THE STRESSES IN AND THE STABILITY OF THE
FLANGES OF ROLLED STEEL BEAMS UNDER
VARIOUS CONDITIONS OF LOADING

BY

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References in the text are made by naming the author and giving the date of the publication referred to.
$G$ = modulus of rigidity taken as $58000 \text{ksi}$ for mild steel.
$I_{xx}$ = moment of inertia of section about minor axis.
$I_{yy}$ = moment of inertia of section about major axis.
$J$ = effective polar moment of inertia of section.
$M$ = applied bending moment.
$P$ = force in flange of beam $= f \times b \times t$.
$R$ = resilience.
$T_0$ = torque applied to ends of beam.
$W$ = work done.
$W_k$ = a point load.

$$a = \frac{\pi^2 I_{yy}}{4 L^2}$$

$b$ = breadth of the flange.
$b_1$ = overhang of flange.
$c$ = of unit length in the center of a span.
$f$ = intensity of stress.
$f_0$ = critical stress.
$h$ = overall height of beam section.
$h_1$ = distance between centroids of flanges.
$k$ = ratio of lateral deflection of top & bottom flanges.
$L$ = length of compression flange of beam
$m$ = thickness of flange at the root.
$n$ = thickness of flange at the toe.
$q$ = span of deflectometer.
$r$ = radius of gyration of beam section.
$s$ = gross length of a curved flange.
$t$ = average thickness of a flange.
$\gamma_0$ = lateral deflection of flange.
$z$ = deflection of beam in direction of applied moments.
$\mu$ = Poisson's ratio.
$\phi$ = torsional deflection of beam.
It is now generally recognised that the presence of eccentricity of loading or non-uniformity of material in a member subject to direct loads will cause secondary stresses which may be equal to if not of greater intensity than the stress due to the direct load. These stresses may bring about the failure of the member at a lower load than that calculated from the direct stress alone. In a beam, where tensile, compressive and shear stresses occur across the same section, the secondary stresses due to the same causes are less easily computed but nevertheless exist except under ideal conditions.

In practice the ideal case is seldom, if ever, met with, and some of the possible causes of secondary stresses are enumerated below:

1. Lack of uniformity in the material of a beam.
2. Unsymmetrical rolling of the section.
3. The presence of initial stress in the beam due to variable rate of cooling throughout the section, rolling below the critical temperature and straightening.
4. Twists or bends in a beam.

The purpose of this investigation is to determine by direct measurement the distribution of stresses in the flanges of rolled steel I beams due to given conditions of loading and to estimate how any irregularities will affect the ultimate strength of the beam.

The stresses and deflections in the web are not considered except insofar as they constrain the flanges.
general method of design until fifty years ago had been 
by rules of thumb based on trials. Sections had been design-
ed and tested by Tredgold, Watt and Fairbairn whose ideal 
had been to obtain "the section of greatest strength".

Hodgkinson, the pioneer of beam testing, found, by a method 
of trial and error, one which was equally strong in tension 
and compression flanges, and he claimed this to be the most 
economical shape.

In 1854 Fairbairn wrote a book in which he gave an 
account of a large number of tests carried out by various 
engineers on cast iron beams and also on wrought iron beams 
of I section which had been rolled first in France by 
Ferdinand Zores during 1847. The tests on the wrought iron 
beams showed that the compression flange was weaker in a 
long beam than the tension flange and in order to increase 
its strength he proposed to make it thicker. He also 
suggested that both flanges should be made wider than was 
then the custom to increase the lateral stiffness.

Bending Theory.

The modern theory of bending was built up by Navier 
and St. Venant from theories, some of which were put forward 
as early as 1688 by Galileo in his 17 propositions and later 
by Harriot, Bernoulli and Euler, but in Fairbairn's time no 
practical use had been made of it. However, Barlow and 
other mathematicians brought it to the fore and engineers 
gradually came to accept it as a basis of beam design.

In using any theory to solve a problem, it is essential 
to bear in mind the assumptions on which the theory is based. 
The following are those made in the theory of simple bending.
normal to the fibres after bending.
(2) That the material is homogeneous, isotropic and
obeys Hooke's law and the limits of elasticity are
not exceeded.
(3) That every layer of material is free to expand or
contract longitudinally and laterally under stress,
as if separate from other layers.
(4) That the modulus of direct elasticity has the same
value for compressive as for tensile strains.

In Fig. 2, PQRS represents a small length of beam and
ABCD and CDEF represent two adjacent fibres. If a pair of
equal and opposite couples be applied to the beam in the
plane of the diagram, it will be bent in a circular arc as
in Fig. 3. The line CD is in both diagrams common to the
two fibres in its entirety and therefore no sliding of the
fibres can have taken place otherwise one fibre would over-
hang the other at the ends and CD would then not be entirely
common to both.

Now consider the case where shear is present, i.e.
where the beam is not bent in a circular arc (Fig. 4). The
radii PO' and QO' are not equal as assumed in the
deduction of the bending formula and the fibres are made
to slide one along the other by the horizontal shear force
which varies from zero at \( PQ \) to a maximum at \( NA \) so that
the lines PS and QR are distorted into curves which are,
in the terms of assumption (1) neither "plane" nor "normal
to the fibres".

If we compare these two conditions of bending with the
assumptions, we find that when (1), which is sometimes called
Bernoulli's assumption, is true, (3) cannot occur while
by Love (1920) and Vivian (1927) for rectangular sections but shear effects in I beams are exaggerated owing to the thiness of the web and H.S. Prichard (1912) has demonstrated that shear distortion of an originally plane section causes a small variation of the fibre stress in the flanges of a beam.

As previously mentioned, Barlow was keenly interested in the bending theory and in 1905 he enunciated what is known as the "beam paradox" in which he pointed out that the ultimate strength of a rectangular cast iron beam is considerably greater than the value calculated from the Bernoulli-Eulerian theory. This has given rise to a great deal of discussion (E.C. Segundo 1883, A.S. Spenser 1923) and E.V. Clarke (1902) pointed out that the discrepancy is probably due to the fact that the stress strain ratio of cast iron is not constant for all stresses as assumed in the theory.

**Bending beyond the Elastic Limit.**

When the stresses pass beyond the elastic limit of the material, the conditions for a steel beam will become similar to those for cast iron. Assuming that the strain of a fibre is still proportional to its distance from the neutral axis, then the distribution of stress will be as Fig. 6A which is really a stress strain diagram for mild steel drawn along the X X axis of the beam. Diagram 7 represents the total force over the section of the beam and is obtained by multiplying the Y Y ordinates of Fig. 5 by those of Fig. 6A and plotting along the X X axis. The moment of resistance is equal to the area of the figure.
half the area of the beam multiplied by the distance between
the insides of the flanges multiplied by the stress at yield
point
i.e. M.R. at yield point = \( f_y \frac{A(d-2t)}{2} \)

This may be assumed to be the ultimate moment of
resistance of the beam since it may be considered to have
failed when the yield load has been reached.

If a beam is bent so that the stress in the flanges
never exceeds the elastic limit, the effective modulus of
elasticity corresponds to the slope of the line O P (Fig.9)
but if the stress is increased above the elastic limit to
Q, the effective Young's modulus as measured by deflections
and strains corresponds to O Q and will continue to do so
until the previous maximum stress is exceeded or until the
beam has been rested for a time. This means that if a beam
is overstrained, the value of E as measured from bending
tests, decreases slightly.

The Buckling of I Beam Flanges:
As an alternative to failure by direct flexure I beams
may fail by buckling of the compression flange and this may
happen either:

(1) By local buckling or wrinkling
or (2) By lateral buckling of the whole beam.
(1) Local flange buckling takes the form of a wrinkling of
the flange and is due to an excessive ratio of overhang to
thickness. T. Box (1903) conducted tests on columns and
beams with outstanding flanges and deduced the formula

\[ f_{wv} = 52 \sqrt{\frac{t}{E_i}} \tan \alpha \]

for the wrinkling stress of wrought iron I beams.
which he deduced from first principles by an approximate mathematical method and he later confirmed it by a series of tests on built up steel plate and angle sections.

Limiting values of overhang have been given by G. Boscarem (See "Columns" by E.H. Salmon), as \( \frac{b}{T} = 7.5 \) for wrought iron while A.F. Thurston (1919) gives that the wrinkling stress for \( \frac{b}{T} = 50 \) is 7.8 T/\( D \) and for \( \frac{b}{T} = 7 \) is 17.5 T/\( D \). These values were for duralium and were found by experiment. The Column Research Committee (1929) reported that a ratio of \( \frac{b}{T} = 20 \) is sufficient to allow the column to develop its full strength.

When an I beam is bent, the cross section is always distorted, and becomes wider at the compression flange and narrower at the tension flange, and at the same time the tips of the flanges bend over slightly as shown in Fig.9. This effect was not allowed for by Roark, but it evidently has some influence on the wrinkling strength of an outstanding flange.

The question of wrinkling in column flanges is discussed by Salmon (1920) and a good illustration of it is given in a report on column tests published in the Engineering News Record of 1929.

(3). Lateral buckling of beams was mentioned by Fairbairn (1870) but no useful information on the matter had been collected until 1883 when Christie carried out tests on beams and columns. He found that when the ratio of \( \frac{L}{b} \) for an I beam exceeded a given value the resistance of the compression flange decreased as a function of that ratio.
on the stability of wood laths of varying proportions. He deduced an empirical formula but admitted in summing up his article that not enough was then known about the subject to design on truly theoretical lines.

