TURBULENCE IN CRYOGENIC LIQUIDS

Thesis prepared for the Degree
of Doctor of Philosophy
by
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A study is presented of the problem of thermally generated turbulence in cryogenic liquid bubble chambers and its effects on the photographically recorded positions of bubbles. Particular emphasis is laid on the optical distortion or root mean square fluctuation in position of light rays emerging from a turbulent fluid at low temperature. Previous work has concentrated on studies of temperature fluctuations in non-cryogenic fluids. Extrapolation is made to the low temperature region to predict turbulence parameters for the design of an experiment to measure optical distortions, turbulence eddy sizes and velocities in a model bubble chamber liquid in a state of thermal turbulence.

An electro-optical probe system is described which was used to measure the parameters of interest and hence check the validity of the model used.

Results of the study are presented and shown to validate the model and these are then used to predict conditions in a real bubble chamber situation. Dynamic effects arising from the cycling of the chamber are also considered.

An improved design of the system for localised rather than path integrated measurements within the fluid, is described.
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LIST OF SYMBOLS AND ABBREVIATIONS

a - turbulence eddy diameter

A - amplitude

A_r - constant

B - signal bandwidth

C_p - specific heat at constant pressure

C, C_1, C_2 - constants

c - velocity of light

D - depth of fluid layer, detector diameter, net electrical displacement

D_1, D_2 - detectors

d - deviation of a light ray in direction normal to propagation direction

\( \frac{1}{d^2} \) - optical distortion
\[
\frac{1}{d_x} \quad \text{optical distortion in the horizontal plane}
\]
\[
\frac{1}{d_z} \quad \text{optical distortion in the vertical plane}
\]
\[
e_{1} \quad \text{fluctuations about the mean positions}
\]
\[
e_{2} \quad \text{fluctuations about the mean positions}
\]
\[
E \quad \text{electric field vector}
\]
\[
F \quad \text{function}
\]
\[
f \quad \text{frequency}
\]
\[
f_r \quad \text{random frequency modulation}
\]
\[
\Delta f \quad \text{detector bandwidth}
\]
\[
f_M \quad \text{maximum amplitude modulation frequency}
\]
\[
g \quad \text{acceleration due to gravity}
\]
\[
H \quad \text{vertical heat flux}
\]
\[
h_i \quad \text{contribution to heat flux}
\]
\[
h_o \quad \text{height of boundary layer}
\]
\[
\frac{1}{2} i_n \quad \text{mean square noise current}
\]
$K$ - thermometric conductivity

$k$ - Boltzmann's constant

$k_1$, $k_2$ - constants

$L$ - path length

$\ell$ - beam separation, length

$m$ - modulation depth

$N$ - cross-correlation coefficient

$Q$ - flux of buoyancy

$q$ - velocity fluctuation

$R$ - Rayleigh number, cross correlation coefficient

$R_c$ - critical Rayleigh number

$R_{ei}$ - inner Reynolds number

$R_N$ - normalised cross correlation coefficient

$r$ - separation between points in space
\( S \) - heat flux per unit length, phase shift, unit vector tangential to ray direction

\( T \) - temperature, length or duration of recorded signal

\( T_0 \) - temperature at height \( h_0 \)

\( t \) - time

\( t_c \) - correlation time

\( t_1 \), \( t_2 \) - independant parameters

\( u \) - fluid velocity fluctuation

\( u_0 \) - dimensionless velocity scale

\( V \) - voltage

\( V_{in} \) - input voltage

\( V_o \) - output voltage

\( V_r^2 \) - mean square flicker noise voltage
\( x \), \( y \), \( z \) - spatial coordinates.

\( Z \) - height of a point in the fluid layer above the lower boundary.

\( z_o \) - dimensionless length scale.

AM - amplitude modulation.

FM - frequency modulation.

S/N - signal to noise ratio.

SNR - signal to noise power ratio.

\( P_S / P_N \) - signal to noise power ratio.

R.M.S. - root mean square.

\( \Delta T \) - temperature difference between upper and lower boundaries of fluid layer.

min - minimum.

\( \beta \) - coefficient of cubical expansion.

\( \epsilon \) - error, angular fluctuation.
\[ \theta \] - phase, fluctuation in temperature about mean value

\[ \theta_1 \] - angle between direction of heat flow and normal to heater plate

\[ \theta_0 \] - dimensionless temperature scale

\[ \lambda \] - wavelength

\[ \rho \] - density

\[ \nu \] - kinematic viscosity, frequency

\[ \sigma \] - optical path

\[ \tau \] - time delay

\[ \mu \] - refractive index, refractive index fluctuation about mean

\[ \phi_E \] - spatial covariance of electric field

\[ \phi_{\Delta\Delta} \] - spatial covariance of amplitude fluctuations

\[ \phi_S \] - spatial covariance of phase fluctuation
CHAPTER 1

Introduction, Summary and Conclusions

1.1 Introduction

The invention of the bubble chamber by Glaser in 1952 signalled the beginning of a period of intense activity in elementary particle research which has continued at an increasing pace. The simultaneous development of accelerators capable of producing particle energies of the same order of magnitude as those found in cosmic rays has resulted in an enormous growth in our understanding of particle interactions.

The two main trends in particle research have been the increase in the particle energies available from accelerating machines and the growth in bubble chamber volumes. The most powerful accelerators now in operation are the 400 GeV machine at the Fermi laboratory and the centre of mass intersecting storage ring system at CERN using the 28 GeV proton synchrotron. A super proton synchrotron is under construction at CERN with a potential capability of 1000 GeV in a laboratory reference frame. Bubble chambers have expanded in size from the few cm. diameter Glaser model to the 4.5 metre diameter liquid hydrogen chamber recently completed at the National Acceleration Laboratory in Batavia. Gargamelle, a freon-propane liquid chamber completed recently at CERN is a cylindrical chamber 4.8 metres long and 1.9 metres in diameter. A liquid hydrogen chamber of similar shape and size has been built at Saclay for operation at the Serpukov laboratory.
The liquid volumes used in these chambers are of the order of 25,000 litres. The most recent chamber to be built at CERN contains 37,000 litres. A much larger chamber 25 metres in diameter and 7 metres deep containing 3.5 million litres of liquid hydrogen has been proposed by Thomas (1).

The present study has its origin in the possibility that there may be a limit to the useful size of a bubble chamber. In order to see why this may be, it is worth reiterating the basic principles of the bubble chamber technique.

A bubble chamber consists of a vessel containing a liquid whose pressure is held above the vapour pressure of the liquid by means of a piston. High energy particles from an accelerator are injected into the chamber and the pressure on the liquid is reduced by releasing the piston. The liquid is thus raised to a temperature above its normal boiling point and boiling occurs preferentially in regions where energy is deposited by the charged particles which lose energy progressively as they collide with liquid atoms. Thus the passage of a charged particle produces a track of small bubbles along its path. The collision of an incoming particle with a liquid atom can produce other new particles, some of which are charged and in turn leave bubble tracks. A high magnetic field in the chamber produces characteristic curvatures of the charged particle tracks. The bubble tracks are photographed by stereoscopic cameras and the pictures are scanned for events of particular interest. The tracks of such events are then scanned in detail by automatic track following devices which store the coordinates of track points in
computer storage. Conservation laws are then applied to the stored data to determine whether it fits an event hypothesis. The computer programs involve sophisticated techniques to cope with the various errors which occur.

A typical bubble chamber experiment may involve the more or less detailed analysis of up to 250,000 stereoscopic records, hence the need for the analysis to be as automated and as fast as possible.

The combination of high energy accelerator bubble chamber and data collection and analysis systems involves a wide range of techniques including vacuum technique, cryogenics, magnet technology, radiofrequency techniques, optics and computing, materials science and mechanical engineering. The continuously accelerating pace of particle physics research has stimulated advances in all of these areas.

The increasing size of bubble chambers has been justified hitherto for a number of reasons (2). On average a high energy proton passing through a chamber will collide with a target proton once in eight metres of track length. Processes considered rare about twenty years ago occurred once in 300 metres while ten years ago an event would be rare if it occurred in $10^6$ metres of track. Neutrino collisions are rarer still by a factor of $2 \times 10^8$. Thus an increase in chamber dimensions leads to an increase in the rate of occurrence of rare events.

In some experimental situations it is found that a better than linear increase in effectiveness is obtained with an increase in linear dimensions. For example in low energy neutrino
interactions it is the chamber volume which determines the effectiveness. Also experiments involving the production of neutrals are difficult to analyse unless these neutrals interact or decay within the chamber. Thus an event involving three neutrals has a probability of unambiguous analysis which is roughly proportional to the third power of the chamber's linear dimensions.

For all experiments, a larger chamber provides more information about the events. Many important advances have been made by studying event statistics.

Now the measurements required are the position of an event vertex, angles between event prongs, track curvature, range and bubble density. These give information about the energies, momenta, velocities and charges of the particles.

The accuracy with which these measurements can be made depends on a number of factors. These are the optical distortions introduced by the wide angle camera lenses and the chamber windows, the resolution of the recording medium, and the effects of thermal turbulences in the chamber liquid on the recorded positions of track bubbles. It is this lattermost aspect of bubble chamber technology which has stimulated this study because, as will be seen, the effect becomes more serious as the depth of liquid increases through which the tracks are observed. Thus the primary concern here is with the optical effects of thermal turbulence in bubble chamber liquids.

1.1.1 Thermal Turbulence

Thermal turbulence in bubble chamber liquids arises in two ways. One is because of a vertical negative temperature
gradient across the chamber liquid. The other is due to irreversible work done on the chamber liquid during the expansion recompression cycle which generates internal heat. The effects are twofold. Firstly a light ray scattered by a track bubble undergoes random angular deviations from its rectilinear path as it encounters the refractive index variations associated with turbulent eddies en route to the chamber window. Secondly the random movement of a turbulent eddy displaces any bubble within the eddy during the bubble growth time. As will be seen the first effect is much the more serious resulting in random fluctuations of the bubble position as recorded on film.

Steps may be taken to eliminate thermal turbulence by deliberate circulation of the liquid to remove the internal heat via the side walls of the chamber and by introducing a positive vertical temperature gradient. However the first modification causes track distortion since the liquid flow is in opposite directions at the walls and centre of the chamber. The second modification also causes gross track distortion due to the positive vertical refractive index gradient resulting from the positive temperature gradient.

Consequently it may be preferable to measure the optical distortions due to thermal turbulence, to see if they are tolerable. Very little theoretical or experimental work has been carried out on thermal turbulence in cryogenic liquids. These are the most commonly used liquids in bubble chambers since for unambiguous analysis of events, simple atoms like hydrogen, deuterium or helium are required which are in the liquid state only at low temperatures.
Some theories have been developed to predict thermal turbulence parameters, and experimental results in normal temperature fluids tend to support these predictions although some inconsistencies are found. Following a comparative analysis of previous theoretical and experimental work it was concluded that the semi-empirical approach adopted by Townsend, was the most appropriate for the present study.

Using Townsend's equations and the concept of dynamical similarity, a test rig was designed and constructed so as to produce easily measurable optical distortions in a turbulent cryogenic liquid, subjected to a vertical heat flux or negative temperature gradient. Measurements were made of the root mean square angular deviation of a Helium-Neon laser beam propagating through the liquid, as this deviation is the parameter of the most direct interest to the bubble chamber physicist. Correlation techniques involving multiple laser beams were used to determine the sizes of the turbulent eddies and their velocities. The results are presented as functions of heat flux and liquid depth.

The techniques employed in this project are many and varied including mechanical design, cryogenics, vacuum technique, optical instrumentation, photodetection, signal recording, processing and computer processing using both minicomputers and large machines. Subjects of central importance are covered in detail while those techniques which are well developed and understood will only be briefly described.

1.2 Summary

Chapter 2 is concerned with the prediction of turbulence parameters in cryogenic liquids and their optical effects.
Section 2.1 introduces the problem and 2.2 is a discussion of an idealised bubble chamber model. The Rayleigh dimensionless number characterising the turbulence, is introduced here. Temperature fluctuation as a function of heat flux is discussed in 2.3, the main theories which predict this parameter being outlined in 2.3.1, to 2.3.4. Comparison is made between the theories in 2.4.1 and 2.4.2 in order to choose the most appropriate for the present purpose. In section 2.5 the refractive index fluctuations are determined from the temperature fluctuations. Turbulent eddy sizes are predicted using the appropriate value of the Rayleigh number in section 2.6. An equation for the r.m.s. angular deviation of a light beam in the medium is given in 2.7. Other effects are discussed, namely random frequency modulation due to scanning (2.8.1), random frequency modulation due to phase changes (2.8.2) which also involves the eddy velocities, scintillation (2.8.3) beam break-up, phase variation and polarisation fluctuations (2.8.4). Finally in section 2.9 the results obtained in 2.4, 2.5, 2.6, 2.7 and 2.8.2 are used to derive estimates of temperature and refractive index fluctuations, eddy sizes and velocities in a model turbulence situation, in order to obtain design parameters for the test equipment.

Chapter 3 describes the construction (3.1) and operation (3.2) of a test rig containing liquid nitrogen. The design philosophy is explained in 3.1.1 and 3.1.2. The cell of the rig and it's surrounding chamber are described briefly in 3.1.3. Heater plate construction and performance are discussed in 3.1.4. In 3.1.5 details are given of the construction of the cooling reservoir above the cell.
The thermocouple temperature measuring system is described in 3.1.6. The chamber shell, window housings and insulation are outlined in 3.1.7, 3.1.8, and 3.1.9.

Section 3.2 described the routine operation of the rig, beginning with filling in 3.2.1. Various methods of pumping liquid nitrogen from large dewars are described in 3.2.2. Operational details after the rig has been prepared for experiments, are given in 3.2.3. Temperature measurements are described in 3.2.4. In conclusion 3.3 summarises what has been achieved in the design and construction of the rig.

Chapter 4 considers the correlation technique in more detail and describes possible methods of implementing it for measuring turbulent eddy sizes and velocities. Section 4.1 introduces the topic, distinguishing two main classes of technique. In 4.2 interferometric methods are discussed, particularly a recently developed technique for eddy size measurements. The theory of this method is described in 4.2.1 and equipment details given in 4.2.2. In 4.3 other purely optical techniques are considered.

A simple technique using the position sensitive photodetector with two continuous laser beams, is described in section 4.4. The theory is explained in 4.4.1 with experimental detail in 4.4.2 and 4.4.3. The technique did not prove successful in the present application, for reasons given in 4.4.4. In 4.4.5 aspects of the method requiring closer study are outlined, and its applicability in larger scale turbulence studies is discussed.
Section 4.5 deals with the crossed beam correlation technique already in common use. The introduction (4.5.1) describes the basis of the method. Theoretical aspects are outlined in 4.5.2 and in 4.5.3 consideration is given to the problem of sample size which determines the accuracy of measurement. In 4.5.4 some experimental details are given.

In conclusion (4.6) the limitations imposed by the geometry of the test rig, demand the use of the parallel beam method which is described in detail in Chapter 5.

Chapter 5, is a description of the opto-electronic probe system used in conjunction with the test rig. In the introduction (5.1) the rationale behind the choice of techniques is explained. In 5.2 a simple optical arrangement is described which models the passage of light from a bubble through a turbulent fluid. In 5.3 possible methods of measuring optical distortion are discussed, specifically neutral density wedge and photomultiplier tube (5.3.1), quadrant photodetector (5.3.2), photodetector array (5.3.3) and the method chosen, a position sensitive photodetector which is discussed in detail (5.3.4).

Section 5.4 deals with noise in the detector and amplifiers. Detector shot noise (5.4.1), Johnson noise (5.4.2) and flicker noise (5.4.3) are shown to be negligible in relation to amplifier noise (5.4.4).

In 5.4.5 a value is derived for the total noise equivalent beam displacement which defines the optical resolution of the detector system.
In section 5.5 the calibration curve is given. For the measurement of relevant turbulence parameters namely eddy size and velocity, correlation techniques are needed and these are described briefly in section 5.6. In 5.6.1 methods of producing multiple laser beams for correlation are described particularly the rotating reflecting polygon (5.6.1.1) torsional scanner (5.6.1.2) and rotating prism cube (5.6.1.3).

The aperture system used (5.6.2) has dimensions derived from estimates of eddy sizes.

Electronics for the correlation experiments are detailed in section 5.7. In 5.7.1 a general description is given and the circuits for demultiplexing of signals produced by the multiple laser beams are described in 5.7.2. In 5.7.3 sample and hold circuitry for the reconstituting of demultiplexed signals, is described. Then 5.7.4 describes the filter circuits needed to remove sampling pulse frequency harmonics from the signals.

Section 5.8 deals with the recording and processing of signals and the need for recording rather than real time analysis is explained. Various recording techniques are outlined namely ultra violet recording (5.8.1) digital magnetic tape recording (5.8.2) and F.M. magnetic tape recording (5.8.3). In 5.8.4 the correlator used to extract the required data from the tape recorded signals is described and in 5.8.5 the filter circuitry linking tape recorder outputs to correlator inputs is outlined.

Section 5.9 discusses the calibration of the complete system from laser to tape recorder playback.
In 5.10 the system performance is considered. In the conclusion (5.11) it is seen that the system sensitivity is adequate for its purpose. Reference is made to other possible turbulence diagnostic applications.

Chapter 6 describes the results of the study and the conclusions that may be drawn from it. Section 6.1 presents the results in order of importance beginning with optical distortion as a function of heat flux (6.1.1). In 6.1.2 optical distortion results as a function of liquid depth are given. In 6.1.3 signal bandwidth figures are presented while turbulence eddy sizes are presented as a function of both heat flux and liquid depth in 6.1.4. Section 6.1.5 deals with velocity measurements. Comparison is made with the expected values for all the results. Conclusions are drawn in section 6.1.6.

Section 6.2 is a general discussion. In section 6.2.1 recommendations for further experiments using a modified version of the test rig, are made. In section 6.2.2 the results of the present study are used to predict conditions which might be expected in a real bubble chamber. Steady temperature gradients are considered in section 6.2.2.1 and optical distortion figures are calculated in 6.2.2.1.1. Velocity effects are covered in section 6.2.2.2 and eddy sizes in 6.2.2.3. The dynamic behaviour of a bubble chamber under cycling conditions is briefly discussed in 6.2.3. Finally the implications of the results for the design and operation of a bubble chamber are discussed in 6.2.4.

1.3 Conclusions

Using results for normal temperatures, a test rig has been constructed providing a useful model of a bubble chamber.
This rig has been used to obtain satisfactory measurements at low temperatures. The results presented in this thesis validate the concept of the experiments, and are of immediate application in the design and operation of large volume cryogenic liquid bubble chambers. The concept of the experiment can be used with different test rig geometries to determine the shapes of future bubble chambers which will give minimum optical distortions.

Electronic signal processing recording and computer analysis techniques were used to obtain the results of what are essentially optical experiments. The success of this approach is encouraging as the computing facilities at most bubble chamber installations can be used with the techniques developed here to determine the optical distortion conditions prevailing during a bubble chamber experiment.

The results were obtained using a multiple laser beam probe developed for the purpose. It can measure optical distortions to an accuracy of 1 micrometre. Simultaneously data can be gathered, enabling estimates of turbulent eddy sizes and velocities to be made.

This type of probe system can be used to analyse other turbulence situations such as liquid flow in pipes and air turbulence particularly in jet engine exhausts and in aircraft wakes and the results of this study may also be usefully applied in the design of atmospheric communication links using lasers.
CHAPTER 2

Theory of Turbulent Convection and its Optical Effects

2.1 Introduction

The primary concern in the present work is the optical effects of thermal turbulence, particularly the angular fluctuation of a ray of light propagating in a turbulent fluid which is of importance in bubble chamber applications. The calculation of this angular fluctuation is tedious, requiring information about vertical heat fluxes, r.m.s. temperature fluctuations and refractive index fluctuations, as well as turbulence eddy diameters. The basic parameter is the temperature fluctuation as a function of height in the fluid, which is solely determined by the vertical heat flux \( \dot{q} \).

What follows is a discussion of how this parameter can be determined from the heat flux. Then follows a derivation of the r.m.s. angular deviation of a laser beam propagating in a slab of turbulent fluid.

There are also several other effects on laser propagation due to thermal turbulence. These are random frequency modulation, beam break-up, scintillation (intensity variation) and polarization fluctuations \( (A) \). Calculations of these effects on laser propagation in a turbulent layer of liquid are carried out to evaluate their importance here.

In the present work, orientated towards bubble chamber applications, no attempt was made to measure the effects of turbulence other than beam angular deviation, eddy sizes and
velocities as functions of height in and heat flux through a turbulent fluid layer, because all other effects are of little practical importance here. However, the present study could prove helpful in free space laser communication applications as it may be possible to model a long atmospheric path in the laboratory using a relatively small volume of cryogenic liquid.

2.2 Turbulent Convection and Heat Transfer

In order to predict turbulence parameters and hence optical distortions occurring in a bubble chamber fluid, it is useful to consider an idealised model of a bubble chamber. The simplest model consists of an infinite horizontal layer of fluid contained between two plane parallel boundaries and heated uniformly from below. Theoretical studies have been carried out on such a model (5,6,7) and these show that the governing parameter is the dimensionless Rayleigh number $R$ (5) where

$$R = \frac{\beta g D^3 \Delta T}{K \nu} \quad (2.1)$$

- $\beta$ = coefficient of thermal expansion of the fluid
- $g$ = acceleration due to gravity
- $D$ = depth of fluid layer
- $\Delta T$ = temperature difference between upper and lower boundaries
- $K$ = thermometric conductivity of the fluid
- $\nu$ = kinematic viscosity of the fluid.

