NUCLEAR MAGNETIC RESONANCE ZEUGMATOGRAPHY

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ABSTRACT

The work reported in this thesis describes the first part of a continuing programme of research and development of Nuclear Magnetic Resonance Imaging ('Zeugmatography') at the University of Surrey.

In recent years techniques have been developed for the localised detection of the magnetic resonance of spin I = \frac{1}{2} nuclei in vivo. Research has shown that the technique has considerable clinical potential, combining the attributes of sensitivity to soft tissue and some pathological lesions with the absence of 'dose'.

Zeugmatography also has the ability to define and locate one or more tomographic images with the flexibility which comes from the absence of moving parts within the scanner.

This thesis describes the design and construction of a prototype NMR scanner and presents a critical appraisal of the first images obtained. In addition a new and powerful method for the simultaneous acquisition of multiple tomographic planes is described and demonstrated. Furthermore, analyses of the problems of signal acquisition and reconstruction are given.
"The theory of NMR Imaging is hideously complicated."

-'Medicine Now', BBC Radio 4 (8/9/81)
during a review of medical imaging methods.
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# LIST OF CONTENTS

## ABSTRACT

## ACKNOWLEDGEMENTS

### CHAPTER 1 - NUCLEAR MAGNETIC RESONANCE IN BIOLOGICAL TISSUE

1(i) The Quantum Mechanical Model 1  
1(ii) The Classical Model 7  
1(iii) Molecular Motion and Relaxation Mechanisms in Biological Tissue 14  
1(iv) The Response of Healthy Tissue 22  
1(v) Models of Cell Water 24  
1(vi) Water in Cancerous Tissue 26

### CHAPTER 2 - THE NMR SIGNAL AND ITS MEASUREMENT

2(i) The Nature of the NMR Signal 28  
2(ii) 'Zeugmatography' - The Adaptation of NMR to Take Images 31  
2(iii) Digitisation of the NMR Signal 32  
2(iv) The f.i.d. Sampling Time 34  
2(v) The f.i.d. Sampling Rate 39  
2(vi) F.i.d. filters 39  
2(vii) Additional Problems 40  
2(viii) Digitisation of the Spin Echo 42

### CHAPTER 3 - CONTEMPORARY NMR IMAGING

3(i) The Different NMR Imaging Methods 45  
   a) Projection Reconstruction 45  
   b) Fourier Imaging 47  
   c) Selective Excitation 49
5(iii) Image Artefacts Specific to NMR Imaging by the Method of Projection Reconstruction 122
5(iv) Image Noise 134
5(v) Potential Improvements 145

CHAPTER 6 - APPARATUS
6(i) Crystal Oscillator 149
6(ii) Phase Shifters 151
6(iii) Gate Pulse Generator 153
6(iv) Radio Frequency Gates 155
6(v) Power Amplifier 158
6(vi) Probe
6(vii) Receiver 164
6(viii) ADC 171
6(ix) Gradient Drivers and DACs 174
6(x) Image Display System 177
6(xi) Gradient Coil System 179
6(xii) Computer Algorithms 184
6(xiii) Computer 189
6(xiv) Magnet 191

APPENDIX 1. Comparison of performances of current (1981) EMI CT5005 Whole Body X-Ray Scanner and Theoretical NMR Scanner, producing images of the human brain 192

APPENDIX 2. Back projection algorithm with convolution filter 195

REFERENCES 196
CHAPTER 1

NUCLEAR MAGNETIC RESONANCE IN BIOLOGICAL SYSTEMS
1(i). The Quantum Mechanical Model.

The phenomenon of nuclear magnetic resonance may be described using both quantum and classical mechanics. The former allows a simple introduction to resonance (for example, why energy is absorbed) whilst the latter is particularly useful when discussing dynamic or transient effects. I shall first consider the quantum mechanical description.

A nucleus may possess a nuclear magnetic dipole moment due to the spinning of the charge distribution within it. Such a nucleus interacts with an applied magnetic field with a resultant change in energy which is given by the Hamiltonian:

$$\hat{H} = -\hat{\mu} \cdot \hat{B}$$

where $\hat{\mu}$ is the magnetic dipole moment and $\hat{B}$ is the applied magnetic field.

$B$ is the proper symbol for magnetic induction or 'flux density' in SI units. Virtually all NMR literature continues to use $H$ as well as other units from the CGS system in which $B$ and $H$ are numerically equal. However, $H$ refers to the applied magnetic field or 'force', whilst $B$ is the induced magnetism.

The magnetic dipole moment is shown as an operator which will yield the value of the moment if applied to the wave function of the nucleus. The angular momentum of the nucleus is related to its magnetic dipole by:

$$\hat{\mu} = \gamma \hat{J}_z$$

where $\gamma$ is a scalar called the gyromagnetic ratio, and is specific for the type of nucleus and its state.

The interaction of the magnetic moment with the field is quantised because the associated angular momentum can only assume discrete orientations in space. When the direction of $\hat{B}$ is nominally fixed, customarily in the $\hat{z}$ direction in the laboratory frame of reference, then the values of momentum in this direction can only be discrete.

Using $\hat{\mu}_z = \gamma \hat{J}_z$ then the maximum absolute value of $J_z$ may be
written as $\hbar I$, so that we may use the dimensionless number $I$ as a convenient label, the 'spin' of the nucleus. The allowed values that $J_z$ may have between $-\hbar I$ and $+\hbar I$ are $\hbar m$ where $m = I, I-1, \ldots, 0, \ldots, -I$. The Hamiltonian may be written with the new dimensionless operator $\hat{I}$. This operator is conventionally used in NMR theory.

The interaction energy with the field $B_0$ in the $\hat{z}$ direction is, then,

$$E = -\gamma \hbar B_0 m$$

and the energy difference between states is $(\gamma \hbar B_0)$. Thus a nucleus gains or loses this amount of energy in a single transition between adjacent levels.

The simplest nucleus to examine, and of which the proton is an example, is one with $I = \frac{1}{2}$. It is simple because there are only two possible energy states, corresponding to $m = \pm \frac{1}{2}$, and therefore only one transition energy. The phenomenon of resonance occurs when photons of this energy are applied to the system. The energy levels are shown in Figure 1.1.

![Energy Levels](image)

**Figure 1.1.** Separation of energy levels as an external field $B_0$ is applied.

Transitions will be associated with photons of energy $E = \hbar \omega = \gamma \hbar B_0$. Thus, $\omega = \gamma B_0$ is the frequency of photons which must be applied to the system. They are typically in the radio frequency (rf) range. It is important to note two points:

1. $E \propto B_0$: Higher energy photons will be absorbed/emitted in a higher field.
2. $\omega \propto \gamma$: Dissimilar nuclei will absorb / emit different frequencies.
For a more detailed description of the absorption of energy into a spin system (a collection of spins), consider an isolated system in which \( N^- \) and \( N^+ \) spins exist in the upper and lower states respectively, for nuclei of \( I = \frac{1}{2} \) in a magnetic field \( B_0 \). The stationary state wave functions may be labelled by their values of \( m \).

If an oscillating field \( B_1 \), with a frequency \( \omega \), is applied orthogonal to \( B_0 \), there is the following extra term in the Hamiltonian:

\[
\hat{H}' = 2\mu_0 \cdot B_1 \cos(\omega t)
\]

The theory predicting transition probability between the two states \( m^- \) and \( m^+ \), corresponding to absorption or emission of radiation, is well documented (2). The probability for a single transition from \( m^- \) to \( m^+ \) is:

\[
W = \gamma^2 B_1^2 |\langle m^+ | \hat{I}_x | m^- \rangle|^2 \delta(\omega_{m^+m^-} - \omega)
\]

where the Dirac delta function is single valued at the transition frequency and \( \langle m^+ | \hat{I}_x | m^- \rangle \) is the matrix element of the operator \( \hat{I}_x \). This equation shows that:

(i) The transition probability depends on the strength of \( B_1 \).

(ii) The selection rule for transition (that \( m \) must change by \( \pm 1 \)) is re-established, since the matrix element would otherwise vanish.

(iii) The absorption frequency is single-valued.

In practice, absorption lines are broadened by local weak fields, the nature of which will be discussed later. The delta function is replaced by a line shape function which describes the distribution of final states after transition \( g(\omega) \). This is very often a Lorentzian shape.

The matrix elements may be evaluated (3) and:

\[
|\langle m^+ | \hat{I}_x | m^- \rangle|^2 = |\langle m^- | \hat{I}_x | m^+ \rangle|^2 = \frac{1}{4}
\]

Thus the transition probability is equal for absorption or emission, and the probability for any spin is:

\[
W = \frac{1}{4} \gamma^2 B_1^2 g(\omega)
\]

This fact alone will mean that any population difference between the two states will disappear under the action of the applied perturbation \( \hat{H}'_1 \). (Spontaneous emissions do occur but much more
rarely). The rate of change of $N^+$ is given by:

$$\frac{dN^+}{dt} = W(N^- - N^+)$$

Using $N = N^+ + N^-$, (constant)
and $n = N^+ - N^-$ then $N^+ = \frac{1}{2}(N + n)$ and $N^- = \frac{1}{2}(N - n)$, and this gives:

$$\frac{dn}{dt} = -2Wn$$

the solution of which is:

$$n = n(0)e^{-2Wt} \quad (1.7)$$

The initial population difference $n(0)$ between states will therefore disappear. This means that, unless another mechanism exists for creating population differences, resonance (a net absorption or emission of energy by the spin system) will disappear permanently. Given that the spins do return to an equilibrium state in which there is a population difference, another system must exist to accept the energy balance. This system is called the 'lattice'.

The tendency for nuclei to align with $B_0$ and so give up energy is opposed by thermal motions and the resulting distribution is the usual compromise predicted by the Boltzmann equation:

$$\frac{N^-}{N^+} = e^{-\Delta E/kT} = e^{-\gamma g B_0 / kT} \quad (1.8)$$

Now that the system is no longer isolated, the probabilities of absorption and emission are no longer equal. The rate of change of $N^+$ is

$$\frac{dN^+}{dt} = N^- \omega \downarrow - N^+ \omega \uparrow$$

and is zero in the equilibrium condition; i.e. the number of upward transitions = the number of downward transitions.

Therefore

$$\frac{N^-}{N^+} = \frac{\omega \uparrow}{\omega \downarrow} \quad \text{at equilibrium, and so}$$

$$\frac{\omega \uparrow}{\omega \downarrow} = e^{-\gamma g B_0 / kT} \quad (1.9)$$

The spin system and lattice together allow energy to be conserved during single upward or downward transitions by performing reciprocal transitions (Figure 1.2).
The two reciprocal transitions for a spin system of $I = \frac{1}{2}$. The spin states are labelled $m^-$ and $m^+$. The lattice states are $a$ and $b$.

When the system is outside a polarizing field, $N^+$ and $N^-$ are equal. When the system is placed in the field, the Boltzmann distribution is not established immediately. Similarly, after prolonged exposure to the perturbing Hamiltonian $\hat{H}'$, the population equality relaxes with time. This non-radiative exchange of energy with the lattice is characterised by the decay constant $T_1$, called the 'spin-lattice relaxation time'.

The rate for transition pairs is given by:

$$\text{number per second} = N^- N_b W_{b \to a^+}$$

where $W_{b \to a^+}$ is the probability of such a transition under the condition that the nucleus is in state $m^-$ and the lattice is in state $b$.

In the steady state, the rate of such transitions is equal to the rate of the inverse transition:

$$N^- N_b W_{b \to a^+} = N^+ N_a W_{a^+ \to b^-}$$

Quantum theory requires that, for the whole system at equilibrium,

$$W_{b \to a^+} = W_{a^+ \to b^-}$$

so that

$$\frac{N_-}{N_+} = \frac{N_a}{N_b} \quad (1.10)$$

The nuclear levels have the same relative populations as do the lattice's, so that the spin system is in thermal equilibrium with the lattice. Also,

$$W^\uparrow = N_a W_{a^+ \to b^-}; \quad W^\downarrow = N_b W_{b \to a^+} \quad (1.11)$$

so that $W^\uparrow \neq W^\downarrow$ as predicted earlier.

Rewriting $\frac{dN_+}{dt} = \frac{d}{dt} \frac{1}{2}(N + n)$,
\[
\frac{\text{dn}}{\text{dt}} = N(W_{\downarrow} - W_{\uparrow}) - n(W_{\downarrow} + W_{\uparrow})
\]

which can be rewritten as
\[
\frac{\text{dn}}{\text{dt}} = \frac{n_o - n}{T_1}
\]

where
\[
n_o = N\left(\frac{W_{\downarrow} - W_{\uparrow}}{W_{\downarrow} + W_{\uparrow}}\right) ; \quad 1/T_1 = (W_{\downarrow} + W_{\uparrow})
\]

Since the solution is
\[
n = n_o + A \exp(-t/T_1) \quad (A \text{ is a constant})
\]

it is clear that \( n_o \) represents the equilibrium population difference and \( T_1 \) is the time constant introduced earlier.

Thus, under the influence of an applied perturbation \( \hat{H} \), the equalisation of the populations is being opposed by the spin-lattice relaxation. Using the earlier equations, it can be seen that \( B_1 \) may be increased so as to overcome the spin-lattice relaxation and 'saturate' the sample. However, NMR Imaging uses the method of applying \( B_1 \) in short, high intensity pulses, which are applied in order to produce the coherent precession of spins contained within an artificially shaped energy distribution. In this method, resonance is synonymous with a ringing bell after being struck, rather than the beating of an applied frequency with a natural resonance. The motion of spins under a pulsed \( B_1 \) is best described using well-developed Classical models, and by introducing the concept of the rotating frame of reference. I will discuss these in the next section.
1(ii). The Classical Model.

Classically, the magnetic moment of a single nucleus $\vec{\mu}$ in a static magnetic field $\vec{B}$ experiences a torque, $\vec{\mu} \times \vec{B}$, equal to the rate of change of angular momentum. This causes $\vec{\mu}$ to precess about $\vec{B}$ with an angular frequency equal to $-\gamma \vec{B}$:

(Figure 1.3)

$$\frac{d\vec{\mu}}{dt} = \vec{\mu} \times (\vec{B} \cdot \gamma)$$  

(1.14)

Figure 1.3. Precession of moments in the laboratory frame ($x y z$).

We may transform this equation into a reference frame rotating at angular frequency $\omega$:

$$\frac{d\vec{\mu}}{dt} = \vec{\mu} \times (\gamma \vec{B} + \omega)$$  

(1.15)

Hence by comparison with eqn. 1.14 we see that the transformation to an observer in the rotating frame has replaced the field $\vec{B}$ by an effective field $\vec{B}_{\text{eff}}$, where

$$\vec{B}_{\text{eff}} = \vec{B} + \omega / \gamma$$

Now if $\omega = -\gamma \vec{B}$ then $\vec{B}_{\text{eff}} = 0$ and the magnetic moment $\vec{\mu}$ is stationary in the rotating frame (equation 1.15). This value of angular frequency, $\omega_0$, is called the Larmor frequency and is equal to the frequency of precession of $\vec{\mu}$ about $\vec{B}$. When $\vec{B}$ is a static field in the $\hat{z}$ direction, $B_z$, we have $\omega_0 = -\gamma B_z$.

$\omega_0$ corresponds exactly to the resonance frequency of the previous brief quantum mechanical description.

Resonant transitions are induced by the application of the oscillating field $B_{\perp}$ perpendicular to $B_z$. The equation of motion then becomes:

$$\frac{d\vec{\mu}}{dt} = \vec{\mu} \times \gamma [B_z + B_{\perp}(t)]$$  

(1.16)
If the frame of reference rotates about \( \hat{z} \) with the same frequency that \( B_1 \) has then \( B_1 \) will be stationary and, for example, in the \( \hat{x} \) direction. Thus:

\[
\frac{d\mathbf{\mu}}{dt} = \mathbf{\mu} \times \mathbf{B} - \gamma \mathbf{\mu} \times \left( \mathbf{B} - \omega_0 \mathbf{g} \right) \hat{z} + \mathbf{B} \hat{z}
\]

At resonance \( \omega = \omega_0 \), and

\[
\frac{d\mathbf{\mu}}{dt} = \mathbf{\mu} \times \left( \mathbf{B} \right) \hat{z}
\]

This means that, in the rotating frame, the only field that \( \mathbf{\mu} \) experiences is \( B_1 \), about which it precesses with an angular frequency equal to \( \gamma \mathbf{C} \) (Figure 1.4). If \( \mathbf{\mu} \) is initially aligned along the \( \hat{z} \) direction then it will precess about \( B_1 \) in the \( y-z \) plane. If the field is applied for a limited time \( t_p \) such that the angle of rotation \( \phi = \gamma B_1 t_p \), then we see that \( \mathbf{\mu} \) may be rotated through any angle by varying the pulse length.

Figure 1.4. A moment, initially stationary in the rotating frame \( (x', y', z') \), precesses about \( B_1 \).

The link between Quantum and Classical mechanics is that the expectation value of the magnetic moment follows an equation of similar form to equation (1.14): (Reference (4))

\[
\frac{d\langle \mathbf{\mu} \rangle}{dt} = \langle \mathbf{\mu} \rangle \times \left( \gamma \mathbf{B} \right)
\]

Hence for an assembly of identical non-interacting spins,

\[
\frac{d\mathbf{M}}{dt} = \gamma \mathbf{M} \times \mathbf{B}
\]

where \( \mathbf{M} = \sum \langle \mathbf{\mu}_i \rangle \)

is the total magnetic moment of the system. Figure 1.5 shows the macroscopic precession of the magnetisation about \( B_1 \).
Consider the magnetisation under the influence of $B_0$ and $B_1$. In the classical picture, a pulse which tips the magnetisation into the $x'y'$ plane is called a $90^\circ$ pulse. This leaves the system with no net magnetisation along $\hat{z}$. This might be incorrectly compared with saturation in the Quantum mechanical model, but in the latter the populations are not only equal but there is no net magnetisation in any direction. The difference between this and the pulse model is essentially to be found in the magnitude of the applied field $B_1$. The pulse is a very strong field applied for a short time, during which almost no relaxation can occur; the magnetisation precesses about it. The field pictured in the Quantum model is a weak one, to which the probability of transition is proportional; the magnetisation may be pictured as turning slowly to align along it, in the $x'y'$ plane.

We now consider what happens to the magnetisation immediately after a $90^\circ$ pulse. The magnetic moments composing $\mathbf{M}$ each have a slightly different precession frequency, and are therefore not completely stationary in the rotating frame. The distribution of frequencies is described by the line shape function $g(\omega)$. There is a resultant dephasing of the magnetic moments and a decay to zero of the net magnetisation in the $x'y'$ plane. This is called the 'free induction decay' (FID). It is the Fourier transform of the absorption lineshape; if the lineshape is the simple Lorentzian shape mentioned earlier, then the FID is purely an.
Figure 1.6. Decay of the bulk magnetisation as a function of time in (a) a frame rotating at the resonance frequency and (b) a frame rotating at another frequency. (c) and (d) are the respective Fourier transforms of the transverse magnetisation.

The decay or relaxation of the transverse (i.e., in the \( x' y' \) plane) magnetisation is due to the exchange of energy between spins, and not between spins and the lattice. It is these interactions which cause there to be a range of resonant frequencies, but most of them are transient interactions and the frequencies lose coherence in a way that prevents the signal being rephased. \( T_2 \) processes are called 'spin-spin' relaxation processes, although both \( T_1 \) and \( T_2 \) are influenced by interactions between spins.
For NMR Imaging applications, inhomogeneities in \( B_0 \) dominate the shape of \( \gamma(\omega) \), the local interactions causing comparatively small broadening. In this case, the time constant is denoted by \( T_2^* \). The decay is not exponential, however, as the lineshape in this case is the distribution of the magnetisation throughout the sample. Analysis of the shape forms the basis of the Imaging technique.

Because there is a range of fields present, all spins dephase quickly in the rotating frame. In all cases, the condition for all spins to turn coherently during a pulse is that \( B_1 \) must be sufficiently powerful that it is the only significant field. The condition on its magnitude is:

\[
\mathbf{B}_0 \gg \frac{1}{T_2^*}
\]

(1.19)

The applied \( B_0 \) inhomogeneities are usually static, so that the transverse magnetisation can be rephased, provided that the \( T_2 \) processes have not permanently dephased the spins. The precessions are reversed by the further application of a 180° pulse (5) and the rephasing magnetisation is called a 'spin echo'.

Bloch Equations.

The behaviour of the spin system after a pulse is described accurately in a Classical model proposed by Bloch (6), in a series of equations now called the Bloch Equations. They are modelled on the following assumptions based on the macroscopic observation of the behaviour of the magnetisation.

(1) \[
\frac{d\mathbf{M}}{dt} = \mathbf{M} \times \mathbf{B}_0 \quad \text{in the laboratory frame, for uniform } B_0. \quad \text{cf.}(1.18)
\]

(2) In a static field \( B_0 \), following a 90° pulse or the introduction of unmagnetised samples, the component of \( \mathbf{M} \) in the \( z \) direction tends to an equilibrium value

\[
\mathbf{M}_z = \chi_m B_0 / \mu_m,
\]

where \( \chi_m \) is the magnetic susceptibility and \( \mu_m \) is the magnetic permeability of the sample.

\[
\frac{d\mathbf{M}}{dt} = - (\mathbf{M}_z - \mathbf{M}_0) / T_1
\]

and, for instance, a long \( T_1 \) indicates a slow return to equilibrium. The relaxation has an exponential behaviour.
(3) The 'transverse' components of \( M \) must decay, exponentially, to zero. The time constant is \( T_2 \) or \( T_2^* \), whichever is smaller (more detailed analysis shows that both constants in fact combine).

\[
\frac{dm_x}{dt} = \frac{m_x}{T_2} \quad ; \quad \frac{dm_y}{dt} = \frac{m_y}{T_2} \quad ; \quad \frac{dm_z}{dt} = \frac{m_z}{T_2}
\]

Note that the transverse decay is said to exist even though this analysis assumes \( B_0 \) to be uniform; in this sense, the equations are approximate, although \( \Delta \theta / B_0 \approx 1% \) at worst in Imaging.

(4) It is assumed that the motion of spins around \( B_0 \) and the relaxation motions may be superimposed.

Hence:

\[
\frac{dm_x}{dt} = (m_x \times \gamma B_0)_x - \frac{m_x}{T_2} \\
\frac{dm_y}{dt} = (m_x \times \gamma B_0)_y - \frac{m_y}{T_2} \\
\frac{dm_z}{dt} = (m_x \times \gamma B_0)_z - \frac{(m_z - m_0)}{T_1}
\]

Since, however,

\[
(m_x \times \gamma B_0)_x = \gamma (m_y B_z - m_z B_y) \hat{i} + \gamma (m_z B_x - m_x B_z) \hat{j} + \gamma (m_x B_y - m_y B_x) \hat{k}
\]

and, after the pulse, \( B_\alpha = B_\beta = 0 \), we have:

\[
\frac{dm_x}{dt} = m_y \gamma B_0 - \frac{m_x}{T_2} \quad ; \quad \frac{dm_y}{dt} = -m_x \gamma B_0 - \frac{m_y}{T_2} \quad ; \quad \frac{dm_z}{dt} = -\frac{(m_z - m_0)}{T_1}
\]

The first two terms are often written for the magnetisation in the rotating frame. If the frame rotates at the frequency \( \omega \), then the precession in the rotating frame takes place at \( \omega_0 - \omega = \Delta \omega \). The value of \( \omega_0 \) varies between spins within the range \( 1 / \pi T_2^* \). Conventionally, \( M_\alpha \) is written as \( \omega \),
and $M_y$ as $v$. Also, the rotational sense of the spins should be taken into account, since $\omega_0 = -\gamma B_0$. Hence:

$$
\begin{align*}
\frac{du}{d\tau} &= -v(\Delta \omega) - u/T_2 \\
\frac{dv}{d\tau} &= +u(\Delta \omega) - v/T_2
\end{align*}
$$

(1.22)

Because this is in the rotating frame, the spread of resonant frequencies is important and $\Delta \omega$ here may only represent the precession of localised spins in the constant field $B_0$. 
Molecular Motion and Relaxation Mechanisms in Biological Systems.

Although NMR is a well-established technique for the analysis of many chemical compounds, it is the relatively new applications in biology which are of interest here. Initial studies have focused on the strong and abundant proton response, although other \( ^1H \) nuclei are present. Protons are found in biological systems in all compounds, but those of interest for NMR Imaging form part of small molecules, predominantly \( \text{H}_2\text{O} \). The factors which dictate the spin relaxation are linked to the degree of motional freedom of the molecules. In this section, relaxation mechanisms will be discussed which are applicable to protons in biological systems, although it is acknowledged that (at least) one other nucleus, \( ^{31}\text{P} \) (\( I = \frac{1}{2} \)) is worth studying, and reference to this nucleus will be made where appropriate.

The speeds, directions and collisions of molecules in a fluid are entirely random. It is possible, however, to know the distribution of times between collisions with other molecules, (i.e. the most likely duration of a fixed state of motion) even though the range of times can be very great. The average time is called the 'correlation time', \( \tau_C \).

The intensity of motional frequencies at both the resonant frequency \( \omega_0 \) and at low frequencies are of great interest. For spin-lattice relaxation to occur, energy exchange and therefore transitions must be available with the frequency \( \omega_0 \), and for line broadening there must also be steady local fields. In the rotating frame, this can be shown as follows.

If \( M \) is to be perturbed, it must have a torque acting on it due to the influence of a neighbouring nucleus. This torque must persist for a significant perturbation, so the perturbing field must be stationary in the rotating frame. It is assumed that the magnetisation is unrelaxed and has components in the \( \hat{x}', \hat{y}' \) and \( \hat{z} \) directions. \( M_x \) and \( M_y \) are transverse components, and processes relaxing these are \( T_2 \) processes. \( M_z \) is the longitudinal component and it is increased (so as to align with \( B_0 \)) by \( T_1 \) processes.
Equation (1.18) stated that:
\[ \frac{d\mathbf{M}}{dt} = \mathbf{M} \times \mathbf{v} \times \mathbf{b}. \]

In the case of the microscopic perturbing fields, denoted by \( \mathbf{h} \), components \( h_x, h_y \) and \( h_z \) all exist. The vector product is calculated in full:

\[
(m \times h) = (\hat{h}_x m_y - \hat{h}_y m_x) + (\hat{h}_y m_z - \hat{h}_z m_y) + (\hat{h}_z m_x - \hat{h}_x m_z).
\]

This shows that \( M_x \) and \( M_y \) interact with both transverse and longitudinal fields, whereas \( M_z \) interacts with transverse fields only.

Clearly, any field that is stationary in the \( x', y' \) plane must be oscillating at frequency \( \omega_0 \) in the laboratory frame, and hence \( M_z \) is only affected by these high frequency local fields. \( M_x \) and \( M_y \) are also affected by these nuclei, and also by nuclei that are stationary in the \( z \) direction, i.e. that are static or slow moving in the laboratory frame.

Generally, then, \( T_1 \) processes are affected by high frequency motions and \( T_2 \) processes are affected by both high and low. This shows immediately why \( T_2 \) is never greater than \( T_1 \), and is usually less than \( T_1 \).

The contribution of the motion of the water molecules to relaxation is thus indicated. The system will relax most rapidly when

\[ \tau_c \approx \frac{1}{\omega_0}. \]

**Correlation function and spectral density.**

The magnitude of the relaxation processes at any frequency depends on the mechanism(s) by which the nuclei interact.

Although the frequencies of motion are known to lie within a finite range, it is not simply true that the distribution of frequencies should have, for example, a Gaussian shape whereby low frequencies occur more than high ones. It is therefore not
intuitively obvious that the frequency $\omega_o$ should occur more often under one set of circumstances than under another. We have introduced the 'correlation time' $\tau_c$ as the time that randomly moving molecules spend between collisions. This term is usually defined as the decay constant for a function called the 'correlation function', $k(\tau)$, which defines the position of a molecule at a time $\tau$ relative to its position at an initial time $\tau = 0$. The equation

$$k(\tau) = k(0) e^{-\tau/\tau_c} \quad (1.24)$$

describes the likelihood that the position will be different (uncorrelated) after a time $\tau$; after a long time, it shows that the chances of the two positions being correlated (the same) is very small.

The range of motional frequencies that are present in the spin system may be found from the Fourier transform

$$J(\omega) = \int_{-\infty}^{\infty} d\tau \ k(\tau) e^{-i\omega \tau} \quad (1.25)$$

where $J(\omega)$ is called the 'spectral density function'. (Figure 1.7)

Note that

$$J(\omega) = \int_{-\infty}^{\infty} d\tau \ k(0) e^{-\tau/\tau_c} e^{-i\omega \tau}$$

$$= \mathcal{A} \left[ \frac{\tau_c}{1 + \omega^2 \tau_c^2} \right]$$

where the constant $\mathcal{A}$ takes a value depending on the nature of the motion, translational or rotational.

![Figure 1.7. The Spectral Density function for various correlation times $\tau_c$. (From Slichter (17))](image)

Because the correlation function has a fixed value at $\tau = 0$ (which depends on the average instantaneous proximity of the spins) then the area under $J(\omega)$ is constant. $J(\omega)$ has been drawn for three different values of $\tau_c$; long, medium and short.
These terms are relative to $\sqrt{\omega}$; long means $\omega_0 \tau_c \gg 1$, medium $\omega_0 \tau_c \sim 1$ and short $\omega_0 \tau_c \ll 1$. The latter is known as the 'extreme narrowing limit', where motion is so fast that $T_1 = T_2$; note that under these conditions, $J(0) = J(\omega_0)$, showing that the motional frequencies $0$ and $\omega_0$ do, indeed, occur equally as often. The most rapid relaxation of the system, however, occurs when $\omega_0 \approx \sqrt{\omega}$, because $J(\omega_0)$ has the highest intensity and so $T_1$ is a minimum.

A value of $\tau_c$ was estimated for water by Pople et al. Intra-molecular motion was assumed to be characterised by a rotational correlation time, and was estimated using the Debye model for the drag on a sphere rotating in a viscous medium:

$$\tau_c = \frac{4\pi a^2 \gamma}{3 \kappa T}$$

where $a$ is the radius, $\gamma$ is the viscosity coeff.

Inter-molecular motion was characterised by a translational correlation time, which was assumed to be the time for a molecule to diffuse across a relative distance $r$:

$$\tau_c = \frac{r^2}{2D}$$

where $D$ is the diffusion coefficient (introduced later).

This is then investigated for all values of $r$.

The value of $\tau_c$ estimated from the rotational model was $0.35 \times 10^{-11}$ s, verifying that, for water, the extreme narrowing condition is in operation. $(1/T_1)_{\text{intra}}$ was found to be $0.26^{-1}$ s and $(1/T_1)_{\text{inter}}$ was $0.15$ s. The total value of $T_1 \approx 2.5$ s is in agreement with experiments.

These figures also indicate that, at resonant frequencies encountered in NMR imaging (typically $10^8$ rad s$^{-1}$ ) that relaxation of larger molecules will be faster, since the condition $\omega_0 \approx \sqrt{\tau_c}$ will be approached.

In general, because the transition probability between states is analogous with the relaxation mechanisms, we expect the rate of relaxation $R$ to be

$$R = E^2 f(\tau_c)$$

where $E$ represents the interaction strength and $f$ is some function of $\tau_c$. 

(1.26)
Relaxation mechanisms.

The efficiency of relaxation depends on the interaction mechanisms acting on the nuclei. The relaxation rate is given, if several mechanisms are available, by

\[ 1/T_{v,\alpha} = \sum_j (1/T_{v,\alpha})_j \]

—that is, the relaxation has a contribution from each mechanism. (The subscript indicates that the equation is equally valid for both processes.) Possible mechanisms in biological systems are now discussed.

Dipole-dipole interaction.

The dominant mechanism for protons is the interaction between the dipole moments of the proton and surrounding nuclei, which may be in the same molecule (intramolecular) or another (intermolecular). The proton experiences a magnetic field perturbation which may fluctuate at any frequency described by the spectral density. Rapid fluctuations may lead to \( T_1 \) relaxation, and slow-moving neighbours will contribute to \( T_2 \) relaxation.

Intramolecular interactions involve rotational motion only. A more detailed treatment of the relaxation processes shows that, for like spins,

\[ (R_1)_{rot} = 1/T_1 = \frac{2y^4k^2I(I+1)}{5r^6} \left[ \frac{\tau_c}{1 + \omega^2\tau_c^2} + \frac{4\tau_c}{1 + 4\omega^2\tau_c^2} \right] \]

\[ (R_2)_{rot} = 1/T_2 = \frac{2y^4k^2I(I+1)}{5r^6} \left[ \frac{3\tau_c + 5\tau_c}{1 + \omega^2\tau_c^2} + \frac{2\tau_c}{1 + 4\omega^2\tau_c^2} \right] \]

and when \( \omega_c\tau_c \ll 1 \), and the motions are very free and rapid, as in a liquid,

\[ R_1 = R_2 = \frac{2y^4k^2I(I+1)}{r^6} \tau_c \]

This is known as the 'extreme narrowing limit'. At other times, \( R_2 \gg R_1 \), and dipole-dipole interactions broaden the resonance line above the minimum width dictated by \( T_1 \).

For intermolecular interactions, relative motions are mostly translational. The same reference gives, for like spins:

\[ (R_1)_{trans} = \frac{3}{2}y^4k^2I(I+1) \left\{ J_1(\omega) + J_2(\omega) \right\} \]

Each nucleus has its own spectral density. When unlike spins 'a'
and 'b' are involved, the gyromagnetic ratio term becomes \( \gamma_a^2 \gamma_b^2 \). Paramagnetic impurities contain electrons for which \( \gamma_{\text{electron}} \approx 657 \gamma_{\text{proton}} \) so that when these are present, spin-lattice relaxation is much increased. Oxygen, especially in its molecular form, can relax protons in this way.

