X-RAY SPECTRA FROM EXOTIC ATOMS OF HYDROGEN AND HELIUM

A thesis submitted to the faculty of Mathematical and physical sciences of the University of Surrey for the degree of Doctor of Philosophy.

by

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An experiment [CERN PS165] is described which aimed to study the interaction between a kaon and a proton by measurement of the X-ray spectrum from kaonic hydrogen in a liquid target. The spectra of exotic atoms of helium, also a liquid target, were studied first to test the apparatus. In each case the objective was to measure X-ray intensities and the strong interaction effects, namely energy shift \( [\varepsilon] \) and the line broadening \( [\Gamma] \), induced by the short range nuclear force operating when a negatively charged exotic hadron reaches a low lying atomic state. The related theory and interpretation are also discussed.

The results for kaonic helium, \( \varepsilon_{2p} = (-0.050 \pm 0.012) \text{ keV} \) and \( \Gamma_{2p} = (0.100 \pm 0.040) \text{ keV} \), confirm the previous report of a disagreement with theoretical expectation. The measured 3-2 X-ray transition intensity is about 1% per atom.

The results on \( \bar{p} \)-helium, \( \varepsilon_{2p} = (-0.012 \pm 0.014) \text{ keV} \) and \( \Gamma_{2p} = (0.040 \pm 0.040) \text{ keV} \), are in agreement with theory and contrary to the conclusions of a previous experiment\(^{14}\); the absolute intensity of the 3-2 X-ray transition, measured for the first time in a liquid target, is \((4.4 \pm 2.2)\%\) per atom.
Kaonic hydrogen X-rays are confidently identified in this experiment, but the result

$$\varepsilon_{1s} = 0.200 \pm 0.060 \text{ keV}$$

$$\Gamma_{1s} = 0.080 \pm 0.220 \text{ keV}$$

and relative X-ray intensities are quite different to two previous more tentative reports. All experiments disagree with the result expected from analysis of kaon-nucleon interactions above threshold.

The yield of X-rays was about 0.1% per atom which was, in the circumstances, close to the limit of sensitivity.

We also report the first observations of $\Sigma^-$-He atom X-rays,

$$\varepsilon_{2p} = -0.023 \pm 0.086 \text{ keV and } \Gamma_{2p} < 0.500 \text{ keV, and } \Sigma^-\text{-p atom X-rays,}$$

$$\varepsilon_{1s} = 0.500 \pm 0.060 \text{ keV and } \Gamma_{1s} < 0.500 \text{ keV, observed in coincidence with stopping kaons.
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1.1 Exotic atoms

Exotic atoms consisting of an ordinary nucleus and at least one extraordinary 'stable particle' bound principally by their mutual coulomb interaction are formed when the negatively charged mesons, pion π⁻, or kaon K⁻, the negative muon μ⁻ or heavier particles, the antiproton p or sigma Σ⁻, survive long enough in ordinary matter to stop and replace an electron in a constituent atom. The mass of an exotic particle, many times that of an electron, freedom from Pauli exclusion in the atom and conventional processes of atomic deexcitation, determine that it will eventually achieve large overlap with the nucleus. If it is a hadron [in a HADRONIC ATOM] then this overlap makes probable the relatively fast process of nuclear absorption, dissolving the atomic state. If it is a muon [in a MUONIC ATOM] the weak interactions can mediate in nuclear absorption or natural decay, also dissolving the atom. In any case the exotic atom is an ephemeral object characterized by the processes of rapid deexcitation, the cascade, between its formation and dissolution. Both X-rays and Auger electrons emitted in consequence of this deexcitation are studied to establish the existence and exotic nature of the atom. However it is X-ray observations, favoured by virtue of simple experimental technique and by rapid advances in detector technology which have recently been most conspicuous.
Assuming the formation of exotic atoms, first worked out in detail by Fermi and Teller, a naive description of the essential properties is given by the Bohr model for a hydrogen-like atom. This gives:

\[ R_n = \frac{\hbar^2}{\mu \epsilon^2} \frac{n^2}{Z} \]  

where \( n \), the principal quantum number, takes integer values only. 

\[ E_n = -\frac{Ze^2}{2R_n} \]  

where \( \mu \) is the reduced mass of the exotic particle and the nucleus 
\( e \) is the electronic charge 
\( Z \) is the atomic number \([\text{or charge number}]\) of the nucleus.

The first of these formulae \([1.1]\) shows explicitly that the average orbital radius for the state \( n, R_n \), is inversely proportional to the mass \( \mu \). Consequently for \( n < \sqrt{\frac{\mu}{m_e}} \), where \( m_e \) is the electron mass, the average separation between the exotic particle and nucleus is less than that typical of the electron distribution in the same atom. In this case surviving electrons play only a minor role, there being a slight effective screening \([\text{reduction}]\) of the nuclear charge which weakens the
electromagnetic binding. This must be calculated in a complete theory, but for \( n \ll \frac{\mu}{m_e} \) the problem is essentially hydrogen-like, to be treated in Dirac or Klein-Gordon formalism according to the spin of the exotic particle. In this context it is convenient to introduce the following notation for the interaction energy, \( W \), between the exotic particle and the nucleus:

\[
W = V + U \tag{1.4}
\]

Let \( V \) be the detailed electromagnetic interaction potential including all established significant contributions.

and \( U \) be a complex optical potential accounting for nuclear-hadron strong interactions, of which part is absorptive.

The weak interaction is negligible during the time of the cascade and \( W = V \) for muonic atoms.

Solution of the full Klein Gordon or Dirac equation including 1.4 is a complicated problem which we shall review more thoroughly later.

Returning briefly to the Bohr model, equation 1.2 gives the discrete bound state [negative] energy level structure and 1.3 is a version of the Balmer formula for radiative transitions between levels \( n' \) and \( n \). The usual notation of atomic physics is applied also to exotic atom X-ray series. Thus K, L, M... series also known as Lyman, Balmer, Paschen...
series comprise X-rays from transitions ending on \( n = 1, 2, 3 \ldots \), and also by convention greek alphabet subscripts label individual X-ray lines:

\[
\begin{align*}
\text{K-series} & \quad \{ K_\alpha \ n = 2 \text{ to } n = 1 \\
\text{or} & \quad \{ K_\beta \ n = 3 \text{ to } n = 1 \\
\text{Lyman series} & \quad \{ K_\gamma \ n = 4 \text{ to } n = 1
\end{align*}
\]

Further labelling is regarded as too often superfluous - fine and hyperfine structure are rarely resolved by experiment.

1.2 Formation and cascade

Fermi and Teller\(^{(1)}\) worked out a detailed semi-classical theory for formation of muonic atoms in iron and graphite which enabled them to calculate the typical time for moderation of a slow muon, followed by the cascade. They concluded that after \(~ 10^{-13} \text{ secs} \) the muon would be in its lowest orbit having considerable overlap with the nucleus. Exacting early justification for the study of exotic atoms they concluded from the observed decay of \( \mu^- \) particles \([T_{\text{life}} \sim 10^{-6} \text{ sec}]\) at rest in emulsions, that if the muon always forms an exotic atom and yet survives long enough to decay \([\text{see for example Wheeler}^{(3)}]\) it could not be hadronic in character - this evidence settled the debate over the nature of the muon.

The Fermi-Teller model applies equally to any particle with lifetime greater than the order of the cascade time \([\text{see Table 1}]\) and as extended by Leon and Seki\(^{(4)}\) provides a basic model for the complete description of exotic atom formation.
## Table 1

<table>
<thead>
<tr>
<th>Particle</th>
<th>Mass MeV/c(^2)</th>
<th>B</th>
<th>I</th>
<th>S</th>
<th>J(^P)</th>
<th>Lifetime sec</th>
<th>Principal % decays</th>
<th>Production Threshold* MeV/c</th>
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</thead>
<tbody>
<tr>
<td>(\mu^-)</td>
<td>105.66</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>J=(\frac{1}{2})</td>
<td>2.00.10(^{-6})</td>
<td>e(^-) (\overline{\nu}) (\nu) 98.6</td>
<td>450</td>
</tr>
<tr>
<td>(\pi^-)</td>
<td>139.57</td>
<td>0</td>
<td>-1</td>
<td>0</td>
<td>1(^-)</td>
<td>2.60.10(^{-8})</td>
<td>(\mu^-) (\overline{\nu}) 100</td>
<td>600</td>
</tr>
<tr>
<td>(K^-)</td>
<td>493.67</td>
<td>0</td>
<td>-(\frac{1}{2})</td>
<td>-1</td>
<td>0(^-)</td>
<td>1.24.10(^{-8})</td>
<td>(\mu^-) (\overline{\nu}) 63.5</td>
<td>2500</td>
</tr>
<tr>
<td>(\overline{\Sigma})</td>
<td>938.28</td>
<td>-1</td>
<td>-(\frac{1}{2})</td>
<td>0</td>
<td>(\frac{1}{2})(^-)</td>
<td>Stable</td>
<td>(\pi^-) (\pi^0) 21.2</td>
<td>5630</td>
</tr>
<tr>
<td>(\Sigma^-)</td>
<td>1197.34</td>
<td>1</td>
<td>-1</td>
<td>-1</td>
<td>(\frac{1}{2})(^+)</td>
<td>1.48.10(^{-10})</td>
<td>n (\pi^-) 100</td>
<td>7850</td>
</tr>
</tbody>
</table>

Properties of particles suitable for the formation of exotic atoms

Data from The Particle Properties Data Booklet
Rev Mod Phys. 52 No. 2 1980 (Ref. 2)

*Production thresholds for particle-antiparticle pair produced from proton-proton [fixed target] collision
It is assumed that from an incident beam, or secondary particles arising from reaction or decay, a negatively charged particle may displace one or more electrons from an atomic or molecular constituent of the target sample. In the semiclassical approach an incident particle loses energy when it scatters an electron into an unoccupied state [the analog of auger deexcitation]. Assuming that atomic electrons form a 'Fermi sea' the division between occupied and unoccupied states defines the Fermi momentum $p_F$, and vice versa. Energy loss $\frac{dE}{dt}$ is given by averaging over all possible collisions with the constraint that in any collision the electron is promoted above the Fermi momentum. With some approximation the result is:

$$- \frac{dE}{dt} = \frac{4}{3\pi} V^2 \lambda n \sqrt{\frac{p_F}{m_e}}$$

where $V$ is the velocity of the incident particle and other \textit{[radiative]} losses are small in comparison.

Slowing down to velocities $< 10^6$ ms$^{-1}$ the incident particle would cross an ion formed after ejection of an electron in time $> 10^{-16}$ sec, long enough for conventional electromagnetic capture to take place forming the precursory bound state. Except in very light elements this initial state corresponds to $n >> \sqrt{\frac{n}{m_e}}$ with the exotic particle somewhere 'at the top' of the electron distribution. At this early stage it moves in the same average potential that binds the electrons and a rigorous treatment
of the motion including the many interacting pairs is not tractable. However the physical environment is likely to have most effect while the exotic particle resides in the periphery of the atom or molecule and because it could have important consequences, affecting the relative atomic capture probabilities and the appearance of the subsequent cascade, this must be subjected to further investigation.

In this connection Fermi and Teller deduced the Z-law for differential capture on the different atomic species of a molecule. According to this rule the fractional time spent about each constituent varies in proportion to the relative Z-values and so capture probability is at least proportional to Z. This simple Z-law is found NOT to hold and most authors would naively add a Z²-dependence, arising from the binding energy, so that we might more nearly have a net Z³-dependence, discussed in detail by Daniel⁵. Consequently it is unlikely that the formation of exotic atoms of hydrogen could be established except in pure hydrogen.

For example Batty et al⁶ observed X-rays from kaonic lithium in lithium (Li) and Lithium hydride (LiH) with no evidence for any kaonic hydrogen X-ray. They thus estimated that a single kaon stopping in LiH yielded less than 2.40.10⁻⁴ probability of a 2p-1s kaonic hydrogen X-ray.

The exotic particle continues to lose energy to surrounding electrons and the semi-classical approach is applied until it reaches n = \frac{\sqrt{\frac{\mu}{m_e}}}{\mu m_e}. Now
because of the preceding uncertainty the complete prescription of the initial hydrogen-like state \((n, \lambda)\) [i.e. including the orbital quantum number \(\lambda\)] cannot be known precisely, nor is the form of the distribution over orbital states, conventionally designated \(P_n(\lambda)\), well known.

Usually the 'statistical distribution' is assumed ab initio, in which the weighting is proportional to the number of available orbital states:

\[
P_n(\lambda) = \frac{2\lambda+1}{n^2}
\]

normalized so that \(\sum_{\lambda=0}^{n-1} P_n(\lambda) = 1\)

Thereafter calculation of the cascade can proceed in a fairly rigorous manner. We find that the processes which are expected to dominate in medium and heavy atoms are the Auger ejection of atomic electrons, radiative [electric dipole] transitions and nuclear capture. Taking these in order, Auger and electric dipole transition rates are computed on the basis of formulae first derived for the one-electron atom\(^{(7)}\). The details vary between authors, but two important facts have consistently emerged:

1) Auger and electric dipole transitions both prefer to populate circular orbits \([\lambda = n-1]\) by a tendency for transitions to end on a circular orbit which is then self propagating - the proof of this rests on the selection rules which favour \(\Delta \lambda = -1\) while \(\Delta n\) is usually large; any transition under these conditions naturally ends on a final state which is nearer to a circular orbital state than the initial state.
2) The Auger process predominates with proximity to orbital electrons and accelerates the early part of the cascade in heavy atoms. The exotic particle might otherwise 'dwell' in high orbits and in this time strong absorption, decay or dissociation of the atom might compete with deexcitation.

Continuing the cascade, the nuclear capture probability is given in a typical and simple representation due to Eisenberg and Kessler\(^{(8)}\) by:

\[
P_c = \frac{J}{\tau}
\]

where \(\tau\) is a parameter identified as the mean lifetime of the exotic particle in nuclear matter,

\[
\tau = \frac{\hbar}{2U_0}
\]

where \(U_0\) is the strength of the imaginary part of the interaction, and

\[
J = 4\pi \int \rho(r)|\psi(r)|^2 r^2 dr
\]

is the overlap integral.

\(\rho(r)\) is the nuclear density function.

\(\psi(r)\) is the particle wavefunction.

The overlap typically falls off very quickly with increasing \(\lambda\) which has the consequence that S-states practically do not exist while hadrons in circular orbits can survive the cascade quite well. Consequently,
regardless of the initial $P_n(\lambda)$ an observable fraction of the population reaches low lying levels where X-ray emission is dominant.

Generally however the **absolute** X-ray yields in hadronic atoms are affected by both the strength of nuclear capture, $\tau$ in equ. 1.8, and the distribution over angular momentum states. In muonic atoms nuclear absorption is practically absent and the relative rates of radiative and non-radiative [Auger] transitions are only affected by changes to the initial population. Thus it is possible to study the phenomenology of nuclear absorption by using muonic atom data as a control.

For example Eisenberg and Kessler\(^{(8)}\) and subsequent authors have started the cascade in $n > 14$ with

$$P_n(\lambda) \propto (2\lambda + 1)\ eta^\lambda$$

which has become known as the "modified statistical distribution" discussed by Batty\(^{(9)}\), reducing to the statistical distribution for $\eta=0$. The parameter $\eta$ is the only provision for adjusting the initial population to best suit a variety of data. Fitting $\mu^-$ and $\pi^-$-atom data simultaneously Eisenberg and Kessler, using in the latter case their prescription [eqn. 1.8] for nuclear absorption, find the optimum $\eta=0.2$. This fits the trend averaged over $Z$ quite well but there is rapid $Z$-dependence in the data \(^{(10)}\) which is not reproduced by such calculations. This and other problems may be solved by attempts to calculate $P_n(\lambda)$ in a theory for formation of exotic atoms if they can be made reliable. However because there exists such difficulty in testing the hypotheses by the long and complicated procedure of a cascade calculation, this is a subject for lengthy discussion which we leave to a specialized review\(^{(5)}\).
1.3 Light exotic atoms

1.3.1 Yields

An important addition to the cascade calculations for hydrogen and helium is the possibility of transitions between different orbital states of the same principal quantum number - sliding transitions or shuffling, negligible in heavier atoms. This mixing between angular momentum states may lead to a cascade through non-circular orbits and hence the relative and absolute yields are strongly affected.

The most likely mechanism inducing sliding transitions is that of STARK EFFECT as first discussed by Day et al\(^{(11)}\) for exotic hydrogen formed in liquid hydrogen bubble chambers. It is postulated that in recoil or thermal motion the exotic atom formed in gas or liquid encounters the electromagnetic field within a second, most likely ordinary atom. This induces sliding transitions in analogy with the manner that an external electric field mixes orbital states of the ordinary hydrogen atom - the stark effect.

The effect is most severely noticed in hydrogen where the small neutral exotic atoms can penetrate the space between proton and electron and 'feel' directly their mutual electric fields. In helium the exotic atom is less likely to be a neutral system because the auger process disposes of the 'spare electron', but a positively charged object [the exotic helium ion] will interact by polarizing the charge distribution of a neighbouring helium atom and there already exists evidence to suggest the X-ray spectra from muonic helium\(^{(12)}\) pionic helium\(^{(13)}\) and antiprotonic helium\(^{(14,15)}\) may be significantly affected by some shuffling during the cascade.
Other processes significant are:-

The External Auger effect – deexcitation by ionization of a neighbouring atom;
Chemical dissociation – deexcitation by break up of a neighbouring molecule.

While these last two can be beneficial, by accelerating the cascade, the stark effect in particular will always cause losses from the cascade when \( n \) is still quite large. This it does by causing the cascade to go through low angular momentum states.

1.3.1.1 Hydrogen

Leon and Bethe\(^{(16)}\) introduced the stark effect and the other external processes into the calculation of the cascade for mesons stopping in liquid hydrogen. Their results suggest that external interactions are required in order to be in better agreement with experiment, particularly with observed capture times, and this established the importance of lateral transitions in the hydrogen cascade.

We note in passing that for kaonic hydrogen these calculations predict less than 0.1% of kaons captured into atomic states pass through the \( n=2 \) level, which sets an upper limit on the yield of the \( K_\alpha \) X-ray. This figure is derived from 0.6% of the initial population expected to reach the \( n=3 \) level, 0.3% of the initial population expected to be absorbed from the 3s-state and 0.2% from the 3p-state. Part of the 0.1% left makes the transition \( n=3 \) to \( n=2 \). The 'missing' 99.4% of the initial population is lost by nuclear capture from higher atomic levels.
Recently calculations in the style of Leon and Bethe have been extended by Borie and Leon\textsuperscript{(17)} to predict in more detail the X-ray yields of mesic hydrogen and protonium [antiprotonic hydrogen] in liquid and gaseous targets, and to investigate the dependence on target density and pressure. They find that the influence of non-radiative [external] processes including stark effect diminishes as atom-neighbour collisions become less likely, and the X-ray yield per atom increases. So in the absence of experimental limitation we expect a gas target to provide the optimum X-ray yields. This is true 'per atom' but because of range straggling the distribution of particle stops from any practical beam will be far more diffuse than if the same beam were stopped in a liquid target. For example Batty\textsuperscript{(18)} has used the yields of Borie and Leon and the estimated fraction of kaons stopped in hydrogen in a 1 m long gas target at 1 atm and 10 atm pressure, compared to 10000 stops in a 10 cm liquid target, to predict in Table 2 the total yields for the $K_\alpha$ X-ray. The 1m target would have to be totally enclosed to achieve efficient X-ray collection while the liquid target could practically achieve the same X-ray count using a single high resolution Si(Li) detector.

In response to these predictions experiments have been performed with large gas targets using proportional chambers with nearly $4\pi$ angular acceptance but poor resolution [1.2keV at 5.5keV], and with liquid targets and Si(Li) detectors - small but with the desirable advantage of high resolution. In the first category, J Bailey et al\textsuperscript{(19)} have observed the Lyman series for pionic hydrogen in 4 atm gas target, with a yield $0.40 \pm 0.04$ X-ray per pion captured by a proton. This figure and the experimental capture time, $\tau_{\text{cap}} = (2.3 \pm 0.6) \times 10^{-12}$ sec are fitted fairly well by Borie and Leon if a large stark mixing rate is assumed.
On kaonic hydrogen a preliminary experiment by Davies et al\(^{(20)}\) tentatively claimed observation of the \(K^-p\) 2-1 X-ray with a yield 0.1\% per kaon stopped in liquid hydrogen. This would be roughly in agreement with yields calculated in refs. 16, 17. However Izycki et al\(^{(21)}\) report an upper limit on the yield in liquid hydrogen which is smaller by approximately an order of magnitude.

The latter group [Izycki et al\(^{(22)}\)] have also reported results of a search for antiprotonic hydrogen (protonium) X-rays; their upper limit of \(3.5 \times 10^{-4}\) K-series X-rays per stopped antiproton is again an order of magnitude smaller than the nearest prediction amongst those due to Borie and Leon. Similarly Auld et al\(^{(23)}\) also report a low yield. They have measured the yield of L-series X-rays from protonium in gas at 4 atm:

\[
Y_L(4\ atm) = 0.06 \pm 0.03
\]

and set a limit on K-series X-rays:

\[
Y_K(4\ atm) < 5 \times 10^{-3}
\]

This implies rapid disappearance of antiprotons from the 2p level by decay channels other than 2p − 1s X-ray transitions. Formally, \(\Gamma_{2p}\) \(^{(\text{ann})}\) [total annihilation width] must be much greater than the radiative width \(\Gamma_{2p}\) \(^{(\text{rad})}\). As these are unknown quantities Borie and Leon\(^{(17)}\) have
calculated X-ray yields with the ratio \( \frac{\Gamma_{2p}^{(\text{ann})}}{\Gamma_{2p}^{(\text{rad})}} \) as a variable. Their results are in agreement with experiment for \( \frac{\Gamma_{2p}^{(\text{ann})}}{\Gamma_{2p}^{(\text{rad})}} > 100 \) and moderate stark mixing [see figs. 1b and 2b of ref. 17]. However it is not clear if this is consistent with other experimental information concerning p-state annihilation in \( \bar{p}p \). Therefore better results are required all round.

