L.W. Stubbs.

SOME CHEMICAL ENGINEERING ASPECTS OF

LOSS OF HEAT FROM SURFACES.

Thesis for M.Sc. degree 1928.
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LOSS OF HEAT FROM SURFACES.

M.Sc.

STUBBS, L.W.

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INTRODUCTION.
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Every chemical reaction and almost every process in the manufacture of chemicals involves some heat transfer problem. It is thus essential that in chemical engineering accurate information concerning the laws of heat transfer should be available.

The main problems of heat transmission may usually be classified under two headings. The one, where heat is transferred through a boundary from a heating agent to the material to be heated. The other, where heat is dissipated unavoidably into space from a surface, dependent upon the excess temperature of the surface above its surroundings. The two classes are not capable of being separated by a definite border-line, as many problems may involve consideration of both. Such a problem may be instanced in the case of air-blast heaters, where the natural convection from the heater surface is considerably supplemented by the air stream which is being heated.

For the purpose of this dissertation, however, only the natural losses peculiar to any surface surrounded by air are considered.

The natural losses from a surface in air will depend upon two effects only - radiation and convection. Conduction, however, will play a necessary part in carrying a constant supply of heat to a surface which is constantly dissipating heat by radiation and convection; moreover
Consequently in a review of the laws of radiation and connection, it will be necessary to include a partial review of conduction also.

It may be necessary to increase the 'emission power' of a surface, as in the case of heaters and radiators, or machinery which it is required to keep as cool as possible. On the other hand, it is usually the case for the heat thus lost to be incidental and undesirable: it is then the effort of the chemical engineer to reduce the 'emission power' to a minimum.

Methods of reducing the heat loss are twofold - the lowering of the surface temperature, and the lowering of the 'emission power' of the surface. The temperature is usually lowered by the application of an insulating layer: this does not strictly lower the actual surface temperature, but presents a lower temperature surface to the air.

To determine whether any decrease can be obtained in the loss or not, requires an accurate knowledge of the laws of heat transfer. It may be that the cost of lagging and maintenance would be greater than the consequent gain in heat energy for a given pipe-line or apparatus; such can only be determined by the application of definite laws to the problem.

There is an enormous amount of information published in technical and academic journals concerning heat transfer. Unfortunately so much experimental work has been empirical or of limited application, and theoretical results, differing widely among themselves, have not received experimental confirmation, that it is time an attempt was made to corre
It is the attempt of the accompanying work so to correlate the existing data for heat transmission from surfaces.

Workers in the past have been divided in their methods of determination of 'emission power'. One section has considered radiation and connection separately, whilst the other has considered both effects as one.

It is generally known that the radiation and connection effects vary in different ways under the same influences, so that no fundamental result can be obtained unless each is studied separately. Those workers who have determined emission power as a single effect have usually been Americans, and employed at the instance of a business organisation for research upon a particular insulating material under limited circumstances.

Such work is not usually adaptable in any way to fundamental use, and as it is the intention of this dissertation to avoid anything of an empirical nature, the method of separate consideration will be followed.

Separate sections will be found under the headings: 'Radiation', 'Connection', 'Conduction', and 'General Applications'.

Work in each section is considered in historic sequence; the review of each author consists of a description and criticism of his methods and results showing how each contributes to the structure of a final result which is adopted as the best available at the present time.

Other small sections will be found on various minor
but to help elucidate some problems which have hitherto been
imperfectly treated, and which can be solved with the help
of the results obtained in the major sections.

In many cases actual examples of these latter sections
cannot be given without a considerable amount of routine
slide-rule calculation, which it would serve no useful
purpose to publish. The generally applicable equations by
which a given problem may be solved, however, are shown in
each case.
RADIATION.
It is well known that heat and light are of the same nature, both being radiant energy but of different quality or wave-length. That which is called light is the energy of such wave-length that affects the nerves of the retina of the eye, and composes the visible part of the spectrum, whilst that part of the spectrum beyond the red, the infra-red, is commonly called heat, the wave-length of which increases with its distance from the visible region.

Newton made the first experiments on the rate of cooling of a body in air. He found that the rate was proportional to the excess of temperature of the body above that of the surroundings. This law, however, has since been experimentally proved to be only applicable to very small ranges of temperature, and can at best only give the tangent to the heat loss curve at any particular point.

Perhaps the first experimenter to study the effect of radiation from a surface, apart from general heat loss, was Leslie, who determined the relative quantities of radiation from a number of surfaces. His apparatus consisted of the now famous Leslie cube, a hollow metal cube containing boiling water, the sides of which were painted or otherwise prepared in various ways. To measure the heat radiated, he concentrated it by means of a parabolic mirror on to one bulb of a differential thermometer placed at its focus. The radiations from the various surfaces produced different effects upon the thermometer in the ratio of their radiating powers. All his results were obtained for a radiating surface at 100° C. The following are some of his figures, giving lampblack the arbitrary value = 100,
These results were confirmed by Mellon, and also to some extent by De la Provostay and Dessains. The two latter investigators, however, showed that the results for the polished metal surfaces were too high. They obtained the results for radiation at 130°C:-

<table>
<thead>
<tr>
<th>Material</th>
<th>Relative Radiating Power</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lampblack</td>
<td>100</td>
</tr>
<tr>
<td>Indian ink</td>
<td>88</td>
</tr>
<tr>
<td>Platinum</td>
<td>11</td>
</tr>
<tr>
<td>Copper</td>
<td>5</td>
</tr>
<tr>
<td>Silver</td>
<td>2.2 - 5.4 (according to polish)</td>
</tr>
</tbody>
</table>

They further found that the relative radiating powers changed with rise of temperature from 100° - 550°C, a result to be expected in the light of later knowledge. The experiments of De la Provostay and Dessains are more likely to give correct results inasmuch as they measured the radiation direct by means of a thermopile, thus excluding any possibility of absorption of energy from reflecting mirrors.

The first law to be enunciated connecting loss of heat by radiation and temperature of radiator, was due to Dulong.
temperature by running water. The results were corrected for loss by conduction through the thermometer and the residual air etc., and then taken to represent the loss by radiation.

The radiation loss from a body was given by:

\[ H_r = K_a t^1 \]

where \( H_r \) = heat loss in cals per cm\(^2\) per sec.

\[ a = \text{Constant: value} = 1.0077 \]

\[ t = \text{Temp. of surface in °C} \]

\[ K = \text{Constant whose value depended upon nature of surface.} \]

For the radiation from a surface to its surroundings at temperature \( t_0 \) °C

\[ H_r = K (a^1 - a^{t_0}) \]

This law was deduced from experiments using temperatures up to 250 °C, and hence cannot be guaranteed accurate above this temperature.

It will be seen by a glance at Fig. 1 how this formula compares with later ones.

Peclet, accepting the formula of Dulong and Petit, obtained a large number of values for \( K \) for different surfaces. He used a thermopile situated equi-distant from two parallel plane surfaces, and adjusted the temperatures of the surfaces until the pile gave a zero deflection. The surfaces were sides of vessels (e.g. Leslie cubes) containing water, the temperature of which could be altered at will. He assumed that the temperature of the outer surface was the same as that of the water, but, despite vigorous stirring, it is highly
Here then for first surface:\n\[ H_{r1} = K_1 (a^1t_1 - a^0) \]

and

for second surface:\n\[ H_{r2} = K_2 (a^2t_2 - a^0) \]

But as\n\[ H_{r1} = H_{r2} \]

we have\n\[ \frac{K_1}{K_2} = \frac{(a^2 - a^0)}{(a^1 - a^0)} \]

By determining the total heat loss from a body suspended in a constant temperature enclosure, Pécelt derived an equation connecting the total heat loss with temperature excess of body above surroundings. The equation thus obtained was of the nature,
\[ H = at (1 + bt) \]

where \( H \) = Heat loss in cals per cm\(^2\) per sec.
\( t \) = temperature excess of body above enclosure °C.
\( a \) = constants depending upon the shape and surface of the body.
\( b \) = of the body.

Such equations derived for bodies of similar size and shape, but having surfaces corresponding to those in the first experiment, give sufficient data for determining the absolute value of \( K_1 \) etc.

Assuming the same conditions of temperature of body and enclosure we have\n\[ H_1 = H_{r1} + H_c \] for first surface

where \( H_c \) = loss due to connection.

and\n\[ H_2 = H_{r2} + H_c \] for second surface.
A direct loss of heat by convection.

\[ H_1 - H_2 = H_{r_1} - H_{r_2} \]

Now for the same temperature

\[ \frac{H}{r_1} = K_1 \]
\[ \frac{H}{r_2} = K_2 \]

also

\[ H_1 = a_1 t (1 + b_1 t) \]
and

\[ H_2 = a_2 t (1 + b_2 t) \]

\[ a_1 t (1 + b_1 t) - a_2 t (1 + b_2 t) = K_2 (C - 1) \]

As \( a_1 b_1 a_2 \) and \( b_2 \) have been determined

and \( C \) is known,

\( K_2 \) is obtained and hence also \( K_1 \).

As determinations of \( K \) in this indirect manner involve many possible errors, the figures obtained are to be accepted with some reserve. The errors involved in the "total heat loss" experiment are discussed more fully under "Connection".

Péclet's work does not prove the correctness or otherwise of Dulong and Petit's formula, as it has been tacitly assumed correct for the determination of \( K \). It is to be wondered why Péclet did not adapt his thermopile method for the proof of the Dulong and Petit law as well as for determining the constants \( K \).

It is an interesting fact that, as the Dulong and Petit formula corresponds so closely with the later accepted formulas for radiation up to temperatures of 250°C, the relative values of his constants can be used in connection.
A list of relative values for the radiation constant are appended. Peclet's result for lampblack being given the arbitrary value = 100.

<table>
<thead>
<tr>
<th>Material</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lampblack</td>
<td>100</td>
</tr>
<tr>
<td>Powdered Chalk</td>
<td>81</td>
</tr>
<tr>
<td>Paper</td>
<td>92</td>
</tr>
<tr>
<td>Cast Fe (new)</td>
<td>76</td>
</tr>
<tr>
<td>Silk</td>
<td>91</td>
</tr>
<tr>
<td>Glass</td>
<td>71</td>
</tr>
<tr>
<td>Wool</td>
<td>90</td>
</tr>
<tr>
<td>Zinc</td>
<td>59</td>
</tr>
<tr>
<td>Oil Paint</td>
<td>90</td>
</tr>
<tr>
<td>Tin</td>
<td>52</td>
</tr>
<tr>
<td>Cotton</td>
<td>89</td>
</tr>
<tr>
<td>Wrought Fe polished</td>
<td>11</td>
</tr>
<tr>
<td>Sand</td>
<td>88</td>
</tr>
<tr>
<td>Ag Paper</td>
<td>10</td>
</tr>
<tr>
<td>Stone</td>
<td>88</td>
</tr>
<tr>
<td>Brass Polished</td>
<td>6</td>
</tr>
<tr>
<td>Plaster</td>
<td>88</td>
</tr>
<tr>
<td>An Paper</td>
<td>6</td>
</tr>
<tr>
<td>Wood</td>
<td>88</td>
</tr>
<tr>
<td>Cn polished</td>
<td>4</td>
</tr>
<tr>
<td>Sawdust</td>
<td>86</td>
</tr>
<tr>
<td>Ag polished</td>
<td>3</td>
</tr>
<tr>
<td>Charcoal</td>
<td>84</td>
</tr>
<tr>
<td>Wrought Fe oxidised</td>
<td>82</td>
</tr>
<tr>
<td>Cast Fe oxidised</td>
<td>82</td>
</tr>
</tbody>
</table>

Peclet points out that the colour of the paper and cloth does not alter the radiation constant.

Rosetti was next to propose a new law for radiation. By observing the loss from a Leslie cube filled with mercury but not surrounded by a black enclosure, he obtained the formula,

\[ H_R = a T_1^2 (T_1 - T_0) + b (T_1 - T_0) \]

where \( a \) and \( b \) are constants for the particular surface

\( T_1 = \) absolute temp. of surface

\( T_0 = \) " " enclosure.
found in some experiments by Tyndall, from fine wires varied with the fourth power of the absolute temperature of the wire. Upon further examination of Dulong and Petit's results he found that the 'fourth power law' still held, and so, by chance, he enunciated the law which is still accepted as true namely, that a body radiates heat proportional to the fourth power of its absolute temperature. Convincing evidence for the truth of this law was forthcoming, when the same result was obtained by Boltzmann in 1884.

He proved, by making the assumption (since proved) that radiation presses upon a surface, and treating the radiation from a 'full radiator' as the working substance for a Carnot cycle, that the radiation from such a radiator in a vacuum must vary with the fourth power of the absolute temperature.

The law, however, was only proved for a full radiator or so-called "black-body".

Further developments of the thermo-dynamic treatment were then made by Wien, who showed that for a black body the wave-length of maximum radiation ($\lambda_\text{max}$) is given by,

$$\lambda_\text{max} T = \text{constant},$$

and that the energy radiated through a small range at this maximum is

$$E_\lambda d\lambda = \text{constant} \times T^5$$

This was followed by a formula expressing the energy of any wave-length at any given temperature.

For the small range $d\lambda$, it is given by

$$E_\lambda d\lambda = c_1 e^{-\frac{\lambda}{\lambda_0}} d\lambda,$$

where $c_1$ and $c_2$ are constants.
If the enclosure surrounding the hot body is at temperature $T_0$, the latter will receive in exchange a quantity $= K T_0^4$.

Hence the total heat loss from the hotter body will be

$$H_T = K \left( T_1^4 - T_0^4 \right)$$

This relation will only hold, however, for cases where the hot element in the surface can 'see' the surrounding surface through a complete hemisphere; in other words it must subtend a solid angle of at least one hemisphere upon the cooler surface. The condition holds in the case of two parallel infinite planes, and where a body is completely surrounded by another and has no re-entrant angles.

As the law is only strictly true for black bodies it will be necessary to define a black body. The definition of a black body according to Kirchhoff is one which absorbs all radiations of all wave-lengths at all temperatures.

It will be seen from Fig. 1 that the Stefan-Boltzmann law accords fairly well with the Dulong and Petit law for a black body, up to temperatures of about 400°C, after which its values are considerably smaller.

Attempts were now made to prove the fourth power law by experiment. Two typical efforts are those of Schleiermacher and Bottomley who determined the loss from a heated platinum wire, suspended in an evacuated tube.

Schleiermacher used three different wires, two of bright platinum and one covered with a layer of black copper oxide. The wires were about 20 cms. long .2 -.3 m.m.
The wire was electrically heated, the loss of energy being determined from the knowledge of the resistance and current strength, and the temperature, by means of the increase of resistance, due to heating. All the heat (after allowing for conductivity along the metal supports etc.) was assumed to be lost by radiation, to the walls of the tube which were kept at constant temperature in a bath.

Upon applying Stefan-Boltzmann law to his results he found that the constant for the two bright wires rose with temperature, being at 130°C excess temperature about $= 0.14 \times 10^{-12}$ and $0.19 \times 10^{-12}$ for a temperature excess of 1035°C. The two results were not quite the same, a fact which Stefan attributed to varying degrees of polish. The oxide covered wire gave anomalous results, sometimes giving a constant, rising with $T$ and at other times a decreasing value. The average result for the black wire gave a constant $= 0.65 \times 10^{-12}$.

Bottomley used a similar apparatus, but had a copper tube for enclosure. A typical result for a bright platinum wire gave a constant $0.18 \times 10^{-12}$ for temperature 300°C rising to $0.42 \times 10^{-12}$ at 800°C.

Considering the possibility of error in this type of experiment it cannot be claimed as did Schleiemacher & Bottomley that the Stefan-Boltzmann law is proved untrue. Besides the difficulty of accurately determining the loss due to conductivity along the supports, glass tube etc., no account was taken of any possible conductivity of the residual air, or of occlusion of gas by the platinum. The fact that the
for a black-oxide coated wire, cannot be accepted at all, as copper oxide is known to attack platinum at high temperatures, and the radiation observed was from the surface composed of some very doubtful compound of copper oxide and platinum. This would no doubt account for Schleiermacher's variable results with this wire. The outer coat of copper oxide may very possibly have been at a much lower temperature than those shown by electrical observations for the wire: this might account largely for the low figure obtained.

Bottomley made further experiments upon the subject, this time using a heated copper sphere, 4 cms. diameter, suspended concentrically in a hollow sphere of 12 cms. diameter. The outer sphere was kept in a constant temperature bath, and the annular space was evacuated to a fraction of a millionth of an atmosphere. The temperature of the surfaces were taken by thermocouples.

His experimental figures give many unexpected irregularities. Using a polished silvered surface he obtained a value for the constant of \(0.246 - 0.253 \times 10^{-12}\) for temperatures 60 - 90°C. Another similar surface gave a constant decreasing from \(0.031 \times 10^{-12}\) to \(0.073 \times 10^{-12}\) for temperatures 175 to 205°C, and then again rising to \(0.191 \times 10^{-12}\) for temperature 250°C. The same surface the following day gave a steadily decreasing value of \(K\), of 0.35 to 0.27 \(\times 10^{-12}\) for a similar range of temperature.

The different values obtained for what is apparently the same kind of surface were explained by Bottomley as being due to a difference in polish, caused by slight surface
Bottomley further used the same apparatus to determine the loss from sooted surfaces. A typical example gives an average value for $K$ of $0.94 \times 10^{-12}$, which value seems to vary with temperature in no regular manner.

The irregularity of the results of Schleiermacher and Bottomley indicate the need for more delicate means of measurement and more accurate apparatus.

In view of the failure of these investigators to adapt the Stefan-Boltzmann law to any and every surface, an attempt was made by Weber to account for all these by a new law, which he deduced from experiments upon electric lamp filaments. His relation for the radiation lost by a body was,

$$ H_r = C \frac{\pi \sqrt{\pi} b T_1 e^{aT_1}}{2} $$

where $C$ is a constant depending upon the surface condition. $a$ and $b$ are constant for all bodies. Since $\frac{\pi \sqrt{\pi} b}{2}$ is a constant quantity

$$ H_r = C T_1 e^{aT_1} $$

where $C$ will depend upon the surface.

$a$ is given equal to $0.0043$.

In an enclosure, temperature $T_0$

$$ H_r = C (T_1 e^{aT_1} - T_0 e^{aT_0}) $$

This will reduce, by giving $a$ and $e$ their respective values, to

$$ H_r = C (T_1 \times 1.0043^{T_1} - T_0 \times 1.0043^{T_0}) $$

Weber's value of $C$ for a carbon coated surface is $1.70 \times 10^{-5}$. Weber claims that this formula, when applied to Schleiermacher and Bottomley's results gives much better concordance than does Stefan-Boltzmann law. How far this is true may be seen by a glance at the comparative results from Bottomley's
be admitted that Weber's formula fits the observed facts any more accurately than that of Stefan.

