THE USE OF ELECTRICAL STRAIN GAUGES ON
CONCRETE SPECIMENS UNDER SIMPLE AND COMPLEX LOADING. 

by

NORMAN SYDNEY GRASSAM, B.Sc.(Eng.)

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The behaviour of concrete under load has been of interest to engineers since the discovery some fifty years ago that, by the use of reinforcement, it could be used for other than gravity structures. The work of numerous experimenters during this period has established many of the load-deformation characteristics of the material under simple loading conditions, but the more advanced problems on its behaviour under combined stresses, remain unsolved.

The combination of torsional with bending loading is frequently encountered in practice, as for example, in concrete structures due to eccentric loading of the members. This problem has been partially investigated in Sweden from the practical point of view, by Nylander using reinforced concrete sections under bending combined with a small torsional load. A fundamental approach to the problem, which is essential if large factors of ignorance in design are to be avoided, was commenced in 1945 by the author and his coadjutor Mr. D. Fisher, B.Sc.(Eng.). A series of tests were carried out on circular plain concrete specimens under various combinations of bending
and torsional loading. The object of the research was to determine the criterion of failure of the concrete.

This involved two distinct programmes - one dealing with the elastic moduli and strain behaviour of concrete on which a theory of failure could be based, and the other with the ultimate strength of the material. The former is reported in this thesis, and describes experiments in which the strains suffered by specimens during combined bending and torsional loading, as well as by control specimens during simple loading, were measured by means of a new and more powerful method than had been available in the past. The latter is described in a thesis by Mr. Fisher.

The work is arranged in chronological order, as far as possible, so that the early chapters deal with the mechanism for the strain measurements, whilst the later chapters describe the detailed tests on various concrete specimens. The Thesis is presented in two volumes: the first containing the Text and the second the Appendix and Illustrations.

The experimental work was carried out in the Engineering Laboratories of the Battersea Polytechnic, London, between 1945 and 1949. The author is indebted to Mr. V.C. Davies, B.Sc.(Eng.), Head of the Engineering Dept., for the facilities provided, and for valuable advice and criticisms during his supervision of the work. Thanks are also due to Mr. Hamilton
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CONTENTS

PREFACE

CHAPTER 1

STRAIN MEASUREMENTS ON CONCRETE

1.1 Introduction.
1.2 Methods of measuring strain.
1.3 Mechanical methods.
1.4 Previous attempts by mechanical methods.
1.5 Electrical methods.
1.6 The Acoustic Strain Gauge.
1.7 The Capacity Strain Gauge.
1.8 The Wire Resistance Strain Gauge.
1.9 Selection of the method of Strain Measurement.

CHAPTER 2

THE WIRE ELECTRICAL RESISTANCE STRAIN GAUGE.

2.1 Historical Survey.
2.2 Modern types of Resistance Strain Gauge - Flat Grid Zig-zag type - Wound type - Woven type.
2.3 Gauge materials - Paper Carriers - The Wire.

CHAPTER 3

THE MECHANICS OF THE OPERATION OF AN ELECTRICAL RESISTANCE STRAIN GAUGE

3.1 Introduction.
3.2 List of Symbols.
3.3 Basic Assumptions - Former Wound type of Gauge.
3.4 Theoretical Analysis - Strain transmission from parent surface to paper.
3.5 Deductions - Shear stress variation in adhesive.
3.6 Deductions - Strain variation in gauge paper.
3.7 Strain transmission from paper to wire.
3.8 Former Wound gauges in which stiffness is due to wire.
3.9 Basic Assumptions - Flat Zig-zag type of gauge.
3.10 Theoretical Analysis - Strain transmission from parent surface to wire.
3.11 Deductions - Shear stress variation in adhesive.
3.12 Deductions - Strain variation along wire.
3.13 Experimental data from commercial series of gauges.
3.14 Analysis - Results - Deductions.
CHAPTER 4

SELECTION OF TYPE OF STRAIN GAUGE

4.1 Maker and Type.
4.2 Length.
4.3 Resistance.
4.4 Specifications of gauges used.

CHAPTER 5

FIRST TEST WITH STRAIN GAUGES ON CONCRETE

5.1 Test No.1

CHAPTER 6

CHOICE OF CIRCUIT

6.1 General Principles.
6.2 Temperature Compensation.
6.3 Zero Drift.
6.4 Use of deflection or Null method.

CHAPTER 7

CIRCUIT ADOPTED

7.1 Principles of the circuit - High resistance ratio arms - High gauge currents.
7.2 Arrangement of Multi-Channel Strain Gauge Set.
7.3 Construction of Multi-Channel Strain Gauge Set.
7.4 Lay-out of the Set.
CHAPTER 8
BRIDGE CIRCUIT SENSITIVITY

8.1 Introduction.
8.2 List of Symbols.
8.3 Abridged Analysis.
8.4 Deductions - Ideal Galvanometer Resistance for a given circuit.
8.5 Deductions - Sensitivity variation with Galvanometer Resistance.
8.6 Maximum Sensitivity of a given Wheatstone Bridge Circuit.
8.7 Sensitivity variation with Bridge Ratio \( \frac{P}{Q} \) with optimum Galvanometer Resistance.
8.8 Sensitivity of circuits used.

CHAPTER 9
TESTS WITH DIFFERENT ADHESIVES


CHAPTER 10
PREPARATION AND OPERATION OF STRAIN GAUGE

10.1 Introduction.
10.2 Materials on which gauges can be fixed.
10.3 Preparation of gauge surface.
10.4 Choice of Adhesive.
10.5 Author's method of fixing on dry concrete.
10.6 Attachment of leads to gauge.
10.7 Protection of gauge.
10.8 Matching gauges.
10.9 Setting-up gauges.
10.10 Conditioning gauges.
10.11 Balancing circuits.
10.12 Testing procedure.
10.13 Booking Results.
CHAPTER 11
CALIBRATION OF GAUGES ON STEEL SPECIMENS

11.1 Introduction.
11.3 Tension Method - Theory - Apparatus - Method - Results - Conclusions.
11.4 General Conclusions on Calibration Tests.
11.5 Cross Sensitivity - Theory - Apparatus - Method - Results - Conclusions.

CHAPTER 12
INTRODUCTION TO THE TESTS ON CONCRETE

12.1 Synopsis of the Tests.
12.2 Materials; Mix; Placing; Curing; Ultimate strength.

CHAPTER 13
TENSILE TESTS

13.1 Introduction.
13.2 Historical Tests.
13.3 Author's Tension Tests - Objects - Outline.
13.5 Results and Comments on the individual Tests.
13.6 General conclusions on the Tension Tests.
CHAPTER 14

COMPRESSION TESTS

14.1 Introduction.
14.2 Summary of items investigated.
14.3 Outline of Tests.
14.4 Details of Tests - Testing Apparatus - Testing procedure - Location of gauges.
14.5 The effect of end-packing on the eccentricity of loading.
14.5.1 Introduction.
14.5.2 Historical Tests.
14.5.3 Author's Tests.
14.5.4 Conclusions.
14.6 Variation of Strain in the height of Specimen.
14.6.1 Introduction.
14.6.2 Hoop Strain Tests.
14.6.3 Conclusions.
14.6.4 Axial Strain Tests.
14.7 Compression Stress-Strain Curves (Introduction)
14.8 Effect of repeated loading.
14.8.1 Historical Tests.
14.8.2 Author's Tests.
14.8.3 Conclusions.
14.9 The complete Compression Stress-Strain Curve.
14.9.1 Historical Tests
14.9.2 Author's Tests.
14.9.3 Conclusions.
14.10 Hysteresis and Permanent Set.
14.10.1 Introduction
14.10.2 Historical Tests.
14.10.3 Author's Tests.
14.10.4 Conclusions.
14.11 Yielding of specimen under load.
14.12 Ratio of lateral to longitudinal strains.
14.12.1 Introduction.
14.12.2 Historical Tests.
14.12.3 Author's Tests. (Specimen 'C')
14.12.4 Author's Tests. (Specimen 'D')
14.13 Volume changes due to loading.
14.13.1 Historical Tests.
14.13.2 Author's Tests.
14.13.3 Conclusions.
14.14 Failure of Specimen 'D'
14.15 Strain Sensitivity of the gauges.
14.15.1 Tests on Specimen 'C'.
14.15.2 Tests on Specimen 'D'.
14.15.3 Conclusions.
14.16 The Negative Loop Phenomenon.
14.17 Summary of General Conclusions on Compression Tests.
CHAPTER 15

BEAM TESTS

15.1 Introduction.
15.2 Historical Tests.
  15.2.1 Strain Capacity Experiments.
  15.2.2 Strain Distribution Experiments.
15.3 Author's Tests - Objects - Outline.
15.4 Details of the Tests - Testing Apparatus - Testing procedure - Observations.
15.5 Results and Comments on the individual Tests.
  15.5.1 Tests on Specimen 'E'.
  15.5.2 Tests on Specimen 'F'.
  15.5.3 Deflection Curves.
  15.5.4 Calculation of Maximum Tensile Stress.
15.6 General Conclusions on the Beam Tests.

CHAPTER 16

THE COMBINED BENDING AND TORSION MACHINE

16.1 Introduction.
16.2 Specifications.
16.3 Construction.
16.4 Operation.
16.5 Observations.

CHAPTER 17

TORSION TESTS

17.1 Introduction.
17.2 Historical Tests.
17.3 Author's Tests - Objects - Outline.
17.4 Details of Tests - Testing Apparatus - Testing procedure - Observations and Calculations.
17.5 Results and Comments on the individual Tests.
17.6 General Conclusions on the Torsion Tests.
CHAPTER 18
BENDING TESTS ON A CIRCULAR SECTION

18.1 Introduction - Objects - Outline.
18.2 Details of Tests - Testing Apparatus - Testing procedure - Observations and Calculations.
18.3 Results and Comments on the individual Tests.
18.4 General Conclusions on the Bending Tests.

CHAPTER 19
THE COMBINED BENDING AND TORSION TEST

19.1 Introduction.
19.2 Historical Tests.
19.3 Author's Test - Objects - Outline.
19.4 Details of Tests - Testing Apparatus - Testing procedure - Rate of loading - Observations and calculations.
19.5 Results and Comments on the Tests.
19.6 General Conclusions on the Combined Bending and Torsion Test.

CHAPTER 20
COMPARISON OF THE RESULTS OF THE SEVERAL TESTS ON THE CONCRETE

20.1 Introduction.
20.2 Historical Tests.
20.3 The Author's Results.

CHAPTER 21
SUMMARY OF CONCLUSIONS

21.1 The Concrete.
21.2 Testing Arrangements.
21.3 Strain Gauges.
21.4 Final Observations.
CHAPTER 1

STRAIN MEASUREMENTS ON CONCRETE

1.1 INTRODUCTION

This research was undertaken in order to investigate a new method of taking strain measurements on concrete by the use of electrical resistance strain gauges. From the readings obtained, the elastic properties of the concrete used in strength tests by the author and his coadjuítor (26) were to be determined.

The methods already in use for measuring, to a high degree of accuracy, the strains of this material under load are very limited. This is partly due to the smallness of the deformations, and partly to the difficulty of securing a uniform distribution of stress on a test specimen. The common method of obtaining the strain on a surface is to use an instrument which will measure the change of a length, and then to evaluate the strain by relating the reading to the original gauge length. Conversely, the electric strain
The measurement of tensile strains presented the most difficult problem. The order of magnitude of the changes involved may be seen from the following considerations:

The concrete used in the strength tests had an ultimate tensile strength of about 300 lb/in$^2$, and from the nature of the mix and the age at testing, was expected to have a Young's Modulus of roughly $4 \times 10^6$lb/in$^2$. Assuming elastic conditions until rupture, the ultimate strain would be only $7 \times 10^{-5}$. To observe the behaviour as loading progressed, it would therefore be necessary to read a strain of $7 \times 10^{-5}$ which is small even by laboratory standards if a local measurement is required. It was also realised that, for testing purposes, specimens of moderate size, for example, 5 inches typical dimension, should be used in order to avoid scale effects in the concrete, to ease the manufacturing and handling difficulties and, at the same time, to ensure a reasonably large fracture load.

1.2 METHODS OF MEASURING STRAIN.

A solution to the problem of measuring small strains is to magnify, without distortion, the change in length to be measured.

The human eye, aided by a magnifying glass and finely
divided scale, will only detect a change of about $2 \times 10^{-2}$ in.
There are, however, two other ways of magnifying a change of
length, (a) by mechanical and (b) by electrical methods.

1.3 MECHANICAL METHODS.

One of the oldest means of magnification is the
mechanical lever method. This, however, necessitates pivots
and/or bearings, in which that fickle quantity, friction,
acts. Whilst the degree of magnification is infinite
theoretically, yet, in practice, there is a definite limit.
By this method the magnified movement is observed directly by
using a microscope, or an eye-piece scale calibrated by a
micrometer screw-thread on the instrument. Alternatively,
the rack and pinion movement of a dial indicator may be used
to show the displacement.

Typical instruments of the lever type are:--
(a) The Berry and Ruggenberger instruments which measure
only surface movements on one side of a specimen, and work
initially on a simple lever magnification.
(b) The Ewing type of instrument which gives the mean
extension of a specimen by averaging the movement at two
positions $180^\circ$ apart by simple lever magnification.

An extension of the lever method is obtained by the use
of plane mirrors with a beam of light as the pointer. For
linear strains this requires the conversion of the linear
displacement into a rotation of the mirrors. The instrument is, however, no longer self-contained, as separate scales and a telescope must be provided. The Lamb Roller Extensometer is typical of this type of instrument which works on the principle of the Bauschinger instrument, and gives the mean extension of a specimen.

These instruments can, theoretically, be made and set up on concrete specimens to measure the smallest change of dimensions desired over gauge lengths as short as one inch. However, experimentation with them is a tricky and tedious business. One of the difficulties of using any of them on concrete is the method by which they are attached to the surface. Two methods are used; one is to provide small metal plugs in the concrete on to which the instrument may be clamped; the other is to clamp two metal strips around the circumference of the specimen and then pivot the instrument on the strips. Both methods will interfere with the behaviour of the specimen under load. An additional disadvantage of the mechanical method is that a delicate and expensive instrument is required which may easily be damaged when the specimen breaks. On the other hand, this method is perhaps rather more straightforward and convincing to the mechanical engineer than the electrical methods to be described later.
The measurement of torsional strains is comparatively simple provided that a reasonable gauge length is available. The angle of twist over a given length can be obtained quite simply by using a lever arm or by attaching mirrors to the specimen.

1.4 PREVIOUS ATTEMPTS BY MECHANICAL METHODS.

In the past, these methods were the only ones available and have been used by a number of experimenters on concrete in compression. For example:

(a) F.E. Richart, A. Brandtsaeg and R.L. Brown, at Illinois Experimental Station, U.S.A. (48) in a large series of experiments on the failure of concrete in compression, used 4 inch and 8 inch Berry gauges engaging in the conical sockets of steel plugs cast in the test pieces. The probable error claimed for the 8 inch instrument was a strain of $8 \times 10^{-6}$, and for the 4 inch, a strain of $25 \times 10^{-6}$.

Whilst this sensitivity is adequate for measuring compressive strains, it would not be good enough for measuring tensile strains.

(b) A number of experimenters have used compressometers working on the Ewing principle which operate quite satisfactorily and are suitable for control cylinders.

In tension there have been few successful detailed measurements of the stress-strain relationships for concrete.
Mr. V.C. Davies (18) of Battersea Polytechnic, London, designed and obtained consistent results with a mirror extensometer working on the Ewing principle for 3 in. x 3 in. concrete tension specimens over a gauge length of 6 inches. The probable error claimed was of the order of a strain $1 \times 10^{-6}$ (i.e. a stress of 4 lb./in$^2$).

A.N. Johnson (34), R.H. Evans (25), Paul Anderson (1) and others have used Marten's Mirror Type Gauges, 2 inch Berry and Huggenberger gauges, and 1 inch roller-mirror gauges for tensile strain measurements, and have obtained a number of valuable stress-strain curves.

In tension, the lever method has been used satisfactorily by nearly all experimenters on concrete (See Chapter 17). Paul Anderson (1), Marshall and Tembe (39) and Nylander (43), to quote recent experimenters, have used this method on gauge lengths of 20 inches or more.

1.5 ELECTRICAL METHODS

Electrical strain gauges have become increasingly popular in the last few years. One of their chief advantages is that readings can be taken remotely from the strained surface at a central reading station. This is possible as, in all cases, the observation is electrical and the conversion to a change in a gauge length can be obtained at a later date.
from a separate calibration. These gauges can be made to detect the small strains exhibited by concrete, provided that adequate undistorted electrical amplification of the signal can be arranged. The disadvantage from the mechanical engineer's point of view is that, if a large number of gauges is used, the electrical equipment may become very complicated and the services of an electrical expert will then become essential. The strain gauges now available include Acoustic, Capacity and Wire Resistance types.

1.6 THE ACOUSTIC STRAIN GAUGE. (2 and 17)

Strain is obtained indirectly by measuring the change in the frequency of a tensioned vibrating wire stretched between gauge points. The gauge is initially calibrated for tension against frequency so that, if the properties and dimensions of the wire are known, the strain may be calculated.

The gauges were first developed by the German firm of H. Maihak and Co. and, at present, are made with 10 cm. and 5 cm. gauge lengths. It is claimed that the 10 cm. type has a reading accuracy corresponding to a strain of $5 \times 10^{-6}$, and the 5 cm. type to a strain of $15 \times 10^{-6}$.

In 1934 the Building Research Station, Garston, built a modified Maihak gauge (17) to measure strains on relatively
soft materials such as building stone. The difficulty was the crumbling of the stone under the knife edges, but the new design minimised this trouble, and readings of $2 \times 10^{-7}$ strain could be made on a $4\frac{1}{2}$ inch gauge length.

This type of gauge has several advantages over other electrical types. For instance, it has good zero stability and is not very sensitive to a limited amount of dampness. Also, as it is a frequency and not an amplitude measuring instrument, the system is relatively insensitive to stray pick-up and minor insulation faults. The gauge would probably be quite suitable for use on concrete but, as yet, only very limited supplies are available in this country.

1.7 THE CAPACITY STRAIN GAUGE (13)

These gauges operate on the principle that the capacity of a condenser can be altered by varying the air-gap or the effective area of the electrodes. They have been made with gauge lengths as short as $\frac{3}{8}$ inch, but they have several disadvantages. For example, special care must be taken with the details of the pick-up circuit, and extra special care is required in damp conditions. This type of gauge has been rather eclipsed by the advent of the wire resistance type, but it may be used at temperatures and strains beyond the range of the resistance gauge. Its use on concrete would hardly be justified in comparison with the
wire resistance type, and there appears to be no record of
its use for this purpose.

1.8  **THE WIRE RESISTANCE STRAIN GAUGE** (See also Chapter 2)

This type of gauge, which has become so popular
in the last few years, is a true strain measuring device. It
depends for its operation on the fact that the resistance of
a wire varies with the mechanical strain to which it is sub-
jected. Thus, if a wire can be placed in intimate contact
with the test surface, the change of resistance which occurs,
can be used as a measure of the test strain. Gauges having
an overall length of \( \frac{1}{2} \) inch and upwards, are quite easily
obtained, and the circuitry required for resistance measure-
ment need not be exceptionally complicated for good
discrimination. Also these gauges are relatively cheap and
hence are expendable.

There appeared to be no record of their use on concrete
in May 1946, when this work was contemplated.

1.9  **SELECTION OF THE METHOD OF STRAIN MEASUREMENT**

The methods already outlined had to be carefully
considered in relation to the strains sustained by a concrete
specimen under combined bending and torsional loading.
There were also, the measurements on various control specimens
under simple loading conditions to be taken into account.
The following points had therefore, to be borne in mind:

1. As certain moulds were available the dimensions of the specimens were clearly defined for the tests.
2. The changes in the dimensions of the specimens which would take place under load.
3. The necessity of measuring these changes in dimensions accurately.
4. The cost of the gauges and the general expense entailed.
5. The need for obtaining the strain at definite points on the specimens.

The wire resistance type of gauge appeared to offer the greatest possibility of success at a reasonable cost and, it was hoped, would give sufficient accuracy for scientific purposes.

At the time this research was contemplated (May 1946) no published work on the use of this type of gauge on concrete was known. The National Physical Laboratory, which was the pioneer of the gauges in this country, was approached and, whilst being most helpful, they expressed some doubt as to the possibility of the gauges working on a concrete surface owing to the difficulty of sticking the gauge. However, the N.P.L. kindly provided a few of their gauges for a preliminary test, together with a copy of their paper on the 'Construction and Use of N.P.L. type strain gauges' (45).
The Building Research Station, Garston, was next contacted in October 1946, and they were able to show some gauges stuck to a concrete slab in a laboratory under carefully controlled conditions. Although they had had little experience with the gauges, they were able to supply a workable method for preparing and sticking the gauges to a well-dried smooth concrete surface. The problem of attaching the gauges to wet or damp concrete was discussed with them, as this was the condition under which it was desired to test. (The author’s normal practice being to test specimens at 28 days immediately after curing in water). The main difficulty is the water absorption by the glue which would give inaccurate readings on the gauges. It is possible to prevent the water being absorbed from the outside of the specimen by using a suitable wax covering, but to prevent absorption from the inner concrete is a more difficult problem. The possibility of using an insert of another material and attaching the gauge to this, did not appear very sound but a possible method suggested, was to remove the specimen from the water about 1 hour prior to testing. The spot to be tested could be dried with an infra-red lamp which would also dry the concrete to a certain depth below the surface. The gauge could then be affixed to the surface whilst the heat was still being applied, and until the glue had hardened off.
This would enable the test to be carried out before the water began to seep back to the surface under test.

With this limited information, preliminary compression tests were undertaken on a concrete cylinder. The tests, as shown later, gave sufficient promise and encouragement for the method to be followed up.

In view of the newness of the method it was decided that, whenever possible, a mechanical means of strain measurement should be used in addition to the wire resistance strain gauges.
CHAPTER 2

THE WIRE ELECTRICAL RESISTANCE STRAIN GAUGE

2.1 HISTORICAL SURVEY

The discovery that a metal changes its electrical resistance when subjected to mechanical strain was made over one hundred years ago by Lord Kelvin. (Thomson) (55). The phenomenon was not utilized for strain measurement, however, until recently.

The invention of the modern bonded wire resistance strain gauge followed as the next step from the carbon gauge. The latter is made up of a thin flat carbon resistor mounted on a piece of flexible insulating material and cemented to the test surface. Strain of the test surface changes the length of the carbon resistor, which changes its electrical resistance owing to the change in the contact pressure between its carbon particles. The gauge, which works on the same principle as the carbon microphone, can be made to have an extremely high sensitivity, and hence is still
used when this is the overriding consideration.
Unfortunately, it suffers from the inherent defects of all carbon resistors by giving a slow drift of resistance and a reduction in strain sensitivity with temperature.

The next step was clearly to replace the carbon strip by a metal wire. This was taken by Simmons (53) in Feb. 1940, and was first used by Clark and Datwyler in California U.S.A., and further developed by De Forest and Ruge (20) and (49), at Massachusetts Institute of Technology. The invention was patented (U.S. Patent No. 2,232,549 = 1940), with the following claim:— "A strain gauge comprising a filament of strain sensitive material secured throughout its effective length to a member so as to be responsive to strains thereon." It was marketed under the trade name "Baldwin - Southwark S.R.-4 gages" by the Baldwin Locomotive Works, Philadelphia. The earlier types consisted of a grid of fine wire bonded between layers of a bakelized paper, or set into a transparent thermoplastic resin. The overall size was about 4 inches long by ½ inch wide by 0.035 inches thick, which is large and clumsy compared with present types.
2.2 MODERN TYPES OF RESISTANCE STRAIN GAUGE

There are three main types of wire resistance strain gauges used at present:

(a) The flat grid or zig-zag type, in which the fine wire is arranged into the form of a uniplanar zig-zag by drawing it round pins, and then cementing it on to the surface of a paper carrier which can itself be cemented on to the test surface.

![Diagram of Flat Grid Type Strain Gauge]

**FIG. 2.1 Flat Grid Type Strain Gauge**

The American Baldwin Southward SR-4 gage of this type has a cover of felt to provide protection from draughts. The British manufacturer of this type (H. Tinsley and Co., Ltd) leaves the top side of the wire uncovered to allow quicker setting of the cement.
(b) The wound type, in which the wire is wound helically on a thin tubular paper former, which is subsequently pressed flat, and then sandwiched between two further pieces of paper.

FIG. 2.2 Wound Type Strain Gauge.

This method of manufacture was developed by the National Physical Laboratory in 1941 (Patent No. 560,260; Oct. 1942) and is now made commercially in this country by the British Thermostat Co. and Rotol, Ltd.

Its advantage over type (a) is that, due to a closer allowable pitch, more wire can be included in the carrier for the same size of gauge thus giving a higher gauge resistance. On the other hand, the gauge is stiffer and thicker than type (a) and the cement will take longer to dry out.

(c) The woven type in which the covered wire forms the weft with the warp of a textile e.g. rayon. The gauges are bought in bulk in the form of a ribbon, and the desired resistance is obtained by cutting off a suitable length of the ribbon. The method of manufacture was invented by
British Célanese Ltd, (Patent No.623,641) in May 1946. The diagram shows the arrangement for attaching the leads on the British Célanese gauge.

![Diagram of woven gauge](image)

FIG. 2.3  Woven Type Strain Gauge.

This type of gauge is in its infancy, as the marketable form has only just been reached. It has the advantage over the other types, in that it will allow quick setting of the cement, as the air will be able to evaporate the solvent easily, and it is very much cheaper.

2.3  GAUGE MATERIALS

The paper carrier types (a) and (b) are at present the most common and were the only types available when this research was started. Hence details of their materials of construction, and the selection of such gauges,
are included in this thesis.

(i) **Paper Carriers**

Initially, a heavy stiff bakelised or resin impregnated paper was used to achieve good insulation between the wire and the test surface. It has since been realised that, if the paper has a high modulus of elasticity, larger forces than are necessary are imposed on the glue, causing it to fail at comparatively small strains. Thin papers having a lower mechanical strength of either a thin rice paper or filter paper consistency are now used (See Theory of Operation of a Strain Gauge, Chapter 3).

(ii) **The Wire**

The wire used is of as small a diameter as can be conveniently handled, as this gives a large circumference to area ratio which minimises the shear stress required in the adhesive for a given strain. The size normally used is 0.001 inches diameter. This is too fine for easy direct connection to the heavier wire leads of the recording set so that short heavier intermediate strip or wire leads, usually of tinned nickel-chrome for ease of soldering, are spot-welded to the filament during manufacture.

The selection of the most suitable metal for the gauge wire has been the subject of much research. (20) It was shown by Lord Kelvin in his Bakerian Paper delivered to the
Royal Society in 1856 (55) that a wire of copper, and a wire of iron of the same size and shape, subjected to the same strain, suffered different changes in resistance. In 1877 Tomlinson showed to the Royal Society that the resistance change occurring when a wire is strained, is greater than that to be expected from the change of shape. (Analytical explanation see Appendix I). In 1883 Gray described to the Royal Society of Edinburgh his experiments on the effect of elastic and plastic elongations on copper, German silver, and iron, and found definite increases in the specific resistance with strain. Different metals may therefore be expected to show different resistance strain sensitivities depending on their particular rate of variation of specific resistance with strain (See Appendix I, Fig. 2.4). That this is so has been shown recently for a large number of possible strain gauge metals by De Forest and Lederman (20), see Table 2.1
The reason for the dependence of specific resistance on the strain is as yet an unsolved problem in the theory of metals.

In selecting the material for the gauge wire, clearly as high a strain sensitivity factor as possible is desired for ease of measurement, but other factors also arise which necessitate a compromise. For example: (a) the material must be easy to manufacture into a gauge, (b) the specific resistance must be high so that sufficient wire may be incorporated in a reasonably small space to give a

<table>
<thead>
<tr>
<th>Material</th>
<th>Approx. Composition</th>
<th>Strain Sensitivity Factor for a Single Wire.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nickel</td>
<td>Pure</td>
<td>- 12.1</td>
</tr>
<tr>
<td>Manganin</td>
<td>34%Cu, 12%Mn, 45%M1</td>
<td>+ 0.5</td>
</tr>
<tr>
<td>Phosphor Bronze</td>
<td></td>
<td>+ 1.9</td>
</tr>
<tr>
<td>Monel</td>
<td></td>
<td>+ 1.9</td>
</tr>
<tr>
<td>Nichrome</td>
<td>80%Ni, 20%Cr</td>
<td>+ 2.1</td>
</tr>
<tr>
<td>Advance</td>
<td>55%Cu, 45%M1</td>
<td>+ 2.1</td>
</tr>
<tr>
<td>Copel</td>
<td>55%Cu, 45%M1</td>
<td>+ 2.4</td>
</tr>
<tr>
<td>Inelastic</td>
<td>36%Ni, 52%Fe, 8%Cr</td>
<td>+ 3.6</td>
</tr>
<tr>
<td>Platinum</td>
<td></td>
<td>+ 6.0</td>
</tr>
</tbody>
</table>

TABLE 2.1 Strain Sensitivity of different Materials - De Forest and Lederman (20)
comparatively large signal when the gauge is strained. The signal must be large enough so that it will not be swamped by slight circuit changes, e.g. thermo e.m.f.'s, resistance changes in the leads, etc., and large enough so that the amplification required is not so great that unavoidable distortion is introduced. (c) the temperature coefficient of resistance should be as low as possible to minimise heating troubles.

For static work (i.e., the measurement of steady strains under fixed loads), where a low temperature coefficient of resistance is most important owing to time effect, the material chosen is either Advance or Nichrome. The former is popular with American manufacturers as it has a low temperature coefficient of resistance (say $20 \times 10^{-6}$ per $^\circ C$), whereas British manufacturers prefer Nichrome for, although it has a higher temperature coefficient (say $70 \times 10^{-6}$ per $^\circ C$), it also has a much higher specific resistance and is easier to fabricate.

(iii) The Resistance.

The gauges are normally of resistances from 50 to 2500 ohms. Dr. F. Aughtie (3) states that "there is no primary fundamental factor which influences the choice of resistance". There is, however, an optimum ratio of gauge resistance to indicating instrument resistance for maximum power to be
supplied by the strained gauge to the instrument.

The insulation leakage limits the upper resistance value to be used, and in addition, high resistance gauges of short lengths (say 1 inch), will have considerable widths, (say \( \frac{1}{4} \) inch), and hence will be more sensitive to cross strains (See cross sensitivity Para.11.5). The cost of a high resistance gauge will also be higher.

The general practice is to select high resistance (2000 ohm) gauges if a high resistance indicating instrument (e.g. a cathode ray oscillograph) is being used, and a low resistance gauge (200 ohm) for a low resistance indicating instrument (e.g. a galvanometer) in order to render the circuit sensitivity as near optimum as possible. (see Circuit sensitivity, Chapter 3).
CHAPTER 3

THE MECHANICS OF THE OPERATION OF AN
ELECTRICAL RESISTANCE STRAIN GAUGE

3.1 INTRODUCTION

The exact strain and stress distribution in the components of the joint between the parent strained surface and the gauge wire is complex, as it depends, not only on the strain transmission between the strained surface and the gauge paper-carrier, but also on the transmission between the paper-carrier and the gauge wire.

The method of transmission from the parent strained surface to the gauge wire depends on the method of manufacture of the gauge. The operation of the two main types, the former wound type and the flat zig-zag type, are discussed separately.
3.2 LIST OF SYMBOLS

Materials

\[ E_1 = \text{Young's Modulus of Elasticity of the material of the parent surface}. \]
\[ E_2 = \text{Young's Modulus of Elasticity of the material of the gauge paper}. \]
\[ E_3 = \text{Young's modulus of Elasticity of the material of the wire}. \]
\[ C = \text{Shear Modulus of Elasticity of the adhesive}. \]

Geometry

\[ l_2 = \text{axial length of gauge paper carrier}. \]
\[ l_3 = \text{axial length of gauge wire}. \]
\[ b = \text{width of gauge paper carrier}. \]
\[ t = \text{thickness of the gauge paper carrier}. \]
\[ D = \text{diameter of the gauge wire}. \]
\[ \lambda_1 = \text{thickness of the adhesive from the parent surface to the gauge paper}. \]
\[ \lambda_2 = \text{thickness of the adhesive from the parent surface to the wire}. \]
\[ x = \text{distance of the element considered from mid-length of the gauge}. \]

Strains

\[ e_1 = \text{uniform axial strain on the parent surface}. \]
\[ e_2 = \text{axial strain in the paper carrier at distance } x \text{ from the mid-length of the gauge}. \]
Stresses

- $e_3$ = axial strain in the wire at the distance $x'$ from the mid-length of the wire.

- $\phi$ = shear strain in the adhesive at distance $x'$ from mid-length of the gauge.

- $K\phi_3$ = apparent shear strain in the adhesive at the surface of the wire at distance $x'$ from the mid-length of the gauge, where $K$ is a constant.

Symbols

- $m_1 = \sqrt{\frac{C}{\sqrt{\lambda_1 E_2 t}}}$

- $m_2 = \sqrt{\frac{4CK}{\sqrt{DE_3 \lambda_2}}}$
3.5 **THE BASIC ASSUMPTIONS - FORMER WOUND TYPE OF GAUGE**

In the former wound type of gauge the wire is firmly bonded into the paper carrier which, in the N.P.L. type of gauge, has four thicknesses of paper (see sketch below).

![Wire Winding and Paper](image)

**FIG. 3.1** Wound type of Strain Gauge.

If the stiffness of the paper carrier is large compared with the stiffness of the wire, as is the case in gauges having only a few strands of wire in a transverse cross section through the gauge, then the following simplified analysis will show the distribution of strain in the paper carrier, and the variation of shear stress in the adhesive along the length of the gauge.

A typical gauge which meets this requirement is the 200 ohm B.T.C. gauge of 1 inch effective gauge length. This has approximately four strands of 1/1000 inch diameter Nichrome wire in a cross section through the gauge.
From the following figures, a rough calculation shows the relative stiffness of paper to wire to be about 10.

Cross section area of wire = \(3 \times 10^{-3}\) in\(^2\).
Cross section area of paper = \(6 \times 10^{-3}\) in\(^2\).

E for Nichrome wire = \(37.5 \times 10^6\) lb/in\(^2\).(36)
E for acetate impregnated paper = \(2 \times 10^5\) lb/in\(^2\).(32)

The method of the following investigation depends on the under-mentioned simplifying assumptions:

(a) There is no shear set up in the gauge.
(b) There is no tension set up in the adhesive.
(c) There is a uniform strain on the parent surface.
(d) The gauge is of uniform stiffness throughout its length.

These assumptions enable a 'straight line' strain variation diagram to be considered. The results obtained show the nature of the variations of the strains and stresses, the exact values being incorrect, due to the assumptions.
3.4 THEORETICAL ANALYSIS

The Strain Transmission from the Parent Surface to the Paper Carrier

Consider the gauge to be stuck uniformly along its length to a test surface.

Before Strain

Gauge

Adhesive

Test Surface

Sign Convention

FIG. 3.2 Longitudinal Section through Gauge.

Let the test surface be given a uniform tensile strain \( e^i \) in the longitudinal direction of the gauge. Then, under the assumptions, an element originally of width \( dx \) distance \( x \) from the mid-length of the gauge will be distorted as shown:

After Strain

Gauge

Adhesive

Uniformly Strained Test Surface

Fig 3.3 Element in Strained Condition.
Geometry of Strain

\[(1 + e_1) \, dx - (1 + e_2) \, dx = \lambda_1 \, d\phi\]
\[e_1 - e_2 = \frac{d\phi}{dx} \cdot \lambda_1\]

But \(\phi = \frac{\theta}{C}\)

so that \(e_1 - e_2 = \frac{d\phi}{dx} \cdot \frac{\lambda_1}{C}\) \(\ldots \text{Eqn. 3.1}\)

Elasticity

Stress in parent surface \(\frac{f_1}{E_1} = E_1 e_1\)

Stress in gauge paper \(\frac{f_2}{E_2} = E_2 e_2\)

so that \(e_1 - e_2 = \frac{f_1}{E_1} - \frac{f_2}{E_2}\) \(\ldots \text{Eqn. 3.2}\)

Hence Eqn. 3.1 = Eqn. 3.2

\[\frac{f_1}{E_1} - \frac{f_2}{E_2} = \frac{d\phi}{dx} \cdot \frac{\lambda_1}{C}\]

Differentiating

\[\frac{d^2\phi}{dx^2} \cdot \frac{\lambda_1}{C} = \frac{1}{E_1} \cdot \frac{df_1}{dx} - \frac{1}{E_2} \cdot \frac{df_2}{dx}\]

But \(\frac{df_2}{dx} = 0\) by the assumption of uniform strain

So that \(\frac{d^2\phi}{dx^2} \cdot \frac{\lambda_1}{C} = -\frac{1}{E_2} \cdot \frac{df_1}{dx}\) \(\ldots \text{Eqn. 3.3}\)
Equilibrium of Element.

If \( \ell \) = width of gauge, then neglecting the force in the wire:

\[ \frac{\ell}{t} \cdot \ell - q \ell \cdot dx \left( \frac{\ell}{t} + d_{\ell/t} \right) = 0 \]

\[ q = -t \frac{d_{\ell/t}}{dx} \quad \text{... Eqn. 3.4} \]

Substituting in Eqn. 3.3

\[ \frac{d^2q}{dx^2} \cdot \frac{\lambda}{C} = -\frac{1}{E_x} \cdot \frac{q}{t} = 0 \quad \text{... Eqn. 3.5} \]

or

\[ \frac{d^2q}{dx^2} = -\frac{C}{\lambda E_x} \cdot q = 0 \]

General solution of this equation is:

\[ q = Ae^{m_x} + Be^{-m_x} \quad \text{where} \quad m_x = \sqrt{\frac{C}{\lambda E_x}} t \]

Solving for the constants 'A' and 'B'.

(a) From symmetry \( q \) at \( x = \frac{\ell}{2} \) = -q at \( x = -\frac{\ell}{2} \)

\[ Ae^{\frac{m_x \ell}{2}} + Be^{-\frac{m_x \ell}{2}} = 0 \]

\[ A = -B \]

Hence

\[ q = A \left( e^{m_x x} - e^{-m_x x} \right) \quad \text{... Eqn. 3.6} \]

Differentiating

\[ \frac{dq}{dx} = m_x A \left( e^{m_x x} + e^{-m_x x} \right) \]

Substituting in Eqn. 3.1

\[ e_1 - e_2 = \frac{m_x A \lambda}{C} \left( e^{m_x x} + e^{-m_x x} \right) \quad \text{... Eqn. 3.7} \]
(b) At \( x = \frac{L}{2} \), \( f_2 = 0 \), \( : e_2 = 0 \).

\[
A = \frac{C e_1}{m_1 \lambda_1 \left( e_{m_2 x} + e^{-m_2 x} \right)}
\]

Hence,

\[
q = \frac{C e_1 \left( e^{m_2 x} - e^{-m_2 x} \right)}{m_1 \lambda_1 \left( e_{m_2 x} + e^{-m_2 x} \right)} \quad \text{from Eqn. 3.6}
\]

i.e. Shear Stress in adhesive is given by :-

\[
q = \frac{C e_1}{m_1 \lambda_1 \coth \frac{m_2 x}{2}} \cdot \sinh m_2 x \quad \ldots \text{Eqn. 3.6}
\]

Also, substituting in Eqn. 3.7 :-

\[
e_1 - e_2 = \frac{m_2 \lambda_1 \cdot C e_1 \left( e^{m_2 x} + e^{-m_2 x} \right)}{m_1 \lambda_1 \left( e_{m_2 x} + e^{-m_2 x} \right)}
\]

i.e. Strain in gauge paper carrier is given by :-

\[
e_2 = e_1 \left( 1 - \frac{\cosh m_2 x}{\cosh \frac{m_2 L}{2}} \right) \quad \ldots \text{Eqn. 3.9}
\]

where \( m_1 = \sqrt{\frac{C}{\lambda_1 E_2 t}} \)
Eqn. 3.9 may be integrated from $x = -\frac{l_2}{2}$ to $x = +\frac{l_2}{2}$ to give the total extension of the gauge paper for a given parent strain as follows:

$$e_2 = e_1 \left(1 - \frac{\cosh mx}{\cosh \frac{ml_2}{2}}\right)$$

Total Extension of Gauge

$$= 2 \int_0^{l_2} e_2 \, dx$$

$$= 2 \int_0^{l_2} e_1 \, dx - 2 \int_0^{l_2} \frac{e_1}{\cosh \frac{ml_2}{2}} \cdot \cosh \frac{mx}{\cosh \frac{ml_2}{2}} \, dx$$

$$= 2e_1 \frac{l_2}{2} - \frac{2e_1 \sinh \frac{ml_2}{2}}{m_1 \cosh \frac{ml_2}{2}}$$

Provided that $\frac{ml_2}{2} > 3$, $\sinh \frac{ml_2}{2} = \cosh \frac{ml_2}{2}$, closely so that

$$\text{Total Extension of Gauge} = e_1 \left(\frac{l_2 - \frac{2}{m}}{m_1}\right) \quad \text{... Eqn.} \ 3.10$$

and

$$\text{Average Strain in Gauge} = e_1 \left(1 - \frac{2}{m_1 l_2}\right) \quad \text{... Eqn.} \ 3.11$$
3.5 DEDUCTIONS

The Shear Stress Variation in the Adhesive along the length of the Gauge Paper

This is given by Eqn. 3.9:

\[ q = \frac{C e_\theta}{m_1 h_1 \cosh \frac{m_2 \ell_2}{2}} \cdot \sinh mx \]

![Diagram showing shear stress variation in adhesive]

**FIG. 3.5** Shear Stress Variation in Adhesive along Gauge.

The shear stress at any point along the length of the gauge will depend on the value of \( m_1 \),

where \( m_1 = \sqrt{\frac{C}{h_1 E_2 t}} \)
At the ends of the gauge \( x = \frac{L}{2} \), and if \( m \frac{L}{2} \) is larger than, say 3. (It is shown in Para. 3.14 that this is usually the case.)

\[
\sinh \frac{m \frac{L}{2}}{2} = \cosh \frac{m \frac{L}{2}}{2}
\]

Maximum shear stress in the adhesive

\[
\hat{q} = \frac{C e_0}{m_1 \lambda_1}
\]

substituting for \( m_1 \),

\[
\hat{q} = e_1 \sqrt{\frac{C E_2 \lambda_2}{\lambda_1}}
\]

Hence, in order that the maximum shear stress in the adhesive for a given strain shall be as small as possible, this being desirable if the adhesive is not to break down under low strains:

For gauge (a) \( E_2 \) and \( t \) must be as small as possible.

For adhesive (b) \( C \) must be as small as possible.

\( \lambda_1 \) must be as large as possible.

For the gauge in tension, as shown by the shear stress diagram FIG. 3.5, for gauges having a large \( m \frac{L}{2} \), the ends carry most of the shear load, so that it is most important that the ends should be well glued down. This is difficult to achieve as the natural tendency is for the gauges to curl up when damped with acetone for the application of the adhesive.

E. Jones (32), of the R.A.E. Farnborough, states that a reduction in the sensitivity factor of the order of 20%
may be expected for a gauge only stuck down over the length of the wire compared with a gauge stuck down completely over the whole length.

The centre portion of the gauge, in which the wire is situated, is therefore relatively unimportant for taking the shear load, and hence imperfect sticking here will not seriously affect the sensitivity factor in tension.

For the gauge in compression, unless it is well stuck down over the centre portion, strut action may occur. Hence, for the gauge to work equally well in both tension and compression, it should be stuck down over the complete length of the paper backing.

If the gauge paper-carrier is very short so that \( \frac{m_l \ell}{2} \) is small, then the distribution of shear in the adhesive is as shown by the dotted curve in FIG. 5.5. Perfect sticking is therefore more important for a short gauge than for a long gauge in tension.
3.6 DEDUCTIONS

The Strain Variation along the Length of the Gauge Paper

This is given by Eqn. 3.9

\[ e_2 = e_1 \left(1 - \frac{\cosh m_1 x}{\cosh \frac{m_1 l_2}{2}}\right) \]

![Diagram showing strain variation along length of gauge.]

FIG. 3.6 Strain Variation along length of gauge.

3.6.1 The transmitted strain \(e_2\) depends on the value of \(\frac{m_1 l_2}{2}\). If \(\frac{m_1 l_2}{2}\) is small, say less than 1, then \(\cosh m_1 x = \cosh \frac{m_1 l_2}{2}\), and the maximum \(e_2\) is very small compared with \(e_1\). The variation of the transmitted strain with the length is as shown by the dotted curve in the above diagram.
If \( \frac{mL}{2} \) is large, say greater than 6, then \( \frac{\cos \theta \cosh \frac{mL}{2}}{\cosh \frac{mL}{2}} \) is very large, so that until \( x \) is nearly equal to \( \frac{L}{2} \) \( \cos \theta \cosh \frac{mL}{2} \) is small. This results in an almost constant strain being transmitted to the paper over the central section of the length of the gauge, where the wire should be situated, as shown in FIG. 3.6. The maximum value of \( e'_2 \) will then be only slightly less than the parent strain \( e'_1 \), and as shown in equation 3.11, the average strain in the gauge will be close to the value \( e'_1 \). Hence in order to obtain a high efficiency of strain transmission \( \frac{mL}{2} \), i.e., \( \frac{L}{2} \sqrt{\frac{C}{\lambda'_1E'_2t}} \), must be large. This requires:

For Gauge (a) \( E'_2 \) and \( t \) shall be as small as possible.  
\( \lambda'_2 \) shall be as large as possible.

For adhesive (b) \( G \) shall be as large as possible.  
\( \lambda'_n \) shall be as small as possible.

The following figures for some commercial gauges examined show how the requirements for (a) are satisfied.

<table>
<thead>
<tr>
<th>Manufacturer</th>
<th>Paper</th>
<th>Thickness of single sheet</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hotol</td>
<td>Thin rice paper</td>
<td>0.001 inches</td>
</tr>
<tr>
<td>Baldwin Southwark</td>
<td>Heavy rice paper</td>
<td>0.003 inches</td>
</tr>
<tr>
<td>British Thermostat Co.</td>
<td>Filter paper</td>
<td>0.005 inches</td>
</tr>
<tr>
<td>N.P.L.</td>
<td>Bakelised paper</td>
<td>0.003 inches</td>
</tr>
</tbody>
</table>
The values for Young's Modulus $E_2$ for the papers are not known, and in any case the value would depend on the humidity. Some rough tests by the author indicate that the order of $E_2$ is 50,000 lb/in$^2$. However, as the paper is thoroughly saturated with the adhesive, it may be assumed that it possesses similar mechanical properties to the adhesive, e.g. $E = 0.2 \times 10^6$ lb/in$^2$ for cellulose acetate cement (E.R. Jones) (32). It will be noticed that the conditions required of the adhesive for high efficiency of strain transmission are in direct contradiction to those required for the best stress conditions in the adhesive. The conditions usually taken are justified as follows:

Experience has shown that a cellulose acetate adhesive gives reliable working, so that this is normally used. The value of $C$ is thus accepted.

The thickness of the adhesive film is directly under the control of the operator and, in order to achieve as high an efficiency as possible, the thickness is made as thin as possible. A normal thickness is about 0.005 inches. The reduction in the maximum strain which can be transmitted by the adhesive due to the thin film, is not usually a disadvantage.

3.6.2 Creep in the adhesive under load will merely alter the strain distribution at the ends of the paper carrier, so that, providing the paper extends well beyond the gauge wire,
the centre section will be relatively unaffected. This explains the remarkable freedom from hysteresis and creep obtained in the electrical resistance measurements. The greater the length of the paper, the less will be the effect.

3.6.3 Considering equation 3.11:

Average strain in the gauge carrier = \( e_1 \left( 1 - \frac{2}{m_1^2} \right) \)

From this it may be seen that, for a given value of \( m_1 \), the strain sensitivity of the gauge will be smaller for short gauges than for long ones.

In addition, the lead on the wire winding will cause a loss of sensitivity. This will have most effect on short gauges, but as the inclination of the strands to the longitudinal axis of the gauge is very small, the effect will be of the second order.

3.7 THE STRAIN TRANSMISSION FROM THE PAPER CARRIER TO THE WIRE.

The former wound gauge has the wire at two levels in the depth of the paper carrier and therefore, theoretical analysis of this problem would be complex. However, it is reasonable to suppose that the distribution of strain along the wire would obey a 'cosh' law, so that long gauges of few windings may be expected to have a higher strain sensitivity than short ones having many windings.
3.8 **FORMER WOUND GAUGES IN WHICH THE GAUGE STIFFNESS IS DUE TO THE WIRE.**

For gauges in which the wire is appreciably stiffer than the paper carrier it is probable that, as a first approximation, the effect of the paper can be neglected, and the strain transmission may be considered to be straight through from the parent surface to the wire.

An analysis for this type of transmission for wire at one level, is given in the following section on the Flat Zig-zag type of gauge.

3.9 **THE BASIC ASSUMPTIONS - FLAT ZIG-ZAG TYPE OF GAUGE**

In this type of gauge the wire is supported by being set in a layer of adhesive on a single thickness of a thin paper.

![Diagram of Zig-Zag Type Gauge](image)

**FIG. 3.7** Zig-Zag Type Gauge.
The strain transmission sequence is thus from strained surface, through adhesive to paper and through adhesive to the wire. The single leaf of paper has, however, a low stiffness compared with the wire so that as a first approximation, the effect of the paper may be neglected, and the transmission between the parent surface and the wire may be assumed to be through a uniform medium.

The following simplifying assumptions are made:

(a) There is no shear in the gauge wire.
(b) There is no tension set up in the adhesive.
(c) There is a uniform strain on the parent surface.
(d) The influence of neighbouring wires is neglected.
   This is justifiable as the distance between wires is of the order of 40 diameters in normal gauges.
(e) The wire is completely embedded in adhesive.
3.10 THEORETICAL ANALYSIS

Strain Transmission from Parent Surface to Wire.

Consider the gauge to be stuck uniformly along its length to a test surface.

Before Straining

![Diagram of a gauge before straining](image)

**FIG. 3.8** Longitudinal Section through Gauge.

Let the test surface be given a uniform tensile strain \( \varepsilon \), in the longitudinal direction of the gauge.

An element originally of width \( \text{d}x \), distance \( \text{x} \) from the mid-length of the wire will, under the assumptions made, be as shown, for the action of the applied strain is to pull the wire out of the adhesive.

After Straining

![Diagram of an element in strained condition](image)

**FIG. 3.9** Element in Strained Condition.
Geometry of Strain

\[(1 + e_1) dx - (1 + e_3) dx = \lambda_2 \frac{d\phi_3}{dx}\]

\[e_1 - e_3 = \frac{d\phi_3}{dx} \lambda_2\]

Let \(q_3\) = average shear stress around the wire.

and define \(\phi_3 = \frac{q_3}{Kc}\)

where \(K\) = a constant depending on \(\lambda_2\) and \(D\).

\(C\) = Shear modulus of adhesive.

so that \(e_1 - e_3 = \frac{d\phi_3}{dx} \frac{\lambda_2}{KC}\) \hspace{1cm} \text{Eqn. 3.12}

Elasticity.

Stress in parent surface = \(\frac{p}{\delta_1} = E_1 e_1\)

Stress in gauge wire = \(\frac{p}{\delta_3} = E_3 e_3\)

so that \(e_1 - e_3 = \frac{p}{E_1} - \frac{p}{E_3}\) \hspace{1cm} \text{Eqn. 3.13}

Then Eqn. 3.12 = Eqn. 3.13

\[\frac{dq_3}{dx} \frac{\lambda_2}{KC} = \frac{p}{E_1} - \frac{p}{E_3}\]

but \(\frac{dq_3}{dx} = 0\),

Hence differentiating

\[\frac{d^2q_3}{dx^2} \frac{\lambda_2}{KC} = -\frac{1}{E_3} \frac{dq_3}{dx}\] \hspace{1cm} \text{Eqn. 3.14}
Equilibrium of Element

\[ \frac{q_0}{d_{33}} \pi D^2 \quad q_3 \pi D \cdot dx - (\frac{q_0}{d_{33}} + \frac{q_3}{d_{33}}) \pi D^2 = 0 \]

i.e. \[ q_3 = -\frac{D}{4} \cdot \frac{dq_3}{dx} \]

Substituting in Eqn. 3.14

\[ \frac{d^2 q_3}{dx^2} - \frac{4CK}{DE_3 \lambda_2} \cdot q_3 = 0 \quad \ldots \quad \text{Eqn. 3.15} \]

The general solution to this equation is:

\[ q_3 = Ae^{m_2x} + Be^{-m_2x} \quad \text{where} \quad m_2 = \sqrt{\frac{4CK}{DE_3 \lambda_2}} \]

In a similar manner to that given in the previous analysis of the Former Wound type of gauge, constants 'A' and 'B' can be found.

Using: \( q_3 \) at \( x = \frac{d_{33}}{2} \) is equal to \( q_3 \) at \( x = -\frac{d_{33}}{2} \)

and \( \frac{b}{d_{33}} = 0 \) at \( x = \frac{d_{33}}{2} \)

Average Shearing Stress in adhesive around wire \[ = q_3 = \frac{CKe_1 \cdot \sinh m_2x}{m_2 \lambda_2 \cosh m_2 \frac{d_{33}}{2}} \quad \ldots \quad \text{Eqn. 3.16} \]

Strain in Gauge Wire \[ = e_3 = e_1 \left( 1 - \frac{\cosh m_2x}{\cosh m_2 \frac{d_{33}}{2}} \right) \quad \ldots \quad \text{Eqn. 3.17} \]

Average Strain in Wire \[ = e_1 \left( 1 - \frac{2}{m_2 \frac{d_{33}}{3}} \right) \quad \ldots \quad \text{Eqn. 3.18} \]
3.11 DEDUCTIONS

These are very similar to those for the former Wound gauge, and therefore will be discussed only briefly.

Shear stress variation in the adhesive along the length of the wire.

As in the transmission to the paper carrier in the previous analysis, the shear stress variation follows a "sinh" curve with maximum values at the ends of the wire of

\[ e_1 \sqrt{\frac{CKE_3D}{4\lambda_2}} \] if \( \frac{E_3^2}{2} \) is larger than 3. Hence the importance of perfect sticking at the ends of the wire to avoid the breakdown of the adhesive at low strains.

To ensure a low maximum shear stress will require:

- For gauge 'E_3' and 'D' to be as small as possible.
- For Adhesive 'C' to be as small as possible.
- 'C' to be as small as possible.

The value of 'K', which will be a function of the wire diameter and the thickness of the adhesive, should be as small as possible.

3.12 DEDUCTIONS

Strain variation along the length of the wire.

The strain variation follows a 'cosh' curve as in the previous analysis.
The average strain in the gauge will be highest when

\[ \frac{m_3 l_3}{2} \text{ is large,} \]

\[ \frac{l_3 \sqrt{4CK}}{2 \sqrt{DE_3 \lambda_2}} \text{ is large.} \]

This requires that:

For Gauge, 'D' and 'l_3' shall be as small as possible.

'C' shall be as large as possible.

For Adhesive

'C' shall be as large as possible.

'\lambda_2' shall be as small as possible.

The value of 'K' should be large.

It will be seen that a compromise is again required between the transmission efficiency and the minimum shear stress considerations.

3.13 EXPERIMENTAL DATA FROM COMMERCIAL SERIES OF GAUGES

To try to establish the validity of the equations put forward in the theory, the data given in Table 3.1 as obtained from the Maker's catalogues, was analysed.
3.14 ANALYSIS

From equation 3.17 for the Flat-Grid type of gauge

\[ e_3 = e_1 \left( 1 - \frac{2}{m_2 \ell_3} \right) \]

But

\[ S_g = \frac{dR}{e_3} \]

where \( S_g \) = Strain Sensitivity of the gauge as measured.

\[ S_w = \frac{dR}{e_3} \]

where \( S_w \) = Strain Sensitivity of wire.

Hence

\[ \frac{e_3}{e_1} = \frac{S_g}{S_w} \]

So that

\[ S_g = S_w \left( 1 - \frac{2}{m_2 \ell_3} \right) \]

If the theory is correct, by plotting a graph of \( S_g \) against \( \frac{1}{\ell_3} \) for a series of gauges of the same type, a straight line curve should result. The intercept of the graph for \( \ell_3 = \infty \) gives the value of \( S_w \), and the slope gives \( 2 \frac{S_w}{m_2} \) from which \( m_2 \) may be found.

RESULTS.

Two series of Flat Grid type Tinsley Gauges are shown plotted in Graph 3.1, and it can be readily seen that the strain sensitivity of these gauges does vary according to a law of the form shown by the theory.
<table>
<thead>
<tr>
<th>Maker</th>
<th>Gauge Type</th>
<th>Material of Wire</th>
<th>Gauge Resistance (Ωcm)</th>
<th>Overall Gauge Size</th>
<th>Gauge Length of Wire (mm)</th>
<th>Sensitivity Ratio</th>
<th>$\frac{1}{\ell_3}$ (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tinsley</td>
<td>Flat Grid</td>
<td>Nichrome</td>
<td>100 4 x 14</td>
<td>3 mm</td>
<td>2.00</td>
<td></td>
<td>0.125</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>200 9 x 24</td>
<td>17 mm</td>
<td>2.18</td>
<td></td>
<td>0.059</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1000 9 x 32</td>
<td>22 mm</td>
<td>2.25</td>
<td></td>
<td>0.042</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1500 9 x 46</td>
<td>33 mm</td>
<td>2.27</td>
<td></td>
<td>0.026</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>2000 9 x 59</td>
<td>51 mm</td>
<td>2.32</td>
<td></td>
<td>0.020</td>
</tr>
<tr>
<td>Flat Grid</td>
<td>S.E. (of Eureka)</td>
<td></td>
<td>50 4 x 14</td>
<td>3 mm</td>
<td>1.8</td>
<td></td>
<td>0.125</td>
</tr>
<tr>
<td>Flat Grid</td>
<td></td>
<td>Nichrome</td>
<td>100 9 x 24</td>
<td>17 mm</td>
<td>1.95</td>
<td></td>
<td>0.059</td>
</tr>
<tr>
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<td></td>
<td></td>
<td>500 9 x 32</td>
<td>24 mm</td>
<td>2.10</td>
<td></td>
<td>0.042</td>
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<tr>
<td>Flat Grid</td>
<td></td>
<td>Nichrome</td>
<td>750 9 x 46</td>
<td>33 mm</td>
<td>2.11</td>
<td></td>
<td>0.026</td>
</tr>
<tr>
<td>Flat Grid</td>
<td></td>
<td></td>
<td>1000 9 x 59</td>
<td>51 mm</td>
<td>2.15</td>
<td></td>
<td>0.020</td>
</tr>
<tr>
<td>BALDWIN-</td>
<td>Flat Grid</td>
<td>Copper-Nickel</td>
<td>120 3/4 x 2</td>
<td>1/2 in</td>
<td>2.0</td>
<td></td>
<td>0.078</td>
</tr>
<tr>
<td>Southward</td>
<td></td>
<td></td>
<td>120 3/4 x 2</td>
<td>1/2 in</td>
<td>2.0</td>
<td></td>
<td>0.039</td>
</tr>
<tr>
<td>(SR-4)</td>
<td></td>
<td></td>
<td>300 5/16 x 7/2</td>
<td>6 in</td>
<td>2.1</td>
<td></td>
<td>0.007</td>
</tr>
<tr>
<td>Helical</td>
<td>Copper-Nickel</td>
<td></td>
<td>60 3/16 x 9/16</td>
<td>1/16 in</td>
<td>1.7</td>
<td></td>
<td>0.650</td>
</tr>
<tr>
<td>Wound</td>
<td></td>
<td></td>
<td>120 15/32 x 3/4</td>
<td>1/8 in</td>
<td>1.8</td>
<td></td>
<td>0.315</td>
</tr>
<tr>
<td>(B Type)</td>
<td></td>
<td></td>
<td>120 15/32 x 3/4</td>
<td>1/8 in</td>
<td>1.9</td>
<td></td>
<td>0.153</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>60 -</td>
<td>5/8 in</td>
<td>2.0</td>
<td></td>
<td>0.105</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>120 -</td>
<td>1/2 in</td>
<td>2.0</td>
<td></td>
<td>0.078</td>
</tr>
</tbody>
</table>

**TABLE 3.1** Strain Gauge Data from Maker's Catalogues.
NICHROME GAUGES

From Graph 3.1

\[ S_w = 2.37 \]

\[ m_2 = \frac{2 S_w}{\text{Slope of curve}} \]

\[ = \frac{2 \times 2.37 \times 0.124}{0.37 \text{ mm.}} \]

\[ = \frac{1.59}{\text{mm.}} \]

S.F. GAUGES

From Graph 3.1

\[ S_w = 2.19 \]

and similarly

\[ m_2 = \frac{1.42}{\text{mm.}} \]

DEDUCTIONS

1. For the shortest gauges \( l_3 = 8 \text{ mm.} \)

Hence \( \frac{m_2 l_3}{2} = \frac{1.59}{\text{mm.}} \cdot \frac{8 \text{ mm.}}{2} = 6.36 \) for Nichrome gauges

and \( \frac{m_2 l_3}{2} = \frac{1.42}{\text{mm.}} \cdot \frac{8 \text{ mm.}}{2} = 5.67 \) for S.F. gauges.