1.3.1.II Helium

In practice cascade calculations for exotic helium atoms are performed in analogy with those of hydrogen but for helium there are good experimental results including measurements on X-ray spectra which allow an analytic rather than speculative attitude to be adopted. External effects which induce sliding transitions are included but the interactions are weaker, consequently lateral transitions are less frequent and in liquid helium, compared to liquid hydrogen, a hadron is at least an order of magnitude more likely to survive the cascade. Yields are easily measurable.

The best yield measurements are obtained from small targets which stop only a small well-defined fraction of the incident beam, as in the measurement of the yield of kaonic helium by Wiegand and Pehl.\(^{(24)}\) This will be discussed later in connection with other results on helium.

In contrast experiments aiming for high statistics and precision without the need for much repeated calibration of the spectra must achieve high count rates or instrumental stability over a long running time,
preferably both. In this case it is easiest to increase the rate by adopting a large liquid target which will stop as much beam as possible. Then because the number and distribution of stops viewed by the detector is poorly determined, the absolute yield is poorly determined.

1.3.2 Strong interaction effects in light exotic atoms

We have seen that only low lying levels are significantly affected by the strong interaction. This has been expressed formally by the overlap integral eqn. 1.9. In this section we consider the energy level structure of the exotic atom and how the fundamental particle-nucleus interactions affect it.

1.3.2.1 Electromagnetic energies

Because the strong interaction is short ranged compared to the dimensions of an exotic atom most of the energy levels are practically free from its influence, and otherwise where the effects observed are small it has sometimes been considered as a perturbation to be added to the electromagnetic interaction. Consequently it is natural first to solve for electromagnetic energy levels regardless of whether the exotic particle is a hadron or a lepton. Then the difference we aim to observe in a hadronic atom is attributed to the presence of strong interactions. At levels below those for which the effects are small the strong interaction eventually predominates, in this region 'deeply bound' nuclear states may exist but the X-ray series and concept of the exotic atom are terminated.

For point like coulomb interactions solution of the Klein-Gordon and Dirac equations give the formulae for the binding energies:-
However, particularly for low lying orbits with the radii typical of an exotic atom, the point charge approximation is inadequate; it is necessary to include some suitable form factor representing the finite nuclear charge distribution and solve the wave-equation numerically. This is the finite size correction $[FS]$. A number of other corrections, chiefly those for electron screening $[S]$ and vacuum polarization $[VP]$ must also be considered; these have been reviewed by Batty ref. (9). The 'exact' electromagnetic energies are then given by:

$$E_{em} = E_{KG/DF} + \Delta E_{FS} + \Delta E_{S} + \Delta E_{VP}$$

and have been calculated by G T A Squier $^{(25)}$.

1.3.2.II Strong interaction effects

In the study of hadronic atoms the most important and frequently measured effect is the complex energy displacement

$$\Delta E_n = \varepsilon_n + i\Gamma_n$$

of the $n^{th}$ bound state, that is induced when the strong interaction is added to the electromagnetic interaction. Of course there is no measurement for which we can turn off the strong interaction so we define
Table 2

<table>
<thead>
<tr>
<th>Target g/cm²</th>
<th>Yield</th>
<th>No. Kaon stops</th>
<th>Total X-ray yield</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gas(1 atm)</td>
<td>0.13</td>
<td>10</td>
<td>38</td>
</tr>
<tr>
<td>Gas(10 atm)</td>
<td>1.3</td>
<td>5</td>
<td>860</td>
</tr>
<tr>
<td>Liquid</td>
<td>2.1</td>
<td>0.1</td>
<td>10000</td>
</tr>
</tbody>
</table>

Yields for the 2p - 1s X-ray in kaonic hydrogen from Batty ref. 18.
\[ \Delta E_n = E_{\text{em}}^n - E_{\text{had}}^n \]

where \( E_{\text{em}}^n \) is the calculated electromagnetic energy eqn. 1.13 and \( E_{\text{had}}^n \) is the complex energy of the hadronic atom. The latter is obtained either experimentally, then 1.14 defines \( \Delta E_{\text{EXPTL}}^n \), or by solution of the associated wave equation, then 1.14 gives \( \Delta E_{\text{THEORY}}^n \).

Experimental measurement is in two parts corresponding to the real and imaginary parts of \( \Delta E_n \).

**The Real Part:** Exotic atoms are identified mainly on the basis of the calculated electromagnetic spectrum assuming that hadronic shifts are small compared to the spacing of unperturbed levels, and are apparently restricted to the single state identified as the last to be populated before dissolution of the atom. Thus implicitly we assume that only the components of the ultimate transition series will be displaced, equally, from their electromagnetic positions while thus retaining their characteristic 'line pattern'. Therefore the real part of the displacement \( \epsilon_n \) is obtained by identification of this line pattern and measurement of its shift from the electromagnetic position.

It is also possible that an adjacent transition series will be observed, unshifted, to confirm formation of the atom and energy calibration.
The Imaginary Part: If absorption or other channels play any role in the interaction the optical model potential, $U$ in equ. 1.4, is complex with the consequence that the eigenvalues of the full Hamiltonian are generally complex

$$E_{n\,\text{had}} = \alpha_n - i\frac{\Gamma_n}{2}$$  \hspace{1cm} 1.15

In this case we expect\(^{(26)}\) that the state observed is distributed with an energy uncertainty $\Gamma_n$, about the value $\alpha_n$, where $\Gamma_n$ 'the natural linewidth' is the full-width-at-half-maximum [fwhm] of the probability density:

$$L(\omega) = \frac{\Gamma_n}{2\pi} \frac{1}{(\alpha - \omega)^2 + \left(\frac{\Gamma_n}{2}\right)^2}$$  \hspace{1cm} 1.16

called the lorentzian or natural line-shape.

Here $\omega$ is a dummy variable and:

the expectation value is: $\alpha_n = \int_{-\infty}^{+\infty} L(\omega) \omega d\omega$  \hspace{1cm} 1.17

with normalization: $1 = \int_{-\infty}^{+\infty} L(\omega) d\omega$  \hspace{1cm} 1.18

Finally line broadening is connected to the finite lifetime $\tau_n$ of the atomic state by the usual uncertainty relation.

$$\tau_n > \frac{\hbar}{2\Gamma_n} = 6.582.10^{-16}\text{eV sec}$$  \hspace{1cm} 1.19
and through this relation the width is practically synonymous with absorption rate.

In fact because all states of exotic atoms are unstable in someway and because we are ignoring the fine structure all X-ray lines have a finite width, typically less than a few electron volts, which is negligible in comparison with an experimental limit, approximately ten times larger, set by detector resolution. Again adjacent X-ray series or calibration lines may provide a useful reference for calibration of the intrinsic detector resolution.

In summary the complex energy displacement is defined:

\[ \Delta E_n = \Delta E_{\text{EXPTL}}^{n' \rightarrow n} - \Delta E_{n' \rightarrow n}^{\text{em}} \equiv \epsilon_n + i \frac{1}{2} \Gamma_n \]

Note that \( \epsilon_n \) is in fact the expectation value of the real energy shift, \( \epsilon_n = E_n^{\text{em}} - \alpha_n \); in this definition \( \epsilon_n \) is positive if the lower level is more bound with the addition of strong interactions, and negative for weaker binding.

It occasionally happens that a strong interaction width considerably smaller than those directly measurable may be deduced for an excited state of the atom when absorption competes with electromagnetic deexcitation\(^{(27)}\). In this connection it is conventional to define the relative intensity \( Y_n \) of the last transition as follows:

\[ Y_n = \frac{\text{relative intensity of the last transition}}{\text{intensity of the previous transition}} \]
\[ Y_n = \frac{N_x (n+1 \rightarrow n)}{P_{n+1}} \]  

\[ \approx \frac{N_x (n+1 \rightarrow n)}{n'>n+1 \frac{P_{n+1}}{N_x (n' \rightarrow n+1)} + N_{\text{AUG}} (n' \rightarrow n+1)} \]  

\[ \approx \frac{N_x (n+1 \rightarrow n)}{n'>n+1 \frac{P_{n+1}}{N_x (n' \rightarrow n+1)}} \]

where \( N_x \) is the number of X-ray transitions

\( N_{\text{AUG}} \) is the number of Auger transitions

\( P_{n+1} \) is the population of the \((n+1)^{th}\) level

The situation is illustrated schematically by the following diagram:

```
\[
\begin{array}{c}
n + 1 \\
\downarrow \Gamma_{n+1}\text{(rad)}
\end{array}
\begin{array}{c}
n \\
\downarrow \Gamma_{n+1}\text{(abs)}
\end{array}
\]  
```

representing competition between direct absorption and radiative deexcitation of the \((n + 1)^{th}\) level.

The absorption width \( \Gamma_{n+1}\text{(abs)} \) is given by

\[ \Gamma_{n+1}\text{(abs)} = (1 - Y_n) \Gamma_{n+1}\text{(rad)} \]

Equations 1.22 and 1.23 are progressively worse approximations to 1.21 because there are usually other types of transition apart from Auger and radiative processes, which might populate the \((n+1)^{th}\) level. However where as 1.21 necessarily involves the complexity of a cascade.
calculation to estimate missing contributions to $P_{n+1}$, 1.23 can be evaluated directly from the X-ray spectrum, provided adjacent transition series are observed and individual line intensities can be assessed.

1.3.2.II(i) Hydrogen

The exotic atoms of hydrogen offer a unique opportunity to investigate the fundamental interactions between the proton and a few exotic particles, in a fairly model independent way, at an energy which is not accessible to conventional scattering experiments. For the hadronic atoms the most interesting result in this connection is that established by Trueman (28).

Treating the hadron-proton interaction as a modified coulomb potential Trueman [see also ref. (29)] used the method of effective range expansion due to Bethe to obtain an expression, involving energy, for the complex phase shifts. Then the behaviour of the phase shift was used to locate the bound states - in this way the parameters of the effective range expansion for the $l$th partial wave are related to $l$-state energy levels.

For s-states Trueman shows that provided $A/B << 1$ a good approximation to the fractional energy displacement $\frac{\Delta E_{ns}}{E_n}$ is

$$\frac{\Delta E_{ns}}{E_n} = -\frac{4A_c}{nB}$$

where $B$ is the Bohr radius of the exotic atom $\frac{h^2}{\mu e^2}$

and $E_n$ is the Bohr energy level eqn. 1.2
\( A_c \) is the complex scattering length defined by the effective range expansion*
\( \Delta E_n \) is the complex energy displacement eqn. 1.20.

Substituting for \( E_n \) and \( B \) we obtain

\[
\Delta E_{ns} = \frac{2Z^2\mu^2\alpha^4 A_c}{n^3} \tag{1.26}
\]

\( h = c = 1 \)

where \( \mu \) is the reduced mass

\( \alpha = \frac{e^2}{\hbar c} \) is the fine structure constant and substituting current numerical values(2) into the RHS of 1.26 gives:

\[
\Delta E_{\pi^-p}^{1s} = 58.14 \text{ (eV/fm)} \ A_c^{\pi^-p}(\text{fm}) \tag{1.27a}
\]

\[
\Delta E_{\bar{K}^-p}^{1s} = 412.11 \text{ (eV/fm)} \ A_c^{\bar{K}^-p}(\text{fm}) \tag{1.27b}
\]

*The notation for scattering lengths will be as follows:-

\( A_c \) denotes the complex scattering length including coulomb effects implicit to the exotic atom. Whereas \( A_c \), derived from scattering experiments relates to the strong interaction only. The scattering lengths defined in separate isospin channels will be written \( A_I \) where \( I \) takes the value of isospin. Lower case will be reserved for real and imaginary parts. e.g. \( A = a+ib \).
Thus measurement of the complex energy displacement 1.20 is equivalent to measurement of $A_c$.

However to date the accepted definition of hadron-nucleon scattering lengths is not $A_c$, but is based on analysis of conventional scattering data, which is abundant, and is defined at threshold by extrapolation of the scattering amplitude. In comparison results from exotic hydrogen atoms are all difficult and either poor or non-existent. Problems are associated with observation of X-rays either because their energy is inaccessible or because yields are too low. For example $\pi^-p$ atom X-rays are observed\(^{(30)}\) at about 2.5 keV. This energy is too low for high resolution detectors. A proportional chamber was used which did not resolve the K-series X-rays and could not provide a measurement of linewidth. On-the-other-hand the yield from antiprotonic hydrogen is too low for X-ray study in existing beamlines. Both the systems, however, are relatively well known after study of data from conventional scattering experiments.

The situation is slightly different for the $K^-p$ system. Here X-ray energies are accessible to solid state detectors and the problem of low yield could be overcome over sufficiently long running time with stable conditions. Conventional scattering experiments are more difficult to perform and interpret, for although $K^-p$ seems to be dominated by s-wave
### Table 3

<table>
<thead>
<tr>
<th>Reaction Channels</th>
<th>Isospin</th>
<th>Q-value MeV</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K^- + p \rightarrow K^- + p$</td>
<td>0,1</td>
<td>0.</td>
</tr>
<tr>
<td>$\bar{K}^- + n$</td>
<td>0,1</td>
<td>-5.3</td>
</tr>
<tr>
<td>$\Sigma^+ + \pi^-$</td>
<td>0,1,2</td>
<td>103.0</td>
</tr>
<tr>
<td>$\Sigma^- + \pi^+$</td>
<td>0,1,2</td>
<td>95.0</td>
</tr>
<tr>
<td>$\Sigma^0 + \pi^0$</td>
<td>0,2</td>
<td>104.5</td>
</tr>
<tr>
<td>$\Lambda + \pi^0$</td>
<td>0</td>
<td>181.4</td>
</tr>
</tbody>
</table>

**K^- p reaction channels**
interactions at low energy it is complicated by the many channels which contribute [see Table 3]. Also the formalism which has been developed to extrapolate to threshold depends strongly on available constraints to exclude spurious model dependence. In this sense the $K^-p$ atom measurement at threshold could be very restrictive.

There exist two reports of positive findings for kaonic hydrogen X-rays. Both use a liquid target and high resolution Si(Li) detectors, and in both cases the search was conducted over the energy region of $5-10$ keV, close to the electromagnetic energy of the $K_\alpha$ X-ray [6.482 keV].

The first is the result due to Davies et al\textsuperscript{(20)}.

$$\epsilon_{1s} + i\frac{\Gamma}{\Gamma_{1s}} = 0.040 \pm 0.060 + i (0. + 0.115) \text{ keV}$$

which leads to a scattering length by eqn. 1.27b

$$A_{c}^{K^-p} = 0.10 \pm 0.15 + i(0. + 0.28)$$

The second has been reported by Izycki et al\textsuperscript{*}(21).

$$\epsilon_{1s} + i\frac{\Gamma}{\Gamma_{1s}} = 0.370 \pm 0.080 + i(0.280 \pm 0.130) \text{ keV}$$

or $$A_{c}^{K^-p} = 0.90 \pm 0.19 + i(0.68 \pm 0.32) \text{ fm}$$

*There are errors in the text of this paper. The shift is $0.370 \pm 0.080$keV.

L Tauscher (1981); private communication.
The results differ but both were presented showing in the spectra only weakly significant structure which could provisionally be attributed to \( K^-p \) X-rays. As they stand neither result is consistent with the predictions from analysis of \( \bar{K}N \) interactions above threshold.

The analysis of \( \bar{K}N \) interactions is far beyond the scope of this work; even the relation of the derived scattering lengths to the exotic atom result is neither trivial nor complete. These problems have been discussed in ref.\(^{(31)}\). To summarize, in the multichannel approach [again see Table 3] the problem splits naturally into the two isospin channels \( I = 0, 1 \) connected to \( K^-p \), and the strong interaction is assumed to be symmetric with respect to charge exchange, but isospin dependent. The associated scattering lengths are \( A_0 \) and \( A_1 \). With the expedience of all available constraints these are determined\(^{(32)}\),

\[
A_0 = -1.70 + i0.68 \text{ fm} \\
A_1 = 0.37 + i0.60 \text{ fm}
\]

and then the scattering length for \( K^-p \) [equal mix of \( I = 0 \) and \( I = 1 \)] is:

\[
A_{K^-p} = \frac{1}{2} (A_0 + A_1) = -0.67 + i0.64 \text{ fm}
\]

For comparison with \( A_c^{K^-p} \) it is thought the most important corrections are for the breaking of charge independence and long range effects [Rutherford scattering] mediated by the coulomb interaction. McGinley\(^{(33)}\) has discussed these and other corrections at length.
The simplest approach to this problem, originating with Dalitz and Tuan\(^{(34)}\), provides two more values for the scattering length obtained by successive corrections to \(A^{K^-p}\). The first, designated \(A_T\), includes the correction at threshold for the mass difference induced by the coulomb interaction between \((K+p)\) and \((K^0+n)\). Next \(A_T\) is corrected for the effect of the coulomb force beyond the range of the strong interaction. This gives \(A_T\) to be compared with \(A_c^{K^-p}\), the kaonic hydrogen result.

The values of \(A\), \(A_T\) and \(A_{DT}\) and the expressions for them are given in Table 4. [see also the discussion in ref. 18].

The comparison with experiment is:

\[
A_{DT} = -0.99 + i0.64 \text{ fm}
\]

\[
A_c^{K^-p} \text{ [ref. 20]} = 0.10 \pm 0.15 + i(0.0 \pm 0.28) \text{ fm}
\]

\[
A_c^{K^-p} \text{ [ref. 21]} = 0.90 \pm 0.19 + i(0.68 \pm 0.32) \text{ fm}
\]

If we overlook the uncertainty of the experimental results the difference which remains between the values of \(A_c^{K^-p}\) and \(A_{DT}\) is probably due to peculiar behaviour at threshold, not yet taken completely into account. For whereas Trueman's scattering length is inherently model independent the internal coulomb corrections do depend on the wavefunction at short distances. Approximations, such as may be found in the Dalitz and Tuan scheme, are therefore liable to comparison with a more detailed model if this exists. In this connection the most likely cause of difficulty is judged to be from the \(\Lambda(1405)\) Baryon state. The following data is taken from ref. 2.
Table 4

<table>
<thead>
<tr>
<th>Scattering lengths</th>
<th>$A_0 = -1.70 + i0.68 , \text{fm}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>at $K^-p$ threshold</td>
<td>$A_1 = 0.37 + i0.60 , \text{fm}$</td>
</tr>
</tbody>
</table>

Isospin average $A = \frac{1}{2}(A_0 + A_1) = -0.67 + i0.64 \, \text{fm}$

(Symmetric)

Threshold correction

for $K^-p - \bar{K}^0n$ mass difference $A_T = \frac{A + \frac{K_0}{A}A_0A_1}{1 + \frac{K_0}{A}} = -1.04 + i0.76 \, \text{fm}$

$K_0 = 0.297 \, \text{fm}^{-1}$ magnitude of $\bar{K}^0n$ momentum evaluated at $K^-p$ threshold

Coulomb correction

due to Dalitz and Tuan $A_{DT} = \frac{A_T}{1 + 2(2\gamma + \ln(2R/B))A_T/B} = -.99 + 10.64 \, \text{fm}$

$R = 0.4 \, \text{fm}$ cut off radius for $K^-p$ strong interactions
$B = 83.6 \, \text{fm}$ Bohr radius of $K^-p$ atom
$\gamma = 0.57722$ Euler's constant

Corrections to the scattering length due to Dalitz and Tuan\(^{(33)}\)
<table>
<thead>
<tr>
<th>Particle</th>
<th>$\Lambda(1405)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I$</td>
<td>0 established</td>
</tr>
<tr>
<td>$j^P$</td>
<td>$\frac{1}{2}^-$</td>
</tr>
<tr>
<td>Threshold</td>
<td>Below $K^-p$ threshold</td>
</tr>
<tr>
<td>$M$ (Mass (MeV))</td>
<td>$1405 \pm 5$</td>
</tr>
<tr>
<td>$\Gamma$ (Full width (MeV))</td>
<td>$40 \pm 10$</td>
</tr>
<tr>
<td>$M^2 \pm \Gamma M$ (GeV$^2$)</td>
<td>$1.97 \pm 0.06$</td>
</tr>
<tr>
<td>Mode</td>
<td>$\Sigma\pi$</td>
</tr>
<tr>
<td>Partial decay mode</td>
<td>Fraction</td>
</tr>
<tr>
<td>$p$(MeV/c)</td>
<td>142</td>
</tr>
</tbody>
</table>

This is discussed by Dalitz and McGinley in ref. (35). They review the current evidence that might distinguish the $\Lambda(1405)$ as either a $\bar{K}N$ quasi-bound state or a hadron, consisting of three quarks in a bound state. In harmony with previous attempts to resolve this question they decide that the data cannot tell the difference, yet conclude a better result on the shift and width of the is level of the $K^-p$ atom may prove interesting and decisive.

1.3.2.II(ii) Helium - strong interaction effects

The interaction of a hadron with a nucleus $[Z > 1]$ is conveniently and in most cases for exotic atoms adequately described by an optical model potential. This can be one of two types, The Phenomenological optical potential or The Microscopic [Formal] optical potential. The latter is generally the more satisfactory, the form and values of some of the parameters being derived from knowledge of the fundamental hadron-nucleon interaction, structure of the nucleus and behaviour of the hadron in nuclear matter. This information is available for pions in reasonable
Thus Friedman and Gal\textsuperscript{(36)} recently described a potential for the pion-nucleus interaction in terms of a formal optical model, and used this to fit pionic atom data. Earlier methods including many experimental details have been reviewed some time ago by Backenstoss\textsuperscript{(37)}, and Tauscher\textsuperscript{(38)} has discussed scattering lengths, obtained by application of Trueman's formula, where coulomb corrections are calculated after an optical model is assumed for the strong interaction.

A phenomenological potential is usually constructed by taking the analytic form of a formal optical model, so that the values of certain parameters are attributed to the effective hadron-nucleon interaction in the nuclear environment. Infact it is not usually possible to relate their values to the free nucleon-hadron interaction and thus they take on the role of variables in an automatic fitting procedure.