The results for a black body, using Weber's value for carbon are compared with results of other formulas, shown in Fig. 13. Stevens found that the radiation from a clean bright iron surface gave better results with Weber's formula than with Stefan-Boltzmann, for temperatures 15 - 110°C. This however seems to be an isolated case.

Experimenting upon polished platinum, Wilson and Gray found that the radiation, compared with that from a blackened surface, increased with temperature, that is, the constant (applying Stefan-Boltzmann or Weber's law), increased with temperature. To account for the radiation observed in their experiments they deduced a relation

\[ N = a (T^3 - T_0^3) + b (T^4 - T_0^4) \]

where \( a \) and \( b \) are constants, depending upon the surface.

This process of obtaining a formula to fit the results from any particular surface can no doubt be followed ad infinitum: it remains however to produce a formula of universal application.

Petavel, again using a platinum surface, found that his results were quite similar to those of Bottomley and Schleiermacher. Using high temperatures to avoid error due to temperature of the enclosure, he found that Weber's law fitted his results better than Stefan's. Incidentally his figures show that the loss from this particular surface varied almost directly with the fifth power of the absolute temperature.
made to test the fourth power law by accurate experiment, on the basis laid down by Boltzmann.

The experiments of Lümmcr and Pringshcin are typical of the more accurate work done at this period, and a description of their apparatus and methods will apply to all the others except for certain minor modifications.

Briefly the above diagram represents the fundamentals for Lümmcr and Pringshcin's apparatus. C is a constant temperature bath in the case of low temperatures, and an electrically heated muffle for higher temperatures. The temperature of C can be kept stationary at any required value by delicate adjustment. E is a hollow enclosure, in this case a sphere, protruding into the constant temperature container, and having only a narrow opening. T. is a thermo-element for measuring the exact temperature of the enclosure. Exactly opposite the opening of the enclosure a diaphragm D1 is placed, having an opening the same shape and size; this cuts off any but direct rays, and is water-cooled to prevent it affecting the second diaphragm D2. D2 has a similar opening, which can be closed by means
thermocouple. When the shutter is raised, radiation passes
direct from the enclosure to the bolometer B, where its
quantity is measured. The bolometer requires to have its
zero corrected from time to time, for which purpose the radia-
tion can be cut off by dropping the shutter.

It can be proved that a constant temperature enclosure
radiates as a black body whatever its surface condition.
It is therefore more accurate to depend upon a glimpse of
such an enclosure through a comparatively small opening as
a black body, than to prepare such a surface by means of
carbon or platinum black. To get a perfect and permanent
black body by the latter means, requires great skill, and
adherence to definite conditions of preparation. In any
case they are liable to alter when subjected to high temperature.

Lummer and Pringshein used the above apparatus for
temperatures of 100 - 1300°C. The value of the constant
obtained varied between $1.21 - 1.27 \times 10^{-12}$ giving a mean
value of $1.24 \times 10^{12}$. The variation showed no increase
or decrease with temperature, and was assumed to be due to
slight error in experiment.

Many similar investigations have been carried out,
mainly to determine accurately the absolute value of the
Stefan-Boltzmann constant.

Essentially the same apparatus has been used by all
workers, but modifications have been introduced to give
greater accuracy. The black body enclosure has been
improved to give more certain temperatures, and to reproduce
more nearly the black body effect. External effects, such
as reflected rays or stray radiation have been more carefully
strips acting as thermocouple junctions. Whilst these are but refinements, two errors of great importance have been eliminated. The first is the absorption of energy due to the presence of water vapour and carbon dioxide in the apparatus. Both these are known to possess absorption bands corresponding to the wave-lengths of normal temperatures. This fault has been eliminated either by applying a correction for the absorption obtained by separate experiment, or by replacing the air by pure nitrogen, which does not affect the radiation. This method was used by Menderhall and Forsythe to compare Wien's optical scale based on his distribution law with the Stefan-Boltzmann law, for temperatures from 1063 - 1549°C. The second error arose from the fact that the surface of the bolometer, thermopile, etc., was not a perfect black body, i.e., did not absorb the whole of the incident radiation. A correction of about two per cent is usually required to give correct results.

It would appear that to obtain the most perfect 'black' surface, great care is needed. A carbon surface reflects a considerable amount of long waves, and must be kept to a very definite thickness. A layer too thin may not completely cover the metal surface; a layer too thick may give low results owing to bad conductivity to the metal, and consequent re-radiation. Platinum black gives the best surface for long waves and is a good conductor.

Coblentz made a survey of all the past methods, and improved the apparatus to compensate for or eliminate all errors; he obtained a value for the Stefan-Boltzmann constant of 1.370. This is about an average of all the previous
The following is a list of the chief experimenters, and their results for the absolute constant $k$:

<table>
<thead>
<tr>
<th>Experimenter</th>
<th>Year(s)</th>
<th>Value of $k$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kurlbaum</td>
<td>1898</td>
<td>$1.306 \times 10^{-12}$</td>
</tr>
<tr>
<td>Fery</td>
<td>1909</td>
<td>1.509</td>
</tr>
<tr>
<td>Bauer</td>
<td>1909</td>
<td>1.249</td>
</tr>
<tr>
<td></td>
<td>1910</td>
<td>1.362</td>
</tr>
<tr>
<td>Todd</td>
<td>1909</td>
<td>1.312</td>
</tr>
<tr>
<td>Valentiner</td>
<td>1910</td>
<td>1.336</td>
</tr>
<tr>
<td>Fery</td>
<td>1911</td>
<td>1.559</td>
</tr>
<tr>
<td></td>
<td>1912</td>
<td>1.485</td>
</tr>
<tr>
<td>Shakespear</td>
<td>1912</td>
<td>1.335</td>
</tr>
<tr>
<td>Gerlach</td>
<td>1912</td>
<td>1.413</td>
</tr>
<tr>
<td>Westphal</td>
<td>1912</td>
<td>1.327</td>
</tr>
<tr>
<td></td>
<td>1913</td>
<td>1.334</td>
</tr>
<tr>
<td>Keene</td>
<td>1913</td>
<td>1.411</td>
</tr>
</tbody>
</table>

The convincing proofs afforded by the above workers of the Stefan-Boltzmann law applied to black body radiation established its acceptance as a fundamental law. In view of this and before proceeding to consider further experimental work, it will be well to consider other accepted laws and definitions.
energy incident,

\[ \frac{A}{E} = \frac{\text{energy absorbed}}{\text{incident}} \]

For other than black bodies, this value may vary with the type of surface, the temperature and the wavelength of the radiation.

Emissive power \( (E) \) is defined as the energy radiated per unit surface per unit time. This will also depend upon the type of surface, temperature and wavelength of the radiated energy.

The absolute emissive power is the radiant energy given out per cm\(^2\) per second by a surface at 1\(^\circ\)A to an enclosure at 0\(^\circ\)A.

Kirchhoff's law states that for the same wave-length and temperature the ratio between emissive power and absorptive power is the same for all bodies and equals the emissive power of a black body, i.e.

\[ \frac{E_\lambda}{A_\lambda} = \frac{E_{\lambda}^{11}}{A_{\lambda}^{11}} = \frac{E_{\lambda}^{BB}}{A_{\lambda}^{BB}} = \frac{E_{\lambda}^{BB}}{A_{\lambda}^{BB}} = 1 \]

Obviously the law states that any surface having a large emissive power will have a large absorptive power in the same ratio when compared with the respective amounts for a black body, and vice versa.

From the Stefan-Boltzmann law it follows that the absolute emissive power for a black body is equal to the constant of that law \((K)\).

The reflective power of a surface is the ratio between the radiation reflected and the radiation incident.
The emissivity, a term commonly used (and commonly misused) is now understood to denote the ratio of the energy emitted by a surface, and the energy emitted by a black body under similar conditions of temperature and wave-length. By consideration of the previous laws, this will have the same value as the absorptive power.

Efforts were now made to try and make the Stefan-Boltzmann law fit the results for any surface. The methods of Bottomley and Schleiermacher were improved upon by the perfection of the thermopile etc., so that the radiation could be directly measured, instead of being computed from heating effects.

Wiedeburg, obtained values for metal surfaces, all highly polished. The values were obtained by thermopile for temperatures about 100°C. Expressing the emissivity of Ag as 100, he gave the following figures:

<table>
<thead>
<tr>
<th>Element</th>
<th>Emissivity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ag</td>
<td>100</td>
</tr>
<tr>
<td>Cu</td>
<td>101</td>
</tr>
<tr>
<td>ANa</td>
<td>108</td>
</tr>
<tr>
<td>Al</td>
<td>107</td>
</tr>
<tr>
<td>N2A</td>
<td>108</td>
</tr>
<tr>
<td>Cd</td>
<td>116</td>
</tr>
<tr>
<td>Ni</td>
<td>116</td>
</tr>
<tr>
<td>SnX</td>
<td>122</td>
</tr>
<tr>
<td>Pt</td>
<td>123</td>
</tr>
<tr>
<td>Pb</td>
<td>136</td>
</tr>
<tr>
<td>Sb</td>
<td>227</td>
</tr>
</tbody>
</table>

\[ R = 1 - A \]
Langmuir gave a number of results for metallic surfaces obtained incidentally from a study upon loss of heat by connection. Using a flat plate of metal heated electrically, he calculated the radiation loss from polished silver by Hagen and Rubens' formula.

\[ E = 0.365 \sqrt{\sigma A} \]

where \( E \) = energy radiated
\( \sigma \) = electrical conductivity
\( \lambda \) = wave length of maximum radiation for given temperatures.

Knowing the temperature of the metal plate the value of \( \lambda \) can be calculated from Wien's law.

\[ \lambda = \frac{0.29}{T} \]

Hagen and Rubens' formula was deduced from experiments with polished metals. The reflected energy of an arc lamp from a metallic mirror was analysed spectrometrically, each bundle of wave-lengths being focussed separately upon a thermopile situated at the eye-piece of a spectrometer.

Their values of reflectivity for some metals at about 1.5 \( \lambda \) wave-length are:

<table>
<thead>
<tr>
<th>Metal</th>
<th>Reflectivity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ag</td>
<td>93.4%</td>
</tr>
<tr>
<td>Au</td>
<td>97.3%</td>
</tr>
<tr>
<td>Pt</td>
<td>79.0%</td>
</tr>
<tr>
<td>Cu</td>
<td>94.5%</td>
</tr>
<tr>
<td>Steel</td>
<td>71.9%</td>
</tr>
</tbody>
</table>

It is a difficult point to decide to which degree of polish Hagen and Rubens formula will correspond, and which degree of polish obtained at the time of the experiment.
calculation for their accuracy. Furthermore the value of the constant in Wien's law is 0.29 only for black bodies. For polished platinum it is of the order of 0.25 (see Exer 34). Griffiths. Adopting the calculated value of E for his silver plate he found the loss due to convection from this plate.

Assuming equal convection losses from all surfaces, he obtained the radiation loss from each succeeding metal.

Taking the mean value of the constant for temperatures 50-600°C, the following are expressed as percentage emissivity,

- Cu - oxidised 72
- " - calorised 26
- Silver 3
- Cast Fe bright 22
- " " oxidised 62
- Al paint 50
- Aq enamel 37
- Monel metal bright 43
- " " oxidised 43

Langmuir's results do not show any systematic variation with temperature excess, and are averages of rather widely varying figures. The method used is hardly conducive to good results for radiation, though it may be for convection.

The emissivity for a surface of nickel oxide was examined by Randolph and Overholser, who heated a bright nickel disc to 600°C and then measured the radiation from the resulting oxidised surface at temperatures from 200° - 600°. They compared the radiation with that from a hollow cast-iron pipe electrically heated. In each case the radiant energy was received direct by an absorption pyrometer. This investigation was carried further by Burgess and Footes35.
No doubt, in this instance, the effect of the smooth, finely grained layer of oxide formed by mere superficial oxidation, would give a surface reflecting more rays than would a layer of precipitated nickel oxide. This may account for the rather low value at low temperature.

Burgess and Foote further determined the emissivity of iron oxide, such as is produced upon iron and steel by ordinary weathering. The values obtained show it to vary between 85% - 92% for a range of temperature 500 - 1200°C. Using electrically heated iron tubes they found a considerable drop in temperature from the tube to the outer oxide surface, a fact which may give grievous error if it is neglected.

Further experiments by Burgess and Foote gave the emissivity for rusty iron as 100 - 94% for temperatures 800 - 1060°C.

Work by Bidwell has shown that the emissivity of polished metal surfaces when examined in an atmosphere of nitrogen or hydrogen show a fairly constant value for temperature 700 - 1900°C. He also showed that there was no change upon melting when the surface remained free from any oxidation. Stubbs and Prideaux, however, found that a sharp change took place upon change of state.

Work upon metals under such circumstances is rarely applicable in chemical engineering practice, and hence is of little value here.

Walmsler, using an improved modification of Bottomley's concentric spheres apparatus obtained a number of values applicable to common surfaces. He used a vacuum, but allowed for heat lost from the inner hotter sphere to the cooled outer sphere by conduction. The surface of the surrounding sphere was maintained as a black body, being
for the constants:

<table>
<thead>
<tr>
<th>Surface</th>
<th>Reflectivity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lamp black</td>
<td>$1.23 \times 10^{-12}$</td>
</tr>
<tr>
<td>Brass (dull)</td>
<td>$0.295 \times 10^{-12}$</td>
</tr>
<tr>
<td>Copper (lightly polished)</td>
<td>0.22</td>
</tr>
<tr>
<td>Wrought Fe oxidised</td>
<td>1.222</td>
</tr>
<tr>
<td>&quot;&quot; highly polished</td>
<td>0.363</td>
</tr>
<tr>
<td>Zinc</td>
<td>0.271</td>
</tr>
<tr>
<td>Cast Fe (? surface)</td>
<td>$1.218 - 1.278$ (42-246°C)</td>
</tr>
<tr>
<td>Lime mortar</td>
<td>$1.224 - 1.143$ (21-33°C)</td>
</tr>
</tbody>
</table>

Considering his low value for lamp-black (usually about 1.30) the remainder of his results may be similarly low.

A splendid piece of work was done by Coblentz in 1916, when he determined the reflectivity of various surfaces for a wide range of wave-lengths.

He employed a box-shaped container A, housing a hemispherical mirror B; a small hole C piercing the container and the mirror allowed the radiation to enter and impinge upon a small block D upon which the experimental specimen was mounted. D was so arranged that reflected light was again reflected from B, and brought to a focus upon T, a thermopile. If the position of the thermopile is changed over with D, a measure of the total energy is obtained and hence the reflectivity of the specimen. The sources of the radiation used were acetylene and bunsen flames, the rays being
the loss of some reflected rays through the opening.

A list of the reflectivities (expressed as percentages) are given for various pigments, paints, materials, etc.:

<table>
<thead>
<tr>
<th>Wave length max</th>
<th>0.54 μ</th>
<th>0.60 μ</th>
<th>0.85 μ</th>
<th>4.4 μ</th>
<th>8.8 μ</th>
<th>24 μ</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cobalt oxide</td>
<td></td>
<td>3.02</td>
<td>4.00</td>
<td>13.9</td>
<td>11.3</td>
<td>5.9</td>
</tr>
<tr>
<td>Copper</td>
<td></td>
<td>23.5</td>
<td>15.2</td>
<td></td>
<td>4.4</td>
<td></td>
</tr>
<tr>
<td>Chromium</td>
<td>24.1</td>
<td>27.0</td>
<td>44.6</td>
<td>32.9</td>
<td>5.0</td>
<td>8.2</td>
</tr>
<tr>
<td>Lead oxide</td>
<td></td>
<td>51.8</td>
<td>50.6</td>
<td>25.6</td>
<td>9.5</td>
<td></td>
</tr>
<tr>
<td>Red iron oxide</td>
<td></td>
<td>26.3</td>
<td>41.0</td>
<td>29.9</td>
<td>3.7</td>
<td>9.1</td>
</tr>
<tr>
<td>Yttrium oxide</td>
<td></td>
<td>73.8</td>
<td>34.4</td>
<td>11.1</td>
<td>10.0</td>
<td></td>
</tr>
<tr>
<td>Lead chromate</td>
<td>61.2</td>
<td>70.2</td>
<td>41.2</td>
<td>4.7</td>
<td>7.4</td>
<td></td>
</tr>
<tr>
<td>Alum. oxide</td>
<td></td>
<td>84.1</td>
<td>87.7</td>
<td>20.8</td>
<td>2.3</td>
<td>6.5</td>
</tr>
<tr>
<td>Zinc oxide</td>
<td></td>
<td>82.2</td>
<td>86.4</td>
<td>8.5</td>
<td>3.2</td>
<td>5.1</td>
</tr>
<tr>
<td>Magnesium oxide</td>
<td>86.3</td>
<td>16.0</td>
<td>2.5</td>
<td>9.1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Calcium oxide</td>
<td>85.4</td>
<td>22.3</td>
<td>3.3</td>
<td>6.2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Zirconium oxide</td>
<td>82.2</td>
<td>85.8</td>
<td>84.1</td>
<td>23.2</td>
<td>5.1</td>
<td>5.4</td>
</tr>
<tr>
<td>Lead Co₃</td>
<td></td>
<td>86.8</td>
<td>90.8</td>
<td>29.2</td>
<td>8.3</td>
<td>6.9</td>
</tr>
<tr>
<td>M₆O₇Co₃</td>
<td></td>
<td>89.9</td>
<td>92.8</td>
<td>13.2</td>
<td>94.5</td>
<td></td>
</tr>
<tr>
<td>White paper</td>
<td>71.7</td>
<td>74.7</td>
<td>18.2</td>
<td>5.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>White paper(Bond)</td>
<td>75.2</td>
<td></td>
<td></td>
<td></td>
<td>10.3</td>
<td></td>
</tr>
<tr>
<td>Feldspar</td>
<td>86.7</td>
<td>33.2</td>
<td>14.6</td>
<td>9.7</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bluestone</td>
<td>18.4</td>
<td>8.1</td>
<td>17.6</td>
<td>11.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Quartz powder</td>
<td>81.0</td>
<td>41.5</td>
<td>7.9</td>
<td>13.4</td>
<td>9.0</td>
<td></td>
</tr>
<tr>
<td>Red brick</td>
<td>30.1</td>
<td></td>
<td></td>
<td></td>
<td>12.4</td>
<td></td>
</tr>
<tr>
<td>Material</td>
<td>Reflecting Power</td>
<td>Absorptive Power</td>
<td>Emissive Power</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-------------------</td>
<td>------------------</td>
<td>------------------</td>
<td>----------------</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>White marble</td>
<td>53.5</td>
<td>6.4</td>
<td>5.1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Limestone</td>
<td>42.9</td>
<td>20.3</td>
<td>5.0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Slate</td>
<td>6.7</td>
<td>20.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Velvet Black</td>
<td>1.75</td>
<td>3.66</td>
<td>2.7</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Green Leaf (tulip tree)</td>
<td>21.9</td>
<td>38.0</td>
<td>5.6</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

It is readily understood that a high reflecting power signifies a low absorptive power and hence a low emissive power. From a casual glance at the above list it is seen that in general the substances have a fairly high reflecting power in the visible spectrum, the power lessening towards the longer rays, where all the surfaces have a reflecting power of the same order of magnitude. Whilst the dark coloured pigments reflect badly for small wave lengths, and the white pigments have a high reflectivity for the same range, they both finish for the longer rays with the same reflectivity. Although the figures in the last column (the wavelength corresponding to ordinary temperatures) show some variation among themselves, they cannot be accepted too rigidly, as large errors are shown in a number of duplicate determinations.