In 1902 Lilly discussed the question and pointed out that the formulae which were in use at that time took no account of the type of loading and he suggested that an effective column length of \(0.7L\) might be used in the straight line or Rankine-Gordon formula when a beam was subjected to a uniformly distributed load.

The matter is not viewed logically by engineers even at the present date and all formulae used are empirical. H.D. Hess (1909) suggested that a formula based on the Rankine-Gordon formula should be used, such as

\[
f_s = \frac{18000}{1 + \frac{L^2}{3000b^2}}
\]

H.F. Moore (1910 & 1913) after carrying out tests which will be described later, suggested the formula

\[
f_c = f_e - q \frac{r}{r}. \sigma
\]

where
- \(f_e\) = approximately the yield point of the material
- \(q\) = a column constant which is in this case slightly smaller than for ordinary struts to allow for the restraint of the web.
- \(r\) = radius of gyration of the section about the axis of the web.
- \(\sigma\) = the loading co-efficient, the values of which are given in the table below for certain types of loading.
The formula for mild steel beams is \( f_c = 40000 - 60 \frac{E}{r} |b| \).  

In 1918 R. Fleming gave a comprehensive list of the types of formulae used in America and suggests  

\[ f_s = 12000 - 250 \frac{f_s}{L} \text{ lb/in}^2 \]

as a typical formula for the safe stress. The Institution of Structural Engineers recommend \( 1 - 15 \frac{f_s}{L} \text{ T/fm}^2 \) with a maximum value of \( f_s = 8 \text{ T/fm}^2 \) and a limiting value of \( \frac{f_s}{L} = 50 \). It can be seen that none of the foregoing modern formulae consider the effect of the depth of the beam or thickness of its parts and few allow for influence of the type of loading. They assume that all beams of the same flange width will have the same stability irrespective of their stiffness about the other axes. This assumption is obviously incorrect and in order to obtain a truer conception of the subject it is necessary to consider the action of a buckling beam. H.F. Moore has noticed that there is a slight twist in addition to the lateral displacement, and this tendency also occurs in columns tested at Watertown Arsenal. An illustration in the report on column tests published in Engineering News Record 1919 shows a very good example of this kind of failure.

**Torsional Resistance of I Beams**

There is at present no established method of estimating the torsional resistance of any but simple sections and to obtain the value of \( J \) for an I beam, empirical formulae are generally used.
but in (1917) A.A. Griffith and G.I. Taylor made a report on the use of soap films for determining the torsional properties of sections, and in it suggested a method of calculating them by dividing up the section into parts and applying equations of the form

$$ J = \frac{1}{3} \int t^3 \, ds $$

(See also Prescott, 1924).

A formula which is more applicable to large I sections than Gibson and Richie's was proposed by Young and Hughes (1924)

$$ J = \frac{2}{3} b t^3 + \frac{1}{3} h t_i^3 $$

($t_i =$ thickness of web).

Other formulae are:

W.B. Campbell (1923)  \[ J = 0.4 h t_i^3 + 0.1 (m+n)^3 (b-t) \] and

S. Timoshenko (1924) \[ J = \frac{A^4}{4.0(I_{xx}+I_{yy})} \]

The Elastic Stability of I Beams.

Although the effect of torsion is not allowed for in the working formulae at present used in practice, yet the problem has been attacked from a purely theoretical point of view by Mitchell (1899) and later by Prescott (1918, 1920 and 1924) when they considered the action of deep beams under vertical loads. The general case is that of a beam which is subjected to two opposite and equal terminal couples in such a way that they are restrained at the ends just sufficiently to prevent any lateral deflection. The stiffness of the beam in the plane of the bending couples is assumed to be very much greater than that in the plane normal to them so that all deflections in that
deflection is $z$, the lateral deflection $y$, and the torsional deflection $\phi$ when the beam is assumed to be in the act of buckling under the action of the applied moment $M$ (See Figs. 10 and 12). The resisting moment of the beam to lateral deflection at this instant may be represented by $EI_{yy} \frac{d^2y}{dx^2}$ at that section, while the resulting bending moment in that plane due to $M$ the applied moment and to the end restraint equals $M \sin \phi + T_o \frac{dz}{dx}$.

If $\phi$ is small $\sin \phi = \phi$ and if $z$ is small $\frac{dz}{dx}$ is negligible. Now the resisting moment equals the resulting bending moment

\[ : EI_{yy} \frac{d^2y}{dx^2} = M \phi \]  \hspace{1cm} (i)

The torsional resistance of the beam is $GJ \frac{d\phi}{dx}$ and the torque is $-(M \frac{dy}{dx} + T_o)$.

Then

\[ GJ \frac{d\phi}{dx} = -M \frac{dy}{dx} - T_o \]  \hspace{1cm} (ii)

differentiating

\[ GJ \frac{d^2\phi}{dx^2} = -M \frac{d^2y}{dx^2} \]

Eliminating $y$ in equation (i)

\[ -EI_{yy} GJ \frac{d^2\phi}{dx^2} = M \phi \]

\[ : \frac{d^2\phi}{dx^2} + \frac{M^2 \phi}{EI_{yy} GJ} = 0 \]

This is of the form

\[ \frac{d^2\phi}{dx^2} + \beta \phi = 0 \]
Applying the limits of \( x = 0 \) and \( x = L \)

\[
M = \frac{P}{L} \sqrt{GJ E I_{yy}}
\]

This is the critical bending moment for which the beam is stable against lateral buckling.

For a simply supported beam with a central point load \( W \) the critical value of \( W \) is given by

\[
W L^2 = 16.94 \sqrt{GJ E I_{yy}}
\]

and for a uniformly distributed load \( w \)

\[
w L^3 = 28.31 \sqrt{GJ E I_{yy}}
\]

The above two cases are for loads applied at the neutral axis of the beam but when they are applied above or below it, the conditions will be modified and the quantity \( q \) must be added to (ii) for uniformly distributed loads and the result contains a second term of the form

\[
K \frac{q}{L} E I_{yy}
\]

Timoshenko (1924) gives a method for use with I section beams in which he uses the general equation

\[
Q_c = K \sqrt{GJ E I_{yy}}
\]

The critical stress is found by substituting the solution to the above equation with a suitable value of \( K \) for the type of loading and substituting in an expression such as

\[
f_c = \frac{Q_c \times L}{\frac{E}{Z}}
\]

(for a uniformly distributed load)

from which

\[
f_c = K \frac{I_{yy} \sqrt{\beta}}{16}
\]

\[
\alpha = \frac{GJ}{E I_{yy}} \left( \frac{L}{h} \right)^2 \quad \text{and} \quad \beta = \frac{I_{yy}}{I_{xx}} \left( \frac{h}{L} \right)^2
\]
He also worked out the critical stress in cases where the load is applied at the top or the bottom flanges of the beam and has tabulated values of $f_0$ for certain values of $\alpha$ and $\beta$

In 1925 Case evolved a method for use with I section beams by considering the extra torque caused by the forces in the flanges. He represented it by the term

$$\frac{h^2}{2} \cdot E \cdot I_3 \cdot \frac{d^3 \phi}{d \lambda^3}$$

and added it to the left hand side of equation (ii). Thus for a beam bent by terminal couples

$$E \cdot I_{YY} \cdot \frac{d^3 \phi}{d \lambda^2} = M \cdot \phi - - - - - (i)$$

and

$$J \cdot G \cdot \frac{d \phi}{d \lambda} - \frac{1}{2} E \cdot I_3 \cdot h^2 \cdot \frac{d^3 \phi}{d \lambda^2} = -M \frac{d \phi}{d \lambda} - T_0 - - (ii)$$

differentiating and substituting

$$\frac{d^4 \phi}{d \lambda^4} - \frac{2J \cdot G}{E \cdot I_3 \cdot h^2} \cdot \frac{d^3 \phi}{d \lambda^3} - \frac{2M^2 \cdot \phi}{E^2 \cdot I_3 \cdot I_{YY} \cdot h^2} = 0$$

$\phi = A \cdot \sin \cdot \frac{\pi}{L} \cdot \lambda \cdot \frac{h^2}{E \cdot I_{YY} \cdot G \cdot J}$ provided that

$$M^2 = \frac{h^2}{L^2} \left( E \cdot I_{YY} \cdot G \cdot J \right) + \frac{n^2}{L^4} \left( E \cdot I_{YY} \right) \left( E \cdot I_3 \right) \frac{h^2}{2}$$

is indeterminate but equating it to unity

$$M = \frac{L}{L} \sqrt{E \cdot I_{YY} \cdot G \cdot J} \cdot \sqrt{\frac{1 + \pi^2 \cdot E \cdot I_3 \cdot \frac{h^2}{L^2}}{2 \cdot G \cdot J \cdot L^2}}$$

where $I_3$ is the moment of inertia of one flange.

He then works out cases for a uniformly distributed load applied at the neutral axis and at the top and bottom flanges and obtains the equation for the critical loading

$$w \cdot L^3 = K \cdot \sqrt{J \cdot G \cdot E \cdot I_{YY}}$$

the value of $K$ being given in a table computed from the above equations.