The assumption made in the analysis that

\[ \sinh \frac{m_2 l_3}{2} = \cosh \frac{m_2 l_3}{2} \]

is thus justifiable.

2. To obtain an estimate of the value of the constant 'K'

From the analysis

\[ m_2 = \sqrt{\frac{4CK}{DE_3 \lambda_2}} \]
Taking the following values for the quantities involved:

- $D = 0.001$ inches
- $E_3 = 37.5 \times 10^5$ lb/in$^2$ (Ker Wilson(36) for Nichrome wire)
- $\lambda_2 = 0.004$ in. for gauge + 0.004" for lower adhesive
- $= 0.008$ in.
- $C = 0.1 \times 10^2$ lb/in$^2$ for cellulose acetate (i.e. One half value for $E = 0.2 \times 10^3$ lb/in$^2$. E.R. Jones (32)
- $m_2 = 1.59/mm$.

Substituting, $K' = 1.22$ which is not an unreasonable value.

3. The Baldwin-Southwark SR-4 gauges are not made in the Flat Grid type, below $\frac{1}{2}$ inch in length, and the values given for the sensitivity factors for different lengths are not sufficiently accurate to stand analysis. However, the values do show the expected reduction for the short gauge lengths.

4. The results for the series of Baldwin-Southwark SR-4 helical wound gauges are plotted in Graph 3.2 in a similar manner to that used for the Flat Grid type.

A straight line relationship is not revealed, indicating that $m_2'$ is not a constant for the series. However, this is not unexpected, as the mechanism of operation of this type of gauge is very complex, and the construction may be expected to vary with different gauge lengths owing to manufacturing details.
CHAPTER 4

THE SELECTION OF THE TYPE OF WIRE RESISTANCE STRAIN GAUGE TO BE USED IN THE AUTHOR'S TESTS.

4.1 MAKER AND TYPE

The first gauges used by the author on concrete were obtained as samples from the N.P.L. These were of their own manufacture and were therefore of the helical wound type. As these worked satisfactorily in the preliminary tests, the commercial equivalent was sought for use in further tests.

The British Thermostat Co. manufacture a good range of gauges of this type, although the paper carrier has a weaker mechanical strength than the original N.P.L. gauges. Gauges from this source were used throughout this series of experiments.

The flat grid type of gauge was considered for the work, but H. Tinsley's gauges were rejected on the grounds of general flimsiness, and the American Baldwin-Southwark SR-4 gauges were virtually unobtainable in this country.
4.2 LENGTH.

The length selected was the general purpose 1 in. gauge length. Bearing in mind the size of the concrete specimens to be tested, it was thought that this length would give a sufficiently local strain measurement, that is, it could be considered as the strain at a point even under strain conditions which varied slightly over the length of the gauge. Also, as the maximum size of the aggregate was $\frac{3}{8}$ inch, there was a good chance that the gauge would cover a representative selection of the materials making up the concrete, and so record the mean strain of the surface.

On this topic an interesting paper by R.D. Birms and H.A. Mygind (9) appeared recently. On concrete specimens made with different maximum sizes of aggregate, a number of wire resistance strain gauges of different lengths were placed end to end. The total strain as recorded by these gauges was then checked against that recorded by one long gauge suffering the same overall strain. The maximum error involved in each case was then plotted against the aggregate size and the graph overleaf produced.
FIG. 4.1  Experiments by Binns and Mygind.

Unfortunately this work is not statistically complete and the curves obtained are rather idealised from a few scattered results. However, the implication is that, with the $\frac{3}{8}$ in. size aggregate used in the author’s tests with a 1 in. gauge length, the maximum error would be of the order of 10%.

An additional point in the selection of a gauge length is the reliability of a batch of gauges of a given pattern as regards their sensitivity factor. Gauges of 1 in. gauge length have been found to be very reliable, but with very short gauge lengths, the risk of obtaining variable strain sensitivity factors increases.
Balancing the requirements for a local strain measurement against reliability, the 1 in. gauge length seemed to be a fair compromise.

4.3 **RESISTANCE**

Two general values of gauge resistance were used, the 2000 ohm gauge and the 200 ohm gauge. The former was chosen for the first tests as this was the value of the gauges supplied by the M.F.L.; the latter was used for the greater proportion of the work as this was the resistance value around which the multi-channel strain gauge set was designed.

4.4 **SPECIFICATIONS OF GAUGES USED**

<table>
<thead>
<tr>
<th>Type</th>
<th>SE/A/7</th>
<th>SE/A/27</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pattern</td>
<td>Single</td>
<td>Single</td>
</tr>
<tr>
<td>Gauge Length</td>
<td>25 mm</td>
<td>26 mm</td>
</tr>
<tr>
<td>Gauge Width</td>
<td>5 mm</td>
<td>4 mm</td>
</tr>
<tr>
<td>Approx. resistance</td>
<td>2000 ohm</td>
<td>200 ohm</td>
</tr>
<tr>
<td>Sensitivity Factor</td>
<td>2.25</td>
<td>2.25</td>
</tr>
</tbody>
</table>

**FIG. 4.2** British Thermostat Co. "Teddington" Strain Gauges
CHAPTER 5

FIRST TEST WITH RESISTANCE STRAIN GAUGES ON CONCRETE.

It was evident that very little was known as to the use of strain gauges on concrete and that, so far as could be ascertained, no data relating to their use on concrete had been published.

The first problem to be considered was to find the best method of attaching the gauge to the concrete surface. It was decided, to try Durofix, a glue recommended for affixing gauges to metal surfaces.

5.1 PRELIMINARY COMPRESSION TEST No. 1 (2nd Dec. 1946)

A concrete cylinder 10 inches high and 5 inches diameter was used as the test specimen, and two N.P.L. gauges of 2500 ohms nominal resistance were affixed to the surface with Durofix, one in the longitudinal and one in the circumferential direction.
The method of sticking was as follows:

(i) The test surface was smoothed down with sand paper and cleaned with acetone.

(ii) The glue was diluted with about 50%, by volume, of Butyl Acetate.

(iii) The surface of the gauge paper and the test surface, having been degreased, were then given an evenly-spread thin coating of the glue and allowed to dry.

(iv) A further thin coating was applied to the test surface and the gauge dipped in acetone, shaken to remove loose liquid, and pressed firmly with the thumb on to the concrete. Excess glue was squeezed out by pressing outwards from the centre.

(v) After about one minute, when the acetone had evaporated from the gauge paper, the pressure was removed and the gauge left to dry out for at least 7 days.

The gauges were next connected in turn to a normal Wheatstone Bridge circuit rigged up for the occasion. To obtain a measure of the strains, an eight inch gauge length lever compressometer of the Ewing type was fitted to the specimen.

A simple compression test was made to about half the ultimate load, and strain readings taken during loading and unloading.
On the first run the compression gauge was used, and on the second the transverse tension gauge.

The detailed results are given in Appendix II and from them the Graphs 5.1 and 5.2 were plotted.

CONCLUSIONS.

The results showed reasonable consistency and, as the electrical measurement of strain varied almost linearly with the mechanical measurement, it was decided to go ahead with further and more comprehensive experiments. Accordingly, plans were made for building the strain gauge set as described in Chapter 7.
CHAPTER 6

THE CHOICE OF CIRCUIT

6.1 GENERAL PRINCIPLES

In static strain gauge work the change of resistance of a gauge, when strain is applied, is usually measured by the use of a Wheatstone Bridge Circuit. There are many variations of circuits which may be used, but they only differ from each other in the method by which they eliminate or reduce errors. In general, an experimenter will have to decide which circuit will give the best results at a reasonable cost.

Consider the circuit shown

![Diagram of Wheatstone Bridge Circuit]

FIG. 6.1 Wheatstone Bridge Circuit.
Let $P$ represent the gauge, and $G, R, S$ resistances.

The initial balance condition is given by the well known relationship

$$\frac{P}{Q} = \frac{R}{S}$$

If $P$ changes its resistance to $(P + dP)$ due to strain 'e', and the bridge is rebalanced by increasing the variable resistance $Q$ to $(Q + dQ)$:

Then \[
\frac{(P + dP)}{(Q + dQ)} = \frac{R}{S}
\]

$$SP + S.dP = RQ + R.dQ$$

$$\frac{dP}{dQ} = \frac{R}{S} = \frac{P}{Q}$$

But by definition \[
\frac{dP}{Q} = \text{Strain Sensitivity Factor (S.S.F.)}
\]

i.e. Strain 'e' = \[
\frac{dQ}{Q} \cdot \frac{1}{S.S.F.}
\]

If 'Q' is known, and the S.S.F. for the batch of gauges, of which 'P' is one, is also known, then the strain is given at once in terms of 'dQ'.
6.2 TEMPERATURE COMPENSATION

A change in the temperature of the wire may be caused in two ways:

(i) a change in the ambient temperature.

(ii) a change in the heating of the wire due to current.

These will cause a change in the resistance of the wire which is not due to a direct loading strain. For example, a change in the ambient temperature will cause a differential strain between the gauge wire and the test specimen if the two materials have different linear coefficients of expansion.

For the particular use of the gauges on concrete, this latter difficulty was not serious as the coefficients of linear expansion of concrete (10 - 14 x 10^{-6} per°C) and Nichrome wire (12.5 x 10^{-6} per°C) are very similar.

In general, however, the fractional change of resistance of 'P' for a temperature rise from t₁ to t₂ will be given by:

\[ \frac{\Delta P}{P} = (t₂ - t₁) \left\{ x + S.3.F. (a_3 - a_2) \right\} \]

where \( x \) = temperature coefficient of resistance of the gauge wire,

\( a_3 \) = coefficient of linear expansion of the test specimen,

\( a_2 \) = coefficient of linear expansion of the gauge wire.
For example:

For gauges made of Nichrome wire, \( \varepsilon = 70 \times 10^{-6} \) per \( \circC \), and when fixed to concrete, a change in temperature of 1\( \circC \) gives \( \frac{d\varepsilon}{dP} = 70 \times 10^{-6} \) which corresponds to a strain of about 30 \( \times 10^{-6} \).

As this is a large strain compared with the increments of strain which it is desired to read, the need for preventing changes in the temperature of the wire relative to the rest of the circuit, is forcibly brought home.

The simplest way to minimise these temperature effects is to use a second strain gauge as a resistance in an arm of the bridge circuit on the opposite side to the active gauge. (i.e. \( R' \) in Fig. 6.1). If this gauge be stuck to a piece of unstrained material similar to that to which the active gauge is stuck, and if the gauge is of the same construction and of the same batch as its active partner, then the heat dissipation from each should be equal. Also, if this gauge (usually known as the 'dummy gauge') be placed in close proximity to its active partner, then it should suffer equal ambient temperature changes in resistance. The bridge should therefore be compensated for temperature.

The use of a compensating gauge also allows for humidity effects to a first order, although humidity conditions are rarely reproducible enough to cancel out completely the effects in both gauges.
6.3 ZERO DRIFT

The phenomenon is that, over a period of loading and unloading a specimen, the true zero, as indicated by the resistance box or the galvanometer deflection, tends to 'drift'. It occurs mainly in static measurements when the gauge is in circuit for an appreciable period of time.

The magnitude of the drift in terms of strain will be small if reasonable precautions are taken, but when it is desired to measure very small strains, as was the case in the author's experiments on concrete in tension, zero drift may mask the results unless extra special care is taken.

Zero drift can be attributed to two sources :-

1. the gauges themselves and the method of attachment.
2. the bridge circuitry.

In the gauges, drift may be due to :-

(a) Creep in the adhesive and paper backing under strain.

The creep in the adhesive surrounding the wire will depend on the wire temperature, which in turn, depends on the gauge current used. Creep in the adhesive between the paper and the parent surface, will depend on the ambient temperature and on the humidity, as the latter will alter the volume of the adhesive and, therefore, its mechanical and electrical insulation properties. Bad insulation between the gauge and the specimen will cause a variable leakage to earth.
It was not thought that this point would give much trouble when the gauges were attached to dry concrete, as concrete is not a good conductor unless it is very wet. This is confirmed in a paper entitled "Physical Properties of Building Material" (6) which states, "Under normal near dry conditions the resistivity of stones ranges from 10 megohms (sandstones and limestones) to 1000 megohms (marbles and granites)"

Any moisture in the gauge paper may act as a variable shunt resistance in parallel with the gauge.

(b) In the bridge circuitry, drift may be due to:

(i) variable switch contact resistances.

(ii) Thermo-electro effects at the many soldered joints in the circuit.

(iii) Changes in the connecting lead resistance causing a variation in the total effective gauge resistance. This is quite a serious point as the gauge leads are often several yards long.

(iv) Imperfect temperature compensation. The use of a dummy gauge will rarely compensate completely for temperature changes in the wire, as it was found in the present series of tests that, even if the active and dummy gauges formed a matched pair before sticking, their resistance was changed by different amounts after sticking (See Para. 10.3)
With so many factors tending to give zero drift, it is indeed surprising that the electrical resistance strain gauge can be used as a measuring device. In fact, it will only give reliable results if every effort is made to reduce zero drift.

Some of the methods of minimising the phenomenon are clear. For example:

(i) by maintaining the gauges at a constant temperature and humidity by covering them.

(ii) by reducing the temperature of the wire as far as possible by using low currents.

(iii) by using a bridge circuit which reduces the effects of items (b) (i) (ii) (iii).

6.4 THE USE OF A DEFORMATION OR NULL METHOD.

Resistance boxes are comparatively expensive for the very limited accuracy and discrimination which they give. Consequently, considerable thought was given to the idea of measuring strain by noting the galvanometer deflection, as opposed to the practice of rebalancing the bridge.

Advantages of a deflection method.

(i) The galvanometer current in an unbalanced bridge is determined by the proportionate change in resistance in one arm. Over the range usually encountered in strain gauge work, the relationship can be taken as linear, so that the galvanometer deflections will be
closely proportional to the strain. (See Chapter 8.

(ii) The method is quicker than the null method.

Disadvantages of a deflection method.

(1) The resistance of the galvanometer will change with temperature. This will alter its sensitivity, the effect being greatest when the galvanometer is at, or above, its optimum value.

(ii) A high grade variable resistance is required for calibrating the galvanometer deflection scale in terms of the resistance change.

(iii) When the same galvanometer is used for recording from circuit to circuit in a multi-gauge set, because of the considerable variation in the resistance of the gauges of a given batch, the scale factor will probably change from gauge to gauge. This is inconvenient.

Advantages of the Null Method.

(1) The effect of changes in the galvanometer resistance are eliminated.

(ii) An experienced observer can obtain a null balance on a bridge almost as quickly as the galvanometer deflection can be read.
Disadvantage of the Null Method.

It introduces extra contact resistances which may vary, but these can be made negligible by using a suitable circuit.

CONCLUSIONS.

In view of the fact that there would be little saving of time by adopting a galvanometer deflection method, and since such a method introduces extra errors and complication, it was finally decided to use "Null Balance" readings for the measurement of strain as far as was possible with the increments available on the resistance "Q".
At the present stage of strain gauge work, zero drift is by far the most difficult problem. In consequence, it was decided to use a stable bridge which would eliminate the circuit sources of this drift, as far as possible.

The circuit selected was suggested by Urwin & Swainger (56) of City and Guilds College, London, to whom the author is indebted for permission to use the circuit before its official publication.

7.1 PRINCIPLES OF THE CIRCUIT

A modified Wheatstone Bridge Circuit is used which incorporates the Kelvin Double Bridge Principle of measuring with 'potential leads', the potential drop across the gauge due to the current supplied to it by 'current leads'.
This state of affairs is achieved by using high resistance ratio arms.

\[ \text{FIG. 7.1 The Basic Circuit} \]

(a) High Resistance Ratio Arms

If the values of the resistances 'G' and 'S' are made high compared with gauge resistances 'P' and 'R', then the bridge consists of a 'current circuit' containing the gauges, together with a potential circuit.

The potential drop across the gauge is measured by the high resistance circuit. By taking the potential leads to the actual gauge wire, the effect of any changes of switch contact resistances, which are in series with this high resistance circuit, is much reduced in importance. Similarly the effects of changes in resistance in the long connecting leads which will be put into the high resistance circuit are made negligible. The lead between the two gauges 'P' and 'R'
is kept as short as possible, so that, apart from the gauges, there is a minimum of wire in these arms.

(b) High Gauge Currents

The use of the high resistance ratio arms means that the circuit does not operate anywhere near the optimum sensitivity condition. (See paras. 3.5 and 3.7). The use of high gauge currents will improve this.

On the other hand, to minimise zero drift, low gauge currents are desirable and therefore a compromise is necessary.

Swainger (58) finds that the resistance of a typical strain gauge, which usually decreases in value, varies with both time and gauge current.
For example:

British Thermostat Co. Gauge (N.P.I. type) 200 ohm Nichrome-50 gauge wire.

On aluminium sheet.

<table>
<thead>
<tr>
<th>Resistance (ohms)</th>
<th>Time (mins.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>210.083</td>
<td>210.083</td>
</tr>
<tr>
<td>210.215</td>
<td>210.208</td>
</tr>
<tr>
<td>210.355</td>
<td>210.352</td>
</tr>
<tr>
<td>210.401</td>
<td>210.200</td>
</tr>
<tr>
<td>210.418</td>
<td>210.218</td>
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<td>210.424</td>
<td>210.420</td>
</tr>
<tr>
<td>210.425</td>
<td>210.420</td>
</tr>
</tbody>
</table>

Current mA

| 87.0 |
| 70.0 |
| 40.0 |
| 25.0 |
| 13.4 |
| 6.45 |
| 2.15 |

FIG. 7.3 Variation in gauge resistance with time (Swainger)

It appears therefore, that, for these gauges, which were of the same type and make as those used by the author, the best working range of current is between say 10 to 30 mA.

Actually, in the author's tests, a gauge current of about 15 to 20 mA was used compared with 5 mA which is the normal practice in static work in this country.
THE CIRCUIT ADOPTED

Basic Multi-Gauge Circuit Diagram.

Diagram

Active Gauges    Dummy Gauges

Fundamental Circuit Diagram

\[ \text{Fundamental Circuit Diagram} \]

- Pole Switch
- Current Leads
- Potential Leads

Balance Resistance

FIG. 7.4
To be on the safe side it is nevertheless advisable to have the gauges in circuit for as short a time as possible.

The basic circuit diagram is shown in Fig. 7.4, in which the points to note are:

(i) The switches are arranged so that they are not connected in the gauge arms of the bridge.

(ii) To allow 'Q' and 'S' to have the same values for each pair of gauges, a small adjustable resistance 's' is included in series with 'S' for each pair, to permit initial balancing of the circuit when

\[
\frac{P_1}{Q} = \frac{R_1}{S + s}
\]

This is necessary because gauges 'P_1' and 'R_1' are, in general, not quite equal initially.

The balance resistance 's' was not fitted in series with the dummy gauge 'R', as this system suffers two big disadvantages:

(a) The slightest change in the resistance 's' would have been magnified in the same ratio as the measured change in the resistance of the active gauge, due to the strain. As such small quantities are involved, the gauge reading could well be obscured. To avoid this would require 's' to be of a very high grade and therefore very expensive.
(b) To avoid long lengths of wire in series with the dummy gauge, each resistance 's' would have had to be fitted very close to its respective dummy, and hence could not be built into the main panels of the set.

(iii) A pair of resistances 'P₀' and 'R₀' of about the same value as 'P₁' and 'R₁' are built into the circuit so that they can be switched into the set in the same way as the gauges. These enable the bridge to be in balance with a continuous circuit when no measurement is taking place. Consequently the whole circuitry, except the active and dummy gauges, can be kept at constant temperature, and the gauges themselves are only switched in for long enough to take a reading. In this manner it was hoped to avoid temperature effects in the wiring, switches, etc.

7.3 CONSTRUCTION OF THE MULTI-CHANNEL STRAIN GAUGE SET

The original conception

The Urwin and Swaiger circuit Fig. 7.4 was adopted, as it offered the greatest possibility of eliminating unmeasurable errors, and of measuring small changes of resistance quite accurately without too much complication and expense. More expensive apparatus would have given greater accuracy.
The requirements were:

(i) A twelve channel set would provide sufficient scope for the purposes in mind.

(ii) Low resistance strain gauges of the order 200 ohms would normally be used.

(iii) The set had to be made from apparatus and materials which could be obtained commercially as standard items. This was due to the immediate post-war difficulties which rendered special apparatus unobtainable except on a several year delivery basis.

Accordingly orders were placed in December 1946 for the following items:

(a) A Post Office Box, 0 - 10000 ohm range.
(b) A moving coil galvanometer, 1500 ohm resistance.
(c) A milliammeter, 0 - 50 mA range.
(d) A rotary stud switch (4 pole 24 studs).
(e) Multi-strand tinned copper wire plastic covered (1.1 mm. o.d. 0.3 mm. wall).
(f) Rheostats - 50 ohm wire wound potentiometers. 1 doz. 250 " " " " 2 off.
(g) Brass Terminals 1/16 in. dia. 4 doz.
(h) Bakelite sheeting 1/4 in. thick.

Details of the apparatus are given in Appendix III.

These orders were not sufficiently complete for a start to be made in constructing the set before January 1948.
7.4 THE LAY-OUT OF THE SET

The set was designed and constructed by the author and his coadjutor and is shown schematically in Fig. 7.7 and in its final form by the photographs (Plates 7.1 and 7.2)

Description

The complete set was built on a rigid base board.

The auxiliary potentiometers 's' together with the heating rheostats $P_o$ and $R_o$, the milliammeter and the 4 pole selector switch, were mounted on a vertical bakelite panel at the front of the set.

The terminals for the connection of the active and dummy gauges were similarly mounted on a panel at the rear of the set.

The Post Office Box which was used for the resistance measurements, was placed immediately in front of the galvanometer, the latter being rigidly supported on a raised platform.

The battery fitted conveniently beneath the galvanometer platform.

Observations

The set enabled:

(1) Twelve active gauge pairs to be accommodated. (The twenty-four stud switch was used as it was readily available.)
(ii) The high resistance arms 'Q' and 'S' on the P.O. box were about 10,000 ohms, which, for 200 ohm gauges, meant a 50 : 1 magnification of the change of resistance 'dP' in the gauge. (i.e. dQ = 50. dP)

(iii) The adjustable heating resistances enabled the circuitry to be kept warm when no measurements were being taken.

(iv) The circuit current could be noted, and any fluctuation in the battery supply, which would change the sensitivity of the set, could be corrected or an allowance made.

(v) The milliammeter served as a valuable indicator of faulty circuits.

Note re Galvanometer.

A plug switch was normally connected across the galvanometer terminals so that, when the switch was closed, the galvanometer was short circuited and became dead-beat. Thus, the violent oscillations of the galvanometer, which occur due to out of balance resistances, could be avoided.
FIG 7.7 Schematic Circuit Diagram.
CHAPTER 8

BRIDGE CIRCUIT SENSITIVITY

3.1 INTRODUCTION

When preliminary experiments were being carried out with strain gauges of different resistances, it was noted that the deflection produced on the galvanometer for a given strain and current in the gauge, varied with the gauge resistance. Also, when other galvanometers of different resistance, but having similar mechanical sensitivities were substituted into the circuits, again different deflections were produced on the galvanometer for the same strain and current in the gauge.

In order that the most suitable galvanometer for the proposed tests could be secured, the following investigation into the circuit sensitivity was undertaken.
2.2 LIST OF SYMBOLS

Voltages

\[ E = \text{Battery voltage} \]

Currents

\[ I = \text{Battery circuit current,} \]
\[ i = \text{Active strain gauge current,} \]
\[ I_G = \text{Galvanometer current,} \]

Resistances

\[ P = \text{Active strain gauge resistance,} \]
\[ P_0 = \text{Active strain gauge resistance at initial balance condition,} \]
\[ R = \text{Dummy strain gauge resistance,} \]
\[ Q = \text{Measuring resistance of bridge,} \]
\[ S = \text{Fixed resistance of bridge,} \]
\[ G = \text{Galvanometer resistance,} \]
\[ G_{\text{opt}} = \text{Optimum galvanometer resistance,} \]
\[ B = \text{Battery resistance,} \]

Galvanometer Properties

\[ k = \text{Mechanical sensitivity constant of instrument,} \]
\[ k_0 = \text{Mechanical sensitivity constant of Cambridge galvanometer,} \]
\[ k_m = \text{Mechanical sensitivity constant of Marconi galvanometer,} \]
\[ G = \text{Galvanometer deflection,} \]
\[ G = \text{Galvanometer resistance,} \]
\[ G_{\text{opt}} = \text{Optimum galvanometer resistance,} \]
\[ n = \text{Ratio } \frac{G}{G_{\text{opt}}}. \]
Sensitivities.

\[ S \quad = \quad \text{Actual sensitivity of a circuit.} \]
\[ S_{\text{max}} \quad = \quad \text{Maximum sensitivity of a circuit.} \]
\[ S_{e200} \quad = \quad \text{Actual sensitivity of circuit with 200 ohm gauges and Cambridge galvanometer.} \]
\[ S_{e2000} \quad = \quad \text{Actual sensitivity of circuit with 2000 ohm gauges and Cambridge galvanometer.} \]
\[ S_{m200} \quad = \quad \text{Actual sensitivity of circuit with 200 ohm gauges and Marconi galvanometer.} \]
\[ S_{m2000} \quad = \quad \text{Actual sensitivity of circuit with 2000 ohm gauges and Marconi galvanometer.} \]

\[ \eta_g \quad = \quad \text{Ratio} \quad \frac{S}{S_{\text{max}}} \]

Miscellaneous

\[ G \quad = \quad \frac{1}{2\sqrt{2}} \]
8.3 ABRIDGED ANALYSIS

Consider the Wheatstone Bridge Circuit shown:

![Wheatstone Bridge Circuit Diagram]

**FIG. 8.1 Wheatstone Bridge Circuit.**

In practice a measure of the circuit sensitivity is not the current through the galvanometer, but the galvanometer deflection 'dθ' corresponding to a change 'dP' in 'P'.

To make the results non-dimensional, define

\[
\text{Circuit Sensitivity} = \frac{dθ}{dP} = \frac{R \cdot dθ}{dP} \quad \ldots \quad \text{Eqn. 8.1}
\]

The sensitivity to current of moving coil galvanometers differing only in number of turns and diameter of wire (i.e. of the same size and shape of coil) is given by

\[
dθ = k \sqrt{E} \cdot dI_g \quad \ldots \quad \ldots \quad \text{Eqn. 8.2}
\]

(derivation see Appendix IV)
Eqn. 8.2 may be written as:

\[ P \frac{d\Theta}{dP} = \frac{k}{P \sqrt{G}} \cdot \frac{1}{\sqrt{G}} \]

... Eqn. 8.3

Again by considering the currents through the individual resistances and evaluating \( \theta \), this may be expressed as:

\[ P \frac{d\Theta}{dP} = \frac{k}{(P+R)(Q+S) + B(P+Q+R+S)(R+S)(Q(P+R) + B(P+Q))} \]

(derivation see Appendix V) ... Eqn. 8.4

A further manipulation of Eqn. 8.4 gives, in terms of the gauge current \( i \):

\[ \text{Circuit Sensitivity} = \frac{P \frac{d\Theta}{dP}}{\frac{k}{R} \frac{i}{R} + \frac{i}{(P+Q)}} \]

(derivation see Appendix V) ... Eqn. 8.5

In particular, when \( P = R \) as in static strain gauge work:

\[ \text{Circuit Sensitivity} = \frac{kP i}{2\sqrt{G} + (P+Q)\frac{1}{\sqrt{G}}} \]
8.4 DEDUCTIONS

Ideal Galvanometer resistance for a given circuit.

Considering Eqn. 8.5

\[
\frac{P}{G} \frac{d\theta}{dT} = \frac{kP}{(1 + \frac{P}{R})G + (P + Q) \frac{1}{\sqrt{G}}}
\]

For maximum sensitivity the denominator must be a minimum, so that, assuming the galvanometer is selected:

\[
\frac{d}{dG} \left\{ (1 + \frac{P}{R})G + (P + Q) \frac{1}{\sqrt{G}} \right\} = 0
\]

that is

\[
\frac{1}{2} \left( 1 + \frac{P}{R} \right) - \frac{1}{2} \frac{(P + Q)}{G^{3/2}} = 0
\]

that is, Ideal Galvanometer Resistance

\[
G_{\text{opt}} = \frac{R(P+Q)}{(P+R)}
\]

Re-arranging Eqn. 8.5.1

\[
G_{\text{opt}} = \frac{P + Q}{(P + Q)(R + S)} = \frac{R(P+Q)}{(P+R)} = \frac{R(P+Q)}{P + Q + R + S}
\]

That is, the best galvanometer resistance for a given circuit is equal to that of the external galvanometer circuit.

In particular,

If \( P = R \) as in this strain gauge work

\[
G_{\text{opt}} = \frac{P + Q}{2}
\]

... ... Eqn. 8.5.2
8.5. VARIATION IN THE SENSITIVITY OF A GIVEN CIRCUIT WITH THE GALVANOMETER RESISTANCE.

When \( P = R \), Eqn 8.5 gives :-

\[
\text{Circuit Sensitivity} = \frac{kP}{2\sqrt{G} + (P + Q)\frac{1}{\sqrt{G}}}
\]

For constant values of the gauge resistance \( 'P' \), gauge current \( 'i' \) and galvanometer constant \( 'k' \).

Actual Circuit Sensitivity \( (S) \propto \frac{\sqrt{G}}{2G + P + Q}

\[
= \frac{\sqrt{G}}{2(G + G_{opt})}
\]

where \( G_{opt} = \frac{P + Q}{2} \) from Eqn. 8.5.2

Particular maximum circuit sensitivity

\[
S_{max} \propto \frac{1}{2\sqrt{2}(P + Q)}
\]

(from Eqn. 8.5.4 (See Para 8.6)

\[
= \frac{1}{4\sqrt{G_{opt}}}
\]

Hence

\[
\frac{\text{Actual Sensitivity} (S)}{\text{Maximum Sensitivity} (S_{max})} = \frac{\sqrt{G}}{2(G + G_{opt})} \cdot 4\sqrt{G_{opt}}
\]

Galvanometer efficiency \( \eta_g = \frac{S}{S_{max}} = \frac{2\sqrt{G}}{n + 1} \)

where \( 'n' = \frac{G}{G_{opt}} \)

This result was first given by Schuster 1895 (51)
Using this equation, Graph 8.1 is plotted showing the variation in sensitivity with galvanometer resistance.

CONCLUSION

From Graph 8.1 it follows that the sensitivity is not greatly reduced from the maximum for a wide range of galvanometer resistances above and below the optimum figure.

The use of a galvanometer of optimum resistance will usually cause it to be overdamped so that observation is much protracted. A galvanometer of lower resistance will avoid this difficulty.
8.6 MAXIMUM SENSITIVITY OF A GIVEN WHEATSTONE BRIDGE CIRCUIT

In general, substituting in Eqn. 8.5 for $G_{opt} = \frac{R(P + Q)}{(P + R)}$

Maximum Sensitivity $= \frac{\partial \delta}{\partial P} = \frac{kP_i}{(1 + \frac{P}{R})\sqrt{R(P + Q)} + \frac{(P + Q)}{\sqrt{R(P + Q)}}}$

Max. Sensitivity $= \frac{kP_i}{2 \sqrt{(1 + \frac{P}{R})(P + Q)}}$ … Eqn. 8.5.3

CONCLUSIONS.

From equation 8.5.3 it follows that the sensitivity will be largest when, for a fixed 'P' and 'G_{opt}', 'Q' is as small as possible and 'R' is as large as possible compared with 'P'.

This is not entirely possible with static strain gauge work as $P = R$ for temperature compensation, and the smaller 'Q' becomes, the greater is the necessity for an accurate knowledge of its resistance.

In particular

When $P = R$

Maximum Sensitivity $= \frac{kP_i}{2 \sqrt{2(P + Q)}}$ … Eqn. 8.5.4
8.7 VARIATION IN THE CIRCUIT SENSITIVITY WITH THE BRIDGE RATIO \( \frac{Q}{R} \) WHEN THE GALVANOMETER RESISTANCE IS OPTIMUM

For \( P = R \), Eqn. 8.5.3 gives:

Maximum Sensitivity \( (S_{\text{max}}) = \frac{kFq}{2\sqrt{2}(P + Q)} \)

\[ = \frac{k\sqrt{q}}{2\sqrt{2}\sqrt{1 + \frac{Q}{P}}} \]

so that \[ \frac{2\sqrt{2} \cdot S_{\text{max}}}{k \cdot \sqrt{q}} = \frac{1}{\sqrt{1 + \frac{Q}{P}}} \]

Graph 8.2 shows plot of L.H.S against \( \frac{Q}{P} \).

This curve shows the variation in the circuit sensitivity for a particular value of \( P \) as \( Q \) is varied, \( k \) and \( l \) being constant.

The table of figures for this plot is given in Appendix Vo.
3.8 SENSITIVITY OF CIRCUITS USED IN AUTHOR'S EXPERIMENTS

FIG. 3.2 Wheatstone Bridge Circuit

Fixed Conditions :-

(a) In order to achieve temperature compensation for the active gauge 'P', an exactly similar gauge 'R' mounted on similar material to 'P' was used, so that :-

\[ P = R \]

for all tests.

therefore \[ Q = S \] for initial balance condition.

(b) To obtain a magnification for the change of the resistance 'P' on the variable arm of the bridge 'Q', and for other reasons given under Para. 7.1.

Resistance \[ Q = S \] was made \( 10,000 \) ohms approx. for each test.
Desired Conditions.

(a) Optimum Galvanometer resistance for 200 ohm gauges:

\[ P = R = 200 \text{ ohms}. \]

\[ \text{Ideal Galvanometer resistance } = \frac{P + Q}{2} = \frac{10200}{2} = 5100 \text{ ohms}. \]

(b) For 2000 ohm gauges:

\[ P = R = 2000 \text{ ohms}. \]

\[ \text{Ideal Galvanometer resistance } = \frac{12000}{2} = 6000 \text{ ohms}. \]

Actual Conditions.

From the above calculations it can be seen that a galvanometer having a resistance of between 5000 and 6000 ohms would be the most suitable. However, normal commercial moving coil galvanometers having such a high resistance would have a very poor mechanical sensitivity. Hence, a standard commercial type of moving coil reflecting galvanometer of relatively high resistance was ordered. After waiting nearly two years for delivery the instrument arrived having the following specifications:

- **Manufacturer**: Marconi.
- **Type**: Moving coil Spot Galvanometer.
- **Resistance**: 1500 ohms (nominal).
- **Small Scale**: 14 divs. at 6 in. radius where 1 div. = 1/20 in. (i.e. 18mm./microamp)
Whilst awaiting delivery another galvanometer was borrowed which had the following specifications:

<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Type</td>
<td>Spot Galvanometer, d'Arsonval movement</td>
</tr>
<tr>
<td>Resistance at 20°C</td>
<td>632 ohms.</td>
</tr>
<tr>
<td>Small Scale</td>
<td>92.5 div. at 6.1 in. radius where 1 div. = 2 mm. (i.e. 185 mm./microamp)</td>
</tr>
</tbody>
</table>

Another similar instrument by H. Tinsley, the specifications of which were not known, was used in some of the earlier tests and was afterwards found to give about the same sensitivity as the Marconi instrument.

The Relative Sensitivity of the particular circuits used with the different galvanometers.

(A) Theoretically

(i) From equation 3.5.4 and graph 3.3

Attainable fraction of the overall sensitivity

\[ S_{\text{max}} = \frac{k \sqrt{E_i}}{2 \sqrt{2} \sqrt{1 + \frac{E_i}{E_f}}} \]

In all tests \( Q = 10,000 \text{ ohms} \).

For constant gauge current, define \( \frac{I}{2 \sqrt{2}} = G \)

Then 200 ohm gauges, \( S_{\text{max}} = G \cdot 1.98 \, k \)

2000 ohm gauges, \( S_{\text{max}} = G \cdot 18.3 \, k \).
(ii) From Graph 8.1

200 ohm gauges

'\eta_g' Galvanometer efficiency of circuit using Camb. Inst. 
= 63%

'\eta_g' Galvanometer efficiency of circuit using Marconi Inst. 
= 85%

2000 ohm gauges

'\eta_g' Galvanometer efficiency of circuit using Camb. Inst. 
= 59%

'\eta_g' Galvanometer efficiency of circuit using Marconi Inst. 
= 80%

(iii) Actual fraction of the overall sensitivity 
\[ S = S_{\text{max}} \eta_g \]

200 ohm gauges

Cambridge Instrument \( S_c = 1.25 k_c G \cdot 63\% = 1.25 k_c G \)

Marconi Instrument \( S_m = 1.65 k_m G \cdot 63\% = 1.65 k_m G \)

From specifications \( \frac{k_c}{k_m} = \frac{185}{13} = 10 \text{ say} \)

Hence Cambridge Instrument Sensitivity \( S_c^{200} \)

Marconi Instrument Sensitivity \( S_m^{2000} \)

\[ = \frac{1.25 \cdot 10}{1.65} = 7.6 \]
2000 ohm gauges

Cambridge Instrument $S_0 = 13.3 \text{kC.56}\% = 10.8 \text{kC}$

Marconi Instrument $S_m = 13.3 \text{kC.80}\% = 14.6 \text{kC}$

Hence

\[
\text{Cambridge Instrument Sensitivity } \frac{S_0}{2000} = \frac{10.8}{14.6} = 0.73
\]

Note.

(a) Marconi Instrument $\frac{S_m}{2000} = \frac{14.6}{1.25} = 11.7$

(b) Cambridge Instrument $\frac{S_c}{2000} = \frac{10.8}{1.25} = 8.6$

(c) Marconi Instrument $\frac{S_m}{2000} = \frac{14.6}{1.25 \times 10} = 1.17$
<table>
<thead>
<tr>
<th>Test No.</th>
<th>Galvo.</th>
<th>Gauge Resistance (ohms)</th>
<th>Applied Voltage</th>
<th>Gauge Current (m.amps)</th>
<th>Galvo Scale Defln. Dvig. Divs/ohm on 'Q'</th>
<th>ins. 'θ'</th>
<th>θ ins/100m.amp gauge current</th>
</tr>
</thead>
<tbody>
<tr>
<td>Comp. No.12</td>
<td>Cambridge</td>
<td>200</td>
<td>7.06</td>
<td>17.6</td>
<td>2.6</td>
<td>0.205</td>
<td>1.17</td>
</tr>
<tr>
<td>Tension No.5</td>
<td>&quot;</td>
<td>200</td>
<td>6.25</td>
<td>17.1</td>
<td>2.7</td>
<td>0.212</td>
<td>1.24</td>
</tr>
<tr>
<td>Torsion(Trial)</td>
<td>&quot;</td>
<td>200</td>
<td>7.06</td>
<td>17.6</td>
<td>3.0</td>
<td>0.236</td>
<td>1.34</td>
</tr>
<tr>
<td>Beam(Trial)</td>
<td>Marconi</td>
<td>200</td>
<td>10.6</td>
<td>25.4</td>
<td>1.0</td>
<td>0.05</td>
<td>0.189</td>
</tr>
<tr>
<td>Beam No.1</td>
<td>&quot;</td>
<td>200</td>
<td>9.4</td>
<td>23.5</td>
<td>1.0</td>
<td>0.05</td>
<td>0.212</td>
</tr>
<tr>
<td>Beam(Trial)</td>
<td>&quot;</td>
<td>200</td>
<td>13.7</td>
<td>34.3</td>
<td>1.2</td>
<td>0.06</td>
<td>0.175</td>
</tr>
<tr>
<td>Torsion(Trial)</td>
<td>&quot;</td>
<td>200</td>
<td>10.6</td>
<td>25.4</td>
<td>0.8</td>
<td>0.04</td>
<td>0.153</td>
</tr>
<tr>
<td>&quot; No.3</td>
<td>&quot;</td>
<td>200</td>
<td>12.1</td>
<td>30.4</td>
<td>0.9</td>
<td>0.045</td>
<td>0.148</td>
</tr>
<tr>
<td>Tension No.1</td>
<td>Marconi</td>
<td>2000</td>
<td>30</td>
<td>7.5</td>
<td>2.5</td>
<td>0.125</td>
<td>1.67</td>
</tr>
</tbody>
</table>

200 ohm Gauges
\[
\begin{align*}
\text{Cambridge Instrument Sensitivity } S_c &= \frac{1.25}{0.17} = 7.4 \\
\text{Marconi Instrument Sensitivity } S_m &= \frac{1.85}{0.17} = 7.4
\end{align*}
\]

\textbf{Note}:-(1) \quad \frac{\text{Marconi Instrument}}{(\text{Marconi Instrument})} = \frac{1.67}{0.17} = 9.8

(11) \quad \frac{\text{Marconi Instrument}}{\text{Cambridge Instrument}} = \frac{1.67}{1.25} = 1.34
(C) Comparison of Theoretical and Experimental Relative Sensitivities.

Summary of Conclusions from (i) and (ii)

<table>
<thead>
<tr>
<th>Item</th>
<th>Expt.</th>
<th>Theory</th>
</tr>
</thead>
<tbody>
<tr>
<td>Marconi Instrument $S_m2000$ $S_m200$</td>
<td>9.8</td>
<td>8.9</td>
</tr>
<tr>
<td>Cambridge Instrument $S_c200$</td>
<td>7.4</td>
<td>7.6</td>
</tr>
<tr>
<td>Marconi Instrument $S_m2000$ $S_m200$</td>
<td>1.34</td>
<td>1.17</td>
</tr>
</tbody>
</table>

The discrepancies between the experimental and theoretical sensitivity ratios are accounted for by the following points:

(a) The experimental deflections given, are only correct about ± 5%

(b) The theoretical ratios were evaluated for 200 and 2000 ohms gauge resistances, whereas the actual gauges varied from about 200 to 220 ohms, and 3050 to 2150 ohms.

Bearing these facts in mind, the theoretical formulae and graphs are justified, and would be very useful to any experimenter using Wheatstone Bridge Strain Gauge sets.

The superiority of the Cambridge over the Marconi Instrument, which unfortunately had been purchased previously, was evident. The reason for selecting the Cambridge type in all final tests is therefore justified.
The new strain gauge set was ready in Feb. 1948 and preliminary compression tests on the concrete cylinder, which had been made two years previously, were carried out.

The Durofix adhesive had given fairly good results, but it was decided to try other types of glues. A high solids 'Durofix' (I.C.I. cement No.3462) was suggested as being very effective on metals and it had the added advantage of reaching its maximum sensitivity about two days after sticking. It was realised that the dampness from the concrete would affect the gauge, and so waterproof adhesives, such as sealing wax and shellac, were tried.

9.1 PRELIMINARY COMPRESSION TEST No.2

(a) Using Sealing Wax (15th Feb. 1948)

A uniform layer of sealing wax was spread over the concrete surface of the compression cylinder. After it had set it was again melted and an N.P.L. gauge pushed into the wax
while it was still soft. This, however, proved a very unsatisfactory method as the wax set too quickly for the gauge to be securely attached. This method, therefore, had to be abandoned.

(b) Using Sealing Wax Paste (15th Feb. 1948)

A quantity of sealing wax was dissolved in methylated spirit and the paste spread evenly over the concrete. An N.P.I. gauge was pressed into it, but this proved ineffective as the paste did not "go off" quickly enough.

(c) Using a Shellac Paste (20th Feb. 1948)

Method - A paste was made by dissolving shellac powder in methylated spirit, and applied to the specimen. The paste was fired, and as soon as it had burnt itself out, a gauge was pressed firmly into the tacky substance. This eventually became hard. After 14 days the compression cylinder was tested on the Richlé machine to a stress of about 1500 lb/in², and electrical measurements taken on the gauge for three different loading runs.

Observations - The readings are recorded in Appendix VI and the graphical presentation in Graph 9.1.

Conclusions - The method worked satisfactorily, although it was rather messy, and a definite undesirable hysteresis loop was formed on each loading.
(d) Using a Shellac Alcohol Paste

Method - The procedure was identical with that in Test (c) except that alcohol was used as the solvent. The gauge was subjected to test after 10 days, and two loading runs were made.

Observations - The readings are recorded in Appendix VII and the graphical presentation in Graph 9.2

Conclusions - This cement behaved in the same manner as that used in Test (c), and again a hysteresis loop was obtained.

(e) Using High Solids Durafix (I.C.I. Glue No.5482)

23rd Feb., 8th and 15th March, 1948

Method - The gauge was affixed as in Preliminary Compression No.1, Chapter 5. It was subjected to test after three days, when one loading sequence was carried out. Tests were again made after 16 and 23 days respectively, when the Hamilton Compressometer was used.

Observations - The readings are recorded in Appendices VIII, IX and X and a graphical presentation given in Graphs 9.3, 9.4 and 9.5

Conclusions - This glue gave very consistent results even at three days after fixing. The hysteresis loop obtained from the electrical readings was extremely small, and much less than that given by the mechanical Compressometer. The strain sensitivity factor obtained from two different runs was 2.23, thus confirming the N.P.L. figure 2.2 ± .03 for these gauges with a fully dried glue.
The bridge circuit appeared to be very stable, as no drift was observed in the electrical readings when the gauge was switched in for periods of up to 5 minutes, either under load or at no load.

**GENERAL CONCLUSIONS**

This series of tests showed that, of the different adhesives used, Durofix or High Solids Durofix was the most satisfactory.

Thermo-setting adhesives were found difficult to use under ordinary conditions, and special methods of fixing would have to be devised if they were to be satisfactory for use on damp concrete.

Further, thermosetting adhesives with solvents proved to be unsatisfactory. It was noticed that Shellac gave a pronounced hysteresis loop under a cycle of strain. It also proved difficult to handle and did not set quickly enough.

As already stated, High Solids Durofix gave the best results, and so it was decided to use this glue for future tests.
CHAPTER 10

THE TECHNIQUE OF THE PREPARATION AND OPERATION OF THE WIRE
RESISTANCE STRAIN GAUGE

10.1 INTRODUCTION

As stated previously, the operation of the electrical resistance strain gauge is dependent on the efficiency of its attachment to the test surface. When it was first introduced to this country in the early part of the late War, this aspect received considerable attention from various government establishments. A comprehensive report(45) of their findings, covering a large number of different adhesives and methods of attachment, was issued by the N.P.L in 1943.

10.2 MATERIALS ON WHICH THE GAUGES CAN BE AFFIXED.

The above report states that most of the early work was done with gauges affixed to steel-machined surfaces. On the subject of cast surfaces the report states that, "Whilst they had no experience of fixing the gauges to rough
machined or cast surfaces, it would seem likely that useful, but perhaps less accurate results, would be obtained on good sand-cast surfaces).

The tests, carried out by the author and described later, show the possibility of fixing gauges to dry concrete surfaces of specimens which, having been cured in water, were allowed to dry for at least 10 days before any attempt was made to experiment on them. No attempt was made to fix the gauges to wet or damp concrete.

10.3 PREPARATION OF THE GAUGE SURFACE

The following points should be borne in mind:

(i) The test surface should be made smooth and flat and finally slightly roughened with sand paper.

(ii) The surface of the gauge as well as the test surface must be made free from grease. This can be successfully done by the use of acetone just before the glue is applied.

10.4 CHOICE OF ADHESIVE

It is desirable that the adhesive should have the following features:

(i) It must be easy to apply.

(ii) It must achieve constant conditions as soon as possible.

(iii) Above all, it must obey Hooke's Law.
A number of adhesives were tried (See Para. 9.1) as a result of which 'Durofix', a cellulose acetate solution, proved the most satisfactory of the cold-setting types. Of the thermo-setting types the de Khotinsky cement, (basically shellac) is the most popular. It requires the test surface to be heated to 150°C, which would be rather difficult to obtain with large concrete specimens.

Constant conditions can be assumed to have been reached with 'Durofix' in about 7 to 10 days in medium humidity, whereas the thermo-setting types are ready for use on cooling.

Contact with the N.P.I. and the Building Research Station was made by the author in May and October, 1946 respectively. The former considered the prospect of the attachment of gauges to concrete, poor. The Building Research Station on the other hand considered that it might be possible if the testing was carried out on dry concrete.

A criticism of the use of this glue is that the cellulose acetate solution in acetone has a hardening effect on the concrete. This phenomenon was used by the Building Research Station (1934) to harden the surface of concrete under the knife edges of mechanical extensometers, and so prevent crumbling and resultant slip.
Application. The following procedure was carried out:—

(a) A reasonably smooth portion of the specimen was selected in which there were few blow-holes.

(b) The surface was then rubbed gently with sand-paper to roughen the texture and also to make it as flat as possible.

(c) Any small holes in the area selected were filled with plaster of Paris and the surface again rubbed down.

(d) Acetone was next rubbed on to take off any grease.

(e) As thin a coat as possible of High Solids Durofix was applied and left to dry.

Before affixing the strain gauge, the exact position of the wire in the gauge was marked, in ink, on the top surface by holding it up to the light. The gauges supplied were usually pre-coated with ordinary Durofix. However, in this particular case, the selected gauge was wiped with acetone to remove any grease, due to handling, and the under surface was given a thin coating of High Solids Durofix.

After about 20 mins., when the glue on both test surface and gauge had dried, another coating of glue was given to the test surface and the gauge was dipped in acetone so as to render it supple as well as to soften the glue-film. It was then pressed firmly on to the concrete, covered with a piece of blotting paper, and pressed outwards from the centre with the thumb in order to squeeze out surplus glue, and also
to avoid any air-pockets. After about 2 mins., pressure with the thumb was released and the whole left to dry for 7 days.

10.6 ATTACHMENT OF THE LEADS TO THE GAUGE

The circuit employed required two long leads from one gauge wire, and one short one from the other. (See Para 7.1). Different colours were chosen for the short and long leads to facilitate identification. The leads were soldered to the gauge as carefully as possible, and tinned-spade terminals were soldered to their free ends. They were then insulated from the test surface, and also from each other, at the bare-joints by well covering with insulating tape. This method of covering, as distinct from the sheath method helped:

(1) To prevent a difference of temperature between the two soldered joints which would otherwise cause thermo-electric e.m.f.'s

(11) To provide a support for the leads in case of an accidental tug, which might be transmitted to the gauge wire.
10.7 PROTECTION OF GAUGE

When the gauge was fully dried out and all the necessary details regarding position, resistance etc. had been observed, it was covered completely by a thick cartridge paper in order to exclude draughts.

Finally, when the gauges were ready for use, the specimen was shrouded with cloths, and left in position in the testing machine for as long as possible so as to ensure that steady temperature conditions would prevail.

10.8 MATCHING THE GAUGES.

The gauges are supplied stamped with their resistance value, which is presumably correct to the nearest ohm although, occasionally, one or two were found to be incorrectly marked. Matched pairs can thus be identified before affixing to the specimen.

After sticking and allowing a suitable period for the glue to set hard, the gauge resistances were measured in several experiments. Typical results were:

### Torsion Specimen

<table>
<thead>
<tr>
<th>Gauge Number</th>
<th>1</th>
<th>5</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Marked Resistances (ohms)</td>
<td>212</td>
<td>212</td>
<td>210</td>
<td>210</td>
<td>211</td>
<td>211</td>
</tr>
<tr>
<td>Measured Resistances (ohms)</td>
<td>211.7</td>
<td>212.3</td>
<td>209.8</td>
<td>209.8</td>
<td>209.8</td>
<td>212.4</td>
</tr>
</tbody>
</table>
Beam Specimen.

<table>
<thead>
<tr>
<th>Gauge Number</th>
<th>2</th>
<th>6</th>
<th>10</th>
<th>1</th>
<th>3</th>
<th>5</th>
<th>7</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>Marked Resist-ance (ohms)</td>
<td>210</td>
<td>210</td>
<td>210</td>
<td>211</td>
<td>211</td>
<td>211</td>
<td>211</td>
<td>211</td>
</tr>
<tr>
<td>Measured Resistance (ohms)</td>
<td>209.8</td>
<td>210.3</td>
<td>209.4</td>
<td>212.0</td>
<td>213.7</td>
<td>210.9</td>
<td>210.6</td>
<td>211.2</td>
</tr>
</tbody>
</table>

As the balance resistance is will accommodate only 50 ohms, this required that 200 ohm gauge pairs should not differ by more than 1 ohm. Thus it is by no means certain that a pair of gauges marked as equal resistances will be suitable as a pair when affixed to a specimen.

E.R. Jones (33) finds that the resistance of a gauge increases by an amount of the order 0.1 to 0.2% above its original value in sticking to a test surface, because of "the volume changes in the gauge paper involved by the exchange and subsequent evaporation of the solvent in the adhesive". The magnitude of the change is variable due to differences in technique, but two gauges, initially equal, may differ by as much as 0.3% of their original value. The need for affixing all gauges at the same time, and by exactly the same method, is therefore very obvious.

In the early tests, one dummy was made to serve, when possible, for several active gauges, but this arrangement gave considerable zero drift. This was due to the dummy remaining
in circuit for a considerable time and thus becoming 'hot', whereas each active gauge was switched-in 'cold' so that, initially temperature compensation was lost. An economy in gauges was effected by fixing all of them to actual test specimens. In this way the gauges on one specimen could serve as dummy gauges when another specimen was tested, and vice versa, unless unluckily, fracture occurred at one of the gauges.

10.9 SETTING UP

Having found matched pairs, the gauges were numbered to correspond with the terminal numbers on the set. The free ends of the three leads from each gauge had tinned spade terminals soldered to them, and each was labelled with the gauge number. This avoided confusion and saved time in tracing faults.

The active and dummy gauges were then connected to the appropriate terminals on the rear panel of the set, and also to a single common terminal plate mounted on a block of wood and placed as close as possible to the test specimen, as well as to the dummy specimen. This acted as gauge and dummy common point 'X' (See schematic circuit diagram Fig. 7.7)

A voltage, sufficient to give a gauge current of 15 - 18 milliamperes, was then applied to the circuit, and a check made for faulty connections by switching quickly through all gauges
and, at the same time, watching the ammeter reading. For identical gauges this should, of course, have remained constant so that extreme variation indicated a fault.

The set was switched on and left to warm up, for at least an hour, with the heating resistances in circuit. During this period all the gauges were closely covered with paper or felt, and whenever possible, the whole set and specimens were shrouded with a thick cloth to prevent draughts.

10.10 'CONDITIONING' THE GAUGES

It was found that if readings were taken on the first straining and unstraining of the gauges, there was a scatter of the readings and a zero drift, which it seemed unreasonable to attribute to permanent set in the specimen, owing to the low load to which it had been subjected. On the next loading and unloading, the results were more regular and the zero drift smaller. After about four or five repetitions, these effects were negligible and it became the normal practice in all future tests to 'exercise' the gauges before use.

This phenomenon has been observed by others. Gibbons (27) claims that by using a nitro-cellulose cement, on first straining up to a strain of \(2.4 \times 10^{-5}\) and unstraining, there may be a drift of \(2.5 \text{ to } 5.5 \times 10^{-5}\). After the necessary zero correction, the zero drift on the next loading should not be more than \(0.5 \text{ to } 1.0 \times 10^{-5}\). Conditioning of the gauge
will reduce the hysteresis effects to negligible values.

E.R. Jones (32) states that on the first straining and release, the zero will move in the direction of the applied strain, e.g. For a strain of \(2.5 \times 10^{-5}\), the zero drift corresponds to a strain of \(3 \times 10^{-5}\). Repeated loading and unloading some ten times will bring the gauge to a cyclic state and reduce drift to about \(1 \times 10^{-5}\).

This effect may be due to hysteresis in the specimen or to locked-up stresses. Difficulties on this latter point have been found in welded structures where it is useless to test before 'exercising' the specimen. It is possible that a virgin concrete specimen requires a few proof loadings to achieve a state of ease.

10.11 BALANCING THE CIRCUITS.

After conditioning the gauges, a small load was put on the specimen in order to tighten-up all components. The high resistance arms were set at \(Q = 10020\) ohms and \(S = (10000 + s)\) ohms. The value of \('Q'\) was chosen so that normal changes in the gauge resistance, whether up or down, could be accommodated on the P.O. Box without introducing errors by altering the 10,000 or 1,000 ohm dials. This was advisable as the coils were only guaranteed to be correct to within 0.1\% and hence, the use of the 10,000 or 1,000 ohm knobs, when measuring, might lead to considerable inaccuracy.
The balance resistances \( s \) were then adjusted for each gauge pair to obtain initial centre zero reading on the galvanometer scale.

When all the resistances had been set, they were quickly rechecked, after which great care was taken to avoid touching or jarring them.

**10.12 TESTING PROCEDURE**

As each condition of loading was applied to the specimen, the set was switched rapidly through to each gauge pair in turn, and the change of reading in \( Q \) to give zero galvanometer deflection, was noted. The sensitivity of the circuit was such that the absolute zero was not obtainable with the smallest increment possible on resistance \( Q \) (i.e., one ohm). Hence the deflections left and right of zero for \( \pm 1 \) ohm on \( Q \) were also noted, so that interpolation of resistance value was possible. The accuracy of the interpolation depends on the actual sensitivity of the circuit employed. For the principal tests, the accuracy of the measurement of \( Q \) was \( \pm 0.03 \) ohms.

When experienced, an operator with the assistance of a clerk, can read the 12 gauges in about 4 minutes.
10.13 BOOKING OF RESULTS

(i) Clerical Method

During the course of a test, using only a few gauges, it was quite normal to obtain several hundred readings. The systematic booking of the results was therefore essential. The gauge readings had to be booked rapidly, and much miscellaneous data, such as the circuit current, the time, mechanical extensometer reading, behaviour of the specimen etc., had to be recorded as the test proceeded. It was therefore found to be convenient to have a printed form made out with appropriate column headings under which the readings could be booked. In addition columns were provided for reducing the readings. These sheets then formed a convenient complete record of each test.

A typical completed form is shown in Table 10.1

(ii) Operational Method

It was found that operating the strain-gauge set completely occupied one experimenter. The manipulating of the testing machine and the recording of all data was carried out by the co-adjutor.
## Experiment: Combined Bending & Torsion Test

### Mechanical Measurement of Strain

<table>
<thead>
<tr>
<th>Load</th>
<th>Remarks</th>
<th>Voltage</th>
<th>Current (Circuit)</th>
<th>Cambridge Galvo</th>
<th>Dial Gauges</th>
</tr>
</thead>
<tbody>
<tr>
<td>T = 3 divs</td>
<td>Voltage 9 Volts</td>
<td>19.5 m.A.</td>
<td>Cambridge Galvo</td>
<td>1</td>
<td>10,000</td>
</tr>
<tr>
<td>B = 30 lbs.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Upper 24.1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lower 3.5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Resistance Box QML & QMR

<table>
<thead>
<tr>
<th>Gauge No.</th>
<th>Resistance Box QML</th>
<th>Calvo Deflection Divs, L/R</th>
<th>Corrected QM</th>
<th>Strain</th>
<th>GAUGE No.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td>GAGE OUT OF ACTION</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>10028</td>
<td>0.6L</td>
<td>10028.19</td>
<td>-0.01</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>10029</td>
<td>2.6R</td>
<td>10029.20</td>
<td>-0.01</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>10023</td>
<td>3.6L</td>
<td>10023.94</td>
<td>-0.04</td>
<td>4</td>
</tr>
<tr>
<td>5</td>
<td>10029</td>
<td>0.2R</td>
<td>10023.98</td>
<td>-0.04</td>
<td>5</td>
</tr>
<tr>
<td>6</td>
<td>10014</td>
<td>1.1L</td>
<td>10014.35</td>
<td>+0.04</td>
<td>6</td>
</tr>
<tr>
<td>7</td>
<td>10015</td>
<td>2.0R</td>
<td>10014.31</td>
<td>+0.04</td>
<td>7</td>
</tr>
<tr>
<td>8</td>
<td>10015</td>
<td>1.85L</td>
<td>10015.63</td>
<td>+0.06</td>
<td>8</td>
</tr>
<tr>
<td>9</td>
<td>10015</td>
<td>0.9R</td>
<td>10015.67</td>
<td>+0.06</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>10015</td>
<td>0.20</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
CHAPTER 11

CALIBRATION OF GAUGES ON STEEL SPECIMENS

11.1 INTRODUCTION

Normally strain gauges cannot be successfully removed after they have been fixed to a surface. In fact, it has been said that, if a strain gauge can be removed from a test surface without damage, then it is not securely cemented. Hence it is not generally possible to calibrate each gauge before use.