**Scalar coupling.**

This is an indirect mechanism whereby spins, especially \( I > \frac{1}{2} \), can relax protons. The electrons in a covalent bond between two nuclei are first coupled with the perturbing spin, and then via electron-electron coupling, transfer fluctuating fields to the proton. In tissue, a good example of this occurs with the abundant \( ^{14}\text{N} \) (\( I = 1 \)) which occurs in NH, NH\(_2\) or NH\(_4^+\) groups.

The \( ^{14}\text{N} \) nucleus may flip between any of its three states (\( m = -1, 0 \) or \( +1 \)) spontaneously and relax protons to which it is coupled. It is assumed that the nitrogen nucleus has a relaxation time \( T_1^{\text{N}} \) for these transitions which is much shorter than that due to any other interactions. The local field that it produces fluctuates with a correlation time \( \tau_e \). The mechanism provides both \( T_1 \) and \( T_2 \) relaxation, and the rates are given by, for nuclei with spins \( I \) and \( S \):

\[
R_1^I = \frac{2A^3 S(S+1)}{3} \frac{\tau_s}{1 + (\omega_x - \omega_s)^2 \tau_s^2} \tau_x^2
\]

\[
R_2^I = \frac{A^2 S(S+1)}{3} \left\{ \frac{\tau_s}{1 + (\omega_x + \omega_s)^2 \tau_s^2} \right\}
\]

where \( A \) is a constant which represents the spin-spin coupling and \( \omega_x, \omega_s \) are the spin frequencies for the two different nuclei; these will be different as each nucleus has an individual value of \( \gamma \) in \( \omega = -\gamma B \).

Relaxation due to the relaxation of another nucleus is known as 'scalar relaxation of the second kind'. 'Scalar relaxation of the first kind' occurs when, due to chemical exchange, the bond between the two nuclei is broken and re-made. It is immaterial to the nucleus \( I \) whether or not the fluctuation in local field is due to the first or second process, so in the case of chemical exchange the above equations still hold, with \( \tau_s \) being replaced by \( \tau_e \), the exchange time.
Scalar coupling may occur in the small amino acid molecules, as well as in other proteins and the macromolecular cell walls.

**Spin-rotation relaxation.**

This is the interaction of the spin with the magnetic field generated at the nucleus by the rotation of the whole molecule, or a large part of it, and it arises from the electronic structure of the molecule. The effect is proportional to the rotational velocity, so that the smaller the molecule, the more important the interaction. For liquids undergoing isotropic molecular re-orientation the relaxation rate is (9):

$$R_i = \left( \frac{2\pi I \hbar T}{\hbar^2} \right) C_{\text{eff}}^2 \tau_{ij}$$

where $C_{\text{eff}}$ is the average component of the spin-rotation tensor and $\tau_{ij}$ is the angular momentum correlation time, which is a measure of the time a molecule spends in any given angular momentum state. It is known that $\tau_{ij}$ gets longer as the temperature increases. Hence it is generally more important at high temperatures or in gases.

In general, spin-rotation interactions are important with nuclei that have a large range of chemical shifts (discussed below), for example, $^{19}$F (100% abundance, $I = \frac{1}{2}$, but not found in tissue); $^{13}$C (I = $\frac{1}{2}$, a rare isotope); $^{15}$N (I = $\frac{1}{2}$, rare) and $^{31}$P (I = $\frac{3}{2}$, 100% abundance, found in tissue).

This mechanism is therefore not important in the study of tissue water, but may be for $^{31}$P compounds.

**Chemical shift.**

The chemical shift, which is the cornerstone of chemical applications of NMR, occurs because the electronic structure of a molecule can produce localised magnetic screening around a resonant nucleus so as to change the local value of the applied field and so alter, or 'shift', its resonant frequency. Compounds may be analysed by noting the shifts, proportional to the main field, which different spins exhibit. Chemical shift in itself does not provide a mechanism for the exchange of energy between spins in different states, and shifted resonance peaks may be distinguished in the Fourier transform of the FID. Protons do not shift significantly, as previously mentioned.
However, because molecules randomly tumble in solution, the vector addition of $B_0$ and local shielding fluctuates, and therefore provides a $T_1$ process. If the shielding is axially symmetric, then in the extreme narrowing limit, \[ R_1 = \frac{2}{15} \gamma^2 B_0^2 \left( \sigma^\parallel - \sigma^\perp \right) \tau_c \] (1.33) where $\sigma^\parallel$ and $\sigma^\perp$ are components of the shielding tensor parallel and perpendicular to the axis of symmetry.

Again, proton resonance is not affected by this mechanism, but $^{31}$P compounds may be, especially if it is studied at high fields, because of the dependance on $B_0^2$.

**Electric quadrupole relaxation.**

A common relaxation mechanism for nuclei with $I > \frac{1}{2}$, and therefore possessing electric quadrupole moments, is the interaction of their electric fields. This mechanism is not applicable for either protons or phosphorus. It is discussed in any reference on chemical NMR (for example, (1)).

**Diffusion.**

This is not strictly a relaxation mechanism for spins, but it does cause a loss of coherence in the transverse magnetic moments when instrumental factors cause spatial inhomogeneities in the applied field $B_0$. Water molecules undergo translational diffusion due to their random motion, both rotational and translational. Motion through the inhomogeneities causes non-reversible phase changes between spins. The effect is usually seen on the reduction in height of a spin echo. This is given by (10):

\[ A(\text{echo at } \tau) \propto \exp \left[ - \left( \frac{2\tau}{T_2} \right) - \frac{2/3}{\gamma^2 G^2 D \tau^3} \right] \]

where $\tau$ is the pulse spacing;
\[ D \] is the diffusion coefficient;
\[ G \] is the spatial field gradient.

The reduction is most significant for long $\tau$. \[ \] (1.34)
1(iv). The response of Healthy Tissue.

About 70% of the bodies of humans and animals is water. Of this, roughly 10% is in the blood plasma, 20% is between the tissues (extra-cellular) and 70% is in the cells (intra-cellular). The total tissue mass, (including cancerous tissue,) is 60 to 90% water. The rest of the mass is made up by proteins, cell membranes and the like.

Only small molecules (including water) will be mobile enough for their resonance lines to be sufficiently narrow to be usefully detected in imaging applications. (This is because the bandwidth of resonance is kept low in order to exclude random electronic noise.) The percentage of body weight made up from all other small (molecular weight $\sim 100$) organic molecules is between 0 and 1% (11).

The notable exception to the above is adipose tissue. This contains 80 to 85% lipids, 10% water and some other macromolecules (12). Other tissues contain lipids, but in much smaller concentrations. Lipids have quite a low molecular weight ($\sim 750$) and are long, flexible molecules which may be mobile enough to have a detectable, but broadened, line shape. Hazlewood (13) studied the mammary tissue of mice and found that the lipids, comprising 20% of the tissue by weight, contributed to the proton signal. The mammary gland is therefore an interesting mixture of the types of responding tissue.

NMR relaxation times have been found for biological tissue (mostly in vitro) to range from 2 to 40% of those of pure water, in which the extreme narrowing limit applies and $T_1$, $T_2$ are of the order of 2 to 3 seconds. The translational diffusion coefficient is approximately 50% of the value for pure water at room temperature.

Taylor and Bore (14) compiled the results of many such studies. They show that the magnitude of a relaxation time relative to that for some arbitrary standard tissue (they chose the kidney) follows a pattern from tissue to tissue which is effectively independant of resonance frequency and species (between rat and mouse). $T_2$
is consistently of the order of 10 times smaller than $T_1$.

Singer and Crookes (15) demonstrated that $T_1$ of whole blood relates to $T_1$ values for plasma and cells in the following manner:

$$\left(R_1\right)_{\text{whole blood}} = \alpha \left(R_1\right)_{\text{cells}} + (1-\alpha) \left(R_1\right)_{\text{plasma}} \quad \text{(1.35)}$$

Battacletti (16) noted that oxygen and haemoglobin are paramagnetic, whereas in their combined state the molecule is not. It was found that $T_1$ for blood varies with the amount of oxygenation. This dependence for blood in tissues, especially the heart, and the fact that the blood is in motion (in vivo) causes a complex measuring problem for imaging. The heart is an ideal subject for $31P$ NMR. In this application, research has so far concentrated on a chemical NMR study (discussed in section 3(i)) in which the chemical shifts are used to identify various phosphorus compounds in living muscle, and then their concentrations are analysed to measure the physical state of the muscle.
Models of Cell Water.

Water provides nearly all the NMR signal from tissue, and yet the structure and nature of intra-cellular water is not precisely known. The extent to which macromolecules and membranes influence the movement of the neighbouring molecules, and thus affect their relaxation times, has prompted three tentative models:

1) In many fields of research, water is thought to be similar to bulk (pure) water (17), suffering insignificant perturbations by the presence of solute macromolecules. This model cannot explain the magnitude of the reduction of $T_1$, $T_2$ and $D$ in all tissues when compared to pure water.

2) Cell water may be loosely 'structured' (18), implying that molecule mobility is reduced. The structuring includes significant influences from macromolecular surfaces which increase correlation times and so decrease relaxation times.

3) Cell water may exist in two fractions between which protons rapidly exchange (19) with a lifetime between states of $\sim 10^{-11}$ sec. Ling (20) provided evidence for the existence of small amounts of bound water around macromolecules, and these form one fraction; the other molecules exist as free water and form the larger fraction. Protons in the 'bound' layers are restricted in motion and therefore have longer correlation times. Protons in rapid exchange possess an 'average' relaxation time of the two fractions.

The last model is the most satisfactory for explaining NMR phenomena, but in neither model '2' nor '3' is a model for the relaxation mechanism explicit; possibly some of the mechanisms listed in the last section play some part in the bound state, as well as the dipole-dipole interaction. It should be noted that the small concentration of paramagnetic ions in cells is insufficient to account for the reduced relaxation times (21).

Daskiewitcz et al (22) found that relaxation rates increased linearly with protein concentration in water for dilute solutions:

$$\left( R_{1,2} \right)_{observed} = \left( R_{1,2} \right)_{bulk} + k_{1,2}c$$

where $R_{1,2}$ (bulk) is the
relaxation rate for bulk, or pure, water, $c$ is the protein concentration and $k_{1,2}$ is a constant. Writing $\rho = 1-c$ as the water concentration, then:

$$\left( R_{1,2} \right)_{\text{observed}} = \left( (R_{1,2})_{\text{bulk}} + k_{1,2} \right) - k_{1,2} \rho$$  \hspace{2cm} (1.37)

This means that the proportionality is such that the relaxation rate decreases with increasing water concentration.

Taylor and Bore (14) took the observed relaxation rate as a weighted average of two parts, with the weighting of the bound fraction being proportional to the amount of 'dry' cell material. For well-hydrated cells, they verified Daskiewitcz's formula. They also showed that, if the cells are not well-hydrated and the amount of 'dry' matter is significant, that the relaxation rate could be inversely proportional to the water content. However, experimental results show that, almost without exception for biological tissue, $T_1$ and $T_2$ increase with water concentration.

Hazlewood (18) discussed many attempts to verify the two-phase model experimentally, but all proposed models to date fail to explain the reduction by a factor of 2 of the molecular translational diffusion coefficient in tissue. This fact has prompted the following three explanations:

1) Structuring in the water (similar to the water model '2') impedes long-range molecule movement.

2) Intracellular membranes compartmentalise the intracellular water.

3) Intracellular macromolecules 'obstruct' the motion of the water.

(Models '2' and '3' apply to the complicated structures in animal cells. In very simple plant cells, the obstructions may simply be the cell walls.)

Unfortunately, the last two models are not fully supported by detailed experiments, and the first model is unhelpful as any degree of structuring may be assumed to explain given results.
1(vi). Water in Cancerous Tissue.

In 1971 Damadian (23) found evidence that the presence of both benign and malignant tumours elevated the relaxation times $T_1$ and $T_2$ in rat tissues. Other researchers have substantiated this observation for mouse tumours ((24), (25)). All mouse tumours showed longer relaxation times than any healthy tissue. Taylor and Bore (14) collected results from these and other workers and demonstrated this graphically.

In an early study, Hazlewood (13) discovered that normal, pre-neoplastic and neoplastic mouse mammary tissue could be differentiated from one another by means of increasing relaxation time. Hollis (26) observed the same trend for a growing Morris liver tumour which they considered to be a pre-cancerous state. The possibility of early diagnosis of cancer can be seen.

Inch et al (27) observed an increase in the relaxation times of healthy tissues from tumourous mice, and correlated this with a general increase in water content of the whole body. The differentiation of diseased and healthy tissue would be reduced by such an effect.

Only a little work has been done on human tissue, and that is in vitro. Scara et al (28) noted an elevation of $T_1$ for malignant and benign tumours of the thyroid gland. McLachlan (27) showed that the $T_1$ of blood plasma reduced proportionately as patients responded to treatment for cancer and a tumour shrank. This is in agreement with other work, for example (30).

Koivula et al showed that $T_1$ in both blood and plasma are elevated in humans suffering with malignant blood disease (31).

It is difficult to compare results between workers because tissues are excised under variable laboratory conditions and may contain uncertain residual quantities of blood. In vivo measurements of tissues, under the exacting conditions of a
functioning metabolism, are thus required. However, all the results suggest that cancerous tissue has increased relaxation times when compared to healthy tissue. The relationship between water content and relaxation times has been discussed. It has been found (for example (32) ) that cancers have increased water content, and it seems probable that this is the cause of their elevated relaxation times. The inference is that the NMR Imaging technique can be selectively receptive to cancerous material, and this is one of its major advantages. In their review, Taylor and Bore (44) discussed the optimisation of the technique with respect to the differentiation of tissues with different relaxation times.
CHAPTER 2

THE NMR SIGNAL AND ITS MEASUREMENT
THE NMR SIGNAL AND ITS MEASUREMENT.

2(i). The Nature of the NMR Signal.

So far, the behaviour of the net magnetisation of the spin system has been discussed without reference to the method of observing it. The perturbing field $B_1$ is generated in a coil within which the sample is placed, and this, or a similar coil, is used to detect the perturbed spins as they precess in a plane which is perpendicular to the main field $B_0$. The receiver coil forms part of a circuit in parallel electrical resonance at the frequency $\omega_0$, and the magnetic dipoles cut the areas of the coil turns and so induce an oscillating voltage across the coil ends. It is this voltage which is recorded as the observed NMR signal.

The emf generated by the changing magnetic flux through the area bounded by the coil turns is given by the Faraday Induction Law:

$$\oint \mathbf{E} \cdot d\mathbf{l} = -\frac{d}{dt} \oint \mathbf{B} \cdot d\mathbf{a} = -\frac{d\Phi}{dt}$$  \hspace{1cm} (2.1)

where $\mathbf{B}$ is the magnetic induction or flux density of the sample, and $\mathbf{E}$ is the magnetic flux, through the coil bounding the surface $S$.

The induced emf is easily calculated, assuming a uniform magnetisation of the sample. The magnetic induction is replaced by the magnetisation of the sample, $\mathbf{M}$, which is assumed to be a function of time, $\mathbf{M} = \mathbf{M}_0 \cos \omega t$. Since it is uniform, the integration is simply equal to the total coil area crossed by the flux; i.e. $A = \pi \pi r^2$. Differentiation with respect to time is trivial. Therefore, the maximum voltage induced in the coil will be given by:

$$v = \omega_0 m_0 A$$  \hspace{1cm} (2.2)

(In a parallel tuned circuit, $v$ will be $Q$ times this value.)

Abragam (33) gives the maximum degree of coherent magnetisation that can be induced in a volume element containing $N$ spins as:

$$m_0 = N \gamma^2 h^2 I(I+1) B_0 / 2kT_s$$  \hspace{1cm} (2.3)

where $T_s$ is the sample temperature and all other symbols have their conventional meanings.
However, NMR Imaging can discriminate spatially within an object because the magnetisation is a function of position, the proton density varying between tissues. Thus, it is important to demonstrate that all contributions to the net magnetism are detected with equal sensitivity throughout the coil (assuming an ideal coil). Hoult and Richards (34) used the expression:

$$\mathbf{E} = -\left(\mathbf{\partial}/\partial t\right) \left\{ \mathbf{B}_1 \cdot \mathbf{M} \right\}$$  \hspace{1cm} (2.4)

to calculate the emf generated by a single dipole $\mathbf{M}$, and then calculated a result from a uniform magnetisation.

$\mathbf{B}_1$ is the field that would be generated at the position of the dipole by the passage of unit current through the receiving coil. Intuitively, (2.4) is similar to the Faraday Law in that the dipole $\mathbf{M}$ includes the units of area because the dipole moment is generated by a current circulating around a loop in its simplest representation. The area is perpendicular to $\mathbf{M}$. In the laboratory frame of reference already introduced, the coil axis lies along the $y$ axis so that (for a solenoid) $\mathbf{B}_1$ is also in the $\hat{y}$ direction; $\mathbf{M}$ precesses after a pulse in the $x$-$y$ plane. Differentiation of the scalar product in equation (2.4) therefore gives the (time-dependant) emf generated from the region of the dipole.

(There is no discrepancy between siting the coil axis on the $y$ axis and yet supposing that the applied perturbation $\mathbf{B}_1(t)$ is in the $\hat{x}$ direction. The switching of direction along the $y$ axis of the oscillating $\mathbf{B}_1$ is pictured as being composed of the vector addition of two contra-rotating vectors with respective frequencies $+\omega_0$ and $-\omega_0$ which move in the $x$-$y$ plane. In the rotating frame, one of these two vectors is stationary, and it is this which the dipoles are pictured as precessing about. The choice of the direction of the component as it is frozen in the $x'$-$y'$ plane is arbitrary.)

If $\mathbf{B}_1$ and $\mathbf{M}$ are known throughout the coil, the induced emf can be predicted:

$$\mathbf{E} = -\int_{\text{volume}} \left(\mathbf{\partial}/\partial t\right) \left\{ \mathbf{B}_1 \cdot \mathbf{M} \right\}$$  \hspace{1cm} (2.5)

The calculation of $\mathbf{B}_1$ is feasible for most shapes of coil. If
is homogeneous throughout the sample volume, then the integration is simplified. The dipole moment is the same for every proton, but the value of the net magnetisation is a function of position. Furthermore, during the imaging experiment, the magnetisation will precess at a frequency that is a function of position, because (as has already been suggested) the main field $B_o$ is deliberately made inhomogeneous to provide this condition. This means that the signal from one region will not be constantly in phase with that from another. The magnetisation is therefore complex, its motion being described by the Bloch Equations. The scalar resultant between $B_1$ and $M$ cannot be represented by a single resultant and a time dependant angle. We have:

$$
\mathbf{E} = (B_1)_{xy} \int dV \hat{\mathbf{e}} \frac{\partial}{\partial t} (M) \\
\text{where } M = M_0 \exp \left( i \left[ \omega_o + \omega^{xy} \right] t \right)
$$

(2.6)

is induced by the magnetisation which is rotating at a frequency which is equal to that of the rotating frame ($\omega_o$) plus an extra term due to the spatially dependant inhomogeneity, ($\omega^{xy}$).

So:

$$
\frac{\partial}{\partial t} M = \left[ \omega_o + \omega^{xy} \right] \exp \left( i \left[ \omega_o + \omega^{xy} \right] t \right) M_0 \exp \left[ \omega^{xy} \right] t
$$

Experimentally, one phase of $\mathbf{E}$ can be measured with a phase sensitive detector, using a reference signal at $\omega_o$. The output represents the induced emf at the frequency of precession in the rotating frame, that is at the frequencies caused by the inhomogeneities.

$$
\mathbf{E}_{rot} = (B_1)_{xy} \int dV \left[ \omega_o + \omega^{xy} \right] \cos \left( \left[ \omega_o + \omega^{xy} \right] t \right) \cos \left( \omega_o t \right) M_0 \exp \left[ \omega^{xy} \right] t
$$

$$
= (B_1)_{xy} \int dV \left[ \omega_o + \omega^{xy} \right] M_0 \exp \left[ \omega^{xy} \right] t \left\{ \frac{1}{2} \cos \left( 2 \omega_o t + \omega^{xy} t \right) \right\}
$$

The frequencies $2\omega_o + \omega^{xy}$ are filtered out by the receiver.

$\omega_o$ is always much greater than $\omega^{xy}$, and so:

$$
\mathbf{E}_{rot} = (B_1)_{xy} \omega_o \int dV \cos (\omega^{xy} t) M_0 \exp \left[ \omega^{xy} \right] t
$$

(2.7)

The behaviour of the integrated magnetism is therefore synonymous with the behaviour of the induced voltage. In this way, the f.i.d. shape represents the decay of the magnetisation coherence and it may be Fourier transformed to yield information about the spin distribution in the sample.
In the early 1970's the resonance line of protons in water was localised within a phantom by deliberately introducing known inhomogeneity into $B_0$. The resonance frequency was thus made specific to a desired location. In America, Damadian (35) patented a technique which he called 'FONAR', in which the resonance was focused at a sensitive point in a warped $B_0$ by applying a shaped $B_1$. In March 1973, Lauterbur (36) published details of a more efficient technique whereby many different locations could be measured simultaneously. He called the technique 'Zeugmatography' from the Greek for 'That which is used for joining', because two magnetic fields were used simultaneously to image details far smaller than the wavelength of the applied radiation.

The principle is as follows. The lineshape of water in a very homogeneous field is a single Lorentzian line, half height width $\Delta$ Hz (Figure 2.1). If a second field is also applied whose strength varies linearly in one direction, then the resonance frequency varies linearly in that direction. In any orthogonal plane, the field is assumed to be effectively single-valued; the contribution to the new lineshape at the corresponding resonance frequency is the integration of the signal induced in the receiver coil by spins in that plane. The new lineshape, disregarding relaxation effects, is recognisable as the 'projected profile' of the water content (Figure 2.2).

Figure 2.1. In an ideal homogeneous field $B_0$, the resonance line is broadened by $T_2$ processes. It has a half-height width $\Delta$ Hz.
Figure 2.2. Resonance line broadened by a linear gradient in the strength of $B_0$.

$\gamma' = B_0' S/2\pi$

$\gamma'' = B_0'' S/2\pi$

Whereas the f.i.d. decay is described initially by $T_2$, when a field variation or 'gradient' is applied the f.i.d. persists for a much shorter time, described by $T_2^*$, which is of the order of the reciprocal of the frequency width ($\gamma'' - \gamma'$). Just how wide this is depends on the resolution of detail required on the image of the water distribution. (The mathematical reconstruction of the plan of water distribution (image) from the projected profiles is described in detail later).

2(iii) Digitisation of the NMR Signal.

The detected NMR signal voltage is an analogue, continuous signal output from a phase sensitive detector. In order that the information that it carries may be stored and operated on in a computer, it must be measured at discrete points and stored as an array of points. The requirements for digitisation of an f.i.d. will be considered first. Problems of sampling a spin echo will be dealt with later.

We already have a picture of the f.i.d. as a smooth, decaying function which is detected after a $90^\circ$ pulse of $B_1$ (Figure 2.3). However, thermal noise in the coil and pre-amplifier of the receiving system produce noise at all frequencies within the
tuned receiver bandwidth. This noise can be thought of as band-limited Johnson or 'white' noise, and is derived from all stages of the receiver and, predominantly, from the receiver coil. The RMS noise level is constant throughout the signal acquisition time and the RMS level of noise in the transformed frequency domain is constant throughout the spectrum in the absence of band-limiting. (Figure 2.4).

The low frequency output of the phase-sensitive detector is usually in the audio range and is therefore readily amplified and digitised. If a single reference frequency of fixed phase with respect to the rotating frame is used, then \( B_0 \) must be increased or decreased so that all spin frequencies are greater, or less than, the reference frequency, because the direction of precession in the rotating frame cannot be distinguished. This is termed 'off-resonance'. The off-resonance f.i.d. always beats with the reference frequency (Figure 2.5).

If two identical reference frequencies are used in phase quadrature, it is possible to allow frequencies either side of the reference to be distinguished from each other, but in the following discussion only off-resonance detection will be considered.

The problems of discrete sampling are discussed by Brigham (37). The relationship between a time signal and its frequency transform is governed by the following points:

1. Discrete sampling of the time signal at intervals \( T \) causes the continuous frequency transform to be periodic, with period \( 1/T \).
(ii) Sampling the time signal for a finite time $T_q$ causes convolution of the frequency transform with the transform of the box function of length $T_q$ which contains the sampled time signal. The convolution broadens each discrete frequency value with a $\sin(\pi f)/(\pi f)$ function which has nodes at distances $1/T_q$ from the convolved point.

(iii) The transform process assumes that the time signal must be periodic with period $T_q$. This is because the frequency transform must be composed of discrete points for computational purposes. The assumed periodicity is of course $T_q$ and the spacing of points is $1/T_q$.

The following consequences should be noted:

(i) Provided that the signal is allowed to be recorded for time $T_q$, the transform will be completely unaffected by the convolution.

(ii) Brigham points out that the consequence of this last point is not valid if the time signal does not have a period $= T_q$. This obviously cannot be possible for a system where no signal relaxation takes place. However, the f.i.d. is a signal with a finite duration and if it becomes insignificant (with respect to either the noise or the sensitivity of the receiver) by the time $t = T_q$, then this consequence holds.

2(iv). The f.i.d. Sampling Time.

The f.i.d. will be characterised by the decay constant $T_2$ or, if the magnet inhomogeneity is dominant, by $T_2^*$. In order that the frequency points are spaced by the half-height width of the lineshape, the f.i.d. must be sampled for a length of time which is found as follows. Neglecting magnet inhomogeneity, an exponentially decaying f.i.d. is described by:

$$E(t) = E_0 \exp(-t/T_2) \quad (t \geq 0)$$

Because the function is not symmetrical about $t=0$, the transform is complex. Brigham (38) shows that the real part, which is detected if the reference phase is chosen correctly, is:

$$s(\theta) = E_0 R \frac{1}{1 + (2\pi f)^2 / R^2} \quad ; \quad R = 1/T_2$$  \hspace{1cm} (2.8)

The functions are sketched in Figure (2.6).
Figure 2.6. (a) Free induction decay of an NMR signal voltage. (b) Frequency spectrum of (a) with half height width shown. (Real part only).

The half-height of the Lorentzian is \( \frac{E_0}{2\tau} \), which occurs when \((2\pi\tau)^2/\tau^2 = 1\), i.e. \( \tau = R/2\pi \). The half-height width of the whole peak is twice this, \( \rho = \frac{R}{\pi} = \frac{1}{4\pi T_2} \). The f.i.d should be sampled for \( \frac{\pi T_2}{2} \) for this to be the resolution between frequency points.

So far, then, before field gradients are applied, the f.i.d. is not insignificantly small at the end of the sampling time \( T_0 = \pi T_2 \). This is not important, however, as the decay is shortened when the field gradients are applied for imaging.

To estimate the reduction in length, suppose that the profile is triangular (Figure (2.7)). The transform, shown alongside, shows the shape of the f.i.d. envelope. If the profile covers \( n \) line-widths, the modified f.i.d. will have its first node after \( 2/n \times \pi T_2 \) seconds. After a time \( T_0 \), the f.i.d. will have about 0.005 of the amplitude that it had at \( t=0 \), and will probably be below the noise level or receiver threshold.

The signal therefore satisfies the condition that it should decay effectively to zero within \( t=T_0 \). In fact, the signal could be sampled for less time, thus increasing the frequency point spacing if this is required. As increasing the gradient strength requires an increase in the receiver bandwidth, which allows more noise through, it is worth considering what is the minimum gradient strength that needs to be applied so that \( n \) points in the frequency profile are 'independent' from each other, and not blurred together because of natural line broadening. The result of such a calculation gives the frequency point spacing and hence sets
Figure 2.7. The transform pair discussed in the estimate of the shortening of the signal decay when a gradient produces a bounded profile shape.

The sketch in Figure (2.6) shows that the natural linewidth will not have zero height at the position of the next discrete point \( f = \frac{1}{\pi T_L} \) away. If any frequency point is not to be overlapped or influenced by its nearest neighbour in a discrete spectrum, then the contribution from this must be less than the change in amplitude of the point that is deemed significant. For instance, if the signal to noise ratio on a frequency point is 100:1, the amplitude of a contribution must be less than 1/100th of the point amplitude in the absence of any other criteria. The separation in linewidths of points, and the contributions that nearest neighbours give by overlapping, are listed in Table 2.1.

<table>
<thead>
<tr>
<th>Separation in linewidths</th>
<th>Contribution, fraction of amplitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1/5 th</td>
</tr>
<tr>
<td>2</td>
<td>1/17 th</td>
</tr>
<tr>
<td>3</td>
<td>1/37 th</td>
</tr>
<tr>
<td>4</td>
<td>1/65 th</td>
</tr>
<tr>
<td>5</td>
<td>1/101 th</td>
</tr>
</tbody>
</table>

For a uniform object (e.g., a human head) it is the high intensity points on the 'diameter' projections which will contribute most significantly to adjacent points. Whole body or head subjects have a notable degree of radial symmetry, and it is at the centre point that a projection value will vary most slowly; the projections...
are much smoother in the centre than the simple example shown in Figure (2.7). It is therefore impossible for the profile to contain points small enough to be below the RMS noise level near the centre (although this could happen with phantoms, if these are discrete volumes of water surrounded by air, for instance). This being the case, the criterion for separation can be relaxed. We can look at the difference in adjacent profile heights and suggest that points should not contribute more to the difference than the noise does.

Judged by this criterion, it is not possible to make a definitive statement about the separation between frequency points. Separation of one linewidth should be regarded as an absolute minimum which is forced by bandwidth considerations, but separations greater than this should be used where possible. For the remaining analysis a value of one linewidth, $\pi T_2$, will be assumed.

Having decided to digitise the signal for $T_0 = \pi T_2$, the problem is whether or not to assume that the f.i.d. is insignificant after a point $T_0$, and then to replace the digitised samples with zeroes, thus reducing the total noise input to the Fourier transform. This problem can be clarified by referring to the equivalent problem in X-ray CT. Here, the transform of the profile is called the 'spatial frequency distribution'. The f.i.d. is simply a spectral density function for the spatial frequencies present in the image; higher spatial frequencies being equivalent to larger values of $t$. In CT, the higher spatial frequencies are suppressed by shaped filters if edge enhancement is not required; the decay of the spatial frequency function indicates that there are very few high frequencies (sharp boundaries) so the suppression of high frequency noise is used to smooth images.

For an f.i.d., the following should be noted. The time information

***************

* For an introduction to X-ray CT see, for example, (37)
is collected for a time $T_o$, so the resolution on the profile is $1/T_o$. However, given a profile made from discrete points of spacing $1/T_o$, then the highest spatial frequency is clearly that which has a period of two spacings, or $2/T_o$; it is impossible to construct a sampled waveform unless it is sampled at least twice in one period. This means that the highest spatial frequency on the profile is $T_o/2$. In other words, for an f.i.d. collected for a time $T_o = \pi T_2$ for resolution purposes, the second half of the sampled points should be replaced with zeroes.

NB. This is NOT equivalent to only having a signal input to the transform of duration $T_o/2$, as the zeroes are treated as data by the transform. Similarly, the convolution function in profile space is not broadened.

This is also NOT equivalent to the practice of adding extra zeroes to a signal to increase the period to $2T_o$ and thus artificially enhancing resolution (see, for example, (40)).

The spin echo (discussed later at some length) presents an interesting problem. If the echo is sampled on both its rising and falling sides, for a time $T_o$, then the resolution on the profile is $1/T_o$. However, the echo is a 'back-to-back' f.i.d., and as such represents a spatial frequency distribution with values no higher than $T_o/2$. The signal therefore should NOT be zeroed. However, if the echo is folded about its maximum and the two halves added together, thus signal averaging, the result has a duration of $T_o/2$. It should then be followed by zeroes to extend the signal length up to $T_o$.

The aim of further zeroing is to reduce noise without suppressing edge information in the image. To a certain extent, the answer to the problem is self-evident. If there is a lot of fine detail in an object, then the spatial frequency function (ie the f.i.d.) will have significant values at the corresponding points (if the receiving system is sensitive enough); that is, it will be significant at large values of $t$. If, however, the number of sharp edges is small, then these values will be below the noise level. As such, they will not be reconstructed in an image, and the edge frequency noise should be suppressed.
2(v) The f.i.d. Sampling Rate.

If a range of frequencies exists in a time signal, then it is well-known that the signal must be sampled at a frequency of sampling that is at least twice as great as the highest frequency present (the Nyquist sampling rate) \(^{37}\). Failure to do this means that the frequency spectrum periodicity occurs with a spacing that is smaller than the range of frequencies, and the frequency aliases overlap with the observed spectrum, thus distorting it.

However, we know that there are noise frequencies present throughout the receiver bandwidth \(^{47}\). The profile spectrum should be detected by a receiver which introduces no significant phase shifts, but it is impossible to design a filter that can cut off sharply above the profile frequencies without introducing phase distortions and attenuation throughout its band pass. There will clearly be a large amount of noise which violates the Nyquist principle and therefore 'aliases' onto the profile, often several times, making the noise level rise in frequency space.

To avoid this, the time signal may be sampled much faster than the Nyquist rate. If \(T_o\) remains fixed, this will mean a larger number of points in the time and frequency domains. The profile will be described by the same number of points as before, and the noise is 'shared out' over the much larger range of frequencies across the transform bandwidth, thus reducing the noise on the actual profile. The ideal sampling rate is one which takes as many time samples in the period \(T_o\) as can be handled and transformed by the computer and the rate of sampling of the analogue-to-digital converting device. The latter device may be the limiting factor, as an ADC with both high resolution and a very fast sampling rate is very expensive.

2(vi). F.i.d. Filters.

This section will deal very briefly with some of the filters that may be applied to the f.i.d. Champeney \(^{42}\) discusses some of them in detail.
(i) Rapid sampling. This has just been discussed. It is a very effective way of reducing profile noise.

(ii) Integrator. This is an RC electrical filter which is applied to the audio output of the phase-sensitive detector. The device is very simple and passive, but only attenuates by 6dB per octave, and so is far from ideal as a band-pass filter.

