SOME NEW MODIFICATIONS TO, AND USES OF, THE METHOD OF REPEATED BALLISTIC DISCHARGE FOR THE DETERMINATION OF CAPACITANCE, TOGETHER WITH THE THEORETICAL ANALYSIS AND HISTORY OF THE METHOD.

Arthur Ernest Hawkins.
ABSTRACT.

The application of the method of continuous ballistic discharge to the comparison of capacitances, and to the measurement of pressure difference, barometric pressure, wind velocity and resistance is explained.

Instruments and circuits that have been developed for the various purposes are discussed and their characteristics described.

The basic theory of the method is given together with the theory of the various developments. Also included is a short history of the method, together with that of closely allied methods. Experiments on some of the switching devices available are also described.
CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>List of figures</td>
<td>1</td>
</tr>
<tr>
<td>List of tables</td>
<td>5</td>
</tr>
<tr>
<td>Introduction</td>
<td>7</td>
</tr>
<tr>
<td>Section 1. The theory of the method</td>
<td>10</td>
</tr>
<tr>
<td>Section 2. An historical survey, together with several investigations arising out of it</td>
<td>31</td>
</tr>
<tr>
<td>Section 3. A null method for the comparison of capacitance</td>
<td>77</td>
</tr>
<tr>
<td>Section 4. A sensitive, direct reading, mercury U-tube manometer and a direct reading barograph</td>
<td>93</td>
</tr>
<tr>
<td>Section 5. A direct reading vane anemometer</td>
<td>121</td>
</tr>
<tr>
<td>Section 6. The measurement of high resistance values</td>
<td>133</td>
</tr>
<tr>
<td>Section 7. Summary and conclusion</td>
<td>211</td>
</tr>
<tr>
<td>Section 8. References</td>
<td>214</td>
</tr>
</tbody>
</table>
LIST OF FIGURES.

1.1. The original Maxwell circuit. 12
1.2. The basic circuit for the method. 12
1.3. The potential difference across a condenser. 17
1.4. An alternative form of the circuit. 24

2.1. Simplified diagram of Siemens' "wippe". 34
2.2. Simplified diagram of Stoletow's commutator. 36
2.3. Diagram of the Fleming and Clinton commutator. 37
2.4. Diagrams of two alternatives for a commutator. 39
2.5. Diagram of Rosa's tuning-fork switch. 42
2.6. Simplified diagram of the Rosa and Dorsey commutator. 44
2.7. Diagram of the Siemens type 73 relay. 47
2.8. Operation of a relay. 49
2.9. Loge(I_d) plotted against 1/R. 52
2.10. Basic thermionic valve circuit. 57
2.11. Effect of phasing circuits. 57
2.12. Pulse clipping circuit. 59
2.13. Practical thermionic valve circuit. 59
2.14. Log_{10}(I/I_L) plotted against 1/R. 61
2.15. Resistance of a type LP 2 valve. 63
2.16. Theoretical curves of I_c/I_t plotted against C. 66
2.17. Cathode ray oscillograms. 69
2.18. Resistance of a type PX 4 valve. 72
2.19. A modified form of the circuit of figure 2.13. 75
<table>
<thead>
<tr>
<th>Section</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.1</td>
<td>Diagram of the original null circuit.</td>
<td>79</td>
</tr>
<tr>
<td>3.2</td>
<td>Diagram of the second null circuit.</td>
<td>83</td>
</tr>
<tr>
<td>3.3</td>
<td>Diagram of the cylindrical condenser.</td>
<td>91</td>
</tr>
<tr>
<td>4.1</td>
<td>Diagrams of devices depending on pressure difference for the measurement of fluid flow.</td>
<td>95</td>
</tr>
<tr>
<td>4.2</td>
<td>Diagram of the U-tube manometer.</td>
<td>96</td>
</tr>
<tr>
<td>4.3</td>
<td>Apparent capacitance of a tube.</td>
<td>102</td>
</tr>
<tr>
<td>4.4</td>
<td>Apparent capacitance of two tubes.</td>
<td>104</td>
</tr>
<tr>
<td>4.5</td>
<td>Values of $R_2$ plotted against pressure difference.</td>
<td>106</td>
</tr>
<tr>
<td>4.6</td>
<td>The measurement of a slow change of pressure difference.</td>
<td>107</td>
</tr>
<tr>
<td>4.7</td>
<td>Galvanometer deflections plotted against change of pressure difference.</td>
<td>109</td>
</tr>
<tr>
<td>4.8</td>
<td>Galvanometer deflection plotted against change of pressure difference.</td>
<td>110</td>
</tr>
<tr>
<td>4.9</td>
<td>Galvanometer deflection plotted against pressure difference.</td>
<td>111</td>
</tr>
<tr>
<td>4.10</td>
<td>Diagram of direct reading siphon barograph.</td>
<td>114</td>
</tr>
<tr>
<td>4.11</td>
<td>Values of $R_2$ plotted against barometric pressure.</td>
<td>116</td>
</tr>
<tr>
<td>4.12</td>
<td>Values of $R_2$ plotted against barometric pressure.</td>
<td>118</td>
</tr>
<tr>
<td>4.13</td>
<td>The basic construction of the Marvin barograph.</td>
<td>120</td>
</tr>
<tr>
<td>5.1</td>
<td>Diagram of vane unit.</td>
<td>123</td>
</tr>
<tr>
<td>5.2</td>
<td>The vane anemometer.</td>
<td>127</td>
</tr>
<tr>
<td>5.3</td>
<td>Circuit diagram of control unit.</td>
<td>128</td>
</tr>
</tbody>
</table>
5.4. Calibration graphs.
5.5. Meter readings obtained over a plane perpendicular to the axis of a blower.
5.6. Meter readings plotted against time.

6.1. Circuit diagrams for the measurement of high resistance values.
6.3. Simple gas-filled triode (thyatron) relaxation oscillator.
6.4. Circuit diagram of the megohmmeter.
6.5. Discharge current plotted against 1/R.
6.6. Discharge current plotted against 1/R.
6.7. Meter reading plotted against 1/R.
6.8. Meter shunt value plotted against resistance.
6.9. First circuit for the measurement of resistance.
6.10. Graphs of $\log_e(I_d)$ plotted against 1/S.
6.11. Calibration curve of a galvanometer.
6.15. Galvanometer deflection plotted against resistance.
6.16. $\log_e I_s$ plotted against 1/S.
5.4. Calibration graphs. 131

5.5. Meter readings obtained over a plane perpendicular to the axis of a blower. 133

5.6. Meter readings plotted against time. 135

6.1. Circuit diagrams for the measurement of high resistance values. 143

6.2. A modified form of the circuits of figure 6.1. 145

6.3. Simple gas-filled triode (thyatron) relaxation oscillator. 148

6.4. Circuit diagram of the megohmmeter. 148

6.5. Discharge current plotted against $1/R$. 152

6.6. Discharge current plotted against $1/R$. 153

6.7. Meter reading plotted against $1/R$. 154

6.8. Meter shunt value plotted against resistance. 156

6.9. First circuit for the measurement of resistance. 158

6.10. Graphs of $\log_e(I_d)$ plotted against $1/S$. 161

6.11. Calibration curve of a galvanometer. 170

6.12. Second circuit for the measurement of resistance. 171

6.13. Galvanometer deflection plotted against resistance. 180


6.15. Galvanometer deflection plotted against resistance. 185

6.16. $\log_e I_s$ plotted against $1/S$. 188

6.17. Circuit for the null method measurement of resistance. 196
6.18. Resistance value $S$ plotted against $1/\log_e(R_2'/R_2)$. 200
6.19. Resistance value $S$ plotted against $1/\log_e(R_2'/R_2)$. 201
6.20. Diagram of second null circuit. 203
6.21. $S$ plotted against $1/\log_e(R_4'/R_4)$. 204
6.22. Galvanometer deflection plotted against resistance. 208
6.23. Galvanometer deflection plotted against resistance. 209
LIST OF TABLES.

1.1. Maximum values for resistance in charging circuit. 19
1.2. Minimum values of condenser leakage resistance. 19
1.3. Limits of error of measurement of the method. 23
1.4. Percentage change in current for various values of leakage resistance. 28

2.1. Effect of frequency on charging/discharging times. 50
2.2. Maximum frequency for various values of 10. 50
2.3. Ratios of capacitances obtained with Siemens' method. 55
2.4. Leakage resistances of paper condensers. 65
2.5. Experimental results for $I_o/I_t$. 67
2.6. Capacitance ratios determined by three different methods. 70
2.7. Effective value of capacitances. 73

3.1. Capacitance ratios obtained with the null circuit. 85
3.2. Capacitance ratios obtained with the null circuit. 89
3.3. Values of dielectric constants. 92

4.1. Sensitivities obtained with the manometer. 113

5.1. Details of figure 5.1. 124
5.2. Component values used in the control unit. 129
6.1. Details of the circuit of figure 6.4. 149
6.2. Galvanometer readings for various resistances. 162
6.3. The resistance in ohms for values of \( \frac{E}{I} \).
6.4. Possible error in value of resistance. 164
6.5. Resistance values obtained by the first method. 166
6.6. Experimental and corrected readings. 167
6.7. Resistance values obtained by the first method. 168
6.8. Possible error in measurement. 173
6.9. Experimental results for a resistance value. 176
6.10. Experimental results for a resistance value. 177
6.11. Resistance values determined by various methods. 181
6.11a. Details of the circuit of figure 6.14. 184
6.12. Experimental results for a resistance value. 190
6.13. Resistance values obtained with the modified circuit of figure 6.12. 193
6.14. Resistance values obtained using a restricted range of values. 194
6.15. Ratio values. 198
6.16. The possible error (\( \varepsilon \)). 198
6.17. Resistance values obtained. 206
INTRODUCTION.

N.F.Astbury has suggested (0.1.) that the methods of determining capacitance may be classified in the following manner: (i) bridge methods, (ii) resonance methods, (iii) oscillator and valve methods, (iv) impedance, voltage and current methods, (v) D.C. and ballistic methods.

All of these methods tend to be complicated if high sensitivity is required, except for those in the last group which, however, suffer from the drawback that unavoidable errors are produced if absorption is present. Recently transistors have come into use in oscillators and detectors for bridge methods, but the simplification and lowering in cost that results is not very great (0.2.).

For the measurement of mechanical displacement, pressure and similar physical parameters, methods (i) and (ii) are used with a capacitance pick-up and high-frequency alternating current. The complexity of the associated circuits is nowadays considerable as may be seen in a paper by Woodcock (0.3.). The earlier circuits were not quite so involved, for example that of the ultramicrometer (0.4.).

This thesis is a study of the method of repeated ballistic discharge and modifications to it; it shows that the method can be usefully employed with apparatus to measure certain physical parameters. The advantages are simple circuitry, results expressed as
direct currents with no ancillary apparatus, and satisfactory sensitivity.

The U-tube manometer described in section 4 was originally designed to operate with a capillary tube flow-meter for use in a simple differential calorimeter for small animals devised by the author and developed with a colleague. This calorimeter is used for the investigation of the heat output of small, active animals. The calorimeter gives a continuous record of the heat emitted by the animal under investigation whether the animal is quiet or active, and is unique in this respect.

The anemometer in section 5 was designed for use by the Animal Husbandry Department of the Royal Veterinary College for their investigation into the effects on the well-being of domestic animals of draughts.

The work in section 6 on the continuous measurement of high resistance is hoped to lead to (i) a simple method for the continuous measurement of body and skin temperatures of small animals, and (ii) a method for the continuous measurement of humidity in the calorimeter already mentioned.

The work which is believed to be original is distinguished from that of others either by a reference being given to the original worker, or by a reference to a standard text.

The work which has been published and of which reprints are included is as follows:
Hawkins, A.E. A simple vane anemometer giving continuous and direct indication of wind velocity.

An electrical indicator for a mercury U-tube manometer.

A simple method for the measurement of high resistance.

Three simple relay circuits for the comparison of capacitance.

Clarke, E.G.C., and Hawkins, A.E. A simple differential calorimeter for small animals.


The author wishes to thank Professor E.C. Amoroso of the Royal Veterinary College and Dr. S. Marsh of Battersea Polytechnic for their interest and encouragement in this work and for the facilities put at his disposal. The author is also indebted to Mr. V.C. Tindley and his workshop staff for their assistance with the construction of some of the mechanical devices described, and to Miss G. Tomlinson for technical assistance.
SECTION 1.

THE THEORY OF THE METHOD.
The determination of the capacitance of a condenser by means of a commutator regularly repeating the process of charge and discharge of the condenser is sometimes attributed in standard texts, for example that of Starling (1.1.), to Maxwell (1.2.). The circuit given by Maxwell is reproduced in figure 1.1. where the reversing switch S is, in practice, some form of commutator. Maxwell states that the operation of reversing the connections of the condenser can be repeated at regular intervals of time, each interval being equal to T. If this interval is sufficiently long to give complete discharge and recharge of the condenser, the quantity of electricity transmitted through the (moving magnet) galvanometer in each interval will be $2EC$, where $E$ is the electromotive force, and $C$ is the capacitance of the condenser. If the galvanometer is of a long period type, the current indicated by it will be $2EC/T$.

Maxwell goes on to say that the condenser may be removed and a resistance coil substituted, its value being adjusted to give the same current through the galvanometer. If $R$ is the total resistance of the circuit, then, since the current is the same in each experiment, $E/R = 2EC/T$ or $C = T/2R$.

Thus, if $T$ and $R$ are measured, $C$ may be calculated.

(Maxwell shows also that a condenser with its commutator may be substituted for any of the
Figure 1.1.
The original Maxwell circuit.

Figure 1.2.
The basic circuit for the method of repeated ballistic discharge.
resistances of a Wheatstone bridge to avoid the measurement of the resistance $R$. This circuit is known as the Maxwell commutator bridge.

Normally the reversal of current as outlined by Maxwell is not used, the condenser merely being charged and discharged. A circuit for this procedure is given in figure 1.2. The theory of the method does not appear to have been published even by Mascart and Joubert (1.3) who give the circuits equivalent to those of figures 1 and 2 and also that mentioned at the end of this section when dealing with leakage resistance. They do, however, state that if there is a total resistance $p$ in the circuit of figure 1.1, then the current will not be $2EC/T$, but $2EC \left[1 - \exp\left(-\beta/pC\right)\right]/T$ where $\beta$ is the contact time in both positions. As the theory of the method is required in later sections it is given in some detail below.

The commutator $Z$ alternately charges the condenser of capacitance $C$ from a battery of constant E.M.F. $E$ and discharges it through a moving coil galvanometer of resistance $G$. If there are $n$ charges and $n$ discharges of the condenser per second, then the galvanometer indicates a steady current $I$ given by $I = nEC$ provided certain conditions are fulfilled. These conditions will now be determined. It is assumed that there is negligible inductance in the circuit as this would be the case in practice.

If the condenser be charging, then, if at some instant of time $t$ the charge on the condenser is
q and the charging current is \( dq/dt \), by Kirchoff's second law:

\[
E - q/C - (p + r) dq/dt - (p + r)q/CR = 0
\]

where \( p \) is the internal resistance of the battery, \( r \) is the sum of the contact resistance of the commutator (assumed constant) and lead resistance, and \( R \) is the leakage resistance of the condenser.

This may be rewritten as:

\[
\frac{dq}{q - ECR/(p + r + R)} = \frac{-(p + r + R)dt}{CR(p + r)}
\]

On integrating and inserting the condition that the condenser was uncharged initially:

\[
q = \frac{ECR}{(R+p+r)} \left[ 1 - \exp \left[ -(p+r+R)t/CR(p+r) \right] \right]
\]

Thus, if \( t_1 \) is the contact time of the commutator in the charging position, the potential difference between the plates of the condenser at the instant charging ceases is \( V_1 \) given by:

\[
V_1 = \frac{ER}{(R+p+r)} \left[ 1 - \exp \left[ -(p+r+R)t_1/CR(p+r) \right] \right] \quad \cdots \quad (1)
\]

During the transit between the charging and the discharging position of the commutator the condenser will partly discharge through the condenser leakage resistance \( R \).

If the charge on the condenser is \( q \) after \( t \) seconds of leakage and the leakage current is \( dq/dt \) at the same instant then \( dq/q = -dt/CR \).
On integrating and inserting the condition that the charge on the condenser is \( CV_1 \) at the instant charging ceases and leakage commences,

\[
q = CV_1 \exp(-t/CR).
\]

Thus if \( t_2 \) is the transit time from the position where charging ceases to the position where discharge through the galvanometer commences, the potential difference between the plates of the condenser at the moment when discharge commences is \( V_2 \) given by:

\[
V_2 = V_1 \exp(-t_2/CR).
\]

During discharge the load across the condenser is \( S \) given by:

\[
S = \frac{R(q+G)}{(q+G+R)},
\]

where \( G \) is the resistance of the galvanometer and \( q \) is the sum of the contact resistance of the commutator (assumed constant) and lead resistance.

If \( t_3 \) is the contact time of the commutator in the discharge position, the charge left on the condenser at the end of discharge is \( CV_2 \exp(-t_3/CS) \). Thus the charge that has passed through the discharge system is

\[
CV_2\left[1 - \exp(-t_3/CS)\right].
\]

This process is repeated \( n \) times per second, therefore the quantity of electricity through the discharge system per second (that is the current through the discharge system) is \( nCV_2 \left[1 - \exp(-t_3/CS)\right] \). Thus the current through the galvanometer is \( I \) given by:

\[
I = \frac{R \cdot nCV_2\left[1 - \exp(-t_3/CS)\right]}{(q+G+R)}.
\]
On substituting for $V_2$ and $S$,

$$I = \frac{nECR^2}{(q+G+R)(R+p+G)} \left( 1 - \exp \left[ -\frac{(p+r+R)t_1}{CR(p+r)} \right] \right)$$

$$\times \exp \left( -\frac{t_2}{CR} \right) \left( 1 - \exp \left[ -\frac{(q+G+R)t_3}{CR(q+G)} \right] \right).$$

This equation reduces to the simple form $I = nEC$, initially quoted, if the following conditions hold:

1. $(p+r) \ll R$.
2. $CR(p+r) \ll (p+r+R)t_1$.
3. $t_2 \ll CR$.
4. $(q+G) \ll R$.
5. $CR(q+G) \ll (q+G+R)t_3$.
6. The galvanometer has a frequency of oscillation somewhat less than $n$.

The potential difference across the condenser during the cycle is given in figure 1.3. for circuit constants that do not satisfy conditions 1 to 5. With such constants the current indicated by a suitable galvanometer would be 0.875 nEC, the greatest reduction being produced by leakage. The individual effects of not satisfying the conditions are 0.990, 0.993, 0.905, 0.990 and 0.994 respectively.

The significance of the conditions is as follows:

1. The sum of the resistances in the charging circuit must be much less than the leakage
Potential difference across a condenser when $I \neq nEC$.

Circuit constants:

- $C = 0.1 \mu F$.
- $t_1 = 5 \times 10^{-3}$ sec.
- $(p+r) = 10^4$ ohms.
- $R = 10^6$ ohms.
- $t_2 = 10^{-2}$ sec.
- $t_3 = 5 \times 10^{-9}$ sec.
- $(e+G) = 10^4$ ohms.
resistance of the condenser. For an error of less than 0.1%,
$R > 999(p+r)$. As $R$ is normally of the order of $10^9$ ohms this
means that $(p+r)$ must be less than $10^6$ ohms approximately.
This condition is thus easily satisfied.

2. As $(p+r)\ll R$ this condition
reduces to $C(p+r)\ll t_1$, that is the time constant of the
charging circuit must be much less than the time available
for charging the condenser. Reference to tables of $\exp(-x)$,
(1.4.), shows that for an error of less than 0.1%,
$t_1/C(p+r) > 6.9$. The maximum values for $(p+r)$ are given in
table 1.1. for various values of $t_1$. This condition is not
too restricting, but it should be noted that a 120 volt H.T.
battery from stock checked on each tapping gave an average
internal resistance of 0.6 ohm per volt, so that for large
capacitances with short periods of charge a low value of the
E.M.F. $E$ is advisable when using dry batteries.

3. The transit time between charge
and discharge must be very much less than the time constant
of the condenser. For an error of less than 0.1%,
$t_2/CR < 0.001$. The minimum values of the leakage resistance
$R$ are given in table 1.2. for various values of $t_2$. These
minimum values are all less than the pass values claimed by
the manufacturers of, for example, metallised paper
condensers. Condensers from stock gave satisfactory values
when measured by ballistic discharge as described in section
2.

4. This condition, like the first,
<table>
<thead>
<tr>
<th>$t_1$</th>
<th>sec.</th>
<th>0.01</th>
<th>0.005</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>μF</td>
<td>1.0</td>
<td>0.1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.0</td>
<td>0.1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>725</td>
<td>724.5</td>
</tr>
<tr>
<td>$t_1$</td>
<td>sec.</td>
<td>0.001</td>
<td>0.000085</td>
</tr>
<tr>
<td>C</td>
<td>μF</td>
<td>1.0</td>
<td>0.01</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.0</td>
<td>0.01</td>
</tr>
<tr>
<td>Maximum value $E_0$ (per)</td>
<td>Dine</td>
<td>14.5</td>
<td>14.99</td>
</tr>
<tr>
<td></td>
<td></td>
<td>36</td>
<td>360</td>
</tr>
</tbody>
</table>

Table 1.1.

Maximum values for resistance in charging circuit.

<table>
<thead>
<tr>
<th>$t_2$</th>
<th>sec.</th>
<th>0.01</th>
<th>0.005</th>
<th>C:0.01</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>μF</td>
<td>1.0</td>
<td>0.1</td>
<td>0.01</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.0</td>
<td>0.1</td>
<td>0.01</td>
</tr>
<tr>
<td>Minimum value $E_0$ (per)</td>
<td>Dine</td>
<td>10$^7$</td>
<td>10$^7$</td>
<td>10$^6$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>10$^7$</td>
<td>10$^6$</td>
<td>10$^5$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>10$^7$</td>
<td>10$^6$</td>
<td>10$^5$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>10$^7$</td>
<td>10$^6$</td>
<td>10$^5$</td>
</tr>
</tbody>
</table>

Table 1.2.

Minimum values of condenser leakage resistance.
can be satisfied without difficulty.

5. As for condition 2, for an error of less than 0.1%, \( t_3(q+G+R)/CR(q+G) > 6.9 \) which, as \( (q+G) \ll R \), reduces to \( t_3/C(q+G) > 6.9 \) so that the figures for condition 2 given in table 1.1. apply here.

Although for small currents the meter resistance may be of the order of \( 3 \times 10^2 \) ohms, the requirement that \((q+G)\) shall be small only occurs when \( C \) is large, that is when the discharge currents are high and the meter has to be shunted and thus presents a much reduced resistance.

6. A series of discharge pulses of electrical energy passing through a galvanometer will only be indicated as a steady current if the period of the galvanometer is long compared with the time interval between pulses. Some vibration of the galvanometer needle or spot is, however, inevitable and the condition to be obeyed will depend upon what restriction is made to this vibration. If the natural frequency of swing of the galvanometer coil is \( F \) cycles sec.\(^{-1} \) and there are \( s \) divisions of the scale on either side of the zero, then, if the vibration is to be restricted to \( p \) divisions, a pulse must arrive approximately \( p/4Fs \) seconds after a previous one, otherwise the spot or needle will have moved more than \( p \) divisions before the arrival of the pulse. But there are \( n \) charges and \( n \) discharges per second so that the time between consecutive discharge pulses is, on neglecting transit times, approximately
the length of the discharge pulse, that is \( \frac{1}{2n} \) seconds. Thus \( \frac{1}{2n} \leq \frac{p}{4Fs} \) or \( F \leq \frac{np}{2s} \). If, for example, \( n = 50 \), \( p = 1 \), and \( s = 100 \) then the maximum frequency of the galvanometer would have to be less than or equal to 0.25 cycles sec\(^{-1}\).

(If the E.M.F. \( E \) be made high enough, a micro-ammeter is easily used with this circuit, but it is not recommended because (1) the scale divisions are too coarse for accurate readings to be taken, and (2) micro-ammeters have such a short period that blurring of the needle is almost inevitable unless \( n \) is raised to an inconvenient value. The difficulty can be overcome, to a certain extent, with a damping resistance across the meter. This solution is used in the anemometer described in section 5.)

The order of accuracy that can be expected for such a system as here described when the conditions to give \( I = nEC \) are fulfilled can be obtained in the standard manner (1.5).

Suppose that \( I \) can be measured to \( t_1 \), \( n \) to \( t_a \) and \( E \) to \( t_e \), then \( C = \frac{(I+i)}{(n+a)(E+e)} \). The two worst cases are \( C = \frac{(I+i)}{(n-a)(E-e)} \) and \( C = \frac{(I-i)}{(n+a)(E+e)} \) which may be written as

\[
C = \frac{(I+i)}{nE(1-a/n)(1-e/E)} \quad \text{and} \quad C = \frac{(I-i)}{nE(1+a/n)(1+e/E)}.
\]

As \( a/n \ll 1 \) and \( e/E \ll 1 \), if terms with a numerator of a product of two or more small quantities are neglected, \( C = I/nE \) within the limits \( t(eI/nE^2 + aI/n^2E + i/nE) \).
It is evident that the major term is that involving the indeterminacy of I, thus \( C = \frac{I}{nE} \) within the limits \( \pm \beta \) where \( \beta = \frac{i}{nE} \). As the maximum value of \( i \) is \((0.25 \times \text{scale division})\) the limits to which \( C \) can be measured can be calculated. If \( n \) be taken as 50 and \( E \) as 50 volts, and if it be assumed that the meter has 100 scale divisions and full scale readings of 25, 250 and 2500 micro-amperes are available by the use of appropriate shunts so that nearly full scale deflections are obtained, then the figures given in table 1.3. are obtained. The possible error is clearly not negligible even under these favourable circumstances.

Instead of determining the capacitance of a condenser in terms of \( n \), \( E \) and \( I \), it may be determined in terms of two currents and another condenser of known capacitance by substitution. If the conditions mentioned above are satisfied for both condensers, \( \frac{I_1}{I_2} = \frac{C_1}{C_2} \). Both \( n \) and \( E \) must remain constant, but their values need not be known. The method is less precise than the first mentioned as the indeterminacy of \( I \) appears in both the numerator and the denominator.

The difficulty of reading the current accurately when comparing capacitances could be overcome by using the circuit given in figure 1.4. A Rayleigh potentiometer (1.6.), consisting of two resistance boxes \( R \) and \( R' \) in series with their sum kept constant at \( P \) ohms, can be adjusted so that the current with both
Table 1.3.

Limits of error of measurement of the method for
n = 50, E = 50 volts and a meter with 100 divisions
and with full scale deflections for 25, 250 and
2500 micro-amperes.
Figure 1.4.

An alternative form of the circuit given in figure 1.2.
condensers is the same. Then \( I = nR_1E_1/P = nh_2E_2/P \) or
\[
C_1/C_2 = R_2/R_1.
\]

The precision of the determination can be obtained as follows. Suppose that the two settings of \( R_1 \) that can be separated are given by \( N_{1a} = nR_{1a}EC/P \) and
\[
N_{1b} = nR_{1b}EC/P
\]
where \( N_{1a} \) and \( N_{1b} \) are the galvanometer deflections. Then \( N_{1a} - N_{1b} = nEC_1(R_{1a} - R_{1b})/P \) or
\[
(R_{1a} - R_{1b})/R_1 = (N_{1a} - N_{1b})/N_1.
\]
For the greatest accuracy a near full scale deflection should be used and, as
\[
(N_{1a} - N_{1b}) \text{ is of the order of } 0.1 \text{ division at a scale mark, with } 100 \text{ divisions }
\]
\[
(R_{1a} - R_{1b})/R_1 = 1/1000.
\]

If two approximately equal capacitances are being compared then \( C_1/C_2 = X/Y \) and \( X \) and \( Y \) can be each determined to \( \pm a \). Then, by the method used above, \( C_1/C_2 = X/Y \pm 2a/Y \) which comes to \( X/Y \pm 0.002 \) with the potentiometer. With the same galvanometer, with near full scale deflections and reading to \( \pm 0.25 \) division the relation is, with the simple substitution method,
\[
C_1/C_2 = X/Y \pm 0.005.
\]

This theory has been investigated (section 2) and, although the potentiometer method is slightly lengthier, it is superior. The improvement with a good reflecting galvanometer is less than the theory indicates as the scale readings can be estimated to rather better than \( \pm 0.25 \) division.

P must be kept low with condensers of large capacitance to ensure complete charge. This means
that accumulators should be used as the source of E.M.F.
with the advantage of a steady value of E. Such use need not
be inconvenient because any sensitive galvanometer can be
used as calibration is not necessary.

According to Glazebrook (1.7.), a feature of the repeated ballistic discharge method of
determining capacitance is that it can be used to determine
whether or not a condenser has a satisfactory leakage
resistance value. It will now be shown that this is only
true in certain circumstances.

Let the condenser have a leakage resistance R such that \( \exp(-t_2/CR) \) cannot be considered as
unity, but that all other conditions are satisfied. The
current through the galvanometer in the discharge circuit
will be \( I_d \) given by \( I_d = nEC\exp(-t_2/CR) \).

The galvanometer must now be
placed in the charging circuit and the discharge circuit
completed by a wire. It is evident from equation (1) on
page 14 that the charge that passes into the condenser and
thus through the galvanometer in time \( t_1 \) is \( q_1 \) given by

\[
q_1 = \frac{ECR}{(R+p+r+G)} \left[ 1 - \exp \left( \frac{(p+r+R+G)t_1}{CR(p+r+G)} \right) \right]
\]

if the condenser is uncharged initially.

This occurs \( n \) times per second,
therefore the galvanometer indicates a current \( I = nq_1 \).

But \( p+r+G \ll R \), \( C(p+r+G) \ll t_1 \), and
\( CCR/(q-R) \ll t_2 \) so that the current through the galvanometer
in the charging circuit will be $I_C = nEC$.

Thus $I_C - I_d = nEC \left[ 1 - \exp(t_2/CR) \right]$.

If $t_2/CR$ is still small, even though $R$ is not at the pass value, then on expanding and neglecting higher order terms $I_C - I_d = nEt_2/R$. Thus $R = nEt_2/(I_C - I_d)$.

It appears, then, that, as all the values on the right hand side may be determined, $R$ can be calculated. Also that if $I_d$ is found to be equal to $I_C$ then the minimum leakage resistance that must exist can be determined as was done when considering condition 3 above.

If, however, $(I_C - I_d)/I_C = t_2/CR$ be calculated for a particular value of $t_2$, it is found that the method can be difficult to apply. In table 1.4., $(I_C - I_d)/I_C \times 100\%$ is given for various values of $R$ for various values of $C$, $t_2$ being taken as 0.01 seconds, a reasonable figure for the transit time between charge ending and discharge commencing when $n = 50$.

It is noticeable that for higher values of capacitance the leakage resistance has to be very low to be detectable and could not be calculated with any pretense of accuracy until lower still.

In this modification and in the original circuit it is possible for the coil of the galvanometer to be put directly across the battery if a fault develops in the commutator such that the charge and discharge contacts touch. However, in this modified circuit a condenser with an internal short circuit would also put
Table 1.4.

