STABILIZATION AND CONTROL OF
ELECTROSTATIC ACCELERATORS

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Trinity Term 1971
Rakjuk le, hangyaszorgalommal, amit
Agyunk az ihlett orákban teremt.
ABSTRACT

The research carried out on the stabilization control and protection of two Van de Graaff electrostatic nuclear accelerators is described in this thesis. The system at Oxford consists of a single ended and a tandem accelerator with two different kinds of stabilizers; the results presented here are directly applicable to most electrostatic accelerators.

As an end result of the mathematical analysis, the behaviour of the single ended injector with combined liner and spray control is formulated. The stabilizer designed for the injector, based on the analysis, incorporates automatic switch over to the auxiliary generating voltmeter stabilizer loop. The automatic gain control introduced into the slit amplifiers eliminates the necessity of manual gain adjustments for different operating conditions. The energy resolution of the injector was measured by using alpha capture resonance in $^{15}$N. It was $0.65$ keV at 4.46 MeV.

The analysis presented here describes the behaviour of the corona stabilizer as used on the tandem. The predicted limitations due to the time delay and dispersion of the ions in high pressure gas are in good agreement with the measured performance figures. The two loop stabilizer with the auxiliary generating voltmeter loop provides facilities for semi-automatic operation. The resolution of the tandem was $\pm 1$ keV by measuring the $^{28}$Si(p,p)$^{28}$Si resonance at 5.83 MeV.
Multiple breakdowns on both the injector and the tandem were prevented by the introduction of the protection circuit. The circuit design was based on the analysis of the breakdown behaviour of the generators as described. The predicted performance of the circuit and its effect on the accelerators is well verified by the observed behaviour of the generators after total breakdown.

The study of the computer controlled generator operation is described in Chapter 5. For the interfacing the internationally accepted Camac system is recommended.
## CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Abstract</td>
<td>2</td>
</tr>
<tr>
<td>Acknowledgements</td>
<td>13</td>
</tr>
<tr>
<td><strong>CHAPTER 1 Introduction</strong></td>
<td></td>
</tr>
<tr>
<td>1.1 General remarks</td>
<td>14</td>
</tr>
<tr>
<td>1.2 Evolution of electrostatic accelerators</td>
<td>16</td>
</tr>
<tr>
<td>1.3 Principles of operation</td>
<td>19</td>
</tr>
<tr>
<td>1.4 Coupled accelerators</td>
<td>23</td>
</tr>
<tr>
<td>1.5 Magnetic momentum analyzer for electrostatic accelerators</td>
<td>26</td>
</tr>
<tr>
<td>1.6 Sources of energy spread</td>
<td>29</td>
</tr>
<tr>
<td>1.7 Methods of stabilization</td>
<td>33</td>
</tr>
<tr>
<td><strong>CHAPTER 2 Injector</strong></td>
<td></td>
</tr>
<tr>
<td>2.1 Introduction</td>
<td>41</td>
</tr>
<tr>
<td>2.2 Method of stabilization</td>
<td>41</td>
</tr>
<tr>
<td>2.3 Open loop gain requirements</td>
<td>43</td>
</tr>
<tr>
<td>2.4 Equivalent circuit, polar diagram and stability criteria for the injector two loop stabilizer</td>
<td>44</td>
</tr>
<tr>
<td>2.5 Generating voltmeter control of the intershield voltage</td>
<td>52</td>
</tr>
<tr>
<td>2.6 Circuit design</td>
<td>56</td>
</tr>
<tr>
<td>2.6.1 Main stabilizer</td>
<td>58</td>
</tr>
<tr>
<td>2.6.2 Servo system</td>
<td>61</td>
</tr>
<tr>
<td>2.6.3 Generating voltmeter circuit</td>
<td>63</td>
</tr>
<tr>
<td>2.7 Injector tests and their results</td>
<td>65</td>
</tr>
<tr>
<td>2.7.1 Test on stability versus slow and fast loop gain</td>
<td>66</td>
</tr>
<tr>
<td>2.7.2 Automatic gain control calibration</td>
<td>68</td>
</tr>
<tr>
<td>CHAPTER 4</td>
<td>Accelerator control under fault condition</td>
</tr>
<tr>
<td>-----------</td>
<td>-----------------------------------------</td>
</tr>
<tr>
<td>4.1</td>
<td>General remarks</td>
</tr>
<tr>
<td>4.2</td>
<td>Analysis of the tandem behaviour under fault condition</td>
</tr>
<tr>
<td>4.3</td>
<td>Analysis of the injector behaviour under fault condition</td>
</tr>
<tr>
<td>4.4</td>
<td>Automatic recovery system for the accelerators after voltage breakdown</td>
</tr>
<tr>
<td>4.5</td>
<td>Tests and observations</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>CHAPTER 5</th>
<th>Proposal for computer control of Van de Graaff generators</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.1</td>
<td>Introduction</td>
</tr>
<tr>
<td>5.2</td>
<td>Sampled controller for Van de Graaff accelerators</td>
</tr>
<tr>
<td>5.3</td>
<td>Automatic NMR frequency selection</td>
</tr>
<tr>
<td>5.4</td>
<td>Automatic magnet field and terminal voltage selector</td>
</tr>
<tr>
<td>5.5</td>
<td>Remarks on circuit design</td>
</tr>
<tr>
<td>5.6</td>
<td>Conclusions</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>CHAPTER 6</th>
<th>Appendices</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.1</td>
<td>Investigation into improving magnet field stability</td>
</tr>
<tr>
<td>6.2</td>
<td>Transfer functions and stability criteria for the injector two loop stabilizer</td>
</tr>
<tr>
<td>6.3</td>
<td>Transfer function and stability criteria of the tandem single and two loop stabilizer</td>
</tr>
<tr>
<td>6.4</td>
<td>Stabilization of electrostatic accelerators with generating voltmeter</td>
</tr>
<tr>
<td>6.5</td>
<td>Energy modulation of electrostatic accelerators</td>
</tr>
<tr>
<td>6.6</td>
<td>Calculations of the tandem energy resolution</td>
</tr>
<tr>
<td>6.7</td>
<td>Computer programs</td>
</tr>
<tr>
<td>6.7.1</td>
<td>Injector NMR frequencies</td>
</tr>
<tr>
<td>Fig. No.</td>
<td>Captions of figures</td>
</tr>
<tr>
<td>---------</td>
<td>------------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>1</td>
<td>Schematic diagram of the 10 MV injector</td>
</tr>
<tr>
<td>2</td>
<td>Schematic diagram of the 6 MV tandem</td>
</tr>
<tr>
<td>3</td>
<td>Coupled electrostatic accelerators</td>
</tr>
<tr>
<td>4</td>
<td>Straight edge slit system</td>
</tr>
<tr>
<td>5</td>
<td>Signal amplitude versus energy change for straight edge slit system</td>
</tr>
<tr>
<td>6</td>
<td>Equivalent circuit of the injector</td>
</tr>
<tr>
<td>7</td>
<td>Polar diagram of $Z_I/R_I$</td>
</tr>
<tr>
<td>8</td>
<td>Polar diagram of $Z_{II}/R$</td>
</tr>
<tr>
<td>9</td>
<td>Polar diagram of $Z_I/R_I + kZ_{II}/R$</td>
</tr>
<tr>
<td>10</td>
<td>Frequency response of $Z_I/R_I + kZ_{II}/R$</td>
</tr>
<tr>
<td>11</td>
<td>Polar diagram of interelectrode capacitance and stack resistance circuit</td>
</tr>
<tr>
<td>12</td>
<td>Frequency response of interelectrode capacitance and stack resistance circuit</td>
</tr>
<tr>
<td>13</td>
<td>Frequency response of generating voltmeter stabilizer</td>
</tr>
<tr>
<td>14</td>
<td>Main stabilizer circuit diagram</td>
</tr>
<tr>
<td>15</td>
<td>Servo modulator and amplifier circuit diagram</td>
</tr>
<tr>
<td>16</td>
<td>Generator voltmeter circuit diagram</td>
</tr>
<tr>
<td>17</td>
<td>Block diagram for measuring the injector energy stability against loop gain variation</td>
</tr>
<tr>
<td>18</td>
<td>Loop gain versus energy stability for the injector at positive terminal voltage</td>
</tr>
<tr>
<td>19</td>
<td>Loop gain versus energy stability for the injector at negative terminal voltage</td>
</tr>
<tr>
<td>20</td>
<td>Slit amplifier gain versus slit current</td>
</tr>
<tr>
<td>21</td>
<td>Slit opening versus energy stability for negative polarity</td>
</tr>
<tr>
<td>No.</td>
<td>Description</td>
</tr>
<tr>
<td>-----</td>
<td>------------------------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>22</td>
<td>Slit opening versus energy stability for positive polarity</td>
</tr>
<tr>
<td>23</td>
<td>Gamma ray yield curves for (^{15})N(a,(\gamma)) resonance</td>
</tr>
<tr>
<td>24</td>
<td>Energy spread of the injector</td>
</tr>
<tr>
<td>25</td>
<td>Measurement of the differential hysteresis of the injector analyzing magnet</td>
</tr>
<tr>
<td>26</td>
<td>Tandem terminal ripple</td>
</tr>
<tr>
<td>27</td>
<td>Corona counter reading calibration against probe distance from the terminal</td>
</tr>
<tr>
<td>28</td>
<td>Potential profiles through the probe</td>
</tr>
<tr>
<td>29</td>
<td>Block diagram of the experimental arrangement for measuring ion transit time</td>
</tr>
<tr>
<td>30</td>
<td>Ion drift time measurement in the tandem</td>
</tr>
<tr>
<td></td>
<td>a. By using pulse</td>
</tr>
<tr>
<td></td>
<td>b. By using sinewave</td>
</tr>
<tr>
<td>31</td>
<td>Coaxial electrode configuration</td>
</tr>
<tr>
<td>32</td>
<td>Equivalent circuit of the tandem</td>
</tr>
<tr>
<td>33</td>
<td>Polar diagram of (K(f))</td>
</tr>
<tr>
<td>34</td>
<td>Frequency response of (K(f))</td>
</tr>
<tr>
<td>35</td>
<td>Polar diagram of the tandem two loop stabilizer</td>
</tr>
<tr>
<td>36</td>
<td>Tandem frequency response with single and two loop stabilizer</td>
</tr>
<tr>
<td>37</td>
<td>Tandem main stabilizer circuit diagram</td>
</tr>
<tr>
<td>38</td>
<td>Block diagram for measuring the polar diagram of the corona stabilizer</td>
</tr>
<tr>
<td>39</td>
<td>Polar diagram for the tandem corona stabilizer</td>
</tr>
<tr>
<td>40</td>
<td>Tandem sensing plate frequency response</td>
</tr>
<tr>
<td>41</td>
<td>Frequency response of the tandem</td>
</tr>
<tr>
<td>42</td>
<td>Corona current versus probe position</td>
</tr>
<tr>
<td>43</td>
<td>Experimental arrangement for measuring the tandem energy resolution</td>
</tr>
</tbody>
</table>
Excitation curves at $\theta = 90^\circ$ and $167^\circ$ for $^{28}\text{Si}(p,p)^{28}\text{Si}$ resonance

Injector terminal voltage versus time after total discharge

Tandem terminal voltage versus time after total discharge

Breakdown protection circuit diagram

Injector controlled recovery after tube breakdown at 5.5 MV

Injector uncontrolled recovery from tube breakdown at 5.5 MV

Tandem controlled recovery after terminal breakdown at 6.2 MV

Tandem uncontrolled recovery after terminal breakdown at 6.2 MV

Tandem controlled recovery after a persisting breakdown, after what manual intervention was needed to restore the terminal voltage

Flow diagram for NMR frequency selection

Block diagram for NMR frequency control

NMR frequency read in interface

Flag and inhibit circuit diagram for NMR interface

Buffer register circuit diagram for NMR interface

NMR frequency range selector circuit

NMR frequency coarse preselector and coarse selector circuit

Truth table

DAC for fine control

Flow diagram for selecting analyzing magnet setting

Block diagram for analyzing magnet current control

ADC for magnet power supply
Flow diagram for terminal voltage setting 172
Block diagram for terminal voltage control 173
Schematic diagram of the current control for the analyzing magnet 178
Magnet control interface circuit diagram 180
The step function response of the magnet and its stabilizer 181
Equivalent circuit of $Z_I$ 182
Frequency response of $\tau_5$ and $\tau_1 \sim \tau_2 \sim \tau_3$ 184
Block diagram of a servo with velocity feedback 186
Schematic diagram of the generating voltmeter 195
Circuit diagram of the magnet current modulator 200
<table>
<thead>
<tr>
<th>No.</th>
<th>Table Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>$\alpha$ values</td>
<td>94</td>
</tr>
<tr>
<td>2.</td>
<td>Critical frequencies and critical gain figures for different $\alpha$ values</td>
<td>102</td>
</tr>
<tr>
<td>3.</td>
<td>Corner frequencies for different $D$ and $\alpha$ values</td>
<td>103</td>
</tr>
<tr>
<td>4.</td>
<td>Calculated and measured critical frequencies</td>
<td>117</td>
</tr>
<tr>
<td>5.</td>
<td>Injector parameters used in the calculations</td>
<td>191</td>
</tr>
<tr>
<td>6.</td>
<td>Tandem parameters used in the calculations</td>
<td>196</td>
</tr>
<tr>
<td>7.</td>
<td>Parameters used in the calculations of the tandem resolution</td>
<td>205</td>
</tr>
</tbody>
</table>
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CHAPTER 1
INTRODUCTION

1.1 General remarks

The applications of new physical and engineering concepts in the accelerator field has led to the development of several different types of accelerators. Each of these though covering the same energy range, fills a specific role for research in certain branches of physics such as nuclear and solid state physics etc.

The oldest concept to accelerate charged particles was the application of direct potential drop in which the particles were accelerated. The means of producing the high voltage led to a large group of different machines out of which the two most often used are the so-called Cockcroft-Walton voltage multiplier and the Van de Graaff generator\(^1\).

The rapid progress in developing these machines of course would not have been possible without the close collaboration between physicist and specialists in many branches of engineering. In fact simultaneous developments in mechanical design, electronics, control engineering and vacuum techniques have made possible the practical realization of the original ideas\(^2\).

The Van de Graaff electrostatic generators possess special properties which make them unique tools for low energy nuclear research. Their first virtue as compared with the resonant
machines is the 100% duty cycle or D.C. beam produced. The second one is the precision and the highly homogeneous beam energy. This property makes these type of accelerators uniquely suitable tools for many nuclear experiments. Finally their flexibility needs to be listed. It is common knowledge that electrostatic accelerators are capable of accelerating any ion produced by the ion source, and as far as the flexibility in changing the energy of the particles is concerned nothing is easier than to do this within the energy range of the generator.

Against these advantages there is a drawback of this type of machine and that is the voltage limitation of the generator which at present is 10 MV for single ended and 20 MV for tandem Van de Graaff generators.

Since in an electrostatic accelerator the energy of the ions is proportional to the terminal voltage of the generator, it is an obvious need to stabilize the terminal potential in order to reduce its fluctuation below the level of the residual energy spread independent of this potential. The limitations in stabilizing the terminal voltage are set by both technical and economic considerations. The main concern of the research described in this thesis is to cover the control engineering problems associated with the two types of electrostatic accelerators, the single ended and the tandem machines, known today. In doing this the analysis and the test results given here provide guidance, from a control engineering point of view, to users and designers of such accelerators.
At the same time it forms a feasibility study for introducing computer control over the operation of Van de Graaff generators.

1.2 Evolution of the electrostatic accelerators

Although the idea of high voltage generation by means of a belt carrying charges to a terminal was conceived by Righi as early as about 1890 the practical realization had to wait until 1929. It was at Oxford in 1928 that Van de Graaff found a practical way for the charge transfer by belt as conceived by Righi. Working as a Rhodes Scholar between 1927 and 1928 he was inspired by Rutherford's forecast of the need for a device supplying particles more energetic than those produced by radioactive sources.

It was in 1929 at Princeton when Van de Graaff demonstrated the first model of his electrostatic generator operating by today's standard at a modest 80 KV terminal voltage. From here on the advance was very rapid. In 1931 Van de Graaff described an electrostatic generator combined with an accelerating tube producing a beam of protons. This marked the birth of the electrostatic accelerators.

Encouraged by this success many of the leading laboratories started building accelerators based on these principles. There is no intention of listing here all the machines built in parallel at different laboratories only those contributing major advancements to electrostatic accelerator technology will be mentioned.
All the accelerators built, up to 1931 were designed for operating in air under atmospheric pressure. In 1932 Barton Mueller and Van Atta at Princeton started the first experiment with electrostatic high voltage generation in high pressure tanks. Since at atmospheric pressure in air between smooth surfaces the breakdown potential gradient or as commonly known breakdown strength is 30 kV/cm and this for a given geometry is proportional to the gas pressure, the introduction of the high pressure gas insulation made it possible to satisfy the rapidly increasing demand for higher voltages and reduced machine sizes. During the last 30 years several gas mixtures were tried out and to date most of the modern accelerators are using nitrogen - CO₂, freon or sulphur hexafluoride at pressures up to 16 atm. Under these conditions the breakdown strength has increased to 150 kV/cm.

A big step forward in increasing the terminal potential for a given size of generator was made by Herb in 1940 by introducing additional concentric electrodes, or intershields. By adding one intershield between the terminal and the tank so that field on the outside surface of the terminal is the same as the one on the intershield, one can gain 20-25% in terminal voltage for a given breakdown strength. Herb by using three intershields in his design managed to reach a 4 MV terminal potential. The intershield has been adopted in some of the modern accelerators including the 10 MV injector at Oxford.
Until 1958 all the electrostatic accelerators built on the Van de Graaff principle were single ended machines. In this type of accelerator the ions are generated inside the terminal and accelerated once by the electrostatic force due to the charged terminal. In 1932 Dempster\(^7\) put forward a proposal for doubling the energy of the charged particle by changing its charge during acceleration. This method however did not become feasible until 1937 when Bennett and Darby demonstrated it experimentally. From 1937 another twenty-one years had to pass before this principle got practical application. In 1958 the High Voltage Engineering Corporation under the direction of Van de Graaff successfully accelerated protons to 13.4 MeV in the first double ended electrostatic accelerator. This machine, the first EN tandem ever, was built for Atomic Energy of Canada Limited in Chalk River\(^8\).

The practical application of the tandem principle was a landmark in the history of electrostatic accelerators. Since 1958 large numbers of tandem accelerators have been put into operation mostly by HVEC. Going by HVEC classification the smallest member of this breed is the model EN designed for 6 MV terminal voltage. One of this type has been in operation since 1964 in the Nuclear Physics Laboratory at Oxford.

The next size up is called the FN tandem. This is rated at 7.5 - 9 MV terminal potential.

The largest member of this family in operation is the Emperor or MP tandem which reached proton energies of 24 MeV at Yale\(^9\).
Even bigger accelerators are now under consideration for transuranium research. For one of these machines called the XTU tandem by HVEC, the design aim is 20 MV on the terminal. In a British proposal for a heavy ion tandem 30 MV has been put forward as a design figure for the terminal voltage.

1.3 Principles of operation

The generation of high voltage by the Van de Graaff method is simple in principle and easy to describe. The operation can be readily understood by reference to Fig. 1 which shows the schematic diagram of the Oxford injector. The generator consists of a high voltage terminal supported on an insulating column and a moving insulating belt carrying the charge to the terminal. The belt runs between the driven pulley at ground potential and a pulley in the terminal. Electric charge is sprayed on the belt at ground potential from fine wire comb directed at a grounded plate behind the belt. The source from which the charges are sprayed has very high impedance to avoid fluctuation in spray current due to the varying belt impedance. The internal impedance of the supply for the Oxford injector is higher than 3000 MΩ whilst the average belt impedance lies between 30 and 50 MΩ. The belt mechanically carries the charge to the high voltage terminal. At the terminal the charge is transferred from the belt to the terminal by a comb similar to the one at ground potential. The same principle can be used to produce positive or negative terminal voltage.
SCHEMATIC DIAGRAM OF THE 10 MV INJECTOR

FIG. I.
In the case of a single ended electrostatic accelerator, the generator described above is combined with an ion source housed inside the terminal and with the accelerating tube. The uniform voltage gradient in the tube sections is maintained by a tapped resistor chain between the terminal and ground. The ions generated in the source are accelerated in the accelerating tube away from the terminal under the repelling electrostatic force.

The generation of voltage in tandem accelerators is done in exactly the same way as for the single ended machines. The principle is shown in Fig. 2. In this case however the terminal potential is used twice to obtain output energies twice that available in a single acceleration. The principle of tandem acceleration involves reversing the sign of the charge of the accelerated ions in the terminal. Various charge changing processes are worked out for different versions of tandem accelerators.

The ion source for a tandem accelerator is outside the machine and the ions injected into the machine have nearly zero initial energy. The ions injected into a tandem accelerator accelerate towards the terminal under the electrostatic force. In the charge exchanger placed inside the centre terminal the particles reverse the sign of their charges. Then under the repelling electrostatic force they accelerate away from the centre terminal.

The final energy of the ions accelerated through a tandem accelerator is:
where \( E_1 \) is the injection energy

\( Z_1 \) is the charge state of the ions entering the accelerator. Normally this value is unity.

\( Z_2 \) represents the charge state of the ions leaving the accelerator

\( e \) is the charge of the electron

and \( V_T \) is the terminal voltage.

The tandem principle can be extended to more than two stages of acceleration.

1.4 Coupled operation

The demand for higher particle energy is increasing rapidly for nuclear experiments since it became feasible with the introduction of the tandem principle. The obvious way to produce higher energy particles would be to increase the terminal potential. At present great effort is being put into the design of bigger accelerators and with the continued progress in high voltage techniques one can expect a steady increase in the future in accelerator voltage. This process however is slow and the operating terminal potential for existing tandem accelerators is limited to 20 MV.

In 1958 at the First International Accelerator Conference, Van de Graaff came up with an entirely different solution to this problem. He put forward a practical
suggestion for multi-stage acceleration. In practice to date, two-, three-stage tandem operation has been used successfully. The so called two-stage tandem involving one machine only is the operation most commonly encountered. Three-stage tandem operation involves a single ended and a tandem accelerator (alternatively a half and a full tandem machine). Different propositions for multi-stage operation are under consideration.

This thesis however is not concerned with the variety of multi-stage accelerators, only with the three-stage operation as built for our laboratory.

In the system shown in Fig. 3 the negative ions are generated inside the high voltage terminal of the single ended injector. After the first stage acceleration the ion beam is bent through $90^\circ$ by the magnet between the injector and the tandem. The beam is then injected into the tandem with positive potential on its terminal. The negative ions are given a second acceleration towards the positive terminal under the electrostatic force. In the charge exchange canal they lose two or more electrons and become positive ions. The repelling force due to the positive terminal then gives the third acceleration to the ions.

The final energy of the ions accelerated through a three stage system is:

$$E = E_1 + eV_i + (Z + 1)eV_T$$

(1.2)

where $E_1$ is the injection energy

$Z$ represents the charge state of the ions leaving the tandem
$V_i$ is the terminal voltage of the injector
and $V_T$ is the terminal voltage of the tandem.

