INVESTIGATION INTO THE RADIATION PATTERN OF SURFACE WAVE AERIALS AT 150 Mc/s AND AT 9000 Mc/s.

by

W. Hersch B.Sc.(Eng.) A.M.I.E.E.

Thesis submitted for the Degree of Doctor of Philosophy (Eng.) in the University of London. 1957
SUMMARY

The radiation from the open-circuited end of an external dielectric coated waveguide can be controlled by varying the size of the guide, the thickness and/or the dielectric constant of the coating. A new type of aerial designed around this principle is given the name "Surface Wave Aerial", and radiation pattern measurements are used to confirm the theory underlying this type of radiator.

According to its mode of operation it belongs to the category of end-fire aerials which are briefly reviewed to show that "Surface Wave Aerials" occupy a place in their own right amongst the many possible arrangements that utilise the end-fire effect to produce a directional radiation pattern.

The theory of the "Surface Wave Aerial" is developed in detail; a necessary preliminary step being a full theoretical analysis of the properties of the first order cylindrical surface wave. It is shown that a dielectric coated cylinder which is approximately a wavelength in circumference can act as a waveguide for higher order surface waves, of which the first order is an example. The "Characteristic Equation" is determined for the general case from which in turn the cut-off frequency and the propagation constant are derived. Two specific cases are
evaluated numerically and the results are used to calculate the polar diagrams of Surface Wave Aerials operating at 150 Mc/s and at 9000 Mc/s respectively.

Experimental confirmation at 150 Mc/s is obtained by means of semi-automatic polar diagram plotting equipment which is described in detail. The initial choice of frequency, governed by available high frequency generators, did not permit the full exploration of the properties of the 1st order surface wave and a short section is devoted to experiments which were carried out at 9000 Mc/s. The propagation constant was measured directly and the behaviour of very long aerials, approximately 20 wavelength long, was studied. In conclusion an outline of future work is given.
# INDEX

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Introduction</td>
</tr>
<tr>
<td>2.</td>
<td>End Fire Arrays</td>
</tr>
<tr>
<td>2.1</td>
<td>General Remarks</td>
</tr>
<tr>
<td>2.2</td>
<td>Long Wire Aerials</td>
</tr>
<tr>
<td>2.3</td>
<td>Yagi Arrays</td>
</tr>
<tr>
<td>2.4</td>
<td>Unconventional Arrays</td>
</tr>
<tr>
<td>2.5</td>
<td>Helical Aerials</td>
</tr>
<tr>
<td>2.6</td>
<td>Dielectric Rod Aerials</td>
</tr>
<tr>
<td>2.7</td>
<td>Dielectric Tube Aerials</td>
</tr>
<tr>
<td>2.8</td>
<td>The Surface Wave Aerial</td>
</tr>
<tr>
<td>3.</td>
<td>Radiation Pattern Measurements</td>
</tr>
<tr>
<td></td>
<td>at 150 Mc/s</td>
</tr>
<tr>
<td>3.1</td>
<td>General Considerations</td>
</tr>
<tr>
<td>3.2</td>
<td>The System Used for Plotting</td>
</tr>
<tr>
<td></td>
<td>The Radiation Pattern</td>
</tr>
<tr>
<td>3.2.1</td>
<td>Details of the System</td>
</tr>
<tr>
<td>3.2.2</td>
<td>Signal Amplitude and</td>
</tr>
<tr>
<td></td>
<td>Receiver Gain Stability</td>
</tr>
</tbody>
</table>
| 3.2.3 | Frequency Stability, Band-
|      | width and Tuning Range |
| 3.3 | The Transmitter |
3.4. The Receiver
3.5. The Wavemeter.
3.6. Standardisation Routine
3.7. Errors
3.7.1. Assessment of Overall System Accuracy.
3.7.2. Polarisation Errors
3.7.3. Experimental Confirmation of System Accuracy.

4. The First Order Surface Wave. . . . . . . . . . . . . . 29
4.1. Surface Waves, Historical Background
4.2. Theory of the First Order Surface Wave.
4.2.1. The General Problem
4.2.2. The Field Equations
4.2.3. The Characteristic Equation
4.2.4. Cut-off Frequency and Higher Order Modes
4.2.5. The E / H Ratio

5. The Surface Wave Aerial . . . . . . . . . . . . . . . . . . 44
5.1. Methods of Launching 1st Order Surface Wave
GLOSSARY OF SYMBOLS

a       Radius of waveguide without dielectric.
a_2a_3a_4 Coefficients
\( \alpha \) Attenuation constant
A_1A_2.B_1B_2 Co-factors
b       Radius of waveguide including dielectric.
b_2b_3b_4 Coefficients
\( \beta_0 \) Phase constant of wave in free space
\( \beta_\gamma \) Phase constant of guided wave
c       Ratio a/b
\( \Delta \) Laplacian operator
\( \Delta_1 \Delta_2... \) Ratios of Bessel functions, defined on p. 37
\( \nabla \) Determinant
\( \varepsilon_0 \) Absolute permittivity of vacuum
\( \varepsilon, \varepsilon_2 \varepsilon_3 \) Relative permittivity of medium 1,2,3.
\( \varepsilon \) Relative permittivity of a medium
E       Electric field
f       Frequency of wave
\( \gamma \) Propagation constant
H       Magnetic field
H_n     Bessel function of order \( n \)
J_n     Hankel function of order \( n \)
Y_n     Neuman function of order \( n \)

Addition of prime denotes differentiation with respect to argument.
\[ i = \sqrt{-1} \]
\[ k_p = \left( \frac{\omega^2 \mu_0 \varepsilon_p - \gamma^2}{\gamma^2} \right)^{\frac{1}{2}} \]
\[ \lambda_o \] Free space wavelength
\[ \lambda_g \] Guide wavelength
\[ \mu_o \] Absolute permeability of vacuum
\[ \mu_1, \mu_2, \mu_3 \] Relative permeability of medium 1, 2, 3.
\[ \mu \] Relative permeability of a medium
\[ N \] \[ k_3 b \]
\[ \omega \] \[ 2\pi f \]
\[ q \] Ratio \[ b/\lambda_o \]
\[ Q \] \[ \omega^2 \mu_0 \varepsilon_0 / \gamma^2 = \left( \frac{\lambda_g / \lambda_o}{\lambda_o} \right)^2 \]
\[ r \gamma z \] Cylindrical polar co-ordinates
\[ T \] \[ (1/v)^2 + (1/W)^2 \]
\[ \theta \] Angle of direction with respect to longitudinal axis of aerial.
\[ \tan \delta \] Loss tangent of dielectric medium.
\[ v_g \] Velocity of propagation of guided wave
\[ V \] \[ k_2 b \]
\[ W \] \[ iN \]
\[ x y z \] Rectangular co-ordinates
5.1.1. General Remarks . . . . . . . . . . . . . . . . . 44
5.1.2. Launching at Metre Wavelength
(150 Mc/s)
5.1.3. Launching at Centimetre
Wavelength (9000 Mc/s)
5.2. Reflectionless Transmission
5.3. Constructional Details of
Surface Wave Aerial
5.4. The Theoretical Radiation Pattern. 49
5.4.1. General Considerations
5.4.2. The Short Aerial
5.4.3. The Long Aerial
6. Experimental Verification of
Aerial and Waveguide Theories . . 54
6.1. The Microwave Test Gear.
6.2. The Polar Diagram of Short Aerials
6.3. The Polar Diagram of Long Aerials
6.4. Direct Measurement of the
Propagation Constant.
7. Conclusions and Future Work . . . 66

continued
8. Acknowledgements ............................................. 70
Appendix I .......................................................... 71
   Reduction of Observational Errors
   by the Use of Statistical Methods.
Appendix II ........................................................... 72
   The Measurement of Power Fluctuations at Very High Frequencies.
Appendix III ......................................................... 75
   Derivation of the Characteristic
   Equation.
Appendix IV ........................................................... 83
   Derivation of the E/H Ratio
Appendix V ............................................................ 87
   The Theoretical Polar Diagram
   of Short Aerials.
Appendix VI .......................................................... 91
   The Theoretical Polar Diagram
   of Long Aerials.
References ........................................................... 93
1. **Introduction.**

The radiation pattern of aerial arrays can be accurately calculated if the phase and amplitude distribution of the radiating elements is known. (Ref. 1). In general these radiating elements can be discrete radiators or current elements forming part of a continuous current distribution.

The reverse problem, that of determining the "aperture" distribution required to give a specified radiation pattern can only be solved by approximations, theoretically to any desired degree of accuracy. (Refs. 2; 3; 4). In practice it is found that the accuracy is limited as the calculated field distribution may not physically be realisable. (Ref. 2; 5)

However, if an actual radiation pattern has been measured, it is possible to deduce the aperture distribution or the phases and amplitudes of the currents in the radiating elements if the configuration of the array is known.

A polar diagram plotting system was therefore developed, working at 150 Mc/s, to investigate a new type of continuous end-fire aerial, and to confirm the theory which predicts the current distribution as a function of various aerial parameters. When the work was first

* The term "aperture" is used to denote the effective radiating aerea, although in the physical sense such aperture need not exist.
started in 1947, generators of appreciably higher frequencies, like magnetrons and klystrons, had just passed the experimental laboratory phase but were not generally available, nor had microwave test gear been developed to facilitate accurate measurements.

In polar diagram plotting, accuracy and ease of operation increase almost in proportion as the frequency is raised, but the design of the equipment used by the author had to be based on techniques established at the time and the then available frequency bands, with the attendant restrictions that at 150 Mc/s single aerials were limited in size to \(3\lambda\) for purely physical reasons. However, certain properties of higher order surface waves with which most of the investigations in this treatise are concerned are of such intrinsic interest that it was decided to make further studies at frequencies centered around 9000 Mc/s when commercial test equipment became available.

The experimental work is, therefore, divided into two parts, measurements at 150 Mc/s and at 9000 Mc/s respectively.

The new type of aerial described by the author belongs to the category of end-fire arrays. The next section is, therefore, devoted to a survey of this type of aerial from which it will be seen that the new aerial occupies a place in its own right amongst the many arrangements that are possible to produce an end-fire effect.
2. **End Fire Arrays**

2.1. **General Remarks**

The description "End-Fire Array" is applied to all aerials in which there is a progressive phase shift between adjacent radiating elements. (In this treatise a further differentiation will be made by using the term "aerial" instead of "array" when the radiating elements are not discrete radiators but form part of a continuous current distribution). In the majority of applications of this principle this phase shift amounts to $360^\circ$ per free space wavelength. Such arrays will be called "conventional" arrays. Special types of end-fire arrays have been proposed in the past in which the phase shift is slightly greater or smaller than $360^\circ$ per wavelength and the polar diagrams obtained from such arrangements are significantly different from those of conventional end-fire arrays.

2.2. **Long Wire Aerials**

A transmission line consisting of a continuous wire supported above ground and terminated by its characteristic impedance constitutes the simplest form of end-fire aerial. The velocity of a wave travelling along the wire is equal to the free space wavelength and the conditions for a conventional end-fire array are fulfilled. Owing to the fact that the
"far-end" of the wire is terminated in a resistance, radiation in the direction along the wire is zero and maximum radiation is concentrated in two major lobes placed symmetrically about the centre of the wire, as shown in Fig.1. The long wire type of aerial has been used in a variety of combinations to form "V's" and "Rhombics". (See Refs. 6; 7), the main lobe occurs then along the line of symmetry. (Fig.1. Polar Diagram of Long Wire Aerial, 2\textsuperscript{along})

2.3. Yagi Arrays

End-fire arrays consisting of discrete radiators were discussed as early as 1922, when the theory underlying the radiation patterns of aerials was first developed. (Ref. 8).

A special method of construction is due to Yagi (Refs. 9; 10) who arranged half wave length long dipoles along a central support. Only the first dipole has to be energized, the others being excited by mutual coupling (See Fig. 2.) The impedance of the parasitic dipoles called "directors" being arranged in such a manner that a progressive phase shift of 360° per wave-
length takes place along the array. The addition of a reflector behind the first dipole greatly increases the forward gain of Yagi aerials and reduces radiation in the backward direction. The literature on this type of aerial is extensive and many authors have discussed the conditions under which optimum performance can be obtained.

Reid (Ref.11) discusses the hypothetical gain that can be obtained from an idealized Yagi array, assuming an infinite number of closely spaced current elements forming a current sheet.

Fishenden and Wiblin (Ref.12) formulate a design procedure for Yagi aerials of specified performance and infer in the course of their paper that a further phase shift in addition to the 360° per wavelength results in greater gain.

2.4. Unconventional Arrays

An improvement upon the conventional type of end-fire array has been suggested by Hansen and Woodyard (Ref.13) who tried a uniform amplitude distribution and various linear phase distributions and concluded that
an additional phase shift of 180° distributed uniformly over the whole length of the array would result in optimum gain. The increase in gain thus obtained is achieved at the expense of increased side lobes.

A very clear physical explanation of this effect is provided by Goward (Ref.14) whose explanation leads him to suggest a further improvement. The array proposed by him has not only a greater gain but the side lobes are smaller than is the case with a conventional end-fire array.

Basically it retains the 180° additional phase change and superimposes two radiation patterns in such a manner that the side lobes are cancelled except in the direction of the main beam where only partial cancellation takes place, resulting in a beam which is narrower than that due to either pattern alone. A high reactive component of the radiation resistance is the only penalty that has to be paid for the improved performance.

2.5. Helical Aerials

A helix may radiate in many modes. The two radiation modes of greatest interest are the normal mode and the axial mode. The latter type of mode turns the helix into an end-fire aerial, the radiation being practically circularly polarized. The phase shift along the aerial is less than the phase shift
in free-space, however the radiation pattern of helical aerials depends in a complicated manner on the physical dimensions of the helix. This type of radiator is included in this survey for the sake of completeness, (See Fig.3). Kraus (Refs. 15, 16, 17) had dealt most extensively with the theoretical and design aspects of helical aerials.

Fig.3. Helical Aerial

2.6. **Dielectric Rod Aerials**

The study of dielectric rod aerials, like that of helical aerials, dates back only a few years, although chronologically dielectric rods were investigated approximately seven years earlier (1939).

Dielectric waveguides had been treated analytically by Hondros and Debye (Ref.18) who showed in a paper published in 1910 that transmission along a single dielectric rod was possible at sufficiently high frequencies. The dielectric rod had not been used in practice until the war years when Mallach (Ref.19) carried out extensive work as part of a four-year plan. Valuable contributions to the subject of dielectric rod aerials were made by Halliday and Kiely (Refs. 20; 21).
The main virtue of this type of aerial is its great bandwidth, ignoring the limitations imposed by the launching device and its relative freedom from side lobes. As the dielectric rod aerial possesses features which are in many respects similar and in others complementary to the "Surface Wave Aerial" investigated by the author in the following chapters, a brief description of its method of operation is given.

A rod of insulating material unbounded by metal walls, though poor and inefficient as a transmission system can be used as a good directional radiator, the radiation being mainly in a single lobe, coaxial with the rod, with side-lobe amplitude and back radiation relatively small. Fig. 4 shows the general principle.

A dielectric rod without metal walls has many of the properties of a metal-enclosed waveguide. There are E- and H-waves similar to those inside metal waveguides, and there are critical frequencies, except for the fundamental mode.

Owing to the different boundary conditions, however, there are essential differences between waves along
dielectric rods and waves along metal waveguides. For example, it is only in the case of the circularly symmetrical waves, the $E_0$ and $H_0$ series, that $E$- and $H$- waves can exist separately. In the case of other modes, an $E$- wave and an $H$- wave must normally be superimposed in order to satisfy the boundary conditions; the $E$- and $H$- waves are inseparably linked with one another. In particular it is impossible for the $H_1$ wave of a circular waveguide to exist by itself in a dielectric rod. It must be associated with an $E$- wave although the amplitude of the $H$- wave is relatively much the greater, and the field distribution in the rod resembles, for the most part, that of the $H_1$ mode in a metal waveguide.

2.7. **Dielectric Tube Aerials**

The waveguide properties of a dielectric tube are similar to those of a dielectric rod; two types of wave can exist, $E$- waves and $H$- waves, and these can only exist in pure forms in the modes of circular symmetry; when symmetric modes are excited these are hybrid, a subsidiary $E$- mode existing necessarily with the dominant $H$- mode, and vice versa.

The radiation patterns of dielectric tube aerials are critically dependent upon wall thickness. There are three well defined thickness levels and accordingly the tube has either no effect at all on the pattern (very thin wall), the pattern is single-lobed below a certain
thickness, the pattern is multi-lobed for wall thicknesses above that value.