Percentage change in current for various values of leakage resistance.

<table>
<thead>
<tr>
<th>R</th>
<th>ohms</th>
<th>$10^6$</th>
<th>$10^7$</th>
<th>$10^8$</th>
<th>$10^9$</th>
<th>$10^{10}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>$1.0 \mu F$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$I_e - I_o$</td>
<td>$%$</td>
<td>1.0</td>
<td>0.1</td>
<td>0.01</td>
<td>0.001</td>
<td>0.0001</td>
</tr>
<tr>
<td>C</td>
<td>$0.1 \mu F$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$I_e - I_o$</td>
<td>$%$</td>
<td>10</td>
<td>1.0</td>
<td>0.1</td>
<td>0.01</td>
<td>0.001</td>
</tr>
<tr>
<td>C</td>
<td>$0.01 \mu F$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$I_e - I_o$</td>
<td>$%$</td>
<td>/</td>
<td>10</td>
<td>1.0</td>
<td>0.1</td>
<td>0.01</td>
</tr>
</tbody>
</table>
the moving coil across the battery. Because of this the circuit is not recommended for the measurement or comparison of capacitance, even though it does avoid any difficulties regarding leakage resistance.

It was mentioned in the introduction that the values of capacitance obtained by the method described are unreliable because of absorption. It seems sufficient to say here that, if absorption is present, when the charging voltage is applied across the condenser under test, although the greater part of the charge enters according to the theory given above, a small extra quantity moves in slowly and continuously for a considerable time as long as the charging voltage is maintained. This extra, absorbed, charge has not been allowed for nor, at the moment, can it be (1.3.).

At discharge most of the charge emerges according to the theory given. This is known as the free charge. The absorbed charge continues to emerge at a slow and decreasing rate for a long time provided the discharge path is maintained.

These facts were originally discovered by Gaugain (1.9.) and Siemens (1.10), and, because of them, it follows that different values for capacitance can be obtained for a particular condenser if different times for discharge or charge are used. This has been investigated by many workers (1.6.), but is now overcome by the use of alternating current bridges where absorption forms part of the power factor term.
In this work it has been found that, in most applications and to the accuracy accepted, absorption does not give rise to difficulties.
SECTION 2.

AN HISTORICAL SURVEY, TOGETHER WITH
SEVERAL INVESTIGATIONS ARISING FROM IT.
The Maxwell Bridge method, mentioned in section 1, has been used for frequency measurements and for the maintenance of constant speeds by the National Physical Laboratory (1.6.), but, apart from fairly recent valve circuits where it has been used to measure frequency, the method of repeated ballistic discharge has been used only for the determination of capacitance. For this purpose the method has been used by many workers, almost without exception when determining the ratio of electrostatic to electromagnetic units.

Before commencing work on the circuits described later, it was decided to investigate the various methods used to switch the condenser from charge to discharge. A brief account of these methods follows.

Gray (2.1.) and Kohlrausch (2.2.) give brief accounts of the method, which they attribute to W. Siemens. Gray states that for small values of capacitance the charge and discharge times may be as low as $5 \times 10^{-5}$ seconds, while Kohlrausch notes that the smaller the capacitance the faster the switching device may be operated. No details are given of the actual switching devices used.

Siemens' paper on the method (2.3.) considerably precedes the first edition of Maxwell's "Electricity and Magnetism", so that it appears that Siemens should be credited with the original idea. As a commutator Siemens used a modified telegraphic instrument, the principle of which is shown in figure 2.1. A and B were poles of an
electro-magnet, energised when the contacts C and D made. This caused a bar of soft-iron E, which was fastened to an axle, and the long arm F, similarly fastened, to rotate. At each end of the arm F were hemispheres running over a knife edge, (shown side view in inset), so that power was required for the change in position of the arm F, and so that firm contact was made without bounce. The return to the original position was accomplished by the energy stored in a spring G. The contacts forming the part Z of figure 1.2. were H, I and J. The number of charges and discharges per second, \( n \), was determined by the use of a clock together with the indicator K which recorded each clockwise rotation of F. The regulation of \( n \) was not very good as may be seen by the figures quoted by Siemens:

No. of oscillations per second: 60.25, 60.41, 60.12.

A report has been traced of a lecture by C.W. Siemens (2.4.) in which he described an alternative form of commutator. It is only briefly mentioned, and apparently consisted of a rotating switch driven by "an electrical appliance" which produced "a regular succession of charges and discharges at rapid intervals". No details are given as to how the frequency was maintained constant, nor how it was determined.

The earliest, rotating commutator of which full details are available is that of Stoletow (2.5.). The commutator was driven by a small Helmholtz electric motor. A chronograph and a clock gave the number of rotations per minute, and each rotation produced six charges and six
Figure 2.1.

Simplified diagram of Siemens' "Wippe".
discharges. As may be seen from figure 2.2., the commutator consisted essentially of a bar of insulating material (shown hatched), rotating about its centre. From the bar were suspended metal needles which dipped into mercury pools contained in a block of insulating material. The outer two needles dipped into pools arranged in two broken rings while the centre two needles dipped into two complete rings. Each outer needle was connected electrically to its neighbouring inner needle. As the E.M.F. used was only of the order of two volts there was no difficulty with mercury vapour.

The commutator acted as the switch shown in the Maxwell circuit of figure 1.1., but Stoletow may have been unaware that Maxwell had already put forward the circuit as his paper implies that a certain isolation resulted from working in Moscow.

Fleming and Clinton (2.6.,2.7.) used a commutator based on the secohmmerter devised by Ayrton and Perry (2.8.). The secohmmerter was a commutator device driven either by hand or by motor. It supplied a bridge circuit from a battery with what was effectively a square-wave alternating current, and also rectified the out-of-balance current in the galvanometer branch, so that a direct current galvanometer could be used as a detector. It has been shown by the author that two double changeover relays can be used for the same purpose in a simple bridge circuit for the comparison of capacitance (2.9.).

Figure 2.3. shows diagrammatically what the Fleming and Clinton commutator looked like when
Figure 2.2.

Simplified diagram of Stoletow's commutator.
Figure 2.3.

Diagram of the Fleming and Clinton commutator.
seen from above. It consisted of three gunmetal, toothed discs A, B, and C rigidly attached to a driving shaft D but insulated from one another by ebonite bushes and washers. As the shaft rotated, a brass-gauze brush E made contact with the two outer discs alternately. The outer discs were in continuous contact with two further brass-gauze brushes F and H. The centre disc supported the brush E when no contact tooth was under it.

The value of n was kept constant by examining, through slits in plates attached to the prongs of a vibrating tuning-fork, a stroboscopic disc attached to the shaft. The pattern was kept steady by varying a frictional force on the drive. Knowing the number of teeth on the outer discs, n was determined by the use of a clock and a mechanism on the shaft which caused a gong to strike after every hundred revolutions of the shaft. It appears that n was usually 60.

An alternative to the rotating commutator, or the "wippe" of Siemens, is a vibrator of the type which is sometimes found in undergraduate teaching laboratories, and which is shown in figure 2.4.(a). When an alternating current is passed through the coil A, a thin steel rod B, clamped at C, is longitudinally magnetised with a polarity which reverses as the current changes sign. The interaction with the field of a permanent magnet causes alternate attraction and repulsion of the vibrator between the poles of the magnet. The steel rod vibrates with an
Figure 2.4.

Diagrams of two alternatives for a commutator.
amplitude which depends upon (i) the field strength, due to the permanent magnet, in the region of the steel rod, (ii) the current through the solenoid and the geometry of the solenoid/rod system, (iii) the closeness of the natural frequency of the rod, acting as a cantilever, to the frequency of the alternating current.

The originator of this apparatus has not been traced, but, as it has serious disadvantages, this is not of great concern. The disadvantages are (i) the high current required to drive it, (ii) the need for the delay time between charge and discharge to be large in order to avoid short circuiting the charge and discharge contacts, (iii) the likelihood of contact bounce.

Some early work on the circuit was carried out with an electrically maintained tuning-fork to the prongs of which were attached styli which dipped into pools of mercury. Klemencič, for example, used this technique (2.10.), although Thomson in 1870 (2.11.) had found it impossible to obtain reliable contact.

In figure 2.4.(b), is shown a contact breaker designed by Thomson and used by him in Maxwell's commutator bridge method (2.12.). It was Thomson who first used a single vibrator in the bridge in contrast to the double changeover suggested by Maxwell. An electrically maintained tuning-fork with a vibrating contact was used as a means of driving the breaker synchronously, and the system gave a better and longer contact than did a stylus attached
directly to a fork. A current passed first through the tuning-fork interrupter, and then through the coils A of an electro-magnet; D was a strip of brass with a piece of soft iron B attached to it. When there was no current passing through the electro-magnet the elasticity of the strip D made it press against the screw F. When the current passed through the electro-magnet, the magnet attracted the iron B and brought it into contact with the stop C. All the places where electrical connection was made were covered with platinum.

A contact breaker of this type was also used with the Maxwell bridge by Glazebrook (2.13.) who actually used the same instrument as Thomson; and by Fleming and Dewar (2.14.) in the continuous ballistic discharge method.

According to Glazebrook the system was troublesome in practice and, as there was an uncertainty as to the actual duration of contact, it was impossible to determine by calculation whether the condenser under test was being fully charged and discharged. It is interesting to note, however, that Rosa (2.15.) found the commutator and fork system of Thomson satisfactory for the Maxwell bridge with a fork of frequency 32, but that there was insufficient uniformity with a fork of frequency 130. To overcome this, Rosa reverted to the use of a stylus as indicated in figure 2.5. As the prongs of the electro-magnetically maintained tuning-fork separated, B closed and the condenser charged.
Figure 2.5.

Diagram of Rosa's tuning-fork switch.
When the prongs came together A closed and the condenser discharged.

Later, when working with Grover (2.16.) and then with Dorsey (2.17.), Rosa used a rotating commutator both with the Maxwell bridge, and with a modified form of the continuous ballistic discharge method employing a differential galvanometer. A diagram of part of the improved commutator used by Rosa and Dorsey is given in figure 2.6., together with a simple circuit to show the connections.

The instrument was constructed of ebonite with the outer contacts, A, of phosphor bronze. The outer contacts were connected electrically to the segments of the brass ring B; the segmentation being to improve the insulation of the system. Brush-copper brushes were used at C and D, clamped between springs and held by two screws to cut down vibration. It will be noticed that the condenser was air insulated.

Rotation was clockwise, \( n \) was approximately 300, the normal revolution rate was 1200 - 1700 r.p.m. and a special chronograph recorded every fiftieth revolution. The velocity of revolution was constant during experimental runs to a few parts in 100,000 and was adjusted either by a carbon rheostat in the motor circuit, or by friction applied to the fly-wheel rim.

By the time Rosa and Grover carried out their work all the main workers in the field had changed
Figure 2.6.

Simplified diagram of the Rosa and Dorsey commutator.
over to revolving commutators when using the Maxwell bridge. They included Thomson and Searle (2.18.), Glazebrook (2.19, 2.20.), Himstedt (2.16.) and Abraham (2.16.). Diagrams of their commutators are not given because those of Fleming and Clinton and of Rosa and Dorsey sum up the two variations in design and the consistency of n obtained by Rosa and Dorsey was the best recorded.

A paper was published by Jason (2.21.), while this work was being carried out, in which he described the use of a small, high speed, electro-magnetic relay driven by an alternating current, as a replacement for a vibrator or a commutator. The circuit used was that of figure 1.2., the relay in the rest position allowing the condenser to charge. The operation of the relay, which will occur each half cycle of the alternating current, causes the condenser to discharge through a moving-coil micro-ammeter.

A brief account of the principle of the method is given in the paper, but the formula for the current through the meter is incorrectly given as

$$ I = \frac{nE_0 \cdot \exp(-a/nCR)}{G} \int_{t_0}^{t} \exp \left[ \frac{-t(G+R)/CR}{G} \right] dt $$

where $n = \text{no. of discharges and number of charges per second.}$

$\frac{a}{n} = \text{transit time between charge and discharge.}$

$\frac{b}{n} = \text{discharge time.}$

The formula should read

$$ I = \frac{nE_0 \cdot \exp(-a/nCR)}{G} \int_{t_0}^{t} \exp \left[ \frac{-t(G+R)/CG}{G} \right] dt $$

The formula neglects charge time,
contact resistance and battery internal resistance so that
the conditions for giving $I = nE_0C$, though correct ($nCG \ll (1+G/R)$; $G \ll R$; $a \ll nCR$), are not the only conditions that
have to be satisfied.

The most convenient commutator is
undoubtedly that used by Jason, since it requires a low
current to drive it, has a positive contact with no bounce
if adjusted correctly, and is readily available at low cost.
A high speed relay can be obtained from Siemens Bros. & Co.
Ltd., Woolwich. There are several types, but the approximate
performance figures used in this discussion are for the
standard unit type 73 (2.22.). A diagram of this relay is
given in figure 2.7.

The fixture A carries a phosphor
bronze spring B which is channelled for stiffness at C, and
carries platinum contacts D. Welded to this spring is a soft
iron armature E which butts against the soft-iron core F.
The spring B passes through a slot in a buffer spring G and
a tension adjusting screw H bears against it. Normally the
contact D is against the contact screw I, but, when the relay
is energised on the passage of current through the coil J,
the contact D changes over to press against contact K.

A close agreement between the
complete operating time and the complete releasing time is
achieved by the fact that the make contact is closed before
the armature E reaches the pole face of the core. The load
is then considerably increased, owing to the channelled
Figure 2.7.

Diagram of the Siemens type 73 relay.
armature spring being slightly bent, and storing energy sufficient for a high release velocity.

If the relay is driven by a sinusoidal alternating current of frequency \( f \), then it will be energised twice per cycle in the manner indicated in figure 2.8.

If the time origin be taken as zero when the current through the relay is zero, the instantaneous current at time \( t \) is given by \( i = I \sin(2\pi ft) \), where \( I \) is the maximum current that passes through the relay coil. Let \( pI \) be the value of the energising current at which the relay armature starts to move towards the make position, and let \( a \) be the time the armature takes to reach the make position after starting. Let \( qI \) be the value of the energising current at which the relay armature is released and let \( b \) be the time elapsing between release and return to the initial condition. Then the charging and discharging times are \( t_c \) and \( t_d \) given by

\[
\begin{align*}
t_c &= \frac{1}{2f} - \frac{\sin^{-1}p}{2\pi f} - \frac{\sin^{-1}q}{2\pi f} - a \\
t_d &= \frac{\sin^{-1}q}{2\pi f} + \frac{\sin^{-1}p}{2\pi f} - b
\end{align*}
\]

But \( p = q \), and \( t_c = t_d \), therefore

\[
\frac{1}{2f} - \frac{\sin^{-1}p}{\pi f} - a = \frac{\sin^{-1}p}{\pi f} - b
\]

The type 73 relay has \( a = b \), so

\[
\frac{\sin^{-1}p}{\pi} = \frac{1}{4}.
\]

Some values for the charging/discharging times at various frequencies are given in table 2.1., but it is evident that the charging time is zero when
Figure 2.8.

Operation of a relay when energised by a sinusoidal alternating current.
Table 2.1.

Effect of frequency on charging/discharging times.
(The transit time is taken as $10^{-3}$ seconds.)

<table>
<thead>
<tr>
<th>$f$</th>
<th>cycles/sec</th>
<th>15</th>
<th>50</th>
<th>100</th>
<th>200</th>
<th>250</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t_c = t_d$</td>
<td>$x 10^{-3}$ sec.</td>
<td>15.7</td>
<td>4</td>
<td>1.5</td>
<td>0.25</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 2.2.

Maximum frequency for various values of $iG$ for two values of $E$. (The transit time is taken as $10^{-3}$ sec.)

<table>
<thead>
<tr>
<th>$iG$</th>
<th>Volts</th>
<th>$10^{-3}$</th>
<th>$10^{-2}$</th>
<th>$10^{-1}$</th>
<th>$4 \times 10^{-1}$</th>
<th>1</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E = 6$ volts</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Max. $f$</td>
<td>cycles/sec</td>
<td>250</td>
<td>244</td>
<td>192</td>
<td>20</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>$E = 60$ volts</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Max. $f$</td>
<td>cycles/sec</td>
<td>250</td>
<td>260</td>
<td>244</td>
<td>227</td>
<td>192</td>
<td>20</td>
</tr>
</tbody>
</table>
\[ f = \frac{1}{4b}. \] This means that if \( b = 10^{-3} \) seconds as stated by Siemens Bros. Ltd., then the maximum frequency at which the system must fail is 250 cycles sec\(^{-1}\). Jason found that the circuit operated up to 300 cycles sec\(^{-1}\), so it must be possible to shorten the operating time of the relay even further.

The time \( b \) has been determined using known resistances \( R \) across the condenser as a leakage path during the time \( b \). As \( I_d = nEC \exp\left(-\frac{b}{CR}\right) \), plotting \( \log I_d \) against \( 1/R \) gives a straight line from the slope of which \( b \) may be determined if \( C \) is known. The natural leakage resistance of the condenser has been neglected here, but, as is shown in section 6, its inclusion does not affect the calculation.

In figure 2.9, are given the graphs for two adjustments of the relay. The first is a normal adjustment with a small gap and a good note. The second adjustment is highly critical and slightly unsteady and was obtained by adjusting the relay until a particular value of \( I_d \) was at its maximum.

It will be seen that a transit time of \( 10^{-4} \) seconds is possible, but that a reasonable minimum value is of the order of \( 5 \times 10^{-4} \) seconds. Thus a frequency of 500 cycles sec\(^{-1}\) gives zero charging time and the charging/discharging times at 300 cycles sec\(^{-1}\) would be approximately \( 4 \times 10^{-4} \) seconds. As may be seen from table 1.1., the resistance in the charging circuit should be carefully
Log$_e$(I$_d$) plotted against 1/R for two adjustments of a relay. The capacitance of the condenser in each case was 0.05 μF.
considered with larger capacitances with such short charging times.

An expression will now be derived for the maximum frequency which can be used with a particular galvanometer in terms of the transit time and the E.M.F. of the battery if an error of less than 0.1% is required.

For a discharge time \( t \) and a capacitance \( C \) the maximum discharge resistance for a 0.1% error is given by \( t > 6.9 CR \). But \( t = (1/4f - a) \), so that \( f < 1/[4(6.9 CR + a)] \).

If \( G \) is the resistance of a galvanometer giving a full scale deflection for a current \( i \), then the shunt value \( S \) for a full scale reading with a current \( I \) is given by \( i/I = S/(G + S) \) and the equivalent resistance of the galvanometer and shunt in parallel is \( GS/(G + S) \). The resistance in the discharge circuit will be very closely given by \( GS/(G + S) \). Further \( GS/(G + S) = Gi/I \) and, as there are two charges and two discharges per cycle, \( I = 2fEC \). Thus it follows that \( f < 1/[4(6.9 GI/2fE + a)] \) or \( f < (1 - 13.8 GI/E)/4a \). If \( a \) be taken as \( 10^{-3} \) seconds then the maximum value of \( f \) is given by \( f = 250(1 - 13.8 GI/E) \) cycles sec\(^{-1} \).

In table 2.2 are given the maximum values of \( f \) for various values of \( iG \) for two values of \( E \). In the work to be mentioned below, the galvanometer used had \( G = 350 \) ohms and \( i = 5.10^{-6} \) amperes, so that it was perfectly satisfactory for use with \( f = 50 \) cycles sec\(^{-1} \).
It was mentioned in section 1 that one means by which the accuracy of the Siemens method might be improved when used for the comparison of capacitance is to use a Rayleigh potentiometer across the source of the E.M.F. and to charge the condensers from part of the potentiometer as shown in figure 1.4.

It was shown that \( \frac{C_1}{C_2} = \frac{R_2}{R_1} \) where \( R_1 \) and \( R_2 \) are the values of the resistance \( R \) across which the potential differences are developed from which the two condensers charge in turn to give the same galvanometer deflection in the discharge circuit.

To determine whether or not the modification was an improvement, the capacitances of some paper condensers were compared with a Wien (de Sauty) alternating current bridge (2.23.). The capacitances were then compared by the normal Siemens method using a high speed relay driven by the A.C. mains through a 6.3 volt filament transformer, as a commutator. To obtain maximum accuracy a galvanometer was shunted and the value of the shunt adjusted for a near maximum deflection for each pair of condensers. The results obtained did not agree with those obtained with the Wien bridge so the shunted galvanometer was calibrated. When the readings were corrected agreement was obtained between the two methods within \( +0.2\% \), which is the limit imposed by the 0.1% resistance boxes used.

(With the smaller capacitances difficulty was found in obtaining a very definite minimum with the Wien bridge so the comparison was repeated using...
the series resistance bridge as recommended by Hague (2.24.).

The capacitances were then compared using the modified system that has been outlined. An unshunted galvanometer was used, the E.M.F. being increased as the capacitance was decreased. The agreement of the results by this method with the results obtained with the Wien bridge was very slightly better than in the previous experiment as may be seen from table 2.3. The point made in section 1 is thus confirmed.

With the development of the triode valve the method of continuous ballistic discharge has been used for the measurement of frequency. The use of thermionic valves removes such difficulties that arise with mechanical make and break systems as wear of, and dirt on contacts. Also very much less power is taken from the frequency source, but ancillary circuit complications have to be considered. The original paper was by Guarnaschelli and Vecchiachi (2.25.) who used the system in a direct reading frequency meter over the range 20 - 10,000 cycles second⁻¹. The system has also been used by Wilkie (2.26.) in a recording water velocity meter.

As the original paper is short and uncommunicative it was thought necessary to investigate certain aspects of the system. The basic circuit used is given in figure 2.10., where the triode valves are shown as indirectly heated for the sake of clarity. The transformer secondaries are connected 180° out of phase with each other.
<table>
<thead>
<tr>
<th>Siemens' method</th>
<th>Siemens' method</th>
<th>Modified Siemens' method</th>
<th>Wien bridge</th>
<th>Series bridge</th>
<th>Nominal values</th>
<th>pF</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uncalibrated galvanometer</td>
<td>Calibrated galvanometer</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.979</td>
<td>0.978</td>
<td>0.977</td>
<td>0.976</td>
<td>0.976</td>
<td>0.1/0.1</td>
<td></td>
</tr>
<tr>
<td>0.989</td>
<td>0.989</td>
<td>0.936</td>
<td>0.936</td>
<td>0.936</td>
<td>..</td>
<td></td>
</tr>
<tr>
<td>0.555</td>
<td>0.536</td>
<td>0.536</td>
<td>0.536</td>
<td>0.536</td>
<td>0.05/0.01</td>
<td></td>
</tr>
<tr>
<td>0.499</td>
<td>0.476</td>
<td>0.476</td>
<td>0.476</td>
<td>0.476</td>
<td>..</td>
<td></td>
</tr>
<tr>
<td>1.054</td>
<td>1.052</td>
<td>1.056</td>
<td>1.057</td>
<td>1.057</td>
<td>0.01/0.01</td>
<td></td>
</tr>
<tr>
<td>0.945</td>
<td>0.940</td>
<td>0.940</td>
<td>0.941</td>
<td>0.940</td>
<td>..</td>
<td></td>
</tr>
<tr>
<td>0.335</td>
<td>0.880</td>
<td>0.879</td>
<td>0.879</td>
<td>0.879</td>
<td>..</td>
<td></td>
</tr>
<tr>
<td>0.884</td>
<td>0.880</td>
<td>0.876</td>
<td>0.877</td>
<td>0.876</td>
<td>..</td>
<td></td>
</tr>
<tr>
<td>0.90(7)</td>
<td>0.90(3)</td>
<td>0.88(4)</td>
<td>0.88</td>
<td>0.88(2)</td>
<td>0.001/0.001</td>
<td></td>
</tr>
<tr>
<td>0.97(0)</td>
<td>0.96(0)</td>
<td>0.96(1)</td>
<td>0.97</td>
<td>0.96(4)</td>
<td>0.0002/0.0002</td>
<td></td>
</tr>
</tbody>
</table>

Table 2.3.

Ratios of capacitances obtained with Siemens' method, a modified Siemens' method, a Wien (de Sauty) A.C. bridge and a series resistance A.C. bridge.
Figure 2.10.
Basic thermionic valve circuit.

Figure 2.11.
Effect of phasing circuits.
so that, as the grid of $V_1$ goes positive with respect to the cathode, the grid of $V_2$ goes negative with respect to its cathode and vice versa. Thus during one half-cycle of the alternating current in the transformer primary, $V_1$ is conducting and $V_2$ is not and the condenser $C$ charges. During the other half-cycle, $V_2$ is conducting and $V_1$ is not and the condenser discharges through the meter. Providing the conditions mentioned in section 1 are satisfied, the current indicated by the meter will be $I$ given by $I = nEC$ and thus $C$ may be determined if $n$ and $E$ are known.

In practice, the first difficulty encountered was that both valves conducted together for part of the cycle. This was due to the fact that the potential difference between the anode and cathode of, for example, $V_2$ was sufficient to cause anode current to flow in $V_2$ even though the grid was negative with respect to the cathode while the condenser was charging.

Phase-shifting circuits in one or other grid circuit are of no help in avoiding this as may be seen from figure 2.11. At A both grids are negative, which is satisfactory, but at B both grids are positive which means that both valves would be conducting.

Any form of circuit to suppress part of the grid-voltage sine wave, for example that given by Brainerd (2.27.) and shown in figure 2.12, where the bias value is so high that current flows through the diodes only at the peaks of the voltage waves, is unsatisfactory for
Figure 2.12.
Pulse clipping circuit.

Figure 2.13.
Practical thermionic valve circuit.
three reasons: (i) it overelaborates the circuit, (ii) it is advisable to keep the delay time \( t \) between charge and discharge short, (iii) it is not advisable with thermionic valves to decrease the time available for charge and discharge because, as will be shown, the effective resistance of a valve is high and complete charge and discharge are difficult to obtain with capacitances above 0.1 \( \mu \)F.

The method adopted was to permanently bias both valves so that each triode did actually cease to operate just at the change-over. The circuit is given in figure 2.13 where \( R_1 \) and \( R_2 \) are 90 ohm resistances used to examine the charge and discharge current wave-forms with a cathode-ray oscilloscope. The bias on the two valves was adjusted until the charging current wave appeared on the cathode-ray oscilloscope at the previously determined crossover point of the grid wave-form and similarly for the discharging current wave. The bias can be set rather less reliably by adjusting it until no current is indicated by the meter when no condenser is present.

In order to determine the transit time \( t \) between charge and discharge, artificial leaks of known value \( R \) were placed across the condenser and the discharge current noted in the same way as was described earlier. The graph of \( \log_{10}(I/I_D) \) against \( 1/R \) for the lowest value of \( t \) obtained is given in figure 2.14, from which it can be seen that \( t \) was 1.8 milliseconds.

The use of the valves presupposes
$\log_{10}(\frac{I}{I_L})$ plotted against $1/R$ for $C = 0.05\mu F$. 

Figure 2.14.

From slope $t = 1.2 \times 10^{-3}$ sec.
that the charging and discharging conditions can be satisfied. That is that the time constant of the charging circuit is very much less than the charging time and that the time constant of the discharging circuit is very much less than the time available for discharge. As the charging and discharging times are the same at just less than 0.01 seconds, for 0.1% accuracy the maximum resistance in the charging and discharging circuits is as given in Table 1.1. Evidently no hard valve has a low enough resistance to satisfy the requirements for a 1\mu F condenser (that is 1449 ohms) and gas-filled triodes (thyatrons) are excluded because they would extinguish before the condenser was fully discharged (e.g. the Osram, argon filled, type GT1C extinguishes with a potential difference of 16 volts between anode and cathode.) The best that can be done is to choose the type with the lowest effective resistance. This type is the output triode. (With a calibrated frequency meter incomplete charge and discharge do not, of course, matter.)

The experimental work was carried out with two-volt, directly-heated, output triodes type LP 2 (Marconi-Osram.) These valves have a rather high resistance as may be seen from Figure 2.15., where the resistance of such a valve (with the grid connected to one side of the filament by 10^6 ohms) is plotted against the current through the valve over part of its working range. This type of valve is convenient, however, in that it permits the use of 6.3 volt filament transformers to feed the grids, the cut off
Figure 2.15.

Resistance of a type LP 2 valve plotted against the current through the valve.
bias value being only ~3 volts.

Tubular paper condensers were used to test the circuit and their leakage resistance was separately determined by ballistic discharge. That is, the condenser was charged to $q$ and immediately discharged through a ballistic galvanometer to give a deflection $\beta_1$. The condenser was then recharged to $q$, allowed to leak through its leakage resistance $S$ for $t$ seconds and then discharged to give a deflection $\beta_2$. Then $t/CS = \log_e(\beta_1/\beta_2)$, from which $S$ was calculated. This method, probably due to Siemens, gave, as may be seen from table 2.4., a leakage resistance in all cases of greater than $10^9$ ohms. This meant that, as the charge and discharge times are equal at $t$, and the resistance $R$ in the charge and discharge circuits is that of the valves, the discharge current given by a meter of suitable period would be $I_0$ given by

$$I_0 = nEC \left[1 - \exp(-t/CR)\right]^2.$$  

If $CR \ll t$, then the current indicated by the meter would be $I_t$ given by $I_t = nEC$.

In figure 2.16, $I_0/I_t$, that is

$$\left[1 - \exp(-t/CR)\right]^2,$$

is plotted against $C$ for three values of $R$ which approximate to the values for the resistance of the type LP 2 valve as given in figure 2.15., the charging and discharging times being taken as 0.01 seconds. In the same figure are also plotted the experimental values obtained and listed in table 2.5. These results were obtained in the following way. The currents for individual condensers were measured and the sum taken as the current that should be
<table>
<thead>
<tr>
<th>B₁ divisions</th>
<th>B₂ divisions</th>
<th>S \times 10^9 \text{ ohms}</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.5</td>
<td>1.6</td>
<td>1.24</td>
</tr>
<tr>
<td>11.1</td>
<td>9.3</td>
<td>1.99</td>
</tr>
<tr>
<td>12.7</td>
<td>8.6</td>
<td>1.54</td>
</tr>
<tr>
<td>12.7</td>
<td>9.9</td>
<td>1.85</td>
</tr>
<tr>
<td>13.7</td>
<td>10.1</td>
<td>1.96</td>
</tr>
<tr>
<td>12.3</td>
<td>8.6</td>
<td>1.68</td>
</tr>
<tr>
<td>12.2</td>
<td>8.3</td>
<td>1.56</td>
</tr>
</tbody>
</table>

Table 3.4.

Leakage resistances of paper condensers.