At negative values of the Rayleigh number, no convection takes place and the fluid is stratified with $\frac{dT}{dZ}$ everywhere negative, where $Z$ is the height in the fluid layer.
It has also been theoretically (5) and experimentally (8,9) demonstrated that for \( R \) less than about 2,000 heat transport is by molecular conduction alone and no convection takes place. When \( R \) exceeds this value, slow convective motion begins. One may now define \( K_{\text{eff}} \), the effective thermal conductivity, as that due to both convection and conduction (3). The experimental value of \( K_{\text{eff}}/K \) increases with \( R \) and the dependence of temperature \( T \) on \( Z \) becomes nonlinear when \( R \) exceeds 2,000. This is because the temperature gradients in the boundary layers must suffice to transport the vertical heat flux by convection alone as convective motion cannot penetrate to the boundary due to the action of viscous shearing force (3).

Studies have been carried out of turbulence phenomena for \( 10^{4} \) values of \( R \) up to \( 10^{10} \) (9) including heat transport and velocity fluctuation measurements. Temperature profiles have also been obtained for values of \( R \) between \( 10^{4} \) and \( 10^{6} \) (10,11). As \( R \) increases it is clear that the profile tends towards the idealised form shown in Fig. 21, and for \( R > 10^{5} \) almost all the temperature difference across the fluid layer occurs in the two thin boundary layers. At high values of \( R ( > 10^{5} ) \) which are relevant to the present work, the temperature gradient is almost zero away from the boundary layers and heat transport becomes independent of \( D \). The gradient is always just greater than zero because of molecular conduction. It is also found that there is close agreement between turbulence parameters for doubly bounded layers at high \( R \) values and those for a layer.
The ratio of slopes is $\frac{K_{\text{eff}}}{K}$. 

FIG. 23 IDEALISED MEAN TEMPERATURE DISTRIBUTION IN HORIZONTAL FLUID LAYERS AT HIGH RAYLEIGH NUMBERS (from ref. 3)
of fluid heated from below but not bounded above (11) as would be expected from theory where the latter case is one of infinite Rayleigh number.

Most experiments have used large horizontal to vertical aspect ratios in order to approximate the infinite horizontal model as closely as possible but it has been found that heat transfer is not affected appreciably by aspect ratios as low as 1:2 (9,11) although slow horizontal fluid motions may arise.

2.3 Temperature Fluctuations

In general the main requirement is to predict the values of $\theta^2$ where $\theta$ is a small fluctuation in temperature about the mean value at some height $Z$ in the chamber and the horizontal bar denotes averaging over time. Using this parameter one can then obtain values for the corresponding r.m.s. refractive index fluctuations and hence predict the optical distortions which will arise. Several theories which predict values of the temperature fluctuation have been developed. An elegant theory due to Malkus (12) predicts values of the temperature fluctuation averaged over the whole volume of the liquid layer. Theories of turbulent thermal convection have also been developed by Kraichnan (13), Priestley (14) and Howard (15). Of these only Priestley's theory explicitly determines the value of the temperature fluctuation as a function of height in the layer. Townsend (16) has carried out measurements of temperature fluctuations in air as a function of height above a heated horizontal plate and gives an empirical equation for the results.
2.3.1 Malkus Theory (12)

This theory applies to turbulent convection between parallel planes and is thus the most appropriate for any bubble chamber application. All other theories deal only with singly bounded layers. It is assumed that the flow adjusts itself to transfer the maximum amount of heat compatible with the boundary conditions,

a) that the heat flux is everywhere down the gradient of mean temperature,

and b) that the temperature and velocity fluctuations can be represented by finite Fourier series.

The main result of interest in the present context is that the mean square temperature fluctuation $\theta^2$ averaged over the whole layer is given by

$$\theta^2 = \frac{1}{3\pi^2} \ln \left( \frac{QD}{K\delta T} \right)$$

where $Q = \frac{H\delta}{C_p\rho}$

$H$ = vertical heat flux

$\rho$ = density

$C_p$ = specific heat at constant pressure

Unfortunately this is not what is required and $\theta^2$ cannot be obtained directly or indirectly from this equation. The Malkus equation can however be used to check experimental results for the function $\theta^2(z)$ by integration over the fluid depth.

2.3.2 Priestley's Theory (14)

This theory is based on the hypothesis that there is a boundary layer whose structure is solely determined by the heat flux.
The theory was developed, successfully, to account for the meteorological problem of convection from earth or water surfaces and is only valid above a certain height $h_0$ from the boundary. The predicted equation for $\bar{\theta}^2(z)$ is

$$\bar{\theta}^2 = c_1 Q_0^2 g^{-\frac{2}{3}} z^{-\frac{2}{3}}$$

(2.2)

where $c_1$ is an undetermined constant, and $g$ is the gravitational acceleration. Also the temperature $T$ at heights $> h_0$ is given by

$$\beta(T - T_0) = c_2 Q_0^2 g^{-\frac{1}{3}} z^{-\frac{1}{3}}$$

where $T_0 = \text{temperature at height } h_0$, and $c_2$ is a further constant.

The latter equation is borne out by experiments in heat transport in the lower atmosphere (13) but the equation for $\bar{\theta}^2$ is not supported by measurements in air over a heated horizontal plate (14).

2.3.3 Theories of Kraichnan & Howard

Kraichnan's theory (13) is of the mixing length type like Priestley's and predicts that the r.m.s. temperature fluctuation $\theta^2$ is given by $\theta^2 \approx Z^{-\frac{1}{3}}$

which is the dependence obtained by Priestley. The dependence of $\theta^2$ on $Z$ cannot be determined exactly. Howard's (15) work extends Malkus' theory. Comparison is made with the experimental results of Townsend (16) and reasonable agreement is obtained, although there are some important reservations (13). The main conclusion is that some verification for the maximum heat transport hypothesis of Malkus is obtained.

2.3.4 Townsend's Semi-Empirical Theory

Townsend (16) has measured the temperature fluctuations in air over a heated horizontal plate. The signal bandwidth used was about 300 Hz.
Elder (17) has obtained measurements in a liquid layer bounded by vertical walls, while Tritton (18) used an arrangement similar to Townsend's with an inclined plate. All the results are consistent. Townsend uses non-dimensional length and temperature scales derived from the heat flux. He obtains an equation which describes the results for $\theta^2$ over a wide range of height. This equation has the virtue that it can be completely determined for any other fluid since all its constants are known unlike the Priestley equation.

Thomas and Townsend (11) using a doubly bounded layer have also obtained a $\theta^2$ dependance on $Z$ measured from each boundary, which is similar to Townsend's equation.

Thus it appears that the Townsend equation adequately describes the variation of $\theta^2$ with $Z$ near room temperature and could possibly be used to predict $\theta^2$ values for cryogenic liquids.

### 2.4.1. Comparison of Priestley and Townsend Theories

In Priestley's theory (14) the value of $\theta^2$ is given by

$$\beta^2 \theta^2 = C_1 Q^\frac{4}{3} g^\frac{-2}{3} Z^\frac{-2}{3}$$  \hspace{1cm} (2.2)$$

while the empirical equation obtained by Townsend (16) is

$$\beta^2 \theta^2 = 2.56 \theta_0^2 (Z/Z_0)^{-1.2}$$  \hspace{1cm} (2.3)

where $\theta_0 = (Q^3/Kg)^\frac{1}{4}$, (2.4) and $Z_0 = (K^3/Qg)^\frac{1}{4}$ (2.5)

are scales of temperature and length and

$$Q = \frac{H \theta}{C_p} = \frac{K \beta \Delta T}{C_p \rho D} (0.08 R^3)^\frac{1}{2}$$  \hspace{1cm} (3)$$

Values of $\theta^2 (Z)$ can be obtained for a liquid nitrogen layer 10 cm. deep using the two equations (2.2 and 2.3). A value of $Z = 5$ cm. is chosen so as to be well away from the boundaries.
Agreement is not likely considering the differences between the two equations, although the dependence of $\theta^2$ on $Q$ ($\theta^2 = Q^{\frac{4}{3}}$ in the Priestley equation and $\theta^2 = Q^{1.2}$ in the Townsend equation) is approximately the same in each case.

The table below gives the values of $\theta^2$ for the two equations at various values of $\Delta T$:

<table>
<thead>
<tr>
<th>$\Delta T$ (K)</th>
<th>0.1</th>
<th>0.5</th>
<th>1.0</th>
<th>2.0</th>
<th>3.0</th>
<th>4.0</th>
<th>5.0</th>
<th>7.0</th>
<th>10.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta^2_T$ (K)</td>
<td>0.002</td>
<td>0.006</td>
<td>0.010</td>
<td>0.017</td>
<td>0.023</td>
<td>0.029</td>
<td>0.035</td>
<td>0.046</td>
<td>0.061</td>
</tr>
<tr>
<td>$\theta^2_P$ (K)$^1_{C_1}$</td>
<td>0.0001</td>
<td>0.0007</td>
<td>0.0009</td>
<td>0.0016</td>
<td>0.0023</td>
<td>0.0030</td>
<td>0.0036</td>
<td>0.0049</td>
<td>0.0067</td>
</tr>
</tbody>
</table>

The values of $C_1$ for liquid nitrogen can be found assuming the results should be in agreement. These values are as shown in the table below:

<table>
<thead>
<tr>
<th>$\Delta T$ (K)</th>
<th>0.1</th>
<th>0.5</th>
<th>1.0</th>
<th>2.0</th>
<th>3.0</th>
<th>4.0</th>
<th>5.0</th>
<th>7.0</th>
<th>10.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_1$</td>
<td>180</td>
<td>140</td>
<td>130</td>
<td>110</td>
<td>100</td>
<td>99</td>
<td>94</td>
<td>90</td>
<td>84</td>
</tr>
</tbody>
</table>

The mean value of $C_1$ is thus $114 \pm 25$ for $\Delta T < 10K$.

Townsend's results for air turbulence over a heated horizontal plate are not in agreement with the Priestley theory (16).

There is also some doubt about the applicability of the theory to cases where there is a circulatory motion (2), such as arises in fluid layers of finite extent.

However, it is clear from the above results that there is some agreement between Priestley's theory and the Townsend empirical equation for liquid nitrogen, provided that $C_1(\Delta T)$ may be chosen to make the fit.
2.4.2 Comparison between Malkus Theory and Townsend's

Empirical Equation

In order to determine if there is any agreement between Townsend's results and the Malkus theory it is necessary to integrate equation (2.3),

\[ \beta \theta^2 = 1.6 \theta_0 (Z/Z_0)^{0.6} \]

over all values of Z to obtain values for \( \theta^2 \) for comparison with equation (2.1),

\[ \theta^2 = \left[ \frac{2}{3\pi^2} \frac{\Delta T \kappa \theta}{QD} \ln \left( \frac{QD}{K \beta \Delta T} \right) \right]^{1/4} \]

From Townsend (16)

\[ \theta^2 = \left( \frac{1.6}{\theta_0} (Z/Z_0)^{0.6} \right)^2 \]

where \( \theta_0 = [Q^3/K]^{1/4} \) and \( Z_0 = [K^3/Q]^{1/4} \) are non-dimensional scales of temperature and length.

It is also assumed that

\[ \theta^2 \frac{1}{T} = \left[ \frac{2}{T_0} \right] \int_0^{D/2} \left[ \frac{1.6 \theta_0}{\beta} \left( \frac{Z}{Z_0} \right)^{0.6} \right]^2 dZ \]

i.e. that the effect of the upper boundary is included by doubling the integrated value over the range 0 to \( D/2 \). Townsend's equation has been tested only for values of \( Z/Z_0 \) between 0 and 100 but for the present this restriction is ignored.

Taking \( D = 10 \text{ cm} \), with appropriate values of the constants for liquid nitrogen+(19), and assuming that \( \theta^2 \) at \( Z = 0 \) is zero, one obtains

\[ \theta^2 \frac{1}{T} = 1.724 \times 10^{-2} \Delta T^{0.6} \]

It is assumed that the form of the ratio of \( K_{eff}/K \) will be the same in air and in liquid nitrogen; this is true if the Prandtl numbers are the same (2), where the Prandtl number \( \nu/K \) is 0.77 for air and 2.31 for liquid nitrogen.

(+) See Chapter 3 Introduction
The dependance of $K_{\text{eff}}/K$ on $\nu/K$ has been found to be weak for some liquids (3) so the discrepancy here is not likely to be serious.

Now \[ Q = \frac{H\theta}{C_p\rho} = \frac{K\Delta T}{C_p\rho D} \left(0.08R^3\right)^{\frac{1}{3}} \] \hspace{1cm} (2.6)

so the Malkus equation (11) gives

\[ \frac{\theta^2}{\Delta T} = \frac{2\Delta T^2}{3\pi^2} \left[0.08R^3\right]^{\frac{1}{3}} \ln\left[0.08R^3\right] \]

with $D = 10$ cm.

\[ \frac{\theta^2}{\Delta T} = 2 \cdot 46.10^{-2} \Delta T^5 \sqrt{\ln\left[1 \cdot 114 \cdot 10^2 \Delta T^3\right]} \]

By inserting various values of $\Delta T$ one can compare the three theories. The range of values of $\Delta T$ is that likely to occur in an actual experiment to measure turbulence parameters in liquid nitrogen.

<table>
<thead>
<tr>
<th>$\Delta T$ K</th>
<th>0.1</th>
<th>0.5</th>
<th>1.0</th>
<th>2.0</th>
<th>3.0</th>
<th>4.0</th>
<th>5.0</th>
<th>7.0</th>
<th>10.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{\theta^2}{\Delta T}$</td>
<td>0.003</td>
<td>0.010</td>
<td>0.017</td>
<td>0.030</td>
<td>0.042</td>
<td>0.052</td>
<td>0.063</td>
<td>0.082</td>
<td>0.110</td>
</tr>
<tr>
<td>$\frac{\theta^2}{\Delta T}$</td>
<td>0.007</td>
<td>0.020</td>
<td>0.053</td>
<td>0.097</td>
<td>0.140</td>
<td>0.180</td>
<td>0.220</td>
<td>0.290</td>
<td>0.390</td>
</tr>
<tr>
<td>$\frac{\theta^2}{\Delta T}$</td>
<td>0.002</td>
<td>0.009</td>
<td>0.017</td>
<td>0.024</td>
<td>0.050</td>
<td>0.056</td>
<td>0.067</td>
<td>0.089</td>
<td>0.130</td>
</tr>
</tbody>
</table>

The values obtained agree within a factor 3. The discrepancy may be due to the assumption for the Malkus equation that $K_{\text{eff}}/K$ does not depend on the Prandtl number $\nu/K$. The other assumption is that Townsend's equation holds for all possible values of $Z/Z_0$. The results are heavily dependant on the form of the $\frac{\theta^2}{\Delta T}$ dependance near $Z = 0$ which is not accurately known except that $\frac{\theta^2}{Z=0} = 0$.

- 36 -
The third row in the table gives the values of $\tilde{\theta}^{2 \frac{1}{2}}$ using the Priestley theory and integrating over the whole fluid depth as before. It is necessary to assume a value of $C_1$ and this is obtained as described in 2.4. The agreement which results for the values of $\tilde{\theta}^{2 \frac{1}{2}}$ is thus consistent with the assumed agreement for $\tilde{\theta}^{2 \frac{1}{2}}$.

To summarise, fair agreement is obtained for $\tilde{\theta}^{2 \frac{1}{2}}$ between the integrated Townsend equation and Malkus' theory. There is also a fair degree of consistency between Townsend's equation and Priestley's equation although the constant in the latter is undetermined.

The consistency of results leaves us with the clear choice of the Townsend equation to establish design parameters for a test rig for use with cryogenic liquids to measure optical distortions produced by thermal turbulence.

2.5 R.M.S. Refractive Index Fluctuations

In order to determine the optical effects of turbulence it is necessary to use an expression relating temperature and refractive index fluctuations, namely the Lorenz-Lorentz (20) equation

$$\frac{\mu^2 - 1}{\mu^2 + 2} = C_\rho(T).$$

where $\mu$ is the refractive index, $\rho$ the fluid density and $C$ is a constant (0.162 for liquid nitrogen).

Now $\rho(T) = 1.175 - 0.0048 T$ over the temperature range 77°K - 83°K for liquid nitrogen (19), so rearranging and differentiating...
\[
\mu^2 = \frac{2 + 2\zeta \tau}{1 - \zeta \tau}
\]

\[
2\mu \frac{du}{dT} = \left\{ \frac{3C}{1 - \zeta \tau} \right\} \frac{dp}{dT}
\]

\[
\left[ \frac{du}{dT} \right]^2 = \frac{9C^2}{4(1 + 2\zeta \tau)(1 - \zeta \tau)^3} \left[ \frac{dp}{dT} \right]^2
\]

whence substituting \( \mu^2 \) for \( du \) and \( \rho^2 \) for \( dT \)

\[
\mu^2 = 1.27 \times 10^{-3} \rho^2
\]

over the temperature range 77º K to 83º K.

### 2.6 Turbulent Eddy Sizes

It is important to know the length scales of the turbulence structure, as the total angular deviation imposed on a ray of light depends both on the magnitude of the refractive index fluctuations encountered at the interfaces of eddies of different temperatures and the total number of such refractive index fluctuations encountered over a given optical path.

The Rayleigh number \( R \) for a fluid, increases with the vertical temperature difference. When \( R \) rises above 2,000 the fluid layer becomes unstable and laminar convection is set up in the form of prismatic columns of hexagonal cross-section, (3) warm fluid rising up the interior of the prism and cold fluid pouring down the walls. At higher \( R \) values (9) of \( (1-3) \times 10^4 \) this regular structure becomes unstable and the length scale of turbulence decreases to about half the plate separation. Further less well defined transitions occur at higher Rayleigh numbers but the main conclusion is that the length scale always decreases with increasing Rayleigh number.

Thomas (3) considers the situation as analogous to fluid flow in pipes.
As the velocity of flow increases, so does the Reynolds number $Re$ until a critical value is reached when the flow becomes unstable and turbulent eddies are formed and, with further increases in $Re$, the eddies decrease in size.

For each eddy in the flow there is an inner Reynolds number $Re_i$ where $Re_i = \frac{ua}{\nu}$, $\nu$ being the fluid viscosity and $a$ the length scale of an eddy having velocity fluctuation $u$. The value of $Re_i$ corresponding to the smallest eddy size is about $10^3$, larger eddies continuously breaking up into progressively smaller ones till the smallest size predominates.

In the case of thermal turbulence energy is again transferred from larger to smaller eddies until the smallest ones predominate. The length scale $a$ of the latter is given by (2.3)

$$a = \left[ \frac{\nu Re_c}{\sqrt{\frac{\partial T^2}{\partial t}}} \right]^{\frac{1}{2}}$$

(2.8)

$Re_c$ is the critical Rayleigh number for a free fluid interface and $\sqrt{\frac{\partial T^2}{\partial t}}$ is the r.m.s. temperature fluctuation about the mean. $Re_c$ has a value of about 650 (21). Assuming the eddies to be spherical $a$ will be their diameter.

We now know the length scales of the regions of changing refractive index as well as the magnitude of the refractive index fluctuations themselves, given the values of the temperature fluctuations.

2.7 R.M.S. Angular Deviation

Using the information already obtained about eddy sizes and refractive index fluctuations it is possible to determine
the r.m.s. angular deviation of a laser beam propagating in a turbulent fluid. The problem can be considered from the ray optics viewpoint if the wavelength of light used is much smaller than the size of a turbulent eddy (4.22). It will be seen subsequently that for values of $\Delta T$ up to 6K in liquid nitrogen $a$ has a minimum value of about a millimetre so this condition is fulfilled for liquid nitrogen. It is also necessary that the first Fresnel zone diameter $\sqrt{(L\lambda)}$, where $L$ is the path length, be less than $a$. For $L = 90$ cm. and $\lambda = 639.8$ nm $\sqrt{(L\lambda)} = 0.8$ mm so this condition is also fulfilled. Finally the ray transit time must be small compared with the eddy lifetime. The equation for angular deviation is

$$e^2 = 4\mu^2 \sigma \sqrt{\pi/a}$$

(see appendix A)

This equation gives the mean square angular deviation $e^2$ of a laser beam propagating over a path length $a$ in a turbulent medium whose mean square refractive index fluctuations are $\mu^2$ and whose turbulent eddy length is $a$.

Substituting for $\mu^2$ and $a$ from equations 2.7 and 2.8 respectively

$$e^2 = 4(1.27.10^{-3})^2 \sigma \sqrt{\pi/a}$$

(2.9)

Hence $e^2$ for a laser beam propagating over a path $a$ in a fluid at a height $Z$ can be found by inserting the appropriate value of $\sigma$ determined from the Townsend equation (2.3). For a value of $Z$ of 5 cm. and $\Delta T$ of 1K one obtains a value of $\frac{e^2}{\sigma}$ of 0.3 milliradians. The parameter of direct interest in bubble chamber applications is the optical distortion which is defined as the r.m.s. deviation $d^2$ of the light ray in a direction normal to the propagation direction.
This is given by

\[ \frac{d\sigma^2}{\sigma^2} = 3.38 \times 10^{-3} \frac{\sigma^2 \theta^2 / a}{a^2} \]  \hspace{1cm} (2.10)

The present study concentrates on the measurement of this variable as a function of height in a fluid layer and temperature gradient across the fluid layer.