The assumption must therefore be made that all gauges supplied in a particular batch are exactly the same. That is, they are all off the same spool of wire and are made by the same operator under the same conditions.

If one or two gauges from each batch of say 15 to 20 are calibrated, the gauge factor obtained is assumed to apply to the complete batch of gauges.
The following calibration tests were carried out by the author on steel specimens at the Royal Naval College, Greenwich. The gauges were taken from the batches used in the concrete tests.

Object - The strain sensitivity factor (S.S.F.) of a gauge is conveniently defined as the fractional change in resistance per unit strain:

(i.e. \( S.S.F. = \frac{dP}{P} \text{ strain} \))

This factor is normally quoted for the gauge situated along the axis of a uni-direction stress field. If the gauge has an appreciable cross sensitivity, the S.S.F. should be measured on the material on which the gauges are to be used, or a correction made for the different Poisson's Ratios (See Para. 11.5)

Thus to obtain this factor the change in resistance of the gauge must be measured for a known strain.

11.2 (a) BEAM METHOD - Four Point Loading.

Theory

Consider a uniform beam simply supported and loaded with concentrated loads as shown overleaf.
This produces a uniform bending movement over the centre section, and the beam bends to a circular arc over this section.

From the theory of bending \( \frac{f}{h} = \frac{E}{R} \)

Deflection \( \delta \)  
Strain \( \varepsilon = \frac{f}{E} = \frac{h}{R} \)

\[ (2R - \delta)\delta = \left(\frac{L}{2}\right)^2 \]

If the deflection is small

\[ \varepsilon = \frac{3h\delta}{R^2} \]

FIG. 11.2 Geometry of Strained Beam.

\[ \varepsilon = \frac{4d\delta}{L^2} \]  \( \text{Eqn.11.1} \)

where \( d \) = depth of beam.

\( \delta \) = central deflection.

Thus if a strain gauge is affixed to the beam in a longitudinal direction anywhere between the supports on the top
or bottom surface it will suffer the same strain. This is convenient as it avoids careful positioning of the gauge.

Apparatus.

The apparatus consisted of a uniform mild steel beam 30 inches x 2 inches x 0.312 inches which was supported on ground hardened steel knife edges. These were carried on movable supports on a rigid bed plate. Dead loads were applied to the ends of the beam through a distributing bar. The deflection was measured by a 1/10,000 inch Ames Clock Gauge which was mounted on a rigid stand from the bed plate. A multi-channel strain gauge set was used for the electrical measurements.

Method of Testing.

The gauge was situated in the middle span and was connected into the strain gauge circuit in the usual manner. The dummy gauge was a matched partner of the active gauge, and was affixed to a bar of similar material which was placed in close proximity to the beam. The readings of the central deflection, and the electrical strain measurements, were recorded using a null method for each load increment during loading and unloading.

By inverting the beam and repeating the test, the sensitivity factor was obtained for both the gauge in tension and the gauge in compression.
### Observations

The readings for three separate calibrations by this method are given in Appendices XI, XII and XIII. For Graphical presentation see Graphs 11.1 and 11.2

### Results

<table>
<thead>
<tr>
<th>Test No.</th>
<th>2000 ohm B.T.C. gauge</th>
<th>Type SE/A/7</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Gauge in tension</td>
<td>S.S.F. = 2.27</td>
</tr>
<tr>
<td></td>
<td>Gauge in compression</td>
<td>S.S.F. = 2.33</td>
</tr>
<tr>
<td>Test No.2</td>
<td>200 ohm B.T.C. gauge</td>
<td>Type SE/A/27</td>
</tr>
<tr>
<td></td>
<td>Gauge in tension</td>
<td>S.S.F. = 2.27</td>
</tr>
<tr>
<td></td>
<td>Gauge in compression</td>
<td>S.S.F. = 2.27</td>
</tr>
<tr>
<td>Test No.3</td>
<td>200 ohm B.T.C. gauges</td>
<td>Type SE/A/27</td>
</tr>
<tr>
<td></td>
<td>Gauge in tension</td>
<td>S.S.F. = 2.25</td>
</tr>
<tr>
<td></td>
<td>Gauge in compression</td>
<td>S.S.F. = 2.14</td>
</tr>
</tbody>
</table>

### Conclusions

The close agreement between the different calibrations indicates that, under these conditions, the 2000 ohm gauges have an average sensitivity factor of 2.30 and the 200 ohm gauges one of 2.27. The discrepancy in Test No. 3 may be due to a different thickness of glue film.

Faulty initial balancing is revealed by the resistance-load curves not passing through the origin.
11.3 (b) TENSION TEST METHOD

The gauges are affixed to a tensile specimen of known Young's Modulus of Elasticity ($E$).

Then

$$\text{Strain } \epsilon = \frac{\text{Load}}{\text{Area}} \cdot \frac{1}{E}$$

The difficulty of the method lies in obtaining a pure axial pull. However, the effects of bending, due to non-axial loading, may be minimised by mounting two identical gauges on the specimen on opposite faces. The mean of the two electrical strain readings corresponds to the average strain on the specimen.

Apparatus

The mild steel specimen was of rectangular section $\frac{3}{4}'' \times \frac{1}{2}''$ approx. and parallel over a length of 5 inches (See Appendix XIV. Fig.11.10). The ends were enlarged to one inch and threaded to suit the ball and socket joints provided for gripping the specimen in the testing machine. The load was applied through a 5 ton Single lever Buckton Tensile-Compression Testing Machine, which was hand driven through gearing.

A 4 inch Cambridge Extensometer was used as the mechanical means of strain measurement.

Method

Two gauges were situated axially directly opposite one another on the $\frac{3}{4}$ inch faces of the specimen and were connected
into separate circuits on the multi-channel strain gauge set. Dummy gauges from the same batch as the active gauges, were affixed to an unstrained steel bar, and were placed in close proximity to the test specimen. The specimen was fitted into the ball and socket joints in the machine, and a 4 inch Cambridge Extensometer attached to enable Young's Modulus to be determined.

Starting from a small initial tightening load, the resistance changes in the gauges were noted, together with the extensometer reading for each increment of load.

Observations

The readings for a single test are recorded in Appendix XIV and their graphical presentation in Graph 11.3

Result

200 ohm B.T.C. gauges Type SE/4/27
Gauges in Tension S.S.F. = 2.12

Conclusions

The graphs show that the two gauges suffered equal rates of change of resistance with strain, which suggests that the loading was perfectly axial.

The resistance-load curves do not pass through the origin, due to faulty initial balance conditions.
11.4 GENERAL CONCLUSIONS ON CALIBRATION TESTS

It will be noticed that the beam tests for the 200 ohm gauges give a value of the sensitivity factor some 7% higher than for the direct tension test on gauges from the same batch. This is possibly mainly accounted for by the fact that the distance of the gauge wire from the neutral axis in bending is, say 8/1000 inch beyond the surface distance (0.156 inches) used in calculations. The error introduced is 8/156, which is a 5% correction. A deeper beam would have avoided this trouble.

Against this, the makers quote an average sensitivity factor for this type of gauge of 2.25

11.5 CROSS SENSITIVITY OF GAUGES

Introduction

The strain sensitivity factor of the gauges used in the concrete experiments, had been obtained from tests on a steel beam (Poisson's Ratio $\sigma = 0.28$). The question arose as to whether this factor would apply when the gauges were affixed to concrete having $\sigma = 0.1$ say.

The answer to this depends on the "cross sensitivity" of the gauge.

Theory.

Consider a uniform specimen stressed in one direction having two identical strain gauges affixed, one in the direction of the stress and one transverse to this direction.
Let $S =$ strain sensitivity factor to axial strain in the absence of cross strain.

$kS =$ strain sensitivity factor to cross strain in the absence of axial strain.

$e =$ axial strain.

$\nu =$ Poisson's Ratio for the material under test.

$dP_1 =$ axial strain gauge reading.

$dP_2 =$ transverse strain gauge reading.

Then $dP_1 = Se - kSe = Se(1 - \nu)$ \hspace{1cm} (a)

$dP_2 = Se = kSe = Se(\nu - k)$ \hspace{1cm} (b)

So that \[ \frac{dP_2}{dP_1} = \frac{\nu - k}{1 - \nu k} \] \hspace{1cm} (c)

\[ \sigma = \sigma + \sigma^2k + \sigma^3k^2 + \sigma^4k^3 + \] \hspace{1cm} (d)

and \[ k = \frac{dP_2}{dP_1} = \frac{\sigma}{1 - \nu k} \] \hspace{1cm} (e)
From equations (c) and (d) it can be seen that, provided 
'k' is very small compared with \( \sigma \), \[ \frac{\partial P_2}{\partial P_1} = \sigma \] very closely.

In order to make certain that the cross sensitivity factor 
(k,3) of the gauges used, was in fact very small, the following 
calibration test was carried out on a steel beam.

**Experiment.**

The types of gauges used in the tests on concrete had 
nominal resistances of 2000 ohms and 200 ohms. Both types 
were one inch long, but the width of the 2000 ohm gauge was 
approximately 7 mm., whilst that of the 200 ohm gauge was less 
than 4 mm. It was decided, therefore, to test the larger 
resistance gauge which would have the larger cross sensitivity.

**Apparatus**

The gauges were affixed to a mild steel beam (30 inches 
\times 2 inches \times 0.309 inches) which was loaded by the four-point 
loading apparatus described in Para 11.2. A simple strain 
gauge set was used for the electrical measurements.

**Method**

One 2000 ohm gauge was affixed axially along the length 
of the beam, and a second gauge placed at 90 degrees to this 
axis. Both were glued with High Solids Durofix and suitable 
dummy gauges were affixed to a steel specimen.

As the multi-channel strain gauge set was not available, 
the longitudinal and transverse gauges were tested on separate,
but consecutive, loading runs. The resistance measurement was carried out on a simple single station Wheatstone Bridge set. The variable resistance \( \alpha \) in series with the gauge, was used initially to calibrate the galvanometer, so that, during the test, only the galvanometer deflection was noted for each load.

The gauges were tested on the tension face of the beam by the four-point loading method.

Observations

The readings are tabulated in the Appendix XV and the graphical presentation is shown in Graph 11.4

Results

From the graphs,

Tension gauge \( F_1 \), slope \( = 6.2 \) divs/10lb.load \( \mu \)

Compression gauge \( F_2 \), slope \( = 1.75 \) divs/10lb.load \( \mu \)

So that \( \frac{dP_2}{dF_1} \) \( = \frac{1.75}{6.2} = 0.282 \)

Assuming Poisson's Ratio \( = 0.28 \) for steel

Continued -
Assuming Poisson's Ratio $\nu = 0.23$ for steel.

From equation (c)

$$k = \frac{\frac{dP_2}{dF_1} - \sqrt{}}{\frac{dP_2}{dF_1} \cdot \frac{dF_1}{dP_2} \cdot 1}$$

$$= \frac{0.28 - 0.28}{0.28 \cdot 0.28 - 1} = \frac{0.002}{0.921}$$

$$= 0.002$$

i.e. $k = \frac{2}{4} \%$ say, which is very small.

CONCLUSION

For practical purposes the cross sensitivity of the gauges used for the concrete experiments may be neglected. That is, the strain sensitivity factor for the gauges obtained on a steel specimen will apply for the gauges attached to a concrete specimen.
CHAPTER 12

INTRODUCTION TO THE TESTS ON CONCRETE

12.1 SYNOPSIS OF THE TESTS

Investigations into the failure of plain concrete under combined bending and torsion took place between 1946 and 1948, and included two distinct programmes; one on the ultimate strength of the concrete, and the other on the strain suffered by the concrete. The part described in this thesis consists, in the first place, of a series of load-strain measurements on the types of specimens which had been used for control purposes in the ultimate strength tests. These were:

(a) Simple compression tests on concrete-cylinders 10 inches high by 5 inches diameter.
(b) Simple tension tests on specially shaped test pieces having a 3 inch x 3 inch section over a working length of 6 inches.
(c) Simple bending tests on 3 inch x 6 inch x 48 inch beams.
These tests were undertaken in order to obtain some of the fundamental properties of the concrete, as well as to provide experience in the possibilities of the method of strain measurement on concrete with the electrical resistance strain gauges. As will be shown, they also provided some interesting information on the validity of the tests themselves. Two specimens of each type were tested.

When these preliminary tests had been completed, a series of load-strain measurements were carried out on a 'main' specimen having a $\frac{7}{2}$ inch dia. over a 30 inch working length. Experiments on this specimen were made as follows:

(a) Pure bending tests to about 50% of the ultimate strength
(b) Pure torsion tests to about 50% of the ultimate strength
(c) A combined bending and torsion test until fracture occurred.

The machine used for loading this specimen was unorthodox and was designed and constructed by the author and his coadjutor.

To provide an alternative method of strain measurement to the electrical strain gauges, mechanical measurements of the strain were attempted in each test wherever practicable.
12.2 MATERIALS

The materials for the concrete, and the mix used for the specimens on which strain measurements were made, were identical with those used for the general strength tests under combined bending and torsional loading as described in a thesis by Mr. D. Fishor (26).

Cement

Rapid Hardening Portland Cement (Ferrocrete) was used in making all specimens. It was chosen for its recognised consistency, and was used as soon as possible after delivery. To avoid depreciation by storage the cement was ordered in quantities as required.

Aggregate

The Thames Valley Sand and Thames Gravel Ballast used were crushed, washed and graded. Sieve analyses and other properties of these materials are given in the table overleaf.
The size of the coarse aggregate was selected so that it bore a similar relationship to the size of the specimens tested, as would normal coarse aggregate to sections used in practice.

Both aggregates were thoroughly dried before use so that the amount of mixing water would be known exactly.

The Mix.

The proportions were close to a 1:2:4 loose volume mix and the consistency was chosen to obtain a good, uniform, workable mixture. The measuring out of the constituents was always carefully carried out by weight; the weight ratios being 1 of cement, 2.43 of sand and 4.01 of coarse aggregate.
The water-cement ratio by weight was 0.55, and the water-total dry mix ratio by weight was 7.5%.

The Mixing.

Before commencing the weighing out, the graded coarse aggregate supplied was always sieved at its upper and lower limits to remove any over, or undersized material, as well as to render the mixture free from any organic matter. The normal dry weight of batches manufactured was between 100 and 270 lb. All mixing was done by hand on a steel sheet by the author and his coadjutor.

The dry constituents were well mixed before the water was added in two approximately equal portions.

Placing the Concrete.

As all the moulds except those for the compression cylinders were made of wood, the inside of each was wiped with an oily cloth before it was filled with concrete so that water absorption by the wood from the mix would be minimised.

The concrete was well shovel-mixed and then placed in the moulds. Each lift of 2 to 3 inches was well rammed with a tamp of size and shape convenient for the mould concerned. At the same time, the outside of the mould was tapped with a heavy hammer. The ramming was somewhat heavier than that recommended by British Standard Practice, but the mix was on
the dry side, and this method was found by experience to produce a consistent dense concrete having a good surface finish.

The concrete had no slump owing to the dry mix and the small size of the coarse aggregate.

Curing

After placing, the specimens were left for five days under damp sacks in the laboratory where temperature conditions varied by only a few degrees from 55°F. Any specimens, which due to their shape, would be held in tension by the mould when shrinkage of the concrete occurred, were completely released from the mould at 12 hours after placing, although they were, of course, otherwise undisturbed.

At five days, when the concrete had attained sufficient strength to permit handling, the specimens were completely immersed in water for at least two weeks. They were then removed from the water, and allowed to dry off in the laboratory for a minimum of one week, before the preparation of the surface for the attachment of strain gauges was considered. A further period of at least another week elapsed before the gauges were ready for operation. The concrete was thus quite dry when the strain measurements were taken.
Compressive Strength (10" x 5" dia cylinder) = 3130 lb/in^2 at 28 days.
Tensile Strength (3" x 3" specimen) = 284 lb/in^2 at 28 days.

These are average figures obtained from a considerable number of results in the author's strength tests on this concrete.
CHAPTER 13

TESSILE TESTS

13.1 INTRODUCTION

A fundamental property of concrete is its ultimate tensile strength. The direct method of determining this value is by the simple tensile test, but one of the difficulties of carrying out such a test is to obtain a perfectly axial pull on the specimen. The relatively large size of specimen which must be used for concrete, makes the design of suitable end attachments particularly difficult. There are two methods which have been used by many experimenters. One is to cast steel bolts with enlarged heads into the centre of the ends of the specimen and then apply the pull to these. The other is to make the specimen with enlarged ends in the form of a flat dumb-bell shape, and then apply the pull by means of claw type grips. (Details of a number of historical tests are given in the companion thesis) (26). The latter type of specimen was used in the author's tests.
The results of many tensile tests by the author, and those of other experimenters, have shown considerable variation (say ± 20%) in the mean value of the ultimate strength of a particular concrete. As it was considered that eccentricity of loading could account for a large part of this discrepancy, it was decided to investigate this possibility.

Another important property of concrete is its strain capacity. This first became the concern of engineers when Considère (15) put forward his astounding hypothesis that steel in concrete increased the strain capacity before cracking occurred, compared with plain concrete. According to his experiments on beams, cracks appeared at strains between 100 and 200 x 10^-6 in plain concrete, whereas in parallel tests on reinforced concrete, strains of 2000 x 10^-6 were recorded. These results were quickly challenged by others, who found no appreciable difference in the strains at the first crack between plain and reinforced concrete, and so Considère (16) made further tests, after which he modified his original statement.

The measurement of strains suffered by plain concrete is difficult owing to the small quantities involved. In the past, most experimenters have used mechanical mirror extensometers or strain gauges. The wire resistance strain gauge
was thought to offer the possibility of a more comprehensive measure of this property, which may be a criterion of the failure of concrete.

The variation of the strain with applied tensile load is also of importance as, if a linear relationship can be established, then the stresses in a concrete component can be obtained from a knowledge of the strain.

13.2 HISTORICAL TESTS


Experiments by V.C. Davies and Leslie Turner (12) and (56)

Tests were carried out at the Battersea Polytechnic, London in 1926, using crushed Thames gravel, graded $\frac{3}{4}$ inch downwards, with a water content of 7.6% by weight of dry aggregate. The specimens were of flat dumbbell shape 3 in. x 3 in. cross section with a 6 in. parallel portion. Strain measurements were made using a specially designed mirror extensometer working on the Ewing principle and reading to a strain of $2 \times 10^{-6}$ over a 6 in. gauge length. Some typical results of a series of eleven tests on 35 day old
concrete are given in Graph 13.1

The equation to an average stress-strain curve obtained from the series of tests was:

\[
\frac{\text{Stress}}{\text{lb/in}^2} = 244 - 450 \times 10^6 (0.000007 - \varepsilon)^{1.5}
\]

where \(\varepsilon\) denotes the strain.

It can be seen that the initial modulus is approximately constant at \(4.40 \times 10^6\text{lb/in}^2\), but that the load at which an appreciable change of modulus occurred, varied considerably between the tests. The mean strain, at fracture, varied between \(320 \times 10^{-6}\) and \(450 \times 10^{-6}\), and the ultimate tensile strength between 212 and 356 lb/in\(^2\).

The conclusions were that, "in spite of the fact that the loading was applied through spherically seated grips, it is probable that the average ultimate strengths recorded are slightly lower than their true values".

The apparatus and extensometer used for these tests were the same as those used in the author's experiments.

Experiments by A.M. Johnson (34)

Strain measurements were made, during a series of tests, at the University of Maryland in 1929 on concrete tension specimens having a 4\(\frac{1}{4}\) in. dia. and a 9 in. parallel length. The ends of the specimen were enlarged and of pyramidal shape making an overall length of 21 inches. Claw type grips fitted over the enlarged ends, and were attached to the testing
machine through a pinned joint. The concrete was of 1:2:3 mix using Portland Cement and a limestone aggregate of maximum size 3\(^{3/4}\) inch. The extensometer was of the Martens mirror type working over a 4 inch gauge length.

Tests were carried out at various ages and showed that Young's Modulus increased from \(3.75 \times 10^6\) lb/in\(^2\) at 7 days, to \(4.0 \times 10^6\) lb/in\(^2\) at 600 days, whilst the ultimate strength rose from 225 to 250 lb/in\(^2\). A typical tensile stress-strain curve obtained at 30 days is shown in Graph 13.1.

Experiments by A.H. Nylander (46)

Fifty tests were carried out in Stockholm in 1945 on concrete having the following specifications:

**Series I** Mix 1:3.4:4.3, Water/Cement Ratio = 0.69, Cement/Cubic foot of concrete = 16.2 lb/ft\(^3\).

**Series II** Mix 1:5.2:5.6, Water/Cement Ratio = 1.0, Cement/Cubic foot of concrete = 11.2 lb/ft\(^3\).

The specimens had a constant cross section of 15 cms x 15 cms, with a steel head cast in-situ, and leaving a clear length of either 64 cms, or 4 cms.

The ultimate strength results varied between:

**Series I** 158 and 294 lb/in\(^2\) for long specimens, 295 and 365 lb/in\(^2\) for short specimens.

**Series II** 160 and 224 lb/in\(^2\) for long specimens, 180 and 274 lb/in\(^2\) for short specimens.
Strain readings were taken over a 50 cm. gauge length using two dial gauges at sections 180° apart, and over a 20 cm. gauge length, with a mechanical strain gauge. A typical stress-strain diagram for Series I is shown in Graph 15.2.

The secant modulus for stresses from zero to 114 lb/in² for Series I averaged $4.57 \times 10^6$ lb/in², and for Series II, $3.85 \times 10^6$ lb/in².

The average strains at fracture, varied between $48 \times 10^{-6}$ and $35 \times 10^{-6}$.

These were control tests parallel to a main research, and the conclusions were that the clear length of specimen has a marked effect on the ultimate strength. This may be accounted for by the "weakest link" theory of probability which indicates that the longer the specimen, the greater is the possibility of finding a "weak link".

Experiments by R.J. Evans (26)

Several types of plain concrete tension specimens were used in tests carried out at the University of Leeds in 1946. These were:

(a) Standard briquettes 1 in. x 1 in., 1½ in. x 1½ in., and 4 in. x 2½ in., respectively, which were tested using claw type grips.

(b) Circular tension columns 3 in. diameter by 52 in. long, 3 in. diameter by 22 in. long, and 2 in. dia by 15 in. long.
respectively. These were made with enlarged ends to receive set screws cast in the concrete for screwing into the grips of the machine.

The concrete mixes were varied to suit the experiments, being either 1:2:4 (water/cement ratio 0.63 or 0.74) or 1:1.5:3 (water/cement ratio 0.57 or 0.50) with a maximum gravel size of 1 inch.

Briquette and Tension Column Tests

The eccentricity of loading during the tests was measured by using two 1 inch mirror strain gauges attached to the opposite extreme edges of two faces of the briquettes. A typical load-strain result is given in Graph 13.2.

Similarly, load-strain curves taken from opposite sides of the circular tension columns are given for various lengths of specimen.

The measured ultimate tensile strengths are corrected for the eccentricity of loading calculated from the strain measurements just before failure, and the results are given in the following Table 13.1 (see over page).
### TABLE 13.1 Tensile Test Results by Evans (25)

<table>
<thead>
<tr>
<th>Standard Briquettes</th>
<th>Small Briquettes $1\frac{3}{8} \times 1\frac{3}{8}$</th>
<th>Large Briquettes $4\text{ in.} \times 2\frac{3}{4}\text{ in.}$</th>
<th>Circular Tension COLUMNS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Plain reinforced with $\frac{1}{2}\text{ in.}$ rod.</td>
<td>Plain reinforced with $\frac{3}{8}\text{ in.}$ rod.</td>
<td>$2''\text{Dia.}$ $15''$ Long</td>
</tr>
<tr>
<td>Uncorrected for eccentricity of Loading.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>394</td>
<td>343</td>
<td>334</td>
<td>420</td>
</tr>
<tr>
<td>350</td>
<td>329</td>
<td>334</td>
<td>270</td>
</tr>
<tr>
<td>338</td>
<td>408</td>
<td>340</td>
<td>470</td>
</tr>
<tr>
<td>225</td>
<td>296</td>
<td>160</td>
<td>270</td>
</tr>
<tr>
<td>Corrected for eccentricity of Loading.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>394</td>
<td>430</td>
<td>400</td>
<td>420</td>
</tr>
<tr>
<td>410</td>
<td>393</td>
<td>420</td>
<td>440</td>
</tr>
<tr>
<td>438</td>
<td>458</td>
<td>447</td>
<td>470</td>
</tr>
<tr>
<td>506</td>
<td>296</td>
<td>260</td>
<td>236</td>
</tr>
</tbody>
</table>

**Conclusions**

Allowing for the eccentricity of loading, the results are in much closer agreement, even though the correction is made on the readings of two gauges instead of three, which are required for a full knowledge of the strain distribution.

The ultimate strength figures show an increase as the length of specimen is reduced.

**Reinforced Tension Columns.**

Similarly, the eccentricity of loading was found for 4 inch $\times$ 4 inch $\times$ 30 inch tension specimens having a central 1 inch or
\( \frac{5}{8} \) inch steel rod. Some 15 tests were carried out with mixes of \( 1 : 1 \frac{1}{2} : 3 \); \( 1 : 2 : 4 \); \( 1 : 4 : 5 \); and \( 1 : 4 : 8 \) respectively, on specimens air and water cured, the strain capacity at the first crack being found to vary between 55 and \( 135 \times 10^{-6} \). This was greater for those specimens cured in water, owing to the reduced shrinkage stresses. Great care was taken in these experiments in searching for the first crack in the specimens.

The effect of percentage reinforcement and thickness of cover on the strain capacity at the first crack was also investigated.

Conclusions

The strain capacity at first cracking of plain and reinforced concrete is sensibly the same. The large strains observed in the latter by some investigators, is thought to be due to insufficient magnification for the observation of the first crack.

13.3 THE AUTHOR'S TENSION TESTS

Object of the experiments.

The primary objective of the experiments was to obtain the tensile stress-strain curve for the concrete.

A secondary objective was to obtain a measure of the amount of eccentricity of loading which might be present in a normal tensile test with the set-up used.
Outline of the experiments.

Two specimens were tested which are now designated the 'A' and 'B' specimens.

The 'A' specimen was 18 months old when tested and had been stored dry after an initial water-curing of three weeks. Three preliminary tests were made in which strain measurements were taken during loading to a maximum nominal stress of 200 lb/in², and on two of these tests the strains were also recorded during the unloading. The fourth test on this specimen was 'to fracture' which occurred at a nominal stress of 368 lb/in². All strain measurements were by electrical gauges.

The 'B' specimen was 47 days old at test, and had been air-cured for the last 26 days. Two preliminary loading runs were made to 136 lb/in², and the strain measured by mechanical extensometer. The third and final loading was 'to fracture', which occurred at a nominal stress of 301 lb/in². Strain readings were taken using both electrical gauges and the mechanical extensometer.
The dumb-bell shaped specimen was designed by Mr. V.C. Davies (18) for some earlier work, and had a working cross sectional area of 3 inches x 3 inches (see Fig. 15.4). The size was limited by the size of the testing machine available. It was cast in a wooden mould in a horizontal position, and was filled in layers parallel to the plane of loading. To avoid shrinkage cracks, the specimen was completely released from the mould at 12 hours after casting, and was also kept moist during the initial setting, and was later water-cured.

The Testing Machine

The specimens were tested in a 100,000 lb. Richle, screw-powered, compound lever beam testing machine which was fitted with special claw grips made to suit the specimen. One of the claw grips had a spherical seating for adjustment.

To measure the load, a half value jockey-weight was provided so that, with the aid of the screw-indicator, it was theoretically possible to measure to 5 lb. In practice, due to friction etc., the machine read correctly to about ± 50 lb.

To apply the load, the machine was fitted with a four speed gear box which could be belt-driven from overhead shafting. For this test, as such a small strain was required to fracture
FIG. 13.4 The Tension Specimen.
the specimen, it was found to be most convenient to turn the machine over by hand.

The Mechanical Extensometer.

The extensometer used was designed for this size of specimen by Mr. V.C. Davies (18).

The instrument had a 6 inch gauge length and operated on the Ewing principle, obtaining an additional magnification from a bell crank and mirror (see Plate 13.1). Readings were made by observing a scale through a telescope via the mirror. It is claimed that the errors, due to friction and backlash, correspond to much less than one half of $10 \times 10^{-6}$ inches. A micrometer is incorporated to calibrate the instrument. The top and bottom frames each have a detachable side to permit attachment of the instrument to the specimen. The conical screw points do not pivot on the concrete, but on four steel bars which are firmly strapped in pairs at the appropriate gauge length to the outside of the specimen. (See Fig. 13.1 below)

![Diagram of extensometer attachment](https://via.placeholder.com/150)

**FIG. 13.1** Extensometer - Method of attachment.
The Multi-channel strain gauge set.

This set, as described in Chapter 7, was used in conjunction with the Cambridge Galvanometer for reading the electrical strain gauges.

The Strain Gauges.

Specimen 'A'

Four 2,000 ohm 1 inch gauges, Nos. 1, 2, 3 and 4 were affixed by the method described in Para 10.5 to suitably smooth portions of the surface of the specimen, two on each of one pair of opposite faces, and all at approximately the same distance from an end. Gauges Nos. 2 and 4 failed to work reliably in Test No. 1 and were replaced by two similar gauges Nos. 5 and 6. For the final Test No. 4, gauge No. 5 was out of action, and the test was carried through to fracture with gauges Nos. 1, 3 and 6 in operation.

Specimen 'B'

Four 200 ohm 1 inch long gauges Nos. 7, 8, 9 and 10 were affixed to this specimen, two on one flank, one on the other, and one on the surface which was at the bottom of the mould on casting. These gauges all worked satisfactorily, and were in operation for the test.

Note: The positions of all gauges are shown in Fig. 13.5
FIG. 15.5  Gauge Positions on Tension Specimens
Testing Procedure.

In each test the specimen was carefully fitted into the claw type jaws of the machine, and 1/10th inch thick, soft rubber pads were placed on the loading faces. Zero balance on the weighing beam having been obtained, a small tightening load was put on the specimen. The strain gauges were then connected up to the multi-channel set, together with appropriate dummy gauges, which were stuck to a concrete cylinder placed as near as convenient to the active specimen. When used, the mechanical extensometer was fitted to the specimen and a calibration test carried out. Plate 13.2 shows a photograph of the set up. To obtain steady temperature conditions, the whole specimen was then shrouded with cloths, except for a small opening to permit the observation of the extensometer mirror and, with the circuit heating coils switched-on, the set was left undisturbed for at least one hour. The gauge pairs were then carefully balanced, and the test begun.

The times taken varied considerably between the tests, but the two final tests to rupture, were carried through with two operators in about 1 1/2 hours from the time of first loading.

Observations and Calculations

The readings for the several tests, together with the calculations are given in Appendices XVI to XX inclusive and are shown in Graphs 13.3 to 13.10 inclusive.
13.5 RESULTS AND COMMENTS ON THE INDIVIDUAL TESTS

**SPECIMEN 'A'**

**Test No.1**

The test served to perfect the testing procedure. Both gauges were initially reluctant to pick up the strain, but subsequently showed a linear stress-strain relationship both during loading and unloading, (See Graph 15.3), and a permanent set in the concrete of approximately a strain of $15 \times 10^{-6}$. No calculations can be made for the eccentricity of loading as only two gauges were in operation.

**Test No.2**

A linear stress-strain relationship (See Graph 15.3) is shown by three of the four gauges over the load range of the test, indicating that the eccentricity of loading remained constant at a distance $x = 0.06"$, $y = 0$ from the centroid of the section (See Graph 15.11). The maximum stress on the specimen was thus $1.18 \times$ average stress. The value of $E = 5.35 \times 10^6$lb/in$^2$.

**Test No.3**

This test was carried out immediately after Test No.2 without disturbing the set-up of the specimen in the machine. A linear stress-strain relationship was again indicated (See Graph 15.4), the constant eccentricity having been reduced
to a distance $x = 0.05''$, $y = 0$ from the centroid of the section (See Graph 13.11)

Gauges Nos. 5 and 6 showed no appreciable hysteresis or permanent set of the concrete during unloading (See Graph 13.4), but the behaviour of gauges Nos. 1 and 3, which indicated a greater release of strain than expected, must be attributed to a gauge or electrical measuring phenomenon. The value of $E = 5.05 \times 10^6$lb./in$^2$. The difference between this value and that obtained in Test No. 2 is within the range of experimental error.

Test No. 4

The three gauges showed initial instability until a nominal stress of about 50 lb./in$^2$ was reached. In each case a compressive strain was first indicated, so that the "bedding in" of the grips to the specimen, which might cause large initial eccentricity of loading, cannot explain the behaviour. On the assumption that this a gauge phenomenon, or is initial slackness being taken up in the specimen, new datums for the strain readings have been obtained by the extrapolation of the graphs (See Graph 13.5) (See also 'Negative Loop' phenomenon in compression tests Para. 14.16)

Until a nominal stress of approximately 250 lb/in$^2$ was reached, a linear stress-strain relationship and constant eccentricity of loading, was shown by the three gauges. The
eccentricity was a distance \( x = 0.01" \), \( y = 0.10" \) from the
centroid of the section (See Graph 15.11) and this resulted
in the maximum stress at failure being 1.22 \times \text{average stress}.
The value of \( E = 5.68 \times 10^6 \text{lb./in}^2 \) was higher than that
obtained in Tests Nos 2 and 3 and in part, is probably due to
the difficulty of measuring the slopes of the graphs accurately.

Above 250 lb/in\(^2\) all gauges showed a reduction in the
slope of their stress-strain curves indicating that this was
due, not to a change in the eccentricity of loading, but to
a change in the elastic modulus. The average stress-strain
curve (See Graph 15.6) shows the nature of this change.

The equation to the stress-strain curve obtained from
the electrical strain readings using a strain sensitivity
factor of 2.3 is given very closely by :-

\[
\frac{\text{Stress}}{\text{lb/in}^2} = 363 - 110 \times 10^3 (0.000075 - e)^{1.3}
\]

where '\( e \)' denotes the strain.

The start of the final break down at approximately
325 lb/in\(^2\) was clearly reflected in the readings of gauges
Nos 1 and 3. That the eccentricity of loading in the 'y'
direction had changed, possibly due to the formation of a
crack, was shown by the incremental strain readings of gauge
No.1 decreasing, whereas those of gauge No.3 increased. The
eccentricity in the 'x' direction remained constant as shown
by gauge No.6. Assuming that plane cross sections remained plane, the eccentricity just before failure at 354 lb/in² was \( x = 0.08'' \), \( y = 0.08'' \). This indicated a maximum stress at fracture of 1.20 x Average Stress.

The fracture load was held by the specimen for sufficient time for the reading of one of the gauges to be taken before the sudden failure at the upper change of section. This reading gave a steady continuation of the stress-strain curve. The ultimate strength of this 20 month old specimen was 363 lb/in² nominal stress, which shows the expected increase in strength of about 15% over the strength at 28 days.

**SPECIMEN 1B**

Test No.5

The two preliminary and the final loading runs showed linear stress-strain relationships to 136 lb/in², the elastic modulus remaining sensibly constant at \( 5.55 \times 10^6 \text{lb/in}^2 \). The electrical gauge readings showed remarkable consistency after sufficient initial tensioning of the specimen had been applied. (See Graph 13,8). The eccentricity of loading during this period was \( x = 0.03'' \), \( y = 0.29'' \) from the centroid of the cross section (See Graph 13,12) which was rather large. This indicated that the maximum stress was 1.70 x average stress.

At approximately 135 lb/in² a sudden change in the
eccentricity and a change in the elastic modulus occurred. This was shown by the discontinuities in the readings of the mechanical extensometer (See Graph 13.9) and the strain gauges. The latter settled down to a new stress-strain rate more quickly than the former. At 175 lb/in\(^2\) the eccentricity was \(x = 0.02''\), \(y = 0.26''\), and this varied but little, until the final stage of breakdown commenced at 270 lb/in\(^2\) when the eccentricity was \(x = 0.03''\), \(y = 0.23''\) (See Graph 13.12).

The position of the starting point of the failure may be inferred, from the gauge readings, to have commenced on the flank to which the gauge No.10 was attached. Bending action about the 'y' axis was increasing as gauges Nos. 8 and 7 showed equal increases in the stress-strain rate, whereas gauge No.10 showed a decrease and gauge No.9, which was approximately on the neutral axis, remained unaffected.

The eccentricity at failure was \(x = 0.05''\), \(y = 0.24''\), so that the maximum stress = 1.64 \* average stress.

The agreement between the average strain as measured electrically and as measured mechanically, was exceptionally good (See graph 13.10). The strain sensitivity factor for the gauges from these measurements was 2.09, which agrees very well with the values of 2.12 obtained by a separate test of these gauges on a steel tension specimen.

The average stress-strain curve is plotted on Graph 13.9.
The equation to the stress-strain curve obtained from the readings of the mechanical extensometer is given very closely by:

\[
\frac{\text{Stress}}{\text{lb/in}^2} = 295 - 21.05 \times 10^6 (0.00006 - \varepsilon)^{1.15}
\]

where \( \varepsilon \) denotes the strain.

### 13.6 GENERAL CONCLUSIONS ON THE TENSION TESTS

1. The electrical strain gauges behaved extremely well and gave a measure of the strain to an accuracy of \( 2 \times 10^{-6} \) when used with the Cambridge Galvanometer, and \( 10 \times 10^{-6} \) with the Marconi instrument. This is quite as high an accuracy as has been obtained with mechanical gauges over a 1 inch gauge length, and the electrical method has the advantage in that it does not interfere with the concrete in any way.

2. The close agreement between the strain sensitivity factor for the gauges, as measured by tests on a steel specimen with that obtained on a concrete specimen, gives added confidence in the electrical method.

3. The tests show that the eccentricity of loading for the set-up used for tension tests, may be very large (i.e. anywhere within the 'middle third rhombus'). Therefore to obtain reliable results for the U.T.S. of a concrete
specimen, the greatest care must be taken in fitting the specimen into the machine. It is suggest that, if possible, some strain readings should be taken at loads well below the ultimate, so that any eccentricity of loading could be corrected by adjusting the position of the jaws.

The large discrepancies obtained in the author's ultimate tensile strength tests (See companion thesis) (26) may be said to be mainly due to this cause, rather than to variable concrete.

4. The equation to the tensile stress-strain curve is a parabola of the form:

\[ f = U.T.S. - \text{constant} (\text{ultimate strain} - e)^n \]

where 'f' is the stress for strain 'e'.

For the two specimens tested, the index 'n' was found to be 1.15 and 1.3.

This is a flatter curve than that given by Davies and Turner (58) who found the average 'n' to be 1.5.

5. The initial value of the Modulus of Elasticity in Tension for this concrete is \(5.6 \times 10^6\text{lb/in}^2\). The value is not definite owing to the difficulty of measuring the slope of an 'irregular' straight line.

From the limited data obtained, the age of the concrete
at test does not alter the value of \( E \) appreciably, but previous loading does appear to increase its value slightly.

6. The load at which the initial elastic modulus changes was found for both specimens to be almost exactly the maximum load to which the specimen had previously been loaded. No virgin specimens were tested, as it was necessary before commencing a test, to make certain that the delicate extensometer was, in fact, working, that the gauges were 'exercised', and that the specimen had achieved an initial 'state of ease'.

7. The effects of creep under load were ignored in these tests. Each load was maintained for about 2 to 3 minutes and no creep was observed.

8. Calculating the maximum strain in the concrete from the gauge readings just prior to failure, the strain capacity for both specimens tested was 90 to 95 x 10^-6, which is within the range of values found by other experimenters on similar concrete.

9. Whilst the readings of the small number of gauges used can be considered reliable, it would be interesting to verify the assumption that plane cross sections before straining remain plane after straining, by placing a larger number of gauges around one cross section.
CHAPTER 14

THE COMPRESSION TEST

14.1 INTRODUCTION

As concrete is strong in compression and very weak in tension, it is used principally for components subjected to compressive loads, or with reinforcement when its compressive strength is the important factor in design. The crushing strength is therefore usually accepted as being the criterion of its quality.

Test specimens are normally cylinders or cubes, the size of which depends on the capacity of the testing machine. The height of the cylinder is usually twice the diameter in order to avoid strut action. The test result for the ultimate strength of the concrete will depend, not only on the mix, but also on the method of testing. For example :-

(a) The load should be applied axially to prevent additional bending stresses.

(b) The packing between the compression plates and the specimen may affect the result.
To investigate the accuracy of the compression test as used in the author's strength tests (26) the following experiments were undertaken on compression cylinders, and incidental to these tests, the stress-strain properties of the concrete were measured.

14.2 SUMMARY OF THE ITEMS INVESTIGATED.

The following items were investigated and are dealt with separately:

(1) The effect of end-packing on the eccentricity of loading.

(2) The variation of strain in the height of the specimen.

(3) The compression stress-strain curve.
   (a) The effect of repeated loading to 50% of the ultimate strength.
   (b) The complete stress-strain curve on initial loading.

(4) Hysteresis and Permanent Set.

(5) The yielding of the specimen under load.

(6) The ratio of lateral to longitudinal strains.

(7) The volume changes due to loading.

(8) The failure of a specimen.

(9) The strain sensitivity factor for the strain gauges.

(10) The 'negative loop phenomenon' for tension gauges.
14.3 OUTLINE OF THE TESTS

Strain measurements were taken on two specimens, which will now be known as the 'C' and 'D' specimens.

Specimen 'C'

(a) The specimen was 18 months old when this series of tests was started, and had been stored dry after an initial water-curing of three weeks. As it was desired to carry out a number of measurements at loads well below the ultimate and over a period of several weeks, an 'old' specimen was used, to avoid the changes in strength of the concrete which occur during initial ageing.

The following compression tests took place over a period of 8 weeks and, in each test, the specimen was loaded to a nominal stress of about 1800 lb/in², which was 50% of its ultimate strength. Strain readings were taken during the loading and unloading, as was appropriate, with resistance strain gauges and an 8 inch gauge length mechanical compressometer.

Tests Nos. 1 to 4

These were simple compression tests with eight or nine gauges in operation, and readings were taken during the loading and unloading of the specimen.
Test No. 5

To investigate the 'negative loop' phenomenon observed previously with the tension gauges, a test was carried out having sixteen load increments in which electrical strain readings were taken on three transverse tension gauges.

Test No. 6

Two consecutive loading tests were carried out with six transverse tension gauges in operation, to determine the variation of transverse strain along the length of the cylinder, due to end constraint.

Test No. 7

To determine Poisson's Ratio, four consecutive loading runs were made using two pairs of suitably placed gauges.

Tests Nos. 8 to 11

The effect of seven different types of packing placed between the concrete cylinder and the steel platens of the testing machine on the eccentricity of loading, was investigated in these tests by recording on four gauges placed axially at the mid-height of the specimen.
Specimen 'D'!

(b) The specimen was 45 days old at test, and had been air-cured for the last 26 days. Two preliminary loading runs were made to 1500 lb/in² to 'exercise' the gauges, and to put the specimen in a 'state of ease'. Strain measurements were made by mechanical compressometer only. The third and final loading was by thirty-three load increments until failure occurred at a nominal stress of 3560 lb/in². Strain measurements were taken by four electrical gauges situated at the mid-height of the specimen, and also with a mechanical compressometer reading over an 8 inch gauge length. This test is now known as Test No. 12.

DETAILS OF THE TESTS

14.4 TESTING APPARATUS.

The Specimen.

The cylindrical specimen, 10 inches high by 5 inches diameter, was cast in a machined steel mould consisting of a flanged base plate, together with a cylindrical section having a longitudinal joint which was kept closed by two bolts inserted through lugs provided for this purpose.

The manufacturing procedure was to place a layer of about ½ inch of a cement-sand grout in the bottom of the mould, and then fill in with four approximately equal layers each of
which was adequately tamped. The top surface was completed with a cement sand grout and trowelled smooth and level.

To obtain a smooth outer surface, the mould was hammered during filling.

With specimens on which it was desired to fit a mechanical compressometer, four steel screws, as shown in Fig. 14.1, were inserted into holes provided in the walls of the mould to give cavities approximately ½ inch deep \( \times \) ½ inch diameter in the concrete, at the desired gauge length.

![Diagram of Moulding Screws]

**FIG. 14.1 - Moulding Screws.**

Specimens were kept under damp sacks for the first four days after casting, and then water-cured.

**The Testing Machine**

The specimens were tested in a 100,000 lb Richle screw-powered, compound-lever beam testing machine. The load could theoretically be measured to 10 lb., but due to friction, backlash etc., the load reading was only correct to about ± 50 lb.

Testing was carried out between two substantial machined steel platens, one of which was placed on the table and the other attached to the loading head of the machine.
Four rates of loading could be obtained, but for the incremental loading used in these tests, this feature was not important.

The Mechanical Compressometer

The compressometer was designed for this size of specimen by Mr. Hamilton of the Battersea Polytechnic, London.

The instrument has an 8 inch gauge length and operates on the Ewing principle giving a 2 to 1 lever magnification. The movement of the upper lever depresses the plunger of a 1/10,000 inch dial gauge rigidly attached to the lower frame (See photograph Plate 14.1) No calibrating screw is incorporated.

The attachment to the specimen is through conical screw points which, for these tests, pivoted on special brass screws. (See Fig. 14.2) These were tightly screwed into the four small holes left in the specimen during casting, which had previously been Rawlplugged.

FIG. 14.2 Compressometer-Pivot Screws.
The Multi-Channel Strain Gauge Set.

This set, as described in Chapter 7, was used in conjunction with the Marconi and Cambridge Galvanometers for reading the electrical strain gauges.

The Strain Gauges

Specimen 'C'

Ten 2000 ohm nominal resistance, 1 inch long gauges Nos.1 to 10 inclusive, were affixed at the selected positions by the method described in Para. 10.5 to the very smooth surface of the specimen (See Fig. 14.3).

After Test No.5, gauge No.8, which had failed to work satisfactorily was replaced, and two additional gauges Nos.11 and 12 were added. For the remaining tests all the gauges were serviceable.

Specimen 'D'

Four, 200 ohm nominal resistance, 1 inch long gauges, Nos. 13, 14, 15 and 16 were affixed to this specimen, and were arranged in pairs at the mid-height on opposite ends of a diameter. Their position is indicated in Fig. 14.4.

Testing Procedure.

For each test the compressometer was attached firmly to the specimen before the latter was placed in the machine. Dependant on the nature of the test, suitable pads were placed
Developed views of 5" dia. compression cylinder

FIG. 14.3 Gauge Positions - Compression Specimen 'C'
(see also Table 14.1 overleaf)

FIG. 14.4 Gauge Positions - Compression Specimen 'D'
O Compressometer Screws.
<table>
<thead>
<tr>
<th>Gauge No.</th>
<th>Circular measurement from Gauge No. 2</th>
<th>Angular Displacement from Gauge No. 2</th>
<th>Distance from Cap of cylinder</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$\frac{3}{8}$ in.</td>
<td>$83^\circ$</td>
<td>$4\frac{3}{4}$ in.</td>
</tr>
<tr>
<td>2</td>
<td>0 &quot;</td>
<td>$0^\circ$</td>
<td>$1\frac{7}{8}$ &quot;</td>
</tr>
<tr>
<td>3</td>
<td>$\frac{5}{16}$ &quot;</td>
<td>$116^\circ$</td>
<td>$9\frac{1}{8}$ &quot;</td>
</tr>
<tr>
<td>4</td>
<td>$\frac{1}{2}$ &quot;</td>
<td>$12^\circ$</td>
<td>$4\frac{3}{4}$ &quot;</td>
</tr>
<tr>
<td>5</td>
<td>$2\frac{1}{4}$ &quot;</td>
<td>$52^\circ$</td>
<td>$4\frac{3}{4}$ &quot;</td>
</tr>
<tr>
<td>6</td>
<td>$12\frac{7}{8}$ &quot;</td>
<td>$278^\circ$</td>
<td>$5\frac{1}{16}$ &quot;</td>
</tr>
<tr>
<td>7</td>
<td>$12\frac{3}{4}$ &quot;</td>
<td>$292^\circ$</td>
<td>$9\frac{1}{16}$ &quot;</td>
</tr>
<tr>
<td>8</td>
<td>9 &quot;</td>
<td>$206^\circ$</td>
<td>$4\frac{5}{8}$ &quot;</td>
</tr>
<tr>
<td>9</td>
<td>$7\frac{1}{16}$ &quot;</td>
<td>$170^\circ$</td>
<td>$4\frac{1}{16}$ &quot;</td>
</tr>
<tr>
<td>10</td>
<td>$\frac{7}{16}$ &quot;</td>
<td>$172^\circ$</td>
<td>$8\frac{1}{4}$ &quot;</td>
</tr>
<tr>
<td>11</td>
<td>$1\frac{5}{8}$ &quot;</td>
<td>$37^\circ$</td>
<td>$2\frac{1}{4}$ &quot;</td>
</tr>
<tr>
<td>12</td>
<td>$9\frac{1}{2}$ &quot;</td>
<td>$216^\circ$</td>
<td>$1\frac{13}{16}$ &quot;</td>
</tr>
</tbody>
</table>

**Table 14.1** Measured Position of Gauges.
on the top and bottom surfaces of the specimen, which was then set carefully between the steel plates of the machine.

The active strain gauges were connected up to the multi-channel set, and appropriate 'dummy' gauges obtained from a concrete specimen placed as near as convenient to the specimen under test. A small 'tightening' load of about 1000 lb. was next applied to the specimen, and the complete set-up shrouded with cloths. The heating coils of the gauge circuit were switched on, and the set-up left undisturbed for at least one hour. The gauge pairs were then carefully balanced and the test begun. To avoid 'drift' in the strain gauge readings with the 2000 ohm gauges, one minute was allowed to elapse after each gauge was switched into circuit before its reading was noted.

The times taken for the test varied considerably, as this was a function of the number of gauges in operation, and the number of loading conditions tested. A typical example of the rate of loading was for Test No.12, when for thirty-four loading conditions, strains were measured on four strain gauges and a compressometer in 90 minutes, with two operators.
14.5 THE EFFECT OF END-PACKING ON THE ECCENTRICITY OF LOADING

14.5.1 INTRODUCTION

To allow for the irregularities in the loading surfaces of the specimen, and to distribute the pressure uniformly over the surfaces, some experimenters introduce packing between the specimen and the compression blocks of the testing machine.

14.5.2 HISTORICAL TESTS

Unwin (57) has shown that the crushing strength, and the type of fracture, are considerably affected by the type of packing. Packing which will flow outwards under load as for example lead, will cause a big reduction in the ultimate strength due to a tension failure resulting from the extra lateral tension produced by the radial friction force at the ends. To obtain plane parallel faces, Unwin suggests the application of thin layers of plaster of Paris. Millboard, which is nearly incompressible, may be used to obtain a uniform bearing surface, although "it does no harm, and it is doubtful if it does any good".

Groneman (30) carried out an extensive series of tests in Chicago in 1924 to determine the effect of various end conditions on the ultimate strength of concrete compression
cylinders 6 inches dia. x 12 inches high. Some 2,100 specimens were tested at 28 days and had mixers of 1 : 7, 1 : 5, or 1 : 3 1/2 Portland Cement to mixed aggregate, which was graded 1 1/2 inches downwards. Various end attachments were tried, together with different types of cement or plaster caps. A series of sheet materials which could be used as a substitute for capping were also tested, giving the results shown in the following Table 14.2

<table>
<thead>
<tr>
<th>Material</th>
<th>Strength ratio as % of Average</th>
<th>Mean Variation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1:7 mix</td>
<td>1:5 mix</td>
</tr>
<tr>
<td>Beaver Board</td>
<td>3/16&quot; thick</td>
<td>93</td>
</tr>
<tr>
<td>White Pine</td>
<td>1/8&quot;</td>
<td>92</td>
</tr>
<tr>
<td>Millboard</td>
<td>1/16&quot;</td>
<td>101</td>
</tr>
<tr>
<td>Leather</td>
<td>9/32&quot;</td>
<td>93</td>
</tr>
<tr>
<td>Blotting Paper</td>
<td>1/32&quot;</td>
<td>82</td>
</tr>
<tr>
<td>Sheet Lead</td>
<td>1/16&quot;</td>
<td>95</td>
</tr>
<tr>
<td></td>
<td>1/8&quot;</td>
<td>91</td>
</tr>
<tr>
<td></td>
<td>3/16&quot;</td>
<td>89</td>
</tr>
<tr>
<td>Cork carpet</td>
<td>5/32&quot;</td>
<td>93</td>
</tr>
<tr>
<td></td>
<td>1/16&quot;</td>
<td>81</td>
</tr>
<tr>
<td>Rubber sheet</td>
<td>1/8&quot;</td>
<td>83</td>
</tr>
<tr>
<td></td>
<td>1/4&quot;</td>
<td>83</td>
</tr>
<tr>
<td>No bedding</td>
<td></td>
<td>95</td>
</tr>
</tbody>
</table>

**TABLE 14.2 Effect of End-Packing - Results by Gonneman (30)**

From these tests it was concluded that Beaver Board was the best form of packing, whereas rubber sheeting was the least satisfactory.
The French Ministry of Reconstruction and Town Planning in their latest "Regulations for the use of Reinforced Concrete", Dec. 1945, specify the use of cardboard packing about 1 mm. thick in the determination of the compressive strength of concrete.

14.5.3 THE AUTHOR'S TESTS ON SPECIMEN 'C'

Object:

The author's strength tests were carried out in a machine in which no spherical seating was provided for the compression platens, so that, in an effort to obtain a uniform pressure on the ends of the specimen, millboard sheets were used for packing. To determine the effects of different forms of packing on the eccentricity of loading, the following tests were undertaken.

The Tests:

Assuming that plane cross sections before straining remain plane after straining, the position of the leading point for a given load may be determined if the strain is known at three points on a cross section (See Appendix XXX)

In this series of tests the four gauges Nos. 1, 4, 6 and 9 which were placed axially and situated at the mid-height of the specimen, provided suitable data for the calculation of
the eccentricity of loading. Seven different types of packing were used and, from the stress-electrical strain \( \varepsilon \) curves (see Graph 14.1) and (Appendix XX) for gauges Nos. 1, 4, and 9, the eccentricity was calculated at a nominal stress of 1275 lb/in\(^2\). The results are given in Table 14.3 below:

<table>
<thead>
<tr>
<th>Test No.</th>
<th>Type of Packing</th>
<th>( x/\text{ins.} )</th>
<th>( y/\text{ins.} )</th>
<th>( z/\text{ins.} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>9 (b)</td>
<td>None</td>
<td>-0.089</td>
<td>-0.043</td>
<td>0.104</td>
</tr>
<tr>
<td>3</td>
<td>Billboard (1/8&quot; thick)</td>
<td>-0.109</td>
<td>-0.053</td>
<td>0.123</td>
</tr>
<tr>
<td>11 (b)</td>
<td>Corrugated Cardboard (1/8&quot; thick)</td>
<td>-0.065</td>
<td>-0.114</td>
<td>0.131</td>
</tr>
<tr>
<td>11 (a)</td>
<td>Cartridge Paper (5 thicknesses)</td>
<td>-0.063</td>
<td>-0.113</td>
<td>0.129</td>
</tr>
<tr>
<td>10 (c)</td>
<td>1 Soft Rubber Sheet (1/16&quot; thick)</td>
<td>-0.072</td>
<td>-0.096</td>
<td>0.130</td>
</tr>
<tr>
<td>10(a)&amp;(b)</td>
<td>2 Soft Rubber Sheets (1/16&quot; thick)</td>
<td>-0.033</td>
<td>-0.002</td>
<td>0.023</td>
</tr>
<tr>
<td>11(c)</td>
<td>&quot;Sorbo&quot; rubber pads (2/8&quot; Free, 3/8&quot; Loaded)</td>
<td>-0.080</td>
<td>-0.010</td>
<td>0.080</td>
</tr>
</tbody>
</table>

**TABLE 14.3** Effect of End-Packing — Author’s Results.

(See Fig. 14.5 overleaf)
FIG. 14.5 Arrangement of Specimen - Eccentricity Tests.

Accuracy of Results

The accuracy of these results may be measured by the compatibility of the four gauge readings with the Navier hypothesis. The reading of gauge No. 6 calculated from the readings of gauges Nos. 1, 4 and 9, is compared with the actual reading obtained from the graphs in the table overleaf.
Actual Gauge Reading $\times 10^4$

<table>
<thead>
<tr>
<th>Test No.</th>
<th>9 (b)</th>
<th>3</th>
<th>11 (b)</th>
<th>11 (a)</th>
<th>10 (c)</th>
<th>10 (a) &amp; (b)</th>
<th>11 (c)</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.1</td>
<td>9.3</td>
<td>8.7</td>
<td>8.7</td>
<td>8.1</td>
<td>7.7</td>
<td>7.5</td>
<td></td>
</tr>
</tbody>
</table>

Calculated Gauge Reading $\times 10^4$

<table>
<thead>
<tr>
<th>Test No.</th>
<th>9 (b)</th>
<th>3</th>
<th>11 (b)</th>
<th>11 (a)</th>
<th>10 (c)</th>
<th>10 (a) &amp; (b)</th>
<th>11 (c)</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.8</td>
<td>7.7</td>
<td>9.4</td>
<td>9.3</td>
<td>9.3</td>
<td>7.7</td>
<td>8.4</td>
<td></td>
</tr>
</tbody>
</table>

**TABLE 14.4 Compatibility of Results with Huygens Hypothesis.**

The discrepancy is of the order of 10%, but it may be accounted for, in most cases, by errors in the exact location of the gauges, and the inaccuracy of the $\frac{\sigma}{E}$ value selected from the graphs and used in the calculations. These values are not accurate to more than $\pm 0.2 \times 10^{-4}$, as insufficient experimental readings were obtained to completely specify the load-strain curve. Lack of homogeneity in the material may also account for some of the discrepancy.

To measure the effect of this, the eccentricity has been evaluated for the four possible combinations of three readings for Test Sb, in which no packing was used. The results are given overleaf.
TABLE 14.5 Compatibility of Results - Eccentricity of Loading.

<table>
<thead>
<tr>
<th>Gauge Combination</th>
<th>x/ins.</th>
<th>y/ins.</th>
<th>z/ins.</th>
</tr>
</thead>
<tbody>
<tr>
<td>4, 1, 9</td>
<td>-0.039</td>
<td>-0.048</td>
<td>0.104</td>
</tr>
<tr>
<td>4, 6, 1</td>
<td>-0.052</td>
<td>-0.018</td>
<td>0.056</td>
</tr>
<tr>
<td>4, 6, 9</td>
<td>-0.078</td>
<td>+0.014</td>
<td>0.081</td>
</tr>
<tr>
<td>1, 6, 9</td>
<td>-0.109</td>
<td>-0.006</td>
<td>0.109</td>
</tr>
</tbody>
</table>

FIG. 14.6 Loading Points using various gauge combinations - Test No. 9b.
The tests indicate that there was, in general, little advantage, if any, in introducing packing between the specimen and the compression plates.

However, two sheets of soft rubber sheeting gave only a small eccentricity of loading, and was the best form of packing tried. This is in direct opposition to the findings of Connerman (50), although his conclusions were based on the ultimate strength. The Sorbo rubber pads gave a reduced eccentricity of loading, but were too springy.

An eccentricity of loading of up to ½ inch is probable for the set-up used in the compression tests. Assuming that this is maintained until fracture, the ultimate strength would be reduced by 20%. It is shown in the companion thesis on the strength of concrete, that the results from a large number of compression tests were within about 20% of each other.

The experimental readings obtained were not sufficient to permit the accurate calculation of the movement of the loading point as the test progressed but, from the shape of the graphs, it may be inferred that the eccentricity changed considerably for stresses under 500 lb/in², but then remained sensibly constant.
14.6 THE VARIATION OF STRAIN IN THE HEIGHT OF THE SPECIMEN.

14.6.1 Introduction

The friction between the specimen and the compression platens of the testing machine constrains the ends of the specimen from radial movement. Hence, under load, the specimen becomes barrel-shaped and there will be a variation in the strain with distance from the ends.

