SMALL ANGLE PROTON SCATTERING FROM HYDROGEN AND LIGHT NUCLEI

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Abstract

This thesis describes the study of proton scattering from protons and light nuclei and the development of a self-supporting target of solid hydrogen for nuclear physics experiments.

Differential cross-sections for the scattering of 144 MeV protons from hydrogen, deuterium, helium, lithium-6, lithium-7 and Carbon-12 around the Coulomb interference region (2° to 20°) have been measured. The measurements of the cross-section for proton-proton scattering were an extension of the measurements made by G.F. Cox et al.\(^1\) and also a test for the feasibility of using a solid self-supporting hydrogen target, developed as part of this work, in Nuclear physics experiments. The measurements at cross-sections for light nuclei were made in order to test some simple nuclear models normally used to predict nuclear scattering, namely, the Born Approximation, W.K.B. Approximation and the construction of the optical model potentials from the two nucleon scattering matrix and the way the Coulomb interaction is included.

Nucleon-nucleon scattering is reviewed in Chapter one together with the types of experiments required to determine the nucleon-nucleon interaction uniquely. Chapter two describes the construction of nucleon-nucleus interaction from nucleon-nucleon interactions and presents a set of formulae used for the final analysis of proton scattering from light nuclei. Chapter three deals with the experimental layout and the measurements of the quantities that finally lead to cross-sections. Chapter four describes the production and the development of a solid self-supporting hydrogen target. Finally chapter five gives the results of the experimental measurements and
and interprets them in terms of the theoretical predictions.

A separate study carried out during the first year at Surrey University is given in Appendix A and deals with the variation of optical model potential with the incident proton energy scattered from carbon in the range 100 MeV to 1000 MeV. A published report on this subject is also provided at the end of Appendix A.

An incidental study on the production of K-shell X-rays by 160 MeV protons is attached to the thesis as an Appendix B.
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NUCLEAR SCATTERING IN A GENERAL CONTEXT

In 1911 Lord Rutherford's experiments on the scattering of $\alpha$-particles by thin foils led to the first qualitative evidence about the nucleus. His results showed that the positive charge is concentrated within a volume almost negligible in comparison with the atomic volume. Two years later Niels Bohr put forward a naive picture of atomic structure in which the electrons revolve around this positive charge at the centre called the nucleus.

This work and the work that followed increased our understanding of atomic structure. It also opened up a new field of nuclear structure. Interest was created about the constituents of the nucleus, and its size and about the nature of forces experienced between the constituents.

Early work on the nucleus was concentrated on the measurements of its radius. Various techniques were utilised. Elastic scattering of $\alpha$-particles, $\alpha$-decay, X-rays emitted by negative $\mu$-mesons captured into orbits around nuclei and electron scattering were amongst the techniques adopted for the measurements of radius of the nucleus.

Electron scattering experiments were initially by far the most extensive for probing the individual nucleons of the nucleus as well defined intense monoenergetic electrons beams were easily available and also their electromagnetic scattering mechanism was well understood. Experiments with protons and neutrons at energies capable of interacting with individual nucleons of the nucleus were carried out as soon as such beams became available in order to learn about the nature of forces experienced by nucleons.
Much of the knowledge about nucleon forces comes from nucleon-nucleon scattering. The information is not complete as nucleon-nucleon scattering gives the on-energy-shell matrix elements of the nuclear force. The off-energy-shell matrix elements can be learned from nucleon-nucleus scattering but so far such efforts have not been fruitful. (Better experiments for studying off-shell behaviour are bremsstrahlung and knock-out reactions such as p-2p). However a number of nuclear properties such as the binding energy of nucleons inside a nucleus, and saturation of nuclear forces have been explained with some success by use of nucleon-nucleon forces represented by a potential which, though obtained by fitting the on-the-energy-shell scattering of two nucleons, contains some information regarding off-the-energy-shell scattering.

Historically, the nucleon-nucleon scattering and the nucleon-nucleus scattering data have been fitted phenomenologically. The nucleon-nucleon scattering data are reduced to a set of phase-shifts for the various partial waves contributing to the scattering. Initially more than one set of phase-shifts could be found to fit the data at a given energy but latter, a multi-energy analysis, where the phase shifts are assumed to vary smoothly with energy, has led to a unique set of phase-shifts to be determined.

The nucleon-nucleus scattering has usually been analysed using an optical model potential. At first the optical potential U(r) was represented by a form

\[ U(r) = V(r) + iW(r) = U_c(r) \]

where \( U_c(r) \) represents the central part of the potential and \( V(r) \) and \( W(r) \) are the real and imaginary parts. Predictions from such a
potential gave poor fits to the data and an improvement was later made by including a spin-orbit potential, introduced by Fermi\(^{(10)}\), in the form of

\[
\frac{1}{r} \frac{dU_c}{dr} \sigma \cdot L
\]

where \(U_c\) is the central complex potential defined above and \(\sigma\) and \(L\) are the spin and orbital angular momentum of the incident nucleons.

Parallel to the phenomenological development, the explanation of nucleon-nucleon and nucleon-nucleus scattering data in terms of more fundamental concepts has been pursued. Attempts have been made to explain the nucleon-nucleon scattering results in terms of meson exchanges\(^{(11)}\), whilst the nucleon-nucleus scattering has been described in terms of the interaction of an incident nucleon with individual nucleons in the nucleus\(^{(12)}\). The optical model potential can be represented in terms of two nucleon interactions (upon which topic we expand further in Chapter 2). The form for the central and spin-orbit parts of the optical potential used in phenomenological analyses can be shown to follow from these theoretical considerations\(^{(13)}\). The fact that the optical potential is energy dependent also follows in a simple manner\(^{(14,15)}\) and is attributed to the energy and momentum dependence of the two nucleon interaction.
1.1 Introduction

Nucleon-nucleon scattering has been a subject of intensive study for many years \(^{11, 16}\). The reason for such concentrated study is two-fold. Firstly, the neutrons and protons form the building blocks for a nucleus and hence the knowledge of the nucleon-nucleon interaction could in principle enable one to understand nuclear properties. Such efforts have not been very successful in the past \(^{17}\); although nucleon-nucleus scattering has been explained successfully in terms of nucleon-nucleon phase-shifts \(^{12, 18, 19}\). Secondly, Yukawa's fundamental assumption that the forces between nucleons are due to the exchange of mesons opened up an entirely new field of elementary particle physics and it became necessary to explain the inter-nucleon forces in terms of exchanges of elementary particles between the nucleons.

Most of the knowledge about the nucleon-nucleon interaction comes from the study of scattering experiments. The only bound state of the two-nucleon system, the deuteron, is weakly bound and, therefore, does not give much information concerning the short range forces. The information from nucleon-nucleon scattering experiments is condensed into sets of phenomenological phase-shift parameters which are well known for all energies up to the inelastic threshold energy at about 350 MeV. The theoretical description of the nucleon-nucleon interaction in terms of exchanges of elementary particles has not been very successful in explaining medium and short range forces but has succeeded in predicting the long-range part of the force accurately by assuming that it arises from the exchange of
one pion. Calculations have also been made using phenomenological potentials constructed under certain physical symmetry conditions and incorporating a repulsive core required to explain the change in sign of $S_0 (T = 1)$ phase-shift at 240 MeV. Hard-core potentials have been fitted to scattering data by Hamada, Johnston\(^{(20)}\), a Yale group \(^{(21)}\) and by Reid\(^{(22)}\). Finite-core potential has also been used by Bressel and Kerman\(^{(33)}\) and a Yukawa-core by Reid\(^{(22)}\).

1.2 General Formalism

For two-nucleon scattering processes all physical information is contained in the elements of the scattering matrix $M$. The $M$-matrix and the quantities related to it will form the general framework for the phenomenological methods. A complete spin-formalism for the scattering of two spin-$\frac{1}{2}$ particles has been given in an excellent review article by L. Wolfenstein\(^{(24)}\). The approach is non-relativistic. Stapp\(^{(25)}\) has shown that the non-relativistic spin formalism can be retained provided that an additional rotation of the polarization vector is added at each scattering angle. The cross-sections $\sigma(\theta)$ and the polarization $p(\theta)$ are not affected but the definitions of some triple-scattering and other quantities are modified.

The spin formalism uses a scattering matrix in the composite spin space of the two nucleons. Each nucleon is described by a Pauli spinor and, thus, the combined spin space has four dimensions. The wave function describing the scattering in the centre-of-mass system has four components\(^{(24)}\)

$$
\Psi_i = a_i e^{ikr} + e^{ikr} \sum M a_i
$$
where the four $a_i$ refer to the incident spin states, $M$ is the scattering matrix, $r$ is the relative distance of the two nucleons and $h\kappa$ is the incident momentum in the centre-of-mass system. The scattering matrix is $4 \times 4$ and thus has 16 complex elements in the general case. However, taking account of involving several symmetry properties which a strong interaction is thought to possess, the complex interaction of the $M$-matrix reduces to five. These symmetry properties are:

(i) Invariance under translation in time and space and under rotation (conservation of energy, momentum and angular momentum, respectively). These hold exactly.

(ii) Invariance under reflection in space (parity conservation). This holds partially. Weak interaction part of the nuclear force violates parity (for example, the $\beta$-decay). However, experiments show that the parity-violating part of the interaction is very small.

(iii) Invariance under time-reversal (principle of reciprocity). This is confirmed by experiment, though less precisely.

(iv) Invariance under rotations in isobaric-spin space (charge independence). Nucleon-nucleon interactions are not exactly charge independent. Apart from the Coulomb interaction between protons, there are probably small violations in the nuclear part of the forces between nucleons. Experiments suggest that these terms are small.

By applying these restrictions, Wolfenstein shows that the scattering matrix may be written as
VECTOR SYSTEMS USED IN THE WOLFENSTEIN FORMALISM

\[ \vec{N} = \frac{\vec{K}_i \times \vec{K}_f}{|\vec{K}_i \times \vec{K}_f|} \]
\[ \vec{k} = \frac{\vec{K}_f - \vec{K}_i}{|\vec{K}_f - \vec{K}_i|} \]
\[ \vec{p} = \frac{\vec{K}_f + \vec{K}_i}{|\vec{K}_f + \vec{K}_i|} \]

(c) PROTON–PROTON SCATTERING IN LAB. FRAME.
\[ M = A + B \left( \sigma_1 \cdot n \right) \left( \sigma_2 \cdot n \right) + C \left( \sigma_1 \cdot n + \sigma_2 \cdot n \right) \]

\[ + E \left( \sigma_1 \cdot k \right) \left( \sigma_2 \cdot k \right) + F \left( \sigma_1 \cdot n \right) \left( \sigma_2 \cdot n \right) \]

where \( \sigma_1 \) and \( \sigma_2 \) are the Pauli spin matrices for the two nucleons and A, B, C, E, and F are complex functions of energy and scattering angle \( \Theta \). The vectors \( n, p \) and \( k \) form the usual right handed set as shown in Figure 1.

This shows that at any given energy and angle ten independent experiments are needed (eleven for each isotopic-spin space, if charge independence is not assumed) to determine the matrix \( M \) unambiguously. However, considering that the conservation of particles must hold for the energies below the inelastic threshold (Unitarity), the optical theorem reduces the minimum number of experiments required to five (six if charge independence is not satisfied) at a given energy and over all the angles in the range \( 0-\pi \) in the centre-of-mass system for n-p scattering and \( 0-\pi/2 \) for p-p and n-n scattering due to particle identity. In practice, however, the minimum number of experiments required must exceed five as experimental data is acquired neither with infinite precision nor continuously over all angles.

1.3 The Density Matrix and Experimentally Measurable Quantities

Up to now only pure initial spin states have been considered. However, in actual experiments, the spin states of the beam and the target are completely or partially incoherent mixture of pure spin states. To describe such states one is required to use a statistical density matrix \( \rho \). This contains the information about
the state of polarization of the beam and the target as well as the correlation between the spins of the beam nucleons and the target nucleons. If the initial density matrix is \( \rho_i \), then the density matrix after the scattering is

\[
\rho_f^\dagger = M^\dagger \rho_i^* M
\]

If the initial beam and the target is unpolarized the differential cross-section is given by

\[
I(\theta) = \frac{1}{4} \text{Tr}(M^\dagger M) \quad \cdots \quad (1.2)
\]

An average over all particles of any spin operator \( \theta \) is given by

\[
\langle \theta \rangle = \frac{\text{Tr}(\rho \theta)}{\text{Tr}(\rho)} \quad \cdots \quad (1.3)
\]

The property of the \( M \)-matrix given previously and the relation (1.3) give rise to important conclusions for nucleon-nucleon scattering

(i) Space-reflection considerations show \( (2n) \) that the polarization arising from the scattering of an unpolarized beam from an unpolarized target has a component in the direction perpendicular to the scattering plane only.

(ii) The differential cross-section for the scattering of a polarized beam from an unpolarized target is

\[
I(\theta, \phi) = I_0(\theta) \left[ 1 + P_{\perp \parallel} \right] P(\theta)
\]
where $I_0(\theta)$ is given by equation (1.2). $P_\perp$ is the polarization of the beam, $P(\theta)$ is the polarization produced and $N$ is a unit vector perpendicular to the scattering plane.

Time reversal invariance states that $\epsilon = P_\perp P(\theta)$ where $\epsilon$ is the asymmetry defined as

$$\epsilon = \frac{I(\theta, 0) - I(\theta, \pi)}{I(\theta, 0) + I(\theta, \pi)}$$

(iii) The polarization, $P_f$, arising from the scattering of a polarized incident beam from an unpolarized target is given by

$$P_f = \frac{I_0(\theta)}{I(\theta, \varphi)} \left[ (P + D P_\perp \cdot n) n + (A P_\perp \cdot k + R P_\perp \cdot n \times k) n \times k \right. + \left. (A' P_\perp \cdot k + R' P_\perp \cdot n \times k) k \right]$$

where $D, R, A, R'$ and $A'$ are functions of the scattering angle and energy and are called the five Wolfenstein triple-scattering parameters. They relate the final polarization to the initial polarization.

(iv) When the target is also polarized, the differential cross-section is given by

$$I = I_0 \left[ 1 + (P_\perp + P_T \cdot P_\perp + P_\perp P_T C_{nm} \right]$$

where $P_\perp$ is the polarization of the incident beam, $P_T$ is the target polarization and $C_{nm}$ is the spin correlation parameter.
Determination of these measurable quantities leads to the determination of $M$-matrix \(^{(37)}\).

1.4 Experimental Analysis

The lack of any precise theory of the basic N-N interaction means that it is not possible to reproduce the experimentally measured quantities with sufficient accuracy and this situation has led to a phenomenological reduction of data in the form of parameters. The parameterization is performed within the framework of quantum mechanics and justifiable physical assumptions. They form the basis for eventual attempts to explain the nucleon-nucleon interaction in terms of more fundamental concepts and also helps in the planning of further experiments so as to determine the matrix of the interaction uniquely. One such phenomenological parameterization is the phase-shift analysis. Nucleon-nucleon scattering data is reduced to phase-shifts of the various angular momentum states allowed by the nucleon-nucleon interaction and a mixing parameter which couples two states of same angular momentum and parity. The angular momentum states allowed for a two nucleon system is

\[
\begin{align*}
1S_0, & \; 3S_1, \; 1P_1, \; 3P_0,1,2, \; 1D_2, \; 3D_{1,2,3}, \; \ldots, \; \text{etc.}
\end{align*}
\]

where the notation is \(2S + 1L_J\). \(S\) is the total spin of the two nucleons, \(L\) is the relative orbital angular momentum in the spectroscopic notation and \(J\) is the total angular momentum. In the isospin formalism the Pauli principle requires that the \(T = 1\) states should be symmetric in space-spin co-ordinates while the \(T = 0\) states are anti-symmetric. The \((p,p)\) and \((n,n)\) systems have only \(T = 1\)
states while both $T = 0$ and $T = 1$ states can occur for the $(n,p)$ system. The first few allowed states are

\[
T = 1 : ^1S_0, ^3P_0, 1, 2, ^1D_2, \ldots \ldots
\]

\[
T = 0 : ^3S_1, ^1P_1, ^3D_1, 2, 3 \ldots \ldots
\]

The presence of the tensor force in the two-nucleon interaction has the effect that the total angular momentum, $J$, is conserved but the total orbital angular momentum, $L$, is not. Hence, there is mixing between states of same $J$ but different $L$. Conservation of parity requires that the mixing of states such as $^1S_0$ and $^3P_0$ is not allowed, while mixing of states such as $^3P_2$ and $^3P_2$ is allowed and can be associated with a mixing parameter $\epsilon_2$.

The presence of Coulomb interaction in the $(P,P)$ scattering can be incorporated in a simple manner if one uses a representation called the "bar" phase-shifts and mixing parameters. In this representation the bar phase-shifts can be written as a sum of nuclear and Coulomb parts. The charge independence means that the nuclear part of the bar phases and mixing parameters for $(P,P)$ scattering are equal to the $T = 1$ bar phases for $(n,p)$ scattering.

The phase-shift analysis incorporates all the symmetry principles satisfied by the scattering matrix $M$ as given by equation (1) and, hence, the two representations are equivalent. The invariance principles like rotation invariance, reflection invariance are accounted for by requiring that the total angular momentum and parity are conserved and invariance under time-reversal is assumed by the
definition that a single parameter \( \epsilon_2 (\epsilon_4, \epsilon_6 \ldots \) etc.) describes the mixing of the pairs of states \( 3P_2 \) and \( 3F_2 \) (\( 3P_4 \) and \( 3H_4 \) etc.)\(^{(32)}\). Unitarity is satisfied by requiring that the phase-shifts are real. A further physical knowledge namely that the nuclear forces are of short-range, is usually incorporated by terminating the number of partial waves contributing in phase-shift analysis to be determined by experiment at some fixed value \( (L_{\text{max}}) \) and substituting the phase-shifts given by an exchange of one pion\(^{(33)}\). This is termed the modified phase-shift analysis (see Moravcsik\(^{(11)}\)).

From the relations between the observables and the co-efficients of the \( M \)-matrix\(^{(11)}\) and the relations between the co-efficients and the phase-shifts can be established. However, this relationship is complicated and, further, the number of partial waves required is infinite, in the general case, rendering an analytical deduction very complicated. In principle, a \( \chi^2 \) fit to the experimental data is done for the phase-shift analysis. \( \chi^2 \) is defined as

\[
\chi^2 = \sum \left[ \frac{O_i - B_i(\delta)}{\sigma_i} \right]^2
\]

where \( O_i \) is the measured value of a quantity, \( B_i(\delta) \) is the value of the quantity calculated from the phase-shift set \( \delta \) and \( \sigma_i \) is the experimental error associated with the measurement of \( O_i \). The sum extends over all the experimental data. Phase-shifts corresponding to a large relative orbital angular momentum \( l > L_{\text{max}} \) are set at a value required by the one-pion-exchange, OPE, model\(^{(33)}\) and phases with small \( l \leq L_{\text{max}} \) are adjusted to give a best \( \chi^2 \) fit to the
experimental data. The pion-nucleon coupling constant may also be taken as a parameter in the search procedure. Above the inelastic threshold allowance for the relaxation of the unitarity restriction has to be made. The most recent phase-shift analysis are by Signell\(^{(34)}\) and MacGregor et al\(^{(35)}\). Briefly, the situation is as follows.

(i) The phase-shifts for \((P,P)\) scattering are well determined up to 350 MeV. Assuming charge independence this gives the \(T = 1\) part of the scattering matrix.

(ii) The \((n,p)\) scattering data is not complete enough to be analyzed by itself. Taking the \(T = 1\) phase-shifts from the analysis of \((P,P)\) scattering data, the \(T = 0\) phase-shifts can be obtained from \((n,p)\) data. These \(T = 0\) phases are not as accurate as the \(T = 1\) phases.

(iii) The best value of the OPE coupling constant is \(g^2 = 14 \pm 2\) \(^{(35)}\) in good agreement with the value found from pion-nucleon scattering experiments, enhancing confidence in the assumption of charge independence.

1.5 Theoretical Analysis

The hypothesis advanced by Yukawa some 25 years ago led to the general belief that the forces between nucleons have their origin in meson exchange processes. Tests of the validity of such ideas can be achieved along two lines; the potential theory and the dispersion relations.

1.5.1 Potential Model

The representation of the nucleon-nucleon interaction in terms of a potential is essentially historic in origin. The success in
describing the electro-magnetic and gravitational forces through a potential led to a natural intuition of representing the two nucleon interaction by a potential. Potentials derived from the meson theory and phenomenological potentials formulated under certain symmetry restrictions are inserted in the two-body Schrödinger equation to yield phase-shift parameters. These parameters can then be compared with those obtained from experimental phase-shift analyses to determine the validity of the model.

A most general phenomenological potential\(^{(11)}\) satisfying the customary symmetry principles of charge independence, time reversal and parity, has the form

\[
V = V_{oi} + V_{sc} \sigma_1 \cdot \sigma_2 + V_t \left[ \frac{3(\sigma_1 \cdot r)(\sigma_2 \cdot r) - (\sigma_1 \cdot \sigma_2)}{r^2} \right] + V_{ls}(\sigma_1 + \sigma_2) \cdot \mathbf{r} \cdot \mathbf{p} + V_q \left[ (\sigma_1 + \sigma_2) \cdot \mathbf{r} \cdot \mathbf{p} \right]^2,
\]

where \(V_{oi}\) is the spin independent central potential, \(V_{sc}\) is the spin dependent central potential, \(V_t\) is the tensor potential, \(V_{ls}\) is the spin-orbit potential and \(V_q\) is the quadratic spin-orbit potential. The most recent phenomenological potential to fit the experimental data reasonably well, is the Hamada-Johnston\(^{(20)}\) potential which reduces to a one-pion potential at large distances and has a repulsive core at small distances.

A potential is also obtained from the meson theory. Various coupling schemes between the meson and the nucleon fields can be assumed leading to differing potentials. One of the most significant contributions from meson theory is that the long range part of
the two nucleon interaction is dominated by the lightest meson, the pion, the intermediate region has contributions from the exchanges of two pions (and possibly exchanges of heavier mesons may contribute) while the innermost region is dominated by heavy meson exchanges.

Meson theory has been a complete success in the outermost region where one-pion-exchange, OPE, process dominates but only has had a partial success for the intermediate region. (For a list of potentials derived under various coupling models see page 93 of reference 11.) Semi-phenomenological potentials have, thus, been derived, incorporating the partial success of pion fields, to explain the experimental data. One of these is the inclusion of a phenomenological spin-orbit potential \( \text{Signell}^{34} \) which improved the experimental fit. Another is the boundary condition model in which one- and two-pion exchange potentials are taken from meson theory and added to a boundary condition at around 0.7 Fermi \( ^36 \).

Disregarding the partial success of a potential model, a potential is essential for nuclear physics calculations as "off the energy shell" properties are involved. However, for nucleon-nucleon scattering problems, the whole philosophy of a potential can be in doubt as a potential is essentially a non-relativistic concept and, consequently, a more direct approach of using dispersion relations may be much more useful.

Recently, it has become common to calculate the phase-shifts directly by dispersion theory where relativistic effects can more easily be incorporated. This theory deals directly with the scattering amplitudes and is based on the three properties of
causality, unitarity and the crossing symmetry\(^{(11)}\). The scattering amplitudes are expressed in terms of an analytic function of a variable in the complex energy plane. The singularities of this amplitude corresponds to two nucleon intermediate states. The near singularities correspond to the one-pion-exchange process, while the more distant singularities correspond to multi-pion and heavy meson exchange processes. Again, as in the potential model, the one-pion contribution is determined accurately. A few other processes are determined directly from the experimental data. However, the calculations appear to be prohibitively difficult and little progress has been made along these lines in recent years.
PROTON SCATTERING FROM LIGHT NUCLEI

1. Introduction

One of the main problems of nuclear theory is to deduce the properties of the atomic nuclei from the nucleon interactions. Partial success has been achieved in the calculations of nuclear binding energies and saturation using phenomenological Hamada-Johnston, Reid and Bressel-kernal potentials. Hartree-Fock and shell model calculations have also been performed using nucleon-nucleon potentials, giving reasonable agreement with experimentally observed energy levels. Most realistic two nucleon potentials are tested by fitting only on-energy-shell scattering amplitudes as obtained from nucleon-nucleon scatterings. They do predict, however, a little about off-energy-shell behaviour as they normally have the one-pion-exchange character at large distances as can be seen from the fact that most potentials fit deuteron properties well. To obtain precise information about nuclear properties off-energy shell amplitudes will have to be known properly.

In contrast to the bound properties of a nucleus the continuum properties (i.e. scattering) of the nucleus can be described reasonably well using nucleon-nucleon phase-shifter parameters. Originally, Bethe performed an analysis of the small angle scattering and polarization of 313-MeV protons by Carbon using proton-proton scattering phase-shifts of Stapp et al. and the neutron-proton phase-shifts of Garnet and Thaler and obtained quantitative agreement with experimental data. His intention was to try to select a unique set of phase-shifts from the five different but equivalent
phase-shifts to the nucleon-nucleon problem as found by staff, but
found that in fact all solutions gave essentially the same agreement.
At the present time, however, a unique set of phase-shifts for
proton-proton and proton-neutron has been obtained from the analysis
of nucleon-nucleon scattering data alone.

