AN EXPERIMENTAL INVESTIGATION OF
AN ULTIMATE LOAD THEORY FOR THE
DESIGN OF REINFORCED CONCRETE
ARCHES

by

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Summary

This work is an investigation, both theoretically and experimentally, of the behaviour of r.c. arches at the ultimate load.

A brief review of past contributions in this field is followed by the theoretical investigations. These open with the derivation of an expression for the simple collapse load $W_{SC}$ of an arch.

$W_{SC}$ causes arch collapse with $(n + 1)$ hinges considering bending only neglecting the effects of axial force and deformation. To determine $W_{SC}$ it is necessary to know the hinge positions, methods of doing this graphically and by partial differentiation are shown.

The effect of axial force on $W_{SC}$ is next considered. It is shown that the application of a certain amount of axial force to a typical r.c. section can increase the ultimate M.O.R. of the section. A graphical moment distribution process is demonstrated using the elastic structural relationship between moment and thrust at each hinge section and the physical ultimate M.O.R.-axial force hinge section curve. This operation produces $W_{AC}$ a simple collapse load modified for axial force.

In deriving $W_{SC}$ and $W_{AC}$ the arch is assumed undeformed. The effects of bending deformation on $W_{AC}$ is next considered. To achieve this the deflected shape of the complete arch is determined in one analysis. This is done by a graphical method for use in the elastic and in-elastic ranges.

Three methods are given to determine the effects of deformation on $W_{AC}$. A collapse load $W_{ACD}$ modified for the effects of axial force and deformation is thus obtained. The effects of axial forces on deformations are then considered.
The effects of pre-stressing and abutment spreading on the collapse load are next discussed.

Three methods to determine $W_{ACD}$ are next presented.

The fourth criteria for r.c. collapse design, i.e. that the strain at the hinge points must be within a defined limit is then examined. The rotation required for collapse under $W_{ACD}$ is found by the $\delta_{IK}$ method. The rotations available in r.c. members are discussed and approximate rules proposed to determine the rotation available. Use of stirrups to increase the available rotation is considered.

A discussion of the material and section properties assumed in the previous 'structural' sections ends the theoretical work.

The practical investigations follow. These are split into three parts (a) the Large Arch Tests - a report on the six parabolic fixed-ended arches tested, (b) Small Arch Tests - a report on the small model arches tested, (c) Analysis of arches tested by Jain.

The thesis concludes with general conclusions and recommendations for design practice.
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I. REVIEW OF PREVIOUS WORK

1. "Elastic Behaviour"

A great deal of work, both experimentally and theoretically, has been done on the elastic behaviour of arches.

Structural design methods have been evolved and utilised which enable bending moments, axial and shear forces, and deflections to be calculated. These have been shown to be sufficiently accurate for design purposes whilst the arch stresses are everywhere within elastic range.

These methods have been adequately described elsewhere and it is not proposed to do so here.

2. In-elastic Behaviour

The steel arch has been the centre of attention as far as work in the in-elastic range is concerned, the r.c. arch receiving little or no attention. This is to be expected as the Simple Plastic Theory methods utilising the concept of plastic hinges was first established and accepted in the field of steel structures. Before proceeding further with the arch discussion the plastic hinges theories in steel and r.c. will be briefly discussed.

Simple Plastic Theory

The principles behind the application of the Simple Plastic Theory to steel structures has been described elsewhere. For the correct collapse mechanism and load capacity to be found Horne has shown that the bending moment distribution at collapse which fulfils the following three conditions is unique.

1. **Mechanism.** Plastic hinges must form at a sufficient number of points to produce a mechanism.

2. **Equilibrium.** The bending moment distribution must be in equilibrium with the applied loads.

3. **Yield.** The bending moment must nowhere exceed the allowable plastic moment.
These three criteria are sufficient for collapse design in steel. The assumption is made that the steel is ductile enough to allow rotations to develop at the hinge points whilst carrying the full plastic moment during mechanism formation. This assumption has been shown experimentally to be correct.

Two relationships are generally used in the steel plastic theory between the moment at a hinge point and the rotation occurring there. These are shown in Figs 1.1 and 1.2. Fig 1.1 shows the ideal elastic-plastic behaviour which approximates closely to the actual behaviour illustrated in Fig 1.3. Fig 1.2 shows the rigid plastic behaviour, a more approximate assumption.

Due to the low crumbling strain of a r.c. section a fourth criterion is added to the three used for steel collapse design. This is that at collapse the strain in the hinge sections must not exceed a limited defined amount.

The failure strain of a bound concrete member may be between 1% and 3% whilst a piece of mild steel will undergo a 20% strain before rupture. For the r.c. section the assumed $M\sim \phi$ relationships comparable to Figs 2.1 and 2.2 are shown in Figs 2.4 and 2.5.

Several methods have been evolved for the analysis of steel structures using the plastic hinge theory to find the collapse load. They all involve using two of the three criteria for collapse and modifying the first two to satisfy the third. Neal and Symonds' virtual work method is usually the most convenient. Many tests have shown that the plastic theory methods of estimating the collapse load are reasonably accurate.

The plastic hinge theory for the design and analysis of r.c. structures proposed by Professor A.L. Baker takes regard of the limited strain capacity of the concrete and modifies the steel plastic hinge theory in the following ways:-

Collapse is deemed to occur when the $n^{th}$ hinge, in an $n^{th}$ redundant structure, forms or when the defined upper strain limit of the r.c. is reached at a hinge.
FIG 2:1

Ideal Elastic-Plastic (Steel)


FIG 2:2

Rigid-Plastic (Steel)


FIG 2:3

Moment-Curvature Relationship for a Mild Steel Rectangular Specimen


FIG 2:4

Failure by Concrete Crushing or High Tensile Steel Rupture at 'Limited' φ


FIG 2:5
if this occurs before the \( n^{th} \) hinge forms. This compares with \((n + 1)\) hinges of the mechanism formed when a steel structure collapses and the 'infinite' strain capacity assumed at a steel hinge section.

The Effect of Axial Forces

The simple plastic theory methods in steel and concrete touched on above neglect the effect of axial force. For a steel section it has been shown that the maximum moment which can be developed at a section is reduced when an axial force acts with an applied moment. Fig 2:6 taken from Stevens shows this variation. Here a stress/strain curve of Fig 2:7 is assumed. The area between the axes and the curve perimeter is termed the admissable area. Combinations of \( M \) and \( H \) which are inside this area are statically admissable states of stress. Any points outside the area are inadmissable, whilst a plastic hinge occurs on the perimeter. The \( M \sim \phi \) relationships are influenced by the effect of axial force. Fig 2:8 taken from Stevens shows the effect of axial force on a rectangular steel section. Another effect of axial force is to cause a contraction at a hinge point.

If a rotation \( \Theta \) occurs at a hinge subjected to an axial force, then there will be associated with this rotation an axial deformation \( \delta \). This is clear from Fig 2:9 where it is seen that rotation at the hinge takes place about the neutral axis, which has been displaced by the amount \((d - y)\) from the centre line.
RELATIONSHIP BETWEEN AXIAL FORCE & PLASTIC MOMENT FOR M. STEEL.

\[ m = \frac{M}{M_0} \]

\[ n = \frac{P}{P_0} \]

FIG 2:6
(FROM STUDYNS)

ASSUMED STRESS/STRAIN FOR STEEL

FIG 2:7
STRESS DISTRIBUTION IN A RECTANGULAR STEEL SECTION UNDER MOMENT AND AXIAL FORCE

FIG 2.9

MOMENT-CURVATURE RELATIONSHIP WITH AXIAL FORCE FOR A M.S. RECTANGULAR SECTION (FROM STEVENS)

FIG 2.8
Then

\[ \delta = (d - y) \theta \quad \text{--- 2:1} \]

Onat and Prager have derived the following expressions for the energy dissipated in deformation at a plastic hinge.

\[ U = P\delta + M\theta \quad \text{--- 2:2} \]
\[ = nP_p \delta + mM_p \theta \quad \text{--- 2:3} \]

By using the principle of Maximum Plastic Work, the relationship between \( \delta \) and \( \theta \) is shown to be

\[ \delta = -\theta \frac{M_p}{P_p} \frac{dm}{dn} \quad \text{--- 2:4} \]

since \( \frac{dU}{dn} = 0 = P_p \delta + M_p \theta \frac{dm}{dn} \quad \text{--- 2:5} \)

For a r.c. section the relationship between \( M_{ult} \sim H \) is not so straightforward as in the case of steel. Fig 2:10 shows the \( M_{ult} \sim H \) relationship for a typical r.c. section. It shows that over a large part of the admissable area the addition of an axial load increases the moment-carrying capacity of the section above that due to pure bending. Also shown on Fig 2:10 is a curve of \( H \sim \theta \). The decrease in the available section rotation is most marked for high values of \( H \).

The Effect of Deflections

The relationship between curvature and deflection in the in-elastic range is given by the approximation

\[ \frac{1}{\rho} = \frac{d^2y}{dx^2} \quad \text{--- 2:6} \]
If the stress/strain curve of the member material and the distribution of strain across the member are known then by finding expressions relating $M$ and $\phi$ the deflections can be obtained by using them in equation 216 and integrating. For a steel member Roderick\(^5\) has shown how this integration can be performed analytically. The analytical method is tedious when the material stress/strain curve is not simple. Newmark\(^6\) has discussed arithmetic methods for use in the latter case.

For r.c. members Professor A.L. Baker\(^2\) has shown that provided deflections are small relative to the length of the member slopes and deflections may be obtained by integrating $\frac{E_0}{E_I}$ along the member. Due to cracking and the nature of the material values of $E_0$ and $I$ vary along an r.c. member which make the determination of deflection less straightforward than is the case with the steel member. Professor A.L. Baker\(^2\) has proposed reasonable values for $E_0I_1$ dependent on $M$, $Cu$, $S$, and $n$, ranging from the elastic uncracked to the plastic cracked condition.

The effect of deflections on the simple plastic theory methods for the analysis of steel structures is usually ignored. Only at collapse are the deflections assumed to become 'large'. The effect of deflection on the simple plastic theory for r.c. arches is discussed in Chapter II section 7.

**Arch Analysis by Plastic Theory Methods**

Using upper and lower bound solutions Onat and Prager\(^4\) have applied the principles of the simple plastic theory to the analysis of pin-ended arches with both concentrated and distributed loads. An upper bound to the collapse load is found by satisfying the mechanism and equilibrium conditions. By satisfying the equilibrium and yield conditions only a lower bound solution can be obtained by statics. By using
inequalities these bounds can be narrowed to improve the estimate of the collapse load.

Onat and Prager modified the relationship between $M_0$ and $H$ as shown in fig 2:11. This makes the analysis easier when axial thrust is considered as the relationships between axial contraction, rotation, $M$ and $H$ are considerably simplified.

The approximate locations of the plastic hinges are initially found by neglecting the effect of axial force. This also gives an estimate of the thrust at the simple plastic theory collapse load. Using this thrust and the expressions shown in Fig 2:10 the collapse load estimate is reduced so that the yield criterion is nowhere violated. This gives a lower bound solution for the collapse load.

The arch material is considered to have the $M \sim \phi$ relationship of Fig 2:2. The upper bound method is also applied to the same problem; the material being considered as rigid-plastic with the $M \sim \phi$ relationship shown in Fig 2:2. When the loaded arch is given a small virtual displacement, the relationship between rotations and contractions must obey equation 2:5. The energy dissipated at the hinges for these deformations is then equated to the work done by the load moving through the corresponding distance. From this equation the upper bound solution for the collapse load can be obtained. This satisfies the mechanism and equilibrium conditions for correct collapse design but unless the correct hinge positions and thrust have been used, the yield condition will not be satisfied and the estimate will be an upper bound. Onat and Prager point out, however, that in most cases a small error in the location of yield hinges will produce a very small change in the estimate of the load capacity.

By using these two methods together, the upper and lower bound solutions can systematically be made to approach each other and hence obtain as close an estimate of the plastic theory collapse load as is desired.
In their investigation no tests were performed to check the accuracy of the analysis, nor was any analysis made on the effect of deformation. It was pointed out that the "limit analysis must be supplemented by a study of structural stability whenever compressive forces of sufficient intensity exist in a structure." However, no indication is given as to what is considered a sufficient intensity, or how this study could be done.

An analytical investigation by Eickhoff considered the effect of axial thrust on sections having only one axis of symmetry. These results he applied to the analysis of a centrally loaded parabolic fixed ended steel arch. Upper and lower bound solutions are discussed and a method presented of finding the collapse load from successive lower bounds. No consideration was made of the effects of deformation. No tests were carried out.

Johansen and Hansen have investigated the behaviour of steel arches and rings both theoretically and experimentally. They applied the simple plastic theory methods to determine the collapse loads.

A graphical method is shown which enables the plastic hinge positions to be fixed when single concentrated loads are applied to an arch with pinned or fixed ends. With the known hinge positions the collapse load is then calculated by statics or virtual work. It is assumed that axial forces in the arch are small and there is a negligible reduction in the fully plastic moment.

Seven tests on semi-circular fixed-ended arches of mild steel having a rectangular cross section were carried out. The material curve was close to Fig. 21 but with some strain hardening.

The influence of deformation on the simple collapse load was recognised and was calculated by a semi-empirical method.
from measured behaviour. No purely theoretical method is suggested for determining the reduction in load capacity due to change in geometry. The method suggested is only applicable after a test has been performed on the particular arch considered.

Hendry has investigated the possibility of determining collapse loads of steel arches by the simple plastic theory and has also performed tests to check the methods proposed.

Assuming that the full plastic moment is developed at the hinges, a graphical method is presented for determining the hinge positions for concentrated and distributed loads acting on pin-ended arches. This is applied to a parabolic arch under a two-point load system, and the conclusion is made that for the rise to span ratios usually found in bridge design the collapse load is independent of the rise, since the axial thrust is always small. This is shown to be true for arches of rectangular section under single point loads, but in the application of the theory to the design of a highway bridge with considerable uniform loading, the axial stress at collapse is found to be about 5 kips/in². This corresponds to a value of n of about 0.32, and a reduction of the fully plastic moment to 80% of its pure bending value is recommended.

To check these theoretical investigations, a series of tests were made on pin-ended parabolic arches of rectangular cross-section under single and two-point concentrated loads. The collapse loads found from these tests were in good agreement with that predicted by the simple plastic theory, the maximum difference being for a collapse load 94.1% of that predicted. Another test was performed on an I type section fabricated from mild steel plate, cut, bent, and welded, to form a parabolic arch with a rise to span ratio of 1/4. A single concentrated load was applied at the third point, and a collapse load 8% greater than that predicted by the simple plastic theory was obtained. This increased load capacity was attributed to
strain hardening increasing the moment of resistance of the hinge under the applied load.

The axial stress in all these tests was considered to be sufficiently small for its effect upon the plastic moment to be neglected. It is concluded that the strength of a two pinned mild steel arch rib can be estimated with sufficient accuracy by the simple plastic theory, with suitable reductions being made to the allowable value of the plastic moment if the thrusts are high.

Stevens has carried out a theoretical and experimental investigation on the behaviour of steel arches in the inelastic range. This is the most complete work on the subject to date. He calculated the simple plastic collapse load and determined independently the effects of axial force and deformation on this load. The combined effects of axial load and deformation on the simple collapse load are appraised. In his method of determining the simple plastic collapse load a method is presented for the location of the hinge positions. The variation of these hinge positions as arch loading changes are dealt with and some charts presented for typical arches as the arch loading position changes. An analytical method is presented for determining the effect of axial forces. These are shown in Figs 2:12 and 2:13. These show that the reduction in load capacity due to axial forces are important for low \( \frac{1}{2d} \) ratios and flat arches.

In his method for determining the effects of deformation Stevens presented a graphical method for the determination of the deformed shape of an arch under bending. This method is applicable to any material \( M \sim \sigma \) curve.

Two methods for determining the effects of deformation on the simple plastic collapse load are presented. One is termed the "Fixed Load Method" and the other the "Fixed Deformation Method". Both methods are discussed in Chapter II Section 7 herein. The latter method is the more direct and recommended
Reduction in load capacity caused by axial force

M.S. pin-ended circular arches with rectangular section
 Curve (a) \( \frac{h_c}{L} = \frac{1}{2} \)
 Curve (b) \( \frac{h_c}{L} = \frac{1}{8} \)

FIG 2:12

Reduction in load capacity caused by axial force

M.S. fixed-ended arch
 Curve (a) rectangular sect
 Curve (b) R.S.T. beam sect

FIG 2:13
by him for use. Graphs are presented showing the effect of geometry change on the load capacity. These are also shown in section 7.

The effect of axial forces on deformation is considered. He shows that for high values of axial thrust the deformation caused by the thrust may negate that due to bending and prevent the stipulated mechanism forming. Recommendations are made as to the approximate values of the rise, ratio span and the size of axial force in the arch to determine whether axial force has a significant effect on the simple collapse load. These are: if the rise > \( \frac{0.25}{\text{span}} \) and the axial force < \( \frac{0.4H_0}{\text{span}} \) then assumed arch collapse can occur.

The effects of prestraining and abutment spreading are also considered. It is shown that in most practical applications the effect of abutment spreading can be ignored.

The effects of axial force and deformation on the simple collapse load are combined to form an assessment of their total effect on the simple collapse load. This is presented in graphical form Fig 2:14 for a typical arch. He points out that the reductions due to axial thrust and deformation do not combine directly to give the collapse load as they are inter-dependent. If the collapse load is reduced by deflexions then the thrust at collapse is also reduced. Hence, the reduction caused by the axial force is less than that obtained when it is assumed that no deformation occurs. Curve (a) Fig 2:14 therefore over-estimates the reduction. Also, if it is assumed that the M-Ø relationships are still ideally elastic plastic when axial force acts, Fig 2:15, the deflections at collapse are less since the moments and curvatures between hinges are reduced. The reduction in load capacity caused by deflection is therefore modified and curve (b) Fig 2:14 gives an over-estimation of the reduction. He goes on to say that an accurate analysis taking into account both these effects is complex due
Fig 2:14

Curves:
1. Reduction caused by axial force
2. Reduction caused by deformation
3. Combination of (1) and (2)

Reduction of load capacity for a section with ideal elast-plastic M-φ curve.

\[
\frac{W_c}{W_{cc}}
\]

Fig 2:15

Idealized M-φ curve for a section with applied axial force.
to their interaction and too laborious for practical use. As already pointed out curves (a) and (b) Fig 2:14 cannot be added directly to give the combined effects. He suggests, however, that their simple addition should provide a lower bound to the load capacity if the $M = \phi$ relationship is that of Fig 2:15. This lower bound is shown as curve (c) Fig 2:14. The actual load capacity lies on the shaded area with the curves (a) and (b) forming the upper bounds.

To experimentally verify his theory Stevens carried out two series of tests. One with approximately thirty small 10 inch span \( \times \) 25inch square section mild steel arches of five types. The second series consisted of 2 - 18' - 6" span 3' - 6" rise fixed ended parabolic 6" x 5" I arches and 1 - 15' - 0 span 3' - 9" rise fixed ended circular braced angle arch. The results obtained compared reasonably well with predicted behaviour. In the case of the small arches concentrated and distributed loadings were used. For the large arches concentrated loads were used.

Kooharian presented a limit analysis method for the analysis of voiseur and unreinforced concrete arches. Again upper bound and lower bound solutions were suggested on the lines proposed by Onat and Prager. No experimental investigation was carried out.

Jain tested thirty r.c. arches in work on redundant structures. His arches were two-pinned with the end thrust experimentally known. Using the known force distribution he was thus able to predict arch failure. Methods were presented for determining the rotation at a plastic hinge and, based on the tests results, envelope values of $EI$ proposed to calculate deflections prior to collapse. The method proposed for the hinge rotation calculation was too laborious for practical use. He did not attempt to present an analysis of arch structural behaviour but used the arch with known moments and thrusts at a section as a means to verify his ultimate strength of a section theory.
The review of previous contributions to the study of inelastic arch behaviour in the preceding section shows that considerable theoretical and experimental work has been performed on the behaviour of steel arches in the inelastic range. Stevens\(^3\) has presented an acceptable design method which for ease and speed compares favourably with elastic design methods. His method is structurally more realistic than those employing elastic concepts and results in more economic design.

As far as the author is aware no comparable work has been done either theoretically or experimentally in formulating an ultimate load design and analytical method for r.c. arches. It is the purpose of this work to carry out such an investigation.

The line of approach followed in this work was outlined in the summary following the title page.

A deliberate attempt has been made, as far as some of the structural considerations are concerned at least, to follow the pattern set by Stevens\(^3\) in his steel arch work. This was done firstly because his ideas were sound and secondly because it was felt that by so doing some assistance would be given to designers who might use either medium.

The main differences between the steel and r.c. arch collapse cases are:-

1. For steel the collapse mechanism is based on \((n + 1)\) fully formed plastic hinges. For r concrete, collapse is defined herein to occur just before the formation of the \((n + 1)\)\(^{th}\) hinge. It will be seen later that this is not strictly true as some plasticity may have in fact occurred at the \((n + 1)\)\(^{th}\) hinge before defined
collapse. A more accurate definition is that collapse occurs in a r.c. arch when the concrete at the \((n + 1)\)th hinge is about to crush. For deflection and rotation analyses herein continuity is assumed to obtain across the \((n + 1)\)th hinge at collapse. This is shown to be a reasonable assumption.

2. For the r.c. arch a 4th criteria for correct collapse design is added to the three required to be fulfilled for the steel case. This is that at collapse the strain in the concrete must not exceed a limited defined amount. This is the reason for stipulating that the collapse 'mechanism' in the case of a r.c. structure occurs prior to the formation of the \((n + 1)\)th hinge.

3. Importantly the effect of a certain amount of axial force on a r.c. arch section is generally to increase its ultimate M.O.R. \((M_{\text{ULT}})\). In the case of steel the application of axial force to a section decreases its available plastic moment.

4. In considering the effect of deflection on the collapse load of a r.c. arch it is shown that although in most cases it lowers the permissible collapse load by a few per cent in some cases deflection actually increases the collapse load. As an example consider the arch Fig 3:1 on which are shown plotted the \(M_{\text{ULT}}\sim H\) characteristic curve of the section and the \(M\sim H\) structural relations for hinge points B and D. As the load increases from zero a stage is reached where a hinge forms at B. When this occurs the arch changes structurally and in so doing the \(M\sim H\) relationships at B and D change from the elastic to the 1st hinge range. As loading proceeds until a hinge is just about to form at D so points B and D follow the dotted lines Fig 3:1. For point B if the \(M\sim \phi\) curve were that of Figs 2:4 or 2:5 the moment at B would remain constant.
Mohr Structural Elastic Relationship for B

Mohr Curve of Section

Mohr Elastic Ins. Cyl. for D

Moment, $N_0 = 13750$ lbs

Axial Thrust, $10^3$ lbs

Figure 3.1

Figure 3.1(a)
until collapse. It is assumed however that whilst other hinges are forming the moment at B continues to rise up the $M_{ULT} \sim H$ curve under the new 1st hinge $M \sim H$ structural curve. This is felt to be a reasonable assumption as what effectively happens at B is that the increased axial thrust increases the N.A. depth and hence more concrete is taken into the compression zone. Thus over a limited length a rising $M \sim H$ curve is assumed Fig 3:1(a). The effect of this for arch Fig 1:3 is that the collapse load including the effects of deflection is $>\$ that without deflection. (N.B. Arch Fig 3:1 is fully analysed in Chapter III Section 16 - Jain's Arch Type I).

The effect of deflection on steel arches is invariably to lower the simple collapse load.

The theoretical investigations now follow.
In this chapter only bending moments acting on the arch are considered. The effects of axial force, shear force and deformation on the simple collapse load are ignored. The arch is treated as a structure retaining its original shape to collapse. The arch sections are assumed to exhibit a rigid plastic characteristic and are assumed to have sufficient rotation available at the hinge sections to allow a mechanism to form.

Definition of Simple Collapse Load. - WeC.

This is defined as the minimum load causing arch failure by the formation of sufficient plastic hinges to cause a collapse mechanism. The hinge formation being brought about by bending action alone. The effects of axial force, shear force and deformations are ignored.

Definition of a Hinge

To fulfill one of the basic criteria for the plastic or collapse design of a structure it is necessary to determine the position of those zones within it which effectively become hinges and thus transform the structure into an unstable mechanism. The term Zone will be used to describe these positions for these practically occur in a reinforced concrete member. The concept of a plastic hinge is that of the rotation of two adjacent parts of a member relative to each other. However due to its material nature the reinforced concrete member presents its rotation over a finite length which varies according to the physical properties of the member and the variation in bending moment in the vicinity of the hinge. These hinge characteristics are discussed more fully in Chapter 11.12.

These points within the structure which become centres of high stress leading to local yielding of the tension steel, compressively stressed concrete, and possibly compression steel, if present, will be termed "Plastic Hinges".
Plastic hinge is a suitable term for these zones as before failure there is usually some plastic flow as outlined above, coupled with some rotation within the zone to achieve the quasi static mechanism leading to collapse.

In what follows the term "Mo" will be used to denote the ultimate M.O.R. of a section where failure is brought about by bending alone.

Consider a fixed ended parabolic arch of uniform section shown in Fig.4:1, vertically loaded so as to cause collapse by hinge formation at A B D E.

The least number of hinges required in the arch to cause collapse is 4, i.e. in this simple case the number of redundancies + 1. This rule applies to all simple redundant structures when the possibility of local collapse is excluded. By simple is meant single storey frames. Frames of more than one storey often require less hinges than given by the above rule, to form a minimum collapse load mechanism. Generally the rule for the minimum number of hinges for collapse of a frame of n > 1 storeys and m > 1 bays is: \[ \text{No. of hinges} = \text{No. of redundancies in structure} - m(n-1) \]

In most cases to form a mechanism adjacent hinges must take on opposite rotations. In this case hinges at which the lower arch face is in tension will be called + ve and those with a compressive lower face - ve. The pair of arrows used at each hinge point also shows the type of hinge formed there. The two arrow heads appearing on the compression face.

For the majority of arch shapes at their minimum collapse load, due to the bending moment distribution, hinges will form at both abutments. The cases when one abutment only is a hinge point will be discussed later in the chapter.
Referring to Fig. 4:1 the parabola's origin is taken at C, the arch crown, the x and y co-ordinates being measured as shown.

Writing down the expressions for the moments at the hinge points we obtain:

\[- M_A + V_A L - W_s f(L) = + M_E\]
\[+ M_E + V_B L - W_s f_1(L) = - M_A\]
\[- M_A + V_A \frac{L}{2} (1-r) - H_o h_B - W_s f_2(L) = + M_B\]
\[+ M_E + V_B \frac{L}{2} (1-q) - H_o h_d - W_s f_3(L) = - M_D\]

\[f(L) \text{ to } f_3(L) \text{ being functions of the load position.}\]

These reduce to:

\[
\begin{align*}
V_A L &- W_s f(L) - (M_A + M_E) = 0 \\
V_B L &- W_s f_1(L) + (M_A + M_E) = 0 \\
V_A \frac{L}{2} (1-r) - H_o h_B &- W_s f_2(L) - (M_A + M_B) = 0 \\
V_B \frac{L}{2} (1-q) - H_o h_d &- W_s f_3(L) + (M_E + M_D) = 0
\end{align*}
\]
For the simple collapse case $M_A = M_B = M_E = M_D = M_P$ where $M_P = M_d$ at A B D E.

Solving for $W_{sc}$ in determinantal form we obtain:

$$W_{sc} = - M_p \begin{vmatrix} L & 0 & 0 & -2 \\ 0 & L & 0 & +2 \\ \frac{L}{2} (1-r) & 0 & -h_B & -2 \\ 0 & \frac{L}{2} (1-q) & -h_D & +2 \end{vmatrix} = F(r,q.) - 4:2$$

For the given loading system the collapse load $W_{sc}$ depends on the hinge positions. A basic criterion of the collapse method of design is that the correct mechanism is the one requiring the minimum load for collapse. This principle can be used to determine $W_{sc}$ in this case.

Applying conditions for maximum and minimum of a function:

$$\frac{\partial F(r,q.)}{\partial r} = 0 \quad \text{and} \quad \frac{\partial F(r,q.)}{\partial q} = 0$$

(The physical nature of the problem makes it unnecessary to discriminate mathematically any further)

These two equations will give values of $r$ and $q$ which when substituted in (2) will give the minimum load $W_{sc}$ required.
As an example consider the pin-ended parabolic arch of uniform rib section loaded at the 1/3 point, Fig. 4:2.

By taking moments about B and D the expression for the collapse load $W_{sc}$ is:

$$W_{sc} = \frac{-18 M_p}{L} \begin{vmatrix} -h_B & -1 \\ -h_D & +1 \\ -h_B & 4 \\ -h_D & 3(l - q) \end{vmatrix}$$

The load at B fixes a hinge there and (3) reduces to:

$$W_{sc} = \frac{-18 M_p}{L} \left[ \frac{(1 + \frac{q}{2} (1-q^2))}{\left(\frac{q}{2} (1-q^2) - 3(l-q)\right)} \right] = F(q)$$
To find the minimum load for collapse \( \frac{\partial W_{sc}}{\partial q} = 0 \)

This gives \( q = 0.446 \) and \( W_{sc} = 23.6 \frac{M_p}{L} \)

Taking another example, a fixed ended parabolic arch, uniform rib, loaded to collapse, Fig. 4:3.

\[ W_{sc} = -32 \frac{M_p}{L} \left\{ \left[ \frac{(1-r^2)(1+2q)}{1-q^2} \right] + \frac{1+r}{2} \right\} \frac{1}{\left( \frac{1}{3} + \frac{3r}{3} - 8r^2 - 3\frac{1-r^2}{1+q} \right)} \]

which on differentiating with respect to \( q \), and \( r \), give fourth order equations which are tedious to solve.

In the case of some other arch shapes, e.g. circular, the resulting expressions for \( W_{sc} \) are difficult to differentiate. This difficulty is encountered in all except the very simple loading conditions and arch shapes which are easy to represent mathematically. An easier solution to these cases can be found graphically by an iteration method. The method will be illustrated by solving Equation 4:4.
By drawing the elastic B,M,D the approximate positions of the hinges at B and D can be found.

Assume a reasonable value for \( r \). Keep this constant and for some values of \( q \) find \( \frac{W_{sc}}{N_p} \). Change the value of \( r \) and repeat. A graph will be obtained as Fig. 4:4 (a). The bottom point of each graph on Fig. 4:4 (a) satisfies the condition \( \frac{\partial W_{sc}}{\partial r} = 0 \). By transferring these bottom points and plotting them against their respective \( q \) values a graph will be obtained, Fig. 4:4 (b), all points of which satisfy the \( \frac{\partial W_{sc}}{\partial q} = 0 \) condition with the lowest point also satisfying the \( \frac{\partial W_{sc}}{\partial q} = 0 \) condition. Thus \( q, r \) and \( W_{sc} \) can be found. In Figs. 4:4 (a) and (b) \( \frac{W_{sc}L}{N_p} \) is plotted against \( r \) and \( q \) respectively.

It will be observed that the variation of \( W_{sc} \) as \( q \) goes from .35 to .4 is .15 and from .4 to .45 is .25. The latter gives 0.55% error in the actual collapse load. This is a negligible error when compared with the possible errors involved in making the normal reinforced concrete design assumptions. The small difference in collapse load with this hinge movement is due to the bending moment remaining nearly constant for quite a large segment of the rib about the hinge. This is clearly shown in the bending moment diagram for the arch at collapse, Fig. 4:5.

This small difference in collapse load only applies to the movement of the hinge along the rib. Movements away from the rib, e.g. by the rib deflecting from its initial shape, can seriously affect the value of the collapse load. This will be discussed in a later chapter.

By plotting \( q \) and \( r \) against \( \frac{W_{sc}L}{N_p} \), Figs. 4:4 (a) and (b) can
thus be used to determine the hinge positions and simple collapse loads of any arch of this shape and type of loading. Let $\frac{w_{sc} L}{F_X}$ the collapse load parameter.

Graphs can be drawn for any arch shape, using the above method showing, the variation of hinge position and simple collapse load with the load position. Some typical arch shapes and simple loading conditions are shown in Figs. A.14 and A.15.

It is particularly useful when a shape not easily expressed mathematically is used. A drawing of the arch can be made from which the geometrical relationships can be scaled. The method can be made as accurate as required.

In Figs. 4.14, 4.15, and in most practical cases hinges will form at the abutments by virtue of the arch shape and loading distribution. Thus the condition $\frac{\partial X_{sc}}{\partial q} = 0 = \frac{\partial X_{sc}}{\partial r}$ only has to be satisfied to determine the unknown hinge positions and collapse load. Under some combinations of arch loading and arch shape three hinges form within the arch, Fig. 4.14. Thus the condition $\frac{\partial X_{sc}}{\partial q} = \frac{\partial X_{sc}}{\partial r} = \frac{\partial X_{sc}}{\partial F_X} = 0$ has to be satisfied. This complicates the problem, but it can be solved applying the same graphical principles as those stated above. The method of solution is as follows:

For a constant value of $F_X$ vary $r$ and $q$ to obtain graphs similar to Fig. 4.4 (a) and (b) repeat this for several values of $F_X$. A graph can be drawn of $F_X$ against $X_{sc}$, the $X_{sc}$ values corresponding to the $\frac{\partial X_{sc}}{\partial q} = 0 = \frac{\partial X_{sc}}{\partial r}$ points on the Fig. 4 (b) graphs. Thus a graph can be drawn all points of which satisfy

$\frac{\partial X_{sc}}{\partial q} = \frac{\partial X_{sc}}{\partial r} = 0$ condition. The minimum value of this graph also satisfies $\frac{\partial X_{sc}}{\partial F_X} = 0$ and this point will give the values of $X_{sc}$, $r$, $q$, and $F_X$ required.
In most practical cases it will be possible to establish the hinge positions to close limits simply by drawing the elastic bending moment diagram. Thus reasonable values of \( P, q\) and \( r\) can be chosen and the graphical work outlined used to refine these values. It will also be found that small variations of any two \( P, q\) and \( r\) has a very small effect on the third; thus the method can be made as accurate as required. It is important to draw the bending moment diagram when the collapse mechanism has been established. This will reveal whether the third plastic collapse criteria is fulfilled, viz., at all sections between the hinges points the bending moment is below the \( M_0\) value of the sections. If this is satisfied the correct simple collapse load has been found. If not, the hinge positions must be adjusted accordingly.

**Arch Ribs of Varying Section**

Many arch ribs are constructed of varying cross section. Thus the bending ultimate M.O.R., i.e. \( M_0\), varies from section to section round the arch. This complicates the determination of the collapse load by introducing another variable in the numerator of eqn 4:2.

Looking at eqns. 4:1 and considering Fig. 4:1 the first step towards a solution is to interlate the \( M_0\) moments at ABCDE so that the 'common factor' \( M_p\) can be taken out and equation 4:2 developed. A way of doing this is to call the crown bending ultimate M.O.R. \( M_p\), work out the plastic moment for some sections round the arch, and plot graphically the variation in \( M_p\), e.g. Fig. 4:6.

\[ \begin{align*}
M_0 \text{ CROWN } &= M_p \\
M_p \text{ VARIATION OF SECTION } M_0 \\
\text{CROWN AROUND ARCH}
\end{align*} \]

**Fig. 4:6.**
It is unlikely that this variation in $M_p$ can be expressed as a simple function of $L$. To overcome this difficulty the values appropriate to points $ABDE$ are inserted in equation 4:1 when the initial bending moment diagram is drawn to determine the appropriate hinge positions. When the hinge positions are found they should be checked with the initial assumptions. If they are within the limits outlined in the previous example, i.e. 2.5% of the arch span, a sufficiently close estimate of $W_{sc}$ will be obtained. If not the process can be repeated until the required accuracy is attained. In many practical arch shapes the variation in $M_o$ over 5% of arch span, segments is small and the effect of guessing a hinge at one end of a segment and finding it at the other is well within the limit of error accepted in reinforced concrete work. It must be stressed, however, that each case must be considered on its merits. Obviously with an arch type possessing a rapidly changing $M_o$ the analysis will probably require two or three closer approximations.

As an example the parabolic arch type tested, No 4L, will be considered loaded at the 1/4 point. It is required to find the hinge positions at collapse and $W_{sc}$. This example will be followed through the thesis.

From an inspection of the elastic BMD Fig. 4:7, hinges would be expected to form under the load, at both abutments and at $D$. The expression for $W_{sc}$ eqn. 4:2 becomes:—
\[ W_{sc} = -M_p \]

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<tr>
<td>( \frac{L}{2} (1-q) )</td>
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<td>(- h_B )</td>
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\[ W_{sc} = -2.37 \times 4M_p \]

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<td>( \frac{L}{2} (1-q) )</td>
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<td>(- 2 (1-q^2) )</td>
<td>+ .933</td>
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\[ W_{sc} = \frac{-2.37 \times 4M_p}{10} \]

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Which on substitution of the known quantities reduces to:

\[ F(q) = \sum F(q) \]

To find \( q \) apply \( \frac{2F}{dq} = 0 \)

This gives \( q = .267 \) and on substitution \( W_{sc} = 10.02 \).

Estimated \( q \) was .27. Thus the error in \( q \) in this case is small and the error in the collapse load negligible.
Elastic B.M. Diag. for Parabolic Arch Loaded at \( \frac{1}{4} \) Pt. Arch 4L

**Fig 4.7**

First Estimate \( M_{0} \) at 'D' = 6055 Nm

\( M_{0} \) at D from Calc

Note. Arch 4L. Depth AT D = Depth AT B

Variation of \( M_{0} \) Around Arch

**Fig 4.9**
So far the hinge determination procedure has dealt with the analysis of an arch and has led to the calculation of the simple collapse load.

For design purposes the loads to be carried are usually known or estimable. The shape of the arch and the variation of $M_0$ around the arch to carry these loads have to be determined. Assuming $W_{sc}$ Fig. 4:1 is the load to be carried by the arch shown Equation 4:2 can be rewritten to give $M_p$ thus

$$M_p = -W_{sc}$$

$$\left| \begin{array}{ccc} L & 0 & 0 \\ 0 & L & 0 \\ \frac{L}{2}(1-r) & 0 & -h_B \\ 0 & \frac{L}{2}(1-q) & -h_D \end{array} \right|$$

Equation 4:2(a) can then be solved for $M_p$, $q$, and $r$ in the same way as outlined above for the determination of $W_{sc}$.

**Determination of Horizontal End Thrust at Collapse.**

To check that the second and third criteria for correct collapse design have been satisfied, viz. that statical equilibrium exists immediately prior to collapse and that nowhere between the hinges is the $M_0$ value of the section exceeded, it is necessary to draw the B.M.D at collapse.

To do this it is necessary to know the horizontal thrust at the abutments. This can be obtained for the simple collapse
load by re-writing equation 4.2 in terms of the end thrust at simple collapse $H_{sc}$.

$$H_{sc} = -M_p$$

\[
\begin{bmatrix}
L & 0 & -f(L) & -2 \\
0 & L & -f_1(L) & +2 \\
\frac{L}{2}(1-r) & 0 & -f_2(L) & -2 \\
0 & \frac{L}{2}(1-q) & -f_3(L) & +2 \\
\end{bmatrix}
\]

In this equation $q$, $r$ and $M_p$ are known from the previous $H_{sc}$ or $M_p$ calculation and hence $H_{sc}$ can be determined.

As an example the end thrust for arch $4L$, Fig. 4:8, is

$$H_{sc} = -2.37M_p$$

\[
\begin{bmatrix}
1 & 0 & -3 & -1 \\
0 & 1 & -1 & +1 \\
\frac{1}{4} & 0 & 0 & -0.938 \\
0 & \frac{1-9}{2} & 0 & +0.938 \\
\end{bmatrix}
\]

which on substituting the known values of $q$, $r$ and $M_p$ reduces to $H_{sc} = 8.14T$. Knowing the $M_0$ values at $A$, $B$, $D$ and $E$, Fig 4:8, the vertical end reactions $V_A$ and $V_E$ can be calculated and the collapse BMD for $H_{sc}$ drawn.
**FIG 4:10**

**EXAMPLES OF VARIATION IN HINGE POSITIONS**
**IN RECTANGULAR M.S. ARCHES Figs. 4:10-13 Inc.**

**FIG 4:11**

- HINGE POSITIONS FOR CIRCULAR FIXED ENDED ARCHES WITH HALF UNIFORM LOADS.

- $P=1$, for range of $\frac{h_c}{L}$ shown.

**FIGS 4:10 TO 4:13 ARE REPRODUCED FROM STEVENS WORK**
Hinge positions for circular arches loaded at the quarter point.

FIG 4:12

Hinge positions for circular arches centrally loaded.

FIG 4:13
II.5. Consideration of Effect of Axial Force on Simple Collapse Load

Examining equation 4:2, it can be seen that the numerator is independent of the load distribution terms, \( f(L) \). It depends on the arch geometry and the moments at the specified hinge points. The denominator is independent of the moments, depending on the load distribution. If the latter is fixed for a particular case then:

\[
\begin{vmatrix}
L & 0 & 0 & -(M_A + M_E) \\
0 & L & 0 & +(M_A + M_E) \\
\frac{L}{2} (1-r) & 0 & -h_B & -(M_A + M_B) \\
0 & \frac{L}{2} (1-q) & -h_D & +(M_E + M_D)
\end{vmatrix}
\]

Constant

In the following it is assumed that the hinge positions are fixed and the effect of deflections on the simple collapse load is ignored.

The effect of the axial thrust on the moment carrying capacity of the hinge sections will be considered. Figure 5:1 shows the Mult-H curve for the \( \frac{1}{4} \) point section of Arch 4L. This shape is typical for a reinforced concrete section. It will be seen that the application of a certain amount of horizontal thrust, over the ascending part of the curve, increases the moment carrying capacity of the section above \( M_0 \), its moment carrying capacity under pure bending.

If the moment and thrust could be varied independently it would obviously be profitable, from the point of view of load carrying capacity, to increase the thrust on the section to enable a higher value of Mult to be carried. In the case of the arch and all
FIG 5.1

M = H ELASTIC STRUCTURAL RELATIONSHIP FOR SECTION

M ULTIMATE M.O.R. POINT OF FORMATION OF HINGES IN ARCH

SLOPE \( \delta m \)

SLOPE \( \delta \text{elast} \)

H. AXIAL THUST

ULTIMATE AXIAL LOAD WITH NO BENDING
practical structures due to the nature of the loadings and their shape, this is not possible. For the arch the structural relationship between \( M \) and \( B \) is usually expressed in the form \( M = H \cdot e \) where \( 'e' \) is the eccentricity of thrust at the section considered. This relationship varies as an arch is loaded to the ultimate load. This variation occurs as follows:

In the elastic range as the arch load is increased so the arch deflects. At the critical moment points, i.e. potential hinge positions, the thrust line is generally pushed further away from the arch centreline. As the deflections are small in this range when compared with those occurring after hinge formation, it is usually assumed that \( M = H \) const. until moment re-distribution starts. On the formation of the first hinge the arch changes its behaviour structurally. The movement of the thrust line away from the arch centreline at the hinge and potential hinge sections is generally increased per unit load increase over that in the elastic range.

Considering figure 5:1, if \( C \) is the point corresponding to the formation of the 1st. hinge in the arch and \( CC \) represents the elastic \( M \sim H \) curve, then from \( C \) with increasing load the \( M \sim H \) curve will assume a steeper slope, shown dotted - i.e. 'e' plast 'e' elast. Applying the same reasoning as before it will be assumed that this new slope will represent the relationship between \( M \) and \( H \) for a section until a further hinge or hinges form, in which case the slope 'e' will again further increase and so on until the arch collapses.
For collapse design the slope 's' of the M-H structural relationship for the sections will be such that generally the ascending part of the M-H characteristic curve is used.

It should be noted that the cases considered here are those for simple bending collapse, i.e., cases where the number of hinges for collapse is greater than \( n + 1 \) are ignored. (\( n = \) No. of redundancies). This does not preclude a condition which may be the minimum collapse case because invariably where more than \( n + 1 \) hinges are required to form for collapse, the amount of work done on the arch is more than that required for a certain \( n + 1 \) hinge case. The latter thus represents the minimum load for arch collapse.

The argument outlined above can be used to find the collapse load under the \( n + 1 + X \) hinge case, if required. More care is required, however, in setting out the section relationships as 's' in these cases may change from one side of the arch centreline to the other as the arch load is increased from zero to the ultimate.

It will be seen then that for all stages to collapse of the arch, the M and H acting on each section are related. In this chapter, the relationship between M and H to arch collapse will be taken as that existing under elastic conditions and assumed to be linear, e.g., OB fig 5:1.

Ignoring deflections the initial M-H relationship is assumed to hold until section failure, i.e., for the section considered pt B figure 5:1.

Thus, if on Fig. 5:1 the section has an axial force \( x_f \) acting on it, the corresponding ultimate moment available in the section \( = M_o \). In chapter II 4 it was assumed that the ultimate section moment \( = M_o \). Thus, a reserve of \( 6m_1 \) is available before this section fails. Hence, it appears that the simple collapse load modified for the effect of axial
force on the hinge sections, will be increased.

If we take a point $X_2$, Fig.5:1, at this axial thrust a moment equal to $M_2$ would be required to satisfy the $M \sim H$ structural relationship for this particular section under the loading considered. At Axial thrust $X_2$ the maximum moment the section can carry is $M_2$. Thus, assuming this is not the last hinge to form for collapse, a total moment of $\delta M_2 = \delta M$ must be re-distributed to other parts of the structure.

For a fixed ended arch, a minimum of 4 sections become hinge points. Thus 4 section $M_{ult} \sim H$ failure curves are used on each of which the initial $M = eM$ line is plotted. These vary in slope. The final simple collapse load modified for the effects of the axial thrust is obtained by balancing out, in a manner shown below, the $\pm \delta M$ moments between the hinge points until a sufficiently accurate modified estimate of the simple bending collapse load allowing for axial thrust, is obtained.

Considering Arch 41, the simple plastic collapse load is given by:

$$W_{sc} = \begin{vmatrix}
L & 0 & 0 & -(M_A + M_E) \\
0 & L & 0 & +(M_A + M_E) \\
\frac{L}{2} (1-r) & 0 & -h_B & -(M_A + M_B) \\
0 & \frac{L}{2} (1-q) & -h_D & +(M_E + M_D)
\end{vmatrix}$$

Assuming the hinge positions and loading fixed, and ignoring deflections as previously stated, the change on
due to the consideration of axial thrust at A B D E will be confined to the numerator in 4:2. The denominator will remain fixed.

\[ \text{Now } W_{sc} = \frac{N}{D} \text{ and } EW_{sc} = \frac{\sum \frac{2N}{\Delta_{MK}}}{D} \]

\[ \therefore \frac{EW_{sc}}{W_{sc}} = \frac{\sum \frac{2N}{\Delta_{MK}}}{N} \] \hspace{4cm} 5:2

Considering the numerator in 4:2 taking out \( L^2 \) and substituting the known values of \( r \) and \( q \), we obtain:

\[ N = \begin{bmatrix} 1 & 0 & 0 & - (M_A + M_E) \\ 0 & 1 & 0 & + (M_A + M_E) \\ 0.25 & 0 & -1.5 & - (M_A + M_B) \\ 0 & 0.392 & -1.906 & + (M_E + M_D) \end{bmatrix} \]

which reduces to \( +0.455 M_A + M_B + 0.819 M_D + 0.274 M_E \). \( \hspace{4cm} 5:3 \)

Now in this case let \( M_p = M_o \) centre section = 114,000 lb ins., \( M_A \) and \( M_E = 1.185 M_p \), \( M_B \) and \( M_D = 1.055 M_p \). (See Fig 4:9).

Thus on substitution Eqn 5:3 becomes \( 2.782 M_p \)

and Eqn 5:2 : \[ \frac{EW_{sc}}{W_{sc}} = \frac{0.455 \Delta M_A}{M_p} + \frac{\Delta M_B}{M_p} + \frac{0.819 \Delta M_D}{M_p} + \frac{0.274 \Delta M_E}{M_p} \]

\[ \frac{2.782}{2.782} \]

The solution of this is best obtained by setting out in Tabular fashion as in Table 5:1 below.
Figs 5:2 and 5:3 show the hinge section $M_{ult} \sim H$ characteristics and the $M \sim H$ elastic structural relationships for these sections. The latter are obtained as follows:

The variation in the axial thrust and the $B.M.$ for the elastic condition are shown in Figs 5:4 and 5:5. To determine the structural $M \sim H$ relationship at sections A, D and E, for plotting on the $M_{ult} \sim H$ section characteristics, Figs 5:2 and 5:3, it is only necessary to read off the requisite values as shown on the thrust and $B.M.$ diagrams. At the load point, $B$, however the thrust changes, in this case from $9242\%$ on the right-hand side to $553\%$ on the left-hand side. The $B.M.$ at $B$ is a discontinuous function but is sensibly the same, at least within the 'hinge length', for points either side of $B$. Thus at $B$ there are two possible $M \sim H$ structural relationships for use in the analysis. These are both shown on Fig 5:2. The problem of choosing the correct one to use is straightforward. The correct one is that which when used results in the smallest collapse load when the adjustment for the effect of axial thrust on the simple collapse load is complete.

In most cases it will not be necessary to carry out two analyses to determine the correct $M \sim H$ relationship to use. An inspection of the relative moment carrying capacities of the section, when subjected to the alternative $M \sim H$ relationships, as the load is increased will usually show which should be used to achieve the overall minimum collapse load.

Taking section B fig 5:2 as an example at an applied load of 12 tons line $B_1$, the left hand side, of less thrust, has an excess of moment above what the section can carry of 23,000 lb ins. This has to be distributed to other sections of the arch. Line $B_2$, with the higher thrust has an excess moment of 6,000 lb ins. As increasing the load this trend
Figure 5.2

- Multiaxial Section Characteristic for Hinges B & D
- Line B_2
- Hinge D: Elastic M - θ
- Structural Relationship
- Hinge B: Ditto Line B_1

- M_θ = 120,000

- E(AXIAL TREATMENT) = 155 x 10^6
M: ELASTIC BMD FOR ARCH AL.

MAX RELATIONSHIPS AT HIGHEST POINTS:
A1: N = -58 W, H = -635 W
B1: N = -82 W, H = -883 W & +924 W
D: N = -34 W, H = -647 W
E: N = -41 W, H = -625 W

H: VARIATION OF AXIAL THRUST AROUND ARCH

VECT SCALE: 1 cm = 1 kN}
continues. Thus line B1, of the smaller thrust, is the one to use as its greater excess moment requiring distribution will bring about a smaller collapse load than if line B2 were used.

Considering the section characteristic, as previously stated an increase in axial thrust on a section pushes up its moment carrying capacity provided to ascending part of the \( M_{\text{ult}} - H \) curve is used. The latter proviso applies in this case and thus the \( M - H \) structural relationship which gives the smallest \( H \) for the same moment presents the weaker section and is the one to be used in the analysis. This effect can be seen in the crack pattern formed about B near collapse, see Fig. 14. More cracks formed on the left hand side i.e. less thrust than on the right hand i.e. a greater thrust, side.