14.6.2 Hoop Strain Tests on Specimen 'G'

The magnitude of the variation of the hoop strain was investigated in Tests No. 6(a) and (b) using millboard packing. Six gauges, Nos. 3, 7, 11, 12, 5 and 8 were used, arranged in diametrically opposite pairs, which were placed at three different distances from the ends of the specimen.

For Test No. 6(b) the electrical strain per unit stress was obtained from the mean slope of the curves, Graph 14.2, ignoring any initial 'negative loop'. To eliminate the effect of eccentricity of loading, the mean readings of the gauge pairs were plotted against distance from the ends of the specimen. (See Graph 14.3 and Appendix XXXI)
14.6.3 CONCLUSIONS

The Tests indicate that there is a considerable barrelling of the specimen, which is such that the hoop strain at the mid-height is about five times that at the ends. It also appears that the hoop strain is approximately constant for only a short section at the mid-height.

The three strain values obtained are considered to be "good" values.

14.6.4 Axial Strain Tests on Specimen 'C'

The difference in the axial strain at two levels using millboard packing may be found from Test No.4, when the four gauges Nos. 1, 4, 6 and 9, which were situated at mid-height, were used together with the two gauges Nos. 2 and 10 situated at approximately one-fifth of the height from the ends (See Graph 14.4).

To eliminate the effect of eccentricity of loading, the mean strains at the sections considered were calculated for a nominal stress of 1275 lb/in². (See Appendix XXXI) The axial strain at the mid-height is then 3% larger than that at one-fifth of the height from the ends. The difference is, however, within the limits of the strain gauge readings and hence no specific conclusion can be reached, other than that the axial strain variation is negligible. The strain measured
over an 8 inch gauge length; using a mechanical compressometer, should therefore agree closely with strain readings taken over a 1 inch gauge length.
Introduction

It is well known that the complete compression stress-strain curve for natural or artificial stones is not, in general, linear. The variations in the shape of the curve under repeated loading have been investigated by a number of early experimenters on natural stones and concrete. The present tests were designed to show the behaviour of concrete, if loaded straight through to fracture, and the effect on the deformations of repeated loading to 50% of the ultimate strength.

14.8(1) THE EFFECT OF REPEATED LOADING

14.8.1 Historical Tests

Experiments by Baushinger (7)

Baushinger, in 1875, states that stone has no elastic limit and takes permanent set at very low loads. In particular, with very hard and dense stone, the compressions are nearly proportional to the load until fracture. For all others, particularly the weaker stones, the greatest departure from proportionality is at the smaller stresses. E.g. Sandstones and soft granites give compressions which, at first, increase for equal load increments, but afterwards decrease. With repetitions of loading, the curvature gradually disappears
Experiments by Bach (4)

Bach subjected marble specimens to a small number of repetitions of loading and found that, after the fifth loading, the deformations for a constant increment of load were larger at lower loads, and smaller at higher loads, than on the first loading.

Experiments by Van Ornum (59)

Tests on concrete cylinders at Washington University, St. Louis, 1907, using compressometers of 8 inch gauge length, showed that the stress-deformation curve for the first loading was a straight line at lower stresses, becoming convex upwards for the higher stresses. The second loading gave a straight line to the maximum load previously applied. Further loading reduced the slope of the straight line (See Graph 14.5), but the effect of continued repetition of loading depended on the maximum stress applied.

Experiments by Mehmel (40)

An extensive series of tests was carried out in Karlsruhe, 1926, on concrete compression specimens 7 x 7 x 28 cms., of 1:6 cement-gravel mix, with a water-cement ratio of 0.63, at an average age of 1 year. Deformations were measured on a 20 cm. gauge length with a Marten's Mirror compressometer. Mehmel concluded that, for repeated stressing to below
50% of the ultimate, the relation between stress and strain was not merely a function of stress, but was dependent on the method of applying the stress, the number of times the stress was applied, the previous history of stressing, and other factors. When a specimen has been repeatedly loaded many times, it is possible for it to become elastic within the limits of the stress to which it has been subjected. Within these limits Hooke's Law is true.

The original stress-strain curve was convex upwards, but subsequently became a straight line.

Repeated stressing to above 50% of the ultimate, gave curves which were initially convex upwards, but which soon became concave upwards (See Graph 14.6).

An explanation of the changing stress-strain behaviour is that the elastic behaviour of concrete is at first controlled by the cement, it being well known that neat cement gives a stress-strain curve which is convex upwards. Repeated loading, when this is sufficiently high, fatigues the cement which takes less and less of the elastic strain energy and becomes plastic, after which it acts merely as a binder. The stress-strain curve of the concrete is then determined mostly by the elastic behaviour of the aggregate.
14.3.2 THE AUTHOR'S TESTS ON SPECIMEN 'C'

Stress-strain loading curves were obtained using electrical gauges together with the 3 inch gauge length mechanical compressor, for some thirteen repetitions of loading to a maximum of about 50% of the ultimate static strength (See Graph 14.8). An exact check on the number of loadings suffered by the specimen prior to this series of tests was not kept, but any changes in the stress-strain curves during the series may be seen.

The mean electrical strain \( \frac{\varepsilon}{d} \) obtained from the readings of the four gauges Nos. 1, 4, 6 and 9, was evaluated for each stress in the appropriate tests, and is presented in Graph 14.7. The regularity of the electrical readings is very good, but inspection of the curves shows that there was no consistent change in the behaviour of the concrete over the number of loadings carried out. The average slope of the mean electrical strain \( \frac{\varepsilon}{d} \) curves, gives a value of Young's Modulus = 3.75 x 10^6 lb/in^2 taking a strain sensitivity factor of 2.1.

The compressor stress-strain curves are similar to those obtained electrically, but slackness in the instrument presumably occurred in Tests Nos. 9(a) and 10(a), as the large deformation for the first load increment is not shown by the electrical gauges. The mean slope of these stress-strain curves (See Graph 14.8) gives Young's Modulus = 3.65 x 10^6 lb/in^2.
Again no consistent change in the behaviour of the concrete is indicated.

Both methods of strain measurement show the stress-strain curve to be concave upwards which agrees with the findings of Mehmol (40).

14.3.3 CONCLUSIONS:

1. There was no consistent change in the stress-strain curve of the concrete for some twenty-one repetitions of loading to 50% of the ultimate strength after the changes which occur on the first few loadings had taken place.

2. The Young's Modulus in compression, based on the mean slope of the graphs, for the concrete in the 'as-tested' condition was $3.7 \times 10^6$ lb/in$^2$.

3. The general shape of the stress-strain curves was concave upwards, which agrees with the work of earlier experimenters.

14.9 (11) THE COMPLETE COMPRESSION STRESS-STRAIN CURVE

14.9.1 Historical Tests.

Experiments by A.N. Johnson (55)

A carefully conducted series of tests were carried out at the Maryland University, 1929, on concrete
compression cylinders 9 inches high by 2\(\frac{3}{4}\) inches diameter, with the object of comparing the elastic moduli in compression with those obtained in tension. The concrete was of 1:2:3 Portland cement - limestone mix, with a maximum size of aggregate of 2\(\frac{3}{4}\) inch. Axial deformations were measured using two Marten's type mirror compressometers placed across a diameter over a 4 inch gauge length. A typical stress-strain curve for concrete 50 days old is shown in Graph 14.9. It may be seen that the curve is linear to approximately 35% of the ultimate load.

Experiments by Richart, Brandtsaeg and Brown (48)

Tests on five plain concrete columns 40 inches high by 10 inches diameter were made at the University of Illinois, 1929, and strain measurements were made with Berry and Howard strain gauges. As a result of these tests it was suggested that the behaviour of concrete in compression consisted of three distinct stages (See "Volume changes due to load", for further details - Para 14.13).

A theory of failure by Brandtsaeg (10), accounts for the behaviour of the concrete during these stages as follows:

"The first departure from elastic action of the material corresponds to the beginning of the second stage, and is considered to be due to the starting of plastic sliding on elementary planes at scattered points, and in every direction,
within the specimen. The spreading and increase of this plastic deformation, in turn, sets up lateral pressures which are resisted by the tensile strength of the elements still intact. Tensile failure of the latter elements marks the beginning of the third stage. The loss of lateral restraint results in a rapid increase in the plastic deformation, and further loading soon leads to failure. With very strong concrete, the splitting failure is the most noticeable, whereas with weak concrete the general crumbling and very gradual plastic yielding are the familiar phenomena of failure.

Opinions on the law of the initial compression stress-strain curve were expressed by:—

(a) C. Bach (4) in 1905, proposed that the compression stress-strain curve for different types of stones was of the form:—

\[ e = \frac{f^n}{E} \]

where \( e \) = strain,\n\( f \) = stress,\n\( E \) = initial Modulus of Elasticity and \( n \) depends on the properties of the material.

He found that \( n = 1.09 \) for pure cement \( = 1.13 \) for granite.

(b) Talbot (54) in 1906, in deriving formulae for the design of reinforced concrete beams, assumed the stress-strain diagram for concrete to be a parabola with the vertex at the ultimate.
(c) Stanton Walker (60) in 1919, from a study of a large number of stress-strain curves for concrete, suggested a curve of the type:

\[ s = k d^n \]

where \( s \) = stress in concrete, 
\( d \) = strain in concrete, 
\( k \) = a constant depending on the strength, 
\( n \) = an approximately constant exponent.

14.9.2 The Author's Tests on Specimen 'D'

The 45 day old specimen, which was loaded twice to 40\% of its ultimate strength just before the final test to fracture, gave the hoop and axial stress-strain curves as shown in Graphs 14.10 and 14.12. The strains were measured both electrically and mechanically.

The mean electrical strains \( \frac{\varepsilon}{a} \) were obtained by taking the mean of the readings of the diametrically opposite gauges Nos. 14 and 16 for the axial strains, and of Nos. 13 and 15 for the hoop strains (See Graph 14.11). From these curves, three stages in the behaviour of the specimen can be seen, which agrees with the findings of Richart (48) and others. During the first stage, which occurred from zero to 50\% of the ultimate load, the deformations were closely proportional to the applied load. The value of Young's Modulus was \( 4.4 \times 10^6 \text{lb/in}^2 \) taking a sensitivity factor of 2.1. The second stage of the deformations occurs from 50 to 90\% of the
ultimate load, and shows that the concrete gradually becomes more plastic. The average tangent modulus during this period was $3 \times 10^6$ lb/in$^2$. The third stage, from 90% of ultimate load to fracture, is most clearly noticed by the abrupt change on the transverse gauge readings. This may be seen from the individual axial gauges, but is indistinct on the mean axial strain curves. The rapid increase in the transverse strain compared with the axial strain during this stage, supports the views of Brandtsaeg (10), that this behaviour is due to an internal splitting of the material.

The compressometer measurements of strain over an 8 inch gauge length, show a similar stress-strain curve to that given by the electrical gauges except at stresses below 250 lb/in$^2$ (See Graph 14.12). Over this range, the mechanical strain readings are abnormally large, and are not considered to represent the normal deformation of the concrete. Possibly this was due to slackness of the instrument, or of the pivot screws.

The two preliminary loading runs to 40% of the ultimate, showed that the concrete was stiffening-up due to the repetition of load. Ignoring the initial readings, the strain was proportional to the load to 50% of the ultimate, and the value of Young's Modulus of Elasticity was $4.4 \times 10^6$ lb/in$^2$. The change between the second and third
stages of the deformation as described above, is not apparent from these results.

The analytical expression for the compression stress-strain curve obtained, using the electrical gauges is:

\[
\frac{\text{Stress}}{\text{lb/in}^2} = 3550 - 443 \times 10^6(0.00285 - \varepsilon_1)^3
\]

where \(\varepsilon_1\) = the electrical strain \(\frac{\varepsilon}{l}\)

Or, in terms of the concrete strain, taking a sensitivity factor of 2.1:

\[
\frac{\text{Stress}}{\text{lb/in}^2} = 3550 - 19.5 \times 10^8(0.00125 - \varepsilon)^2
\]

where \(\varepsilon\) = the axial compressive strain.

The stress-strain curve obtained from this equation is shown in Graph 14.12.

The parabolic shape for the curve agrees with the findings of Talbot (54).

14.9.3 CONCLUSIONS

1. There are three distinct stages before fracture in the deformation of a specimen under compressive loads.

2. The compression stress-strain curve is closely represented by a parabola with its vertex at the ultimate.
3. The initial value of Young's Modulus obtained by
electrical and mechanical means is $4.4 \times 10^6$ lb/in$^2$
and this holds to 50% of the ultimate strength.

4. Repeated loading to 40% of the ultimate gives a slight
stiffening of the concrete compared with its behaviour
on the first loading. This is in agreement with
Van Crum's (59) experiments (See Graph 14.5)
14.10 HYSTERESIS AND PERMANENT SET

14.10.1 Introduction

It has been shown by many experimenters that concrete takes permanent set after the application of loads well below its ultimate strength. The magnitude of the hysteresis and permanent set was investigated in the following tests for a specimen loaded to 50% of its ultimate strength.

14.10.2 Historical Tests

Experiments by Mehmel (40)

Experiments on concrete in compression at Karlsruhe, 1926, led Mehmel to conclude that "the actions in a stress-deformation curve which at first are not reversible, may become reversible through repeated loading, provided that a certain critical stress (approx. 50% of the ultimate) is not exceeded."

Experiments by F.G. Lea (38)

Experiments were carried out at Sheffield University, 1934 on concrete compression specimens of various mixes, of both prismatic and cylindrical shapes, the size of the latter being 12 inches high by 6 inches dia. A special optical compressometer of 3 inch gauge length was used to measure the deformations. To ensure that the ends of the specimen were plane and parallel before testing, the ends were carefully
A typical repeated loading test gave the hysteresis loops shown in Graph 14.13.

14.10.3 The Author's Tests on Specimen 'C'

The variation in the compressive strain of the concrete during loading, to approximately 50% of the ultimate, and unloading, was measured electrically in Tests Nos. 2, 3, 4 and 8, and with the compressometer in Tests Nos. 2 and 4.

From the readings of gauges Nos. 1, 4, 6 and 9, the mean electrical strains \( \frac{\Delta e}{e} \) were calculated for each stress, the results of which are shown in Graph 14.14. Definite hysteresis was shown in each test, and a permanent set equal to a strain of \( 0.3 \times 10^{-4} \) was obtained on each loading.

The readings of the mechanical compressometer show a larger hysteresis loop than the electrical readings (See Graph 14.14), and a permanent set equal to a strain of \( 0.8 \times 10^{-4} \). This discrepancy between the electrical and mechanical measurements may be due to back-lash in the mechanical instrument.

An indication of the magnitude of the variation in the hoop strain during loading and unloading, is obtained by inspection of the curves for the transverse gauges Nos. 3, 7 and 5 (See Graph 14.15) Gauge No. 5, situated at mid-height on the specimen, showed a thin hysteresis loop of average
width corresponding to a strain of $0.1 \times 10^{-2}$. Gauges Nos 3 and 7, which were situated near the ends of the specimen, gave rather erratic readings, but unfortunately, the possible error in the readings was of the same order as the size of the loop.

14.10.4 CONCLUSIONS

These experiments were only incidental to the main theme but, although they were not carried out with a high degree of precision and thoroughness, they do represent the behaviour of the material fairly definitely. The main conclusions are that:

1. Repeated loading, a small number of times to 50% of the ultimate strength, produces initially a permanent set on each loading of the order of a strain of $0.3 \times 10^{-2}$.

2. The size of the hysteresis loop becomes smaller with repeated loading. This is not proved conclusively however, owing to the small number of repeated-loading tests undertaken.
14.11 THE YIELDING OF THE SPECIMEN UNDER LOAD - SPECIMEN 1P!

During Test No.18 it was noted that, in the one and a half minutes taken to make the electrical observations the strain, as indicated by the mechanical compressometer, increased slightly, and that the beam of the testing machine dropped on to its "stop". As it was not a practical proposition to measure the release of load on the Riehlè machine due to backlash errors, the reading of the compressometer was taken immediately on the application of the load, and then again prior to further loading. The general form of the creep stress curve for approximately constant time increments of one and a half minutes is given in Graph 14.16. As would be expected, the creep was rapid at loads near failure and at 30% of the ultimate strength, amounted to an increase of 2% of the strain existing. Self release of load by the machine, due to flowing of the grease, is not thought to account for more than a small percentage of this creep.

CONCLUSIONS

1. At loads above 70% of the ultimate, creep of the concrete during the one and a half minutes of load application became noticeable.

2. The maximum error, due to creep, in the strain readings obtained is not greater than 2% at 30% of the ultimate load, increasing to 6% just before failure.
14.12 THE RATIO OF LATERAL TO LONGITUDINAL STRAINS

14.12.1 Introduction

In simple theories of elasticity, Poisson's Ratio for a material is normally assumed to be constant with increase of load over a considerable range. The validity of this assumption for concrete was investigated in the following tests.

14.12.2 Historical Tests

Tests by A.N. Johnson (33)

As part of a series of compression tests carried out at the University of Maryland, 1924, the ratio of lateral to longitudinal strains was measured. The specimens were of cylindrical shape 9 inches high by 4½ inches dia., of various concrete mixes, and they were tested at different ages. To measure the mean longitudinal strain on a 4 inch gauge length, two Marten's type mirror compressometers were fixed, one on each side of a diameter of the specimen, and read to an accuracy of a strain of $2.5 \times 10^{-6}$. The lateral extensometer worked on the principle of the Lamb Roller extensometer, and was clamped on to steel pads provided on the surface of the specimen.

The results for a typical specimen of 1 : 2 : 4 mix,
tested at 6 months, are shown in Graph 14.17. These show that Poisson's Ratio increased linearly with load to about 40% of the ultimate, changing in this range from 0.150 to 0.162 Above this load the ratio increased at a steadily increasing rate until fracture, the value being 0.3 just before failure.

The ultimate tensile strain at failure was $300 \times 10^{-6}$ compared with $30 \times 10^{-6}$ obtained by Johnson in tension tests on similar concrete (See Para. 13.2)

**Tests by Richard, Brandtsaeg and Brown (49)**

Tests were carried out at the University of Illinois, 1939, on five concrete columns 40 inches high by 10 inches dia. using Berry and Howard type gauges over 4 inches, 8 inches, and 30 inches. These showed that Poisson's Ratio increased at a gradually increasing rate from approximately 0.11 (at 28 days), but due to cracking within the large gauge lengths employed, values of the ratio greater than 0.5 were obtained at high loads (See Graph 14.18). Such values are inconsistent with the rational behaviour of a homogeneous body.

**Tests by Davis and Troxell (19)**

Some 300 specimens 6 inches dia. by 12 inches high, of Portland Cement-sandstone concrete, of mixes 1 : 3$\frac{1}{2}$, 1 : 4$\frac{1}{2}$ and 1 : 6, and maximum aggregate size of 1$\frac{1}{2}$ inches, were
tested. Using Johnson's (34) compressometers, Poisson's Ratio was shown to increase with age exponentially with time (as shown in Graph 14.17) from 0.17 to 0.20 at 500 days. Their findings, that Poisson's ratio decreased with increasing stress, were discounted.

Tests by Withy (62)

In a series of experiments on reinforced concrete columns carried out at the University of Wisconsin, 1909, Withy finds that for stresses under 25% of the ultimate strength, the value of Poisson's Ratio for concrete two months old, increased from 8 to 13%.

14.12.3 The Author's Tests on Specimen 'C'

The ratio of lateral to longitudinal strains is evaluated from the results of Test No.7, in which two lateral gauges Nos. 5 and 8, and two longitudinal gauges Nos. 4 and 9, all of which were situated at the mid-height of the specimen, were used.

At a nominal stress of 1275 lb/in², by taking the mean of the readings of gauges Nos. 5 and 8, and the mean of gauges Nos 4 and 9, the hoop and axial strains were evaluated at a point within the specimen. The "Poisson's Ratio" = 0.24, which is a typical value for a concrete 18 months old.

Over the load range investigated (i.e. 0 - 1800 lb/in²)
ignoring the initial 'negative loop' on the tension gauge No. 5, the "Poisson's Ratio" was increasing linearly (See Fig.14.7) from 0.20 to 0.26

CONCLUSIONS

There is an increase in Poisson's Ratio with increase of load. For the concrete in the 'as tested' condition, this increase was from 0.20 to 0.26 for a load increase from zero to 50% of the ultimate.

14.12.4 The Author's Tests on Specimen 'D'

In Test No.12, the ratio of the lateral to the longitudinal strain may be found from the readings of the gauge pairs Nos. 13 and 14, or 15 and 16, as each one of a pair was subjected to almost the same longitudinal strain. More accurately, the ratio may be obtained from the mean strains at the centre of the cross section of the specimen,
using the mean of the readings of the diametrically opposite gauges Nos. 13 and 15, and 14 and 16.

The results from the three possible methods of calculation from no load until fracture, are given in Appendix XXIX, and are shown graphically against percentage of the ultimate stress in Graph 14.19.

Inspection of the gauge readings shows that there was a large eccentricity of loading which produced very small transverse strains on gauge No.13 at low loads. The values of the ratio calculated using the readings of this gauge are therefore not reliable until a nominal stress of approximately 40% of the ultimate was exceeded. However, the gauge pair 15 and 16 indicate the law of the variation of this ratio below this stress.

Defining Poisson's Ratio as the ratio of lateral to longitudinal strain immediately on the application of a load, then for this concrete, this ratio varied uniformly with stress until 87% of the ultimate strength was reached. During this period the value increased from 0.139 to 0.142. At stresses above 87% of the ultimate strength, the value of Poisson's Ratio increased rapidly, though uniformly, to 0.32 just before failure.

The agreement between the two gauge pairs during this period was perfect, and indicated that the eccentricity of
loading had been reduced to zero. The point of breakdown is very well defined, a change which is not obvious from consideration of the axial deformation alone.

CONCLUSIONS

1. Poisson's Ratio increased linearly with load to 87% of the ultimate strength of the concrete. The increase for the concrete tested was from 0.120 to 0.142. The experimental work of Johnson(33) and Withey(62) is thus confirmed.

2. The apparent increase in the ratio at loads above 87% of the ultimate, is due to a discontinuity in the material which causes large transverse strains. The increase is thus due to the method of loading, and is not a property of the concrete.
14.13 VOLUME CHANGES DUE TO LOADING

A body subjected to a compressive strain \( \varepsilon_1 \) in one direction due to a load, will suffer a volumetric strain \( \varepsilon_v = \varepsilon_1 - 2\varepsilon_2 \), where \( \varepsilon_2 \) = lateral strain accompanying \( \varepsilon_1 \).

14.13.1 Historical Tests.

Tests by Richart, Brandtsaeg and Brown (43)

As part of the research into the failure of plain and spirally reinforced concrete as described in Para. 14.9 and 14.12, the longitudinal and lateral deformations of five plain concrete columns, 40 inches high by 10 inches dia., were measured under load as described in Para. 14.12.2 under 'Poisson's Ratio'. The volume changes of the material during test were calculated from the strain readings, and gave a typical curve of volume variation with stress as shown in Graph 14.20.

Three distinct stages in the deformation are identified. During the first stage, which occurs at stresses from 0 - 60% of the ultimate, the decrease in volume is proportional to load. The second stage occurs to 30% of the ultimate load, and shows an increasing rate of decrease of volume with load. The third stage indicates an internal splitting of the material which causes a rapid increase in the lateral plastic...
yielding of the concrete, and the volume to increase.

Tests by A.N. Johnson (34)

Analysis of the tests described in Para. 14.12.2 on 'Poisson’s Ratio’, shows that a similar change in the volume of the specimen was obtained to that found by Richard, Brandtzaeg and Brown. The results of a typical test are shown in Graph 14.20

14.15.2 The Author’s Test on Specimen 'D'

Using the mean electrical transverse and axial strain measurements from Test No.18 with a strain sensitivity factor of 2.1, the variation in the volumetric strain with stress is shown in Graph 14.20

The three stages in a compression test as noted by Richard etc. may be seen in this test. In the first stage, the volume decreases linearly with stress to about 50% of the ultimate strength. Above this stress is the second stage, and in it the rate of change of volumetric strain with stress increases gradually to a maximum at 90% of the ultimate. The third stage is marked by the rapid increase of volume until fracture.

14.15.3 CONCLUSIONS

The volume variation with load indicated three distinct stages in the behaviour of the concrete :-
1. The material obeyed Hooke's Law to about 50% of the ultimate load.

2. Between 50% and 90% of the ultimate load the concrete deformed more easily than before.

3. Above 90% of the ultimate load, the increase in volume of the specimen indicated that a fault had developed in the material.
14.14 THE FAILURE OF THE SPECIMEN 'D'

No cracks were observed on the specimen until the maximum load was reached. The rupture consisted mainly of four longitudinal cracks equally spaced round the circumference, and extending from top to bottom of the specimen. Two of these were along the line of the compressometer screws. Otherwise there was little sign of cracking at mid-height. The spalling of thin flakes of concrete occurred at the upper end, and when the specimen was removed from the machine, the top and bottom surfaces were found to be covered with many fine cracks, which had been prevented from spreading by the end constraint of the packing.

The maximum tensile strain recorded by a gauge before failure was $550 \times 10^{-6}$, compared with an ultimate tensile strain in the tension test on concrete of the same batch of $90 \times 10^{-6}$. A similar large increase in the tensile strain capacity in compression, compared with its behaviour in tension, was observed by A.N. Johnson (34) who measured a strain of $80 \times 10^{-6}$ in tension, and $300 \times 10^{-6}$ in compression.

CONCLUSIONS

1. The specimen failed by hoop tension, the crack starting at the compressometer screws.

2. There is a considerable increase in the tensile strain capacity of concrete when tested in simple compression, compared with that obtained in simple tension.
STRAIN SENSITIVITY FACTOR OF THE GAUGES

14.15.1 Tests on Specimen 'C'

An estimate of the strain sensitivity factor of the 2000 ohm gauges may be obtained by comparing the mean electrical strain \( \frac{dQ}{Q} \) at a particular cross section, with the corresponding mechanical measure of the strain.

The mean electrical strain \( \frac{dQ}{Q} \) at mid-height, has been evaluated from gauges Nos. 1, 4, 6 and 9, and is plotted against the compressometer measure of strain for two typical tests, viz. Test No. 9(a), using no packing between specimen and the compression head, and Test No. 10(b) using two rubber sheets as packing (See Graph 14.21). These give strain sensitivity factors of 2.15 and 2.25 respectively, which is a lower figure than the 2.3 obtained in the tests on a steel beam.

14.15.2 Tests on Specimen 'D'

The strain sensitivity factor of the 200 ohm gauges used in Test No. 12, is estimated by comparing the mean axial strain obtained electrically with that obtained using the 8 inch mechanical compressometer.

As the axial gauges Nos. 14 and 16 were diametrically opposite each other, the mean of their readings gives the mean electrical strain on the section. The variation in
the electrical readings with those obtained mechanically is shown in Graph 14.22.

This shows that, except at the very small strains which occur below 250 lb/in², and at large strains which occur above 20% of the ultimate load, the relationship between the electrical and mechanical readings was linear. The strain sensitivity factor of the gauges as measured, was 2.0 as compared with 2.12 obtained in tests on a steel tension specimen, and 2.08 obtained on the concrete tension specimen.

The very low sensitivity indicated at small stresses is, no doubt, due to the large strains shown by the mechanical compressometer (See Graph 14.12). This was not confirmed by the electrical gauges, and, as it is unusual for concrete to behave in this manner on first loading, the mechanical gauge readings are considered suspect over the initial range.

At high loads, due to a discontinuity in the material, the specimen was barrelling considerably. The electrical gauges were recording high local strains, whereas the mechanical compressometer measured an average strain over an 8 inch length. The increased sensitivity factor shown in this range may thus be ignored.
14.15.3 CONCLUSIONS

The compression test is not suitable for measuring the strain sensitivity factor of electrical gauges owing to the variation of strain along the length of the specimen, unless a mechanical measure of the local strain over the length of the gauge can be made.

However, it will be noted that the values obtained are lower than was obtained on steel specimens, whereas, if the mechanical gauge suffered a lower average strain than the electrical gauges, then the sensitivity would appear higher. This indicates a small reduction in the efficiency of sticking the gauges. Alternatively this may be due to a lack of homogeneity in the material.

The actual figures obtained for the sensitivity factors in these tests are not of much value as they may be expected to vary with the end-packing used, but the response of the gauges to strain is confirmed by the compressometer.
14.16 THE ‘NEGATIVE LOOP PHENOMENON’

During the first few compression tests it was noticed that the transverse gauges which were expected to go into tension, initially indicated compression. When the compressive stress had reached approximately 400 lb/in², the gauges began to show tensile strains, so that the stress-strain curve formed a ‘negative loop’. As it is unlikely that concrete behaves in this manner, this phenomenon was made the subject of Test No.5, when the readings of the transverse gauges Nos. 5, 5 and 7 were recorded for a large number of small increments of load (See Graph 14.25).

In an attempt to explain this phenomenon the following points were noted and considered:--

(1) The trouble was only experienced with 2000 ohm gauges and was observed in the simple tension tests (See Graph 13.5) as well as the compression tests.

(2) The magnitude of the compression strain indicated, varied between the tests.

(3) Large initial eccentricity is not a satisfactory explanation, as no axial gauge indicated tension.

(4) The readings of the gauges were taken at each load 1 minute after being switched into circuit, so that any drift, due to imperfect temperature compensation, would be constant for a given gauge.
(5) A change in the galvanometer resistance, due to self heating, is an unsatisfactory explanation as a null method was used.

(6) Changes in the resistance of the dummy gauge or the Post Office Box resistances could explain the phenomenon, but some effect should also be noted on the compression gauges.

(7) The gauges indicated a compressive permanent set in the concrete on release of load, which is most improbable. The general slope of the loading and unloading curves is the same, so that, if the initial loop is ignored, then the loading and unloading curves have much the same strain value at zero load.

(8) The cross sensitivity of the gauges has been shown to be negligible (See Para 11.5)

(9) The shape of the curve obtained, suggests that it may consist of the addition of a 'tension phenomenon' which is directly proportional to load, and a 'compression phenomenon' which increases exponentially with time.

CONCLUSIONS:

A satisfactory explanation of this phenomenon has not been found.

However, from the evidence given in items (2), (5) and (8), it may be concluded that the negative loop is not a
measure of the behaviour of the concrete.

The circuit considerations given in items (4), (5) and (6) do not satisfy the problem, as all gauges would be affected.

The phenomenon may therefore be a function of the gauge itself, or its method of attachment, which is peculiar to very small tensile strains.

As the gauges behaved linearly with stress after the initial compression, the 'negative loop' has been ignored in all calculations, and tensile strain is assumed to be directly proportional to stress over the load range considered.

Post-Script

During the writing of this thesis, an interim report issued by the R.A.E. Farnborough, describes tension tests on fine wires in which the electrical resistance changes were measured for various strains. In particular, for nickel wire, the fractional resistance change \( \frac{\Delta Q}{Q} \) was found to be negative for small tensile strains, but subsequently to become positive as the strain was further increased (See Fig. 14.3)

It is quite reasonable to suppose that a similar phenomenon occurs in the nichrome wire which is used in the B.T.C. Gauges. This gives a suitable explanation of the 'negative loop' observed in the author's tests.
FIG. 14.8 Variation in Resistance with Strain of Fine Nickel Wire
14.17 SUMMARY OF THE GENERAL CONCLUSIONS ON THE COMPRESSION TESTS

1. In general, the introduction of packing between the specimen and the compression platens of the testing machine gave little, if any, reduction in the eccentricity of loading. Thin rubber sheets, however, did give some improvement.

2. Eccentricity of loading sufficient to cause a reduction of 20% in the ultimate strength is probable for the set-up used.

3. There was considerable barrelling of the specimen under load, which was such that, with millboard packing, the hoop strain at the mid-height of the specimen was about five times that at the ends. The axial strain variation along the length of the specimen was negligible.

4. With repeated loading to 50% of the ultimate strength, after the first few loadings, the compression stress-strain curve was unchanged by a further twenty-one repetitions of loading. The shape of the curve was then concave upwards.
5. For the virgin concrete 45 days old, loaded through to fracture, the stress-strain curve was given by the parabola

\[
\text{Stress} \quad \frac{\text{lb/in}^2}{\text{in}^2} = 3550 - 19.5 \times 10^8 (0.00135 - e)^2
\]

where 'e' denotes the compressive strain.

6. The value of Young's Modulus of Elasticity in compression was \(3.7 \times 10^6\)\(\text{lb/in}^2\) for concrete 18 months old, which had been repeatedly loaded to 50% of its ultimate strength. The low value was presumably due to the repeated loading.

7. From zero to 50% of the ultimate load, Young's Modulus was \(4.4 \times 10^6\text{lb/in}^2\) for the virgin concrete 45 days old.

8. Concrete in compression undergoes three distinct changes before fracture. For the test conditions, these consisted of a Hookes Law stage to 50% of the ultimate, followed by a semi-plastic stage to 90% of the ultimate, when a discontinuity occurred within the specimen which soon caused fracture.

9. The volume changes in the concrete during loading were a valuable guide to its behaviour, and clearly indicated the three stages in the deformations before fracture mentioned in (8).
10. Hysteresis occurred on each loading to 50% of the ultimate strength, and left a permanent set equal to a strain of $0.5 \times 10^{-2}$ each time.

11. Creep of the concrete in the two minutes under each load was noticeable above 70% of the ultimate load. It increased rapidly with higher loads, and just before fracture amounted to 6% of the total strain.

12. The Poisson's Ratio for the concrete 18 months old which had been repeatedly loaded, increased from 0.20 to 0.23 over the load range from zero to 50% of the ultimate.

13. The virgin concrete 45 days old had a Poisson's Ratio which increased linearly from 0.120 to 0.142 over a load range from zero to 87% of the ultimate load.

14. There was an increase of the order of 600% in the tensile strain capacity of the concrete when tested in simple compression, compared with that obtained in simple tension.

15. The 2000 ohm resistance strain gauges indicated compressive strains when subjected to very small tensile strains. The precise reason for this is uncertain, although it may be a property of the wire.
CHAPTER 15

BEAM TESTS

15.1 INTRODUCTION

The quality of a concrete may be judged by its tensile resistance. This property is best measured by a direct tension test but, owing to the difficulty of obtaining truly axial loading, the results from such tests are rather variable. A transverse bending test on a plain concrete beam is therefore often used as a measure of this property, because it is simpler to carry out and gives more consistent results. In addition as, in practice, a bending action is the most common form of loading, it is claimed that this forms a better criterion of its strength.

The reliability of the results of the beam tests will depend on the method of loading. For instance, if the bending moment is a maximum at only one section of the beam, then the results will probably be higher than if a considerable length is subjected to this bending moment. This follows from the
'weakest link theory of probability'. On the other hand, if the loading is such that some torsion is also put into the specimen, then low results will prevail. It is therefore advisable to obtain some measure of the accuracy of loading for a particular set-up.

The ultimate tensile strength is sometimes calculated from the results of a beam test. For example, "The regulations for the use of reinforced concrete" (Dec.1945) (41) in France, specify that the tensile strength shall be found by testing a square section of side 'b' and length '4b'. The beam shall be simply supported at its ends and centrally loaded until fracture. The tensile strength shall be determined by taking 0.6 of the breaking stress calculated elastically. However, it should be noted that the calculation of the ultimate tensile strength from a flexural test is dependent on the stress and strain distributions assumed.

The strain variation in the depth of a plain concrete beam is commonly assumed to be linear. Experiments have been carried out in the past, to measure the change in length of different layers of the beam over lengths of 8 inches or more. The usual method has been to measure between plugs inserted in the concrete, although this would interfere, if only slightly, with its behaviour. The electrical gauge offered the possibility of measuring more local strains without interfering with the concrete, and thus of justifying or condemning the
assumption of the Navier hypothesis. The question as to whether the tensile strain capacity of concrete is a property of the material, or whether it depends on the method of loading, has been debatable for many years. In the past, it has been necessary to measure over large gauge lengths and hence it has been extremely difficult, in bending tests, to avoid errors due to the formation of very small hair cracks within its length. The resistance strain gauge enables the mean strain to be measured over a very short length and thus gives a greater chance of avoiding a crack, and of obtaining a reliable measure of the strain capacity of the concrete.

15.2 HISTORICAL TESTS.

The tensile strain capacity of concrete has been studied by many experimenters since the discovery of reinforced concrete construction. Considère startled the engineering world in 1893 by stating that reinforced concrete had twenty times the strain capacity of plain concrete and for the last fifty years others have sought to disprove his original findings, including Kleinlogel (1903), Bach (1907) and Evans (1946). The strain variation in the depth of a concrete beam has been investigated by Berry and Goldbeck (Pennsylvania, 1910), F.C. Lea (Sheffield, 1934), and R.H. Evans (Leeds, 1935).
15.2.1 STRAIN CAPACITY EXPERIMENTS

Considère (15), who carried out the first reported tests on the strain capacity of concrete in 1888-9, measured tensile strains up to \(2000 \times 10^{-6}\) in reinforced concrete beams before cracks appeared, whereas in parallel tests on plain concrete beams, the breaking strain was 100 to 200 \(\times 10^{-6}\). Others at about the same time, in tests carried out for Messrs. Wayss and Freytag (61) disagreed with these findings.

Kleinlogel (37) found in 1905 that, for all quantities of reinforcement, the strain before a crack appeared was the same, that is, 143 to 196 \(\times 10^{-6}\).

Considère (16) repeated his tests in 1905 with larger reinforced concrete beams, and found the strain capacity to be 220 to 500 \(\times 10^{-6}\) for damp-cured specimens, and 560 to 1070 \(\times 10^{-6}\) for water-cured specimens.

Bach (5), also found in 1907 that the effect of water-curing was to increase the breaking strain, and that, for air-cured plain concrete, the strain capacity was 150 to 180 \(\times 10^{-6}\).

Evans (25) carried out tests on a number of plain and reinforced concrete beams in 1946, using 8 inch mirror extensometers and 1 inch roller mirror extensometers on the tension surface. Great trouble was taken in searching for the
first cracks to appear, and microscopes giving 250 magnification were used.

Tests were also carried out on similar concrete tension specimens, and the strain capacity measured.

He concludes that there is no sensible difference between the strain capacity of plain and reinforced concrete in tension or bending. The greater extensibility observed in reinforced concrete by some earlier experimenters, was entirely due to insufficient magnification and preparation of the concrete surface, which resulted in the recording of the opening of minute cracks.

15.2.2 STRAIN DISTRIBUTION EXPERIMENTS

Tests by Goldbeck (29)

From tests at the University of Pennsylvania in 1910 on plain rectangular concrete beams of 1:2:4 mix, Goldbeck concluded that the strain-depth curves are straight lines with a tendency to become concave upwards above the neutral axis, and concave downwards below the neutral axis, as loading progressed.

Micrometer screw extensometers of 24 inches gauge length were used, which measured to $1 \times 10^{-4}$ inches.

Further tests by Berry and Goldbeck (8) on reinforced concrete beams showed that the position of the neutral axis was not changed by repetitions of load.
Tests by F.C. Lea (38)

Tests were made at the University of Sheffield in 1934 on a reinforced concrete beam 11 inches deep, by 7 inches wide, by 10 ft. long of 1:2:4 mix by weight, at an age of 3 months. The concrete had been water-cured for one week, followed by air-curing.

A four point loading method was used with a central span of 36 inches, and strain measurements were taken on both vertical faces of the beam, using special mirror compressometers, with the scales so placed that the accuracy of strain measurement was $1.4 \times 10^{-6}$. Two vertical lines of steel pins were let into the beam 18 inches apart, each having nine pins. Using these as the gauge marks, the change in length at nine different levels, was measured with the compressometer. The central deflection of the beam was measured, using an 18 inch saddle resting on two of the pins, and a dial gauge attached to the specimen.

The results show that:

(a) The strain at each layer was proportional to the load, up to a maximum bending moment of 103 ton inches.

(b) There was distinct hysteresis and permanent set on each loading, the size of the loop remaining almost constant after the first three loadings (See Graph 14.16)

(c) The strain distribution above the neutral axis is proportional to distance from the axis, whereas below,
The strains are greater and the curve is convex downwards to the axis. (See Graph 15.1)

Tests by R.H. Evans (23)

Experiments at the University of Leeds in 1935 were carried out on plain rectangular concrete beams, using a four point loading arrangement and a special portable 8 inch gauge length extensometer, engaging in pins suitably positioned on the side of the beam.

A typical result is shown in Graph 15.1 and indicates that the strain distribution is linear until failure, and that the neutral axis does not rise as in reinforced beams.

He concluded that, as the stresses are small, the only reason for a change of the neutral axis, would be a change in the plastic properties of the concrete in tension and compression. Therefore plane sections remain plane.

Using mirror extensometers reading to $1 \times 10^{-6}$ inches over an 8 inch length, Evans also showed that the creep in the concrete is sensibly the same in tension and compression at the same stresses. This verified the experiments described in the Building Research Station Report 1930.

The maximum tensile strain was found to be somewhat greater than the compressive strain.

From further tests on reinforced beams (24) Evans concluded that the strain curves above the neutral axis, at high loads, are generally convex to the axis, and those below, approximately linear.
THE AUTHOR'S BEAM TESTS

OBJECTS OF THE EXPERIMENTS

The object of these experiments was to ascertain:

(a) If plane sections of the beam did, in fact, remain plane on bending, which would enable a simple but rational calculation to be made for the U.T.S. of the concrete.

(b) The tensile strain capacity of the concrete in bending, and to compare this with the strain capacity in simple tension.

(c) The degree of perfection of loading for the set-up used for the beam tests.

Outline of the Beam Experiments.

Two plain concrete beams, approximately 6 inches deep by 3 inches wide by 48 inches long, were tested which are now designated the 'E' and 'F' specimens.

The 'E' specimen was 14\(\frac{1}{2}\) weeks old when first tested, and had been water-cured for 10 weeks, and air-cured for the remaining time. Two preliminary tests were carried out 7 weeks apart, in which the strains on the top and bottom surfaces were measured electrically, whilst the beam was loaded to about 50\% of its capacity. The final test through to fracture took place when the specimen was 7\(\frac{1}{2}\) months old, and electrical strain measurements were made by six gauges for sixteen loading conditions. The modulus of rupture (or transverse rupture Stress)
was 506 lb/in².

The 'E' specimen was also 14.5 weeks when first tested, and had had the same curing as the 'F' specimen. Six preliminary loading tests to about 66% of the ultimate load were carried out, but no reliable strain readings were obtained owing to electrical troubles. A seventh test provided strain readings at four different depths on the beam. The final test to fracture was made when the specimen was 7.5 months old using a new set of electrical gauges, and strain measurements were obtained at eight different depths on the beam for thirteen loading conditions. The modulus of rupture was 413 lb/in². The central deflections of the beams were measured with a dial gauge in both final tests.

DETAILS OF THE BEAM TESTS

15.4 TESTING APPARATUS

The Specimens.

The beams were cast in specially constructed collapsible wooden moulds, giving specimens nominally 6 inches deep, by 3 inches wide, by 48 inches long. Filling was carried out with the mould horizontal, in four approximately equal layers of 1.25 inches. To obtain smooth outer surfaces, the mould was hammered during the tamping of each layer. The specimens were kept damp during initial curing, and at 5 days were removed from the moulds for water-curing.
The Beam was tested on the saddle of a Buckton 10 ton single lever Testing Machine, which was hand driven through a worm and wheel. It was symmetrically supported on steel rollers of \( \frac{3}{4} \) inch radius over a 27 inch span. In order to test a considerable length of the beam, four point loading, to give a uniform bending moment over the central section, was arranged by placing a 3 inch wide by 1 inch thick steel bar symmetrically between the supports on the upper side of the beam. This made line contacts with the beam at two sections 15 inches apart, through \( \frac{3}{4} \) inch diameter steel rods which were welded to it. The steel bar was centrally loaded. As the upper and lower surfaces of the beam were not perfectly plane, pads of 1/16 inch thick, soft rubber sheeting, were placed between the concrete and the four loading points, to avoid twisting the specimen by eccentric loading.

The jockey weight and lever scale of the machine were not suitable for measuring the relatively small loads involved, owing to backlash in the machine and creep of the concrete. A better arrangement, giving a continuous record of the load, was obtained by placing a large proving ring between the centre of the steel bar and the moving head of the testing machine. To ensure that the steel bar was loaded centrally in both the transverse and longitudinal directions, and that the proving
ring did not spring out under load, \( \frac{1}{2} \) inch diameter steel balls were placed at the top and bottom of the ring in dimples made for location in the steel bar and in the platen of the testing machine. For ease of setting up and for safety, the balls were loosely held in 'tight' holes in pieces of thin metal plate. The dead weight of the ring and steel bar was 40 lb.

The photograph, Plate 15.1, shows the general arrangement with the electrical gauges in position.

The Proving Ring.

The details of the steel proving ring are shown in Fig.15.1.

The arm was arranged to suit the fixing lug of a standard plunger type 1/10,000 inch dial gauge.

The calibration of the ring was carefully carried out in a standardised testing machine to a maximum load of 2 ton, and it was found to have a linear load deflection characteristic of 47.1 lb. per 1/10,000 inch. It was therefore possible to read loads to an accuracy of \( \pm 5 \) lb.

This method of measuring the load on the beam enabled the amount of the release of load, due to creep in the concrete, to be measured.

Continued :-
The Deflectometer.

Owing to limitations imposed by the size of the testing machine, there was only about 2 in. available under the beam for the setting up of a deflectometer. The lever mechanism shown in Fig. 15.2, was borrowed from the Royal Naval College, Greenwich to overcome this difficulty.

Continued :-
The Multi-Channel Strain Gauge Set.

This set, as described in Chapter 7, was used in conjunction with the Cambridge and Marconi Galvanometers for reading the electrical strain gauges.

The Strain Gauges

'E' Specimen

For the preliminary tests, four 200 ohm, 1 inch long gauges, Nos. 1, 2, 3 and 4, were affixed within the central 15 inches of the beam to suitably smooth portions of the top and bottom surfaces, by the method described in Para. 10.5. Gauges Nos. 1 and 2 were on one face, and Nos. 3 and 4 on the other.
As a further check, two extra gauges Nos. 11 and 12 were affixed for the final test, one to each face. All the gauges functioned.

**'F' Specimen**

Initially, six gauges of 200 ohm nominal resistance, 1 inch long, and designated Nos. 5, 6, 7, 8, 9 and 10, were affixed to the beam, one on each of the top and bottom faces, and four at various depths on a vertical face of the specimen. Over a period of six weeks, during which some six preliminary loading runs were made, three of the gauges, Nos. 7, 8 and 10 broke down. For the final test to fracture, the broken gauges were replaced and additional ones, Nos. 13, 14, 15 and 16, were fixed on to the specimen, three on the vertical face and one on the top surface of the beam. During this test, gauges Nos. 5 and 14 failed to record.

**Note:** The exact location of all gauges is shown in Figs. 15.4 and 15.5.
FIG. 15.3 General Arrangement of Beam Specimen

FIG. 15.4 Gauge Positions - Beam Specimen "E"
(See also Appendix XXXII)

FIG. 15.5 Gauge Positions - Beam Specimen "F"
(See also Appendix XXXII)
Testing Procedure

The Test beam was set up in the machine as described in Para. 15.4, and a small tightening load applied. A central span of the beam of 27 in. was chosen so that the uniform bending moment over the central 15 in. was produced by a large force at a short lever arm, and the bending moment induced in the beam due to its own weight, was a minimum. This arrangement reduced any errors in reading the load. When used, the deflectometer as described in Para. 15.4, was fitted in position, and a small check loading carried out to make certain that it was recording.

The strain gauges were connected up to the multi-channel strain gauge set, together with the appropriate dummy gauges obtained from the companion beam. This was placed as close as possible to the test beam.

Considerable trouble was experienced in some preliminary tests due to fluctuations in the temperature of the laboratory. This caused disturbing phenomena such as, all the tension gauges recording compressions, or vice-versa. The trouble was finally traced to a "finned" steam pipe running near the machine which formed a "blew-off" for the main heating system of the building, and which went from hot to cold in the space of a quarter of an hour. The remedy was to insulate the pipe, to completely cover the gauges with a thick cartridge paper, and to shroud the test beam and the "dummy" beam completely with
After the set had been left to warm up on the heating coils for at least an hour, the gauge pairs were carefully balanced and the test begun. The loads were maintained for periods of up to ten minutes, depending on the number of gauges in operation, and any release of load due to creep in the concrete, was noted.

Observations

The readings and incidental results for the several tests are given in Appendices XXXII to XXXVII inclusive. The graphical presentation of the readings is given in Graphs 15.2 to 15.8 inclusive.

15.5 RESULTS AND COMMENTS ON THE INDIVIDUAL TESTS

15.5.1 Tests Nos. 1, 2 and 3 Specimen 'B'

1. Preliminary Tests Nos. 1 and 2, showed that the maximum compressive and tensile bending strains were proportional to the load to 50% of the ultimate strength. (See Graphs 15.2 and 15.3).

2. Although all the gauges were attached to concrete suffering the same bending moment, the two compression gauges gave different load-strain rates from one another, as also did the two tension gauges. These differences
may be attributed to:

(a) Strain variations due to local areas of cement or aggregate which have different elastic properties. This was most probable on the compression surface which was uppermost on casting. It had been coated with a thin cement grout during manufacture to improve the surface, which would otherwise have been very poor, as the concrete was on the dry side and it was very difficult to tamp the final layer sufficiently to bring any water to the surface.

(b) Variation in the strain sensitivity factor of the gauges from say 1.6 to 2.1, due to poor sticking, could account for some of the differences, but it is not considered to be the primary cause.

(c) The tension gauges may be expected to show differences due to the opening up of hair cracks under the gauges.

3. The tension gauges showed higher strains than either of the compression gauges.

4. In Test No.5 greater precision was obtained in the strain measurements, as may be seen from the smooth load-strain curves (See Graph 15.4)

5. No change is discernable in the load strain rate due to repeated loading of the gauges to 50% of the ultimate strength.
6. Of the additional gauges used for Test No.3, gauge No.12 on the compression side agreed with the readings of gauge No.2. It is therefore reasonable to suppose that the reading of gauges No.12 or 2 is representative of the maximum compressive bending strain in the beam. Gauge No.11 on the tension side recorded very much larger strains than either of gauges Nos. 3 and 4, so that to select a tension gauge as giving a fair reading is more difficult. However, gauge No.5 gave an ultimate tensile strain of three times that obtained in the simple tension test, and gauge No.11 five times that given in the tension test. Gauge No.4, on the other hand, indicated an ultimate electrical strain of approximately $300 \times 10^{-6}$ compared with $200 \times 10^{-6}$ obtained in the tension test, and is therefore preferred. Presumably, gauges Nos. 3 and 11 were situated over hair-cracks which gradually opened, causing the decreasing load-strain rate obtained. The ratio of the tensile to the compressive load-strain rate initially based on the readings of gauges Nos.4 and 12, was 1.65

7. Graph 15.4 shows that all tension gauges indicated a reduction in the slope of their load-strain curves as soon as the maximum bending moment, to which the beam had previously been loaded, was exceeded. The concrete, in tension, was therefore behaving more plastically than before.
At about 90% of the ultimate strength, the rate of increase of the tensile and compressive strains with load, increased distinctly. Failure occurred on the section through gauge No.4, and the adhesive under this gauge broke down at 90% of the ultimate load. It may therefore be concluded that a crack started at this load under gauge No.4, and spread quickly thus causing the failure of the beam.

The release of load and increase of deflection, due to creep of the concrete, became noticeable above the maximum load to which the beam had previously been subjected. (See Appendix XXXV). Near failure, over a period of nearly three minutes, the relaxation of load was 1% and the increase in deflection 1%.

15.5.2 Tests Nos. 4 and 5 Specimen - F

Test No. 4 was carried out after the beam had been loaded some six times to approximately 66% of its ultimate strength. The beam was loaded in the reverse direction from the previous loadings so that the concrete, which was at the bottom on moulding, was on the upper compressive surface in the test.

Inspection of Graph 15.5 shows that the strains were proportional to the load to 65% of the ultimate. The
curves do not pass through the origin due to the faulty balancing of the bridge.

The four gauges in operation were situated at various depths on the beam, and an idea of the strain variation over the depth can be obtained from Graph 15.5. It will be seen that gauge No. 3, which was at the mid-depth, was very close to the neutral axis of bending. The position of this axis obtained experimentally agrees closely with that calculated on the assumption that plane sections remained plane, and that $\frac{E_t}{E_0} = 1.23$ (the value obtained in other tests). The readings of gauges Nos. 2 and 6 suggest that, over the middle portion of the depth of the beam, the compressive and tensile strains were proportional to the distance from the neutral axis. The strain reading on the lower surface was, however, higher than is required by this proposition.

II. In Test No. 5 the beam was loaded in the opposite direction from that in Test No. 4.

Graph 15.6 shows that the strain at various depths on the beam, was proportional to load until the maximum bending moment which had previously been applied, was exceeded, that is, at approximately 5250 lb in.

The position of the neutral axis was closely fixed by the readings of the eight gauges, and was constant at
0.47 x depth from the top surface until the limit of the first stage of the test was reached at 5250 lb.in. bending moment. The experimental position of this axis, even at low loads, is higher than that calculated, and is possibly due to the effect of the alternating stressing on the $E_t/E_0$ ratio. Just before failure, the neutral axis rose to about 0.4 x the depth of beam from the top surface. The rise is clearly shown by the increasing rate of strain variation with load on gauge No.9, and the decrease on gauge No.15. (See Graph 15.7)

The results obtained do not verify the assumption of plane sections remaining plane on bending, as at loads below 66% of the ultimate, the measured strain on the upper and lower surfaces was greater than is required by this hypothesis. Unfortunately however, the evidence is limited.

12. The measured tensile strain capacity of the concrete was $93 \times 10^{-6}$, taking a strain sensitivity factor of 2.1. This agrees closely with the values obtained in the simple tension tests on this concrete (Chapter 13).

13. Gauge No.9 gave consistently low readings until 75% of the ultimate, when it broke away from the concrete surface. Inefficient sticking to the rather poor surface was no doubt responsible for this.
14. Failure occurred on the section of gauge No. 13, and the high reading of this gauge just before failure, is probably due to local yielding at this section prior to the fracture.

15. A measure of the amount of creep in the concrete under load, is obtained from the release of load and increase of deflection, which occurred after the initial application of the load. (See Graph 15.8)

It may be seen that creep became noticeable above the load to which the beam had previously been subjected. Just prior to failure, the load relaxation amounted to $2\frac{1}{3}$ and the increased deflection to $1\frac{3}{4}$ over a period of 5 minutes.

15.5.3 Deflection Curves

The deflection curves obtained, are considered to be a failure. Whilst the results plotted graphically are smooth curves, the magnitude of the deflections is very much greater than that calculated from the elastic formula using a reasonable value for Young's Modulus. For example, allowing for errors at low loads, the measured deflections for both beams, are of the order of six times the calculated values. The difference is almost certainly due to the extra deflection obtained by the rotation of the beam on the half roller supports. The
necessity for measuring deflections with instruments situated on the beam, is now fully appreciated.

15.5.4 The Calculation of Maximum Tensile Stress

Further information on the behaviour of a plain concrete beam at various loads, can be obtained by endeavouring to balance the internal forces in the beam, calculating the moment of resistance, and equating to the applied bending moment.

At low loads it has been shown, in both the tension and compression tests, that stress is proportional to strain. In Beam Test No.5, the tensile strains are smaller than the compressive strains at equal distances from the neutral axis, which is slightly above the mid-depth of the beam. The tensile modulus 'E_t' must therefore be greater than the compressive modulus 'E_c' to obtain a balance of forces, which agrees with the Author's tests on tension and compression specimens. On the other hand, if plane sections are assumed to remain plane, and the neutral axis is high, then the tension modulus must be less than the compression modulus. A.M. Johnson (34) and R.H. Evans (23) have found E_t < E_c in beam tests, but Prof. Moorsch (44) indicated that 'E_t' may sometimes be slightly higher than 'E_c'. Evans agreed also, that E_t/E_c varies between 0.85 and 1.1. The assumption of plane sections remaining plane is therefore in some doubt.
On the Navier assumption, various writers have suggested forms of stress distribution at the ultimate load, which enables equilibrium conditions to be satisfied, and a value for the ultimate tensile stress to be obtained. Evans (26) for example, has evaluated the U.T.S. for the concrete of a beam assuming a straight line, a hyperbola, a parabola, a cubic parabola, a quarter ellipse, and a fifth degree parabola for the stress distribution, in an effort to obtain a figure which agreed with his value in simple tension. The only rational equation put forward is by Turner and Davies (56), who suggest that, as the compressive stresses are low, the stress distribution on the compression side is linear. In tension, the curve will vary as for a simple tension test. To maintain equilibrium, the neutral axis will rise from its original position. Using an equation of the same form as their tensile test curves, the ratio of the U.T.S. calculated elastically to that obtained by this theory, was 1.24.

A linear stress distribution gives results which, according to Moench (44) are 2.2 times those obtained in simple tension, according to Gommerman and Shuman (31) vary between 1.5 and 1.8 and according to Evans (25) vary between 1.3 and 1.6. The Author's experiments give 1.4 to 1.5.

It appears therefore that, until it has been established very much more conclusively that plane sections do remain plane until fracture, an empirical value of about 1.4 should be used
to relate tension test to beam test results.

The Author's experiments in Beam Test No.5, throw considerable doubt on the assumption that plane sections remain plane, especially at high loads where the neutral axis clearly rises. It would be a very valuable contribution if this fundamental point could be established by using the new electrical strain gauges, which provide an opportunity for accurate strain measurement not previously available.

15.6 GENERAL CONCLUSIONS ON THE BEAM TESTS

1. Repeated loading, some eight times, renders strain proportional to applied load at every layer in the depth of the beam until the load exceeds the maximum to which it has previously been subjected. This has been established for loads of 66% of the ultimate. The neutral axis therefore remains in a fixed position during this stage, and it was found to be a distance of \(0.47 \times \) depth of the beam from the compression surface.

2. At loads above the maximum to which the beam has previously been subjected, the neutral axis rises. For the beam tested, the position of the neutral axis just before fracture, was about \(0.4 \times \) depth of the beam from the compression surface. The rise of the axis agrees with the theory put forward by Turner and Davies (56) and
disagrees with Evans (23).

3. It is considered to be extremely doubtful whether plane sections do remain plane on bending. The strain distribution at loads well below the ultimate was found to be concave to the neutral axis on the compression and tension sides of the beam. At loads near the ultimate, the distribution was concave to the neutral axis on the compression side, and convex to the axis on the tension side. This disagrees, completely, with Evans (23) and Goldbeck (23).

4. At the present time there appears to be no method of obtaining the ultimate tensile strength of concrete from the result of a flexural test. Variation in the ratio between the modulus of rupture and the tensile strength is thus inevitable. An empirical value of 1.4 is suggested by these experiments.

5. The tensile strain capacity of the concrete is the same in bending tests as in the simple tension tests, viz. $90 \times 10^{-6}$ for the concrete used. This is in agreement with Evans (25).

6. Creep of the concrete becomes important at loads above that to which the beam has previously been subjected.
Just prior to failure, a reduction of load of approximately 2% will give stable conditions over 5 mins.

7. As both fractures occurred away from the centre of the beam, and also away from the leading points, the bending moment is considered to be quite uniform over the central 15 inch section.

Articulated half-roller supports would be an improvement on those used, providing that no end constraint was offered to the bending. The method of measuring the load was quite satisfactory and the rubber pads assisted in avoiding eccentric loading.

8. The electrical resistance strain gauge gives a more sensitive method of measuring the strain distribution on concrete beams than has been available in the past, and hence the disagreement with the findings of earlier experimenters is not surprising.
16.1 INTRODUCTION

The machine was designed in 1945 by the author and his coadjutor in order to carry out research on the strength of concrete under various combinations of bending and torsional loading. So far as was known there was no published work on this topic, and the principle of operation of machines which had been used in the past for testing metals under this type of loading, as for example, the Coker Machine (14) would have been expensive if used for a testing machine suitable for large concrete specimens. Subsequently, work in Stockholm by Nylander (46) on the "Torsion and Torsional Restraint of Concrete Structures", was published in September, 1945 and described experiments on the effect of combined bending and torsion on reinforced-concrete rectangular and tee-sections. The testing arrangements are shown in Fig. 16.1, and appear to be more extravagant and complicated than the machine designed by the author, a description of which is given below.
Testing Arrangements for Combined Bending and Torsion Tests

By
A.H. Nylander

Fig. No. 161
16.2 SPECIFICATIONS

The original specifications were dictated by the following considerations:

(1) It had been decided to test specimens of circular cross-section in the first instance as, under torsional loading, this shape was most convenient for theoretical consideration.

