \)

\[
= 0.35, 0.2, 0.25, 0.4, 0.2, x 10 \times 10^{-3}
\]

\[
= 1.4 \times 10^{-5} \text{ amps}
\]

Therefore \( I_{amp} = (1.4 \times 10^{-5} - 25 \times 10^{-9}) \text{ amps} \)

\[
1.4 \times 10^{-5} \text{ amps}
\]

\( R_f \) is 500 K

\[
V_o = I_{amp} R_f
\]

\[
= 7 \text{ volts}
\]

Thus set voltages of the order of volts are possible using an L.E.D. as light source.
13.5.0 LIGHT SOURCE DRIFT & NOISE

Because the basic pressure transducing system discussed relies on detecting changes in radiant flux falling on a photo-detector, then one obvious cause for concern is the drift in d.c. light source output. 'Noise' within the light source output has the same significance of other noise sources and must be kept within the levels described earlier.

For an incandescent source a d.c. supply is preferred to eliminate 'ripple', the use of rectifiers necessitates smoothing circuits, and mains transformers are needed as the input to the rectifiers. This is because Q1 lamps generally used rate at 12 or 15 volts, their current consumption being between 2 - 10 amps. Now because the resistance of the filament alters on switching on, and in the subsequent warm up period, then the power supply should be of the constant current type. Stabilised Power supplies are available commercially and can be used to drive a Q1 lamp; their main problem is not noise generation but the effect which temperature has on their output which results in drift which results in light source flux drift.

For semi-conductor source (L.E.D.) a d.c. power supply must be used. The problem now becomes related to the power supply's noise content, drift being a less of a problem. The reason for the opposite nature of significances of power supplies for the two light source types is very broadly

1) A Q1 lamp consumes more power, this on being transformed and rectified dissipates more heat within supply, and consequently the temperature sensitivity coefficient becomes critical.

2) An L.E.D., because of its relative low output flux, implies a need for a better signal to noise ratio for the total system (refer to Chapter 12) and consequently noise becomes a problem.
13.5.1 DRIFT COMPENSATION - CONTINUOUS SOURCE

When the initial practical investigation was performed, Chapter 5, problems due to light source drift were encountered. The method considered to compensate for drift is discussed below, where it can be seen mathematically, that such a fundamental comparative compensation system accounts for drift of the light source in a fibre optic pressure transducer dependent on the detector mode.

Consider FIG. 93 which shows a hybrid schema of a fibre optic pressure transducing system (and is based on FIG. 18 and 86), using symbols as defined earlier together with a few additional symbols which will be defined.

The figure shows the initial conditions for the system, i.e. with $R_F$ as $F_0$ the initial value of reflection factor. The full lines depict the normal system configuration, the broken lines the compensation system. This later system samples a fraction $t_c$ of the radiant flux from the light source and as shown, is transmitted by optical fibres of transmission coefficient $t_{cc}$ to a variable optical coupling (iris diaphragm) and thence to a photo-detector which is assumed to be of identical performance and features as the normal detector.

Initially, the variable coupling is set to give (as shown) the same input to the compensating detector (input 2), as that received by the normal detector (input 1), thus $K_{STOP}$ actual, is set to be $\left( \frac{C_F t_{o c}^2}{C_{t_{cc}}} \right)$.

Only the inputs (1) and (2) need to be considered if a linear mode detector is considered, i.e. '$f$' is a linear function when initially

\[
\begin{align*}
\text{input (1)} &= F_{o c f_d}^2 C_{c f d_i} \\
\text{and input (2)} &= F_{o c f_d}^2 C_{c f d_i} \\
\text{thus input (1) - input (2) } &= 0
\end{align*}
\]
FIG. 93  COMPENSATING SYSTEM FOR LIGHT SOURCE DRIFT (continuous source)
Now when the reflection factor changes by say \( +\Delta F_o \)

\[
\text{input (1)} = (F_o + \Delta F_o) t_{C_f C_d e_i}^2
\]

and input (2) remains = \( F_o t_{C_f C_d e_i}^2 \)

Therefore \( \text{input (1)} - \text{input (2)} = e_i t_{C_f C_d}^2 \Delta F_o \)

\[185a\]

This is the normal desired result; however, if simultaneously the radiant input flux changes by \( +\Delta e_i \) using equation \( 185a \) we have

\[
\text{input (1)} - \text{input (2)} = (e_i + \Delta e_i) t_{C_f C_d}^2 F_o \]

\[185b\]

Thus instead of the difference between inputs (1) and (2) remaining constant for a subsequent change of input radiant flux (for any other reflector factor apart from the initial value), we have, by the ratio of equation \( 185b \) to equation \( 185a \), the effect of flux changes on signal difference, i.e. (input to following stage).

\[
= \frac{e_i + \Delta e_i}{e_i}
\]

\[186\]

Thus, there is a direct change in input to following stage (amplifier) proportional to change in radiant flux from source. This can be seen to occur because a change in reflection factor modifies only the value of input (1), and if accompanied by a change in source flux, only modifies this input.

However, if the detectors are considered to operate in a logarithmic mode, i.e. where detector transducing function 'f' is logarithmic, then with just a change in reflection factor \( \Delta F_o \), the output from input (1) is say (considering outputs now)

\[
= \log \left( t_{C_f C_d e_i}^2 (F_o + \Delta F_o) \right)
\]

and the output from input (2)

\[
= \log \left( t_{C_f C_d e_i}^2 F_o \right)
\]

Therefore the differences emerging from the amplifier, assuming gains \( G = 1 \)
As before consider $e_i$ to also change by $\Delta e_i$.

Therefore output from input (1)

$$D' = \log \frac{F_o + \Delta F_o}{F_o}$$

and output from input (2)

$$= \log \left( t^2 c_f c_d (e_i + \Delta e_i) (F_o + \Delta F_o) \right)$$

whereby the difference now emerging from amplifier

$$d' = \log \frac{F_o + \Delta F_o}{F_o}$$

Consequently, the effect of light source drift for a logarithmic mode detector may be compensated by the method shown on FIG. 93, because 'D' from equations 187 and 188 are identical. It is the logarithmic nature of the response (Chapter 12) which allows for the elimination of the effect of light source flux from the value of signal output.

In a similar manner, a drift of light source may be eliminated from a linear mode by addition of a logarithmic function differential amplifier to replace the simple type shown. (However, then the previous discussion on resolution effects of the linear mode detector must be reconsidered).
13.5.2 PULSED SOURCE

The use of a light source which gives a continuous (constant) output of radiant flux has been considered this far. For reasons to be discussed now there could be advantages by using a pulsed source. A pulsed source is one which is effectively being continuously switched on and off. Prior to discussing why a pulsed source could be advantageous let us consider how a pulsed source of radiant flux could be achieved.

Incandescent sources, by virtue of their principle of operation, are not suitable for rapid electrical pulsing; this is because, as discussed earlier, radiant flux is emitted as being a function of the temperature of the filament, and to achieve repeatable outputs from a rapidly pulsed incandescent source is not practicable. True alternating voltage or current do, of course, produce a good output from an incandescent source, however, a pulsed source here is assumed to be a true pulse with a finite mark/space ratio (see FIG. 94).

Semi-conductor sources on the other hand, are far more suitable for electrically pulsed operation. In fact, by pulsing, an increase in permissible diode current can be achieved, which results in greater radiant flux output from the device; with the proviso that the pulse duration and mark space ratio allow it to operate within its maximum average power dissipation capabilities. For example, the GAL 2 can operate with a diode current of 20A with 100 pulses per sec. and a pulse duration of 1μs, resulting in pulsed power of 250 mW. Thus, one advantage of pulsing is to increase the effective radiant flux output of the device; however, the pulse repetition rate then limits the true frequency response of the system.

However, mechanical means can be utilized for effectively producing a pulsed source. This can be achieved with either an incandescent or semi-conductor source by interposing a rotating disc between the source and efferent fibres, the disc presenting its flat face towards the source and having many aperture cut-outs in its periphery.
The main disadvantage of a mechanical 'chopper' is of added mechanical system size, also drift of light source output cannot be compensated for in the manner to be described, unless two aperture sizes are provided in the disc as will be discussed.

Generally with respect to FIG. 94, an idealised electrical input pulse to source is shown where

\[ a \] is pulse duration
\[ \frac{a}{b} \] is mark space ratio
\[ c \] is pulse amplitude
\[ d \] is 'set offset'

when \( a + b = T \) as pulse period
\[ e \] = pulse height

and \( I_c \) is maximum continuous current rating

Thus for average power conditions described above to hold

\[
\frac{I_c^2}{T} \left[ \int_0^a (c + d)^2 dt + \int_0^T d^2 dt \right] \frac{1}{T}
\].................................. 189

noting that 'd' should be \(< c\)

Another proviso for an electrically pulsed source is that the pulse duration is significantly greater than the LED rise time which is of the order of nano-secs and that the photo-detector has a similar rise time.

Now if one assumption is made, that is that the pulse amplitude of the emanating flux remains constant, i.e. some proportion of 'c', then it is possible to compensate for the 'dc' drift of the light source, photo-detector and amplifier (of which it is the detector whose drift is difficult to compensate for in other ways).

The manner in which this is accomplished is now discussed.
13.5.3 DRIFT COMPENSATION USING PULSED SOURCE

By the method of electrically pulsing the light source the resulting system output becomes effectively a pulsed amplitude modulation system where only the pulse amplitude of the signal is of consequence. The d.c. component, although it may vary due to drift of any of the system components can be filtered from the output by suitable circuitry. The resulting output pulse carries the system information and is effectively a square wave alternating about zero, FIG. 95 of amplitude 'c'; 'c' only varying as a function of the pressure on the diaphragm.

This amplitude can, by various means, be displayed and consequently system errors due to drift (of a continuous nature) are eliminated.

Mechanically chopped systems though, because of the absence of a light controlled base level, i.e. equivalent to 'd' the pulse offset value, cannot allow for compensation of the drift of the light source, even though allowing for photodiode drift compensation. One method of producing this light source offset voltage would be to provide a much smaller aperture slot in the chopping wheel, alternating between the longer apertures.
FIG. 94

IDEALISED INPUT PULSE TO L.E.D

FIG. 95

IDEALISED COMPENSATED OUTPUT FROM TOTAL SYSTEM WHEN OPERATING IN PULSE AMPLITUDE MODULATION MODE.
13.6.0 DISCUSSION

Two types of light sources have been discussed, incandescent and semi-conductor. From considerations of size and ruggedness, the semi-conductor sources are desirable. The calculations performed to ascertain an approximate value for the minimum value of radiant flux from a source which will satisfy both the amplifier and system resolution requirement has shown that on L.E.D. would suffice. Also from considerations of pulsed sources L.E.D.s are indicated, which are electrically pulsed.

Drift compensation systems of a simple nature have been discussed. For a 'continuous source', a simple comparator system only functions if the photo-detector is operating in effectively a logarithmic mode; then if both detectors (FIG. 93) are identical, drift from light source and detector can be compensated. A pulsed source also enables light source and photo-detector drift to be accounted for; (if electrically driven); however, this necessitates more involved circuitry for both pulse generation and detection.

Also, the pulse duration must enable source and detection response to be compatible, and the pulse period to allow the frequency response of the system to be high enough to discern the maximum pressure change frequency desired.

Mechanically chopped light source systems, however, suffer from additional disadvantages, namely, compensation for light source basal drift is not achieved unless two sized apertures are provided in the disc in order to simulate the amplitude basal level of the light source signal, rather than a dark basal level; and probably more important the added physical size of the system which would result.
CHAPTER 14

THEORETICAL APPRAISAL - TOTAL SYSTEM
14.0.0 INTRODUCTION

The individual system elements comprising the fibre optic intravascular pressure transducer have been described in detail. Furthermore, two pairs of these elements have been discussed in the context of composite systems; viz the optical fibre/diaphragm unit (Chapter 11) and the effective light source, detector and amplifier system (Chapter 13). The results of the discussion of the latter system has allowed for an approximate quantitative assessment of the minimum value of the input flux to the efferent fibres ($e_i$) and the resulting requirement for the reflection factor total ratio $\left( \frac{F_{pm}}{F_o} \right)$ and initial value. Also, by assessing a maximum value of $'e_i'$ for an L.E.D., the minimum change in reflection factor which will allow for resolution requirements to be met has been found. These calculations can be seen to form the two boundary conditions by which the system will function to a desired specification. The optical fibre/diaphragm system parameters can then be found to satisfy the specified value of $\left( \frac{F_{pm}}{F_o} \right)$ as defined, and has been discussed in Chapter 11.

In a practical system though, two features of the devices are likely to be best obtained experimentally; these are

1) total system noise
2) static calibration curve

Bearing these factors in mind now, which in no way invalidates the exactitude of the following theoretical appraisal, a general approach for the prescribing of a functional system specification will be discussed. The first part considers the general approach used to evolve a simple method which will enable a system specification to be achieved and includes a description of system parameters and the logic relationship between those parameters (FIG. 96).
However, it should be remembered that as a philosophical point, there are many ways of approaching a problem. In the previous chapters it has been prudent to, for example, consider conditions of input to the photo-detector rather than output from the detector amplifier (i.e. system output) to obtain a minimum value for $e_i$.

The following part describes the method for system specification which utilizes the fundamental relationships derived for the fibre optic pressure transducer, and presents them into what is felt to be the most useful theoretical and practically orientated manner for system specification.

There then follows an investigation into two aspects of the system

1) does an optimum initial separation distance exist and
2) if so, is there a maximum acceptable initial separation.

Subsequent to this discussion a 'proposed theoretical system' is evaluated using the graphical methods described for specification of system parameters.
14.1.0 SUMMARY OF THEORETICAL CRITERIA FOR SYSTEM ELEMENT SPECIFICATION

The signal output from a fibre optic pressure transducing system was initially in Chapter 4, shown as a functional relationship by equation 31 which is shown here

\[ y_{op} = f_1(F_1 d_1 D, s.c.), f_2(e_i C_f, C_d, t_c, G) \] .......................... 31

where the first function \( f_1 \) is a dimensionless fraction and represents the effect on relative changes in the modulation of light by the optical fibre/diaphragm system. Subsequent to the analysis of the system, Chapter 12, defines the system output in a more specific manner (equation 129) which is

\[ y_{op} = G f(R_F, t^2_c, C_f, C_d, e_i) \] .......................... 129

where \( f' \) is the photo-detector transducing function, and \( RF' \) is the general reflection factor expression which is equivalent to the first function in equation 31 above (\( f_1 \)). A further point to stipulate is that the manner prescribed for specifying the fibre spatial configuration (S.C.), as described in equation 31 is to introduce the term \( A' \) (\( R_F \)) Chapter 8, which can be seen included in the relationship for \( RF' \) as given by equation 110 in Chapter 11.1.2 which is shown

\[ F_p = \frac{C}{2r^2} \left( R^2 - r^2 + 4 \sqrt{r^2 d^2_p + d^4_p} - \sqrt{(r^2 + R^2 + 4d^2_p)^2 - 4R^2 r^2} \right) \] .......................... 110

where \( F_p \) is the 'reflection factor' at a pressure (differential) \( p \) on the diaphragm.

With \( d_p \) the separation of the optical fibre/diaphragm at a pressure \( p \), which for the type of diaphragm discussed in Chapter 9 and is mentioned in Chapter 11 equation 111

\[ d_p = d_o - \frac{3P (1 - z^2) R^4_{dm}}{16Et^3} \] .......................... 111

With the constant due to assuming a 'flat diaphragm' of

a) \( d_p \max \gg \frac{(A + 1)r}{2 \tan 60} \)

b) and that all the response occurs on the same slope of the static response curve.
Now it is possible to combine these relationships to form a
near total definitive description

\[ y_{op} = G.f. \left( t_c^2 C_f C_d e_i \right) \left[ \frac{c}{2r^2} \left[ \frac{2}{r^2 - r^2 + 4 \sqrt{r^2 (d_o - 3P(1-Z^2) R_{dm}^4)}^2} \right] \right. \\
+ \left. \left( d_o - \frac{3P(1-Z^2)}{16E_t^3} R_{dm}^4 \right)^4 \right] \sqrt{\left( r^2 + R^2 + 4(d_o - 3P(1-Z^2) R_{dm}^4)^2 \right) - 4R^2r^2} \]


The form of the function \( f \) in the above equation is dependent on the
mode in which the detector is functioning (Chapter 12). This function
for a linear mode detector is a constant, of value \( 'K RF' \), thus the
system output expression can be written considering a linear mode detector.

\[ y_{op} (\text{volts}) = G.K.R_f t_c^2 C_f C_d e_i \frac{c}{2r^2} \left[ \frac{2}{r^2 - r^2 + 4 \sqrt{r^2 (d_o - 3P(1-Z^2) R_{dm}^4)}^2} \right. \\
+ \left. \left( d_o - \frac{3P(1-Z^2)}{16E_t^3} R_{dm}^4 \right)^4 \right] \sqrt{\left( r^2 + R^2 + 4(d_o - 3P(1-Z^2) R_{dm}^4)^2 \right) - 4R^2r^2} \]

where all the symbols represent parameters as described within the
relevant chapters. However, it is worth re-stating all the parameters
together now

- **G** is auxiliary amplifier gain (volts/volts)
- **K** is detector responsivity (amp/watt)
- **R_f** is feedback resistance (ohms) of current amplifier
- **t_c^2** is transmission coefficient of afferent and efferent fibres (watts/watts)
- **C_f** is coupling factor: light source to efferent fibres (watts/watts)
- **C_d** is coupling factor: afferent fibres to detector (watts/watts)
- **e_i** is the radiant flux emitted by the light source (watts)
- **C** is the coefficient of reflection of the diaphragm for the
  radiant flux striking it
is the radius of the central emitting disc within the sender/receiver system in the optical fibre system (mm)

\( R \) is the radius of the outer diameter of the sender/receiver system (mm)

\( d_0 \) is the initial separation between the fibre end face and the diaphragm, i.e. when the diaphragm is subjected to an applied differential pressure of zero (mm)

\( P \) is the applied differential pressure to which the diaphragm is subjected (gm/mm\(^2\)) and is the true variable

\( Z \) is Poisson's ratio for the diaphragm (\( \frac{\text{lat. strain}}{\text{long. strain}} \))

\( R_{dm} \) is the effective radius of the diaphragm (mm)

\( E \) is Young's modulus of elasticity for the diaphragm material (gm/mm\(^2\))

\( t \) is the diaphragm thickness (mm)

One term with equation 191 is worth simplifying for reasons which will become apparent. This is the term describing the actual deflection of the diaphragm and has been shown in Chapter 10 to be described by equation 91.

\[
y_{\text{max}} = \frac{3P (1-Z^2) R_{dm}^4}{16Et^3}
\]

................... 91

and if \( P \) is a maximum, \( y_{\text{max}} \) is written \( y'_\text{max} \) which upon substitution into equation 191 becomes

\[
y_{\text{op}} = G.K.R_f t_c^2 c_f c_d e_i \frac{C}{2r^2} \left( R^2 - r^2 + 4 \sqrt{r^2(d_o - y_{\text{max}})^2 + (d_o - y_{\text{max}})^4}
- \sqrt{(r^2 + R^2 + 4(d_o - y_{\text{max}})^2)^2 - 4R^2 r^2}ight)
\]

................... 192

equation 191 and 192 allow for the same determinations of system response to be ascertained. However, the description for the variable 'P' included in equation 192, i.e. \( y_{\text{max}} \) will allow for two methods for specification of \( d_o', y_{\text{max}} ' \) and the resolution \( N \) achievable for a given system noise value \( V_n \). These methods are briefly discussed here
as a preface to the detailed description in the following section.

The main requirements for a system of specification are

1) defining the true dependent parameters and consequently, there shortly follows a list of the parameters which can be given values.

2) defining the critiques by which system performance can be evaluated in order to assess whether the system goals are achieved.

Let us consider the latter aspect first. Chapters 12 and 13 deal primarily with the concept of the system output change resulting from a change of the value of the pressure to be resolved and the comparison with actual noise value, and consider a boundary condition for system acceptance of \( V_n \) \( \leq \frac{y_o}{N} \left( \frac{F_{pm}}{F_o} - 1 \right) \) noting that the RHS of this inequality is the actual system output change corresponding to one \( N^{th} \) of the total pressure to be registered. The ratio \( \frac{F_{pm}}{F_o} \) is utilised to enable the boundary values of \( \frac{F_{pm}}{F_o} \) to be determined for various values of the set output level \( y_o \) for specific \( N \) values. Such a relationship can be plotted in isolation, for the actual system itself, but it should be noted that the above inequality only holds for a linear mode detector.

Thus it is possible to specify, knowing a set output value and a desired resolution value, what the boundary value of \( \frac{F_{pm}}{F_o} \) is required, (noise level specified). This can later be related to actual total reflection factors which are achievable for different total deflections at various initial separation values (giving the desired set output value also) and will be termed the GASSP-I method.

Another approach which can be made is to reconsider the description for the system output change for the linear mode detector.
\[ \Delta y = \frac{y_0}{N} \frac{(F_{pm} - F_0)}{F_0} \]

thus total output change
\[ ' \text{total} \Delta y' = N \Delta y = y_0 \left( \frac{F_{pm}}{F_0} - 1 \right) \]

however as 'y_o' the set output value is actually
\[ y_0 = G.K.R_f e_i C_f C_d t_c^2 F_0 \]

The above equation may be re-written
\[ ' \text{total} \Delta y' = N \Delta y = G.K.e_i C_f C_d t_c^2 (F_{pm} - F_0) \]

where obviously the RHS is just the difference in system output between
when \( P_{\text{max}} \) and \( P_{\text{min}} \) are applied.

So an alternative method of system specification will be to
obtain the total system output change and ascertain whether it is greater
than \( 'V_N' \), the noise resolution product; if so, the system performance
requirement is achieved.

The boundary conditions and maximum deflection \( 'Y_{\text{max}}' \) limitations
are applied to both this second method (GASSP 2) and the former method.

Each method is actually based on the signal to noise
requirements.

Thus with a knowledge of all parameters which can be defined,
the following methods using the critiques just described, will be employed
to assess system performance. These are approaches:

1) As it is possible to ascertain the noise level of a
system (either theoretically or practically), a
relationship between reflection factor total ratio and
resolution for a linear mode detector (equation 158 and 159)
can be related to the system noise for the boundary condition
where the noise is equal to the change in signal output
caused by \( \frac{1}{N} \) of the total pressure to be registered.

This boundary condition is
\[ 'V_n' \ (\text{noise}) = \frac{y_0}{N} \left( \frac{F_{pm}}{F_0} - 1 \right) \]
In terms of signal to noise ratio of this boundary condition, FIG. 89 depicts the minimum acceptable signal to noise ratio for different reflection factor total ratios and values of 'N' for different values of \( \frac{F_{pm}}{F_0} \).

A more useful graphical representation which is shown in the next section is based on expression 193 and will show, for a set system noise level and different resolutions, the boundary condition variation of \( \frac{F_{pm}}{F_0} \) for different values of set output values.

The set output level is described by equation 192, where a relationship between initial separation distance (or ratio) and set output value could be plotted. This has been termed 'static calibration curve'. From this static calibration curve it is possible to obtain from equation 192 the reflection factor total ratio resulting from a specific diaphragm maximum deflector \( (Y_{max}) \). This results from a pressure \( P_{max} \), to be maximally determined at any initial separation. It is also possible to obtain the relative relationship, i.e. a normalised deflection curve. However, considering the previous relationship, it can be seen that using the three graphical relationships discussed it is possible to assess:

(i) whether, at a specific initial separation \( d_0 \) and when considering a specific diaphragm total deflection \( Y_{max} \), the resolution required is achieved.

(ii) or at what value of \( d_0 \) and \( Y_{max} \) can the resolution requirement be met.

Furthermore, because each of these three extracted representations have at least one common parameter, they lend themselves towards a combined graphical aid for specification of system parameters (GASSP-1).
Approach 2) The alternative approach considers directly, the value of total system output change for various initial separations and values of total deflections. This is best shown graphically in a normalised form with system total output change plotted against initial separation ratio for specific values of normalised deflection ratios. One must then employ the critique that the noise x resolution product $V_n N$ must be less than the value of total system change in output graphically determined for a specific $B_0$ and $\Delta B$. This method will, in essence, fulfil the same basic assessments as the previous method discussed. The construction details of this method GASSP 2 is given later.

As mentioned, there are certain system parameters which can be assigned values within a certain tolerance band. (The following section 14.2.0 taking specific values for a proposed system specification). This is so because of constraints on the system due to either physical or commercial availability restrictions. Later it will be seen that the above method for system specification can be justified by virtue of those 'known' parameters. They are listed below.

$e_i$ - the approximate value of light source flux for an L.E.D. which can be specified due to the limitations imposed by commercial availability (10 mW).

$\lambda$ - although not actually appearing in the relationship for system output (equation 192), it is required for determination of the fibre transmission coefficient, $t_c$ and the detector response characteristics. However, as the light source is an L.E.D, this wavelength is known (for a GAS device) thus $t_c$ can be found. Most detectors have a peak response at the wavelength emitted by a Gallium Arsenide device. Nevertheless as detectors usually have their response specified in terms
of radiant flux rather than luminous flux, the necessity for knowledge of \( \lambda \) for the detector is redundant, considering this requiremental aspect for \( \lambda \) (8 - 9 um).

\( C_f \) & \( C_d \) - The coupling factors for the light source and efferent and afferent fibres to photo-detectors respectively can be approximated for their maximal values. This is primarily due to geometric and optical constraints for the system (.40 and .20 respectively).

\( t_c \) - The transmission coefficient for the optical fibres used can be approximately specified as the wavelength of radiation to be transmitted is known, and although there are different optical fibres commercially available, one is to be chosen which has a high transmission in the region specified. There is noticeably little difference for transmission at wavelength .8 - 9 mu of different commercially available fibres. The further specification is the numerical aperture required which then limits the fibre type and transmission coefficient because of the different types of core and sheath glass combinations necessary for altering the N.A., (c.f. Chapter 2).

It should be borne in mind that the total length of optical fibres is considered to be 1 metre (a system specification). Consequently, within this specification for \( t_c \) there are several sub-parameters, which have been discussed in Chapter 3 (equation 16), and must be fixed. Fortunately, all of them are what can be called primary variables and can be fixed. They are \( r_f \), \( L \), N.A. (.25).

\( K \) - the responsivity of a photo-detector has a value which can be ascertained approximately by the manufacturer's specification and as the highest value is to be preferred, the range of value is small (\( \frac{2}{4} \) amps \( \frac{\text{amps}}{\text{watts}} \)).
R_{dm} - the value of the effective radius of the pressure sensing diaphragm can be given a maximum value due to system's design requirements (1.5 mm).

\( r_f \) - the radius of fibre used within the optical fibre system can be given a minimum value and maximum value due to commercial availability, constructional constraints, transmission restraints and finally mechanical constraints (20 \( \mu m \) - 200 \( \mu m \)).

\( \frac{R(A)}{r} \) - this is the ratio of a central emitter to surrounding annular receiver radius. For reasons of and due to practical construction details and theoretical limitations (11.2.0) a value of 3 is imposed when considering micro-systems. Macro systems could have any value only if the central emitter is of adequate size to enable an efficient coupling factor to result. (\( \frac{3 \text{ micro}}{\sqrt{3} \text{ macro}} \)) (Chapter 14)

\( r \) - the radius of the central emitter; For a micro system with \( A = 3 \), \( r \) is simply the fibre radius ' \( r_f \)' with the proviso that the sensitivity provided, by virtue of the finite size of ' \( r \)' is adequate and has been shown to be a function of \( \frac{\gamma_{\text{max}}}{r} \) (11.2.0) (20 - 40 \( \mu m \)).

\( C \) - the coefficient of reflection for the diaphragm and the wavelength '\( \lambda \)' of incident light. This is dependent on the material and surface finish of the diaphragm and also the wavelength of incident light. This has been assumed to be unity prior to this juncture. However, the value of this coefficient is fairly well defined. (0.8 - .96).