In principle, the effective two particle interaction for nucleon-
nucleus scattering can be derived from the nucleon-nucleon interaction. Following Kerman, McManus and Thaler such an effective potential
can be expressed in terms of a multiple-scattering expansion where the
first term represents single and multiple scatterings not involving
virtual intermediate states, the second term involves scattering to
one virtual intermediate state and so on. Scattering to virtual
intermediate states requires a knowledge of correlations between
nucleons and these are not well known. In practice, one makes multiple-
scattering approximations, which is to say, ignore contributions from
all virtual intermediate states scattering. Under this approximation
the theory is accurate only for the forward scattering angles. For
high energies a further approximation is normally made, namely the
impulse approximation, first devised by Chew, where the two nucleon
interaction inside the nucleus is replaced by free two nucleon inter-
action; that is, the binding energy of the nucleons is ignored. Under
these approximations one could describe the nucleon-nucleus scattering
through a simple Born-approximation calculation or through the w.k.B.
approximation where a partial account of multiple-scattering involving
only the ground state as an intermediate state is taken or by solving
the Schrodinger equation which takes a complete account of multiple
scattering involving only the ground state as an intermediate state.
In each case, however, scattering to intermediate excited states is ignored through the virtual "multiple-scattering" Approximation.

For proton-nucleus scattering a further complication arises because of the long range of Coulomb force. The calculations of the potential assume that the forces involved are short-ranged. The Coulomb force is not. The question is then whether it is acceptable to split the nuclear and the Coulomb forces and treat them separately. Bethe\(^{(12)}\) has demonstrated that at high energies the individual Coulomb phase-shifts are approximately the same for all partial waves and hence, may be taken out of the expansion for the total nuclear amplitude and be treated as a multiplying phase-factor. Under these circumstances the total nuclear amplitude can be built up from the nucleon-nucleon interaction and the Coulomb interaction. It is one of the purposes of this work to discover whether Coulomb interaction can indeed be treated in the manner proposed by Bethe. The complication is not so obvious if one builds up an effective proton-nucleus potential from the two-nucleon interaction first and then substitutes this potential in the Schrodinger equation or an equivalent framework for describing the nucleon-nucleus scattering. The Coulomb force can then be included in the usual manner.

Early attempts were made by Bethe\(^{(12)}\) and others\(^{(43)}\) to obtain the nucleon-nucleus scattering amplitudes in the forward direction using proton-nucleus data in the Coulomb interference region. Later Cromer and Palmieri\(^{(18)}\) performed a more comprehensive analysis on proton scattering in the Coulomb interference region at 140 MeV for \(\text{He}^4\), \(\text{Be}^9\), \(\text{C}^{12}\) and \(\text{Al}^{27}\) to obtain the scattering amplitudes. The neutron differential cross-section and polarization data\(^{(44)}\) were
also included in the analysis as available. The high energy scattering approximation was then used to determine the central and spin-orbit optical potential parameters from these amplitudes and these were related to the nucleon-nucleon scattering amplitudes.

In the present work differential cross-sections for proton scattering in the angular range $2^\circ$ to $20^\circ$ in the Laboratory system have been measured from $^2\text{D}$, $^4\text{He}$, $^6\text{Li}$, $^7\text{Li}$ and $^{12}\text{C}$. An analysis of these data has been made including the polarization data from other workers$^{(45)}$. The total absorption cross-sections for protons have also been included in the analysis (where available) through the optical theorem. The analysis was made as follows:

i) The Born Approximation scattering amplitudes were calculated from the two-nucleon scattering matrix and compared with the data.

ii) The spin-independent forward scattering amplitudes were then calculated in the W.K.B. Approximation and were used to renormalize the B.A. scattering amplitudes, but retaining the B.A. angular dependence. The reason for this is that W.K.B. Approximation takes a partial account of multiple-scattering, hence, is a better approximation.

iii) Finally, the optical model potential was constructed from the nucleon scattering amplitudes through the relations given by Kerman, McManus and Thaler$^{(15)}$ and an optical model prediction was made by solving the Schrödinger equation exactly. The parameters of the optical-model were then allowed to vary and a $\chi^2$ fit was made to the data. These parameters were then related to the two-nucleon amplitudes and were compared with the values obtained by Cromer and Palmieri and the values obtained by J. Perring$^{(46)}$, from the analysis of proton-proton scattering.
2.2 Theory

2.2.1 The scattering amplitude

The scattering amplitude is defined through the asymptotic wave-function representing the nucleon-nucleus scattering. The asymptotic wave function can be written as:

$$\psi(k,r) = e^{ik\cdot x} + f(\theta) \frac{e^{ikr}}{r}$$

where the first term represents the incident wave with momentum $k$ in centre-of-mass system and the second term represents an outgoing spherical wave. $f(\theta)$ is the scattering amplitude and the differential cross-section is given by:

$$\frac{d\sigma}{d\Omega} = |f(\theta)|^2$$

For the scattering of a spin-$\frac{1}{2}$ particle from an even-even nucleus with $J=0$ and $T=0$, the scattering amplitude can be written in terms of momentum transfer defined by:

$$q = 2k \sin \theta/2$$

as

$$f(q) = g(q) + h(q) \sigma \cdot n$$

where $\sigma$ is the Pauli spin matrix and $n$ is a unit vector in the direction $k_i \times k_f$. Here $k_i$ and $k_f$ are the initial and final momentum of the scattered nucleon and $k = k_i = k_f$ for elastic scattering.

The differential cross-section, polarization and the total cross-section can be written as:

$$\frac{d\sigma}{d\Omega} = |g|^2 + |h|^2$$

$$P(\theta) = 2 \text{ Re} (g^*h) / \left[ |g|^2 + |h|^2 \right]$$

and

$$\sigma_T = \frac{4\pi}{k} \text{ Im} \left[ g(\theta) \right]$$

from the optical theorem.
2.2.2 Coulomb Interaction

In the presence of the Coulomb field, the scattering amplitude can be written as

\[ f(q) = f_c(q) + \chi(q) f_n(q) \]  \hspace{0.5cm} (2.5)

where \( f_c(q) \) is the pure Coulomb scattering amplitude, \( f_n(q) \) is the nuclear scattering amplitude and \( \chi(q) \) is the modification to \( f_n(q) \) due to the presence of the Coulomb field.

For the purpose of building up the nuclear scattering amplitude from the scattering amplitudes for two-nucleon scattering contributions by individual nucleons of the target nucleus, it is convenient to separate the Coulomb scattering from the nuclear scattering amplitude. As mentioned earlier Bethe has shown that at high energies the modification \( \chi(q) \) is approximately constant for all partial waves and hence, only represents a multiplicative phase-factor. As one is interested only in relative phases, similar physical properties can be obtained by modifying the Coulomb scattering amplitude \( f_c(q) \). The results given by Bethe are written for \( f_c(q) = g_c(q) + h_c(q) \) and \( \chi(q) \) of formula 2.5 as

\[ g_c(q) = -\frac{2nk}{q^2} \exp \left[ 2 \ln \left( \frac{2k}{q} - 0.577 \right) \right] F(q^2) \]

\[ h_c(q) = -\frac{4k^2}{2m^2} (\mu - \frac{1}{2}) g_c(q) \]

and

\[ \chi(q) = \exp \left[ 2 \ln \left( \frac{\ln ka_c + 0.058}{\ln a_c + 0.058} \right) \right] \]

where \( m \) is the rest mass of the incident proton, \( \mu \) is the magnetic moment, \( n = Z/137 \beta \), \( \beta \) is the relative velocity of the incident proton in the laboratory system and \( a_c \) is the radius parameter obtained.
from the electron-nucleus scattering data. In the deviation of the above formulae, the Coulomb phase-shifts were obtained by a W.K.B. Approximation, corrected for relativistic effects, from a point charge distribution. The effect of an extended charge distribution is taken into account by incorporating the Coulomb form factor of the nucleus $P_c(q^2)$.

In the Optical model analysis, the Coulomb interaction is taken into account in the usual manner. The Coulomb scattering amplitude is now due to a point charge distribution

$$f_c(q) = -n \exp \left[ -i \ln \left( \sin^2 \frac{\theta}{2} \right) + 2 i \cdot \sigma_c^o \right] / 2k \sin^2 \frac{\theta}{2}$$

and the modified nuclear scattering amplitude is

$$f_n(q) = \frac{1}{2ik} \sum (2l+1) e^{2i\sigma_n^l} (e^{2i\delta_n^l} - 1) P_n(\cos \theta)$$

where $\sigma_n^l$ is the pure Coulomb point-charge phase-shift and $\delta_n^l$ is the nuclear phase-shift and contains modification due to the actual extended Coulomb-charge distribution. $f_n(q)$ now is obviously not the same for protons and neutrons.

### 2.2.3 Born Approximation

Following Kerman, McManus and Thaler\(^{(15)}\), and using the impulse approximation one obtains the Born Approximation and the expression for the nuclear scattering amplitude is

$$f_{nB}(q) = N^{(k/k_0)} \bar{M}(q) F(q) \quad ...... (2.6)$$

where the suffix B stands for B.A., $N$ is the total number of nucleons in the nucleus, $k$ is the momentum in the nucleon-nucleus centre-of-mass system, $k_0$ is the momentum in the nucleon-nucleon centre-of-mass,
\( M(q) \) is the average of the two nucleon scattering matrix over all the isotopic spin states of the target nucleons, \( F(q) \) is the nuclear form factor, and \( k \) and \( k_0 \) are related by equating the momentum transfer in the two reference systems

\[
2k \sin \theta/2 = 2 k_0 \sin \theta_{0/2}
\]

The isotopic spin average values of the coefficients of the \( M \)-matrix are given as

\[
\bar{A} = \frac{1}{2} (3A_1 + A_0) + \frac{N - 2Z}{4N} (A_0 - A_1)
\]

\( \bar{B}, \bar{C}, \bar{D} \) and \( \bar{F} \) are obtained from similar expressions.

For even-even nuclei all coefficients linear in target spin cancel out leaving a simple expression

\[
\overline{M}(q) = \overline{A}(q) + \overline{C}(q) \sigma \cdot n
\]

The differential cross-section is given by

\[
\frac{d\sigma}{d\Omega} = |f_B|^2
\]

where \( f_B(q) = f_C(q) + f_{nB}(q) \) for proton scattering and

\[
f_B(q) = f_{nB}(q) \quad \text{for neutron scattering.}
\]

For the special case of the deuteron with spin 1, the terms involving the target spin do not average to zero one obtains \( (68) \)

-24-
for neutron scattering

\[ \frac{d\sigma}{d\Omega} = N^2 \left( \frac{k}{k_0} \right)^2 \left| \hat{M}(q) \right|^2 |F(q)|^2 \]

and

\[ P = 2 \text{Re} \left| \overline{A} \ast \overline{C} + \frac{2}{3} \overline{B} \ast \overline{C} \right| / \left| \hat{M} \right|^2 \]

with

\[ \left| \hat{M} \right|^2 = \left| \overline{A} \right|^2 + \left| \overline{C} \right|^2 + \frac{2}{3} \left[ \left| \overline{B} \right|^2 + \left| \overline{C} \right|^2 + \left| \overline{E} \right|^2 + |F|^2 \right] \]

Similar non-vanishing expressions for the nuclei Li\(^6\), Li\(^7\) and Be\(^9\) exist for the terms dependent on target spin. The contribution from these terms is less than 2% for momentum transfer \( q \leq 2 \text{ fm}^{-1} \) and is neglected in comparison with the experimental uncertainties.

### 2.2.4 Form Factors

The form factors used in above expressions are well described by an expansion such as

\[ F(q) = \frac{4\pi}{q} \int_0^\infty \rho(r)r \sin qr \, dr = 1 - \frac{1}{6} q^2 < r^2 > \]

For small angle scattering that we are concerned in this work \( q \) is small and hence, one can use simple gaussian density distribution

\[ \rho(r) = \frac{e^{- (r/a)^2}}{\pi^{3/2} a^3} \]

which yields

\[ F(q) = \exp \left\{ - \frac{1}{4} q^2 a^2 \right\} \]

where \( a^2 = 2/3 < r^2 >_e \) is the radius parameter to be determined from the electron scattering values for \( < r^2 >_e \). When shell-model wavefunctions are used for analysing electron scattering data, finite electromagnetic size of the proton has to be included to obtain \( < r^2 >_e \) \((47)\).
The mean-square radii for nuclear matter distribution \( \langle r_N^2 \rangle \) and effective charge distribution \( \langle r_{c,\text{eff}}^2 \rangle \) for the scattering of a proton is given by

\[
\langle r_N^2 \rangle = \langle r_e^2 \rangle - \langle r_p^2 \rangle_e
\]

and

\[
\langle r_{c,\text{eff}}^2 \rangle = \langle r_e^2 \rangle + \langle r_p^2 \rangle_e
\]

where \( \langle r_p^2 \rangle_e \) is the mean-square radius for proton charge distribution as obtained from electron scattering experiments.

When form factors are obtained from shell-model wave-function a correction by a factor \( \exp\left(\frac{q^2a^2}{4A}\right) \) has to be applied in order to take account of the fact that the origin of the coordinates of the shell model is not the same as the centre-of-mass \( (47) \). Making this correction the r.m.s. radii used in this calculation are given in Table IX.

### 2.2.5 W.K.B. Calculations

Bethe \( (12) \) has performed W.K.B. calculations for the spin-independent scattering amplitude. He assumed that the difference between \( \delta_1^+ \) and \( \delta_1^- \) for the phase-shifts of the partial waves \( l, j = l + \frac{1}{2} \) and \( j = l - \frac{1}{2} \) respectively, is small so that they can be represented by an average \( \delta_1', A \).

Using the W.K.B. approximation he evaluates \( \delta_1', A \) from

\[
\delta_1', A = \frac{2\pi Q_E}{k} \int_0^\infty \frac{\delta}{\left(\frac{2}{y} - y^2\right)^{\frac{1}{2}}} \rho(r) \, dr
\]

where \( y = (l + \frac{1}{2})/k \) and \( Q = q_B(0) \), the spin-independent forward Born scattering amplitude.
He then replaces the summation over all partial waves for the spin-independent scattering amplitude $g$ by an integral and arrives at the final result

$$
g(\phi) = \left( \frac{N}{N-1} \right)^{\frac{1}{2}} \frac{i k a^2}{\gamma^{\frac{1}{n}} \exp \left( -\frac{x}{n} \right)} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n} \gamma^{n} \exp \left( -\frac{x}{n} \right)
$$

where

$$\gamma = -2i \left( \frac{N-1}{N} \right) \frac{k a^2}{\gamma}$$

and

$$x = \frac{1}{4} Q^2$$

\[ (2.9) \]

2.6 Optical Model Calculations

For a spin-zero nucleus, the optical potential is normally written as

$$V(r) = V_{\text{coul}}(r) + V_{c}(r) + \frac{1}{r} \frac{d}{dr} V_{s}(r)$$

where $V_{\text{coul}}(r)$ is the Coulomb potential. Kerman, MacManus and Thaler desired a relationship between the central potential $V_{c}(r)$ and the spin-orbit potential $V_{s}(r)$ and the two nucleon amplitude. The relations are

$$V_{c}(r) = -\frac{\hbar^2}{E_L} (N-1) \frac{k_L}{k_0} \frac{1}{(2\pi)^2} \int \exp \left( i \mathbf{q} \cdot \mathbf{r} \right) F(q) \tilde{C}(q)$$

and

$$V_{s}(r) = \frac{\hbar^2}{E_L} (N-1) \frac{k_L}{k_0} \frac{1}{(2\pi)^2} \frac{\mu c}{n} \left( \frac{\mu c}{n} \right)^2 \int \exp \left( i \mathbf{q} \cdot \mathbf{r} \right) F(q) \tilde{C}(q)$$

\[ (2.10) \]

where $\tilde{C}(q)$ is related to $\bar{C}(q)$ by

$$\tilde{C}(q) = \frac{\bar{C}}{k_2 \sin \theta}$$

and $E_L$ is the total relativistic energy of the incident nucleon in the laboratory frame.
Taking \( F(q) = e^{-\frac{4}{3} a_q^2 q^2} \) and parametrizing \( \overline{A}(q) \) and \( \overline{c}(q) \) as

\[
\overline{A}(q) = \overline{A}_r(0) \exp \left( -\frac{1}{3} a_1^2 q^2 \right) + i \overline{A}_i(0) \exp \left( -\frac{1}{3} a_2^2 q^2 \right)
\]

and

\[
\overline{c}(q) = \overline{c}_r(0) \exp \left( -\frac{1}{3} a_3^2 q^2 \right) + i \overline{c}_i(0) \exp \left( -\frac{1}{3} a_4^2 q^2 \right)
\]

The integrals can readily be evaluated analytically to give

\[
V_{cr} = \frac{2h^2}{\pi^2 E_L} \frac{k_L}{k_0} \left( \frac{N-1}{a_1^3} \right) \overline{A}_r(0) \exp \left( -\frac{r^2}{a_1^2} \right)
\]

\[
V_{ci} = \frac{2h^2}{\pi^2 E_L} \frac{k_L}{k_0} \left( \frac{N-1}{a_2^3} \right) \overline{A}_i(0) \exp \left( -\frac{r^2}{a_2^2} \right)
\]

\[
V_{sr} = \frac{2h^2}{\pi^2 E_L} \frac{k_L}{k_0} \left( \frac{N-1}{a_3^3} \right) \left( \frac{\mu e}{\hbar} \right)^2 \overline{c}_i(0) \exp \left( -\frac{r^2}{a_3^2} \right)
\]

and

\[
V_{si} = -\frac{2h^2}{\pi^2 E_L} \frac{k_L}{k_0} \left( \frac{N-1}{a_4^3} \right) \left( \frac{\mu e}{\hbar} \right)^2 \overline{c}_r(0) \exp \left( -\frac{r^2}{a_4^2} \right)
\]

\[\ldots (2.11)\]

The radii \( a_i \) are given by

\[ a_i^2 = a_n^2 + a_i^2 \text{ for } i = 1 \text{ to } 4 \]

Following Wilson (74)

\[ a_n^2 = \frac{2}{3} \left[ \langle r_c^2 \rangle - \langle r_p^2 \rangle \right] \]

where \( r_c \) is the charge radius of nucleus and \( r_p \) is the proton radius from electron scattering.
The above expression for $V(r)$ has been corrected by a factor \( \frac{m}{2E} \frac{k_L}{k_0} \) found by Kerman, McManus and Thaler for the use of the Schrödinger equation at relativistic energies. Elton\(^{(48)}\) also arrives at a similar expression with which the potential has to be multiplied when solving a Schrödinger equation rather than Klein-Gordon or Dirac equation and shows that the wave number appearing in the Schrödinger equation should also be relativistic.

The Coulomb term appearing in the optical potential was represented by an expression

\[
V_{\text{coul}}(r) = \frac{Ze^2}{r} \exp\left(\frac{r}{a}\right)
\]

which is appropriate for a Gaussian charge distribution\(^{(42)}\) and thus is similar to the distributions appearing above for the nuclear optical potential.
Chapter 3

EXPERIMENTAL PROCEDURE

3.1 General Consideration

Elastic differential cross-sections were measured for the scattering of approximately 144 MeV incident energy protons in the Laboratory system from hydrogen, deuterium, helium, lithium-6, lithium-7 and carbon-12. A previous experiment was carried out by Cox et al. (1), hereafter referred to as CEZJR, who measured the differential cross-section and polarization of 143 MeV protons scattered from hydrogen in the angular range extending down to 8° in laboratory system. The present work is in essence an extension of the differential cross-section measurements of CEZJR. The measurements were taken in the angular range between 2° to 20° in laboratory system for proton scattering from hydrogen and other light nuclei.

The Harwell synchrocyclotron can now provide a beam of 160 MeV protons with an intensity of about 30 nA as a result of the recent installation of a magnetic regenerative extraction system (50). This represents a considerable improvement in intensity over CEZJR (of about \(10^8\) protons per sec). However, the beam energy is also higher and, therefore, to make the hydrogen differential cross-section measured in this work compatible with CEZJR, the average beam energy was reduced to 144 MeV by degradation in 2.4 gm/cm² of polythene. The degradation, however, increased the beam divergence due to Coulomb multiple scattering in polythene. Use of bending magnets and collimation at sensible places gave the final beam spot at the target position of 1.0 cm in diameter with a beam divergence of 0.2 degrees. Large losses in beam intensity resulted as a consequence of collimation.
and losses in the degradation process. The final intensity was 3 nA. with the use of an additional "cee" electrode, recently incorporated (51), which takes up the last stage of acceleration of the protons from the 'D' and stretches out the otherwise pulsed beam, the final beam intensity was 0.5 nA with a duty-cycle of about 20%. This repre­

sented an acceptable count rate of about 10,000 per second at 1.75° laboratory system, for proton scattering from hydrogen without saturating the electronics.

Two forms of hydrogen targets (whose development is described in the next chapter) were used. One was a solid self-supporting cylindrical target of diameter 1 cm and length 6 cm. This target had no material other than hydrogen in the path of the beam. The other target was made of solid hydrogen contained in a pod of diameter 5 cm and thickness 2.0 cm. The faces of the pod were araldited with 0.025 mm thick mylor film. Targets of deuterium and helium used similar sized pods: these pods had 0.010 mm and 0.050 mm mylar windows for solidified deuterium and liquid helium respectively. Targets of lithium-6 and lithium-7 used were 288 mg/cm² thick 95% lithium-6 and 2.12 mg/cm² thick 92.6% lithium-7 while two targets of carbon-12 of thickness 118 mg/cm² and 562 mg/cm² were used. The thin carbon-12 target was used for small angle measurements where multiple Coulomb scattering is appreciably large while the thicker target was used for larger angles to give acceptable statistics.

In practice it was found that the solid self-supporting cylinder of hydrogen was irregular in shape while the targets of solid hydrogen and deuterium and liquid helium in the pod were of a shape whose volume was difficult to calculate. This led to the problem of establishing
the amount of these materials in the path of the beam and hence made the computation of absolute cross-sections difficult. Thus, for these cryogenic targets only relative differential cross-section measurements were made and the large angle data in the literature were used for normalization purposes. Absolute cross-sections were computed for the case of lithium-6, lithium-7 and carbon-12.

3.2 Experimental Layout

The general layout is shown in figure 2. The extracted beam was bent through $15^\circ$ and was directed into stable II. The first pair of quadrupoles focussed the beam onto a polythene degrader and the second pair re-focused the beam onto the target. The beam was collimated by a 6 cm long brass collimator with 2 cm circular aperture placed at the centre of the second pair of quadrupoles. This enabled the protons with only small divergence to reach the target. Two more collimators were used downstream to act as baffles to the scattered protons from the defining collimator. The beam was guided along inside an aluminium tube of diameter 10 cm. This aluminium tube was coupled onto a scattering vacuum tank. The latter had one 20 cm hole for the cryostat and two insertions on either side of the beam making an angle of $44^\circ$ with the beam to carry the $(P-2P)$ monitor. The scattering vacuum tank and the beam pipe were both pumped down to a vacuum of about $10^{-5}$ mm of Hg with the cryostat warm and about $10^{-6}$ mm of Hg with the cryostat cooled to liquid helium temperature. With this arrangement a fairly sharp beam spot was obtained at the target position. The down-stream face of the scattering vacuum tank was closed with a large 0.050 mm thick mylar window.
Beneath the scattering vacuum tank was placed a steel platform (of diameter 4 feet) onto which two scattering arms were attached. The steel platform was so placed that the axis of rotation of the scattering arms was directly beneath the centre of the target. The scattering arms were rotated along the steel platform by stepping motors from a controlling unit placed in the counting room. The rotations of the arms could be initiated from the unit. The smallest rotation that could be achieved with the unit was $\frac{1}{100}$ of a degree. A digital shaft encoder was also attached to the scattering arms and indicated the position of the arms in terms of degrees. The smallest unit read by the encoder was $\frac{1}{20}$ of a degree. This remote control of the scattering arms proved invaluable for experiments with the solid self-supporting hydrogen target whose life time was between 2 to 3 hours.

3.3 The Detecting Telescope

The detection of scattered protons was done by means of a telescope consisting of a series of plastic scintillators. Each scattering arm carried four scintillators numbered 1, 2, 3 and 4 for the right arm and 1', 2', 3' and 4' (see figure 2) for the left arm. The two telescopes were, in principle, identical and most of what follows applies equally to both. The two combinations 124 and 134 were operated in coincidence. The scintillators 2 and 3 defined two solid angles. The solid angle defined by 2 was chosen to be approximately twice the solid angle defined by 3. The purpose of this was to enable a check for edge effects in the scintillators to be made and to check the uniformity of the sensitivity of the scintillators. The constancy of the ratio $3/2$ over the range of scattering angles investigated in the experiment afforded a useful check on the proper functioning of
FIG. 3  BLOCKDIAGRAM OF COUNTING ELECTRONICS
the apparatus. The size of the defining counter 3 was chosen so as to give a good angular resolution and was taken to subtend an angle of 0.20 degrees in the horizontal direction. The counter 1 was used to limit the region from which the telescope detected the scattered protons. Its size was chosen to be such that the protons scattered from the collimators and the material of the pod used to contain solidified targets were not viewed by the telescope. Counter 4 was used to enable energy discrimination on the scattered protons to be made by placing a copper absorber between it and the defining counters. It was fairly large and was placed as close as possible to the defining counters in order to limit losses due to the elastic scattering in the absorber and the defining counters.