C J Batty has described the phenomenological optical model potential and compiled data for analysis in the cases of $K^{-}$, $\bar{p}$ and $\Sigma^{-}$-atoms\textsuperscript{(39)}. The potential taken is the form:-

$$U(r) = -\frac{2\pi}{\mu} \left(1 + \frac{M}{M_N}\right) \rho(r) \tilde{A}$$

Where $M_N$ is the mass of the nucleon

$M$ is the mass of the exotic hadron

$\mu$ is the reduced mass of $M$ and the nucleus

$\rho(r)$ is the nuclear matter distribution

$\tilde{A} = a_R + ia_I$, is the complex effective scattering length.
This successfully describes a wide variety of data on nuclei heavier than helium with only two fitting parameters, $a_R$ and $a_I$, for each type of hadron-nucleus interaction. In Table 5 we give the predictions for helium, for comparison with the following experimental results:

Antiprotonic helium has been studied by Poth et al$^{14}$. They measured

$$\epsilon_{2p} = -0.050 \pm 0.018 \text{ keV}$$
$$\Gamma_{2p} = 0.090 \pm 0.070 \text{ keV}$$

The shift $\epsilon_{2p}$ is clearly at variance with the prediction of Table 5. The width is not sufficiently determined to be conclusive.

Kaonic helium measurements are due to Wiegand and Pehl$^{24}$ and Batty et al$^{40}$.

$$\epsilon_{2p} = -0.040 \pm 0.030 \text{ keV}$$
and $\epsilon_{2p} = -0.035 \pm 0.012 \text{ keV}$

$$\Gamma_{2p} = 0.030 \pm 0.030 \text{ keV}$$

There is disagreement again with the predictions of Table 5. This will be either confirmed or resolved by successful remeasurement. The alternatives are:

Reanalysis: the helium results have been included with the data on heavier nuclei and optical model analysis repeated. The fit is not satisfactory$^{41}$. 

- 33 -
<table>
<thead>
<tr>
<th>Atom</th>
<th>Transition</th>
<th>$\alpha$ (fm)</th>
<th>$E_R$ (keV)</th>
<th>$I$</th>
<th>$\Gamma$ (keV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\text{K}^4\text{He}$</td>
<td>3-2</td>
<td>3.4 ± 0.03</td>
<td>6.464</td>
<td></td>
<td>0.0019</td>
</tr>
<tr>
<td>$\text{P}^-\text{He}$</td>
<td>3-2</td>
<td>5.4 ± 0.03</td>
<td>11.134</td>
<td></td>
<td>0.0097</td>
</tr>
<tr>
<td>$\Sigma^-\text{He}$</td>
<td>3-2</td>
<td>3.6 ± 0.05</td>
<td>13.463</td>
<td></td>
<td>0.0200</td>
</tr>
</tbody>
</table>

Predicted (41) strong interaction effects in exotic helium atoms using results from the analysis of C. J. Batty (39).

The nuclear matter distribution was a 3-parameter Fermi distribution; see eqn. 1.28:

$$\rho(r) = \rho_0 \left(1 + \frac{\omega^2}{c^2}\right)^{-1} \left(1 + \frac{r-c}{a}\right)^{1-a}\rho_0$$

Where $\omega = 0.517$, $a = 1.531$ fm, $c = 0.322$ fm.
New theory: this is unlikely so long as the data remain uncertain by up to 10%. In view of the qualitative success of the phenomenological optical model in fitting this data it is not desirable to add to it any microscopic detail which might exhibit model dependence. Helium therefore may be the first kaonic or antiprotonic atom to be treated in a formal approach, but this will only be worthwhile if the experimental results are confirmed.

1.4 The present experiment

It is clear that there exist already many complementary observations which are related to the theory of exotic atoms. Yet there is much that could be resolved by better experimental results all round. The present experiment was proposed to check measurements on kaonic hydrogen and helium which were obtained on the NIMROD proton synchrotron [PS] at The Rutherford Laboratory, Chilton, England. Indeed the apparatus used here was ready prior to closure of NIMROD, in 1978, and already provided the preliminary observations: Davies et al\(^{(20)}\) and Batty et al\(^{(40)}\).

The remainder of this thesis will describe the experimental apparatus used in work at the CERN PS and the new results obtained. Measurements were made of X-ray spectra from pionic, kaonic, antiprotonic and sigma atoms of helium, and kaonic hydrogen. The work was performed by collaborating physicists from The University of Birmingham, The Rutherford Laboratory and The University of Surrey. The author was responsible for calibration of the X-ray detector and analysis of the X-ray spectra.
2.1 Experimental background

Though in principle the formation of exotic atoms is relatively simple there are only a few accelerator facilities which can provide high intensity beams of the secondary particles listed in Table 1. In most cases a dedicated beam line has been set aside for exotic atom studies. Thus the number of experimental groups contributing data, particularly for $K^-(\Xi^-)$ and $\bar{p}$ atoms, is limited [see recent reviews, ref. (9,27,37)]. Opportunity is further limited for the study of atoms formed in hydrogen and helium, a purpose built target being required.

This collaboration, Birmingham University-Rutherford Laboratory-Surrey University, previously (20) reported an attempt to observe the $K^-p$ $2p - 1s$ X-ray, using the same target [Fig. 1] described in this chapter, at the Rutherford Laboratory's 7 GeV Proton Synchrotron - NIMROD. The data was collected just before the closure of NIMROD in 1978 and showed evidence of an X-ray at $6.52 \pm 0.06$ keV, coincident with kaons stopping in liquid hydrogen; the yield was found to be about 0.1% per stopped kaon. A separate measurement (40) with helium filling the target provided the opportunity to test the apparatus in data taking by observation of the $K^-\text{He } 3-2$ X-ray at similar energy, approximately 6.5 keV.
FIGURE 1.1
schematic diagram of the target assembly
FIGURE 1.2
Detail of the target vessel and Si(Li) detector
These experiments were discussed in chapter 1, where we stressed some apparent reasons for their importance, and the case for remeasurement. They were repeated at the CERN 26 GeV proton synchrotron [CERN PS], experiment number PS165, which is described in the remainder of this thesis.

2.2 The beamline

Protons at 26 GeV/c extracted in straight-section 62 of the CERN PS main ring [$>10^{12}$ protons per bunch] were delivered to the east hall area along slow ejection branch e15 [known as slow ejection 62 or SE62] see figure 2. In normal running approximately 40% [maximum] of the primary beam supplied by SE62 was incident on a tungsten production target [dimensions: $2 \times 5 \times 25$ mm], the remainder being shared by other beamlines.

In practice the thresholds for production given in Table 1 were far exceeded so that all particle species were produced in abundance. Then beam separation [momentum and mass selection] was accomplished in the secondary beamline, K23 Fig. 3 and Table 6, leading to the focus where at best the transported beam would converge to an image of the production target. Duty cycle varied according to the prevailing PS supercycle, typically duration of the beam spill from SE62 was 400 - 600 msec at the rate of 1 pulse train [Beam Burst] per 3.6 secs. Performance of K23, monitored throughout running time was normalized to the signals "SE62" - ejection intensity, and "K23/TELESCOPE" - a measure of secondary particle production. Both were standard NIM pulse trains [$1 \text{ pulse} = 10^9$] supplied by the PS Machine Control Room [MCR].
FIGURE 2
Schematic diagram of PS beams 1979/80
from CERN annual report 1979
Ejection branch, el5 SE62

Maximum design momentum. 1 GeV/c

Production angle 0°

Solid angle acceptance 8.7 mstr

Momentum resolution at 1 GeV/c < 10%

Length production target to focus 22 m

Beam spot size at focus

600 MeV/c kaons:--
vertical 3cm

Beamline elements:--

Quadrupoles Q1,2,5,8 0.5 m PS quadrupoles
Q3,7 0.5 m QNPO2 quadrupoles
Q4,6 0.25 m quadrupoles
Q9 Figure-of-eight quadrupoles

Bending magnets BM1 1 m magnetic field; 17 cm gap
beam deflection 24.4°

BM2 1 m magnetic field; 11 cm gap

Electrostatic Separator [ES] 3 m long with standard compensating magnets
FIGURE 3
Floor plan of the K23 beamline

KEY
ES Electromagnetic separator
BS Beam stop
BM Bending magnet
Q Quadrupole
C1 Momentum slit
C2 Mass slit
B1
B3 Beam defining counters
C_IF
Momentum selection was obtained by setting the current in the first bending magnet. Elsewhere in the beamline a vertical [momentum] slit accepted only particles close to the direction of the axis and in so doing restricted the transmitted beam to a small but finite range of momenta.

Mass separation, in the vertical plane, was accomplished in the mass separator in Fig. 3, by combined magnetic and electrostatic fields as follows. A horizontal magnetic field followed by vertical electrostatic field [between 3m long separator plates] separated the mass components of the beam. Then a second horizontal magnetic field restored the desired component to the path along the beam axis. A horizontal [mass] slit performed the rôle analogous to that of the momentum slit. At low momenta the mass differences between π's, K's and p's were easily resolved and the setting was not critical. On-the-other-hand both slits acted to shield the beamline down-stream and thus reduced the random incidence of unwanted contaminants.

An exception among the particles listed in Table 1 was the Σ− which was too short lived to form an intense low momentum beam. Therefore Σ− atoms were studied in conjunction with stopping K− particles, as Σ− hyperons are produced in approximately 25% of all ΛK reactions. Even for the longer lived particles losses due to decay in-flight and scattering out of the beam pipe had to be minimized by transporting the beam bunch to its focus at relativistic velocities \[p > M_0c\]. This necessitated a copper degrader [see Fig. 1.1], being inserted at the theoretical focus as a moderator to optimize the fraction of the beam stopping in the target.
The appropriate thickness of copper [8.95 gcm⁻² per cm] was determined from a 'degrader curve' described later. In practice the degrader also removed a large part of the beam by absorption and dispersion, consequently the selection of beam momentum was a compromise aimed to minimize beam lost in-flight and beam lost due to the thickness of degrader. A momentum of 600 MeV/c was adopted here for kaons and antiprotons and 200 MeV/c for pions.

2.3 The target

The target assembly was suspended from a transom bridge at the end of K23. The assembly, Figures 1.1-2, conjoined, in a large aluminium vacuum-pressure vessel, the last four detectors of the beam defining telescope [to be described later], the aluminium target vessel [called a spinning' from the way it was made] and a special Kevex 300 mm² x 5 mm Si(Li) detector which viewed the spinning vertically. The target liquid was condensed from gas through heat exchangers supplied with liquid helium by internal reservoirs. The same supply of helium cooled the aluminium spinning. Thermal contact was made by structural supports from underneath the helium reservoir to the top-plate which was consequently the coldest part of the target vessel. Thus any vapour rising from the liquid condensed back to liquid on the top-plate. By the addition of an aluminium heat-shield around the spinning, also cooled by contact with the helium reservoirs, cryogenic performance was such that hydrogen boil-off was completely contained. The target volume could then be closed. [This was not possible when the target was filled with helium which had to be allowed some boil-off, this loss being continuously replenished from a external supply]. All helium from the cryogenic system and target was recycled by the CERN recovery plant.
In the interest of safety the vacuum-pressure vessel was tested up to 2atm prior to installation in K23 and comprehensive safety measures were built into the automatic control systems. These have been described in detail elsewhere (43).

The spinning was 99.9% pure aluminium, 270 mm in diameter, 1 mm wall thickness with 15 mm thick aluminium top-plate. This enclosed a total volume of 12 litres, viewed by the Si(Li) detector through a conical beryllium window, 0.25 - 0.50 mm thick Fig. 1.2. The heat shield was 0.9 mm thick.

The target was assembled and maintained by colleagues from the support section of the High Energy Physics Division and the Technology Division of the Rutherford Laboratory.

The design and material were specified to minimize contamination from two major sources of unwanted X-rays.

Firstly, the constituents of the target were chosen to be of low atomic number, \( Z < 25 \). Then fluorescence of the target materials induced by the high levels of radiation around the beam did not cause any interference above 6 keV.

Secondly, exotic atoms would be formed in surrounding material by loosely-defined incident particles or reaction products due to an event in the target. Consequently their characteristic X-rays would contaminate the spectrum. The problem cannot be eliminated but was relatively mute.
in the pure low-Z environment of the aluminium spinning which simultaneously minimized the variety and yield of contaminants. Also the large dimensions and thick top-plate ensured that side walls were not viewed directly by the detector, while the top-plate and bottom of the target lay far from the path of the beam and the majority of stopped particles. Yet contaminant X-rays from exotic atoms of aluminium and beryllium have been identified amongst discrete X-ray contributions to the background. We concluded that these were most likely to be from the edge of the top-plate around the beryllium target window and the window itself.

2.4 The X-ray detector

A Si(Li) [silicon-lithium drifted] detector approved in design and specification to operate as an integral part of the target apparatus was supplied by The Kevex corporation Burlingame, California. Details are given in Table 7.

The principle of a Si(Li) detector rests on the process of drifting lithium ions into a thin volume of hyper-pure silicon. This compensates the natural limiting prevalence of charge traps [acceptor sites] which make even the purest silicon p-type and otherwise prevent an extended depletion region from being established at a junction. When the drift is complete the compensated volume [designated i-type for its similarity under normal conditions to intrinsic semiconductor] is left sandwiched between a p-type interior and, due to an excess of lithium, n-type surface-dead layer. Subsequently the surface is covered by a thin gold contact which improves and protects electrical characteristics - the
device is identical to a p-i-n junction diode. Under a reasonable reverse bias the i-type layer is characterized by the presence of a strong electric field and is depleted of free charge carriers [i.e. No current flows].

A low energy photon, above a few keV, is practically unaffected by the gold layer [typically \(<0.1 \, \mu m\) thick] and the dead layer \(< 1 \, \mu m\) yet is totally absorbed in a fraction of the thickness of the i-type layer. [Compare the length required for 95\% attenuation of 6.0 keV X-rays in silicon, which is about 0.1 mm\(^{(44)}\), with 5 mm depletion depth]. The photon interaction liberates electron-hole pairs [e-h pairs] in proportion to energy deposited, which drift to opposite contact layers under the influence of the electric field. Because the volume is relatively small, while the lifetimes of free charge carriers are long [no traps] charge collection is fast and efficient. Thus this device produces an output pulse with fast rise-time, and amplitude proportional to X-ray energy. These are characteristics which make possible an efficient detector with good energy resolution up to high count rates [1000 counts per sec].

An excellent comprehensive review of this subject, with further discussion covering the relative merits of different detector systems, is the book by Knoll\(^{(45)}\).

So far as the active detector component, the diode, is concerned, all Si(Li) detectors are much alike. They are most commonly planar in configuration and cylindrical in shape. However several of the
**Table 7**

Characteristics of the Si(Li) detector

<table>
<thead>
<tr>
<th>Type</th>
<th>Downward looking, large area Si(Li)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Resolution</td>
<td>250eV at 5.9keV, 1000cps</td>
</tr>
<tr>
<td>Active area</td>
<td>300 mm²</td>
</tr>
<tr>
<td>Depletion depth</td>
<td>5 mm</td>
</tr>
<tr>
<td>Beryllium window area</td>
<td>700 mm²</td>
</tr>
<tr>
<td>thickness</td>
<td>.05 mm</td>
</tr>
<tr>
<td>External dimensions of the finger</td>
<td>21cm long x 4.4cm diam</td>
</tr>
<tr>
<td>Preamp</td>
<td>Charge sensitive with pulsed optical feedback</td>
</tr>
</tbody>
</table>
important characteristics are affected by the dimensions of the active volume. Notably the capacitance increases and with this the resolving power of the detector deteriorates, if the surface area is increased without increasing the thickness of the depletion layer. Further, while it is difficult to drift the lithium very much deeper than about 5mm, it is anyway undesirable to increase without limit the detector volume as this increases in proportion the occurrence of background events due to the ambient radiation. Thus, though the power of X-ray collection would be increased by increasing the detector area, it is limited by the acceptable resolution.

To reduce electronic noise and deterioration of the characteristics of the diode both diode and preamp are cooled to liquid nitrogen temperature [about 77K] via a thermal contact - the cold finger. For the detector that we have been using this was connected with the diode front face downward looking from a 7.5 litre cryostat containing liquid nitrogen. The cold finger was enclosed in an aluminium tubular casing, 5 mm thick and 210 mm long, which was sealed at the top around the cryostat and at the bottom by a beryllium window 0.05mm thick. Then the interior was evacuated. The position and dimensions of the diode behind the beryllium endcap window have been verified using a finely collimated X-ray source to scan across the face and along the outside of the aluminium tube.

The aluminium casing provided protection and important collimation for the diode which was thus shielded from low energy background originating outside the restricted field of view. So the detector 'saw'
only the spinning and liquid as sources of X-rays below 20 keV. However, high energy $\gamma$-rays and particles from the beam scattered throughout the volume of the diode and could deposit any fraction of their energy. This gave rise to a continuous background which appeared to tail-off exponentially towards larger energy losses.

At the commencement of data taking the detector was sealed in a special emplacement completing the integrity of the vacuum-pressure vessel. It was not removed until data taking finished six months later. The aluminium heat shield was continued down around the finger of the detector with a beryllium window 0.025mm thick through which the detector viewed the target. [See Fig. 1]. The total thickness of beryllium due to the target window and heatshield between the detector encap window and the target liquid was less than $0.12 \text{ gcm}^{-2}$.

2.5 **Beam defining telescope and electronics**

The detectors which comprised the trigger telescope, listed in Table 8, were fast scintillator detectors and Cerenkov detectors. The electronic logic is represented schematically in Figure 4, it illustrates the basic operations which are required in the event trigger, discussed below. A more detailed plan including additional logic for double ended detectors, pulse discrimination and timing not shown in Fig. 4 may be found in Appendix I.

Note that both Cerenkov detectors act in anticoincidence mode to veto fast particles [$\beta > 1/n$]; they were included principally to separate pions resulting from in-flight decay of kaons and reaction products of kaons and antiprotons.
<table>
<thead>
<tr>
<th>Name</th>
<th>Type</th>
<th>Material</th>
<th>Function</th>
<th>(b) Dims.</th>
<th>(c) L</th>
<th>(d) Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>B1</td>
<td>Fast Scint.</td>
<td>NE102</td>
<td>Time of Flight</td>
<td>120x50x6</td>
<td>5.0</td>
<td>Before BM2 double ended left and right</td>
</tr>
<tr>
<td></td>
<td>Liquid Cerenkov</td>
<td>FC75</td>
<td>Veto fast particles</td>
<td>120x50x40</td>
<td>0.30</td>
<td>double ended left and right not used on π⁻ beams</td>
</tr>
<tr>
<td>B3</td>
<td>Fast scint.</td>
<td>NE102</td>
<td>Time of flight + Beam spill monitor</td>
<td>80x80x6</td>
<td>0.15</td>
<td>TIME-OF-FLIGHT [TAC] spectrum is recorded event-by-event</td>
</tr>
</tbody>
</table>
Table 8 (Cont.)

Followed by copper degrader and aluminium pressure-vessel wall then detectors:

<table>
<thead>
<tr>
<th>Name</th>
<th>Type</th>
<th>(a) Material</th>
<th>Function</th>
<th>(b) Dims. mm</th>
<th>(c) L m</th>
<th>(d) Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>Plastic</td>
<td>Pilot 425 n=1.49 ρ=1.19</td>
<td>Veto fast particles</td>
<td>70x70x50</td>
<td>0.03</td>
<td>Double ended left and right situated inside the pressure-vacuum vessel not used on π⁻ beams</td>
</tr>
<tr>
<td></td>
<td>Cerenkov</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B4A</td>
<td>Fast Scint.</td>
<td>NE102</td>
<td>Trigger (optional)</td>
<td>80x80x6</td>
<td>0.10</td>
<td>Pulse height spectrum recorded event-by-event</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B4B</td>
<td>Fast Scint.</td>
<td>NE102</td>
<td>Trigger</td>
<td>80x80x6</td>
<td></td>
<td>Pulse height spectrum recorded event-by-event</td>
</tr>
</tbody>
</table>
Table 8 (cont.)
Followed by aluminium target vessel [and heat-shield] then detector:-

<table>
<thead>
<tr>
<th>Name</th>
<th>Type</th>
<th>(a) Material</th>
<th>Function</th>
<th>(b) Dims.</th>
<th>(c) L</th>
<th>(d) Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>B5</td>
<td>Fast Scint.</td>
<td>NE102</td>
<td>Veto counter on-line stops</td>
<td>100x100x10</td>
<td>0.45</td>
<td>Pulse height spectrum recorded event-by-event</td>
</tr>
</tbody>
</table>

(a) \( n \) = refractive index  \( \rho \) = density in g cm\(^{-3}\)
(b) dimensions given X x Y x Z. X horizontal Y vertical Z along beam axis
(c) \( L \) = distance from degrader [i.e. focus] approximate
(d) Phototubes: RCA 8575; single tubes unless stated double ended.
FIGURE 4

Schematic plan of detector logic and event trigger
Here we consider the TRIGGER and EVENT signals formed for a stopping kaon. They were unchanged for antiprotons but the cerenkov detectors were omitted when forming the pion trigger.

2.5.1 The trigger

The trigger, which signaled that a kaon or antiproton \([P-\text{TRIG}]\) or pion \([\pi-\text{TRIG}]\) was present [it is convenient to refer generally to a trigger particle as a Trigger], was formed in the third logic unit shown in Fig. 4. It was a 3-fold logic condition requiring simultaneously:

particle-in-flight AND detector B4B AND \(\text{NOT-}\)detector C \([\bar{C}]\);

\[
P-\text{TRIG} = P_{\text{IF}} \cdot B4B \cdot \bar{C} \\
\pi-\text{TRIG} = \pi_{\text{IF}} \cdot B4B
\]

Particle-in-Flight was itself a compound signal:

\[
P_{\text{IF}} = (B1 \cdot B3)_{\text{TOF}} \cdot \bar{C}_{\text{IF}} \\
\pi_{\text{IF}} = (B1 \cdot B3)_{\text{TOF}}
\]

formed in the two preceding logic units between signals from detectors, B1, B3 and \(C_{\text{IF}}\), before the copper degrader. The first logic unit performed the operation B1-AND-B3 requiring B1 coincident with B3. The timing of this coincidence depended on the Time-of-Flight \([\text{TOF}]\) of the particle over fixed distance \([\sim 6\text{m}]\) between B1 and B3, and the adjustment of delay boxes T1 and T3. This was therefore equivalent to velocity selection. The minimum overlap of signal pairs coincident at a logic
unit which resulted in a normal output pulse was nominally 2.5ns. Therefore time-of-flight difference greater than 2.5ns was resolved. [Over 6m we could resolve $\beta < 0.89$ from ultra-relativistic particles $\beta = 1$.]