\( R_f \) - the value of the feedback resister for the linear mode detector; this has been shown to be of an optimal value of approximately the effective source impedance of the photo-detector (current generator) and can be approximately fixed (500K).
P - the pressure (differential) to which the diaphragm is subjected is assumed to be the applied pressure external i.e. a vented gap existing between the fibres' face and diaphragm. The maximum value is arbitrarily set as being significantly greater than is to be physiologically encountered. This parameter is actually the basic variable; from which a minimum value to be recognised is chosen to be 1mm Hg, consequently enabling the specification of 'N' the resolution factor (300 mm Hg).

N - this is the denominator of the fraction of the maximal pressure which is to be resolved (300).

$y_{max}$ - this is the derived variable for the pressure variable 'p' and is the central deflection a diaphragm suffers when subjected to the differential pressure 'p'. Its value for different diaphragm materials, and radii ($R_{dm}$), when subjected to the maximal pressure $P_{max}$ has been tabulated (FIGS. 6 & 7) and is to be termed $y_{max}$.

The following FIG. 96 displays the fundamental logic employed which enables the two forms of GASSP to be established.
14.1.1 GRAPHICAL METHODS FOR SPECIFICATION OF SYSTEM PARAMETERS

The methods outlined in the previous section for aiding system parameter specification require:-

1. GASSP 1) The graphical plotting on a system of three axes originating from the same origin the three relationships pertaining to the fibre optic pressure transducer as discussed.

2. GASSP 2) a conventional single graphical representation.

Considering these, in turn:

GASSP 1 Static Calibration Curve: (FIG. 97)

This is the relationship between the initial separation \( (d_o) \) of a diaphragm and the resulting system output, and can be obtained theoretically from equation 192 or from a practical evaluation as discussed in Chapter 5 and obtained from a set-up shown in FIG. 24.

The same relationship could have been depicted with the 'separation axis' as a 'separation ratio' axis, i.e. \( \frac{d_o}{r} \). The choice of method is dependent on the initial tolerance band given to 'r'. However, if plotted in this manner then the next graphical relationship to be described viz. the 'deflection curve' should use normalised deflections as conditional boundaries.

In FIG. 97 a generalised static calibration curve is shown where two features should be noted

a) an offset in output is given to the curve for 'contact' conditions; this has been performed simply to enable the system to become more general and in effect is obtained by simply adding some constant to equation 192.

b) only the positive going slope is shown to enable, at this time a more lucid description of the use of this graphical aid for system parameter specification. However, it should be borne in mind that the exact portion of the theoretical static calibration curve used is dependent on condition 1) on Page 277:
in other words, the initial commencement of the usable theoretical part of the curve is a function of $Y_{\text{max}} \& \frac{R}{r}$ (as $d_{p \text{ max}} = d_0 - Y_{\text{max}}$). However, for the example figure shown (FIG. 97) the condition for non-contact of the diaphragm is to be utilized, i.e.

$$d_0 - Y_{\text{max}} \gg 0$$

and only depends on $Y_{\text{max}}$, and it can be seen that all 'action' does occur on the same 'slope'.

**Deflection and Curve.**

This curve is primarily a theoretically derived curve, obtained from that above. Its formation is achieved by considering, at different initial separations 'd$_0$' (or separation ratios B), various maximum diaphragm deflections to occur ($Y_{\text{max}}$), (or normalised deflection ratios $\Delta B$) to occur. Consequently, it is possible to determine the reflection factor total ratio achieved at one initial separation for a specific total deflection. This is achieved by using equation 192 with 'y$_{\text{max}}$', initially as zero and then as 'Y$_{\text{max}}$' using the same value for initial separation 'd$_0$'. Then the reflection factor total ratio becomes the value for the second computation divided by the first. This manoeuvre has already been discussed and is described by equation 89.

The family of curves shown, depict for one value of $Y_{\text{max}}$ (or $\Delta B$) which occurs at a value of initial separation $d_0$ (or B), the resulting value of reflection factor total ratio.

Thus, the curves reading from right to left on FIG. 97 (4-1) actually depict decreasing values set for the deflection parameter. The explanation of the discontinuous nature of them being the previously mentioned boundary condition for this particular GASSPI, that contact between fibres and diaphragm should not occur

i.e.  

$$d_0 \gg Y_{\text{max}}$$

but as will be seen and has been mentioned the cut-off will occur such that
\[ d_0 - Y_{\text{max}} > \frac{(A + 1)r}{2 \tan 60} \]

or \[ d_0 - Y_{\text{max}} > 'B'_t \times r \]

Resolution Curve

The axes of this curve share one common axis with each of the aforementioned curves. The relationship which it represents is given by equation 193, which describes the boundary condition when the system noise equals system output change. The system output change results from the applied pressure changing by an amount equal to the pressure resolution interval

\[ i.e. \quad 'V_n' = \frac{Y_0}{N} \left(1 - \frac{F_{\text{pm}}}{F_0}\right) \]

where \( 'V_n' \) is the output noise from the system and can be determined theoretically from equation 175a which gives the input N.E.P. conditions of detector and amplifier. This value can then be used as \( 'e_i \times 2C_iC_F' \) in equation 160 to calculate \( 'V_n' \) which also could be found practically.

The relationship between \( \frac{F_{\text{pm}}}{F_0} \) and the set output level \( 'y_0' \) are plotted from equation 193.

\[ i.e. \quad \frac{F_{\text{pm}}}{F_0} = 1 - \frac{V_N}{y_0} \]

with the set output level ordinate sharing the system output level axis of the static calibration curve, and the boundary value of \( \frac{F_{\text{pm}}}{F_0} \) computed, sharing the \( \frac{F_{\text{pm}}}{F_0} \) achieved axis of the deflection \( \frac{F_{\text{pm}}}{F_0} \) curve, using various values for \( V_n \) or \( N \) (i.e. \( V_N \)).

The figure shows four values used for \( 'N' \) or \( 'V_N' \) and reading from right to left, \( N_1 \) through to \( N_4 \) goes from large \( N \) values (or \( V_N \) products) to smaller values. In other words, from small pressure resolution requirements to large (more easily attainable) ones.

It should be noted that, in actual fact, it is the product \( 'V_N' \) seen in 194 which could be used as a value for prescribing the individual curves composing this graph, (allowing a more general interpretation of system specification).
Static Calibration Curve

From Equations 82 & 160 (192)
(Or from Practical Calibration)
cf. Fig. 20

Sep. D Microns
OR Sep. Ratio D/R

Fig. 97 Graphical aid for specification of System parameters-linear mode detector
The second mentioned graphical aid for the specification of system parameters (GASSP 2), uses more directly the basic critique that the system output change must be adequate to cover the resolution noise product

\[ \text{total} \Delta y' \gg V_n N \]

The total change in system output has been described earlier by equation 154

\[ \text{total} \Delta y' = KGR_f C_f C_d e_i t_c^2 \text{ (total} \Delta F) \]

Then with respect to equation 192 for the system output, and writing in normalised terms:

\[ \text{total} \Delta y' = GKR_f t_c^2 C_f C_d e_i C_x \frac{C}{2} \times \]

\[ \frac{(B_0 - \Delta B)^2 + (B_0 - \Delta B)^4 - (B_0^2 + B_o^4 - \sqrt{(A^2 - 1)^2 + 16 (B_0 - \Delta B)^4}}}{+ 8(B_0 - \Delta B)^2(A^2 + 1) + \sqrt{(A^2 - 1)^2 + 16B_0^4} + 8B_0^2(A^2 + 1)} \]

Thus 'total \Delta y' may be plotted against initial separation ratio 'B_0' for specific normalised deflection ratios \( \Delta B \).

Then by using the inequality shown above, it can be ascertained whether the parameters of \( B_0 \) & \( \Delta B \) allow the requirement for noise/resolution product to be met.

\[ \ldots \ldots \ldots \]

The formation and significance of each individual graph having now been discussed, we proceed to a description of how one can generally manipulate the graphical aid for specification of system parameters (specifically the GASSP 1).

With reference to FIG. 97, consider the static calibration curve to represent the output conditions for a specific system, i.e. 'r' is known. Then the output resulting from any initial separation can be found; let the output at an initial separation \( d_o \) be considered, which is \( y^1 \). This output value becomes the set output value for the 'resolution' curve, whereby the minimum or boundary condition for a required reflection factor total ratio \( \frac{F_m}{F_o} \) can be determined for any of the resolution lines (N_1 etc.)
These are shown as

\[
\begin{pmatrix}
\frac{F_{pm}}{F_0} \\
\frac{d_0}{N_1}
\end{pmatrix} = F_4^1, \ F_3^1, \ F_2^1, \ F_1^1
\]

The most stringent resolution is \(N_1\), as discussed earlier, and consequently initial use is made of the \(\frac{F_{PM}}{F_o}\) boundary value \(F_1^1\) corresponding to the set output level \(y_1\) with a resolution \(N_1\) as the one ordinate of the deflection curve (diamond shape graph). The point of intersection 'A' of this ordinate and 'd_0^1', the other ordinate, then allows the specification of which diaphragm deflections will satisfy the resolution requirements.

In the figure shown all deflections of the diaphragm except for the smallest \(Y_{max(1)}\) will suffice, as the reflection factor total ratio for all the others is less than the boundary value \(F_1^1\) (considering \(\frac{F_{pm}}{F_o}\) L.U. conditions).

Furthermore, if the deflection of the diaphragm can be large \(Y_{max(4)}\), then a greater initial separation can be allowed for (perhaps of practical significance), by projecting the \(F_1^1\) boundary value until it intersects the curve representing the situation for a deflection \(Y_{max(4)}\), point 'B', then reading at what initial separation the condition holds up to i.e. \(d_0^2\). Strictly speaking, by increasing 'd_o' the complete procedure should be repeated, i.e. re-determination of the set output value, then determination of the boundary conditions \(\frac{F_{pm}}{F_o}\) for resolution \(N_1\) say. However, when only a forward giving slope is considered, the resulting value of \(d_0^2\) can be seen always to err on the more correct side (as \(d_0^2 > d_0^1\)), which is also the case if the portion of the resolution curve corresponding to the set output values \(y^1\) and inferred \(y^2\) is nearly flat.
If, on the other hand, the smallest deflection $Y_{\text{max}(1)}$ is preferred, and it can be seen that, at an initial separation $'d''\hat{\omega}_1$, a resolution $'N''\hat{\omega}_1$ is not attainable then two courses of action can be taken, graphically. First, the resolution which is attainable can be determined by checking the other resolution boundary condition reflection factor total values $F''\hat{\omega}_4$, $F''\hat{\omega}_3$ & $F''\hat{\omega}_2$ to ascertain which of these can be satisfied for a deflection $Y_{\text{max}(1)}$, which from the graph it can be seen that the lowest resolution $N''\hat{\omega}_4$ is just achievable. Secondly a change in initial separation $'d''\hat{\omega}_0$' can be investigated and the complete procedure repeated. In this case, of a forward going slope a smaller value of initial separation $'d''\hat{\omega}_0$ is indicated. This allows for the deflection response to occur on a 'steeper slope' on the static calibration curve. This results in a greater change in reflection factor, i.e. a smaller value for $F''\hat{\omega}_d$ which gives rise to a set output value $'y''$ reduces which in turn necessitates more stringent boundary conditions. However, as is shown by an initial separation of $d''\hat{\omega}_3$ which gives rise to a set output level $y''$ which, in turn, allows the boundary conditions $F''\hat{\omega}_3$ to be ascertained, it can be seen that even with the minimum deflection conditions $Y_{\text{max}(1)}$, the most stringent resolution requirement could be met (intersection point C).

Furthermore, prescribed manipulations of the GASSP.1. are discussed later in the context of specifying an optimum initial separation distance for a specific total diaphragm deflection, and the limiting or maximal initial separation distance whereby the resolution requirements can be met. The GASSP 2 will be shown.
14.1.2 THE SIGNIFICANCE OF 'd_{opt}' AND VALUES OF THE REAL OPTIMUM INITIAL SEP. RATIO 'b_{opt}'

The concept of an optimum initial separation distance 'd_{opt}' for a specific value of a diaphragm's total deflection 'y_{max}' was introduced in 8.2.2 and further mentioned in 11.2.0. The significance of 'd_{opt}' in the context in which introduced was simply that, at this initial separation distance a maximum value for $\frac{p_{m}}{F_o}$ results (note only for G.U. conditions) as can be seen from FIG. 55.

However, its merits as an optimum separation distance to actually be used as an assigned condition ('real d_{opt}') in the GASSP.1 must be investigated. A superficial appraisal of how 'd_{opt}', as defined, actually affects the other system parameters can be made by considering two figs, FIG. 41, the theoretical normalised static calibration response curve $A = 3$, and FIG. 55 showing the ratio $\frac{p_{m}}{F_o}$ for different initial separations having specific normalised deflections. It can be seen here that the value of reflection factors $F_p$ & $F_o$ at this separation distance 'd_{opt}' are lower than is possible if a smaller value of initial separation distance is considered, but then the value of $\frac{p_{m}}{F_o}$ at this smaller separation distance is lower than for 'd_{opt}'.

Consequently, the actual set output value at a separation 'd_{opt}' is also lower than would be possible by using a lower value for initial separation. It is this factor, of 'set output values', and the 'ratio $\frac{p_{m}}{F_o}$', which determine the resolution capabilities of the system. As has been mentioned earlier, the greater the set output, the greater the pressure resolution possible, because the change in system output corresponding to the pressure change increases with increases in set-output levels (12.1.2)

Thus, although at 'd_{opt}' a better $\frac{F_p}{F_o}$ results then at a smaller initial separation, the set output value is smaller, implying some form of 'trade off' in actual resolution capabilities. It is this 'trade off' which will be discussed.
Now the output change for a linear mode detector is from equation 159 \( \frac{F_{pm}}{F_0} \gg 1 \)

\[
\Delta y = \frac{y_0}{N} \left( \frac{F_{pm}}{F_0} - 1 \right)
\]

but \( y_0 \) is the set output value where \( \Delta y \) is \( \frac{1}{N} \)th the total change in output which from equation 129 becomes

\[
y_0 = R_f G K t^2_c C_f C_d e_i F_o
\]

thus re-writing the above expression 159

\[
N \Delta y = R_f G K t^2_c C_f C_d e_i F_o \left( \frac{F_{pm}}{F_0} - 1 \right)
\]

\[
= R_f G K t^2_c C_f C_d e_i \left( F_{pm} - F_o \right)
\]

(N.B.) this is of the same form of equation 154 introduced in Chapter 12)

and substituting for reflection factor \( F_{pm} \) and \( F_o \) using the normalised expression 82 for reflection factor

\[
'total \Delta y' = G K R_f t^2_c C_f C_d e_i \frac{c}{2} \frac{c}{2} \left[ (A^2 - 1 + 4 \sqrt{(B_o - \Delta B_o)^2})
\right.
\]

\[
+ (B_o - \Delta B_o)^4 - \sqrt{(A^2 - 1)^2 + 16(B_o - \Delta B_o)^4} + 8(B_o - \Delta B_o)
\]

\[
\frac{1}{(A^2 + 1) - (A^2 - 1 + 4 \sqrt{B_o^2 + B_o^4} - \sqrt{(A^2 - 1)^2 + 16B_o^2(A^2 + 1)})}
\]

Where \( B_o \) is the initial separation ratio \( 'd' \) and \( '\Delta B_o ' \)

is the normalised deflection ratio \( \frac{\text{max}}{r} \).

Thus the output change \( '\Delta y' \) can be shown by the proportionality below; (as values for the magnitude parameters have not yet been assigned).

\[
'total \triangle y' \propto \left[ 4 \sqrt{(B_o - \Delta B_o)^2} + (B_o - \Delta B_o)^4 - 4 \sqrt{B_o^2 + B_o^4} - (A^2 - 1)^2 + 16(B_o - \Delta B_o)^4 + 8(B_o - \Delta B_o)^2(A^2 + 1) \right]
\]

\[
16(B_o - \Delta B_o)^4 + 8(B_o - \Delta B_o)^2(A^2 + 1) + \sqrt{(A^2 - 1)^2 + 16B_o^4 + 8B_o^2(A^2 + 1)}
\]

\[
\]

\[
\]

\[
\]

\[
\]

\[
\]
Thus by plotting 'total Δy' against 'B_o' for different ΔB_o values, it will be possible to establish whether an optimum value for 'B_o' exists which gives rise to a maximum value of 'total Δy'. (Note that differentiation for determination of maximal values could be used also i.e. a maximum value for the noise signal/ratio.).

Firstly, let us consider the boundary value conditions which must be stipulated.

1) The prime proviso is that the initial separation ratios both lie in the same 'slope' as each other (8.2.2.). This can be achieved by making reference to FIG. 44 (etc.) from which the separation ratio at the point of inflection i.e. when a peak value of reflection factor occurs can be found using equation 87

\[ B_{\text{peak}} = \frac{A - 1}{2} \]

thus

\[ B_o - \Delta B_o \geq B_{\text{peak}} \] - ve slope ................. 199

\[ B_o \geq B_{\text{peak}} \] + ve slope ................. 200

2) The minimum value for 'B' due to the flat diaphragm assumption (11.1.2, Page 277 ) is

\[ B_o - \Delta B_o \geq \frac{A + 1}{2 \tan 60} (B_t) \] ................. 201

Thus for this theoretical treatise, it is the relative difference between the transition point separation ratio 'B_t' and that for peak reflection factor 'B_{\text{peak}}' which defines whether there will be a need to consider conditions 199 and 200 above which ensure that the 'action' occurs on the same slope. Earlier in Chapter 8 FIG. 46a allows for any 'A' value and 'θ' value the determination of the relative magnitudes of 'B_t' and 'B_{\text{peak}}'. Due to the flat diaphragm assumption θ = 60° it can be seen from FIG. 46a that the transition point separation 'B_t' always lies past the point where the 'standard' peak value of reflection factor occurs, and consequently the above condition for the diaphragm assumption...
\( B_0 - \Delta B_0 \geq B_t \) inherently satisfies the initial condition of action occurring on the same slope.

Let us now consider FIG. 98, which depicts the relationship (equation 198) plotted with only the boundary condition

\[ B_0 \geq B_t \quad (\theta = 60^\circ) \]

in other words, so that the initial separation is always greater or equal to the transition point.

With \( A = 3 \)

whereby \( B_0 \geq 1.1547 \), 1.1547 being the separation ratio at which transition occurs. Now from FIG. 44 in equation 87 the value of \( B_{\text{peak F}} \) with \( A = 3 \) is approximately .7.

From the figure in question it can be seen that an optimum initial separation ratio does apparently exist, let it be termed \( B_{\text{app.opt.}} \) which results in a maximal change in output.

However, for it to be theoretically acceptable, condition 201 must hold, i.e. all response occurring past transition point

\[
B_{\text{app.opt.}} - \Delta B_0 \geq 1.1547
\]

\[
\therefore B_{\text{app.opt.}} \geq 1.1547 + B_0
\]

which can be seen not to be true for any of the values for \( \Delta B_0 \) shown except when \( \Delta B_0 = .25 \)

as for \( \Delta B_0 = 1 \quad B_{\text{app.opt.}} = 1.99 \)

\[
\therefore 1.99 \geq 1.1547 + 1
\]

\[
\Delta B_0 = .75 \quad B_{\text{app.opt.}} = 1.85
\]

\[
1.85 \geq 1.1547 + .75
\]

\[
\Delta B_0 = .5 \quad B_{\text{app.opt.}} = 1.66
\]

\[
1.65 \geq 1.1547 + .5
\]

\[
\Delta B_0 = .25 \quad B_{\text{app.opt.}} = 1.51
\]

\[
1.51 \geq 1.1547 + .25
\]

thus only the value above of \( B_{\text{app.opt.}} \) is a theoretically true 'real d_opt' value.
The only other consideration to be verified is to ensure that condition 199 holds which can be seen to be valid for all cases (response occurs on same slope) of (Chapter 8).

This situation can be clarified by considering FIG. 99 which incorporates the correct boundary condition for $B_o$ namely

$$B_o - \Delta B_o \gtrsim 1.1547$$

where the following real values for optimum initial separations except when $\Delta B_o$ is .25 can be seen to be simply the value

$$'real d_{opt}' = \frac{A + 1}{2 \tan 60} + \Delta B_o$$

That is the closest separation allowed within the theoretical boundary conditions.

Thus with $A = 3$

- $\Delta B = 1$  \hspace{1cm}  $'real B_{opt}' = 2.1547$
- $\Delta B = 0.75$  \hspace{1cm}  $'real B_{opt}' = 1.9047$
- $\Delta B = 0.5$  \hspace{1cm}  $'real B_{opt}' = 1.6547$
- $\Delta B = 0.25$  \hspace{1cm}  $'real B_{opt}' = 1.51*$$

* the only case plotted where graphical assessments of the maximal value for $B$ must be assessed graphically.

It should be noted that $'real d_{opt}'$ does not necessarily signify that the resolution requirements will be achieved, rather that it is at this 'separation' that an optimum resolution for these pairs of 'separations' and 'deflections' is achieved by the system.

Now comparison of the above values of $'real optimum initial separation ratios'$ with those depicted in FIG. 55, where maximal values of $\frac{F_{pm}}{F_o}$ occur, shows that the former ($real B_{opt}$) are all lower than these when considering the ratio concept for $'d_{opt}'$. 
FIG. 98 GRAPHICAL DETERMINATION OF APPARENT OPTIMUM INITIAL SEP. RATIO $d_{app \, opt}$
FIG. 99 GRAPHICAL DETERMINATION OF REAL OPTIMUM SEP. RATIO "real $d_{opt}$"

$A = 3 \left( \frac{r}{R} \right)$

- $A' = \Delta B_0 = 1.0$
- $B' = \Delta B_0 = 0.75$
- $C' = \Delta B_0 = 0.50$
- $D' = \Delta B_0 = 0.25$

Initial Sep. Ratio $B_0$
Summarising then: values for real optimum initial separation
values have been determined for $A = 3$ with four values of normalised
deflection ratios. These values for 'real $B_{opt}$', differ from those
initially introduced in 8.2.2, which are arrived at by considering a
maximum value for $\frac{F_{pm}}{F_0}$, and signify a maximal change in system occurring for
the specific normalised deflection. These values of 'real $B_{opt}$' will
later be shown in GASSP. 1 for the proposed theoretical system 14.2.1.

However, the linearity of the response may be of primary
importance. Reference must then be made also to the slope factor
variation through $\Delta B_0$ or the resulting output and pressure graph FIG. 103
and 104. (Static pressure calibration curves.)
14.1.3 MAXIMUM VALUE FOR INITIAL SEPARATION \( d_{o \text{ max}} \) AND THE GASSP 2

At any initial separation 'd' and for a deflection 'y_{max}'
two features of the system can be determined

1. the set output value
2. the reflection factor total ratio

Now from the set output level as has been discussed, it is
possible to determine the boundary condition for the minimum value of
the ratio \( \frac{F_{pm}}{F_{o}} \) (when \( \gg 1 \)) for which the noise and resolution
requirements are met. Graphically using the GASSP 1 it is possible to
determine the value of 'd_{o \text{ max}}' for which the actual value of \( \frac{F_{pm}}{F_{o}} \) is
equal to the boundary value. This is shown later in 14.2.1, FIG. 101.

Mathematically this condition can be described using the
system output description (equation 192) and the resolution boundary
condition equation from equation 194.

\[
\begin{align*}
i.e. \quad y_{op} &= G.K.R_{f}t_{c}^{2}C_{f}d.e_{i}C_{e}^{2}R_{2}^{2} \left[ R^{2} - r^{2} + 4 \sqrt{r^{2}(d - y_{max})^{2} + (d - y_{max})^{4}} \right] \\
& \quad - \sqrt{(r^{2} + R^{2} + 4(d - y_{max})^{2})^{2} - 4R^{2}r^{2}} \quad \text{................. 192}
\end{align*}
\]

Boundary Value \( \frac{F_{pm}}{F_{o}} = 1 + \frac{VnN}{y_{o}} \quad \text{.......................... 194} \)

now expressing 192 in normalised terms, with \( \Delta B = \frac{y_{max}}{r} \)

\[
\begin{align*}
y_{op} &= G.K.R_{f}t_{c}^{2}C_{f}d.e_{i}C_{e}^{2}R_{2}^{2} \left[ \frac{A^{2} - 1 + 4(B_{o} - \Delta B)^{2} + (B_{o} - \Delta B)^{4}}{(A^{2} - 1)^{2} + 16(B_{o} - \Delta B)^{4} + 8(B_{o} - \Delta B)^{2}(A^{2} - 1)} \right] \\
& \quad \text{.......................... 201a}
\end{align*}
\]

Thus from 201a the value of \( y_{op} \) at a specific value of \( B_{o} \) can be found;
this value \( (y_{o}) \) can be used in equation 194 to determine the boundary value
of \( \frac{F_{pm}}{F_{o}} \), which is then compared to the actual value of \( \frac{F_{pm}}{F_{o}} \) obtained by
using a specific value corresponding to 'F_{pm}' found for a value of \( \Delta B \) in
201a, which is the numerator of the required ratio; the denominator
being the initial value.
A simpler method is to realise that at the boundary condition

\[ \text{total} \Delta y' = VnN \]

i.e. the change in output is the noise times the resolution required, and 'total \( \Delta y' \) can be determined from equation 197 if the values of 
G.K.R.t^2_c.f_{c, e}.c.A. are known. From this, values of initial separation ratio (minimum) and corresponding values of the normalised deflection ratio can be determined. This will be performed for the proposed system of 14.2.0. in normalised terms, and can be considered the second form of GASSP. i.e. GASSP 2, from which also the value of 'real \( d_{opt} \)' can be found (as this graph is a finite relationship rather than a proportional relationship as discussed in the previous section, cf. (FIGS. 98 and 99).
14.2.0 PROPOSED 'THEORETICAL SYSTEM' - SPECIFICATION

There now follows a theoretically biased specification for a fibre optic intravascular pressure transducer. 'Theoretically' here is meant to describe a specification which is dependent on values being assigned to certain parameters for reasons discussed earlier and in the previous section, i.e. commercial availability, size, performance, etc. but is based on a theoretically derived 'static calibration curve' and 'resolution curve'.

The primary objective then, is to assign optimum values to as many of the parameters in the system output expression (192) as is possible, and also to define a value for the system noise 'Vn' for use in equation 194; then by using values permissible for $Y_{\text{max}}$ the diaphragm central deflection (P being 300 mm Hg), a GASSP (1 or 2) may then be plotted. The majority of assigned values have already been mentioned within the preceding chapters, but to clarify any misapprehensions they are briefly discussed as members within each of the system elements in which they appear. Each element/s is considered individually and its effect on the simplification of the system output expression shown, together with any system conditions.

Optical Fibre/Diaphragm System

The optical fibre system is to be of the micro-format, with a discrete central emitter of 20 $\mu$ radius ($r = r_f$) and an annular receiver of 60 $\mu$ outer radius ($R = 60$), (11.2.0). There is a further stipulation due to the flat diaphragm assumption that the minimum initial separation of the diaphragm from the optical fibre face is

$$d_{o(\text{min})} = Y_{\text{max}} + \frac{r + R}{2 \tan \theta}$$  \hspace{1cm} 115

\[ d_{o(\text{min})} = Y_{\text{max}} + 23.094 \mu \]  \hspace{1cm} 202

Now, from the previous section which discusses whether an optimum value for the initial separation exists:-- for an emitter radius of '20 $\mu$' the following deflections 'Y$_{\text{max}}$' result in the below specified 'real d$_{\text{opt}}$' values. These have been found using the normalised conditions
The reflection coefficient for the diaphragm is taken as .95, as the wavelength $\lambda$ is known (section 14.1.1.). At this time it will be possible to assume a maximum reflective condition (i.e. as a gold surfaced diaphragm is assumed which may well be vacuum deposited).