3.4 Alignment of the Apparatus

The alignment of the apparatus was done with the help of a laser beam. The pencil beam was passed through the centre of the carefully aligned beam pipe and the scattering vacuum tank, the scintillators and the Faraday cup were adjusted so that the laser beam went through the centre. Thus the proton beam axis was defined and any deviations of the proton beam from this direction was off-set by the use of switching magnets.

3.5 The Electronics

The counting electronics consisted of standard Harwell units. The block diagram for the electronic arrangement is shown in figure 3. The outputs from the photo-multiplier tubes were fed directly into 10 MHz discriminators via delay boxes. The outputs from these discriminators were then fed to double and triple coincidence units and finally into 10 MHz scalars to count the number of 12, 13, 124
and 134 coincidences (and similarly for the primed arm). The resolving time of the coincidences was set at about 24 ns. Randoms were measured by taking the coincidences of 12 and 13 with a 50 ns (the mean period of the cyclotron Rf) delayed 4. The randoms were less than 1% of the true rate.

3.6 The Beam Monitoring

The beam was monitored by a Faraday cup (which contained a cylindrical graphite beam dump 10 cm in diameter and about 20 cms in length, under vacuum within a 12 cm diameter Faraday cup with a 0.025 mm mylar entrance window to the cup). The graphite was capable of stopping all the protons in the beam and the associated charged particles induced by the reactions of protons with the graphite. The charge deposited by the beam in graphite was integrated by a standard unit and a pulse was generated after a charge of $10^{-8}$, $10^{-9}$, $10^{-10}$ or $10^{-11}$ coulombs had been accumulated as desired. These pulses were used to control the counting electronics.

An electrostatic guard ring was provided for the suppression of very low energy electrons emitted from the surface of the graphite. Putting -100 Volts on the ring changed the efficiency of the Faraday cup by about 1%. An electromagnet was also placed between the mylar window and the graphite to prevent the delta-rays emitted by the window from reaching the graphite. The effect of the electromagnet was to raise the apparent efficiency of the Faraday cup by about 2%.

The distance of the Faraday cup from the target was large so that the multiple coulomb scattering of the beam by the target and the air present between the target and the Faraday cup, resulted in a
fraction of the beam missing the Faraday cup. The situation was not serious for experiments with cryogenic targets as only relative differential cross-sections were sought. For lithium and carbon, the amount of beam lost had to be determined. This was done by placing another small portable Faraday cup close to the scattering vacuum tank face and performing the differential cross-section scan for large angles. Comparison showed that for the two lithium targets and for thin carbon targets about 2% of the beam was lost while for thick carbon target as much as 10% of the beam was lost.

The beam monitoring by Faraday cup for hydrogen experiment was found to be very poor. Both the self-supporting solid cylinder and the solid in the pod evaporated continuously. (We did a later pod run which did not evaporate and used both (p-2p) and Faraday cup as monitors for counting rates from hydrogen and Faraday cup to monitor backgrounds from the pod). Thus, the mass of hydrogen present at any time was not known. This difficulty was overcome by the use of a (p-2p) monitor consisting of two plastic scintillator counters placed at about 44° on either side of the beam. These counters recorded the scattered protons and the recoil protons from the target. The scintillator sizes chosen were such that one was about twice the area of the other and were such that a reasonable counting rate was achieved. Copper absorbers were placed in front of both the counters so that only protons with energy greater than 55 MeV gave (p-2p) coincidences. As both, the (p-2p) counts and the counts recorded by the telescope, are proportional to the mass of hydrogen present at that time, relative differential cross-sections could be calculated with respect to the (p-2p) counts.
3.7 The Mean Energy of the Experiment

The energy of the beam was determined by measuring a range curve with the detection telescope placed in the direct path of the beam, much reduced in intensity. The coincidences 134 were plotted against the amount of copper absorber placed in front of counter 4. A typical curve is shown in figure 4.

3.8 Zero-Angle-Scan

The position of the beam with respect to the telescope of the two scattering arms was obtained by scanning the scattering arms across the beam, again much reduced in intensity. The zero-angle was obtained by plotting the counting rate at the telescope versus the angle setting. A typical plot is shown in figure 5. No monitor was used for counting as the Faraday cup used for monitoring purposes was hidden by the telescope. The counting was done for a period of ten seconds for each angle setting and it was assumed that the machine conditions remained the same throughout the scan. As the positioning of the telescope was done remotely, the machine conditions were undisturbed. In fact, this is verified by the smooth curve for the zero angle scan in figure 5. The zero angles for the two arms were determined to $\pm 0.010$ and were close to the settings obtained with the laser beam. The effects of any small drifts in the beam direction were averaged by making all small angle counting measurements with the telescopes of both arms set at equal angles on each side of the beam and taking an average of the two telescopes. There was some error involved in taking the average of the two telescopes and this was taken as random for the final evaluation of the total uncertainty of the counts at each angle.
3.9 Attenuation in Absorber

The copper absorber produces attenuation of the scattered protons through nuclear reactions and by elastic scattering through angles wide enough to cause protons to miss counter 4. The loss of the protons in copper was determined experimentally by placing the detection telescope in the direct path of the beam suitably reduced in energy and intensity to simulate the scattering conditions. The ratio of triple coincidences $^{134}$ to double coincidences (i.e. $\frac{^{134}}{^{13}}$) was determined. The loss in copper was then given by the ratio $\frac{^{134}}{^{13}}$ Absorber In/$^{134}$ Absorber out subtracted from unity. This absorption loss was then used to correct the counting rates obtained with the use of the absorber.
Chapter 4

SOLID HYDROGEN TARGET

4.1 Introduction

Hydrogen targets have reached quite a degree of sophistication since the early days of gas targets. These gas targets were only useful for low energy scattering, and for higher energies, where the range of incident protons is large, targets with a high density of hydrogen became necessary. Early experiments were performed by scattering from polyethylene \((\text{CH}_2)^n\) targets. Here, however, the background of scattered particles from carbon had to be assessed separately by scattering from carbon. The hydrogen scattering may be increased relative to the carbon by taking advantage of the two-body kinematics which ensures that the protons scattered from the hydrogenous material have different energy from those scattered from carbon. Furthermore, the recoil protons from the hydrogenous material under \((p-2p)\) interaction process come out at a separation angle of approximately 90° to the scattered protons while those from the carbon come out at different separation angles which depend on their internal motions within the carbon nuclei. Thus, in \(p-p\) scattering two protons may be detected in coincidence at the appropriate separation angle or the energy of one proton may be accurately measured. Both these methods fail at small angles where the recoil proton has an energy too low to be detected or to influence the kinematics.

Because of the need to make measurements at small angles and to obtain a high density of hydrogen, liquid hydrogen targets are now universally used. Typically, in medium energy \(p-p\) scattering
experiments a liquid hydrogen target consists of an upright cylinder made of Mylar film, the upper and the lower ends being closed with metal plates which also carry connections to a liquid hydrogen reservoir. The hydrogen target is surrounded by radiation shields and the whole assembly is supported within a vacuum enclosure. It is desirable to keep the thickness of the container walls as thin as possible to cut down the background scattering from materials other than hydrogen. For small angles the differential cross-section is roughly proportional to the square of the atomic number of the material, and thus, even a small amount of material in the container walls (which is mainly carbon by weight and is usually less than 20 mg/cm²) gives rise to an appreciable background scattering.

The presence of non-hydrogenous material becomes much more important for experiments such as the study of bremsstrahlung production in nucleon-nucleon scattering where the detection of the protons in the reaction $(p,p)$ is obscured by the presence of the protons from the knock-out reactions $(p,2p)$ arising from the material of the container walls. The $(p,2p)$ cross-section for most elements is about two orders of magnitude greater than that for the $(p,p)$ process. Break-up of the deuteron $^2D(p,2p)n$ and the triton $^3H(p,2p)2n$ are further experiments where $(p,2p)$ events from the walls complicates the results.

It is, of course, possible to design counter telescopes so that they see only the hydrogen liquid inside the container and not the walls. There is, however, a drawback in the sense that the amount of hydrogen liquid in the direct path of the beam viewed by the telescope varies with the angle making the choice of geometry to
optimize counting rate difficult. Another method of target construction can be adopted in which a double container (57, 58) is used. The walls of the inner container can be made of very thin material (reference 83 uses 200 g/cm² aluminium foil) as it is required to withstand only the hydrostatic pressure of the liquid hydrogen that it contains. The isolation from the surrounding vacuum is provided by the outer container which has thick walls. This second container can be made as large as desired so that it is not viewed by the detecting telescope and the space between the outer and the inner containers is filled with hydrogen gas at such a pressure that it is in equilibrium with the liquid in the inner vessel. In this manner both the thickness of the target and the minimum angle which can be investigated are reduced.

However, neither of these techniques is suitable at very small scattering angles where the multiple Coulomb scattering becomes very important. The Coulomb multiple scattering depends strongly on the target thickness and in order to cut down this multiple scattering, the target thickness has to be reduced and this in turn increases the relative thickness of the target walls making the background scattering much more important.

It has long been appreciated (59) that a self-supporting solid hydrogen target could, in principle, be constructed and in such a target there would be no container to produce background events. This chapter describes work which has resulted in the translating this idea into a practical target.
4.2 Previous Solid Hydrogen Cryostats

Hydrogen targets in the solid form have been produced recently for the study of plasma production by laser irradiation. The literature describes the cryostats used to produce small quantities of solid hydrogen within a vacuum enclosure. One of these succeeded in producing a solid hydrogen foil 2 mm in diameter and about 1 mm thick. It was obtained by introducing hydrogen gas into a glass chamber which was surrounded by a vacuum enclosure. Inside this glass chamber was a condensation plate 1 mm thick with a hole 2 mm in diameter and in thermal contact with liquid helium. The gas condensed onto the condensation plate and covered the hole through the action of surface tension; the glass chamber was then removed and so exposed the solid hydrogen foil to bombardment with light from a laser. The lifetime of this foil was short, of the order of 10 minutes, as a result of sublimation owing to a lack of radiation shielding. The second cryostat was more exotic in that a narrow strip of solid hydrogen was extruded. The gas was condensed and the resulting solid compacted in a cylindrical chamber and from which it was, subsequently, extruded as a flat ribbon. A metal pin, accelerated by a spring, punched a pellet out of the ribbon and ejected it across the vacuum chamber to act as moving targets for a laser gun.

The method adopted in the original design was for the extrusion of a solid hydrogen sheet of 1 cm wide by a few millimeters thick. This represented an increase in target volume over those of references and by a factor of a thousand or more. In the following paragraphs a brief mention of the difficulties encountered...
Fig. 6

A diagram illustrating the components of a cryostat, including:
- Liquid inlet
- Gas inlet
- N₂ heat exchanger coils
- Liquid helium reservoir
- Copper plunger
- Support plate (cryostat can be dismantled at this point)
- Nitrogen cooled radiation shield
- Helium cooled radiation shield
- Small observation window
- Glass observation window
- Solid H₂ Target
Fig. 7

- Liquid Helium Reservoir
- Plunger
- Nozzle
- Retractable Pedestal
in the pursuit of the original method is given and the subsequent modifications made arrive at the final target design.

4.3 The Cryostat

The cryostat, with later modifications is shown in figure 6. In order to reduce the size of the cryostat no liquid nitrogen reservoir was provided; instead, the nitrogen radiation shield was cooled by a continuous flow of liquid nitrogen through the spiral heat exchange coils. This also reduced the weight of the cryostat and proved very convenient for handling. The volume of the liquid helium reservoir was 1 litre. In use, the helium reservoir required refilling after about two hours because of large radiation heat input from the slits left in the radiation shields for viewing purposes. If the radiation shielding was complete, refilling was necessary only after eight hours.

In the original design hydrogen gas was permitted to enter the central 2.54 cm diameter tube for subsequent solidification. Figure 7 shows an enlarged section of the original design. At the bottom of the tube a nozzle was provided with a suitably shaped slot cut through it from which the solidified hydrogen was to be extruded. Before the hydrogen gas was permitted into the tube, the whole cryostat was evacuated, cooled and filled with liquid helium. The slot in the nozzle was then closed by means of the pedestal, inserted up through the base of the vacuum vessel, which pressed a knife-edge into the lower face of the nozzle. Hydrogen gas was then allowed to enter the inner tube which condensed and solidify at the bottom of the nozzle. The pedestal was then removed and the solidified hydrogen pushed through the slot in the nozzle by forcing the plunger down.
In practice it proved impossible, while admitting the hydrogen gas, to obtain an adequate vacuum seal across the slot in the nozzle by simply pushing the pedestal onto the face of the nozzle. It was envisaged that a perfect seal would not be achieved in this manner but reliance on the fact that whatever small vacuum leaks which might be present would be blocked by the solidifying hydrogen was not justified. Even though, most of the hydrogen was pumped away, the small amount of solid hydrogen that was left behind was difficult to extrude through the nozzle by forcing the plunger down owing to the shape of the nozzle. The extrusion principle was, therefore, abandoned in favour of simply pushing solid hydrogen out of the tube (no nozzle being fitted) once the pedestal had been withdrawn. This at least permitted access to all the solid hydrogen that had deposited on the tube walls. In this manner a small quantity of solid hydrogen (at most 0.5 cm$^2$) was, in fact, obtained on the bottom face of the plunger and could be positioned in the centre of the vacuum chamber for viewing. It was discovered during this phase that the bonding by the solid hydrogen between the plunger and the tube walls was considerable. This binding can be reduced if the temperature of the system is raised and for this purpose heaters were provided. The permissible working temperature was, however, limited by the vapour pressure of the solid. The vapour pressure of solid hydrogen is about 4 x 10$^{-7}$ mm of Hg at liquid helium temperature rising rapidly to 2 x 10$^{-5}$ mm of Hg at about 5$^\circ$K. Warming the lower end of the tube freed the plunger but the rise in temperature and, consequently, of the vapour pressure of the hydrogen usually led to a very rapid loss of helium from the reservoir. The use of the heaters was, therefore, discontinued.
The inability to form a vacuum seal at the mouth of the tube prompted the use of a seal, usable once-only.

4.4 The Final Target

The modifications made were the addition of a soft-soldered copper flange to the mouth of the tube with a 1 cm diameter tube inserted inside the existing 2.54 cm diameter tube. The hydrogen, it was hoped, would solidify in the 1 cm diameter tube to give a solid hydrogen rod of 1 cm in diameter. On to the copper flange a chamber of 1 cm diameter was fastened using an Indium seal and the chamber was provided with a viewing port covered with mylar film. The bottom of the chamber was closed by a 0.012 mm thick sheet of indium clamped firmly between two copper flanges. Hydrogen was solidified successfully in this chamber and was pushed right through the indium sheet. No leaks appeared in the chamber as indium acts as a very good seal. The amount of indium used to seal the mouth of the chamber required that the pressure at which the hydrogen gas is transferred into the chamber must be much below atmospheric pressure so as not to burst the thin sheet of indium. In a preliminary experiment it was found that the sheet of indium used, withstood a pressure of 1/3 of the atmospheric pressure when warm and 1/2 of the atmospheric pressure when cooled to liquid helium temperature. A large gas reservoir was, thus, provided which was filled with hydrogen gas at 1/3 of the atmospheric pressure and from this reservoir the hydrogen gas was fed to the solidifying chamber.

The typical procedure for making solid hydrogen was as follows. The plunger was retracted above the hydrogen gas inlet, and liquid nitrogen was forced through the heat exchanger coils. The helium
vessel was pre-cooled with liquid nitrogen and then filled with liquid helium. Hydrogen gas was then admitted from the large gas reservoir at reduced pressure but above the triple-point. The gas first liquified and filled the target forming chamber through the action of gravity and when viewed through the viewing port, via slits left in the radiation shields, a colourless liquid could be seen rising in level. When the chamber was full of liquid, the hydrogen gas was stopped and the liquid could be seen to change into a milky colour. The temperature (monitored by a carbon resistance) can be seen to show a fall in temperature at the same time. When the temperature has fallen to about the temperature of liquid helium and when the helium reservoir is full, the plunger is slowly lowered to the top of the solid hydrogen, thus formed, in small steps so that the copper head of the plunger cools down to liquid helium temperature. It is then gradually forced downwards (with gentle taps from a hammer) so as to burst the indium seal and to push the solid hydrogen cylinder out into the vacuum chamber. Cylinders of up to 5 cm in height have been formed in this manner. A typical example is shown in the photograph (figure 8).

The solid hydrogen cylinder shown in figure is 2 cm in length and approximately 1 cm in diameter. The shape is irregular and bears the features of the method of production. The bottom of the cylinder is hemispherical due to the deformation of indium seal during the gas introduction phase. The upper half of the cylinder is thinner than the
lower half for the reason that the part of the solid hydrogen trapped in the viewing port scraped some of the solidified hydrogen from the cylinder while the latter was pushed down by the plunger. The scrapings can be seen to be retained in the viewing port. Once confidence for the production of solid hydrogen had been gained, the viewing port was discarded and the resulting solid hydrogen cylinders obtained were more uniform in thickness. The striations seen are due to the jerk imparted on the plunger by the use of a hammer. The indium sheet can be seen to be largely retained within the volume of the target forming chamber while a small piece can be seen poking out.

Provided the plunger is not forced out below the bottom of the target-forming chamber, the bonding between the solid hydrogen and the copper head of the plunger is so good that the cylinder shows no apparent tendency to fall off, despite the vibrations in the system from a backing pump. With just a small hole in the radiation shield for viewing purposes, the mass of solid hydrogen cylinder fell to half its original volume over a period of 5 hours, through sublimation, with the loss of material appearing as a uniform reduction in diameter of the cylinder. After this period, the remaining solid would suddenly become detached and fall on the warmer part of the cryostat resulting in a rapid evaporation. The life-time of the solid hydrogen could be increased by pumping over helium in the liquid helium reservoir and, thus, reducing the temperature of the solid hydrogen cylinder with the net result that the vapour pressure decreases sharply thus cutting down the rate of sublimation. The thermal conductivity also falls.
4.5 Use in the Experiment

The cryostat was successfully used in the experiment for the measurement of very small angle differential cross-sections for proton-proton scattering at 144 MeV. For this purpose the cryostat was coupled to a large scattering chamber and about 6 meters of beam pipe. Even though the whole assembly was pumped by one large diffusion pump the pressure was higher than that obtained in the test rig. For the purpose of letting the direct beam pass straight through the cryostat unhindered, holes of about 4 cm diameter were left in the radiation shield; as a result a large amount of radiation fell on the solid hydrogen cylinder increasing the sublimation rate of the hydrogen and also the boil-off of helium liquid in the reservoir. Under these conditions, the life time of solid hydrogen cylinder was only between 2 and 3 hours and the liquid helium reservoir was found to require filling at intervals throughout the run; this was done remotely.

In the actual production of solid hydrogen targets for the experimental runs, difficulties arose from the use of indium seal. It was found that small pieces of indium were torn off and were stuck to the sides of the hydrogen target instead of being pushed to one side as in figure 8. These pieces of indium were detected by observing that the cross-section for proton scattering at small angles was extremely high and had a different shape than expected. It was found that the diameter of the flange, used for clamping the sheet of indium onto the lower end of the target forming chamber, was not wide enough in diameter to hold both the thickness of the hydrogen target and the indium sheet. Making the diameter of the flange larger resulted in a production of hydrogen targets without the adhering pieces of indium.
sheet, which were instead pushed to one side and largely retained within the target forming chamber. This absence of indium was demonstrated by making several sets of runs which agreed within statistics indicating that no indium was present on these occasions.

If this situation had been considered serious it would have been possible to use a solid state Si(Li) detector to look for 24.2 keV indium to X-rays emitted during the proton bombardment. The total cross-section for K-shell ionization induced by 160 MeV proton is very large\(^{(62)}\) and, hence, any traces of indium left on the hydrogen target could be easily detected.

The major disadvantage of producing hydrogen targets in this manner is that only a single attempt can be made to produce a target since a new indium seal must be installed before making a further attempt. However, the probability of producing a good sample was high and a run was made every 8 hours and which lasted for about 2 to 3 hours giving a reliable set of data in the course of a few days. One of the advantages of the present technique is the possibility of extending, without much difficulty, the production of either large or small samples of solid hydrogen with any shape. It would also be a considerable advantage if a vacuum seal at liquid helium temperature is designed which can be re-established without the need to dismantle the cryostat.

4.6 Solid hydrogen target within a container

A second form of hydrogen target was also successfully produced within a container. The container was made as a 5 cm diameter copper annulus, 1 cm thick possessing 0.025 mm thick mylar sheet clamped firmly on the two faces using indium seals. This will hereafter be called the 'pod'. This pod replaced the target forming chamber used
for the production of the self supporting solid hydrogen target. The piston was also removed. The apparatus and the procedure were otherwise similar to the previous case. The hydrogen gas permitted first liquified and filled the pod under gravity and later solidified when the hydrogen gas inlet was closed. It was possible to use very thin sheet of melinex, as the hydrogen gas was introduced at a reduced pressure and, hence, the sheet of melinex was not required to withstand a pressure of more than, say, \( \frac{1}{2} \) an atmosphere.

The large slots left in the radiation shields made the pod relatively warmer than the regions not affected by the radiations from the slits. This gave rise to two phenomena. The solidified hydrogen in the pod gradually sublimed to cooler parts, leaving an ever-growing depression in the target and, hence, requiring that the pod be filled with solid hydrogen completely so that the solidified hydrogen in the direct path of the beam (at the middle) was not affected. Secondly, the warming up of the cryostat at the end of the run invariably led to the exploding of the pod. During the run, the sublimed hydrogen from the pod deposited and blocked the inlet to the pod (being at a lower temperature) with the result that the hydrogen in the pod liquified quickly due to the direct radiations from the slits in the radiation shields. This meant that the background counting rate from the pod windows and elsewhere had to be measured before filling the pod and these measurements could not be checked after the run to see if all the machine conditions were the same. However, good agreement was obtained with the results of the measurements with self-supporting target (where such difficulties did not exist) showing that the backgrounds were accounted for correctly. This was important for later experiments with
4.7 Solid Deuterium Target

An attempt to produce solid deuterium as a self-supporting target was made but turned out to be fruitless. The vapour pressure of deuterium is very small at $4.2^0K$ (10^{-11} mm of Hg). This made the solid, produced inside the target forming chamber in the similar manner to the hydrogen, very hard indeed. The pushing of the solid deuterium out of the target chamber with the plunger consequently proved very difficult. Use of large pressures on the plunger caused small pieces of solid deuterium to shoot across the vacuum chamber. These small flying pieces of deuterium were, obviously, useless for a nuclear physics experiment. Raising the temperature of the walls surrounding the solid deuterium would help in pushing latter out of the target chamber but a large rise in temperature would be needed which in turn would evaporate the helium liquid in the reservoir.

Clearly refinement to the cryostat is needed in which the thermal coupling to the helium reservoir could be reduced while warming the target forming chamber for the solid deuterium extraction procedure to be carried out. Once the deuterium is pushed out the thermal coupling could again be re-established. The cryostat as described above did not allow such a technique to be adopted and, thus, the production of self-supporting solid deuterium was not pursued. A target of solid deuterium contained in a pod such as described in the hydrogen work was in fact used.
### Table I

Some Physical properties of H<sub>2</sub> and D<sub>2</sub>

<table>
<thead>
<tr>
<th>Property</th>
<th>T&lt;sup&gt;0&lt;/sup&gt;K</th>
<th>H&lt;sub&gt;2&lt;/sub&gt;*</th>
<th>D&lt;sub&gt;2&lt;/sub&gt;*</th>
</tr>
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<tbody>
<tr>
<td>Boiling point</td>
<td>20.39° K</td>
<td>23.6° K</td>
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</tr>
<tr>
<td>Melting point</td>
<td>13.96° K</td>
<td>18.72° K</td>
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</tr>
<tr>
<td>Specific heat (liquid)</td>
<td>~4 Cal mol&lt;sup&gt;-1&lt;/sup&gt;°K&lt;sup&gt;-1&lt;/sup&gt;</td>
<td>~5 Cal mol&lt;sup&gt;-1&lt;/sup&gt;°K&lt;sup&gt;-1&lt;/sup&gt;</td>
<td></td>
</tr>
<tr>
<td>(solid)</td>
<td>~1 Cal mol&lt;sup&gt;-1&lt;/sup&gt;°K&lt;sup&gt;-1&lt;/sup&gt;</td>
<td>~2 Cal mol&lt;sup&gt;-1&lt;/sup&gt;°K&lt;sup&gt;-1&lt;/sup&gt;</td>
<td></td>
</tr>
<tr>
<td>Latent heat of Fusion</td>
<td>28.0 Cal mol&lt;sup&gt;-1&lt;/sup&gt;</td>
<td>47.0 Cal mol&lt;sup&gt;-1&lt;/sup&gt;</td>
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</tr>
<tr>
<td>Triple point</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tempt.</td>
<td>13.96°K</td>
<td>18.72°K</td>
<td></td>
</tr>
<tr>
<td>Pressure</td>
<td>54.0 mm of Hg</td>
<td>128.6 mm of Hg</td>
<td></td>
</tr>
<tr>
<td>Critical point Tempt.</td>
<td>33.24°K</td>
<td>38.35°K</td>
<td></td>
</tr>
<tr>
<td>Pressure</td>
<td>9735.6 mm of Hg</td>
<td>12487.0 mm of Hg</td>
<td></td>
</tr>
<tr>
<td>Thermal conductivity</td>
<td>~1 watts/cm/°K</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Vapour pressure</td>
<td>4.2</td>
<td>1.23 x 10^{-11}</td>
<td></td>
</tr>
<tr>
<td>(mm of Hg)</td>
<td>4.5</td>
<td>1.49 x 10^{-10}</td>
<td></td>
</tr>
<tr>
<td></td>
<td>7.2</td>
<td>4.52 x 10^{-5}</td>
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</tr>
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<td>8.0</td>
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<td></td>
<td>9.0</td>
<td>7.56 x 10^{-3}</td>
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</table>

*Equilibrium at room temperature.*
Chapter 5

RESULTS AND DISCUSSION

5.1 Results

Relative, but not absolute, differential cross-section for the cryogenetic targets hydrogen, deuterium and helium were measured as amounts of these target surface densities in the path of the incident beam were difficult to assess. Absolute cross-sections were obtained by normalising to the large angle data available in the literature. For the targets of lithium and carbon, absolute cross-sections were calculated directly from the measured counting rates and the measured target thicknesses.

