ON THE MAGNETIC PROPERTIES OF SHEET STEELS.

A STUDY OF THE EFFECTS OF PREVIOUS MAGNETISATION ON THE PERMEABILITY AND MAGNETIC LOSSES OF CERTAIN MAGNETIC MATERIALS IN THE FORM OF THIN SHEETS.

Thesis submitted for the degree of Master of Science in the Faculty of Engineering of the University of London.

by J.C. Fleming Bsc., A.M.I.E.E.
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The experimental work described in this paper was carried out on sheet specimens of Stalloy and Mumetal, together with a few tests on Lohys.

The main object was to determine the effect of magnetic polarisation due to previous magnetisation on the magnetisation curves and magnetic losses.

The specimens were tested with D.C. by means of ballistic galvanometer, and with 50 cycle A.C. supply, by means of A.C. potentiometer and synchronous rectifier voltmeter.

The maximum reduction in permeability, at a given flux density, which was recorded in the D.C. tests amounted to about 45% with Stalloy and Lohys compared with 31% with Mumetal. The A.C. figures were about the same for Stalloy and somewhat lower for Mumetal.

The D.C. hysteresis loss increased by a maximum amount of 36% for Stalloy compared with 4% for Mumetal. The A.C. losses, including eddy currents, showed smaller increases, which became hardly measurable in the case of Mumetal. These extreme figures are obtained at very low flux densities, and following polarisation at very high magnetising force. No appreciable effect results from polarisation unless the normal working flux density is low and the polarising magnetising
force is at least 5 times normal.

The effect of polarisation on the shape and location of the hysteresis loops was investigated.

The degree of stability of the changes was investigated and was found to be surprisingly high with Stalloy. The polarisation in Mumetal could be reduced somewhat by vibration, A.C. magnetisation etc., but only to the extent of less than 50%. It has been shown that very serious polarisation can be produced in Mumetal by mechanical shock when the magnetising current is flowing.

Comparison of D.C. and A.C. magnetisation curves drew attention to the discrepancy between these, a subject which has received the attention of many investigators recently. The curves are discussed and certain suggestions are made.

In an appendix, the theory of the ballistic galvanometer is considered, with particular reference to the use of the instrument for iron testing when shunted to give critical damping.
INTRODUCTION.

The research work, of which the following thesis is a description, originated in a very minor investigation. This set out to find the magnitude of the increase in the errors of a current transformer, due to open circuiting its secondary circuit when the primary winding was carrying current.

It soon became apparent that it would be desirable to extend the work to cover a much larger field, and deal with the general effects of residual magnetism in magnetic materials working with low alternating flux densities.

The term "Magnetic Polarisation", or simply "Polarisation" is used hereafter to denote a magnetisation of higher magnitude than normal, which leaves the material with a residual magnetism sufficient to alter its magnetic properties.

It is immaterial whether this polarisation is due to alternating or direct currents, since, if the alternating current is interrupted suddenly, without gradual reduction in magnitude, the same degree of polarisation is obtained as would result from magnetisation with direct current of an equal maximum value.
The theoretical aspect of the problem is comparatively simple. Figure 0 shows a hysteresis loop with a fairly high maximum flux density. A ring sample which has been polarised to this flux density will, on the removal of the magnetising force, possess a retained magnetism or retentivity or. Subsequent alternating magnetisation of a much lower magnitude will result in the ring following a displaced hysteresis loop such as ACA'C', having its negative tip A' slightly below the corresponding point on the main hysteresis loop. This displaced loop will require a larger magnetising force, for the same range of flux density, than the corresponding loop for the demagnetised specimen shown as DED'E' in the figure. In general also the area of the displaced loop, and therefore the hysteresis loss in the polarised condition will be greater than in the demagnetised state. It has been found that both the shape and position of the displaced loop are remarkably stable, being unaffected by the duration of the alternating magnetisation, after the first few cycles, and also little affected by vibration or mechanical shock.

Some attempts were made in the early stages of the work to trace the reduction in polarisation during the first few cycles of alternating magnetisation, but as the total effect was very small and the results somewhat
erratic, they are not reproduced in this paper.

The extension of the investigation to the general case was largely due to the fact that author could not discover any previous work directly dealing with the subject. It is evident that polarisation of a high order may result from short circuit or other abnormal condition in any piece of apparatus which employs a magnetic material, normally working at low flux density. The subject seemed therefore of sufficient importance to justify a fairly extensive investigation of the effects of polarisation. Evidence that the effects of polarisation were negligible would be welcomed as useful information, though not of course as interesting as results which might lead to alteration of present methods.

The experimental work described in this paper has been carried out in the Electrical Laboratories of Battersea Polytechnic, London, chiefly during the period 1930 to 1934. The author's normal duties, and the use of laboratories and apparatus for classes has necessitated the experiments being carried out in a fragmentary manner, to a large extent during vacation periods. Moreover, much time has been wasted and experimental work rendered valueless due to disturbance of apparatus which had to be used for class work.
The author would like to acknowledge his indebtedness to the Principal and Governing Body of Battersea Polytechnic for affording facilities for the work, and particularly to Mr. A.T. Dover for his advice and inspiration.
EXPERIMENTAL METHODS.

SPECIMENS.

Stalloy Large Rectangular specimen.

This specimen, which is illustrated in Fig.1 is made up of rectangular Stalloy stampings 0.5 mm. thick, and was used for most of the ballistic tests on this material.

Data:

Area of cross section, 18.2 sq.cm.
Length of mean line, 122 cm.
Number of turns in magnetising winding 1000
Number of turns in secondary winding 25 or 5.

Stalloy ring specimen.

This specimen, illustrated in Fig.2 is a small current transformer core consisting of ring stampings, 0.5 mm. thick, having a permanent secondary winding of 59 turns. Various additional windings were provided during the course of the experiments. This specimen was used for the whole of the alternating current tests on Stalloy.

Data:

Area of cross section, 9.9 sq.cm.
Length of mean line, 25.9 cm.

It is realised that, with this specimen, the value of H must vary slightly from the inside to the outside of the
Fig. 1

RECTANGULAR STALLOY SPECIMEN.
ring, owing to the small mean diameter compared with the radial thickness. No correction has been made for this however as percentage changes rather than absolute values are the aim of this work.

Lohya Specimen.

This specimen is very similar to the large rectangular Stalloy specimen.

Data:

<table>
<thead>
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<th>Description</th>
<th>Value</th>
</tr>
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<tbody>
<tr>
<td>Area of cross section</td>
<td>17 sq.cm.</td>
</tr>
<tr>
<td>Length of mean line</td>
<td>122 cm.</td>
</tr>
<tr>
<td>No. of turns of magnetising winding</td>
<td>957</td>
</tr>
<tr>
<td>No. of secondary turns</td>
<td>Various</td>
</tr>
</tbody>
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Mumetal Specimen.

This specimen, illustrated in Fig. 3, is also a current transformer core in the form of a ring. In this case the ring is made of a long strip of mumetal made up in a tight spiral.

Data:

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<tr>
<td>Area of cross section</td>
<td>10.1 sq.cm.</td>
</tr>
<tr>
<td>Length of mean line</td>
<td>24.0 cm.</td>
</tr>
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The specimen has been provided with various magnetising and secondary windings, details of which are given in the description of the experimental work.
**Fig. 3.**

**Mumetal Specimen.**
APPARATUS.

BALLISTIC GALVANOMETER TESTS.

The instrument employed for most of this work was specially made by H. Tinsley & Co., and has an exceptionally long periodic time of swing, with comparatively high sensitivity and a high value of parallel resistance for critical damping.

The constants of the instrument are as follows:

- **Sensitivity**, 2800 mm. deflection per microampere at 1 metre.
- **Periodic Time of Swing** (undamped), 16.5 sec.
- **Damping resistance** 18000 ohm.

The long periodic time is obtained by the use of a suspension 22 cm. long and by the use of small balance weights mounted on arms under the galvanometer case.

Some of the earlier work was done on a Gambrell cable testing galvanometer having a periodic time of 10 sec. This galvanometer was made from a low period instrument by suspending a cylindrical lead weight below the coil, out of the field of the magnet. Both instruments were of the permanent magnet, moving coil type.

The connections employed are shown in Fig. 4. The two ammeters used for measuring the magnetising current were:

1. Weston precision ammeter 0 - 10 amperes
2. Siemens precision ammeter with shunts for
CIRCUIT DIAGRAM FOR BALLISTIC GALVANOMETER TEST

12 VOLT BATTERY

67A. Resistance

10Ω

12Ω

MERCURY REVERSING SWITCH

CAMAGELL STANDARD VARIABLE MUTUAL INDUCTION
0.15, 0.3, 0.75, 1.5, 3.0 and 7.5 amperes, which can be very easily changed. The supply is taken through either of the two ammeters by a two-way plug switch, which also alters the arrangement of the resistances. With the larger ammeter in circuit, the heavy current rheostats only are connected and these will carry currents up to 10 amperes. With the lower range ammeter, the 67 ohm rheostat may be used either in series or as a potentiometer, providing currents from 1.5 amperes down to zero.

The reversing switch is of the mercury cup type, and has two extra cups, by means of which an additional rheostat may be inserted in the circuit when the main rocker is on the right hand side. These extra cups are normally shorted by means of a link.

When a B-H curve is to be obtained, the link is in place, and the magnetising current reversed, the ballistic galvanometer reading giving the corresponding change in linkage or flux density. In the case of a Hysteresis Loop the intermediate points are obtained either by a sudden increase in the resistance in the magnetising current circuit, without reversal; or by reversing the current and, at the same time, inserting the extra resistance. In the first case the link is removed to effect the desired change, the rocker being on the right hand side.
In the second case the change is effected by moving the rocker from the left to the right hand side, the link having previously been removed. In either case the resulting value of $B$ is found from the change from the initial positive or negative maximum value.

In the case of the loops in the polarised state, it is necessary to obtain a complete set of points for both positive and negative currents, since the loops are distorted. In the demagnetised condition, as the loops are symmetrical about either axis, it is necessary to obtain points with positive values of current only.

A reversing switch is placed in the secondary circuit so that all deflections may be in the same direction, thus obviating zero creep of the galvanometer.

The galvanometer was calibrated by means of a Campbell Variable Mutual Inductance having a maximum value of 11,000 microhenry. The fixed resistance secondary of this was permanently in series with the secondary winding on the specimen, and the primary of the mutual inductance could be connected in place of the specimen primary by means of a two-way key. In most of the work the galvanometer series resistance was set so as to make the galvanometer direct reading in change of flux density. By having two secondary windings, with numbers
of turns in the ratio of 5 to 1, a large range of flux density changes can be covered with one value of series resistance.

BALLISTIC GALVANOMETER DAMPING.

A considerable amount of attention has been devoted to this question. It seems to be usual in this country to arrange the galvanometer with as small an amount of damping as possible, and work was commenced with this arrangement. With the very delicate suspension employed, trouble was experienced due to irregular vibration in the building. The galvanometer had to be kept shorted until immediately before taking the reading, and erratic readings were periodically experienced due to this cause, necessitating repeating every reading two or three times. Further the open circuit zero generally differed from the shorted zero due to thermal or contact E.M.F's.

It was decided to experiment with the galvanometer approximately critically damped, obtained by means of shunt resistance. It was first ascertained by careful experiment that the relation between deflection and change of linkage remained linear with different values of flux density and that the calibration with the standard mutual inductance could be relied upon. The problem of the critically damped and shunted galvanometer was also
investigated mathematically together with other aspects of ballistic galvanometer theory. An account of this work appears in Appendix I.

As a result of this change the speed of testing was considerably increased, due largely to the galvanometer automatically returning to zero in the minimum time. It was also found that the readings were much more consistent as vibration had much less effect on the galvanometer. With regard to the accuracy with which the deflection could be read, although the time taken to reach the maximum value is less, the rate of change of deflection with time, with critical damping, is very much the same, in the neighbourhood of the maximum reading, as with negligible damping. This is illustrated in the curves of Figs. 44 showing the calculated deflection time curves for the two cases. There is, of course, a considerable reduction in sensitivity, but this is rarely of much importance in iron testing where it can be compensated for by decrease in series resistance or increase in secondary turns.