It is also possible to specify the type of optical fibre which satisfies these parameters; 'Schott type W fibre'. This enables a further parameter to be specified, namely the transmission of 1 metre of the fibre $t^2_c$, which is 25% for a radiant energy of .8 - .9 $\mu$ incident wavelength. This can be seen from Schott Specification sheet 7008/IE where it can also be seen that the polar distribution within the N.A. of .866 is uniform up to $55^\circ$. ½ angle.

The parameters thus far assigned in the specification are:

$$r_f = r = 20 \text{ microns} \quad A = 3$$

$$R = 60$$

with certain specified optical initial separation distances (page 401)

$$N.A. = .866$$

$$C = .96$$

$$t^2_c = .25$$
Now for convenience it is simpler to modify the system output expression such that the reflection factor component therein is in normalised form prior to substitution of the values for the above parameters. Then using equation 82 the system output expression can be written

\[ y_{o/p} = G.K.R.E \cdot C_c \cdot C_f \cdot C_d \cdot e_i \cdot c \cdot \frac{c}{2} \left[ A^2 - 1 + 4 \left( \frac{d - y_{\text{max}}}{r} \right)^2 + \left( \frac{d - y_{\text{max}}}{r} \right)^4 \right] \]

\[ - \sqrt{\left( A^2 - 1 \right)^2 + 16 \left( \frac{d - y_{\text{max}}}{r} \right)^4 + 8 \left( A^2 + 1 \right) \left( \frac{d - y_{\text{max}}}{r} \right)^2} \]

\[ \left( \frac{64}{20} + 16 \left( \frac{d - y_{\text{max}}}{20} \right)^4 + 80 \left( \frac{d - y_{\text{max}}}{20} \right)^2 \right) \]

on substitution of \( t^2_c, C, A \) & \( r \)

\[ y_{o/p} = G.K.R.E \cdot C_c \cdot C_f \cdot C_d \cdot e_i \cdot 0.25 \cdot 0.48 \left[ 8 + 4 \left( \frac{d - y_{\text{max}}}{20} \right)^2 + \left( \frac{d - y_{\text{max}}}{20} \right)^4 \right] \]

\[ - \sqrt{64 + 16 \left( \frac{d - y_{\text{max}}}{20} \right)^4 + 80 \left( \frac{d - y_{\text{max}}}{20} \right)^2} \]

where the term within the square brackets (and \( 0.48 \)) is simply the value of reflection factor for a specific applied pressure \( 'P' \) \((y_{\text{max}})\) the central deflection resulting).

**Light Source**

The light source to be specified is a commercially available semi-conductor source (L.E.D.) and radiates 10 mW radiant flux at .925 \( \mu \) (TABLE 8). It is manufactured by PLESSEY LTD. and is termed a GAL 2 (13.4.0.)

\[ e_i = 10 \text{ mW} \]

and may be substituted into equation 197 after discussion of the amplifier and detection parameter specification.
Amplifier & Detector

A detector/amplifier system operating in a linear mode (current to voltage follower, the detector in "short circuit configuration") is preferred (12.3.0). Initially two discrete elements are specified. The detector, a Plessey Sc/100, "Photo-Voltaic Cell - Silicon' Planar Diffused and the amplifier, an Analogue Devices 184L operational amplifier. Technical Data sheet P5 1319 describes the detector and '184J/K/L' the a amplifier. From these specifications and earlier discussion in the thesis (references given) the following parameters and boundary conditions can be stated

\[ K = 0.128 \text{ amps/watt} \]

TOTAL SYSTEMS N.E.P. = \( 25.8 \times 10^{-11} \) Watts (13.40 from equation 185) noting

\[ R_f \approx 500 \text{ K} \]

\[ G = 1 \]

Also the value of \( V_n \) the output system noise can be defined as

\[ V_n = K R_f N.E.P. \]

(from equation 160)

Noting that \( V_n \) is not a variable with changes in reflection factor.

A stipulation on the system response is due to the amplifier input bias current (12.2.2.) described by 176; the value of this input current is

\[ I_B = 25nA \]

That is that the input current from the detector to the amplifier must be greater than the value \( I_B \). This condition has been shown to be achieved for practical systems with an L.E.D. producing 10mW radiant flux.

Two further parameters defined earlier are included here; the coupling factors \( C_f \) & \( C_d \) are dependent on geometry and optical conditions (13.3.2). They are

\[ C_f = .4 \]

\[ C_d = .2 \]

(Note: see later comments) Page 459
The definitive values can only be determined by final specification of active areas of efferent and afferent fibres (see Chapter 15).

Thus the system output expression upon substitution of all assigned parameters is

\[ Y_{o/p} = 0.128, \ 5 \times 10^5, 0.4, 0.2, \ 10 \times 10^{-3}, 0.25, 0.48 \times \]

\[ 8 + 4 \sqrt{\left(\frac{d - y_{\text{max}}}{20}\right)^2 + \left(\frac{d - y_{\text{max}}}{20}\right)^4} - \sqrt{64 + 16 \left(\frac{d - y_{\text{max}}}{20}\right)^4} \]

\[ + 80 \left(\frac{d - y_{\text{max}}}{20}\right)^2 \]

\[ = 12.79 \ \& \ .48 \]

N.B. by increasing \( R_f \) an increase in output is achievable; however, this simply results in a directly proportional increase in \( V_n \) (from equation 205) and no gain in this manoeuvre theoretically results.

Now from the above conditions stipulated from equation 205

\[ V_n = 0.128, \ 5 \times 10^5, 25.8 \times 10^{-11} \]

\[ = 1.65 \times 10^{-5} \]

At this time it is worth noting that the apparent signal to noise ratio is very good, i.e. the 'signal' value being from equation 207 and assuming an RF of say .5

signal = 12.79 x .5 volts

= 6.39 volts

\[ \text{signal} \over \text{noise (dB)} = 20 \log \frac{6.39}{1.65 \times 10^{-5}} \]

= 111 dB
14.2.1 DETERMINATION OF OPTIMAL INITIAL SEPARATION DISTANCE AND DIAPHRAGM DEFLECTION ENABLING RESOLUTION ACHIEVEMENT

The objective here is to form the GASSP (1 & 2) for the system specified in order first to determine the optimal values for the initial separation and maximum deflection of the diaphragm under maximum differential pressure (P = 300 mm Hg) such that the system achieves the resolution required and secondly, to determine if there is a maximum value of range for \( d_0 \). From section 14.1.2 it appears that for any value of deflection there is an optimal separation, with regards to the best resolution possible. This will be verified and consequently using the GASSP's shown it will be determined whether the resolution achieved is satisfactory and what total deflection is required.

The formation of each of the graphs comprising the GASSP's are discussed.

**Static Calibration Curve**

This may be formed directly from the system output expression equation 206, noting that as this curve is in effect an initial separation calibration curve; \( y_{\text{max}} \) is zero. Thus

\[
y_{op} = \left[ 6.14 \left( 8 + 4 \sqrt{\left( \frac{d_0}{20} \right)^2 + \left( \frac{d_0}{20} \right)^4} - \sqrt{64 + 16 \left( \frac{d_0}{20} \right)^4 + 80 \left( \frac{d_0}{20} \right)^2 \text{volts}} \right) \right]
\]

\[
d_0 \text{ (microns)} \quad \text{.................} \quad 208
\]

with the boundary condition for the absolute minimum value of \( d_0 \) the initial separation of

\[
d_0 \text{ (abs. min)} = \frac{r + R}{2 \tan 40^\circ} \quad \text{..................}(115)
\]

\[
d_0 \text{ (abs. min)} = 23.08 \text{ microns (from 201) as } Y_{\text{max}} = 0
\]

Thus the static calibration curve commences at a separation distance of 23.08 microns, the vertical ordinate representing the voltage output from the system at any initial separation greater than this.
It can also be seen that, as the working part of the static calibration curve is of 'negative slope' then the conditions of \[ \frac{F_{pm}}{F_o} \] (G.U.) prevail.

**Deflection and \( \frac{F_{pm}}{F_o} \) Curve**

The formation of this graphical representation is achieved by first extracting the 'reflection factor' expression from the system output expression (204 & 207) before simplification when

\[
RF = 0.48 \left[ 8 + 4 \sqrt{\left(\frac{d - Y_{\text{max}}}{20}\right)^2 + \left(\frac{d - Y_{\text{max}}}{20}\right)^4} - \sqrt{64 + 16 \left(\frac{d - Y_{\text{max}}}{20}\right)^4 + 80 \left(\frac{d - Y_{\text{max}}}{20}\right)^2} \right]
\]

Thus with an initial separation \( d_o \) and a deflection \( Y_{\text{max}} \) for the diaphragm, for maximal pressure applied to a diaphragm the value of \( F_{pm} \) the reflection factor resulting at maximum pressure conditions can be computed. Similarly by allowing \( Y_{\text{max}} \) to be zero at this separation \( d_o \) a value of \( F_o \) can be computed:

Generally then

\[
\frac{F_{pm}}{F_o} (\text{at } d_o) = 8 + 4 \sqrt{\left(\frac{d - Y_{\text{max}}}{20}\right)^2 + \left(\frac{d - Y_{\text{max}}}{20}\right)^4} - \sqrt{64 + 16 \left(\frac{d - Y_{\text{max}}}{20}\right)^4 + 80 \left(\frac{d - Y_{\text{max}}}{20}\right)^2}
\]

Thus by giving \( Y_{\text{max}} \) finite values (5, 10, 15 & 20 \( \mu \)) the ratio \[ \frac{F_{pm}}{F_o} \] can be computed for 'incremental d' values and the curve plotted as \[ \frac{F_{pm}}{F_o} \] against \( d_o \).

The boundary condition for \( d_o \) and \( Y_{\text{max}} \) have been discussed in the previous section by equation 202.
\[ d_0(\text{min}) = Y_{\text{max}} + 23.08 \mu \] .............................. 202

(transition point condition)

and this condition will set the lower limit on the incremented \( d_0 \) computing procedure for calculation of \( \frac{F_{\text{pm}}}{F_0} \).

\[ \text{Resolution Curve} \]

The resolution curve is obtained from inequality 194

\[ \frac{F_{\text{pm}}}{F_0} \gg 1 + \frac{V_nN}{Y_0} \] .............................. 194

by considering this boundary condition, noting that \( V_n \) is now considered to be the actual output change for a linear mode system.

\( V_n \) is given a value of the noise level of the system and does not vary with changing values of reflection factor.

\[ \frac{F_{\text{pm}}}{F_0} = 1 + \frac{V_nN}{Y_0} \] .............................. 211

where \( V_n \) is the system noise level.

\( Y_0 \) is the set output value.

and \( N \) is the resolution specified.

Now \( V_n \) is \( 1.65 \times 10^{-5} \) volts (page 404) and the relationship between \( \frac{F_{\text{pm}}}{F_0} \) and \( Y_0 \) can be computed for various values of \( N \) the resolution, which are 300, 3000 and 30,000. The range over which \( Y_0 \), the set output, is incremented in equation 210 can be ascertained by considering its practical maximum value as deduced from the modified value of the system output expression 207 shown

\[ Y_{\text{op}} = 12.79 \times \text{R.F. volts} \] .............................. 211

Now the maximum value for the reflection factor possible for a system \( A = 3 \) is from FIG. 44 approximately .48. Thus the maximum value of \( Y_{\text{op}} \) to be used in equation 210 is

\[ Y_{\text{op}} (\text{max}) = 12.79 \times .48 \]
\[ = 6.14 \text{ volts} \]

consequently the set o/p range over which the value of the boundary condition for \( \frac{F_{\text{pm}}}{F_0} \) is computed is 0 - 7 volts.
Again it can be seen that the actual signal to noise ratio will be very high.

For the proposed system two GASSP.1's are shown, FIG. 100 and 101, the latter having an expanded scale, and FIG. 102 shows the GASSP.2 for the system.

Considering first, FIG. 102, GASSP.2, from which the following can be deduced

1) maximum separation ratio possible for a specific normalised deflector ratio fulfilling the resolution requirement of output charge greater than $V_{n}^N$.

Whereby for condition 29 shown with $V_{n}^N$ as .3 gives, i.e. .01 mm Hg.

<table>
<thead>
<tr>
<th>Cond.</th>
<th>$\frac{B_{o}}{\Delta B}$</th>
<th>$\Delta B$</th>
<th>and if $r = 20 \mu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>29</td>
<td>2.9</td>
<td>0.25</td>
<td>58 $\mu$</td>
</tr>
<tr>
<td>29</td>
<td>4.2</td>
<td>0.5</td>
<td>84 $\mu$</td>
</tr>
<tr>
<td>29</td>
<td>5.7</td>
<td>1</td>
<td>114 $\mu$</td>
</tr>
</tbody>
</table>

and the optimum initial separation ratio for each of the $\Delta B$ values can be seen (apart from graphical errors) to be the same as discussed on pages 394 & 401 and shown graphically on FIG. 100 in finite terms.

FIG. 101 the GASSP.1 also shows the conditions for $d_{o \ max}$ and can be seen to agree with these obtained from the GASSP.2 when accounting for the normalised description given in this figure.

It should be noted here, that the GASSP.1 as shown, is a specific aid for a specific system with $r \ & R$ assigned whilst the GASSP.2 (as shown) is a general aid in normalised terms. The relative merits of these aids will be discussed later.
"E" - \( N = 3 \times 10^4 \)  \( V_n = 10^{-5} V \), \( NV_n = 0.3 V \)

"F" - \( N = 3 \times 10^3 \)  \( V_n = 10^{-5} V \), \( NV_n = 0.03 V \)

"G" - \( N = 3 \times 10^2 \)  \( V_n = 10^{-5} V \), \( NV_n = 0.003 V \)

**FIG. 100. GASSP.1**

THEORETICAL PROPOSED SYSTEM-SHEWING 'real d_opt' CONDITIONS

\( A = 3, \ r = 20 \) microns

\( A \) - \( Y_{max} = 5 \) microns

\( B \) - \( Y_{max} = 10 \) microns

\( C \) - \( Y_{max} = 15 \) microns

\( D \) - \( Y_{max} = 20 \) microns
FIG. 102  G.A.S.S.P 2 - FOR "A"(r/R)=3.
It is possible with the system as specified to achieve a resolution of \( 300 = N \) i.e. 1 mm Hg for any deflection \( Y_{\text{max}} \) even at very large separation \( 'd_o' \). This can be seen either from FIG. 101 or FIG. 102 when a \( V_n = .003V \) gives a maximum value for initial separation greater than the scale shown. Now let us choose quite arbitrarily a total deflection diaphragm of 10 \( \mu \), which can be approximately achieved with a diaphragm of Be/Cu of 30 \( \mu \) thickness and radius 1.5 mm (Chapter 10, TABLE 7). Let this diaphragm be considered to be located first at a critical separation distance \( '\text{real } d_{\text{opt}}' \) and then at the maximum for achieving the resolution of .01 mm \( (V_n = .3) \)

\[
'\text{real } d_{\text{opt}}' = 33.094 \mu
\]

\[
'd_{\text{max}}' = 84 \mu \quad \text{(from condition 29C. FIG. 102)}
\]

For each of these initial separations FIG. 103 represents the variation of system output against applied pressure and has been computed using equation 207 and is the theoretical static pressure calibration curve for the proposed fibre optic intravascular pressure transducer. From which, for case 'A' shown i.e. the optimum conditions, it can be seen that the total output change does indeed agree with that shown by FIG. 102 with respect to curve 'C' i.e. \( \Delta B = .5 \) when at the optimum initial separation ratio shown.

FIG. 104 shows an alternative diaphragm system which incorporates a 35 \( \mu \) thick glass diaphragm of 1.5 mm radius with two positions for initial separations; again the optimum value and the maximum value, noting that as a diaphragm such as described, has a deflection of 14.25 \( \mu \) (Chapter 10 TABLE 7) and consequently as \( r = 20 \), \( B \) is \( \Delta .75 \) and \( '\text{real } d_{\text{opt}}' \) and \( 'd_{\text{max}}' \) becomes 38 and 98 \( \mu \) respectively.

The suitability of a macro system will be briefly re-assessed due to the apparent ease of system specification accomplished. It should be noted that the sole factors for enabling such a diverse nature for the value \( d_o \) and \( Y_{\text{max}} \) is the lower noise figure used \( 'V_n' \) of \( \approx 10^{-5} \) volts; \( 'V_n' \) is low and the high comparative value of the set output value i.e. giving a very good signal to noise ratio.
FIG. 103. THEORETICAL STATIC PRESSURE CALIBRATION CURVE FOR PROPOSED SYSTEM. -Be/Cu DIAPHRAGM.

Be/Cu Diaphragm  Thickness = 30 microns  Radius = 1.5 mm.

Applied differential pressure on diaphragm mm.Hg.

SYSTEM OUTPUT VOLTS

"A"  "B"

5.7  1.8
5.4  1.6
5.1  1.4
4.8  1.2
4.2  0.8
3.9  0.6
3.6  0.4
3.3  0.2

50  100  150  200  250
FIG. 104  THEORETICAL STATIC PRESSURE CALIBRATION CURVE FOR PROPOSED SYSTEM—GLASS DIAPHRAGM.
However, now let us reconsider; let one factor remain constant namely the active area of the efferent fibres.

Now if \( A_{\text{out}} \) is the area of the distal tip fibre face and \( A_{\text{in}} \) is the area of the input efferent fibres, then the aim will be to preserve \( A_{\text{in}} \) as for the micro-system proposed before, and ascertain what the \( A' \left( \frac{r}{R} \right) \) ratio then becomes.

Now let \( N_i \) be no. of efferent fibres

& \( N_o \) be no. of afferent fibres

then for a micro-system having more than 20 fibres on a diameter the ratio of \( N_i/N_o \) for the simplest micro system is (see Appendix 2)

\[
\frac{N_i}{N_o} = \frac{1}{2}
\]
The figure at first may seem large considering that for the simplest micro-system one efferent is surrounded by six afferent. However, in Appendix 2 it can be seen that due to the sharing of afferent fibres by different sender/receiving systems this figure is valid.

Thus \( A_{\text{in}} = \frac{1}{3} A_{\text{out}} \)

Now for a macro-system of outer receiving annular radii similar to the total outside radii of the '\( \mu \)' system, it follows that if \( A_{\text{in}} \) remains constant also;

\( r_{\text{in}} \) radius of efferent group = \( \sqrt{3} \) outer radius

\( R = \frac{R_l}{\sqrt{3}} \)

\( A = \sqrt{3} \)

Specifically the radius \( R^* \), effectively the outside radius of the distal tip, is 1.5 mm the value for the \( \mu \) system proposed earlier. Consequently, a macro system will have a central emitter of \( \frac{1.5}{\sqrt{3}} \) mm radius and an outer radius of 1.5 mm

i.e. \( r = 866 \mu \)

\( R = 1500 \mu \)

\( A = \sqrt{3} \)
The main constraint has been to ensure that the central emitting fibres are of the same active area as for the earlier proposed micro-system, as this is the limiting area for the coupling factor $C_f$. The exact diameter of the individual fibres comprising the single sender/receiver system may be assumed to be of the same nature as for that proposed.

To determine acceptable values for the parameters of initial separation $d_o$ and diaphragm deflection $Y_{\text{max}}$ for the system to function to a desired resolution (N) FIGS. 105 and 106 are shown. These are a GASSP.2 for a system with $A = \sqrt{3}$ and a GASSP.1 (shown in finite terms) respectively. It should be noted that an identical detector/amplifier system as the initial proposed system has been assumed.

Considering first FIG. 105; the normalised deflections depicted are as for the previous GASSP.2 with $A = 3, 1, .75, .5, .25$ plus a value for $\Delta B = .025$. Now as the value of $r$ is $866\mu$ and a maximum possible practical diaphragm deflection is $20\mu$ (TABLE 7) a maximum value for deflection in normalised terms is then; $\Delta B$ is $= .025$. Consequently, it is the curve designated 'E' on FIG. 105 which must be considered to determine values for 'real $d_{\text{opt}}$' and $d_{\text{o max}}$. If, as before, a value for the noise-resolution production of .3 is a set requirement, then it can be seen that the system will not fulfil this requirement. 'd_{\text{opt}}' does indeed have a value but does not fulfil the resolution requirement. Consequently 'd_{\text{o max}}' is non-existent for this resolution requirement. But it can be seen that at 'real $d_{\text{opt}}$' (con. 30) at approximately $(B = .9) .650 \mu$, a value for $V \text{N'}$ of about .06 is achieved. Thus as $V \text{N'}$ is considered to be $1.6 \times 10^{-5}$ volts, a resolution of approximately .1 mm Hg is achievable. If a resolution of .1 mm Hg is required, a value for 'd_{\text{o max}}' does exist and is about $1470 \mu (1.7 = B)$ condition 31. Thus the GASSP.2 shown does show system fulfilment which could be acceptable.
The following FIG. 106 shows the GASSP.1 for the system. However, although it is difficult to determine as precise values for 'real d opt' and 'd o max' a questimate could be made. This figure does show, however, at what set output level the system will operate at, and this is of the order of 3.2 (32) volts at the optimum initial separation of 'real d opt' = 650 µ (from FIG. 105) and 1.4 volts (33) at the maximum separation possible 'd o max' = 1470 (from FIG. 105) for a resolution/noise product of .03 to be achieved.
FIG. 105 G.A.S.S.P 2 - FOR "A" (r/R) = √3
FIG. 106. GASSP.1 - FOR MACRO-EQUIVALENT SYSTEM (\( \lambda = \sqrt{3} \))

\( (r=866 \text{ microns}) \)

SHEWING "real \( d_{opt} \) & "\( d_{opt} \)"
FOR \( V_n = .03 \).

\( \gamma_{max} \) = 5 to 20 microns
Two methods for graphical assessment of specification of system parameters for a fibre optic pressure transducer have been discussed (GASSP.1 and GASSP.2). They have been utilised for the determination of the suitability of the proposed theoretical transducing system from which the following can be determined:

1) the optimum value of initial separation 'real $d_{opt}$' (from either GASSP 1 or 2) for a specific value of total diaphragm deflection.

2) at the optimum separation, whether the required resolution is achievable (GASSP 1 & 2) and what is the actual resolution achieved (GASSP 2)

3) the value (if existing) of a maximum value for initial separation which, for a specific total deflection of the diaphragm, forms the boundary separation condition for satisfaction of the resolution requirement (GASSP 1 & 2)

4) the set output level for the chosen initial separation to be utilised (GASSP 1)

5) the actual change in system output caused by the total deflection occurring at the chosen initial separation (GASSP 1 & GASSP 2)

The fundamental differences between using a GASSP 1 or GASSP 2 for system specification are

1) GASSP 2 allows for the assessment of whether any value of resolution can be achieved by the system; however, using this aid alone no indication of set output is possible.

2) GASSP 1 only allows for the resolution curves plotted, the assessment of which (if any) are achievable. However both set output level and output difference can be determined from this aid.

The system actually proposed has been shown to satisfy the requirements for an intravascular pressure transducer, and is summarised
\[ A = 3 \text{ (micro)} \]
\[ r_f = r = 20 \mu \]
\[ NA = 60^\circ \]

SCHOTT TYPE 'W' fibre

GAL 2 GALLIUM ARSENIDE SOURCE

ANALOQUE DEVICES 184 OP. AMP

using a Be/Cu diaphragm, 30 \( \mu \) thick of 1.5 mm diameter (giving a natural frequency of 27 KHz (FIG. 65) and located at between 33 and 84 microns.

The overall active diameter of the distal tip fibre face is 1.5 mm. For this system FIG. 103 depicts the theoretical static pressure calibration curve.

Because of the proposed systems inherent value of signal to noise ratio which appears to be excellent (117dB) for the system mentioned, a much lower value for \( \frac{F_p}{F_0} \) can be tolerated. Consequently a macro.

equivalent system was discussed such that the overall active diameter of the distal fibre face and the efferent fibres was kept constant. This resulted in a system with \( A = \sqrt{3} \), \( 'r' = 866 \), with the same system otherwise, a GASSP 1 & 2 curves are shown, FIGS. 105 & 106, from which it can be seen that this system would also satisfy the resolution requirements \( \frac{V_n}{N} = .03 \).

However, because practically the noise and signal output will always be respectively higher and lower than the theoretical, the initial proposed system is preferred.
SECTION FOUR

FIBREOPTIC PRESSURE TRANSDUCER

PRACTICAL SYSTEM.
CHAPTER 15

THE PRACTICAL SYSTEM

THE PHILOSOPHY OF APPROACH
The previous chapter concluded with a proposed specification for a fibre optic pressure transducer; this specification utilises and relies on:

1) The theoretically derived static response characteristics of the optical fibre system
2) Theoretical values for system noise
3) Theoretical diaphragm deflection characteristics

Now, because of assumptions made in 1) above, namely

a) light is emitted in a Lambertian manner from the efferent fibres
b) the diaphragm deflects with a 'flat' profile

Therefore if the theoretically derived expression for the static response curve is to be valid, a constraint on the optical fibre system must be imposed, namely that the fibres N.A. must not be greater than .866, and that a minimum boundary condition exists for the initial separation, \(d_f\) the point where the transition occurs. In other words, the theoretical description of the response curve is not valid up to the initial separation '\(d_f\)' because the N.A. is less than unity (.866).

However, a response will result prior to this separation '\(d_f\)', and in practice can be determined for a practical system. Also although for the 'flat' diaphragm assumption a maximum value for N.A. is given, this does not preclude in practice, the utilisation of fibres of greater N.A.'s, as the total static response curve could be assessed practically, (c.f. page 277) using the set-up shown on FIG. 24.

Furthermore, the deflections of 'thin', 'small', diaphragms as discussed in Chapter 10 are best determined practically as is the system noise level also. As all the theoretically derived relationships may be ascertained for a practical system by experimentation, the verification of the theoretical analysis can be performed. The logical method for producing a functioning system is to construct the various system elements utilising the theoretical proposals of Chapter 14 and then by experiment ascertain first:
1) The actual static calibration curve for the fibre optic probe
2) The actual system noise

which may then be compared to that theoretically expected. Then, with knowledge of the practically determined expression for the fibre optic probe's static calibration curve and system noise, the same logic as used in Chapter 14 (GASSP 1 and 2) for the specification of the diaphragm's central deflection resulting at maximal applied differential pressure ($Y_{\text{max}}$), and the initial separations ($d_o$) of the diaphragm which satisfies the system's resolution requirements can be made.....

Secondly, rather than fabricating a 'finished' diaphragm located at the specified initial separation '$d_o$' which would be a final commitment to the acceptance of the theoretical description for diaphragm deflection characteristics, these parameters have been computed using the value of '$Y_{\text{max}}$' extracted from the GASSP 1. A simulated static pressure calibration determination is indicated in order to verify the theoretical description of diaphragm deflection characteristics. This involves producing a diaphragm as specified, and devising a jig to hold it such that

1) a differential pressure may be applied to it
2) the jig may be precisely moved to and from the distal fibre face of the fibre optic probe

The jig is initially positioned so that the diaphragm it carries lies at the initial separation '$d_o$' as has been specified from the GASSP 1 - practical; pressure applied to the diaphragm then enables a static pressure calibration to be performed for the specific initial separation '$d_o$'.

Consequently it is possible, using the practically obtained static pressure calibration curve to verify the diaphragm deflection relationship by either considering the GASSP 1 - practical or a computed static pressure calibration curve derived from it.
The practically simulated static pressure calibration curve thus fulfils two functions:

1) it allows for a final assessment of the initial separation of the diaphragm.

2) it allows assessment of the validity of the diaphragm deflections as predicted in theory.

There is an additional problem though, which must be assessed and which may be caused through the utilisation of a jig to position the probe and the diaphragm. This is the drift due to temperature on the system output caused by the jig. This will be discussed later.

It is useful at this point to re-state that the 'fibre-optic probe' comprises the 'optical fibre system', ensheathing catheters and ferrules. The diaphragm unit, amplifier/detector and the fibre-optic probe form the fibre-optic pressure transducer.
CHAPTER 16

FIBREOPTIC PROBE AND DETECTOR/AMPLIFIER

AND LIGHT SOURCE DETAILS
16.0.0 INTRODUCTION

This chapter discusses the fabrication details of the fibre optic probe, and the circuit design and details of the amplifier/detector unit, the basic specification of which, is that as proposed in the conclusion of Chapter 14.