(2) A suitable wooden mould was available for producing specimens having prismatic ends of 8 inch square cross-section which, by a smooth transition, gave way to a 50 inch cylindrical section of \( \frac{7}{8} \) inches diameter. (See Fig. 17.1)

(3) It was estimated that to fracture a plain concrete specimen of these dimensions, would require a torque of 40,000 lb.in. The machine, therefore, must incorporate a means of measuring this torque as accurately as possible.

(4) It must be easily adaptable to take various shapes of specimen of approximately the dimensions given above and, owing to limited accommodation, it had to be collapsible.

Accordingly, after careful consideration, the principle of operation of the machine, as shown in Fig. 16.2, was decided upon. It will be noted that this arrangement enables a pure torque and a pure bending moment to be applied simultaneously to the specimen, without undesirable shear force. Also the weight of the specimen itself does not induce additional bending moment.
THE AUTHOR’S MACHINE FOR COMBINED BENDING AND TORSION

Fig. No. 162
16.3 CONSTRUCTION

The mould was extended to give 8 inch cubical ends to the specimen, and, in the plane of bending, at each end of the specimen, two 7\(\frac{1}{2}\) inch lengths of 1 inch bore gas piping were cast in-situ. Through this piping \(\frac{1}{8}\) inch bolts were inserted for the rigid attachment of the steelwork to the specimen.

New rolled steel sections were extremely difficult to obtain at the time owing to the immediate post-war difficulties. However, this problem was overcome by purchasing a "Morrison Table Type Air Raid Shelter" which provided all the steel sections required.

The footing consisted of two 6" x 6" x \(\frac{5}{8}\)" equal angle sections 2'6" long which, by means of cross bolts, gripped two sides of the specimen and could be firmly fixed to the floor by four 1" bolts screwed into Rawlplug foundation-screwed sockets. The specimen was held in the other direction by four cleats. These were attached to the specimen by bolts through the gas-piping, and to the footing, by two bolts. To provide an anchor for downward vertical loading, two 3" x \(2\frac{1}{2}\)" x \(\frac{1}{2}\)" x \(\frac{1}{2}\)" long extension arms were also bolted to the footing (See Plate 16.1).

The two leading arms were of 3" x \(2\frac{1}{2}\)" x \(\frac{1}{2}\)" angle section 6 ft long. These gripped the top of the specimen by means of cleats, in the same manner as was used at the footing. Two
gas pipe spacers were forced between the two arms at points an equal distance on each side of the specimen, and 65 inches apart. To obtain torsional and/or bending actions, the loads were applied horizontally and vertically respectively, to the loading arms through the gas pipe spacers.

The loads were applied by using a turnbuckle connected in series with a proving ring and a rigid support, by \( \frac{3}{4} \) in. steel rods. The turnbuckle was of novel design consisting of a thrust ball-bearing, welded into one end of a 9 in. length of gas pipe, and a fine-pitched screwed rod working in a nut at the other. This gave a very smooth application of load.

Three proving rings of \( 7\frac{5}{8} \) in. O.D. \times 6\( \frac{3}{8} \) in. I.D. \times \( \frac{5}{8} \) in. thick mild steel, were specially made in the College workshops for the purpose. They were found to have almost equal linear load-deflection characteristics of 4.36 lb. per 1/10,000 inch diametral change, over a load range of 0 to 600 lb. Their safe capacity is about 1000 lb. Further details are to be found in the companion thesis (26). When bending and torsional load was required simultaneously, dead weights were suspended on a hanger for the downward load. The anchorage for the horizontal loading was provided by two heavy brackets suitably positioned (See Plate 19.1). The upward load was carried by a convenient rolled steel joist overhead.
16.4 OPERATION

The machine required four operators to maintain the appropriate dial gauge readings on the proving rings.

The normal operational procedure was to apply the smaller action first, and then maintaining this constant, the other action was gradually increased until failure. To avoid direct forces on the specimen, the loads of a pair must always be kept equal. This was achieved by verbal contact between the two operators concerned, and it was an easy matter to keep the readings on the two dial gauges equal to within a half of one division, i.e. to within 2 lb.

With the existing proving rings, the capacity of the machine is 65,000 lb.in. bending or twisting moment, although as far as the steel-work is concerned, this could be considerably increased. The accuracy of measurement was ± 100 lb.in.

16.5 OBSERVATIONS

The apparatus was used successfully for a large number of strength tests carried out by the author.

It was found desirable to reinforce the upper and lower changes of section of the specimen against bending failure. This was done by using two straight 6 inch lengths of ½" steel rod placed, one on the compression and one on the tension side of the specimen, and extending approximately equally on each side of the change of section.
The results of electrical and mechanical strain measurements made on a specimen in this machine are given in the following chapters.
CHAPTER 17

TORSION TESTS

17.1 INTRODUCTION

Except in the case of reinforced-concrete screw-piles, concrete is rarely deliberately put into torsion. However, longitudinally reinforced concrete beams are often eccentrically loaded, as a result of which they are in torsion. The knowledge of the torsional stiffness of such a section is therefore valuable. The design factor of the concrete, which enables this to be calculated, is the Shear Modulus of Elasticity and, as the stress and strain distribution in a rectangular section in torsion is complicated by plane sections not remaining plane, it is best determined from a torsion test on a circular specimen.

During the torsion of such a section, the shear stresses induce tensile and compressive stresses on planes at 45 degrees to the axis of twist. Concrete, however, has a lower strength in tension than in shear, so that a tension failure occurs on a 45° helix. From a knowledge of the ultimate torque, it should
therefore be possible to evaluate the ultimate tensile strength of the concrete. An observation on this topic is given in the companion thesis (26). In the tests to be described, special reference is made to the various methods of carrying out torsion tests and of measuring the shear strains. New methods have been used in tests carried out by the author.

17.2 HISTORICAL TESTS

The problem of calculating the ultimate torsional strength of a plain or reinforced-concrete section from a knowledge of the ultimate tensile strength of the concrete, has engaged the attention of numerous experimenters since the work of Prof. E. Heorsch (45) (Stuttgart 1903) and is dealt with in the companion thesis (26).

The torsional stiffness of the sections has been obtained by most of these experimenters by measuring the angle of twist over a given length. Experiments by T. Miyamoto (42) (Tokio 1926), Leslie Turner and V.C. Davies (56) (London 1926-28), Paul Anderson (1) (U.S.A. 1935), F.M. Russell (50), W.T. Marshall and N.R. Tombe (39) (London 1940) are amongst the most interesting on this topic.

Experiments by Takanosuke Miyamoto (42)

An exhaustive series of torsion tests were carried out in the Japanese Home Dept. Laboratories, Tokio 1926, on sixteen plain and seventy-eight reinforced-concrete circular sections.
These were of 1.5 metres overall length, and 30 cms. diameter over the 30 cms. central portion, and had enlarged prismatic ends 40 x 40 x 15 cms. The Portland Cement concrete was of 1 : 2 : 4 mix, having a water-dry aggregates ratio of 9%, and a maximum size of aggregate of \( \frac{3}{8} \) inch. The specimens were damp-cured and tested at 45 days.

During the tests, which were carried out in a conventional type of horizontal torsion machine, the angle of twist was observed over the central 50 cms. using mirrors, telescopes and scales. From the ninety-four specimens tested, many of which had spiral or hooped reinforcement, it was concluded that:

(a) The failure of plain and reinforced sections followed, without exception, 45 degree helices.

(b) The shear modulus of elasticity decreased as the stress intensity increased, and was initially \( 1.24 \times 10^6 \text{lb/in}^2 \)

(c) Reinforcement, irrespective of the type, had little influence on the stiffness of a section as compared with that of a plain section.

Experiments by Turner and Davies (56)

A series of tests were carried out at the Battersea Polytechnic, London in 1926-28, on eight square, two rectangular and two tee-shaped plain and reinforced-concrete sections of 1 : 2 : 4 mix, using crushed gravel graded \( \frac{3}{8} \) inch downwards,
and Rapid Hardening Portland Cement. The specimens were
water-cured and tested wet at 35 days. The square sections were
4 ft. overall length with a 5 x 5 inch working section. A
standard horizontal Olsen 60,000 lb.in. capacity torsion machine
was used with special end attachments. The angle of twist was
measured over a 30 inch length by noting the relative movement
of two projecting arms which were attached to the specimen by
clamps. The movement of the arms was measured by using the
eye-piece scale of a microscope attached to one of the clamps.
Torque-twist curves for plain and reinforced-sections are
shown in Graph 17.1. From a study of these it may be concluded
that :=

(a) On first loading, Hooke's Law holds to about 50% of the
ultimate load.

(b) The effect of reinforcement does not greatly affect the
stiffness of the section compared with that of a plain
section.

(c) Re-loading gives elastic conditions until the maximum load
previously applied, is reached.

(d) The value of the Shear Modulus 'C' was 1.775 x 10^6 lb/in^2
giving a ratio \( \frac{C}{E_t} = 0.334 \), where \( E_t \) = Young's Modulus
in tension.
Experiments by Paul Anderson (1).

Tests were carried out on six plain and forty-two reinforced concrete sections in 1935. The former were of 28 inches overall length, with a central section 14 inches long of 8 inches diameter. The ends were of 10 inch square section and were lightly reinforced to prevent failure. The Portland Cement concrete was machine-mixed, and gravel graded ½ to 1 inch was used. Specimens were moist-cured for 28 days before testing. A standard type of horizontal torsion testing machine of 250,000 lb.in. capacity was used.

The angular distortion was measured by noting the deflection between two frames, 16 inches apart, with the aid of an attached connecting bar and screw micrometer. The principal strains were also measured with 2 inch Berry strain gauges and extensometers. Steel plugs for the extensometers, and drilled holes for the Berry gauges, were made in the specimens. The gauges were placed in the 45 degree directions on three sides of the square reinforced sections, and two extensometers on the fourth side.

The relevant conclusions were :-

(a) The torque-twist curves for plain specimens were substantially linear.
(b) The value of the Shear Modulus 'G' was found to increase with the compressive strength and the ultimate tensile strength of the concrete. e.g.

<table>
<thead>
<tr>
<th>Compression Cylinder Strength (lb/in²)</th>
<th>Ultimate Tensile Strength (lb/in²)</th>
<th>Shear Modulus 'G' (lb/in²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2000</td>
<td>250</td>
<td>1.42 x 10⁶</td>
</tr>
<tr>
<td>5200</td>
<td>420</td>
<td>2.34 x 10⁶</td>
</tr>
</tbody>
</table>

TABLE 17.1 Properties of Concrete in Tests by Anderson.

(c) A 50% difference was observed between the tensile and compressive extensometer strains. As the principal stresses must be equal, Anderson assumed that the concrete must have different values of Poisson's Ratio, or different elastic moduli, in tension and compression (See Graph 17.1).

Experiments by F.M. Russell (50)

Tests on specimens loaded to between 40 and 60% of the ultimate at 7, 28, and 90 days, and restored to water after each test, showed an increase in the Shear Modulus 'G' on each occasion. There was no apparent effect on the ultimate strength due to the partial loading. Other tests showed that loading and unloading twice before loading to fracture, does not affect the elastic properties.
A series of torsion tests on plain and reinforced-concrete of circular, rectangular, and tee sections, were carried out at the City and Guilds College, London in 1940. The concrete was of 1:2:4 mix, using Rapid Hardening Portland Cement with a water-cement ratio of 0.55. All specimens were water-cured.

The circular specimens were of 3ft. 6in. overall length, 5 in. diameter, and a 30 inch central section with 6 inch cubical ends. The change of section was lightly reinforced to prevent local failure at the ends. The testing machine was of a conventional horizontal type, with a wheel and rope torque measuring device. The angles of twist were measured during the tests with mirrors, telescopes and scales, and for the circular specimens, these were taken over a 20 inch length.

From ten tests on plain circular specimens at ages of 7, 28, 60 and 90 days, the torque-twist curves indicated a linear relationship to 90% of the ultimate load. The mean value of the Modulus of Rigidity was $2.3 \times 10^6$ lb/in$^2$, which would increase slightly with the age of the concrete. The times for the tests were less than 15 minutes. It is therefore considered that, in practice, to allow for creep under continuous loading a value of $1.8 \times 10^6$ lb/in$^2$ should be used.

Reinforced-concrete beams showed elastic behaviour to between $\frac{1}{2}$ and $\frac{3}{4}$ of the ultimate load, and during this stage,
the stiffness was almost the same as for a plain concrete beam. This indicated that, during the early part of the test, the concrete alone was resisting the torque.

It is suggested that concrete behaves elastically until the stress reaches the U.T.S. when a re-distribution occurs until failure.

17.3 THE AUTHOR'S TORSION TESTS

Objects of the Experiments

The objective was to obtain the initial Shear Modulus for the concrete by twisting a circular specimen and measuring the strains by the conventional angle of distortion method, and also directly, by using electrical strain gauges recording the principal tensile and compressive strains induced by the complementary shear stresses.

Outline of the Experiments.

The 7\frac{1}{2} inch dia. solid circular specimen was 6 weeks old when first tested, and had been stored dry after an initial wet-curing of 5 days damp, followed by 12 days in water.

Torsion Test No.1 was carried out on the virgin specimen and strain measurements were taken on four electrical gauges, placed in pairs, at opposite ends of a diameter, and arranged at 45 degrees to the axis of the specimen, to record the tensile and compressive strains. The maximum torque applied was approximately 40% of the ultimate torsional strength of
the section.

When the specimen was eight weeks old, after a bending test to approximately 45% of its ultimate bending strength, Torsion Test No. 2 was carried out. This was identical with Test No. 1 except that a mechanical measurement of the stiffness of the section was also made.

One week later, after two further bending tests to 45% of the ultimate bending strength had taken place, Torsion Tests Nos. 3, 4 and 5 were made to 40% of the ultimate torsional strength, when only mechanical measurements of the distortion were made.

DETAILS OF TESTS

17.4 TESTING APPARATUS

The 'G' Specimen.

The specimen was of 7½ inches diameter over a working length of 50 inches, and had cubical ends 8 inches square for loading purposes, as described in Para. 16.5. The change of section at each end was reinforced on the tension and compression sides, by two straight steel rods extending approximately three inches on either side. Two gas pipes of one inch bore were cast in each end transversely to the axis of the specimen and in the plane of bending (See Fig. 17.1).

Casting was carried out in a wooden moulding box, which
FIG. 17.1  Specimen 'G', showing dimensions and gauge positions.
was made in two longitudinal sections bolted together. With the bore completely blocked with a ram, one end of the mould was filled, the reinforcement and gas pipes inserted, and the concrete well tamped. A sheet of wood was then screwed over this end of the mould and the box inverted. The ram was removed, and the bore filled in layers not thicker than 4 inches. Each layer was well tamped by hand with a 6 inch diameter steel disc mounted centrally on a steel rod.

Twelve hours after casting, the specimen was released from the mould but not otherwise moved, and was covered with damp sacks until it was strong enough to be handled at about 5 days. It was subsequently water and air-cured. The weight of the specimen was approximately 216 lb.

The Testing Machine

The combined bending and torsion machine as described in Chapter 16 was used.

The Deflectometer.

The rig-up was improvised, and consisted of two small pieces of tin-sheet fixed with plasticine approximately 20 inches apart, normal to the surface, in line, and parallel with the axis of the specimen. A rigid stand, well ballasted with weights, was brought as close as possible to the specimen, and on it, on short firm clamps, two 1/10,000 inch plunger type dial gauges were mounted to press against the metal pads
provided. As the difference between the two gauge readings and the radius of the plungers was known, the angle of twist over a known length could be evaluated.

There is an objection to this method in that bending distortion would also cause deflection readings, but at the time these tests were carried out, bending was not thought to be caused by loading in torsion.

The Multi-Channel Strain Gauge Set.

The set, as described in Chapter 7 was used in conjunction with the Cambridge Galvanometer.

The Strain Gauges

1G Specimen.

Originally six 200 ohm, 1 inch long gauges Nos. 1 to 6 inclusive, were affixed to the specimen at suitably smooth portions of the surface, by the method described in Para. 10.5. They were arranged in two rosettes on opposite ends of a diameter in the plane of bending. The gauges were orientated at zero, 45° and 135° to the axis of the specimen (See Plate 17.1). The axial gauges Nos. 1 and 6 were uncertain in operation in the first test, and were replaced by fresh gauges, and supplemented by two new ones, Nos. 7 and 8, placed vertically below them. In spite of this, gauge No. 1 failed to record in the final test of the specimen to destruction.

The positions of all gauges are shown in Fig. 17.1.
Testing Procedure.

The specimen was fitted into the machine and was left undisturbed except for the leading arms which were removed after each test.

The gauges were carefully covered with a thick cartridge paper to prevent damage, and remained connected to the strain gauge set. The dummy gauges were attached to a beam specimen and were placed as close as possible to the main specimen. The gauges on both were completely shrouded with a heavy black cloth and, as far as possible, were left undisturbed. Before use, the set was allowed to warm up for at least one hour.

The deflectometer rig was fitted up as required. A minimum of three operators was necessary for the tests in which electrical readings were taken.

Observations and Calculations

The readings for the several tests, together with the calculations, are given in Appendices XXXVIII to XLII. Graphical presentation of the readings is given in Graphs 17.2, 17.3 and 17.4.
17.5 RESULTS AND COMMENTS ON THE INDIVIDUAL TESTS

Tests Nos. 1 and 2

1. Torsion Test No. 1 was on the virgin specimen, and the individual gauges Nos. 2, 3, 4 and 5, showed that the strains were proportional to the load to 40% of the ultimate torque (See Graph 17.2)

A similar conclusion follows from Torsion Test No. 2 (See Graph 17.3)

2. The 'electrical' shear strains, which are given by the algebraic difference between the readings of the 45° gauges of a pair, were found to be the same for each pair, in both tests. (See Graphs 17.2 and 17.3)

The rate of electrical shear strain per 10,000 lb.in. torque was \(1.05 \times 10^{-4}\) from which:

Shear Modulus of Elasticity 'G' = \(2.4 \times 10^{6}\) lb/in\(^2\)

3. A remarkable feature of the tests was that the principal tensile strains at 45° to the axis of the specimen accompanying the state of shear, were about 70% greater than the corresponding compressive strains. (See Appendices XXXVIII and XXXIX) The slight difference between the two tension gauges and the two compression gauge readings, is most probably due to errors in the assumed angles of orientation of the gauges. These are correct to about \(\pm 2°\)
This is exactly the same phenomenon as was observed by Paul Anderson (1). However, it is difficult to imagine how the tensile and compressive strains can be different in a state of pure torsion, without there being a discontinuity in the material. Assuming that the difference in strains is possible, then as Young's Modulus in tension has been found in other tests to be greater than in compression, Poisson's Ratio in compression must be greater than in tension, but this is contrary to the results of the other tests.

It should be noted that the difference of strains cannot be caused by the loading, for even if the torsion loads were slightly unequal, the bending action resulting would not be recorded on the gauges, as they would be situated on or near to the neutral axis of the bending.

Unfortunately, the axial gauges Nos. 1 and 6 were not in operation during these tests, so that there is no check on the principal strains.

Tests Nos. 3, 4 and 5

4. The mechanical stress-strain curves show a linear relationship in all tests. (See Graph 17.4). It is probable that the deflections obtained in Test No.3 are different from the others, due to the movement of the specimen in the footing. A properly designed
torsionmeter attached to the specimen would have been more satisfactory, as bending deflections must have been measured in these tests. However, the consistency of the results of a particular test is not in doubt.

5. The average Shear Modulus from the mechanical tests, was found to be $2.1 \times 10^6\text{lb/in}^2$, which is of the same order as the values obtained electrically. However, it is probable that the deflections are slightly high due to movement of the specimen in the footing so that the true modulus would be greater than the above value.

17.6 GENERAL CONCLUSIONS ON THE TORSION TESTS

1. The average value of the Shear Modulus of Elasticity for this concrete at 6 - 8 weeks old was $2.4 \times 10^6\text{lb/in}^2$. This is a typical value for concrete of this strength, and it provides particularly useful information, as it has been shown by several experimenters, that it may be applied to stiffness calculations on longitudinally reinforced-concrete beams.

2. Hooke's law is obeyed over the range tested, that is from 0 - 40% of the ultimate strength, which agrees with the findings of Turner and Davies (56) and Marshall and Turnbe (39)
3. A 70% difference was observed between the principal tensile and compressive strains at 45° to the axis of the specimen accompanying a state of pure shear, which agrees with the findings of Paul Anderson (1). As this state of affairs is not compatible with the behaviour of a homogenous material, more evidence is required before the idea can be accepted as a general torsion phenomenon. The experiments carried out were quite definite on this point, however.

4. Torsional loading, repeated a few times to 40% of the ultimate with intermediate loading in bending to 45% of the bending strength, does not affect the elastic properties.

5. No creep under load was observed during the tests, each of which was completed in under 30 minutes.

6. It is necessary to reinforce, lightly, the change of section at the ends of the circular torsional specimens, to prevent failure here. This is in agreement with the findings of Paul Anderson (1) and Marshall and Tambe (39). Information on the best form of this reinforcement is given in the companion thesis (26).

Continued -
7. It would have been most interesting to have completed strain measurements in a pure torsion test to fracture, in order to determine the ultimate tensile strain but, owing to shortage of time, this was unfortunately not possible. The results obtained to 40% of the ultimate are, however, the more significant as they are virtually within the elastic range of the material, and are not based on observations when the specimen is deforming rapidly.
CHAPTER 18

BENDING TESTS ON A CIRCULAR SECTION

18.1 INTRODUCTION

Bending tests on concrete specimens of circular cross-section are unusual, although this is a standard shape for transverse strength tests on cast iron, which is another brittle material. So far as is known, there was no record of any such experiments on concrete. Tests by the author are described below.

Object of the Experiments.

The object was to determine the 'Modulus of Bending' of the concrete, and also, to determine Poisson's Ratio in tension and compression, by measuring the strains in three known directions at given points on the specimen.

Outline of the Experiments

Specimen 'G' which was used for the torsion tests (See Chapter 17), was also used for this series of tests in the
combined bending and torsion machine.

Bending Test No. 1 was carried out when the specimen was seven weeks old, after it had been loaded to 40% of its ultimate torsional strength. During loading to approximately 45% of its ultimate bending strength, strain measurements were taken on two strain gauge rosettes, which were placed at opposite ends of a diameter in the plane of loading.

At an age of eight weeks, after a further torsion test to 40% of its ultimate strength had been made, Bending Test No. 2 was carried out to 45% of the ultimate bending load. The deflection of the specimen was measured, together with the principal bending strains.

Tests Nos. 3 and 4, in which only the deflections of the specimen were noted, took place one week after Test No. 2. After two more torsion tests to 40% of the ultimate, Bending Test No. 5 was carried out. The specimen was loaded to 50% of its ultimate bending strength and strain measurements on the rosettes, as well as the deflections of the specimen, were noted.

13.2 DETAILS OF THE TESTS

The testing apparatus and the testing procedure were identical with those used in the torsion tests and are fully described in para. 17:4
18.3 RESULTS AND COMMENTS ON THE INDIVIDUAL TESTS

1. From an electrical standpoint, Tests Nos. 1 and 2 were not very satisfactory (See Graph 18.1). In Test No.1, the 45° gauge No.5 on the tension side of the specimen failed to record, thus spoiling the rosette readings. On the compression side, the 45° gauges Nos. 2 and 3 behaved erratically, whilst axial gauge No.1 recorded smaller strains than were expected. The readings were not compatible with the normally accepted principle that Poisson's Ratio is negative.

In Test No.2, when only the axial gauges Nos. 1, 7, and 8 were in operation, the tension gauge failed to record, leaving the two gauges on the compression side showing equal values, as was expected. However, they did give an estimate of the strain rate probable for Test No.5

2. The Tests Nos. 3 and 4, in which the deflection over a 20.7 inch gauge length was measured, gave consistent readings, (See Graph 18.5), but calculations from them are not reliable, as small changes in the reading of the lower dial gauge make very large differences to the result. (See Appendices XLIV and XLVI). Better results would have been obtained by raising the position of the lower gauge so that larger readings would be indicated. Also, owing to the method of supporting the dial gauges, any
bodily shift of the specimen in its support during loading, would have been recorded as deflection. A deflectometer attached to the specimen would have avoided this difficulty. The measurements showed that the specimen was bending about a section in the footing approximately 2 inches below the clamping bolts. The Modulus of Bending from these results is approximately $3 \times 10^6$lb/in$^2$, but the value is unreliable for the reasons given above.

3. In Test No.5, the $45^\circ$ gauges on the compression side of the specimen showed equal strains, which were 30% greater than those of the corresponding gauges on the tension side. (See Graph 18.2).

The two axial tension gauges agreed to within 5%. The axial compression gauge No.1 failed to operate, and its companion gauge No.8 showed a change at about 20% of the ultimate load, that is, 3,000 lb.in. This change was also reflected in the $45^\circ$ compression gauges, and was probably due to a local breakdown which was followed by a reorganisation of the material. This however, dis-organised the absolute strain readings of the rosette, but initially, gauge no.8 behaved as in Test No.2 and, using this value, the direct compression strains were about equal to those in tension.
Evaluating Poisson's Ratio \( \nu \) from the relationship

\[ \epsilon_{45} = -\frac{a}{2}(1 - \nu), \]

\( \nu \) in tension was 0.35 and in compression 0.12.

4. Assuming that plane sections remain plane at loads below 40% of the ultimate, the "Modulus of Bending" from electrical readings of Test No.5 was \( 5.06 \times 10^6 \) lb/in\(^2\). This is reasonable, as it lies between the values obtained for the Young's Modulus of this concrete in tension and compression. The value, which is for a circular section may, however, be slightly different from that obtained in a bending test on a rectangular section. For example, Bach (4) in transverse bending tests on cast iron, gives the ratio of the modulus of rupture to the tensile strength as 2.12 for a circular section, and 1.73 and 1.75 for a square and rectangular section, respectively. Although the effect of shape will be greatest at loads near to fracture, some small effect may be expected at lower loads.

5. From the several tests it may be seen that Hooke's Law was obeyed to 40% of the ultimate bending strength. There appeared to be a small reduction in the Modulus above this load, which was confirmed by both the electrical and mechanical readings.
18.4 GENERAL CONCLUSIONS ON THE BENDING TESTS

1. Poisson's Ratio for this concrete, in the condition tested, was found to be 0.35 in tension, and 0.12 in compression. The tension value, in particular, is very reliable.

2. The bending 'Modulus of Elasticity' calculated on the assumption of plane sections remaining plane to 40% of the ultimate load, was, from the electrical readings, $5.05 \times 10^6\, \text{lb/in}^2$ which is in between the value obtained for Young's Modulus in the tension and compression tests on this concrete.

3. The deflection measurements must be made by instruments attached to the specimen.

4. Hooke's Law was obeyed by the concrete in bending to 40% of the ultimate bending strength. Previous torsional loading to 40% of the ultimate torsional strength apparently had no effect on the behaviour of the concrete in bending.
CHAPTER 19

THE COMBINED BENDING AND TORSION TEST

19.1 INTRODUCTION

A knowledge of the behaviour of a material under combined stresses is of the utmost value in design, since only rarely, is a structural member subjected to simple loading.

The available experimental evidence suggests that brittle materials fracture at a definite maximum principal stress, when this is a tension. Early tests were mostly on cast iron. Tests by Scoble (52) under combined bending and torsion, for example, showed agreement with the maximum principal stress theory. However, in 1913 a Committee set up by the British Association to investigate the problem of the complex stress distribution in engineering materials reported, (11), that "few experiments have been made with the materials now classed as brittle, and those already made, should mostly be repeated."

The solution to the problem for brittle materials has not progressed greatly since this date. So far as is known,
there have been only three major researches on stone or concrete under combined stresses, two of a fundamental and one of a practical nature.

The fundamental problem of the behaviour of plain concrete under the very practical type of loading of combined bending and torsion, which is of particular interest in reinforced-concrete engineering, had not been investigated. A series of tests, using this type of loading, were carried out by Mr. D. Fisher (26) and the author (1945-47). As a part of this programme, the test described in this chapter was made to investigate the deformation of the concrete under compound loading.

19.2 HISTORICAL TESTS.

Since 1913 experiments on stone or concrete under combined stresses, have been made by Kármán and Böker (Göttingen 1915); Richart, Brandtzaeg and Brown (Illinois 1928); and Nylander (Stockholm 1945).

Tests by Kármán and Böker (55) (see comment #)

A comprehensive series of tests, on a very uniform quality of marble under two conditions of stress, were undertaken at the University of Göttingen. There were:

1. Cylinders subjected to various combinations of axial load and lateral fluid pressure.
(11) Cylinders subjected to torsion in addition to axial load and lateral fluid pressure.

Tests by Richart, Brandtsaeg and Brown (47) (see comment *)

A large number of concrete cylinders were tested under two, and three, dimensional compression at the University of Illinois in 1923. The two dimensional compression was supplied by oil pressure.

* It should be noted, that in these tests, only the overall deformations along the axis of the specimens were measured, owing to the lateral fluid pressure apparatus. The strain measurements, therefore, were of limited value in explaining the behaviour of the material. A discussion on the conclusions from these tests with regard to the theories of failure, is given in the companion thesis (26).

Tests by Nylander (46)

As part of a very large number of experiments on the torsion and torsional restraint of concrete structures carried out in Stockholm in 1945, tests were made under combined bending and torsional loading on twenty-six reinforced-concrete beams. Ten were of 20 x 20 cms. square section, six of 9.5 x 20 cms. rectangular section, and ten of tee section 6.3 x 50 cms. flange by 9.5 x 20 cms. web. The beams had a working length of 100 cms. and were tested in the machine shown in Fig. 16.1. The concrete mix varied between the different series, a typical
one being 1 : 3.7 : 4.0 Portland cement, sand, gravel mix, with a water-cement ratio of 0.75. The tests were made at 28 days.

Deflection measurements were taken with Ames dial gauges and torsional rotations by using plane mirrors, scales and telescopes over a 100 cm. gauge length.

Rectangular and tee sections, with strong reinforcement, were tested to study the concrete failure; square and tee sections, with low or normal reinforcement, were tested to study the bending failure of the reinforcement when subjected to simultaneous torsion.

Interesting graphs (See Graph 19.1) were produced, showing the variation in the secant Shear Modulus of Elasticity with bending moment for four different twisting moments, and with twisting moment for three different bending moments. It appears, from the results of the four specimens tested, that the secant Shear Modulus diminishes by about 70% over the range of the applied bending moment for the given maximum twisting moment. The specimens were subjected to each load for 1 minute and the load was then completely removed for 2 minutes. Any effect from the repeated loading is neglected. Similarly, the secant Modulus diminishes by about 30% over the range of the applied torque for a given bending moment. However, it is noted that the conditions of these tests were all very similar, and were primarily a large bending moment,
plus a small torque of approximately 15\% of the ultimate torsional strength of the section.

19.3 THE AUTHOR'S COMBINED BENDING AND TORSION TEST

Object of the experiment.

The object was to determine the effect of an applied bending moment on the torsional stiffness of a plain concrete specimen of circular section. In addition by measuring the strains in three directions at a definite point on the specimen a measure of the tensile strain capacity of the concrete was to be made. This would give some indication as to whether the principal tensile strain is the criterion of the failure of concrete under combined stresses.

Outline of the experiments.

The 'G' specimen was tested in the combined bending and torsion machine at an age of nine weeks, after it had been used for the simple torsion and bending tests as described in Chapters 17 and 18.

The specimen was loaded by a pure bending moment to approximately 50\% of its ultimate bending strength, and then, with this bending moment maintained, it was subjected to gradually increasing twisting moments until fracture occurred. During loading, measurements were made on two electric strain gauge rosettes, placed at opposite ends of a diameter in the plane of bending. Deflection measurements were also taken.
over a gauge length of 20.7 in.

19.4 DETAILS OF THE TESTS.

The testing apparatus and testing procedure were identical with those used in the simple torsion and bending tests, and are fully described in Para. 17.4. Four operators were required to carry out this test.

Rate of Loading.

The bending moment was applied in twelve equal increments, and the torsional load in eighteen increments. Each load condition was maintained for approximately 5\(\frac{1}{2}\) minutes and, of this time, two minutes were required to read the seven electric gauges.

Observations and Calculations

The readings for the test, together with the calculations, are given in Appendices XLVII and XLVIII. Graphical presentation of the readings is given in Graphs 19.2 and 19.5

19.5 RESULTS AND COMMENTS ON THE TEST

1. The behaviour of the specimen during bending is discussed fully in Chapter 18. However, it should be noted that the four gauges on the tension side were extremely consistent, and their readings and calculations made from them are considered to be very reliable. The
45° compression gauges were not as consistent as those of the tension gauges, and the axial gauge No. 6 was indicating lower readings than was compatible with a rational value of Poisson's Ratio. This may have been due, in part, to inefficient sticking of the gauge. Calculations from the actual readings of the compression gauge group are, therefore, dependent on an assumed value of the axial gauge reading.

When torsion was applied to the specimen, the four 45° gauges all indicated the same rate of strain with torque to 66% of the ultimate torsional loading (See graph 19.2) This was contrary to the 70% difference between the tensile and compressive 45° strains observed in the pure torsion tests (Para. 17.5)

The Shear Modulus 0° evaluated from this rate, was 2.41 x 10^6 lb/in², which is the same as the value obtained in simple torsion.

For all practical purposes, Hooke's Law was obeyed during the bending and, as is shown by the torque-shear strain diagram, Graph 19.3, during torsion up to 85% of the ultimate load. A discontinuity in the material on the tension side of the specimen occurred at 66% of the ultimate torque, as indicated by the sudden change in the readings of the 45° gauges Nos. 4 and 5. However, this
was a purely local effect and did not affect the shear strains.

4. The two 45° gauges which, when the shear strain was applied, were subjected to strains in the same direction as during the bending, were reluctant to change their rate of strain on the first torsional load. The gauges, which suffered a change in the direction of strain, responded immediately to the shear strain. The phenomenon was probably hysteresis in the concrete, rather than in the gauges.

5. The gauge readings from the tension rosette, have been evaluated at a few typical loads to give the principal strains and also the value of Poisson's Ratio. (See Appendix XLVIII.)

Poisson's Ratio, in tension is clearly 0.35 and, as the ratio is very sensitive to small changes in the strain $\varepsilon_y$, the constancy of this value in the five calculations, is a measure of the accuracy of the strain gauge readings.

6. The gauges on the compression side of the specimen were very consistent during the torsional loading. A very slight increase in their rate of strain with load was recorded when the breakdown was commencing on the
tension side, which possibly indicated a rise in the neutral axis of bending.

Unfortunately, no calculations for the principal tensile strains on the compression side are reasonable, as the absolute value of the axial strain from gauge No. 8 was found to be low in the bending test. The 45° gauges on the compression side indicated an absolute shear strain which was slightly lower than that obtained from the tension group, but the rate of increase of shear strain with load was the same for both groups.

7. The release of load, due to creep, during the 3½ minutes under constant load, was first observed at a torque of 85% of the maximum. The phenomenon was a useful indication of imminent failure. The amount of the release as measured by the change in the reading of the torque proving rings, is shown in the Table 19.1. The bending moment was maintained constant throughout the test.

<table>
<thead>
<tr>
<th>Nominal Torque (lb.in)</th>
<th>Release of Torque (lb.in)</th>
</tr>
</thead>
<tbody>
<tr>
<td>19837</td>
<td>110</td>
</tr>
<tr>
<td>20971</td>
<td>230</td>
</tr>
<tr>
<td>22104</td>
<td>530</td>
</tr>
<tr>
<td>22954</td>
<td>1200</td>
</tr>
<tr>
<td>23804</td>
<td>Fracture</td>
</tr>
</tbody>
</table>

**TABLE 19.1** Effect of Creep - Author's Combined Bending and Torsion Test.
8. The deflection readings obtained were not very satisfactory in either the bending (See Graph 18.5), or in the torsional part of the test. This was due, no doubt, to the improvised nature of the deflectometer.

9. The maximum tensile strain recorded, was 67 x 10^-6 on a 45° gauge. Unfortunately, the ultimate principal tensile strain cannot be evaluated, as discontinuities in the material had disorganised the strains on the rosettes by that time. In the pure tension tests, (Chapter 13), the ultimate tensile strain was found to be 90 - 95 x 10^-6.

10. The strains on six of the seven gauges had been recorded before the specimen fractured under this load.

The photograph (Plate 19.2) of a specimen subjected to a similar combination of bending and twisting moments, shows the type of fracture obtained.

10.6 GENERAL CONCLUSIONS ON THE COMBINED BENDING AND TENSION TEST.

1. For all practical purposes Hooke's Law was obeyed in bending and to 85% of the ultimate torsional load. Breakdown commenced on the tension side at this load. This is contrary to the 30% reduction in the secant Shear Modulus found by Nylander(46) under a fixed bending moment. However, the previous loading history, the relative magnitudes of the bending and twisting moments,
and possibly, the direction of filling in the manufacture, affect this property.

The Shear Modulus of Elasticity was found to be constant at $2.41 \times 10^6$ lb/in$^2$ which is the same as that obtained in the pure torsion tests. Hence an applied bending moment of 50% of the ultimate bending strength has no effect on the torsional stiffness of a concrete section.

2. Poisson's Ratio for the concrete in tension was 0.35, which is about twice that observed previously for compression.

3. No creep of the concrete under load, was observed in the test until 85% of the maximum torsional load was reached, and this only appreciably affected the results just prior to fracture.

4. The specimen fractured by tension, along a helix at an angle modified from the 45° torsion angle by the bending moment.

5. Although the ultimate principal tensile strain could not be calculated from the results, it is probable that it was close to the value obtained in the simple tension tests on this concrete.
CHAPTER 20

COMPARISON OF THE RESULTS OF THE SEVERAL TESTS ON THE CONCRETE

20.1 INTRODUCTION

The ratios between the elastic moduli and between the ultimate strains obtained in the various tests, give particularly useful information about the concrete.

20.2 HISTORICAL TESTS

An extensive series of tests on concrete specimens was carried out by Gilkey and Vogt (28) at the Bureau of Reclamation, University of Colorado in 1954 in connection with the construction and the interpretation of results from tests on models of arch Dams.

The dimensions of the specimens were as follows:

(a) Compression Specimens were cylinders 3 in. dia. x 6 in high

(b) Beam Specimens were 3 in. x 3 in. x 40 in. loaded at the centre of a 58 in. span.

(c) Tensile specimens were cylinders 3 in. dia. x 12 in. long, with the ends clamped in spherically seated grips.
(d) Torsion specimens were 3 in. dia. x 12 in. long.

The concrete mixes were:

Series 1

1 : 3.25 by weight of cement to mixed aggregate of \( \frac{3}{8} \) in. maximum size of crushed granite; water-cement ratio = 1; slump = 8 in.

Series 2.

1 : 2\( \frac{1}{2} \) : 2 by weight of cement; 0 = 4 standard sieve sand; and \( \frac{3}{8} \) in to \( \frac{1}{2} \) in. pea gravel; water-cement ratio = 1; slump = 7 in.

The results given in Table 20.1 below, are the mean of at least five specimens for compression, and of two specimens for the other types of loading. All specimens were tested wet at 28 days after moist curing.

<table>
<thead>
<tr>
<th>Ratio of Moduli</th>
<th>Strain at 50% ultimate load</th>
<th>Strain at ultimate load</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>TorsoN compression</td>
<td>TorsoN bending</td>
</tr>
<tr>
<td>Series 1</td>
<td>0.38</td>
<td>0.54</td>
</tr>
<tr>
<td>Series 2</td>
<td>0.51</td>
<td>0.47</td>
</tr>
</tbody>
</table>

TABLE 20.1 Comparison of results of tests by Gilkey & Vogt (23)
The flexural modulus of elasticity was calculated on elastic assumptions from the deflection of the beams. Similarly, the flexural and torsional strains were calculated from the deflection measurements.

The relevant conclusions were:

(i) The ratio of the torsional to compression moduli is between 0.4 and 0.5, but the value for torsion to flexure, and torsion to tension is practically the same. This agrees with the general conclusion that the modulus of elasticity of concrete is sensibly the same in compression, tension and flexure.

(ii) The maximum torsional and the maximum tensile flexural deformations are roughly three times the maximum simple tensile deformations, but this is probably because only the outer surface is fully stressed.

20.3 THE AUTHOR'S RESULTS

The following specimens were roughly of the same age at test, and had been subjected to similar conditions before test:

Tension Specimen 'B', Compression Specimen 'D', Main Specimen 'G'
TABLE 20.2 Comparison of results of tests by the Author.

The above results are expressed as ratios of the torsional values for comparison with the experiments of Gilkey and Vogt (28) in Tables 20.3 and 20.4 below:–

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Age (Days)</th>
<th>Modulus of Elasticity ( \times 10^{-6} ) lb/in²</th>
<th>Strain at 50% ultimate load ( \times 10^6 )</th>
<th>Tensile Strain at Fracture ( \times 10^6 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tension 'B'</td>
<td>47</td>
<td>5.68</td>
<td>28</td>
<td>95</td>
</tr>
<tr>
<td>Compression 'D'</td>
<td>45</td>
<td>4.4</td>
<td>comp. 330</td>
<td>1550</td>
</tr>
<tr>
<td>Bending 'G'</td>
<td>63</td>
<td>5.06</td>
<td>38</td>
<td>-</td>
</tr>
<tr>
<td>Torsion 'G'</td>
<td>56</td>
<td>2.4</td>
<td>comp. 23</td>
<td>-</td>
</tr>
</tbody>
</table>

TABLE 20.3 Comparison of Moduli of Elasticity from Author's Tests.

<table>
<thead>
<tr>
<th>Torsion Tension</th>
<th>Torsion Bending</th>
<th>Torsion Compression</th>
<th>Torsion Comp(Tensile)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.55</td>
<td>0.47</td>
<td>0.42</td>
<td></td>
</tr>
</tbody>
</table>

TABLE 20.4 Comparison of the Strains at 50% ultimate load from Author's Tests.

<table>
<thead>
<tr>
<th>Torsion Tension</th>
<th>Torsion Bending</th>
<th>Torsion Compression</th>
<th>Torsion Comp(Tensile)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.7</td>
<td>2.0</td>
<td>0.20</td>
<td>1.6</td>
</tr>
</tbody>
</table>
COMMENTS.

1. Comparing these ratios with those of Gilkey and Vogt, it can be seen that they are in general agreement.

2. The torsional deformation at 50% of the ultimate torsional load, is about twice the tensile strain obtained at a similar load in bending, tension and compression.

3. The Ratio of $G/E$ cannot be greater than 0.5 for rational values of Poisson's Ratio ($\nu$), so that the ratio of 0.55 between the torsional and compressive moduli must indicate that either the compression modulus is low or the torsion modulus is high. However, it is probable that the ratio $G/E$ is close to 0.5 so that the average Poisson's Ratio for the concrete is small.

4. The ratio between the tensile and compressive moduli $\frac{E_t}{E_c} = 1.29$, which is larger than has been found by other experimenters. For example, Evans (25), states that the ratio may vary from 0.65 to 1.1. However, as the strains were measured in both tests by mechanical and electrical methods, the ratio must stand.

5. The ultimate tensile strains at failure obtained in all tests, are given in Table 20.5 overleaf.
<table>
<thead>
<tr>
<th>Specimen</th>
<th>Age(Days)</th>
<th>Ultimate Tensile Strain $x 10^6$</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tension 'A'</td>
<td>550</td>
<td>90.95</td>
<td>Allow for eccentricity of loading</td>
</tr>
<tr>
<td>Tension 'B'</td>
<td>47</td>
<td>90.95</td>
<td></td>
</tr>
<tr>
<td>Compression'D'</td>
<td>45</td>
<td>550</td>
<td>Transverse to load.</td>
</tr>
<tr>
<td>Beam 'E'</td>
<td>225</td>
<td>143</td>
<td></td>
</tr>
<tr>
<td>Beam 'F'</td>
<td>225</td>
<td>93</td>
<td></td>
</tr>
<tr>
<td>Main 'G'</td>
<td>63</td>
<td>67</td>
<td>Max. recorded under combined bending and torsion. (Principal Strains not obtainable at failure.)</td>
</tr>
</tbody>
</table>

**TABLE 20.3** Ultimate Strain capacity of the concrete in the Author's tests.

The tensile strain capacity of the concrete in the beam tests was practically the same as under simple tensile loading, but the tensile strains accompanying direct compressive loading were about six times those in simple tension before failure occurred.

Under combined bending and torsional loading it is probable that the tensile strain capacity is the same as in simple tension. If this is so, then this may well be the criterion of failure of the concrete.
In the foregoing chapters an effort has been made to determine the strain characteristics of a 1:2:4 mix concrete, and to correlate the results of measurements made on specimens loaded in the four fundamental manners of tension, compression, bending and torsion, as well as with torsion combined with bending. This has involved a thorough investigation into the possibilities of electrical resistance strain gauges on concrete and has in some cases, resulted in a re-statement of well-known principles simply for the sake of obtaining a new viewpoint.

Whilst some of the conclusions which follow, must be looked upon as tentative until verified by further and more detailed experiments, yet the indications are such that the facts in the following summary of the important phenomena may be regarded as fairly well established.
THE CONCRETE

1. The equation to the tensile and compressive stress-strain curves is a parabola of the form:

\[ f = \text{U.T.S.} - \text{constant (ultimate strain } - e)^n \]

where 'f' is the stress for strain 'e'.

In Tension, index 'n' was found to be 1.15 and 1.5 for concrete 45 and 600 days old respectively, and which had been previously loaded two or three times to 50% of the ultimate.

In Compression, index 'n' was found to be 2.0 for concrete 45 days old which had been previously loaded twice to 40% of the ultimate.

2. Young's Modulus of Elasticity for concrete 45 days old, and previously loaded twice to about 45% of its ultimate strength, is 5.55 \( \times 10^6 \)lb/in\(^2\) in tension, and 4.4 \( \times 10^6 \)lb/in\(^2\) in compression. Hooke's Law holds closely over the load range 0 - 45% of the ultimate.

3. The Shear Modulus of Elasticity of concrete at 50 days is 2.4 \( \times 10^3 \)lb/in\(^2\), and Hooke's Law is obeyed from 0 - 40% of the ultimate.

4. An applied bending moment of 50% of the ultimate bending strength does not affect the torsional stiffness of a concrete section, and Hooke's Law is obeyed under
5. Poisson's Ratio for the 45 day old concrete in compression increases from 0.120 to 0.142 over the load range from 0 - 87% of the ultimate. Above this load, owing to the development of discontinuities within the material, the apparent Poisson's Ratio increases rapidly to 0.32 just before failure.

   In tension, from the results of bending tests, the ratio is 0.35 over the load range from 0 - 45% of the ultimate bending strength.

6. From both simple tension and bending tests the tensile strain capacity of the concrete is 90 to 95 x 10^-6.

   It is probable that under combined bending and torsional stresses, the ultimate principal tensile strain is the same as in simple tension, so that this may well be the criterion of failure under compound loading.

   The ultimate tensile strains accompanying compressive loading are about 6 times those in simple tension.

7. In a plain rectangular concrete beam having a depth to breadth ratio of 2, the position of the neutral axis of bending is close to the mid-depth of the beam for loads up to 70% of the ultimate, but above this load it rises to about 0.4 of the depth from the compression surface just prior to fracture.
The distribution of strain in the depth of the beam is not perfectly linear even at low loads, so that plane sections do not remain plane on bending. In the limited tests carried out, the strains on the compression and tension surfaces were greater than was required for a linear strain distribution.

At loads near to fracture, the strain distribution curve was found to be concave to the neutral axis on the compression side and convex to this axis on the tension side, thus indicating plasticity in the concrete in tension.

8. The ratio of the bending modulus of rupture to the ultimate tensile strength of the concrete is about 1.4.

9. The effect of repeated loading of the concrete a few times is that Hooke's Law is obeyed until the load exceeds the maximum to which it has previously been subjected. Above this load the elastic modulus diminishes. This phenomenon has been observed in tension, compression and bending tests.

10. The loading of a circular concrete section to 45% of its ultimate bending strength followed by unloading, does not affect the torsional stiffness.

11. Creep of the concrete under load is not important below 70% of the ultimate load over periods of up to half-an-hour, in compression or bending tests.
Concrete in compression undergoes three distinct changes before fracture. These are an elastic stage to about 50% of the ultimate, followed by a semi-plastic stage to 90% of the ultimate, when a discontinuity in the material precipitates fracture.

**TESTING ARRANGEMENTS**

The author's combined bending and torsion machine for testing concrete specimens of various shapes of cross-section under pure bending, pure torsion, or in any combination of bending and torsion, was found to be very satisfactory in operation. It offers a relatively simple and cheap method for carrying out more comprehensive researches into the behaviour and strength of concrete.

The set-up used for compression tests may give an eccentricity of loading sufficient to cause a 20% reduction in the ultimate strength. This is confirmed in recent researches, which showed that the machine used does give low results in compression tests.

The eccentricity of loading in compression tests is not appreciably affected by packing of the cardboard type, but it may be reduced by introducing thin rubber sheeting between the platens of the machine and the specimen.
16. The hoop strains at the mid-height of a compression cylinder, of height equal to twice the diameter for loads up to 50% of the ultimate, are, when millboard packing is used, about five times those at the ends.

17. The loading point in the set-up used for tension tests may be only just within the middle third rhombus of the specimen, even when the latter appears to be satisfactorily positioned. This was probably the reason for the variation in the ultimate tensile strength results obtained in tests on this concrete by the author and his coadjutor (26).

It is suggested that, to make a tensile test with this arrangement, a few strain measurements should be taken at low loads, and in the light of these, suitable adjustments made to the shackles in order to correct the eccentricity. Alternatively, a device to centre the specimen in the claw grips should be developed.

18. The use of proving rings for measuring the load in beam testing is a convenient method of obtaining a continuous record of the load when self-indicating testing machines are not available.
21.3 STRAIN GAUGES

19. Wire Resistance Strain Gauges can be firmly affixed to a smooth dry concrete surface with High Solids Durofix, and will then be responsive to strains thereto.

20. Strains of $1 \times 10^{-6}$ can be detected quite easily on a 1 inch length. This corresponds to about 1% of the ultimate tensile strain capacity of concrete.

21. The strain sensitivity of the 1 in. B.T.C. gauges used, was found to be about 5% lower on concrete of $\frac{3}{8}$ in. maximum aggregate size, than on steel specimens. This was probably due to the increased thickness of the glue film required for sticking to concrete, although it might be caused by lack of homogeneity of the material over the gauge lengths used for the mechanical gauges.

22. Thermo-setting adhesives are difficult to use on concrete specimens, although it is probable that a technique could be developed which would enable measurements to be taken on damp concrete surfaces.

23. The N.P.L. type 2000 ohm B.T.C. gauges exhibited, initially, a reduction in resistance when subjected to small tensile strains. This may be a property of the Nichrome gauge wire, but the phenomenon warrants further detailed investigation.
24. A new theory of the operation of a strain gauge has been evolved which, so far as can be ascertained, shows the connection between the major variables in the problem of strain transmission.

25. It is desirable to 'exercise' the gauges by a proof loading of the specimen, before embarking on a test programme to bring the gauges to a cyclic state and so reduce zero drift.

21.4 FINAL OBSERVATIONS

To sum up, the investigations have justified the use of electrical resistance strain gauges on concrete. The gauges have made possible the determination of the strain characteristics of a concrete, by measuring the deformations over a smaller area of surface than has been possible in the past.

It is generally recognised that an exact knowledge of the behaviour of plain concrete is essential if a rational method for the design of reinforced concrete is to be found. The information presented in this thesis and summarised in the above conclusions will, it is hoped, lead to a better understanding of the behaviour of plain concrete under various types of loading and, at the same time, suggest a number of lines of research in which the existing knowledge is either still in doubt or completely lacking.
"THE USE OF ELECTRICAL STRAIN GAUGES ON
CONCRETE SPECIMENS UNDER SIMPLE AND COMPLEX LOADING!"

by

NORMAN SYDNEY GRASSAM, B.Sc.(Eng.)

A Thesis presented for

THE DEGREE OF Ph.D.

in the Faculty of Engineering
University of London.

JUNE 1950
CONTENTS

LIST OF APPENDICES

I  Wire Electrical Resistance Strain Gauge—Theoretical Sensitivity Factor.

II Preliminary Compression Test No. 1

III Details of Electrical Apparatus.

IV Galvanometer Deflection Formulae.

V Derivation of Circuit Sensitivity.
   (a) Sensitivity in terms of Galvanometer Current.
   (b) Sensitivity in terms of Gauge Current.
   (c) Variation in Circuit Sensitivity with the Bridge Ratio when the Galvanometer resistance is optimum.

VI Preliminary Compression Test No. 2 (c)

VII Preliminary Compression Test No. 2 (d)

VIII Preliminary Compression Test No. 2 (e) i

IX Preliminary Compression Test No. 2 (e) ii

X Preliminary Compression Test No. 2 (e) iii

XI Beam Calibration Test of a Strain Gauge No. 1

XII Beam Calibration Test of a Strain Gauge No. 2

XIII Beam Calibration Test of a Strain Gauge No. 3

XIV Tensile Calibration Test of a Strain Gauge

XV Beam Calibration Test (Cross Sensitivity)

XVI Tension Test. Calculation of Eccentricity.

XVII Tension Test No. 1

XVIII Tension Tests Nos. 2 and 3

XIX Tension Test No. 4

XX Tension Test No. 5

XXI Compressions Nos. 1, 2, 3 and 4

XXI(a) Compression Test No. 1 (Results).

XXI(b) Compression Test No. 2 (Results)
LIST OF APPENDICES (Cont.)

XXI(c) Compression Test No.3 (Results)
XXI(d) Compression Test No.4 (Results)
XXII Compression Test No.5
XXIII Compression Test No.6
XXIV Compression Test No.7
XXV Compression Test No.8
XXVI Compression Test No.9
XXVII Compression Test No.10
XXVIII Compression Test No.11
XXIX Compression Test No.12
XXX Compression Test, Eccentricity of Loading.
XXXI Compression Test, Variation of Strain in height of Specimen.
XXXII Beam Tests. Details of Gauges.
XXXIII Beam Test No.1
XXXIV Beam Test No.2
XXXV Beam Test No.3
XXXVI Beam Test No.4
XXXVII Beam Test No.5
XXXVIII Torsion Test No.1
XXXIX Torsion Test No.2
XL Torsion Test Nos. 3, 4 and 5
XLI Torsion Test. Calculations.
XLII Bending Test No.1
XLIII Bending Test No.2
XLIV Bending Test Nos. 3 and 4
XLV Bending Test No.5
XLVI Bending Test. Calculations.
XLVII Combined Bending and Torsion Test.
XLVIII Combined Bending and Torsion Test. Calculations.
XXX Principal Strain Calculations from a Rosette.

Bibliography.
LIST OF GRAPHS

3.1 Variation in Strain Sensitivity with length of Gauge. 
   (Flat Grid Type Gauges)

3.2 Variation in Strain Sensitivity with length of Gauge. 
   (Helical Wound Type 'B')

5.1 Electrical and Mechanical Stress-Strain Curves. 
   (Preliminary Compression Test No.1. by Author).

5.2 Strain Sensitivity of N.P.L. 2500 ohm Gauge. 
   (Preliminary Compression Test No.1. by Author).

8.1 Variation in Circuit Sensitivity with Galvanometer Resistance.

8.2 Variation in Circuit Sensitivity with Bridge Ratio for optimum Galvanometer Resistance.

9.1 Stress-Axial Strain Curves. 
   (Preliminary Compression Test No.2e by Author)

9.2 Stress-Axial Strain Curves. 
   (Preliminary Compression Test No.2d by Author)

9.3 Stress-Axial Strain Curves. (Electrical) 
   (Preliminary Compression Tests No.2e by Author)

9.4 Stress Axial Strain Curves. (Mechanical) 
   (Preliminary Compression Tests No.2e by Author)

9.5 Strain Sensitivity of N.P.L. 1300 ohm Gauge. 
   (Preliminary Compression Tests No.2e by Author)

11.1 Gauge Calibration Tests on Steel Beams. 
   (Load-Electrical Strain Curves by Author)

11.2 Gauge Calibration Tests on Steel Beams. 
   (Load-Deflection Curves by Author)

11.3 Gauge Calibration on a Steel Tension Specimen. 
   (Load-Strain Curves by Author)

11.4 Gauge Calibration Test on Steel Beam. 
   (Cross Sensitivity Investigation by Author)

13.1 (a) Tension Tests by L. Turner and V.C. Davies. 
   (b) Tension Tests by A.N. Johnson.

13.2 (a) Tension Tests by A.H. Nylander. 
   (b) Tension Tests by R.H. Evans.

13.3 Tensile Load-Strain Curves 
   (Tension Tests Nos. 1 and 2 by Author.)

13.4 Tensile Load-Strain Curves. 
   (Tension Tests No.3 by Author.)
<table>
<thead>
<tr>
<th>Section</th>
<th>Graph Title</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>13.5</td>
<td>Tensile Load-Strain Curves</td>
<td>(Tension Test No. 4 by Author.)</td>
</tr>
<tr>
<td>13.6</td>
<td>Tensile Stress-Strain Curve</td>
<td>(Tension Test No. 4 by Author.)</td>
</tr>
<tr>
<td>13.7</td>
<td>Extensometer Calibration Curve</td>
<td>(Tension Test No. 5 by Author.)</td>
</tr>
<tr>
<td>13.8</td>
<td>Tensile Load-Strain Curves</td>
<td>(Tension Test No. 5 by Author.)</td>
</tr>
<tr>
<td>13.9</td>
<td>Tensile Stress-Strain Curves</td>
<td>(Tension Test No. 5 by Author.)</td>
</tr>
<tr>
<td>13.10</td>
<td>Strain Sensitivity of B.T.C. 200 ohm Gauges</td>
<td>(Tension Test No. 5 by Author.)</td>
</tr>
<tr>
<td>13.11</td>
<td>Strain Distribution during Tension Tests</td>
<td>(Tension Tests Nos. 2, 3 and 4 by Author.)</td>
</tr>
<tr>
<td>13.12</td>
<td>Strain Distribution during Tension Test</td>
<td>(Tension Test No. 5 by Author.)</td>
</tr>
<tr>
<td>14.1</td>
<td>Compression Load-Strain Curves with various end-packings</td>
<td>(Compression Tests by Author.)</td>
</tr>
<tr>
<td>14.2</td>
<td>Compression Load-Hoop Strain Curves</td>
<td>(Compression Tests No. 6 b. by Author.)</td>
</tr>
<tr>
<td>14.3</td>
<td>Variation in Hoop-Strain in height of Specimen</td>
<td>(Compression Test No. 6 b. by Author.)</td>
</tr>
<tr>
<td>14.4</td>
<td>Compression Load-Axial Strain Curves</td>
<td>(Compression Test No. 4 by Author.)</td>
</tr>
<tr>
<td>14.5</td>
<td>Concrete under repeated Compression</td>
<td>(Tests by Van Grumm)</td>
</tr>
<tr>
<td>14.6</td>
<td>Stress-Strain Curves for Concrete after cycles of Compression</td>
<td>(Tests by Mehnel.)</td>
</tr>
<tr>
<td>14.7</td>
<td>Effect of repeated loading on Compression Stress-Strain Curves</td>
<td>(Compression Tests by Author.)</td>
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<tr>
<td>14.8</td>
<td>Effect of repeated loading on Compression Stress-Strain Curves</td>
<td>(Compression Tests by Author.)</td>
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<tr>
<td>14.9</td>
<td>Complete Compression Stress-Strain Curve</td>
<td>(Test by A.F. Johnson)</td>
</tr>
<tr>
<td>14.10</td>
<td>Compression Load-Axial and Lateral Strain Curves</td>
<td>(Compression Test No. 12 by Author.)</td>
</tr>
</tbody>
</table>
LIST OF GRAPHS (Cont.)

14.11 Compression Stress-Axial and Lateral Strain Curves.
    (Compression Test No.12 by Author.)

14.12 Compression Stress-Strain Curves.
    (Compression Test No.12 by Author.)

    (Tests by F.C. Lea.)

14.14 Hysteresis and Permanent Set in Compression-Strain Curves.
    (Compression Tests by Author.)

14.15 Hysteresis in Hoop Strains.
    (Compression Test No.4 by Author.)

14.16 Variation in Creep Strain with Load.
    (Compression Test No.12 by Author.)

14.17(a) Variation in Poisson's Ratio with Age.
    (Tests by Davis and Troxell).