Some attempt was made to use a "Null" method, by opposing the E.M.F.'s. induced in the secondary windings of the specimen and mutual inductance. The difference in the time constants of the two circuits resulted in a double deflection and in view of this together with the
longer time required to obtain a reading, the method was abandoned. It is possible that, if the two primary windings could be connected in series, and with a long period galvanometer or preferably a fluxmeter, the "Null" method would give a greater accuracy.

An attempt was made at a certain stage to use a Cambridge Grassot fluxmeter in the place of the ballistic galvanometer. A preliminary calibration of the instrument showed that the accuracy obtainable with it was not as good as could be obtained with the ballistic galvanometer, and that the fluxmeter had no advantages over the latter instrument.
ALT ERNAT I NG C U R R EN T POTENT I O M E TE R T ES T S.

In addition to Ballistic Galvanometer tests, it was thought desirable to obtain magnetisation curves and curves of iron losses, now including eddy current losses, with alternating magnetisation at 50 cycles per second. This investigation, while not giving as much information on the nature of the changes produced by polarisation, deals with the normal practical conditions.

For these measurements the Call Tinsley Co-ordinate type of A.C. Potentiometer was chosen. The A.C. potentiometer has the advantage over other methods that its ability to measure very small voltages and powers enables small specimens having windings of comparatively few turns to be used.

The general scheme was to measure the vectors of primary current and E.M.F. induced in a secondary winding. From these the total iron losses and the R.M.S. value of the magnetising current can be obtained directly. In order to get the maximum values of magnetising current and flux density, it is necessary to have certain information about the wave forms of the current and induced voltage. In the present work, the primary current was constrained to follow a sine wave by keeping a large non-inductive resistance in series with the
primary winding. As a result of this the secondary E.M.F. will necessarily be distorted. The A.C. potentiometer gives the R.M.S. value of the fundamental component of this wave, whereas what is required for calculation of the maximum value of flux density is the Mean Value of the half wave of E.M.F.

Various devices were tried for measuring this Mean Value. Metal and Thermionic Valve rectifiers were discarded on account of the non-linearity of their characteristics at low voltages, and finally a half-wave mechanical rectifier was built and used in connection with a sensitive unipivot galvanometer. A description of this instrument is given later in this section. The mechanical rectifier voltmeter was calibrated on a sinusoidal voltage consisting of the D.D across a non-inductive resistance, and was then used to obtain a curve relating the R.M.S. value of the fundamental component of the induced E.M.F. with its true mean value, with each specimen in both demagnetised and polarised states. The rectifier was not used for every reading, because, firstly, it was not available for the earlier work, and secondly the load imposed by the synchronous motor rather upset the circuit conditions.

The complete circuit diagram employed for these tests is given in Fig. 5. It is not proposed to explain
in detail the theory, and method of calibrating and using the potentiometer, as far as this is common knowledge. There are however some special points of interest in connection with the arrangements employed and experience gained. In connection with the theory, it was felt that the phase splitting circuit employed with this potentiometer might, with advantage, receive some investigation. The vector diagram for this, with all the vectors drawn to scale and in their correct positions, is given in Appendix 2.

The A.C. supply was obtained from a small three phase alternator of the rotating pole type driven by a D.C. shunt motor. The D.C. motor and the alternator field were supplied from a large 100 volt battery. Even with this arrangement it was exceedingly difficult to keep the alternator voltage and frequency sufficiently steady. The commutator and slip rings required frequent cleaning, and the set had to be run for at least an hour to warm up before readings could be taken. More trouble was experienced with the slip rings than with the commutator. The reflecting dynamometer which indicates the current in the In Phase potentiometer has a sensitivity such that 1 mm. on the scale corresponds to a variation of 1 part in 10,000 in voltage; and it was found quite difficult to keep the spot for any length of time within
1 cm. on either side of the zero. It was found, however, that if the potentiometer were balanced without paying attention to variations of voltage, provided these were reasonably small, when the voltage was brought back to the correct value, the balance was maintained, or at the worst a very small readjustment was required.

The wave form of the alternator employed was naturally rather poor, containing harmonics of the order of the 11th. and 13th due to tooth ripples. It was found possible to eliminate these and other harmonics practically completely by employing a tuned circuit, consisting of an air coil of about 0.3 henry in series with a variable condenser, in the supply leads feeding both potentiometer and specimen. The resulting wave form of voltage, taken with a Joubert contact maker and D.C. potentiometer, is shown in figure 6. The difference between starting and finishing voltages in this wave is due to variation in Alternator E.M.F. which was falling slowly due to temperature rise of the field. As the total current taken from the alternator was never more than 2 amperes, the voltage drop in the tuned circuit was never more than about 20 volts.
The measurement of frequency constituted a problem of some difficulty which was eventually solved in a very simple way. With the Geiss Potentiometer, the frequency must not only be kept constant, but, for the calibration of the quadrature potentiometer, must be known. The accuracy of frequency setting should be of the same order as the accuracy of adjustment of the current in the In Phase potentiometer, that is about 1 part in 5000. The final solution of the problem involved the use of the electrically maintained tuning fork vibrator of a Robertson stroboscope. In this instrument, the two arms of the fork carry light vanes which have slots so placed that, when the forks are vibrating at 50 cycles per second, 100 glimpses per second are obtained through the slots. A neon lamp supplied from the alternator through a small step up transformer was viewed through these slots, that is 100 times per second. When the frequency was exactly 50 cycles per second, the lamp appeared to be either slight or out the whole time. It was quite easy to adjust the frequency to give a departure from 50 cycles per second of not more than 1 cycle in 3 or 4 minutes, representing an accuracy of say 1 part in 10,000. The accuracy of the tuning fork itself was checked by timing with a stop watch the revolutions of a synchronous motor, the frequency being adjusted so that a stroboscopic
disc, attached to the motor, appeared stationary, when viewed through the vibrator. The stop watch was then checked against an accurate clock. The resulting error in the vibrator was found to be 1 part in 1250 (fast) at a temperature of 17.5°C. This was considered to be sufficiently accurate and the effect of temperature variation was also ignored as the room was maintained at a fairly constant temperature.

**Calculation of Results.**

The quantities derived from the A.C. potentiometer tests are the following:

- Millivolts per turn induced in the secondary winding
- Milliwatts of iron losses
- Magnetising amperes turns

If the readings are represented by:

- E: In phase component of the secondary voltage
- E': Quadrature...
- V: In phase component of the voltage drop across the non-inductive resistance R in the primary
- V': Corresponding quadrature component.

\[
\text{Millivolts per turn} = \frac{1000}{n_2} \cdot \sqrt{E^2 + E'^2}
\]

which is worked out on the slide rule in the form

\[
\frac{1000 \cdot E}{n_2} \cdot \sqrt{1 + \left(\frac{V'}{E}ight)^2}
\]
Milliwatts iron losses = \( \frac{1000 \cdot N_1}{R \cdot N_2} \left( E \cdot v + E' \cdot v' \right) \)

Magnetising Amp. turns = \( \frac{N_1}{R} \left\{ \frac{v \cdot E' - v' \cdot E}{E^2 + E_1^2} \right\} \)

\[ = \frac{1000 \cdot N_1}{R \cdot N_2} \left\{ \frac{v \cdot E' - v' \cdot E}{mV. \ per \ turn} \right\} \]

where \( N_1 \) and \( N_2 \) are the primary and secondary turns respectively.

For comparison with the ballistic tests the values of millivolts per turn have to be converted to the equivalent flux densities, and the ampere turns to maximum values.

The rectifier voltmeter used for obtaining the mean values of secondary E.M.F. was calibrated by means of the potentiometer with voltages of sine wave form. When used with the specimen its readings give 1.11 times the true mean value of the E.M.F. The tests with the rectifier were used to obtain the ratio of this quantity to the E.M.F. obtained from the potentiometer readings; and this ratio is called the "wave form correction factor" and is denoted by \( K_w \). \( K_w \) would be unity with a sine wave and is inversely proportional to the form.
factor.

The flux density is then given by:

\[ B = \frac{\varepsilon_w \cdot 10^5}{4.44 \cdot A \cdot f} \times \text{mV. per turn} \]

This figure is of course the maximum value of the flux density on the hysteresis loop traversed.

Since the magnetising current is sinusoidal, the maximum value of the ampere turns is 1.414 times the P.M.S. value.

**RECTIFYING VOLTMETER.**

An existing small 3 phase synchronous motor driving a Joubert contact maker was adapted for this purpose. A brass ring about 6 in. diameter and \( \frac{1}{2} \) in. radial thickness was mounted on the side of the ebonite disk carrying the rotating contact. A small square section copper-Morganite brush was arranged to bear on the side of this ring. The brush was rigidly attached to a spring which is fixed to an insulating arm which can be rotated by means of a worm and worm wheel. The brass ring is cut into four sectors by narrow saw cuts, two opposite segments being short of the quadrant by twice the brush width. The brush thus makes contact with these segments for exactly 90 degrees of rotation. These two segments are connected through a slipring on
the shaft to another brush and the remaining two segments are insulated. The result is that the two brushes are connected for exactly half of each cycle of the supply frequency and insulated for the other half. The secondary winding on the specimen was connected through the brushes to a unipivot galvanometer which gives a full scale deflection with 240 microamperes, the resistance of the circuit being made up to, generally, 3500 ohm. The galvanometer gives a deflection proportional the mean value of the induced E.M.F. when the brush position is adjusted to give maximum reading.

Possible causes of error in this rectifier voltmeter are lack of accuracy in the period of contact and brush contact voltage drop. It was found very difficult to ensure that contact was made for exactly 90 degrees; but, in measuring the mean value of an alternating voltage, at the points of making and breaking contact, the voltage is practically zero and the error due to this cause must be exceedingly small. Moreover the error will tend to be compensated by a similar error made in calibrating on a sine wave of voltage. With regard to brush contact drop, calibration curves are absolutely straight lines passing through zero.
with voltages ranging from 0.15 to 1.25 volts, showing that the contact drop must also be negligible.

The rectifier voltmeter does, of course, impose a load on the secondary winding, and is not used therefore to obtain absolute readings of E.M.F., but only the wave form correction factor. The values of E and E' employed in the calculation are taken with the rectifier disconnected.

Although under ideal conditions the calibration curves give a straight line, a considerable amount of trouble was experienced with brush contacts. As soon as the ring became slightly dirty, which happened after a few minutes running, the reading fell slightly and then became unsteady. This trouble could be avoided by frequent cleaning of the ring, and eventually it was made the standard procedure to clean the ring half a minute before each reading. By doing this the readings could be reproduced to a very high degree of accuracy, but not of course approaching that of the A.C. potentiometer readings. Occasionally readings were obtained differing from the curve by as much as 1 percent. The actual curves of wave form correction factor, being drawn from a large number of points may be taken to be reasonably accurate.
EXPERIMENTAL WORK.

STABILITY. BALLISTIC CALVANOMETER TESTS.

Large rectangular specimen.

The results of these tests are plotted in Figs. 7 to 17 inclusive.

Fig. 7 shows the B-H curves of the specimen in the Demagnetised and Polarised states. Demagnetisation was effected by a process of reversals of the magnetising current, which was at the same time gradually reduced in value. The specimen was polarised by passing a current of 5 amperes through the magnetising winding, reversing two or three times then switching off. The values of H and B corresponding to this current are 51.6 and 14,500 respectively. The Polarised B-H curve was then taken in the ordinary way, readings being taken at each value of current after 20 reversals. The curve thus obtained gives, not the actual value of flux density in the specimen, but the increase and decrease above and below a mean residual flux density due to polarisation. On the same graph (Fig. 7) are plotted a number of points, each one corresponding to a hysteresis loop taken later. The good agreement between these points and the polarised curve is interesting as in the case of the hysteresis loops, each separate loop was preceded by
polarisation as opposed to one polarisation for the whole B-H curve.

It is interesting to notice the very low value of Initial Permeability. In the Demagnetised case the permeability corresponding to the lowest reading is about 500. There is of course present in this case a superposed cross magnetisation due to the earth's field.