Other optical effects will now be considered and evaluated to show that they are unimportant in the present context.

2.8 Other Effects

Apart from the angular deviation of the laser beam, also known as beam scanning, several other optical effects of turbulence are known to occur. These have been dealt with by Hodara (4) and, following his derivations, some figures are calculated for a one metre path in liquid nitrogen.

2.8.1 Random Frequency Modulation (due to scanning)

The first effect is a random frequency modulation due to beam scanning. The expression for the r.m.s. frequency shift \( \Delta f^2 \) has been derived by Hodara (4) as

\[ \Delta f^2 \approx \frac{L^2}{4 \lambda^2} \frac{e^4}{t_c^2} \]

Where \( L \) is the path length, \( \lambda \) the laser wavelength, \( e \) the instantaneous angular deviation of the beam about the mean and \( t_c \) a correlation time which equals the eddy size (or correlation length) divided by the eddy velocity. Since the statistics of angular deviation are Gaussian however, \( e^4 \) will be zero and no parasitic frequency modulation will occur.

2.8.2 Random Frequency Modulation (due to phase changes)

A second random frequency modulation effect occurs because
of the random phase changes causing degradation of time coherence. The expression for this is (4)

\[ \Delta f_r^{2} = \sqrt{\frac{L}{\lambda^2}} \frac{\mu^2}{t_c^2} \]

\( \Delta f_r \) being the random frequency shift, \( a \) the eddy size, \( \mu \) the refractive index fluctuation about the mean, and \( t_c \) again being the correlation time. To find \( \mu^2 \) it is necessary to turn to Townsend's equation (eq. 2.3) from which \( \theta^{2} \) and hence \( \mu^2 \) (eq. 2.7) can be estimated. A value is obtained of \( \theta^{2} \) of 5.0.10^{-3}K midway between the boundaries. Hence (eq. 2.7) one obtains a value of \( \mu^2 \) of 3.5.10^{-11} and (eq. 2.8) eddy sizes a of 3mm.

Townsend (16) assumes that the temperature gradient and r.m.s. velocity fluctuations are statistically independent and obtains

\[ \langle \delta \theta \delta f \rangle^2 = q^2 \langle \delta \theta \delta \theta \rangle^2 \]

whence for \( Z/Z_o = 100 \)

\[ q^{2} = 7u_o \]

with \( u_o = (QgK)^{1/4} \) (2.12)

a velocity scale.

Hence one obtains \( q^{2} t_c = a \)
or \( t_c = a/q^{2} \)

Thus we find the velocity fluctuations \( q^{2} \) are about 6mm/sec.

Hence

\[ \Delta f_r^{2} = \sqrt{\frac{L}{\lambda^2}} \frac{q^2}{a^2} \mu^2 \]

and a value of around 1 Hz is obtained for the parasitic frequency modulation on a laser beam of wavelength 632.8 nm. propagating over a 1.0 metre path of the liquid. At a laser frequency of more than 10^{14} Hertz, this effect is insignificant.
2.8.3 Scintillation

Random changes in the cross sectional area of the beam impose a lower limit on the percentage modulation depth required to obtain a specified signal to noise ratio in amplitude modulation applications. For relatively short path lengths specified by the ratio being much less than $10^4$ Hodara (4) obtains the following for the modulation depth $m$.

$$2 \sqrt{\frac{P_S}{P_n}} \sqrt{\left( \frac{L}{a} \right)^3 \mu^2} \leq m \leq 1$$

in order to obtain a signal to noise power ratio $P_S/P_n$

Using the figures given above, the modulation depth would need to be almost 100 per cent in order to achieve a signal to noise power ratio of 10. This is a rather serious limitation which becomes more stringent with increasing $\Delta T$.

The implication is that amplitude modulation would be very impractical as a means of obtaining information. Thus for example one might think of using a neutral density filter and photomultiplier where the filter density varies uniformly across its width, as a position sensitive detection system. This would not be practical in view of the severe limitations imposed by the above result.

2.8.4 Beam Break-Up, Phase variation and Polarization Fluctuation

The first of these effects, due to degradation of spatial coherence across the detector aperture, imposes an upper limit on the detector diameter $D$ for effective optical heterodyning (4)

$$D \ll \frac{\lambda}{2} \sqrt{\frac{a/L}{2\mu^2}}$$

One finds for the figures given in 2.8.2 that an upper limit for $D$ is about 3 mm. Random angular fluctuations of the beam could cause it to miss a detector of this size, when optical heterodyning is being used.
Phase variations across the detector aperture limit the maximum amplitude modulation rate that can be used. The time delay between the arrival at the aperture edges of parts of the same original wavefront must be less than the period of the maximum modulation frequency $f_M$. Hodara (4) obtains

$$f_M = \frac{10^{-1}C}{2La \mu^2}$$

giving a value of $6.0 \times 10^{13}$ Hz which imposes no practical limitation.

Random fluctuations occur in the polarization angle which is defined as the angle whose tangent is the ratio of the laser wave amplitudes in orthogonal directions. These angular fluctuations are found to be of the same magnitude as the angular deviations of the beam itself ($\theta$) if the eddies are assumed to be spherical.

In summary it is clear that the scintillation effect is the only serious one, prohibiting as it does the use of amplitude modulation detection systems.

2.9 Predictions of Turbulence Parameters and Optical Effects

The foregoing theory and calculations enable predictions to be made from which a suitable cell can be designed for experimental measurements. Thus for a layer of liquid nitrogen 0.1 metre deep, and 1 metre long with a stable temperature difference between top and bottom surfaces of 1.0 K the Rayleigh number $R$ will be $R = 3.6 \times 10^9$ where $R$ greater than $10^4$ corresponds to fully developed turbulence. The maximum value of the r.m.s. temperature fluctuations (i.e. near the boundary surfaces) will be $2.5 \times 10^{-1}$ K calculated from Townsend's equation (2.3) at $Z/Z_0 = 6$.

The length scale of the turbulence near the middle of the
layer will be 3.4 mm producing optical distortion (eq. 2.10) $\bar{d}_{\text{rms}}^{\frac{1}{2}}$ of 0.3 mm where optical distortion is defined as the r.m.s. transverse deflection of the laser beam from its rectilinear path. The velocity fluctuations calculated from equation 2.11 are of order 6.0 mm/sec. For higher values of $\Delta T$ the optical distortion increases steadily to 3 mm at $\Delta T = 6$ K. Optical distortions of these magnitudes are easily measured and an order of magnitude either way can be accommodated. Accordingly it was decided to use the above dimensions in the design of a cell to measure turbulence parameters. This will be discussed in the next chapter.
3.1 Design and Construction

3.1.1 Introduction

The geometry of most modern bubble chambers does not correspond very well to the ideal Rayleigh model (3) which consists of an infinite layer of fluid contained between plane horizontal boundaries. Many chambers also have undesirable arrangements of the observation windows for the type of experiments envisaged, unless mirrors are used. Such optical modifications in the interior of a chamber can only be carried out when it is emptied for general maintenance and inspection. There is also some difficulty in carrying out measurements while a chamber is in use for particle experiments although Leutz et. al. (24) have succeeded in carrying out measurements in a parasitic manner during normal chamber operation.

The estimated cost of a purpose built chamber with appropriately positioned windows to be used exclusively for the present experiments was not justified. Such a chamber would probably have to be at least as large (1 metre long) as many chambers presently in operation. In addition the design and operation of a liquid hydrogen chamber requires considerable attention to safety procedures which the departmental laboratories could not provide at short notice.
Accordingly it was decided to design and build a test cell specially for this work. This approach enabled the Rayleigh model to be realised more exactly. Instead of using liquid hydrogen or helium, liquid nitrogen was used because it is more readily available, cheaper, and does not require the same attention to safety as many bubble chamber fluids. In addition, if the results obtained proved to agree with theory, it should be possible to scale the results using dynamical similarity, to obtain quantitative values for the optical distortions and fluctuating velocities to be expected in many bubble chamber fluids.

3.1.2 Design Philosophy

The design was based on sample calculations of optical distortions and fluctuating velocities in liquid nitrogen using the Townsend equation. It was found (Ch. 2) that if a laser beam is propagated through a 1 metre path in a liquid nitrogen layer 10 cm. deep with a vertical temperature difference of 1° K, an optical distortion of 0.3 mm can be expected. A distortion of this order is easily measured and even if it proves through experiment to be too high by a factor of 10, no great difficulty arises as the multiple passage of the beam can be employed to increase the distortion effect.

3.1.3 Description of Test Cell and Chamber

The purpose of the test cell and chamber is to isolate a horizontal layer of liquid nitrogen of known dimensions. The liquid is placed in a state of turbulent convection by heating it from below and cooling from above.
A laser beam is passed through a window at one end of the cell at a measured height above the base. It then propagates through the fluid, undergoing progressive optical distortion, to emerge through an exit window at the opposite end and is picked up on a displacement measuring detector.

The construction of the cell and chamber is shown schematically in fig. 3.1, and details are seen in plates 1 and 2. The cell containing the fluid layer of interest is made from perspex, the vertical side walls being 3.0 mm. thick, and 11.7 cm. high, the end walls 10.0 mm. thick and 11.7 cm. high. The base is 10 mm. thick. A 10.0 cm. diameter circular hole is cut in each end wall of the cell, concentric with the main axis of the cell. The cell width of 40 cm thus gives a horizontal to vertical aspect ratio of 4:1. It is found (9,11) for low aspect ratios (e.g. 1:2 and 1:6), that while general circulatory motion (i.e. having a component in the horizontal direction) is set up there is no noticeable increase in heat transfer. Hence it can be assumed that the theory holds for layers of fluid of finite dimensions.

3.1.4 Heater Plate

The heater plate (90 cm x 40 cm x 1.7 cm) is laid on the base of the cell. It consists of two copper sheets each 2.0 mm thick explosively welded to the upper and lower surfaces of a stainless steel plate 1.3 cm thick. Soldered to the lower copper surfaces is a 16 metre length of Pyrotenax heating cable laid as a continuous winding of 14 longitudinal strips. The high thermal conductivity of the lower copper plate allows
FIG. 3.1. LIQUID NITROGEN TURBULENCE TEST RIG
diffusion from local hot spots in the horizontal plane. This process is assisted by the relatively low conductivity of the stainless steel section so that hot spots diffuse sideways rather than upwards. The upper surface copper plate enhances the uniformity of normal heat flux still further.

The low thermal conductivities of the liquid nitrogen under the heater plate and of the perspex base of the cell, ensure that very little of the heat produced will travel downwards (see 6.1.1).

It is assumed that the heat flux from each section of the cable is uniform across any cylindrical surface coaxial with the cable itself. Assuming there is a constant heat flux $S$ per unit length of heating cable in direction normal to the cable, (which is true if the temperature and cross section of the cable are uniform) then it is possible to calculate the variation in heat flux normal to the upper surface of the plate.

The maximum difference in heat flux normal to the heater plate within the diameter of the windows occurs between two points one in the centre vertically above a length of heater cable, the other above a point midway between two lengths of cable, corresponding to the window edge (see 3.1.8). The vertical heat flux from each of these points consists of contributions from each of the lengths of cable.

Referring to fig. 3.2 the contribution $h_i$ to the total heat flux $H$ normally from a point $A$, from any cable is given by

$$h_i = S \cos \theta_i$$

assuming a Lambertian dependance, where $\theta_i$ is the angle between
the direction of heat flow from the i-th cable and the normal to the plate.

The cable lengths are 3 cm apart and there are 14 of them. Heat flux $H$ from a point above the centre of the plate is thus given by

$$H = 2S \frac{6}{\sum_{n=0}^{1} \frac{1.7}{\sqrt{1.7^2 + (3n+1.5)^2}}} = 3.59S$$

since this point receives contributions from 7 pairs of cable length located symmetrically with respect to it.

At a point almost under the edge of a window the flux $H'$ will be given by

$$H' = S + 2S \frac{5}{\sum_{n=1}^{5} \frac{1.7}{\sqrt{(3n)^2 + 1.7^2}}} + S \frac{8}{\sum_{n=6}^{8} \frac{1.7}{\sqrt{(3n)^2 + 1.7^2}}} = 3.65S$$

since this point receives symmetrical contributions from 5 pairs of cable length as well as contributions from three unpaired lengths.

The percentage variation in heat flux is given by

$$\frac{H - H'}{H} \times 100 = 1.7\%$$

Thus the maximum heat flux variation from points at the centre of the plate and under the window extremities is less than 2%.

3.1.5 Cooling Reservoir

The cooling reservoir is positioned above the perspex cell and is made from aluminium alloy. It is 5 cm deep and is formed from two aluminium sheets again (90 x 40)cm$^2$ in area welded to extruded aluminium shaped sections forming end and side walls.

The base is thin (2.0 mm) so that the heat flux through the liquid in the cell is efficiently conducted away by the liquid.
in the reservoir. The tank is internally supported by extruded \( J \) sections welded to the sides top and bottom, see fig. 3a. During experiments, the pressure in the main cell can rise substantially, usually to a permitted maximum of \( 10^6 \text{ Kgf} \). Hence the need for the internal supports. The reservoir must also be reasonably well evacuated during the filling of the cell so that its inlet and outlet lines cannot seize up as the cell is filled. For this reason the welding of the reservoir parts has to be thoroughly free from gaps or weak points. Circular holes 2.5 cm in diameter were bored at intervals in each internal section to allow free passage of liquid nitrogen into and out of the reservoir via filling and vent lines.

3.1.6 Temperature Measurement

For measurement of \( \overline{\Delta T} \), the mean temperature difference across the cell, six copper-constantan thermocouples are fixed to the underside of the cooling tank and six more vertically below these, on the heater plate. The method of fixing was to solder the thermocouple junction on to a thin flat copper plate about 1 cm square which was then fixed to the surface whose temperature was to be measured by means of a thin layer of epoxy resin adhesive, such as Bostik. Such resins tend to become brittle and flaky after being cycled through the temperature range 300\(^\circ\) K to 77\(^\circ\) K a number of times and it was frequently necessary to renew the adhesive layer after an experiment. It was found helpful to smear a thin layer of Devcon EK-40 adhesive around the edges of the thin copper plates.
FIG 33: SECTIONS THROUGH RESERVOIR

(a) end  4cm.

(b) side

Al alloy sheet

40 cm

Lugs

Alloy section

Sheet

90 cm welding
This adhesive does not become brittle like other adhesives which were tried, though it does develop a rubbery texture and can easily peel away at low temperatures. Care in making thin smooth layers of adhesive emerged as the most important consideration in obtaining good thermocouple mountings (25).

It was also desirable to measure the heat flux downwards from the heater plate to obtain a measurement of the heat transfer to the bulk liquid nitrogen jacket so as to assess the actual total upward heat flux. To this end six thermocouples were mounted on the lower surface of the heater plate, directly below those on the upper surface. A further six thermocouples were mounted on the perspex base of the cell, three on the upper surface, three on the lower, to investigate the heat flux through the perspex layer.

All the constantan leads from the thermocouples are passed through a leak proof channel in the front flange of the chamber (3.1.7) and into a small dewar of liquid nitrogen at normal boiling point, where the ends are twisted together. A copper-constantan thermocouple junction is also immersed in the dewar and is used as the cold reference junction. Each copper lead from a thermocouple is connected to one way of a set of two, 2 pole, 6 way switches. A third, 2 pole, 4 way switch selects the particular set of thermocouples to be read. These switches are mounted at one end of a hollow stainless steel cylinder which is mounted on the front flange of the chamber. The cylinder was in direct contact with the chamber flange, being fixed to it by socket screws and made vacuum tight by
Indium wire seals. This meant that it was continually losing heat by conduction to the chamber walls, with ice forming all around it and clogging the switching mechanism. This occurred even with the cylinder continuously pumped by a rotary vacuum pump since in practice it proved difficult to maintain the cylinder at sufficiently low pressure to prevent ice forming inside it. It was found possible to maintain relatively smooth switching by playing the exhaust from a fan heater over the switch housing at intervals during an experiment.

Temperatures were read to within 0.05K by connecting the output from the switches and Cu-Cn reference thermocouple to a Comark 1603 electronic thermometer, switched to external cold-junction reference, yielding individual AT values accurate to 0.1K.

3.1.7 The Chamber

The cell, reservoir and heater plate are supported on a double beam cantilever which in turn is welded to a disc-shaped flange of stainless steel 1.5 cm thick and 42 cm diameter. This flange also contains all the filling and vent lines to the cell and the reservoir, as well as the thermocouple switch housing and the exit window for the laser beam.

The normally free end of the cantilever is supported on a vertical pillar with a wheeled base so that the whole flange, cell and reservoir assembly can be withdrawn from the 1.5 cm thick stainless steel cylindrical shell of the chamber, for maintenance and cleaning.
The cylinder is about 110 cm long, has a hole 15 cm in diameter bored into it at the far end concentric with the cylinder and cell axes, to accept the entrance window assembly. The flange is held in position by 24 bolts which screw into the cylindrical shell of the chamber. An indium wire double seal is sandwiched between the flange and the cylinder.

3.1.8 Window Housings

Each window housing consists of a stainless steel cylinder 10 cm inner diameter with flanges at each end, one grooved to accept indium wire seals, the other to accept a rubber O-ring seal. Clean optical flats 12 cm in diameter and 1.0 cm thick are placed in position against the seals, and stainless steel ring flanges having appropriate seals in position are placed over the windows and fixed to the cylindrical housing by socket screws. The complete housings are then mounted in the 15 cm diameter chamber ports again using Indium seals. The inner window is thus in contact with the liquid nitrogen in the cell when filling is completed. A section through the window housing is shown in fig. 3.4.

A vacuum pipeline is brazed into a hole bored in the cylindrical wall of each housing, halfway between the two windows. The window spaces were evacuated to a pressure of approx. $10^{-4}$ torr during experiments, to prevent heat conduction from the outside of the chamber causing boiling of the fluid near the windows which would seriously affect any measurements.

3.1.9 Insulation

All external metal surfaces of the chamber are covered by a
FIG. 3.4 SECTION THROUGH WINDOW HOUSING

- Chamber flange
- Socket screws
- Rubber o-rings
- Windows
- Indium seals
- Vacuum pump
10 cm thick layer of expanded polystyrene foam to insulate the system. Also all parts of the chamber outside the cell and reservoir are kept filled with liquid nitrogen during experiments. This extra liquid goes to provide additional insulation of the cell and reservoir from the outside. The action of the reservoir and the heater plate is to provide a stable uniform vertical temperature difference across the liquid layer in the cell.

3.2 Operation

3.2.1 Filling

The operation of filling the chamber requires about 4 to 5 hours and a total volume of liquid nitrogen of 400 litres including that required to cool down the system to 77 K and for subsequent topping up purposes. The filling operation has to be slow to avoid excessive thermal shock to the perspex cell. The cell and insulating volume between the cell and cylindrical wall of the chamber are filled together through a filling line mounted in the chamber flange, above the level of the reservoir top surface. This feeds to a copper pipe inside the chamber along which holes are bored at intervals. Thus filling is uniform along the length of the chamber. The vent line from the cell is open to atmosphere during filling. The reservoir is kept evacuated by a rotary pump meanwhile to prevent ice forming in it or in its filling and venting lines. When liquid nitrogen emerges from the cell venting pipe, the reservoir is then filled, requiring about 15 to 20 litres of liquid nitrogen.
3.2.2 Methods for Pumping Liquid Nitrogen

Three different techniques for pumping liquid nitrogen were tried based on pressurisation, compression and heating of the liquid nitrogen.

3.2.2.1 Pressurisation

The liquid nitrogen dewar (65 litres capacity) is pressurised by a cylinder of oxygen free nitrogen gas usually 4 to 6 p.s.i. being applied. The initial gas pressure in the cylinder is 2,500 p.s.i. The method is economical but rather laborious as the cylinders have to be disconnected from dewars during changeover from one dewar to another.

3.2.2.2 Compression

The principle of the compression or Haynes pump is that the nitrogen gas above the liquid level in a dewar is compressed and hence liquified (26). This liquid is then forced back into the dewar causing overpressure on the liquid surface. A unit manufactured by Union Carbide was claimed to produce over-pressure of 4 to 5 p.s.i. and a delivery rate of 15 litres per minute which would have been quite adequate for the purpose. However, in practice delivery rates of less than 0.25 litres per minute were obtained, insufficient to keep pace with boiloff inside the chamber.
3.2.2.3 Heating

In this case an electric heating element coiled around the delivery tube inside the dewar was used to boil the liquid causing over-pressure. The main problem with this technique is the time taken for the liquid flow rate to respond to changes in power level. However coiled cable cartridge heaters using Pyrotenax mineral insulated heating cables have been developed to supply uniform heating throughout any required depth of liquid. A cartridge heater of internal diameter 20 mm and coiled length 600 mm was used, incorporating glass to metal seals which can sustain thermal shock from 300 K to 77 K without deterioration. It had a maximum load capability of 1500 watts, 240 volts. The coiled heater surrounds the flow pipe which passes through a rubber stopper in the neck of the dewar along with the cable conductor wires. A pressure gauge is also inserted through the stopper into the dewar. This pumping system was found to work very well. Control of the heat load was applied by a variable transformer and with practice, it was found that quite precise control of the pressure and flow rates could be achieved. Response time of the flow rate to power level changes was found to be a few
seconds once a suitable pressure of 4 to 6 p.s.i. was established.