The cross-sections were corrected for the multiple Coulomb scattering using the method of Cormack and corrections were also made for losses in the copper absorber used for energy discrimination for beam divergence, for zero-angle misalignment and for inelastic contamination where applicable. Typical corrections at several angles are listed in Table II for the case of hydrogen and $^6\text{Li}$. Table III gives the contribution of the various sources to the total error in the differential cross-sections.

5.1.1 Hydrogen

Cross-sections were obtained at angles extending from $1.75^\circ$ to $20.0^\circ$ (Lab.). They were normalized to the small angle data of CEZJR by minimizing $\chi^2$ resulting from data points between angles $8^\circ$ to $20^\circ$ (Lab.). The minimum $\chi^2$ value obtained was 0.65 per point showing no significant statistical difference between the two sets of data. The cross-sections are presented in table IV. The normalization error is 0.88%. The cross-sections from the two targets, the solid self-supporting
<table>
<thead>
<tr>
<th>$\theta_{\text{LAB}}$</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.0</td>
<td>0.884</td>
<td>0.909</td>
<td>1.211</td>
<td>0.977</td>
<td>0.997</td>
</tr>
<tr>
<td>4.0</td>
<td>0.972</td>
<td>0.930</td>
<td>1.213</td>
<td>0.990</td>
<td>0.941</td>
</tr>
<tr>
<td>6.0</td>
<td>0.987</td>
<td>0.994</td>
<td>1.215</td>
<td>0.997</td>
<td>0.940</td>
</tr>
<tr>
<td>8.0</td>
<td>0.993</td>
<td>0.998</td>
<td>1.218</td>
<td></td>
<td>0.944</td>
</tr>
<tr>
<td>10.0</td>
<td>0.995</td>
<td></td>
<td>1.222</td>
<td></td>
<td>0.940</td>
</tr>
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<td>12.0</td>
<td></td>
<td></td>
<td>1.228</td>
<td></td>
<td>0.917</td>
</tr>
<tr>
<td>14.0</td>
<td></td>
<td></td>
<td>1.234</td>
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<td>0.883</td>
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<td>16.0</td>
<td></td>
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<td>1.241</td>
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<td>0.828</td>
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<td>20.0</td>
<td></td>
<td></td>
<td>1.258</td>
<td></td>
<td>0.704</td>
</tr>
</tbody>
</table>

Note a) Multiplicative Corrections to $\frac{d\theta}{d\Omega}$

* A = Multiple Coulomb scattering
  B = Beam divergence
  C = Losses in Copper absorber
  D = Zero-angle misalignment
  E = Inelastic contamination
TABLE II (Continued)

Typical Correction Factors a) for Hydrogen

<table>
<thead>
<tr>
<th>$Q_{LAB}$</th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>P</td>
<td>S.S.</td>
<td>P</td>
</tr>
<tr>
<td>2.0</td>
<td>0.923</td>
<td>0.983</td>
<td>0.909</td>
</tr>
<tr>
<td>3.0</td>
<td>0.972</td>
<td>0.992</td>
<td>0.960</td>
</tr>
<tr>
<td>4.0</td>
<td>0.985</td>
<td>0.995</td>
<td>0.980</td>
</tr>
<tr>
<td>5.0</td>
<td>0.990</td>
<td>0.998</td>
<td>0.990</td>
</tr>
<tr>
<td>6.0</td>
<td>0.993</td>
<td>0.999</td>
<td>0.994</td>
</tr>
<tr>
<td>10.0</td>
<td>0.997</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>15.0</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>20.0</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Note: a) Multiplicative Corrections to $\frac{d\theta}{d\Omega}$

* P = target contained within a pod

* S.S. = target in the form of a solid self-supporting cylinder

A = Multiple Coulomb scattering

B = Beam divergence

C = Losses in copper absorber

b) Zero-angle misalignment corrections same as $^6$Li
TABLE III

Contribution to the Total Error* for $^6\text{Li}$

<table>
<thead>
<tr>
<th>$\theta_{\text{LAB}}$</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.0</td>
<td>8.5%</td>
<td>1.8%</td>
<td>2.3%</td>
<td>-</td>
<td>0.17%</td>
<td>9.0%</td>
</tr>
<tr>
<td>4.0</td>
<td>1.8</td>
<td>0.2</td>
<td>.1.0</td>
<td>2.6%</td>
<td>0.83</td>
<td>3.4</td>
</tr>
<tr>
<td>6.0</td>
<td>0.8</td>
<td>-</td>
<td>0.3</td>
<td>3.2</td>
<td>0.91</td>
<td>3.4</td>
</tr>
<tr>
<td>8.0</td>
<td>0.5</td>
<td>-</td>
<td>-</td>
<td>3.2</td>
<td>0.90</td>
<td>3.3</td>
</tr>
<tr>
<td>10.0</td>
<td>0.3</td>
<td>-</td>
<td>-</td>
<td>2.7</td>
<td>0.94</td>
<td>2.8</td>
</tr>
<tr>
<td>12.0</td>
<td>0.1</td>
<td>-</td>
<td>-</td>
<td>3.0</td>
<td>1.05</td>
<td>3.1</td>
</tr>
<tr>
<td>14.0</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>3.5</td>
<td>1.22</td>
<td>3.6</td>
</tr>
<tr>
<td>16.0</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>5.3</td>
<td>1.41</td>
<td>5.4</td>
</tr>
<tr>
<td>20.0</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>14.0</td>
<td>1.89</td>
<td>14.0</td>
</tr>
</tbody>
</table>

* A = Multiple Coulomb scattering,

B = Beam divergence

C = Zero-angle misalignment

D = Inelastic contribution

E = Statistics and

F = Total error.

Absolute Error:

- Target thickness = 2.0%
- Solid angle of Counters = 2.0%
- Copper absorber = 0.5%

Total = 3.0%
### TABLE III (Continued)

**Contribution to the Total Error* for Hydrogen**

<table>
<thead>
<tr>
<th>$Q_{LAB}$</th>
<th>A P</th>
<th>S.S. P</th>
<th>B P</th>
<th>S.S. P</th>
<th>C P</th>
<th>S.S.</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.0</td>
<td>4.0%</td>
<td>1.8%</td>
<td>1.8%</td>
<td>1.0%</td>
<td>0.22%</td>
<td>0.40%</td>
<td>2.3%</td>
</tr>
<tr>
<td>3.0</td>
<td>1.0</td>
<td>0.4</td>
<td>0.7</td>
<td>0.3</td>
<td>0.42</td>
<td>0.84</td>
<td>1.5</td>
</tr>
<tr>
<td>4.0</td>
<td>0.5</td>
<td>0.2</td>
<td>0.3</td>
<td>-</td>
<td>0.62</td>
<td>1.70</td>
<td>1.0</td>
</tr>
<tr>
<td>5.0</td>
<td>0.3</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.69</td>
<td>1.60</td>
<td>0.6</td>
</tr>
<tr>
<td>6.0</td>
<td>0.2</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.63</td>
<td>1.85</td>
<td>0.3</td>
</tr>
<tr>
<td>10.0</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.62</td>
<td>1.60</td>
<td>-</td>
</tr>
<tr>
<td>15.0</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.62</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>20.0</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.62</td>
<td>1.45</td>
<td>-</td>
</tr>
</tbody>
</table>

*P = target contained within a pod

S.S. = target in the form of a solid self-supporting cylinder

A = Multiple Coulomb scattering

B = Beam divergence

C = Statistics

D = Zero-angle misalignment
<table>
<thead>
<tr>
<th>Scattering Angle (c.m.)</th>
<th>$\sigma(\theta)$ (mb/sr)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.63</td>
<td>285.3 ± 10.2</td>
</tr>
<tr>
<td>4.15</td>
<td>151.2 ± 4.5</td>
</tr>
<tr>
<td>4.67</td>
<td>89.8 ± 2.5</td>
</tr>
<tr>
<td>5.19</td>
<td>56.4 ± 1.4</td>
</tr>
<tr>
<td>5.71</td>
<td>37.47 ± 0.80</td>
</tr>
<tr>
<td>6.23</td>
<td>23.44 ± 0.44</td>
</tr>
<tr>
<td>6.74</td>
<td>16.12 ± 0.29</td>
</tr>
<tr>
<td>7.26</td>
<td>11.42 ± 0.19</td>
</tr>
<tr>
<td>8.30</td>
<td>6.99 ± 0.13</td>
</tr>
<tr>
<td>8.82</td>
<td>5.84 ± 0.07</td>
</tr>
<tr>
<td>9.34</td>
<td>4.85 ± 0.05</td>
</tr>
<tr>
<td>10.38</td>
<td>3.943 ± 0.036</td>
</tr>
<tr>
<td>11.41</td>
<td>3.525 ± 0.030</td>
</tr>
<tr>
<td>12.45</td>
<td>3.388 ± 0.022</td>
</tr>
<tr>
<td>13.49</td>
<td>3.351 ± 0.021</td>
</tr>
<tr>
<td>14.52</td>
<td>3.410 ± 0.022</td>
</tr>
<tr>
<td>15.56</td>
<td>3.504 ± 0.022</td>
</tr>
<tr>
<td>16.60</td>
<td>3.569 ± 0.022</td>
</tr>
<tr>
<td>18.67</td>
<td>3.731 ± 0.023</td>
</tr>
<tr>
<td>20.74</td>
<td>3.791 ± 0.023</td>
</tr>
<tr>
<td>22.81</td>
<td>3.896 ± 0.024</td>
</tr>
<tr>
<td>24.88</td>
<td>3.977 ± 0.024</td>
</tr>
<tr>
<td>26.95</td>
<td>3.990 ± 0.024</td>
</tr>
<tr>
<td>29.01</td>
<td>4.034 ± 0.025</td>
</tr>
<tr>
<td>31.08</td>
<td>4.006 ± 0.025</td>
</tr>
<tr>
<td>36.24</td>
<td>4.042 ± 0.025</td>
</tr>
<tr>
<td>41.38</td>
<td>3.985 ± 0.025</td>
</tr>
</tbody>
</table>

Normalization = 1.00 ± 0.0088
and the solid in a pod, agreed within their statistical errors, but
the results for small-angle apply only to the self-supporting target
as the uncertainties were significantly smaller than for the pod.

5.1.2 Deuterium

The same counter telescopes as in the hydrogen work were used
for the deuterium measurements. The energy discrimination was
provided by the copper absorber placed in front of the counter 4.
The absorber chosen for the deuterium work was such that the tele­
scope detected elastically scattered protons as well as protons with
an energy about 15 MeV. below that of elastically scattered protons.
A large number of quasi-elastically scattered protons were thus
detected. The amount of such inelastic contamination was estimated
from the measurements made by the Orsay group\(^{(64)}\) using a large
magnetic spectrometer. They obtained the inelastic energy spectrum
for several angles but it was necessary to interpolate between these
angles to deduce the inelastic contributions appropriate to the
present work. This was done by drawing a smooth line through the
points. The inelastic contributions were found to be large and were
of the order of 30 to 40\%. The error involved in smoothing out the
inelastic spectrum at various angles was large and an uncertainty of
30\% of each inelastic correction was taken.

The normalization of the data was made through 155 MeV Orsay
elastic scattering results\(^{(65)}\). Because of the energy difference
between the two experiments compatibility was obtained by scaling the
Orsay elastic data by a factor obtained from the theoretical calcul­
ations of p-d scattering (see section 5.3.2). The inelastic
TABLE V
Small-angle proton cross-section from $^2$H and $^4$He at 144 MeV

<table>
<thead>
<tr>
<th>$Q_{\text{LAB}}$</th>
<th>$Q_{\text{c.m.}}$</th>
<th>$dQ/d\Omega$ (mb/sr)</th>
<th>$Q_{\text{c.m.}}$</th>
<th>$dQ/d\Omega$ (mb/sr)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.75</td>
<td>2.72</td>
<td>$424.0 \pm 34.0$</td>
<td>2.24</td>
<td>$2016.0 \pm 160.0$</td>
</tr>
<tr>
<td>2.00</td>
<td>3.11</td>
<td>$227.0 \pm 15.0$</td>
<td>2.56</td>
<td>$1115.0 \pm 66.0$</td>
</tr>
<tr>
<td>2.25</td>
<td>3.50</td>
<td>$129.0 \pm 7.0$</td>
<td>2.89</td>
<td>$628.0 \pm 30.0$</td>
</tr>
<tr>
<td>2.50</td>
<td>3.89</td>
<td>$75.5 \pm 3.8$</td>
<td>3.21</td>
<td>$381.0 \pm 15.0$</td>
</tr>
<tr>
<td>2.75</td>
<td>4.27</td>
<td>$47.2 \pm 2.5$</td>
<td>3.53</td>
<td>$236.0 \pm 7.0$</td>
</tr>
<tr>
<td>3.00</td>
<td>4.66</td>
<td>$31.6 \pm 1.7$</td>
<td>3.85</td>
<td>$151.0 \pm 3.7$</td>
</tr>
<tr>
<td>3.25</td>
<td>5.05</td>
<td>$22.0 \pm 1.2$</td>
<td>4.17</td>
<td>$101.9 \pm 2.5$</td>
</tr>
<tr>
<td>3.50</td>
<td>5.44</td>
<td>$16.8 \pm 1.2$</td>
<td>4.49</td>
<td>$70.5 \pm 1.4$</td>
</tr>
<tr>
<td>3.75</td>
<td>5.83</td>
<td>$14.1 \pm 1.1$</td>
<td>4.81</td>
<td>$52.4 \pm 1.0$</td>
</tr>
<tr>
<td>4.00</td>
<td>6.22</td>
<td>$12.6 \pm 1.1$</td>
<td>5.13</td>
<td>$42.9 \pm 0.8$</td>
</tr>
<tr>
<td>4.50</td>
<td>6.99</td>
<td>$12.0 \pm 1.1$</td>
<td>5.77</td>
<td>$35.1 \pm 0.5$</td>
</tr>
<tr>
<td>5.00</td>
<td>7.77</td>
<td>$12.6 \pm 1.1$</td>
<td>6.41</td>
<td>$34.6 \pm 0.6$</td>
</tr>
<tr>
<td>5.50</td>
<td>8.55</td>
<td>$13.3 \pm 1.1$</td>
<td>7.05</td>
<td>$36.0 \pm 0.4$</td>
</tr>
<tr>
<td>6.00</td>
<td>9.32</td>
<td>$14.1 \pm 1.2$</td>
<td>7.69</td>
<td>$38.7 \pm 0.5$</td>
</tr>
<tr>
<td>6.50</td>
<td>10.10</td>
<td>$14.8 \pm 1.2$</td>
<td>8.33</td>
<td>$41.6 \pm 0.5$</td>
</tr>
<tr>
<td>7.00</td>
<td>10.87</td>
<td>$15.2 \pm 1.2$</td>
<td>8.97</td>
<td>$43.2 \pm 0.4$</td>
</tr>
<tr>
<td>7.50</td>
<td>11.65</td>
<td>$15.6 \pm 1.2$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8.00</td>
<td>12.42</td>
<td>$16.6 \pm 1.3$</td>
<td>10.25</td>
<td>$46.5 \pm 0.4$</td>
</tr>
<tr>
<td>9.00</td>
<td>13.97</td>
<td>$16.6 \pm 1.3$</td>
<td>11.53</td>
<td>$46.9 \pm 0.4$</td>
</tr>
<tr>
<td>10.00</td>
<td>15.52</td>
<td>$16.0 \pm 1.3$</td>
<td>12.81</td>
<td>$46.6 \pm 0.4$</td>
</tr>
<tr>
<td>12.50</td>
<td>19.38</td>
<td>$13.6 \pm 1.2$</td>
<td>16.00</td>
<td>$39.8 \pm 0.4$</td>
</tr>
<tr>
<td>15.00</td>
<td>23.23</td>
<td>$10.4 \pm 0.9$</td>
<td>19.19</td>
<td>$32.2 \pm 0.3$</td>
</tr>
<tr>
<td>17.50</td>
<td>27.06</td>
<td>$8.2 \pm 1.2$</td>
<td>22.36</td>
<td>$23.9 \pm 0.2$</td>
</tr>
<tr>
<td>20.00</td>
<td>30.88</td>
<td>$6.1 \pm 1.2$</td>
<td>25.53</td>
<td>$16.8 \pm 0.1$</td>
</tr>
</tbody>
</table>

Normalization: 1.00 ± 0.07  1.00 ± 0.05
contributions (determined as above) were then added to the Orsay data so that both sets of data now have the same inelastic contamination. The present data were then normalized to the Orsay elastic plus inelastic data and the inelastic contributions were finally subtracted to give elastic cross-sections as given in Table V.

5.1.3 Helium

The helium data were normalized to the 147 MeV. Harvard data. Allowance was made for the later re-normalization by the Harvard group. No inelastic corrections were necessary for the helium data. For the purpose of normalization, the small energy difference between this experiment and the Harvard measurements was ignored because of the large uncertainty (5%) in the normalization of the Harvard data.

5.1.4 Lithium and Carbon

Absolute cross-sections were obtained with an uncertainty in normalization of ± 3% arising from the target thickness errors and the errors involved in the measurements of the sizes and distances of the defining counters. The carbon data was further re-normalized to the absolute scale of Jarvis and Cox for which the absolute normalization was particularly precise (0.51%).

The inelastic contribution to $^6$Li was obtained from the work of the Orsay group and for $^7$Li and carbon the inelastic contribution was obtained from earlier Orsay work. The contribution from the 0.418 MeV level of $^7$Li not analysed by the Orsay group was obtained from reference 81. The cross-sections for $^6$Li, $^7$Li and carbon are given in Table VI.
TABLE VI  
Small-angle proton cross-section for $^6\text{Li}$, $^7\text{Li}$ and $^{12}\text{C}$  
at 144 MeV

<table>
<thead>
<tr>
<th>$\theta_{\text{lab}}$</th>
<th>$^6\text{Li}$</th>
<th>$^7\text{Li}$</th>
<th>$^{12}\text{C}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta_{\text{c.m.}}$</td>
<td>$d\theta/d\Omega$ (mb/sr)</td>
<td>$d\theta/d\Omega$ (mb/sr)</td>
<td>$d\theta/d\Omega$ (mb/sr)</td>
</tr>
<tr>
<td>2.0</td>
<td>2.38</td>
<td>2958.0 ± 266.0</td>
<td>2.33</td>
</tr>
<tr>
<td>2.5</td>
<td>2.97</td>
<td>1086.0 ± 54.0</td>
<td>2.91</td>
</tr>
<tr>
<td>3.0</td>
<td>3.57</td>
<td>449.0 ± 18.0</td>
<td>3.49</td>
</tr>
<tr>
<td>3.5</td>
<td>4.16</td>
<td>213.0 ± 7.0</td>
<td>4.07</td>
</tr>
<tr>
<td>4.0</td>
<td>4.76</td>
<td>133 ± 4.0</td>
<td>4.65</td>
</tr>
<tr>
<td>4.5</td>
<td>5.35</td>
<td>106 ± 5.0</td>
<td>5.23</td>
</tr>
<tr>
<td>5.0</td>
<td>5.95</td>
<td>101.0 ± 4.0</td>
<td>5.81</td>
</tr>
<tr>
<td>5.5</td>
<td>6.54</td>
<td>104.0 ± 4.0</td>
<td>6.40</td>
</tr>
<tr>
<td>6.0</td>
<td>7.14</td>
<td>110.0 ± 4.0</td>
<td>6.98</td>
</tr>
<tr>
<td>6.5</td>
<td>7.73</td>
<td>113.0 ± 4.0</td>
<td>7.56</td>
</tr>
<tr>
<td>7.0</td>
<td>8.32</td>
<td>117.0 ± 4.0</td>
<td>8.14</td>
</tr>
<tr>
<td>7.5</td>
<td>8.92</td>
<td>118.0 ± 4.0</td>
<td>8.72</td>
</tr>
<tr>
<td>8.0</td>
<td>9.51</td>
<td>118.0 ± 4.0</td>
<td>9.30</td>
</tr>
<tr>
<td>9.0</td>
<td>10.70</td>
<td>112.0 ± 4.0</td>
<td>10.46</td>
</tr>
<tr>
<td>10.0</td>
<td>11.89</td>
<td>105.0 ± 3.0</td>
<td>11.62</td>
</tr>
<tr>
<td>11.0</td>
<td>13.07</td>
<td>96.7 ± 2.8</td>
<td>12.78</td>
</tr>
<tr>
<td>12.0</td>
<td>14.76</td>
<td>85.4 ± 2.8</td>
<td>13.94</td>
</tr>
<tr>
<td>13.0</td>
<td>14.63</td>
<td>64.3 ± 2.3</td>
<td>16.26</td>
</tr>
<tr>
<td>14.0</td>
<td>19.00</td>
<td>44.3 ± 2.4</td>
<td>18.57</td>
</tr>
<tr>
<td>15.0</td>
<td>21.36</td>
<td>30.0 ± 2.5</td>
<td>20.89</td>
</tr>
<tr>
<td>16.0</td>
<td>23.72</td>
<td>20.4 ± 2.9</td>
<td>23.19</td>
</tr>
<tr>
<td>Normalization</td>
<td>1.00 ± 0.03</td>
<td>1.00 ± 0.03</td>
<td>1.000 ± 0.006</td>
</tr>
</tbody>
</table>
TABLE VII
Summary of other data used in the analysis of small-angle proton scattering

<table>
<thead>
<tr>
<th>Nucleus</th>
<th>Energy (MeV)</th>
<th>Type of Data</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^1_H$</td>
<td>143</td>
<td>Proton Cross-section and polarization</td>
<td>Harwell, ref. 70</td>
</tr>
<tr>
<td>$^2_H$</td>
<td>155</td>
<td>Proton cross-section and polarization</td>
<td>Orsay, ref. 65</td>
</tr>
<tr>
<td></td>
<td>146</td>
<td>Proton cross-section and polarization&lt;sup&gt;a)&lt;/sup&gt;</td>
<td>Harvard, ref. 71</td>
</tr>
<tr>
<td></td>
<td>155</td>
<td>Proton inelastic cross-section</td>
<td>Orsay, ref. 64</td>
</tr>
<tr>
<td></td>
<td>144</td>
<td>Neutron total cross-section: $\sigma_T = 77.5 \pm 1.3 \text{ mb}^c)$</td>
<td>Harvard, ref. 72</td>
</tr>
<tr>
<td>$^4_He$</td>
<td>147</td>
<td>Proton Cross-section and polarization&lt;sup&gt;a)&lt;/sup&gt;</td>
<td>Harvard, ref. 67</td>
</tr>
<tr>
<td></td>
<td>144</td>
<td>Total Cross-section: $\sigma_T = 119.5 \pm 4.0 \text{ mb}^c)$</td>
<td>Harvard, ref. 73, 74</td>
</tr>
<tr>
<td>$^6_Li,^7_Li$</td>
<td>155</td>
<td>Proton Cross-section and polarization</td>
<td>Orsay, ref. 75</td>
</tr>
<tr>
<td></td>
<td>155</td>
<td>Proton inelastic cross-section</td>
<td>Orsay, ref. 80, 82</td>
</tr>
<tr>
<td>$^9_Be$</td>
<td>141.5</td>
<td>Proton and neutron cross-section and proton polarization</td>
<td>Ref. 76&lt;sup&gt;d)&lt;/sup&gt;</td>
</tr>
<tr>
<td></td>
<td>*</td>
<td>Neutron total cross-section: $\sigma_T = 284 \pm 14 \text{ mb}^c)$</td>
<td>Ref. 77,78,79</td>
</tr>
<tr>
<td>$^{12}_C$</td>
<td>143</td>
<td>Proton and neutron cross-section and polarization and neutron total cross-section $\sigma_T = 343 \pm 5 \text{ mb}^c)$</td>
<td>Ref. 76&lt;sup&gt;d)&lt;/sup&gt;</td>
</tr>
</tbody>
</table>

Notes:
<sup>a)</sup> Renormalised in accordance with ref.  
<sup>b)</sup> " " " " " "  
<sup>c)</sup> The total cross-section are interpolated  
<sup>d)</sup> Reference 76 contains data tabulation and source references
### TABLE VIII