Two interesting items in the above list call for attention. The first is the reason for using white paints for covering buildings, roofs, etc. which are exposed to direct rays. Most such paints are made with zinc oxide or lead carbonate as base; these two show a high reflectivity for waves received in greatest quantity from the sun, and a high emissivity for radiation of normal temperatures. Hence any radiation absorbed by the paint
will depend upon the relative values for the reflectivities given in the above table. The second is the fact that ordinary roofing slate, absorbs nearly all the solar radiation and is but a poor radiator of the longer heat rays. It is to be noted how constant the value remains throughout the spectrum for black velvet.

Coblentz, in his commentary, says that substances of highly metallic nature reflect to a greater extent than others in the 7-9μ region. This applies to silicates, carbonates and sulphates, pigments of which should be avoided for painting exteriors. Further, he points out that these metallic compounds depend to some extent upon the fineness of grain in the layer, whilst others, e.g. ZnO, do not suffer change of emissivity with size of grain. He shows also that a highly reflecting surface can be produced from a layer of transparent substance: this is accounted for by the assumption that the ray is internally reflected and refracted so many times that the chances are that the ray will eventually escape from the surface. This, needless to say, will depend upon the thickness of the layer.

The main point of interest now is to investigate the possibility of accounting for the result of the various experiments by some common formula. The main issue is
experiments of Bottomley Schleiermacher, Petavel and others on the noble metals, show an increase in emissivity with temperature. As the increase is found to be fairly regular there is no doubt that an equation could be found for each particular metal. This in fact was done by Wilson and Gray (mentioned previously) for the metal platinum. For chemical engineering practice, however, very little is seen of the noble metals being employed. Most metals of general applicability are liable to tarnish very easily when exposed to air, especially when heated at the same time. It was shown by Bidwell that such metals showed little alteration in emissivity when protected from oxidation but such a condition is rare, so it is necessary to consider the oxidised surface. It has been shown by Coblentz, Randolph and Overholser, Burgess and Foote, Walmsley, Peclet, and others that the commoner metal oxides have a high emissivity at low temperatures, usually about 90%, a figure which remains fairly constant over a large range of temperatures. For metals such as silver, brass and aluminium and their alloys which oxidise only to a slight extent by forming a very thin oxide layer, the emissivity apparently varies with the extent of oxidation or tarnish, most probably eventually remaining fixed when the oxidation has gone as far as it can under the existing conditions.

The large emissivity found by most workers for rusted iron etc., is probably due to the oxide forming on the surface innumerable small cavities, each functioning as a black body. Such an explanation would certainly account for the low reflecting of white pigments for the longer rays: colour
the irregular surface.

The miscellaneous class of surfaces have in general a large emissivity at ordinary temperatures. This is evidenced by the results of Peclet and Coblentz, who found such surfaces to have an emissivity of 85-95% for ordinary temperatures. There is, however, this difference from metallic emissivity, that the value apparently decreases with increased temperature. Very little work shows any evidence for this, beside the comprehensive results of Coblentz. Walmeler, however, shows a decrease for lime mortar of 90% to 83% for temperatures up to 83°C.

However possible it may be to obtain empirical formulas to fit the bright surfaces of the noble metals individually it has been shown that no common formula has been found to account for them all (c.f. Weber and Stefan Boltzmann law applied to Bottomley’s and Schleiermacher’s results). Furthermore it is conclusively proved by the results of Coblentz that no common formula can cover the results obtained with the particular surfaces used in his investigations. It is evident from a glance at his figures that the various surfaces have absorptions bands of different strengths, occurring at various points of the spectrum.

It should be noted, however, that although a surface may show a low emissivity for a given wave-length, the general emissivity for that surface, at a temperature which would produce radiation containing that wave-length as a maximum using a black body, may still be fairly high.

In the Fig. V, this phenomenon is shown diagrammatically.
Curves I, II and III etc. are energy curves for temperatures $T_1$, $T_2$, $T_3$ etc., for a particular surface. Assuming two absorption bands at L and N, near the maximum $\lambda$ for $T_1$, it is seen that they occupy only a portion of the total energy as denoted by the area beneath the curve. These particular bands will have less effect upon the absorption at higher temperatures $T_2$ and $T_3$ as they will appear to the right of the maximum. Furthermore it cannot be assumed that the proportion of absorption at a given wavelength will remain constant at all temperatures.

Whilst the emissivity may in general remain fairly constant over a small range of temperature, no universal law can obtain.
Stefan-Boltzmann law will be applicable to that surface when a modified constant is used.

In the Stefan-Boltzmann equation:

\[ H_r = C \left( T_1^4 - T_0^4 \right) \]

C is equal to the absolute emissive power of a black body (proved above).

\[
\text{Emissivity} = \frac{\text{actual emissive power}}{\text{black body emissive power}} = \frac{H_r}{H_r^b} = \frac{E_m}{E_m^b}
\]

where \( H_r^b \) is the emissive power of a given surface.

i.e. \( H_r^b = E_m^b H_r \)

i.e. \( H_r = E_m C (T_1^4 - T_0^4) \)

The new constant for the surface \( C_1 \) is therefore equal to the "black body" constant times the emissivity.

It will only be legitimate to apply the Stefan-Boltzmann formula to universal use, when the emissivity is known for each range of temperature, and the requisite value used. That the required values for the emissivity for any surface are not known is obvious from the scanty published data. Except by direct determination, the only way to arrive at a value for an emissivity is by intelligent interpretation of the previously described observations, and the subsequent choice of an approximate figure.
Having accepted the Stefan-Boltzmann law for general application, it will be necessary to consider its applicability to heat transfer problems, such as occur in technical practice with any kind of surface.

Primarily the Stefan-Boltzmann law concerns the radiation between two "black" surfaces such as two infinite parallel planes, in a vacuum. That these conditions rarely occur in chemical engineering is obvious; it is therefore necessary to apply the formula to the existing conditions. For chemical engineering plant situated in a factory workshop the conditions are usually favourable to the application of Stefan-Boltzmann law: that is to say, that each point on the apparatus will subtend a solid hemispherical angle to a constant temperature enclosure. For apparatus not complying with this requirement, a separate formula is discussed later.

To consider, then, an infinite plane surface $P_1$ having a radiation constant $C_1$ and at absolute temperature $T_1$ opposed to an infinite parallel plane $P_0$, surface constant $C_0$ and absolute temperature $T_0$; $(T_1$ is greater than $T_0)$: it is required to find the heat exchange between the two.

Let the absorptive and reflective powers of the two surfaces be $A_1$, $R_1$, $A_0$, and $R_0$ respectively.
The radiation $Q_1$ from $P_1$ striking $P_0$ will not be fully absorbed, and an amount $R_0 Q_1$ will be reflected from $P_0$ back to $P_1$. Thus $P_1$ will receive from $P_0$ a quantity $Q_0 + R_0 Q_1$.

Now this radiant energy upon striking $P_1$ will again suffer partial reflection to the extent $R_1 (Q_0 + R_0 Q_1)$. This amount upon returning again to $P_0$ will once more be reflected to the extent $R_0 R_1 (Q_0 + R_0 Q_1)$. Obviously this process will proceed ad infinitum.

Summing up the successive losses from $P_1$ we get

$$Q_1 + R_1 (Q_0 + R_0 Q_1) + R_1^2 R_0 (Q_0 + R_0 Q_1)$$

and the total gains from $P_2$ will be

$$= (Q_0 + R_0 Q_1) + R_1 R_0 (Q_0 + R_0 Q_1)$$

The sum of (1) to infinity is:

$$Q_1 + R_1 (Q_0 + R_0 Q_1)$$

and the sum of (2) to infinity is:

$$Q_0 + R_0 Q_1$$

Hence the nett loss from $P_1 = H_r$

$$= Q_1 + R_1 (Q_0 + R_0 Q_1) - (Q_0 + R_0 Q_1)$$
Since $R_1 = 1 - A_1$ and $R_0 = 1 - A_0$

$$
\frac{R_1 - R_0}{A_1 + A_0 - A_1 A_0} = \mathbf{(6)}
$$

Now since the emissive powers $E_1$ and $E_2$ of two surfaces under the same conditions will be in the same ratio as their respective radiation constants $C_1$ and $C_2$, Kirchhoff's law may be written:

$$
\frac{C_1}{A_1} = \frac{C_0}{A_0} = C_{bb},
$$

i.e. $A_1 = \frac{C_1}{C}$ and $A_0 = \frac{C_0}{C}$

also $Q_1 = C_1 T_1^4$ and $Q_0 = C_0 T_0^4$

" by substituting the proper values of $Q_1, Q_2, A_1$ and $A_2$

in (6)

$$
\frac{H_r = C_1 C_0 (T_1^4 - T_0^4)}{C_1 C_0 + C_0 C_1 - C_0 C_1} = \mathbf{(7)}
$$

dividing throughout by $C_1 C_0$

$$
\frac{H_r = (T_1^4 - T_2^4)}{C_1 + C_0 - C} = \mathbf{(7)}
$$
or more for normal temperatures. Hence \( C_0 \) will almost
always be nearly the value of \( C \) the black body constant.

\[
\frac{1}{C_0} - \frac{1}{C} \quad \text{will be small by comparison with } \frac{1}{C_1}
\]

when \( C_1 \) itself has a low value.

The equation then reduces to:

\[
H_r = C_1 (T_1^4 - T_0^4) \quad \text{......................... (8)}
\]

This will be the more accurate, the greater the value of
\( C_0 \) and the less the value of \( C_1 \), but considering the doubt
concerning the actual values of \( C_1 + C_0 \) it is as good an
approximation as can be obtained. Equation (8) will in
these circumstances tend to give a high result but this
may be cancelled by the extra loss involved by absorption of
a certain amount of the radiation by the water vapour and
carbon dioxide present in the surrounding atmosphere.

It has been shown by Musselt, using a calculation
similar to the above, that the heat lost from a sphere
surrounded by a cooler surface is given by:

\[
H = A_1 \left( T_1^4 - T_0^4 \right) \quad \text{.......................... (9)}
\]

\[
\frac{1}{C_1} + \frac{A_1}{A_0} \left( \frac{1}{C_0} - \frac{1}{C} \right)
\]

where \( H \) = Total radiation loss from inner surface
per unit time

\( A_1 \) = Area of inner surface

\( T_1 \) = Abs. temperature of inner surface

\( C_1 \) = Radiation constant for outer surface

\( A_0 \) = Area of outer surface

\( T_0 \) = Abs. temperature of inner surface

\( C_0 \) = Radiation constant of outer surface
If the simplifications made above to arrive at the same ultimate result for both cases are not permissible, then either equation (7) or (9) must be used.

The choice of the appropriate equation in that case will depend upon whether the surfaces concerned in the heat exchange, approximate to parallel planes or not. For a comparatively small object, such as a pipe running through a room equation (9) reduces to equation (8) by reason of the resultant smallness of the factor $A_1/A_0$.

Having obtained equations suitable for application to general problems, it is necessary to have an accepted list of constant values to use in conjunction with them. The figures for emissivity given above from various workers are scanty and much more work is needed. Most work has been done at one particular temperature, and with no regard to the possible effects of degree of oxidation, roughness of surface, thickness of surface layer, etc., etc. The figures given below are supported to some extent by the empirical work of Norton, Allen and Rowley, and Griffiths and Davis.

41.
Norton, using a steam pipe, the surface of which was
<table>
<thead>
<tr>
<th>Condition of surface</th>
<th>Heat loss</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cast Fe rusty</td>
<td></td>
</tr>
<tr>
<td>&quot; black painted</td>
<td></td>
</tr>
<tr>
<td>&quot; clean</td>
<td></td>
</tr>
<tr>
<td>Aluminium paint</td>
<td></td>
</tr>
</tbody>
</table>

Griffiths and Davis, at the National Physical Laboratory, using a polygonal shaped vessel containing water at a constant temperature, measured the radiation from each face by a thermopile. Each face, of course, was prepared in a different way from its neighbours. A list of their comparative results is:-
The following table is a list of the average results from the more reliable data, and can be considered the best available figures for the respective constants in the Stefan-Boltzmann equation for temperatures of the order of 100°C. In general it appears that the value for the metallic surfaces rises with increase of temperature, whilst from the work of Coblentz the value for the other surfaces seems to decrease. For want of better information, an increase or decrease of 20% for a range of 500°C may be assumed. This range of temperature will usually cover all problems in chemical engineering involving heat loss.

<table>
<thead>
<tr>
<th>Surface</th>
<th>Value of $C_{at}$ temp. 100°C</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Polished silver</td>
<td>$0.04 \times 10^{-12}$</td>
<td></td>
</tr>
<tr>
<td>Tarnished silver</td>
<td>0.08</td>
<td></td>
</tr>
<tr>
<td>Polished copper</td>
<td>0.06</td>
<td>Increases with temperature owing to oxidation</td>
</tr>
<tr>
<td>Tarnished copper</td>
<td>0.20</td>
<td>Depending upon degree of tarnish</td>
</tr>
<tr>
<td>Black oxidised copper</td>
<td>0.95</td>
<td></td>
</tr>
<tr>
<td>Polished Platinum</td>
<td>0.15</td>
<td>Rising to 0.19 @ 1000°C</td>
</tr>
<tr>
<td>Surface</td>
<td>Peclet Constant</td>
<td>Remarks</td>
</tr>
<tr>
<td>------------------------------</td>
<td>-----------------</td>
<td>--------------------------------------------</td>
</tr>
<tr>
<td>Polished Gold</td>
<td>0.15</td>
<td></td>
</tr>
<tr>
<td>Polished Iron</td>
<td>0.15</td>
<td></td>
</tr>
<tr>
<td>Tarnished Iron</td>
<td>0.30</td>
<td>Rising to value for oxid. iron depending upon tarnish.</td>
</tr>
<tr>
<td>Oxidised Iron</td>
<td>1.25</td>
<td>Normal oxidised state</td>
</tr>
<tr>
<td>Polished lead</td>
<td>0.25</td>
<td></td>
</tr>
<tr>
<td>Tarnished lead</td>
<td>0.70</td>
<td>Normal state of oxidation.</td>
</tr>
<tr>
<td>Polished Brass</td>
<td>0.08</td>
<td></td>
</tr>
<tr>
<td>Tarnished Brass</td>
<td>0.30</td>
<td>Normal state of oxidation.</td>
</tr>
<tr>
<td>Polished Zinc</td>
<td>0.25</td>
<td></td>
</tr>
<tr>
<td>Tarnished Zinc</td>
<td>0.70</td>
<td>Normal state of oxidation.</td>
</tr>
<tr>
<td>Tarnished</td>
<td>0.60</td>
<td>Normal state of oxidation.</td>
</tr>
<tr>
<td>Aluminium paint</td>
<td>0.80</td>
<td></td>
</tr>
<tr>
<td>Glass</td>
<td>1.00</td>
<td></td>
</tr>
<tr>
<td>Oil paints</td>
<td>1.25</td>
<td>Value larger for dull and smaller for glossy paints.</td>
</tr>
<tr>
<td>Lampblack surface</td>
<td>1.30</td>
<td></td>
</tr>
<tr>
<td>Non-metallic surfaces</td>
<td>1.25</td>
<td></td>
</tr>
<tr>
<td>Powdered materials etc.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

It is a matter for considerable surprise that no extensive investigation has been carried out to determine the radiation from various commonplace surfaces by direct measurement, or in a word, to re-determine the Peclet constants. There seems no reason, if the surface temperature can be accurately determined, why this should not be done.
equal surface temperatures the least heat is lost by polished surfaces, or by surfaces treated with some metallic application. Hence in exposed metal work, it behoves the user to keep the surfaces as clean and polished as possible; for the non-metallic surfaces the use of aluminium paint will reduce the heat lost by radiation by about 40%. To promote heat dissipation it will obviously be the effort of the chemical engineer to increase the value of \( C \) to the utmost extent, either by permitting normal oxidation or by painting.

**CONCLUSION.**

The various laws suggested to account for radiation from any kind of surface are examined and found to be deficient in every case.

The Stefan-Boltzmann law, accurate for black bodies, can give approximate results under limited circumstances.

A list of constants applicable to the Stefan-Boltzmann law for the surfaces generally found is given; these figures are average values from the more reliable data.
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2. MELLONI, See POYNTING and THOMSON's 'Heat'.
3. DE LA PROVOSTAYE, and DESAINS, See POYNTING and THOMSON's 'Heat'.
4. DULONG and PETIT, Ann de Chemie 7, 1817.
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17. PRESTON'S 'Heat'.
    Jour. de Phys. 1, 1911, p. 551.
    Compt. Rend. 165, 1912, p.1239.
22. BAUER, Compt. Rend. 149, 1909, p. 238
    150, 1910, p. 167.
    " " (4) 39, 1912, p. 469.
   "     "     "  91, 1915, p. 190.
30. WIEDEBURG, Poggend Annalen, 66, 1898, p. 92.
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34. GRIFFITHS "Methods of Measuring temperature", 1918.
36. BURGESS and FOOTE, Bureau of Standards 2, Bulletin No. 1.
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CORRECTION.
The loss of heat from a body by connection is caused by the movement of streams of the surrounding fluid due either to density differences set up by the hot body itself, or by mechanical agitation. As the flow of the fluid and hence the heat transfer from the body, is governed by many physical properties of the fluid, (all of which may vary with conditions in different ways) and the shape and orientation of the body, the laws of connection loss are complicated and necessarily approximate. So many assumptions and approximations have to be made to simplify the relationship that results obtained cannot be accepted as accurate for general application.