In the second logic unit ($B_1, B_3$)\textsubscript{TOF} is taken with $C_{IF}$ in veto mode giving NOT-$C_{IF}$; fast charged particles [$\beta > 0.78$] fired the $C_{IF}$ detector which was mainly required to veto products of in-flight decay not resolved by time-of-flight. It was positioned before $B_3$, allowing $B_3$ to be moved back where it provided the best possible definition of the beam incident on the degrader.

The signal $P_{IF}$ was also used to monitor the length of the beam spill and to generate a 'machine-gate', $M/G$, which defined the presence of beam. When $M/G$ was present priority was allocated to data acquisition. The complement $\overline{M/G}$ was used to veto spurious events which might be generated at other times. It is illustrated in Fig. 4 by a veto in the trigger but, conferring no essential information is omitted from equations 2.1. In much of this discussion $M/G$ is omitted where it may be considered an implicit requirement.

The second Cerenkov detector was included principally as a veto for the products of $K^-N$ and $\bar{p}N$ reactions in the degrader and the wall of the vacuum-pressure vessel. By choice of the radiating medium it did not veto a stopping kaon. This may be demonstrated using range curves, ref(2) with the known thicknesses and composition of materials in detectors and target after the degrader. We find approximately that $\beta < 0.5$ as the kaon enters detector $C$, otherwise it would not stop before the far side of the
liquid contents of the target. In comparison the critical velocity for the Cerenkov medium, Pilot-425\(^{(46)}\), is \( \beta_c = \frac{1}{n} = 0.67 \).

B4B - with the two earlier B-detectors B1 and B3 defined the fraction of beam entering the target spinning.

2.5.2 Optimized beam
2.5.2.I Beam optics

Beam optics, magnet and separator settings, were optimized prior to data taking to maximize the scaler reading for particles-in-flight per beam burst \([P_{IF}/B]\) under conditions of stable SE62-ejection intensity. The mass and momentum slits were adjusted to obtain an acceptable value for the ratio 'others-in-flight' to 'particle-in-flight' [e.g. \( \mu_{IF}/\pi_{IF} \), \( \pi_{IF}/K_{IF} \) and \( \pi_{IF}/P_{IF} \)] without reducing too much the transmitted beam intensity.

Nominally others-in-flight were identified by B1-and-B3 coincidence for which the time-of-flight condition was relaxed. This was achieved by stretching one or both of the B1, B3 signals to ensure overlap at the coincidence unit, this coincidence was designated \((B1,B3)_{LONG}\) to distinguish it from \((B1,B3)_{TOF}\).

2.5.2.II B5 and degrader curve

The purpose of the degrader curve was to establish the dependence of degrader efficiency on degrader thickness, thereafter set at the optimum. The measure of this efficiency was defined as the fraction of particles-in-flight stopped in the target vessel. For this the detector
B5 [see Fig. 4] is essential and is taken in anticoincidence to veto triggers with coincident signal from B5. Thus a stop is signalled by:

\[ P_{ST} = P - \text{TRIG} \cdot B5 \quad 2.3.1 \]

\[ \pi_{ST} = \pi - \text{TRIG} \cdot B5 \quad 2.3.2 \]

The degrader curve for kaonic hydrogen is shown in Fig. 5; \( P_{ST}/P_{IF} \) is plotted against degrader thickness [mm Cu], the curve reaches the optimum [maximum] at 88-90 mm. Below this triggers transmitted to B5 were self vetoing. Above, the fraction of beam stopped in the degrader increased and trigger rate declined.

Note that \( B5 \) was not part of the trigger because reaction products from the target volume might veto valid events. Also \( P_{ST} \) is not a trustworthy indicator of particles stopped in the liquid. There must first be correction for particles missing B5, in-flight decay of slow unstable particles and for absorption by the aluminium of the heat-shield and spinning. [The thickness of aluminium between B4 and B5 was 1gcm\(^{-2}\) compared to 2gcm\(^{-2}\) liquid hydrogen or 3.4gcm\(^{-2}\) liquid helium]. These corrections are awkward and may not be accomplished with confidence because transmitted beam was weak, implying large statistical error, and slow, in which case an estimate for fraction of transmitted beam [based on particle range] becomes uncertain. A better measure of stopping beam was obtained by correction of the trigger count. Typically we expect between 20% and 50% of triggers to stop in the liquid. The figures are predicted by the beam transport program DEGRAD described in section 2.7.
FIGURE 5

DEGRADER CURVE

\[ K_{if} = (B1.B3)_{TOF}\cdot C_{if} \]

\[ K_{st} = K-TR1G.B5 \]

data points joined to illustrate continuity
<table>
<thead>
<tr>
<th>Beam Particle-Momentum</th>
<th>$P_{\text{IF}}$/RB</th>
<th>Others/$P_{\text{IF}}$</th>
<th>Degrader mm Cu</th>
<th>$P_{\text{TRIG}}$/BB</th>
<th>$P_{\text{ST}}$/BB</th>
<th>STOP/TRIG</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi^-$ 200 MeV/c</td>
<td>40000</td>
<td>2.4</td>
<td>13</td>
<td>12500</td>
<td>1000</td>
<td>0.40</td>
</tr>
<tr>
<td>$K^-$ 600 MeV/c</td>
<td>5000</td>
<td>35</td>
<td>90</td>
<td>800</td>
<td>600</td>
<td>0.30</td>
</tr>
<tr>
<td>$\bar{P}$ 600 MeV/c</td>
<td>1800</td>
<td>60</td>
<td>3.5</td>
<td>800</td>
<td>470</td>
<td>0.50</td>
</tr>
</tbody>
</table>
**Table 9 (Cont.)**

**Hydrogen**

<table>
<thead>
<tr>
<th>Beam Particle-momentum</th>
<th>P_{IF}/BB</th>
<th>Others/P_{IF}</th>
<th>Degrader mm Cu</th>
<th>P-TRIG/BB</th>
<th>P_{ST}/BB</th>
<th>STOP/TRIG</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\pi^-) 200 MeV/c</td>
<td>40000</td>
<td>1.7</td>
<td>14.5</td>
<td>9000</td>
<td>8000</td>
<td>0.40</td>
</tr>
<tr>
<td>(K^-) 600 MeV/c</td>
<td>5000</td>
<td>20</td>
<td>88</td>
<td>1050</td>
<td>690</td>
<td>0.30</td>
</tr>
<tr>
<td>(p) 600 MeV/c</td>
<td>1800</td>
<td>30</td>
<td>2.0</td>
<td>610</td>
<td>500</td>
<td>0.50</td>
</tr>
</tbody>
</table>

**TABLE 9** - Typical performance of K23 monitored over periods of helium and hydrogen data-taking.

The ratio STOP/TRIG in column 7 is taken from monte carlo prediction - see section 2.7.
2.5.2.11 Beam statistics

CAMAC scalers which count various signals formed throughout the electronic logic [see diagram appendix I] were displayed at the end of each beam burst. The most important were $P_{IP}$, eqns. 2.2, TRIG, eqns. 2.1, and $P_{ST}$ eqns. 2.3. These and the ratio OTHERS/$P_{IP}$, defined above, are illustrated by typical values in Table 9.

2.5.3 Event

The trigger defined above started a clock and a 0-480 ns TAC [Time-to-Amplitude converter] which was connected to a CAMAC ADC giving digital read-out for the time elapsed between trigger [START] and a signal from the X-ray detector [STOP] - see Fig. 4. Note that a separate fast circuit TFA, timing-filter amplifier, and CFD, constant fraction discriminator was incorporated for better timing resolution.

Provided an X-ray, STOP, followed within 480ns of START the TAC generated "VALID STOP" - a signal to indicate the arrival of an X-ray. If VALID STOP was received then EVENT was signalled:

$$\text{EVENT} = \text{TRIG. X-RAY [VALID STOP]. COMPUTER READY} \quad 2.4$$

If 'computer ready' was not present the event was lost. In fact the event rate was approximately 4 per second, and losses were not expected to be significant.
2.5.3.1 Timing

X-ray timing was set up prior to data taking, before the Si(Li) was sealed in position in the vacuum-pressure vessel. Using cable lengths exactly the same as for normal data taking and a reference source of Selenium-75 \(^{75}\text{Se}\) a coincidence was formed between 10.5 keV X-rays in the Si(Li) and 136 and 265 keV \(\gamma\)-rays in B3. Delay boxes were adjusted so that the 10.5 keV X-rays were distributed about the centre of the TAC gate [the width of the 'prompt' time spectrum distribution was observed to be about 30 ns].

2.5.4 Data acquisition and display

Analog signals from the Si(Li) preamp were transformed by a special Kevex 4520 pulse processor into square shaped pulses and digitized into 4K \([1K = 1024]\) channels by a Laben amplitude-to-digital converter [LABEN ADC in Fig. 4]. The amplification gain was set prior to data taking to match 0-50 keV X-rays to the 0-8V input range of the Laben, [thus 1 energy channel \(\approx 12.5\) eV].

The outputs from the Laben ADC and Si(Li) TAC were read by the data acquisition program running in a GEC 4080 computer. Similarly coincident with an event, outputs from detectors B4A, B4B and B5 and the Time-of-Flight TAC, were digitized by CAMAC ADC's. These, too, were read by the computer. All the digitized information mentioned here was written event-by-event to magnetic tape, the event-by-event tape, and used for the construction of data histograms. Data acquisition and display programs were written by G J Pyle and P Sharman.
A list of all the data histograms is given in Table 10. Of these the most important was the two-dimensional Time-Energy spectrum. This was constructed using the output from the Laben ADC [energy information] and Si(Li) timing to histogram 'number of counts' against Time and Energy in an array of up to 64K channels. In the case of a 64K spectrum we used 16 time sections [1 time section = 30 ns] each containing 4K energy bins [1 bin = 12.5 eV].

We have already noted that the prompt time spectrum of the Si(Li) covers about 30ns therefore several adjacent time sections were added to ensure that the proper coincidence region was covered. In fact most events were observed to be clustered around the middle time section with a tail falling into later time sections [the time distribution of events is illustrated in Fig. 6]. Therefore we have added later time sections, 150-450ns, to be taken as the coincidence spectrum. Others were assumed to consist entirely of random events, mostly background - Randoms spectrum.

2.5.4.1 Histograms ON-Line

All events were histogrammed ON-line during data taking to provide a cumulative display of monitor spectra, see table 10. Periods of data taking were divided into RUNS of 5000 beam bursts each. At the end of each Run the latest monitor spectra were written to magnetic tape, the monitor tape. Also recorded were the scaler totals [including count of the number of beam bursts]. The monitor tape therefore provides a complete record of the data taking period [most importantly the scaler totals for P_{IF}, P_{TRIG} etc].

- 64 -
Table 10

Summary of histograms constructed by the data acquisition program

<table>
<thead>
<tr>
<th>Spectra</th>
<th>Size in No. of channels</th>
<th>On-line monitor spectra</th>
<th>Off-line event-by-event replay</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time energy [Et]</td>
<td>1K = 1024</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Time B4B [B4xT]</td>
<td></td>
<td>32K</td>
<td>64K</td>
</tr>
<tr>
<td>as Et for B4B pulse height</td>
<td></td>
<td>4K</td>
<td>8K</td>
</tr>
<tr>
<td>B4A Pulse height spectrum</td>
<td></td>
<td>256</td>
<td>512</td>
</tr>
<tr>
<td>B4B Pulse height spectrum</td>
<td></td>
<td>256</td>
<td>512</td>
</tr>
<tr>
<td>T Si(Li) TAC spectrum</td>
<td></td>
<td>~ 256</td>
<td>512</td>
</tr>
<tr>
<td>B5 Pulse height spectrum</td>
<td></td>
<td>256</td>
<td>512</td>
</tr>
<tr>
<td>TOF TAC spectrum</td>
<td></td>
<td>512</td>
<td>1K</td>
</tr>
<tr>
<td>E energy spectrum</td>
<td></td>
<td>2K</td>
<td>4K</td>
</tr>
<tr>
<td>[all events histogrammed]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ECAL CO-57 calibration spectra</td>
<td></td>
<td>2K</td>
<td></td>
</tr>
</tbody>
</table>
FIGURE 6 - TIME SPECTRA
Figures for each principal data set represent the distribution of events binned into 16 Time Sections
2.5.4.II Histograms OFF-line

Histograms could be constructed OFF-line using the digital information stored on the event-by-event tape; event-by-event replay. The principal advantage was the availability of more core space and computer time, otherwise taken up ON-line by the processes of data acquisition.

Options provided within the OFF-line histogramming procedure were used to set software thresholds on beam defining detectors. If the default options were allowed all events were histogrammed exactly as they would have been to construct monitor spectra. However hardware thresholds [set by discriminator units] were just high enough at the lower limit to exclude electronic noise, and generously high at the upper limit; therefore the quality of the histogrammed data could be improved by more rigorous event selection.

2.5.5 Calibration spectra

Calibration data was taken from an X/γ - reference source in coincidence with the machine-gate, already described, which established coincident beam. [BEAM-ON]

EVENT = M/G . X-RAY

Dependence of the detector response on the beam was observed to be marked and unmistakable as inferred by comparison between the calibration in π, K and p beams, and 'Beam-off' calibration.
Beam-off was obtained by the interposition of a beam stop
(see Fig.3). To replace the $P_{IF}$ pulse train an internal machine gate
required in eqn 2.5 was generated by a pulse generator.

The reference isotope was cobalt-57 \(^{57}/\text{Co}\). This was retractable,
being connected to a twisted cable, and was viewed by the detector only
when in the 'source lowered' position (see Fig. 1.2). At all other times
the source was raised into a recess in the aluminium top-plate where it
was completely shielded, preventing contamination of other data. The
reference X-rays are:

\[
\begin{align*}
\text{\(^{57}\text{Fe} X\text{-rays}\)} & : \begin{cases}
K_\alpha : 6.40 \text{ keV} \\
K_\beta : 7.05 \text{ keV}
\end{cases} \\
\text{\(^{57}\text{Fe }\gamma\text{-ray}\)} & : 14.413 \text{ keV}
\end{align*}
\]

X-ray energies from reference (47).

The data was histogrammed into a spectrum of 2K channels.
Calibration Runs were performed between the normal periods of data taking
with a frequency of one calibration for three data runs of 5000 beam
bursts each. They were alternately beam-on and beam-off and were written
to the monitor tape with the monitor spectra. They were subsequently
recalled for analysis described in the next chapter.

2.6 Response and characteristics of the Si(Li) detector
2.6.1 Linearity

Linear calibration of the detector:

\[
E(N_{\text{CH}}) = \varepsilon_0 + \varepsilon_1 \times N_{\text{CH}}
\]
where $\varepsilon_0$ is a constant off-set,

$\varepsilon_1$ is the energy gradient per channel,

and $N_{CH}$ is channel number

has been established before the detector was installed in the target and
after it was removed using reference sources in the region of 5-35 keV.

Also, during data taking accurate calibration was obtained between 6.4 and
14.4 keV, using the cobalt-57 reference source, and beyond this region we
have observed the positions of prominent contaminant X-ray lines in
agreement with previously measured or calculated energies. No
improvement in the quality [chisquared per degree of freedom], of the fit
to calibration data has ever been obtained by addition of a non-linear
term to eqn. 2.6 for our detector.

2.6.2 Detector resolution

We assumed that the pulse height spectrum recorded in response to
monochromatic X-rays is well approximated by a Gaussian distribution about
a centre $X_o$:

$$G(X-X_o) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(X-X_o)^2}{2\sigma^2}}$$

Then the full-width-at-half-maximum, fwhm, quoted as the characteristic of
detector resolution was simply related to the variance, $\sigma^2$, of the
gaussian distribution [$\sigma = 1$ standard deviation]
This variance was derived from:

1) A constant contribution due to electronic noise.

2) Random variation in charge collection efficiency, particularly related to non-uniformity over the detector volume [e.g. edge effects: loss of e-h pairs near edges; and strong-field effects: multiplication of e-h pairs in regions of strong electric field]. These effects were not manifestly energy dependent over the energy range of 6-50 keV. If charge collection efficiency were energy dependent this might cause non-linearity between X-ray energy and detector output, whereas no non-linearity has been observed for the detector used here.

3) Statistical fluctuation in the production of e-h pairs: The interacting X-ray produces an average number of e-h pairs in proportion to energy deposited:

\[ N = \frac{E}{\varepsilon} \]

where \( \varepsilon = 3.7 \text{ eV} \), is the average energy required per e-h pair.

By analogy with a poisson process the variance is proportional to \( N^{(45)} \).

The Fano factor \( F \), is defined as the constant of proportionality:

\[ \sigma^2 = (\text{CONST.} + \frac{FE}{\varepsilon}) \varepsilon^2 \]
The constant term and coefficient of energy were determined by fit to the beam-on calibration spectra. Note that $\sigma$ has the dimensions of energy whereas $N$ is the number of e-h pairs.

2.6.3 X-ray detection efficiency

The measurement of X-ray yields required an assessment to be made of the efficiency of the detector viewing the target. For this we have assumed a simple model to represent the absolute efficiency $\varepsilon_{\text{abs}}$. This was a function dependent on position of the X-ray source [exotic atom] within the target [co-ordinates $r, \theta, \phi$ from the geometric centre of the face of the Si(Li) diode] and X-ray energy, $X$. Subsequently $\varepsilon_{\text{abs}}$ was averaged over the distribution of exotic atoms in the target to obtain the overall detection efficiency per exotic atom X-ray. [See appendix II].

2.6.3.1 Model

The intrinsic detector efficiency $\varepsilon(x)$, which is assumed to depend only on the energy $X$, was defined as:

$$\varepsilon(x) = \frac{\text{No. of counts recorded}}{\text{No. of photons of energy } X \text{ incident on detector}}$$

where the detector was defined by its active area. This was the 300 mm$^2$ surface of the Si(Li) diode, and the solid angle subtended by the detector at a point on the axis, $\theta=0$, distance $s$ from the end cap window was:

$$\Omega(r) = 2\pi(1- \frac{r}{r^2+a^2})$$

where $a = 9.7$ mm, is the radius of the Si(Li) diode, $r = s+d$, and $d = 15$ mm is the distance of the diode behind the endcap window.
The absolute detection efficiency was defined as:

\[
\varepsilon_{\text{abs}}(r,X) = \frac{\text{No. of counts recorded}}{\text{No. of photons, of energy } X, \text{ emitted by source at } r}  \tag{2.13}
\]

For a point source on axis

\[
\varepsilon_{\text{abs}}(r,X) = \frac{\Omega(r)}{4\pi} \cdot \varepsilon(X) \cdot I(r,X) \tag{2.14}
\]

where \(\Omega(r)\) is given by 2.12, \(\varepsilon(X)\) is defined by 2.11, and

\[
I(r,X) = e^{-\sum \mu_i(X) t_i} \tag{2.15}
\]

represents those absorbing layers which were not considered to be intrinsic parts of the detector.

\(t_i\) is the thickness along a straight line to the detector, of the \(i^{th}\) absorbing layer

\(\mu_i(X)\) is the absorption coefficient.

The model was completed by the inclusion of angular dependence for a source off-axis. Axial symmetry was assumed after it had been shown experimentally that this was reasonable.

Thus:

\[
\varepsilon_{\text{abs}}(r,\theta,X) = \frac{\Omega(r,\theta)}{4\pi} \cdot \varepsilon(X) \cdot \alpha(\theta,X) \cdot I(r,\theta,X) \tag{2.16}
\]
where the function $a(\Theta, X)$ was introduced to represent the angular dependence of the intrinsic efficiency 2.11.

For small angles solid angle varies as $\cos(\Theta)$, therefore

$$\Omega(r, \Theta) \approx \Omega(r) \cos(\Theta)$$  \hspace{1cm} 2.17

this should be a reasonable approximation when angular dependence is dominated by $a(\Theta, X)$, which depended strongly on $\Theta$ because the field of view of the detector was restricted by collimators.

It has also been observed\(^{(48)}\) that solid angle depends on X-ray energy, but this correction would be small and difficult to determine.

$I(r, \Theta, X)$ is defined as in 2.15 but the path lengths, $t_i$, were computed with allowance made for their possible dependence on $\Theta$, unless otherwise indicated.

2.6.3.II Measurements

Measurements were performed in the east hall at CERN in May 1980. The same cable lengths and electronics as data taking were used, therefore the detector was triggered by an internal machine-gate exactly as for beam-off calibration. The pulse heights were histogrammed into 2K channels and a gaussian peak shape was fitted to determine the integrated area. Integration by hand, summing channels over the region which included a photpeak, was tested and found adequate where the background was small and could be approximated by a constant, or straight line.
Standard sealed X/γ-reference sources were supplied by The Radiochemical Centre, Amersham, they are listed in Table 11. The nominal activity was quoted for the active material implanted in the surface of a 1mm diameter bead. The bead, a good approximation to the ideal point source, being sandwiched [for safety and ease of handling] between rectangular polystyrene windows 0.5mm thick. To overcome the difficulty of estimating self absorption of low energy X-rays, photo-activities [i.e. activity of X-ray emission] for X-rays below 20 keV were measured independently using a proportional chamber in the Radiation Laboratory at the University of Surrey.

Photo-activities above 20 keV were derived, assuming negligible self-absorption, from the nominal activity of the source and previously published values for secondary X-ray intensities:

\(^{109}\)Cd was calibrated [by the manufacturer] according to emission of 88 keV γ-rays from de-excitation of \(^{109}\)Ag. The X/γ-ratio was taken to be \(26.02^{(49)}\), and the relative X-ray intensities were taken from Lederer and Shirely\(^{(47)}\).

\(^{133}\)Ba was calibrated for the nominal activity of the material in the bead. The yield of \(^{133}\)Cs X-rays, per \(^{133}\)Ba disintegration, was taken to be \(~123^{(50)}\), and relative X-ray yields were again from ref\(^{(47)}\).

A small plastic holder was made into which the rectangular sources fitted securely. Once the holder was positioned the source-detector geometry remained fixed inspite of the interchange of sources.
\( \varepsilon(X) \), given by eqn. 2.14, was measured with the bead sources on-axis 80\,mm from the Be endcap of the detector [95\,mm from the diode]; the solid angle was:

\[
\frac{\Omega(r)}{4\pi} = 0.0026
\]

Correction for absorption in 80\,mm air at 2 atm 293 K was calculated from data of ref (44), results presented in Table 11 and Fig. 7.