\( T = 1 \) Phase-shifts at 140 MeV

<table>
<thead>
<tr>
<th></th>
<th>( ^1S_0 )</th>
<th>( ^1D_2 )</th>
<th>( ^3G_4 )</th>
<th>( ^3P_0 )</th>
<th>( ^3P_1 )</th>
<th>( ^3P_2 )</th>
<th>( E_2 )</th>
<th>( ^3P_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>OLD*</td>
<td>17.58 ± 0.62</td>
<td>5.22 ± 0.22</td>
<td>0.58 ± 0.09</td>
<td>6.74 ± 0.52</td>
<td>-16.61 ± 0.18</td>
<td>13.71 ± 0.12</td>
<td>-2.77 ± 0.09</td>
<td>0.40 ± 0.31</td>
</tr>
<tr>
<td>NEW‡</td>
<td>17.42 ± 0.67</td>
<td>5.28 ± 0.21</td>
<td>0.62 ± 0.08</td>
<td>6.71 ± 0.60</td>
<td>-16.43 ± 0.20</td>
<td>13.86 ± 0.11</td>
<td>-2.73 ± 0.09</td>
<td>0.65 ± 0.39</td>
</tr>
</tbody>
</table>

* Pering 1967
‡ Pering 1971
TABLE VIII (Continued)

<table>
<thead>
<tr>
<th></th>
<th>$3_{F3}$</th>
<th>$3_{F4}$</th>
<th>$E_4$</th>
<th>$3_{H4}$</th>
<th>$3_{H5}$</th>
<th>$3_{H6}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>OLD*</td>
<td>$-1.91 \pm 0.20$</td>
<td>$0.52 \pm 0.18$</td>
<td>$-0.70 \pm 0.14$</td>
<td>$0.18 \pm 0.14$</td>
<td>$-0.38 \pm 0.16$</td>
<td>$0.11 \pm 0.09$</td>
</tr>
<tr>
<td>NEW‡</td>
<td>$-1.84 \pm 0.21$</td>
<td>$0.67 \pm 0.21$</td>
<td>$-0.66 \pm 0.06$</td>
<td>$-0.03 \pm 0.17$</td>
<td>$-0.12 \pm 0.17$</td>
<td>$-0.02 \pm 0.12$</td>
</tr>
</tbody>
</table>

*Pering 1967  ‡Pering 1971
TABLE IX

Predicted and Optimized parameters for the Optical potential

<table>
<thead>
<tr>
<th></th>
<th>(^4\text{He})</th>
<th>(^6\text{Li})</th>
<th>(^7\text{Li})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(v_R)</td>
<td>20.6</td>
<td>10.3</td>
<td>Th</td>
</tr>
<tr>
<td>(v_I)</td>
<td>16.7</td>
<td>23.5</td>
<td>Th</td>
</tr>
<tr>
<td>(V_{SR})</td>
<td>4.34</td>
<td>2.57</td>
<td>Th</td>
</tr>
<tr>
<td>(V_{SI})</td>
<td>-0.52</td>
<td>-0.82</td>
<td>Th</td>
</tr>
<tr>
<td>(a_1)</td>
<td>1.82</td>
<td>2.27</td>
<td>Th</td>
</tr>
<tr>
<td>(a_2)</td>
<td>1.83</td>
<td>1.55</td>
<td>Th</td>
</tr>
<tr>
<td>(a_3)</td>
<td>1.55</td>
<td>1.75</td>
<td>Th</td>
</tr>
<tr>
<td>(a_4)</td>
<td>1.92</td>
<td>1.92</td>
<td>Th</td>
</tr>
<tr>
<td>(\sigma_0)</td>
<td>1.45</td>
<td>2.22</td>
<td>Th</td>
</tr>
<tr>
<td>Norm.</td>
<td>1.013</td>
<td>0.996</td>
<td>Th</td>
</tr>
<tr>
<td>(\sigma_T)</td>
<td>15.6</td>
<td>11.6</td>
<td>Th</td>
</tr>
<tr>
<td>(\chi^2)</td>
<td>988.8</td>
<td>2.57</td>
<td>Th</td>
</tr>
<tr>
<td>(\sigma_T^{(exp)})</td>
<td>11.9±0.4</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Dimensions: potentials in MeV, radii in fm and \(\sigma_T\) in fm\(^2\).
<table>
<thead>
<tr>
<th></th>
<th>$^9\text{Be}$</th>
<th></th>
<th>$^{12}\text{C}$</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Th</td>
<td>Exp.</td>
<td>Th</td>
<td>Exp.</td>
<td></td>
</tr>
<tr>
<td>$V_R$</td>
<td>26.8</td>
<td>18.3</td>
<td>30.9</td>
<td>19.5</td>
<td></td>
</tr>
<tr>
<td>$V_I$</td>
<td>21.4</td>
<td>12.6</td>
<td>25.4</td>
<td>28.2</td>
<td></td>
</tr>
<tr>
<td>$V_{SR}$</td>
<td>4.22</td>
<td>4.08</td>
<td>4.42</td>
<td>4.52</td>
<td></td>
</tr>
<tr>
<td>$V_{SI}$</td>
<td>-0.56</td>
<td>-0.61</td>
<td>-0.72</td>
<td>-0.35</td>
<td></td>
</tr>
<tr>
<td>$a_1$</td>
<td>2.30</td>
<td>2.64</td>
<td>2.45</td>
<td>2.77</td>
<td></td>
</tr>
<tr>
<td>$a_2$</td>
<td>2.32</td>
<td>2.64</td>
<td>2.46</td>
<td>2.23</td>
<td></td>
</tr>
<tr>
<td>$a_3$</td>
<td>2.10</td>
<td>2.10</td>
<td>2.26</td>
<td>2.26</td>
<td></td>
</tr>
<tr>
<td>$a_4$</td>
<td>2.48</td>
<td>2.48</td>
<td>2.52</td>
<td>2.52</td>
<td></td>
</tr>
<tr>
<td>$\sigma$</td>
<td>2.03</td>
<td></td>
<td>2.19</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Norm</td>
<td></td>
<td>0.999</td>
<td>0.996</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma_T$</td>
<td>34.8</td>
<td>28.2</td>
<td>48.7</td>
<td>34.4</td>
<td></td>
</tr>
<tr>
<td>$X^2$</td>
<td>152.7</td>
<td>0.48</td>
<td>561.6</td>
<td>1.34</td>
<td></td>
</tr>
<tr>
<td>$\sigma_T$ (exp)</td>
<td>28.4 ± 1.4</td>
<td></td>
<td>34.3 ± 0.5</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
**TABLE X**

Nucleon-Nucleon amplitudes derived from phase-shifts and from the Optical potential

<table>
<thead>
<tr>
<th>Nucleus</th>
<th>$A_R$</th>
<th>$A_I$</th>
<th>$C_R$</th>
<th>$C_I$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Theoretical</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$^7$Li</td>
<td>0.483</td>
<td>0.390</td>
<td>0.072</td>
<td>0.372</td>
</tr>
<tr>
<td>$^9$Be</td>
<td>0.485</td>
<td>0.394</td>
<td>0.074</td>
<td>0.371</td>
</tr>
<tr>
<td>$^4$He, $^6$Li, $^{12}$C</td>
<td>0.490</td>
<td>0.409</td>
<td>0.083</td>
<td>0.365</td>
</tr>
<tr>
<td></td>
<td>Experimental</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$^4$He</td>
<td>0.479</td>
<td>0.349</td>
<td>0.131</td>
<td>0.309</td>
</tr>
<tr>
<td>$^6$Li</td>
<td>0.486</td>
<td>0.400</td>
<td>-0.063</td>
<td>0.339</td>
</tr>
<tr>
<td>$^7$Li</td>
<td>0.483</td>
<td>0.405</td>
<td>0.112</td>
<td>0.260</td>
</tr>
<tr>
<td>$^9$Be</td>
<td>0.494</td>
<td>0.344</td>
<td>0.088</td>
<td>0.360</td>
</tr>
<tr>
<td>$^{12}$C</td>
<td>0.449</td>
<td>0.339</td>
<td>0.039</td>
<td>0.371</td>
</tr>
<tr>
<td>Average</td>
<td>0.478 ± 0.015</td>
<td>0.367 ± 0.029</td>
<td>0.061 ± 0.069</td>
<td>0.328 ± 0.040</td>
</tr>
</tbody>
</table>

Note: The nucleon-nucleon radii are $a_1 = 1.41$, $a_2 = 1.43$, $a_3 = 1.43$, $a_4 = 1.54$ (fm).

The nuclear radii are $a_N = 1.15(^4$He), 2.02($^6$Li and $^7$Li), 1.82($^9$Be) and 2.01($^{12}$C) (fm).
5.2 Other Data Used in the Analyses

Small angle data for the targets considered in the present work have been measured by other workers but the data were poor in the Coulomb interference region and analyses to study the interference of Coulomb interaction with nuclear interaction was not made.

To make the present analyses complete data not measured in this work are taken from the literature. Table VII contains a listing of the data considered in the present analyses. Reference 76 discusses how some of the set of data listed in Table VII is chosen and gives further references from where the data is actually taken. To mention a few important points, the total cross-sections for helium and beryllium were taken as an average of proton and neutron total cross-sections. The Harvard proton and neutron total cross-sections for helium, for example, differed by about 7% well outside the quoted errors. Since, in this work, charge independence is assumed (this means, for even-even nuclei the proton and neutron total cross-sections should be the same), an average of proton and neutron total cross-sections has been taken.

The neutron differential cross-section and polarization data for carbon were not taken in the analyses but were fitted by the predictions made by the Optical Model parameters obtained from the analyses of the proton data.

5.3 Discussion of Results

5.3.1 Hydrogen

A phase-shift analysis has been performed by Perring\(^{(83)}\) on the present small angle p-p data. Other data included in the analysis were the large angle cross-sections and polarization of CEZJR at
P-P DIFFERENTIAL SCATTERING CROSS SECTION NEAR 140 MeV

CROSS-SECTION (MB sr⁻¹)

COX et al (Present Work) - 144-1 MeV
NORMAL FACTOR 0.996 SMALL ANGLE
0.999 LARGE ANGLE

PALMIERI et al (HARVARD) - 147 MeV
NORMAL FACTOR 0.876 SMALL ANGLE
0.936 LARGE ANGLE

CAVERSAZIO et al (ORSAY) - 155 MeV
NORMAL FACTOR 1.023

ALL LINES FROM RECENT ANALYSIS OF P-P DATA BY J.K. PERRING.

Fig. 9
Fig. 10
144 MeV. The triple scattering parameters $P, D, R, A$ and $R'$ of Harwell and $D$ and $R$ of Harvard at 142 MeV, the spin correlation parameter $C_{NN}$ of Harwell at 142 MeV, the cross-sections and polarization for angles greater than 14° (Lab) of Harvard at 147 MeV and the 155 MeV Orsay cross-sections for angles greater than 14° (Lab). The energy variation of the phase-shifts were taken from the Livermore multi-energy analysis (84). A fourteen phase parameter search (varying phases up to and including $3H_6$, higher order phases being set to the values obtained from the one-pion-exchange contribution with $g^2 = 14.00$) gave a value for $\chi^2$ of 36.5 for 27 points. $\chi^2$ for the first four smallest angles was 15.2 reflecting a statistically significant fit to the data for angles greater than 5.5° cms. Figure 10 shows the predicted fit to the data. The interference region between nuclear and Coulomb interaction is well described.

In the analysis Perring used the nuclear bar phase-shift representation (31) and took the total phase-shifts as the sum of the nuclear phase-shift and the Coulomb phase-shift for a point charge. A relativistic correction was applied by modifying the Coulomb parameter $n = Z/137\beta$ so that $\beta$ is the relativistic velocity of the incident proton in the laboratory frame. Magnetic moment terms were also included in the analysis for $f$ and higher partial waves.

The new phase-shifts obtained are compared with phase-shifts obtained from an earlier analysis (85), in Table VIII. Inclusion of the present data has made very little difference to the phase shifts, all the changes lying within the uncertainties.
The $T=1$ phase-shifts (p-p scattering) are now well determined at 140 MeV and there is no apparent need for further work unless an increase in precision is required by advances in the theoretical interpretation of the data.

5.3.2 Light Nuclei

The data for light nuclei together with the theoretical predictions are shown in figures 11 to 19. Initial predictions were made using Born Approximation (formulae 2.6, 2.6.1, 2.7 and 2.8). As usual the polarizations are fitted well but the cross-sections are over-estimated except for deuterium where a reasonably good description of the data is attained as shown by the dashed curve in figure 11. The theoretical fits were much improved by modifying the forward scattering Born amplitudes by the value obtained from W.K.B. calculations so that some account of multiple scattering processes is taken. For deuterium the prediction is well within the experimental errors as shown by the full curve in figure 11 apart from the small angle region where the difference might be entirely due to insufficient allowance for inelastic scattering. The value for the total cross-section obtained at 144 MeV is 78.8 mb, in good agreement with the value of $77.5 \pm 1.3$ mb interpolated from measurements of Measden and Palmieri\(^{72}\). The polarization is also well reproduced (figure 12). For other nuclei, although improvements are made, the predictions are still high. However, the W.K.B. estimate of $\sigma_n$ is also too high. This can be corrected by use of the optical theorem which reduces the imaginary part of the forward scattering amplitude. With this modification the predictions are further improved. The Coulomb interference region is fairly well reproduced.
Fig. 11

CROSS-SECTION (mb/ster)

ORSAY 155 MeV (x100)

p-D CROSS-SECTION

144 MeV

SCATTERING ANGLE (c.m.)
Fig. 12

\textbf{p-D POLARIZATION}

\textit{ORSAY 155 MeV}

\textit{HARVARD 146 MeV}

\textbf{SCATTERING ANGLE (c.m.)}

\textbf{POLARIZATION}
p-He CROSS-SECTION AT 144 MeV

Fig. 13
Fig. 14

p-He CROSS-SECTION AT 144 MeV
Fig. 15
Fig. 16

Scattering Angle (c.m.) vs. Cross-Section (mb/sr) for different isotopes:
- $^6\text{Li}$
- $^7\text{Li} \times 10$
- $^{9}\text{Be} \times 10^2$
POLARIZATION

SCATTERING ANGLE (c.m.)

Fig. 17
Fig. 19

$\text{p}^{-12}\text{C POLARIZATION AT 144 MeV}$
as seen in figure 13 for the case of helium. Other nuclei behave similarly and have not been shown.

To take a complete account of the multiple scattering on-the-energy-shell, cross-sections can be calculated by solving the Schrodinger equation. A program called SEEK(86) was used for this purpose which solves the Schrodinger equation exactly and has the facility of performing a search over the optical model parameters to give a minimum $\chi^2$. The optical model parameters were calculated using formulae 2.11 and the Schrodinger equation was solved to give the predictions of cross-sections and polarizations. The parameters were then allowed to vary to give a best fit to the data. The calculated and the optimized parameters are given in Table IX together with the value of the normalization constant (obtained when used as one of the adjustable parameters in the search), the $\chi^2$ per point and the total cross-sections predicted by optimised potentials. Apart from the cross-sections measured in this work, polarizations and total cross-sections as listed in Table VII were used in the search. The predictions were found to be insensitive to both the spin-orbit radius parameters $a_3$ and $a_4$ and these, therefore, were held constant at their calculated values (except for $a_3$ in the case of helium).

The predictions from the calculated and the optimised potentials are shown in figures 14 to 19 where the dashed curves correspond to the predictions from calculated optical potential parameters and the full curves correspond to the optimised parameters. The predictions from the calculated optical model parameters for $^6$Li and $^7$Li were remarkably good as shown in figure 16. For other light nuclei
excellent fits to the data were obtained with the optimised parameters. The high value of $X^2$ for helium is due to large fluctuations in the polarization data. It was found that the cross-sections had to be normalized by a factor of 0.90 in order to fit the neutron cross-section data from carbon at 137 MeV to the predictions made by using the proton optimized parameters at 144 MeV. The experimental uncertainty was ± 0.04 and when allowance is made of the energy differences, involving a scaling factor of 0.95, the value of 0.90 does not seem unreasonable. The neutron polarization predictions are well in agreement with the data, provided the normalization factor 0.78 (obtained from Ref. 76, Jarvis) is applied.

The optimized potential depth and radius parameters differed considerably from the theoretical values but when expressed in terms of the nucleon-nucleon amplitudes $\bar{A}(0)$ and $\bar{C}(0)$, which are proportional to the integrated potentials (as can be seen by taking an inverse fourier transform of equation 2.10), a meaningful picture emerges. Table X gives a list of the $\bar{A}(0)$ and $\bar{C}(0)$. Their values for individual nuclei fluctuate but their average value is in good agreement with the values obtained from the nucleon-nucleon phase-shifts of Perring (85). Cromer and Palmieri (18) have done a similar analysis but have fitted the forward scattering amplitudes directly to the data through $X^2$ minimization and then deduced $\bar{A}(0)$ and $\bar{C}(0)$ from these scattering amplitudes. The resulting $\bar{A}$ and $\bar{C}$ agree well except for the imaginary value of $\bar{A}(0)$. The present analysis gives the average $\bar{A}_I(0)$ to be 0.367 ± 0.029 which is much closer to the theoretical average value of 0.398 ± 0.010 than the value (0.24 ± 0.01).
obtained by Cromer and Palmieri. The discrepancy between the present analysis and that of Cromer and Palmieri for $A_1^{(0)}$ could be due to the fact that Cromer and Palmieri uses a high energy approximation (for evaluating the scattering amplitudes) which assumes $E \gg v$ and $ka \gg 1$. However, for 140 MeV protons $ka \gg 5$ only and therefore the second assumption is not strictly satisfied.

The confidence in the present analysis is further reflected by the recent calculations made by Tatischeff [97] using a more basic approach. He calculates the scattering amplitudes directly from the multiple scattering expansion [19] and includes terms to all orders (on-the-energy-shell only) and obtains results for $^4$He, $^6$Li, $^7$Li and $^{12}$C which are in very good agreement with the present Optical Model predictions.

A more complete test of the agreement between the nucleon-nucleon amplitudes obtained from nucleon-nucleus scattering and those obtained from nucleon-nucleon scattering could be made if one also includes the neutron cross-sections and polarizations as well as the corresponding proton data for all nuclei to form a complete analysis. The present analysis, as it stands, appears to show that the nucleon-nucleus interaction can in fact be built up from the nucleon-nucleon interactions and that the Coulomb interaction can be satisfactorily incorporated in the manner described.
APPENDIX A

OPTICAL MODEL POTENTIAL AT 100 MeV - 1000 MeV

In Chapter 2 it was shown that an optical model potential can be derived from nucleon-nuclear scattering amplitudes (formulae 2.11). These amplitudes are assumed to be functions of energy and momentum transfer and therefore it follows that the optical model would be energy dependent.

Here the optical model is treated phenomenologically and energy variations of the model are determined by analysing proton scattering from carbon for incident proton energy from 100 MeV to 1000 MeV. The phase-shifts are calculated from optical model potential through a high energy approximation and are fed into the expressions for the scattering amplitudes expanded into partial waves.

A.1 Optical Model Phase-Shift Analysis

The differential cross-section for the elastic scattering of spin-half particles from spin-zero nuclei is given by

$$\frac{d\sigma}{d\Omega} = |A(\theta) + B(\theta)\sigma \cdot n|^2 = |A(\theta)|^2 + |B(\theta)|^2$$

where $\theta$ is the scattering angle in the centre-of-mass system and $A(\theta)$ and $B(\theta)$ are equivalent to $g(g)$ and $h(g)$, respectively, as defined in formula and can be written as

$$A(\theta) = f_c(\theta) + \frac{1}{2ik} \sum_l \left[ (l+1) \eta^+_{l+1} - l \eta^-_{l-1} - (2l+1) \right] x e^{2i\sigma_l} \frac{p_l(\cos \theta)}{\sqrt{2l+1}}$$

$$B(\theta) = \frac{1}{2ik} \sum_l \left[ \eta^+_{l+1} - \eta^-_{l-1} \right] e^{2i\sigma_l} \frac{p_l(\cos \theta)}{\sqrt{2l+1}}.$$
where $\sigma^\dagger_1$ is the Coulomb phase-shift due to a point charge and $f_c$ is the Coulomb scattering amplitude given by

$$f_c = -\frac{n}{2k} \left( \csc \frac{\vartheta}{2} \right)^2 \exp \left[ -2 \ln \left( \sin \frac{\vartheta}{2} \right) + 2i \sigma_0^\dagger \right]$$

with

$$n = \frac{Z_1 Z_2 e^2}{\hbar v}$$

The polarization is given by

$$P(\theta) = \frac{2 \text{Im}(A B^*)}{|A|^2 + |B|^2}$$

and the reaction (or "absorption") cross-section is given by

$$\sigma_{\text{re}} = \frac{\pi}{k^2} \sum_{l} \left[ (l+1)(1-|\eta^+_l|^2) + 1(1-|\eta^-_l|^2) \right]$$

The reflection coefficients or the S-matrix elements $\eta^+_l$, $\eta^-_l$, correspond to the two possible values of the total angular momentum of the incident nucleons, $j = l + \frac{1}{2}$ and $j = l - \frac{1}{2}$ respectively. For spinless projectiles $\eta^+_l = \eta^-_l$ and the polarization is consequently zero. The reflection coefficients are related to the nuclear phase shift $\delta^+_l$

$$\eta^+_l = e^{2i \delta^+_l}$$

For low and medium energy scattering it is customary to compute the reflection coefficients by numerical integration of the Schrodinger equation with an optical potential of the form

$$V(r) = V_0(r) + V_0 f_1(r) + i W f_2(r)$$
where $V_c(r)$ is the Coulomb potential due to the nuclear charge distribution \(^{(88)}\).

At incident energies above 150 MeV it becomes necessary to take account of departures from non-relativistic kinematics. This may be done within the framework of the non-relativistic Schrödinger equation by modifying the wave-number and optical potential \(^{(48)}\). In the very high energy region nucleon scattering may be described by solving the Klein-Gordon equation either exactly \(^{(89)}\) or approximately \(^{(90)}\). Calculations which involve the numerical integration of the Schrödinger equation or the Klein-Gordon equation become rather lengthy at high energies owing to the large number of partial waves required, while those calculations which use approximate expressions for the amplitudes 'A' and 'B' do not allow the examination of the reflection coefficients which are often of interest \(^{(91)}\). In the following calculation, therefore, we have used the semi-classical or high energy approximation \(^{(87)}\) to obtain the phase shifts $\delta^+_1$ and have then calculated the reflection coefficients, differential cross-section, polarization and reaction cross-section according to the standard formulae given above. The accuracy of the method has been tested \(^{(92)}\) and at 180 MeV has been shown to be accurate out to about 70°.
A.2 Method of Calculation

According to the semi-classical approximation the complete phase shift is given by

\[ \delta(b) = -\frac{1}{\hbar V} \int_0^\infty V(r) \, dz \]

Where \( V \) is the velocity of the incident particle in c.m. system and \( b \) is the impact parameter defined by:

\[ kb = 1 + \frac{1}{2}, \quad r^2 = b^2 + Z^2 \]

The nuclear phase shift \( \delta \) also contains that part of the Coulomb phase shift which arises because the nucleus is not a point. Hence, if the coulomb potential is written in the form:

\[ V_c(r) = \frac{Z_1 Z_2 e^2}{2R_c} \left( 3 - \frac{r^2}{R_c^2} \right) \quad r < R_c \]

\[ = \frac{Z_1 Z_2 e^2}{r} \quad r > R_c \]

the required phase-shift is given by

\[ \delta^+(b) = -\frac{1}{\hbar V} \int_0^\infty \frac{V_N^+(r)}{\epsilon_N} \, dz \]

\[ + n \int_0^{Z_c} \frac{dz}{\epsilon_r} - \frac{n}{2R_c} \int_0^{Z_c} \left( 3 - \frac{r^2}{R_c^2} \right) \, dz \]

where \( Z_c^2 = R_c^2 - b^2 \), \( \epsilon_r = \frac{Z_1 Z_2 e^2}{\hbar V} \) and \( \epsilon_N \) is the potential

\[ V_N^+ = V_0 f_1(r) + i W_0 f_2(r) + \frac{1}{2} \left[ V_{S0} f_3(r) + i W_{S0} f_4(r) \right] \]

\[ V_N^- = V_0 f_1(r) + i W_0 f_2(r) - (l+1) \left[ V_{S0} f_3(r) + i W_{S0} f_4(r) \right] \]

-95-
The functions $f_1(r)$ and $f_2(r)$ are taken to be of the Saxon-Woods form.

$$f_i(r) = \left\{\begin{array}{l}
1 + \exp\left[-\frac{R_i}{a_i}\right]^{-1} \quad i = 1, 2
\end{array}\right.$$  

while the functions $f_3$ and $f_4$ are taken to be of the derivative form

$$f_j(r) = \frac{1}{a_j} \left\{\frac{\exp\left[-\frac{R_j}{a_j}\right]}{1 + \exp\left[-\frac{R_j}{a_j}\right]}\right\}^2 \quad j = 3, 4$$

If $W_{S0}$ is non-zero it is necessary to ensure that

$$W_0 f_2(r) + W_{S0} f_4(r) \leq 0$$
$$W_0 f_2(r) - (1 + 1) W_{S0} f_4(r) \leq 0$$

for all important values of $l$, otherwise the imaginary spin-orbit potential will act as a source.