(\( C = 0.1 \mu F; \ t = 60 s \). )
Theoretical curves of $I_0/I_t$ plotted against $C$, together with the experimental results obtained.
<table>
<thead>
<tr>
<th>Nominal value of capacitance</th>
<th>$I_t$</th>
<th>$I_o$</th>
<th>$I_o/I_t$</th>
<th>Nominal value of capacitance</th>
<th>$I_t$</th>
<th>$I_o$</th>
<th>$I_o/I_t$</th>
<th>Nominal value of capacitance</th>
<th>$I_t$</th>
<th>$I_o$</th>
<th>$I_o/I_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu F$</td>
<td>mA</td>
<td>mA</td>
<td>Av</td>
<td>$\mu F$</td>
<td>mA</td>
<td>mA</td>
<td>Av</td>
<td>$\mu F$</td>
<td>mA</td>
<td>mA</td>
<td>Av</td>
</tr>
<tr>
<td>0.1</td>
<td>840</td>
<td>0.1</td>
<td>250</td>
<td>0.1</td>
<td>240</td>
<td>0.1</td>
<td>242</td>
<td>0.1</td>
<td>242</td>
<td>0.1</td>
<td>242</td>
</tr>
<tr>
<td>0.05</td>
<td>180</td>
<td>0.1</td>
<td>237</td>
<td>0.1</td>
<td>238</td>
<td>0.1</td>
<td>238</td>
<td>0.1</td>
<td>238</td>
<td>0.1</td>
<td>238</td>
</tr>
<tr>
<td>0.15</td>
<td>360</td>
<td>0.1</td>
<td>249</td>
<td>0.1</td>
<td>249</td>
<td>0.1</td>
<td>249</td>
<td>0.1</td>
<td>249</td>
<td>0.1</td>
<td>249</td>
</tr>
<tr>
<td>0.1</td>
<td>234</td>
<td>0.1</td>
<td>739</td>
<td>0.2</td>
<td>474</td>
<td>0.1</td>
<td>946</td>
<td>0.4</td>
<td>946</td>
<td>0.1</td>
<td>946</td>
</tr>
<tr>
<td>0.2</td>
<td>481</td>
<td>0.1</td>
<td>734</td>
<td>0.1</td>
<td>240</td>
<td>0.1</td>
<td>240</td>
<td>0.1</td>
<td>240</td>
<td>0.1</td>
<td>240</td>
</tr>
<tr>
<td>0.1</td>
<td>244</td>
<td>0.1</td>
<td>234</td>
<td>0.1</td>
<td>234</td>
<td>0.1</td>
<td>234</td>
<td>0.1</td>
<td>234</td>
<td>0.1</td>
<td>234</td>
</tr>
<tr>
<td>0.1</td>
<td>237</td>
<td>0.1</td>
<td>726</td>
<td>0.1</td>
<td>236</td>
<td>0.1</td>
<td>236</td>
<td>0.1</td>
<td>236</td>
<td>0.1</td>
<td>236</td>
</tr>
<tr>
<td>0.2</td>
<td>471</td>
<td>0.1</td>
<td>240</td>
<td>0.2</td>
<td>474</td>
<td>0.1</td>
<td>946</td>
<td>0.4</td>
<td>946</td>
<td>0.1</td>
<td>946</td>
</tr>
<tr>
<td>0.1</td>
<td>237</td>
<td>0.1</td>
<td>248</td>
<td>0.1</td>
<td>240</td>
<td>0.1</td>
<td>240</td>
<td>0.1</td>
<td>240</td>
<td>0.1</td>
<td>240</td>
</tr>
<tr>
<td>0.1</td>
<td>229</td>
<td>0.1</td>
<td>226</td>
<td>0.1</td>
<td>226</td>
<td>0.1</td>
<td>226</td>
<td>0.1</td>
<td>226</td>
<td>0.1</td>
<td>226</td>
</tr>
<tr>
<td>0.2</td>
<td>466</td>
<td>0.1</td>
<td>236</td>
<td>0.1</td>
<td>236</td>
<td>0.1</td>
<td>236</td>
<td>0.1</td>
<td>236</td>
<td>0.1</td>
<td>236</td>
</tr>
<tr>
<td>0.1</td>
<td>231</td>
<td>0.1</td>
<td>242</td>
<td>0.1</td>
<td>240</td>
<td>0.1</td>
<td>240</td>
<td>0.1</td>
<td>240</td>
<td>0.1</td>
<td>240</td>
</tr>
<tr>
<td>0.1</td>
<td>243</td>
<td>0.1</td>
<td>236</td>
<td>0.1</td>
<td>248</td>
<td>0.1</td>
<td>248</td>
<td>0.1</td>
<td>248</td>
<td>0.1</td>
<td>248</td>
</tr>
<tr>
<td>0.2</td>
<td>474</td>
<td>0.1</td>
<td>237</td>
<td>0.2</td>
<td>466</td>
<td>0.1</td>
<td>940</td>
<td>0.4</td>
<td>940</td>
<td>0.1</td>
<td>940</td>
</tr>
<tr>
<td>0.1</td>
<td>249</td>
<td>0.1</td>
<td>248</td>
<td>0.1</td>
<td>240</td>
<td>0.1</td>
<td>240</td>
<td>0.1</td>
<td>240</td>
<td>0.1</td>
<td>240</td>
</tr>
<tr>
<td>0.1</td>
<td>227</td>
<td>0.1</td>
<td>236</td>
<td>0.1</td>
<td>226</td>
<td>0.1</td>
<td>226</td>
<td>0.1</td>
<td>226</td>
<td>0.1</td>
<td>226</td>
</tr>
<tr>
<td>0.05</td>
<td>183</td>
<td>0.1</td>
<td>242</td>
<td>0.1</td>
<td>250</td>
<td>0.1</td>
<td>250</td>
<td>0.1</td>
<td>250</td>
<td>0.1</td>
<td>250</td>
</tr>
<tr>
<td>0.25</td>
<td>696</td>
<td>0.35</td>
<td>255</td>
<td>0.998</td>
<td>946</td>
<td>0.1</td>
<td>946</td>
<td>0.4</td>
<td>946</td>
<td>0.1</td>
<td>946</td>
</tr>
<tr>
<td>0.1</td>
<td>242</td>
<td>0.1</td>
<td>242</td>
<td>0.1</td>
<td>242</td>
<td>0.1</td>
<td>242</td>
<td>0.1</td>
<td>242</td>
<td>0.1</td>
<td>242</td>
</tr>
<tr>
<td>0.1</td>
<td>236</td>
<td>0.1</td>
<td>236</td>
<td>0.1</td>
<td>236</td>
<td>0.1</td>
<td>236</td>
<td>0.1</td>
<td>236</td>
<td>0.1</td>
<td>236</td>
</tr>
<tr>
<td>0.05</td>
<td>133</td>
<td>0.1</td>
<td>250</td>
<td>0.1</td>
<td>238</td>
<td>0.1</td>
<td>238</td>
<td>0.1</td>
<td>238</td>
<td>0.1</td>
<td>238</td>
</tr>
<tr>
<td>0.25</td>
<td>496</td>
<td>0.485</td>
<td>838</td>
<td>0.985</td>
<td>946</td>
<td>0.1</td>
<td>946</td>
<td>0.4</td>
<td>946</td>
<td>0.1</td>
<td>946</td>
</tr>
<tr>
<td>0.1</td>
<td>240</td>
<td>0.1</td>
<td>240</td>
<td>0.1</td>
<td>240</td>
<td>0.1</td>
<td>240</td>
<td>0.1</td>
<td>240</td>
<td>0.1</td>
<td>240</td>
</tr>
<tr>
<td>0.1</td>
<td>256</td>
<td>0.1</td>
<td>242</td>
<td>0.1</td>
<td>242</td>
<td>0.1</td>
<td>242</td>
<td>0.1</td>
<td>242</td>
<td>0.1</td>
<td>242</td>
</tr>
<tr>
<td>0.05</td>
<td>120</td>
<td>0.1</td>
<td>235</td>
<td>0.1</td>
<td>235</td>
<td>0.1</td>
<td>235</td>
<td>0.1</td>
<td>235</td>
<td>0.1</td>
<td>235</td>
</tr>
<tr>
<td>0.25</td>
<td>616</td>
<td>0.485</td>
<td>838</td>
<td>0.985</td>
<td>946</td>
<td>0.1</td>
<td>946</td>
<td>0.4</td>
<td>946</td>
<td>0.1</td>
<td>946</td>
</tr>
</tbody>
</table>

Table 2.5.

Experimental results for $I_o/I_t$. ($n = 50$; $E = 47.5$ volts.)
observed \(I_t\) when the condensers were put in parallel. The current obtained \(I_0\) with the condensers in parallel was then measured.

It will be noted that, as the capacitance value is increased the value of \(I_0/I_t\) experimentally obtained moves to the curves of lower resistance. This would appear to be due to the increased maximum current that passes through the valve with increased capacitance and which, from figure 2.15, would mean that the valve would offer a lower effective resistance.

In figure 2.17. are reproduced tracings from the tube face of the cathode ray oscilloscope showing the effect on the current pulse of increased capacitance. (Tracing paper was used which is considered to be superior to the cellulose tape technique used by Rathgeber (2.28) as such tape rapidly deteriorates.) The E.M.F. used as the source of charge was kept at 47.5 volts.

The main conclusion to be drawn from this series of experiments is that the upper limit of capacitance value, for which true measure can be obtained using the valve circuit, can be deduced from the approximate resistance determined by static measurement, provided that the static measurement be made with the appropriate potential difference across the valve. Some actual ratios determined with condensers of capacitances near the upper limit for the type LP 2 valve are given in table 2.6. The agreement between the methods used is of the same order as was obtained
Figure 2.17.

Cathode ray oscillograms showing the effect of capacitance values on the charging (left) and discharging current pulses.
Wien (de Sauty) bridge.  1.020  1.040  0.970  0.990  
Schering bridge.  1.020  1.040  0.970  0.990  
Valve circuit.  1.016  1.042  0.970  0.992  

<table>
<thead>
<tr>
<th>Method</th>
<th>Capacitance ratio 1</th>
<th>Capacitance ratio 2</th>
<th>Capacitance ratio 3</th>
<th>Capacitance ratio 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wien (de Sauty) bridge</td>
<td>1.020</td>
<td>1.040</td>
<td>0.970</td>
<td>0.990</td>
</tr>
<tr>
<td>Schering bridge</td>
<td>1.020</td>
<td>1.040</td>
<td>0.970</td>
<td>0.990</td>
</tr>
<tr>
<td>Valve circuit</td>
<td>1.016</td>
<td>1.042</td>
<td>0.970</td>
<td>0.992</td>
</tr>
</tbody>
</table>

Table 2.6.

Capacitance ratios determined by three different methods.

(Nominal capacitance of condensers = 0.1μF.)
with the relay circuits.

The output triode type PX 4 (Cossor) was investigated and, as can be seen from figure 2.18., the resistance value found was about 3000 ohms. This meant that the circuit described would fail for values of capacitance greater than 0.4\( \mu \)F with \( n = 50 \) and, as the PX 4 has one of the lowest resistances in the small power triode class, some other solution had to be sought if valves were to be used over a wide range of capacitance.

One solution is to use two or more valves in parallel in place of each of the valves in the circuit of figure 2.13. This will be referred to again later.

Another solution is to use an alternating current of lower frequency, but this would require a special oscillator. A possible solution which still uses the mains frequency is to put a condenser (known to have a capacitance \( C_1 \) such that complete charge and discharge occurs) in series with each of the condensers whose capacitances \( C_2 \) and \( C_3 \) are to be compared. This ensures that the effective value of the two condensers in series is below the maximum permissible value. A set of values is given in table 2.7. to illustrate the point. Three currents are thus measured being \( I_1 = nE C_1, \quad I_2 = nE C_1 C_2/(C_1 + C_2), \quad I_3 = nE C_1 C_3/(C_1 + C_3) \). Thus \( C_1/C_2 = (I_1 - I_2)/I_2 \) and \( C_1/C_3 = (I_1 - I_3)/I_3 \). Thus \( C_2/C_3 = I_2(I_1 - I_3)/I_3(I_1 - I_2) \).

With three currents to be measured and differences used in the calculations the
Figure 2.18.

Resistance of a type PX 4 valve plotted against the current through the valve.
Effective value of capacitance when two condensers are in series.

<table>
<thead>
<tr>
<th>$C_2$</th>
<th>μF.</th>
<th>10</th>
<th>1</th>
<th>0.5</th>
<th>0.1</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>μF.</td>
<td>0.099</td>
<td>0.091</td>
<td>0.083</td>
<td>0.050</td>
</tr>
<tr>
<td>$C_1$ as 0.1 μF.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
possible error must be rather high so that the arrangement
of figure 2.19. which incorporates a Rayleigh potentiometer
must be preferable.

The condenser \( C_1 \) is the fixed one
known to completely charge and discharge and \( C_2 \) is one of
the condensers under test. With the switch \( S \) closed, so
that \( C_2 \) is short-circuited, the Rayleigh potentiometer,
consisting of two non-inductive resistance boxes \( R \) and \( R' \),
is adjusted for some convenient meter reading \( I \). Then
\[
I = nV_1 C_1 \quad \text{where} \quad V_1 \quad \text{is the potential difference across the}
\text{resistance box} \quad R. \quad \text{The switch} \quad S \quad \text{is then opened and the}
\text{potentiometer adjusted for the same meter reading} \quad I \quad \text{as was}
\text{obtained with} \quad C_1 \quad \text{alone. Then} \quad I = nV_2 C_1 C_2 / (C_1 + C_2) \quad \text{as the}
\text{condensers are now in series. Thus, if the Rayleigh}
\text{potentiometer has a total resistance} \quad P, \quad C_1 / C_2 = (R_2 - R_1) / R_1.
\text{Similarly, with} \quad C_3 \quad \text{in series with} \quad C_1, \quad C_1 / C_3 = (R_3 - R_1) / R_1.
\text{It follows that} \quad C_2 / C_3 = (R_3 - R_1) / (R_2 - R_1).

Let the resistance values be \((R_1 + a), (R_2 + a)\) and \((R_3 + a)\) where \( a \) is the change of
resistance that just causes a detectable change in the meter
reading. Then the two worst possible cases are
\[
\frac{C_2}{C_3} = \frac{(F_3 + a) - (R_1 + a)}{(R_2 - a) - (R_1 + a)}
\quad \text{and} \quad \frac{C_2}{C_3} = \frac{(F_3 - a) - (R_1 + a)}{(R_2 + a) - (R_1 - a)}.
\]
\text{As} \quad a \ll (R_2 - R_1), \quad \text{on neglecting terms}
\text{involving} \quad a^2,
\[
\frac{C_2}{C_3} \quad \frac{(R_3 - R_1)}{(R_2 - R_1)} \quad \left\{ \frac{2a + 2aR_3}{(R_2 - R_1)(R_2 - R_1)^2(R_2 - R_1)} \right\}.
\]
Figure 2.19.

A modified form of the circuit of figure 2.13.
Expressed as a percentage the possible error is $\pm 200.\alpha \left[ \frac{1}{(R_3 - R_1)} + \frac{1}{(R_2 - R_1)} \right] \%$
or, in terms of capacitance, $\pm 200.\alpha \left( \frac{C_3 + C_2}{R_1 C_1} \right) \%$.

This possible error is so high that the method is clearly impracticable.

It is considered that this analysis shows that the only way in which thermionic valves can be used in the Siemens method for the comparison of capacitances over any range is to use valves connected in parallel or to use a low frequency of operation. The only advantage compared with a mechanical system apart from preventing wear of the switching device would be if the time available for leakage between charge and discharge were lower. As it is not, the use of thermionic valves with the resultant complexity of apparatus does not appear warranted except in circuits already involving such valves. An example of this is a circuit described by Walker (2.29) where a vane attached to a shaft, whose speed of rotation is required, interrupts a beam of light falling on an emission type photo-cell. As the cell requires an amplifier, the use of valves in the frequency part of the circuit is not inconvenient as the requisite power supplies are already necessary. A similar example occurs in section 6 when dealing with resistance measurement.
SECTION 3.

A NULL METHOD FOR THE COMPARISON OF CAPACITANCE.
A disadvantage of the Siemens method of determining capacitance or of comparing capacitance is that it is not a null method. This disadvantage has been overcome in the Maxwell bridge, as was mentioned earlier, by the double commutator bridge of Campbell (3.1, 2.9) and also by the use of a differential galvanometer (2.15, 3.2, 3.3, 3.4, 3.5). In this latter modification there are two coils instead of one in the galvanometer and, effectively, a steady, known current is passed through one coil to counterbalance the current due to the discharge pulses from the condenser in the other. It is then possible, knowing the steady current values, to calculate the capacitance values.

Siemens Bros., produce a type 100 relay which closes at the same high speed as the type 72, but closes two circuits at once and together (3.6). If the capacitances of two condensers were to be compared, it seemed that it would be possible to charge the two condensers with such a relay and to discharge them, at the same instant, but with opposite polarity, through a galvanometer. If the potential differences from which the condensers charge be adjusted so that there is no deflection of the galvanometer at discharge, it should be possible to relate the potential differences and thus the capacitances. It appeared that such a system would be very sensitive and also accurate provided there was no absorption of charge by the condensers.

The first circuit used is shown in figure 3.1. \( C_1 \) and \( C_2 \) are the two condensers whose
Figure 3.1.

Diagram of the original null circuit for the comparison of capacitance.
capacitances are to be compared and A and B are the two units of the high-speed, double changeover relay.

The multiple switch $S_1$ has three positions; position 1 is the off position; position 2 is for preliminary adjustment as explained below and position 3 is for the actual comparison. At position 3 the relay causes a repetition of the following sequence of events.

The two condensers begin to charge at the same instant and are on charge for the same length of time. The condensers then begin to discharge at the same instant and are able to discharge for the same length of time through the galvanometer. The times may not be exactly equal, but this can be ignored because (a) the transit time between charge and discharge is short ($10^{-3}$ seconds) therefore any difference in the transit time of the two tongues will be too small, compared with the charging and discharging times, to cause error, (b) the time constants are low therefore charge and discharge are completed well before the end of the periods allowed by the relay.

The polarities of the batteries $E_1$ and $E_2 + E_3$ are so arranged that the discharge currents are in opposite directions through the galvanometer. If the potential differences to which $C_1$ and $C_2$ charge are $V_1$ and $V_2$ respectively, then the current through the galvanometer is given by $I = nC_1V_1 - nC_2V_2$, where $n$ is the number of charges per second and thus twice the frequency of the alternating current driving the relay.
Across $E_1$ are two resistance boxes $R_1$ and $R_2$ used as a Rayleigh potentiometer with their sum kept at $P$. The current through the galvanometer can, therefore, be written as $I = nC_1E_1R_1/P - nC_2V_2$. Thus if $V_2$ has previously been made equal to $E_1$ (position 2 of switch $S_1$), $R_1$ can be varied to give zero deflection of the galvanometer. In this case $C_2/C_1 = R_1/P$.

If $P$ be made $10^2$, $10^3$ or $10^4$ ohms (its maximum value is limited by the requirement that the time constant of the charging circuit must be low enough to ensure complete charge) then the ratio of the capacitances is given directly in decimal form by $R_1$.

The preliminary setting of $V_2$ equal to $E_1$ is made in the following way when the switch $S_1$ is at position 2. $R_2$ is made zero and $R_1$ set to the value $P$ so that the full value of $E_1$ is available and acting through its normal load. A series circuit exists consisting of the galvanometer, a high resistance $R_3$ and the battery $E_1$ with its polarity in opposition to that of $E_2$ and to that part of $E_3$ developed across the potentiometer system $R_5$, $R_7$. $R_6$ and $R_5$ are adjusted until there is no deflection of the galvanometer. To make this adjustment more accurate, $R_3$ is steadily reduced to about $10^3$ ohms, $R_6$ and $R_7$ being adjusted all the time to retain the zero deflection. $R_3$ should not be reduced too far in case cross charging of the batteries occurs, and with $R_3$ at the above value the system gives $E_1 \sim V_2 \lesssim 5 \cdot 10^{-6}$ volts, using a galvanometer of sensitivity
A variable shunt $R_4$ is provided to protect the galvanometer when comparing capacitances although a series safety resistance can be used even though it affects the time constants of the two discharge circuits. The shunt may be removed by the switch $S_2$ for final balance.

As a deflection of 0.2 mm could be detected with the galvanometer above, it follows that a difference of current of $10^{-8}$ ampere could be detected. Thus the difference in two settings of $R_1$ that could just be differentiated is given by $R_{1a} - R_{1b} = 10^{-8}P/nC_1E_1$. Thus $(R_{1a} - R_{1b})/R_1 = 10^{-8}/nC_2E_1$. This expression has a very low value if $n$ and $E_1$ are sufficiently high. For example with $C_2 = 0.1 \mu F$, $E = 6$ volts, $n = 100$, $R_1$ can be determined to 2 parts in $10^4$. With small capacitance values, the E.M.F. values can be increased as $P$ can be increased as soon as the time constant has been reduced.

The circuit gave agreement of capacitance ratios with other methods to ± 0.2% with high sensitivity, but it was unsatisfactory in practice as the E.M.F. of the batteries drifted. The drift made each determination a slow process and liable to serious error if great care was not taken.

To overcome the difficulty recourse was made to the circuit shown in figure 3.2. which is similar to the D.C. circuit of Thomson (5.7.). The circuit is much simpler than that of figure 4.1. and if the
Figure 3.2.

Diagram of the second null circuit.
battery E.M.F. varies it causes no error.

$C_1$ and $C_2$ are the two condensers whose capacitances are to be compared and $A$ and $B$ are the two units of the high speed relay. The relay uses a common yoke and the coil is connected to a 6.3 volt filament transformer $T$ through a variable resistance $R_3$. The relay produces the same sequence of events as it did in the previous circuit and the same considerations apply.

If the potential differences to which $C_1$ and $C_2$ charge are $V_1$ and $V_2$ respectively, then the current through the galvanometer is $I = nC_1V_1/nC_2V_2$ where $n$ is again the number of charges per second and equal to twice the frequency of the alternating current driving the relay. As the same current from the battery $E$ passes through $R_1$ and $R_2$, $V_1/V_2 = R_1/R_2$ even if the value of the source of E.M.F. varies.

$R_1$ and/or $R_2$ are adjusted until there is no deflection of the galvanometer, and then

$$C_1/C_2 = R_2/R_1.$$  

As both condensers must charge completely it is necessary that $R_1$ and $R_2$ be kept sufficiently low for this to occur. From the figures given in table 1.1., it is evident that, as a rule of thumb, with 1.0, 0.1 and 0.01 $\mu$F condensers the resistance should not exceed 500, 5000 and 50,000 ohms respectively.

Usually a tapping key, with a one megohm resistance in parallel with it, has been
included in the galvanometer lead for protective purposes. The final adjustment of the ratio is most conveniently made with the key kept closed.

The sensitivity of this circuit, as with the previous one, depends upon both the E.M.F. of the battery and the sensitivity of the galvanometer. The galvanometers used had sensitivities of 22 mm/µA and 2480 mm/µA. For nearly all purposes the former was sufficiently sensitive.

Results obtained with paper condensers (and, for small capacitances, mica condensers) were compared with those obtained with an alternating current bridge. As a further check on the performance of the circuit, four condensers were compared in turn with a fifth. The four condensers were then put in parallel and compared with the fifth. The sum of the individual ratios was found to be the same as the sum determined experimentally, within the limits imposed by the resistance boxes used as $R_1$ and $R_2$. A typical set of results is given in table 3.1.

Although the resistance boxes used were only reliable to 0.1%, the results show that a sensitivity of 1 in $10^4$ can be obtained with the circuit (except with very low capacitances) when the ratio is approximately 1:1. This is evident from the theory of the circuit as if $R_2$ be kept fixed the difference in two settings of $R_1$ that give a detectable current difference is
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>μF.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2/4</td>
<td>2</td>
<td>0.542</td>
<td>0.555</td>
</tr>
<tr>
<td>1/1</td>
<td>..</td>
<td>0.839</td>
<td>0.890</td>
</tr>
<tr>
<td>0.1/0.1</td>
<td>12</td>
<td>0.9750</td>
<td>0.976</td>
</tr>
<tr>
<td>0.05/0.1</td>
<td>..</td>
<td>0.9836</td>
<td>0.986</td>
</tr>
<tr>
<td>..</td>
<td>..</td>
<td>0.5363</td>
<td>0.536</td>
</tr>
<tr>
<td>0.9709</td>
<td></td>
<td>2.974</td>
<td></td>
</tr>
<tr>
<td>(0.1+0.1+0.05 +0.05)/0.1</td>
<td>..</td>
<td>2.9704</td>
<td>2.98(0)</td>
</tr>
<tr>
<td>0.01/0.01</td>
<td>120</td>
<td>1.0550</td>
<td>1.057</td>
</tr>
<tr>
<td>..</td>
<td>..</td>
<td>0.9388</td>
<td>0.941</td>
</tr>
<tr>
<td>..</td>
<td>..</td>
<td>0.8776</td>
<td>0.879</td>
</tr>
<tr>
<td>3.7459</td>
<td></td>
<td>3.754</td>
<td></td>
</tr>
<tr>
<td>(0.01+0.01+0.1 +0.01)/0.01</td>
<td>..</td>
<td>3.7577</td>
<td>3.75(0)</td>
</tr>
<tr>
<td>0.001/0.001</td>
<td>240</td>
<td>0.8740</td>
<td>0.875</td>
</tr>
<tr>
<td>0.002/0.01</td>
<td>..</td>
<td>0.2026</td>
<td>0.202</td>
</tr>
<tr>
<td>0.0002/0.0002</td>
<td>..</td>
<td>1.025</td>
<td>1.03</td>
</tr>
</tbody>
</table>

Table 5.1.

Capacitance ratios obtained with the null circuit and with an alternating current bridge at 1000 cycles second⁻¹.
\[(R_{1a} - R_{1b}) = (R_1 + R_2)(I_a - I_b)/nEC\] because \((R_1 + R_2)\) can be taken as constant. For example, if \(C = 1\mu F\), \(E = 2\) volts, \(n = 100\), \((I_a - I_b) = 10^{-8}\) ampere, and \(R_1 = R_2 = 100\) ohms, then \((R_{1a} - R_{1b}) = 10^{-2}\) ohms.

With paper condensers of higher capacitance than \(1\mu F\), it has been found that the circuit is only reliable to approximately \(\pm 2\%\). This may be due to absorption as changing \(n\) from 100 to 50 with the normal Siemens method changed the apparent capacitance of such a condenser from 1.26 to 1.24 \(\mu F\). Such condensers have also been found unsatisfactory for the measurement of high resistances by leakage (section 6).

The relay is easily adjusted. The lower contacts are brought up to give the smallest gaps visible with the naked eye and then, with 0.5 or 1.0 \(\mu F\) condensers as \(C_1\) and \(C_2\), the tension screws are adjusted to give a steady deflection of the galvanometer spot. This ensures that the relay tongues are closing together as, when they are not, with such capacitances the spot swings about the scale in an erratic manner. Adjustment is further simplified if the two tongues are waxed together.

By means of the techniques described in section 6, it has been found that the time interval between the closure of one contact and the closure of the other is less than \(10^{-3}\) seconds when the relay is adjusted as above.

The relays take a few minutes to
achieve steady operation. This can be seen in the figures given in table 3.2. These figures were obtained over one and a half hours using \( n = 50 \), a reading being taken every three minutes with the less sensitive galvanometer. In each case the reading was obtained by reducing down from \( 10^4 \) ohms. It will be noticed that there is a certain amount of wandering, but only to the extent of less than 0.1%. The standard deviation (4.7.) is very low. Even with 0.0001\( \mu \)F condensers with the above conditions the wander was less than 0.5% with a galvanometer of sensitivity 2480 mm/\( \mu \)A.

It was mentioned that the circuit is similar to that due to Thomson. With this latter circuit two-way keys are used instead of relays and the condensers are charged and then their charges allowed to mix. Any charge remaining on one or other of the condensers then causes a transient current when the galvanometer key is closed. The relationship at balance is the same as that above and the sensitivity is as high, but Thomson's method is slower to use, is not continuous reading and more susceptible to leakage of charge from the condensers. The circuit described is, in fact, two Siemens circuits operating "back-to-back" rather than a Thomson circuit operated by relays.

Although the results obtained when comparing the capacitances of commercial condensers agreed well with those obtained by other methods, it was felt that a severer test for errors due to absorption of charge was necessary. The best test that could be devised was the
### Table 3.2.

Capacitance ratios obtained with the null circuit over a period of time. (Readings taken every three minutes. Nominal values of capacitance = 0.001/0.001 μF. Ratio by series resistance A.C. bridge = 1.19(0). Value of $R_1$ in the null circuit = 10,000 ohms.)

<table>
<thead>
<tr>
<th>$R_2$ ohms</th>
<th>Null circuit ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>8430</td>
<td>1.186</td>
</tr>
<tr>
<td>8430</td>
<td></td>
</tr>
<tr>
<td>8420</td>
<td>1.188</td>
</tr>
<tr>
<td>8420</td>
<td></td>
</tr>
<tr>
<td>8410</td>
<td>1.189</td>
</tr>
<tr>
<td>8415</td>
<td>1.188</td>
</tr>
<tr>
<td>8415</td>
<td></td>
</tr>
<tr>
<td>8410</td>
<td>1.189</td>
</tr>
<tr>
<td>8415</td>
<td>1.188</td>
</tr>
<tr>
<td>8415</td>
<td></td>
</tr>
<tr>
<td>8410</td>
<td>1.189</td>
</tr>
<tr>
<td>8415</td>
<td>1.188</td>
</tr>
<tr>
<td>8415</td>
<td></td>
</tr>
<tr>
<td>8417</td>
<td></td>
</tr>
<tr>
<td>8415</td>
<td></td>
</tr>
<tr>
<td>8417</td>
<td></td>
</tr>
<tr>
<td>8416</td>
<td></td>
</tr>
<tr>
<td>8416</td>
<td></td>
</tr>
<tr>
<td>8417</td>
<td></td>
</tr>
<tr>
<td>8419</td>
<td></td>
</tr>
<tr>
<td>8417</td>
<td></td>
</tr>
<tr>
<td>8416</td>
<td></td>
</tr>
<tr>
<td>8419</td>
<td></td>
</tr>
<tr>
<td>8420</td>
<td></td>
</tr>
<tr>
<td>8420</td>
<td></td>
</tr>
<tr>
<td>8420</td>
<td></td>
</tr>
<tr>
<td>8419</td>
<td></td>
</tr>
<tr>
<td>8420</td>
<td></td>
</tr>
</tbody>
</table>
measurement of the dielectric constant of some liquids. For this two condensers were used. The first, which had a capacitance of the order of 0.0001 \( \mu F \), was a simple cylindrical condenser consisting of a solid brass inner cylinder surrounded by a brass tube. Perspex end pieces were used as is shown in figure 3.3. The ratio of the capacitance with the liquid between the electrodes to the capacitance with air as the dielectric was taken as the dielectric constant. The values obtained are given in table 3.3, together with the values obtained using a parallel plate condenser with a capacitance of the order of 0.001 \( \mu F \). This second condenser was constructed from two 1/8" thick plates of mild steel, milled flat to within \( \pm 0.001" \), separated by very small pieces of X-ray film. It will be seen that the values for the dielectric constant determined with the null method were low compared with the values obtained by other methods and from tables (3.8.). The disagreement between the null method values would seem to be due to different charging/discharging times as different relays were used with the two condensers. This is evidence that the low values were due to absorption effects. The subject was not investigated further as the circuit was not primarily intended for the comparison of condenser capacitances, but to provide a sensitive and precise circuit for use with the U-tube manometer described in the next section. Moreover previous circuits of this type have been found unreliable with condensers exhibiting absorption effects (1.7.).
Figure 3.3.