Fig. 6 shows the corresponding Retentivity curves. In the polarised condition, over a considerable range, there are two values for each current. These correspond to densities the one positive and the other negative to the mean axis of the displaced loop. The Retentivity value in the direction of polarisation is the higher. It would appear from these curves that the distortion of the loop, as shown by the difference between the two Retentivity values, might be used as a convenient and sensitive test of demagnetisation.

It is interesting to note the absolute saturation point reached in the Retentivity value at a flux density of 6950 Gauss. The value of $H$ at which this saturation occurs is 11.35, corresponding to a maximum flux density of 12,100 Gauss.
Fig. 8


Retentivity Curve

Full line - Demagnetised
Dotted - After polish
Bm = 19

(To givewow)
Figures 9 to 15 show hysteresis loops taken at maximum flux densities of 300, 500, 700, 1000, 1500, 2000 and 3000 Gauss. In each case a loop is given with the specimen in the demagnetised state and one following polarisation to a flux density of 14500 Gauss. In place of H as abscissae a scale of amperes is used for these curves, but a constant is given by means of which the currents can be converted to values of H in lines per sq. cm. In the cases of the 500 and 1000 Gauss loops, polarised loops are also given for a lower degree of polarisation. A series of loops were also taken at higher flux densities than 3000, but these are not shown as the demagnetised and polarised loops were practically coincident. In all cases between polarisation and the taking of the polarised loop, the current was given 50 reversals at the value required for the loop concerned. The stability of these polarised loops and the effect of more drastic intervening treatment is discussed later.

In figure 16 curves of hysteresis loss in Ergs per cycle per cu. cm. are given, obtained from the areas of the preceding hysteresis loops. A curve of percentage increase in losses due to polarisation plotted against flux density is also given in this
**STALLOY SPECIMEN**

**Loops at** $B_{max} = 500$

- Full Line - Demagnetised
- Dotted - Polarised to $B = 14,500$
- Chain Dotted - Polarised to $B = 3000$

---

**Fig 10**

Multiply by $10^{-32}$ to get...
STALLOY SPECIMEN

Loops at $B_{max} = 1000$

Full Line - Demagnetised

Chain - Polarised to $B = 5000$

Dotted - Polarised to $B = 14,000$

Multiply by $10^{-32}$

to give $H$.  

Fig. 12.
**Fig. 13**

**Stalloy Rect. Specimen**

- Full line: Demagnetised.
- Dotted: Polarisated to 14,500 Gauss.

Multiply by \(10^{-3}\) to give \(H\).

Date: 29.1.32

P
Fig. 14. Stalloy Rect. Specimen

Full line - Demagnetised

Dotted - Polarised to 19500 Gauss.

Amperes.

Multiply by $10^{-3}$ to give $H$. 

B (Gauss)
**Fig. 16**

**Stallloy Rectangular Specimen**

**Hysteresis Loss**

*From D.C. Hysteresis Loops*
The percentage increase in hysteresis loss decreases rapidly from a maximum amount of 36% at a flux density of 300 Gauss.

The curves discussed above (figures 9 to 15) give no indication of the absolute position of the polarised loops, that is of the true values of flux density concerned. The loops are plotted about an arbitrary zero of flux density midway between the two maximum values. In order to obtain information about these absolute values, the following procedure was adopted:— After each loop had been taken, assuming that the specimen had originally been polarised in the positive direction, the magnetisation was changed suddenly from the retentivity value on the small polarised loop to the negative maximum value on the main polarising loop. This required a change of ampere turns from zero to minus 590. The ballistic galvanometer deflection corresponding to this change gives the jump from a point on the polarised loop to the maximum value on the main loop. This value was checked by a reversal of the magnetising force. In figure 17 each of the polarised loops is plotted in its correct position with respect to the main loop. This diagram shows...
that in each case the negative tip of the small polarised loop has dropped very little from its theoretical position on the main loop. Figure 17a shows the displacement of the axes of the polarised loops plotted against amplitude.

Certain tests were taken at this stage to investigate the degree of stability of the polarised loops. The results of these are summarised in Table (1). Tests 1 and 14 refer to the demagnetised condition and are given for comparison with the other results. Tests 2 to 5 were taken under the conditions normally employed for the B-H and Retentivity-H curves and for the hysteresis loops when the specimen is in the polarised state. After polarisation the current required to give the desired small loop was found, and is reversed 50 times before the readings were taken. Tests 6 and 7 show the effect of a larger number of reversals accompanied by a slight vibration of the bench. In test No.8 after polarisation the specimen was tapped sharply with a hammer several times. In tests 9 to 13 the specimen was subjected to 50 frequency alternating current magnetisation of the same peak or maximum value as is required for the normal polarised loop. In none of these cases was there any marked reduction in the effects of polarisation. Tests 15, 16 and 17 deal with
Fig 17.a

Stalloy Rect Specimen

Displacement of Axes
of Polarised Loads.
### TABLE (1).

Stalloy Rect. Specimen.  
Stability of polarised loops.

All tests taken with loops of Flux Density amplitude 2x1000 Gauss.

<table>
<thead>
<tr>
<th>Test No.</th>
<th>Mag. Current</th>
<th>Retentivity</th>
<th>Displacement of Zero</th>
<th>Polarisation Flux Density</th>
<th>Conditions of test</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.0480</td>
<td>447</td>
<td>447</td>
<td>-</td>
<td>Demagnetised condition</td>
</tr>
<tr>
<td>2</td>
<td>0.0596</td>
<td>620</td>
<td>500</td>
<td>14500</td>
<td>From curves of figs.7 &amp; 8</td>
</tr>
<tr>
<td>3</td>
<td>0.0607</td>
<td>503</td>
<td>423</td>
<td>14900</td>
<td>From loop of fig.12 after 50 reversals.</td>
</tr>
<tr>
<td>4</td>
<td>0.0600</td>
<td>435</td>
<td>414</td>
<td>14500</td>
<td>After 50 reversals.</td>
</tr>
<tr>
<td>5</td>
<td>0.0598</td>
<td>495</td>
<td>3750</td>
<td>14500</td>
<td>Ditto</td>
</tr>
<tr>
<td>6</td>
<td>0.0593</td>
<td></td>
<td></td>
<td>14500</td>
<td>After 500 reversals, with vibration from electric motor.</td>
</tr>
<tr>
<td>7</td>
<td>0.0590</td>
<td></td>
<td></td>
<td>14500</td>
<td>After tapping with hammer</td>
</tr>
<tr>
<td>8</td>
<td>0.0590</td>
<td></td>
<td></td>
<td>14500</td>
<td>Subjected to Alternating current (max. value 0.059 A) slowly raised and lowered. A.C. on for 15 mins. Ditto</td>
</tr>
<tr>
<td>9</td>
<td>0.0603</td>
<td>475</td>
<td>388</td>
<td>14500</td>
<td>A.C. (max. 0.059 A.) switched on and off 20 times</td>
</tr>
<tr>
<td>10</td>
<td>0.0608</td>
<td>480</td>
<td>403</td>
<td>14500</td>
<td>Demagnetised (from loop fig.12.</td>
</tr>
<tr>
<td>Test No.</td>
<td>Mag.</td>
<td>Retentivity Pos.</td>
<td>Retentivity Neg.</td>
<td>Displacement of Zero</td>
<td>Polarisation Flux Density</td>
</tr>
<tr>
<td>---------</td>
<td>------</td>
<td>-----------------</td>
<td>-----------------</td>
<td>----------------------</td>
<td>--------------------------</td>
</tr>
<tr>
<td>15</td>
<td>0.055</td>
<td>507</td>
<td>444</td>
<td>3043</td>
<td>14500</td>
</tr>
<tr>
<td>16</td>
<td>0.050</td>
<td>506</td>
<td>466</td>
<td>2244</td>
<td>14500</td>
</tr>
<tr>
<td>17</td>
<td>0.051</td>
<td>496</td>
<td>455</td>
<td>2304</td>
<td>14500</td>
</tr>
<tr>
<td>18</td>
<td>0.048</td>
<td></td>
<td></td>
<td></td>
<td>2000</td>
</tr>
<tr>
<td>19</td>
<td>0.0495</td>
<td>453</td>
<td>437</td>
<td></td>
<td>3000</td>
</tr>
<tr>
<td>20</td>
<td>0.053</td>
<td>466</td>
<td>416</td>
<td></td>
<td>5000</td>
</tr>
<tr>
<td>21</td>
<td>0.056</td>
<td>482</td>
<td>410</td>
<td></td>
<td>7000</td>
</tr>
</tbody>
</table>
an alternating magnetisation of slightly higher value. This is seen to reduce considerably the effect of polarisation on a loop taken subsequently at the lower current. The flux densities corresponding to these alternating currents are about 1900 and 3000 respectively, these are about twice and three times the density at which the polarised loop was afterwards taken. Tests 18 to 21 refer to much smaller polarising flux densities. Test 20 applies to the intermediate loop of figure 12. These tests show that to produce an appreciable alteration in the permeability or hysteresis loss the polarising flux density has to be at least five times the normal density.

It is noticeable that the figures Retentivity and Displacement of the axis on the polarised loops are subject to considerable variations. The Retentivity values in Test 2 are much higher than in all the other tests. This peculiarity applies to all the values of the polarised Retentivity curves of figure 8, and is possibly due to the somewhat different conditions under which these values were obtained. The values for displacement have all through been found to be rather erratic and are therefore considered as approximate.
In order to compare the behaviour of Lohys with that of Stalloy, a set of B-H curves was obtained for this material by exactly the same method as was employed for the Stalloy specimen.

These are shown in Figure 13 with the Retentivity curves in Figure 19. The effect of polarisation on the B-H curve is very similar to that obtained with Stalloy. Curves of percentage increase in ampere turns due to polarisation plotted against flux density for both materials are given in Figure 20. The two curves are almost identical. The Retentivity curves again show a difference between positive and negative values when in the polarised state.

No further tests were taken on the Lohys specimen.
Lohys Specimen.

Reentivity Curves.

Full Line - After Demagnetisation
Dotted Line - After Polaring (Pos. v.
Chain Dotted - Ditto (Neg.

Fig 19.

Multiply by 360
STALLOY. TESTS WITH ALTERNATING CURRENT.

by A.C. Potentiometer at 50 cycles.

Small Ring Specimen.

The method of carrying out these tests has already been described in the section on Experimental Methods (page 27).

The test results are summarised in the curves of figures 21 to 23a.

Figures 21 and 21a give the A.C. magnetisation curves for the Stalloy ring specimen. In the place of flux density B, the ordinates chosen for these curves is millivolts per turn. This figure, which is obtained from the A.C. potentiometer measurement of E.M.F. induced in the secondary winding, will give the R.M.S. value of the fundamental component of the E.M.F. wave form. It has been explained in a previous section how the true values of flux density have been obtained from these by means of a mechanical rectifier (page 35). In the place of H the figures plotted are Ampere turns per cm. The R.M.S. currents obtained from the A.C. potentiometer readings have been multiplied by \( \sqrt{2} \) to give the peak value of the wave, for comparison with the D.C.
Fig 21.

Stalloy Ring Specimen

Magnetisation Curves

with A.C. 50 Frequency

Full Line - Demagnetised

Dotted Line - @ Polarised to B = 12,300

(1) Polarised to B = 5,000

Magnetising Amp-Turns

Max. Value

Multiply by 0.00910 give H.
A.C. Tests 50 Cyc.

Stalloy Rings.

Specimen Demagnetised.

Specimen - Polarised.
To B = 12,300.

Multiply by 0.099 to give H.
figures. The current wave form throughout is practically sinusoidal. The graphs of figures 21 and 21a show the collected results from a number of tests, the first set of which were taken in 1932 and the remainder in 1934. The points for the various tests are distinguished from one another and the dates on which they were taken are given. As the author had had no previous experience with the Call Potentiometer it was considered that taking a number of independent tests of this sort would afford a useful check on the accuracy of measurements with this instrument. The very good general agreement in the readings gives confidence in the A.C. potentiometer method and also in the stability of the specimen.

In addition to the normal polarised curve, a second polarised curve is given in figure 21 for a lower polarisation of 5000 Gauss.