(iii) Several standard functions exist with which the f.i.d. may be multiplied to achieve a weighting out of noise during the later parts of the signal where the noise is predominant. The essence of a good filter is that the shape of its transform, with which the frequency profile is convoluted, is known to give no great distortion to the profile. One such function is the first order Bessel Function.

(iv) A matched filter or a correlator filter cannot be used. Both of these require a knowledge of the shape of the function which is expected beneath a layer of noise; since each f.i.d. is a unique record of the magnetisation throughout a sample, is cannot be predicted in advance.

2(vii). Additional problems.

(i) Receiver recovery. The tuned receiver coil and receiver pre-amplifier are hit by the excitation pulse and must therefore be protected. However, the pre-amplifier must necessarily be very sensitive. It is very difficult to build a protection circuit for the pre-amplifier input that can immediately dissipate the charge accumulated by the capacitance of the circuit during the pulse. More seriously, perhaps, the RC filter on the receiver may limit the slew rate of the receiver output. In either case, the receiver output should swing to the initial peak voltage of the f.i.d. in a time that is short compared to the ADC sampling rate. Normally it cannot, and the first one or two points are distorted. Since the phase of the profile is governed by the first point of the transform, this can lead to serious errors. To avoid the problem, many experimenters rephase the f.i.d. with a 180° pulse and record the spin echo.

(ii) Often the f.i.d. is recorded in the presence of a d.c. level,
which may be caused by the operation of the phase sensitive detector. This is usually of no consequence, as it is transformed to be the first (zero frequency) point on the profile. However, if the level is a significant fraction of the dynamic range of the ADC analogue input, then it will limit the range remaining for the f.i.d. A large first profile point could also introduce spurious scaling of the profile amplitudes in the computer. D.c. levels should therefore be removed; measurement of the offset on the signal is quite easy.

(iii) Diffusion. In the presence of applied gradients, a spin-echo height may be reduced by translational diffusion (see section 1(iii)). Because intracellular water is effectively trapped within cells, it may be thought that the application of gradients across a whole tissue or body would be without effect, as the water cannot move appreciable distances as it can in a solution sample. However, the technique is very effective with studies of restricted diffusion.

Tanner (43) measured $D$ in yeast cells and, using a modification of equation (1.34), actually measured the dimensions of the barriers preventing motion, i.e. the cell walls. Therefore it is worth calculating the magnitude of the effect on the spin echo in an imaging experiment.

Equation (1.34) requires several parameters. $T_2$ will be often set to the value for the shortest $T_2$ present in the sample, say 30ms (see next section). $D$ is about $1.2 \times 10^{-5}$ cm$^2$/sec, half the value for water. A machine operating at 4 MHz with a homogeneity of (typically) 1 part in $10^5$, requiring 128 points to be distinguishable across a profile, would need a frequency spread of about 6 khz; across a 20cm head, this would require a gradient of about 0.08 G cm$^{-1}$. $\gamma$ is $2.67 \times 10^4$ rad s$^{-1}$ G$^{-1}$. The resultant reduction in echo height given by these parameters is about $10^{-3}$, and is not likely to be significant.
2(viii). Digitisation of the Spin Echo.

Figure (2.8) shows a simple on-resonance spin-echo.

In a $90^\circ - \gamma - 180^\circ - \gamma$-spin echo sequence, the echo should be sampled for $2\gamma$. If the total echo sampling time is $T_o$, then the resolution in frequency space is $1/T_o$, as before. However, the echo is now in the centre of the acquired signal, and it is treated as a time-shifted function. It can be folded about its centre-point and 'signal averaged' by adding the two halves, but this requires that both halves are identical which may not be the case, especially if the phases of spins are manipulated in sophisticated measurements. A better way of handling it is to transform it directly, which yields the profile modulated by a cosine of period $2/T_o$. This is, in fact, simply the profile with every other point inverted. It is a simple matter to invert every other point to recover the profile.

If the echo is misplaced in its digitisation, however, so that it is not quite symmetrical about $T_o/2$, then it will no longer be modulated with the same period as it is demodulated by, and it will exhibit a slow oscillation across the frequency transform. The effect is equivalent to the phase error that can occur by poor synchronisation between the ADC and the signal which results in early or late sampling of an f.i.d.

The optimum position for the echo is found as follows. If $\gamma$ is the spacing of the pulses, then the height of the echo is, after $2\gamma$, $E_o e^{-2\gamma/\tau}$. The echo is acquired for a time $2\gamma$, so the resolvable frequency is $1/2\gamma$. If the signal is acquired for less time, the resolvable frequency rises and the receiver bandwidth must be increased for the same number of points across a profile. In other words, the receiver bandwidth $\propto 1/2\gamma$. In frequency space, the signal:noise ratio varies as $1/(\text{bandwidth per profile point spacing})$. This is because the noise power, or RMS noise voltage, is constant throughout a receiver bandwidth if no attenuation is present, and if the profile bandwidth is doubled then the profile encompasses twice as much noise power.
and spin echo that are observed when a gradient is applied. The dotted line shows the decay if 
$T_2$ is the only decay constant.

A small signal follows the 180° pulse (see text).
The pulse timing is only shown.

Therefore the signal:noise ratio $\propto 2\gamma$.

Also, the area under the profile is proportional to the echo height at $t = \frac{2\tau}{T_2}$ so that if the number of profile points is kept constant the profile height (maximum value) must be proportional to the echo height.

Using these two facts, the signal:noise ratio on the profile $\propto 2\tau e^{-2\gamma/\tau_2}$. The maximum value of this function is found as follows:

$$\frac{d}{dt} \left( 2\tau e^{-2\gamma/\tau_2} \right) = 2e^{-2\gamma/\tau_2} + 2\gamma \left(-\frac{2\gamma}{\tau_2} e^{-2\gamma/\tau_2} \right)$$
$$= 2e^{-2\gamma/\tau_2} - \frac{4\gamma^2}{\tau_2} e^{-2\gamma/\tau_2}$$
$$= e^{-2\gamma/\tau_2} \cdot 2 \left(1 - \frac{2\gamma}{\tau_2} \right)$$

$d/d\tau = 0$ at maximum.

The optimum position for the echo to occur is at $t = T_2$ (If more than one value of $T_2$ is present in a compound, then this refers to the shortest of them.)

Only an infinitely long solenoid has a uniform $B_0$ throughout its length. The flux density in a real coil rolls off towards the coil ends. This means that the pulse length is not constant throughout the coil. Any sample which extends past the ends of the coil will be irradiated in parts by the weaker pulses. In the spin echo sequence, there will be a significant NMR signal following the second '180°' pulse if it is shorter than this value. Clearly, it is undesirable to sample this signal, and so the echo should be moved by increasing $\tau$ fractionally to $\tau'$. The echo is then sampled for $\tau'$ before the maximum and $\tau$ afterwards, the first
number of points in the range \((t' - \tau)\) being subsequently disregarded. \(\tau'\) should still be of the order of \(T_2\).
CHAPTER 3

CONTEMPORARY NMR IMAGING
CONTEMPORARY NMR IMAGING.

3(i). The Different NMR Imaging Methods.

The induced NMR signal is integrated throughout the whole of
the receiver coil volume, and the need to reduce the information
produced to one or two dimensions, as well as to convert it into
a representation of spin density, has produced a variety of imaging
methods. As a general rule, the more the response is localised then
the simpler the mapping of the spins becomes. A simpler experiment
disregards information, but takes longer to map the whole of the sample.
The pursuit of a shorter scan time requires the more complicated
methods. In between simplicity and complexity, a number of novel
approaches have been adopted. All the techniques currently being
tried will be outlined here.

A comprehensive bibliography covering the formative years of NMR
Imaging was given by Lauterbur (45). A list of all published images
was given by Bottomley in 1979 (46).

Mallard (47) published a theoretical $T_1$ map of the human
body, using relaxation times taken from rabbits. Mansfield and
Pykett (48) produced a representation of the human body in cross-
section using known water concentrations of tissue. The data
gives a useful guide to the contrast between tissues, but not every
method is capable of measuring the pure relaxation or density
parameters.

a) Projection Reconstruction, (PR) 'Zeugmatography'.

This method of reconstruction was proposed by Lauterbur (36).
Any method that produces a projection profile (eg. the f.i.d.
Fourier transform) of the object can be used to collect the data,
although strictly speaking the information should be collected
from NMR signal under static planar field gradients to be called
'Zeugmatography'.

The field gradient vector may be made to point in any direction
by the vector addition of three orthogonal gradients $G^x$, $G^y$
and $G^z$. If the gradient is made to point in a complete set of discrete
directions in three dimensions then images in many plane slices
may be reconstructed. Lauterbur (49,50) used this method to produce
slice images of a nectarine, a pigs heart, and a human heart and brain (excised).

In two dimensions, that is in the image plane, the reconstruction of an image from projections may be described fairly simply, and this is done in section 3(iii). The profile \( P \) is a function of both the vector angle in the plane, \( \theta \), and the position along in integer multiples of the discrete point spacing, \( a \). Thus the profile is written as \( P(p \alpha, \theta) \) where \( p \) is an integer.

In polar coordinates, the profile will be a function of \( r \) and \( \theta \). When the image is created in polar coordinates, positions on it are described by multiples of discrete polar coordinates \( r_0 \) and \( \phi_0 \). The most common form of image reconstruction is called 'Filtered Back Projection' (described in section 3(iii) ) which is described by the following equation for the image of spin density \( M \):

\[
M(r, \theta) = \sum_{t=1}^{N} P'(i r_0 \cos(k \phi_0 - t \Delta \theta), t \Delta \theta) \Delta \theta
\]  

(3.1)

where \( P' \) is the profile, the prime indicating that the function has been modified or 'filtered';

\( t \) is the projection number in the range 1 to \( N \);

\( N \) is the total number of projections;

\( r_0, \phi_0 \) are the polar coordinate intervals;

\( \Delta \theta \) is the vector angular increment \( \pi/N \); \( j,k,t \) and \( N \) are integers.

Note that the vector is only swept around a total plane angle of \( \pi \) to prevent duplication of projection profiles.

PR was used by Doyle et al (51,52) and Young (53) to take images produced on an EMI built machine installed in the Hammersmith Hospital, London. An inversion recovery technique was used to make the images 'T_1 dependant'. The simplest form of T_1 dependance is seen in signals obtained using a 180°-τ-90°-τ sequence so that the magnetisation \( M_0 \) is initially inverted along the \( \hat{z} \) axis at \( t = 0 \). Quantitively, the decay of \( M_z \) is given by the Bloch equations (equation 1.21):

\[
\frac{dM_z}{dt} = - \frac{(M_z - M_0)}{T_1}
\]  

(3.2)

Integration of this with \( M_z = -M_0 \) at \( t = 0 \) gives

\[
M_z(t) = M_0 \left( 1 - 2 \exp \left( -t/T_1 \right) \right)
\]  

(3.3)
Thus \( M_z \) is a function of the local \( T_1 \). The problem with this method is in the accuracy of finding \( M_o \). If \( t \) is sufficiently large (\( t \gg 5 T_1 \)) then \( M_z \approx M_o \). However, if \( M_o \) is small, the uncertainty on its measurement, caused by electronic noise, causes the ratio \( M_z / M_o \) to be subject to very significant error, and so \( T_1 \) can only be measured where \( M_o \) is large.

Doyle took cross-sectional images of the human head in vivo and presented these next to X-ray CT pictures taken with the same patient. The NMR picture quality is fair in comparison to the CT pictures, and contrast exists between some tissues which are not discriminated by the X-ray method.

Work such as Doyle's is only concerned with obtaining 2-D slice images, and often it is time consuming to obtain information for 3-D reconstruction when it is not required. Notwithstanding that there are efficient methods of obtaining 3-D information (for example the Polytomic method, which I developed and is described later at length) Zeugmatographic techniques often use a method for the elimination of signal from outside the plane of interest called 'selective excitation'. Since this forms the basis of a different imaging technique, discussion will be postponed until selective excitation imaging is described.

b) Fourier Imaging.

Kumar et al (54) used transient gradients with controlled durations to frequency encode spins as a function of position, and then Fourier transformed the signals to obtain an image. This method differs from Zeugmatography in that static gradients are not used, so that 'projections' are not obtained; a three-dimensional Fourier transform obviates the need for back projection. The timing diagram is shown in Figure 3.1.

The observed signal is composed of the integrated response throughout the sample:

\[
\mathcal{E}(r) = \iiint m(r) f(r, t) \, dV
\]

(3.4)

where \( \mathcal{E}(r) \) is the observed signal;
\( m(r) \) is the spatial spin density function;
Figure 3.1. The timing diagram for the three gradients which are applied during the Fourier imaging method (5^). \( t_x \), \( t_y \) and \( t_z \) are varied individually until a complete set of signals have been collected, before transforming.

\[
f(\mathbf{r},t)\ dV\ 
\]
is the contribution from volume element \( dV \) at position \( \mathbf{r} \).

The three dimensional Fourier transform of the signal is:

\[
s(\omega) = \int \int \int \mathcal{E}(r) \exp(-i\omega \cdot r) \ dV_x \ dV_y \ dV_z \tag{3.5}
\]

The duration of the three gradients are thus varied until enough information is acquired to perform the transform.

Kumar shows that \( s(\omega) = \tilde{m}(r) \), where \( \tilde{m}(r) \) is a 'filtered' spin density function, in which \( m(\mathbf{r}) \) is convoluted with the lineshape function.

A similar method was proposed by Lai (55) which is properly called 'Zeugmatography' because it uses static gradients, combined with a three-dimensional Fourier transform, this time in spherical coordinates. Denoting the coordinate system in the laboratory frame by \( (x,y,z) \) and that in the 3D transform space by \( (u,v,\omega) \), then the 3D transform of \( m(x,y,z) \), the spin density, is

\[
m(u,v,\omega) = \int \int m(x,y,z) \exp(-i2\pi[ux + vy + wz])
\]

This is written as:

\[
m(x,y,z) = \int \int \int s(u,v,\omega) \exp(-i2\pi[ux + vy + wz])
\]

The inverse relation is:

\[
s(u,v,\omega) = \int \int m(u,v,\omega) \exp(i2\pi[ux + vy + wz])
\]

In terms of spherical coordinates in \( uv\omega \) space,

\[
s'(\rho,\alpha,\beta) = s(u,v,\omega)
\]

and equation (3.6) becomes:
\[ M(\alpha, \beta, \gamma) = \iint_S s'(\rho, \alpha, \beta) \exp\left(-i2\pi \rho S_{z\beta}\right) \rho^2 \sin \alpha \, d\alpha \, d\beta \, d\rho \]  

(3.8)

where \(\alpha\) and \(\beta\) are the spherical angles, \(\rho\) is the distance to the origin and \(S_{z\beta}\) is the plane at angle \(\alpha, \beta\).

Lai shows that
\[ s'(\rho, \alpha, \beta) = \int P_{\alpha\beta}(R) e^{i2\pi \rho R} \, dR \]  

(3.9)

showing that \(s'(\rho, \alpha, \beta)\) and \(P_{\alpha\beta}(R)\), the Zeugmatographic projection at position \(R\) in the laboratory frame, are a 1D FT pair; he derives this result in three dimensions. He then finds a value for \(s'(\rho, \alpha, \beta)\) by deriving \(P_{\alpha\beta}(R)\) for a simple time signal:
\[ s'(\rho, \alpha, \beta) = 2\pi \frac{R}{G} \cdot h_{\alpha\beta} \left( 2\pi \rho \frac{S_{z\beta}}{G} \right) \]  

(3.10)

where \(G\) is the linear static field gradient strength and \(h_{\alpha\beta}(\rho)\) is the f.i.d.

Using this in equation (3.8), the density of spins can be found from the 3D FT:
\[ M(\alpha, \beta, \gamma) = \left(\frac{G}{2\pi}\right)^2 \int \sin \alpha \, d\alpha \int h_{\alpha\beta}(\rho) e^{-i\rho GS_{z\beta}/\rho^2} \, d\rho \]  

(3.11)

In practice, the f.i.d. is multiplied by a smoothing filter, and the integration is replaced by finite summation.

Lai has termed the method 'Fourier Reconstruction Zeugmatography' or 'FRZ'. Because it chooses the spherical coordinates to produce a constant increment in solid angle, which the FT requires, it uses less orientations than a 3D projection reconstruction, in which many angles become redundant when they are small, and so is more efficient.

c). Selective Excitation techniques.

In his first experiments, Mansfield (56) was concerned with removing information from the experiment so as to map the sample with a set of 1D line measurements, thus reducing the experimental complexity. To this end he developed the selective excitation technique. The experiment went as follows:

(i) A field gradient is first applied in the \(\hat{z}\) direction and a selective pulse is applied which irradiates only those spins
which are outside the plane of interest (which is orthogonal to the
gradient). The 90° pulse rotates the spins and equalises the
spin populations. The pulse is made spatially selective by
modulating ('tailoring') its shape so that the spectral distribution
of the pulse excites only those spins (outside the plane) that
are required. The spectral shape is Fourier transformed and the
resulting function is the pulse envelope. Negative amplitudes in
this function are achieved with a 180° phase shift of the pulse.
For instance, if one requires a pulse which has a discrete rectan-
gular spectral distribution (used in part (ii) of this method)
then the time domain pulse is shaped according to:

$$B_c(t) = 2B_m \Delta \omega_p \text{sinc}(\Delta \omega_p mt) \cdot \exp(i\omega t)$$

where the 'sinc' function is the FT of the square distribution, and
is of the form \(\sin(\omega t)/(\omega t)\), of width \(2m \Delta \omega_p\) (m integer)
centered at the frequency \(\omega\). \(\Delta \omega_p\) is the angular frequency per
point.

(ii) The gradient is replaced by another one, \(G^Y\), and
a second selective pulse excites a line across the previously
undisturbed layer of spins.

(iii) \(G^Y\) is replaced by \(G^X\), which frequency-encodes the
spins along the line. The f.i.d. from the line yields the spin
distribution along the line. No reconstruction is necessary, as
the whole plane can be mapped by covering the plane by a raster
of sensitive lines in successive experiments. The process is
drawn in Figures 3.2 and 3.3.

Figure 3.2. Switching sequence in the sensitive line experiment.
(a) Applied gradients; (b) r.f. pulse sequence; (c) NMR signal.
Figure 3.3. The selective line is examined as follows:
(a) a slice is left undisturbed in the x-y plane;
(b) a line is excited using a selective pulse;
(c) the line is frequency-encoded using a 'read gradient' $G^x$.

The combined effects of pulse tailoring and the appropriate field gradients is represented by a spatially selective operator $(1 - \hat{S}_z^x)$. If the spin density distribution is $M(x,y,z)$ then $\hat{S}_z^x M(x,y,z)$ represents the undisturbed spins in stage (i) and $(1 - \hat{S}_z^x) M(x,y,z)$ is the distribution which recieves the first $90^\circ$ pulse. A second operator, $\hat{S}_y^x$, selects all the spins which are nutated by the second pulse in stage (ii). Mansfield solves the operator equations to show that the only non-vanishing signal following $t_y$ is:

$$E_x(t) = \int \hat{S}_y^x \hat{S}_z^x M(x,y,z) \cos(y x G^x t) \, dv$$

thus formally showing that the model will work in practice.

Morris et al (57) used line scanning to image a human abdomen in vivo, and also a cadaver thigh and excised breast. Hansen et al (58) used the technique to image rats in vivo, and saw high contrast and evidence of flow in vessels. In this case, the pulse sequence was fast enough to cause $T_1$ dependance in the signal, as each line was measured with a series of repeated measurements in order to signal average, and the repetition time was less than the spin systems required for complete spin-lattice relaxation. His equipment was designed by Crooks (59).

Mansfield's next step was to saturate in one dimension only and image the whole plane simultaneously (60) with what was called
Planar Imaging. After the initial stage of plane selection, the spectral distribution of $B_1$ is calculated so as to excite a set of parallel discrete lines, instead of only one. In this experiment, the final 'read gradient' is not orthogonal to the second gradient $G^y$ but is angled in the x-y plane by vector addition with $G^y$. The angle is such that all the frequency distributions along the lines are projected onto a single 1D spectrum where they can be formed into an array to represent the plane image (Figure 3.4).

![Figure 3.4](image)

Mansfield has pointed out that Planar Imaging can be extended for simultaneous 3D imaging, but without developing this idea the new Multiplanar Echo Imaging was introduced (61). In two dimensions, the discreteness in the y direction is not introduced in the spectral density of $B_1$ but rather by causing $G^y$ to reverse in direction periodically. This causes a discreteness to be imposed on the profiles caused by $G^y$ alone. The orthogonal gradient $G^x$ is applied statically throughout, so that the whole plane spectrum may be projected in 1D, as before. In three dimensions, $G^z$ may also be switched simultaneously, but at such a rate as to cause discreteness in the $G^z$ profiles that fall between the discrete lineshapes caused by $G^y$. The poor signal:noise figures accompanying this very fast method have so far prevented any significant clinical images being published.

In both Multiplanar and Planar Imaging (the latter extended to three dimensions), in the limit that the ranges $\Delta z$ and $\Delta y$ become zero, the three spatial sampling operators yield a discrete lattice of points (disregarding $T_2$ broadening):

$$\hat{S}_x \hat{S}_y \hat{S}_z m(\alpha, y, z) \to m(la, mb, nc) = \delta_{mn} (1, m, n)$$

In this limit, the f.i.d. signal becomes:
\[
\varepsilon = \sum m_{lmm} \cos k [ l \Delta \omega_x + m \Delta \omega_y + n \Delta \omega_z ] \Delta n_{lmm}
\]  \hspace{1cm} (3.15)

where \(\Delta n_{lmm}\) is the volume of spins at a lattice point contributing to a signal, the lattice spacings being \(a, b\) and \(c\), not necessarily equal. When \(M\) and \(N\) are the largest values of \(m\) and \(n\), all points in the frequency domain are uniquely defined if:

\[
N \Delta \omega_x \leq \Delta \omega_y \leq \Delta \omega_z / M
\]  \hspace{1cm} (3.16)

The angular increments given by \(\Delta \omega = \gamma G \delta w\) etc.

d) Steady-state Free Precession (SFP) Techniques.

Two methods for data collection and image formation were suggested by Hinshaw (62) which use the technique for inducing a signal which was proposed by Carr (63) called SFP. Hinshaw solved the Bloch equations for a string of pulses and showed that, for \(90^\circ\) pulses and \(T_1 = T_2\), the magnetisation just after each pulse when a steady state has been reached, \(m_y^+\), is given by:

\[
m_y^+ = m_0 \frac{1 - E_1 \cos \beta}{1 + E_1 (1 - \cos \beta) + E_1 \alpha}
\]  \hspace{1cm} (3.17)

where \(\beta = \omega_0 T\), \(T\) is the inter-pulse interval and \(\delta \omega\) is \(\omega_0 - \omega\). \(E_1\) is \(\exp (-T/T_1)\).

If the rf pulses are off resonance by an amount \(\delta \omega = k\pi / T\) where \(k\) is an odd integer, then, for closely spaced pulses and using the approximation \(E_1 = 1 - T / T_1\), the equation 3.17 becomes:

\[
m_y^+ = \frac{1}{2} m_0
\]  \hspace{1cm} (3.18)

Thus half of the full equilibrium magnetisation is continuously in the \(x-y\) plane and induces a large continuous signal. The time required for the steady state to build up is of the order of \(T_1\).

A requirement for the steady state to be achieved is that the field \(B_0\) should remain static. Hinshaw proposed that time-dependance of the field, using oscillating gradients, could limit the signal spatially. He showed that, for a homogeneous line-width that is much smaller than the modulation frequency, that if a gradient is sinusoidally modulated, the observed signal is proportional to:

\[
\varepsilon \propto \gamma B_1 T_2 m_0 \cdot J_0^2 \left( \frac{\gamma G \delta \omega}{\omega m} \right)
\]  \hspace{1cm} (3.19)
where \( J_0 \) is the zeroth order Bessel function which describes the envelope of responding spins in the \( \hat{x} \) direction when the \( G^x \) gradient is modulated. \( \omega_m \) is the modulation frequency.

Three orthogonal gradients define a 'Sensitive Point'. This yields a signal that is very easily reconstructed, as the signal intensity is simply recorded as a point height on an image array. The point is scanned over the whole image array by gradient current manipulations. Unfortunately, simplicity is matched by a long total scan time.

Two orthogonal gradients are oscillated and the third is static in the version known as 'Multiple Sensitive Point' imaging. The repetition of the pulses causes discreteness in the profile that is formed along the static gradient. The image is reconstructed in the same way as the Mansfield line scan. The MSP technique was used with success by Hinshaw (64) but it is not as efficient as techniques which image a whole plane. Hawkes et al (65) used SFP with one time dependant and one static gradient to obtain an MSP projection profile of an object which is reconstructed by projection reconstruction. A series of papers, of which one is referenced here, show the results of the clinical trials of the method, showing coronal, sagital and axial image slices of the human head in vivo. The repetition rate of the pulses can be varied in the experiment to enhance contrast between tissues with different \( T_1 : T_2 \) ratios.

Because the steady-state response is independent from lineshape considerations (equation 3.18) and localisation is a function of applied gradients (equation 3.19) the SFP technique is most useful with magnets that have poor homogeneity, such as the large, whole body machines in medical research.

e) FONAR

FONAR was invented by Damadian (35). A deliberately warped \( B_0 \), with a single 2D saddle point, is applied to a sample. Only at the saddle point will the field be uniform enough for a strong, monochromatic signal to be obtained. A \( B_1 \) is applied at the resonance frequency of the sensitive point. The point is scanned across an image plane by physically moving the magnet.
Some in vivo human images have been presented of the thorax, but resolution appeared to be poor (66).

f) Rotating Frame Zeugmatography.

This novel technique was proposed by Hoult (67). A homogeneous field plus one field gradient are applied in the laboratory frame. A field gradient is also applied in the rotating frame by applying a deliberately inhomogeneous B\textsubscript{1}, so that the precession angle is dependent on position in one laboratory frame coordinate. The amplitude of the received signal depends on the pulse length t\textsubscript{p} and on the laboratory position, say, in the \( \hat{x} \) direction. Resolution in \( x \) is therefore obtained by varying t\textsubscript{p}. Resolution in the other direction is frequency dependent in the static field.

Applied pulses are sufficiently long that the spins which experience the highest B\textsubscript{1} field flip between \( 90^\circ \) and \( (2\pi n) + 90^\circ \) for the minimum and maximum values of t\textsubscript{p}. The flip angle as a function of \( x \) can be seen in Figure 3.5.

![Figure 3.5. The spins in the rotating frame experiment. The precession angle varies along the x' axis. The 'pitch' of the 'screw' changes as t\textsubscript{p} is altered.](image)

Plane definition can be achieved using a second B\textsubscript{1} gradient, but at the expense of complicated probe coils. It can also be accomplished by phase modulation (68).

The aim of the research is to image premature babies, but so far no results have been published.

g) Spin Warp Imaging.

This technique, introduced by Edelstein (69) uses phase encoding to produce resolution in one of the two directions in the image.
plane. This is achieved by having a transient gradient pulse of controlled magnitude during the pulse sequence. Because the other gradient in the imaging plane is static during data collection, and does not rotate as in Zeugmatography, the $B_0$ inhomogeneities can be corrected, and the frequency point spacing requirements relaxed. Consequently, a lower $B_0$ can be tolerated, which makes the experimental rf requirements more lenient. The image is reconstructed using one phase-decoding FT and one projection transform. The use of phase encoding is described in Chapter 4 as it is the basis for the Polytonic Imaging method for plane definition, so it will not be described further here.

The Spin Warp method is unlike the Polytonic method for several reasons, not least in that the phase-encoding takes place within the image plane. (Spin Warp planes are selected by selective excitation.) Another difference is that Spin Warp uses an inversion recovery technique (preceding the $90^\circ$ pulse with a $180^\circ$ inversion pulse) which is used when $T_1$ information is required. Data is collected from a spin-echo which is caused by the inversion of the gradient which is subsequently held static in the imaging plane.

The $T_1$ technique was fully described in the earlier work (70) in which images were reconstructed line by line. (The shortcomings of the simple $T_1$ dependant methods was outlined earlier.) Spin Warp has produced pictures of the human chest, abdomen and legs.

h) Polytonic Imaging.

This is the name given to the multiple plane imaging technique which I have developed and which is described in detail in Chapter 4. ('Poly' - many; '-tonic' - tomographic images.) Each plane is reconstructed by the projection reconstruction technique, and the planes are differentiated from each other by transient gradient phase encoding in the direction orthogonal to the planes.

i) Other Related Work.

The work on Topical Magnetic Resonance (71) is not an imaging technique, yet it does produce a localised NMR response in vivo. A warped field, similar to the FONAR field, is applied, although because of sensitivity problems, the saddle point is
quite broad, so as to encompass a large volume. The resulting frequency spectrum is a high resolution experiment, so that the various chemical compounds present may be identified, and their concentrations measured so as to gauge the metabolic state of the tissue. The work is concerned with $^{31}$p compounds.

A novel approach to imaging was suggested by Johnson (72). He demonstrated that signals, albeit weak, could be detected in a magnetic field using a receiver coil held against the subject, the coil not surrounding the object in the conventional way. The induced magnetic field is thought to have a return path outside the body, which is detected. The work suggests that the signal may be far too weak for clinical use.
3(ii). Relative Efficiency of the Imaging Techniques.

Most of the techniques discussed in the last chapter were compared in terms of sensitivity and performance time by Brunner and Ernst (73). The theoretical signal:noise ratio was calculated for each method, and defined as a relative quantity between methods which reflects the accumulated signal energy per unit time. Because each technique will fail to reach its theoretical best performance, these figures are not an exact measure. However, measurements of minimum performance (scan) time and of relative sensitivity highlight the two salient features of the methods by which they should be compared. These are:

a) the number of dimensions discriminated in one measurement;

b) the number of spins that are measured in one measurement.

To demonstrate point (a), the Echo Planar method simultaneously forms the complete 3D image per measurement, whereas the line scan and projection reconstruction methods only measure one dimension, and the sensitive point measures only a single point. In terms of minimum performance time, the greater the number of dimensions measured simultaneously, the faster the total scan will be. Clearly, with relaxation times in the order of hundreds of milliseconds which must be allowed between measurements, it is only the 2D or 3D methods that will be able to compete with current X-ray CT scanners in minimum performance time.

Point (b) is demonstrated by comparing the line scan methods with projection reconstruction. Both measure information in 1D only, and the line scan will require \( n \) lines for an image where the PR technique will need \( n \) angles; both therefore have the same scan time. Where they differ is that PR measures the spin signals from all over the plane, whereas the line scan measures spins in a single line after those spins outside the line have been ignored. The PR signal is of the order of \( n \) times larger, and it is therefore more sensitive.

Most techniques measure 1D information, but with varying efficiencies. The selective excitation experiments (not including Echo Planar) restrict information to narrow bands across the image and hence miss information between the bands (see Figure 3.4). Fourier
Imaging measures the whole plane or sample, but the gradient manipulations before data collection restrict the sampling time and thus increase the noise contribution. Line scanning and sensitive point measurements are wasteful because they throw away data.

The most efficient techniques measure the whole sample simultaneously. These include Planar imaging (in 2D) and Echo Planar Imaging, and also projection reconstruction methods when selective excitation is not used to define planes. 3D Zeugmatography, FRZ and Polytomic Imaging waste no signal energy, and the latter is particularly easy to implement and is flexible. The fastest technique is certainly the Echo Planar method, and it approaches what Brunner and Ernst call 'the optimum imaging technique'. It is, however, complicated to implement.

The poor sensitivity of NMR may well mean that the minimum performance times of the fastest methods (a 'single shot' experiment) can never be achieved, as signal averaging to improve signal:noise may be the rule. If a few minutes for a complete scan is tolerable, then there is no method more efficient than projection reconstruction.

Between techniques which are equivalent when judged by these criteria, one method may be more able to distinguish between dissimilar tissues than another because of a greater contrast between the individual signals. Taylor and Bore (14) indicated that the complex dependance of the NMR signal on relaxation times may mean that the pulse repetition rate has an optimum value for contrast between particular tissues. In comparing the signal strengths following repeated 90° pulses with that obtained form the SFP technique using a higher pulse repetition rate, they also showed that the signal strengths per unit time may depart from the simple proportionality with water content as the pulse repetition rate increases towards the SFP condition. In so doing, they have shown that the pulse sequence may necessarily have to be carefully chosen to suit a particular tissue contrast requirement.
3(iii). The Mathematical Reconstruction of Images from Projections.

The problem in many forms of 2D scanning is to reconstruct a two-dimensional image from one dimensional projections of information about the object. Although some NMR techniques do not produce projections, the subsequent work in this thesis is based upon Projection Reconstruction in common with many other medical imaging techniques, and it is this type of reconstruction that will be considered now.

The picture of a 2D slice is called a tomogram, and the art of obtaining it is 'tomography'. If the image is constructed using digital computation, the process is known as computed tomography or 'CT'. NMR Imaging is one of the many medical CT techniques.

The reconstruction of a picture from 1D projections was first solved by Bracewell and Riddle (7?) in the field of astrophysics, based on early work by J. Radon. The local activity of the sun was to be measured using the integrated intensity from strips across its surface. The equivalent measurement in contemporary research is that of the total radioactive emission from a line through a subject, the total attenuation of an X-ray or the integrated induced proton signal along a line.

The ray-sum in X-ray CT is the height of a single projection profile point and it is inversely proportional to the total attenuation along the ray path. A complete set of ray-sums is called a 'projection', which is called in this thesis a 'projection profile' or 'profile' to distinguish it from a ray-sum, which is sometimes confusingly also called a 'projection'. This is what the x-ray scanner measures. The NMR scanner, on the other hand, measures a time signal which is composed of a band of resonance frequencies, called the f.i.d.