During 1929 a number of articles appeared in the Proceedings of the American Society of Civil Engineers on
the point of view of minimum energy and for a cantilever with a point load applied at a distance $q$ from the neutral axis they found that

$$W_q \left( \frac{l}{2EI_{xx}} + \frac{1}{2EI_{yy}} \right) \left( \frac{L^3}{6} \frac{L^3}{\pi} \right) + W_q - \frac{EI_{yy}n^2\pi^2}{252L^3} \frac{G\sqrt{\pi}}{16L} = 0$$

They then plotted values of $W$ against $L$ for beams of different shapes and deduced from the curves the empirical formula

$$f_S = \frac{f_{max}}{1 + \left( \frac{L}{C_{VY}} \right)^2 X \left( \frac{r_{xx}}{r_{yy}} \right)^2 \frac{C}{K}}$$

Where $C$, $C^{i}$ and $K$ are constants depending on the shape factor of the section. For similar sections, i.e. where the ratio of $r_{xx}$ and $r_{yy}$ is constant this formula may be reduced to

$$f_S = \frac{f_{max}}{1 + \left( \frac{L}{b} \right)^2 X \frac{1}{K}}$$

which is similar to the Rankine Gordon form for struts.
be given of previous tests made during recent years, together
with the conclusions which have been drawn from them.

In 1904 Burr and Elmore gave an account of tests they had
made on wrought iron and mild steel beams. They paid special
attention to the manner of loading and the conditions of
lateral restraint and after due consideration they reported
that:

(1) The maximum allowable stress in the compression
flange decreased as the ratio $f_n$ increased after a given
value had been exceeded.

(2) The modulus of elasticity of the material of the
beams varied from $14 \times 10^6$ to $27 \times 10^6$ lbs/\(\square\) as computed
from deflections (partly because they neglected to take
account of deflection due to shear).

J. Christie carried out tests (with 3', 6', 10' & 12'
deep I-section beams) in 1899 on the effect of length of
span on the lateral strength of beams. His conclusions with
regard to buckling were similar to those of Burr and Elmore
and he devised an empirical formula of the type now
generally used. Being in doubt as to homogeneous nature of
the steel in the beams, he cut test pieces from the flanges
and web and carried out tensile and compression tests. He
found that the elastic modulus of the mild steel was not
the same in tension as compression and these again were
different from the modulus calculated from the bending tests,
but he had neglected the deflection due to shear which
might have accounted for part of the discrepancy.

The tests which have been most noticed by engineers
are perhaps the Marburg (1909) tests. He used in all
simply supported at the ends with a minimum of lateral restraint and the loading was varied in such a way that different kinds of failures were obtained.

Take for example the tests on the 15" deep beams over a span of 15 feet. The Bethlehem shape which had a ratio of $\frac{L}{b}$ of 17.1, when loaded at the centre by a point load failed by flange wrinkling at a computed stress of 22.5 tons/" but similar beams when loaded at quarter points failed by lateral buckling at a stress of 18.5 tons/". The standard beams which had a ratio of $\frac{L}{b}$ of 32.7 when tested over the same span failed in both cases of loading by lateral buckling at calculated stresses of 17.23 T/" for central loading and 15.55 T/" for loads at quarter points.

He had strain gauges attached to the flanges of the beams and took readings from which he computed the stresses at different loads. He found that the values so obtained agreed fairly closely with the stresses calculated by the usual method from the loading but the strain readings were found not to be in some cases constant across the breadth of the flange. He makes especial mention of this fact in his paper but does not attempt to give an explanation of the reasons for its occurrence.

On completing the bending tests, he cut specimens from the flanges and webs of all the beams and tested them in tension and compression. He reported that the value of the elastic modulus for the material in one particular beam was constant for direct stress but varied considerably for different beams. He also obtained a discrepancy between the measured value of $E$ and that computed from
and yield point varied considerably throughout the section of any one beam and was lower at the root than in the web and flanges.

During 1910 and 1913 M.F. Moore conducted a series of tests on 40 I beams with several conditions of loading and side restraint. He lays special stress on the twisting action of beams under test and includes in his results a summary of tests by Burr and Elmore, Tetmager, Lanza, Christie and Marburg. His conclusions were

(1) The yield point should be regarded as the ultimate fibre stress of structural steel in flexure.

(2) The slight inelastic action which may be observed in steel I beams at stresses as low as those used in practice is in general local and does not affect the load carrying capacity of the I beam if the load is not reversed in direction.

(3) The yield point of structural steel in compression is about the same as the yield point in tension.

(4) The resistance of a beam against buckling depends on the stiffness of the beam and on the amount of torsional fixity of the bearings. When the length of the beam passes a certain limit, its stability is a function of the elastic modulus rather than the yield point.

(5) The value of $E$ as calculated from deflections is 10% less than the value obtained by tensile or compressive tests on the material of the beam.

Gibson and Richie (1914) carried out tests on 6 small I beams, bending them with webs vertical and horizontal. Their deductions were that the calculated values of $E I$
same for each axis. The observed value of $E$ when the web was vertical was $30.7 \times 10^6$ lbs./\(\text{in}^2\) and with the web horizontal $26.4 \times 10^6$ lbs./\(\text{in}^2\). In the former case the web provided 14.5% and in the latter case 8.4% of the moment of inertia, and they suggested that the discrepancy in the value of $E$ was due to the fact that the elastic modulus of the material in the web was greater than that in the flanges due to heavier working in the rolling.

The supposition that the elastic modulus varies throughout the section is not supported by recent reports on tests of the material of rolled sections. Hancock (1910 & 1912) carried out tests on steel cut from I beams and came to the conclusion that neither the tensile strength nor the percentage elongation are affected much by chemical composition or treatment during and after rolling but the yield point and elastic limit are affected. The effect of the rolls on the web tends to increase its elastic strength.

Moore and Wilson (1914) made tests on the material of six 12" deep and two 24" deep I beams and found that the material at the root had a lower elastic limit than that in the flanges and web.

In 1923 W.E. Dalby states that mild steel has practically the same $E$ in tension as in compression. The results of 20 tensile tests on specimens cut from I beams are reported by the structural Steel Research Committee (1923). These tests show that the material at the root had a considerably higher yield point than material from flanges and web and that the material in the toe had a higher yield point than that in the web.
results as follows:

(1) Young's modulus is approximately the same in tension as compression for any given beam but varies in different beams.

(2) The ultimate strength is the same in any given beam but different for material of different beams.

(3) The proportional limit varies considerably throughout the section.

(4) The yield point varies considerably throughout the section but is the same in tension as compression for specimens cut from similar parts of the section.

Dr. Hadfield in a report on steel columns (1929) states that the maximum variation of $E$ throughout the material of any one beam was only 3.9% in the series of tests carried out in connection with that research.

It may, therefore, be assumed quite reasonably that the elastic modulus is constant for the material distributed throughout the section of any one beam even though the yield point and elastic limit may vary considerably. It is, therefore, unlikely that any irregularities of stress which may occur in the flanges of a beam are due to the elastic properties of the mild steel.

The procedure generally adopted in the tests noted above has been to bend several beams of the same section under the same conditions of loading and lateral restraint and then to average out the results so obtained. This method has resulted in the general verdict that an I section beam behaves in accordance with the bending theory provided that the yield point is not exceeded and that the laterally unsupported length of flange is not excessive. Very little
or to variation in the material. It was, therefore, decided to carry out a number of tests on one I-section beam and discover any irregularities in stress, if any, which might occur when it is loaded under conditions of slight lateral restraint. As it was intended to deal with the flange stresses only, it was considered advisable to use four point loading so that, the middle portion of the beam being free from vertical shear stresses, a state of pure bending was obtained.
The machine used for the bending tests on the beams is a 10 ton Buckton single lever machine, the bending table of which is 6 feet long and the head room under the top movable ram is 3'-0". Fig. 18 shows how the beam under test was set up on the machine and it will be seen from the diagram that a second beam had to be used in order to apply the load at two points on the beam equidistant from the supports; as stated before, this arrangement had to be used in order to apply a constant bending moment to the middle portion of the beam where all the readings of stress and deflection were taken.

The end bearings used were half round roller bearings which may be oiled, thus giving a practically constant span for all inclinations of the ends of the beam and a minimum of end restraint. The load was applied to the beam by means of two knife edges which were carefully placed equidistant about the ram of the machine.

The instruments used were chiefly a deflectometer and a strain gauge. The deflectometer is shown in position for vertical deflections in Fig. 18 and consists of a wooden beam of 1 inch square section and 2'-6" long carrying an Ames dial mounted on a small bracket at the centre of the beam and two knife edges 2'-0" apart and equidistant about the Ames dial. One of the knife edges is adjustable by means of a screw and nut so that on setting up the instrument the plunger of the dial may be brought into contact with the beam. However, when the plunger was depressed, it was found that a small deflection of the wooden beam took place and in order to allow for this in the tests, the deflectometer was set up on two knife edges at 2'-0" centres and
any deflection could be detected. The plunger of the dial belonging to the deflectometer was then depressed and readings of both dials were taken simultaneously. It was found that for a deflection of 50-thousandths of an inch as recorded by the deflectometer dial, the beam deflected .001 inches and that for other readings the deflection was pro rata within the limits of accuracy of the dial. The deflectometer was fastened to the beam by means of four springs and four wire stirrups. The latter passed two over the joist and two under the deflectometer and the springs were hooked on to them and held the knife edges hard up against the beam.

The strain gauge is shown in Fig.19 and consists of a frame having two gauge points 3/16" apart, one being fixed to the frame and the other carried on a lever which is pivoted at its lower end. The top of the lever is fitted with a screw which bears on the plunger of an Amsa dial which reads in thousandths of an inch. The multiplication of the lever is 5 to 1 so that the instrument reads to 1/5000 inch but considerably smaller deflections may be estimated since the graduations are fairly coarse (about 1/3 of an inch). The frame is drilled with 3/8" diameter holes through which special clamps may be fixed, enabling the instrument to be clamped to the beam in any position.