    (b) Variation in Poisson's Ratio with Compressive Load.
    (Tests by A.N. Johnson.)

14.18 Variation in Poisson's Ratio with Compressive Load.
    (Tests by Richard, Brandtzaeg and Brown.)

14.19 Variation in Poisson's Ratio with Load.
    (Compression Test No.12 by Author.)

14.20 Volume Changes during Compressive Loading by Author.

    (Compression Tests by Author)

14.22 Strain Sensitivity Factor of 200 ohm Gauges.
    (Compression Test No.12 by Author)

14.23 The 'Negative Loop' Phenomenon with 2000 ohm Gauges
    (Compression Test No.5 by Author)

15.1 Strain Variation in depth of Rectangular Beams.
    (a) Reinforced Concrete Beam Test by F.C. Lea.
    (b) Rectangular Plain Concrete Beam Test by R.H. Evans.

15.2 Load-Strain Curves for Plain Concrete Beam.
    (Beam Test No.1 by Author).

15.3 Load-Strain Curves for Plain Concrete Beam.
    (Beam Test No.2 by Author).

15.4 Load-Strain Curves for Plain Concrete Beam.
    (Beam Test No.3 by Author).
LIST OF GRAPHS (Cont.)

15.5 Strain Distribution in depth of Beam by Author.
15.6 Load-Strain Curves for Plain Concrete Beam.
   (Beam Test No.5 by Author)
15.7 Strain Distribution in depth of Beam.
   (Beam Test No.5 by Author)
15.8 Effect of Creep in Beam under Load.
   (Beam Test No.5 by Author)

17.1 (a) Torsion Tests by L. Turner and V.C. Davies
   (b) Torsion Tests by Paul Anderson.
17.2 Torque-Principal and Shear Strain Curves.
   (Torsion Test No.1 by Author.)
17.3 Torque-Principal and Shear Strain Curves.
   (Torsion Test No.2 by Author.)
17.4 Torque-Deflection Curves.
   (Torsion Tests by Author.)
18.1 Load-Bending Strain Curves.
   (Bending Tests Nos. 1 and 2 by Author.)
18.2 Load-Bending Strain Curves.
   (Bending Test No.5 by Author.)
18.3 Load-Bending Deflection Curves.
   (Bending Tests by Author.)
19.1 Combined Bending and Torsion Tests by A.H. Nylander.
19.2 Load-Strain Curves.
   (Combined Bending and Torsion Test by Author.)
19.3 Torque-Shear Strain Curves under a constant Bending Moment.
   (Combined Bending and Torsion Test by Author.)
### LIST OF PHOTOGRAPHIC PLATES

<table>
<thead>
<tr>
<th>Plate</th>
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<tbody>
<tr>
<td>7.1</td>
<td>The Strain Gauge Set.</td>
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<tr>
<td>7.2</td>
<td>The Strain Gauge Set.</td>
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<tr>
<td>13.1</td>
<td>Tension Specimen 'A'</td>
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<td>13.2</td>
<td>Tensile Testing Arrangement.</td>
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<tr>
<td>14.1</td>
<td>Compression Specimen 'C'</td>
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<td>15.1</td>
<td>Beam Testing Arrangement.</td>
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<td>16.1</td>
<td>Combined Bending and Torsion Testing Arrangement.</td>
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<td>17.1</td>
<td>The Strain Gauge Rosette on Specimen 'C'.</td>
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<tr>
<td>19.1</td>
<td>Specimen 'C' in the Combined Bending and Torsion Machine.</td>
</tr>
<tr>
<td>19.2</td>
<td>Typical Fracture under Combined Bending and Torsion.</td>
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Appendix I

WIRE ELECTRICAL RESISTANCE STRAIN GAUGES

THEORETICAL SENSITIVITY FACTOR.

The resistance of a uniform wire is given by

\[ R = \frac{\rho L}{A} \]

where \( \rho \) = the specific resistance of the material of the wire,
\( L \) = the length of wire
\( A \) = the cross sectional area.

This may be written as

\[ \log R = \log \rho + \log L = \log A \]

\[ \frac{dR}{R} = \frac{d\rho}{\rho} + \frac{dL}{L} = \frac{dA}{A} \]

But \[ \frac{dA}{A} = -2\sigma \frac{dL}{L} \]

where \( \sigma \) = Poisson’s Ratio of the wire.

Hence letting strain \( \varepsilon = \frac{dL}{L} \)

\[ \text{Strain Sensitivity Factor} = \frac{dR}{R} = \frac{d\rho}{\rho} + 1 + 2\sigma \]

If the specific resistance of a metal was independent of strain (i.e. \( \frac{d\rho}{\rho} = 0 \)), and assuming that Poisson’s Ratio
Appendix I (Cont.)

for a fine wire is the same as for a larger piece of the same material then, for a metal \( \sigma = 0.3 \)

Strain Sensitivity Factor = \( 1 + 2\sigma = 1.6 \)

However, this is not so as may be seen in the Table 2.1 where the strain sensitivity factor varies from -12 to +6, but fortunately the factor does remain sensibly constant for a considerable range of strain, so that the variation of specific resistance with strain must be linear. All metals in the purely plastic state do not change their specific resistance with strain, and hence the shape of the strain -rate of change of specific resistance curve, must be as shown if Poisson's Ratio remains constant.

\[
\frac{d\rho}{\rho} > 1 + 2\sigma \quad \text{S.S.F.} > 1 + 2\sigma
\]

\[
\frac{d\rho}{\rho} < 1 + 2\sigma \quad \text{S.S.F.} < 1 + 2\sigma
\]

Fig. 2.4 Variation of Specific Resistance with Strain.
Appendix I (Cont.)

The Poisson's Ratio of solids tends to 0.5 in the plastic region, so that the strain sensitivity factor for all metals should tend to +2.0
Appendix II

PRELIMINARY COMPRESSION TEST, No. 1

Details

Date . . . . . . 2nd December, 1946

Specimen . . . . 5 in. dia. by 10 in. high concrete cylinder several months old.

Gauges . . . . . N.P.I. resin impregnated 2500 ohm gauges.

Glue . . . . . 'Durofix'

Compressometer . 'Hamilton' Instrument.

Testing Machine. Richle 100,000 lb. Testing Machine

Electrical Measurements. Test-rig Wheatstone Bridge Circuit

Abstract of Readings (Plotted on Graphs 5.1 and 5.2)

Run (i)

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<tr>
<td>Stress ( \frac{\text{lb.}}{\text{in}^2} )</td>
<td>Tension Gauge</td>
</tr>
<tr>
<td>Strain ( \times 10^5 )</td>
<td>Electrically Measured ( \frac{\Delta Q}{Q} \times 10^4 )</td>
</tr>
<tr>
<td>Stress ( \frac{\text{lb.}}{\text{in}^2} )</td>
<td>Electrically Measured ( \frac{\Delta Q}{Q} \times 10^4 )</td>
</tr>
<tr>
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<tr>
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<td>322</td>
<td>61.12</td>
</tr>
<tr>
<td>0</td>
<td>18.13</td>
</tr>
</tbody>
</table>

Note: This was the first test with the gauges on concrete and there were therefore none of the refinements of the later tests.
Appendix III

DETAILS OF APPARATUS

The details of the apparatus were:

Post Office Box

(For circuit diagram see Fig. 7.6)

Maker . . . . . Gambrell Bros. & Co., Ltd., Pott. 2036 No. 41

Range (as used) . . Fixed arm 'S' = 10,000 ohms
Variable arm 'Q' = 1 to 11,110 ohms
by increments of 1 ohm.

Accuracy . . . . each coil to 0.1%

A check on the value of the fixed 10,000 ohm resistances revealed that the left hand resistance was accurate, whereas the right hand resistance was in error. Hence as shown in the Fig. 7.6, the left hand resistance was used for 'S' in the strain gauge set.

Galvanometer

Type . . . . . Reflecting beam, moving coil galvanometer.

Maker . . . . . Marconi Instrument Co.

Resistance . . 1500 ohms (nominal).

Sensitivity . . 14 divs/micro amp. at 6 inch radius
(1 div = 1/20 inch.)

Features . . . . (1) A shunt is incorporated in the instrument variable in steps
1 : 10, 1 : 100, 1 : 1000 & short

(ii) A transformer is incorporated for the galvanometer light from 220 volt A.C.

(iii) An adjusting knob is provided for zero setting.
Appendix III (Cont.)

Disadvantages . . . (i) The shunt switch is too strong so that the galvanometer zero is disturbed by its use.

(ii) Four rubber pads are provided as feet, hence in general, the galvanometer is not firm unless wedged.

(iii) Dust can too easily enter the movement.

Rotary Stud Switch.

Maker . . . . . . Muirhead and Co Ltd.

Type . . . . . . (D 223 A) 4 pole switch 24 studs.

![Diagram of Rotary Stud Switch Contacts]

FIG. 7.5 Rotary Stud Switch Contacts.

The switch operates through pressure contact as shown. The changes in potential are assumed to be negligible.
Appendix III (Cont.)

Rheostats

Type . . . . . Wire wound potentiometers 0-50 ohms.

Disadvantages . . . . These were of poor construction and were not very satisfactory. The contact was not firm and therefore they were easily disturbed.

Heating Resistances

Type . . . . . Wire wound potentiometers 0-250 ohms.
These proved quite satisfactory.

Milliammeter

Type . . . . . Moving coil.
Range . . . . . 0-50 milliamps, reading to +1 milliamp.

Wire

Type . . . . . Multi-strand tinned copper wire, plastic covered 0.3 mm. wall, 1.1 mm.o.d.
Maker . . . . . Duratube and Wire Co Ltd.
Note . . . . . This was obtained in 2 colours for ease of tracing circuits. More colours would have been useful, but the supply position made this out of the question at the time.

Battery

Type . . . . . Ever-Ready 135 volt. H.T. Dry Battery.
Note . . . . . Numerous tapping points were provided within this voltage range, and these proved most convenient when operating the set with gauges of different resistance e.g. 200 or 2000 ohm.
FIG. 7.6  Post Office Box Circuit.
Appendix IV

GALVANOMETER DEFLECTION FORMULAR

The magnetic intensity at the centre of a rectangular coil is directly proportional to the number of turns per unit area of cross section of the coil.

If the thickness of the insulation is negligibly small and the shape and the volume of the coil are unaltered, the number of coils, and therefore the length of wire, is inversely proportional to the cross sectional area of the wire.

But Resistance \( \propto \frac{\text{length}}{\text{area}} \)

and volume = length \( \times \) area \( \times \) constant.

Hence Resistance \( \propto \text{length}^2 = (\text{number of turns})^2 \)

i.e. Number of turns \( \propto \sqrt{\text{Galvo Resistance}} \).

So that, in galvanometers, the coils of which are wound in similar channels and contain the same mass of wire, the electro-magnetic force on the needle, and therefore the deflection, is proportional to the current and to the square root of the galvanometer resistance.

\[ \theta = k_1 \cdot \sqrt{\Omega} \] \hspace{1cm} \text{Eqn. 3.2}

where \( k_1 \) = constant of galvanometer.
Appendix V

DERIVATION OF CIRCUIT SENSITIVITY FORMULAE

(a) Sensitivity in terms of Galvanometer current.

\[ \text{(Active Gauge)} \quad \text{Gauge Current } I_i. \]
\[ \text{(Dummy Gauge)} \quad \text{Current } (I-i). \]
\[ \text{Circuit } I. \]
\[ \text{Voltage } E. \]
\[ \text{Battery Resistance } B. \]

**FIG. 8.3 Wheatstone Bridge Network**

Considering closed circuits of the network shown :=

\[ a,b,c. \quad E = 1P + (1 - i_g)R + IB \]
\[ a,b,d. \quad 0 = 1P + i_g \cdot G - (I - i)Q \]
\[ b,c,d. \quad 0 = (I - i_g)R - (I - 1 + i_g)S - i_g \cdot G \]

which may be re-written as :=

\[ - R i_g + BI + (P + R)I = E \]
\[ + GI_g + QI + (P + Q)I = 0 \]
\[ -(R + S + G)i_g - SI + (R + S)I = 0 \]
When $f = 0$, which is the usual condition for balance of the bridge, then

\[ S \cdot P = Q \cdot R = 0 \]

\[ \frac{P_o}{Q} = \frac{R}{S} \quad \text{the well known balance equation.} \]

when $P_o$ has changed to $P_o + dP = P$

Then numerator = $E \left\{ S(P_o + dP) - QR \right\}$

= $E \cdot S \cdot dP$

Denominator :=

\[ = -R \left\{ -Q(R + S) + S(P + Q) \right\} - B \left\{ G(R + S) + (R + S + G)(P + Q) \right\} + (P + R) \left\{ -GS - Q(R + S + G) \right\} \]

\[ = R(I(R + S) - RS(P + Q) - BS(R + S) - BS(P + Q) - B(R + S)(P + Q) - GS(P + R) - Q(P + R)(R + S) - QG(P + R) \]

\[ = -G \left\{ B(R + S) + P + Q \right\} - G \left\{ S(P + R) + Q(P + R) \right\} - B(R + S)(P + Q) + R^2Q + RQS + RSP + QSQ \]

\[ -PRQ - R^2Q - ESQ - PSQ. \]
Appendix V (Cont)  

Assuming \( S = \frac{QR}{P} \) (i.e. that the bridge is not greatly out of balance.)

Then \( RSP = R^2 q \) 

denominator 
\[
= \left[ G\left( P+Q+R+S \right) + (S+Q)(P+R) + B(R+S)(P+Q) + RSP + PR + ESQ + PSE \right]
\]
\[
= \left[ G\left( P+Q+R+S \right) + (S+Q)(P+R) + (R+S)\left( E(P+Q)+Q(P+R) \right) \right]
\]

Then 
\[
\frac{di}{dp} = \frac{E \cdot S}{\left[ G\left( P+R \right) \left( Q+S \right) + B\left( P+Q+R+S \right) \right] + \left( R+S \right) \left( Q(P+R) + B(P+Q) \right)} \quad \text{Eqn. 3.4.1}
\]

Substituting for \( \frac{di}{dp} \) in equation 3.5 which gives 

Sensitivity, \( \frac{P \cdot \partial Q}{\partial P} = k F \sqrt{S} \cdot \frac{di}{dp} \)

We get 
\[
\frac{P \cdot \partial Q}{\partial P} = \frac{k F S E \sqrt{S}}{G\left( P+R \right) \left( Q+S \right) + B\left( P+Q+R+S \right) + \left( R+S \right)\left( Q(P+R) + B(P+Q) \right)} \quad \text{Eqn. 3.4}
\]

(b) Sensitivity in terms of gauge current \( i^* \)

Gauge Current \( i^* = \frac{E}{\text{Total Resistance of Circuit}} \cdot \frac{(Q+S)}{(P+R+Q+S)} \)

Total resistance of Circuit  
\[
= \frac{(P+R)(Q+S)}{(P+R+Q+S)} + B
\]

\( i^* = \frac{E(P+R+Q+S)}{(P+R)(Q+S) + B(P+R+Q+S)} \cdot \frac{(Q+S)}{(P+R+Q+S)} \)
Dividing both sides of equation by $G(1 + \frac{P}{R}) + (P + Q)$

$$R.H.S. = \frac{E(Q + S)}{\{(P + R)(Q + S) + B(P + R + Q + S)\}} \{G(\frac{P + R}{R}) + (P + Q)\}$$

$$= \frac{E(Q + S)R}{\{(P + R)(Q + S) + B(P + R + Q + S)\}} \{G(P + R) + R(P + Q)\}$$

$$= \frac{ER(Q + S)}{G(P + R)(Q + S)(P + R) + E(G(P + R)(P + Q + R + S) + R(P + Q)(P + R)(Q + S) + BR(P + Q + R + S)}$$

Assuming that $P = P_o$ and using the following equalities

$$\frac{P + R}{R} = \frac{Q + S}{S}$$

$$\frac{P + Q}{Q} = \frac{R + S}{S}$$

this expression becomes

$$R.H.S. = \frac{E S}{G\{(P + R)(Q + S) + B(P + Q + R + S)\} + (R + S)\{Q(P + R) + B(P + R)\}}$$

$$= \frac{dE}{dP} \quad \text{see Eqn. 8.4.1}$$

$$= L.H.S. = \frac{1}{G(1 + \frac{P}{R}) + (P + Q)}$$

Hence circuit sensitivity, $P \frac{d\theta}{dP} = k P\sqrt{G} \frac{dE}{dP}$
(c) Variation in the circuit sensitivity with the bridge ratio $Q'$, when the galvanometer resistance is optimum.

Eqn. 8.5.4 gives for $P = R$

\[
\text{Maximum sensitivity } S_{\text{max}} = \frac{kF}{2\sqrt{Q(P+Q)}}
\]

which may be arranged as

\[
\frac{2\sqrt{Q} S_{\text{max}}}{kF} = \frac{1}{\sqrt{1 + \frac{Q}{P}}}
\]

<table>
<thead>
<tr>
<th>(Q/P)</th>
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See Graph 8.2.
Appendix VI

PRELIMINARY COMPRESSION TEST No. 2(c)

Details

Date . . . . . . . . . 20th February, 1948

Specimen . . . . . . 5 inch dia. by 10 inches high concrete cylinder. Age 18 months.

Gauges . . . . . . N.P.I. resin impregnated

P = 1189 ohm gauge.

Cement . . . . . . Shellac dissolved in methylated spirits. (Applied 14 days).

Testing Machine . . Richle 100,000 lb. Testing Machine

Electrical Measurements . . Multi-strain gauge set.

(Gauge current 25 milliamps.)

Abstract of Readings (Plotted on Graph 9.1)

Run (1) \( Q = 9115 \text{ ohms.} \)

<table>
<thead>
<tr>
<th>Load (lb)</th>
<th>Stress lb./in²</th>
<th>Resistance change ( dQ ) (ohms)</th>
<th>( \frac{dQ}{Q} \times 10^4 )</th>
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</table>
Appendix VI (Cont.)

Run (ii)  \( Q = 9115 \text{ ohms} \)

<table>
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<th>Stress lb/in(^2)</th>
<th>Resistance change ( dQ ) (ohms)</th>
<th>( \frac{dQ}{Q} \times 10^4 )</th>
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<tbody>
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<td>0</td>
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<td>18330</td>
<td>935</td>
<td>7</td>
<td>7.7</td>
</tr>
<tr>
<td>11300</td>
<td>601</td>
<td>5</td>
<td>5.5</td>
</tr>
<tr>
<td>5020</td>
<td>266</td>
<td>2</td>
<td>2.2</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Run (iii)  \( Q = 9115 \text{ ohms} \)

<table>
<thead>
<tr>
<th>Load (lb)</th>
<th>Stress lb/in(^2)</th>
<th>Resistance change ( dQ ) (ohms)</th>
<th>( \frac{dQ}{Q} \times 10^4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
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<tr>
<td>2570</td>
<td>131</td>
<td>1</td>
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</tr>
<tr>
<td>2040</td>
<td>409</td>
<td>3</td>
<td>3.3</td>
</tr>
<tr>
<td>12750</td>
<td>648</td>
<td>5</td>
<td>5.5</td>
</tr>
<tr>
<td>16390</td>
<td>860</td>
<td>6.5</td>
<td>7.1</td>
</tr>
<tr>
<td>21590</td>
<td>1100</td>
<td>8</td>
<td>8.8</td>
</tr>
<tr>
<td>25750</td>
<td>1311</td>
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<td>9.9</td>
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<td>30780</td>
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<td>22100</td>
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<td>17540</td>
<td>833</td>
<td>7</td>
<td>7.7</td>
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<tr>
<td>10240</td>
<td>522</td>
<td>5</td>
<td>5.5</td>
</tr>
<tr>
<td>5800</td>
<td>288</td>
<td>3</td>
<td>3.3</td>
</tr>
<tr>
<td>0</td>
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<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Note: Test carried out with a low sensitivity Tinsley Reflecting Galvanometer so that no interpolation was required. No zero error was observed on the galvo. at the completion of the test.
Appendix VII

PRELIMINARY COMPRESSION TEST, No. 2(a)

Details

Date ............... 20th February, 1948
Specimen ........... 5 inch dia. by 10 inches high concrete cylinder. Age 12 months.
Cement ............. Shellac dissolved in alcohol. Tested at 10 days.
Testing Machine ..... Riehle 100,000 lb. Testing Machine
Electrical Measurements ..... Multi-strain gauge set. (Gauge current 25 milliamps).

Abstract of Readings (Plotted on Graph 9.2)

Run (1) Q = 9983 ohms.

<table>
<thead>
<tr>
<th>Load (lb)</th>
<th>Stress (lb/in²)</th>
<th>Resistance change (ohms)</th>
<th>(\frac{dQ}{Q} \times 10^4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>5250</td>
<td>263</td>
<td>-1</td>
<td>-0.35</td>
</tr>
<tr>
<td>10360</td>
<td>527</td>
<td>-3</td>
<td>-0.55</td>
</tr>
<tr>
<td>15240</td>
<td>777</td>
<td>-4</td>
<td>-0.66</td>
</tr>
<tr>
<td>25340</td>
<td>1302</td>
<td>-5</td>
<td>-0.72</td>
</tr>
<tr>
<td>28760</td>
<td>1367</td>
<td>-7</td>
<td>-0.84</td>
</tr>
<tr>
<td>31120</td>
<td>1588</td>
<td>-8</td>
<td>-0.92</td>
</tr>
<tr>
<td>20210</td>
<td>1031</td>
<td>-5</td>
<td>-0.95</td>
</tr>
<tr>
<td>15170</td>
<td>670</td>
<td>-3</td>
<td>-1.03</td>
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<tr>
<td>6690</td>
<td>342</td>
<td>-1</td>
<td>-1.04</td>
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<tr>
<td>0</td>
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<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
Appendix VII (Cont.)

Run (11) \( Q = 9937 \text{ ohms} \)

<table>
<thead>
<tr>
<th>Load (lb)</th>
<th>Stress (lb./in(^2))</th>
<th>Resistance change (dQ) (ohms)</th>
<th>( \frac{dQ}{Q} \times 10^4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>6090</td>
<td>310</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>10280</td>
<td>523</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>14960</td>
<td>763</td>
<td>3.5</td>
<td>3.5</td>
</tr>
<tr>
<td>19440</td>
<td>908</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>24500</td>
<td>1252</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>29440</td>
<td>1500</td>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td>34520</td>
<td>1884</td>
<td>6</td>
<td>6</td>
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<td>39750</td>
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<td>11030</td>
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<td>2</td>
</tr>
<tr>
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<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
Appendix VIII

PRELIMINARY COMPRESSION TEST No. 2(c) 1

Details

Date . . . . . . . . 23rd February, 1948
Specimen . . . . . . 5 inch dia. by 10 inches high concrete cylinder. Age 18 months.
Gauge . . . . . . . N.F.L. resin impregnated P = 1303 ohms.
Glue . . . . . . . High Solids Durofix (I.C.I. glue No. 5463). Test at 3 days.
Electrical Measurements . . Multi-strain gauge set (Gauge current 16 milliams)

Abstract of Readings (Plotted on Graph 9.3)

\[ Q = 9982 \text{ ohms}. \]

<table>
<thead>
<tr>
<th>Load (lb)</th>
<th>Stress lb./in²</th>
<th>Resistance change dQ (ohms)</th>
<th>( \frac{dQ}{Q} \times 10^4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
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<td>0</td>
<td>0</td>
</tr>
<tr>
<td>6450</td>
<td>329</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>9120</td>
<td>464</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>13610</td>
<td>694</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>20610</td>
<td>1051</td>
<td>8</td>
<td>8</td>
</tr>
<tr>
<td>25470</td>
<td>1300</td>
<td>9</td>
<td>9</td>
</tr>
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<td>27610</td>
<td>1408</td>
<td>10</td>
<td>10</td>
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<td>22130</td>
<td>1135</td>
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<td>3</td>
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<td>536</td>
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<td>4</td>
</tr>
<tr>
<td>6120</td>
<td>315</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
Appendix IX

PRELIMINARY COMPRESSION TEST No.2 (e) 11

Details

Date . . . . . . . . . 8th March 1948.

Specimen, gauge, Testing Machine . . As for Test II,(e) 1


Electrical Measurements . . . (Gauge current 24 milliams)

Mechanical Extensometer . . . Hamilton.

Abstract of Readings (Plotted on Graph 9.3)

\[ Q = 10,018 \text{ ohms} \]

<table>
<thead>
<tr>
<th>Load (lb)</th>
<th>Stress Lb./in²</th>
<th>Resistance change ( \Delta R ) (ohms)</th>
<th>( \frac{\Delta R}{Q} \times 10^4 )</th>
<th>Mechanically Measured Strain ( \times 10^4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>5500</td>
<td>228</td>
<td>( 2\frac{1}{2} )</td>
<td>( 2\frac{1}{2} )</td>
<td>0.7</td>
</tr>
<tr>
<td>9770</td>
<td>496</td>
<td>( 3\frac{1}{2} )</td>
<td>( 3\frac{1}{2} )</td>
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</tr>
<tr>
<td>14250</td>
<td>726</td>
<td>4</td>
<td>4</td>
<td>1.9</td>
</tr>
<tr>
<td>18800</td>
<td>956</td>
<td>( 6\frac{1}{2} )</td>
<td>( 6\frac{1}{2} )</td>
<td>2.5</td>
</tr>
<tr>
<td>21100</td>
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<td>( 7\frac{1}{2} )</td>
<td>( 7\frac{1}{2} )</td>
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</tr>
<tr>
<td>22250</td>
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<td>( 8\frac{1}{2} )</td>
<td>( 8\frac{1}{2} )</td>
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</tr>
<tr>
<td>25580</td>
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<td>( 9\frac{1}{2} )</td>
<td>( 9\frac{1}{2} )</td>
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</tr>
<tr>
<td>26000</td>
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<td>( 10\frac{1}{2} )</td>
<td>( 10\frac{1}{2} )</td>
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</tr>
<tr>
<td>14080</td>
<td>716</td>
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<td>( 1\frac{1}{2} )</td>
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</tr>
<tr>
<td>6370</td>
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<td>( 2\frac{1}{2} )</td>
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<tr>
<td>1670</td>
<td>35</td>
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<td>( 3\frac{1}{2} )</td>
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<tr>
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<td>1\frac{1}{2}</td>
<td>0.4</td>
</tr>
</tbody>
</table>

Note: The Tinsley reflecting galvanometer was used, and with the gauge current increased to 24 milliams, interpolation was possible to \( \frac{1}{2} \) ohm.
Appendix X

PRELIMINARY COMPRESSION TEST No.2 (e) 111

Details

Date . . . . . . . . . 15th March 1943.

Data . . . . . . . . . as per Test 2 (e) ii

Glue . . . . . . . . . 23 days old.

Abstract of Readings (Plotted on Graph 9.3)

\[ Q = 10,022 \text{ ohm}. \]

1st Run

<table>
<thead>
<tr>
<th>Load (lb)</th>
<th>Stress, lb./in(^2)</th>
<th>Resistance change</th>
<th>dQ (\times 10^4)</th>
<th>Mechanically Measured Strain (\times 10^4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
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<td>0</td>
<td>0</td>
<td>0</td>
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<td>239</td>
<td>1.7</td>
<td>1.7</td>
<td>0.6</td>
</tr>
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<td>5.4</td>
<td>1.3</td>
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</tr>
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<td>8.7</td>
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</tr>
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<td>10.1</td>
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<td>10.1</td>
<td>10.1</td>
<td>4.4</td>
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<td>4.1</td>
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<td>2.6</td>
<td>1.2</td>
</tr>
<tr>
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<td>1.9</td>
<td>0.9</td>
</tr>
<tr>
<td>1750</td>
<td>23</td>
<td>0.9</td>
<td>0.9</td>
<td>0.4</td>
</tr>
<tr>
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<td>0</td>
<td>0.4</td>
<td>0.4</td>
<td>0.2</td>
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</table>
### 2nd Run

<table>
<thead>
<tr>
<th>Load (lb)</th>
<th>Stress (lb./in²)</th>
<th>Resistance change (Ω)</th>
<th>$\frac{\Delta \Omega}{\Omega} \times 10^4$</th>
<th>Measurably Measured Strain $\times 10^2$</th>
</tr>
</thead>
<tbody>
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<td>0</td>
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<td>0</td>
</tr>
<tr>
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<td>0.9</td>
<td>0.3</td>
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<td>2.4</td>
<td>0.9</td>
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<td>9.0</td>
<td>4.0</td>
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</tr>
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<td>2.9</td>
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<td>4.7</td>
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<td>2.4</td>
<td>1.2</td>
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<td>1.7</td>
<td>1.7</td>
<td>0.8</td>
</tr>
<tr>
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<td>0.6</td>
<td>0.6</td>
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<td>0.1</td>
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</tr>
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</table>
Appendix XI

BEAM CALIBRATION TEST OF A STRAIN GAUGE No.1

Details

Date . . . . . . . 5th May, 1943

Specimen . . . . Mild Steel Beam 30 inches x 2 inches x 0.312 inches.

Gauge . . . . . . B.T.C. Type SE/A/8 2095 ohms nominal.

Glue . . . . . . High Solids Durofix (I.C.I. glue No. 3462) Tested at 14 days.

Testing Machine . . . 4-point loading apparatus.

Electrical Measurements . . Single-station strain gauge set. (Gauge current 5½ millamps.)

Set Up

[Diagram of beam test arrangement]

FIG. 11.4 Beam Test Arrangement.

Test Beam section

[Diagram of beam section]

FIG. 11.5 Beam Section.
Appendix XI (Cont.)

Abstract of Readings (Plotted on Graph 11.1)

\( Q = 10,012 \text{ ohms} \)

<table>
<thead>
<tr>
<th>Load W (lb)</th>
<th>Gauge in Tension</th>
<th>Gauge in Compression</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Deflection ( \delta \times 10^4 \text{in.} )</td>
<td>Resistance change ( \delta Q ) (ohms)</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>12( \frac{1}{2} )</td>
<td>0.15</td>
</tr>
<tr>
<td>4</td>
<td>24( \frac{3}{10} )</td>
<td>0.30</td>
</tr>
<tr>
<td>6</td>
<td>37( \frac{3}{10} )</td>
<td>0.40</td>
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<tr>
<td>10</td>
<td>65( \frac{3}{10} )</td>
<td>0.70</td>
</tr>
<tr>
<td>14</td>
<td>91</td>
<td>1.00</td>
</tr>
<tr>
<td>16</td>
<td>103( \frac{1}{2} )</td>
<td>1.10</td>
</tr>
<tr>
<td>22</td>
<td>146</td>
<td>1.54</td>
</tr>
<tr>
<td>24</td>
<td>159</td>
<td>1.68</td>
</tr>
</tbody>
</table>

Calculations

From Eqn. 11.1

\[
\text{Strain } \varepsilon = \frac{4 \delta \delta}{L^2}
\]

Gauge in tension.

For a 20 lb. central load from Graph 11.2

\[
\varepsilon = \frac{4 \times 0.312 \text{ inch} \times 154.5 \times 10^{-4} \text{ inch}}{16.5^2 \text{ inches}^2} = 0.0000615
\]
Appendix XI (Cont.)

Gauge in tension (cont.)

also at this load \( \frac{dQ}{\Omega} = \frac{1.40 \text{ ohm}}{10012 \text{ ohm}} = 1.4 \times 10^{-4} \)

Strain Sensitivity Factor in tension \( = \frac{\delta_{e}}{\varepsilon} = \frac{1.4 \times 10^{-4}}{0.615 \times 10^{-4}} \)

\( = 2.27 \)

Gauge in compression

For a 20 lb. central load

\( \varepsilon = \frac{4 \times 0.312 \text{ inch} \times 156.5 \times 10^{-6} \text{ inch}}{16.5^2 \text{ inch}^2} \)

\( = 0.0000624 \)

At this load \( \frac{dQ}{\Omega} = \frac{1.46 \text{ ohm}}{10012 \text{ ohm}} = 1.46 \times 10^{-4} \)

Strain Sensitivity Factor in compression \( = \frac{\delta_{e}}{\varepsilon} = \frac{1.46 \times 10^{-4}}{0.624 \times 10^{-4}} \)

\( = 2.33 \)
Appendix XII

BEAM CALIBRATION TEST OF A STRAIN GAUGE NO. 2

Details

Date . . . . . . . . . November 1948
Specimen . . . . .  Mild Steel beam 30 inches x 2 inches x 0.312 inches.
Gauge . . . . . . . .  B.T.C. Type SE/A/27, 200 ohms nominal.
Testing Machine .  4-point loading apparatus.
Electrical Measurements . Multi-station strain gauge set.

Set-up

FIG. 11.6 Beam Test Arrangement.

Test Beam Section.

FIG. 11.7 Beam Section.
Appendix XII (Cont.)

Abstract of Readings (Plotted on Graph 11.1)

\( Q = 10,050 \, \text{ohm} \)

<table>
<thead>
<tr>
<th>Central Load ( W ) (lb.)</th>
<th>Central Deflection ( \delta \times 10^4 \text{in.} )</th>
<th>Tension</th>
<th>Compression</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Resistance change</td>
<td>Resistance change</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( dQ ) (ohms)</td>
<td>( dQ ) (ohms)</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>11.5</td>
<td>+0.05</td>
<td>-0.3</td>
</tr>
<tr>
<td>6</td>
<td>25</td>
<td>+0.25</td>
<td>-0.6</td>
</tr>
<tr>
<td>9</td>
<td>41</td>
<td>-</td>
<td>-0.9</td>
</tr>
<tr>
<td>12</td>
<td>53</td>
<td>+0.8</td>
<td>-1.1</td>
</tr>
<tr>
<td>15</td>
<td>66</td>
<td>+1.10</td>
<td>-1.4</td>
</tr>
<tr>
<td>18</td>
<td>80</td>
<td>+1.45</td>
<td>-1.7</td>
</tr>
<tr>
<td>21</td>
<td>95</td>
<td>+1.7</td>
<td>-2.0</td>
</tr>
<tr>
<td>24</td>
<td>108</td>
<td>+2.0</td>
<td>-2.2</td>
</tr>
<tr>
<td>27</td>
<td>123</td>
<td>+2.3</td>
<td>-2.5</td>
</tr>
<tr>
<td>30</td>
<td>137</td>
<td>+2.5</td>
<td>-2.8</td>
</tr>
</tbody>
</table>

Calculations

From Eqn. 11.1

\[ \text{Strain } \varepsilon = \frac{4 \, d \delta}{L^2} \]

For a 10 lb. load \( W \) from Graph 11.2

\[ \varepsilon = \frac{4 \times 0.312 \, \text{inch} \times 46.5 \, \text{inch} \times 10^{-6}}{12^2 \, \text{inch}^2} \]

\[ = 0.0000403 \]

Also at this load in both tension and compression

\[ \frac{dQ}{Q} = \frac{0.92 \, \text{ohm}}{10050 \, \text{ohm}} \]

Strain Sensitivity Factor \( \frac{dQ}{\varepsilon} = \frac{0.92}{10050 \times 0.0000403} \)

\[ = 2.27 \]
Appendix XIII

BEAM CALIBRATION TEST OF A STRAIN GAUGE No. 3

Details

Date . . . . . . . . 1st December 1948
Specimen . . . . Mild Steel beam 30 inch x 2 inch x 0.312 inches.
Gauges . . . . . B.T.C. Type SE/A/27 213 ohm. One affixed to each side of the beam.
Glue . . . . . . High Solids Durofix (I.C.I. glue No.3462) Tested at 14 days.
Testing Machine . . 4-point loading apparatus.
Electrical Measurements . . Multi-station strain gauge set.

Set-up

FIG. 11.8 Beam Test Arrangement

Test beam section

FIG. 11.9 Beam Section
Appendix XIII (Cont.)

Abstract of Readings (Plotted on Graph 11.1 and 11.2)

\[ Q = 10,050 \text{ ohm} \]

<table>
<thead>
<tr>
<th>Central Load W (lb)</th>
<th>Central Deflection ( \delta \times 10^4 \text{in} )</th>
<th>Gauge (1) ( \frac{dQ_1}{Q} ) ohms</th>
<th>Gauge (2) ( \frac{dQ_2}{Q} ) ohms</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>32.0</td>
<td>+ 0.35</td>
<td>0.45</td>
</tr>
<tr>
<td>10</td>
<td>64.8</td>
<td>+ 0.75</td>
<td>0.85</td>
</tr>
<tr>
<td>15</td>
<td>97.6</td>
<td>+ 1.20</td>
<td>1.20</td>
</tr>
<tr>
<td>20</td>
<td>130.0</td>
<td>+ 1.57</td>
<td>1.60</td>
</tr>
<tr>
<td>25</td>
<td>162.5</td>
<td>+ 2.00</td>
<td>2.00</td>
</tr>
<tr>
<td>30</td>
<td>197.0</td>
<td>+ 2.30</td>
<td>2.45</td>
</tr>
</tbody>
</table>

Calculations

From equation 11.1

Strain \( \varepsilon = \frac{4 d\delta}{L^2} \)

For 10 lb load \( W \) from Graph 11.2

\[ \varepsilon = \frac{4 \times 0.312 \text{ inch} \times 65.6 \text{ inch}}{15^2 \text{ inch}^2 \times 10,000} = 0.0000363 \]

Also for this load, Gauge No.1 (tension) \( \frac{dQ_1}{Q} = \frac{0.32 \text{ ohm}}{10050 \text{ ohm}} \)

Gauge No.2 (compression) \( \frac{dQ_2}{Q} = \frac{0.78 \text{ ohm}}{10050 \text{ ohm}} \)

Gauge No.1 (tension) Strain Sensitivity Factor

\[ \frac{dQ_1}{Q} = \frac{0.32}{10050 \times 0.0000363} = 2.25 \]

Gauge No.2 (compression) Strain Sensitivity Factor

\[ \frac{dQ_2}{Q} = \frac{0.78}{10050 \times 0.0000363} = 2.14 \]
Appendix XIV

TENSILE CALIBRATION TEST OF A STRAIN GAUGE

Details

Date ................ June 1948

Specimen ............. Mild Steel Test Piece (Area 0.346 in²)
(See fig. 11.10)

Gauges ............... B.T.C. Type SE/A/27 211 ohm and
212 ohm nominal. Affixed on
opposite faces of specimen.

Glue ................. High Solids Durofix (T.C.I. glue
No.3462) Tested at 14 days.

Testing Machine ...... 5 ton Buckton Single Lever
Testing Machine (Ball & socket
shackles to grip specimen.)

Electrical Measurements Multi-station strain gauge set.

Mechanical Strain Measurements

Cambridge 4 inch Extensometer.

Cross Section of test piece

![Diagram of test piece with gauges and dimensions]

FIG. 11.10 Tension Specimen.
Appendix XIV (Cont)

Abstract of Readings (a) Electrical (Plotted on Graph 11.3)

<table>
<thead>
<tr>
<th>Load (lb)</th>
<th>211 ohm Gauge dΩ ohms</th>
<th>212 ohm Gauge dΩ ohms</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1000</td>
<td>2.2</td>
<td>1.5</td>
</tr>
<tr>
<td>2000</td>
<td>4.5</td>
<td>3.5</td>
</tr>
<tr>
<td>3000</td>
<td>6.5</td>
<td>5.5</td>
</tr>
<tr>
<td>4000</td>
<td>8.5</td>
<td>7.5</td>
</tr>
<tr>
<td>5000</td>
<td>10.5</td>
<td>9.6</td>
</tr>
<tr>
<td>6000</td>
<td>12.5</td>
<td>11.6</td>
</tr>
<tr>
<td>7000</td>
<td>14.3</td>
<td>13.7</td>
</tr>
</tbody>
</table>

(b) Mechanical

<table>
<thead>
<tr>
<th>Load (lb)</th>
<th>Extension on 4&quot; O.L. x 10^4 inches</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>2.3</td>
</tr>
<tr>
<td>1310</td>
<td>7.7</td>
</tr>
<tr>
<td>2110</td>
<td>11.0</td>
</tr>
<tr>
<td>3100</td>
<td>14.5</td>
</tr>
<tr>
<td>4100</td>
<td>18.0</td>
</tr>
<tr>
<td>5360</td>
<td>22.7</td>
</tr>
<tr>
<td>6130</td>
<td>26.0</td>
</tr>
</tbody>
</table>

Calculation

From the graphs, as the slopes of the load-change of resistance curves are identical for each gauge:

Then, for 1,000 lb load, dΩ = 2 ohms.

\[
\text{Strain } \varepsilon = \frac{3.76}{2} \times 10^{-4}
\]

\[
\text{Strain Sensitivity Factor} = \frac{2 \times 4}{10050 \times 3.76 \times 10^{-4}} = 2.12
\]
Appendix XV

BEAM CALIBRATION TEST (Cross Sensitivity)

Details

Date ............. 2nd December 1943.
Specimen .......... Mild Steel beam 30 inch x 2 inch x 0.509 inch.
Gauges .......... B.T.C. Type SE/A/9, F₂ = 2450 ohms
                F₁ = 2449 ohms
Glue ............. High Solids Durefix (I.C.I. glue No.3462) Tested at 14 days.
Testing Machine... 4-point loading apparatus.
Electrical Measurements .......... Single station Wheatstone Bridge set.

Set-up

![Beam Calibration Test Diagram]

Plan View of Beam

FIG. 11.11 Testing Arrangement.

Initial Balance P₂ = 2450 ohms  P₁ = 2449 ohms

Galvanometer Calibration

<table>
<thead>
<tr>
<th>dq (ohms)</th>
<th>Galvanometer Deflection</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>0</td>
</tr>
<tr>
<td>0.2</td>
<td>5.15</td>
</tr>
<tr>
<td>0.3</td>
<td>6.2</td>
</tr>
<tr>
<td>0.4</td>
<td>9.3</td>
</tr>
<tr>
<td>0.5</td>
<td>12.4</td>
</tr>
<tr>
<td>0.6</td>
<td>15.5</td>
</tr>
</tbody>
</table>

Hence :-

Galvanometer Rate = \frac{31 \text{ diva.}}{\text{ohm}}

(this applied for both gauges.)
**Leading Test**

**Abstract of Readings (Plotted on Graph 11.4)**

<table>
<thead>
<tr>
<th>Load W (lb)</th>
<th>Axial Gauge</th>
<th>Transverse Gauge</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Galvo Deflection</td>
<td>dP1 (ohms)</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>+1.15 divs</td>
<td>0.037</td>
</tr>
<tr>
<td>4</td>
<td>+2.40</td>
<td>0.077</td>
</tr>
<tr>
<td>6</td>
<td>+3.65</td>
<td>0.113</td>
</tr>
<tr>
<td>8</td>
<td>+4.85</td>
<td>0.156</td>
</tr>
<tr>
<td>10</td>
<td>+6.05</td>
<td>0.195</td>
</tr>
<tr>
<td>12</td>
<td>+7.25</td>
<td>0.234</td>
</tr>
<tr>
<td>14</td>
<td>+8.45</td>
<td>0.272</td>
</tr>
<tr>
<td>16</td>
<td>+9.75</td>
<td>0.314</td>
</tr>
<tr>
<td>14</td>
<td>+8.55</td>
<td>0.276</td>
</tr>
<tr>
<td>12</td>
<td>+7.35</td>
<td>0.237</td>
</tr>
<tr>
<td>10</td>
<td>+6.15</td>
<td>0.198</td>
</tr>
<tr>
<td>8</td>
<td>+4.85</td>
<td>0.156</td>
</tr>
<tr>
<td>6</td>
<td>+3.65</td>
<td>0.113</td>
</tr>
<tr>
<td>4</td>
<td>+2.45</td>
<td>0.079</td>
</tr>
<tr>
<td>2</td>
<td>+1.35</td>
<td>0.044</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

**Note:** The calculation of the cross sensitivity factor is given under the results section of this topic.
Appendix XVI

TENSION TEST

Calculation of Eccentricity from Strain Distribution.

Assume plane cross sections before straining remain plane after straining.

Let specimen be loaded at point 'X' \((x,y)\) as shown, by a vertical load.

Strain at point \(B\) = Average Strain on cross section \((1 + \frac{6x}{d})\)

Strain at point \(A\) = Average Strain on cross section \((1 - \frac{6x}{d})\)

\[
\frac{12x}{d} = \frac{\text{Strain at point } B - \text{Strain at point } A}{\text{Average strain on cross section}}
\]

when \(d' = 3\) in.

\[
x = \frac{\text{Strain at point } B - \text{Strain at point } A}{4 \times \text{Average strain on cross section}}
\]

Similarly

\[
y = \frac{\text{Strain at point } C - \text{Strain at point } D}{4 \times \text{Average strain on cross section}}
\]

Maximum Stress on section due to eccentric loading.

\[
\text{Maximum Stress} = \text{Average Stress} \left(1 + \frac{6x}{d} + \frac{6y}{d}\right) \quad \cdots \quad \text{Eqn. 13.1}
\]

For \(d = 3\)"

\[
\text{Maximum Stress} = \text{Average Stress} \left(1 + 2x + 2y\right) \quad \cdots \quad \text{Eqn. 13.2}
\]
Appendix XVII

TENSION TEST No. 1

Details

Date ........ 28th June 1948
Specimen .... Plain concrete, 'A', section 3" x 3"
Age 18 months.
Gauges ........ B.T.C. Type SE/A/8, 2000 ohm nominal
Glue ........ High Solids Durofix (I.C.I. glue
No. 5462). Tested at 14 days.
Testing Machine ... Riehle 100,000 lb. screw-powered,
compound lever beam type machine.

Electrical Measurements ...
Multi-channel strain gauge set,
Marconi Galvo, Gauge current
3 milliamps.

Abstract of Readings (Plotted on Graph 13.3)

\[ Q = 10,150 \text{ ohms} \]

<table>
<thead>
<tr>
<th>Load (lb)</th>
<th>Stress lb/in²</th>
<th>Gauge No. 1</th>
<th></th>
<th>Gauge No. 3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Resistance change dQ (ohms)</td>
<td>( \frac{dQ}{Q} \times 10^4 )</td>
<td>Resistance change dQ (ohms)</td>
</tr>
<tr>
<td>0</td>
<td>4</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>325</td>
<td>36.1</td>
<td>+0.17</td>
<td>+0.17</td>
<td>-0.08</td>
</tr>
<tr>
<td>705</td>
<td>78.3</td>
<td>+0.25</td>
<td>+0.25</td>
<td>+0.17</td>
</tr>
<tr>
<td>1035</td>
<td>115.0</td>
<td>+0.44</td>
<td>+0.43</td>
<td>+0.37</td>
</tr>
<tr>
<td>1300</td>
<td>144.5</td>
<td>+0.64</td>
<td>+0.63</td>
<td>+0.70</td>
</tr>
<tr>
<td>1590</td>
<td>176.6</td>
<td>+0.81</td>
<td>+0.80</td>
<td>+0.69</td>
</tr>
<tr>
<td>1860</td>
<td>205.5</td>
<td>+0.53</td>
<td>+0.52</td>
<td>+0.43</td>
</tr>
<tr>
<td>1175</td>
<td>130.5</td>
<td>+0.29</td>
<td>+0.29</td>
<td>+0.16</td>
</tr>
<tr>
<td>610</td>
<td>67.7</td>
<td>+0.17</td>
<td>+0.17</td>
<td>-0.02</td>
</tr>
<tr>
<td>200</td>
<td>22.2</td>
<td>+0.68</td>
<td>+0.08</td>
<td>-0.11</td>
</tr>
</tbody>
</table>

Note (i) Gauges Nos 2 and 4 were unreliable, due to poor sticking.

(ii) Galvo. error at end of test corresponded to +0.13 ohms
Appendix XVIII

TENSION TESTS Nos 2 & 3

Details

Date ........ 22nd November 1943

Gauge Current ........ 12 ½ milliamps

Remaining data as per Test No.1

Abstract of Readings (Plotted on Graphs 15.3 and 15.4)

Nominal Q = 10,000 ohms

<table>
<thead>
<tr>
<th>Load (lb)</th>
<th>Stress 1b/in²</th>
<th>Fractional Resistance Change $\frac{dQ}{Q} \times 10^4$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Gauge No.1</td>
</tr>
<tr>
<td>Test No.2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>33</td>
<td>3.7</td>
<td>0</td>
</tr>
<tr>
<td>383</td>
<td>43.1</td>
<td>0.43</td>
</tr>
<tr>
<td>633</td>
<td>72.5</td>
<td>0.55</td>
</tr>
<tr>
<td>925</td>
<td>102.5</td>
<td>0.76</td>
</tr>
<tr>
<td>1193</td>
<td>133.0</td>
<td>0.87</td>
</tr>
<tr>
<td>1493</td>
<td>166.0</td>
<td>0.88</td>
</tr>
<tr>
<td>1798</td>
<td>200.0</td>
<td>1.20</td>
</tr>
<tr>
<td>33</td>
<td>3.7</td>
<td>0.36</td>
</tr>
<tr>
<td>Test No.3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>33</td>
<td>3.7</td>
<td>0</td>
</tr>
<tr>
<td>543</td>
<td>60.4</td>
<td>0.07</td>
</tr>
<tr>
<td>963</td>
<td>109.2</td>
<td>0.32</td>
</tr>
<tr>
<td>1268</td>
<td>141.0</td>
<td>0.52</td>
</tr>
<tr>
<td>1568</td>
<td>176.2</td>
<td>0.62</td>
</tr>
<tr>
<td>1863</td>
<td>207.0</td>
<td>0.72</td>
</tr>
<tr>
<td>1328</td>
<td>147.2</td>
<td>0.32</td>
</tr>
<tr>
<td>1013</td>
<td>113.0</td>
<td>0.25</td>
</tr>
<tr>
<td>648</td>
<td>72.0</td>
<td>+0.04</td>
</tr>
<tr>
<td>33</td>
<td>3.7</td>
<td>-0.05</td>
</tr>
</tbody>
</table>
Appendix XVIII (Cont.)

TENSION TEST No.2 (Cont.)

Calculations

Gauges No's 5, 6 and 1 varied consistently with the applied load, and the electrical strain for these gauges is taken from the curves (Graph 13.3) and used for calculations. Gauge No.3 behaved erratically and therefore its readings are not used.

At 200 lb/in$^2$ nominal stress, electrical strains $\times 10^4$ were:

- Gauge No.6 ... ... 1.0
- Gauge No.1 ... ... 1.0
- Gauge No.5 ... ... 0.7

Eccentricity of Loading

Using the formulae derived in Appendix XVI, co-ordinates of load-point from centroid of specimen:

$$x = 0.09''; y = 0$$ (See Fig. 15.6)

Young's Modulus

Taking Strain Sensitivity Factor of Gauges = 2.27

Average strain on specimen at 200 lb/in$^2$ = \( \frac{0.35 \times 10^{-4}}{2.27} \)

Then \( E = \frac{\text{stress}}{\text{strain}} = 200 \times 2.27 \text{ lb} \)

\( \text{strain} = 0.35 \times 10^{-4} \text{ in}^2 \)

\( = 5.55 \times 10^6 \text{lb/in}^2 \)

Note: Owing to the particular positioning of the gauges Nos. 5 and 6, the mean of their electrical readings gives average electrical strain \( \frac{\circ}{\circ} \) on the specimen very closely.
Appendix XVIII (cont)

TENSION TEST No.3 (Cont.)

Calculations

Gauges Nos. 6, 5, 3 and 1 varied consistently with the applied load, and the electrical strain values for the gauges are taken from the curves (Graph 13.4) and used for calculations At 200 lb/in² nominal stress, electrical strains x 10⁻⁴ were:

\[
\begin{array}{ccc}
\text{Gauge No.} & \text{...} & 1.0 \\
\text{Gauge No.1} & \text{...} & 1.0 \\
\text{Gauge No.5} & \text{...} & 0.8 \\
\text{Gauge No.3} & \text{...} & 0.8 \\
\end{array}
\]

Eccentricity of Loading

Using the formulae derived in Appendix XVI, co-ordinates from load-point from centroid of specimen:

\[ x = 0.06'' \; \; ; \; y = \text{zero} \; \text{(see Fig. 13.7)} \]

Young's Modulus

Taking Strain Sensitivity Factor of gauges = 2.27

Average strain on specimen at 200 lb/in² = \( \frac{0.9 \times 10^{-4}}{2.27} \)

Then \( E = \frac{\text{stress}}{\text{strain}} = 5.05 \times 10^8 \text{lb/in}^2 \)

Note: Owing to the particular positioning of the gauges Nos. 5 and 6, and 1 and 3, the mean of their electrical readings gives the average electrical strain \( \frac{\Delta \varepsilon}{\varepsilon} \) on the specimen very closely.
### Appendix XIX

**TENSION TEST No. 4**

**Details**

Date . . . . . . . . 23rd December 1948

Gauge Current . . . 1\(\frac{3}{4}\) milliamps.


**Abstract of Readings** (Plotted on Graph 13.5).

Nominal \(Q = 10,000\) ohms

<table>
<thead>
<tr>
<th>Load (lb)</th>
<th>Nominal Stress (lb/in(^2))</th>
<th>Fractional Resistance change (\frac{\Delta R}{Q} \times 10^4)</th>
<th>Mean Electric Strain (\frac{\Delta R}{Q} \times 10^4)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Gauge No. 1</td>
<td>Gauge No. 3</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>168</td>
<td>19</td>
<td>-0.04</td>
<td>-0.16</td>
</tr>
<tr>
<td>265</td>
<td>23</td>
<td>+0.06</td>
<td>+0.04</td>
</tr>
<tr>
<td>398</td>
<td>44</td>
<td>+0.15</td>
<td>+0.03</td>
</tr>
<tr>
<td>533</td>
<td>59</td>
<td>+0.16</td>
<td>+0.03</td>
</tr>
<tr>
<td>530</td>
<td>79</td>
<td>+0.17</td>
<td>+0.05</td>
</tr>
<tr>
<td>718</td>
<td>80</td>
<td>+0.16</td>
<td>+0.03</td>
</tr>
<tr>
<td>855</td>
<td>95</td>
<td>+0.19</td>
<td>+0.11</td>
</tr>
<tr>
<td>933</td>
<td>110</td>
<td>+0.25</td>
<td>+0.22</td>
</tr>
<tr>
<td>1113</td>
<td>124</td>
<td>+0.23</td>
<td>+0.28</td>
</tr>
<tr>
<td>1213</td>
<td>135</td>
<td>+0.23</td>
<td>+0.28</td>
</tr>
<tr>
<td>1513</td>
<td>146</td>
<td>+0.40</td>
<td>+0.42</td>
</tr>
<tr>
<td>1448</td>
<td>161</td>
<td>+0.43</td>
<td>+0.47</td>
</tr>
<tr>
<td>1588</td>
<td>176</td>
<td>+0.52</td>
<td>+0.46</td>
</tr>
<tr>
<td>1688</td>
<td>188</td>
<td>+0.54</td>
<td>+0.53</td>
</tr>
<tr>
<td>1828</td>
<td>203</td>
<td>+0.56</td>
<td>+0.67</td>
</tr>
<tr>
<td>2045</td>
<td>227</td>
<td>+0.63</td>
<td>+0.75</td>
</tr>
<tr>
<td>2188</td>
<td>244</td>
<td>+0.75</td>
<td>+0.82</td>
</tr>
<tr>
<td>2225</td>
<td>255</td>
<td>+0.78</td>
<td>+0.92</td>
</tr>
<tr>
<td>2418</td>
<td>269</td>
<td>+0.86</td>
<td>+1.02</td>
</tr>
<tr>
<td>2518</td>
<td>280</td>
<td>+0.93</td>
<td>+1.12</td>
</tr>
<tr>
<td>2622</td>
<td>291</td>
<td>+0.93</td>
<td>+1.18</td>
</tr>
<tr>
<td>2768</td>
<td>308</td>
<td>+1.01</td>
<td>+1.25</td>
</tr>
<tr>
<td>2868</td>
<td>319</td>
<td>+1.05</td>
<td>+1.30</td>
</tr>
<tr>
<td>2978</td>
<td>331</td>
<td>+1.19</td>
<td>+1.52</td>
</tr>
<tr>
<td>3088</td>
<td>345</td>
<td>+1.19</td>
<td>+1.60</td>
</tr>
<tr>
<td>3188</td>
<td>354</td>
<td>+1.24</td>
<td>+1.69</td>
</tr>
<tr>
<td>3508</td>
<td>363</td>
<td>Fracture</td>
<td>+1.81</td>
</tr>
</tbody>
</table>
Appendix XIX (Cont.)

TENSION TEST No.4 (Cont.)

Calculations

The gauges Nos. 1, 6 and 3 were in operation and, except for faulty initial balance and zero instability, gave steadily varying readings with the applied load. Using their electrical strain values from the curves (Graph 15.5) the following calculations were made:

At 200 lb/in² nominal stress, electrical strains $\times 10^4$ were:

<table>
<thead>
<tr>
<th>Gauge No.</th>
<th>...</th>
<th>...</th>
<th>0.70</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gauge No.6</td>
<td>...</td>
<td>...</td>
<td>0.80</td>
</tr>
<tr>
<td>Gauge No.3</td>
<td>...</td>
<td>...</td>
<td>0.85</td>
</tr>
</tbody>
</table>

Eccentricity of Loading

Using the formulae derived in Appendix XVI

Co-ordinates of load-point from centroid of specimen:

\[ x = 0.01 \text{ in.}; \quad y = 0.10 \text{ in.} \]  (Graph 15.11)

Young's Modulus

Taking Strain Sensitivity Factor of gauges = 2.27

Average strain on specimen at 200 lb/in² = $\frac{0.80 \times 10^{-4}}{2.27}$

Then $E = \frac{\text{stress}}{\text{strain}} = \frac{200 \times 10^4 \times 2.27}{0.80} \text{ lb/in}^2$

\[ = 5.63 \times 10^6 \text{ lb/in}^2 \]
Appendix XIX (Cont.)

Maximum Stress due to eccentric loading.

As shown in Appendix XVI

Maximum stress = Average Stress \( (1 + 2x + 2y) \)

At elastic breakdown \((x = 0.01 \text{ in.}, \ y = 0.10 \text{ in.})\)

Maximum Stress = \(1.22 \times\) Average Stress

At fracture load (Nominal Stress = 354 \(\text{lb/in}^2\))

From Graphs, electrical strains \(\times 10^4\) were:

<table>
<thead>
<tr>
<th>Gauge No.</th>
<th>(\times 10^4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. 1</td>
<td>1.33</td>
</tr>
<tr>
<td>No. 6</td>
<td>1.55</td>
</tr>
<tr>
<td>No. 3</td>
<td>1.92</td>
</tr>
</tbody>
</table>

Eccentricity of Loading

Using above values:

\[ x = 0.02 \text{ in.}, \ y = 0.03 \text{ in.} \]

Maximum Stress due to eccentric loading:

Using above values:

Maximum Stress at fracture = \(1.20 \times\) Average Stress

\[ = 436 \text{ lb/in}^2 \]

Note: Mean strain on the cross section.

Owing to the particular positioning of the gauges, Nos 5 and 6, the mean of their electrical readings gives the average electrical strain \(\varepsilon_0\) on the specimen very closely.
Appendix XX

**TENSION TEST No. 5**

**Details:**

<table>
<thead>
<tr>
<th>Date</th>
<th>19th Feb. 1949</th>
</tr>
</thead>
<tbody>
<tr>
<td>Specimen</td>
<td>Plain concrete &quot;B&quot; section 3&quot; x 2.9&quot;</td>
</tr>
<tr>
<td>Age</td>
<td>47 days. Curing: damp 7 days, water 14 days, air 26 days.</td>
</tr>
<tr>
<td>Gauges</td>
<td>B.T.C. Type SE/A/27(200 ohm nominal)</td>
</tr>
<tr>
<td>Glue</td>
<td>High Solids Durefix (I.C.I. glue No.3462) Tested at 9 days.</td>
</tr>
<tr>
<td>Testing machine</td>
<td>Riehle 100,000 lb. screw-powered, compound lever beam type machine.</td>
</tr>
<tr>
<td>Mechanical Extensometer</td>
<td>V.G. Davies instrument 6&quot; gauge length.</td>
</tr>
<tr>
<td>Electrical Measurements</td>
<td>Multi-channel strain gauge set, Cambridge Galvanometer. Gauge Current 17.5 milliamps.</td>
</tr>
</tbody>
</table>

**Extensometer Calibration** (Plotted on Graph 13.7)

<table>
<thead>
<tr>
<th>Micrometer Reading (x 10³in.)</th>
<th>Fixed Scale Reading (mm)</th>
<th>Moving Scale Reading (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.05</td>
<td>64.77</td>
<td>73.37</td>
</tr>
<tr>
<td>1.75</td>
<td></td>
<td>71.71</td>
</tr>
<tr>
<td>1.55</td>
<td></td>
<td>70.98</td>
</tr>
<tr>
<td>1.30</td>
<td></td>
<td>70.24</td>
</tr>
<tr>
<td>1.00</td>
<td>Constant</td>
<td>69.00</td>
</tr>
<tr>
<td>0.75</td>
<td></td>
<td>67.90</td>
</tr>
<tr>
<td>0.50</td>
<td>at</td>
<td>66.69</td>
</tr>
<tr>
<td>0.30</td>
<td></td>
<td>66.01</td>
</tr>
<tr>
<td>0</td>
<td>64.77</td>
<td>64.70</td>
</tr>
<tr>
<td>+ 0.25</td>
<td></td>
<td>63.55</td>
</tr>
<tr>
<td>+ 0.50</td>
<td></td>
<td>62.30</td>
</tr>
<tr>
<td>+ 0.75</td>
<td></td>
<td>61.51</td>
</tr>
<tr>
<td>+ 1.00</td>
<td></td>
<td>60.12</td>
</tr>
<tr>
<td>+ 1.25</td>
<td></td>
<td>59.10</td>
</tr>
<tr>
<td>+ 1.50</td>
<td></td>
<td>58.30</td>
</tr>
<tr>
<td>+ 1.70</td>
<td></td>
<td>57.47</td>
</tr>
<tr>
<td>+ 2.00</td>
<td></td>
<td>56.70</td>
</tr>
</tbody>
</table>
Appendix XX (Cont.)