Diagram of the cylindrical condenser.
Benzene.  Olive oil.  Linseed oil.
(a)  (b)  (a)  (b)  (a)  (b)

Null circuit.  1.82  2.21  2.43  2.95  1.90

Campbell bridge.
(relay type.)  2.2(6)  2.28  3.0(9)  3.0(8)  3.3(7)

Relay bridge.  2.2(4)  2.28  3.0(7)  3.12  3.4(0)

Series resistance
a.c. bridge.  2.3(6)  2.30  3.1(0)  3.10  3.3(8)

Tables.  2.29  3.11  3.35

Table 3.5.

Values of dielectric constants determined at room temperature.
(a) = cylindrical condenser, (b) = parallel plate condenser.
SECTION 4.

A SENSITIVE, DIRECT READING, MERCURY U-TUBE MANOMETER AND A DIRECT READING BAROGRAPH.
A standard method for the measurement of pressure differences is a U-tube containing a liquid, generally of low vapour pressure, as the manometric indicator. Such a manometer is often used with devices for the measurement of fluid flow where the difference in pressure may be generated in several ways. These include a simple constriction, the Venturimeter principle, the Pitot tube principle and the orifice principle (4.1.). Diagrams of these are given in figure 4.1.

It is not always convenient nor easy to read such liquid manometers. Float devices, to give a record of pressure variations, so increase the inertia of the system that it becomes unreliable in use if variations are rapid. This is particularly so when the U-tube manometer is used in physiological experiments where the shape of the recorded pressure-time curve can be of importance.

This section deals with a mercury U-tube manometer which operates in conjunction with the circuit of the previous section. The combination gives a very sensitive system which has the inertia of the U-tube manometer alone. Moreover, the menisci effects can be minimised by a differential method of operation.

The circuit is given in figure 4.2., together with a diagram of the U-tube. Each limb of the U-tube consists of a metal tube coated on the inside with a thin layer of dielectric. The limbs are joined at the bottom by a tube of insulating material, with a short length
Constriction.  Venturi.

Pitot tube.  Orifice.

Figure 4.1.

Diagrams of devices depending on pressure difference for the measurement of fluid flow.
Figure 4.2.

Diagram of the U-tube manometer and associated circuit.
of metal tubing somewhere in its length, to enable electrical contact to be made with the mercury.

The mercury acts as a variable inner conductor of two cylindrical condensers, and the metal tubes forming the limbs act as the outer conductors. A pressure difference across the U-tube produces an increase in capacitance of one limb and a decrease in capacitance of the other.

Evidently such a system as this could be used in conjunction with circuits such as the D.C. bridges of de Sauty, Thomson and Groot (1.6.), the A.C. bridges of Wien or Schering (2.22.) or the relay bridge (2.9.), but the null circuit has various advantages. These are simplicity, precision, high sensitivity and, provided the deflection of the galvanometer is proportional to the current through it, an out of balance deflection directly proportional to the change in capacitance after balancing. In this manometer, the change in capacitance is a direct measure of the change in pressure difference. Moreover, in a.c. bridges it is necessary to adjust two components since both the phase angles and the magnitudes of the bridge-arm impedances must be suitably balanced.

When the U-tube is combined with the null circuit, there are several ways in which the apparatus can be used. These will now be considered.

1. The resistances $R_1$ and $R_2$ are replaced by a potentiometer which is adjusted for no
deflection of the galvanometer when there is no difference of pressure. The galvanometer is shunted so that full scale deflection is obtained for maximum possible pressure difference required. The system used in this way would have the following advantages: (a) the apparatus would be direct reading, (b) a linear scale would be possible, (c) recording would be possible by passing film down past the galvanometer face, (d) meniscus effects would be minimised by the differential effect.

2. This method is similar to method 1, but the potentiometer is adjusted for no deflection of the galvanometer when the pressure difference is at the value required and then the shunt across the galvanometer removed. With this arrangement maximum sensitivity would be available and the instrument would be useful for studying variations in pressure difference about some mean value.

3. The potentiometer is replaced by a fixed resistance $R_1$ and the value of a resistance box $R_2$ adjusted for no deflection of the galvanometer at the various values of pressure difference. This method gives maximum sensitivity at all pressure differences, but cannot be used for recording and there is not a linear relationship between pressure difference and the value of $R_2$. This latter point follows from the theory.

At initial balance $C_1/C_2 = R_2/R_1$. If one column rises by $h$ then the other falls by $h$ so that, if the dielectric layer has a uniform thickness, the
capacitance on one side increases by c and, on the other, the capacitance falls by c. Thus, at balance,
\[(C_1 + c)/(C_2 - c) = R_2/R_1.\] Thus \[c = (R_2C_2 - R_1C_1)/(R_1 + R_2).\]

4. A linear relationship between the value of \(R_2\) and \(h\) can be obtained if the sensitivity is reduced below that of method 3 for the same value of the E.M.F. Only one limb of the U-tube is used, the other being replaced in the circuit by a fixed condenser of about the same capacitance. Then, at balance, when the capacitance of the active limb has been, for example, increased by c,
\[(C_1 + c)/C_2 = R_2/R_2 \text{ or } c = C_2R_2^2/R_1 - C_1.\]

It is evident that the last two methods are the most accurate and it is these that have been investigated most fully.

The capacitance per unit length of a cylindrical condenser, if end effects are neglected, is given by \(K/2\log_e(r_2/r_1)(1.1.)\), where \(K\) is the dielectric constant of the material between the conductors and \(r_2\) and \(r_1\) are the external and internal radii of the dielectric. A displacement from zero of a reflecting galvanometer of ±0.2 mm can certainly be detected. If the sensitivity of the galvanometer is \(p\) mm/pA, then the indeterminacy of the apparatus \(d\) is given by
\[nEKd/2\log_e(r_2/r_1) = 0.2 \times 10^{-6}.9/p \times 10^{-11} \text{ with method 3 and}\]
\[nR_2EKd/(R_1-R_2)2\log_e(r_2/r_1) = 0.2 \times 10^{-6}.9/p \times 10^{-11} \text{ with method 4.}\]

As \(d\) is constant with method 3, a
typical figure will be given for that method. With \( E = 45 \) volts, \( r_2/r_1 = 1.02, K = 20, n = 100, p = 22 \text{ mm}/\mu \text{A} \) then 
\[ d = \pm 0.04 \text{ mm}. \] This assumes that there are no surface tension effects. In practice the performance is not as good as the figure indicates.

Mercury has been used as the manometric fluid and various materials as the dielectric. It was hoped to produce a manometer equivalent to a water manometer by using an electrolyte, but several months of work finally showed convincingly that it was not practicable. Eventually dielectric breakdown due, probably, to pin holes was overcome, but there remained the difficulty that any time up to 30 seconds was required for the conducting fluid to become free from the dielectric when a column was falling.

With mercury these difficulties did not arise. The mercury, due to its angle of contact, did not penetrate pin holes and fell cleanly. The following materials were used as dielectrics: paraffin wax, beeswax and resin, resin varnish, various cellulose enamels, cerisin wax and a commercial lacquer "Valspar". The two most satisfactory to work with, in that manipulation was easy, a reasonably uniform and thin layer was produced and the dielectric constant was high, were brushing cellulose and Valspar. However, brushing cellulose did not give reliable single coats.

Coating was most easily performed by filling the tube with the material and then allowing the
tube to drain in a vertical position.

The junction tube was of rubber in all cases. It was found initially that the dielectric was sometimes removed accidentally when attaching the rubber tube and sometimes the dielectric did not cover the bottom edge of the tubes properly. These faults were overcome by allowing the lacquer to dry, dipping the bottom two or three centimetres of the tubes into smoking beeswax and resin, allowing the wax to harden, and inserting an inner rubber sleeve before the rubber junction tube was attached.

Using method 4, the apparent capacitance per centimetre of some tubes coated as described above was determined. Measurement of the length of the mercury column was made with a glass tube as the other limb of the U-tube. Typical results for Valspar are given in figure 4.3, where the smoothing out of irregularities by a second coat is very clear. Volume measurements were unavoidably inaccurate but gave an average layer thickness of 0.012 cm., with a standard deviation of ±16%. Direct measurement with a vernier travelling microscope was only possible at the end from which the lacquer had drained where the layer was thinnest. Tubes were filled with benzene and the films observed through the microscope as they came away. This gave a value of 0.005 cm., with a standard deviation of ±20%. Using the average layer thickness value, the apparent dielectric constant was 24 with a standard deviation of ±17%.
Figure 4.3.

Apparent capacitance of a tube plotted against length of mercury column.
(Narrow tube. Upper curve is for one coat of lacquer. Lower curve is for two coats.)
As may be seen, the thickness of the layers was uniform for 5 to 10 cm., but experiments have been carried out to see if this can be improved. The most promising technique seems to be to allow excess lacquer to drain and then to rotate the tubes about their axis at about 750 r.p.m. with the axis horizontal for 1 to 2 hours while the lacquer dries. The curves for two tubes produced in this way are given in figure 4.4. It will be seen that there was little difference between the two tubes and little variation over a considerable length.

The experimental work was carried out with brass tubes of internal diameters 1.02 cm., and 2.51 cm. respectively. The galvanometer had a sensitivity of 22 mm/μA and, normally, the E.M.F. was kept at 45 volts. A very sensitive galvanometer did not improve the performance of the circuit to any great extent as it began to oscillate due to occasional transient currents. This made the determination of the balance point difficult and nullified the increased sensitivity.

A carbon tetrachloride manometer was originally used for calibration purposes, but this had to be replaced by a water manometer when very cold weather prevented adequate ventilation being used to remove the vapour.

Most of the exploratory work was carried out using both limbs of the U-tube as it was felt that the differential effect would partially eliminate errors.
Figure 4.4.

Apparent capacitance of two tubes plotted against height of mercury column. (Narrow tubes. Single coat of lacquer. Rotated whilst drying.)
due to meniscus shape. This was found to be so except when the pressure difference was changing very slowly. For slow changes of pressure difference it was therefore necessary to calibrate the manometer by changing the pressure difference slowly. This is clearly shown in the curves (a) and (b) of figure 4.5. The curves were obtained with the narrow tubes with two coats of lacquer. The tubes were inclined to each other at $120^\circ$. The manometer was calibrated by setting up a particular pressure difference and then allowing the columns to settle down after being caused to oscillate by squeezing the tube connecting the two manometers. The calibration was then checked by setting up pressure differences at random, and without squeezing the connecting tube. However, when the pressure difference was allowed to fall slowly from the maximum value (10 cm.Hg/hour) the lower curve was obtained. Exactly the same effect was obtained when the pressure difference was slowly increased. The effect was not so marked when the limbs were vertical. This is shown in the pair of curves (c) and (d) of figure 4.5.

It will be noticed that the pressure difference had to change by an appreciable amount before the discrepancy arose, so that for very slow changes the main calibration curve could be used over short periods of time. In either case the apparatus could be used to measure slow changes of pressure very conveniently and in considerable detail. This is shown in figure 4.6., where part of three long runs with a constant leak are shown. The
Figure 4.5.

Values of $R_2$ plotted against pressure difference. (Narrow tubes. Curves (a) and (b) are for a V-manometer. Curves (c) and (d) are for a U-manometer. $\Delta$ = calibration; $\bullet$ = random; $\bigcirc$ = slow fall of pressure difference; $\times$ = slow rise of pressure difference.)
Figure 4.6.
The measurement of a slow change of pressure difference.
(○ = water manometer; ◦ = galvanometer deflection from balance; × = values of $R_2$; Δ = water manometer values derived from values of $R_2$ by calibration curve.)
change in pressure difference with time was determined with a water manometer and then the original pressure difference restored. The leak was reopened and the resistance readings with time taken. The equivalent pressure differences were determined from a normal calibration curve and finally the galvanometer deflections with time were taken as the pressure difference fell. The rapidity with which readings can be taken is clearly shown.

It has been mentioned that theoretically the deflections from zero are proportional to the change of pressure difference. This was investigated with both sizes of tube coated by draining and is illustrated in figures 4.7 and 4.8. It will be noted that the use of a single calibration line could involve an error of the order of ±1.0 mm Hg, but that individual calibrations would give a much lower possible error. It was thought that the error might be reduced with tubes coated by spinning, but, in fact, the improvement was negligible.

The sensitivity of the apparatus to change of pressure difference is shown by the curves so far given to be very high, but it was made still higher. In figure 4.9, is given the curve for the narrow tubes, with a single layer of Valspar, an E.M.F. of 120 volts and the tubes inclined to one another at 120°. With this arrangement a change of pressure of less than 0.1 mm Hg was detected. To plot the calibration curve of the value of $R_2$ against pressure difference requires 4.5 metres of graph paper. As
Figure 4.7.
Galvanometer deflections plotted against change of pressure difference from its initial value.
(Narrow tubes. Two coats of lacquer. U-tube. $\Theta =$ initial value 1.8 cm.Hg; $\Delta =$ initial value 4.5 cm.Hg; $X =$ initial value 7.0 cm.Hg; $\nabla =$ initial value 9.5 cm.Hg)
Galvanometer deflection plotted against change of pressure difference from its initial value.

(Wide tubes. One coat of lacquer. U-tube. $\odot$ = initial value 0.8 cm.Hg; $\triangle$ = initial value 3.3 cm.Hg; $\bigcirc$ = initial value 5.5 cm.Hg; $\triangledown$ = initial value 8.5 cm.Hg. Individual calibration lines are indicated as $\triangle$ ———, $\bigcirc$ ——— and $\triangledown$ ———.)
Galvanometer deflection plotted against pressure difference.

(Narrow tubes. One coat of lacquer. V-tube.)
this sensitivity is masked by the meniscus errors it is of little use for measurement, but it could be of use as a detector of minute pressure changes. The sensitivity to change remains appreciably constant whatever the initial pressure difference may be. It is this feature of the apparatus that renders it different from any other type of sensitive manometer working over the same wide range of pressure difference.

The sensitivity increased by approximately the expected amount on sloping the tubes while the maximum scatter remained substantially the same at ±1.0 mm.Hg near zero pressure difference and ±0.7 mm.Hg over the rest of the range where it remained fairly constant. Both the sensitivity and the maximum error increased when the diameter of the tubes was increased. The scatter rose to ±1.5 mm.Hg and ±1.0 mm.Hg.

The sensitivities obtained are given in table 4.1.

One application of this system that is still being developed is a direct, continuous reading barograph. The barograph is of the siphon type. This type was chosen because it is possible, by giving it proper dimensions, to eliminate the influence of temperature for all practical purposes (4.2.). It is shown in figure 4.10.

A glass tube A is bent into the form shown and waxed into a brass tube B. The inner surface of the brass tube is coated with lacquer and spun while
<table>
<thead>
<tr>
<th>Tube size</th>
<th>Coats of lacquer</th>
<th>Arrangement</th>
<th>Sensitivity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Small</td>
<td>1</td>
<td>U</td>
<td>2.7</td>
</tr>
<tr>
<td>Small</td>
<td>1</td>
<td>V</td>
<td>4.6</td>
</tr>
<tr>
<td>Small</td>
<td>2</td>
<td>U</td>
<td>2.1</td>
</tr>
<tr>
<td>Small</td>
<td>2</td>
<td>V</td>
<td>3.6</td>
</tr>
<tr>
<td>Large</td>
<td>1</td>
<td>U</td>
<td>3.5</td>
</tr>
</tbody>
</table>

Table 4.1.

Sensitivities obtained with the manometer.

(Sensitivities in mm deflection of galvanometer per mm Hg change of pressure difference. Galvanometer sensitivity = 22 mm/μA., battery E.M.F. = 45 volts. V = tubes inclined to each other at 120°.)
Figure 4.10.
Diagram of the direct reading siphon barograph.
drying in the manner explained earlier. A small, rubber bung
is waxed into the other end of the brass tube and the wax
joints painted over with shellac varnish to prevent air
leaking into the system. The shorter arm of the barometer
U-tube is divided into two parts joined by a short length
of brass tube waxed into position. The brass tube enables
electrical contact to be made with the mercury. The end unit
C is identical with B except that the rubber bung is not
waxed into position as it is only used to prevent the entry
of dirt. Several small holes are drilled in the brass tube
for connection with the atmosphere.

A change of atmospheric pressure
causes an increase in capacitance of one cylindrical
condenser and a decrease in capacitance of the other. The
two condensers being connected to the null circuit described,
the change is indicated by a deflection of the galvanometer.
The galvanometer current is returned to zero by adjusting the
resistance \( R_2 \), the resistance \( R_1 \) being kept fixed. The
apparatus must be calibrated by the use of a standard
barometer and atmospheric pressures then determined by the
value of \( R_2 \), if a resistance box is used, or the scale
reading, if a precision, helical potentiometer is used.

Readings were taken at intervals
over several weeks of the values of \( R_2 \) for balance at
various values of the atmospheric pressure as measured by a
Fortin barometer. The values obtained are plotted in figure
4.11. In the same figure are the values of \( R_2 \) plotted
Figure 4.11.

Values of $R_2$ plotted against barometric pressure.

- ○ Uncorrected barometer readings
- △ Corrected (Mehrke) readings.
against the corrected barometric reading using Mehmke's method for brass scales graduated at 0°C (4.3.).

The results show that, although the arrangement operates, there is such a scatter that it is not satisfactory. Although no fault could be detected in the relay circuit, it was decided to try the effect of connecting the barometer to the less sensitive, modified Campbell circuit mentioned earlier (2.9.). With this circuit it is also possible to plot atmospheric pressure against a resistance value for balance and the results are shown in figure 4.12. The scatter is again apparent.

The scatter may be due wholly or in part to the effect of temperature on residual air and/or water vapour above the mercury column because the points well off the general curve have generally been those taken when the temperature of the mercury column was a degree or two above or below the average temperature of 18.5°C. (The temperatures were measured with a mercury-in-glass thermometer the bulb of which was wrapped in aluminium foil and held against the vertical tube A with more foil wrapped round the tube and wrapped bulb.) The shape of the barometer is such that it is difficult to fill with mercury and the normal method of removing the air from the mercury by heating is precluded because of the waxed joints.

It is hoped to remove the fault by remaking the apparatus in a form similar to that of Marvin's float operated barograph (4.4.). This uses conical joints.
Figure 4.12.

Values of $R_2$ plotted against barometric pressure.
at the foot of the limbs as indicated in figure 4.13. The long limb of the barograph can thus be filled in the normal manner for barometer tubes with alternate pumping and admission of dry air while the tube is heated prior to the admission of warm mercury. The long limb is then inserted into the mercury filled female unit with no loss of vacuum.

If the fault is cleared the apparatus could be used in the manner described above or else the galvanometer deflections from zero utilised in a direct reading version.
Figure 4.13.

The basic construction of the Narvin barograph.
SECTION 5.

A DIRECT READING VANE ANEMOMETER.
This anemometer was designed in an attempt to produce a portable, direct reading and inexpensive instrument for measuring draught velocities in animal houses where knowledge of the extreme conditions over twenty-four hour periods may be required.

It was evident that for draught velocities (which may be considered to be in the range 0 - 10 m.p.h.) the instrument could not use any method where a reaction is passed back to the vane by the recording device. This fact eliminated the standard tachometer device of magnetic linkage. Photo-electric methods were eliminated by the requirement that the apparatus be simple and inexpensive and the same considerations eliminated the use of sensitive manometers with some form of capacitance change.

Siemens' method of measuring capacitance was utilised because, as \( I = nEC \), if \( E \) and \( C \) are kept constant then \( I \propto n \). That is, the current is proportional to the number of charges or discharges per second which can be arranged to be produced by the vane rotating. The only restraint exerted on the vane system is that of the contacts which the vane operates as it rotates and which produce the charge and the discharge of the condenser.

The apparatus as constructed consists of a vane unit (figure 5.1.) and a control unit.

The vane unit, which is essentially a rotary switch, has an aluminium vane system \( A \) fastened perpendicularly to a steel axle. As the apparatus was
Figure 5.1.
Diagram of vane unit.
A = vanes, 30 S.W.G. aluminium, 4.3 cm. diameter.
B = brass spider, arm 0.5 cm.
C = platinum contact wire, 3 cm. long, 0.006 in. diameter.
D, E, F, G = platinum wire brushes, 2.5 cm. long, 0.006 in. diameter.
H = plastic sleeve, 1.5 cm. long, 0.02 in. inside diameter.

Table 5.1.

Details of figure 5.1.
required to measure the draught velocities from small holes and pipes the vanes had to have a fairly full operating area so that the design was rather restricted. The optimum number of blades was found by trial and error and was found to be twelve with an external diameter of 4.3 cm., and an internal diameter of 0.5 cm.

Perpendicular to the axle are two brass spiders B that carry between them three platinum wires C parallel to the shaft at 120° intervals around the shaft. As this system rotates it alternately joins the brushes D and E, thus causing the condenser to charge, and the brushes F and G, thus causing the condenser to discharge through the micro-ammeter.

This rather unorthodox 'commutator' was adopted after exhaustive trials with cylindrical devices made it clear that sound contact with the necessary very low frictional restraint was impossible. The open structure of the rotating system was chosen to reduce both the moment of inertia and the air resistance of the system, to give quick response to variations in wind velocity. It has done this, the instrument detecting changes in wind velocity of 6 ft. min.\(^{-1}\) or more and the response time is less than 1 second for changes in wind velocity of less than 75 ft. min.\(^{-1}\), with a maximum response time of 5 seconds.

The brushes each consist of two platinum wires held at one end between brass plates, the pair of brushes on each side being mounted on perspex carried by
the frame of the unit. The position of each brush can be adjusted. Vibration of the brush wires could cause intermittent loss of contact and thus unsteady readings. It could also cause premature contact with the possibility of the meter being put across the battery. Both troubles were met with in the first designs constructed so damping was introduced by plastic sleeves fitted over the wires and waxed to the brass plates. The possibility of premature contact was further precluded by varnish on the brush wires except at their tips.

The steel axle is tapered at each end and fits into conical indentations in hardened brass. The indentation at the vane end is in a thin strip to avoid obstruction to wind flow. The position of the strip relative to the frame may be adjusted for alignment purposes. The indentation at the other end is in a small cylinder sliding in the frame to provide an adjustment for the bearing pressure. The whole vane unit is enclosed in an open-ended box 2" x 4.5" x 4" for protection of the unit (figure 5.2.).

The E.M.F. applied across the condenser must remain at the value used for calibration so a step-by-step adjustment has been incorporated in the control unit (figure 5.3.), the meter being used as a standardizing voltmeter. Adjustment to the correct E.M.F. is made with the anemometer running, a resistance \( R_1 \) (\( R_1 \propto G \)) replacing the meter to act as a load. A potentiometer system could be used for adjusting the charging voltage, but would cause a drain on the battery while the increase in the
Figure 5.2.

The vane anemometer.
Figure 5.3.

Circuit diagram of control unit.
$R_1 = 50 \text{ ohms.}$
$R_2 = 3 \cdot 10^5 \text{ ohms.}$
$R_3 = 3 \cdot 10^3 \text{ ohms.}$

$C = 0.1 \mu F, \text{ range A; 0.05 } \mu F, \text{ range B.}$

$E_1 = 90 \text{ volt miniature H.T. battery.}$

$E_2 = 9 \text{ volt grid bias battery.}$

$G = \text{ Ferranti, } 0 - 500 \mu \text{A, moving coil meter.}$

$S_{1,2,3} = 3 \text{ pole/3 way wafer switch.}$

$S_4 = 1 \text{ pole/7 way wafer switch.}$

Table 5.2.

Component values used in the control unit of figure 5.3.
The accuracy of setting of \( E \) would be wasted as the calibration is only to \( \pm 1\% \).

The meter is shunted by \( R_2 \) to reduce needle flutter and the range of the instrument may be varied by a switch selecting different values of \( C \).

The instrument was calibrated with \( C = 0.05 \mu F \), by carrying it along a corridor at speeds giving steady meter readings. The actual speed was determined by timing over a measured distance. As this procedure involved unavoidable inaccuracies it was repeated many times and in figure 5.4 the mean velocities obtained are plotted against the meter readings with the scatter shown in the normal manner.

To check the calibration, a standard vane anemometer was used to determine the wind velocity at a point in front of a blower. The standard vane anemometer was then replaced by the vane unit of the instrument described and the meter reading noted. The results of this calibration check are also shown in figure 5.4. A discrepancy was noted at wind velocities greater than 300 ft.min\(^{-1}\). This was thought to be due to the air from the blower having a greater velocity at the centre of the beam than at the periphery and to the fact that the vanes of the two anemometers were of different construction. The standard anemometer had vanes of diameter 6.4 cm., with a central dead space of diameter 2.8 cm., whereas, as already mentioned, the vanes of the anemometer described had diameters of 4.3
Figure 5.4.

Calibration graphs.
and 0.5 cm. respectively. Due to this difference in the vanes, as the anemometers were brought closer to the blower the standard anemometer indicated a lower velocity than the anemometer described.

This explanation of the discrepancy was confirmed by calibrating the continuous reading anemometer with a dry kata-thermometer (5.1.) which had a bulb of smaller dimensions than the vanes of either anemometer. The results are also given in figure 5.4. and it will be noted that, as expected, the kata-thermometer gave a higher value than the anemometer.

A further confirmation are the scale readings obtained with the direct reading anemometer over a plane perpendicular to the axis of the blower and 65 cm. away from the exit orifice. These are given in figure 5.5., where the velocity gradient across the beam is very evident. It is also noticeable that the blower projects its jet of air slightly upwards and towards the right. This off-axis projection is not sufficient, however, to affect the calibration check.

The calibration curve does not pass through the origin because a certain minimum velocity is required to turn the shaft of the instrument and also because, below a wind velocity of 150 ft.min⁻¹, n is too low (about 10) for a continuous current to be indicated by a microammeter of relatively short period.

The quick response of the instrument
<table>
<thead>
<tr>
<th>100</th>
<th>150</th>
<th>225</th>
<th>100</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>225</td>
<td>275</td>
<td>325</td>
</tr>
<tr>
<td>100</td>
<td>200</td>
<td>325</td>
<td>375</td>
</tr>
<tr>
<td>75</td>
<td>225</td>
<td>375</td>
<td>400</td>
</tr>
<tr>
<td>150</td>
<td>300</td>
<td>325</td>
<td>350</td>
</tr>
</tbody>
</table>

- = center of vanes.
@ = axis of blower.

Figure 5.5.

Meter readings obtained over a plane perpendicular to the axis of a blower. The diagram is to scale; the dots represent the axis of the vanes in their various positions. The blower is behind the diagram.
has already been mentioned. To illustrate this, in figure 5.6, are given graphs of the meter readings recorded on the most sensitive range every five seconds for four horizontal positions of the vanes in a plane perpendicular to the blower axis and moving away at right angles to the axis. This was carried out in a room with the windows and door open to create air movements. The variations in the readings with time were due to light air currents (detected by smoke) moving the jet of air from the blower across the vanes. The graphs illustrate the detail obtainable with the instrument.

At the time the anemometer that has been described was designed, photo-electric cells that were available all required a high intensity of illumination and amplifiers to produce outputs suitable for the operation of switching devices. A truly portable, robust anemometer using a shutter attached to the vane shaft and designed to 'chop' a light beam falling on a photo-cell was, therefore, impracticable. Recently several new types of photoconductive cells have become available. These cells when connected to a low voltage source pass a relatively large current when exposed to a relatively low intensity of illumination. For example, the Hilger cells, which have a polycrystalline, photoconductive layer of cadmium-selenide on an insulating base have a sensitivity of several amperes per lumen and that of Standard Telephones and Cables, which is a germanium junction cell, has a sensitivity of some 30 mA per lumen. These figures can be compared with the maximum
Figure 5.6.
Meter readings plotted against time with the anemometer vanes in an air stream of variable velocity.
available from a Mullard, gas-filled, emission cell of 150 µA per lumen.

The new type cells can be used to operate a relay directly and, particularly the Hilger type, the requisite intensity of illumination does not require to be very high. Evidently it should be possible to produce a simple, portable anemometer using one of these photo-cells. Such an anemometer should have the following advantages:

(i) no contacts in the anemometer head requiring cleaning,
(ii) a negligible restraining torque and hence a very rapid response to wind velocities over the whole of the requisite range 0 - 10 m.p.h. It was envisaged that an electro-mechanical counter would be used for the very low wind velocities and a micro-ammeter for the values where n was high enough for steady readings.

Some progress has been made with the system using a shutter on the vane shaft and an Elliott type R 350 B moving coil relay operated by the current passed by the photo-cell. Unfortunately a difficulty has arisen with the relay circuit. When the current value has been adjusted, by means of the value of the low voltage source and/or the intensity of illumination, so that the relay responds to the high chopping rate at high wind velocities, the relay will not 'drop out' (i.e. break circuit) during the dark periods at low wind velocities. Likewise when adjusted to operate in a satisfactory manner at low wind velocities, the relay misses impulses at high
wind velocities which leads to very erratic deflections of the micro-ammeter.

The solution would appear to be to use a polarised relay. The delivery time of these relays is so long that it has not yet been possible to test the correctness of this supposition. There is little doubt, however, from the work so far carried out that the arrangement will eventually be successful, but will remain more expensive and probably less robust than that described earlier.
SECTION 6.

THE MEASUREMENT OF HIGH RESISTANCE VALUES.
The measurement of skin and rectal temperatures of small laboratory animals by methods depending on resistance change is not always satisfactory due to the small size of the sensitive element required and, therefore, its small resistance. As variations in the resistance of sliding or rotating contacts would be a source of large errors, temperature measurement in mobile animals is virtually impossible. It seemed possible that the difficulty might be overcome if the sensitive element were made of an insulating material of extremely high specific resistance so that a tiny unit might have a resistance of the order of $10^8 - 10^9$ ohms. The resistance of most insulators decreases rapidly with increase of temperature. A variation of a few degrees in temperature may produce a 25 to 50% change in resistance (6.1., 6.2.). Some simple method of measuring such a high resistance would be required, preferably a method giving continuous readings and capable of measuring to better than $\pm 1\%$ which would be equivalent to a temperature measurement accurate to better than $\pm 0.4^\circ\text{C}$. (Since this work was started, Randall and Stow (6.3.) have shown that a thermistor bead can be mounted in a hypodermic needle and that the arrangement combined with a Wheatstone bridge will measure temperatures to $\pm 0.1^\circ\text{C}$ with very great rapidity. Addink (6.4.) has used a ceramic semi-conducting material with a high negative temperature coefficient of resistance with a Wheatstone bridge with very similar results. Neither
arrangement, however, enables temperature measurements to be made via a movable link so that small animals would need to be restrained.)

This section deals with modifications to the continuous ballistic method for capacitance measurement which enable the method to be used to measure high resistance values simply, reasonably accurately, and continuously.

The Siemens' circuit has been used in two ways:— (1) by using the circuit to measure the repetition frequency of a thyatron relaxation oscillator, the frequency of the oscillator depending on the value of the high resistance and (2) by leaking charge away through the high resistance during the period of time between charge ending and discharge commencing. As some of the circuits described are not entirely satisfactory the many measurements made when developing these circuits have been, in general, omitted and only those which illustrate particular points included.

Before this new work there follows a short account of the various methods which have been used to measure high resistance values and the reasons why they were not considered suitable for the purpose outlined above.