In figure 21a, which deals with the higher values of flux density, the demagnetised and polarised curves are plotted separately. This is to show clearly the points corresponding to tests taken on different days. The two curves at these higher flux densities are practically identical.
The difference between the two magnetisation curves, resulting from polarisation in the one case, is shown up in Figure 21b in which the percentage increase in magnetising current due to polarisation is plotted against millivolts per turn. In order to render this scale in more familiar terms, a number of points on it are marked with their corresponding values of flux density. The graph is divided into two sections in order to show to a better scale the lower percentage increases at high flux densities.

In figures 22 and 22a the iron losses in the small Stalloy specimen are plotted. The values, which were obtained from the A.C. potentiometer tests, are for A.C. magnetisation at 50 cycles per second. In this case also, the results from a number of tests are shown, with the dates on which they were taken indicated. For the sake of clearness the demagnetised and polarised curves are shown on separate sheets. Figure 22b shows the percentage increase in these losses resulting from polarisation.

For the sake of comparison the A.C. iron losses have been reduced to Ergs per cycle per cu.cm. and are shown in this form for the demagnetised state, together with the corresponding D.C. hysteresis losses, in Figure 22c. The curves do not apply to the same
Stalloy Ring - A.C. Tests.

Percent Increase in A.M.A. Turns Due to Polarisation.

Percent Increase in Iron Losses Due to Polarisation.
**Fig. 22**

**Stalloy Ring Specimen.**

**Iron Losses**

**With A.C. 50 Cyc.**

**Demagnetised State.**

Iron Losses - Watts.

9 mV. per turn.

Millivolts per turn.
Fig. 22.a.

Stalloy Ring Specimen.
Iron Losses.
with A.C. 50 cyc.
Polarized to B = 12.300
Fig. 22c

Stalloy Ring

Comparison of AC and DC Iron Losses

Max. Flux Density.
specimen, but show quite a reasonable agreement, and
give an approximate idea of the relationship between
the hysteresis and eddy current losses on alternating
current.

Figures 23 and 23a give comparative B-H curves
for the two Stalloy specimens, showing both D.C. and
A.C. curves for the small ring. There is a considerable
difference between the D.C. curves for the two
specimens. Neither specimen is ideal in form, and
errors due to variation of H with radius in the ring,
and non uniform flux density in the rectangular specimen
may account for some of the difference between the two
curves. It is quite likely however that, as the two
samples were purchased at very different times and
places, they may differ slightly in constitution.

The difference between the D.C. and A.C. curves
for the small ring specimen is more important and
more interesting. The considerably higher permeability
with alternating magnetisation over practically the
whole range is very remarkable. The whole question
is discussed fully in the final section of the thesis,
but it may be remarked at this stage that, if the total
primary ampere turns are taken instead of the, so called,
magnetising component, the difference practically disappears.
FIG. 23.

**Stalloy Specimens.**

**Comparative Magnetization Curves.**

*Full Line - Chain Dotted*

*Dotred - Chain Dotted*
The results of these tests are very similar to those obtained with the Stalloy specimen and it will be sufficient to catalogue briefly the graphs in which they are represented.

Figure 24 shows the magnetisation curves in demagnetised and polarised states.

Figure 25 gives the curves of Retentivity plotted against magnetising ampere turns. The difference between the positive and negative retentivity values in the polarised state is more marked with this material than with Stalloy. A very small difference was also noticed in the demagnetised state, indicating some slight residual magnetisation very likely due to the earth's field. Of course the horizontal component of the earth's field is capable of producing a flux density of about 3,500 Gauss in this material, or rather would do so if it were exerted in a closed magnetic circuit.

The earth's field must certainly produce a cross magnetisation comparable with the circumferential magnetisation due to the magnetising current. The effect of this additional field on the circumferential
Fig 24.

Flux Density $B$

- Full line: Demag
- Dotted: Polaris $B = 7000$

Points from Hysteresis loops

Ampere turns multiplied by 0.0025 to give
flux was the subject of some tests which are described later.

Figures 26 to 30 show hysteresis loops in demagnetised and polarised states at maximum flux densities of 1000, 1500, 2000, 2500 and 3000 Gauss. All of the polarised loops follow polarisation to a fairly high flux density. The distortion due to polarisation being much less with this material no intermediate degrees of polarisation were attempted.

Figure 31 shows the hysteresis loss, as obtained from the hysteresis loops, in Ergs per cycle per cu.cm. plotted against flux density. The percentage increase in this loss due to polarisation is also shown.

Figure 32 shows the various polarised loops in their correct positions in relation to the main loop. The subsidiary loops again lie with their negative tips just within the main loop, but with Mumetal the drop from the main loop is somewhat greater than with Stalloy. Two positions are shown for the 1000 Gauss loop. The loop shown in dotted line corresponds to a later determination and is the correct position.

Figure 32a shows the displacement of the axes of these loops from that of the main loop plotted against their semi-amplitudes in flux density.

Figure 33 gives the percentage increase in
Fig 26

Mumetal Specimen

Full line — Demagnetised
Dotted — Polarised to $B = 8000$ Gauss

Multiply by 0.0525 to give $H$.
Fig. 27

Mumetal Specimen

Full line - Demagnetised
Dotted - Polarised to $B = 8000$ Gauss
Fig 31.

Mumetal Specimen

D.C. Hysteresis Loss
From Hysteresis Loops

Erms per Cycle per cm²

10

20

30

B max (Gauss)

1000

2000

3000

Increase Percent
Fig 32a
Mumetal Specimen
Displacement of Axes of Polarised Loops
Fig. 33.

** Mumetal Specimen **

*Percent increase in magnetising current due to polarisation*
magnetising current, resulting from polarisation, the values being obtained from the curves of figure 24.

Apart from these direct results, certain incidental details of the magnetic properties of Mumetal were investigated in the course of the ballistic tests.

*Variation of Permeability with time.*

The tests described above were taken over a period of three years, and it happened that $B-H$ curves were taken in 1931 and again in 1933. The 1933 values, which are those used in Figure 24, show a lower permeability than the earlier values, at all flux densities above 1000 Gauss. The two curves for the demagnetised state are given in Figure 38, the maximum decrease in permeability being of the order of 6 percent. Both curves were thoroughly verified by check tests, the earlier one by a complete $B-H$ curve taken on the 12-3-31 and the later one by the hysteresis loops as well as sundry odd points. Each curve has a corresponding polarised curve taken at about the same time.

It is considered that the decrease in permeability is due to a change in the properties of the material, which may be partly due to ageing and partly due to intervening mechanical treatment. It is well known that Mumetal is adversely affected by mechanical strain.
Effect of the Earth's Magnetic Field.

The magnetisation curves and hysteresis loops which were obtained with Mumetal are derived from values of mean circumferential flux, with two superposed magnetic fields; the circumferential field due to the magnetising current and the transverse field of the earth. Owing to the extremely high permeability of Mumetal, the circumferential magnetising forces are of the same order as, and indeed sometimes much lower than the horizontal component of the earth's field. To get absolute values of permeability at fairly low densities, it would be necessary to eliminate the earth's field. In the present case this refinement was considered unnecessary as the behaviour under normal working conditions was of more interest. It is, however, necessary to establish whether the orientation of the specimen in the earth's field modifies the "Effective" magnetisation curves.

The earth's field will generally have a component along the axis of the ring, which will be at right angles to the circumferential field; and a component at right angles to the axis of the ring, which will add to the circumferential field on one side of the ring and subtract from it on the other. In special cases, one only of these two components need be present.
Most of the ordinary tests were taken with the axis of the ring vertical, and both components of the earth's field therefore present. A number of check readings in different positions were however taken and are dealt with below.

Some preliminary tests were taken on the 22-4-33 at magnetising ampere turns of 0.82 and 0.47, corresponding to flux densities of 2500 and 1100 Gauss respectively. In the first case the specimen had the following positions:

Axis vertical - sec. leads on top
Axis horizontal - sec. leads pointing North
South
East
West

The variation in flux density did not exceed 1 in 250. In the second case in the above positions and also with the axis vertical and leads pointing downwards the variation did not exceed 1 in 100.

The flux densities employed in the above tests were perhaps rather high and no treatment was given in the various positions before taking the readings.

In a further series of tests, taken on the 30-8-33, a considerable range of flux densities was employed, and in each position, before taking the readings,
the specimen was put through the ordinary demagnetising process. The results of these tests are given in Table 2 below:

**TABLE 2.**

<table>
<thead>
<tr>
<th>Mag. Amp. turns</th>
<th>Flux Density</th>
</tr>
</thead>
<tbody>
<tr>
<td>Col. 1</td>
<td>Col. 2</td>
</tr>
<tr>
<td>0.2</td>
<td>254</td>
</tr>
<tr>
<td>0.4</td>
<td>766</td>
</tr>
<tr>
<td>0.6</td>
<td>1696</td>
</tr>
<tr>
<td>0.8</td>
<td>2400</td>
</tr>
<tr>
<td>1.2</td>
<td>3360</td>
</tr>
<tr>
<td>1.6</td>
<td>3970</td>
</tr>
<tr>
<td>2.0</td>
<td>4400</td>
</tr>
</tbody>
</table>

**Col. 1.** Axis of ring in line with earth's field specimen demagnetised in this position.

**Col. 2.** Axis of ring at right-angles to earth's field specimen demagnetised in this position.

**Col. 3.** Axis of ring vertical, specimen demagnetised in this position.

**Col. 4.** Standard curve of Fig. 24 (25-8-33). Axis vertical, demagnetised.
Col. 5.  Axis at right-angles to earth's field; specimen knocked on bench in this position after demagnetising.

Col. 6.  Axis in line with earth's field; specimen knocked on bench as above.

In all cases of straightforward demagnetising the variations with position are negligible. The figures of Cols. 1, 2, 3 differ as a whole from those of Col. 4 taken on a different day.

Cols. 5 and 6 deal with a slightly different condition. Here the specimen is knocked sharply to assist in the production of a cross flux by the earth's field. It is difficult to assure uniformity of treatment in these cases, but the effect seems to be much larger when the axis of the ring is at right-angles to the earth's field. A cross flux at right-angles to the main circumferential flux has very little effect on it.

Polarisation due to Mechanical Shock.

It was noticed at a very early stage in the work on Mumetal that, if the specimen were tapped sharply with the current flowing in the magnetising winding, the increase in flux density due to this tapping produced considerable subsequent polarisation. The change in flux density produced by subsequent reversal of this current was considerably reduced compared with
the value obtained before tapping.

The effect of this polarisation is illustrated by the small loops of Figure 34a. These are sketched in from four points on each loop. The polarised loops were taken after sharp tapping with a hammer followed by twenty reversals of magnetising current. The absolute positions of these polarised loops were obtained in the usual way by returning to a main loop at high flux density, which can be relied upon to be symmetrical.

In figure 34 is shown the percentage decrease in flux density, for a given magnetising current, resulting from polarisation by tapping. The points are naturally somewhat irregular but the curve gives a good idea of the magnitude of the effect.

In addition to these tests a complete B-H curve was taken in which each reading was preceded by tapping followed by 20 reversals. This is not reproduced but closely follows the normal polarised curve over a range of flux density from 250 to 3000 Gauss. A mechanical blow, while the normal current is flowing, can, within this range of flux density, produce polarisation of the same order as that produced by very heavy excess current.
Mametal Specimen.

Reduction in Flux Density Produced by Tapping.

Fig. 34.
Stability of the Polarised State.

Although the question of stability did not receive so much attention with Numetal as with Stalloy, sufficient was done to show that the polarisation produced by excess current is fairly stable and likely to persist under normal mechanical and electrical treatment.

Tests were taken at three different flux densities, the results being set out below. The treatment in each case is given in the order in which it was applied. The specimen was not repolarised between items of treatment except where stated.

