ON THE ENTRAINMENT INTO AN AXI - SYMMETRIC
CIRCULAR JET DISCHARGING FROM A GROUND BOARD

THIS THESIS IS SUBMITTED TO THE UNIVERSITY OF SURREY IN PARTIAL FULFILMENT OF THE REQUIREMENT FOR THE DEGREE OF PH. D. IN THE DEPARTMENT OF MECHANICAL ENGINEERING.

BY

ALI HADI - KHADEM

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SUMMARY

A review of experimental data on jets exhausting into a crossflow is presented with particular interest for the case of a circular jet exhausting normally into a crossflow. This is followed by a survey of a number of proposed models attempting to describe this jet interference phenomenon.

As a result of this survey a vortex sheet model, in which the jet is replaced by a pair of trailing vortices connected by a bound vortex sheet, is developed to predict the jet flow properties. Comparisons with experimental data, and in particular the collapse of data presented in similarity form, gives valuable support to this model.

Computer programs, in Algol 60, have been developed for the calculation of flow properties and for plotting of the induced pressure distribution on a flat plate.
ACKNOWLEDGEMENTS

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Mechanical Engineering Department Laboratory staff of the University of Surrey for the construction of the rig.
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<tr>
<td>$A_j$</td>
<td>Jet exit area.</td>
</tr>
<tr>
<td>$A_a$</td>
<td>Area of annulus ring.</td>
</tr>
<tr>
<td>$A_c$</td>
<td>Area of core.</td>
</tr>
<tr>
<td>$A$</td>
<td>Constant.</td>
</tr>
<tr>
<td>$a$</td>
<td>Jet nozzle radius-Thermal diffusivity.</td>
</tr>
<tr>
<td>$B$</td>
<td>Constant.</td>
</tr>
<tr>
<td>$b$</td>
<td>Width of the tab.</td>
</tr>
<tr>
<td>$b$</td>
<td>Bar width of the screen.</td>
</tr>
<tr>
<td>$C$</td>
<td>Constant.</td>
</tr>
<tr>
<td>$D$</td>
<td>Jet nozzle diameter.</td>
</tr>
<tr>
<td>$D_e$</td>
<td>Equivalent circular nozzle diameter.</td>
</tr>
<tr>
<td>$D_a$</td>
<td>Diameter of annulus jet.</td>
</tr>
<tr>
<td>$D_c$</td>
<td>Diameter of core jet.</td>
</tr>
<tr>
<td>$D_p$</td>
<td>Planform diameter.</td>
</tr>
<tr>
<td>$d$</td>
<td>Jet nozzle diameter of calibration tunnel.</td>
</tr>
<tr>
<td>$E_XP$</td>
<td>Exponential.</td>
</tr>
<tr>
<td>$f$</td>
<td>Frequency.</td>
</tr>
<tr>
<td>$H$</td>
<td>Height of the tabs.</td>
</tr>
<tr>
<td>$L$</td>
<td>Velocity confidence interval.</td>
</tr>
<tr>
<td>$L_x$</td>
<td>Longitudinal scale of turbulence.</td>
</tr>
<tr>
<td>$L_y$</td>
<td>Lateral scale of turbulence.</td>
</tr>
<tr>
<td>$T$</td>
<td>Length.</td>
</tr>
<tr>
<td>$M$</td>
<td>Screen mesh length.</td>
</tr>
<tr>
<td>$M$</td>
<td>Mass flow in the jet.</td>
</tr>
<tr>
<td>$M_0$</td>
<td>Mass flow of the jet in the nozzle.</td>
</tr>
<tr>
<td>$m$</td>
<td>Mass flow of the entrainment.</td>
</tr>
<tr>
<td>$N$</td>
<td>Number of samples-Number of tabs.</td>
</tr>
<tr>
<td>$P_{loc}$</td>
<td>Local dynamic pressure.</td>
</tr>
<tr>
<td>$P_j$</td>
<td>Dynamic pressure at the jet exit.</td>
</tr>
<tr>
<td>$P_{st}$</td>
<td>Static pressure.</td>
</tr>
<tr>
<td>$P_{tot}$</td>
<td>Total head pressure.</td>
</tr>
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</table>
\textbf{P}  
Constant.

\textbf{Q}  
Flow rate.

\textbf{q}  
Constant.

\textbf{q}^2  
Total turbulence energy = \((u^2 + v^2 + w^2)\)

\textbf{R}_s  
Radius of sphere.

\textbf{r}_c  
Co-ordinate of the outside edge of the potential core region.

\textbf{r}_{0.5}  
Radius at which the velocity is one half of the velocity on the centre-line, i.e.

\(u_{x,r} = 0x,0/2\).

\textbf{R}_{xx}  
Auto-correlation function of a waveform \(x(t)\).

\textbf{R}_{xy}  
Cross-correlation function.

\textbf{R}(r)  
Spatial cross-correlation function.

\textbf{S}  
Ground board or planform area.

\textbf{s}  
Distance of tabs away from the jet nozzle plane.

\textbf{St.}  
Strouhal number = \(f.D/U\).

\textbf{T}  
Total thrust of a jet.

\textbf{T}  
Time of flight in case of pulsed-wire anemometer.

\textbf{t,T}  
Time.

\textbf{t}  
Parameter, student's 't' distribution.

\textbf{U}  
Velocity component parallel to X-axis of both cartesian and cylindrical co-ordinates.

\textbf{U}_{x,r}  
Velocity in the cylindrical co-ordinates \((x,r,\phi)\) with the origin at the centre point of the jet nozzle.

\textbf{U}_{x,0}  
Velocity on the centre-line axis of the jet at a distance away from the jet nozzle plane.

\textbf{U}_{0,0}  
Velocity at the centre-point of the jet nozzle = \(U_j\).

\textbf{U}_a  
Velocity at the nozzle of annular jet.
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<th>Description</th>
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<tr>
<td>$U_c$</td>
<td>Velocity at the nozzle of a core jet.</td>
</tr>
<tr>
<td>$U_1$</td>
<td>Free stream velocity.</td>
</tr>
<tr>
<td>$U_i$</td>
<td>Instantaneous data value.</td>
</tr>
<tr>
<td>$U_{cont}$</td>
<td>Mean value of continuous data.</td>
</tr>
<tr>
<td>$U, V, W$</td>
<td>Mean velocity values of $U, V, W$ velocity components, $(U = (\bar{U} + u))$.</td>
</tr>
<tr>
<td>$u, v, w$</td>
<td>Fluctuation part of $U, V, W$.</td>
</tr>
<tr>
<td>$(u'^2)^{1/2}$</td>
<td>Turbulent intensity.</td>
</tr>
<tr>
<td>$V$</td>
<td>Velocity component parallel and along $Y$-axis in Cartesian co-ordinates and along $r$ in cylindrical co-ordinates.</td>
</tr>
<tr>
<td>$V_r = V_c$</td>
<td>Entrainment flow velocity.</td>
</tr>
<tr>
<td>$V_0$</td>
<td>Zero velocity voltage.</td>
</tr>
<tr>
<td>$V_{out}$</td>
<td>Output voltage from a Hot-Wire Anemometer Unit.</td>
</tr>
<tr>
<td>$V_s$</td>
<td>Velocity directed to the centre of a sphere.</td>
</tr>
<tr>
<td>$W$</td>
<td>Velocity component parallel and along $Z$-axis.</td>
</tr>
<tr>
<td>$X, Y, Z$</td>
<td>Cartesian co-ordinates with the origin at the middle point of the jet nozzle.</td>
</tr>
<tr>
<td>$X$</td>
<td>Jet centre-line axis.</td>
</tr>
<tr>
<td>$X_0$</td>
<td>Constant.</td>
</tr>
<tr>
<td>$X_c$</td>
<td>The length of the potential core.</td>
</tr>
<tr>
<td>$x$</td>
<td>Distance from the jet nozzle.</td>
</tr>
<tr>
<td>$x_0$</td>
<td>Virtual origin.</td>
</tr>
<tr>
<td>$x, r, \phi$</td>
<td>Cylindrical co-ordinate with the origin at the centre-point of the jet nozzle.</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Yaw angle.</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Pitch angle.</td>
</tr>
<tr>
<td>$\delta_{0.5}$</td>
<td>Radius at which the velocity is one half of the velocity on the centre-line = $r_{0.5}$.</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Momentum - deficit thickness.</td>
</tr>
<tr>
<td>$\Delta\eta$</td>
<td>Induced load.</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Parameter = $r/r_{0.5}$</td>
</tr>
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\( \rho \)  
Density.

\( \delta^* \)  
Displacement boundary layer thickness,

\( \delta = (1 - D_{0,r}/D_{0,0}). \)

\( \phi \)  
Non-dimensional radial distance,

\( \phi = (r - D/2)/x. \)

\( \lambda \)  
Constant.
1. Introduction

An important factor in the aerodynamics of V/STOL aircraft is the aerodynamic interference between the lifting jets exhaust flows and the mainstream flow over the aircraft. It is, therefore, necessary to carry out model tests on such aircraft in order to properly simulate these exhaust flows. The purpose of this investigation was related to just one aspect of the simulation of the jet exhaust flows.

There have been numerous tests performed on models of VTOL aircraft in the hovering phase of the flight. It was found that the entrainment of air into the jet exhausts gives rise to an aerodynamic force on the model which is opposite in direction to the jet thrust. This loss of lift is usually only a few percent of the jet main thrust but it has been found that the precise value is very dependent on the type of arrangements used to simulate the exhaust flow of the lifting jet. In the hovering phase of flight, it is only the entrainment arising from the spreading of the jet that gives rise to the lift loss. It is, therefore, clear that different configurations of jet nozzle and the way air is supplied to these nozzles will have an influence on the rate of spreading of the jet flow. The purpose of this investigation is to study the various factors which will influence the rate of spreading of a single jet issuing normally from a plane wall. It is worth stressing that the lift loss in the hovering stage of flight is a few percent, and, as this may also be influenced by entrainment, it is generally important to obtain some information about the factors influencing the rate of entrainment flow, hence the spread of jet.
In carrying out model tests to study various aspects of the aerodynamics of VTOL aircraft in the hovering phase of flight, it has been noted that there are significant differences in the decay of the centre-line velocity in the various axi-symmetric jets used in the experiments. These differences are important because they imply that the jets were spreading at different rates and that the entrainment into them was different. Good examples of these differences can be seen in Figure 37 which shows the decay of the jet centre-line mean velocity from experiments of Kuhn (1959), Gentry & Margason (1966) and Bradbury (1972). Although the jet nozzle details in each of these tests were differed from one another, there was nothing in the experimental arrangements which would have led one to believe that the differences in the jet development would be as marked as they would appear to be.

It is obviously important to try to establish the cause of these differences, and in this experimental investigation, attempts have been made to explain the discrepancies that have been observed in the spread of nominally axi-symmetric jets. A number of experimental tests have been carried out in which the effects of a number of factors which it was thought might be important to jet development have been studied. These factors include the nozzle boundary-layer thickness, turbulence level and convergence. However, over the limited range of these tests, it was found that none of these factors had a very strong influence on the jet development. By contrast, the insertion of small rectangular tabs into the jet flow on the nozzle perimeter was found to have a very profound effect on the jet development. In particular, it was found that just two tabs produced gross distortions in the jet development resulting in the jet almost splitting in two, with high velocity regions on either side of the diameter joining the tabs. Some explanations for this effect based on further tests with wedges are put forward.
In addition to the measurements of the mean flow field, a number of spectrum and correlation measurements were made. Some of these measurements are reported for jet flows issuing from a clean nozzle and also from a nozzle with two tabs. In the former tests, additional evidence to the results of past measurements was found which showed the existance of flow structures which have some coherence around the entire circumference of the jet. This could suggest that these "vortex rings" or "puffs" may be of some importance in producing jet noise and it seems that the effect of inserting tabs is to prevent the occurrence of these structures.

In order to highlight the importance of the initial conditions of the nozzle and flow characteristics upstream from the nozzle and show that their effects on the jet development is considerably larger than has been appreciated in the past, the results of a few jet investigators are represented and discussed in Chapter II. In section 2.2 of this chapter the structure of an axi-symmetric jet emerging from a so called "clean nozzle" was discussed. The effects of boundary layer thickness, the pressure ratio, the form of contraction duct, non-uniform exit velocity distribution across the jet nozzle, plenum chamber configurations, presence of acoustic waves in the jet exit and geometrical form of the jet nozzle is considered in the remaining of Chapter II.

The details of the experimental facilities are described in Chapter III. In this chapter the use of desk calculator for on-line hot-wire and pulsed-wire measurements are described.

In Chapter IV, a detailed theoretical and experimental approach has been made of the effects of non-uniform jet exit velocity profile on the flow downstream from the jet nozzle. Using one of the very early models of turbulence, namely Reichardt's inductive theory, attempts are made to predict the velocity profiles downstream of a circular jet with non-uniform jet
nozzle velocity profile. The predicted profiles are compared with a number of experimental distribution obtained from tests with both uniform and non-uniform nozzle flows. Chapter V then outlines the results of tests on the rate of spreading of the jets in the case when the velocity profile is uniform from a clean nozzle, with the exception of the nozzle boundary layers, and when the jet nozzle turbulence level is low (i.e. about 1.5 percent). These results show a small but unexpected influence of Reynolds number. However, by comparison with results obtained from other investigators, it would appear that this effect is almost certainly due to the change in nozzle boundary layer thickness that inevitably accompanies a change in jet Reynolds number rather than a direct influence on the Reynolds number effect.