In order that the points should not move when the gauge was clamped to the beam, special holes were drilled in the particular parts of the beam where strain measurements were to be taken. After a few trials, it was found best to drill the holes with a 54 Stubbs gauge drill to the depth of about one eighth of an inch and then the edges of the holes were slightly countersunk with an ordinary drill.
determined from tests with an extensometer of known characteristics. Details of the calibration will be given later on but the constant of the strain gauge was found to be \( \frac{1}{12330} \) and this figure is used throughout the tests in order to calculate the value of unital strain from the gauge readings.

Before putting the beam on to the machine, it was first of all set up on a marking off table and centre lines were scribed along the flanges and the web. The mid point of the beam was found and marked and all other marks were set out symmetrically about it. The gauge holes were set out as shown on Fig. 18, there being three sets, and each set consisting of eight holes. One set was in the middle of the beam and the holes bb and cc were on the centre line of the flanges and \( 2\frac{3}{4} \) inches apart. Holes aa, cc, dd and ff were on lines \( 1\frac{3}{4} \)" either side the centre lines and each pair on one flange was \( 2\frac{3}{4} \) inches apart. This allowed the gauge to be set up across the flange in order that transverse stress readings might be taken. Two holes were drilled on the centre line of the web in order to measure strains, if any, on the neutral axis. The other set of holes were identical with the set at the middle of the beam and were marked out \( 9\frac{3}{4} \) inches on either side of the centre line. The \( 9\frac{3}{4} \)" was the maximum distance possible from the centre that the instrument could be set up without fouling the heads of the knife edges. On the other hand the deflectometer was made \( 2'-0" \) gauge in order that no deflections due to local stresses under the loads should be included if any should occur, dispersion angles of \( 45^\circ \) being considered sufficient in allowing for this.

When the beam had been marked off each set of holes
directly under the ram and the flange edges and upper beam were placed in position. The webs of the two beams were then lined up vertically by suspending plumb lines at the ends of the beams and adjusting the positions until the lines coincided with the centre lines of the webs. This precaution ensured concentric loading of the lower beam and avoided as far as possible the application of twisting moments which might cause unstable results.

In the first set of tests the beam was subjected to bending on both axes. At first loads below the elastic limit were applied in both cases and this will be called "test I". Then loads above the elastic limit but below the yield point were applied and this is called "test II". Finally the beam was stressed up to the yield point so that a small permanent set remained when the load was removed and this was "test III".

TEST I.

Owing to the fact that the bending table of the machine is only 6'-0" long and that the headroom is 2'-0" under the ram, only small sections could be used if sufficient space to fix the strain gauge above and below the beam was to be allowed. Consequently, a 3" or 4" deep beam seemed the best size while a breadth of 3" over the span of 5'-6" was desirable if the yield point was to be reached without buckling. In addition to this consideration, the fact that the strain gauge was 2½" made it desirable to use this width if transverse measurements were to be used. It was therefore decided to test first of all a 3" x 3" x 8.5 I beam which was kindly supplied by Messrs. Dorman Long & Co.
of 1000 lbs. was applied and then instruments were fixed in position. When their readings had been recorded the load was increased to 6000 lbs. and more readings were taken. In order to obtain an average reading for the maximum load of 6000 lbs. a slight increase was made and then another reading was taken as the load was brought back again to 6000 lbs. this time being a decreasing load. Then the strain was taken off and a reading was taken at 1000 lbs. again.

This procedure was carried out several times for each setting of the strain gauge which was set up in all the gauge holes in turn. The beam was then set up with the web horizontal and loads of 500 lbs and 2000 lbs. were applied and sets of readings of strain and deflections were recorded. When complete sets of readings had been obtained for three settings of the beam in each position of the web, averages were taken of each set of three tests.

In order to evaluate the stresses it was necessary to know the Young’s Modulus for the material. This was calculated from the deflection measurements in the following manner. The portion of beam between the knife edges was bent into a circular arc of radius \[ R = \frac{EI}{M} \]

If \( q \) (Fig.17) is the length of the deflectometer, then the deflection \[ \Delta = \frac{q^2}{5R} \]

\[ E = \frac{Mq^2}{8I \Delta \text{deflection}} \]

The value for \( I \), used in calculating the stress from the loading and the value of \( E \), was taken from a section
The stress readings for a, a, c, c, d, d, and f, f, were not exactly proportional to the load and there was a noticeable variation between the measured stresses at these points for loading and those for unloading. Another small variation occurred between the stresses in the top and bottom flanges. The average stress in the top flange was slightly less than that in the bottom flange; this, however, in this case was a very small difference and since no stress could be measured along the centre line of the web, no importance may be attached to it. The value of Young's Modulus as calculated from deflections was 31 x 10^6 lbs/\( \text{in} \) which seems to be rather high since the stresses at b, b, and e, e, calculated from the measured strains and using this figure, are slightly higher than the calculated stress and in order to obtain an exact result a value of 30.3 x 10^6 lbs/\( \text{in} \) would be required.

For testing with the web horizontal it was found that the stresses at b, b, and e, e, were zero while the other stresses varied considerably in all the flange toes due to slight errors in the position of the gauge holes relative to the neutral axis or slight eccentricity of rolling in the
The calculated stresses in this case were not the maximum stresses since the gauge holes were only 1.25 inches from the neutral axis. The maximum stresses, however, may be obtained by multiplying by the ratio 1.5 : 1.25.

In both cases the strain gauge and deflectometer pointers came back to the initial readings when the loads were removed showing that the elastic limit had not been exceeded.

TEST II. In this test the beam was stressed to just below the yield point and similar readings of stress and deflection were taken as in the previous test. Starting with a load of 1000 lbs the machine was run up to 10,000 lbs and then brought back to 1000 lbs again, readings of stress and deflection being taken twice for each load. A permanent set of .001 inch occurred in the deflection showing that the elastic limit had been exceeded but when the load was run up and down again no further set was measured at all. It will be observed on examining table I that the stresses vary across the breadth of the flanges as previously noticed in Test I, the variation in the top flange being 3000 lbs per square inch out of a total mean stress of 33000 lbs per sq. inch and in the bottom flange the variation was 2000 lbs per square inch. These figures are about three times those for the variations in the previous test, the loads being in the ratio of 9 to 5, i.e. less than twice. They do not therefore increase proportionally to the load.

On taking readings for loading and unloading, it was again noticed that the stresses in the flange toes were lower when the load was increasing than when it was decreasing. Young's Modulus works out to $30.5 \times 10^6$ lbs.
lower than those found from the lighter loading and this fact bears out the point demonstrated in Fig. 3 where it is shown that when the proportional limit of mild steel is exceeded the apparent value of the Young's Modulus is slightly reduced.

**TEST III.** This time the load was taken up to the yield point of the beam with a setting similar to that of the two previous tests. Testing with the web vertical, the load was taken to 7000 lbs; readings were taken, it was removed, and readings were again taken for stress and deflection. This was done for other loads increasing by 1000 lbs until the beam began to sag. This point was detected by observing the pointer on the testing machine lever and when it began to drop without any increase of load no further readings were taken. This occurred at 11,750 lbs.

The beam was then loaded with the web horizontal and in this case readings were first taken at 2000 lbs per sq. inch and increments of 500 lbs were made until the load reached 4000 lbs when the lever began to sink, indicating the yield point.

The results of this test are tabulated in table II and table III from which it can be seen that permanent set first occurred at a load of 9000 lbs giving a calculated fibre stress of 32,000 lbs per sq. inch. No permanent set was noticed on the strain gauge until the load reached 10,000 lbs and at this load it was found to be \( \frac{1}{10000} \) inch unital strain. The measured stress for loading with the web horizontal was not the maximum but the table IV gives a column showing the maximum stress at the toes and it will
loaded on the weak axis is greater for a given stress than when it is loaded on the strong axis.

In order to complete this series of tests it was necessary to standardise the instrument and to carry out a tensile test on a specimen cut out of the I beam flange. The strain gauge was standardised by first of all testing the tensile specimen with a Cambridge extensometer and then testing it with the same loading with the strain gauge attached. Accordingly a piece of steel 14" long and 1" broad was cut out of one flange of the beam and was planed down until it had a rectangular section with the dimensions as given in Fig. 21.

The Cambridge extensometer has a gauge length of 100 mm. and reads to .001 mm. and with this attached to the specimen the load was applied by a tensile testing machine. The first loading gave rather unstable results and a certain amount of permanent set remained when the load had been taken off but this was thought to be due to initial stress in the flange material. The subsequent loadings, however, gave very uniform results, the average of which was taken and is tabulated in column 2 of table IV. The special strain gauge was then substituted for the Cambridge instrument and readings were taken for the same loads and their average is given in column 4 of table IV.

Column 3 shows the unital strain as calculated from the results given in column 2 and by dividing the differences of column 4 by the corresponding unital strains in column 3 constants for the gauge were obtained. Several constants were obtained but one was found which is the average and used throughout (the values in column 5 are really the reciprocals of the constant) and as stated before
The gauge length is .153 square inches and the maximum load was 2 tons while the minimum load was .02 tons. The change of stress intensity was therefore $1.93 \div .153$ which equals 15.0 tons per square inch. The unital strain for this load is $0.000988$ inches so that Young's Modulus has a value of $13200$ tons per square inch or $29.5 \times 10^6$ lbs per sq. inch.

The average value of $E$ deduced from deflections of the beam was about $30.5 \times 10^6$ tons per square inch, but this was calculated on a nominal moment of inertia of 3.787 ins.$^4$.