1.5 Magnetic momentum analyzer for electrostatic accelerators

In order to separate from an ion beam a component of specific energy and charge, some form of analysis is required. To date two kinds of analyzer are used for this purpose i.e. the electrostatic and the magnetic\textsuperscript{14}). In both cases the selection is based upon the effect, suffered by the ions of different energy, under the influence of electrostatic or magnetic field. The electrostatic field accelerates the charged particles in the direction of the field whilst the magnetic field deflects the particles at right angles to the direction of the particle motion and also the magnetic field. The effect of both the electrostatic field $E$ and the magnetic field $B$ on a particle of charge $e$ and mass $m$ travelling at $v$ velocity can be described in vector notation as

$$\frac{d}{dt}(mv) = eE + ev \times B \quad (1.3)$$

At Oxford both accelerators are furnished with $90^\circ$ sector shaped magnets for beam analysis. If the radius of curvature of the path of the ions is known the field strength can be calculated for ions of any given momentum. The slit systems placed in the object and image focal point of the magnet complete the system as a beam analyzer.

The energy resolution of this kind of $90^\circ$ sector magnet with symmetrical double focussing between $2R$ and $2R$, assuming that the object slit separation is smaller than
the size of the beam is

\[ \frac{\Delta E}{E} = \frac{\Delta r_1 + \Delta r_2}{2R} \]  

(1.4)

where \( E \) is the energy of ions,

\( R \) is the radius of the magnet,

\( \Delta r_1 \) and \( \Delta r_2 \) are the object and image slit separations respectively.

As has been shown above, the field in the magnet forms a primary energy reference for the accelerator; it is desirable to hold it more stable than the required energy stability.

In the further analysis the instability in the magnet field will be taken as negligible to other factors. In Chapter 6.1, however, the work done on the analyzing magnet stabilization at Oxford will be presented.

It is important of course to know the magnetic field strength to the same accuracy. For measuring the magnetic field to this sort of accuracy the most accurate devices are the ones based upon the Nuclear Magnetic Resonance principle \(^\text{16}\). In these devices - based upon the strict relationship between Larmor frequency of precession, the gyromagnetic ratio and the magnetic flux intensity - the measured field is converted directly into frequency.

In the Oxford system the D - XK - 310 - 004 NMR Fluxmeter manufactured by HVEC is being used. This can also be part of the magnet field stabilizer as described in Chapter 6.1.
As the high degree of energy stability and resolution of electrostatic accelerators became possible the need for accurately known energy calibration grew stronger. The calibration at this accuracy can only be done by the use of nuclear reactions within the energy range of the accelerator. The reactions used for energy calibration are (p,n) thresholds or sharp nuclear resonances of known energy. The energy of the nuclear reaction is related to the NMR frequency by the equation below:

\[ \nu = \sqrt{\frac{m - m_e}{kZ^2}} \cdot E \left( 1 + \frac{E}{(m - 2m_e)C^2} \right) \]  

(1.5)

where \( \nu \) is the NMR frequency in MHz, 
\( m_0 \) is the atomic rest mass of the isotope related to the mass of C\(^{12}\),
\( m_e \) is the rest mass of the electron,
\( E \) is the reaction energy,
\( Z \) is the ion charge state,
\( C \) is the velocity of light,
and \( k \) is a constant dependent only on the momentum analyzer.

During a calibration experiment the measured NMR frequency is related to the known reaction energy \( E \) so \( k \) can be calculated. In order to check the linearity of the analyzer several reactions need to be used within the energy range of the accelerator. For the calculation of the NMR frequency versus ion energy of the Injector and the Tandem see Appendix 6.7.1 and 6.7.3.
1.6 Sources of energy spread

The prime concern of the research was the minimization of the energy spread of the ions due to the generator itself. All the sources of energy spread however need to be considered in order to find out how far it is worth going with the stabilization of the terminal voltage. The prime consideration should be the targets used for these experiments. This of course is not part of the accelerator and normally it is left to the experimenting physicist to select and prepare to suit his own purpose. It is however important to know the upper limits of the energy spread imposed by the target.

Targets fall into three categories: gaseous, liquid and solid. Solid targets with thickness equivalent to 1 keV can be fabricated with care but there are technological limitations for reducing this thickness below 1 keV equivalent energy. Gas or liquid targets can be used for some experiments but their use is limited to materials available in gas or liquid form. Great effort is being put into developing better targets in order to minimize their effect on the accuracy of the data obtained by the experiment.

The effect of the beam energy spread in nuclear experiments was assessed by Newson on nuclear threshold energy measurements. He found that if the energy spread of the beam used was δ the threshold shifted by approximately δ/2 from the true threshold energy.

The factors affecting the energy spread of the ions come under the following major categories:

1. Energy spread due to ion source
2. Energy spread caused by the fluctuation of the
3. The effect of the beam optics on energy stability
4. The effect of the limits of the resolution of the analyzing magnet and field instability.

The overall effect of all these factors should not be bigger than the upper limit in energy spread due to the finite target thickness.

1. Ion source:

Since the birth of the accelerators, great attention has been paid to the generation of ions. Due to the diverse demands on performance a number of ion sources working on different principles have been developed, each of these having certain advantages and disadvantages.

It is not part of this thesis to go into details of ion sources only to point out the range of energy spread one can expect from them since this has some bearing on the final energy spread. Once the type of the ion source is chosen the initial energy spread of the ions is also fixed. In order to overcome difficulties inherent in any particular source in many laboratories more than one ion source is provided for the accelerators, each suited for different requirements.

The energy fluctuation of the ions produced by ion sources can range from less than 50 eV for radio frequency and duo-plasmatron ion sources to 20 KV for cold-cathode canal-ray tube type ion sources\(^1\).
On the grounds of long life, reliability and capability or producing variety of ions of high intensity (10-20 μA of negative ions) a mercury pool cathode source was chosen for the injector despite its relatively large energy spread. The presence of impurity beams spreading over a 5 keV energy range (amongst other factors) necessitated the use of a 7° analyzing magnet, which has an energy resolution of ±1 keV with 0.15" aperture.

2. Terminal voltage:

The drift and fluctuation in the terminal voltage can be attributed to several factors. Firstly, the effect of fluctuation and irregularities of the charging and charge transport system. Secondly, the presence of leakage and corona discharge current partially bypassing the stack. Finally, the loading effect of the beam on the terminal. These are the effects - some periodic and some random - which can be eliminated or greatly reduced as shown in the later paragraphs.

3. Beam optics:

The slit setting and the focusing conditions of the beam should be carefully considered for each experiment. It is obvious for instance that the beam diameter at the stabilizer slits must not be smaller than the separation of the slit jaws. Otherwise the beam tends to oscillate between the slit edges. Alternatively, when the terminal voltage shows unidirectional drift it may bounce on one
of the slits.

Under these conditions any instability in the supplies to the elements of the beam optics causing positional deviation of the beam will be interpreted by the analyzing magnet as energy modulation. The stabilizer will then force a compensating energy variation on the terminal to maintain the beam centrally on the analyzing slits. Angular deviation can cause similar effects due to the aberration of the magnet and the fringe field effect as given by Cross\(^{19}\) and Enge\(^{21}\) and also the second order coefficients in the magnet transfer matrix as calculated by Enge and other authors\(^{20}\).

4. Analyzing magnet:

The effect of the limitation in the resolution of the analyzing magnet on the energy spread of the ions has been formulated in section 1.5. Since the energy reference for the accelerator is the field in the analyzing magnet one would like to have the field absolutely stable. In practice, however, one has to be content with some finite field stability. Any instabilities in the field \(H\) represent a certain change in particle energy \(E\) and these two quantities are linked by the formula below:

\[
\frac{dH}{H} = \frac{1}{2} \frac{dE}{E} \quad (1.6)
\]

Since the required energy stability is 1 part in \(10^4\), for the magnetic field stability 5 parts in \(10^5\) was
considered satisfactory.

The study and its results carried out on the magnet current stabilizer to achieve this stability are described in section 6.1.

1.7 Methods of stabilization

The first electrostatic accelerators ran without the stabilization of the generator voltage. Experimenting physicists were content with the short term stability of the new tool and only the long term drift in the terminal potential was corrected by applying manual control to the current sprayed on the belt. This method, however, soon proved not only tedious but insufficiently accurate and people started looking for means of controlling and stabilizing the generator voltage.

To date several ways of stabilization have been used successfully, differing in the following two major ways:

1. The means of generating the error signal.
2. The means of correcting the terminal voltage.

The methods for generating an error signal proportional to the change in terminal voltage use either the slit system of the electrostatic or electromagnetic analyzer or the generating voltmeter.

In the first case the beam traverses narrow slits at the entrance and the exit of the analyzer. At constant magnetic field any variation of the terminal potential makes the beam move on the exit slits. If the slits are insulated the difference of the intercepted slit currents
will indicate the change in particle energy. The sense of change will represent the direction of deviation in the terminal voltage. The amplified error signal generated this way is used to correct the terminal potential.

The merit of using the analyzer slit current to stabilize the terminal potential is that its speed and sensitivity is only limited by the amplifier in the loop. Furthermore, the error signal for small changes is proportional to the change in energy of the ions.

![Diagram of a straight edge slit system.]

During the years, slits with jaws of different shape have been developed, nowadays however almost exclusively straight edged slits are used in energy analyzers. It can be shown that assuming a cylindrical beam profile and
uniform distribution of ions in the cross section of the beam (see Fig. 4) the signal versus energy change characteristic of such slits is described by the following equation:

\[ \frac{\Delta T}{\Delta E} = \frac{E}{R} \frac{(d+g)^2}{\frac{d}{2r}} \frac{\sqrt{1 - (\frac{d-g}{2r})^2} - (\frac{d-g}{2r}) \sqrt{1 - (\frac{d+g}{2r})^2}}{3 \sqrt{1 - \frac{2dr+2s^2}{4r^2} + \left(\frac{d^2-s^2}{4r^2}\right)^2}} \]

(1.7)

where \( E \) is the energy,
\( T \) is the error signal amplitude,
\( d \) is the slit separation,
\( s \) is the deviation from the central position,
\( r \) is the radius of the beam profile,
and \( R \) is the radius of the analyzing magnet.

Signal amplitude versus energy change for straight edge slit system.

Fig. 5
The signal versus energy plot is shown in Fig. 5 in arbitrary units. It is clear that although this system produces an error signal proportional to $\Delta E$ for small energy changes, the response becomes highly nonlinear for large changes. In fact when the beam leaves the range of the slits due to breakdown or larger energy shift the error signal will diminish to zero and normally the generator will not recover without manual intervention. The second disadvantage is that the stabilizer can lock on to a spurious beam of similar magnetic rigidity.

The second way of generating an error voltage is by using the generating voltmeter. This device has been used for years for measuring the terminal voltages of electrostatic generators. Its history goes back to as early as 1933 and a number of descriptions can be found in the literature\textsuperscript{22).}

The device consists of stationary metal segments periodically shielded by a rotary cruciform of identical shape. The stationary segments charge up when exposed to the HT terminal and discharge when covered by the earthed rotary segments. The amplitude of the AC signal generated this way is proportional to the potential on the HT electrode.

In spite of its great advantage of covering the full range of the terminal voltage this method has several drawbacks. The biggest probably is the difficulty in producing such a device with long term mechanical stability better than 1 part in $10^8$. There are reports of a stability of 1 part in $10^4$ for 12 hours under well controlled conditions\textsuperscript{23}, being achieved. This order of stability,
however, is difficult to reproduce in day to day operation. Further disadvantages are its narrow frequency response, limited virtually to a range between 0 and about 20 Hz, and its reading being proportional to the terminal voltage and not to the beam energy. It is also prone to spurious readings after a terminal breakdown. Recently Polga and others\textsuperscript{24}) at San Paulo University produced a differential generating voltmeter system which achieved 0.2\% resolution on a 1 MeV accelerator.

To date the following five major ways of applying the correction signal on the terminal have been used:
1. Controlling the belt charge current.
2. Using electron beam to load the terminal.
3. Capacitive coupling to the terminal, using the liner.
4. Corona current loading of the terminal.
5. Extract/focus voltage control by using a light beam for control (only for single ended accelerators).

Historically the first method, using to stabilize the terminal voltage of electrostatic accelerators, was to control the current sprayed on to the belt\textsuperscript{25}). The system works well for slow drift but the finite time required to transport the charge to the terminal which for large generators is of the order of 0.2 sec may cause oscillation above a certain loop gain.

Miller\textsuperscript{26}) recently suggested controlling the down spray current on the belt. This method reduces the time delay
described above at the price of using rather a complicated way for transmitting the error signal into the HT terminal.

The limitations of the spray current control method soon made the accelerator designers look for other ways. In one method electrons were injected from an electron gun into the vacuum tube and accelerated towards the terminal\(^{27}\). The delay time of this method is of the order of \(10^{-6}\) sec but can be used only on generators running at a positive terminal potential. Such a system can also be a source of high intensity X-ray radiation\(^{28}\) and requires a second tube and differential pumping.

The first successful application of the liner stabilization method was on the accelerator at the University of Notre Dame in 1950\(^{29}\). Here the inside of the pressure tank is lined with metal tube surrounding the terminal insulated from ground. The potential of the terminal voltage is in this case with respect to the liner potential and not to the ground. With varying liner voltage the terminal potential will vary with respect to the ground. A liner system is capable of compensating for very fast changes into the KHz range but its limited low frequency response makes it impossible to use on its own.

In the so called "corona" control method the terminal voltage is maintained by a constant belt charge current and the control is effected by varying a component of the load current. If a sharp point or group of points is inserted through the pressure tank wall it will become a source of
current to the terminal. This current can be adjusted by mechanically setting the distance between the point and the terminal or by forcing the potential of the point above ground potential. Usually the latter is done by connecting the point to the anode of a high voltage thermionic valve. By changing the grid bias of the tube one can control the current drawn by the probe.

Accelerators with corona control have been in operation since 1942 \(^{30}\); not all the problems associated with this kind of stabilization have yet been fully investigated and explained. These problems are associated with the following three phenomena:

1. The finite drift time for charges to move from the probe to the terminal.
2. The dispersion caused by the collisions between the gas molecules and the charge carrier, as well as the electron diffusion and the probability of electron attachment to molecules of the gas during transit. This manifests itself in a loss of high frequency content of the transmitted signal.
3. The field distortion due to the presence of the probe (near ground potential) in the space between tank and terminal, and its effect on the transit time of the charges.

Both the drift velocity and the dispersion depend on the gas conditions in the tank such as the gas mixture, pressure and the water vapour content. Although the drift time was measured on several accelerators no effort was made
to analyze the effect of the two other factors on the performance of the stabilizer 18).

The type of stabilization last on the list was developed by Block and others in 1967 31). This system modulates the potential on the focus electrode and the control signal is transmitted by a light beam from ground potential. This system has a fast response but its voltage range is limited to part of the total focus voltage. The application of the system is limited to single ended accelerators where the focus electrode is available for control purposes. Generally it cannot be used on its own as it is essentially an auxiliary system which extends the frequency response of other stabilizers.

It was shown above quite clearly that no method is superior to the other. Each has its merits and drawbacks and the designer is usually influenced by large numbers of factors in selecting the best stabilizer system for an accelerator. In practice the combination of two or more of the methods described here are used on one accelerator.
2.1 Introduction

The Oxford electrostatic generator project was initiated in early 1960. The specification called for the development of a compound system consisting of two accelerators. The aim of the project was to produce a system capable of generating protons up to 20 MeV and heavier ions like oxygen and sulphur in the 60-80 MeV region. Both accelerators were required to be capable of operating independently, the vertical injector at positive or negative and a tandem with positive terminal voltage into two separate target rooms\(^{32-35}\). The objective was to be achieved by the design and installation of a vertical machine with 10 MV on its terminal which can be coupled with a standard HVEC EN tandem accelerator with its usual range of facilities. The work was carried out by a team from the National Institute for Research in Nuclear Science with the co-operation of the Oxford Electrostatic Generator Group.

2.2 Method of stabilization

Out of the many different ways of stabilization described in section 1.7 the combination of the belt spray control and liner control was considered most suitable for the injector. This choice was made on the following grounds:
1. Since the injector is designed to work at positive or negative terminal voltage, the stabilization is independent of the polarity of the terminal voltage.

2. Having indirect capacitive coupling to the terminal cap through the intershield provides instantaneous control over the terminal voltage without the time lag of the alternative corona system.

3. Difficulties experienced with corona stabilization in Van de Graaff generators with negative terminal voltage.

In order to produce this capacitive coupling to the terminal the tank was lined with an insulated coaxial metal tube, called the liner constructed of five panels each of 23 oval shaped highly polished metal tubes 4" wide and 1" deep.

Fig.1 shows the schematic diagram of the injector stabilization. The system operates in the following way. Ions generated by the ion source inside the terminal cap are accelerated in the accelerating tube. The magnet after the accelerator is used to analyze the momentum of the particles entering the magnet gap from the accelerator. The magnet transmits only particles having the right rigidity to be deflected through 90°. A slit system placed at the image focal point of the magnet serves to detect any small changes in particle momentum. Comparing the beam current intercepted by the "lower" and "upper" slits a difference signal occurs when the particle momentum is low or high. The error signal formed in this way is used to stabilize the voltage on the terminal by changing the current sprayed on the belt and also changing the potential
of the intershield around the terminal\textsuperscript{36}).

This form of stabilization only works when a beam is present on the slits. When, however, the beam is interrupted or drifts outside the range of the momentum analyzer it provides no control over the generator voltage. In order to overcome this difficulty a third auxiliary loop was introduced in the system using the signal supplied by the generating voltmeter\textsuperscript{37}). During slit operation this control loop is paralyzed and only used for visual indication of the terminal voltage.

2.3 \underline{Open loop gain requirements}

Supposing that the ripple of the terminal is of the same order of magnitude as that of the intershield we can determine the required open loop gain for the system. Since factors other than the machine stability as described in paragraph 1.6 limit the energy resolution to about ± 1 KV it seems that any attempt to stabilize the terminal voltage better than 1 KV will not result in much better overall performance. This ± 1 KV fluctuation on the terminal at 5 MeV terminal voltage will result in ± 0.030 inch linear movement on the analyzing slits. In the case of very low current operation (minimum in the order of 10 nA) this movement produces a difference current in the slits of about 1 nA. If 10 MΩ is selected for the input impedance of the slit amplifier this 1 nA will produce a 10 mV error signal with reference to the input of the amplifier. It can be seen that the attenuation of the sensing element is
$10^5$ i.e. 100 dB. Since we want to reduce the ripple from 60 KV to 1 KV the required open loop gain, applying Bode's principle, is 60 or 36 dB. Thus the open loop gain is a minimum of 136 dB for the conditions above.

In the following sections the stability criteria will be calculated for the two loop stabilizer using the injector equivalent circuit, and also the analysis of the auxiliary generating voltmeter loop will be given.

2.4 Equivalent circuit polar diagram and stability criteria for the injector two loop stabilizer

The injector as an electrical circuit consists of essentially three electrodes: the terminal, the intershield and the liner, a pressure vessel and a resistor chain connected to the internal electrodes. All of these are fed by two current sources, one of them being the charge belt which has an infinitely high source impedance and the other the control valve driving the liner. The second one has a Norton equivalent circuit with a finite source impedance. The equivalent circuit of the system is shown in Fig. 6. Here $C_1$, $C_2$ and $C_3$ are the inter-electrode capacitance of terminal to intershield, intershield to liner and liner to tank respectively. $R_2$ and $R_3$ are the upper and lower stack resistors, and $R_4$ is the parallel combination of the control valve anode resistance $R_L$ and valve dynamic plate resistance.
Equivalent circuit of the injector

Fig. 6

The circuit in Fig. 6 shows that the voltage induced on the terminal by any one of the signal sources is independent of the other, thus one can apply Maxwell's theorem of superposition and write:

\[ V_0 = Z_{II} i_2 + Z_I i_1 \]  \hspace{1cm} (2.1)

where \( V_0 \) is the terminal voltage, \( Z_I \) and \( Z_{II} \) are the corresponding feed impedances to \( i_1 \) and \( i_2 \) generator currents.

From Fig. 6 the following expressions can be derived for \( Z_I \) and \( Z_{II} \) feed impedances:
The polar diagram and the frequency response of $Z^R$ as a function of $\omega$ is shown in Fig. 7 and Fig. 10 respectively, while the polar diagram and the frequency response of $Z^{^R}$ is plotted in Fig. 8 and 10. For the derivation of the expressions given here for $Z^T$ and $Z^{^T}$ feed impedances and also for the symbols, see Appendix 6.2.

After substitution of expressions 2.2 and 2.3 into 2.1 we can write the terminal voltage in the following form:

$$V_o = \frac{R_1A}{(1+\tau_2)(1+\tau_1)} \varepsilon(p) + \frac{e^{-\tau_6p}}{p(1+\tau_5)} \varepsilon(p)$$

where $\varepsilon(p)$ represents the error voltage.

Using the numerical values listed in Table 4 the frequency dependent part of the combined transfer function is plotted in Fig. 9 by adding vectorially the polar diagrams of $Z_T$ and $Z^{^T}$ and its frequency response is shown in Fig. 10.

From the analysis above the following conclusions can be drawn:

1. The frequency response of the $Z_T$ feed impedance is dropping at the rate of $6 \text{ dB/octave}$ between the corner
frequency 0.0632 Hz and 0 and also 55.6 Hz and infinity. The response is flat between the two corner frequencies.

2. The polar diagram of $Z_I$ does not encircle the critical (-1,0) point.

3. The theoretical gain of $Z_{II}$ at $f = 0$ is infinitely high.

4. The polar diagram of $Z_{II}$ can encircle the critical (-1,0) point when the open loop gain exceeds 23.2 dB at $f_c = 0.0938$ Hz.

5. The frequency response of $Z_{II}$ drops at the rate of 6 dB/octave between $f = 0$ and $f = 0.0127$ Hz. From then on it drops at a rate of 12 dB/octave.

6. The transfer function of $Z_I/R + K Z_{II}/R$ becomes infinite at $f = 0$.

7. The frequency response of the two loop system is falling at 6 dB/octave between 0 and the cross-over frequency $f_c = 0.0127$ Hz. The response curve is flat between $f_c$ and $f_1 = 55.6$ Hz. From then on it falls at the rate of 6 dB/octave again.

Summarizing the results above we can say the following:

The servo driven belt charge control used as a single loop stabilizer can provide some stability for the injector at D.C. and also at low frequencies. The loop gain, however, is limited to 23.2 dB at $f = 0.0938$ Hz by the conditional stability of the loop. The loop has a poor response for higher frequencies. On the other hand the liner stabilizer loop has the necessary wide frequency response except for D.C. and for low frequencies. The loop on its own forms
POLAR DIAGRAM OF $\frac{Z_\Pi}{R}$

FIG. 8.
POLAR DIAGRAM OF $\frac{Z_1}{R_1} + K \frac{Z_II}{R}$

FIG. 9.
an unconditionally stable system which, however, cannot compensate for D.C. changes. The combination of the two loops gives a right frequency response from D.C. up to the KHz range and the open loop gain is not limited by oscillatory conditions.