The Fourier transform of the f.i.d. yields the 1D spin frequency distribution which is the 1D projection profile. This is equivalent to the spatial projection profile in X-ray CT. The FT of the X-ray CT profile yields the spatial frequency distribution of the profile. This is equivalent to the time signal f.i.d. Thus there is considerable
scope for misunderstandings when the existing CT reconstruction terminology is applied to the 'new' NMR experiment.

The available methods of reconstruction from projections may be classified as follows:

1) Back projection.
2) Iterative reconstruction.
3) Analytical reconstruction: - Filtered back projection;
   - 2D Fourier reconstruction.

1) Back projection is the simplest reconstruction method, but it does not produce good results. For each profile, the values of points on the image array are increased by the value on the profile that is normal to the point. Its shortcomings are illustrated in Figure 3.6, where the profiles of a small object are back-projected. The coincidence of back projections show the position of the original object, but information is added to the whole image array, and the background intensity rises. Furthermore, information from surrounding objects will be superimposed on the reconstruction. These shortcomings are overcome with 'filtered back projection'.

Figure 3.6. (a) shows the projection profiles, or shadows, of an object, taken at three angles. (b) shows the simple back-projection of profiles from a point source, showing the 'star-burst' artefact on the image.
2) The term 'iterative' refers to a method of successive approximations in which an arbitrary starting image is chosen and corrections are applied to it to bring it into better agreement with the measured projections. This is done by comparing the actual projections with calculated projections from the image array as it is at any particular stage in the reconstruction. There are three variations of iterative reconstruction currently used, and they differ in the sequence in which corrections are made during the iteration:

**ILST (Iterative Least Squares Technique).** In this simplest approach, all projections are calculated before any corrections are made. The simultaneous correction of all the image points according to the difference between measured and calculated projections causes over-correction on the array, and this is reduced by the application of a damping factor to the corrections. The damping factor is often chosen to produce the best least squares fit after each iteration. This correction was introduced by Goitein (75).

**ART (Algebraic Reconstruction Technique).** In this method, the corrections are applied to each ray-sum path across the image sequentially. Thus, corrections for subsequent rays always embody the corrections that were made for previous ray-sums and whole profiles. The method was named by Gordon (76).

**SIERT (Simultaneous Iterative Reconstruction Technique).** In this variation, each point on the image array is corrected for all the rays that pass through it, the array then being corrected point by point. Corrections to subsequent points embody the corrections that were made on previous points. The method was introduced by Gilbert (77).

3) **2D Fourier reconstruction.** The 2D Fourier reconstruction is intuitively clear in the NMR experiment. The f.i.d. is the 1D FT of the projection profiles, and a 2D FT using all the f.i.d.s will yield an image. FRZ (section 3(i)) is a more complicated version of this, in which a 3D FT is performed in spherical coordinates, in which the angular intervals are chosen to obviate the need for interpolations in Fourier space which will be shown to be normally necessary.
Let \( M(x,y) \) be the two-dimensional spin distribution and 
\( P_{\theta}(R) \) be the projection onto a line at an angle \( \theta \) in the image plane. 
The function \( M(x,y) \) can be found by 2D FT of its transform 
\( s(X,Y) \), which is in the same Fourier space as \( P_{\theta}(R) \):
\[
M(x,y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} s(x,y) e^{i2\pi(xX+yY)} dx dy
\]
(3.20)

Unfortunately, \( s(X,Y) \) is only known on all points on lines 
through the origin; along these lines, at angle \( \theta \), the values are 
given by \( P_{\theta}(R) \). It is necessary to interpolate between the 
set of radial lines described by the set of \( P_{\theta}(R) \) to obtain the 
values of \( s \) at all \( X \) and \( Y \) points. Much discussion regarding 
the Fourier method of reconstruction concentrates on the choice of 
interpolation method. These will not be discussed further here.

**Filtered Back Projection.** If the object shown in Figure 3.6 
is infinitely thin, and the profiles are taken at all possible 
angles, the background appears to be a \( 1/r \) blurring function. 
Any heterogeneous distribution of protons will have each point 
blurred by this function in the back-projection. The image is in 
fact the convolution of the spin distribution and the \( 1/r \) function :
\[
\mu''(r', \theta) = \int \mu((r-r'), \theta) \frac{1}{r'} dr'
\]
(3.21)

where \( \mu \) is the correct image (The symbol \( \mu \) is used to indicate 
a map of any parameter; if the image is purely one of 
spin density, then \( \mu \) could be replaced by \( M \); however, 
\( \mu \) could be a complicated mixture of \( M \) and relaxation 
time information in a real experiment.)

\( \mu_{bp} \) is the back-projection, convoluted image.

The filtering in the filtered back projection algorithm is 
meant to deconvolute the back projection image by modification 
of the profiles. (Convolution implies that each point is combined 
with every other profile point where the others have been weighted 
by the correcting function.)

The discrete filtered profiles are given by:
\[
\rho'_{l}(na) = \sum_{q=-\infty}^{q'=\infty} P_{\theta}(qa) f((n-q)a)
\]
(3.22)
where \( P_\theta (qa) \) is a projection profile composed of discrete points, 
\( a \) is the spacing between points and 
\( q \) is an integer.

The Fourier transform of the function \( f(qa) \) is a ramp function, 
\[ F(t) = t, \quad t \leq |T|; = 0, \quad t > |T| \quad (T = 1/a) \]. The discrete values which it has in frequency (projection) space are:
\[
\begin{align*}
\hat{f}(qa) &= \frac{1}{4a} \quad (q=0) \\
&= -\frac{1}{\pi^2q^2a} \quad (q \text{ odd}) \\
&= 0 \quad (q \text{ even})
\end{align*}
\] (3.23)

These are the values given by Ramachandran and Takshminarayan (78) except that a factor of \( 1/a \) has been omitted. This is because in deriving the above equation 3.22 from the case for a continuous profile, a factor of \( a \) is introduced as a constant. When the values are used in a computer algorithm, the spacing between points is unity. The substitution of the values thus obtained gives the following formula for computation:
\[
P_\theta'(n) = P_\theta(n)/4 - \frac{1}{\pi^2} \sum_{q \text{ odd}} P_\theta(n-q)/q^2
\] (3.24)

\( n \) and \( q \) now represent the 1D array positions in the computation.

The image is formed by the following summation of the filtered projection profiles, now included at all \( N \) angles:
\[
\mu(r, \theta) = \mu(jr_0, k\phi_0) = \sum_{t=1}^{N} P'(jr_0 \cos(k\phi_0 - t\Delta\theta), t\Delta\theta) \Delta\theta
\] (3.25)

where \( t \) is the projection number, in the range 1 to \( N \); 
\( \Delta\theta \) is the angular increment between projections, \( = \pi/N \); 
\( r_0, \phi_0 \) are the polar coordinate intervals; 
\( j, k, t, N \) are integers.

Interpolation is necessary to find the values on a square array \( \mu(x,y) \).

An alternative approach is to multiply the profile transform (ie. the f.i.d.) with the ramp function \( F(t) \). This is equivalent to the convolution in profile space. In this instance, there is a chance of reduced computational time. The convolution of two functions, each of \( N \) points, requires at least \( N^2 \) multiplications.
as each point on the filtered profile is combined with the weighted values of all the other points. This is, however, equivalent to the multiplication of two functions in the other Fourier domain. For a profile produced by any CT system, this way of filtering will use $N$ multiplications in Fourier space plus two $N$-point Fourier transforms, each of which requires $N \log_2 N$ multiplications (77). To enter and leave the Fourier space and perform the multiply will therefore take $N + 2N \log_2 N$ multiplications. This is less than $N^2$ for large $N$. In NMR CT, where the f.i.d. is already available, the multiply and transform to profile space will take $N + N \log_2 N$ multiplications. As the transform is inevitable in NMR CT, this means that the savings are a factor of $N$ if the convolution is done by multiplying the f.i.d.

If the time signal is multiplied, care must be taken in the selection of the details of the ramp function. The NMR time signal has a continuous noise baseline which buries the signal as it decays, and since the ramp will accentuate this noise, a cut-off frequency is desirable (discussed in section 2(iv)). Hutchinson (90) has used filtering of spin echoes in the time domain.
3(iv). Advantages and Limitations of NMR Imaging.

Advantages.

a) Signals are induced by the magnetic resonance of protons (or other suitable nuclei) in the tissue molecules. After the transient measurement, the nuclei relax to their initial energy states. There is no lasting damage (as with ionisation due to Compton scattering in X-ray CT) or substitution of nuclei (as with radioactive labelling).

b) The energy deposited in the tissue during irradiation by $B_1$ is very small. The physiological effect of energy deposition involves photon energies of the order of $10^{-7}$ eV, compared to an X-ray beam energy of around 70 keV. (The absorption of energy is discussed in section 3(v).)

c) Within the limits for static magnetic field strength, rate of change of field and r.f. exposure (discussed in section 3(v)) there is no documented pathological effect at all, and no accumulated dose.

d) The restriction of information to a single plane could be achieved at any angle to the receiver coil axis. The image is therefore not restricted to being a cross-section. Longitudinal planes or any choice of coronal, sagittal and axial planes may be chosen. Also, some techniques allow the restricted investigation of volumes of interest within the subject, for example Sensitive Point imaging (62) and TMR (71).

e) Restriction of the signal isotropy is achieved by the differential selection of current magnitudes passing through the gradient-producing coils (with the exception of FONAR (35)) so that an NMR Imaging device needs no moving parts. This gives advantages in both reliability and ultimate performance time.

f) The protons which are detected occur mostly in intracellular water. The technique therefore measures parameters of 'soft' tissues, in contrast to X-ray experiments. NMR may therefore produce contrast between tissues (for instance in the brain) which is not available with other non-invasive methods (65).
g) The NMR signal is usually not only a function of spin density, but also of the relaxation times \( T_1 \) and \( T_2 \). The range of water content in tissues is between 60 and 90%, but the range of relaxation times is much greater; Taylor and Bore (14) show a range of \( T_1 \) from about 300 msec for the liver up to about 900 msec for a mammary tumour. They also indicate how all the parameters may be combined together to obtain high contrast between two types of tissue, for example, one healthy and one cancerous. Other techniques aim to obtain pure \( T_1 \) images (19) which should exhibit a wide range of contrast.

h) There are other nuclei of spin \( I = \frac{1}{2} \) that can be investigated. Whichever nucleus is chosen, the NMR signal is specific to it because of the unique values of \( \gamma \). However, against this versatility must be set the disadvantage of relative sensitivity, which depends on two things:

1. All nuclei have \( \gamma < \gamma_{proton} \) and sensitivity to detection has \( \gamma^3 \) dependence. This is because \( \mathcal{E}_r \), the voltage induced in the receiver coil, is proportional to \( \omega_0 m_0 \), and \( m_0 \propto \gamma^2 \) (section 2(i)).

2. The natural abundance of other nuclei in tissue, and the natural abundance of the isotopes of these nuclei with \( I = \frac{1}{2} \), is much less than that of protons.

I have calculated the relative signal voltages induced in a coil by protons and other nuclei occurring in muscle and bone in a constant field \( B_0 \) (Table 3.1). Note, however, that the other nuclei occur in large molecules and may therefore have very broad resonance lines. This may restrict their use to a chemical study in a small localised region of interest.

Only two other nuclei have been studied to date. One is \( ^{31}P \), which has been studied in vivo using Topical Magnetic Resonance (71). The other is \( ^{19}F \), which does not naturally appear in tissue. However, it has been used in the compound perfluorotributylamine to perform some imaging with phantoms (82). \( S_{19F} = 4.00 \), so it is a very sensitive nucleus.) This fluorine compound can be used as a 100% blood replacement in vivo, and laboratory animals have survived ventilation with oxygenated fluorocarbon liquid. \( ^{19}F \) may be used in the future as a 'tracer' compound to investigate vascular functions.
Table 3.1. Receptivity of nuclei found in muscle and in bone tissue, as a % of the receptivity of $^1$H.

<table>
<thead>
<tr>
<th>ELEMENT</th>
<th>% of atoms in tissue by number.</th>
<th>% abundance of $I=\frac{1}{2}$ isotope.</th>
<th>% of isotope in tissue by number</th>
<th>$\gamma_{\text{H}_2\text{O}}$</th>
<th>% of receptivity of proton.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>MUSCLE</td>
<td>BONE</td>
<td>$^1$H</td>
<td>99.9</td>
<td>MUSCLE</td>
</tr>
<tr>
<td>H</td>
<td>63.2</td>
<td>53.1</td>
<td>63.1</td>
<td>53.0</td>
<td>4.26</td>
</tr>
<tr>
<td>C</td>
<td>6.5</td>
<td>19.2</td>
<td>0.07</td>
<td>0.21</td>
<td>1.07</td>
</tr>
<tr>
<td>N</td>
<td>1.6</td>
<td>1.6</td>
<td>0.006</td>
<td>0.006</td>
<td>0.43</td>
</tr>
<tr>
<td>P</td>
<td>0.04</td>
<td>1.9</td>
<td>0.04</td>
<td>1.9</td>
<td>1.72</td>
</tr>
</tbody>
</table>

OTHERS: $^{14}$N: Abundance of isotope $I=1$ is 99.63%, $I=\frac{1}{2}$ is 0.36%. Only 1.6% of tissue nuclei are nitrogen. $\gamma$ is 0.31, and so the receptivity of either isotope is very low.

$^{17}$O: Oxygen has only one isotope with spin, for which $I=\frac{5}{2}$. Although 25% of tissue nuclei are oxygen, the natural abundance of the isotope with spin is only 0.037%. Hence although $|\gamma|$ = 1.89, the receptivity is very low.

* The number of nuclei was calculated from the % composition by weight of each of the atomic species given in the ICRP Reference Man (8/) for muscle and bone.
i) Blood flow through the image plane during a pulse sequence causes a reduction in the spin-echo amplitude because of the loss of excited protons from the plane (S3); it also prevents the steady-state build up of an SFP signal. In the spin-echo case, the rate of flow may be estimated from the observed diameter of the blood vessels and the signal reduction. I have participated in the production of NMR images of H$_2$O flow through tubes (§4) in which velocity profiles through the tubes could be seen.

j) Many pathological lesions have different relaxation times to healthy tissue (Section 1(vi)). The NMR Imaging method therefore has considerable diagnostic potential.

Disadvantages.

a) The most obvious drawback to NMR Imaging is its lack of sensitivity. The distribution of nuclei between the upper and lower states is described by the Boltzmann equation (1.8) and the ration upper:lower is, at room temperature and in a typical NMR Imaging field, about 1:1.0000006. The energy given up by each spin as it relaxes is about $2.5 \times 10^{-27}$ J. Consequently, NMR is fairly insensitive.

The induced signal must therefore be detected in the presence of the minimum amount of electrical noise. Unavoidable noise arises from the thermal motion of the electrons in the receiver coil and the highly sensitive pre-amplifier. However, the receiver coil for medical imaging is very large and is a very effective aerial, picking up all available electrical noise in the vicinity from other apparatus and services.

Libove and Singer (85) calculated the theoretical signal available from a volume which would generate the amplitude of an image pixel (an discrete image cell). They also estimated the electrical noise on an optimised apparatus. Their conclusions were in the form of graphs of spatial resolution vs. 'system integration time', which is the reciprocal of the (optimised) bandwidth of the system per pixel of the image, which is the system acquisition time. Appendix (1) shows that, for an imaging system in which projection profiles are obtained, the NMR 'scanner' in the optimised form would produce images of a signal:noise quality, or contrast, that could be comparable with
those empirically obtained from the latest EMI Whole Body scanner. Realistically, the NMR images are unlikely to achieve a similar 'quality' unless total scan times are raised to obtain signal averaged projection profiles. It may be, however, that the contrast between adjacent tissues will be greater, given suitable combinations of proton density and relaxation time, as discussed earlier. Because NMR acts differently to X-ray CT or a gamma camera, say, it necessarily measures different parameters. Many of the known parameters measured by other techniques have been proved to be diagnostically useful already, and so NMR may ultimately be only a complimentary technique to those already in use. Comparisons between head scans taken by NMR and by X-ray CT have already been referenced: Hawkes et al (65), Doyle (51,52) and Young (53).

It is possible that long scan times may be tolerable in the presence of heart and lung motion. Because the duration of a measurement is several milliseconds every second or so, the pulse sequence may be synchronised with, say, an electrocardiogram so as to capture the heart between contractions.

b) The possible attenuation and phase-shifting of the r.f. radiation was investigated by Bottomley and Andrew (86). Attenuation is greater at higher frequencies, used at higher fields. However, since attenuation necessarily means energy deposition, the limits on raising the strength of $B_0$ to increase sensitivity are dependant on safety levels for power absorption as well as on image degradation. The results are discussed in section 3(v).

c) NMR Imaging is the newest technique to be applied to medical imaging, and has to prove its worth in the clinical environment. In order to be acceptable, it must satisfy several conditions. It must be safe, or reasonably so, and this must be shown in the absence of long-term epidemiological studies. It must be beneficial enough diagnostically to warrant the expense of purchase. It should also be aesthetically acceptable to patients, and not intimidating or impersonal.
3(v). Safety.

There are three recognised hazards associated with NMR Imaging. The effects of these and the 'exposure' limits for each are discussed here.

a) R.f. radiation. The absorption of energy from the rf radiation is due to conductive losses in the subject, associated with the magnetic component of the radiation. In terms of the amount of energy deposited, the absorption is only significant from the irradiating pulse, although both this and the induced field suffer attenuation.

The equation for an electromagnetic wave propagating in a homogeneous, isotropic, stationary medium is:

$$\nabla^2 H - \frac{\varepsilon \mu \partial^2 H}{\partial t^2} - \sigma \mu \frac{\partial E}{\partial t} = 0$$

where:
$$\frac{H}{E} = \frac{\varepsilon}{\mu};$$

The $\varepsilon$ component has a similar equation.

$$\sigma$$ is conductivity; 
$$\varepsilon$$ is permittivity.

Equation (3.26) can be solved for conducting and non-conducting media. In a non-conductor (dielectric) $\sigma$ is zero, and the wave travels with very little attenuation. $E_\varepsilon$ induces the displacement current, whereby the equivalent of a conduction current flows, charge being moved by the transient formation of electric dipoles in the medium. The formation of dipoles increases the electric field flux density, given by $D = \varepsilon E$. The only losses experienced by the wave are in the resistance of the dipoles to turn, and these are very small.

In a conductor, $\sigma$ is significant. Conduction current density $J = \sigma E$ flows, and is attenuated by ohmic resistances. The impedance of a conductor varies as $1/\omega C$, where $C$ is its capacitance and $\omega$ is the wave frequency. The capacitance of the human body is such that at low frequencies it behaves as a dielectric, but at radio frequencies it behaves more as a conductor. As its conductivity rises, the attenuation of the rf pulse also rises. The consequence is that there is a finite 'skin' depth for the radiation, which is the distance traversed by the wave after which both $E$ and $H$ have been attenuated to $(1/e)$ times their initial values upon entering the tissue. The skin depth for various human organs as a function of frequency was investigated.
by Bottomley and Andrew (86), and their results are shown in Figure 3.7 for a slab of tissue. Clearly an image based on this model would be distorted due to non-uniform excitation of deeply-situated tissues.

Figure 3.7. Skin depth at 37°C as a function of resonance frequency. (Planar tissue model) (From Bottomley and Andrew (86)).

Examples of signal amplitude at centre of human body at 4 MHz using this model (ie at a depth of 20 cm):

- MUSCLE : 61% of maximum signal
- LUNG : 80% of maximum signal

A more realistic model was used by the same workers for the results in Figure 3.8, this being a tissue cylinder. On this model, most tissues behave similarly at frequencies which are commonly used (around 4 MHz). The reduction in amplitude is offset by refraction of the wave in the medium which causes a degree of focusing towards the centre of the cylinder and thus
maintains the field. Although in the medium the wavelength and wave velocity are changed, the frequency remains unchanged and the detection of proton position is therefore unaffected. However, the power deposition still occurs near the surface of the tissue more than it does toward the centre.

The ratio of the energies associated with the electric and magnetic components of the wave is given by Lorrain and Corson (87):
\[ \frac{V_2 E \cdot E^2}{\sqrt{\mu H^2}} = \frac{1}{(1 + 1/Q^2)^{1/2}} \]

where \( Q = \frac{\text{displacement current}}{\text{conduction current}} \)

In dielectrics, \( Q \to \infty \) and the energy is equally shared between the components. In conductors, \( Q \ll 1 \), and most of the energy is associated with the \( H \) component. The human body is a conductor in this respect.

A cylindrical model can be used to calculate the power absorption associated with the magnetic component in a conductor. From Faraday's Induction law, the electromotance induced in a circuit around a changing magnetic field is (28):

\[ \oint \frac{\partial E}{\partial t} \cdot dl = - \int \frac{\partial B}{\partial t} \cdot dA \]

where \( B = \mu H \).

The greatest electromotance is induced on the largest loop in the conductor, which for the human body is around its circumference. The model therefore predicts the absorption of the majority of energy in the surface. Eddy currents which are induced to flow around the loop are dependant on the conductivity of the medium; the currents are responsible for Joule heating. The power dissipated is given by \( P = \frac{1}{2} \sigma E^2 \). Figure 3.9 shows the absorbed power at the surface of a cylinder of radius 20cm (representing the human body) for an experiment in which a 90° pulse lasting 10\( \mu \)s is repeated every 10 ms (as it might be in an SFP experiment) giving an average rf power of 0.1% of the pulse power. Techniques using f.i.d.s would produce an average power of about 0.1 times this much. The dotted line shows the equivalent of the power dissipated per unit mass by the body at rest (the basal metabolic rate). This power is enough to raise the body temperature by 1°C.

As the frequency increases, the skin depth reduces, and above 10 MHz the flux through the cylinder starts to decrease; the absorbed power stops linearly increasing with frequency. Eventually, the body becomes opaque to rf and the flux is severely reduced. The
Figure 3.9

Absorbed power at the surface of a 20cm radius cylinder.
(Fixed $B_1$ strength)

From Bottomley & Andrew (86)

A - LUNG
B - BRAIN
C - KIDNEY
D - MUSCLE

--- is the NRPB limit.
\[\leq 1 \text{ W/kg}, \text{ the basal metabolic rate.}\]

Absorbed power in muscle at 10 MHz.
Curves have a maximum, therefore, and this occurs at about 70MHz for man; it is known as the body resonance (67). The basal metabolic heating of 1W/kg equals the heating rate for the human body at the 70MHz resonance when the incident rf exposure is 10 mW/cm². This exposure has been adopted by the U.S. Occupational Health and Safety Administration as the exposure limit for all rf frequencies, below which the effects are considered reversible. The National Radiological Protection Board in Britain has reiterated that the heating should not exceed that due to the basal metabolic rate, and put a limit on the averaged absorbed power for the whole body of 70W; this is 1W/kg for an average person (90).

In conclusion, Figure 3.9 indicates that the power absorption will limit the rf frequency to 5 MHz for a pulse duty cycle of $10^{-3}$, or to 15 MHz for a duty cycle of $10^{-4}$. Independantly, Figure 3.8 indicates that picture distortion is likely to impose a limit of about 10MHz, unless detailed knowledge of tissue attenuation 'coefficients' is used.

b) Time dependant $B_0$ field. Most techniques employ switched or time-dependant gradients $\frac{\partial B_0}{\partial t}$. The frequencies involved are always audible. Since $\sigma \rightarrow 0, E \rightarrow 0$, there is no longer a problem with power dissipation. The effects are those of physiological stress caused by induced currents. Table 3.2 lists some of the effects of low current densities (from Budinger, (87)).

<table>
<thead>
<tr>
<th>Table 3.2. Physiological effects of some current densities (from Budinger (87))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Blood/brain barrier permeability changes</td>
</tr>
<tr>
<td>(Current density of naturally occurring membrane ion currents)</td>
</tr>
<tr>
<td>Minimum current needed to be passed through the eye for visual sensation (phosphenes)</td>
</tr>
<tr>
<td>Phosphen threshold due to currents induced by external field</td>
</tr>
</tbody>
</table>
Budinger took the limit for induced current density as $3 \mu A/cm^2$. He showed that this current is induced by a change of $3T \, s^{-1}$ in a head, or $0.5 T \, s^{-1}$ in the body, the currents being induced around the subject perimeter. In contrast, the NRPB recommend a limit of $20 T \, s^{-1}$ in any part of the body. Both references agree that larger rates of change may be tolerated in pulses of very short duration, since the physiological effects do not appear over very short periods ($< 0.1 \, ms$).

As an example of magnitudes involved, consider the application of a single 'read' gradient after exciting a spin system in the absence of the gradient (eg. in a selective excitation experiment (60) or Fourier Imaging (54), or if a gradient is only applied after a $90^\circ$ pulse in order to reduce the pulse bandwidth requirements). For a magnet of typically $1:10^5$ homogeneity operating at 4 MHz. If 128 separate points are to be distinguishable along the profile, the total gradient would need to produce a spread of about 6kHz; this corresponds to roughly a $1.5G$, or $1.5 \times 10^{-4} \, T$. If this was switched on in the shortest time for a physiological effect, $0.1 \, ms$, then the rate of change of field would be $1.5 \, T \, s^{-1}$. This rate of change would only be possible if an air core magnet with negligible hysteresis was used. Continually oscillating fields of the same order of magnitude would be used at much lower frequencies than this. There are unlikely to be any effects, therefore, for any of the techniques being used at present.

c) Static fields. This problem was also reviewed by Budinger in the same paper.

Theoretically, the permeability (referring here to the ease of passage of small molecules and ions) of membranes depends on the orientations of the molecules of which they are formed. Sufficiently high fields could re-orientate cellular or sub-cellular structures because of the positioning of unpaired electrons in the molecules; this would then change the permeability. Changes would require a field of 10,000G (1T) or more, corresponding to $42MHz$ for protons, and these fields are beyond the limit given by the other considerations.

Not enough consistent results have been reported of physiological
or behavioural changes in higher fields. One possible effect is that of the induction of a nerve impulse in one nerve due to an action potential, equivalent to a moving charge, being transmitted down an adjacent nerve. It is thought that this process could explain electrophysiological changes in the hearts of animals that have been reported. Budinger reported that this was unlikely below 3000G, or 12.5 MHz. The NRPB reported that, above 25,000 G, there could be cardiac spasms due to the flow of ions in blood vessels causing additional motion. However, both of these effects occur outside the range of acceptable frequencies. Barthony et al. observed a drop in the leucocyte level in blood taken from mice that had been kept in a field of 4000G for several weeks. There are numerous tests of this nature that could be carried out on biological sub-systems like this to attempt a more detailed study of the effects of fields.

One other significant hazard is the high voltage that is generated across the tuned coil on a whole body or head system. Whereas the impedance of the tuned circuit is of the order of 50\(\Omega\) regardless of coil size, the larger systems require high power pulses in order to generate the high \(B_1\) fields over large volumes. Several hundred volts would typically be needed. This is a hazard which should be remembered during the construction and maintainance of imaging machines.
CHAPTER 4

POLYTOMIC (MULTIPLANAR) IMAGING
POLYTOMIC (MULTIPLANAR) IMAGING.

4(i). Introduction.

For clinical applications, the inherent low sensitivity of NMR imaging makes it desirable to measure simultaneously as many individual locations as possible (73). The method of projection reconstruction has therefore been adopted by many workers in preference to point or line measurements in which much of the sample is ignored, and to other planar techniques because of its technical simplicity. However, PR requires a means of removing the response from locations outside a plane (slice of finite thickness) of interest. I have developed a transient field gradient technique that allows the simultaneous measurement of a set of planes so that information wastage is minimised.

The technique is a simple yet powerful variant of the phase-encoding concept utilised by Edelstein (69) and Kumar (54) in their imaging methods. However, unlike the former it does not restrict data collection to a single slice, and unlike the latter it does not require the multiple rapid switching of gradients which imposes stringent conditions of the gradient instrumentation. The technique utilises the $90^\circ - \tau - 180^\circ$ echo pulse sequence coupled with controlled phase shifts within the sequence produced by transient magnetic field gradients orthogonal to the planes. The spin-echo data collection sequence gives superior signal discrimination over single $90^\circ$ pulses in many situations (14). Projection reconstruction is used within each plane.

I will firstly derive the theory of the separate frequency and phase Fourier transforms which allow the three-dimensional location of spin density. The consequences of the use of a finite number of transient gradient strengths will then be discussed, with reference to the profile of the plane discrimination function and the minimum number of gradients required. The efficiency of the method will be discussed, followed by consideration of non-uniformity of the receiver coil sensitivity and of the transient gradient drivers. Finally the derivation of the discrete terms in the phase transform will be considered.

The Bloch equations (equations 1.22) describe the motion of the bulk magnetisation of the sample which interacts with the applied magnetic field to generate the detected signal. Consider the case in which the magnetically induced signal is detected with a single phase-sensitive detector whose reference signal is stationary in the rotating reference frame. Static and transient gradients are applied to the sample, and the bulk magnetisation is perturbed by the pulse sequence shown here:

\[
\begin{align*}
\text{R.f. pulses} & \\
\text{Transient } & \frac{\partial B}{\partial x} + \frac{\partial B}{\partial z} \\
& \text{Signal voltage} \\
0 & \tau & 2\tau & 3\tau
\end{align*}
\]

Figure 4.1. The spin-echo pulse timing used in Polytomic imaging.

The precession frequency in the rotating frame is composed of a static and a transient part, \( \Delta \omega' = \Delta \omega + \omega_r \). The solution of the Bloch equations, for a time \( t' > \tau \), includes a phase term which results from the accumulation of phase angle with respect to the frame. This must be treated separately before and after the 180° pulse as this introduces a reversal of phase accumulation. This is shown as follows. Equation 1.22 gives us:

\[
\begin{align*}
\frac{d\omega}{dt} &= -\frac{\omega}{\tau_2} - (\Delta \omega + \omega_r) v \\
\frac{dv}{dt} &= -\frac{\nu}{\tau_2} + (\Delta \omega + \omega_r) u
\end{align*}
\]
Substituting $M = u + iv$, the complex magnetisation,

$$
\frac{dm}{dt} = -\frac{u}{T_2} - (\Delta \omega + \omega_r) v - iv/T_2 + i(\Delta \omega + \omega_r) u
$$

Integrating for the period $0 \leq t \leq \tau$,

$$
m(\tau) = m(0) \exp [\left(-\frac{1}{T_2} + i\Delta \omega \right) \tau] \exp \left[ \int_0^\tau i\omega_r \, dt \right]
$$

After a $90^\circ$ pulse, $M(0) = M_0$, the total magnetisation in the region of the sample volume with the single resonance frequency,

$$
m(\tau) = m_0 \exp \left[\left(-\frac{1}{T_2} + i\Delta \omega \right) \tau\right] \exp \left[ \int_0^\tau i\omega_r \, dt \right]
$$

At the time $t' = \tau$, a $180^\circ$ pulse is applied. After this, the same procedure yields:

$$
m(t) = -m^*(\tau) \exp \left[\left(-\frac{1}{T_2} + i\Delta \omega \right) t' \right] \exp \left[ \int_{\tau}^t i\omega_r \, dt \right]
$$

The precession of the magnetisation is reversed by the pulse and $M(\tau)$ has been converted into its complex conjugate. This value is taken as the magnetisation at time $t = 0$ in this equation. Substituting for $M^*(\tau)$, the magnetisation as a function of time is given by:

$$
m(t) = -m_0 \exp \left[ i\Delta \omega (t - \tau) - \frac{1}{T_2} (t - \tau) \right] \exp \left[ \int_\tau^t i\omega_r \, dt - \int_0^\tau i\omega_r \, dt \right]
$$

The same expression was derived by Allerhand (92) for the case when $t = \tau$, i.e. at the expected position of a rephased spin-echo in the absence of $\omega_r$.

In the proposed multiplanar imaging method, the following points are important:

a) The axis orthogonal to the planes, along which the transient gradient is applied, depends on the magnet used. In a whole body magnet, this axis is the same as that of $B_0$, that is the $\hat{z}$ axis. This analysis is performed with a conventional magnet in mind, in which planes are orthogonal to the receiver coil axis, which is itself orthogonal to $B_0$. Figure 4.2 illustrates the orientation.

![Figure 4.2. The coordinate system used in the analysis. A single plane is showed shaded in.](image)
b) A transient gradient is applied during the $90^\circ - \tau - 180^\circ$ interval only. The applied field $B_\tau = \omega_\tau / g$ is held constant, with a value $G^* y$, so that $\omega_\tau = \gamma G^* y$. The bulk magnetisation is a function of $G^*$ and is integrated over all $y$.


c) In an image plane defined by circular coordinates $(r, \theta)$, the magnetisation is also integrated throughout the plane. In the PR method, a gradient is applied at an angle of $\theta$ and with a strength $G^* \theta$ per unit distance along $r$. (This strength is the same for all projection angles.) The static part of the precession frequency is:

$$\Delta \omega = \gamma B^* \theta$$

where $B^* \theta = G^* \theta \cdot r \cdot \theta$

and $r = r' + r_o$, where $r'$ is the radial co-ordinate, $r_o$ is the coil radius.

The addition of the constant $r_o$ to $r'$ represents the adjustment of the strength of the main magnetic field $B_o$ so that all spins have positive resonant frequencies (i.e., they are detected 'off resonance'). This is always necessary if the signal is detected with a single reference phase. As will be shown, the phase decoding of the spins along the coil axis does not require the use of two reference phases in quadrature.

To simplify the analysis, transverse relaxation will be ignored. This will later be shown to be valid. The pulse sequence will be retimed so that the echo occurs at $t = 0$ so that it is not time shifted. Also the reference phase is chosen to detect the positive cosine component of $M(t)$.