Little work has been carried out on this type of aerial so far; a summary of the theoretical and experimental knowledge that has accumulated to date is given by Kiely (Ref. 21)

2.8. The Surface Wave Aerial

A metal rod coated with a dielectric skin can support a first order surface wave if excited by a suitable launcher. The mode, although essentially an E-wave, is a hybrid mode with an attenuation sufficiently low to enable end-fire aerials to be built. It was the author's idea to study this type of radiator which was given the name "Surface Wave Aerial". The general principle and field distribution are shown in Fig. 5. It possesses many of the virtues of a dielectric rod aerial such as great band-width and relative freedom from side-lobes. In addition, it is considerably stronger as it is constructed from a metal rod or tube which allows aerials of greater length, and hence
greater directivity, to be made than is possible with dielectric rods.

The field of the surface wave aerial is mainly confined to one plane, the E-plane, and is zero in a plane at right angles. This is a unique property exhibited by no other end-fire aerial. Groups of such radiators can, therefore, be closely spaced in the H-plane to produce narrow, wafer-like radiation patterns.
3. Radiation Pattern Measurements At 150 Mc/s

3.1. General Considerations

Measurements of field strength are notorious for their inaccuracy and any attempt to utilize a polar diagram plot to determine the phase and current distribution along an aerial involves first of all a careful study of the factors that are responsible for this inaccuracy. The next step is to design a system consisting of a transmitter and a receiver with their associated aerials and aerial sites in which the errors arising from the various causes are known and controlled so that the overall system accuracy can be assessed.

Preliminary considerations showed that, excluding polarization errors, an inherent overall system accuracy of 10% was a desirable target. Improvements on that figure, if required, could be obtained by using statistical methods, see Appendix I, based on a large number of observations. Individual, permissible errors within the system were in turn derived from that assumption.

The causes which reduce the accuracy in field strength measurements are listed below:

i) Frequency drift of transmitter
ii) Variation of power output of transmitter
iii) Detuning of receiver
iv) Change in receiver gain
v) Poor "Signal to Noise" ratio
FIG. 6 AERIAL MAST
vi) Misalignment between transmitter and receiver aerials.

vii) Positional (angular) variation of transmitter and/or receiver aerial.

viii) Polarization errors arising as a result of:-
   a) Two way reception of signal via direct and indirect ray.
   b) Reflections from surrounding trees and buildings.
   c) Unbalance in the transmitting aerial
   d) Unwanted pick-up by the receiver input lead.

Based on the above considerations a system for the accurate plotting of polar diagrams is described in the following sections and the steps taken to reduce the various types of error are discussed.

The nature of the work precluded the help of assistants and with the exception of the receiver the system had to be fully automatic in operation.

3.2. The System Used For The Plotting Of Radiation Patterns

3.2.1. Details Of The System

The arrangement used to measure polar diagrams at 150 Mc/s consisted of an 8 watt crystal controlled transmitter which generated an unmodulated signal of constant amplitude and frequency. The signal was fed to the aerial via a coaxial cable.

The aerial in turn was mounted on top of a rotatable aerial mast (Fig.6) the rotation of which was controlled by a motor operated by a timing mechanism in the transmitter.
The function of the timing mechanism (incorporating a cam driven by a synchronous electric motor) was three fold.

Every 25 seconds it energized the aerial drive motor and caused it to rotate the aerial by 5°. While this operation was taking place a modulator in the transmitter, also controlled by the timing mechanism, produced a 3,000 c/s audio modulation of the 150 Mc/s carrier thus indicating to a distant observer that the aerial position was changing.

After completion of 180° rotation, the timing mechanism started a small electric motor which at the end of a built-in gear box carried a "Call Signal Disc". When revolving, this Call Signal Disc turned the 3,000 c/s modulator on and off in accordance with the call sign symbols (G9KH) translated into Morse code.

This last function of the timing mechanism had to be included in order to satisfy a requirement of the G.P.O. (Radio Branch) Licencing Authority which stipulates that an experimental transmitter station must send out its call sign three times every fifteen minutes. This period determined the choice of the 25 second time interval between successive movements of the aerial mast. It proved to be a convenient time in which a reading could be taken and a 5° rotation every 25 seconds added up to 180° every 15 minutes.

The call sign requirement, although at first unwelcome was turned to advantage. The angular position of the aerial mast at which the call sign was transmitted could be chosen at will. The inclusion of this distinctly
modulated signal every 180° enabled a distant observer to check the exact position of the aerial once every 15 minutes. On receiving the call sign one could also make sure that 36 readings had been taken since the last call sign had been received and no points had therefore been missed.

The rotation of the aerial was not continuous but the direction of rotation was reversed after every complete sweep, normally 360°. In order to allow small sections of the polar diagram to be explored an adjustment on the motor drive reduced the rotation per step from 5° to 1 degree.

This facility together with an adjustment of the reversal stops enabled an arc of 36° to be swept repetitively, the time taken per sweep and the interval between call sign transmissions (both 15 minutes) remaining unchanged.

The receiver consisted of a crystal controlled double superhet with built in calibrating oscillator to standardize the overall gain; it could measure inputs as low as 1 μV and by means of attenuators as great as 100 mV.

The receiving aerial was fixed and of the straightforward λ/2 dipole and reflector type.

A temperature compensated, coaxial wavemeter was used as a portable transfer standard to ensure that the transmitter and receiver frequencies remained within the design limits.
The transmitter with its associated rotatable mast and supporting structure was mounted on top of a high tower ("Ruxley Towers", the headquarters of N.A.A.F.I) in Claygate, Surrey.

The receiver was installed in different points at various times, the distances from the transmitter ranging from two miles to half a mile for reasons explained in Section 3.7.2.

The ratio between transmitter power and receiver sensitivity might appear to be in excess of the normal requirement for satisfactory point-to-point communication for the distances involved in polar diagram measurements, but it will be seen later that not all the transmitter power was effectively radiated.

3.2.2. Signal Amplitude And Receiver Gain Stability

The frequency drift between transmitter and receiver can be compensated by tuning the receiver provided the sensitivity remains unchanged; it can also be taken care of by the "flat-topped" bandwidth of the receiver itself.

Constancy of transmitter signal amplitude can be assured by using stabilized power supplies or a voltage or current sensitive output monitor which measures and rectifies a fraction of the transmitter output and compares it with a standard reference voltage. The difference voltage,

*"Flat-topped" bandwidth is here defined as the frequency band over which the receiver response varies by less than 0.1 dB."
suitably amplified in a feed-back network, can be used to reduce the power output variations to any desired degree depending solely upon the gain of the feed-back network and the response characteristic of the controlled parameter.

Fortunately these refinements were not needed in the transmitter, as preliminary experiments showed that the short-term amplitude stability (1 hour) was better than \( \pm 0.3 \text{ dB} \) after an initial warming up period.

A remote check can be carried out by comparing the field strength measured by the receiver every time the aerial is known to pass through the same point of the polar diagram.

Receiver gain stability can be achieved by the judicious choice of temperature compensating circuit components, such as thermistors and condensers possessing a negative temperature coefficient. The absolute gain can only be established by injecting a known signal into the receiver.

Having decided that the transmitter output would be allowed to drift slightly, provided it was continuously monitored by a calibrated receiver, it was considered essential to provide the receiver with its own built-in standard signal generator. Temperature compensation was, therefore, not required.

In order to ensure that the frequency of the injected calibrating voltage was correct and corresponded to that of the transmitter frequency, a temperature com-
pensated coaxial type wavemeter was used as a transfer standard between the transmitter and the receiver.

The wavemeter had to be capable of resolving a frequency spectrum smaller than the total frequency range over which the receiver was designed to operate (Tuning range + "flat-topped" bandwidth).

3.2.3. Frequency Stability, Bandwidth and Tuning Range

The bandwidth and tuning range of the receiver are intimately linked with the frequency stability of the transmitter. Both are determined by the maximum frequency drift that is likely to occur between transmitter and receiver over the range of temperature variations to be expected within the equipments.

Size and complexity of the receiver increases in proportion to the frequency divergence. The actual stability of crystals with nominally zero temperature coefficient is shown in Fig. 7, from which it will be seen that freedom from drift occurs only over a very narrow temperature range. Unless temperature stabilisation is employed, frequency drift must be allowed for.
FIG. 8  FREQU.  DRIFT  OF  RECEIVER  AND  TRANSMITTER  CRYSTALS
The transmitter and receiver crystals had to operate at different frequencies. The plots of temperature versus frequency shown in Fig.8 are experimental verifications of the performance data quoted by the manufacturers.

As no adjustment of the transmitter frequency was possible once the transmitter was installed, frequency drift had to be entirely compensated by the receiver. For the receiver, therefore,

\[
\text{Bandwidth plus Tuning Range} = \text{Maximum Frequency Divergence} \times \text{Signal Frequency}
\]

Assuming unequal warming up periods for transmitter and receiver over the temperature range 10° to 60°C, see Fig.8, the minimum

\[
\text{Bandwidth plus Tuning Range} = 80 \text{ c/Mc/s} \times 150 \text{ Mc/s} = 12 \text{ Kc.}
\]

3.3. The Transmitter

The transmitter which was the first item to be constructed, consisted at first of a 20 watt unstabilised, self-excited power oscillator tuned to 150 Mc/s, preceded by a modulator which by means of plate modulation produced a 3 Kc/s amplitude modulation of the oscillator output when required, as described earlier in Section 3.2.1. Timing circuits, an electric clock, relays and a thyratron and the necessary power supplies completed the self-contained transmitter.
FIG. 9 FREQUENCY DRIFT OF FREE-RUNNING TRANSMITTER.
FIG. 10 BLOCK SCHEMATIC OF TRANSMITTER.

- AERIAL MAST
  - 150 M/s DRIVER
  - MULTIPLIER STAGES
  - POWER SUPPLIES
    - ALL HEATERS & DC SUPPLIES
  - 3000-9/s GENERATOR
  - POWER AMPLIFIER
  - MODULATOR
  - 25 M/s CRYSTAL
  - CLOCK
  - TIMING UNIT

- MAINS
FIG. 12 TRANSMITTER (WITHOUT COVER) FRONT.
FIG. 13 TRANSMITTER (WITHOUT COVER) REAR. SUB-UNITS LIFTED FOR INSPECTION.
Two cables connected the transmitter with the aerial mast. One of them supplied power to the aerial mast drive motor and also provided the link between the drive gear and the call signal circuits. The other cable was used to feed r.f. power to the aerial.

The frequency drift of the free-running transmitter as indicated by the coaxial wavemeter, appeared to settle down approximately 55 minutes after switching on in the manner shown in Fig.9. It was, therefore, thought that experimental work could begin one hour after switching on as the receiver tuning would cope with any residual drift.

It was not until the crystal controlled receiver had been build that this arrangement was found to be quite unworkable as minor, rapid frequency fluctuations were found to be present in the transmitter to which the wavemeter had not responded.

It was, therefore, decided to alter the transmitter and to add a crystal controlled driver unit, using a 25 Mc/s 3rd. overtone crystal and several multiplier stages. A block schematic of the transmitter is shown in Fig.10. and the complete circuit diagram after conversion in Fig.11. The constructional details can be seen from Figs. 12 and 13 respectively, which show the transmitter without cover.
FIG. 14 BLOCK SCHEMATIC OF RECEIVER.
FIG. 15 BALANCE-TO-UNBALANCE TRANSFORMER

FIG. 16 CROSS-SECTION THROUGH ABOVE
The conversion entailed a reduction of the transmitter output power to 8 watts. The constancy of the output was checked by means of a novel type of dummy load, developed by the author, which allowed very rapid power fluctuations to be observed with great accuracy. Details of the method and the dummy load, which used a gold film resistor immersed in carbon-tetrachloride, are described in Appendix II.

After an initial warming up period, the power remained constant within 0.3 dB over any 15 minute time interval.

3.4. The Receiver

The receiver consisted of a crystal controlled double superhet, one crystal by suitable multiplication of its frequency determining the first as well as the second intermediate frequency. Fig. 14 shows the block schematic of the receiver.

The balanced dipole and reflector type receiving aerial supported on an eight-foot mast, fed the signal via a twin screened cable to a "balance - to - unbalance" transformer, the construction of which is shown in Figures 15 and 16 respectively. This arrangement was essential to ensure that no signal was picked up by the aerial cable. (Subsequent measurements showed that in fact less than 60dB of the signal was induced in the feeder).
FIG. 17 CIRCUIT DIAGRAM OF RECEIVER.
The transformer was plugged straight into a six position input selector switch-cum-attenuator on the receiver which could reduce the signal by as much as 60 dB in steps of 20 dB, the output meter (M2) of the receiver covering a 10:1 range. A fifth position on the selector switch enabled the receiver input to be short-circuited so that the output meter circuit could be set to zero. In the sixth position the output from a built-in calibrating oscillator was injected into the receiver to enable the gain to be standardized.

Two low noise grounded grid stages and a push-pull amplifier raised the signal a total of 26 dB before it reached the first mixer. (Frequency change from 150 Mc/s to roughly 24½ Mc/s). Two stages of 1st I.F. amplification increased the signal by a further 30 dB before it was reduced a second time in frequency (from 24½ Mc/s to 460 Kc/s). Three stages of 2nd I.F. amplification followed with a further gain of 74 dB before the signal was finally rectified and registered on a meter. A further stage of audio frequency amplification made the 3,000 c/s modulation audible in a small monitor loudspeaker.

The complete circuit diagram with the actual local oscillator and intermediate frequencies is shown in Figure 17; only the few adjustable controls such as condensers, potentiometers and the input selector as well as the two
FIG. 18 EXTERNAL VIEW OF RECEIVER.
FIG. 19. CONSTRUCTIONAL DETAILS OF RECEIVER.
FIG. 20 FREQUENCY RESPONSE OF RECEIVER.
meters have been designated by letters for future reference. An external view of the receiver can be seen in Figure 18, and the inside construction is shown in Figure 19.

Tuning was achieved by frequency pulling of the local oscillator crystal which, working on a fundamental of 8.3592 Mc/s could be reduced by 500 c/s using a 25 pF variable shunt condenser, resulting in a tuning range for the receiver of approximately 10 Kc/s. The bandwidth was virtually flat over a 6 Kc/s band as a result of carefully aligned, stagger-tuned 2nd I.F. Stages. Frequency misalignment between transmitter and receiver due to temperature changes were, therefore, adequately catered for.

Figure 20 shows the frequency response of the receiver, measured by injecting a known signal into the first I.F. amplifier stage from a standard signal generator.

The output meter was calibrated directly in µVolts by applying a known signal to the input of the receiver through an accurately adjustable piston attenuator. Corrections for the input attenuator were obtained at the same time. There was no need to determine the calibration in terms of absolute µVolts, as only relative field strength matters in polar diagram measurements.
FIG. 21 CROSS SECTION THROUGH COAXIAL WAVEMETER.
3.5. The Wavemeter

A coaxial type, quarter wavelength wavemeter which was direct reading was designed to be used as a transfer standard. It was fully temperature compensated, part of the length of the inner conductor being held constant by an "Invar" rod.

A cross section and an external view are shown in Figures 21 and 22 respectively, from which it can be seen that a section of the inner conductor consisted of metallic bellows to accommodate the fine adjustment. By carefully proportioning the compensated and the uncompensated parts of the inner conductor in accordance with the coefficients of expansion of brass and "Invar", the materials used for its construction, complete temperature compensation was theoretically obtained over an 80°C degree temperature range.

Provision was made for fine and coarse adjustment; the fine adjustment, six turns of the tuning dial, covered a total of 1 Mc/s and any frequency could be reset with an error not greater than 500 c/s, although the accuracy with which any relative frequency measurement could be made did not exceed ± 1.8 Kc/s (at 150 Mc/s). This could be achieved by operating slightly off resonance. The measured "Q" of 3,000 very nearly approached the theoretical "Q".
FIG. 23 CIRCUIT DIAGRAM OF RECEIVER INPUT AND CALIBRATING OSCILLATOR.
The wavemeter was calibrated at the National Physical Laboratory where many of its characteristics were determined.

3.6. **Standardisation Routine**

Before making any field strength measurements the gain of the receiver was standardised. Figure 23, which shows the receiver input circuits and the calibrating oscillator on an enlarged scale illustrates the principle of gain standardisation.

In position 1. of the input selector switch, SI, the receiver input was short-circuited. Resistor, R3, (see Figure 17) was adjusted until the output meter M2 showed zero deflection. In position 2, the H.T. supply was connected to the calibrating oscillator and a 150 Mc/s signal injected into the receiver. The output level of the calibrating oscillator was standardised by adjusting the grid current taken by the oscillator to a fixed value by means of R1 and meter M1. The frequency of the calibrating oscillator could be set accurately to the transmitter frequency by means of a trimmer condenser C1. The correct frequency was in turn verified periodically by checking against the wavemeter which could be coupled to the calibrating oscillator via a monitor socket on the receiver.