**Test 1.** Magnetising Amp. Turns 0.48 19-8-31
Demagnetised value from curve of 12-8-31 1120
Polarised .. .. .. 13-3-31 710
Taken directly after polarisation 680
Sharp tapping - 20 reversals 738
Ditto 740
A.C. magnetisation (max. value = D.C.) 738
Slowly raised and lowered.
A.C. as above accompanied by tapping 815
Repolarise with opposite polarity 650
Lifted up and replaced - 20 reversals 701
Tapping 730
A.C. for 1 hour 727
A.C. accompanied by tapping 848
A.C. (max. value = 1.25 x D.C.) 832
Test 2. Magnetising Amp. Turns 0.3 28-8-33.

Demagnetised value from curve of 25-8-33 2380
Polarised .. .. .. .. 28-8-33 2120
Direct after polarisation - 20 reversals 2140
Further 20 reversals 2130
.. 50 ..
2140
Turned over twice 2140
Knocked on bench 2150
Ditto 2160

Test 3. Magnetising Amp. Turns 2.0 21-4-33.

Demagnetised value from curve of 19-4-33 4310
Polarised .. .. .. .. 21-4-33 4200
Direct after polarisation - 20 reversals 4230
Turned over twice - 10 reversals 4230
Knocked on bench - 10 reversals 4230
Turned over with current on - 10 r. 4230
Knocked on bench with current on - 10r. 4240

Tests 2 and 3, particularly the latter, have flux densities which are rather high side and therefore do not provide a marked polarisation which is capable of being affected by treatment. Test 1 however provides satisfactory evidence that the reduction of permeability resulting from polarisation is maintained to the extent of 70% in spite of very severe treatment.
NUMETAL. TESTS WITH ALTERNATING CURRENT.

By A.C. POTENSIOMETER at 50 cycles.

The D.C. tests on Numetal were, as in the case of Stalloy, supported by tests with A.C. magnetisation at 50 cycles per second.

As the method of carrying out these tests has already been described fully, it is necessary here only to explain the results obtained, which are given in the form of graphs, as before.

Figures 35 and 36 show the demagnetised and polarised A.C. magnetisation curves. These graphs have millivolts per turn plotted in the place of $B$ and ampere turns in the place of $H$. Each curve is plotted from three sets of experimental figures obtained on different dates as indicated on the graphs. The three sets of figures show good agreement in both curves, but it will be noticed that a point of discontinuity occurs in the curves where the 120 turn primary was substituted for the primary of 6 or 10 turns. The reason for this discontinuity has not been fully established, though it was found on dismantling the specimen recently that the winding had only 119 instead of 120 turns. This alteration is not sufficient to account for the
Mumetal Specimen

A.C. Magnetisation Curves

Demagnetised State - Collected Results

- [Symbol] 4.34
- [Symbol] 259.26 - 3.34
- [Symbol] 224.12 - 5.24

Magnetising Amp. Turns

Max. Value

Multiply by 0.0545 to give H.
**MUMETAL SPECIMEN**

**A.C. MAGNETISATION CURVES**

Polarised State - Collected Results

- 8 8 24
- 22 2 34
- 23 22 32

**Magnetising Amp.-Turns**

Max. Value

Multiply by 0.0525 to give H.
full difference between the two curves, the rest of
which may be due to the different positions of the
two windings on the core. The accuracy of the
readings obtained with the lower turn primary was
well established by the good agreement obtained
with a number of different windings of 6, 10, 12 and
15 turns which were employed at various times.
The cause of the error with the 120 turn primary was
not cleared up, as this would have involved setting
up the whole apparatus again. The error is confined
to high flux densities at which there is no difference
between the Demagnetised and Polarised curves, and
which is not of main interest in the present investigation.

The values for the Demagnetised and Polarised states
are compared in figures 37a and 37. In figure 37a,
the two curves of figures 35 and 36 are reproduced on
one sheet, without the experimental points. In order
to make the curves applicable to any specimen of the
same material, the scales are reduced to millivolts
per turn per sq. cm. of core section and ampere turns
per cm. length of core. In both cases the R.M.S.
values are used, which in the case of the E.M.F. is
the R.M.S. value of the fundamental component only.
A scale of maximum flux density is added on the right
hand side of the graph. This scale is correct for
the Demagnetised curve, being slightly high for the
Fig 37

Mumetal

A.C. Magnetisation Curves

% Increase in Mag. Current due to Polarisation
Fig. 37a

Mumetal Specimen

Effect of Polarisation on Magnetisation Curve

Full line - Demagnetised
Dotted line - Polarised

Millivolts per turn per sq. cm. section.

Flux Density (D.C. tangent, 10 bars)

Amp. Turns per cm. (R.M.S.)

Amp. Turns per cm. (R.M.S.)
Polarised state for which the E.M.F. wave is a little more peaked.

The conversion from secondary E.M.F., as measured on the potentiometer, to flux density is carried out by means of a "Wave form correction factor". This factor gives the relationship between the R.M.S. value of the fundamental component of the E.M.F. wave and 1.11 times the mean value of the half wave. The method of calculation and the synchronous, mechanical rectifier by means of which the wave form correction factor were obtained have been described on pages 35 to 38.

Figure 37 shows the percentage increase in magnetising ampere turns in the Polarised as compared with the Demagnetised state, for the same induced E.M.F. This is plotted against millivolts per turn per sq. cm. of core area, with the addition of a scale of flux density in the Demagnetised state.

In figure 38, the wave form correction factors are employed to obtain a B-H curve with A.C. magnetisation, which is compared with the D.C. curves obtained by the ballistic galvanometer method. All the curves are for the demagnetised state. The one A.C. curve is obtained from experiments taken between Aug. 1932 and Apr. 1934. The two D.C. curves are for Sep. 1931 and Aug. 1933 respectively. These dates suggest that the
change in magnetic properties indicated by the two D.C. curves must have taken place between Sep. 1931 and Aug. 1932. As the specimen was obtained in 1931, this period corresponds to its first year of existence. During this period, it was subjected to some knocking in connection with tests of stability in the polarised state.

It is again very interesting to compare the D.C. and A.C. magnetisation curves. In the A.C. curve shown in figure 38 the values of ampere turns are the magnetising component of the total ampere turns. This is the component in quadrature with the induced EMF. In this curve, at values of flux density below 2000 Gauss, the A.C. permeability is lower than the D.C. value. At about this flux density, the two curves cross over and the A.C. permeability remains higher than the D.C. up to the highest density obtained. If, however, the maximum value of the total primary ampere turns is employed for the A.C. curve, the A.C. permeability is lower than the corresponding D.C. value up to a flux density of about 6000 Gauss at which point the two curves cross. The matter is further discussed in the final section under the heading of "Conclusions".
Arising out of a desire to account for the difference between the D.C. and A.C. magnetisation curves, a point was investigated which is worthy of notice. The A.C. tests were all taken with substantially sinusoidal current, and, therefore, with distorted secondary E.M.F. The fundamental component only of this was balanced in the potentiometer, and, therefore, currents of harmonic frequencies were certainly present in the secondary circuit. These currents might add appreciably to the magnetising ampere turns. At high flux densities, it was noticed that a good balance could not be obtained on the vibration galvanometer, when measuring the secondary E.M.F. At the point of balance an elongated spot was obtained in which two or more modes of vibration could be distinguished. As one decreased, the others increased. In order to find out whether such harmonic currents had any appreciable effect on the magnetisation curve, a high inductance smoothing choke, tuned to the fundamental frequency by means of a condenser, was introduced into the secondary circuit. No appreciable difference in the readings could be detected with the tuned circuit in and shorted. About half of the readings from which the points of curves on figures 35 and 36 were plotted were taken with the tuned secondary circuit.
Figure 38a gives curves of the wave form correction factors for both Stalloy and Mumetal. The figures from which these curves were obtained were not as consistent as might be desired, due to variable brush contact resistance. It is not surprising that this effect should give trouble in a circuit in which the total E.M.F. is from 0.2 to 1.5 volts. However a procedure was finally adopted which gave readings, rarely departing from the mean curve by more than 0.5%. It must be admitted that insufficient, reliable figures were obtained to confirm the large difference between the polarised and demagnetised figures for Mumetal, at the higher values of E.M.F. It seems certain that the two curves concerned should approach one another at densities at which the polarisation ceases to affect the magnetisation curve. This is another matter which time does not permit of a satisfactory investigation. It has however been clearly established that wave form does not account for the large difference between the A.C. and D.C. magnetisation curves. The curves for Stalloy are more definite, but here the curves at low values of E.M.F. are somewhat uncertain.
Figures 39 and 40 show the iron losses in the demagnetised and polarised states, plotted on different sheets to show up all the experimental points clearly. Figure 39 shows a few points taken with the 120 turn primary winding. These do not agree with the main curve, the discrepancy having already been discussed in connection with the magnetisation curves of Fig. 35. The demagnetised and polarised curves are plotted on the same sheet on figure 41. Actually this is an early graph plotted from the 1932 figures only. The flux density scale, which is employed as abscissa, is obtained from the millivolts per turn without taking the wave form correction factor into account, that is on the assumption of a sine wave of E.M.F. The correction in the case of Mumetal is extremely small (see Fig. 38a). Figure 42 is given to show the relative values of A.C. and D.C. losses. In this case, the D.C. losses are small compared with the A.C. losses, showing that the eddy current loss is considerably greater than the hysteresis loss. This fact accounts for the very small effect of polarisation on the A.C. losses.
Mumetal Specimen
A.C. Iron Losses
Demagnetised State
Collected Results

Fig. 39
MUMETAL SPECIMENT
IRON LOSSES ON A.C.
Polarised State.
Collected Results.

Fig 40.
**Fig. 41.**

*MUMETAL SPECIMEN*

IRON LOSSES AT 50 CYCLES

Full line = Demagnetised.

Points X = After Polarisation to 8000 Gauss.

**Flux Density (Gauss)**
Fig. 42

Monometal Specimen

Comparison of A.C. and D.C. Iron Losses
CONCLUSIONS.

Each of the individual tests has already been discussed in detail, and it is proposed in this section to consider the general conclusions which can be drawn from the results which have been obtained.

The original object with which the work was undertaken was to determine the effects of previous magnetisation or polarisation. Polarisation first of all effects a reduction in permeability to subsequent alternating magnetisation. The magnitude of the change, as shown by ballistic galvanometer tests, for Stalloy, Lohys and Numetal is shown in Figures 20 and 33 (pages 62 and 87). These curves show the percentage increase in magnetising current for given values of flux density. In all cases the maximum change is found with low densities, and the percentage increase falls off rapidly as the density increases. With both Stalloy and Lohys the maximum increase in magnetising current obtained was about 80% at a density of 200 Gauss, falling off to 5% at about 4000 Gauss. Numetal showed a smaller increase of about 40% at 200 Gauss falling to 5% at about 4000 Gauss. The corresponding curves obtained with 50 cycle alternating magnetising current are given in figures 21b and 37 (pages 68 and 82) for Stalloy and Numetal respectively.
In the case of Stalloy, the reduction in permeability with A.C. magnetisation agrees fairly well with the corresponding D.C., or ballistic values. At the lowest and highest densities shown there is very good agreement. At the middle of the range, the change is smaller with A.C. magnetisation, due possibly to the effect of polarisation being less stable over the steep part of the B - H curve. With Mumetal, the change in permeability with A.C. magnetisation is generally less than with D.C., except at the higher densities. This might be expected, as the polarisation has been demonstrated to be rather less stable with this material.

The effect of polarisation has also to be considered in relation to the iron losses. The polarised loop, for a given maximum flux density, has a greater value of $H_{\text{max}}$, being longer and thinner than the corresponding demagnetised loop. It is also drawn up in the centre in the direction of polarisation, having unequal values of retentivity for the two directions of magnetisation. The net effect on the area of the loop, and therefore on the losses, is an increase, the percentage increase again falling off with increase in maximum flux density.

The curves of percentage increase in hysteresis loss, due to polarisation, are given in figures 16 and 31 (pages 67 and 84) respectively for Stalloy and Mumetal.
Stalloy shows the larger increase in loss, falling from 36% at $B_{\text{max}}$ of 300 to 4% at $B_{\text{max}}$ of 3000. Mumetal shows an increase in loss of 4% at $B_{\text{max}}$ of 1000 falling to 1% at $B_{\text{max}}$ of 3000 Gauss.