3.2.3 Operational Technique

The chamber vent lines were provided with precise pressure control systems requiring compressed air supplies of 30 p.s.i. As this facility was not available to the department, the following procedures were normally adopted.

The vent valve for the cell was closed and a measured power ranging between 50 and 300 watts was applied to the heater plate. The vent line from the reservoir was left open. After a few minutes the temperatures at each of the thermocouples on the heater plate and reservoir base were noted on the thermometer and when the mean temperature difference attained a steady value, measurements of optical distortion were made. Optical distortions were measured at various heights in the fluid and using multiple beam scanning techniques, velocity fluctuation and eddy size information could be acquired simultaneously. About two hours were available for experiments during which the slowly rising pressure (see figs. 3.5 and 3.6) in the test cell was monitored on a gauge in the venting line. At the end of this period the pressure rose quickly and had to be released. This sudden pressure rise coincided with the fall in liquid level in the insulation region to the top of the cell, leaving the reservoir exposed to higher temperature nitrogen gas. The reservoir liquid then boiled off quite quickly and at this point the chamber and reservoir required refilling.

It was found that condensation appeared on the windows
FIG. 36: PRESSURE IN CELL AT TIME t AFTER 294W HEAT INPUT APPLIED TO HEATER PLATE
due to the housing being in contact with the very cold metal of the chamber flange. This problem was eliminated by playing jets of hot air blowers onto the window housings.

3.2.4 Temperature Measurements

The original concept of temperature measurement was not strictly quantitative. It was merely expected that monitoring would show the temperature at the reservoir base to be everywhere lower than the temperature at the heater plate whose design was such as to ensure uniform heat flux in the vertical direction. The accuracy of the electronic thermometer was about 0.5 K at 77 K but it was found that the standard deviation taken on the mean temperature difference across the cell, was greater than this. Plots are given of $\Delta T$ against time for two heat inputs in fig. 3.7. It is seen that $\Delta T$ rises to a steady final value within about 20 minutes, but there is little difference between the final values of $\Delta T$ attained for the different heat inputs. Further discussion of the unexpectedly low measured values of $\Delta T$ at higher heat inputs is given in Chapter 6 and an explanation for the discrepancy between expected and measured values is offered.

3.3 Conclusions

The available volume of fluid for experimental purposes, is a cylinder 10 cm in diameter and almost a metre long. The fluid layer has a good aspect ratio and can be uniformly heated from below and cooled from above to produce thermal turbulence. As can be seen from fig. 3.6 the time available
FIG.37 Mean temperature difference $\Delta T$ across cell versus time $t$ after heat input $W$ applied.

(Vertical bars represent the standard deviation on six measurements: horizontal bars represent the time taken for six measurements.)
for experiments, varies between one and two hours depending on the heat flux level.

The equipment is adequate for its primary purpose of measuring optical distortions. Consideration is next given to the electronic systems used for measurement of optical distortions, and turbulent eddy sizes and velocities.
CHAPTER 4

Techniques for the Measurement of Turbulence Parameters

4.1 Introduction

The most important quantity to be measured in this study is the optical distortion, or r.m.s. spatial fluctuation, of a light ray about its mean position. Fortunately this parameter is also the easiest to measure by simple techniques which are described in Chapter 5. This present chapter is concerned with techniques for the measurement of other variables, specifically the sizes of turbulence eddies and their velocities. Eddy size plays a large part in determining optical distortion (eq. 2.10) and so ideally should be measured simultaneously so as to check if the results are consistent. Eddy velocity may also be important since this will determine how far a track bubble will move during its growth time.

Some methods for acquiring data on turbulence parameters are interferometry, holography, light scattering Doppler velocimetry and various correlation techniques.

Light scattering offers some possibilities, but cryogenic fluids would require seeding to produce measurable effects. This could not be conveniently done here and in any case only small angle scattering can be observed with the present rig. Laser Doppler velocimetry has been used in turbulence velocity measurements but the hardware tends to be expensive. A variation of the Laser Doppler Velocimetry technique is the light
scattering fringe anemometry technique of Jones (27) used for particle sizing. With this method however high constant visibilities and scattered intensities are required as well as constant phase over a finite angular range centred around a scatter angle of several tens of degrees, more than the test rig geometry permits. For large scattering centres i.e. transparent turbulent eddies, the polar diagram of light intensity scattered at low angles, becomes too complex for analysis.

Interferometric techniques, while very accurate, are time consuming and require a vibration-free environment. Also the presence of higher frequencies could render interferometry of little use except for phase measurements (28). This problem would also arise with holographic methods.

Various techniques are described in this chapter, some of which were attempted, usually with little success. Reasons for these failures are discussed. The techniques fall into the two broad groups of interferometric and opto-electronic. In the former group the spatial coherence of laser beam is estimated after propagation of the beam through the turbulent medium. The coherence can then be related to the size of the refractive index fluctuations (eddies). In the second group the size of the eddies influence the degree of correlation between electrical signals produced in photodetectors by the angular fluctuations of two or more parallel beams with fixed mean separations. The beams may be crossed perpendicularly for localised investigation or be coplanar for path integrated measurements. Time delayed cross-correlation gives information
Finally a system of measurement is described which involves the cross-correlation of position fluctuations of multiple parallel coplanar laser beams. While a disadvantage of this system is that it only measures eddy sizes averaged over the whole optical path of the beams, it has the great virtue of providing size, velocity and optical distortion information simultaneously.

4.2 Interferometric Techniques

4.2.1 Theory

An interferometer produces fringe patterns whose visibilities are related to the sizes of the turbulence eddies (refractive index fluctuations) averaged over the laser path in the turbulent fluid. Two forms of interferometer are common. In the conventional type the turbulent path constitutes one arm of the interferometer; this technique is well known and will not be dealt with here. In the other type a single laser beam after propagating through the fluid is passed through a two-arm interferometer consisting of a beamsplitting cube and two right angle prisms. This causes the lightfield to interfere with its own inversion causing fringes of varying visibility (see fig. 4.1). The theory of this technique will now be outlined, following the treatment of Wesseley and Bolstad.

Consider an electric field vector \( \mathbf{E}(x,t) \), \( x \) being the position coordinate in the entrance aperture plane of the interferometer and \( t \) the time. If the laser beam is split into
FIG. 4.1 INTERFEROMETRIC MEASUREMENT OF TURBULENT EDDY DIAMETER
two equal intensity beams, one part then inverted and the components recombined, then the net field D at \((x,t)\) is given by

\[ D \propto E(x,t) + E(-x,t) \]

Squaring and averaging gives the irradiance as

\[ \langle E^2(x,t) \rangle + \langle 2E(x,t)E(-x,t) \rangle + \langle E^2(-x,t) \rangle \]

and assuming the irradiance average across the entrance plane to be uniform then the spatial distribution of irradiance in the exit plane is

\[ 2E^2 + 2\phi_E(2x) \]

\(\phi_E\) being the spatial covariance of the electric field.

Now for plane polarized wave illumination

\[ E(x,t) = A(x,t) \cos(2\pi vt - \Theta(x,t)) \]

\(v\) being the frequency, \(A\) the amplitude and \(\Theta\) the phase.

Hence the irradiance distribution in the exit plane is

\[ \langle 2A^2 \rangle + 2(\langle A \rangle^2 + \phi_{A \Delta A}(2x)) \langle \cos(\Theta(x,t) - \Theta(-x,t)) \rangle \]

\(\phi_{A \Delta A}\) being the spatial-covariance function of the amplitude fluctuations. The phase function \(\Theta(x,t)\) includes the aberrations of the optical system and beam misalignment, the source phase fluctuations, and the phase fluctuations due to refractive index variations in the turbulent fluid. The source phase fluctuations will be negligible for a laser beam. Optical aberrations do not vary with time and have no effect if they are symmetrical about the centre of the irradiance field.

If \(\beta\) is the angle between the laser beam axis and the optical axis, a systematic phase variation occurs across the entrance
plane of form $2\pi k \sin B$ where $k$ is the wavenumber.

Then the phase function reduces to

$$\theta(x,t) = 2\pi k \sin B + S(x,t)$$

$S(x,t)$ being the phase shift due to turbulence. In the exit plane of the interferometer the irradiance distribution is now

$$<2A^2> + 2(<A>^2 + \phi_{AA} (2x)) <\cos(4\pi k \sin B + \Delta S(x,t)>$$

with

$$\Delta S(x,t) = S(x,t) - S(-x,t).$$

Assuming the phase difference to be symmetrically distributed about zero the irradiance becomes

$$2A^2 + 2(<A>^2 + \phi_{AA} (2x)) <\cos \Delta S(x,t)> \cos (4\pi k \sin B)$$

consisting of the nominal fringe pattern $\cos (4\pi k \sin B)$ with the envelope $<A>^2 + \phi_{AA} (2x)) <\cos \Delta S(x,t)>$

Thus the spatial correlation function of the electric field is related directly to the fringe visibility. If the phase fluctuations produced by the turbulence at any two points follow a bivariate gaussian distribution with mean zero and covariance $\phi_S(x)$, the envelope becomes (28)

$$<A>^2 + \phi_{AA} (2x))e^{-4\pi^2[(\phi_S(0) - \phi_{AA}(2x)]}$$

Similarly if amplitude fluctuations at each pair of points have gaussian distribution, then the correlation function is $\phi_{AA}^2(2x)$. Thus one can distinguish between the effects of amplitude fluctuations and phase fluctuations and both of these quantities can be related to turbulence parameters (23).

4.2.2 Implementation

An expanded He-Ne laser beam of 1 cm diameter and of divergence 0.1 milliradians was used. After propagation through
the chamber the beam was incident on the interferometer, as shown in fig. 4.1. The only important restriction on the right angle prisms is that the roof angle tolerance be ± 0.05 milliradians to avoid having too many fringes in the pattern.

With no turbulence, fringes of constant visibility are observed in a vibration-free environment. With a turbulent path, the incident wavefront is randomly tilted about the optical axis, varying the fringe spacing and hence reducing fringe visibility towards the edges of the exit plane. It is important that the interferometer be rigidly mounted as vibrations can affect the fringe visibility in the same way as turbulence.

In principle the correlation diameters estimated from the integrated fringe pattern size are determined by exposing a photographic plate to it for a suitable length of time, this latter being the correlation time of the turbulence.

In practice however, severe vibration problems were encountered due to the vacuum pumping equipment and the fringe patterns were rarely visible in the absence of turbulence. Anti-vibration mounts were employed but with little success. The experiments were carried out on the 5th floor of the department building and this added to the problem. Ideally the experiment should be done in a basement with the interferometer mounted some distance from the chamber in a vibration-free environment.

4.3 Other Optical Techniques for Turbulence Diagnosis

Several optical techniques for turbulence parameter
measurement have been developed by Bertolotti et al. (29, 30, 31) which involve the interference of a set of laser beams. All these methods involve photographic processing and are somewhat laborious. Many of them permit measurements of turbulence eddy diameters only, and ideally one would like to obtain optical distortion information simultaneously. The method of Carlson (33) uses the intensity fluctuation of a laser beam to obtain the spatial spectrum of the turbulence eddies considered as scattering points. The technique is applicable over a range of eddy sizes from a few mm to tens of cm. By using suitable instrumentation, optical distortions could probably be measured simultaneously.

A method due to Consortini et al. (32) is rather similar to the technique used in the present study involving multiple beams and correlating the instantaneous optical distortion signals produced by each of them. A laser beam is split into several parallel beams using a multiple beam splitter. The beams propagate through the turbulent medium and are allowed to fall on a translucent screen having a millimeter grid reference. This is photographed at regular intervals using exposure times of about 1 millisec. The coordinates of the laser beam spots are then read and computer processed to give measurements of the turbulence eddy sizes. The essential difference between this method and our technique is in the means used to detect and record the instantaneous positions of the beams (see Chapter 5).

Consortini's technique is extremely simple and seems to give much more satisfactory results than the methods of
Bertolotti in that good agreement with theory is obtained. The anisotropy of the turbulence structure was successfully measured as well. Again the technique involves photographic processing and laborious tabulating of beam coordinates.

4.4 A C.W. Technique Using a Position Sensitive Photodetector

4.4.1 Theory

Based on the principle that a position sensitive photodetector determines the coordinates of the centroid of whatever light intensity distribution is incident upon it, it is possible to measure the correlation length (though not velocity) of the turbulence structure using two c.w. laser beams whose separation can be continuously varied.

Thus if two parallel beams propagate through a turbulent medium and are incident on a position sensitive photodetector the instantaneous output voltage of a difference amplifier connected across the Z axis detector outputs is given by

\[ V = \frac{k_1 z_1 + k_2 z_2}{k_1 + k_2} \]

\( k_1, k_2 \) being directly related to the respective beam intensities, \( z_1, z_2 \) the instantaneous Z coordinates of the beam centroids on the detector face.

Let

\[ z_1 = z_{20} + e_1 \]
\[ z_2 = z_{20} + e_2 \]

\( e_1 \) and \( e_2 \) being fluctuations about mean positions \( z_{10} \) and \( z_{20} \), these fluctuations being caused by thermal turbulence. By high pass filtering, the mean position components are removed. Squaring

\[ V^2 = \frac{k_1^2 e_1^2 + 2k_1 k_2 e_1 e_2 + k_2^2 e_2^2}{(k_1 + k_2)^2} \]
Averaging over a time long compared with the periods of the fluctuating signals

$$\bar{v}^2 = (k_1^2 \bar{e}_1^2 + 2k_1k_2 \bar{e}_1 \bar{e}_2 + k_2^2 \bar{e}_2^2) / (k_1 + k_2)^2$$

If the beams were originally separated by distance $\ell$ in the horizontal plane, and if the turbulent process is both isotropic and stationary

$$\bar{e}_1 \bar{e}_2 = R(\ell_x)$$

where $R(\ell_x)$ is the correlation coefficient of the spatial structure of the turbulent fluid. With the beams completely coincident, the value of $\bar{v}^2$ is given by

$$\bar{v}^2 = k_1^2 \bar{e}_1^2 + k_2^2 \bar{e}_2^2 + 2k_1k_2 R(0) / (k_1 + k_2)^2$$

and as the beams separate so the correlation term vanishes at values of $\ell_x$ large compared with the turbulence eddy diameter leaving

$$\bar{v}^2 = (k_1^2 \bar{e}_1^2 + k_2^2 \bar{e}_2^2) / (k_1 + k_2)^2$$

Thus if $k_1$ and $k_2$ are determined by calibrating the detector separately for each of the beams one can obtain both the mean square beam deviation (optical distortion squared) and plots of $R(\ell_x)$ to determine eddy diameters.

**4.4.2 Implementation**

It is convenient if one of the beams can be caused to move automatically relative to the other so $R(\ell_x)$ may be plotted automatically. The velocity of movement is determined by the signal bandwidth, circuit bandwidth and required resolution.

Assuming a signal bandwidth of 100 Hz and a circuit output
bandwidth of 1 Hz (improving the signal to noise ratio by 20 dB) with a spatial resolution of 1 mm in the system, the velocity of the moving beam would be 1 mm s\(^{-1}\). Fig. 4.2 shows the optical system for producing parallel beams. The mirror is mounted on a metal block which moves on a motor-driven worm screw. The block is braced by three grub screws to minimise play in the system. When the block reaches the end of its travel it activates an electro-mechanical switch \(S_1\) which in turn reverses the motor via a relay (fig. 4.3). Simultaneously the power transistor is turned hard on (or off) to maintain the reverse direction of motion until switch \(S_2\) is activated to begin the cycle over again.

4.4.3 Circuit Details

The detection circuit is shown in fig. 4.4. The high pass filter is a single pole CR type having a 3 dB point at 0.005 Hz. On switching on the supply voltages, transients of up to ±15V are initiated and these are attenuated by the 5 M and 10 M resistor chain to ±5V to avoid overloading the mosfet inputs. These mosfets are needed as buffers between the filter and the 741 operational amplifier inputs to prevent attenuation of the higher frequency components of the signal. The gain of the amplifier stage can be switched to ensure an input voltage to the squaring circuit of a few volts.

The squarer is a Motorola MC1594L integrated circuit 4-quadrant multiplier at the output of the second amplifier stage. It has an output voltage \(V_o\) given by

\[ V_o = k V_{in}^2 \]

\(k\) being about 0.1.
FIG. 4.2 ELECTRONIC MEASUREMENT OF EDDY DIAMETER IN THE VERTICAL PLANE

FIG. 4.3 ELECTRONICS FOR REVERSING MIRROR DRIVE
(S₁ & S₂ electromechanical switches)
FIG. 44. ELECTRONICS FOR MEASUREMENT OF TURBULENT EDDY DIAMETER
4.4.4 Operation

In operation, the output of the circuit of fig. 4.4 is passed through a low pass filter \( f_{1/3} \text{ dB} = 1 \text{ Hz} \) and thence to the \( Y \) plates of an oscilloscope while the \( X \) axis signal from the photodetector is applied to the \( X \) plates. The \( X \) axis signal has to be low-pass-filtered to remove the amplitude fluctuations caused by optical distortion due to thermal turbulence. Hence the correlation curve is traced on the oscilloscope screen as the beams separate.

From section 4.4.1 it is seen that when the beams have separated by a distance which is much greater than the correlation length of the turbulence, it is expected that the amplitude of the signal on the \( Y \) plates will have fallen by at least one half. In practice it was found that no change in correlation signal amplitude was observed with change in beam separation. This was found to be due to the frequency content of the signals from the photodetector; significant frequency components exist only up to about 15 Hz (Chapter 5) but very substantial components exist at frequencies well below 1 Hz (Chapter 5).

In order to allow for a proper correlation measurement, it is necessary to average the output of the squaring circuit using an integrator with time constant about 10 seconds. This reduces the maximum velocity of laser beam motion to about 0.1 mm s\(^{-1}\). Thus it would take about 1 minute for the oscilloscope trace to reach the asymptote at a beam separation of say 6 mm.
On this basis it was considered easier to set the beam separation manually and photograph the oscilloscope trace for each beam separation using long exposure times. This meant that play in the system was eliminated completely. The results while not quantitatively successful, indicated that the correlation amplitude did fall off with beam separation. However, inevitable d.c. drifts in the circuitry caused severe problems often to the extent of swamping the variation in correlation amplitude.

4.4.5 Conclusions

In spite of the lack of success with the continuous beam technique in the present application, the author believes it could prove extremely useful as a diagnostic tool in turbulence studies, particularly in atmospheric work where turbulence eddy sizes are an order of magnitude greater than in the present context (29, 30, 31) and hence optical geometry easier to set up and align.

4.5 Crossed Beam Correlation Technique

4.5.1 Introduction

The crossed beam correlation technique allows measurement of eddy size and velocity at localised points in the turbulent fluid. The concept is depicted in fig. 4.5. Two mutually perpendicular beams are passed through the fluid. The beams may intersect or be separated by a minimum distance $l$. Upon emerging from the fluid the beams are incident on two position-sensitive photodetectors $D_1$ and $D_2$ producing voltages directly related to the beam coordinates in the detector planes.
FIG 4.5. CROSSED BEAM CORRELATION
The optical distortions (r.m.s. beam deviations) measured by the detectors, result from integration of the instantaneous beam deviations along the optical paths. The technique isolates the time averaged local fluctuations of beam position within a correlation or covariance volume of the signals from the detectors. The correlation volume is determined by the eddy size and not by the laser beams. The components of the signals due to fluctuations in beam position outside the correlation volume are not correlated and appear only as noise.

4.5.2 Theory

Theoretical aspects of the method have been described in detail elsewhere (34, 35, 36). The main result is that the cross-correlation coefficient of the fluctuating signals from the detectors is

\[ R(\xi, \tau) = \frac{1}{T} \int_0^T v_1(t)v_2(t + \tau) dt \]

where \( T \) is the integration time over which the correlation is carried out, \( v_1 \), the voltage signal produced by \( D_1 \), \( v_2 \) the voltage signal produced by \( D_2 \), and \( \tau \) the time delay introduced between the two detector signals and \( \xi \) the minimum separation of the beams.

When spatial correlation coefficients are being measured the time delay \( \tau \) is set at zero and the minimum beam separation \( \xi \) is varied from zero to some value at which \( R \) becomes zero. It is assumed of course that the turbulent process is stationary and isotropic.

Two techniques are used to obtain measurements of \( R \). In the analogue method, one signal is time delayed with respect to the other, the signals are then multiplied and the result
integrated over the time $T$.

If a digital technique is used, as with the Hewlett Packard 3721A correlator, the signals are separately sampled every $t$ seconds and summing a finite number $N$ of the sample products. The result is then divided by $N$.

Thus one has

$$R(\ell, \tau) = \frac{1}{T} \int_0^T v_1(t)v_2(t + \tau) dt$$

$$= \frac{1}{N} \sum_{k=1}^{N} v_1(\kappa \Delta t)v_2(\kappa \Delta t + \tau).$$

$R$ is computed for several values of $\Delta t$.

The range of $\tau$ over which $R(\ell, \tau)$ is of interest, depends on both $\ell$ and the bandwidths of signals $v_1$ and $v_2$. If, for example, $\ell$ is small compared with the correlation length of the turbulent fluid and the signal bandwidth is 100 Hz, then $R$ would be adequately determined using values of $\Delta t$ from 0 to 100 ms and 1 ms resolution. The interval $\Delta t$ between pairs of samples can be large in relation to the resolution in $\tau$.