The overall gain of the receiver was standardised by adjusting R2 until the injected signal produced full scale
deflection of the output meter M2. The signal required to standardise the receiver was of the order of microvolts. In order to obtain such a small signal it was found more convenient to operate the calibrating oscillator at 37\(\frac{1}{2}\) Mc/s and to inject a fraction of the much reduced fourth harmonic. Tuning was also found to be less critical with this arrangement.

3.7. Errors

3.7.1. Assessment of Overall System Accuracy

The inherent accuracy, excluding polarisation errors, of the system as described, was determined by two types of error. Firstly those due to the absolute accuracy with which a basic standard could be established and secondly those due to the relative accuracy with which a measurement could be repeated.

Transmitter power output variations and receiver gain calibration belong to the first category, whereas backlash in the aerial mast drive, alignment of receiver and transmitter aerials belong to the second category.

A summary of all errors is given below:-

Field strength, due to transmitter output fluctuations ................................. 3\(\frac{1}{2}\)%
Receiver gain calibration (initial)........ 2 %
Receiver gain standardisation .......... 3 %
Meter error (output meter) ............ 1 %
FIG. 24 GEOGRAPHICAL LAYOUT OF A SUITABLE SITE
Misalignment between transmitter and receiver aerials ................. $1^\circ$
Backlash in aerial mast drive ............ $4^\circ$

3.7.2. **Polarisation Errors**

Polarisation errors remained the only unknown factor in the estimate of the overall accuracy of the system. They were, however, minimised by a careful choice of the position of the receiver relative to the transmitter.

A survey of the area around the transmitter established a number of sites where only the direct ray could be received and where in addition reflections from far away trees were so small that the cross-polarisation was better than 40 dB when the transmitter aerial pointed in certain directions. Figure 24 shows the general geographical layout of a suitable site.

3.7.3. **Experimental Confirmation of System Accuracy**

The overall accuracy of the system was finally checked against the polar diagram of a thin half wavelength long dipole by mounting it horizontally on the aerial mast. Figure 25 shows the polar diagram which is a circle in the H-plane and for sinusoidal current distribution the E-plane polar diagram is given by (Ref.22, Chapter 5, Page 142.)

$$G_\theta = \frac{\cos (\pi/2 \cos \Theta)}{\sin \Theta}$$
The agreement between the theoretical and the measured polar diagrams was remarkably good. It was found possible to repeat measurements on different days with a maximum scatter of 5%. Taking the polar diagram of the standard dipole as an example, the error could be reduced to 2%, with 95% confidence, by taking a minimum of eighteen readings and using statistical methods to obtain better accuracy, see Appendix I.

It was evident that still greater accuracy would require a prohibitively large number of readings and this was, therefore, never attempted. Most results at 150 Mc/s presented in the following chapters are based on statistical averages.
4. The First Order Surface Wave.

4.1. Surface Waves, Historical Background.

The theoretical study of surface waves dates back to the year 1899 when Sommerfeld (Ref. 23) predicted that a straight cylindrical conductor of finite conductivity and having a smooth surface could act as a guide for electromagnetic waves.

Two of his pupils carried out further theoretical studies into the properties of cylindrical surface waves. Harms (Ref. 24) calculated the velocity of propagation of Sommerfeld's wave (the zero order mode using present day nomenclature) for a dielectric coated wire. Hondros (Ref. 25) examined the existence of higher order modes on metallic wires and concluded that they were possible but could never be observed because they were too heavily damped.

More recently, Stratton in his comprehensive book on "Electromagnetic Theory" (Ref. 26) considered higher order surface waves on a cylinder embedded in a dielectric and proved that they die away in the space of a few hundredths of a wavelength.

So far as the author is aware only the zero order mode has received any attention up to now because it had been demonstrated that it could propagate on a bare wire, small in diameter compared with a wavelength, with very
little attenuation. In fact many papers have appeared in recent years dealing with various aspects of this mode.

The application of the single wire waveguide to military purposes formed the subject of confidential American reports in which it was referred to as G-string waveguide. Officially, Goubau (Ref. 27) was the first to show renewed interest in this type of waveguide by pointing out that the addition of a thin dielectric layer to the wire considerably reduces the radial spread of the wave and that losses due to the scattering from nearby objects can thus be made negligibly small. Methods of launching the wave were studied by Dyott (Ref. 28) and Goubau himself (Ref. 29) and experimental work carried out by Goubau and his design data were critically reviewed by Barlow and Karbowiak (Ref. 30).

Higher order surface waves which were dismissed as being of no practical significance by the few authors who have made reference to them, have in fact a low attenuation, provided they are allowed to propagate along a dielectric coated cylinder which is approximately a wavelength in circumference. Under those conditions a higher order surface wave is not a pure mode, but this is only of academic interest, as there are also other types of waveguide which support waves of one predominant
mode and in addition require a subsidiary mode to enable propagation to take place, as was pointed out in Section 2.6.

In the following chapters the properties of the first order surface wave are studied, expressions for the field equations and propagation constant are derived and the results are experimentally verified.

4.2. Theory of the First Order Surface Wave

4.2.1. The solution of the general problem of wave propagation along a dielectric coated metallic cylinder, which includes the case of asymmetric hybrid modes, has not been attempted so far. The first order surface wave is a specific case of such a mode and the method used to determine the propagation constant of this mode as a function of the diameter of the cylinder and the thickness and dielectric constant of the coating, follows the method employed by Astrahan (Ref. 31) for the case of a dielectric tube, as reported by Kiely (Ref. 21).

Maxwell's equations expressed in vector notation are:

\[
\begin{align*}
\text{Curl } E &= -i\omega \mu H \\
\text{Curl } H &= i\omega \epsilon E \\
\text{Div } E &= 0 \\
\text{Div } H &= 0
\end{align*}
\]

\(1\)

Fig. 26. Co-ordinate System.
for regions of zero conductivity and no charge, a time
dependence of $e^{i\omega t}$ is assumed. By taking the curl of either
of the first two equations (1) and substituting from the
others, the well-known wave equations for a region with
constant $\mu$ and $\varepsilon$ are obtained, where $\omega$ and $\varepsilon$ are the per­
meability and the permittivity respectively of the medium.

$$\Delta E_z = -\omega^2 \mu \varepsilon E_z$$
$$\Delta H_z = -\omega^2 \mu \varepsilon H_z$$

(2)

Solutions of these wave equations which satisfy Maxwell's
equations at all boundary surfaces will satisfy Maxwell's
equations everywhere. It is assumed that the field depen­
dence on $t$ and $z$ is of the form $e^{i(\omega t - \gamma z)}$, where $\gamma$ the propa­
gation constant is real and positive. This form represents
a wave propagating without attenuation in the positive $z$
direction. From the vector wave equations (2) the following
scalar equations for the $z$ field components are derived:

$$\frac{\partial^2 E_z}{\partial r^2} + \frac{1}{r} \frac{\partial E_z}{\partial r} + \frac{1}{r^2} \frac{\partial^2 E_z}{\partial \phi^2} = -k^2 E_z$$

(3)

$$\frac{\partial^2 H_z}{\partial r^2} + \frac{1}{r} \frac{\partial H_z}{\partial r} + \frac{1}{r^2} \frac{\partial^2 H_z}{\partial \phi^2} = -k^2 H_z$$

where

$$k^2 = \omega^2 \mu \varepsilon - \gamma^2$$

(4)

The most general solutions of equations (3) are linear com­
binations of:

$$E_{zn} = [A_n J_n(k_p r) + B_n Y_n(k_p r)] e^{i(n\phi - \gamma z)}$$

$$H_{zn} = [C_n J_n(k_p r) + D_n Y_n(k_p r)] e^{i(n\phi - \gamma z)}$$

(5)
for different values of \( n \), where a time variation of \( e^{i\omega t} \) is assumed; the A's, B's, C's and D's are arbitrary constants and \( J_n \) and \( Y_n \) are Bessel functions of the first and second kinds respectively and n'th order.

In order that the fields might be unchanged when \( \phi \) is increased by \( 2\pi \), the values of \( n \) must be restricted to positive and negative integers, including zero. It can be shown that for any value of \( n \) Maxwell's equations can be satisfied at the boundary surfaces and each value of \( n \) corresponds therefore to a wave mode that can be guided by the system of Figure 26.

In the following derivations only the first order mode is considered and the subscript \( n = 1 \) will therefore be omitted, with the exception of the Bessel and Hankel functions of the first and zero order. If \( \partial / \partial z \) is replaced by \(-i\gamma\) and \( \partial / \partial t \) by \( i\omega \) in the six curl relations of \( \mathbf{E} \) and \( \mathbf{H} \) (1), the \( \mathbf{E} \) and \( \mathbf{H} \) components can then be expressed in terms of the \( z \) components,

resulting in:

\[
\begin{align*}
E_\phi &= \frac{1}{k^2} \left[ -i\gamma \frac{\partial F_z}{\partial \phi} + i\omega \mu \frac{\partial H_z}{\partial r} \right] \\
E_r &= \frac{-1}{k^2} \left[ i\gamma \frac{\partial F_z}{\partial r} + i\omega \mu \frac{\partial H_z}{\partial \phi} \right] \\
H_\phi &= \frac{-1}{k^2} \left[ i\omega \varepsilon \frac{\partial F_z}{\partial r} + i\gamma \frac{\partial H_z}{\partial \phi} \right] \\
H_r &= \frac{1}{k^2} \left[ i\omega \varepsilon \frac{\partial F_z}{\partial \phi} - i\gamma \frac{\partial H_z}{\partial r} \right]
\end{align*}
\]

(6)
The most general form of equations (5) are now determined for each of the three regions I, II and III. The two components of E and H are matched at the boundary surfaces to the corresponding $\phi$ obtained from equations (6) where $\varepsilon E_z$ and $\omega H_z$ must be continuous. For region I:

\[
\begin{align*}
E_z &= a_1 J_1(k_1 r) F \\
H_z &= b_1 J_1(k_1 r) F
\end{align*}
\]

where $F = e^{i(\phi + \omega t - yz)}$. The $Y_1$ solution is discarded because the fields must remain finite and $Y(r)$ becomes infinite as $r$ approaches zero. In the case of a dielectric coated cylinder for which infinite conductivity is assumed, the field components $E_z$ and $H_z$ are zero inside and on the surface of the metallic cylinder. For region II

\[
\begin{align*}
E_z &= [a_2 J_1(k_2 r) + a_3 Y_1(k_2 r)] F \\
H_z &= [b_2 J_1(k_2 r) + b_3 Y_1(k_2 r)] F
\end{align*}
\]

For region III:

\[
\begin{align*}
E_z &= a_4 H_1^{(0)}(k_3 r) F \\
H_z &= b_4 H_1^{(0)}(k_3 r) F
\end{align*}
\]

where $H_1^{(0)}(r) = J_1(r) + i Y_1(r)$ is the Bessel function of the third kind or Hankel function. Now from equation (4)

\[
k_3^2 = \omega (\omega \varepsilon_3 - \gamma^2 = \left(\frac{\omega}{V_3}\right)^2 - \left(\frac{\omega}{V_g}\right)^2
\]
where \( V_3 \) is the velocity of a uniform plane wave in medium III, i.e. free space, and \( v_g \) is the velocity of propagation of the guided wave; \( v_g \) is the same for all three regions. A guided wave cannot travel faster than a wave propagating in free space and \( k_3^2 \) must therefore be less than or equal to zero and consequently \( k_3 \) must be either zero or pure imaginary.

Now the field must approach zero at large distances from the axis and the boundary conditions can be met by either using \( H_1^{(1)} \) or \( H_1^{(2)} \). In the following treatment \( H_1^{(1)} \) with positive imaginary argument is used throughout.

For large values of \( r \) one can write: \( H_1^{(1)}(kr) \sim e^{ikr} \).

To ensure that the exponential tends to zero, \( k \) must be of the form \( i\eta \), where \( \eta \) is positive and real, i.e. one may write \( H_1^{(1)}(kb) = H_1^{(1)}(i\eta b) \).
4.2.2. The Field Equations

Substituting equations (7), (8) and (9) into equations (6) to obtain the $r$ and $\phi$ components results in the following set of field equations:

\[ E_{\phi} = 0 \]
\[ H_{\phi} = 0 \]
\[ E_{\phi 2} = \frac{1}{k_2} \left\{ \frac{\omega}{r} \left[ a_2 J_y(k_2 r) + a_5 Y_1(k_2 r) \right] + i\omega k_2 \left[ b_2 J_y(k_2 r) + b_3 Y_1(k_2 r) \right] \right\} F \]
\[ H_{\phi 2} = -\frac{1}{k_2} \left\{ \omega k_2 \left[ a_2 J_y(k_2 r) + a_5 Y_1(k_2 r) \right] - \frac{\omega}{r} \left[ b_2 J_y(k_2 r) + b_3 Y_1(k_2 r) \right] \right\} F \]
\[ E_{\phi 3} = \frac{1}{k_3} \left\{ \frac{\omega}{r} \left[ a_4 H_y(k_3 r) + i\omega k_3 b_4 H_1(k_3 r) \right] \right\} F \]
\[ H_{\phi 3} = -\frac{1}{k_3} \left\{ \omega k_3 a_4 H_1(k_3 r) - \frac{\omega}{r} b_4 H_y(k_3 r) \right\} F \]

\[ E_{r 1} = 0 \]
\[ H_{r 1} = 0 \]
\[ E_{r 2} = -\frac{1}{k_2} \left\{ i\frac{\omega}{r} k_2 \left[ a_2 J_y(k_2 r) + a_5 Y_1(k_2 r) \right] - \omega k_2 \left[ b_2 J_y(k_2 r) + b_3 Y_1(k_2 r) \right] \right\} F \]
\[ H_{r 2} = \frac{1}{k_2} \left\{ -\frac{\omega}{r} k_2 \left[ a_2 J_y(k_2 r) + a_5 Y_1(k_2 r) \right] - i\frac{\omega}{r} k_2 \left[ b_2 J_y(k_2 r) + b_3 Y_1(k_2 r) \right] \right\} F \]
\[ E_{r 3} = \frac{1}{k_3} \left\{ i\frac{\omega}{r} k_3 a_4 H_1(k_3 r) - \omega k_3 b_4 H_y(k_3 r) \right\} F \]
\[ H_{r 3} = \frac{1}{k_3} \left\{ -\omega k_3 a_4 H_1(k_3 r) - i\frac{\omega}{r} k_3 b_4 H_y(k_3 r) \right\} F \]
4.2.3. The Characteristic Equation

The properties of the first order surface wave can be determined from the field equations (11) and (12) by equating the \(\phi\) and \(z\) components at the boundaries \(r = a\) and \(r = b\) where the fields must be continuous. The complete derivation is given in Appendix III where it is shown that the Characteristic Equation is

\[
\begin{align*}
Q\left\{ (\Delta_3 - \Delta_6) (\Delta_1 - \Delta_4) - (\Delta_5 - \Delta_6) (\Delta_7 - \Delta_4) \right\} & + \varepsilon \left\{ (\Delta_3 - \Delta_6) (\Delta_7 - \Delta_4) - (\Delta_5 - \Delta_6) (\Delta_7 - \Delta_4) \right\} \\
& - T^2 (\Delta_3 - \Delta_6) (\Delta_7 - \Delta_4) = 0
\end{align*}
\]

(13)

The various symbols being defined as follows:

\[
\begin{align*}
\Delta_1 &= \frac{J_i'(cV)}{cV J'(cV)} & \Delta_4 &= \frac{J_i(V)}{Y_i(V)} & \Delta_7 &= \frac{Y_i''(V)}{V Y_i'(V)} \\
\Delta_2 &= \frac{J_i'(V)}{V J_i(V)} & \Delta_5 &= \frac{H_i'(N)}{N H_i(N)} & Q &= \frac{\omega^2 \mu \sigma}{Y^2} \\
\Delta_3 &= \frac{J_i(cV)}{Y_i(cV)} & \Delta_6 &= \frac{Y_i'(cV)}{cV Y_i(cV)} & T &= \left(\frac{1}{V}\right)^2 - \left(\frac{1}{N}\right)^2
\end{align*}
\]

(14)

where the following abbreviations were introduced to simplify the writing:

\[
c = a/b \quad k_2 a = cV \quad k_2 b = V \quad k_3 b = N \quad \text{or} \quad iW
\]

(15)

where \(W\) is real and positive, since \(k_3\) must be imaginary.
FIG. 27
THREE DIMENSIONAL PLOT OF THE CHARACTERISTIC EQUATION (c=0.9, ε=2.3)
The significance of the Characteristic Equation will now be discussed using some of the relationships obtained earlier on. From equation (4)

\[ \gamma^2 = \omega^2 (\mu_2 \varepsilon_2 - \left( \frac{V}{b} \right)^2 = \omega^2 (\mu_3 \varepsilon_3 + \left( \frac{W}{b} \right)^2 ) \] (16)