The values of \( \Delta M_A, \Delta M_B \) etc are obtained from Figs 5:2 and 5:3 and Table 5:1 compiled below.

<table>
<thead>
<tr>
<th>Stage</th>
<th>Position</th>
<th>Axial Thrust (lbs)</th>
<th>Available Moment above ( M_0 ) 1b ins</th>
<th>( \Delta M ) ( M_p )</th>
<th>( \eta )</th>
<th>( \Delta M ) ( M_p )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>A</td>
<td>23650</td>
<td>+ 36000</td>
<td>( .316 )</td>
<td>.455</td>
<td>( .144 )</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>12600</td>
<td>+ 2000</td>
<td>( .008 )</td>
<td>1.0</td>
<td>( .008 )</td>
</tr>
<tr>
<td></td>
<td>C</td>
<td>15270</td>
<td>+ 22000</td>
<td>( .193 )</td>
<td>.819</td>
<td>( .158 )</td>
</tr>
<tr>
<td></td>
<td>D</td>
<td>14250</td>
<td>+ 22000</td>
<td>( .193 )</td>
<td>.274</td>
<td>( .053 )</td>
</tr>
</tbody>
</table>

\[ \frac{SW_{w_{ec}}}{W_{w_{ec}}} = + .363 \]

\[ \frac{2.782}{2.782} = + .131 \]

<table>
<thead>
<tr>
<th>Stage</th>
<th>Position</th>
<th>Axial Thrust (lbs)</th>
<th>Available Moment above ( M_0 ) 1b ins</th>
<th>( \Delta M ) ( M_p )</th>
<th>( \eta )</th>
<th>( \Delta M ) ( M_p )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>A</td>
<td>25800</td>
<td>+ 40000</td>
<td>( .355 )</td>
<td>.455</td>
<td>( .162 )</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>14230</td>
<td>- 18000</td>
<td>( .158 )</td>
<td>1.0</td>
<td>( .158 )</td>
</tr>
<tr>
<td></td>
<td>C</td>
<td>17250</td>
<td>+ 24000</td>
<td>( .215 )</td>
<td>.819</td>
<td>( .176 )</td>
</tr>
<tr>
<td></td>
<td>D</td>
<td>16100</td>
<td>+ 25000</td>
<td>( .219 )</td>
<td>.274</td>
<td>( .060 )</td>
</tr>
</tbody>
</table>

\[ \frac{SW_{w_{AC1}}}{W_{w_{AC1}}} = + .24 \]

\[ \frac{2.782}{2.782} = + .086 \]
### Table 5.1 (Continued)

<table>
<thead>
<tr>
<th>Hinge Position</th>
<th>Axial Thrust (lbs)</th>
<th>Available Moment above H₀ (lb ins)</th>
<th>$\Delta M_\text{K} / H_\text{P}$</th>
<th>$n$</th>
<th>$\Delta M_\text{K} / H_\text{P}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>29100</td>
<td>+ 45000</td>
<td>0.394</td>
<td>0.455</td>
<td>+0.179</td>
</tr>
<tr>
<td>Stage 3 B</td>
<td>15500</td>
<td>- 29000</td>
<td>0.254</td>
<td>1.0</td>
<td>-0.254</td>
</tr>
<tr>
<td>$W_{AC_2} = 12.55$</td>
<td>D</td>
<td>18700</td>
<td>+ 26000</td>
<td>0.223</td>
<td>0.819</td>
</tr>
<tr>
<td>$W_{AC_2}$</td>
<td>E</td>
<td>17500</td>
<td>+ 27000</td>
<td>0.236</td>
<td>0.274</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$\varepsilon = 0.177$</td>
<td></td>
</tr>
<tr>
<td>$\frac{EW_{AC_2}} {W_{AC_2}}$</td>
<td>(= \frac{0.177}{2.782} )</td>
<td></td>
<td></td>
<td></td>
<td>$\frac{0.064}{0.064}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$\varepsilon = 0.117$</td>
<td></td>
</tr>
<tr>
<td>Stage 4 B</td>
<td>16450</td>
<td>- 39000</td>
<td>0.342</td>
<td>1.0</td>
<td>-0.342</td>
</tr>
<tr>
<td>$W_{AC_3} = 13.35$</td>
<td>D</td>
<td>19900</td>
<td>+ 28000</td>
<td>0.246</td>
<td>0.819</td>
</tr>
<tr>
<td>$W_{AC_3}$</td>
<td>E</td>
<td>18600</td>
<td>+ 29000</td>
<td>0.254</td>
<td>0.274</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$\varepsilon = 0.117$</td>
<td></td>
</tr>
<tr>
<td>$\frac{EW_{AC_3}} {W_{AC_3}}$</td>
<td>(= \frac{0.117}{2.782} )</td>
<td></td>
<td></td>
<td></td>
<td>$\frac{0.042}{0.042}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$\varepsilon = 0.021$</td>
<td></td>
</tr>
<tr>
<td>Stage 5 B</td>
<td>17200</td>
<td>- 48000</td>
<td>0.421</td>
<td>1.0</td>
<td>-0.421</td>
</tr>
<tr>
<td>$W_{AC_4} = 13.92$</td>
<td>D</td>
<td>20850</td>
<td>+ 29000</td>
<td>0.254</td>
<td>0.819</td>
</tr>
<tr>
<td>$W_{AC_4}$</td>
<td>E</td>
<td>19500</td>
<td>+ 31000</td>
<td>0.272</td>
<td>0.274</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$\varepsilon = 0.021$</td>
<td></td>
</tr>
<tr>
<td>$\frac{EW_{AC_4}} {W_{AC_4}}$</td>
<td>(= \frac{0.021}{2.782} )</td>
<td></td>
<td></td>
<td></td>
<td>$\frac{0.0075}{0.0075}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$\varepsilon = 0.021$</td>
<td></td>
</tr>
</tbody>
</table>
This process can be continued until the degree of accuracy required is achieved. The difference between the assumed collapse load and that indicated at Stage 5 for this particular case being approximately 1%. It is possible, and admissible, to cut out some of these stages by using judgement as to the increase of the simple collapse load allowing for axial load. An inspection of the graphs, Figures 5:2 and 5:3, will assist in this aim and it should be possible to arrive at the modified collapse load \( W_{AC} \) in less stages than in the more rigorous method, depending on the degree of accuracy required. In the above case, three stages would probably suffice. Thus, it can be seen that when the effect of the axial force on the section moment carrying capacity is considered, the ultimate load is increased. This will be generally true for all arches under minimum plastic collapse conditions. The slope of the critical hinge sections \( M = H_e \) line for these conditions, will be such that a reserve of moment exists in the arch over that assumed for simple collapse. If a particular arch is considered under a form of loading which results in the thrust line lying close to the rib at the hinge sections, thus resulting in a small slope and a diminution of the simple collapse load when axial force is considered, then by definition the simple plastic conditions no longer exist and cannot be used for analysis. Generally, an arch will carry a greater total load under conditions where the thrust line lies always close to the rib centreline than it will if it is loaded to create conditions leading to simple collapse. The latter loading conditions are those the proposed theory is concerned with. The simple collapse load modified for the effect of axial load gives an upper bound solution for the minimum collapse load. As will be seen later the effects of deformations are to reduce this modified simple collapse load.
In the preceding sections, the assumption was made that the arch retained its initial shape up until the last hinge formed and then became a mechanism, i.e., there was no deformation until the collapse mechanism was formed. Thus the load-deformation relationship assumed for all points on the arch was as shown in Fig 6:1.

Deformation does occur, however, in both the elastic and inelastic ranges due to bending, shear and axial forces. In this section deformation by bending alone will be considered, the effects of shear and axial forces on the deformation are assumed to be small and are ignored.

To determine the actual load-deformation curve followed by the arch up to collapse, it is necessary to assume a $M-\theta$ relationship for the individual arch sections.

Fig 6:2 shows the relationship assumed in this case. (This is discussed in Chapter II-1). The rotation $\theta$ increases linearly with $M$ until the value corresponding to $M_0$ is reached when $\theta$ increases horizontally until the section fails by excessive straining of the concrete or steel. Considering the assumed $M-\theta$ curves for the four hinge sections of Arch 4L, Fig 6:3 (a) (b) (d) (e) (f). As the load increases from zero to the formation of the first hinge, $W_1$ Fig. 6:3 (b), $\theta$ at all points on the arch increases linearly. Hence the load-deformation relationship will be linear. After the formation of the first hinge however and with increasing load, the load-deformation relationship changes both $M$ and $\theta$ Fig. 6:3 (b) remaining constant at $B$ whilst at all other sections the linear part of the $M-\theta$ diagrams are still followed. This new relationship persists until the next hinge is formed when the relationship is again changed and so on until the arch collapses.
Failure of concrete or steel by overstraining.

**FIG 6:1**

LOAD

DEFLECTION

RIGID - PLASTIC

**FIG 6:2**

Elastic - Plastic

**FIG 6:3**

Failure of concrete or steel by overstraining.
To illustrate this behaviour consider the deflection of the load point of the arch shown in Fig. 6:4. The arch fails by hinge formation at B, C and D. Hinge C forms first followed by B and D together.

Stage 1 Fig. 6:5 shows the elastic load—deflection curve which is linear up to the load at which the movement at C reaches its $M_0$ value.

As the load is increased further the moment at C remains constant and the behaviour of the arch can be considered to be the addition of the two systems; Fig. 6:4 (b) a statically determinate three-pinned arch with load W plus Fig. 6:4 (c) a three-pinned arch with a constant couple at C. The final deflected shape of the arch being the resultant of the two individual arches Fig. 6:4 (b) and (c).

As for Stage 1 the deflection of C in Stage 2 can be found using strain energy (see Fig 6:5). The arch continues to follow the linear relationship for Stage 2 until hinges form at B and C. The load—deflection curve then becomes horizontal and the arch collapses as shown by Stage 3 Fig. 6:5

Other types of arches and loadings can be dealt with in the same way.

A further example is shown in Fig 6:6. The load—deflection curves of the various stages being obtained by superposition of the individual parts shown.

To draw the collapse B.M.D, which is required to show that nowhere between the hinges the yield condition is violated, it is necessary to know the complete deformed shape of the arch just prior to collapse.

To do this it is necessary to know which hinge forms last. Neal and Symonds have shown how to determine this
ELASTIC STAGE 1:

\[ \delta_c = 8.25 W D^2 \times 10^{-3} \]

PIN ENDED SEMICIRCULAR ARCH

\[ EI_c = \text{const} \]

PLASTIC HINGE AT C - STAGE 2

\[ \delta_c = 8.35 W D^2 \times 10^{-3} - \frac{71.4 D^2 \mu t_c}{E I_c} \times 10^{-3} \]

STAGE 3

COLLAPSE

Fig 6.4

Fig 6.5
ARCH 4L  THE VARIOUS STAGES TO COLLAPSE ARE SHOWN ON FIG 6.6 TO (A).
for Portal Frames and their analysis can be applied to arches. Their method is to carry out deflection analyses assuming continuity over each of the hinges in turn with the other hinges formed. The last hinge to form is that one which when continuity is assumed across it gives the maximum estimate of some particular one of the arch deflections. This means that for the correct order of hinge formation the work done on the arch is a maximum. Applying the second criterion for the correct collapse load and mechanism, viz the arch is in equilibrium just before collapse, the internal strain energy \( U_T \) must equal the external work done \( W \).

Now \( U_T = U_{\text{elastic}} + U_{\text{plastic}} \) \hspace{1cm} 6.1.

Since there is a unique B.M. distribution at collapse \( U_{\text{elastic}} \) is independent of the order of hinge formation.

Also \( U_T = \sum_{\text{all } K} M_0 \theta_k \) \hspace{1cm} 6.2.

Hence the condition that the work done is a maximum requires that \( \sum \theta_k \) must be a maximum.

The rotation at the hinges can be found either by strain energy, the \( S_{IA} \) method, or by the semi-graphical method, the latter two are described later. It has been shown that strain energy can be used to determine the deformed shape of the arch in the elastic and inelastic ranges. The method consists in finding deflections one point at a time and is tedious to apply when used to find the complete deformed shape of the arch.

A method will now be developed for finding the complete deformed shape in one analysis. It can be used in the elastic and inelastic ranges.
The method proposed for the calculation of arch deformations is based on Mohr's Area-moment method. All effects other than bending moment are ignored. Assume the moments around it vary linearly as shown. \( \ell \) is a small segment of the arch assumed straight. Assume M.O.I of section varies as shown and \( E_c \) is constant. \( d_B, d_c \) and \( d_d \) are movements at right angles to the original \( \theta \). \( \theta_B, \theta_c \) are angular rotations, between the initial tangents to the \( \ell \) at each end of an individual segment, brought about by the moment acting along that segment.

Consider section \( AB \). Using Mohr's theorem deflection at

\[
B = \delta_B = \left[ \frac{M_B \ell^2}{2} + \left( \frac{M_A - M_B}{3} \right) \ell^2 \right] \frac{1}{EI_A}
\]

and

\[
\theta_B = \left[ \frac{M_B \ell}{2} + \left( \frac{M_A - M_B}{6} \right) \ell \right] \frac{1}{EI_A}
\]
From these relationships it can be seen that, apart from the abutment section which is assumed fixed at A, the end deflection of a segment is composed of two parts (a) that due to bending within the segment itself and (b) that due to the sum of the rotations which have taken place in all the segments up to but not including the one considered.

In the case of segment $AB$ the end deflection $d_B$ is caused by bending action in the segment only.

The end deflection of the $k$-th segment can be written as

$$d_k = \frac{1}{EI_k} \left[ \Theta_k \ell + \left( \frac{M_{k-1} \ell^2}{2} + \left( \frac{M_k - M_{k-1}}{3 \ell} \right) \ell^2 \right) \right].$$

For convenience it has been assumed that the moments have decreased going around the arch. In practice the moments will vary up and down, i.e. if $M_k > M_{k-1}$, $d_k$ becomes

$$d_k = \frac{1}{EI_k} \left[ \Theta_k \ell + \left( \frac{M_{k-1} \ell^2}{2} + \left( \frac{M_k - M_{k-1}}{3 \ell} \right) \ell^2 \right) \right].$$

The change occurs in part (a) above and is merely due to the
different configuration of the \( \frac{M}{I} \) diagram. It can be seen that for any large number of segments the determination of the segment end deflections involves a considerable amount of arithmetic. To decrease the work a further simplifying assumption is made, i.e., the moment acting on a segment is the mean of the moments at its ends.

Re-writing the equations for \( \theta_B, \theta_C \) and \( \theta_D \) Fig.6:6 we obtain

**Section AB**

\[
\theta_B = \frac{M_{NA} \ell}{EI_A}, \quad d_B = \frac{M_{NA} \ell^2}{2EI_A}
\]

**Section BC**

\[
\theta_C = \frac{M_{NB} \ell}{EI_B}, \quad d_C = \frac{M_{NA} \ell^2}{2EI_A} + \frac{M_{NC} \ell^2}{2EI_B}
\]

**Section CD**

\[
\theta_D = \frac{M_{NC} \ell}{EI_C}, \quad d_D = \frac{\ell}{EI_A} \left( \frac{M_{NA}}{I_A} + \frac{M_{NB}}{I_B} + \frac{M_{NC}}{2I_C} \right)
\]

For the \( k \)-th segment \( d_n = \sum_{k=1}^{n-1} \theta_k \ell + \frac{M_{NN} \ell^2}{2I_N} \)

Where \( M_{NA}, M_{NB} \) etc are the mean moment values acting on the segments.

So far we have considered a case where the moment is of the same sign. For simple collapse conditions both B.M and deflections vary round the arch. To ensure that the deflection is in accordance with the loading conditions the following sign convention will be established. B.M's will be plotted on the same side as the axial thrust line. Those above the arch will be termed +ve, those below -ve, e.g., see Fig. 6:8.
In applying the analysis a tabular method can conveniently be employed.

The method is best illustrated by an example.

Consider the arch shown in Fig. 6:9 loaded so as to be in the elastic range. As the loading is symmetrical it is only necessary to consider half the arch. The left hand side is divided into 20 segments of chord length \( l = 3.28" \).

The various steps required for the deflection analysis are set out in Table 6:1 (see Fig. 6:9).

The movements at the ends of each segment are calculated in terms of \( k \) and the factor \( K \). The latter is introduced to compensate for the difference between the theoretical values of \( E_c I \) and the actual values in the arch. It transforms the theoretical \( E_c I \) value into an 'effective' one. The value of \( I \) for the arch Fig 6:9 is the uncracked full section, plus steel, \( m = \frac{E_s}{E_c} \), M.O.I. The actual resistance to deflection is less than given by this value due to cracking in various sections around the arch. This means that if I 'uncracked' is used with an assumed
usual value of $E_c$ between $E_c = 1200 \text{ Cu for Cu } = 3500 \text{ lbs ins}^2$ and $E_c = 1000 \text{ Cu for Cu } = 5500 \text{ lbs ins}^2$ then the segment end deflections will be underestimated. The value of $K$ will be less than unity and will be discussed later.

The segment end deflections are then plotted, normal to the chord across each segment, on a Williot - type diagram.

Because the loading in Fig. 6:9 is symmetrical it is only necessary to plot a diagram for half the arch. This is shown in Fig. 6:10. The tangent at A is fixed and thus the vertical and horizontal deflections of the segment ends are read from point O. These deflections are plotted to obtain the deformed shape of the arch Fig. 6:11.

Taking a further example consider the semi-circular arch shown in Fig. 6:12. The arch is on the point of collapsing under the load shown. Hinges have formed at A, C and E and are about to form at B and D. It is required to find the deformed shape of the arch under these conditions.

As the loading is symmetrical it is only necessary to determine the deformed shape of half the arch. The deflections of the segment ends are calculated and tabulated in Table 6:2 (see Fig. 6: ). The half arch is divided into 20 segments.

The calculated deflections are plotted as before to obtain the deflection diagram Fig 6:13. This diagram shows a horizontal movement of the centre point equal to $\delta HA$. As the arch is symmetrically loaded there can be no horizontal movement at this point. To achieve this on the diagram the half arch is rotated into the position shown thus eliminating the centre horizontal movement. This rotation is in the nature of a Mohr correction to the Williot diagram. On drawing the original deflection diagram, Fig. 6:13, it was assumed that the tangent at A was fixed. There is in fact a
Parabolic Arch Fixed-Ends

$y = \frac{2}{3} x^2$

$I = 127 \text{ in}^4$, uncracked
$E_c = 5 \times 10^6 \text{ lbs} \text{ in}^2$

Deflection Diagram

$M_o = 926 \times 10^-3 \text{ kips} \text{ ft}$

Scale 1" = $2.16 \times 10^3$ kips
### Table 6.1

<table>
<thead>
<tr>
<th>Segment No.</th>
<th>$F_a$</th>
<th>$F_b$</th>
<th>$F_a + F_b$</th>
<th>$\frac{F_a + F_b}{2}$</th>
<th>$\frac{F_a + F_b}{2} / I_3$</th>
<th>$(A - F_a + F_b + \ldots)^{12} / (L_0 - \delta T_A T_B)$</th>
<th>$I_{12}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>+ 4.24</td>
<td>+ 2.8</td>
<td>+ 3.52</td>
<td>+ 276.5</td>
<td>+ 138.25</td>
<td>+ 138.25</td>
<td>+ 2.93</td>
</tr>
<tr>
<td>2</td>
<td>+ 2.8</td>
<td>+ 1.55</td>
<td>+ 2.18</td>
<td>+ 171.5</td>
<td>+ 85.75</td>
<td>+ 362.3</td>
<td>+ 7.65</td>
</tr>
<tr>
<td>3</td>
<td>+ 1.55</td>
<td>+ 0.7</td>
<td>+ 1.11</td>
<td>+ 87.5</td>
<td>+ 43.75</td>
<td>+ 491.8</td>
<td>+ 10.55</td>
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<td>4</td>
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<td>+ 0.325</td>
<td>+ 25.6</td>
<td>+ 12.6</td>
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<td>+ 11.8</td>
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<td>- 0.8</td>
<td>- 0.425</td>
<td>- 33.4</td>
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<td>+ 11.7</td>
</tr>
<tr>
<td>6</td>
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hinge at A and thus the tangent rotates. This rotation is anti-clockwise, as shown by the clockwise rotation of the half arch Fig. 6:13, and the angle of rotation, $\theta_A$:

$$\theta_A = \frac{60}{60} = 28.5\text{ radians.}$$

The deformed shape of the arch is shown in Fig. 6:14.

As a further example consider Arch 4L Fig. 6:15 loaded with the modified simple collapse load ($W_{AC}$) 13.9T as determined from the previous section.

It will be seen later that the effect of bending deformation on the modified simple collapse load ($W_{AC}$) is to reduce it. Thus the deflections obtained using $W_{AC}$ will be on the high side and hence the reduction in $W_{AC}$ due to these 'higher' deflections will be on the safe side.

In this case hinge D is the last to form and it is assumed that at a load of 13.9T it is just about to form. The arch has three plastic hinges at A, B and E and the arcs between them are considered to act elastically. As hinges are located at both A and E, tangent rotation will occur at these points. Two diagrams will be drawn, one for arc EE, the other for arc AB. They will be joined at B using the fact that the horizontal and vertical movement at B must be the same on both diagrams.

The arch is broken up into 40 segments. Tables 6:3 (see Fig. 6:21) and 6:4 (see Fig. 6:22) tabulate the calculations for the segment end deflections of arcs EE and AB respectively.

Using the calculated values from Tables 6:3 and 6:4 the segment end deflections are plotted as before to obtain the deflection diagrams for the arcs EE and AB figs 6:16 and 6:17 respectively. The arcs are then rotated and drawn to a convenient scale so as to make the horizontal deflection
 ARCH 4L PARABOLIC FIXED-ENDS

Fig 6:15

DEFLECTED SHAPE ARCH 4L 13.9 T ¼ PT. LOAD

Fig 6:18
**FIG. 6:21**

**Table 6:3**

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ARCH 4L FIG 6.15
DEFLECTION DIAGRAM ARC AB

FIG 6.17

SCALE:
1" = 1.96 x 10^-2 lns/K

\[ S_{AB} = 6.06 \times 10^{-2} \text{ lns} \]
of point 'B', Fig. 6:16, equal to the horizontal deflection of point 'B', Fig. 6:17, and similarly the vertical deflections of the point B on both diagrams equal.

This being done, horizontal and vertical movements of all points on the arch can be read from these diagrams. It will be noted that the rotations exhibited by both A and E are anti-clockwise, both arcs being moved in a clockwise direction. A physical examination of the collapse mechanism about to form in Fig. 6:15 confirms this.

The deformed shape of the arch is shown in Fig. 6:13.
II 7. Effects of Deformation

In this section only the effects of deformation by bending will be considered. The deforming effects of shear and axial force are ignored.

Consider the arch shown in Fig. 7:1. With no deformation it is on the point of collapse with the modified simple collapse load \( W_{AC} \) applied. At the critical section C a hinge has formed and hinges are about to form at sections B and D.

The deformed shape is also shown. It can be seen that the arch in deforming produces the following effects:

1. The load in attempting to flatten the arch causes the horizontal end thrust to increase to resist this.
2. At the points B C and D, and generally, the arch moves away from the thrust line.
3. There is a rotation of the thrust line about A and E.

The change in geometry caused by the arch deforming thus changes the system of forces acting.

Two general methods, No's 1 and 2, have been advanced for determining the effects the change in geometry have on arch behaviour and a further method, 3, is now suggested.

1. Fixed Load Method

In this method the load is fixed and the deflected shape found which satisfies the conditions of compatibility of moments and deformation and statical equilibrium.
For slender arches within the elastic range methods have been developed, by Rowe and Ketchum, for determining the effects of changes of geometry on arch behaviour. As its name implies, at a fixed load, primary deflections are found assuming no change in geometry. The change in force system and the additional B M's caused by these deflections are then calculated. These additional B M's are then assumed to act on the arch and in turn produce secondary deflections. These secondary deflections again produce additional B M's and the process is continued until the additional moments produced by the preceding set of deflections have a negligible effect. If the process is not convergent it indicates that an equilibrium position does not exist for the load considered. If the \( M \sim \varphi \) curve of the arch material is the elastic-plastic one of Fig. 6:2 the method can also be used in the in-elastic range.

Consider the arch, Fig. 7:2. A hinge exists at \( C \). The load is such, say 90\% of \( W_{AC} \), that hinge formation is near at \( B \) and \( D \) to form the collapse mechanism.

The moment of any point on the undeflected arch can be written as \( M_{xy} = Vx - Hy \). Under the load the arch deflects. This deflection can be found as described in section 6. On deflection, \( V \) remains constant and to maintain the \( M_{ult} \) value required at \( C \), \( H \) increases to \( \frac{Hy}{\delta_c} \). The moment at the same point on the arch on the primary deflected shape is given by

\[
m_{1xy} = V(x - x_1) - H(1 + \frac{yc}{yc - \delta_c})(y + y_1)
\]

therefore the increase in moment

\[
M_{1xy} = -Vx_1 - H\left[y_1 + \left(\frac{yc}{yc - \delta_c}\right)(y + y_1)\right]
\]

The increase in moment can similarly be found for all points on the arch and these additional B M's used themselves
in the semi-graphical deflection method of section 6 to find secondary deflexions. This procedure is repeated until the additional deflexions produced are negligible.

The total moments at B and D are then summed, i.e. the primary plus secondary and subsequent. The result is examined to see whether hinges have formed, as assumed, or not. It is usually necessary to carry out several complete analyses with different values of $K \cdot W_{AC}$ before the correct load causing hinge formation at B and D can be found. This can be achieved by interpolation, e.g. Fig. 7:3. Thus the reduction in $W_{AC}$ due to deflection can be found.

The method is tedious to apply since several full analyses are required in order to determine the collapse load.

If the arch material possesses a non-linear $M - \phi$ curve, e.g. Fig. 1:2; the problem is more difficult. Changes in moment found from the primary deflections must be used with that part of the curve which corresponds to the total moment.

Due to the difficulties in practical application no further discussion of this method will take place. The remaining simpler methods will now be investigated.

2. The Fixed Deformation Method

This was developed by Stevens. The method is to fix the final deformed shape and then find the corresponding load system which will produce this shape.

The deflected shape $X_n Y_n$ is arrived at by applying a set of curvatures $\phi_xn \ y_n$ to the undeflected form $X_0 Y_0$. 
Corresponding to these curvatures there will be a unique set of moments \( M_{X_n} Y_n \). The correct relationship of load and deflexion is obtained when the load system produces moments \( M_{X_n} Y_n \) on the deflected shape \( X_n Y_n \) and everywhere the values of \( M \) and \( \phi \) are compatible.

When changes in geometry are neglected a force system \( F_0 = f(W^0 H^0) \) applied to the undeflected shape \( X_0 Y_0 \) will produce moments \( M_{X_0} Y_0 \) with the corresponding changes in curvature \( \phi_{X_0} Y_0 \). The primary deflected shape \( x_1 y_1 \) can be obtained for this condition by employing the semigraphical method of section 6.

If some other force system \( F_1 \), which satisfies the requirements of statics, is applied to the shape \( x_1 y_1 \), it will produce moments \( M_{X_1 Y_1} \) and corresponding curvatures \( \phi_{X_1 Y_1} \). If \( F_1 \) is so chosen that at all pts

\[
M_{X_1 Y_1} = M_{X_0 Y_0} \quad 7.4
\]

then

\[
\phi_{X_1 Y_1} = \phi_{X_0 Y_0} \quad 7.5
\]

but \( \phi_{X_0 Y_0} \) applied to \( x_0 y_0 \) produces \( x_1 y_1 \)

\( \therefore \phi_{X_1 Y_1} \) applied to \( x_0 y_0 \) produces \( x_1 y_1 \) and this deflected shape, and load system \( F_1 \), satisfy statics and compatibility of moment and curvature. It is not usually possible to satisfy eqn 52 exactly at all points when the same type of load system \( f(W^H) \) is used for both \( F_0 \) and \( F_1 \). However, by satisfying it as many points as possible, and choosing these points at maximum values of moment a good approximation can usually be obtained.

This method will not be illustrated as it is very similar to that now proposed.
3. **Fixed Moment Method.**

In this method the B.M. distribution just prior to collapse is fixed. The load and deflection compatible with this B.M. distribution and with statical equilibrium is then found.

Consider the arch, Fig. 7.4

Let the load $W_{AC}$ produce moments throughout the undeflected arch so that it is on the point of collapse.

Let the moment of any point $x_0y_0$ on the arch be represented by $M_{x_0y_0} = -H_0y_0 + f(W_{AC})$. Allow the arch to deflect so that the general movement of point $x_0y_0$ is $u_1$ and $v_1$. The deflected shape is arrived at by applying the semigraphical method of section 6.

Taking moments about the same point on the deflected arch with the new load $W_{AC_1}$ applied

$$M_{x_1y_1} = -H_1(y_0 + v_1) + f(W_{AC_1})(x_0 + u_1)$$
Now \( M_{x_0y_0} = M_{x_1y_1} \).

\[ H_0y_0 + f(w_{AC}) = -H_1(y_0 + v_1) + f(w_{AC})(x_0 + u_1). \]

Using this relationship the load \( w_{AC_1} \) can be found. \( w_{AC_1} \) is then the first approximation to establish the reduction in \( w_{AC} \) due to deformation. To obtain a closer approximation the load \( w_{AC_1} \) is applied to the arch, the deflected shape with this load found, and a new load \( w_{AC_2} \) found which produces compatible moments and deflections throughout the arch. This procedure can be repeated until the difference between successive loads is negligible.

In most practical cases it is only necessary to perform one analysis. The initial reduced load \( w_{AC_1} \) is a lower estimate of the modified \( w_{AC} \) because it operates on a deflected shape which is the greatest which can be obtained in the successive steps. \( w_{AC_2} \) for example operates on the deflected shape due to \( w_{AC_1} \) which as \( w_{AC_1} < w_{AC} \) the corresponding deflections are smaller. Thus to produce the same moments in the arch \( w_{AC_2} \geq w_{AC_1} < w_{AC} \).

Thus performing one analysis results with a reduced \( w_{AC} \) on the safer side and considerably reduces the analytical work required. As in the Fixed Deformation Method it is not possible to satisfy the assumption that the moments in the undeflected and deflected shapes are the same throughout the arch. Only the points of maximum moments, i.e. the hinge points, are held constant in the analysis. It will be shown
however, by plotting the B.M.D with the reduced $W_{AC}$, using the initial deflected shape, and comparing it with the initial assumed B.M.D that reasonable agreement is reached throughout the arch. As an example consider the arch 4.L Fig. 7:5 under $W_{AC} = 13.9_T$. The horizontal and vertical deflections at B and D are shown read from Fig.6:6 with $K$ assumed = 0.25.

Re-writing equation 4:5 with the known value of $q$ and revised values of $h_B$, $h_D$ a revised simple plastic collapse load $W_{sc1}$ can be found.

\[
W_{sc1} = -4 \times \frac{2.37 \times 114,000}{10} \begin{vmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & +1 \\ \cdot25 & 0 & -1.46 & -\cdot938 \\ 0 & \cdot367 & -1.9 & +\cdot938 \\ \end{vmatrix}
\]

which reduces to $W_{sc1} = 9.68_T$. This compares with $10.02_T$, the initial estimate for $W_{sc}$.

The deflection thus reduces $W_{sc}$ by 3.4%.

It will be assumed that at the hinge points the $M \sim H$ structural relationships remain the same in the undeformed and deformed shapes.

The % reduction in $W_{sc}$ immediately gives the reductions to be applied to $W_{AC}$ to include the effect of deformation. This is the same % reduction as the only difference between equation 7:7 and one which could be written for $W_{AC1}$ lies in the value of the hinge moments.
Thus $W_{AC}$ modified for deformation, $W_{ACD} = 13.4_T$, in this case.

To investigate the validity of the initial assumption i.e. that the B M's throughout the arch are the same in the deflected and undeflected shapes, the B M's acting on the arch at $W_{ACD} = 13.4_T$ are compared with those in the arch at $W_{AC} = 13.9_T$. This is shown in Fig. 7:6. It will be seen that agreement is good except in the region of D where the fourth hinge is expected to form. Even here the error is only approximately 4.0%.

That the major divergence of the diagram appears around D is due to the method of determining the horizontal end thrust $H$, for the $13.4_T$ load on the deflected arch.

To hold the moment at the load point constant as previously stipulated at 140,000 lb ins, on the deflected arch, $H$ was found by taking moments about B. This $H$ was then used in the B.M.D calculations. This value of $H$ was greater than that which could be obtained by attempting to make the difference in moments between the old and new B.M.D's, at B and D the same. This could be done by suitably adjusting the value of $H$.

It will also be noted that the value of the moment to be carried at D in both the $13.9_T$ and $13.4_T$ cases is > that given by the point where the elastic $M\sim H$ structural curve meets the $M\sim H$ section characteristic curve for $D$, Fig 5:12. The value of $H$ is very sensitive to vertical arch movements and in these two loading cases the deflections considered are > those under a $13.4_T$ load.

Considering a $13.4_T$ load on the undeflected arch the moment at point 15.'D' is 174,000 lb ins. Assuming on deflection that this moment increases in the same ratio as on the application of the previous analysis, i.e. $\frac{192.5}{192}$ the moment at D on the $13.4_T$ deflected arch case will be
very nearly 180,000 lbs ins which is the moment given at
the elastic failure point previously mentioned on Fig 5:2.

In the assumption that the primary deflected shape
is maintained by the secondary load with the primary
bending moments acting throughout it is implicit that the
internal S E's in the primary and secondary cases are
equal.

If the internal S E's are equal it follows that the
external energies are equal, i.e.

$$E_p = E_s$$  \(7:8\)

The internal SE is given by

$$U = \int_0^1 \int_0^1 bM d\theta$$  \(7:9\)

The external SE

$$E = \int_0^1 W d\delta$$  \(7:10\)

where \(\delta\) is the vertical movement of the load point as N
primary > W secondary. \(\therefore E_p > E_s\). This is illustrated
in Fig. 7:7 below.

\[\text{FIG. 7:7}\]
In Fig. 7.7 $E_p$ is represented by area $OCA$ and $E_s$ by $OBA$. Hence $E_s < E_p$ by the area $OCD$ and equation 7.3 is not satisfied.

The source of this error lies in the deflection calculation method. In the primary analysis $δ_B$ is found by drawing a Williot diagram in which it is assumed that the changes in curvature are small enough to allow the relative deflections to be plotted perpendicular to the segment concerned. The tacit assumption is made that if curvature $φ_{xy}$ produces deflections $Δ_{xy}$ then $nφ_{xy}$ will produce $nΔ_{xy}$. This is an approximation which becomes increasingly inaccurate as the curvatures $nφ_{xy}$ become larger. The actual deflections produced in an arch will usually be greater than those predicted by assuming that the relationship between $φ_{xy}$ and $Δ_{xy}$ is linear.

In order to satisfy equation 7.3 an approximate correction can be made by altering the load-deflection diagram Fig 7.7. This is done by moving point $B$ to point $F$ so as to make the area $OFK = area OCA$. Thus $E_p = E_s$. In performing this operation the load point deflection was increased. This will alter the value of the secondary load as $h_B$ is altered in equation 7.7. This is, however, a second order effect and may be neglected. In any particular case the error can be assessed by measuring the area between the two curves.

It has been seen that effects of deformation are to reduce the modified collapse load $W_{AC}$. The deformation experienced by a particular arch depends, amongst other factors, upon its value of $\frac{L}{k}$. Hence for long slender arches the decrease in $W_{AC}$ will be greater than for short stocky types.

Stevens in his work on mild steel arches produced some graphs showing the effects of deformation on the simple plastic collapse load of arches with varying ratios...
of $\frac{L}{\text{depth}}$. They are reproduced here in Figs 7:8 & 7:19.

Similar graphs could be produced for r.c arches with given steel percentages, concrete strengths, and dimensions.

Generally due to the shape and size of the comparative r.c. member the effect of deformation on the collapse load is less in the case of the latter than in the comparative mild steel arch.
THE EFFECT OF CHANGE OF GEOMETRY ON LOAD CAPACITY NEGLECTING AXIAL FORCE

CIRCULAR FIXED-ENDED ARCS
RECTANGULAR SECTION WITH IDEALLY ELASTIC - PLASTIC M - \phi CURVE (FIG 6:2)

FIG 7:8

FIG 7:9

THE EFFECT OF CHANGE OF GEOMETRY ON LOAD CAPACITY NEGLECTING AXIAL FORCE

(a): RECTANGULAR SECTION
(b): TYPICAL I SECTION
(a) & (b) FOR IDEALLY ELASTIC PLASTIC M - \phi CURVES
II 8. The Effects of Axial Forces on Deformation

In Section 6 the effects of axial force on the deformation, and then in Section 7 on the collapse load of the arch, were ignored. The arch depends for its high load carrying capacity on a high axial thrust acting close of the arch \( f \). The effects of this axial force will depend on the shape of the arch, the loading conditions and the physical make-up of the arch sections. When using the elastic analysis for arches a "rib-shortening" term is often introduced to make allowance for this axial force. This term is employed as a correction, it being recognised that deformation by bending is usually predominant. In this section the effects of axial force on deformations in the in-elastic range are considered. The effects of the axial force on the moment carrying capacity of the section and on the collapse load are ignored.

Fig. 8:1 shows a section of a fixed ended r.c. arch divided into small segments of length 1 as Fig. 6:1 so that the chord and curve lengths are sensibly equal. The member is assumed to possess the ideal elastic-plastic characteristic of Fig. 6:2.

\[ \delta_1 = \frac{M_1}{E_c A_c + E_s A_s} \]
Due to this shortening an additional deflection is introduced at the segment ends, Fig. 8:1. To obtain the resulting deflected shape of the arch under the loading considered a Weilbett-Kohr diagram can be plotted similar to those of Section 6. In this case the deflection of each segment end is obtained by plotting the moment deflection component perpendicular to the arch segment chord and adding to it, parallel to the chord, the change in length due to the axial contraction. Thus for arch Fig. 6:9 in the deflection diagram Fig. 6:10 the deflection of segment 1 would be equal to the bending component shown, plus a component from the point 1 thus obtained, equal to the axial shortening, and extending to the left downwards at right angles to it. From this new point 1(a) say the diagram is continued as before and point 2(a), etc. obtained in a similar manner. For the arch Fig. 6:9 the horizontal components obtained are very small compared with those due to bending and produce a negligible effect on the diagram. This arch is in the elastic range. As an example in the in-elastic range, consider the semi-circular arch Fig. 6:2 loaded so as to be on the point of collapse. Hinges have formed at A, C and E and others are about to form at B and D. In this case a further problem arises. This lies in the determination of the contraction at the plastic hinges. Onat and Prager found that the relationship between the contraction and rotation at a hinge in a rectangular steel section exhibiting the ideal elastic-plastic relationship of Fig. 6:2, could be expressed as follows:

\[ \delta = -\frac{\Theta \, dm}{d H} \]

where \( \delta \) is the contraction and \( \Theta \) the rotation at a hinge point. \( M_p \) and \( H_p \) are the ultimate moment of resistance under pure bending, and the pure axial failing load capacity of the section respectively. For a rectangular steel section a graph between \( M \) and \( H \) expressing the perimeter of the admissible area for the section takes the form of a parabola, Fig. 6:4. This perimeter expresses the failing conditions of the section under varying values of \( M \) and \( H \). If \( m = \text{actual moment} \) and \( n = \text{actual thrust} \)

\[ \frac{m}{M_p} \quad \text{and} \quad \frac{n}{H_p} \]
Then the curve from $M_p$ to $H$, Fig. 8.4 is expressed as
\[ m = 1 - n^2 \]  
\[ \therefore \frac{dm}{dn} = -2n \]

Thus for the rectangular steel section considered
\[ S = \frac{Q^2n M_p}{H_p} \]  

The values of $Q$ and $n$ used in the above equations are calculated independently by assuming that the simple plastic collapse loads are unaffected by the effects of axial force and change of geometry.

It is not possible to develop so straightforward a relationship between $S$ and $Q$ in the case of an r.c. section for the following reasons. The $M_{ult} - H_{axial}$ section characteristic curve does not follow a simple mathematical form, e.g. Fig. 8.5 hence $m$ and $n$ cannot easily be related and the fact that concrete takes little or no stress in tension breaks down the simple configuration of the stress blocks leading to equation 8.2.

To determine the axial deformation at a hinge in an r.c. arch member the following two methods are suggested.

1. Eqn. 8.1 can be used directly provided that $l$, now equal to the hinge length, is known and that a reasonable value for $E_s$ at plasticity can be assessed. The latter presents a considerable difficulty. Lee, Morris and Jain considered it $-\epsilon$ at the ultimate strain in their assumed stress/strain concrete relationships, whilst Chan considered it zero in his curve. As an example consider the section shown in Fig. 8.5 under an axial thrust of 74,000 lbs. with the corresponding moment acting of 220,000 lb. ins. to bring the section to the point of failure. It is required to find the axial deformation at this hinge point assuming the hinge length $l_p = 3"$.

(Hinge lengths will be discussed later.) $E_s$ assumed = 0.

Now in this case
\[ S_1 = \frac{H_l}{A_s E_s} \]  

\[ \therefore S_1 = \frac{74,000 \times 3}{2 \times 0.588 \times 30 \times 10^6} = 0.0063" \]

This result will be compared with that obtained by method 2.

Method 2

Consider the section Fig. 8.5 subject to two different sets of
$M_0 = \text{Section failing moment under pure bending}$

$m = 1 - \eta^2$

Curves showing relationship between $m$ & $n$ for a rectangular M.S. section

$P_0 = \text{Section failure under pure axial thrust}$
conditions, (1) Fig. 8:6(a) under a pure bending moment $M$
and (2) under the same bending moment with an axial thrust
$H$ applied, Fig. 8:6(b). Initially let the value of $M$ and $H$
be chosen so that when either condition applies the section
remains elastic.

For bending only, Fig. 8:6(a) the extension of a fibre at the
beam centre, $C_1 = \frac{(d - n_1 d)}{2}$  

For bending plus axial thrust $C_2 = \frac{(d - n_2 d)}{2}$  

Thus the contraction at the centre due to the application of
axial thrust = $C_1 - C_2$

This relationship holds good for the elastic range of the section.

For plastic hinges a further problem arises. If the contraction
at a section is required which is under $M$ and $H$ conditions so as
to be represented at point $J$ say Fig. 8:5 it can be seen that the
moment the section reaches with axial thrust is > the moment it
can sustain under pure bending. This applies to all points
above the line $AB$ Fig. 8:5. Hence the elastic analysis cannot be
applied. The following assumption will be made in an effort to
produce a solution. As $M_0$ is the highest moment the section can
sustain under pure bending for all points between $A$ and $B$ Fig. 8:5
the strain distribution analogous to Fig. 8:6(a) for the plastic
hinge case will be assumed to be equal to that at $M_0$. The result-
ing centre fibre extension will then be multiplied by the ratio

$$\frac{M_{AB}}{M_d}$$
For the case analogous to Fig. 8:6(b) the application of the method is the same as outlined for the Fig. 8:6(b) case: the contraction for the elastic range example of Fig. 8:6 was considered to occur in unit length. For plastic hinges, however, contraction occurs over the finite length of the hinge. The latter varies and will be discussed later. As an example of the contraction at a hinge consider the section Fig. 8:5(b) under M and H conditions represented by point J Fig. 8:5(a). Fig. 8:7(a) and 8:7(b) represent the strain distribution across the section when it is under the action of $M_d$ and $M_j$ and $H_j$ respectively. At J $H = 74,000$ lbs, $M = 220,000$ lb. ins. and $M_d = 120,000$ lb. ins. 1 $p$ the hinge length = 3".

![Diagram](image)

Under $M_d$

$C_1 = \theta_1 \left( \frac{n_1}{2} - n_1 d \right) \frac{p}{M_d} \quad \text{8:8}$

And $M_j$

$C_{1j} = \theta_j \left( \frac{n_2}{2} - n_2 d \right) \frac{M_j}{M_d} \quad \text{8-9}$

which in this case reduces to $C_{1j} = \frac{0.0038}{91} \times 1.75 \times 3 \times \frac{2.2}{1.91} = 0.0434$ ins.

For the $M$ plus $H$ case Fig. 8:7(b)

$C_2 = 2 \left( \frac{n_2}{2} - n_2 d \right) \frac{p}{M_d}$

which reduces to $C_2 = \frac{0.0038}{0.91} \times 0.59 \times 3 = 0.00353$ ins.

The contraction due to the application of $H_j$ is thus .0592 ins.

Returning to the example of arch Fig. 8:2 the contractions at the hinge points can now be calculated and the Williot diagram plotted. The axial contractions are small, however, when compared with the bending deformations. To avoid plotting vectors of different orders on the same diagram the axial contractions for Fig. 8:2(a) are plotted alone on Fig. 8:2(b).
Deflections due to axial contraction

Scale 1cm = 1x10^-3 in

FIG 8.2
a Mohr rotation diagram is used to allow for the rotation of the tangent at A. In this case the hinge lengths have been restricted to 1 in. The table showing the segment axial contractions is tabulated below in Table 8:1.

<table>
<thead>
<tr>
<th>Segment No.</th>
<th>Axial Thrust $\text{lbs x 10}^4$</th>
<th>Axial Contraction $\text{ins x 10}^{-3}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hinge A</td>
<td>1.0</td>
<td>10</td>
</tr>
<tr>
<td>1</td>
<td>1.08</td>
<td>.55</td>
</tr>
<tr>
<td>2</td>
<td>1.2</td>
<td>.61</td>
</tr>
<tr>
<td>3</td>
<td>1.3</td>
<td>.66</td>
</tr>
<tr>
<td>4</td>
<td>1.37</td>
<td>.7</td>
</tr>
<tr>
<td>5</td>
<td>1.41</td>
<td>.72</td>
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<td>1.41</td>
<td>.72</td>
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<td>7</td>
<td>1.37</td>
<td>.7</td>
</tr>
<tr>
<td>8</td>
<td>1.3</td>
<td>.66</td>
</tr>
<tr>
<td>9</td>
<td>1.2</td>
<td>.61</td>
</tr>
<tr>
<td>10</td>
<td>1.08</td>
<td>.55</td>
</tr>
<tr>
<td>$\frac{1}{2} \times \text{Hinge C}$</td>
<td>8</td>
<td>5</td>
</tr>
</tbody>
</table>

Comparing the vertical deflection at C due to bending only from Fig. 6:12 ($K = .25$) with that due to axial thrust only the former = 1.61 ins. whilst the latter = .024". The hinge contraction at A contributes 43% towards this deflection. Had the hinge length been assumed equal to 4 ins. say (a reasonable length in this case) this % would have been greater. It can be seen, therefore, that the contribution of hinge contraction towards arch deflection dominates the contribution made by the elastic sections between the hinges. In this case even if the hinge were 4 ins. long the deflection at C due to bending would still be small. Hence with the slenderness ratio used here, $\frac{L}{d} = 24$, and, more importantly, the rise to span ratio of 1, the additional deflection due to axial contraction round the arch is small. To satisfy the condition that there should be no horizontal movement at point C to half arch was rotated anti-clockwise into the position shown on Fig. 8:2(b). This implies a clockwise rotation of the tangent at A. This is in opposite direction to that
occurring under bending alone. Hence another important point arises that is that the rotation at a hinge due to axial contraction may be in opposition to that under bending and hence in opposition to the direction required for the formation of the stipulated collapse mechanism. If this opposing rotation is large enough it can cancel out the bending rotation and hence a hinge will not form as required and the plastic hinge ultimate load theory cannot be applied. In the case of arch Fig. 6:2 the rotation at A due to the axial thrust = .0015 rads whilst that due to bending Fig. 6:3 = .019 rads, i.e. in that case $\theta_{A, Axial} = .08 \theta_{A, Bending}$

Looking at the problem physically, it can be seen that as the ratio $\frac{\text{rise}}{\text{span}}$ decreases the axial forces increase relatively and hence the effects of axial thrust on deflection and hinge rotation become more pronounced.

As an example of an arch in this range consider the arch shown in Fig. 8:6(a) loaded as shown to be on the point of collapse with hinges at A, C and E, and hinges about to form at B and D. The bending only segment end deflection diagram is shown in Fig. 8:6(b). The axial thrust deflection diagram is shown in Fig. 8:6(c). A hinge length of 4 ins. is assumed. Comparing the vertical deflections due to bending and axial thrust at point C. Deflection due to bending = 1.65 ins. " " axial thrust = 92 a ratio of 1.8. Hence the deflection at C due to axial thrust is 56% of that due to bending. To form the collapse mechanism postulated it is necessary for the rotation of hinge A to be anti-clockwise. Examining Fig. 8:6(b) and (c), in both cases the tangents at A have been rotated anti-clockwise to complete the diagrams and hence the actual rotation at A is clockwise.

$\theta_A$ due to bending = .005 rads.

$\theta_A$ due to axial thrust = .0014 rads.

In this case the axial thrust supplies 22% of the total rotation at A. Again it can be seen Fig. 6:6(b) that the axial contractions at the hinge are responsible for the major share of the deflections and hinge rotations.
DEFLECTION DIAGRAM
BENDING ONLY SCALE: FULL SIZE

$S_{HC_B} = 0.4''$

$S_v C_B = 1.65''$

$S_{HC_A} = 0.315''$

(C) DEFLECTION DIAGRAM AXIAL THRUST ONLY SCALE 1" TO 0.1"
For arch Fig. 8:8(a) then the collapse mechanism anticipated will not form and hence the moments and thrusts shown are not real. However, the example does serve to illustrate the important effect the axial thrust has on the deflections and the hinge rotations for low values of rise/span.

It has been shown that for an arch of high rise ratio, the span

effect of axial force on the deflections and hinge rotations is small whilst for an arch with a low value of rise the effect is span

marked and can lead to the breakdown of the plastic hinge approach. Stevens presented some curves for the variation of the ratios

\[ \frac{\Delta_{\text{Axial}}}{\Delta_{\text{Bending}}} \]  and \[ \frac{\Delta C_{\text{Axial}}}{\Delta C_{\text{Bending}}} \]

arch of constant rise. They are reproduced in Fig. 8:9. These span
give an indication of how a similar r.c. arch might vary.

It should be noted, however, that an r.c. arch of the same shape and "strength" will not necessarily develop the same \( \frac{\Theta_A}{\Theta_B} \) and \( \frac{\Delta A}{\Delta B} \) relationships. This is because of the difference in the plastic hinge characteristics. For a steel section these depend mainly upon the quality of the steel, established when rolled, and the loading conditions. For an r.c. section the hinge characteristics are subject to variation. They depend upon the two conditions mentioned for the steel hinge plus others. Predominant amongst these are the quality of the concrete and the \% of tensile and compressive steel. By varying the latter it is possible to make a hinge adopt a compressive or tensile mode at failure. This in turn affects the hinge length which again affects the total hinge contraction.