Since signal statistics are of interest, the detailed signal waveforms do not matter and the Nyquist sampling criterion need not be met. In fact assuming the signal statistics are stationary (36) it does not matter how infrequently signal samples are taken. The statistical error on $R$ decreases with increasing $N$. The only restriction is that the number of samples $N$ must be taken within a time over which the signal statistics can be considered stationary.

It is usually desirable to convert the results of analyses into standard values so as to be able to compare the results from different experimental conditions. Hence one defines the
normalised cross-correlation coefficient as

\[ R_N(\ell, \tau) = \frac{\frac{1}{T} \int_0^T v_1(t)v_2(t + \tau)dt}{\left( \frac{1}{T} \int_0^T v_1(t)dt \right) \left( \frac{1}{T} \int_0^T v_2(t + \tau)dt \right)^{\frac{1}{2}}} \]  \hspace{1cm} (37)

\[ R_N(\ell, \tau) = \frac{\frac{1}{N} \sum_{k=1}^N v_1(k \ell t)v_2(k \ell t + \tau)}{\left( \frac{1}{N} \sum_{k=1}^N v_1(k \ell t)^2 \right)^{\frac{1}{2}}} \]

The properties of the function \( R_N(\ell, \tau) \) for \( \ell = 0 \) are given in references 37 and 38.

4.5.3 Sample Size Requirements

The normalised standard error \( \epsilon \) (39) for a cross correlation function estimate \( \hat{R}(\ell, \tau) \) from two signal records \( v_1(t) \) and \( v_2(t) \) with zero mean values and true cross correlation function \( R(\ell, \tau) \) is

\[ \epsilon = \frac{\text{standard devn.}(\hat{R}(\ell, \tau))}{R(\ell, \tau)} = \frac{1}{\sqrt{2BT}} \left[ 1 + \frac{R_1(0)R_2(0)}{R^2(\ell, \tau)} \right]^{\frac{1}{2}} \]

\( T \) being the signal record length and \( R_1(0) \) and \( R_2(0) \) being the autocorrelation functions of the two signals at \( \tau = 0 \).

For spatial correlation measurements, \( \tau \) is always zero and

\[ \epsilon = \frac{1}{\sqrt{2BT}} \left[ 1 + \frac{R^2(0)}{R^2(\ell, 0)} \right]^{\frac{1}{2}} = \frac{1}{\sqrt{2BT}} \left[ 1 + \frac{1}{R^2(\ell, 0)} \right]^{\frac{1}{2}} \]

Thus the error \( \epsilon \) increases with \( \ell \), the beam separation, becoming very large at values of \( \ell \) much greater than the turbulence correlation length or eddy diameter. When the beams are at zero separation, and signal bandwidths (sec. 6.1.1) are around 10 Hz
then for $T$ of the order of 80 seconds, the error on the spatial
cross-correlation function estimate will range from about
3.5\% at zero beam separation to 5\% when $\ell$ is near the correlation
length of the turbulent process.
At higher values of $\ell$ problems arise because of the conflicting
requirements of accuracy in estimates of spatial correlation
coefficient (i.e. large values of $T$) and the desirability of
acquiring enough data during a given experimental run.

4.5.4 Experimental Details

In the present application the use of crossed beams
proved to be difficult since the windows are coaxial and only
10 cm in diameter. It would be possible to arrange for beams
to cross at an angle of at most 12°. This would substantially
lower the spatial resolution of the system. Alternatively it
would be possible to insert mirrors into the turbulent fluid
so as to direct beams in mutually perpendicular directions
within the fluid layer. However, because of the difficulty of
mounting mirrors at various points on the heater plate and
the likelihood of disturbing the turbulent field, this was not
attempted. In practice the cross-correlation coefficients
for two beam signals are obtained using an analogue or digital
correlator for a series of different beam separations, with
the time delay set at zero. A plot of $R_N$ at $\tau = 0$, versus
beam separation $\ell$, will fall from a maximum ($R_N = 1$) at $\ell = 0$
to zero at $\ell = \infty$. In the present case the decay of $R_N$ may be
of the general form (22), $R_N = \exp(\ell^2/a^2)$
where $a$ is the correlation length or eddy diameter.
By cross-correlating signals from two beams at a fixed separation \( l \) a repetitive correlogram is obtained with peaks separated by a characteristic time \( \tau \). Then the turbulent velocity \( v \) is given by \( v = l/\tau \).

i.e. \( \tau \) is the time taken for turbulent eddies to travel a distance \( l \) at velocity \( v \).

### 4.6 Conclusions

Because of the difficulty of using the crossed beam technique due to the awkward geometry of the present cell and chamber, it was decided to arrange for the beams to be in a single plane, propagating parallel to one another.

This of course had the advantage that only one position sensitive photodetector was required. The theory of the method is as outlined in section 4.5.2. The only distinction that must be made is that the eddy size and velocity estimates obtained using this method are now averaged over the total path of the beams in the turbulent fluid. The problem of D.C. drift encountered with the C.W. technique (4.4.4) is eliminated by using sampling methods.

In the next chapter methods of implementing this system are described along with electronics for signal processing and recording. Data processing techniques are also described.
5.1 Introduction

In practice bubble chambers are illuminated by high intensity flash tubes and the track bubbles scatter light into the camera lenses. Since the bubbles are of the order of a hundred microns in diameter when illuminated, they may be considered as point sources of light. For the purposes of optical distortion measurement, a laser may be used as a point source of light. Thus a laser light beam propagating through the turbulent fluid behaves as a ray of light, undergoing random angular fluctuations.

With the test rig as designed it is a simple matter to pass a laser beam into the turbulent fluid, and the laser source then simulates a bubble observed through the 90 cm length of the turbulent layer.

For optical distortion measurements, some kind of position sensitive photodetector is required, preferably with a linear characteristic. A number of possible approaches are considered. The detector finally chosen was also very suitable for use with a multiple beam probe for correlation experiments.

Ultimately a system of measurement was developed which produced measurements of optical distortions at the same time as information on turbulence eddy sizes and velocities.
Optical designs are considered in sections 5.2 and 5.6. Opto-electronics are covered in 5.3, and 5.4 while electronic design and data processing are dealt with in 5.7 and 5.8. System calibration is described in 5.5 (simple system) and 5.9 (multiple beam system) and the chapter ends with a discussion of system performance in 5.10 and 5.11.

5.2 Optics for Simple Beam Deviation Experiments

A well collimated, stable monochromatic light source is needed to produce a ray of light whose distortion is to be measured when passed through a turbulent fluid layer. A Spectra Physics 132 Helium Neon laser was used for this purpose, with an output power of 1.0 milliwatt in the TEM\textsubscript{0,0} mode, a beam diameter of 1.00 mm at the 1/e\textsuperscript{2} points and a beam divergence of 1.0 milliradians. The wavelength of the laser was 632.8 nanometres, to which wavelength liquid nitrogen is transparent. Using 3-axis adjustable mirrors, the laser beam was passed into the chamber propagating in a direction normal to the chamber window planes as shown in fig. 5.1. The beam encountered the turbulent fluid and underwent random angular fluctuations so that the emerging beam spot fluctuated randomly about its mean position.

5.3 Techniques for Measurement of Optical Distortion

The random fluctuations of the beam emerging from the chamber must be converted to a signal whose root mean square value is a measure of the optical distortion. Typical methods for converting are a neutral density wedge and photomultiplier, a quadrant photodetector and proportional control system,
FIG 5.1: OPTICS FOR DISTORTION MEASUREMENT
a two dimensional photodetector array or a continuously-sensitive position photodetector.

5.3.1 Neutral Density Wedge and Photomultiplier

This technique involves placing a graded neutral density filter in front of a photomultiplier detector so that the intensity measured by the detector, is a simple function of beam position. Continuously graded filters were only obtainable with log characteristics, so a step wedge filter was made by piecemeal exposure of photographic film or plate.

The technique was successfully used in preliminary experiments (40), but had the disadvantages of not being continuously sensitive and also being sensitive in only one direction of beam deviation. It has also been found (see 2.8.3) that the turbulent medium produces amplitude modulation of the laser beam giving poor signal to noise ratios.

Using a beam splitter and two wedges with photomultiplier tubes, optical distortions can be measured in two orthogonal directions.

5.3.2 Quadrant Photodetector

The laser beam is allowed to illuminate a 4 quadrant photodetector on emerging from the chamber. If the laser spot diameter is large enough to illuminate all 4 quadrants, then by comparing the electrical outputs the positional fluctuations of the laser beam can be determined. However the need to have all 4 quadrants illuminated imposes a serious limitation as the spot diameter has to be greater than the maximum spot fluctuation. Otherwise the spot will frequently
be wholly in one quadrant when a measurement of its position fluctuation will not be possible.

5.3.3 Photodetector Array

Photodetector arrays can be used to measure optical distortions. A 25 diode array arranged as a 5 x 5 square matrix was employed by Leutz et al. (24) with an angular resolution of 0.05 milliradians with an optical path of 23 metres, to measure optical distortion in a liquid hydrogen bubble chamber. The method was to feed signals of fixed frequency and amplitude into a 25 channel counter hodoscope. When laser light impinged on a particular diode the gating signal produced blocked the corresponding hodoscope channel. The difference between the maximum and recorded count rates in a channel is proportional to the dwell time of the laser on the corresponding diode of the array. However while the time resolution of the system was about 1 microsecond, the spatial resolution was only about 0.6 mm which is inadequate for the optical distortions likely to be encountered in the present study. Also the very long optical path required to achieve this resolution was rather prohibitive.

5.3.4 Position Sensitive Photodetector

Position sensitive photodetectors have been successfully developed (41, 42) having resolutions of about 25 microns over the whole detector surface which can be up to 4 cm² in area. The devices are sensitive to the centroid of the light spot illuminating the photosensitive surface provided the entire spot falls within the active area. Fig. 5.2 shows the signals produced when a laser beam is incident on the detector face.
position sensitive photodetector

current difference $\propto z_1 P$

position of laser intensity $P$

current difference $\propto x_1 P$

sum of all currents $\propto P$

FIG 5.2 DETECTOR OUTPUTS
Sensitivities ranging from 40 to 600 mV/mm/mW (42) have been reported and under proper conditions of reverse bias and loading, the variation in sensitivity to uniform illumination can be made negligible (43).

The device is shown in cross section in fig. 5.3. The Schottky barrier (44) is formed by evaporation of a thin layer (~15 nanometres) of gold on to the surface of a slice of semiconductor, normally N-type silicon.

The laser beam incident on the detector surface passes through the almost transparent metal layer into the depletion layer where electron - hole pairs are generated. The majority carriers travel through different lengths of depletion region (44), depending on the point of impact of the laser, into the undepleted region where they are collected and pass through the external load resistances $R_1$ (see fig. 5.4). The minority carriers are swept to the junction under the action of a reverse bias. The detector equivalent circuit is shown in fig. 5.5.

Variations in the resistivity of the bulk material can give rise to errors in the measurement of the beam's position but these are minimised by evaporating a resistive layer (44), usually aluminium, onto the lower surface of the semiconductor between the ohmic lateral contacts. This then controls the distributing resistance between these contacts.

By summing the currents obtained in all resistances $R_1$ one can obtain a measure of the laser intensity. The difference between the currents in resistors $R_1$ give an instantaneous measure of the beam's position in a single or dual axis coordinate frame in the plane of the detector surface.
Schottky barrier electrode
inversion region

deployment region

ohmic lateral contacts
(four define 2 axis coord.
system)

bias

n-type

FIG5.3 POSITION SENSITIVE PHOTO-DETECTOR

FIG5.4 DETECTOR-DIFFERENCE AMPLIFIER CIRCUIT

Rs = sheet resistance
C_d = detector capacitance
R_l = load resistance
i_s = signal current = i_1 + i_2
i_1 = i_2 at null

FIG5.5 DETECTOR EQUIVALENT CIRCUIT
The device satisfies the requirements of continuous uniform sensitivity and is independent of laser spot diameter, so it was used for all subsequent measurements. Also it can be used in multiple beam applications as will be seen in section 5.7.

5.4 Detector and Amplifier Noise Considerations

Noise will obviously affect the accuracy of measurement. The detector is only sensitive to fluctuations in the position of the centroid of the laser beam so major sources of noise are those generated in the detector itself, namely shot noise, thermal or Johnson noise and flicker noise.

5.4.1 Shot Noise

The responsivity of the UDT SC/50 detector is around 0.25 mA (45) per mW of incident laser power at 632.8 nm, resulting in the passage through the device of $0.25 \times (1.6 \times 10^{-19}) \times (39.3 \times 10^3)$ carriers per second per mW laser power. With a reverse bias of 15 V d.c. the device capacitance is approximately 6000 pF for a sheet resistance of around 15000 Ohms, so the number of carriers produced within the detector integration time of $(6 \times 10^{-9})(1.5 \times 10^4)$ seconds will be

\[
0.25 \times 9 \times 10^{-8} \quad \frac{1}{1.6 \times 10^{-19}} = 1.4 \times 10^{11}
\]

Hence the shot noise (46) = $(1.4 \times 10^{11})^{1/2} = 3.8 \times 10^5$

Then the signal to shot noise ratio = 100 dB.

5.4.2 Johnson Noise (47)

The mean square Johnson noise current $\frac{1}{R} = 4kT\Delta f/R$
where \( k \), Boltzmann's constant = \( 1.38 \times 10^{-23} \) J/K,

\( T \), the temperature in K = 300 K,

\( \Delta f \), the detector bandwidth (integration time)\(^{-1} \) =

\( (9 \times 10^{-5})^{-1} \) s\(^{-1} \),

and \( R \), the load resistance = 1kΩ

Hence \( \frac{1}{n} \) \( \frac{1}{R} \) = \( 3.8 \times 10^{-10} \) A, and so

\[
\frac{S/N}{(Johnson)} = \frac{2.5 \times 10^{-4}}{3.8 \times 10^{-10}} = 120 \text{ dB}.
\]

### 5.4.3 Flicker Noise

Radeka (45) has determined the flicker noise contributions in field effect transistors and obtained for \( v_f^2 \) the mean square flicker noise voltage per Hertz bandwidth, \( v_f^2 = A_f \Delta f/f \)

where \( A_f = 10^{-13} \) V^2, while \( \Delta f \)

is the bandwidth and \( f \) the centre frequency in the band.

Van der Ziel (47) points out that flicker noise in other solid state devices is due to surface states as in the case of F.E.T.s.

Assuming then that flicker noise voltage for Schottky barrier diodes is given by Radeka's equation one obtains

\[
\bar{v}^2 = \int_{f_0}^{1\text{kHz}} 10^{-13} df/f
\]

The upper frequency limit of 1kHz is given by (45). Obviously the above equation cannot hold down to D.C. and one assumes that \( v_f^2 \) is constant and equals \( A_f \) over the range D.C. to say 10Hz. Hence

\[
\bar{v}^2 = \int_{0}^{10} 10^{-13} df + \frac{1000}{10} \int_{10}^{10} 10^{-13} df/f = 1.5 \times 10^{-12} V^2
\]

\[
\bar{v}^2 = 1.5 \times 10^{-12} \quad \text{V}^2
\]

\[
\cdot \cdot \cdot \bar{v}^2 = 1.5 \times 10^{-12} \quad \text{V}^2
\]

\[
\cdot \cdot \cdot \bar{v}^2 = 1.5 \times 10^{-12} \quad \text{V}^2
\]

\[
\cdot \cdot \cdot \bar{v}^2 = 1.5 \times 10^{-12} \quad \text{V}^2
\]

\[
\cdot \cdot \cdot \bar{v}^2 = 1.5 \times 10^{-12} \quad \text{V}^2
\]
which is almost as low as the shot or Johnson noise contributions. Clearly system noise contributions due to shot, Johnson and flicker noise in the detector, are negligible.

5.4.4 Amplifier Noise

With detector noise negligible, the major source of noise is now the amplifier.

The complete circuit for r.m.s. beam deviation (optical distortion) measurement is shown in fig. 5.4. The source resistance to each amplifier input is 3 kΩ. Using appropriate feedback for a voltage gain of 40 dB, a 709 operational amplifier connected in the difference mode has a passband extending from D.C. to 500 kHz followed by a decrease in gain of 20 dB per decade \((48)\). Typical results for the total noise of the µA 709 amplifier referred to the input, derived from ref. \((49)\), are given in fig. 5.6. Extrapolating from these curves for 3 kΩ source resistance one obtains for the total noise input \(e_T\)

\[ e_T = 0.8 \mu V \]
due to shot, thermal and flicker noise in the amplifier. This noise voltage leads to uncertainty in laser beam position measurements.

5.4.5 Noise Equivalent Displacement

The position sensitivity of the UDT SC/50 detector was measured as 0.016 mA/mW/mm at a laser wavelength of 632.8 nm. With the \(\frac{1}{2}\) kΩ load resistors, this results in an offset voltage of 32 mV per mm displacement of 1 mW He-Ne laser beam from the null position. Reflection losses at the windows and scattering
FIG 56 TOTAL NOISE REFERRED TO I/P OF μA709 V
SOURCE RESISTANCE $R_S$ AND BANDWIDTH $f_2 - f_1$

$V_s = \pm 15V$
$T_a = 25^\circ C$

- $f_s = 100kHz$
- $f_s = 30kHz$
- $f_s = 10kHz$
- $f_s = 3kHz$
- $f_s = 1kHz$
- $f_1 = 1Hz$

$R_S$ = source resistance (ohms)
in the turbulent liquid nitrogen reduce the laser power incident on the detector to around 0.3 mW, so the offset voltage will be about 10 mV per mm displacement of the beam. According to the predictions of equation 2.3 for \( \sigma^2 \) the r.m.s. beam deviation due to a 1 metre path of liquid nitrogen 0.1 metre deep whose vertical temperature difference is 0.1K (the smallest \( \Delta T \) likely to be obtained), is around 0.025 mm, giving an r.m.s. voltage output from the detector of 250 uV. This is substantially larger than the total amplifier noise voltage of 8uV, and this latter figure indicates a system minimum detectable displacement or noise equivalent displacement of about \( (8/250) \times 0.025 \) mm, i.e. approximately 1 micron. Thus the detector noise does not contribute significantly to signal degradation.

5.5 Detector Calibration

It is necessary to calibrate the detector/amplifier in terms of voltage output per mm of laser beam displacement across the detector surface. This was done by mounting the detector normal to the laser beam in an X,Z traverse with clock gauges mounted so as to monitor the coordinates of the detector. The detector was carefully positioned so that the outputs from the X and Z difference amplifiers were both zero. The detector was then moved along its positive horizontal axis and the voltage and position coordinates noted at intervals. The process was repeated for the negative X axis and the positive and negative Z axes. Voltages were measured on a vacuum tube voltmeter. The slope of the calibration curve obtained (fig. 5.7) is 0.79 volts/mm with a noise equivalent beam deviation of
FIG 57 CALIBRATION CURVE OF DETECTOR/AMPLIFIER xAXIS
(1mW LASER BEAM)
approximately 1 micron which is smaller than the minimum optical distortion expected due to turbulence (at ΔT = 0.1K) by a factor of 25. Thus the laser detector and difference amplifier combination should prove more than adequate for optical distortion measurements over a wide range of vertical temperature difference.

The Z-axis calibration proved to give similar values of the slope factor and noise equivalent beam deviation.

In practice single laser beams were not much used to measure optical distortion because of the short time available for experiments once filling of the chamber was complete. Instead a multiple beam probe, as described later in this chapter, was used.

5.6 Correlation Techniques Using Multiple Beams

Correlation experiments are required in order to measure turbulence eddy sizes and velocity fluctuations. Several techniques are available for such experiments (see Ch. 4). They all involve the production of multiple parallel beams with variable separations between them. The beams can be in the same plane, parallel or perpendicular to the heater plate for integrated measurements over the whole optical path, or they can be perpendicularly crossed for measurements at specific locations in the cell.

The general idea is to record beam deviation signals and apply spatial and temporal cross correlation techniques to the signals obtained. In the former case the cross correlation coefficient is obtained as a function of beam separation.
In the latter case the cross correlation coefficient is obtained as a function of the time delay between the signals. The theory of correlation is dealt with more fully in Chapter 4.

Using the Townsend equation to obtain $\frac{\delta^2}{\Delta T}$ from $\Delta T$ the sizes of turbulence eddies can be estimated from equation 2.8. For $\Delta T = 6 \text{ K}$ in a 0.1 metre layer depth of liquid nitrogen, the eddy size should be about 3mm. The beams must lie in the same plane to within a fraction of an eddy diameter for correlation purposes. This fact enables tolerance criteria to be established for the design of multiple-beam producing optics (see 5.6.1.1 and 5.6.1.3).

5.6.1 Production of Multiple Parallel Beams

Multiple laser beams may be produced by means of rotating optical polygons, torsional scanners or rotating prism cubes, all of these being used in conjunction with a set of apertures or slits. Beams may be produced simultaneously using several lasers, or fibre optics with one laser, but these methods are costly. As will be seen it is preferable anyway to have consecutive beams for the purpose of time-division multiplexing.

5.6.1.1 Rotating Polygon

A rotating polygon may be used to produce multiple beams by mechanical scanning. The polygon may comprise of a set of front surface mirrors mounted on the rim of an aluminium polygon which must be carefully machined (see fig. 5.8). A more common method is to machine a polygon from a slab of crown glass (50) and after careful polishing the faces are vacuum coated with aluminium and silicon monoxide. Twelve sided
FIG 5.8 MULTIPLE BEAMS USING A REFLECTING POLYGON
polygons have been successfully manufactured by this method but they are extremely costly. The reflected beam produced by the rotating polygon scans in a circular sector. The beam is made parallel by placing a lens in the reflected beam, its focus coinciding with the point of incidence of the beam on the polygon. The beam now passes through an aperture system with parallel slit apertures having unequal separations with the slits perpendicular to the plane of the laser beam. After passing through the liquid nitrogen the beams fall on the detector.