\( \alpha(\Theta,X) \), see eqn. 2.16, was determined by a series of measurements for which the X-ray source was moved on a radial arc radius \( r_0 \), about the coordinate origin. Solid angle was given by eqn. 2.17. Intrinsic angular response of the detector was then deduced by comparing count rate observed for angle \( \Theta \), with count rate for \( \Theta=0 \).

\[
\alpha(\Theta,X) = \frac{\varepsilon_{\text{abs}}(r_0,\Theta,X)}{\varepsilon_{\text{abs}}(r_0,\Theta=0,X) \cos \Theta} \quad 2.18.1
\]

\[
= \frac{\text{count rate for X-rays, } X, \text{ at angle } \Theta}{\text{count rate for X-rays, } X, \text{ at } \Theta=0} \cdot \frac{1}{\cos \Theta} \quad 2.18.2
\]

Infact we have not used the intrinsic angular dependence given by 2.18. To reproduce the effect of the beryllium window and aluminium top-plate a dummy rig was assembled as illustrated by Fig.8. The dimensions were exactly as we believed they had been in the target. Transmission through 0.25 \,mm beryllium was included in the normalization but no attempt to calculate the angular dependence of the absorption in the beryllium window.
was included. This becomes part of the relative dependence-with-collimator which we denote \( \alpha_c(\theta,X) \). Using eqn. 2.16

\[
\alpha_c(\theta,X) = \frac{\varepsilon_{\text{abs}}(r_0,\theta,X)}{\varepsilon_{\text{abs}}(r_0,\theta=0,X) \cos \theta} e^{-0.025\mu}
\]

where \( \mu \) is the absorption coefficient [units cm\(^{-1}\)] for X-rays of energy \( X \) in beryllium, taken from ref. (44).

The advantage of this definition was that the attenuation due to the top-plate and beryllium window were contained in eqn. 2.19. Thus using \( \alpha_c(\theta,X) \) in place of \( \alpha(\theta,X) \) in eqn. 2.16 the absolute detection efficiency was easily obtained. This required only the attenuation in other interposing layers, for which the geometry was usually simple, to be evaluated - these were the layer of liquid and the planar 0.025 mm beryllium window of the heat shield.

The angular measurements were performed at five X-ray energies corresponding to \(^{57}\text{Co} \) and \(^{109}\text{Cd} \) reference sources. Results in Fig. 9.

An important correction, that has not been mentioned was for dead-time. Dead time was assumed to arise mostly from the analog electronics and time delay while an event was read by the computer. The fast timing pulses would not be affected but 'EVENT' would have to wait for 'COMPUTER READY'. Therefore dead-time correction was provided by the factor TFA/EVENT [where TFA is the count in the timing amplifier and EVENT the event count], which multiplied the integrated X-ray count to give the dead-time corrected count total.
Table 11

<table>
<thead>
<tr>
<th>Source (Half life)</th>
<th>Ser. No.</th>
<th>Nominal activity (Date)</th>
<th>X/γ-rays</th>
<th>Photoactivity 5 May 1980 1μCi=3.7x10⁴ S⁻¹</th>
<th>Measured intrinsic efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>⁵⁴Mn (312.5d)</td>
<td>1V487</td>
<td>11.24μCi (1 Apr 1980)</td>
<td>Cr X-rays 5.5keV</td>
<td>.371μCi</td>
<td>.632</td>
</tr>
<tr>
<td>⁵⁵Fe (2.7yr)</td>
<td>1E059</td>
<td>50μCi (29 Feb 1980)</td>
<td>Mn X-rays 5.95keV</td>
<td>4.16 μCi</td>
<td>.847</td>
</tr>
<tr>
<td>⁵⁷Co (270.9d)</td>
<td>1T882</td>
<td>11.20μCi (1 Apr 1980)</td>
<td>Fe X-rays 6.5keV</td>
<td>1.37 μCi</td>
<td>.890</td>
</tr>
<tr>
<td>¹⁰⁹Cd (453d)</td>
<td>1B049</td>
<td>1.7x10⁴ S⁻¹* (1 May 1980)</td>
<td>Ag Ka 22.1keV</td>
<td>9.86 μCi</td>
<td>.889</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Ag Kβ₁ 24.9keV</td>
<td>1.76 μCi</td>
<td>.833</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Ag Kβ₂ 25.5keV</td>
<td>.32 μCi</td>
<td>.780</td>
</tr>
<tr>
<td>¹³³Ba (10.8yr)</td>
<td>1R332</td>
<td>11.60μCi (1 Apr 1980)</td>
<td>Cs Ka₁** 30.63keV</td>
<td>4.06 μCi</td>
<td>.652</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Cs Ka₂ 30.97keV</td>
<td>7.50 μCi</td>
<td>.624</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Cs Kβ₁ 35.00keV</td>
<td>2.21 μCi</td>
<td>.502</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Cs Kβ₂ 35.80keV</td>
<td>0.50 μCi</td>
<td>.507</td>
</tr>
</tbody>
</table>

*88 keV γ-rays produced by the source
**relative X-ray intensities from Lederer and Shirley(47)
FIGURE 7

Intrinsic detector efficiency $\xi_{\text{in}}$ (dots) and fractional transmission through 0.25 mm beryllium (open circles). Lines drawn to interpolate between data points.
FIGURE 8

Detail of dummy rig set up for assessment of detector efficiency
Fit to angular response of the Si(Li) detector measured at five energies corresponding to X-rays from cadmium-109 and cobalt-57 references sources.
DEGRAD is a monte-carlo program which generates a statistical summary and momentum analysis for a beam of particles traversing the elements of the detector telescope and the target [degrader elements]. The incident particles, assumed charged, are specified by their mass, decay length, incident momentum profile and beam divergence. The 'degrader elements' are specified by mass number, atomic number [averaged for compounds], mean excitation potential [as defined for the Bethe-Block energy loss formula], density and dimensions.

The program follows each particle in a random sample through the degrader elements, until it decays-in-flight, scatters-out, is absorbed or is kinematically stopped [kinetic energy < 0], or it has traversed the entire ensemble. The eventual fate of each particle, what it was and where it happened, is logged in a storage array which is printed at the end of the run. Nominally particles in flight are any that have traversed B1, CTF and B3. Triggers are any that enter B4B, and stops are those whose eventual fate is to be kinematically stopped in the volume of target liquid. The latter two were used to compute the ratio STOP/TRIG in Table 9.

DEGRAD calculations were the responsibility of S Baird and K R Parker. S Baird has also modified a version of DEGRAD to print energy loss in B4B, in two lists:

1) For any particle which traversed B4B - i.e. TRIGGERS.
2) For any particle stopped, in the liquid - i.e. STOPS.
FIGURE 10.1

B4B pulse-height distributions recorded event-by-event for incident kaons. The region plotted and arbitrary horizontal scale are the same in each case, events between the markers were accepted in event-by-event replay.
FIGURE 10.2
Energy loss in B4B - Monte carlo simulation. The distribution of energy loss in B4B (arbitrary units) in a typical DEGRAD sample is plotted for all particles traversing B4B (TRIGGERS) and then only if the particle stops in the target liquid (STOPS).

From the comparison it is clear that the tails of the B4B spectrum can be excluded from the trigger.
A typical result is represented in Fig. 10.2. Degrad clearly predicted that the distribution of pulse heights in the B4B spectrum would be less diffuse than the trigger distribution that we observed, if particles were selectively stopped in the target volume.

In addition Degrad listed the co-ordinates of each stop in the volume of the last degrader element. This option was used to provide the coordinates of stops in the target volume.

It is impossible to extract a rigorous theoretical assessment of the accuracy of these predictions, the problem is too deeply convoluted. The two major sources of uncertainty were the ratio of stops to triggers, and the distribution of stops. The latter, not entirely disinterred from the former, was in all cases predicted to be diffuse after range straggling in the material before the target. A uniform distribution would be a good approximation. To make provision for all uncertainty 50% standard deviation in the number of stops was assumed. This has been included, with statistical uncertainties, in the absolute yield.

2.8 Summary of the data-taking period.

Data collection is summarized in Tables 12-13. Periods of data taking were naturally interrupted by changes to particle type and target, and by periods for machine development, M/D, and maintenance, during which the PS was not available for physics [approximately 1 M/D period per 2 weeks physics]. We used these for any changes to the target-beam configuration that were considered necessary.
The measurements on $K^-\text{He}$ and $\bar{p}\text{He}$ were each performed over single continuous periods of PS operation. $\pi^-\text{He}$ data was collected in about 1 day. The target contained 12 litres of liquid helium.

For hydrogen the circumstances were different; $K^-\bar{p}$ data extends over six periods which we have labelled A-F in table 13. Each corresponds to one period of continuous PS operation. There were also periods dedicated to pion and antiproton beams. This data has been analysed to eliminate contaminant X-ray lines which might be wrongly associated with kaons.

Degrader thickness and the liquid contained in the target have been given in the tables. Also we note the introduction of lead shielding. First above the B3 detector to reduce the random interactions involving particles which form a halo around the focussed beam. Later more lead was introduced around the beam pipe before the C$_{IF}$ detector for the same reason. A significant improvement in the standard measure of contamination of the beam, namely $\text{OTHERS}/P_{IF}$, is seen when comparing statistics for like-beams in Table 9 [Helium data was collected before the lead shielding was installed, hydrogen data afterwards]. $P_{IF}$ is formed before the target so it is not affected by the contents.

The only unforseen interruption to data-taking arose during period D, upon failure of the mechanism which aligned the tungsten production target in the primary beam. This was preceded by running time during which it was impossible to move the production target to its correct station. In these circumstances the position of the target and presumably the focus
may have been changed detrimentally. We have discovered that the Data Sets A–C and D–F are different beyond any reasonable random change of conditions, or the effect of the small change to the thickness of copper degrader. Therefore we have analysed A–C and D–F as separate data sets.
### Table 12

**Summary of Helium Data**

<table>
<thead>
<tr>
<th>Data taken</th>
<th>Beam Momentum</th>
<th>Degrader Thickness (Copper)</th>
<th>No. Hours</th>
<th>No. In-flight</th>
<th>No. Triggers</th>
<th>No. Stops</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi^-$ He</td>
<td>200 MeV/c</td>
<td>13 mm</td>
<td>5</td>
<td>2.14.10^8</td>
<td>6.7.10^7</td>
<td>(2.68 ± 1.34).10^7</td>
<td>About 1 day was required for this measurement. 12 litres liquid helium</td>
</tr>
<tr>
<td>$K^-$ He</td>
<td>600 MeV/c</td>
<td>90 mm</td>
<td>100</td>
<td>5.0.10^8</td>
<td>7.5.10^7</td>
<td>(2.25 ± 1.1).10^7</td>
<td>12 litres liquid helium</td>
</tr>
<tr>
<td>$P^-$ He</td>
<td>600 MeV/c</td>
<td>3.5 mm</td>
<td>50</td>
<td>1.0.10^8</td>
<td>4.07.10^7</td>
<td>(2.0 ± 1.0).10^7</td>
<td>12 litres liquid helium</td>
</tr>
</tbody>
</table>

- 86 -
<table>
<thead>
<tr>
<th>Data Taken</th>
<th>Beam Momentum</th>
<th>Copper Degrader Thickness</th>
<th>No. hours</th>
<th>No. in-flight</th>
<th>No. Triggers</th>
<th>No. Stops</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>K⁻p (A)</td>
<td>600 MeV/c</td>
<td>88 mm</td>
<td>150</td>
<td>0.72.10⁹</td>
<td>1.51.10⁸</td>
<td>(4.54±2.27).10⁷</td>
<td>Initial lead shielding introduced 8.2 litres liquid hydrogen</td>
</tr>
<tr>
<td>(B)</td>
<td></td>
<td></td>
<td>90</td>
<td>0.44.10⁹</td>
<td>9.25.10⁷</td>
<td>(2.78±1.39).10⁷</td>
<td>7.8 litres liquid hydrogen</td>
</tr>
<tr>
<td>(C)</td>
<td></td>
<td></td>
<td>210</td>
<td>1.09.10⁹</td>
<td>2.29.10⁸</td>
<td>(6.88±3.44).10⁷</td>
<td>Additional lead shielding to beamline. 9.4 litres liquid hydrogen</td>
</tr>
<tr>
<td>(D)</td>
<td></td>
<td>90 mm</td>
<td>200</td>
<td>1.08.10⁹</td>
<td>1.96.10⁸</td>
<td>(5.88±2.94).10⁷</td>
<td>Breakdown at production target. 8.4 litres liquid hydrogen</td>
</tr>
<tr>
<td>(E)</td>
<td></td>
<td></td>
<td>140</td>
<td>0.72.10⁹</td>
<td>1.31.10⁸</td>
<td>(3.92±1.96).10⁷</td>
<td>8.0 litres liquid hydrogen</td>
</tr>
<tr>
<td>(F)</td>
<td></td>
<td></td>
<td>160</td>
<td>0.78.10⁹</td>
<td>1.44.10⁸</td>
<td>(4.31±2.16).10⁷</td>
<td>8.8 litres liquid hydrogen</td>
</tr>
</tbody>
</table>

Continued on next page...
<table>
<thead>
<tr>
<th>Data Taken</th>
<th>No. Hours</th>
<th>No. Triggers</th>
<th>No. Stops</th>
<th>Copper Deg affairs</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>π⁻ p</td>
<td>65</td>
<td>2.60.10⁹</td>
<td></td>
<td>14.5 mm</td>
<td></td>
</tr>
<tr>
<td>p p</td>
<td></td>
<td></td>
<td></td>
<td>2 mm</td>
<td></td>
</tr>
<tr>
<td>π⁻ p</td>
<td>185</td>
<td>3.33.10⁸</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>p p</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Data for assessment of background contamination (C) above. Collected between (B) and (C) above. 7.8 litres liquid hydrogen.

(5.64±2.82).10⁷

8.8 litres liquid hydrogen.
3.1 Introduction

In this chapter the analysis of the X-ray spectra is described. The division of the data being as given in Tables 12-13. The ExT spectra were constructed in 16 time sections and sections 6-15 [inclusive] were added. In the beginning all events were histogrammed exactly as read from event-by-event tape - uncut replay. But subsequently we have attempted to set a 'window' [i.e. between lower-level and upper-level discriminator cuts] on the pulse-height spectrum of detector B4B, to discriminate in favour of trigger particles, Triggers, which stop in the target liquid-Stops. [See also section 2.5.4.II].

The optimum B4B window was decided by trial replay of the K-helium data to maximize the ratio of peak area to integrated background. The final cut excluded the tails of the B4B pulse-height spectrum, rejecting about 30% of the events read from tape, and was in good agreement with monte carlo simulation - see Fig. 10. Similar results were obtained for p-helium and \( \pi^- \)-helium spectra. It should be noted that for all helium data the B4B spectra recorded event-by-event were biased by the conditional 'Trigger.X-ray' which defines EVENT [eqn. 2.4], and therefore defines the data which will be written to tape. The bias which arises favours stops because stops in helium produce X-rays relatively abundantly. Thus a sample of triggers associated with EVENT incorporates a greater proportion of stops than a similar sample of triggers chosen from the 'trigger singles' spectrum [i.e. all triggers due to incident beam].
For K-hydrogen the same approach did not work so well. It seems likely that in choosing the bulk of the B4B spectrum fast kaons were favoured. The optimum result was obtained by setting the B4B window to exclude only the particles grouped about minimum energy loss - minimum ionizing particles. These are by definition highly relativistic, probably the products of kaon decay; they would not stop in the liquid contents of the target. Simultaneously discriminator levels on the B5 - detector pulse height spectrum were used to set the complementary veto for transmitted particles. The lower level was set so that all events above minimum ionizing were vetoed. This strategy accepted events for which highly relativistic particles were observed in B5 [these were supposed to be K"p reaction products - relativistic particles in the incident beam having been excluded at B4B], but excluded kaons, which would be less than relativistic after traversing the target. Approximately 40% of recorded events were rejected by these cuts.

3.2 X-ray spectra

Events were initially binned into 4K [4096] channel spectra but were then rebinned by adding adjacent channels. A channel width of 12.5eV became 50eV [4x12.5eV channels added] or 100eV [8x12.5eV channels added]. No detail was lost by this procedure provided the final channel width was small compared to genuine structure in the spectrum. A minimum of three channel widths to the detector resolution determined the maximum bin width that could be tolerated. The advantage was that random point-to-point variations in the spectra were reduced when the statistics per channel were increased.
The aim of analysis was to account for the observed distribution of events in each spectrum, by showing that it was consistent with a supposed analytic fit, optimized according to the principle of least squares. The observed count of events over one bin width, denoted \( N_i \) for the \( i^{th} \) channel, the average value of the fit over the same region and the midpoint value of the fit, denoted \( Y_i \), were practically synonymous therefore the histogram was replaced by discrete data points with 1 channel-width separation. Poisson statistics was assumed - i.e. for an expected count, \( y \), the standard deviation of measurement is \( \sqrt{y} \). It is not strictly accurate to use \( \sqrt{N_i} \) where \( N_i \) is the observed count but with high statistics the difference is negligible. This is an accepted approximation which gains some saving of computer time.

Chisquared for the fit is then given by

\[
\chi^2 = \sum_{i=1}^{n} \frac{1}{N_i} (Y_i - N_i)^2 \tag{3.1}
\]

where \( n \) is the number of channels in the region of fit.

If there are \( m \) variables to be determined the difference \( f = n - m \) is the number of degrees of freedom and the quality of fit is measured by the reduced chisquared, \( \chi^2/f \)

3.2.1 Analysis

Each event is an output pulse from the detector [X-ray or any other] in coincidence with the trigger. There is no method which selects target-atom X-rays as the only category of event but they would dominate
the spectrum in an ideal experiment. Other events are due to sources of contamination and chance interactions of ionizing radiation in the active volume of the detector. These three causes of event were mentioned in Chapter 2. A fourth category is that of Compton events - i.e., exotic atom X-rays which are detected after Compton scattering. They have by definition lost a fraction of their initial [characteristic] energy and form a 'Compton tail' behind the direct events. The yield of Compton events, calculated in proportion to the full energy photopeak is defined in appendix II.

The fit to a spectrum comprises contributions from all these four categories of event. The principal part of analysis being to identify their likely origins and invent reasonable analytic representations, corresponding to detector response for each. Having calibrated the response, where this is meaningful, the analysis program, ANGELA\textsuperscript{(52)}, extracts a precise parameter fit and integrated count for each component in the spectrum. The method is regression analysis over a many variable-parameter space, to minimize the value of chi-squared [eqn. 3.1]. The current version of ANGELA handles up to 200 channels of data with 20 variables in the fit. Beyond 200 channels the maximum number of variables reduces, limited by computer core space, to 10 as the data increases to the maximum, limited by the data array, 400 channels. ANGELA runs in the IBM195 computers at the Rutherford Laboratory.

The origin of each X-ray line, if it was not a component in a characteristic line pattern, could often be guessed with confidence from
its characteristic energy. This was greatly facilitated by the compilation ELIST\(^{25}\) due to G T A Squier and C J Batty. ELIST lists electromagnetic transition energies and calculated X-ray yields for a range of exotic atoms, many never having been measured. All exotic atoms and fluorescence likely to occur in and around the target were considered and the final listing included X-rays in the range of 4-50 keV with calculated yields greater than 0.1% per atom. The cascade did not include the effect of sliding transitions and the results were used only to indicate the relative importance of related X-rays. Hadronic atom measurements\(^{53}\) were used to verify ELIST particularly where a transition might be strongly displaced.

3.2.2 Components of an analytic fit

3.2.2.1 The Background

The shape of the background appearing to tail-off exponentially was mentioned in Chapter 2. We found that this was well approximated by a simple exponential for broad regions of the high energy part of the spectrum, being the dominant shape certainly above 10 keV and adequate above 6 keV.

\[
\text{BGD}(X') = Ae^{-\lambda X'}
\]

where \(A\) and \(\lambda\) are the adjustable parameters and \(X'\) is the energy variable, usually offset to the start of the fitting region for convenience.

Below 6 keV the background shape, initially rising towards lower energies, turned down and eventually cuts off near 3 keV. An analytic
form which worked well, with the correct limiting behaviour at high energies was:

$$B GD(X') = Ae^{-\lambda X'} (1 - Be^{-\gamma X'}) \quad \gamma > \lambda$$  \hspace{1cm} 3.2.2

where $\lambda$, $\gamma$, $B$ and $\gamma$ are the adjustable parameters.

$X'$ is the energy variable [see 3.2.1].

It was not possible to obtain the same quality of fit [judged by chisquared per degree of freedom] from any background shapes more simple than 3.2.1 or 3.2.2 [i.e. analytic functions or polynomials with equal or smaller number of parameters]. Also it was impossible for these forms of background to show sudden deviations from the trend of the data, and they extrapolated quite well beyond the region of fit. This was not true for polynomials which were strongly influenced by single data points, and extrapolated poorly.

3.2.2.II Monoenergetic X-rays

Detector response to monoenergetic X-rays was given by equations 2.7 and 2.10

$$G(X-X_0) = \frac{G_0}{\sqrt{2\pi} \sigma} e^{-\frac{(X-X_0)^2}{2\sigma^2}}$$  \hspace{1cm} 3.3.1

$$\sigma^2 = C + FX$$  \hspace{1cm} 3.3.2
where $X_0$ is the centre, $C$ and $F$ are fixed parameters predetermined by analysis of the calibration spectra and $G_0$ is the amplitude, equivalent to integrated peak area.

Values were to be assigned to the three parameters $\sigma$, $X_0$ and $G_0$, corresponding to each Gaussian photopeak. Generally these could be treated as independent parameters [i.e., there is no correlation between peak width, centre and amplitude] and all could simultaneously be optimized by the fitting routine provided it could accommodate their number. In fact if $\sigma$ was predetermined, then only $X_0$ and $G_0$ were variables - i.e., two independent variable parameters per photopeak.

If two or more photopeaks were related the average number of independent parameters might yet further be reduced. Especially important cases were:

(i) If the separation of related X-rays was accurately known, as for a characteristic line pattern, but a common displacement was expected or suspected; then the values of $X_0$ for one or more peaks might be fixed in relation to one for which $X_0$ was varied.