The corresponding curve for Stalloy, with A.C. magnetisation, is given in figure 22b. The percentage increases are slightly lower than with D.C. This would be expected as the loss includes a small amount of eddy current loss. In the case of Mumetal the increase in loss due to polarisation is so small, with A.C. magnetisation, that it could not be expressed as a percentage with any accuracy. The corresponding curves of loss in the demagnetised and polarised states are shown in figures 41 (page 113). The hysteresis loss being a smaller proportion of the total A.C. loss, in the case of Mumetal, the small increase in hysteresis loss is swamped.

What is the practical significance of these results? Firstly, the effects of polarisation can be serious only in cases of apparatus working with low flux densities, say below 2000 Gauss. Furthermore, it is unlikely that the resultant increase in magnetising current would be appreciable in any case of a magnetic circuit with an air gap. As long as the ampere turns for the gap are large compared with those for the iron, an increase of
say 50% in the latter will have a negligible effect on the total ampere turns. The percentage increase in iron losses will not be affected by the presence of the air gap. Secondly, the polarising current must be very large compared with the normal current. It is unlikely that ordinary switching operations could produce any appreciable polarisation. The full effects of polarisation could be produced by a short circuit or by such a condition as open circuiting the secondary of a current transformer on load. It has been shown that polarisation, once set up, will persist for an indefinite period, under normal conditions. To reduce the iron to its normal condition, careful demagnetisation from a high flux density is necessary. Mumetal in addition to being less affected by polarisation is somewhat less stable in this state. There is however the additional possibility, with this material, of a knock or jar, while the current is flowing, causing serious polarisation, though the current is of normal magnitude.

Apart from the initial object of investigating the effects of polarisation, certain of the results obtained in this work have led the author to study the question of the connection between D.C. and A.C. magnetisation curves.
The attitude of engineers and designers in the past seems to have been that it was quite legitimate to base A.C. designs on the D.C. curves. Moreover, the magnetising current in the A.C. case was taken as the component of the current in phase with the flux and in phase quadrature with the induced E.M.F. It was realised that either the current or the flux or both must be non-sinusoidal in wave form, and could not therefore be completely represented by a vector in a vector diagram. Throughout the present work, what is referred to as the magnetising current or ampere turns is the component in phase with the flux. In order to get over the difficulty of having to calculate a component of this current, the current was arranged to be approximately sinusoidal and the flux allowed to depart from this wave form. The A.C. magnetisation curves on this basis, together with the corresponding D.C. curves, are shown in figures 23 and 23a for Stalloy and figure 38 for Mumetal (pages 73, 74 and 105). In neither case is the A.C. permeability approximately the same as the D.C. value over any considerable range.

Until recently there has been no real agreement between different investigators as to the relation between A.C. and D.C. magnetisation curves. Within the last two years, however, a number of valuable papers have been published on this subject. It is proposed
to review three of these very briefly, in so far as they throw light on this question of the relation between A.C. and D.C. permeabilities.

D.C. Call and L.G.A. Sims (1), starting with the conventional vector representation of alternating magnetisation, show that this involves an elliptical hysteresis loop and a complex permeability. Eddy currents are allowed for by a component of current in phase with the E.M.F., it being considered that the quadrature component due to this cause was negligible. This power component of current due to eddy current losses was separated in the usual way from tests at two frequencies. A series of tests were taken on a small transformer, having a core of Lohys, measuring the current with a dynamometer ammeter and iron losses with a wattmeter. The results of these tests show agreement between the A.C. and D.C. curves up to a flux density of 3000 Gauss, provided that, in the A.C. case, the values of H are obtained from the total current, corrected for eddy current losses. Above this flux density the apparent A.C. permeability is higher than the D.C. As the peak value of the current was taken as $\sqrt{2}$ times the R.M.S. value, the difference is probably accounted for by the

peaked nature of the current wave at higher flux densities. A further series of tests were taken with the Gill co-ordinate type of A.C. potentiometer in conjunction with a special form of Joubert contact maker, which enabled the harmonics in the current wave to be obtained as well as the fundamental. An A.C. magnetisation curve was obtained from these tests taking the true maximum value of the current, built up from its harmonic components. This curve showed closer agreement with the D.C. curve at the higher flux densities. As the eddy current losses were not allowed for there was some difference between the curves at low flux densities. There remained unexplained a tendency for the A.C. permeabilities to be slightly higher that the D.C. values at high densities. The flux wave form was arranged to be as nearly sinusoidal as possible throughout the whole of their work.

C.E. Webb and L.H. Ford (2) employed a direct method of measuring the maximum values of both B and H. A synchronous mechanical rectifier was used to obtain the meanvoltages induced in a secondary winding on the specimen and in the secondary of an air cored mutual inductance, the primary of which carried the magnetising

(2) Alternating current permeability and the bridge method of magnetic testing. (I.E.E. Journal, Vol.76, p 185)
current. With a specimen of 0.3 mm. Stalloy sheet, they found no difference between the D.C. curve and the A.C. curve taken at 50 cycles. Their readings range from $B_{\text{max.}}$ 6000 to 15,000 Gauss. At 500 cycles, the A.C. permeability is lower than the D.C. below a $B_{\text{max.}}$ of 11,000 Gauss, and is in agreement with it above this figure. With a specimen of 0.1 mm. Stalloy sheet, agreement between the D.C. and 500 cycle A.C. curves was obtained above a $B_{\text{max.}}$ of 9000 Gauss. With a specimen of 0.4 mm. Lohys, the 50 cycle A.C. curve gave lower permeability values up to a $B_{\text{max.}}$ of 12,000 Gauss.

Arrangements were made for varying the wave shape between the two extremes of sinusoidal flux and sinusoidal current. With the 0.4 mm. Lohys specimen, at flux densities below 10,000 Gauss, the permeability decreases slightly with increased flux distortion. With the 0.1 mm. Stalloy sheet, wave shape had no measurable effect.

E. Hughes (3) was concerned with Mumetal and Permalloy, working with sinusoidal current. His first A.C. tests

were carried out with an asynchronous mechanical rectifier at frequencies of 21 and 30 cycles. In a second series of tests, the cathode ray oscillograph was employed to obtain cyclograms of current and secondary voltage and also of current and flux. From these additional B-H curves were obtained at 20.6, 50 and 75 cycles. All his A.C. curves were coincident with or closely approached the D.C. curve at the highest density shown. Over the range of high permeability, all the A.C. curves showed a lower flux density and therefore permeability than the D.C. curve. This reduction in permeability increased with increasing frequency.

It should be clearly emphasised that the above summaries are concerned with a part of the field covered in each of the papers. For the conclusions of each author and for an appreciation of the full scope of each work reference should be made to the original papers.

From a study of these three papers and from the very small amount of work which he has himself done, the present author is led to the following conclusions on the question of A.C. and D.C. magnetisation curves.

Firstly, to obtain any correlation at all between the two curves, the total magnetising current must not be split up into two components in phase and in quadrature
with the E.M.F.

Secondly there is the effect of eddy currents in the A.C. case. These increase the magnetising current, but it is suggested do not very greatly affect its peak value, except at low flux densities. The reduced A.C. permeability compared with D.C., which has been found in all the materials tested, with flux densities below and in the range of maximum permeability is probably, but not necessarily entirely, accounted for by eddy currents. It seems likely that no strictly accurate for the eddy currents can be made; but the quite satisfactory results, at low flux densities, of the correction for the power component only of the eddy currents made by D.C. Gall and L.C.A. Sims (1) in the case of their Lohys core is of great practical interest. It would be interesting to know whether a similar correction would be satisfactory in the case of Munetal.

Thirdly, there is the question of wave form. The A.C. curves which correspond most nearly with the D.C. are obtained from the true peak values of both current and flux. It is not obvious that peak values should be employed for both as cyclograms and mechanical rectifier show that the two peak values do not occur simultaneously. It seems probable however that the phase displacement between the peak values is due to eddy currents, and that if the eddy currents are
Fig. - B-H Curves for Mum

from Various Sources

D.C. Ballistic Measurement
negligible or can be allowed for, the peak values with alternating current will correspond to the D.C. values irrespective of phase.

Finally is there any practical value whatever in attempting to correlate the A.C. and D.C. magnetisation curves. At low flux densities, where the difference is appreciable, it seems that it would be impossible to calculate the A.C. performance from the D.C. curves. It is suggested therefore that A.C. methods should be generally employed for obtaining the magnetisation curves on which the design of A.C. apparatus is to be based; and that each curve would apply to a particular frequency. A.C. methods are normally employed to obtain curves of iron losses and could be generally extended to give data for the magnetising ampere turns as well. The practical A.C. magnetisation curves would not be $B-H$ curves. The author has employed in the place of $B$ the quantity "Millivolts per turn per sq. cm. of core section. This is a clumsy term and may be improved, but has the advantage of being perfectly definite. D.C. Call and L.C.A. Sims (1) make a suggestion for expressing the magnetising data which would take the place of $H$. They suggested that the designer requires iron loss in watts and the reactive volt-amperes required to magnetise the core. It can be shown that the iron loss watts per cu. cm of core, and the reactive magnetising
volt-amperes per cu. cm. are independent of number of
turns on the winding. It is suggested therefore that
A.C. magnetisation curves might be in the form of these
two quantities plotted against millivolts per turn per
sq. cm. It is necessary to consider how the wave form
distortion, which is inherent in the magnetisation of
iron, may be taken into account. In most cases in practice,
the E.M.F. and therefore the flux wave is approximately
sinusoidal. As the current will be distorted, what is
to be taken as the reactive magnetising volt-amperes?
An A.C. potentiometer or bridge will give the R.M.S.
value of the fundamental, ignoring the harmonics.
Where the magnetising current is to be added to load
currents, which will be approximately sinusoidal, this
is probably the most satisfactory value to employ, as
the harmonic content of the resultant will be negligible.
The R.M.S. value of the complex wave of magnetising
current can be measured on an ammeter of suitable type,
but there does not seem much point in basing an effective
reactive component on this. If a knowledge of the
separate components is required a complete analysis is
necessary. The work of C.E. Webb and L.H. Ford
suggests that figures obtained with sinusoidal flux
could be used, with very little error, when the flux is
distorted.
APPENDIX 1.

**Ballistic Galvanometer Theory.**

The main contribution to the theory of the ballistic galvanometer which the author desires to make in this section is to investigate the theory of the instrument when critically damped, particularly when this is accomplished by shunting the instrument with a non-inductive resistance.

The ordinary simple theory of the instrument shows that, when critical damping is employed, the amplitude of the swing is still proportional to the total quantity of electricity which has passed. This amplitude of swing is of course reduced in magnitude as compared with case of small damping.

If the constants of the galvanometer are expressed as follows:

- $c = \text{Torque per unit current in dyne-cm.}$
- $k = \text{Torque per radian deflection in dyne-cm.}$
- $D = \text{Damping torque per radian per sec. of angular velocity in dyne-cm.}$
- $I = \text{Moment of Inertia of movement in gm\cdot cm^2.}$
- $q = \sqrt{\frac{k}{I}} = 2\pi \times \text{natural frequency of swing (undamped).}$
- $\delta = \frac{D}{2I} = \text{decrement factor}$
- $Q = \text{Total quantity of electricity.}$
The deflection with small damping

\[ E = \frac{\sqrt{2}}{4} \tan^{-1} \left( \frac{CQ}{\sqrt{I}} \right) \]

With critical damping the deflection becomes

\[ \frac{1}{E} \left( CQ \right) \frac{\sqrt{I}}{} \]

This simple theory is based on the assumption that the instrument deflection during the passage of the current is zero. It is evidently necessary to investigate the errors involved by this assumption with both negligible and critical damping.

It appears to be impossible to do this without a knowledge of the mathematical expression of the current surge through the instrument. When this current is derived from the change of flux in an iron sample, and this change is brought about by alteration of the magnetising current by hand, the form of the surge is naturally somewhat indefinite. It was however considered that it would be interesting to get some idea of its nature, and arrangements were therefore made to obtain oscillograph records of the current by means of the low frequency Cambridge "Duddell" oscillograph. The mumetal specimen was used for this purpose, the usual hand operated mercury reversing switch being employed for reversing the magnetising current. One of the elements of the oscillograph was connected across a
shunt resistance in the magnetising current circuit, the second to a 50 cycle timing wave and the third through a two valve, approximately straight line, amplifier to a resistance in the galvanometer circuit. The amplifier, in its final form, consisted of a screened grid valve resistance coupled to a power valve (TX4) having an anode current of 50 mA. In the earlier records the wave forms for the two directions of reversal were dissimilar. This was overcome by reducing the amplitude and was probably due to saturation of one of the valves. Some tracings of these oscillograms are shown in Figure 43.