It follows, therefore, that for an r.c. arch, the relative effects of axial thrust and bending upon the deflections and hinge rotations depend on the physical "make-up" of the section and their
FIG 8:9

RELATIONSHIPS BETWEEN DEFORMATIONS CAUSED BY AXIAL FORCE & BENDING JUST PRIOR TO COLLAPSE FOR M.S. ARCHES.
TAKEN FROM STERNANS.

$\Delta_c(A)$: DEFORMATION CAUSED BY AXIAL FORCE ONLY
$\Delta_c(B)$: DEFORMATION CAUSED BY BENDING ONLY
material characteristics as well as the arch shape. Without further investigation it is not possible to establish precise limits as to when the effects of axial force are negligible and when they become overriding. Each case must be treated on its merits. As an approximate guide, however, if for an arch \( \frac{d}{l} > 20 \) and the \( \frac{rise}{span} > .25 \) the effects axial on the deflections and hinge rotations can be assumed to be negligible.
II 9.  **Effect of Prestressing and Abutment Spreading**

9A  **Effect of Prestressing**

If by some means sections of the arch, or the whole arch, are pre-stressed then this prestressing may increase or decrease the load at which the first hinge will form. In this part of the section it will be assumed that prestressing has no effect on the ultimate load capacity of the arch. The effect on deflexion and hinge formation will be considered.

As a simple example consider the arch Fig 9:1(a) loaded as shown. It is required to increase the load at which the first hinge, at C, forms. One way of doing this is to initially bring the abutments closer together as in Fig 9:2(b). The stresses induced at C by this movement are opposite to that due to the load and hence hinge formation is delayed. It follows that if hinge formation is delayed then the load-deflection behaviour of the arch is altered. In this case the elastic range is increased. Arch Fig 9:1(a) collapses with hinges at B C and D. From Fig 9:1(b) it can be seen that the effect of moving in the abutments is to produce additional stresses at B and D of the same sign as those existing in the plastic hinges formed at these points. Thus by a certain choice of inward movement $SH$ it may be possible to make hinges at B, C and D form simultaneously. This produces elastic behaviour up to collapse. Fig 9:1(c) curve 3 shows this graphically for arch 9:1(a) of constant cross-section and a material $M=0$ curve of Fig 6:2.

The initial deflection of any point of the arch when a 'lack of fit' $\delta H$ is introduced at each abutment can be found, in the elastic range using Maxwell's reciprocal deflexion theorem. Thus for Fig 9:1(b) $\delta CH = \frac{2\delta H}{W}$

Curves 1 to 4 Fig 9:1(c) illustrate the effect that various initial movements of the abutments have on the load deflection behaviour at point C.
FIG 9.1
The curves are drawn as follows. Curve 2 illustrates the arch behaviour with no initial movement $\delta H$. Curve 3 illustrates the behaviour when the abutments are 'pushed in' $\delta H$ and Curve 1 when they are initially 'pulled out' by $\delta H$. The initial intercepts on the deflection axis are determined by Equation 9:1, i.e. $\delta CH = 2 \delta H$. This difference in deflection is maintained until the first hinge forms for each case. The arch then becomes 3-pinned. The value $y$, again the difference in deflection between the initially unstrained and strained arches for this load stage, is found on consideration of the geometric properties of the arch. In this case

$$y = 2 \delta H$$

--- 9:2.

Curve 4 illustrates arch behaviour when an initial inwards movement $\delta H$ is introduced sufficient to cause the three hinges to form simultaneously. It will be noted that the collapse load is the same for all the cases considered.

Where deflection is an important criterion in design its effects may be reduced by applying an initial lack of fit as shown above. However this is not possible for all types of arches with variable loadings.

Under a fixed loading condition by pre-stressing arch sections locally to delay section failure it may be possible to increase the collapse load of an arch. This will not be investigated as pre-stressed arches are outside the scope of the present work.

**Shrinkage**

This plays an important part in practical arch construction and should not be overlooked in design. As an example consider the arch Fig 9:2 constructed in one piece. Shrinkage of the concrete as curing proceeds introduces tensile stresses in the concrete and compressive stresses in the steel. These tend to
flatten the arch out and introduce undesirable cracks. Thus shrinkage is analogous to an initial outward movement $SH$ at each abutment. The effects of shrinkage can be alleviated to a varying extent by allowing sections of the arch to shrink independently for at least about a month then joining the section together. E.g. the crown section C could be poured after the ribs AC and BC had shrunk. The reinforcement at C could be lapped and not continuous to assist in relieving the shrinkage stresses.
9B. Abutment Spreading

In the first part of this section the effect of pre-straining on the load capacity was ignored. It has already been shown in section 7 that change in geometry can bring about a reduction in the ultimate load capacity. Thus the curves 1, 3 and 4 are not correct in that $W_{sc}$ was regarded as constant for all cases.

Thus the effect of pre-straining and abutment spreading is to modify the ultimate load. Consider the effect of abutment spreading on the simple collapse load $W_{sc}$ for the arch Fig 9:3. A hinge has formed at C and others are about to form at B and D. The $M$-$\varphi$ diagram for the material is that assumed for the simple collapse load $W_{sc}$, i.e. the rigid-plastic form of Fig 6:1. Thus to permit abutment movement a hinge within the arch is necessary. In this case the hinge forms at C and as the arch continues to be loaded to collapse outward movement at the abutments occurs.

Using equation 4:2 a value of $W_{sc}$ with no abutment spreading can be written down as can $W_{sc\perp}$ allowing for abutment spreading. For arch Fig 9:3(a)

\[
W_{sc} = \begin{vmatrix}
-h_B + M_B \\
-h_C - M_C \\
-h_B - PL \\
-h_C + \gamma L
\end{vmatrix}
\]

and

\[
W_{sc\perp} = \begin{vmatrix}
-h_{B_1} + M_B \\
h_{C_1} - M_C \\
-h_{B_1} + B_1(L + 25H) \\
-h_{C_1} + \gamma_1(L + 25H)
\end{vmatrix}
\]
The ratio \( \frac{W_{SC}}{W_{SC}} \) gives the change in the simple collapse due to the abutment spreading shown. As an example consider the arch Fig 9:4 with a central load. Abutment movement allows the arch to deform as shown with a hinge at C and others about to form at B and D.

Using the values shown, the reduction in \( W_{SC} \) due to abutment movement is approximately 1%. The effect in this case of a 2% outward abutment spread is a negligible decrease in \( W_{SC} \).

As a further example consider the arch Fig 9:5 taken from Stevens' work. The effect of abutment spread on this arch of low rise to span ratio is quite marked and is shown graphically in Fig 9:7 curve (a). The reduction in load capacity is found in the same way as described for arch Fig 9:3(a).

From Fig 9:7 curve (a) it can be seen that the reduction in load capacity is considerable even for small amounts of abutment spread. At a value of \( \frac{26H}{L} \) of 3.08%, the arch crown is on the same level as the supports and simple beam collapse occurs with a hinge forming at C only.

Curve (b) Fig 9:7 is also taken from Stevens and shows the reduction in load capacity for abutment spreading for arch Fig 9:6.

It has been shown that for a high arch the effects of abutment spreading on \( W_{SC} \) are very small. It has also been shown that for practical slenderness values there is a considerable reduction in \( W_{SC} \) for flat arches.

If the condition of moment and thrust at the critical sections are assumed equal for the undeformed and 'abutment - spread' deformed conditions, then the remarks made above for \( W_{SC} \) are applicable to \( W_{ACD} \).

As for the effects of axial force on the arch deformations, it is not possible to lay down precise limits concerning the
FIG 9:5

CIRCULAR PIN-ENDED ARCH

FIG 9:6

EFFECT OF SPREADING OF ABUTMENTS ON THE SIMPLE PLASTIC THEORY ESTIMATES OF LOAD CAPACITY.

FIG 9:7
effects of abutment spread. Each case must be treated on its merits and should be reduced to an equation of the form of 4:2, due allowance being made in the case of $W_{ACD}$ for the change in axial force and moment between the two conditions considered. However if the rise to span ratio is $> .25$ and the total abutment spread is less than 0.5% of the span the effect of abutment spreading can be considered negligible.
Combination of Effects

In section 7 a modified collapse load $W_{AC}$ was arrived at which included the effects of axial force and deformation.

In this section two methods will be presented combining the effects of axial load and deformation on the simple collapse load. The first method employs the $M_{ult} - H$ axial characteristics of the sections combined with $M - H$ structural section relationships. The changes in the latter are deduced as the arch goes from the no-load to failure. The point where the last hinge section $M - H$ structural curve cuts the $M - H$ characteristic is adjudged the modified collapse load.

The second method presented is a variation of that used in section 5 for the determination of the effects of axial force on $W_{sc}$.

Method 1

Consider the arch $4L$, Fig 10:1(a) as it is loaded to collapse. As the load is gradually increased until the formation of the first hinge at B the arch behaves elastically. The $M - H$ structural relationships for this stage, Stage 1, for the critical sections A B D and E are shown in Figs 10:3 and 10:4. By inspection it is found that the first hinge forms at B. At this point $H$ at $B = 11,000$ lbs and this corresponds to a load $W = 19000$ lbs. The points corresponding to this one for the other hinges can be found from their elastic $W - M$ and $w - H$ relationships. This point is marked 1 on the $M - H$ curves for A D E, Figs 10:3 and 10:4.

On the formation of this hinge the arch changes structurally and becomes as shown in Fig. 10:1(b) with a hinge of constant moment at B. On further application of the load the $M - H$ structural relationships for A B D and E change. These can be found by analysing the two separate arches shown in Figs 10:1(c) and (d) and adding the individual arches to obtain the total effect. The value of $H$ and $M$ at a critical section in this stage
is thus composed of two parts. The equations for M and H of a typical section can be written as follows:-

\[ H_{s2} = f_1(w) + F_1(M_{ult})B \quad \text{--- 10:1} \]

\[ M_{s2} = f_2(w) + F_2(M_{ult})B \quad \text{--- 10:2} \]

The first terms of the right hand side of equations 10:1 and 10:2 are derived from Fig. 10:1(c) and increase as the load increases. The second term is from Fig. 10:1(d) and remains constant throughout load Stage 2. The relationship \( M \sim H \) is thus no longer linear. To illustrate this Fig. 10:2 shows a typical \( M \sim H \) curve for a critical section in an arch. From point 0 to point \( M \) the relationship is assumed linear. At \( M \) the first hinge is formed and load Stage 2 commences. Depending upon the relative weights of the \( F(M_{ult}) \) terms in 10:1 and 10:2 so the stage 2 \( M \sim H \) curve will take the form of curve (a) or (b) Fig. 10:2. The new curves for Stage 2 for Arch 4L are shown on Figs 10:3 and 10:4.

These new relationships will persist until the next hinge forms. This is determined by the next \( M \sim H \) structural line to cut the \( M_{ult} \) axial characteristic. For arch 4L Fig. 10:1(a) this is section A. The load at which this occurs can be found by an iteration process. This involves assessing the load at which the next hinge will form assuming the load stage 1 \( M \sim H \) relationships. Establishing a new set of relationships for load Stage 2 with the new hinge formed the arch deflected and the assumed load applied. With these new relationships the load at which the hinge forms is calculated and this load compared with the original estimate. This process is repeated until the original estimated and final calculated loads agree. The \( M \sim H \) structural relationships are thus known for load Stage 2 for all the hinge points. With the load at which hinge A forms known the points corresponding to this can be plotted for section B, D and E.
The points on all the curves representing the formation of the second hinge are shown as $\theta$.

As loading continues the arch enters load stage 3 represented by Fig 10:1(e). For the purposes of finding the $M-H$ relationships for the critical sections the arch can now be split into the three individual components represented by Figs 10:1(f)(g)(h). The addition of the three components give the desired results.

The values of $H$ and $M$ for load stage 3 can be written as:

\[ H_{S3} = f_3(w) + F_3(Mult)_B + G_1(Mult)_A \quad \text{--- 10:3} \]

\[ M_{S3} = f_4(w) + F_4(Mult)_B + G_2(Mult)_A \quad \text{--- 10:4} \]

Repeating the argument of load stage 2 it can be seen that the $M_{S3}-H_{S3}$ structural relationships are again non-linear. The same reasoning as for load stage 2 is applied to find the load at which hinge 3 at $E$ occurs and the $M-H$ relationships for this load stage. For arch 4L these new relationships are shown plotted on Figs 10:3 and 10:4.

The point of this hinge formation is shown by $\theta$, for points A B D and E arch 4L, on Figs 10:2 and 10:3. The arch is now represented by Fig 10:1(f) which can be split down into individual components as shown in Figs 10:1(x)(l)(m)(n).

For load stage 4 further $M_{S4}-H_{S4}$ relationships can be established. They can be written as:

\[ H_{S4} = f_5(w) + F_5(Mult)_B + G_3(Mult)_A + J_1(Mult)_E \quad \text{--- 10:5} \]

\[ M_{S4} = f_6(w) + F_6(Mult)_B + G_4(Mult)_A + J_2(Mult)_E \quad \text{--- 10:6} \]

Thus for load stage 4 the $M-H$ structural relationships are again non-linear. As before the load at which the fourth hinge
ARCH 41
Fig 10:3
M (MOMENT) = 25 IMS x 10^5
H (AXIAL THRUST) = 100 x 10^3
MUlt. ~ H_axial section characteristics
for hinges A & E
MULTI-HAUL SECTION CHARACTERISTIC FOR HINGES B & D

ARCH 4L

FIG 10:4
at D forms and the $M-H$ relationships for load stage 4 can be found by iteration.

The load stage 4 relationships are shown plotted for arch 4L on Figs 10:3 and 10:4. They are continued until the last hinge is formed at D. This is shown by points on the diagrams. In this case load stage 4 follows closely on load stage 3 indicating that hinges at E and D occur at about the same load. The collapse load modified for the effects of axial force and deformation is given by the point where line D crosses the $M-H$ characteristic. If the curves given by equations 10:1, 10:2, 10:3, 10:4, 10:5 and 10:6 were plotted the process of establishing the hinge formation loads would be extremely tedious. It is sufficiently close an approximation to draw straight lines between the hinge formation points on the $M-H$ curves. The hinge formation points can be found by estimating the next hinge formation load and substituting direct in the $M$ and $H$ equations 10:1 to 10:6. This is equivalent to stepping from one end of the curve to the other, between hinge points, and thus avoiding working round it.

For arch 4L the $M-H$ structural relationships were not calculated for each load stage but a simplified approximate procedure followed. In this the $M-H$ structural relationships for section A, B, D and E are found under elastic conditions and under the deflected near collapse condition occurring under the load $W_{AC}$ given by section 7. Graphs are then drawn, Figs. 10:5(a) to (h) showing the relationship between $W-M$ and $W-H$ over the load range considered. The variation is assumed to be linear. In doing this allowance is made for the gradual deformation of the arch from no load to near collapse. The procedure for finding the requisite $M-H$ relationships for the various load stages is as follows:-

Load Stage 1

The section $M-H$ curves are examined to see which hinge forms first. For arch 4L Fig 10:1(a) this occurs at B. The point where the $M-H$ structural line for B
cuts the $M_{ult}$ $H$ characteristic is translated into a load and this load used to determine the points on the $A$, $D$ and $E$, $M\sim H$ curves. For first hinge formation -

- $H$ at $A = 20,600$ lbs
- $H$ at $B = 11,000$ lbs
- $H$ at $D = 13,200$ lbs
- $H$ at $E = 12,400$ lbs

This corresponds to a load $W$ of 8.87$T$.

**Load Stage 2**

On re-examining the $M\sim H$ curves, section $A$ forms the second hinge. As before assume this forms at the point where the curves cross. Using the elastic, stage 1, $W\sim H$ relationship translate this point into a load. Use this load on diagrams, Figs 10:5(a) to (h), to establish the load stage 2 $M\sim H$ relationships.

Section $A$ fails at $H = 30,000$ lbs. For stage 1 $W\sim H$ relationships this corresponds to $W = 12.9$ $T$. This is an estimated value of the load at second hinge formation. From Figs 10:5(a) to (h) the $M\sim H$ relationships are determined.

These are: -

<table>
<thead>
<tr>
<th>Section</th>
<th>$M$</th>
<th>$H$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>-0.512$W$ T.F.</td>
<td>1.13$W$ T.</td>
</tr>
<tr>
<td>$B$</td>
<td>-0.466$W$ T.F.</td>
<td>1.78$W$ T.</td>
</tr>
<tr>
<td>$D$</td>
<td>-0.45$W$ T.F.</td>
<td>1.78$W$ T.</td>
</tr>
<tr>
<td>$E$</td>
<td>-0.466$W$ T.F.</td>
<td>1.7$W$ T.</td>
</tr>
</tbody>
</table>

On plotting these curves for L.S.2 and using these new relationships at $A$ the load causing failure is found.
Ill to be 11.95\(^T\). Thus the first estimate of this load is high.

To obtain a closer agreement between the load stage 2 \(M-H\) relationships and the second hinge formation load the \(M-H\) relationships can be revised. Assume the hinge A forms at 12.5\(^T\).

Load stage 2 revised \(M-H\) relationships are:

<table>
<thead>
<tr>
<th>Section</th>
<th>(M)</th>
<th>(H)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>(-.514W) T.F.</td>
<td>(1.12W) T.</td>
</tr>
<tr>
<td>B</td>
<td>(.418W) T.F.</td>
<td>(.64W) T.</td>
</tr>
<tr>
<td>D</td>
<td>(-.436W) T.F.</td>
<td>(.78W) T.</td>
</tr>
<tr>
<td>E</td>
<td>(.46W) T.W.</td>
<td>(.69W) T.</td>
</tr>
</tbody>
</table>

These give \(W = 12.55\(^T\)\) therefore the estimate that \(W = 12.5\(^T\)\) is reasonable.

At 12.5\(^T\)
- \(H\) at A = 31,500 lbs
- \(H\) at B = 18,000 lbs
- \(H\) at D = 21,900 lbs
- \(H\) at E = 19,500 lbs

These points are marked on Figs 10:3 and 10:4.

Load Stage 3

The next hinge to form is at E and using the load stage 2 \(M-H\) relationship at E as a guide, \(W\) at this point = 14.5\(^T\).

At 14.5\(^T\)

<table>
<thead>
<tr>
<th>Section</th>
<th>(M)</th>
<th>(H)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>(-.506W) T.F.</td>
<td>(1.16W) T.</td>
</tr>
<tr>
<td>B</td>
<td>(.556W) T.F.</td>
<td>(.695W) T.</td>
</tr>
</tbody>
</table>
Using these relationships at third hinge formation
\( H \) at \( E = 22,000 \) lbs. This corresponds to \( W = 13.7T \).

To obtain a closer approximation assume \( W = 14T \)
when hinge \( E \) forms.

At \( 14T \)

At hinge formation given by these relationships
\( E = 22,500 \) lbs. This corresponds to \( W = 14.15T \).

At this load

\( H \) at \( A = 36,500 \) lbs
\( H \) at \( B = 21,500 \) lbs
\( H \) at \( D = 25,400 \) lbs
\( H \) at \( E = 22,500 \) lbs

These are plotted as points on Figs 10:3 and 10:4.

Load Stage 4.

By inspection of Fig 10:4 it can be seen that
hinge \( D \) forms immediately after that for \( E \). The
simple collapse load modified for the effects of
axial force and deformation using the graphical
approximate $M-H$ relationships for arch 4L is $14.15T$. This compares with $13.4T$ obtained in section 7.

Examining the $M-H$ relationships for the hinge sections on Figs 10:3 and 10:4 the following observations can be made.

**Hinge A.**

The $M-H$ relationship for this abutment hinge exhibits very nearly the same slope in all load stages. In load stages 2 and 3 eccentricity $e'$ has decreased in comparison to $e'$ elastic load stage 1. This is due to the fact that the horizontal end thrust increases with respect to the load as the arch load increases beyond point 1. Hinge A being located at the abutment suffers no deflection, hence no increase in $e'$ on this account.

**Hinge B.**

The comments made on Hinge A apply to this hinge also.

**Hinge C.**

This is the load point hinge and the first to form. In load stages 2 and 3 $e'$ at B decreases below $e'$ elastic. At this point, therefore, the increasing axial thrust with load on the assumed constant moment more than compensates for the increase in $e'$ due to the deflection occurring.

**Hinge D.**

The effect of deflection produces a marked increase in $e'$ after the first hinge has formed.
Method 2

As previously stated this is based on the method outlined in Section 5. The method used in that section involved a moment distribution process using the Moment section characteristic curves and the Elastic structural relationships for the hinge sections. In this revised method to allow for the effects of deflection the Elastic relationships for hinge positions within the arch length, i.e. excluding abutments, will be modified. The modifications will be made as follows. The deflection of these hinge positions at collapse are known from Section 6. Let these be $e^1_D$. Let the elastic eccentricity of $H$ at a typical hinge point be $e^1_E$. The assumption is now made that the slopes of these 'within-arch' hinge positions after the first hinge formation is linear and equal to $e^1_D + e^1_E$. The analysis of Section 5 is then carried out using the Elastic slopes for the abutment hinge sections and the new slopes for the hinge sections within the arch. As an example consider arch 4L Fig. 10:1(a). Hinges from at A, B, D and E. B and D are within the arch. The deflections at B and D just prior to collapse are taken from Fig. 6:6 with $k = 0.25$ they are $\delta_B = 0.25\ in$ Total $= 0.25\ in$. (These total deflections are assumed to be perpendicular to the arch).

For $B$, $e^1_E$ Elastic $= 12.6\ in$. $\delta_D + e^2_E = 13.2\ in$.

For $D$, $e^1_E = 4.32\ in$. $e^1_D + e^2_D = 4.82\ in$.

The revised Moment relationships plus the elastic M-$H$ curves for A and E are shown in Figs. 10:6 and 10:7. Using these the following analysis is re-applied in tabular form and set out in Table 10:1 below.

$$\delta_{sg} = 0.455 \frac{\Delta H}{MP} + \frac{\Delta M}{MP} + 0.819 \frac{\Delta W}{FP} + 0.274 \frac{\Delta M}{FP} \quad 5:2$$

$$2.782$$
M. H. A. W. L. T U G U S F. I B S > X L O ^


\[ M_p = 135,000 \]

H | UGLE A | E L A S T I C S T R U C T U R A L R E L A T I O N S H I P

H | UGLE E | D I T T O

ARCH 4 L

FIG 10:6

H. (AXIAL THRUST) 1bs x 10^3
M (MOMENT) kips x 10^3

POINT OF 1ST HINGE FORMATION

HINGE B: DISS. ELASTIC STRUCTURAL RELATIONSHIP, M & H

HINGE D: REVIS. M = H STRES. RELATIONSHIP INCLUDING DEFLECTION JUST PRIOR TO COLLAPSE

HINGE B: DISS. HINGE

ARCH 4L

FIG 10:7

H (AXIAL THRUST) IBS

M_0 = 120,000
From Section 7, $W_{ACD} = 13.4T$. This will be used as the first estimate in the analysis.

Table 10:1

<table>
<thead>
<tr>
<th>Hinge Position</th>
<th>Axial Thrust</th>
<th>Available Moment</th>
<th>$\Delta M_K$ MP</th>
<th>$\Delta M$ MP</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>31,000</td>
<td>+48,000</td>
<td>+ .42</td>
<td>- .455</td>
</tr>
<tr>
<td>B</td>
<td>16,550</td>
<td>-48,000</td>
<td>- .421</td>
<td>- .421</td>
</tr>
<tr>
<td>Stage 1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>W_{ACD1} = 13.4T</td>
<td>20,000</td>
<td>+27,000</td>
<td>+ .236</td>
<td>- .819</td>
</tr>
<tr>
<td>D</td>
<td>18,700</td>
<td>+30,000</td>
<td>+ .263</td>
<td>+ .274</td>
</tr>
<tr>
<td>E</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$$\frac{\Delta W_{ACD1}}{\Delta W_{ACD}} = + \frac{.035}{2.782} = .0125 \quad \Sigma = .035$$

Thus $W_{ACD1}$ is within 1.3% of 13.4T using this method. The only difference affecting the above analysis compared with those of Section 5 is the slope of the $M$-$H$ relationship for point B. Care is needed in setting out the $M$-$H$ lines especially when, as in the case of point B, their slope is considerable.

Three methods have been advanced which combine the effects of axial force and deformation on the simple collapse load.

These are the two methods in this section plus that of Section 6.

For any particular problem it is suggested that the one of Section 6 be employed together with any one of the two advanced in this Section.

Of the latter, the first is more correct analytically but could be tedious to apply whilst the second is advanced as a quick method producing an approximate result best used before a design is finalised.

To fulfill the first three basic criteria for collapse, or in the case of r.c. "near-collapse", design, it is necessary to draw the B.M.D. just prior to collapse. This will establish whether the assumption that sections between hinge points remain elastic and that the hinge point $M_{ult}$'s have not been exceeded.

This will now be done for Arch 4L under $W_{ACD} = 13.4T$. Hinges are assumed at A, B and E and one is just about to form at D, Fig. 101(a).

The arch has deflected and for the purposes of this check the deflections are assumed to be $\frac{13.4}{13.9}$ X deflection on Fig. 6:16 and 6:17 with $K = .25$. 


The collapse B.M.D. obtained is shown, as are the values of hinge moment assumed, on Fig. 10:8. The latter were assessed by inspecting Figs. 5:3, 10:3 and 10:4. On examining this B.M.D. it can be seen that in the region of D the moment existing is $M_{\text{ult.D}}$. Therefore the $3.4T$ load arrived at is $> M_{\text{ult.D}}$. The reason for this is as follows. In eqn. 7:7 from which the % change in $W_a$ due to deflection was found the values of the hinge moments were in the same ratio as the $M_0$ values of the sections. This is not correct for the collapse condition. To obtain a revised $W_a$ including the probable value of the hinge moments at collapse eqn. 7:7 will be re-written using these moment values and the deflections under the $13.9T$ load as used previously.

Thus:

\[
W_{a \text{cd1}} = \frac{-4 \times 3.55 \times 100,000}{10} = -1.25 \times 100,000
\]

Here $M_A = 185,000$, $M_B = 145,000$, $M_D = 160,000$ and $M_E = 170,000$, are the assumed hinge moment values just prior to collapse.

This reduces to $W_{a \text{cd1}} = 12.7T$.

With this revised $W_{a \text{cd}}$ value, hinge moments as before and using the deflected shape equal to $12.7 \times$ those of Fig. 6:16 and 6:17 with $K = .25$. The horizontal end thrust can be calculated for this collapse condition.

\[
H_{a \text{cd1}} = \frac{-3.55 \times 100,000}{12 \times 2,240} = -1.46
\]

Which reduces to $H_{a \text{cd}} = 9.85T$. 

\[
\begin{array}{ccc|c}
1 & 0 & 0 & -1 \\
0 & 1 & 0 & +1 \\
.25 & 0 & -1.46 & -0.93 \\
0 & .367 & -1.9 & +0.93 \\
\hline
1 & 0 & 0 & -3 \\
0 & 1 & 0 & -1 \\
.25 & 0 & -1.46 & 0 \\
0 & .367 & -1.9 & 0 \\
\end{array}
\]

\[
\begin{array}{ccc|c}
1 & 0 & -3 & -1 \\
0 & 1 & -1 & +1 \\
.25 & 0 & 0 & -0.93 \\
0 & .367 & 0 & +0.93 \\
\hline
1 & 0 & -3 & 0 \\
0 & 1 & -1 & 0 \\
.25 & 0 & 0 & -1.46 \\
0 & .367 & 0 & -1.9 \\
\end{array}
\]
B.M.D AT COLLAPSE WAC D = 13.4 t

ARCH 41

W = 30,000 lb
Mx = 145,000
N = 170,000
V = 46,000 lb
V_A = 253,600 lb

FIG 10.8

ScaLES:
Vert = 1 cm = 10,000 lb ins
Horiz = 1 cm = 5 lbs
All moments in lb ins
Using this value and taking moments about point B Fig. 10:1(a) this moment is found to be 155,000 lb. ins. which is 10,000 lb. ins. above the original estimate. Hence \( W_{ad} \) and \( H_{ad} \), the hinge moments and the deflected shape of the arch are not compatible. In an effort to bring about compatibility the hinge moment at B is assumed equal to 150,000 lb. ins. This has the effect of increasing the vertical reaction at A, Fig. 10:1 (a) in a greater proportion than the horizontal end thrust is increased. Fig. 10:4 shows that at arch collapse a value of \( M_B = 150,000 \) lbs. ins. is reasonable.

The assumed hinge moments are now: \( M_A = 183,000 \) lb. ins. \( M_B = 150,000 \) lb. ins. \( M_D = 160,000 \) lb. ins. \( M_E = 170,000 \) lb. ins. Using these values, \( W_{ad} \) is further revised again using the deflected shape at 13.9T.

\[
W_{ad} = -4 \times \frac{3.55 \times 100,000}{10} \\
0 \quad 0 \quad 0 \quad -1 \\n0 \quad 1 \quad 0 \quad +1 \\n.25 \quad 0 \quad -1.46 \quad -.958 \\
0 \quad .367 \quad -1.9 \quad +.33 \quad 10:9
\]

which reduced to \( W_{ad} = 12.85T \).

Under the same conditions \( H_{ad} = 10.2T \)

With this end thrust acting on a deflected shape \( = 12.85 \times 13.9 \) of Figs. 6:16 and 6:17 (\( K = .25 \)). The moment at B = 153,000 lb. ins. This is close enough to the revised estimate. The hinge moments \( W_{ad} \) and \( H_{ad} \) and deflected shape are now compatible and the final collapse BMD can be drawn. This is shown in Fig. 10:9. From this the maximum moment at D is seen to be 152,000 lb. ins. This is within the \( M_{ult} \) estimated for D of 160,000 lb. ins. \( W_{ad} \) thus gives an estimate of the modified collapse load sufficiently close for design purposes, the section at D being near to failure.

The \( M_{ult} \) values given to A, B and E were assessed from Figs. 10:5 10:4, 5:3 and 5:2. For each hinge the value where the \( M \) line cuts the \( M_{ult} \) characteristic curve is very nearly the same on
B.M.D. AT COLLAPSE \( \text{WACD}_t = 12.85T \)

**ARCH 41**

- \( M_e = 170,000 \) lbs
- \( V_e = 4470 \) lbs
- \( V_d = 24330 \) lbs
- \( W = 23,800 \) lbs
- \( M_A = 183,000 \) lbs

**FIG 10-9**

Scales as Fig 10-5A
Moments in 1000 Pounds
both diagrams. Judgement has to be exercised in the choice of these moments particularly in this case in the choice of that allowable at D.

These moment values directly affect the collapse load but a 5 to 10% variation in their value can be tolerated without altering the collapse load beyond the limits of error normally acceptable for this type of r.o. design, i.e. ± 15%. In the case considered the moment at D was decreased by 20% (13.4₀ of 12.8₅₀ loads). This change only decreased the collapse load by 4.1%.

Four estimates of the collapse load have been presented viz:

1. 13.4₀ Section 7
2. 14.1₅₀ This Section Method 1
3. 13.4₀ This Section " 2
4. 12.8₅₀ This Section revised eqn. 7:7

Assuming 12.8₅₀ as the lowest to be correct, the highest, No. 2, of 14.1₅₀ gives a % increase of 10.2% above the lowest. This is within the limits of error specified above. The importance of drawing the final B.M.D. cannot be over emphasised. It should be drawn at the completion of every analysis and the data assigned to the arch, i.e. deflected shape and hinge moments, must be carefully checked, from the previous steps in the analysis, to verify their reasonableness.

In the final analyses, Wₐₙₜ at B was taken as 153,000 lb. ins. An inspection of Fig. 10:4 shows that this section fails at a moment of 136,000 lb. ins. It is assumed that in spite of failure occurring at this moment as the axial thrust is increased so the moment carrying capacity of the section is pushed up a greater depth of the section being utilised as the compressive zone.

The same argument can be applied to hinge A, Fig. 10:3 where the moment at arch collapse > the 183,000 lb. ins. assumed in the final analyses. This greater moment available will, in effect, relieve the moment at B. Using this new value of Wₐₙₜ A, a closer approach to the actual Wₐₙₜ could be obtained. It is considered, however, that Wₐₙₜ obtained is a close enough estimate of the collapse load. The collapse load obtained thus satisfies
the first three conditions for correct collapse design, viz. Mechanism (hinges) Yield and Equilibrium.

The remaining criterion, i.e. are the required hinge rotations for collapse available from the hinge sections, is dealt with in the next section.
II 11. (A). Determination of Hinge Rotation Required for Collapse

In section 10 a collapse load modified for the effects of axial thrust and deflection was arrived at. Further by drawing the B.M.D at collapse it was found that the first three criteria for correct collapse design were satisfied, i.e. Mechanism (the n-1 hinge was just about to form), Yield and Equilibrium.

The fourth criterion, whether or not sufficient rotation is available at the hinge sections for form the collapse mechanism prescribed will now be investigated.

This section deals with the calculation of the hinge rotation required whilst the next considers the amount of rotation available at a particular hinge point.

The δik (influence coefficient) is used here for the calculation of the required hinge rotations.

A full explanation of this method will be found in Professor Baker's work, and it will not be described here.

A simple example will show the difference between the application of the method in the elastic and inelastic ranges.

Fig 11:1 shows an arch loaded to be in the elastic range. It is required to determine the B.M.D. On applying the δik method the arch is made statically determinate and the determinancies replaced by the unknown forces and moments X₁ to X₃ as shown in Fig 11:2. To simplify the calculation these unknowns are placed at the elastic centre of the arch and act on the arch through infinitely rigid arms joined to the abutments.

The following routine is then carried out.

Each unknown is taken in turn made equal to unity and applied alone to the statically determinate structure. The bending moments thus produced are multiplied in turn by those produced by the other
FIG. 11:1

FIG. 11:2

FIG. 11:3

FIG. 11:4

FIG. 11:5
FIG. 11:4
unknowns and the load system similarly all acting alone again on the statically determinate system. The equations thus evolved are equated to the movement in the direction of the particular unknown considered, e.g. if $X_1$ is a force this movement is a distance, if $X_2$ a movement this movement is a rotation. For the purposes of obtaining the B.M.D's required the unknowns, $X_1$, $X_2$, $X_3$ are all assumed equal to unity.

In the elastic range these movements are assumed to be zero and the aforementioned equations are equated to zero, e.g.

$$
\delta_{10} + X_1 \delta_{11} + X_2 \delta_{12} + X_3 \delta_{13} = 0 \quad --- \quad 11:1
$$

$$
\delta_{20} + X_1 \delta_{21} + X_2 \delta_{22} + X_3 \delta_{23} = 0 \quad --- \quad 11:2
$$

$$
\delta_{30} + X_1 \delta_{31} + X_2 \delta_{32} + X_3 \delta_{33} = 0 \quad --- \quad 11:3
$$

$\delta_{10}$ to $\delta_{33}$ are known hence $X_1$ to $X_3$ can be found.

In the inelastic range the movements are not zero and become the unknowns. When the system is statically determinate $X_1$, $X_2$, $X_3$ are known. Consider the arch Fig 11:3. It is on the point of collapse. The three hinge moments $\overline{X}_1$, $\overline{X}_2$, $\overline{X}_3$ are known and it is desired to find the angle of rotations at these hinges $\theta_1$, $\theta_2$, $\theta_3$. These are plastic rotations. The procedure is similar to that previously outlined. The arch is considered statically determinate with hinges at $\overline{X}_1$, $\overline{X}_2$, $\overline{X}_3$. The load system, $\overline{X}_1$, $\overline{X}_2$, $\overline{X}_3$ are then applied, in their respective positions, one at a time to the statically determinate arch and the resulting B.M.D's plotted. The B.M.D for $\overline{X}_1$ = unity is then multiplied, as before, by all the B.M.D's, i.e. for $\overline{X}_2 = 1$, $\overline{X}_3 = 1$ and that for the load system.

In this case three equations are produced. As there is in fact plastic rotation at each hinge point, the movements due to
\( \bar{X}_1, \bar{X}_2, \bar{X}_3 \) acting are not zero but equal to \(-\theta_1, -\theta_2, -\theta_3\) respectively. These equations are:

\[
\begin{align*}
\delta_{10} + \bar{X}_1 \delta_{11} + \bar{X}_2 \delta_{12} + \bar{X}_3 \delta_{13} &= -\theta_1 \quad -11:4 \\
\delta_{20} + \bar{X}_1 \delta_{21} + \bar{X}_2 \delta_{22} + \bar{X}_3 \delta_{23} &= -\theta_2 \quad -11:5 \\
\delta_{30} + \bar{X}_1 \delta_{31} + \bar{X}_2 \delta_{32} + \bar{X}_3 \delta_{33} &= -\theta_3 \quad -11:6
\end{align*}
\]

It will be noted that the sum of the rotations at each hinge point is in opposition to the moment acting. In causing the arch to collapse the load does a certain amount of work \( W \Delta w \) say. It was shown in section 6 that this external work done equalled the internal work done. The latter \( U_T = U_{\text{elastic}} + U_{\text{plastic}} \). The internal work done in the plastic range is assumed to be performed by the moments acting at the hinge points. These moments perform this work by resisting the angle change taking place and hence the signs of moment and rotation are opposite.

The method will now be directly applied to Arch 4L at the final modified collapse load of 12.85 \( T \) from section 10. The arch, Fig 11:4(a) is on the point of collapse with plastic hinges at A, B and E. Continuity exists across D. The moments and rotations at A, B and E are denoted by \( \bar{X}_1 \) to \( \bar{X}_3 \) and \( \theta_1 \) to \( \theta_3 \) as shown, Fig 11:4(b). The arch is assumed to have retained its initial shape up to the point of collapse.

Assuming \( \bar{X}_1 \) to \( \bar{X}_3 = 1 \), Fig 11:4(b) is split up into the systems shown in Fig 11:4(c)(d)(e)(f). In this case the M.O.I varies round the arch in a way not easy to represent mathematically as shown in Fig: 11:5. To establish the equations for \( \theta_1 \) etc it is necessary to plot the M diagrams for the four loading systems.
instead of the B M D's, if the M.O.I were constant.

Figs 11:6(a)(b) show the $\frac{M}{I}$ diagrams with the load system acting on the statically determinate arch. The labour involved in multiplying each segment of the $\frac{M}{I}$ diagram by corresponding segments, as required, of the $\frac{M}{I}$ diagrams for the $\bar{X}_1$, $\bar{X}_2$ and $\bar{X}_3$ systems, is large and tedious.

To simplify and reduce the arithmetic work it will be assumed that the varying M.O.I can be replaced by an "effective" I for the arch and thus the B M D's regain their simpler form.

The 'effective' I is found, Fig 11:10(a) by establishing a triangle of the same area as the $\frac{M}{I}$ diagram as shown. The varying M.O.I of the $\frac{M}{I}$ diagram is related to that at the arch centre so that the effective I is equal to the ratio of the heights of the new triangle and the $\frac{M}{I}$ diagram at the load point multiplied by the M.O.I at that point. In this case the M.O.I at the centre and load point are 125.4 in $^4$ using the uncracked section and transformed steel. The Effective I = 1.125 x I centre = 141 in $^4$.

To check that this value is reasonable the $\frac{M}{I}$ effective diagram is superimposed on the original $\frac{M}{I}$ diagram for the Ho component of the external loading case, Fig 11:10(b). The area under the new diagram is very close to that under the old. Hence in this case the assumed I effective is found to be reasonable.

In applying the method use is made of the heights of the diagrams at various points. The modified diagram's heights vary from those of the actual $\frac{M}{I}$ diagrams but this variation is assumed small enough to be neglected.

Using the effective I found above B M D's are drawn for the loading cases 11:4(c)(d)(e)(f). These are shown on Figs 11:6 (a)(b)(c), for the load W, Figs 11:7(a)(b)(c) for $\bar{X}_1$, Figs 11:8(a)(b) for $\bar{X}_2$, & Figs 11:9(a)(b)(c) for $\bar{X}_3$. In these diagrams bending moments are plotted as follows: - bending causing tension in the top arch surface are plotted above the datum line and those causing compression in this face, below it.
Fig 11:10

(a) V component of \( M_I \) diagram for \( W \)

(b) H component of \( M_I \) diagram for \( W \)

\( W = 28,500 \text{ lb} \)

Actual \( M_I \) diag
Effective \( M_I \) diag

57,000 units

770,000 units
In this case it is assumed that $E_0 = 8000$ for $Cu = 6900$ lb in$^2$. Thus $E_0 = 5.52 \times 10^6$ lb in$^2$. Applying the $\delta$ik analysis to obtain $\theta_1$, $\theta_2$ and $\theta_3$, Fig 11:4(b) the following equations are obtained:

\[
\frac{10^{-6}}{K \times 141 \times 5.52} \left[ -17.3 \times 10^6 + 170,000 \times 17.7 + 153,000 \times 35.1 + 183,000 \times 20 \right] = -\theta_1 ^{11:7}
\]

\[
\frac{10^{-6}}{K \times 141 \times 5.52} \left[ -45.9 \times 10^6 + 170,000 \times 35.1 + 153,000 \times 142 + 183,000 \times 53.3 \right] = -\theta_2 ^{11:8}
\]

\[
\frac{10^{-6}}{K \times 141 \times 5.52} \left[ -23.2 \times 10^6 + 170,000 \times 20 + 153,000 \times 53.3 + 183,000 \times 26.6 \right] = -\theta_3 ^{11:9}
\]

Assuming as in the case of section 6 that $K = .25$ these equations reduce to:

- .0254 rad = $-\theta_1 ^{11:7}$
- .0432 rad = $-\theta_2 ^{11:8}$
- .0349 rad = $-\theta_3 ^{11:9}$

From the signs of the L.H. sides of equations 11:7 to 11:9 it can be seen that the stipulation that the rotation at a hinge must be in opposition to the moment acting there is satisfied. Thus Fig 11:4(b) represents an admissible collapse mechanism from the point of view of hinge formation. Equations 11:7 to 11:9 thus give the plastic rotations required at the hinge points A, B and E, Fig 11:4(a), for collapse.

As the application of the $\delta$ik method is straightforward and whether hinges form as initially assumed is self-checking no further examples will be given here. Further examples appear in Chapter III.
Fig 11:1(a)

The following alternative method of determining the rotations required for collapse is suggested as an alternative to the $S_\text{LX}$ method. Its advantages are that it is simple to apply and gives an indication of the amount of rotation expected at the $(n+1)^{th}$ hinge for arch collapse. Fig 11:1(a) will be used to illustrate the method.

Considering Fig 11:1, $\Theta_1$ to $\Theta_4$ are found as follows:

\[
\begin{align*}
\Theta_1 \times 43.5 &= .51 \quad \therefore \Theta_1 = .0117 \text{ rads.} \\
\Theta_3 \times 30 &= .62 \quad \therefore \Theta_3 = .0207 \text{ rads.} \\
\Theta_4 \times 22.7 - \Theta_2 \times 18.1 &= 0 \quad \text{i.e. } \Theta_2 = 1.25 \Theta_4
\end{align*}
\]

For collapse \( \sum_{n} \Theta_n = 0 \).

\[
\therefore .0117 + .25 \Theta_4 - .0207 = 0 \quad \therefore \Theta_4 = .036 \text{ rads & } \Theta_2 = .045 \text{ rads.}
\]
Comparing these rotations with those obtained under the $\delta_{LK}$ method:

<table>
<thead>
<tr>
<th>Rotation</th>
<th>$\delta_{LK}$</th>
<th>This method</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 rads</td>
<td>.0254</td>
<td>.0117</td>
</tr>
<tr>
<td>2 rads</td>
<td>.0432</td>
<td>.045</td>
</tr>
<tr>
<td>3 rads</td>
<td>.0349</td>
<td>.0207</td>
</tr>
<tr>
<td>4 rads</td>
<td>0</td>
<td>.036</td>
</tr>
<tr>
<td>Total Rotation</td>
<td>.1035 rads</td>
<td>.1134</td>
</tr>
</tbody>
</table>

This method shows that a comparatively high rotation is required at $D$ for the stipulated collapse case, i.e. the $(n + 1)^{th}$ hinge about to form. This is not shown in the $\delta_{LK}$ method as collapse here is defined as the load at which plasticity is about to start in $(n + 1)^{th}$ hinge. The table above shows reasonable agreement for the rotations required at $\theta_1$, $\theta_2$ and $\theta_3$ and for the total rotation required in the arch.

Assumptions are made that the hinges are 'compressible' at collapse and that the arch segments between the hinges are rigid. It was shown in Section 8 that considerable contraction can occur at a hinge and therefore the first assumption is considered reasonable. The second assumption implies that the deflection due to bending shear or axial force between the hinges is small in comparison to that caused by the hinge rotation. Again this is considered to be a reasonable assumption.

This method will be used in the arch tests described herein.
In the previous section the plastic rotation at hinge sections required for collapse was found. This section deals with the rotation available within a hinge due to its physical make-up, the moments and forces acting on it and their distribution.

The rotation \( \phi_E \) at a section in the elastic range under pure bending, Fig 12:1(a) is given by \( \epsilon_E \) where \( \epsilon_E \) and \( nd \) are determined by the normal elastic methods. The rotation \( \phi_U \) at failure of the same section under bending only is given by \( \epsilon_u \) where \( \epsilon_u \) is taken here to be the strain occurring at the compressive edge when the concrete crushes. The values of \( \epsilon_u \) and \( nd \) vary considerably with types of stress and strain distribution assumed to occur at failure. These fundamental section properties are discussed in section 13.

Consider an r.c. member, Fig 12:2(a) under a B.M. varying as shown. Assuming a constant EI in the elastic range the variation of \( \phi \), Fig 12:2(b) will be linear and proportional to the moment at a particular point. As the moments are proportionally increased a stage is reached where plasticity begins to develope.
either in the tensile steel, the compressive concrete or both together. When this point is reached the variation of $\phi$ in that part of the member is no longer linear. Fig 12:2(a) shows the distribution of $\phi$ along the member when the moment has increased to cause plasticity at A. The hinge length at 'A' is $\overline{AB}$. Points B and C correspond to point E on Fig 12:2(d). Point E represents the moment where yielding actually commences in the section as against the ideal elastic plastic case, Fig 12:2(e) where plasticity does not occur until $M_{\text{ULT}}$ is developed. Fig 12:2(e) shows the assumed concept of the point plastic hinge whilst Fig 12:2(d) shows that practically the hinge is a zone whose length depends, amongst other factors, upon the section $M$-$\phi$ characteristic.

The difference between the practical and ideal $M$-$\phi$ curves for the section, Figs 12:2(d) and (e), lies in the fact that some yielding of a constituent part, i.e. a compressive concrete or tensile steel, develops before $M_{\text{ULT}}$ is reached. The adjustment which then occurs in the lever arm as the applied moment is increased between initial yielding and section failure accounts for the difference between point E and $M_{\text{ULT}}$, Fig 12:2(d). As an example, if the section exhibits a tensile failure point E, Fig 12:2(d), represents the point where the tensile steel yields. As the applied moment increases from this point the N.A rises and the M.O.R increases by increasing the concrete stress. At the same time the rate of increase of $\phi$ with $M$ increases. At some stage $M$ applied $= M_{\text{ULT}}$ section and the straight $M$-$\phi$ curve becomes horizontal, point G Fig 12:2(d).

Hinge length $L_p$. (Bending only).

In Fig 12:2(a) assume the variations in moment either side of A, to B and C are the same. Let $M = f(s)$ $\Rightarrow \frac{dM}{ds} = f'(s)$
For a linear B.M.D. \( M_x = \frac{M_{ud}}{x} \)

and \( dM = \frac{M_{uds}}{x} \)

\[
\therefore \frac{\ell_{P}}{2} = x \left[ 1 \frac{M_y}{M_{ud}} \right] \quad 12:1
\]

Similar expressions can be developed if the moment variations either side of A, Fig 12:2 are linear but dissimilar. The total hinge length is then the sum of those either side of \( A \) to \( B \) and to \( C \).

For concentrated loads it is reasonable to assume that the variation in moment in the hinge length is linear.

When the moment variation about a hinge point \( A' \) Fig 12:2(f) is parabolic, as in the case of a U.D load, the hinge length \( \ell_{P} = 2x \sqrt{\frac{M_y}{M_u}} \). The B.M.D. parabola is assumed regular about \( A' \).

From Fig 12:2(c) the plastic rotation is seen to be equal to the shaded area, i.e. -

\[
\varphi_{PR} = \varepsilon_{yds} \quad 12:2
\]

\( \varphi_{PR} \) is the actual rotation available for moment re-distribution within the member. \( \varepsilon_{yds} \) can be determined as follows:-
Find $\phi_{PL}$ at a section within the hinge length, at the same section find $\phi_E$, which is the rotation which would exist if the section remained elastic, then the plastic rotation at the hinge is given by:

$$\int_0^L \left( \phi_{PL} - \phi_E \right) ds$$  \hspace{1cm} 12.3$$

This equation can be used to find the available rotation for any section under bending whether it fails as a tensile or compressive hinge if the section stress and strain distributions at failure are known.

From Fig 12.2(a) it can be seen that hinge rotation depends on several factors. These are:

1. The difference between $M_{YIELD}$ and $M_{ULT}$ of the hinge.
2. The variation in bending moment at the hinge.
3. The hinge length which in turn depends on (1) and (2).

(1) depends upon the physical construction of the hinge, i.e. tensile and compressive steel, its disposition, concrete strength and concrete crushing strain.

(2) is dependent upon the loading and the variation of stiffness around the arch member.

For a particular design the loading is normally fixed and thus available rotation at a hinge can be varied by changing the hinge physical make-up and the arch member stiffness.

The maximum length a hinge can occupy in an arch is (a) between successive points of contraflexure for a hinge within the arch and (b), for abutment hinges, between the abutment and the nearest point of contraflexure in the arch.
It follows therefore that the rotation available at a particular hinge may not be sufficient to develop the stipulated mechanism. It is then necessary to re-value the hinge by changing its physical make-up or the surrounding members flexural rigidity or by down-rating the ultimate load.

Chung has shown that the addition of closely-spaced binders at a hinge allows the compressive concrete to develop considerably higher strains before failure than those developed under similar conditions in an unbound hinge.

Thus if the use of $\sigma_u$, in the case of an unbound hinge, produces available rotations which are too small binders can be added to increase the hinge rotational capacity. Chung assumes that the actual cross-sectional area of the concrete available to carry moment in a bound hinge, at strains above $\sigma_u$, is the core contained by the binders.

So far consideration has been given to sections under bending. In the arch, sections are under bending and axial force. Fig 12.3(a) shows the variation of $\phi_y$ (being the section rotation at the instant the compressive edge fails) with $H_{AXIAL}$ for a section of Arch 4L. Failure here is defined as when the compressive edge reaches a strain of $0.0038$ under the loading conditions applied. It can be seen that the presence of an axial force has a marked effect on the rotation available. To achieve a given rotation with $M$ and $H$ acting $\phi,M$ alone a greater hinge length is required.