16.1.0 FABRICATION DETAILS - FIBRE OPTIC PROBE

The design details of the finished probe are shown in FIG. 107, where its specification is as listed below:

- **Probe catheter material**: P.V.C. 2.5mm od by 1.8mm id
- **Probe useable length**: 600 mm
- **Probe calibre**: 2.5 mm
- **Distal tip ferrule**: 14mm long by 2.99mm od
- **Efferent ferrule**: 20mm long by 4.5mm od
- **Afferent ferrule**: 20mm long by 5.0mm od
- **Optical fibres**: 'Schott type W' - 0.04mm od.

There are three phases in the construction of the probe:
- **Phase 1** - the construction of the 'useable probe section' (FIG. 107) i.e. from the distal tip of the probe to where it is encapsulated within the bifurcation;
- **Phase 2** - the separation (choosing) of efferent and afferent fibres from those comprising the useable probe section. In other words performing the fibre orientation in the distal tip group by selection at the bifurcation. Then encapsulating this bifurcation. Finally:
- **Phase 3** - the construction of the proximal efferent and afferent fibre groups. These phases are discussed below.

**Phase 1.**

Loose optical fibres, those not ensheathed within a close fitting catheter were obtained from 'Schott'. They were two metres in length and of 40 microns in diameter. Protection of the fibres whilst in transit to the author was afforded by packaging within a cardboard box. In order to
eliminate 'kinks' or tangles developing the fibres themselves were kept inside a plastic bag 2 metres long and 10 cms. wide and wrapped around the periphery of a 20 cm. diameter former.

A 'Rank Precision Industries' optical fibre ensheathing catheter (P.V.C.) was chosen to be the fibre optic probe catheter (any such catheter fulfilling the requirements as discussed in Chapter 4.1.0. would suffice). In fact, this catheter originally contained 'Rank' fibres which were removed. The catheter length cut for use as the useable probe section was 590 mm. The lumen of the probe, 1.8 mm, allowed approximately 1500 fibres to be threaded through. A 5 cms. length of fibres were protruding from the distal end. Using a scalpel blade, the proximal end of the fibres were cut such that about a 20 cms length protruded from the proximal end of the catheter, (the junction end of the useable probe section catheter).

The brass distal tip ferrule has a bore of 1.65 mm and is counter bored to take a 5 mm length of the catheter. Prior to threading the 5 mm length of protruding fibres into their ferrule a mix of epoxy resin was prepared (Araldite MY 750 & HY 956 hardener in 4:1 proportions) and the ends of the fibres carefully dipped into this medium viscosity resin. When, by capillary action and gentle rubbing with gloved fingers, the resin was dispersed evenly over the fibre ends. Subsequent to the removal of surplus resin they were then threaded into the ferrule such that the 5 mm of the catheter passed into the ferrule counterbore.

A period of twenty-four hours was allowed for resin cure at room temperature prior to the optical grinding and polishing operation which must be performed on the face of this ferrule. This optical grinding and polishing operation is critical. The reasons for this are discussed:
**FIG. 107** DETAILS OF FIBREOPTIC PROBE

- **Scrap View "A" (Afferent Ferrule Face)**
  - Dia. 1
  - 4:5 Dia.
  - 2.99 Dia.

- **Scrap View "B" (Afferent Ferrule Face)**
  - 1.3 Dia.

- **SECTION X-X**
  - 10 Dia.
  - 20 Dia.

- **"Bifurcation of Efferent and Afferent Optical Fibres"**
- **"Epoxy Casting"**

- P.V.C. Catheter 2.5 Dia.

All dimensions in mm.
All end ferrules Epoxy Casting.
✓ - denotes optical polish.
1) the better the polish the lower the light losses.

2) In order to produce the system described theoretically earlier, it is of prime importance that the distal ferrule face resulting from the grinding operation, is perpendicular to the ferrule axis as:

a) the location of the diaphragm necessitates this condition and

b) otherwise oblique angled fibres result in the distal tip group for which no theoretical account has been made. (The emergent cone of light for oblique ended fibres is not symmetric about the fibre's axis).

The grinding operation was performed on an 'Auto-flow Engineering' spherical generator, Model 124 Mk. III. This machine tool has provision for clamping of the workpiece on to a table which rotates slowly, above which a diamond impregnated grinding wheel revolves at speed (1800 rpm).

FIG. 108 shows a schematic of the spherical generator, the relative axis of rotation of the workpiece and cutter can be adjusted from in line (vertical cutter axis) to 45 degrees. Provision is also made for providing an initial offset of the workpiece from a central horizontal position. Consequently, by using a cupped grinding wheel and with an angular offset in the axis of rotation of the cutter which also has an offset in central position, a spherical surface may be generated on the workpiece. To generate the flat surface required on the distal fibre face the axis of the cutter has to be set 'in line'. Then, using a 180 grit cupped wheel with the ferrule clamped into the workpiece chuck, which is offset to give complete coverage of the fibre face by the cutter, a flat fibre surface which was perpendicular to the ferrule axis was generated, (the hole in the workpiece spindle taking the 'hanging catheter' FIG. 108).

The subsequent polishing operation does not require jigging to ensure perpendicularity because during optical polishing, effectively no material is removed. The polishing operation was performed using a basically more
Diamond impregnated Cupped wheel.

Work-piece chuck taking "distal tip ferrule" "Auto-feed"

"Manual feed"

Hollow work-piece spindle taking "Useable Probe section" catheter

FIG. 108 GRINDING OPERATION SCHEMATIC - DISTAL TIP FERRULE
simple mechanical system comprising:

- a rotating felt pad and a drip feed water supply.

Precision grade ceri-rouge (Auto-flow 90) was impregnated into the felt pad and polishing achieved by holding the ferrule between thumb and forefinger on to the damp felt pad. This phase thus resulted in the fabrication of the useable probe section of the fibre optic pressure transducer.

Phase II

The epoxy junction which encapsulates the efferent fibres, afferent fibres and mixed efferent and afferent fibres of the useable probe section is termed the 'bifurcation'. The initial operation in producing the bifurcation is to separate the fibres in the useable probe section into efferent and afferent groups (or branches). This was performed by viewing the distal fibre face through a 'Zeis dissection microscope'. Then by using two light sources, one red and the other green, to illuminate what are initially chosen arbitrarily as the ends of the efferent and afferent branches, sorting was achieved manually to select which fibres should be 'efferent' (appear as red dots through the microscope) and which should become afferent (appear green through the microscope). This was a long and tedious task, especially as the aim was to produce a Micro system with 'A = 3', in other words, to produce a fibre format of one red dot surrounded by six green dots when viewed through the microscope. An alternative method is discussed in the conclusions to this thesis and approaches the problem of producing a defined spatial arrangement of fibres in the distal tip group, by constructing the distal tip group by using a method which then allows for a mathematical definition of which fibres (proximal) should be efferent and which afferent.
On completion of this sorting operation PVC catheters similar to those used for the useable probe section, were immediately used to ensheath the efferent and afferent fibre branches. A silicon rubber mould was used to cast the bifurcation and was in two parts. The lower half was placed on the bench and the bifurcation area laid in position in the mould, allowing a few millimetres of each of the limbs of catheters of the probe to lay within the encapsulation area. The top of the mould was then fitted and the mould filled with epoxy casting resin (and black filler).

Phase III.

The two ferrules required for the efferent and afferent fibre branches are of different dimensions because:-

a) Of different inside diameters - as there are more afferent than efferent fibres

b) Of different outside diameters - to enable a distinction to be easily made for the discrimination of each type of fibre.

The actual sizes of the inside diameters through which the fibres were to pass was determined by trial and error, although in theory their values may be computed by assuming a certain value for the packing factor. These ferrules, like the distal tip ferrule, are counterbored to take a short length of the ensheathing catheters. Fixation of the fibres within these ferrules was achieved, as has been described for the distal tip ferrule, and the optical polishing performed as previously described. The grinding operation is not as critical as for the distal tip ferrule as perpendicularity of their end faces is not a prime requisite. Consequently the grinding operation was performed on the same equipment as for the subsequent polishing operation but using a series of diamond impregnated wheels prior to using the felt pad for polishing.

The following FIG. 109 shows two photo-micrographs taken of the
Efferent Fibres—Bright
Afferent Fibres—Dim

FIG. 109  Photo-micrograph of Distal Tip Face

Efferent Fibres Only
The distal tip face of the pressure transducer probe constructed (all black and white photographs were taken and processed by the author).

The upper picture shows both efferent and afferent fibres appearing as white dots, the lower showing only efferent fibres as white dots.

These photographs were obtained by illuminating the ends of the fibres in the efferent and afferent branches with separate light sources in a manner to be described below, then taking photographs using the Zeiss dissection microscope and associated camera attachment on FP 4 at $1/125$ sec shutter speed.

An explanation of and discussion of these photographs follows:

1) The outer boundary of these photographs was produced by using a 10 cm diameter mask, during enlargement and printing.

2) The inside diameter of the distal tip ferrule is actually 1.65 mm, and from the photograph can be seen to be the maximum diagonal diameter across the fibres; this measures 90 mm.

Thus, the overall magnification of the distal face shown is 54.5 times. The fibre diameter measured from the photograph is 2.1 mm, which represents, when accounting for the magnification factor, a fibre diameter of 38 microns. This compares favourably with the fibre's nominal diameter of 40 microns.

3) Two photographs are shown; the upper was intended to enable the discrimination between efferent and afferent fibres, as the light source supplying the efferent fibres was set to give a greater output than that supplying the afferent fibres.

Consequently, it was intended that those fibres appearing bright would be the efferent fibres and those appearing dim, the afferent fibres, whence the fibre orientation would become obvious. To some limited extent this is so, however, from comparison with the lower picture, which shows only the efferent fibres, as these alone were illuminated from their proximal ends, it can be seen that some efferent fibres appear dim and some afferent fibres appear bright. Such fibres may be identified as briefly
discussed below:-

a) dim efferent fibres; these can be spotted from the study of the lower picture and then related to the upper one where their true nature can be determined. One example shewn is that of fibre "a".

b) bright afferent fibres: these can be spotted by a process of comparison between upper and lower pictures to decide which of the bright spots in the upper picture do not appear in the lower one. One example shewn is that of fibre "b".

The causes of this variation in illumination of either the efferent or afferent fibres will be due to the following:-

a) The variation in the transmission of individual fibres due to factors as discussed in Chapter 3
b) The presence of 'oblique' ended fibres which may be caused due to the skewing of fibres in the distal tip ferrule
c) A variation in the quality of the end face polish.
d) The occurrence of cross-talk between fibres

A detailed examination of the relative significance of the above to the observations made could be performed, but would be of only academic interest. However, these photo-micrographs clearly show that the required arrangement of fibres (basic Micro format) was not achieved. To define accurately the arrangement of fibres attained can be seen to be an almost impossible task, as any definition would have to be couched using the 'R/r' concept. Furthermore, it is questionable that even if this were possible, any quantitative use could be made of the abstracted 'R/r' ratio. The reasons for which are dealt with later.

Because the 'dim-bright' technique for assessing the fibre arrangement, is not totally satisfactory, another photo-micrograph FIG.110 is shown overleaf. This shows the distal tip fibre face when the face of the efferent fibre branch is illuminated with red light and the afferent
FIG. 110 PHOTO-MICROGRAPH OF DISTAL TIP FACE (colour)
fibres with green light. Thus, all the red dots are efferent fibres and all the green dots are afferent fibres.

The unsuspected occurrence of 'bright afferent fibres' and 'dim efferent fibres' can also be spotted from this figure, and the same fibres shown as examples on the previous figure have been labelled on FIG. 110, i.e. fibres "b" and "a", respectively. Now, from this figure it may be more clearly seen that the basic Micro format was not achieved (cf. FIG. 26), although the mix is actually good. (A subjective comment).

For the purpose of assessing how well the mathematical description for the probe's practically obtained static calibration curve compares to that theoretically expected (Chapter 17), it is necessary to assess the fibre orientation of the probe as constructed because, as can be seen from FIG. 110 it is definitely not of the form described by 'R/r = 3', the 'A' ratio. Now a rigorous mathematically arrived at description for the geometrical fibre orientation would, as previously intimated, not prove useful alone. This is because the reflection factor expression into which one would immediately presume to use the 'A' ratio computed, actually assumes that each fibre within the group transmits equally. This has been illustrated not to be true.

In other words, although for each 'group' it may be possible to obtain an 'A' value and thus a reflection expression of that group, because the transmission of some of the fibres in that group may differ, a quantitative account of the effects of this variation must be made. And in fact, it infers a complete re-assessment of the theoretical derivation. Furthermore, if one assumes that the transmissions of all the fibres comprising one group are identical and it is only the transmissions of the groups comprising the total which differ, it is not possible to simply combine their respective reflection factor expressions to obtain an effective reflection factor expression for the whole system for the following reasons:-
1) The reflection factor expression is by definition, the ratio of the light detected to the light emitted, and as it can be seen that the light emitted from each group varies, then quantitative values for the light emitted from each group must be known; this is difficult to achieve.

Now mathematically the system output has been shown to be described by equation 129:

\[ y = G f (R_F \text{ etc } t_c^2 C_f C_d e_i ) \] .......................... 129

Thus, although one could obtain for groups 1, 2, 3, etc., their respective reflection factor expressions \( R_{F1}, R_{F2}, R_{F3}, \) etc., it is the determination of the value of \( t_c(1) \) etc. which presents difficulties.
The components used as amplifier, detector and light source are respectively: Analogue Devices 184L operational amplifier, Plessey Ltd. SC 100 Silicon Planar Diffused Photovoltaic Cell and a Plessey Ltd. Gallium Arsenide GAL 2. These are the components as proposed and discussed in Chapter 14.

The amplifier/detector circuit configuration is shown in FIG. 111. The detector is effectively short circuited, consequently, as discussed earlier, it operates in a linear mode, the amplifier functioning as a current to voltage follower with an effective gain which is in proportion to the value of the feedback resistors shown. From the figure it can be seen that three values for $R_f$ were provided. On the same page, FIG. 112 shows the power supply and current stabilisation circuit used for driving the Gallium Arsenide Source, the nominal current stability of which is 1% up to a current load of 1 amp. from 0 - 2 volts. This unit is based on a LM 320 commercial voltage regulator (5 volts) driven from a simple unregulated source using RS. component parts.

The voltage regulator used is a 3 port device which senses voltage error in the output voltage by comparison with a reference voltage. The comparator error signal drives the regulator signal transistor. In order to use this device to enable constant current conditions to prevail, the common terminal is floated and a fixed load connected across the output, which defines the current between the floating point and the upper rail within the operating limits of the regulator.

It should be noted that the amplifier circuit configuration effectively allows for three response bandwidths according to what 'gain' is set. These are on a 3dB power point basis; 500, 100 and 50 Hz, corresponding to the feedback resistors value of .470, 2.3 and 4.7 M ohms respectively.
FIG. 111  AMPLIFIER/DETECTOR CIRCUIT CONFIGURATION

FIG. 112  LIGHT SOURCE - POWER SUPPLY AND CURRENT STABILISATION CIRCUIT
The amplifier/detector unit is housed in a cast iron box (a modified electrician's junction box) and appears virtually as full size in the following photograph, FIG. 113, of the unit. The photodetector lies within the unit, directly behind the afferent proximal ferrule holder. The range switch is located on the top side of the unit together with the coaxial output socket. There is provision for centralising the photo-detector and afferent source output, not visible in the Fig. This is achieved by means of a slideway to which the photodetector bracket is clamped. The following FIG. 114 shows the amplifier/detector unit coupled to the light source by the pressure transducer probe.

The light source itself is mounted on RS components heat sink which is rated for $2.1^\circ\text{C/}w$ thermal resistance but is much larger than was thought necessary bearing in mind the light source power consumption.

A fully adjustable efferent ferrule holder allows for the centralisation of the efferent fibres and the light source. The source itself is obscured from view in the Fig. by this holder.

The discussions concerning the operating performance of these systems (noise and drift characteristics) are to be found in Chapter 17.1.0, where the first practical use of the system is described.
FIG. 114 LIGHT SOURCE AND AMPLIFIER/DETECTOR UNIT COUPLED TO PRESSURE TRANSDUCER

Coaxial Output Switch

"Gain" Switch

Centralising Jig
CHAPTER SEVENTEEN

THE DETERMINATION OF SYSTEM NOISE & DRIFT

AND

THE SPECIFICATION OF DIAPHRAGM PARAMETERS

& INITIAL SEPARATIONS
17.0.0. INTRODUCTION

As has been discussed in Chapter 15, in order to define the parameters of a diaphragm and the value of the initial separation it must allow for the fulfilment of the system performance required, i.e. pressure resolution; first, a static calibration curve for the probe must be determined. Secondly, the system noise (and drift) must be assessed. Then, to be in a position to apply the GASSP 1 or 2 for the specification of diaphragm parameters and initial separations, a mathematical curve fit for the practically determined static calibration curve is ideally required. This mathematical description can then be compared with that theoretically expected. The 'A' ratio as discussed in Chapter 16.

Consequently, the GASSP 1 or 2 can then be utilised to obtain values of the central deflection $Y_{max}$ of a diaphragm and its initial separation $d_0$ which satisfy the system resolution requirements.

However, there are causes for concern in applying the results of a static calibration curve for a probe obtained by using the set-up shown by FIG. 24 as the basis for the GASSP 1 and 2. The concern is due to the following:

1) Use of a different diaphragm (reflecting surface) to that actually to be utilised, i.e. different reflection co-efficients 'c'.

2) The effective distal tip area is not totally enclosed as is the case for the final probe, which will have a small but finite cylindrical extra reflecting surface between the fibre face and the diaphragm.

3) The real diaphragm will not deflect with a flat profile (flat diaphragm assumption).
17.1.0. STATIC CALIBRATION OF PRESSURE TRANSDUCING PROBE AND SYSTEM NOISE DRIFT DETERMINATION

The static calibration of the pressure transducing probe was determined by using the set-up as described earlier (and shown again overleaf, FIG. 24). Movement of the gold deposited glass reflecting surface was again effected using the micrometer screwgauge. This was calibrated in 10 micron divisions. It was necessary for care to be taken, due to backlash effects, when performing the calibration.

The display unit used was a digital voltameter supplied by Exel Electronics, model XL 200. This meter has an effective band width of 10 Hz. Improved response necessitates the use of a pen recorder (100 Hz.) or a U.V. recorder (100 Hz.)

Five successive determinations of the probe's static calibration characteristics were performed using gainsetting (1) on the amplifier, i.e. 470 K ohms as the feedback resistor. After each calibration the system output for 'contact' was noted (see following drift discussion p.p. 450). The repeatability of the system output for identical initial separations over the five calibrations was within the system noise level, i.e. less than 0.1 mV.

System noise being assessed by two methods

1) from the display D.V.M.

2) by replacing the D.V.M. by an oscilloscope whereby, it was concluded that the system noise level is of the order of 0.1 mV. p.p. with a band width of the order of 500 Hz.

Prior to discussing the static calibration characteristics determined, there follows a discussion on an investigation into the drift characteristics of the system.
The system's drift characteristics were studied over a period of one week, using a direct 'light line' from the light source to the amplifier/detector unit. Two different features were studied

1) with the detector 'dark'
2) with the detector illuminated

With the amplifier on gainsetting 1, readings were taken of the system output as shown by two digital displays (the Exel meter and a Solartron D.V.M., as the Exel meter had developed a fault prior to use here and although repaired, was under suspicion), the ambient temperature ($\pm$ 0.05 °C) and the time.

The results are tabulated in FIG. 114a and are discussed below.

1) with the detector 'dark': the 'dark' output drift can be seen from FIG. 114b to be a function of ambient temperature, increasing temperature caused a directly proportional increase in system output, amounting to approximately $0.186 \text{ mV/°C} + 0.002$

The readings denoted $\Theta$ signify the first of each day, and the spread of readings obtained for certain identical temperatures is due to thermal inertia effects in the system. The critical feature demonstrated is the linear fashion in which the detector/amplifier output changes with temperature.

As will be seen it is the determination of drift which, although no quantitative use of the drift value can be made, allows for an explanation of the total system drift in the next paragraph (2). In other words, the detector/amplifier and display have been considered as a 'black box' for the purposes of its qualitative characteristics. A further series of tests with the photo-detector in short circuit conditions would allow a quantitative assessment to be achieved, but for the purposes
<table>
<thead>
<tr>
<th>TIME (hours)</th>
<th>TEMPERATURE (°C)</th>
<th>SYSTEM OUTPUT (mV)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td><strong>DARK</strong></td>
</tr>
<tr>
<td>15.0</td>
<td>21.5</td>
<td>25.5</td>
</tr>
<tr>
<td>17.83</td>
<td>21.0</td>
<td>25.5</td>
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<td>33.93</td>
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<td>23.5</td>
<td>24.8</td>
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**FIG. 114a**  
**TABLE SHOWING SYSTEM DRIFT CHARACTERISTICS**
FIG. 114b  TEMPERATURE DEPENDANT "DARK" OUTPUT DRIFT OF AMPLIFIER AND DETECTOR
With the detector illuminated: a direct line (light guide) was coupled between the light source and the detector/amplifier unit. The output from the detector/amplifier was set by suitable positioning of the light guide ferrule in the light source to give a value which was thought to be the lowest that would be met with when actually performing the static pressure calibration tests (Chapter 18).

FIG. 114c shows how the system output varied with time and temperature, the dots representing the variation of output with time and the crosses, the variation of ambient temperature with time. Immediately two features of this graph are apparent:

a) there appears to be a continuous decay of system output with time,
b) that during the period of test, the temperature varied cyclically and all values increased over the period of test. To assess quantitatively the possible values for any drift with time which is independent of temperature, necessitates the consideration of system outputs obtained at different times but at the same temperature.

Time drift is considered on the next page for four ambient temperature conditions and is best shown in tabulated form.
FIG. 114c  TEMPERATURE AND OUTPUT DRIFT OF AMPLIFIER/DETECTOR AND LIGHT SOURCE

NB  * signifies temp d.
* signifies output
and those shown within a circle are the first readings of the day
<table>
<thead>
<tr>
<th>TEMPERATURE (°C)</th>
<th>TIME (hours)</th>
<th>OUTPUT (mV)</th>
<th>TIME DRIFT (mV / hour)</th>
</tr>
</thead>
<tbody>
<tr>
<td>21.5</td>
<td>15.00</td>
<td>110.3</td>
<td>0.032</td>
</tr>
<tr>
<td></td>
<td>59.75</td>
<td>108.9</td>
<td></td>
</tr>
<tr>
<td>22.6</td>
<td>44.25</td>
<td>108.6</td>
<td>0.044</td>
</tr>
<tr>
<td></td>
<td>105.60</td>
<td>105.9</td>
<td></td>
</tr>
<tr>
<td>23.4</td>
<td>68.00</td>
<td>107.9</td>
<td>0.047</td>
</tr>
<tr>
<td></td>
<td>108.50</td>
<td>106.0</td>
<td></td>
</tr>
<tr>
<td>23.6</td>
<td>92.28</td>
<td>106.6</td>
<td>0.042</td>
</tr>
<tr>
<td></td>
<td>111.32</td>
<td>105.8</td>
<td></td>
</tr>
</tbody>
</table>

Where the mean value is 0.041 mV / hour.

Standard Deviation S.D. = 0.007

The first three readings (conditions), refer to readings taken upon entering the room where these tests were conducted, that is on entering a stable environment. However, the accuracy of the determination of the ambient temperature is ± 0.05°C. Consequently, in the assumption that the temperatures noted were equal is an error of about ± 0.25 % / °C. Thus the computed value of 'time drift' may also consist of a temperature drift of this order. The magnitude of this oversight is to be discussed overleaf.

For the determination of the system's temperature drift, pairs of system outputs are identified, the difference in output then noted. By using the deduced value of the time drift obtained by using the previously computed value drift/hour and the time period elapsed between the taking of the said readings, and subtracting it from the actual difference in readings, a value of the drift which is due to the temperature results. Thus, from the noted ambient temperature for each reading a value for the system drift in mV / °C can be computed.
This determination and results is shewn below in tabular form:-

<table>
<thead>
<tr>
<th>TIME (hours)</th>
<th>OUTPUT (mV)</th>
<th>DIFFERENCE (mV)</th>
<th>COMPUTED TIME DRIFT (mV)</th>
<th>TEMP. (°C)</th>
<th>COMPUTED TEMP. DRIFT. (mV / °C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>33.93</td>
<td>110.5</td>
<td>- 4.6</td>
<td>- 2.95</td>
<td>19.7</td>
<td>- 0.57 ± 0.010</td>
</tr>
<tr>
<td>105.60</td>
<td>105.9</td>
<td></td>
<td></td>
<td>22.6</td>
<td></td>
</tr>
<tr>
<td>59.75</td>
<td>108.9</td>
<td>- 1.0</td>
<td>- 0.86</td>
<td>21.6</td>
<td>- 0.35 ± 0.040</td>
</tr>
<tr>
<td>80.66</td>
<td>107.9</td>
<td></td>
<td></td>
<td>22.0</td>
<td></td>
</tr>
<tr>
<td>33.93</td>
<td>110.5</td>
<td>- 2.6</td>
<td>- 1.92</td>
<td>19.7</td>
<td>- 0.30 ± 0.005</td>
</tr>
<tr>
<td>80.66</td>
<td>107.9</td>
<td></td>
<td></td>
<td>22.0</td>
<td></td>
</tr>
<tr>
<td>33.93</td>
<td>110.5</td>
<td>- 3.3</td>
<td>- 2.14</td>
<td>19.7</td>
<td>- 0.29 ± 0.003</td>
</tr>
<tr>
<td>86.00</td>
<td>107.2</td>
<td></td>
<td></td>
<td>23.7</td>
<td></td>
</tr>
</tbody>
</table>

Where the accuracy in temperature determination is also subject to an error of ± 0.05°C. Thus the effect of this error for each of the cases shewn above must be considered, e.g. the first case where a temperature difference of 2.9°C. is used in the computation of temperature drift. The possible error then, in using this figure becomes ± 0.05 / 2.9; (1.72 %). Thus the value of the temperature drift arrived at is subject to an error of 1.72 %, i.e. 0.010 mV / °C. Tabulated above are the error bands due to this effect.

Let us consider the worst possible case of determined temperature drift, e.g. 0.58 mV /°C, and assess the possible effects the accuracy of temperature determination may have had on the previously discussed time drift computations. The effect of this has been given earlier as 0.25 % of the temperature drift, and now we can give this figure a quantitative value. Hence the error inherent in the determination of time drift is 0.0025 x 0.58 mV. i.e. 0.0014 mV. This error is only of the order of 3% of the values of time drift computed.

Summarising, the drift of the total system, i.e. light source and detector / amplifier fall into two categories. These are

a) drift which is a function of time alone (- 0.04 ± 0.0014 mV. / hr.)
b) drift which is a function of temperature (pessimistic value - 0.58 mV./°C.)

these values are with a set output level of about a hundred mV.

These tests were performed so that the effects of the system's drift could be compensated for when analysing the results of the static pressure calibration tests, Chapter 18. In Chapter 18 we can quantify the effect of system drift in terms of base line stability and sensitivity stability.

The most interesting feature demonstrated is that the light source contributes both the continual decrease in system output with time and the negative temperature drift coefficient. This has been shown, as for the 'dark' conditions, the detector / amplifier has a positive temperature drift coefficient with no detectable time dependent drift. It should be noted that further tests on the system drift show that they were in direct proportion to the gainsetting used.

In conclusion, these tests were performed not primarily to be considered as a final system specification, for which they will be based, but to be used when evaluating the static pressure calibration results.

Considering now the results of the probe's static calibration characteristics. These tests were performed over a period of time and temperature range such that as has been mentioned, the repeatability of the system output for identical initial separations over the period of test was within the system noise level, i.e. less than 0.1 mV. Re-determination of the static calibration curve characteristics was performed on two subsequent days, and due primarily to the time drift, it was necessary to, in effect, reset the 'base line' by setting the
light guide ferrule slightly closer to the photo-detector. \( C_d \) change.