(ii) If the relative intensity of X-ray lines was known but the absolute yields were not; then the values of $G_0$ for one or more peaks might be fixed in relation to one for which $G_0$ was varied.

(iii) Combining (i) and (ii): if relative intensities and separations were known for a series of photopeaks, then a fit could be reduced to
dependence on two variables \( G \) and \( X \) of any one transition, other parameters being fixed in relation to them.

(iv) If all energies were accurately known the centres \( X \) could be fixed. Linear calibration could then be extracted by optimizing the parameters \( \epsilon \) and \( \epsilon' \) [eqn. 2.6]. Similarly to extract detector resolution parameters \( C \) and \( F \) in eqn. 3.2.2 would be optimized [along with amplitudes on any others except \( X \)].

(v) Following (iv) any relative intensities that might be known could also be fixed. However it should be noted here and in (ii) that before fixing any X-ray yields correction for detection efficiency is required to obtain the correct relation between photopeak area. Because this and also yields, relative or absolute, are not usually well known, the option to fix them in the fit is used only when economy of variables is essential.

Fixing all peak areas rigidly must be avoided, as this action would interfere with the free fit of the background.

3.2.2.III Voigtian photopeak

When the natural lorentzian distribution

\[
L(X-X_0) = \frac{L_0}{\pi} \frac{(\Gamma/2)}{(\Gamma/2)^2 + (X-X_0)^2}
\]

centre \( X_0 \), fwhm \( \Gamma \) and integrated area \( L_0 \), is "viewed" through a gaussian instrumental resolution the result is the voigtian distribution being
the convolution of a gaussian with a lorentzian weight function [see appendix III].

\[ V(X - X_0) = \int_{-\infty}^{+\infty} G(X - y) L(y - X_0) \, dy \]  
\[ = \frac{L_0}{\sqrt{2\pi} \sigma} \, \Re W(U + ia) \]  

where \( W(U + ia) \) is the complex error function defined in appendix III

\[ U \equiv \frac{X - X_0}{\sqrt{2} \sigma} \quad \text{and} \quad a \equiv \frac{\Gamma}{2 \sqrt{2} \sigma} \]

\( \sigma \) is the detector resolution - for simplicity this was assumed to be constant in the neighbourhood of the lorentzian distribution; it was evaluated at \( X = X_0 \).

The discussion in section II regarding the parameters \( \sigma, X_0 \) and \( G_0 \) applies to \( \sigma, X_0 \) and \( L_0 \) here. The fourth parameter \( \Gamma \) which is defined for a voigtian occurred as a variable only in the special case of lines broadened by the strong interaction. In this case we assumed only one level was broadened. Then the value of \( \Gamma \) was common to all transitions of a series ending on that level. Otherwise \( \Gamma \) was set to an arbitrarily small value of 1eV [see comments in chapter 1] or gaussians were used.
3.2.2. IV Compton tails

The relative yield of compton events was calculated by the Target program described in appendix II. For high energies they were distributed relatively diffusely. At low energies they were almost indistinguishable from the main photopeak, the maximum single-compton energy loss being less than the half-width of the detector resolution. It was at intermediate energies, 8 - 14 keV, when the compton distribution extended back 1 - 2 resolution widths from the peak centre that the compton tails were most important. Then they gave to the photopeak an asymmetry, broadening the low energy side, which if not correctly subtracted would contribute to a false measurement of the width and centroid.

A method which gave an effective analytic representation of the comtons was to add them in bins 50 eV wide, smearing each bin out in gaussian or voigtian satellite distributions, at 50 eV intervals behind the main peak centre. The ratio of the number of counts in each bin to the total normalized compton yield [both as predicted by the Target Program], multiplied the area of the main photopeak to determine the area under each satellite. Also a parameter, normally set to unity, was introduced to multiply the total area of the compton tail - this could be optimized by the fitting routine to check the normalization of the compton yield. No significant change from unity resulted though it was noted that the natural width and the value of this normalization parameter were in some cases strongly correlated. This suggests that the quality of fit could be preserved over a range of widths by compensating adjustment to the compton yield, and vice versa. Therefore compton yields must be predetermined for a precise width measurement.
3.3 X-ray spectra from exotic atoms of helium

3.3.1 Pionic Helium

A fit to the spectrum of pionic helium is shown in Fig. 11, the Lyman series [K-series] for pionic and muonic helium are clearly established. The most likely origin of the very strong muon contamination being π-decay in which muons are produced with a range about 1 cm in liquid helium. No other contamination is observed.

The extracted photopeak areas and strong interaction shift are given with the statistical uncertainties in Table 14. The absolute uncertainty of the strong interaction measurement, shift and width, was obtained by duly adding, in quadrature, the statistical uncertainty of the linear calibration and detector resolution. The presence of the muon contamination provided satisfactory verification of the detector calibration and gaussian response but was not sufficiently precise to reduce the absolute uncertainty. The result is given and compared with previous measurements in Table 20.1. Here it is clear that the statistical average is dominated by the precision results due to Backenstoss et al.

In Table 15 the X-ray yield is assessed. The detector efficiency was evaluated by the Target program described in Appendix II, the distribution of stops being supplied by DEGRAD [Section 2.7]. The stops distribution was almost uniform, only slightly peaked about the beam axis, so that it did not critically affect the calculation. Therefore without serious fault it also served as the basis of calculation for μ-helium X-rays - probably the products of decay or interaction would be
distributed more diffusely than incident beam but the difference between using the correct distribution and any reasonable, approximate, distribution should be negligible. However the absolute number of particles comprising secondary distributions could not be assessed and only relative yields were extracted.

The yields are compared [Table 15] with those obtained by previous measurements [cols. II and III] and by cascade calculation [col. I]. The latter clearly does not account for the phenomenology of sliding transitions and in both absolute and relative terms provides an interesting contrast to experiment. The obviously low absolute yield is accompanied by modification of the relative yields also attributed to the non-circular character of the cascade. Thus when the 3-\( \text{l}_s \) transition is stronger than the 2-\( \text{l}_s \) transition it indicates, because of the \( \Delta \ell = \pm 1 \) selection rule, an uncommonly high population of the non-circular \( 3\text{p} \)-state. A similar, but in most cases, less pronounced redistribution towards non-circular transitions [high \( n \) to \( 1\text{s} \)] is commonly observed throughout the exotic atom spectra of helium and hydrogen.

Comparison between the experimental results in Table 15 reveals that the measured relative yields are approximately in agreement but that the total K-series yield per atom, \( Y_K \), in this experiment:

\[
Y_K = 0.123 \pm 0.062
\]

and that due to Backenstoss et al:

\[
Y_K = 0.195 \pm 0.049
\]
disagree beyond the supposed [absolute] uncertainties. To add to the above there are two more measurements; the first due to Wetmore

\[ Y_K = 0.095 \pm 0.014 \]

and the second due to Berezin et al. \(^{(56)}\)

\[ Y_K = 0.180 \pm 0.050 \]

These results span the range \( Y_K = 0.080 \) to \( Y_K = 0.225 \), a factor of 3 difference which reflects the great experimental difficulty in establishing absolute yields.

### 3.3.2 Kaonic Helium

Fits to the X-ray spectrum of kaonic helium are shown in Fig. 12.1 [50eV per channel width] and Fig. 12.2 [100eV per channel width] with results in Table 16. The Balmer series is clearly observed; Lyman series X-rays are not observed, the limit on their yield from this experiment is \(< 0.002 \) X-rays per atom. There are also small contributions due to stopping \( \pi^- \) and \( \Sigma^- \) particles. Other contaminants were \( \pi^- \) and \( K^- \) aluminium, observed in small quantities over the running time required for this measurement, at 30 keV [80 counts] and 26 keV [40 counts] respectively, but no other contamination was observed in the low energy region.
The K-He results quoted in Table 16 were extracted from the 50 eV per channel spectrum; the shift extracted from the 100 eV per channel spectrum was in exact agreement but the width measured disagreed with the quoted result. This was attributed to loss of accuracy with the coarse 100 eV per channel binning. The principal advantage of this spectrum was that the fitting region easily included the energies 6-20 keV, therefore it was possible to search for the line pattern of sigma helium.

All the photopeaks shown are voigtians plus, for each of the π^-He and K^-He peaks, the duly calculated contribution of compton events. The shift and width of the π^-He transition series were fixed according to the average of the results given in Table 20.1 and the relative peak areas were fixed as observed in Table 14.1. Thus the pionic helium contribution was practically constructed to be in agreement with the previously observed spectrum. A possible variation in the relative detector efficiency which might have changed the relative peak areas has been overlooked. Such is likely to be related to the difference of the distribution of π-stops from kaon reaction products and decay, to that of π-stops from the 200 MeV/c pion beam. This correction is expected to be negligible, all stops-distributions being fairly uniform. This strategy achieves essential economy of variables and resolves certain ambiguity which would arise in the present spectra because the π^-He 2-1 transition overlaps K^-He 6-1, and π^-He 4, 5, 6-1 transitions overlap Σ He 3-2. Only the π^-He 3-1 amplitude may be freely fitted to normalize the π^-He X-ray count; this totals ~ 80 counts, which is smaller than 10% of the kaonic helium total.
Amplitudes and strong interaction shift and width were freely fitted to the K^-He photopeaks. The yield is assessed in Table 17 where the distribution of K-stops was predicted by DEGRAD. Similarly amplitudes and shift were freely fitted to Σ-He while for the width the appearance of the photopeak gives an indication [less than 500eV] which, because of low statistics could not otherwise be extracted. The kaon stops distribution was assumed for Σ particles, thus relative yields could be assessed in Table 17.

Also in Table 17 are previous experimental measurements of the K-He yield. There is good agreement between this experiment and the Wiegand and Pehl value given for the absolute yield of the 3-2 X-ray but between other X-rays [compare relative yields] there is disagreement which probably results from incorrect assessment of detector efficiency. Which is to say that in ref. (24) the relative areas for the fitted 3, 4, 5-2 photopeaks are 100:88:44; in this experiment 100:31:22. This difference is due to the different experimental conditions, target configurations, detectors etc., and a discrepancy which is observed after correction indicates an error in the assessment of these factors in one or both of the experiments.

3.3.3 Antiprotonic helium

The spectrum of antiprotonic helium is shown in Figs. 13.1 and 13.2; both M-series and L-series X-rays are observed with results given in Tables 18 and 19 - the layout is similar to the tables already discussed. The only contaminant is π-He fitted as described for the K-He spectrum. The total count was 80 π-He X-rays.
All photopeaks are visible with Compton contributions added [easily seen in the L-series]. However the n=3 level, thus all M-series, are not expected to be broadened or shifted noticeably by the strong interaction so the width was fixed to 1 keV; the shift observed [-0.008 ± 0.011 keV] is consistent with statistical anomaly.

For the L-series, shift and width are freely fitted. The strong interaction effect and absolute uncertainty of the measurement are presented in Table 20.4 with the previous measurement and prediction from Table 5. This experiment does not confirm the disagreement with theory that was previously reported [Poth et al(14)]. An important factor in this result being the identification of Compton events associated with the dominant Lα photopeak. If the Compton distributions were omitted the result was a shift $\epsilon_{2p} = (-0.030 \pm 0.012)$ keV and width $\Gamma_{2p} = (0.030 \pm 0.030)$ keV.

Yields are assessed in Table 19. The relative yields are normalized to the 3-2 transition [Lα] at 11.1 keV, and there is comparison with the results of the experiment due to Poth et al [col. (II)] and a recent calculation of the cascade due to Wodrich et al(57) [col. (I)]. For the L-series the agreement of the relative yields is quite good, certainly a qualitative agreement. This does not extend throughout the M-series nor is there agreement about the absolute yield which is calculated to be 0.013 Lα X-rays per atom [in liquid helium].
For the M-series there is pronounced disagreement about the relative yield of the $M_\alpha$ X-ray. However this is a very difficult X-ray to measure with a Si(Li) detector, being at very low energy. Efficiency has to be extrapolated from the last data point at 5.5 keV, and this was performed very crudely. Because the efficiency is so small and steeply changing in this region there could easily be a factor 3 in the uncertainty.
Table 14

Table 14.1 Results for pionic helium - Figure 11

<table>
<thead>
<tr>
<th>PIONIC HELIUM</th>
<th>Electromagnetic energy (keV)</th>
<th>Measured energy (keV)</th>
<th>Instrumental resolution (eV)</th>
<th>Relative peak area</th>
<th>Total peak area (counts)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2-1</td>
<td>10.773</td>
<td>10.682±.007</td>
<td>316.± 18.</td>
<td>100.</td>
<td>481 ± 29</td>
</tr>
<tr>
<td>3-1</td>
<td>12.764</td>
<td>12.673</td>
<td>322.± 19.</td>
<td>116. ± 8.</td>
<td>558 ± 38</td>
</tr>
<tr>
<td>4-1</td>
<td>13.460</td>
<td>13.369</td>
<td>325.± 19.</td>
<td>32. ± 4.</td>
<td>154 ± 19</td>
</tr>
<tr>
<td>5-1</td>
<td>13.783</td>
<td>13.692</td>
<td>326.± 20.</td>
<td>12. ± 4.</td>
<td>58 ± 19</td>
</tr>
<tr>
<td>6-1</td>
<td>13.958</td>
<td>13.867</td>
<td>326.± 20.</td>
<td>5. Fixed to 2-1</td>
<td>24</td>
</tr>
</tbody>
</table>

\[ \epsilon = (-0.091 ± 0.007) \text{ keV} \]

\[ \Gamma = (0.035 ± 0.018) \text{ keV} \]
Table 14 (cont.)

<table>
<thead>
<tr>
<th>MUONIC HELIUM</th>
<th>Electromagnetic energy (keV)</th>
<th>Measured energy (keV)</th>
<th>Instrumental resolution (eV)</th>
<th>Relative peak area</th>
<th>Total peak area (counts)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2-1</td>
<td>8.224</td>
<td>8.221±.012</td>
<td>307. ± 16</td>
<td>100.</td>
<td>230 ± 21</td>
</tr>
<tr>
<td>3-1</td>
<td>9.744</td>
<td>9.741</td>
<td>312. ± 17</td>
<td>59. ± 9.</td>
<td>136 ± 21</td>
</tr>
<tr>
<td>4-1</td>
<td>10.276</td>
<td>10.273</td>
<td>314. ± 17</td>
<td>9. ± 7.</td>
<td>21 ± 16</td>
</tr>
<tr>
<td>5-1</td>
<td>10.522</td>
<td>10.519</td>
<td>315. ± 17</td>
<td>3. Fixed to 2-1</td>
<td>7</td>
</tr>
</tbody>
</table>

Table 14.2  Results for muonic helium - Figure 11
Table 15

Table 15.1 X-Ray yields for pionic helium

<table>
<thead>
<tr>
<th>Transition</th>
<th>(I) calculated yield</th>
<th>Previously measured (II) from $\sim 10^9$ $\pi$ stops</th>
<th>Previously measured (III) from $\sim 10^8$ $\pi$ stops</th>
<th>This Expt. ((2.68 \pm 1.34 \times 10^7 , \pi \text{ stops}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi$-He 2-1</td>
<td>0.422</td>
<td>0.071 ± 0.018 ((100 \pm 5))</td>
<td>((100 \pm 4))</td>
<td>3.69\times 10^{-4} (481 \pm 29) ((100 \pm 6))</td>
</tr>
<tr>
<td>$\pi$-He 3-1</td>
<td>0.183</td>
<td>0.087 ± 0.022 ((124 \pm 6))</td>
<td>((112 \pm 7))</td>
<td>3.92\times 10^{-4} (558 \pm 38) ((109 \pm 10))</td>
</tr>
<tr>
<td>$\pi$-He 4-1</td>
<td>0.044</td>
<td>0.030 ± 0.007 ((42 \pm 2))</td>
<td>((37 \pm 3))</td>
<td>3.95\times 10^{-4} (154 \pm 19) ((30 \pm 4))</td>
</tr>
<tr>
<td>$\pi$-He 5-1</td>
<td>0.066</td>
<td>0.007 ± 0.002 ((9 \pm 1))</td>
<td>((10 \pm 3))</td>
<td>3.96\times 10^{-4} (58 \pm 19) ((12 \pm 4))</td>
</tr>
</tbody>
</table>
Table 15 (Cont.)

Table 15.2 X-ray yields for muonic helium

<table>
<thead>
<tr>
<th>Transition</th>
<th>(I) calculated yield</th>
<th>Previously measured (II) from $\sim 10^9 \pi$ stops</th>
<th>Previously measured (III) from $\sim 10^8 \pi$ stops</th>
<th>This Expt. $(2.68 \pm 1.34 \times 10^7 \pi$ stops)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Yield per atom (relative yield)</td>
<td>(relative yields)</td>
<td>detector efficiency Peak area (relative yields)</td>
</tr>
<tr>
<td>$\mu$-He 2-1</td>
<td>0.694</td>
<td>$0.454 \pm 0.114$ $(100 \pm 5)$</td>
<td>$(100 \pm 6)$</td>
<td>$3.39 \times 10^{-4}$ 230 $\pm$ 21 $(100 \pm 9)$</td>
</tr>
<tr>
<td>$\mu$-He 3-1</td>
<td>0.152</td>
<td>$0.246 \pm 0.062$ $(54 \pm 3)$</td>
<td>$(43 \pm 5)$</td>
<td>$3.60 \times 10^{-4}$ 136 $\pm$ 21 $(56 \pm 10)$</td>
</tr>
<tr>
<td>$\mu$-He 4-1</td>
<td>0.023</td>
<td>$0.049 \pm 0.012$ $(11 \pm 2)$</td>
<td>$(14 \pm 4)$</td>
<td>$3.65 \times 10^{-4}$ 21 $\pm$ 16 $(9 \pm 7)$</td>
</tr>
<tr>
<td>$\mu$-He 5-1</td>
<td>0.003</td>
<td>$0.012 \pm 0.007$ $3 \pm 2$</td>
<td>$(6 \pm 4)$</td>
<td>$3.67 \times 10^{-4}$ 7 $(3 \pm 3)$</td>
</tr>
</tbody>
</table>

References
(I) G T A Squier and C J Batty ELIST\(^{(25)}\) : yield per atom
### Table 16

**16.1 Kaonic helium: L-series**

<table>
<thead>
<tr>
<th>Kaonic Helium</th>
<th>Electromagnetic energy (keV)</th>
<th>Measured energy (keV)</th>
<th>Instrumental resolution (eV)</th>
<th>Relative peak areas</th>
<th>Total peak area (counts)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3-2</td>
<td>6.464</td>
<td>6.414 ± 0.010</td>
<td>366. ± 13</td>
<td>100.</td>
<td>673 ± 40</td>
</tr>
<tr>
<td>4-2</td>
<td>8.723</td>
<td>8.674</td>
<td>373. ± 15</td>
<td>31. ± 4.</td>
<td>209 ± 27</td>
</tr>
<tr>
<td>5-2</td>
<td>9.768</td>
<td>9.719</td>
<td>376. ± 15</td>
<td>22. ± 4.</td>
<td>148 ± 27</td>
</tr>
<tr>
<td>6-2</td>
<td>10.331</td>
<td>10.282</td>
<td>378. ± 16</td>
<td>5. ± 3.</td>
<td>34 ± 20</td>
</tr>
</tbody>
</table>

\[ \varepsilon_{2p} = (-0.050 ± 0.010) \text{ keV} \]

\[ \Gamma_{2p} = (0.100 ± 0.038) \text{ keV} \]
<table>
<thead>
<tr>
<th>Sigma Helium</th>
<th>Electromagnetic energy (keV)</th>
<th>Measured energy (keV)</th>
<th>Instrumental resolution (eV)</th>
<th>Relative peak areas</th>
<th>Total peak area (counts)</th>
<th>$\varepsilon_{2p}$</th>
<th>$\Gamma_{2p} &lt; 0.500$ keV</th>
</tr>
</thead>
<tbody>
<tr>
<td>3-2</td>
<td>10.163</td>
<td>13.440 ± 0.066</td>
<td>387 ± 18</td>
<td>100</td>
<td>38 ± 19</td>
<td>-0.023 ± 0.086</td>
<td>0.500 keV</td>
</tr>
<tr>
<td>4-2</td>
<td>18.163</td>
<td>18.140</td>
<td>401 ± 21</td>
<td>62 ± 54</td>
<td>24 ± 21</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
## Table 17
X-ray yields for kaonic helium

<table>
<thead>
<tr>
<th>Transition</th>
<th>(I) calculated yield</th>
<th>Previously measured (II) from $\sim 10^6$ Kstops</th>
<th>Previously measured (III) $\sim 2.0.10^7$ Kstops</th>
<th>This Expt. $(2.25 \pm 1.2).10^7$ Kstops</th>
</tr>
</thead>
<tbody>
<tr>
<td>K-He 3-1</td>
<td>0.152</td>
<td>0.092 ± 0.020 (100 ± 11)</td>
<td>100 ± 4</td>
<td>3.36.10^{-4} 673 ± 40</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(relative yield)</td>
<td></td>
<td>0.089 ± 0.045 (100 ± 6)</td>
</tr>
<tr>
<td>K-He 4-2</td>
<td>0.075</td>
<td>0.052 ± 0.013 (56 ± 10)</td>
<td>37 ± 5</td>
<td>4.05.10^{-4} 209 ± 27</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(relative yields)</td>
<td></td>
<td>0.023 ± 0.012 (26 ± 4)</td>
</tr>
<tr>
<td>K-He 5-2</td>
<td>0.043</td>
<td>0.024 ± 0.007 (26 ± 5)</td>
<td>20 ± 4</td>
<td>4.19.10^{-4} 148 ± 27</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(relative yield)</td>
<td></td>
<td>0.016 ± 0.008 (18 ± 3)</td>
</tr>
<tr>
<td>K-He 6-2</td>
<td>0.016</td>
<td>-</td>
<td>-</td>
<td>4.26.10^{-4} 34 ± 20</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(relative yield)</td>
<td></td>
<td>0.004 ± 0.003 (4 ± 3)</td>
</tr>
<tr>
<td>Σ-He 3-2</td>
<td>0.104</td>
<td>-</td>
<td>-</td>
<td>4.58.10^{-4} 38 ± 19</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(relative yield)</td>
<td></td>
<td>(100 ± 50)</td>
</tr>
<tr>
<td>Σ-He 4-2</td>
<td>0.033</td>
<td>-</td>
<td>-</td>
<td>5.15.10^{-4} 24 ± 21</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(relative yield)</td>
<td></td>
<td>(57 ± 57)</td>
</tr>
</tbody>
</table>