**Hinge Length under B.M. and Axial Thrust**

Expressions similar to those for bending only can be developed for members under combined bending and axial thrust. For a linear variation in $BM$ at a hinge as in Fig 12.2(h)
which is also under axial force $E$, the hinge length

$$
\ell_p = \ell_B + \ell_C = x \left[ 1 - \frac{e_B}{e_A} \right] + x_1 \left[ 1 - \frac{e_C}{e_A} \right] \quad 12.2(a)
$$

where $M_{ULT} = He_A$, $M_{YIELD,B} = He_B$, $M_{YIELD,C} = He_C$

$e_A$, $e_B$ and $e_C$ are the nominal eccentricities plus the deflection at the particular section.

For a hinge with symmetrical parabolic B.M. variation Fig 12.2(k)

the hinge length

$$
\ell_p = 2x' \sqrt{\frac{e_B}{e_A}} \quad 12.2(b)
$$

where $e_A'$ and $e_B'$ are as in equation 12.2(a).
To find the rotation available in a section under bending and axial load Jain used equation 12.3, viz:

\[ \sum_{0}^{l} \left( \theta_{PL} - \theta_{E} \right) ds \]

He arrived at the total available rotation by splitting the hinge length into convenient segments and applying equation 12.3 to each segment. He then summed the segment rotations. This is a tedious method especially if more than one trial per hinge is necessary.

Another method of solving equation 12.3 is graphically. Curve (b) Fig 12.3 shows the variation of \( \theta_{E} \) with \( H \) at failure. The values of \( \theta_{U} \) and \( \theta_{E} \) can be read direct from the graph and used in equation 12.3 to determine the available rotation.

As an example consider the unbound hinge at load point B Arch 4L Fig 12.3. In this figure curves (a) and (b) show the variation of \( \theta_{U} \) and \( \theta_{E} \) with \( H \) at failure. Fig 12.5 shows the B.M.D. for Arch 4L at collapse. To find the available rotation it is necessary to know the hinge length, Curves (c) and (f) Fig 12.3 show the \( M_{\text{YIELD}} \) and \( M_{\text{ULT}} \) section characteristics. Yield is assumed to occur when either a maximum strain of .001 occurs in the concrete or .0015 in the tensile steel.

At B, Fig 12.5, \( M_{\text{ULT}} \) is 153,000 lb ins. Under the same \( H, M_{\text{YIELD}} = 142,000 \) lb ins. On Fig 12.5 these points are marked as b and b, either side of B and hence \( l_{p} = bb' = 2a \).

The assumption is now made that the total available rotation

\[ l_{p} \times \text{average} \left( \theta_{U} - \theta_{E} \right) \text{ in } l_{p} \]  

12.4

In this case available rotation = \( 2 \times \frac{3.55 \times 10^{-3}}{2} = .00358 \) radians.
This compares with 0.0432 radians required for collapse, section 11A.

It appears therefore that to obtain the required rotation binders are necessary at this hinge.

Chan in his work produced relationships between stirrup volume and ultimate strain. These are shown in Fig 12:6. From his test results a reasonable maximum value of concrete strain in a bound hinge is 0.015. These strains are achieved in the bound core of the hinge due to the tri-axial compression exerted by the binders on the core. At strains above the unbound failure strain the concrete cover is assumed to spall off.

A curve using a maximum strain of 0.015 for section B is shown in Fig 12:3(c). In this case the concrete above the compressive steel is assumed to be intact and 0.015 is the strain at the compressive edge. Also shown on Fig 12:3 is a similar curve for a maximum strain of 0.01 again with compressive cover assumed intact. This particular maximum strain value is recommended by Professor Baker in his Plastic Hinge Theory for a bound hinge. In his theory, however, the concrete cover is assumed to have spalled off at strains > 0.002.

Using the same assumptions as embodied in equation 12:4

1) For a maximum strain of 0.015

\[ \Theta_p \times \text{ave.}(\phi_p - \phi_E) = 2 \times 7.3 \times 10^{-3} = 0.0156 \text{ rads.} \]

-- 12:5

2) For a maximum strain of 0.010

\[ \Theta_p \times \text{ave.}(\phi_B - \phi_E) = 2 \times 5.0 \times 10^{-3} = 0.010 \text{ rads.} \]

-- 12:6

Neither 12:5 or 12:6 produce the required rotation. By adding binders the section rotational capacity has been increased to an
Relation between ultimate concrete strain and lateral binding ratio.

Taken from Chan

Fig 12.6
assumed maximum, the only other variable is the hinge length which must be increased if the required rotation is to be obtained. Whether an increase in \( \delta_p \) is justified will now be discussed. Fig 12:5 shows a peak to the B.M.D. at B. In practice this will not occur as the loading platform or column will occupy a finite length, at least 3 to 4 inches long and hence the B.M.D. 'peak' will become a 'plateau'. Throughout this plateau \( M_{ULT} \) will act as will the maximum section rotation.

A further important point is that the effect of axial thrust is to spread the hinge along a member. As an example of this consider a member under the action of axial thrust and moment as shown in Fig 12:7.

![Fig. 12:7](image)

If the section at F has yielded under \( H \) alone then the whole of the member has yielded since fibres to the left and right of F carry higher strains than those at F. Hence the plastic zones spread from A to F and from F to B.

Fig 12:8 shows two curves taken from Chan for a column section showing the relationship between \( \delta_p \) and the ratio of axial load. As expected in the unbound case, \( \delta_p = 0.035 \), the hinge length extends to the point of contraflexure. For the bound case the hinge length increases for increasing axial loads at low load values only, whilst at higher values of \( P \) the value of \( \delta_p \) is decreased for further axial load increments. This is because in this example the value of \( P_{yield} \) after spalling \( < P_{yield} \) without spalling hence yield could not occur.
RELATION BETWEEN LENGTH OF PLASTIC ZONE AND TENSION STEEL RATIO FOR MEMBERS IN BENDING ONLY

FIG 12.9

Note: \( h' / x \) = LENGTH OF PLASTIC ZONE WHERE, \( x \) IS LENGTH OF HINGE BM SEGMENT

RELATION BETWEEN LENGTH OF PLASTIC ZONE & RATIO OF AXIAL LOAD

FIG 12.8

Note: FIGS 12.8 & 12.9 TAKEN FROM CHAN
at the point of contraflexure and \( \lambda' \) could not equal \( \lambda \).

Chan goes on to say that in most practical columns, the steel, depth of cover, concrete strength, and amount of lateral bending used are such that \( \lambda' \) will usually be less than \( \lambda \).

The evidence of the present series of tests suggests that for most hinges provided there is a sufficient segment of an arch under the same sign \( B M \) for a hinge to develop, and generally this is the case, then the hinge under thrust and moment spreads itself out, the amount of spread being largely dependent upon the hinge \( B M \) distribution, until sufficient rotation is available. The available arch length for hinge development may be inadequate where a load is very close to an abutment. This case, however, is not likely to be the minimum collapse load case.

Considering the width of the loading platform and the effect of axial force the hinge length at \( B \) arch \( 4L \) can be re-valued. It is assumed that a 3 inch length at \( B \) is fully plastic and 1½ inches either side of this as varying from \( M_{ULT} \) to \( M_{YIELD} \), i.e. \( p \) = 6 inches.

With these new hinge conditions the rotation available at \( B \) for bound sections becomes:

(1). For a maximum strain of \( .015 \)

Available rotation = \( 3 \times 15.6 \times 10^{-3} + 3 \times 7.8 \times 10^{-3} \)

= .0702 rads.

(2). For a maximum strain of \( .010 \)

Available rotation = \( 3 \times 10 \times 10^{-3} + 3 \times 5 \times 10^{-3} \)

= .045 rads.

These compare with .0432 rads required to develop the stipulated mechanism of section 11.

It should be noted that for hinge \( B \) it is assumed that as rotation develops the hinge moment carrying capacity actually
increases due to increased axial force (see Fig 10:4). The N.A depth at failure is at the level of the compressive steel and the cover is assumed to remain intact as rotation proceeds.

Generally it would be safer to assume that for all strains \( \varepsilon_u \) at the compressive edge if spalling were assumed to occur with consequent reduction in available moment capacity.

However, the effect of reducing the available M.O.R. after failure \( (\varepsilon_u = 0.0038 \text{ assumed in this work}) \) would be to introduce discontinuity into the \( M_{\text{ULT}} \) section characteristics, i.e., Figs 12:3 and 12:4. The author does not consider that the resulting small increase in accuracy which would be obtained in the determination of \( M_{\text{EG}} \) of \( M_p \) merits the increase of work required to do this when the collapse load may be 10 to 15% in error due to the nature of r.c. work. In any case it should also be noted that the actual \( M_{\text{ULT}} \) section characteristics obtained generally undervalue a section from the point of view of its ultimate strength. Thus the falling off in \( M_{\text{ULT}} \) resultant on spalling at a section is in part at least allowed for. Throughout this work hinges will be assumed to remain unspalled retaining their full section to arch collapse.

The discussion of hinge B arch 4L can be carried further. Fig 10:4 shows that the section initially failed at \( M_B = 135,000 \text{ lb.ins} \). Fig 12:3 shows that yielding commenced (assuming the same \( H \) as at 135,000 lb.ins) at \( M_B = 115,000 \text{ lb.ins} \). Using this last value to fix the hinge length, (marked C-C, Fig 12:5) when \( M_B = 153,000 \text{ lb.ins} \), \( \ell_p \) from Fig 12:5 = 6 ins. This is the same as estimated previously.

Using this \( \ell_p \) with a 3 inch fully plastic length, as before, the available rotation at B becomes:

\[
\begin{align*}
\text{when (1) } \varepsilon_u &= 0.0038 \\
&= 3 \times 3.46 \times 10^{-3} + 7 \times 1.73 \times 10^{-3} = 0.0225 \text{ rads} \\
(2) \varepsilon_u &= 0.0150 \\
&= \text{as before} = 0.0702 \text{ rads} \\
(3) \varepsilon_u &= 0.010 \\
&= \text{as before} = 0.045 \text{ rads}
\end{align*}
\]
Continuing with Arch 4L, hinges at A and E Fig 12:4 will be investigated to see if the rotations required for collapse in section 11 are available.

**Hinge A.**

At collapse $M_{ULT} = 183,000$ lb.ins. From Fig 12:4 $M_{YIELD}$ at this $H$ value = 173,000 lb.ins. Using this value to determine $\ell_p$. From Fig 12:5 hinge length at $A = 2$ in. Using this $\ell_p$, rotation available:

When (1) $\epsilon_u = .0038 = \ell_p \times \text{ave}(\varphi_u - \varphi_e) = 2 \times 1.2 \times 10^{-3} = .0024$ rads.

(2) $\epsilon_u = .015 = \ell_p \times \text{ave}(\varphi_p - \varphi_e) = 2 \times 5.25 \times 10^{-3} = .0105$ rads.

(3) $\epsilon_u = .010 = \ell_p \times \text{ave}(\varphi_B - \varphi_e) = 2 \times 3.6 \times 10^{-3} = .0072$ rads.

Required rotation for collapse is .0254 rads. To achieve this binders will be used in accordance with Fig 12:6 and the axial force assumed to spread the hinge until sufficient rotation is made available. Fig 12:5 shows that the B.M. segment available for hinge A is 18.5 inches long. Assuming that the hinge can spread out without restriction. The lengths required using an unbound hinge $\epsilon_u = .0038$ and bound hinges with $\epsilon_u = .015$ and .01 are:-

(1) $\epsilon_u = .0038$. $\ell_p$ required = 21.2 inches

(2) $\epsilon_u = .015$. $\ell_p$ required = 4.85 inches

(3) $\epsilon_u = .01$. $\ell_p$ required = 7.56 inches

Chan found that for bending only the hinge length occupied between .2 and .4 of the B.M. segment. Fig 12:9 taken from his work shows for a medium strength concrete and different %'s tensile steel this variation. Fig 12:8 also shows this variation, as previously mentioned. He also states that generally in an r.c. member under bending and axial load the hinge will spread to between .4 and .7 of the B.M. segment. For the determination of hinge rotations in an arch it will be assumed that maximum $\ell_p = .4 \times \text{B.M. segment}$. 
Of these (1) exceeds the B.M. segment and is therefore inadmissible whilst (2) and (3) give lengths which are reasonable and either can be used.

**Hinge E.**

At E, Fig 12:5, $M_{ULT} = 170,000$ lb.ins. From Fig 12:4 $M_{YIELD}$ at this $M_{ULT} = 165,000$ lb ins. Fig 12:5 gives $l_p = .75$ in using these values. The required rotation for collapse, section 11, is $0.0349$ rads. By inspection it can be seen that the required rotation will not be available in $0.75$ in with or without binders.

The B.M. segment length is $14.5$ in which is sufficient for the required rotation to be developed with binders using either $\epsilon_l = 0.01$ or $0.015$.

1. $l_p$ required for $\epsilon_l = 0.01 = \frac{34.2}{3.83} = 9$ ins. — 12:7

2. $l_p$ required for $\epsilon_l = 0.015 = \frac{34.2}{5.7} = 6.14$ ins. — 12:8

Equations 12:7 and 12:8 give hinge lengths $> 0.4 \times$ B.M. segment.

For hinges A and E it has been tacitly assumed that the B.M variation is linear and reaches a peak at A and E. The rotation at each section throughout the hinge has been assumed to equal half that available at $M_{ULT}$. It has further been assumed that maximum plasticity occurs at a point whereas in fact maximum plasticity occurs for a finite length along a member. The minimum length of this spread of maximum plasticity from the present series of tests would appear to be 1 inch.

Using this fact the rotation available at E can be re-evaluated for the $\epsilon_l = 0.01$ and $0.015$ cases.
The maximum permitted hinge length = \(0.4 \times 14.5 = 5.8\) ins.

Using this length, a 1 inch length spread of maximum plasticity, and assuming a linear distribution of moment on the hinge from the 1 inch \(M_{\text{ULT}}\) plateau to \(M_{\text{YIELD}}\), the rotations available are:

1. For \(\varepsilon = 0.01\) Rotation available = \(0.00776 + 4.8 \times 0.00388 = 0.0265\) rads
   
2. For \(\varepsilon = 0.015\) Rotation available = \(0.01 + 4.8 \times 0.0057 = 0.0372\) rads

Equation 12:8 gives insufficient rotation whilst 12:9 is satisfactory.

For hinge E, binders based on Fig 12:6 to give \(\varepsilon = 0.015\) should be used in order to obtain the required rotation.

It has been shown that the required rotations stipulated for the collapse condition of section 11 can be obtained by placing suitable binding at each hinge point. The fourth criterion for collapse design, viz that hinge sections must have sufficient rotation available for the stipulated collapse mechanism, has thus been fulfilled.

The 'near collapse' mode of failure assumes no rotation at D, Fig 12:3. The moment at D at stipulated collapse is 153,000 lb.ins. For n + 1 hinge collapse the moment at D would be 157,000 lb.ins, Fig 10:4.

From Fig 12:3 \(M_{\text{YIELD}} = 146,000\) lb.ins at \(M_{\text{ULT}} = 157,000\) lb.ins. Therefore at \(M_D = 153,000\) lb.ins. some yielding at D has occurred. The length over which this occurs is given by dd, Fig 12:5. The parabolic form of the BMD at D gives a long yield length = 13 inches. As the concrete at D is theoretically not crushed at arch collapse it is not strictly necessary to bind this yield zone. Practically, however, it is advisable to do so. In most cases the shear reinforcement necessary in the arch at D would be placed as binders and would be sufficient to minimise...
spalling and cracking in the yield zone. In other cases
nominal binding should be provided.

The effect of yielding occurring at D prior to failure
is to increase the rotations required at A B and E for
collapse. These additional rotations are considered small
and are neglected.

The methods employed to calculate the available hinge
rotations above are laborious. A certain number of practical
qualifications, based on experimental observations, were
introduced to increase the rotation available when the
theoretical approach gave too little. This shows that the
determination of available hinge rotation is not an exact
science. Professor Baker in his Plastic Hinge Theory stresses
this and proposes empirical expressions based on test results
for determining available hinge rotations.

He proposes 'reasonable safe' values for tensile and
compressive hinges. The following is a summary of these
proposals taken from his work.

1. For Tensile Hinges. Referring to Fig 12:10(a)(b)(c)

\[
\theta = \frac{\ell_p + s_p(1-n)\ell_p}{d + r} = \frac{\ell_p - s_p\ell_p}{r}
\]

within safe limits, hence \( \theta = \frac{s_p\ell_p}{n_d} \).

--- 12:11
In equation 12:11 $n_h d$ is the depth of the N.A. the instant the concrete crushes, $\ell_p$ the hinge length and $Sp$ the average strain at the compressive edge between tensile steel yielding and concrete crushing. This plastic hinge concept is one where conditions throughout the hinge length are uniform, e.g. N.A. depth, and safe limiting values are assigned to $Sp$ and $\ell_p$. Professor Baker suggests that $Sp$ be made equal to .001 for sections with no special binding and equal to .01 where special binding is provided. Tests have shown that these are reasonable values. The value of $\ell_p$ is assumed equal to $d$ for most cases. The width of the support in a continuous beam may will cause a spread-out of the hinge and in cases like this $\ell_p$ may be $> d$.

Concrete failure is assumed to occur when the maximum concrete compressive strain reaches .002 in either a tensile or compressive hinge.

2. **Compressive Hinges**

For compressive hinges Fig 12:11(a) and (b) $Sm$ is the maximum compressive strain, $Sd$ the strain difference across the section and other terms are as before.

From Fig 12:11(a) \[ \frac{n_h d}{n_h d - d} = \frac{Sm}{Sm - Sd} \Rightarrow Sd = \frac{Sm}{n_h} \]
Values of $p$ ranging from $\frac{d}{2}$ to $d$, where $d$ is the column width, are proposed with reference to plastic hinges formed due to sway close to beam intersections.

Professor Baker goes on to say that values of $\ell_p$ cannot at present be precisely predicted. He says that although it would be useful to have experimental evidence on the variation of $\ell_p$ this is not necessary as the values proposed for $S_d$, .01 for bound columns and .001 for unbound columns, are sufficiently low to take care of any error in the assumed value of $\ell_p$.

Professor Baker points out however that the ultimate strength of a structure can be affected if large errors are made in the assumed strains. When strains $>.002$ are assumed he recommends that the value of the concrete cover be ignored.

Professor Baker in choosing safe limiting values for $\ell_p$ and $S_d$ obviates the need to deduct the elastic rotation from the plastic rotation at each hinge section. Thus in Fig 12:4 if curve (d) represents the variation of $\varphi \sim H$ at $\ell_p = .01$ the rotation available from a hinge of length $\ell_p$, represented by point K Fig 12:4 say, $= \ell_p \varphi_{EK}$ which is considerably simpler than the methods employed for hinges A B and E, Arch 4L.

Professor Baker refers to Chan's series of tests on bound prisms to establish the relationship between binder volume and concrete ultimate strain. These have been referred to previously in Fig 12:6.

For the purposes of checking whether sufficient hinge rotation is available at a hinge in an arch it is proposed that basically Professor Baker's approach be adopted with the following modifications:

\[
\Theta = \ell_p \frac{(1 - 5m - S_d)}{r} = \ell_p \frac{(1 - 5m)}{r + \ell_p}
\]

Hence $\Theta = \frac{\ell_p S_d}{d}$

--- 12:12
(1) For either tensile or compressive hinges the maximum \( \ell_p \) to equal \( 0.4 \times B.M \) segment.

(2) Spalling be ignored.

It is suggested that Chan's curves Fig 12:6 be used to determine the volume of binding required until possibly more accurate information is available.

It is important to note that in many practical arches the shear reinforcing provided in the form of stirrups will often be adequate to provide the necessary hinge binding.

Using the above assumptions the rotations at A B and E Arch 4L Fig 12:3 and 12:4 can be re-calculated.

**Hinge B**  A tensile hinge \( d = 4.25 \). Assume \( S_p = 0.001 \)

Available rotation \( \varphi_B = \frac{S_p \ell_p}{n_d} = \frac{0.001 \times 4.25}{0.23 \times 4.25} = 0.00435 \text{ rads.} \)

On providing binding

\( \varphi_B = 0.0435 \text{ rads } \sim \text{satisfactory} \)

**Hinge A**  A tensile hinge \( d = 4.69 \text{in.} \). On using suitable binding

the available rotation = \( \frac{0.01 \times 4.69}{0.28 \times 4.69} \)

When \( n = 0.28 \) = 0.0358 rads.

\( \sim 0.0349 \text{ rads required.} \)

**Hinge E**  A tensile hinge \( d = 4.69 \)

Using binding available rotation = \( \frac{0.01 \times 4.69}{0.27 \times 4.69} \)

\( = 0.0372 \text{ rads} \)

\( \sim 0.0254 \text{ rads required.} \)
For each hinge binding has been provided to achieve a strain of .01. From Fig. 12:16 for a rectangular section, volume of binder required = .005 x core vol.

For arch 4L, core vol = 30 x length in^3

Assume binding spacing 3" c - c. Binder vol required = 90 x .005 in^3. 3" Ø binders @ 3" c - c are adequate for all hinges.

Checking on Shear Reinforcement Required.

As far as the author is aware, no simple design basis for shear in the form of an ultimate load theory is in use. Current practice is to use elastic methods to determine shear reinforcement requirements when the ultimate load theory is used.

For arch 4L with the maximum allowable elastic load at B, maximum shear force at B is approximately 5000 lbs. To provide for this force, binders are required at B and A. 3 in Ø double binders at 3 in c - c are adequate for this force.

At hinge E, maximum elastic SF is approximately 1500 lbs. 3/4 in Ø single binders are adequate in this case.

In most practical cases, however, the shear reinforcement provided would be symmetrical about the arch crown.

For this case, it can be seen therefore that the shear reinforcement provided is more than adequate for hinge binding purposes, and this will probably be true for most practical arches.

The effect of Shear on Hinge Moment Capacity.

It has been tacitly assumed in this section that shear has no effect on the ultimate M.O.R. of a section. As far as the author is aware, there is no data available on the effect of shear on the ultimate M.O.R. of plastic hinges in the presence
of an axial force.

Work has however been carried out combining the effects of shear and bending on an ultimate load basis. This will be reviewed in the next section. Tests have shown that the formation of a plastic hinge in simple bending does not weaken the hinge's resistance to shear.

Jain reported nominal shear stresses defined as

\[
\text{Total Shear Force Across Section} = 0.17 \times \text{cylinder lever arm} \times \text{breadth}
\]

strength with no apparent effect on the hinge section's moment carrying capacity.

In the present large arch series shear failure occurred once only on Arch 6L. In this case the nominal shear stress \( q = 2300 \text{ psi or } 0.35\text{cu} \). In this case shear failure occurred at a hinge after formation. It is problematical whether this failure would have occurred if shear reinforcement capable of taking shear at the failure had been placed in the arch. Using the elastic approach and assuming all the shear taken by the stirrups and that the latter yield at the ultimate load the shear stirrups required would be \( \frac{1}{3} \text{in } \Phi \) double at 2in c-c. The stirrups provided were \( \frac{1}{4} \text{in } \Phi \)double at 3in c-c. The latter at 45,000 psi could take 11,000 lbs acting alone. This compares with the S.F. at failure of 40,000 lbs. Just before failure therefore the compression concrete must have been taking approximately 75% of the total S.F. (Ignoring any dowel action of the main reinforcement).

During the testing of the other arches in this series nominal shear stresses of the order of \( 0.15\text{cu} \) were carried by hinges under moment and load without apparent ill effect.

In practice using the elastic theory the effect of axial force, when shear is also present with bending, is often ignored. This is assumed to be on the safer side but as Hognestad states there is almost no experimental evidence in this field.

For present design purposes it is suggested that at a hinge under moment and axial thrust the effect of shear force on hinge
formation be ignored if the nominal shear stress is \(0.3 \sigma_u\) or lower, using present elastic rules regarding the allocation of the S.F. to the concrete and stirrups.

The design of a hinge section under shear moment and axial thrust will be dealt with in the next section.
II 13. Material Properties. Section and Member Characteristics

In determining the section characteristics for use in sections 5 to 11 certain basic assumptions regarding material properties and section characteristics have been made. These will now be discussed.

Many theories have been advanced as to the conditions existing at failure in an r.c. section. The relative merits of the various theories have been fully discussed elsewhere and it is not intended to do so here.

The theory for the ultimate load capacity of a section used in this work is due to Hognestad. He carried out two series of tests on columns under bending and axial load. From the results of these tests he proposed a theory for the ultimate M.O.R of an r.c. section under bending and axial load.

It should be emphasized that other ultimate M.O.R. theories could be used for the determination of the $M_{\text{ULT}}$ section relationships upon which the proposed arch structural analysis is based. The effect of using these other theories would be to change the $M_{\text{ULT}}$ section curves from those obtained herein. This in turn would alter the collapse load $W_{\text{ACD}}$. The difference in the final collapse obtained using other theories c.f. that obtained herein using Hognestad's method would probably be $\pm 15\%$ as generally this is the difference between the $M_{\text{ULT}}$ section curves obtained using these other theories. The method due to Hognestad was adopted because the nature of his experimental effort was closely related to the arch work. The basic assumptions in Hognestad's theory are briefly outlined in the following. A full account of these basic assumptions will be found in the two papers mentioned above.
Material Properties

(1) **Distribution and Magnitude of Compressive Stresses in Concrete**

Fig 13:1 shows the results of compression tests on 3 x 6 inch cylinders.

Fig 13:2 shows the assumed stress-strain curve in bending. The initial curved part of the diagram is fairly similar to the relation in direct compression, Fig 13:1. Tests on 6 in x 12 in cylinders showed that Ritter’s parabola was a good approximation to this curved part when expressed in the following form:

\[
fc = fc'' \left( \frac{2\varepsilon}{\varepsilon_0} - \left( \frac{\varepsilon}{\varepsilon_0} \right)^2 \right) \quad 13:1
\]

Which with \( \varepsilon_0 = \frac{2fc''}{E_c} \) may be written as

\[
f_c = \varepsilon E_c \left( 1 - \frac{\varepsilon E_c}{4fc''} \right) \quad 13:2
\]

The initial modulus of elasticity \( E_c \) was determined by cylinder tests. Satisfactory agreement was found with Inge Lyse’s equation

\[
E_c = 1.8 \times 10^6 + 460fc' \quad 13:3
\]

The average value of \( \varepsilon_U = .33\% \) Fig 13:1 was determined from the column tests.

The descending part of the stress-strain curve was assumed linear and the value of \( .15fc'' \) found to give the best agreement with the test results.

The values assumed for \( k_1 \), \( k_2 \) and \( \alpha \) for various cylinder strengths \( fc' \) are shown in Table 13:1 below. Fig 13:3 and 13:4 show the relationship between \( k_1 \) and \( fc' \) and \( k_2 \) and \( fc' \).
<table>
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<th>$f'_0$ (psi)</th>
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<th>$k_2$</th>
<th>$\alpha = k_1 k_2$</th>
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<tr>
<td>6000</td>
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<td>0.417</td>
<td>0.550</td>
</tr>
</tbody>
</table>

Table 13:1 taken from Hognestad 20.
In the '51 series of tests the columns were cast vertically and the best value of \( k_x \) found to be \( .85 \). In the '55 series however the columns were cast horizontally and the best value of \( k_x \) found to be \( 1 \). The difference in these values lies in the fact that in the '51 series water gain as concreting proceeded reduced the concrete strength at the top of the column. Failure occurred in the top section in every case. In the '55 series the columns were cast horizontally and water gain had no appreciable effect on the column's strength.

For the purposes of arch work \( k_x \) has been assumed equal to \( .85 \). This may give a lower estimate of the ultimate M.O.R. but is on the safe side and helps to compensate for the assumption made in the previous section, viz. that the effect of spalling on the M.O.R. of a hinge section is ignored.

(2) **Tensile Stresses in Concrete.**

These are ignored as is any precompression in the tensile steel due to concrete shrinkage.

(3) **Bernoulli's Hypothesis**

In the elastic and in-elastic regions under combined bending and axial force plane sections are assumed to remain plane.

(4) **Absence of Slip**

It is assumed that no general slip occurs between reinforcement and concrete. It is recognised that at cracks local bond failure must occur. These slips are assumed small and are ignored.
(5) Stress - Strain Relation for Reinforcing Steel

In Hoggestad's theory this is assumed as the usual trapezoid with a yield level at the yield point stress. No account is taken of the strain hardening range, i.e., above approximately 1.5% strain as it is only in lightly reinforced sections that strains above this will be encountered before failure is reached. It is further assumed that the stress-strain relationships in tension and compression are equal.

Section Characteristics

In Hoggestad's theory the stress and strain conditions at a section subject to eccentric loading are shown in Fig 13:5. The basic assumptions behind these have already been outlined.
(a) **Tension Failures**

A rectangular section with reinforcement at opposite faces is considered loaded in the plane of symmetry as shown in Fig 13:1(a). Hognestad assumed that since $\varepsilon_u = .38\%$ whilst the yield point of the steel is generally .1 to .2\%, both tension and compression reinforcement may be assumed to be strain'd over the yield point at failure. This has not been assumed in this work as it was found that at failure the compression steel was often still elastic. In this work, therefore, the following equations were used to determine section M.O.R. at failure.

1. Taking moments about the tension steel

$$He'_c = k_1 f'_c n_1 b d^2 (1 - k_2 n_1) + A_s d' f_{cs} \quad \text{--- 13:4}$$

2. Equilibrium of forces gives

$$H = k_1 f'_c n_1 b d + A_s f_{cs} - A_s f_{yp} \quad \text{--- 13:5}$$

3. Strain compatibility equation gives

$$f_{cs} = \frac{E_s \varepsilon_u (n_1 d - c)}{n_1 d} \leq f_{yp} \quad \text{--- 13:6}$$

Where $f_{cs} = \text{stress in compression steel}$ $f_{yp} = \text{yield point stress}$ in compression or tension steel. See Fig 13:5 for other symbols.

(b) **Compression failures**

A. Neutral axis inside member.

Considering Fig 13:5.

1. Taking moments about the tension steel

$$He'_c = k_1 f''_c n_1 b d^2 (1 - k_2 n_1) + A_s d' f_{yp} \quad \text{--- 13:7}$$
2. Equilibrium of forces gives

\[ H = k_1 f'' n_1 b d^2 (1 - k_2 n_1) + A_s ' f''_p - A_s f_{ts} \]  

--- 13:8

3. Strain compatibility gives

\[ f_{ts} = \frac{E_s e}{E_s u} \left( \frac{1-a}{n} \right) \leq f''_p \]  

--- 13:9

Where \( f_{ts} \) is the stress in the tension steel.

B. **Neutral Axis outside section**

If the eccentricity \( e \) is very small the neutral axis will fall outside the section 13:5(c). The equilibrium equations then become -

1. \[ E_s e' = k_1 f'' n_1 b d^2 (1 - k_2 n_1) + A_s ' d' f''_p + A_{bs} \]  

--- 13:10

2. \[ H = k_1 f'' n_1 b d + A_s ' f''_p - A_s f_{ts} - A_{bs} \]  

--- 13:11

and strain compatibility gives

3. \[ f_{ts} = \frac{E_s e}{E_s u} \left( \frac{n - 1}{n} \right) \leq f''_p \]  

--- 13:12

This case will not be considered further and the remarks that follow refer to tension and compression failures with the neutral axis inside the member.

To allow for the area of concrete replaced by the compression bars it is advisable to substitute \( A_s ' (f_{cs} - f''_c) \) for \( A_s ' f_{cs} \) in equations 13:4 and 13:5 and \( A_s ' (f''_p - f''_c) \) for \( A_s ' f''_p \) in equations 13:7, 13:10 and 13:11. At \%'s of compression steel > 2 and relatively small eccentricities the error in neglecting to subtract the area of concrete displaced by the steel may be 4 to 6 \% on the unsafe side. This substitution
has been used in this arch work. In solving for tension and compression failures the section dimensions, material constants and eccentricity e' are known. It is required to find H and \( n_1 \).d.

The solution of equations 13:4 to 13:6 and equations 13:7 to 13:9 for \( n_1 \)d and H yields a cubic equation the solution of which is tedious.

The method followed is one of trial and error. A value of \( n_1 \) is selected and substituted in equations 13:9 and 13:8 for a compression failure or 13:5 and 13:6 for a tension failure. From these a value of H is established. This is then compared with the value of H obtained from 13:7 for a compression or 13:4 for tensile failure. This procedure is repeated until satisfactory agreement is reached. As soon as a 'feel' for the method is acquired it is simple to determine whether or not a section under a given eccentricity fails in tension or in compression and whether or not the tension and compressive steels have exceeded \( f_{yp} \), before commencing an analysis.

3. Balanced Failures

A given section is considered balanced when it is loaded with such an eccentricity that the tension steel reaches its yield point and the concrete its ultimate strain at the same load. When, for a given eccentricity and concrete section, both limiting conditions are reached at the same load, the reinforcement is referred to as being balanced. The limiting condition between tension and compression failures, which is defined as a balanced failure may be referred to in terms of the ratio \( k = \frac{n_1 d}{d} \). For the balanced condition this ratio is:

\[
k_b = \left( \frac{n_1 d}{d} \right)_b = \frac{\epsilon_u}{\epsilon_u + \frac{f_{yp}}{E_s}} \quad 13:13
\]
A tension failure will result if \(0 < k < k_b\) and compression failure will take place if \(k_b < k \leq \infty\). The various balanced quantities, \(R_b\), \(\varepsilon_b\), \(P_b\), and \(P_b'\) may then be computed from equations 13:4 and 13:5, if the concrete section is given.

Any two of the four quantities may be chosen, and the remaining two computed.

Section \(M_{ULT-H}\) Characteristics

In the proposed arch structural analysis use is made of the \(M_{ULT-H}\) section characteristics.

This is obtained by using equations 13:4 to 13:9. In using these equations to determine this characteristic a trial and error method is not necessary. The method followed is best illustrated by an example. Consider the arch section at the load point B, Fig 13:6. This is a hinge point in the arch. It is required to find its \(M_{ULT-H}\) characteristic for use in section 5.

The values of the section properties are:
- \(f'' = 5400\) psi
- \(A_s = A_s' = 0.588\) in
- \(A_p = 46,000\) psi
- \(E_s = 30 \times 10^6\) psi
- \(d' = 3.5\) in
- \(M_1 = 0.754\)
- \(k_2 = 0.415\)
- \(b = 10\) in
- \(e = 1.75\) in.

To establish a point on the curve the following procedure was adopted, e.g. taking the point corresponding to \(n_1 = 0.64\). Equation 13:9 shows that this is a tension failure.

\(H\) is then found from equations 13:6 and 13:5.

At Section B, \(H = 107,000\) lbs. ——— 13:14

Using \(n_1\), a value is found for \(H_c'\) equation 13:4. In this case —

\[H_c' = 428,200\ \text{lb ins.} ——— 13:15\]
The value of \( a' \) required to make \( H \), equation 13.15 the same as equation 13.14 is 4in. Compatibility is thus achieved between the three relevant equations without a trial and error process.

By choosing various values of \( n \), the \( M_{\text{ULT}} \) curve for section B, Fig 13.6, was obtained.

Examining the \( M_{\text{ULT}} \) curve for section B, Fig 13.6, and for section A & E, Fig 13.7, of Arch 4L it can be seen that the slope of the diagrams from just after the point of maximum moment to the \( H \) axis is almost a straight line. This was pointed out by Whitney, Hognestad and Thomas when they produced interaction diagrams for use in column design. Fig 13.8 shows an interaction diagram produced by Thomas for two square columns, one of which is a high strength concrete used with a low \% of reinforcement and the other a low strength concrete with a high \% of steel. In both cases the tension and compression reinforcement are equal. It will be observed that the parts of the curves relating to compression failure for columns reinforced with mild steel with \( f_y \) equal to 36,000 to 40,000 psi do not deviate greatly from the line \( P + M = 1 \). It should be noted that \( P_0 \) is the ultimate load in pure compression whilst \( M_{OB} \) is the ultimate moment that would be attained in pure bending if the area of tensile reinforcement were increased so as to give a balanced section in which the ultimate concrete strain \( \varepsilon_u \) is reached simultaneously with the yield of the tension bars.

Curves such as Fig 13.8 could be developed for use in the arch design and analyses proposed in this work. For this purpose however the balanced section moment \( M_{OB} \) should be replaced by the actual failing moment under pure bending. Using a diagram of this type the methods outlined in section 5 and 10 would still apply.

For most practical sections the straight line portion of the interaction diagrams includes the cases where the N.A. is outside the section.
FIG 13.6
Fig 13:8(a) shows an interaction diagram presented by Whitney. The ultimate strength curve was derived using his expressions for tensile and compressive failures. These were

(a) for tensile failure

\[ H = 0.85 \beta f_c \left\{ \sqrt{\left( \frac{a}{t} - 0.5 \right)^2 + \frac{d}{t} f_p m - \left( \frac{a}{t} - 0.5 \right)} \right\} \quad \text{Eqn 4 Fig 13:8(a)} \]

and (b) for compressive failure

\[ H = \frac{2A_t f_y p}{2t + 1} + \frac{b_t f_y}{3t f_c} + 1.178 \quad \text{Eqn 3 Fig 13:8(a)} \]

In these expressions \( e \) = eccentricity from column \( C \),
\[ P_t = \frac{\text{Total steel area}}{\text{Total concrete area}} \text{, } m = \frac{f_y}{0.15f_c}, \text{ and } t = \text{column width} \]

On Fig 13:8(a) he shows the standard straight line, i.e. elastic method as dotted and points out that this bears no consistent relation to the actual strength.

\( h \sim e \) and \( H \sim n_1 \) diagrams.

To assist in the evaluation of the practical work these two curves were developed for various sections where strain measurements were taken. Fig 13:9 shows these curves for section B Arch 4L. As required, the actual strain measurement across the section gave the position of the neutral axis and hence \( e \) from curve (a) Fig 13:9. Curve (b) Fig 13:9 then gave the value of \( H \) at the section and this together with \( 'e' \) gave the moment acting. Hence the forces on the section were readily determined.
interaction diagrams for eccentrically loaded columns from thomas

fig 3.8(a)

column interaction diagram from whitney

fig 13.8

for set of curves A $c = 6000$
$P_e = 8\%$

for set of curves B $c = 1250$
$P_e = 15\%$

tension + compression reinforcement

$\sigma' = 2630$ psi
$\sigma_{y, steel} = 53700$ psi

$A_s = 1.246^2$
$A_c = 1.246\sigma_c$

$15.75''$

$e = 0$
$e = 0.5t$
$e = 1.75t$
$e = 2.25t$
$e = 3t$

safe load limits

straight line eqn

eqn 3

eqn 4

t = full section depth

test results

moment $\frac{10^3}{14.45} = M_e$

compression failure

"balance" points

tension failure

limiting compression failure lines

ultimate strength

$10^3$
ARCHAL SECT B CHARACTERS

H = P, H_o = 217,000 lbs.
Member Characteristics

Bond Strength.

The bond strength of concrete is variable and depends upon the concrete mix, the compaction of the concrete around the steel, the surface condition of the steel, the curing of the concrete, and the lateral stresses existing between the steel and the concrete.

In the large arch tests reported herein the arch main reinforcement was anchored in the abutments by means of hooks for sufficient length to prevent steel slippage. The local bond stress in arch 6 as given by:

\[
\text{Local bond stress} = \frac{\text{Total shear force across section}}{\text{Arm of resistance} \times \frac{\text{Sum of bar}}{\text{moment}} \times \frac{\text{perimeters}}{\text{in tensile reinforcement}}}
\]

reached 4250 psi without apparent ill effect on the load carrying capacity of the sections concerned.

That no ill effect resulted from this high bond stress may have been due to the fact that the main reinforcement was carried from one abutment through to the other without a break. In a practical arch where steel is lapped special attention should be paid to high bond stresses especially in the region of the laps.

Vora and Wilkins have carried 'pull-out' tests on plain bars in bond strength investigations. However, for ultimate load design purposes at present the elastic methods have still to be used for guidance.

Shear Strength

When using the elastic theory an empirical method embodying the concept of a nominal shear stress is usually employed in the design of an r.c. section to resist shear. For a normal beam
this nominal stress is defined as:

\[ q = \frac{\text{Total shear force on section}}{\text{Lever arm} \times \text{breadth}}. \]

Hognestad has comprehensively reviewed the main contributions to the theory of shear action in r.c. members. This will not be attempted here.

Although the nominal stress approach is written into many design codes there have been many attempts, both theoretically and experimentally, to provide alternative and more rational approaches to the problem. Some of these will be presently mentioned.

The effect of shear acting on a Hinge section under moment and axial thrust

The basis of the arch structural analysis is the reduction of the arch to a statically determinate structure with approximately constant ultimate M.O.R.'s acting at a number of hinges. It is important to know what effect, if any, the presence of shear has at an arch hinge. In an arch a hinge section is under bending, axial and shear forces. As previously stated to the author's knowledge no data is available as to the effects produced by their interaction.

Khan carried out a series of tests on ten r.c. beams, under simple bending and shear, in which he determined the shear resistance of a hinge section after the hinge had formed. Half his beams had stirrups and the others had none. From these tests he came to the important conclusion that, at least for the range of variables included in his tests, the formation of a plastic hinge did not decrease the resistance to shear of the hinge section. His test series included only tension failure type hinges.

Khan used the nominal shear stress approach and assumed that the shear stress

\[ = \frac{\text{Shear Force}}{0.875 \times \text{d x b}} \]

From his tests he evolved a general expression for the shear strength of a plastic hinge. He also states, as did Laupa previously, that an
important function of the shear reinforcement is to resist the extension and widening of shear cracks. Failure, which was sudden, occurred by the formation of shear cracks and the destruction of the compression zone concrete.

Laupa, Siess and Newmark introduced a new theory in 1953 which attempted to treat the combined effects of bending and shear on an ultimate load basis. This theory gives four ways in which a beam may fail:

(1) In bending unaffected by shear.

(2) In shear-compression. This is similar to a bending failure except that the effect of shear tends to restrict the compression zone and lead to a lower moment than case (1).

(3) In 'shear proper'. This will only occur due to heavy loads very close to the supports and such failure exhibits a different mechanism from (1) and (2). Little is known about this type of failure.

(4) A type of failure between (1) and (2).

This theory introduced the idea of a shear compression moment. Zwoyer, Moody and Viest have also developed formulae relating bending and shear at the ultimate moment. Zwoyer accepted Laupa's shear compression idea whilst Moody and Viest did not.

Subbiah and Smith carried out tests on over 100 beams to determine the effect of shear on the ultimate M.O.R. of r.c. beams. They introduced a shear deterioration factor F, which they show can be <1, to allow for the effect of shear on a section's ultimate M.O.R. Thus the permissable ultimate moment at a section under shear = F x theoretical moment available under bending only. Their tests were limited to simply supported beams loaded with 2 point loads placed symmetrically within the beam. They show that using the nominal shear stress concept for beams with no shear reinforcement F is often <1. They
conclude, as does Whitney, that the nominal shear stress is not a reliable criterion of shear capacity.

They recognise that their results are to a certain extent at variance with those due to Khan and suggest further experiments. It is not intended to elaborate on their work and further details can be found in the reference given.

The concept of shear compression moment and shear deterioration factor seem to be important steps to obtaining a more realistic appraisal of the effects of shear on a beam under bending but the results to date are not sufficiently comprehensive to enable them to be used for design purposes.

Design of Arch Sections under Moment, Thrust and Shear.

In the absence of sufficient reliable ultimate load data it is suggested that:— (1) the effect of axial force on a section under moment and shear be ignored, and (2) the section be designed for shear using the elastic nominal stress concept i.e. \[ q = \frac{\text{Shear Force}}{\text{Lever arm} \times \text{breadth}} \]. This Shear Force on the section should be that acting at arch collapse.

The criterion for the provision of shear reinforcement can be taken from the normal elastic design methods, e.g. for C.P.(114) 1957 \[ q = 0.033 \times \frac{\text{Cu}}{F} \] for \( F < 3000 \text{ lbs in}^2 \).

If shear reinforcement is necessary, as will be the case in most practical arches, it can be assumed to reach a stress equal to \( 0.9 \times \text{its yield stress} \) at the ultimate load. This is a safe assumption as tests show that the concrete compression zone will take some of the shear force up until the zone disintegrates. This provision will also guard against the incidence of a sudden shear failure which is a far less desirable type of failure, from the point of view of safety, than that when the sections fail in bending.
Subbiah and Smith record that from the point of view of attaining the ultimate M.O.R. of a section under shear it is desirable that hinge sections exhibit a tension failure. Laupa and others have demonstrated that beams with high ratios of tensile reinforcement are more prone to shear failures as the value of $n_1$ is increased.

**Flexural Rigidity - $E \times I$.**

As an r.c. member is loaded to its ultimate load the value of the neutral axis throughout the member continually changes. The tension zone in the concrete either grows larger or smaller hence the M.O.I. of the section, however defined, is continually changing. This happens throughout the member. Also as the loading changes so the stress at every section in the concrete will generally change. Fig 13:5 shows that $E_c$ varies with the stress and hence $E_c$ will vary continually throughout the member.

From these simple considerations it can be seen that throughout the loading history of an r.c. member the flexural rigidity of every section is changing. From the practical point of view the important considerations are the incidence of serious cracking, i.e. (a) cracking which may through corrosion reduce the strength of the structure or (b) 'lighter' cracking which may be aesthetically unacceptable, and the deflection under load.

Broadly there are two avenues of approach to the problem, either (1) a detailed analytical study of the structure can be made in which values of $n_1 \times E_c$ and $I$ are carefully made around the structure and then used for deflection analyses or (2) a reasonable value of $E \times I$ for the member or structure can be made based on past deflection experience.

Following the first approach Professor Baker has drawn up a table of reasonable values of $E_c$ and $I$ for use in cracked and uncracked sections in the elastic and plastic regions.
For the purpose of ultimate load arch design the author favours the second approach. It is generally recognised that at each section as the stress varies so does $E_c$, $m$ and $I$ and so any careful calculation is likely to be only an approximation to the truth at best. The second method is straightforward and easy to apply. Use is made of a practical relationship between $E_c$ and $C_u$, see Fig 13:2, and of the uncracked transformed $I_T$ of the section. It has been used in section 6 but will be re-illustrated here. E.g. Assume a fixed ended arch of constant cross-section, $C_u = 3000$ lb in$^2$. From Fig 13:2 $E_c = 6 \times 10^6$ lbs in$^2$. To determine the arch deflections just prior to collapse the effective arch $EI = 6K_I$, where a reasonable value of $K = .25$, (from experiment) and $I_T$ is the uncracked transformed section $I$.

Values of $K$ for use with $E_cI_T$ will be discussed in the Chapter III conclusions.

Jain tested 30 arches and found that the average value of the effect $E_cI_T$ as defined above was $.5E_cI_T$. However he designated failure to occur just before the $(n + 1)^{th}$ hinge began to form, hence his arches were stiffer at failure than those in this current series where at failure some plasticity had in fact occurred in the $(n + 1)^{th}$ hinges.

Secondary Effects

Creep. Hognestad showed in his second series of tests on columns that rapid loading to the ultimate is a safe criterion for structural design.

He showed that columns could sustain loading of loads very near to their 'fast-loading' ultimate load for many months with large increases in strain, due to creep, without failing. After this period the columns were loaded to failure and showed only a slight reduction in load capacity when compared with their fast-loading ultimate load.
For ultimate load arch design purposes therefore creep can be ignored.

**Shrinkage**

This should be eliminated by suitable construction methods wherever possible. If not shrinkage cracking will occur which may lower member strength and lead to corrosion with a further diminution of strength.

**Temperature**

If the temperature range is high it must be considered in design as causing arch expansion and contraction from the ambient condition. This effects the forces acting and arch deflection and hence can effect the arch ultimate load.
III Practical Investigations

Introduction

The practical investigations fell into three parts, arch tests, model arch tests and the analysis of arches previously tested by others.

The main purpose of the tests was to compare the ultimate loads hinge positions etc obtained practically with those predicted by the proposed theory. The most important secondary purpose was to obtain more information on the formation of plastic hinges in members under bending and axial load.

To fulfil these purposes it was decided to test six similar parabolic arches under different loading conditions and a number of small model arches. The latter would broaden the experimental base of the investigation without materially adding to the total laboratory effort required.

Jain tested thirty arches in his work on redundant structures. Some of his arches were analysed using the proposed theory and these theoretical results were compared with his test results.
III. 14. Large Arch Tests

Discussion of Testing Programme

These tests comprised the loading to failure of 6 similar fixed ended parabolic \( I_x = I_{crown} \) r.c. arches. Four of the arches were loaded so as to induce conditions leading to simple bending mechanism collapse, i.e. conditions conducive to \( n + 1 \) hinge formation. These tests are important as they simulate failure conditions likely to be encountered practically. They are characterised by the fact that the plastic hinges formed were all of a tensile nature.

The other two arches were loaded symmetrically at the third points. This loading condition was not conducive to the formation of \( (n + 1) \) hinges for simple bending mechanism collapse. Also, with the apparatus available, it was the nearest approach which could be made to induce the formation of a compressive hinge in the arch.

Choice of Type of Arch

The fixed-end arch was chosen mainly because there appeared to be no data available on this type at ultimate loads. Another reason was that this type, with its three redundancies, gave a better appreciation of the degree of moment re-distribution to be expected in an arch at failure than would the study of a two-pinned arch say.

Choice of Arch Shape

From the considerations of the usefulness of experimental evidence and from that of ease of fabrication, it was decided to test six similar arches. By similar is meant the same physical shape, concrete strength, % and type of compression, tension and binding steel. The classical parabolic arch, i.e. \( I_{section} = I_{crown} \) with a \( \frac{rise}{span} \) ratio of 0.4 and \( \frac{span}{depth} \) ratio of approximately 24 was considered suitable.

From testing and handling considerations the span was fixed at 10 ft., the rise thus 2 ft. The overall depth at the centre was chosen as 5". For ease of fabrication, it was decided to run the top and bottom steel at the same position relative to the arch through the arch. Using \( I_x = I_c \) this gave an overall arch depth of 5 \( \frac{11}{16} \)" at each springing.
The breadth was chosen as 10". Again this was considered to give a practical breadth value and importantly obviated any problems of lateral instability. To facilitate the handling of the arch ribs, 2 - \(\frac{1}{2}\)" rectangular eyes were case in the arch approximately at the \(\frac{1}{3}\)rd points on the rear face. Fig. 14.1 shows the three component parts of the arch structure used in the arch tests.

**Arch Reinforcement**

(a) **Main Reinforcement**

It was decided to run the same \% reinforcement top and bottom through the length of the arch. This made fabrication easy. \(6 - \frac{1}{2}\)" dia. bars were used, 3 as the top and 3 as the bottom reinforcement. This gave a \% tensile and \% compressive steel of 1.2\%, again a practical figure.