It is essential that the beams be within planar registration to about 0.2 of an eddy diameter on emerging from the liquid. Thus the permitted angular spread of the beams relative to each other for an eddy size of 3.0 mm and path length 1 metre is

\[ 3.0 \times 10^{-3} \times 0.2 = 0.6 \text{ milliradians} \]

This tolerance is increased by the effect of the lens used to make the beams parallel. Given a typical focal length of 15 cm the angular tolerance becomes 4.0 milliradians. Thus the reflecting faces of the polygon have to be in planes perpendicular to the polygon plane to within an angle of 2.0 milliradians.

The number of faces required on the polygon depends on the signal bandwidth in each channel (sampling rate) and the speed of rotation of the polygon. For example a 12 sided polygon would need to rotate at 100 r.p.m. for a signal bandwidth of 100 Hz. Unfortunately the estimated cost of manufacture of an optical polygon with 12 faces to the above tolerance was found to be prohibitive. Attempts were made in the departmental
workshops to produce an aluminium alloy polygon, and while excellent tolerances on the machine faces and bearing movements were achieved, problems arose with the secure mounting of front surface reflectors on the faces. No manufacturer could be found willing to undertake accurate polishing and vacuum reflective coating of the faces as an alternative, so this approach was reluctantly abandoned.

5.6.1.2 Torsional Scanner

A torsional scanner consists of a single beam torsional fork mounted between the pole pieces of an electromagnet (see fig. 5.9). Fixed to the torsional beam is a small reflector. The beam oscillates in a plane parallel to the direction of magnetic flux (\(\phi_1, \phi_2\)) so that the reflector scans through a small angle. Oscillation frequencies up to 10 kHz and scanning angles of up to 20° peak to peak are possible. The use of such a device would require full cycle decoding i.e., if the apertures through which the beam passes in turn are numbered 1,2,3,4, then to decode the signals using a demultiplexer after the position sensitive photodetector and amplifier, the channels would have to be selected in the sequence 1,2,3,4,4, 3,2,1,1,2, etc. This presents no difficulty using count up – count down logic, but can make correlation analysis more difficult.

Torsional scanners are however expensive and again require a lens to make the beams parallel after reflection.

5.6.1.3 Rotating Prism

It was decided to use an optical quality cubical prism to
FIG59 MULTIPLE BEAMS USING TORSIONAL SCANNER
produce a laser beam scanning parallel to itself without the need for additional lenses. Careful design of the prism mount was required to obtain an uninterrupted scan (see plate 3 and fig. 5.10). A total scan width of 1.3 cm was obtained using a prism 2.5 cm on a side. The prism mount was in turn centrally mounted on the axle of a 90 Watt, 50 Hz, 4000 r.p.m. motor which was firmly fixed to a heavy metal table (plate 4).

As can be seen the laser beams lie in a plane perpendicular or parallel to the base of the test cell. The aperture system (4.7.3) produces 4 samples of each beam signal per revolution of the prism scanner. Single laser beam experiments showed that the maximum frequency of signals from the detector due to turbulence was around 10 Hz measured on an oscilloscope. To record the signals adequately requires a minimum sampling rate of 30 per second according to the Nyquist criterion. At a prism revolution rate of 4000 r.p.m., each aperture is scanned 265 times per second with an interval between samples of 3.7 milliseconds which is more than adequate.

The use of a 45°, 45°, 90°, prism oriented at 45° to the plane of the beams, causes scanning in the horizontal plane when required as shown in plate 4.

Tests showed that planar registration of the scanning beams was obtained to an angular accuracy of 0.5 milliradians, better than required (5.6.1.1). The simplicity, accuracy and economy of this technique led to its adoption for the present purpose.
PLATE 3  ROTATING PRISM SCANNER
PLATE 4 ARRANGEMENT FOR PRODUCING PARALLEL LASER BEAMS IN THE HORIZONTAL PLANE
5.6.2 Aperture System

A series of 4 parallel slots each 1 cm long and 0.5 mm wide were cut in a thin sheet of aluminium with separations of 1, 2 and 4 mm respectively (see fig. 4.10 and plate 4), thus giving spatial correlation measurements at 1, 2, 3, 4, 6 and 7 mm beam separations. The aluminium was coated with matt black paint to eliminate stray laser reflections.

5.7 Electronics for Multiple Beam Work

5.7.1 General Description

The effect of the optical scanner and aperture system was to multiplex signals from 4 beams onto a single photo-detector channel. It was, therefore, necessary to demultiplex the detector output to obtain 4 separate channel signals corresponding to beam deviations parallel to the heater plate, and 4 more for the perpendicular direction.

The detector outputs are added together in the 1 kΩ load resistor (see fig. 5.4), to produce a voltage signal corresponding to the total light intensity falling on the detector. Since this signal will appear each time the laser passes through one of the apertures, it can be used to gate a demultiplexer. A block diagram of the complete arrangement is shown in fig. 5.11. The amplifiers are D.C. coupled for reasons which are explained in section 5.9. This means that the Z signals contain substantial D.C. levels which could saturate the corresponding amplifiers when the beams are in the vertical plane. Hence only X signals are recorded in this case. The reverse is also true when the beams are in the horizontal plane.
FIG 511 CORRELATION EXPERIMENT
The total signal current was used to drive the output enable and channel select gates of an integrated circuit demultiplexer as described in section 5.7.2. The demultiplexer signals were low pass filtered and recorded on a Precision Instruments 61061 FM - tape recorder having a bandwidth from D.C. to 100 Hz. On playback the signals were fed two at a time to a Hewlett-Packard Correlator type 310601, to obtain the correlation coefficients.

5.7.2 Logic Circuits

The logic circuitry for demultiplexing is shown in fig. 5.12 and the associated waveforms presented in fig. 5.13. The total signal current from the detector, flowing in a 1k load resistor develops a voltage at the negative going input of the 741 integrated circuit operational amplifier. The positive going input to this stage is offset by -10 mV by a potential divider to compensate for the 10 mV offset at the negative going input, thus the output (A) of the stage has a baseline at 0V and peak voltage about 5V, the peaks corresponding in time to the peaks of the 4 signals in the multiplexed channel. Waveform (A) is then differentiated to obtain zero crossings (B) at its peaks. This is done using a 741 integrated circuit differentiator with 6 dB per octave roll off asymptotes intersecting at 100 kHz.

This frequency was chosen as a compromise between the need to have a stable system with reduced high frequency noise and to avoid significant phase shift error (53) in the frequency range of waveform (A) which extends from about 270 Hz.
FIG533 LOGIC WAVEFORMS FOR DEMULTIPLEXING
to around 2.5kHz for a prism scanner rotation rate of 4000 r.p.m.

The maximum slope of the input signal is about $5 \times 10^4$ V per second. If the input capacitance and resistance are denoted $C_{in}$ and $R_{in}$ respectively (in series) and feedback components $C_f$ and $R_f$ in parallel

$$V_{out} = C_{in} \frac{R_f}{dt} \frac{dV_{in}}{dt}$$

for a 5V output

$$5 = C_{in} \frac{R_f}{5 \times 10^4}$$

Then if $R_f = 10^4 \Omega$ then $C_{in} = 10nF$

also $C_f = 180 \text{ pF}$ and $R_{in} = 180 \Omega$

The positive feedback loop is resistance controlled for the negative going input of the next stage which produces a rectangular wave output (C) whose falling edges correspond to the zero crossings of the waveform (B). The 10kΩ potentiometer is used to adjust the timing of the rising edge of (C) i.e., the rectangular waveform is triggered at around the 1 volt level of (B). A 709 integrated amplifier is used for the production of (C). With positive feedback, no frequency compensation is required. The 709 is preferred to the 741 as its slew rate is 50V/µS compared with 0.5V/µS for the 741.

One now has waveform (C) with falling edges corresponding in time to the peaks of the original signal and it is these edges which are used to control the output enable of the demultiplexer. The output enable is switched off when peak
signal levels are reached so that if capacitors of appropriate values are connected between the demultiplexer outputs and ground, the end result is an effective sample and hold circuit for each channel.

Pulse train (C) is fed to a differentiator circuit of time constant 500 nS. The BC 507 transistor is held on normally and the positive going pulses resulting from differentiation (D) of (C), do not affect it. The negative going pulses however momentarily switch off the transistor producing positive going pulses (E) about 1 microsecond wide, whose leading edges coincide with the peaks of the waveform (A). Waveform (E) is then passed to NOR gate along with the output (F) from a 74121 (54) monostable integrated circuit. This monostable is driven by the Q₂ output (I) from a 7476 (54) dual masterslave JK flip-flop whose clock (Cle) inputs are both driven by the NOR gate output (G).

With the J and K inputs for one flip-flop at +5V and all clear (Cle) and preset (P) inputs also at +5V the trailing edges of the clock pulses cause output Q₁ to change state. Q₁ is directly connected to the J and K of the second flip-flop so the state of Q₂ only changes when Q₁ has been high immediately before the arrival of a clock pulse trailing edge. Thus the 7476 acts as a counter from 1 to 4 to select the appropriate output channel of the 3705 CD demultiplexer, using outputs Q₁ and Q₂ connected to the 2⁰ and 2¹ channel select gates of the demultiplexer.

The 74121 monostable produces a pulse to the NOR gate.
This pulse is designed to appear at the negative going edge of $Q_2$ (I) and is short enough to appear between laser scans of the 4 aperture system, but too long to fit between pulses due to any one scan. The effect is to reset the counting system when a scan is completed, so that optical signal samples by a particular aperture will always be fed to the same electrical channel output of the demultiplexer. The $Q_2$ output (I) of 7476 is inverted by applying it to the commoned inputs of the NAND section of the 74121. The NAND output is applied to one input of the AND section of the 74121, the other input being held at +5V. This triggers the monostable output when $Q_2$ goes low i.e., after the fourth channel of the demultiplexer has been switched off.

5.7.3 Sample and Hold

Pulse waveform (c) operates the output enable of the demultiplexer, turning it on shortly after the appropriate signal sample pulse (A) arrives, and turning it off when the signal sample pulse has just reached its peak. Waveform (c) corresponds to the total signal waveform (A), its falling edges occurring at the same time as the peaks in (A), while the (A) peaks correspond to peaks in the signals, produced by the deviation of the laser beams.

The ON resistance of each channel is 100 Ohms, so with a capacitance to ground of 0.1 uF, the charging time constant is $10^{-5}$ seconds, fast enough for each channel output to reach the signal sample level each time the latter is updated.

The OFF resistance is $10^{10}$ Ohms so that the discharge time
is $10^3$ seconds and the updated signal level is held easily until the next update $3.7 \times 10^{-3}$ seconds later. Thus the logic system used incorporates excellent sample and hold circuitry requiring only simple low-pass filtering circuits.

5.7.4 Low-Pass Filters

Originally it had been thought, based on rough measurements, that signal frequencies from D.C. up to 100 Hz would be present but subsequent measurements of the half widths of autocorrelation curves showed that significant frequency components up to only about 10 Hz were present. Thus the circuits described here are not appropriate for their purpose as the noise spectrum in each channel is wider than required.

The sampling rate of the system is 270 per second for each channel and it was necessary to eliminate this frequency and its upper harmonics. This was done by passing the demultiplexed signals through low pass filters (fig. 5.14). Each consists of three stages. The first two are straightforward single pole RC types having 3dB turning points at 200 and 400 Hz respectively. Transistors are required for buffering purposes. The third stage is a controlled source low pass active filter (53), whose response peaks at 87.5 Hz with a gain of +1 dB at that point. The response then rolls off at 12 dB per octave. The complete response of the three filter sections is shown in fig. 5.15.

5.8 Signal Recording and Processing

The necessity for recording the signals in some form arises because as mentioned earlier, only about two hours are available for measurements and because of the low frequencies
FIG. 5.15 LOW PASS FILTER CHARACTERISTIC
it is necessary to collect optical distortion and correlation information over periods of several tens of seconds. It is also desirable to carry out measurements at various heights in the liquid nitrogen layer. Generally the procedure followed was to set a fixed power input to the heater plate for a given experimental run, and use the multiple laser beam probe and detector/demultiplexer system to obtain recorded signals corresponding to the four different mean beam positions. The beams can be in a horizontal plane for measurement of eddy lengths and velocities in the X direction, or in the vertical plane for measurements in the Z direction.

Several methods of signal recording are possible, including magnetic tape (analogue or digital) and ultra-violet recording. The method of recording to be used depends on the technique chosen for data processing.

5.8.1 Ultra Violet Recording

The technique of direct writing on ultra violet sensitive paper is useful for visualisation purposes and probability distribution measurements although the analysis is extremely laborious. An SE Laboratories model 2005 uV recorder with 6 channels was available with galvanometer bandwidths from D.C. to 100 Hz. Timing and amplitude scale marks can be written onto the recording paper to facilitate measurement of signal voltage/time coordinates. In practice the use of six channels simultaneously led to much confusion in the tabulation of data for processing. Measurements were made of optical distortion using a single channel and the results agreed well with those
obtained subsequently using more satisfactory techniques.

5.8.2 Digital Magnetic Tape Recording

A proposal was made by the Industrial Electronics Group within the department for an 8 channel magnetic tape recorder to record in analogue form, feeding the data in digital form to a paper tape punch for paper tape processing on an ICL 1905 computer. This system was necessary as it was not possible either to install a direct link or to provide a compatible digitally recorded magnetic tape for computer processing. The system requirement was for 8 channels each of 100 Hz bandwidth so that data rate would be 1600 samples per second. This is beyond the capability of most available punch tape units so the data was to be recorded on a commercial audio tape recorder at 7.5 inches per second with an electronically controlled speed reduction of 8:1 to produce a playback data rate of 25 samples per second per channel. The output format was to be 7 binary bits and one parity bit.

To identify and label the data samples a marker track was to be recorded simultaneously with the data track. Data could be recorded in blocks and on playback a particular channel, block number and sample number could be selected using a set of switches and punching would begin at a sample corresponding to the channel block and sample numbers selected and would continue until the end of the block. Sample counting to 100,000 and block counting to 10 were considered appropriate. An FM system was to be used with signal amplitudes encoded as bursts of a carrier signal of appropriate frequency.
The development of the system took much longer than anticipated and satisfactory performance was never, in fact, achieved. In view of these development difficulties, attention was turned to other techniques for recording and processing the data.

5.8.3 Analogue F.M. Recording

The use of a comparatively simple instrumentation recorder was considered. Again the idea was to record at high speed and playback at slow speed, compatible with a punch tape unit for processing by computer. A Precision Instruments 61061AM-FM tape recorder with a 100:1 tape speed ratio and 4 channels was made available.

The use of this machine meant the sacrifice of 4 of the data channels i.e., optical distortions could be measured in one direction at a time, for the 4 beam channels.

The specification for the recorder at its slowest speed of 0.375 inches per second was:

- frequency response D.C. to 100 Hz ± 1dB (fig. 5.16.)
- signal to noise ratio 30 dB relative to 1 Volt peak signal (2 mm laser beam deviation)
- flutter 4% peak to peak 0.02 Hz to 100 Hz.

The signal to noise ratio was rather low and this would increase the minimum detectable optical distortion to about 0.07 mm which is too large. At the next higher speed of 3.75 i.p.s. the signal to noise ratio was 35 dB for a frequency response from D.C. to 1kHz ± 1dB. If the 100 Hz low pass filter were introduced, this would increase the signal to noise ratio to
FIG 5.16 TAPE RECORDER FILTER CHARACTERISTIC

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45dB giving a minimum detectable distortion of 0.01 mm which would be tolerable. This of course would reduce the speed ratio available to 10:1 recording at 37.5 i.p.s. and playback at 3.75 i.p.s. Recording data at 37.5 i.p.s. would in any case mean that a great deal of tape would be used in a given experiment, typically about 100,000 feet. Editing, replay and processing were attempted but proved extremely laborious. It did not therefore seem to be worthwhile to adopt this approach.

5.8.4 Correlator

A Hewlett-Packard 3721A correlator was obtained on loan and, using this, r.m.s. signal measurements could be made using the autocorrelation coefficient of each signal. Also cross correlation coefficients could be determined by playing back two tape recorder channels at a time. Using a recording/playback tape speed of 3.75 i.p.s., an S.N.R. of 45 dB referred to 1 volt signal level is possible using the 100 Hz low pass filter. This gives a noise equivalent displacement of 0.01 mm. While still very laborious, each tape having to be replayed a total of 10 times, apart from calibration (see section 5.9), this method obviated the need for computer processing. It was found that an experiment could be carried out and results processed in a period of about 45 hours. Consequently this general approach was adopted as being the best compromise available at the time.

5.8.5 Filtering of Tape Recorder Output

Two problems arise in connection with the playback of signals from the tape recorder into the correlator. One is that
there are substantial D.C. levels on the signals whose purpose will be explained later (5.10) and these can effectively swamp any variations in the correlation coefficient. They can be removed by switching in the correlator filters which are high pass having 3 dB points at 1 Hz, but this was considered rather too high as there are frequency components below 1 Hz in the signals. Accordingly high pass filters with 3 dB point at 0.04 Hz were inserted between the tape recorder and correlator which was then D.C. coupled.

Another difficulty is the 100 Hz ripple on the laser output due to the ripple on the laser power supply voltage. This ripple appears as a sinewave modulation on the auto and cross correlograms displayed by the correlator and leads to uncertainty in the measurement of the correlation coefficients. Accordingly twin-T networks with notch frequencies at 100 Hz were cascaded with the high pass filters described above. The transfer function of the resulting filter section is shown in fig. 5.17. Finally the complete system transfer function is shown in fig. 5.18.

5.9 System Calibration

Because of possible differences in gain between the 4 channels, it is necessary to calibrate each of them. The optical transmission of liquid nitrogen at 632.8 nm, is almost 1.0 and in principle the calibration could be carried out with the cell empty.

However, it was found during experiments with the cell filled with liquid nitrogen, that small traces of vacuum pump oil and
FIG 517 RESPONSE OF FILTER SECTION BETWEEN TAPE RECORDER AND CORRELATOR
other pollutants tended to condense on the vacuum sides of the windows which were in contact with the liquid nitrogen. The windows were in fact acting as cold traps for oil vapour and other impurities present even under high vacuum conditions. This resulted in substantial variations in the window transmission over its surface.

This problem only matters when one is measuring optical distortions. For cross-correlation work, normalisation of the coefficients eliminates variations in the transmitted laser intensity between beams. If a calibration is carried out at the beginning of an experiment (i.e. after filling the cell) then that calibration holds good until the beams are moved to a different height in the cell. Then a new calibration must be carried out. In general the procedure adopted was to carry out a calibration at the same time as a recording of the 4 signals was being made on the FM tape recorder. This is the reason for maintaining the D.C. levels of all signals right through to the recording stage.

Each recording was made for a period of two minutes duration to provide an adequate signal length for subsequent optical distortion and correlation analysis. Then the detector was moved to a new position along the horizontal axis (d^2 x) measurements) and a new recording made. Up to 8 detector positions were set in this way (giving 8 calibration points) care being taken that the combined D.C. (calibration) and A.C. (data) signals from the demultiplexer and filter outputs, did not overload the recorder. This required careful setting up of
the recorder gain controls at the outset for a given calibration/data collection procedure.

On playback the signals from each of the 4 channels were in turn fed to a data logger and punch tape unit (55). This samples the signals at a rate of up to 9 per second outputting the results in computer compatible paper tape form.

A simple machine code programme was used with a high-speed reader and PDP8 minicomputer to produce the mean and standard deviations for each signal record. Then using a Hewlett-Packard 9100A desk calculator the system calibration curves were computed. A flow diagram of the complete data collection and processing procedure is shown in fig. 5.19.

5.10 System Performance

Typical system calibration curves are plotted in fig. 5.20. The table below gives the slopes of the curves and the correlation coefficients defining the closeness with which the curves approximate straight lines. A value of 1.0 indicates a perfectly straight line.

<table>
<thead>
<tr>
<th>Channel No.</th>
<th>Slope V. mm⁻¹</th>
<th>Correlation Coeff.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.463</td>
<td>0.994</td>
</tr>
<tr>
<td>2</td>
<td>0.558</td>
<td>0.998</td>
</tr>
<tr>
<td>3</td>
<td>0.514</td>
<td>0.998</td>
</tr>
<tr>
<td>4</td>
<td>0.630</td>
<td>0.999</td>
</tr>
</tbody>
</table>

Root mean square noise levels were 0.025 Volts on all channels, resulting in noise equivalent optical distortions of 0.05, 0.04, 0.05 and 0.04 mm respectively for the 4 channels. In fact if the beam is maintained within ± 2.0 mm of the detector centre
FIG 59

DATA COLLECTION AND PROCESSING

- optical distortions

- calculate

- determine calibration coefs (V/nm)

- cross correlation coefs

- normalise cross correlation coefs

- auto (channel) cross correlation & coeffs (V/rnm)

- auto (channel) cross correlation & coeffs (V/rnm)

- data logger & PTU

- FM record & replay

- apertures & test cell
FIG. 5.20 SYSTEM CALIBRATION
(or null position), noise equivalent optical distortions of less than 0.03 mm can be obtained which compares well with the expected value of 0.01 mm (see 5.8.4).