Extensometer Calibration (Cont.)

From Graph 13.7  0.001 in. on micrometer = 4.55 cm on scale

Calibration Condition.

Micrometer Pivot

\[
0.001 \times \frac{4.7}{3.3} = 0.001425''
\]

= 4.55 cm on scale.

Test Condition

Pivot Specimen

\[
\frac{0.001425}{4.55} = 1 \text{ cm. on scale}
\]

\[8''
\]

FIG. 13.5

Extension at Specimen = \[
\frac{0.001425}{4.55} \times \frac{5.3}{8} = 0.0001292 \text{ in.}
\]

\[8''
\]

1 cm. at scale = 0.0001292 inches extension.

= 0.0000215 strain.
Appendix XX (Cont.)

Abstract of Readings (Electrical Gauge Readings)

(Plotted on Graph 15.8)

Nominal $Q = 10,000$ ohms.

<table>
<thead>
<tr>
<th>Nominal Stress (lb/in²)</th>
<th>Fractional Resistance Change $\frac{\Delta R}{R} \times 10^4$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Gauge No.7</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
</tr>
<tr>
<td>24</td>
<td>+ 0.06</td>
</tr>
<tr>
<td>34</td>
<td>+ 0.10</td>
</tr>
<tr>
<td>46</td>
<td>0.020</td>
</tr>
<tr>
<td>58</td>
<td>0.30</td>
</tr>
<tr>
<td>75</td>
<td>0.50</td>
</tr>
<tr>
<td>93</td>
<td>0.57</td>
</tr>
<tr>
<td>107</td>
<td>0.57</td>
</tr>
<tr>
<td>113</td>
<td>0.57</td>
</tr>
<tr>
<td>132</td>
<td>0.59</td>
</tr>
<tr>
<td>146</td>
<td>0.68</td>
</tr>
<tr>
<td>162</td>
<td>0.78</td>
</tr>
<tr>
<td>174</td>
<td>0.84</td>
</tr>
<tr>
<td>186</td>
<td>0.94</td>
</tr>
<tr>
<td>198</td>
<td>1.03</td>
</tr>
<tr>
<td>211</td>
<td>1.06</td>
</tr>
<tr>
<td>224</td>
<td>1.13</td>
</tr>
<tr>
<td>238</td>
<td>1.24</td>
</tr>
<tr>
<td>250</td>
<td>1.36</td>
</tr>
<tr>
<td>254</td>
<td>1.40</td>
</tr>
<tr>
<td>276</td>
<td>1.49</td>
</tr>
<tr>
<td>287</td>
<td>1.59</td>
</tr>
<tr>
<td>296</td>
<td>1.69</td>
</tr>
<tr>
<td>301</td>
<td>Rupture at upper change of section</td>
</tr>
<tr>
<td>After fracture</td>
<td>+ 0.16</td>
</tr>
</tbody>
</table>
Appendix XX (Cont.)

Abstract of Readings (Average Strain Measurements)
(Plotted on Graph 13.9)

**Conditioning Run No. 1**

<table>
<thead>
<tr>
<th>Nominal Stress ( \text{lb/\text{in}^2} )</th>
<th>Mechanical Measurement of Mean Strain ( \times 10^5 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>0</td>
</tr>
<tr>
<td>40</td>
<td>0.54</td>
</tr>
<tr>
<td>56</td>
<td>0.86</td>
</tr>
<tr>
<td>76</td>
<td>1.43</td>
</tr>
<tr>
<td>102</td>
<td>1.76</td>
</tr>
<tr>
<td>117</td>
<td>2.10</td>
</tr>
<tr>
<td>135</td>
<td>2.36</td>
</tr>
<tr>
<td>4</td>
<td>0.32</td>
</tr>
</tbody>
</table>

**Conditioning Run No. 2**

<table>
<thead>
<tr>
<th>Nominal Stress ( \text{lb/\text{in}^2} )</th>
<th>Mechanical Measurement of Mean Strain ( \times 10^5 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>0</td>
</tr>
<tr>
<td>38</td>
<td>0.49</td>
</tr>
<tr>
<td>63</td>
<td>0.69</td>
</tr>
<tr>
<td>94</td>
<td>1.43</td>
</tr>
<tr>
<td>125</td>
<td>1.95</td>
</tr>
<tr>
<td>136</td>
<td>2.41</td>
</tr>
<tr>
<td>4</td>
<td>0.17</td>
</tr>
</tbody>
</table>
Appendix XX (Cont.)

Abstract of Readings (Average Strain Measurements) (Plotted on Graph 13.9)

<table>
<thead>
<tr>
<th>Nominal Stress (lb/in²)</th>
<th>Mechanical Measurement of mean strain x 10⁻²</th>
<th>Electrical Measurement of mean strain (dQ/Q) x 10⁻²</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>24</td>
<td>0.01</td>
<td>0.10</td>
</tr>
<tr>
<td>34</td>
<td>0.02</td>
<td>0.07</td>
</tr>
<tr>
<td>46</td>
<td>0.03</td>
<td>0.17</td>
</tr>
<tr>
<td>58</td>
<td>0.10</td>
<td>0.24</td>
</tr>
<tr>
<td>75</td>
<td>0.14</td>
<td>0.30</td>
</tr>
<tr>
<td>83</td>
<td>0.16</td>
<td>0.33</td>
</tr>
<tr>
<td>107</td>
<td>0.19</td>
<td>0.42</td>
</tr>
<tr>
<td>113</td>
<td>0.21</td>
<td>0.46</td>
</tr>
<tr>
<td>122</td>
<td>0.28</td>
<td>0.52</td>
</tr>
<tr>
<td>146</td>
<td>0.37</td>
<td>0.55</td>
</tr>
<tr>
<td>162</td>
<td>0.33</td>
<td>0.60</td>
</tr>
<tr>
<td>174</td>
<td>0.39</td>
<td>0.65</td>
</tr>
<tr>
<td>186</td>
<td>0.55</td>
<td>0.71</td>
</tr>
<tr>
<td>193</td>
<td>0.37</td>
<td>0.76</td>
</tr>
<tr>
<td>211</td>
<td>0.41</td>
<td>0.81</td>
</tr>
<tr>
<td>224</td>
<td>0.45</td>
<td>0.83</td>
</tr>
<tr>
<td>238</td>
<td>0.47</td>
<td>0.84</td>
</tr>
<tr>
<td>250</td>
<td>0.49</td>
<td>0.93</td>
</tr>
<tr>
<td>264</td>
<td>0.51</td>
<td>1.05</td>
</tr>
<tr>
<td>276</td>
<td>0.56</td>
<td>1.08</td>
</tr>
<tr>
<td>287</td>
<td>0.56</td>
<td>1.15</td>
</tr>
<tr>
<td>293</td>
<td>0.60</td>
<td>1.23</td>
</tr>
<tr>
<td>301</td>
<td>Fracture</td>
<td></td>
</tr>
</tbody>
</table>

Note: Owing to the particular positioning of the gauges Nos. 8 and 10, the mean of their electrical readings gives the average electrical strain (dQ/Q) on the specimen very closely.
Appendix XX (Cont.)

TENSION TEST No. 5 (Cont.)

Calculations

The four electrical gauges worked satisfactorily, but as gauges Nos. 8, 9 and 10 were all situated at approximately the same cross section of the specimen, their readings were used for all calculations. The readings of gauge No. 7, which are included in some instances, are a little higher than is compatible with the readings of the other gauges, but this may well be due to its close proximity to a shoulder of the specimen where stress concentrations occur.

(a) Eccentricity of loading.

The eccentricity may be calculated at any load from the readings of the three gauges using the relationship derived in Appendix XVI. The readings used are corrected for the zero error when necessary. (Plotted in Graph 15.12)

<table>
<thead>
<tr>
<th>Nominal Stress lb/in²</th>
<th>Fractional Resistance Change $\frac{dQ}{Q} \times 10^4$</th>
<th>Eccentricity</th>
<th>x (in.)</th>
<th>y (in.)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Gauge No. 7</td>
<td>Gauge No. 8</td>
<td>Gauge No. 9</td>
<td>Gauge No. 10</td>
</tr>
<tr>
<td>120</td>
<td>-</td>
<td>0.40</td>
<td>0.19</td>
<td>0.55</td>
</tr>
<tr>
<td>175</td>
<td>0.84</td>
<td>0.56</td>
<td>0.31</td>
<td>0.75</td>
</tr>
<tr>
<td>260</td>
<td>1.37</td>
<td>0.92</td>
<td>0.55</td>
<td>1.16</td>
</tr>
<tr>
<td>296</td>
<td>1.63</td>
<td>1.20</td>
<td>0.65</td>
<td>1.26</td>
</tr>
</tbody>
</table>
Appendix XX (Cont.)

(b) Strain Sensitivity of Gauges

From Graph 13.10 of the "electrical strain \( \frac{dQ}{Q} \)" against the mechanical measurement \( \varepsilon \):

\[
\text{Strain Sensitivity Factor} = \frac{\frac{dQ}{Q}}{\varepsilon} = 2.08
\]

Note: A previous calibration of this type of gauge on a steel tension specimen gave factor = 2.12 (See Para. 11.3)

(c) Young's Modulus

From Graph 15.9 of average stress-strain.

By mechanical extensometer:

Initial Modulus \( E = 5.55 \times 10^6 \text{lb/in}^2 \)

By electrical gauges:

Initial Modulus \( E = 5.68 \times 10^6 \text{lb/in}^2 \) using

S.S.F. = 2.03 or using an independent calibration factor, S.S.F. = 2.12

Initial Modulus \( E = 5.68 \times 10^6 \text{lb/in}^2 \)
Appendix XXI

COMPRESSION TEST No. 1

Details

Date ........ 23rd April, 1948
Specimen .... Plain Concrete 'C', 10 in. x 5 in. dia. cylinder. Age 18 months.
Gauges ........ B.T.C. Type SE/A/3, 2100 ohms nominal.
Glue ........ High Solids Dureflex (I.C.I. glue No.3462) Tested at 14 days.
Packing .......... Millboard pads between specimen and platens.
Testing Machine .......... Rischio 100,000 lb. screw powered, compound lever type testing machine.
Electrical Measurements .......... Multi-channel strain gauge set.

Measurements .......... Marsoci Galvanometer,
Gauge Current, 15 milliamps.

COMPRESSION TESTS No's 2, 3 and 4

Details

Dates ........ 30th April, 3rd and 12th May 1948, respectively.
Mechanical Compressometer .......... Hamilton Instrument.
Electrical Measurements .......... Gauge Current, 14 milliamps.
Remaining Data .......... As per Test No.1 (above).

Note :- For Abstracts of Results see next four pages.
### ABSTRACT OF READINGS (Compression Test No.1)

<table>
<thead>
<tr>
<th>Nominal Stress 1b/in²</th>
<th>Fractional Resistance Change $\frac{\Delta R}{R} \times 10^4$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Gauge No.1</td>
</tr>
<tr>
<td><strong>LOADING</strong></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>243</td>
<td>-1.5</td>
</tr>
<tr>
<td>431</td>
<td>-2.6</td>
</tr>
<tr>
<td>705</td>
<td>-3.6</td>
</tr>
<tr>
<td>960</td>
<td>-4.6</td>
</tr>
<tr>
<td>1290</td>
<td>-5.0</td>
</tr>
<tr>
<td>1550</td>
<td>-7.0</td>
</tr>
<tr>
<td>1800</td>
<td>-7.7</td>
</tr>
<tr>
<td><strong>UNLOADING</strong></td>
<td></td>
</tr>
<tr>
<td>1370</td>
<td>+6.6</td>
</tr>
<tr>
<td>1015</td>
<td>+6.5</td>
</tr>
<tr>
<td>794</td>
<td>+5.9</td>
</tr>
<tr>
<td>471</td>
<td>+4.9</td>
</tr>
<tr>
<td>161</td>
<td>+3.4</td>
</tr>
<tr>
<td>0</td>
<td>+1.5</td>
</tr>
</tbody>
</table>
### ABSTRACT OF READINGS (Compression No. 2)

<table>
<thead>
<tr>
<th>Nominal Stress (lb/in²)</th>
<th>Fractional Resistance Change $\frac{dR}{R} \times 10^4$</th>
<th>Mech. Measure of Strain $\delta$ $\times 10^4$</th>
<th>*Mean Electric Strain $\varepsilon$ $\times 10^4$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>LOADING</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>42</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>771</td>
<td>-0.56 -2.00 +0.60 +4.00 -0.49 -3.85 -0.62 -4.86 -3.17</td>
<td>2.70</td>
<td>-4.47</td>
</tr>
<tr>
<td>1810</td>
<td>-9.14 -5.14 +1.42 -8.28 +1.72 -9.61 +1.72 -10.33 -8.00</td>
<td>5.70</td>
<td>-9.93</td>
</tr>
<tr>
<td><strong>UNLOADING</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>970</td>
<td>-5.44 -2.58 +0.58 -5.72 +0.86 -5.77 +0.86 -6.72 -4.33</td>
<td>3.92</td>
<td>-6.33</td>
</tr>
<tr>
<td>715</td>
<td>-4.22 -1.72 +0.35 -4.50 +0.55 -4.52 +0.55 -5.14 -3.14</td>
<td>3.20</td>
<td>-4.86</td>
</tr>
<tr>
<td>462</td>
<td>-2.70 -1.00 0 -3.38 +0.14 -3.30 +0.23 -3.57 -2.00</td>
<td>2.42</td>
<td>-3.49</td>
</tr>
<tr>
<td>53</td>
<td>0 +0.72 0 -0.42 +0.28 -0.55 -0.14 -0.33 -0.28</td>
<td>0.75</td>
<td>-0.58</td>
</tr>
</tbody>
</table>

*Note: Obtained from Readings $\frac{(4) + (9)}{2} + \frac{(6) - (1)}{10}$*
## ABSTRACT OF READINGS (Compression Test No.5)

<table>
<thead>
<tr>
<th>Nominal Stress</th>
<th>Fractional Resistance Change $\frac{dQ}{Q} \times 10^4$</th>
<th>Mean Electric Strain $\frac{dQ}{Q} \times 10^4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>61 lb/in²</td>
<td>Gauge No.1: 0</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gauge No.2: 0</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gauge No.3: 0</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gauge No.4: 0</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gauge No.5: 0</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gauge No.6: 0</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gauge No.7: 0</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gauge No.8: 0</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gauge No.9: 0</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gauge No.10: 0</td>
<td></td>
</tr>
<tr>
<td>UNLOADING</td>
<td>1500: 0</td>
<td>-0.86</td>
</tr>
<tr>
<td></td>
<td>1077: 0</td>
<td>-0.86</td>
</tr>
<tr>
<td></td>
<td>630: 0</td>
<td>-0.86</td>
</tr>
<tr>
<td></td>
<td>434: 0</td>
<td>-0.86</td>
</tr>
<tr>
<td></td>
<td>81: 0</td>
<td>-0.86</td>
</tr>
</tbody>
</table>

**Note:** Final Galvanometer Error $= +0.28 \times 10^{-4}$ Electrical Strain

$\#$ obtained from Readings $(4) + (6) + (6) - (1)$
# ABSTRACT OF READINGS (Compression Test No. 4)

<table>
<thead>
<tr>
<th>Nominal Stress</th>
<th>Fractional Resistance Change $\frac{dQ}{Q} \times 10^4$</th>
<th>Mech. Measure of Strain $\frac{dQ}{Q} \times 10^4$</th>
<th>Mean Elect. Strain $\frac{dQ}{Q} \times 10^4$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Gauge No. 1</td>
<td>Gauge No. 2</td>
<td>Gauge No. 3</td>
</tr>
<tr>
<td><strong>LOADING</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>250</td>
<td>2.17</td>
<td>1.67</td>
<td>1.00</td>
</tr>
<tr>
<td>550</td>
<td>2.17</td>
<td>1.67</td>
<td>1.00</td>
</tr>
<tr>
<td>948</td>
<td>2.17</td>
<td>1.67</td>
<td>1.00</td>
</tr>
<tr>
<td>1423</td>
<td>2.17</td>
<td>1.67</td>
<td>1.00</td>
</tr>
<tr>
<td>1890</td>
<td>2.17</td>
<td>1.67</td>
<td>1.00</td>
</tr>
</tbody>
</table>

| **UNLOADING**  |             |             |             |             |             |             |             |             |             |             | 1500             | -6.14         | +0.93          | -8.86          | +0.57          | -8.50          | -9.43          | 5.62          | -6.18          |
| 1000           | -7.00       | -5.50       | 0           | -7.00       | +0.33       | -6.99       | 0           | -6.67       | -7.77        | 4.37          | -6.65          |
| 685            | -5.23       | -4.23       | -0.31       | -5.23       | -0.14       | -4.94       | -0.23       | -4.93       | -5.43        | 4.26          | -4.88          |
| 38             | -1.58       | -0.98       | -0.87       | -1.58       | -1.00       | -0.55       | -0.57       | -0.50       | 0.73          | -0.58         |

# Note: Obtained from Readings \[
\frac{(4) + (3)}{2} + \frac{(8) - (1)}{10}
\]
Appendix XXII

COMPRESSION TEST No.5

Details

Date . . . . . . . 13th May, 1948

Electrical Measurements . . Gauge Current, 15 milliamps.

Remaining Data . . . As per Test No.1

Abstract of Readings (Plotted on Graph 14.15)

\[ Q = 10,000 \text{ ohms} \text{ (nominal)} \]

<table>
<thead>
<tr>
<th>Nominal Stress (\text{lb/in}^2)</th>
<th>Fractional Resistance Change (\frac{dQ}{Q} \times 10^4)</th>
<th>Gauge No.5</th>
<th>Gauge No.6</th>
<th>Gauge No.7</th>
</tr>
</thead>
<tbody>
<tr>
<td>47</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>129</td>
<td>0.33</td>
<td>-0.33</td>
<td>-0.33</td>
<td>-0.17</td>
</tr>
<tr>
<td>217</td>
<td>0.75</td>
<td>-0.44</td>
<td>-0.31</td>
<td></td>
</tr>
<tr>
<td>292</td>
<td>1.00</td>
<td>-0.87</td>
<td>-0.31</td>
<td></td>
</tr>
<tr>
<td>385</td>
<td>-1.60</td>
<td>-0.67</td>
<td>0.33</td>
<td></td>
</tr>
<tr>
<td>485</td>
<td>-1.15</td>
<td>-0.54</td>
<td>-0.33</td>
<td></td>
</tr>
<tr>
<td>555</td>
<td>-1.12</td>
<td>-0.50</td>
<td>0.33</td>
<td></td>
</tr>
<tr>
<td>640</td>
<td>-1.08</td>
<td>-0.33</td>
<td>-0.31</td>
<td></td>
</tr>
<tr>
<td>720</td>
<td>-1.00</td>
<td>-0.23</td>
<td>-0.33</td>
<td></td>
</tr>
<tr>
<td>803</td>
<td>-1.17</td>
<td>-0.39</td>
<td>-0.15</td>
<td></td>
</tr>
<tr>
<td>890</td>
<td>-1.08</td>
<td>-0.17</td>
<td>0.03</td>
<td></td>
</tr>
<tr>
<td>982</td>
<td>-1.00</td>
<td>-0.17</td>
<td>-0.03</td>
<td></td>
</tr>
<tr>
<td>1073</td>
<td>-1.00</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>1280</td>
<td>-0.86</td>
<td>+0.03</td>
<td>+0.20</td>
<td></td>
</tr>
<tr>
<td>1732</td>
<td>-0.87</td>
<td>+0.67</td>
<td>+0.67</td>
<td></td>
</tr>
</tbody>
</table>
### COMPRESSION TEST NO. 6

**Details**

Date .............. 25th May, 1948

Mechanical
Compressometer * * Hamilton Instrument.

Electrical Readings.  Gauge Current, 15½ milliamps.

Remainng Data . . . As per Test No. 1

**Abstract of Readings (Plotted on Graph 14.2)**

$Q = 10,000$ ohms (nominal.)

<table>
<thead>
<tr>
<th>Nominal Stress (lb/in²)</th>
<th>Fractional Resistance Change $\frac{dQ}{Q} \times 10^4$</th>
<th>Mech. Measure of Strain $x 10^4$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>TEST 6(a)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>61</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>161</td>
<td>-0.14</td>
<td>-0.32</td>
</tr>
<tr>
<td>511</td>
<td>0</td>
<td>0.32</td>
</tr>
<tr>
<td>436</td>
<td>-0.42</td>
<td>-0.28</td>
</tr>
<tr>
<td>792</td>
<td>-0.03</td>
<td>0.33</td>
</tr>
<tr>
<td>1245</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1700</td>
<td>+0.54</td>
<td>1.14</td>
</tr>
<tr>
<td><strong>TEST 6(b)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>33</td>
<td>+0.15</td>
<td>-0.15</td>
</tr>
<tr>
<td>171</td>
<td>0</td>
<td>-0.39</td>
</tr>
<tr>
<td>322</td>
<td>-0.23</td>
<td>-0.36</td>
</tr>
<tr>
<td>434</td>
<td>-0.14</td>
<td>-0.31</td>
</tr>
<tr>
<td>607</td>
<td>0</td>
<td>0.14</td>
</tr>
<tr>
<td>1245</td>
<td>+0.23</td>
<td>0.50</td>
</tr>
<tr>
<td>1763</td>
<td>+1.00</td>
<td>1.38</td>
</tr>
<tr>
<td>52</td>
<td>+0.31</td>
<td>0</td>
</tr>
</tbody>
</table>

### Appendix XXIII

-55-
### Appendix XXIV

**COMPRESSION TEST No. 7**

**Details**

- **Date**: 31st May 1943
- **Mechanical Compressometer**: Hamilton Instrument
- **Electrical Readings**: Gauge Current, 15 milliamps
- **Remaining Data**: As per Test No. 1

#### Abstract of Readings

\( Q = 10,000 \text{ ohms (nominal)} \)

<table>
<thead>
<tr>
<th>Nominal Stress (lb/in²)</th>
<th>Fractional Resistance Change ( \frac{Q}{Q_0} \times 10^4 )</th>
<th>Mechanical Measure of Strain ( \times 10^4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Gauge No. 4</td>
<td>Gauge No. 5</td>
</tr>
<tr>
<td><strong>TEST 7(a)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>55</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>155</td>
<td>-0.38</td>
<td>-0.50</td>
</tr>
<tr>
<td>341</td>
<td>-1.36</td>
<td>-0.23</td>
</tr>
<tr>
<td>423</td>
<td>+2.00</td>
<td>-0.21</td>
</tr>
<tr>
<td>766</td>
<td>+5.44</td>
<td>+0.27</td>
</tr>
<tr>
<td>1205</td>
<td>+5.64</td>
<td>+0.26</td>
</tr>
<tr>
<td>1715</td>
<td>+7.79</td>
<td>+1.55</td>
</tr>
<tr>
<td><strong>TEST 7(b)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>51</td>
<td>+0.28</td>
<td>-0.22</td>
</tr>
<tr>
<td>513</td>
<td>+3.41</td>
<td>+0.14</td>
</tr>
<tr>
<td>1630</td>
<td>+7.72</td>
<td>+1.55</td>
</tr>
<tr>
<td><strong>TEST 7(c)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>50</td>
<td>+0.14</td>
<td>-0.33</td>
</tr>
<tr>
<td>527</td>
<td>+3.42</td>
<td>0</td>
</tr>
<tr>
<td>1715</td>
<td>+7.68</td>
<td>+1.54</td>
</tr>
<tr>
<td><strong>TEST 7(d)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>51</td>
<td>0</td>
<td>-0.46</td>
</tr>
<tr>
<td>523</td>
<td>-3.61</td>
<td>-0.23</td>
</tr>
<tr>
<td>1760</td>
<td>-8.00</td>
<td>+1.38</td>
</tr>
</tbody>
</table>
Deductions from Test 7(a).

<table>
<thead>
<tr>
<th>Stress (lb/in²)</th>
<th>Mean Electrical Longitudinal Strain $\frac{dE}{E} \times 10^4$</th>
<th>Mean Electrical Lateral Strain $\frac{dY}{Y} \times 10^4$</th>
<th>Poisson's Ratio $\nu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>55</td>
<td>0</td>
<td>0</td>
<td>0.145</td>
</tr>
<tr>
<td>153</td>
<td>+0.55</td>
<td>+0.03</td>
<td>0.202</td>
</tr>
<tr>
<td>341</td>
<td>+1.68</td>
<td>+0.34</td>
<td>0.310</td>
</tr>
<tr>
<td>423</td>
<td>+2.29</td>
<td>+0.71</td>
<td>0.234</td>
</tr>
<tr>
<td>766</td>
<td>+4.10</td>
<td>+0.96</td>
<td>0.238</td>
</tr>
<tr>
<td>1203</td>
<td>+6.50</td>
<td>+1.55</td>
<td>0.238</td>
</tr>
<tr>
<td>1715</td>
<td>+8.89</td>
<td>+2.29</td>
<td>0.238</td>
</tr>
</tbody>
</table>

# Obtained from mean readings of gauges Nos. 4 and 9.

# Obtained from mean readings of gauges Nos. 5 and 8

Note: Readings of gauge No.5 have been corrected for zero error.
Appendix XXV

COMPRESSION TEST No.8

Details

Date . . . . . . 7th June, 1948

Electrical Readings. Gauge Current, 15.6 milliamps.

Remaining Data . . . As per Test No.1

Abstract of Readings

Q = 10,000 ohms (nominal)

<table>
<thead>
<tr>
<th>Nominal Stress (lb/in²)</th>
<th>Fractional Resistance Change ( \frac{\delta Q}{Q} \times 10^4 )</th>
<th>( \frac{\delta Q}{Q} \times 10^4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Gauge No.1</td>
<td>Gauge No.4</td>
</tr>
<tr>
<td>LOGGING</td>
<td></td>
<td></td>
</tr>
<tr>
<td>51</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>155</td>
<td>-0.69</td>
<td>-0.38</td>
</tr>
<tr>
<td>257</td>
<td>-1.14</td>
<td>-0.67</td>
</tr>
<tr>
<td>374</td>
<td>-1.75</td>
<td>-1.23</td>
</tr>
<tr>
<td>463</td>
<td>-2.45</td>
<td>-1.85</td>
</tr>
<tr>
<td>1000</td>
<td>-5.46</td>
<td>-4.33</td>
</tr>
<tr>
<td>1450</td>
<td>-7.72</td>
<td>-6.88</td>
</tr>
<tr>
<td>1595</td>
<td>-8.62</td>
<td>-7.17</td>
</tr>
<tr>
<td>UNLOADING</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1350</td>
<td>-7.69</td>
<td>-6.33</td>
</tr>
<tr>
<td>1040</td>
<td>-6.33</td>
<td>-5.14</td>
</tr>
<tr>
<td>360</td>
<td>-2.31</td>
<td>-1.63</td>
</tr>
<tr>
<td>57</td>
<td>-0.45</td>
<td>-0.64</td>
</tr>
</tbody>
</table>

* Note: Obtained from Readings \( \frac{(4) + (9)}{2} \) + \( \frac{(6) - (1)}{10} \)
Appendix XXVI

COMPRESSION TEST No.9

Details

Date ........... 11th June, 1948
Packing ........ No packing between the specimen and the steel platens.
Mechanical Compressometer .... Hamilton Instrument.
Electrical Readings .... Gauge Current, 14 milliamps.
Remaining Data .. As per Test No.1.

Abstract of Readings (Plotted on Graph 14.1)

\[ Q = 10,000 \text{ ohms (nominal)} \]

<table>
<thead>
<tr>
<th>Nominal Stress ( \text{lb/in}^2 )</th>
<th>Fractional Resistance Change ( \frac{d\Omega}{Q} \times 10^4 )</th>
<th>Mech. Measure of Strain ( \frac{d\Omega}{Q} \times 10^4 )</th>
<th>Mean Elect. Strain ( \frac{d\Omega}{Q} \times 10^4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \text{TEST 9(a)} )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>220</td>
<td>-1.00</td>
<td>-1.00</td>
<td>-0.96</td>
</tr>
<tr>
<td>350</td>
<td>0.00</td>
<td>-2.00</td>
<td>-3.50</td>
</tr>
<tr>
<td>1790</td>
<td>8.67</td>
<td>9.56</td>
<td>-7.07</td>
</tr>
<tr>
<td>1705</td>
<td>8.64</td>
<td>7.82</td>
<td>-3.80</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( \text{TEST 9(b)} )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>24</td>
<td>-0.64</td>
<td>-0.97</td>
<td>-0.87</td>
</tr>
<tr>
<td>56</td>
<td>-0.64</td>
<td>-0.97</td>
<td>-0.59</td>
</tr>
<tr>
<td>155</td>
<td>-1.14</td>
<td>-1.22</td>
<td>-1.44</td>
</tr>
<tr>
<td>257</td>
<td>-1.04</td>
<td>-1.64</td>
<td>-2.09</td>
</tr>
<tr>
<td>510</td>
<td>2.07</td>
<td>2.97</td>
<td>-3.55</td>
</tr>
<tr>
<td>1025</td>
<td>-5.31</td>
<td>-5.94</td>
<td>-6.65</td>
</tr>
<tr>
<td>1850</td>
<td>-8.64</td>
<td>-8.22</td>
<td>-3.60</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( \text{TEST 9(c)} )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>520</td>
<td>-2.27</td>
<td>-2.11</td>
<td>-3.32</td>
</tr>
<tr>
<td>1275</td>
<td>-6.75</td>
<td>-5.86</td>
<td>-7.79</td>
</tr>
<tr>
<td>1700</td>
<td>-9.25</td>
<td>-8.03</td>
<td>-10.73</td>
</tr>
</tbody>
</table>

Load accidentally increased to 2450 \( \text{lb/in}^2 \) before unloading.
\( \rho \) Obtained by readings \( \frac{(4) + (9) + (6) - (1)}{10} \).
Appendix XXVII

COMPRESSION TEST No. 10

Details

Date . . . . . . . . 14th June, 1943
Mechanical Comparator . . Hamilton Instrument
Packing . . . . . . Soft red rubber sheets approx. 1/8" thick were placed between the specimen and the steel platens.
Electrical Readings . . Gauge Current, 15 milliamps

Abstract of Readings (Plotted on Graph 14.1)

\[ Q = 10,000 \text{ ohms (nominal)} \]

<table>
<thead>
<tr>
<th>Nominal Stress (lb/in²)</th>
<th>Fractional Resistance Change ( \frac{dQ}{Q} \times 10^4 )</th>
<th>Mech. Measure of Strain ( \frac{dQ}{Q} \times 10^4 )</th>
<th># Mean Elect. Strain</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Gauge No. 1</td>
<td>Gauge No. 4</td>
<td>Gauge No. 6</td>
</tr>
<tr>
<td>TEST No. 10(a) Packing - Two thicknesses of rubber</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>153</td>
<td>-0.90</td>
<td>-1.00</td>
<td>-1.06</td>
</tr>
<tr>
<td>1510</td>
<td>-5.25</td>
<td>-3.75</td>
<td>-5.67</td>
</tr>
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<td>1080</td>
<td>-6.50</td>
<td>-6.77</td>
<td>-7.05</td>
</tr>
<tr>
<td>1515</td>
<td>-9.16</td>
<td>-8.55</td>
<td>-8.94</td>
</tr>
<tr>
<td>0</td>
<td>-1.50</td>
<td>-0.16</td>
<td>+0.31</td>
</tr>
<tr>
<td>TEST No. 10(b) Packing - Two thicknesses of rubber</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>115</td>
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<td>-0.66</td>
<td>-1.15</td>
</tr>
<tr>
<td>420</td>
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<td>-3.61</td>
<td>-3.60</td>
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<tr>
<td>930</td>
<td>-5.83</td>
<td>-5.66</td>
<td>-6.49</td>
</tr>
<tr>
<td>1552</td>
<td>-8.53</td>
<td>-8.50</td>
<td>-9.96</td>
</tr>
<tr>
<td>0</td>
<td>-0.58</td>
<td>-0.53</td>
<td>-0.62</td>
</tr>
<tr>
<td>TEST No. 10(c) Packing - One thickness of rubber</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>152</td>
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<td>-1.15</td>
</tr>
<tr>
<td>573</td>
<td>-2.97</td>
<td>-3.09</td>
<td>-3.75</td>
</tr>
<tr>
<td>1560</td>
<td>-5.22</td>
<td>-5.75</td>
<td>-7.05</td>
</tr>
<tr>
<td>1550</td>
<td>-7.55</td>
<td>-8.09</td>
<td>-9.91</td>
</tr>
</tbody>
</table>

Note: Obtained from readings \( \frac{(4) + (9) + (6) - (1)}{10} \)
### Appendix XXVIII

**COMPRESSION TEST No.11**

**Details**

**Date** . . . . . . . . . . . . . . . . . . 17th June, 1948

**Packing** . . . . . . . . . . . . . . . (a) Five thicknesses cartridge paper
(b) Corrugated Cardboard $\frac{1}{8}$ thick.
(c) Sorbo Pads, compressed from $\frac{3}{8}$" to $\frac{1}{4}$"

**Mechanical Compressor** . . . Hamilton Instrument.

**Electrical Readings**. Gauge Current, 16½ milliamps.

**Remaining Date** . . . As per Test No.1

**Abstract of Readings** (Plotted on Graph 14.1)

\[ Q = 10,000 \text{ ohms (nominal)} \]

<table>
<thead>
<tr>
<th>Nominal Stress (lb/in²)</th>
<th>Fractional Resistance Change $\frac{dR}{R} \times 10^4$</th>
<th>Mech. Measure of Strain $\frac{dE}{E} \times 10^4$</th>
<th>Mean $\frac{dE}{E}$</th>
<th>% Mean $\frac{dE}{E}$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>TEST 11(a)</strong> Packing - Cartridge (five thicknesses)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>205</td>
<td>-1.00</td>
<td>-2.00</td>
<td>-1.29</td>
<td>-0.75</td>
</tr>
<tr>
<td>460</td>
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<td>-2.90</td>
<td>-3.05</td>
<td>-2.91</td>
</tr>
<tr>
<td>816</td>
<td>-4.82</td>
<td>-4.62</td>
<td>-5.53</td>
<td>-5.14</td>
</tr>
<tr>
<td>1670</td>
<td>-7.69</td>
<td>-3.75</td>
<td>-11.30</td>
<td>-10.00</td>
</tr>
</tbody>
</table>

**TEST 11(b)** Packing - Corrugated Cardboard ($\frac{1}{8}$ thick)

| | | | | |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 225 | -1.51 | -1.61 | -1.61 | -1.15 | 1.15 | -1.40 |
| 596 | -3.06 | -3.56 | -4.01 | -3.76 | 3.62 | -3.76 |
| 948 | -4.61 | -5.46 | -6.50 | -6.15 | 3.87 | -5.97 |

**TEST 11(c)** Packing - Sorbo (2½" Free) ($\frac{1}{4}$" Loaded)

| | | | | |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 173 | -1.07 | -0.97 | -0.97 | -1.56 | -1.56 | -1.24 |
| 423 | -2.92 | -2.61 | -2.72 | -4.23 | -4.23 | -5.40 |
| 765 | -5.15 | -4.61 | -4.98 | -6.61 | -6.61 | -5.57 |
| 1334 | -7.33 | -7.05 | -7.52 | -9.23 | -9.23 | -8.11 |

*Note: Obtained from readings $\frac{(4) + (9) + (6) - (1)}{2}$*
Appendix XXIX

COMPRESSION TEST No.12

Details

Date ........... 21st February, 1949.
Specimen ........ Plain Concrete 'D' 10" x 5" dia. cylinder. Age 45 days.
Curing : Damp 7 days, water 14 days, air 28 days.

Packing ........ One rubber sheet 1/16" thick.

Gauges ........... E.T.C. Type SB/A/27, 200 ohm nominal.

Glue .............. High Solids Durofix (I.C.I. Glue No.3462) Tested at 9 days.

Testing Machine .... Richle 100,000 lb screw-powered compound lever-beam type

Mechanical Compressometer ........ Hamilton Instrument, 3" gauge length.

Electrical Measurements .......... Multi-channel strain gauge set, Cambridge Galvanometer.
Gauge Current, 18 milliams.

Time of Test ........ 90 minutes.

Abstract of Readings (Plotted on Graph 14.12)

Preliminary Loading Runs

<table>
<thead>
<tr>
<th>1st Loading Run</th>
<th>2nd Loading Run</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stress</td>
<td>Mech. Measure of Strain x 10^4</td>
</tr>
<tr>
<td>lb/in^2</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>0</td>
</tr>
<tr>
<td>271</td>
<td>1.98</td>
</tr>
<tr>
<td>499</td>
<td>2.39</td>
</tr>
<tr>
<td>754</td>
<td>3.38</td>
</tr>
<tr>
<td>1010</td>
<td>4.06</td>
</tr>
<tr>
<td>1250</td>
<td>4.61</td>
</tr>
<tr>
<td>1500</td>
<td>5.29</td>
</tr>
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</table>
### COMPRESSION TEST No.12

#### Abstract of Readings (Plotted on Graph 14.10)

#### 3rd Loading Run

<table>
<thead>
<tr>
<th>Nominal Stress lb/in²</th>
<th>Fractional Resistance Change</th>
<th>( \frac{dP}{Q} \times 10^4 )</th>
<th>Mech. Measure of Strain x 10²</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Gauge No.13</td>
<td>Gauge No.14</td>
<td>Gauge No.15</td>
</tr>
<tr>
<td>0</td>
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<td>0</td>
<td>0</td>
</tr>
<tr>
<td>75</td>
<td>-0.05</td>
<td>+0.34</td>
<td>+0.10</td>
</tr>
<tr>
<td>163</td>
<td>-0.03</td>
<td>+0.11</td>
<td>+0.22</td>
</tr>
<tr>
<td>239</td>
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<td>+0.15</td>
<td>+0.29</td>
</tr>
<tr>
<td>326</td>
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<td>+0.45</td>
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<td>+0.69</td>
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<td>691</td>
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<td>+1.48</td>
<td>+0.69</td>
</tr>
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<td>+0.15</td>
<td>+2.04</td>
<td>+0.79</td>
</tr>
<tr>
<td>1014</td>
<td>+0.27</td>
<td>+2.82</td>
<td>+0.90</td>
</tr>
<tr>
<td>1552</td>
<td>+0.42</td>
<td>+3.92</td>
<td>+1.08</td>
</tr>
<tr>
<td>1498</td>
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<td>+1.39</td>
</tr>
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<td>+5.83</td>
<td>+1.59</td>
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<td>+1.47</td>
</tr>
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<td>1930</td>
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<td>+7.41</td>
<td>+1.60</td>
</tr>
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<td>+1.68</td>
</tr>
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<td>+1.75</td>
</tr>
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<td>+9.29</td>
<td>+1.86</td>
</tr>
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<td>+2.08</td>
</tr>
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<td>+11.45</td>
<td>+2.18</td>
</tr>
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<td>+1.63</td>
<td>+12.19</td>
<td>+2.29</td>
</tr>
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<td>+12.94</td>
<td>+2.43</td>
</tr>
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<td>+13.66</td>
<td>+2.45</td>
</tr>
<tr>
<td>3082</td>
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<td>+2.74</td>
</tr>
<tr>
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<td>+2.62</td>
<td>-17.39</td>
<td>+2.95</td>
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<td>+5.67</td>
</tr>
<tr>
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<td>+6.54</td>
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<td>3662</td>
<td>+1.29</td>
<td>-12.65</td>
<td>+0.06</td>
</tr>
</tbody>
</table>
### Appendix XXIX (Cont.)

**Densities from Test No. 12**

<table>
<thead>
<tr>
<th>Percentage of Ultimate</th>
<th>Poisson's Ratio Gauge No. 15</th>
<th>Poisson's Ratio Gauge No. 12</th>
<th>Mean Poisson's Ratio</th>
<th>Mean Electrical Strain $\delta_0 \times 10^6$</th>
<th>Mean Electrical Strain $\delta_0$</th>
<th>Volumetric Strain $\delta_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.1</td>
<td>0.110</td>
<td>0.114</td>
<td>0.116</td>
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<td>0.02</td>
<td>0.17</td>
</tr>
<tr>
<td>4.6</td>
<td>0.122</td>
<td>0.132</td>
<td>0.127</td>
<td>0.12</td>
<td>0.05</td>
<td>0.18</td>
</tr>
<tr>
<td>6.7</td>
<td>0.124</td>
<td>0.134</td>
<td>0.129</td>
<td>0.13</td>
<td>0.07</td>
<td>0.20</td>
</tr>
<tr>
<td>9.1</td>
<td>0.123</td>
<td>0.132</td>
<td>0.128</td>
<td>0.23</td>
<td>0.06</td>
<td>1.13</td>
</tr>
<tr>
<td>10.0</td>
<td>0.143</td>
<td>0.153</td>
<td>0.148</td>
<td>0.23</td>
<td>0.06</td>
<td>1.20</td>
</tr>
<tr>
<td>15.7</td>
<td>0.125</td>
<td>0.135</td>
<td>0.130</td>
<td>0.27</td>
<td>0.17</td>
<td>2.20</td>
</tr>
<tr>
<td>25.4</td>
<td>0.125</td>
<td>0.135</td>
<td>0.130</td>
<td>0.27</td>
<td>0.17</td>
<td>2.20</td>
</tr>
<tr>
<td>35.1</td>
<td>0.125</td>
<td>0.135</td>
<td>0.130</td>
<td>0.27</td>
<td>0.17</td>
<td>2.20</td>
</tr>
<tr>
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<td>0.135</td>
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<td>0.27</td>
<td>0.17</td>
<td>2.20</td>
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<td>0.147</td>
<td>0.133</td>
<td>1.00</td>
<td>0.73</td>
<td>5.25</td>
</tr>
<tr>
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<td>0.120</td>
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<td>1.07</td>
<td>0.83</td>
<td>5.49</td>
</tr>
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<td>0.84</td>
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<td>6.95</td>
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<td>8.35</td>
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<td>9.20</td>
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<td>9.73</td>
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<td>0.141</td>
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<td>80.3</td>
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<td>0.142</td>
<td>0.140</td>
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<td>0.142</td>
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<td>0.142</td>
<td>0.142</td>
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<td>12.33</td>
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<td>0.143</td>
<td>0.143</td>
<td>2.58</td>
<td>-18.07</td>
<td>12.91</td>
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<tr>
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<td>0.153</td>
<td>0.151</td>
<td>2.78</td>
<td>-18.47</td>
<td>13.31</td>
</tr>
<tr>
<td>90.4</td>
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<td>0.167</td>
<td>3.36</td>
<td>-20.15</td>
<td>13.43</td>
</tr>
<tr>
<td>95.5</td>
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<td>0.185</td>
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<td>-21.50</td>
<td>15.50</td>
</tr>
<tr>
<td>94.0</td>
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<td>0.203</td>
<td>4.65</td>
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<td>15.26</td>
</tr>
<tr>
<td>96.5</td>
<td>0.234</td>
<td>0.233</td>
<td>0.233</td>
<td>5.78</td>
<td>-24.78</td>
<td>15.22</td>
</tr>
<tr>
<td>99.1</td>
<td>0.260</td>
<td>0.262</td>
<td>0.261</td>
<td>7.08</td>
<td>-27.10</td>
<td>12.94</td>
</tr>
<tr>
<td>100%</td>
<td>0.283</td>
<td>0.284</td>
<td>0.283+</td>
<td>8.39</td>
<td>-29.20+</td>
<td>11.23</td>
</tr>
</tbody>
</table>
Appendix XXX

COMPRESSION TEST

Eccentricity of Loading from strain gauge readings

If the bridge circuit is initially balanced when gauge \( P \) is unstrained, then under strain, \( P \) will change to \( P + dP \), and to maintain the balance condition, let \( Q \) change to \( Q + dQ \).

Then Strain in gauge = \( \frac{dQ}{Q \times \rho} \)

where \( \rho \) = Strain Sensitivity Factor of gauges.

FIG. 14.9 Wheatstone Bridge Circuit

Consider a circular cylinder, radius \( r \), area \( A \), loaded by a vertical load \( W \) at point \((x, y)\) from the centre \( O \).

Let \( A \), \( B \), and \( C \) be three strain gauges situated at angular positions \( \alpha \), \( \beta \), and \( \gamma \) to the \( 'x' \) axis and, due to strain, let changes \( d\gamma_A \), \( d\gamma_B \) and \( d\gamma_C \) respectively be required in \( Q \) to maintain the balance of the bridge circuit.

FIG. 14.10 Gauge positions on Compression Cylinder
Appendix XXX (Cont.)

Using the simple Theory of Bending :-

Strain at A = \( \frac{dQ_A}{Q_A} = \frac{W}{AE} \left( 1 + \frac{4x}{r} \right) \) \( \text{Eqn. 14.1} \)

Strain at B = \( \frac{dQ_B}{Q_B} = \frac{W}{AE} \left( 1 + \frac{4x \cos \alpha + 4y \sin \alpha}{r} \right) \) \( \text{Eqn. 14.2} \)

Strain at C = \( \frac{dQ_C}{Q_C} = \frac{W}{AE} \left( 1 + \frac{4x \cos \beta + 4y \sin \beta}{r} \right) \) \( \text{Eqn. 14.3} \)

From Eqn. 14.1 and 14.2 :-

\[
\frac{dQ_a}{dQ_A} \left( 1 + \frac{4x}{r} \right) = \left( 1 + \frac{4x \cos \alpha + 4y \sin \alpha}{r} \right) \text{ Eqn. 14.4}
\]

Similarly

\[
\frac{dQ_c}{dQ_A} \left( 1 + \frac{4x}{r} \right) = \left( 1 + \frac{4x \cos \beta + 4y \sin \beta}{r} \right) \text{ Eqn. 14.5}
\]

Re-arranging Eqn. 14.4 to simplify :-

\[
\frac{dQ_B}{dQ_A} \left( \frac{r + 4x}{r} \right) = \frac{4}{r} \left( \frac{r}{4} + x \cos \alpha + y \sin \alpha \right)
\]

\[
\frac{dQ_A}{dQ_A} \frac{4x}{r} + \frac{dQ_A}{dQ_A} x = \frac{r}{4} + x \cos \alpha + y \sin \alpha
\]

\[
\frac{r}{4} \left( \frac{dQ_A}{dQ_A} - 1 \right) = x \left( \cos \alpha - \frac{dQ_A}{dQ_A} \right) + y \sin \alpha \text{ Eqn. 14.4.1}
\]

Similarly Eqn. 14.5 becomes

\[
\frac{r}{4} \left( \frac{dQ_B}{dQ_A} - 1 \right) = x \left( \cos \beta - \frac{dQ_B}{dQ_A} \right) + y \sin \beta \text{ Eqn. 14.5.1}
\]

Solving equations 14.4.1 and 14.5.1 simultaneously, the position of the leading point \((x,y)\) may be found from the strain gauge readings.
Appendix XXX (Cont)

COMPRESSION TESTS.

Calculations of Eccentricity of loading.

From the curves (Graph 14.1) at a stress of 1275 lb/in² gauges Nos 1, 4 and 9 indicated the following electrical strains \( \frac{\Delta Q}{Q} \) from which the eccentricity was calculated, using Equations 14.4.1 and 14.5.1. See Appendix XXX

<table>
<thead>
<tr>
<th>Test No.</th>
<th>Fractional Resistance Change ( \frac{\Delta Q}{Q} \times 10^4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Gauge No.1</td>
</tr>
<tr>
<td>9(b)</td>
<td>6.4</td>
</tr>
<tr>
<td>3</td>
<td>6.0</td>
</tr>
<tr>
<td>11(b)</td>
<td>6.3</td>
</tr>
<tr>
<td>11(a)</td>
<td>6.2</td>
</tr>
<tr>
<td>10(c)</td>
<td>6.2</td>
</tr>
<tr>
<td>10(a)&amp;(b)</td>
<td>7.4</td>
</tr>
<tr>
<td>11(c)</td>
<td>7.3</td>
</tr>
</tbody>
</table>
Appendix XXXI

COMPRESSION TESTS

The Variation of the Strain in the Height of the Specimen

Axial Strains

From the curves (Graph 14.2) for Test 6(b), the average slopes of the stress-electrical strain \( \frac{\partial e}{\partial Q} \) curves were:

<table>
<thead>
<tr>
<th>Gauge No.</th>
<th>3</th>
<th>7</th>
<th>11</th>
<th>12</th>
<th>5</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{\partial e}{\partial Q} \times 10^8 \text{ ohm.in}^2 / \text{ohm.lb} )</td>
<td>5.7</td>
<td>5.7</td>
<td>9.2</td>
<td>6.3</td>
<td>13.7</td>
<td>14.7</td>
</tr>
<tr>
<td>Equivalent mean Strain ( \frac{\partial e}{\partial Q} \times 10^8 \text{ ohm.in}^2 / \text{ohm.lb} )</td>
<td>5.7</td>
<td>7.8</td>
<td>14.2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gauge position from one end</td>
<td>9(\frac{1}{16}) in.</td>
<td>2 in.</td>
<td>4(\frac{3}{8}) in</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Axial Strains

From the curves (Graph 14.4) for Test No.4 at a nominal stress of 1875 lb/in\(^2\) the electrical strain \( \frac{\partial e}{\partial Q} \) readings for the gauges were:

<table>
<thead>
<tr>
<th>Gauge No.</th>
<th>1</th>
<th>4</th>
<th>6</th>
<th>9</th>
<th>2</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{\partial e}{\partial Q} \times 10^8 \text{ ohm.in}^2 / \text{ohm.lb} )</td>
<td>72</td>
<td>60</td>
<td>50</td>
<td>66</td>
<td>49</td>
<td>72</td>
</tr>
<tr>
<td>Equivalent mean Strain</td>
<td>62</td>
<td>60.5</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \frac{\partial e}{\partial Q} \times 10^8 \text{ ohm.in}^2 / \text{ohm.lb} )</td>
<td>62</td>
<td></td>
<td>60.5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gauge position from one end</td>
<td>5 in.</td>
<td>1(\frac{3}{8}) in.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
### Appendix XXXII

#### BEAM TESTS

### Details of Gauges

#### Beam Specimen "E"

<table>
<thead>
<tr>
<th>Gauge No.</th>
<th>Resistance (ohms)</th>
<th>Distance from L.H. end. (inches)</th>
<th>Distance from front face (inches)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Compression Gauges</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>211</td>
<td>28.1</td>
<td>1.5</td>
</tr>
<tr>
<td>2</td>
<td>210</td>
<td>18.5</td>
<td>0.6</td>
</tr>
<tr>
<td>12</td>
<td>209</td>
<td>29.2</td>
<td>1.3</td>
</tr>
<tr>
<td>Tension Gauges</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>211</td>
<td>25.6</td>
<td>0.8</td>
</tr>
<tr>
<td>4</td>
<td>212</td>
<td>21.5</td>
<td>2.5</td>
</tr>
<tr>
<td>11</td>
<td>210</td>
<td>30.3</td>
<td>2.8</td>
</tr>
</tbody>
</table>

#### Beam Specimen "F"

<table>
<thead>
<tr>
<th>Gauge No.</th>
<th>Resistance (ohms)</th>
<th>Distance from L.H. end. (inches)</th>
<th>Distance from front face (inches)</th>
<th>Distance from top face (inches)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>211</td>
<td>26.3</td>
<td>2.25</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>210</td>
<td>22.5</td>
<td>0</td>
<td>0.70</td>
</tr>
<tr>
<td>7</td>
<td>211</td>
<td>24.0</td>
<td>0</td>
<td>1.85</td>
</tr>
<tr>
<td>8</td>
<td>212</td>
<td>26.0</td>
<td>0</td>
<td>2.85</td>
</tr>
<tr>
<td>9</td>
<td>211</td>
<td>26.9</td>
<td>0</td>
<td>4.80</td>
</tr>
<tr>
<td>10</td>
<td>210</td>
<td>24.0</td>
<td>1.05</td>
<td>5.75</td>
</tr>
<tr>
<td>13</td>
<td>210</td>
<td>23.1</td>
<td>0.50</td>
<td>0</td>
</tr>
<tr>
<td>14</td>
<td>214</td>
<td>23.2</td>
<td>0</td>
<td>1.15</td>
</tr>
<tr>
<td>15</td>
<td>209</td>
<td>19.5</td>
<td>3.05</td>
<td>1.70</td>
</tr>
<tr>
<td>16</td>
<td>214</td>
<td>26.1</td>
<td>0</td>
<td>4.80</td>
</tr>
</tbody>
</table>
Appendix XXXIII

BEAM TEST No. 1

Details.
Date . . . . . . . 18th Oct. 1949
Specimen . . . . Plain Concrete 'E' section 3" wide
x 6" deep. Age 102 days. (Cured
70 days in water, 32 days dry)
Gauges . . . . . . B.T.C. Type SE/A/27 (200 ohm
nominal.)
Glue . . . . . . . High Solids Durofix (I.C.I. glue,
No.3462) Tested at 14 days)
Testing Machine . . Buckton 10 Ton, single lever
machine, fitted with proving ring
to measure load.
Electrical
Measurements . . Multi-channel strain gauge set,
Marceni Galvanometer. Gauge
current 25.5 milliamps.

Abstract of Readings (Plotted on Graph 15.3)
Nominal Q = 10,000 ohms

<table>
<thead>
<tr>
<th>Bending Moment (lb.in)</th>
<th>Fractional Resistance Change $\frac{\Delta Q}{Q} \times 10^4$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Gauge No.1</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1545</td>
<td>+0.6</td>
</tr>
<tr>
<td>2256</td>
<td>+0.7</td>
</tr>
<tr>
<td>3102</td>
<td>+1.2</td>
</tr>
<tr>
<td>3849</td>
<td>+1.3</td>
</tr>
<tr>
<td>4410</td>
<td>+1.5</td>
</tr>
<tr>
<td>4956</td>
<td>+1.4</td>
</tr>
<tr>
<td>0</td>
<td>+0.3</td>
</tr>
</tbody>
</table>
Appendix XXXIV

**BEAM TEST No. 2**

**Details**

Date ....... 6th December, 1948

Specimen ...... Plain Concrete "B". Age 151 days.

Gauge Current ...... 20 milliamps.

Remaining data ...... As per Test No. 1

**Abstract of Readings (Plotted on Graph 15.3)**

<table>
<thead>
<tr>
<th>Bending Moment (lb. in)</th>
<th>Fractional Resistance Change $\frac{\Delta R}{R} \times 10^4$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Gauge No. 1</td>
</tr>
<tr>
<td>Test 2 (A)</td>
<td></td>
</tr>
<tr>
<td>120</td>
<td>0</td>
</tr>
<tr>
<td>1632</td>
<td>+0.47</td>
</tr>
<tr>
<td>2109</td>
<td>+0.30</td>
</tr>
<tr>
<td>2739</td>
<td>+0.92</td>
</tr>
<tr>
<td>3504</td>
<td>+1.22</td>
</tr>
<tr>
<td>4152</td>
<td>+1.72</td>
</tr>
<tr>
<td>4569</td>
<td>+1.99</td>
</tr>
<tr>
<td>120</td>
<td>+0.22</td>
</tr>
<tr>
<td>Test 2 (B)</td>
<td></td>
</tr>
<tr>
<td>120</td>
<td>0</td>
</tr>
<tr>
<td>840</td>
<td>+0.33</td>
</tr>
<tr>
<td>1688</td>
<td>+0.70</td>
</tr>
<tr>
<td>2559</td>
<td>+1.00</td>
</tr>
<tr>
<td>3405</td>
<td>+1.45</td>
</tr>
<tr>
<td>3999</td>
<td>+1.53</td>
</tr>
<tr>
<td>4569</td>
<td>+1.80</td>
</tr>
<tr>
<td>120</td>
<td>+0.10</td>
</tr>
</tbody>
</table>
Appendix XXXV

BEAM TEST No. 3

Details
Date .......................... 26th February 1949
Specimen ......................... Plain Concrete "E". Age 233 days.
Gauge Current .................. 18 milliams.
Deflections ...................... Ames Dial Gauge 1/10,000 inch.
Remaining data ................. As per Test No. 1.

Abstract of Readings (See next page)

Calculations

The initial strain rates per 5000 lb.in bending moment are given below:

<table>
<thead>
<tr>
<th>Gauge No.</th>
<th>Compression Gauges</th>
<th>Tension Gauges</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2</td>
<td>12</td>
</tr>
<tr>
<td>Electrical Strain Rate</td>
<td>0.58</td>
<td>0.59</td>
</tr>
<tr>
<td>( \frac{\Delta Q}{Q} /3000 \text{ lb.in} \times 10^4 )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Observations

Release of load and increase of deflection.