(b) **Shear**

For ease of fabrication this was constant for all the arches. \(\frac{1}{4}\)" \(\phi\) double stirrups at 3" c-c were used continuously between springings. The amount of shear reinforcement was arrived at by using the elastic nominal stress concept and by assuming that the maximum S.F. on the arch at failure would be approximately 5T. At this load the stress in the stirrups was assumed to be 45,000 psi, i.e. yield. It was expected that the S.F. taken by the compression concrete plus that in the binders and the dowel action of the main reinforcement, aided by the axial force present would be sufficient to resist the S.F.'s occurring in the arch at failure. This proved correct except in the case of Arch 6L. At the design stage it was felt that had reinforcement been provided strictly according to the elastic nominal stress idea for S.F.'s at failure, the amount of this reinforcement would have been excessive, even if it were assumed to be stressed to yield at arch failure. Figs. 14.2 and 14.3 show details of the arch reinforcement. All reinforcement joints were wired.

**Arch Tie Member**

As will presently be explained the arch tie member fig. 14.4 was used as a means of cancelling out any arch abutment rotation which occurred whilst a load or further load was applied to the arch. This member was made 18½" deep by 10" wide and reinforced with
SYMMETRICAL ABOUT ARCHA

6'-6"  6'-6"

1'-6"  5'-0"

SPRINGING
ARCH MEMBER PROJECTS
1" to 2" H TO ABUTMENT FOR KEY

TIE MEMBER

ARCH

ABUTMENT SECTION

2'-3"

2'-3"

4'-3"

4'-3"

5'-0"

13"

1'-6"

10"

ALL SECTIONS 10" WIDE.

PLAN

COMPONENT PARTS, ARCH STRUCTURE

ARCH SHAPE

FIG 14:1
Reinforcement Cage
As Arch Shape
Springing, Fig 14:1

1/4" Double Stirrups, Mk 1.
@ 3" c-c Continuous
Around Arch - 108 off

Symmetrical About Arch C
Cover Varies
1 3/4" Constant Along Rib.
Overall Length Top Steel = 14'-6"
" Bottom " = 13'-8"

Straight

ARCH REINFORCEMENT

Fig 14:2

5" 5"
3 7/8 3 7/8

ARCH PROFILE

3 1/2" BARS
Top & Bottom

Sect AA

Fig 14:3

1/4" 3 7/8

4"

Detail Mk 1

Total Length = 1'-9"
4 1\(\frac{1}{2}\)" bars top and bottom. Shear reinforcement was provided by stirrups continuously along the member. These were 7/16" at 3" c-c. The tie member was designed to remain elastic throughout the loading history of the arch. The tie beam main steel as shown on fig. 14.4 projected from the end of the beam into the abutments. The concrete in the tie beam was a nominal 1.2.4 mix with 28 day nominal cube strength of 3000 psi and an actual one of 5900 psi at 7 months. For reasons of economy it was decided to utilise the same tie member for all the tests.

**Abutments**

Fig. 14.1 shows the shape of the 10" wide abutment sections. They were given these sizes both from the point of view of moment carrying capacity (the abutments were assumed to remain elastic throughout the arch loading history) and because these sizes were suitable for use in the measurement of abutment rotation. Fig. 14.5 shows the reinforcement adopted for the abutment sections. The abutment concrete was a nominal 1.2.4 mix with a nominal 28 day cube strength of 3000 psi.

**Arch Structure Preparation Procedure**

As indicated in fig. 14.1 the arch structure was divided into 3 parts, the arch proper, the abutments and the tie member. The procedure for preparing each arch structure was as follows:

The tie member, with some abutment stirrups close to the end face of the member already in place, and arch were first levelled and lined up on the floor with the aid of a wooden template, fig. 14.6. Where applicable the extended arch reinforcement was then hooked around the projecting bottom bars of the tie reinforcement. The rest of the abutment reinforcement was then wired into position. Next the abutment forwork was oiled and wedged into position. The abutment concrete was then placed and cured. After each test the arch structure was lifted from the testing frame back into a horizontal position suitably wedged about 1\(\frac{1}{2}\)" above the floor. The abutments were then broken off with the aid of a pneumatic concrete road breaker. The tested arch was then discarded. The ends of the tie member were then cleaned off by hammer and chisel and thus made ready for the next arch to be placed in position.
Symmetrical about

4 - 1" Ø bars

4' - 3" 4' - 3" 11' - 4"

1/8 cover

6" 6'

37 - 7/8" Ø stirrups @ 3" c-c

1/16" cover

6' - 4" to O/S

4 - 1/2" Ø bars

Top & Bottom

Reinforcement Details
Arch Tie Member

Fig 14:4

Abutment reinforcement details showing (dotted) tie-in with arch & tie member steel

Fig 14:5

Fig 14:6
Arch Materials

Concrete

The concrete used in the arch ribs was a 5:1 mix, by weight, broken down as follows:

1 cwt. ordinary Portland cement (to BS 12)
250 lbs. sand (Heston)
300 lbs. 3/8" to 3/16" natural gravel aggregate
nominal w/c ratio .5.

The specified cube strength at 28 days was 5500 lbs. At the time of each arch test the actual cube strength was between 5000 and 6000 psi.

Details of the actual cube and cylinder strengths and workability factors will be found in the record of each arch test.

Cube and Cylinder Strengths

On casting an arch 9 cubes and 3 cylinders were also cast. The cubes were tested at 7 days, 28 days and at arch test. They were stored in a water tank between casting - testing. The cylinders were not stored in water before casting and testing, but in a dry laboratory atmosphere. The difference between these storage methods can be seen by comparing the results obtained in each case. The relevant cylinders were tested within two weeks of each arch test.

Reinforcement

This was of normal quality mild steel (to BS 15). Tensile tests to rupture were carried out on 3 specimens of the 1/4" main arch steel bars for each arch. These results are recorded with the arch test concerned.

Formwork

(A) Arch Rib

Fig. 14.7 shows the formwork for the ribs. It was essentially a wooden braced rib structure with a waterproofed hardboard contact surface to the concrete built on a baseboard. The hardboard stood up well to 6 uses. To facilitate the removable of the cast arch ribs the inside section of the arch formwork faces was made removable by suitably attaching it to the base board with wood screws.
(E) **Tie Beam and Abutments**

These were of normal 1" thick wrought timber with normal double wedging for location and firmness. Fig. 14.8 shows the tie formwork. Fig. 14.9 shows the abutment formwork.

**Arch Construction**

Some difficulty was initially experienced in bending the main bars to the correct shape. These bars were carried continuously from one end of the rib to the other with projections, for tying into the abutment, at either end. The bars were formed with the aid of the template fig. 14.6. The reinforcing cage was made, placed in the formwork, and positioned correctly with the aid of a small concrete cubes. Just previously the inside of the formwork had been oiled. The requisite volume of concrete was then placed and compacted by hand rodding and with the aid of a Kango hammer operating on the formwork. The lifting hooks were pushed into the top surface which was then floated off. Damp sacks were placed over the arch which stood for 2 days before the formwork was struck. For a further week or so, the arch was kept covered with damp sacking. The arch ribs were then stored in the open, without shelter, until required for test.

**Tie Member Construction**

This was essentially the same as that for the arch construction.

**Abutment Construction**

This has been dealt with under Arch Structure Preparation Procedure just previously. In this case most of the concrete was hand rodded due to the closeness of the reinforcing bars to each other. In only one case Arch 6 was a Kango hammer used to assist compaction and this was not very successful.
Testing Apparatus

Basic Testing Principle

Fig. 14:10 illustrates the basic testing principle. With no restraint from Jack 2, through the bottom pull down frame, a load is applied to the arch causing the abutments to rotate as shown dotted. The term fixed ended implies that there is no rotation at the arch ends. To achieve this, a load is applied using Jack 2, fig. 14:10 through the "pull-down" frame to the tie member. This load brings the abutments back to a position of "no-rotation". Hence, fixed ended conditions are obtained. Whilst carrying out this operation some adjustment usually occurs in the actual load applied. This is then adjusted and the procedure repeated until the desired balance, i.e. load and abutment "no-rotation" is achieved. Fig. 14:10 shows the principle applied to a symmetrical loading case. Fig. 14:10 illustrates the principle when an eccentric load is applied. In this case the load causes the abutments to rotate in the same direction. In this case 2 jacks and 2 "pull-down" frames are used. It will be noted that the right-hand "pull-down" frame is placed over the right-hand abutment. The operating procedure is the same as outlined for the symmetrical loading case.

Testing Frame and Equipment.

Fig 14:12 shows a typical test set up.

(A) Testing Frame

Fig 14:13 shows the testing frame which was designed for the arch testing programme. The top and bottom main beams are joined by 4 - 6" x 3" I's as tie members which complete a "closed-loop" force system. Only the dead weight of the frame and the arch were thus transmitted, through the 22 x 7 I supports to the floor.

The frame was of bolted construction, for ease of erection and dismantling. Turned and fitted bolts were used at load
FIG 14:0
TESTING PRINCIPLE: SYMMETRICAL LOAD

FIG 14:1
TESTING PRINCIPLE: ASYMMETRICAL LOAD
A TYPICAL 'SET-UP'

FIG 14:12

VIEW OF DEFLECTION FRAME

FIG 14:12(a)

END VIEW OF TEST SET-UP

FIG 14:12(b)
NOTE. ALL BOLTS 3/4" D.
UNLESS NOTED.
ALL BOLTS TURNED & FITTED.

8 - 1" H.D. BOLTS

22" 7/"" STEIFFENING L'S WELLED IN

3" 3/2" STEIFFENING L'S WELLED IN

GENERAL ARRANGEMENT ARCH TESTING FRAME

FIG 14:13  HALF SECT 'B-B'

SECT 'AA'

SYMETRICAL ABOUT C

B

1' 6"
10"

2' 10"
carrying connections to take advantage of their higher shear value c.f. black bolts. The 22" x 7" I frame supports were also used as supports for a removable wooden walkway. This gave easy access to the arch.

(B) Deflection Frame

This is shown in Fig 14:14. This was constructed of nominal 2 in tubing and 'welded-up'. The top continuous member was made to the same contour and elevation as the middle of the arch, when the latter was in the frame. From it were attached, by means of arms, Fig 14:15, fixed on by two 'jubilee clips', the 11 dial gauges to the underside of the arch proper as shown in Fig 14:12.

Initially the frame was held rigidly in position, by Rawlbolts, to the floor at the 4 vertical bases and by two horizontal members bolted to a near wall. After tests it was found that a heavy load applied to the floor through the jacks to the underside of the test frame bottom beam, see Fig 14:12, caused the floor to sink and thus affect the arch dial gauge readings to a small extent. To alleviate this the bolts were withdrawn from the outside supports A and B, Fig 14:14 during the tests.

'Pull-Down' Frames

Figs 14:16 and 14:17 show these frames used in the tests. Fig 14:16 is the one illustrated in Fig 14:10 used directly on the tie member whilst 14:17 is the one used across the top of the abutment, Fig 14:11. They were constructed of B.S. sections.

Arch Tie-Bars

These are shown in Fig 14:18. These were 1/4 in Ø mild steel bars with a stiffened 6" x 3" C yoke at each
FIG 14:15
**Symmetrical Floor Plate**

- 4-1" Bolts
- 6' 3" [ ]

**Sect to Right of Line Cut Off for Tests A to C Inc.**

**Cont. Weld Top & Bottom Jigs**

- 5' 3' 4"
- Straps

**VIEWS B & B'-B**

Note: Views C & C'-C' Similar but No Straps

**Limit of 1/4 Plate**

**VIEW DD**

(1/4" omitted)

**Pull-Down Frames Details**

**FIG 14:16 & FIG 14:17**
end. The latter transferred the pull in the ties to the abutment ends. They were introduced on to the arch structure after the abutment of Arch 3L had failed. They were used on Arch 3L Re-test and all subsequent arches. They provided a means of taking some of the arch horizontal end thrust and were also used in one or two instances to assist the pull down jacks in returning the abutments to the 'no-rotation' position after an arch load application. The tie bar position for all tests were as shown in Fig 14:18(b). To measure the strain hence the forces in the bars 2 paper-backed E.R.S.G.'s were fixed in series to the top and bottom of each bar at its centre. In this way the bending strain was eliminated and only the direct strain measured.

**Loading Apparatus**

This is detailed under the record of each arch test.

**Proving Rings**

25, 50, and 10 circular steel proving rings were used in the tests. They were all carefully calibrated before use. Also used in some tests were proving cylinders of 3in schedule 80 pipe cut to 1lin and 2in lengths. These are shown in the arch tests concerned. Paper-backed E.R.S.G.'s were employed as the strain measuring medium on these cylinders. The cylinders were calibrated before use in a 50 Denison machine.

**Hydraulic Jacks**

Those used were standard items of equipment.

**Strain Measuring Devices**

Two methods of strain measurement were employed in the tests, - (A) Electrical and (B) Mechanical.
FIG 14:18
(b) Position of tie-bars on arch structures for tests 4L to 6L inc.

FIG 14:19
strain gauge location frame
Electrical

This method employed E.R.S.G's. For all measurements on the arch, with one exception mentioned later, 1 in linear foil gauges, either of Ferry or Advance metal, were used. The foil gauge was chosen because of its growing reputation as a method of strain measurement on concrete and on direct advice from the Cement and Concrete Association Research Department. The reason for their use was twofold. Firstly they offered an opportunity of measuring strain over a shorter length than that available by simple mechanical means and secondly they could be used in locations where mechanical strain measurement was difficult. Dealing with the first point, from the point of view of section analysis, it was important to determine as accurately as possible the maximum strain at the critical sections. The 8 in mechanical gauge length used summed the strain over 8 ins and hence a lower maximum value at a section should have been recorded than that given by a 1 in gauge about the same section. Provided of course that the strain did not remain constant over the 8 in length.

At this point it may be noted that previous workers have found that there is a minimum length over which the measurement of strain on an r.c. member is meaningful. Below this length the gauge may measure a strain which is predominantly that of a piece of aggregate. The error resulting from this occurrence depends upon the relative stress/strain moduli of the aggregate and the matrix.

From a curve derived from results obtained by Binns and Mygind, Peattie and the Building Research Station, Cooke and Seddon deduce that to obtain results which contain less than 5% error, due to this effect, the gauge length should be at least four times the nominal maximum gravel size and at least eight to ten times this size for errors of less than 2½%.

For the arch the maximum aggregate size was 3/4" nominal and the gauge length used 1".
The foil gauges were fixed to the arch by using two epoxy-resin fixatives. For half the arches Araldite 'B' with hardener 901 was used to apply the gauges. After the third arch was tested an Araldite strain-gauge cement was introduced to the market, by the same company, and this was used for the remainder of the tests.

The method of fixing the gauges to the arch with Araldite B was as follows:

1. Emery cloth - to smooth surface of concrete.
2. Clean off grease with acetone.
3. Use 2% ZnCl₂ Zinc Chloride.
   + 3% H₃PO₄ Phosphoric Acid
   Dilute 4 times in water.
4. Wash with distilled water.
5. Allow surface to dry.
6. Heat Araldite to 50°C (not more, hardens if above 50°C) for ten minutes. Spread a thin layer of Araldite pressing it into the surface very hard so as to leave it smooth and thin.
7. Wait one day and apply Araldite to gauge and surface excepting leads.
   Place a pad of P.V.C. over. Place steel square over P.V.C. and apply a pressure of 5 lbs.
   When cured, solder on leads and cover all with Araldite.

Fig 14:19 shows the 'handy-angle' frames used when applying the foil E.R.S.G's using Araldite B. The top and bottom screws shown held the frame in the correct vertical
position. The horizontal screws, one at the arch front face and the other at the rear, supplied the pressure to force the gauges on to the arch. The force from the front screw was transmitted through a steel bearing plate to a rubber block then on to a P.V.C. pad bearing on the top surface of the gauge. The P.V.C. did not adhere to the Araldite. At first, difficulty was experienced in keeping the gauges in their correct position as the horizontal screws were tightened up but this was overcome after some experience had been acquired. The frame was left in position whilst the araldite dried out and cured. This process took about a week at laboratory temperature. The frame was then removed and the gauges were ready for the leads to be affixed.

The application of the gauges to the arch was a much simpler matter using the strain-gauge cement. The arch surface was prepared as for the araldite B case. The strain gauge cement was then applied to the arch and to the underside of the gauge which was then pressed firmly into position. Any 'bumps' in the intersurface that appeared were eased out with the fingers. The gauges were then allowed to dry out naturally. As far as the cement was concerned the gauges were ready for use within twenty-four hours of fixing.

For all foil gauges after the leads had been soldered to the gauge ears a covering of araldite either 'B' or the strain gauge cement was applied over the gauge body to prevent the ingress of moisture and to protect the foil from draughts which might have effected the gauge temperature.

For some gauges the ears were tinned prior to fixing to the arch, others were tinned after fixing.

The gauge resistances varied from 100 to 60 ohms with a gauge factor of about 2.2. (See test results for details).
Performance of Foil Gauges

Generally the performance of the gauges was poor. As previously stated it was hoped to record more precisely, than was possible with the mechanical method used, strains at the critical points. These strains should have been larger than those recorded for the same section with the longer mechanical gauge length. In very many cases this was not the case. (This can be seen by inspecting the test results). The 8in mechanical strain length often gave a higher value of strain at a hinge point than was recorded by the comparative E.R.S.C. All reasonable precautions were taken with the fixing, and soldering of the gauges. They were covered to prevent the ingress of moisture and the guard against draughts both of which may have caused a change in gauge resistance. During the test the current was constantly passed through the gauges to maintain a steady temperature. As far as possible, allowance was made for drift. But despite this careful treatment inconsistent results were obtained. These could have been due to slippage between the gauge and arch surface due to gauge cement failure but externally at least there was no evidence of this.

As previously mentioned there was an instance where foil type gauges were not used. This was at the load point for Arch 1. Here in an attempt to record the shear strain paper backed rosettes of 3 - 1 in long wire gauges were used. These were in the shape of an equilateral triangle (see results Arch 1). They were applied to the arch by immersing them in acetone and holding firmly in place until the acetone evaporated. The arch surface was previously cleaned as for the foil gauges. On test the gauges did not perform satisfactorily, in this case mainly due to the lack of experience of the author with the gauges. No significant results were obtained. This attempt to find the shear strain was not repeated.

On ancillary items of equipment, e.g.Proving Cylinders, 1\frac{1}{4}in \Ø tie bars and 6 x 12in concrete test cylinders *paper-
backed* wire E.R.S.G's were used. They were fixed on a cleaned surface by immersing in acetone and firmly holding in position until dry.

**Operation of E.R.S.G's.**

The gauges were employed in a normal Wheatstone Bridge circuit. This has been adequately described in other works. A 50 way Savage Parsons Strain Measuring Box, Fig 14:20 was used in all the tests with the exception of one test where two of them were required due to the number of gauges used in the test.

**Mechanical Strain Recording.**

The 'Demec' gauge developed by the Cement and Concrete Association and described by Morice and Base was used in these tests. The Demec gauge measures the change in distance between two round mild steel Metzer points rigidly fixed to the member. These points are fixed to the member 8 ins apart by a standard centering bar. Fig 14:21 shows the Demec gauge, centering bar, and dummy invar bar. The use of the Demec is adequately described in the two publications reference above.

Some thought prior to commencing the experimental work was given to the possibility of using a mechanical gauge of a similar type but with a shorter gauge length. However it was considered that the accuracy of a proposed gauge would be doubtful and the idea abandoned.

For the most part the Metzger points were fixed to the concrete with Durofix. The concrete was smoothed, cleaned with acetone and a coat of Durofix 'pushed' into the top surface and allowed to dry. A further coat was applied and when tacky the Metzger point pushed hard on to the concrete surface and the Durofix allowed to set. Very little trouble was experienced with either the Metzger points or the Demec gauge. With the points very few became dislodged during the tests even when very close to cracks.
Some points were applied using sealing wax and a soldering iron. This method was used when it was required to use the point immediately. They were confined to the concrete cylinder tests.

Performance of the Demeo Gauge

The performance of this gauge was excellent. The metzger points stayed on the arch right to collapse and strain measurements could be made on nearly every gauge length up to this stage.

Deflexion Dial Gauges

These were generally set out as shown in Fig 14:22 for all the arch tests. Any deviations or additions are shown in the particular arch test results. As shown two types were used. On the arch proper \( \frac{1}{1000} \) th in division gauges were used. To measure the abutment rotation \( \frac{1}{10,000} \) th inch gauges were used. On the arch the gauge actuating shafts were placed to record movement at \( 90^\circ \) to the arch.

As the test proceeded, due to the actual arch deflection, the dial gauge did not actually measure the deflection of the point it had measured at the test start. In most cases the positional error was within \( \frac{3}{4}" \) to \( \frac{1}{4}" \). This deviation from the original position was ignored and it was assumed that the dial gauges recorded the deflection of the same point of the arch throughout the test.
Test Preparation Procedure

The arch was lifted, by means of 2 blocks and tackle attached to an overhead beam, from the horizontal position on the floor into the testing frame. To allow this to be done the two front 6" x 4½" ties, see Fig. 14:13 had previously been removed. To assist in moving the arch two 'U' hooks had been cast at approximately the 1/3rd points into the rear arch face. The arch was set on roller supports and levelled up by means of steel packing pieces, and an engineering level, in both directions. To prevent the roller biting into the underside of the tie member a steel plate was placed between them.

The arch was set up so that its vertical centreline coincided with that of the testing frame. The roller supports were set at equal distances, one each side of this centre line. These distances were checked when the arch was finally in position. Any 'patching up' required on the abutments was then carried out with cement mortar. Then followed the marking out of the positions of the loading platforms. The arch top where these were located was roughed up by hammer and chisel and the platforms set level in cement mortar. Care was taken to see that this mortar was surrounded by a damp cloth for a week. This prevented cracking and maximised the mortar strength.

The whole of the front face of the arch, plus other surfaces, to which E.R.S.G's and Metzger points were to be fixed were then carefully cleaned off with acetone or carbon tetrachloride.

The positions of the E.R.S.G's and Metzger points were then marked out with the aid of a plumb-bob, a 3ft builders level and an engineering set square. The E.R.S.G's were then fixed to the arch as described previously. After the gauges had dried out the araldite film covering the lead-in ears was carefully removed with a razor blade and the ears tinned.

The Metzger points were then fixed on with two coats of Durofix.
Leads were then run from the right hand end of the deflection frame, (where they were bundled before fixing to strain measuring instrument) to the E.R.S.G's. Leads were run in pairs to each gauge and in groups to each group of gauges located on the arch.

These grouped leads were taped at intervals to form an easily moveable cable. Leads were tagged numerically in pairs at both ends.

The lead ends for attachment to the E.R.S.G's had previously been cut back approximately \(\frac{3}{16}\) in. and tinned. They were soldered to the gauges. The other ends of the leads were cut back approximately \(\frac{3}{4}\) in.

The cables were then temporarily attached to the arch and the deflection frame, to prevent movement, with 'Scotch' tape.

Each gauge was tested for resistance and continuity with an Avometer. Where faults were discovered, i.e. high or low resistance, corrective action was taken. In many cases this meant re-soldering a joint.

To expedite the work leads were run straight from the gauges to the measuring instruments. These leads were larger than the small wires which are often run from E.R.S.G's to a terminal block then, by lead, to the measuring instrument.

There was some tendency, on re-soldering a badly made joint, for the relatively heavy lead to pull off a gauge ear. This was overcome by taping the requisite leads to the arch.

After checking the E.R.S.G's an araldite coat was placed over each one and the gauge leads fixed to arch, at the gauge, by applying some araldite.

The actual shape of the arch and its width were then recorded at 10 in. centres either side of the arch centreline.
At the same time the underside of the arch was marked for deflection dial gauges positions.

The positions for the dial gauges under the abutments, two at either end, were then marked out. Due to the fact that the surface of the underside of the abutments was generally too rough for accurate deflexion measurement, aluminium alloy pads, approximately 3/16 in. x 3/16 in. x 3/32 in. thick, were fixed at these positions by Durofix. Where the 4 - 1¾ in. dia. horizontal tie bars were used, i.e. Arches 3RL to 6L, these were then placed in position and the end 6" x 3" R.S.C. yokes levelled up. The nuts were tightened to be little more than hand tight. The bars were placed so that the two paper backed E.R.S.G's fixed at the centre of each bar were diametrically opposite, one on the top and one on the bottom of the bar. These gauges had previously been attached and wired up in series, to remove the bending effect, when strain measurements were taken. They were protected with araldite and Scotch tape.

The two front 6" x 4½" ties were then re-placed in the frame and bolted up.

The "pull-back" frame (or frames) was then placed in position and its top cross piece bolted into position, see Fig 14.172. In some tests a 2 in. high 3 in. Sch 40 pipe proving cylinder was inserted between the underside of the cross piece and the packing on top of the tie member. The "pull-back" jack (or jacks) was then inserted and lined up. The E.R.S.G's leads were then connected to the Savage Parsons 50 way strain measuring box and balanced against suitable similarly connected 'dummies'. The latter had previously been fixed to a reinforced concrete block in the same way as those fixed to the arch. The dummy gauges were carefully labelled with their resistance value as three different resistance foil gauges were used in the tests.

The dummy blocks were placed in the tray underneath the body of the strain measuring box. In some cases it was found
that balance could not be obtained with the 2 ohm balancing resistance in the strain gauge box. When this occurred other dummies or active gauges of suitable resistance were tried in an effort to achieve balance. If these also failed an examination was made of the soldered and other joints and if necessary these were remade. If this too failed then a variable radio type resistance was placed in the requisite circuit and balance established.

The loading apparatus, i.e. jacks, proving rings and any attachments were then placed in position and lined up.

Four mirrors, approximately 4 in. x 1½” depth, were then firmly placed, by Scotch tape, on the abutment ends and left hand wall, see Fig 14:24. Two telescopes and two scales were set up to read the abutment rotation. They were placed as shown in Fig 14:24.

The dial gauges to be used were then examined for freedom of movement, cleaned as required, and placed in position by firmly attaching them to the locating arms.

The arch was then ready for test.
III. LARGE ARCH TESTING PROCEDURE

The tests were usually carried out during three consecutive days. The first was spent in final preparations and ironing out snags which occurred in the set up of the apparatus. On the second day a "bedding-down" load was applied to the arch and the loading stages for approximately two-thirds of the elastic range applied. On the third day a check point in the elastic range was taken and the loading stages continued until arch collapse.

Three men ran each test. One devoted his time to reading and recording the E.R.S.G.'s, whilst the other two were responsible for reading and recording the mechanical strains over the Metzger point lengths, deflection gauges, crack patterns and recording other observations as necessary. They also ensured that the loading conditions remained sensibly constant throughout each loading stage.

The detailed testing procedure now follows:-

The telescopes and scales reading on each abutment were focussed and a zero mark placed on both scales coincident with the telescopes horizontal hair line. Any rotation of the abutments was thus discernible whilst abutment sinking had no effect on the reading.

A reading on the dummy invar bar was taken with the Demec gauge. With no load on the arch, and with the nuts of the 4 - 1\(\frac{1}{2}\)" dia tie-bars, where used, slackened off, the zero readings of all the 8" Metzger point strain lengths were taken and recorded. As this was taking place, the E.R.S.G.'s were balanced. The E.R.S.G. measuring box had been switched on previously and allowed to warm up for at least an hour before use to keep the gauges at a constant temperature and
thus help to preserve their state of balance, the box was left continuously switched on during the whole course of the test.

To bed the arch down a small load of approximately $2\ fraction$ was then applied to the arch. This was treated as a load stage and the procedure for a load stage, as described later, carried out. This checked the equipment, i.e. return and levelling jacks, dial gauges, etc., was functioning satisfactorily. This load was retained for approximately half an hour then released. The E.R.S.G.'s were then re-zeroed, if necessary, and the nuts of the 4 - $1\frac{1}{2}$" dia tie bars tightened up to put a small load into them. To check that the load had a negligible effect on the arch a few check measurements of strain were made at or near the arch crown with and without the ties tightened up.

Readings were taken and recorded of the deflection dial gauges.

The first load stage was then applied by hydraulic jack through a proving ring (see individual arch tests for arrangements) and allowed to remain steady for approximately five minutes.

Under the action of the load the bottom frame beam fig. 14:13 may have deflected. This was checked by inspecting dial gauges 16 and 17 fig. 14:22. If deflection had occurred the beam was brought back to its original position by applying a load to its underside through the jacks shown in fig. 14:22, reacting on the floor. The telescopes' readings were then observed to determine if abutment rotation had occurred. If it had then a load was applied through the pull down jack(s) and frame(s) until the rotation was again sensibly zero. This procedure usually led to a change in the applied load which was then re-adjusted. The whole procedure was repeated as necessary until the applied load
required was obtained with no bottom beam deflection and no rotation. To check on the last point the readings of dial gauges 12 to 15 inc. were taken and recorded at this stage. The arch was then considered to be under load stage 1. Readings were then taken and recorded of the mechanical strain lengths and deflection dial gauges by a pair of the testing team whilst the other member recorded the strains in the E.R.S.G.'s.

The first pair also recorded any relevant data concerning the load stage, e.g. cracks, creaking, leaking jacks, time for load stage. This was done in a load stage log. A crack pattern log was also kept by this pair. This consisted of a set of arch profiles, one for each load stage, on which were recorded the location and size of the cracks. To assist in the general descriptive work a tape-recorder was used in half the tests. This helped considerably from the points of view of time and labour. It was found much easier to record a description than to write it down.

During the load stage, it was sometimes necessary to "pump up" the load jacks slightly to maintain the loads.

At the completion of the readings, which took between three-quarters and one and a quarter hours for a load stage, photographs and film were taken as thought fit. The next load was then applied to the arch and load stage 2 achieved by the same procedure, outlined for stage 1. Readings were taken as previously and the same procedure followed.

Generally four to six load stages were used in the elastic range. This gave a firm base for the consideration of the "in-elastic"
stages. The elastic range had been approximately determined previ­
ously. As the load stages proceeded, deflections near one or two
critical points in the arch were plotted to indicate how the arch was
behaving. As the load stages left the elastic for the in-elastic
range of the arch the applied loads were allowed to remain constant
for about ten minutes before readings were taken. At near-ultimate
loads it was found extremely difficult to maintain a pre-determined
load for a load stage without large increases in deflection occurring
as the load jack(s) were "pumped-up" to maintain the load. Under
these conditions the method adopted was to attempt to "pump-up" two
or three times and if this did not succeed on stabilising the load
allow the load to settle at its own level and then take a set of read­
ings at this load. The usual load stage procedure was carried out as
before.

It was difficult to determine exactly the ultimate load as
the deflection increases were relatively large just below it and no
sudden large reduction in load carrying capacity was noticed (with the
exception of Arch 6) when it was passed. It was thus also difficult
to measure strains exactly at the ultimate load.

After the ultimate load was passed, the deformed shape of the
arch was measured at 10" spacing each side of the vertical as a
height from the horizontal line between the arch springings.

The load jacks were then "pumped-up" and the variation in the
load noted as the arch behaved as a mechanism.

The loads were then released and final observations as to
cracking and spalling, etc. noted. The test was then concluded.
Large Arch Test Results

The six arches of this series were loaded as follows:

- Arch 1L and 2L: Central Load Point
- Arch 3L and 6L: 2 - 4rd Point Loads
- Arch 4L: % Point Load
- Arch 5L: Loads at % Point and 1/6 Point

Arch 1L was considered a 'guinea-pig' arch and was used as a means to get the feel of the experimental work.

Arch 2L. The left hand abutment of this arch failed on the first run-up to the ultimate load. After repairs the test was continued with a hinge already in the arch.

Arch Concrete Strength

As previously stated there was some discrepancy between the cylinder and cube strengths obtained for the arches. The cube results have been taken in every case as an indication of arch concrete strength. This was done as the cube results were higher and gave better correlation with the experimental results, i.e. $M_{ULT}$ - H section characteristics. The cylinder results were also subject to error due to poor capping.

The conversion from cube to cylinder strengths was carried out using Evans' curve, Fig 14:25(4)

Arch Main Steel Characteristic

For arches 1L to 3L inclusive the yield point and ultimate strengths were determined for three specimens for each arch by simple tension tests in a Denison machine. For these arches 2" long compression specimens were also tested but these were not satisfactory as failure was in nearly every case brought about by buckling of the specimen. For arches 4L to 6L inclusive stress-strain curves were plotted for three specimens for each
arch as they were tensile tested. Three typical curves obtained are shown in Fig. 14:26.

The yield point stress (nominal) and the ultimate stress for the reinforcement are shown with each particular arch.

Construction Errors

Fig 14:1 shows the theoretical arch rib size. Due to initial errors of +1/16" in formwork size and formwork shrinkage and change of shape due to repeated handling errors in rib depth generally of the order of ± 1/16" to ½" occurred in the finished arch. The actual size at the critical sections has been used in the computations.

The tension and compression steel were also out of position relative to each other by generally = 1/16" to ½". The position of the steel cage in the rib was also generally in error by = 1/16" to ½". These errors were checked and not allowed to be cumulative.
6x12 Cylinder Crushing Strength (psi)

6" Cube Crushing Strength (lb/in²)

R.H. Evans Curve showing relationship between cylinder & cube strengths

FIG 14.25(a)
Large Arch Tests  Arch 11

This was centrally loaded. Fig 14:25 shows the test set up.

Due to an error in the design of the abutment reinforcement the abutments could not take the moment required for arch collapse.

The test was treated as a 'guinea-pig' test to find out how long a set of reading of the electrical and mechanical strain gauges on the arch took and to ascertain the behaviour of the testing equipment. This was achieved and the results used in establishing the number of strain measuring points for the succeeding arches.

After three load stages the right-hand abutment failed under a central load of seven tons. Five △ paper-backed wire gauges were used in this test to the left of the load point in an effort to determine the shear strain acting. This attempt was not successful. Several of the fifteen elements in the △'s did not function at the test start and the results obtained were of little value. The △ gauges were not used in the remainder of the tests.
This arch was centrally loaded. Fig 14:2 shows the reinforcement. Fig 14:39 shows the actual variation in arch thickness.

**Theoretical Analysis**

![Diagram of arch under load](image)

Fig 14:30 shows the arch under a central load \( W \). Under elastic conditions, allowing for rib shortening \( M_A = M_E = 3.84 \) \( W \) ins, \( M_B = 6.04 \) \( W \) ins and \( H = 1.16 \) \( W \).

Fig 14:31 shows the elastic B.M.D for this loading condition. This shows the maximum -ve moment at D.

For collapse, if symmetry obtained throughout the loading history, five hinges would form, at A D B D' and E, Fig 14:31. This is one more than required for arch collapse. It is unlikely that symmetrical distribution of bending moment will exist, up to collapse, due to the fact that sections either side of the \( \theta \) are not equally strong. Hence for design purposes the 4 hinge collapse mode is the one to be considered.

Before proceeding further with the structural analysis the hinge section characteristics will be obtained.

From the preceding analysis and the elastic B.M.D. it appears that hinges will form at A D B and E. Section characteristics for these will now be obtained.
Material and Section Properties

\[ C_u = 6110 \text{ psi} \quad f'_{c} = 4610 \text{ psi} \quad f_{yp} = 43,000 \text{ psi} \quad E_t = 30 \times 10^6 \text{ psi} \]
\[ \varepsilon_0 = 5.5 \times 10^6 \text{ psi} \quad m = 5.45 \quad A_t = A'_t = .583 \text{ in}^2 \quad k_1 = .754 \]
\[ k_2 = .55 \quad I_q \text{ varies.} \]

For abutment sections A & E
\[ d = 4.57 \text{ ins} \quad d' = 3.5'' \quad g = 1.07'' \]

\( M_{ULT \sim H} \) section characteristic for A & E.

This is shown in Fig. 14:32.

For Load Point B
\[ d = 4.25 \quad d' = 3.5'' \quad g = .75'' \]

\( M_{ULT \sim H} \) section characteristic for B is shown in Fig 14:33.

For Hinge point D
\[ d = 4.375'' \quad d' = 3.5'' \quad g = .875'' \]

\( M_{ULT \sim H} \) section characteristic for D is shown in Fig 14:34.

Determination of Hinge Positions and \( W_{SC} \)

Assuming five hinges form in the arch at collapse as shown in Fig 14:35 the hinge positions and \( W_{SC} \) are determined as follows:

\[ \text{FIG 14:35} \]
Let \( M_0 \) at B be datum moment = \( M_p \). \( M_0 \) at B = 102,000 lb ins

\( M_0 \) at A & E = 116,500 lb ins \( \Rightarrow M_A \) & \( M_E = 1.135 \) \( M_p \).

\( M_0 \) at D = 106,500 lb ins = 1.045 \( M_p \).

The expression for \( W_{SC} \) becomes:

\[
-1 \begin{vmatrix}
-1 & 1 \\
-(1-q^2) + 1.02 & -(1-q^2)
\end{vmatrix}
\]

14:1

To find \( q; \frac{\partial W_{SC}}{\partial q} = 0 \). This gives \( q = .532 \) and \( q_2 = 32^\circ \).

This checks with the max - ve moment as given by the elastic BMD Fig 14:31.

On substituting in 14:

\[ W_{SC} = 22.2 \]

Fig 14:36 shows the probable hinge positions at collapse. With four hinges failure can occur in two ways, either with hinges at A D B and B' or at A D B and E. A guide to the probable hinges forming for collapse and their order of formation can be obtained by plotting the \( M-H \) structural elastic relationships for the hinge points on the \( M_{ULF}-H \) characteristic curves for these points. This is done on Figs 14:32, 14:33 and 14:34. The elastic \( M-H \) relationships are:

- \( B: M = 6.04 \) W ins \( A & E: M = 3.84 \) W ins \( D: M = 2.05 \) W ins
- \( H = 1.16 \) W \( H = 1.22 \) W \( H = 1.26 \) W
An inspection of Figs 14:32, 14:33 and 14:34 shows that the first hinge forms at B and the second likely to form at A and E. Collapse then occurs with a hinge forming at D or D' with A or E changing the sign of its rotation to form the fourth hinge.

**Determination of $W_{AC}$**

For this the M-H elastic structural relationships are superimposed on the $M_{ULT}$-H section characteristics for the hinge points and the moment distribution process outlined in Chapter 2 Section 10 carried out. If the initial M-H structural elastic relationships are used in this case the $W_{AC}$ obtained will be that assuming the formation of five hinges at collapse. This will be $\geq$ than that occurring with a four hinge collapse mode. The simple approach of Section 10 is based on the supposition that the bending moments at hinge points in the arch retain the same sign from no-load to collapse.

For the 4 hinge collapse mode considered here this is not the case. Hinge E, Fig 14:35, changes sign during the loading to collapse and hence the structural elastic M-H relationships for the hinge points employed in the $W_{AC}$ analysis must also change at some stage in the analysis.

It is assumed that the initial M-H structural elastic M-H relationships for A B D & E, Fig 14:35 hold until the second hinges at A and E form. From this point to collapse the hinge at E is assumed to carry no moment. This is considered to be a reasonable and a safe assumption. Actually the moment at E varies as follows: - Under elastic conditions assuming symmetry $M_E$ develops exactly as $M_A$. This behaviour continues after 1st hinge formation at B until 2nd hinge formation at A and E. As loading proceeds the stage has now been reached where asymmetry develops and the moment at D increases above that at D'. The moment at E decreases and finally changes sign to form a hinge as hinge D develops.
To upright
Folded Temp. at 10°

AT T = 69.600 lb

M-H LINE used.
For W_all

High pressure point C

N.H LANCE FORMATION AT B^'

V = 16.000 lb/ft

ARCH 21 ABUTMENT SECT.

FIG 14-32
HIGH LINE PULLED FOR WAG 4

TO ANGLE FORMATION AT D1
AT 4F = 69,000 LBS.

2nd ANGLE FORMATION AT A

MAXIMUM CURVE

M = \frac{105,000}{15} in.

AECA 2L LOAD PT.

FIG. 14-33
The moment \( E \) can develop in this opposite sense is limited due to the fact that what finally is the compression zone was formerly the tension zone and hence probably suffered previous cracking. It is assumed in fact here that the section at \( E \) is so badly cracked as to be incapable of taking moment in this opposite direction. After the formation of hinge \( E \) therefore up to collapse \( M_E = 0 \). Hence to determine \( W_{AC} \) it is firstly necessary to determine the load at which the second hinge forms. This is done by employing Method 1, Section 10. Assuming at collapse five hinges form \( W_{AC} \) is found. At this load \( M \sim H \) relationships are found. These are termed the collapse \( M \sim H \) section relationships. The approximate graphical method of Section 10 is employed and a linear variation in \( M \) and \( H \) for the hinge points assumed to obtain between the initial elastic and final collapse conditions. By employing the trial and error process of Section 10 the load causing 2nd hinge formation and final collapse can be found.

**Determination of \( W_{AC} \) for five hinge collapse mode.**

The expression for \( W_{AC} \) reduces to:

\[
\frac{2W_{SC}}{W_{AC}} = \frac{0.716 \frac{\Delta M_A}{H_0} + 0.716 \frac{\Delta M_B}{H_0} + \frac{\Delta M_D}{H_0} + \frac{\Delta M_E}{H_0}}{3.71}
\]

The determination of \( W_{AC} \) is set out in Table 14:1 below.

**TABLE 14:1**

<table>
<thead>
<tr>
<th>Hinge</th>
<th>Thrust (lbs)</th>
<th>( \Delta m ) (lb ins)</th>
<th>( n )</th>
<th>( \frac{n \Delta m}{H_0} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( W_{AC_1} = 50,000 \text{ lbs} )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>61200</td>
<td>+ 99000</td>
<td>.716</td>
<td>+ .74</td>
</tr>
<tr>
<td>B</td>
<td>58000</td>
<td>- 26000</td>
<td>.716</td>
<td>- .18</td>
</tr>
<tr>
<td>D</td>
<td>64100</td>
<td>+ 102000</td>
<td>1</td>
<td>+ 1.0</td>
</tr>
<tr>
<td>E</td>
<td>61200</td>
<td>+ 99000</td>
<td>1</td>
<td>+ .74</td>
</tr>
</tbody>
</table>

\[
\frac{\delta}{W_{AC_1}} = \frac{2.3}{3.71} = .62
\]
### TABLE 14-1 (continued)

<table>
<thead>
<tr>
<th>Hinge</th>
<th>Thrust (lbs)</th>
<th>$\Delta m$ (lb ins)</th>
<th>$n$</th>
<th>$n\Delta m$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$W_{AC_2} = 81,000$ lbs</td>
<td>A 99000</td>
<td>+83,000</td>
<td>.716</td>
<td>+.53</td>
</tr>
<tr>
<td></td>
<td>B 94000</td>
<td>-171,000</td>
<td>.716</td>
<td>-1.2</td>
</tr>
<tr>
<td></td>
<td>D 102000</td>
<td>+123,000</td>
<td>1</td>
<td>+1.21</td>
</tr>
<tr>
<td></td>
<td>E 99000</td>
<td>+83,000</td>
<td>1</td>
<td>+.82</td>
</tr>
</tbody>
</table>

$$\frac{\delta W_{AC_2}}{W_{AC_2}} = \frac{1.41}{3.71} = .33$$

| $W_{AC_3} = 110,000$ lbs | A 136000 | -42,000 | .716 | -1.3 |
|          | B 129500 | -318,000 | .716 | -2.23 |
|          | D 141000 | +96,000 | 1 | +.94 |
|          | E 136000 | -42,000 | 1 | -.41 |

$$\frac{\delta W_{AC_3}}{W_{AC_3}} = -\frac{2}{3.71} = -.54$$

$W_{AC}$ lies between 81,000 and 110,000 lbs.

| $W_{AC_4} = 90,000$ lbs | A 110,000 | +57,000 | .716 | +.40 |
|          | B 104,000 | -210,000 | .716 | -1.43 |
|          | D 113,000 | +120,000 | 1 | +1.17 |
|          | E 110,000 | +57,000 | 1 | +.56 |

$$\frac{\delta W_{AC_4}}{W_{AC_4}} = +.17$$

i.e. $W_{AC}$ lies between 90,000 and 110,000 lbs.

| $W_{AC_5} = 95,000$ lbs | A 115,000 | +41,000 | .716 | +.29 |
|          | B 110,000 | -245,000 | .716 | -1.73 |
|          | D 119,500 | +117,000 | 1 | +1.15 |
|          | E 115,000 | +41,000 | 1 | +.40 |

$$\frac{\delta W_{AC_5}}{W_{AC_5}} = +.11$$

$W_{AC} = 95,000$ lbs within 3% which is close enough in this case.
Determination of $M$-$H$ relationships for hinge point
under $W_{AC} = 95000$ lbs

In an effort to allow for the asymmetric development of the bending moment which occurs with the assumed 4 hinge collapse mode as loading proceeds the $M$-$H$ relationships assumed for points A and D at collapse will be found as follows: for A assume that this is the last hinge to form, with hinges already at D, B and E; Fig 14:35; under $W_{AC} = 95,000$ lbs. This set of conditions will give a $M$-$H$ relationship at collapse for A which will be used to set the variation in $M_A$ and $H_A$ from elastic conditions to collapse. Similarly the $M$-$H$ relationship for D at collapse will be found by assuming it is the last hinge to form under $W_{AC} = 95,000$ lbs, other hinges already existing at A, B and E. This $M$-$H$ at collapse relationship will be used to set the variation in $M_D$ and $H_D$ from elastic to collapse conditions.

(a) Assuming hinge A forms last.

![Diagram showing force distribution with hinge A about to form on the undeformed arch. Values of $M_A$ and $M_D$ are assessed by inspecting Figs 14:33 and 14:34. Under these conditions $M_A = 10.3$ W ins, $M_B = 1.79$ W, $H_A = 1.19$ W, $H_B = 1.23$ W, $H_D = 1.32$ W. The variation of $M_A$ and $H_A$ with W is shown on Fig 14:37.](image)
(b) Assuming hinge D forms last.

**Fig 14:33**

Fig 14:33 shows conditions with hinge D about to form. Under these conditions $M_D = 5.9\ W\ ins$, $H_A$, $H_B$ and $E_D$ are as for case (a) above.

The variation of $M$ and $H$ with $W$ at $B$ and $D$ is shown on Fig 14:39. The $M \sim H$ relationships shown on Figs 14:37 and 14:39 can now be used to determine the load of 2nd hinge formation and the collapse load.

**Determination of Collapse Load**

The 1st hinge using point B elastic $M \sim H$ relationship occurs at $W = 24800\ lbs$. This point is marked 1 on the $M \sim H$ lines for A B D on Figs 14:32, 14:33 and 14:34.

The next hinge to form is at A. By continuing the elastic $M \sim H$ relationship for D until it cuts the $M_{ULS} \sim H$ section curve a value of $M_A = 212,000\ lb\ ins$ is obtained corresponding to a central load at this point of 102,000 lbs. This load is obviously too high so from Fig 14:37 the values of $M_A$ and $H_A$ at 95,000 lbs will be used. These are $M = 10.3\ W\ ins$ and $H = 1.19\ W$. Plotting these on Fig 14:32 they give $M_A$ at 2nd hinge formation of 180,000 lb ins and corresponding load 17,500 lbs. As a 2nd trial taking value of $W$ as 60,000 lbs from Fig 14:37 this gives $M_A = 6.7\ W\ ins$ and $H_A = 1.2\ W$. Plotting this line of Fig 14:32 gives $M_A$ at 2nd hinge formation $= 190,000\ lb\ ins$ and corresponding $W = 23,500\ lbs$. As a 3rd
trial use \( W = 43,000 \) lbs, \( M_A = 5.4 \) \( W \) ins and \( H_A = 1.2 \) \( W \) which on plotting give \( M_A = 200,000 \) lb ins and \( W = 37,000 \) lbs. As a 4th trial use \( W = 40,000 \) lbs giving \( M_A = 5.25 \) \( W \) ins and \( H_A = 1.2 \) \( W \) which give on plotting \( M_A = 199,000 \) lb ins and \( W = 38,000 \) lbs which is close enough to the 4th trial.

At \( W = 38,000 \) lbs, \( M_D = 2.82 \) \( W \) ins, \( H_D = 1.28 \) \( W \), \( M_B = 5.2 \) \( W \) ins, \( H_B = 1.18 \) \( W \). These are shown plotted on Figs 14:33 and 14:34.

2nd hinge to collapse stage

Projecting 1st hinge stage \( M-H \) line for \( D \) to cut \( M_{ult} \) curve, Fig 14:34, this occurs at \( M_D = 230,000 \) lb ins giving \( W \) (1st hinge stage) = 77,000 lbs. From Fig 14:39 for \( W = 77,000 \) lbs \( M_D = 4.9 \) \( W \) ins and \( H_D = 1.31 \) \( W \). Plotting this line on Fig 14:34 gives \( M_D = 227,000 \) lb ins and corresponding \( W \) (3rd hinge formation) = 46,000 lbs.

As 2nd trial use \( W = 60,000 \) lbs Fig 14:39 gives \( M_D = 4.0 \) \( W \) ins and \( H_D = 1.3 \) \( W \) for this load. Plotting this line on Fig 14:34 gives \( M_D = 229,500 \) lb ins and corresponding \( W = 57,500 \) lbs which is close enough to the 2nd trial. Therefore collapse occurs at 57,500 lbs = 25.6 T. The relationships at \( W = 57,500 \) lbs for \( A \) and \( B \) for the 2nd hinge stage are shown plotted on Figs 14:32 and 14:34.

In the foregoing the effects of deformation has been ignored. This is considered reasonable in view of the other assumptions made.

Determination of \( W_{AC} \) with 4 hinges

From the preceding analysis the 2nd hinge forms at \( W = 33,000 \) lbs. Using elastic relationships at 33,000 lbs \( H_D = 48,000 \) lbs, \( H_A = 45,000 \) lbs and \( H_B = 44,000 \) lbs.

\( W_{AC} \) is determined as set out in Table 14:2 below.
From Fig 14:37 and 14:39. At 60,000 lbs, \( H_B = 1.5 \) W, \( H_A \) and \( H_B = 1.2 \) W.

The points of 2nd hinge formation for the purpose of this analysis are marked \( \sigma \) on Figs 14:32, 14:33 & 14:34 and the 2nd stage \( M-I \) relationships for A, B & D are shown dotted on these figs.

**Table 14:12**

<table>
<thead>
<tr>
<th>Hinge</th>
<th>Thrust (lbs)</th>
<th>( m ) (10 lbs)</th>
<th>( n )</th>
<th>( \frac{m-n}{n} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>72000</td>
<td>82,000</td>
<td>.715</td>
<td>+ .625</td>
</tr>
<tr>
<td>B</td>
<td>72000</td>
<td>13,000</td>
<td>.716</td>
<td>+ .001</td>
</tr>
<tr>
<td>C</td>
<td>78000</td>
<td>112,000</td>
<td>1</td>
<td>+ 1.33</td>
</tr>
<tr>
<td>D</td>
<td>72000</td>
<td>116,000</td>
<td>1</td>
<td>= 1.14</td>
</tr>
</tbody>
</table>

\[
\frac{W_{AC_4}}{W_{AC_4}} = .13
\]

Continuing \( W_{AC} = 75,000 \) lbs within 2L.

**Effects of Deformation**

To determine the deflection just prior to collapse, \( W_{AC} = 57,500 \) lbs is placed on the arch with hinges at A, B and D, Fig 14:35(a), with moments at these points assumed from the previous analysis of the collapse load.