The Coulomb (point) phase shifts are calculated from the recurrence relation

$$\sigma_l = \sigma_{l+1} - \tan^{-1}\left(\frac{n}{l+1}\right)$$

with the starting value for large $l$ obtained from the series

$$\sigma_l = \alpha(l + \frac{1}{2}) + n(\log \beta - 1)$$
$$+ \frac{1}{\beta} \left(\frac{\sin \alpha}{12} + \frac{\sin \frac{3\alpha}{2}}{360 \beta^2} - \frac{\sin \frac{5\alpha}{2}}{1260 \beta^4} + \cdots\right)$$

where $\alpha = \tan^{-1}\left(\frac{n}{l+1}\right)$, $\beta = \left\{n^2 + (l+1)^2\right\}^{\frac{1}{2}}$
The Legendre polynomials are computed from the recurrence relations

\[ P_1(\cos \theta) = \left( 2 - \frac{1}{\sin^2 \theta} \right) \cos \theta P_{l-1} + \frac{1}{l-1} P_{l-2} \]

\[ P_{l-1} = \cos \theta P_{l-1} - \sin \theta P_{l-1} \]

Using the starting values of \( P_0 = 1, P_1 = \cos \theta P_0 \)

The wave number and other required constants are calculated from the usual formulae of relativistic kinematics. Thus, using the notation:

Kinetic energy of incident particle in Lab system = \( E_{\text{LAB}} \)

Total energy of incident particle in Lab system = \( E_{\text{LAB}} + m_1c^2 \)

Total energy of target particle in Lab system = \( m_2c^2 \)

Total energy of target + incident particle in Lab system = \( E_{\text{LABT}} \)

Total energy of target + incident particle in C.M. system = \( E_{\text{CMT}} \)

Total energy of incident particle in C.M. system = \( \epsilon_1 \)

Total energy of target particle in C.M. system = \( \epsilon_2 \)

Wave number of either particle in C.M. system = \( K \)

Relative velocity of target and incident particle

in C.M. system = \( V \)

One has,

\[ E_{\text{LABT}} = E_{\text{LAB}} + m_1c^2 + m_2c^2 \]

\[ E_{\text{CMT}} = \epsilon_1 + \epsilon_2 = \left( (m_1c^2 + m_2c^2)^2 + 2m_2c^2E_{\text{LAB}} \right)^{\frac{1}{2}} \]

\[ \epsilon_1 = \left[ m_1c^2 + E_{\text{LAB}} - \frac{\hbar^2c^2K^2}{m_2c^2} \right] \frac{E_{\text{CMT}}}{E_{\text{LABT}}} \]
\[ \mathcal{E}_2 = \left[ M_2 c^2 + \frac{\hbar c^2 K^2}{M_2 c^2} \right] \frac{E_{\text{CM}}}{E_{\text{LAB}}} \] ..... (1)

\[ \hbar \nu = \hbar c^2 K \frac{1}{\mathcal{E}_1} \]

\[ \hbar \kappa K = \left[ E_{\text{LAB}} + 2 M_1 c^2 E_{\text{LAB}} \right]^{\frac{1}{2}} \left[ \left(1 + \frac{M_1^2}{M_2^2}\right)^2 + \frac{2 E_{\text{LAB}}}{M_2 c^2} \right]^{\frac{1}{2}} \]

In order to compare the potential used in this calculation with that obtained using the Klein-Gordon equation\(^{(48)}\) it is necessary to multiply the potential used in this calculation by

\[ D = \frac{M_2 c^2}{M_1 c^2} \left[ 1 - \frac{M_2 c^2}{E_{\text{CM}}} \right] \] ..... (2)

Hence, we replace the Coulomb parameter \( n \) and the nuclear potential \( V_N(r) \) by

\[ n' = Dn, \quad V_N^1(r) = D V_N(r) \]

In the non-relativistic limit \( D\hbar \nu = \left[2\hbar c^2 / E_{\text{CM}} / \mu \right]^{\frac{1}{2}} \) where \( \mu \) is the reduced mass \( M_1 M_2 / (M_1 + M_2) \) and \( E_{\text{CM}} \) is the kinetic energy of the particle of mass \( \mu \) in the C-M system.

A.3 Procedure

A computer program originally written in Algol by K. Knight\(^{(94)}\) for an ICL 503 Computer was run to give an approximate value of the parameters \( V_0, W_0, V_{SO}, W_{SO}, R_1 = R_3, R_2 = R_4, a_1 = a_2 = a_3 = a_4 = a \) and with a fixed value of \( R_c \) given by electron scattering for data at various energies. Later, the program was rewritten in Fortran and
a $\chi^2$ minimization subroutine incorporated. The program was then
run on IBM 360/65 at the Atomic Energy Establishment, Harwell. This
utilised the preliminary parameters from the ICL 503 computer runs
together with the experimental differential cross-sections, polariza­
tions and absorption cross-sections. The program computes theoretical
cross-sections and polarizations according to the method described
in A.2 and compared them with the experimental values by calculating
a $\chi^2$ for each experimental data point and fed the information into
the $\chi^2$ minimization subroutine. A total of some 20 calculations of
the cross-sections and polarizations were made with the values of
the parameters set by the $\chi^2$ subroutine before the final values of
the parameters corresponding to a minimum $\chi^2$ were reached by the
$\chi^2$ subroutine. These parameters and the corresponding cross-sections
and polarizations were then printed out.

A.4 Results

The set of best values obtained for the parameters by the $\chi^2$
minimization procedure are given in Table.XI.

The Coulomb radius was kept fixed at $1.3 \text{ A}^{1/3}$ fermi for analyses
at all energies. The diffuseness parameters were taken to be
$a_1 = a_2 = a_3 = a_4 = 0.5$ fermi for all energies.

Runs were made in which the normalization was one of the
parameters which could be varied to give a good fit. If all the
parameters were optimised simultaneously, it turned out, however,
that the values for the other parameters and the corresponding
cross-sections and polarizations were unreasonable. The theory is
not accurate for large scattering angles and the $\chi^2$ minimization
procedure, in effect, tried to balance out the large and the small
### TABLE XI

Optical Model Parameters for $^{12}$C

<table>
<thead>
<tr>
<th>$E_2$ (MeV)</th>
<th>$V_0$ (MeV)</th>
<th>$W_0$ (MeV)</th>
<th>$V_{SO}$ (MeV fm$^2$)</th>
<th>$W_{SO}$ (MeV fm$^2$)</th>
<th>$R_1$ (fm)</th>
<th>$R_2$ (fm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>182.8</td>
<td>-16.0 ± 2.0</td>
<td>-10.0 ± 2.0</td>
<td>5.0 ± 1.0</td>
<td>2.0 ± 1.0</td>
<td>2.3 ± 0.05</td>
<td>3.15 ± 0.1</td>
</tr>
<tr>
<td>310.0</td>
<td>-9.5 ± 2.5</td>
<td>-18.0 ± 2.5</td>
<td>4.0 ± 1.0</td>
<td>0.0 ± 0.5</td>
<td>2.3 ± 0.05</td>
<td>2.6 ± 0.1</td>
</tr>
<tr>
<td>424.0</td>
<td>-3.0 ± 3.0</td>
<td>-25.0 ± 3.0</td>
<td>2.0 ± 1.0</td>
<td>0.0 ± 0.5</td>
<td>2.2 ± 0.05</td>
<td>2.5 ± 0.1</td>
</tr>
<tr>
<td>725.0</td>
<td>13.0 ± 4.0</td>
<td>-42.0 ± 4.0</td>
<td>2.0 ± 1.0</td>
<td>-1.0 ± 0.5</td>
<td>2.1 ± 0.05</td>
<td>2.1 ± 0.05</td>
</tr>
<tr>
<td>970.0</td>
<td>18.0 ± 2.0</td>
<td>-60.0 ± 4.0</td>
<td>1.0 ± 1.0</td>
<td>-1.0 ± 0.5</td>
<td>2.05 ± 0.05</td>
<td>2.05 ± 0.05</td>
</tr>
<tr>
<td>1000.0</td>
<td>20.0 ± 3.0</td>
<td>-65.0 ± 5.0</td>
<td>0.0 ± 1.0</td>
<td>0.0 ± 1.0</td>
<td>2.02 ± 0.05</td>
<td>2.02 ± 0.05</td>
</tr>
</tbody>
</table>
TABLE XI (Continued)

<table>
<thead>
<tr>
<th>$E_2$ (MeV)</th>
<th>$R_3$ (fm)</th>
<th>$R_4$ (fm)</th>
<th>$a$ (fm)</th>
<th>$\sigma_R$ (mb)</th>
<th>$\sigma_{R \text{expt}}$ (mb)</th>
</tr>
</thead>
<tbody>
<tr>
<td>182.8</td>
<td>2.3 ±0.05</td>
<td>3.15±0.1</td>
<td>0.5</td>
<td>220±10</td>
<td>212 ± 4.8</td>
</tr>
<tr>
<td>310.0</td>
<td>2.3 ±0.05</td>
<td>2.6 ±0.1</td>
<td>0.5</td>
<td>215±10</td>
<td>193 ± 4.2</td>
</tr>
<tr>
<td>424.0</td>
<td>2.2 ±0.05</td>
<td>2.5 ±0.1</td>
<td>0.5</td>
<td>215±10</td>
<td>210 ± 4.6</td>
</tr>
<tr>
<td>725.0</td>
<td>2.1 ±0.05</td>
<td>2.1 ±0.05</td>
<td>0.5</td>
<td>221±10</td>
<td>-</td>
</tr>
<tr>
<td>970.0</td>
<td>2.05±0.05</td>
<td>2.05±0.05</td>
<td>0.5</td>
<td>248±10</td>
<td>254 ± 37</td>
</tr>
<tr>
<td>1000.0</td>
<td>2.02±0.05</td>
<td>2.02±0.05</td>
<td>0.5</td>
<td>248±10</td>
<td>258 ± 17</td>
</tr>
</tbody>
</table>
scattering angle data by shifting the absolute scale of the cross-
sections. Optimization of the normalization was consequently,
discarded and the $\chi^2$ minimization was performed on the potential
depths and the radius parameters $R_1$ and $R_2$ only for different
predetermined values of normalization constant.

In some cases the results of the $\chi^2$ search failed to fit the
data at an interesting region, for example, the negative polarization
around 18° lab at 424 MeV data. For such cases the $\chi^2$ minimization
procedure was discarded and the values of the parameters were determined
by visual inspection of many sets of predictions.

The errors quoted in Table XI thus, do not represent the true
upper and lower limits to the parameters but rather the order of
magnitude of the errors involved estimated visually, as no reliable
mathematical prescription exists.

The theoretical predictions obtained using the set of parameters
given in Table XI are shown in figures 20 to 30 for elastic differen-
tial scattering cross-sections and polarizations; and the real and
imaginary reflection coefficients for the case of 182 MeV and 970 MeV
protons (figure 31 and 32 respectively) to show the number of waves
involved in the calculations and the transparency of the carbon nucleus
at the energies considered. The imaginary reflection coefficients at
970 MeV are essentially zero for all partial waves while the real
reflection coefficients contribute almost equally to the scattering
for the first ten partial waves. These two facts show that the nucleus
is behaving in a strongly absorbing manner. In contrast at 180 MeV
the nucleus is seen to be relatively transparent because the contribu-
tions of the real and imaginary reflection coefficients to the
X - EXPT. AT 182 Mev [3]
UN-NORMALIZED

\[ \frac{d^2 \sigma}{d\Omega} \text{(mb/sr)} \]

\[ \theta \text{ c.m.} \]

FIG. 20
scattering decrease gradually with increasing partial wave number.

A.5 Discussion of results

Fits to small angle differential scattering cross-sections (for angles less than the Coulomb interference region) for 310 MeV and 970 MeV were not achieved initially. At these energies, the incident proton velocity is relativistic and thus one should treat the Coulomb interaction relativistically. The nuclear part of the interaction was already being treated relativistically as explained in section A.1 but the Coulomb part did not include any relativistic corrections. Following the argument of Elton\(^{(48)}\) that when solving a Schrödinger equation, relativistic effects can be incorporated if the wave vector is taken to be relativistic and the potential is replaced by

\[
\vec{V}(r) = D \cdot \vec{V}(r) \cdot \frac{Z_1 Z_2 e^2}{\hbar \nu}
\]

where \(D\) is defined in equation A.2, the Coulomb parameter \(n = \frac{Z_1 Z_2 e^2}{\hbar \nu}\) was modified to \(n' = Dn\). Making this change led to good agreement with the smallest angle data. In detail a comparison with the data shows:

(a) The prediction for the 182 MeV differential cross-section and polarization data (figure 20 and 21) is good except for large angles but here the theory would not hold so well as for small angles. The predicted value of the absorption cross-section given in table XI is in good agreement with the experimental value, within the errors involved.

(b) The 310 MeV data is well fitted. Figures 22 and 23 show the predictions of differential cross-sections and polarization.
\( \bar{\text{EXPT AT 310 MeV}} \) [4]
EXPT AT 424 MeV [5]
X EXPT. AT 725 Mev [6]  
WITH 15 Mev RESOLUTION
EXPT AT 725 MeV [6]
WITH 15 MeV RESOLUTION

FIG. 27
EXPT AT 970 MeV [7]

FIG. 28
EXPT. AT 1000 MeV [8]
(c) The fit to the polarization at 424 MeV, figure 25, seems to be good. The small angle shape is reasonably reproduced and the sign and minimum is at the right place. The differential cross-section was taken to be an average of the right and left cross-sections.

(d) The 725 MeV data was taken with an energy resolution of 15 MeV. The theory fits the differential cross-sections very well at small angles where one would expect small inelastic contributions but deviates wildly at large angles. The fit to the polarization (figure 27) is reasonable.

(e) The fit to 970 MeV differential cross-section data is excellent (figure 28). The polarization is shown in figure 29 and reflects the amount of polarization to be substantial at such a high energy.

(f) At 1000 MeV the agreement is again good up to the first diffraction minimum (figure 30).

A.6 Variation of parameters with Energy

Figures 33 and 34 show the variation of the real central potential depth $V_0$ and the imaginary central potential depth $W_0$. Both $V_0$ and $W_0$ include the factor D.

As can be seen from the figures, the agreement with the values predicted for $V_0$ by C. Batty (95) and Palevsky (8) is very good. $V_0$ changes sign between 400 MeV and 500 MeV incident proton energy. Figure 34 shows a good agreement between present and Batty only for $W_0$ at about 1000 MeV but for incident proton energies less than 500 MeV. Batty's values are higher than the present analyses. Palevsky's value is high at 1000 MeV. He did not use relativistic corrections for the potential and when this is done his value agrees well with the present analyses.
REFLECTION COEFFICIENTS

180 Mev

\( \text{Re} \eta^+ \)
\( \text{Re} \eta^- \)
\( \text{Im} \eta^+ \)
\( \text{Im} \eta^- \)

PARTIAL WAVE No.

FIG. 31
To compare the present analyses with that of Batty who uses gaussian form for the radial distribution of the potentials, we should, in fact, compare volume integrals of the potentials. This quantity would be rather more independent of the radial form and radius chosen for the potentials (Feshbach\textsuperscript{96}). In figure 35 and 36 the volume integrals of $V_o$ and $W_o$ are compared with Batty's and again for the real central potential the results are in agreement with Batty. For the imaginary central potential the agreement is also fairly good.

Batty used one radius parameter for all the central and spin-orbit potentials. Thus, only the radius parameter for the real part of the potential of the present analyses is compared with his results (figure 37). The comparison is done through an equivalent radius $R_{EQ}$ defined as:\textsuperscript{(47)}

$$R_{EQ}^2 = R^2 \left[ 1 + \frac{10}{3R^2} \frac{2a^2}{2} + \frac{7}{3R^4} \frac{4a^4}{4} \right] / \left[ 1 + \frac{2a^2}{R^2} \right]$$

where $R$ is the radius parameter used in the Saxon-Woods form-factor and $a$ is the diffuseness parameter. The agreement with Batty and Palevsky is within the uncertainties.
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THE OPTICAL POTENTIAL FOR $^{12}$C BETWEEN
100 MeV and 1 GeV.

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We have analysed the elastic scattering, polarization and reaction
cross-sections for proton scattering from $^{12}$C at 182, 310, 424, 725, 970 and
1000 MeV, using a Saxon-Woods potential of the form

$$V(r) = V_0 f(r, R_1, a) + i W_0 f(r, R_2, a) + V_{\text{Coulomb}} \frac{1}{r} \frac{d}{dr} f(r, R_1, a)$$

$$+ i W_{\text{so}} \frac{1}{r} \frac{d}{dr} f(r, R_2, a)$$

where

$$f(r, R, a) = \left[1 + e^{(r-R)/a}\right]^{-1}$$

We use a modified form of the non-relativistic Schrödinger equation which
takes account of relativistic kinematics and use the semi-classical approx­
imation to obtain the phase shifts which are then inserted into the usual
partial wave expansions for the differential cross-section, polarization and
reaction cross-section. The object of this work was to examine the energy
dependence of the parameters and to make a comparison with the results
obtained by Batty (1) who used a semi-classical approximation for the scat­
tering amplitude and an optical potential with the same radial form as the
nuclear matter distribution calculated from oscillator wavefunctions.
Satisfactory fits to the data were obtained with a diffuseness $a = 0.5$ fm at all energies. The parameters obtained are summarised in Figures 1-3 where they are compared with the results of Batty, and of Palevsky et al$^{(2)}$ at 1 GeV. It can be seen that our results confirm the
energy dependence observed by Batty and are in full agreement with his result that the real potential changes sign at about 400 MeV. For energies above 700 MeV we obtain fits to the data with \( R_1 = R_2 \) but for energies below this we require \( R_2 > R_1 \). Batty took the same size parameters for the real and imaginary parts of the potential at all energies, and this accounts for the difference between the values of \( W_0 \) obtained at the lower energies. The volume integrals of the potentials are in reasonable agreement. The magnitudes of the parameters \( V_0, W_0 \), found at 1 GeV, are consistent with those calculated using impulse approximation.

Figure 2.
Figure 3

In Batty's work the size parameters was the oscillator length parameter $a_1$. However, this parameter was allowed to vary in the analysis, and the ratio of the parameter for the nuclear matter distribution required to fit the proton scattering data and the value of $a_1 = 1.64$ fm for the proton distribution required to fit the electron scattering data is shown in Figure 3. (It is assumed that the proton and neutron distributions are identical, which seems very reasonable for $^{12}$C.) For our Saxon-Woods shape the relevant size parameter is the equivalent uniform radius $R_{eq}$ defined by the relation
\[ R_{EQ}^2 = R^2 + \frac{7}{3} \pi^2 a^2 \]

The value of \( R_{EQ} \) for the proton distribution is \( \approx 3.10 \) fm. It can be seen from Figure 3 that the values of \( a_1/1.64, R_{EQ}/3.10 \) for the real part of our potential and for the potential of Palevsky et al at 1 GeV, are in satisfactory agreement. These results also suggest that the nuclear matter distribution can be investigated through high energy elastic scattering.

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A SELF-SUPPORTING TARGET OF SOLID HYDROGEN
FOR NUCLEAR PHYSICS EXPERIMENTS

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A cryogenic hydrogen target is described in which the hydrogen is in the form of a self-supporting cylinder of solid hydrogen, 1 cm diameter and 5 cm high. With large holes cut through the radiation shields to permit the unhindered passage of charged particle beams, useful target lifetimes of about three hours have been achieved.

1. Introduction

In medium energy proton-proton scattering experiments a typical liquid hydrogen target consists of an upright cylinder made of mylar film, the upper and lower ends being closed with metal plates provided with connections to a liquid hydrogen reservoir. This hydrogen target is surrounded by radiation shields and the whole is supported within a vacuum enclosure. The design aim is, naturally, to minimize the amount of material other than hydrogen in the direct path of the incident proton beam. Although the amount of such non-hydrogenous material is small (less than 20 mg/cm², if we exclude the windows of the vacuum enclosure) there are at least two types of experiment where even this small amount of material may be considered intolerable. These experiments are the study of bremsstrahlung production in nucleon-nucleon scattering and the study of very small angle proton-proton elastic scattering.

In bremsstrahlung experiments it is essential to minimize the amount of non-hydrogenous material in the target region because the (p, 2p) cross section of most elements is about two orders of magnitude greater than that for the (p,py) process. An obvious solution to this problem is to use counter telescopes designed so that they cannot see the walls of the hydrogen target directly but view only the liquid contained within. Unfortunately, this technique is not suitable for small scattering angles (less than, say, 20°). A second solution is to use a double container target construction in which the walls of the inner vessel need withstand only the hydrostatic pressure of the liquid hydrogen that it contains and which, therefore, may be constructed of very thin material; one such target used 200 µg/cm² aluminium foil. The inner vessel is surrounded by a second much larger vessel containing hydrogen gas in pressure equilibrium with the liquid in the inner vessel. The outer vessel possesses relatively thick walls and provides the mechanical isolation from the vacuum of the cryostat. Because the outer vessel contains gas rather than liquid it may be quite large so that the counter telescope design can ensure that only the thin walls of the inner vessel are viewed directly. In this manner both the thickness of the target and the minimum scattering angle which can be investigated are reduced.

Neither of the techniques used for bremsstrahlung experiments is suitable at very small scattering angles where the importance of target wall thickness is enhanced by the need to use thin hydrogen targets, which in turn is dictated by a consideration of the multiple Coulomb scattering problem.

It has long been appreciated that a target of solid hydrogen could, in principle, be constructed such that the hydrogen would be self-supporting and would, therefore, need no container to produce background events. The present paper describes work which has resulted in translating this idea into a practical target. Although developed for the small angle scattering experiment, the desirability for similar targets in bremsstrahlung experiments will be obvious.

2. Previous solid hydrogen cryostats

There have been two recent publications which describe cryostats constructed for thermonuclear research purposes in which small quantities of solid hydrogen are produced within a vacuum enclosure without the support provided by a thin-window container. One of these is of possible interest for low-energy nuclear physics experiments because it provided a solid hydrogen foil 2 mm diameter and about 1 mm thick initially. It was obtained by intro-
Fig. 1. The solid hydrogen target.
ducing hydrogen gas into a vacuum enclosure containing a cold finger at liquid helium temperature. The gas condenses onto the finger, covering a 2 mm diameter hole through the action of surface tension before solidifying. The cover isolating the cold finger from the vacuum of the cryostat may then be removed so exposing the solid hydrogen foil to bombardment with light from a laser. The lifetime of this foil was short – a few minutes – as a result of sublimation owing to a lack of radiation shielding. The second cryostat\(^3\) was more exotic in that a narrow strip of solid hydrogen was extruded; from this strip pellets 0.25 mm long by 0.25 mm diameter were mechanically punched and projected across the vacuum chamber to act as moving targets for a laser gun. Neither of these cryostats satisfy the requirements of the nuclear physicist but they do demonstrate the current state of the technology of such devices.

3. The original concept

We first outline the method by which we originally hoped to produce a solid hydrogen target and mention briefly the difficulties encountered before continuing to describe the final target design and operating procedure.

The original intention was to adopt the method of ref. 8 and to extrude a sheet of solid hydrogen 1 cm wide by a few millimetres thick. This represents an increase in target volume over those of refs. 7 and 8 by a factor of a hundred or more. A small cryostat was constructed to act as a test vessel. This cryostat, with later additions, is shown in fig. 1. In order to reduce the size of the cryostat no liquid nitrogen reservoir was provided; instead, the nitrogen radiation shield was cooled by a continuous flow of liquid nitrogen through the heat exchanger coils. The volume of the liquid helium reservoir was 1 litre. In use, the helium reservoir required refilling after about two hours because large slits were left in the radiation shields for viewing purposes, but when the radiation shielding was complete refilling was necessary only after eight hours.

Later additions to the cryostat include the innermost vertical tube (1 cm diameter) and the target forming appendage. In the prototype the 2.54 cm diameter tube was closed at the bottom with a plug or nozzle. A suitably shaped slot was cut in the nozzle through which the hydrogen was to be extruded. The slot was closed initially by means of the pedestal – inserted up through the base of the vacuum vessel – which pressed a knife-edge into the lower face of the nozzle. The operating procedure envisaged was as follows. The vertical plunger was to be retracted, the vessel evacuated, cooled and filled with liquid helium and the pedestal pressed firmly against the slot in the nozzle. Hydrogen gas would then be permitted to enter the inner cylinder; the gas would condense and solidify at the bottom of the nozzle. The pedestal would then be removed and by forcing the plunger down onto the hydrogen a ribbon of solid hydrogen would be extruded through the slot in the nozzle. It was appreciated that this extrusion would work best at an elevated temperature and heaters were provided. The permissible working temperature is, of course, limited by the vapour pressure of the solid. The vapour pressure of solid hydrogen is about $4 \times 10^{-7}$ torr at liquid helium temperature, but rises very rapidly with temperature reaching $2 \times 10^{-5}$ torr at about 5 K.

In practice it proved impossible (without major modifications) to obtain an adequate vacuum seal across the slot in the nozzle during the period when hydrogen gas was being admitted. It had been expected that the hydrogen would quickly solidify and block any small vacuum leaks there may have been; instead, the gas was pumped away with remarkable efficiency. As it was not possible to fill the extrusion nozzle with solid hydrogen the extrusion principle was abandoned in favour of simply pushing solid hydrogen out of the tube (no nozzle being fitted) once the pedestal had been withdrawn. This as least permitted access to all the solid hydrogen that had deposited on the tube walls. In this manner small quantities of solid hydrogen (at most 0.5 cm\(^3\)) were obtained on the bottom face of the plunger and could be positioned in the centre of the vacuum chamber for viewing. It was discovered during this phase that the bonding by the solid hydrogen between the plunger and the tube walls was considerable. Warming the lower end of the cylinder freed the plunger but the rise in temperature and, consequently, of the vapour pressure of the hydrogen usually led to a very rapid loss of helium from the reservoir. Consequently, the use of heaters was discontinued. The small quantities of solid hydrogen obtained in this way were of no interest as a practical target.