2.5 Generator voltmeter control of the intershield voltage

The two loop stabilizer as described in the previous paragraph works only when a beam is present on the analyzing slits. Sometimes, however, it is desirable to stabilize the terminal voltage of the accelerator when the beam is interrupted. Since the generating voltmeter measures the intershield potential it seems logical to use this to provide voltage stabilization for the machine when there is no beam on the slits. One cannot expect, however, a great degree of energy stability from the generating voltmeter control loop because the voltage distribution between the terminal and the intershield depends on factors such as the tolerance and condition of the stack resistors, the condition of the accelerating tube and also the loading on the accelerator by the beam.

The effect of the presence of the intershield was analyzed by using the equivalent circuit of the injector and it was found that the transfer function between the terminal and intershield voltage was

\[
\frac{V_o}{V_s} = \frac{R}{R_3} \cdot \frac{1 + \tau_s p}{1 + \tau_5 p}
\]  

(2.5)
where $V_o$ is the terminal voltage

$V_s$ is the intershield voltage

$\tau_s$ and $\tau_5$ are time constants (see paragraph 6.4)

$R$ and $R_3$ are shown in Fig.6

The polar diagram and the frequency response of this transfer function is shown in Figs. 11 and 12 respectively.

The relationship between generating voltmeter output $V_g$ and the terminal voltage is given below taking into account the parameters of the circuit following the generating voltmeter.

$$\frac{V_g}{V_o} = K \frac{1 + \tau_5 p}{(1+\tau_f p)(1+\tau_s p)}$$

(2.6)

where $K$ is a constant

and $\tau_f$ is the time constant of the low-pass filter following the AC-DC converter for filtering out the carrier frequency.

After plotting the frequency response of the system using the generating voltmeter as a source of error signal (see Fig.13) it is clear that the difference between the two ways of stabilization is in the frequency range only.

So the conclusions drawn from the analysis of the two loop main stabilizer are applicable to the generating voltmeter stabilizer as well. The differences in performance are as follows:

1. The additional phase-shift caused by the intershield capacitances and the stack resistor circuit is negligible compared with that of the other circuits in the loop.
POLAR DIAGRAM OF INTERELECTRODE CAPACITANCE AND STACK RESISTOR CIRCUIT

FIG. II.
\( \tau_s = 24 \text{ sec} \)

**Fig. 12.**

**Frequency response of interelectrode capacitance and stack resistor circuit.**
2. The maximum additional attenuation for frequencies higher than 0.01 Hz is 10 dB.

3. The overall frequency range of the combination of the transfer functions given in (2.4) and (2.6) is determined by $\tau_f = 32$ msec. The frequency response of the generating voltmeter stabilizer is plotted in Fig. 13. For detailed analysis see paragraph 6.4.

2.6 Circuit design

The circuit design described here forms an integral part of the research work presented in this thesis and it is based on the detailed analysis presented in the earlier sections.

The design of the stabilizer is based on the use of operational amplifiers. This choice was made on the easy serviceability of such a system. It was considered necessary for the operational amplifiers particularly in the slit amplifier circuit to satisfy the following conditions:

- Open loop D.C. gain: $> 100$ dB
- Average (max) drift: $< \pm 30 \mu V/\degree C$
- Input bias current: $< \pm 0.10$ nA
- Input noise D.C. to 10 KHz: $< 10$ dB
- Input impedance: $> 10^8$ $\Omega$
- Output impedance: $< 1 K\Omega$
- CMRR: $> 60$ dB
On this ground out of the number of makes investigated, the range manufactured by Burr-Brown was chosen, out of which the type 1552/15 selected for the input stage satisfies the requirements above.

2.6.1 Main stabilizer

The detailed circuit diagram for the two loop stabilizer is shown in Fig. 14 and the functional description of the circuit is as follows.

The two slit amplifiers $A_1$ and $A_2$ have fixed voltage gains of 20 dB and variable input impedances. This is achieved by the use of the double emitter transistor $TR_1$ in the input circuit. The sum of the two slit currents provides bias for the base of the transistor while the input signal to the slit amplifiers is formed across the emitter-collector junction by the slit current. In this way the open loop gain becomes inversely proportional to the total beam current intercepted by the slits. This circuit refinement published by Gere, Lie and Miller in 1967\textsuperscript{39} was adopted to help automatic operation. The performance test carried out on the circuit is described in section 2.7.3.

$A_3$ is a differential amplifier which forms the error signal from the two slit signals and its external circuit also incorporates the polarity switch $SW_1$ for positive and negative generator operation. The gain of the amplifier is set to unity.

$A_4$ is the main amplifier with gain ranging between
0 and 40 dB. The optimum gain the system can be preset by the gain selector switch SW₂ for a wide range of operating conditions. The output from A₄ amplifier is split into low and high frequency components. The D.C. and the low frequency components of the signal are fed into the servo modulator while the higher frequencies are fed into A₅ the high signal level amplifier.

A₅ is a fixed gain amplifier with a gain of 20 dB. It is capable of providing maximum 80 V peak to peak signal for the driver stage.

The driver stage is formed by TR₂ and TR₃ and has a gain of 14 dB. The circuit is capable of handling 400 V peak to peak output swing with transistors designed for much less collector voltage. The circuit was designed originally for CRT plate amplifiers. The output voltage from the driver stage provides D.C. bias and the drive signal for the grid of the HT7 liner valve. The HT7 valve and the associated circuit elements are accommodated in an approximately 150 gal capacity oil tank to provide the required insulation and cooling. The oil is cooled by water pipes lining the walls inside the oil tank. The cooling is to reject the 1.5 kW electrical power turned into heat by the circuit. The HT7 valve is powered from a 60 kV HT power supply manufactured by Sorensen.

A₆ amplifier forms the sum of the two slit currents and energises relay RLA/1 when the slits intercept some beam current. TR₄ transistor forms a power driver for the relay. This circuit provides an automatic change-over
between the slit and the generating voltmeter stabilizer. The generating voltmeter loop is paralyzed under normal operation when the accelerator is stabilized by the beam. The change-over is provided by the relay contact RIA/1 between stages A3 and A4.

2.6.2 **Servo system**

The circuit diagram of the low frequency servo loop is shown in Fig. 15. The operation of this circuit is as follows.

The outputs of A4 in opposite phase are fed into the low frequency modulator circuit TR4 and TR6. In the circuit formed by transistors TR4-TR5 and TR6-TR7 the low frequency components of the error signal are converted into 50 Hz A.C. modulated in amplitude by the low frequency fluctuation. This signal is fed into the PD 86 T/1 servo amplifier driving the servo motor-generator set. The output of the generator is fed back to the summing point of TR10 and TR11 transistors to give the required servo response.

An A.C. servo was chosen in preference to a D.C. servo as it is less subject to drift and also it has higher efficiency. For driving the servo, a high efficiency servo amplifier, sometimes known as class D amplifier, was selected which does not require any D.C. power supply. This type of amplifier is characterised by its small size and the low power dissipation.
2.6.3 Generating voltmeter circuit

In Fig. 16 the circuit diagram of the auxiliary generating voltmeter stabilizer is shown. The link between the main stabilizer and the generating voltmeter loop is formed by $A_6$ as described earlier. This amplifier sums the slit currents and energises a relay via transistor $TR_3$ when the beam is present on the slits. Under this condition the generator is controlled by the main stabilizer. When the beam drifts off the slits the relay becomes de-energised and the generating voltmeter control takes over. The level of the switching can be preset by the gain control switch of $A_6$ amplifier.

In this circuit, stage $A_7$ and $A_8$ provide buffering between the plates of the G.V. and $A_9$ summing amplifier. The purpose of $A_9$, apart from summing the signal from $A_7$ and $A_8$, is to eliminate the dependence of the output voltage of the G.V. on the speed of the motor by integrating the signal from the generating voltmeter as described in section 6.4.

Stage $A_{10}$ is an A.C. to D.C. converter which separates the terminal voltage fluctuation from the carrier frequency of the generating voltmeter. The output signal of this stage is used to monitor the terminal voltage via transistors $TR_4$ and $TR_5$.

Stage $A_{11}$ is a differential amplifier which forms the error signal between the output of $A_{10}$ and the reference voltage provided by the reference source $A_{12}$. The error
signal formed in this way is fed into the stabilizer loop when relay RL₄ is not energised to stabilize the generator terminal voltage as described above.

2.7 Injector tests and their results

In order to prove the performance of the stabilizer the following tests were carried out on the injector.

1. Test on stability versus slow and fast loop gain.
2. Automatic gain control calibration.
3. The response of the automatic gain control circuit versus slit current.
4. Energy resolution measurement and energy calibration of the injector.

Block diagram for measuring the injector energy stability against loop gain variation.

Fig. 17
2.7.1 Test on stability versus slow and fast loop gain

For measuring the relative energy stability the arrangement shown in Fig. 17 was used. First the machine was operated at positive polarity and 5 MeV proton beam was used. The entrance slits of the analyzing magnet were set to 0.300" square while the exit slit was set to 0.03". The total beam current at the entrance slits was 830 nA, out of which 360 nA left the magnet analyzed. The output of the differential amplifier was recorded by a Devices M4 recorder. During the experiments the gain of the LF and the HF loop was varied independently. The results of this test are shown in fig. 18. The diagram is constructed by joining the points of equal relative stability. The 0 dB point represents the best operating conditions at 136.7 dB equal loop gain at the cross-over frequency. It can be seen that all stable operating conditions fall inside an ellipse with 6 dB and 5 dB major half axes. If the higher HF loop gain region the high gradient indicates that although the energy stability is slightly better (-1.2 dB) a small disturbance can take the beam outside the range of the slits. This is due to the non-linear behaviour of the energy analyzer as described in section 1.7.

In the second part of the test the terminal voltage was negative and negative H beam of 5 MeV energy was produced. The slits were set to the same settings as for the positive run. The current was 1.1 μA before the magnet and 240 nA analyzed. Then the test was repeated in the same way
LOOP GAIN VERSUS ENERGY STABILITY FOR THE INJECTOR AT POSITIVE TERMINAL VOLTAGE.

FIG. 18.
as described before. The results are shown in Fig. 19. The difference in the shape of the two diagrams is due to the difference in the way of charging the terminal for positive and negative operation. At negative operation the terminal voltage is generated by spraying charges on the belt inside the terminal and the spray at ground potential, approximately 30 - 50 \( \mu \text{A} \), only used for stabilization. In positive operation, however, the charges are carried by the belt from ground potential while the terminal belt charge power supply is switched off. The terminal spray power supply was not stabilized and this instability shows up in higher terminal ripple. It can be seen from the diagram that the optimum operating condition is shifted towards the lower HF loop gain and higher LF gain region.

2.7.2 Automatic gain control calibration

The results described above are only applicable for all operating conditions when the loop gain of the control loops is independent of the current intercepted by the slits. The first stages of the slit amplifiers are designed to provide this automatic gain control as described in section 2.6.1. The calibration curve of this stage was measured (see Fig. 20) by injecting a constant AC signal on both slit inputs parallel with DC current from a high impedance source (100M\() and measuring the open loop AC gain of the system for injected DC currents ranging from 0.5 nA to 10 \( \mu \text{A} \). The test was carried out at the frequency of 20 Hz.
LOOP GAIN VERSUS ENERGY STABILITY FOR THE INJECTOR AT NEGATIVE TERMINAL VOLTAGE.

FIG. 19.
The circuit visibly gives a good linear gain control for slit currents between 10 nA and 2 μA. Because of the flat frequency response of the amplifier between DC and 50 KHz, it was not considered necessary to repeat the test at other frequencies.

2.7.3 The response of the automatic gain control circuit versus slit current

After the stabilizer was installed the dependence of the gain on the analyzer slit setting was measured in the following way. Protons were accelerated to 5 MeV energy and the beam current at the entrance slit of the analyzing magnet was set to 2 μA, out of which a maximum of 1.4 μA came out of the magnet analyzed. The analyzing slit opening was then varied between 0 and 0.15". The current measured after the slit was 50 nA for 0 setting and 1.4 μA for 0.1" setting. For each slit setting the output of the differential amplifier was measured. For 0 to 0.1" slit opening, all error signal figures fell between ±1 dB as shown in Fig. 21.

The experiment was repeated for negative terminal polarity with 7.025 MeV O\(^{16}\) beam. The result was the same as before. The measured points are plotted in Fig. 22.

In both cases when the slit separation was increased beyond the diameter of the beam sudden large fluctuation occurred due to the beam moving between the slit jaws.

Due to the slight beam intensity fluctuation from the ion source the error in the measurement was in the order of 1 dB. Great care was taken during both experimental runs to retain identical focusing conditions during the experiment.
SLIT OPENING VERSUS ENERGY STABILITY FOR NEGATIVE POLARITY.

FIG. 21.

SLIT OPENING VERSUS ENERGY STABILITY FOR POSITIVE POLARITY.

FIG. 22.
2.7.4 Energy resolution measurement and energy calibration of the injector

The resonance chosen for checking the energy calibration of the injector and its analyzing magnet was produced by alpha capture in $^{15}\text{N}$. The target was produced in $\text{Ta}^{15}\text{N}_2$ form on Au backing and its thickness was found to be 10 keV by measuring the total width of the resonance. The $^{15}\text{N}(\alpha,\gamma)$ reaction emits gamma rays which were detected in the ranges of 5.1 - 8 MeV and 6.3 - 8 MeV.

In order to reduce the error due to the differential hysteresis the field in the analyzing magnet was raised in opposite polarity near to saturation and slowly reduced to zero. Then the field was gradually increased in the normal direction and the resonant energy was approached from below. The magnet entrance slits separation was kept during the whole experiment at 0.120 inch in both east-west and north-south directions. The gamma ray yield curves were first measured at 0.12 inch analyzing slit separation, then the slit separation was reduced to 0.03 inch and the experiment was repeated. The leading edges of the resonance measured in this way are plotted in Fig. 23. After graphical differentiation of the leading edge of the resonance curves the energy spread in the incident alpha beam was plotted (see Fig. 24) for 0.12 and 0.03 inch analyzing slit separation by fitting a Gaussian curve to the experimental points. The full width at half full height representative of the energy spread was found to be 0.97 keV and 0.65 keV respectively for the two slit settings.
COUNTS FOR 100 µC x 1000

ANALYSING SLIT SEPARATION
0.03" --- --- ---
0.12" --- --- ---

4.4477 MeV

WINDOW 5.1-8 MeV

4.4475 MeV

WINDOW 6.3-8 MeV

4.4414

4.4474

4.4539 ENERGY MeV

NMP f kHz

γ RAY YIELD CURVES FOR 15N(α,γ) RESONANCE.

FIG. 23.
CURVES SHOWING THE ENERGY SPREAD IN THE BEAM AT TWO ANALYSING SLIT SETTING.

FIG. 24.
This method assumes that both the contribution of the target thickness and the finite width of the resonance are negligible compared to the beam energy spread. In this particular experiment the measured target thickness was 10 keV and the thickness of the resonance quoted in the literature is less than 0.1 keV.

In order to check the differential hysteresis of the momentum analyzer the experiment was repeated weeks later. During this experiment first the demagnetizing procedure was repeated on the magnet as described earlier and the resonance was plotted in the direction of increasing energies. Then the magnet current was increased to near saturation in positive direction and the resonance was plotted again in the direction of decreasing energies. The yield curves are shown for the gamma rays between 3.9 - 8 and 5.1 - 8 MeV in Fig. 25.

Comparing the two sets of curves in Fig. 23 and Fig. 25 taken in the direction of increasing energies the following conclusions can be drawn:

1. Provided the analyzing magnet is demagnetized and the resonance is approached from the direction of the increasing energies the reproducibility of the yield curve is within the error of the measurement. In fact all measured resonant particle energies fall within 0.5 keV between 4.4472 MeV and 4.4477 MeV.

2. When, however, the resonance is approached from the
MEASUREMENT OF THE DIFFERENTIAL HYSTERESIS OF THE INJECTOR ANALYSING MAGNET.

FIG. 25.
direction of high energies the differential hysteresis of the momentum analyzer is 9 keV.

3. The beam energy spread for 0.12 inch slit separation was found very reproducible.

The energy of this resonance according to published data is $4.4675 \pm 0.0002$ MeV. The energy, however, where this resonance occurred in this experiment according to our earlier calibration was $4.4475$ MeV. This 20 keV difference is likely due to the changes introduced in the system particularly in the slits and the NMR probe.

By using equation (1.5) $k$ the magnet constant was calculated. The value for $k$ resulted from the calculation is $0.09023 \pm 0.00005$ against the figure of $k = 0.08984 \pm 0.00004$ based upon earlier calibration runs.

On this evidence more calibration points are needed within the energy range of the injector to check the calibration of the momentum analyzer.

2.8 Conclusion

The method of stabilizing the voltage of the injector as a generator chosen originally certainly fulfilled the expectations. The two loop stabilizer - the combination of the belt charge control and the liner capacitively coupled to the terminal - has got fundamentally wide enough frequency response to cover the frequency spectrum of the disturbances occurring in the generator.

The mathematical analysis presented in section 2.4
fills a gap in the theory on the stabilization of Van de Graaff accelerators with one intermediate electrode. In doing so it gives a better understanding of the system for designers and users.

The belt charge control on its own as it was shown suffers with two shortcomings. These are the loop limited frequency response and the delay in transferring the charges to the terminal due to the finite speed of the transporting belt. The limited frequency response prevents the stabilizer from eliminating disturbances above the frequency of 0.01 Hz. The delay in the charge transfer at the same time limits the loop gain to $23.2 \times 20^3$ by the oscillatory conditions at 0.0938 Hz. The loop, having infinite gain at $f = 0$, is capable of eliminating all DC drifts.

The capacitive liner loop has wide enough frequency response to cover the spectrum of the disturbances occurring in the accelerator with the exception of a limited frequency range between $f = 0$ and 0.001 Hz. As such it is not suitable to use on its own.

The combination of the two loops, however, has a wide frequency response between $f = 0$ and 200 Hz with 6 dB drop in gain. The system is unconditionally stable so there is no theoretical limit to the open loop gain.

The introduction of the generating voltmeter as a sensing element represents a great step forward towards easing the operator of the injector. With the introduction of automatic changeover between the slit stabilizer and the generating
voltmeter stabilizer the system becomes semi-automatic. The generator is continuously under the control of one or other closed loop. The generating voltmeter control, however, does not provide a satisfactory accurate control over the terminal voltage. This is simply due to the fact that the generating voltmeter measures the voltage of the intershield and not directly the terminal potential. The ratio of upper and lower stack currents depends on such factors as the condition of the resistors and the accelerating tube, loading due to the beam etc. Experience shows that the difference between the reading of the generating voltmeter and the actual terminal voltage in extreme cases such as high beam current (30 - 40 μA) can induce an error as large as 100 KV.

As was demonstrated by the tests described in section 2.7.3 the automatic gain control circuit sets the optimum loop gain for slit currents between 8 nA and 2 μA. This range covers approximately 90% of all operating beam intensities but in practice the injector was operated above and below the given limits with very little deviation from the optimum conditions.

The electrical design and the construction of the control system proved satisfactory. The components in all were reliable, the servicing and testing was easy.

The analysis as described in the earlier sections describes the system well and predicts the behaviour of the generator very closely. The calculated open loop gain required to reduce the terminal ripple to 1 KV was 136 dB. In practice the best relative stability was achieved with 136.7 dB at the cross-over frequency. The limitation in
achieving higher stability lies in the highly non linear nature of the sensing elements, the slits as shown in section 1.7.
3.1 Introduction

The EN tandem forms part of the Oxford electrostatic accelerator project and provides the second and third stages of acceleration for the particles injected from the 10 MV single-ended machine. The tandem accelerator, designed and manufactured by High Voltage Corporation, is capable of producing protons of energy up to 12 MeV and heavier ions up to higher energies depending on their charge state if multiply charged ions are produced. When it is coupled to the injector the total energy of the ions is determined by the charge state of the ions leaving the accelerators and the terminal voltages, as described in equation (1.2).

3.2 Energy spread due to the generator

The major components of the unstabilized terminal voltage ripple were found by analyzing the photographs taken of the output signal of the sensing plate looking directly at the terminal. A typical set of photographs are shown in Fig. 26.

The likely origin of these major frequencies in the spectrum is as follows:

2.5 Hz is the belt rotational frequency

5 Hz is the first harmonic of the belt rotational frequency
TANDEM TERMINAL RIPPLE

a. TIME BASE : 200 msec/cm
   CALIBRATION : 2 kV/cm

b. TIME BASE : 20 msec/cm
   CALIBRATION : 2 kV/cm

FIG. 26.
15 Hz is the 6th harmonic of the belt rotational frequency probably due to irregularities on the surface of the belt.

50 Hz is the mains pick-up.

400 Hz is the frequency of the terminal alternator.

The last two components are most probably induced by the magnetic fields of the alternators and they are unlikely to contribute to the beam energy fluctuation.

3.3 Factors involved in corona stabilization

All HVEC EN tandems are designed to use a corona stabilizer. This stabilizer consists of three major parts: the momentum analyzer, the amplifier system and the corona probe. The corona stabilization is described in detail in section 1.7 and the schematic diagram of the system is shown in Fig. 2.

Since the corona probe is connected directly to the anode of a thermionic valve, the grid voltage fluctuation produced in the way described in section 1.7 governs the field emission of the corona probe.

The corona probe itself is a semi-spherical device with a number of sharp needles protruding from its face. The probe is mounted on a long arm and the position of the needles relative to the terminal can be adjusted by mechanical means, using an electric motor. The position of the probe is indicated on a counter. The calibration of the counter against probe distance for the Oxford tandem is given in Fig. 27.
25" MAX. STROKE
245 TURNS TOTAL
0.102" / TURN

D = \frac{\pi}{8}

Fig. 27.

INCH

D = 1

D = 0.75

D = 0.5

2 5/8

200 180 160 140 120 100 80 60 40 20 0

COUNTER READING

CORONA COUNTER READING CALIBRATION AGAINST PROBE DISTANCE FROM THE TERMINAL.
The tandem as a control circuit consists of a capacitor formed between the centre terminal and the pressure vessel and the two resistor chains connected in parallel between the high voltage terminal and earth. The system is fed from two current sources one of which is the belt which has a high source impedance and the other one is the corona load. The former one provides a constant current which is not governed by the control loop, and thus as far as the control loop calculations are concerned does not need to be taken into account. On the other hand the corona current source delivers negative current consisting of DC and AC components to the terminal which tends to lower the positive terminal potential. The DC component of this current does not play any part in the control of the terminal ripple.

The difficulty in analyzing this system arises mostly from the following three factors:

1. The effect of the high pressure gas on the charge carriers.

2. The presence of the probe in the electrostatic field between the terminal and the tank, which disturbs the axial symmetry of the field.