Since \( \gamma^2 = \left( \frac{2\pi}{\lambda_g} \right)^2 \) and \( \omega^2 (\mu_0 \varepsilon_0) = \left( \frac{2\pi}{\lambda_o} \right)^2 \) where \( \lambda_g \) is the guide wavelength and \( \lambda_o \) is the free space wavelength, \( Q \) can also be written as \( Q = \left( \frac{\lambda_g}{\lambda_o} \right)^2 \), and Eqn. (16) can be written in the forms \( (\varepsilon_2 = \varepsilon \varepsilon_0, \varepsilon_3 = \varepsilon_0), (\mu_3 = \mu_2 = \mu_0) \)

\[ \left( \frac{2\pi}{\lambda_o} \right)^2 (\varepsilon - 1) = V^2 + W^2 \] (17)

\[ \left( \frac{2\pi}{\lambda_g} \right)^2 = \varepsilon \left( \frac{2\pi}{\lambda_o} \right)^2 - \left( \frac{V}{b} \right)^2 \] (18)

It is thus found that

\[ \left( \frac{b}{\lambda_o} \right)^2 = \frac{V^2 + W^2}{(2\pi)^2 (\varepsilon - 1)} \] (19)

\[ \left( \frac{\lambda_g}{\lambda_o} \right)^2 \frac{V^2 + W^2}{V^2 + \varepsilon W^2} \] (20)

In the Characteristic Equation the \( \Delta \)'s are either dependent upon \( V \) or \( W \), \( Q \) as well as \( T \) are both functions of \( V \) as well as \( W \), so that each pair of \( V \) and \( W \) which satisfies equation (13) for a given \( \varepsilon \) and \( c \) determines a dimensionless pair \( b/\lambda_o \) and \( \lambda_g/\lambda_o \). The method of solution by systematic trial and error is also described in Appendix III. Figure 27 shows a plot of the manner in which the value of the Charac-
FIG. 28 CHARACTERISTICS OF HE_{II} MODE
teristic Equation changes as a function of different \( V \)'s and \( W \)'s for the case of a polythene coated cylinder \((\varepsilon = 2.3)\) for which \( \sigma = 0.9 \). The value of the Characteristic Equation goes to infinity whenever \( J_1 \) or \( Y_1 \) goes to zero. The \( \Delta \)'s in which \( J_1 \) occurs in the denominator are always associated with other \( \Delta \)'s where \( J_1 \) occurs in the numerator so that cancellation takes place. The only two \( \Delta \)'s which make the value of the Characteristic Equation display discontinuities are, therefore, \( \Delta_3 \) and \( \Delta_4 \) and infinities occur when either \( Y_1(V) \) or \( Y_1(cV) \) is zero.

Now \( Y_1(V) \) and \( Y_1(cV) \) are related by the factor \( c \) and whenever there is a \( V \) for which the function \( Y_1(V) \) has a zero there will be a second zero when the function is \( 1/c \) times greater. Reference to tables of Bessel functions (Ref. 32) shows that zeros occur at 2.197; 5.430; 8.596 etc.

4.2.4. Cut-Off Frequency and Higher Order Modes

The intercept of the family of \( W \) curves with the \( V \) axis in Figure 27 defines points for which the Characteristic Equation is satisfied. These points are replotted in Fig. 28 as explained in the previous section to show the relationship between \( \lambda_0/\lambda \) and \( b/\lambda_0 \). Two specific cases with which the experimental work has been concerned were evaluated numerically viz. \( c = 0.96; \varepsilon = 23 \) (Titanium Dioxide loaded Polystyrene) and \( c = 0.90; \varepsilon = 2.3 \) (Polythene). It is instructive to examine the Characteristic Equation analyti-
cally, in conjunction with Figure 27. The first zero is most important since it indicates that the Characteristic Equation has no solution for cV smaller than 2.197/c. Putting \( W = 0 \) in Equation (19) one obtains the cut-off condition for the first order surface wave

\[
\frac{b}{\lambda_o} = \frac{2.197(k)}{2\pi \sqrt{\varepsilon - 1}}
\]  

(21)

The relationship of Equation (21) holds so long as the Characteristic Equation has no zeros in the range for which \( \nu \) lies between the limits \( 2.197 < \nu < 2.197/c \). For practical values of c which are never likely to be as small as 0.5, so that \( 2.197/c < 5.430 \) (the next zero), Equation (21) applies to all cases of the HE_{11} mode.

The dependence of the cut-off frequency on the dimensions of the waveguide are better appreciated if Eqn. (21) is re-arranged to

\[
2\pi b = \frac{2.197(k)}{\sqrt{\varepsilon - 1}} \lambda_o
\]  

(22)

where \( 2\pi b \) represents the circumference of the guide. Examination of Eqn. (22) clearly shows that the first order surface wave cannot exist in the absence of a dielectric coating. For the case of air (\( \varepsilon = 1 \)) the guide would need to be of infinite circumference. Low-loss solid dielectrics, mixtures and foams excepted, have a dielectric constant greater than 2 so that in general the circumference of a
surface waveguide will have to be at least twice the free space wavelength. The result expressed by Equation 22 was confirmed experimentally at 150 Mc/s by covering a surface wave aerial \( \frac{1}{2} \lambda_0 \) in circumference with several layers of glass tape \( (\varepsilon = 6) \) when no directive radiation could be observed. The tape was subsequently replaced by a coating of Titanium Dioxide loaded Polystyrene \( (\varepsilon = 23) \) of the same thickness; the structure now clearly acted as a directive aerial.

In the first case the guiding surface was well below the cut-off condition, in the second case well above it. Further experiments, carried out at 9000 Mc/s, are described in Chapter 6.

It was pointed out in Chapter 4.2.2 that the Characteristic Equation can be satisfied by more than one value of \( V \) for the same \( W \), owing to the behaviour of the Bessel function. The second, third and subsequent zeros of this function determine the onset of higher order modes of the first order surface wave, i.e. \( HE_{12} \), \( HE_{13} \), etc. which can be analysed in a similar manner by modifying the numerical constant in Eqn. (22).

So far one set of limiting conditions applicable to Figure 28 has been found. A second set of limiting conditions can be obtained from Eqn. (20) If \( W \) is large compared with \( V \), the value of \( \left( \frac{\lambda_3}{\lambda_0} \right)^2 \) will
tend to $1/\sqrt{\varepsilon}$ and if $V$ is large compared with $W$, then
$(\lambda_g/\lambda_o)^2$ will tend to 1. It follows therefore that the
ratio of guide wavelength to free space wavelength is
confined to the range $\frac{1}{\sqrt{\varepsilon}} < \lambda_g/\lambda_o < 1$

4.2.5. The E/H Ratio

The ratio of the $E_z$ and $H_z$ field components can
be derived from Eqn. (9) if the coefficients $a_4$ and $b_4$
are known. In Appendix IV these coefficients have
been determined and it is shown that

$$\sqrt{\frac{\varepsilon_o}{\mu_o}} \frac{a_4}{b_4} = \frac{i\sqrt{Q}}{T} \left[ \frac{H_1'(iW)}{iW H_1'(iW)} + \frac{J'_1(cV)Y_1'(V) - J'_1(V)Y'_1(cV)}{J_1'(cV)Y_1'(V) - J'_1(V)Y'_1(cV)} \right] \quad (24)$$

where $\sqrt{\frac{\varepsilon_o}{\mu_o}}$ is the impedance of free space. Substitution
of a particular solution of a pair of $V$ and $W$ into Eqn. (24)
which satisfies the Characteristic Equation can be used to
obtain the numerical value of this ratio for any specific
case of $c$ and $\varepsilon$. By using Eqn. (9) to determine the E/H
ratio, it has been assumed that most of the energy of the
surface wave resides in the space surrounding the dielec­
tric. Inspection of the $\lambda_g/\lambda_o - b/\lambda_o$ curves, Fig. 28, shows
that for large $c$'s this assumption is justified so long as
the ratio $b/\lambda_o$ is less than 0.8. The ratio $\lambda_g/\lambda_o$ lies
then more closely to unity than to $1/\sqrt{\varepsilon}$. A large radia­
ting aperture is an essential requirement for a high gain
aerial and surface wave aerials must therefore always operate under those conditions. Introducing the symbols previously defined by Eqns. (14) simplifies Eqn. (24) to:

\[
\sqrt{\frac{\varepsilon_0}{\mu_0}} \frac{a_4}{b_4} = \frac{i\sqrt{Q}}{\Delta_5} \left[ \Delta_5 - \frac{\Delta_1 \Delta_3 \Delta_7 - \Delta_2 \Delta_4 \Delta_6}{\Delta_1 \Delta_3 - \Delta_4 \Delta_6} \right]
\]

(25)

The numerical values of the \( \Delta^s \) have been computed for a complete range of possible solutions of the Characteristic Equation; they are tabulated at the end of Appendix III.

In the case of aerials for which the aforementioned conditions apply, it is found that solutions of the Characteristic Equation occur when \( W \) is very much smaller than \( V \). In practice, therefore, the second term in Eqn. (25), which is a function of \( V \) only, is negligibly small compared with \( \Delta_5 \), which depends on \( W \) and is large when \( W \) is small.

The value of \( Q \) is nearly unity and \( \Delta_5 \) and \( T \), although large numerically, are almost equal. The E/H ratio is therefore,

\[
\sqrt{\frac{\varepsilon_0}{\mu_0}} \frac{E_{\text{z}}}{H_{\text{z}}} = \sqrt{\frac{\varepsilon_0}{\mu_0}} \frac{a_4}{b_4} = i\sqrt{Q} \frac{\Delta_5}{\Delta_5} \approx i
\]

(26)
5. The Surface Wave Aerial

5.1. Methods of Launching 1st Order Surface Wave

5.1.1 In order to excite a particular surface wave on a
dielectric coated cylinder it is in general necessary to
create at the end of the cylinder a field configuration as
closely similar to that of the required mode as possible.
From Figure 5 (Page 10) it is seen that the field distri-
bution of the 1st order surface wave along a line perpen-
dicular to the cylinder resembles that of a \( \lambda/2 \) dipole
which can, therefore, be used to launch the wave.

![Diagram of E and H Lines](image)

Fig. 29 E and H Lines of
a.) Surface Wave
b.) Dipole

The similarity between the electric and magnetic
fields of the surface wave and the dipole respectively
is illustrated in Figures 29a and 29b.

In order that power may conveniently be fed to the
exciting dipole, a surface wave aerial is best constructed
as a dielectric coated tube. Two alternative methods of
feeding the dipoles are described in the following sections.
5.1.2. Launching at Metre Wavelength (150 Mc/s)

A suitable arrangement is illustrated in Figure 30. The dipole is excited in conventional manner by feeding power directly through a twin, balanced cable to the centre of the dipole. In order to obtain greater launching efficiency a reflector is placed behind the dipole, \( \lambda/3 \) away. Owing to the fact that part of the radiation from the dipole is trapped inside the aerial tube and has to be dissipated by some lossy packing material, the efficiency of this method of launching is not very great.

5.1.3. Launching at Centimetre Wavelength (9000 Mc/s)

At microwave frequencies the aerial tube can be used as a waveguide and energy can be converted in several ways. The direction of the electric field of the \( H_{\text{II}} \) mode at the centre of a circular waveguide is perpendicular to the axis of the guide as shown in Figure 31a. Two \( \lambda/4 \) dipoles projecting into the tube through insulating bushes as shown in Figure 31b can therefore be used to transfer...
FIG. 32 DIPOLE TRANSITION (TOP)
FIG. 33 WAVEGUIDE TRANSITION (BOTTOM)

SURFACE WAVE LAUNCHERS.
energy from the inside of the tube to the outside. An adjustable, reflecting piston inside the guide can be used to achieve maximum power transfer. Figure 32 shows the principle applied to a 1st order surface wave launcher operating at 9000 Mc/s.

An altogether different method is shown in Figure 33. The $H_{II}$ mode in a circular metal guide is used to excite the $H_{II}$ mode in a dielectric rod. The rod is then continued as a thin dielectric sleeve surrounding the metal rod along which it is desired to propagate the surface wave. Tapered transitions will ensure smoother conversion from one mode to another and hence result in better launching efficiency.

5.2. Reflectionless Transmission

Reflections from the end of a surface wave aerial result in a standing wave and thereby destroy the wideband properties of this type of aerial which is then limited by
FIG. 35 CROSSED SECTION THROUGH SURFACE WAVE AERIAL

- Counterpoise
- Dipole
- Guy Wire
- Spacers
- Aerial Mast
- Joining Sleeve
- Dielectric
the input impedance variations. If the dielectric is gradually reduced in thickness so that nothing but the bare metal cylinder remains, the 1st order mode changes from a travelling wave into an evanescent mode and the remaining energy is lost in non-directional radiation from the end. Figure 34 shows two forms of reflectionless transmission; (a) the gradual reduction of the thickness of the dielectric coating and (b) a V-shaped incision into the dielectric layer.

5.3. Constructional Details of Surface Wave Aerial

For operation at Centimetre wavelength a Surface Wave Aerial can easily be constructed from thin metal tubing which is self supporting, in the manner shown in Figure 33, the outside being coated with the appropriate amount of dielectric.

At Metre wavelength the aerial is best sectionalised for ease of manufacture and assembly, each section consisting of a lightweight fibre shell (cardboard) with an outer layer of tin foil. If the thickness of this
Fig. 36 Dielectric const. of polystyrene/titanium dioxide mixtures.
foil is chosen to be at least five times the skin depth at the operating frequency, there is practically no loss of power due to insufficient conducting material. At 150 Mc/s a thickness of only 0.0014" meets this requirement. Figure 35 shows a cross section through a typical aerial assembly. The construction relied on a central Duralumin tube, suitably counter-balanced to achieve mechanical strength. Balsawood spacers were used to support the outer shell which was coated with a layer of Titanium Dioxide loaded Polystyrene. Physical considerations made it desirable that the dielectric constant be as high as possible in order to reduce the overall size of the aerial.

When a quantity of specially prepared Titanium Dioxide powder became available it was decided to construct the aerial around this material, which has a very low loss, (\(\tan \delta = 0.0003\) approx. at 150 Mc/s) and a dielectric constant approaching 100 when solid. Using Polystyrene as a binder, the dielectric constant of the mixture could be maintained consistently at \(\varepsilon = 23\). The manner in which the dielectric constant of the mixture changes is depicted in Figure 36.
5.4. The Theoretical Radiation Pattern.

5.4.1 General Considerations.

In the preceding Chapter the conditions were derived under which the first order surface wave can exist. The radiation pattern of an unterminated surface waveguide, i.e. a surface wave aerial, is the result of a variety of factors, varying in relative importance, depending upon the size and the length of the aerial as well as upon the launching conditions.

Without further experimental work it is not possible to put forward a single, unified theory to predict the radiation pattern of surface wave aerials, as a number of assumptions have to be made concerning the mechanism of radiation and it is therefore possible to account for an observed polar diagram in more than one way, depending upon the nature of those assumptions. The radiation pattern can be due to a combination of:

a.) direct radiation from the dipole
b.) continuous radiation along the whole length of the aerial.
c.) radiation from the open end.

The direct radiation from the launching dipole tends to make the pattern omnidirectional in the H-plane, though in general this direct contribution to the field can be kept small.

Continuous radiation from the whole length of the aerial was
specifically excluded when the Characteristic Equation was solved, it being assumed that the 1st order mode is a non-radiating mode. Continuous radiation can nevertheless be present, either due to surface irregularities, the launching of higher order modes, reflections from the end of the aerial resulting in standing waves along the guide, or be due to the fact that propagation of the 1st order mode is accompanied by continuous radiation.

It is difficult to treat this case mathematically as it involves Hankel functions with complex argument, and to the author's knowledge existing tables only deal with complex arguments having equal real and imaginary components. This case, however, is further removed from the true state of affairs than the assumption of 'no radiation', as experimental evidence indicates that radiation loss is small compared with the total amount of energy transported by the guide; the effect on the radiation pattern would barely be noticeable in the case of short aerials.