It should be noted that the above figures were obtained with the cell empty of liquid nitrogen. The slope factors used in the computation of turbulent optical distortions, were obtained as described (5.9) with the cell filled with liquid nitrogen. Obviously the latter figures had larger standard deviations due to the effects of turbulence. The above figures are presented in order to indicate the ultimate resolution of the system which is the same whether or not the cell is full.

5.11 Conclusions

Referring to the predicted figures for optical distortion (section 2.10), it is clear that the system will accurately measure optical distortions for values of ΔT down to less than 0.1K. Since the heater system is unlikely to admit of such fine temperature control, the optical probe system as described should be more than adequate and indeed capable of coping with a factor of 10 either way on the predicted values of optical distortion.

It is also possible to use the probe for measurement of optical distortions in any system where turbulence could be expected to produce similar effects. Examples of interest are in atmospheric turbulence and its effects on optical communication links, the measurement of eddy velocities and sizes in turbulent pipe flows and rocket exhausts, as well as
engine exhausts and aircraft wakes. The system could also be used with some modifications to test rotating optical and mechanical components for errors in pitch, yaw and roll as well as surface finish defects.
6.1 Results

6.1.1 R.M.S. Beam Deviation As A Function of Temperature Gradient and Heat Flux

Experiments were carried out using a single c.w. laser beam propagating through the liquid nitrogen parallel to the cell base, at a height of 5.0 cm. The r.m.s. beam deviation was measured at various heat fluxes, and measurements were made simultaneously of the thermocouple temperatures. A plot of $d^{2\zeta}$ (r.m.s. beam deviation in a direction parallel or perpendicular to the cell heater plate) against $\Delta T$ (mean vertical temperature difference taken over six thermocouple pairs) is shown in figure 6.1.

As can be seen there is some, albeit small, measure of agreement between the experimental points and the curve obtained from Townsend's (16) empirical equation for temperature fluctuation

$$\beta^2 \sigma^2 = 2.56 \sigma_0^2 (Z/Z_0)^{-1.2}$$

whence one obtains using equations 2.6, 2.7, 2.8.

$$d^{2\zeta} = 4.48 \times 10^{-4} \Delta T^{2.8/3} Z^{-0.7} L$$

for the cell dimensions used. But it is clear that at higher values of $\Delta T$ the results obtained diverge increasingly from the expected values.
FIG. 6.1 R.M.S. LASER BEAM DEV n d^2 \frac{1}{2} v. MEAN TEMP DIFFERENCE \Delta T

BEST FIT

TOWNSEND (ref 16)
An examination of the final values of $\Delta T$ obtained for a given input heat flux showed that $\Delta T$ was much lower than expected. Some of the reduction in $\Delta T$ was due to heat loss through the perspex sides of the cell and downwards from the heater plate although measurements of temperatures at the upper and lower surfaces of the plate showed a negligible temperature gradient across it. It was not possible to measure the temperature gradients across the cell walls but these were considered to be negligible. It was found that certain pairs of thermocouples gave consistently higher values of $\Delta T$ than others at a given heat flux. This suggested a variation in the thickness of the adhesive layers between the thermocouples and the heater plate or reservoir base. A typical value of thermal conductivity $K$ of the resin adhesive (56) is $3.0 \times 10^{-3}$ W/cm.K.

Whence $Q = KA \frac{dT}{dl}$

$A = \text{cross sectional area of the layer, } \frac{dT}{dl} \text{ is the temperature gradient across the layer and } Q \text{ is the heat flux through it, per unit time.}$

Hence $\delta T = \frac{Ql}{KA}$

For a layer thickness of 0.5 mm and a thermocouple mounting plate (copper) of area 1 cm$^2$ and a total heat flux of 0.08 W/cm$^2$ (at 300 W input), $\delta T$ is 1.5°, so that a total error of 3°K is introduced by the two resin layers. In other words for a given pair of thermocouples, $\Delta T$ may be underestimated by up to 3K depending on the heat flux and the thickness of the adhesive. However, the main purpose of the thermocouples was to establish that a negative temperature gradient was obtained without
necessarily being able to measure it accurately and this aim was satisfactorily achieved.

A plot of $d^2$ versus $\Delta T$ corrected for temperature loss, is shown in fig. 6.2 and much closer agreement is obtained between the experimental results and those obtained from the Townsend equation.

Assuming no significant heat losses present, $d^2$ was next plotted against $\Delta T$ as derived from the expected value based on the heat input

$$\Delta T = 0.048 W^\frac{1}{4}$$

(see eq. 2.6)

where $W$ is the total heat input. Much closer agreement between experiment and theory has again been obtained (see fig. 6.3).

The vertical error bars represent errors in the measurement of beam height ($\pm 0.1$ cm) and statistical errors.

Statistical errors arise from two sources. One is the fact that calibration of the position sensitive photodetector was necessarily carried out during the experiment, so each point on the calibration curve is subject to a statistical error because of the turbulence itself, resulting in an error on the value of the calibration factor (volts per cm of beam deviation) of about 5%. There is also a statistical error $\epsilon$ in the measurement of the r.m.s. voltage output from the detector which depends on $T$ the duration of the recorded signal (37) given by

$$\epsilon = 1/\sqrt{BT}$$

where $B$ is the bandwidth of the signal, about 10 Hz, and $T = 80$ seconds usually. Therefore $\epsilon$ varies between 4 and 5%.
FIG. 6.2. $d^2$ v $\Delta T$ (corrected)
FIG. 6.3. $\vec{q} \cdot \nabla (\Delta T) \quad \text{(derived from heat flux)}$

TOTAL HEAT $I/P$ (Watts)

- 50
- 100
- 200
- 300
- 350
- 400
- 450
- 500

HEAT FLUX Wcm$^{-2}$

- 40
- 80
- 60
- 40
- 20
- 10
- 20
- 30

$X$
The total error on any measurement of $\frac{1}{\sqrt{\sigma^2}}$ is around 10%.

It was found that the signals were stationary over periods of 20 minutes in that all values of the autocorrelation function of the signal were within the statistical error over that period. This of course was only true when a steady value of $\Delta T$ was attained, usually 10 to 15 minutes after application of a given heat flux.

6.1.2 R.M.S. Deviation As A Function of Beam Height

Using Townsend's equation (2.3) for $\theta^2$ it can also be predicted that $\frac{1}{\sqrt{\sigma^2}} = 2^{-0.7}$ (see eq. 6.1) where $Z$ is the height beam in the fluid layer. The equation strictly applies to a layer heated from below, and extending indefinitely upwards. It is necessary to make some assumption about the effect of the upper boundary (the cooling reservoir). It has already been shown that there is some agreement between the Townsend results and the predictions of the Malkus theory (12) if the assumption is made that in a two boundary system, the functional dependance of $\theta^2$ is mirrored in a plane at $Z = D/2$ where $D$ is the depth of the fluid layer. The results for a two boundary system, obtained by Thomas and Townsend (11) also support the assumption. It was used again to obtain the theoretical plot of $\frac{1}{\sqrt{\sigma^2}}$ versus $Z$.

Values of $\frac{1}{\sqrt{\sigma^2}}$ were measured at various heights in the fluid, 4 values of $Z$ being available at each setting, by using the multiple beam technique described earlier in Chapter 5. The main purpose of the multiple beam system was for correlation work and the beams were closely spaced (maximum separation 7.0 mm)
so it was necessary to move the beams several times during an experiment, in order to cover an adequate range of Z values. Thus 4 or 5 groups of closely spaced points were plotted for each heat flux (figs. 6.4, 6.5, 6.6, 6.7). The results show fair agreement with what one would expect from the Townsend equation (2.3) particularly in the upper half of the cell. Nearer the heated plate, the agreement is not so good. The results imply that, particularly for lower heat fluxes, fully developed turbulence only begins, some distance away from the heater plate, though there is no theoretical basis for this. Visual study of the turbulence did at times indicate that turbulence was relatively confined to the upper half of the cell.

It is expected that turbulence effects and consequent optical distortion should be at a minimum in the central part of the cell away from the boundaries and this was found to be roughly the case.

6.1.3 Autocorrelation Measurements

The autocorrelation function is \( (37) \)

\[
R_{ii}(\tau) = \frac{1}{T} \int_{0}^{T} v_i(t)v_i(t + \tau)dt
\]

where \( v_i(t) \) is the instantaneous value of the voltage appearing at a demultiplexer output \( i \) \( (i = 1, 2, 3, 4) \) corresponding to a fluctuation in position of the laser beam \( i \) at the detector. The autocorrelation plots were obtained using the Hewlett Packard Correlator and display unit by playing back the recorded signals \( v_i(t) \) from the tape recorder.

The autocorrelation function can be used to obtain information about the bandwidths of the signals because the signal power spectral density \( G_{ii}(f) \) and autocorrelation function form
FIG. 6.4 $\frac{1}{2}d^2v \frac{1}{Z}$ (30W I/P)

- Townsend Theory
a Fourier transform pair \((27)\),
i.e. \[ G_{ii}(f) = 2 \int_{0}^{\infty} R_{ii}(c) \cos 2\pi f c \, dc \]
\[ R_{ii}(c) = \int_{0}^{\infty} G_{ii}(f) \cos 2\pi f c \, df. \]
The autocorrelation function is usually normalised so that its value is 1 at \( c = 0 \). The normalised value is thus
\[ R_{Nii}(c) = \frac{R_{ii}(c)}{R_{ii}(0)}. \]
Normalised plots of experimentally obtained autocorrelation curves are shown in fig. 6.8. Their general form appears to be
\[ R_{Nii}(c) = \exp(-\omega_0^2 c^2) \]
The Fourier transform of this function is
\[ G_{ii}(f) = (\sqrt{\pi}/2\omega_0) \exp(-\omega^2/4\omega_0^2) \quad (\omega_0 = 2\pi f_0) \]
It is found in practice that wideband signals have narrow autocorrelation functions i.e. the autocorrelation function only has an appreciable value over a small range of values of the time delay \( c \). The converse is also true. Assuming that the width of the exponentially decaying autocorrelation function is given by the value of \( c \) which satisfies the relation
\[ R_{Nii}(c) = (1/e)R_{Nii}(0) \]
one can estimate values of the signal bandwidth \( f_0 \) from the autocorrelation plots.

The following table shows the relevant values of the height \( Z \) of the laser beam in the fluid, the heat input and the value of the bandwidth as derived from fig. 6.8.
FIG 6.8 NORMALISED AUTOCORRELATION FUNCTIONS
<table>
<thead>
<tr>
<th>Input (Watts)</th>
<th>Height Z (cm)</th>
<th>( f_0 ) (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>75</td>
<td>2.15</td>
<td>1.1</td>
</tr>
<tr>
<td>75</td>
<td>3.35</td>
<td>0.6</td>
</tr>
<tr>
<td>75</td>
<td>5.50</td>
<td>0.7</td>
</tr>
<tr>
<td>75</td>
<td>7.85</td>
<td>0.6</td>
</tr>
<tr>
<td>150</td>
<td>8.40</td>
<td>1.0</td>
</tr>
<tr>
<td>225</td>
<td>2.10</td>
<td>1.3</td>
</tr>
</tbody>
</table>

Table 6.1

One would expect the bandwidths to be greatest near the horizontal boundaries of the layer where the turbulence intensity is greatest and also to increase with heat input. Neither of these trends is confirmed by the results obtained. In fact the signal bandwidths contain frequencies up to 10 Hz but the amplitudes at these frequencies were found to be about 5 dB down on the amplitudes at around 0.5 Hz, as oscilloscope traces of the signals showed.

The accuracy of measurement of the autocorrelation function \( R_{Nii}(\tau) \) is given by

\[
\varepsilon = \left( \frac{1}{2 f_0^2} \right) \left( 1 + \frac{1}{R_{Nii}^2(\tau)} \right)^{\frac{1}{2}}.
\]

\( T \) = record length in seconds (80 seconds)
\( \varepsilon \) = normalised standard error

When \( R_{Nii}(\tau) = 1/\varepsilon \) one obtains for the error \( \varepsilon \) a value of about 0.1. This error is rather large and in order to reduce it larger record lengths would be required.

6.1.4 Correlation Length and Its Dependence

Using a Hewlett Packard Correlator and display unit, the tape recorded signals from the 4 demultiplexed channels,
were cross-correlated to obtain the cross-correlation function

\[ R_{ij}(\tau) = \frac{1}{T} \int_{0}^{T} v_i(t)v_j(t + \tau)dt \quad (i \neq j = 1, 2, 3, 4) \]

\( v_i(t) \) and \( v_j(t) \) being instantaneous values of the voltages appearing at two of the demultiplexer outputs \( i \) and \( j \), corresponding to fluctuations in position of laser beams \( i \) and \( j \) at the detector. The autocorrelation functions \( R_{ii}(\tau) \) were also obtained since these are required to obtain normalised cross-correlation coefficients \( R(\ell) \) where

\[ R(\ell) = \frac{R_{ij}(0)}{R_{ii}(0)R_{jj}(0)} \]

Plots were then obtained of \( R(\ell) \) versus \( \ell \) both as a function of heat input and height in the fluid (figs. 6.9, 6.10, 6.11). It is clear from the experimental geometry that the spatial correlation curves obtained as described (see Chapter 5) represent values of \( R(\ell) \) integrated over the optical path. It was not possible to examine the correlation length as a function of distance from a window as this would require the use of perpendicularly crossed beams. The vertical error bars represent the statistical errors on the mean value of \( R(\ell) \) taken over 6 to 8 measurements, each consisting of 80 seconds duration of recorded signal for each channel. In all cases a value of \( R(0) \) of 1.0 was assumed.

From Tatarski (23) \( R(\ell) = \exp(-\ell^2/a^2) \)

where \( a \) is the correlation length or eddy diameter. Curves of this form were fitted to the experimental points to determine the values of \( a \), making the assumption that \( R(\ell) \) decays to 0 at large \( \ell \). In fact non-zero asymptotic values of \( R(\ell) \) are obtained but these are probably due to correlated
FIG. 6.9 \( R(l_z) \) vs. \( l_z \) at various heights \( Z \) (beams in vertical plane)
FIG. 6-9 (cont'd): $R(l_x) v l_x$ at various heights $Z$  
75W I/P

(beams in horizontal plane)
FIG. 610 $R(l_Z) \nu l_Z$ at various heights $Z$

196W I/P

(beams in vertical plane)
FIG. 610 (cont'd) $R(l_x)$ v $l_x$ at height $Z=19$ cm. 196W I/P

(beams in horizontal plane)
FIG. 6.11  $R(l_z)$ vs. $l_z$ at various heights $Z$. 294 W I/P

(beams in vertical plane)
FIG. 611 (contd) $R(l_x) \times l_x(mm)$ at various heights $Z$ 294W I/P

(beams in horizontal plane)
The value of $a$ is obtained from the relation
\[ \exp\left(-\frac{x^2}{a^2}\right) = \frac{1}{e}. \]

The table below gives the values of the parameters obtained.

The $Z$ values given correspond to the lowest beam when the 4 beams lie in the vertical plane; in this case it is the horizontal fluctuations of the beams which are cross-correlated to give the vertical dimensions of the eddies. When the beams are in the horizontal plane, the vertical fluctuations of the beams are cross-correlated to give the horizontal dimensions of the eddies.

<table>
<thead>
<tr>
<th>Input ($w$)</th>
<th>$Z$ (cm)</th>
<th>$a$ (Theory)</th>
<th>$a_x$ (Expt)</th>
<th>$a_x$ (Expt)</th>
</tr>
</thead>
<tbody>
<tr>
<td>75</td>
<td>1.8</td>
<td>2.6</td>
<td>2.8</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3.0</td>
<td>2.9</td>
<td>3.0</td>
<td></td>
</tr>
<tr>
<td></td>
<td>5.2</td>
<td>3.2</td>
<td>2.9</td>
<td></td>
</tr>
<tr>
<td></td>
<td>7.5</td>
<td>2.8</td>
<td>2.8</td>
<td></td>
</tr>
<tr>
<td></td>
<td>5.0</td>
<td>2.5</td>
<td>2.5</td>
<td>2.5</td>
</tr>
<tr>
<td></td>
<td>1.9</td>
<td>3.0</td>
<td></td>
<td>3.0</td>
</tr>
<tr>
<td>196</td>
<td>1.4</td>
<td>2.1</td>
<td>3.1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2.8</td>
<td>2.4</td>
<td>4.0</td>
<td></td>
</tr>
<tr>
<td></td>
<td>4.5</td>
<td>2.6</td>
<td>3.8</td>
<td></td>
</tr>
<tr>
<td></td>
<td>7.9</td>
<td>2.3</td>
<td>3.8</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1.9</td>
<td>2.0</td>
<td></td>
<td>3.8</td>
</tr>
<tr>
<td>294</td>
<td>1.7</td>
<td>2.0</td>
<td>3.4</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3.4</td>
<td>2.3</td>
<td>3.2</td>
<td></td>
</tr>
<tr>
<td></td>
<td>5.0</td>
<td>2.4</td>
<td>3.7</td>
<td></td>
</tr>
<tr>
<td></td>
<td>8.4</td>
<td>2.0</td>
<td>3.4</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1.8</td>
<td>2.5</td>
<td></td>
<td>4.6</td>
</tr>
<tr>
<td></td>
<td>7.5</td>
<td>2.5</td>
<td></td>
<td>4.3</td>
</tr>
</tbody>
</table>

Table 6.2
It is clear that there is no obvious trend with either height $Z$ or input flux except at the lowest heat input of 75W. One would expect $a$ to be maximum both at lowest heat input and in the middle of the cell, decreasing towards the boundaries, according to the Rayleigh stability criterion

$$a^3 \theta^2 = 658 \frac{Kv}{\beta g}$$

where $\theta^2$ is the r.m.s. temperature fluctuation at height $Z$, $K$ is the thermometric conductivity, $\beta$ is the coefficient of cubical expansion, $v$ is the kinematic viscosity and $g$ is the acceleration due to gravity. Combining this equation and equation 2.3 one finds that

$$a \propto Z^{0.2}$$

and

$$a \propto W^{-0.2}$$

i.e. the dependence of $a$ on $Z$ and $W$ is very weak, and may well be swamped by the variation in $a$ along the beam path.

There is some indication that the correlation coefficient is maintained over a greater distance in the horizontal plane than in the vertical plane. This may be due to the fact that $a$ only varies with height, and is expected to be constant in the horizontal plane. Thus when the beams are in the vertical plane they cover a vertical range of 7 mm and the value of $a$ will vary, albeit slightly, over this range.

The results in general are quite encouraging as they agree with expected figures within the resolution limit of the system. It is desirable that in future experiments, perpendicularly crossed beams be used to determine the values of $a$ at various
positions in the cell, particularly in the window, heater plate and reservoir boundary regions.

6.1.5 Turbulence Velocities

Again using the multiple parallel beam techniques described earlier the velocity fluctuations were measured by time-delayed cross correlation of signals produced by two beams of known separation. The cross-correlation coefficient

\[ R_{ij}(\tau) = \frac{1}{T} \int_0^T v_i(t) v_j(t + \tau) dt \]

where \( v_i(t) \) is the signal at the detector produced by positional fluctuations of beam \( i \) and \( v_j(t + \tau) \) is the signal produced by positional fluctuations of beam \( j \) at the detector, with \( i \neq j = 1, 2, 3, 4 \) and \( \tau \) is the time delay between the two signals.

Figures 6.12 and 6.13 show typical traces of the cross-correlograms. Vertical scales are arbitrary. The time delay between peaks in the traces corresponds to the time taken for eddies to travel between the two beams \( i \) and \( j \) and hence knowing the separation distance between the beams, eddy velocity estimates may be made. The following table gives some typical values of the velocity of eddies obtained by measurements on cross-correlation peaks. Again the value of \( Z \) corresponds to the height of the lowest beam. The error on a given estimate of \( R_{ij}(\tau) \) is given by

\[ \varepsilon = \left( \frac{1}{\sqrt{2T^2}} \right) \left( 1 + \frac{1}{R_{ij}(\tau)} \right)^{\frac{1}{2}} \]

i.e. for a single sample of 80 seconds duration (\( T \)), \( \varepsilon = 0.15 \) when \( R_{ij}(\tau) = 0.2 \) of the peak value. For this reason it was often difficult to decide whether a given cross-correlation peak was significant. Much longer signal samples would be required to improve these measurements.
FIG. 612 CROSS-CORRELATION COEFFICIENT $R_{ij}(\tau)$ versus $\tau$(sec)
b) I/P 196W  Z=14 cm. 
    beam sepn. = 4 mm.

a) I/P 196W  Z=1.4 cm. 
    beam sepn. = 4 mm.

COCO

c) I/P 196W  Z=19 cm. 
    beam sepn. = 7 mm. 
    (in horizontal plane)

d) I/P 196W  Z=28 cm. 
    beam sepn. = 3 mm.

e) I/P 294W  Z=4.65 cm. 
    beam sepn. = 4 mm.

f) I/P 294W  Z=8 cm. 
    beam sepn. = 2 mm.

FIG 6-13 CROSS-CORRELATION COEFFICIENT $R_{ij}(\tau)$ v $\tau$(sec)
The results agree within a factor of 2, for the smaller beam separations, with the predictions of Townsend's equation which states that for $Z/Z_0 = 100$, the velocities are of order $7u_0$ (see section 2.8.2). Thus for $Z$ about 1.0 cm and heat input ranging from 75 - 300 W, velocities range from 6.8 to 9.6 mm.s$^{-1}$. Malkus' (12) theory predicts values of velocities averaged over the total cell volume to be between 4.0 and 10.0 cm.s$^{-1}$. Thus the Townsend equation seems to be the more appropriate.
There is also evidence (Fig. 6.13 c and Table 6.3) of a slow horizontal component of velocity, indicating circulatory motion as expected with horizontal fluid layers of finite extent (see Chapter 2).