3. The variable position of the probe between the tank and the terminal.

The behaviour of charge carriers in a high pressure gas travelling under the influence of electric fields, has been the subject of studies as far back as 1922\textsuperscript{(43)} and since then a steady stream of theoretical and experimental data has been published. An outstanding attempt was made by Uman and Warfield\textsuperscript{(44)} to follow up the problem.
mathematically with the results, as they call it "... mathematical approximations of questionable validity" to apply their theory to simple cases. All authors, however, do agree that the effect of the gas on the charge carriers can be split up into two parts, namely the delay caused by the finite drift time and the "smearing" or dispersion. The latter is caused by the collisions between the gas molecules and the charge carrier as well as the diffusion of electrons and the possibility of electron attachment to the molecules of the gas. The effect of this dispersion on the signal is the change in its frequency content. Although in equilibrium over a given time the total charge collected on the terminal is the same as that injected by the probe, the signal collected is somewhat distorted with higher low frequency and lower high frequency content. In mathematical form:

\[
\int_{T_1}^{\infty} \left\{ \int F(\omega)e^{i\omega t} \, d\omega \right\} \, dt = \int_{T_2}^{\infty} \left\{ \int F(\omega)e^{i\omega t+\alpha \omega t} \, d\omega \right\} \, dt
\]

(3.1)

where \( F(\omega) \) is the Fourier transform of the injected current signal and \( \alpha \) is the dispersion coefficient.

Although the published flight time and drift velocity measurements are in good agreement with the theoretically predicted data\(^5,\,^6\) the dispersion coefficient needs to be measured experimentally in each individual case. Both the drift velocity and the dispersion depend on gas conditions such as the components of the tank gas, pressure and the water vapour content\(^7\) as well as the HT electrode potential and the position of the probe.
The dependence of the drift velocity $v$ on the factors mentioned is well described by the following formula\(^{48}\):

$$v = k\left(\frac{E}{P}\right)^{\frac{1}{2}} \quad (3.2)$$

where $k$ is a constant

$P$ is the gas pressure

and $E$ is the electric field.

Similarly to other dispersive transmission systems in the analysis presented in section 3.5 the dispersion is represented by an exponential term whose power is a linear function of the frequency, the flight time and the dispersion coefficient $\alpha$.

3.4 The field distortion caused by the probe

Special attention was paid to an equivalent electrode arrangement in the tandem to obtain the best possible approximation in the analysis. The problem of concentric cylindrical electrode systems with a third electrode in the space in between them is well known and reasonably well documented in the literature\(^{49}\). The electrostatic field profile was calculated in the plane perpendicular to the accelerator axis and going through the centre of the corona probe. This is shown in Fig. 28. From these calculations on the effect of the presence of the corona probe in the vessel one can reach the following conclusions:

1. The effect of the position of the probe relative to the terminal on the drift time of the charge carriers is not negligible, thus it needs to be considered.
POTENTIAL PROFILES THROUGH THE PROBE.

FIG. 28.
2. The mathematical expression for the field distortion is too complicated to be used in the analysis and its effect on the final velocity of the charge carriers is smaller than other factors such as the varying gas pressure.

3. The effect of the space-charge due to the drifting ions is at least an order of magnitude smaller than other factors involved, since the current representing the space charge does not exceed 50 μA under normal operating conditions.

3.5 Tests of the behaviour of the corona probe

As it was pointed out in the previous section, before the corona system can be analyzed certain operational parameters need to be determined. The tests described here were to determine the following two parameters:

1. Ion transit time from the probe to the terminal
2. Dispersion coefficient.

3.5.1 Ion transit time

In order to find out the transit time of the ions between probe and terminal in the high pressure gas, two sets of tests were carried out on the tandem. The block diagram of the experimental arrangement is shown in Fig. 29. First a burst of charge with a shape of a square pulse was injected into the gas via the corona probe and the waveform of the pulse induced on the terminal was photographed via
Block diagram of the experimental arrangement for measuring ion transit time.

Fig. 29

the sensing plate. (See Fig. 30a). By measuring the interval while the displacement current is present, the ion drift time in the gas can be measured. In this experiment the probe was at 21" distance from the terminal and the drift time measured was 20 msec at 4 MV terminal voltage and 175 lb/in² gas pressure. To cross-check the results obtained that way a sinusoidal signal was injected into the tank via the corona probe and the phase shift was measured between the input and the terminal signal. The result of this test was within 1 msec of that of the previous experiment. (See Fig. 30b).

Then the probe was withdrawn to 25" from the terminal and the test was repeated. The measured ion transit time was 26.3 msec ± 0.5 msec.
ION DRIFT TIME MEASUREMENT IN THE TANDEM
a. BY USING 20 msec DURATION PULSE
b. BY USING SINE WAVE
TIME BASE: 20 msec/cm

FIG. 30.
3.5.2 Dispersion coefficient

For determining the dispersion coefficient \( \alpha \) the same experimental arrangement was used as before. Since the attenuation due to the gas can be described by an exponential term the conditions for one single frequency of the Fourier spectrum of the corona current can be described in the following mathematical form:

\[
I(t) = I_0 \sin(\omega(t-\tau_a)e^{-\alpha \omega \tau_a})
\]  

(3.3)

where \( \tau_a \) is the ion transit time and 

\( I \) is the ion current.

If the effect of the gas were not present the machine and the corona system would behave as a simple integrator with \( \tau_1 = R_C C_o \) time constant where \( R_o \) is the stack resistor and \( C_o \) is the terminal to tank capacitance. The attenuation versus frequency for a simple integrator was calculated and plotted in Fig. 34. \( (\alpha = 0) \) Considering the assumptions made in section 3.3, that the effect of the gas on the delay and the dispersion is separable it can be said that the deviation in attenuation from the ideal case at any frequency is attributed to the dispersion of the gas. In order to measure the attenuation a sinusoidal signal was injected via the corona probe and the response was measured via the sensing plate. Since it was assumed that the attenuation in logarithmic units is linearly proportional to the frequency, the expected deviation from the ideal attenuation is small at low frequencies. At high frequencies, however, the signal to noise ratio is low, thus the possible experimental error is high. For these reasons the frequency
at which the measurement was carried out had to be very carefully considered. Eventually the measurement of this attenuation was carried out at 100 Hz where the expected difference in attenuation was much larger than the expected error in the measurement. During the experiment the terminal voltage was kept constant at 4 MV and the gas conditions unaltered. The attenuation measured this way was 10 ± 0.5 dB. The test was repeated at different operational gas conditions and all figures measured were within 15 dB. During the experiments the recorded moisture content of the gas was less than 20 ppm.

Finally to obtain an upper limit for α, the attenuation was measured at 180 lb/in² and 60 ppm moisture content. We must note, however, that this condition is outside the normal operating limits. The attenuation measured then was 28.5 ± 0.5 dB.

The obtained attenuation figures and the α values calculated from them are listed below:

<table>
<thead>
<tr>
<th>P lb/in²</th>
<th>100</th>
<th>150</th>
<th>175</th>
<th>180</th>
</tr>
</thead>
<tbody>
<tr>
<td>A dB</td>
<td>10</td>
<td>13</td>
<td>15</td>
<td>28.5</td>
</tr>
<tr>
<td>α</td>
<td>0.08</td>
<td>0.091</td>
<td>0.1</td>
<td>0.2</td>
</tr>
</tbody>
</table>

Table 1

From these figures one can see that α normally does not exceed 0.2 and for most operating conditions it is below 0.1.
3.6 Transfer function and polar diagram of the corona stabilizer

The tandem can be considered as a coaxial system with electrodes as shown in Fig. 31. Then the electric field at any point in x distance from the centre is:

$$E = \frac{V}{x \log \frac{b}{a}}$$  

(3.4)

where V is the potential difference between the inner and the outer electrode, a and b are the radii of the inner and the outer electrode respectively.

Coaxial electrode configuration

Fig. 31

A charge Q moving between the electrodes induces a q(x) on the terminal which has the following relation to Q:
\[ q(x) = Q \frac{\log \frac{b}{x}}{\log b/a} \]  

(3.5)

where \( x \) is the distance of the charge from the centre.

As it was shown in section 3.3, the ions drifting between the two electrodes suffer delay so the current collected by the inner electrode will be late by \( \tau \). In addition to this the collision between the ions and the gas molecules causes dispersion in the current waveform. This dispersion makes the frequency complex

\[ p = p^* + \alpha \omega \]  

(3.6)

where \( \alpha \) is dispersion coefficient, and \( p^* = j\omega \).

Considering these above, the total charge on the terminal will be given by the following integral for \( x \) between the limits of \( a \) and \( b \).

\[ q(t) = \frac{1}{kV^2 (\log b/a)^2} \int_{a}^{b} \log \frac{b}{x} x^{\frac{1}{2}} f(t-\tau) dx \]  

(3.7)

For the detailed analysis see section 6.3.

It can be shown that integral (3.7) after neglecting the second order terms gives the following results:

\[ q(p) = \frac{I_0}{\log b/a \tau_a} \left[ \frac{-p(2D-1)^2 \tau_a}{p^2} + \right. \]

\[ (1-D) \tau_a \left. \frac{1-2(2D-1)e^{-p(2D-1)\tau_a}}{p} \right] \]  

(3.8)
where $D = x_o/b$ is the probe distance relative to the radius of the tank and $\tau_a$ is the ion drift time from the probe to the terminal.

**Equivalent circuit of the tandem**  
Fig. 32

From the tandem equivalent circuit shown in Fig. 32 the relationship between $q(p)$ charge and $V_o$ the potential of the terminal is

\[
V_o(p) = q(p) \frac{R_o p}{C_o R_o p + 1} \tag{3.9}
\]

where $C_o$ and $R_o$ are the terminal to tank capacitance and the parallel combination of the high and low energy column resistors respectively.

After substituting (3.8) into (3.9) the expression for
the terminal voltage change subject to the corona load will be the following:

\[
V_o(p) = \frac{I_R R_o}{\log 2} \left[ \frac{1-e^{-p(2D-1)\tau_a}}{\tau_a p(\tau_1 p+1)} \right]
\]

\[
(1-D) \frac{1-2(2D-1)e^{-p(2D-1)\tau_a}}{\tau_1 p+1}
\]

where \( \tau_1 = R_o C_0 \) the machine time constant and \( b \sim 2a \).

The corona load transfer function from (3.10) will be:

\[
K(p) = \frac{V_o(p) \log 2}{R I_0}
\]

In the following sections detailed numerical analysis of the transfer function will be given for different conditions, which have practical significance.

a. Ideal case \((a = 0, D = 1)\)

The polar diagram of the ideal case, when the attenuation is zero and the probe is fully withdrawn, can be calculated from the expression of \(K(p)\) by substituting \(a = 0\) and \(D = 1\) values. Then the transfer function becomes:

\[
K(p) = \frac{1-e^{-p \tau_a}}{\tau_a p(\tau_1 p^*+1)}
\]

b. High terminal voltage \((a \neq 0, D \sim 1)\)

In practice, one of the most important operating
conditions is when the generator is operating at or near its maximum terminal voltage and the probe is fully or nearly fully withdrawn. For this condition the parameters are \( a \neq 0 \) and \( D = 1 \). Then the expression for \( K(p) \) becomes:

\[
K(p) = \frac{1-e^{-pt}}{\tau_a \rho (\tau_1 \rho + 1)}
\]  

Equation (3.13) looks similar to (3.12) here, however, \( p \) is complex as given in equation (3.6).

c. General case \((a \neq 0, D \leq 1)\)

Depending on the terminal voltage of the generator, the probe can be set anywhere between the centre terminal and the tank. (From \( D = 0.5 \) to \( D = 1 \).) However, for practical operation the probe is never nearer than half way between the two electrodes \( (D = 0.75) \). This general case when \( a \neq 0 \) and \( D < 1 \) is described by equation (3.10).

The expressions (3.10) and (3.13) are far too complicated for easy evaluation so the numerical analysis was carried out on the PDP-10 computer (see Appendix 6.6.2).

The polar diagrams and frequency responses for the cases above are shown in Fig. 33 and Fig. 34 respectively.

The following conclusions can be drawn from this analysis:

1. The open loop gain is finite at \( f = 0 \).
2. When \( a = 0 \) and \( D = 1 \) the frequency response drops at the rate of slightly higher than 6 dB/octave between the corner frequency 0.0658 Hz
and infinity. The response is flat between 0 and the corner frequency.

3. The polar diagram oscillates around the negative real axis and doing so cuts it more than once. Because of this the system is conditionally stable.

4. At $\alpha = 0$ and $D = 1$ the polar diagram can encircle the critical ($-1,0$) point the first time when the open loop gain exceeds 52.6 dB at the critical frequency $f_c = 19$ Hz.

5. The corner frequency (see ideal case) shifts to $f_T = 0.0623$ Hz when $\alpha = 0.1$, to 0.0554 Hz when $\alpha = 0.2$.

6. The critical frequencies increase as $\alpha$ increases. The critical frequencies and gain figures are tabulated below for different $\alpha$ values:

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>$f_c$ Hz</th>
<th>$A_c$ dB</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>19</td>
<td>52.6</td>
</tr>
<tr>
<td>0.1</td>
<td>21.5</td>
<td>56.86</td>
</tr>
<tr>
<td>0.2</td>
<td>25.95</td>
<td>62.4</td>
</tr>
</tbody>
</table>

Table 2

7. While $D$ changes from 1 to 0.75 at $f = 0$ the open loop gain drops by 12 dB irrespective of the attenuation.

8. The corner frequency of the frequency response
shifts towards the high frequencies with decreasing D values. The corner frequencies are tabulated below for different \( \alpha \) and D values.

<table>
<thead>
<tr>
<th></th>
<th>( \alpha )</th>
<th>0</th>
<th>0.1</th>
<th>0.2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f_c )</td>
<td>D=1</td>
<td>0.0658</td>
<td>0.0623</td>
<td>0.0554</td>
</tr>
<tr>
<td>Hz</td>
<td>D=0.75</td>
<td>0.0657</td>
<td>0.0588</td>
<td>0.0518</td>
</tr>
</tbody>
</table>

Table 3

3.7 Analysis of the two loop stabilizer

As it was demonstrated by the analysis in the previous paragraph the corona stabilizer system suffers from two major shortcomings both connected with its frequency response. Firstly its gain is finite at \( f = 0 \), secondly its response for frequencies above a few Hz drops sharply. The finite gain at DC prevents the accelerator recovering from long term drift without manual intervention. Because of this it can be regarded as the more serious of the two. Due to this condition the system is not suitable for automatic, or semiautomatic operation without the introduction of automatic DC correction. In order to overcome this it was necessary to introduce a second control loop which increases the open loop gain to infinity at \( f = 0 \).

It was shown in section 3.5 that the current drawn by the probe depends not only on the grid potential of the corona valve but also on the position of the probe relative
to the terminal. This feature of the system can be used to correct for DC changes and increase the open loop gain to infinity at \( f = 0 \).

When the error signal not only changes the bias on the grid of the corona valve but also changes the position of the probe via a servo, a system of this kind can achieve the control as required. This then becomes a two loop stabilizer system for which the corona current can be formulated in the following form:

\[
I_0 = \mu V_g + \eta D
\]  

(3.14)

where \( \mu \) is the amplification factor of the corona tube

\( V_g \) is the grid potential

\( \eta \) is the derivative of the \( I_0(D) \) function

and \( D \) is the relative distance of the probe from the terminal.

By substituting (3.4) into (3.10) and considering the servo response which is described in detail in section 6.2 the following expression for the transfer function has been derived:

\[
K_2(p) = \left[ \frac{1-e^{-p(2D-1)\tau_a}}{\tau_a p (\tau_1 p + 1)} + (1-D) \frac{1-e^{-p(2D-1)\tau_a}}{\tau_1 p + 1} \right] \left[ 1 + \frac{K_0}{p} \right]
\]  

(3.15)

The polar diagram of the system and its frequency response was computed and the curves are shown in Fig. 35 and Fig. 36 respectively. Visibly the transfer function
POLAR DIAGRAM FOR THE TANDEM TWO-LOOP STABILIZER.
FIG. 35.
Figure 36.

Response with single and two loop stabilizer.
$K_2(p)$ is composed of two terms one of which is the single loop transfer function and the other is the servo response applied to the same function.

The conclusions one can draw from this analysis are as follows:

1. The open loop is infinite as required at $f = 0$ and it drops at the rate of 6 dB/octave between $f = 0$ and the corner frequency.

2. Below the frequency of $f = 0.2$ Hz the system follows the same pattern as described in section 3.6.

3.8 Generating voltmeter control of the terminal voltage

The mathematical analysis of the generating voltmeter circuit is given in detail in section 6.4 so here only the specific application will be described using the formulae derived in that section. The relationship between the terminal voltage and the DC output voltage developed by the GV circuit is

$$V_G(p) = V_o(p) \frac{R_3 C_3}{\tau_1} \frac{1}{\tau_f p + 1}$$

(3.16)

where $C_3$ is the maximum capacitance of the segments of the generating voltmeter

$R_3$ is the load impedance on the generating voltmeter segments

$\tau_1$ is the integrator time constant

and $\tau_f$ is the time constant of the low pass filter.

After the substitution of expression (3.10) into (3.16)
the GV output voltage will be

\[ V_G(p) = \frac{I_o R C_3}{\log 2 \tau_i} \left[ \frac{1-e^{-p(2D-1)\tau_a}}{\tau_a p(\tau_1 p+1)(\tau_2 p+1)} \right] - \frac{-p(2D-1)\tau_a}{(1-D) \tau_1 p+1)}(\tau_2 p+1) \]  

(3.17)

Here the \((\tau_1 p+1)\) and \((\tau_2 p+1)\) terms represent two integrating circuits. Since, however, \(\tau_1 \gg \tau_f\) the second term is negligible and for all practical purposes equation (3.17) only differs from (3.10) by a constant. This makes it possible to apply all the conclusions drawn in sections 3.6 and 3.7 to the generating voltmeter control loop.

3.9 Circuit design

The circuits described in the following sections have been designed and built as a part of the research described in this thesis and this work was based upon the results of the analysis presented in the earlier paragraphs.

For convenience, operational amplifiers identical to those used in the injector stabilizer were selected for building the tandem stabilizer.

3.9.1 Main stabilizer

The functional description of the circuit shown in Fig. 37 is as follows. Just as for the injector, the two slit amplifiers here \(A_1\) and \(A_2\) have fixed voltage gains of
20 dB and variable input impedances. This is achieved by the use of the double emitter transistor TR₁ in the input circuit. The sum of the two slit currents provides bias for the base of the transistor while the input signal to the slit amplifiers is formed across the emitter-collector junction by the slit current. In this way the open loop gain becomes inversely proportional to the total beam current intercepted by the slits.

A₃ is a differential amplifier which forms the error signal from the two slit signal. The gain of the amplifier is set to unity.

A₄ is the main amplifier with gain ranging between 0 and 40 dB. The optimum gain of the system can be preset by the gain selector switch for a wide range of operating conditions. The output of this stage controls the current of V₁ corona valve via its grid.

3.9.2 Servo system

For the description of the servo circuit see section 2.6.2. The circuit diagram is shown in Fig. 15.

3.9.3 Generating voltmeter circuit

For the description of the circuit see section 2.6.3 and for the circuit diagram Fig. 16.

3.10 Tests on the tandem and their results

The following tests were performed on the tandem to test the stabilizer performance and to check the results
of the analysis described in the earlier sections:

1. Measurement of the polar diagram
2. Measurement of the frequency response
3. Measurement of the critical frequencies
4. Measurement of the corona current versus probe position
5. Measurement of the tandem energy resolution.

3.10.1 Polar diagram

To find out how closely the theory describes the practical conditions, first it was necessary to measure the polar diagram of the corona system and compare it with the calculated curve.

Block diagram for measuring the polar diagram of the corona stabilizer

Fig. 38
Reference to the block diagram in Fig. 38, the test was carried out in the following way. The probe position was set to $D = 0.95$ i.e. 26.1 inches from the terminal. The current sprayed on the belt was 170 $\mu$A and 50 $\mu$A probe current was drawn. These conditions maintained 4 MV on the terminal. Then by modulating the grid of the corona valve a sinusoidal signal was injected into the gas via the probe. The modulation on the terminal was monitored via the sensing plate and the input and the output signal were compared in the L.F. Resolved Component Indicator Type VP253.2 manufactured by Solartron Ltd.

By measuring the in-phase and quadrature components the polar diagram of the system was determined. The test was carried out for frequencies between 1 and 100 Hz. The lowest frequency in the experiment was determined by the lower frequency limit of the servo resolver and the measurable highest frequency was limited by the noise in the system. The result of the test is shown in Fig. 39 with the calculated curve. The bars on the calculated points represent the limits in the position of the end of the vector sweeping through the polar diagram. These upper and lower limits for the total resistor chain do not exceed $\pm 3\%$. The estimated error in the measured figures was $\pm 5\%$.

It was necessary in the experiment to make corrections for frequency response of the sensing plate. This was measured preceding the experiment by injecting a sinusoidal signal of constant amplitude and varying frequency to the terminal and analyzing the response on the output of the sensing plate.
$f$ in Hz

$D = 0.95$

$\alpha = 0.1$

**Polar Diagram for the Tandem Corona Stabilizer.**

Fig. 39.
amplifier. That was analyzed by using the servo component resolver. The measured frequency response is shown in Fig. 40.

Up to about 50 Hz the measured and the calculated figures are within ±1 dB. Above 50 Hz the noise in the system tended to increase the measured signal amplitude and the error at the measurement. The accuracy of the measurement below 50 Hz was ±1 dB.

3.10.2 Frequency response

The frequency response of the system was determined by taking the absolute values of the vectors as measured above and relating them to the vector at \( f = 0 \). Both the measured and the calculated curves are shown in Fig. 41. The two curves are in agreement within ±2 dB between 1 and 40 Hz. Above this frequency the mains frequency and its higher harmonics tend to increase the positive error in the measurement.

3.10.3 Critical frequencies

Since the critical frequency plays an important role in the system behaviour tests were carried out to determine the system critical frequencies for different probe settings. For this test the experimental arrangement was the same as the one shown in Fig. 38. The critical frequencies were determined in the following way. A sinusoidal signal was injected into the tank via the corona probe. Then the signal frequency was varied until the output signal of the
$V = 4 \text{MV}$

$\alpha = 0.1$

$D = 0.95$

- EXPERIMENTAL
- CALCULATED

FREQUENCY RESPONSE OF THE TANDEM.

FIG. 41.
sensing plate amplifier was lagging 180° behind the phase of the input sinewave. The frequency satisfying this condition is the critical frequency for that particular probe setting. Then the probe position was changed and the test repeated. The measured and the calculated frequencies are listed in the table below. The measured value for the dispersion coefficient \( \alpha \) at the experiment was 0.1. (See section 3.5.2)

\[\begin{array}{cccccc}
D & 0.86 & 0.9 & 0.92 & 0.94 & 0.97 \\
\hline
f_{\text{calc}} \text{ Hz} & 19.9 & 17.4 & 17.0 & 15.5 & 12.6 \\
\hline
f_{\text{exp}} \text{ Hz} & 20.5 & 19.3 & 18 & 16.3 & 13 \\
\end{array}\]

Table 4

The agreement between the calculated and the measured values, on average is better than 1 Hz.

3.10.4 Measurement of corona current versus probe position

In section 3.7 it was assumed that for small position variation the current of the corona probe is proportional to the change in the probe position. In order to prove the validity of this assumption—see equation (3.14).

\[\eta = \frac{dI_D}{dD} = \text{const} \quad (3.18)\]

a series of tests were carried out to measure the current
as the function of the distance of the probe from the terminal.