**References**

(I) G T A Squier and C J Batty ELIST\textsuperscript{25}: yield per atom


(III) Batty et al. Nuc. Phys. A\textsuperscript{326} (1979) 455 - 462
Table 18

Table 18.1 Results for antiprotonic helium: M-series

<table>
<thead>
<tr>
<th>Transition</th>
<th>Electromagnetic energy (keV)</th>
<th>Measured energy (keV)</th>
<th>Instrumental resolution (eV)</th>
<th>Relative Peak area</th>
<th>Total Peak area (counts)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4-3</td>
<td>3.884</td>
<td>3.876 ± 0.011</td>
<td>304. ± 19.</td>
<td>45. ± 8.</td>
<td>110 ± 20</td>
</tr>
<tr>
<td>5-3</td>
<td>5.625</td>
<td>5.617</td>
<td>310. ± 20.</td>
<td>100.</td>
<td>244 ± 22</td>
</tr>
<tr>
<td>6-3</td>
<td>6.660</td>
<td>6.652</td>
<td>313. ± 21.</td>
<td>42. ± 8.</td>
<td>105 ± 20</td>
</tr>
<tr>
<td>7-3</td>
<td>7.249</td>
<td>7.241</td>
<td>315. ± 21.</td>
<td>8. ± 7.</td>
<td>20 ± 17</td>
</tr>
<tr>
<td>8-3</td>
<td>7.623</td>
<td>7.615</td>
<td>316. ± 22.</td>
<td>3. Fixed to 5.3</td>
<td>7</td>
</tr>
</tbody>
</table>
Table 18.2 Results for antiprotonic helium: L-series

<table>
<thead>
<tr>
<th>Transition</th>
<th>Electromagnetic energy (keV)</th>
<th>Measured energy (keV)</th>
<th>Instrumental resolution (eV)</th>
<th>Relative Peak area</th>
<th>Total Peak area (counts)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3-2</td>
<td>11.131</td>
<td>11.119 ± 0.009</td>
<td>328. ± 25.</td>
<td>100.</td>
<td>414 ± 25</td>
</tr>
<tr>
<td>4-2</td>
<td>15.017</td>
<td>15.005</td>
<td>339. ± 30.</td>
<td>22. ± 4.</td>
<td>91 ± 17</td>
</tr>
<tr>
<td>5-2</td>
<td>16.815</td>
<td>16.803</td>
<td>345. ± 32.</td>
<td>22. ± 4.</td>
<td>91 ± 17</td>
</tr>
<tr>
<td>6-2</td>
<td>17.790</td>
<td>17.778</td>
<td>348. ± 33.</td>
<td>23. ± 4.</td>
<td>95 ± 17</td>
</tr>
<tr>
<td>7-2</td>
<td>-</td>
<td>18.378</td>
<td>350. ± 33.</td>
<td>8. ± 3.</td>
<td>33 ± 12</td>
</tr>
</tbody>
</table>

\[ \varepsilon_{2p} = (-0.012 \pm 0.009) \text{ keV} \]

\[ \Gamma_{2p} = (0.020) \text{ keV} \]
Table 19
Antiprotonic helium X-ray yields
19.1 M-series

<table>
<thead>
<tr>
<th>Transition</th>
<th>(I) Calculated* yield</th>
<th>(II) Previously measured from $3.5 \times 10^{-7}$ p stops (relative yields)*</th>
<th>This Expt $(2.03 \pm 1.0) \times 10^{7}$ p stops</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Absolute efficiency</td>
<td>Fitted Peak area</td>
<td>Yield per atom (relative yield)*</td>
</tr>
<tr>
<td>4-3</td>
<td>250</td>
<td>$(74 \pm 8)$</td>
<td>$1.38 \times 10^{-4}$</td>
</tr>
<tr>
<td>5-3</td>
<td>70</td>
<td>$(33 \pm 4)$</td>
<td>$2.54 \times 10^{-4}$</td>
</tr>
<tr>
<td>6-3</td>
<td>25</td>
<td>$(13 \pm 3)$</td>
<td>$3.76 \times 10^{-4}$</td>
</tr>
<tr>
<td>7-3</td>
<td>10</td>
<td>$(7 \pm 2)$</td>
<td>$4.00 \times 10^{-4}$</td>
</tr>
<tr>
<td>8-3</td>
<td>5</td>
<td>$(5 \pm 2)$</td>
<td>$4.10 \times 10^{-4}$</td>
</tr>
<tr>
<td><strong>Total M-series</strong></td>
<td><strong>0.103 ± 0.033</strong></td>
<td><strong>239 ± 22</strong></td>
<td></td>
</tr>
</tbody>
</table>

*From Table 19.
<table>
<thead>
<tr>
<th>Transition</th>
<th>(I) Calculated* yield</th>
<th>(II) Previously measured from $3.5 \times 10^7 \bar{p}$ stops (relative yields)*</th>
<th>This Expt $(2.03 \pm 1.0) \times 10^7 \bar{p}$ stops</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Absolute efficiency</td>
<td>Fitted Peak area</td>
</tr>
<tr>
<td>3-2</td>
<td>100</td>
<td>$(100 \pm 18)$</td>
<td>$4.68 \times 10^{-4}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4-2</td>
<td>15</td>
<td>$(19 \pm 4)$</td>
<td>$4.96 \times 10^{-4}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5-2</td>
<td>15</td>
<td>$(13 \pm 4)$</td>
<td>$5.02 \times 10^{-4}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6-2</td>
<td>11</td>
<td>$(17 \pm 4)$</td>
<td>$5.04 \times 10^{-4}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7-2</td>
<td>-</td>
<td>-</td>
<td>$5.06 \times 10^{-4}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

References
(I) Wodrich et al. CERN EP/82-02
*Relative yields normalized to the 3-2 transition
Table 20
Resume of helium measurements
20.1 Pionic helium

<table>
<thead>
<tr>
<th>Measurement</th>
<th>This Expt.</th>
<th>Nimrod(40)</th>
<th>Backenstoss(13)</th>
<th>Statistical* Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\epsilon_{1s}$ (keV)</td>
<td>-0.091±0.012</td>
<td>-0.079±0.005</td>
<td>-0.076±0.002</td>
<td>-0.076±0.002</td>
</tr>
<tr>
<td>$\Gamma_{1s}$ (keV)</td>
<td>0.035±0.026</td>
<td>0.045±0.003</td>
<td>0.045±0.003</td>
<td>0.045±0.002</td>
</tr>
</tbody>
</table>

*Results of refs. (13) and (40) only

20.2 Kaonic helium

<table>
<thead>
<tr>
<th>Measurement</th>
<th>This Expt.</th>
<th>Nimrod(40)</th>
<th>Wiegand(12)</th>
<th>Statistical Average</th>
<th>Theory Table 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\epsilon_{2p}$ (keV)</td>
<td>-0.050±0.012</td>
<td>-0.035±0.012</td>
<td>-0.040±0.030</td>
<td>-0.043±0.008</td>
<td>-0.0002</td>
</tr>
<tr>
<td>$\Gamma_{2p}$ (keV)</td>
<td>0.100±0.040</td>
<td>0.030±0.030</td>
<td>-</td>
<td>0.065±0.024</td>
<td>0.002</td>
</tr>
</tbody>
</table>
### 20.3 Sigma helium

<table>
<thead>
<tr>
<th>Measurement</th>
<th>This Expt.</th>
<th>Theory Table 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\varepsilon_{2p}$ (keV)</td>
<td>-0.023±0.086</td>
<td>-0.010</td>
</tr>
<tr>
<td>$\Gamma_{2p}$ (keV)</td>
<td>Less than 0.500</td>
<td>0.020</td>
</tr>
</tbody>
</table>

### 20.4 Antiprotonic helium

<table>
<thead>
<tr>
<th>Measurement</th>
<th>This Expt.</th>
<th>Poth et al (14)</th>
<th>Theory Table 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\varepsilon_{2p}$ (keV)</td>
<td>-0.012±0.014</td>
<td>-0.050±0.018</td>
<td>-0.008</td>
</tr>
<tr>
<td>$\Gamma_{2p}$ (keV)</td>
<td>0.40±0.040</td>
<td>0.090±0.070</td>
<td>0.028</td>
</tr>
</tbody>
</table>
FIGURE 11

PIONIC HELIUM

COUNTS/50eV

ENERGY keV

$\kappa$-He

2-1 3-1

4, 5-1 4, 5, 6-1
FIGURE 12.1

COUNTS/50eV

ENERGY keV

K-He
4-2 5,6-2

π-He
2-1 3,4,5-1

Σ-He
3-2
FIGURE 12.2
FIGURE 13.1

ANTIPROTONIC HELIUM M-SERIES

ENERGY keV

COUNTS/50eV
FIGURE 13.2
3.3.4 Concluding remarks on helium

The first objective of the measurements on helium was to establish the viability of the apparatus to measure X-ray spectra from exotic atoms formed in a liquid target, helium being in many respects an excellent substitute for hydrogen. For example the time required for the test was long enough to establish stability for the beam and electronics in the experimental circumstances and target geometry that prevail during data taking. Yet this time was short compared to that required to get a spectrum from hydrogen.

The second objective was to check previous measurements on $K^-$ and $\bar{p}$-helium atoms which have shown discrepancy with the theory discussed in Chapter 1. None of the helium spectra collected in this experiment continued up to very high statistics being by nature preparatory to the dedicated aim of the experiment, namely the measurements on hydrogen. Never-the-less we have achieved a marginal statistical improvement over the previous data: $K^-$-He due to Batty et al\(^{(40)}\) and $\bar{p}$-He due to Poth et al\(^{(14)}\). [Some of this advantage was lost due to poor resolution of the detector and absolute uncertainty of the calibration associated with prevailing beam conditions]. On $\pi^-$-He the measurement was hurried, of relatively low statistics and performed in possibly the worst beam conditions that we experienced [due to the very high rate of pions per beam burst]. Consequently the precision has suffered. The result in Table 20.1 is 1.5 standard deviations from the definitive result due to Backenstoss et al\(^{(13)}\). This is good enough to prove that the target is viable and though the Si(Li) electronics may have been affected by the beam the measurement of the yield should remain valid.
On K-helium the 2p-shift is found to be consistent with the two previous measurements [see Table 20.2]. However the width is grossly uncertain and must be presented with the following reservation. For the duration of data-taking on helium we experienced the problem with charged particle background that has already been mentioned several times. This was the cause of instrumental drift and deterioration of resolution so that detector response was stable to the extent that stable beam conditions prevailed. Further, contrary to the best policy, regular periods devoted to checking beam-on calibration were missed, being left to a late stage in the data-taking, to save time and the helium which was being consumed by the target. Therefore small random drift of the beam, going unnoticed, might have induced random electronic drift which could fake line-broadening in the histogrammed data. Therefore some confidence in the width measurement is lost but it is probably greater than 0.030keV being between 0.030keV and 0.100keV.

In conclusion the discrepancy with theory is confirmed. The measured shift and width are both larger, by 1-2 orders of magnitude, than predicted. For the future we can look forward to precise measurements on K and Σ helium which, we have shown, could already be achieved within a dedicated experiment using existing apparatus and beamlines. On the theoretical side it remains to be seen whether or not a formal optical model such as has worked so well for pion nucleon interactions can be applied to kaon nucleon interactions.

Finally, on p- helium we experienced generally better conditions of beam, lower random incidence of charged particles, possibly due to the more effective separation of ps from other particles in the mass
separator. The results contradict the conclusions found by Poth et al, [see Table 20.4] seeming to be in agreement with the optical model predictions. Further in a very recent preprint Wodrich et al\(^{(57)}\) obtain \(\varepsilon_{2p} = (-0.014 \pm 0.008)\) keV from analysis of the atomic cascade. We have shown, therefore, that the simple optical model for antiprotons has not yet broken down. For the future we can expect the comming Low Energy Antiproton Ring [LEAR], at CERN, to \textbf{facilitate} more stringent measurements on \(\bar{p}\)-nucleus interactions which will test the theory to greater accuracy.

3.4 X-ray spectrum of kaonic hydrogen

3.4.1 Resumé

Following the data collected from the helium filled target it was supposed that charged particles from the beam, interacting in the detector, were overloading the operations of X-ray signal processing. The cure for this was to install lead shielding, already mentioned, sited to protect the Si(Li) diode from the direct line of the incoming beam, \([\pi^-, K^- \text{ or } \bar{p}]\).

Pion and antiproton X-ray data were taken as a control to show up contamination that was not associated with kaons, particularly fluorescence \([\text{if any}]\) induced by charged particles. This is especially relevant to the antiproton data where the higher energy and numbers of particles from \(\bar{p}p\) annihilation might more efficiently induce fluorescence. Infact no discrete structure was observed in the \(\bar{p}p\) data - the yield of unbroadened \(\bar{p}p\) X-rays was deduced as being less than 0.001 per \(\bar{p}p\) atom. Pionic hydrogen X-rays were not in the energy range of this experiment, however contamination was observed from stopping pions. \([\text{See Fig. 14 and 15}].\)
Kaonic hydrogen data is analysed in two sets labelled A-C and D-F [see Table 13]. The X-ray spectrum mostly comprises contamination from particles stopped in the aluminium. A few of these contaminants are unmistakable [see Tables 21-24] occurring repeatedly with integrated areas up to several hundred counts. Their measured energies were used to test the detector calibration. Once the strongest X-rays were firmly established others, with lower yield, were considered. Generally a photopeak, apparent to the eye in Figs. 14 to 19, was fitted by a gaussian and could be attributed to a single 'probable origin'. In other cases more than one likely origin existed, and possibly more than a single source of X-rays contributed to the photopeak. The purpose of the accompanying tables is to summarize the evidence and conclusions, the principal techniques for resolving the origin of each discrete photopeak being study of the correlations between X-rays, produced in aluminium, beryllium and other constituents of the target, and comparison of π-hydrogen and K-hydrogen data.

In particular relative yields [see Tables 21-25], derived from absolute yields compiled in ELIST\(^{(25)}\), were used to sort unlikely sources from probable sources of contamination. The yield of each of the strongest X-rays in the range 0-50keV is normalized to 100 arbitrary units; the yields of related X-rays being scaled correspondingly. A list of some likely contaminants, including those observed in any part of the present experiment, is compiled in Appendix IV.

At the conclusion of the analysis most of the X-rays have been identified within a tolerance of the order of 1 channel width [credible when combining poor statistics with coarse binning]. Yet other
'photopeaks' have no likely origin at all. These were observed at 7.214keV, in the π-hydrogen data set, 38.345keV and 37.581keV in K-hydrogen A-C. These last two are close to Σ-Al 7-6 at 38.062, intensity 100 units [i.e. expected to be the strongest Σ-Al X-ray]. This particular Σ-Al X-ray has not previously been the subject of measurement but evidence of it is found at about the right energy in a K-Al spectrum due to Barnes et al(58). It might be the origin of either of the two lines we have observed.

An alternative explanation that was considered at length involved π-copper X-rays: π-Cu 6-5, 38.206keV, intensity 100 units, and π-Cu 8-6, 37.897keV, intensity 8 units. However they are first, not expected equal in amplitude, and second no other copper contamination is observed in this spectrum. But then by coincidence the 7.214keV photopeak in π-hydrogen is close to π-Cu 10-9, 7.299keV, intensity 14 units. Infact a source of copper might exist in the thermal contact and mounting to the Si(Li) diode but a search has shown nothing of any other X-rays attributed to copper. In particular copper fluorescence X-rays(47) [Kα at 8.04keV and Kβ at 8.91keV] would be induced by any incident beam particularly p, yet they are not observed in any data. In addition a test was later conducted by irradiating the Si(Li) detector with electrons from a 106Ru beta source - again no evidence was found of copper fluorescence X-rays. The 7.214keV photopeak remains, unidentified.

The line pattern of kaonic hydrogen X-rays is observed in data set A-C only, [see Fig. 18 and Table 26.1] most of the total X-ray count
being attributed to the 3-1 photopeak. The fit also includes contributions due to exotic atoms of aluminium and beryllium fixed in position but whose amplitudes are freely optimized. The background is of type eqn. 3.2.2. In comparison data set D-F [Fig. 19] does not resemble A-C. There are small photopeaks at 8.3keV and 14.8keV which cannot be precisely identified. Also the relative proportions of high energy contaminants [ 20keV Figs. 16 and 17] show increased pion contamination particularly the appearance of a small structure at 42.320keV identified as π-Be 2-1 [compare Figs. 16.2 and 17.2]. Similarly compare with the π-hydrogen data where π-Be 2-1 was observed at 42.320keV with π-Be 3-2 at 8.134keV. The different levels of contamination in the data sets A-C and D-F indicate deterioration of experimental conditions between periods C and D which cannot be definitely explained, but which might be related to the small changes of degrader thickness and level of liquid in the target and the breakdown of the production target. [See Table 13].

Two other photopeaks in Data A-C have been tentatively attributed to Σ−p X-rays; results in Table 26.2. Σ− being produced in 45% of K−p reactions (59) with momentum about 170 MeV/c [range about 2cm in liquid hydrogen (2)]. Because stark mixing causes K−p interactions almost as many Σ− particles stop in the liquid as K− particles, therefore the yield of X-rays from the Σ−p atom has only to be similar to the yield of K−p X-rays for the photopeaks to be similar in amplitude. It is apparent that the 3-1 transition is again stronger than the 2-1, as with K−-p and π−-He atoms. This is the first observation of Σ−-p X-rays and coming as it does adds to the evidence of kaon stops in the target.
using voigtian photopeaks with freely optimized width. Compton contributions were not included. The statistics of the photopeaks being too low to observe significant effect.

Other line patterns which have been considered to fit the observed spectrum have not worked so well. In particular π-helium contamination, postulated to account for the Σ^−p candidate lines did not fit, nor were these photopeaks observed in π-hydrogen data which was collected at the end of period B. Fluorescence X-rays of iron and copper were also eliminated in this way, throughout the data.

Similarly, an attempt to fit the K^−p line pattern to the spectrum collected in the π-hydrogen data failed. Showing that it was not in this case faked by pionic background. No better methods exist for this experiment which can establish that the observed K^−p line pattern is uniquely associated with kaons in hydrogen.
<table>
<thead>
<tr>
<th>Energy (keV)</th>
<th>No. counted</th>
<th>Transition</th>
<th>Energy (keV)</th>
<th>Yield</th>
</tr>
</thead>
<tbody>
<tr>
<td>44.625±0.035</td>
<td>200±30</td>
<td>(\pi)-Al 5-3</td>
<td>44.509</td>
<td>22</td>
</tr>
<tr>
<td>42.282±0.117</td>
<td>55±20</td>
<td>(\pi)-Be 2-1</td>
<td>42.320</td>
<td>100</td>
</tr>
<tr>
<td>33.600±0.050</td>
<td>160±40</td>
<td>((\mu)-Al 5-3, (\mu)-Be 2-1)</td>
<td>33.721</td>
<td>21</td>
</tr>
<tr>
<td>30.502±0.013</td>
<td>590±40</td>
<td>(\pi)-Al 4-3</td>
<td>30.447</td>
<td>100</td>
</tr>
<tr>
<td>23.058±0.062</td>
<td>100±30</td>
<td>(\mu)-Al 4-3</td>
<td>23.016</td>
<td>0.290</td>
</tr>
</tbody>
</table>

See appendix IV for listing of likely contaminant X-rays
### Table 22

**Pion-hydrogen [contaminants] Fig.15**

<table>
<thead>
<tr>
<th>Energy (keV)</th>
<th>No. counted</th>
<th>Transition</th>
<th>Energy (keV)</th>
<th>Yield</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>13.942±0.064</td>
<td>120±40</td>
<td>π-α 5-4</td>
<td>14.070</td>
<td>41</td>
<td></td>
</tr>
<tr>
<td>8.332 fixed</td>
<td>105±45</td>
<td>μ-Be 4-2</td>
<td>8.332</td>
<td>7</td>
<td></td>
</tr>
<tr>
<td>8.134 fixed</td>
<td>90±50</td>
<td>π-Be 3-2</td>
<td>8.134</td>
<td>69</td>
<td></td>
</tr>
<tr>
<td>7.214±0.072</td>
<td>95±40</td>
<td>unknown</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6.179 fixed</td>
<td>160±45</td>
<td>μ-Be 3-2</td>
<td>6.179</td>
<td>100</td>
<td>π-Cu 10-9 unlikely π-Cu absent at higher energies.</td>
</tr>
</tbody>
</table>

- 132 -
Table 23
K^-p A-C Contaminants 20 keV Fig. 16

<table>
<thead>
<tr>
<th>Energy (keV)</th>
<th>No.counted</th>
<th>Transition</th>
<th>Energy (keV)</th>
<th>Yield</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>49.249±0.019</td>
<td>490±40</td>
<td>K-Al 5-4</td>
<td>49.233</td>
<td>100</td>
<td>listed in ref. (53) at 49.270±0.070 keV.</td>
</tr>
<tr>
<td>44.695±0.072</td>
<td>115±30</td>
<td>π-Al 5-3</td>
<td>44.509</td>
<td>22</td>
<td>π-Cu considered</td>
</tr>
<tr>
<td>42.852±0.092</td>
<td>90±30</td>
<td>K-Al 7-5</td>
<td>42.795</td>
<td>18</td>
<td></td>
</tr>
<tr>
<td>38.345±0.100</td>
<td>80±30</td>
<td>Σ-Al 7-6</td>
<td>38.062</td>
<td>100</td>
<td>see section 3.4.1</td>
</tr>
<tr>
<td>37.581±0.102</td>
<td>80±30</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>30.457±0.024</td>
<td>390±40</td>
<td>π-Al 4-3</td>
<td>30.447</td>
<td>100</td>
<td></td>
</tr>
<tr>
<td>26.636±0.028</td>
<td>300±40</td>
<td>K-Al 6-5</td>
<td>26.685</td>
<td>90</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>K-Al 8-6</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>24.850±0.079</td>
<td>100±40</td>
<td>Σ-Al 8-7</td>
<td>24.682</td>
<td>80</td>
<td></td>
</tr>
<tr>
<td>22.822±0.079</td>
<td>100±40</td>
<td>μ-Al 4-3</td>
<td>23.016</td>
<td>100</td>
<td></td>
</tr>
</tbody>
</table>
Table 24
K^-p D-F contaminants  20 keV  Fig. 17

<table>
<thead>
<tr>
<th>Observed X-ray</th>
<th>Probable origin</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Energy (keV)</td>
<td>No. counted</td>
<td>Transition</td>
</tr>
<tr>
<td>49.257±0.021</td>
<td>380±40</td>
<td>K-Al 5-4</td>
</tr>
<tr>
<td>44.618±0.089</td>
<td>80±30</td>
<td>π-Al 5-3</td>
</tr>
<tr>
<td>42.916±0.099</td>
<td>70±30</td>
<td>K-Al 7-5</td>
</tr>
<tr>
<td>42.320 fixed</td>
<td>45±30</td>
<td>π-Be 2-1</td>
</tr>
<tr>
<td>30.472±0.017</td>
<td>480±40</td>
<td>π-Al 4-3</td>
</tr>
<tr>
<td>26.707±0.029</td>
<td>260±35</td>
<td>K-Al 6-5</td>
</tr>
<tr>
<td></td>
<td></td>
<td>K-Al 8-6</td>
</tr>
<tr>
<td>24.582±0.112</td>
<td>80±35</td>
<td>Σ-Al 8-7</td>
</tr>
</tbody>
</table>

Compare data A - C
Fig. 17.2 and Fig. 16.2

but Σ-Al 7-6 is not observed in this data?
Table 25
Kaonic hydrogen A–C  Fig. 18

Contaminant X-rays

<table>
<thead>
<tr>
<th>Energy (keV)</th>
<th>No. counted</th>
<th>Transition</th>
<th>Energy (keV)</th>
<th>Yield</th>
</tr>
</thead>
<tbody>
<tr>
<td>14.154±0.078</td>
<td>80±30</td>
<td>π- Al 6-5</td>
<td>14.070</td>
<td>41</td>
</tr>
<tr>
<td>12.981±0.060</td>
<td>105±33</td>
<td>Σ- p 3-1</td>
<td>12.500</td>
<td>-</td>
</tr>
<tr>
<td>11.200±0.167</td>
<td>35±31</td>
<td>Σ- p 2-1</td>
<td>10.553</td>
<td>-</td>
</tr>
<tr>
<td>9.678 fixed</td>
<td>20±35</td>
<td>K- Be 4-3</td>
<td>9.678</td>
<td>100</td>
</tr>
<tr>
<td>7.150 fixed</td>
<td>45±40</td>
<td>K- Al 9-8</td>
<td>7.150</td>
<td>10</td>
</tr>
<tr>
<td>6.179 fixed</td>
<td>35±40</td>
<td>μ- Be 3-2</td>
<td>6.179</td>
<td>100</td>
</tr>
</tbody>
</table>

Evidence of kaons stopping in the center of the target volume.