Resistance bridge methods are generally considered unsuitable for the measurement of resistance values above $10^6$ to $10^7$ ohms (1.7., 2.2., 6.5.) although one method which seems quite satisfactory, but appears to have been forgotten, is to have a condenser and
a galvanometer to detect an out of balance condition. The condenser is allowed to charge from the out of balance potential difference for a length of time sufficient to collect a reasonable charge, and then discharged through the galvanometer. The sensitivity would appear to be virtually independent of the resistance values if complete charge is allowed to occur. The method is described by Price (6.6.). In its simple form the method is not satisfactory for resistance values above $10^7$ ohms because of the length of time required to obtain balance. It has been shown by the present author (6.7.) that this can be overcome by automatic, regular switching and the system has been used to measure resistance values up to $10^{10}$ ohms. However, if the resistance short circuits serious damage to the galvanometer results and the method is not, therefore, suitable for work on experimental thermometers.

Another method of extending the range of the Wheatstone bridge has been developed by Glass and Abele (6.8.). This involves the use of a valve voltmeter as the out of balance detector. However, very great care is required concerning the materials used in the construction of this voltmeter in order to avoid leakage, and its construction is not possible in a small laboratory. The modified bridge is said to measure resistances up to $10^{13}$ ohms with 1% accuracy.

There are four simple, standard methods for the determination of high resistance values.
(i) Direct deflection using a source of known E.M.F. and a calibrated sensitive galvanometer of resistance G in series with the high resistance X. If the E.M.F. of the battery is E and its internal resistance B, then, if the current is I, \( X = \frac{E}{I} - (B + G) \). The method cannot be said to be as accurate as that which follows although Scroggie (6.9.) and others (6.10.,6.11.) describe analogous methods which enable values between \( 10^6 \) and \( 10^{10} \) ohms to be measured to \( \pm 1\% \). These involve a carefully designed and constructed valve voltmeter put across a known resistance in series with the unknown high resistance and the battery.

(ii) This method was devised by T. and A. Gray (2.1.), and Loeb (6.5.) considers it to be reliable to about \( 1\% \). Two measurements are made. A battery of between 100 and 200 volts E.M.F. is connected in series with the high resistance X and a sensitive galvanometer as in figure 6.1.(a) and the deflection of the galvanometer noted. The circuit of figure 6.1.(b) is then connected up using the same source of E.M.F. with a known resistance R in series. R must be very much greater than B, the internal resistance of the battery. As \( R \ll X \) a known shunt S is placed across the galvanometer and adjusted to give the same deflection as was obtained before. Then it follows that \( X = R + G(B + R)/S \) or, as \( R \gg B \), without sensible error, \( X = R + RG/S \).

A modification of this method has
Figure 6.1.

Circuit diagrams for the measurement of high resistance values as devised by T. and A. Gray.
been described (6.7.) which involves the use of a Rayleigh potentiometer as shown in figure 6.2. The potentiometer is adjusted for a convenient deflection of the galvanometer with the unknown resistance $X$ in series and the value of $R_2$ noted. The known resistance $R$ is substituted and the potentiometer adjusted for the same galvanometer deflection as was obtained with $X$. $R'_2$ is noted. Then, if $Z$ is the resistance of the potentiometer,

$$\frac{R_2}{Z(X + G) + R_1R_2} = \frac{R'_2}{Z(R + G) + R'_1R'_2}$$

As $G$ is very much less than $X$ and $R$, and $R_1R_2$ and $R'_1R'_2$ very much less than $ZX$ and $ZR$ respectively this expression reduces to $X = \frac{R_2R}{R'_2}$.

This modification avoids the excessive damping of the galvanometer which results if high resistances of known value are not available for the Gray method, but has been found unsatisfactory above $10^3$ ohms. Moreover, it would be unwise to use such a circuit with experimental resistance thermometers liable to short circuit.

(iii) This is a method which seems to have been devised by W. and C.W. Siemens (6.12.). A condenser is charged to a particular potential difference and placed in parallel with the unknown resistance $X$ and an electrostatic voltmeter. As charge leaks away through the high resistance the potential difference indicated by the voltmeter falls from $V_0$ to $V_t$ in time $t$. Then it can be
Figure 6.2.

A modified form of the circuits of figure 6.1.
shown that, if the natural leakage resistance is so much greater than $X$ that it can be neglected, $X = t \log_e \left( \frac{V_0}{V_t} \right) / C$.

Normally the natural leak would have to be determined and allowed for, or eliminated by repeating the experiment with a known resistance $R$.

Loeb considers the method to be good to 1-2%, but it has been found that some care in the choice of condenser is necessary to achieve this, presumably because of the absorption current due to the rearrangement of charges in the dielectric. With resistances of $10^9$ and $10^{10}$ ohms completely false values were obtained using large $1 \mu F$ paper condensers. Normal results were obtained with small $0.1 \mu F$ paper condensers.

(iv) The ballistic discharge method probably worked out by Siemens and, later, Alemenčič (6.13.).

This method is briefly described in section 2 and is probably best known for its use by Sabine (6.14.) in finding the velocity of projectiles. The natural leak of the condenser has to be allowed for or eliminated with a known high resistance, but the method is generally preferred to (iii) and Loeb states that it is good to 0.5%, but that it is only useful for resistances above $10^8$ ohms and requires capacitances of the order of microfarads. This is to enable reasonably long values of $t$ to be used so that they may be measured accurately. A difficulty with this method is the limitation on the voltage that can be applied across the resistance. Most high value resistances have
marked voltage coefficients and these may be required.

Methods (i) and (ii) become less and less useful as the value of resistance increases and it is impossible with methods (iii) and (iv) to obtain continuous readings of resistance.

Only one valve megohmmeter seems to have been devised (6.15.). This, when calibrated with known resistances, is claimed to give an accuracy of $\pm 2\%$ up to $10^{10}$ ohms, but the complication of the circuit would be unwarranted for the purpose in mind here. This is also true of a valve bridge devised by Smith (6.16.) which is more elaborate still with an accuracy of $\pm 1\%$ to $10^{12}$ ohms. The methods used in standardizing laboratories (6.17.) and the electrometer methods used to measure the electrical resistance of insulating materials (6.18.) are precise, but very elaborate and slow.

As has already been mentioned, the variation in the frequency of a relaxation oscillator with the variation in resistance value has been investigated as a possible means of comparing high resistance values. The frequency of a gas-filled triode (thyatron) relaxation oscillator of the type shown in figure 6.3. is given by

$$f = \frac{1}{RC \log_{e} \left\{ (E_b - V_a)/(E_b - V_f) \right\} } \quad (2.27.)$$

where $E_b =$ battery voltage, $E_a =$ tube drop (= ionisation voltage of the gas in the valve), $V_f =$ firing voltage which depends on $E_g$ the bias voltage. This is so only if the discharging time of the condenser $C$ is negligible in comparison with the
Figure 6.3.
Simple gas-filled triode (thyratron) relaxation oscillator.

Figure 6.4.
Circuit diagram of the megohmmeter.
\( V_1 \) = GT1C.
\( V_2 \) = GT1C.
\( R_1 \) = 47,000 ohms.
\( R_2 \) = unknown.
\( R_3 \) = 47,000 ohms.
\( R_4 \) = 1000 ohms.
\( T \) = 3:1 intervalve transformer.
\( P_1 \) = metal rectifier.
\( P_2 \) = metal rectifier.
\( C_1/C_2/C_3 \) = interelectrode capacitance/0.00005\( \mu F \)/

0.0005\( \mu F \).
\( C_4 \) = 0.05 or 0.1\( \mu F \).
\( E_1 \) = 9 volt battery.
\( E_2 \) = 220 volt battery.
\( E_3 \) = 16.5 volt battery.
\( E_4 \) = 48 volt battery.
\( M \) = Ferranti. 0-100 \( \mu A \). 596 ohms.

Table 5.1.

Details of the circuit of figure 6.4.
charging time. This condition is satisfied because the discharging current is limited only by a small circuit impedance in series with the valve, whereas charging occurs through a high resistance \( R \). As \( C \) and the logarithmic term can be kept constant, the frequency of the oscillator can be made inversely proportional to \( R \). The frequency can be determined in terms of a discharge current \( I \) using the Siemens' circuit, and if \( I_X \) and \( I_R \) are the respective currents for an unknown resistance \( X \) and a known resistance \( R \), then \( I_X/I_R = f_X/f_R = R/X \).

Some difficulty was experienced in producing a circuit which operated in a satisfactory manner for resistance values from \( 10^6 \) to \( 10^9 \) ohms, but the reasonably straight-forward circuit given in figure 6.4. has been developed and investigated. The values of the oscillator capacitor can be varied to cover the required range of resistance values. To simplify the circuit the frequency meter used is not that of Guarnaschelli and Vecchiachi (2.25.) but a modified form of that described by Walker (2.29.). The condenser \( C_4 \) charges while the gas-filled triode \( V_2 \) is non-conducting and discharges through \( V_2 \) when that valve is triggered by a positive pulse from the oscillator through the transformer. When the potential difference across \( C_4 \) falls to a value approximately equal to the ionisation voltage of the gas in \( V_2 \) that valve extinguishes. The two metal rectifiers \( P_1 \) and \( P_2 \) ensure that only the discharge pulses pass through the meter. A steady
current is indicated provided the frequency is high enough.

To obtain a high enough frequency with the higher resistance values it was found necessary to have an oscillator capacitance as low as 50 pF with the result that $E_2$ had to be at least 200 volts to provide a trigger voltage high enough for stable operation of the frequency meter. $E_4$ was found to have a maximum value of 60 volts otherwise the frequency meter became independent of the trigger frequency.

Meter readings were taken with various combinations of resistors in series and parallel. For values above $3 \times 10^7$ ohms, Weimey glass enclosed resistors were used and carbon resistors for lower values. In figure 6.5, the current as indicated by a Universal Avometer, model 7, (a suitable long period meter) is plotted against the reciprocal of the resistance value in the oscillator. The linear relation is clearly confirmed. A careful investigation at high values of resistance with a micro-ammeter shows, however, (figure 6.6.) that there is some curvature. This means that a calibration curve should be used with the circuit if an extended scale is being used to obtain accuracy at high resistance values. If the circuit is only being used to obtain an indication of the resistance value then the simple relation can be used as is further confirmed in figure 6.7, where the micro-ammeter readings obtained with the meter shunted are shown. Again straight lines very nearly passing through the origin were obtained.
Figure 6.5.

Discharge current plotted against $1/R$.

(Oscillator capacitance = 0.00005 $\mu$F., frequency meter capacitance = 0.1 $\mu$F.)
Figure 6.6.

Discharge current plotted against $1/R$.

(Oscillator capacitance = 0.00005 μF., frequency meter capacitance = 0.05 μF.)
Figure 6.7.

Meter reading plotted against $1/R$.

(Oscillator capacitance = inter-electrode, frequency meter capacitance = 0.05 μF. ⊘ = 300 ohm shunt across meter, □ = 200 ohm shunt across meter.)
A series of experiments of this nature with various oscillator capacitances (including the inter-electrode capacitance alone), various settings of the bias $E_1$ and various shunt values across the meter $M$ showed that although the circuit had great flexibility the inevitable cramping of the scale at the high resistance end made it unsuitable for use with a high resistance thermometer.

As a possible means of obviating this it was decided to investigate the possibility of using the requisite shunt value across the meter for a constant meter reading as a measure of the resistance in the oscillator. In figure 6.8, the necessary shunt value obtained with a constant meter reading of 10 $\mu$A is plotted against the value of the resistance $R_2$. The result was as hoped, a straight line with the sensitivity decreasing at the low resistance end where it does not matter. Unfortunately near $10^9$ ohms the straight line changed to a pronounced curve and no adjustment of the circuit would remove this fault. However, if a thermometric unit can be constructed with a resistance of the order of $5.10^8$ ohms at 30°C then the circuit may prove useful. The circuit has the advantages that it is silent in operation and does not require a sensitive galvanometer as the indicating instrument. Moreover, if the discharge current is used to indicate resistance as in figure 6.5, the current is high enough to operate a pen recorder.
Figure 6.8. Meter shunt value plotted against resistance.
(Oscillator capacitance = inter-electrode, frequency meter capacitance = 0.05 pF.)
The other approach that has been tried has been to utilise the fact that, in the Siemens method for the determination of capacitance, a resistance placed across the condenser reduces the indicated current. This is because charge is lost through the resistance between charge ending and discharge commencing.

If the leakage resistance of the condenser be $R$ it was shown in section 1 that, provided the other conditions were satisfied, the discharge current would be

$$I_d = nEC \cdot \exp\left(-\frac{t}{CR}\right),$$

if $t$ is the transit time from charge ending to discharge commencing. If $S$ is the value of the resistor placed across the condenser in parallel with $R$, the current indicated by the galvanometer would be

$$I_s = nEC \cdot \exp\left[-\frac{t(R+S)}{KSC}\right].$$

It is evident that various values for $S$ could be used and the scale of the galvanometer thus calibrated in terms of $S$. If $S$ should short circuit no damage would be done to the galvanometer as the condenser would not charge.

To determine whether the theory held, and also to investigate various ways of applying it, known resistance values up to $10^7$ ohms were used with very simple relay circuits. The circuit shown in figure 6.9. was the first to be used. (The broken lines refer to a modification described later.)

Two Siemens single action, high speed relays $A$ and $B$ are driven from a filament transformer
Figure 6.9.

First circuit for the measurement of resistance.
T through small metal rectifiers $M_1$ and $M_2$ so connected that the relays operate on alternate half-cycles of the alternating current. As was mentioned in section 2, the relays do not operate immediately the current through the coils rises from zero. This means that the relay A will switch the condenser from the charging position and then, some milliseconds later, the relay B will operate and connect the condenser to the galvanometer G. If reference be made to figure 2.8, it will be seen that the delay time between charge ending and discharge commencing is $(b+t_d+a)$ which should be approximately $5\times10^{-3}$ seconds. During this time the condenser will discharge through its leakage resistance $R$ and also the resistance $S$ to be measured in parallel.

Unless the galvanometer scale is calibrated with known resistors, before resistances can be measured it is necessary to know the delay time $t$. This was determined with a $10^5$ ohm decade resistance box and a 0.1 µF condenser. Ideally this should be a condenser with no absorption, but the work described in sections 2 and 3 has shown that good quality commercial condensers show little variation in the value of their capacitance whether determined by methods affected by absorption or by a.c. bridges to the limits of accuracy required.

Readings of the current were taken for various values of the resistance box which was in the position of the resistance $S$ in figure 6.9.
As $I_d = nEC \exp \left[ -t(R+S)/RS \right]$, it follows that $\log e I_d = \log e nEC - t/CS - t/CR$. Thus, if $\log e I_d$ be plotted against $1/S$, a straight line should result. From the slope, $t$ may be calculated if $C$ is known. No error in the value of $t$ is produced by natural leakage from the condenser.

It is important that $S$ must not be so low that the conditions 1 and 4 of section 1 are not satisfied. The lowest value is about 40,000 ohms if an error of less than 0.1% is required when a 350 ohm galvanometer with a sensitivity of 22 mm/µA is used.

An early graph obtained with the apparatus is shown in figure 6.10. It will be seen that a good straight line was obtained with a transit time of $8.7(0) \times 10^{-3}$ seconds. This was obtained with the gaps of the relays large so that the apparatus was noisy. When the relays were adjusted to give a good, steady note graphs of the type also shown in figure 6.10. were obtained. The graph given indicated a transit time of $5.43 \times 10^{-3}$ seconds.

The constancy of the value of $t$ is good. To illustrate this in table 6.2. galvanometer deflections taken during the day are listed, the apparatus being switched off in between each run.

Once $t$ has been obtained, the apparatus can be used to determine any unknown resistance within the calibration range or of greater value. It is convenient to have a series of condensers of known value and to use a slightly different technique from that used
Figure 6.10.

Graphs of $\log_e I_d$ plotted against $1/S$ for two adjustments of relays with a 0.1 $\mu$F condenser.
### Table 6.2.

Galvanometer readings for various resistances across a condenser.

<table>
<thead>
<tr>
<th>Resistance across 0.1 μF condenser ($10^5$ ohms)</th>
<th>Series 1</th>
<th>Series 2</th>
<th>Series 3</th>
<th>Series 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.00</td>
<td>5.05</td>
<td>5.00</td>
<td>5.00</td>
<td>5.00</td>
</tr>
<tr>
<td>0.95</td>
<td>4.90</td>
<td>4.85</td>
<td>4.90</td>
<td>4.90</td>
</tr>
<tr>
<td>0.90</td>
<td>4.70</td>
<td>4.70</td>
<td>4.70</td>
<td>4.70</td>
</tr>
<tr>
<td>0.85</td>
<td>4.55</td>
<td>4.50</td>
<td>4.50</td>
<td>4.50</td>
</tr>
<tr>
<td>0.80</td>
<td>4.30</td>
<td>4.30</td>
<td>4.30</td>
<td>4.30</td>
</tr>
<tr>
<td>0.75</td>
<td>4.10</td>
<td>4.05</td>
<td>4.10</td>
<td>4.10</td>
</tr>
<tr>
<td>0.70</td>
<td>3.85</td>
<td>3.80</td>
<td>3.80</td>
<td>3.85</td>
</tr>
</tbody>
</table>
for the determination of $t$.

The procedure is to determine the current $I$ without the unknown resistance in place, and the current $I_p$ with the resistance $P$ in place across the condenser of capacitance $C$. Then $I = nEC \exp(-t/CR)$ and $I_p = nEC \exp \left(-t(R+P)/CRp \right)$. Thus $\log_e(I/I_p) = t/CP$ or $P = t/ \left[C \log_e(I/I_p) \right]$.

In table 6.3 are given the resistances that will produce currents $I_p$, which are certain fractions of $I$, with condensers of various capacitance. The delay time has been taken as $5 \times 10^{-3}$ seconds in the calculations.

It will be noticed that the ranges overlap which means that a decade set of condensers enable resistances to be measured over the complete range. It also means that, where the resistance values change rapidly with deflection, the scale can be extended by switching to the next smaller condenser.

An interesting point is that the galvanometer can be made to read resistance values directly by the use of a suitable scale and a condenser selector indicating for each condenser the factor by which the scale has to be multiplied.

For maximum accuracy, or if a direct reading scale is being used, it is necessary to vary $E$ to give a full scale reading with the particular condenser being used when there is no resistance across the condenser.
<table>
<thead>
<tr>
<th>C μF</th>
<th>( I_p/I )</th>
<th>0.9</th>
<th>0.8</th>
<th>0.7</th>
<th>0.6</th>
<th>0.5</th>
<th>0.4</th>
<th>0.3</th>
<th>0.2</th>
<th>0.1</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Resistance</td>
<td>48</td>
<td>22</td>
<td>14</td>
<td>9.7</td>
<td>7.2</td>
<td>5.5</td>
<td>4.2</td>
<td>3.1</td>
<td>2.2</td>
</tr>
<tr>
<td>1.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.1</td>
<td></td>
<td>( \times 10^3 )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.01</td>
<td></td>
<td>( \times 10^4 )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.001</td>
<td></td>
<td>( \times 10^5 )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.0001</td>
<td></td>
<td>( \times 10^6 )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.00001</td>
<td></td>
<td>( \times 10^7 )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.000001</td>
<td></td>
<td>( \times 10^8 )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 6.3.

The resistance in ohms necessary to produce certain values of \( I_p/I \) with particular values for C. (\( t = 0.005 \) seconds.)
This method of using the circuit is very convenient but the least accurate, so only a few experimental results will be given. If there is an indeterminacy in reading the galvanometer scale of \( \pm \beta \), then

\[
P = \left[ \frac{t}{C \log_e(I/I_p)} + \beta (1/I - 1/I_p) \right].
\]

As \( I_p \) is of the order of \( I/2 \) the method cannot be considered an accurate one. Figures are given in table 6.4. of the possible error, \( I \) being taken as equivalent to a full scale deflection of 100 mm and \( \beta \) as \( \pm 0.2 \) mm.

To check the performance of the circuit, resistance values were determined using it and also using the methods mentioned in the beginning. The results are given in table 6.5.

The necessity of calibrating the galvanometer was immediately apparent when the results were first calculated. The use of a calibration curve, even when it is plotted on a large scale, renders the results even less reliable. In order to eliminate the effect of the calibration curve error as far as possible, the galvanometer was calibrated twice using a known, fixed resistance in series with the galvanometer and measuring the potential difference across the resistance with a Baldwin D.C. potentiometer. Two large calibration curves were drawn. The readings were corrected on each of these and the average of the corrected readings used in the calculations. The readings are given in table 6.6. and one of the calibration curves is shown on a small scale with only a few points
<table>
<thead>
<tr>
<th>$I_p$ (mm)</th>
<th>Possible error (±%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>95</td>
<td>8</td>
</tr>
<tr>
<td>90</td>
<td>4</td>
</tr>
<tr>
<td>80</td>
<td>2</td>
</tr>
<tr>
<td>70</td>
<td>1.3</td>
</tr>
<tr>
<td>60</td>
<td>1</td>
</tr>
<tr>
<td>50</td>
<td>0.9</td>
</tr>
<tr>
<td>40</td>
<td>0.8</td>
</tr>
<tr>
<td>30</td>
<td>0.7</td>
</tr>
<tr>
<td>20</td>
<td>0.7</td>
</tr>
</tbody>
</table>

Table 6.4.

Possible error in value of resistance $P$ due only to estimating parts of divisions. (Full scale = 100 mm., $\beta = \pm 0.2$ mm.)
Resistance by experiment. Expected Actual

<p>| | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1.151.10^6</td>
<td>0.1</td>
<td>1.22.10^6</td>
<td>1.20.10^6</td>
<td>8</td>
<td>+4</td>
</tr>
<tr>
<td>1.054</td>
<td>1.14</td>
<td>1.10</td>
<td>+5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.058</td>
<td>1.11</td>
<td>1.06</td>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.05</td>
<td>1.18</td>
<td>1.11</td>
<td>4</td>
<td>-3</td>
<td></td>
</tr>
<tr>
<td>1.03</td>
<td>1.08</td>
<td>1.04</td>
<td>-1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.09</td>
<td>1.05</td>
<td>-1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.01</td>
<td>1.22</td>
<td>1.14</td>
<td>1</td>
<td>-1</td>
<td></td>
</tr>
<tr>
<td>1.13</td>
<td>1.05</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.14</td>
<td>1.06</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4.20.10^6</td>
<td>0.05</td>
<td>4.40</td>
<td>4.16</td>
<td>8</td>
<td>-1</td>
</tr>
<tr>
<td>4.117</td>
<td>4.57</td>
<td>4.48</td>
<td>+7</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4.08</td>
<td>4.07</td>
<td>4.00</td>
<td>-2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.01</td>
<td>4.48</td>
<td>4.36</td>
<td>4</td>
<td>+4</td>
<td></td>
</tr>
<tr>
<td>4.41</td>
<td>4.26</td>
<td>+2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4.28</td>
<td>4.14</td>
<td>+1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.86.10^7</td>
<td>0.01</td>
<td>0.90.10^7</td>
<td>0.85.10^7</td>
<td>8</td>
<td>-1</td>
</tr>
<tr>
<td>1.03</td>
<td>1.02</td>
<td>0.99</td>
<td>-3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.99</td>
<td>1.00</td>
<td>0.96</td>
<td>-3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.001</td>
<td>0.98</td>
<td>0.87</td>
<td>1</td>
<td>+1</td>
<td></td>
</tr>
<tr>
<td>1.11</td>
<td>1.03</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.08</td>
<td>0.99</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 6.5.
Resistance values obtained by the first method using the first circuit.
<table>
<thead>
<tr>
<th></th>
<th>Resistance</th>
<th>C. Deflection</th>
<th>Corrected deflection</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>1.</td>
<td>2. Ave.</td>
</tr>
<tr>
<td>μF</td>
<td>cm.</td>
<td>cm.</td>
<td>cm.</td>
</tr>
<tr>
<td>B 1</td>
<td>0.1</td>
<td>9.6</td>
<td>9.55</td>
</tr>
<tr>
<td>B 3</td>
<td></td>
<td>9.56</td>
<td>9.51</td>
</tr>
<tr>
<td>B 1</td>
<td>0.05</td>
<td>9.15</td>
<td>9.10</td>
</tr>
<tr>
<td>B 2</td>
<td></td>
<td>9.08</td>
<td>9.04</td>
</tr>
<tr>
<td>B 3</td>
<td></td>
<td>9.09</td>
<td>9.05</td>
</tr>
<tr>
<td>B 1</td>
<td>0.01</td>
<td>6.5</td>
<td>6.2</td>
</tr>
<tr>
<td>B 2</td>
<td></td>
<td>6.26</td>
<td>6.0</td>
</tr>
<tr>
<td>B 3</td>
<td></td>
<td>6.29</td>
<td>6.02</td>
</tr>
<tr>
<td>A 1</td>
<td>0.01</td>
<td>9.43</td>
<td>9.4</td>
</tr>
<tr>
<td>A 2</td>
<td></td>
<td>9.5</td>
<td>9.5</td>
</tr>
<tr>
<td>A 3</td>
<td></td>
<td>9.49</td>
<td>9.47</td>
</tr>
<tr>
<td>A 1</td>
<td>0.001</td>
<td>6.28</td>
<td>5.95</td>
</tr>
<tr>
<td>A 2</td>
<td></td>
<td>6.74</td>
<td>6.45</td>
</tr>
<tr>
<td>A 3</td>
<td></td>
<td>6.64</td>
<td>6.35</td>
</tr>
<tr>
<td>C 1</td>
<td>0.05</td>
<td>9.76</td>
<td>9.76</td>
</tr>
<tr>
<td>C 3</td>
<td></td>
<td>9.75</td>
<td>9.75</td>
</tr>
<tr>
<td>C 1</td>
<td>0.01</td>
<td>8.89</td>
<td>8.85</td>
</tr>
<tr>
<td>C 2</td>
<td></td>
<td>8.87</td>
<td>8.82</td>
</tr>
<tr>
<td>C 3</td>
<td></td>
<td>8.84</td>
<td>8.80</td>
</tr>
</tbody>
</table>

Table 6.6.

Experimental and corrected readings. (Full scale deflection of 10 cm. with leakage resistance alone.)
plotted in figure 6.11.

The condenser values given are only nominal. The actual ratios of capacitance between the condenser used for determining $t$ and those used with the particular resistances were determined both with a series resistance a.c. bridge at 1000 cycles second$^{-1}$ and with the null circuit of section 3.

It was found that it was not necessary to use the calibration curve when determining $t$ when sufficient points were taken. For example, when $t$ was determined for some measurements it was found to be $6.41 \times 10^{-3}$ seconds using uncorrected values and $6.39 \times 10^{-3}$ seconds using corrected values.

The results indicate that practice follows theory and that the method is reliable to 1-2% from $10^5$ to $10^7$ ohms provided a calibration curve is used and a condenser value chosen so that $I_p/I < 0.8$.

The circuit had the disadvantage that there was a stray capacitance effect. All readings had to be taken when the operators hands were not touching the apparatus. To eliminate this the circuit of figure 6.12., using a Siemens double changeover relay type 100, was devised. The two tongues were so arranged that the condenser $C_1$ instead of being discharged through the galvanometer shares its charge with a large capacitance condenser $C_2$ through the galvanometer. (The broken lines refer to a modification described later.)
Figure 6.11.

Calibration curve of a galvanometer.
Second circuit for the measurement of resistance.
If the amount of charge that passes from $C_1$ to $C_2$ be $q$ and the charge on $C_1$ at the moment sharing commences is $Q$, then $q/Q = C_2/(C_1 + C_2)$. This expression gives the following values if $C_2$ be 1.0 µF:

<table>
<thead>
<tr>
<th>$C_1$ (µF)</th>
<th>0.1</th>
<th>0.01</th>
<th>0.001</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q/Q$</td>
<td>0.90</td>
<td>0.990</td>
<td>0.9990</td>
</tr>
</tbody>
</table>

It is evident that when $C_1$ is small the presence of the large condenser does not introduce an appreciable error in the discharge current. The large condenser is discharged during the half-cycle in which $C_1$ is being charged and can be shorted out by the switch $S$ if desired.

The adjustment of the relay is rather more difficult than in the first circuit but is very satisfactory, though noisy, when so adjusted. To obtain a reasonable value for $t$ it has been found that the gap $A$ should be larger than the gap $B$. With the relay used for this work the gaps, as determined with a vernier microscope, were 0.52 and 0.48 mm.

Some results obtained with this circuit are given in table 6.7. and it will be seen that they have a similar accuracy to those given in table 6.5.

An alternative method of using the circuit, which avoids using the leakage resistance reading and thus wasting scale space, is to work with a single, known resistance of value $S$ say. The transit time $t$ of the relay system having been obtained as described above, the
<table>
<thead>
<tr>
<th>Resistance value, ohms.</th>
<th>$uF.$</th>
<th>Corrected. ohms.</th>
<th>$%$</th>
<th>$%$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.151$\cdot 10^6$</td>
<td>0.1</td>
<td>1.13$\cdot 10^6$</td>
<td>7</td>
<td>-2</td>
</tr>
<tr>
<td>1.054</td>
<td></td>
<td>1.04</td>
<td>-1</td>
<td></td>
</tr>
<tr>
<td>1.058</td>
<td></td>
<td>1.04</td>
<td>-2</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.05</td>
<td>1.16</td>
<td>6</td>
<td>+1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.03</td>
<td>-1</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.03</td>
<td>-3</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.01</td>
<td>1.16</td>
<td>1</td>
<td>+1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.06</td>
<td>+1</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.07</td>
<td>+1</td>
<td></td>
</tr>
<tr>
<td>4.20$\cdot 10^6$</td>
<td>0.05</td>
<td>3.94</td>
<td>7</td>
<td>-6</td>
</tr>
<tr>
<td>4.17</td>
<td></td>
<td>3.24</td>
<td>-5</td>
<td></td>
</tr>
<tr>
<td>4.08</td>
<td></td>
<td>3.94</td>
<td>-3</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.01</td>
<td>4.38</td>
<td>4</td>
<td>+4</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4.38</td>
<td>+5</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>4.17</td>
<td>+2</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.001</td>
<td>4.27</td>
<td>1</td>
<td>+2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4.28</td>
<td>+3</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>4.12</td>
<td>+1</td>
<td></td>
</tr>
<tr>
<td>0.86$\cdot 10^7$</td>
<td>0.01</td>
<td>0.87$\cdot 10^7$</td>
<td>7</td>
<td>+1</td>
</tr>
<tr>
<td>1.03</td>
<td></td>
<td>1.05</td>
<td>+2</td>
<td></td>
</tr>
<tr>
<td>0.99</td>
<td></td>
<td>0.93</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

Table 6.7.

Resistance values obtained by the first method using the second circuit. ($t = 5.52\cdot 10^{-3}$ seconds.)
current for the unknown resistance $P$ across the condenser is measured. If this current be $I_p$ and the current with a known resistance $S$ across be $I_S$, then, if the leakage resistance of the condenser is $R$, $I_p = nEC \exp \left[ -t(R+P)/RPC \right]$, and $I_S = nEC \exp \left[ -t(R+S)/RSC \right]$. Dividing and collecting up terms gives $I_S/I_p = \exp \left[ t(S-P)/CSP \right]$ and thus it follows that $P = tS/\left[ t + CS \log_3(I_S/I_p) \right]$. The reciprocal form of this equation is, in practice, rather more convenient. That is $1/p = 1/S + \left[ C \log_e(I_S/I_p) \right] /t$.