Thus, the magnetisation at angle $\theta$ and with a transient gradient $G^*_\theta$ is:

$$m^\theta(t, G^*_\theta) = \int dy \int dr \, M^\theta_\circ(\theta, y) \cos \left[ \gamma \left( G^* \theta r + G^*_y y \right) \right]$$

(4.5)

It is required to find the magnetisation at radial position $r'$ and axial position $y'$ by finding the Fourier transformed magnetisation $s^\theta(r', y')$. ($s^\theta$ is used to designate one or two dimensionally transformed magnetisation.)

The first complex transform is:

$$m^\theta(t, G^*_\theta) \leftrightarrow s^\theta(t', y')$$

(4.6)
The analysis is clearer if a continuum of transient gradient strengths is assumed; the case for discrete values will be considered later. For the real part we have:

\[
\text{Re} [ s^0 (r, y') ] = \int dG^* \, m^0 (r, G^* y') \cos (G^* y', \tau)
\]

\[
= \int dy \int dr \int dG^* \, m^0 (r, y) \cos [G^* y, r + G^* y', \tau] \cos (G^* y', \tau)
\]

\[
= \int dy \int dr \int dG^* \, m^0 (r, y) \left[ \cos (G^* y, r) \cos (G^* y', \tau) - \sin (G^* y, r) \sin (G^* y', \tau) \right] \cos (G^* y', \tau)
\]

\[
= \int dy \int dr \int dG^* \, m^0 (r, y) \left\{ \frac{1}{2} \cos (G^* y, y+y') \cos (G^* y', \tau) \right. \\
\left. + \frac{1}{2} \cos (G^* y, y-y') \cos (G^* y', \tau) \right\}
\]

Immediately, all the sine terms disappear as the functions are odd.

Isolation of the response from axial position \( y' \) begins as follows. Using:

\[
\int dG^* \cos (G^* y, y+y') \tau = \delta \left[ \frac{1}{2\pi} (y+y') \tau \right] \quad \text{(Papoulis (93))}
\]

\[
\int dG^* \cos (G^* y, y+y') \tau = \delta \left[ \frac{1}{2\pi} (y+y') \tau \right] \quad \text{(4.9)}
\]

to provide Dirac delta functions, we are left, after evaluating \( \int dG^* \), with:

\[
\text{Re} [ s^0 (r, y') ] = \int dr \cos (G^* y, r) \left[ \frac{m^0_0 + y'}{2} \delta (r) + \frac{m^0_0 - y'}{2} \delta (-r) \right]
\]

Here, for example, \( m^0_0 \delta (r) \) is the magnetisation at angle \( \theta \), axial position \( +y' \), which is a function of \( r \).

The imaginary part of this transform is performed similarly.

\[
\text{Im} [ s^0 (r, y') ] = \int dG^* \, m^0 (r, \frac{m^0_0 + y'}{2} \delta (r) - \frac{m^0_0 - y'}{2} \delta (-r))
\]

\[
= \int dr \sin (G^* y, r) \left[ \frac{m^0_0 + y'}{2} \delta (r) - \frac{m^0_0 - y'}{2} \delta (-r) \right]
\]

\[
(4.11)
\]

So far, then, the signals from axial positions \( \pm y' \) have been isolated but they are mixed together. They separate when the second transform:

\[
s^0 (r', y') \leftrightarrow s^0 (r, y')
\]

is performed. Because \( \text{Re} [ s^0 (r, y') ] \) is even
and $s^\theta(r',y')$ is odd, the function $s^\theta(r',y')$ will be purely real.

$$s^\theta(r',y') = \int dt \ Re \left[ s^\theta(r,y') \right] \cos(\gamma G^\theta r')$$

$$- \int dt \ Im \left[ s^\theta(r,y') \right] \sin(\gamma G^\theta r')$$

(4.13)

The first part of this is:

$$\int dt \ Re \left[ s^\theta(r,y') \right] \cos(\gamma G^\theta r')$$

$$= \int dt \int dr \cos(\gamma G^\theta r') \left[ \frac{m_0^{\theta,y} \gamma(r) + m_0^{\theta,-y}(r)}{2} \right] \cos(\gamma G^\theta r')$$

$$= \int dt \int dr \left[ \frac{m_0^{\theta,y} \gamma(r) + m_0^{\theta,-y}(r)}{2} \right] \left[ \frac{1}{2} \cos(\gamma G^\theta (r+r') \gamma) + \frac{1}{2} \cos(\gamma G^\theta (r-r') \gamma) \right]$$

(4.14)

and again there were odd terms from the expansion which disappeared.

Again using equation 4.9 to evaluate $\int dt$, we obtain:

1st part: $\int dt \left[ \frac{m_0^{\theta,y} \gamma(r) + m_0^{\theta,-y}(r)}{2} \right] \left[ \frac{1}{2} \left( \delta(r+r') + \delta(r-r') \right) \right]$}

$$= \frac{1}{4} \left[ m_0^{\theta,y}', + r' + m_0^{\theta,-y}', + r' \right]$$

(there are no spins at $r'$) (4.15)

2nd part: $\int dt \left[ \frac{m_0^{\theta,y} \gamma(r) - m_0^{\theta,-y}(r)}{2} \right] \left[ \frac{1}{2} \left( \delta(r+r') + \delta(r-r') \right) \right]$}

$$= \frac{1}{4} \left[ + m_0^{\theta,y}', + r' - m_0^{\theta,-y}', + r' \right]$$

(no spins at $r'$) (4.16)

The two parts add together as both are real, and the choice of position $+y'$ in the first transform has yielded the signal from the plane at position $+y'$. Experimentally, the magnetisation across the whole plane is simultaneously found by performing a fast Fourier transform (FFT) on the time signal, and similarly all planes of interest would be found using the first transform.

The sign in equation 4.13 is of interest here. The choice of sign in the sine transform is dictated only by convention, and affects only which of the two planes $-y'$ or $+y'$ is detected for a given transform method. I have adopted the convention that the transformation from time to frequency space ,

$$F(\omega) = \int dt \, \Phi(t) e^{-i\omega t}$$

uses a negative exponent but I have chosen a positive exponent in the first transform so that the resulting magnetism is at the position $+y'$. The result is, then:

$$s^\theta(r',y') = \frac{1}{2} m_0^{\theta,y}', + r'$$

(4.17)
The factor of $\frac{1}{2}$ is due to the absence of the magnetism that is out of phase with the reference signal. The doubling of NMR signal energy that occurs when detecting with two reference phases in quadrature represents the recovery of this factor of 2. The coordinate system would be repositioned so as to introduce spins to the position $-r'$ as they would be distinguishable from spins at $+r'$.

4(iii) The Effect of Transverse Relaxation.

This discussion pre-supposes that the final image will be independent of transverse relaxation after due consideration to signal bandwidth.

If $T_2$ is included, equation 4.5 becomes:

$$m^0(t,G^y) = \int dt \int dr \, m^0_0(r,y) \cos[\gamma G \cdot r + (-2\pi t/T_2) + iG \cdot y]$$

The first transform, which integrates over G and y leaves the relaxation term unchanged, and

$$s^0(r',y') = \int dt \int dr \, \cos[\gamma G \cdot r + (-2\pi t/T_2)] \cos[\gamma G \cdot r' + (-2\pi t/T_2)]$$

$$\times \left[ m^0_0 + m^0_0 \cos^2(r) \right] \left[ m^0_0 - m^0_0 \cos^2(r) \right]$$

(4.18)

In rearranging the expression as before so as to evaluate $\int dt$, notice that:

$$\int dt \int dr \, \frac{1}{2} \cos[\gamma G \cdot (r+r') + (-2\pi t/T_2)]$$

$$+ \frac{1}{2} \cos[\gamma G \cdot (r-r') + (-2\pi t/T_2)]$$

$$= \int dt \int dr \, \frac{1}{2} \cos(t \left[ \gamma G \cdot (r+r') - 1/T_2 \right] - 2\pi t/T_2)$$

$$+ \frac{1}{2} \cos(t \left[ \gamma G \cdot (r-r') - 1/T_2 \right] - 2\pi t/T_2)$$

(4.19)

The terms in the square parenthesis are both frequency terms and the transverse relaxation can be seen to represent the expected resonance shift or broadening. The term $-2\pi t/T_2$ is a constant. Using $\cos(A-B) = \cos A \cos B + \sin A \sin B$, where B is in this case the constant, the integration $\int dt$ is of the form:

$$(\text{constant}) \times \int dt \cos A + (\text{constant}) \times \int dt \sin A.$$

The second part is odd and therefore zero. Thus, the expression for $S^0(r',y')$ is changed only by the reduction in echo amplitude...
due to $T_2$ relaxation which is irreversible, and by the associated frequency broadening.


In practice, both $\int dG'$ and $\int db$ are finite summations. The latter is the result of the discrete sampling of the time signal. The former, the transient gradient summation, represents the application of a set of equally spaced discrete strengths whose values are governed by the number and thickness of the image slices. Before considering what restraints these values have, the effect of integrating between finite limits will be examined.

Equation 4.9 becomes:

$$\sum_{G_0} dG' \cos \left( \frac{\gamma G' (y + y') \tau }{G_0} \right) = 2 G_0 \sin \left( \frac{\gamma (y + y') \tau }{G_0} \right) \left( \frac{G'}{G_0} \right)$$

(4.20)

The $\sin(\pi y)/\pi y$ appearance has been emphasised. This is the weighting factor which $M_0(y)$ has in the integration $\int dy$. It is, therefore, the axial profile of the magnetisation; the detected signal originates from locations that lie in the volume swept out around the coil axis by the $\sin(\pi y)/\pi y$ function, which is centred on $y = y'$.

The function first becomes zero where:

$$y + y' = \left( \frac{\gamma G_0 \tau }{\pi} \right)$$

and the thickness of a slice of spins may be defined as the distance between nodes,

$$\rho = \left( \frac{\gamma G_0 \tau }{2\pi} \right)$$

The effect of choosing discrete values of $G'$ within a range is as follows. It is well known that sampling in one domain with frequency $f$ causes periodicity of the Fourier transform, each reproduction or 'alias' being spaced $1/f$ from its neighbour (37). The aliases of the function $\cos(\gamma G' y \tau )/\pi G'$ or $\cos(2\pi \left[ \frac{\gamma G' y \tau }{2\pi} \right])$, occur at intervals $\left( \frac{\gamma G' \Delta \tau }{2\pi} \right)^{-1}$ when the spacing between values of $G'$ is $\Delta G'$.

If the coil is sensitive over an axial distance $y_0$ then:
\[
\left( \frac{\delta \Delta G}{2\pi} \right)^{-1} \gg y_o
\]

for no detectable aliases, i.e.:
\[
\Delta G \leq 2\pi / (y_o \gamma c)
\]

If \( m \) planes are to be imaged (the central peaks will not overlap) then the plane thickness
\[
p = y_o / m = \left( \frac{\delta \Delta G}{2\pi} \right)^{-1}
\]
i.e. \( G_o = m \times 2\pi / (y_o \gamma c) \)

(4.22)

Therefore, since the range of values for \( G^y \) is \(-G_o \leq G^y \leq +G_o\)
there must be at least \((2m + 1)\) values of \( G^y \), including zero.
The number of values can be reduced to \((m + 1)\) if \( G^y \) is only
given positive values. \( G^y = G_o \) should be omitted to prevent
duplication of the term for \( G^y = 0 \), and so only \( m \) values are
in fact required.

However, for positive \( G^y \) only, the odd terms in equation 4.8
do not disappear, and the analysis requires further examination.

Equation 4.20 is still applicable, with a modification to the
limits of integration (assuming a continuum of values):
\[
\int_{G_o}^{G_o} \ dG^y \cos \left( \sqrt{G^y (y+y')} \gamma c \right)
= \frac{G_o \sin \left( \sqrt{G_o (y+y')} \gamma c \right)}{\sqrt{G_o (y+y')} \gamma c}
\]

(4.23)

So far, this result is the same except that the magnitude is
reduced by a factor of 2. Odd terms do not disappear, and must be
evaluated. For example:
\[
\int_{G_o}^{G_o} \ dG^y \sin \left( \sqrt{G^y (y+y')} \gamma c \right)
= \left( \frac{\cos \left( \sqrt{G_o (y+y')} \gamma c \right) - 1}{\sqrt{G_o (y+y')} \gamma c} \right)
\]

(4.24)

Designating equation 4.23 as \( m_o^{s(y')} \) and equation 4.24 as \( m_o^{c(y')} \)
then equation 4.10 becomes:
\[
R \gamma \left[ S^t (r, y') \right] = \frac{1}{2} \int dr \cos \left( \sqrt{G^t r} \gamma c \right) \left[ m_o^{\delta, s(y')} (r) + m_o^{\delta, s(-y')} (r) \right]
- \frac{1}{2} \int dr \sin \left( \sqrt{G^t r} \gamma c \right) \left[ m_o^{\delta, c(y')} (r) + m_o^{\delta, c(-y')} (r) \right]
\]

(4.25)

The result of the second cosine transform on the second line of
this gives the term:
\[
\int dr \int dt \sin (2 G_r r \cdot r') \cos (2 G_r^0 r' \cdot r')
\]
which is zero if the summation \(\int dt\) is continuous, since the product is an odd function. The end result is unchanged, therefore, if the change from a continuous integral to a discrete sum does not affect this step. In practice this is true because the signal is sampled a large number of times, and the resonance frequency broadening ensures that the echo is small towards the ends of the sampling period.

The sine transform can be similarly dealt with, so there are no problems caused by ignoring negative values for \(G^y\).

It is interesting that equation 4.23, and the above demonstration that the terms from equation 4.24 can be ignored, show that the plane thickness remains unchanged even though the range of \(G^y\) values is halved. Usually, the width of the \(\sin(\pi y)/\pi y\) function is inversely dependant on the range of values in the other Fourier space. This unexpected result is due to the effect of the second transform.

\(4(v)\) Efficiency of the technique.

The efficiency of this technique of plane definition is defined as:

\[
\frac{(\text{signal strength from plane of thickness } y_0 / m)}{(\text{maximum signal strength from cylindrical volume, thickness } y_0 / m)} \times 100\%
\]

- both as a result of \(m\) measurements.

(The measured signal voltage is proportional to the magnetism within the two volumes.)

The two values will be calculated separately.

(a) When \(G^y = 0\), the whole magnetisation has the same phase at position \((r', \theta')\) and equation 4.5 becomes:

\[
M^o_0 (r') = \int dr' \int dy' \cos (2 G_r^0 r' \cdot r') M^o_0 (r, y')
\]

If \(M^o_0 (r')\) is the magnetisation per unit axial distance in an axially uniform sample, then the value of the magnetisation in a plane,
thickness $y_0/m$, which has been signal averaged $m$ times, is:

$$\int dr \cos(\gamma G^0 r \cdot t) m_0^0(r) \cdot y_0 \quad (4.27)$$

(b) When transient gradients are used within a finite range, equation 4.10 becomes:

$$\text{Re} \left[ s^0(t, y') \right] = \int dr \cos(\gamma G^0 r \cdot t) \times \frac{1}{2} \int dy m_0^0(r, y) G_0 \times \left[ \frac{\sin(\gamma G_0(y+y') \tau)}{(\gamma G_0(y+y') \tau)} + \frac{\sin(\gamma G_0(y-y') \tau)}{(\gamma G_0(y-y') \tau)} \right] \quad (4.28)$$

If we choose $y' = 0$, the imaginary transform vanishes as $\sin(\gamma G^0 y \cdot \tau)$ is zero.

If the first Fourier transform in fact involves discrete summation, with sampled values of $G^0$, then the magnitude of $s^0(t, y')$ is reduced, being scaled by a factor of $1/\Delta G$. We therefore have a factor of $G_0/\Delta G$, which is optimally equal to $m$. Therefore, again assuming axially uniform magnetisation,

$$s^0(t, 0) = \int dr \cos(\gamma G^0 r \cdot t) m_0^0(r) m \int dy \frac{\sin(\gamma G_0 y \cdot \tau)}{\gamma G_0 y \cdot \tau} \quad (4.29)$$

The integral $\int dy$ is not easily solved between arbitrary limits. However, a simple solution exists for the definite integral:

$$\int_0^\infty \frac{\sin(\alpha y)}{y} dy = \pi/2 \cdot \text{sign} \alpha .$$

Strictly, the lower limit $y = 0$ is included and the result

$$\int_0^\infty \frac{\sin(\alpha y)}{\alpha y} dy = \pi/\alpha \quad (\alpha \text{ positive})$$

is not true; however, the function changes smoothly at $y = 0$. Since the plane thickness

$$P = y_0/m = \left( \frac{\gamma G_0 \tau}{2\pi} \right)^{-1}$$

we may substitute

$$\gamma G_0 \tau = m 2\pi / y_0$$

and

$$s^0(t, 0) = \int dr \cos(\gamma G^0 r \cdot t) m_0^0(r) \cdot \frac{y_0}{2\pi m} \cdot \frac{y_0}{2} \quad (4.30)$$

Comparing equations 4.27 and 4.30, it is clear that the non-ideal plane profile induces a loss of signal, causing the efficiency, defined earlier, to be 50%. The choice of limits $\int_{-y_0/2}^{y_0/2}$ is not necessary even when considering non-uniform $m_0^0(r, y)$ as the
magnetisation is confined within the sample. It has been shown, by computation of the integral, that the \( \frac{\sin(\pi y)}{\pi y} \) function is well contained within the coil even for low values of \( m \), the small positive and negative peaks outside being 'pairable' so that the efficiency is within 2\% of the 50\% figure derived by integration between infinite limits.

In conclusion, after \( m \) signals have been collected, all of the \( m \) planes may be reconstructed, and each yields a response that is about \( m/2 \) times as great as the 'single shot' response.

4(vi) Single Plane Definition.

To obtain a profile from a desired plane by this method obviously requires a lot of computational time and storage space. There is one plane, however, that can be obtained without some of these stages, this being the one around \( y' = 0 \). The term \( \cos(\gamma G \sin \phi) \) in the first transform is unity for every value of \( \gamma \), so the transform is just equivalent to adding the \( m \) signals together without modification. This may be done as part of the signal averaging of the signal at the projection angle \( \theta \). (D.G. Taylor and D.J. May, \( \gamma \).

Provided that the condition \( \Delta G \ll \left( \frac{2\pi}{y_0 \gamma \tau} \right) \) is satisfied, there is no reason why the \( G^\gamma \) values should be accurately calculated by computer; in the early experiments with this technique, \( G^\gamma \) was made to oscillate incoherently with the pulse sequence repetition rate, being driven by a simple signal generator. The incoherence ensured that any phase angle \( \gamma G^\gamma \gamma \) was as likely to occur as any other, and provided a sufficient number of signals were added together, the criterion \( \Delta G \ll \left( \frac{2\pi}{y_0 \gamma \tau} \right) \) was met. Because of the reliance on the incoherency, however, the number of accumulated time signals was of the order of \( 3m \) for a plane thickness \( y_0 / m \).

4(vii) Non-uniform Coil Sensitivity.

Although receiver coils are constructed so as to have uniform sensitivity across the image plane, it is diminished towards and beyond the coil ends. Normally one expects the axial integration
of magnetisation to be dominated by the central peak of the
\[ \frac{\sin(n\pi y)}{n\pi y} \] function. The modification of the plane signal by the
inclusion of adjacent lower peaks and troughs is small, and if
necessary the signal or final image may be improved by the addition
of small fractions of the adjacent signals or images. However,
end planes produce smaller signals which may contain too much
detail from adjacent (more sensitive) locations to be meaningful.
(Figure 4.3). Deconvolution of the coil response from the plane
shape is not possible, since neither the continuous values of the
axial magnetisation, nor the integrated complex magnetisation in
the adjacent planes, are known; the latter because the period of the
\[ \frac{\sin(n\pi y)}{n\pi y} \] function halves outside the plane of interest.

![Plane at coil end vs Plane in coil centre](image)

Figure 4.3. The plane profile modified by the coil response (dotted
line). In (a), the plane around position A has been selected. However,
contributions from positions B and C are relatively large.

A solution to this would be to confine the set of images within
the most sensitive regions by increasing the strength of the gradient
limit \( G_o \). However, the increase of the interval \( \Delta G \) would cause
aliasing of information into the planes from weaker parts of the
coil. In other words, the information for the reconstruction of the
end planes must be collected, even if these planes are never used,
unless other measures are taken. These include:

a) Some form of selective excitation, omitting regions near the
ends of the coil. (The spin-echo sequence itself effectively does
this, as will be shown later.)
b) Increasing the number of transient gradients. This can be
used to compact the planes of interest without causing aliasing,
although the end plane information is still collected. Alternatively,
the set of signals may be weighted, for example, by a triangular
function so that the plane thickness is unchanged but the profile
becomes a \( \sin^2(\pi y/\alpha y)^2 \) function, which falls away quickly
along the axis.

The plane profile shape may be observed at the position \( y' = 0 \)
by applying a single, static gradient \( \partial \gamma_y \) on top of which the
set of transient gradients \( \gamma_y \) may be applied. This (purely real)
time transform of the signal, from a homogeneous sample, yields a
projection profile where frequencies (axial positions) are modified
in phase so that the plane profile is seen. This allows the gradient
strength \( G_0 \) to be properly selected so that \( p \times m = y_0 \). This is
adjusted by changing the power output from the gradient driver.

The echo received after a \( G_y \) pulse is unchanged except that it
exhibits a time shift. This is because the phase shift \( (\omega G_y \tau) \) in
equation 4.5 linearly increases as a function of frequency (axial
position) with the result that, following the 180° pulse, all the
spins rephase together, later than expected when \( G_y = 0 \). Because
the time shift is proportional to \( G_y \), it is possible, by measuring
the time shifts occurring due to the set of \( G_y \), to verify that the
transient gradient values are equally spaced. Non-linear computer
DACs and/or gradient drivers, or magnet hysteresis, could impede
the switching. If the values of \( G_y \) do not increase linearly, the
values applied to the DACs can be modified.

One advantage of the spin echo technique is that the coil sensi-
tivity is contained reasonably well. If the pulse length is correct
so as to give a flip angle of 90° at the coil centre, then as
the strength of \( B_1 \) decreases towards the coil ends so the pulse
angle decreases. Figure 4.4 shows that the perturbed magnetisation
after a flip angle of \( \alpha < 90° \) can be dealt with as a component
\( M_0 \cos \alpha \) which has not moved, and a component \( M_0 \sin \alpha \) which
precesses in the \( x'-y' \) plane.

After the second pulse \( 2\alpha \), the magnetisation in the \( x'-y' \)
plane flips and the amount remaining in the plane is \( M_0 \sin \alpha \cos 2\alpha \). This
becomes zero when \( \alpha = 45° \) and for \( \alpha < 45° \) the magnetisation is
not flipped far enough for its precession to reverse; consequently
no signal is obtained from regions where the \( B_1 \) is to weak to
cause a 45° flip.
Figure 4.4. The precession of the magnetisation in the rotating frame when the pulse length $\alpha$ is less than $90^\circ$. (A small difference between the frequencies of the reference frame and the resonance is assumed.)

a) After a pulse $\alpha$, the magnetisation can be broken up into vector components. $M_0 \sin \alpha$ lies in the $x'$-$y'$ plane and is therefore detectable.

b) Following a second pulse $2\alpha$, which is supposed to induce a spin echo, if $\alpha < 45^\circ$ then the direction of precession (shown arrowed) is unchanged and the spins in the $x'y'$ plane continue to dephase.

c) Following a second pulse where $\alpha > 45^\circ$ then the precession direction (shown arrowed) is reversed and spins will rephase and cause a spin echo.

The fraction $M_0 \cos \alpha$ also flips after the second pulse. The amount $M_0 \cos \alpha \sin 2\alpha$ turns into the $x'$-$y'$ plane. This produces a signal directly after the second pulse. The problem of sampling to ignore this signal was discussed in section 2(viii).
4(viii). Discrete Coefficients for the Transient Gradient Transform.

Experimentally, the first transform is performed by multiplying the signals by discrete values of $\exp(i\Delta G^+ y, \tau)$, which can be calculated and stored as a table. Just as equation 4.22 gives:

$$\tau G_0 \tau = m \cdot 2\pi/y_0$$

so equation 4.21 gives:

$$\tau G^+ \tau = n \cdot 2\pi/y_0 \text{ whe } n \Delta G = G^+,$$

and we find values of

$$\exp(i2\pi n/y_0, y').$$

If the plane thickness $p = \text{unit distance}$, then $y'$ has $m$ integer values (including 0) and $y_0$ is numerically equal to $m$. The expression is tabulated in Table 4.1 for $m = 8$. 
Table 4.1. Coefficients for the real and imaginary GY transforms (8 planes).

\[ \cos\left(2\pi \frac{n \cdot y'}{y_0}\right) \]

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\[ \sin\left(2\pi \frac{n \cdot y'}{y_0}\right) \]

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CHAPTER 5

EXPERIMENTAL RESULTS
EXPERIMENTAL RESULTS

The apparatus which was used is detailed in Chapter 6. Chapter 5 describes the operation of the equipment and gives a critical evaluation of the results which were obtained.

5(i). Experimental settings.

All the research was carried out at a resonance frequency of 15 MHz using a spin-echo technique to collect data. The transverse decay of the signal was dominated by the inherent inhomogeneity of the magnet over the sample volume, which gave a value for $T_2^*$ of 10 ms. The linewidth of 30 Hz gives the magnet homogeneity of 2 parts in $10^6$.

The very first step is to obtain the exact magnetic field strength which is required for resonance. It was arranged for the resonance frequencies to be above the reference frequency used by the phase sensitive detector. A short term flux stabiliser was then activated.

The single, dual purpose probe coil was tuned to a parallel resonance at 50Ω using two methods. Whilst the probe network was being built, a Hewlett Packard vector impedance meter was used which gave information on both phase angle and impedance. The probe was tunable for both resonance and resonance impedance, both of which had to be correct. When the probe was connected to its 'T' network of coaxial cable and the (small) impedance loading of the vector impedance meter was removed, the resonance conditions changed slightly. In this instance, the probe was tuned by the observation of the $B_1$ pulse magnitude across the probe, a maximum indicating a parallel resonance. A pulse height equal to half that available on the unloaded transmitter output indicated an impedance match.
The correct pulse length for the 90° pulse was estimated by observing the maximum initial height of the f.i.d. whilst the pulse length was increased from zero. During this stage the repetition rate of the pulses was made greater than 5 $T_1$ for the sample, as shorter pulses can produce a maximum value at more rapid repetition rates. The contribution from the sample in the region of the coil ends where the pulse length is reduced introduces no significant error at this stage. The pulse length was 15 s.

The second pulse in the spin-echo sequence was set to occur after a time equal to the transverse relaxation time where this was known in advance. The pulse length was then optimised for a 180° pulse by observing the echo height as the pulse length was increased from zero. The contribution from the coil ends produces a significant signal after this pulse even when it produces a 180° flip in the coil centre. Attempts to obtain a 180° pulse by minimising this signal will therefore cause an error.

For the optimisation of the 180° pulse length and for all subsequent signal observations, the pulse repetition rate was increased to be about equal to the sample $T_1$. When a water sample was used, this would be doped with CuSO$_4$ whose paramagnetic ions reduced the $T_1$ dramatically. $T_1$ was often roughly estimated as being the spacing of 90° pulses when the f.i.d.s produced by a repetition of these pulses fell to $m_e(1-1/e)$.

To reduce the amount of noise in the audio-frequency signal produced by the phase-sensitive detector, the output R-C filter bandwidth was reduced until the spin-echo height was observed to perceptibly reduce, thus indicating that the highest frequencies present were being reduced in amplitude or phase-shifted by the filter. The bandwidth was then increased until no reduction occurred.

In order to exploit the full dynamic range of the Analogue to Digital Converter (ADC) which was ±5 v, the audio signal was amplified to fill this range, regardless of the signal to noise ratio, i.e. the signal strength. Care was needed to make allowances for any d.c. level on the signal so as not to overflow the ADC input.
The spin echo was observed during the correction of phase of the reference signal to the phase sensitive detector to obtain a signal consisting of purely real cosine frequencies. To this end the echo was balanced so as to appear even and symmetrical about its centre peak, with due regard to the asymmetry which the $T_2$ decay could cause.

The spin-echo was digitised using all the guidelines in Chapter 2. The sampling was made as fast as possible, and this, together with the sampling time available within the transverse relaxation, set the total number of points that could be acquired as 512. The maximum sampling rate was determined by the speed at which the computer could interrogate the ADC, and therefore this was determined by the execution time for the algorithms. The maximum sampling rate was found by measuring the width of the frequency spectrum of the digitised signal as a function of demanded signal digitisation rate; when the digitisation rate could no longer be increased, the spectral width reached its maximum. Signals were usually digitised at the rate of 1 point every $30\,\mu s$, and thus sampling took approximately $15.4\,\text{ms}$. The spacing of points in the frequency domain was thus $65\,\text{Hz}$, or about twice the magnet linewidth. 65 points were used for each discrete profile. The profile thus had a width of $65 \times 65\,\text{Hz} = 4.225\,\text{kHz}$. The frequency spectrum of the $B_1^\circ$ pulse, lasting $29\,\mu \text{s}$, was considered to be flat over this range. Only 65 of the 256 positive frequency points in the discrete spectrum were used for the profile, the other points containing only noise frequencies in the bandwidth of the audio signal for filtering purposes (section 2(v)). (The negative points in the discrete spectrum are identical to the positive ones.)

The ADC, described in the next chapter, began sampling directly after the $180^\circ$ pulse. The signal from the coil ends was therefore sampled, but was subsequently ignored, the data being extracted from the 512 sampled points immediately after this; about 100 points were usually ignored.

It is important that the spin echo should lie in the exact centre of the sampled signal. This was approximately achieved by timing
the interval between the 90° and 180° pulses very accurately and comparing this with the ADC sampling rate. The echo was then positioned accurately by fine adjustment of the inter-pulse interval whilst observing the centre points of the digitised signal.

The orthogonal transverse gradients (i.e. those in the image plane) were then optimised. The action of the two digital to analogue converters (DACs) that were used to control the magnitudes of the gradient coil currents was checked by the observation of a spot on an oscilloscope screen which represented the tip of the gradient vector. However, the gain of each gradient coil driver was set by measuring the spectral width of the projection profiles of a cylindrical sample. For this, the two coils were energised independently. The shape of the profiles was also used as a check of gradient linearity, attenuation of the signal as a function of frequency (in the filter, for instance) and possible phase errors. At 65 Hz per point in the digitised frequency spectrum, the gradient strength required to form a profile across 65 points, using the receiver coil described in the next chapter, was 1.2 G cm⁻¹.

The shape of the echo should remain unchanged when the DACs are instructed to provide no current, and periodically this was checked to make sure that there was no gradient driver zero offset.

If the experiment to be run was to use plane definition, the strength of the third gradient would be set at this stage by observation of the plane profile shape, as described in section 4(vii).

All the images that were taken employed N= nπ/4 projection angles, where n is the number of discrete points across a profile, usually 65, as recommended by Brooks and Di Chiro (95). The inside of the glass tube around which the receiver coil was wound had a diameter of 8.5 mm, so in all cases the diameter of the objects shown in the next section is slightly less than this. Other relevant experimental details are discussed with the results in the next section. The experiment was under overall computer control, and the sequence of steps through the various stages of the experiment is outlined in the description of the computer programs, shown in
5(ii). Images

Plate 1(a) shows an f.i.d. as seen on the oscilloscope used to monitor the received signal voltage after a 90° pulse. A static gradient has been applied. Without the gradient, the signal would decay, exponentially, across the full width of the photograph, so the dramatic shortening of the decay can be seen when a typical imaging gradient is applied. The initial maximum voltage is negative, which would cause the maximum of the rephased spin echo (when applied) to be positive. The profile of an object contains a complicated mixture of frequencies, and this is reflected in the complexity of the shape of this decay.

Plate 1(b) shows a spin echo, after digitisation by the ADC. This signal is ready for a discrete Fourier transform. The phase is incorrectly set; this is clear by the absence of a single, positive central peak. The reference phase would have to be changed in this case.

Both plates have a very low level of baseline noise.

The photographs in Plate 2 show spin echo signals that were obtained before a Fourier transform routine had been implemented. These were part of a set which were used to check that the gradient vector was rotating under computer control in the image plane. Two capillary tubes of water were placed side by side. When the gradient vector was parallel to the line joining the tube centres, the resonance frequency distributions of the two tubes was identical. (a) As the gradient rotated, the resonance frequency distributions began to beat. Calculation of the beat frequencies showed the the gradients were rotating correctly (b).

Plate 3(a) shows the first NMR image taken with the apparatus. The dark areas represent water between two concentric glass tubes. The image was drawn on a dot-density plotter. The odd shape was a consequence of non-optimisation of most of the equipment at that time, and not least to the fact that the field \( B_0 \) drifted during the experiment. Later pictures incorporated a routine to correct for this, described later in this chapter.
Plate 1(a). F.i.d. signal voltage under an applied gradient.

Plate 1(b). Digitised spin echo.
Plate 2(a). Spin echo from two capillary tubes. Applied gradient is applied along a line through the tube centres.

Plate 2(b). Spin echo from two capillary tubes. Applied gradient is orthogonal to a line through the tube centres.
Plate 3.
(a) The first NMR image. Two concentric glass tubes. The dots represent water.

(b) Cross sectional photograph of a daffodil stem.

(c) Cross sectional NMR image of a daffodil stem. Dark areas represent water.
Plate 3(c) shows the cross-sectional image of a daffodil stem, without plane definition (ie. the plane thickness equals the sensitive length of the receiver coil). Plate 3(b) is a photograph of a similar stem. At this stage of development the stem is hollow. The image array is 85 x 85 pixels. The outline is crisp, and this is aided by the windowing of the image which, in this case, allowed the full dynamic range of the picture to represent between 60 and 90% of the maximum values on the image; this also revealed some changes of intensity at different positions in the stem. This and other plant pictures shown here were taken with typically 16 signal averages per projection angle, with a pulse sequence repetition rate of the order of 1 repetition per second, which makes allowance for the $T_1$ of the plant cells. Plate 3(c) was taken with 65 projection angles and therefore took 17 minutes to acquire. All the other images were taken with 50 projection angles, and a signal acquisition time of 13 minutes using these parameters.