This manoeuvre is an important one for, as will be shown later, it is the simplest method for compensating for base line stability effects and sensitivity stability. The following FIG. 115 tabulates the values of system output and respective 'initial' separation distances practically obtained for the pressure transducer probe, and FIG. 116, the plotted static calibration curve. The separation axis scale employed in this figure (0 - 4 mm.) clearly shows that the probe's most sensitive response occurs at initial separations which lie on its ascending slope. Two further figures are shown FIGS. 117 and 118 which depict the ascending and descending portions of the static calibration curve respectively.

It can be seen from these figures that at the 'contact' position, the system output does not as closely approach the 'dark' output as at the greatest separation distance. This will be due partially to cross-talk between the fibres, as is the case for the greatest separation conditions, and also due to

1) the effect of having a 'wedge' gap between the mirror and distal fibre face.

2) oblique fibres in the distal tip.

With information of only the system outputs that occur at various initial separations, no comments can be made regarding the 'reflection factor' values achieved, i.e. the ratio of the emitted flux from the distal efferent optical fibres and that which is received by the afferent fibres in that group. This can be achieved most readily by assessing the reflection factor associated with the peak value of system output. This occurs at about a separation of 100 microns and is 290 mV. This value of system output is then compared to that which results by replacing the afferent fibre ferrule in the photo-detector input socket by the probe's actual distal tip. By so doing the peak value of reflection
factor attained by the probe was 0.1. However, this figure also includes the losses which result on transport of the radiant flux from the distal afferent group to its proximal end, and losses equivalent to 'C\text{\textsubscript{d}}', which infers that the actual peak value of reflection factor is 0.2, if a fibre transmission of 50% is assumed. Now, as the value of the efferent fibre output was 2.9 V we can now establish what the radiant flux equivalent value is by use of equation 153 which describes the relationship for a linear mode system between radiant flux and electrical output. The value of radiant flux becomes 0.05 mW. From this figure and the knowledge that the light source emits 10 mW total flux, we can say that the value of the 'collection factors' $C\text{\textsubscript{f}} \cdot t\text{\textsubscript{c}} \cdot C\text{\textsubscript{d}}$, amount to .5% when it was expected that they would be of the order of 5%. This discrepancy can be partially attributed to incorrect centralization of the fibres and light source, but the other causes will be of more significance. These are: the presence of low transmittance fibres, poor packing of fibres in the proximal efferent group, and a much lower value of $C\text{\textsubscript{d}}$ for reasons to be described. Because of the lower effective values of $'C\text{\textsubscript{f}} \cdot t\text{\textsubscript{c}} \cdot C\text{\textsubscript{d}}'$ one immediately expects the sensitivity of the probe to be at least $5 / 0.5$ times less than predicted. This and the other points made are further discussed later.

On consideration of the causes of these discrepancies between $C\text{\textsubscript{d}} \cdot C\text{\textsubscript{f}} \cdot e\text{\textsubscript{i}}$ expected and achieved, it was discovered that a practical factor in the assessment made earlier of the value of $'C\text{\textsubscript{d}}'$ had been overlooked. This factor was that for the SC 100 photo-detector, the fact that it is enclosed within a TO 5 can limits the closest position at which one can locate the afferent fibre face to 6 mm. Now, as the effective active diameter of this detector is 2mm. then one arrives at, from page 351, a value for the actual $'C\text{\textsubscript{d}}'$ to be 0.03, this is 15% of the value assumed earlier for this coupling factor. Let us now calculate as before, the value of radiant flux from the efferent fibres, this time using the just practically determined value for $'C\text{\textsubscript{d}}'$. When the expected output becomes, using the values of $C\text{\textsubscript{f}} \cdot e\text{\textsubscript{i}} \cdot t\text{\textsubscript{c}}$, that assumed previously:
10 \times 0.03 \times 0.4 \times 0.5 \text{ mW}, which is 0.6 \text{ mW}, this figure then can be compared with that obtained, i.e. 0.5 \text{ mW}. One can then deduce that the errors incurred in using the above values for C_f, e_i \& t_c can only amount to 20\%.
<p>| SEPARATION | OUTPUT | SEPARATION |</p>
<table>
<thead>
<tr>
<th>Microns x10</th>
<th>Mv.</th>
<th>Microns x10</th>
<th>Mv.</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>54.7</td>
<td>130.0</td>
<td>117.4</td>
</tr>
<tr>
<td>1.0</td>
<td>84.6</td>
<td>140.0</td>
<td>107.1</td>
</tr>
<tr>
<td>2.0</td>
<td>161.4</td>
<td>150.0</td>
<td>97.8</td>
</tr>
<tr>
<td>3.0</td>
<td>205.8</td>
<td>160.0</td>
<td>89.2</td>
</tr>
<tr>
<td>4.0</td>
<td>239.5</td>
<td>170.0</td>
<td>81.5</td>
</tr>
<tr>
<td>5.0</td>
<td>259.0</td>
<td>180.0</td>
<td>74.8</td>
</tr>
<tr>
<td>6.0</td>
<td>271.1</td>
<td>190.0</td>
<td>68.8</td>
</tr>
<tr>
<td>7.0</td>
<td>281.0</td>
<td>200.0</td>
<td>63.3</td>
</tr>
<tr>
<td>8.0</td>
<td>285.5</td>
<td>210.0</td>
<td>58.3</td>
</tr>
<tr>
<td>9.0</td>
<td>290.0</td>
<td>220.0</td>
<td>53.8</td>
</tr>
<tr>
<td>10.0</td>
<td>293.5</td>
<td>230.0</td>
<td>49.8</td>
</tr>
<tr>
<td>15.0</td>
<td>299.3</td>
<td>240.0</td>
<td>46.1</td>
</tr>
<tr>
<td>20.0</td>
<td>297.8</td>
<td>250.0</td>
<td>42.8</td>
</tr>
<tr>
<td>30.0</td>
<td>286.0</td>
<td>260.0</td>
<td>39.8</td>
</tr>
<tr>
<td>40.0</td>
<td>268.4</td>
<td>270.0</td>
<td>37.0</td>
</tr>
<tr>
<td>50.0</td>
<td>248.6</td>
<td>280.0</td>
<td>34.5</td>
</tr>
<tr>
<td>60.0</td>
<td>228.9</td>
<td>290.0</td>
<td>32.2</td>
</tr>
<tr>
<td>70.0</td>
<td>209.0</td>
<td>300.0</td>
<td>30.1</td>
</tr>
<tr>
<td>80.0</td>
<td>190.4</td>
<td>320.0</td>
<td>26.5</td>
</tr>
<tr>
<td>90.0</td>
<td>172.4</td>
<td>340.0</td>
<td>23.3</td>
</tr>
<tr>
<td>100.0</td>
<td>157.0</td>
<td>360.0</td>
<td>20.7</td>
</tr>
<tr>
<td>110.0</td>
<td>142.3</td>
<td>380.0</td>
<td>18.4</td>
</tr>
<tr>
<td>120.0</td>
<td>129.4</td>
<td>400.0</td>
<td>16.4</td>
</tr>
</tbody>
</table>

FIG.115  INITIAL SEPARATION AND SYSTEM OUTPUT ACHIEVED

\[(R_f = 470 \text{ K ohms }, \text{ Flat Gold Deposited Mirror})\]
FIG. 117

STATIC CALIBRATION (EXPANDED SCALE) (ascending)
17.1.1  MATHEMATICAL DESCRIPTION OF STATIC CALIBRATION CURVE

To express mathematically the static calibration curve obtained practically, a curve fitting method employing a least square technique was utilised.

This method was obtained in programme form from Hewlett Packard Ltd. for use with their desk calculator, plotter and digitiser (9820 series). It was written by F. Yockey of that company whom in turn extracted it from a Fortran programme written by KUO SHANS (1965).

The programme determines from a set of input data points the coefficients of a polynomial

\[ P(x) = a_0 + a_1 x + ... + a_m x^m \]  (m is the degree set by the user)

which passes near or through the input data points which lie at equal intervals \( x_i \). The programme determines \( a_i \) by considering \( P(x) \) as a linear combination of Chebyshev polynomials

\[ T_i (x), P(x) = c_0 T_0 (x) + c_1 T_1 (x) + ... + c_m T_m (x) \]

and applying the least squares criterion to the expression

\[ S = \sum_{i=1}^{n} (y_i - P(x))^2 \]

\( n \) is the number of data point pairs to give a set of simultaneous equations

\[ \frac{\partial S}{\partial c_j} = 0, \quad j = 0, 1, ..., m \]

from which \( c_j \) is obtained as follows. Because of the orthogonality properties of Chebyshev polynomials, \( P(x) \) is evaluated at special points i.e.

\[ x_i = \frac{\cos (2i - 1)}{2(m + 1)} \]

within the interval \([-1, 1]\)

which allows for off-diagonal elements to be zero.
The corresponding values of \( \bar{y}_i \) are also needed for the system of equations to enable the solution for \( c_i \), and this is obtained by applying a linear transform to \( x_i \) to bring it within the integral -1,1. Then using these values after applying the Lagrange interpolation formula to obtain \( \bar{x}_i \), to obtain \( \bar{y}_i \). Solution of the system of equations then follow after which \( \bar{x} \) is linearly transformed to allow for the final form of \( P(x) = a_0 + a_1 x + \ldots + a_m x^m \) to be outputed.

Two curve fittings were carried out, one for the ascending portion (positive slope) of the static calibration curve, the other for the descending portion. These are shown graphically by FIGS. 119 and 120 respectively. The 'x' marks denoting data points, the full curve signifying the curve fit of which the mathematical expression is shown on each graph. These are:

ascending portion (positive slope) from: contact to 100 microns
\[
y (mV) = 37.078 + 7.9466 d_o - 8.7078 \times 10^{-2} d_o^2 + 3.3303 \times 10^{-4} d_o^3
\]

descending portion from: 200 to 3000 microns
\[
y (mV) = 364.434 - 2.845634456 \times 10^{-1} d_o + 8.571065937 \times 10^{-5} d_o^2
- 9.341688434 \times 10^{-9} d_o^3
\]
in all cases 'd_o' is in microns

The above mathematical expressions were derived not considering the static calibration curve as one entity but as ascending and descending portions. This was because

a) the programme could not, using the data points obtained experimentally, fit the total curve.

b) by assessing the static calibration curve into the two portions described, the mathematical expressions which result represent the useful sections of the response curve i.e. the portion of the response curve from 100 to 200 microns has been omitted as in this region, the sensitivity is low and of both positive and negative nature.
FIG. 119 MATHMATICAL CURVE FIT (Ascending portion of static calibration curve)

\[ y = 37.078 + 7.3456x - 0.07078x^2 + 0.000333x^3 \]
FIG. 120  MATHMATICAL CURVE FIT (Descending portion of static calibration curve)

\[ y(\text{mV}) = 364.4345849 - 2.845634456 \cdot 10^{-1} d_0 + 8.571065937 \cdot 10^{-3} d_0^2 - 9.341688434 \cdot 10^{-9} d_0^3 \]
Furthermore, considerations of these expressions are summarised below.

1) Certain inexactitudes between mathematical fit and experimental values will be due to the somewhat false boundary values which these portions attribute.

2) From the following FIG. 121, the fundamental accuracy of the mathematical fit can be seen to be better than 2% between separations of 20 to 300 microns. However, although this figure appears good when expressed as a percentage, the actual finite differences, (the inherent error) may be quite large. This is so especially when realising the experimental error for each determined output is better than $\pm 0.05 \text{ mV}$.

As a result, depending on what is expected of the mathematical description of the static calibration curve, it may prove adequate. In other words, if values of system output, as defined by this expression, are intended to represent the actual, this will only occur at specific separations as shown in FIG. 121. Furthermore, as the relationship will be used later to determine system output changes which result from various diaphragm output deflections, $Y_{\text{max}}$ values, which take place at various initial separation distances, it is the value of $Y_{\text{max}}$ which must be taken into account to assess the validity of the mathematical expression and the allowable range of initial separations used.

It is at this point necessary to discuss why and for what purpose a mathematical fit to the experimental curve is required:

1) The next section aims to compare the experimentally obtained static calibration curve to that obtained from the theoretical expression derived in Chapter 14 (equation 192), with the optical fibre parameters $R'$ and $r'$, first as proposed in Chapter 14 and secondly, using $R'$ and $r'$ as deduced for the actual probe constructed FIG. 110. Such a
<table>
<thead>
<tr>
<th>Separation Microns</th>
<th>Experimental Output Mv E</th>
<th>Mathematical Output Mv M</th>
<th>Difference M-E</th>
<th>Ratio M/E</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>84.6</td>
<td>108.1</td>
<td>23.5</td>
<td>1.28</td>
</tr>
<tr>
<td>20</td>
<td>161.4</td>
<td>163.8</td>
<td>2.4</td>
<td>1.015</td>
</tr>
<tr>
<td>30</td>
<td>205.8</td>
<td>206.1</td>
<td>0.3</td>
<td>1.001</td>
</tr>
<tr>
<td>40</td>
<td>239.5</td>
<td>236.9</td>
<td>-2.6</td>
<td>0.989</td>
</tr>
<tr>
<td>50</td>
<td>259.0</td>
<td>258.4</td>
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</tr>
<tr>
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<td>-0.3</td>
<td>0.999</td>
</tr>
<tr>
<td>100</td>
<td>293.5</td>
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<td>0.4</td>
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<td>1.044</td>
</tr>
<tr>
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<td>286.5</td>
<td>0.5</td>
<td>1.002</td>
</tr>
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</tr>
<tr>
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<td>242.4</td>
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<td>0.975</td>
</tr>
<tr>
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<td>228.9</td>
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<td>0.972</td>
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</tr>
<tr>
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<td>190.4</td>
<td>186.8</td>
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</tr>
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<td>0.991</td>
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<td>157.0</td>
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<td>-0.7</td>
<td>0.995</td>
</tr>
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<td>0.1</td>
<td>1.000</td>
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<td>1.012</td>
</tr>
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<td>107.1</td>
<td>108.4</td>
<td>1.3</td>
<td>1.012</td>
</tr>
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<td>98.9</td>
<td>1.1</td>
<td>1.011</td>
</tr>
<tr>
<td>1600</td>
<td>89.2</td>
<td>90.3</td>
<td>1.1</td>
<td>1.012</td>
</tr>
<tr>
<td>1700</td>
<td>81.5</td>
<td>82.5</td>
<td>1.0</td>
<td>1.012</td>
</tr>
<tr>
<td>1800</td>
<td>74.8</td>
<td>75.5</td>
<td>0.7</td>
<td>1.009</td>
</tr>
<tr>
<td>1900</td>
<td>68.8</td>
<td>69.2</td>
<td>0.4</td>
<td>1.005</td>
</tr>
<tr>
<td>2000</td>
<td>63.3</td>
<td>63.5</td>
<td>0.2</td>
<td>1.003</td>
</tr>
<tr>
<td>2100</td>
<td>58.3</td>
<td>58.4</td>
<td>0.1</td>
<td>1.002</td>
</tr>
<tr>
<td>2200</td>
<td>53.8</td>
<td>53.9</td>
<td>0.1</td>
<td>1.002</td>
</tr>
</tbody>
</table>

**FIG.121**  COMPARISON BETWEEN MATHMATICAL AND EXPERIMENTAL VALUES OF SYSTEM OUTPUT
comparison could be achieved graphically using the experimental plotted values of the static calibration curve and overplotting the theoretical curve. Numerate comparisons, if desired, would then become difficult to interpolate at separations which were not experimentally determined. The realisation of a mathematical fit will simplify the procedure. The requirement for the 'fit' in this case being that it ideally falls within the experimental error band. This can be seen not to be true (FIG. 121) but corrections for various separations could be made.

2) The section after next then develops a GASSP 1 & 2. These are graphical methods which enable the establishment of at which values of initial separation and diaphragm total deflection, the resolution noise product is met (Chapter 14). Both aids ideally require a mathematical relationship for the static calibration curve which then allows for the continuous computation or plotting of the value of the reflection factor total ratio \( \frac{F_{pm}}{F_0} \) (GASSP 1) or \( y_1 - y_2 \), total system output difference (GASSP 2), for specific values of diaphragm total deflection.

If, rather than using a mathematical fit, the actual experimental values of system output and initial separation are used, the following limitations result:

a) the values of \( \frac{F_{pm}}{F_0} \) or \( y_1 - y_2 \) can only be found for total deflections of the diaphragm which are equivalent to the interval spacing experimentally utilised.

b) as mentioned, continuous computation of \( \frac{F_{pm}}{F_0} \) or \( y_1 - y_2 \) is not possible as they too can only be computed for the values of initial separation experimentally utilised.

In effect the purpose of the GASSP-1 and the GASSP-2 is to determine whether at the specific value of initial separation and diaphragm total deflection the noise resolution product is satisfied (Chapter 14).
In order to ascertain whether the mathematical fit is sufficiently accurate to enable such a system specification to be extracted necessitates a comparison between the actual experimentally obtained differences in output occurring for two specific separations and the differences when considering the mathematical fit values. This can be seen for initial separations as shown in FIG. 121 and 'deflections' which are multiples of the interval spacing experimentally utilised. This is accomplished by subtracting the respective values of 'Difference M - E' shown in FIG. 121. For example if a total deflection of 10 microns is considered to occur from 50 - 60 microns separation, then the difference in output when considering the mathematical fit and the experimental values is -1.8 mV. (1.2 - - 0.6). The significance of this comparison becomes apparent when values are assigned to the resolution noise product \( N.V_n = V.N_n \), i.e. if \( V.N_n = 0.03 \) then an error of -1.8 mV is -6% of this boundary value for total output change, similarly, as 13.9 mV. is the expected total output change (from FIG. 121), there will be a +13% error incurred by using the 'fit'. In other words, a +13% error results in the expected sensitivity (mV. per mm. Hg.) Let us consider another case where 10 micron deflection occurs, say from 30-20 microns, when the error is +2.1 mV. and the total system output change expected is 42.3, whereby the percentage error in sensitivity when using the mathematical fit is -5. In other words the errors incurred using a mathematical fit for the determination of system resolution decrease with decreasing values of separation. For further illustration, let us consider a deflection of 20 microns occurring first at an initial separation of 60 microns and secondly at a separation of 40 microns. From FIG. 121 (and with reference to FIG. 126) the percentage errors are (-3.8 / 35.4) or -10%, and (-5.0 / 78.1) or -7%. It is apparent that it will be necessary to consider the errors described for a specific value of initial separation and total deflection. In Chapter 17.2.1, values for the diaphragm total deflection and initial separation which
satisfy the system requirements are extracted from the GASSP 1 and 2, At the same time the errors incurred in the expected sensitivity (due to using the mathematical fit) are calculated using the same technique as above. The mathematical fit may thus contribute errors of up to 14%, in expected sensitivity, although by simply comparing finite values of output gives errors of less than 1% in the system output values.

In the next and subsequent sections both descriptions for the static calibration curve are to be used to clarify these points.
COMPARISON BETWEEN PRACTICALLY OBTAINED STATIC CALIBRATION CURVE AND THEORETICAL PREDICTIONS

As just discussed, there are two methods open which enable the comparison between the static calibration curve obtained experimentally and that expected from the theoretical treatise performed earlier (generally described by equation 192). The direct method compares the value of system output computed from equation 192 with the actual value obtained experimentally, although this is limited to comparison of outputs obtained at the separation intervals practically utilised. Secondly, there is the indirect method, which compares the theoretically ascertained output with that described by the 'mathematical fit'. This method allows a continuum of comparisons to be performed. To complicate matters further, there are two descriptions for the theoretical predictions which should be used, both using the basic reflection factor expression. One uses the probe optical fibre arrangement as proposed in Chapter 14, the other, the arrangement in the constructed probe (FIG. 110). This value being a guesstimate.

Now the theoretical description for system output is given by equation 192

\[ y_{op} = G.K.R_i t^2 C_f C_d e^{-R_{2i}} \left[ \frac{R^2 - r^2 + 4 \sqrt{r^2(d - y_{max})^2 + (d - y_{max})^2}}{2r^2} \right] - \sqrt{(r^2 + R^2 + 4(d - y_{max})^2)^2 - 4R^2r^2} \] .... 192

where \( y_{op} \) is the system output when the separation between the distal fibre face and the diaphragm is \( d \) - \( y_{max} \), i.e. the initial separation minus the diaphragm deflection assigned \( y_{max} \). If the system output is to be determined for various values of initial separation \( d \) they can be used as \( d \) - \( y_{max} \) as is shown below. Now for the practical system the above equation reduces to

\[ y_{op} \text{ (volts)} = 12.79 \times \frac{0.48}{r^2} \left[ \frac{2r^2 - r^2 + 4 \sqrt{r^2d + d^2} - \sqrt{(r^2 + r^2 + 4d^2) - 4R^2r^2}}{2r^2} \right] \]

where all terms except the first constitute the reflection factor expression of equation 78.
As mentioned, the parameters defining the optical fibre arrangement for use in the system output expression (214), were taken for the two cases: a) the actual probe when \( A = 1.2 \) and \( r = 120 \) micron, b) that for the proposed system when \( A = 3 \) and \( r = 20 \). The following two figures 122 and 123, show for the ascending and descending portions of the experimentally determined static calibration curve the curves resulting from

1) plotting the mathematical fit to the experimental curve
2) plotting the theoretical static calibration curve by considering the fibre arrangement to be as that proposed
3) plotting the theoretical static calibration curve by assessing from FIG. 110 the probe's actual fibre arrangement.

From these figures the following comments can be made

1) The mathematical fit can be seen to adequately represent the actual static calibration curve for the purpose of assessing the theoretically derived response curves. This applies to both ascending and descending portions.

2) When considering the comparison between the probe's actual response and that from the theoretical considerations, a limitation over which the range of comparison can be made is necessary. This is because the theoretical plot represents the situation using optical fibres having an N.A. of unity, but as the fibres used have an N.A. of 0.866 then strictly speaking a comparison of system outputs can only be made at separations greater than the transition point. For the case of the proposed system this is approximately 23 microns. Nevertheless, in this case, the actual response can be seen to be of a much reduced sensitivity and output than of that desired (by about a factor of 40). The sensitivity aspect can be explained by the realisation that the fibre arrangement was not as desired,
FIG. 122 COMPARISON BETWEEN (ascending)
PRACTICAL AND THEORETICAL
"STATIC CALIBRATION CURVES"

Theoretical (as determined from actual probe Fig. 110)
Scale "C"
r = 120 microns "A" = 1.2

Theoretical (as Proposed Ch 14)
r = 20 microns "A" = 3

N.B ref. scale B

Mathematical fit to "practical"

Scale "C" Practical points

Separation Microns

(43) (42)
FIG. 123 COMPARISON BETWEEN PRACTICAL (descending) AND THEORETICAL "STATIC CALIBRATION CURVES"

Curve "X" - Theoretical (as Proposed, Ch14) r=20 microns "A"=3
Curve "Y" - Theoretical (as determined from actual probe Fig.1100 r=120 microns
Curve "Z" - Mathematical fit to "Practical" where the points are the actually determined
the quantitative aspect being caused by lower values of 
\( C_d, C_f, t_c^2, e_i \) than expected, as has been shown on Page 459 
by a factor of 10.

3) The curve depicting the theoretical response which is 
based on the actual probe fibre arrangement may only be 
compared, as discussed above, at or beyond the transition point 
which in this case is 76 microns. Consequently, the section 
of the curve shown in FIG. 112 cannot, up to a separation of 
76 microns, be fairly compared with the values actually achieved, 
extcept to note that the outputs are of the correct order. The 
differences in form may be due to a) the modification of the 
basic theoretical assessment of reflection factors by working 
closer than the transition point  b) the fact that the probe's 
fibre arrangement is not as the theory demands (Chapter 17). 
The descending portion of FIG. 123 also shows that, although 
the outputs are of the same order there is a difference 
in form which will be due to (b) as discussed above.

Because neither of the theoretical plots agree with that 
practically obtained, (although explanations of why this is so have been 
mentioned) a further question can be raised. How valid is the 
theoretical expression for reflection factor? In the theoretical analysis 
section the various assumptions made were discussed and the effect of these 
assumptions would not produce in neither form nor magnitude the errors as 
depicted in FIGS. 122 and 123. Because it was thought that this section 
would, in effect, allow for the verification of the theoretical analysis 
sections' reflection factor expression and this was not accomplished for 
the reasons mentioned, a special probe was built. This probe has a 
macro fibre arrangement and Appendix 1 discusses it, and the results of 
the comparison between the static calibration curve practically obtained 
and that predicted theoretically.
Thus, summarising, the fibre arrangement used is not as theoretically specified or desired. The quantitative values of the parameters $C_f, C_d, t_c^2$ are also not as expected, (Chapter 17). The accuracy of the mathematical fit is adequate only because the differences in the theoretically expressed curves are far greater.
17.2.0 THE GASSP'S 1 & 2 FOR THE PRACTICAL SYSTEM AND THE EFFECT USING THE 'MATHEMATICAL FIT' (OF THE STATIC CALIBRATION CURVE) HAS ON THE RESULTING SYSTEM SPECIFICATION

One of the requirements for the plotting of these graphical aids for the specification of system parameters, is a description of the probes static calibration curve. We have two descriptions, one a co-ordinate description (the actual experimental values) and also a mathematical, fitted, expression. The errors incurred by using the mathematical fit rather than the experimental values for the static calibration curve on the expected resolution (sensitivity) have been discussed in 17.1.1 by considering the differences in total system output change from inspection of FIG. 121. To illustrate the points made there of the manner in which one assesses the error incurred in using the 'fit' and its magnitude, we will consider the two graphical aids used for specification of system parameters.

The method by which these graphical aids are constructed has been described earlier. The following GASSP 1 & 2 being based on both mathematical description of the static calibration curve (ascending portion equation 212) and the actual experimental values.

Consider first FIG. 124, which shows the GASSP 1 for the practical system where all the full curves in sectors two and three are computed from the mathematical fit. The curves marked 'gain factor 1' represent the actual system gain outputs, i.e. with a feedback resistor $R_f = 470$ K ohms. The upper two curves are the assumed response for system output when increasing the system gain by the factors shown. The 'points' shown result from usage of the experimental values for the practically determined static calibration curve. From this figure the following comments can be made with respect to the limitations imposed by using the mathematical fit rather than the actual experimental values.

1) The differences between the finite outputs (system) are;
   - for the scales shown, well within an interpretation accuracy.
FIG. 124 GASSP 1- PRACTICAL SYSTEM: SHEWING EFFECT OF USING "MATHMATICAL FIT" ON SPECIFICATION.

N.B all curves from mathematical expressions

"A" - $V_n = 0.03V$
"B" - $V_n = 0.015V$
"C" - delta $d = 10$ microns
"D" - delta $d = 20$ microns
The discernable effect on the boundary value of the ratio $F_{pm}/F_o$ is small, and consequently it is only the error in the achieved ratio which will cause an error in the assessment of the maximum value for the initial separation at which the system resolution requirements are just met. The error incurred is the 'x' ordinate in Sector 2 from point to curve which can be seen in most cases to be about 2 microns. This is well within the practical accuracy within which a diaphragm could be positioned. It is worth noting that the GASSP 1 has not been designed for the determination of system resolution given an initial separation and total deflection. Whereas the GASSP 2 has. This is shown in the following FIG. 126 where the full curve shows the system total output change as computed from the mathematical 'fit'. Whilst for two values of diaphragm total deflection, 'points' represent the values as computed from the actual experimentally obtained static calibration curve. It is worth recalling that the GASSP 1 and 2 are in fact only different presentations of the system's characteristics. Consequently, any limitations of the mathematical 'fit', considering the GASSP 2, should confirm those just mentioned. This will be shown to be so but with one further consideration (the effect on expected system resolution as discussed on Page 472). With respect to FIG. 126, the effect of the mathematical 'fit' can be considered either, by considering a specific boundary level for $V_n$ and determining the horizontal ordinates between curve and point, i.e. an increase or decrease in described initial separation (this aspect has been discussed in context with the effects caused on the GASSP 1 specification); or by assuming a set value for initial separation and determining the vertical ordinate between curve and point, i.e. the decrease or increase in effective system resolution (this aspect has been
"A" - delta d = 5 microns
"B" - delta d = 10 microns
"C" - delta d = 15 microns
"D" - delta d = 20 microns

N.B 'points' shown result from experimental values i.e. actual static calibration and are shown for two values of delta d = 10 & 20 microns.