Two methods of approaching the problem are general. One is to obtain by thermodynamic and hydrodynamic reasoning, an equation of universal application, and then perhaps to narrow it down to a required isolated case, or to obtain suitable constants from certain practical experiments.

The second method is to perform practical experiments under known varying conditions and to deduce an equation which can translate all the observed results.

It has been found impossible to give a universal equation for every possible case of connection, and it is usual to find that the experiments performed have been too few or of a too particular nature to give adequate proofs to the mathematical results.

As it is the effort of this work to obtain an expression capable of practical application to chemical engineering problems, it is necessary to examine closely the practical and theoretical work already published, with a view to establish an amicable agreement between the two.
is usually termed "natural connection" to distinguish it from "forced connection", that is, connection caused by mechanical draught. As the two effects are necessarily inseparably connected, it will be necessary to include a partial review of forced connection.

Connection losses from surfaces are usually dealt with under two headings:

1) From plane surfaces.
2) From cylindrical surfaces.

In practice this usually reduces to the losses from the sides and top of large apparatus, and the losses from conduit pipes.

Remarkably little work has been done upon natural connection loss from any shape surface; much more work has been done upon "forced connection", although it is mostly of a theoretical nature. Many experimental results are applicable only to the peculiar cases which the authors considered.

**VERTICAL PLANE SURFACES.**

From experiments performed upon the rate of cooling of a thermometer in air and in vacuo, Dulong and Petit deduced an equation connecting heat lost by natural connection with the temperature difference between the cooling body and its surroundings.
where \( H_c \) = loss of heat in unit time from unit area

\[ H_c = K (t_1 - t_0) 1.233 \]

For the purpose under consideration it may be assumed that air under atmospheric pressure is the connective fluid hence \( p^x \) can be considered constant. The equation will then reduce to

For the purpose under consideration it may be assumed that air under atmospheric pressure is the connective fluid hence \( p^x \) can be considered constant. The equation will then reduce to

This equation does not deal exclusively with plane surfaces, but is intended to apply to all shapes, the only variable dependent upon the shape being \( K \).

Peclet confirmed the above equation by means of his apparatus partly described under Radiation. It consisted essentially of a water cooled enclosure in which was suspended bodies of various shapes and sizes. These latter were hollow, and were made to contain hot water, the falling temperature of which was measured by means of a thermometer. Each vessel contained an arrangement of paddles which stirred the water vigorously while it cooled. Great care was taken to ensure accurate readings and a large number of different cooling bodies was used. Whilst it can be accepted that the thermometer was perfectly accurate, it cannot be admitted that it showed the outside surface temperature; despite the vigorous stirring, it is fairly certain that a small drop in temperature occurred across the water-wall boundary. The water-cooled chamber measured 0.8
space connection would be considerably restricted. In this case the results given for the taller bodies may not be so accurate. Dulong and Petit consider their formula to be accurate for temperature excesses up to 250°C but Peclet did not check it beyond 65°C excess.

From his experiments Peclet deduced values for K for vertical planes, spheres and cylinders.

For planes K is given by the relation:

$$10^5 K = 4.90 + 17.9 \frac{1}{\sqrt{h}}$$

when $H_c$ = heat lost in cals. per cm$^2$ per sec.

$h$ = height of plane in cm.

Lorentz, considering vertical walls, obtained a theoretical equation:

$$H_c = 0.548 \frac{C_p g k^3}{u h T} \sqrt{d} \cdot (t_1 - t_0)^{5/4}$$

where (in C.G.S. units).

$C_p$ = specified heat of gas at constant pressure.

$g$ = gravity constant

$k$ = conductivity of gas.

$u$ = viscosity of gas

$h$ = height of plane

$T$ = Absolute Gas temperature

$d$ = density of gas

$t_1$ = Temperature of body °C

$t_0$ = Temperature of surroundings °C

For air at 300°C the following values of the constants are applicable (Kaye and Laby)
Replacing in the original equation, the following is obtained:

\[ H_c = 10.0 \times 10^{-5} \ (t_1 - t_0) \frac{5}{4} h^{-1/4} \]

Lorentz tested his formula experimentally by means of an electrically heated square plate 11 cm square. He seems to have proved his formula to his satisfaction, the only variance shown by experiment being that his constant was too small. His experiments were interpreted by the equation

\[ H_c = 12.5 \times 10^{-5} \ (t_1 - t_0) \frac{5}{4} h^{-1/4} \]

Experiments made with a plate of one size only, however, could not prove that \( H_c \) depended upon the fourth root of the height, and the error in the constant may be accounted for by inaccuracy in the height factor.

Langmuir\(^5\) used an electrically heated plate of polished metallic silver of one size only, 15 cm square. By calculating the radiation as previously described, finding total heat lost by current and potential measurements, and allowing for conduction losses, he deduced the loss due to connection.

Langmuir regarded connection from the film conception point of view, namely that the hot surface is bordered by a layer of gas of definite thickness through which the heat is conducted to the surrounding air. At the outer surface of the gas film the heat is assumed to be carried away by
ductivity of the gas, which itself will vary with the temperature.

Excluding the possible effects of the change of viscosity, specific heat, and density of the gas with temperature, Langmuir obtained an equation allowing for the change of conductivity with temperature, and assuming a constant film thickness.

\[
\frac{H_c}{B} = \int_{T_0}^{T_1} \kappa dt - \int_{T_0}^{T_0} \kappa dt
\]

where \( B \) = film thickness in cms.

By replacing observed values of \( H_c \) and \( T \) in the above equation Langmuir found that \( B \) remained constant at all temperatures, and was equal to 0.43 cm. The equation therefore reduces to

\[
\frac{H_c}{K} = \int_{T_0}^{T_1} \kappa dt - \int_{T_0}^{T_0} \kappa dt.
\]

By plotting Langmuir's experimental results a curve is obtained giving

\[
H_c \propto (t_1 - t_0)^{1.23}
\]

If the values of \( H_c \) and \( t \) given by his formula are plotted

\[
H_c \propto (t_1 - t_0)^{1.24}
\]

is obtained.

As Langmuir did not determine the effect of height upon the heat loss his results can only be accepted as valid for the height of 15 cms.

Hence Langmuir's results may be represented by the simpler formula -
By plotting his experimental figures $H_c$ against $(t_1 - t_0)^{5/4}$, a value of $K = 4.7 \times 10^{-5}$ is obtained.

$$H_c = 4.7 \times 10^{-5} (t_1 - t_0)^{1.24}$$

Langmuir compares his experimental results with figures given by application of Lorentz's formula and he finds his are lower than Lorentz's theoretical formula. Unfortunately Langmuir has used the value for $C_v$ in Lorentz's equation instead of $C_p$ which Lorentz required: substituting $C_v$ by $C_p$ has the effect of increasing the theoretical value by about $8\%$, and of leaving Langmuir's results still further behind those of Lorentz.

The constant thickness film theory requires that the film remains the same thickness not only at all temperatures but for any height of the plane. As previously stated, Langmuir did not examine this point.

The variation of heat loss was more closely studied by Griffiths and Davis, who experimented upon vertical planes and vertical cylinders of heights varying between 5 and 260 cms. They found that

$$H_c = K (t_1 - t_0)^{5/4}$$

$K$ varying with the height of the plane but found that $H_c$ did not vary with the height of the plane according to any simple law, such as that given by Lorentz. By using a 9ft wall consisting of separate electrically heated elements, it was found that the loss per unit area up to a height of 20 cms. gradually decreased, then slightly increased to a constant loss independent of height. The loss obtained from each independent element is shown by the dotted line, whilst
By measuring the velocity streams in the neighbourhood of the wall, it was shown that velocity increased up to the point where constant heat loss obtained and then remained constant. Griffiths and Davis accounted for the two phenomena by supposing that the rising cold air gradually became warmer, (thus taking less heat from the wall), and increased in velocity until a turbulent state was set up, with consequent increased heat loss.

As the maximum air velocity at a height of 20 cms, was of the order of 25-30 cm/sec, it seems quite probable that their conjecture is right; the critical velocity for air at atmospheric pressure travelling along a 2" diam. pipe is about 27 cm/sec. (Walker Lewis and McAdams).

These two workers show that the film theory, even if it can fit the experimental results, does not by any means give a sound account of the actual mechanism of connection. It is found that outside the 0.43 limit there are considerable velocity streams, which extend further from the plate with increasing height of plane. Moreover, whilst the temperature drop at the base of the plane occurs wholly within 0.5 cm, it extends much further from the plane at greater heights. It seems quite certain that the film theory has not been given all the consideration that it merits.

Griffiths and Davis give a theoretical relationship connecting natural connection loss with the properties and conditions of the connecting medium, which they claim fits the results of their experiments. It is obtained by the application of the principle of Similitude: it is:
where \( t \) = temp. excess plane above air.
\( l \) = linear dimension of cooling body.
\( c \) = capacity of heat of the air per unit vol.
\( a \) = coefficient of expansion of the fluid.
\( g \) = acceleration due to gravity.
\( v \) = kinematical viscosity.

\( n \) & \( x \) are unknown constants.

According to the kinetic theory of gases \( \frac{c}{k} v \) is a constant quantity, hence it may be eliminated from the equation.

\[
H_c = \left( \frac{k}{l} \right) K \left( \frac{c^2 l^3 a g}{k^2} \right)^n
\]

If, for a moderate temperature range, we assume \( c, g, a \) and \( k \) to be constant, then \( t \) and \( l \) are the only variables,

\[
H_c = \frac{t}{l} k (t l^3)^n \quad \ldots \ldots \quad (1)
\]

It is said that this equation does fit the facts of the experiment, hence it can be said that \( H_c \sim t^{5/4} \)

Replacing in the above equation

\[
H_c = K l^{-1/4} t^{5/4}
\]

which is the law of Lorentz again.

Hence it would seem that if the above does fit the results, then the heat loss must vary with \( l^{1/4} \), despite the statement to the contrary.

The curve obtained by plotting equation (1) shows that at its upper extremity \( H_c \sim t^{4/3} \), and Griffiths and Davis's results for very high planes show this to be true for the highest points. If now this value is replaced in equation (1)
showing $H_o$ to be independent of height at this point.

This seems strong supporting evidence that the heat loss remains independent of height above a certain height.

It is highly probable from a consideration of all the results that a law similar to that of Lorentz holds good for small heights up to 20 cms., beyond which the index of the height factor increases from $-\frac{1}{3}$ to zero, at a height of 50 cms., after which the height has no effect upon the mean heat loss per unit area.

The values of $H_c$ given by the various equations mentioned above are plotted against the height of a plane for a temperature excess of 100°C in Fig. VII.

It is interesting to note from the figure that for small heights up to 10 cms. Peclet's and Griffiths and Davis's formulas give closely corresponding results. Above this height Peclet gets higher results decreasing, however, to the same constant loss at great heights as obtained by Griffiths and Davis. The theoretical results of Lorentz give a curve similar to that of Griffiths and Davis, but considerably displaced to the left; the curve given by his corrected formula almost coincides with that of Griffiths and Davis in the region of 30 cms. height, but gives lower values for heights under 20 cms. The point marked 'A' shows the position of Lorentz's one experimental result from which the whole curve is predicted.

It is noteworthy that for heights beneath 50 cms. Griffiths and Davis's formula gives a curve similar in shape to that of Lorentz, which shows further evidence of a $h^{-1/4}$ law holding: Lorentz, however, persists in the height factor beyond 50 cms whereas Griffiths and Davis show that $H_c$ remains definitely independent of height above 50 cms.
mentioned no height factor so it is assumed that he intends his results to apply to any height. Despite the remarkably low results obtained with a plate of 15 cms. height, it is seen that when produced vertically upward the value of $H_c$ approximates to the constant value of Griffiths and Davis, and Peclet. This may be quite incidental or may be due to the possibility that Langmuir exposed his plate to air which, having already been heated by some means nearer the floor of the room, was already in the turbulent state mentioned by Griffiths and Davis.

In view of the fact that the two most comprehensive sets of results agree together so well, and that all experimenters find that $H_c$ varies with $(t_1 - t_0)^{1.25}$ or thereabouts, it is intended for this work to accept the formula offered by Griffiths and Davis. This is accepted in preference to that of Peclet because of its more recent determination, and that the experimental methods are open to less criticism than those of Peclet.

Hence for vertical planes losing heat by connection to surrounding still air, the heat loss is represented by:

$$H_c = K (t_1 - t_0)^{1.25}$$

where $K$ is given by: $-\log (A - 50) = 2.22 - \frac{h}{20}$

A curve connecting the value of $K$ with $h$ is given for convenience. (Fig. VIII).
Langmaid, using the same plate as described above, found a decrease of 50% in the convection loss from a horizontal plate, with heated side facing downwards, when compared with the loss from the same surface in the vertical. With heated surface facing upwards an increase of 9% was obtained.

For a plate 100 cms. square, Griffiths and Davis found an increase of 25% when the hot surface faced upward, and a decrease of 35% when it faced downward.

It cannot be accepted that $H_c$ from a horizontal surface, either upward or downward, will be independent of the linear dimension, because air streams of varying types will be set up, much in the same way as those observed for vertical planes.

To consider a surface $S$ facing upward; initially the heated air will rise as $A$, to be replaced by incoming air from $B$ and $B'$, which will sweep horizontally on to the plate, increasing in temperature, until the streams meet in the centre, forming a kind of vortex motion. The limits of this condition will no doubt be governed to some extent by the linear dimensions of the surface.

The movement of the air from a surface facing horizontally downwards will no doubt follow some similar scheme as in Fig. X.
It seems fairly certain that the loss per unit area cannot be constant over the whole surface as streams of air increasing in temperature will be passing from the centre to the edges of the plane which temperature rise will depend upon the dimension of the plane.

By applying the results of Griffiths and Davis's observations to horizontal surfaces, it can safely be assumed that $R_c$ will decrease with extent of surface somewhat in the same way as with vertical heights. As no work has been published dealing with this point, it only remains to accept Griffiths and Davis's figures (given above) for surfaces of any extent, until further evidence is produced.

Hence, unsatisfactory as it may be, a decrease of 35% on vertical values applying to the particular linear dimension is accepted for horizontal surfaces facing downward, and an increase of 25% for those facing upward.

For a baring surface that is not square, presumably an average value of the boundaries will apply.

Included on Fig. VIII are given curves connecting the tentative value of $K$ with the average boundary dimension for the two kinds of surface.

Experimental work is urgently required to settle the above problems, before any reliable rules can be given.
that the same laws apply to vertical cylinders as for vertical planes. As the air streams produced around a heated cylinder would be essentially the same as along the surface of a plane, provided the cylinder does not assume too small proportions, it is fairly safe to accept the above generalisation as true.

**Horizontal Cylinders.**

The only published work of any importance concerning natural connection from horizontal cylinders has been done by Peclet. Nearly all the work in this direction has been done with a view to determining heat-transmission for problems connected with tubular heaters, boilers, super-heaters, etc., under the special conditions required by the experimenters. Most of these experiments have been for the requirements of industrial concerns, and few are fundamentally of any practical use.

For the losses from cylindrical pipes most work has been done with mechanically produced draughts running parallel to the direction of the pipe, and frequently inside the pipe. Many equations offered are extremely complex and unwieldy.

A fair amount of information is published concerning
whether results deduced from these experiments can be applied with any accuracy to larger diameters.

To consider work dealing primarily with natural connection, it will be convenient to deal with each worker individually and to correlate their results later.

The only comprehensive research upon connection from horizontal cylinders of a size generally met with in practice is due to Peclet. Using the same apparatus as previously described, he determined the losses from cylinders varying in diameter from 3 to 30 cms.

In proving that Dulong and Petit's connection law was correct, Peclet evaluated the constant, and its variation with the radius of the cylinders. He obtained the following:—

\[ 10^5 V = 5.717 + \frac{10.61}{r} \]

where \( r \) is in centimetres.

Peclet's results are here subject to the errors enumerated above, and can be accepted with the same degree of confidence as his figures for plane surfaces.

S. Kennelly, using fine wires electrically heated of 0.011, 0.026 and 0.069 cm. diameter, obtained a number of results for natural connection under varying circumstances.

Mainly his work was to find the relation between the connection loss and the pressure of the connective fluid; he gives a complicated equation involving terms of dimension, temperature and pressure for atmospheric air. As air of normal pressure is under consideration at the moment it is necessary to abstract figures from his list of results, which relate to the same pressure conditions.
plotting the connection loss against the temperature excess on a logarithmic curve, it is found that \( H_c \) varies directly with \((T_1 - T_0)\) (Fig. XI). Further, by obtaining from the curves so drawn, values of \( H_c \) for each wire for one given temperature excess (e.g. 100°), the relation between \( H_c \) and the radius is obtained. By drawing a line through the three points so given, the most reliable result obtainable with so few experimental points, is procured. In this \( H_c \) varies with \( r^{3/4} \).

Hence \[ H_c = \frac{K(t_1 - t_0)}{r^{3/4}} \]

By applying the above formula to Kennelly's results an average value of \( K \) is obtained.

It is equal to \( 6.9 \times 10^{-13} \)

In experiments with fine wires it is very possible that errors may be present due to the vibration of the wire caused by the connection currents, or to lack of proper calibration of the wire resistance with regard to its change with temperature, thus causing error in the temperature reading. Langmuir considers that Kennelly used too great a value for his radiation connection, namely 0.94 that for a black body. As it is difficult to know exactly what the surface condition of the wire was at the time of the experiment, and as radiation accounts for only a small proportion of heat loss in any case from such small wires, it is useless to tamper with the published results.

Davis has applied his formula (mentioned above under "Connection from planes"),

\[ H_c = \frac{k(t_1 - t_0)}{r} \left[ \frac{c^2 r^3 a g(t_1 - t_0)}{k^2} \right] \]
From the above equation it is clear that if values of $(t_1 - t_0)$ and $r$ are taken to make the right hand side of the equation remain constant, then it would be expected that the corresponding value of $\frac{H - r}{t_1 - t_0}$ would remain constant.

For temperatures between 60 and 7.5°C the relative value of $\frac{H - r}{t_1 - t_0}$ increases by 13%, whilst the corresponding values for spheres and vertical planes decrease for the same range of temperature by about 4%.