The arch is divided into twenty segments for the deflection analysis. Table 14:3 below tabulates the deflection of the segment ends in terms of moment units.
The actual deflections are obtained from Fig 14:35(b) by multiplying the vertical and horizontal movements by 

\[ \frac{Q^2}{K E_0 I_T} \]

where \( Q^2 = 43.6 \), \( I_T = \text{Effective I of 140 in}^4 \), 

\( E_0 = 5.5 \times 10^6 \text{psi from Fig 13:10 and K} = .25 \).

Fig 14:35(b) shows the deflection diagram. From it \( \delta_y \mathbf{B} \) and \( \delta_y \mathbf{D} \) at 57500 lbs = .75" and .13" respectively, \( \delta_y \mathbf{D} = .15" \).

An inspection of Table 14:3 shows that at point D the moment is 280,000 lb ins which exceeds the section capacity at D, Fig 14:34. 

\( W_{AC} = 57500 \text{ lbs therefore exceeds the arch load capacity.} \)

**Effect of Deformation on Load Capacity.**

Using the deflection obtained from Fig 14:35(b) a revised expression for \( W_{SC} \) can be obtained using this deformed arch.

\[
W_{SC} = \frac{-213500 \times 4}{2240 \times 120} = \frac{-1 \times 1.02}{-1 \times 1 + 1} = \frac{-775 + 468}{-775 + 1.02} = \frac{18.5}{14.12}
\]

which reduces to 18.5\( \text{T} \), i.e. a reduction in 16.7% of the previous
$W_{SC}$ of 22.2. Applying this reduction to $W_{AC} = 57500$ lbs gives $W_{AC_1} = 48,000$ lbs.

Applying $W_{AC_1} = 48,000$ lbs to the arch the deflection analysis is set out in Table 14:4 below and the deflection diagram shown in Fig 14:36(b). The deflected shape of the arch being shown in 14:36(c).

### TABLE 14:4.

<table>
<thead>
<tr>
<th>Segment No.</th>
<th>$m_a$ lb ins $\times 10^3$</th>
<th>$m_b$ lb ins $\times 10^3$</th>
<th>$m_a + m_b$ lb ins $\times 10^3$</th>
<th>$\frac{m_a + m_b}{4}$ lb ins $\times 10^3$</th>
<th>$\sqrt{\frac{m_a + m_b}{2}}$ lb ins $\times 10^3$</th>
<th>$m_{na} + m_{nb}$ lb ins $\times 10^3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>+ 210</td>
<td>+ 116</td>
<td>+ 163</td>
<td>+ 82</td>
<td>+ 201</td>
<td>+ 101</td>
</tr>
<tr>
<td>2</td>
<td>+ 116</td>
<td>+ 36</td>
<td>+ 76</td>
<td>+ 38</td>
<td>+ 204</td>
<td>+ 104</td>
</tr>
<tr>
<td>3</td>
<td>+ 36</td>
<td>- 103</td>
<td>- 70</td>
<td>- 35</td>
<td>+ 204</td>
<td>+ 101</td>
</tr>
<tr>
<td>4</td>
<td>- 103</td>
<td>- 168</td>
<td>- 136</td>
<td>- 68</td>
<td>- 60</td>
<td>- 30</td>
</tr>
<tr>
<td>5</td>
<td>- 168</td>
<td>- 203</td>
<td>- 186</td>
<td>- 93</td>
<td>- 60</td>
<td>- 30</td>
</tr>
<tr>
<td>6</td>
<td>- 203</td>
<td>- 185</td>
<td>- 200</td>
<td>- 100</td>
<td>- 253</td>
<td>- 126</td>
</tr>
<tr>
<td>7</td>
<td>- 105</td>
<td>- 143</td>
<td>- 164</td>
<td>- 82</td>
<td>- 435</td>
<td>- 217</td>
</tr>
<tr>
<td>8</td>
<td>- 143</td>
<td>- 75</td>
<td>- 109</td>
<td>- 55</td>
<td>- 572</td>
<td>- 286</td>
</tr>
<tr>
<td>9</td>
<td>- 75</td>
<td>+ 45</td>
<td>- 12</td>
<td>- 6</td>
<td>- 632</td>
<td>- 316</td>
</tr>
<tr>
<td>10</td>
<td>+ 45</td>
<td>+ 170</td>
<td>+ 103</td>
<td>+ 52</td>
<td>- 586</td>
<td>- 293</td>
</tr>
</tbody>
</table>

An inspection of Table 14:4 shows that the moment at D does not exceed that available at that section.

The actual deflections are those shown on Fig 14:36(b) $\times 24 \times 10^{-6}$. Fig. 14:40 compares the actual deflections just prior to collapse with those obtained theoretically in Fig. 14:38(c).

As a final check the collapse B to D under a load of 48,000 lbs is drawn on Fig 14:41. The moment at B is increased to 200,000 lb ins, this being a reasonable figure from Fig 14:33.
$H_a \sim W$

$W \sim H_a$

Central load: $W = \text{Box} \times 10^3$

Graphs showing variation of $W - H_a$

Fig. 4.31
Graphs showing variation of \( W \) - moment & thrust at hinge points from the elastic condition to collapse.

\( M_o \sim W \)

\( H_o \sim W \)

\( \frac{1}{2} W \)

\( \frac{1}{1.6} W \)

Central load \( W \) 163 x 10^3

\( 2.08 \text{ Wins} \)

\( 5.9 \text{ Wins} \)

\( 4 \)

\( 5 \)
The B M D shows that the arch can take 48,000 lbs just prior to collapse, which is therefore an admissible collapse load and shows that a hinge will form next at D (or D') and hence the arch will collapse in the 4 hinge mode stipulated.

Determination of Rotation required for Collapse

Using the method of section 11 (B), Fig 14:42 shows the arch deflected prior to collapse as determined in Fig 14:36(c).

\[ W = 48,000 \]

\[ \sum \Theta_n = 0 \text{, i.e. } \Theta_1 - \Theta_2 + \Theta_3 - \Theta_4 = 0 \text{, which gives } \]

\[ \Theta_3 = 0.0123 \text{ rads.} \]

Rotation Available at hinge points

Sufficient binding has been provided for \( p \) to be taken as 0.01.

Available rotations at collapse

At \( B \)

\[ n = 0.22 \quad l = 15'' \quad d = 4.25'' \quad \Theta_3 = \frac{0.01 \times 4 \times 15}{0.22 \times 4.25} = 0.0641 \text{ rads} \]
At D

\[ n = 0.65 \quad l = 48" \quad d = 4.375" \quad \theta_2 = \frac{0.01 \times 0.4 \times 48}{0.65 \times 4.375} = 0.0641 \text{ rads} \]

At A and E

\[ n = 0.35 \quad l = 4" \quad d = 4.57" \quad \theta_1 = \frac{0.01 \times 0.4 \times 4}{0.35 \times 4.57} = 0.01 \text{ rads} \]

These available rotations are greater than those required.

The stipulated collapse mechanism can therefore develop satisfactorily.
Practical Results. Arch 2L

Fig 14:40(a) shows the experimental set up. The load was applied to the arch centre by means of a jack acting on a 2" x 1" x 10" steel loading strip resting on the arch.

The strain gauge layout is shown in Fig 14:40(b).

The dial gauge arrangement was that of Fig 14:22.

Material Properties

Reinforcement

Average f_{yp} of 3 specimens = 43,000 psi (nominal)

Average U.T.S of 3 specimens = 57,700 psi

Concrete

Grading of sand and \( \frac{3}{8} - 3/16" \) gravel aggregate (% passing)

(Typical for all arches in series)

<table>
<thead>
<tr>
<th>Sieve Size</th>
<th>Sand</th>
<th>( \frac{3}{8} - 3/16&quot; ) Gravel</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{3}{8}&quot; )</td>
<td>100</td>
<td>86</td>
</tr>
<tr>
<td>3/16&quot;</td>
<td>100</td>
<td>4</td>
</tr>
<tr>
<td>No. 7</td>
<td>92</td>
<td>0</td>
</tr>
<tr>
<td>No. 14</td>
<td>80</td>
<td>0</td>
</tr>
<tr>
<td>No. 25</td>
<td>61</td>
<td>0</td>
</tr>
<tr>
<td>No. 52</td>
<td>16</td>
<td>0</td>
</tr>
<tr>
<td>No. 100</td>
<td>2</td>
<td>0</td>
</tr>
</tbody>
</table>

Both materials were supplied by Feltham Sand and Gravel Co. Ltd. from their works at Heston.
Slump 3/8" 6" Cube Strength.

<table>
<thead>
<tr>
<th>Age</th>
<th>Weight Kg.</th>
<th>Conda. at test</th>
<th>Ultimate Strength lb/sq.in.</th>
</tr>
</thead>
<tbody>
<tr>
<td>6 months</td>
<td>8.328</td>
<td>Wet</td>
<td>6,110</td>
</tr>
<tr>
<td></td>
<td>8.343</td>
<td></td>
<td>6,330</td>
</tr>
<tr>
<td></td>
<td>8.351</td>
<td></td>
<td>5,750</td>
</tr>
</tbody>
</table>

12" x 6" Cylinder Strength

At 6 months ave value = 4100 psi

Loading Log

Eleven load stages were employed, they were: 2, 4, 6, 7, 8, 9, 10, 14, 18, 22.5 and after failure 20.

<table>
<thead>
<tr>
<th>Load</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>From 0 to 7_T</td>
<td>No distress.</td>
</tr>
<tr>
<td>7.75_T</td>
<td>First hair crack under centre load.</td>
</tr>
<tr>
<td>8_T</td>
<td>Fine hair cracks under centre. One 2&quot; to right and other 2&quot; to left of Q.</td>
</tr>
<tr>
<td>10_T</td>
<td>Centre cracks growing. At centre 1/64&quot; wide.</td>
</tr>
<tr>
<td>18_T</td>
<td>Cracks appear top of arch at D and D'. Slight cracks appear bottom both abutments. Load falls off approximately 2 (in 365) divisions in twenty minutes. Centre crack within 1&quot; of top. Crack pattern 1.2 L. (Fig 14:43a).</td>
</tr>
<tr>
<td>19.75_T</td>
<td>Slight spalling at centre.</td>
</tr>
<tr>
<td>22.5_T</td>
<td>Cracks at abutments, D and D' spreading.</td>
</tr>
<tr>
<td>23_T</td>
<td>Failure with hinge D visibly forming.</td>
</tr>
<tr>
<td></td>
<td>Crack pattern 22 L (Fig 14:43(b))</td>
</tr>
<tr>
<td></td>
<td>Fig 14:40(a) shows the arch just after collapse.</td>
</tr>
</tbody>
</table>

Generally the apparatus behaved very well. The pull-back frame returned the abutments to their original position sensibly after each load application.
Deflection Curves

Fig 14:44 is a plot deflection of D.G.17.

Fig 14:45 is a plot of centre point vertical deflection load. This shows predicted and actual behaviour.

Fig 14:46 are plots of D.G's at points D and D' respectively. These indicated the growth of asymmetry during the loading.

Section n-H and n-e Curves

These are shown for the hinge points A B D & E in Figs 14:47, 14:48 and 14:49.

Strain at Hinge Sections

Figs 14:50, 14:51 and 14:52 show the measured strains, thrusts and moments at the hinge points A B D & E and for D' under a load of 22.5T. Fig 14:51(b) and Fig 14:50(b) show the strain variation as measured by the E.R.S.G's and metzger point lines respectively at E. As would be expected the E.R.S.G's show a higher N.A. being closer to the actual abutment.

The moments and thrust line at 22.5T as shown by the strain gauge lines around the arch is shown on Fig 14:55.

Variation of maximum compressive strain at hinge sections

This is shown graphically on Fig 14:53. At 22.5T it shows that only at B has the strain reached the assumed strain at collapse of 38%.
ARCH 2L: HINGE 'D'  
AFTER COLLAPSE  
FIG. 14:40(d)

ARCH 2L CENTRE HINGE  
AT COLLAPSE  
FIG. 14:40(e)

ARCH 2L L.H ABUTMENT HINGE AT COLLAPSE  
FIG. 14:40(f)
PREDICTED failure at 21.44
3rd hinges

ACTUAL load

PREDICTED failure mode

VERTICAL DEFLECTION: LOAD POINT B ~ W

(DG. G)

CENTRE LOAD TONS

DEFLECTION: INCH X 10^3

100 200 300 400 500 600

FIG 4.45
Comparison of deflections occurring at the 3rd hinge points D and D'.
Diagram showing strain distribution at hinge points under load. Figure 14.52

Variation of max. coup strain with load at hinge points. Figure 14.53

Scales as 14.51

Line A at 'A'

Line E at 'B'

Line G at 'D'

Line K at 'E'

H = 15,000 ft
W = 155,000 lbs.

Load in tons

12

22.5 ft

38 ft

50.5 ft

150 ft

Diagram showing strain distribution at hinge points under load. Figure 14.52

Variation of max. coup strain with load at hinge points. Figure 14.53

Scales as 14.51

Line A at 'A'

Line E at 'B'

Line G at 'D'

Line K at 'E'

H = 15,000 ft
W = 155,000 lbs.

Load in tons

12

22.5 ft

38 ft

50.5 ft

150 ft
Arch 6L.

Arch loaded at 2 - 1/3 points with equal loads.
Reinforcement as Fig 14:2. Actual variation in thickness as shown in Fig 14:70.

Material and Section Properties

\[ C_u = 6000 \text{ psi} \quad f'_c = 5300 \text{ psi} \quad E_c = 5.14 \times 10^6 \text{ psi} \quad f_{yp} = 45,000 \text{ psi} \]
\[ E_s = 30 \times 10^6 \text{ psi} \quad A_t = A'_t = 0.588 \text{ in}^2 \quad k_1 = 0.766 \quad k_2 = 0.55 \quad k_1k_2 = 0.42 \]
\[ m = 5.85 \]

Elastic B M D This is shown in Fig 14:58.

From this probable hinge points are at A B D & E, Fig 14:58. M\text{ULT} section characteristics for A B & D are shown in Figs 14:59 and 14:60.

Determination of Hinge Points and WSC

The arch is symmetrically loaded. To cause collapse without assymetry developing six hinges are required at A D B B' D' & E, Fig 14:58. Practically however collapse can occur with the formation of four hinges. As this represents the minimum collapse condition only this case will be considered.

Fig 14:63 shows the assumed mechanism at collapse.
Taking $M_B$ as datum moment $M_{OB} = 105,000$ lb ins. $M_{OD} = M_{OB}$.

$M_{OA}$ and $M_{OE} = 1.07$

\[
\begin{array}{ccc}
1 & 0 & 0 \\
0 & 1 & 0 \\
.33 & 0 & -1.778 \\
0 & (\frac{1-q}{2}) & -2(1-q^2) + 1 \\
\end{array}
\]

\[
W_{SC} = -\frac{2.07 \times 105000}{120 \times 2240} = 14.3
\]

Which gives $q = .67$ (which checks with elastic B M D) and $W_{SC} = 21.8$.

**Determination of $W_{AC}$**

The elastic relationships for A, B and D are:

$M_A = 2.84$ W ins  $H_A = 1.99$ W,  $M_B = 5.4$ W ins  $H_B = 1.69$ W,

$M_D = 1.7$ W ins  $H_D = 2.01$ W. These are shown plotted on Figs 14:59 and 14:60.

The expression for $W_{AC}$ reduces to:

\[
S_{WSC} = 0.137 \frac{\Delta M_A}{M_O} + 1.1 \frac{\Delta M_B}{M_O} + 1.778 \frac{\Delta M_D}{M_O} + 0.815 \frac{\Delta M_E}{M_O}
\]

$W_{AC}$ is found in tabular form below:
### TABLE 14:5

<table>
<thead>
<tr>
<th>Hinge</th>
<th>Thrust (lbs)</th>
<th>( \Delta m ) (lb ins)</th>
<th>( n )</th>
<th>( \frac{nM}{M_0} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>156,000</td>
<td>+ 80,000</td>
<td>.137</td>
<td>+ .105</td>
</tr>
<tr>
<td>B</td>
<td>132,500</td>
<td>- 225,000</td>
<td>.55</td>
<td>- 1.18</td>
</tr>
<tr>
<td>D</td>
<td>157,500</td>
<td>+ 55,000</td>
<td>1.778</td>
<td>+ .93</td>
</tr>
<tr>
<td>E</td>
<td>156,000</td>
<td>+ 80,000</td>
<td>.815</td>
<td>+ .625</td>
</tr>
</tbody>
</table>

\[
\frac{\delta w_{AC1}}{w_{AC1}} = .15
\]

continuing until

<table>
<thead>
<tr>
<th>Hinge</th>
<th>Thrust (lbs)</th>
<th>( \Delta m ) (lb ins)</th>
<th>( n )</th>
<th>( \frac{nM}{M_0} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>165,000</td>
<td>+ 35,000</td>
<td>.137</td>
<td>+ .05</td>
</tr>
<tr>
<td>B</td>
<td>140,000</td>
<td>- 102,000</td>
<td>.55</td>
<td>- 1.07</td>
</tr>
<tr>
<td>D</td>
<td>167,000</td>
<td>+ 45,000</td>
<td>1.778</td>
<td>+ .76</td>
</tr>
<tr>
<td>E</td>
<td>165,000</td>
<td>+ 35,000</td>
<td>.815</td>
<td>+ .27</td>
</tr>
</tbody>
</table>

\[
\frac{\delta w_{AC4}}{w_{AC4}} = .028
\]

i.e. \( w_{AC} = 83,000 \) lbs within 3\% which is close enough.

Note factor 'n' for B is halved to allow for the fact that hinge B acts effectively at B and B' i.e. total rotation at 'B' is assumed divided equally between B and B' and therefore excess moment at B' is half that occurring if hinge 'B' were in one location.

**Effect of Deformation on \( w_{AC} \)**

**Deflection Analysis**

Fig 14:64 shows the arch split into twenty segments for the deflection analysis.

Inspection of Figs 14:59 and 14:60 shows that hinges A and E are probably the second hinges to form.
For the deflection analysis it is assumed that section BB' Fig 14:64(a) descends vertically to the stage just prior to collapse and hence $\delta_{BB'} = 0$.

Assuming just prior to collapse, under $W_{AC} = 83000$ lbs, $M_A = 200,000$ lb ins and $M_B = 170,000$ lb ins, the deflection of the segment ends in terms of moment is set out in Table 14:6 below.

**TABLE 14:6**

<table>
<thead>
<tr>
<th>Segment No</th>
<th>$m_a$ (lb ins x $10^3$)</th>
<th>$m_b$ (lb ins x $10^3$)</th>
<th>$\frac{m_a + m_b}{2}$</th>
<th>$\frac{m_a + m_b}{4}$</th>
<th>$\sum^n_{o} \frac{m_a + m_b}{2}$</th>
<th>$\sum^n_{o} \frac{m_a + m_b}{4}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>+200</td>
<td>-11</td>
<td>+95</td>
<td>+48</td>
<td>+8</td>
<td>+16</td>
</tr>
<tr>
<td>2</td>
<td>-11</td>
<td>-205</td>
<td>-108</td>
<td>-54</td>
<td>-4</td>
<td>-8</td>
</tr>
<tr>
<td>3</td>
<td>-205</td>
<td>-264</td>
<td>-235</td>
<td>-113</td>
<td>-13</td>
<td>-131</td>
</tr>
<tr>
<td>4</td>
<td>-264</td>
<td>-320</td>
<td>-292</td>
<td>-146</td>
<td>-39</td>
<td>-394</td>
</tr>
<tr>
<td>5</td>
<td>-320</td>
<td>-133</td>
<td>-227</td>
<td>-114</td>
<td>-65</td>
<td>-654</td>
</tr>
<tr>
<td>6</td>
<td>-133</td>
<td>+50</td>
<td>-92</td>
<td>-46</td>
<td>-81</td>
<td>-813</td>
</tr>
<tr>
<td>7</td>
<td>-50</td>
<td>+17</td>
<td>-16</td>
<td>-8</td>
<td>-86</td>
<td>-867</td>
</tr>
</tbody>
</table>

Fig 14:64(b) shows the deflection diagram. The factor $\frac{q^2}{KE}$ in this case reduces to $0.24 \times 10^{-6}$ with $K = 0.25$ and $I_e = \text{effective I of } 140\text{in}^3$.

The deflection diagram gives

$$\delta_{BB'} = 0.21" \quad \delta_{DD'} = 0.21" \quad \delta_{HH'} = 0.12"$$

**Effect of deformation on $W_{SC}$**

Using $h_B$ and $h_D$ as the deformed heights in equation 14:3 the revised value of $W_{SC} = W_{SC_1} = 21.1_T$ i.e. % reduction of 3.2%.

$W_{AC_1} = 36_T$. 
Method I Section 10.

The M~H elastic curves give the first hinge at B.

Assuming hinges form at A and E before D the variation in M and H for the hinge points at A B & D from the elastic condition to collapse at W = 83000 lbs is shown on Fig 14:65.

Using the elastic M~H relationships hinge A forms at 230,000 lb ins, from Fig 14:60. This gives W = 81,000 lbs. At 80,000 lbs from Fig 14:65 M_A = 2.67 W ins and H_A = 2.6 W. Plotting these on Fig 14:60 gives M_A = 200,000 lb ins and W = 75,000 lbs. As the 2nd trial for W at formation of hinge A use W = 75,000 lbs. At this load from Fig 14:65 M_A = 2.69 W ins H_A = 2.48. On plotting on Fig 14:60 these give M_A = 205,000 lb ins and W = 76,000 lbs which is close enough to trial value of 75,000 lbs.

At W = 75,000 lbs Fig 14:65 gives M_D = 3.45 W ins and H_D = 2.52 W. On plotting this line on Fig 14:59 M_D at hinge formation at D = 155,000 lb ins with W = 45,200 lbs i.e. this indicates that hinge D forms first. Assuming hinge D forms first and still using Fig 14:65 relationships M_D using elastic relationships = 140,000 lb ins at hinge formation. This gives W = 82,500 lbs. As first trial use W = 60,000 lbs. At this load from Fig 14:65 M_D = 2.72 W ins and H_D = 2.32 W. These, on plotting on Fig 14:59 give M_D = 157,000 lb ins and W = 58,000 lbs which is near enough to the trial value of 60,000 lbs. Hence hinge D forms at 60,000 lbs.

At W = 60,000 lbs from Fig 14:65 M_A = 2.72 W ins and H_A = 2.15 W. Plotting these from the first hinge point the line cuts the M_ULT~H curve at M_A = 205,000 lb ins. This gives W at hinge A formation = 75,000 lbs. At this load M_A = 2.68 W ins and H_A = 2.48 W. Plotting this line from 2nd hinge formation gives M_A at failure = 202,000 lb ins.

∗∗ collapse load using this method = 75,500 lbs.
Collapse B M D

Under a load of $W_{ACD_1} = 75,500$ lbs the B M at D at collapse is > 160,000 lb ins which from inspection of Fig 14:59 is about the BM taken at D at collapse.

Reducing W to 70,000 lbs gives the collapse B M D shown in Fig 14:66 and shows that this load and the collapse mechanism stipulated give admissible collapse conditions.

Rotations required for collapse

\[ \theta_1 = \frac{2.5}{20} = 0.125 \text{ rads} \]
\[ \theta_4 = \frac{2.5}{80} = 0.032 \text{ rads} \]
\[-\theta_1 \times 40 + \theta_2 \times 20 = 0.25 \]
\[ \text{i.e. } \theta_2 = \frac{0.75}{20} = 0.0375 \text{ rads} \]
\[ \theta_1 - \theta_2 + \theta_3 - \theta_4 = 0. \]
\[ \text{i.e. } \theta_3 = 0.0272 \text{ rads.} \]

[Diagram showing deformation]
Rotations available for collapse

Sufficient binding has been placed at all hinges to permit $\varepsilon_p$ to be taken as 0.01.

At A, ($\Theta_1$) Available rotation $= \frac{2 \times 0.01}{0.4 \times 4.5} = 0.0112$ rads

At D ($\Theta_2$) Available rotation $= \frac{26 \times 0.01}{0.95 \times 4.25} = 0.0645$ rads

At B ($\Theta_3$) Available rotation $= \frac{16 \times 0.01}{0.4 \times 4.25} = 0.0945$ rads

The available rotation at A is slightly under the value required for collapse but is close enough to it for it to be assumed that the required hinge rotation will in fact develop.

The final collapse load and mechanism thus fulfill the 4 basic criteria for correct collapse design.
Arch 6L - Practical Results

Fig 14:71 shows the experimental set up. The load was transferred to the arch by means of two mild steel platforms the underside of which were serrated, see Fig 14:71(a). These platforms were set in a neat grout, about ¾" thick, on the top surface of the arch. This surface had previously been roughened up with a hammer and chisel. The behaviour of the platforms and grout during test up to arch collapse was very good. They were used on arches 4L to 6L and in no case did the platform dissociate itself from the grout right through the test.

The strain gauge layout is shown in Fig 14:70. The dial gauge layout was that of Fig 14:22 plus a gauge added to measure the vertical movement of the underside centre of the tie beam.

Material Properties

Reinforcement

Average f_y of 3 specimens = 45,000 psi (nominal)
Average U.T.S of 3 specimens = 59,200 psi

Concrete

Mix used:-

1 cwt O.P. cement
250 lbs damp sand (4% moisture content)
310 lbs ⅛ = 3/16 (5% moisture content)
⅜ gallons added water
Estimated W/C ratio 0.56

Workability:-

Slump ¾ inch
Compacting Factor $\frac{23\frac{3}{4}}{20\frac{3}{4}} = 0.83$
<table>
<thead>
<tr>
<th>Weight (gm)</th>
<th>Condition</th>
<th>Crushing strength lb/sq.in.</th>
</tr>
</thead>
<tbody>
<tr>
<td>8327</td>
<td>Wet</td>
<td>5,960</td>
</tr>
<tr>
<td>8327</td>
<td>Wet</td>
<td>6,350</td>
</tr>
<tr>
<td>8263</td>
<td>Wet</td>
<td>5,610</td>
</tr>
</tbody>
</table>

**12" x 6" Cylinder Strength**

At arch test date ave. value = 4000 psi

**Loading Log**

The arch was subjected to 13 load stages spread over two days. During the first day elastic stages to 2 x 15 tons were applied. The loads were removed overnight, the next day a check load stage was taken and the test continued to arch collapse. The load stages were 2 x (3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 35 Failure 37 tons).

<table>
<thead>
<tr>
<th>Load Stage</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>5. 2 x 15\text{T}</td>
<td>Slight hair crack across top of arch at point D (Fig 14:63). Crack disappeared on load removal</td>
</tr>
<tr>
<td>5(a) 2 x 15\text{T}</td>
<td>Next day. Crack as above.</td>
</tr>
<tr>
<td>6. 2 x 18\text{T}</td>
<td>Cracks appear on L.H and R.H abutments, under both 1d pts and on top of arch at 4th hinge point D.</td>
</tr>
<tr>
<td>7. 2 x 21\text{T}</td>
<td>R.H abutment crack spread to near bottom of tie beam. See crack record 611 Fig 14:63.</td>
</tr>
<tr>
<td>8. 2 x 24\text{T}</td>
<td>D.Gauge 13 removed due to approach of underside tie beam and bottom 12 x 8 I frame member.</td>
</tr>
<tr>
<td>9. 2 x 27\text{T}</td>
<td>Cracking approximately the same. Largest crack at R.H abutment which at widest was 1/64&quot;.</td>
</tr>
</tbody>
</table>
Load Stage (continued) | Remarks (continued)
---|---
11. 2 x 33_T | Cracks widen. Both load jacks dropped approximately 1/70 of load during 3/4 hour. Load maintained by pumping. Crack pattern 6L2 Fig 14:68.
12. 2 x 35_T | On applying load a great deal of creaking emanated from the arch. More pumping was required than used previously to steady the loads. Heavy flaking under L.H load pt. Slight flaking R.H. load pt. Cracking to D increased very slightly. Crack pattern 6L3 Fig 14:69.
13. 2 x 37_T | On attempting to apply a steady higher load the arch failed at 37_T with a 'shear-type' failure at L.H load pt.

Generally the apparatus behaved well. The centre pull-back frame returned the abutments to sensibly their original position after each load application.

Deflection Curves

Fig 14:72 shows four plots of dial gauge readings against loads. The lots of DG's 4 & 8 illustrate the almost identical load-deflection curves of the 2 load points. Also shown is the predicted behaviour.

The plots of actual deflections at D and D' (DG's numbers 10 & 2 respectively) show the development of assymetry in the later loading stages.

Fig 14:76 shows the calculated deflections just prior to collapse compared with those actual occurring at W = 35_T. The
actual deflected shape deviates considerably from that predicted. This is probably due to the high axial force causing rib shortening and hence negating the upward bending deflections at D and D'. The actual shape shows that the assumption that section EB moves down vertically is reasonable.

Strain at Hinge Sections.

Figs 14:73 and 14:74 show the variation of strain at the hinge points under $W = 35 \text{T}$. From these and using Figs 14:61 and 14:62 are deduced the moments and thrusts acting at these sections, these are shown on Fig 14:75. These show some agreement with those assumed to occur prior to collapse in the analytical work with the exception of the moment at A. At A the thrust as given by the strain gauges is approximately 5 times that at E. This would result in an unbalanced horizontal end thrust of about $8 \text{T}$. This would not be sufficient to move the arch sideways. It is unlikely that the strain readings at A were incorrect as they show a near straight line strain variation. This strain distribution probably indicates that the potential hinge at A has already changed sign from the elastic condition to that necessary to form a collapse mechanism at A B D & E, Fig 14:75.

Fig 14:73(a) compares the strain distribution as given by the metzger point line L and 2 E.R.S.G. lines at load point B.

Fig 14:77 shows the variation of maximum measured strain with load for the hinge points. This shows that up to near failure these strains were nearly equal again showing the symmetrical arch behaviour up to this point.
FIG 14.64(a)

DEFLECTION DIAG. SCALE:
1" = 2 \times 10^4 \text{ in units}
Assumed Arch Shape & Moment Distribution Just Prior To Collapse

$M_B = 170,000$ lbs

$M_D = 220,000$ lbs

$H_D = 1350$ lbs
Diagrams showing variation of maximum measured strains at critical points, Arch Cl. Fig 1477.
Arch 5L

Arch loaded at ¼ and 1/10 points. Load at 1/10 point
4 x that at ¼ point.

Reinforcement as Fig 14:12. Actual variation in arch
thickness shown in Fig 14:93.

Material & Section Properties

\[ C_u = 6633 \text{ psi} \quad f' = 5100 \text{ psi} \quad E_c = 5.47 \times 10^6 \text{ psi} \quad f_{yp} = 46,000 \text{ psi} \]
\[ E_s = 30 \times 10^6 \text{ psi} \quad A_t = A_t' = .538 \text{ in}^2 \quad k_1 = .758 \quad k_2 = .55 \]
\[ m = 5.43 \]

Elastic B M D  This is shown in Fig 14:78. From this probable
hinge points are at A, B, B', D & E. \( M_{UL} \sim H \) section character-
istics for these points are shown in Figs 14:79, 14:80 and 14:81.
The elastic \( M \sim H \) structural relationships for sections A B B' D
and E are:

\[ M_A = 32.2 \text{ W ins} \quad M_B = 13.05 \text{ W ins} \quad M_D = 5.8 \text{ W ins} \]
\[ H_A = 3.92 \text{ W} \quad H_B = 1.075 \text{ W} \quad H_D = 1.216 \text{ W} \]
\[ M_E = 8.48 \text{ W ins} \quad M_{B'} = 12.93 \text{ W ins} \]
\[ H_E = 1.194 \text{ W} \quad H_{B'} = 1.452 \text{ W} \]

These are shown plotted on Figs 14:79, 14:80 and 14:81.

Determination of Hinge Points and \( W_{SC} \).

From an inspection of the elastic \( M \sim H \) structural lines of
B and B' and where they cut their respective \( M_{UL} \sim H \) curves it
appears that the four hinges for the minimum collapse case will
form at A B D & E, Fig 14:78.

The determination of \( W_{SC} \) with these four hinge points
follows:

Taking \( M_{OB} \) as datum moment = 105,000 lb ins. \( M_{OA} = M_{CE} = 1.05 M_{OB} \)
\( M_{OD} = M_{OB} \).
The expression for \( W_{SC} \) reduces to:

\[
W_{SC} = -\frac{2.05 \times 10^5}{120 \times 2240 \times .6} \frac{1}{1 - q^2} 0 - 1.33(1 - q^2) + 1 \]

which gives \( q = .23 \) (this checks with elastic EMD) and \( W_{SC} = 3.48 \).

**Determination of \( W_{AC} \)**

Using the numerator of equation 14:4, the expression to determine \( W_{AC} \) reduces to:

\[
\frac{S_{W_{SC}}}{W_{SC}} = -\frac{.559 \Delta M_A}{M_A} + 1.226 \frac{\Delta M_B}{M_B} + \frac{\Delta M_D}{M_D} + .33 \frac{\Delta M_E}{M_E}
\]

\[
\frac{n \Delta m}{M_0} = \frac{3.162}{3.162}
\]

\( W_{AC} \) is found as set out in the table below:

**TABLE 14:7**

<table>
<thead>
<tr>
<th>Hinge</th>
<th>Thrust (lbs)</th>
<th>( \Delta m ) (lb ins)</th>
<th>( n \Delta m )</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>35,300</td>
<td>-47,000</td>
<td>.559</td>
</tr>
<tr>
<td>B</td>
<td>9,800</td>
<td>+21,000</td>
<td>1.226</td>
</tr>
<tr>
<td>D</td>
<td>11,000</td>
<td>+20,000</td>
<td>.333</td>
</tr>
<tr>
<td>E</td>
<td>10,750</td>
<td>+19,000</td>
<td>.060</td>
</tr>
</tbody>
</table>

\[
\frac{S_{W_{AC}}}{W_{AC}} = \sum n \Delta m + .253
\]

\[
\frac{n \Delta m}{M_0} = + .08
\]
Table 14:7 (Continued).

<table>
<thead>
<tr>
<th>$W_{AC_2}$ = 9,700 lbs</th>
<th>Hinge</th>
<th>Thrust (lbs)</th>
<th>$\Delta m$ (lb ins)</th>
<th>$n$</th>
<th>$n \Delta m$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>33,000</td>
<td>-125,000</td>
<td>.559</td>
<td>-.666</td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>10,450</td>
<td>+15,000</td>
<td>1.226</td>
<td>+.13</td>
<td></td>
</tr>
<tr>
<td>D</td>
<td>11,800</td>
<td>+21,000</td>
<td>1</td>
<td>+.2</td>
<td></td>
</tr>
<tr>
<td>E</td>
<td>11,600</td>
<td>+20,000</td>
<td>.333</td>
<td>+.063</td>
<td></td>
</tr>
</tbody>
</table>

$\frac{\varepsilon_{W_{AC_2}}}{W_{AC_2}} = -0.075$

i.e. $W_{AC}$ lies between 9000 and 9700 lbs

<table>
<thead>
<tr>
<th>$W_{AC_3}$ = 9350 lbs</th>
<th>Hinge</th>
<th>Thrust (lbs)</th>
<th>$\Delta m$ (lb ins)</th>
<th>$n$</th>
<th>$n \Delta m$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>36,600</td>
<td>-90,000</td>
<td>.559</td>
<td>-.47</td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>10,050</td>
<td>+20,000</td>
<td>1.226</td>
<td>+.234</td>
<td></td>
</tr>
<tr>
<td>D</td>
<td>11,350</td>
<td>+20,500</td>
<td>1</td>
<td>+.198</td>
<td></td>
</tr>
<tr>
<td>E</td>
<td>11,150</td>
<td>+20,000</td>
<td>.333</td>
<td>+.063</td>
<td></td>
</tr>
</tbody>
</table>

$\frac{\varepsilon_{W_{AC_3}}}{W_{AC_3}} = +0.0064$

i.e. $W_{AC} = 9350$ lbs within 1%

Effect of Deformation on $W_{AC}$

Fig 14:83 shows the arch split into twenty-two segments for the deflection analysis with reasonable ultimate moments at A B & E assessed from Figs 14:79 and 14:80.

Table 14:8 gives the deflection of the segment ends in terms of moment.
### Table 14:8

<table>
<thead>
<tr>
<th>Segment</th>
<th>$m_a$ lb ins x 10^3</th>
<th>$m_b$ lb ins x 10^3</th>
<th>$m_a + m_b$ lb ins x 10^3</th>
<th>$m_a + m_b$ lb ins x 10^3</th>
<th>$m_a + m_b$ lb ins x 10^3</th>
<th>$m_a + m_b$ lb ins x 10^3</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>+ 150</td>
<td>+ 93</td>
<td>+ 122</td>
<td>+ 61</td>
<td>+ 61</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>+ 93</td>
<td>+ 52</td>
<td>+ 73</td>
<td>+ 37</td>
<td>+ 159</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>+ 52</td>
<td>+ 14</td>
<td>+ 33</td>
<td>+ 17</td>
<td>+ 212</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>+ 14</td>
<td>- 25</td>
<td>- 6</td>
<td>- 3</td>
<td>+ 225</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>- 25</td>
<td>- 62</td>
<td>- 44</td>
<td>- 22</td>
<td>+ 200</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>- 62</td>
<td>- 79</td>
<td>- 71</td>
<td>- 36</td>
<td>+ 142</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>- 79</td>
<td>- 94</td>
<td>- 87</td>
<td>- 44</td>
<td>+ 63</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>- 94</td>
<td>- 107</td>
<td>- 101</td>
<td>- 52</td>
<td>- 32</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>- 107</td>
<td>- 106</td>
<td>- 107</td>
<td>- 104</td>
<td>- 185</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>- 106</td>
<td>- 97</td>
<td>- 101</td>
<td>- 52</td>
<td>- 230</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>- 97</td>
<td>- 65</td>
<td>- 81</td>
<td>- 41</td>
<td>- 320</td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>- 58</td>
<td>- 26</td>
<td>- 42</td>
<td>- 21</td>
<td>- 443</td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>- 26</td>
<td>- 12</td>
<td>- 19</td>
<td>- 10</td>
<td>- 474</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>- 12</td>
<td>+ 64</td>
<td>+ 26</td>
<td>+ 13</td>
<td>- 496</td>
<td></td>
</tr>
<tr>
<td>16</td>
<td>+ 64</td>
<td>+ 120</td>
<td>+ 92</td>
<td>+ 46</td>
<td>- 411</td>
<td></td>
</tr>
</tbody>
</table>

### Arch AB

<table>
<thead>
<tr>
<th>Segment</th>
<th>$m_a$ lb ins x 10^3</th>
<th>$m_b$ lb ins x 10^3</th>
<th>$m_a + m_b$ lb ins x 10^3</th>
<th>$m_a + m_b$ lb ins x 10^3</th>
<th>$m_a + m_b$ lb ins x 10^3</th>
<th>$m_a + m_b$ lb ins x 10^3</th>
</tr>
</thead>
<tbody>
<tr>
<td>21</td>
<td>- 175</td>
<td>- 40</td>
<td>- 108</td>
<td>- 54</td>
<td>- 54</td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>- 40</td>
<td>+ 119</td>
<td>+ 40</td>
<td>+ 20</td>
<td>+ 83</td>
<td></td>
</tr>
<tr>
<td>19</td>
<td>+ 119</td>
<td>+ 69</td>
<td>+ 94</td>
<td>+ 47</td>
<td>- 21</td>
<td></td>
</tr>
<tr>
<td>18</td>
<td>+ 69</td>
<td>+ 152</td>
<td>+ 126</td>
<td>+ 63</td>
<td>+ 92</td>
<td></td>
</tr>
<tr>
<td>17</td>
<td>+ 152</td>
<td>+ 76</td>
<td>+ 114</td>
<td>+ 57</td>
<td>+ 212</td>
<td></td>
</tr>
<tr>
<td>16</td>
<td>+ 76</td>
<td>+ 120</td>
<td>+ 93</td>
<td>+ 49</td>
<td>+ 313</td>
<td></td>
</tr>
</tbody>
</table>

In this case factor $\frac{g^2}{K E \frac{1}{\ell}} = \frac{37.3 \times 10^{-6}}{25 \times 5.47 \times 140} = 1.94 \times 10^{-6}$
From Fig 14.83(b) and (c) deflections just prior to collapse are:

\[ \delta_y B = 0.153" \quad \delta_y D = 0.254" \]
\[ \delta_h B = 0.080" \quad \delta_h D = 0.034" \]

**Effect of Deformation on \( W_{AC} \)**

Using the deformed shape of the arch under \( W = 9350 \) lbs in equation 14.4, \( W_{SC1} = 3.4T \) i.e. \% reduction due to deflection is 2.3\%. Using ratio \( \frac{W_{SC1}}{W_{SC}} \) applied to \( W_{AC} = 9350 \) lbs, \( W_{ACD1} = 9150 \) lbs.

**Method 1 Section 10**

![Diagram](image)

Fig 14.84 above shows the assumed force distribution at collapse under \( W = 9350 \) lbs.

From Fig 14.79 the first hinge forms at A at \( W = 4500 \) lbs. Hence the arch elastic range ends at this load. Fig 14.85 shows the variation in M and H against W for A B D & E from \( W = 4500 \) lbs to 9350 lbs. \( W = 4500 \) lbs gives the first hinge points on Figs 14.79, 14.80 and 14.81. Inspection of Figs 14.79 and 14.80 shows that the 2nd hinge to form is at E. Following the elastic line for E to cut the \( M_{ULT} \) curve the former cuts the latter at \( M_E = 145,000 \) lb ins. This gives \( W = 17,100 \) lbs. Try \( W = 9,000 \) lbs. From Fig 14.85 at 9000
lbs $M_E = 15.2 \ W \ \text{ins}, \ H_E = 1.68 \ W$. Plotting this line on Fig 14:79 gives $M_E = 140,000 \ \text{lb ins}$ and $W = 9220 \ lbs$. As 2nd trial use $W = 9100 \ lbs$. Fig 14:85 gives $M_E = 15.5 \ W \ \text{ins}$ and $H_E = 1.7 \ W$. Plotting this line on Fig 14:79 gives $M_E = 140,000 \ \text{lb ins}$ and $W = 90,500 \ lbs$ which is near enough to the 2nd trial load.

At $W = 9050 \ lbs$

$M_B = 13.33 \ W \ \text{ins} \quad M_D = 11.2 \ W \ \text{ins} \quad M_A = 19.5 \ W \ \text{ins}$

$H_B = 1.62 \ W \quad H_D = 1.92 \ W \quad H_A = 4.41 \ W$

At 9050 lbs

$H_B = 14650 \ lbs \quad H_D = 17,350 \ lbs \quad H_A = 39,900 \ lbs$

On plotting these lines and points on Fig 14:79 and 14:80 it is seen that a hinge forms at $B$ just prior to $E$. For collapse therefore only the hinge at $D$ is now required to form.

Continuing the 1st hinge line at $D$ to cut the $M_{ULF-H}$ curve $M_D$, at this point, $= 150,000 \ \text{lb ins}$ and $W = 13,380 \ lbs$. As 1st trial assume $W = 11,000 \ lbs$. Extending Fig 14:85 $M_D$ and $H_D$ lines as shown, at $W = 11,000 \ lbs \quad M_D = 13.8 \ W \ \text{ins}$ and $H_D = 2.2 \ W$. Plotting this line to cut $M_{ULF-H}$ curve gives $M_D = 152,500 \ \text{lb ins}$ and $W = 11,000 \ lbs$ at this point. Thus the collapse load $= 11,000 \ lbs$. One reason for the large discrepancy between $W_{AC} = 9350 \ lbs$ and this collapse load is due to the fact that the moment at $D$ Fig 14:84 is 110,000 lb ins which is approximately 40,000 lb ins below the actual failure moment at $D$. This fact affected the graph of $M_D - W$ Fig 14:85 giving the 2nd hinge stage for $D$ a smaller slope than it should have had and hence a higher $M_D$ at collapse and correspondingly higher collapse load. Another reason for this discrepancy may lie in the fact that the present collapse mode is incorrect. This can be checked by finding the moment at $B'$ at collapse.
Collapse B M D.

Under $W = 11,000$ lbs and the present collapse mode the moment at $B'$ is $188,000$ lb ins. This moment is $> \text{than the M}_{\text{ULT}}$ available at $B'$ under these conditions, see Fig 14:81. Hence the present collapse mode appears incorrect, a hinge occurring at $E'$ as well as $B$. Practically before collapse it is likely that hinges will form both at $B$ and $B'$. However from a design point of view the minimum collapse case basically has four hinges and an analysis will now be made assuming hinges at $A$ $B'$ $D$ & $E$. Fig 14:78.

Deflection Analysis

Fig 14:86 shows the arch about to collapse under $W_{AC} = 9350$ lbs with hinges at $A$ $B'$ & $E$. A reasonable value of $M_{\text{ULT}}$ at $B'$ of $150,000$ lb ins is assumed from Fig 14:81.

Fig 14:87(a) shows the arch broken into twenty segments for the analysis and Table 14:9 below gives the deflection of the segment ends in terms of moment.

**TABLE 14:9.**

<table>
<thead>
<tr>
<th>Arch Segment EB</th>
<th>$m_a + mb$</th>
<th>$m_a + mb$</th>
<th>$\frac{n \cdot m_a + mb}{4}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Segment No.</td>
<td>lb ins x 10$^3$</td>
<td>lb ins x 10$^3$</td>
<td>$\frac{m_a + mb}{2}$</td>
</tr>
<tr>
<td>1</td>
<td>-150</td>
<td>+87</td>
<td>+118</td>
</tr>
<tr>
<td>2</td>
<td>+87</td>
<td>+30</td>
<td>+59</td>
</tr>
<tr>
<td>3</td>
<td>+30</td>
<td>-19</td>
<td>+5</td>
</tr>
<tr>
<td>4</td>
<td>-19</td>
<td>-60</td>
<td>-40</td>
</tr>
</tbody>
</table>
TABLE

.
i
Segment
Ho

19 (continued)

-

\

®a
lb ins x 10-7

,fT ma 42m.b ' “a 4* ° b

mb
.
lb ins x 10

♦ V;

2
mm<m ®|!la * ^Hb

0

.
-...T".

5

- 60

- .100 .

-

6

- 100

- 125

7

• 125

- 40

4 129

- 113

~

57

4 32

- 139

— 132

—

66

-

139

— 146

- 143

-

72

- 22a

9

• 146

- 138

- 142

- 370

10

- 138

- 128

- 133

? ’ 71
- 6?

- 508

11

- 12$

- 114

— 121

-

61

~ 635

12

-114

-

58

-

86

*» 43

- 738

13

-

.

6

-

32

-

16

- 797

*

58

♦

26

4 13

- 803

84

4

42

- 745

8

—

58

14

I
|

do

90

15

♦

58

4 107

+

16

♦ 10?

♦ 122

4 115

4 58

- 645

17
IS

♦ 122

♦ 147
4 145

4 135

4

68

- 519

4 146

4 73

- 378

♦ 147

Arch Segment AS*
20

-175

-

39

- 107j

•

54

-

54

19

-

4 124

4 82j

4 41

*

66

39

-

Fig I4j87(b) shows the deflection diagram*
la

this ease the factor

•ff2

(with

K< * *25 end I0 a 14 ia 4)

The deflection diagram gives the following deflections under
W » 9350 lbs

la this case Method

S VB* * .09”

SyD » .64«

S HB* * .05”

S gB « ,14«

1 Section 10 will be omitted and the collapse

B.M.D. proceeded with.

>

•


Collapse B M D

With $W = 9350$ lbs on the deformed arch the moment at D = 163,000 lb ins, this exceeds that which the section may be expected to carry under these conditions, see Fig 14:79.

On reducing $W$ to 8750 lbs the collapse B M D Fig 14:89 is obtained. This shows that at this load an admissible collapse condition exists satisfying the mechanism equilibrium and yield conditions. It should be noted that the deflection diagram for this case shows a slight upward movement at B whilst the collapse B M D shows no hinge occurring at this point. Practically the deflection at B would be expected to be downwards. If this were so it would diminish the upward movement of D as given by Fig 14:87. Again as the elastic moments at B and B' are so close if a hinge forms at B' another would be expected at B.

From the point of view of the diminution of $W_{AC}$ due to deflection the hinge at B' collapse mode gives a lower estimate of the collapse load than the hinge at B mode and will be used to determine $W_{ACD}$.

On Fig 14:81 are shown the $M$ and $H$ against $W$ relationships for B'. As a check that $M_{B'}$ at collapse is approximately 150,000 lb ins, as assumed, the $M$-$H$ structural line for B' is drawn from the 1st hinge point. This cuts the $M_{ULT}$-$H$ curve at 145,000 lb ins hence the 150,000 lb ins assumption is reasonable.

Calculation of rotations required and available for collapse

Rotations required under $W = 8750$ lbs

![Diagram of rotations and angles](image-url)
From Fig 14:90

\[ \theta_4 = \frac{.09}{12} = .00756 \text{ rads} \]

\[ \theta_1 = \frac{.6}{43.2} = .0139 \text{ rads} \]

\[ -\theta_1 \times 90 + \theta_2 \times 46.8 = +.09 \]

\[ \theta_2 = .0286 \text{ rads} \]

\[ \sum_{n=0}^{n} \theta_n = 0 \quad \text{i.e.} \quad +.0139 - .0286 + \theta_3 - .0076 = 0. \]

\[ \text{i.e.} \quad \theta_3 = .0233 \text{ rads}. \]

**Rotations available.**

Sufficient binding is provided to enable \( s_p \) to be taken as .01.

Hinge A  Available rotation = \[ \frac{7.3 \times .4 \times .01}{.23 \times 4.5} = .0283 \text{ rads}. \]

Hinge B' Available rotation = \[ \frac{5.5 \times .4 \times .01}{.25 \times 4.375} = .201 \text{ rads}. \]

Hinge D Available rotation  By inspection satisfactory.

Hinge E Available rotation = \[ \frac{15 \times .4 \times .01}{.24 \times 4.25} = .0586 \text{ rads}. \]

The stirrups were placed all round arch hence any rotation required at B for collapse is available.

Hence required rotations are available.

The collapse mode with hinges at A B' D & E and \( W = 8750 \) lbs is thus an admissible collapse condition.
Fig 14.92 shows the experimental set up. The load was applied to the arch by hydraulic jacks through two loading platforms similar to those described for Arch 6L. In this case the platform at the 1/10 load point was braced, by two solid m.s. pins, with spherical ends, against a reinforced m.s. angle connected to the R.H. test frame 6” x 4½” I ties. These struts were to prevent the platform from sliding down the arch if the grout bed broke. They proved unnecessary, however, as the grout bed remained intact up to collapse.

Due to the limitations of the jacks available the 1/10 load point loading device was constructed so that the jack reacted against the u/s of the bottom 12” x 8” I test frame member. From here the load was transferred by a yoke, consisting of 4 - 1” Ø bars connecting 2 - 8” x 4” I top and bottom cross sections, to the top of a proving ring and then on to the arch.

One object of this test was to ascertain the hinge behaviour in the neighbourhood of two closely spaced loads on the arch. The ratio of W at ¼ pt and 4W at 1/10 pt was fixed to give approximately the same B M at these points.