In the experiment the terminal voltage was set to 5 MV by spraying 220 μA on the belt and drawing 20 μA corona current via the probe and the position of the probe was recorded. Then the probe position was changed whilst the terminal voltage was kept constant by altering the spray current on the belt. By varying the probe position the current drawn by the probe changed between 20 and 90 μA. The measurement was repeated for 2, 3 and 4 MV on the terminal. The probe current versus probe position is plotted in Fig. 42.

Normally the corona current is about 50 μA with a maximum variation of ±5 μA. As it can be seen from the graph in Fig. 42, the corona current versus probe position may be considered linear for real operating conditions for up to ±50 KV changes in the terminal voltage.

3.10.5 Measurement of the tandem energy resolution

As described in sections 1.5 and 1.6 the way to prove the system energy stability is to measure the energy resolution by using some nuclear reactions of suitably narrow width.

In recent years quite a lot of effort has been devoted to studying the analogues of the ground states, which are usually found to stand out from a smoothly varying background in proton-scattering experiments. These resonances usually are characterised by their narrow total widths which make them very suitable for measuring the energy spread of the
incident beam\textsuperscript{50}).

In order to measure the energy resolution of the tandem accelerator one of these resonances, namely the ground state analog of \textsuperscript{28}Al in \textsuperscript{28}P, was studied by scattering protons from \textsuperscript{28}Si. This resonance which occurs at a proton energy of 5.83 MeV has a true width of about 170 eV. The target used for the experiment was natural Si evaporated on a 0.005 in thick Au backing. The target thickness was 10 \textmu m/cm\textsuperscript{2}.

The block diagram of the experimental arrangement is shown in Fig. 43. In the target chamber two Nuclear Enterprises Si(Li) detectors with 2 mm depletion depth were positioned at 167\textdegree{} and 90\textdegree{} from the target. The protons in both directions were counted by scalers after pulse shaping. The observed half width of the narrow peaks in the cross sections are all approximately equal to the energy spread of the proton beam combined with that introduced by the energy loss in the target. The actual peaks may be much narrower and higher than the observed peaks. The theoretical curve is degraded by the target thickness and the beam energy spread.

When the cross section is measured at an energy of E the number of incident protons with energies between E' and E' and dE' is Qg(E-E')dE' where Q is the total number of protons incident and g(E-E') is the energy distribution of protons in the beam. When the entrance slit opening is uniformly illuminated by the beam, function g is triangular in shape.
EXPERIMENTAL ARRANGEMENT FOR MEASURING $^{28}\text{Si} (p, p)^{28}\text{Si}$ RESONANCE.

FIG. 43.
In order to ensure uniform illumination 0.03 inch object slits separation were selected in each direction. The total beam measured at this point was 800 nA, out of which 300 nA beam current got through the 0.03 in x 0.03 in square aperture. Assuming Gaussian charge density distribution in the cross section of the beam one can find out the actual intensity variation in the illumination of the object slits.

It can be shown that when 500 nA current out of 800 is intercepted on the slits the actual intensity variation in the whole area of the slit aperture is in the order of 13%. This variation, however, is small and the assumption earlier that the intensity distribution against momentum at the image slit is a triangular function is justified.

After a proton of energy \( E \) has penetrated a distance \( x \) into the target its energy has decreased to the value \( E - ax \) where \( a \) is the stopping power of the target material. The energy distribution of protons at the depth \( x \) in the target is thus \( g(E-v-E') \) where \( v = ax \).

The measured differential cross-section at the nominal energy \( E \) and angle \( \theta \) is

\[
\sigma_\theta(E,\theta) = \frac{1}{a E} \int_{E-v-\delta}^{E-v+\delta} \int_{E-v-E'}^{E'} g(E-v-E') \sigma(E',\theta) dE' \ dv \text{ (3.19)}
\]

where \( a \), is the target thickness in energy units

\( \delta \) is the maximum deviation of protons in the beam from the energy \( E \) and

\( \sigma(E',\theta) \) is the theoretical cross-section at the energy \( E' \).
It is convenient to change the order of integration and perform the integration over v first.

The triangular function \( g \) can be separated conveniently into two pieces

\[
g^L = \left( \frac{1}{\delta} \right)^2 (y + \delta + v) \quad \text{when} \quad -\delta - v \leq y \leq -v \quad (3.20)
\]
and

\[
g^R = \left( \frac{1}{\delta} \right)^2 (-y + \delta - v) \quad \text{when} \quad -v \leq y \leq \delta - v \quad (3.21)
\]

\[y = E' - E\]
\[v = ax\]

\( F(E' - E) \) the integral of \( g^L \) and \( g^R \) gives the distribution of energies available for collision in the target.

The integrand in expression (3.119) is obtained by reading the corresponding values of \( F(E' - E) \) and \( \sigma(E', \theta) \) from their graphs and multiplying them together by repeating the integration for a series of energies \( E \) similar to those used in the experiment, one obtains a cross section curve to be compared with the one actually observed. The result of this treatment is that the qualitative shape of the resonance is preserved even though the energy distribution function is several times wider than the resonance but that peaks are spread and flattened. It follows from the normalization of the function \( F(E' - E) \) that the area between the energy axis and the cross section curve of a resonance is conserved.

The measured and the calculated resonance curves at 90° and 165° are shown in Fig. 44. The calculated curve which made the best fit to the measured curves corresponds to an
EXCITATION CURVES AT $\theta=90^\circ$ AND $167^\circ$ FOR $^{28}\text{Si}(p,p)^{28}\text{Si}$ RESONANCE.

FIG. 44
energy spread in the proton beam of ±1 KV. The resonance occurred at the energy of 5.830 ± 0.0005 MeV.

From the excitation energy above k the magnet constant was calculated by using equation (1.5). The value of k resulted from the calculation is 0.02270 ± 0.00003 against the figure of k = 0.02270 ± 0.00002 based upon earlier calibration runs.

3.11 Conclusions

The mathematical analysis of the Van de Graaff tandem accelerators with corona stabilization covers unexplored fields in the theory of electrostatic accelerators. The analysis as described in the earlier sections describes the system well and predicts the system behaviour very closely. It shows that this type of stabilization suffers from the following shortcomings:

1. The delay, caused by the finite drift velocity of the ions between probe and the terminal, limits the loop gain to values between 52.6 dB and 62.4 dB depending on the operating conditions. Higher gain exceeding these figures causes oscillation at some frequencies in the range of 19 and 25.9 Hz.

2. The frequency response is limited to the range of 0 to 0.0623 Hz between the 6 dB points.

3. The limited gain in the system at D.C. makes the generator prone to slow drift.

4. The system is only operational when some of the beam intercepted on the slits.
5. The loop gain depends on the intercepted beam current and as such it needs frequent manual adjustments to suit the varying conditions.

The third shortcoming was eliminated by the introduction of the servo controlled probe position. This second loop in the system made the gain infinite at zero frequency and as such the system became free of DC drift. The improvement was well predicted theoretically and up to now the Oxford tandem is the only one with this facility.

The introduction of the generating voltmeter as a sensing element represents another step forward towards easing operation. With the introduction of the automatic changeover between the slit stabilizer and the generating voltmeter stabilizer the system became semi-automatic. The generator is continuously under the control of one or the other closed loop. By doing this the listed fourth shortcoming was eliminated.

As was demonstrated by the experimental results the automatic gain control circuit sets the optimum loop gain for slit currents between 8 μA and 2 nA without manual intervention. This current range covers most operation beam intensities and only in extreme cases does the system need manual gain adjustment. This represents a great improvement on the conditions outlined under point five.

The limitation in achieving higher stability lies in the finite drift time of the ions between probe and terminal which is inherent in the system and only an additional
loop can eliminate.

Considering all these above on the corona system as a stabilizer for Van de Graaff generators one can see that there is still room for further improvements.
4.1 General remarks

In order to produce higher energy ions for research the electrostatic accelerators are not only operated most of the time near to their upper design limits but often they run beyond them. Considering the two accelerators discussed here the EN tandem can store about 2000 Joule and the injector about 10000 Joule at near to their maximum voltage limits. When this energy is dissipated within m secs by a few components or parts of the machine during a partial or complete breakdown they may suffer substantial damage. Since the causes of a voltage breakdown are numerous (such as poor vacuum, adverse tank gas conditions, beam focussing, faulty components, etc.) the breakdown is usually unpredictable. It is, however, possible and also very desirable that after a major voltage breakdown the generator should return again relatively slowly to the operating voltage. By doing this the ions developed during the breakdown have time to neutralize, the vacuum in the accelerating tube to improve and these usually prevent multiple breakdowns.

As was described in the earlier sections both the injector and the tandem are kept under control by the
multiple loop control system irrespective of whether the beam is present or not on the slits. Furthermore, since the changeover between generating voltmeter control and slit control is done automatically it should be possible to keep the generator under control in all conditions. During and after the breakdown the accelerator is controlled by the error signal generated by the generating voltmeter control loop and the system only returns to slit control, which is regarded as the normal control condition, when the terminal voltage reaches the required value smoothly.

In the following paragraphs a mathematical analysis of a total voltage breakdown is given separately for each machine in an attempt to find out what is to be done to control the generators under fault conditions. A circuit based on the results of the analysis which was installed on both machines is also described.

4.2 Analysis of the injector behaviour under fault condition

The behaviour of the injector and its stabilizer as a servo system under all conditions is fully described mathematically by equation (2.4). Here, however, the first term representing the contribution of the liner stabilizer to the total voltage swing is about 1% and may be neglected when the generator suffers a major breakdown and the terminal is fully discharged. In the analysis full terminal discharge is assumed as a condition which puts the most severe demand on the stabilizer.
Assuming that the terminal is fully discharged in the first instance after breakdown the current sprayed on the belt will stay constant for $\tau_6$ sec which is the time needed to transport the charges from the spray combs to the terminal. The large error signal after breakdown saturates the servo system and the signal developed in the closed loop after $\tau_6$ sec will take the following form:

$$\epsilon(p) = \text{const for } 0 < t < \tau_6 \quad (4.1)$$

Here $\epsilon(p)$ is the Laplace transform of the error signal and $t$ starts at $\tau_6$.

After a time $\tau_0$ has elapsed the servo comes out of saturation and the error signal controlling the spray current becomes again proportional to the voltage of $V_o - V(t)$ where $V_o$ is the required terminal voltage and $V(t)$ is its value at the moment of $t$. During the period of $0 < \tau_0$ the Laplace transform of the terminal voltage takes the following form:

$$V(p)_1 = \frac{V_o}{p(1+\tau_5 p)} + A_1 \frac{1-e^{-\tau_0 p}}{p^2(1+\tau_5 p)} e^{-\tau_6 p} \quad (4.2)$$

After a time $\tau_0$ has elapsed the additional voltage developed on the terminal can be described by the formula below:

$$V(p)_2 = A_2 \frac{e^{-\tau_0 p}}{p(1+\tau_5 p)} \left\{ \frac{V_o}{p} - V_1(p) - V_2(p) \right\} e^{-\tau_6 p} \quad (4.3)$$
By adding (4.2) to (4.3) an expression can be formed which describes the Laplace transform of the terminal voltage for all $t$-s.

$$\frac{V(p)}{V_o} = \frac{1}{p(1+\tau_5 p)} + A_3 \frac{e^{-\tau_6 p}}{p^2(1+\tau_5 p)} \cdot A_3 \frac{e^{-(\tau_0+\tau_6)p}}{p^2(1+\tau_5 p)} +$$

$$+ A_4 \frac{e^{-(\tau_0+\tau_6)p}}{p(p-p_1)(p-p_2)} - A_5 \frac{e^{-(\tau_0+\tau_6)p}}{p(1+\tau_5 p)(p-p_1)(p-p_2)} -$$

$$-(\tau_0+2\tau_6)p$$

$$A_6 \frac{e^{-(2\tau_0+2\tau_6)p}}{p^2(1+\tau_5 p)(p-p_1)(p-p_2)} +$$

After the application of the inverse Laplace transformation to (4.4), the terminal voltage as a time function will be described by the following equation:

$$\frac{V(t)}{V_o} = 1 - e^{-\frac{1}{\tau_5} t} A_3 (t-\tau_6) - A_3 \tau_5 \left\{ 1 - e^{-\frac{1}{\tau_5} (t-\tau_6)} \right\} -$$

$$- A_3 (t-\tau_0-\tau_6) + A_3 \tau_5 \left\{ 1 - e^{-\frac{1}{\tau_5} (t-\tau_6-\tau_0)} \right\} -$$

$$- A_4 e^{d(t-\tau_6-\tau_0)} \cos e(t-\tau_6-\tau_0) +$$

$$+ A_5 e^{d(t-\tau_6-\tau_0)} \sin e(t-\tau_6-\tau_0) - A_4 -$$

$$\frac{1}{\tau_5} (t-\tau_0-\tau_6) d(t-\tau_0-\tau_6) - A_6 e^{-\frac{1}{\tau_5} (t-\tau_0-\tau_6)} - A_7 e^{\frac{1}{\tau_5} (t-\tau_0-\tau_6)} \cos e(t-\tau_0-\tau_6) -$$
\[ - A_8 e^{(t-\tau_0-\tau_6)} \sin e(t-\tau_0-\tau_6) - A_9(t-2\tau_6-\tau_0) - \]
\[ - A_{10} - A_{11}\left\{1 - e^{\frac{1}{\tau_5} (t-2\tau_6-\tau_0)}\right\} - \]
\[ d(t-2\tau_6-\tau_0) - A_{12} e^{-(t-2\tau_6-\tau_0)} \cos e(t-2\tau_6-\tau_0) - \]
\[ d(t-2\tau_6-\tau_0) - A_{13} e^{-(t-2\tau_6-2\tau_0)} \sin e(t-2\tau_6-\tau_0) + A_9(t-2\tau_6-2\tau_0) + \]
\[ + A_{10} + A_{11}\left\{1 - e^{\frac{1}{\tau_5} (t-2\tau_6-2\tau_0)}\right\} + \]
\[ d(t-2\tau_6-2\tau_0) - A_{12} e^{-(t-2\tau_6-2\tau_0)} \cos e(t-2\tau_6-2\tau_0) + \]
\[ d(t-2\tau_6-2\tau_0) - A_{13} e^{-(t-2\tau_6-2\tau_0)} \sin e(t-2\tau_6-2\tau_0) \quad (4.5) \]

Out of these terms all but the ones which are linear functions of the time are convergent. Because of these terms - four in number - the generator after a major breakdown will not recover and return to its normal terminal voltage, but it will overshoot causing multiple breakdown of rapidly increasing periodicity. The numerical analysis of this time function was carried out in our PDP-10 computer (see section 6.6.5) and the curves for 5 and 9 MV terminal voltages are plotted in Fig. 45. From the curves one can see that for voltages between 5 and 9 MV the system after a total terminal discharge reaches the original terminal voltage within 28 and 34 seconds respectively and shows no
INJECTOR TERMINAL VOLTAGE VERSUS TIME AFTER TOTAL DISCHARGE

FIG. 45.
tendency to converge towards the original terminal voltage asymptotically. The voltage is not plotted beyond the 1.3 \( V_o \) because in practice the terminal will be discharged by another breakdown before the voltage would reach it. However, theoretically the system will oscillate with increasing amplitude. This runaway tendency caused by the four time-linear terms and the successful elimination of these terms from \( V(t) \) time function can lead to slower and satisfactory recovery.

It was assumed that it ought to be possible not only to eliminate the terms in question but also to control their signs and amplitudes by electronic means in order to adjust the recovery rate to near its optimum value. It will be shown in paragraph 4.4 that such solution is possible and the circuit described there is capable of controlling the recovery rate within the safe limits of the operation. Based on this later verified assumption a safe recovery rate was computed for the system and the recovery curve as a time function was plotted in Fig. 45. The recovery curve for all voltages between 5 and 9 MV came within a few per cent from each other so only one curve is shown for clarity.

4.3 **Analysis of the tandem behaviour under fault condition**

In order to describe the behaviour of the tandem and its stabilizer after a voltage breakdown we have to return to equation (3.15) and substitute the error signals
present in the system during the recovery time. Since normally terminal breakdown only occurs during high voltage operation the second term in expression (3.15) can be neglected which will greatly ease the analysis. Further simplification can be applied to the transfer function by neglecting the effect of the delay in ion collection on the terminal due to the finite ion drift velocity in the high pressure gas. This is fully justified considering that the delay \( \tau_a \) is in the order of 20m sec and the recovery time - as it will be shown later - is in the order of tens of seconds. Mathematically this implies the following:

\[
\lim_{\tau_a \to 0} \frac{1 - e^{-p\tau_a}}{\tau_a p(\tau_1 p + 1)} = \frac{1}{p(\tau_1 p + 1)} \tag{4.6}
\]

Assuming now that during the breakdown the terminal has been totally discharged, in the first instance the recovery will be described by the following expression:

\[
V_1(p) = I_o \frac{R}{p(\tau_1 p + 1)} - A_1 \frac{1}{(\tau_1 p + 1)} \left( \frac{V_o}{p} - V_1(p) \right) + A_2 \frac{1}{(\tau_2 p + 1)} V_1(p) + A_3(1 - e^{-\tau_0 p}) \frac{1}{p^2(\tau_2 p + 1)} \tag{4.7}
\]

where the first term represents the voltage due to the constant belt current, the second term describes the voltage changes due to the corona valve and the third one is the effect of the saturated servo controlled probe inside the tank.
After a time \(\tau_0\) has elapsed the signal on the servo input becomes small enough for the servo output to become proportional to the error signal and the recovery function above will become combined with signal of the proportional servo control thus:

\[
V_2(p) = A_3 \frac{e^{-\tau_0 p}}{p(p\tau_1+1)} \left\{ \frac{V_o}{p} - V_1(p) - V_2(p) \right\} \quad (4.8)
\]

In equation (4.7) and (4.8) \(A\)-s are constants, \(I_o\) is the belt current and \(R\) is the resistor between terminal and earth.

The combination of equations (4.7) and (4.8) will now describe the Laplace transform of the function of recovery for the generator which takes the following form:

\[
\frac{V(p)}{V_o} = \frac{1}{p(\tau_2 p+1)} + A_4(1-e^{-\tau_0 p}) \frac{1}{p^2(\tau_2 p+1)} + \\
A_5 e^{-\tau_0 p} \frac{1}{p(p-p_1)(p-p_2)} - \\
A_6 e^{-\tau_0 p} \frac{1}{p^2(\tau_2 p+1)(p-p_1)(p-p_2)} - \\
A_7 e^{-\tau_0 p} (1-e^{-\tau_0 p}) \frac{1}{p^2(\tau_2 p+1)(p-p_1)(p-p_2)} \quad (4.9)
\]

where

\[
\tau_2 = \frac{1}{1+A_1+A_2} \quad (4.10)
\]
By applying the inverse Laplace transformation to (4.9) the recovery function as a time function can be calculated. This will come out in the following form:

\[
\frac{V(t)}{V_0} = 1 - e^{-\frac{t}{\tau_2}} + A_4 t - A_4 \tau_2 (1 - e^{-\frac{t}{\tau_2}}) + \\
\frac{d(t-t_o)}{1-e^{-\frac{t-t_o}{\tau_2}}} + A_5 - A_5 e^{-\frac{t-t_o}{\tau_2}} \cos e(t-t_o) + \\
d(t-t_o) + A_6 e^{-\frac{t-t_o}{\tau_2}} \sin e(t-t_o) - \\
A_3 (t-t_o) + A_3 \tau_2 (1 - e^{-\frac{t}{\tau_2}}) - \\
- \frac{1}{\tau_2} (t-t_o) \\
- A_5 - A_5 e^{-\frac{t-t_o}{\tau_2}} - A_{10} e^{-\frac{t-t_o}{\tau_2}} \cos e(t-t_o) + \\
d(t-t_o) + A_{11} e^{-\frac{t-t_o}{\tau_2}} \sin e(t-t_o) - A_{12} (t-t_o) - \\
- \frac{1}{\tau_2} (t-t_o) \\
A_{13} - A_{14} (1 - e^{-\frac{t-t_o}{\tau_2}}) - \\
d(t-t_o) + A_{14} e^{-\frac{t-t_o}{\tau_2}} \cos e(t-t_o) - A_{15} e^{-\frac{t-t_o}{\tau_2}} \sin e(t-t_o) + \\
d(t-t_o) + A_{12} (t-t_o) + A_{13} + A_{14} (1 - e^{-\frac{t}{\tau_2}}) + \\
- \frac{1}{\tau_2} (t-2\tau_o) \\
+ A_{14} e^{-\frac{t-2\tau_o}{\tau_2}} \cos e(t-2\tau_o) + \\
d(t-2\tau_o) + A_{15} e^{-\frac{t-2\tau_o}{\tau_2}} \sin e(t-2\tau_o) \tag{4.11}
\]

With the exception of the terms which are linear
functions of the time all other terms are convergent. This recovery function (4.11) was calculated numerically by programming the PDP-10 computer (see section 6.6.6) and the curves are plotted in Fig. 46. From the unprotected recovery curves plotted for 4 and 6 MV operation it can be seen that after the breakdown the terminal voltage does not approach the operating voltage but gradually increases while oscillating around an increasing mean value. This function, however, can be made convergent by eliminating the terms in expression (4.11) whose value increases linearly with time. The recovery curve predicted for all operating conditions after it is converted into a convergent function is shown in Fig. 46. This calculation shows that the recovery rate after a major voltage breakdown can be slowed down to such an extent that the generator can reach energy after breakdown without overshoot or oscillation. The safe recovery curve shows the same character for all terminal voltages.

The circuit designed on the result of these calculations is described in section 4.4.

4.4 Automatic recovery system for the accelerators after voltage breakdown

Based on the results of the analysis presented in the previous two sections a circuit was designed and installed on each of the accelerators. The function of the circuit shown in Fig. 47 can be described in the following way.
Fig. 46. Tandem terminal voltage versus time after total discharge.

With protection circuit

4 MV

6 MV
BREAKDOWN PROTECTION CIRCUIT DIAGRAM.

FIG. 47.
The generating voltmeter error signal is formed in amplifier $A_{11}$ (see Fig. 36) by comparing the reference signal with the generating voltmeter reading. The output of this amplifier which is fed into the main amplifier in both cases is also fed into a comparator circuit type 910M after buffering. In the circuit shown in Fig. 47 the first two stages formed by two OC201 transistors provide the buffering and zeroing facilities for the comparator circuit. This circuit keeps the generating voltmeter error under continuous surveillance. The function of the circuit is to energize the relay RLD when the error signal reaches a predetermined level. The relay then reverses the polarity of the signal from $A_{11}$. By doing that the signs of all non-convergent terms in the recovery function will be changed so the system will be stopped from running away. This is now in complete agreement with the theoretically predicted conditions. At the end of the $\tau_0$ period when the terminal voltage reached the same predetermined value the relay becomes de-energized and the terminal voltage will rise under the control of the proportional servo.

The operating range called the window of the comparator can be set within a wide range by altering the feedback of the comparator. In the present circuit as shown here the feedback is determined by 200K and 1K resistors. Setting the window is to a certain extent an arbitrary decision. Two practical factors, however, need to be considered. If the
time to reach the lower limit of the window is too long the
servo in this time may turn the belt charge current down so
much that the terminal voltage never reaches the lower
limit, so the generator will not recover its original voltage.
If, however, the limit is reached too early, i.e. the window
is set too wide the circuit will not provide adequate
protection for the accelerator. In practice it was found
convenient to set the window to the voltage corresponding to
the maximum error signal for the servo working in its
proportional region. The loop gain was lowered by 30 dB
at the same time in order to set the servo to the speed
corresponding to the calculated recovery rate.