The effects of introducing a turbulence grid into the jet nozzle to produce an initial jet flow with an intensity of about 5.5 percent are also discussed in this chapter. These tests show that modest turbulence levels do not appear to have a measurable influence on the jet rate of spreading. Section 5.3 gives the results of the influence of small tabs inserted into the jet around the circumference of the jet nozzle. The reason for investigating the influence of these tabs is discussed at the beginning of this section but it was found that they produced an unexpectedly large effect on the rate of spreading. These results are certainly the most interesting obtained and the velocity variations appears very similar to those produced in other investigations of V/STOL aerodynamics (see, for example, Kuhn (1959) and Gentry & Margason (1966)) which provided the stimulus for the present work. It seemed, therefore, that the mechanism by which these tabs produced the much more rapid decay of the jet centre-line velocity might also explain the anomalies that have arisen in other work on V/STOL jet aerodynamics.
In Chapter VI both the calculations of the entrainment into an axi-symmetric turbulent jet by representing of the jet entrainment by a line distribution of sinks are compared with direct induced flow measurements using a Pulsed Anemometer Unit. The entrainment measurements were also carried out for a circular jet with two tabs which resulted, as was expected, in increased entrainment into the jet.

A number of tests were carried out to study the vortex rings produced in a jet from a uniform "clean nozzle" which are reported in Chapter VII. These vortex rings have some intrinsic interest of their own and a few spectra and correlation measurements were made with two hot-wire anemometers to determine whether the vortex motion were rings or spirals.
2. Comments on existing work

2.1 Introduction

When a circular jet exhausts from the lower surface of a wing or fuselage, a pressure field is set up which invariably gives rise to a loss of lift compared to the case when the jet is not present. This phenomenon is obviously of interest in the design of V/STOL aircraft, where the lifting jets must support the aeroplane in hovering and during the transition and, therefore, requires a detailed knowledge of the losses in thrust due to lifting jets. A loss of only three percent in the total lifting capacity in hovering would mean a reduction of three percent in gross weight and, in turn, a reduction of over ten percent in the fuel that could be carried and, therefore, a large reduction in the range of flight. (*

These lift losses include those common to the conventional jet aircraft such as the inlet and nozzle losses but more important are the lift losses due to the recirculation of relatively hot air back into the engine (+) and the aerodynamic interference effects which arise from the jet efflux. These can have a marked influence on the installed lift and the stability of the aircraft during the hovering manoeuvres especially near the ground. The performance and stability of the aircraft during the transition can be also effected due to forward speed and natural wind because the lifting jet efflux can seriously constrain the relative mainstream flow over the air frame surface. (see Bradbury & Wood (1964)).

(*) The amount of fuel being carried at the start is usually as much as thirty to forty percent of the gross weight of the aircraft.

(+) Hot air ingestion and the disadvantage effect on the ingestion of flying particles caused by ground erosion due to the jet impact pressure and the temperature.
In order to study these lift losses, it is necessary to simulate the jet flows in wind tunnel model tests of V/STOL aircraft. However, since it is not possible to establish exact dynamical similarity with a full scale lifting jet, it is necessary to understand the important parameters which are properly represented on the VTOL lifting jet. There are many difficulties with this problem and the present investigation is concerned with only one of them, namely, a study of those factors which influence the entrainment into a single axisymmetric jet issuing into still air. This entrainment of ambient air which induces a flow over the airframe surface in the hovering condition gives rise to a loss of lift often accompanied by undesirable pitching movements. This phenomenon is important in dealing with the performance and control of direct jet lift VTOL aircrafts in the hovering phase of the flight. It is also intended in the present investigation to correlate this kind of lift loss to the rate of velocity decay on the centre-line of axisymmetric lifting jet.

Under hovering conditions well away from the ground, a reduction in pressure due to entrainment effect occurs on the underside of the fuselage and the wings, especially surrounding the lifting-jet-exits of a VTOL aircraft. According to Williams and Wood (1966) this sort of reduction in lift can be about two percent of the total thrust for a jet exit to planform - diameter ratio of

\[
\frac{D}{D_p} = \left( \frac{\text{Jet Exit Area } A_j}{\text{Planform Area } S} \right)^{\frac{1}{2}} = 0.1
\]

which is about the minimum for a pure jet fighter installation. The lift loss reduces steadily as the ratio (D/Dp) increases, becoming about 0.5 percent for D/Dp = 0.3 which is more representative of turbo-fan or high by-pass-ratio engine installation. When D/Dp = 0.5 it was shown that the lift loss due to entrainment was very small and could be neglected.

The lift-loss due to aerodynamic interference effects of multi jet configurations which were investigated by Gentry and Margason
(1966), Williams and Wood (1966) and others is of a greater percentage than for the corresponding single jet of the same equivalent surface ratio \((-\frac{A_j}{S})\). The four lifting jet configuration of Otis (1962) indicated lift losses of the order between three and four percent. This increase is partly due to the increase in mixing rate (which will be discussed in Chapter five), but also to the additional depression produced on the lower surface between the jets because of the constricting effect between the jet themselves on the entrained flow. This appeared to be a strong effect when the jet-exits were arranged in rows or elongated narrow slots, thus tending to enclose a significant amount of the planform area (Williams and Wood (1966)).

There might be a close relationship between the rate of velocity decay on the centre-line of the jet and the entrainment flow which leads to induced loads on the planform because both of these parameters are functions of the amount of the air drawn into the jet.

Fundamental investigation of the lift loss due to the effect of entrainment flow with the method of direct lift-loss measurement have been reported by Gentry and Margason (1966) and led to the development of an empirical correlation expression:-

\[
\frac{\Delta L}{T} = (-0.009) \left[ \frac{S}{A_j} \left( \frac{\partial (P_{1oc}/P_j)/\partial (x/D)}{(x/D)_{max}} \right) \right]^{\frac{3}{2}} \quad (2.1)
\]

Where \(\Delta L\) is the induced loads due to the effect of entrainment, \(T\) is the total thrust of the lifting jets, \(P_{1oc}\) is the local dynamic pressure, \(P_j\) is the dynamic pressure of the jet at the exit, \(S\) is planform area, \(A_j\) is the jet area, \(\partial (P_{1oc}/P_j)/\partial (x/D)_{max}\) is the maximum rate of the dynamic pressure on the centre-line of the jet and \((x/D)_{max}\) is the distance downstream from the jet exit divided to the jet exit diameter at which this maximum decay rate occurs. This correlation expression was based on the data.
which was obtained from single, four and eight circular jet configurations and also on eight slot jet configuration. Therefore (by considering Eq. 2.1) in order to study lift losses due to the effect of entrainment with the method of direct lift-loss measurements, attempts were made to appraise the velocity decay rates on the centre-line of circular jet of air discharging at subsonic velocities into stagnant air. Attempts were also made to highlight the influence of the jet nozzle characteristics and initial flow conditions on the rate of the centre-line mean velocity decay, which is practically an indication of entrainment flow and in turn lift losses.

Many jet flow investigators and specially those pioneers such as Tollmien (1926) and others studied the flow of a single jet, far away downstream from the jet nozzle in the region of fully turbulent flow without considering the initial conditions at the exit and the state of the flow itself within the jet nozzle. However, the initial condition of the jet nozzle as well as the condition of the flow within the orifice, as will be shown later, have a significant effect on the flow characteristics, particularly in the region of nonturbulent core near the jet exit and also further downstream in the fully turbulent region. The entrainment which is responsible for VTOL lift loss which is of particular interest in this experimental work arises in the region near the nozzle.

Because it has generally been believed that the greater portion of the noise from a jet emanates from the regions covered by the first one to eight diameters from the nozzle, it is interesting to know the full description of the structure of turbulent jet flow in this region and the influences of the initial conditions of the nozzle on the characteristic of the flow in this region.

If we consider a single subsonic jet issuing normally from a large plane wall, there are a number of factors which might influence its subsequent rate of spreading and therefore the entrainment flow into the jet. The more obvious ones are:-
(i) Geometrical form of the jet exit such as circular, square, rectangular, slot, etc.
(ii) Jet Reynolds Number.
(iii) Boundary layer at the orifice. In other words, the ratio of the boundary layer thickness on the walls of the jet nozzle to the nozzle diameter (D), and whether the boundary layer is laminar or turbulent.
(iv) The plenum chamber, the aspect ratio and geometrical shape of contraction duct.
(v) The intensity and length scale of the turbulence in the initial jet stream.
(vi) The shape of the mean velocity profile at the nozzle, such as those of a by-pass-lifting-fan and a tip driven engine.
(vii) The presence of velocities in the directions other than the jet centre-line axis in the jet nozzle, such as converging or diverging jets.

There are also many less obvious factors such as the influence of acoustic waves in the jet nozzle and, as found in this research programme, the influence of small restrictions such as tabs inserted into nozzle throat or around the circumference of the nozzle. At the outset, it is not clear which of these factors are necessarily the most significant and the aim of this research effort was therefore to discover those factors which strongly influence the rate of spread of an axi-symmetric jet issuing into free air.

In order to make it clear that the effects of the initial conditions are considerably larger than has been generally appreciated in the past, the results of a few jet investigators are represented and discussed in this chapter. The influence of some other factors which were experimentally investigated in the present study are discussed in chapter five.
The major objectives of the remainder of this chapter is therefore to discuss the previous works carried out on jet flows and highlight some of the effects of the initial conditions at the jet nozzle on the jet flow downstream from the exit. At the end of this chapter attempts are made to discuss and correlate the results and indicate those factors which have a large effect on the jet velocity decay on the centre-line.
2.2 Structure of an axi-symmetric jet emerging from a 'clean nozzle'

Interest in a full description of an axi-symmetric turbulent jet emerging from a "clean circular nozzle" has been a long standing one. Because of the introduction of the turbo-jet engine, this interest has taken on a new and enlarged dimension within the last decade and a half.

For a uniform jet issuing from a nozzle, there are several well defined regions, as shown in Figure 3. The nonturbulent core is a cone-shaped region having its base on the nozzle opening and its apex on the centre-line axis of the flow about four nozzle diameters from the nozzle. In this initial region, the flow domain has been found to contain three separate regions with well-defined characteristics which are divided by what are perhaps less well-defined boundaries. These regions are the potential core, the mixing region, and the entrainment region (Figure 3). In this initial core region, the intensity of turbulence is low, the velocity is substantially uniform and equal to the velocity at the discharge. Surrounding this core is the wedge-shaped annular mixing-region with its apex at the lip of the nozzle. Here the flow is highly turbulent and the velocity decreases rapidly away from the nonturbulent core.

In the region of flow more than 8 nozzle diameters from the nozzle, the turbulence is said to be fully developed, that is, the mean velocity profiles in this region are closely geometrically similar and this is well illustrated in Figure 4 (see also Bradbury (1967)). The many types of mathematical functions that have been proposed for fitting the profiles of velocity in the region of fully developed turbulence include power series (Tollmien (1926)) trigonometric series (Squire and Trouncer (1944)) and the probability function (Albertson et al (1948)). A good fit to these profiles is given by
\[ f (\gamma) = \frac{\bar{U}_{x,r}}{U_{x,0}} = \exp \left[ -0.6749 \gamma^2 (1+0.0269\gamma^4) \right], \tag{2.2} \]

where \( \bar{U}_{x,0} \) is the velocity on the centre-line at a distance \( x \) from the jet nozzle, \( \bar{U}_{x,r} \) is the velocity at a distance \( x \) from the nozzle and at a distance \( r \) from the centre-line, \( \gamma = r/\delta_{0.5} \) and \( \delta_{0.5} \) is the radius at which the mean velocity \( \bar{U}_{x,r} \) is equal \( \bar{U}_{x,0}/2 \).

Between the nonturbulent core and the region of fully developed turbulence is a transition region.
2.3 Effects of boundary layer thickness

In a fully turbulent 2-D mixing region, self-preserving can occur in which the structure of the flow at all streamwise stations is similar. It is easy to show that in this flow,

\[ \delta_{0.5} \propto x \quad \frac{\bar{u}_{x,0}}{u_{x,0}} \propto x^{-1/2} \]

where \( \delta \) is the momentum deficit thickness of the jet. From experiments on self-preserving jets (Bradbury (1967)) it is found that taking \( C = 2.5 \) gives good agreement with the result.

In the case of mixing layers originating at the edge of an axisymmetric jet, the flow is similar to the 2-D self-preserving flow, only here

\[ \delta_{0.5} \propto x \quad \frac{\bar{u}_{x,0}}{u_{x,0}} \propto x^{-1} \]

\[ \frac{\bar{u}_{x,0}}{u_{x,0}} = C \left( \frac{x - x_0}{\delta} \right)^{-1} \]

From experiments on jets issuing into still air, it is found that good agreement is obtained with \( C = 7.7 \).