It follows then, that the radiation pattern is mainly determined by the radiation from the open end of the guide for aerials that are comparatively short. For long aerials different considerations must apply. A small amount of continuous radiation gives rise to a polar diagram of decreasing beam width as the length of the aerial is increased, at the same time the power radiated from the end is reduced.
5.4.2. **The Short Aerial.**

In the case of short aerials all radiation is assumed to emanate from the end of the aerial. The fields $E_r$ and $E_\varphi$ at the end of a surface waveguide are given by Equations (11 & 12) and the radiation pattern is the sum of all the fields in that plane. Let the launching as well as the receiving dipole be in the $x$-$z$ plane, so that only $E_x$ is of interest. Converting $E_\varphi$ and $E_r$ to rectangular components, omitting the suffix '3' results in:

$$E_x = E_r \cos \varphi - E_\varphi \sin \varphi$$

The radiation pattern in the $x$-$z$ plane ($E$-plane) and the $y$-$z$ plane ($H$-plane), is found by summation, viz.

$$E_{\varphi} = \int \int E_x \, dx \, dy \, e^{i(\omega t - yz + \varphi - f(\Theta))}$$

where $f(\Theta)$ denotes the phase difference at a point 'P' in the end plane relative to the point 'O'.
Using Eqns. (11 & 12) and the relationship expressed by Eqns. (26) and (27) and making use of mathematical identities, it is shown in Appendix V that the E-plane polar diagram is given by:

\[ E_\theta = \text{const} \int_1^\infty \frac{r}{b} H_0(iW \frac{r}{b}) \times J_0 \left( \frac{2\pi}{q} \frac{r}{b} \sin \Theta \right) d\left( \frac{r}{b} \right) \]  \hspace{1cm} (29)

where \( q = \frac{\lambda_o}{b} \) and the other symbols are defined by Fig. 27.

The above integral can readily be evaluated graphically, using the normalised parameter \( r/b \) and varying \( \sin \Theta \) in discrete steps, since it is the product of an oscillating function \( J_0(z) \) and a rapidly decreasing function \( H_0(iz) \) both of which are tabulated.

The value of \( W \) is known from the particular solution of the Characteristic Equation from which \( b/\lambda_o \) was calculated. It is also shown in Appendix V that the H-plane polar diagram is similar to that for the E-plane.

5.4.3. The Long Aerial.

In the case of long aerials the effect of continuous radiation must be included. The radiation pattern is the vector sum of this continuous radiation and the radiation from the end of the aerial.
The latter has been determined in the previous section. When continuous radiation takes place, the amplitude decreases as the wave progresses towards the end. Considering the radiation to originate from two line sources in the x-z plane on either side of the surface waveguide, it is shown in Appendix VI that the E-plane (and H-plane) polar diagram is given by:

\[
E_\theta = e^{i\pi \lambda_o (\cos \theta - \frac{\lambda_o}{\lambda_g})} \left[ \sin \frac{\pi l}{\lambda_o} (\cos \theta - \frac{\lambda_o}{\lambda_g}) \cosh \frac{\alpha l}{2} + i \cos \frac{\pi l}{\lambda_o} (\cos \theta - \frac{\lambda_o}{\lambda_g}) \sinh \frac{\alpha l}{2} \right]
\]

\[
\alpha - i \frac{2\pi}{\lambda_o} (\cos \theta - \frac{\lambda_o}{\lambda_g})
\]

(30)

where \( \alpha \) is the radiation loss per unit length in Nepers.

When performing the vector addition, the 'phase' reference plane is assumed to be at the centre of the aerial rod for the contribution from the continuous radiation and at the end of the aerial for the contribution from the end plane. Similarly, the contribution from the launching dipole could be added if it was thought to be of sufficient magnitude to warrant its inclusion.
FIG. 40 SURFACE WAVE LAUNCHER.
6. Experimental Verification of Aerial and Waveguide Theories.

6.1. The Microwave Test Gear.

As an introduction to this Chapter a brief description of the microwave equipment, for measuring the properties of surface waves at 9000 Mc/s, is given, so that the results at the two frequencies that were used can be discussed together.

A schematic diagram of the experimental set-up is shown in Figure 39. A klystron tuneable from 8000 to 10,000 Mc/s by means of an external cavity was used as the source of r.f. power. By inserting a rectangular to circular waveguide transition the 

\[ \text{HE}_2 \]

surface wave was launched from the end of the internally short-circuited waveguide, as described in Section 5.1.3. and illustrated in Figure 32.

In order to avoid unwanted reflections from supports, that part of the wave that travelled away from the desired direction of propagation, was absorbed by a screen of carbon-loaded rubber ("Space Cloth") which was placed on one side of the launching dipoles. Fig.40 shows the general arrangement. The receiving termination was constructed in an exactly identical manner, the received energy after being transformed back from circular to rectangular waveguide being detected by a probe and crystal
Fig. 41 Polar Diagram of 2λ Aerial (h/λ₀ = 0.076, c...
diode whose output was indicated on a galvanometer. A calibrated attenuator was inserted in the waveguide and by keeping the galvanometer deflection constant, gain or loss relative to a standard signal could be measured directly.

Propagation measurements were carried out by applying dielectric coatings of different materials to the outside of the guide. In Figure 40 one layer of Polystyrene tape is shown in position. When radiation patterns were measured, the launcher in Figure 39 was replaced by a horn which was used to illuminate the aerial under observation. In this case the aerial, used as a receiver, resembled the launching arrangement of Fig. 40, suitably extended to the appropriate length. The whole assembly was supported on a long vertical pole, which was fixed to the centre of a turntable to facilitate the plotting of polar diagrams.

6.2. The Polar Diagram of Short Aerials.

The effect of varying the diameter of the aerial, the length of the aerial and the effect of the launching dipoles on the resulting radiation pattern was studied on comparatively short aerials at 150 Mc/s.

The E and H-plane polar diagram for a surface wave aerial, 2 wavelength long, for which \( b/\lambda_o = 0.076 \) is shown in Figure 41. There are two sidelobes on either side of the main beam in the E-plane and the polar diagram in the H-plane shows little directivity. The theoretical
FIG. 43 E-PLANE POLAR DIAGRAM SHOWING EFFECT OF DIPOLE HEIGHT ON SIDELOSES
radiation pattern, which is shown dotted, contains no side-lobes when \( b/\lambda \), is small. The departure of the actual radiation pattern from the ideal pattern is due to the direct radiation from the launching dipoles. Assuming two sources, 2 wavelengths apart (i.e. the dipoles and the end plane of the aerial) radiating in phase, it can be seen from Figure 42 that the fields at a distant point 'P' are in phase whenever \( \frac{2\pi L \cos \theta}{\lambda} = 2n\pi \) and that they are out of phase whenever \( \frac{2\pi L \cos \theta}{\lambda} = (2n-1)\pi \), where \( n = 1, 2, 3 \) etc. Hence minima should occur at 41.6° and at 75.5°, and maxima at 0°, 60° and 90°. Examination of Figure 41 shows that this is in fact the case.

The polar diagram of the main beam agrees well with the theoretical diagram as calculated with the aid of Eqn.(29). The length of the launching dipoles projecting from the aerial was kept to 0.1\( \lambda \), this being a compromise between a reasonable launching efficiency and the desire to keep the effect of direct radiation from the launcher as low as possible. Figure 43 shows the result of reducing the height of the launching dipoles, so that they only projected 0.02\( \lambda \). The magnitudes of the sidelobes is much
FIG. 44 THE FUNCTION 

\[ \frac{r}{b} \mathcal{H}_0(iW \frac{r}{b}) \times J_0 \left( \frac{2\pi r}{q} \frac{r}{b} \sin \theta \right) \]
reduced as would be expected from the conclusions reached earlier on, that the field from the launching dipoles can materially contribute to the radiation pattern.

Physical limitations prevented increasing the length of the aerial at 150 Mc/s. The effect of increasing the ratio \( b/\lambda_0 \) was studied on an aerial 1 wave-length long. The ratio \( b/\lambda_0 \) was increased from 0.076 to 0.09. Theoretically, the beam width decreases as the ratio \( b/\lambda_0 \) is increased. This can be understood by making reference to Figure 44 which shows one step in the graphical integration of Eqn.29 for \( \theta = 0^\circ, 20^\circ \) and \( 50^\circ \).

The ratio of the net area under the curve for any particular angle \( \theta \) to that of the area belonging to \( \theta = 0^\circ \) corresponds to the ratio of the field strength in the direction under consideration, to that in the direction \( \theta = 0^\circ \). Converted to dB, these ratios are used to plot the polar diagram. Increasing the ratio \( b/\lambda_0 \) is equivalent to increasing the argument of \( J_0(\frac{2\pi}{b} r \sin \theta) \) and as \( r/b \) is varied from unity to some large value, the periods of oscillation of the \( J_0 \) function are compressed.

This results in a reduction of the net area under the curve for any given angle but the area under the curve \( \theta = 0^\circ \) remains unchanged. A physical explanation can be sought in the fact, that the phase across the aerial aperture changes more rapidly as one moves away from
Fig. 45 E-plane polar diagram showing effect on 2λ aerial of varying $b/\lambda_0$. 
FIG. 46 POLAR DIAGRAM OF 2λ AERIAL ($b/\lambda_0 = 0.375$, c)
the centre, resulting in a reduced beam width. The appearance of sidelobes is the penalty that has to be paid if an optimum beam width is exceeded. Fig. 45 illustrates the point; the reduction in beam width is clearly perceptible as $b/\lambda_o$ is increased from 0.076 to 0.090. Further confirmation of the relationship between the ratio $b/\lambda_o$ and gain was obtained at 9000 Mc/s. Figure 46 shows the polar diagram of an aerial 2 wavelengths long, as before, but with $b/\lambda_o = 0.375$. Owing to the method of launching, sidelobes are again prominent, however the same degree of agreement between the theoretical and the measured polar diagram exists. (One small sidelobe is due to the large ratio of $b/\lambda_o$.) Although Figure 41 as well as Figure 46 refers to an aerial 2 wavelengths long, the two diagrams cannot be compared directly, as Titanium dioxide was used as the dielectric in one case and Polythene in the other. (This affects the value of $W$ in Eqn. 29.)

The effect of shortening the aerial from 2 wavelengths to 1 wavelength results in a broadening of the main beam, as can be seen from a comparison of Figures 41 and 46. However, this is not a property of the surface wave aerial, but is due to the combined field of launcher and end-plane radiation. The diagrams of long aerials are discussed in the next section.
FIG. 47. POLAR DIAGRAM OF 10 λ AERIAL (b/λ₀ = 0.375, c = 1)
6.3. **The Polar Diagram of Long Aerials.**

The properties of long aerials were studied at 9000 Mc/s, using aerials constructed from brass tubing which had been overwound with Polythene tape, the number of layers being adjusted as required to produce different values of $c = a/b$. The ratio of $b/\lambda_0$ was approximately 0.375 and was not varied. Figure 47 shows the polar diagram of an aerial 10 wavelength long, with $c = 0.90$. The reduction in beam width of the major lobe is very noticeable compared with a short aerial.

This can only be explained by the presence of continuous radiation, since the pattern due to the end plane has not changed. Assuming that the 'phase' reference plane of the continuous radiation is at the centre of the aerial, 5 sidelobes on either side of the main lobe can be expected when the radiation from a 10 wavelengths long aerial is combined with that from the end plane.

The position of the sidelobes depends upon the exact length of the aerial in terms of $\lambda_0$. By the same argument that was used in the case of the short aerial, where the radiation from the dipole and that from the end plane were assumed to be in phase, it can be shown (Ref.1) that maxima occur whenever $\xi + \frac{2\pi L}{\lambda} \cos \theta$ amounts to $n\pi$ and that minima are observed whenever $\xi + \frac{2\pi L}{\lambda} \cos \theta$ is equal to $(2n - 1)\pi$, where $\xi$ is the phase angle between
FIG. 48 E-PLANE POLAR DIAGRAM SHOWING EFFECT OF VARYING THICKNESS OF DIELECTRIC.
the two equivalent sources. The phase angle $\phi$ is determined by Eqn. (30), and as can be seen it is a function of $\alpha$, the radiation loss per unit length. The amplitude distribution expressed by Eqn. (30) obeys a $(\sin n\alpha)/\alpha$ law, where $n$ denotes the number of wavelengths, provided $\alpha$ is small and $n/\alpha_n$ does not depart appreciably from unity.

In Figure 47 the theoretical polar diagram has been superimposed dotted on the measured polar diagram. The radiation from the dipole was neglected and in adding the two fields it was assumed that the power is divided in the ratio 30% to 70%, 30% being the contribution from the continuous radiation. This figure is based on measurements described in the next Section, which indicate that power along a surface waveguide of the type used is lost at the rate of approximately 4 dB/mtr.

The position of the sidelobes as measured are in reasonable agreement with the theoretical polar diagram. It is difficult to predict the sidelobes very accurately, as a small change in the length of the aerial leads to a disproportionately large change of $\phi$. Furthermore, the effect of the launching dipoles has not been taken into consideration.

The effect of increasing the ratio $b/\lambda_0$ is demonstrated more clearly in Figure 48 which shows the polar diagrams of two aerials, 10 wavelengths long, one aerial
FIG. 49  E-PLANE POLARDIAGRAM SHOWING EFFECT OF INCREASING LENGTH OF AERIAL
having a dielectric thickness twice that of the other. The actual values are: \( b/\lambda_0 = 0.375 \); \( c = 0.90 \) and \( b/\lambda_0 = 0.410 \); \( c = 0.85 \) respectively. The reasons for the reduced beam width have been given before. The reduced sidelobe level indicates that a greater portion of the total energy reaches the end plane in the second case and that a smaller amount of energy is lost by continuous radiation.

Lastly, in Figure 49 a comparison is shown between two identical aerials, 10 wavelengths and 20 wavelengths long respectively. A further reduction in the width of the major lobe is apparent as the length of the aerial is increased, with an attendant increase in the number of sidelobes. The apparent improvement in gain is not as great as one would expect; this can be attributed to the additional field from the dipole which cannot be entirely eliminated with this method of launching.
6.4. **Direct Measurement of the Propagation Constant.**

When a dipole is used to launch a surface wave in the manner shown in Figure 33, the launching efficiency is not very great and the field propagating outwards consists of a guided surface wave of wavelength \( \lambda_g \) and an unguided "free space" wave of wavelength \( \lambda_0 \). The radiation field in the vicinity of a dipole consists of two components (Ref. 34) which diminish as \( \frac{1}{d} \) and \( \frac{1}{d^2} \) respectively, where 'd' is the distance from the dipole.

The guided wave suffers attenuation as a result of loss in the dielectric, resistive loss in the conductor and radiation loss due to a variety of reasons, the decay being exponential. Figure 50 shows qualitatively how the amplitudes change. Along the waveguide the two waves are periodically in and out of phase and
FIG. 51 INTERFERENCE PATTERN BETWEEN 'DIRECT' AND GUIDED WAVE.
this effect can be used to determine the guide wavelength directly by measuring the resultant field at a point along the guide until a phase change of $2\pi$ has occurred, by either varying the frequency or by altering the separation between launcher and receiver.

Experimentally it was found more convenient to choose the second method as this ensured that the klystron power remained constant, the conditions at the receiving end did not vary and the launching efficiency of the transmitting and receiving dipoles remained unaltered.

Sections of waveguide of different lengths, arranged in various combinations, were used to change the guide length in discrete steps. They consisted of thin-walled (telescopic) brass tubing coated respectively with Polythene tape or Titanium Dioxide loaded Polystyrene, making $b/\lambda_o = 0.38$ (at 9,000 Mc/s) and $c = 0.90$ and 0.96 respectively.

The results obtained are plotted in Figure 51 and it can be seen that a $2\pi$ phase change between the two waves takes place every 66.6 cm. in the case of the Polythene covered guide and every 36.6 cm. in the case of the TiO$_2$ coating.

Since the free space wavelength at 9,000 Mc/s is $\lambda_o = 3.33$ cm, these distances correspond to $20\lambda_g$ and $11\lambda_g$ respectively.
The guided wave, travelling at a reduced velocity, occupies one additional cycle in the same space, so that \( n \lambda_o = (n + 1) \lambda_g \). Hence the ratio \( \lambda_o / \lambda_g \) is 20:21 in the first case and 11:12 in the second. Below, these results are compared with the calculated values (See Figure 28) for the case \( b / \lambda_o = 0.38 \).

<table>
<thead>
<tr>
<th>Material</th>
<th>Theoretical</th>
<th>Measured</th>
</tr>
</thead>
<tbody>
<tr>
<td>Polythene</td>
<td>0.90</td>
<td>0.99</td>
</tr>
<tr>
<td>TiO₂ loaded Polystyrene</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( c = 0.96 )</td>
<td>0.97</td>
<td>0.92</td>
</tr>
</tbody>
</table>

Although the general trend shows agreement with the predicted behaviour, considering the order of magnitude involved, the difference between the theoretical and the measured values is rather large. The explanation can be sought in the fact that dielectric and resistive losses as well as radiation losses were neglected in solving the Characteristic Equation.

Losses along the surface of the guide provide additional retardation of the wave and this would account for \( \lambda_o / \lambda_g \) to be smaller than the calculated value.

Since only a very small portion of the wave travels in
the dielectric coating, the effect of dielectric loss is negligible, particularly in view of the fact that the \( \tan \delta \) of the material used is small. Resistive losses can account for some of the retardation, although radiation is believed to be the main cause.