The value of $P$ which should be used to determine $S$ depends on the precision required from the determination. This can be seen in the following manner.

The condensers must be considered to have no absorption and the galvanometer to be exactly calibrated so that the problem is reduced to the effect of the indeterminacy of taking readings on the galvanometer. The effect can be neglected in determining $t$ as $t$ is calculated from the slope of a straight line graph.

Suppose $I_S$ and $I_p$ can be determined to $\pm a$, then, as in section 1, $I_S/I_p$ must be replaced by $I_S/I_p \pm a(I_S/I_p + 1)/I_p$. Thus the equation above must be written as $I_S/I_p \pm a(I_S/I_p + 1)/I_p = \exp \left[ t(S-P)/CSP \right]$.

Now, if $(A+B) = \exp(y)$, then

$$\log_e A + \log_e (1 + B/A) = y.$$ Therefore it follows that

$$\log_e(I_S/I_p) + \log_e \left[ 1 + a(I_S/I_p + 1)/I_S \right] = t(S-P)/CSP.$$ 

On using the logarithmic series

$$\log_e(1 + x) = x - x^2/2 + x^3/3 \ldots \ldots\ldots\ldots ( -1 < x \leq 1 ),$$ and
neglecting powers greater than the first as negligible, this reduces to \( \log_e(I_s/I_p) + a(I_s/I_p + 1)/I_s = t(S-P)/CSP \). This may be rewritten in the reciprocal form

\[
\frac{1}{P} - \frac{1}{S} = C \log_e(I_s/I_p)/t + a(1/I_p + 1/I_s)/t.
\]

The possible error, expressed as a percentage, is, therefore,

\[
a(1/I_p + 1/I_s)
\]

\[
\times 100 \%
\]

\[
\frac{t}{CS + \log_e(I_s/I_p)}
\]

Table 6.3. has been calculated on the assumption that \( t = 5 \times 10^{-3} \) seconds, \( P = 10^9 \) ohms, the full scale deflection of the galvanometer is 100 mm and can be read to ±0.2 mm. It is also assumed that the E.M.F. is adjusted to give \( I_s \) as a full scale deflection and that \( I_s/I_p = 2 \). It can be seen that the resistance \( P \) could be determined with a \( 7 \times 10^6 \) ohm resistance as \( S \) but the value obtained would be virtually useless. With a capacitance of \( 10 \mu F \), however, the value could be determined with a \( 4.2 \times 10^8 \) ohm resistance and the value so obtained should be within approximately ±1%.

To determine if this theory regarding the galvanometer readings gave figures in agreement with those obtained in practice a series of readings was made using a \( 0.1 \mu F \) condenser and a decade resistance box accurate to ±0.1% as the resistance across the condenser.

The fact that the galvanometers available did not have deflections proportional to current meant that calibration curves had to be used. This reduced the precision of the results which are given in table 6.9.
### Table 6.8.

Possible error in measurement of a $10^9$ ohm resistance.

<table>
<thead>
<tr>
<th>C</th>
<th>S</th>
<th>Possible Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>FARADS</td>
<td>$10^6$ohms</td>
<td>$\pm\frac{2}{3}$</td>
</tr>
<tr>
<td>$10^{-9}$</td>
<td>7.1</td>
<td>60</td>
</tr>
<tr>
<td>$10^{-10}$</td>
<td>67.1</td>
<td>12</td>
</tr>
<tr>
<td>$10^{-11}$</td>
<td>912.4</td>
<td>1.2</td>
</tr>
<tr>
<td>$10^{-12}$</td>
<td>877.4</td>
<td>0.1</td>
</tr>
</tbody>
</table>
C = 0.1 \mu F.

Actual value of resistance \( P = 1.00(0) \times 10^5 \) ohms.

Galvanometer deflection for \( P = 100 \) mm.

<table>
<thead>
<tr>
<th>( 10^4 ) ohms</th>
<th>mm.</th>
<th>mm.</th>
<th>( 10^4 ) ohms</th>
<th>( \pm % )</th>
<th>( % )</th>
</tr>
</thead>
<tbody>
<tr>
<td>9.5</td>
<td>97.0</td>
<td>96.5</td>
<td>9.98</td>
<td>1</td>
<td>-0.2</td>
</tr>
<tr>
<td>9.0</td>
<td>93.5</td>
<td>92.0</td>
<td>10.13</td>
<td>1.3</td>
<td>-1.3</td>
</tr>
<tr>
<td>8.5</td>
<td>90.0</td>
<td>88.5</td>
<td>10.06</td>
<td>0.6</td>
<td>-1.6</td>
</tr>
<tr>
<td>8.0</td>
<td>86.5</td>
<td>84.5</td>
<td>10.00</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>7.5</td>
<td>82.5</td>
<td>80.5</td>
<td>9.89</td>
<td>1.1</td>
<td>-1.1</td>
</tr>
<tr>
<td>7.0</td>
<td>78.0</td>
<td>75.0</td>
<td>9.98</td>
<td>0.2</td>
<td>-0.2</td>
</tr>
</tbody>
</table>

Table 6.9.

Experimental results for a resistance value determined by the use of another resistance of known value.
which are in substantial agreement with theory. These were obtained with the first circuit.

More results are given in table 6.10. These were obtained using a 0.001 μF condenser and carbon resistances previously measured with a Wheatstone network. The second circuit was used and, again, substantial agreement between theory and practice was obtained.

A reasonably sensitive galvanometer is required when measuring high resistances. Using the figures for a $10^9$ ohm resistance above and assuming that the insulation resistance of the condenser is $10^{10}$ ohms and that the value of $E$ is 130 volts the galvanometer would have to have a current sensitivity of 1923 mm/μA. This is not exceptional as may be seen from some specimen figures:

<table>
<thead>
<tr>
<th>Manufacturer</th>
<th>Sensitivity (mm/μA)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cambridge</td>
<td>6000</td>
</tr>
<tr>
<td>Pye</td>
<td>6000</td>
</tr>
<tr>
<td>Sullivan</td>
<td>2000</td>
</tr>
<tr>
<td>Sullivan</td>
<td>8000</td>
</tr>
</tbody>
</table>

In practice it was found that a longer discharge time than is available with simple relay circuits was required if resistance values above $10^8$ ohms were to be measured. This is illustrated in figure 6.13, where galvanometer deflection is plotted against average resistance values (table 6.11.) determined by leakage, by the modified Wheatstone bridge and by the potentiometer method mentioned earlier. (Welmeg glass enclosed resistors
C = 0.001 μF.

Actual value of resistance P = 0.99 \times 10^7 \text{ ohms}.

<table>
<thead>
<tr>
<th>S.</th>
<th>Corrected deflections</th>
<th>P.</th>
<th>Expected calculated</th>
<th>error.</th>
<th>Actual</th>
</tr>
</thead>
<tbody>
<tr>
<td>10^7\text{ohms.}</td>
<td>mm</td>
<td>mm</td>
<td>10^7\text{ohms.}</td>
<td>±%</td>
<td>%</td>
</tr>
<tr>
<td>0.420</td>
<td>47.8</td>
<td>93.5</td>
<td>0.991</td>
<td>1</td>
<td>+0.1</td>
</tr>
<tr>
<td>0.525</td>
<td>60.3</td>
<td>93.6</td>
<td>0.993</td>
<td>+0.3</td>
<td></td>
</tr>
<tr>
<td>0.631</td>
<td>70.2</td>
<td>92.7</td>
<td>0.983</td>
<td>-0.7</td>
<td></td>
</tr>
<tr>
<td>0.825</td>
<td>84.3</td>
<td>93.2</td>
<td>0.994</td>
<td>+0.4</td>
<td></td>
</tr>
</tbody>
</table>

Actual value of resistance P = 2.01 \times 10^7 \text{ ohms}.

| 0.837 | 69.0 | 96.6 | 1.980 | 2 | -1.5 |
| 0.870 | 70.6 | 96.7 | 1.972 | -2 |
| 1.278 | 84.0 | 96.6 | 2.012 | +0.1 |
| 1.695 | 92.6 | 96.6 | 1.988 | -1 |

Table 6.10.

Experimental results for a resistance value determined by the use of another resistance.
Figure 6.13.

Galvanometer deflection plotted against resistance.
<table>
<thead>
<tr>
<th>Nominal value. (± 10%)</th>
<th>Potentiometer method.</th>
<th>Leakage.</th>
<th>Wheatstone bridge.</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.3</td>
<td>3.3(7)</td>
<td>3.4(0)</td>
<td>3.3(5).10^7</td>
</tr>
<tr>
<td>5</td>
<td>5.0(2)</td>
<td>5.1(3)</td>
<td>5.0(2).10^7</td>
</tr>
<tr>
<td>8.3</td>
<td>8.4</td>
<td>8.4</td>
<td>8.4 .10^7</td>
</tr>
<tr>
<td>1</td>
<td>1.03</td>
<td>0.99</td>
<td>1.02 .10^8</td>
</tr>
<tr>
<td>1.5</td>
<td>1.5(2)</td>
<td>1.5(6)</td>
<td>1.5(4).10^8</td>
</tr>
<tr>
<td>3.3</td>
<td>3.0(3)</td>
<td>2.9(0)</td>
<td>3.0(6).10^8</td>
</tr>
<tr>
<td>5</td>
<td>-</td>
<td>4.5(3)</td>
<td>4.5(6).10^8</td>
</tr>
<tr>
<td>6</td>
<td>-</td>
<td>6.0(1)</td>
<td>5.9(0).10^8</td>
</tr>
<tr>
<td>1</td>
<td>-</td>
<td>0.90</td>
<td>0.91 .10^9</td>
</tr>
<tr>
<td>1.5</td>
<td>-</td>
<td>1.4(0)</td>
<td>1.37 .10^9</td>
</tr>
</tbody>
</table>

Table 6.11.

Resistance values determined by three different methods.
were used for all values above $3 \times 10^7$ ohms.)

The circuit of figure 6.12 was used and the lack of sensitivity at the higher values is very evident although the air condenser used had a capacitance of only 5 pF and the relay was adjusted for the maximum delay time possible.

In view of this a more elaborate relay circuit was designed to give a long, adjustable delay. The circuit is shown in figure 6.14. A thyatron relaxation oscillator sends a pulse through the coil of the relay $B_2$ and the relay closes allowing the air condenser $C_2$ to charge. The upper tongue of $B_2$ causes $B_3$ to close. The lower tongue of $B_3$ locks $B_3$ closed so that the next pulse from the oscillator passes through the upper tongue of $B_3$ and closes $B_1$. This closure of $B_1$ discharges the condenser $C_2$ through the galvanometer and also breaks the holding circuit of $B_3$ so that the system is ready to repeat the whole sequence.

In this way the time available for discharge depends on the frequency of the oscillator which is adjusted in this application by the value of the grid bias of the thyatron.

As may be seen in figure 6.15, the modification has been found satisfactory. Resistance values from $5 \times 10^7$ to $10^{10}$ ohms can be measured to an accuracy of approximately $\pm 2\%$ over the greater part of the range of the calibration curve.

With the low frequencies involved
Figure 6.14.

Relay circuit with a variable delay time.
\[ R_1 = 15 \text{ k} \Omega. \]
\[ R_2 = 47 \text{ k} \Omega. \]
\[ R_3 = 285 \text{ k} \Omega. \]
\[ C_1 = 2 \mu\text{F}. \]
\[ C_2 = 5 - 100 \text{ pF air condenser. Ceramic insulation.} \]
\[ B_1 = \text{P.O. relay type 3000. 1000A.} \]
\[ B_2 = \text{P.O. relay type 3000. 1000A.} \]
\[ B_3 = \text{Siemens relay type 100. 100A.} \]
\[ G = \text{Sullivan portable galvanometer, 460n, 2490 mm/\muA.} \]
\[ E_1 = 120 \text{ volt dry battery.} \]
\[ E_2 = 6 \text{ volt accumulator.} \]
\[ E_3 = 45 \text{ volt dry battery.} \]
\[ V = \text{gas filled triode GT1C.} \]

Table 6.11c.

Details of the circuit of figure 6.14.
Figure 6.15.

Galvanometer deflections plotted against resistance values.
it is evident that ideally the galvanometer should be of a long period, ballistic type, but it has been found that a sensitive reflecting galvanometer of two second period can be used provided it is shunted with a resistance of about double the value of its coil resistance. The results given have, in fact, been obtained in this way.

It has been shown in section 2 that the use of a Rayleigh potentiometer improves the precision of the Siemens method when capacitance ratios are determined and also removes the necessity of calibrating the galvanometer. It seemed possible that the method of determining resistance might also be improved by the use of the same device. The modification to the earlier circuits used is shown by broken lines in figures 6.9., and 6.12., the lead \( \ell ' \) being omitted and the resistance boxes \( X \) and \( Y \) forming a Rayleigh potentiometer of constant resistance \( Z \) inserted.

With the unknown resistance \( P \) across the condenser, the Rayleigh potentiometer is adjusted to give a particular current \( I \). Then, if \( X_p \) is the value of \( X \),

\[
I = \left\{ nX_pEC.exp\left[-t(R+P)/RBC\right]\right\}/Z \quad \text{provided that } X_p \text{ and } Z \text{ are much greater than the internal resistance of the battery.}
\]

With the known resistance \( S \) across the condenser the potentiometer is adjusted to give the same scale reading as with \( P \). Then, if \( X_s \) is the value of \( X \), it follows that

\[
I = \left\{ nX_sEC.exp\left[-t(R+S)/RSC\right]\right\}/Z. \quad \text{On dividing, collecting up terms and taking logs one has } 1/S - 1/P = C \cdot \log_e(X_s/X_p)/t.
\]
t can be determined by this method of operation. If $X_1$ is the value of $X$ with only the leakage resistance across the condenser and $X_S$ the value with a particular known resistance $S$ across, then
\[
\log_e X_1 - \log_e X_S = -t/SC -t/RC
\]
so that plotting $\log_e X_S$ against $1/S$ should give a straight line from the slope of which $t$ could be determined if $C$ be known. As it is not necessary to estimate parts of scale divisions the experimental scatter on the straight line should be diminished by this method. Figure 6.16 illustrates this point. It is plotted on a very large scale to illustrate the smallness of the scatter. The value of $t$ obtained from the slope is $5.41 \times 10^{-3}$ seconds which agrees favourably with the value $5.43 \times 10^{-3}$ seconds obtained some weeks previously when using the earlier method.

As the capacitance $C$ is small, $Z$ and $X$ can be made large. That $Z$ be high is necessary since $E$ has to be of the order of 100 volts and would be, presumably, a dry battery from which only a small current may be taken if the voltage is to remain constant. The fact that $X$ can be made high is of importance to the precision of measurement as will now be shown.

It is evident that, if $X_p$ and $X_S$ can each be determined to $\pm b$, then
\[
1/S - 1/P = \left[ C \log_e (X_S/X_p) \right] / t = bC(1/X_S + 1/X_p)/t.
\]

To determine $b$ it may be assumed that, with a full scale deflection, $I$ and $I + I/1000$ can
Log$_e$ $x_s$ plotted against $1/S$ for the determination of $t$. 
just be distinguished. This assumes that 0.1 mm. can be observed as a deflection from a scale mark and that the whole scale length of 100 mm. is being used. Thus
\[ I = AX \cdot \exp(-B) \] and \[ I + I/1000 = A(X + b) \cdot \exp(-B) \]
where \( A \) and \( B \) are constants. Evidently \( b = X/1000 \).

The precision of the boxes should also be included so that \( b \) has to be taken as \( 2X/1000 \). The average value of \( X \) might be 5000 ohms in which case \( b = 10 \) which may be taken as a fair value. With the same figures as before, so that a comparison can be made, that is
\[ P = 10^9 \text{ ohms}, \quad n = 50, \quad E = 180 \text{ volts}, \quad S = 4.2 \cdot 10^8 \text{ ohms}, \]
\[ C = 10^{-11} \text{ farad}, \quad t = 5 \cdot 10^{-3} \text{ seconds}, \quad R = 10^{10} \text{ ohms} \]
and with \( Z = 10^4 \text{ ohms} \) and taking \( I \) as 0.02 \( \mu \text{A} \) for full scale deflection, then \( X_p = 3850 \) and \( X_s = 7700 \text{ ohms} \). From these values the calculated possible error is \( \pm 0.8\% \), showing a small but definite improvement.

This method of operation is slightly less convenient in that a more sensitive galvanometer is required. In the case considered the sensitivity necessary is 5000 mm/\( \mu \text{A} \). Results obtained using this technique are given in table 6.12. It will be seen that theory and practice agree reasonably well when it is realised that the values are known at best to \( \pm 0.5\% \).

As with the previous method of using the circuit it is not essential to have a known resistance available. The value of \( X \) may be determined with only the leakage resistance across the condenser. If this
<table>
<thead>
<tr>
<th>S.</th>
<th>$X_s$</th>
<th>$X_p$</th>
<th>P.</th>
<th>Expected Determined</th>
<th>Actual error value.</th>
<th>Actual error value.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$10^7$ohms.</td>
<td>ohms</td>
<td>ohms</td>
<td>$10^7$ohms</td>
<td>$10^7$ohms</td>
<td>%</td>
</tr>
<tr>
<td>0.837</td>
<td>5550</td>
<td>3675</td>
<td>2.82</td>
<td>2.89</td>
<td>2</td>
<td>-2.4</td>
</tr>
<tr>
<td>2.31</td>
<td>5840</td>
<td>3590</td>
<td>2.86</td>
<td>2.89</td>
<td>3</td>
<td>-1.0</td>
</tr>
<tr>
<td>2.73</td>
<td>3720</td>
<td>3590</td>
<td>2.55</td>
<td>2.89</td>
<td>4</td>
<td>-1.4</td>
</tr>
<tr>
<td>0.837</td>
<td>5550</td>
<td>4020</td>
<td>1.88</td>
<td>1.89</td>
<td>1</td>
<td>-0.5</td>
</tr>
<tr>
<td>0.870</td>
<td>5430</td>
<td>4040</td>
<td>1.85</td>
<td>1.89</td>
<td>2</td>
<td>+2.1</td>
</tr>
<tr>
<td>1.24</td>
<td>4620</td>
<td>4020</td>
<td>1.93</td>
<td>1.89</td>
<td>2</td>
<td>-2.1</td>
</tr>
<tr>
<td>1.29</td>
<td>4545</td>
<td>4040</td>
<td>1.88</td>
<td>1.89</td>
<td>2</td>
<td>-0.5</td>
</tr>
<tr>
<td>1.71</td>
<td>4150</td>
<td>4040</td>
<td>1.88</td>
<td>1.89</td>
<td>2</td>
<td>-0.5</td>
</tr>
</tbody>
</table>

Table 6.12.

Experimental results for a resistance value determined by the use of another resistance. ($C = 0.001 \mu F$)
value of $X$ be $X_r$ and the value for the same deflection when the unknown resistance $P$ is across be $X_p$ then

$$P = \frac{t}{C \log e \left(\frac{X_p}{X_r}\right)}.$$  

If allowance be made for errors this must be written as

$$P = \frac{t}{C \left[ \log e \left(\frac{X_p}{X_r}\right) \pm \frac{b}{X_p + \frac{1}{X_r}} \right]}.$$  

In table 6.13 are given the results obtained with some known carbon resistors singly, in parallel and in series using the second circuit with a 0.001 μF condenser. The theoretically possible errors and the actual errors are given for two series of experiments and it will be seen that the results are superior to those given in tables 6.5., and 6.7., although the possible error is generally exceeded. This is to be expected as the resistance values are known at best to ±0.5% and, probably, only to ±1.0%. When values are taken to ±1.0% there is complete agreement in 75% of the measurements. The resistors were chosen in a random manner so that duplication of measurement occurs only by chance.

If a smaller range of resistance values be measured then the accuracy of the system should be increased as the value of $X$ can be made somewhere near $Z/2$ by suitable adjustment of the E.M.F. used. In practice no significant improvement was found. This is shown in table 6.14, where the experimental errors of both techniques are given for some typical results.

As with the method using galvanometer deflections, above $10^8$ ohms difficulty was found in measuring resistance values due to the short delay
<table>
<thead>
<tr>
<th>Resistance</th>
<th>Experimental</th>
<th>Actual</th>
<th>Experimental</th>
<th>Actual</th>
<th>Expected</th>
</tr>
</thead>
<tbody>
<tr>
<td>value.</td>
<td>value.</td>
<td>error.</td>
<td>value.</td>
<td>error.</td>
<td>error.</td>
</tr>
<tr>
<td>(1)</td>
<td>(2)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$10^7$ ohms.</td>
<td>$10^7$ ohms.</td>
<td>%</td>
<td>$10^7$ ohms.</td>
<td>%</td>
<td>±%</td>
</tr>
<tr>
<td>0.408</td>
<td>0.403</td>
<td>+1.0</td>
<td>-</td>
<td>-</td>
<td>0.3</td>
</tr>
<tr>
<td>0.413</td>
<td>0.421</td>
<td>+2.0</td>
<td>-</td>
<td>-</td>
<td>0.4</td>
</tr>
<tr>
<td>0.417</td>
<td>0.415</td>
<td>-0.5</td>
<td>-</td>
<td>-</td>
<td>0.4</td>
</tr>
<tr>
<td>0.420</td>
<td>0.420</td>
<td>0</td>
<td>-</td>
<td>-</td>
<td>0.4</td>
</tr>
<tr>
<td>0.513</td>
<td>0.516</td>
<td>+0.6</td>
<td>-</td>
<td>-</td>
<td>0.4</td>
</tr>
<tr>
<td>0.523</td>
<td>-</td>
<td>-</td>
<td>0.530</td>
<td>+1.3</td>
<td>0.4</td>
</tr>
<tr>
<td>0.619</td>
<td>0.627</td>
<td>+1.3</td>
<td>-</td>
<td>-</td>
<td>0.6</td>
</tr>
<tr>
<td>0.628</td>
<td>-</td>
<td>-</td>
<td>0.633</td>
<td>+0.8</td>
<td>0.6</td>
</tr>
<tr>
<td>0.734</td>
<td>0.743</td>
<td>+1.2</td>
<td>0.746</td>
<td>+1.6</td>
<td>0.6</td>
</tr>
<tr>
<td>0.825</td>
<td>0.824</td>
<td>-0.1</td>
<td>0.836</td>
<td>+1.2</td>
<td>0.8</td>
</tr>
<tr>
<td>0.870</td>
<td>0.878</td>
<td>+0.9</td>
<td>0.887</td>
<td>+1.9</td>
<td>0.8</td>
</tr>
<tr>
<td>0.930</td>
<td>0.934</td>
<td>+0.4</td>
<td>-</td>
<td>-</td>
<td>0.9</td>
</tr>
<tr>
<td>0.940</td>
<td>-</td>
<td>-</td>
<td>0.951</td>
<td>+1.2</td>
<td>0.9</td>
</tr>
<tr>
<td>0.990</td>
<td>0.998</td>
<td>+0.8</td>
<td>-</td>
<td>-</td>
<td>0.6</td>
</tr>
<tr>
<td>1.030</td>
<td>1.027</td>
<td>-0.3</td>
<td>-</td>
<td>-</td>
<td>0.6</td>
</tr>
<tr>
<td>1.036</td>
<td>1.042</td>
<td>+0.6</td>
<td>-</td>
<td>-</td>
<td>0.1</td>
</tr>
<tr>
<td>1.045</td>
<td>-</td>
<td>-</td>
<td>1.055</td>
<td>+1.0</td>
<td>0.1</td>
</tr>
<tr>
<td>1.151</td>
<td>1.158</td>
<td>+0.6</td>
<td>-</td>
<td>-</td>
<td>0.1</td>
</tr>
<tr>
<td>1.245</td>
<td>1.240</td>
<td>-0.4</td>
<td>1.257</td>
<td>+1.0</td>
<td>0.1</td>
</tr>
</tbody>
</table>

Table 6.13.

(continued on page 193)
<table>
<thead>
<tr>
<th>Resistance</th>
<th>Experimental</th>
<th>Actual</th>
<th>Experimental</th>
<th>Actual</th>
<th>Error</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.290</td>
<td>-</td>
<td>1.299</td>
<td>+0.7</td>
<td>0.1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.360</td>
<td>1.378</td>
<td>1.354</td>
<td>-0.5</td>
<td>0.2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.465</td>
<td>1.442</td>
<td>1.479</td>
<td>+1.0</td>
<td>0.2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.571</td>
<td>1.573</td>
<td>-</td>
<td>-</td>
<td>0.2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.707</td>
<td>1.710</td>
<td>1.696</td>
<td>-0.6</td>
<td>0.2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.890</td>
<td>1.857</td>
<td>1.852</td>
<td>-1.5</td>
<td>0.2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.440</td>
<td>2.478</td>
<td>-</td>
<td>-</td>
<td>0.2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.727</td>
<td>-</td>
<td>2.782</td>
<td>+2.0</td>
<td>0.3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.845</td>
<td>2.860</td>
<td>-</td>
<td>-</td>
<td>0.3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3.005</td>
<td>3.063</td>
<td>-</td>
<td>-</td>
<td>0.3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3.135</td>
<td>3.130</td>
<td>-</td>
<td>-</td>
<td>0.3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3.310</td>
<td>3.314</td>
<td>3.374</td>
<td>+1.9</td>
<td>0.4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3.727</td>
<td>3.763</td>
<td>3.740</td>
<td>+0.3</td>
<td>0.4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4.135</td>
<td>4.162</td>
<td>4.102</td>
<td>-0.7</td>
<td>0.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4.461</td>
<td>4.543</td>
<td>-</td>
<td>-</td>
<td>0.5</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 6.13.

Resistance values obtained with the modified circuit of figure 6.12., using a Rayleigh potentiometer. \(t = 5.52 \times 10^{-3} \text{s.}\)
<table>
<thead>
<tr>
<th>Resistance value (10^7 ohms)</th>
<th>Experimental value</th>
<th>Actual error (%)</th>
<th>Actual error (percent)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.523</td>
<td>0.524</td>
<td>+0.2</td>
<td>+1.3</td>
</tr>
<tr>
<td>0.628</td>
<td>0.634</td>
<td>+0.6</td>
<td>+0.8</td>
</tr>
<tr>
<td>0.825</td>
<td>0.832</td>
<td>+0.8</td>
<td>-0.1 &amp; +1.2</td>
</tr>
<tr>
<td>0.940</td>
<td>0.951</td>
<td>+1.2</td>
<td>+1.2</td>
</tr>
<tr>
<td>1.245</td>
<td>1.255</td>
<td>+0.8</td>
<td>-0.4 &amp; +1.0</td>
</tr>
<tr>
<td>1.360</td>
<td>1.373</td>
<td>+1.0</td>
<td>+1.3 &amp; -0.5</td>
</tr>
</tbody>
</table>

Table 6.14.

Resistance values obtained using a restricted range of values. The errors are compared with the errors that resulted when an extended range of values was used.
time. When the Rayleigh potentiometer was inserted in the circuit of figure 6.14., however, no improvement in the precision was obtained. The low frequency of the discharge pulses through the galvanometer prevented the setting of the potentiometer to an accuracy any greater than the accuracy to which the galvanometer could be read in the galvanometer reading method. Thus the inconvenience of the potentiometer was not warranted, although there does remain the possibility that the use of a very long period galvanometer might warrant it.

With all the circuits so far described it is imperative that the value of the E.M.F. remain constant. For this reason it was decided that an investigation of a null method was worthwhile. The circuit derives from that of the null method for the comparison of capacitance described in section 3 and is given in figure 6.17.

A Siemens type 100 double change-over relay is used to charge two condensers $C_1$ and $C_2$ from the potential differences developed across a fixed resistance $R_1$ and a resistance box $R_2$. The condensers are then discharged together and in opposite directions through a moving-coil galvanometer. The resistance $S$ whose value is to be determined can be placed across $C_2$.

When $S$ is not connected and $R_2$ is adjusted so that no current flows through the galvanometer then

$$\frac{nR_1EC_1 \exp(-t_1/C_1P_1)}{(R_1+R_2)} = \frac{nR_2EC_2 \exp(-t_2/C_2P_2)}{(R_1+R_2)}$$
Figure 6.17.
The circuit for the null method measurement of resistance.
where $t_1$ and $t_2$ are the times the tongues of the relay take to travel from the charge position to the discharge position and $P_1$ and $P_2$ are the leakage resistances of the two condensers. The circuit constants are such that complete charge and discharge occur.

If $S$ be now connected across $C_2$, and $R_2$ adjusted to a new value $R'_2$ so that balance is again obtained then

$$\frac{nR_1EC_1 \cdot \exp\left(-t_1/C_1P_1\right)}{(R_1+R_2)} = \frac{nR'_2EC_2 \cdot \exp\left[-t_2(P_2+S)/P_2C_2S\right]}{(R_1+R'_2)}.$$

On dividing these two equations and taking logs there results the expression

$$S = \frac{t_2}{C_2} \cdot \log_e \left(\frac{R'_2}{R_2}\right).$$

The time interval $t_2$ may be determined using $0.1 \mu F$ condensers as $C_1$ and $C_2$ and a $10^5$ ohm box as $S$ or, alternatively, $t_2/C_2$ may be determined if some known high resistances in the required range are available. Typical ratio values are given in table 6.15, and it will be noted that there is an overlap enabling the scale to be opened out by switching to a lower value of capacitance when necessary.

To obtain a value for $t_2$ greater than $3 \times 10^{-3}$ seconds it was found necessary to use a metal rectifier to give half-wave rectification of the alternating current operating the relay. It was found possible in this way to obtain $t_2$ greater than $6 \times 10^{-3}$ seconds. To reduce stray capacitance effects it was found convenient to earth
Table 6.15.
Ratio values required for various values of resistance with a given capacitance. \((t = 5 \times 10^{-3} \text{s})\)

<table>
<thead>
<tr>
<th>(\frac{R_2}{R_1})</th>
<th>150</th>
<th>10</th>
<th>2.7</th>
<th>1.65</th>
<th>1.1</th>
<th>1.05</th>
</tr>
</thead>
<tbody>
<tr>
<td>(C = 0.001 \mu F)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(S = 10^6 \text{ohms})</td>
<td>1.0</td>
<td>2.0</td>
<td>5.0</td>
<td>10.0</td>
<td>50.0</td>
<td>500.0</td>
</tr>
<tr>
<td>(C = 0.0001 \mu F)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(S = 10^7 \text{ohms})</td>
<td>&quot;</td>
<td>&quot;</td>
<td>&quot;</td>
<td>&quot;</td>
<td>&quot;</td>
<td>&quot;</td>
</tr>
<tr>
<td>(C = 0.00001 \mu F)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(S = 10^8 \text{ohms})</td>
<td>&quot;</td>
<td>&quot;</td>
<td>&quot;</td>
<td>&quot;</td>
<td>&quot;</td>
<td>&quot;</td>
</tr>
</tbody>
</table>

Table 6.16.
The possible error (\(\beta\)) in determining resistance values for various capacitances and galvanometers.
the centre point of the circuit. The two galvanometers shown in the circuit switched by the key $K_2$ were of different sensitivities. Normally a $10^6$ ohm resistor was kept across the key $K_3$ to protect the galvanometers.