The circuit described here basically is the injector
protection circuit. For the tandem protection an identical
circuit was built and the only significant difference
between its operation and the description of the one given
above is that the servo here is not being used to control
the belt current but the position of the probe relative to
the terminal. This produces, however, the same overall
effect, that is to say controlling the sum of all charges
reaching the terminal.

4.5 Tests and observations

The evaluation of the results and the test of the
circuit described in paragraph 4.4 was carried out by
observation of breakdowns occurring during normal use of
the accelerators. The system after installation was put
under close watch and the recovery of the generator
was observed after the natural breakdowns. The results of these observations are only statistical within approximately ±15% tolerances.

The observed injector recovery time was about 120 sec. The terminal voltage did not overshoot and the control circuit recovered the terminal voltage smoothly until the voltage reached the limits of the slit stabilizer when the system switched over and locked on the beam.

The rate of rise of the voltage on the terminal without the protection circuit was measured by the following experiment. The terminal was charged up to 5 MV and without altering the belt charge current and then the belt charge was switched off. After the terminal voltage dropped to zero the belt spray current was switched on again and the time taken by the terminal voltage to reach 3.5 MV was measured. Repeating the same measurement several times, all figures fell between 18 and 20 sec. The test was repeated for 6 and 7 MV terminal voltages and the charging up time was measured at 4.5 and 5.5 MV respectively. For 6 MV all figures measured fell between 21 and 22.5 sec and for 7 MV they grouped closely around 24.5 sec. Considering that the calculated figures for the voltages above are 18.5, 21.5 and 24.5 secs one can say that the experimentally and the theoretically obtained figures are in close agreement.

Finally recoveries from breakdowns occurring naturally were recorded with and without using the protection circuit by using a chartrecorder. Fig. 48 shows a controlled recovery from a breakdown caused by an accelerator tube spark.
INJECTOR CONTROLLED RECOVERY AFTER TUBE BREAKDOWN AT 5.5 MV

a. DIFFERENTIAL AMPLIFIER OUTPUT
b. TERMINAL VOLTAGE
c. VACUUM

FIG. 48.
alongside with the recordings of the change in pressure in the tube and the error signal. The breakdown has occurred at 5.5 MV terminal voltage. Fig. 49 shows a recording of a series of breakdowns from which the system recovered in an uncontrolled way. The recording shows clearly the diminishing time interval between breakdowns. The second breakdown has occurred 27 sec after the first one which is in good agreement with the calculated 28 sec.

The recovery time of the tandem was also observed with and without protection circuit after natural breakdown. The voltage on the terminal reached its set value in 15 sec without and in 28 sec with the protection circuit in operation.

The recovery curves of the tandem after total terminal discharge are shown in Figs. 50 and 51. In the controlled way (see Fig. 50) the voltage recovers in two distinct steps. The first lasts for 19 secs. When the voltage recovered in an uncontrolled way after breakdown the voltage overshot without breaking down immediately. After 29 sec, however, the machine developed multiple breakdowns. In both cases the pattern of recovery was in good agreement with the calculated one.

In cases when the generator was heavily loaded or when cascaded voltage breakdown discharged the terminal the protection circuit kept the voltage below the operating voltage and manual intervention was needed to restore the operating conditions. A recording of one such case is shown in Fig. 52. The recovery has been recorded on the tandem after a breakdown at 6.2 MV terminal voltage.
INJECTOR UNCONTROLLED RECOVERY FROM TUBE BREAKDOWN AT 5.5 MV.

a. DIFFERENTIAL AMPLIFIER OUTPUT
b. TERMINAL VOLTAGE
c. VACUUM

FIG. 49.
TAINED CONTROLLED RECOVERY AFTER TERMINAL BREAKDOWN AT 6.2 MV.

a. TERMINAL VOLTAGE
b. DIFFERENTIAL AMPLIFIER OUTPUT

FIG. 50.
TANDEM UNCONTROLLED RECOVERY AFTER TERMINAL BREAKDOWN AT 6.2 MV.

a. TERMINAL VOLTAGE
b. DIFFERENTIAL AMPLIFIER OUTPUT

FIG. 51.
TANDEM CONTROLLED RECOVERY AFTER A PERSISTING BREAKDOWN AFTER WHICH MANUAL INTERVENTION WAS NEEDED TO RESTORE THE TERMINAL VOLTAGE

a. TERMINAL VOLTAGE

b. DIFFERENTIAL AMPLIFIER OUTPUT

FIG. 52.
CHAPTER 5

PROPOSAL FOR COMPUTER CONTROL OF VAN DE GRAAFF GENERATORS

5.1 Introduction

In laboratories using Van de Graaff type of accelerators - although the presence of on-line computers in the control room is long accepted - very little has been done to use computers to control operational parameters.

Considering a Van de Graaff accelerator under computer control there are natural dividing lines which split the overall problem into three parts; that is, the control of the ion source, the control of the beam handling by the computer and the control of the parameters of the generator. Although these are closely interlinked through machine operation they can be dealt with independently.

The computer control of the ion source involves great technical difficulties such as complicated remote controlling and monitoring of the beam parameters, change of programming and interfacing when type of ion source is changed etc. So the problems to be solved here are too great and often require human intuition which is difficult to formulate. As such the price to be paid for a computer controlled ion source would be too high and the return in time saving would not justify the effort and the expenditure.

The computer control of the beam handling on the other hand would have more justification on the grounds of cutting down the time lost through human error. Programs to help
designers of beam transport systems have been written and the adaptation of these programs for computer controlled operation should not be very difficult, although it may require a vast effort on interfacing and monitoring of the beam parameters.

The proposal described in this chapter is solely devoted to the control of the operational parameters of the generators. These are easier to formulate while still giving an idea of the magnitude of the overall technical problems involved. A control of this kind would help the operators and users especially running under critical conditions such as tube conditioning or running the accelerator near to breakdown point.

Although an attempt was made to study the problem as closely as possible one cannot claim by any means that answers are given in this chapter to all problems associated with generator control by the computer.

5.2 Sampled controller for Van de Graaff generators

As its name indicates in a sampled-data controlled operation the signal somewhere in the system undergoes a process of periodic sampling. A typical case for the sampled-data system is the one using a time shared digital computer as a controller. In our laboratory a PDP7/PDP10 on-line computer complex is available for sampling the vital operational parameters of the accelerators.

Although in each case these parameters are controlled
by their own built-in closed loop circuits the sampled control
system would form an additional loop with over-riding power
in order to set automatically the NMR frequency, the
analyzing magnet field, and the terminal voltage, to make
corrections for small errors during operation and to provide
adequate protection for the accelerators in case of a
voltage breakdown.

This computer controlled automatic operation would take
us nearer to achieve the following objectives:

1. An increase in the long term energy stability.
2. Reduction of the manual intervention from the operating
   procedure with an increase in efficiency and saving of
time.
3. An increase in the degree of protection of the accelerators
during operation.
4. Automatic return to the required conditions after
   interruption.
5. Simplification in fault monitoring.

The introduction of a sampled-data control system
brings up new problems such as the effect of the sidebands
caused by the sampling on the data and the effect of the
sampling on the stability criteria of the system. Both of
these problems need careful attention in order to predict the
behaviour of the sampled system.

The problem of removing the high frequency complementary
components due to the periodic sampling is eased, if not
completely solved, by the use of an nth order holding
circuit, which holds the value of the sampling pulse until the next pulse arrives. In the case of the sampled-data control for the accelerators the holding of the data will be done by the output buffer interface forming a first order holding circuit or "boxcar" generator as it is normally referred to in the literature. Furthermore, all the controlled devices, in this particular case, will act as natural lowpass filters since their frequency responses are limited to the 0-100 Hz range and the data transfer time of the PDP-10 computer is 1.75 µsec for an 18-bit word. Consequently it is evident that transients caused by the transfer will be outside the frequency bands of the controlled devices.

In order to evaluate the effect of sampling on the system stability the method of constructing a graphical approximate of the Nyqvist plot was used. This method is based upon the graphical representation of $M^*(j\omega)$ the open-loop frequency response of the sampled-data control system, expressed in a series thus:

$$M^*(j\omega) = \frac{1}{T} \sum_{n=\infty}^{\infty} M(j\omega + n\omega_s) \quad (5.1)$$

where $\omega_s$ is the sampling frequency,

$$T = \frac{2\pi}{\omega}$$

and $M(j\omega)$ is the open-loop frequency of the system with continuous control.
It can be shown that when $M(j\omega)$ is of a low pass nature the higher order terms can be neglected and the method gives good approximation for $n = 0, 1, -1$.

5.3 Automatic NMR frequency selection

The prime energy reference for the Van de Graaff accelerators is the NMR frequency indicating the magnetic field in the analyzing magnet. The problem of selecting the required frequency with the aid of the computer will be dealt with first. The flow chart is shown in Fig. 53 and the description of the step by step operation is as follows:

The operation starts at the teleprinter allocated for machine operation.

1. The operator by raising the flag asks for servicing from the computer.
2. Computer asks for specification of the operation (injector, tandem, coupled.)
3. Computer asks for data ($E$, $m$, $Z$).
4. Computer calculates parameters ($\text{NMRF}$, $V_T$, $V_M$). At this point computer calls for NMRIC subroutine.
5. Computer reads in NMRF displayed by the TF2401A counter through the NMRF read in interface. Here the computer calls for the NMR frequency read in subroutine.
6. Computer calculates NMRF.
7. Data is transferred into NMR buffer register.
8. The 6 most significant bits are fed into the selector decoder to operate the range selector.
9. The 10 most significant bits are fed into the coarse
FLOW DIAGRAM FOR NMR FREQUENCY SELECTION.
FIG. 53.
preselector decoder to operate the preselector.

10. The 12 most significant bits are fed into the coarse selector decoder to operate the coarse selector.

11. The rest with the exception of the four least significant bits (bits 14-4) are channelled into a DAC whose analog output signal operates the fine frequency control.

12. When frequency is set the computer skips to set the terminal voltage.

13. The computer supervises the frequency and resets the fine frequency control ADC when necessary.

The block diagram of the circuit performing the operation as described above is shown in Fig. 54. Here the NMRF read in interface (see Fig. 55) operated by the program called "NMR FREQUENCY READ IN ROUTINE" (see Appendix 6.6.7) in the following way.

The scaler (Marconi TF 2401A) sampling the NMR frequency, during display time gives a print command in the form of -6v on one of its output pins. This voltage raises the flag by triggering a R200 flip-flop (see Fig. 56). When the PDP-10 computer is ready to read in the frequency, it inhibits the reading of the scaler for the read in period. Then the computer reads in the NMR frequency digit by digit and the number of digits is counted by a binary counter made of R202 units. This counts up to eight which is the maximum number of digits from the scaler. The output from the binary counter is converted into octal numbers by unit R151. The output levels of this converter after the
BLOCK DIAGRAM FOR NMR FREQUENCY CONTROL

FIG. 54.
DATA FROM
TE2401A SCALER

1ST
DIGIT

1 2 4 8

7 OTHER GATES

R141 R113

R107

R650

7 OTHER GATES

R141 R113

R107

R650

7 OTHER GATES

R141 R113

R107

R650

7 OTHER GATES

R141 R113

R107

R650

7 OTHER INVERTERS

0 1 2 3 4 5 6 7

CLEAR

R202

R202 BCD COUNTER

END

DATA TO PDP-10

NMR FREQUENCY READ IN INTERFACE.

FIG. 55.
FLAG AND INHIBIT CIRCUIT DIAGRAM FOR NMR INTERFACE.

FIG. 56.
necessary buffering are used to enable the gates (R141 and R113) for reading the relevant digit into the PDP-10. At the end of this read in procedure the computer clears the binary counter and sets it to 0, clears the flag by resetting the R200 flip-flop and then removes the inhibit from the scaler. Then the interface is ready for the read in command (see Fig. 56).

The NMR flux meter suggested for measuring the field in the magnet is manufactured by HVEC. The principle and the design described above, however, can easily be adapted to any other types of NMR flux meters. After the computer has calculated the NMR frequency for one particular operation, the figures are transferred and stored in the buffer circuit shown in Fig. 57. This circuit is a parallel buffer to store the octal coded binary numbers from the computer. To the "Read" command it reads all 27 bits representing the NMR frequency. The "Clear" command sets all flip-flops to zero ready for receiving the next set of figures. Out of the 26 bits the 6 most significant bits operate the frequency range selector on the NMR flux meter control unit. The interface between the computer and the range selector switch, with the Boolean functions, is shown in Fig. 58.

The coarse frequency preselection and the coarse selection is done by homing stepping motors. Once the computer has calculated the required frequency the buffer storing the 26 bits via the decoders marks one position on
BUFFER REGISTER CIRCUIT DIAGRAM FOR NMR INTERFACE.  

FIG. 57.

BITS 0-26 FROM PDP 10
 BITS 0-14 TO DAC
 BITS 15-26 TO DECO
 RANGE SELECTOR
 BITS 17-26 TO DECO
 COARSE PRESELECTOR
 BITS 21-26 TO DECO
 COARSE SELECTOR
FIG. 58. NMR FREQUENCY RANGE SELECTOR CIRCUIT.
both the coarse preselector and the coarse selector switch. Both stepping motors rotate until they find the marked positions corresponding to the calculated frequency. The shafts of the stepping motors are mechanically coupled to the NMR oscillator tuning capacitor. The circuit diagram of the selectors is shown in Fig. 59. In order to ease understanding only the connections for a single position are shown. In Fig. 60 as a sample the truth table for decoding coarse preselector position No. 1 is shown. The stepping motors selected for the purpose are capable of the maximum speed of 160 steps/sec so the minimum selection time for the maximum number of steps (24 x 24) would be just over 3.5 sec. In practice the expected frequency selection time is 5-10 sec.

For the final fine frequency adjustment the least significant 14-4 bits are used. These bits set the ADC which controls the varactor diode tuning the NMR oscillator. The circuit is shown in Fig. 61. The circuit analogue accuracy is 0.08%.

Since it was impossible to formulate the frequency behaviour of the NMR flux meter tests were carried out to determine the periodicity and the amplitude of the change in the oscillator frequency. These tests showed a maximum amplitude fluctuation of ±2.5 parts in 10⁴ with a period between 1 and 2 hours.
FIG. 59. NMR FREQUENCY COARSE PRESELECTOR & COARSE SELECTOR CIRCUIT.

COARSE SELECTOR

DECODER

DRIVER

9904131
03003

STEPPING MOTOR

9904-N2-08001

10 MOST
SIGNIFICANT
BITS

COARSE SELECTOR

DECODER

+2 V

+2 V
TRUTH TABLE.

**FIG. 60.**

<table>
<thead>
<tr>
<th>No of Steps</th>
<th>No of Bits</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>26 25 24 23 22 21 20 19 18 17 16</td>
</tr>
<tr>
<td>2</td>
<td>0 0 0 0 0 1 0 0 0 0 1</td>
</tr>
<tr>
<td>3</td>
<td>0 0 0 0 0 1 0 0 0 1 0</td>
</tr>
<tr>
<td>4</td>
<td>0 0 0 0 0 1 0 0 0 1 1</td>
</tr>
<tr>
<td>5</td>
<td>0 0 0 0 0 1 0 0 1 0 0</td>
</tr>
<tr>
<td>6</td>
<td>0 0 0 0 0 1 0 0 1 0 1</td>
</tr>
<tr>
<td>7</td>
<td>0 0 0 0 0 1 0 1 0 1 0</td>
</tr>
<tr>
<td>8</td>
<td>0 0 0 0 0 1 0 1 0 1 1</td>
</tr>
<tr>
<td>9</td>
<td>0 0 0 0 0 1 0 1 0 1 0</td>
</tr>
<tr>
<td>10</td>
<td>0 0 0 0 0 1 0 1 0 0 1</td>
</tr>
<tr>
<td>11</td>
<td>0 0 0 0 0 1 0 1 0 1 0</td>
</tr>
<tr>
<td>12</td>
<td>0 0 0 0 0 1 0 1 0 1 1</td>
</tr>
<tr>
<td>13</td>
<td>0 0 0 0 0 1 0 1 1 0 0</td>
</tr>
<tr>
<td>14</td>
<td>0 0 0 0 0 1 0 1 1 1 0</td>
</tr>
<tr>
<td>15</td>
<td>0 0 0 0 0 1 0 1 1 1 1</td>
</tr>
<tr>
<td>16</td>
<td>0 0 0 0 0 1 0 1 1 1 1</td>
</tr>
<tr>
<td>17</td>
<td>0 0 0 0 0 1 1 0 0 0 0</td>
</tr>
<tr>
<td>18</td>
<td>0 0 0 0 0 1 1 0 0 0 1</td>
</tr>
<tr>
<td>19</td>
<td>0 0 0 0 0 1 1 0 0 1 0</td>
</tr>
<tr>
<td>20</td>
<td>0 0 0 0 0 1 1 0 0 1 1</td>
</tr>
<tr>
<td>21</td>
<td>0 0 0 0 0 1 1 0 1 0 0</td>
</tr>
<tr>
<td>22</td>
<td>0 0 0 0 0 1 1 0 1 0 1</td>
</tr>
<tr>
<td>23</td>
<td>0 0 0 0 0 1 1 0 1 1 0</td>
</tr>
<tr>
<td>24</td>
<td>0 0 0 0 0 1 1 0 1 1 1</td>
</tr>
</tbody>
</table>
DAC FOR FINE CONTROL.

FIG. 61.
5.4 Automatic magnetic field and terminal voltage selection

The ions, leaving the accelerators undergo momentum analysis in order to select the required energy, as described in section 1.5. In the following a description is given how to select the field to the accuracy required by using our PDP-10 computer. The process of automatic field selection can be best understood with the help of the flow chart and the block diagram as shown in Figs. 62 and 63.

The step by step operation is as follows:


2. Computer reads in the content of the ADC through interface. (Here the computer calls for VM read in subroutine.)

3. Computer compares read in data to calculated VM and transfer difference data to buffer register. (Here the computer calls for VM subtraction subroutine.)

4. DAC converts data given by the computer and sets magnet stabilizer.

5. When NMR signal is found computer inhibits magnet reference and the power supply locks to NMR phase signal.

6. Computer supervises the conditions of the magnet power supply in every 10 sec.

The loop as shown in Fig. 63 is operated by a program called "VM READ IN ROUTINE" and it is a modified version of the read in programs described earlier. The circuit also operates as described in section 5.3. The buffer register chosen for this loop is also the same as the one shown in Fig. 57.
FLOW DIAGRAM FOR SELECTING ANALYSING MAGNET SETTING.

FIG. 62.
BLOCK DIAGRAM FOR ANALYSING MAGNET CURRENT CONTROL.

FIG. 63.
The DAC designed for the magnet power supply is capable of providing $1:10^5$ resolution for the magnet current setting and it is operated directly by the buffer register. Its circuit diagram is shown in Fig. 64 for one decimal digit only for simplification and the other digits are derived from similar circuits with altered resistor values. Here the computer sets the relays by the BCD numbers through the buffer register. Each relay supplies a voltage to the summing point of the summing amplifier, which is proportional to its binary value. The voltage is supplied by a reference source stable to minimum of $3:10^6$. The resistors in the circuit are selected for accuracy and stability of 0.01% and the relays have very low contact resistance.

The energy of the ions leaving the accelerator, is determined by the terminal voltage of the generator and their charge state. (The energy variation due to the ion source and the charge exchange is neglected.) The operation of selecting and controlling the terminal voltage can be understood from the flow chart and the block diagram as shown in Figs. 65 and 66.

The operation starts when the PDP-10 computer skips in the program from the NMR frequency to the terminal voltage, then:

1. Computer inhibits ADC from generating voltmeter read out.
2. Computer reads in content of ADC through interface.
   (The computer here calls for the VT read in subroutine.)
3. Computer compares reading to calculated VT and transfers data into buffer.
FLOW DIAGRAM FOR TERMINAL VOLTAGE SETTING.
FIG. 65.
BLOCK DIAGRAM FOR TERMINAL CONTROL.

FIG. 66.
4. DAC converts and feeds reference signal to differential amplifier in generating voltmeter circuit. (This part of the system has been fully described in section 2.6.3.)

5. When the output of the differential amplifier is zero the computer skips to other operations.

The program operating this part of the system is called "VT READ IN ROUTINE" and it is derived from the "NMR READ IN ROUTINE" by appropriate modifications. The computer, after the necessary analog-digital conversion, reads in the output of the differential amplifier comparing the reference and the generating voltmeter readings. (See section 2.6.3). The circuits designed for read in interface, DAC, and the buffer register are similar to those shown in Figs. 55, 57 and 64.

The stability criterion of the system was determined by applying the graphical method described in section 5.2. As was demonstrated earlier, the injector and its G.V. stabilizer forms an unconditionally stable system. It also can be shown that the locus of the two phasors $M(j\omega \pm \omega_s)$ will never cut the negative real axis. It is evident now that the resultant locus can only describe a stable system.

5.5 Remarks on the circuit design

The study of the interfacing between the computer and the accelerator was based upon the DEC FLIP CHIP modules supplied by Digital Equipment Corporation. The reason for this was rather historical and circumstantial, since they
were held as spares and also for further development purposes.

In late 1969, however, our laboratory in line with other nuclear laboratories adopted the new instrumentation standard recommended and developed by the European Standard of Nuclear Electronics (ESONE). This system of highway for the transfer of digital data and control information was made independent of computer types and called CAMAC system\(^{55-57}\). The advantage in using this system can be fully realized by considering the following specific features:

1. The data from the accelerators could be handled like other nuclear data.

2. The scheme could be adapted by other laboratories with ease.

3. It simplifies the programming since complex autonomous operations may be initiated inside the system.

4. Error checking, display and indication of the transferred data can be done within its own boundaries.

5. Being assembled of standard plug-in units to expand the capabilities of the system requires no modification to standard units.

6. Servicing can be done with minimum interruption to operation by replacing standard units.

The system would be linked to central processor through the General Users Area which would provide two way flow of data between the computer and the control systems of the accelerators.
5.6 Conclusions

The study above presents a proposal for an automated Van de Graaff generator. It shows how the three major operational parameters of the system, namely the NMR frequency, the magnetic field and the generator voltage can be met by the computer. Since the controls of the generator in Oxford were designed with the automated operation born in mind the system would not need modification only additions to the existing system. This would also guarantee the quick return to the semi-automatic operation as described in the earlier paragraphs, in case of a computer or interface failure.

Although the circuitry suggested in the study satisfies both theoretical and the practical objectives, due to the introduction of the more advanced interfacing electronics it would be disadvantageous to continue using the flip-chip modules. By using the CAMAC highway, however, for interfacing the circuit development would be reduced since the accelerator data would be handled on equal terms with the nuclear data through the General Users Area using identical modules. Since the CAMAC system can be interfaced to any computer the suggested system would be easily adaptable to other Van de Graaff accelerators.
6.1 Investigation into improving magnet field stability

As a momentum analyzer the 90° bending magnet provides an absolute energy reference for the accelerators. In most cases, therefore, the inadequacy in field stability presents an upper limit in the achieved energy stability and resolution. In order to increase the stability to the required level a circuit was designed to lock the magnet field to the NMR signal measuring the field in the analyzing magnet. This was a logical step since the HVEC NMR used in the system provides phase output on its control unit which produces a voltage proportional to the position of the resonant signal on the CRT's screen.