The section of the beam was measured to the nearest .01 inch and the 6 feet length was weighed and was found to be 51.6 lbs as against 51 lbs nominal. This meant that the area of section was larger than the required 2.5 square inches in the proportion of $31.6:51$, and gives an area of cross section of 2.53 sq. inches. Now the web was found to be of the required thickness so the extra material must have been in the flanges. This being the case an additional moment of inertia of $0.03 \times 1.5 = 0.068$ ins.$^4$ must be added to the nominal of 3.787 ins.$^4$ making it 3.855 ins.$^4$. Using this value instead of 3.787, Young's Modulus as found by deflections becomes $29.9 \times 10^6$ tons per sq. inch. It is likely that the difference between $29.9 \times 10^6$ and $29.5 \times 10^6$ may be due to small errors in the deflectometer which were noted in test I causing the measured value of the stresses to be greater than the calculated.

The ratio of transverse to longitudinal stress measured in test I was .297. Using the expression $\frac{E}{G} = 2 + 2\mu$ the ratio of the moduli for this beam is 2.59. This value will be used later when torsional rigidity is considered.
conclusions have been drawn with regard to the bending of rolled steel I beams:

(1) The modulus of elasticity of the material of the beams which have been tested has usually been found to be:
   (a) Constant throughout the section.
   (b) The same in tension as compression.
   (c) Has the same value for bending.

(2) When the elastic limit stress is exceeded to any extent a permanent set takes place. If, however, the load is removed, the beam will then behave in a perfectly elastic manner until the previous stress is exceeded when a further permanent set will occur, but the modulus of elasticity will be slightly reduced.

(3) The yield point is the maximum stress at which a rolled steel beam can be of any practical value, especially if it be loaded with the web vertical.

(4) Though a beam may be loaded concentrically and under the action of no lateral loads, yet variations of stress across the breadths of the flanges have been observed.

Secondary stresses as noted in 4 were described by Marburg (1909) and seem to indicate a lateral deflection of the flanges. The compression flange may be expected to deflect more than the tension flange since the stress differences are greater in that flange, in which case the beam, in addition to deflecting, also twists a little about the polar axis of the section. Accordingly an attempt was made to measure the torsional deflection of a beam subject to vertical loads and for this purpose a torsion meter which is shown in Fig.33 was constructed. It consisted of a wooden frame carrying a
microscope calibrated in tenths of a millimetre was fastened to the lower end of the frame and 50 centimetres from the bridge piece. Readings on the edge of the plumb line were taken and the twist in radians was found by multiplying the difference of two readings by \( \frac{1}{5000} \). The frame was clamped to the beam and readings for various series of applied loads were taken. On comparing the results of a number of tests on the same beam no connection could be traced between the various sets of readings, due probably to slight rotational movements of the beam on the supports and the method was finally abandoned.

An attempt was then made to measure the lateral deflections directly and in order to do this, the deflectometer was fixed up by means of springs and clamps into a horizontal plane. The knife edges of the instrument bore on the edge of the flange near to the point loads and the dial plunger rested on a flat plate fastened across the flanges. It was hoped by measuring deflections within the load points to avoid the effects of any lateral restraint which might be imposed by the rigidity of the beam and knife edges by means of which the load was applied.

The 3\(^n\) x 3\(^n\) x 8.5 lbs per foot I beams used in the previous tests was first of all set up in the machine and simultaneous readings of flange stress and lateral deflection were taken for loads varying from 1000 lbs to 10,000 lbs.

Figs. 33 and 34 show the variation of stress across the flanges at mid span and near to the point loads. The variation is greater at the mid span than at the load points which seems to indicate that the knife edges restrain the beam slightly. If, however, they restrained it absolutely, the stress differences would probably be of opposite sign at the load points because the action would be similar to that of a continuous beam.
term loops for loading and unloading and as subsequent diagrams will show, the unloading curve seems to give a truer indication of the action of the beam. The stresses also showed a slight loop but to a less degree and it was not possible to read the small differences between loading and unloading stresses, and they are therefore not shown on the diagrams.

The ratio of the lateral deflections of the tension and compression flanges is 2.0/3.8 = .73. It is assumed that this is the result of the initial lateral forces caused by some slight inequalities in the beam section making an initial deflection, followed by the combined action of this initial defect and the consequent eccentricity of the direct forces in flanges which will now deflect further. In order to verify this assumption the dimensions of the beam section were carefully taken in a number of places along its length and it was found that the web was 1/100 inch eccentric. This small defect was thought to be responsible directly and indirectly for the comparatively large lateral deflections and secondary stresses in the flanges.

In order to verify these conclusions a second 5" x 3" x 8.5 lbs per foot I beam was marked off and drilled for gauge holes as before and tested in a similar manner for lateral deflection and flange stresses. This beam was of older manufacture than the first one and 8000 lbs was the maximum load it would carry without permanent set. The results of this test are given in tables V and VI and it can be seen that the variation of stress across the flanges was very small. The maximum lateral deflection of the compression and tension flanges were .001" and .0005" respectively so that the deflection ratio was .5 in this case.
indicate that the relative lateral deflection of the flanges is due more to the buckling action of the flange forces than was the case in the previous tests.

The 3" x 3" x 9.5 lbs per foot I section is one of comparatively big lateral stiffness and will not buckle under ordinary circumstances over a span of 5'-8" and although it is a convenient section when it is required to verify the theory of bending, yet when the object of the test is to investigate the causes of lateral deflections of beams, a more slender section may possibly give more satisfactory results. It was therefore decided to test a 4" x 1 1/2" x 5 lbs. per foot I beam. It was set up on a 5'-8" span with the point loads 2'-8" apart and the deflectometer was set up as before on the edges of the flanges. The deflection of the compression flange increased at first rather slowly but when a load of 2000 lbs had been applied it became much greater. On the other hand, the tension flange first of all deflected in the opposite direction but as the load increased it moved in a reverse direction until at a load of 2000 lbs it had returned to its original shape and then began to deflect in the same direction as the compression flange. The result of this test is shown in Fig.29 where the loading and unloading curves for each flange are plotted. The tension flange did not deflect in a negative direction while being unloaded but simply returned to zero displacement when the load had been entirely removed but the negative deflection occurred when the load was again applied. No measurable eccentricities were found in the section although the deflections were in this case much greater than in previous tests but a slight twist was observed in the beam and it is thought that this was the cause of the initial
an I beam may affect the action in one of two ways. Either they may cause it to move laterally or they may constrain it to rotate but in both cases another deflection caused by the action of the direct bending forces in the flanges will be superimposed on the initial one. The first may be called the eccentric lateral deflection and is directly proportional to the load while the second may be called the characteristic lateral deflection and is governed by the shape of the section.

In the first 3" x 3" x 6.5 lbs per foot I beam test both kinds of deflection were acting in the same direction so that the ratio of lateral flange deflections was in all probability greater than the theoretical characteristic ratio for a perfectly true beam. On testing the second 3" x 3" x 6.5 lbs per foot I beam, the ratio was found to be 0.5 and it is likely that this value is nearer the characteristic ratio since the section was practically symmetrical. In the test on the 4" x 1\frac{1}{2}" I beam the deflections were evidently in opposition to one another and at first the eccentric deflection was most in evidence. As the load increased, however, another factor which increased at a greater rate than the bending forces in the flanges took control of the situation and the initial tendency of the beam to rotate was overcome. It is probable that in this case the actual ratio of lateral deflections which was 0.47 is slightly less than the characteristic ratio since the forces are now acting in opposition to each other. The definition of the characteristic lateral deflection ratio as deduced from the above observations may be stated as follows:

When a symmetrical I beam is subjected to given conditions of loading the ratio of lateral flange deflections,
Unless a beam section is slightly unsymmetrical no lateral deflections of the flanges may occur within the elastic limit and therefore it would not be possible to find the characteristic ratio by direct test. If, however, the conditions of loading are such that the beam will fail by buckling, then, and only then, would the flanges instantaneously deflect laterally in accordance with the above definition.

If, therefore, the ratio of flange deflections of a buckling beam can be calculated for given conditions of loading, this ratio will be the characteristic ratio for that particular case.

The following section contains an attempt to determine the conditions of lateral stability of I beams under varying conditions of vertical loading and given conditions of lateral end fixity.
first begun by Euler and has been carried on by his method until comparatively recent times. Unless the problem is very simple this method, which consists of the formation of an equation from the applied moments and the elastic resistance moment of the member when slightly bent, results in a differential equation which may be difficult to solve.

In recent years other methods have been used involving the assumption of a given form which the member takes up when it commences to buckle. One of these is suggested by Morley in Strength of Materials (1905) page 388, as an approximate solution of a simple column problem.

The column is supposed to deflect according to the equation \( y = \frac{x^2 a}{L^2} \) and from this the work done on the member is calculated from the expression \( P \left( \int_0^L \frac{dL}{dx} dx - L \right) \) which is merely the load multiplied by the reduction in length due to buckling. The work done is equated to the resilient energy of the bent columns which is \( \frac{1}{2} \int \frac{M^2}{EI} dx \) neglecting shear and direct compression.

In the following analysis it is assumed that the compression flange buckles in the form of a sine curve in all cases. It is evident that the shape of the curve will vary according to the loading and properties of the section but any variation from the true curve will result in an error of the order of probably only 2-3%. This point is dealt with by W.M. Wallace (1912) and Salmon (1920) in connection with the buckling of columns of varying section or under uniformly increasing or decreasing loads.
$S$ is the gross length of the beam, and $L$ is the distance between the ends at a given moment.

$\gamma_0$ is the maximum deflection at the centre and $\gamma_X$ is the deflection at a distance $X$ from the centre of the beam.