The precision of the method can be determined as follows. If the sensitivity of the galvanometer is $z$ mm/µA and a displacement of ±0.1 mm from zero can be detected, it follows that the difference in values of $R'_2$ that can be detected, i.e. $R_{2a}' - R_{2b}'$, is given by

$$I_a - I_b = \frac{nEC_2 \exp \left[ -t_2 \frac{(P_2 + S)}{P_2C_2S} \right] (R_{2a}' - R_{2b}')}{(R_1 + R_2')} = 10^{-6}/5z.$$  

as $R_{2a}' = R_{2b}'$. As the exponential term is close to unity (> 0.95) this can be written as $nEC_2 (R_{2a}' - R_{2b}') = 10^{-6}/5z$. 

Thus the possible error, expressed as a percentage, is given by

$$\pm \left[ (R_{2a}' - R_{2b}').100 \right]/R_2' = \pm \left[ (R_1 + R_2'), 10^{-6}.100 \right]/5znEC_2R_2' = \pm \beta\%.$$  

Typical values for $\beta$ are given in table 6.16.

To $\beta$, however, must be added the possible error $\alpha$ in determining $R_2$. This is given by

$$\pm (R_1 + R_2).10^{-6}.100/5znEC_2R_2 \%.$$  

As $R_2 < R_2'$, $\alpha$ is always greater than $\beta$. Normally $R_2 = R_1$ so that the value of $\alpha$ is given by the second half of table 6.16.

It has been found in practice that the possible error does not decrease so rapidly with increase of galvanometer sensitivity as is indicated in the table.
This is because the time $t_2$ varies slightly and a very sensitive galvanometer responds to the occasional pulse of out of balance current and oscillates. This prevents adjustment to zero closer than approximately $\pm 2.5$ mm. with the most sensitive galvanometer mentioned, thus the figures in table 6.16. should be multiplied by a factor of 25 for this galvanometer.

A long series of experiments was carried out with this circuit, resistance values being determined using galvanometers of sensitivities of 22 and 2480 mm/$\mu$A respectively and comparing the results obtained with other methods. For conciseness, some of these results are presented in figures 6.18., and 6.19. in the form of the resistance value $S$, as determined by other methods, plotted against $1/\log_e(\frac{R_2}{R_3})$. The lines representing a $\pm 2\%$ variation from the best straight line that can be drawn through the points are given.

Theoretically with a capacitance of 0.001 $\mu$F and a value of $S$ of $2 \times 10^6$ ohms there should be a possible error of $\pm 1\%$, and with a value of $S$ of $5 \times 10^7$ ohms a possible error of $\pm 1.6\%$ when a galvanometer of sensitivity 22 mm/$\mu$A is used. It will be seen that, allowing for a small indeterminacy for the value of $S$, the points are within these limits.

When the galvanometer of higher sensitivity was used it was found that there was no increase in precision. This may be seen in figure 6.19. This meant
Resistance value $S$ plotted against $1/\log_e(R_2'/R_2)$.

(Calorimeter sensitivity: 0.02 cm/mW; $S = 2.001$ cm.)
Resistance value $S$ plotted against $1/\log_e(R_2/R_1)$.

(Galvanometer sensitivity = 2480 mm/μA., $C = 0.001 \mu F$.)
that not only was the factor 25 already mentioned too low, but also that the apparatus would be unsatisfactory for resistance values in the $10^9$ ohms region where the sensitive galvanometer would have to be used.

It was evident that the variation in the transit time was producing errors apart from causing vibration of the galvanometer coil. In the earlier methods described, the sensitivity is not sufficient for such a variation to have any noticeable effect. It was decided that the time during which leakage could occur had to be increased.

This was done by using the circuit of figure 6.14., with an extra make contact on both relay $B_1$ and relay $B_2$ to enable two condensers to be used. The circuit thus produced is given in figure 6.20., where the resistance $S$ to be measured is placed across $C_2$. As before, $B_2$ and $B_1$ close on alternate pulses from the oscillator. $C_1$ and $C_2$ are charged at one pulse, then $C_1$ loses charge through its leakage resistance $P_1$ while $C_2$ loses charge through its leakage resistance $P_2$ and $S$ in parallel until the next oscillator pulse when $C_1$ and $C_2$ discharge through the galvanometer $G$. In contrast with the circuit of figure 6.14., it was found more convenient with a wide range of resistance values to keep $R_5$ constant and adjust $R_4$ for balance. This is done first with no resistance and then with known resistance values as $S$. Then, as $S = t_2/C_2 \log_e(R_4/R_4')$, $S$ plotted against $1/\log_9(R_4/R_4)$ should give a straight line. An experimental graph is shown in figure 6.21., from which
Figure 6.20.

Diagram of second null circuit.
Figure 6.21.

$S$ plotted against $1/\log_e(R_4/R'_4)$.
t_2/C_2 may be determined. As the higher resistance values were only reliable to \(\pm 2\%\) the scatter is not excessive.

Unless the frequency of the oscillator is stabilized by injecting a small voltage, at mains frequency, to the grid by means of a transformer it is best to measure resistance by means of a comparison with a known resistance. If the known and unknown resistances are \(S_1\) and \(S_2\) respectively and the values of \(R_4\) are:

- \(R_4\) with no resistance across \(C_2\),
- \(R'_4\) with \(S_1\) across \(C_2\) and
- \(R''_4\) with \(S_2\) then

\[
\begin{align*}
R_4 \exp(-t_1/C_1 P_1) &= R_5 \exp(-t_2/C_2 P_2) \\
R'_4 \exp(-t_1/C_1 P_1) &= R_5 \exp[-t_2(P_2+S_1)/C_2 P_2 S_1] \\
R''_4 \exp(-t_1/C_1 P_1) &= R_5 \exp[-t_2(P_2+S_2)/C_2 P_2 S_2]
\end{align*}
\]

from which it follows that

\[
\left[\log_e(R_4/R'_4)\right]/\left[\log_e(R_4/R''_4)\right] = S_2/S_1.
\]

This comparison can be carried out so rapidly that the frequency of the oscillator, and thus the delay time, may be taken as constant.

In table 6.17 are given the results of some comparisons and it will be seen that an agreement within \(\pm 2\%\) was obtained between the values obtained in this way and those obtained with a modified Wheatstone bridge. This agreement is obtained provided the ratio between the resistance values is within the range 5 to 0.2. With a 10 : 1 ratio the agreement has been found to be within \(\pm 5\%\). These values agree well with those to be expected from the precision with which the balance can
### Modified Wheatstone Bridge Values.

<table>
<thead>
<tr>
<th>'Known' ohms</th>
<th>'Unknown' ohms</th>
<th>(i) ohms</th>
<th>(ii) ohms</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.37 \times 10^9</td>
<td>0.9(1) \times 10^{10}</td>
<td>0.94 \times 10^{10}</td>
<td>0.96 \times 10^{10}</td>
</tr>
<tr>
<td>1.01 \times 10^9</td>
<td>0.9(1) \times 10^{10}</td>
<td>0.96 \times 10^{10}</td>
<td>0.95 \times 10^{10}</td>
</tr>
<tr>
<td>4.56 \times 10^8</td>
<td>1.37 \times 10^9</td>
<td>1.37 \times 10^9</td>
<td>1.36 \times 10^9</td>
</tr>
<tr>
<td>1.02 \times 10^8</td>
<td>1.37 \times 10^9</td>
<td>1.35 \times 10^9</td>
<td>1.35 \times 10^9</td>
</tr>
<tr>
<td>1.37 \times 10^9</td>
<td>1.01 \times 10^9</td>
<td>1.00 \times 10^9</td>
<td>1.03 \times 10^9</td>
</tr>
<tr>
<td>1.02 \times 10^8</td>
<td>1.01 \times 10^9</td>
<td>1.01 \times 10^9</td>
<td>1.00 \times 10^9</td>
</tr>
<tr>
<td>1.37 \times 10^9</td>
<td>5.68 \times 10^8</td>
<td>5.63 \times 10^8</td>
<td>5.71 \times 10^8</td>
</tr>
<tr>
<td>1.01 \times 10^9</td>
<td>5.68 \times 10^8</td>
<td>5.75 \times 10^8</td>
<td>5.62 \times 10^8</td>
</tr>
<tr>
<td>5.06 \times 10^8</td>
<td>3.48 \times 10^8</td>
<td>3.50 \times 10^8</td>
<td>3.49 \times 10^8</td>
</tr>
<tr>
<td>4.56 \times 10^8</td>
<td>3.48 \times 10^8</td>
<td>3.49 \times 10^8</td>
<td>3.48 \times 10^8</td>
</tr>
<tr>
<td>5.06 \times 10^8</td>
<td>1.02 \times 10^8</td>
<td>1.04 \times 10^8</td>
<td>1.03 \times 10^8</td>
</tr>
<tr>
<td>4.56 \times 10^8</td>
<td>1.02 \times 10^8</td>
<td>1.04 \times 10^8</td>
<td>1.02 \times 10^8</td>
</tr>
</tbody>
</table>

Table 6.17.

Resistance values obtained (a) with the null circuit using a known resistance value for comparison and (b) with a modified Wheatstone bridge. (C = 40pF. (i) t = 0.08 s., (ii) t = 0.10 s.)
be set.

There are three further advantages of this circuit. These are (i) although the sensitivity of the circuit is dependent on \( E \) the balance ratios are independent of \( E \). Thus the voltage applied across \( C_2 \) (and therefore that across the resistor under test) can be set to any desired value and measured.

(ii) When the system has been balanced any small deflections of the galvanometer are proportional to the change in resistance causing them. This is illustrated in figure 6.21., and may prove to be useful when the circuit is used for temperature measurement.

(iii) If the resistance under test should break down no harm can come to the galvanometer as the short circuit is applied across the galvanometer during discharge.

It is evident that, if the circuit is balanced with no resistance across \( C_2 \), the deflections with known resistors can be noted and the apparatus then used in a direct reading manner. A typical calibration curve is given in figure 6.22. The shape of the curve and its position can be altered by adjustment of \( C_2 \) and \( t_2 \). The method, however, is only useful over the approximately linear portion of the curve.

A relatively simple method for the comparison of high resistance values has been developed which is certainly reliable to ±2%. This would give
Figure 6.22.

Galvanometer deflection plotted against resistance.
Figure 6.23.
Galvanometer deflection plotted against resistance.
temperature readings with a suitable transducer to better than ±0.7°C in the temperature range required. As the instrument would be calibrated over the temperature range it is almost certain that the accuracy would be better than is indicated by this figure.
SECTION 7.

SUMMARY AND CONCLUSION.
It has been shown that:

(1) for the comparison of capacitance the precision of the method of continuous ballistic discharge can be improved by (a) the use of a Rayleigh potentiometer and (b) a null circuit consisting of two Siemens circuits working 'back-to-back'. It follows that the discrimination of the Jason capacitance hygrometer (7.1.) would be improved by the use of (a) in place of the simple relay circuit described in section 2 as is at present used.

(2) using commercial condensers of capacitances between 0.001 and 1.0 μF., agreement to ±0.2% was obtained for capacitance ratios determined by alternating current bridge methods at 1000 cycles second⁻¹ and by the continuous ballistic discharge methods described.

(3) there is no real advantage to the use of thermionic valves as the switching device in the method, and that the resistance of the valves is a disadvantage.

(4) the mercury in a U-tube manometer can be used as the moving electrode of a variable condenser and that a system can be arranged both to measure pressure difference over a very wide range and to detect very small changes of pressure difference anywhere in that wide range.

(5) the method of continuous ballistic discharge can be utilised for the continuous measurement of low wind velocities. The system is sensitive
and quick to respond to changes of wind velocity.

(6) the method can be modified to provide a reliable, safe and simple means of measuring resistance values up to $10^{10}$ ohms, provided that an accuracy greater than $\pm 2\%$ is not required at the higher values. It has been shown that, if a lower precision is acceptable, it should be possible to indicate high resistance values on a pen recorder. This would be a unique arrangement.

The conclusion which is drawn is that, although the method of continuous ballistic discharge is now no longer required for its original purpose of determining the ratio of electrostatic to electromagnetic units, it can be applied successfully to the measurement of a number of physical parameters. The method is particularly useful since it has the advantages of simplicity in circuit construction and in presentation of data compared with other methods.
SECTION 8.

REFERENCES.
0.1. Astbury, N.F. Report on Progress in Physics, 6, 1940.
0.4. Whiddington, R. Phil. Mag., 40, 1920.


2.4. Siemens, C.W. Journal of the Royal United Service Institution, 10, 1867.
2.5. Stoletow, M.A. Jour. de Physique, 10, 1881.
2.6. Fleming, J., and Clinton, W. Phil. Mag., 5, 1903.
2.11. Lord Rayleigh. Phil. Mag., 21, 1886.
2.15. Rosa, E.B. Phil. Mag., 28, 1889.
2.20. Glazebrook, R.T. The Electrician, 25, 1890.


4.2. Goulier, C.M. Comptes Rendus, 84, 1877.


A simple vane anemometer giving continuous and direct indication of wind velocity


[Paper first received 6 May, and in final form 9 June, 1954]

The anemometer has been designed in an attempt to produce a portable and inexpensive instrument for measuring draught velocities in animal houses where knowledge of the extreme conditions over twenty-four-hour periods may be required. It has the normal characteristics of a vane anemometer to which have been added continuous and rapid indication of wind velocity by a moving-coil microammeter.

**THEORY**

The instrument uses the alternate charge and discharge of a condenser, suggested by Maxwell* for the measurement of capacitance, to register speed of rotation as a pseudo-direct current. Suppose that a condenser has a capacitance \( C \) and a high leakage resistance and that \( R \) and \( G \) are the resistances in series with the condenser at charge and discharge respectively. Then, if \( a \) and \( b \) are the charging and discharging times, \( E \) is the constant charging e.m.f., and if there are \( n \) charges and \( n \) discharges per second, the mean discharge current is given by:

\[
I = nEC\left[1 - \exp\left(-a/CR\right)\right]\left[1 - \exp\left(-b/GC\right)\right]
\]

If \( a > CR \) and \( b > CG \), then

\[
I = nEC
\]

and the current through a meter in the discharge circuit is independent of the meter resistance, and as \( C \) and \( E \) are constant, the meter deflexion is proportional to \( n \).

**DETAILS OF APPARATUS**

The apparatus consists of a vane unit (Fig. 1) and a control unit.

The vane unit, which is essentially a rotary switch, has an aluminium vane system \( A \) fastened perpendicularly to a steel axle. Also perpendicular to the axle are two brass spiders \( B \) that carry between them three platinum wires \( C \) parallel to the shaft at 120° intervals around the shaft. As this system rotates it alternately joins the brushes \( D \) and \( E \), thus causing the condenser to charge, and the brushes \( F \) and \( G \), thus causing the condenser to discharge through the microammeter. The open structure of the rotating system has been chosen as it reduces both the moment of inertia and the air resistance of the system, thus giving quick response to variations in wind velocity.

The brushes each consist of two platinum wires held at one end between brass plates, the pair of brushes on each side being mounted on perspex carried by the frame of the unit. The position of each brush can be adjusted. Vibration of the brush wires could cause intermittent loss of contact and thus unsteady readings. It could also cause premature contact with the possibility of the meter being put across the battery. Damping is, therefore, introduced by plastic sleeves \( H \) fitted over the wires and waxed to the brass plates. The possibility of premature contact is further precluded by varnish on the brush wires except at their tip.

---

The steel axle is tapered at each end and fits into conical indentations in hardened brass. The indentation at the vane end is in a thin strip to avoid obstruction to wind flow. The position of the strip relative to the frame may be adjusted for alignment purposes. The indentation at the other end is in a small cylinder sliding in the frame to provide an adjustment for the bearing pressure. The whole vane unit is enclosed in an open-ended box $2 \times 4\frac{1}{2} \times 4$ in. for protection of the unit.

The e.m.f. applied across the condenser must remain at the value used for calibration so a step-by-step adjustment has been incorporated in the control unit (Fig. 2), the meter being used as a standardizing voltmeter. Adjustment to the correct e.m.f. is made with the anemometer running, a resistance $R_1$ ($R_a = G$) replacing the meter to act as a load. The meter is shunted by $R_2$ to reduce needle flutter. The range of the instrument may be varied by a switch selecting different values of $C$. The dimensions of the control unit are $5 \times 4\frac{1}{2} \times 4$ in.

**PERFORMANCE**

The instrument was calibrated with $C = 0.05 \mu F$, by carrying it along a corridor at speeds giving steady meter readings. As this procedure involved unavoidable inaccuracies it was repeated five times and in Fig. 3 the mean velocities obtained are plotted against the meter readings with the scatter shown in the usual manner.

To check the calibration, a standard vane anemometer was used to determine the wind velocity at a point in front of a blower. The standard vane anemometer was then replaced by the vane unit of the instrument described and the meter readings noted. The results of this calibration check are also shown in Fig. 3. A discrepancy will be noted at wind velocities greater than 150 ft/min. This is due to the air on first emerging from the blower having a higher velocity at the centre of the beam than at the periphery and to the fact that the vanes of the two anemometers are of different construction. The standard anemometer has vanes of diameter $6.4$ cm, with a central dead space of diameter $2.8$ cm. The vanes of the anemometer described are of diameter $4.3$ cm, with a dead space of only $0.5$ cm, to enable the instrument to measure draughts from pipes, small holes, etc. Due to this difference in the vanes, as the anemometers are brought closer to the blower the standard anemometer indicates a lower wind velocity than the anemometer described.

This explanation of the discrepancy has been confirmed by calibrating the continuous reading anemometer with a dry kata-thermometer having a bulb of smaller dimensions than the vanes of the anemometer. The results are also shown in Fig. 3 and it will be noted that, as expected, the kata-thermometer gives a higher value than the anemometer. The calibration curve does not pass through the origin by a factor of $0.05 \mu F$ by doubling the current for any particular wind speed in the appropriate range.

The calibration curve for $C = 0.1 \mu F$ is derived from that for $C = 0.05 \mu F$ by doubling the current for any particular wind speed in the appropriate range.

The instrument described detects changes in wind velocity of 6 ft/min or more and the response time is less than 1 s for changes in wind velocity of less than 75 ft/min, with a maximum response time of 5 s.

**CONCLUSION**

Although the instrument has been designed for a particular purpose it is suggested that with slight adaptations it is superior to the normal vane anemometer for many purposes.

It will provide direct, continuous readings and/or permanent recordings (by pen recorder) of wind velocities with a high sensitivity to slight changes in the velocity.

**ACKNOWLEDGEMENTS**

The author wishes to thank Professor E. C. Amoroso for his interest and encouragement in this work and Mr. V. C. Tindley for his machining of various parts during the construction of the instrument.
An electrical indicator for a mercury U-tube manometer

By A. E. Hawkins, B.Sc., A.R.C.S., A.Inst.P., Department of Physiology, Royal Veterinary College, University of London

[Paper first received 21 March, and in final form 4 June, 1956]

The metal limbs of a U-tube act as outer conductors of two cylindrical capacitors. The tubes are coated internally with a thin layer of a dielectric material. The mercury acts as a variable inner conductor. The system can be made very sensitive and has a general scatter of less than \( \pm 1 \) mm of mercury.

A Siemens high-speed, double changeover relay can be used in Thomson's method of mixtures to give a very sensitive null method for the comparison of capacitance. The circuit has been utilized in the construction of a simple mercury manometer of very high sensitivity which gives a continuous indication of pressure difference.

**PRINCIPLE OF THE APPARATUS**

Each limb of the U-tube consists of a metal tube coated on the inside with a thin layer of a dielectric material. The limbs are joined at the bottom by a tube of insulating material, with a short length of metal tubing somewhere in its length to enable electrical contact to be made with the mercury. The mercury acts as a variable inner conductor of the two cylindrical capacitors, and the metal tubes forming the limbs act as the outer conductors. A pressure difference across the U-tube produces an increase in capacitance of one limb and a decrease in capacitance of the other. The U-tube is used with Thomson's circuit rather than with an alternating current bridge, as a null indication is obtained by adjusting only one component. The arrangement gives an out-of-balance direct current proportional to the change in capacitance after balancing. In this manometer, the change in capacitance is a direct measure of the change in pressure difference.

**DETAILS OF THE APPARATUS**

The U-tube and the associated circuit are shown in Fig. 1. Various materials were used as the dielectric; the most successful was found to be the commercial lacquer Valspar. This lacquer covered bare metal very well with a single, thin coat. Brass tubes were thoroughly cleaned with fine emery paper wound on a rod, the rod being rotated within the tubes by an electric drill or on a lathe. In order to coat the tubes, they were filled with lacquer and then allowed to drain in a vertical position. When dry the tubes were inverted for use in the manometer. Only once did a single layer of lacquer fail to cover the surface in a satisfactory manner.

When a layer of uniform thickness was required in a tube, the tube was rotated for an hour at approximately 750 rev/min. The rotation was in a horizontal plane about the longitudinal axis of the tube and was commenced as soon as lacquer ceased to drip from the tube when in the vertical position.

The junction tube was of rubber in all cases. It was found initially that the dielectric material was sometimes removed accidentally when attaching the rubber tube and sometimes it did not cover the bottom edges of the tubes properly. This was overcome by allowing the lacquer to dry, dipping the tubes into smoking beeswax-and-resin, allowing the wax to harden, and inserting an inner rubber sleeve before the rubber junction tube was attached.

Supporting the tubes presented no difficulty. They were bolted to a board using strips of ebonite, or were held by Terry clips. Wide tubes, carrying a large volume of mercury, were held with clamps with rubber covered jaws, and, for safety, the junction tube was tied and waxed on to the manometer tubes.
METHODS OF USING THE APPARATUS

There are several ways of using the manometer, but the two described below are considered to be the most satisfactory and generally useful. For both of them the resistance $R_1$ is kept constant and the value of a decade resistance box $R_2$ is varied until balance is obtained. As, at balance, $C_1/C_2 = R_2/R_1$ it is evident that a linear relationship does not exist between pressure difference and the value of $R_2$.

For experimental work on this apparatus two sets of tubes were used. These had internal diameters of 1.02 and 2.51 cm, respectively.

With the draining technique described, it was not expected that the layers would be uniform over their entire length. Using the circuit described, the capacitance (i.e., the apparent capacitance) of the tubes with various heights of mercury was measured and the non-uniformity confirmed. It was generally constant, however, over 10 cm lengths. The spinning of tubes up to 50 cm in length produced layers which were uniform except for 5 cm at each end.

The sensitivity of the apparatus depends on the internal radius of the tubes, the thickness of the dielectric layer, the inclination of the tubes, the sensitivity of the galvanometer and the electromotive force used. The galvanometer had a sensitivity of 22 mm/\mu A and, except in one case, the electromotive force was kept at 45 V. A very sensitive galvanometer (2300 mm/\mu A) did not improve the performance of the circuit to any great extent since, with its longer period, it began to oscillate owing to occasional transient currents, for example, when the tongues of the relay did not close together exactly. This made the determination of the balance point difficult and partially nullified the increased sensitivity.

A carbon tetrachloride manometer was originally used for calibration purposes, but this had to be replaced by a water manometer when very cold weather prevented adequate ventilation being used to remove the vapour.

Most of the exploratory work was carried out using both limbs of the U-tube as it was felt that the differential effect would partially eliminate errors due to meniscus shape. This was found to be so except when the pressure difference was changing slowly. For slow changes of pressure difference it was therefore necessary to calibrate the manometer by

Fig. 1. Circuit diagram of manometer

$T = 6.3$ V filament transformer; $P =$ Siemens type 100 high-speed, double changeover relay, 53 $\Omega$; $R_1 = 10$ k$\Omega$; $R_2 = 100$ k$\Omega$ decade box; $R_3 = 1$ M$\Omega$; $E = 120$ V dry battery; $G =$ Tinsley portable reflecting galvanometer, type SSI, 22 mm/\mu A, 520 $\Omega$.

If the dielectric layer is uniform throughout the length of the tubes, the same deflexion is obtained for the same change of pressure difference from a balance point anywhere in the range of the manometer, since the out-of-balance current is proportional to $C V$, where $C$ is the capacitance change on each side and $V$ is the voltage applied to the bridge.

If a linear relation between pressure difference and the value of $R_2$ is essential it can be obtained with a reduced sensitivity for the same value of electromotive force. One limb is disconnected and replaced by a fixed capacitor of the same order of capacitance. Then, provided the dielectric layer is uniform, there is a linear relation between pressure difference and the value of $R_2$.

Fig. 2. Values of $R_2$ plotted against pressure difference. Curves (a) and (b) are for V-manometer; curves (c) and (d) are for U-manometer.

\[ \sqrt{C \text{ cm Hg}} \]

$\triangle$ calibration; $\cdot$ random; $\circ$ slow fall of pressure difference; $\times$ slow rise of pressure difference.
An electrical indicator for a mercury U-tube manometer

changing the pressure difference slowly. This is shown in curves (a) and (b) of Fig. 2. The curves were obtained with the narrow tubes with two coats of lacquer. The tubes were inclined to each other at 120°. The manometer was calibrated by setting up a particular pressure difference and then allowing the columns to settle after being caused to oscillate by squeezing the tube connecting the two manometers. The performance was then checked by setting up pressure differences at random, and without squeezing the connecting tube. The results were found to be very satisfactory, as may be seen in curve (b). However, when the pressure difference was allowed to fall slowly (10 cm of mercury/hour) curve (a) was obtained. A similar effect was observed when the pressure difference was increased slowly. As may be seen in curves (c) and (d) of Fig. 2, the effect was not so marked when the limbs were vertical.

Provided the calibration has been carried out under the appropriate conditions the apparatus can be used to measure slow changes of pressure very conveniently and in considerable detail. This is demonstrated in Fig. 3 where part of three long runs with a constant leak are shown. The change in pressure difference with time was determined with a water manometer and then the original pressure difference restored. The leak was re-opened and the resistance readings with time taken. The equivalent pressure differences were obtained from a normal calibration curve and finally the galvanometer deflexions with time were taken. The rapidity with which pressure readings can be taken is clearly shown. The superiority of resistance readings over galvanometer deflexions is also evident.

It has been mentioned that theoretically the deflexions from zero are proportional to the change of pressure difference. This was confirmed with both sets of tubes and with single and double layers of Valspar. It was found that the use of a single calibration line could involve an error up to about ±1 mm of mercury, but that individual calibrations would give a lower indeterminacy of the order of ±0.2 mm of mercury. The scatter was approximately the same for tubes with uniform and non-uniform layers. It appears to be due to meniscus effects.

The sensitivity of the apparatus to change of pressure difference can be made very high. With the narrow tubes, a single layer of Valspar, an electromotive force of 120 V and the tubes inclined to one another at 120°, a change of pressure difference of less than 0.1 mm of mercury can be detected. As this sensitivity is masked by the meniscus errors it is of no use when measurements of pressure difference are being made, but it could be of use as a detector of minute pressure changes.

The sensitivity to change remains the same whatever the initial pressure difference may be. It is this feature of the apparatus which renders it different from any other type of sensitive manometer working over the same wide range of pressure difference.

The sensitivity increased by approximately the expected amount on sloping the narrow tubes, whilst the maximum scatter remained substantially the same at ±1 mm of mercury near zero pressure difference and ±0.7 mm of mercury over the rest of the range, where it remained fairly constant. The sensitivity increased when the diameter of the tubes was increased, but the maximum scatter rose to ±1.5 mm of mercury and ±1 mm of mercury. The sensitivities obtained are given in the table.

<table>
<thead>
<tr>
<th>Indicator sensitivities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tube size</td>
</tr>
<tr>
<td>Small</td>
</tr>
<tr>
<td>Small</td>
</tr>
<tr>
<td>Small</td>
</tr>
<tr>
<td>Small</td>
</tr>
<tr>
<td>Large</td>
</tr>
</tbody>
</table>

Sensitivities in mm deflexion of galvanometer per mm of mercury change of pressure difference. Galvanometer sensitivity = 22 mm/µA, battery e.m.f. = 45 V. V = tubes inclined to each other at 120°.

CONCLUSION

It has been shown that the system is satisfactory for mercury. It was not found possible to produce a satisfactory water manometer using an electrolyte in the water to act as the variable conductor. Dielectric breakdown was finally overcome but there remained the difficulty that over thirty seconds was required for the conducting fluid to become free from the dielectric when a column was falling.

ACKNOWLEDGEMENTS

The author wishes to thank Prof. E. C. Amoroso for his interest and encouragement in this work and Miss G. Tomlinson for technical assistance.
A simple method for the measurement of high resistance values

By A. E. HAWKINS, B.Sc., A.R.C.S., A.Inst.P., Department of Physiology, Royal Veterinary College, University of London

[Paper received 3 July, 1956]

A modified Wheatstone network is described for the measurement of resistance values up to $10^9$ Ω. The accuracy is better than ±2%.

Price outlines a procedure whereby a Wheatstone network may be used to measure high resistance values. Any small, out-of-balance potential difference instead of being applied across a galvanometer is allowed to charge a high capacitance condenser and the energy stored in the condenser is discharged through a galvanometer. If the condenser is given sufficient time to charge completely, then the sensitivity of the apparatus is independent of the values of the resistances comprising the network.

In this simple form the network was found difficult and tedious to use. Complete charging had to be permitted to obtain an accurate balance. This made each determination very slow indeed. When incomplete charging was attempted, slight variations in the charging period caused incorrect adjustments to be made to the network. This, again, slowed up each determination and the difficulty in obtaining balance produced results with a 5% scatter.

Automatic switching at regular intervals made the arrangement almost as quick and easy to use as a normal network. The circuit used for testing the method is shown in Fig. 1. The switching is carried out by a simple thyratron relaxation oscillator driving a double changeover relay. The double changeover is necessary to avoid confusion arising near the balance position due to a slight drift of the galvanometer spot when one side of the galvanometer is left connected to one side of the condenser.

To protect the galvanometer, a rough balance is obtained with 100 Ω across the galvanometer and a charging time of 3 s. The charging time and the value of the shunt are increased by adjusting $R_6$ and $R_7$ respectively and the bridge brought into more accurate balance. The key is kept closed during the final stages.

It was not found necessary to increase the charging time beyond 10 s with a battery electromotive force of 45 V when measuring resistance values up to the order of $10^9$ Ω, despite the fact that complete charging did not occur. It was found essential to determine the ratios for the two current directions because of thermoelectric effects.

The values of some Welmeg, glass enclosed, high value resistors (by Welwyn Electrical Laboratories Ltd.) singly, in parallel and in series were determined with the circuit described. The values were determined by leakage and also by a constant galvanometer deflexion method using a Rayleigh potentiometer. The circuit for the latter method is shown in Fig. 2. The values of the resistance boxes $R_1$ and $R_2$ are adjusted for a near full-scale deflexion of the galvanometer with the resistance $P$ in circuit. The values are then adjusted for the same deflexion with $Q$ in circuit. Then, as the sum of the boxes is kept constant, $P/Q \approx R_1/R_2$. The potentiometer method failed at about $10^8$ Ω so that the accuracy of the bridge method was determined above this figure by the comparison of the calculated and experimental values of the combinations and the leakage values.