It can be seen that the duration of the discharge is of the order of 0.01 sec. The current recorded is the sum of galvanometer and shunt current and it is realised that the galvanometer current would be of longer duration due to the inductance of the instrument. The wave form naturally consists of two peaks corresponding to the make and break of the current. Some sign of fairly high frequency oscillation can be seen, but the main body of the wave has a very steep front, a flat top and a fairly steep tail.

Three very approximate expressions for the current wave have been taken as being of simple nature and therefore within the scope of the authors mathematics.
1. A rectangular wave of amplitude $i_m$ and duration
$$t_1 = \frac{\pi}{p}$$

2. A sine wave of amplitude $i_m$ and pulsance $p$ between limits of $pt = 0$ and $pt = \pi$.

3. A cosine wave of amplitude $i_m$ and pulsance $p$ between limits of $pt = 0$ and $pt = \frac{\pi}{2}$.

As will be seen later, the wave form assumed has very little influence on the errors which depend chiefly on the duration of the discharge.

As an example of the method of working, case 3 is given in full below, the results for the other two cases being given without proof.

**Case 3.**

$$i = i_m \cos pt$$

between limits $pt = 0$ and $pt = \frac{\pi}{2}$

Equating the torques acting on the galvanometer movement gives the differential equation:

$$I \dddot{\theta} + D \ddot{\theta} + k \theta = c i_m \cos pt$$

Differentiating twice and substituting for $c i_m \cos pt$

$$I \dddot{\theta} + D \ddot{\theta} + k \dot{\theta} + p^2(I \dddot{\theta} + D \ddot{\theta} + k \theta) = 0$$

Putting $A \xi^\lambda t$ for $\theta$

$$A \xi^\lambda (\lambda^2 + p^2) (I \lambda^2 + D \lambda + k) = 0$$

giving the following values for $\lambda$:

$$\lambda_1, \lambda_2 = \pm j p$$

$$\lambda_3, \lambda_4 = -\frac{D}{2I} \pm \sqrt{\frac{p^2}{4I^2} - \frac{k}{I}}$$
Considering the case when \( \frac{k}{I} > \frac{D^2}{4I^2} \)

Let \( \frac{k - \frac{D^2}{I^2}}{I} = q \) so that \( \lambda = -q + iq \)

Substituting these values for \( \lambda \) and using the trigonometrical form of

\[ \phi = A \cos pt + B \sin pt + e^{-qt} (M \cos qt + N \sin qt) \]

When \( t \) is sufficiently large

\[ \phi = A \cos pt + B \sin pt \]

Substituting this in the differential equation,

\[ -Ip^2(A \cos pt + B \sin pt) + Dp(B \cos pt - A \sin pt) + k(A \cos pt + B \sin pt) = c \ i_m \ \cos pt. \]

Hence

\[ -Ip^2A + DpB + kA = c \ i_m \]

\[ -Ip^2B + DpA + kB = 0 \]

and

\[ A = \frac{c \ i_m \ (k - Ip^2)}{D^2p^2 + (k - Ip^2)^2} \]

\[ B = \frac{c \ i_m \ Dp}{D^2p^2 + (k - Ip^2)^2} \]

When \( t = 0 \), \( \phi = 0 \), therefore

\[ 0 = A + M \] and \[ M = -A \]

\[ \frac{da}{dt} = Ap \cos pt - Ap \sin pt \]

\[ + e^{-qt} q(N \cos qt - M \sin qt) - \delta(M \cos qt + N \sin qt) \]

When \( t = 0 \), \( \frac{da}{dt} = 0 \), therefore

\[ 0 = Ap + Nq - M \delta \] and \[ N = -\left( \frac{hp + M\delta}{q} \right) \]
At the end of the discharge, when $t = \frac{\pi}{2p}$

$$\phi = B + \mathcal{E}^{\frac{-5p}{2}} \left\{ M \cos \frac{\pi}{p^2} + N \sin \frac{\pi}{p^2} \right\}$$

$$\frac{d\phi}{dt} = -Ap + \mathcal{E}^{\frac{-5p}{2}} \left\{ (qM + SN) \cos \frac{\pi}{p^2} - (qM + SN) \sin \frac{\pi}{p^2} \right\}$$

In the case of small damping $\frac{5}{p}$ is very small compared with unity.

Also, in the case of the instrument concerned,

$q$ is of the order of $\frac{1}{1000}$

Neglecting secondary terms, $\phi$ reduces to

$$B \left(1 - \frac{\pi}{2}\right) - A$$

Substituting for $A$ and $B$ and again neglecting small terms

$$\phi = \frac{c i_m}{Ip^2 - k} = \frac{c i_m}{I(p^2 - q^2)}$$

Similarly

$$\frac{d^2\phi}{dt^2} = -Ap + Nq - \frac{q^2 \pi p}{p^2}$$

$$= \frac{c i_m p}{(k - Ip^2)} \left( \frac{\pi p^2}{p^2} - 1 \right)$$

$$= \frac{c i_m}{Ip} \left( \frac{p^2 - q^2 \pi}{p^2 - q^2} \right)$$

The quantity of electricity flowing during the discharge

is $i_m \cdot \frac{\pi}{2p} = \frac{i_m}{p} = q$
Critical Damping.

To obtain this condition $D^2$ must equal $4I_k$

that is $\delta = \frac{k}{I} = q$

$q$ being the pulsation of the instrument with negligible damping.

The equation for $\varphi$ now becomes

$$\varphi = A \cos pt + B \sin pt + e^{-qt} (M + Nt)$$

and

$$\frac{d\varphi}{dt} = Bp \cos pt - Ap \sin pt + e^{-qt} \left\{ (N - q(M + Nt)) \right\}$$

Substituting for $\varphi$, the constants $A$ and $B$ become

$$A = \frac{c I_m (k - Ip^2)}{(k + Ip^2)^2}$$

$$B = \frac{c I_m 2 Ik p}{(k + Ip^2)^2}$$

From the initial conditions

$$M = -A \quad \text{and} \quad N = -(Bp + Aq)$$

At the end of the discharge, when $t = \frac{\pi}{2p}$

$$\varphi = B - \left\{ A + (Bp + Aq) \frac{\pi}{2p} \right\} e^{-\frac{q\pi}{p}}$$

$$\frac{d\varphi}{dt} = -Ap - e^{-\frac{q\pi}{p}} \left\{ (Bp + Aq) \left( 1 - \frac{q\pi}{2p} \right) + Aq \right\}$$

$$= -Ap - e^{-\frac{q\pi}{p}} \left\{ Aq^2 \frac{\pi}{p} + B (p - q\pi) \right\}$$

Since $\frac{q}{p}$ is small compared with unity,

$$e^{-\frac{q\pi}{p}}$$ can be substituted by $1 - \frac{q\pi}{p}$.
\[ \omega = -A \left\{ 1 - \frac{a^2 \pi^2}{p^2} \right\} + B \left\{ 1 - \frac{\pi \cdot p^2}{2 \cdot 4 \cdot p} \right\} \]

Substituting for \( A \) and \( B \) and neglecting terms involving higher powers of \( \frac{a}{p} \) than the first,

\[ \omega = \frac{c m}{I p^2} \left\{ 1 - \frac{a}{p} (\pi - 2) \right\} \]

\[ \frac{d\omega}{dt} = -Ap \left\{ 1 + \frac{a^2 \cdot \pi^2}{p^2} - \frac{a^3 \cdot \pi^2}{p^3} \right\} - Bp \left\{ 1 - \frac{a \cdot \pi^2}{p} \right\} \]

Substituting for \( A \) and \( B \) and neglecting higher powers of \( \frac{a}{p} \),

\[ \frac{d\omega}{dt} = \frac{c m}{I p} \left\{ 1 - \frac{2a}{p} \right\} \]

\[ \frac{I_m}{p} = Q \text{ the total quantity of electricity} \]

**Case 1.**

Rectangular wave \( I = I_m \) between limits \( t = 0 \) and \( t = \frac{\pi}{p} \).

Negligible damping,

\[ \omega = \frac{c q}{I p} \frac{\pi}{2} \]

\[ \frac{d\omega}{dt} = \frac{c q}{I} \left\{ \frac{\sin \frac{q \pi}{p}}{\frac{q \pi}{p}} \right\} \]
Critical Damping.

\[ \theta = \frac{c Q}{I} \cdot \pi \]

\[ \frac{d\theta}{dt} = \frac{c Q}{I} \left(1 - \frac{q}{p} \right) \]

Case 2.

Negligible Damping.

\[ \theta = \frac{c Q}{I} \cdot \pi \]

\[ \frac{d\theta}{dt} = \frac{c Q}{I} \left(1 - \frac{q^2(p^2 - 4)}{p^2} \right) \]

Critical Damping.

\[ \theta = \frac{c Q}{I} \left(\frac{\pi}{2} - \frac{q}{p} \left(\frac{p^2 - 4}{2} \right) \right) \]

\[ \frac{d\theta}{dt} = \frac{c Q}{I} \left(1 - \frac{q\pi}{p} + \frac{q^2(p^2 - 6)}{p^2} \right) \]

These values compare with the simple case in which \( \theta \) is assumed to be zero and \( \frac{d\theta}{dt} \) to be equal to \( \frac{c Q}{I} \).

It is now necessary to investigate the effect of these changes in the initial values on the amplitude of the galvanometer swing.
Theory of the Ballistic Galvanometer.

Ordinary theory extended to take into account an initial deflection $\theta_0$, in addition to the initial angular velocity $\omega_0$.

The differential equation of the motion:

$$I \ddot{\theta} + D \dot{\theta} + k \theta = 0$$

is integrated by:

$$\theta = \left\{ A \cos qt + B \sin qt \right\} e^{-\delta t}$$

giving:

$$\frac{d\theta}{dt} = \left\{ Bq \cos qt - Aq \sin qt - \delta(A \cos qt + B \sin qt) \right\} e^{-\delta t}$$

When $t = 0$, $\theta = \theta_0$.

$$\therefore A = \theta_0$$

When $t = 0$, $\frac{d\theta}{dt} = \omega_0$.

$$\therefore \omega_0 = Bq - \theta_0 \delta$$

and $B = \frac{\omega_0 + \delta \theta_0}{q}$

$$\theta = e^{-\delta t} \left\{ \theta_0 \cos qt + \left( \frac{\omega_0 + \delta \theta_0}{q} \right) \sin qt \right\}$$

$\theta$ is a maximum when $\frac{d\theta}{dt} = 0$

that is when

$$\tan qt = \frac{Bq - A \delta}{Aq + B \delta}$$

When $\frac{\delta}{q}$ is very small this reduces to

$$\frac{B}{A}$$

since $\frac{A}{B}$ is also small compared with unity

Since $\tan qt$ is large compared with 1,

$\sin qt$ can be taken as 1 and $\cos qt$ as

$$\frac{1}{\tan qt} = \frac{B}{A}$$
whence
\[ \theta_{\text{max}} = e^{-\delta t} \left\{ \frac{\theta_0^2 q}{\omega_0 + \delta \theta_0} + \frac{\omega_0 + \delta \theta_0}{q} \right\} \]

with negligible damping this reduces to
\[ \theta_{\text{max}} = \frac{\omega_0}{q} + \frac{\theta_0 q}{\omega_0} \]

In the case of critical damping the equation for \( \theta \) becomes
\[ \theta = e^{-\delta t} (M + N t) \]

and
\[ \frac{d\theta}{dt} = e^{-\delta t} \left\{ N - \delta (M + N t) \right\} \]

when \( t = 0 \), \( \theta = \theta_0 \), \( M = \theta_0 \)
\[ \frac{d\theta}{dt} = \omega_0 \]
\[ N = \omega_0 + \delta \theta_0 \]

Also for critical damping \( \delta = q \)
\[ \theta = e^{-qt} \left\{ \theta_0 + (\omega_0 + q \theta_0) t \right\} \]

The maximum deflection occurs when \( \frac{d\theta}{dt} = 0 \)
that is when
\[ t = \frac{1}{q} \frac{\omega_0}{N} = \frac{\omega_0}{q(\omega_0 + q \theta_0)} \]
\[ \theta_{\text{max}} = \left\{ \theta_0 + \frac{\omega_0}{q} \right\} e^{-\frac{\omega_0}{\omega_0 + q \theta_0}} \]

The corresponding values of \( \theta_{\text{max}} \) according to the ordinary simple theory are:

Negligible damping \( \theta_{\text{max}} = \frac{\omega_0}{q} \)

Critical damping \( \theta_{\text{max}} = \frac{\omega_0}{q} \cdot e^{-t} \)
where \( \omega_o' = \frac{c_o}{I} \), the initial value of \( \omega \) according to the simple theory.