Testing the resolving power of the apparatus involves the manufacture of phantoms which contain very small compartments of water. The simplest phantom, shown photographed in Plate 4(b), consisted of a bunch of fine capillary tubes (1mm outside diameter, 0.5 mm bore) inside a 5 mm bore glass tube filled with doped water. The tubes fill by capillary action. The larger tube was positioned inside the 8.5 mm bore sample space within the probe. Plate 4(a) shows the NMR image, without plane definition, the light rings representing the capillary tube walls. In some places there appears to be no water between the outermost tubes and the surrounding tube; in some tubes the grey colour suggests that the tubes did not fill completely. Some tubes appear to be oval, which may be due to their being non-parallel with the receiver coil axis, or may be caused by field drift or other similar problems which are discussed in section 5(v). This and other phantom pictures shown here were taken with typically 8 signal averages per projection angle, and, because water doped with copper sulphate was used, a pulse rate of the order of 3 repetitions per second was used. All images were taken with 50 projection angles, giving a total signal acquisition time of 2½ minutes.
Plate 4(a). NMR image of a bunch of fifteen capillary tubes immersed in water within a larger tube. Dark areas represent water.

Plate 4(b). A photograph of the capillary tubes.
Plate (5) shows some early images of a small leek, again taken without plane definition. 5(a) shows the cross-section of the bulb, clearly showing the characteristic division of the structure into two whorls, which lie either side of a central modified shoot. The leaf structure is tightly packed in concentric rings which become green above the bulb. Plate 5(b) is a section taken at this position. Plate 5(a) was windowed so that the lowest 25% of the image intensities were removed, so as to suppress background noise. 5(b) was similarly windowed, with the dynamic range of the picture representing between 15 and 50% of the maximum image values, in order to enhance the low intensity ring structure around the sample.

Plate 6 demonstrates the ability of the NMR imaging technique to investigate very small systems. The image is of a single grain of wheat. In its dormant state the water content of the grain was too low to be detected by the apparatus, but after soaking overnight in water (representing wetting prior to germination) the grain had absorbed sufficient water to be detected. The general features of the structure are sketched beneath the image. Water has been absorbed preferentially in the 'embryo' of the seed, which is the tiny young plant consisting of a root, shoot and one or two leaves. The rest of the seed is visible in outline, possibly owing to the absorption of some water on the outer surface. The characteristic crease along one side of the seed is marked by a line along the length of the grain; this may be due to surface water being trapped after soaking. The further study of seeds would be possible with a much smaller receiver coil volume in this experiment. To offset the low signal strength, 50 signal averages were taken per projection angle, giving a total signal acquisition time, with pulses repeated every second, of 42 minutes.

Plate 7 demonstrates the versatility in the choice of image plane orientation. 7(a) is an image of the tail of a rat. The major details are sketched below it. 7(b) is a longitudinal section through the same point. Both pictures were taken in vitro, and without plane definition. Like many pictures taken at this stage of development of the apparatus, the plane definition of a thin slice would have improved resolution of detail, but there would not have been sufficient signal available to produce a clear image.
Plate 5(a). Cross section of a leek bulb.

Plate 5(b). Cross section of the leaves just above the leek stem.
Plate 6. A single wheat seed after overnight soaking in water. The seed embryo is at the lower left hand end.

Sketch of the wheat seed components.
Plate 7(a). Cross section of a rat tail in vitro. Below is a sketch of the main features.

Plate 7(b). Longitudinal view of the tail at the same point (without plane definition).
The application of the plane definition technique developed by the author and described in Chapter 4 begins with the setting of the image plane thickness. To do this a static gradient is applied in the direction orthogonal to the image planes and the transient gradients are then applied on top of this. Plate 8(a) shows the frequency spectrum or profile of a water sample in this direction. Signals are collected during the application of a set of transient gradient strengths and the profile is displayed again, Plate 8(b). The width of the central peak of the \( \sin \left( \frac{m_y}{A} \right) \) function in 8(b) is measured as a fraction of the profile width of 8(a) which is equal to the sensitive length of the coil. This length was measured to be 12mm for the apparatus used. A plane profile width of 1/8 th of the width of the coil sensitive length would therefore represent a plane 1.5 mm thick. Where multiple planes are measured, the set of planes should be adjusted in thickness so that the sensitive length is just filled by planes.

Plate 9(a) shows another image of a daffodil stem, this time imaged on a 65x65 pixel array which has been mathematically interpolated onto a 128x128 array in order to smooth the detail. This stem was used to demonstrate single plane definition (section 3(vi) in a simple experiment. One side of the stem was removed as a slot 4mm long. Plate 9(b) shows the cross-section of the cut stem without plane definition. Plate 10(a) shows the profile of the cut stem along the coil axis, and the cut is clearly seen. Plate 10(b) shows the cross-sectional image with a thick plane defined through the region of the slot; the signal from the upper half of the stem is all but removed.

Plate 11 shows another pair of images that were used to demonstrate that the plane definition was possible. 11(a) is the cross-section, without plane definition, of a spirally formed glass tube, which is sketched alongside. The tube was positioned so that it spiralled through the axial position of the image plane. When plane definition was applied, the signal remained from one arc of the spiral only. For this early experiment, the transient gradient strengths were obtained from a free-running oscillator, running asynchronously with the pulse sequence.
Plate 8(a). Profile of the NMR response along the coil axis.

Plate 8(b). Axial profile of the image plane.
Plate 9(a). Cross section of a daffodil stem before plane definition.

Plate 9(b). The daffodil stem cross-section after a slot was cut. No plane definition.
Plate 10(a). The axial profile of the cut stem.

Plate 10(b). The plane-defined cross-section of the cut stem.
Plate 11(a). Un-plane defined cross section of a spiral tube of water, sketched alongside.

Plate 11(b). Plane defined cross-section of the tube revealing a single arc.
For these early experiments, the usual number of signal averages was increased so that at least 30 signal averages were taken at each projection angle, so that the required set of transient gradient strengths was certain to be acquired, despite the method of obtaining these from a free-running oscillator running asynchronously with the pulse sequence. Later experiments placed the selection of gradient strength under computer control, so that the minimum number of signals needed to be collected (section 4(iv)).

Plate 12 shows the results from an experiment in which only one transient gradient strength was applied at each projection angle. This was meant to represent the limit of the case in which a complete set of gradient strengths was not obtained at each projection angle. One signal was measured per projection angle whilst the free-running oscillator provided random gradient strengths throughout the experiment. 12(a) shows the arc in a thick image plane, using the spiral tube, for a conventional experiment; 12(b) shows the same plane thickness after one signal per angle. Although the result is noisy, the plane definition can be seen to have worked.

A further demonstration of plane definition is shown in Plate 13. Because the phantom used (a bunch of capillary tubes) has cylindrical symmetry, a longitudinal slice has been chosen by superimposing the transient gradients on one of the two gradients in the image plane.

When the transient gradients are under computer control, it is possible to detect the set of signals and apply a second Fourier transform, as described in Chapter 4, so as to recover a set of image planes stacked on top of one another. The technique has been called 'Polytomic Imaging'. Plate 14 shows the first set of images taken with the method. A single glass tube was shaped so as to lie at an angle across a larger tube which was filled with doped water. The arrangement is sketched alongside the images. Each image was then displayed in 'perspective' by staggering each line of pixels. They were further compressed by ignoring every other line. Thus staggered, the eight images were stacked on top of each other and displayed in a single picture on the television monitor.
Plate 12(a). A thick plane-defined cross section of the spiral tube.

Plate 12(b). The same image, taken with one transient gradient strength per projection angle.
Plate 13(a). Cross-sectional image of a phantom of seven capillary tubes.

Plate 13(b). Plane definition of 13(a) orthogonal to the image plane.
Plate 14. Polytomic images of a single tube set at an angle within a second tube of water, sketched alongside.

Plate 15 (overleaf). Set of Polytomic images of numbers cut from rubberised cork sheet and stacked within a tube of water.
8 planes were taken, which meant that the number of signal averages at each projection angle (initially eight averages were needed) were further multiplied by 8. The signal acquisition time for the experiment was therefore increased by a factor of 8; comparing the time with the $2\frac{1}{2}$ minutes needed for Plate 4(a), the time increased to 18 minutes.

The image planes were arranged to fill the whole of the sensitive volume of the receiver coil, so that the end planes were taken outside the coil which is only 8mm long but sensitive over 12mm. All images were normalised to the same brightness, but those planes at the coil ends were very noisy, especially the top plane shown. In the second and third planes from the top, a dark area appears to the left which was due to the presence of an air bubble.

The most attractive demonstration of the technique is shown on Plate 15. The numbers '1' to '8' were roughly cut from 1.5mm thick sheet rubberised cork and stacked on top of each other in a tube. The tube was filled with doped water and the air bubbles shaken out in an ultrasonic cleaning bath. The numbers were thus surrounded by water and occupied the full 12mm of sensitive axial coil length. Each image plane was therefore an image of a number. In Plate 15 the images are shown, the water represented as light areas and the cork as black. Again, the end planes are noisy, but all the numbers are recognisable. They number '1' to '4' down the left hand column and '5' to '8' down the right hand column. The set of images are shown stacked in Plate 16(a).

Plate 16(b) shows another set of images, this time of a solid plastic rod lying across the tube of doped water. In this case the end planes were too noisy and have not been reproduced here.

The low signal to noise figures for the small sample size in the apparatus was a problem with all the images presented here, especially the multiplanar ones in which the volume contributing to each image is so small. In these, the range of contrast is so limited that it is only possible to decisively say whether water is present or not. Nevertheless, there is sufficient resolution to prove that the Polytomic method works.
Plate 16(a). (Right). The set of planes in Plate 15 stacked on top of each other.

Plate 16(b). (Below). A set of planes of a plastic rod set at an angle within a tube of water.
5(iii). Image Artefacts Specific to NMR Imaging by the Method of Projection Reconstruction.

An artefact is something that is not what it appears to be. As such, an image artefact is a misleading piece of information or detail, but more often than not the word is used in CT to describe a flaw. The latter description is implied here. There are some such artefacts which may occur on an NMR image that are caused by processes unique to the NMR method. They may be classified as:

a) Distortion due to gradient imbalance.
b) Artefacts due to $B_0$ drift.
c) Artefacts caused by poor gradient performance.
d) Artefacts caused by poor spectrometer performance.

These will be considered in turn. A discussion of statistical image noise is in the next section.

a) The gradient vector used to obtain projection profiles is rotated in the image plane by means of vector addition of two orthogonal components. If these are not balanced, that is if the gradient power amplifiers do not produce the correct current so that the gradient coils do not produce the same gradient magnitudes for the same input, then the entire image will be elliptical rather than circular. It will be correct in every other respect, however. Plate 17 shows four glass capillary tubes placed one within another and all within a fifth tube which was filled with doped water. Gradient imbalance has caused the elliptical shape of each tube.

b) In order to analyse subsequent problems of information mis-placement, reference should be made to Figure 5.1. Ideally, information on a projection profile which is derived from a single point will always be back-projected through that point on the image array during reconstruction. Simple back-projection produces the reconstruction of the point convoluted with the $1/r$ 'star-burst' function. (The deconvolution process, carried out on the profiles before projecting back, removes the artefact.) This is shown in sketch A. However, a misplacement error that is a function of projection angle causes a smear of information along a line that is tangential to the back-
Plate 17. Elliptical image of four glass tubes immersed in doped water. The distortion is due to gradient imbalance.
Figure 5.1. The coordinate system for gradient rotation in the image plane and two reconstructions; (A) Star-burst artefact on a single point. (B) Smeared reconstruction from misplaced profiles.

Furthermore, the deconvolution of the image array is incorrect but in a very complicated way. Deconvolution on a profile takes the form of the weighted suppression of points adjacent to the object position. Thus, image array points adjacent to the back-projected rays in sketch A are made negative, whilst the points cut by the ray are positive. The back-projection of all filtered profiles onto the image array ideally produces the reconstructed point or object, surrounded by a 'flat' image plane. However, when the rays are mis-placed, the 'star-burst' loses its symmetry. This is shown in Figure 5.2. Areas on the image array around the point A where the rays are more dense (at the top) are oversuppressed and will have negative values; areas where the rays are less dense (at the bottom) are undersuppressed and information will be present on the
Figure 5.2. Reconstruction of the projection profiles of a single point which have been systematically mis-placed as a function of projection angle. Positions of under- and over-correction of the deconvolution are labelled.

The appearance of this kind of artefact is best demonstrated in the case in which $B_0$ drifts throughout the experiment. The resonance frequency for any point in the object changes from its correct value by an amount which changes from projection angle to projection angle. The reconstruction of the information from any point is smeared in the way described. Plate 18 shows two cross-sectional images of a lemon which demonstrate the appearance of the artefact for an extended sample. These images were taken with gamma-ray CT in which linear sourcetracking is misplaced with respect to the centre of object rotation on a turntable, a problem which is analogous with field
Plate 18(a). Gamma-ray CT picture of a lemon (Foster, 96). The misplacement streak artefacts can be seen.

Plate 18(b). The same image as 18(a) after 'Centre of Mass' correction to the image, developed by the author to remove field drift artefacts from NMR images.
drift in NMR (96). The appearance of the artefact is clearer on these pictures than on any available NMR picture.

Plate 18(a) shows that, on an extended sample, the artefact occurs as two streaks from the extreme ends of the object which appears broader in the lower half than in the upper. Plate 3(a) has the same appearance, but the streaks have been suppressed by windowing the image levels. The artefact appears to be most noticeable for objects which are well-bounded and of high intensity, such as water and glass phantoms, and are uncommon in biological systems in which boundaries and intensity changes are less severe.

'Centre of Mass' correction.

The Author has developed a simple computer routine to correct for this problem of the displacement of whole profiles which are otherwise undistorted. It is most easily visualised if the profiles are thought of as the 1-D projections of the mass of a 2-D flat object whose mass is heterogeneously distributed. We can then invoke a simple rule in Statics which says that the 'centre of mass' of the projection profiles is the projection of the 'centre of mass' of the image. The correction applied to each profile in turn first treats the profile as a 1-D non-uniform beam and finds its 'centre of mass', using the formula:

\[ \bar{x} = \frac{\sum m_i x_i}{\sum m_i} \]

where \( m_i \) are the masses along the beam
\( x_i \) are the positions of the masses from the profile centre
and \( \bar{x} \) is the position of the c.of m.

The profile is then moved so that \( \bar{x} \) is zero. The 'centre of mass' will thus back-project through the image plane centre. If this is done, it does not matter where the profile had originally drifted to on the profile axis. As the centre of mass of the reconstructed object lies on the projected paths of all the centre of masses of the profiles, the object will always be correctly reconstructed in the centre of the image array, provided that the field drift is less than one linewidth during each profile accumulation.

The 'Centre of Mass' correction may also be used to remove translational motion artefacts when a sample is imaged 'in vivo'.

Plate 18(b) shows the image of 18(a) after correction by this method.

(c) The artefacts which are caused by poor gradient performance may be split into those which cause image smearing, and those which cause individual image points to be correctly reconstructed but in the wrong place, thus causing an image 'warp'.

(i) Image smears will result if a gradient current driver is required to produce more than its maximum current or if it responds non-linearly to its input. A simple example is shown in Figure 5.3. A point object lies at the position \( \mu(0,-z') \). The \( \partial G_z/\partial z \) gradient should increase to a maximum at the projection angle of 90°, but the power amplifier is assumed to produce no more than the current required for an angle of 45°. After this, the current output is limiting. The result is a misplacement artefact.

\[ A \]

Figure 5.3. Reconstruction of a point object at position 'A' when the \( z \) gradient driver is limiting.
The $\partial g/\partial x$ gradient in this example is required to produce both positive and negative gradient directions. If the outputs are offset so that a zero input results in a finite output, then the vector sum of the $\partial g/\partial z$ and $\partial g/\partial x$ gradients will describe a circle whose centre does not correspond with the centre of reconstruction of the image and data will be misplaced on the array.

The positive and negative gradients may be produced by separate current drivers and in this case it is important that both drivers have identical output magnitude for a given input. If this is not the case, the gradient vector will not describe a circle, although the centre of rotation will be correct, and information will again be misplaced.

(ii) The best example of image warping has already been described and shown in Plate 17. The imbalance of the $\partial g/\partial z$ and the $\partial g/\partial x$ gradient driver gains causes the gradient vector to describe an ellipse which is nevertheless centred on the centre of rotation of the reconstruction. A circle is just a special case of an ellipse and, as can be seen in Plate 17, the reconstruction is correct apart from an elliptical shape.

A less obvious case for image warping, which is an extension of gradient imbalance, is when the individual gradient coils cause gradient non-linearity. Figure 5.4 shows that a warped $\partial g/\partial z$ gradient misplaces a point reconstruction, but consistently so, the misplacement being proportional to the sine of the projection angle. Similarly, Figure 5.5 shows this for a non-linear $\partial g/\partial x$ gradient from both positive and negative directions. The result is a distorted image when all points are considered, but not one with smeared information.

(d) Several potential faults which would be caused by the spectrometer could be identified by studying the profile shape from a standard phantom such as a tube of water.

A problem arises when the received signal is not completely in phase with the reference signal in the phase-sensitive detector. The resulting frequency spectrum contains a mixture of the real, even cosine spectrum and the imaginary, odd sine spectrum. The
Figure 5.4. The effect of a non-linearity in the z gradient. Information is misplaced by successive back-projected rays by amounts which are proportional to the projection angle.

Below, (i) shows the non-linear field, (ii) a simple homogeneous object and (iii) shows the profile which is distorted towards the higher frequency end because most of the object is in a high field.
Figure 5.5. The non-linear x gradient (shown as a solid line; the dashed line shows the linear gradient) has both positive and negative directions during the scan. In addition, the gradient both adds to and subtracts from the main field $B_0$. A point object 'A' in the centre of the object has profiles which are shifted by the warped gradient. The figure shows the back projections at the angles $0^\circ$ and $180^\circ$, both of which misplace the object but in a coherent fashion.
latter odd function is sketched in Figure 5.6. The problem can be corrected either by adjustment of the reference signal phase or by a suitable combination of the real and imaginary frequency spectra.

\[ S(f) \]

Figure 5.6. The sine spectrum of a resonance line, showing positive and negative parts with a central discontinuity.

Phase errors that are a function of frequency are more difficult to remove. These can be caused by the non-linear frequency response of the receiving equipment, or they may occur on signals collected from a simple 90° pulse in which receiver recovery following the pulse masks the correct signal value at the first points in time. The latter problem can be avoided by using a spin-echo. Mis-timing the collection of data can mean that the initial point at which all frequencies are in phase does not correspond with the first sampled value. The constant time-shift represents a phase-shift which is a function of frequency.

The pulse must contain all the required resonance frequencies and so must be sufficiently short (equation 1.19). If the r.f. power amplifier cannot provide such a pulse, whose length is a function of r.f. power, then high frequencies will not precess through a complete 90° arc and may also begin to align with the pulse, thus introducing phase errors which will have a very complicated frequency dependance.

The effect on the image of any phase error is likely to be an inversion of image information from positive to negative values within the boundary of an object.

The receiver is susceptible to picking up stray radiation in its vicinity because of its high sensitivity. A constant frequency spike in the profile spectrum will remain unmoved on the profiles.
as a function of projection angle and the reconstruction artefact is a semicircular ring on the image array. Large frequency spikes occurring once during an experiment can appear as a single straight line across the image.
In this section the ratio of signal-to-noise on an image pixel will be examined as a function of the signal-to-noise ratio on the f.i.d. time signal. The noise on the signal can be assumed to be thermal noise caused by the random motion of 'free' electrons in conductors in the receiving circuit. Libove and Singer (85) derived the following equation which can be used, for a proposed design of receiver coil and pre-amplifier, to evaluate the signal-to-noise ratio which will be produced. The signal originates from the volume of spins within the coil, and the coil r.f. resistance is calculated from the coil dimensions:

\[ R.M.S. \text{ Signal:Noiseratio} = \frac{1}{\sqrt{2}} \left( \frac{K_{\mu_0} n V_s N \gamma h \nu_0 I(I+1) \omega_0^{7/4}}{10.1 k^{3/2} T_s (\mu \rho)^{1/4}} \left( \frac{P (\alpha^2 + \rho^2)(F T_c L B)}{(\alpha^2 + \rho^2)(F T_c L B)} \right) \right)^{1/2} \]

where

- \( K \) is an inhomogeneity factor which depends on the sample placement in the coil, and is approximately unity;
- \( \mu_0 \) is the permeability of free space;
- \( n \) is the number of coil turns (solenoidal coil);
- \( V_s \) is the sample volume at resonance;
- \( N \) is the number of resonant spins per unit volume;
- \( \gamma \) is the nuclear gyromagnetic ratio;
- \( h \) is Planck's constant / \( 2\pi \);
- \( I \) is the spin;
- \( \omega_0 \) is the resonant frequency in radians s\(^{-1}\);
- \( k \) is Boltzmann's constant;
- \( T_s \) is the sample temperature in degrees Kelvin;
- \( \mu \) is the magnetic permeability of the coil;
- \( \rho \) is the resistivity of the coil at temperature \( T_c \);
- \( P \) is the wire circumference;
- \( \alpha \) is the coil radius and \( \gamma \) is half its length;
- \( F \) is the noise amplification of the pre-amplifier, eg. the factor equals 2 for a noise figure of 3dB;
- \( T_c \) is the coil temperature;
- \( L \) is the total length of wire in the coil;
- \( B \) is the receiver bandwidth.

This equation gave the correct order of magnitude for the solenoid used in this research. For an existing coil, the equation can be compared with measured figures, and a discrepancy could indicate

\* see p.142.
poor performance in the phase sensitive detector or the audio frequency amplifiers which follow.

A more usual problem is the calculation of the image noise level given the signal:noise ratio from a water sample in an existing coil. The Author has examined this problem in two stages; (a) the s:n ratio on the frequency spectrum as a function of that in time, and (b) the ratio on the image as a function of that on the frequency profiles. Both are initially investigated without plane definition.

(a) The voltage spectral density $G(f)$ is defined as the mean square noise voltage with frequency $f$. Connor[97] defines it as:

$$G(f) \, df = \overline{V_n^2}$$

The power spectral density of white noise is independent of frequency up to about $10^{13}$ Hz and is therefore constant over the range of NMR resonance frequencies. It is therefore called white noise, or 'Johnson noise'. The resistance of the conductor within which the power is generated is constant, and so $G(f)$ is constant also. Therefore, over the finite bandwidth $B$,

$$G(f) \int_{-B}^{B} df = \overline{V_n^2}$$

or, in the case in which negative frequencies are indistinguishable from positive ones because only one phase sensitive detector is used, the limits of integration are $df_{\pm}$ and $G(f)$ is twice as large:

$$G(f) = \frac{\overline{V_n^2}}{2B}$$

On the discrete frequency spectrum, points are spaced $\Delta f$ apart. The average mean square voltage in the range $\Delta f$ is $G(f) \Delta f$.

If the magnitude of the noise voltage at any discrete frequency point is represented by $\sigma_f$, the standard deviation about the correct value (in volts), then $G(f) \Delta f$ is the variance at the point, or $\sigma_f^2$.

The subscript 'P' indicates that this quantity is measured on the projection profile. Writing $\sigma_P^2$ for $\overline{V_n^2}$ in time, we have:
Now the signal voltage is considered without noise. The area under the frequency spectrum, or projection profile, equals the magnitude of the f.i.d. at the origin (44), say \( V \) volts. The area under a continuous rectangular profile of width \( B' \) Hz and constant height \( V_p \) is \( V_p B' \). Discrete sampling in the frequency domain implies that the time signal, and hence the area under the frequency spectrum, is scaled by \( 1/\Delta f \) (37). Hence:

\[
V = V_p B'/\Delta f
\]

ie.,

\[
V_p = \frac{V \Delta f}{B'}
\]

A rectangular profile is not a realistic case so we use the profile of a circular object. The profile height is proportional to the area of an ellipse whose major axis is \( B' \) and whose minor axis is \( V_p \). The area of such an ellipse is given by \( \pi B'/2 \times V_p/2 \). The area under the frequency spectrum is, after frequency sampling:

\[
V = \pi \frac{B'}{2} \times \frac{V_p}{2} \times \frac{1}{\Delta f}
\]

ie.,

\[
V_p = \frac{4\pi}{\pi} \frac{V \Delta f}{B'}
\]

where \( B' \) is the width of the profile and \( V_p \) is its maximum height.

We now have expressions for the maximum profile height and the standard deviation on the profile points. Note that neither derivation has involved the discrete sampling of the time signal, which results in scaling of the frequency spectrum; however, both signal and noise are subject to this scaling so that the ratio of signal:noise in frequency space is unaffected. Therefore:

\[
\frac{V_p}{\sigma_p} = \frac{4}{\pi} \frac{V}{B'} \Delta f \frac{1}{\sigma} \sqrt{\frac{B}{\Delta f}}
\]
Equation 5.9 relates the signal to noise on the profile, measured as the maximum profile height from a circular object over the R.M.S. noise level, to the signal to noise in time, measured as the maximum signal voltage (at the beginning of an f.i.d. or the centre of a spin-echo) over the R.M.S. noise level estimated from the signal baseline.

The analysis is simplified if a sample of water is considered that will have a bandwidth \( B' \) equal to the width of the frequency spectrum which is back-projected onto the image array. In this way, we can investigate the signal:noise ratio on every image point. We shall see that, although real objects will have smaller signals and profiles, the scaling of signal:noise in the reconstruction process is unchanged, so that the figure that we obtain for the largest sample gives the signal:noise ratio on the image that any individual image pixel will have if occupied by spins.

Equation 5.9 may be re-written, substituting \( m = \frac{B'}{\Delta \phi} \), the number of points over which the profile extends, as:

\[
\frac{V_p}{\sigma_p} = \frac{V}{\sigma} \times \frac{4}{\pi} \sqrt{\frac{\Delta \phi}{B'}} \quad 5.10
\]

Note that \( \sigma^2 \propto B \) (Connor, (41)) so that, for a given sample and value of \( m, \frac{V_p}{\sigma_p} \) is independent of \( B \).

(b). The amplification of profile noise during image reconstruction can be thought of as having two separate processes. Part of the loss of signal integrity is due to 'algorithm noise'. This refers to computational errors, which can be grouped as:

(i). Loss of precision in arithmetic operations because of operation in integer arithmetic and/or short length words (eg. 8-bit);

(ii). Errors in interpolation between points on the profile which is necessary for the back-projection of profiles expressed in circular coordinates onto a cartesian array;

(iii). Errors due to partial mis-alignment with part- or whole pixels on the image array during the subsequent back-projection.
As all these errors are dependant on the structure of the reconstruction program, their magnitudes cannot be estimated for a general case. Although they cannot be removed entirely, their contribution will be assumed to be negligible in the following analysis. An example of a reconstruction routine is given in Appendix 2.

The statistical propagation of noise in the reconstruction process cannot be ignored. It will be examined here for the filtered back projection method which was used by the Author. We will first consider the reconstruction of an image from a profile which contains only noise with no signal.

Before back projection, the noise level on any profile point is compounded during the convolution or 'filtering' process. The discrete filtered profiles were given by equation 3.22:

\[ p'_o(n) = \sum_{q=-\infty}^{q=\infty} p_o(q) \cdot f(n-q) \]  

(3.22)

where \( p_o \) is a projection profile; 
\( q, n \) are integer, 
and the spacing between points is unity. The discrete values for the filter function \( f(q) \) was given by equation 3.23:

\[
\begin{align*}
    f(q) &= \frac{1}{4} & \quad (q=0) \\
          &= -\frac{1}{\pi^2 q^2} & \quad (q \text{ odd}) \\
          &= 0 & \quad (q \text{ even})
\end{align*}
\]

To find the variance of a filtered projection point we use the general formula for the variance of a compound function (eg. Topping, (98)): 

\[
\sigma^2 \langle \rho, \sigma_r, \ldots \rangle = \sum_k \left( \frac{\partial \rho}{\partial r_k} \right)^2 \sigma^2 \langle \sigma_r \rangle 
\]

5.11

Therefore:

\[
\sigma^2 \langle p'_o(n) \rangle = \sum_{q=-\infty}^{q=\infty} |f(n-q)|^2 \sigma^2 \langle p_o(q) \rangle 
\]

5.12

We have already seen that \( \sigma^2 \langle \rho(q) \rangle \) has the same value \( \sigma_p \) at all profile points and this obviously holds true for any projection angle.
too. The value of \( \sum_{q=0}^{\infty} |P_0(q)|^2 \) at the centre of the profile where \( n = 0 \) is:

\[
\sum_{q} |P_0(q)|^2 = \left(\frac{1}{4}\right)^2 + 2 \left(\frac{1}{\pi^2 q^2}\right)^2 \text{ for } q = \text{even}, \\
\text{or } q = \text{odd}, \\
\rightarrow \frac{1}{12}
\]

The sum converges to this value very quickly, and on a profile with a finite number of points, it can be shown numerically that the sum converges to within 0.01\% of this figure when \( |q| \gg 7 \).

Therefore:

\[
\sigma_r^2 \langle P_0'(0) \rangle = \frac{1}{12} \sigma_r^2
\]  

5.13

On a profile of finite width the variance on the filtered profile varies slightly with position. However, because the above sum converges so quickly, it is only at the edges of the profile that \( q \) is not summed both positive and negative. Even here, it is only on the very first and last point that the sum is significantly different from the above result, and then only by the amount 1/96. Hence the filtered noise profile can be thought of as also having a constant value, given by equation 5.13, \( = \sigma_r^2 \) filtered.

The back-projection of the constant variance of the profiles across every image pixel, from all projection angles, means that the image array pixels have a constant variance in the area where the circular object image is reconstructed. The variance is found using the equation 5.11 again. The back projection reconstruction was given by equation 3.25 (without any allowance for the interpolation which will be necessary to locate the array points exactly):

\[
\mu(r, \theta) = \mu(j r, k \phi) = \sum_{k=1}^{N} P'(j r_0 \cos(k \phi_0 - k \Delta \theta), k \Delta \theta) \Delta \theta
\]  

3.25

where \( \Delta \theta \) is the angular increment between projections, = \( \pi/N \); 
\( \Delta \theta \) is the angular increment between projections, = \( \pi/N \);  
\( r_0, \phi_0 \) are the polar coordinate intervals;  
\( j, k, h, t, N \) are integers.
Therefore:
\[
\sigma^2 \langle \mu(j_\circ, k_\phi) \rangle = \sum_{\theta=1}^{N} \sigma^2 \langle p'(j_\circ \cos k_\phi + i i \Delta \theta, + \Delta \theta) \rangle \Delta \theta^2
\]
\[
= \sigma_p^2 \frac{\pi^2}{N^2} \times N
\]
\[
= \sigma_p^2 \frac{\pi^2}{N}
\]

This verifies that the image variance is constant if \( \sigma_p \) is constant. We may substitute from equation 5.13 to give:
\[
\sigma^2_p = \frac{\pi^2}{12} N 
\]

We now consider the noiseless circular profile again. The height of the central point of the profile after convolution is found by numerical evaluation of equation 3.22. In this case, each profile value has a different value and the sum must be evaluated up to some finite number of points away from the central point. The number, \( q \), is the number of profile points to the edge of the bandwidth of frequency points which are back-projected, where the bandwidth may be entirely filled by the circular profile, as in case A in Figure 5.7, or it may only be partly occupied by the profile of a smaller object, case C. In either sum, the sum needs to be calculated as it does not converge. The convolution involves the weighted subtraction of values from each profile point, and since less profile points are subtracted in the case of a smaller

Figure 5.7. Similar filtered profile heights from different sizes of sample when \( q \) is constant. Convolution of A yields B; the central profile height is reduced by many subtracted components. In contrast, C yields D, in which only a few values are subtracted.
profile, the profile height is reduced by proportionally less. Thus the filtered profile heights of both large and small circular objects are the same; this is because the addition of the values back onto the image array must result in image pixels which represent the presence of spins having the same value, irrespective of the size of the object within which they occur. This scaling can be proved numerically.

The consequence of this is that the signal height on an image in any pixel can be estimated from the profile height, and therefore from the f.i.d. magnitude, of a water phantom which fills the receiver coil and image array, regardless of the size of the object which is studied.

It is most convenient to study the point in the centre of the circular profile. The reduction in height on the filtered profile is a function of the number \( q \). If a profile is spread over more points then the reduction is greater. The values are listed in Table 5.1 for a central profile height before filtering of \( P_\theta(0) \), on a profile that fills the back-projected bandwidth.

<table>
<thead>
<tr>
<th>Popular image array size (pixels)</th>
<th>Nearest profile width (odd number), ( n = (2q - 1) )</th>
<th>Multiplying factor, ( F )</th>
</tr>
</thead>
<tbody>
<tr>
<td>8 x 8</td>
<td>9</td>
<td>0.0334</td>
</tr>
<tr>
<td>16 x 16</td>
<td>17</td>
<td>0.0180</td>
</tr>
<tr>
<td>32 x 32</td>
<td>33</td>
<td>0.0094</td>
</tr>
<tr>
<td>64 x 64</td>
<td>65</td>
<td>0.0048</td>
</tr>
<tr>
<td>128 x 128</td>
<td>129</td>
<td>0.0025</td>
</tr>
</tbody>
</table>

Table 5.1. Evaluation of the equation: \( \rho_\theta'(0) = \sum_{q=-\infty}^{q=\infty} P_\theta(0) f(q) \)

This gives the multiplying factor which is applied to the central profile point \( n = 0 \) in convolution. The first column gives the popular sizes of image arrays. The convolution and back-projection routines (equation 3.22 and Appendix 2) require an odd number of points on a projection profile, listed in column 2. The multiplying factor is in column 3.
The filtered profile height is added $N$ times to the array points within the boundary of the circular object as $N$ projection angles are used. Therefore the signal height on the pixels can be written as a function of the maximum profile height:

$$V_{\mu} = FNV_p$$  \hspace{1cm} 5.16

where $V_{\mu}$ is the image signal height; $V_p$ is the profile signal maximum height as before; $N$ is the number of projection angles; $F$ is the factor from Table 5.1.