$A0B$ is a sine curve whose equation is

$$y = \gamma_0 \cdot \cos \frac{\pi x}{L}$$

The work done by $P$ in suddenly bending the beam into the shape indicated by the diagram

$$W_K = P(S - L)$$

$$= 2\int_0^\frac{L}{2} \sqrt{1 + \left(\frac{dy}{dx}\right)^2} \, dx$$

$$= 2\int_0^\frac{L}{2} \left(1 + \frac{1}{2} \left[\frac{dy}{dx}\right]^2\right) \, dx$$

if $\frac{dy}{dx}$ is small.

Now $\frac{dy}{dx} = -\frac{\pi x}{L} \cdot \gamma_0 \cdot \sin \frac{\pi x}{L}$

$$W_K = 2P \int_0^\frac{L}{2} \left(1 + \frac{1}{2} \gamma_0^2 \frac{\pi^2}{L^2} \sin^2 \frac{\pi x}{L} - 1\right) \, dx$$

$$= \frac{2P\pi^2\gamma_0^2}{2L^2} \int_0^\frac{L}{2} \frac{1}{2} \left(1 - \cos 2\frac{\pi x}{L}\right) \, dx$$

$$= \frac{P\pi^2\gamma_0^2}{4L}$$

and if $\gamma_0$ is the mean flange stress.

$$= \frac{P\pi^2\gamma_0^2}{4L}$$

and if $\gamma_0$ is the mean flange stress.

This is for flanges having uniform stress throughout their whole lengths and in order to obtain an expression for the work done by a force increasing uniformly from zero
from the equation \( P = P_0 \left(1 - \frac{2x^2}{L^2}\right) \) and:

\[
W_k = 2P_0 \int_0^{\frac{L}{2}} \left(\frac{\pi^2 y_0^2 \sin^2 \frac{\pi x}{L} - \frac{2\pi^2 y_0^2 \sin^2 \frac{\pi x}{L}}{L^2}}{L^2} \right) dx
\]

\[
= 2P_0 \frac{\pi^2 y_0^2}{L^2} \int_0^{\frac{L}{2}} \left[ \frac{x}{4} - \frac{1}{8\pi} \sin 2\pi x - \frac{x^2}{4L^2} + \frac{x}{4\pi} \sin 2\pi x \right]
\]

\[
+ \frac{1}{8\pi^2} \cos 2\pi x \right]
\]

\[
= \frac{P_0 \pi^2 y_0^2}{4L} \left[ \frac{5}{2} - \frac{3}{\pi^2} \right]
\]

\[
= \frac{3b + \pi^2 y_0^2 \cdot f_c}{4L} - - - - - - - (1A)
\]

If the force in the flanges follows a parabolic law such as \( P = P_0 \left(1 - \frac{4x^2}{L^2}\right) \) which gives zero force at the ends and a force of \( P_0 \) at the centre, then:

\[
W_k = 2P_0 \int_0^{\frac{L}{2}} \left(\frac{\pi^2 y_0^2 \sin^2 \frac{\pi x}{L} - \frac{2\pi^2 y_0^2 \sin^2 \frac{\pi x}{L}}{L^4}}{L^4} \right) dx
\]

\[
= 2P_0 \frac{\pi^2 y_0^2}{L^4} \int_0^{\frac{L}{2}} \left[ \frac{x}{4} - \frac{1}{8\pi} \sin 2\pi x - \frac{x^3}{3L^2} + \frac{x^2}{2\pi L} \sin 2\pi x \right]
\]

\[
+ \frac{1}{2\pi^2} \cos 2\pi x \right] - \frac{1}{4\pi^3} \sin 2\pi x \right]
\]

\[
= \frac{2P_0 \pi^2 y_0^2}{L^2} \left( \frac{L}{6} - \frac{1}{24} - \frac{1}{4\pi^2} \right)
\]

\[
= \frac{P_0 \pi^2 y_0^2}{4L} \left( 1 - \frac{1}{3} - \frac{2}{\pi^2} \right)
\]

\[
= \frac{47b + \pi^2 y_0^2 \cdot f_c}{4L} - - - - - - - (1B)
\]
perhaps to a system of loading similar to that shown in Fig. 17
the force at the centre of the beam where \( x = 0 \) and the
point load where \( x = \frac{qL}{2} \) may be represented by \( P_0 \).

Between \( x = \frac{qL}{2} \) and \( x = \frac{L}{2} \) the force is

\[
\frac{P_0}{1-g} \left( 1 - 2x \frac{L}{2} \right)
\]

and the work done by the total forces

\[
W_k = 2P_0 \int_{\frac{qL}{2}}^{\frac{L}{2}} \left( \frac{1}{2} \frac{v_0^2 \pi^2}{L^2} \sin^2 \frac{\pi x}{L} \right) dx
\]

\[
+ 2 \frac{P_0}{1-g} \int_{\frac{qL}{2}}^{\frac{L}{2}} \left( \frac{1}{2} \frac{v_0^2 \pi^2}{L^2} \sin^2 \frac{\pi x}{L} - \frac{2}{3}v_0^2 \sin^2 \frac{\pi x}{L} \right) dx
\]

\[
= 2P_0 \frac{v_0^2 \pi^2}{L^2} \left\{ \left[ \frac{x}{4} - \frac{1}{4} \frac{L}{\pi} \sin 2\pi x \right] + \frac{1}{1-g} \left[ \frac{x}{4} - \frac{1}{8\pi^2} \sin 2\pi x \right]
\right. \\
- \frac{L}{8\pi^2} \sin 2\pi x \left( \frac{x^2}{4L} - \frac{x}{2} \sin 2\pi x \right)
\)

\[
= 2P_0 \frac{v_0^2 \pi^2}{L^2} \left\{ \frac{qL}{8} - \frac{L}{8\pi} \sin \pi \gamma + \frac{1}{1-g} \left( \frac{L}{8} - \frac{L}{16} + \frac{L}{8\pi^2} \right)
\right.
\\
- \frac{qL}{8} + \frac{1}{8\pi} \sin \pi \gamma + \frac{q^2}{16} - \frac{qL}{8\pi} \sin \pi \gamma - \frac{L}{8\pi} \cos \pi \gamma
\)

\[
= 2P_0 \frac{v_0^2 \pi^2}{L(1-g)} \left\{ \frac{L}{8} + \frac{1}{16} - \left( \frac{1+\cos \pi \gamma}{8\pi^2} - \frac{q}{8} + \frac{q^2}{16} + \frac{(1-q)q}{8} \right) \right. \\
\left. \right\}
\]

\[
= \frac{P_0 v_0^2 \pi^2}{4L} \left( \frac{1}{1-g} \left( \frac{1}{2} - \frac{L+\cos \pi \gamma}{\pi^2} - \frac{q^2}{4} \right) \right)
\]

48.
slightly so that the compressive force in the flange has
to do work against the beam in two ways. The nett work done
on the beam must be equal to the sum of the resilient
energy of bending and twisting.

The resilience of a beam bent laterally about the minor
axis into a sinusoidal curve whose maximum ordinate is \( y_0 \)
may be represented by

\[
R = 2 \frac{EI}{L^2} \int_0^L \left( \frac{d^2y}{dx^2} \right)^2 \, dx,
\]

where \( y = y_0 \cos \frac{\pi x}{L} \)

\[
\frac{dy}{dx} = -y_0 \frac{\pi}{L} \sin \frac{\pi x}{L}
\]

and \( \frac{d^2y}{dx^2} = -y_0 \frac{\pi^2}{L^2} \cos \frac{\pi x}{L} \)

\[
\therefore R = \frac{EI \pi^4 y_0^2}{4L^4} \int_0^L \cos^2 \frac{\pi x}{L} \, dx
\]

\[
= \frac{EI \pi^4 y_0^2}{4L^3}
\]

\[\text{(2)}\]

The resilience of a beam twisted as in Fig.13 may be
expressed as follows:

\[
R = 2 \frac{GJ}{2} \int_0^L \left( \frac{d\phi}{dx} \right)^2 \, dx
\]
CASE I. A beam is bent by two opposite and equal terminal couples acting in a vertical plane, otherwise it is free to move laterally except at the ends where sufficient lateral restraint is applied to keep the web vertical. (See Fig. 14). The only forces tending to increase or restrain lateral deflection are considered and in a perfectly symmetrical section may be numerated as follows:

(1) The compressive force in one flange tending to make the beam deflect laterally.

(2) The resistance of the beam against lateral deflection.

(3) The resistance of the beam to torsional deflection.

(4) The tensile force in the other flange tending to prevent lateral deflection.

The work done by (1) is equal to the resilience of 2 & 3 plus the work done by 4.

\[
\frac{f_c b + \frac{\pi^2}{4} y_c^2}{4L} = \frac{f_c b + \frac{\pi^2}{4} y_c^2 (1+k)^2}{4L} + \frac{G J \pi^2 y_c^2 (1-k)^2}{4L h^2} + \frac{E J \pi^2 y_c^2 (1-k)^2}{4L h^2}
\]

\[
\frac{f_c b + \frac{\pi^2}{4} y_c^2 (1-k)^2}{4L} = \frac{y_c^2 E}{4L} \left\{ \frac{I_{yy}}{L^2} \left( 1+k^2 \right) + \frac{G J}{E h^2} (1-k)^2 \right\}
\]

If \( E = 2.5G \) then

\[
f_c = \frac{E}{b + \left( \frac{I_{yy}}{4L^2} \left( 1+k^2 \right) + \frac{J}{2.5 h^2} \left( 1-k^2 \right) \right)}
\]

\[
L_{ef} = \frac{\pi^2 I_{yy}}{4L^2} = \alpha \quad \text{and} \quad \frac{J}{2.5 h^2} = C
\]
The value of $K$ substituted in equation 5 gives the critical stress for any given value of $L$. If $L$ is very large the term \( \frac{\pi^2 I_{yy}}{4L^2} \) becomes very small and "$a$" in equation 6 approaches zero so that the solution of the equation is $K = 1$ and $f_c = 0$.