This shows either, that the theoretical equation does not apply so well to horizontal cylinders, or that Peclet's formula does not translate the results for horizontal cylinders so well as for other shapes.

By making experiments with electrically heated wires of .0633 cm and .0155 cm. diameter, suspended in liquids, Davis found that his equation

$$\frac{H - r}{(t_1 - t_0) k} \sim \frac{(c^2 g r^3 a (t_1 - t_0)^x}{k^2 \left( \frac{c v}{k} \right)^y}$$

was satisfied by $x = 1/6$ to $1/10$

$y = -1/8$ to $-1/10$

depending upon the radius of the wire. These being so close he assumed that $x = -y$

$$\frac{H - r}{(t_1 - t_0) k} = k \left( \frac{c a g r^3 (t_1 - t_0)^x}{k v} \right)$$
If the value \( x = 1/9 \) is adopted, then assuming \( c = g \) and \( k \) to be constant over a small range of temperature

\[
H_c = K (t_1 - t_0)^{10/9} r^{-2/3}
\]

for fine wires.

This changes to

\[
H_c = K (t_1 - t_0)^{5/4} r^{-1/4}
\]

for larger cylinders, (probably of the order of greater than 10 cms. diameter)

Langmuir, following his 'film' theory, made experiments upon the natural connection from fine wires, at temperatures varying between 1100 - 2300°C. Using the formula

\[
H_c = \frac{2 \pi}{\log_3 b/a} \int_{T_0}^{T_1} K dT
\]

where \( b = \) radius of gas "film"

\( a = \) radius of wire

an adaptation of his formula mentioned under "Plane Surfaces", applicable to cylinders, he calculated the thickness of the "film". He obtained results varying from 0.27 - 0.60 cm, but as the variation did not correspond in any regular manner with temperature or radius, he took the average value, namely 0.41 as the constant film thickness.

Langmuir further, assumed that the film thickness would only vary with viscosity, which caused the film, and density, which determined the thickness, whether the variation was caused by changes in temperature, pressure or nature of the gas.

However on applying his 'film' equation to Kennelly's connection results in still air at various pressures,
An average result of \( \bar{B} = 5 \) was obtained (where \( B \) is the film thickness).

Griffiths and Davis, did not publish any results of experimental work on horizontal cylinders, nor yet is there any mention of such work having been done. It seems that Davis's formula \( q = \alpha \sqrt{T} \) has been accepted as true for horizontal cylinders, after its fairly successful application to Péclet's results, and tables of values of \( \alpha \) are given pertaining to such bodies, assuming \( H_c \sim K T^{5/4} \). These values are plotted on Fig. XII for comparison with Péclet's results. It will be seen that they agree fairly well together, especially in that \( H_c \) is independent of the radius above a certain maximum radius, \( (10 \text{ cms.}) \).

For values below \( r = 1 \text{ cm} \) it is seen that Griffiths and Davis give \( H_c \sim r^{-3/2} \).

As the origin of these results is not divulged very little reliability can be placed in them.

No more work dealing with natural connection from horizontal cylinders can be found, but recourse is made to some results tabulated by Petavel and McMillan. These are obtained from work connected with general heat loss from cylinders, and although no attempt was made to derive any fundamental issue, it is possible to use the tabulated data to determine the connection-temperature-difference relation.

Petavel used pipes of \( 1/4 - 12 \text{ inches in diameter} \), and determined the total heat loss by passing in steam at a definite pressure and measuring the amount of condensate produced. Assuming the surface to be at the same
The method of determining heat lost by steam condensation method is open to great criticism. In the first place it is difficult to entrap the whole of the condensate so that it can be measured without losing any of the heat it contains. Moreover, whatever precautions are taken to eliminate moisture, it is very difficult to obtain perfectly dry steam, and any "spray" passing through into the experimental pipe, will increase the amount of apparent condensate. It seems that more than 20% moisture may be carried over in the form of fine spray. Although with ordinary steam there is little temperature drop across the steam-pipe-air boundaries, it has been shown by McMillan and others that a drop of about 1° - 2°C occurs with an uncovered pipe for steam at temperatures 60° - 200°C.

By taking Petavel's value for the radiation constant for the surface of his pipes i.e. 5/16 of black body, and subtracting the resulting radiation (H_r) from his values, the value of H_c is obtained for each diameter pipe, over the same range of temperature. By plotting H_c against temperature excess logarithmically, a series of straight lines are obtained the average slope of which gives

$$H_c = K (t_1 - t_0)^{1.12}$$  (Fig. XIII)

Further by plotting H_c against the radius for given temperature excesses it is shown for small cylinders (below 2.5 cm radius) that H_c and r^{-3/2} (Fig. XIV). The dependence upon r gradually decreases until the radius equals 10 cms., or thereabouts, when H_c is independent of r.

Hence for small cylinders

$$H_c = \frac{K (t_1 - t_0)^{1.12}}{r^{3/2}}$$

where K = 2.8 x 10^{-4}. 
to about 7.8 inches. The pipe was electrically heated, and temperatures inside and on the surface of the pipe and insulation were taken in a large number of places by means of thermocouples. Great care was taken to eliminate errors, but it is probable that the readings given by the thermocouples on the surface were too low. Although no results of fundamental use are given, results for connection may be obtained by assuming a radiation constant of 0.90 black body for all the insulator surfaces, and subtracting the radiation from the total loss. By plotting his figures and drawing an average curve it is found that (Fig. XV.)

\[ H_c = K (t_1 - t_0)^{1.5} \quad (K = 1.1 \times 10^{-5}) \]

As the same pipe was used throughout, no information is obtainable with regard to the effect of the radius.

Owing to the fact that such a small amount of reliable information is available concerning the natural connection from pipes, it is intended to review some of the more important work upon forced connection, with a view to finding further evidence of the variation of \( H_c \) with temperature, radius etc.

Ser,\(^1\) by passing cool air along the surface of a vertical tube containing hot water obtained the relation

\[ H_c = K \sqrt{v} (t_1 - t_0) \]

Corrections for radiation were made by using Peclet's constants.

An interesting sequel to this equation is demonstrated by Ser, who shows that gas velocity in chimneys varies with the square root of the temperature difference.
Hence his original equation assumes the form:

\[ H_c = K (t_1 - t_0)^{5/4} \]

which is Peclet's original connection equation.

The natural connection from a horizontal pipe will obviously be perpendicular to the axis of the pipe, whilst the above result has been deduced from experiments with forced connection parallel to the axis of the pipe.

Osborne Reynolds\(^{14}\) regarded connection as the conduction of heat through a surrounding gas film.

If \( b \) is the thickness of the film then

\[ H_c = \frac{K (t_1 - t_0)}{b} \]

The thickness \( b \) is obtained from a consideration of the resistance to the flow of the air. The velocity gradient is \( \frac{v}{b} \) and the resistance to flow therefore is

\[ R = \frac{u v}{b} \]

Now \( R \) for a fluid flowing inside a pipe is given by

\[ R = k d \frac{v^2}{u} \left( \frac{v r d}{u} \right)^{n-2} = \frac{u v}{b} \]

\[ \therefore \frac{1}{b} = \frac{K}{r} \left( \frac{v r d}{u} \right)^{n-1} \]

and

\[ H_c = \frac{K}{r} \left( \frac{v r d}{u} \right)^{n-1} (t_1 - t_0) \]

Apparently for internal flow \( n = 1.8 \).

Hence

\[ H_c = K \frac{v^{0.8} d^{0.8} k (t_1 - t_0)}{u^{0.8} r^{0.2}} \]

A theoretical equation for forced connection obtained by Boussinesq\(^{15}\) may be written in the form
If it can be assumed, with Ser and others, that $n = 0.5$.

Hence

$$H_c = K \frac{(t_1 - t_0) k^{0.5} \nu^{0.5} \rho^{0.5}}{r^{0.5}}$$

Another theoretical equation was deduced by Russell, who considered that the fluid had no viscosity, was incompressible and moreover that the density and conductivity did not vary with temperature.

$$H_c = K \frac{c d k v r}{\nu} (t_1 - t_0)$$

This formula can only be considered approximate, but Langmuir found that it fitted the results of Kennelly fairly well if $K$ was varied with the radius of the pipe or wire. Hence $H_c \propto \sqrt{r}$ does not satisfy the observed results as the equation requires.

It is necessary to note the observations published by Carrier. Whilst the work of Carrier may be useful to manufacturers of blast heaters etc., it is in no way of any fundamental use. He assumed that heat was lost by conduction through an air-film, and hence varied directly with the temperature drop, pipe to air.

To test his formula he blew air across a series of steam pipes arranged as in a heater, and found the heat lost by the amount of water condensed. The errors coincident with this method of calculating heat loss are discussed above. Had the experiment been conducted upon a single pipe and the results corrected for radiation, his figures might have been of some use. However, his experiments gave values $8\%$ too high according to his theoretical deductions so he promptly assumed $8\%$ error in his velocity readings, and obtained a good curve.
0.01 - 0.07 cms. diameter at temperatures excesses up to 180°C. For connection perpendicular to the axis of the wire he deduced the following equation:

\[ H_c = (a + b r) \left( t_1 - t_0 \right) \sqrt{v + 25}. \]

where \( a \) and \( b \) are constants, and \( a \) very small compared to \( b \).

Here it is seen that approximately

\[ H_c = K(t_1 - t_0) \sqrt{v} \]

When considering forced connection results from fine wires it must be remembered how large the experimental error may be, due especially to the vibration of the wire in the air-stream.

Another experimenter who determined losses from fine wires was King. He used platinum wires and obtained a series of equations of the type

\[ H = B \sqrt{v} + c \]

where \( B \) and \( c \) are constants.

An equation of this type was obtained for each wire at each temperature difference, but no attempt was made to correlate the whole series into one comprehensive equation involving the temperature and dimensional factors. From specimen sheets of King's results an attempt has been made to find the relation between the temperature difference and the heat loss for each particular velocity. King used very great care in the performance of his experiments, and no doubt his published figures are very reliable. He did not, however, correct his results for radiation losses, which in the case of fine polished platinum wires is nevertheless very small. To obtain the \( H_c \propto T^R \) relation, the figures
but apparatus \(T^2\) for higher velocities. (Fig. XVI).

Since from King's equations and values of \(B\) and \(C\) it is found that \(H_c \propto \sqrt{v}\) for the higher velocities, it can be assumed that for these velocities

\[ H_c = K \sqrt{v} (t_1 - t_0) \text{ (approx.).} \]

For lower velocities the effect of velocity is diminished and the temperature effect rises to \(T^{1.25}\).

Langmuir performed no experimental work upon forced connection losses from cylinders, but attempted to apply his film theory to the results of Kennelly. In this he admitted failure, and said finally, that the film theory breaks down under forced connection conditions. It is a serious drawback to the film conception method of calculating connection loss, that it is only applicable under given conditions.

Langmuir stated that \(H_c\) varies directly with density, whether density changes are caused by temperature or pressure changes or by the composition of the connective fluid. This statement, however, is not backed up by any experimental data.

A series of experiments using cylinders of intermediate sizes viz., 0.43 - 15.5 cms diameter, was performed by Hughes. Here, however, the cylinders were vertical, but as forced connection was caused to act in a transverse direction, the results are considered here. The apparatus, unfortunately, was very elementary, and the results can only be considered with very great reserve.

Primarily the apparatus consisted of the cylinder suspended across a horizontal channel along which the
an inefficient water-trap, and any condensate was caught beneath in a weighed beaker. Besides the inherent errors of this method, Hughes assumed the outer walls of his cylinders to be at 100°C, and gave no correction for losses from the exposed ends. Moreover no air-temperature was recorded.

By plotting the connection loss (after suitable radiation correction) for each diameter against the velocity, he found that in the relation $H_c \propto v^n$, the value $n$ varied from 0.55 to 0.98, according to increasing diameters. Hughes thinks this may explain why some workers obtain $H_c \propto v^{1.0}$ for large bodies, and other workers find $H_c \propto v^{0.5}$ when dealing with fine wires.

<table>
<thead>
<tr>
<th>diameter</th>
<th>$n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.43 cms</td>
<td>0.55</td>
</tr>
<tr>
<td>0.81 &quot;</td>
<td>0.60</td>
</tr>
<tr>
<td>1.93 &quot;</td>
<td>0.65</td>
</tr>
<tr>
<td>5.06 &quot;</td>
<td>0.70</td>
</tr>
<tr>
<td>15.50 &quot;</td>
<td>0.98</td>
</tr>
<tr>
<td></td>
<td>(0.60 (at low velocity))</td>
</tr>
</tbody>
</table>

By converting Hughes' tabulated figures for $H_c$ and plotting the value against the radius at given velocities, (Fig. XVII), a relation between the two is obtained.

For radii between 0.2 and 2.5 cms, the value of $n$ in $H_c \propto r^n$ varies between $-0.8$ and $-4.8$, with increasing velocities. The largest diameter cylinder gives results quite out of keeping with the other four, which may be accounted for by the fact that the large cylinder was too
are unexpectedly high, the high value of \( n \) in the velocity equation given by it may be explained, and shown to be incorrect. This causes a little surprise when the crudeness of the method is taken into account.

General conclusions from Hughes' works show that

\[
H_c = \frac{K v^{0.6}}{r^{0.55}}
\]

Nusselt obtained a formula by hydrodynamic and thermodynamic reasoning, connecting the heat loss from a pipe by a stream of gas flowing along inside it.

This is only Reynolds' equation re-stated however.

\[
H_c = \frac{K (t_1 - t) k_{wall}}{r} \left( \frac{v dr}{u} \right)^n \left( \frac{u gp}{k} \right)^m
\]

where \( k_{wall} \) = conductivity of fluid at wall temperature.

According to the kinetic theory \( \frac{u C^2}{k} \) is a constant hence

\[
H_c = K \frac{(t_1 - t) k_{wall}}{r} \left( \frac{v dr}{u} \right)^n
\]

By experiment he obtained the formula

\[
H_c = K \frac{(t_1 - t) k_{wall}}{r} \left( \frac{v dr}{u} \right)^{0.786}
\]

Hence for small temperature differences may be written,

\[
H_c = K \frac{(t_1 - t_0)}{r^{0.72}}
\]

Nusselt applied his fundamental equation to the results of Hughes (mentioned above). To apply a formula produced from consideration of parallel fluid flow inside a pipe to the conditions of an external fluid flow perpendicular to the axis of the pipe is certainly a reprehensible practice.

Nevertheless, by making the assumptions necessary owing to
connection.

\[ H_c = K (t_1 - t_0) \frac{k_r \left( \frac{C + r v d}{u} \right)}{r^{0.72}} \]

where \( C \) is a constant.

\( k_r \) is the fluid conductivity at the mean temperature of the wall and gas.

Despite this crude method of obtaining a formula, Nusselt's work was partly corroborated by Reiher\(^{22}\), who arrived at the same formula for external perpendicular connection from theoretical considerations.

\[ H_c = K (t_1 - t_0) \frac{k_m (v r d_m)^n}{r \left( \frac{u_m}{r} \right)^n} \]

where \( d_m \) = the mean gas density

\( u_m \) = the mean gas viscosity.

By passing an air blast across pipes heated by running water he evaluated \( n = 0.56 \). The pipes used varied between 0.5 and 3.0 cm diameter.

Hence the equation reduces to

\[ H_c = K (t_1 - t_0) \frac{k}{r^{0.44} u^{0.56}} \]

and for small temperature differences

\[ H_c = K (t_1 - t_0) \frac{v^{0.56}}{r^{0.44}} \]

A further equation of the Reynolds' type is published by McAdams and Frost\(^{23}\).

\[ H_c = K (t_1 - t_0) \frac{(r v d)^{0.80}}{r \left( \frac{u}{r} \right)} \]

That this type of equation fits the experimental facts fairly well is evident, but whilst the external connection
that the respective exponents will be the same, or in the same ratio, for both cases. \( \text{Nusselt's application of his formula, to Hughes' results is not quite successful as is evidenced by the somewhat clumsy resulting equation.} \)

\( \text{Reihen, however, shows that the equation is directly applicable in both cases, with a modified index value.} \)

From previously reviewed work it is shown that the dependence upon \( r \) changes greatly below a certain maximum for \( r \), but neither Reynolds, Nusselt nor McAdams and Frost, give any indication of radius limits for their equations, or any possible changes in the respective index values.

\( \text{Reihen, on the other hand, states definitely the size of pipe to which his results apply.} \)

\[
\text{There is a certain confusion as to whether } H_c = K \left( t_1 - t_0 \right)^{1.00} \text{ for natural connection, or } = K \left( t_1 - t_0 \right)^n \text{, where } n \text{ is some number between } 1.00 \text{ and } 1.25.
\]

\( \text{Now, fundamentally connection loss is controlled by the conductivity of the connective fluid: if there was no conductivity no heat would be transmitted despite velocity, density, specific heat, etc. Hence it is a safe theoretical basis to say that connection loss varies directly with temperature. That } H_c \text{ is found empirically to increase at a greater rate than this, is due to the incidental change of the properties of the fluid. In a general way the heat transfer would increase with increasing density, conductivity} \)
specific heat increase with rising temperature, density decreases, so a constant change of total effect is taking place. Furthermore while these properties do not vary with temperature in a constant manner, their total effect may vary over the scale of temperature.

The issue is still further complicated by the velocity effect, which is not properly known, and whilst applicable chiefly to forced connection is nevertheless present to some extent in natural connection. The attempt by Ser to correlate his 'velocity' equation with Peclet is interesting, but may not be universally applicable.

It is possible, by plotting the values of viscosity, density, conductivity and specific heat of air against the temperature on a logarithmic curve (Figs. XVIII, XIX, XX), to find how each property depends upon the temperature in a given temperature range, and thus to see whether the increased temperature index can be accounted for from the individual indices. When these values are substituted in any of the preceding equations, for fairly large bodies, it is seen that the temperature effects cancel out, the higher the temperature the more nearly they cancel. This fact justifies the elimination of these variables from the said equation, and shows that the increased temperature index must be due to the velocity effect.

From the results of forced connection nearly always \( \frac{h}{c} \sim v \), so Ser's suggestion may yet be accepted as true, and be taken to explain the index value of the temperature.

For small cylinders and fine wires the issue becomes more complicated however. Davis found that his formula required a lower index figure the smaller the radius of the
By reference to the figures of temperature effect upon the variables, it is found that the whole of the above equation becomes

\[ H_c = k (t_1 - t_0)^{1.23} \]

which shows that for small radius wires it is not permissible to neglect the remaining variables as it is in the case of the larger bodies.