)tentative; first observation Σ- p X-rays.
Table 26

Results for kaonic hydrogen. Fig. 18

<table>
<thead>
<tr>
<th>Transition</th>
<th>Electromagnetic energy (keV)</th>
<th>Measured energy (keV)</th>
<th>Instrumental resolution (eV)</th>
<th>Relative peak are</th>
<th>Total peak area (counts)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2-1</td>
<td>6.482</td>
<td>6.675±0.060</td>
<td>308±8</td>
<td>22±29</td>
<td>36 ± 48 - 36</td>
</tr>
<tr>
<td>3-1</td>
<td>7.679</td>
<td>7.872</td>
<td>315±8</td>
<td>100±31</td>
<td>164 ± 51</td>
</tr>
<tr>
<td>4-1</td>
<td>8.098</td>
<td>8.291</td>
<td>317±8</td>
<td>12±30</td>
<td>20 ± 50</td>
</tr>
<tr>
<td>5-1</td>
<td>8.288</td>
<td>8.481</td>
<td>318±8</td>
<td>5 fixed to 3-1</td>
<td>8</td>
</tr>
</tbody>
</table>

\[ \epsilon_{1s} = (0.193 ± 0.060) \text{ keV} \]

\[ \Gamma_{1s} = (0.082 ± 0.220) \text{ keV} \]
### Table 26 (Cont.)

#### 26.2 Sigma Hydrogen Candidates

<table>
<thead>
<tr>
<th>Transition</th>
<th>Electromagnetic energy (keV)</th>
<th>Measured energy (keV)</th>
<th>Instrumental resolution (eV)</th>
<th>Relative peak area</th>
<th>Total peak area (counts)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2-1</td>
<td>10.553</td>
<td>11.200±0.170</td>
<td>332±9</td>
<td>35±31</td>
<td>37±33</td>
</tr>
<tr>
<td>3-1</td>
<td>12.500</td>
<td>12.981±0.060</td>
<td>341±10</td>
<td>100±31</td>
<td>105±33</td>
</tr>
<tr>
<td>4-1</td>
<td>13.182</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>5-1</td>
<td>13.497</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

$\varepsilon_{1s} = (0.500 \pm 0.057)$ keV

$\Gamma_{1s} = 0.500$ keV
Table 27.1

K⁻p X-rays

<table>
<thead>
<tr>
<th>K⁻p X-rays</th>
<th>previously measured (I)</th>
<th>This Expt. (1.42±0.43).10⁸ K stops</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Measured energy (keV)</td>
<td>Peak area</td>
</tr>
<tr>
<td>2-1</td>
<td>6.96±0.09</td>
<td>78±34</td>
</tr>
<tr>
<td>3-1</td>
<td>7.99±0.07</td>
<td>102±34</td>
</tr>
<tr>
<td>4-1</td>
<td>8.64±0.10</td>
<td>64±33</td>
</tr>
<tr>
<td>5-1</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Sigma hydrogen X-ray yields

<table>
<thead>
<tr>
<th>$\Sigma^{-p}$</th>
<th>Measured energy (keV)</th>
<th>detector efficiency</th>
<th>Peak area</th>
<th>Yield relative to $K^{-p} \text{ Table 27.1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2-1</td>
<td>11.200±0.170</td>
<td>$3.81 \times 10^{-4}$</td>
<td>37±33</td>
<td>(21 ± 20)</td>
</tr>
<tr>
<td>3-1</td>
<td>12.981±0.060</td>
<td>$3.89 \times 10^{-4}$</td>
<td>105±33</td>
<td>(58 ± 26)</td>
</tr>
</tbody>
</table>
FIGURE 14

ENERGY keV

COUNTS/100eV

IONIC HYDROGEN

FIGURE 14
COUNTS/100eV

PIONIC HYDROGEN

FIGURE 15
FIGURE 16.1
FIGURE 16.2

CONTRASTS/100eV

K-AL
7-5
π-AL
5-3

K-AL
5-4

ENERGY keV

KP CONTAMINANTS A-C
KAONIC HYDROGEN D-F CONTAMINANTS

COUNTS/150eV

ENERGY keV

FIGURE 17.1
FIGURE 17.2
FIGURE 18

KAONIC HYDROGEN A-C
3.4.2 Concluding remarks on kaonic hydrogen

In Table 27.1 the yield of $K^-p$ X-rays is assessed and compared with the experiment of Izycki et al. The discrepancy is of the order of a factor of 10 in the yields which is much greater than the tolerances of the two experiments will allow. For example, given the total $K$-series yield 0.005 X-rays per $K^-p$ atom, measured in the present experiment, Izycki et al would have counted 2000 X-rays. In view of this anomaly it would be wise to leave the discussion to be resolved by future experiments.

Regarding the present experiment the following evidence supports the hypothesis that the $K^-p$ X-ray pattern observed is associated with kaons stopping in hydrogen.

To start, this line pattern does not appear in any other than the $K$-hydrogen data-set A-C. Coincidentally $\Sigma$-hydrogen X-rays are observed, for the first time, only in this data-set, and there is a contribution attributed to $K$-$Be$ X-rays which is evidence, albeit speculative, of kaons having about the right range and timing. Finally refs. (16) and (17) predicted a yield about 0.001 $K^-p$ X-rays per atom in good agreement with the measured yield, the viability of the target assembly having been established by the results on helium.

The greatest uncertainty in the assessment of the data and of the yield, which may also be realized in the success or failure of the experiment, occurs in the definition of a particle stop and is derived from poor momentum resolution and spatial containment of available beam.
Infact the only explanation that can be put forward to account for the apparently void data-set, D-F, is deterioration of the beam conditions compared to the earlier data.

Now consider the NIMROD experiment \cite{Davies 20}. A single photopeak at \((6.52 \pm 0.06)\) keV was attributed to the \(K^-p\) 2-1 X-ray; the yield was \((0.0011 \pm 0.0006)\) X-rays per stopped kaon but the 3-1 X-ray was not observed. The upper limit on the yield of the 3-1 was '40\% of the supposed 2-1 yield'. In comparison the same pattern of X-rays has not been observed in the present experiment [the relative yields are reversed] and the measured shift is quite different [see Table 28]. Yet the same target apparatus was used and the main sources of contamination should have been the same. The kaon stops totals too were very similar: \(1.13 \times 10^8\) at NIMROD and \(1.42 \times 10^8\) for the data-set A-C. Nor should the change of beamline have been very consequential except for the comparison of background levels. In this respect we observe that the continuous background was at a much greater level and was more steeply rising in the latest data. This is relevant to the ratio of the peak to background amplitudes of the 3-1 and 2-1 photopeaks, but does not alter the conclusion of the present experiment, that the 3-1 X-ray should have been observed in the NIMROD data. It is very difficult to reconcile this with the suggestion that the origin of the X-rays is the same in these two experiments.

In conclusion we feel that the latest experiment is in part \cite{Davies 20} the strongest so far, showing more detailed structure than has been observed in any other data. In comparison data-set D-F illustrates the difficulty with which all the experiments have had to contend. Namely the
current situation regarding experimental facilities, combined with the low X-ray yield from $K^-p$, determines that their detection is close to the present limit of sensitivity. In these circumstances the setting up of the experiment is both difficult [there is no event rate to optimize, hence the value of helium data to check the viability] and critical [a factor-of-two effect in the event rate can determine success or failure].

For future experiments we can expect improved beam conditions—more intense, better resolution and better spatial confinement. These should arrive with kaon factories now at an advanced stage of planning. In the shorter term it might be possible to improve the definition of an event in the target by 'tagging'—triggering on characteristic reaction products; or reduce the randoms by background suppression—the X-ray detector is enclosed in a 'veto box'. These techniques are currently in planning or early stages of application, but not for kaonic hydrogen, with no further experiments yet scheduled.

On the theoretical side we have EITHER to explain a positive energy level shift, $\epsilon_{1s} = (0.200 \pm 0.060)$ keV from the present experiment, which contradicts the predictions due to extrapolation from scattering data taken above the $K^-p$ threshold, and existing attempts to describe the $K^-p$ bound state problem \(^{(60)}\); OR postpone acceptance of the current results. Pursuing the first possibility, the suggestions that detailed coulomb corrections are responsible for the discrepancy are, as currently contrived, not widely accepted \(^{(33)}\). Therefore, if no alternative interpretation of the $K^-p$ scattering data, or different result on kaonic hydrogen is forthcoming, resolution of the theoretical problem may eventually be found within the framework of a fundamental theory of the strong interaction, as its formal principles become known.
Table 28

Resumé of results on kaonic hydrogen

<table>
<thead>
<tr>
<th>Measurement</th>
<th>This experiment</th>
<th>NIMROD J D Davies et. al(^{(20)})</th>
<th>Izycki et. al(^{(21)})</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. K stops</td>
<td>(1.42 \times 10^8)</td>
<td>(1.13 \times 10^8)</td>
<td>(0.90 \times 10^8)</td>
</tr>
<tr>
<td>(\varepsilon_{1s}) (keV)</td>
<td>(0.200 \pm 0.060)</td>
<td>(0.040 \pm 0.060)</td>
<td>(0.370 \pm 0.080)</td>
</tr>
<tr>
<td>(\Gamma_{1s}) (keV)</td>
<td>(0.080 \pm 0.220 - 0.080)</td>
<td>(0. + 0.230 - 0.)</td>
<td>(0.560 \pm 0.260)</td>
</tr>
<tr>
<td>Yield per atom</td>
<td>(~ 0.45%)</td>
<td>(~ 0.1%)</td>
<td>(~ 0.05%)</td>
</tr>
</tbody>
</table>
References

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electron atoms (Springer-Verlag) 1957.
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Appendix I — plan of electronic logic

Key

F/O = Fan out
Disc. = Discriminator unit
Coinc. unit = coincidence unit

Beam spill monitor

88 ns
Time of flight TAC (Stop)

79 ns
Time of flight TAC (Start)

Scaler \((B_1B_3)_{\text{TOF}, \bar{C}_F}\)

Scaler \((B_1B_3)_{\text{TOF}}\)

Scaler \((B_1B_3)_{\text{LONG}}\)

Scaler \((B_1B_3)_{\text{TOF}}\)

Scaler \((B_1B_3)_{\text{LONG}}\)

Nominal "others-in-flight"
The purpose of the TARGET PROGRAM is to compute the probability that an X-ray event will be recorded in coincidence with formation of an exotic atom which is responsible for X-ray emission - an X-ray source. For the calculation the program is supplied with the following data: Target dimensions including the level of the liquid, and position of the geometric center; position of the beryllium windows and detector [the origin of co-ordinates used here is the geometric center of the front face of the Si(Li) diode]; X-ray energy $X_0$ and corresponding photon cross-sections $\sigma^{\text{TOT}}, \sigma^{\text{INCOH}}$ [Incoherent, Compton, scattering cross-sections], and $\sigma^{\text{COH}}$ [coherent, Rayleigh, scattering cross-sections] for the target liquid; density of the liquid; $\sigma^{\text{TOT}}$ for beryllium and density of beryllium. [Discussion and compilation of photon cross-sections is given by Storm and Israel (44)]; finally the detector efficiency in the form of equation 2.12 which was approximated by simple rules of interpolation between measured efficiencies.

For an X-ray source, energy $X$, co-ordinates $(r, \theta, \phi)$, the program evaluates the absolute detector efficiency given by the product of the factors $\alpha_c(\theta,X), \theta \geq 40^\circ$, and $\epsilon(X), 2\text{keV} < X < 35\text{keV}$; the solid angle $\Omega(r,\theta)$ given by eqn. ; and attenuation due to the liquid intervening between source and detector, and the beryllium window of the heatshield, given by:

$\alpha_c(\theta,X)$ includes the attenuation by the target window and collimation due to the aluminium top-plate.
\[
\frac{I}{I_0} = e^{-\left(\sum_{i} \mu_i t_i\right)}
\]

where
\[
\mu_i = \frac{N_0 \rho_i}{A_i} \sigma_i \text{TOT}(x)
\]

- is initial intensity
- \(N_0\) is Avogadro's number
- \(A_i\) is the mass number and
- \(t_i\) is the thickness of the \(i\)th absorber
- \(\sigma_i \text{TOT}\) is the total absorption cross-section which depends on energy \(X\),
and densities \(\rho_i\), are:

0.0708 g cm\(^{-3}\) for liquid hydrogen
0.125 g cm\(^{-3}\) for liquid helium
1.846 g cm\(^{-3}\) for beryllium.

Events which fire the detector are of three types - random background due to the ambient radiation, direct events and scattered events. Randoms will not be discussed here.

1) Direct events

For X-ray source, energy \(X_0\) co-ordinates \((r_0, \theta_0, \phi_0)\), direct events are registered with probability given by the absolute detector efficiency described above. This probability is averaged over all the stops in the distribution supplied by DEGRAD [see sec. 2.7] thus forming the absolute
detector efficiency, X-rays detected per X-ray per stop. Results are plotted in figure A2. For convenience we shall refer to this as $\epsilon'_{\text{abs}}$ where prime denotes average over the distribution of stops.

2) Scattered events

For the small fraction, $\epsilon'_{\text{abs}}$, of X-rays which are registered directly by the detector $(1-\epsilon'_{\text{abs}})$ encounter another alternative fate. Some of this fraction may by chance scatter in the volume of the target, the probability of any interaction being given by $\sigma_{\text{TOT}}$, and the probability of scattering being given by the partial cross-sections, $\sigma_{\text{COH}}$ for Rayleigh scattering and $\sigma_{\text{INCOH}}$ for compton scattering. The angular distribution of compton scattering is given by the Klein-Nishina formula. A routine which is part of the Target Program evaluates these terms integrating up the probabilities over the distribution of stops and the target volume, to estimate the fraction of all X-rays emitted that are counted by the detector after an interaction, which scatters them, in the target. An earlier version of this routine written by J Egger (61) has been described by Backenstoss (62). It has been adapted to our detector and target geometry by S F Biagi.*

The method adopted is monte carlo integration. Each stop, $P$, is considered to be a source of X-rays of energy $X_0$, then steps in the algorithm are roughly:

(i) For each stop generate a number of unit cells distributed uniformly throughout the target volume.

*Present address: Queen Mary College, London.
(ii) For each cell, \( S \), calculate the probability that an X-ray photon, \( X_o \), from \( P \) will interact at \( S \).

(iii) Calculate the probability that the interaction scatters the photon into solid angle \( \Omega(s) \) subtended by the cell at the detector. As scattering must be distinguished from absorption \( \sigma_{\text{COH}} \) and \( \sigma_{\text{INCOH}} \) are required for this calculation.

(iv) Calculate attenuation from \( S \) to the detector, eqn. A2.1.

(v) Evaluate the detector efficiency \( \varepsilon(X_s) \alpha(\theta_s, X_s) \) where \( X_s \) is the scattered X-ray.

The product of all these terms represents the minute probability that an event \( X_s \) is scattered into the detector. Bin the scattered event according to its energy \( X_s \). Repeat for many cells and many stops.

Note that Rayleigh scattering does not affect the energy or trajectory of a photon and is therefore relevant only for those interaction cells which lie along a path to the detector.

In Compton scattering there is energy loss \( [X_s < X_o] \) thus Compton events are distributed in a tail behind the direct photopeak. The length of the tail increases with photopeak energy according to the formula

\[
\Delta E_{\text{max}} = -\frac{2X_o}{M \cdot c^2 + 2X_o} X_o
\]

A2.2

- A.II.4 -
The binned distribution of scattered events is then normalized to: compton events per direct event and is added to the peak shape fitted to data, as described in chapter 3.
Detector efficiency averaged over particle stops in the target
The Voigtian distribution results from the convolution of Lorentzian line shape and Gaussian detector resolution. Its application to the measurement of natural line widths has, for example, been established by Batty et al. (63). An earlier discussion, including nomograms drawn up to facilitate evaluation of the integrated area, $L_o$, and fwhm, $\Gamma$, by hand, was given by Wilkinson (54). When computerized analysis is possible the Voigtian can be evaluated very efficiently using its association to the complex error function which follows:

The Lorentzian is defined as in Chapter 3.

$$L(X - X_0) = L_0 \frac{\Gamma}{2\pi} \frac{1}{(\Gamma/2)^2 + (X - X_0)^2}$$

Centre $X_0$, fwhm $\Gamma$ and integrated area $L_0$

$$L_0 = \int_{-\infty}^{+\infty} L(X - X_0) \, dX$$

The Gaussian probability distribution is:

$$G(X - X_0) = \frac{1}{\sqrt{2\pi} \sigma} e^{-(X - X_0)^2 / 2\sigma^2}$$

$$1 = \int_{-\infty}^{+\infty} G(X - X_0) \, dX$$

- A.III.1 -
and the Voigtian is the convolution

\[ V(X-X_0) = \int_{-\infty}^{+\infty} G(X-y) L(y-X_0) \, dy \]

\[ = (2\pi)^{-\frac{3}{2}} \frac{L_0}{\sigma} \frac{\Gamma}{\sigma} \int_{-\infty}^{+\infty} \frac{e^{-(x-y)^2/2\sigma^2}}{(\Gamma/2 + (y-X_0)^2)} \, dy \quad \text{A3.5} \]

a more convenient final form than A3.5 is obtained by making the following substitutions:

let \( \Theta = \sqrt{\frac{3}{2}} \sigma \)

and put \( Z = \frac{X-Y}{\Theta} \), \( dy \rightarrow -\Theta \, dz \)

let \( a = \frac{\Gamma}{2\Theta} \) and \( U = \frac{X-X_0}{\Theta} \)

Then \( V(U) = (2\pi)^{-\frac{3}{2}} \frac{L_0}{\sigma^2} \frac{\sqrt{2} \, \Gamma}{\Theta} \int_{-\infty}^{+\infty} \frac{e^{-Z^2}}{a^2 + (U-Z)^2} \, dZ \quad \text{A3.6} \)

finally let

\[ I(U,a) = \int_{-\infty}^{+\infty} \frac{e^{-Z^2}}{a^2 + (U-Z)^2} \, dZ \quad \text{A3.7} \]

Let \( \omega = U + ia \) and A3.7 is given by

\[ I(\omega) = \pi \Re \{ e^{\omega^2} \left[ 1 + \frac{2i}{\sqrt{\pi}} \int_{0}^{\omega} e^{q^2} \, dq \right] \} \quad \text{A3.8} \]

- A.III.2 -
See Abramowitz and Stegun\textsuperscript{(64)} sec. 7.4.13

Therefore:

\[ V(U) = \frac{L}{\sqrt{\pi}} \text{Re} \{W(U + ia)\} \]

\[ U = \frac{X - X}{\Theta} \quad a = \frac{\pi}{2\Theta} \]

where $W(Z)$ is called the complex error function.
### APPENDIX IV

**COMPILATION OF X-RAY ENERGIES AND YIELDS FROM REF. (25)**

<table>
<thead>
<tr>
<th>Energy (keV)</th>
<th>Atom transition</th>
<th>Yield*</th>
<th>Comment</th>
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*Yields are not corrected for detector efficiency.*

Energy listed in ref (53)

Yield = 0.102 per atom

Yield = 0.110 per atom
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<th>Energy (keV)</th>
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<th>Yield</th>
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