Measurements to determine the attenuation along the guide lent support to this supposition. It can be seen from Figure 51 that the attenuation corresponds to 4.0 dB/mtr which cannot all be due to resistive and dielectric losses, particularly since most of the energy travels in the space surrounding the guide. (The contribution of the "direct" wave in these measurements was small. This was established by measuring the received signal with and without the guide. The "free space" signal being approximately 14 dB down on the guided signal, except in close proximity of the dipole.)
7. Conclusions and Future Work.

The existence of a first order cylindrical surf-
face wave has been proved and the feasibility of designing
d-end-fire aerials, utilising this mode, has been success-
fully demonstrated.

The properties of this wave, which is a hybrid
mode, as well as that of any other higher order mode can
be studied by examining the "Characteristic Equation",
which has been derived for the general case. In solving
the Characteristic Equation for specific cases, it was
assumed that the wave mode is non-radiating i.e. the rad-
ial propagation constant is a pure imaginary quantity and
that the longitudinal propagation is loss free.

These assumptions, although substantially cor-
rect, limit the validity of the results. A more rigorous
treatment would necessitate solving the Characteristic
Equation, by systematic trial and error, for complex argu-
ments of Bessel and Hankel functions, a stupendous task,
which is more suitably undertaken by a team of people
than by an individual.

The range of conditions for which the Characte-
ristic Equation can be solved, even allowing for the above
simplifying assumptions, is at present severely restricted
owing to the limitations imposed by available tables. In
particular it is not possible to predict the behaviour of
cylindrical surface waveguides which are coated with dielectric materials having a substantially higher dielectric constant than was used by the author, i.e. compounds of Titanium dioxide and Barium and/or Strontium which have been developed in recent years. (Ref. 35)

The experimental work has only been concerned with basic, straightforward Surface Wave Aerials and adequate theories have been put forward to predict their radiation patterns. However, different constructions can be envisaged. Since the size of a Surface Wave Aerial of given gain is proportional to \(\frac{1}{\sqrt{\varepsilon-1}}\), the arrival of high dielectric constant ceramic materials should make it possible to design Surface Wave Aerials for the microwave bands which are physically much smaller than any conventional type of aerial.

A ceramic sleeve supported by a metal tube is one form of construction which is not only rigid, but which is also inherently immune to great heat. Such Surface Wave Aerials can be built into the noses of aircraft flying at supersonic speeds, or guided rockets equipped with forward looking radar, where the effect of aerodynamic heating prevents the use of protective covering which is normally required by microwave aerials forming an integral part of airframe structures. An aerial of tapered construction with a considerably thicker layer of dielectric
near the launching point than at the end plane, would com-
bine desirable electrical properties with improved mechan­
cal strength and reduced wind resistance.

Complete freedom from sidelobes which is charac­
teristic of short Surface Wave Aerials, was never achieved
in practice, mainly because the very ineffective but con­
venient dipole method of launching the wave was used. The
waveguide transition type of launcher described in Section
5.1.3. is superior, as it is possible by proper design, to
eliminate direct radiation from the primary source. Further
work on launching methods and their efficiency needs to be
carried out, if it is desired do establish the propagation
characteristics of higher order surface waves more accura-
tely.

Although the work described in this treatise had
as its primary objective the investigation into a new type
of aerial, it has prompted research into higher order sur­
face waves and these have in turn given rise to many new
problems. Further research into the losses associated
with the propagation of the first order surface wave is
needed, to determine whether or not the wave is a radia­
ting mode, or whether the radiation loss that has been ob­
served is due to surface irregularities which lead to radia­
tion of the very loosely guide wave, particularly when
\[ \frac{\lambda}{\lambda_0} \] is almost unity.
Other higher order modes have not yet been investigated and the properties of the complete range of higher order modes which a dielectric coated cylinder can support remain still to be explored. A paper on the subject, enlarging upon the work done so far, is in course of preparation by the author and it is hoped that its publication will stimulate further research into the field of external surface waveguides.
8. **Acknowledgements.**

I am grateful to Dr. G. L. d'Ombrain, Head of the Electrical Engineering Department, Battersea Polytechnic, for his encouragement and for making available to me the engineering facilities of his Department, and to Mr. E. Wells, Senior Laboratory Steward, whose untiring efforts to overcome the many material difficulties that beset a research student, have been a most valuable help.

Thanks are also due to the Secretary of N.A.A.F.I. who kindly allowed me to install the aerial mast on top of their Headquarters, Ruxley Towers, Claygate, (Surrey) and to the Royal Observer Corps who consented, sometimes under protest, to the continued sharing of the small platform of Ruxley Towers with the aerial structure.

Lastly, I wish to acknowledge the cooperation of countless mystified residents in the Claygate area, who allowed me to use their mains supply.

* * *
Appendix I

Reduction of Observational Errors by The Use of Statistical Methods

If a number of observations are made, \( X_1, X_2, \ldots, X_n \), i.e. the field strength of an aerial measured in a particular direction and the plot of the measured points shows the normal (Gaussian) distribution, the error can be calculated as follows:

Find a.) the mean value from

\[
X_{\text{mean}} = \frac{1}{n} \sum_{r=1}^{n} X_r
\]

and b.) the standard deviation which is defined as

\[
S = \sqrt{\left( \frac{1}{n-1} \sum_{r=1}^{n} X_r^2 \right) - \frac{n}{n-1} X_{\text{mean}}^2}
\]

The error can then be said to lie with 95% confidence* between the limits

\[
\varepsilon_{0.95} = \pm 1.96 \frac{S}{\sqrt{n}}
\]

and with 90% confidence between the limits

\[
\varepsilon_{0.90} = \pm 1.64 \frac{S}{\sqrt{n}}
\]

* See Table II "Percentage Points of the Normal Distribution" (Ref 38).

See also "Statistical Methods in Research and Production" (Ref 37).
FIG. A1 DUMMY LOAD FOR USE AT V.H.F.
Appendix II

The Measurement of Power Fluctuations at Very High Frequencies

A dummy load can be constructed in the form of a coaxial lossy line. The length of such a line can be shortened appreciably if the dielectric is made from a lossy material or if the losses along the inner or outer conductor are artificially increased. By making the inner conductor in the form of a glass rod on to which a very thin metallic film (i.e. gold) a few Ångstrom units thick is deposited, a high resistance per unit length is obtained as well as a comparatively large surface area which can be adequately cooled. Full details of a wide band v.h.f. power measuring device based on this principle, are given in a paper by the author (Ref. 36). Figure A1 shows how this concept was applied to a dummy load in which the variation in input power can be accurately determined.

A thermocouple is sealed into a glass tube, forming the inner conductor of this lossy line so that the junction lies entirely on the surface of the glass tube. The outer surface is coated with a thin gold film which also covers the thermo-couple.

When power is absorbed by the load the resistive film heats up, as practically all the power loss takes place on the inner conductor, the time constant being of the order of milliseconds. In order to assist the heat dissipation, the load is filled with carbontetrachloride. The input end is
sealed against leakage of the coolant with Polytetrafluoroethylene, and terminates in a standard coaxial socket. Both the liquid and the solid dielectric have practically identical dielectric constants (2.10 and 2.18 respectively) and there is therefore no reflection of energy from the boundary surface between the two.

The "far-end" of the coaxial line is terminated in a circular array of resistors whose combined resistance, in parallel, is equal to the characteristic resistance. The division of power between line and termination is immaterial in this particular application although it is desirable that most of the power be dissipated in the resistive film.

In order to achieve uniform power loss along the resistive film, the resistance should vary exponentially so that

$$R_x = R_{in} e^{x/L}$$

where L is the total length of the resistive inner conductor,

x is the distance from the input end,

$R_{in}$ is the resistance per unit length at the input end

$R_x$ is the resistance per unit length at any point x.

The output from the thermo-couple is measured by a galvanometer in conjunction with a "cold junction" in the usual manner.
The device can be calibrated on direct current as the skin effect is virtually absent where the metallic film is used. (If the coolant is circulated and the flow rate and temperature rise between inlet and outlet are measured, the absolute power can be determined.)
Appendix III

Derivation of The Characteristic Equation

In this appendix the General Characteristic Equation of the first order surface wave is determined and the method of solving it by systematic trial and error is described. Solutions are obtained for specific cases for which tables have been compiled to assist with the numerical computations. These tables can be used for the solution of problems not considered in this treatise.

The field equations (11) and (12) in Section 4.3 contain 6 arbitrary coefficients $a_2, a_3, a_4$ and $b_2, b_3, b_4$, which can be found by applying the boundary conditions requiring continuity of $\phi$ and $z$ components at $r=a$ and $r=b$. In order to simplify the writing the following notations are adopted:

\[
a/b = c, \quad k_2 a = cV, \quad k_2 b = V, \quad k_3 b = N \text{ or } iW \quad (A1)
\]

A set of six equations in the $a$'s and $b$'s is then obtained as follows. At $r = a$

\[
\begin{align*}
E_{z2} &= 0, \\
E_{\phi2} &= 0, \\
\text{At } r = b, \\
E_{z2} &= E_{z3}, \\
H_{z2} &= H_{z3}, \\
E_{\phi2} &= E_{\phi3}, \\
H_{\phi2} &= H_{\phi3},
\end{align*}
\]

\[
\begin{align*}
\alpha_2 J_1(cV) + \alpha_3 Y_1(cV) &= 0, \\
\dfrac{1}{cV} \left[ 8 \left[ \alpha_2 J_1(cV) + \alpha_3 Y_1(cV) \right] + i \omega \mu_2 V \left[ b_2 J_1(cV) + b_3 Y_1(cV) \right] \right] &= 0 \\
\alpha_2 J_1(v) + \alpha_3 Y_1(v) &= \alpha_4 H_i(N), \\
b_2 J_1(v) + b_3 Y_1(v) &= b_4 H_i(N), \\
\frac{1}{V^2} \left[ \frac{8}{\pi} \left[ \alpha_2 J_1(v) + \alpha_3 Y_1(v) \right] + i \omega \mu_2 V \left[ b_2 J_1(v) + b_3 Y_1(v) \right] \right] &= \frac{1}{\pi^2} \left[ 8 \alpha_4 H_i(N) + i \omega \mu_3 N b_4 H_i(N) \right], \\
\frac{1}{V^2} \left[ \frac{8}{\pi} \left[ \alpha_2 J_1(v) + \alpha_3 Y_1(v) \right] - i \omega \mu_2 V \left[ b_2 J_1(v) + b_3 Y_1(v) \right] \right] &= \frac{1}{\pi^2} \left[ i \omega \mu_3 N \alpha_4 H_i(N) - \gamma b_4 H_i(N) \right].
\end{align*}
\]
If equations (A2) are written in the standard form so that corresponding terms in a's and b's are placed in the same order on the left hand side of each equation, an array of six equations results in which the right-hand members are zero. As these equations are only consistent if the determinant of the coefficients of the a's and b's is zero, the following determinant must be solved:

\[
\begin{array}{cccccc}
        a_2 & a_3 & a_4 & b_2 & b_3 & b_4 \\
- \mathcal{J}(CV) & - \mathcal{Y}(CV) & 0 & 0 & 0 & 0 \\
- \frac{\mathcal{Y}}{c} \mathcal{J}(CV) & - \frac{\mathcal{Y}}{c^2} \mathcal{Y}(CV) & 0 & - \frac{i \omega}{c} \mathcal{J}(CV) & - \frac{i \omega}{c} \mathcal{Y}(CV) & 0 \\
\mathcal{J}(V) & \mathcal{Y}(V) & - H_i(N) & 0 & 0 & 0 \\
0 & 0 & 0 & \mathcal{J}(V) & \mathcal{Y}(V) & - H_i(N) \\
\frac{\mathcal{Y}}{V^2} \mathcal{J}(V) & \frac{\mathcal{Y}}{V^2} \mathcal{Y}(V) & - \frac{\mathcal{Y}}{N^2} H_i(N) & \frac{i \omega}{V} \mathcal{J}(V) & \frac{i \omega}{V} \mathcal{Y}(V) & - \frac{i \omega}{N} H_i(N) \\
i \frac{\omega}{V} \mathcal{J}(V) & i \frac{\omega}{V} \mathcal{Y}(V) & - \frac{i \omega}{N} H_i(N) & - \frac{\mathcal{Y}}{V^2} \mathcal{J}(V) & - \frac{\mathcal{Y}}{V^2} \mathcal{Y}(V) & \frac{\mathcal{Y}}{N^2} H_i(N) \\
\end{array}
\]

This determinant is simplified in stages, the first one being the introduction of a new set of variables in order to reduce
the total number of variables, thus facilitating the numerical evaluation of the resulting expression, which involves a considerable amount of computational effort, as will be seen later. The following substitutions are made:

\[
\begin{align*}
\Delta_1 &= \frac{J_1'(cV)}{cV J_1(cV)} \\
\Delta_2 &= \frac{J_2(V)}{V J_2(V)} \\
\Delta_3 &= \frac{J_3(cV)}{Y_3(cV)} \\
\Delta_4 &= \frac{J_4'(V)}{Y_4(V)} \\
\Delta_5 &= \frac{H_5'(N)}{N H_5(N)} \\
\Delta_6 &= \frac{Y_6'(cV)}{c V Y_6(cV)} \\
\Delta_7 &= \frac{Y_7'(V)}{V Y_7(V)}
\end{align*}
\]

\[T = \frac{(1/V)^2}{(1/N)^2} \]

Multiplying or dividing the various rows and columns by suitable factors like \(J_1(V), J_2(V), Y_3(cV), \frac{J_1'(cV)}{cV}, \frac{Y_4(V)}{V}, \frac{Y_6'(cV)}{cV} \) where appropriate, so that the above new variables are introduced, results in the following simplified determinant:

<table>
<thead>
<tr>
<th>(\alpha_2)</th>
<th>(\alpha_3)</th>
<th>(\alpha_4)</th>
<th>(b_2)</th>
<th>(b_3)</th>
<th>(b_4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\Delta_3)</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>(\Delta_1\Delta_3)</td>
<td>(\Delta_6)</td>
<td>0</td>
</tr>
<tr>
<td>(\Delta_4)</td>
<td>1</td>
<td>(-1)</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>(\Delta_4)</td>
<td>1</td>
<td>(-1)</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>(T)</td>
<td>(\frac{i\omega_2 \Delta_2 \Delta_4}{8})</td>
<td>(\frac{i\omega_2 \Delta_7}{8})</td>
<td>(-\frac{i\omega_3 \Delta_5}{8})</td>
</tr>
<tr>
<td>(-\frac{i\omega_2 \Delta_2 \Delta_4}{8})</td>
<td>(-\frac{i\omega_2 \Delta_7}{8})</td>
<td>(\frac{i\omega_3 \Delta_5}{8})</td>
<td>0</td>
<td>0</td>
<td>(T)</td>
</tr>
</tbody>
</table>
By a process of "pivotal condensation" the determinant is reduced from a 6 x 6 to a 5 x 5 to a 4 x 4, interchanging rows where appropriate, until finally a 3 x 3 determinant is arrived at which can be solved in the usual manner.