The method appears reliable up to $10^9$ Ω to at least ±2% and certainly to ±1% up to $10^8$ Ω. Moreover, the magnitude of the deflexions obtained at $10^9$ Ω indicates that the upper limit of the method is higher than that figure.

ACKNOWLEDGEMENT

The author wishes to thank Prof. E. C. Amoroso for his interest and encouragement in this work.
THREE SIMPLE RELAY CIRCUITS FOR THE COMPARISON
OF CAPACITANCE.
by A. E. Hawkins, B.Sc., A.R.C.S., A.Inst.P.
Department of Physiology, Royal Veterinary College, University of London

Introduction
Jason (1954) has shown that a high-speed, electromagnetic relay may be used as the commutator in the well-known Siemens (1857) repeated ballistic discharge method for the determination of capacitance and for the comparison of capacitance by substitution. He points out that the accuracy of the method is limited by the accuracy with which a meter may be read and recommends the use of the relay in the Maxwell (1873) commutator bridge for accurate work. As the Maxwell bridge is not very convenient for the comparison of capacitance, three circuits which use high-speed, double changeover relays have been investigated. The circuits have definite advantages as regards sensitivity and precision compared with simple alternating current bridges.

Description of Circuits
Circuit 1 is the most sensitive of the three and is based on a circuit due to Thomson (1873). It is shown in Fig. 1, in which C₁ and C₂ are the two condensers whose capacitance values are to be compared, and P is the relay. The relay is connected to a 6-3 v. filament transformer T fed from the mains. The relay produces a repetition of the following sequence of events. The two condensers begin to charge at the same instant and are connected to the source of charge for the same length of time. The condensers then begin to discharge at the same instant and are able to discharge, in opposite directions, for the same length of time through the moving coil galvanometer G. The times may not be exactly equal, but any error introduced will be small because (a) the transit time between charge and discharge and between discharge and charge is low (less than 1 m. sec.), therefore any difference in the transit times of the two tongues is negligible compared with the charging and discharging times, (b) the time constants are kept low enough to ensure that charge and discharge are completed well before the end of the periods allowed by the relay, and (c) the galvanometer has a long period (2 sec.) compared with the discharge time.

The resistances R₁ and R₂ are adjusted until there is no deflection of the galvanometer, and then \( C_1/C_2 = R_2/R_1 \). As both condensers must charge completely, it is necessary that R₁ and R₂ be sufficiently low for this to occur. As a ‘rule of thumb’ it can be taken that with 1-0, 0-1, 0-01 μF condensers the resistances should not exceed 500, 5,000, and 50,000 ohms respectively.

The tapping key with a high resistance in parallel is included to protect the galvanometer. Decade boxes are used for R₁ and R₂ and it is convenient to keep one of them at 100, 1,000 or 10,000 ohms. If the E.M.F. is increased from 15 v. with 0-1 μF condensers to 45 v. with 0-0001 μF condensers, the sensitivity, with approximately 1:1 ratios, is to one in ten thousand with the former capacitance and to one in one thousand with the latter.

The relay requires care in adjustment. The lower gap is brought up to give the smallest gap visible with the naked eye and then, with 0-5 or 1-0 μF condensers as C₁ and C₂ the tension screws are adjusted to give a steady deflection of the galvanometer spot. With such capacitances if the relays are not closing together the spot swings about the scale in an erratic manner. Waxing the two tongues together simplifies the adjustment.

The circuit has the same limitations as that of the Siemens and the Maxwell circuits, that the condensers

![Fig. 1. Diagram of circuit 1. P=Siemens type 100 relay. T=6-3 v. filament transformer. R₁=R₂=10,000 ohm resistance box. R₃=2-10 Mr fixed resistor. G=Tinsley galvanometer type S.S.122 A/mm. 320 ohm. E=dry battery.](image-url)
The effect of absorption is not so great in circuit 2, which is shown in Fig. 2. It is effectively the double commutator bridge described by Campbell (1912) and tested by him against alternating current methods. Only the charging currents are used, the condensers being short-circuited during the other half cycle of the alternating current driving the relay. It is evident that at balance \( \frac{C_1}{C_2} = \frac{R_2}{R_1} \) as the time constants must then be equal.

As \( G \) is a moving coil galvanometer the method is sensitive and it is also precise. With the same apparatus as for circuit 1, ratios near to 1:1 and an E.M.F. of 45 v. the sensitivity is approximately one third of that of circuit 1.

It is necessary that the relay tongues close simultaneously in order that the condensers start to charge together. Under these conditions the same ratio is obtained irrespective of which tongue switches the individual condensers. The relay is easily adjusted to give agreement between the ratios to better than \( \pm 0.2 \) per cent.

Condensers exhibiting marked absorption effects are normally compared with alternating current bridges. When simple bridges are used the balance point is often very difficult to determine with accuracy and its exact determination is practically impossible. To remove these drawbacks complex networks are required.

Before the advent of the alternating current bridge, Ayrton and Perry introduced the secohmmeter (Glazebrook 1922). This mechanical device feeds a bridge from a battery with what is effectively a square wave alternating current, and also rectifies the out-of-balance current so that a moving-coil galvanometer can be used as the detector.

In circuit 3 two of the relays described are connected to act as a secohmmeter. In this way a simple, quick, and easily operated bridge with a definite balance point is produced. The circuit is shown in Fig. 3. It is of the de Sauty type and, at balance, \( \frac{C_1}{C_2} = \frac{R_2}{R_1} \) as before.

Using this arrangement with ratios near to 1:1 the sensitivity is slightly better than one in one thousand with values of capacitance above 0.001 \( \mu \)F. The equipment used is the same as before with an E.M.F. of 15 v. for the higher capacitances and 45 v. for the lower capacitances.

**Results**

Capacitance ratios have been determined for both mica and paper condensers with all three of the circuits described and also with the series resistance modification of the de Sauty alternating current bridge (Hague 1938) which is considered the best of the simple bridges. The frequency of the bridge is 1,000 c./sec. Agreement between the four methods was always to within \( \pm 0.2 \) per cent with 1:1 ratios and capacitance values above 0.001 \( \mu \)F. With lower capacitance values and other ratios the agreement was to \( \pm 0.5 \) per cent between the three circuits described, while, as the a.c. bridge was only capable of giving values to about \( \pm 1 \) per cent at 0.001 \( \mu \)F, agreement with the bridge was only to that figure.

To test the circuits further, the dielectric constants of some liquids have been determined, first with the simple cylindrical condenser shown in Fig. 4 with a capacitance of the order of 0.0001 \( \mu \)F and then with a parallel plate condenser with a capacitance of the order of...
TABLE 1

<table>
<thead>
<tr>
<th>Circuit 1</th>
<th>Circuit 2</th>
<th>Circuit 3</th>
<th>Series resistance a.c. bridge</th>
<th>Tables</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a)</td>
<td>(b)</td>
<td>(a)</td>
<td>(b)</td>
<td>(a)</td>
</tr>
<tr>
<td>Benzene</td>
<td>1.82</td>
<td>2.21</td>
<td>2.2(4)</td>
<td>2.28</td>
</tr>
<tr>
<td>Olive oil</td>
<td>2.43</td>
<td>2.95</td>
<td>3.0(9)</td>
<td>3.08</td>
</tr>
<tr>
<td>Linseed oil</td>
<td>1.90</td>
<td>—</td>
<td>3.3(7)</td>
<td>—</td>
</tr>
</tbody>
</table>

Values of dielectric constants determined at room temperatures.
(a)=cylindrical condenser, (b)=parallel plate condenser.

0.001 μF (Table 1). The former condenser consisted of a solid brass inner cylinder surrounded by a concentric brass tube with Perspex end pieces, while the latter consisted of two 0.25-mm thick plates of mild steel separated by small pieces of X-ray film. The cylindrical condenser was the easiest to use, but its low capacitance prevented precise measurements being made. The dielectric constants are the ratios of the capacitances of the condenser with and without the liquids as given by circuits 1, 2 and 3 and a series resistance a.c. bridge. Circuit 1 is known to be unreliable for condensers exhibiting absorption effects. Included in the table are standard values of the dielectric constants (Hodgman 1948). It appears that circuit 3 gives true values despite absorption.

Conclusion
The three circuits described are all more sensitive and precise than the series resistance alternating current bridge. From the experimental work summarized above it is apparent that both circuits 1 and 2 are suitable for the detection or measurement of small changes of capacitance, while circuit 3, being unaffected by absorption, is to be preferred for the comparison of capacitance.

Circuit 2 is less affected by absorption than circuit 1 and is easier to adjust than that circuit, but is less sensitive. Circuit 3 is still less sensitive, but is still as sensitive as and far more precise than the series a.c. bridge, particularly at low capacitance values.

Summary
Three simple, sensitive and precise relay circuits for the comparison of capacitance are described. Their performance is discussed.

Acknowledgments: The author wishes to thank Prof. E. C. Amoroso for his encouragement in this work, Mr. V. C. Tindley for constructing the condensers, and Miss G. Tomlinson for technical assistance.

REFERENCES
Clerk Maxwell, J. (1873). Electricity and Magnetism, 2, 776.
Hodgman, C. D. (1948). Handbook of Chemistry and Physics, 30th Ed. 1940
A Simple Differential Calorimeter for Small Animals

By E. G. C. Clarke and A. E. Hawkins
Department of Physiology, Royal Veterinary College, University of London

§ 1. INTRODUCTION

The rate of heat production in an animal is usually measured by indirect methods. Although direct measurements by animal calorimeters are desirable, up to now they have not been satisfactory for small animals (Benedict 1938). The equipment is complicated and expensive, and gives an average value relating to a period of an hour or longer.

The apparatus described here is simple and inexpensive and can be constructed from material available in any laboratory. It has proved useful for both teaching and research. It gives continuous values for heat output with a time lag of less than thirty seconds, which provide a better picture of the minute-to-minute heat output of a small animal under normal conditions than do the more precise figures obtained by indirect methods. These precise figures are, again, only average values owing to the fact that the heat output of a small animal is rarely constant for more than a few minutes at a time (Benedict and Lee 1936).

The experiments to be described were carried out with albino mice.

§ 2. EXPERIMENTAL PROCEDURE

2.1. Apparatus

This calorimeter is similar in principle to that of Benedict and Lee (1937) in which the heat output of the animal in a chamber is balanced by the heat output of an electrically heated coil in an identical chamber. The apparatus is shown in figs. 1 and 2. The inner chambers A and B are covered with aluminium foil to obtain increased thermal conductivity. An uncovered strip is left down one side through which the animal may be observed. Inside each chamber, inserted in a rubber bung C, is an air-exit tube D, containing self-indicating silica gel. This is intended to recapture the heat emitted by the animal in the form of water vapour.

The apparatus is used in a room at 18–22°C which produces in the inner chambers an environmental temperature in the neutral zone (Benedict and Lee 1936). Air passes through a direct reading capillary flow-meter E into a thin-walled copper tubing spiral F in a water tank thermostatically controlled at 20°C. From the spiral the air passes through a drying tower G containing silica gel and thence into the animal chamber A. The air now passes through a second spiral H in the water
tank and another drying tower I before reaching the heater chamber B, whence it passes via a calcium chloride tower to a filter pump.

Equality of heat output is indicated by a reflecting galvanometer connected to two copper-constantan thermocouples arranged differentially. The thermocouples are fastened on top of the chambers by aluminium foil and adhesive. When the heat outputs are equal the temperatures of the identical inner chambers are equal and there is no deflection. The heat output of the coil is measured in terms of watts dissipated, indicated by the voltmeter and ammeter. The arrangement used will detect a difference in temperature of 0.01°C which is sufficient for all practical purposes.

Other thermocouples are inserted at various points to enable temperatures to be compared with that of the water tank.
2.2. Tests of the Apparatus

To test the apparatus a heating coil was fitted into the animal chamber. The watts dissipated in this coil, measured by a second voltmeter and ammeter, were varied by an assistant in a manner unknown to the operator who attempted to balance them. Although there was a tendency to overcorrect for changes in temperature agreement was obtained within ±2%.

To determine whether the silica gel in the inner chambers recaptured the heat given off in the form of water vapour, experiments with mice were interrupted and the silica gel inserted or removed. Adding the silica gel produced an average increase of about 10%. Removing it produced a similar decrease. An exact determination was found extremely difficult due to the variable nature of mouse activity and of excreta produced. Weighing the silica gel before and after the experiments gave an increase in mass of less than 0.1 g per hour which represented less than 12% of heat output. These figures agree with the range quoted by Benedict (1938).

The rate of air flow was found to produce no change in the heat output recorded by the calorimeter over the range of 30 ml min\(^{-1}\) to 250 ml min\(^{-1}\). Normally a flow of 50 ml min\(^{-1}\) was employed.

Measurement of the temperatures of the air entering and leaving either chamber showed a difference of less than 0.05°C which represented a negligible heat loss considering the low specific heat of air.

As an overall check of the apparatus the total heat output of a mouse over a period of two hours was measured with the calorimeter. At the same time the mass of carbon dioxide produced by the mouse was determined by bubbling the expired gases through weighed potash bulbs. The mouse had been kept on a carbohydrate diet in order to obtain as high a respiratory quotient as possible. For the purposes of the experiment it was assumed that this value was unity and the indirect value for the heat produced calculated accordingly. It was found to be 3% higher than the value of the heat output determined directly.

2.3. Technique

The air flow rate is adjusted, the mouse inserted, and the heating circuit switched on. Balance of temperature is attained within 5–10 minutes, provided the mouse is familiar with the apparatus. The mice normally wash at the beginning of a run and then settle down.

Readings may be recorded every half-minute. It is convenient for an assistant, who also notes the behaviour of the mouse, to record readings read out by the operator.

§ 3. Discussion

The layout of the apparatus has been determined after a series of experiments carried on over several years in which different types of
chamber have been used and other differential thermometric systems
such as resistance thermometers employed. The apparatus described in
this paper is the simplest that has been found to give reproducible results.

In this work the activity of mice was arbitrarily classified as follows:
(a) deep sleep, (b) quiet, (c) active. A quiet mouse was taken as one still,
but awake. To obtain reproducible conditions an active mouse was
taken as one vigorously washing.

Parts of records of a sleeping mouse, a quiet mouse and an active mouse
are given in figs. 3, 4 and 5. Several interesting facts may be deduced
from inspection of these records. In the case of a sleeping mouse (fig. 3)

\[\text{Heat output of a sleeping mouse, 29 g.}\]

it will be noted that there is no stationary thermal state typical of this
condition. In fact it is suspected that there is a definite rhythm
unconnected with visible movement.

The variable nature of the thermal state, in this case for a quiet mouse,
is also shown in fig. 4 where even when there is no apparent movement
there are fluctuations of up to 15% in the heat output.

In the case of an active mouse (fig. 5) these variations may amount
to over 50%, but these high values can usually be connected to peaks
of visible activity.
Fig. 2. The animal calorimeter.
Simple Differential Calorimeter for Small Animals

It will be appreciated that these detailed, minute to minute records would be completely lost in an apparatus dependent on indirect methods over a period of time.

Fig. 4

Heat output of a quiet mouse, 16 g.

It is hoped to publish in due course detailed records of the heat output of mice under normal and abnormal conditions.

Acknowledgments

The authors wish to express their gratitude to Professor E. C. Amoroso for the great interest he has taken in this work. They are also greatly indebted to Mr. C. Waterman and Miss G. Tomlinson for technical assistance.

Summary

A simple apparatus for giving a continuous record of the heat output of a small animal is described.
HEAT OUTPUT.
CAL.°/HR

0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1

Heat output of an active mouse, 28 g.

Fig. 5
Résumé

Description d’un simple appareil enregistrant d’une manière continue la chaleur produite par un petit animal.

Zusammenfassung

Ein einfaches Gerät zur ununterbrochenen Aufzeichnung der Wärmeabgabe eines kleinen Tieres wird beschrieben.

References

THE AVOIDANCE OF X-RAY INJURY

By K. M. DYCE, B.Sc., M.R.C.V.S., and
A. E. HAWKINS, B.Sc., A.R.C.S., A.Inst.P.

Departments of Anatomy and Physiology, Royal Veterinary College, London, N.W.1.
THE AVOIDANCE OF X-RAY INJURY

By K. M. DYCE, B.Sc., M.R.C.V.S., and
A. E. HAWKINS, B.Sc., A.R.C.S., A.Inst.P.

Departments of Anatomy and Physiology, Royal Veterinary College, London, N.W.1.

The dangers of over-exposure to X-rays and allied radiations have been stressed in the medical press, and this has drawn attention to those risks which are inseparable from the practice of diagnostic radiology, and to the appropriate protective counter-measures which can be adopted. The only uncertainty that remains concerns the actual amount or dose of radiation which may be tolerated without ill-effects. Authorities differ on this point, but it is noticeable that with accumulated experience successive estimates have become increasingly conservative, and the recommendations on protection correspondingly more severe. The latest publication in this field, the report of the Medical Research Council on "The Hazards to Man of Nuclear and Allied Radiations," is even more stringent than most of its predecessors, since it emphasises the need to consider all the sources of ionising radiation to which the population is exposed before defining a permissible figure for the dosage received from X-radiation alone.

The appearance of this report provides a timely reminder of the imperative need to examine the precautions necessary in veterinary radiography. There is little doubt that many of those concerned are not sufficiently aware of the need for precaution, and that some entirely disregard the risks involved. Whether this imprudence is due to ignorance, or to an optimistic, and ill-founded, belief that the volume of work undertaken is insufficient to justify anxiety is immaterial, since neither attitude is excusable. In particular, it should be remembered that the subordinates called upon to restrain the animals are generally exposed to greater dangers than the person who operates the controls. The veterinary literature is remarkably silent on these matters, and a paper by Henny (1953) appears to be the only relevant contribution in the veterinary periodicals. It is hoped that the present article may be of some value in drawing attention to the dangers that exist, and to the counter-measures that are available to provide protection; it will be obvious that it makes no claim to originality or to special authority, which, indeed, the authors would specifically disclaim. Fortunately, authoritative information and comment are available both in the report already mentioned and in numerous other publications. A short list of useful references is appended and particular mention of some of these is made in this article.

The nature of the injury which may be sustained from over-exposure to X-rays varies greatly and is conditioned by the amount and nature of the radiation, by the portion of the body exposed, and by the period over which the dose was received. It should be recognised that although the effects are cumulative, the response is also influenced by the intervals between exposures.
Descriptions of the resulting lesions are provided by Colwell and Russ (1934) and Abraham (1954), among others, and it is not necessary, in the present context, to describe the various forms. In general, the effects may be defined as local or generalised, and within each category there are acute and chronic varieties. All tissues of the body may be involved but the skin and blood are affected most frequently. In addition to these somatic effects, and in some ways surpassing them in importance, are the genetic consequences which may follow irradiation of the gonads. Great care must be taken to avoid injury to these parts, particularly in young people.

The menace of X-rays is increased by their insidious nature for not only are the rays themselves invisible, but their effects may be delayed. Faulty procedures may be practised for years without producing manifest injury so that when the effects at last appear, perhaps years after the last exposure, it is unlikely that the damage can be arrested, far less reversed. (See, for example, “American Martyrs to Science through the Roentgen Rays,” by P. Brown, and “X-ray and Radium Injuries,” by Colwell and Russ.)

Mention has been made of the efforts undertaken to define a permissible dosage level at and below which radiation will be tolerated without immediate or delayed ill-effect. This cannot be determined with precision since so many variables are involved and, although many estimates exist, none has been universally accepted. A unit, the roentgen or r, has been defined in which dosage may be measured. The definition need not be given in a paper of this nature and it is probable that it is wiser to avoid any quantitative indications, but it is worth noting that the permissible doses recommended by the International Commission on Radiological Protection (1954) is 0.3 r per week on blood-forming organs, gonads and eyes. It has been suggested (Lorenz, et al., 1947) that for women the permissible daily dose delivered to the ovaries should be only 0.02 r and it is stated (e.g., Quimby, et al., 1946) that the local exposure to the fingers and hands should never exceed ten times the permissible dose. (The I.C.R.P. recommended only five times.) In interpreting these figures it will be of assistance to recall that the intensity of the main beam, three feet from the target of an X-ray tube operating at about 50 kv, is of the order of 0.3 r per minute per milli-ampere of tube current.

With average installations three sources of radiation have to be considered—the primary beam, leakage in other directions from the tube, and scattered radiation arising from all objects in the path of the beam. The primary beam constitutes the most obvious danger and for this reason it is most likely to be recognised and guarded against in practice. Its coverage is restricted in the manufacture of the tube. Although rays are emitted in all directions from the focus on the target, only those that impinge upon a restricted area or window in the tube lining escape. Thus the beam is regulated by the size and relative position of the two areas—focus and window; even so, it is too widespread for safety and in practice it is further restricted by the use of limiting cones or diaphragms of absorbent material placed in its pathway. These adjuncts will be referred to again but, as will be seen, they must always be employed to ensure
that no one places his unprotected body within the area covered by the primary beam except in the case of the person undergoing a radiographic examination of that part. Elementary as this precaution may be, it is often ignored as is attested by the frequent appearance on veterinary radiographs of hands and arms grasping and restraining the patient.

In the manufacture of the tube care is taken to ensure that no rays can find exit except through the window, and leakage from other parts should not occur. It is wise to test each unit for reliability in this respect (White, et al., 1943), especially as many installations are already old and possibly faulty. An unsuspected leakage forms a very serious hazard since it permits the passage of a primary beam in a direction and in a region where no danger is anticipated.

The third source of radiation is provided by every object within the path of the beam. Each obstruction, including the patient, gives rise to some scattering of the beam in all directions. The proportion which continues on its course and the proportion which is scattered are determined by the wavelength of the original beam and also by the size and composition of the intervening object. This scattering effect is recognised by a blurring of the image and provides the reason for the use of grids to absorb this radiation when dealing with bulky parts. The potential danger it constitutes is more important than any loss of quality in the radiograph.

Scattered radiation is softer, that is, more readily absorbed, than the primary radiation and therefore protection against it is obtained more easily. It must be recognised that constant or recurring exposure is just as dangerous as exposure to the direct rays. Binks (1943) has determined an approximate expression for the intensity of the scattered radiation. From this expression it is easy to show that the permissible dose can be received in a few minutes at a point close to the patient and this has been confirmed by direct measurement (Osborn, 1955). Fortunately, as the rays diverge, their intensity diminishes rapidly according to the law of inverse squares; hence it follows that distance alone provides a measure of protection. The rapid drop in intensity with distance from a point source is shown in Fig. 1.

With these facts in mind the measures which can be adopted to provide protection may be considered. It will be found that they are of two classes. In the first category are those measures that are appropriate to all classes of radiography; in the second, those peculiar to the special circumstances of veterinary practice.

The first and most obvious aim must be to restrict the number of exposures that are performed. This may be done in two ways: by restricting the use of radiology to those cases in which the technique is essential for diagnosis or prognosis and secondly, by improving the efficiency of each examination. With regard to the first point it must be admitted that X-rays are at present used in many cases when the clinical diagnosis should require no additional support and the technique is used to impress, or more often placate, an over-anxious owner. Such unnecessary examinations should be refused. On the other
hand increased attention to details of exposure and dark room technique will do much to reduce the frequency with which repeat exposures are required. Repetition may be avoided in some cases by more care in positioning and restraint; in others by more attention to past experience. Many operators would derive much advantage, and also effect considerable economy if they compiled a list of exposures and noted the quality of the films obtained. Another useful

\[\text{Fig. 1} \]

The intensity of an X-ray beam from a point source plotted against the distance from the source.

A further general precaution is to reduce the exposure to the minimum by avoiding the use of grids and of slow film unless these are absolutely necessary. In veterinary practice fast screens and film will provide acceptable results on most occasions, perhaps even with improved definition, since the reduction of the exposure time and the corresponding reduction in movement is often of great benefit with apparatus of low power.

The insertion of an aluminium filter close to the source may be considered; this will cut out much of the softer radiation and should considerably reduce the superficial dosage without greatly affecting the film-blackening quality of the residual radiation beam (Martin, 1947). This practice does not appear to be adopted in veterinary radiography very often, but it is worthy of trial.
Perhaps the most important of the general measures concerns the use of cones or diaphragm to reduce the field covered by the beam; this reduces the danger not only from the primary beam but also, in corresponding degree, from the secondary or scattered radiation. Above all, where it is absolutely essential to restrain an animal by hand, a small cone or a restricted diaphragm will at least ensure that with proper centring the assistant is outside the line of direct rays. A valuable routine is to keep the smallest cone in position, and although it must be replaced when larger parts are to be examined, constant supervision should be exercised to ensure its return as soon as possible; painting the larger and more dangerous cones in a conspicuous colour is useful since it draws attention to neglect of this precaution.

The additional difficulties experienced in veterinary radiography arise through the need to control patients who are often unco-operative and frequently aggressive. This also provides the greatest danger for the obvious method of control is the active manual restraint of the animal in the required position. Those called upon to do this will be exposed to radiation. Before considering how to overcome this difficulty it is as well to place the problem in its true perspective and to recognise that in veterinary practice the principal concern must be for the welfare of those taking part in the examination and only to a lesser degree with the patient. Fortunately, it is necessary only rarely to consider the danger to the patient of excessive radiation; most animals are protected against superficial injury to some extent by the covering of hair and the exposures they are likely to receive are rarely sufficient to produce injury; some protection against delayed effect is also provided by the relatively short span of life of many species. When accidental damage has been reported (e.g., Neumann-Kleinpaul and Zeller, 1955) it has been as the result of experimental procedures. The possibility of producing genetic injury might be worth considering when dealing with breeding animals and the effect of X-rays on the foetus by repeated examination of pregnant animals should be remembered (Boddie, 1946).

It is the risk to the personnel to which most attention must be directed. The operator of the controls is relatively safe; he is placed at some distance from the X-ray tube and, if necessary, he can be guarded easily by a protective screen. Those holding the animal are much more vulnerable and it is essential that the practitioner recognises his responsibilities to them. The most obvious and the most effective way to overcome this danger is to administer a sedative or anaesthetic to the animal so that it can be handled and positioned without the need for restraint during the actual exposure. The newer anaesthetic and sedative agents reduce both the risks to the patient and the inconvenience to the operator, and the possibility of employing one or other of these drugs should be considered in every case. The extra inconvenience may be slight since in many cases anaesthesia merely anticipates what would, in any case, be indicated by the result of the X-ray examination.

At present it is often the practice to safeguard assistants by requesting the owner to restrain the animal or by allocating the duty in rotation. Unfortunately
the owner is unlikely to perform this task efficiently and it may, in fact, be beyond the capabilities of one person unaided. Rotation is also not very effective unless there are sufficient helpers available to make it of real benefit. All too frequently, despite good intentions, it becomes the custom for the same people to perform this task. It is difficult to reconcile either practice with recent recommendations advocating the avoidance of radiography in medical practice where only minor conditions are involved. Whoever restrains the patient, it is essential that protective clothing should be provided and worn. It must be admitted that the clothing is not very satisfactory. The gloves are clumsy and prevent adequate control of a struggling animal, while aprons, though less inconvenient, are heavy and cause fatigue when worn for any length of time. Although these articles and, in particular, gloves give only partial protection from the main beam and may contribute to a false sense of security (Osborn, loc. cit.), it is to be regretted that more use is not made of them; often they are only worn when new and discarded when the novelty is gone. (Recent work in America [Archer, 1955] with fabric of spun lead glass promises protection without discomfort.)

A device that may be worth consideration is a lead-covered box which can be placed over the animal, but permits the passage of the head and limbs through a large opening at each end which is guarded by a series of overlapping lead rubber flaps. The tube is centred within the protected area and the walls stop most of the scatter so that the limbs may be held with safety. Considerable use has been made of this device even though it has certain drawbacks. It is heavy and clumsy and, unless rather large, it interferes with the chest when the patient is in the dorsal position and is likely to frighten a nervous animal.

So far all mention of fluoroscopy or screening examinations has been avoided. Undoubtedly these constitute the greatest threat (Barclay, 1934; Taft, 1941; Sievert, 1947, and many others). It is relatively easy to avoid danger when making radiographs but it is difficult to achieve a similar degree of safety when screening unless elaborate and expensive precautions are taken. These restrict the fluoroscopist by their somewhat unwieldy nature (Chamberlain, 1943). These requirements may be mentioned but first it is as well to consider the actual value of screening to the average practitioner. Few veterinary surgeons have either the facilities or the scope to become proficient in its use. It would appear that screening is employed as a rapid and cheap substitute for radiography of such conditions as fractures and the detection of foreign bodies and its real potentialities in the investigation of dynamic processes are largely ignored. In these circumstances it seems wiser to discontinue the practice entirely.

Those who wish to continue should ensure that examinations are made only when absolutely necessary and that they are of the minimum duration. Great attention should be paid to securing proper adaptation of the eyes by using a fully-darkened room and spending at least 15 minutes in total darkness or in a dim red light. If this practice is adhered to it will be found that the beam
intensity may be reduced and yet the image will appear brighter, enabling a diagnosis to be made in a fraction of the time formerly required.

It is important to make sure that the operator is not exposed to radiation for it is easy to forget the danger when engrossed in the study of the screen. The apparatus should be so constructed that the beam cannot overlap the fluorescent screen which is covered in lead glass. To ensure this, the movements of the screen and tube should be co-ordinated and the lighted part of the screen kept within the margin of the glass. There is considerable exposure to scattered radiation, at a point where its intensity is high, and protective material must be constantly between the observer and the source of all radiation, whether primary

or secondary. A number of alternative methods of doing this are illustrated in Fig. 2. These can be used instead of, or in conjunction with, gloves and apron. Palpation of the part under investigation may involve direct exposure of the practitioner’s hands to the primary beam (Stevenson and Leddy, 1937).

It is advisable to maintain a check upon the exposure received. This may be done by wearing a sensitive film which can be examined periodically for evidence of exposure (Quimby, loc. cit.). Alternatively, the facilities offered by the Radiological Protection Service at Sutton, Surrey, may be utilised.

It is a sound precaution to undergo regular blood examinations since quite often over-dosage is indicated first by alterations in the differential and total cell counts. These examinations and their interpretation should be done by experts. Usually where there is evidence of over-exposure it is sufficient to cease X-ray work for a time to restore full health, but competent advice should be sought always. It should be noted that some doubt about the usefulness of blood examinations has been expressed recently (Loutit, 1955).
The authors hope that their remarks do not make it appear that all X-ray work is attended inevitably by frightening risks. In actual fact it is recognised that reasonable alertness and caution are sufficient to avoid any danger, but, as Sievert (loc. cit.) has remarked, propaganda and instruction are the most important measures for preventing radiation injuries, and the most efficient protection arrangements are of little value if those working with X-rays do not appreciate and understand the risks to which they are exposed. The truth of this is illustrated by the records of 70 patients with radiation injuries who were treated in an American hospital between 1939 and 1942 (Uhlmann, 1942). Fifty per cent of the patients were not professional radiologists but had been engaged in X-ray diagnostic work. Only 4 per cent of the injured were radiologists. The latter had been practising their profession for periods ranging from 20 to 40 years.

REFERENCES
International Commission on Radiological Protection (1938) : Radiology, 30, 511.
Quimby, E. H., Stone, R. S., et al. (1946) : "Protection against X-rays." Radiology, 46, 57.