FIG.126 GASSP 2 PRACTICAL SYSTEM - (using both 'mathmatical fit' and experimental values for static calibration curve)
First, let us consider the effects on the expected value on initial separation. If a total deflection of 20 microns is considered, it can be seen that the horizontal ordinate has a value of about ±2 microns maximum as mentioned earlier. Secondly, the effects on the expected resolution. Consider two separations, 60 microns and 40 microns, and assume that these are the initial separations for two diaphragms which have a total deflection of 20 microns (the same examples are given on Page 472). When the diaphragm is initially at 60 microns from FIG.126, it can be seen that there is an error in total system output change of approximately 0.004 volts, this occurring when the expected total system output changes 0.036 volts. Thus the resolution expected

\[ N_{\text{expected}} = \frac{0.036}{0.0001} = 360 \]

whilst the actual resolution is

\[ N_{\text{actual}} = \frac{(0.036 - 0.004)}{0.0001} = 320 \]

which results in an error of less than ±1 mm. of mercury in resolution although expressed as a percentage it is ±12% (c.f. Page 472).

With the diaphragm at 40 microns, using the same procedure

\[ N_{\text{expected}} = \frac{0.073}{0.0001} = 730 \]

whilst the actual resolution is

\[ N_{\text{actual}} = \frac{(0.073 + 0.005)}{0.0001} = 780 \]

which results in a very small resolution error of ±6%.

Thus a mathematical 'fit' enables a system of parameter specification which is adequate and any errors incurred may be defined.
17.2.1 SELECTION OF DIAPHRAGM TOTAL DEFLECTIONS AND INITIAL SEPARATIONS WHICH FULFIL SYSTEM REQUIREMENTS FOLLOWED BY SPECIFICATION OF A DIAPHRAGM

The two graphical aids for specification of system parameters (GASSP 1 & 2) are shown overleaf, FIGS. 125 & 126. The manner in which they are interpreted has been discussed (Chapter 14.1.1) as also the means by which a system specification can be extracted (Chapter 14.2.1).

Using both aids, the values of initial separations and diaphragm total deflections which just satisfy the resolution requirements will be established, and it will also become clearer as to the relative merits of each aid, as will the fact that they are only different representations of the same system characteristics.

Consider now FIG. 125 which shows the GASSP 1 for the practical system. The 'resolution' curves shown are classified for various resolution/noise products, $V_N$ (Chapter 14). The actual system noise has been defined as 0.1 mV. Thus the resolution curves give the boundary conditions for reflection factor total ratio for resolutions $N = 300, 250, 200, 150, 100$ respectively, and as the total deflections considered will be for diaphragms subjected to 300 mm Hg. The actual pressure resolutions are 1, 1.2, 1.5, 2 & 3 mm Hg. respectively.

The maximum value of the initial separation 'd_o' of the diaphragm will be determined for the different values of diaphragm total deflection 'Y_{max}' shown, i.e. 5, 10, 15, & 20 microns. In each case, the. separations give rise to the boundary value of reflection factor total ratio which satisfy a pressure resolution of 1 mm Hg. These values have been graphically extracted using the procedure discussed in detail in Chapter 14 and are shown in FIG. 125 by conditions 32, 33, 34 and 35 respectively, and may be considered as the maximum values for initial separation which just allow for the system resolution requirements to be met for each of the diaphragm total deflections considered. These are shown on page 487 in tabular form.
FIG.125 GASSP 1- PRACTICAL SYSTEM (using mathematical fit for static calibration curve)
The following FIG. 126 shows the system GASSP 2 for which the same conditions for the maximum values of the initial separation distance are shown, and are also labelled 'conditions 32 - 35'. From the GASSP 2 they are rather more easily determined, simply by setting the system output change to the value of the noise resolution product for the system i.e. say 0.03 volts (if a pressure resolution of N = 300 is desired) thus providing a boundary condition for the total system output change (Cond. 36). The intersection of this boundary condition for total system output change with each of the curves formed, by consideration of using different diaphragm total deflections gives the maximum initial separation distance for each case of diaphragm total deflection. Graphically the accuracy of defining these separation distances can be seen to be \( \pm 1 \) micron which is the same as for their determination by GASSP 1, although by use of the GASSP 2 are easily obtained. However, as discussed in Chapter 14 it is only from the GASSP 1 that a finite value of the system output at a specific value of the initial separation distance can be instantly determined, whilst from the GASSP 2 alone can the systems resolution at any value of initial separation be determined.

We are now in a position to state that it will be possible for the total system to achieve its requirements of resolution, but prior to stating at exactly what value of initial separation distance a diaphragm should be located we must specify the diaphragm parameters fully. The probe was designed bearing in mind that the diaphragm should be of 3 mm diameter maximum. Now in Chapter 10, TABLE 7 shows the values of the central deflection of diaphragms (total deflections) of different materials

<table>
<thead>
<tr>
<th>DIAPHRAGM TOTAL DEFLECTION ( \mu ) (microns)</th>
<th>MAX. INITIAL SEPARATION ( \mu ) (microns)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.0</td>
<td>14.0</td>
</tr>
<tr>
<td>10.0</td>
<td>41.5</td>
</tr>
<tr>
<td>15.0</td>
<td>54.0</td>
</tr>
<tr>
<td>20.0</td>
<td>64.0</td>
</tr>
</tbody>
</table>
"A"- delta d = 5 microns
"B"- delta d = 10 microns
"C"- delta d = 15 microns
"D"- delta d = 20 microns

N.B 'points' shown result from experimental values i.e. actual static calibration and are shown for two values of delta d=10 & 20 microns.

FIG. 126 GASSP 2 PRACTICAL SYSTEM- (using both 'mathematical fit' and experimental values for static calibration curve)
and thicknesses when their radius is 1.5 mm when the applied differential pressure is 300 mm Hg. Using this table in conjunction with FIG. 126 it can be seen that a Beryllium Copper diaphragm 25 microns thick will give a 'total deflection' of 17 microns and that with such a value for $Y_{\text{max}}$ a pressure resolution of 1 mm Hg. results for practicable values for initial separation. A 30 micron diaphragm was also considered but was found later to be a non-standard size and consequently its mention will be omitted, i.e. that as proposed in 14.2.1.

It is clear that by positioning such a diaphragm closer than that shown by the maximum initial separation distance, which is approximately 54 microns (cond. 34 on FIGS. 125 & 126), that an improved resolution will result. However, it should be noted that 54 microns is an approximate value for the maximum separation distance, because a 'load line' for a total deflection of 17 microns was not plotted and the separation distance quoted is for a total deflection of 15 microns but by interpolation, it can be seen that it is only likely to be 2 microns in error. Now from FIG. 126 it can be seen that the actual values of total system output change are, for initial separations less than 40 microns, greater than those predicted by use of the mathematical fit. Thus by suggesting that the diaphragm be located at an initial separation distance of 35 microns then a resolution even better than predicted from the curve shown will result.

This value for predicted system resolution can be seen from condition 37 on FIG. 126 and gives a total system output change of greater than 60 mV, in other words a resolution 'N' = 600 or in terms of pressure 0.5 mm Hg. The errors incurred by using the mathematical fit for describing the static calibration curve on expected resolution or sensitivity are discussed earlier and quantitative examples given. Using the same procedure now for our suggested system, one arrives at the conclusions that the predicted values will be 96% of the actual and this is the case.
for the sensitivity when considering the 15 micron 'load line'. Thus
the figure of a resolution of better than 0.5 mm Hg. is a pessimistic
figure. The sensitivity figure is obtained from the predicted system
total output change noting that this is caused by total change in pressure
of 300 mm of mercury, when the predicted sensitivity will become
\[
\frac{60}{300} \text{(mV / mm Hg.)} = 0.2 \text{ mV/mm Hg.}
\]

To determine the system output level which results initially,
we can refer to FIG. 125 when condition (37) shows the set output level
at the initial separation distance of 35 microns which is about 220 mV.
Thus, summarising the parameter specification as determined graphically
we know that:

a) the initial separation distance \(d_0\) will be 35 microns
b) the diaphragm total deflection will be approximately 15 microns

which results in a resolution 'N' of 600

By using a Beryllium Copper diaphragm 25 microns thick and of
1.5 mm. radius a total deflection of 17 microns results when a differential
pressure of 300 mm. Hg. is applied. Consequently, with respect to
condition (b) above the system's pressure resolution will become 0.5 mm Hg.
and its sensitivity 0.2 mV/mm Hg. It is worth noting here that generally
a figure of sensitivity gives no indication of the pressure resolution as
the system noise level must be known. Thus, a determined sensitivity
can be defined as a primary extracted parameter from the GASSP 2 and the
pressure resolution, in effect, a secondary derived characteristic.

It is now possible to plot a predicted static pressure
calibration curve for the system, as we have a description for the static
calibration curve and a relationship for the pressure and deflection of
an edge clamp diaphragm (equation 91). This will be performed and is
shown in the next chapter where it is compared with that practically
obtained.
17.3.0 DISCUSSION

The photo-detector and amplifier were deemed satisfactory, a fact which could be only ascertained after practical assessment of the GASSP 1 and 2.

These displays have enabled a specification to be made for diaphragm total deflection and initial separation which allows for the satisfaction of the system's pressure resolution requirements. The 'diaphragm' used for determining the system's static calibration curve was a flat glass disc, with a mirror surface formed by gold deposition, and was determined using the set up shown by FIG. 24. Now because of the following details the system as specified may not perform as expected:

1) If the reflection coefficient of the diaphragm (mirror surface) as specified and used (Chapter 18) differs from that of the flat diaphragm discussed, which has been used for the determination of the static calibration curve, then the magnitude of the system output for any separation distance will differ from that as defined in this chapter, although being of the same form. It could be argued that the actual diaphragms to be considered for use in the practical system should in fact be used for determination of the static calibration curve. However, in practice, it is always possible to coat any of the diaphragms considered with gold. The actual effect of different magnitudes of system output causing an incorrect assessment of the resolution achievable.

2) Another effect of establishing the static calibration curve using the set up shown in FIG. 24 is that there is no side reflecting surface present as there will be with a real diaphragm mounted in situ in front of the probe. This surface would be formed by the cylindrical metal surface which provides the initial separation distance. The quantitative effects of this is thought to be small, when it is realised that the actual gap will be only 35 microns, as will be deduced in the next chapter.
3) Of more significance perhaps is the 'flat diaphragm effect' assumption (11.1.1) were the diaphragm curvature will modify the form of the response curve.

4) Finally, the expression defining the central deflection of a 'thin diaphragm' is suspect (equation 91 chapter 10). This could be of significance if a value of the diaphragm's total deflection chosen from the GASSP 1 or 2 is compared to the predicted deflection of a real diaphragm using equation 91. If there is any discrepancy in actual central deflection and predicted value the practical system may
   a) display a different resolution or sensitivity than expected
   b) and the form of the static pressure calibration curve may differ.

For the above reasons it is prudent to perform what has been termed a 'Simulated Static Pressure Calibration' determination, this philosophy has been introduced in Chapter 15. After this, apart from being in a position to state the practical performance of the system, it will also be possible to assess the relative effects on the predicted performance of the probable limitations discussed in this section. This approach is followed in Chapter 18.

For the suggested system a Be/Cu diaphragm of 1.5 mm radius and 25 microns thickness is expected to have a sensitivity of 0.2 mV / mm Hg. and a pressure resolution of at least 0.5 mm Hg. when taking up an initial separation of 35 microns from the distal fibre face. For this system the effect of system output drift on baseline and sensitivity drift can now be ascertained. There are two forms of drift, one is time dependent, the other temperature dependent, the former is attributed to the light source alone (0.041 mV / hr.), and the latter is primarily due to the light source also (0.058 mV / °C). Both have a twofold effect on the system's performance. First a baseline change where for one pressure the system
output effectively changes with time and temperature and secondly, that the system sensitivity will change. Consider the system suggested then the following are the baseline and sensitivity stabilities.

a) Baseline stability: $0.041/220 \text{ \% F.S.D. } / \text{ hr.} = \text{ less than } -0.02\%$
   and $0.58/220 \text{ \% F.S.D. } / \text{ }^\circ\text{C} = \text{ less than } -0.3\%$

b) Sensitivity stability:

   again less than $-0.02\%$ / hr
   and less than $-0.3\%$ / $^\circ\text{C}$

from which it can be seen that, the effect on sensitivity is negligible and the baseline stability amounts to initially $\leq -0.2 \text{ mm Hg.}$ and $\leq -3 \text{ mm Hg.}$ for the first hour and $^\circ\text{C}$, respectively.
CHAPTER 18

STATIC PRESSURE CALIBRATION OF
FIBRE OPTIC PRESSURE TRANSDUCER.
18.0.0 INTRODUCTION

The objectives of this penultimate chapter may be grouped into those which are solely practically orientated (A), and those which are concerned with assessing the validity of both the theoretical and practical methodology from which the previous section's suggested system specification results (B).

These are A 1) by using a simulated static pressure calibration jig to obtain a practically determined static pressure calibration curve for the total system.

2) to then determine at what value of diaphragm initial separation the system's pressure resolution requirements are satisfied.

B) 1) to determine over the practical range, whether the static calibration curve as determined using the set up shown in FIG. 24, significantly differs from that which results when the probe is in the static pressure calibration rig.

2) to assess whether there are any discrepancies between the diaphragm deflection equation (91)'s predictions for deflection, and that actually achieved with a diaphragm clamped in the rig.

3) to assess the validity of the 'flat' diaphragm assumption.
The following FIG. 127 shows a photograph of the static pressure calibration jig. The function of the jig is to enable the precise positioning of the distal face of the probe from a 'real' diaphragm which is clamped in position, and to permit the application of a pressure to the diaphragm. With respect to FIG. 128, which shows a sectional view of the jig, it can be seen that the initial separation distance can be set by rotation of the 'initial separation adjuster' which raises or lowers both distal ferrule and dial gauge. The dial gauge bears on a ground surface which is perpendicular to the bearing surface through which the 'modified pin chuck' carrying the distal ferrule passes.

In order to set the dial for the contact position, the upper clamp pad is removed and replaced by one of similar shape but without a hole. The initial separation adjuster is then used to raise the dial gauge and distal ferrule until contact between it and the upper clamp pad results. At this point the dial gauge scale is set to read zero, after which the probe tip is lowered. The diaphragm itself can then be clamped in position having first replaced the upper clamp pad by the one with the 3 mm diameter aperture. This aperture effectively results in the diaphragm being of this size. The clamp pads are themselves surface and cylindrically ground to ensure for satisfactory clamping of the diaphragm between their faces. Note the centralising ring provided, this is internally surface ground to ensure that the apertures in both top and bottom clamp pads line up exactly, otherwise a non-circular diaphragm would result. The pressure line catheter can now be secured in position into the female luer fitting provided on the upper clamp pad.

Prior to performing the static pressure calibration tests, a series of auxiliary tests were carried out to assess whether, by incorporating the pressure transducing probe and 'test jig' into the system rather than a direct link between light source and amplifier/detector unit, a change in system drift characteristics result (Chapter 17). This
Catheter - From Static Pressure Head

FIG. 127  STATIC PRESSURE CALIBRATION JIG

Initial Separation Indicator

Initial Separation Adjuster

Pressure Transducing Probe
proved not to be the case. These tests were performed with the initial separation set to 30 microns which, as has been seen, lies on a very steep section of the static calibration curve, to ensure that if there were any shifts of the probe for any reason, then the effect on system output would be maximal. From the design of the jig the effects of temperature causing expansion and contraction on the initial separation distance were minimised by having the rigid linkages locating the probe in position in the form of a compensating U. For the tests, the system output level which resulted was of the order of 190 mV and this fact will be discussed overleaf. The resulting time and temperature drifts for the system were now - 0.08 mV/hr and - 1.1 mV/°C which, when accounting for the fact that the system output was nearly twice that when the drifts were determined for the system when using a direct line between the light source and amplifier/detector unit, confirmed the fact that the jig has no detectable effect on the system's drift characteristics.
Fig. 128 Sectional View
Static Pressure Calibration Jig

- Catheter from static pressure head
- Toggle action clamp
- Diaphragm
- Diaphragm clamp pads
- Modified 'Pin chuck'
- Ground Bearing Surfaces
- 'Dial gauge for relative gauging - i.e. initial gap distance'
- 'Flexible section'
- Scrap view of chuck & distal ferrule
The static pressure calibrations were performed using a 25 microns thick Berrylium Copper diaphragm which was supplied by Goodfellow Metals Ltd. It was free from pinholes and was described by the manufacturers as 'light tight'. It was used in a heat treated form as described in Chapter 10. The calibrations were determined with initial separations distances of 25, 30, 35, 50, 54 & 70 microns. At each initial separation five calibrations were performed, each time system output readings were taken on increasing the pressure on the diaphragm and also on decreasing the pressure. The pressure head was applied by use of a standard sphynomanometer coupled to the rig via the pressure line catheter. For this reason the pressure applied to the diaphragm could only be read to within ± 0.5 mm Hg. Now on setting the diaphragm in place in the rig, it was found that for each of the initial separations used that the system output was approximately 15% of the value than when using the gold deposited mirror for determination of the probe's static calibration curve (in vitro FIG. 24). It was possible, by slight re-adjustment of the probe's afferent ferrule in the photo-detector unit, to increase the system output at the initial separation of 25 microns, to an output value less than 2% in error compared with that obtained using set-up FIG. 24.

Furthermore, this error decreased with increasing initial separations (c.f. FIG. 129, 130 and 121) and the tests were then performed using this setting of system level. In other words, the effect of having the probe in the 'test rig' on the static calibration curve was minimal. It was found during these tests that the setting of an initial separation could be in error by ± 1.5 micron. The effects of this will be discussed later. The results of these tests are shown in FIGS. 129 and 130.
<table>
<thead>
<tr>
<th>INITIAL SEP.</th>
<th>INITIAL SEP.</th>
<th>INITIAL SEP.</th>
</tr>
</thead>
<tbody>
<tr>
<td>'d₀' = 25 micron</td>
<td>'d₀' = 30 micron</td>
<td>'d₀' = 35 micron</td>
</tr>
<tr>
<td>PRESSURE SYSTEM</td>
<td>PRESSURE SYSTEM</td>
<td>PRESSURE SYSTEM</td>
</tr>
<tr>
<td>APPLIED OUTPUT</td>
<td>APPLIED OUTPUT</td>
<td>APPLIED OUTPUT</td>
</tr>
<tr>
<td>mm. Hg. mV. ±0.1</td>
<td>mm. Hg. mV. ±0.1</td>
<td>mm. Hg. mV. ±0.1</td>
</tr>
<tr>
<td>----------------</td>
<td>----------------</td>
<td>----------------</td>
</tr>
<tr>
<td>0</td>
<td>177.0</td>
<td>0</td>
</tr>
<tr>
<td>30</td>
<td>170.6</td>
<td>30</td>
</tr>
<tr>
<td>40</td>
<td>167.9</td>
<td>40</td>
</tr>
<tr>
<td>50</td>
<td>165.7</td>
<td>50</td>
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<tr>
<td>60</td>
<td>163.0</td>
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<td>160.7</td>
<td>70</td>
</tr>
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<td>80</td>
<td>158.2</td>
<td>80</td>
</tr>
<tr>
<td>90</td>
<td>155.6</td>
<td>90</td>
</tr>
<tr>
<td>100</td>
<td>152.9</td>
<td>100</td>
</tr>
<tr>
<td>110</td>
<td>150.0</td>
<td>110</td>
</tr>
<tr>
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<td>147.3</td>
<td>120</td>
</tr>
<tr>
<td>130</td>
<td>145.0</td>
<td>130</td>
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<td>139.2</td>
<td>150</td>
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<td>160</td>
<td>136.4</td>
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<td>130.5</td>
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<td>190</td>
<td>127.7</td>
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<tr>
<td>200</td>
<td>124.5</td>
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<td>118.5</td>
<td>220</td>
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<td>230</td>
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<tr>
<td>250</td>
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</tr>
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<td>260</td>
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<td>270</td>
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</tr>
<tr>
<td>280</td>
<td>101.0</td>
<td>280</td>
</tr>
<tr>
<td>290</td>
<td>98.1</td>
<td>290</td>
</tr>
<tr>
<td>300</td>
<td>95.3</td>
<td>300</td>
</tr>
</tbody>
</table>

**FIG.129**  
TABLE SHOWING RESULTS OF STATIC PRESSURE CALIBRATION TESTS  
(using a Berrylium copper diaphragm 25 micron thick and 1.5mm rad)
<table>
<thead>
<tr>
<th>PRESSURE APPLIED</th>
<th>OUTPUT</th>
<th>PRESSURE APPLIED</th>
<th>OUTPUT</th>
<th>PRESSURE APPLIED</th>
<th>OUTPUT</th>
</tr>
</thead>
<tbody>
<tr>
<td>mm. Hg.</td>
<td>mV. ±0.05</td>
<td>mm. Hg.</td>
<td>mV. ±0.05</td>
<td>mm. Hg.</td>
<td>mV. ±0.05</td>
</tr>
<tr>
<td>0</td>
<td>254.9</td>
<td>0</td>
<td>258.6</td>
<td>0</td>
<td>277.9</td>
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<tr>
<td>30</td>
<td>252.6</td>
<td>30</td>
<td>256.4</td>
<td>30</td>
<td>277.3</td>
</tr>
<tr>
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<td>251.8</td>
<td>40</td>
<td>255.7</td>
<td>40</td>
<td>276.9</td>
</tr>
<tr>
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<td>251.0</td>
<td>50</td>
<td>255.0</td>
<td>50</td>
<td>276.5</td>
</tr>
<tr>
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<td>250.2</td>
<td>60</td>
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<td>60</td>
<td>276.2</td>
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<tr>
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<td>70</td>
<td>275.8</td>
</tr>
<tr>
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<td>248.7</td>
<td>80</td>
<td>252.7</td>
<td>80</td>
<td>275.3</td>
</tr>
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<td>252.0</td>
<td>90</td>
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<td>246.6</td>
<td>100</td>
<td>251.3</td>
<td>100</td>
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</tr>
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<td>245.8</td>
<td>110</td>
<td>250.4</td>
<td>110</td>
<td>274.1</td>
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<tr>
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<td>244.9</td>
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<td>120</td>
<td>273.7</td>
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<td>130</td>
<td>273.3</td>
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<tr>
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<td>243.1</td>
<td>140</td>
<td>248.1</td>
<td>140</td>
<td>272.8</td>
</tr>
<tr>
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<td>242.1</td>
<td>150</td>
<td>247.3</td>
<td>150</td>
<td>272.3</td>
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<td>241.1</td>
<td>160</td>
<td>246.5</td>
<td>160</td>
<td>271.9</td>
</tr>
<tr>
<td>170</td>
<td>240.1</td>
<td>170</td>
<td>245.7</td>
<td>170</td>
<td>271.4</td>
</tr>
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<td>200</td>
<td>242.9</td>
<td>200</td>
<td>270.0</td>
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<td>210</td>
<td>242.0</td>
<td>210</td>
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</tr>
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<td>233.1</td>
<td>240</td>
<td>239.4</td>
<td>240</td>
<td>268.1</td>
</tr>
<tr>
<td>250</td>
<td>232.0</td>
<td>250</td>
<td>238.5</td>
<td>250</td>
<td>267.5</td>
</tr>
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<td>260</td>
<td>230.9</td>
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<td>229.7</td>
<td>270</td>
<td>236.6</td>
<td>270</td>
<td>266.5</td>
</tr>
<tr>
<td>280</td>
<td>228.8</td>
<td>280</td>
<td>235.7</td>
<td>280</td>
<td>265.8</td>
</tr>
<tr>
<td>290</td>
<td>227.6</td>
<td>290</td>
<td>234.7</td>
<td>290</td>
<td>265.3</td>
</tr>
<tr>
<td>300</td>
<td>226.5</td>
<td>300</td>
<td>233.5</td>
<td>300</td>
<td>264.8</td>
</tr>
</tbody>
</table>

FIG. 130 TABLE SHOWING RESULTS OF STATIC PRESSURE CALIBRATION TESTS.

(using a Berrylium Copper diaphragm 25 microns thick and 3.0 mm diameter)
18.1.1  DOES A PRACTICAL SYSTEM ACTUALLY FULFIL THE REQUIREMENTS FOR A PHYSIOLOGICAL PRESSURE TRANSDUCING PROBE?

Let us consider the results of these tests, first with a view to assessing whether the total system satisfies the requirements of a physiological pressure transducer. FIG. 131 shows the results of the static pressure calibration tests plotted, together with a straight line least squares regression analysis fit for each calibration. From this figure it can be seen that, as theoretically expected, the system sensitivity increases with decreasing values for initial separation. All the characteristics were remarkably linear. The results of the regression analysis are shown below:

<table>
<thead>
<tr>
<th>Initial separation microns</th>
<th>Sensitivity mV / mm Hg.</th>
<th>Baseline mV.</th>
</tr>
</thead>
<tbody>
<tr>
<td>25.0</td>
<td>0.279</td>
<td>180.0</td>
</tr>
<tr>
<td>30.0</td>
<td>0.247</td>
<td>195.5</td>
</tr>
<tr>
<td>35.0</td>
<td>0.186</td>
<td>221.0</td>
</tr>
<tr>
<td>50.0</td>
<td>0.096</td>
<td>256.1</td>
</tr>
<tr>
<td>54.0</td>
<td>0.084</td>
<td>259.4</td>
</tr>
<tr>
<td>70.0</td>
<td>0.045</td>
<td>278.9</td>
</tr>
</tbody>
</table>

in each case the correlation coefficient was 0.999. It can now be seen that the experimental discrepancies noted in the system output for specific pressures which arose during the determination of each of the static pressure calibrations performed, i.e. ± 0.1 & 0.5 mV, can be attributed to the fact that there was an inherent inaccuracy of ± 0.5 mm Hg. in the noting of these pressures and that had the sensitivity value been determined from experimental values a ± 3% error would result.

Thus far, the system noise figure quoted has been 0.1 mV, although as mentioned earlier, it was determined to be .1 mV peak to peak. The former figure has been used to ensure a pessimistic assessment of the resolutions attainable. Thus from the above table it can be seen that
Note: dots represent experimental values and full curves the result of a mathematical fit.

FIG. 131 Static Pressure Calibration Curves—Pрактически Полученные
(Beryllium copper diaphragm 25 micron thick and 3 mm diameter)
the system requirements of resolving to a pressure of 1 mm. Hg. are satisfied up to an initial separation of say 54 microns, i.e. a sensitivity of at least 0.08 mV / mm Hg.

We are now in a position to compare the actual sensitivity for an initial separation of 35 microns with that predicted in Chapter 17.2.1. (Page 490). The predicted value is, that the system, when using a Be/Cu diaphragm 25 microns thick and of 1.5 mm radius set at an initial separation of 35 microns, should allow for a resolution of better than 0.5 mm Hg. and a sensitivity of better than 0.2 mV / mm Hg. which, when compared to that actually achieved, i.e. a sensitivity of 0.186 mV / mm Hg. is a good prediction. The quantitative comparisons between the sensitivities practically obtained and predicted are discussed in the next section; in order to define the predicted sensitivities more accurately the static pressure calibration curves as predicted are plotted and the systems predicted sensitivities are determined by the fitting a least squares regression analysis straight line.