In the design there were two problems to overcome: the first was the coupling between the NMR with fixed earth connection to the floating magnet supply and the second was the provision of infinitely high gain at the frequency of \( f = 0 \). Both were overcome by the introduction of a floating, servo controlled reference potential, proportional to the NMR signal phase deviation. The circuit diagram is shown in Fig. 67 and the operation is as follows.

The current in the analyzing magnet is determined by a reference voltage \( V_R \) (see Fig. 67) which is compared in a differential amplifier with the voltage \( V_1 \).
SCHEMATIC DIAGRAM OF THE CURRENT CONTROL FOR THE ANALYSING MAGNET.

FIG. 67.
developed across a standard resistor through which the magnet current passes. The output from this amplifier controls the current of the analyzing magnet and thus the magnetic field.

The stabilization of the magnet field to the NMR signal is achieved by varying the reference voltage VR proportionately to the output of the NMR phase signal in opposing phase. To do this a varying current is passed through a standard resistor in the line of AB. The current for the stabilization is supplied by an emitter follower in which the base of the transistor T₄ is connected to a ten turn helipot P₁ controlled by a servo motor. The circuit is shown in Fig. 68.

In order to find the stability criteria for the analyzing magnet and its stabilizer some experimental data was needed. First the system response to a step function input signal was measured by changing the reference voltage by 10A and observing the change in the magnet current. The response was displayed on the screen of an oscilloscope and photographed. The resulting response is shown in Fig. 69. As a second step a number of transfer functions was tried in the attempt to find a good fit to the response. The continuous transfer function found this way describing the system to a few per cent accuracy is:

\[
M(p) = \frac{1}{p^2+2\xi_0 p+\omega_0^2} \frac{1}{p^2+0.17p+0.08} \quad (6.1)
\]
The step function response of the magnet and its stabilizer.  

Fig. 69

Finally the transfer function describing the conditions of the magnet stabilizer with the NMR phase loop in operation, was calculated and it was found as follows:

\[ M_2(p) = \frac{K \times 0.08}{1.18 \times 10^{-3} p^3 + 0.17 p^2 + 0.17p + 0.08} \]  

(6.2)

It can be shown that this system is conditionally stable and the loop gain is limited by oscillatory conditions occurring around the frequency of 0.013 Hz. By using this method of stabilization measured best magnet field stability was 6dB below the stability of the NMR oscillator.
6.2 The feed impedances for the injector two loop stabilizer

Reference to Fig. 6 one can see that the belt as a current generator feeds into an impedance as shown in Fig. 70. In order to ease the circuit analysis some simplifications can be made. It will be shown here that these simplifications are justified.

Equivalent circuit of $Z_I$

Fig. 70

Solving Kirchoff's equations for $i_1$ one can show that when the condition

$$\tau_1 << \tau_2$$

where $\tau_1 = R_1 C_3$ and $\tau_2 = R_2 C_2$

is fulfilled circuit (a) in Fig. 70 can be approximated with circuit (b). In the case of the injector
$\tau_1 = R_1 C_3 = 0.25 \ \Omega \cdot 14000 \ \text{pF} = 3.5 \ \mu\text{sec}$

and $\tau_2 = R_3 C_2 = 36000 \ \Omega \cdot 700 \ \text{pF} = 25.2 \ \text{msec}$

It will be shown later that the substitution of one integrating circuit with

$\tau_5 = (R_2 + R_3)(C_1 || C_2) = 12.54 \ \text{sec}$

time constant in the place of the two formed by $R_2 C_1$ and $R_3 C_2$, introduces a negligible error. This simplification brings us to circuit (c). The last step the transformation of circuit (c) into (d) is only justified when

$R_1 \ll R_2 + R_3$

Since $R_1 = 0.25 \ \Omega$

$R_2 = 21000 \ \Omega$

$R_3 = 36000 \ \Omega$

these values satisfy the condition above so the use of circuit (d) in the analysis is fully justified.

It was shown above that the equivalent circuit of the accelerator can be substituted with the combination of three integrating circuits as shown in Fig. 70. The substitution of this again with one single integrating circuit would simplify the analysis greatly. In order to investigate the justification of this step the frequency response of both circuits was calculated and plotted (see Fig. 71). The graph shows that the maximum error introduced at any point is under 0.8 dB. Since an error of this order of magnitude can be expected from the change in components value due to ageing, the introduced simplification in the analysis can be regarded as justified. The expression of
FREQUENCY RESPONSE OF $\tau_5$ AND $\tau_1\sim\tau_2\sim\tau_3$ INTEGRATING CIRCUITS.

FIG. 71.
this \( Z_{II} \) impedance is as follows:

\[
\frac{Z_{II}}{R} = \frac{1}{1+\tau_5 p}
\]  

(6.3)

where \( \tau_5 = (R_1+R_2+R_3)(C_1||C_2||C_3) \)

and \( R = R_1+R_2+R_3 \).

This is the impedance seen by the belt carrying the charges to the terminal as a current generator.

To control the charges on the belt an AC servo system has been considered. The final aim here is to provide a control for the belt charge, proportional to the error signal and to simulate the required transfer function

\[
f(p) = \frac{K}{p}
\]  

(6.4)

The linearized differential equation describing the servo system chosen is

\[
\frac{d\theta}{dt} = K_v \epsilon(p) - \tau_m'' \frac{d^2\theta}{dt^2}
\]  

(6.5)

where \( \theta \) is the angular position

\( K_v \) is the velocity constant of the motor

and \( \tau_m'' \) is the motor time constant. Its value depends on the operating point of the servo motor.

Assuming that the initial conditions are \( \frac{d\theta}{dt} = \frac{d^2\theta}{dt^2} = 0 \)

we can get the Laplace's transform of (6.5) in the following form

\[
\frac{\theta(p)}{\epsilon(p)} = \frac{K_v}{p(\tau_m''p+1)}
\]  

(6.6)

\( K_v \) and \( \tau_m'' \) also can be calculated from the motor diagrams.
In order to make the servo transfer function that of a linear integrator, velocity feedback should be applied. The feedback signal can come from a generator coupled to the servo motor, so its signal will be proportional to the speed of the motor. The block diagram of the system is shown in Fig. 72. The open loop transfer function deduced from Fig. 72 is as follows:

\[
G(p) = \frac{K_v}{p(pT_m''+1)} K_T K_A \quad (6.7)
\]

The closed loop transfer function derived from (6.7) when

\[
\frac{pT_m''}{1+K K_v K_T} \ll 1
\]

goes into the form below:

\[
\frac{\theta(p)}{\epsilon(p)} = \frac{K_o}{p} \quad \text{q.e.d.} \quad (6.8)
\]
where
\[ K_0 = \frac{K_V K_T}{1 + K_V K_T}. \]

In the further analysis the belt and the belt charge system will be regarded as a current generator of infinite impedance. It will be also assumed that all the charges deposited on the belt will be collected at the terminal.

After all these assumptions the transfer function of the belt on the Laplace's transform p plane is
\[ f(p) = e^{-\tau_6 p}, \quad (6.9) \]

This implies that charges injected on the belt will be collected after \( \tau_6 \) delay in the terminal. This delay is due to the distance between spray and collecting combs and the finite speed of the belt.

Combining equations (6.3), (6.4), and (6.9) we arrive at the final form for the \( Z_{II} \) feed impedance
\[ \frac{Z_{II}}{R} = \frac{K e^{-\tau_6 p}}{p(1 + \tau_{5}p)}. \quad (6.10) \]

Let us normalize on \( \tau = \sqrt{\tau_1 \tau_2} = 0.297 \) sec. Then
\[ \tau_5 = 12.54 \text{ sec} = 42.3\tau \]
and
\[ \tau_6 = 0.2 \text{ sec} = 0.673\tau. \]

After separating the real and imaginary parts, (6.10)
\[ \frac{Z_{II}}{R} = K \left\{ -\frac{42.3 \cos 0.673\omega}{(42.3\omega)^2 + 1} - \frac{\sin 0.673\omega}{[(42.3\omega)^2 + 1]^{\frac{1}{2}}} \right\} \]
\[ + j \frac{42.3 \sin 0.673\omega}{(42.3\omega)^2 + 1} - j \frac{\cos 0.673\omega}{[(42.3\omega)^2 + 1]^{\frac{1}{2}}}. \quad (6.11) \]
Starting from Fig. 71 with the already justified approximation we can write for $Z_I$ feed impedance,

$$Z_I = R_1 \frac{\tau_2 p}{(1+\tau_2 p)(1+\tau_1 p)}$$

(6.12)

After separating the real and imaginary parts it becomes

$$\frac{Z_I}{R_1} = \frac{\tau_2 + \tau_1 \tau_2}{(1+\tau_2^2 \omega^2)(1+\tau_1^2 \omega^2)} \omega^2 + j \frac{\tau_2 - \tau_1 \tau_2 \omega^2}{(1+\tau_2^2 \omega^2)(1+\tau_1^2 \omega^2)}$$

(6.13)

Let us now normalize this expression to the frequency of $\omega = \frac{1}{\sqrt{\tau_1 \tau_2}} = 3.37 \text{ sec}^{-1}$ at which the imaginary part of the above expression becomes zero. With this choice of frequency the relative time constants become

$$\tau_1 = 11.8 \times 10^{-3}$$

and

$$\tau_2 = 85$$

After substitution of these values into equation (6.13) it becomes

$$\frac{Z_I}{R_1} = \frac{(7200 + 1) \omega^2}{(1+7200 \omega^2)(1+139.1 \times 10^{-6} \omega^2)} + j \frac{85(1 - \omega^2)}{(1+7200 \omega^2)(1+139.1 \times 10^{-6} \omega^2)}$$

(6.14)

Now the expression of the terminal voltage with $Z_I$ and $Z_{II}$ feed impedances is

$$V_o = R_1 A \frac{\tau_2 p}{(1+\tau_2 p)(1+\tau_1 p)} \epsilon(p) + R K_1 \frac{e^{-\tau_2 p}}{p(1+\tau_5 p)} \epsilon(p)$$

(6.15)
where $e(p)$ is the error signal.

In order to make the combined gain function monotonic in the whole frequency range the absolute value of the two impedance vector $Z_I$ and $Z_{II}$ was made equal at $\omega = 1/\tau_5$.

Thus

$$|R_1^A \frac{\tau_2 p}{(1+\tau_2 p)(1+\tau_1 p)}| = |R_1^K \frac{e^{-\tau_6 p}}{p(1+\tau_5 p)}|$$

for $\omega = 1/\tau_5$. \hspace{1cm} (6.16)

The numerical solution of equation (6.16) gives

$$\frac{R_1^K}{R_1^A} = 0.1015 \text{ sec}^{-1}$$

6.3 Transfer function for the tandem single and two loop stabilizer

In a coaxial system as shown in Fig. 29 charges move with the velocity

$$v(x) = k\left(\frac{V}{x \log b/a}\right)^{\frac{1}{2}}$$ \hspace{1cm} (6.17)

When ion current $I_o f(t)$ is injected by the probe and the ion drift velocity is $v(x)$, the charge in the layer of thickness $\Delta x$ at distance $x$ from the centre is:

$$\Delta Q = \frac{dQ}{dt} \cdot \frac{dt}{dx} \cdot dx = I_o f(t) \frac{1}{v(x)} \int dx$$ \hspace{1cm} (6.18)

Then, the charge induced by $\Delta Q$ on the terminal, calculated from (6.17) and (6.18) is: .
\[ \Delta q = \frac{\log b/a}{\log b/a} \int_0^\infty I(t) \frac{1}{v(x)} \, dx \quad (6.19) \]

The relationship between the drift velocity \( v(x) \) and the drift time \( \tau \) from

\[ x = \int_0^\tau v(x) \, dt \quad (6.20) \]

\[ \tau = KP^\frac{1}{2} \left( \frac{d^q}{V} \right)^{\frac{1}{2}} = \frac{2}{3M} \, d^{3/2} \quad (6.21) \]

where

\[ M = k\left( \frac{V}{P \log b/a} \right)^{\frac{1}{2}} \quad (6.22) \]

Considering the \( \tau \) delay suffered by the ions the total charge on the terminal will be given by the integral of (6.19) for \( x \) between the limits of \( a \) and \( b \).

\[ q(t) = \frac{1}{kV^2(\log b/a)^2} \int_a^b \log b/x \cdot x^{\frac{1}{2}} \, I_0 f(t-\tau) \, dx \quad (6.23) \]

Since the Laplace's transform of the current as a time function is

\[ L[I_0(t)] = L[I_0(t-\tau)] = I_0(p)e^{-p\tau} \quad (6.24) \]

equation (6.23) becomes

\[ q(p) = \frac{I_0}{M \log b/a} \int_a^b \log b/x \cdot x^{\frac{1}{2}} \, e^{-p} \frac{2}{3M} x^{3/2} \, dx \quad (6.25) \]
List of injector parameters used in the calculations

\[ C_1 = 475 \text{ pF} \quad \tau_6 = 0.2 \text{ sec} \]
\[ C_2 = 700 \text{ pF} \quad \omega_6 = 5 \text{ rad/sec} \]
\[ C_3 = 14000 \text{ pF} \quad f_6 = 0.795 \text{ Hz} \]
\[ R_1 = 300 \text{ k\Omega} \quad \tau = 0.297 \text{ sec} \]
\[ R_2 = 21.10^3 \text{ M\Omega} \quad \omega = 3.37 \text{ rad/sec} \]
\[ R_3 = 36.10^3 \text{ M\Omega} \quad f = 0.536 \text{ Hz} \]
\[ \tau_1 = 3.5 \text{ msec} \quad \tau_c = 17.78 \text{ sec} \]
\[ \omega_1 = 286 \text{ rad/sec} \quad \omega_c = 0.0563 \text{ rad/sec} \]
\[ f_1 = 55.6 \text{ Hz} \quad f_c = 0.00894 \text{ Hz} \]
\[ \tau_2 = 25.2 \text{ sec} \quad C_s = 666 \text{ pF} \]
\[ \omega_2 = 0.0397 \text{ rad/sec} \quad R_s = 36.10^3 \text{ M\Omega} \]
\[ f_2 = 0.00632 \text{ Hz} \quad \tau_s = 24 \text{ sec} \]
\[ \tau_3 = 10 \text{ sec} \quad \omega_s = 0.0416 \text{ rad/sec} \]
\[ f_3 = 0.0159 \text{ Hz} \quad f_s = 0.0066 \text{ Hz} \]
\[ \omega_3 = 0.1 \text{ rad/sec} \quad \tau_f = 32 \text{ msec} \]
\[ \tau_5 = 12.54 \text{ sec} \quad \tau_f = 32 \text{ msec} \]
\[ \omega_5 = 0.0797 \text{ rad/sec} \quad \omega_5 = 0.0797 \text{ rad/sec} \]
\[ f_5 = 0.01271 \text{ Hz} \quad f_5 = 0.01271 \text{ Hz} \]

Table 5
This integration can only be carried out for the first order terms. Then we have the following expressions for \( v(x) \) and \( \tau \).

\[
v(x) = - \frac{v(x) - v(a)}{x - a} \cdot x \quad (6.26)
\]

and

\[
\tau = - \frac{x - a}{x} \cdot \frac{x - x}{v(x) - v(a)} \quad (6.27)
\]

After introducing the following parameters

\[
A = - \frac{x - a}{v(x) - v(a)} \cdot \frac{1}{\log b/a} \quad (6.28)
\]

and

\[
B = - \frac{x - a}{v(x) - v(a)} \quad (6.29)
\]

equation (7.25) becomes

\[
q(p) = A I_0 \int_{x_a}^x \log \frac{b}{x} \cdot \frac{1}{x} e^{-pB \cdot x - x/x} dx \quad (6.30)
\]

It can be shown that integral (6.30) after neglecting the second order terms gives the following results:

\[
q(p) = \frac{I_0}{\log b/a \cdot \tau_a} \left[ \frac{1 - e^{-p(2D-1)^2\tau_a}}{p^2} \right. \\
\left. + (1-D)\tau_a \cdot \frac{1-2(2D-1)e^{-p(2D-1)\tau_a}}{p} \right] \quad (6.31)
\]
where \( D = \frac{x_0}{b} \) is the probe distance relative to the radius of the tank and \( \tau_a \) is the ion drift time from the probe to the terminal.

Substituting (6.31) into (3.9) and separating the real and imaginary parts of the frequency dependent part we get for \( K(\omega) \) the following expression:

\[
K(\omega) = \frac{[1-\cos(2D-1)^2\tau_a \omega e^{-\tau_a(2D-1)^2\omega}]}{\tau_a \omega (\alpha-\omega+\alpha^2 \omega)^2 + \tau_a \omega (1+2\omega)^2} \cdot \left( \alpha^2 + \alpha^2 \omega^2 \right)
\]

\[
+ \frac{-\tau_a(2D-1)^2\omega}{\tau_a \omega (\alpha-\omega+\alpha^2 \omega)^2 + \tau_a \omega (1+2\omega)^2} \cdot (1+2\omega)
\]

\[
+ \frac{(1-D)[1-2(2D-1)\cos(2D-1)^2\tau_a \omega e^{-\tau_a(2D-1)^2\omega}]}{(1+\omega)^2+\omega^2} \cdot (1+\omega)
\]

\[
+ \frac{\omega(1-D)\sin(2D-1)^2\tau_a \omega e^{-\tau_a(2D-1)^2\omega}}{(1+\omega)^2+\omega^2}
\]

\[
+ \frac{j}{\tau_a \omega (\alpha-\omega+\alpha^2 \omega)^2 + \tau_a \omega (1+2\omega)^2} \cdot \left( \alpha^2 + \alpha^2 \omega^2 \right)
\]

\[
- \frac{j}{\tau_a \omega (\alpha-\omega+\alpha^2 \omega)^2 + \tau_a \omega (1+2\omega)^2}
\]

\[
+ \frac{(1-D)[1-2(2D-1)\cos(2D-1)^2\tau_a \omega e^{-\tau_a(2D-1)^2\omega}]}{(1+\omega)^2+\omega^2}
\]

\[
+ \frac{j}{\tau_a \omega (\alpha-\omega+\alpha^2 \omega)^2 + \tau_a \omega (1+2\omega)^2} \cdot \left( \alpha^2 + \alpha^2 \omega^2 \right)
\]
When the probe position is controlled by a servo, the corona current charge becomes the function of the probe distance from the terminal thus

\[ I_o = \mu V_g + \eta D \]  \hspace{1cm} (6.33)

where \( \mu \) is the amplification factor of the corona tube
\( V_g \) is the grid potential
\( \eta \) is the derivative of the \( I_o(D) \) function
and \( D \) is the probe relative distance from the terminal.

Introducing \( K(p) \) for noting the single loop transfer function (see equations (3.10) and (3.11)) and using equation (6.4) for the servo transfer function the two loop transfer function becomes

\[ K_1(p) = K(p) \left[ 1 + \frac{K_o}{p} \right] \]  \hspace{1cm} (6.34)

where
\[ K_1 = \eta K_o \cdot 1/\mu \]  \hspace{1cm} (6.35)

After the separation of the real and imaginary part of \( K_1(p) \) we get the following expression...
\[ K_1(\omega) = F_1(\omega) + \frac{K F_2(\omega)}{\omega} \]
\[ + j F_2(\omega) - j \frac{K F_1(\omega)}{\omega} \]  
(6.36)

where \( F_1(\omega) \) and \( F_2(\omega) \) are the real and imaginary parts of \( K(\omega) \) respectively as described in equation (6.32).

6.4 Stabilization of electrostatic accelerators with generating voltmeter

Fig. 73 shows the schematic diagram of the generating voltmeter. Here the rotating blades screen the stationary segments periodically from the H.T. electrode, thus inducing periodically changing current into the load resistors.

Schematic diagram of the generating voltmeter

Fig. 73
List of tandem parameters used in the calculations

\[ a = 21 \text{ in} \]
\[ b = 47.7 \text{ in} \]
\[ \tau_a = 26.3 \text{ msec} \]
\[ R_o = 34.6 \times 10^3 \Omega \]
\[ C_o = 80 \text{ pF} \]
\[ \tau_1 = 2.3 \text{ sec} \]
\[ \omega_1 = 0.435 \text{ rad sec} \]
\[ f_1 = 0.0692 \text{ Hz} \]
\[ \alpha = 0, 0.1, 0.2 \]
\[ f_c = 19, 21.5, 25.95 \text{ Hz} \]
\[ C_3 = 0.3 \text{ pF} \]
\[ R_3 = 1 \text{ M} \]
\[ \tau_i = 0.6 \text{ msec} \]
\[ \tau_T = 0.144 \text{ sec} \]
If $V_T$ is the voltage on the H.T. electrode and $C_4$ is the capacitance of the segment to the electrode the total charge collected on the segment is

$$Q = C \cdot V_T \quad (6.37)$$

The capacitance of the segment depends on the position of the rotary blade and it changes periodically as the blade turns around.

When

$$C_4 = C_0 \left[ 1 + \frac{A^C}{\ell} \cdot f(p) \right] \quad (6.38)$$

where

$$f(p) = \frac{8 \pi \omega}{\rho^2} \th \frac{D}{16 \pi \omega} \th \frac{D}{8 \pi \omega} \quad (6.39)$$

the Laplace transform of a triangular waveform $62,63$)

$$\omega = N \nu$$

$N$ is the number of segments

$$\nu = \frac{2 \pi n}{60}$$

and $n$ is rotational speed of the motor in rpm.

By using equations (6.37) and (6.38) the current in the load resistors will be

$$I = \frac{dQ}{dt} = pV TC_0 \frac{A^C}{\ell} \cdot f(p) \quad (6.40)$$

So the output signal developed across the load is

$$V_{gen} = R pV TC_0 f(p) \quad (6.41)$$

The $p$ in (6.41) indicates that the amplitude of the output signal is dependent on $\omega$ and thus adversely affected by any change in the speed of the drive motor. In order
to eliminate this effect this signal is put through an
integrator with \( \tau_i = R_i C_i \) time constant. The signal then
becomes

\[
V_{\text{OUT}} = \int V_{\text{gen}} \, dt = \frac{RC_i}{R_i C_i} V_T f(p) \quad (6.42)
\]

When \( V_T \) changes \( V_{\text{OUT}} \) becomes a time function.

If the carrier frequency \( \omega \) is filtered out we end up
with the following transfer function

\[
\frac{V_{\text{OUT}}(p)}{V_T(p)} = \frac{RC_i}{R_i C_i} \frac{1}{\tau_f p + 1} \quad (6.43)
\]

where \( \tau_f \) is the time constant of the low-pass filter.