Having experienced such unexpected difficulty in obtaining a removable vacuum seal at liquid helium temperatures it was natural to turn our attention to using a once-only seal which would be punctured after the hydrogen had solidified in the target-forming chamber.

4. The final target

The target-forming chamber in current use is an appendage soft-soldered onto the bottom of the
cylinder at the position of the nozzle in the original design. The chamber is simply an open-ended cylinder closed at the lower end by a 0.012 cm thick sheet of indium clamped firmly between two copper flanges, thus providing an excellent vacuum seal. The chamber was provided with a viewing port covered with mylar film.

The 1 cm diameter tube inserted down the centre column of the cryostat was incorporated during the early development work when it was decided that the thermal coupling between the helium reservoir and the solid hydrogen was too good. This inner tube has been retained in the final system solely because it has proved to be of a convenient diameter for use in a subsequent experiment.

The operation of the target, resulting in the production of a cylinder of solid hydrogen 1 cm diameter and several cm high (see fig. 2), is essentially trivial. With the indium sheet clamped in position the main vacuum vessel and the inner tube are evacuated simultaneously. The plunger (possessing a copper head) is then retracted above the hydrogen gas inlet. Liquid nitrogen is forced through the heat exchanger coils and the helium vessel is pre-cooled with liquid nitrogen. A measured quantity of hydrogen gas is next admitted from a large gas reservoir whilst liquid helium is being transferred to the helium reservoir. Liquid hydrogen begins to accumulate in the chamber as the target chamber temperature falls below 20.4 K (the hydrogen liquefaction temperature). A carbon resistance thermometer attached to the target forming chamber is used for monitoring the temperature. The filling of the target chamber and the subsequent freezing of the liquid (at 14 K) may be watched through the viewing port via slits left in the radiation shields. When the liquid helium reservoir has been filled – by which time the hydrogen temperature will have fallen below 5 K – the plunger is slowly lowered to the top of the solid hydrogen and is then gradually forced downwards (with gentle taps from a hammer!) so as to burst the indium seal and to push the cylinder of solid hydrogen out into the vacuum chamber. Cylinders of up to 5 cm height have been formed in this manner. A typical example is shown in the photograph (fig. 2).

Fig. 2 illustrates several features of the method of production. The bottom of the cylinder is hemispherical due to deformation of the indium seal during the gas introduction phase. The indium sheet is largely retained within the volume of the target-forming chamber but a small piece can be seen poking out. The striations caused by the jerky production technique are apparent. The target is of highly irregular shape and there can be no question of measuring the quantity of solid present with accuracy, other than through a nuclear scattering experiment. Actually, the example of fig. 2 is
perhaps the least regular in shape that has been produced and further work demonstrated that the irregularity was caused mainly by the presence of the (non-essential) observation window.

Provided the plunger is not forced out below the bottom of the target-forming chamber the hydrogen cylinder does not fall off, despite vibration in the system from a backing pump.

5. Operational experience

The cryostat has been used in preliminary measurements of very small angle proton-proton scattering at 160 MeV. For this experiment it was necessary to leave 4 cm diameter holes in the radiation shields in order to permit the incident proton beam to pass through the cryostat unhindered. However, the absence of complete radiation shielding results in an increased sublimation rate of hydrogen from the target. By recording the scattering of protons from the target we found that the target volume decreased by about 30% over a period of two hours. The loss of material was manifested as a uniform reduction in diameter of the cylinder. Eventually, the hydrogen cylinder detached itself from the target-forming chamber. For larger scattering angles a very thin – but complete – radiation shield can be tolerated and the lifetime of the target has been extended in excess of five hours for a 30% loss of hydrogen. Alternatively, a considerable extension of this lifetime could be obtained by reducing the temperature below 4.2 K. Loss of hydrogen due to the proton beam is negligible as this provides a heat input of only about a milliwatt. This is to be compared with the remarkably high thermal conductivity of solid para-hydrogen of about 1 W/cm K at 4.2 K.

The major disadvantage with this system is the obvious one that only a single attempt can be made to produce a target since a new indium seal must be installed before a further attempt is possible. Fortunately, the probability of producing a good sample is very high in practice. There would appear to be no difficulty in extending the present technique to the production of either larger or smaller samples of solid hydrogen. Also, it should be possible to design a vacuum seal at liquid helium temperature which can be re-established without the need to dismantle the cryostat, although it is apparent that a considerable force is necessary to effect this vacuum seal.

The authors wish to thank Mr. E. Wood for roughing out the design of the original cryostat and for some of the initial vacuum testing. They are also very grateful to Messrs. K. Done and L. R. Caldecourt for the detailed design work and their continued assistance throughout the development period.

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United Kingdom Atomic Energy Authority
RESEARCH GROUP
Report

PRODUCTION OF K X-RAYS
BY 160 MeV PROTONS

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Production of K X-Rays by 160 MeV Protons

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and

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Abstract

K shell ionization cross-sections have been measured for elements between Z = 26 and Z = 92 using 160 MeV protons as incident projectiles. The results are compared with two theoretical models, the PWBA calculations of Khandelwal, Choi and Merzbacher and the binary-encounter model of Garcia. The results are in qualitative agreement with both these non-relativistic models. The necessity for a relativistic theory is emphasized by comparison with recent high energy electron measurements.

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4. Comparison of the high energy proton induced cross-section data with PWBA theory in the form of the "universal curve" of Merzbacker and Lewis. Note that the 2-28 MeV data of Bissinger et al. are for three selected elements as a function of energy, whereas the 160 MeV proton data are for a range of elements at a single energy. (The 160 MeV data have been reduced by the factor 1.26, see text.)

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Measurement of K-Shell Ionization Cross-Sections for 160 MeV Protons

1. Introduction

Many experimental and theoretical studies have been made of the production of K X-radiation from various materials by heavy particles of very low energy. The early work in this field was reviewed by Merzbacher and Lewis \(^1\) in 1958. Until very recently, subsequent work \(^2\) was also concerned with low energy projectiles. When the present work was initiated the most recent experimental study was that of Sellers et al \(^3\) who used alpha particles in the energy range 1 to 5 MeV. During the course of the present work, however, further measurements have become available including those by Bissinger et al \(^4\) using 2 to 28 MeV protons, and of Richard et al \(^5\) using 6 to 10 MeV protons and 15 to 19 MeV oxygen ions. Of particular related interest are the ultra-relativistic electron measurements of Middleman et al \(^6\) using electrons in the range 150 to 900 MeV.

Previous to the present work all the heavy particle K X-radiation cross-section data have been obtained at quite low projectile energies for which a non-relativistic theoretical treatment is expected to be reasonably accurate. A major incentive for the present measurements was to extend the proton work to high energies in order to investigate the importance of relativistic effects. In particular it was of interest to test whether the production cross-section, which was expected to rise to a maximum for a proton velocity close to the r.m.s. velocity of the K-shell electron, would indeed fall as \(1/E\) on the high energy side of the peak as predicted. The present high bombarding energy readily permits a study of the excitation cross-sections for even the heaviest stable elements. This is in contrast with the other recent measurements (Refs. 2 to 5) in which only light...
elements (€ \leq 29) were studied. Whereas previous work has usually been for a single element as a function of energy, the present work is for a wide range of elements at a single energy - a consequence of the type of particle accelerator used. According to the existing non-relativistic theories the two different approaches provide equivalent results.

2. Experimental Arrangement

The experimental geometry is shown in Fig. 1. The 160 MeV proton beam from the Harwell synchrocyclotron was focussed onto the targets by three pairs of quadrupoles to form a spot of about 1 cm diameter overall. After emerging from the evacuated target chamber the proton beam first passed through a 1 mm thick sheet of polythene used as a target for a p-2p intensity monitor and was then collected in a 10 cm diameter Faraday Cup for normalization purposes. The target chamber was constructed so that the target foils were observed by the X-ray detector at a scattering angle of 155° to the beam direction. This large angle was required to reduce the intensity of high energy protons scattered by the target which might subsequently pass through the detector.

The X-ray detector was a Kevex Si(Li) detector of 1.10 cm$^2$ area and 5 mm thick. This detector was intended for use at room temperature as a detector of heavy charged particles but when cooled to liquid nitrogen temperature it was found to exhibit an energy resolution of 1.3 keV at 6 keV, which was quite adequate for the present work. Si(Li) was chosen instead of Ge(Li) because the narrow region of energy sensitivity of the Si(Li) detector is desirable when used in regions of very high background radiation levels, such as exist in the experimental areas of the synchrocyclotron.

The signals from the Si(Li) detector were amplified using double
delay-line clipping to reduce the overloading effect arising from the very large signals caused by high energy charged particles passing through the detector. The amplified signals were fed into an ADC coupled to the on-line Honeywell DDP-516 computer using the standard CAMAC interface electronics. The computer was used in multiprogramming mode so that the software permitted data reduction, graph plotting and magnetic tape data storage and retrieval, all to be carried on simultaneously with data acquisition. Counting rates were generally in the range 500 to 5,000 cps, using beam currents of between 0.3 and 3.0 nA. The normal duty-cycle of the beam extracted from the synchrocyclotron was increased to about 20% with the use of the auxiliary Cee electrode but nevertheless the dead-time corrections were large (5 to 50%) and their determination was a major problem. To enable dead-time corrections to be made a pulser was fed both into the detector preamplifier and into a separate scaling system. To allow for the pulsed nature of the proton beam this pulser was triggered by signals from the p-2p monitor. However, corrections based on this technique were not entirely satisfactory, presumably because the duty-cycle for the characteristic X-rays is different from that for the backgrounds (which arise from long life induced activity) and consequently measurements were made at several beam levels so that an extrapolation to zero count rate could be made for each target. Additional complications arose from the fact that the effective duty-cycle of the beam varied with time and with beam level. Use of very low beam levels to minimize counting losses was impracticable owing to the difficulty in integrating accurately beam currents below 0.1 namp.

A consequence of the high background in the experimental area was the necessity to use thick targets; the target thickness used lay in the range
to 200 mgm/cm$^2$. Corrections were made for the absorption by the target of its own characteristic K X-radiation. The use of thick targets also introduced the need to allow for the subsidiary effect of characteristic K X-radiation produced by the energetic $\delta$-rays which in turn are produced by the incident protons whilst passing through the target. This contribution was taken into account by making measurements for each element as a function of target thickness. Some care was necessary in the extrapolation to zero thickness because the $\delta$-ray contribution is not expected to be linear with thickness and only in the limits of very thin or very thick targets should a linear dependence be obtained; in the latter case the extrapolation to zero thickness does not give the desired cross-section. The two limiting regions occur because the $\delta$-ray contribution arises from two distinct physical processes. For very thin targets only the direct ionization by the $\delta$-ray is important but for thick targets the secondary process involving bremsstrahlung production followed by photoelectric absorption provides an important contribution. Approximate calculations of the apparent ionization cross-sections due to $\delta$-ray production are presented in the Appendix, together with typical comparisons of these calculations with experiment. Where necessary these calculations were used to guide the extrapolation of the experimental cross-sections to zero target thickness.

Except for the two elements Ba and Tb, the targets were all in the form of self-supporting metal foils. The two exceptions were provided in the form of the oxides painted onto a hydrocarbon backing. Target thicknesses were estimated both by weighing and by measurements of X-ray attenuations using appropriate radioisotope sources and the convenient tabulation of absorption coefficients of Dewey et al.$^9$). Since the accuracy of the coefficients presented in this tabulation is expected to be only $\pm 5\%$ on
average, the attenuations were measured for several different photon energies. A generous uncertainty of $\pm 3\%$ was attributed to the target thickness measurements for the self-supporting foils, but the much larger uncertainty of $\pm 20\%$ was attributed to the two oxide targets as these were noticeably non-uniform.

The detector efficiency was determined over the energy range 6 to 120 keV using radioisotope sources of $^{244}$Am, $^{160}$Tb and $^{57}$Co assuming the relative $\gamma$-ray yields quoted by Lederer et al\textsuperscript{10} and the $X:\gamma$ yield for $^{57}$Co given by Campbell et al\textsuperscript{11}. The shape of the low energy side of the efficiency curve near the 6.4 keV $^{57}$Co line was obtained by calculation from the known materials of the cryostat windows and the assumption that any residual loss of efficiency at 6.4 keV was attributable to a Si dead-layer on the front surface of the detector. The active area of the detector was determined using an $^{55}$Fe source and a collimator of known dimensions. Interpolated efficiencies for each target element are given in Table I.

3. The Experimental Measurements

Typical energy spectra are displayed in Fig. 2. For the heavier elements Ta through U it was necessary to interpose an appropriate filter between the target and detector to absorb preferentially the highly intense L X-radiation. This introduced only a small correction for absorption of the K X-rays.

For all the elements except Fe and Cu the photopeak intensities were extracted from the spectra assuming a linear background dependence. For the two light elements this was inapplicable and the background was estimated by eye; fortunately, the necessary background subtractions were small. Some difficulty was experienced in subtracting the background from the spectra for the thinnest gold target (2 mg/cm$^2$) and for the uranium target.
(207 mg/cm²). The difficulty for uranium was partly due to the small
cross-section, to the high background and to the $\text{K}_\alpha_1$ and $\text{K}_\alpha_2$ radiations
being sufficiently separated as to reduce the apparent cross-section
relative to the lighter targets for which these two radiations were not
resolved. It is pertinent to note that the cross-section for production of
K X-rays from uranium is comparable with the cross-section for induced
fission.

For elements with atomic number below 65 both $\text{K}_\alpha$ and $\text{K}_\beta$ radiations were
summed together but for elements with $Z \geq 65$ only the $\text{K}_\alpha$ contribution was
taken from the spectra and the $\text{K}_\beta$ contribution was estimated from the known
$\text{K}_\beta : \text{K}_\alpha$ ratios as obtained from X-ray fluorescence studies.

It was assumed implicitly in the present experiment that the emission
of characteristic K X-radiation is isotropic. This point has been investi-
gated and found valid by Merzbacker and Lewis for L X-rays from gold and
was again checked in the high energy electron measurements of Middleman et
al.

The final K-shell ionization cross-section data are listed in Table I,
together with the assumed fluorescent yields and the detector efficiency.
There is an overall normalization uncertainty to be applied to these data.
This uncertainty contains contributions from the detector sensitive area,
the solid angle subtended at the target, the Faraday Cup calibration and for
the collection of $\delta$-rays in the Faraday Cup - these $\delta$-rays being produced in
the window of the Faraday Cup vacuum chamber. Summing these contributions
quadratically yields an overall uncertainty of $\pm 7\%$.

Some preliminary measurements were also made using the 85 MeV deuteron
and the 160 MeV alpha particle beams from the synchrocyclotron. These
measurements were bedevilled by counting rate and dead-time problems because
the long duty-cycle facility exists only for the proton beam.

4. Results

4.1 Proton data

The final total cross-section data for 160 MeV protons are listed in Table I and are presented graphically in Fig. 3. The solid curve is derived from the PWBA calculations conveniently tabulated by Khandelwal et al. These calculations were made for low energy projectiles (possessing non-relativistic hydrogenic wave-functions for the atomic states. The total cross-section \( \sigma_K \) is expressed as:

\[
\sigma_K = \frac{8\pi z^2 a_o^2}{Z_K^2 \eta_K^2} f_K(\eta_K, \xi_K)
\]

In this expression \( z \) is the projectile charge, \( Z_K \) is the effective nuclear charge as seen by a K-shell electron and \( a_o \) is the Bohr radius of hydrogen. The quantity \( \eta_K \) is dimensionless and is given by

\[
\eta_K = \frac{\eta_{K}}{M^2 R_{\infty}}
\]

where \( R_{\infty} \) is the Rydberg constant (13,605 eV). Finally, \( f_K(\eta_K, \xi_K) \) is the quantity which is actually tabulated by Khandelwal et al. The parameter \( \xi_K \) is the K-shell screening number

\[
\xi_K = \frac{\xi}{Z_K^2 R_{\infty}} = 1
\]

where \( \xi \) is the observed K-shell ionization potential; \( \xi_K \) is the lower limit of the integration defining \( f_K \). The effective charge is taken to be \( Z_K = z - 0.3 \), as is customary. Rather than use the \( \xi_K \) values.
computed from the simple relationship above we adopt the values given by Walske\textsuperscript{14}) where $\sigma_K$ has been modified so as to partially correct for the relativistic contributions to the binding energy $\xi_K$. In Fig. 3 the theoretical PWBA curve has been multiplied by the factor 1.26 in an attempt to correct at least partially for the relativistic velocity of the 160 MeV protons. It is assumed that the most important kinematic factor in the PWBA formula for $\sigma_K$ is the $\frac{1}{v^2}$ term which enters through the quantity $\eta_K$. The impulse approximation calculation - to be discussed later - also possesses such a factor. Thus, both non-relativistic theories predict $\sigma_K \rightarrow 0$ as $E \rightarrow \infty$, which is an unlikely behaviour. It would be more reasonable if $\sigma_K \rightarrow \text{constant as } E \rightarrow \infty$ (or $v \rightarrow c$). This behaviour is produced by using the true relativistic velocity in the $\frac{1}{v^2}$ term instead of the velocity computed using non-relativistic kinematics; hence the factor 1.26. In practice $\sigma_K$ will begin to increase as the energy moves into the highly relativistic region because of the relativistic enhancement of the transverse component of the electromagnetic field.

The "theoretical" curve in Fig. 3 provides a good qualitative description of the data. Perhaps surprisingly, the fit is best for high Z elements where the lack of a relativistic description for the atomic states would have been thought most serious. Fig. 3 also presents some K-shell ionization data obtained for electrons: these data will be considered later.

In Fig. 4 we again present the ionization data for 160 MeV protons, but this time in a form which permits comparison with data for other heavy projectiles taken over a wide range of energies. Merzbacher and Lewis\textsuperscript{1}) have shown that there is an approximate relationship
\[ f_K(\eta_K, \phi_K) = \theta_K f_K\left(\frac{\eta_K}{\phi_K^2}, 1\right). \]

Since this enables us to write
\[ \frac{\theta_K \frac{\eta_K^4}{\phi_K^2} \sigma_K}{z^2} = 8\pi a_0^2 \frac{\phi_K^2}{\eta_K} f_K\left(\frac{\eta_K}{\phi_K^2}, 1\right) \]
we see that a "universal" curve can be obtained by plotting
\[ \frac{\theta_K \frac{\eta_K^4}{\phi_K^2} \sigma_K}{z^2} \]
against \( \frac{\eta_K}{\phi_K^2} \). This approximation is quite good on the low energy side of the maximum, where existing data for energies such that the abscissa lies between 0.005 and 1.0 cover six orders of magnitude of the ordinate. However, it does not hold very well for the high energy side of the maximum as can be seen (Fig. 4) by the differing results for \( \theta_K = 0.78 \) and \( \theta_K = 0.95 \). In this presentation the 160 MeV proton data have been scaled for the effect of relativity, instead of the theoretical curves. (Also, in this and later figures, the data for the two oxide targets have been omitted for clarity.) Shown together with the 160 MeV data for \( \Phi \) between 26 and 92 are the 2 to 28 MeV proton data of Bissinger et al.\(^4\) for Ca, Ti and Ni. It is clear that the fit of theory to experiment is poorer for the low energy proton data than for the 160 MeV data. In fact the FWBA theory underestimates the Ti and Ni data by about 50%. Perhaps it should be noted that the absolute errors on the low energy data are quite large (11-13%) and that for relatively light elements such as Ca, Ti and even Ni there is a possibility that the fluorescent yields may be substantially in error.

A simpler method of presenting K-shell ionization data has been proposed by Garcia\(^15\). He has shown that a classical binary-encounter model\(^16\) (an impulse approximation calculation) provides a very simple

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[1] A simpler method of presenting \( f_K \) data has been proposed by Garcia. He has shown that a classical binary-encounter model (an impulse approximation calculation) provides a very simple
scaling law such that $\xi_K^2 \sigma_K$ plotted against $E/\xi_K$ defines a universal function. Such a graph is more convenient than the PWBA method which involves the parameters $\eta_K$ and $\sigma_K$. Moreover, this impulse approximation calculation provides a better fit \(^{16}\) to the low energy data ($\eta_K < 1$ or $E/\xi_K < 2000$). Fig. 5 presents the proton data (modified, as before, for relativistic projectiles) on a graph drawn to Garcia's prescription. It is clear that this representation is more truly universal than that provided by the PWBA calculation, that the fit to the 160 MeV data is of similar quality to that shown in Fig. 3 and, furthermore, that the fit to the 2-28 MeV data is now greatly improved.

The two "universal" graphical representations are, of course, remarkably similar. One can approximate the binding energy as 
$$\xi_K = 1.263 \times 10^{-2} \, (\mathcal{Z} - 2)^2 \text{ keV}. \quad \text{The difference in the two methods is expressible as } \sigma_K \xi_K^{14} \rightarrow (\mathcal{Z} - 2)^{14} \text{ as scaling factor for the ordinate, and } \frac{1}{\xi_K^2 \sigma_K^2} \rightarrow \frac{1}{(\mathcal{Z} - 2)^2} \text{ for the abscissa. The result is to depress the results for light elements relative to the heavy.}

4.2 Deuteron and alpha particle data

As mentioned earlier, these results are of a preliminary nature. Therefore, we restrict ourselves to considering only the averaged results for five elements between $\mathcal{Z} = 29$ and $\mathcal{Z} = 79$, expressed as ratios for the different projectiles. The comparison between the 160 MeV alpha-particle and the 85 MeV deuteron data is particularly simple because the velocities of these two projectiles are equal - the theoretical ratio for each element is then just $4/1$, arising from the fact that the alpha particle possesses charge $z = 2$. Experimentally, we have observed a mean ratio of $4.6 \pm 0.8$. For the ratio of alpha-particle to proton cross-section data the situation is more
complicated because the projectile velocities are quite different. Following the PWBA theory, the alpha-particle cross-section is enhanced relative to the proton cross-section by a factor of four through the factor $z^2$, a further factor of 4 (non-relativistically) for the reduction in $\eta_K$ and is reduced by an amount dependent on $g$ through the factor $f_K$. For consistency with the previous section a small correction is also required for the effect of relativistic velocities. It was found that the experimental ratio $\sigma_K(\alpha)/\sigma_K(p)$, divided by the theoretical ratio, taken as an average for all $g$ was $0.96 \pm 0.10$. Since good qualitative agreement has already been demonstrated for the proton data with theory, this result indicates that the alpha-particle data (and, from the previous result, the deuteron data also) are in equally good agreement with theory.

4.3 L X-rays from gold

The energy resolution of the detector used in the present work was inadequate for an accurate study of L X-ray production and, moreover, with the exception of the thin gold target the targets were too thick. Nevertheless, it was desired to check that the L X-ray production was in accord with our preconceptions. Unfortunately, the fluorescent yields for the three L sub-shells are not accurately known and the situation is rendered even more confusing by the existence of the Coster-Kronig re-arrangement transitions. Thus, for the present purpose we have summed the La, Lb and Ly X-ray yields and have adopted the average fluorescent yield $\bar{\sigma}_L$ = 0.40 from Ref. 21. With these approximations we obtain a value for the L-shell ionization cross-section of $1,680 \pm 300$ barns, the large error being assigned in an attempt to include all the uncertainties involved.
For high energy protons as projectiles the particular atomic wave-functions (velocity distributions) should not be important provided the electron velocity in the shell is relatively small. In this case we would expect the L-shell ionization cross-section to be just four times the (experimental) K-shell cross-section for an atom whose K-electron binding energy is the same as the L-electron binding energy of gold (the factor of four being the ratio of the number of electrons in the L and K shells). This argument leads us to predict an L-shell ionization cross-section of $1,460 \pm 100$ barns, in good agreement with the experimental result quoted above.

5. **Comparison with Electron Data**

In any first-order theory of the interaction between a very fast charged particle and an atomic electron the ionization cross-section will be determined essentially by the atomic number of the target atom, the magnitude of the projectile charge and its velocity but should be relatively independent of the projectile mass. This is not true, of course, for low projectile velocities where the various conservation laws impose kinematic restrictions which in turn define the low-energy behaviour of the cross-sections. For electrons the increase in kinetic energy experienced by the electron whilst penetrating the atom is most important and the energy threshold for ionization is determined solely by energy conservation. For protons a purely two-body collision would imply a threshold velocity one-half that for electrons. In fact the proton production of K-shell ionization is appreciable for energies considerably less than are implied by this condition because the internal motion of the atomic electron is important, momentum conservation being satisfied by the recoil of the atom. Nevertheless, in the collision between the sub-system comprising the proton and the
atomic electron both energy and momentum conservation will be satisfied.