<table>
<thead>
<tr>
<th>Bending Moment (lb.in)</th>
<th>Load (lb)</th>
<th>Release of load (lb)</th>
<th>Deflection ( \times 10^{-4} ) in.</th>
<th>Increase of Deflection ( \times 10^{-3} ) in.</th>
</tr>
</thead>
<tbody>
<tr>
<td>4440</td>
<td>1480</td>
<td>0</td>
<td>154.6</td>
<td>-</td>
</tr>
<tr>
<td>5145</td>
<td>1715</td>
<td>33</td>
<td>183.3</td>
<td>-</td>
</tr>
<tr>
<td>5980</td>
<td>1930</td>
<td>33</td>
<td>203.1</td>
<td>0.8</td>
</tr>
<tr>
<td>6560</td>
<td>2182</td>
<td>33</td>
<td>221.3</td>
<td>1.2</td>
</tr>
<tr>
<td>7116</td>
<td>2372</td>
<td>33</td>
<td>235.0</td>
<td>1.7</td>
</tr>
<tr>
<td>7636</td>
<td>2562</td>
<td>33</td>
<td>245.1</td>
<td>1.8</td>
</tr>
<tr>
<td>8250</td>
<td>2750</td>
<td>47</td>
<td>256.9</td>
<td>2.0</td>
</tr>
<tr>
<td>8686</td>
<td>2895</td>
<td>-</td>
<td>263.9</td>
<td>2.3</td>
</tr>
<tr>
<td>9096</td>
<td>3032</td>
<td>-</td>
<td>282.1</td>
<td>-</td>
</tr>
</tbody>
</table>
### Appendix XXXV (Cont)

#### BEAM TEST No. 3

**Abstract of Readings** (Plotted on Graph 15.4)

<table>
<thead>
<tr>
<th>Time (Ind.)</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. 1, 1/4</td>
<td>0</td>
<td>0.11</td>
<td>0.22</td>
<td>0.33</td>
<td>0.44</td>
<td>0.55</td>
<td>0.66</td>
</tr>
<tr>
<td>No. 2, 1/4</td>
<td>0</td>
<td>0.26</td>
<td>0.52</td>
<td>0.78</td>
<td>1.03</td>
<td>1.29</td>
<td>1.55</td>
</tr>
<tr>
<td>No. 3, 1/4</td>
<td>0</td>
<td>0.17</td>
<td>0.34</td>
<td>0.51</td>
<td>0.68</td>
<td>0.85</td>
<td>1.02</td>
</tr>
<tr>
<td>No. 1, 1/8</td>
<td>0</td>
<td>0.04</td>
<td>0.08</td>
<td>0.12</td>
<td>0.16</td>
<td>0.20</td>
<td>0.24</td>
</tr>
<tr>
<td>No. 2, 1/8</td>
<td>0</td>
<td>0.24</td>
<td>0.48</td>
<td>0.72</td>
<td>0.96</td>
<td>1.20</td>
<td>1.44</td>
</tr>
<tr>
<td>No. 3, 1/8</td>
<td>0</td>
<td>0.12</td>
<td>0.24</td>
<td>0.36</td>
<td>0.48</td>
<td>0.60</td>
<td>0.72</td>
</tr>
</tbody>
</table>

**Nominal Q = 10,000 cals**

<table>
<thead>
<tr>
<th>Fractional Resistance Change ( \times 10^{-4} )</th>
<th>0.1</th>
<th>0.2</th>
<th>0.3</th>
<th>0.4</th>
<th>0.5</th>
<th>0.6</th>
<th>0.7</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. 1, 1/4</td>
<td>+0.26</td>
<td>+0.52</td>
<td>+0.78</td>
<td>+1.03</td>
<td>+1.29</td>
<td>+1.55</td>
<td>+1.81</td>
</tr>
<tr>
<td>No. 2, 1/4</td>
<td>+0.17</td>
<td>+0.34</td>
<td>+0.51</td>
<td>+0.68</td>
<td>+0.85</td>
<td>+1.02</td>
<td>+1.19</td>
</tr>
<tr>
<td>No. 3, 1/4</td>
<td>+0.04</td>
<td>+0.08</td>
<td>+0.12</td>
<td>+0.16</td>
<td>+0.20</td>
<td>+0.24</td>
<td>+0.28</td>
</tr>
<tr>
<td>No. 1, 1/8</td>
<td>+0.24</td>
<td>+0.48</td>
<td>+0.72</td>
<td>+0.96</td>
<td>+1.20</td>
<td>+1.44</td>
<td>+1.68</td>
</tr>
<tr>
<td>No. 2, 1/8</td>
<td>+0.12</td>
<td>+0.24</td>
<td>+0.36</td>
<td>+0.48</td>
<td>+0.60</td>
<td>+0.72</td>
<td>+0.84</td>
</tr>
<tr>
<td>No. 3, 1/8</td>
<td>+0.04</td>
<td>+0.08</td>
<td>+0.12</td>
<td>+0.16</td>
<td>+0.20</td>
<td>+0.24</td>
<td>+0.28</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Central Deflection ( \text{in. \times 10^{-4}} )</th>
<th>0</th>
<th>0.1</th>
<th>0.2</th>
<th>0.3</th>
<th>0.4</th>
<th>0.5</th>
<th>0.6</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. 1, 1/4</td>
<td>42.6</td>
<td>127.4</td>
<td>113.4</td>
<td>163.6</td>
<td>183.9</td>
<td>203.1</td>
<td>223.3</td>
</tr>
<tr>
<td>No. 2, 1/4</td>
<td>123.6</td>
<td>143.9</td>
<td>154.2</td>
<td>174.5</td>
<td>194.8</td>
<td>215.1</td>
<td>235.4</td>
</tr>
<tr>
<td>No. 3, 1/4</td>
<td>204.0</td>
<td>224.3</td>
<td>244.6</td>
<td>264.9</td>
<td>285.2</td>
<td>305.5</td>
<td>325.8</td>
</tr>
<tr>
<td>No. 1, 1/8</td>
<td>204.0</td>
<td>224.3</td>
<td>244.6</td>
<td>264.9</td>
<td>285.2</td>
<td>305.5</td>
<td>325.8</td>
</tr>
<tr>
<td>No. 2, 1/8</td>
<td>123.6</td>
<td>143.9</td>
<td>154.2</td>
<td>174.5</td>
<td>194.8</td>
<td>215.1</td>
<td>235.4</td>
</tr>
<tr>
<td>No. 3, 1/8</td>
<td>42.6</td>
<td>127.4</td>
<td>113.4</td>
<td>163.6</td>
<td>183.9</td>
<td>203.1</td>
<td>223.3</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Bending Moment ( \text{(in. \times 10^{-4})} )</th>
<th>100</th>
<th>200</th>
<th>300</th>
<th>400</th>
<th>500</th>
<th>600</th>
<th>700</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. 1, 1/4</td>
<td>123.6</td>
<td>247.2</td>
<td>370.8</td>
<td>494.4</td>
<td>618.0</td>
<td>741.6</td>
<td>865.2</td>
</tr>
<tr>
<td>No. 2, 1/4</td>
<td>204.0</td>
<td>408.0</td>
<td>612.0</td>
<td>816.0</td>
<td>1020.0</td>
<td>1224.0</td>
<td>1428.0</td>
</tr>
<tr>
<td>No. 3, 1/4</td>
<td>42.6</td>
<td>127.4</td>
<td>212.2</td>
<td>297.0</td>
<td>381.8</td>
<td>466.6</td>
<td>551.4</td>
</tr>
<tr>
<td>No. 1, 1/8</td>
<td>123.6</td>
<td>247.2</td>
<td>370.8</td>
<td>494.4</td>
<td>618.0</td>
<td>741.6</td>
<td>865.2</td>
</tr>
<tr>
<td>No. 2, 1/8</td>
<td>204.0</td>
<td>408.0</td>
<td>612.0</td>
<td>816.0</td>
<td>1020.0</td>
<td>1224.0</td>
<td>1428.0</td>
</tr>
<tr>
<td>No. 3, 1/8</td>
<td>42.6</td>
<td>127.4</td>
<td>212.2</td>
<td>297.0</td>
<td>381.8</td>
<td>466.6</td>
<td>551.4</td>
</tr>
</tbody>
</table>
### BEAM TEST No.4

**Details**

- **Date**: 8th December, 1948
- **Specimen**: Plain Concrete 4" section 3" wide x 5.7" deep. Age 153 days. (Cured 70 days in water, 83 days dry)
- **Gauges**: B.T.C. type SE/A/27 (200 ohm nominal)
- **Glue**: High Solids Duraflex (I.C.I. glue No.3462) Tested at 69 days.
- **Testing Machine**: Buckton 10 Ton single lever machine with worm and wheel loading. Proving ring fitted to measure the load.

### Abstract of Readings (Plotted on Graph 15.5)

<table>
<thead>
<tr>
<th>Bending Moment (lb.in)</th>
<th>Fractional Resistance Change $\Delta R / R$ x $10^4$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Gauge No.5</td>
</tr>
<tr>
<td>120</td>
<td>0</td>
</tr>
<tr>
<td>1110</td>
<td>+0.33</td>
</tr>
<tr>
<td>2070</td>
<td>+0.37</td>
</tr>
<tr>
<td>2799</td>
<td>+0.37</td>
</tr>
<tr>
<td>3624</td>
<td>+1.04</td>
</tr>
<tr>
<td>4233</td>
<td>+1.33</td>
</tr>
<tr>
<td>4775</td>
<td>+1.25</td>
</tr>
<tr>
<td>120</td>
<td>-0.15</td>
</tr>
<tr>
<td>120</td>
<td>0</td>
</tr>
<tr>
<td>1092</td>
<td>+0.34</td>
</tr>
<tr>
<td>2235</td>
<td>+0.67</td>
</tr>
<tr>
<td>2955</td>
<td>+0.98</td>
</tr>
<tr>
<td>3771</td>
<td>+1.11</td>
</tr>
<tr>
<td>4543</td>
<td>+1.53</td>
</tr>
<tr>
<td>1775</td>
<td>+1.56</td>
</tr>
<tr>
<td>120</td>
<td>-0.12</td>
</tr>
</tbody>
</table>
Appendix XXXVII

BEAM TEST No. 5

Details

Date . . . . . . . 26th February 1949
Specimen . . . . Plain Concrete 'F'. Age 233 days
Gauge Current . . . 18 milliamps.
Deflections . . . Ames Dial Gauge 1/10,000 inch.
Remaining Data . . As per Test No. 4

Abstract of Readings (See next page)

Calculations

The initial strain rates per 3,000 lb.in. bending moment are given below:

<table>
<thead>
<tr>
<th>Gauge No.</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>13</th>
<th>15</th>
<th>16</th>
</tr>
</thead>
<tbody>
<tr>
<td>Electrical Strain Rate $\times 10^4$</td>
<td>0.55</td>
<td>0.29</td>
<td>0.07(5)</td>
<td>0.27</td>
<td>0.67</td>
<td>-1.13</td>
<td>-0.23</td>
<td>0.45</td>
</tr>
</tbody>
</table>

Observations

Release of load and increase of deflections

<table>
<thead>
<tr>
<th>Bending Moment (lb.in.)</th>
<th>Load (lb)</th>
<th>Release of load (lb)</th>
<th>Deflection $\times 10^4$ in.</th>
<th>Increase of Deflection $\times 10^4$ in.</th>
</tr>
</thead>
<tbody>
<tr>
<td>4170</td>
<td>1590</td>
<td>0</td>
<td>197.7</td>
<td>0</td>
</tr>
<tr>
<td>4710</td>
<td>1570</td>
<td>9</td>
<td>214.1</td>
<td>0.2</td>
</tr>
<tr>
<td>5256</td>
<td>1752</td>
<td>23</td>
<td>230.3</td>
<td>0.7</td>
</tr>
<tr>
<td>5820</td>
<td>1940</td>
<td>33</td>
<td>246.8</td>
<td>1.5</td>
</tr>
<tr>
<td>6396</td>
<td>2152</td>
<td>45</td>
<td>262.3</td>
<td>2.0</td>
</tr>
<tr>
<td>6966</td>
<td>2322</td>
<td>52</td>
<td>283.1</td>
<td>2.8</td>
</tr>
</tbody>
</table>
### Appendix XXXVII (Cont.)  
**BEAM TEST No. 5**  

**Abstract of Readings (Plotted on Graph 15.6)**

<table>
<thead>
<tr>
<th>Time (mins)</th>
<th>No. 3.5 Mo.</th>
<th>No. 3 Mo.</th>
<th>No. 3.2 Mo.</th>
<th>No. 3.4 Mo.</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>+0.05</td>
<td>+0.03</td>
<td>+0.12</td>
<td>+0.31</td>
</tr>
<tr>
<td>16</td>
<td>+0.21</td>
<td>+0.45</td>
<td>+0.79</td>
<td>+1.04</td>
</tr>
<tr>
<td>30</td>
<td>+0.35</td>
<td>+0.67</td>
<td>+1.02</td>
<td>+1.35</td>
</tr>
<tr>
<td>44</td>
<td>+0.50</td>
<td>+0.92</td>
<td>+1.33</td>
<td>+1.63</td>
</tr>
<tr>
<td>60</td>
<td>+0.69</td>
<td>+1.19</td>
<td>+1.70</td>
<td>+2.06</td>
</tr>
<tr>
<td>90</td>
<td>+0.92</td>
<td>+1.45</td>
<td>+2.01</td>
<td>+2.43</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Fractional Resistance Change G x 10^4</th>
<th>No. 3 Mo.</th>
<th>No. 3.2 Mo.</th>
<th>No. 3.4 Mo.</th>
<th>No. 3.5 Mo.</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>+0.03</td>
<td>+0.05</td>
<td>+0.07</td>
<td>+0.09</td>
</tr>
<tr>
<td>16</td>
<td>+0.14</td>
<td>+0.26</td>
<td>+0.38</td>
<td>+0.50</td>
</tr>
<tr>
<td>30</td>
<td>+0.24</td>
<td>+0.47</td>
<td>+0.70</td>
<td>+1.03</td>
</tr>
<tr>
<td>44</td>
<td>+0.33</td>
<td>+0.66</td>
<td>+1.09</td>
<td>+1.42</td>
</tr>
<tr>
<td>60</td>
<td>+0.44</td>
<td>+0.89</td>
<td>+1.32</td>
<td>+1.65</td>
</tr>
<tr>
<td>90</td>
<td>+0.55</td>
<td>+1.12</td>
<td>+1.55</td>
<td>+1.88</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Central Deflection (in. x 10^4)</th>
<th>No. 3.5 Mo.</th>
<th>No. 3 Mo.</th>
<th>No. 3.2 Mo.</th>
<th>No. 3.4 Mo.</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>+0.05</td>
<td>+0.03</td>
<td>+0.12</td>
<td>+0.31</td>
</tr>
<tr>
<td>16</td>
<td>+0.21</td>
<td>+0.45</td>
<td>+0.79</td>
<td>+1.04</td>
</tr>
<tr>
<td>30</td>
<td>+0.35</td>
<td>+0.67</td>
<td>+1.02</td>
<td>+1.35</td>
</tr>
<tr>
<td>44</td>
<td>+0.50</td>
<td>+0.92</td>
<td>+1.33</td>
<td>+1.63</td>
</tr>
<tr>
<td>60</td>
<td>+0.69</td>
<td>+1.19</td>
<td>+1.70</td>
<td>+2.06</td>
</tr>
<tr>
<td>90</td>
<td>+0.92</td>
<td>+1.45</td>
<td>+2.01</td>
<td>+2.43</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Bending Moment (lb. in.)</th>
<th>No. 3.5 Mo.</th>
<th>No. 3 Mo.</th>
<th>No. 3.2 Mo.</th>
<th>No. 3.4 Mo.</th>
</tr>
</thead>
<tbody>
<tr>
<td>56</td>
<td>123.5</td>
<td>124.6</td>
<td>123.8</td>
<td>121.5</td>
</tr>
<tr>
<td>215</td>
<td>424.9</td>
<td>435.0</td>
<td>424.2</td>
<td>415.3</td>
</tr>
<tr>
<td>182</td>
<td>388.4</td>
<td>398.5</td>
<td>387.7</td>
<td>378.8</td>
</tr>
<tr>
<td>214.5</td>
<td>474.0</td>
<td>484.1</td>
<td>473.2</td>
<td>464.3</td>
</tr>
<tr>
<td>291.1</td>
<td>638.6</td>
<td>648.7</td>
<td>637.9</td>
<td>628.0</td>
</tr>
<tr>
<td>345.4</td>
<td>729.8</td>
<td>739.9</td>
<td>729.1</td>
<td>719.2</td>
</tr>
</tbody>
</table>
## Appendix XXXVIII

### TORSION TEST No.1

**Details**

- **Date**: 8th January 1949
- **Specimen**: Plain Concrete 'G' Specimen 7½" dia.  
  Age 6 weeks.  
  Curing: 5 days damp, 12 days water, 25 days air.
- **Gauges**: B.T.C. Type SE/A/27 (200 ohms nominal)
- **Glue**: High Solids Dureflex (I.C.I. Glue No.3462). Tested at 22 days.
- **Testing Machine**: Combined Bending and Torsion Machine
- **Electrical Measurements**: Multi-Channel Strain Gauge Set, Cambridge Galvanometer.  
  Gauge current 27 milliamps.

### Abstract of Readings (Plotted on Graph 17.2)

<table>
<thead>
<tr>
<th>Torque lb.in.</th>
<th>Fractional Resistance Change</th>
<th>$\frac{dR}{R} \times 10^4$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Gauge No.2</td>
<td>Gauge No.3</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1412</td>
<td>-0.15</td>
<td>+0.03</td>
</tr>
<tr>
<td>2825</td>
<td>-0.08</td>
<td>+0.25</td>
</tr>
<tr>
<td>4240</td>
<td>-0.27</td>
<td>+0.31</td>
</tr>
<tr>
<td>5660</td>
<td>-0.31</td>
<td>+0.45</td>
</tr>
<tr>
<td>7070</td>
<td>-0.37</td>
<td>+0.53</td>
</tr>
<tr>
<td>8480</td>
<td>-0.37</td>
<td>+0.52</td>
</tr>
<tr>
<td>9900</td>
<td>-0.37</td>
<td>+0.65</td>
</tr>
<tr>
<td>11320</td>
<td>-0.54</td>
<td>+0.75</td>
</tr>
<tr>
<td>12730</td>
<td>-0.59</td>
<td>+0.75</td>
</tr>
<tr>
<td>14130</td>
<td>-0.70</td>
<td>+0.87</td>
</tr>
<tr>
<td>1413</td>
<td>0</td>
<td>+0.41</td>
</tr>
</tbody>
</table>
Appendix XXXVIII (Cont.)

<table>
<thead>
<tr>
<th>Torque lb.in.</th>
<th>Electrical Shear Strain $\frac{d\Omega}{\Omega} \times 10^4$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Gauges (2) + (3)</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1412</td>
<td>0.18</td>
</tr>
<tr>
<td>2226</td>
<td>0.33</td>
</tr>
<tr>
<td>4240</td>
<td>0.58</td>
</tr>
<tr>
<td>5680</td>
<td>0.76</td>
</tr>
<tr>
<td>7070</td>
<td>0.90</td>
</tr>
<tr>
<td>8430</td>
<td>0.99</td>
</tr>
<tr>
<td>9900</td>
<td>1.02</td>
</tr>
<tr>
<td>11320</td>
<td>1.29</td>
</tr>
<tr>
<td>12750</td>
<td>1.34</td>
</tr>
<tr>
<td>14130</td>
<td>1.57</td>
</tr>
<tr>
<td>1413</td>
<td>-</td>
</tr>
</tbody>
</table>

**Observations**

(a) From the Graph 17.2, for torque range $0 = 14000$ lb.in., mean electrical strains $\frac{d\Omega}{\Omega}$ per 10,000 lb.in. torque were:

<table>
<thead>
<tr>
<th>Fractional Resistance Change $\frac{d\Omega}{\Omega} \times 10^4$ per 10,000 lb.in. torque</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gauge No. 2</td>
</tr>
<tr>
<td>-0.42</td>
</tr>
</tbody>
</table>
Appendix XXXIX

TORSION TEST No. 2

Details

Date . . . . . . . . . . . . . . . . 22nd January, 1949
Gauge Current . . . . . . . . . . 25 Milliamps.
Deflections . . . . . . . . . . . Two plunger type dial gauges,
1/10,000 in., reading at 4.45 in.
radius over a 20.7 in. gauge length.

Remaining Data . . . . As per Torsion Test No. 1

Abstract of Readings (Plotted on Graph 17.3)

<table>
<thead>
<tr>
<th>Torque</th>
<th>Top Deflection</th>
<th>Bottom Deflection</th>
<th>Fractional Resistance Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>11485</td>
<td>12.6</td>
<td>12.6</td>
<td>0</td>
</tr>
<tr>
<td>11510</td>
<td>12.4</td>
<td>12.4</td>
<td>0</td>
</tr>
<tr>
<td>12720</td>
<td>10.5</td>
<td>10.5</td>
<td>0</td>
</tr>
<tr>
<td>14130</td>
<td>30.5</td>
<td>30.5</td>
<td>0</td>
</tr>
<tr>
<td>1412</td>
<td></td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>2625</td>
<td>0.8</td>
<td>0.8</td>
<td>0</td>
</tr>
<tr>
<td>4237</td>
<td>3.3</td>
<td>3.3</td>
<td>0</td>
</tr>
<tr>
<td>5650</td>
<td>6.7</td>
<td>6.7</td>
<td>0</td>
</tr>
<tr>
<td>7070</td>
<td>11.7</td>
<td>11.7</td>
<td>0</td>
</tr>
<tr>
<td>8480</td>
<td>14.8</td>
<td>14.8</td>
<td>0</td>
</tr>
<tr>
<td>9900</td>
<td>28.1</td>
<td>28.1</td>
<td>0</td>
</tr>
<tr>
<td>11310</td>
<td>41.1</td>
<td>41.1</td>
<td>0</td>
</tr>
<tr>
<td>10.8</td>
<td>10.8</td>
<td>10.8</td>
<td>0</td>
</tr>
<tr>
<td>14.0</td>
<td>14.0</td>
<td>14.0</td>
<td>0</td>
</tr>
<tr>
<td>27.5</td>
<td>27.5</td>
<td>27.5</td>
<td>0</td>
</tr>
</tbody>
</table>

Observations

(a) From graphs for mechanical gauges (Graph 17.4)

Mean Deflection per 10,000 lb. in. torque :=

Upper Gauge = 27.5 x 10^-4 in
Lower Gauge = 13.5 x 10^-4 in
Difference = 14 x 10^-4 in
Appendix XXXIX (Cont)

(b)

<table>
<thead>
<tr>
<th>Torque (lb.in)</th>
<th>Electrical Shear Strain $\frac{d\sigma}{\sigma} \times 10^4$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Gauges (2) + (3)</td>
</tr>
<tr>
<td>1412</td>
<td>0</td>
</tr>
<tr>
<td>2625</td>
<td>0.14</td>
</tr>
<tr>
<td>4237</td>
<td>0.37</td>
</tr>
<tr>
<td>5650</td>
<td>0.50</td>
</tr>
<tr>
<td>7070</td>
<td>0.69</td>
</tr>
<tr>
<td>8480</td>
<td>0.77</td>
</tr>
<tr>
<td>9900</td>
<td>1.09</td>
</tr>
<tr>
<td>11310</td>
<td>1.15</td>
</tr>
<tr>
<td>12720</td>
<td>1.19</td>
</tr>
<tr>
<td>14130</td>
<td>1.37</td>
</tr>
<tr>
<td>1412</td>
<td></td>
</tr>
</tbody>
</table>

From Graph 17 for torque range 0 = 14,000 lb.in.
Mean electrical strains $\frac{d\sigma}{\sigma}$ per 10,000 lb.in. torque were:

<table>
<thead>
<tr>
<th>Fractional Resistance Change $\frac{d\sigma}{\sigma} \times 10^4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>per 10,000 lb.in torque</td>
</tr>
<tr>
<td>Gauge No.2</td>
</tr>
<tr>
<td>-0.35</td>
</tr>
</tbody>
</table>
Appendix XL

TORSION TESTS Nos. 3, 4, and 5

Details

Date ................ 29th January, 1949
Specimen ............ Specimen 'G', as per Torsion Test No. 1
Deflections .......... Two plunger type dial gauges 1/10,000 inch reading at 4.45 inch radius over 20.7 inch gauge length

Abstract of Readings (Plotted on Graph 17.4)

<table>
<thead>
<tr>
<th>Torque (lb.in.)</th>
<th>Test No. 3</th>
<th>Test No. 4</th>
<th>Test No. 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deflection</td>
<td>x 10^-4 in.</td>
<td>x 10^-4 in.</td>
<td>x 10^-4 in.</td>
</tr>
<tr>
<td>Top</td>
<td>Bottom</td>
<td>Top</td>
<td>Bottom</td>
</tr>
<tr>
<td>850</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2350</td>
<td>4.7</td>
<td>6.1</td>
<td>5.8</td>
</tr>
<tr>
<td>4250</td>
<td>7.7</td>
<td>8.1</td>
<td>12.0</td>
</tr>
<tr>
<td>5670</td>
<td>12.6</td>
<td>13.0</td>
<td>16.6</td>
</tr>
<tr>
<td>7090</td>
<td>17.9</td>
<td>18.3</td>
<td>19.7</td>
</tr>
<tr>
<td>8500</td>
<td>23.2</td>
<td>19.0</td>
<td>25.0</td>
</tr>
<tr>
<td>9910</td>
<td>28.3</td>
<td>22.1</td>
<td>25.0</td>
</tr>
<tr>
<td>11320</td>
<td>33.5</td>
<td>26.1</td>
<td>29.0</td>
</tr>
<tr>
<td>12730</td>
<td>37.8</td>
<td>31.3</td>
<td>32.5</td>
</tr>
<tr>
<td>14160</td>
<td>43.6</td>
<td>35.0</td>
<td>37.0</td>
</tr>
<tr>
<td>850</td>
<td>11.4</td>
<td>3.5</td>
<td>4.3</td>
</tr>
</tbody>
</table>

Observations

<table>
<thead>
<tr>
<th>Test No.</th>
<th>Mean Deflection per 10,000 lb.in.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Upper Gauge x 10^-4 in.</td>
</tr>
<tr>
<td>3</td>
<td>35</td>
</tr>
<tr>
<td>4</td>
<td>30_2^-1</td>
</tr>
<tr>
<td>5</td>
<td>29</td>
</tr>
</tbody>
</table>
Appendix XLI

CALCULATIONS ON TORSION TESTS

(a) Mechanical Measurements

The mean deflections over a 20.7 inch gauge length at 4.45 inches radius per 10,000 lb.in. torque were found to be:

<table>
<thead>
<tr>
<th>Test No.</th>
<th>Deflection x 10^4 in</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>14</td>
</tr>
<tr>
<td>3</td>
<td>16.5</td>
</tr>
<tr>
<td>4</td>
<td>14.5</td>
</tr>
<tr>
<td>5</td>
<td>14</td>
</tr>
</tbody>
</table>

Using the elastic formula

Shear Modulus \( G = \frac{T \cdot l \cdot r}{J \cdot y} \)

where
- \( T \) = torque = 10,000 lb.in.
- \( l \) = gauge length = 20.7 in.
- \( r \) = deflection radius = 4.45 in.
- \( J \) = Polar 2nd Moment of Area = 311 in^4
- \( y \) = deflection over gauge length.

Taking \( y = 14 \times 10^{-4} \) in.

Shear Modulus \( G = \frac{10,000 \text{ lb.in.} \times 20.7 \text{ in.} \times 4.45 \text{ in.}}{311 \text{ in}^4 \times 14 \times 10^{-4} \text{ in.}} \)

\[ G = 2.11 \times 10^3 \text{lb/in}^2 \]
Appendix XIII (Cont.)

(b) Electrical Measurements

The electrical shear strains $\frac{dQ}{dJ}$ at a particular rosette are obtained by adding numerically the readings of the 45° gauges (See Appendix XXXB) viz. Nos. 2 and 3, and 3 and 4.
These give :-

<table>
<thead>
<tr>
<th>Test No.</th>
<th>$\frac{dQ}{dJ} \times 10^4$ per 10,000 lb.in</th>
<th>Torque</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Gauges Nos. 2 &amp; 3</td>
<td>Gauges Nos. 3 &amp; 4</td>
</tr>
<tr>
<td>1</td>
<td>1.05</td>
<td>1.05</td>
</tr>
<tr>
<td>2</td>
<td>1.05</td>
<td>1.05</td>
</tr>
</tbody>
</table>

Using the elastic formula :-

$$\text{Shear Modulus } G = \frac{Tr}{J\varepsilon_{xy}}$$

where $T = \text{torque} = 10,000 \text{ lb.in.}$

$r = \text{radius of section} = 3.75 \text{ in.}$

$J = \text{Polar 2nd Moment of Area} = 311 \text{ in}^4$

$$\text{Shear Modulus } G = \frac{10,000 \text{ lb.in} \times 3.75 \text{ in.}}{311 \text{ in}^4 \times \varepsilon_{xy}}$$

Assuming a Strain Sensitivity Factor of 2.1

For Tests No. 1 and 2 taking $\varepsilon_{xy} = \frac{1.05 \times 10^{-4}}{2.1}$

$$\text{Shear Modulus } G = 2.41 \times 10^6 \text{ lb/in}^2$$
Appendix XLIII

BENDING TEST No.1

Details

Date . . . . . . . . . 15th January 1949
Specimen . . . . . Specimen 'G'
Gauge Current . . . 51 Milliamps.
Remaining data . . As per Torsion Test No.1

Abstract of Readings (Plotted on Graph 18.1)

<table>
<thead>
<tr>
<th>Bending Moment (lb.in.)</th>
<th>Fractional Resistance Change $\frac{\Delta q}{q} \times 10^4$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Gauge No.1</td>
</tr>
<tr>
<td>280</td>
<td>0</td>
</tr>
<tr>
<td>1690</td>
<td>-0.11</td>
</tr>
<tr>
<td>3150</td>
<td>-0.28</td>
</tr>
<tr>
<td>4520</td>
<td>-0.33</td>
</tr>
<tr>
<td>5940</td>
<td>-0.48</td>
</tr>
<tr>
<td>7360</td>
<td>-0.53</td>
</tr>
<tr>
<td>8780</td>
<td>-0.48</td>
</tr>
</tbody>
</table>

Note. Gauge No.5 failed to record.

Observations

From Graph 18.1 for bending moments 0 = 70000 lb.in
Mean Electrical Strains $\frac{\Delta q}{q}$ per 10,000 lb.in bending moment were :

<table>
<thead>
<tr>
<th>Fractional Resistance Change $\frac{\Delta q}{q} \times 10^4$ per 10,000 lb.in</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bending Moment.</td>
</tr>
<tr>
<td>Compression</td>
</tr>
<tr>
<td>Gauge No.1</td>
</tr>
<tr>
<td>-0.34</td>
</tr>
</tbody>
</table>
Appendix XLIII

BENDING TEST No. 2

Details

Date . . . . . . . . 22nd January, 1949
Gauge Current . . . . 25 Milliamps.
Deflections . . . . Two plunger type 1/10,000 inch
dial gauges 20.7 in. apart.

Remaining Data . . . As per Torsion Test No. 1

Abstract of Readings (Plotted on Graphs 18.1 and 18.3)

Electrical Readings

<table>
<thead>
<tr>
<th>Bending Moment (lb.in)</th>
<th>Fractional Resistance Change ( \frac{\Delta R}{R} \times 10^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Gauge No. 1</td>
</tr>
<tr>
<td>230</td>
<td>0</td>
</tr>
<tr>
<td>1410</td>
<td>-0.06</td>
</tr>
<tr>
<td>2625</td>
<td>-0.22</td>
</tr>
<tr>
<td>4240</td>
<td>-0.59</td>
</tr>
<tr>
<td>5650</td>
<td>-0.59</td>
</tr>
<tr>
<td>7070</td>
<td>-0.37</td>
</tr>
<tr>
<td>230</td>
<td>-0.37</td>
</tr>
</tbody>
</table>

Note: Gauges Nos 6 and 7 failed to record.
Appendix XLIII (Cont)

BENDING TEST No. 2 (Cont)

Mechanical Readings

<table>
<thead>
<tr>
<th>Bending Moment (lb.in)</th>
<th>Top Deflection x 10^4 in.</th>
<th>Bottom Deflection x 10^4 in.</th>
</tr>
</thead>
<tbody>
<tr>
<td>220</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1410</td>
<td>0.2</td>
<td>0.2</td>
</tr>
<tr>
<td>2820</td>
<td>1.0</td>
<td>1.5</td>
</tr>
<tr>
<td>3960</td>
<td>2.0</td>
<td>2.0</td>
</tr>
<tr>
<td>5100</td>
<td>3.2</td>
<td>4.4</td>
</tr>
<tr>
<td>6230</td>
<td>4.4</td>
<td>7.0</td>
</tr>
<tr>
<td>7070</td>
<td>5.5</td>
<td>9.0</td>
</tr>
</tbody>
</table>

Observations

(a) From Graph 18.1 for bending moments 0 = 7000 lb.in.
Mean Electrical Strain \( \varepsilon \) per 10,000 lb.in.

\[ \text{bending moment} = -1.06 \times 10^{-4} \]

(b) From Graph 18.3 for bending moment 0 = 7000 lb.in.
Mean Deflections on top and bottom dial gauges per 10,000 lb.in. bending moment were:

Top Gauge = 106 \times 10^{-4} in.
Bottom Gauge = 26 \times 10^{-4} in.

Ratio \( \frac{\text{Top}}{\text{Bottom}} = 5.26 \)
Appendix XLIV

BENDING TESTS Nos. 3 and 4

Details

Date . . . . . . . . . 29th January 1949

Specimen . . . . . Specimen 'G', as per Torsion Test No.1

Deflections . . . Two plunger type dial gauges 1/10,000 in., 20.7 inches apart.

Abstract of Readings (Plotted on Graph 18.3)

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>650</td>
<td>1.6</td>
<td>0</td>
<td>1.1</td>
<td>0</td>
</tr>
<tr>
<td>1300</td>
<td>12.1</td>
<td>0.4</td>
<td>9.6</td>
<td>1.2</td>
</tr>
<tr>
<td>1950</td>
<td>18.9</td>
<td>1.6</td>
<td>18.9</td>
<td>2.4</td>
</tr>
<tr>
<td>2600</td>
<td>27.2</td>
<td>3.0</td>
<td>27.1</td>
<td>4.0</td>
</tr>
<tr>
<td>3250</td>
<td>34.6</td>
<td>5.9</td>
<td>35.1</td>
<td>5.8</td>
</tr>
<tr>
<td>3900</td>
<td>43.3</td>
<td>8.0</td>
<td>43.5</td>
<td>7.4</td>
</tr>
<tr>
<td>4550</td>
<td>53.6</td>
<td>9.1</td>
<td>50.3</td>
<td>8.3</td>
</tr>
<tr>
<td>5200</td>
<td>63.2</td>
<td>11.3</td>
<td>60.6</td>
<td>10.0</td>
</tr>
<tr>
<td>5550</td>
<td>72.5</td>
<td>14.5</td>
<td>70.0</td>
<td>12.3</td>
</tr>
<tr>
<td>6200</td>
<td>81.2</td>
<td>16.6</td>
<td>79.7</td>
<td>15.3</td>
</tr>
<tr>
<td>7150</td>
<td>92.9</td>
<td>20.6</td>
<td>95.2</td>
<td>21.9</td>
</tr>
<tr>
<td>0</td>
<td>9.2</td>
<td>6.6</td>
<td>5.5</td>
<td>2.3</td>
</tr>
</tbody>
</table>

Observations

From Graph 18.3, the average readings were :

<table>
<thead>
<tr>
<th>Test No.</th>
<th>Mean Deflection per 10,000 lb.in.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Top Gauge x 10^-4 in.</td>
</tr>
<tr>
<td>3</td>
<td>144</td>
</tr>
<tr>
<td>4</td>
<td>135</td>
</tr>
</tbody>
</table>
Appendix XLV

BENDING TEST No.5

Details

This Test formed the first part of the combined bending and torsion test, full particulars of which are given in Appendix XLVII.

Observations

(a) From Graph 18.2, for bending moments 0 - 7000 lb.in.,

Mean Electrical Strains $\frac{dq}{Q}$ per 10,000 lb.in. bending moment were :-

<table>
<thead>
<tr>
<th>Fractional Resistance Change $\frac{dq}{Q} \times 10,000$ lb.in. B.M.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Compression</td>
</tr>
<tr>
<td>Gauge No.2</td>
</tr>
<tr>
<td>-0.40</td>
</tr>
</tbody>
</table>

Note  Gauge No.1 failed to record.

(b) From Graph 18.3, for bending moment 0 - 7000 lb.in.,

Mean Deflections per 10,000 lb.in. bending moment were :-

Top Gauge = $152 \times 10^{-4}$ in.

Bottom Gauge = $23 \times 10^{-4}$ in.

Ratio $\frac{\text{Top}}{\text{Bottom}} = 5.74$
CALCULATIONS ON THE BENDING TESTS

(a) Mechanical Measurements

FIG. 18.1

Considering a uniform section subjected to a uniform bending moment, then it may be shown that the Bending Modulus of the Section is given by:

\[ E = \frac{M}{2I} \left( \frac{l + 20.7''}{y_T} \right)^2 \]...

Eqn. 18.1

and

\[ E = \frac{M}{2I} \frac{\ell^2}{y_B} \]

\[ \frac{y_T}{y_B} = \left( 1 + \frac{20.7''}{\ell} \right)^2 \]

From the various tests the ratios of \( \frac{y_T}{y_B} \) per 10,000 lb.in. bending moment were:

<table>
<thead>
<tr>
<th>Test No.2</th>
<th>Test No.3</th>
<th>Test No.4</th>
<th>Test No.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.26</td>
<td>4.50</td>
<td>5.87</td>
<td>5.74</td>
</tr>
</tbody>
</table>

For Test No.5, using the above formulae:

\[ \ell = 14.8'' \]

\[ E = 3.06 \text{ lb/in}^2 \]

Note: The calculation of the Bending Modulus is dependent on the absolute value of the deflection. The variations obtained are due to errors in the absolute values.
Appendix XLVI (Cont.)

(b) Electrical Measurements

(1) It is shown in Appendix XLIX that \( e_{45} = \frac{e}{2} (1 - \nu) \)

where \( e_{45} \) = strain at 45° to the direct strain 'e',

\( \nu = \) Poisson's Ratio

From the average electrical strain readings of Test No. 5 for 10,000 lb.in. bending moment:

**Compression Rosette:**
- \( e_{45} = 0.42 \times 10^{-4} \)
- \( e = 0.95 \times 10^{-4} \)

Poisson's Ratio in compression = 0.12

**Tension Rosette:**
- \( e_{45} = 0.31 \times 10^{-4} \)
- \( e = 0.97 \times 10^{-4} \)

Poisson's Ratio in tension = 0.25

(2) Assuming that plane sections remain plane, using the simple Theory of Bending:

Bending Stress \( f = \frac{M}{I} \times y \)

For 10,000 lb.in. bending moment, and specimen \( \frac{7}{8} \) in. dia.

\( f = 241 \text{ lb/in}^2 \)

Taking a mean bending strain = \( \frac{1.0 \times 10^{-4}}{3.5 \text{ in.}} \)

and a Strain Sensitivity Factor (S.S.F.) = 2.1

Then Bending Modulus \( \frac{E}{I} = \frac{\text{stress}}{\text{strain}} = \frac{241 \times 2.1}{1 \times 10^{-4}} \text{ lb/in}^2 \)

= 5,06 lb/in^2
Appendix XLVII

COMBINED BENDING AND TORSION TEST

Details

Date . . . . . . . . . . . 29th January 1949
Specimen . . . . . Specimen 'G' (See Torsion Test No.1)
Age 64 days.
Gauge Current . . . 19 Milliamps.
Remaining Data . . . As per Torsion Test No.1
(Appendix XXXVIII)

Abstract of Readings (Plotted on Graph 19.2)

See over page.
<table>
<thead>
<tr>
<th>Bending Moment (lb.in.)</th>
<th>Torque</th>
<th>Top Deflection x 10^4 in.</th>
<th>Bottom Deflection x 10^4 in.</th>
<th>Fractional Resistance Change dΩ/Ω x 10^4</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>650</td>
<td>0</td>
<td>2.9</td>
<td>0</td>
<td>+0.01</td>
</tr>
<tr>
<td>1300</td>
<td>0</td>
<td>8.6</td>
<td>0</td>
<td>+0.04</td>
</tr>
<tr>
<td>1950</td>
<td>0</td>
<td>14.3</td>
<td>1.2</td>
<td>+0.07</td>
</tr>
<tr>
<td>2600</td>
<td>0</td>
<td>21.4</td>
<td>2.2</td>
<td>+0.10</td>
</tr>
<tr>
<td>3250</td>
<td>0</td>
<td>28.7</td>
<td>3.2</td>
<td>+0.14</td>
</tr>
<tr>
<td>3900</td>
<td>0</td>
<td>36.7</td>
<td>4.2</td>
<td>+0.17</td>
</tr>
<tr>
<td>4500</td>
<td>0</td>
<td>45.7</td>
<td>5.3</td>
<td>+0.20</td>
</tr>
<tr>
<td>5200</td>
<td>0</td>
<td>55.5</td>
<td>6.5</td>
<td>+0.23</td>
</tr>
<tr>
<td>5850</td>
<td>0</td>
<td>65.5</td>
<td>7.8</td>
<td>+0.26</td>
</tr>
<tr>
<td>6500</td>
<td>0</td>
<td>76.0</td>
<td>9.2</td>
<td>+0.29</td>
</tr>
<tr>
<td>7150</td>
<td>0</td>
<td>86.2</td>
<td>11.3</td>
<td>+0.32</td>
</tr>
<tr>
<td>7800</td>
<td>0</td>
<td>97.7</td>
<td>13.7</td>
<td>+0.35</td>
</tr>
<tr>
<td>2324</td>
<td>3.3</td>
<td>0</td>
<td>0</td>
<td>+0.38</td>
</tr>
<tr>
<td>4251</td>
<td>3.6</td>
<td>0</td>
<td>0</td>
<td>+0.47</td>
</tr>
<tr>
<td>5668</td>
<td>3.2</td>
<td>0</td>
<td>0</td>
<td>+0.28</td>
</tr>
<tr>
<td>7085</td>
<td>3.3</td>
<td>0</td>
<td>0</td>
<td>+0.55</td>
</tr>
<tr>
<td>8502</td>
<td>3.9</td>
<td>0</td>
<td>0</td>
<td>+0.61</td>
</tr>
<tr>
<td>9919</td>
<td>11.9</td>
<td>0</td>
<td>0</td>
<td>-0.68</td>
</tr>
<tr>
<td>11326</td>
<td>14.5</td>
<td>0.4</td>
<td>0</td>
<td>-0.75</td>
</tr>
<tr>
<td>12753</td>
<td>16.5</td>
<td>1.3</td>
<td>0</td>
<td>-0.81</td>
</tr>
<tr>
<td>14170</td>
<td>18.6</td>
<td>3.3</td>
<td>0</td>
<td>-0.90</td>
</tr>
<tr>
<td>15503</td>
<td>18.6</td>
<td>3.9</td>
<td>0</td>
<td>-0.97</td>
</tr>
<tr>
<td>16437</td>
<td>18.4</td>
<td>5.2</td>
<td>0</td>
<td>-1.00</td>
</tr>
<tr>
<td>17870</td>
<td>18.4</td>
<td>6.6</td>
<td>0</td>
<td>-1.07</td>
</tr>
<tr>
<td>18704</td>
<td>18.2</td>
<td>7.3</td>
<td>0</td>
<td>-1.13</td>
</tr>
<tr>
<td>19537</td>
<td>16.8</td>
<td>9.0</td>
<td>0</td>
<td>-1.20</td>
</tr>
<tr>
<td>20971</td>
<td>14.6</td>
<td>10.9</td>
<td>0</td>
<td>-1.28</td>
</tr>
<tr>
<td>22104</td>
<td>10.1</td>
<td>11.7</td>
<td>0</td>
<td>-1.35</td>
</tr>
<tr>
<td>22324</td>
<td>5.1</td>
<td>13.0</td>
<td>0</td>
<td>-1.45</td>
</tr>
<tr>
<td>23504</td>
<td>-7.5</td>
<td>14.6</td>
<td>0</td>
<td>-1.50</td>
</tr>
</tbody>
</table>

* New data for deflections. 

<table>
<thead>
<tr>
<th>Bending Moment constant at 7800 lb. in.</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.3</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Fractional Resistance Change dΩ/Ω x 10^4</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
</tr>
</tbody>
</table>

ρ Reading increasing.
CALCULATIONS ON THE COMBINED BENDING AND TORSION TEST

(1) Shearing Actions.

The electrical shear strains \( \frac{\Delta Q}{Q} \) are obtained by taking the algebraic difference between the readings of the 45° gauges of a pair. These give:

<table>
<thead>
<tr>
<th>Torque (lb.in)</th>
<th>Electrical Shear Strain ( \frac{\Delta Q}{Q} \times 10^4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Gauges ((2) + (3))</td>
</tr>
<tr>
<td>2834</td>
<td>0.08</td>
</tr>
<tr>
<td>4251</td>
<td>0.30</td>
</tr>
<tr>
<td>5668</td>
<td>0.34</td>
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<tr>
<td>7085</td>
<td>0.46</td>
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<tr>
<td>8502</td>
<td>0.66</td>
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<tr>
<td>9919</td>
<td>0.73</td>
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<tr>
<td>11336</td>
<td>0.90</td>
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<tr>
<td>12757</td>
<td>1.06</td>
</tr>
<tr>
<td>14170</td>
<td>1.22</td>
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<tr>
<td>15533</td>
<td>1.36</td>
</tr>
<tr>
<td>16497</td>
<td>1.44</td>
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<tr>
<td>17570</td>
<td>1.59</td>
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<tr>
<td>18704</td>
<td>1.63</td>
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<tr>
<td>19827</td>
<td>1.62</td>
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<tr>
<td>20971</td>
<td>1.94</td>
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<td>22104</td>
<td>2.04</td>
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<tr>
<td>22294</td>
<td>2.21</td>
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<tr>
<td>23804</td>
<td>2.29</td>
</tr>
</tbody>
</table>

Note: From Graph 19.3, the average electrical shear strain per 10,000 lb.in. torque is \( 1.05 \times 10^{-4} \).

Using the elastic formula (See Appendix XLI)

Shear Modulus \( G = 2.41 \times 10^6 \text{lb/in}^2 \)

(ii) Bending Actions

As shown in Appendix XLVI, the Bending Modulus obtained in this test was \( 5.06 \times 10^6 \text{lb/in}^2 \).
### Analysis of the Tension Rosette

<table>
<thead>
<tr>
<th>Lead (lb.in)</th>
<th>Fractional Resistance Change</th>
<th>Electrical Strains $\frac{d\varepsilon}{Q} \times 10^4$</th>
<th>Principal Electrical Strains $\frac{d\varepsilon}{Q} \times 10^4$</th>
<th>Angles of Principal Planes to $x'$ axis</th>
<th>$\phi_1$</th>
<th>$\phi_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>B.H. Torque</td>
<td>Gauge No. 4</td>
<td>Gauge No. 5</td>
<td>Gauge No. 6 and 7</td>
<td>$\varepsilon_x$</td>
<td>$\varepsilon_y$</td>
<td>$\varepsilon_{xy}$</td>
</tr>
<tr>
<td>7000 Zero</td>
<td>+0.21</td>
<td>+0.21</td>
<td>+0.64</td>
<td>+0.64</td>
<td>-0.22</td>
<td>0</td>
</tr>
<tr>
<td>7800 4000</td>
<td>+0.10</td>
<td>+0.33</td>
<td>+0.73</td>
<td>+0.73</td>
<td>-0.30</td>
<td>+0.28</td>
</tr>
<tr>
<td>7800 8000</td>
<td>-0.10</td>
<td>+0.59</td>
<td>+0.75</td>
<td>+0.75</td>
<td>-0.26</td>
<td>+0.69</td>
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<tr>
<td>7800 12000</td>
<td>-0.30</td>
<td>+0.79</td>
<td>+0.75</td>
<td>+0.75</td>
<td>-0.26</td>
<td>+1.02</td>
</tr>
<tr>
<td>7800 16000</td>
<td>-0.54</td>
<td>+1.01</td>
<td>+0.74</td>
<td>+0.74</td>
<td>-0.27</td>
<td>+1.55</td>
</tr>
</tbody>
</table>

*Note:* The calculations were made using Eqn. 19.1 to 19.5 inclusive, See Appendix XLIX.

$\sigma$ = Poisson's Ratio.
CALCULATION OF THE PRINCIPAL DIRECT AND SHEAR STRAINS
FROM THE READING OF A STRAIN GAUGE ROSETTE

It may be shown that for an element of material under any two-dimensional stress system, the strain in a direction at any angle $\Theta$ to perpendicular reference axes 'x' and 'y' is given by:

$$\varepsilon_\Theta = \varepsilon_x \cos^2 \Theta + \varepsilon_y \sin^2 \Theta + \frac{1}{2} \varepsilon_{xy} \sin 2\Theta \quad \ldots \quad \text{Eqn. 19.1}$$

where $\varepsilon_x$ and $\varepsilon_y$ are the strains in the directions 'x' and 'y', and $\varepsilon_{xy}$ is the shear strain.

Bending Strain at $45^\circ$

$$\varepsilon_{45} = \frac{\varepsilon_x + \varepsilon_y}{2} = \frac{\varepsilon}{2} (1 - \nu) \quad \ldots \quad \text{Eqn. 19.2}$$

where $\varepsilon = \varepsilon_a$ = axial strain.

For a $0^\circ$, $45^\circ$ and $135^\circ$ rosette,

$$\text{Shear Strain } \varepsilon_{xy} = \varepsilon_{45} - \varepsilon_{135} \quad \ldots \quad \text{Eqn. 19.3}$$

The maximum and minimum values of equation 19.1 are obtained when

$$2\Theta = \tan^{-1} \left( \frac{\varepsilon_{xy}}{\varepsilon_x - \varepsilon_y} \right) \quad \ldots \quad \text{Eqn. 19.4}$$

and are given by:

Principal Strains

$$\varepsilon_{\text{max}} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{1}{2} \sqrt{(\varepsilon_x - \varepsilon_y)^2 + \varepsilon_{xy}^2} \quad \ldots \quad \text{Eqn. 19.5}$$

$$\varepsilon_{\text{min}} = \frac{\varepsilon_x + \varepsilon_y}{2} - \frac{1}{2} \sqrt{(\varepsilon_x - \varepsilon_y)^2 + \varepsilon_{xy}^2} \quad \ldots \quad \text{Eqn. 19.5}$$
<table>
<thead>
<tr>
<th>Ref. No.</th>
<th>Author</th>
<th>Title</th>
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<tr>
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</tr>
</tbody>
</table>

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Baldwin-Southwark SR-4 Gauges

(HEXICAL-WOUND TYPE B)

VARIATION IN STRAIN SENSITIVITY WITH LENGTH OF GAUGE.

Strain Sensitivity Factor

Graph No. 3-2
Electrical and Mechanical Stress-Strain Curves

Graph No. 5-1

Nominal Stress (lb/in²)

-105-
Strain Sensitivity of N.P.L. 2500 Ohm Gauge.

Preliminary Compression Test No. 1

by

THE AUTHOR

Graph No. 52
Variation in Circuit Sensitivity with Galvanometer Resistance

Graph No. 81
VARIATION IN CIRCUIT SENSITIVITY WITH BRIDGE RATIO FOR OPTIMUM GALVANOMETER RESISTANCE

Graph No. 8-2
STRESS-AXIAL STRAIN CURVES
PRELIMINARY COMPRESSION TEST No. 2 (C)
(GAUGES AFFIXED WITH SHELLAC DISSOLVED IN METHYLATED SPIRITS)
BY THE AUTHOR
Graph No. 92.

**Stress-Axial Strain Curves.**

Preliminary Compression Test No. 2(a)

*Gauges affixed with shellac dissolved in alcohol* by the author.
STRESS-AXIAL STRAIN CURVES
PRELIMINARY COMPRESSION TESTS N° 2 (c)
(GAUGES AFFIXED WITH HIGH SOLIDS DUROFIX AND TESTED AT VARIOUS AGES)
THE AUTHOR
Strain-Sensitivity of N.P.L. 1300 Ohm Gauge
Preliminary Compression Tests No. 2(e)
by The Author

Graph No. 9-5
Gauge Calibration Tests on Steel Beams.
Load - Electrical Strain Curves
by
The Author

Graph No. 111
Gauge Calibration Tests on Steel Beams.
Load-Deflection Curves
by the Author

Graph No. 112
Gauge Calibration on a Steel Tension Specimen
Load-Strain Curves.

by the Author

Graph No. 11-3
GAUGE CALIBRATION TEST ON STEEL BEAM
CROSS SENSITIVITY INVESTIGATION.

by THE AUTHOR

GRAPH No. 11-4
TENSION TESTS BY L. TURNER AND V. C. DAVIES

1:2:4 Concrete at 35 Days

1:2:3 Concrete at 30 Days

TENSION TESTS BY A. N. JOHNSON

Graph No. 13.1
TENSION TESTS BY A.H. NYLANDER

TENSION TESTS BY R.H. EVANS

Graph No. 132
TENSILE LOAD-STRAIN CURVES
TENSION TESTS NO. 1 & 2
by
THE AUTHOR
TENSILE LOAD-STRAIN CURVES
TENSION TEST NO. 3.
by THE AUTHOR
TENSILE LOAD - STRAIN CURVES
TENSION TEST NO. 4
by THE AUTHOR
GRAPH NO. 135

Nominal Stress (lb/in²)

Electrical Strain $\Delta Q/Q$

0.50 x 10^-4

Specimen 'A'

Gauge Positions

Gauge No. 1
Gauge No. 2
Gauge No. 3
Gauge No. 4
Law of Curve:

\[
\text{Stress (Lb./in.\(^2\))} = 363 - 175(1.75 - e)^{13}
\]

where \( e \) = electrical strain \( \times 10^{-4} \)

TENSILE STRESS-STRAIN CURVE

TENSION TEST NO. 4

by THE AUTHOR

GRAPH No. 13-6
Extensometer Calibration Curve
Tension Test No. 5
by
The Author

Graph No. 137
TENSILE LOAD-STRAIN CURVES
TENSION TEST NO. 5
by THE AUTHOR

Graph No. 13-8
TENSILE STRESS-STRAIN CURVES
TENSION TEST No. 5
by THE AUTHOR

Graph No. 13-9
Strain Sensitivity of B.T.C. 200 Ohm Gauges
Tension Test No. 5
by The Author
Graph No. 13-10
TENSION TEST No.2

TENSION TEST No.3

TENSION TEST No.4

STRAIN DISTRIBUTION DURING TENSION TESTS

by THE AUTHOR

Note:

- Strains quoted at 200 lb/ins²
- Units: Electrical strain "\(\varepsilon'\) x 10⁻⁶"
Strain Distribution During Tension Test

Tension Test No. 5

by

The Author

Graph No. 13-12

Note: Units - Electrical Strain \( \frac{\Delta \sigma}{\sigma} \times 10^6 \)
Compression Load-Hoop Strain Curves

Compression Test No. 6B

by the author
Mean Electrical Strain 
40° PER 1276 LB/IN²

Variation in Hoop Strain in Height of Specimen
Compression Test No. 6B
by The Author

Graph No. 43
COMPRESSIVE LOAD - AXIAL STRAIN CURVES
COMPRESSION TEST No. 4
THE AUTHOR
NUMBER OF REPETITIONS

STRESS (LB./IN.²)

DEFLECTION ON 8" GL.

DEFORMATION ON 8" GL.

CONCRETE UNDER REPEATED COMPRESSION
TESTS BY VAN ORNUM

GRAPH No. 45
STRESS-STRAIN CURVES FOR CONCRETE AFTER CYCLES OF COMPRESSION TESTS BY MEHMEI

GRAPH No. 4-6
Effect of Repeated Loading on Compression Stress-Strain Curves

by

The Author
EFFECT OF REPEATED LOADING ON COMPRESSION STRESS-STRAIN CURVES

Graph No. 148
COMPLETE COMPRESSION STRESS-STRAIN CURVE
TEST BY AN. JOHNSON

Graph No. 14·9
COMPRESSIÓN LOAD—AXIAL & LATERAL STRAIN CURVES

COMPRESSIÓN TEST No. 12

by
THE AUTHOR

Graph No. 14:10
COMPRESSION STRESS-AXIAL & LATERAL STRAIN CURVES

COMPRESSION TEST NO. 12

by

THE AUTHOR

GRAPH NO. 14-11
Compression Stress-Strain Curves
Compression Test No. 12
by
THE AUTHOR

Graph No. 14.12
EFFECT OF REPEATED COMpressive LOADING ON THE STRESS-STRAIN CURVE

GRAPH No. 14-13

Nominal Stress (lb/in²)

Compression on 0'' Inch Length

Concrete 5'/year old

Cylinder 12'' x 6.4''

1st Run
2nd Run
3rd Run

5 x 10⁴ lb/in²
Hysteresis and Permanent Set in Compression Stress-Strain Curves

by

THE AUTHOR
Hysteresis in Hoop Strains

Graph No. 14:15
INCREASE IN STRAIN X 10^4
IN 1 1/2 MINUTES

VARIATION IN CREEP STRAIN WITH LOAD
COMPRESSION TEST NO. 12
BY THE AUTHOR

GRAPH NO. 14-16
VARIATION IN POISSON'S RATIO WITH AGE
TESTS BY DAVIS & TROXELL

Note: Results for Sandstone Concrete at 800 Lb./in.²

VARIATION IN POISSON'S RATIO WITH COMRESSIVE LOAD
TEST BY A.N.JOHNSON

Graph No. 14-17
Variation in Poisson's Ratio with Compressive Load

Tests by Richart, Brandtzaeg & Brown

Graph No. 14-18
VARIATION IN POISSON'S RATIO WITH LOAD
COMPRESSION TEST NO. 12
by
THE AUTHOR
GRAPH NO. 14-19
Volume Changes during Compressive Loading.

Graph No. 14-20
**Mechanical Strain**

**Strain Sensitivity Factor of 2000 Ohm Gauges.**

_Figures 14-21_

- **Compressometer:** 8 G.L.
- **Specimen:** 10 x 5 dia.

_Note:_

- Test No. 9A
- Test No. 10A
- S.S.F. = 2.15
- S.S.F. = 2.24

**Graph No. 14-21**
Strain Sensitivity Factor of 200ohm Gauges
Compression Test No. 12
by The Author

Graph No. 14.22
THE 'NEGATIVE LOOP' PHENOMENON WITH 2000 OHM GAUGES.
REINFORCED CONCRETE BEAM TEST BY F.C. LEA

RECTANGULAR PLAIN CONCRETE BEAM TEST BY R.H. EVANS

STRAIN VARIATION IN DEPTH OF RECTANGULAR BEAMS

GRAPH NO. 151
LOAD-STRAIN CURVES FOR PLAIN CONCRETE BEAM

BEAM TEST NO. 1
by
THE AUTHOR
LOAD - STRAIN CURVES FOR PLAIN CONCRETE BEAM
BEAM TEST NO. 2
BY THE AUTHOR
LOAD-STRAIN CURVES
BEAM TEST NO. 4
by
THE AUTHOR

Specimen F
6" x 3" Beam

Compression
Top of Beam.

Electrical Strain
$\varepsilon_0 \times 10^4$

N.A. Calculated
elastically $\frac{E}{E_c} = 12.5$

Tensile Strains
Bottom of Beam.

Strain Distribution in Depth of Beam
Graph No. 15-5
LOAD-STRAIN CURVES FOR PLAIN CONCRETE BEAM
BEAM TEST NO. 5
by
THE AUTHOR

GRAPH No. 15-6
STRAIN DISTRIBUTION IN DEPTH OF BEAM

BEAM TEST NO. 5
by
THE AUTHOR

GRAPH NO. 15-7
Increase of Deflection due to Creep

Release of Load due to Creep

Release of Load in 5 minutes

Increase of Deflection in 5 minutes

Effect of Creep in Beam under Load

Beam Test No. 5
by
The Author

Graph No. 15-8
Torsion Tests by L. Turner & V. C. Davies

Note:
1) Specimen: 10' Square
2) Strains measured over 2' gauge length

Torsion Tests by Paul Anderson

Graph No. 171
Graph No. 172

Torque-Principal & Shear Strain Curves
Torsion Test No. 1
by
The Author
The diagram presents torque - principal and shear strain curves for a torsion test No. 2 by the author. The graph shows the relationship between applied torque (lb in) and electrical strain (μ). The data points are marked for different gauge pairs located at 45° to the axis. The graph includes specifications for specimen 'Q' with dimensions and notes on shear strains at opposite ends of a diameter. The author's name is indicated at the bottom of the graph.
LOAD-BENDING STRAIN CURVES
BENDING TESTS NO. 1 & 2
by THE AUTHOR
LOAD-BENDING STRAIN CURVES
BENDING TEST NO. 5
by THE AUTHOR

Compression Gauges
No. 2
No. 3
No. 4
No. 5

Tension Gauges
No. 6
No. 7

ELECTRICAL STRAIN \( \varepsilon_0 \)

Specimen 'G'
7/2 dia.

3\(\frac{1}{2}\)"
LOAD-BENDING DEFLECTION CURVES
BENDING TESTS
BY THE AUTHOR

GRAPH No. 18.3
Secant Shear Modulus of Elasticity ($\text{kg/cm}^2 \times 10^3$)

- Bending Moment = 1158 Kg.M.
- = 1088 Kg.M.
- = 948 Kg.M.
- = 878 Kg.M.

Graph No. 19-1

Combined Bending and Torsion Tests by A.H. Nylander
LOAD-STRAIN CURVES
COMBINED BENDING AND TORSION TEST

Specimen 'G' 7\frac{1}{2} dia.

Graph No 192
Torque-Shear Strain Curves under a Constant Bending Moment.
Combined Bending and Torsion Test
by THE AUTHOR
Graph No. 19-3
TENSILE SPECIMEN 'A'

The Davies Extensometer and two strain gauges are shown in position.

Plate 13.1
Plate 13.2

TENSILE TESTING ARRANGEMENT

The Specimen is shown in the Riehle Machine with Extensometer and Strain Gauge attached.
COMPRESSION SPECIMEN 'C'

The Hamilton Compressometer and strain gauges are shown in position.

Plate 14.1
BEAM TESTING ARRANGEMENT

Test Specimen 'F' with Strain Gauges attached is shown in the Buckton Machine with the proving ring and distributing bar in position. 'Dummy' Specimen 'E' is shown in the foreground.
A STRAIN GAUGE ROSETTE ON SPECIMEN 'G'
SPECIMEN 'G' IN COMBINED BENDING AND TORSION MACHINE

Proving Rings and a Strain Gauge Rosette are shown.
TYPICAL FRACTURE UNDER COMBINED BENDING AND TORSION