To apply these expressions into a practical problem, it is necessary to know the shift in the effective origin of the flow, \( x_0 \). If the flow at the nozzle is uniform and the contributions to the net momentum flux from the boundary layers on the nozzle walls are small,
then \( x_0 \) will be a function of the jet velocity, \( \bar{U}_{0,0} \). Unfortunately, these conditions are often not met by a practical nozzle configuration and it is difficult in such cases to see how the effective origin shift due to the presence of boundary layers on the nozzle walls can be determined other than by experiment.

An investigation of the boundary layer effect on the transition process of the jet have been reported by Bradshaw (1966). It was reported that virtual origin of the mixing layer developing from turbulent boundary layer of about 0.05 inch thick in a two inch diameter nozzle at a speed of the order of 300 feet per second was about 150 momentum-deficit thickness (\( \delta \)) downstream of the nozzle. When the boundary layer was laminar, \( x_0 \) varied from 200 momentum deficit thickness upstream to 350 (\( \delta \)) downstream from the nozzle depending on the speed at the exit from 150 to 600 feet per second respectively.

The significant effect of the boundary layer thickness on the components of the turbulent intensity, in particular near the orifice upto two jet diameters downstream from the nozzle, are shown by Bradshaw (1966).

Wille (1963) and his collaborators also discussed the effect of the initial boundary layer thickness on the fully developed flow. They conducted experiments concerning different forms of nozzle including the end of a fifty diameter long tube. The mean velocity decay curves of these experiments along the central-line of the jet are reproduced in Figure 5. A comparison is also made between their results and the result of the present work on a clean axi-symmetric circular nozzle.
2.4 Effects of the pressure ratio and the form of contraction duct

Gentry and Margason (1966) and other investigators such as Stephenson (1968) studied the jet velocity decay rate on the centre-line of the flow for different pressure ratios, that is:-

Static pressure at the plenum chamber
Static atmospheric pressure

They reported comparatively small changes in the velocity decay rate on the centre-line for different pressure ratios and constant jet nozzle diameter. These small changes may have been occurred due to the compressibility effect because the tests were carried out in a pressure range of upto 2.8 atmospheric pressure. They might also have been resulted owing to the influence of the rate of exhausting velocity on the thickness of boundary layer at the nozzle lip (see "effects of boundary layer thickness" section 2.3.). However, these results show relatively small changes in the rate of mean velocity decay on the centre-line of the flow. On the other hand, as will be discussed in more detail later in Chapter V, the form of the contraction duct indicates a more significant effect on the spread of the jet and hence the mean velocity decay rate on the centre-line of the jet.

Wille (1963) investigated the effects of contraction duct on the centre-line mean velocity decay rate which are reproduced and illustrated in Figure 5.
2.5 Effects of non-uniform exit velocity distribution on the jet flow.

Another highly significant parameter effecting the rate of velocity decay is the exit velocity distribution.

The modern turbine driven lifting jet engines have a rather non-uniform exit velocity distribution. This non-uniformity ranges from a high velocity annular type of distribution from a tip driven fan to the high velocity in the core of the high by-pass ratio lift fan.

Stephenson (1968) carried out a general investigation into the jet induced interference on the circular ground board for a range of core and annulus velocities and area ratios. The co-axial jet rig used by Stephenson consisted of concentric nozzles which were supplied by two separate plenum chambers. Air was supplied to the plenum chambers through flexible hoses with individual control to the annulus or core of the jet. Various diameter core tubes could be fitted or removed, or replaced by a solid base equipped with static pressure tubes. Interference loads were measured using a single strain gauged plexture attached to the plenum chamber. Using the same test rig, Mayson et al (1972) measured the velocity profiles at successive positions downstream from the exit. Typical results are reproduced in non-dimensional form which are compared with theoretical estimations of the velocity profiles discussed in Chapter IV. The curves in Figure 23 represent the ratio of dynamic pressure distribution divided by the dynamic pressure at the annulus nozzle. These were plotted for various positions away from jet nozzle plan as a function of non-dimensional distance from the jet nozzle co-axial centre-line (r/Da). In these studies the areas of the core-jet and annulus ring were taken to be approximately equal.
\[ A_c = A_a \]
\[ D_c/D_a = 0.69 \]

The variation of interference load ratios (\( \triangle L/T \)) with the ratio of annular to core velocity (\( U_a/U_c \)) by keeping the velocity of core constant and varying the velocity of annulus and vice versa are shown in Table 1.

<table>
<thead>
<tr>
<th>Annulus jet velocity ( U_a ) in feet per sec.</th>
<th>1020</th>
<th>400</th>
<th>1020</th>
<th>400</th>
</tr>
</thead>
<tbody>
<tr>
<td>Core jet velocity ( U_c ) in feet per sec.</td>
<td>1050</td>
<td>1050</td>
<td>300</td>
<td>300</td>
</tr>
<tr>
<td>Interference loads ( \triangle L ) in pounds</td>
<td>0.14</td>
<td>0.138</td>
<td>0.035</td>
<td>0.012</td>
</tr>
<tr>
<td>Total jet thrust ( T ) in pounds</td>
<td>40</td>
<td>22</td>
<td>22</td>
<td>3.5</td>
</tr>
</tbody>
</table>

\[ L/T \ (\%) \]
| 3.5% | 62.7% | 16   | 34.4 |

Table 1: The variation of interference load ratio with the ratio of annular to core velocity of a jet with "square" velocity distribution.

A survey of the results shown in Table 1 indicates a significant variation of interference loads divided by total jet thrust of as much as sixty percent with a change of the condition of the velocity distribution at the jet nozzle.
2.6 Effects of plenum chamber configuration.

In the investigation of Gentry and Margason (1966) into induced lift losses due to the entrainment, it appeared that the loads induced on a circular ground board mounted on a small rectangular plenum chamber, (which was designed to fit inside the fuselage of a VTOL model aircraft) were four to five times larger than those induced on the same size plate mounted on an ideal circular plenum chamber.

A study of the rate of velocity spread on the jet centre-line issuing from these two plenum chamber configurations indicated a more rapid velocity decay for rectangular plenum chamber compared with the ideal circular one (see Figure 6). Simple tuft investigation seemed to suggest that the flow from the rectangular plenum chamber was extremely turbulent and it was assumed that this might be responsible for the more rapid velocity decay and, hence, the higher induced loads.

Surveys of the exit flow from the rectangular plenum chamber also indicated a distorted exit velocity profile with the low velocity region at the centre, whereas the flow from the circular plenum chamber had a flat velocity distribution.

In an attempt to check this hypothesis, various modifications were made to both circular and rectangular plenum chamber but they were not comprehensive. The effects of these changes on the centre-line mean velocity decay downstream of the nozzle and the shape of velocity distribution at the nozzle exit are also shown in Figure 6. Hence, the original rectangular plenum chamber exhibited the most rapid decay of dynamic pressure and produced the highest lift losses. The circular plenum chamber had the lowest decay rate and the lowest lift losses. It was assumed that the lower decay rate in the circular plenum chamber is due to smoothness of the flow at the nozzle. The modified rectangular plenum
chamber and the circular chamber with a strut restriction of fifteen percent of the jet nozzle diameter in width, placed at about one diameter upstream from the nozzle mounting had similar lift losses and similar decay curves on the centre-line of the flow.

In a later discussion in chapter V, it will be shown that this similarity of the two decay curves is purely accidental and had nothing to do with the two plenum chamber configurations.

The interesting feature of Gentry and Margason's results is that their tests are rather typical of those carried out on models of VTOL aircraft. Their results, which are partly shown in Figure 6, demonstrate the type of inconsistency that can arise in the model tests. However, the results of some tests which are carried out in the present study on a reproduced double size version of their plenum chamber are reported later in chapter V. The main objective of these tests were to investigate the cause and origin of such a rapid mean velocity on the centre-line of the flow due to the shape of the rectangular plenum chamber.
2.7 Effects of the presence of acoustic waves in the jet exit on the jet flow.

Another less obvious factor which influences the jet velocity decay downstream from a jet nozzle, is the presence of acoustic waves in the jet exit.

Crow and Champagne (1971) published the results of their experiments on "the orderly structure in jet turbulence". The technological motivation of their study was jet noise. They showed that the orderly structure in jet turbulence could be enhanced and controlled by a slight periodic surging imposed at the jet nozzle exit by means of a twelve inch diameter loudspeaker which was installed in the setting chamber supplying the jet. A train of vortex rings could thus be observed in the flow visualisation of the jet flow between the exit and as far as eight diameters downstream, whereas the vortex puffs observed, without any periodic surging, appeared to grow abruptly about four diameters downstream, and the average Strouhal Number (St)

\[
St = f \cdot \frac{D}{U_j}
\]

based on the puff counts, were shown to have values of around 0.3. In the above equation (f) is the number of puffs per second, D is the jet diameter and \(U_j\) is the velocity at the exit of the jet.

Under a surging imposed at the exit, it was shown that there was a comparatively strong response of the u-component turbulent intensity on the centre-line of the flow, in particularly up to eight diameters downstream, and also on the rate of the centre-line velocity decay. These results are reproduced in Figure 7 and are compared with those of the present study measured on an axi-symmetric circular jet "clean nozzle" without any periodic surging.
2.8 Effects of geometrical form of the jet nozzle

As part of a study of ground impingement problems of VTOL aircraft, a number of tests have been carried out by various investigators on nozzles with a wide range of geometrical configurations. Some of these results are of interest to the current investigation.

A study of the potential of various lifting nozzle configurations and the investigation of exhaust nozzle design factor which, by increasing the rate of mixing of jet flow with ambient air will alleviate the ground impingement problems, was carried out by Higgins (1964) and also by Szlenkier (1967). Higgins designed and tested twelve basic converging nozzle configurations including circular, five rectangular-slot and three multiple-segment nozzles. The circular nozzle also had provisions for incorporation turbulence-generating inserts consisting of sand paper lining, vortex generators, and concentric rings. The rectangular slot nozzles provided variations in exit aspect ratio and wall angle. The multiple-segment nozzles had the exit area divided equally into two, four or twelve sectors. All nozzles were designed to have the same physical exit area as that of a three inch diameter circular nozzle.

Surveys of the dynamic pressure downstream from the nozzle were completed for all nozzles (operated pressure ratio was 1.5). The values of dynamic pressure ratio are represented for each nozzle configuration as a function of distance from the nozzle divided by the equivalent circular nozzle diameter \((D_e)\) and compared with the results of a clean axi-symmetric circular nozzle obtained in the current investigation and shown in Figure 8.

The results in Figure 8 show, therefore, the large variation in the decay rates of the centre-line velocity that can be obtained from different type of nozzle configurations.
2.9 Comparison between the velocity decay rates on the centre-line of a small-scale jet and a full-scale jet engine

Variation of velocity ratio on the jet centre-line axis with distance from jet origin has been investigated by Fleming (1946). The tests were carried out on a full-scale engine with 18 and 19½ inch diameter tail pipe nozzles at pressure altitudes from 10,000 to 40,000 feet. The result of velocity ratios at various positions downstream, which were compared with those of an axially symmetrical heated jet of air of Corrsin (1943), are reproduced in Figure 9. For comparison the mean velocity decay rate on the centre-line of the axi-symmetric cold jet exhausting uniformly, apart from the boundary layer, at a speed of 120 feet per second from an eight inch diameter clean nozzle, obtained in the present investigation, are also given in Figure 9. These results demonstrate the type of inconsistency that can arise between the full-scale jet engine, the model tests of a heated jet of air and a cold jet. This in turn indicates the large effects of the nozzle and initial flow conditions at the exit on the flow characteristics downstream from the jet nozzle.
2.10 Discussion

The result of surveys by the above mentioned investigators of the mean velocity decays and the u-component turbulent intensities on the centre-line of free subsonic jets performed for various conditions of the jet nozzle and states of flow within the nozzle itself, are reproduced for reference in Figures 3 to 9. These results indicate that the effects of the initial conditions are considerably larger than has been generally appreciated in the past.

A number of factors have been discussed by these investigators which have contributed to an increase in the rate of spreading of subsonic jet flow. Some of these factors have a significant increase in the induced loads which are of sound interest in the design of lifting jets in the V/STOL aircrafts. The more significant ones, in the order of effectiveness are:-

i) The geometrical shape of the nozzle.
ii) The form of plenum chamber.
iii) Introduction of a restriction in the nozzle throat.
iv) The curvature and the aspect ratio of the contraction duct.
v) The shape of the mean velocity profile at the nozzle exit.
vi) Boundary layer thickness on the walls of the jet nozzle.
vii) Acoustic waves in the jet nozzle.

A number of tests were carried out to introduce and evaluate the contribution of other factors effecting the jet flow characteristics downstream from the jet nozzle. These are fully reported in chapter V.
3. Description of experimental set up

3.1 Jet rig and the ground board

In order to be able to carefully control the jet nozzle conditions, the jet development was studied on a much larger scale than is usual in laboratory experiments. The jet flow rig is shown in Figures 1 and 2. It consisted of an eight inch diameter jet nozzle supplied with air from a centrifugal fan driven by a five horse power variable speed d.c. motor. The speed of the motor was regulated by thyristor control over a range from zero to two thousand revolutions per minute and the speed holding was better than ten revolutions per minute of the maximum speed.