If $a = c$ then $K = 0$ and the beam buckles as shown in Fig. 13, and if $a$ is greater than $c$, $K$ is a minus quantity and the flanges deflect as shown in Fig. 15. For any given condition of span and beam section the two terms which may be represented by \( u \left( \frac{1 + K}{1 - K} \right)^2 \) and \( c \left( \frac{1 - K}{1 - K^2} \right) \) are equal.

If the force in the flange is not uniform but varies uniformly from the ends to a maximum at the centre as in the case of a beam under a central point load applied at the neutral axis of the section, the work done on the beam is

\[
3 \times \frac{b + \frac{\pi^2 I_{yy}}{4L^2}}{b} \quad \text{(see 1A)}
\]

and \( f_c = 3.33 \frac{E}{b} \left\{ \frac{\pi^2 I_{yy}}{4L^2} \left( \frac{1 + K}{1 - K} \right)^2 + \frac{J}{25I_{yy}} \left( \frac{1 - K}{1 - K^2} \right) \right\} \) (54)

but $K$ still has the same value that it had before.

For a uniformly distributed load applied in the same way

\[
2.13 \frac{E}{b} \left\{ \frac{\pi^2 I_{yy}}{4L^2} \left( \frac{1 + K}{1 - K} \right)^2 + \frac{J}{25I_{yy}} \left( \frac{1 - K}{1 - K^2} \right) \right\} \quad \text{(55)}
\]
point load applied to the top flange (See Fig. 16).

Let the point load be \( W \) and if it is assumed that it does not restrain the action of the beam, the buckling action of the compression flange will be just the same, but will be assisted by the load tending to cause more torsional deflection.

The work done by the load in a direction parallel to \( XX \) axis is zero and therefore the only work which affects the buckling is the twisting of the beam about the axis normal to \( XX \) and \( YY \).

The maximum torque = \( W y_{o} \frac{(1-k)}{2} \) and the angle of twist

\[ \theta = \frac{W y_{o} (1-k)}{2} \]

The work done = \( \frac{1}{2} W y_{o} (1-k) y_{o} (1-k) = \frac{1}{4} \frac{W y_{o}^2}{h} (1-k)^2 \)

This in terms of maximum stress is

\[ W_{K} = \frac{2 f_{c} I_{xx} y_{o}^2 (1-k)^2}{L h^2} \]

Equating work done to strain energy as before.

\[ \frac{3 f_{c} b \pi^2 y_{o}^2}{4 L} + \frac{2 f_{c} I_{xx} y_{o}^2 (1-k)^2}{L h^2} = E I_{yy} \frac{\pi^2 y_{o}^2 (1+k)^2}{4 L^2} + G J \frac{\pi^2 y_{o}^2 (1-k)^2}{4 L h^2} \]

\[ \frac{f_{c} x}{4 L} \left[ \frac{3 b \pi^2 (1-k)^2}{4 L} + \frac{2 I_{xx} (1-k)^2}{L h^2} \right] = \frac{E h^2}{4 L} \left[ \frac{I_{yy} \pi^2 (1+k)^2}{4 L^2} + G J (1-k)^2 \right] \]

\[ f_{c} = E \left\{ \frac{\pi^2 I_{yy} (1+k)^2}{4 L^2} + \frac{J (1-k)^2}{2 h^2} \right\} - 3 b + (1-k)^2 + \frac{8 I_{xx} (1-k)^2}{\pi^2 h^2} \]

\[ \therefore f_{c} = E \left\{ \frac{\pi^2 I_{yy} (1+k)^2}{4 L^2} + \frac{J (1-k)^2}{2 h^2} \right\} - 3 b + (1-k)^2 + \frac{8 I_{xx} (1-k)^2}{\pi^2 h^2} \]
\[ K^2 + 2K \left( \frac{ad+cd}{ad-2ae-cd} \right) + \frac{ad+2ae-cd}{ad-2ae-cd} = 0 \quad (8) \]

where \( a = \frac{\pi^2 I_{yy}}{4L^2} \), \( c = \frac{J}{2.5h^2} \), \( d = 3b + \), and \( e = \frac{8I_{xx}}{\pi^2 h^2} \).

If "L" is very large then "a" becomes very small and the equation becomes \( K^2 - 2K \times 1 = 0 \) so that \( K = 1.0 \) as before and equation 7 then simplifies to \( f_c = 0 \).

If the load is applied to the bottom flange the last term "e" becomes negative so that

\[
f_c = E \left\{ \frac{\pi^2 I_{yy}(1+k)^2}{4L^2} + \frac{J(1-k)^2}{2.5h^2} \right\} \quad (7A)
\]

and the value of \( K \) is obtained from the equation

\[ K^2 + 2K \left( \frac{ad+cd}{ad+2ae-cd} \right) + \frac{ad-2ae-cd}{ad+2ae-cd} = 0 \quad (8A) \]

The buckling stress for a load on the bottom flange is higher than that for a load on the top flange while the value for a load applied at the neutral axis as calculated from Case I.A is between the two though not always the mean.

**CASE III.**

A beam simply supported and restrained at the ends from twisting with a distributed load applied at the top flange.

Let the distributed load be \( w \) units per foot length of beam so that the load on a length \( dx \) is \( wdx \).
Expressing in terms of flange stress

\[ W_k = 2 f_c \frac{I_{xx}}{L} y_0^2 \frac{(1-k)^2}{L} \]

Equating the terms of work and resilience as in Case IV and solving for \( f_c \)

\[ f_c = E \left\{ \frac{\frac{\pi^2 I_{yy}}{4L^2} \frac{(1+k)^2}{L} + \frac{J(1-k)^2}{25b^2}}{47b^2(1-k^2) + \frac{8 I_{xx}}{\pi^2} (1-k)^2} \right\} \tag{9} \]

The value of \( k \) being found from equation (9) where:

\[ q = \frac{\pi^2 I_{yy}}{4L^2}, \quad c = \frac{J}{25b^2}, \quad d = 47b \text{ and } e = \frac{8 I_{xx}}{\pi^2 b^2} \]

If the load is applied at the bottom flange the term "e" becomes negative as in (7 A) and (8 A) in case II.
from twisting with two equal point loads applied at the top flange equidistant from the supports and at a distance $g$ apart.

Let the point loads be $W/2$.

The work done by the loads in twisting the beam

$$W_k = \frac{1}{2}x^2 + \frac{y^2}{2} \left( \frac{I-k}{L} \right)^2 \cos^2 \frac{\pi g}{L}$$

$$W = \frac{8fI_{xx}}{hL(1-g)}$$

$$\therefore W_k = \frac{2fI_{xx}y^2}{Lh^2} \left( 1-k \right)^2 \cos^2 \frac{\pi g}{L}$$

If the loads are at quarter points $g = .5$ and 10 becomes

$$W_k = \frac{2fI_{xx}y^2}{Lh^2} \left( 1-k \right)^2$$

and 10 becomes

$$548 \frac{f_c}{b + \frac{1}{2}b} \frac{b}{2}$$

$$f_c = E \left\{ \frac{\pi^2 I_{yy}(1+k)^2}{4L^2} + \frac{J(1-k)^2}{2.5h^2} \right\}$$

In the tests the value of $g$ was .455.

and

$$f_c = E \left\{ \frac{\pi^2 I_{yy}(1+k)^2}{4L^2} + \frac{J(1-k)^2}{2.5h^2} \right\}$$

the value of $K$ in both cases is found from equation 8.
that expressions 7, 9, 11 and 12 contain four main terms each modified by "K", but the terms "a" and "c" occur in expression 5 also and since this is simpler than the others a detailed consideration of its parts will first be made. The first term \( \frac{E \Sigma_{1} \pi^{2}}{4b+1L^{2}} \) is the Euler crippling stress which the beam is capable of taking as a pin ended strut. The second term which may be written as \( \frac{JG}{b+h^{2}} \) is a torsional constant for the particular beam section and is independent of the length. If "K" were a constant, \( \sigma_{0} \) might be represented by an expression of the form \( \sigma_{0} = \frac{A}{L^{2}} + B \) where \( A \) and \( B \) are constants. The value of \( \sigma_{0} \) is directly proportional to the elastic constant \( E \), but this is so only for stresses within the elastic limit. The value of "K" may vary between + 1 and - 1 but if it nearly reaches either of these values, one or other of the terms "a" or "c" is multiplied by a very small quantity and is negligible so that the effect of "K" is considerable. It is not affected by the type of loading which is applied at the neutral axis although the critical buckling stress may be considerably altered.

The application of loads to the flanges of a beam may alter considerably the conditions of stability and the value of "K" which is governed by the conditions of loading and the dimensions of the beam. The expressions for \( \sigma_{0} \) are much more complicated and may not be reduced to a simple combination of two factors. The value of \( K \) may still vary between the limits - 1 and + 1 and has a large influence on
of buckling stresses with an actual case, take for example the 15" deep beams tested by Marburg and mentioned previously in an account of his tests. The dimensions were as follows:

- \( h = 15" \)
- \( b = 5.5" \)
- \( m = 234" \)
- \( n = .41" \)
- \( I_{xx} = 441.8 \text{ ins}^4 \)
- \( I_{yy} = 14.6 \text{ ins}^4 \)
- \( J \) (from Richmond 1929) = 1.45 & \( L = 180 \) ins.