The 3 x 3 determinant

\[
\begin{array}{c|c|c}
\frac{i\omega}{8}\varepsilon_3\Delta_3\Delta_5(\Delta_3-\Delta_4) & 0 & \Delta_3(\Delta_3-\Delta_4)T \\
\frac{i\omega}{8}\varepsilon_2\Delta_3(\Delta_3\Delta_7-\Delta_2\Delta_4) & & -i\omega\mu_3\Delta_5 \\
T & i\omega\mu_2(\Delta_3\Delta_7-\Delta_2\Delta_4) & \Delta_1\Delta_3-\Delta_6\Delta_4 \\
0 & & -1
\end{array}
\]

after putting \(\mu_2=\mu_1=\mu_0\), \(\varepsilon_2=\varepsilon_0\), \(\varepsilon_2=\varepsilon\varepsilon_0\), and introducing \(Q=\omega^2\mu_0\varepsilon_2\) is expanded to:

\[
\begin{align*}
&[Q(\Delta_3-\Delta_4)\Delta_3\Delta_5 - \varepsilon Q\Delta_3(\Delta_3\Delta_7-\Delta_2\Delta_4)] [\Delta_3\Delta_7-\Delta_2\Delta_4] \\
&-Q\Delta_3\Delta_5(\Delta_3-\Delta_4)\Delta_3-\Delta_4 + T^2\Delta_3(\Delta_3-\Delta_4)(\Delta_3-\Delta_4) \\
&+\varepsilon Q\Delta_3\Delta_5(\Delta_1\Delta_7-\Delta_2\Delta_4)(\Delta_3\Delta_7-\Delta_2\Delta_4) = 0
\end{align*}
\]  

(Simplifyng and re-arranging equation (A4) yields the "Characteristic Equation" from which the properties)
of the first order surface wave can be derived for any particular set of conditions.

\[ \begin{align*}
Q \{ (\Delta_1 \Delta_3 - \Delta_4 \Delta_6)(\Delta_3 - \Delta_4) \Delta_5^2 & - (\Delta_1 \Delta_3 \Delta_7 - \Delta_2 \Delta_4 \Delta_6)(\Delta_3 - \Delta_4) \Delta_5 \\
& + \varepsilon \left[ (\Delta_1 \Delta_3 \Delta_7 - \Delta_2 \Delta_4 \Delta_6)(\Delta_3 - \Delta_4) \right] \\
& - (\Delta_1 \Delta_3 - \Delta_4 \Delta_6)(\Delta_3 - \Delta_4) \Delta_5 \Delta_6 \} \\
& - T^2 (\Delta_1 \Delta_3 - \Delta_4 \Delta_6)(\Delta_3 - \Delta_4) = 0
\end{align*} \]  

(A5)

All the \( \Delta \)'s with the exception of \( \Delta_5 \) are functions of \( V \) and \( \Delta_5 \) depends only on \( W \); a re-grouping of Equation (A5), leaving \( \Delta_5 \) as the main variable, facilitates the finding of solutions for which the Characteristic Equation becomes zero. Introducing

\[ \begin{align*}
\mathcal{G} &= (\Delta_3 - \Delta_4)(\Delta_1 \Delta_3 - \Delta_4 \Delta_6) \\
\mathcal{G} &= (\Delta_3 - \Delta_4)(\Delta_1 \Delta_3 \Delta_7 - \Delta_2 \Delta_4 \Delta_6) + \varepsilon(\Delta_1 \Delta_3 \Delta_7 - \Delta_2 \Delta_4 \Delta_6)(\Delta_3 - \Delta_4 \Delta_6) \\
\tau &= (\Delta_3 - \Delta_4 \Delta_6)(\Delta_1 \Delta_3 - \Delta_4 \Delta_6) \varepsilon
\end{align*} \]  

(A6)

results in a very much simplified version of Equation (A5) for the purpose of numerical evaluation, viz

\[ Q(\mathcal{G} \Delta_5^2 - \mathcal{G} \Delta_5 + \tau) - \mathcal{G} T^2 = 0 \]  

(A7)
Using the relationship $Z_j(x) = Z_0(x) - \frac{1}{x} Z(x)$ reduces the $\Delta$'s as defined by equations (A3) to functions which are tabulated (Refs. 33, 34). Thus

$$\begin{align*}
\Delta_1 &= \frac{J_0(cV)}{cV J_1(cV)} - \frac{1}{(cV)^2} \\
\Delta_2 &= \frac{J_0(V)}{V J_1(V)} - \frac{1}{V^2} \\
\Delta_3 &= \frac{J_0(cV)}{Y_1(cV)} - \frac{1}{(cV)^2} \\
\Delta_4 &= \frac{J_0(V)}{Y_1(V)} \\
\Delta_5 &= \frac{H_0(iW)}{iW H_1(iW)} + \frac{1}{W^2} \\
\Delta_6 &= \frac{Y_0(cV)}{cV Y_1(cV)} - \frac{1}{(cV)^2} \\
\Delta_7 &= \frac{Y_0(V)}{V Y_1(V)} - \frac{1}{V^2}
\end{align*}$$

For any particular value of $c$ the above $\Delta$'s can now be determined with the aid of tables. Next, $\varphi$, $\sigma$ and $\tau$ are evaluated for a selected $V$ and the value of expression (A7) is calculated for a series of judiciously chosen $V$'s until a change of sign occurs. One pair of $V$'s and $W$'s is thus found which can be determined very accurately if the incremental steps by which $W$ is changed are reduced, or by plotting the results obtained in order to find where a zero occurs between two $W$'s for which the value of expression (A7) is of opposite sign.

Tables of $\Delta$'s calculated in accordance with the foregoing remarks are given on the next page; they can be used to solve problems involving dielectric constants other than those specifically given in the text.
### Tables of \( \Delta \)'s (Dependent on "c")

<table>
<thead>
<tr>
<th>( V )</th>
<th>( \Delta_1 )</th>
<th>( \Delta_3 )</th>
<th>( \Delta_6 )</th>
<th>( \Delta_1 )</th>
<th>( \Delta_3 )</th>
<th>( \Delta_6 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.2</td>
<td>-0.050</td>
<td>-4.871</td>
<td>-2.416</td>
<td>-0.111</td>
<td>-13.85-</td>
<td>-6.242</td>
</tr>
<tr>
<td>2.4</td>
<td>-0.105</td>
<td>-28.80-</td>
<td>-12.61-</td>
<td>-0.147</td>
<td>9.450</td>
<td>3.588</td>
</tr>
<tr>
<td>2.6</td>
<td>-0.156</td>
<td>7.300</td>
<td>2.880</td>
<td>-0.197</td>
<td>3.520</td>
<td>1.254</td>
</tr>
<tr>
<td>2.8</td>
<td>-0.215</td>
<td>3.181</td>
<td>1.103</td>
<td>-0.262</td>
<td>1.991</td>
<td>0.619</td>
</tr>
<tr>
<td>3.0</td>
<td>-0.284</td>
<td>1.941</td>
<td>0.613</td>
<td>-0.386</td>
<td>1.171</td>
<td>0.376</td>
</tr>
<tr>
<td>3.2</td>
<td>-0.317</td>
<td>1.322</td>
<td>0.376</td>
<td>-0.400</td>
<td>0.913</td>
<td>0.230</td>
</tr>
<tr>
<td>3.4</td>
<td>-0.395</td>
<td>0.930</td>
<td>0.236</td>
<td>-0.527</td>
<td>0.619</td>
<td>0.135</td>
</tr>
<tr>
<td>3.6</td>
<td>-0.511</td>
<td>0.650</td>
<td>0.143</td>
<td>-0.795</td>
<td>0.379</td>
<td>0.053</td>
</tr>
<tr>
<td>3.8</td>
<td>-0.713</td>
<td>0.423</td>
<td>0.075</td>
<td>-1.451</td>
<td>0.189</td>
<td>0.011</td>
</tr>
<tr>
<td>4.0</td>
<td>-0.122</td>
<td>0.230</td>
<td>0.022</td>
<td>31.73-</td>
<td>-0.008</td>
<td>-0.037</td>
</tr>
<tr>
<td>4.2</td>
<td>-5.130</td>
<td>0.051</td>
<td>-0.023</td>
<td>1.201</td>
<td>-0.196</td>
<td>-0.080</td>
</tr>
<tr>
<td>4.4</td>
<td>1.926</td>
<td>-0.126</td>
<td>-0.064</td>
<td>0.555</td>
<td>-0.400</td>
<td>-0.057</td>
</tr>
<tr>
<td>4.6</td>
<td>0.735</td>
<td>-0.311</td>
<td>-0.104</td>
<td>0.316</td>
<td>-0.649</td>
<td>-0.170</td>
</tr>
<tr>
<td>4.8</td>
<td>0.400</td>
<td>-0.518</td>
<td>-0.148</td>
<td>0.199</td>
<td>-0.956</td>
<td>-0.229</td>
</tr>
<tr>
<td>5.0</td>
<td>0.279</td>
<td>-0.768</td>
<td>-0.193</td>
<td>0.125</td>
<td>-1.401</td>
<td>-0.310</td>
</tr>
<tr>
<td>5.2</td>
<td>0.954</td>
<td>-1.097</td>
<td>-0.280</td>
<td>0.071</td>
<td>-2.160</td>
<td>-0.447</td>
</tr>
<tr>
<td>6</td>
<td>-0.012</td>
<td>-34.10-</td>
<td>-6.264</td>
<td>-0.074</td>
<td>2.953</td>
<td>0.492</td>
</tr>
<tr>
<td>7</td>
<td>-0.196</td>
<td>0.858</td>
<td>0.122</td>
<td>-0.638</td>
<td>0.234</td>
<td>0.033</td>
</tr>
<tr>
<td>8</td>
<td>0.736</td>
<td>-0.185</td>
<td>-0.025</td>
<td>0.796</td>
<td>-0.776</td>
<td>-0.109</td>
</tr>
<tr>
<td>9</td>
<td>0.058</td>
<td>-1.185</td>
<td>-0.236</td>
<td>-0.012</td>
<td>22.85-</td>
<td>2.625</td>
</tr>
<tr>
<td>10</td>
<td>-0.053</td>
<td>2.343</td>
<td>0.254</td>
<td>-0.167</td>
<td>0.643</td>
<td>0.061</td>
</tr>
<tr>
<td>11</td>
<td>-0.367</td>
<td>0.279</td>
<td>0.025</td>
<td>0.246</td>
<td>-0.405</td>
<td>-0.043</td>
</tr>
<tr>
<td>12</td>
<td>0.124</td>
<td>0.721</td>
<td>-0.070</td>
<td>0.016</td>
<td>-4.302</td>
<td>-0.376</td>
</tr>
<tr>
<td>13</td>
<td>0.001</td>
<td>-20.40-</td>
<td>-1.747</td>
<td>-0.075</td>
<td>1.120</td>
<td>0.086</td>
</tr>
<tr>
<td>14</td>
<td>-0.093</td>
<td>0.882</td>
<td>0.068</td>
<td>0.646</td>
<td>-0.116</td>
<td>-0.011</td>
</tr>
<tr>
<td>15</td>
<td>0.414</td>
<td>-0.178</td>
<td>-0.016</td>
<td>0.035</td>
<td>-1.850</td>
<td>-0.130</td>
</tr>
<tr>
<td>16</td>
<td>0.035</td>
<td>-1.840</td>
<td>-0.130</td>
<td>-0.034</td>
<td>2.005</td>
<td>0.129</td>
</tr>
<tr>
<td>20</td>
<td>0.001</td>
<td>-22.90-</td>
<td>-1.273</td>
<td>-0.119</td>
<td>0.441</td>
<td>0.022</td>
</tr>
<tr>
<td>25</td>
<td>-0.168</td>
<td>0.266</td>
<td>0.011</td>
<td>0.014</td>
<td>-2.880</td>
<td>-0.118</td>
</tr>
</tbody>
</table>
### Tables of Δ's (Independent of "c")

<table>
<thead>
<tr>
<th>V</th>
<th>Δ₂</th>
<th>Δ₄</th>
<th>Δ₇</th>
<th>V</th>
<th>Δ₂</th>
<th>Δ₄</th>
<th>Δ₇</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.2</td>
<td>-0.116</td>
<td>370.0</td>
<td>200.0</td>
<td>5.0</td>
<td>0.068</td>
<td>-2.220</td>
<td>-0.560</td>
</tr>
<tr>
<td>2.4</td>
<td>-0.171</td>
<td>5.175</td>
<td>1.944</td>
<td>5.2</td>
<td>0.285</td>
<td>-4.330</td>
<td>-0.841</td>
</tr>
<tr>
<td>2.6</td>
<td>-0.227</td>
<td>2.500</td>
<td>0.832</td>
<td>6.0</td>
<td>-0.119</td>
<td>1.581</td>
<td>0.247</td>
</tr>
<tr>
<td>2.8</td>
<td>-0.291</td>
<td>1.555</td>
<td>0.462</td>
<td>7.0</td>
<td>-9.142</td>
<td>0.016</td>
<td>-0.008</td>
</tr>
<tr>
<td>3.0</td>
<td>-0.367</td>
<td>1.042</td>
<td>0.276</td>
<td>8.0</td>
<td>0.075</td>
<td>-1.484</td>
<td>-0.192</td>
</tr>
<tr>
<td>3.2</td>
<td>-0.481</td>
<td>0.705</td>
<td>0.161</td>
<td>9.0</td>
<td>-0.053</td>
<td>2.352</td>
<td>0.254</td>
</tr>
<tr>
<td>3.4</td>
<td>-0.685</td>
<td>0.447</td>
<td>0.081</td>
<td>10.0</td>
<td>-0.574</td>
<td>0.174</td>
<td>0.012</td>
</tr>
<tr>
<td>3.6</td>
<td>-1.217</td>
<td>0.230</td>
<td>0.022</td>
<td>11.0</td>
<td>0.080</td>
<td>-1.080</td>
<td>-0.102</td>
</tr>
<tr>
<td>3.8</td>
<td>-8.330</td>
<td>0.031</td>
<td>-0.028</td>
<td>12.0</td>
<td>-0.025</td>
<td>3.912</td>
<td>0.322</td>
</tr>
<tr>
<td>4.0</td>
<td>1.440</td>
<td>-0.166</td>
<td>-0.073</td>
<td>13.0</td>
<td>-0.232</td>
<td>0.321</td>
<td>0.023</td>
</tr>
<tr>
<td>4.2</td>
<td>0.590</td>
<td>-0.377</td>
<td>-0.116</td>
<td>14.0</td>
<td>0.087</td>
<td>-0.801</td>
<td>-0.060</td>
</tr>
<tr>
<td>4.4</td>
<td>0.232</td>
<td>-0.643</td>
<td>-0.165</td>
<td>15.0</td>
<td>-0.051</td>
<td>9.730</td>
<td>0.646</td>
</tr>
<tr>
<td>4.6</td>
<td>0.203</td>
<td>-0.940</td>
<td>-0.225</td>
<td>16.0</td>
<td>-0.113</td>
<td>0.508</td>
<td>0.030</td>
</tr>
<tr>
<td>4.8</td>
<td>0.125</td>
<td>-1.400</td>
<td>-0.311</td>
<td>17.0</td>
<td>0.123</td>
<td>-0.328</td>
<td>-0.021</td>
</tr>
</tbody>
</table>

### WΔ₅

<table>
<thead>
<tr>
<th>W</th>
<th>Δ₅</th>
<th>W</th>
<th>Δ₅</th>
<th>W</th>
<th>Δ₅</th>
<th>W</th>
<th>Δ₅</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>102.470</td>
<td>1.1</td>
<td>1.478</td>
<td>2.1</td>
<td>0.618</td>
<td>4.0</td>
<td>0.286</td>
</tr>
<tr>
<td>0.2</td>
<td>26.835</td>
<td>1.2</td>
<td>1.305</td>
<td>2.2</td>
<td>0.577</td>
<td>5.0</td>
<td>0.222</td>
</tr>
<tr>
<td>0.3</td>
<td>12.600</td>
<td>1.3</td>
<td>1.186</td>
<td>2.3</td>
<td>0.551</td>
<td>6.0</td>
<td>0.182</td>
</tr>
<tr>
<td>0.4</td>
<td>7.525</td>
<td>1.4</td>
<td>1.053</td>
<td>2.4</td>
<td>0.523</td>
<td>7.0</td>
<td>0.154</td>
</tr>
<tr>
<td>0.5</td>
<td>5.116</td>
<td>1.5</td>
<td>0.958</td>
<td>2.5</td>
<td>0.498</td>
<td>8.0</td>
<td>0.134</td>
</tr>
<tr>
<td>0.6</td>
<td>3.772</td>
<td>1.6</td>
<td>0.879</td>
<td>2.6</td>
<td>0.475</td>
<td>9.0</td>
<td>0.118</td>
</tr>
<tr>
<td>0.7</td>
<td>2.979</td>
<td>1.7</td>
<td>0.811</td>
<td>2.7</td>
<td>0.453</td>
<td>10.0</td>
<td>0.105</td>
</tr>
<tr>
<td>0.8</td>
<td>2.382</td>
<td>1.8</td>
<td>0.753</td>
<td>2.8</td>
<td>0.410</td>
<td>11.0</td>
<td>0.095</td>
</tr>
<tr>
<td>0.9</td>
<td>1.988</td>
<td>1.9</td>
<td>0.702</td>
<td>2.9</td>
<td>0.427</td>
<td>12.0</td>
<td>0.087</td>
</tr>
<tr>
<td>1.0</td>
<td>1.700</td>
<td>2.0</td>
<td>0.657</td>
<td>3.0</td>
<td>0.399</td>
<td>13.0</td>
<td>0.080</td>
</tr>
<tr>
<td>1.1</td>
<td>1.438</td>
<td>2.1</td>
<td>0.622</td>
<td>3.1</td>
<td>0.373</td>
<td>14.0</td>
<td>0.069</td>
</tr>
</tbody>
</table>
Derivation of the $E/H$ Ratio