The centrifugal fan discharged the air into a two feet square by three feet long settling chamber which contained a high loss filter unit for removing dust particles down to five microns in diameter. This filter enabled hot wire anemometers to be used for long periods of time in the region immediately downstream of the jet exit without changes in calibration being caused by dust accumulation. However, further downstream in the fully developed turbulent region, its influence on the stability of the hot wire anemometer calibration was negligible as most of the air within the jet flow in this region had been entrained from the surroundings.

The settling chamber also contained an aluminium honeycomb screen to help further smooth the flow.

The square cross sectional settling chamber was followed by a smooth contraction to an eight inch diameter nozzle. The contraction shape was designed using the method of Whitehead, Wu and Waters (1951). This method is known to give a satisfactory contraction in which flow separation are avoided.
The maximum upstream pressure obtained in the contraction duct was about 140 mm water and the pressure holding was better than 0.5 percent at maximum pressure.

The parallel part of the nozzle was 15" long and finally the jet issued from an eight inch diameter hole which was set flush in the middle of an eight foot square plane wall carefully set normal to the jet axis.

A theoretical calculation of the entrainment flow velocities \( V_r \) has been carried out in order to minimise the effects of the finite wall. This theoretical calculation is fully reported in Chapter VI. Also reference has been made to previous works such as Wyganski (1964) on the flow induced by axisymmetric turbulent jets issuing normally from an infinite plane surface.

From both theoretical calculations and the results of Wyganski (1964) it was found that an eight feet (12 jet diameters) square plane wall was sufficient to simulate an infinite plane wall. This was because the induced velocity on and close to the plane wall at a distance of six jet diameters away from the jet central axis was small in comparison to the induced velocity near the jet nozzle lip. Hence the induced flow over the edge of the wall was considered to have no significant effect.

The entire apparatus was mounted on a Mezzanine floor and the jet was arranged to discharge in as unconstrained manner as possible (see Figures 1 and 2). With this arrangement the floor and the ceiling were both about twelve feet away from the jet axis and the nearest wall was nine feet away. The nearest surface on which the jet could impinge was about fifty feet downstream.
3.2 Traversing gear

The traversing gear was by normal standards a gargantuan affair constructed from scaffolding poles. Measurements could be taken up to twenty-five nozzle diameters downstream in the direction of the jet axis in steps of one inch (that is 1/8 of the jet nozzle diameter). The probes could also be traversed in the other two directions of the coordinates (Y and Z) and, for fine positioning, a rack and pinion traversing gear with a travel of fifteen inches was mounted on the top of the main traversing table enabling radial measurements up to four feet from the jet centre-line to be taken. The resolution of this gear was 0.1 mm or 1/200 nozzle diameter (see Figures 1 and 2).
3.3 Instrumentation

The measurements of all mean velocities and static pressures were carried out generally with a separate pitot and static tube. These were made from 0.028 inches outside diameter hypodermic tubing, the inside diameter of the pitot tube being 0.0165 inches. The static tube used contained four holes which were placed diametrically opposite one another. The readings of both the pitot and static tubes were checked against a standard N. P. L. pitot static tube and were found to be satisfactory. The pressures were measured by means of an electrical micro-manometer that enabled measurements of pressure to be made down to 0.1 mm of water.

There is a considerable amount of work that has been done on the use of these instruments in turbulent flows. The high turbulence levels and the wide range of flow directions encountered in the present tests causes, of course, errors in the velocities and static pressures deduced from pitot and static tube readings respectively, but these do not have any significant effect on the interpretation of some of the gross effects on jet development observed in the present tests.

However, some attempts to assess the effects of turbulence on the mean velocity measurements taken in the course of the present investigation by pitot-static tubes can be made.

The total head readings \( P_{\text{tot}} \) of the pitot tube is given by the equation

\[
P_{\text{tot}} = P_{\text{St}} + \frac{1}{2} \rho (\bar{u}^2 + \bar{q}^2)
\]

(3.1)

where

\[
\bar{q}^2 = (\bar{u}^2 + \bar{v}^2 + \bar{w}^2)
\]

(3.2)

and \( P_{\text{St}} \) is the true static pressure.
Equation (3.1) was suggested by Goldstein and it is only strictly valid for low turbulence levels when the velocity vector does not deviate more than 20 degrees from the pitot tube axis.

The turbulent intensities encountered were generally low enough to ignore the effect of turbulence on the mean velocity measurements with the pitot tubes. In the region near the nozzle, the turbulence intensities on the centre-line axis of the jet (as shown in Figure 16) varied from two percent at the nozzle lip to five percent at a distance of four jet diameters downstream. The effect of turbulence was small, since

\[ U^2 \gg u^2, v^2, w^2 \]

In the region of fully turbulent flow the turbulent intensities were of the order of a maximum of twenty five percent of the longitudinal mean velocity, U. Therefore, the equation (3.1) takes the following form

\[ P_{\text{tot}} = P_{\text{St}} + \frac{1}{2} \rho (1.03U)^2 \]

A correction of a maximum of three percent should be deducted from the measured values of the mean velocities in this region.

On the other hand the turbulence effect on the measurements of static pressure coefficients proved to be significant (see Bradbury (1963)).

Hence, due to significant difference between the measured and true static pressure coefficients it is probably more accurate in the cases of velocity measurement with a pitot tube to assume that the mean static pressure gradient in the direction of the jet flow is everywhere negligibly small, compared to other mean forces in the jet flow.
In the course of the present investigations, it was necessary to make measurements of flow direction. This was carried out by using a five-hole pitch and yaw tube. This tube, which is shown in Figure 10, consisted of a bundle of five hyperdermic tubes (0.0165 inches inside diameter and 0.028 inches outside diameter) inserted inside a larger tube (0.084 inches inside diameter and 3/32 inches outside diameter) and being bend in the form of a claw in order to allow angular rotation to take place about axes passing through the fixed point at which the local flow direction is required.

The pressure type claw form yaw tube was used to measure the two orientation angles $\alpha$ and $\beta$, in the specification of flow direction in the three dimensional condition of axi-symmetrical jet flow. Two distinct methods can be used, namely:

a) The null reading method, in which use is made of the equality of pressure at symmetrically opposite points of the probe. In other words, the probe has been oriented until corresponding readings are equal in pairs, the flow direction therefore simply being related to the geometry of the probe. Ideally when $\alpha$ and $\beta$ are equal to zero, it would coincide with the probe axis. The use of this method necessitates somewhat elaborate orientation gear.

b) In the second method, which was used in the present study, no attempt was made to align the probe accurately. Instead, the pressure differences between the symmetrically opposite holes were measured (not equalized) and then divided by the centre hole pressure as a reference pressure. The flow direction was then deduced from them by means of prior calibration. This calibration (Figure (10)) shows that the ratio of pressure differences to the centre hole pressure were related approximately to the angles $\alpha$ and $\beta$ made by the probe axis to the flow direction.

Turbulence measurements were made using a DISA Hot-Wire Anemometer and DISA R.M.S. - Meter. The accuracy of the hot-wire measurements, made with the constant temperature method, is discussed in Lawrance
& Landes (1952) and the DISA Instrumentation Manuals. In measuring highly turbulent flow a Time Domain Analyser was used with integral times up to 300 seconds in order to obtain averaged readings.

When the fluctuation of the U-component becomes large, there is a possibility of errors being introduced due to the interaction of u, v and w on U and on each other. The magnitude of these errors have been evaluated by Sandborn (1955), however it was decided, as these errors were small in comparison with the turbulence levels, not to make any allowance for them in this investigation. Also no correction was made to any of the hot wire measurements for the finite length of the wire. The accuracy of the turbulent measurements using the DISA Hot-Wire Anemometer was checked against the measurements made by Pulsed-Wire Anemometer on the centre-line of the jet. These had good agreement in the lower turbulent region. The probes used for the Hot-Wire Anemometer were DISA miniture hot-wire probes (platinum plated tungsten wires of 5 microns diameter and a sensitive length of 1.25 mm). Owing to the effects of turbulence, the signal of the anemometer was true averaged by means of a Time Domain Analyser (sometimes up to 180 secs) in order to obtain the mean value of the signal voltage, $\bar{V}$.

Two methods could be used to evaluate the power spectra density functions namely:-

a) The analogue method.

b) The digital method.

a) The analogue method, which was used in this investigation, involved the use of a B & K Audio Frequency Spectrum Analyser. The output signal from the anemometer was made to pass through 1/3 octave bandwidth filters which enabled the centre frequency to be varied from two Hertz to sixteen kilo-Hertz and the intensity of each band was used directly.
In order to obtain an accurate value of the energy spectrum at the lower frequencies, the output from the B & K Audio Frequency Spectrum Analyser was measured on a Time Domain Analyser with integral times upto 300 seconds.

b) The second method is based on a Fourier Integral relationship between the power spectral density function and the Auto-correlation function. The autocorrelation function of the anemometer output signal, which was evaluated by using a Hewlett Packard Correlator, could have been run through a H. P. Calculator or a mini computer (such as M. C. S. Mini-Computer) using a standard programme to evaluate the power spectral density function (see Appendix I).

The Hewlett-Packard (3721 A) Correlator unit was used in the present study to evaluate the auto and cross correlations as well as the probability density distribution of the random signal received from the single hot-wire anemometer (in the case of autocorrelation and probability density function) or a pair of two hot wires, one fixed and the other movable (in the case of cross correlation function). The displayed results (one hundred points) were recorded on a H - P X-Y-Plotter. The digital method of obtaining the probability density function with the help of a hot-wire anemometer and H - P calculator will be discussed in detail later in this chapter, in 3.4.

In order to measure the statistical quantities in highly turbulent and low velocity regions such as the entrainment flow region of a jet, the pulsed-wire anemometer technique was used. The advantage of this technique, which was developed for flows with large changes in the flow direction (Bradbury (1969)), is being able to make measurements in highly turbulent flows. This includes regions in which flow reversals occur which cause the hot-wire anemometer to suffer from errors due to yaw response.
A Pulsed-Wire Anemometer consists of two "Sensor Wires" operating as resistance thermometers, are either side of the electrically transmitting "Pulsed Wire" and at right angles to it. The response of a Pulsed-Wire-Anemometer is not linear, i.e. the time of flight measurements are proportional to the reciprocal of the velocity. The pulsed wire instrument is capable of displaying the time of flight in micro seconds, after each pulse in the form of four digit numbers. It can also indicate the direction of flow in the form of a positive or negative sign. The instrument was controlled "on-line" by the Hewlett-Packard 9810 calculator which enabled the calibration as well as the statistical quantities to be worked out automatically.
3.4. Use of desk top calculator for on-line hot-wire and pulsed-wire measurements

3.4.1. Calculator controlled pulsed wire measurements

There are two methods which have so far been used to evaluate 'flight times' using Pulsed Wire Anemometer techniques (Bradbury and Castro (1971)).

The simplest method was to display the sensor-wire signal on a storage oscilloscope and obtain the time of flight by estimating the intercept position from these traces. Due to this process being quite slow, this method is used if only a few samples are required.

However, to speed up the process of measurement, an automatic means of recording the time of flight was proposed. These two systems are fully described by Bradbury and Castro (1971).

Further development of the automatic method of measuring the flight time led to the construction of a Pulsed-Wire Anemometer instrument unit. The use of this instrument was significantly improved by interfacing it to the H.P. 9810 calculator.

The advantages are as follows:

i) Sample taking is automatic under the control of the calculator.

ii) The calculation of the mean velocity, turbulent intensity or probability distribution are performed immediately after each run.

iii) Linearization can be applied to each sample.

iv) Great amount of experimental and calculation time can be saved.
The block diagram of the interface is shown in Figure 11. The output of the pulsed wire instrument is in the form of a 4 digits parallel BCD and sign. The calculator, required on the other hand, a serial A.S.C.II code, hence the interface converted the parallel data into a serial form. To make up the A.S.C.II code, two of the output lines were always at logic "1". Each block of data was preceded by a sign and was followed by a "/" to indicate to the calculator the end of the message. The control line activated the pulsed wire instrument to take a sample. When sampling was finished and the data was ready to be transferred, the pulsed wire instrument signal led back so that the data transfer could take place.

Similarly for hot-wire anemometer measurements each pulsed-wire-probe was individually calibrated. Calibration plots were made in each case for both sensor-wires. For velocities between 5 - 50 ft/sec. the calibration was performed by comparing the pulsed wire anemometer with pitot readings, when both were on the centreline of the jet and were within the potential core of the axisymmetric jet. The calibration curve was of the form:

\[ U = \frac{A}{T - B} \quad \text{for } 5 \text{ ft./sec} > U > 5 \text{ ft./sec.} \quad (3.4) \]

For low velocities (less than 5 ft./sec.), the calibration was such that the reciprocals of the flight times were related linearly to the velocity, (see Figure 12):

\[ \frac{1}{T} = A + BU \quad \text{for } U < 5 \text{ ft./sec.} \quad (3.5) \]

The low velocities could not be measured accurately by means of a pitot tube, therefore, the calibration was carried out in a low velocity calibration tunnel which was specially designed and built for this purpose. In this tunnel the velocities were calculated from the readings of the total flow rate measured by a gas flow meter, (according to Q=U.A,
where A is the area of the jet nozzle of the calibration tunnel, which was 2 inches diameter \( A = 3.14 \text{ in}^2 \), and Q was the flow rate.