Moreover, it is shown that the temperature index remains fairly constant around 1.25 for every size of cylinder, when using Davis's formula.

By making use of the Reynolds's type of equation involving velocity

\[ H_c = \frac{K k (t_1 - t_0) (v d r)^n}{r} \]

a similar state of affairs can be shown to exist.

From the general average of both theoretical and experimental results of the reviewed workers, it is seen that the effect of the radius grows with decreasing radius, and for quite small wires \( H_c = K r^{-0.66} \) or thereabouts.

If it is assumed that this value for \( r \) holds good, then in the Nusselt equation -

\[ H_c = \frac{K k (t_1 - t_0) (r d v)^{0.33}}{r} \]

Extending this to include only temperature effects then,

\[ H_c = \frac{K (t_1 - t_0)^{1.10} v^{0.33}}{r^{0.66}} \]

If now considering somewhat larger cylinders when \( H_c = K r^{-0.5} \).
Thus with increasing temperature index there is an accompanying lowering of the velocity index, and although the actual relationship between temperature and velocity is not properly known, it seems likely from Ser's work, that the temperature index would remain somewhere in the region of 1.25 in an empirical equation involving only temperature and dimension.

Of course, the introduction of the forced connection equations and data to support thenatural connection laws is wholly dependent upon the supposition that the velocity effect will be the same whether set up by mechanical means or by the heated body itself. Whether this application is permissible or not must be determined by separate experiment. The empirical equation

$$H_c = K (t_1 - t_0)^{1.25}$$

can only apply when the acting velocities are produced wholly by temperature differences.

It is interesting to note that King's results showed that

$$H_c = K (t_1 - t_0)$$

when the velocity of air was very low, of the order of natural connection velocities.

As previously mentioned the effect of \( r \) increases with decreasing radius; it varies from \( r^{-0.25} \) for large cylinders up to \( r^{-0.66} \) for fine wires. It would be confidently expected that the effect of \( r \) would have no connection with the temperature or velocity effects.

To evaluate the value of \( n \) for the various sizes of cylinder it is necessary to glance at the previously discussed results again.
work, and, perhaps, the Reynolds's type equations. As these latter account primarily for a different kind of connection, it has been shown that they are applicable to the present problem when modified somewhat. Above 10 cms radius $r$ has no effect as shown by Griffiths and Davis Peclet and Petavel.

For values of $r$ between 0.10 and 1 cm, $n$ has the value 0.5. This is approximately the value given by Boussinesq's theoretical formula, and Hughes's and Davis's practical results. Griffiths and Davis give values for $K$ in their theoretical equation which vary with $r^{-0.5}$ below a radius of 1 cm. They, however, assume 0.5 to be the index number of all cylinders below 1 cm radius, but experiment seems to show that the dependence upon $r$ changes beyond this for fine wires. Rother found $H_c$ to vary with $r^{-0.44}$, but as the size of his pipes varied between 0.5 and 3.0 cms, that is to say, were somewhat on the large side, this lower figure may be accounted for. Petavel gives a figure $-0.25$ for cylinders below 2.5 cm radius, but his experimental methods as previously indicated may have led to some error in calculation.

Another experimenter, Russell, gives $H_c$ to vary directly with the radius, and not inversely, as generally proved true. This formula, which is intended to be true for cylinders of all radii, has been shown by Langmuir to be quite wrong as far as the radius factor is concerned.

Finally, for fine wires, the radius index appears to increase to about $-0.66$. This is supported by Davis's theoretical formula, and his practical results, which requires a value of $-2/3$ for wires; and further by
A good average value is about 0.66.

Hence \( n \) may be given the values below for the particular radius range:

<table>
<thead>
<tr>
<th>Radius of wire</th>
<th>Value of ( n )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Above 10 cms</td>
<td>0</td>
</tr>
<tr>
<td>From 1 cm to 10 cms</td>
<td>.25</td>
</tr>
<tr>
<td>&quot; 0.1 &quot; &quot; 1 &quot;</td>
<td>.50</td>
</tr>
<tr>
<td>Below 0.1 cm</td>
<td>.66</td>
</tr>
</tbody>
</table>

Of course these radius ranges are bounded by quite arbitrary divisions which can only be fixed by more complete experimental evidence. It is not known whether there are any sharp changes in the index, or the index changes smoothly from 0 to -.66.

If we assume with Langmuir, for a moment, that the film thickness around a heated body is of the order 0.5 cm, then it is seen that beyond 10 cms, that is where the film thickness is fairly insignificant, \( r \) has no effect. When the film thickness becomes comparable with the radius, there is a sharp change. With still smaller radii the film assumes much greater relative importance and the index again increases. For wires when the radius is insignificant compared with the film thickness, the radius effect again increases to a constant maximum.

By including the \( r \) factor in the constant \( K \), the equation assumes the Peclet form

\[
H_c = K \left( t_1 - t_0 \right)^{1.25}
\]

where \( K = \frac{1}{r^n} \)
r against an arbitrary value of \( K \), and accepting the average constant value for large cylinders as given by Griffiths and Davis and Peclet as true. Connection of heat is unaffected in any way by the actual condition of the surface, (as is radiation) below a certain velocity. The critical velocity given by Marshall\(^{24}\) is 1000 cm per sec., and so does not affect the work under consideration.

**CONCLUSION.**

Loss of heat by natural connection can be expressed mathematically by the same equation for any shaped body. The magnitude of the constant will depend only upon the shape of the heated body.
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CONDUCTIVITY.
Although all the standard books of reference give lists of accurate conductivity values for metals and other materials which can be obtained in a pure condition and in a definite state, it is with difficulty that reliable figures for the normal insulating materials can be found. It is rare to find two authorities agree on any one material, and more rare is it to find that the particulars of the sample considered are published. As will be seen later from the tables, wholly different values for an apparently similar substance are obtained, each by a quite reliable method of determination.

In measurements of conductivity the difficulties are not met with, that are present in radiation and connection experiments.

Conductivity can be rigidly defined and directly determined, hence modern methods are but refinements of the older methods, and results obtained by various observers should agree closely, assuming a definite conductivity value is possible.

Upon closer examination of the work published, it is found that each material has not a definite conductivity peculiar to itself, but that its conductivity value varies with many conditions, the effects of which are not thoroughly known.

Before a review is made of these effects, a general criticism of the methods for determination of conductivity will be of assistance.

First of all, conductivity is defined as the amount of heat transmitted in unit time between two parallel walls.
Although he mentions experiments made by Despretz, Peclet appears to have been the first to make detailed investigations concerning the insulating properties of poor conductors.

Making every due allowance for the knowledge and apparatus at his disposal, Peclet seems without a rival for the care and patience shown in his conductivity experiments.

For insulating materials he devised three distinct methods.

The first consisted essentially of a layer of the material packed into the annular space between two thin copper spheres. The inner sphere contained hot water, whilst the whole arrangement was immersed in a bath of cold water. The water, both inside and outside, was stirred vigorously by paddles which moved as close as possible to the copper surfaces. The loss of heat from the inner to the outer water, was determined by noting the fall and rise of
Whilst the sphere is theoretically the best shape available for conductivity experiments, practically, errors arise from the difficulty of enclosing the source of heat completely. In the above case some heat would inevitably be lost by conduction through the metal supports of the spheres (which projected from the sides of the bath) and along the stems of the thermometer and stirring apparatus. The violent agitation of both the hot and cold water would give wrong indications of the temperature changes. Again, the temperature indicated by the thermometers would be almost certain to differ from the actual temperature of the surfaces by reason of the temperature gradient at the liquid-solid boundaries, despite the vigorous agitation of the liquid. It is evident that for bad conductors, the losses due to these causes will be far greater in proportion than for good conductors, e.g. the metals. Lastly, there is the difficulty of obtaining homogeneous distribution of the material even in powder form, when it is present in an annular space of any kind.

Péclet's second method was essentially similar, the material being packed in the space between two vertical cylinders of paper or glass. The inside was heated by passing in steam, the temperature of the steam being taken as the same as that of the walls. The outer surface temperature was taken by means of a copper resistance thermometer. The errors of this apparatus are even greater than those of the first, for in addition to those mutually applicable, there would be loss along the uninsulated inlet and outlet for the steam, the error due to the steam entraining spray, and
by placing one around the outer cylinder and keeping the other in a variable temperature bath, there seems considerable doubt concerning its accuracy.

His third method was most unusual, especially in the matter of finding the surface temperature. A rectangular box had one side composed of a slab of the insulator and otherwise fitted with inlet and outlet for hot water or steam, and a mercury thermometer. Another box, all metal, but of similar external shape, contained water which could be raised to any desired temperature. The two were placed equidistant from a thermopile, the surfaces facing it being covered with a layer of paper to give equal radiating powers. When equilibrium was reached in the test box by passing in constant temperature water, the temperature of the water in the other vessel was raised until a zero reading was obtained upon the thermopile. Although this doubtless gave the true surface temperature the same, it cannot be admitted that the thermometers in the boxes gave the true temperatures of the surfaces, inasmuch as no stirring arrangement of any kind was used.

Thus although Peclet's experiments on conductivity are valuable as a pioneer work, their accuracy is questionable, and it must be left to later work to give more accurate results.

2. Next in the field appears to be Ordway, who in 1884 published a method using condensing steam as the heating agent. The steam was passed into an insulated pipe and the
surrounded by a large calorimeter containing water, the
temperature of the water giving the temperature of the outside
of the insulation. Besides the difficulty of isolating
a portion of the steam pipe completely, there was the danger
of the steam carrying over fine spray and incondensible
gases with it. Ordway, himself, admits finding in the steam
any quantity of spray varying between 7 and 57%, after
passing it through an ordinary water-trap. The figures given
by steam-condensation methods must, therefore, be greatly in
error and of no use whatever.

The use of the insulated steam pipe, however, was a
step in the right direction, as the insulation could be
applied to the pipe in exactly the same way as it would
be applied in practice.

In many of the tables published today values of
conductivity are given which were obtained by Lees\(^3\) in 1892
by means of an adaptation of the classical Forbes' Bar
method for metals. The bar was divided at one part and
a small bar of the material conforming to the general
shape was interposed. This method is itself difficult
and susceptible of much error, in addition to which it is
unsuitable for insulators when the smallness of the actual
test piece is considered. Moreover, Lees gives no information
concerning the condition of his specimens.

A great improvement was made in 1898 by Norton. He
employed electrical heating, which has the great advantage
that the energy supply can be accurately determined and
easily regulated. His apparatus consisted of a steam
pipe 13 inches long and 4 inches diameter, with the insulating
a thermometer, and insulated end-cap completed the apparatus. The errors, though doubtless smaller than those met with in previous methods, are still considerable. The temperature gradient between the oil and the pipe-wall would be much greater than similar gradients for hot water or saturated steam, owing to the higher viscosity of the oil. The corrections for loss due to conduction through the thermometer, insulated ends, etc., are difficult to apply, and were not mentioned by Norton in his paper.

With one or two unimportant exceptions all the modern experimenters have employed electric energy for heating purposes. As in this essential all the methods are the same it is necessary only to discuss details affecting the usefulness of the results obtained for commercial purposes.

There are two fundamental methods of attacking the problem. The first - the Diffusion or Fourier method: the second - the Equilibrium method.

The diffusion method depends upon Fourier's equation for the movement of heat within a body:

$$\frac{dT}{dt} = k \left( \frac{d^2 v}{dx^2} + \frac{d^2 v}{dy^2} + \frac{d^2 v}{dz^2} \right)$$

where x, y and z are the ordinates of any given point along the three dimensional axes, and $dT/dt$ is the temperature increase with time.

This method is not extensively used, mainly, no doubt, because of the experimental difficulties. Typical attempts were made by Green and Green and Edwards, who arranged for heat to pass in one direction through a rectangular block, and noted the rate of increase of temperature along the heat stream. In this case, only one axis is considered, and the
Other workers to use the Fourier diffusion method were Hessel and VanRinsum, who both used an electrically heated sphere. The sphere was built up in sections of the insulator around the central heating ball, thermocouples being introduced at suitable distances from the centre, and the sections cemented together with insulation paste. The possibility of error, however, in these experiments is greater than in the equilibrium method, and both the above workers used an equilibrium method to supplement their diffusion results.

As previously indicated the equilibrium method is more adaptable and therefore more popular. The general lay-out of the experiment may be similar to that of the Fourier method, but a certain time is allowed to elapse, until the temperatures throughout the insulator are steady. Shortly, the energy which is required to maintain a definite temperature gradient across the poor conductor is measured.

The particular mode of experiment best suited for testing an insulator is obviously the one in which the material is tested under the same conditions as those under which it will eventually be used. Thus, for pipe insulations, methods have been worked out consisting generally of electrically heated pipes with the insulation applied in the technical manner, and with elaborate precautions for eliminating errors due to the ends of the pipe. Among the pioneers in this field is Heilmann, who has made hosts of tests of commercial insulations with his "guard-coil" apparatus.
socially along the pipe. When temperatures along the central portion of the pipe enclosing the middle heating coil are all the same, it is assumed that all the heat given out by the middle coil is dissipated radially from the barrel of the pipe. His temperatures were taken at several points in the inside of the barrel and on the outer surface by specially tested thermocouples.

This apparatus seems to comply with all the requirements of a satisfactory test, and by taking the temperature of the outer surface after equilibrium is set up, all the necessary factors for calculation of conductivity are obtained.

A similar method is given by Van Dusen, who gives a diagrammatic temperature field, showing the equal temperature distribution around the central heating coil.

Another method of eliminating end correction is that of having the test pipe made with insulated ends, in conjunction with a shorter pipe with exactly similar ends, the two being heated to the same temperature as far as the inside barrel is concerned, and assuming that the extra heat from the larger is dissipated radially wholly from the extra central portion. The assumption that the loss from the small pipe is equal to that from the ends of the bigger one is not fully justified, because of the difficulty of assuring the same temperature distribution, and seems to be less satisfactory than the self-contained Heilmann guard-coil method. Such a method was used by McMillan, and an anonymous contributor to 'Engineering'.

For insulation required for walls of constant temperature chambers etc., other methods are available, the chief of which is the "pile" method, as exemplified by the
Here the material to be tested is pressed tightly between two surfaces of known temperature, and the energy required to maintain the temperature difference is measured. The heating coil is surrounded by other coils forming a 'guard-ring', the currents through all the coils being adjusted to give a constant temperature distribution over the whole of the central coil, when the heat from it is assumed to pass through the tested material only.

![Diagram of the apparatus](image)

Fig. 13

It is pointed out by Griffiths that different values of conductivity are given for various positions of the apparatus, the value being greatest for a vertical test. This indicates a connection effect in the granular materials; consequently if the material is to be subjected to connection through its substance when in use, provision must be made to incorporate the same effect in the preliminary test.

The apparatus of Griffiths seems to fulfil all the necessary conditions for technical experiment better than any other.

The conductivity of the normal insulating materials does not follow the law of change with temperature as does that of the metals. For metals the conductivity invariably decreases with rising temperature, but for poor conductors
Moreover, in general, the conductivity increases with increased density for a given material. For a granular or finely powdered insulator, the conductivity increases with the degree of packing.

It is apparent that the conductivity is not due wholly to the actual material, but to a large extent to the air enclosed in the pores of the structure, or the interstices of the powder. Inasmuch as the value for still air is about half that of the lowest recorded values for any insulator, and that the greater the density (i.e. less air enclosed) the greater the figure becomes, it can be assumed that the air does play a large part in the insulation. Unfortunately no work has been done upon conductivity in vacuo, such that the air effect is removed entirely: neither is anything known about the actual conductivity values of the single fibres or strands of the materials.

Kellor and others have suggested that radiation plays a large part in the so-called conductivity of bad conductors. They suppose that most of the heat transmitted is by radiation across the pores. This might account for the increase with temperature, as radiation is known to increase with the fourth power of the absolute temperature.

There is further the effect of the actual condition and size of the pores to consider. Much speculation has been rife on this subject, but no organised work has been done. Workers disagree concerning the effect of porosity, several admitting that they can find no connection between porosity and conductivity. Porosity, however, is a property
seems that the 'apparent' conductivity will depend not so much upon the porosity, density, etc., but upon the shape, number and size of the pores. Large pores may favour connection of the air contained and radiation from wall to wall, whilst small ones, if present in large numbers may cause the conductivity to approach the value for still air. Experiment upon materials applied under varying conditions would most probably elucidate these matters.

Hallmann suggests a crystalline structure might give a low conductivity because the crystals would touch in relatively few spots, and radiation across the interstices would be considerably reduced.

It is shown by the following results, that a great difference in conductivity is observed between the insulators of a fibrous nature, and the refractory type of non-conductor. This is no doubt due to the greater percentage of solid "wall" for the heat to traverse.

It is obvious, in face of the above considerations, that it is idle to attempt to obtain more concordant results by improving existing methods of determination. From a glance at the following tables it can be seen how impossible it is to set up standard values for given insulators: to make more than approximate calculations, the conductivity of any material under consideration must be determined individually under actual working conditions: it is most probable that different batches of the same material will have different conductivities.

Briefly, the main sources of variation may be summarised as follows:-
2. Size of pores or interspaces.
3. Density.
4. Porosity (a combination of 2 and 3).
5. Moisture present.
6. Method of application to insulated body.
7. Temperature.