The theoretical value of \( K \) for loading the top flange at the centre is - .24 and the buckling stress works out at 17.5 tons per sq. inch, the actual value for the tests being 17.25 tons per sq. inch. Other formulae give the following results for the same condition of loading:

- E.F. Moore's formula gives \( f_c = 15.8 \) tons per sq. inch.
- Timoshenko's formula gives \( f_c = 13.8 \) tons per sq. inch.
- Case's formula is given for a load applied at the centre line of the section and gives \( f_c = 20.4 \) tons per sq. inch. This case is not given by Timoshenko or Case. A working formula used in America and recommended by Hess is

\[
\frac{f_c}{f_s} = \frac{120000}{1 + \frac{L}{300b^2}} = 6 \text{ tons per sq. inch}
\]

and the formula recommended by The Institution of Structural Engineers also gives \( f_s = 6 \) tons per sq. inch.

This working stress has a factor of safety of 2.87 tons per sq. inch on the result obtained by experiment for the centrally loaded beam and 2.6 tons per sq. inch for the one
described in this dissertation the value of $K$ is all that is required. The measurements of transverse stress across the flanges of the $3'' \times 3'' \times 3.5$ lbs/foot I beam gave a computed ratio of elastic moduli to be 2.69 instead of 2.5 as assumed in the previous calculations (See page 36). The value of $g$ in this case was .455 and the dimensions of the beam are as follows:

- $L = 60''$
- $I_{xx} = 3.873 \text{ in}^3$
- $I_{yy} = 1.262 \text{ in}^3$
- $t = .332$
- $J = .1$ (from Gibson & Richie)
- $h = 3''$

The equation in $"K"$ is $K^2 - 2.19 K + .55 = 0$ giving $K = .23$.

Fig. 27 gives the values of $K$ as obtained by experiment and computed from Fig. 25. It varies from .45 at small loads to .77 at maximums while the load was being applied. When the load was being reduced $K$ is almost constant at about .7.

The ratio of $L/b$ in this case was 22 and it is not likely that buckling effects of any importance would come into play at stresses within the elastic limit, especially if there was any appreciable eccentricity in the section. The tension flange would undoubtedly restrain the beam a little which accounts for the "$K"$ value being .7 and not 1.0.

The tests on the second $3'' \times 3''$ I beam where there was very little eccentricity in the section, gave a $K$ value of .5. This approaches nearer to the theoretical ratio but such large divergences are not a satisfactory confirmation of the formulae and show that the broad flange I beams with ordinary defects are not affected by buckling forces over moderate spans and will in most cases fail by direct flexure. This is borne out by the fact that although during
yet no excessive lateral deflection took place, the displacement being approximately proportional to the load for all loads, especially while unloading (See Fig. 23).

The tests on the 4" x 1 3/4" x 5 lbs/ft. I beam gave results of an entirely different character. The ratio of \( \frac{L}{D} \) in this case was 59.5 and the lateral deflections of the flanges were in consequence much larger.

The dimensions of the beam were as follows:

\[
L = 66^2, \quad I_{xx} = 3,663, \quad I_{yy} = 186, \quad t = .24, \quad h = 4''
\]

\[
J = .058 \quad \text{(from Gibson & Richie)} \quad \text{(See Appendix).}
\]

The equation in \( K \) is \( K^3 - 2.04K + .73 = 0 \), and \( K = .507 \).

Fig. 23 shows the values of \( K \) obtained from actual measurement during loading and unloading. When the load was first applied the ratio of lateral deflections was about -2.0. Now it is impossible for \( K \) to be greater than -1.0, when no forms of eccentricity are present and it is obvious in this case that the initial deflections were due more to inequalities in the section or twists in the beam. As more energy was imparted to the compression flange this initial tendency was overcome and \( K \) quickly changed from -2.0 to +.4 and then gradually increased to a maximum of .52 which is slightly more than the theoretical one of .507 obtained above. After a load of 6000 lbs had been applied \( K \) decreased to .47 at 7000 lbs. The bending stress at this load was 15.7 tons per sq. inch and it is likely that the secondary stress due to the large lateral deflection was at least 10\% more so that one toe of the compression flange would probably be stressed beyond the elastic limit or even the yield point so that the flange would deflect more and the value of \( K \) would consequently be decreased.
the latter occurred. In this case the value of \( k \) did decrease at the lower loads because the twist in the beam began to influence the action of the flanges to a relatively greater extent. For this reason it was not possible to obtain a nearly constant ratio down to small loads as was the case in the tests on the 3" x 3" I beams.

GENERAL CONCLUSIONS:

It is true within fairly fine limits that when a beam of perfectly symmetrical section is subjected to ideal conditions of loading the actual stresses will be equal to the calculated stresses if they are not beyond the elastic limit of the material. In the case of mild steel, this state of affairs obtains to the yield point also, especially if the beam has once been stressed to the same extent on a previous occasion. If the ratio of the length of span to breadth of flange should be large, the beam may fail by buckling before the yield point of the material is reached but the conditions of loading and the torsional stiffness of the beam also have a considerable influence in this respect.

In practice, however, such ideal conditions are rarely realised and many factors may be present to modify the action of a beam. Variations in the elastic properties of the material of a commercial section do not seem to affect the strength or stability of a beam to a marked degree under the usual conditions of loading but unsymmetrical rolling and twists in the beam may have serious consequences under certain conditions of loading. These defects will cause a beam to deflect laterally under vertical concentric loading and once the beam starts deflecting the direct bending forces in the flanges will tend to produce further deflection.
in the broad flange type. In the former type of beam lateral deflection will increase at a greater rate as the load is applied and will eventually lead to buckling.

The broad flange type will continue to deflect as the load increases but almost in direct proportion to it. Secondary stresses will be set up in the flange to the extent of sometimes 10% of the main stress, but the beam is not seriously weakened owing to the accommodating way which mild steel has of yielding in highly stressed areas and so distributing the load more evenly over the section. These secondary stresses would be much more dangerous in high carbon steels or cast iron.

Under given conditions of loading, each type of beam will have a tendency to assume a certain characteristic position depending on the mathematical properties of the section. Eccentricity or deformity in beams will tend to modify this characteristic position and in broad flange beams will often have more influence on the behaviour than the said mathematical properties. Such eccentricities when present in slender I section beams will have a slight effect on the behaviour of a beam at high stresses when it will take up the characteristic position but they undoubtedly will cause initial deflection and the column action of the compression flange does the rest.

In addition to the lateral stiffness, torsional stiffness and the system of loading another factor affects the elastic stability of a beam. This is the ratio of lateral flange deflection which at higher stresses often is identical with the characteristic position of the beam referred to above. This ratio is a function of the three other factors in a perfectly symmetrical section and may be obtained by actual measurement for beams which are slightly distorted, so that
deduced for the buckling stresses and for the "K value" are of no direct use to the practical engineer but they demonstrate that an estimate of elastic stability of a beam based on its ratio of length to breadth only does not deal fairly with all types of section. The allowance for slight defects in beams must be made in the factor of safety but strength or weakness in a section due to its fundamental properties should be known to the engineer and allowed for in his designs. This can only be done by acquiring a more accurate knowledge of the elastic stability of I beams and will be more necessary if higher working stresses than are now generally used are in the future adopted for structural work.

In conclusion the author thanks Mr. Curnock, M.Sc. Etc. for his supervision and advice, and also Mr. V.G. Davies, B.Sc., who designed and constructed the special strain gauge for his assistance in testing and help in the mathematical portions.
Since this paper has been written the results of torsion tests made at Battersea Polytechnic have been made known to the author. The results of the tests agree very closely with the formulae deduced by Griffith and Taylor from their soap bubble film method (1917). The effective polar moment of inertia (J) for a 3" x 5" x 3.5 lbs/foot I beam was found to be 0.103 inches\(^2\) which is almost exactly the value calculated from the formulae. The effective polar moment of inertia of a 4" x 1\(\frac{3}{4}\)" x 5 lbs/ft. I beam was measured to be 0.0346 inches\(^2\) which also agrees with the calculated value. Now the experimental values of Gibson & Richie for the same sections were 0.1 and 0.058 respectively. The discrepancy in the values for the 4" x 1\(\frac{3}{4}\)" x 5 lbs/ft. I beams is considerable and the new value of 0.1 using an effective polar moment of inertia of 0.0346 inches\(^2\) is 0.39. This does not agree at all well with the author's tests although it would seem that it is more reliable than the result given in the text (page 57).
Fig. 19

Berry Strain Gauge
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**Table II**

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<td>200</td>
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<tr>
<td>6,000</td>
<td>60</td>
<td>120</td>
<td>180</td>
<td>240</td>
<td>300</td>
<td>360</td>
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**Table III**


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<th>Load (kN)</th>
<th>Extensometer Reading</th>
<th>Steam Gauge Reading</th>
<th>Error</th>
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<td>3.2</td>
<td>2.0</td>
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<td>7.9</td>
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<tr>
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<td>163.8</td>
<td>5.0</td>
<td>1.00</td>
</tr>
</tbody>
</table>

**Table V**

Tensile test of asbestos and from the fibers of 3.6 & 8.6 %

Gauge length of Cambridge Extensometer = 100 mm
Scale reading = 0.0001
Average constant (divided) for steam gauge = 0.2270

**Fig. 2 a.**