Due to non-uniformity of the exit velocity profile at the nozzle in the calibration tunnel, which was caused by the boundary layer, the following corrections were made to the calculated velocity, \( U \);

\[
U_{\text{correct}} = U \left( 1 - \frac{4\delta^*}{d} \right)
\]

where \( d = 2'' \) was the diameter of the circular jet nozzle and the displacement boundary layer thickness \( \delta^* \) was given by;

\[
\delta^* = \int_0^R \left( 1 - \frac{U_{0,r}}{U_{0,0}} \right) \cdot dr
\]

\( U_{0,0} \) is the velocity on the centre-line of the jet at \( x/d = 0 \).

\( \delta^* \) was found to be 0.062 inches and 0.063 inches for jet centre-line velocities at the nozzle of 1.58 and 0.775 ft./sec. respectively. It is apparent from the above definition that \( U_{0,0} \cdot \delta^* \) represents the total defect in the rate of flow in the boundary layer.

The calibration curves obtained by the method mentioned above are shown in Figure 11. These are typical of the many calibration curves that have been obtained.

The constants A and B of the calibration curve were calculated by the least square fit method and were plotted automatically on a x-y-plotter. The values of the intercept and slope of the calibration curve were also printed by the calculator. The steps of the calculations is given in Appendix II.

The velocity measurements were taken when the values of A and B became available. Linearization of the data was done by decoding the time of flight, using the stored calibration constants, after each sample was taken.
The continuously averaged values of velocity in ft./sec., the turbulent intensity in percent of the mean velocity, and the number of samples already taken were displayed in the calculator. When the averaged values became reasonably constant the sampling process was stopped.

A second method which was used, to decide when to stop the sampling process, was to pre-determine the number of samples (N). This is described in Appendix III. The sample size selected was usually greater than 300.

The probability density histogram of the velocity could also be obtained using the described method in Appendix II. The 20 stepped histogram as well as the value of the equal spaced interval of each step (step size) was printed at the end of each run.

In addition to the printed output the measurement results were also plotted on a H.P. x-y plotter which was interfaced to the calculator on line.

3.4.2 Calculator controlled Hot-Wire Anemometer measurement

The conventional method Hot-Wire Anemometer measurement employs a digital voltmeter, a linearizer unit, a true averager (in the case of higher turbulence level) and a RMS-meter to measure the mean velocity and the intensity of turbulence in a flow. A probability density function analyser, such as a H.P.-Correlator, is also necessary to analyse the probability distribution of the velocity.

The method of calculator controlled hot wire anemometer measurement described below is capable of measuring and calculating the mean velocity, the turbulent intensity and the probability distribution of the velocity in a shorter time and at a lower cost of instrumentation than the conventional method.
Digitizing the continuous analogue signals of a hot wire anemometer into discrete numbers consisted of two main steps. First was sampling which can be defined as the points at which the data is observed and second was the matter of quantization which was the actual conversion of the observed values into numerical form.

The equi-spaced sampling at points which are too close together in time will yield correlated and highly redundant data and increases greatly the time of calculation; on the other hand, if the sampled values are separated too much, it constitutes a source of error known as "aliasing". The rate of sampling used was rather low (about 2 cps), but due to the measurements of the mean values and root-mean square values the amplitude characteristics of the signals were of concern and not the frequency distribution characteristics, therefore the error caused by "aliasing" did not occur.

The number of samples (N) to be taken was determined on the basis of preconsidered accuracies. For the estimate of sample size (N), refer to Appendix III.

The second step in the digitization procedure was quantization which was the conversion of the observed values into integer numerical forms. No matter how fine the scale was, a choice between two consecutive values was required, i.e. between the two closest numerical integer numbers.

The process of digitization as shown in Figure 13 was as follows;

The zero velocity voltage ($V_0$) was subtracted from the output voltage ($V_{out}$) of the constant temperature hot-wire anemometer and at the same time it was amplified, by means of a differential amplifier (amplification gain = 5), so that the maximum possible voltage became +10 volts. This led to a reduction of error caused by the digitization procedure.
Due to a voltage restriction of Analogue to Digital Converters (ADC) the amplification of the analogue voltage was restricted to 0-10 volts.

The last voltage value was held by means of a Sample-and-Hold Module (SHM). The one used in the present study was compatible with a 12-bit analogue to digital conversion system accuracy. When it was in the "Sample" mode, the module acted as an amplifier with unity gain. When in the 'Hold' mode the drop rate was a maximum of 50 V./msec. and was therefore capable of holding an input signal to 0.01 percent of full scale (20 V. P-P) for 40 msecs. This holding time was more than sufficient for 8-bit analogue to digital conversion which took place in 25 msecs.

The held voltage was now converted by means of ADC which utilized the technique of successive approximation. The continuous comparison of the signal with the most significant bit (MSB), second significant bit (2SB) ......................(8SB) is shown in the diagram of Figure 13. The analogue voltage in the range 0-10 volts was therefore converted into a binary number. The binary coded decimal (BCD) module converted it to a 3 digital parallel BCD number. Since the calculator required the information to be presented in the form of A.S.C. II code further conversion took place in the computer interface which was a serialization of digits.

To synchronize the ADC with the sample-and-hold amplifier, the latch "status" and its complement, which are provided on the ADC, were used. The sample and hold module was commanded to "Hold" by the control line from the ADC. Logic "1" was "Sample" and logic "0" was "Hold". Digitization was under the command and control of the calculator. All calculations took place between two samples.
The digitization between steps 0-1, 1-2, 255-256, 509-510 and finally 510-511 was calibrated against a D.C. voltage generator. The intercept and slope values of the linear calibration were evaluated by averaging the above measurements (see Figure 13).

Further advantages of using this system are as follows:-

i) The time of hot-wire probe calibration using this method is very much reduced compared to the conventional method of analogue data processing.

ii) The calibration can be carried out more frequently and the errors caused by the change of surrounding temperature and dirt deposition on the probe can be therefore very much reduced.

iii) Calculation time will be reduced.

The calibration obtained by this method is shown in Figure 14 which is typical of many calibration curves that have been obtained. The constants A and B of the calibration curve \(v^2 = A + BU^n\) were calculated by the least squares fit method, described in Appendix III.

The linearized and non-linearized version of the mean velocity and turbulence intensity were measured on the centre-line of the axisymmetric jet and were compared with those of the conventional method and are illustrated in Figures 15 and 16. Figure 15 shows the mean velocity ratio decay on the centre-line axis of the 8 inch diameter 'clean nozzle' jet and Figure 16 indicates the u-component of turbulence intensity on the centre-line axis of the jet. By comparing the results of linearized and non-linearized method of measurement mentioned above with those of conventional method deduced from the pitot tube readings, there was a fair agreement in the results.

The probability distribution of velocity was also evaluated using the above mentioned method.
Probability density functions are theoretically described as the probability that the data will assume a value within some defined range. An estimate of probability density histogram \( P(U) \) was obtained digitally by dividing the range of the difference of the maximum value and minimum value, \( (U_{\text{max}} - U_{\text{min}}) \), into appropriate number of intervals. The number of observations and the percentages of the number of sampled data \( U_i \) which occurred in each interval were tabulated and accumulated. The programme written for this purpose generated a 20-cell histogram (occurrence distribution) for a set of data values \( (U_i) \). A typical example of the many histograms that have been obtained for probability density distribution of the jet velocity measured by the method described in 3.4.1. and 3.4.2. is shown in Figure 17. This illustrates a typical sample of many curves that have been obtained at several positions downstream of the jet exit on the centre-line axis of the jet. The probability distribution of velocity at 8 jet diameter downstream from the nozzle is given in Figure 17 using 20,000 samples which is compared with the results obtained using the H.P. correlator. The results are in fair agreement.
4. Theoretical and Experimental Approach to the Effects of Non-Uniform Jet Exit Velocity Profile

4.1 Introduction

A difficulty with entrainment problems is that there is little theoretical basis on which estimates of entrainment can be made. However, in the case of the effect of non-uniform nozzle velocity profiles, it is possible to study the problem using one of the very early models of turbulence, namely, Reichardt's inductive theory which is purely phenomenological. It is, of course, well known that this theory, as will be discussed later in this chapter, is not very soundly based - see for example Hinze (1959) - but it is capable of being applied to quite complicated jet problems and it is the intention of this chapter to examine the usefulness of Reichardt's method when applied to these jet flows.

In recent times far more comprehensive theories of turbulent flow have been developed of which Bradshaw and Spalding are rather typical. However, it would be difficult to apply these types of methods to complicated jet flows such as non-uniform jet exit conditions, and if it can be demonstrated that Reichardt's method leads even to moderately good agreement with experimental results, this simpler method would provide a useful guideline for predicting complicated jet flows.

Reichardt's hypothesis leads to a Linearisation of the equations of motion in the axial direction when applied to free jets. This proves to be useful when dealing with jets of complicated initial conditions at the exit or a number of jets of parallel central axis in predicting the momentum distribution across free jets.
4.2 Reichardt's Theory and Application

An empirical approach to the problem of free turbulence was proposed by Reichardt (1941). Experiments have shown that the velocity distributions across the mixing zones follow Gaussian probability functions rather closely. These functions are also from solutions of the one dimensional heat diffusion equation.

\[ \frac{\partial \theta}{\partial t} = \alpha \frac{\partial^2 \theta}{\partial x^2} \]  

(4.1)

where \( \theta \) denotes the temperature, \( t \) denotes the time, \( \alpha \) is thermal diffusivity and \( x \) is distance measured in the direction of the diffusion.

Reichardt investigated the conditions under which such functions become solutions of the equation of motion. For an axi-symmetric incompressible stationary free turbulent jet, the equation of motion in cylindrical coordinates \((x,r,\phi)\) takes the form:

\[ \frac{\partial \bar{u}^2}{\partial x} + \frac{1}{r} \frac{\partial}{\partial r} (r \bar{u} \bar{v}) = 0 \]  

(4.2)

where

\[ \bar{u}^2 = \bar{u}^2 + \bar{v}^2 \]

and

\[ \bar{u} \bar{v} = \bar{u} \bar{v} + \bar{u} \bar{v} \]

The origin of cylindrical coordinates lies on the central point of the nozzle (in the case of a circular jet). The pressure term and the terms containing the viscosity are neglected. This equation is the conservation of momentum flux \( \bar{u}^2 \) (incorporating the mean and turbulent components) and if it follows the Gaussian probability law, it must satisfy the differential equation which after integration gives the following form:-
\[ \bar{U}^2 = \frac{K}{b} e^{-\left(\frac{r}{b}\right)^2} \]  
\[ (4.3) \]

where \( K \) is a constant, depending upon the strength of the jet; 
\( b \) is a parameter depending only on \( x \), and \( b \) is independent of \( r \). The condition under which such functions become solutions of the equation of motion (4.2) is

\[ \frac{\bar{U}^2}{r} = -\lambda \frac{\bar{U}^2}{r} \]  
\[ (4.4) \]

where \( \lambda \) is a parameter with the dimensions of length and Reichardt assumed that this parameter was constant on each cross section of a jet but was variable along the jet, in other words;

\[ \lambda = \frac{b}{2} \frac{db}{dx} \]  
\[ (4.5) \]

After substitution of equation (4.4) into the equation of motion (4.2), it gives

\[ \frac{\partial \bar{U}^2}{\partial x} - \frac{\lambda}{r} \frac{\partial}{\partial r} \left( r \frac{\partial \bar{U}^2}{\partial r} \right) = 0 \]  
\[ (4.6) \]

The relation (4.4) which Reichardt calls the momentum transfer law indicates that the rate of transfer of \( U \) - momentum with the velocity \( V \) is proportional to the change of momentum flux \( \bar{U}^2 \) in the lateral direction.

This law was found in fact to be incorrect because Gaussian probability equations such as equation (4.1) are invariant under translation at constant velocity of the coordinate system, whereas, this appears not to be true of the momentum - transfer equation (4.4) which therefore conflicts with Newton's relativity (see Hinze (1959)).

It has been observed that the successive profiles of the momentum flux density ratio are similar.
i.e.

\[
\frac{\mathcal{F}U_x^2}{\mathcal{F}U_0^2} = f\left(\frac{r}{x}\right) \quad (4.7)
\]

The subscript notations used are:

- \(x, r\) indicates the conditions at points \((x, r)\) of the cylindrical coordinates \((x, r, \phi)\) in the jet flow for any \(\phi\).
- \(x, 0\) indicates the conditions at point \(x\) on the centre-line axis of the jet.
- \(0, 0\) indicates the conditions at the middle point of exhausting jet nozzle.