For approximate work, where no actual determination can be made, the following figures have been selected from among the sounder experimental data:

<table>
<thead>
<tr>
<th>Material</th>
<th>Conductivity</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Cals per cm. per cm per °C</td>
</tr>
<tr>
<td>Cork</td>
<td>$1.2 \times 10^{-4}$</td>
</tr>
<tr>
<td>Slag Wool</td>
<td>2.0</td>
</tr>
<tr>
<td>Cellular Rubber</td>
<td>1.2</td>
</tr>
<tr>
<td>Kieselguhr</td>
<td>2.5</td>
</tr>
<tr>
<td>Charcoal</td>
<td>1.2</td>
</tr>
<tr>
<td>Asbestos</td>
<td>3.0</td>
</tr>
<tr>
<td>65% Magnesia</td>
<td>2.0</td>
</tr>
<tr>
<td>Wool or cotton</td>
<td>1.2</td>
</tr>
<tr>
<td>Concrete</td>
<td>25.0</td>
</tr>
<tr>
<td>Silica bricks</td>
<td>20.0</td>
</tr>
<tr>
<td>Fireclays</td>
<td>20.0</td>
</tr>
<tr>
<td>Magnesite</td>
<td>16.0</td>
</tr>
<tr>
<td>Material</td>
<td>Density (g/cm³)</td>
</tr>
<tr>
<td>--------------------------</td>
<td>-----------------</td>
</tr>
<tr>
<td>Cork</td>
<td>1.15 x 10^-4</td>
</tr>
<tr>
<td>&quot;</td>
<td>1.28</td>
</tr>
<tr>
<td>&quot; Slab</td>
<td>1.07-1.18</td>
</tr>
<tr>
<td>Slag Wool</td>
<td>1.00-1.21</td>
</tr>
<tr>
<td>Cellular Rubber</td>
<td>1.06-1.18</td>
</tr>
<tr>
<td>Charcoal</td>
<td>1.2</td>
</tr>
<tr>
<td>Kieselguhr</td>
<td>2.2</td>
</tr>
<tr>
<td>Concrete</td>
<td>20.3-29.3</td>
</tr>
<tr>
<td>Bitumen Slabs</td>
<td>20.2</td>
</tr>
<tr>
<td>Van Rumsed Asbestos</td>
<td>2.0-3.0</td>
</tr>
<tr>
<td>Kieselguhr mixtures</td>
<td>2.0-3.3</td>
</tr>
<tr>
<td>85% Magnesia</td>
<td>1.2-25</td>
</tr>
<tr>
<td>Grüber</td>
<td></td>
</tr>
<tr>
<td>Asbestos</td>
<td>1.96-3.92</td>
</tr>
<tr>
<td>Wool</td>
<td>0.84-1.68</td>
</tr>
<tr>
<td>Heilmann</td>
<td></td>
</tr>
<tr>
<td>85% Magnesia</td>
<td>1.47-2.32</td>
</tr>
<tr>
<td>Asbestos</td>
<td>1.17-4.21</td>
</tr>
<tr>
<td>Nusselt</td>
<td></td>
</tr>
<tr>
<td>Kieselguhr</td>
<td>1.68-2.18</td>
</tr>
<tr>
<td>Asbestos</td>
<td>4.68-5.21</td>
</tr>
<tr>
<td>Portland Cement</td>
<td>21.8</td>
</tr>
<tr>
<td>Cotton</td>
<td>1.5</td>
</tr>
<tr>
<td>Wool</td>
<td>1.2</td>
</tr>
<tr>
<td>Lees</td>
<td></td>
</tr>
<tr>
<td>Para Rubber</td>
<td>3.84</td>
</tr>
<tr>
<td>Cork</td>
<td>1.29</td>
</tr>
<tr>
<td>Cotton</td>
<td>5.48</td>
</tr>
<tr>
<td>Flannel</td>
<td>2.30</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>----------</td>
<td>----------</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>6. Green</td>
<td>Silica bricks</td>
</tr>
<tr>
<td></td>
<td>Fireclays</td>
</tr>
<tr>
<td></td>
<td>Magnesite</td>
</tr>
<tr>
<td>7. Green and Edwards</td>
<td>Kieselguhr</td>
</tr>
<tr>
<td></td>
<td>Slag wool</td>
</tr>
<tr>
<td>21. Thomas</td>
<td>85% Magnesia</td>
</tr>
<tr>
<td></td>
<td>Kieselguhr</td>
</tr>
<tr>
<td></td>
<td>Asbestos</td>
</tr>
<tr>
<td></td>
<td>Slag Wool</td>
</tr>
<tr>
<td>12. McMillan</td>
<td>85% Magnesia</td>
</tr>
<tr>
<td></td>
<td>Asbestos</td>
</tr>
</tbody>
</table>

**CONCLUSION.**

The methods of determining conductivity values of insulating materials are reviewed, and the best methods indicated and suggested for general practice.

It is shown that no definite values can be set down for given insulators, owing to variations in physical properties.

The best average values published are tabulated for use in approximate calculations.
1. FECLET, "Traite de la Chaleur, 1878".
5. FOURIER, "Analytical Theory of Heat".
7. GREEN and EDWARDS, "Gas Journal", 1925.
14. LEE, Phil. Trans. 1898.
18. GRIFFITH, Food Investigation Board Report, No. 5.
20. MAUVE, J.S.C.I., 1927, 158, T.
GENERAL APPLICATIONS.
The conductivity of heat through a body may be represented by:

\[ Q = -kA \frac{dt}{dL} \]  

(1)

where \( Q \) is the heat conducted per unit time.

\( k \) " conductivity.

\( A \) " area of the surface.

\( \frac{dt}{dL} \) " temperature gradient across the body.

In the usual case of a wall with parallel sides equation (1) when integrated between the limits of temperature \( t_2 \) and \( t_1 \) gives

\[ Q = kA \frac{(t_2 - t_1)}{L} \]  

(2)

\[ \text{H (cond)} = k \frac{(t_2 - t_1)}{L} \]  

(2a)

In the case of conduction through the insulating material around a cylindrical pipe equation (1) may be written

\[ Q = -kA \frac{dt}{dr} \]  

(3)

where \( \frac{dt}{dr} \) is the temperature gradient across the insulation.

In the figure consider a section \( dr \) at a distance \( r \) from the centre; considering the heat conducted across this small annular ring, (3) becomes

\[ Q = -k \frac{2 \pi r l}{dr} \]  

(4)

where \( l \) is the length of the pipe.

Hence by integration between the limits \( t_2 \) and \( t_1 \).
where \( r_2 \) is the external radius of the insulation and \( r_1 \) is the radius of the pipe.

This can be written

\[
Q = \frac{2\pi k_1 (t_2 - t_1) (r_2 - r_1)}{(r_2 - r_1) \log 2 \frac{r_2}{r_1}} \tag{6}
\]

where \( \frac{r_2 - r_1}{\log e \frac{r_2}{r_1}} \) may be termed the logarithmic mean radius, this quantity does not differ from the arithmetic mean by more than 1.5% when \( r_2 \) is less than 1.5 \( r_1 \). In this circumstance equation (5) can be written,

\[
Q = \frac{n k_1 (r_2 + r_1) t_2 - t_1}{(r_2 - r_1)} \tag{7}
\]

\( Q \) is the amount of heat conducted through to the surface of the insulation; therefore to obtain the quantity which will necessarily escape from unit area of surface, it is necessary to divide (5) or (7) by the surface area.

\[
\therefore \quad H_{(\text{cond})} = \frac{k (t_2 - t_1)}{r_2 \log e \frac{r_2}{r_1}} \tag{8}
\]

or \( H_{(\text{cond})} = \frac{k (r_2 + r_1) t_2 - t_1}{2 r_2 (r_2 - r_1)} \tag{9}\)

For the case of other shapes, e.g. the sphere, similar equations can be obtained, but as the two above, the rectangular slab and the cylinder, are of paramount importance in technical calculations, they alone are given here.
temperature range through the insulation. If, however, the temperature is so great that the conductivity cannot be assumed constant, then an expression \( \int_{t_1}^{t_2} \frac{dk}{dt} \) must be substituted for \( k \).

This naturally presupposes an accurate knowledge of the value \( k \), and its change over a given temperature range.

As the conductivity in general gives a straight line curve, the average value at the two extremes of temperature is usually taken for \( k \).

If the conducting wall be composed of several layers of different conductivity the problem may be approached as follows:

Let \( L_1, L_2, L_3, \ldots, L_n \) be the thicknesses of the layers and \( k_1, k_2, k_3, \ldots, k_n \) be their respective conductivities.

and \( (t_1 - t_2), (t_2 - t_3), \ldots, (t_n - t_0) \) be the temperature drop across the respective layers.

Then, as the same quantity of heat passes through each layer, we may write,

\[
\begin{align*}
    t_1 - t_2 &= \frac{Q}{k_1A} L_1 \\
    t_2 - t_3 &= \frac{Q}{k_2A} L_2 \\
    &\quad \text{etc. to} \\
    t_n - t_0 &= \frac{Q}{k_nA} L_n
\end{align*}
\]
and \[ Q = A \frac{(t_1 - t_0)}{\left( \frac{L_1}{k_1} + \frac{L_2}{k_2} + \cdots + \frac{L_n}{k_n} \right)} \] (11)

i.e. \[ H_{\text{cond}} = \frac{(t_1 - t_0)}{\left( \frac{L_1}{k_1} + \frac{L_2}{k_2} + \cdots + \frac{L_n}{k_n} \right)} \] (12)

By a similar calculation for the parallel case of an insulated cylinder equations (9) and 9) become

\[ H_{\text{cond}} = \frac{(t_1 - t_0)}{r_0 \left\{ \log e \frac{r_2}{r_1} + \log e \frac{r_3}{r_2} + \cdots + \log e \frac{r_0}{r_n} \right\}} \left( \frac{1}{k_1} + \frac{1}{k_2} + \cdots + \frac{1}{k_n} \right) \] (13)

\[ H_{\text{cond}} = \frac{(t_1 - t_0)}{2r_0 \left\{ \frac{(r_2 - r_1)}{k_1 (r_2 + r_1)} + \frac{(r_3 + r_2)}{k_2 (r_3 + r_2)} + \frac{(r_0 - r_n)}{k_n (r_0 + r_n)} \right\}} \] (14)
the insulating material, and the loss from the surfaces of
the insulation. That a certain drop in temperature must
occur across the insulation is obvious from the laws of
conductivity. That the outer surface temperature must be
higher than that of the air is also obvious. Therefore
it is necessary that an equilibrium be set up between the
two effects, such that the amount of heat conducted to unit
surface must be equal to the amount dissipated by radiation
and connection from unit surface. That is to say

$$H_{\text{cond.}} = H_R + H_c$$

To take the simplest case, that of a wall with parallel
sides, the surface of the insulation being in the normal
rough state the following equations apply

$$H_{\text{cond}} = \frac{k(t_2 - t_1)}{L}$$

$$H_R = \frac{K_R(T_1^4 - T_0^4)}{R}$$

$$H_c = \frac{K(t_1 - t_0)^{1.25}}{c}$$

Example.

If it is assumed that the insulation is an asbestos
of $k = 3.0 \times 10^{-4}$, that the surface is rough and has a
radiation constant of 90% black body, and that the wall
is more than 20 cms. high then the equations assume a
more definite nature. Let the thickness, $L$, be equal
to 10 cms.

$$H_{\text{cond}} = \frac{3.0 \times 10^{-4} (t_2 - t_1)}{10} \quad \text{......... (1)}$$
\[ H_c = 5.0 \times 10^{-5} \frac{t_1 - t_0}{t_1 - t_0}^{1.25} \]  

If the temperatures of the interior and the outside air are known, then the equation \( H_{\text{cond}} = H_R + H_C \) can be solved by assuming values of \( t_1 \) and drawing curves of \( H \) against \( t_1 \); where the two intersect the required values of \( H \) and \( t_1 \) are obtained.

When the internal temperature \( t_2 \) is taken to be 200°C, and the air temperature is 20°C then from the curves in Fig. XXV the values at the point of intersection are:

\[ H = 0.0048 \text{ cal/s cm}^2 \text{ per sec.} \]
\[ t_1 = 39.6°C. \]

Hence it is possible to solve all such problems for any shape and surface provided the conductivity and radiation constants are known, together with the internal and air temperatures.
BOUNDARY RESISTANCES.
The term 'boundary film' has been used frequently in the preceding text, and a word of explanation is necessary at this juncture.

Imagine a vessel wall A, bounded on one side by liquid at constant temperature and on the other by 'still' air, which is free to move under connection influences. The temperature of the liquid \( t_1 \) is assumed to be higher than that of the air \( t_0 \).

Heat will now be lost from the water across the wall to the air. The heat will be conducted through the wall material, and dissipated from the outer surface into the surrounding space by radiation and connection. These various effects can only function when there is a temperature difference across the wall and from the surface to the surroundings, hence the outer surface temperature of the wall must be higher than that of the surroundings, say at \( t_2 \).

Conduction has been shown to depend upon the energy required to maintain a definite temperature potential, and to follow the analogy further the material across which the potential is maintained can be regarded as a resistance. In this instance the boundary wall will assume the role of a resistance.
as conduction through a given though variable layer of comparatively motionless gas, hence for problems of heat transmission of this nature it has become convenient to regard the whole of the surface loss effect as a resistance effect.

The temperature change across the wall boundary is therefore represented by Fig. XXVI.

There is also evidence of a similar film on the liquid side of the boundary which will occasion a similar resistance and a consequent temperature difference. If the inner wall surface temperature is indicated by $t_3$, the figure shows the course of the temperature change from the liquid to the outer air.

The relative temperature gradients will naturally depend upon the relative resistance effects. In the case of a metal vessel containing water, by far the greatest drop will occur in the air film, owing to the small thickness and large conductivity of the metal, and the relatively large conductivity of water when compared with the value for air.

Nevertheless, it is apparent that despite vigorous stirring, there is a definite temperature drop at the liquid boundary, which if not taken into account, may lead to serious error.

Some figures taken from various published works serve to indicate the extent of the errors involved.

<table>
<thead>
<tr>
<th>Observer</th>
<th>Boundary</th>
<th>Temp. drop</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Carter &amp; Cope</td>
<td>Superheated steam/wall</td>
<td>4-10°F</td>
<td>For insulated pipe</td>
</tr>
<tr>
<td>Eberle</td>
<td>&quot;</td>
<td>6°F</td>
<td>&quot;</td>
</tr>
<tr>
<td>McMillan</td>
<td>Saturated steam/wall</td>
<td>1°F</td>
<td>&quot;</td>
</tr>
<tr>
<td>&quot;</td>
<td>&quot;</td>
<td>2-4°F</td>
<td>&quot; Uncovered pipe</td>
</tr>
<tr>
<td>Beastie</td>
<td>Water/wall</td>
<td>4°F</td>
<td>&quot;</td>
</tr>
</tbody>
</table>
a steam/wall/water boundary, depending upon the total temperature difference.

4. HEASTIE, J.S.C.I., 1923, 445T.
EFFECT OF SURFACE CONDITION OF INSULATION.
A great deal of discussion has taken place concerning the best surface for the insulation on an insulated body. The main contention is, that a polished surface would decrease radiation losses and therefore save heat. At first this seems admissible, but consideration of the equilibrium state shows that for a lower surface loss, there is a corresponding lower conductivity loss, which can only be brought about by a smaller temperature difference across the insulation. As the inner temperature ($t_2$) is constant, the only variable is $t_1$, which must rise to satisfy the lower value of $H_{cond}$. But if the surface temperature increases, the connection and radiation will increase with it, so that another equilibrium must be set up between heat conducted and heat dissipated from the surface.

The solution of the problem, therefore, depends upon whether the new heat loss will be greater or smaller after polishing the surface than before.

In a previous section it was shown how the heat loss can be determined from a knowledge of all the constants involved.

In Fig. XXVI, the case for a wall under typical conditions was considered. It is proposed to use the same constants and conditions to solve the new problem, with the exception that the radiation constant shall be $0.04 \times 10^{-12}$ instead of $1.25 \times 10^{-12}$. This constant corresponds to a polished silver surface, and is used to give as great a contrast as possible.

The curves given by both surfaces are drawn together on Fig. XXVII. It can be seen that the introduction of a
C.C.S. units to $4.36 \times 10^{-3}$. This does not seem to be a very great saving in heat, but it must be noticed, from the slope of the 'conductivity' curve, that the greater the conductivity and the smaller the thickness of the insulation, the greater will the saving effect be. The saving effect will be greatest when the conducting wall is of comparatively thin metal only.

The saving obtained from any surface treatment can be determined with a high degree of approximation by the simple process of drawing the conductivity and emissivity curves, and find the conditions of equilibrium.
SURFACE TEMPERATURE MEASUREMENT BY

THERMOCOUPLE.
There is one problem which permeates the whole of surface heat loss experiments. That is the accurate measurement of the surface temperature.

Some devices have been resorted to which avoid the necessity of the direct determination of the surface temperature. In the case of wires the temperature may be computed when an accurate knowledge of the variation of the electrical conductivity with the temperature has previously been obtained. Other workers have tried to overcome the difficulty by assuming the surface to take the same temperature as a hot fluid at its back. That this method is fallacious, and leads to varying errors is shown in another section under the heading "Boundary Films". In any case this latter method fails completely when outer temperatures of insulated pipes are required.

In face of the difficulty many workers, notably in the United States, have carried out experiments with insulating coverings, and expressed their results in an empirical or comparative manner, which has no fundamental basis whatever, and is only of limited practical use under the peculiar circumstances of the experiment. Such work has been published by Heilmann, McMillan, Ordway, Norton, Brill, and Barrus.

The more reliable date have been produced by measurements with thermocouples. There is, however, considerable difference of opinion concerning the appropriate construction of a thermocouple for surface work.

The fundamental requirements for a surface temperature
for temperature measurement, nothing must interfere with the radiation and connection from the surface. If a thick slab or plate is placed on the surface, immediately the apparent thickness of the insulator is increased at this point, and a fresh equilibrium will gradually be set up in accordance with the rules of insulators in series. This will tend to increase the temperature of the true surface, whilst the radiation and connection will apply to the new surface formed by the slab.

The solution to this difficulty is to have a thin metal plate to act as the thermocouple junction placed in intimate contact with the insulation, and having its outer surface prepared to give the same radiation constant as the surrounding surface. By the rules of insulators in series it will be seen that the addition of the comparatively thin metal disc to the outer surface will make but a very small difference to the outer temperature, which should anyhow be within the error of the reading of the thermocouple. In this way the metal disc should not interfere with the original equilibrium, and should attain the constant temperature of the surface. It will be necessary to ensure a perfectly intimate contact between the surface and the metal; this no doubt could best be done by scraping the original surface, and applying the disc with the help of a paste made of the same material as the insulation.

A further difficulty with the surface thermoelement, which is shared by all other thermoelements, is the avoidance of heat conduction along the thermocouple wires themselves. This has been eliminated largely in the case of thermocouples used for internal temperatures by placing the wires along a constant
difficulty could be overcome in a similar manner for surface elements. By trailing the wires along the surface for a distance of 4 inches or more, the conductivity effect should be eliminated; of course, the wires should best be treated in a like manner with the metal disc.

In practice there are two methods of determining the surface temperature: (a) by embedded thermocouple; (b) by padded thermocouple.

By embedding a thermocouple is usually meant the process of making the junction flush with the surface, or of placing it just beneath the surface, such that the same radiation surface will be exposed and the process of heat dissipation will not be modified in any way. The method of placing the junction flush with the surface is quite the better method, always provided the conditions mentioned above are adhered to, namely – thin plate – same radiation surface – good contact. By placing the junction beneath the surface a slight error may be introduced on account of the difference in temperature between the thermocouple and the actual surface.