The strain gauge layout is shown in Fig 14.93(a).

Material Properties

Reinforcement

Average fyp of 3 specimens 46,000 psi

Concrete

Aggregate Grading as Arch 2L

Workability Slump 4” Compacting Factor .81
6" cube strength

<table>
<thead>
<tr>
<th>Age (months)</th>
<th>Weight Kg</th>
<th>Condition at test</th>
<th>Ultimate Strength psi</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td>8392</td>
<td></td>
<td>6970</td>
</tr>
<tr>
<td></td>
<td>8382</td>
<td>Wet</td>
<td>6300</td>
</tr>
<tr>
<td></td>
<td>8352</td>
<td></td>
<td>6280</td>
</tr>
</tbody>
</table>

12" x 6" Cylinder Strength

At 9 months, stored dry, average value 4060 psi.

Loading Stages

Eleven were employed, these were \(w = \frac{1}{2}, \frac{1}{4}, 0.75, 1, 1.5, 2, 2.5, 3, 4, 4.1\) and after failure \(3.5T\). The test was carried out over two days.

Loading Log

<table>
<thead>
<tr>
<th>Load</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\frac{3}{4}) pt 1/10 pt</td>
<td>No rotation at ends, i.e. pull back jacks and frames not required. No cracks.</td>
</tr>
<tr>
<td>1.25T</td>
<td>Slight crack on top R.H. abutment, i.e., point A, also slight hair crack under 1/10 pt load.</td>
</tr>
<tr>
<td>2.5T 10T</td>
<td>Two cracks approximately 1/64&quot; wide at R.H. abut and 2&quot; away, approximately 1&quot; deep. Two hair cracks appear between load points, (6\frac{1}{2}&quot; and 9\frac{1}{2}&quot; from 1/10 pt towards (\frac{3}{4}) pt, on u/s of arch.</td>
</tr>
<tr>
<td>4T 16T</td>
<td>Cracks appeared at all hinge points. See crack pattern 5L1 Fig 14:94(a)</td>
</tr>
</tbody>
</table>
An attempt was made to reach 4.5 and 18\(\text{T}\). 

\(W = 4.1\text{T}\) was the highest steady load attained. Both proving rings showed very slow release of load. R.H. abutment rotated - could not be pulled back by R.H. pull back jack - was pulled back to near zero by operating L.H. pull down jack. Result L.H. abutment horizontal R.H. abutment slightly out of horizontal. Heavy flaking at R.H. abutment and 1/10 py. Light flaking L.H. abutment and 1/10 pt. Loose pieces from cracks under 1/4 and 1/10 pts. Crack pattern.

On attempting again to reach 4.5 and 18\(\text{T}\) the arch failed with flaking at L.H. abutment and D as shown on Fig 14:94(b). After collapse the arch was able to take 3.5 and 16\(\text{T}\) without undue increase in deflection.

Generally the apparatus behaved well. The pull back frames returned the abutments to sensibly their original positions except at \(W = 4.1\text{T}\). At this load stage readings were taken with the R.H. abutment rotated anticlockwise .008 rads. This rotation was considered to have insignificant effect on the arch behaviour.

**Deflection Curves**

Fig 14:95 shows plots of DD’s near point D and at B.

Fig 14:96 compares the predicted deflections just prior to collapse at \(W_{\text{AC}} = 9350\) lbs with those actually occurring at \(W = 9150\) lbs. It shows that with a hinge assumed to occur at B without B', or at B' without B the resulting analytical curve is considerably away from the actual curve particularly in the region of B and B'. From the point of view of ultimate load capacity the assumed collapse mode i.e. hinge a B' gives a higher value of \(S_{V,D}\) than actual and as the moment at D is the final determining factor for the collapse B.M.D. this deflection curve is satisfactory.
Section n=H and n=e curves

These are shown for hinge points A B B' D & E in Figs 14:97 and 14:98.

Strain at Hinge Sections

Fig 14:99 shows the strain distribution moments and thrusts at A B B' D & E at W = 4.1T.

Fig 14:100 shows the variation of the thrust line around the arch at W = 4.1T.

Fig 14:101 shows the variation of strain in the compression edge at the hinge points with load. Also shown are the points where the tension steel yielded. These indicated that plasticity started at E first followed by A then B', B and finally D at approximately 4.1T.
ARCH 5L
TEST SET-UP
FIG 14:92

ARCH 5L
COLLAPSE CONDITION
FIG 14:92(a)

ARCH 5L, CRACKING
AT B&B' NEAR CCLL
FIG 14:92(b)
5 MAY 58

ARCH 5L
HINGE POSITIONS
JUST PRIOR TO COLL
FIG 14: 93k

5 MAY 58

4TH HINGE POSITION
LOAD STAGE 11
ARCH 5 MAY 58

R.H. ABUTMENT
LOAD STAGE 11
ARCH 5 MAY 58
3'' Dia 2'' High Proving Cylinder

3/4 PT. LD. Horizontal Supp.
Device M.S. Pins Removed
FIG 14:93(d)
 ARCH SE

FIG 14:80

AXIAL TIELOST KN x 10^3
FIG 1485
SEGMENTS 1-2

ARCH SL HINGE AT B

ARCH SECTOR AB

ARCH SECTOR EB

DEFLECTION DIAGRAM

SCALE 1 = 1 x 10^-6 IN UNITS 13
Arch 4L

Arch loaded at ¼ pt.

Theoretical Analysis

This has been carried out through Chapter II. Summarising, 4 hinges required for collapse at A B D & E Fig 14:104, predicted collapse load 12.9 tons.

Practical Results.

Fig 14:102 shows the experimental set up. The load was applied by a jack, supported from the top test frame member. The load was then transferred through a 50T proving ring to a m.s. platform set in grout on the arch top surface as described for Arch 6L. Fig 14:102(a) shows the strain gauge layout.

Material Properties.

Reinforcement

Average f_y of 3 specimens = 45,000 psi
" UTS " 3 " = 62,000 psi

Concrete

Aggregate - as Arch 2L.

Workability Slump ½" Compacting Factor \( \frac{26}{28\%} = 0.92 \)

6" Cube Strength

<table>
<thead>
<tr>
<th>Weight (gm)</th>
<th>Condition at test</th>
<th>Crushing Strength P.S.L.</th>
</tr>
</thead>
<tbody>
<tr>
<td>8241</td>
<td>Wet</td>
<td>7050</td>
</tr>
<tr>
<td>8236</td>
<td>Wet</td>
<td>6690</td>
</tr>
<tr>
<td>8260</td>
<td>Wet</td>
<td>7140</td>
</tr>
</tbody>
</table>

Average strength = 6960 psi

Stored in water from casting to test.
**12" x 6" Cylinder Strength**

At 5½ months average value of 3 cylinders = 5380 psi.

**Loading Log**

The test was spread over two days. Thirteen load stages were employed, they were: - 2, 3.5, 5, 6.25, (re 5ₜ check load 2nd day), 7, 8, 9, 10, 11.3, 13, 13.6ₜ and after failure 13.6ₜ.

<table>
<thead>
<tr>
<th>Load (ₜ)</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 - 5ₜ</td>
<td>No cracks. No significant rotation of abutments on load application.</td>
</tr>
<tr>
<td>6.25ₜ</td>
<td>Three fine hair cracks under load point.</td>
</tr>
<tr>
<td>re 5ₜ</td>
<td>Deflections and some strains checked with previous 5ₜ load stage, found to be almost exactly the same.</td>
</tr>
<tr>
<td>7ₜ</td>
<td>Three cracks under load widen slightly. Slight hair crack to U/S arch at junction with L.H. abutment at E appears.</td>
</tr>
<tr>
<td>8ₜ</td>
<td>After 8ₜ load, load released to 2ₜ and all cracks closed up.</td>
</tr>
<tr>
<td>9ₜ</td>
<td>Barely perceptible rotation of abutments on load application. Crack pattern 4L1 Fig 14:103(a).</td>
</tr>
<tr>
<td>11.5ₜ</td>
<td>Cracks under load point deepening. Crack pattern 4L2 Fig 14:103(b)</td>
</tr>
<tr>
<td>13ₜ</td>
<td>New cracks appear top R.H. abutment section. Load jack was pumped up for some time to maintain a steady load. This increase deflections.</td>
</tr>
<tr>
<td>13.6ₜ</td>
<td>Crack pattern 4L3 Fig 14:103(c). On increasing the load in an attempt to reach 1₄ₜ the highest steady load attainable was 13.6ₜ without excessive deflection.</td>
</tr>
</tbody>
</table>
13.6\textsuperscript{T} load required from R.H. pull down jack after 13.6\textsuperscript{T} achieved deflections remained constant. Load dropped 2 divisions in 280 of the proving ring.

13.6 after Crack pattern 4L 4 Fig 14:103(d). Maximum load failure taken by arch by continued pumping was 14.5\textsuperscript{T} however this quickly dropped to a steady 13.6\textsuperscript{T} and final readings taken. After the reading for this load stage completed load fell to 13.1\textsuperscript{T} where it remained constant with no increase in deflections for ½ hour before load was released. On release of load the arch sprang back but not to its original shape. On re-application of load arch took 12.7\textsuperscript{T} with fast increasing deflections.

The apparatus behaved very well during the test.

**Deflection Curves**

Fig 14:104 shows plots of deflections at the load point and near D. The predicted behaviour is also shown.

Fig 14:105 compares the actual with predicted arch shapes just prior to collapse. Reasonable agreement is shown.

**Section n-H and n-e Curves**

These are shown for hinges B and D in Fig 13:9 and for A and E in Fig 14:106.

**Strain at Hinge Sections**

Figs 14:107 and 14:107(a) show the variation in strain with load at the hinge points A B D & E. These plots illustrate the validity of the assumption that plane sections remain plane even after plasticity has developed at the section.

Fig 14:108 shows plots of maximum comp. strains at the hinge points against load. Also shown are the points where the tension steel yielded, as shown by the strain measurements, at the hinge sections.
FIG 14:103 (CONTD)
Strain Variation at Hinge Points E, D (6-8 Load)
ARCH

ABUT. A

Scales vert full size

At 13 ft
N = 404
I = 68,000
E = 251,660 ksi
C = 37 in
QA = 0.076 rads

Plots of max comp strains

Strain Variation at A = Load Arch 4L

FIG 14:10a

FIG 14:10b
Arch 3L

Arch loaded at 2 - 1/3 points with equal loads. Reinforcement as Fig 14:2. Actual variation in thickness as shown in Fig 14:113.

Material and Section Properties

\[ C_u = 6910 \text{ psi} \quad f'_c = 5400 \text{ psi} \quad f_{yp} = 45,000 \text{ psi} \quad E_a = 30 \times 10^6 \]

\[ A_t = A_t' = 0.588 \text{ in}^2 \quad k_1 = 0.765 \quad k_2 = 0.55 \quad E_c = 5.52 \times 10^6 \text{ psi} \quad m = 5.44. \]

Elastic B M D  This is shown in Fig 14:58.

This loading case is the same as Arch 6L. From that analysis hinge points are probable at A B B' D D' & E, Fig 14:58.

The \( M_{ULT-H} \) characteristics for these points are shown in Figs 14:109 and 14:110.

As previously stated under a test load of \( W = 25T \) the L.H. abutment of this arch failed. At the same time a hinge formed, i.e. compression concrete had crushed, at B'. At this point the loads were released. The arch was subsequently tested with a new L.H. abutment and the existing hinge at B'. This test was continued until a hinge had formed at D' Fig 14:58, the loads were again removed and the arch re-tested with hinges at both B' and D'. Loading was then continued until arch collapse. In this analysis an attempt is made to evaluate the collapse load of the arch under 2 - equal 1/3rd point loads when (a) one hinge exists at B' and (b) when hinges exist at B' and D'. These are termed conditions, \( 3R_1 \) and \( 3R_2 \). The effects of deformation will be ignored and the \( W_{AC} \) found in each case will be called the collapse load.

Determination of Hinge Points and \( W_{SC} \)

As for Arch 6L the ratio of \( \frac{M_{oA}}{M_{oB}} = 1.07. \)

In this case \( M_{oB} = 122,000 \text{ lb ins.} \).
Adapting Equation 14:3 gives \( q = 0.67 \) and \( W_{SC} = 25.3T \).

**Determination of \( W_{AC} \)**

This will not be determined for the unbroken arch. The elastic relationships for \( A, B, \) and \( D \), are shown plotted on Figs 14:109 and 14:110 for the unbroken arch.

**Determination of \( W_{AC} \) for case 3R, i.e. hinge at \( B' \)**

On re-applying the load, due to the hinge at \( B' \), a different moment distribution obtained in the arch in the new 'elastic' range. For the purposes of determining the revised elastic \( M-H \) relationships at the hinge points the arch was split up into ten segments as shown in Fig 14:111(a). The arch segment around \( B' \) was given a weight of \( \frac{I}{3} \) to allow for the decrease in \( I \) at this point due to cracking and crushing. Using the \( \chi_{IK} \) method the revised arch elastic centre was found as were the moments and thrusts at the hinge sections.

These were:

\[
\begin{align*}
M_E &= 4.02 \text{ W ins} \\
H_E &= 2.17 \text{ W} \\
M_{E'} &= 2.72 \text{ W ins} \\
H_{E'} &= 2.17 \text{ W} \\
M_{D'} &= 2.18 \text{ W ins} \\
H_{D'} &= 2.2 \text{ W} \\
M_A &= 3.88 \text{ W ins} \\
H_A &= 2.17 \text{ W}
\end{align*}
\]

These are shown plotted on Fig 14:109 and 14:110.

Due to the crushing at \( B' \) which occurred under the first test, the \( M_{ULT} \sim H \) curve for this section under 3R, differed from that at the first test.

A probable \( M_{ULT} \sim H \) curve for \( B' \) is arrived at as follows: -

On Fig 14:109 is shown dotted the \( M_{ULT} \sim H \) curve for the reinforcement
Although the section has less ability to resist moment than previously with axial thrust applied the uncrushed concrete will still assist in carrying moment. Hence the curve for $3R_1$ lies between the 'steel' curve and the original $M^{ULT} \sim H$ curve for $B'$. It is assessed for $3R_1$ to lie between the two.

Using this curve and the revised 'elastic' $M \sim H$ relationship for the hinge section $W_{AC}$ for $3R_1$ can be found.

The revised expression $\frac{8_{W_{SC}}}{W_{SC}}$ for $W_{AC}$ from Arch 6L with $M_{OB} = 106,000 \text{ lb ins}$ is:

$$\frac{8_{W_{SC}}}{W_{SC}} = \frac{\Delta M_A + 1.1 \frac{\Delta M_B}{M_O} + 1.778 \frac{\Delta M_D}{M_O} + 0.315 \frac{\Delta M_E}{M_O}}{3.752}$$

$W_{ACR_1}$ is determined as set out in Table 14:10 below:

<table>
<thead>
<tr>
<th>Hinge</th>
<th>Thrust $\Delta m$</th>
<th>$\Delta m$ ins</th>
<th>$\frac{n}{M_O}$</th>
<th>$\frac{n\Delta M}{M_O}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$W_{AC_1} = 75,000 \text{ lbs}$</td>
<td>A 163,000 + 95,000</td>
<td>.137</td>
<td>.105</td>
<td></td>
</tr>
<tr>
<td></td>
<td>B' 163,000 - 190,000</td>
<td>1.1</td>
<td>- 1.72</td>
<td></td>
</tr>
<tr>
<td></td>
<td>D' 165,000 + 85,000</td>
<td>1.778</td>
<td>+ 1.24</td>
<td></td>
</tr>
<tr>
<td></td>
<td>E 163,000 + 75,000</td>
<td>.315</td>
<td>+ .5</td>
<td></td>
</tr>
</tbody>
</table>

$\sum + .125$

$$\frac{8_{W_{AC_1}}}{W_{AC_1}} = .033$$

i.e. $W_{AC}$ for $3R_1 = 75,000 \text{ lbs}$ within 4% using this method.

How near this value was to the actual collapse load under these conditions was not checked as $3R_1$ was stopped when a second hinge had formed at $D'$. 
Determination of $W_{AC}$ for 3R2

The ratio of the M.O.I for the arch segments assumed for this test with hinges at B' and D' are shown in Fig 14:111(b).

Re-applying the $E_{IK}$ method the revised 'elastic' $H\sim H$ structural relationship for hinge points A B' D' & E are:

\[
M_E = 8.61 \text{ W ins} \\
M_B' = 1.41 \text{ W ins} \\
M_D' = 1.0 \text{ W ins} \\
M_A = 9.74 \text{ W ins} \\
H_E = 2.32 \text{ W} \\
H_B' = 2.36 \text{ W} \\
H_D' = 2.4 \text{ W} \\
H_A = 2.32 \text{ W}
\]

These are shown plotted on Figs 14:109 and 14:110.

For test 3R2 both sections B' and D' have suffered crushing and cracking. For B' the $M_{ULT}$ curve is assumed further reduced as shown on Fig 14:109 whilst the curve assumed for D' is that previously assumed for B' for test 3R1.

The expression for $W_{AC}$ becomes:

\[
\frac{\delta W_{SC}}{W_{SC}} = \frac{0.137 \frac{\Delta M_A}{M_0} + 1.1 \frac{\Delta M_B}{M_0} + 1.778 \frac{\Delta M_D}{M_0} + 0.815 \frac{\Delta M_E}{M_0}}{3.257}
\]

$W_{AC}$ for 3R2 is obtained as set out below.

<table>
<thead>
<tr>
<th>Hinge</th>
<th>Thrust lb</th>
<th>$\Delta m$ lb ins</th>
<th>$\frac{n \Delta m}{M_0}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>81,500</td>
<td>-35,000</td>
<td>0.137</td>
</tr>
<tr>
<td>B</td>
<td>82,500</td>
<td>-30,000</td>
<td>1.1</td>
</tr>
<tr>
<td>D</td>
<td>84,000</td>
<td>+15,000</td>
<td>1.778</td>
</tr>
<tr>
<td>E</td>
<td>81,500</td>
<td>0</td>
<td>0.815</td>
</tr>
</tbody>
</table>

\[\frac{\delta W_{AC}}{W_{AC1}} = 0.028\]

\[\Sigma = 0.091\]

\[W_{AC} = 15.6 \text{ T within 3\%}\]
Arch 3L Practical Results

Fig 14:112 shows the experimental set up. The load was applied to the arch by jacks, as shown, through two knife edges joined by 2 - 4" x 3" I sections. The strain gauge layout is shown in Fig 14:113. The L.H. load measuring device was a 11" long piece of 3" schedule 40 pipe. To this were attached 3 paper backed E.R.S.G's along the cylinder in the centre at 60° to each other. The cylinder had previously been calibrated against the E.R.S.G reading in a testing m/c. The E.R.S.G's were connected in series.

Material Properties

Reinforcement

Average f yp of 3 specimens = 45 430 psi
Average U.T.S of 3 specimens = 56 450 psi

Concrete

Aggregate grading As for Arch 2L

Moisture content of sand by drying

<table>
<thead>
<tr>
<th>Description</th>
<th>Wt. (g)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wt. of tray</td>
<td>261</td>
</tr>
<tr>
<td>Wt. of tray + wet sand</td>
<td>376</td>
</tr>
<tr>
<td>Wt. of tray + dry sand</td>
<td>347</td>
</tr>
<tr>
<td>Wt. of water</td>
<td>29</td>
</tr>
</tbody>
</table>

Moisture content = $\frac{29}{586} = 5\%$

Moisture content $\%$ - 3/16" aggregate = 4.5% (approx).

Mix used

1 cwt. cement
250 lb. wet sand
310 lb. wet $\%$ - 3/16"
Vol. of C.F. cylinders

Wt. empty 7840 g.
Wt. filled with water 13360 g.
Wt. of water 5520 g.

Volume = \( \frac{5520}{16.336 \times 1728} = 0.195 \) cu.ft.

Workability

Slump = \( \frac{1}{2} \)"

Wt. of C.F. cylinder and slings = 16½ lb.
Wt. + uncompacted concrete = 41 lb. 24½ lb.
Wt. + compacted concrete = 45 lb. 28½ lb.

C.F. = \( \frac{24.5}{28.5} = 0.86 \)

6" Cube Strength

<table>
<thead>
<tr>
<th>Age</th>
<th>Weight gm</th>
<th>Condition at test</th>
<th>Crushing Strength psi</th>
</tr>
</thead>
<tbody>
<tr>
<td>17 months</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8374</td>
<td>Wet</td>
<td></td>
<td>6570</td>
</tr>
<tr>
<td>8438</td>
<td>&quot;</td>
<td></td>
<td>6390</td>
</tr>
<tr>
<td>8335</td>
<td>&quot;</td>
<td></td>
<td>7280</td>
</tr>
</tbody>
</table>

Average strength = 6913 psi

12" x 6" Cylinder Strength

At 17 months average value of 3 cylinders = 5600 psi.

Loading Log

First Test

Eight load stages were employed. On application of the eighth load the L.H. abutment failed. The load stages were: 2 x \( (2.5, 3, 5, 7, 11, 17, 20, 25) \)T.
Load | Remarks
--- | ---
0 - 11\text{T} | No distress shown on arch. Slight hair cracks on L.H. abutment.
17\text{T} | Slight hair cracks under L.H. and R.H. load points.
20\text{T} | Crack grows under R.H. load point. L.H. abutment cracking increased.
25\text{T} | L.H. abutment fails by cracking in many places. At R.H. load point crushing occurs, and tension crack 1/16" wide to within 1/8 inch of top. L.H. end of arch moves out approximately 1/4 in. Crack pattern for arch only as shown in 3L 1 Fig 14:114. Loads released on failure.

On conclusion of first test the arch was jacked from the u/s to remain in its correct position whilst the L.H. abutment was removed by a road concrete breaker. The L.H. abutment was then re-cast using 'Ciment Fondu' for rapid hardening. To prevent the re-occurrence, abutment failures 4 - 1/4" ø bars and yokes were fabricated and placed from abutment to abutment, as previously described.

The arch was then ready for re-test.

Re-test 1: 3R

**Loading Log**

Four loading stages were employed. These were 2 x (3, 12, 21, 24)\text{T}.

<table>
<thead>
<tr>
<th>Load</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>Crack pattern 3L 1 Fig 14:114.</td>
</tr>
<tr>
<td>3\text{T}</td>
<td>Cracks on R.H. abutment open slightly, 1/100&quot; wide.</td>
</tr>
<tr>
<td>12\text{T}</td>
<td>Crack appears through E.R.S.G. 33 on top of arch 2&quot; deep approximately 1/100&quot; wide. Increase of cracking at R.H. abutment.</td>
</tr>
</tbody>
</table>
**Load** | **Remarks**  
---|---  
$24_T$ | Point D' fails. Large shear type cracks between B' and D'. Crack pattern 3L 2 Fig 14:114. Loads released.  

---

**Re-Test 3P_2**  

Six load stages $2 \times (3, 6, 9, 13.5, 15, & 16)_T$.  

**Loading Log**  

| Load | Remarks  
---|---  
$3_T$ | R.H. load point $\frac{3}{4}''$ lower than left.  
$6_T$ | No appreciable new cracking.  
$13.5_T$ | Abutment rotation corrected by pulling up on 4 - $1\frac{1}{8}''$ Ø tie rods. This closed up abutment cracks to some extent.  
$16_T$ | Failure at E section adjacent to R.H. abutment L.H. side arch then became cantilever and abutment section A fails. Final crack pattern 3L 4 Fig. 14:114.  

After the abutment had failed on the first test it was decided to treat the arch as a means of observing hinge behaviour under the 1 hinge and 2 hinge test runs.  

Accordingly no detailed analysis was attempted of the structural behaviour under these two sets of conditions.  

The following are shown to illustrated how the arch did behave.
Deflection Curves

Fig 14:115 shows load-deflection curves for two points on the arch. These illustrate that a permanent set did not develop in the arch until 3R₁ was completed.

Fig 14:116 shows the deflected shape at the end of the first test, 3R₁ and 3R₂.

Strains

Fig 14:117 shows the change in strain at some points in the arch during the tests.
FIG 14:09

ARCH 3L
LD PTS B & B'
C PTS D & D'

M = H ORIGINAL SECT CURVE

M = H FOR B
1st TEST

M = H FOR B' B1

M = H FOR D = 3R1

M = H FOR D' = 3R1

ASSUMED M = H CURVE FOR B' FOR 3R1

ASSUMED M = H CURVE FOR B FOR 3R2

ASSUMED M = H CURVE FOR B' FOR 3R2

FOR REINFORCEMENT ONLY

AXIAL THRUST 105 x 10^3
Fig 14:110

M = H for A
3 R₁

M = H for E = 3 R₁

N = A for A = 3 R₁

N = A for E = 3 R₁

NOT CURVE
FOR SECTIONS ADE

Mₑ = 131,000 ° ins

AXIAL THRUST 6B + 10⁷
VARIATION IN $I$ ASSUMED FOR TEST 3 \( \text{A} \)

**FIG 4: III(a)**

VARIATION IN $I$ ASSUMED FOR TEST $3R_2$

**FIG 4: III(b)**
DEFLECTION = LOAD, ARCH BL

FIG. 14:115

DEFLECTION = LOAD, ARCH 3L

LOAD W
Tests 1, 2, 3

DG 1
1st Test

DG 2
1st Test

DG 3
1st Test

LU ARST FAILED 1st Test

WOT FOR 3R^2
ARCH SECTION 2 ROTATION CHARACTERISTICS

ARCSG LINE 8/11

HOR SCALE 1/2 FULL SIZE

NETZGER POINT LINES

MAX COMPR. STRAIN  

$\frac{\%}{\text{RAD} \times 10^{-2}}$

$\phi$  

$W$  

$H$  

$E$  

$F$  

$G$  

$H$  

$I$  

HINGE B CHARACTERISTICS AT $W = 35T$ ARCH GL

FIG 14.125
Plastic Hinges

Fig 14: 14 to 14: 26 show the variation of maximum compressive strain, $\varepsilon$, and rotation defined as $\frac{E}{nd}$ measured at the hinge points of Arch 2L to 6L, and photographs of some of the hinges.

From the curves the following points can be noted:

1. At abutments the hinge lengths were very short, the maximum strain usually dropping off rapidly within 1 to 3 inches from the abutment.

2. At load points the curves were generally peaky with a flat top of about 1 inch length indicating a plastic compression zone of this length. The load point hinges were about twice as long as those occurring at the abutments.

3. At hinge points of long B M segment e.g. hinge D Arch 4L the hinge spread itself out to provide the required rotation. This gave a lower maximum compressive strain than occurred at the abutments or under a load point. The strain was also fairly constant over the hinge length.

4. Crushing occurred generally over a 1 to 3 inch length. The exception to this occurred in Arch 3 where the hinges at B' and D were subjected to 2 & 1 respectively, loading cycles with the hinge already formed. Re-loading a formed hinge caused extensive crushing.

5. Fig 14: 118 shows the only instance in the test where the concrete cover has largely spalled off to leave the moment and rotation to be carried by the bound concrete. Again this is due to the cycle of load and unload which was followed on Arch 3L. Another cause of this cover spall-off was the advent of the shear crack on test 3R as shown in Fig 14: 119. Generally there was little spalling of concrete at any of the hinge points of the remaining
arches the full compression section remaining operative up to arch collapse.

6. From the measured maximum compressive strains the figure of .0038 used for the determination of the section $M_{ULP}$ seems reasonable. To provide the required rotation for collapse over the hinge lengths as indicated by the strain measurements the actual maximum compressive strains occurring in many cases must have been considerably greater than .0038.
Large Arch Ancillary Tests

Arch Section Tests

To determine the actual $M_0$ of sections of the arch and to ascertain the practical $M\sim \theta$ section curve two uncracked pieces of the arches were tested as simply supported beams. The two pieces selected were about 40 inches long and were removed from their arches when the latter were broken up after the arch test. They were tested in bending and shear with a central point load over an unsupported length of 30 inches. Fig 14: 125 shows the test set up.

Fig 14: 126 shows the details of Arch Section 1

Fig 14: 127 shows the details of Arch Section 2

Fig 14: 129 shows the load-deflection curve for Section 1

Fig 14: 130 shows the load-deflection curve for Section 2

Also shown on these figures are the $M_0$ values as given by the Hognestad theory. These curves show that the ideal elastic $M\sim \theta$ curve assumed theoretically is close to the practical behaviour.

The previous section on plastic hinges shows variation of $\theta$ and $\epsilon$ for Sections 1 and 2 at collapse.
ARCH SECTION 1
IN TEST MACHINE
FIG. 14:124

ARCH SECTION 1
AFTER FAILURE
FIG. 14:125
STRAIN GAUGE LAYOUT ARCH SECT 2  (b)

STRAIN GAUGE LAYOUT ARCH SECT 1  (a)

Actual failing Moment Sect 1 = 130,000 lbs
This compares with Hoestadt theory value M_o = 120,000 lbs
For Arch Sect 2, actual M_o = 140,000 lbs.
Theoretical M_o = 125,000 lbs.

FIG 14:128
Summary of Results, Large Arch Tests.

1. The hinge points as indicated by strain readings and cracking occurring practically were in all cases within 1 to 4 inches of those predicted theoretically. The long hinges, e.g. D Arch 4L were liable to exhibit their maximum compressive strains at a stirrup position and this fact may be in part responsible for the discrepancy between theory and practice at these hinges. Hinge points occurred at all load points and abutments as indicated theoretically.

2. The deflected shapes of the arches as calculated compared reasonably well with those occurring under test. This is not true of Arch 5L where the simple theory broke down due to its lack of recognition of plasticity just prior to collapse occurring at both load points. The predicted deflection in this case however gave a modified load on the safe side and hence the predicted deflected shape from the point of view of load-carrying capacity could be considered satisfactory. For Arch 6L the predicted and actual shapes are also at variance but again the predicted shape gives a safe collapse load prediction.

3. For those arches loaded so that moments at hinge points remained the same sign from very light load to collapse, i.e. Arch 4L & 5L the simple collapse load modified for the effects of axial force \( W_{AC} \) gave a very close estimate of the collapse load.

4. The simple theory broke down under the symmetrical loading arrangements and gave a value of \( W_{AC} \) considerably higher than the final predicted \( W_{ACD} \). However in most practical collapse cases the minimum value of the collapse load will be such that hinge points will retain the same moment sign throughout their loading history. Also for most practical cases \( W_{AC} \) will give a close enough approach to the arch collapse load for design purposes.
5. The effect of deflection on $W_{AC}$ for these arches was to lower its value by a few per cent, i.e. below 5.

6. The values adopted for $E$, $I$ and $K$ gave reasonably accurate results in the deflection analysis.

7. With the exception of Arch 3L the cracks occurring in the arch were not considered serious from a corrosion standpoint until a load of approximately 70% of the ultimate was reached.

8. The collapse loads of the arches were predicted to within 5% of the actual by the methods proposed.

9. From strain observations a considerable amount of moment re-distribution sufficient for mechanism formation occurred prior to collapse.
III. 15. Small Model Arch Tests

Background

The object of carrying out these tests was to supplement and broaden the experimental data gained from the larger model tests. Due to the time factor however it was not possible to test approximately thirty small arches of varying type and end conditions as planned, in fact six of a similar type were tested.

Model Material

Previous experimental workers have found difficulty, mainly due to shrinkage and the difficulty in obtaining the same relative particle size distribution in the model mix as found in practical mixes, in the use of concrete or cement mortar for model material. In view of this and after a study of the requisite material properties it was decided to attempt to develop a mixture of Araldite Resin 'B' and sand for use in the models.

Araldite 'B' is an epoxy resin material in hard lump form.

The sand used was ordinary river sand passing G.18. B.S sieve.

After some work with araldite and sand, as presently outlined, due to the difficulty experienced in obtaining proper compaction two arches were made of a 2.1 sand/cement mortar.

Araldite Models

As one aim of the tests was to correlate the small model and large model arch tests hence it was necessary to determine a suitable araldite B/sand mix to give crushing strengths around 5000 psi.

A rectangular m.s. beam mould 12" x ½" wide x 1" deep was available and this was used to cast beams to acquire the necessary
casting technique. These beams were then tested to find out the required material properties.

Two 12" x ½" x 1" unreinforced beams and four similar beams singly reinforced on the tension side with an ⅛" dia m.e. bar were tested. The method of fabrication using an araldite B and sand mix was as follows:

The araldite, hardener, and sand were carefully weighed out on laboratory scales. The ratio of sand to araldite B was varied between 4 and 10 to 1. Hardener equal to 30% of the araldite weight was used in the mixes. The mould was clean and greased with a silicone grease to prevent the mix from sticking to it. The reinforcement when used was wedged in position. The araldite, sand, beam mould and three similarly treated 2" x 1" brass cylinders, serving as cylinder moulds, were placed in an oven and heated to 120 - 140° C. The araldite was placed in a saucepan and at the oven temperature the araldite became liquid. The hardener was then stirred into the araldite which was then re-heated. When the hardener had dissolved the hot sand was then thoroughly stirred into the mixture and the new mixture re-heated. The moulds were then taken from the oven and the hot araldite/sand mixture 'punned' into place. The mixture hardened very quickly and it was always necessary to re-heat the half-filled mould and remaining mixture at least once to obtain good compaction. After the beam and three cylinder moulds were filled they were replaced in the oven and cured for twenty-four hours at 100° C. The heat was then switched off and they were allowed to cool down slowly to room temperature. After removal from the moulds the models were then available for immediate test, if required. In the case of the four arch specimens curing was effected by subjecting them to a temperature of 200° C for 1½ hours and then allowing them to cool slowly in the oven. Fig 15:1 shows the mould reinforcement and cylinders.

Araldite Beam Results

The araldite beams were tested in simple bending with a centre point load over a 9" span as shown in Fig 15:2.
Two unreinforced 4:1 sand/araldite mix beams were tested initially to get the 'feel' of the work.

The load-deflection curve for beam 1 is shown in Fig 15:3. From this, assuming the material acts in tension as well as compression, $E_{\text{elastic}} = 0.91 \times 10^6$ psi. Both beams were of a 10:1 sand/araldite mix.

For beam 1 average 1" x 2" cylinder strength was 2880 psi. Unreinforced beam 2 gave very similar results. The load was supplied by hanger and ordinary dead weights.

Four 12" x 1" x ½" beams reinforced with one ⅛" Ø m.s. bar were tested with a single centre point load. The sand/araldite mixtures were:- 4:1 5:1 7:1 and 10:1.

Load-deflection curves for these four beams are shown on Fig 15:4.

Summary of Results

Beam 3. 4:1 mix. ave. Cyl = 17,820 psi $E_{\text{AR}} = 2.4 \times 10^6$ psi (approx)
Beam 4. 5:1 " " = 17,400 psi $E_{\text{AR}} = 2.5 \times 10^6$ " "
Beam 5. 7:1 " " = 6,240 " " = 1.5 $\times 10^6$ " "
Beam 6. 10:1 " " = 4,375 " " = 1.0 $\times 10^6$ " "

To calculate $I$, a modulus ratio of 15 was assumed.

Model Arch Tests

The beam results were considered sufficiently encouraging to justify the manufacture of some model arches of reinforced sand/araldite mixtures. It was decided to use the 10:1 mix as this gave crushing strengths about those of the larger arch models. Practically it was difficult in any case to go above a 12:1 sand/araldite mixture as mixing became very difficult with insufficient liquid araldite to moisten the sand.
Arch Mould

This was of m.s. with removable sides and end blocks as shown in Fig 15:5. It was made 1/10 the size of the large model arch, of parabolic shape and of varying cross section so as to give the relationship \( I_x = I_c \sec \Theta \). The arch proper was extended in a straight length 2\( \frac{3}{4} \)" each end to allow a solid fixing in the end anchors to form the fixed ends.

Arch Reinforcement

As this arch was intended to be a scale model of the larger arches tested the same % reinforcement was provided. This was achieved by using for the main reinforcement three continuous BWG 18 m.s mild drawn (20 - 32 T/in\(^2\) U.T.S) wires top and bottom. These were placed symmetrically in the section .35 in between centres top and bottom. The 18 G. wire gave a cross-sectional area of .049 in\(^2\) against the area required of .0498 in\(^2\). BWG 20 m.s. mild drawn wire was used to form single stirrups placed at 0.3 in around the arch which represented approximately the same % volume of stirrups as used in the larger arch. Across the arch the main steel was spaced at \( \frac{3}{4} \)" c-c. Liberal hooks were provided from the arch proper right into the 2\( \frac{3}{4} \)" straight end pieces. Fig 15:6 shows the reinforcement cage, mould stirrup former and components of 10:1 sand/araldite mix.

Arch Fabrication

The reinforcement cage was made first. This was done by cutting the main wires to the correct lengths cleaning with emery cloth and tinning completely in a molten bath of solder. A suitable length of stirrup wire was taken cleaned and the stirrups formed on a 4 peg m.s. former. When the stirrups were completed they were tinned by immersion in the molten solder. The cage was then fabricated by soldering approx 70% of the joints between the main steel and stirrups. This was a tedious operation. In the case of the araldite arches the
mould was then cleaned, greased and the reinforcement inserted. The 10:1 sand/araldite mix was then prepared, as described previously for the beam tests. In this case the mould was also inserted in the oven whilst the araldite mix was prepared. The point of heating the mould as in the case of the beam mould was to make them heat centres and thus prevent the mix from going hard and lumpy whilst it was placed. As in the case of the beam specimens three 1" dia x 2" high test cylinders were cast at the same time. The arch mould was then filled with the hot sand/araldite mixture. It was difficult, due to the sticky nature of the substance and its tendency to harden into lumps within a few minutes of removal from the oven, to obtain good compaction. To obtain reasonable compaction it was necessary to reheat the mix and partially filled mould usually twice before the mould was filled. Rapid curing of the arch and cylinders was then effected by heating in the oven for 1½ hours at 200°C. After cooling to room temperature the arch and cylinders were then removed from the moulds and were ready for test.

Arch Fabrication using Cement/Sand Mix

Due to the difficulties experienced in compacting the sand/araldite mix it was decided to attempt to fabricate some arches of a neat cement/sand mix. A 1:2 cement/sand mix was used. To eliminate, as far as possible, shrinkage effects the mortar was 'knocked-up' in the following way: - the cement and sand were thoroughly mixed dry, just sufficient water was then added and mixed in to make the mix homogeneous, i.e. without small lumps. This mix was then allowed to stand for an hour occasionally turning to ensure that it did not take its first set. The remainder of the water was then added, in this case to give a w.c. ratio of about .5, and mixed in. The mix was then placed and compacted in the arch and three cylinder moulds. These were then covered with a wet cloth for three days before stripping the moulds. The cement used was rapid
hardening. The sand passed the 7 gauge BS sieve. For the quantity of mix used for one arch and three cylinders ¼ pint of water was added at the first mix and a further ¾ pints one hour later. The compaction obtained with the sand/cement mix was far better than with the sand araldite mix.

**Testing Apparatus**

This is shown in Fig 15:7. The hydraulic jack supplied the load to the arch by rods connected to m.s. yokes over the load points. It was a closed loop force system, the reaction being supplied from the underside of the support frame and transmitted to the jack top by means of the \( I_p \), force measuring, proving ring. To seat the loading yokes on the arches blobs of araldite strain gauge cement were used and filed to form horizontal platforms. This worked very well.

Deflection readings were taken usually at the load point and another point. The end blocks were adjusted to take the arch ends without any pre-straining. The slots in the end support blocks were made .015 in smaller in depth and breadth than the arch ends. The latter were made a close press fit by filing the excess material away.

**Testing Cylinder Moulds**

These were of brass tube with one split to facilitate cylinder removable. The split was closed for pouring by a 'jubilee' clip.

**Model Analysis**

Let suffixes 'm' and 'p' refer to model and prototype respectively. Then the relationship between the collapse loads of the model and prototype assuming similar conditions obtain in both throughout their loading histories is given by:

\[
W_p = W_m \frac{L_m}{L_p} \frac{f_p}{f_m} \frac{b_m d_p^2}{b_p d_m^2} \quad 15:1
\]

This expression has been used to compare the model and large arch results.
Practical Arch Results

Arch 1 10:1 sand/araldite mix. Badly compacted specimen. Centrally loaded to fail at 202 lbs. Tested seven days after manufacture.

Arch 2 10:1 sand/araldite mix. Reasonable but not good compaction. Centrally loaded to fail at $144_T$. Average Ccyl = 4250 psi. Average Ccyl Arch 2L = 5400 psi. Using Equation 15:1. Calculated collapse load of large arch = $17.8_T$. This compares with actual collapse load of approximately $23_T$. A plot of centre deflection against load and the hinge formation indicated by cracking are shown on Fig 15:8.

Arch 3 10:1 sand/araldite mix. Loaded at the 2 - 1/3 points to collapse at a total (i.e. 2W) load of $585_T$. Average Ccyl = 4560 psi, Average Ccyl Arch 6L = 6250 psi. Using Equation 15:1. Large arch collapse load = $60_T$. This compares with the estimated collapse load of $63_T$ using the large arch data and that realised of $74_T$ with the shear failure of Arch 6L. Fig 15:9 shows a load-deflection plot and the mode of failure.

Arch 4 2:1 sand/cement. Centrally loaded to fail at $174_T$. Average Ccyl = 6410 psi. Average Ccyl Arch 2L = 5400 psi. Using Equation 15:1 predicted large arch collapse load = $14.6_T$. This compares with the actual collapse load of approximately $23_T$. The arch failed suddenly with a shear type failure at the centre point. A load-deflection curve is shown in Fig 15:10. **N.B. This cylinder strength is suspect due to incorrect setting of testing machine.

Arch 5 2:1 sand/cement. Loaded at 2 - 1/3 points to fail at $588_T$ total load.
Average Ccyl = 5060 psi Average Ccyl Arch 6L = 6250 psi Using Equation 15:1 predicted large arch collapse load = $72.5_T$. Fig 15:11 shows a load-deflection curve.
Arch 6 10:1 sand/araldite mix. Good compaction. Centrally loaded to fail at .254 T. Average Ccyl = 6150 psi Average Ccyl Arch 2L = 5400 psi Predicted Arch 2L collapse load from Equation 15:1 = 22.4 Tons.
Fig 15:12 shows a load - deflection curve.

Conclusions

Generally the hinge positions on the arches corresponded to those calculated and occurring in the large arches. The load - deflection behaviour of the small arches was similar to that of the large. It is most likely that in the case of the araldite arches the weak sections at D' due to improper compaction was the main cause of the 4th hinge forming there and not at the abutments.

It is also probable that the end fixing and support blocks may have allowed a small but sufficient end rotation to transfer moment from the abutment to D' to cause hinge formation at D'.

For the arches where a reasonable degree of compaction was obtained the small arches show quite good correlation with the large ones.

The sand/araldite material exhibited a certain amount of tensile strength. No tests were carried out to determine this strength. The observed failures in the araldite arches were of a 'concrete' type.

Although the number of tests were small it is believed that both the sand/araldite or sand/cement mixes when used in model work of this kind will give a good indication of the failing mode and load of a prototype. Fig 15:15 shows the arches after test.
Small Arch Mould & Items for Fabrication of Arch

Fig 15:5

Small Arch Testing Apparatus

Fig 15:6

Small Arches After Test

Fig 15:13
FAILURE BEAM 3

FAILURE BEAM 5 - 308°

W [lbs]

Load-Carrying Capacity of Beams 3-6 [in]

Beam 6 (1x1 mix)

Beam 4 (9x1 mix)

Beam 5

Load-Deflection Curves for Araldite Beams

Center Deflection 1 in. = 1/3
PLAN

PUSH FIT REMOVABLE BLOCK: 1/16" x 1/4" x 1/4"

SIDE VIEW

CONTINUOUS 1/2" x 1/4" MS STRIP
VARYING WIDTH

CROSS-SECTION

SMALL MODEL ARCH MOULD

FIG 15:5
In his work on the ultimate load of r.e. redundant structures Jain tested thirty 2-pinned arches. His theory was concerned with the ultimate strength of a section and the arch tests were used primarily as a means to verify this.

The thirty arches were divided into ten types. Three similar specimens of each type were tested. Of these ten types five have been chosen for analysis using the methods proposed in this work. The results obtained by this analysis are then compared with the test results.

The following have been calculated for each arch type:

1. Collapse load, modified for axial thrust and deflection, i.e. \( W_{ACD} \)
2. Deflection at the hinges.
3. Required rotation at the hinges.
4. Available rotation at the hinges.

The following Table 16:1 shows those of Jain's arches analysed:

<table>
<thead>
<tr>
<th>Type</th>
<th>Cu psi</th>
<th>Reinforcement</th>
<th>( \text{MOMENT THRUST} )</th>
<th>Depth</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>( e ) at 1st hinge point</td>
<td>span</td>
</tr>
<tr>
<td>1</td>
<td>3000</td>
<td>( 4 \times \frac{3}{8} &quot; )</td>
<td>3 D</td>
<td>30</td>
</tr>
<tr>
<td>2</td>
<td>3000</td>
<td>( 4 \times \frac{3}{8} &quot; )</td>
<td>3 D</td>
<td>30</td>
</tr>
<tr>
<td>3</td>
<td>5500</td>
<td>( 4 \times \frac{3}{8} &quot; )</td>
<td>3 D</td>
<td>30</td>
</tr>
<tr>
<td>6</td>
<td>3000</td>
<td>( 4 \times 5/16 &quot; )</td>
<td>2 D</td>
<td>20</td>
</tr>
<tr>
<td>7</td>
<td>3000</td>
<td>( 4 \times \frac{3}{8} &quot; )</td>
<td>3 D</td>
<td>30</td>
</tr>
</tbody>
</table>

All the above arches were circular.

In the determination of the \( H \) section characteristics the test \( Cu \) has been converted to an equivalent cylinder strength by using Evans' curve Fig 14.25(a).
The maximum concrete stress has been assumed = \(0.85 C_{	ext{Cyl}}\). Throughout the analysis of these arches deformation refers to that caused by bending.

All the arches are such that secondary effects, e.g. lateral instability, the effects of axial force on deformation and hinge formation can be ignored.

In the following analyses three methods suggested in Chapter 2 Section 10 will be employed. For each analysis they will be referred to by title only and they are:

**Method 1**

This is Method 1 of Section 10, for the arches considered here there are only two hinges to form for collapse and the following procedure is adopted. The collapse load modified for the effect of axial force is applied to the arch. An estimate of the value of the first hinge moment at arch collapse is made. The force system on the deformed arch is determined and from it revised 1st stage \(M-H\) structural relationships for the two hinge points are obtained. These are then plotted on the \(M_{\text{ult}}-H\) characteristic curve and the results obtained compared with the original estimated hinge moment values at collapse. This procedure is repeated until the original estimated moments and collapse load are satisfactorily close to the final moments and collapse load obtained.

**Method 2**

This is Method 2 of Section 10. This utilises the change in distance between the thrust line and arch at the hinge points as loading proceeds. Due to the high initial slopes at the hinge points in Jain's arches this method is of little practical value in these cases.

**Method 3 (a)**

This corresponds to that method suggested in Section 10 where the ratio of \(W_{sc}\) with and without deflection is applied
to $W_{AC}$. In this method the value of the hinge moments in equations similar to 4:2 are taken as $M_o$.

Method 3 (b)

This is similar to 3(a) except that in this case the moments used are:— (1) for the undeflected arch where the hinge point $M\sim H$ elastic lines cut the $M_{ULF}\sim H$ sect. curve and (2) for the deflected arch the moments used are those estimated to occur at the hinge points in the arch at collapse, from Method 1 above.
Jain Arch Type I

Fig 16:1 shows the shape, reinforcement and loading condition of Type I.

Material and Section Properties.

- $C_u = 3200 \text{ psi}$
- $f''_c = 2500 \text{ psi}$
- $E_c = 3.84 \times 10^6 \text{ psi}$
- $E_t = 29.4 \times 10^6 \text{ psi}$
- $A_t = A_t' = 0.1 \text{ in}^2$
- $f_{yp} = 45000 \text{ psi}$
- $D = 4''$, $d = 3.375''$, $d' = 2.75''$
- $b = 4''$, $k_1 = 0.826$, $k_2 = 0.55$

Using the section properties given above, the section $M_{ULT} \sim H$ curve is obtained in Fig 16:3.

Elastic B.M.D.

Fig 16:2 shows the elastic B.M.D. for this loading condition.

Hinge Points

These are determined to be at B and D in Fig 16:2. Under elastic conditions, the relationships between $W$ and $H$ and $M$ at B & D are:

$$
B. \begin{cases}
M = 9.41W \text{ in} \\
H = 5.49W
\end{cases} \quad \text{--- 16:1}
$$

$$
D. \begin{cases}
M = 5.075W \text{ in} \\
H = 7.19W
\end{cases} \quad \text{--- 16:2}
$$

Fig 16:3 shows the elastic $M \sim H$ structural relationships for B and D.

$W_{sc}$, the simple collapse load for this case is given by:

$$
W_{sc} = \frac{-h_B - 1 \quad -h_B + 0.75}{-h_D + 1 \quad -h_D + 0.25} \quad \text{--- 16:2(a)}
$$

which on substitution reduces to 917 lbs.
The equation to determine the change in load carrying capacity due to axial force reduces to

\[
\frac{\delta W_{wc}}{W_{wc}} = \frac{\Delta M_B}{M_0} + \frac{\Delta M_D}{M_0}
\]

Equation 16:3 is set out in Tabular Form in Table 16:1 below. Use is made of Fig 16:3.

<table>
<thead>
<tr>
<th>Hinge</th>
<th>Thrust (lbs)</th>
<th>$\Delta M$ (lb ins)</th>
<th>$\frac{\Delta M}{M_0}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$W_{AC_1} = 1500$ lbs</td>
<td>B: 825</td>
<td>$\Delta M_B$: 1200</td>
<td>$\Delta M_D$: 1400</td>
</tr>
<tr>
<td></td>
<td>$\delta W_{AC_1} = \frac{\Delta M_{AC_1}}{M_0} = +0.096$</td>
<td>$\delta W_{AC_1} = 0.191$</td>
<td></td>
</tr>
<tr>
<td>$W_{AC_2} = 1640$ lbs</td>
<td>B: 925</td>
<td>$\Delta M_B$: 1200</td>
<td>$\Delta M_D$: 1600</td>
</tr>
<tr>
<td></td>
<td>$\delta W_{AC_2} = \frac{\Delta M_{AC_2}}{M_0} = +0.103$</td>
<td>$\delta W_{AC_2} = 0.205$</td>
<td></td>
</tr>
<tr>
<td>$W_{AC_3} = 1810$ lbs</td>
<td>B: 990</td>
<td>$\Delta M_B$: 400</td>
<td>$\Delta M_D$: 1850</td>
</tr>
<tr>
<td></td>
<td>$\delta W_{AC_3} = \frac{\Delta M_{AC_3}}{M_0} = +0.082$</td>
<td>$\delta W_{AC_3} = 0.164$</td>
<td></td>
</tr>
<tr>
<td>$W_{AC_4} = 1960$ lbs</td>
<td>B: 1080</td>
<td>$\Delta M_B$: 800</td>
<td>$\Delta M_D$: 2000</td>
</tr>
<tr>
<td></td>
<td>$\delta W_{AC_4} = \frac{\Delta M_{AC_4}}{M_0} = +0.044$</td>
<td>$\delta W_{AC_4} = 0.088$</td>
<td></td>
</tr>
<tr>
<td>$W_{AC_5} = 2060$ lbs</td>
<td>B: 1130</td>
<td>$\Delta M_B$: 2000</td>
<td>$\Delta M_D$: 2100</td>
</tr>
<tr>
<td></td>
<td>$\delta W_{AC_5} = \frac{\Delta M_{AC_5}}{M_0} = 0.003$</td>
<td>$\delta W_{AC_5} = 0.006$</td>
<td></td>
</tr>
</tbody>
</table>

**Note:** $W_{AC} = 2060$ lbs within 1%
Effect of Deformation on $W_{AC}$

Fig 16:4 (a) shows the arch split into 22 segments for the deflection analysis. Table 16:3 shows the calculation of the deflection of the segment ends in terms of unit load applied.