Furthermore, the value of the maximum initial separation distance at which the systems pressure resolution requirements are met practically have been stated to be 54 microns, this confirms the figure predicted on Page 487. Let us now consider the errors incurred by using the 'straight line' fit to the practically determined static pressure calibration curve when the initial separation is 35 microns. In other words, to define for a system output, the difference between the related pressure for the 'fitted line' and the actual characteristics (or vice versa).

The mathematical fitted expression has been stated on Page 503 and is:

\[ y_{op} = -0.186 \, P + 221.0 \quad \text{P in mm. Hg.} \quad y_{op} \text{ in mV.} \]

To aid the assessment of errors incurred in using the mathematical fit FIG. 132 shows graphically the experimentally and mathematically defined static pressure calibration curves. If the system output is 200 mV (Cond. 39) then the pressure applied to the diaphragm as defined by the mathematical
fit is 114 mm Hg., whilst it is actually shown to be $120 \pm 0.1$ mV.
causing an error of $6 \pm 0.5$ mm Hg. under estimation when using the
mathematical fit. By nature of the experimental curve if a system
output level of 206 mV (Cond. 38) is considered, there is only a $\pm 0.5$ mm Hg.
error incurred. In other words, for arterial pressure measurement there
would be a maximum error of a 6 mm. Hg. under estimation of
pressure at 120 mm Hg., and this linearly decreases to zero at 80 mm Hg.
For the total pressure range the non-linearity is $\pm 2\%$ full scale reading
(notting that from $60 - 115$ mm Hg. there is only an under estimation of
3 mm Hg.).

For the practical system with the diaphragm positioned at an
initial separation of 35 microns let us now consider the effect that the
system's drift would have on its baseline and sensitivity stability.
The system's drift characteristics were determined 'in the jig' with a
set level of system output of 192 mV. (Page 496). This is close
enough to the value of the set level actually obtained for the case in
point, that no corrections to the practically determined values of time
and temperature drift are indicated.

Thus, the baseline stability is

\[
\begin{align*}
-0.08 / 220 \% \text{ F.S. / hr.} &= \langle -0.04 \% \text{ F.S./ hr.} \\
-1.1 / 220 \% \text{ F.S. / hr.} &= \langle -0.5 \% \text{ F.S./ } ^\circ \text{C.}
\end{align*}
\]

or in other words initially $\langle -0.5 \text{ mm. Hg. initially / hr}$
and $\langle -6.0 \text{ mm. Hg. initially / } ^\circ \text{C.}$
and the sensitivity stability is also

\[
\begin{align*}
\langle -0.04 \% / \text{ hr} \\
\langle -0.5 \% / ^\circ \text{C.}
\end{align*}
\]

which in finite terms are insignificant

These figures for baseline and sensitivity stability are virtually
twice those as defined for the predicted system (Ch. 17). At that time
the fact that system drift would increase proportionately with the system
set output level, was overlooked.
In conclusion, it has been clearly demonstrated that the fibreoptic pressure transducer will fulfil the requirements of a physiological pressure transducer. (See Chapter 19, The Conclusion).
FIG. 132 STATIC PRESSURE CALIBRATION CURVE-PRACTICAL, WHEN INITIAL SEPARATION IS 35 microns.

note: dots represent the experimental values and the curve the mathematical fit.
18.1.2 DO THE STATIC PRESSURE CALIBRATION CURVES DERIVED FROM GASSP 1 & 2 PREDICTIONS AGREE WITH THOSE PRACTICALLY ATTAINED?

In Chapter 17, a range of initial separation values were extracted from the GASSP 1 & 2 which, for a diaphragm deflection of approximately 15 microns, would enable the system resolution requirements to be met. It is possible for each of these initial separations to compute an expected static pressure calibration curve. This requires the use of the mathematical fit to the practically obtained static calibration curve (ascending portion) equation 212, and the expression relating diaphragm central deflections to the applied pressure to which it is subjected (equation 91).

Whereby the system output at any pressure 'P' is given by

$$y_p = 37.078 + 7.9466 \frac{d_p}{P} - 8.7078 \times 10^{-2} \frac{d_p^2}{P} + 3.3303 \times 10^{-4} \frac{d_p^3}{P}$$

where $d_p$ (in microns) the separation between distal fibre face and diaphragm is

$$d_p = d_o - \frac{3P (1 - Z^2) R_{dm}^4}{10^3.16Et^3}$$

'd_o' is the initial separation in mm.

- $R_{dm}$ the diaphragm radius mm.
- $P$ the applied differential pressure gm/mm²
- $Z$ is Poisson's ratio
- $E$ is Young's modulus gm/mm²
- $t$ is the thickness of the diaphragm mm.

and for the diaphragm considered

$$R_{dm} = 1.5 \text{ mm.} \quad Z = 0.3 \quad E = 13.3 \times 10^6 \text{ gm/mm}^2 \quad t = 0.025 \text{ mm.}$$

Prior to discussing FIG. 133 which presents graphically the static pressure calibration curves obtained practically and in the manner described above, it is profitable to re-state the probable causes for any discrepancies arising.
These are:

1) For the predictions the 'in-vitro' static calibration curve is used.
2) This curve assumes the diaphragm will deflect with a flat profile
3) The expression describing the central deflections of a thin diaphragm may not hold
4) The coefficient of reflection of the real diaphragm may differ from that used in the in-vitro static calibration determination. (the effect of this inconsistency had in fact been circumnavigated as mentioned earlier).

Consider now FIG. 133, the dots represent the practically determined values of system output and the full curves, those as predicted. A qualitative comparison between 'actual and predicted' shows:

a) that at any initial separation distance the actual sensitivity achieved is less than that predicted.

b) and that for all values of initial separation the initial system output level is practically less than that predicted.

Noting that, as stated before, there is a $\pm$ 1.5 microns uncertainty in the positioning of the diaphragm.

These comments are now to be quantitively qualified

a) That the sensitivity at any initial separation is less than that predicted.

For each of the experimentally determined static pressure calibration curves, a straight line least squares regression analysis fit was made to assess the sensitivity in each case, and has been mentioned in the previous section where they are documented and shown in FIG. 131. Similarly now straight lines are fitted to the predicted static pressure calibration curves, these being shown on FIG. 134 where the (dots) represent extracted co-ordinates from the predicted curve and the 'full curve' the straight line fit. The results of this straight line
FIG. 133 PRACTICAL AND PREDICTED STATIC PRESSURE CALIBRATION CURVES.

Note: dots represent experimental values and full curves the 'Predicted' values (see text)

Initial Sep. $d_0 = 70$ microns

$d_0 = 54$

$d_0 = 50$

$d_0 = 35$

$d_0 = 30$

APPLIED DIFFERENTIAL PRESSURE mm. Hg.
Note: 'dots' represent values extracted from predicted curve (fig. 133) and full curve represents a mathematical fit. (see text)

FIG.134 STATIC PRESSURE CALIBRATION CURVE—PREDICTED AND A MATHEMATICAL FIT TO IT.
The results of this straight line least squares regression analysis (29 data pairs) and that for the actual static pressure calibration curve are shown below.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>25.0</td>
<td>-0.279</td>
<td>-0.306</td>
<td>0.911</td>
<td>180</td>
<td>190.4</td>
<td>0.94</td>
<td>0.999</td>
</tr>
<tr>
<td>30.0</td>
<td>-0.247</td>
<td>-0.267</td>
<td>0.925</td>
<td>195.5</td>
<td>209.6</td>
<td>0.93</td>
<td>0.999</td>
</tr>
<tr>
<td>35.0</td>
<td>-0.186</td>
<td>-0.231</td>
<td>0.805</td>
<td>221.0</td>
<td>226.1</td>
<td>0.97</td>
<td>0.999</td>
</tr>
<tr>
<td>50.0</td>
<td>-0.096</td>
<td>-0.140</td>
<td>0.685</td>
<td>256.1</td>
<td>260.7</td>
<td>0.98</td>
<td>0.999</td>
</tr>
<tr>
<td>54.0</td>
<td>-0.084</td>
<td>-0.121</td>
<td>0.694</td>
<td>259.4</td>
<td>266.9</td>
<td>0.97</td>
<td>0.999</td>
</tr>
<tr>
<td>70.0</td>
<td>-0.045</td>
<td>-0.059</td>
<td>0.762</td>
<td>278.9</td>
<td>282.3</td>
<td>0.98</td>
<td>0.997</td>
</tr>
</tbody>
</table>

The feature observed graphically, that for any value of initial separation considered, the actual sensitivity is less than predicted, can be quantitively seen above. For the separations of 25, 30, 35, 50, 54 and 70 microns the under-estimation between the actual and predicted sensitivities is approximately 8, 7.5, 19, 31, 30.6 and 24% respectively.

One possible cause for these errors is due to the fact that the baseline levels are less than predicted (B /a /B ). This will reduce the levels of practically obtained sensitivities by 6, 7, 3, 2, 3, and 2% respectively which, when considering the actual under-estimations stated, still leaves the practical values to be 2, 0.5, 16, 29, 27.6 and 22% less than expected.

This is interesting because, as has been mentioned, the positioning accuracy of the initial separation distances is subject to a ± 1.5 micron tolerance and due to the nature of the static calibration curve (ascending portion), one would expect greater errors to occur at small separations. This can be seen not to be the case, and consequently it is not the positioning accuracy which is the prime cause of the under-estimation of sensitivities. It will be recalled that the 'flat diaphragm' assumption...
has been suggested as a possible cause of errors in a predicted static pressure calibration curve. In Chapter 11.1.1., it was shown, using a ray tracing technique, that for rays leaving the distal fibre face which are axially offset from $\pm$ 20 micron having a maximum angle of $60^\circ$, that the errors resulting are minimal up to separations of 60 micron. This was shown for a 30 micron thick Berrylium Copper diaphragm subjected to 300 mm. Hg., For our practical case, though, the diaphragm deflects 1.7 times that of the example given in Chapter 11.1.1., and no account was made then for the situation pertaining to more offset emergent rays.

Chapter 11 considered how, for a single ray, the actual radius $R_{\text{pre \ (TRUE)}}$ at which it returns to on the plane of the distal fibre face, compares to that at which it returns $R_{\text{pre \ (FLAT)}}$ when considering the diaphragm to deflect 'piston-like' a distance which the real diaphragm would suffer as a central deflection. But as mentioned already this comparison was only considered for near axial rays. With respect to the comments made in the previous paragraph, it is prudent to consider these effects for more axially offset rays and, assuming a diaphragm of parameters as for the practical system, FIG. 135 shows for the suggested practical system the value $R_{\text{pre \ (TRUE)}}$ and the ratio $R_{\text{pre \ FLAT}}$ plotted against values for the initial separation distance for various values of offset of a ray emerging at an angle of $60^\circ$. Let us consider condition (40) which shows the computed radius of the ray, which is offset 0.6 mm, which accounts for the diaphragm curvature, when the initial separation is 70 microns. $R_{\text{pre \ TRUE}}$ is 0.813 mm. and from condition (41) the ratio $R_{\text{pre \ FLAT}}$ resulting is 0.96. From these figures, the expected return radius when assuming a flat diaphragm is 0.780 mm. The difference between these values of returned ray is 33 microns, which is of the order of a fibre diameter for the practical system. To illustrate that the curves allow for an accurate assessment of the ratio mentioned let us compute $R_{\text{pre \ FLAT}}$ from the mathematical description for it given by equation

$$R_{\text{pre \ (FLAT)}} = r_e + 2 (d_o - 3P R_{dm}^4) (1 - z^2) / 16 Et^3 \tan \alpha$$
Be/Cu diaphragm t=25 micron P=300mm Hg R=1.5 mm when θ = 60°

Initial Separation 'd₀' microns

"R. pre TRUE" group

% Ratio R. pre TRIP

"R. pre TRUE" group

Graphical representation of the effect of initial separation has on the "Flat Diaphragm Assumption" errors - For practical system.
where \( r_e = 0.6 \, \text{mm} \) \quad \text{\( E = 13.3 \times 10^6 \, \text{gm/\text{mm}^2} \)}
\[ d_0 = 0.07 \, \text{mm} \] \quad \text{\( \angle = 60^\circ \)}
\[ P = 300 \, \text{mm Hg. (4.08 \, \text{gm/\text{mm}^2})} \]

when \( R_{\text{pre}}^{(\text{FLAT})} = 0.783 \, \text{mm} \) which is only 0.3\% in conflict with that graphically determined.

The effect of accepting the 'flat diaphragm assumption' for an initial separation of 70 microns is for an emergent ray 0.6 mm offset and angle of 60\(^\circ\) to, in practice, not actually return on to the fibre adjacent to it. Theoretically it is complex to quantitively define, for rays emerging at a multitude of angles, the overall effect of the 'flat diaphragm' assumption. However, it is clear that the prime cause of the actual sensitivities being less than predicted, is due to the fact that the diaphragm does not deflect with a flat profile and the maximum errors attributable are those as stated on page 513.

Another cause for these sensitivity differences may be due to 'the linear' description for diaphragm deflection when the true deflection/pressure relationship is not linear, but we can see that the non-linearity of the system output is minimal (FIG. 123). We must consider, however, whether the values of finite deflections, i.e. when the applied differential pressure is 300 mm Hg., can be another cause of sensitivity error.

FIG. 64a shows the ratio of the diaphragm's central deflection computed using the 'Timoshenko' more precise description (equation 99b) for diaphragm deflection compared to that computed from the linear description (equation 91). For a computed value of \( Y_{\text{max}} \) (from linear description) of say 17 micron as for our practical diaphragm, it can be seen that the more accurate value is 15 micron. Consequently, for any value of initial separation, the actual diaphragm deflection could be 2 microns less than expected, which will not explain why the sensitivity error has been determined to increase with increasing separation. The error in positioning the diaphragm practically assessed is of the same order as the
expected discrepancy in the described methods for computing the diaphragm central deflection, and therefore it is not possible to verify this aspect of the diaphragm deflection expressions.

b) The initial system output is less than for any initial separation distance than predicted.

It will be recalled that, prior to commencing the static pressure calibration tests, the system output level was reset by adjusting the position of afferent fibre ferrule in the amplifier / detector unit with the initial separation distance set to 25 microns. This was done in order to give a value of system output 2% less than expected from the probes in-vitro static calibration curve. Now, from the table shown on Page 513 where comparisons are made between actual and predicted system features, it can be seen that the maximum error of actual to predicted initial system outputs is 7%. This immediately infers that the static calibration curve as determined by the simple 'in-vitro' set up (FIG. 24) is not significantly in error, especially when the figure of 7% also includes the possible errors incurred in the fitting of the straight lines to each of the static pressure calibration curves (actual and predicted). Following on from this argument, if one were to use the experimental values of initial system output and the actual predicted values, the just quoted errors would decrease. The following table shows the ratio $B_a / B_p$ when computed in this manner.

<table>
<thead>
<tr>
<th>Initial Separation microns</th>
<th>$B_a / B_p$</th>
<th>Error %</th>
</tr>
</thead>
<tbody>
<tr>
<td>25.0</td>
<td>177 / 186.5</td>
<td>-5.0</td>
</tr>
<tr>
<td>30.0</td>
<td>192.5 / 206.1</td>
<td>-6.0</td>
</tr>
<tr>
<td>35.0</td>
<td>218.9 / 222.8</td>
<td>-2.0</td>
</tr>
<tr>
<td>50.0</td>
<td>254.9 / 258.4</td>
<td>-2.0</td>
</tr>
<tr>
<td>54.0</td>
<td>258.6 / 264.7</td>
<td>-2.0</td>
</tr>
<tr>
<td>70.0</td>
<td>277.9 / 281.0</td>
<td>-1.0</td>
</tr>
</tbody>
</table>
The errors shown are, in effect, the maximum errors arising from the static calibration curve determined 'in-vitro' rather than 'in-vivo'. It should be noted that -2% is accounted for by the fact that initially at 25 microns the system level was set 2% down and as has already been mentioned the differences in initial level cannot account for the sensitivity discrepancies between actual and predicted conditions.
18.2.0 DISCUSSION

The discussion here will be brief as certain of the points made will be repeated and elucidated in the next chapter, the conclusion. First and most significant is that by using a Berrylium Copper diaphragm 25 microns thick and of 1.5 mm radius, a pressure resolution of at least 1 mm Hg. can be achieved for the pressure transducing system when the diaphragm is positioned no further than 54 microns from the distal fibre face. Second, for the suggested system with the diaphragm positioned at 35 microns from the distal fibre face, the practically determined system sensitivity resulting is 0.186 mV / mm. Hg, which is a pressure resolution of 0.5 mm Hg. The baseline drift is \(-0.5 \text{ mm.Hg. / hr}\) initially \((-0.04\% \text{ F.S. / hr.})\) and \(-6.0 \text{ mm Hg. / } ^\circ\text{C initially}\) \((-0.5 \% \text{ F.S. / } ^\circ\text{C})\), sensitivity drift being negligible. The system's non-linearity is for full scale \(+2\%\) noting that for the pressure range of 60 - 115 mm. Hg. the non-linearity is \(-3 \text{ mm Hg. maximal.}\)

Considering now the following expectations:-

a) the 'in-vivo' static calibration curve to match the 'in-vitro' curve
b) the 'flat diaphragm' assumption to hold
c) the theoretical description for the diaphragm deflection to hold

It has been shown that:

a) a \(-4\%\) maximum discrepancy results from assuming the 'in-vitro' curve to apply for 'in-vivo' conditions
b) due to the errors inherent in the 'flat diaphragm' assumption at initial separations greater than 50 microns an over-estimation in sensitivities of at least 30% result.
c) any system output non-linearity arising from the diaphragm deflection characteristics not being linear are within that figure for system non-linearity stated above, and that if there are in practice lower deflections than predicted
c) continued/....
these can only amount to a few microns, which is within
the practical positioning accuracy of the initial
separation distance.

Now, although for the sensitivities at separations up to 35 microns,
one could attribute the over-estimation in the predicted values to there
being lower diaphragm deflections than predicted, this is only conjecture.

The GASSP 1 has shown itself to be an acceptable method for
predicting, within the practicable positioning accuracy of a diaphragm,
the maximum permissible values for the initial separation distance which
enables the system resolution requirements to be met. The GASSP 2 allows
a specification of the sensitivities at separation distances up to 70 microns
which are over-estimates of 30% maximum.

Let us now define the predicted sensitivity of the actual probe
in terms of mV / micron; first from that derived from its static calibration
curve and secondly, from that figure which results from the predicted
static pressure calibration curve which has an initial separation of
35 microns. From FIG. 122 condition 42 gives the system output arising
at this initial separation and graphically the 'slope' is determined to be

$$S_{p} = \frac{(347 - 166.6)}{50} = -3.61 \text{ mV / micron} \text{ i.e. } -1.6\% \text{ F.S. / micron}$$

and that value as predicted from the predicted static pressure calibration
curve is with respect to the table on page 513

$$S_{p} = 0.231 \text{ mV / mm Hg}.$$  
Now as the diaphragm deflection involved
in this prediction is 17 microns, and the pressure range was 300 mm Hg.
then this figure on conversion to the units of mV / micron is

$$S_{p} = .231 \times \frac{300}{17}$$

$$= 4.07 \text{ mV / micron i.e. } -1.8\% \text{ F.S. / micron}$$

There is an error of 11% which has been incurred in trying to
fit a tangent to the curve as shown in FIG. 122. However, if a tangent
to the predicted static calibration curve for the system as proposed in
Chapter 14 is fitted, FIG. 122 condition 43, then the resulting sensitivity
is
S_p = -125 mV / micron or -2.5% F.S. / micron

which is, in finite sensitivity terms, nearly 40 times that of the actual system.

Let us go one stage back in this thesis (8.1.2.1.) to arrive at a possibly more accurate determination of the above mentioned sensitivity for the system as proposed in Chapter 14; FIG. 49 shows for various optical fibre system arrangements what is, in effect, the finite change in reflection factor / micron plotted against initial separation value and the reflection factor achieved. Condition 44, for an initial separation of 35 microns, gives the sensitivity as .01 / micron when the 'set level' of reflection factor is shown by Condition 45 as 0.34. Thus the sensitivity when expressed as % F.S. / micron is

\[ S_p = -0.01 / 0.34 \times 100 \% / \text{micron} \text{ which is } -2.9\% \text{ F.S. / micron} \]

which compares favourably with that obtained from FIG. 122.

Now it has been demonstrated that there will be errors in graphical determination of the sensitivities, but nevertheless, the prime aim is to show that the sensitivities as determined from the figures shown much earlier, i.e. FIG. 49, and those which show 'slope Factor' are correct and this has been demonstrated.

The significant feature for the actual probe now becomes apparent, that is, that its sensitivity when described as a percentage of the full scale value (+1.5) is comparable to that for the proposed system as the RF values for each are .36 and \( \equiv .2 \) (page 459). Consequently, the prime cause of the lower finite sensitivities are the quantitative discrepancies between system outputs, which confirms what was deduced earlier. The re-introduction of this approach for the consideration of system sensitivity by use of the graphical presentations of 'slope factor and sensitivity' against 'initial separation and reflection factor', shows clearly that it is a combination of a fibre arrangement exhibiting a good response and a high set output level which is prerequisite for the system. This approach will also be made use of to establish the viability of a
'single fibre' transducer.
CHAPTER 19

THE CONCLUSIONS
The conclusions to a thesis are, in a way, the beginnings of other work. This is the way things should be, for we have learned that there is no full stop to be found in the search for knowledge. This thesis is the first to consider in depth, the analysis of the various system elements which form a fibre optic pressure transducer, and the first to define a method for the specification of the position in which a diaphragm of specific parameters should be located, in order to achieve the physiological requirements of range and resolution.

The initial aim of this thesis was to define theoretically what has been termed the 'reflection factor' expression from which a static calibration curve for a fibre optic probe can be formed. This is the relationship between the fraction of light received by the 'afferent fibres' in the probe's distal tip with that emitted by the 'afferent fibres', and the separation between the distal tip of the probe and a 'flat diaphragm'. The resulting expression has two limitations, first that it is derived for a system with an N.A. of unity and second, that the reflecting surface was considered to be flat. Let us consider the first limitation. Practically, it has been shown that for a probe comprised of optical fibres with an N.A. of 0.52, the errors in sensitivity and reflection factor for separations greater than the 'transition point' are acceptable, (Appendix 1). Due to the difficulty experienced in obtaining a small quantity of fibres having an N.A. of unity, it was not possible to ascertain experimentally the validity of the 'reflection factor' expression for the conditions for which it was derived.

The second limitation mentioned is that it was derived by considering the reflecting surface to be 'flat'. Ideally one requires a reflection factor expression which can be related to a fibre optic probe/diaphragm unit. Then, by considering the diaphragm to be initially placed at a specific separation from the probe tip 'd₀' and of specific parameters, it would be possible to predict the reflection factors achieved for various applied differential pressures across the diaphragm. If the
diaphragm did truly deflect with a flat profile then the reflection factor expression would need no qualification. A ray tracing technique was employed to ascertain approximately the errors which would result by assuming, in the reflection factor expression, that the central deflection of a 'real diaphragm' is equivalent to the deflection of a 'flat diaphragm'. The results of this analysis showed that for micro systems, rays which are emergent at small axial offsets and of up to an angle of 60° axial inclination, negligible errors result for separations of up to 100 microns. But for more offset rays there will be significant errors. A quantitative analysis was not performed. Practically it was concluded that, for the system constructed, the effect of this 'flat diaphragm' assumption resulted in a 30% error in sensitivity at a separation of 70 microns. The parameters upon which these errors are dependent will be numerous, but as a general comment smaller errors result with:

- increasing diaphragm radius and 'A' ratio
- and decreasing N.A., diaphragm separation and diaphragm deflection.

In retrospect, the use of the reflection factor expression to describe the fibre optic probe/diaphragm unit is justified. The method, by which one can specify the initial separations at which diaphragms exhibiting specific total deflections satisfy the pressure and resolution requirements of a physiological pressure transducer has been defined, and is of a graphical presentation (GASSP 1 & 2), and is based on the theoretically defined static calibration curve.

By defining theoretically the proposed system's noise level and static calibration curve, a GASSP 1 & 2 was constructed, from which the diaphragm and its initial separation was chosen. However, the probe constructed to the theoretical specification exhibited a much differing static calibration curve than expected in theory. This was because the fibres were not arranged in the micro fashion prescribed, the fibres' transmissions varied and the systems coupling factors were not as expected.
Consequently as the GASSPs constructed were partly based on an 'imaginary' static calibration curve and not that of the constructed probe, a second set of GASSPs were produced. This time the total system noise could also be determined practically. The practically determined characteristics of the system, as finally arrived at from these graphical aids are:

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diaphragm material</td>
<td>Berrylium copper</td>
</tr>
<tr>
<td>Diaphragm thickness</td>
<td>25 microns</td>
</tr>
<tr>
<td>Diaphragm radius</td>
<td>1.5 mm.</td>
</tr>
<tr>
<td>Sensitivity</td>
<td>0.186 mV / mm. Hg.</td>
</tr>
<tr>
<td>Pressure resolution</td>
<td>0.5 mm. Hg.</td>
</tr>
<tr>
<td>Baseline drift</td>
<td>(&lt;0.5) mm. Hg. / hr.</td>
</tr>
<tr>
<td></td>
<td>((\leq0.04% \text{ F.S.} / \text{hr.}))</td>
</tr>
<tr>
<td></td>
<td>(&lt;6.0) mm. Hg. / °C.</td>
</tr>
<tr>
<td></td>
<td>((\leq0.5% \text{ F.S.} / °C))</td>
</tr>
<tr>
<td>Sensitivity drift</td>
<td>(&lt;0.04% \text{ F.S.} / \text{hr.})</td>
</tr>
<tr>
<td></td>
<td>(&lt;0.50% \text{ F.S.} / °C)</td>
</tr>
<tr>
<td>Linearity</td>
<td>(\pm2% \text{ F.S.})</td>
</tr>
<tr>
<td></td>
<td>((-3\text{ mm. Hg. max. 60 to 115 mm. Hg.}))</td>
</tr>
</tbody>
</table>

It may be stated that if a fibre optic pressure transducer probe uses fibres of less than 60° aperture, then the philosophy to be followed in the construction of the system's GASSP 1 & 2 will depend on whether or not the fibre arrangement within the probe is truly 'micro'. If it is, and small fibres are used, then the theoretical description for its static calibration curve can be used. This is because it is only applicable to separations subsequent to the transition point, but for a true micro system, it has been shown that even on the descending slope of the curve, the sensitivity resulting is likely to be adequate for operating the pressure sensing diaphragm. However, if a probe does not show a fibre arrangement as specified then a practical determination of the probe's static calibration curve will be indicated.
For a system which uses fibres of an N.A. of unity and a fibre arrangement as specified, then it is in order to use the 'reflection factor' expression which will be valid for any value of initial separation. The limitation with regard to the 'flat diaphragm assumption' has been stated to increase with increasing N.A., but the contribution to the flux received with respect to extreme rays, when considering micro systems at the separations likely to be used, will be minimal.

By following the above procedural logic, the rationale for corroborating the specification of the final system specification extracted from the system's GASSP 1 & 2 by performing a simulated static pressure calibration determination is obviated. The next step is to fabricate the distal tip diaphragm unit and encapsulate the probe tip with it.

Lindstroem, Morikawa and De La Croix have been mentioned as other authors of work concerning a fibre optic pressure transducing probe. De La Croix discussed the system in very superficial terms. He assumed that the light emitted from an optical fibre emerges as a parallel beam which invalidates his description for what is in effect, reflection factor, a fact which he demonstrates practically. Furthermore he investigates theoretically and practically the effects of changing the parameters R, r and d on the static calibration curve, but makes no mention of the significance of the concept of 'initial separation ratio' and the 'A' ratio. All the systems he investigated are of the Macro type and consequently, possess poor response characteristics, they being typically with 'A = 3' and 'r = 800 microns' cf. Appendix 1.