From equation (4.3) it can be realized that

\[
\frac{\mathcal{F}U_x^2}{\mathcal{F}U_0^2} = f\left(\frac{r}{b}\right) \quad (4.8)
\]

the two equations (4.7) and (4.8) are compatible only if:

\[
b = c \cdot x \quad (4.9)
\]

Integration over a section through the flow field of a free jet in a plane perpendicular to the axis of flow leads to

\[
\int_0^\infty U_{x, r}^2 \, 2 \pi r \, dr = U_0^2 \cdot A_0 \quad (4.10)
\]

where, \(A_0\) is the area of a point source jet. This states the conservation of the flux in which the entrainment is being ignored.

Substituting equation (4.3) into equation (4.10) and after integration leads to;
Therefore, for a free jet exhausting from a point source the equation (4.3) takes the form

\[ \frac{f U^2_{x,r}}{f U^2_{0,0}} = \frac{A_0}{\Pi c^2 x^2} \cdot e^{-(r/c \cdot x)^2} \] (4.12)

This equation which gives the distribution of the momentum flux ratio is based on the following assumption:-

i) The jet issues from a point source.

ii) Successive distribution profiles are similar.

iii) Equation (4.4) is valid.

This equation is linear in \( U^2 \), hence, a linear combination of particular solutions is also a solution of the differential equation. This fact can therefore be applied to predict the distribution of the momentum flux of a jet which can be imagined to be formed of \( N \) point source jets of parallel axis, in other words;

\[ \bar{U}^2_{x,r} = \sum_{n=1}^{N} \frac{(U^2_{0,0} \cdot (A_0)_n)}{\Pi c^2 x^2_n} \cdot e^{-(r_n/c \cdot x_n)^2} \] (4.13)

In this summation the point source jets do not necessarily lie in the same plane, provided that no point source impedes the flow from a source lying behind it. The jets can also possess various strength.

If a finite source may be taken as an infinite number of elemental point sources, the summation may then be replaced by the integration.
If all the point source jets are in the same plane perpendicular to x-axis such as an axi-symmetrical jet, the summation form can be replaced by integration

\[
\bar{U}_{x,r}^2 = \int_{0}^{A_j} \left( \frac{\bar{U}^2}{\pi c^2 x^2} \right) \cdot e^{-\left(\frac{r_d A/c_x}{c} \right)^2} \cdot dA
\]  

(4.14)

where Aj is the area of the axi-symmetrical jet nozzle.

In the case of a circular jet exhausting at a non-uniform velocity distribution the integral of equation (4.14) takes the following form:-

\[
\bar{U}_{x,r}^2 = \int_{0}^{D/2} \int_{0}^{2\pi} \left( \frac{\bar{U}^2}{\pi c^2 x^2} \right) \cdot e^{-\left(\frac{r^2 + s^2 - 2rs \cos \phi}{c^2 x^2} \right)} \cdot U(s,\phi)sd\phi
\]  

(4.15)

Where a system of polar coordinates is introduced at the jet nozzle plane with the origin coinciding with the centre of the circular jet nozzle and the velocity at the nozzle is a function of (s,\phi).

Considering the circular jet issuing uniformly across the jet nozzle, that is, \(U(s,\phi) = U_{0,0} = \text{const}\), then equation (4.15) takes the following simplified form:

\[
\frac{\bar{U}_{x,r}^2}{\bar{U}_{0,0}^2} = \frac{1}{\pi c^2 (x/D)^2} \int_{0}^{0.5} \int_{0}^{2\pi} \text{EXP} \left( -\left(\frac{(r/D)^2 + (S/D)^2 - 2rD \cos \phi}{c^2 (x/D)^2} \right) \right) \cdot S/D \cdot d(S/D) \cdot d\phi
\]  

(4.16)
In the case of a 'square' velocity profile at the nozzle where the jet consists of two concentric jet (annular and core jet) with two various uniform velocity distributions (annular velocity = $U_a$ and core velocity = $U_c$), the integration will be carried out first for the core and finally for the annular jet which subsequently will be added for each point $(x,r)$ in the jet flow.

This procedure may be applied for the case of a jet exhausting with a non-uniform but axi-symmetric velocity distribution, where the function of velocity profile across the jet nozzle, $U(s)$, is not mathematically defined, but is known experimentally (see two examples of non-uniform velocity profile measured in the present investigation shown in Figure 21).

There are several ways of calculating the integral of equation (4.16). It is more convenient to evaluate this integral by graphical methods than to use the integrated equation. However, another simple method which was applied in this investigation was numerical integrations which was carried out in a data processing computer. The procedure of this method and the programme are given in Appendix IV.

Constant $c$ in the equation (4.9) indicates the slope of the spread of the jet. This is the only constant which had to be evaluated experimentally in the Reichardt's theory.

Reichardt obtained the following value of the empirical constant, $c$, from a comparison of theoretical curves with the experimental data and gave

$$c = 0.072 \text{ for an axially symmetrical jet}$$
$$c = 0.095 \text{ for a plane jet}$$

Alexander, Baron and Comings (1953) evaluated this constant in a uniform axi-symmetrical jet at successive sections downstream
from the jet exit. They showed that the value of $c$ decreases and then increases with increasing the distance $x$ from the nozzle. The deviation was within 4 percent. They calculated the mean value of $c$ which was equal to:

$$c = 0.075$$

A simple expression for the decays of the momentum flux on the centre-line axis of the jets may be obtained by setting $(r)$ in the equation (4.19) equal to zero which after integration for a uniform axi-symmetrical circular jet gives:

$$\frac{\overline{u^2}}{u_{0,0}^2} = 1 - e^{-\left(\frac{1}{4c^2 (x/D)^2}\right)}$$  \hspace{1cm} (4.17)

and for a circular jet with a non-uniform 'square' velocity distribution gives:

$$\frac{\overline{u^2}}{u_{0,0}^2} = \left(\frac{\overline{u^2}}{u_a^2} - 1\right) e^{-\left(\frac{(Dc/2a)^2}{4c^2 (x/D)^2}\right)} + \left(1 - \frac{\overline{u^2}}{u_c^2}\right) e^{-\left(\frac{1}{4c^2 (x/D)^2}\right)}$$ \hspace{1cm} (4.18)

It is well known in a uniform axi-symmetric circular jet that almost immediately after the disappearance of the potential core, the velocity profile are closely similar and, as shown by Bradbury (1965), a good representation of the profiles is given by:

$$\frac{\overline{U_{x,r}}}{\overline{U_{x,0}}} = f(\gamma) = \exp\left[-0.6749 \gamma^2 + (1 + 0.0269 \gamma^4)\right]$$ \hspace{1cm} (4.19)
where \( \gamma = r/\delta \) and \( \delta \) is defined as the radius at which the mean velocity \( \bar{U}_{x,r} = \frac{U_{x,0}}{2} \), which was found to be proportional to the distance \( x \) from the jet nozzle.

\[
\frac{\delta_{0.5}}{D} = 0.089 \frac{x}{D} \tag{4.20}
\]

It was also shown that the experimental results for the mean velocity decay along the centre-line axis are reasonably well fitted by the expression

\[
\frac{\bar{U}_{x,0}}{\bar{U}_{0,0}} = \frac{6.82}{x/D} \tag{4.21}
\]

for values of \( x/D > 8 \)

After substituting (4.21) into equation (4.19) and squaring both sides of the equation, this leads to the following equation

\[
\frac{\bar{U}_{x,r}^2}{\bar{U}_{0,0}^2} = \frac{1}{4(0.07331 \frac{x}{D})^2} \exp \left[ -\frac{(r/D)^2}{(0.0766 x/D)^2} \left( 1 + 0.0269 \frac{(r/D)^4}{(0.089 x/D)^4} \right) \right] \tag{4.22}
\]

Neglecting powers of \( (x/D) \) greater than 2 and comparing with Reichardt's theory for a source jet, equation (4.22) can be written as:

\[
\frac{\bar{U}_{x,r}^2}{\bar{U}_{0,0}^2} = \frac{1}{4 (c \frac{x}{D})^2} \exp \left( -\frac{(r/D)^2}{(c \frac{x}{D})^2} \right) \tag{4.23}
\]

The constant \( c \) which was found to be equal to 0.075 is close to the value of the constant found by Bradbury (1965).
4.3 Presentation and discussion of results

i) Single uniform axi-symmetric jet

An axi-symmetric circular jet was considered discharging uniformly at subsonic velocity into stagnant air. Assuming that the fluid density remains constant, then the ratio of momentum flux ($\frac{\bar{\rho} U^2_{X,r}}{\rho U^2_{O,0}}$) was determined applying equation (4.15). The computed results are given in Figures (18 and 19) where the ratio of the mean velocity divided by jet velocity at the centre of the nozzle ($\frac{U_{X,r}}{U_{O,0}}$) are plotted against the non-dimensionised distance from the centre-line at successive positions of $(X/D)$ values of 1, 3, 6 and 10.

A comparison is also made in Figures (18) and (19) between the results of Reichardt's hypothesis with those of experimental investigation of Sami (1967). The largest discrepancy occurs along the central axis of the jet. However, there is fair agreement further away from the centre-line and in the sections further downstream. These kind of results were of course expected because equation (4.12) was based on the assumption that successive distribution profiles are similar. The velocity decay rate on the centre-line of an axi-symmetric jet downstream of a circular nozzle was also computed which is shown together with the experimental results of the present investigation on an 8 inch diameter jet nozzle, the results of Bradbury (1972) on a 1 inch diameter nozzle and also the results of Sami (1967) in Figure 41. The agreement between experimental and theoretical results as mentioned above is not very good along the central axis.

ii) Square and triangle jet nozzle

The dynamic pressure decay along the centre-line axis of a triangle (of equal sides) and a square jet nozzle exhausting
uniformly into still air was obtained by the method of Reichardt. The results are compared with the experimental results of Soforza et al (1965) on a Logarithmic graph paper in Figure 20. The agreement between the theory and experiment is fair. These kind of results as mentioned before were expected because equation (4.12) was based on the assumption that successive profiles are similar.

iii) Axi-symmetric circular jet with exit non-uniform "square" velocity distribution

The continuing development of gas turbines for aviation purposes has led to a need to understand more fully the effect of varying the shape of the velocity profile at the exhaust nozzle on the development of the jet flow as well as the jet entrainment. Velocity distributions of a non-linear nature occur in practice resulting, for instance, from the pressure of the cone-shaped rear bearing housing within an engine exhaust pipe, or in the mixing zone of the hot and cold jets of the bypass engine.

It is the purpose of this part of investigation to attempt to develop a method of evaluating mathematically the effects of various $U_a/U_c$ ratio of a "square" velocity distribution at the exit of a circular jet on the development of the jet flow downstream and the entrainment in the jet. In order to check the theoretical results, attempts were made to produce a "square" velocity profile at the 8 inches diameter nozzle exit. This was done by using a cylindrical porous blockage placed concentric inside the parallel part of the nozzle exit. A bundle of fine plastic drinking straws (diameter, $d=0.015\text{"}$) were used as the porous medium, placed within a cylindrical container, such that the axes were parallel with that of the jet nozzle. The porous blockage was regular in size and shape with thin walls and narrow so that the total surface area presented to the flow within the bundle was large. The total gross sectional area of the bundle
were chosen to be equal to the annular core area. The two resulting velocities are defined as \( U_a \), a fast moving annulus and \( U_c \) as a slower moving core.

Two bundles of 5.5 inch diameter were used; one having tubes of \( L/d = 27.3 \) and the other with tubes of \( L/d = 54.6 \).

This cylindrical porous blockage was positioned about 0.5 jet diameter upstream from the exit to allow the flow to be settled down to an average velocity in the core region at the time of issuing from the jet exit. The total head pressure measurements along the radial directions of various sections perpendicular to the jet centre-line axis were taken. They were carried out at distances 0.0625, 1, 2, 4, 8, 12 and 16 jet annular diameters away from the concentric jet nozzle axis. The static pressure was assumed to be ambient everywhere in the jet flow. The results are shown in Figure 21 and 22 for the annular to core dynamic pressure ratio of \( \frac{U^2_a}{U^2_c} = 5 \) at the exit, in which the ratio of local total head pressure divided by the maximum dynamic pressure of annular jet at the exit \( \frac{U^2_{loc}}{U^2_a} \) is plotted against the ratio of radial distance from the jet centre-line axis divided by the annular jet diameter \( (r/D) \).

The velocity profile resulting from the use of a bundle of tubes concentric with the nozzle, displays a peak for the maximum velocity rather than an ideal "square" distribution at the annular section. This is due to boundary layer growth in the annular flow on the surfaces of both the nozzle inside wall and outside of tube bundle. Also a non-uniform initial velocity distribution at the core section of the jet nozzle, as shown in Figure 21, can be realized. This was assumed to be due to the flexibility of the drinking straws which caused a non-uniform tube bundle.

The mathematical approach of Reichardt's theory into circular jets with axi-symmetrical non-uniform "square" velocity distribution across the jet nozzle can be dealt with as follows:
This problem may be faced in three different forms, namely:

a) When the shape of velocity profile at the exit of the jet is defined mathematically, i.e. $U=U(s)$. In this case the equation (4.14) will be applied.

b) When the velocity profile at the nozzle is in form of two distinct constant annulus and core velocity which was defined above as a 'square' form of velocity distribution. A problem of this sort will be solved by integrating the double integral of equation (4.15) twice, namely from $0$ to $R_c$ (core radius = $R_c$) with a uniform jet velocity $U_c$ and again from $R_c$ to $R_a$ (annulus radius = $R_a$) with a uniform jet velocity $U_a$.

c) When the shape of velocity profile is already evaluated experimentally. In this case it may be assumed that the circular jet consists of $N$ concentric jets of step-wise velocity distribution and the integration of equation (4.15) will be numerically computed $N$ times.