Combining this with equation 5.15 we have an expression for the signal to noise on the image as a function of that on the circular profile:

$$\frac{V_{\mu}}{\sigma_{\mu}} = FNV_p \times \sqrt{\frac{12N}{\pi \sigma_p}}$$

$$= \frac{V_p}{\sigma_p} \times \sqrt{\frac{12N^{3/2}F}{\pi}}$$  \hspace{1cm} 5.17

Brooks and Di Chiro (95) have suggested that the number of projection angles used should be at least $\pi/4$ times the number of discrete points across the image array. For the 65x65 pixel arrays commonly reconstructed by the Author, 50 angles were used. These figures give $F = 0.0048$ and $N = 50$, and equation 5.17 gives:

$$\frac{V_{\mu}}{\sigma_{\mu}} \approx 1.9 \frac{V_p}{\sigma_p}$$

so that in the absence of algorithm noise, there should be some improvement in signal:noise as the image is reconstructed.

Finally, equation 5.17 can be used with equation 5.10 to give:

$$\frac{V_{\mu}}{\sigma_{\mu}} = \frac{V}{\sigma} \times \frac{4N^{3/2}F}{\pi^2 \sqrt{12 \frac{B}{B'}}}$$  \hspace{1cm} 5.18

where $m$ is the number of points across both the array, and the projection profile, whose bandwidth is $B'$. 
Following on with the example of the 65x65 array, we have $m = 65$ and, if $V/m$ was obtained with an ideal receiver where $\mathcal{B}' = \mathcal{B}$:

$$\frac{V_m}{\sigma_m} \leq 1.9 \times \frac{4}{\pi} \sqrt{\frac{1}{m}} \times \frac{V}{\sigma}$$

$$\approx 0.3 \times \frac{V}{\sigma}.$$ 

This, then, relates the signal to noise on any image point containing water to the signal to noise measured on an f.i.d. which is obtained using a pure water phantom which would completely fill the width of the image array under identical conditions.

Throughout this proof the relaxation effects and non-uniform density of water in a sample has been ignored, in as much as pixels have contained either 0 or 100% water.

Equation 5.10 predicts the correct noise level only in the presence of white noise, and also provided that the time signal has been sampled fast enough to measure all the frequencies in the receiver bandwidth. If the sampling is not sufficiently fast, high frequency noise will be folded back onto the low frequency spectrum including the profile bandwidth, thus increasing the noise level (see section 2(v)).

**Plane Definition.**

The derivation of equation 5.18 has assumed that no plane definition has been employed. When a plane is defined, the following points should be remembered:

a) The signal voltage $V$ which is the initial f.i.d. height will be shared amongst all the planes which are produced. Furthermore, it was shown in section 4(v) that each plane yields only 50% of its potential signal because of the spatial response of the plane-defining function. Thus the signal to noise on each of $m$ planes would be decreased by a factor of $\frac{1}{2} \times 1/m$. However, the technique requires that $m$ signal 'averages' are taken at each projection angle, so that after the experiment, the response from each plane is improved by simple signal averaging by $m^{1/2}$. 
The signal to noise ratio would therefore be decreased by a factor of $0.5 / \sqrt{m}$.

b) Although the signal height has been referred to as being read from the f.i.d., of course in a spin-echo experiment it is the maximum echo height which must be measured. If plane definition is used, the echo height changes as transient gradients are applied because the spins do not rephase coherently. The spin-echo should therefore be measured in the absence of any transient gradient.

In conclusion it must be said that the above calculations have assumed both a perfect receiver and a perfect reconstruction without algorithm 'noise'. The whole NMR Imaging experiment can be summed up as two distinct phases once a technique has been developed; firstly, the signal must be obtained without a loss of integrity caused by hardware; and secondly, the image must be reconstructed without a loss of integrity caused by software. It is because neither can be achieved in practise that the above calculations can only provide an estimate for a 'best case' image signal to noise figure.

In this section some of the factors which may be optimised to improve the image contrast and/or imaging time are discussed.

For an imaging experiment, it is important to use a method which does not discard any resonance signals, and a full 3-D experiment is often preferable unless information is required from a specific plane or point (section 3(ii)). There are, however, times when these latter experiments would be more useful. For instance, it has been shown (section 4(v)) that the selection of a plane whose orthogonal profile is a \(\sin(\pi y)/(\pi y)\) function rather than a rectangle reduces the signal in that plane by 50%. If a single plane is required, then the selective excitation method may be used in which the spatially selective pulse is tailored to excite all spins in the plane equally, thus yielding a higher response. (Note that this method is not efficient if more than one plane is required.) Also, a sensitive point experiment may be useful if it yields the data for relaxation times from within an organ, as it is not only a rapid experiment but also the size of the sensitive point may be increased to fill the organ volume with the subsequent increase in signal strength and accuracy. Both of these variations demand flexibility in the control of both pulses and gradient magnitudes, both of which would have to be under computer control.

Equation 5.2 suggests many ways of obtaining more signal to noise. These are:

a) The use of a higher \(B_0\) field strength, within the limits of r.f. safety (section 3(v)). This yields a \(\omega_0^{7/4}\) improvement.

b) Reduce the volume of the receiver coil as far as possible within the restrictions of sample volume and \(B_1\) homogeneity.

c) The use of quadrature detection (using two reference phases and phase sensitive detectors in phase quadrature) enables the profile to be centred on the zero frequency point and the receiver bandwidth to be halved, resulting in a \(\sqrt{2}\) improvement.

d) The coil may be cooled, possibly to superconducting temperatures.
However, Libove and Singer (85) indicate that the improvement in signal to noise, accomplished by reducing the series resistance of the coil, is limited by the finite conduction and resistance of the sample within the coil, and the maximum improvement is likely to be only of the order of $\sqrt{2}$. Thermal noise may also be reduced in the preamplifier by cooling, thus reducing the noise factor $F$.

Many other improvements may be made by optimising the specifications of the components of the imaging system, and these are outlined in the next chapter for each specific piece of apparatus. Such improvements include the removal of $B_o$ fluctuations, the use of a filter which defines a sharp band-pass, and improved computer storage and arithmetic precision.
CHAPTER 6

APPARATUS
APPARATUS.

In this chapter the component parts of the apparatus used by the Author are described. The descriptions are preceded by a list of the requirements which each particular component should have for its successful inclusion in the overall design.

Where it is appropriate, the specifications of the performance of pieces of equipment are given.

A block diagram of all the component parts and their interactions is shown in Figure 6.1.
Figure 6.1. Block diagram of the apparatus.
6(i). Crystal oscillator.

Requirements. The oscillator is the source of the r.f. resonance frequency of $B_1$ and should have stability that exceeds the requirements of the magnet homogeneity by several orders of magnitude. A square-wave output should be converted to a pure sine wave with the capacity to drive all the other spectrometer circuits.

Equipment. A Cathodean voltage controlled, temperature stabilised crystal oscillator, No. FS5953 Version 01, was used to produce a TTL - compatible 15MHz output.

A 15MHz tuned amplifier was used to obtain a sinusoidal signal. The circuit is shown in Figure 6.2. The square wave forms the input to a 'long tailed pair' of transistors. The collector load of one of these is a tuned transformer. The transformer output is fed back to the base of the transistor which acts as the high impedance tail resistor, thus tuning the gain of the pair, and also to the output transistor stage. All the transistors are located on a single CA 3046 integrated circuit which ensures close electrical and thermal matching.

The output of the tuned stage was split to feed the pulse gating circuits and the reference circuit for the phase sensitive detector. Each signal was amplified by a separate r.f. amplifier, the circuit for which are shown in the receiver section (6.(vii)).

Specification.
Crystal oscillator, Cathodean FS5953 Version 01.
Frequency 15 MHz, finely tuned by control voltage.
Frequency stability 0.1 ppm 0°C to +60°C
Supplies 5V ±10%, oscillator; 9V ±10%, oven
Output TTL compatible.
Figure 6.2. Tuned amplifier producing sinusoidal r.f. All transistors are CA 3046.
6(ii). Phase shifters.

Requirements. The phase of the r.f. signal may be shifted on the reference signal, the signal supplied to the pulse gates, or on both of these, so as to shift the phase of the received signal. Phase shifter(s) should be capable of covering the full $360^\circ$ of phase angles, and should not distort the signal shape or amplitude as a function of angle.

Equipment. Two phase shifters were used, each being built to the design shown in Figure 6.3. The 733 differential output amplifiers provide two r.f. outputs which are exactly opposite in phase. These are balanced by 100Ω preset potentiometers. In the first stage, one output is loaded by a small capacitance, and the other by a FET which acts as a voltage-controlled resistance. The impedances of these two loads are always at right angles in phase. The equal and opposite output signals are loaded by this combined load whose complex impedance therefore is the locus of a semicircle. The signal which is fed to the second stage has a constant magnitude but a phase which is a function of the FET gate voltage. This is sketched in Figure 6.3.

The two opposite phases produced by the second 733 are selected by a two way switch. The two stages together ensure that a full $360^\circ$ range of angles is available. It is of course desirable that the required phase is not at the end of the range of adjustment, and the use of two shifters ensures that this is not the case.
Figure 6.3. Radio frequency phase shifter circuit. The sketch (right) shows the vector addition of the impedances of the capacitor and FET which are at right angles, thus always loading the 733 outputs with a total vector impedance whose magnitude is the locus of a semicircle.
6(iii). Gate pulse generator.

**Requirements.** The pulse generator produces the 90° and 180° pulse length TTL signals which open and close the r.f. gating circuitry. The pulse generator should therefore have two channels which have individual pulse length and delay adjustments. Each pulse cycle should be started by a trigger pulse which is available as a separate output to reset the ADC. The pulse lengths should be stable to within 1 or 2°; the pulse delays should be stable to within the time spacing between ADC sampled points, i.e. $1/(\text{sampling frequency})$. A 'single shot' facility is useful.

In order that the pulse separation be accurately reproduced, it should be monitored with a timer which has a very high order of accuracy.

**Equipment.** Initially, a simple timer was built using 8T22 retriggerable monostable multivibrators to provide both pulse delays and lengths. However, in the interests of repeatability this was superseded by a Farnell PG5222 pulse generator. This has double channel output with pulse period, delay and width continuously variable over 6 decades by means of switched ranges and fine potentiometer controls. The pulse output voltages and polarities are adjustable.

Accurate pulse separations were achieved by firstly setting the delay of the second (180°) pulse on a range of 10's of milliseconds, and then setting the delay of the first pulse on a 'fine tuning' range of 10's of microseconds. Although the separation thus depends on the precision of the potentiometer which controls the delay of the second pulse, in practice this was found to be very stable (see specifications).

Pulse separation was measured with a Gould Advance TC314 timer counter.

The monitored 90° gating pulse was lengthened with an 8T22 monostable so that it could be detected from within a program loop.
Specifications.

Pulse Generator.  
Farnell PG5222  
Ranges, period & delay: 200nS to 200mS.  
Range, width : 100nS to 100mS.  
Output: 30mV to 20V into 50Ω, either polarity.  
Protection: Against open or short circuits.  
Stability of pulse separation: About ± 3 μs  
at a separation of 10 mS (empirical)  
Trigger output: 2.5V to 20V adjustable.

Timer Counter.  
Gould TC314  
Clock frequency: 10 MHz  
Range: 100nS to 10^{-7}S  
Accuracy: ± 1 count ± timebase accuracy  
± trigger level error, 1st pulse  
± trigger level error, 2nd pulse  
( At 10 mS, 1 count represents .01μs.  
Timebase stability 2 parts in 10^{7}.  
Trigger error less than 1% at 1V,1kHz, but  
Farnell pulse rises in 10 nS so this is  
negligible.)  
Input impedance: 1MΩ.
6(iv). Radio frequency gates.

Requirements. The radio frequency should be gated into pulses so effectively that the leakage between pulses, when amplified by the power amplifier, irradiates the coil with a signal that is much smaller than the induced signal, otherwise the leakage will saturate the receiver input. The pulse size may be $10^7$ times larger than the NMR signal, and gate isolation should therefore exceed 140 dB.

Equipment. Three r.f. gates were developed in the quest for a high level of signal isolation. These are described in chronological order.

a) An MC1445G differential output amplifier was used initially. This has a very wide bandwidth and can be gated to select either of its inputs by using external logic levels. When used as a gate, one input was connected to the r.f. and the other to ground. Unfortunately, although it is capable of 130 dB channel separation at low frequencies, this falls to about 40 dB at 15 MHz.

b) The second design employed SD210 FET's, used in pairs, to gate the signal. R.f. conduction across one FET was controlled by a second, whose gate was controlled by the input pulse voltage and whose drain was connected to the gate of the first. Furthermore, this arrangement was then repeated to obtain higher isolation. The output voltage waveform was then converted to a current drive for the power amplifier input by a Burr Brown 3553 wideband buffer amplifier with zero voltage gain.

This arrangement worked well, but there was still a significant leakage. Although too small to be observed directly, its presence was confirmed by the observation of the noise baseline of the receiver output in the absence of an NMR signal. As the receiver reference phase was adjusted, the gain of the noise was seen to vary, and the d.c. shift to change. This indicated the presence of a large signal, at the same frequency as the reference signal.
c) The third design proved to have sufficient isolation. The circuit is shown in Figure 6.4. The design was purchased, half assembled, from Polaron Ltd., who had abruptly ceased production of the unit. It was completed and tested at the University of Surrey.

The circuit operates as follows. A pair of npn transistors have a common collector output and are fed by two separate inputs. One is the gate pulse, which is cleaned up by the Zener diode and then fed to one of the two bases. The other input is the r.f. signal. The output of this stage is the r.f. gated signal, raised on a 'pedestal' above the baseline of zero volts.

At subsequent stages, the transistors only conduct in the presence of the positively raised signal, and do not conduct when the baseline leaves them unbiased. Isolation thus increases stage by stage, and is helped by the individual stages being screened from one another. The d.c. level is removed by the final output capacitor.

Isolation from this design is claimed to be 140dB. In practice, the isolation was further helped by series crossed diodes in the probe transmission line (section 6(vi) ) and no leakage was detectable.
Figure 6.4  Radio frequency gates. The dotted lines represent earthed screens. Transformer T1 is 5 turns of 50A line on a small ferrite ring.
6(v). Power amplifier.

Requirements. The power amplifier should be powerful enough to produce the required $B_1$ strength in the tuned transmitter coil so that spins may be turned coherently. This requires as short a 90° pulse as possible. The output impedance of the amplifier should match the impedance of the probe network for the efficient transfer of power to the coil. Distortion should be low, but there is some room for compromise as the load is a tuned circuit.

Equipment. A class AB broadband amplifier, described in a Mullard application note (79) was built. Its specifications are given below. To provide a flat frequency response with, at the same time, a reasonably constant input impedance, a frequency compensating circuit is built into the base supplies of the BLX15 pair. A temperature compensated base bias is provided. The circuit for this is shown in Figure 6.6. The main amplifier is shown in Figure 6.5.

The power transistors were required to be attached to a heat sink. An aluminium block was designed for this purpose. The cross-section was shaped like the letter 'H', with the transistors being bolted through the arms on one side. The other side was left free for the optional addition of a second, identical amplifier. Four water channels were machined through the block, each one close to the base of the arms of the 'H', with water looping outside the block and thus flowing through all four channels before being drained off. The operation of the sink was satisfactory.

Specifications.

Gain: 33dB max. into 50Ω, equivalent to a signal voltage increase of a factor of 45 (input and output impedances equal).
Frequency range: 1.6 to 30 MHz.
Maximum power output: 300 W into 50Ω.
Empirical output voltage using r.f. gates described: 80V p.e.p.
Empirical 90° pulse length using coil described: 15μs.
Figure 6.5. Mullard radio frequency power amplifier used by the Author.
Figure 6.6. The circuit for the temperature compensated base bias circuit is shown above. Below, the transformer windings and unmarked inductances are illustrated.

All components other than these listed here are explicitly described in the original reference.

T1: 3 turns, twisted pair 0.5mm en.cu., wound on FX2249 Ferroxcube (illus.)
T2: 25μ to 52; 4C6 Ferroxcube toroid, 23x14x7mm, Philips type 4322 020 91070
T3: 5 turns, twisted pair 1mm en.cu., wound on 4B1 Ferroxcube rod, 50x10mm, part of Philips type 3122 104 91250
T4: 12.5Ω to 50Ω; 4C6 Ferroxcube toroid, 36x23x15mm, Philips 4322 020 91090
L2: 25 turns, 0.35 en.cu., closewound on FX1898 Ferroxcube core
L7,8: 3B Ferroxcube 6 hole bead, Philips type 4312 020 31500
L9,10: As for L7,8.
6(vi). Probe.

Requirements. The probe network here includes the cables which transmit the excitation pulses and the induced signal.

The tuned coil and capacitor(s) which make up the NMR probe should be shielded from airbourne radiation. The unit should be impedance matched to the transmitter and receiver.

The probe coil (and here it is assumed that the same coil is used for both transmitting and receiving) should have a high Q factor in the tuned parallel resonance circuit, because this directly affects the signal to noise ratio for the experiment (100). The ideal design for a coil was investigated by Hoult and Lauterbur (101). A solenoidal coil should be slightly shorter than it is wide, and turns should be spaced by 3/2 times the wire radius. The Q should not be so high that the circuit rings for an unacceptably long time after the pulse, or the bandwidth that it can excite is over-limited.

\( B_1 \) should be homogeneous. A method for checking this was given by Farrah and Becker (102).

The capacitors in the probe design should be capable of withstanding the peak pulse voltages without arcing across their dielectrics.

The whole of the probe assembly that is in the magnetic field should be non-paramagnetic.

Equipment. A probe coil was wound with a diameter of 10.5 mm, a length of 8mm and 7.5 turns of 20 swg copper wire. It was found to be impossible to obtain an impedance of \( Z = 50 \Omega \), using this coil in parallel resonance, as the inductance was too high for a physically realisable tuning capacitor to be used, as it would need to be very small. A variable capacitor was therefore placed in series with the coil and the pair tuned so as to be only slightly inductive. The circuit is drawn in Figure 6.7.

At the resonance frequency of 15MHz, the absolute values of inductance and capacitance of any component in the unit were in doubt. The Q
Figure 6.7. Probe network with $\lambda/4$ transmission lines.

Note that the variable capacitors should be rated at $\approx 2 \times 10^{-8}$ F for the coil described, but because of the physical size of the coil, the capacitor leads are long enough to have significant inductance at 15 MHz and the actual value of the capacitor can be much less than its rating, so that larger capacitors may be necessary. Flat trimmer capacitors were used, with suitable fixed capacitors in parallel to increase the value factor could therefore not be calculated from the usual equation, $Q = \omega L / R$. Furthermore, the value of the resistance of the coil at r.f. frequencies is very difficult to estimate (103). Most of the current flows in the surface of the conductor, and the cleanliness of the surface is therefore important. Furthermore, the surface resistance is still very small so that contact resistances at soldered joints in the parallel circuit are significant. An empirical estimate of $Q$ is therefore probably the most reliable. After the $B_1$ pulse, the resonance decays with a time constant given by:

$$t_d = \frac{2Q}{\omega_0} \quad 6.1$$

$t_d$ was about 1 $\mu$s for the coil described here, so that $Q$ is of the order of 50. The signal to noise ratio obtained with the largest water sample possible was about 100:1.
The 'T' network connecting the probe to the transmitter was based on a design by Low and Tarr (184). The quarter wavelength cables have the same characteristic impedance as the (nominal) impedances of the transmitter, receiver and probe. The exact lengths are not critical, and may be calculated from the fact that the velocity of propagation of signals in the transmission lines is approximately 2/3 rd's that of light (185). The cables are 'transparent' to signals over half-wavelength sections, thus removing the chance of reflections if, for example, the probe does not exactly match the transmitter impedance. The transmission efficiency is given by:

\[ \eta = (100 - 2l) \% \]

where \( l \) is the length of line in wavelengths. The efficiency is therefore 99% on the received signals.

The cables act as transformers over quarter wavelength sections, and a short circuit at one end will present an open circuit at the other. The presence of the crossed diode pairs can thus be explained. They have two functions:

a) During a \( B_\perp \) pulse, the diodes at \( B \) present a short circuit as they are conducting in either polarity. The quarter wavelength cable transforms this to an open circuit on the receiver side of the \( T \) branch, so that all the pulse power is directed to the probe and the sensitive preamplifier is protected. Similar diodes may perform this function within the preamplifier circuit.

b) After the pulse, the diodes require a small voltage (as do all semiconductors) to conduct forwards. The induced signal is too small to go back to the transmitter through the diode pair(s) at \( A \), and is directed to the receiver. Also, these diodes do not allow any leakage from the r.f. gates to pass, and for this latter function, several pairs may be used in series.

The forward impedance of the diodes at \( A \) may be significant, and to avoid reflections a large parallel resistance may be placed in front of them to retain the impedance of the line.
6(vii). Receiver.

Requirements. The signal should be detected by a very sensitive preamplifier which should have some front end protection against the transmission pulse. It should recover from the pulse in a time which is very much less than the ADC sampling interval if the f.i.d. is to be acquired. The input impedance should match that of the probe circuit. The whole receiver should be capable of operating at all the frequencies that the system may be required to produce, for the measurement of different nuclei or measurements over a range of field strengths.

There should be no phase shift as a function of frequency in the amplified signal caused by the receiver and/or filtering. There should be a negligible d.c. level on the audio output of the phase sensitive detector. Ideally, there should be a very effective filter for all frequencies outside the profile width.

The preamplifier should have a very low noise figure.

Equipment. Because of the stringent requirements above, a complete receiver was purchased from Polaron Ltd. at an early stage in the research. Its three main stages are described here and the circuit diagrams reproduced.

The preamplifier (Figure 6.8) has crossed diode protection at its input which shorts to ground any r.f. B$_1$ pulses; the low level NMR induced signal is not affected. Switch-selectable pairs of L and C provide a band-pass filter which is tuned by a small variable capacitor. 733 - series amplifiers provide linear gain, with emitter-follower outputs providing low impedance between stages. Each amplifier is positioned in a hole in an earthed sheet which reduces radiation between stages.

The main amplifier again uses 733 -series amplifiers. The 50Ω r.f. output is supplemented by a demodulated output provided by a CA3002 balanced differential amplifier. (Figure 6.9).

The phase sensitive detector (Figure 6.10) has 50Ω inputs for both the signal and the reference. The reference signal is
Figure 6.10. NMR wide band phase sensitive detector manufactured by Polaron Ltd.
the input to a 'long-tailed pair' of transistors whose common output is amplified by further stages. Diodes prevent the pair from producing opposing voltages at the common point. A third transistor acts as the 'tail resistor'. The gain of the pair is high if the 'tail resistor' is very large, that is when the current through the third transistor's collector is unaffected by the voltage across it. Because the current through the third transistor is a function of its base current (which in turn depends on the input voltage) it provides a high impedance in a pattern which matches the reference when the input signal is the same as the reference. Hence the output is high when the signal is in phase with, and of a similar frequency to, the reference.

The reference signal derived from the oscillator tuned circuit required more amplification, although it was not important that it be a perfect sinusoid. The amplifier which was used is shown in Figure 6.11. Similarly, the receiver output of 1 volt peak to peak was insufficient to fill the ±5v dynamic input range for the ADC so a second amplifier was built (Figure 6.11). This amplifier was not linear below 200 Hz and this meant that signals were moved slightly in frequency so as to avoid distortion.

The receiver output had a simple R-C filter to reduce noise. This could have been replaced to advantage by an active filter with a more sharply-defined band pass.

Radio-frequency and audio frequency signals were observed with a Gould OS3300B oscilloscope which combines sensitivity with fast time-bases. External z modulation was used in early experiments to provide an image display system with grey scale.

Specifications.

Preamplifier: Polaron RF-460-RPA-WB

Gain: 50dB
Noise figure: 3dB
Recovery time from overload: 3-8 μs, depending on frequency.
Input & output impedances: 50 Ω.
Bandwidth: 4 - 60 MHz selectable.
Main Amplifier: Polaron
RF-460-RA WB

Gain: 60dB.
Recovery from overload: 2μs.
Input and output impedances: 50Ω.
Max. linear output swing: 1V
Figure 6.11. Receiver circuits.

Above: M.F. amplifier to increase the p.s.d. output voltage swing so as to fill the ADC dynamic range.

6(viii). Analogue to digital converter (ADC).

Specifications. The ADC may have to sample the NMR signal at a sampling frequency that is much higher than the maximum resonance frequency after phase sensitive detection in order to prevent noise folding onto the projection profile (section 2.v). It should also have as many bits resolution as are necessary to fully digitise the signal to noise ratio which is expected in a single signal acquisition. The ADC should be self-timed and self-supporting, requiring only timing pulses from the pulse generator and read requests from the host computer.

Equipment. The ADC which was used is shown in Figure 6.13 complete with all other control circuitry. A Datel Systems Inc. SHM2 sample and hold unit, a Hybrid Systems Corp. 592-8 8 bit ADC, and an 8-bit 7475 latch were used. The circuit operation is as follows. (Note that inputs marked @ are tied up to +5V.)

The polarities of the various control pulses are shown below:

- Gate pulse input, 90° or 180° pulse.
- Master reset or stop pulse.
- Read request flag from ADC to computer.
- Read done acknowledgement from computer.

Figure 6.12. ADC control pulse polarities.

The period between sampled points is set on five thumb-wheel switches ('T.W.S.') which pre-set the start counts of the 74190 up/down counters. A reset pulse via the master reset NAND gate and a NOR gate provides a high-to-low pulse which loads the period into the counters. The reset also causes the 74S74 flip-flop above the counters to set Q low, Q high. The 74S27 triple input NOR gate connected to the gate pulse inputs has three low inputs, thus forcing the flip-flop 'D' input low, thus ensuring that Q stays
low. The two A inputs to the 74121 monostable are simultaneously held high by $\overline{Q}$, so that it is insensitive to its B input.

When a high-to-low input pulse is received on either of the gate pulse inputs, the triple input NOR produces a low output, so that the flip-flop D input goes high. The Q output now stays high and the $\overline{Q}$ low. Three things now happen:

a) The two A inputs to the 74121 monostable go low, thus making it receptive to its input.

b) The high Q biases the 2N3705 npn transistor, thus making it conduct, so that the monostable B input is dragged low.

c) The counter enable input of the first counter is made low, thus causing it to start counting.

Each counter, from left to right, counts under the 10MHz clock until it overflows. It then sends a maximum/minimum count on the MM output, which is a high level, to the 74S182 look-ahead carry generator, which starts the next counter to the right. The max/min pulse is also sent to the pair of quad-input NAND gates below.

When the last counter is full, two things happen:

a) Both quad-input NANDS output a low pulse, thus causing the 74S74 flip-flop below them to re-load the counters with their original inputs, thus resetting them;

b) The last counter sends a ripple clock (RC) output which is a negative pulse. This biases the 2N 5771 pnp transistor, thus making it conduct, which drags the monostable B input high. This causes a low pulse from $\overline{Q}$.

This pulse will strobe the ADC to accept the next input if its status is ready for this step. The ADC instructs the sample and hold to take a reading, and the latch is enabled to hold the binary conversion.

The latch enable pulse also clocks the flip-flop which controls the computer control lines. The read request flag is put high. When a read done acknowledgement is received, a low pulse is applied to the clear input, which immediately forces the request flag down.
Figure 6.13. Analogue to digital converter with control circuitry.
6(ix). Gradient drivers and digital to analogue converters.

Specifications. The computer sets up binary representations of the required gradient strengths and the DACs and drivers must accurately provide the required gradient current. The output current should therefore be linearly related to the input. The drivers must be able to drive low impedance coils with d.c. currents and in both polarities. The output should be balanced so that no current is produced when the input is zero. The DAC resolution should be great enough for a set of discrete gradient strengths to reproduce the required gradient vector angle accurately.

Equipment. An 8-bit ZN425E DAC was used to provide an analogue voltage signal (all positive voltages). A 741 operational amplifier acts as a buffer amplifier which removes the inherent offset voltage (with the 10k potentiometer) and sets the gain (with the 2.2k potentiometer). Two more 741's increase the voltage to required levels without inversion.

The NE540 is an audio line driver which is required to provide base currents for the pair of power transistors which form a complementary symmetry emitter pair. Its input is offset so as to cover a range of voltages, positive and negative.

Positive voltages cause the npn transistor to conduct whilst turning off the pnp transistor, which then acts as a high impedance emitter load. Negative voltages have the same effect, with the roles of the transistors reversed. Hence both positive and negative currents are available for the gradient coils. It is of course important that the two transistors have matched gains.

The circuit is shown in Figure 6.14.

A different driver was used for the transient gradient $\frac{\partial B_z}{\partial y}$. Due to the development of a much larger imaging system within the NMR group at the University of Surrey, two 2-channel professional power amplifiers were obtained which were capable of driving high currents into larger gradient coil systems. They are ideally suited to the task of driving d.c. currents for this application. The specifications are given here for these drivers.
Figure 6.14. Gradient DAC and current driver.
Power amplifier/gradient current
H&H S500-D

Maximum output power: 500W per channel into 2.5Ω.
900W into 5Ω, bridged mono.

Frequency response: ± 0.2dB, d.c. to 20kHz
Stability: Unconditionally stable, any impedance.

Noise: 100dB below 180W into 8Ω, above 10Hz
Sensitivity: 0.75V input gives 300W into 4Ω.

Protection: Full protection against open and closed circuit loads.
6(x). Image display system.

Requirements. The image display system should be linked with the main computer in such a way that images, once reconstructed, can be rapidly displayed. It should ideally be able to store a large number of pictures independently, and be able to process them (i.e. window, interpolate etc.) without help from the computer. Data reception from the main computer should not interrupt the display. The method of display should be able to show the full range of values of pixel intensity, and should produce a picture without visual distortion.

Equipment. Initially, images were displayed using a Gould OS3300B oscilloscope and a South West Technical Products M6800, later updated to M6809. Images were received from the computer via an RS232 serial interface operating at a high baud rate, the pixel intensities being encoded into 16 ASCII characters and sent serially, image row by image row. These were displayed using programmed I/O. Each image point was sent to the display labelled with its vertical position and its intensity. These two parameters were sent to two DACs (as described in section 6(ix)) whose analogue voltage outputs controlled the oscilloscope's y and z (intensity) controls. The time base of the oscilloscope provided the scan across each image line as each row was transmitted. The whole picture was transmitted and continuously refreshed rapidly enough for serious flicker to be avoided.

Each image line on the display was broadened vertically by combining the scanning spot with a high frequency signal whose amplitude was chosen so that lines just touched those above and below. Examples of the display, which proved to be very clear and undistorted, are Plates 3(c), 4(a), 5 (a) & (b), 7(a) & (b), 11(a) & (b) and 12 (a) & (b). They can be identified by the graticule lines.

One advantage of this display is that, by expanding the y gain and selectively displaying a portion of the input signal (possible with the OS3300B) any portion of the image can be blown up to fill the whole screen.

The disadvantages of the system are the small screen size, and
the commissioning of an otherwise useful and valuable oscilloscope. Also, the M6809 is tied up in the display refresh routine and must turn the image off to receive an update.

The system was improved by the acquisition of a Matrox RGB-256 imaging system, which is integrated onto a single PC board. Its specifications are listed below. The board can hold and display a single picture without refreshment, thus freeing the M6809 for further reception or other work. A pair of 8" floppy disc drives (DMAF2) were added to give the system the ability to store images. Images were displayed on a Hitachi Denshi VM-910 video monitor which provided high contrast monochrome pictures and which could display large areas of white without distortion.

**Specifications.**

**Image display card.**

Matrox RGB-256

<table>
<thead>
<tr>
<th>Resolution: 256x256 dots picture possible</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grey scale: 16 shades for the above array</td>
</tr>
<tr>
<td>DAC: Built-in high speed 8 bit DAC</td>
</tr>
<tr>
<td>and 8 bit color encoder</td>
</tr>
<tr>
<td>TV standard: NTSC or PAL</td>
</tr>
<tr>
<td>Interface: Intel SBC</td>
</tr>
</tbody>
</table>
6Cxi. Gradient coil system.

Requirements. The gradient coils should produce magnetic field gradients that are linear throughout the sensitive volume of the receiver coil. Gradients should be available in three orthogonal directions; two in the desired image plane and one to facilitate plane definition. Gradient coils and leads to the coils should be shielded if necessary from airborne radiation, so that stable gradients can be maintained. Total resistances should match the requirements of the gradient current drivers.

Equipment. The set of gradient coils which were built were based on a design by Parker et al (106). Each gradient is produced by an array of four current carrying wires placed next to the magnet pole faces. Expansion of the Biot-Savart law for the current flowing in each wire, plus that which flows in each of the images of the wires in the pole faces, yields the field produced by each current. Parker shows how the field strengths may be represented by a power series, in which the most significant, first order terms may be made dominant by the appropriate choice of wire separation as a function of pole gap. The required spacings and current directions are shown in figure 6.15.

The expansions are:

\[
B_z = \begin{cases} 
\frac{\mu_0 I}{2g} C_{2v+1} \Re\left(\frac{\gamma}{2g}\right)^{2v+1}, & \text{for } \partial B_y / \partial y \\
\frac{\mu_0 I}{2g} K_{2v+1} \Re\left(\frac{i \gamma}{2g}\right)^{2v+1}, & \text{for } \partial B_z / \partial z 
\end{cases} \tag{6.3}
\]

where \( \mu_0 \) is the permeability of free space,
\( I \) is the current,
\( g \) is half the pole gap separation
\( C, K \) are coefficients
\( \gamma = g + iy \)
\( i = \sqrt{-1} \)
\( y \) is the displacement in the \( \hat{y} \) direction.