For proton energies well above the "classical" threshold of approximately $460 \varepsilon_K$ we would expect proton and electron produced ionization cross-sections to be equal provided the projectile velocities are the same. We therefore compare the present 160 MeV proton data with 80 keV electron data. Very few electron measurements have been made, and Fig. 3 presents the only data which are for Ni, Ag and Sn. That these electron data are smaller than the proton data is not surprising, since we really require data for elements of very low $Z$ for which the electron production cross-sections are determined by momentum conservation rather than energy conservation. However, the electron and proton data appear to be converging as $Z$ decreases.

Also shown in Fig. 3 are the 150 MeV electron data of Middleman et al.¹⁶, the three high $Z$ points being extrapolated downwards from their 300 to 900 MeV data. The remarkable agreement with our 160 MeV proton data is entirely fortuitous. The variation with $Z$ for electrons is a good fit to $Z^{-n}$ with $n = 2.70 \pm 0.02$. The proton data does not possess such a simple $Z$ dependence, but for comparison purposes a reasonable representation can be found with $n \approx 3.3$.

The behaviour of the electron data for Ag with energy is shown in Fig. 6. This figure covers an energy range from 25 keV to 900 MeV and clearly the fit to the revised relativistic theory of Kolbenstvedt¹⁷ is excellent. For the non-relativistic energy range the cross-sections may be expressed¹⁸ as

$$\frac{\varepsilon_k^2 \sigma_k}{\varepsilon_k} = 7.92 \times 10^{-20} \frac{\varepsilon_k}{E} \ln \left( \frac{E}{U_k} \right) \text{ cm}^2 \text{ keV}^2$$
This relationship provides a "universal" curve bearing a very strong resemblance to the prescription of Garcia\textsuperscript{15}, but the universality fails as the energy becomes relativistic, as is indicated by the difference in the predictions for elements as close as Ag($Z = 47$) and In($Z = 49$).

We expect the cross-section data to be the same for protons and electrons at the same velocity when in the ultra-relativistic energy region. Fig. 7 has been drawn in an attempt to gain some feeling for the behaviour of the proton data in the relativistic energy region, with particular reference to the manner in which the non-relativistic theories must break down as the energy increases. This figure should be viewed not as an attempt to provide a universal curve for all $Z$ at any energy, as previously considered, but rather as an indication of the energy dependence for each particular element. The curves labelled "electron theory" have been adapted from the theory of Kolbenstvedt by the simple expedient of using as abscissa the proton energy corresponding to the same velocity of the electron to which the Kolbenstvedt theory applies. The "proton theory" curves are adapted from the theory of Garcia et al\textsuperscript{16}. Three such curves are shown, one being the uncorrected theory and the other two - for Ag and Au - being obtained by making the crude correction for projectiles of relativistic velocities. The collapse of the "universal" curve and the expected crossover of the true cross-sections from the behaviour given by the non-relativistic proton theory to that of the relativistic electron theory can be readily imagined.

6. Discussion and Conclusions

Cross-sections for the K-shell ionization produced by 160 MeV protons have been measured for elements in the range $26 \leq Z \leq 92$. These cross-
sections have been shown to be in qualitative agreement with both the PWBA calculations of Khandelwal et al\textsuperscript{13}) and the binary collision model calculations of Garcia et al\textsuperscript{16}), provided approximate correction is made for the relativistic velocity of the protons. For lower energies the binary collision model provides a better fit to the data than the PWBA method and any existing discrepancies between this theory and the low energy experiments are as likely to be due to uncertainties in the various experimental measurements as to the approximations of the theory.

Preliminary measurements with 85 MeV deuterons and with 160 MeV alpha particles are also found to be in good agreement with theory. The L-shell ionization cross-section for gold was found to be determined essentially by the L-shell binding energies, as expected.

Attention has been given to the manner in which the proton cross-sections vary with energy as the energy moves into the relativistic region. The non-relativistic theories are clearly on the verge of breaking down at 160 MeV. The need for a fully relativistic proton theory is therefore apparent. Since the Kolbenstvedt theory fits the ultra-relativistic electron data remarkably well without recourse to a relativistic description of the atomic wave-functions it is probable that this complication will also prove unnecessary for the proton work.

At the start of this work it had been hoped that proton ionization might provide a sensitive method for detecting small concentrations of very heavy elements in a medium-weight matrix. This hope was dashed by the high background levels and the discovery that, for high Z, the "backgrounds" originate from nuclear interactions within the target itself. For low or medium Z elements the signal to noise ratio can be quite impressive but for such elements the high energies provided by a synchrocyclotron are quite
unnecessary and remarkable results have been obtained\textsuperscript{19} using a Van de Graaff accelerator where the background problem is almost non-existent.

Acknowledgements

The authors are particularly grateful to Mr. C. G. Clayton for stimulating their interest in this problem. They are also very grateful to Mr. A. C. Sherwood for his assistance in the preparation for and execution of the data taking and to the cyclotron crew for their efficient running of the machine.
Appendix

1. Introduction

In the study of K-shell ionization by heavy ions of high energy there is a competing ionization process which must be considered. This competing process arises from the copious production of knock-on electrons, or $\delta$-rays, of sufficient energy to eject a K-shell electron either in a direct collision or indirectly by producing bremsstrahlung radiation which is photoelectrically absorbed. In this Appendix we present an approximate calculation of the cross-section for K-shell ionization by $\delta$-rays.

Ideally, difficulties arising from these secondary ionization processes would be avoided by the use of extremely thin targets. However, when relatively thick targets are required - as in the present experiment - it becomes necessary to measure the excitation cross-section as a function of target thickness and to extrapolate to zero thickness. A linear extrapolation may not be appropriate. For very thin target the slowing down of the $\delta$-rays may be ignored, in which case the bremsstrahlung production would be negligible. This "thin target" approximation yields an effective $\delta$-ray ionization cross-section which increases linearly with target thickness. For thicker targets the energy loss suffered by the $\delta$-rays in their passage through the target becomes significant and the bremsstrahlung production and absorption (proportional to the square of the target thickness) may be important. When the target is thick enough to stop the majority of the $\delta$-rays the "direct" ionization cross-section saturates. However, because of the bremsstrahlung absorption the total cross-section continues to rise (approximately linearly) with thickness. Finally, for very thick targets the absorption of the bremsstrahlung radiation will be essentially complete.
and the measured ionization cross-section will appear independent of target thickness. (As we are discussing ionization cross-sections it is assumed that corrections for self-absorption of the characteristic K X-rays will have been applied to the experimental data.) It is clearly essential to calculate the magnitude of the "direct" and "indirect" δ-ray cross-sections in order to decide what precisely is meant by "thin" and "thick" targets.

A complete and accurate calculation of the δ-ray ionization cross-section is likely to be prohibitively difficult as it involves a knowledge of the δ-ray energy spectra and angular distributions, the slowing and scattering of electrons in the target medium and the consequent bremsstrahlung energy spectra and angular distributions. Fortunately, a precise calculation is unnecessary as we wish mainly to determine the region where the effect is small and over which the extrapolation to zero thickness can be made, rather than to obtain accurate corrections for the two δ-ray processes. Therefore, we consider just the two extreme cases:

(a) thin target approximation: the δ-rays pass through the target without loss of energy.

(b) thick target approximation: the δ-rays are all stopped within the target medium.

Notwithstanding the simplifications implied by considering only these two extreme cases, further assumptions are necessary. These are:

(i) that the production of δ-rays is strongly peaked in the forward direction of the heavy particle beam - a reasonable assumption as we are concerned only with the most energetic δ-rays;

(ii) that the scattering of the δ-rays within the target medium may be neglected;
(iii) that only the primary $\gamma$-ray contributes to K-shell ionization (i.e. we ignore the occurrence of electron cascades);

(iv) that either the bremsstrahlung radiation is strongly peaked in the forward direction or that it is isotropic. The forward peaking assumption is reasonable for very high energy $\gamma$-rays but isotropic production may be more appropriate to very low energy $\gamma$-rays. However, low energy $\gamma$-rays can contribute only to the ionization of low-$Z$ elements for which the bremsstrahlung process is relatively unimportant. Consequently, we expect the forward peaking or "relativistic" approximation to be more realistic than the "isotropic" approximation. The true cross-section will presumably lie between these two limiting cases.

2. The Production of $\delta$-rays by heavy particles

The cross-section for the production of $\delta$-rays of energy $\varepsilon$ by a particle of mass $M$, charge $z$ and velocity $v = \beta c$ traversing a thin target of atomic number $Z$ is given by\(^{20}\)

$$\frac{d\sigma(\varepsilon)}{d\varepsilon} = \frac{2\pi e^2 z^2 \varepsilon}{m^2 c^2} \frac{1}{\varepsilon^2} \left[1 - \beta^2 \frac{\varepsilon}{\varepsilon_{\text{max}}} \right]$$

where $\varepsilon_{\text{max}} = \frac{2mc^2}{1-\beta^2} \left[1 + \frac{2m}{M(1-\beta^2)^{1/2}} + \left(\frac{m}{M}\right)^2 \right]^{-1}$

$$\simeq \frac{4mM}{(m+M)^2} E$$

where $E = \frac{1}{2} Mv^2$.

In the present calculations it is adequate to work in the non-relativistic limit, which implies the retention of just the first term in equation (1).
3. The Thin Target Approximation

3.1 Direct ionization

Following Green and Cosslett we take the K-shell ionization cross-section of an atom by an electron of energy \( \varepsilon_\delta \) to be \( \sigma_K \), where

\[
\sigma_K \varepsilon_K^2 = 7.92 \times 10^{-26} \left( \frac{\varepsilon_K}{\varepsilon_\delta} \right) \text{bn} \left( \frac{\varepsilon_\delta}{\varepsilon_K} \right) \text{ cm}^2 \text{ MeV}^2
\]

where \( \varepsilon_K \) is the K-shell electron binding energy, given approximately by the formula

\[
\varepsilon_K = 1.265 \times 10^{-5} (Z-2)^2 \text{ MeV}
\]

The probability that a \( \delta \)-ray of energy \( \varepsilon_\delta \) will ionize an atom whilst passing through a thin foil of surface density \( S \) is given by

\[
P(\varepsilon_\delta) = \frac{6.023 \times 10^{23}}{A} \times \frac{S}{2} \times \sigma_K(\varepsilon_\delta)
\]

where \( S/2 \) is used, rather than \( S \), to allow for the fact that on average the \( \delta \)-ray will be created at the centre of the target. The "direct" ionization cross-section is obtained from the product of the \( \delta \)-ray production cross-section (equation 1) and the ionization probability (equation 5) summed over all permissible \( \delta \)-ray energies.

3.2 Indirect Ionization

The cross-section for production of a bremsstrahlung photon of energy \( \varepsilon_\gamma \) by an electron of energy \( \varepsilon_\delta \) is given by

- 20 -
\[\frac{d\sigma(\varepsilon_Y)}{d\varepsilon_Y} = \left(\frac{6.023 \times 10^{23}}{A} \times \frac{S}{2}\right) \times \frac{4.86 \varepsilon^6 \varepsilon^2}{mc^2 \xi_o \xi_{ic}} \times \frac{1}{\varepsilon_Y} \quad \text{for } \varepsilon_Y < \varepsilon_\delta \]

\[= 0 \quad \text{for } \varepsilon_Y \geq \varepsilon_\delta \] \hspace{1cm} (6)

where, as before, the effective target thickness is \(S/2\).

The probability that a photon will ionize an atom in the K-shell we write as

\[\left(\frac{r_K^{-1}}{r_K}\right) f \mu(\varepsilon_Y)S = 0.85 f \mu(\varepsilon_Y)S \quad \text{..... (7)}\]

Here \(r_K\) is the ratio of the photoelectric absorption coefficients on the high and low sides of the K absorption edge, \(\mu(\varepsilon_Y)\) is the absorption coefficient at energy \(\varepsilon_Y\) (by implication, \(\varepsilon_Y > E_K\)) and \(f\) is a quantity dependent both on the bremsstrahlung angular distribution and on target thickness. The "indirect" cross-section is obtained from the product of equations (1), (6) and (7) summed over all permissible bremsstrahlung and \(\delta\)-ray energies.

3.3 Total cross-section for K-shell ionization

The total cross-section is given by

\[\sum(\text{thin}) = \left(\frac{6.023 \times 10^{23}}{A} \times \frac{S}{2}\right) \frac{2\pi e^4 z^2 \varepsilon^2}{m^2 c^2} \times \]

\[\left\{ \int_{E_K}^{E_{\text{max}}} \frac{7.92 \times 10^{-26}}{\xi_2} \left(\frac{\xi}{\xi_2}\right) \ln \left(\frac{\xi_\delta}{\xi_K}\right) \frac{d\xi}{\xi_\delta^2} \right\}
\]

\[+ \int_{\xi_K}^{E_{\text{max}}} \frac{4.86 \varepsilon^6 \varepsilon^2}{mc^2 \xi_{ic}} \frac{d\xi}{\xi_\delta^3} \int_{\xi_K}^{E_{\text{max}}} \frac{0.85 f \mu(\varepsilon_Y)S}{\xi_\delta} \frac{d\xi}{\xi_\delta} \right\} \quad \text{..... (8)}\]
If we assume \( \mu(\varepsilon_{\gamma}) = \mu(\varepsilon_K) \left( \frac{\varepsilon_K}{\varepsilon_Y} \right)^2 \), where \( \mu(\varepsilon_K) \) is taken on the high side of the absorption edge, the second term in equation 8 is integrable. Using equation 4, and simplifying, yields for \( \Sigma(\text{thin}) \) in cm\(^2\):

\[
\Sigma(\text{thin}) = 0.76 \times 10^{12} \frac{\varepsilon^2 S_0}{AB^2 (S-2)^6} \left\{ 1 - \left( \frac{\varepsilon_K}{\varepsilon_{\max}} \right)^2 \left[ 1 + 2 \ln \left( \frac{\varepsilon_{\max}}{\varepsilon_K} \right) \right] \right. \\
+ 0.770 \times 10^{-7} f \mu(\varepsilon_K) S_0^2 (S-2)^2 \left[ 1 - \frac{5}{3} \left( \frac{\varepsilon_K}{\varepsilon_{\max}} \right)^2 + \frac{2}{3} \left( \frac{\varepsilon_K}{\varepsilon_{\max}} \right)^5 \right].
\]

\[ \ldots \ldots \text{(9)} \]

If we assume that the production of bremsstrahlung radiation is strongly forward peaked, then it can easily be shown that the photon absorption probability is given by

\[
f \mu(\varepsilon_K) S = 1 - \left\{ \frac{1 - \exp \left( - \mu(\varepsilon_K) S \right)}{\mu(\varepsilon_K) S} \right\}
\]

\[ \ldots \ldots \text{(10)} \]

whereas, if the bremsstrahlung production is isotropic then

\[
f \mu(\varepsilon_K) S = 0.4614 \mu S - \frac{1}{2} \mu S \ln \mu S + \frac{1}{6} (\mu S)^2 - \frac{1}{48} (\mu S)^3 + \frac{1}{360} (\mu S)^4 - \ldots \ldots \text{(11)}
\]

The quantity \( f \) should strictly be kept within the integrand of equation 9 but the additional complications of doing so are not warranted considering the other uncertainties in the calculation.

It is interesting to note that the direct excitation term in equation 9 predominates for low \( Z \) atoms, but for high \( Z \) atoms it is
the indirect (bremsstrahlung) term which is dominant.

4. The Thick Target Approximation

The starting point for this calculation is the formula of Green and Cosslett\(^{18}\) for the production of K X-radiation produced by monoenergetic electrons incident on a thick target. From their formula we deduce the number of K-shell ionizations for each electron of energy \(\varepsilon_0\) produced within the target to be

\[
N = \left( \frac{9.54 \times 10^4}{A_K} + 2.92 \times 10^{-8} \, \theta (\theta - 2)^2 \right) \left[ 1 - \frac{\varepsilon_0}{\varepsilon_K} + \frac{\varepsilon_0}{\varepsilon_K} \ln \left( \frac{\varepsilon_0}{\varepsilon_K} \right) \right]
\]

\[\ldots \ldots (12)\]

\(K\) is the quantity in the experimental Thomson-Whiddington energy loss relation for electrons

\[
\varepsilon_{\text{incident}}^2 - \varepsilon_{\text{emergent}}^2 = K \rho x
\]

Approximately, \(\ln K = 11.52 + 0.379 \ln \varepsilon\) with \(K\) in units of \(\text{keV}^2 \text{ cm}^2 \text{ gm}^{-1}\) and \(\varepsilon\) in keV. Since \(K\) varies slowly with energy we may treat it as a constant. The first term in equation 12 represents the "direct" ionization process. The second term represents "indirect" ionization.

To obtain the thick target cross-section \(\Sigma \text{(thick)}\) we must average equation 12 over the \(\delta\)-ray yield (equation 1) and multiply the indirect term by \(f \mu_{\text{eff}} S\) to permit only partial absorption of bremsstrahlung radiation.
We obtain

\[ \Sigma_{\text{thick}} = 1.92 \times 10^{-15} \frac{e^2 g}{A \beta^2 K(g-2)^2} \left[ 1 + 3.08 \times 10^{-13} \frac{gA(g-2)}{K} \mu_{\text{eff}} S \right] \times \]

\[ x \left\{ 1 - \left( \frac{\varepsilon_K}{\varepsilon_{\text{max}}} \right) - \ln \left( \frac{\varepsilon_K}{\varepsilon_{\text{max}}} \right) + \frac{1}{2} \ln \left( \frac{\varepsilon_{\text{max}}}{\varepsilon_K} \right)^2 \right\} \]

\[ \text{(13)} \]

Before using equation 13 we need to know at what energy \( \bar{\varepsilon} \) to obtain a value for \( K = K(\bar{\varepsilon}) \). It appears reasonable to take the average of \( \varepsilon_K \) and the mean energy of \( \delta \)-ray production between \( \varepsilon_K \) and \( \varepsilon_{\text{max}} \),

i.e. \( \bar{\varepsilon} = \frac{1}{2} \left[ \varepsilon_K + \frac{\varepsilon_K \varepsilon_{\text{max}}}{\varepsilon_{\text{max}} - \varepsilon_K} \ln \left( \frac{\varepsilon_{\text{max}}}{\varepsilon_K} \right) \right] \)

The effective value of the absorption coefficient (\( \mu_{\text{eff}} \)) is also required.

The bremsstrahlung energy spectrum in a thick target is given as

\[ \frac{dN}{d\varepsilon_Y} = 2.76 \times 10^{-6} g \left( \frac{\varepsilon_\delta - \varepsilon_Y}{\varepsilon_Y} \right) \]

Assuming, as before, \( \mu(\varepsilon_Y) = \mu(\varepsilon_K) \left( \frac{\varepsilon_K}{\varepsilon_Y} \right)^3 \) enables us to calculate

\[ \mu_{\text{eff}} = \mu(\varepsilon_K) \left[ \frac{1}{3} \ln \left( \frac{\varepsilon_{\text{max}}}{\varepsilon_K} \right) + \frac{1}{18} \left\{ 1 - \left( \frac{\varepsilon_K}{\varepsilon_{\text{max}}} \right)^3 \right\} - \frac{1}{2} \left( 1 - \frac{\varepsilon_K}{\varepsilon_{\text{max}}} \right) \right] \]

\[ \left\{ 1 - \frac{\varepsilon_K}{\varepsilon_{\text{max}}} + \ln \left( \frac{\varepsilon_{\text{max}}}{\varepsilon_K} \right) \left[ \frac{1}{2} \ln \left( \frac{\varepsilon_{\text{max}}}{\varepsilon_K} \right) - 1 \right] \right\} \]

The bremsstrahlung radiation angular distribution is approximated as in
section 3C, through equations (10) and (11). In the case of complete bremsstrahlung absorption we replace $f \mu_{\text{eff}} S$ by unity. This gives the "very thick target" approximation which is an upper limit to the $\delta$-ray induced ionization.

5. Comparison with Experiment

In Fig. 8 we show the experimentally determined K-shell ionization cross-sections for 160 MeV protons on copper and gold as a function of target thickness. These data have been corrected for self-absorption of the K X-radiation in the targets. For copper the "isotropic" and "relativistic" calculations give very similar results, a reflection of the insignificance of the indirect (bremsstrahlung production) process relative to the direct collision process for low Z elements. The copper data clearly favour the "relativistic" calculations. For gold the difference between the "isotropic" and "relativistic" calculations is very considerable, but - again, as expected - the data strongly favour the relativistic calculations. From these two comparisons we conclude that the "relativistic" calculations may be used to make surprisingly precise corrections for the $\delta$-ray ionization processes this, in turn, implies that the limiting factor in choice of target thickness need not be an awareness of the $\delta$-ray ionization process but simply the desire to keep the self-absorption correction to an acceptable value. For most of the measurements in the present work the magnitude of the $\delta$-ray ionization corrections are less than the uncertainties in target thickness.
Table I

<table>
<thead>
<tr>
<th>Element</th>
<th>Z</th>
<th>Target thickness range (mg/cm²)</th>
<th>$\omega_K$</th>
<th>$\eta$</th>
<th>$\sigma_K$ (barns)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fe</td>
<td>26</td>
<td>1 - 3</td>
<td>0.293</td>
<td>0.36 ± 0.03</td>
<td>780 ± 65</td>
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<tr>
<td>Cu</td>
<td>29</td>
<td>2.5 - 41.1</td>
<td>0.393</td>
<td>0.62 ± 0.04</td>
<td>560 ± 40</td>
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<tr>
<td>Zr</td>
<td>40</td>
<td>5.5 - 16.0</td>
<td>0.70</td>
<td>0.89 ± 0.02</td>
<td>193 ± 13</td>
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<tr>
<td>Mo</td>
<td>42</td>
<td>2.8 - 52.0</td>
<td>0.73</td>
<td>0.88 ± 0.02</td>
<td>162 ± 7</td>
</tr>
<tr>
<td>Rh</td>
<td>45</td>
<td>5.8 - 16.4</td>
<td>0.78</td>
<td>0.81 ± 0.03</td>
<td>149 ± 7</td>
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<td>Ag</td>
<td>47</td>
<td>11.9 - 7.7</td>
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<td>0.76 ± 0.03</td>
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<tr>
<td>Sn</td>
<td>50</td>
<td>8.0 - 24.2</td>
<td>0.84</td>
<td>0.68 ± 0.03</td>
<td>112 ± 6</td>
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<tr>
<td>Ba</td>
<td>56</td>
<td>7.0 - 21.0</td>
<td>0.88</td>
<td>0.52 ± 0.02</td>
<td>48 ± 10</td>
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<tr>
<td>Sm</td>
<td>62</td>
<td>10.0 - 55.0</td>
<td>0.91</td>
<td>0.35 ± 0.02</td>
<td>50 ± 2.8</td>
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<tr>
<td>Tb</td>
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<td>8.8</td>
<td>0.92</td>
<td>0.27 ± 0.02</td>
<td>32 ± 6</td>
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<tr>
<td>Ta</td>
<td>73</td>
<td>91.7 - 185.9</td>
<td>0.94</td>
<td>0.132 ± 0.008</td>
<td>24.3 ± 1.6</td>
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<tr>
<td>Pt</td>
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<td>22.5</td>
<td>0.95</td>
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<td>19.2 ± 1.4</td>
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<td>Au</td>
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<td>0.95</td>
<td>0.076 ± 0.004</td>
<td>17.7 ± 1.0</td>
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<tr>
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<td>25 - 75</td>
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<td>0.059 ± 0.005</td>
<td>19.2 ± 1.7</td>
</tr>
<tr>
<td>U</td>
<td>92</td>
<td>207</td>
<td>0.96</td>
<td>0.027 ± 0.005</td>
<td>10.5 ± 1.9</td>
</tr>
</tbody>
</table>

Overall accuracy: ± 7%

$\sigma_K = $ total K-shell ionization cross-section assuming isotropic emission

$\eta = $ interpolated detector efficiency

$\omega_K = $ fluorescent yield$^{10}$ assumed in deducing $\sigma_K$; no allowance has been made for uncertainties in $\omega_K$. 
References


   quoted by Middleman et al., Ref. 6.
   Meth. 84, (1970) 141.
    (1966) 513.
General layout of the machine and experimental area.
Typical pulse height spectra of the characteristic X-radiation produced by bombardment with 160 MeV protons.
The total K-shell ionization cross-sections for a range of elements between $Z = 26$ and $Z = 92$. The 160 MeV proton data are compared with a PWBA theory (which has been scaled by $x 1.26$, see text). The data are compared with results for electrons of the same velocity as the protons (80 keV electrons) and with results for 150 MeV electrons (for which the agreement with PWBA theory is entirely fortuitous).

AERE - R 6612  Fig. 3

HL 80657
Comparison of the high energy proton induced cross-section data with PWBA theory in the form of the "universal curve" of Merzbacker and Lewis. Note that the 2-28 MeV data of Bissinger et al. are for three selected elements as a function of energy, whereas the 160 MeV proton data are for a range of elements at a single energy. (The 160 MeV data have been reduced by the factor 1.26, see text.)

HL 80658
Comparison of the high energy proton induced cross-section data with the binary encounter model of Garcia. (The 160 MeV data have been reduced by the factor 1.26, see text.)
AERE - R 6612  Fig. 6
The total K-shell ionization cross-sections for indium and silver using electrons as incident projectiles, as a function of electron energy. Comparison is made with the theory of Kolbenstvedt\(^{(17)}\).
An attempt to predict the variation of the proton-induced total K-shell ionization cross-sections in the highly relativistic energy region.
The apparent K-shell ionization cross-sections for copper and gold, measured as a function of target thickness.