This equation describes the output voltage of a
generating voltmeter exposed directly to the H.T. terminal.
However, when the generating voltmeter facing the intershield
as in the injector it picks up the voltage fluctuations on
the intershield, besides sensing the D.C. potential. Any
frequency present in the ripple of the terminal voltage is
proportional in amplitude to that of the intershield and
attenuated by the stack resistors and the electrode capacitances.
The transfer function can be derived from the electrode arrange­
ment as shown in Fig. 70b. Using the same argument as before
in section 6.2 the two bottom RC circuits can be made into
one. Then for the new circuit

\[
R_s = R_3 + R_1 = 36.10^6 \Omega
\]

\[
C_s = C_2 || C_3 = 666 \, \text{pF}
\]

and

\[
\tau_s = R_2 C_s = 24 \, \text{sec.}
\]

Now the transfer function for this circuit is

\[
\frac{V_s}{V_T} = \frac{R_3}{R} \frac{1 + \frac{\tau_s}{p}}{1 + \frac{\tau_s}{p}} \quad (6.44)
\]
where $V_S$ is the intershield voltage and $V_T$ is the terminal voltage.

After the substitution of (6.44) into (6.43) the transfer function for the generating voltmeter of a machine with intershield, becomes

$$\frac{V_{\text{OUT}}(p)}{V_T(p)} = \frac{RC_oR_s}{R_1C_1R} \frac{1 + \tau_s p}{1 + \tau_s p} \cdot \frac{1}{\tau_f p + 1} \quad (6.45)$$

6.5 Energy modulation of electrostatic accelerators

The method of achieving a beam energy modulation is closely associated with the voltage stabilization of the accelerators. The beam after passing through a 90° bending magnet is focussed through a vertical analyzing slit (see Fig. 2.) If the beam travels centrally through the slit, and assuming axial symmetry in the beam profile, both sides of the slit intercept an equal fraction of the beam. If, however, the beam travels off axis, then the currents picked up by the two sides of the slit will be different. These intercepted beam currents, after suitable amplification, are fed into a differential amplifier. The output signal produced by this amplifier is used to control the corona loading of the centre terminal, which so changes the terminal voltage that the accelerated beam is restored to a central position through the analyzing slit.

In order to change the beam energy cyclically the current in the 90° analyzing magnet was modulated. As a result, the stabilization system then continuously
changed the beam energy by minimising the output of the differential amplifier. Schematic diagrams of the experimental apparatus used to modulate the magnetic current are shown in Fig. 67.

The current in the analyzing magnet is determined by a reference voltage $V_R$, which is compared in a differential amplifier with the voltage $V_4$ developed across a standard resistor, through which the magnet current passes. The output from this amplifier controls the current of the analyzing magnet and so the magnetic field.

The modulation of the magnetic field is achieved by slowly varying the reference voltage $V_R$. To do this, a varying current is passed through a standard resistor inserted in the line AB (see Fig. 67), so inducing a variable potential. The varying current is supplied by an emitter follower (see Fig. 74) in which the base of the transistor T1 is connected to a cyclically driven ten-turn helipot P1. The period of the energy modulation is determined by the cycle.

![Circuit diagram of the magnet current modulator.](Fig. 74)
of the base voltage change. The period of change can be varied from ten seconds upwards. The peak amplitude of the voltage sweep across the standard resistor, and so the depth of the energy modulation, is set by the potentiometer $R_3$. If the average energy is to be altered from one experiment to another the setting of the potentiometer $R_3$ needs to be changed in order to keep the modulation depth constant. The reason for this is that the energy of the beam $E$ is proportional to the square of the magnetic field $H$ and therefore the square of the magnet current, i.e.

$$E = H^2 = I^2$$

$$\Delta E = I \Delta I = E^2 \Delta I$$

(6.46)

so for a constant $\Delta E$, $\Delta I$ and so $R_3$ needs to be varied inversely with $\Delta E$.

The differential hysteresis of the analyzing magnet was determined using an NMR probe and it was found to be small enough not to cause any significant error. The system worked well for energy resolution widths from 5 to 400 keV. It was possible to achieve such a wide resolution function, while maintaining a good focussed beam, since the beam focussing elements were synchronized to the current of the analyzing magnet.

6.6 Calculation of the tandem energy resolution

Following Koestler's treatment the measured differential cross section at the nominal energy $E$ and angle $\theta$ can be described by the following formula:
\[
\sigma_0(E, \theta) = \frac{1}{at} \int \int_{E \geq v, E' \geq v} g(E-v-E') \sigma(E', \theta) dE'
\]

(6.47)

where \( at \) is the target thickness in kV

\( \delta \) is the maximum spread in proton energy from the \( E \) nominal

and \( \sigma(E', \theta) \) is the theoretical cross section at energy \( E' \).

The theoretical expression for the differential cross section calculated by Richards is the following

\[
\sigma_T = A + B \sin^2 (\beta + \gamma) \]

(6.48)

where

\[
\beta = \tan^{-1} \frac{\Gamma}{2(E_R + E)}
\]

(6.49)

Here \( A, B \) and \( \gamma \) are constants depending only on \( \theta \). \( E_R \) and \( \Gamma \) are the resonant energy and total width respectively.

It is convenient to change the order of integration and perform the integration over \( v \) first. Since the triangular function \( g \) can be separated one can write the following expressions

\[
g_L = \frac{1}{\delta}(y + \delta + v) \text{ for } -\delta-v \leq y \leq -v \] (6.50)

and

\[
g_R = \frac{1}{\delta^2}(-y + \delta - v) \text{ for } -v \leq y \leq \delta-v \] (6.51)

where \( y = E'-E \)

and \( v = ax \)
Following now Teitelman and Temmer treatment when

\[ 0 \leq \alpha \leq \delta \]

\[ F(y) = \delta^{-2} \left[ \frac{1}{2}(y+\delta)^2 + \alpha(y+\delta) + \frac{1}{2}(\alpha)^2 \right] \text{ for } -\alpha - \delta \leq y \leq -\delta \]

\[ = \delta^{-2} \left[ \alpha(y+\delta) + \frac{1}{2}(\alpha)^2 \right] \text{ for } -\delta \leq y \leq \alpha \]

\[ = \delta^{-2} \left[ -y^2 - \alpha y + 8\alpha \delta - \frac{1}{2}(\alpha)^2 \right] \text{ for } -\alpha \leq y \leq 0 \]

\[ = \delta^{-2} \left[ \alpha(-y+\delta) - \frac{1}{2}(\alpha)^2 \right] \text{ for } 0 \leq y \leq -\alpha + \delta \]

\[ = \delta^{-2} \left[ \frac{1}{2}(-y+\delta)^2 \right] \text{ for } -\alpha + \delta \leq y \leq \delta \]

The target thickness was calculated from the geometry of the experimental arrangement and the number of counts obtained at the experiment. The figures calculated and used in the calculations are tabulated in Table 7.

With the combination of equation (6.47) and (6.52) we arrive at the following formula for the calculated resonance curve widened by the energy spread \( \delta \) of the protons.

\[ \sigma_{\text{meas}}(E_{\text{NMR}}, \theta) = \frac{1}{\alpha \delta} \left[ \int_{-\alpha}^{-\delta} \sigma_T(E+y, \theta) F_1(y) \, dy \right. \]

\[ + \int_{-\delta}^{-\alpha} \sigma_T(E+y, \theta) F_2(y) \, dy \]

\[ + \int_{-\alpha}^{0} \sigma_T(E+y, \theta) F_3(y) \, dy \]

\[ + \int_{0}^{\alpha-\delta} \sigma_T(E+y, \theta) F_4(y) \, dy \]

\[ + \int_{\alpha-\delta}^{\delta} \sigma_T(E+y, \theta) F_5(y) \, dy \]
Since the integration in closed form was not practical, numerical integration was carried out on the PDP10 computer. By altering the energy spread $\delta$ as an input data the best fit was obtained to the measured resonance curve. The computer program and the subprograms for this calculation are shown in Appendix 6.7.4. The experimental and the calculated resonance curves are plotted in Fig. 44.
<table>
<thead>
<tr>
<th>Θ</th>
<th>90°</th>
<th>167°</th>
</tr>
</thead>
<tbody>
<tr>
<td>d</td>
<td>13.97 mm</td>
<td>11.43 mm</td>
</tr>
<tr>
<td>D</td>
<td>4 mm</td>
<td>5 mm</td>
</tr>
<tr>
<td>dΩ</td>
<td>0.643 \times 10^{-3}</td>
<td>0.1684 \times 10^{-2}</td>
</tr>
<tr>
<td>dσ/dΩ</td>
<td>65 mb/sr</td>
<td>100 mb/sr</td>
</tr>
<tr>
<td>γ</td>
<td>6767</td>
<td>19102</td>
</tr>
<tr>
<td>at</td>
<td>0.603 kV</td>
<td>0.6608 kV</td>
</tr>
<tr>
<td>dE/dξ</td>
<td>50 keV/mg/cm²</td>
<td>50 keV/mg/cm²</td>
</tr>
<tr>
<td>A</td>
<td>5.7 mb/sr</td>
<td>30.0 mb/sr</td>
</tr>
<tr>
<td>B</td>
<td>90.67 mb/sr</td>
<td>330.0 mb/sr</td>
</tr>
<tr>
<td>γ</td>
<td>125.0°</td>
<td>27.0°</td>
</tr>
<tr>
<td>Γ</td>
<td>0.192 keV</td>
<td>0.143 keV</td>
</tr>
</tbody>
</table>

θ is the laboratory angle

d is the distance between target and detector

D is the diameter of the aperture in the front of the detector

dΩ is the solid angle between target as a point source and the detector

γ is the number of experimental count

at is the target thickness

dE/dξ is the specific energy loss in the target

A, B and γ are constants

Γ is the total width of the resonance

Table 7
C INJECTOR NMR FREQUENCY (NMRFI)
2 READ (5,4)E,POL,TM,Z
4 FORMAT (F12.6)
F=SQRT(E*(T-M*POL*Z*5.486*10,0**(-4,0))/(0.089839
 1 *Z**2,0))*(1+E/((T-M*POL*Z*5.486*10,0**(-4,0))
 1 *931.478*2,0))
8 WRITE (5,10)F
9 TV=E/(0.36364+Z*0.63636)
14 WRITE (5,10)TV
10 FORMAT (F12.6)
GO TO 2
END
C MACHINE CONTROL
C TANDEM POLAR DIAGRAM (TANPOD)
DIMENSION F(4),G(8),H(4),ALFA(2),Z(2),DB(2)
READ (5,4)X,D,W
A=(X,W)*X/(D*W,5)
T=(2,3*2.10,0*3.0)/(2,3*2.10,0*3.0)
FORMAT (F14.4)
G(1)=ABS(SIN((2,0*D-1,0)*2,0*T*A+W)*A*W)/(D*SIN((2,0*D-1,0)*2,0)*T*A+W)
G(2)=ABS(SIN((2,0*D-1,0)*2,0*T*A+W)*A*W)/(D*SIN((2,0*D-1,0)*2,0)*T*A+W)
H(1)=G(1)+G(2)/T
WRITE (5,10)H(1)
F(1)=H(1)+H(2)
WRITE (5,10)F(1)
F(3)=F(1)+1.73*F(2)/W
F(4)=F(2)-1.73*F(1)/W
WRITE (5,10)F(3)
WRITE (5,10)F(4)
ALFA(1)=57,4*ATAN(F(2)/F(1))
WRITE (5,10)ALFA(1)
GO TO 24
Z(1)=SQR(T(F(1)**2,F(2)**2,F(2)**2))
DB(1)=20,0*ALOG10(Z(1))
WRITE (5,10)DB(1)
WRITE (5,10)Z(1)
DB(2)=20,0*ALOG10(Z(2))
WRITE (5,10)DB(2)
WRITE (5,10)H(2)
ALFA(2)=57,4*ATAN(F(4)/F(3))
WRITE (5,10)ALFA(2)
WRITE (5,10)Z(2)
WRITE(5,10)DB(2)
28   GO TO 2
30   END
TANDEM NMR FREQUENCY (NMRFT)

READ (5,4)E,TM,Z

FORMAT(F12.6)
F=SQRT(E*(5,4*10,0/Z*(1+E/((5,4*10,0/Z)))*931.478/Z))
TV=E/(1+Z)
WRITE (5,10)F,TV
GO TO 2
END
C TANDEM ENERGY RESOLUTION (TANER)
DIMENSION DATA(200)
READ (5,4) DEL,A,B,C,D,E,CON
FORMAT (F12.6)
ER=5830.0
DO 40 J=1,29
TOTAL=0.0
VALUE=0.0
ENMR=5825.5*(J-1.0)*0.3928
Y=-(E+DEL)
DO 3 K=1,5
DO 30 L=0,199
Y=Y+DELX(DEL,E,J,K)
ARG=A/(2.0*(ER-(ENMR+Y)))
BET=ATAN(ARG)
SIG=B*C*(SIN(BET+3.14/180.0))*2.0
DATA(L)=SIG*F(DEL,E,J,K,L,Y)
CONTINUE
CALL GRINTG (DATA, VALUE, DELX(DEL,E,J,K))
TOTAL=TOTAL+VALUE
CONTINUE
SIM=CON*TOTAL/E
WRITE (5,10) SIM
FORMAT (F12.5)
CONTINUE
GO TO 2
END
FUNCTION F(DEL,E,J,K,L,Y)
IF (K,EQ,1) GO TO 1
IF (K,EQ,2) GO TO 2
IF (K,EQ,3) GO TO 3
IF (K,EQ,4) GO TO 4
IF (K,EQ,5) GO TO 5
1 F=(((1/DEL)**2,0)*(0,5*(Y+DEL)**2,0)+
1 E*(Y+DEL)+0,5*(E**2,0))
RETURN
2 F=(((1/DEL)**2,0)*((E*(Y+DEL))+0,5*(E**2,0)))
RETURN
3 F=(((1/DEL)**2,0)*(-(Y**2,0)-E*(Y-DEL))-
1 0,5*(E**2,0))
RETURN
4 F=(((1/DEL)**2,0)*((E*(-Y+DEL)-0,5*(E**2,0)))
RETURN
5 F=(((1/DEL)**2,0)*(0,5*(DEL-Y)**2,0))
RETURN
END
FUNCTION DELX(DEL,E,J,K)
IF (K,EQ,1) GO TO 1
IF (K,EQ,2) GO TO 2
IF (K,EQ,3) GO TO 3
IF (K,EQ,4) GO TO 4
IF (K,EQ,5) GO TO 5
1  DELX=(-DEL+DEL+E)/199,0
   RETURN
2  DELX=(-E+DEL)/199,0
   RETURN
3  DELX=E/199,0
   RETURN
4  DELX=(DEL-E)/199,0
   RETURN
5  DELX=(DEL-DEL+E)/199,0
   RETURN
END
SUBROUTINE GRINTG (DATA, VALUE, DELX)
DIMENSION DATA (200)
VALUE=DATA(1)+DATA(200)
DE=0,0E0
DM=0,0E0
DO 1 I = 2, (200-1), 2
   DM=DM+DATA(I)
   CONTINUE
1
DO 2 I = 3, (200-2), 2
   DE=DE+DATA(I)
   CONTINUE
2
VALUE=(VALUE+4,0E0*DM+2,0E0*DE)*DELX/3,0E0
RETURN
END
INJECTOR BREAKDOWN (INBRAKE)

DIMENSION D(3), VT(5), E(12), F(3)

T = 0
VT(1) = 0
VT(2) = 0
VT(3) = 0
VT(4) = 0
VT(5) = 0

READ (5, 4) V, TIM, A, B, C

FORMAT (1H, E14.8)

FORMAT (F12.8)

T = C*(15.5 + (V - 4)*3.0) - 8*35.21*ALOG(1.5/V)
D(1) = 35.21 + 0.031*(T + 0.2)*2.0/2.0
D(2) = 1 - 0.031*(T + 0.2)
D(3) = 0.031

E(1) = D(2)/(2*D(1))
E(2) = SQRT(-D(2)*2.0 + 2.0*D(2)*D(3))/2.0*D(3)
E(3) = 1/(E(1)*2*0) + E(2)*2.0
E(4) = 2.0*E(1)*E(3)
E(5) = E(3)
E(6) = 1/((-35.21*(35.21*2.0 + 2.0*E(1)*35.21)
1 + 1/E(3)))
E(7) = 2.0*E(3)*E(1) - 35.21 - E(6)/E(3)
E(8) = (-E(3)*35.21 - E(6))/35.21
E(9) = 2.0*E(3)*E(1)*E(3) - E(3)*35.21
E(10) = 1/((35.21*2.0 + 2.0*E(1)*35.21)
1 - E(3))
E(11) = -E(10)/E(3) - E(9) + E(3)*2.0*E(9)*E(1) +
2.0*E(3)*E(1)*E(1)*35.21 - E(3)
E(12) = (-E(10) - E(9) + E(3)/35.21)^21

WRITE (5, 6) VT(1)

VT(2) = VT(1) + (0.200*A*1.5/V)*((TIM - 0.2) -
1*35.21*(1.0 - EXP(-0.284*(TIM)))
WRITE (5, 6) VT(2)

VT(3) = VT(2) + (0.200*(-A*1.5/V))*((TIM - T-0.2) -
1*35.21*(1.0 - EXP(-0.284*(TIM - 0.2)))
WRITE (5, 6) VT(3)

VT(4) = VT(3) - (0.200*0.031*A*1.5/V)*((1.0 - EXP(-0.284*(TIM - T-0.2)))
1*35.21*(1.0 - EXP(-0.284*(TIM - T-0.2))))

WRITE (5, 6) VT(4)

VT(5) = VT(4) + (0.200*0.031*(A*1.5/V))*((1.0 - EXP(-0.284*(TIM - T-0.2)))
1*35.21*(1.0 - EXP(-0.284*(TIM - T-0.2))))
\[ (E(12) \ast (\exp(E(1) \ast (TIM - 2,0 \ast T - 0,4)) \ast \cos(E(2) \ast (TIM - 2,0 \ast T - 0,4)) + ((E(11) / E(12)) - E(1)) / E(2) \ast \exp(E(1) \ast (TIM - 2,0 \ast T - 0,4)) \ast \sin(E(2) \ast (TIM - 2,0 \ast T - 0,4)))) \]

WRITE (5,6) VT(5)
IF (TIM - 0,4 - 2,0 \ast T) > 22, 22, 32
IF (TIM - 0,4 - T) > 26, 26, 30
IF (TIM - 0,2 - T) > 14, 14, 20
IF (TIM - 0,2) > 16, 16, 18
WRITE (5,6) VT(1)
GO TO 2
WRITE (5,6) VT(2)
GO TO 2
WRITE (5,6) VT(3)
GO TO 2
WRITE (5,6) VT(4)
GO TO 2
WRITE (5,6) VT(5)
GO TO 2
END
TANDEM BREAKDOWN (TANBRAKE)
DIMENSION D(3), VT(3), E(12)

T = 0
VT(1) = 0
VT(2) = 0
VT(3) = 0
READ (5, 4) V, T, A, B, C, R

FORMAT (1H, E14.8)

FORMAT (F12.8)

T = C* ((V-1,5)/(1.5+V)) * 5,0 - B * (5,0/1,5) * ALOG (1,5/V)
D(1) = 5,0 + R * (T*2,0)/2,0
D(2) = -1,0 - R * T
D(3) = R
E(1) = -D(2)/(2,0*D(1))
E(2) = SQRT(-D(2)*2,0 + 4,0*D(1)*D(3))/(2,0*D(1))
E(3) = 1,0/(E(1)*2,0 + E(2)*2,0)
E(4) = 2,0*E(1)*E(3)
E(5) = -E(3)
E(6) = 1,0/((-5,0*2,0)+2,0*E(1)*5,0)
1 + 1,0/E(3))

E(7) = 2,0*E(3)*E(1)-5,0-E(6)/E(3)
E(8) = (-E(3)+5,0-E(6))/5,0
E(9) = 2,0*E(3)*E(1)-E(3)-E(3)*5,0
E(10) = 1,0/((5,0*2,0)*((-5,0*2,0)+2,0*E(1))
15,0+1,0/E(3))

E(11) = -E(10)/E(3) = E(9)*5,0/E(3) + 2,0*E(9)*E(1) +
12,0*E(3)*E(1)-5,0-E(6)/E(3)
E(12) = (-E(10)+E(9)*5,0)/5,0
WRITE (5, 6) E(6), E(7), E(8), E(9), E(10), E(11), E(12)

8 VT(1) = 1,0 - EXP(-((1,5/5,0)*TIM))
WRITE (5, 6) VT(1)

VT(2) = VT(1) + (R*A*1,5/V) * ((TIM-T)-
1 (5,0/1,5) * (1,0-EXP(-((1,5/5,0)*(TIM-T))))
WRITE (5, 6) VT(2)

VT(3) = VT(2) + (R-A*1,5/V) * ((TIM-T)-
1 (5,0/1,5) * (1,0-EXP(-((1,5/5,0)*(TIM-T))))
1+R*(-EXP(E(1)*(TIM-T)))*
1 COS(E(2)*(TIM-T)) + (E(1)/E(2))
1+EXP(E(1)*(TIM-T)) + SIN(E(2)*(TIM-T)-
1 (E(6)/E(3)) + EXP(-((1,5/5,0)*(TIM-T))-
1 (E(6)/E(3)) + EXP(E(1)*(TIM-T)) + COS(E(2)*
1 (TIM-T) + ((E(7)/E(8)*E(3)) + E(1)/E(2)) +
1 EXP(E(1)*(TIM-T)) + SIN(E(2)*(TIM-T))-
1 - ((R*2,0)*A*1,5/V)*((1,0/E(3)) +
1 (E(3)*(TIM-T) + E(9)*E(10) * (1,0-
1 EXP(-(1,5/5,0)*(TIM-T)) + E(12)
1+EXP(E(1)*(TIM-T)) + COS(E(2)*(TIM-T)) +
1 (E(11)/E(12)) - E(1)) + E(2)) +
1 EXP(E(1)*(TIM-T)) + SIN(E(2)*(TIM-T)))

WRITE (5, 6) VT(3)

VT(4) = VT(3) + ((R*2,0)*A*1,5/V) * (1,0/E(3)) +
1 (E(3)*(TIM-T) + E(9) * E(10) * (1,0-
1 EXP(-(1,5/5,0)*(TIM-T)) + E(12)
1+EXP(E(1)*(TIM-T)) + COS(E(2)*(TIM-T)) +
1 (E(11)/E(12)) - E(1)) + E(2)) +
1 EXP(E(1)*(TIM-T)) + SIN(E(2)*(TIM-T)))

WRITE (5, 6) VT(4)

IF (TIM-T) 26, 26, 30

IF (TIM-T) 18, 18, 20
18  WRITE (5,6)VT(2)
    GO TO 2
20  WRITE (5,6)VT(3)
    GO TO 2
30  WRITE (5,6)VT(4)
    GO TO 2
END
FREQUENCY READ IN ROUTINE

/* THIS IS TO READ IN TF2A01 OUTPUT */

777770

DAC C0UNT
EZM NMRA
EZM NMRB
LAC NMRA
EZM NMRA
NIL
TEN
LACR
DAC NMRA
LAC NMRR
EZM NMRR
NIL
TEN
ADP NMRA
EZM NMRA
DAC NMRA
LACR
DAC NMRB
KSF
JMP -1
KRR
AND C 17
TAC NMRB
EZM NMRB
DAC NMRB
S2L
ISZ NMRA
ISZ COUNT
JMP LOOP
HLT
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