For displacements where \( |\gamma| < \frac{2\pi g}{g} \), only the first and third terms are significant. The coefficients for these are:
Figure 6.15. Gradient coil design. Above: spacings and current directions for linear orthogonal gradients (Parker et al. [106]). $\partial B_0/\partial x$ is produced using the design for $\partial B_0/\partial y$, rotated through 90°. Below, a sketch of the probe design, showing the gradient coils with return paths omitted. Tuned circuit is shown by dotted lines.

(2g = 22 mm)

Wire is 20 s.w.g. Cu
\[ C_1 = \frac{-1}{\cosh^2 \alpha} \quad C_3 = \frac{-1}{\cosh^2 \alpha} \left( \frac{2}{3} - \frac{1}{\cosh^2 \alpha} \right) \]

\[ K_1 = -\frac{\tanh \alpha}{\cosh \alpha} \quad K_3 = -\frac{\tanh \alpha}{\cosh \alpha} \left( \frac{1}{6} - \frac{1}{\cosh^2 \alpha} \right) \]

where \( \alpha = \frac{\pi a}{2g} \)

Thus:

\[ C_3 = 0 \quad K_3 = 0 \quad \alpha = \cosh^{-1} \left( \frac{3}{2} \right)^{\frac{1}{2}} \]

\[ \therefore \quad \frac{\partial B_z}{\partial y} \quad \alpha/2 = 0.418 \]

\[ \therefore \quad \frac{\partial B_z}{\partial z} \quad \alpha/2 = 0.983 \]

Therefore, for \( \frac{\partial B_z}{\partial y} \), \( \alpha/2 = 0.418 \) and for \( \frac{\partial B_z}{\partial z} \), \( \alpha/2 = 0.983 \)

The \( \frac{\partial B_z}{\partial y} \) gradient has the same conditions as \( \frac{\partial B_z}{\partial y} \).

It is formed by a similar wire array, rotated through 90° against the pole faces.

The gradient magnitudes, when these requirements are met, are:

\[ \frac{\partial B_z}{\partial y} = \frac{\mu_0 \pi I}{(2g)^2} \times \frac{2}{3} \]

\[ \frac{\partial B_z}{\partial z} = \frac{\mu_0 \pi I}{(2g)^2} \times \frac{5}{6} \]

Given the magnet linewidth, or \( T_2 \) linespread (whichever is greater), the required gradient strength in terms of required frequency profile bandwidth can be estimated. Given that a field gradient of 1G across the sample will yield a spread of 4.2 kHz (this is calculated from the gyromagnetic ratio) the required gradient current can be estimated and the gradient wire size chosen accordingly. The wire should be as thin as possible so that, where the \( y \) coils cross the \( x \) and \( z \) coils (see Figure 6.15) the wires are not forced an unnecessary amount away from contact with the pole faces. Laquered copper wire gives an insulated line with a minimum diameter.
The return paths which are necessary for the coils in which all wires run in the arrays in the same direction must be far enough away from the sample coil as to make no contribution to $B_z$. Gradients $\partial B_z/\partial y$, $\partial B_z/\partial x$ require return paths. All terms contributing to these in all powers contain the scaling factor $1/\cosh^2 \alpha$, $\alpha = \pi a / 2g$. For $\alpha = 4$, $\cosh^2 \alpha \approx 700$. This occurs when $a \approx 3g$. Return paths should run at least $a = 3g$ away from the receiver coil therefore. Typical windings are shown in Figure 6.16.

Figure 6.16. Complete gradient windings for $\partial B_z/\partial \omega$. Return paths are shown drawn lightly, and are spaced apart by at least 3 times the pole gap spacing.
The gradient coils as described, wound from copper wire, will have a resistance of about 0.1Ω. Load resistors were therefore required. It was found to be desirable to screen these resistors and all leads as induced a.c. signals, especially on the \( G^y \) gradient coils whose field links the receiver coil, can induce signals in the coil.

**Specifications.**

(Values given are for the design used by the Author, using the magnet and probe described in this chapter.)

Gradient strengths:
- \( \frac{\partial B_z}{\partial x_y} : 0.055 \text{ G mm}^{-1} \text{ per amp} \)
- \( \frac{\partial B_z}{\partial z} : 0.030 \text{ G mm}^{-1} \text{ per amp} \)

Profile bandwidth: From section 5(i), 4,225 kHz.

Required gradients: 1G across 8.5 mm (inside diameter of glass receiver coil former)

Required currents:
- \( \frac{\partial B_z}{\partial x_y} : 2.1 \text{ A} \)
- \( \frac{\partial B_z}{\partial z} : 3.9 \text{ A} \)

Switching time: Hysteresis will oppose the change of image current magnitudes and hence will cause a finite switching time for gradient strength changes. Parker suggests that this will be no more than 5 ms for a complete removal of a large gradient.

Linearity: Provided that displacements \(|\mathbf{L}| < \sqrt{3} \cdot 2' \times 9^\circ\), the gradients should be linear. In the sample space 8 mm in diameter and 8 mm long, the maximum \( \gamma \) is \( \sqrt{3} \), and the condition is met. However, the coil was known to be sensitive over a total length of 12 mm, so that in the very extreme positions there may have been some non-linearity. There was not sufficient signal strength above noise to check if this was the case with the end-plane images taken with the multi-planar technique.
The procedures listed here outline the computer algorithms which were used to control and analyse the imaging experiments. No programs are listed, except for a simplified version of the filtered back-projection reconstruction, listed in Appendix 2. The algorithms are presented in three groups:

a) An imaging experiment without plane definition;
b) A detailed description of the signal digitisation;
c) Additional stages required for multiple plane imaging.

a). An imaging experiment using spin-echo signals.

At every projection angle:

Calculate the $\frac{\partial B_z}{\partial x}$ and $\frac{\partial B_y}{\partial z}$ gradient components of the gradient vector. (Note 1)

Write the components to the gradient digital to analogue converters so as to set the gradient vector.

Run the signal acquisition subroutine. (Note 2)

Correct any d.c. shift on the digitised spin echo. (Note 3)

Compute the complex fast Fourier transform (Note 4).

Demodulate the projection profile (Note 5).

Use 'centre of mass' correction on profile (Note 6).

Store the profile for subsequent reconstruction.

After all signals have been collected at all angles:

Reconstruct image (Section 3.iii and Appendix 2)

Window maximum and minimum image values if required (Note 7)

Store image on disk and/or transmit to image display.

Note 1. The magnitudes of the orthogonal magnetic field gradients at an angle should be $A \cos \Theta$, $A \sin \Theta$. The 16-bit parallel interface of the MP200 general purpose interface was split into upper and lower words, to supply 2 8-bit DACs.

Note 2. The signal acquisition subroutine was as follows:
Do this loop \( n \) times if \( n \) signals are to be averaged for each projection angle:

- Disable all CPU interrupts so that the CPU does not miss any calls from the ADC.
- Look for the 90° gate pulse which indicates that the ADC is about to start sampling.

Do this loop \( m \) times where \( m \) is the number of discrete points to be collected:

- Look for the ADC read request flag.
- Read the ADC level.
- Send back a read done flag.
- Remove the leading points on the signal which follow the 180° pulse.
- Return the digitised signal to the main program.

**Note 3.** The d.c. shift was calculated from the average offset from the baseline of the last 20 points measured. Every point on the digitised signal was then corrected.

**Note 4.** Early images were taken using a 256 point FFT routine in 16 bit precision which was obtained from E.O. Brigham (37). This was written in FORTRAN and was slow (30 seconds per transform). A much faster routine, transforming 1024 points in about 2 seconds, was written in assembly language and based on a routine by Cooper (107).

**Note 5.** The spin echo appears in the centre of the acquired points and is therefore 'time shifted' signal (section 2(viii)). Its transform is easily demodulated by inverting every other point.

**Note 6.** The 'centre of mass' correction (section 5(iii)) is applied to all profiles to counteract long term field drift.

**Note 7.** Image windowing levels were set as percentages of the maximum image values. Pixel values below the minimum threshold were zeroed, including any negative values. Those above the maximum level were made equal to the maximum. Points with levels between the maximum and minimum were then rescaled. Windowing often improves the clarity of otherwise noisy images.
A modified version of the imaging algorithm was used prior to any experiment to set up gradient magnitudes, field strength, phase, pulse spacing and pulse widths, etc. Signals were acquired and Fourier transformed but not reconstructed. Instead, the digitised signals and projection profiles were alternately displayed on a 'storage' oscilloscope whilst either gradient could be selected to be fully on or off.

b) Acquisition of the spin echo for digital computation.

The actions of the computer during the signal acquisition have already been outlined. In order to communicate, the ADC and the computer had one control line each, on which they could set a logic level 'flag'. The ADC could tell the computer to read its digital output on its 8-bit parallel interface, whilst the computer could tell the ADC that it had been read. The ADC also had its analogue input, a control line from the pulse programmer by which it knew that a pulse sequence had begun and another control line reading the $180^\circ$ gating pulse. The computer was also able to detect the $90^\circ$ gating pulse.

The computer, ADC and pulse programmer all ran with reference to their own internal clocks, so that synchronisation was essential. The computer had to read the ADC, beginning at the correct number of points before the echo so that the echo was central on the acquired signal. Once the ADC had started to sample, it ran under its own very precise timing in order to sample the signal at regular intervals. The computer had to process the data within each sampling period, and finally tell the ADC to stop sampling after the correct number of points had been taken; it did this by ceasing to request data.

The ADC was reset every time a new pulse sequence started. It then began to sample data by taking its first point after the $180^\circ$ pulse. The computer read each point after detecting the 'read request' flag, and sent a 'read done' pulse after each point to reset the flip-flop which holds the 'read request' flag. Clearly, if the computer began a program by looking for a read request, it would find one at any time between the $180^\circ$ pulse and the next reset. It
was therefore desirable that the computer should always look for its first read request immediately after the 90° pulse and so this was always looked for first.

Since the computer sampled m points starting at \( t = \tau \) where \( \tau \) was the pulse spacing, the sampling rate had to be such that m times the sampling rate = 2\( \tau \). In practise this was done by monitoring the pulse separation and finely adjusting it whilst observing the digitised signal.

c) Additional stages required for Polytomic imaging.

At the beginning of data collection at every projection angle the computer set up the static gradient vector and then waited to monitor the 90° pulse. Immediately after the pulse, the computer set the magnitude of \( \theta \) to the first of the set of discrete values (usually zero) by placing the appropriate number on the parallel interface controlling the third gradient DAC. The computer then waited for the first read request flag from the ADC which comes immediately after the 180° pulse. \( \theta \) was set to zero again before the first point was read.

To collect m planes, the computer used m transient gradient values (signal averaging signals with each value if required) at each projection angle.

Limited RAM space in the computer required that the data was placed on disk after each projection angle. For processing at each angle after the complete scan, the following algorithm was used:

Read up array of discrete transform coefficients from disk (section 4(viii)).

For each projection angle:

Read up one point at a time from each of the m signals. Then:

For each of the m planes, eg. the ith:

Perform m-point cosine transform for the ith plane;
Perform m-point sine transform for the ith plane;
Store these m complex time signal points.

Repeat for all other points to obtain m complete complex time signals.
Conventional fast Fourier transformation of the complex time signals associated with the \textit{i}th plane for all projection angles yields a set of profiles which will reconstruct the data on the \textit{i}th plane.
Requirements. In order that high signal to noise ratio signals may be processed in integer form, a computer should have 16 or 32 bit precision or should have true double precision operation if it is an 8-bit machine. Similarly, round-off errors in the fast Fourier transform will be very significant if 8-bit operations are used (10%).

Image arrays can occupy large amounts of RAM, especially if double precision means that 4 or more bytes are used to represent each pixel. A memory large enough to take a 128x128 image array plus programs and operating system should be used. A Winchester technology hard disk unit may be useful for the rapid saving and reading of data, especially with multiplane imaging.

There should be the equivalent of three parallel interfaces for controlling gradient DACs. Data must be read from the ADC so either at least one parallel interface must be bi-directional or another must be used. Serial output to an image display system is necessary.

The clock frequencies of the CPU and peripheral devices in the laboratory should be different from the NMR resonance frequency. The CPU clock, and its instruction set, should be able to perform the repetitive calculations very rapidly.

Equipment. The computer used was a Data General MP200 mini-computer which superceded an earlier Data General MicroNova, used in the earliest work presented here, because of its superior execution times.

Specifications.

Word length: 16 bit, capable of double precision arithmetic.
RAM storage: 32k words.
Interfaces: 3 parallel I/O general purpose interfaces.
                 Serial RS232 compatible interface.
                 VDU interface.
Hard storage: Twin 8" floppy disks, single sided single density holding 250 kbytes each.
Operating System: Data General DOS with assembler and FORTRAN IV compiler.

Typical instruction times:
MUL, 4.92 μs; ADD, 0.84 μs; DIV, 6.60 μs.

VDU: Lear Siegler Inc. ADM3A was the only one of several tried whose clock did not interfere with 15MHz resonance. Also equipped with RG-512 graphics card.
6(xiv). Magnet.

Requirements.
For the spin-echo technique, the minimum receiver bandwidth, and therefore minimum noise, is determined by the need for a number of linewidths to be detected side by side on the frequency spectrum. There is little point in improving the homogeneity of $B_0$ much beyond the point at which transverse relaxation times are longer than those caused by the $T_2$ processes in the samples. The shortest $T_2$ in tissue is likely to be about 20ms. The homogeneity should be better than this throughout the coil volume.

The $B_0$ field should be free of short and long term fluctuations that are significant compared to the inhomogeneity.

The pole gap or sample volume should be large enough to take both sample with sample coil and the surrounding gradient coils, which may not produce linear gradients unless they are spaced some way away from the sample volume.

Equipment. A Varian V3601-I magnet was used for all the imaging work presented in this thesis. This had 12" diameter pole faces and a 22mm pole gap. A field of 3600 G gave a proton resonance of 15MHz. Homogeneity was about 2 parts in $10^6$, which means a decay constant of about 10 ms; this means that the transverse decay was dominated by the magnet rather than the sample $T_2$ processes.

Current was supplied by a V2608 regulated power supply, and the field was shimmed using electrical shims. Short term field drift was corrected by a Varian flux stabiliser. Long-term field drift was significant, because of malfunction of the water cooling regulator which meant that the coils were permanently being cooled and not regulated at the design temperature of about 30°C.

The cooling problems, and the transplantation of the magnet from one laboratory to another with possible mechanical stress changes which were not shimmed out, meant that the magnet performance was below specification. It was also operated for this work at well below its optimum field strength.
APPENDIX 1


a) EMI CT5005 Series 2.

Scan time 20 sec. Image array 160x160 pixels. Slice thickness 13 mm.

The EMI number of air is -500, and that for brain matter is between 14 and 19 \(^1\) \(^{10}\). The uncertainty of the number for an area 3x3 mm on the image is estimated at ±1.8 units \(\text{(10)}\). The 'signal to noise' of the brain image pixel is thus about 250:1 above the background of the air measurement. (Note that this is not a measure of contrast between brain tissues.)

b) NMR scanner.

The relationship between signal to noise on the NMR image and the achievable resolution was investigated by Libove and Singer. For a single pixel being detected, they derived the following expression for the length of the side of this volume (assumed cubic) which they termed the 'spatial resolution' \(d\):

\[
d = \frac{7 \cdot \Psi \left(\frac{\Psi}{\omega_0^{1/2}}\right)^{1/2} \left(\frac{Q_{\text{empty}}}{Q_{\text{full}}}\right)^{1/4} \exp\left(\frac{\omega}{3 \delta}\right)}{\gamma_{\text{sys}}}
\]

where \(\Psi\) is the signal:noise ratio required on the image for the distance \(d\) to be resolved;
\(\omega_0\) is the resonant frequency;
\(\gamma_{\text{sys}}\) is the system integration time, \(\gamma_{\text{sys}} = (1/2 \times \text{receiver bandwidth})\)

This equals the signal acquisition time when only one pixel is detected.

\(\left(\frac{Q_{\text{empty}}}{Q_{\text{full}}}\right)\) is the ratio of the coil \(Q\) for the empty coil to the \(Q\) for the coil with the conducting sample inside. (\(Q\) decreases due to conductive losses in the induced currents in the sample.)

\(\exp\left(\frac{\omega}{3 \delta}\right)\) shows the reduction in signal due to r.f. attenuation.
\(x\) is the furthest distance that the signal must traverse,
in the worst case the radius of the head. $\delta$ is the skin depth of the r.f.

Libove and Singer plotted $d$ against $\tau_{\gamma}$. 

A decrease in imaging time is obtained by measuring many pixels simultaneously, but the bandwidth must be increased and is no longer matched to the acquisition time. The equation is altered by a decrease in $\tau_{\gamma}$. For instance, if an image is to contain 128x128 pixels, then the bandwidth must be increased (at least) 128 times; this means that $d$ is increased by a factor of $\left(\frac{128}{1/6}\right)$, or 2.2. For a fixed signal:noise ratio, this means that the resolution is poorer.

The bandwidth per pixel on the Oxford Instruments 1m bore superconducting imaging magnet, operating at 4MHz with a homogeneity of 5 parts in $10^6$ is 20 Hz. This corresponds to a complete decay time of about 50 ms. The signal must be acquired for this time for the resolution on the profile to be 20Hz. In addition, the resolvable $d$ given for a single pixel must be increased by 2.2.

The graphs show that a voxel of side 4.2 mm could be imaged with the simultaneous collection of 128 profile points, with a signal:noise ratio of 10:1 at 4MHz. A voxel of side length 8.8mm may be imaged with a ratio of 100:1. The volume of EMI CT voxel is somewhat larger than the 10:1 voxel.

NOTES.

a) The signal for the NMR scanner has been defined as the induced voltage from a voxel containing 80% water. This water density has been the only NMR parameter considered.

b) The contrast between different tissues depends can be a function of proton density, $T_1$ and $T_2$, depending on the method used. The appearance of detail on images is a function of the difference in signal intensities referenced to the background noise level. Contrast will occur between different tissues on an NMR image that will not occur on the CT image, and vice-versa. However, the
NMR method has the capacity to change the signal dependence on the three parameters and therefore adjust the available contrast between adjacent tissues.

c) An NMR scanner cannot achieve the minimum performance time of 20s to make it comparable to the CT scanner if it utilises a projection reconstruction method, because the signal acquisition time of 50 mS must be followed by a period of about $T_1$ seconds for spin-lattice signal relaxation. The total scan time does, however, vary between techniques and at least one method (Multiplanar Echo Imaging, \textit{(61)}) could take a complete image in one shot. Forced methods of relaxation also exist \textit{(112)}.

These calculations have been based on a hypothetical NMR machine. Comparisons between head scans taken by NMR and X-ray CT are now available from clinical trials \textit{(65,51,52,53)}, and more images are now being produced all the time.
APPENDIX 2. Back projection algorithm with convolution filter.

(TORTRAN)

THERE ARE M PROJECTION ANGLES
THE FOLLOWING STEPS ARE DONE FOR EACH PROJECTION ANGLE.
THE PROFILE P HAS BEEN READ FROM THE PROJECTION STORE.
THE PROJECTION ANGLE IS $\phi$, AND IS BETWEEN $0$ AND $\pi$.
THE PROJECTION $P$ AND THE FILTERED PROJECTION $P^{\star}$ ARE 65 POINTS LONG.
THE IMAGE IS $65 \times 65$ PIXELS.

\[ SINE = \sin(\phi) \]
\[ COSINE = \cos(\phi) \]

C CALCULATE THE FILTERED PROFILE:
\[
\begin{align*}
DO& \ 30\ I=1,65 \\
Q& = P(I) \times 2.467201 \\
JO& = 1 + \text{MOD}(I,2) \\
DO& \ 20\ J = JO,65,2 \\
20\ Q& = Q - P(J)/(I-J)^2 \\
30\ P^{\star}(I) = Q/M
\end{align*}
\]

C BACK PROJECT ONTO THE IMAGE ARRAY $F$
\[
\begin{align*}
DO& \ 50\ J=1,65 \\
IMIN& = J*65 - 32 - \\
& \text{INT(SQRT(1024.0 - (33-J)^2))} \\
IMAX& = (2*J - 1) - IMIN + 1 \\
X& = 33 + (33 - J)*SINE + \\
& (IMIN - J*65 + 31)*COSINE \\
DO& \ 50\ I = IMIN, IMAX \\
X& = X + COSINE \\
IX& = X + 0.5 \\
50\ F(I) = F(I) + P^{\star}(IX)
\end{align*}
\]

If algorithm noise is to be reduced, then the position $X$ on the profile $P^{\star}$ should be found by interpolation and the line 50 should be:

\[
50\ F(I) = F(I) + P^{\star}(IX) + ((X-IX)*(P^{\star}(IX+1)-P^{\star}(IX)))
\]

INFORMATION

For each profile point.
\[ 2.467201 = \frac{\pi}{4} \]
Step every other $J$ to avoid $I=J$.
Weight $P(J)$ and subtract. This is just scaling.

For each image line, calculate first and last points on $J$th line of circular image array.
Find 1st profile point.

For each point on image line, step along profile and add it to the image.
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Specification by Oxford Instruments for their 1m bore superconducting imaging magnet over 280 mm transverse disc is \( \frac{1}{10^5} \) part in 10^5.

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MULTIPLANAR NMR IMAGING

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For clinical applications, the inherent low sensitivity of NMR imaging makes it desirable to measure simultaneously as many individual locations as possible (Brunner and Ernst 1979). The technique of projection (PR) has therefore been adopted by many workers in preference to point or line measurements in which much of the sample is ignored, and to other planar techniques due to its technical simplicity. However, limitations of computer storage and of uniform spatial sensitivity along the receiver coil often require that the information be restricted to a slice or set of slices through the object. In this compromise, the PR technique requires a means of removing the response from locations outside the 'plane' (slice of finite thickness) of interest. The possible techniques fall into two broad categories: selective excitation methods, and oscillating field gradient techniques. In this abstract, we describe an oscillating field gradient technique that allows both variable plane thickness and shape, and, more importantly, the simultaneous measurement of a set of planes.

The technique is a simple yet powerful variant of the phase encoding concept utilised by Edelstein et al. (1980) and Kumar et al. (1975) in their imaging methods. However, unlike the previous applications of this concept, it does not require the rapid switching of large orthogonal magnet field gradients nor does it restrict data collection to a single slice. The basic technique proposed utilises the 90°-x-180° echo pulse sequence for data collection coupled with controlled phase shifts within the sequence produced by weak time-varying magnetic field gradients. To obtain plane definition, spins in the direction perpendicular to the slice of interest are phase encoded by a time variant gradient. The resulting spatial response function can be qualitatively understood as follows. Following the 90° pulse, a single spin experiencing a field perturbation will accumulate a phase angle. At the expected time of the echo (2x) the angle will be zero unless the perturbation was different between 0 < t < x and x < t < 2x. For an oscillating field, (3B/3t) the phase angle t = 2x will depend on the history of the perturbation. Along a line of spins in the direction of the time-varying gradient, (3B/3y), say, the accumulated phases at t = 2x vary linearly with distance. The amplitude of the signal read with a phase-sensitive detector from consecutive points along the line will oscillate sinusoidally along the line. This spatial frequency may be anything from zero (when all the spins accumulate no net phase) up to a maximum which will depend on the gradient strength.

The signal averaging of responses modulated by different spatial frequencies can be thought of as the superposition of these frequencies, resulting in a modified response whose spatial form depends on the frequency distribution. The simple averaging of successive signals in fact discards useful information. If, under computer control, known spatial frequencies are imposed on the sample, then by means of a discrete Fourier transform multiple image planes may be reconstructed from the same information collected for the single plane technique.

The complex magnetisation at time 2x in the spin-echo sequence, including a time dependent magnetic field perturbation, B_t is given by

\[ M(2x) = M_0 \exp(-2x/T_2) \exp \left[ \int_0^{2x} \omega dt - i \int_0^{2x} \omega dt \right] \]

where

\[ \omega = y B_x. \]

The magnetisation at any t time in the spin-echo sequence, for static gradient G_0 applied at projection angle ø and a transient gradient G_y applied for a time t is therefore

\[ M^0(G_y,t) = \int dy M^0(r,y) \exp(-iG_y \cdot \hat{r}) \exp(iyt) \]

Taking a discrete Fourier transform over G_y:

\[ \text{Re} \left[ S_x(t,y',y) \right] = \frac{1}{2} \int dy M^0(r,y') \exp \left[ i G_y \cdot \hat{r} \right] \]

\[ \text{Im} \left[ S_x(t,y',y) \right] = \frac{1}{2} \int dy \sin(G_y \cdot \hat{r}) \exp \left[ i G_y \cdot \hat{r} \right] \]

followed by a discrete transform over \( t \):

\[ S_y(r',y') = \frac{1}{2} \int dy M^0(r,y') \]

\[ = \frac{1}{2} \int dy M^0(r,y') \]

\[ S_y(r',y') = \frac{1}{2} \int dy M^0(r,y') \]

\[ \text{terms in } r' \text{ are not present with off-resonance detection).} \]

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WORLD CONGRESS ON MEDICAL PHYSICS AND BIOMEDICAL ENGINEERING 1982, HAMBURG
An efficient method of plane definition for NMR imaging by the method of reconstruction from projections

The Editor,

Sir,

For clinical applications, the inherent low sensitivity of NMR imaging makes it desirable to measure simultaneously as many individual locations as possible (Brunner and Ernst 1979). The method of projection reconstruction (PR) has therefore been adopted by many workers in preference to point or line measurements in which much of the sample is ignored, and to other planar techniques due to its technical simplicity. However, limitations of computer storage and of uniform spatial sensitivity along the receiver coil often require that the information be restricted to a slice or set of slices through the object. In this compromise, the PR technique requires a means of removing the response from locations outside the 'plane' (slice of finite thickness) of interest. The possible techniques fall into two broad categories: selective excitation methods, and oscillating field gradient techniques. In this letter, we describe an oscillating field gradient technique that allows both variable plane thickness and shape, and the simultaneous measurement of a set of planes.

The technique is a simple yet powerful variant of the phase encoding concept utilised by Edelstein et al (1980) and Kumar et al (1975) in their imaging methods. However, unlike the second of these techniques, it does not require the rapid switching of large orthogonal magnet field gradients nor does it restrict data collection to a single slice as in the first. The technique proposed utilises the 90°-τ-180° echo pulse sequence for data collection coupled with controlled phase shifts within the sequence produced by weak time-varying magnetic field gradients. The spin-echo data collection sequence, we believe, gives superior tissue discrimination per unit time over single 90° pulses, comparable discrimination with the steady state free precession technique (SSFP) on whole-body magnetic systems, and may surpass SSFP on high-homogeneity magnets such as our 17 cm bore superconducting magnet system (Taylor and Bore 1981).

To obtain plane definition, spins in the direction perpendicular to the slice of interest are phase encoded by a time variant gradient. The resulting spatial response function can be qualitatively understood as follows. Following the 90° pulse, a single spin experiencing a field perturbation will accumulate a phase angle. At the expected time of the echo (2τ) the angle will be zero unless the perturbation was different between 0 < t < τ and τ < t < 2τ. For an oscillating field, (dB₀/dt), the phase angle at t = 2τ will depend on the history of the perturbation. Along a line of spins in the direction of the time-varying gradient, (dB₀/dy), say, the accumulated phases at t = 2τ vary linearly with distance. The amplitude of the signal read with a phase-sensitive detector from consecutive points along the line will oscillate sinusoidally along the line. This spatial frequency may be anything from zero (when all the spins accumulate no net phase) up to a maximum which will depend on the gradient strength.
The signal averaging of responses modulated by different spatial frequencies can be thought of as the superposition of these frequencies, resulting in a modified response whose spatial form depends on the frequency distribution. Just as the shape of the FID is the Fourier transform of the distribution of spin frequencies, so the spatial response is the Fourier transform of the envelope of spatial frequencies. For example, a band of equally spaced frequencies has a rectangular envelope and thus the signal averaged response will be a $\sin(\pi x)/\pi x$ function. The central peak of this function, occurring at the null position, indicates the response of the plane.

When discrete frequencies are used to describe the envelope shape, the continuous Fourier transform has a periodicity in space. Aliases of the spatial function are produced with a spacing equal to the inverse of the discrete frequency separation. For a coil length $L$ cm, the first alias must be at least $L$ cm from the null plane. Hence the envelope must contain frequencies spaced not more than $1/L$ cm$^{-1}$ apart. As an example, if the central peak is required to be $L/8$ cm across, the maximum spatial frequency is known to be $8/L$ cm$^{-1}$, and so the envelope between 0 and $8/L$ cm$^{-1}$ must contain at least eight discrete frequencies, spaced $1/L$ cm$^{-1}$ apart.

Any plane shape is possible in theory, provided that the required distribution of frequencies can be engineered. Free running square or sine wave oscillators, asynchronous with the spectrometer, and with periods between $2\pi$ and $4\pi$, can be used to create plane shapes of typically $\sin(\pi x)/\pi x$ or $J_0(x)$. In this application, it is the difference in phase between the oscillator and the pulse sequence at the 90° pulse which determines the spatial frequency. With the gradient driven under computer control, the frequencies may be chosen by switching a gradient of constant magnitude at a known time during the pulse sequence, or by changing the magnitude at a fixed time during the sequence. By this means the plane may be made more sharply defined than the shape of the sine or $J_0$ peaks. Further field gradient switching and hence induced currents are reduced substantially from the free running case.

The spatial response shape is very easily measured. If a static gradient is applied along the sample length with a superimposed, smaller time-dependent gradient in the same direction, the response of such point along the sample length is described by a unique frequency. The variation in phase is seen as a modulation of the profile thus obtained in the y-direction. The plane shape is simply the profile obtained from a number of averaged signals (figure 1).

**Figure 1.** (a) The longitudinal profile of a tube of water within the RF coil. The asymmetry arises due to an inhomogeneity in the $B_1$ field along the coil. (b) The modification of the longitudinal profile resulting from the averaging of eight random spatial frequencies arising from the time-varying component of the longitudinal magnetic field gradient. ($f_0 = 15$ MHz, coil length = 8 mm).
Figure 2. (a) A cross-sectional image of a hexagonal arrangement of glass capillary tubes (1.5 mm id) within a tube of water. (b) As in (a) with a longitudinal slice selected by means of a time-varying component on one of the two orthogonal magnetic field gradient in the image plane. ($f_0 = 15$ MHz, information bandwidth = 2.0 kHz, 64 x 64 pixels interpolated to 128 x 128).

The simple averaging of successive signals in fact discards useful information. If, under computer control, known spatial frequencies are imposed on the sample, then by means of a discrete Fourier transform multiple image planes may be reconstructed from the same information collected for the single plane technique. This obviously substantially reduces the imaging time per plane. Furthermore if spatial frequencies are imposed in more than one direction then data collection may be restricted to a volume of variable shape and size.

Acknowledgment

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19 May 1981

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Reviews of books


Most physicists who work in the field of Nuclear Medicine are not only competent regarding the technical aspects of the apparatus but also like to acquire an intelligent appreciation of the physiological behaviour of the radiopharmaceuticals used and the distribution of these materials in normal and diseased states. They are often concerned with the assessment of results and particularly in the mathematical analysis of data. Although this book is mainly intended to guide physicians in the medical interpretation of radionuclide images it will also be very useful to technical staff wishing to gain knowledge about the appearance of clinical images in healthy and ill patients. Most physicists and technicians in nuclear medicine will, therefore, find this book interesting and informative.

True to its title, the book is an atlas consisting of over 2800 illustrations in eleven chapters which cover all the main aspects of radionuclide imaging (though adrenal and pancreatic images are omitted). It presents the combined experience of five nuclear medicine specialists from different centres in the USA.

Each chapter deals with a particular body organ and commences with a concise description of the physiological behaviour of the radiopharmaceuticals used and brief general notes on the interpretation of resulting images. The large number of images which then follow have explanatory text which amplifies the initial information and interprets the findings in normal cases and in various pathological conditions. Each chapter concludes with an extensive bibliography and there is a useful 11-page index at the back of the book.

The images are chiefly from gamma cameras but some rectilinear scans are reproduced and for gallium imaging the rectilinear scanner is recommended. Some images are positives (white on a black background) whereas others are presented as negatives (black on white) but both are adequately clear. Sometimes the descriptive text follows a few pages after a series of sequential images and this entails continual page-turning—a minor annoyance in what is generally a very clearly written clinical interpretation of the images.

The examples given represent a wide coverage of routine imaging procedures though inevitably there are some gaps. The use of gallium in the differentiation of loose and infected orthopaedic prostheses is not included, nor is the aerosol technique for lung ventilation imaging. On the other hand the salivary glands occupy a short chapter and there is a good chapter on radionuclide cisternography. It becomes apparent that doses of radiopharmaceuticals given to patients in the USA are often much higher than the DHSS would authorise. This may be because the book emphasises the usefulness of blood flow studies at the time of injections in liver, brain, kidney and lung perfusion examinations. There is also a separate chapter on other blood flow studies.

It is perhaps a pity that the authors have confined themselves to images, with reference to computer-derived graphical and numerical results only in the renal chapter. There is no mention of Left Ventricular Volume curves in the chapter on nuclear cardiology, nor to the usual parameters derived from these curves, neither is there any form of functional imaging displayed (such as phase imaging or regional ejection fraction imaging). Perhaps it is too much to expect this in a book which is primarily concerned with the interpretation of routine images and in which it succeeds in no small measure.

This book will be a great asset to any physician who is seeking to increase his skill in the reporting of radionuclide images. It will also be of considerable interest to physicists and technicians who are closely associated with this work, and it sets a high standard to aim for in nuclear medicine imaging. It has an initial educational value and will also serve as a departmental reference book covering most of the current imaging techniques in routine use.

J L Birks


This is a revised and up-to-date version of a booklet first published in 1973. It is intended for a wide readership and provides brief information on the nature, origins and effects of ionising radiation. The various