<table>
<thead>
<tr>
<th>Segment No</th>
<th>$m_a$</th>
<th>$m_b$</th>
<th>$m_a + m_b$</th>
<th>$\frac{m_a + m_b}{2}$</th>
<th>$\sum_{n=0}^{m} \frac{m_a + m_b}{2}$</th>
<th>$\frac{m_a + m_b}{2}$</th>
<th>$\frac{m_a + m_b}{2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>.31</td>
<td>.15</td>
<td>.08</td>
<td>.08</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>+.17</td>
<td>+.31</td>
<td>+1.6</td>
<td>+1.93</td>
<td>+4.63</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>+.31</td>
<td>+.31</td>
<td>+.64</td>
<td>+.31</td>
<td>+.31</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>+.31</td>
<td>+.31</td>
<td>+.64</td>
<td>+.31</td>
<td>+.31</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>+.31</td>
<td>+.31</td>
<td>+.64</td>
<td>+.31</td>
<td>+.31</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>+.31</td>
<td>+.31</td>
<td>+.64</td>
<td>+.31</td>
<td>+.31</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>+.31</td>
<td>+.31</td>
<td>+.64</td>
<td>+.31</td>
<td>+.31</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>+.31</td>
<td>+.31</td>
<td>+.64</td>
<td>+.31</td>
<td>+.31</td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>+.31</td>
<td>+.31</td>
<td>+.64</td>
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<td>+.31</td>
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<td>11</td>
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<td>+.31</td>
<td>+.64</td>
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<td>13</td>
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<td>+.64</td>
<td>+.31</td>
<td>+.31</td>
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<td>+.31</td>
<td>+.64</td>
<td>+.31</td>
<td>+.31</td>
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<tr>
<td>15</td>
<td>+.31</td>
<td>+.31</td>
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<td>+.31</td>
<td>+.31</td>
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</tr>
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<td>16</td>
<td>+.31</td>
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<td>+.64</td>
<td>+.31</td>
<td>+.31</td>
<td></td>
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</tr>
<tr>
<td>17</td>
<td>+.31</td>
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<td>+.64</td>
<td>+.31</td>
<td>+.31</td>
<td></td>
<td></td>
</tr>
<tr>
<td>18</td>
<td>+.31</td>
<td>+.31</td>
<td>+.64</td>
<td>+.31</td>
<td>+.31</td>
<td></td>
<td></td>
</tr>
<tr>
<td>19</td>
<td>+.31</td>
<td>+.31</td>
<td>+.64</td>
<td>+.31</td>
<td>+.31</td>
<td></td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>+.31</td>
<td>+.31</td>
<td>+.64</td>
<td>+.31</td>
<td>+.31</td>
<td></td>
<td></td>
</tr>
<tr>
<td>21</td>
<td>+.31</td>
<td>+.31</td>
<td>+.64</td>
<td>+.31</td>
<td>+.31</td>
<td></td>
<td></td>
</tr>
<tr>
<td>22</td>
<td>+.31</td>
<td>+.31</td>
<td>+.64</td>
<td>+.31</td>
<td>+.31</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

These deflections are plotted on Figs 16:4(b) and (c). The following deflections were obtained for points B and D:

\[ S_{vB} = 72 \text{ units} \quad S_{H^B} = 61 \text{ units} \]
\[ S_{vD} = 71 \text{ units} \quad S_{H^D} = 2 \text{ units} \]
DIAGRAM

scale: 1 cm = 10 lb in. units

FIG 16:4

\[ \delta_{VB} = 72 \text{ units} \]

\[ \delta_{OB} = 61 \text{ units} \]
To obtain the actual deflections these are multiplied by
\[
\frac{l^2}{Ks_c^2}.
\]
In this case assuming \( K = 0.25 \). This factor reduces to \( 1.64 \times 10^6 \).

Using \( W = 2060 \) lbs actual deflections just prior to collapse are:

\[
\begin{align*}
S_v^B &= .244'' \\
S_h^B &= .21'' \\
S_v^D &= .24'' \\
S_h^D &= .007''
\end{align*}
\]

These compare quite well with those obtained experimentally, see Fig 16.5, except for \( S_h^D \).

At failure Jain reported horizontal movements of the order of .4 in at B and D. This discrepancy is due to the fact that the deflection analysis assumes continuity at D when in fact at the 'near to collapse' condition, defined herein as collapse, some plasticity has occurred at D. This allows D to move sideways more freely than the analysis shows.

The errors in the horizontal movements are not significant as they are usually small compared with the horizontal distances involved. The error in the calculated and actual horizontal distances are therefore considered to have a negligible effect on the collapse load and are ignored.

The effect of deformation on \( W_{AC} \)

(a) Method 1

Fig 16.6 shows the deformed arch at collapse, under load \( W_{AC} = 2060 \) lbs, split into two component parts. From Fig 16.3 a reasonable value of \( M_B \) at collapse is seen to be 16000 lb ins.
From Fig 16:6 the value of $H$ at collapse = 1670 lbs and $M_D = 16150$ lb ins.

Under these conditions the 1st hinge stage relationships between $W$ and $H$ & $M$ at $D$ become

$$M = 7.85W \text{ in}$$
$$H = 0.85W$$  —— 16:4

and at $B$

$$M = 7.54W \text{ in}$$
$$H = 0.68W$$  —— 16:5

These new relationships are shown plotted on Fig 16:3. They give $M_D$ at collapse = 17000 lb and $W_{ACD} = 2170$ lbs. This revised collapse load is now used and the above procedure repeated.

Using 2170 lbs as the collapse load the distribution of forces on the arch is as shown in Fig 16:7.
From Fig 16:7 the value of $H$ at collapse = 1765 lbs and $M_D = 17100$ lb ins. The revised 1st hinge stage structural relationships for $B$ and $D$ are:

at $B$

$$
\begin{align*}
M &= 7.4 W \text{ ins} \\
H &= 0.69 W
\end{align*}
$$

--- 16:4(a)

at $D$

$$
\begin{align*}
M &= 7.9 W \text{ ins} \\
H &= 0.86 W
\end{align*}
$$

--- 16:5(a)

Plotting these relationships from the 1st hinge points on Fig 16:3 at collapse they give $M_D = 16900$ lb ins and $W_{ACD_1} = 2140$ lbs. These are considered close enough to the initial estimates of the 2nd trial for the collapse load $W_{ACD}$ to be defined as 2140 lbs.

**Collapse B.M.D.** As a final check Fig 16:8 shows the collapse B.M.D. and illustrates that the collapse load and mechanism are satisfactory. It also shows that hinge D may move in slightly towards the arch $A$ but not sufficiently to affect the collapse load.

(b) **Method 2**

This method utilises the increase in distance between the thrust line and the arch $A$ at the hinge points in the arch as loading proceeds. An analysis similar to that applied to find $W_{AC}$ is then carried out.

In this case after the formation of the first hinge:

at $B$

$$
\begin{align*}
\varepsilon_B' &= \varepsilon_{\text{elastic}}_B + \delta B \\
\text{TOTAL} &= 17.15 + 0.24 = 17.39 \text{ ins}
\end{align*}
$$

--- 16:6

at $D$

$$
\begin{align*}
\varepsilon_D' &= \varepsilon_{\text{elastic}}_D + \delta D \\
\text{TOTAL} &= 7.06 + 0.24 = 7.30 \text{ ins}
\end{align*}
$$

--- 16:7

Unless a very large scale is employed it is difficult to record on Fig 16:3 any significant change in the $M-H$ line of $B$. 

The same remarks, to a lesser degree, apply to hinge point D. When this occurs the modification in $W_{AC}$ to allow for the effects of deformation using this method is difficult to determine. In this case using Fig 16:3 $W_{ACD}$ would equal $W_{AC}$.

This is due to the initially high slopes of the $M-H$ curves for B and D.

**Method 3 (a)**

This method is the application of the ratio between the $W_{SC}$'s, found when deformation is and is not considered, to $W_{AC}$.

In this case $W_{SC1}$ when deformation is considered is equal to 890 lbs. This is equivalent to a 2.9% reduction in the original $W_{SC}$ of 917 lbs. Hence $W_{ACD}$ using this method = 2078 lbs.

**Method 3 (b)**

Replacing $W_o$ by the actual moments at B and D at collapse in equation 16:2(a) when $h_B$ and $h_D$ are those at no deflection at collapse a further $W_{SC}$ ratio can be obtained. Following this method with no deflection $W_{SC1} = 1100^w$ with deflection $W_{SC2} = 1070^w$ % reduction in $W_{ACD}$ due to deflection = 2.7%

Revised $W_{ACD}$ using this method = 2082 lbs.

Methods 2 and 3 are approximate guides to the effect of deformation on $W_{AC}$. The collapse load will therefore be defined in every case as that obtained by Method 1.

Fig 16:3 shows that the $M-H$ relationship for point B after a hinge has formed there slopes less than in the elastic range. Thus assuming, as is done so herein, that the hinge
moment capacity can be increased after failure by increasing $H$ and following the $M_{ULT} \sim H$ section characteristic, hinge $B$ moment capacity per unit increase in $H$ is greater in the '1st hinge range' than in the elastic range. Thus a higher moment occurs at $B$ at failure than that given by the point where the elastic line cuts the $M_{ULT} \sim H$ curve.

**Determination of Rotation required at $B$ for failure**

Fig 16:9 (a) and (b) show the arch at collapse split into two component parts for the $\delta_{IK}$ analysis.

Using $k = .25$ in the expression for $\theta_B$, similar to Equations 11:7 to 11:9,

$$-\theta_B = \frac{10^{-6}}{.25 \times 3.84 \times 23.75} (-89 \times 10^6) = -.0392 \text{ rads}.$$
Stirrups have been provided using the elastic theory to take the S,F. at the ultimate load. These are sufficient to allow the section to take a maximum compressive strain of

\[ \epsilon_p = 0.1 \]

B.M. segment length = 38". \( d = 3.375" \) \( n = 0.22 \)

Available rotation = \( \frac{4 \times 38 \times 0.01}{0.22 \times 3.375} \) = 0.208 rads

Hinge at B can develop satisfactorily.

As previously noted plasticity occurs at D before failure. In this case the binders are carried throughout the arch at the same pitch and therefore the rotation required at D for collapse can develop satisfactorily.

The average maximum load taken by the three specimens tested was 2544 lbs. This compares with the ultimate load by analysis of 2140 lbs. From Fig 16.5 it can be seen that the actual deflection at the load point at collapse is approximately three times that at the analytical collapse load. The actual deflections start to become large at about 2250 lbs.
Jain's Arch Type 2.

Fig 16:9 shows shape reinforcement and loading condition.

The Elastic B.M.D. is shown on Fig 16:2. Hinge points are the same as type 1.

Material and Section Properties

\[
\begin{align*}
C_u &= 2750 \text{ p.s.i.} \quad f^\prime = 1740 \text{ p.s.i.} \quad A_t = A_t^\prime = .22 \text{ in}^2 \\
E_0 &= 3.3 \times 10^6 \text{ p.s.i.} \quad E_t = 29 \times 10^6 \text{ p.s.i.} \quad m = 8.79 \\
d &= 3.3 \quad d^\prime = 2.6 \quad f_{yp} = 45000 \text{ p.s.i.} \\
k_1 &= .843 \quad k_2 = .55 \quad I_T = 20.09 \text{ in}^4
\end{align*}
\]

MULT~\(H\) Section Characteristic

This is shown on Fig 16:11. Also shown on this figure are the elastic \(M~\!\!\!\!H\) structural relationships for B and D which are the same as those of type 1.

\(W_{SC}\) is found to be 1835 lbs.

Using the curves of Fig 16:11 \(W_{AC}\) is determined as follows in Table 16:1.

<table>
<thead>
<tr>
<th>TABLE 16:4</th>
</tr>
</thead>
<tbody>
<tr>
<td>(W_{AC_1}) = 3700 lbs</td>
</tr>
<tr>
<td>B</td>
</tr>
<tr>
<td>D</td>
</tr>
<tr>
<td>(\frac{\delta W_{AC_1}}{W_{AC_1}}) = +.032</td>
</tr>
<tr>
<td>(\delta W_{AC_2})</td>
</tr>
<tr>
<td>(\frac{W_{AC_2}}{W_{AC_1}}) = .009</td>
</tr>
<tr>
<td>(\delta W_{AC_2}) = +.018</td>
</tr>
<tr>
<td><strong>(W_{AC} = 3830 \text{ lbs within 1}%)</strong></td>
</tr>
</tbody>
</table>
Effect of Deformation on $M_{AC}$

Fig 16.4 shows the deflected shape of the arch just prior to collapse with a hinge at B.

Using 4420 lbs as the collapse load the deflections at B and D from Fig 16.4 are:

\[
\begin{align*}
\delta_Y^B &= 72 \times 3830 \times 1.6 \times 10^{-6} = .44 \text{ ins} \\
\delta_H^B &= 61 \times 3830 \times 1.6 \times 10^{-6} = .37 \text{ ins} \\
\delta_Y^D &= 71 \times 3830 \times 1.6 \times 10^{-6} = .43 \text{ ins} \\
\delta_H^D &= 2 \times 3830 \times 1.6 \times 10^{-6} = .01 \text{ ins}
\end{align*}
\]

Where $K = .25$ and \[
\frac{E^2}{Kg I_T} = \frac{37.3 \times 10^{-6}}{.25 \times 3.3 \times 28.09} = 1.6 \times 10^{-6}
\]

Method 1 Chapter II Section 10

Assuming $M_B = 34,000$ lb ins at collapse Fig 16.12 shows the forces acting on the arch just prior to collapse.

With these forces the moment at $D = 30200$ lb ins and $H$ at collapse = 3070 lb. At collapse the $M_H$ structural relationships for $B$ and $D$ are:

at $B$ : $M = 7.85 \ W \ ins$ \\
$H = .67 \ W$

at $D$ : $M = 7.9 \ W \ ins$ \\
$H = .84 \ W$
M.H. STRUCT. REL. FOR HINGE B', ELASTIC
1st HINGE STAGE

M.A. STRUCT. REL. FOR HINGE D', ELASTIC
1st HINGE STAGE

M.H. SECTION CHARACTERISTIC

M.H. 27350 I.B.U.S

1st HINGE FORMATION

ARCH TYPE 2

FIG. 16-11
On plotting these 1st hinge relationships on Fig 16:12 at collapse \( M_D = 31600 \text{ lb ins} \) and \( W_{ACD} = 4000 \text{ lbs} \). As a 2nd trial take the collapse load as the average of 4000 lbs and 38130 lbs, i.e. 3920 lbs. With this collapse load acting the force distribution at collapse is shown in Fig 16:12(a).

These forces give \( M_D = 31400 \text{ lb ins} \) and \( W \) at collapse = 3170 lbs. The revised 1st hinge structural relationships for B and D are:

at B: \[-\]
- \( M = 7.66 W \text{ ins} \)
- \( H = 0.68 W \)

at D:
- \( M = 8.02 W \text{ ins} \)
- \( H = 0.85 W \)

Plotting these relationships on Fig 16:11 give \( M_D = 31600 \text{ lb ins} \) and \( W_{ACD} = 3940 \text{ lbs} \). These are close enough to the 2nd trial estimate for the collapse load to be defined as 3920 lbs.

Collapse B.M.D.

Fig 16:13 shows that the collapse load of 3920 lbs is admissible and the assumed mechanism correct.

Method 3 (a) \( W_{SC} \) ratio approach.

Without deformation \( W_{SC} = 1835 \text{ lbs} \)
With deformation \( W_{SC_1} = 1750 \text{ lbs} \)
With reduction 4.6\% applying same % reduction to \( W_{AC} = 3830 \text{ lbs} \) \( W_{ACD_1} = 3660 \text{ lbs} \).
Method 3 (b)

\[ W_{SC_2} \] without deflection = 2050 lbs

\[ W_{SC_3} \] with deflection = 1970 lbs

\[ \% \text{ reduction} = 3.9\% \] Applying this reduction to \[ W_{AC} = 3830 \text{ lbs} \]

\[ W_{ACD_2} = 3680 \text{ lbs}. \]

Method 2 Chapter 2 Section 10

Again as for Arch Type 1 the initial slopes of the \( M \sim H \) structural lines for B and D Fig 16:11 are too large to make the application of the method practical without considerably enlarging Fig 16:11.

Hinge Rotation required for collapse

Rotation required at B = \[ \frac{1.72}{.25 \times 3.3 \times 20.09} \] = .104 rads.

Rotation available

Sufficient binders have been provided for \( \varepsilon_p \) to be taken as .01.

Available rotation = \[ .4 \times \frac{.01 \times 33.5}{.28 \times 3.3} \] = .166 rads

which is satisfactory. It should be noted that special close binding has been provided over a 21 in section about B. Practically this means that \( \varepsilon_p \) at B could be taken as .015 or .02 if required.

Binders have been provided throughout the arch and therefore any rotation required at D is available.

Jain tested three specimens of arch Type 2. The average maximum load taken 4280 lbs. It should be noted that the tested specimens were subjected to two cycles of loading to
ultimate conditions before being finally tested to failure. This repetition of ultimate loading conditions probably diminished the final collapse load. Fig 16:14 shows the actual load-deflection curve for Arch 2:1 and also shows the sequence of loading and unloading. The predicted behaviour to collapse is also shown.
SVBAT COLLAPSE - .89''

PREDICTED COLL. LO
3/120 E

PREDICTED COLL. BEHAVIOR

LOAD POINT VERT. DEF.

FIG 16:14
Jain's Arch Type 3

Fig 16:1 shows the shape reinforcement and loading condition of Type 3.

Material and Section Properties

\[ \begin{align*}
C_u &= 5500 \text{psi} \quad f_y = 4070 \text{ p.s.i.} \quad E_d = 5.5 \times 10^6 \text{ p.s.i.} \\
E_t &= 29.4 \times 10^6 \text{ in} = 5.35 \quad A_t = A_t' = 0.1 \text{ in}^2 \quad d = 3.375 \\
d' &= 2.75'' \quad b = 4'' \quad k_1 = 0.785 \quad k_2 = 0.55.
\end{align*} \]

Hinge Point characteristic

This is shown in Fig 16:16.

Elastic B.M.D.

This is shown in Fig 16:2.

Hinge Points

These are determined to be at B and D, Fig 16:2.

The \( M \sim H \) elastic structural relationships for B and D are the same as type 1 and are shown plotted on Fig 16:16.

\[ \frac{W_{SC}}{W_{AC}} \]

This is found to be 1015 lbs.

\[ \frac{W_{AC}}{W_{AC}} \]

This is found as set out in Table 16:5 below.

<table>
<thead>
<tr>
<th>( \Delta M / M_0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.139</td>
</tr>
</tbody>
</table>

\[ \frac{W_{AC}}{W_{AC}} \]

<table>
<thead>
<tr>
<th>Table 16:5.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hinge</td>
</tr>
<tr>
<td>--------</td>
</tr>
<tr>
<td><strong>B</strong></td>
</tr>
<tr>
<td><strong>D</strong></td>
</tr>
</tbody>
</table>

\[ \frac{\Delta W_{AC_1}}{W_{AC_1}} = 0.07 \]

\[ \frac{\Delta W_{AC_2}}{W_{AC_2}} = 0.033 \]
TABLE 16:5 (Continued)

<table>
<thead>
<tr>
<th>Hinge</th>
<th>Thrust (lbs)</th>
<th>$\Delta M$ (lb ins)</th>
<th>$\frac{\Delta M}{M_0}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$W_{AC} = 2240$ lbs</td>
<td>B</td>
<td>1225</td>
<td>-2100</td>
</tr>
<tr>
<td></td>
<td>D</td>
<td>1610</td>
<td>+2300</td>
</tr>
</tbody>
</table>

\[
\frac{\delta W_{AC}}{W_{AC}} = +.006
\]

i.e. $W_{AC} = 2240$ lbs $\approx W_{AC}$ within 1%.

Effect of Deformation on $W_{AC}$

The term $\frac{\rho^2}{K_{E_1}L_T} = \frac{37.5 \times 10^{-6}}{.25 \times 5.5 \times 22.94} = 1.19 \times 10^{-6}$ units.

With $W_{AC} = 2240$ lbs the following deflections of B and D are obtained from Fig 16:4.

$\delta_B = .196$ ins
$\delta_D = .167$ ins
$\delta_D = .188$ ins
$\delta_D = .005$ in.

The actual load-deflection curve for B is shown in Fig 16:17.

$W_{ACD}$

The reduction in $W_{AC}$ caused by deflection as found by the three methods are:

(1) Method 1

![Fig 16:8](image-url)
Fig 16:8 shows the forces on the arch under $W_{AC} = 2240$ lbs. When these act $M_D = 17600$ lb ins.

At collapse at B

\[
\begin{align*}
H &= 0.68 W \\
M &= 7.6 \text{ W ins}
\end{align*}
\]

At D

\[
\begin{align*}
H &= 0.85 W \\
M &= 7.86 \text{ W ins}
\end{align*}
\]

These are drawn on Fig 16:16. Line D gives $M_D = 18,200$ lb ins at failure which corresponds to a $W_{ACD}$ of 2320 lbs.

Taking as the next trial value of $W_{ACD}$ 2320 lbs. With this as the collapse load the forces on the arch are as in Fig 16:19 below.

At $W_{ACD} = 2700$ lbs as the collapse load.

At B

\[
\begin{align*}
H &= 0.68 W \\
M &= 7.85 \text{ W ins}
\end{align*}
\]

At D

\[
\begin{align*}
H &= 0.8 W \\
M &= 7.76 \text{ W ins}
\end{align*}
\]

On plotting equations 16:9 on Fig 16:16, the following are obtained:

$M_D$ at collapse = 18200 lb ins. and $W_{ACD} = 2340$ lbs.
Thus the collapse load modified for axial thrust and deformation can be defined as 2320 lbs. The average maximum load taken by the three specimens tested was 2450 lbs. Deflections became large at approximately 2300 lbs, see Fig 16:17.

**Collapse B.M.D.**

This is shown in Fig 16:20. It indicates that the collapse case chosen is admissible.

(2) **Method 3 (a)**

By ratio $W_{SC} \%$ decrease in $W_{SC}$ due to deformation = 3%  

$\therefore W_{ACD_1} = 2170$ lbs.

**Method 3 (b)**

By ratio $W_{SC}$ where the actual moments are used in equation 16:2(a) with and without arch deformation. No change in the $W_{AC}$ is shown as a result of deformation.

(3) **Method 2**

Impractical to apply. Same remarks apply as for types 1 and 2.

**Rotation required at B**

By inspection of Fig 16:20 and comparing with type 1 it can be seen that the rotation required at B and D for collapse is available in the arch.

The collapse load of 2320 lbs and mechanism chosen are thus admissible collapse conditions.
COLLAPSE B.M.D. FOR TYPE B

Scales: Vert. 1" to 10,000 lbs
4 ft. to 1 ft.

W

M = 17,400 lbs

D

M = 18,000 lbs
Jain's Arch Type 6

Fig 16:21 shows the shape reinforcement and loading condition of arch type 6.

Material and Section Properties:

- $C_u = 2870 \text{ psi}$
- $E_c = 3.44 \times 10^6 \text{ psi}$
- $E_t = 30.6 \times 10^6 \text{ psi}$
- $f''_c = 1875 \text{ psi}$
- $m = 8.9$
- $A_t = A_t' = 0.154 \text{ in}^2$
- $d = 5.35''$
- $d'' = 4.7''$
- $b = 3''$
- $k_1 = 0.81$
- $k_2 = 0.55$
- $I_T = 67.4 \text{ in}^4$

Elastic B.M.D. This is shown in Fig 16:2

Mult-H section characteristic. This is shown in Fig 16:22

Hinge positions. These are determined to be at B and D, Fig 16:2.

The M-H elastic structural relationships for B and D are shown plotted on Fig 16:22.

$W_{SC}$ This is found to be 2280 lbs.

$W_{AC}$ This is found as set out in Table 16:7 below:

**TABLE 16:7**

<table>
<thead>
<tr>
<th></th>
<th>Hinge</th>
<th>Thrust (lbs)</th>
<th>$\Delta M$ (lb ins)</th>
<th>$\frac{\Delta M}{M_0}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$W_{AC1} = 7000 \text{ lbs}$</td>
<td>B</td>
<td>3840</td>
<td>-13500</td>
<td>-0.395</td>
</tr>
<tr>
<td></td>
<td>D</td>
<td>5030</td>
<td>+10500</td>
<td>+0.300</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$\sum = -0.087$</td>
<td></td>
</tr>
<tr>
<td>$W_{AC2} = 6700 \text{ lbs}$</td>
<td>B</td>
<td>3680</td>
<td>-10500</td>
<td>-0.308</td>
</tr>
<tr>
<td></td>
<td>D</td>
<td>4810</td>
<td>+10000</td>
<td>+0.292</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$\sum = -0.016$</td>
<td></td>
</tr>
</tbody>
</table>

i.e. $W = 6700 \text{ lbs within 1\%}$.
Hinge B: Max Elasticity

Maximum Relationship between First Hinge Formation and Calculated Moment

Hinge A: Sector Characteristic

\[ M = 3420 \text{ lbf-in} \]

Arch Type G

Fig. 16"22
Effect of Deformation on $W_{AC}$

$$\frac{2}{K\varepsilon g I_T} = \frac{37.3 \times 10^{-6}}{0.25 \times 3.44 \times 67.4} = 0.644 \times 10^{-6} \text{ units (where } K = 0.25)$$

Using Fig 16:4 again the deflections at B and D under a load of 6700 lbs are:

- $\delta_B = 0.31''$
- $\delta_D = 0.27''$
- $\delta_D = 0.31''$
- $\delta_H = 0.01''$

The actual load-deflection curve for B is shown in Fig 16:24.

**The Reduction in $W_{AC}$ due to Deformation**

(1) **Method I**

**FIG 16:24**

Fig 16:24 shows $W_{AC} = 6700$ lbs applied to the deformed arch. The forces as shown result. Under these conditions $M_B = 61400$ lbs ins (which is high, see Fig 16:22) and the $M-H$ relationships at collapse are:

- B: $M = 6.4 \ W \ \text{ins}$
  $H = 0.74 \ W$
- D: $M = 9.16 \ W \ \text{ins}$
  $H = 0.91 \ W$

---
When the lines given by equations 16:9 are plotted from the first hinge points on Fig 16:22 the following results are obtained: \( M_D = 46,500 \) lb ins and \( \frac{W_{ACD}}{\Delta} = 5060 \) lbs. Therefore the first trial collapse load of \( \frac{W_{AC}}{\Delta} = 6700 \) lbs is high.

For the second trial \( \frac{W_{ACD}}{\Delta} \) the average of 6700 lbs and 5030 lbs, i.e. 5900 lbs will be used.

Fig 16:25 shows the distribution of forces with this load acting just prior to collapse.

![Diagram showing forces and moments](image)

This force system gives \( M_D = 48,300 \) lb ins and the following \( M-H \) relationships for B and D:

at B: \( M = 7.3 \) W \( H = 0.69 \) W

at D: \( M = 8.2 \) W \( H = 0.87 \) W

On plotting equations on Fig 16:22 at collapse \( M_D = 47500 \) lb ins and \( \frac{W_{ACD}}{\Delta} = 5820 \) lbs. These results are considered sufficiently close to the second trial load for 5820 lbs to be defined as the collapse load.

The maximum average load taken by the three specimens tested was 7300 lbs.

**Collapse B,M,D.** This is shown in Fig 16:26 for a collapse load of 5820 lbs. It shows that the collapse mechanism
chosen is correct and the collapse load admissible.

(2) Method 2

Due to large initial slopes of $M-H$ curves impractical to apply.

(3) Method 3 (a)

$W_{SC}$ with deflection = 2250 lbs  \( \Rightarrow \) 3% reduction due to deflection = 1.3%  \( \Rightarrow \) $W_{ACD_1} = 5750$ lbs.

Method 3 (b)

$W_{SC_1}$ without deflection = 29,200 lbs

$W_{SC_2}$ with deflection = 29,100 lbs

i.e. reduction negligible.

Rotation required at B

This = \[
\frac{2.55}{3.44 \times 67.4} = .044 \text{ rads}
\]

Stirrups have been provided to enable $s_p$ to be taken as .01.

Available rotation at B = \[
.01 \times \frac{.4 \times 37}{.17 \times 5.35} = .155 \text{ rads}
\]

which is sufficient.

Binding has also been provided to make available any rotation at D required for collapse.
Sain's Arch Type 7

Fig 16:27 shows the shape reinforcement and loading condition of Arch Type 7.

Material and Section Properties

\[ C_u = 2330 \text{ psi} \quad f'_{c} = 1850 \text{ psi} \quad E_c = 3.4 \times 10^6 \text{psi} \]

\[ E_t = 29.4 \times 10^6 \text{psi} \quad m = 8.65 \quad A_{t} = A_{t}' = 0.1 \text{in}^2 \quad d = 3.375'' \]

\[ d' = 2.75'' \quad b = 4'' \quad I_t = 23.75 \text{in}^4 \]

Elastic B.M.D. This is shown in Fig 16:28.

Multi-H section characteristic This is shown in Fig 16:29.

Hinge Positions These are determined to be at B and D, Fig 16:28.

The M-H elastic structural relationships for B and D are shown plotted on Fig 16:29. They are:

- For B \[ M = 7.65 \text{ W ins} \quad H = 0.93 \text{ W} \]
- For D \[ M = 3.5 \text{ W ins} \quad H = 1.06 \text{ W} \]

\[ W_{SC} \] This is found to be 1400 lbs.

\[ W_{AC} \] The expression for the change in \( W_{SC} \) for the effect of axial force reduces to \( \frac{\delta W_{SC}}{W_{SC}} = \frac{0.582 M_B + M_D}{1.582} \). \( W_{AC} \) is obtained by setting this expression out in Table 16:8 below.

<table>
<thead>
<tr>
<th>TABLE 16:8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hinge</td>
</tr>
<tr>
<td>--------</td>
</tr>
<tr>
<td>W ( AC_1 ) = 4000 lbs</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>( \frac{\delta W_{AC_1}}{W_{AC_1}} )</td>
</tr>
</tbody>
</table>
TABLE 16:8 (Continued)

<table>
<thead>
<tr>
<th>Segment</th>
<th>$\xi_{W_{AC}^2}$</th>
<th>$\xi_{W_{AC}^3}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>B</td>
<td>4100</td>
<td>-8500</td>
</tr>
<tr>
<td>D</td>
<td>4350</td>
<td>+5900</td>
</tr>
<tr>
<td>$\xi_{W_{AC}^2}$</td>
<td>+.043</td>
<td></td>
</tr>
<tr>
<td>$\xi_{W_{AC}^3}$</td>
<td>-.013</td>
<td></td>
</tr>
</tbody>
</table>

i.e. $W_{AC} = 4600$ lbs within 1%.

Effects of Deformation

Fig 16:30(a) shows the arch under a unit central load. As the load is symmetrical it is only necessary to consider half the arch. In this case the half arch is divided into eleven segments. Table 16:10 below sets out the deflection of the segment ends perpendicular to the original arch.

TABLE 16:10

<table>
<thead>
<tr>
<th>Segment</th>
<th>$\xi_{lin}$</th>
<th>$\xi_{lin} + M_{NI}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>-1.75</td>
</tr>
<tr>
<td>2</td>
<td>-1.75</td>
<td>-2.47</td>
</tr>
<tr>
<td>3</td>
<td>-2.47</td>
<td>-3.2</td>
</tr>
<tr>
<td>4</td>
<td>-3.2</td>
<td>-3.6</td>
</tr>
<tr>
<td>5</td>
<td>-3.6</td>
<td>-3.1</td>
</tr>
<tr>
<td>6</td>
<td>-3.1</td>
<td>-2.5</td>
</tr>
<tr>
<td>7</td>
<td>-2.5</td>
<td>-1.2</td>
</tr>
<tr>
<td>8</td>
<td>-1.2</td>
<td>+.4</td>
</tr>
<tr>
<td>9</td>
<td>+.4</td>
<td>+2.4</td>
</tr>
<tr>
<td>10</td>
<td>+2.4</td>
<td>+4.8</td>
</tr>
<tr>
<td>11</td>
<td>+4.8</td>
<td>+7.65</td>
</tr>
</tbody>
</table>
ARCH TYPE 7

M = 15,000 lbs.

MAX H. Stet eel

MAX I. beam steel eel.

M = M. I beam steel eel.

MAX I. beam steel eel.

MAX H. Stet eel.

MAX I. beam steel eel.

MAX H. Stet eel.

MAX I. beam steel eel.

MAX H. Stet eel.

MAX I. beam steel eel.

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MAX I. beam steel eel.

MAX H. Stet eel.

MAX I. beam steel eel.

MAX H. Stet eel.
These deformations are shown plotted on Fig 16:30(b).

For this arch the factor \( \frac{\xi^2}{K E I_T} = \frac{37.3 \times 10^{-6}}{0.25 \times 3.4 \times 23.75} = 1.85 \times 10^{-6} \).

From Fig 16:30(b) and using this factor the deflections at B and D under a 4600 lbs central load are:

- at B: \( \delta_B = 0.24'' \)
- at D: \( \delta_D = 0.16'' \)
- \( \delta_{BD} = 0.09'' \)

The actual test load-deflection curve is shown in Fig 16:31.

**W_{ACD}**

1. **By ratio \( W_{SC} \)** with deflection = 1340 lbs.
   - % decrease in \( W_{SC} \) due to deflection is 4% and \( W_{ACD_1} = 4400 \) lbs.
2. **By Method 2**
   - \( e_B \) elastic = 8.23 ins, \( \delta_B \) at coll = 0.24 ins
   - \( e_B \) 1st hinge = 8.47 ins
   - \( e_D \) elastic = 3.3 ins, \( \delta_D \) at coll = 0.16 ins
   - \( e_D \) 1st hinge = 3.46 ins

The 1st hinge relationships for B and D are plotted on Fig 16:29. Re-applying the \( W_{AC} \) analysis \( W_{ACD_2} = 4500 \) lbs.

3. **Method 1**

   ![Diagram](FIG16:32)
Fig 16:32 shows the distribution of forces on the arch under a collapse load of 4600 lbs and \( M_B \) assumed = 19000 lb ins. This gives \( M_D = 26200 \) lb ins and the following \( M \sim H \) structural relationship for B and D after first hinge formation.

\[
\begin{align*}
\text{For B} & \quad M = 4.12 W \text{ ins} & \quad H = 1.08 W \\
\text{For D} & \quad M = 5.7 W \text{ ins} & \quad H = 1.2 W
\end{align*}
\]

Plotting equations 16:11 on Fig 16:29, \( M_D \) at collapse = 21,000 lb ins and \( W_{ACD_1} = 3700 \) lbs. \( \therefore \) \( W_{ACD} = 4600 \) lbs is too low.

As second trial use 4200 lbs. Fig 16:33 shows the force distribution on the arch with this load.

With these forces acting \( M_D = 21,600 \) lb ins and the \( M \sim H \) structural 1st hinge relationships for B and D.

\[
\begin{align*}
\text{For B} & \quad M = 4.75 W \text{ ins} & \quad H = 1.06 W \\
\text{For D} & \quad M = 5.15 W \text{ ins} & \quad H = 1.17 W
\end{align*}
\]

Plotting equations 16:12 on Fig 16:29, \( M_D = 21,500 \) lb ins and \( W_{ACD_2} = 4200 \) lbs.

Collapse B,M,D. This is shown on Fig 16:34 and shows that the collapse mode and load defined are satisfactory.

The maximum average load taken by the three specimens tested was 4745 lbs. From Fig 16:31 it can be seen that deflections became large at about 4200 lbs. Fig 16:31 shows the predicted mode of failure. It shows that hinges at B and D form about the
same load, B forming at 4000 lbs and D at 4000 lbs. The fact that plasticity occurs at B and D before hinge B forms accounts for the under-estimation of $\delta_yB$ in the deflection analysis.

**Determination of Rotation required and available.**

If it is assumed that the first hinge to form is at B Fig 16:35(a) then the $\delta_yB$ method does not give the rotation required.

The previous work has shown that plasticity develops at B and D about the same time. For the purpose of determining rotation required at B and D it is assumed that hinge D forms first.
Applying the $\delta_{LK}$ analysis the rotation required at $D = 0.0272$ radians. The stirrups provided and the hinge length available are adequate for this rotation to develop. The rotation required at $B$ will be of the same order as at $D$ and again by inspection this is available from the section.
FIG 16:31

- Arch 7:1
- Δ = 7:2
- □ = 7:3

Vertical deflection of load point B in.

Load W (lbs)

5000
4000
3000
2000
1000
0

VECTICAL DEFORMATION AT B

Prior to collapse

Elastic

Predicted failure mode

W_{UC} = 4200 lbs
Summary of Results on Jain’s Arches

1. In every case the predicted collapse load is below that shown by Jain. This may be due to the way the load-deflection curves were taken experimentally. The loads recorded in the later stages of the test may not have been allowed to settle down to a steady value before deflection readings were taken. In the large arch tests at load stages near collapse some time elapsed with an inevitable increase in deflection occurring before a steady load stage was obtained.

2. The predicted deflection of the load points at collapse agree quite closely with the point, as indicated on the actual deflection plot, where the arch deflections began to show large increases with load.

3. In some cases the effect of deflection was to increase $W_{AC}$. The effect of deflection was to alter $W_{AC}$ by under 5%. Hence for design purposes $W_{AC}$ could be used as the collapse load.
Conclusions and Application to Structural Design

In the light of results obtained from the experimental work reported herein the following conclusions are drawn.

Generally tests have shown that arches develop sufficient hinges to form a mechanism before collapse. They are thus suitable for the application of the 4 collapse design principles as set out in the theoretical work. Sufficient moment redistribution took place in all the test arches, prior to collapse, with the exception of the 2 shear failures to allow the stipulated mechanism to form and the required hinge moments to develop. The 4th collapse design criterion, viz that at collapse, the strain at a hinge point should not exceed a defined amount, in the test cases .01, did not limit the complete formation of the collapse mechanism. Sufficient rotation was available at each hinge to develop that required for collapse. Only in the exceptional case of Arch 3L did the development of this rotation entail any real loss of section by spalling or crushing, up to collapse. Generally where a hinge formed at some load below the collapse load the amount of 'taking-off' in the compression zone was less than % in wide across the arch. The hinge sections therefore can be considered to maintain their full compressive sections up to collapse.

Commenting on the experimental results in relation to the various sections of the proposed theory.

1. The collapse load $W_{sc}$ underestimates the load carrying capacity of r.c. arches under minimum collapse conditions, i.e. for loads causing a minimum number of hinges to form for collapse, such hinges retaining the same sign B.M. through to collapse. In the large arch test series this under-estimation varied from about 15 to 50%. The hinge positions as calculated were very close to those occurring in the arch, as indicated by cracking and strain measurement.
2. The collapse load $W_{AC}$, again for minimum collapse conditions, (in the large arch tests these were fulfilled by Arches 4L and 5L) gave a load within 2.5% of the actual collapse load. In every case the presence of an axial force acting in the arch increased the arch load capacity. The calculated $W_{AC}$'s for Jain's arches averaged 10 to 15% below the collapse load as given by Jain. As previously pointed out however it is not known how Jain's load-deflection curves were taken. The calculated $W_{AC}$'s are within about 5% of the point on his curves where the deflection increases begin to become large which indicated in the large arch tests that failure was imminent.

3. The calculated deflected shapes just prior to collapse agree fairly well with the actual shapes. Arch 4L, the minimum collapse case for the series of large tests, showing good agreement. From this it can be concluded that the proposed deflection method is satisfactory and the values of $E_u$ related to $C_u$ and of $I_T$ and $K = .25$ also satisfactory. It is suggested that these values can be used to determine deflections in other r.c. members.

4. The effect of deflection on the collapse load $W_{AC}$ for the arches tested and reported was shown to be small causing generally a reduction of the order of 3% in $W_{AC}$ to arrive at $W_{ACD}$. In a few cases from Jain's arches the deflection caused a small (about 3%) increase in $W_{AC}$ to arrive at $W_{ACD}$. The effect of deflections on $W_{AC}$ for 'long slender' arches may be more pronounced. In these cases also lateral instability may bring down the collapse load. Long slender arches have not been considered here.

5. Of the three methods presented in Section 10 for the determination of $W_{ACD}$ the first, the application of the ratio of $W_{SC}$'s with and without deflection to $W_{AC}$' is the simplest to apply but the results show varying degrees of accuracy. This inconsistency is however not material as the reduction in $W_{AC}$ due to deflection is small anyway.

The method using the graphs of $W\sim H$ and $M$ gave quite good results and has the advantage over the other two that the order
of hinge formation and the load at which they form are calculated.

The method of adding the deflection to the $M \sim H$ structural lines for hinges within the arch and then carrying out another $W_{AC}$ analysis was shown to be applicable only to those cases where the initial slope 'e' of the $M \sim H$ line was not large. This is an arbitrary method but where applicable it does make a suitable reduction in $W_{ACD}$ and is therefore considered adequate.

Excluding rotation consideration

The final arbiter of whether a load and mechanism give acceptable collapse conditions is the drawing of the collapse B.M.D. In this context it is necessary to exercise some judgement in using the $M_{ULT} \sim H$ section curves in conjunction with the $M \sim H$ section structural lines as to what moment exists at a section at collapse. This is however relatively easy to decide. To eliminate this judgement altogether a flat-topped $M \sim \phi$ section characteristic could be assumed. This would however lead to an underestimation of the arch load carrying capacity.

6. Throughout the tests it was not considered necessary to make any allowance for the effect of axial force on the deformations due to the size of these forces.

7. Of the two methods presented for the determination of hinge rotation required for collapse the second is easier to apply; the deflection analysis already being completed, and recognises the existence of some plasticity in the last hinge to form which the $\delta_{LK}$ method does not.

8. To determine the rotation available at hinge sections an empirical rule based on Chan's work and the present experimental data was proposed. This was that the rotation available at a hinge $= \frac{1}{4} \times \text{BM segment} \times \text{max strain permitted at hinge}$. For a bound...
hinge maximum strain was taken as .01, as recommended by Professor Baker. This expression gives reasonable results and allows rotations which do not take off a considerable per cent of a hinge compression zone as loading proceeds to collapse. For very short hinges this expression could be modified to allow the hinge to spread out over the whole EM segment. In these cases a reduced compression section available for moment carrying might be used.

It was shown that the shear reinforcement required in an arch would normally be sufficient to provide the necessary binding to allow the necessary hinge rotation.

9. The two arch section tests showed that the M~Ø ideal elastic-plastic curve assumed was reasonable. They also showed that Hognestad’s section failure theory gave reasonable results for M₀, on the safe side by about 10%.

Summing up the proposed theory offers a relatively straightforward way of designing by analysis r.c. arches. The amount of work involved is less than that generally carried out to design an arch elastically. A structural and economic advantage of the method is that the collapse load is determined very closely. This cannot normally be done using elastic methods. A suitable factor of safety can be fixed before design starts, another unknown quantity in elastic design, and an arch produced which will usually be more economic in the material sense, and probably overall financially, than if elastic design methods are used.

**Application to Structural Design**

The following steps are suggested in applying the method to structural design.

1. Decide span, rise and variation of section, e.g. Lₓ = L₀secØ to be used.

2. Decide suitable factor of safety against collapse.
3. Determine minimum collapse position(s) for load(s). This may need several trials.

4. Draw elastic B.M.D. for (3) and from this determine hinge positions.

5. Assume a reasonable relationship between the $M_0$'s at the hinge positions. Write out the statical equations for the arch under $0.7 \times$ actual loads $\times$ factor of safety and solve for $M_0$.

N.B. Steps 4 and 5 can also be carried out using the determinantal form as shown in the theory.

6. With $M_0$ determine a suitable section.

7. Draw the $M_{\text{ULT}} \sim H$ curves for the hinge sections and using their $M \sim H$ elastic structural line determine $W_{\text{AC}}$. This will be close enough to the collapse load for most practical arches.

8. Check that $W_{\text{AC}}$ corresponds reasonably closely to the factor of safety $\times$ worse loading. If not repeat analysis.

9. Determine order of hinge formation. Generally this can be done by inspection of $M_{\text{ULT}} \sim H$ hinge section curves and $M \sim H$ hinge elastic structural relationships. If not use Method 1 section 10 (this will give a $W_{\text{ACD}}$ as well).

When order of hinge formation known load arch with $W_{\text{AC}}$ with last hinge about to form but assumed still elastic, determine deflected shape using the method of section 6. The moments at the hinges can be assessed by using the $M_{\text{ULT}} \sim H$ and $M \sim H$ section curves.

10. With the deflected shape determine $W_{\text{ACD}}$ by any of the three methods of Section 10.

11. Draw collapse B.M.D. to check that $W_{\text{ACD}}$ and mechanism admissible, if they are check that $W_{\text{ACD}}$ and factor of
safety x worse load are in fairly close agreement. If not repeat for revised $W_{ACD}$.

12. Reinforce for shear.

13. Determine rotations required by Method 2 section 11.

14. Determine available rotations with shear reinforcement first checking with Fig 12:6 that sufficient volume is provided to enable $\delta_p$ to be taken at $0.01$. If not either (a) binding can be increased until $\delta_p$ can be taken as $0.01$ or (b) reduce $\delta_p$ according to binding present.

If available rotations are satisfactory then collapse load and mechanism are admissible and design satisfactory. If rotations not available (this will seldom be the case practically) firstly if $\delta_p < 0.01$ from 14(b) then increase bending to make $\delta_p = 0.01$. If rotations still not available reduce collapse load and/or change section until they are. For change of section this may mean complete re-analysis.
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Notation and Abbreviations

\( A_0, A_0' \) area of compression reinforcement

\( A_s \) total area of reinforcement

\( A_t \) area of tensile reinforcement

\( \alpha, (\ ), B' ( ) \) function of ( )

\( b \) breadth of section

B.M.D. Bending Moment Diagram

\( C \) total force from concrete in compression

\( C' \) total load in compression steel

\( C_u \) crushing strength of concrete cubes

\( c - c \) centre to centre

\( C_{cyl} \) crushing strength 6" x 12" cylinders

\( c \) centreline

\( d \) effective depth of section

D.G. Dial Gauge

\( d' \) distance between compression and tension reinforcement

\( D \) Total depth of section

\( E_p \) primary strain energy

\( E_c \) modulus of elasticity of concrete, \( E_s \) modulus of elasticity of steel or secondary strain energy

\( e \) strain or eccentricity in expression \( M = eH \).

E.R.S.G. Electrical resistance strain gauge, foil or paper backed

\( \mathcal{E} \) strain

\( \mathcal{E}_E \) strain at compression edge in the elastic range

\( \mathcal{E}_U \) strain at compression edge when concrete crushes

\( \mathcal{E}_o = \frac{2f^*}{E_g} \)

\( \phi_E \) rotation at section under elastic conditions

\( \phi_B \) rotation at hinge section with maximum compressive strain of .01

\( \phi_p \) rotation at hinge section with maximum compressive strain of .015

\( \phi_U \) rotation at hinge section with maximum compressive strain of .0038

\( \phi \) section rotation or diameter

\( f(\ ), F(\ ), G_n(\ ), J_n(\ ) \) functions of ( )
$f$  
stress

$f_{c'}$  
cylinder crushing strength

$f_c'' = k_3 f_{c'}$  

$f_{cs}$  
stress in compression reinforcement

$f_{ts}, f_{ts}$  
stress in tension reinforcement

$f_{yp}$  
tension reinforcement yield point stress

$f_{yp}'$  
compression reinforcement yield point stress

$h'$  
hinge length

$B.M.$  
segment length

$h_n$  
height to point $n$ on arch from springing line

$H_{AC}$  
horizontal end thrust on arch with load $W_{AC}$ on arch

$H$  
al axial thrust or horizontal end thrust

$H_{0^*} H_{ULT}$  
ultimate section capacity under direct axial thrust

$H_{ACD}$  
horizontal end thrust with load $W_{ACD}$ on arch.

$I$  
moment of Inertia

$I_T$  
Uncracked transformed section moment of inertia

$k$  
status of gyration

$k_1$  
coefficient defining the magnitude of the internal compressive force in concrete as defined by Fig 13:2

$k_2$  
coefficient defining the position of the internal compressive force in concrete as defined by Fig 13:2

$k_3$  
ratio of flexural compressive strength $f_{c'}$ to cylinder strength $f_c''$

$k_d = k_1, k_2$  

$k_b$  
'balanced' failure ratio $n_d$

$L$  
length

$\ell_p$  
hinge length

$\ell_n$  
length of hinge at $n$

$m$  
ratio $\frac{M}{M_0}$

$M_{yield}$  
section moment at which tension steel yields

$M_{ULT} M_0 M_p$  
ultimate section M.O.R under pure bending
M.O.R
\( \Delta m, S_m \)
N.A.
n
\( n_1, n_h \)
\( \phi \)
\( P_u, P_o \)
p.s.i.
p.q.r.
q
r.c.
\( S_m \)
\( S_d \)
\( S_p \)
T
u/s
U, S.E.
U.T.S.
W
W_{SC}
W_{AC}
W_{ACD}
w/c
\( \bar{X}_n \)
\( \delta \)
\( \varepsilon_{VN}, \varepsilon_{HN} \)
\( \Theta \)

Moment of Resistance
moment available at section above \( M_0 \)
Neutral Axis
ratio axial thrust or no. of redundancies \( P_o \)
N.A. factors
plate
section ultimate load under axial thrust alone
lbs per square inch
location of hinge position factors, e.g. \( P_2 \)
nominal shear stress
reinforced concrete
strain on the minimum compressive section
difference of strain across the section
at failure due to plasticity
the average strain of the concrete at the compressive edge which occurs under increasing load between the yielding of the steel and the crushing of the concrete.
axial force in tension steel
underside
strain energy
Ultimate Tensile Strength
Load
simple plastic collapse load considering bending only.
modified for effects of axial force
modified for effects of axial force and bending deformation
water/cement (ratio)
plastic moment value at \( n \)
axial contraction, or deflection
vertical and horizontal deflections of point \( N \)
total rotation at hinge