From the original determinant of the coefficients $a_2, a_3, a_4, b_2, b_3, b_4$ on page 76, the individual $a$'s and $b$'s can be determined. Once these are known, the ratio of the $E_z$ and $H_z$ components of the surface wave can be found from either Equation (8), or (9) in Section 4.2. It is preferable to choose Equation 9 as this requires only a knowledge of $a_4$ and $b_4$. Expressing all coefficients in terms of $b_4$, leaves five unknowns to be determined which can be obtained by solving a $5 \times 5$ determinant; the last row of the original determinant is therefore omitted, resulting in:

$$\begin{bmatrix}
J_0(V) & Y_0(V) & 0 & 0 & 0 \\
\frac{\kappa}{2} J_1(V) & \frac{\kappa}{2} Y_1(V) & 0 & \frac{i \omega \mu_2 J_0'(V)}{c V} & \frac{i \omega \mu_2 Y_0'(V)}{c V} \\
J_1(V) & Y_1(V) & -H_1(N) & 0 & 0 \\
A_1 & A_2 & A_3 & A_4 & A_5 \\
0 & 0 & 0 & J_1(V) & Y_1(V) \\
B_1 & B_2 & B_3 & B_4 & B_5 \\
\frac{\kappa}{2} J_0(V) & \frac{\kappa}{2} Y_0(V) & -\frac{\kappa}{N} H_0(N) & \frac{i \omega \mu_2 J_0'(V)}{V} & \frac{i \omega \mu_2 Y_0'(V)}{V} \\
\end{bmatrix} \begin{bmatrix}
a_2 \\
a_3 \\
a_4 \\
b_2 \\
b_3 \\
\end{bmatrix} = \begin{bmatrix}
a_2 \\
a_3 \\
a_4 \\
b_2 \\
b_3 \\
\end{bmatrix} = \begin{bmatrix}
b_4 \\
H_1(N) \\
\frac{i \omega \mu_3 H_0(N)}{N} \\
\end{bmatrix}$$
Since the a's and b's are required, it is necessary to find the reciprocal matrix. The first three elements of the column matrix are zero, and the first three columns of the reciprocal matrix are therefore not needed. The co-factors of the last two rows, denoted by $A_1$, $A_2$, $A_3$, $A_4$, $A_5$, and $B_1$, $B_2$, $B_3$, $B_4$, $B_5$ respectively are then as follows:

\[
-A_1 = \begin{bmatrix}
Y_i(cV) & 0 & 0 & 0 \\
\frac{\delta}{c^2 V^2} Y_i(cV) & 0 & i\omega \mu_2 \bar{J}_i'(cV) & i\omega \mu_2 Y_i'(cV) \\
Y_i(V) & -H_i(N) & 0 & 0 \\
-\frac{\delta}{V^2} Y_i(V) & -\frac{\delta}{N^2} H_i(N) & \frac{i\omega \mu_2}{V} \bar{J}_i'(V) & \frac{i\omega \mu_2}{V} Y_i'(cV)
\end{bmatrix}
\]

\[
= Y_i(cV) H_i(N) \left[ (i\omega \mu_2 \bar{J}_i'(cV) \frac{i\omega \mu_2 Y_i'(cV)}{cV} - (i\omega \mu_2 \bar{J}_i'(cV) \frac{i\omega \mu_2 Y_i'(cV)}{cV}) \right]
\]

\[
= -\omega^2 \mu_2^2 Y_i(cV) H_i(N) \left( \bar{J}_i'(cV) Y_i'(cV) - Y_i'(cV) \bar{J}_i'(V) \right)
\]

Similarly,

\[
A_2 = -\omega^2 \mu_2^2 \frac{\bar{J}_i(cV)}{cV^2} H_i(N) \left( \bar{J}_i'(cV) Y_i'(cV) - Y_i'(cV) \bar{J}_i'(V) \right)
\]

\[
-A_3 = \omega^2 \mu_2^2 \left( \bar{J}_i(cV) Y_i'(cV) - Y_i'(cV) \bar{J}_i(V) \right) \left( \bar{J}_i'(cV) Y_i'(cV) - J_i'(V) Y_i'(cV) \right)
\]
\begin{align*}
A_4 &= \frac{i\omega_1 c V}{\varepsilon} Y_i(c V) H_1(N) \left( J_i(c V) Y_i(V) - Y_i(c V) J_i(V) \right) \\
-A_5 &= \frac{i\omega_2 c V}{\varepsilon} J_i(c V) H_1(N) \left( J_i(c V) Y_i(V) - Y_i(c V) J_i(V) \right)
\end{align*}

\begin{align*}
B_1 &= \frac{i\omega_1 c V}{\varepsilon} Y_i(c V) H_1(N) \left( J'_i(c V) Y_i(V) - Y'_i(c V) J_i(V) \right) \\
-B_2 &= \frac{i\omega_2 c V}{\varepsilon} J'_i(c V) H'_1(N) \left( J'_i(c V) Y_i(V) - Y'_i(c V) J_i(V) \right) \\
B_3 &= -\frac{i\omega_2 c V}{\varepsilon} \left( J_i(c V) Y_i(V) - Y_i(c V) J_i(V) \right) \left( J'_i(c V) Y_i(V) - Y'_i(c V) J_i(V) \right) \\
-B_4 &= 0 \\
B_5 &= 0 \\
\end{align*}

whence

\[ \begin{bmatrix}
\alpha_2 \\
\alpha_3 \\
\alpha_4 \\
\beta_2 \\
\beta_3
\end{bmatrix} = \begin{bmatrix}
0 & 0 & 0 & A_1 & B_1 \\
0 & 0 & 0 & A_2 & B_2 \\
0 & 0 & 0 & A_3 & B_3 \\
0 & 0 & 0 & A_4 & B_4 \\
0 & 0 & 0 & A_5 & B_5
\end{bmatrix} \begin{bmatrix}
\omega_1 \varepsilon \\
H(N) \\
\omega_2 \varepsilon \\
N
\end{bmatrix} \]

where "$\Delta$" denotes the original, simplified determinant.
It can readily be solved and is found to be equal to:

\[ D = \frac{i \omega \mu_2}{c_0 v} \sin \left[ \sin^{-1} \left( \frac{\mu_2}{\mu_1} \right) \right] \]

The individual \( a \)'s and \( b \)'s are then as follows:

\[
\begin{align*}
\begin{bmatrix}
a_2 \\
a_3 \\
a_4 \\
b_2 \\
b_3
\end{bmatrix} &= \begin{bmatrix}
A_1 H_1(N) + B_1 \frac{i \omega \mu_3}{N} H'_1(N) \\
A_2 H_1(N) + B_2 \frac{i \omega \mu_3}{N} H'_1(N) \\
A_3 H_1(N) + B_3 \frac{i \omega \mu_3}{N} H'_1(N) \\
A_4 H_1(N) + B_4 \frac{i \omega \mu_3}{N} H'_1(N) \\
A_5 H_1(N) + B_5 \frac{i \omega \mu_3}{N} H'_1(N)
\end{bmatrix}
\end{align*}
\]

Thus the ratio of \( \frac{a_4}{b_4} \) after simplification becomes

\[
\frac{a_4}{b_4} = \frac{i \omega}{\sin \left[ \sin^{-1} \left( \frac{\mu_2}{\mu_1} \right) \right]} \left[ \frac{\mu_3}{N} H'_1(N) + \frac{\mu_2}{V} H_1(N) \left( J'_1(cV) Y'_1(V) - J'_1(Y'_1 cV) \right) \right]
\]

Putting \( \mu_2 = \mu_3 = \mu_0 \), as in Appendix III and remembering that \( Q = \frac{\omega^2 \varepsilon}{c_0 \mu_0} \) and \( N = iW \), yields

\[
\sqrt{\frac{\varepsilon}{\mu_0}} \frac{a_4}{b_4} = \frac{i \sqrt{Q}}{T} \left[ \frac{H'_1(iW)}{iW H_1(iW)} + \frac{J'_1(cV) Y'_1(V) - J'_1(Y'_1 cV)}{V(J'_1(Y'_1 cV) - J'_1(cV) Y'_1(V))} \right]
\]
Appendix V

The Theoretical Polar Diagram of Short Aerials.

Radiation takes place from the end of an unterminated surface waveguide i.e. a surface wave aerial. The fields $E_r$ and $E_\varphi$ are given by Equations 11 & 12, page 36. Let the launching as well as the receiving dipole be in the $x$-$z$ plane, so that only $E_x$ is of interest.

The polar diagram in the $x$-$z$ plane is found by summation:

$$E_\varphi = \oint \oint E_x \, dx \, dy \, e^{i(\omega t - \mathbf{z} + \varphi - f(\theta))} = \oint \oint E_x \, dx \, dy \, e^{i\varphi - if(\theta) i(\omega t - \mathbf{z}^2)}$$

where

$$E_x = E_r \cos \varphi - E_\varphi \sin \varphi$$  \hspace{1cm} (A 13)

and $f(\theta)$ is the phase difference at a point 'P' in the end plane relative to the point 'O'. Hence $f(\theta)$ can be written

for the E-plane \hspace{1cm} $f(\theta) = \frac{2\pi r}{\lambda} \cos \varphi \sin \varphi$  \hspace{1cm} (A 15)

for the H-plane \hspace{1cm} $f(\theta) = \frac{2\pi r}{\lambda} \sin \varphi \sin \varphi$  \hspace{1cm} (A 16)
Substituting for \( f(\theta) \) and \( E_x \) from Equations (A 14 and A 15) respectively and introducing polar co-ordinates yields:

\[
E_\theta = \int_0^{2\pi} \left( E_r \cos \varphi - E_0 \sin \varphi \right) (\cos \varphi + i \sin \varphi) e^{i(\omega t - r - \varphi)} \, dr \, d\varphi \quad (A 17)
\]

The time variable \( e^{i(\omega t - \varphi)} \) has been omitted. Using the relationship \( \cos^2 \varphi = \frac{1}{2}(1 + \cos 2\varphi) \) and \( \sin^2 \varphi = \frac{1}{2}(1 + \sin 2\varphi) \) transforms Equation (A 17) to:

\[
E_\theta = \int_0^{2\pi} \left[ \frac{E_r(1+\cos 2\varphi + i \sin 2\varphi)}{2} - \frac{E_0(i+\sin 2\varphi - i \cos 2\varphi)}{2} \right] e^{i(\omega t - r - \varphi)} \, dr \, d\varphi \quad (A 18)
\]

Equation (A 18) can be simplified to:

\[
E_\theta = \int_0^{2\pi} \left[ \frac{E_r}{2} - iE_\varphi + \frac{E_r e^{i\varphi}}{2} + iE_\varphi e^{i\varphi} \right] e^{i(\omega t - r - \varphi)} \, dr \, d\varphi \quad (A 19)
\]

Using the mathematical identity

\[
\int_0^{2\pi} e^{iz \cos \varphi} \sin \varphi \, d\varphi = \frac{2\pi}{i} J_1(z) \quad (A 20)
\]

where \( z = \frac{2\pi r \sin \theta}{\lambda_0} \), reduces Equation (A 19) to:

\[
E_\theta = \int_b^\infty \left[ \frac{2\pi (E_r - iE_\varphi)}{2} J_0\left(\frac{2\pi r \sin \theta}{\lambda_0}\right) + \frac{2\pi (E_r + iE_\varphi)}{2} J_2\left(\frac{2\pi r \sin \theta}{\lambda_0}\right) \right] \, dr \quad (A 21)
\]

Now \( E_r \) and \( E_\varphi \) as defined by Eqns (11 & 12) can be simplified
by using the relationship \( \frac{a_4}{b_4} = i\sqrt{\frac{\mu_3}{\varepsilon_0}} \) (Eqn.26) which was derived earlier on. With this substitution \( E_r \) becomes

\[
E_r = \frac{1}{k_3} \left[ -i\omega a_4 k_3 H_1'(k_3r) + \frac{\omega b_4 \mu_3}{r} H_1(k_3r) \right]
\]
\[
= \frac{b_4}{k_3^2} \left[ \sqrt{\frac{\mu_3}{\varepsilon_0}} k_3 H_1'(k_3r) + \frac{\omega \mu_3}{r} H_1(k_3r) \right]
\]
\[
= \frac{\sqrt{\mu_3}}{\varepsilon_0} k_3 H_1'(k_3r) + \frac{\omega \mu_3}{\varepsilon_0} \frac{1}{r} \sqrt{\frac{\mu_3}{\varepsilon_0}} H_1(k_3r)
\]

Since \( \mu_3 = \mu_0 \) and \( \sqrt{\varepsilon} = \omega \sqrt{\mu_0} = 1 \), the above expression simplifies to

\[
E_r = \frac{\sqrt{\mu_0}}{k_3^2} \left[ k_3 H_1'(k_3r) + \frac{1}{r} H_1(k_3r) \right] \quad (A \ 22)
\]

Similarly, it can be shown that

\[
E_\phi = i \frac{\sqrt{\mu_0}}{k_3^2} \left[ k_3 H_1'(k_3r) + \frac{1}{r} H_1(k_3r) \right] \quad (A \ 23)
\]

It follows then from Eqns.(A 22 & A 23) that

\[
E_r + iE_\phi = 0 \quad (A \ 24)
\]
and similarly by subtracting Equations (A 23 & 24), one obtains

\[ E_r - iE_\varphi = 2E_r = \frac{2\chi b^4}{K^3} \sqrt{\frac{\mu_o}{\varepsilon_o}} \left[ k_3 H_i'(k_3r) + \frac{i}{r} H_i(k_3r) \right] \]

Making the substitution \( H_i'(z) = H_0(z) - \frac{1}{z} H_i(z) \) finally results in

\[ E_r - iE_\varphi = \frac{2\chi b^4}{K^3} \sqrt{\frac{\mu_o}{\varepsilon_o}} H_0(k_3r) \quad (A 25) \]

Putting the results of Eqns. (A 24 & 25) into Equation (A21) and introducing the normalised parameter \( r/b \), leads to the ultimate expression for the field \( E_\theta \), replacing \( k_3 \) by \( iW \),

\[ E_\theta = \text{const.} \int_0^\infty \frac{r}{b} H_0 \left( iW \frac{r}{b} \right) \times J_0 \left( \frac{2\pi}{b} \frac{r}{b} \sin \theta \right) \frac{d(\frac{r}{b})}{\left( \frac{r}{b} \right)^2} \quad (A 26) \]

The \( H \)-plane polar diagram can be derived in a similar manner by using Eqn. (A 16) instead of Eqn. (A 15). The mathematical identity expressed by Equation (A 20) does not change if \( e^{iz \cos \theta} \) is replaced by \( e^{iz \sin \theta} \) and hence the \( E_\theta \) and the \( H_\theta \) polar diagrams are identical.
Appendix VI

The Theoretical Polar Diagram of Long Aerials.

The polar diagram of a long aerial which is radiating continuously along its length is made up from the radiation of the end plane as well as the continuous radiation. In this Appendix only the contribution from the continuous radiation is considered.

Let the phase constant of a wave in free space be \( \beta_0 = \frac{2\pi}{\lambda_o} \) and let the wave along the aerial be retarded so that \( \beta' = \frac{2\pi}{\lambda'} \) then the field in any direction \( \theta \) is given by (See Ref. 39 p.152)

\[
E_\theta = \int_0^1 e^{i(\beta_0 \cos \theta - \beta')z} \, dz = \frac{e^{i(\beta_0 \cos \theta - \beta')z} - 1}{i(\beta_0 \cos \theta - \beta')} 
\]  

(A 27)

If the amplitude is not constant, but decreases exponentially, either due to resistive loss or radiation loss and \( \alpha \) is the loss per unit length in Nepers, than Eqn. (A 27) is modified to

\[
E_\theta = \int_0^1 e^{i(\alpha + \beta_0 \cos \theta - \beta')z} \, dz = \frac{e^{i(\alpha + \beta_0 \cos \theta - \beta')z} - 1}{i(\alpha + \beta_0 \cos \theta - \beta')} 
\]  

(A 28)

Since \( e^x - 1 = 2i \sin \frac{x}{2} e^{ix} \) and

\[
\sin(a+b) = \sin a \cosh b + i \cos a \sinh b
\]
Equation (A 28) can be re-written as follows:—

\[
E_\theta = \frac{\sin \frac{\pi}{\lambda_o} (\cos \theta - \frac{\lambda}{\lambda_o}) \cosh \frac{\alpha}{2} + i \cos \frac{\pi}{\lambda_o} (\cos \theta - \frac{\lambda}{\lambda_o}) \sinh \frac{\alpha}{2} \frac{i \lambda (\cos \theta - \frac{\lambda}{\lambda_o})}{\lambda_o}}{i \alpha + 2 \pi (\cos \theta - \frac{\lambda}{\lambda_o})} 2i e^{i \phi} \]

The above expression applies to an infinitely thin wire. It does, however, also apply to the case of the surface wave aerial.

The radiation can be assumed to originate along two line sources, in the E-plane, on either side of the aerial rod. Owing to the shielding effect of the metal tube, there can be no interference pattern in the E-plane; one line source can be thought of as covering + θ and the other - θ. In the H-plane the two line sources are always in phase. The foregoing assumes that the launching dipole lies in the E-plane.

* * *
LIST OF REFERENCES