Lindstroem, chronologically the next original worker in the field, describes mathematical expressions for reflection factor when considering a system of two fibres, one emitter and one receiver in close proximity, with a numerical aperture of 0.56.
The result of which is that two expressions are derived, one for separations which are "much less than the fibre's radius", the other "when they are much greater". These expressions may be applied to what we have termed 'Macro' systems (with 'A = 3') when written as

\[ R_F^d (d \ll r) = 0.375 \left( \frac{d}{r} \right)^{3/2} \]

and

\[ R_F^d (d \gg r) = 1.06 \left( \frac{d}{r} \right)^{-2} \]

The limits imposed by Lindstroem on the applicability of these expressions, can be seen by the work carried out here, to apparently exclude their application to the working range of a fibre optic pressure transducing probe. However, in order to examine exactly what Lindstroem means by his terms "much greater than" and "much less than", the following FIG. 135a shows graphically the theoretical static calibration curves after Lindstroem (which apply for a system with an N.A. of 0.56) and this author for an N.A. of unity and 0.56 considering a macro system with 'A = 3'. Prior to discussing this figure, it must be clarified that first, Lindstroem did not define the significance of the concept of separation ratio or 'R/r' ratio, and secondly, that the mathematical descriptions shown above are this author's modifications to Lindstroem's original for reflection factor. Lindstroem's equation for reflection factor considers only two fibres and the above modifications are a factor of 6x 1.25 times Lindstroem's originals. This enables them to represent the case where a Macro system prevails, the factor '6' accounting for the additional fibres present and the factor 1.25 accounting for the packing error (Chapter 9).

Consider now FIG. 135a, it can be seen that for large values of the separation ratio, the curve, after Lindstroem, approach that of this author, but there are two aspects to be considered in any comparison between what are actually static calibration curves: finite values of reflection factor and sensitivities as predicted by each method. From this
FIG. 135a THEORETICALLY PREDICTED STATIC CALIBRATION RESPONSE CURVES, AFTER 'BRAND & LINDESTROM' FOR A MACRO SYSTEM WITH 'A = 3''.'
figure the values of reflection factor from a separation ratio of '5'
remain as a ratio to one another, almost constant, with Lindstroem's
value at 0.7 of this author's value. The more interesting feature is
that of sensitivities. The following FIG. 135b, shows the value of what
has been termed earlier 'slope factor' for each author plotted against
separation ratio. It shows that a separation ratio of say 5 there is
no practically significant difference between the values as predicted by
these authors. Now bearing both the above points in mind, and with
reference to Lindstroem's expression for reflection factor at "much greater
values of separation than fibre radius", we can confirm this statement,
especially as this author's predictions have been predominantly
substantiated for separation ratios which are beyond the transition point
(Appendix 1). It is also significant that Lindstroem's condition for
acceptance of this expression for the descending slope effectively consider
the systems N.A. to be unity.

Considering now his first expression for the ascending portion
of the static calibration curve, we are unfortunately not in a position
to compare either practically or theoretically up to what value of separation
ratio his expression is valid. This is because this author's predictions
for a system's static calibration curve when the N.A. is less than unity
is known to be grossly inadequate up to the transition point after which
the standard expression holds, and also because the practical evaluation
of a system with an 'A' value of 3 was not carried out, although a
practical system where 'A' was 2.5 has been investigated (Appendix 1).
It is from the considerations of the results stated there, that one can
forecast that it is likely that Lindstroem is correct in limiting his
relationship to separations much less than the radius of the central emitter.
Furthermore, the results from the calibration of the 'standard probe'
constructed for the validation of this author's predictions which is
discussed in Appendix 1, would serve as an ideal basis for the comparison
at pre-transition point separation ratios, but Lindstroem's basic
expressions cannot be applied to any system other than one which is either
of a type described here as micro or macro with \( A = 3 \), or as it was originally derived, to represent one with only two fibres.

He makes a significant comment concerning his practically attained coupling factor \( C_f \), that is that it is only 0.05 which is in fact very similar to that achieved here. As he points out, improvements could possibly be made by the provision of an optically matched coupling and closer packing of the efferent fibres in the proximal group. His final system specification incorporates a Berrylium Copper diaphragm of only 6 microns thickness and 0.5 mm. radius, set at an initial separation of 50 microns. The salient features of the system's performance characteristics are; linearity -2.5 \% F.S., baseline drift -2.5 mm. Hg. per hour, system noise -0.5 \% F.S. (46 dB). The fibre diameter was assessed pictorially as 50 microns. The arrangement of fibres was not specified.

These performance characteristics compare favourably with the system worked on here, noting that our noise-figure is superior, whilst linearity and base-line drift are not. However, he has compensated for the effects of temperature on the photo-detector by use of a reference detector, and that of light source drift by use of a compensating system whose principle has been discussed - 13.5.1. (a logarithmic mode detector is used). Unfortunately, although for his system we know, can deduce or calculate \( A, d_o, Y_{max} \) and \( r \), because our theory is valid only for separation subsequent to the transition point (Appendix 1) and his system operates within this system value which also excludes using his theoretical description for the static calibration curve, and as no static calibration curve is given, it is not possible to assess the value of \( F_{pm}/F_o \) achieved. Whereas if the value of the input flux was known it would be possible to determine the minimum requirements for his system's signal to noise ratio.
FIG. 135b  GRAPHICAL REPRESENTATION OF 'SLOPE FACTOR' THEORETICAL "AFTER "BRAND AND LINDSTROEM" FOR A SYSTEM WITH 'A = 3' MACRO.
Morikawa describes a system whose probe was constructed for him by the American Optical Company. No indication is given as to the fibre diameter, N.A. or fibre arrangement used—In fact, a misprint of significance can be seen when he states that a 20 microns glass diaphragm of 1.47 mm. diameter is set 6 microns from his probe's distal tip face. This would virtually result in 'bottoming' at around 300 mm. Hg. On consideration of the only static calibration curve shown ('FIG. 1') it is more likely that this separation is 60 microns, if so the sensitivity at that point has been found by the author to be 0.0086 % F.S. per micron. For a diaphragm which would deflect 6 microns the total output change resulting as a percentage of full scale is 0.052, i.e. reflection factor total ratio is 0.9995 which is extremely low. The just mentioned deductions are made from an unspecified static calibration curve and are questionable but are included here to suggest that a significantly more powerful light source was used. One can be more specific in comparison with this author's work from consideration of Morikawa's two figures showing practically determined static pressure calibration curves (FIG. 4 and 7). We can arrive at sensitivity values of 20 mV./micron and 5 mV./per micron. These compare favourably with that of the practical system produced here, i.e. 3.6 mV./micron. However, for these static calibration curves no mention has been made of fibre arrangements, etc. so that we cannot predict his practically determined static pressure calibration curve. It is worth noting that the static pressure calibration curve for his most sensitive system (FIG. 4) shows that his experimental points oscillate in apparent sinusoidal fashion about the 'straight line fit' as do the author's equivalent FIG. 132.
Summarising, De La Croix approaches the problem of the analysis and design of a fibre optic pressure transducer unaware of the optical characteristics of optical fibres, and although his methodology used for predicting the response both of static and pressure types are beyond criticism, his underlying assumptions invalidate the resulting theoretical predictions. The following workers, Lindstroem and Morikawa, both produced functioning systems which cannot be faulted, but it is their appreciation of the principles upon which their systems are based which are questionable. Lindstroem derives reflection factor expressions which are not valid over a probe's useful range, and Morikawa gives little information of an analytical nature and, like the others, his probe was manufactured by the American Optical Company.

Now let us consider the value of the reflection factor total ratio achieved by the optical fibre system constructed, and derive this value by different approaches. First from the figures given for the predicted values of baseline (B) and sensitivity (S) values (mV / mm.Hg.) given on page 513 and secondly from the use of its sensitivity when expressed as a percentage change of system output / micron.

First then

\[
\frac{F_{pm}}{F_o} = \frac{B - S}{P} \times 300
\]

and is

\[
\frac{F_{pm}}{F_o} = \frac{(226.1 - 300 \times 0.231)}{226.1} = 0.70
\]

and secondly as the sensitivity at 35 microns has been determined to be 1.8 % F.S. / micron, then if the reflection factor at 35 microns is \( RF_{35} \) and the diaphragm deflection is 17 microns then

\[
\frac{F_{pm}}{F_o} = \frac{RF_{35} - 0.018 \times 17 \times RF_{35}}{RF_{35}}
\]

\[
= 1 - 0.018 \times 17 = 0.70
\]
The first point to be made is that, from the knowledge of the sensitivity expressed as \( \frac{\text{y \% F.S.}}{\text{micron}} \), the ratio can be determined without reference to the reflection factor as set. Secondly, the results achieved from use of these methods should and indeed do agree as the value of sensitivity used in the second example described was originally obtained from the values used in the first method. It is the manner in which one can make use of the sensitivity when expressed as \( \% \text{ change F.S.} / \text{micron} \) which is of interest as will be seen later, i.e. \( \frac{F_{\text{pm}}}{F_0} = 1 - \left( \frac{\% \text{ F.S.}}{\text{micron}} \right) \times \text{deflection} \), which is no more or less than a linear approximation using the 'tan' relationship for defining the total change in system output (RF or mV). It should be noted that the form of sensitivity in \( \% \text{ F.S.} / \text{micron} \) may be obtained from FIG. 49, etc.

Let us now consider whether a pressure transducing probe using a 'single fibre' (6.1.2) can satisfy the system's requirements. FIG. 48 shows, using normalised terms, the 'slope factor and reflection factor' achieved, plotted against the initial separation ratio value for a fibre of N.A. of unity (which are obtainable). Then consider condition 46 which sets the achieved value of the reflection factor initially as 0.2 (a semi-arbitrary choice as will be discussed). The initial separation ratio than becomes defined as 0.94 and the 'slope factor' (condition 47) becomes 0.3. To convert the value slope factor to sensitivity (\( \% \text{ change F.S.} / \text{micron} \)), we must divide by the fibre radius in microns and the value of the set reflection factor (0.2). Below is shown the sensitivities resulting by consideration of several fibre radii.

<table>
<thead>
<tr>
<th>'single fibre' radius</th>
<th>% F.S. / micron</th>
<th>'Y' max</th>
<th>'d' o</th>
<th>'R' dm</th>
<th>F pm/F o</th>
</tr>
</thead>
<tbody>
<tr>
<td>micron</td>
<td>micron</td>
<td>micron</td>
<td>micron</td>
<td>mm</td>
<td></td>
</tr>
<tr>
<td>20.0</td>
<td>7.5</td>
<td>4.08</td>
<td>18.8</td>
<td>1.05</td>
<td>0.7</td>
</tr>
<tr>
<td>35.0</td>
<td>4.2</td>
<td>7.28</td>
<td>32.9</td>
<td>1.21</td>
<td>0.7</td>
</tr>
<tr>
<td>50.0</td>
<td>3.0</td>
<td>10.3</td>
<td>47.0</td>
<td>1.32</td>
<td>0.7</td>
</tr>
<tr>
<td>75.0</td>
<td>2.0</td>
<td>15.3</td>
<td>70.5</td>
<td>1.46</td>
<td>0.7</td>
</tr>
<tr>
<td>100.0</td>
<td>1.5</td>
<td>20.4</td>
<td>94.0</td>
<td>1.57</td>
<td>0.7</td>
</tr>
</tbody>
</table>
Thus, with an initial reflection factor of 0.2, all the sensitivities shown are greater than that for the practically constructed probe, and when bearing in mind that the reflection factor set initially for the 'pressure transducing probe' was of the order of 0.2 also. If one then assumes that an equivalent light flux were to enter the single fibre set up, as for the practically produced system, then one could state that if a diaphragm were to be used such that its deflection would enable a reflection factor total ratio identical to that achieved for the practical system, then the 'single fibre' would be in equivalence with the 'practically produced' one. The preceding table thus also shows the diaphragm total deflections necessary for this condition to be met, these figures being calculated in the manner described on page 534 and the preceding page, accompanied by the radius of a diaphragm which of Berrylium Copper.

This brief feasibility study is given because the constructional problems involved in the 'optical fibre' arrangement would disappear, although as mentioned, there would be more stringent limits placed on the 'light' source requirements. It can be seen that the example given is not an optimum for the single fibre system, as by reducing the initial separation distance for each of the radii mentioned a significant increase in both the 'sensitivity and initial reflection factor' result.

By taking a safe figure of 0.3 for coupling factor $C_f$ achieved in our practical system, if a single fibre system were to be utilised, a light source capable of providing a flux level of 0.3 mV would be required, for a fibre of 75 microns this would mean an equivalent flux density of 170 mV/mm$^2$.

Another method by which the probe's construction could be simplified, yet still result in a well defined fibre arrangement was considered. The concept on which it is based is that, if one produced a 'long fibre tape', (a sheet of fibres one fibre thick which lie with their axes parallel and are held together by some form of lacquer), and
if their distal ends are wound into a tight spiral and the fibres are of adequate length so that their proximal ends can be fanned out, then the effective fibre arrangement can be accomplished by a process of splitting of these fibres. The relative numbers of fibres in each separated band will increase as one moves from one side of the tape to the other.

Mathematically, one can define the progressive increments of fibres required to produce at their distal end a fibre arrangement which will be effectively composed of annular receivers and emitters. Use of such a system would necessitate a modified theoretical assessment. However, if such a fabrication technique were successful it would warrant further work. Due to time limitations in carrying out this work a true diaphragm was not fabricated in place on to the practical probe. Consequently, no in-vitro or in-vivo tests on a finished system were performed. In fact, another probe would have been constructed either of the single fibre type or of the form already constructed, but this time a venting catheter would have been incorporated.
APPENDIX 1

AN ASSESSMENT OF THE VALIDITY OF

THEORETICALLY DERIVED EXPRESSION

FOR REFLECTION FACTOR.
In order to assess the validity of the theoretically derived expression for reflection factor (equation 78), and the approximate method suggested which accounts for the optical fibres within the probe having a numerical aperture of less than unity (equation 80), a 'probe' having a well defined fibre arrangement in the distal tip group was constructed in a similar manner to that described in 16.1.0. Ideally several 'probes' would have been fabricated, each of different numerical apertures, however, due to limitations of availability and time, only one probe was constructed. This 'probe' uses a single, large optical fibre (clad rod) of 1 mm diameter as a central emitter and this is surrounded by an annulus of fibres each 80 μ in diameter (supplied by R.T.H.), the effective outside diameter of this 'macro' system being 2.5 mm. Thus in the terms used in this thesis 'A = 2.5' and 'r = 500 microns'. The numerical aperture of the system is 0.53, i.e. the half angle of acceptance is 32°. Let us term this probe the 'standard probe'. Determination of the practically assessed static calibration curve of this standard probe was achieved by coupling it to the 'light source and amplifier/detector unit', and using the set up as depicted by FIG. 24. Due to the fact that a rigid efferent group was employed, the calibration of the system required care in order to avoid the fracture of the single fibre which was held at one end into the amplifier/detector unit with the other in a clamp on the optical bench. The results of the static calibration determination are shown in FIG. 136. To convert the values of system output into equivalent values of 'reflection factor' it was necessary to obtain a value of system output which would be equivalent to a reflection factor of unity. This was achieved by setting the distal tip of the standard probe into the photo-detector unit in place of the afferent fibre group. The value of system output resulting 'Eₐ' then represented the baseline to which, by referring all the values of adjusted system output achieved during static calibration determination, one obtains the values of equivalent reflection factor.
<table>
<thead>
<tr>
<th>Separation Distance (microns)</th>
<th>System Output (mV.) ± 0.05</th>
<th>Reflection Factor (rationalised) ± 5 x 10^-5</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2.0</td>
<td>0.0022</td>
</tr>
<tr>
<td>100</td>
<td>16.0</td>
<td>0.0175</td>
</tr>
<tr>
<td>200</td>
<td>32.1</td>
<td>0.0351</td>
</tr>
<tr>
<td>300</td>
<td>49.2</td>
<td>0.0538</td>
</tr>
<tr>
<td>400</td>
<td>66.6</td>
<td>0.0729</td>
</tr>
<tr>
<td>500</td>
<td>83.5</td>
<td>0.0914</td>
</tr>
<tr>
<td>600</td>
<td>96.7</td>
<td>0.1058</td>
</tr>
<tr>
<td>700</td>
<td>104.4</td>
<td>0.1143</td>
</tr>
<tr>
<td>800</td>
<td>107.7</td>
<td>0.1179</td>
</tr>
<tr>
<td>900</td>
<td>109.6</td>
<td>0.1202</td>
</tr>
<tr>
<td>1000</td>
<td>108.1</td>
<td>0.1183</td>
</tr>
<tr>
<td>1100</td>
<td>104.9</td>
<td>0.1148</td>
</tr>
<tr>
<td>1200</td>
<td>100.4</td>
<td>0.1099</td>
</tr>
<tr>
<td>1400</td>
<td>89.5</td>
<td>0.0979</td>
</tr>
<tr>
<td>1500</td>
<td>83.5</td>
<td>0.0914</td>
</tr>
<tr>
<td>1600</td>
<td>77.9</td>
<td>0.0853</td>
</tr>
<tr>
<td>1700</td>
<td>72.4</td>
<td>0.0792</td>
</tr>
<tr>
<td>1800</td>
<td>67.5</td>
<td>0.0740</td>
</tr>
<tr>
<td>1900</td>
<td>62.7</td>
<td>0.0686</td>
</tr>
<tr>
<td>2000</td>
<td>58.2</td>
<td>0.0637</td>
</tr>
<tr>
<td>2100</td>
<td>54.1</td>
<td>0.0592</td>
</tr>
<tr>
<td>2200</td>
<td>50.6</td>
<td>0.0554</td>
</tr>
<tr>
<td>2400</td>
<td>44.4</td>
<td>0.0486</td>
</tr>
<tr>
<td>2600</td>
<td>39.0</td>
<td>0.0427</td>
</tr>
<tr>
<td>2800</td>
<td>34.4</td>
<td>0.0376</td>
</tr>
<tr>
<td>3000</td>
<td>30.5</td>
<td>0.0334</td>
</tr>
<tr>
<td>3200</td>
<td>27.3</td>
<td>0.0298</td>
</tr>
<tr>
<td>3700</td>
<td>21.0</td>
<td>0.0229</td>
</tr>
<tr>
<td>4200</td>
<td>16.8</td>
<td>0.0184</td>
</tr>
<tr>
<td>4700</td>
<td>13.7</td>
<td>0.0150</td>
</tr>
<tr>
<td>5200</td>
<td>11.4</td>
<td>0.0125</td>
</tr>
</tbody>
</table>

FIG.136 THE RESULTS OF THE STATIC CALIBRATION DETERMINATION FOR THE "STANDARD PROBE"
The system output must be adjusted to that level which 'falls' on to the distal tip face. In other words, an allowance must be made for the losses of radiant flux which arise during its transport through the afferent fibres. For the length of the fibres used (0.650 m). The fibre transmission is between 45 and 53%, (from the supplier's catalogue). Taking a value of 50%, then each of the values of system output must be 'adjusted' by multiplying by a factor of '2'. When the ratio of the adjusted value of system output to the value of $E_f$ gives the value of the equivalent reflection factor resulting for the standard probe, the actual value of $E_f$ measured was 1830 mV, and the values of reflection factor achieved are to be found on FIG. 136 also, and have been termed there as 'reflection factor rationalised'. The experimental error incurred in the values of system output and RF are seen to be small, but it should be noted that the latter are derived by assuming the afferent fibre losses to amount to 50%. As the actual transmission band is 45 - 53, then one can say that all the values stated for experimental values of reflection factor are within the bounds of being 11% greater than shown or 5% less than shown. Furthermore, the experimental error is of insignificant proportions compared to the finite values of RF calculated. The effect of this effective baseline shift will be to either increase the probes sensitivity or decrease it, by the values stated. This will be mentioned later.

For the present, consider the loss to be 50%. Then the following FIG. 137 graphically presents the static calibration curve for the standard probe as shown by the 'dots', the theoretically defined relationship for an N.A. of unity and for an N.A. equal to that for the standard probe (0.53) is also shown here by the full curves. From this figure the following comments can be made:-

1) as expected the response of the standard probe does not follow the predictions from the theory for a system with an N.A. of unity (equation 78).
the simple method of accounting for a system with an N.A. of less than unity (equation 80) is with provision, valid after the transition point has been passed.

In order to compare quantitatively, from the transition point, what the errors between theory and practice amount to, we require to define for the actual and predicted conditions at specific values of separation distance, the resulting values of both reflection factor and sensitivity. For the actual conditions, it is permissible to consider these to occur at initial separations subsequent to the transition point, which are midway between the interval spacing used during the determination of the response. This assumes linear characteristics to prevail between the interval spacings. The values of reflection factor predicted theoretically and sensitivity are computed for these separations from equations 80 and 85 respectively. The following FIG. 138 shows the results of the calculations. From this figure it can be seen that, at separations greater than 2050 microns, the error incurred theoretically is at maximum 25% (at 2050 microns). Now as was mentioned on the preceding page, the defined sensitivities practically determined are all subject to a baseline shift of between +11% and -5%. Thus, at best, the predictions regarding sensitivity will allow for acceptance with practical achievement (25% error) from a closer initial separation, i.e. 1850 microns and at worst, extend this distance to 2300 microns.

These errors are considered by the author to be acceptable for the validation of the theoretical descriptions for reflection factor when the transition point has been passed, especially as the assumptions which are made for the theoretical analysis are known in practice not to be satisfied, i.e. the polar distribution within the N.A. is not uniform the total area of the distal tip is not an active receiver.
Note: full lines show theoretical plots. 'dots' show experimental values.

FIG 137  STATIC CALIBRATION RESPONSE CURVE
PRACTICAL AND THEORETICAL, when 'A= 2.5'
AND THE EMITTER RADIUS IS 500 microns (macro...
<table>
<thead>
<tr>
<th>Initial Separation (microns)</th>
<th>Sensitivity (\times 10^{-4}) (RF/micron)</th>
<th>Reflection Factor</th>
<th>(R_{a-p})</th>
<th>(S_{a/p})</th>
</tr>
</thead>
<tbody>
<tr>
<td>1450</td>
<td>0.650 ±1.5% 1.387</td>
<td>0.0946±1.5% 0.1253</td>
<td>0.0307 0.468</td>
<td></td>
</tr>
<tr>
<td>1550</td>
<td>0.610 ±1.6% 1.194</td>
<td>0.0883±1.6% 0.1125</td>
<td>0.0242 0.510</td>
<td></td>
</tr>
<tr>
<td>1650</td>
<td>0.610 ±1.6% 1.032</td>
<td>0.0823±1.6% 0.1013</td>
<td>0.0190 0.591</td>
<td></td>
</tr>
<tr>
<td>1750</td>
<td>0.520 ±2.0% 0.897</td>
<td>0.0766±2.0% 0.0907</td>
<td>0.0141 0.579</td>
<td></td>
</tr>
<tr>
<td>1850</td>
<td>0.540 ±1.8% 0.783</td>
<td>0.0710±1.8% 0.0833</td>
<td>0.012 0.689</td>
<td></td>
</tr>
<tr>
<td>1950</td>
<td>0.490 ±2.0% 0.686</td>
<td>0.0662±2.0% 0.0760</td>
<td>0.0098 0.714</td>
<td></td>
</tr>
<tr>
<td>2050</td>
<td>0.450 ±2.2% 0.604</td>
<td>0.0615±2.2% 0.0696</td>
<td>0.0081 0.745</td>
<td></td>
</tr>
<tr>
<td>2150</td>
<td>0.380 ±2.6% 0.534</td>
<td>0.0573±2.6% 0.0639</td>
<td>0.0060 0.711</td>
<td></td>
</tr>
<tr>
<td>2300</td>
<td>0.340 ±1.4% 0.448</td>
<td>0.0520±1.4% 0.0565</td>
<td>0.0045 0.758</td>
<td></td>
</tr>
<tr>
<td>2500</td>
<td>0.295 ±1.6% 0.358</td>
<td>0.0456±1.6% 0.0485</td>
<td>0.0029 0.824</td>
<td></td>
</tr>
<tr>
<td>2700</td>
<td>0.255 ±2.0% 0.291</td>
<td>0.0402±2.0% 0.0420</td>
<td>0.0018 0.876</td>
<td></td>
</tr>
<tr>
<td>2900</td>
<td>0.210 ±2.3% 0.239</td>
<td>0.0355±2.3% 0.0368</td>
<td>0.0013 0.878</td>
<td></td>
</tr>
<tr>
<td>3100</td>
<td>0.180 ±2.7% 0.198</td>
<td>0.0316±2.7% 0.0324</td>
<td>0.0008 0.909</td>
<td></td>
</tr>
<tr>
<td>3450</td>
<td>0.138 ±1.4% 0.147</td>
<td>0.0264±1.4% 0.0264</td>
<td>0.0 0.938</td>
<td></td>
</tr>
<tr>
<td>3950</td>
<td>0.090 ±2.2% 0.0997</td>
<td>0.0206±2.2% 0.0204</td>
<td>-0.0002 0.900</td>
<td></td>
</tr>
<tr>
<td>4450</td>
<td>0.068 ±2.9% 0.0707</td>
<td>0.0167±2.9% 0.0161</td>
<td>-0.0006 0.960</td>
<td></td>
</tr>
<tr>
<td>4950</td>
<td>0.050 ±4.0% 0.0519</td>
<td>0.0137±4.0% 0.0131</td>
<td>-0.0006 0.970</td>
<td></td>
</tr>
</tbody>
</table>

**FIG. 138** TABLE SHOWING SENSITIVITIES AND REFLECTION FACTORS FOR PRACTICAL AND THEORETICAL CONDITIONS
APPENDIX 2

THE RATIO OF EFFERENT TO AFFERENT FIBRES

FOR A MICRO SYSTEM WERE 'A = 3'
If the fibres are arranged in the distal tip of a probe such that one efferent fibre is surrounded by six afferent fibres throughout, then the numbers of efferent and afferent fibres used may be determined as follows. Consider FIG. 139 which shows such a tip arrangement of fibres. All the digits represent efferent fibres and the 'corners of the surrounding hexagons signify the afferent fibres. It is evident that a sharing of afferent fibres by different efferent ensues in such an arrangement, with the result that the overall ratio of the number of efferent fibres \( (N_{ip}) \) to the number of afferent fibres \( (N_{op}) \) will be greater than one sixth.

Let us designate 'n' as the number of rings of concentric efferent fibres present, then from the figure shown it follows that

when \( n \) is 0, 1, 2, 3, 4, 5, 6,

\[
\begin{align*}
N_{ip} & = 1, 7, 19, 37, 61, 91, 127, \\
N_{op} & = 6, 24, 54, 96, 150, 216, 294, 
\end{align*}
\]

these numbers obtained by inspection of FIG. 139
d from which the series representing \( N_{ip} \) and \( N_{op} \) can be written

\[
\begin{align*}
N_{ip} & = 6 \left( n + (n - 1) + (n - 2) + (n - 3) + (n - n) \right) + 1 \\
N_{op} & = 6 \left( n (2n + 1) - 2((n - 1) + (n - 2) + (n - 3) + (n - n)) + 6 \right)
\end{align*}
\]

which on simplifying the arithmetic progressions become

\[
\begin{align*}
N_{ip} & = 3n (n + 1) + 1 \\
N_{op} & = 6n (n + 2) + 6
\end{align*}
\]
FIG. 139 EFFERENT AND AFFERENT FIBRE GROUPING FOR A MICRO SYSTEM WITH 'A = 3'
The following FIG. 140 shows using the last mentioned equations the value of the numbers of efferent fibres and afferent fibres for a basic micro system plotted against the 'n' value applicable. From this figure or the equations it can be seen that for 'n' values of 15 or more the ratio of \( N_{ip} / N_{op} \) approximates to 1/2.

Now if the outside radii of the active fibres in the distal tip is \( R_o \) and the 'n' value for a close packed hexagon arrangement is

\[
n = R_o / (2r \sqrt{3})
\]

which for the case in point is \( n = 22 \) : when \( R_o = 1.5\text{mm} \) and \( r = 20 \text{microns} \).
FIG. 140 GRAPH SHOWING THE NUMBERS OF INPUT AND OUTPUT FIBRES FOR A SYSTEM WITH 'A=3 micro'
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