The first attempt made to study the agreement between the theoretical prediction of Reichardt's hypothesis with the experimental results of the momentum flux, in a jet flow with a non-uniform velocity distribution at the exit, was the theoretical evaluation of total head pressure ratio and comparison with the experimental results of Mayson, Ogilvie and Harris (1972) which was obtained downstream of a co-axial jet issuing with two distinct uniform core and annulus velocities ('square' velocity distribution) into stagnant air. They produced a series of graphs showing the variation of the ratio of local dynamic pressure to dynamic pressure at the jet exit centre ($\frac{U_x^2}{U_o^2}/\frac{U_o^2}{U_o^2}$) along the radial direction at successive positions downstream of the co-axial jet nozzle obtained at various annulus to core velocity ratios. The areas of the annular and core jets were nearly equal (core/annular jet diameter = 0.69) and the diameter of annular jet was equal to 1.57 inch.
Typical examples of these diagrams are reproduced in Figures 23 and 24 indicating an annular to core velocity ratio of 831 (ft/sec) divided by 761 (ft/sec) at the jet exit. For comparison the predicted values of dynamic pressure distribution in the radial direction are also plotted in this diagram at positions 1.5, 3, 4.5, 6, 7.5 and 9 inches away from the jet exit.

The agreement between theoretical and experimental values are not very good. However, this discrepancy may have been occurred because of simplifications made by simulating the dynamic pressure ratio profile at the exit as a simple 'square' dynamic pressure distribution profile, neglecting the boundary layer section.

In the application of Reichardt's hypothesis into the experimental results of "square" velocity profiles, as mentioned above and illustrated in the form of total head pressure ratios in Figures 21 and 22, time and effort has been spent to:-

a) Evaluate an exact total head pressure distribution profile near the jet exit (x/D = 0.0625) by producing more points.

b) The annulus jet radius was divided into 20 equal parts and it was assumed that each part formed a ring jet of uniform exit velocity equal to the mean velocity of that part. Reichardt's hypothesis was then applied to compute the ratio of the dynamic pressure divided by maximum dynamic pressure at the exit of 20 concentric ring jets. The results are shown for comparison with the experimental results, in Figures 21 and 22. There is little discrepancy between the theoretical and experimental results, as shown in the illustrations. The agreement is fair especially at distances from the jet exit greater than four annulus diameters downstream from the exit. On the other hand, the results of shorter bundle of
L/d = 27.3 with the annular/core dynamic pressure ratio of 2.5 showed that the agreement between the theoretical and experimental results were not as good as the longer one with L/d = 54.6. This large discrepancy was believed to originate from geometrical non-symmetry in the bundle due to short length of straws.

iv) Multi-jet nozzle configuration

In the prediction of the flow characteristics downstream of a combination of jet sources, issuing at the same velocity (U_j) from jet nozzles of equal diameter (D) into stagnant air, the equation (4.13) was applied. The plane of these jet sources were all lying in the same plane perpendicular to the jet axis.

Attempts were made on two different jet configurations, to compute velocity profiles downstream from the jet exit by applying the theoretical computation results and the theoretical method of Reichardt's hypothesis.

First the four jet configuration of NASA, Gentry and Margason (1966), was considered. The geometrical lay out of this jet configuration is shown in Figure 25. The origin of the cylindrical coordinates (x, r, \( \phi \)) was placed at the diagonal cross point of the four jets. The Equation (4.12) may be developed in the following form, from which the dynamic pressure ratio can be obtained as follows:

\[
\frac{U_{x, r, \phi}}{U_{j}^2} = \frac{1}{(2C_m \frac{X}{D})^2} \left[ \exp \left( \frac{-((r/D)\sin \phi - \frac{B_j}{D})^2 - (r/D \cos \phi - \frac{A_j}{D})^2}{C_m (X/D)^2} \right) + \exp \left( \frac{-(r/D \sin \phi + \frac{B_j}{D})^2 - (r/D \cos \phi - \frac{A_j}{D})^2}{C_m (X/D)^2} \right) + \exp \left( \frac{-(r/D \sin \phi + \frac{B_j}{D})^2 - (r/D \cos \phi + \frac{A_j}{D})^2}{C_m (X/D)^2} \right) \right] \tag{4.24}
\]
In Figure 25 the following values are given

\[ D = 28.65 \text{ mm}, \quad B = 21.5 \text{ mm} \quad \text{and} \quad A = 114.6 \text{ mm}. \]

The dynamic pressure ratio along the x axis was obtained from the above equation by setting \( r \) and \( \phi \) equal zero; hence,

\[
\frac{\overline{U^2_{x,00}}}{U^2_J} = \frac{1}{(C_m x/D)^2} \exp \left( \frac{-(B/D)^2 - (A/D)^2}{C_m (x/D)^2} \right)
\]

(4.25)

Velocity profiles on the planes perpendicular to x-axis along the horizontal axis (\( \phi = 0 \)) and vertical axis (\( \phi = \pi/2 \)) were obtained using the H.P. calculator and are shown in Figure 25, in which the squared velocity/jet source exit velocity ratio are plotted against the radial distance from the centre-line divided by jet diameter \((r/D)\).

The validity of Reichardt's hypothesis in this case is beyond the point of the interaction of the jet flows, where the flow is well established.

Williams and Wood (1966) reported that the reduction in lifting thrust \((-\frac{\Delta L}{T})\) exceeds 5 percent when the jet exits are arranged in rows and thus tending to enclose a significant amount of the planform area in a VTOL aircraft.

Consider a row of four-jets with equal nozzle diameters issuing in parallel with one another with equal velocity \( U_j \). The jet nozzles are lying in the same plane perpendicular to the axis of flow and the origin of cylindrical coordinates \((x, r, \phi)\) is placed midway between the second and third jet nozzle. The distance between each nozzle is equal two jet diameters \((2D)\), (See Figure 26).

In this case the local dynamic pressure/jet dynamic pressure ratio of the equation (4.12) takes the following form:-
The dynamic pressure ratio along the x-axis may be obtained by setting the values of r and θ in the above equation equal to zero which then takes the following form:-

\[
\frac{U_x^2}{U_j^2} = \frac{1}{2(C_m^2)^2} \left[ \exp \left( -\left( \frac{r}{D} \right)^2 \sin^2 \theta - \left( \frac{r}{D} \cos \theta - 3 \right)^2 \left( \frac{C_m^2}{(x/D)^2} \right) \right) + \exp \left( -\left( \frac{r}{D} \right)^2 \sin^2 \theta - \left( \frac{r}{D} \cos \theta - 1 \right)^2 \left( \frac{C_m^2}{(x/D)^2} \right) \right) + \exp \left( -\left( \frac{r}{D} \right)^2 \sin^2 \theta - \left( \frac{r}{D} \cos \theta + 1 \right)^2 \left( \frac{C_m^2}{(x/D)^2} \right) \right) + \exp \left( -\left( \frac{r}{D} \right)^2 \sin^2 \theta - \left( \frac{r}{D} \cos \theta + 3 \right)^2 \left( \frac{C_m^2}{(x/D)^2} \right) \right) \right] \quad (4.26)
\]

The dynamic pressure ratio profile along the x-axis (Equation (4.27)) and the dynamic pressure ratio profiles (Equation (4.26)) at various sections downstream from the jet exits, on the planes perpendicular to x-axis and parallel to the line joining the centre of four nozzles (θ=0) has been computed and the results are shown in Figure 26.

v) Segmented jet nozzle configuration

Reichardt's theory was also applied in another example of theoretical prediction of dynamic pressure decay on the centre-line of 2, 4 and 12 segmented nozzle configuration which was designed and tested by NASA (Higgins (1964)). The areas of all 3 nozzle configurations were chosen to be equal. For the dimensions and geometrical form of these nozzles refer to Figure 27.
The dynamic pressure variation along the central-axis of these three nozzles are reproduced in Figure 27 from the experimental results reported by Higgins (1964).

The theoretical predicted values of dynamic pressure decays along the centre-line of all three nozzles may be obtained from equation (4.15) by setting \( r = 0 \) which after integrations over the area (which is equal for all three nozzles) gives:

\[
\frac{\overline{u_x^2}}{\overline{u_j^2}} = \frac{1}{5} \left( 1 - e^{-\frac{5}{4c^2_m(x/D_e)^2}} \right)
\]  

(4.28)

where \( D_e \) is the diameter of circular nozzle of equal area.

The above equation is also plotted in Figure 27. The discrepancy between the theory and experimental results may have occurred because of boundary layer effects at the nozzle walls.

The experimental evidence discussed above indicates that Reichardt's hypothesis may be applied to the problems of complicated initial flow conditions. The practical utility of this principle can therefore be extended to more complex situations.
4.4 Effects of 'square' velocity distribution on entrainment flow

One of the major factors effecting the entrainment flow into a jet flow, hence the jet induced loads, is the velocity distribution at the exit.

Attempts are made in this section to show the influence of the rate of annular/core velocity ratio of a 'square' velocity distribution at the jet exit on the entrainment flow into the axi-symmetric jet.

This kind of velocity distribution occurs more frequently in the modern turbine driven lift engines. The distributions vary in practice from a higher velocity outer annulus, such as may arise from a tip driven fan or from the presence of the cone shaped bearing housing within the engine, to a higher velocity core jets such as the by-pass ratio engine.

Therefore, in the study of the effects of the annulus to core velocity ratio in the present theoretical approach, it was decided to attempt to vary the annulus to core velocity ratio from an extreme case of zero annulus velocity ($U_a = 0$) to the opposite extreme case of a ring jet, i.e. with zero core velocity ($U_c = 0$).

The velocity profile in the radial direction downstream of an axi-symmetrical jet with 'square' velocity distribution at the exit was computed by the method of Reichardt at positions of 1, 2, 3, 4, 5, 6, 8, 10, and 20 annulus diameters downstream from the exit. The areas of the core and annulus jet nozzles were chosen to be nearly equal and the ratio of core/annulus jet diameter was then equal to 0.69. The computations were carried out for the annulus/core velocity ratios of 0, 0.5, 0.8, 1, 2, 3, 5, 10, 20, 50, 100 and $U_c = 0$. Some typical results are shown in Figure 28 to 30 in which the ratio of
the velocity in the jet flow divided by the core velocity at the exit \((U_x/r/U_{0,0})\) is plotted against the non-dimensionalized distance from the centre-line axis divided by annulus jet diameter \((r/D_a)\).

The mass flow rate of the jet increases with the axial distance from the origin by drawing in the surrounding air (entrainment).

The change of mass flow in the axial direction is equal to the local entrainment rate \(\frac{dM}{dx}\).

In case of an axi-symmetrical jet with a 'square' velocity distribution at the exit the mass flow \((M)\) is given by:

\[
M = 2 \pi \int_0^\infty U_x, r \cdot r \cdot dr \quad (4.29)
\]

the integral can be written as:

\[
M = 2 \pi \int_0^\infty U_x, r \cdot r \cdot d(r/D) \quad (4.30)
\]

where \(\rho\) is the fluid density of air; and \(D\) is the diameter of annular jet.

The initial mass flow at the axi-symmetrical nozzle of the jet \((M_0)\) is according to:

\[
M_0 = \frac{\pi D_c^2}{4} \cdot U_{0,0} + \left( \frac{\pi D_a^2}{4} - \frac{\pi D_c^2}{4} \right) \cdot \bar{U}_a \quad (4.31)
\]

where \(D_c\) is the diameter of the core.

Dividing Equation (4.30) by Equation (4.31) leads to:

\[
\frac{M}{M_0} = C_R \int_0^\infty \frac{U_x, r}{U_{0,0}} \cdot r/D \cdot d(r/D) \quad (4.32)
\]
where

\[ C_R = \frac{8}{\left( \frac{D_c}{D_a} \right)^2 + \frac{U_a}{U_c} - \left( \frac{D_c}{D_a} \right)^2 \frac{U_a}{U_c}} \]  \hspace{1cm} (4.33)

or

\[ C_R = \frac{8}{K_A^2 + K_S - K_A^2 K_S} \]  \hspace{1cm} (4.34)

The integration of equation (4.32) has been carried out by the use of Simpson's rule. The ratio of \((M/M_0)\) was computed directly at the end of each run, as the velocity profiles at successive positions downstream of the jet were calculated. The results of the theoretical evaluation of velocity profiles downstream from a circular jet, with "square" velocity distribution profile of different \(U_a/U_c\) ratios at the exit, are shown in Figures 28-30. The results of mass flow rates \((M/M_0)\) are plotted against the length distance away from the jet divided to jet annular diameter \((X/D)\) in Figure 31.