The percentage correction factors for the two cases, by which is meant

\[
\left\{ \frac{\text{Correct value of } \theta_{\text{max}}}{\text{Simple theory value}} - 1 \right\} \times 100
\]

are as follows:

**Negligible damping**

\[
\left\{ \frac{\omega_o - 1 + \theta_o q}{\omega_o' - \theta_o q} \right\} \times 100
\]

**Critical damping**

\[
\left\{ \left( \frac{\omega_o - 1 + \theta_o q}{\omega_o'} \right) \varepsilon^{\frac{q\theta_o}{\omega_o + q\theta_o}} - 1 \right\} \times 100
\]

If \( \theta_o q \) is small compared with \( \omega_o \)

\[
1 + q \frac{\theta_o}{\omega_o + q\theta_o}
\]

can be used for \( \varepsilon^{\frac{q\theta_o}{\omega_o + q\theta_o}} \),

when the percentage correction factor reduces to

\[
\left\{ \frac{\omega_o - 1 + 2 q \theta_o}{\omega_o'} \right\} \times 100
\]

Table overleaf gives values for the various terms and for the percentage correction factors for the three cases of wave form already discussed.

An inspection of this table shows that, whereas in all cases of negligible damping the correction factor involves only the second power of \( \theta_o \), with critical damping, normally, the first power appears.
<table>
<thead>
<tr>
<th></th>
<th>Case 1</th>
<th>Case 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I = I_0$ between $t = 0$ and $\pi/p$</td>
<td>$I = I_0 \sin pt$ between $t = 0$ and $\pi/p$</td>
<td></td>
</tr>
<tr>
<td>Negligible damping</td>
<td>Critical damping</td>
<td>Negligible damping</td>
</tr>
<tr>
<td>$\omega_0$</td>
<td>(-\frac{c_0 \cdot \pi}{I \cdot 2p})</td>
<td>(-\frac{c_0 \cdot \pi}{I \cdot p})</td>
</tr>
<tr>
<td>$\omega_0' = \frac{c_0 \cdot \omega_0}{I}$</td>
<td>(-\frac{c_0 \cdot \pi}{I \cdot \omega_0^2} \cdot (1 - 1.65q^2))</td>
<td>(-\frac{c_0 \cdot \pi}{I \cdot \omega_0^2} \cdot (1 - \frac{q \pi}{p}))</td>
</tr>
<tr>
<td>$\frac{\omega_0^2 - 1}{\omega_0^2}$</td>
<td>(-\frac{1.65 \cdot q \cdot q}{p^2})</td>
<td>(-\frac{3.14q^2}{p})</td>
</tr>
<tr>
<td>$\frac{\omega_0^2 \cdot q^2}{\omega_0 \cdot \omega_0'}$</td>
<td>(2.48 \cdot \frac{q^2}{p^2})</td>
<td>(2.48 \cdot \frac{q^2}{p^2})</td>
</tr>
<tr>
<td>$\frac{2q \cdot \omega_0}{\omega_0'}$</td>
<td>(6.28q)</td>
<td>(\frac{6.28q}{p})</td>
</tr>
<tr>
<td>Percent correction factor</td>
<td>(\frac{83q^2}{p^2})</td>
<td>(\frac{314q}{p})</td>
</tr>
</tbody>
</table>
In case 3, critical damping, the first power of \( q^2 \) appears in each of the two terms involved but cancels out in the percentage correction factor. This is no doubt peculiar to the particular wave form concerned. The magnitude of \( q \) depends on the duration of the discharge and the periodic time of swing of the galvanometer when undamped. Taking these values as 0.01 sec. and 16 sec. respectively, the value of \( q \) is \( \frac{1}{800} \) for cases 1 and 2 and is \( \frac{1}{400} \) for case 3.

The largest percent correction factor given by these figures is 0.4 percent, being obtained in case 1 with critical damping. The correction in all cases of negligible damping is extremely small. This figure of 0.4%, small as it is, does not represent an error of this magnitude in the value of flux density determined by a critically damped galvanometer, since the galvanometer is calibrated by a standard mutual inductance, also in the critically damped state. The figures in Table represent, rather, limiting values due to possible differences in wave form between the calibration and test conditions.

There remains to be considered the effect of the parallel circuit consisting of shunt and galvanometer.
Figure below represents the circuit involved, the electrical constants of the various components being marked on the diagram.

![Circuit Diagram]

If \( e \) is the voltage induced in the secondary winding.

\( i_1 \) is the current in the shunt.

\( i_2 \) is the current in the galvanometer.

\( v \) is the P.D. across both shunt and galvo.

we get

\[
v = e - R_1(i_1 + i_2) - L_1 \frac{d}{dt}(i_1 + i_2)
\]

\[
= i_1 R_2
\]

\[
= i_2 R_3 + L_3 \frac{d}{dt}i_3
\]

Integrating all three equations between limits \( t = 0 \) and \( t = t_1 \), \( t_1 \) being the time at the end of the discharge.

\[
\int_0^{t_1} v \cdot \delta t = \int_0^{t_1} e \cdot \delta t - R_1 \int_0^{t_1} (i_1 + i_2) \cdot \delta t
\]

\[
= R_2 \int_0^{t_1} i_1 \cdot \delta t
\]

\[
= R_3 \int_0^{t_1} i_2 \cdot \delta t
\]

all terms involving \( L \) disappearing since the currents are zero at both limits.
Substituting $Q_1$ & $Q_2$ for $i_1 \cdot t$ and $i_2 \cdot t$ respectively we get

$$\int_0^t e \cdot st = \left\{ \frac{R_1R_2 + R_1R_3 + R_2R_3}{R_2} \right\} Q_2$$

since

$$Q_1 = \frac{Q_2 R_3}{R_2}$$

This result shows that the presence of inductance in one of the parallel branches does not affect the division of the total quantity between them. The accuracy of a ballistic galvanometer will be affected by the shunt therefore, only in respect to that part of its deflection which depends on the current wave form. It is admitted that the difference in time constants of shunt and galvanometer circuits will affect the wave form of the galvanometer current to a small extent, but in view of the results of the previous section, it can safely be assumed that the possible errors due to this cause are negligible.

As a matter of interest the inductance of the galvanometer movement was measured by means of a Heaviside Campbell equal ratio bridge at 2000 cycles shunting the unknown inductance with a non-inductive resistance as it was beyond the range of the variable mutual inductometer. The value obtained was 0.19 H, giving a time constant of $0.4 \times 10^{-4}$
Fig. 44a

Effect of Damping on
Ballistic Calorvimeter Deflection.

Fig. 44b

Full line: Negligible Damping.
Dotted line: Critical

Deflection %

TIME, Secs.
A small objection to the use of critical damping is the shorter time which the instrument takes, in this condition, to reach its maximum deflection. Figs. 44 & 45 are concerned with this point. The curves of deflection against time which are shown in these figures are calculated from the equations on the assumption of equal maximum values. Fig. 44 is the more interesting as this compares the variation of deflection with time in the two cases in the immediate neighbourhood of the maximum values, the two maxima being superposed. It can be seen that the time for which the deflection is within 0.1% of its maximum value is practically the same for both negligible and critical damping. It is suggested that this feature is more important in connection with the accuracy of observing the deflection than the time interval between start and maximum.
### Appendix 2.

**Gall A.C. Potentiometer - Phase Splitting Circuit.**

The circuit diagram and vector diagram for the phase splitting system of the Gall potentiometer are given in Figure 45. The vector diagram, which refers to a 50 cycle supply, was obtained from the readings tabulated below which were taken on high grade ammeter, voltmeter and wattmeter.

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Supply voltage</td>
<td>$V$</td>
<td>99 volts</td>
</tr>
<tr>
<td>Primary</td>
<td>$V_1$</td>
<td>78.5</td>
</tr>
<tr>
<td>Resistance</td>
<td>$V_2$</td>
<td>45.0 (45.5)</td>
</tr>
<tr>
<td>Secondary</td>
<td>$V_3$</td>
<td>37.5</td>
</tr>
<tr>
<td>Output</td>
<td>$V_4$</td>
<td>57.5</td>
</tr>
<tr>
<td>$V_1 + V_2$</td>
<td>V1 + V2</td>
<td>118.0 (116.0)</td>
</tr>
<tr>
<td>Primary current</td>
<td>$I_1$</td>
<td>0.86 amperes</td>
</tr>
<tr>
<td>Resistance</td>
<td>$I_2$</td>
<td>0.675</td>
</tr>
<tr>
<td>Cos. of phase angle</td>
<td>$\cos \theta_1$</td>
<td>0.765</td>
</tr>
<tr>
<td></td>
<td>$\cos \theta_2$</td>
<td>0.639</td>
</tr>
</tbody>
</table>

Of these quantities, $V_2$ and $(V_1 + V_3)$ are redundant and the good agreement between the measured values and those obtained from the diagram are evidence of its accuracy. The latter values are shown in brackets in the above table. An additional check is given by the phase angle between supply and output voltages, $V$ and $V_A$, which angle in the vector diagram is $91.4^\circ$. The angle should be
Fig. 45

Gall Potentiometer Phase Splitting

Vector Diagram

Circuit Diagram
Fig 46.

For 100 V, 50 cycle supply.

\[ R = 66.7 \, \Omega \]
\[ L = 0.37 \, \text{H} \]
\[ M = 0.117 \, \text{H} \]
about 90°.

Inspection of the vector diagram of Figure 45 shows that the secondary current of the phase shifting transformer (I₂) is small compared with the primary current (I₁), the latter being nearly all magnetising current. In order to get a simplified equivalent circuit, this transformer might be considered to be a mutual inductance with open circuited secondary. The vector diagram shows that the secondary voltage of this transformer (V₃) lags the current in the resistance (I₂) by practically 90°, the actual value being 87°, giving additional justification for the simplified circuit. A further simplification is obtained by ignoring the resistance of the primary of this transformer, so that the equivalent mutual inductance has self inductance but no resistance in its primary. The angle between the transformer primary voltage (V₁) and the current (I₂) is actually 77°.

The simplified circuit and its vector diagram are shown in Figure 46. The loci of the various voltages are now circles, these loci being shown in the figure. The voltage vectors are drawn for the condition giving 90° phase displacement between supply and output voltages (full line), and also for changes in frequency of plus and minus 15 per cent.
It is interesting to compare this method of phase splitting with the arrangement of capacitance and resistance in series with the quadrature isolating transformer. The rate of change of phase angle with frequency for the condenser and mutual inductance systems has been worked out, employing the symbolic method. It was assumed that the isolating transformer used with condenser phase splitting would require the same voltage and current input (\(V_A\) and \(I_2\), Fig. 45) as the actual transformer supplied with the Hall potentiometer. The values of rate of change of phase angle with frequency for the two cases are 0.017 radians per cycle for the mutual inductance circuit and 0.025 radians per cycle for the condenser circuit, a result which is rather surprising as the mutual inductance is generally supposed to be superior in respect of frequency variation. The very small advantage which the mutual inductance method possesses with regard to phase angle changes is offset by slightly larger changes in quadrature voltage with frequency.