The practice of padding a thermocouple is absolutely wrong in principle. By padding is meant a covering over of the thermo-junction with an insulator, the assumption being that it will not lose any heat, and hence retain its temperature. However, by padding, the original surface relinquishes its function of a surface, that of radiating and allowing connection, and becomes but an insignificant point in a solid wall, therefore a new equilibrium will be set up and the temperature indicated by the thermocouple will most certainly rise to a certain maximum determined by the new state of equilibrium.
the surface of an insulator cannot. For the latter he suggests that the temperature gradient across the insulation should be measured by building in a suitable number of thermocouples, and the surface temperature found by extrapolation. Whilst this is a perfectly sound idea, it is laborious, and seemingly unnecessarily complicated. It is difficult to understand from the published paper why the surface temperature cannot be measured directly.

A further difficulty arising out of the question of measuring the surface temperature of a metal, is the proper insulation of the leading wires from the metallic surface. Possibly this could be overcome by using some other method of measurement.

Numerous modifications of the two methods of applying the thermocouple have been put to practical use, although only those conforming to the basic requirements can be considered to give accurate results.

The above requirements can quite easily be carried out when the thermocouple is to have a permanent or semi-permanent position, but when an instrument is required to measure any and every type of surface, both stationary and mobile, errors are introduced which must be cancelled by some compensating device.
One was used by Van Rimsven during his work on conductivity. His apparatus consisted of a metal disc, A, acting as the thermojunction, with the wires B leading from it in a vertical direction through an insulated stem C, which served as a handle. To conceal the effect of conduction along the wires, a sliding metal strip D, was attached to the base of the handle which apparently restricted connection from the plate. If, when the plate was planted on a surface, the temperature gradually rose or fell, the slide was adjusted until a steady temperature could be obtained at once. To be able to measure temperature on any type of surface it is anticipated that a large compensating range would be necessary, unless the junction plate surface could be treated according to its requirements.

![Diagram](image)

Fig. 28

Another adaptation, this time of the 'padded' type, was published by Boyer and Buss and used chiefly for measurements on moving strips of paper, etc.

Here two thermocouples were used, one above the other. A. was the measuring couple supported by an insulating block C, and B. was a compensating couple, heated by a coil D, such that no heat was lost from A. When the apparatus was applied A took up the
As this apparatus was only to be used on moving surfaces, where it came into contact with no portion of the surface for more than a fraction of second, it is apparent that no marked change in the surface equilibrium can take place. If, therefore, the measuring couple is bounded on each side by the same temperature, its own temperature will remain constant. The inventors admitted that a building up of temperature was observed in time, which no doubt was due to the difficulty of registering the same temperature in both couples. Some kind of naked junction might possibly give better results; even then the effect of friction due to good contact would affect the results obtained. The apparatus, however, could obviously not be used for stationary surfaces, as the equilibrium would be seriously disturbed by the insulated block and especially the heating coil.

It should not be difficult to apply a thermocouple complying with all the necessary requirements to any apparatus used in experimental work, or in any position of which a permanent temperature record is required: it is feasible, however, that in many special cases, compensated instruments must be used.

There is still an unaccountable tendency on the part of many instrument designers to ignore the general rules of equilibrium, and who do their utmost to prevent any loss whatever from the thermocouple junction. That such instruments will show results too high is obvious from the preceding sections.

It must be assumed in the absence of any specific information that the results given by the various workers which have already been discussed have been obtained with
may be susceptible to error from this cause.

SURFACE TEMPERATURE MEASUREMENT BY
RADIATION METHODS.
It might be considered that the easiest way to measure a surface temperature would be by some method depending upon the radiation from the surface.

It has been shown conclusively, however, that there are no existing surfaces which obey the Stefan-Boltzmann Law of Radiation, with the exception of some specially prepared 'black' surfaces. If the radiation from any particular surface was found to obey a given law absolutely, then no doubt radiation measurements would give the most accurate temperatures.

The required conditions are demonstrated fully in the usual methods for registering the temperature of furnaces or similar cavities which conform strictly with black body conditions. The most common instruments in use in this connection are the "Total Radiation Pyrometers", and the "Optical Pyrometers".

The total radiation pyrometers are thermopiles arranged to give quick and direct readings, and to be independent of distance from the hot body. Their action depends essentially upon the radiation being focussed on to the hot junction of a thermocouple which registers the temperatures directly upon the galvanometer scale. A good pyrometer should give deflections proportional to the intensity of the radiation, which again depends upon the difference of the fourth powers of the absolute temperatures of the hot body and the instrument: therefore, for a black body, a single calibration of the instrument should suffice for the complete scale to be determined.

It would be possible under limited circumstances to apply this instrument to the measurement of other surfaces.
no two surfaces can be depended upon to have exactly the same radiation constant. Moreover, the instrument would have to be calibrated over the whole of the required temperature range, as it is known that the 'constant' changes very appreciably with the temperature. Unfortunately, even with complete calibration the effect of temperature, time and general conditions may alter the value of the 'constant' very appreciably, as witness the effect of tarnishing on bright copper surfaces.

If this type of radiation instrument is ever to give any but approximate values of surface temperatures, it will require a complete calibration with regard to the surface under consideration, every time it is used. As this involves much labour and also some other means of determining the surface temperature itself (for calibration) there is little wonder at its not being used more extensively.

The optical pyrometer depends upon Planck's Law

\[ E_\lambda \, d\lambda = c_1 \lambda^{-5} \ \frac{1}{c_2 \lambda^2 + 1} \]

The amount of energy radiated of a particular wavelength range is matched for intensity against a standard intensity corresponding to a known energy expenditure. From the energy thus obtained, the temperature is obtained from Planck's Law.

The method is necessarily only applicable to surfaces which are hot enough to emit radiations within the visible region of the spectrum, and would only be applicable to ordinary surfaces under the same conditions as for the total radiation instruments.

Unless some new universal law of radiation is found,
INFLUENCE OF STRUCTURE OF INSULATORS UPON
MECHANISM OF HEAT TRANSFER.
Much has been said in a previous section concerning the effect of porosity upon the conductivity of an insulator.  

Griffiths and Davis, in their report, say that radiation can have but little effect on the 'conduction' or heat transfer across a porous insulator, mainly on account of the small size of the individual pores. Mellor, on the other hand, states quite definitely that radiation plays a very large effect especially at the higher temperatures; he gives temperatures for various size pores at which radiation across the pores is equal to the conductivity. Consequently Mellor questions the statement that porosity favours insulating properties.

Griffiths and Davis base their conclusion upon the theoretical result obtained by supposing two planes, at temperatures $t_2$ and $t_0$ respectively at equilibrium as far as radiation is concerned; if now a third plane, infinitely thin, is placed between the two, a new equilibrium will be set up, in which the centre plane will receive as much energy from the plate at $t_2$ as it will radiate to the plate at $t_0$, and will take up an intermediate temperature $t_1$. (It must be assumed that $t_2$ and $t_0$ remain constant, and all planes radiate as black bodies).

Now the centre plane receives radiation

$$H_{R_1} = K_R (T_2^4 - T_1^4)$$

and radiates

$$H_{R_2} = K_R (T_1^4 - T_0^4)$$

but

$$H_{R_1} = H_{R_2}$$
To take a numerical example, suppose \( t_2 = 100^\circ C \)
\[ t_0 = 10^\circ C \]

Then \( T_1 = 337^\circ A \) i.e. 64\(^\circ\)C.

\[ \text{. . the amount of heat lost from the hot plane} \]
\[ K_R = K_R (373^4 - 337^4) = 8.78 \times 10^{-3} \]

whereas before the intervention of the centre plane the loss
\[ = K_R (373^4 - 283^4) = 17.55 \times 10^{-3} \]

which is exactly double.

Hence by the intervention of a single plane, that is by dividing the original radiation effect into two separate radiation effects, the total radiation loss is halved.

In a similar way it can be shown that with two planes, the radiation is only one third of the original, and so on for any number of planes.

If now it is imagined that an insulating wall is made up of a large number of parallel planes, the radiation effect will depend upon the number of planes. In this case, the whole of the heat transmitted will be by radiation across the "pores", and by conduction and connection through the air, that is to say the structure will be that of a perfectly porous body.
no connection. Therefore, if the planes in the above problem are set at a distance of 0.1 cm, it can be said that all the heat transfer due to the air will take place by conduction alone.

The state of equilibrium of the wall will be given by the equations:

\[
Q_1 = \frac{k_{\text{air}} (t_2 - t_1)}{l} + \frac{K_R (T_2^4 - T_1^4)}{n}
\]

\[
Q_2 = K_R (T_1^4 - T_0^4) + K_c (t_1 - t_0)^{1.25}
\]

where \( l \) = thickness of total wall
\( n \) = number of 'pore' spaces.

To find the relative effects of radiation and conduction it is necessary to take numerical examples.

Assume thickness = 10 cm.
number of pores = 100
width of pore = 0.1 cm.
\( t_2 \) (variously = 100, 200, 300, 400°C
\( t_0 \) = 20°C
\( K_R \) = \( 1.36 \times 10^{-12} \)
\( K_c \) = \( 5.0 \times 10^{-5} \)
\( k_{\text{air}} \) = given by Fig. XXX.

By the process of giving \( t_2 \) the various values indicated, values of \( t_1 \), and \( Q \) corresponding to the two equations can be obtained, and hence the quantities of heat lost by radiation and conduction.
50% radiation for the particular conditions under consideration is found.

(As the working involved is all performed by slide-rule, and as a similar example has already been shown fully, the complete figures are not shown here).

The temperature required in this case is 450°C.

By reducing the thickness of the total wall to 5 cms. the temperature required drops to 400°C.

By reducing the pore size to 0.05 cms. and retaining the total width = 10 cms, the corresponding temperature is raised to 800°C.

These temperatures are only valid for the considered structures, and to determine similar temperatures for a structure including thick walls would require a knowledge of the actual arrangement of the walls. As this is not known, an example can be worked for an assumed simple case.
Imagine the insulator to be made up of cubes of insulating material and cubical pores of the same cubic capacity arranged "chess-board" fashion.

At equilibrium there will be a constant heat flux, which will be the same across any plane perpendicular to the heat flow. If, as a first approximation, the temperature gradient across the insulator is assumed to be straight, then the heat flowing across the first row of cubes will be the same as across the whole thickness, and temperature difference will be \( \frac{1}{n} (t_2 - t_1) \), where there are \( n \) rows.

Again it will be necessary to take a numerical example.

Assuming the wall is 10 cm thick, and has an overall conductivity (apparent conductivity) of \( 3.0 \times 10^{-4} \) then for \( t_2 = 200^\circ \text{C} \) and \( t_0 = 20^\circ \text{C} \)

\[ t_1 = 40^\circ \text{C} \]

\[ Q = 4.8 \times 10^{-3} \]

Hence the quantity passing 1 sq. cm area is \( 4.8 \times 10^{-3} \), which is made up of heat conducted through the solid material, heat conducted through the air in the pores, and heat radiated across the pores.

Assume the layers to be, as before, 0.1 cm thick

\[
4.8 \times 10^{-3} = \frac{k_m \times 1.60}{0.1 \times 2} + \frac{k_R (473^4 - 471.4^4)}{2} + \frac{k_{air} \times 1.60}{2 \times 0.1}
\]
The above equation demonstrates many interesting points. One is that the radiation effect in this case is less than 10% of the whole, whereas for similar conditions with the more porous wall it accounted for 33% of the total.

Another point is that the actual conductivity of the cell walls of an insulator is much greater than the overall conductivity, in this case about 50% greater.

The assumption of a straight line temperature gradient across the insulator is not strictly valid, owing to the change of conductivity of the air and the material. If the straight line is assumed the loss due to radiation will become less as the cooler side is approached, the loss by conductivity through the air will decrease, and the loss by conductivity through the material will (probably) increase. To maintain the necessary constant heat flux, these effects must balance, or a curved temperature gradient must exist.

That the radiation effect depends largely upon conditions of temperature, thickness and conductivity of the insulator, is obvious from the preceding examples, and no definite figures can be given without qualification.

Mellor's results were given as follows:

<table>
<thead>
<tr>
<th>Pore Size</th>
<th>Temp. giving 50% radiation</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1 cm</td>
<td>1,400°C</td>
</tr>
<tr>
<td>0.5 cm</td>
<td>730°C</td>
</tr>
<tr>
<td>0.01 cm</td>
<td>3,000°C</td>
</tr>
</tbody>
</table>

No note was taken of connection or of change of conductivity with temperature.
The general conclusions obtainable from the examples are that the more porous material transmits a greater percentage of heat by radiation than the denser material; nevertheless for low temperatures the total amount of heat transmitted is less for the more porous material. The rate of increase of total heat transmitted is greater with the porous material than with the denser, and total heat loss from the former tends to catch up and overtake the total from the denser material.

That is to say, the apparent conductivity is less for the porous material at normal temperatures, but it increases at a greater rate.

The actual temperature at which the porous structure becomes disadvantageous cannot be determined, without an actual knowledge of the arrangement of the pores, the conductivity of the 'cell-walls', and the temperature gradient. In general, therefore, the less the pore size the greater the insulating capacity, irrespective of porosity.

Porosity itself will decrease the apparent conductivity up to a certain temperature, when it will become disadvantageous which temperature however is greatly increased for lowering the pore-size.

Previous workers have not been careful to differentiate between 'pore-size' and 'porosity'. It will be understood that the same porosity may be obtained for two materials comprising aggregates of different size pores.

1. GRIFFITHS and DAVIS, Food Investigation Board Report, No.9.
THERMOMETER ERRORS DUE TO RADIATION

CONNECTION AND CONDUCTION.
It is frequently necessary to measure the temperature of gases which are surrounded by walls of a different temperature. In the case of a stack the gases are bounded by walls at a lower temperature, whilst in a preheater the cool gases are passing across hot surfaces. Very often the difference in temperature amounts to many hundreds of degrees Centigrade, and measurements taken by inserting a mercury thermometer or thermocouple in the stream of gas are liable to be very wide of the actual temperature.

To take one case, that of a thermocouple, suitably encased, inserted in the stack of a boiler installation.

It will be readily understood from the section on Boundary Resistances, that the temperature of the inside of the chimney wall will be lower than that of the free flowing gases, a necessary circumstance to allow heat to be conducted to the outer surface. The thermocouple casing will therefore lose heat by radiation from its surface, and will consequently lose a certain amount of temperature.

Let the absolute temperature of the gases be $T_2$, the temperature of the wall $T_0$, and the observed temperature $T_1$. 
the thermometer will lose at such heat is the wall, and gains from the gases. It will radiate to the walls an amount depending upon its size and its surface condition. It will lose to the wall a certain amount of heat by conduction along itself, depending upon its position, size, construction and method of insertion into the wall.

The other side of the balance will be made up by connection from the gases, an effect dependent upon its size, and the velocity and composition of the gases, together with an amount radiated from the compound gases present in the fluid.

These two latter effects are outside the intended scope of this dissertation, consequently the subject can only be discussed in a general way.

The 'loss' side of the equation may be represented by

\[ Q_1 = K_R (T_1^4 - T_0^4) + K_{\text{cond}} (T_1 - T_0) \]

and the 'gain' side by

\[ Q_2 = \text{Connection} + \text{Radiation (both from gases at } T_2 \text{ to thermometer at } T_1) \]

As \( Q_1 = Q_2 \), \( T_2 \) can be determined provided a knowledge of the wall temperature \( T_0 \), the values of the constants \( K_R \) and \( K_{\text{cond}} \), and the form of the two expressions in the second equation is first obtained.

After long insertion in a stack, no doubt \( K_R \) would be about 95\% black body. \( K_{\text{cond}} \) is an overall figure depending upon the construction of the thermometer casing. The connection effect may be obtained from some existing formula, though no such of universal application presents itself to the mind of the writer. It is doubtful whether the laws
monoxide, and water vapour are well enough known for application to a problem similar to the present one.

In a general way it can safely be stated that the smaller is $Q_1$, then the smaller will be the difference $(T_3 - T_1)$. $Q_1$ can be reduced by preparing the surface of the thermometer, so as to reduce the radiation to a minimum. Both the radiation and conduction effects will be reduced the higher $T_0$ can be raised. It can easily be seen that the thicker the wall of the stack, and the lower the conductivity of the bricks, the higher will become; consequently greater error can be expected in an iron than in a brick chimney.

Hence the best way to reduce error in cases of the kind under consideration is to surround the silvered bulb of the thermometer or thermocouple by a metal shield, silvered inside and out. The radiation loss will then be reduced to a minimum. Conduction effects are far more difficult to eliminate, if the thermometer must pass through the wall, but may perhaps be overcome by suspending the instrument in the gas stream by fine insulating threads.
Fig. 1
Plotting $H_\alpha$ against temperature for various equations. (Enclosure at $0^\circ C$.)
Fig. 11 Comparative curves showing variation of the 'constant' in the Stefan-Boltzmann and Weber laws for Bottomley's results.
Fig. 6. Convection Loss from Vertical Plane Surface. (Temperature excess ±0°C)

--- Mean loss per unit area for plane of height 'z'.

Actual "..." element of plane at
from lower edge.
Plotting $h_c$ against temperature difference. Kennelly's results $H_c \propto T_1$.
becomes independent of $r$ above $r = 1 \text{cm}$.

$H_c \propto 1/r^2$ below $r = 2.5 \text{cm}$, and

plot $H_c$ against $r$ against radius at various temperatures for Reyleigh results.
Plotting $\alpha - \beta$ against temperature for various velocities, for King's results.

For higher velocities:

$H \propto T^{1.22}$

For lower velocities:

$H \propto T^{0.06}$
Plotting $H$ against radius at various velocities for Hughes' results.
Fig 18. Plotting density against temperature. °C
Fig. 21. Curve giving value of $K$ in $H_e = KT^{1.25}$ equation for any radius

(Horizontal Cylinders)
Fig. 25
Plot showing solution of heat loss equations.

\[ H = 4.8 \times 10^{-3} \text{ cgs.} \]

\[ T_i = 39.6^\circ C \]
Effect of surface Condition of Insulation

For bright surface: $H = 1.34 \times 10^{-3}$, $T_i = 54.6^\circ C$

For rough surface: $H = 4.80 \times 10^{-3}$, $T_i = 39.7^\circ C$
Fig. 3a
Plotting conductivity of air against temperature (°C)