6.1.6 Discussion and Conclusions

The experimental measurements of $\overline{d^2}$ the r.m.s. laser beam deviation, in directions parallel and perpendicular to the heater plate are in fair agreement with predicted values based on Townsend's empirical results for r.m.s. temperature fluctuations in air over a heated plate. The agreement is found both for $\overline{d^2}$ as a function of $\Delta T$ and as a function of height $Z$ in the fluid.

Results for turbulence eddy sizes and fluctuating velocities are also in quite good agreement with expected values, although no significant trends in eddy size or velocity with $\Delta T$ or $Z$ have been detected.

The main implication is that one can extrapolate from results obtained at room temperatures in air, to predict turbulence parameters and optical effects in bubble chamber fluids.

It has also been found, as expected, that in the centre of the cell well away from all boundaries, that the optical distributions are minimised. There is also some suggestion that turbulence effects near the warm boundary may not be so serious as those at a corresponding distance from the upper, cooled boundary, but further experiments are needed to verify this.
There is also some evidence that a limited amount of free boiling is permissible in order to reduce the optical distortion at a given heat flux. This is indicated by the fall off in $\frac{1}{d^2}$ at higher heat fluxes, from the predicted values. Boiling was observed at these higher fluxes, usually well away from the region of optical access. It is believed that some of the energy normally used to maintain the vertical temperature gradient, is lost through boiling.

6.2 General Discussion

6.2.1 Modifications to Present Equipment and Recommendations for Further Work

The minimum r.m.s. beam deviation which could be measured with the present equipment is about 0.03 mm. At the lowest heat input of $20 W \ (\Delta T = 0.5 \ K) \frac{1}{d^2}$ at $Z = 5 \ cm$ will be about 0.1 mm. This is for a total path length of about 140 cm (including 90 cm in liquid nitrogen). Thus the r.m.s. angular deviation is 0.07 milliradians and this deviation is proportional to the square root of the optical path in liquid nitrogen.

The minimum r.m.s. angular beam deviation that can be detected at present is about 0.02 milliradians. It follows that the path length in liquid nitrogen could be reduced by a factor of 3.

Also the signal channel bandwidth is at present about 100 Hz., and the maximum signal frequency is around 10 Hz at 300 W heat input, so channel bandwidths would be reduced by a factor of six or more. This would increase the signal to noise ratio or if the SNR is maintained at its present value, the length of the cell could be further reduced.
To increase the degree of optical access, it would be worth while to increase the window diameter to about 15 cm. The total result would be to halve the volume of liquid nitrogen in the cell and it might be possible to have a continuously refrigerated system. This would eliminate the laborious process of filling the test rig and greatly increase the time available for experiments.

To carry out crossed beam correlation experiments (see Chapter 4, section 4.5) windows should be built into the cylindrical walls of the chamber. Careful geometrical arrangement of the windows would be necessary so that crossed beam correlation measurements could be carried out as close to the windows as possible. Figure 6.14 shows one possible design of a new test rig, to enable crossed beam correlation to be carried out. The cell has a horizontal to vertical aspect ratio of almost unity and this may result in some circulatory motion of the turbulent fluid (9,11). The overall length of the rig would be as low as 50 cm making far greater ease of operation as well as a reduction in running costs. The filling and venting lines are not shown, being arranged as in the original design. Filling could be done by atmospheric pressure siphoning. After cooling down to 77 K, 14 litres would be required for filling. A 65 litre dewar of liquid nitrogen would probably suffice for cooling and filling.

In the original system, the time available for experiments after a single filling was about two hours, after which the pressure in the test cell rose sharply (figs. 3.5 3.6).
FIG. 6K REDESIGN OF LIQUID NITROGEN TEST RIG
This happened when the liquid level in the main cylinder and test cell had fallen below the reservoir base. The total volume of liquid nitrogen boiled off in the two hour period was about 18 litres i.e. 9 litres per hour. In the new version of the rig this volume would be about 1 litre and the heat input would be reduced by a factor of 8. So the time available for experiments would be around one hour if continuous refrigeration were not possible.

The smaller total volume of the rig suggests the possibility of enclosing it in an outer jacket having windows for optical access and evacuating the space between to provide effective thermal insulation from the external environment. This would help to increase the experiment time. Another possibility would be to use all the space above the cell as the reservoir volume, by inserting a shelf into the cylinder above the cell. This would need support against the pressure in the cell and presents serious problems in fabrication.

Probably the easiest method of achieving suitable experimentation time is to have liquid nitrogen flowing continuously into and out of the reservoir. The reservoir volume is about 1 litre and a flow rate of 0.25 litre per minute would suffice to maintain a stable temperature difference for as long as required. The total usage of liquid nitrogen would be 15 litres per hour plus 65 litres for initial filling.

The redesigned rig would enable much more comprehensive data to be obtained on laser beam deviations as functions of height in the fluid and heat input, as well as providing data on...
eddy sizes and velocities as functions of positions in the
cell including the window and heat exchanger regions.

6.2.2 Scaling to Bubble Chamber Liquids

The primary purpose of this work is to determine the
magnitudes of optical distortions occurring in a liquid hydrogen
bubble chamber. Consequently the results obtained must be
scaled using dynamical similarity, from liquid nitrogen, the
fluid used, to liquid hydrogen. Now it has been shown that
the empirical equations obtained by Townsend (16) predict with a
good degree of accuracy the conditions which prevail in
turbulent liquid nitrogen. It follows that the Townsend equations
can be used with confidence to estimate the effects in liquid
hydrogen as this involves an extrapolation over a further 50 K.
This is reasonable as an extrapolation of about 220 K has been
justified by the results obtained.

The effects in a hydrogen bubble chamber are twofold. The
first is due to turbulence arising from static temperature
gradients within the chamber and the second due to the internal
heat generated by irreversible work done on the liquid as the
chamber is cycled.

6.2.2.1 Steady Temperature Gradients

6.2.2.1.1 Optical Distortion

Townsend's equation for $\theta^2$ was quoted for a range of $Z/Z_0$ (16)
between 6 and 100. The results for liquid nitrogen suggest that
the range of validity of the equation can be extended up to 500
and it is reasonable to assume that the range may extend further.
The main point which is borne out by the present results, is that
$\frac{\sigma}{2}$ does not tend towards an asymptotic value far from the boundaries, as suggested by Thomas (5).

As an example the Batavia National Accelerator Laboratory liquid Hydrogen Chamber has a diameter of approximately 4.6 metres. The author has been unable to obtain any data on the temperature control of the chamber but assuming that it is possible to maintain a constant vertical temperature difference of 0.1°K between the top and bottom of the chamber then the constant upward flux of buoyancy

$$Q = \frac{\Delta T}{c_p} \left( \frac{K_B}{\beta g} \right)^{\frac{1}{2}} (0.08) = 2.97 \times 10^{-5} \text{ cm/sec.} \quad \text{(see eqs. 2.1, 2.6)}$$

$$Z_0 = \left( \frac{K^2}{Q g} \right)^{\frac{1}{2}} = 1.66 \times 10^{-2} \text{ cm}$$

and if a bubble appears in the centre of the chamber at a height of 200 cm, then

$$Z/Z_0 = 1.2 \times 10^4$$

$$\theta_0 = \left( \frac{3}{gR} \right)^{\frac{1}{2}} = 3.78 \times 10^{-4} \text{ K}$$

Therefore $\theta_0 = 8.05 \times 10^{-5} \text{ K} \quad \text{(see eq. 2.3)}$ and the r.m.s. refractive index fluctuation

$$\mu_2^{\frac{1}{2}} = 2.5 \times 10^{-3} \quad \text{and} \quad \sigma_2^{\frac{1}{2}} = 2.01 \times 10^{-7} \text{ K} \quad \text{(see eq. 2.7)}$$

The diameter $a$ of an eddy is given by

$$a = \left[ \frac{658 K \nu}{\beta g \sigma_2} \right]^{\frac{1}{2}} = 0.8 \text{ cm.}$$

and the r.m.s. angular deviation of a ray of light reflected from the bubble in a deviation parallel to the chamber base.

$$\epsilon_2^{\frac{1}{2}} = \left( \frac{4 \pi}{\mu_2} \right)^{\frac{1}{2}} = 6.3 \times 10^{-6} \text{ radians} \quad \text{(eq. 2.9)}$$

Thus the optical distortion $d_2^{\frac{1}{2}}$ in the film plane (assuming unity magnification) is 14 microns. This is negligible compared with the size of the bubble, 700 microns.
However the situation can be expected to be much worse nearer the boundaries; at a height of 2.5 cm above the base the r.m.s. angular deviation would be 0.236 milliradians so the optical distortion over a path of 2.3 metres would be 544 microns. The size of the bubbles in a large modern chamber is about 700 microns.

Of course the effect of these distortions is greatly reduced since measurements are made on extended bubble tracks rather than single bubbles, and such distortions may not matter in practice. The problem would be rather more serious for short particle tracks particularly in the region of vertices. Highly curved tracks such as would arise with the use of high flux superconducting magnets may prove difficult to measure accurately through a large depth of fluid.

6.2.2.2 Velocity Effects

It has already been established that the Townsend equation (16) for velocity fluctuations, gives reasonable agreement with the experimental results in liquid nitrogen. For a temperature difference of 0.1 K in liquid hydrogen the value of

\[ u_o = (Qgk)^{\frac{1}{4}} = 7.8 \times 10^{-2} \text{ cm.s}^{-1} \] (see eq. 2.12).

Townsend's equation predicts that the velocity fluctuations will be about \( 7u_o = 5.5 \text{ mm/sec} \). Modern bubble chambers using Scotchlite brightfield systems require bubble growth times of about 4 millisec before photographic records are made. Thus there would be a r.m.s. bubble displacement of 22 microns before the photograph is taken. This is insignificant compared to the optical distortions calculated above. It can be assumed
therefore that velocity fluctuation effects on photographically recorded particle tracks will be negligible.

6.2.2.3 Eddy Sizes

While this aspect is of secondary importance, as the effect of eddy size is already included in the optical distortion calculations, some figures are presented here for completeness. As already mentioned in section 6.2.2.1.1 the diameters of turbulent eddies in the centre of a chamber subjected to a vertical temperature difference of 0.1 K should be around 8 mm and at a height of 2.5 cm the eddy diameter would be about 3 mm. This latter figure compares well with Thomas' observations near the base of the 1 metre R.H.E.L. chamber. He reports great difficulty in observing eddies near the middle of the chamber.

6.2.3 Dynamic Behaviour of a Bubble Chamber

The dynamic heat load in a bubble chamber is mostly due to irreversible work performed during formation and collapse of track and spurious bubbles while the chamber is cyclically expanded and recompressed. Most of the heat is produced in the piston region and transferred to the cooler walls by turbulent convection.

Turbulence is also produced when laminar flow of the expanding liquid degenerates into turbulence. This latter turbulence should not itself give rise to optical distortions but will produce movement during bubble growth. The eddy velocities \( v \) depend on the critical Reynolds number \( R_c \) given by

\[
R_c = \frac{va}{v}
\]
where \( a \) is the eddy size and \( \nu \) the fluid viscosity. The eddy size depends on the chamber and piston geometry assuming \( R_c \approx 1000 \) and \( a \) to be of the order of mm as for thermal turbulence then the velocity fluctuations will be between 3 and 5 cm sec\(^{-1} \). Assuming these fluctuations are maintained during bubble formation then bubble displacements of 120 to 150 microns will result during the bubble growth time of about 4 millisec. However the effect is still not significant.

The thermal turbulence effects should be the same as in the static case, for the same heat flux. It is difficult to estimate the effects without knowing the chamber geometry or thermal conditions. Thomas (3) has observed the turbulent intensity in the R.H.E.L. 1.5 metre chamber at a cycling rate of 0.5 Hz with a static flux of 5 watts over the whole chamber. No increase in turbulent intensity was observed, indicating an insignificant increase in heat flux due to the expansion and contraction of the chamber.

6.2.4 Implications for Bubble Chamber Design and Operation

The main direct conclusion from the present work is related to the fact that the optical effects of thermal turbulence are smallest at the centre of the chamber, if particle tracks are viewed in a direction perpendicular to that of the temperature gradient. With most modern chambers the particle beam enters the chamber horizontally and in order for adequate spread to be obtained in the recorded event, the optical axis of the cameras should be roughly perpendicular to the beam axis. This arrangement of axes means that for a vertical
temperature gradient rays of light from track bubbles near 
the chamber centre will travel to the cameras through regions 
where turbulent effects are minimum.

It is of great importance that the camera lenses do not 
themselves act as heat exchangers. Thermal radiation 
transmitted by optical glass but absorbed by liquid hydrogen 
could be eliminated by depositing infra-red reflective coatings 
on the lens surfaces in contact with the liquid hydrogen. 
Refrigeration of the expansion chamber itself would help to 
reduce the thermal turbulence effects of the cyclic expansion 
process.

Stable stratification would completely eliminate thermal 
turbulence \( R < 0 \) but there is still the problem of determining 
the optical paths of rays of light from the track bubbles. 
In general these paths will be curved and the transformation 
from chamber to film space could prove laborious.

Thomas (57) describes a simple technique for integrating 
over a number of realisations of the turbulent field, using large 
annular apertures. The effect is to reduce optical distortion.

Another possibility is to use the technique of double exposure 
holography, one hologram being taken just before the particle 
event and the second after the appearance of track bubbles. 
This could not entirely eliminate the effect of velocity 
fluctuations during bubble growth and the time between exposures 
must be short compared with the lifetime of a given turbulence 
situation.
APPENDIX A

Derivation of the Equation for Mean Square Angular Deviation of a Light Ray Propagating in a Turbulent Medium.

The derivation given here closely follows that of Chernov (22).
The diagram below illustrates the problem.

![Diagram of light ray propagation through a turbulent medium.]

Fig. A.1. Propagation of Light Ray through a Turbulent Medium.

The ray equation is by Fermat's principle (22)

\[ \int_{A}^{B} \frac{d\sigma}{c} = \text{minimum} \]

A and B being the end points of a ray propagating through path elements \( d\sigma \), and \( c \) the velocity of light in the element of fluid. The refractive index at a point \((x,y,z)\) is \( \mu(x,y,z) \) and hence since \( \mu(xyz) = \frac{1}{c} \)

\[ \int_{A}^{B} \mu(x,y,z)d\sigma = \text{minimum} \]

The ray paths are assumed to belong to a family of curves
\[ x = x(t), y = y(t), z = z(t) \]
passing through A and B

\[ \therefore \frac{d\sigma}{dt} = \left[ \frac{dx}{dt} \right]^2 + \left[ \frac{dy}{dt} \right]^2 + \left[ \frac{dz}{dt} \right]^2 \right]^{\frac{1}{2}} dt \]
and \( t = t_1 \) at \( A \), \( t = t_2 \) at \( B \).

Hence

\[
\int_{t_1}^{t_2} \mu(x,y,z) \left( \frac{dx}{dt}^2 + \frac{dy}{dt}^2 + \frac{dz}{dt}^2 \right)^{\frac{1}{2}} \, dt = \text{min.}
\]

defining \( \frac{dx}{dt} = x', \frac{dy}{dt} = y', \frac{dz}{dt} = z' \)

we have

\[
\mu(x,y,z)(x'^2 + y'^2 + z'^2)^{\frac{1}{2}} = F(x,y,z,x',y',z')
\]

and by Euler's equations,

\[
\frac{d}{dt} \left( \frac{\delta F}{\delta x} \right) - \frac{\delta F}{\delta x} = \frac{d}{dt} \left( \frac{\delta F}{\delta y} \right) - \frac{\delta F}{\delta y} = \frac{d}{dt} \left( \frac{\delta F}{\delta z} \right) - \frac{\delta F}{\delta z} = 0
\]

\[
\therefore \frac{d}{dt} \left[ \mu x'/\left( x'^2 + y'^2 + z'^2 \right)^{\frac{1}{2}} \right] - (x'^2 + y'^2 + z'^2)^{\frac{1}{2}} \frac{\delta \mu}{\delta x} = 0
\]

\[
\frac{d}{dt} \left[ \mu y'/\left( x'^2 + y'^2 + z'^2 \right)^{\frac{1}{2}} \right] - (x'^2 + y'^2 + z'^2)^{\frac{1}{2}} \frac{\delta \mu}{\delta y} = 0
\]

\[
\frac{d}{dt} \left[ \mu z'/\left( x'^2 + y'^2 + z'^2 \right)^{\frac{1}{2}} \right] - (x'^2 + y'^2 + z'^2)^{\frac{1}{2}} \frac{\delta \mu}{\delta z} = 0
\]

Defining unit vector \( S \) tangential to the ray as

\[
S = (x'/\sqrt{x'^2 + y'^2 + z'^2}, y'/\sqrt{x'^2 + y'^2 + z'^2}, z'/\sqrt{x'^2 + y'^2 + z'^2})
\]

one obtains

\[
\frac{\delta (\mu S)}{\delta \sigma} - \nabla \mu = 0
\]

Let \( \mu(x,y,z) = 1 + \Delta \mu(x,y,z) \) with \( |\Delta \mu| \ll 1 \) i.e. the refractive index has mean value 1 and undergoes small fluctuations.

Integrating over a path extending from \( 0 \) to \( \sigma \),

\[
\mu' S' - \mu S = \int_{0}^{\sigma} \nabla \mu \, d\sigma
\]

where \( \mu' \) is refractive index at end of path, \( \mu = \) refractive index at beginning.

Assuming \( \mu = \mu' = 1 \) and squaring

\[
(S' - S)^2 = 2(1 - \cos \epsilon) = \epsilon^2 = \int_{0}^{\sigma} \int_{0}^{\sigma} \nabla_1 \nabla_2 (\mu_1 \mu_2) \, d\sigma_1 \, d\sigma_2
\]

\( \epsilon \) being the angle between \( S \) and \( S' \).
Averaging over the correlation coefficient \( N \) of the refractive index fluctuations

\[
\bar{\varepsilon}^2 = \frac{\mu}{\rho} \int_{-\infty}^{+\infty} \nabla_1 \nabla_2 N(x_1-x_2, y_1-y_2, z_1-z_2) \sigma_1 d\sigma_2 d\sigma_2
\]

Integration can be carried out along the ray path assuming the ray curvature to be small

\[
\bar{\varepsilon}^2 = \frac{\mu}{\rho} \int_{-\infty}^{+\infty} \nabla_1 \nabla_2 N(x_1-x_2, y_1-y_2, z_1-z_2) dr_1 dr_2
\]

Let

\[
x = x_1-x_2, y = y_1-y_2, z = z_1-z_2, r = r_1-r_2, x_0 = (x_1+x_2)/2, y_0 = (y_1+y_2)/2, z_0 = (z_1+z_2)/2
\]

\[\nabla_1 \nabla_2 N(x_1-x_2, y_1-y_2, z_1-z_2) = -\nabla^2 N(x, y, z)\]

\[
\bar{\varepsilon}^2 = -\frac{\mu}{\rho} \int_{0}^{\infty} \nabla^2 N(x_0, y_0, z_0) dr_0
\]

since \( \sigma \gg a \) one can integrate with respect to \( r \) between the limits \(-\infty\) to \(+\infty\). \( N \) is even so that

\[
\bar{\varepsilon}^2 = -2\mu^2 \sigma \int_{0}^{\infty} \nabla^2 N dr
\]

If the turbulence is isotropic then \( N = N(r) \) and

\[
\varepsilon^2 = -2\mu^2 \sigma \int_{0}^{\infty} \frac{1}{r^2} \frac{\delta N}{\delta r} (r^2 \frac{\delta N}{\delta r}) dr
\]

We now assume \( N = e^{-r^2/a^2} \), as commonly taken for atmospheric turbulence (23). (The equation \( N = e^{-r^2/a^2} \) is used rather than one of the type \( N = e^{-r/a} \) as this would make the integrand

\[
1/r^2 \frac{\delta}{\delta r} (r^2 \frac{\delta N}{\delta r}) = (1/a^2)e^{-r/a} - (2/ar)e^{-r/a}
\]
causing the integral to diverge logarithmically at \( r = 0 \).

This is because the correlation coefficient \( N = e^{-r/a} \)
corresponds to discontinuous changes in the refractive index \( \mu \).

In this case the original ray equation is inappropriate since \( \nabla \mu \)
becomes infinite at the discontinuities.)

Hence

\[
\overline{e^2} = 2\overline{\mu^2} \sigma/a^2 \int_0^\infty \frac{1}{r^2} \frac{\delta}{\delta r} (\frac{r^3 e^{-r^2/a^2}}{a^2}) dr
\]

\[
= 4\overline{\mu^2} \sigma 2 \int_0^\infty \frac{1}{r^4} (3e^{-r^2/a^2} - 2r^2/a^2 e^{-r^2/a^2}) dr
\]

\[
= 4\overline{\mu^2} \sigma/a^2 \int_0^\infty (3e^{-r^2/a^2} - 2r^2/a^2 e^{-r^2/a^2}) dr
\]

\[
\therefore \overline{e^2} = 4\overline{\mu^2} \sigma \sqrt{\pi}/a
\]
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