This diagram indicates the linear proportionality of mass flow \((M)\) after about 4 diameters downstream in the region of fully turbulent flow. The typical diagram shown in Figure 32 indicates that the value of mass flow rate \((M/M_0)\) in the case of a uniform jet exit velocity is minimum. In other words, jets with a uniform exit velocity distribution entrain the minimum quantity of surrounding air, hence minimum induced loads.

The non-dimensional rate of entrainment \(c_e = \frac{dM}{dx} \frac{D}{M_0}\) was evaluated by differentiating the computed numerical results at successive positions downstream of the jet. The results are plotted against the distance away from jet divided by the jet annular diameter \((X/D)\) in Figure 32. These curves can now be represented as the strength of the line sink distribution to simulate the entrainment flow downstream of the nozzle for various \(U_a/U_c\) configurations.
The line sink distribution of the uniform jet is reproduced in Figure 33 and is compared with the theoretical results of Bradbury and Wood (1964) and experimental results of Hill (1972).

The sink distribution line and its image, as will be fully described in chapter VI, give rise to an induced velocity directed towards the jet centre-line.

If \( V_S \) is the velocity directed to the centre of a sphere of radius \( R_S \) with a sink point in its centre, the mass flow \( (m) \) towards the centre is given by the equation:-

\[
2m = V_S \cdot 4\pi R_S^2
\]  

(4.35)

The induced velocity \( (V_r) \) directed towards the jet centre-line takes the form

\[
V_r = V_S \frac{r}{R_S}
\]

(4.36)

Substituting Equation (4.35) into Equation (4.36) leads to

\[
V_r = \frac{2m}{4\pi R_S^2} \frac{r}{R_S}
\]

(4.37)

From the geometrical relationship

\[
R_S^2 = x^2 + r^2
\]

(4.38)

after substituting Equation (4.38) into Equation (4.37) takes the form:-

\[
V_r = \frac{mr}{2\pi (x^2 + r^2)^{3/2}}
\]

(4.39)
differentiating equation (4.39) by \( dx \) leads to:

\[
\frac{dV_r}{dx} = \frac{r}{2 \pi \rho} \frac{dm/dx}{(x^2 + r^2)^{3/2}}
\] (4.40)

or

\[
V_r = \int_0^\infty \frac{r}{2 \pi \rho} \frac{dm/dx}{(x^2 + r^2)^{3/2}} \, dx
\] (4.41)

Equation (4.41) can also be written in the following form by considering equation (4.31)

\[
\frac{V_r}{U_{0,0}} = C_{EN} \int_0^\infty \frac{dm/dx}{((x/D)^2 + (r/D)^2)^{3/2}} \frac{D}{Mo} \, d\left(\frac{x}{D}\right)
\] (4.42)

where

\[
C_{EN} = \frac{1 - K_A^2}{8} K_s + K_A
\] (4.43)

where \( K_A = \frac{D_c}{D_a} \) and \( K_s = \frac{U_a}{U_c} \)

The integration indicated in equation (4.42) was performed by the use of Simpson's rule and the results for various annular to core jet velocity ratios are shown in Figure 34. In this diagram the pressure coefficient \( (C_p) \) which is equal to the square of the radial velocity or entrainment flow velocity to the square of the jet nozzle core velocity is plotted for various annular/core velocity ratios against the radial distance from the centre-line divided by annular nozzle diameter \( (r/D_a) \). The result for a uniform axi-symmetrical jet is shown in Figure 35 which is compared with those of the theoretical results of Wyganski (1964) and the experimental results of Stephenson (1968) and indicate a fair
agreement. The entrainment pressure coefficient of the two extreme cases of core jet \((U_a = 0)\) and annular ring jet \((U_c = 0)\) are replotted again in Figure 36 which indicates a high entrainment flow rate in case of a ring jet.
5. Presentation and discussion of experimental results

5.1 Introduction

In carrying out model tests to study various aspects of the hovering phase of flight, it has been noted that there are significant differences between the decay of the centre-line velocities in the various axi-symmetric circular jets used in the experiments. These differences are important because they imply that the jets are spreading at different rates and that the entrainment into them is different. Good examples of these differences have been discussed in the previous chapter and can be seen in Figure 37, which shows the decay of the jet centre-line velocity from experiments of Kuhn (1959) Gentry and Margason (1966) and Bradbury (1972). Although the jet nozzle details in each of these tests differed from one another, there was nothing in the experimental arrangements which would have suggested that the differences in the jet development would be as marked as they appear to be. It is obviously important to try and establish the cause of these differences and this chapter contains the results of the experiments and studies the variety of factors which it was thought might be responsible for the variations in the development of axi-symmetric jets.

Therefore, the major objective of the present investigation on the axi-symmetric circular subsonic turbulent jet was to highlight the importance of the influence of the initial conditions of the jet nozzle and the state of flow within the jet exit, as well as further downstream in the region of fully turbulent flow. From the description and discussion of the results of the present investigation it appears that the initial characteristics of an exhausting jet have a larger influence on the jet flow
than has been generally appreciated in the past. This influence appears to be more effective in the near field region, where it is generally believed that the excitation of turbulence as well as the re-shaping of the mean velocity profile occurs. Also the greater portion of the noise from a jet comes from this region which is covered by the first one to eight diameters from the nozzle.

The purpose of the present investigation reported in the remainder of this chapter is therefore to study the various factors which will influence the rate of spreading of a single jet issuing normally from a plane wall. In the hovering phase of flight of a VTOL aircraft, the entrainment arising from the spreading of the jet gives rise to the lift loss, and it is, therefore, clear that different configurations of jet nozzle and air supply to them will have an influence on the rate of spreading of the jets issuing from them.

All the measurements except those indicated in section 5.6 were carried out on an axi-symmetric circular subsonic free turbulent jet of eight inches diameter, on the main rig described in details in Chapter three. The results of the experiments discussed in section 5.6 were obtained from the tests carried out on a circular jet with an unusual rectangular plenum chamber.

In order to examine whether the flow in the parallel part of the jet nozzle and at the jet exit itself is well established and the velocity at the jet nozzle is uniform, the following tests were carried out during the "build-up" process of the main rig.

1) Before mounting the parallel part of the jet nozzle and the plane wall, the mean velocity as well as the turbulent intensity across the contraction duct mounting were measured. Apart from the boundary layers on the wall, the velocity distribution across the exit of contraction duct was found
to be uniform. The turbulence was of the order of about 1.4 percent of the local velocity.

ii) After the parallel part of the jet nozzle (a tube of fifteen inches length and eight inches inside diameter) was attached, the mean velocity distribution as well as the intensity of turbulence across the exit of this extension tube were measured. The results which were obtained after traversing across the jet nozzle several times, were plotted in non-dimensionalized form of \( \frac{U_o r}{\bar{U}_o} \) and \( \sqrt{\frac{u'^2}{U_{loc}}} \) against the radial distance from the centre-line divided by the jet nozzle diameter \( r/D \), as shown in Figures 38 and 39. The subscript notations used above indicates the conditions at points \((x,r)\) of the cylindrical coordinates with the origin at the centre point of the jet nozzle. \( o, r \) indicates the condition at point \( 0 \) on the centre-line axis of the jet and a distance \( r \) away from the centre-line.

The velocity profile across the jet nozzle was found to be uniform within 2% apart from the boundary layer on the nozzle walls. The turbulent intensity level in the plane of the jet nozzle, apart from the boundary layer, varied from 1.3 to 2.1 percent of the local mean velocity, depending on the jet Reynolds number based on the jet diameter. Figure 39 shows the turbulent intensity levels at the jet exit with velocities of 18.8, 70 and 120 feet per second, corresponding to jet Reynolds numbers based on jet diameters of \( 1.05 \times 10^5 \), \( 3.85 \times 10^5 \) and \( 6.61 \times 10^5 \). The influence of Reynolds number on the development of jet flow will be fully reported in this chapter.

The plane wall was then mounted flush to the jet nozzle and the axi-symmetry of the jet was then checked by a pitot tube traverse on the main traversing gear and traversed both in the vertical and horizontal directions at two positions of six and fifteen jet diameter downstream from the jet nozzle. In this test the jet was issuing with a uniform mean velocity of 120 feet per second. The results of these measurements, which were plotted in form of total
head pressure $P_t$ against the radial distance from the centre-line divided by the jet nozzle diameter ($r/D$), are shown in Figure 40, and indicate that the jet flow is indeed axi-symmetrical.

The mean velocity profile and turbulence intensity across the jet nozzle were checked again after the mounting of the plane wall flush to the jet nozzle and no change was found in the flow characteristics at the jet nozzle with and without the plane wall. The velocity distribution apart from the boundary layer on the walls of the nozzle was uniform and the turbulence level was 1.47 percent of the local velocity.

The boundary layer displacement thickness was measured for various jet diameter Reynolds Numbers which will be described in more detail later in this chapter.

The peaks of the total head pressures were found by traversing the pitot tube along Y and Z directions on two positions at six and ten diameters downstream from the exit, in order to obtain the centre-line of the axi-symmetric more accurately. The pitot tube was traversed until a maximum was detected.

The centre-line mean velocity measurements and also mean velocity profiles in the planes perpendicular to the axis of symmetry were all taken by means of a pitot tube. These total head measurements were carried out with the assumption that the static pressure was ambient everywhere.

The flow in the established region was rather turbulent and no corrections were made for the influence of turbulence on the pitot tube measurements. It was shown in Chapter three that the influence of low level turbulence on the total head readings of a pitot tube is small and may be ignored. On the other hand its effects on the measurement of static pressure coefficient is significant. Bradbury (1963) showed that the measured static pressure coefficient differs significantly from the true static pressure coefficients by the presence of low level turbulence in the flow.
5.2 Uniform circular jet with "clean nozzle"

A "clean nozzle" is generally considered to be a circular jet nozzle with no restrictions and no disturbances either on its circumference or in its parallel section. The flow at the exit of this nozzle should be uniform with low turbulent intensity level.

A question of immediate interest for this part of exploratory investigation on an axi-symmetric subsonic jet with a relatively large circular "clean nozzle" of eight inches in diameter was to obtain a set of results, such as mean velocity decay along the jet centre-line, mean velocity profiles and turbulent intensity in the core region and fully turbulent flow region, all to be considered as a set of patterns. The test results of other jet configurations could then be compared with these experimental results.

A broader aim was the collection of data on the characteristics of the flow downstream of a considerably larger scale of jet nozzle dimensions, in other words, a larger jet diameter Reynolds Number with a lower velocity (Re = 7 x 10^5), especially in the vicinity of the efflux section, where the results may be determined with sufficient details.

In such a free jet of air discharging at subsonic velocities into stagnant air, there are several well defined regions which are shown in Figure 3. Firstly, there is the initial region in which total head on the jet centre-line remains constant. This so called "potential flow region" or "potential core region" can extend up to about five jet diameters from the nozzle, but its length depends on the precise nozzle and upstream flow condition, which have an influence on the state of flow at the exit. In some cases this region can be much shorter than five jet diameters. This non-turbulent core is a cone-shaped region having its base
on the nozzle opening and its apex on the central axis of the jet flow. In this region the intensity of turbulence is low, the velocity is substantially uniform and equal to the velocity at the discharge. Surrounding this core is a wedge-shaped annular region with its apex at the tip of the nozzle. Here the flow is highly turbulent and the velocity decreases rapidly away from the turbulent core. Secondly, in the region of more than eight nozzle diameters downstream from the nozzle the turbulence is said to be fully developed, that is, the profiles of the velocity versus radius are similar at successive sections taken perpendicular to the axis of flow. The relative intensity of the turbulence may become 25 percent at some points in this region. Thirdly, in the entrainment region, where surrounding air around the jet flow is entrained into the jet (see Figure 3).

According to the results reported by Ko and Davis (1971), there is a similarity of the mean velocity ratio profiles \( \left( \frac{U_x}{U_{x0}} \right) \) in the potential core region, when plotted against the non-dimensional radial distance \( \varphi \)

\[
\varphi = \frac{r - D/2}{x} \tag{5.1}
\]

The geometrical similarity is true only between axial distances of one to four jet diameters downstream from the nozzle, that is, \( 4 > \frac{x}{D} > 1 \). The nearer to the nozzle exit, the more deviation of the velocity ratio profile from the similarity curve.

The potential core is followed by the "fully turbulent flow region" in which it is well known that the mean velocity profiles are closely geometrically similar. It is well known from the equation of motion that when an axi-symmetric jet is exhausting into a stationary atmosphere, self preservation of the jet flow can occur in which the structure of the flow at all streamwise stations is similar (see Figure 4) and therefore in the case of an axi-symmetric jet the following proportionality can be found.
\[ \delta_{0.5} \propto (x - x_0) \quad (5.2) \]

\[ \tilde{u}_{x,0} \propto \left( \frac{1}{(x - x_0)} \right) \quad (5.3) \]

where \( \delta_{0.5} \) is the radii at which the velocity is one half of the value of the velocity on the centre-line and \( x_0 \) is a shift in the apparent origin of the self-preserving flow due to the particular nozzle condition. In the case of an axi-symmetric jet issuing uniformly into still air, the origin of self-preserving flow seems to roughly coincide with the jet nozzle centre point, that is,

\[ \frac{x_0}{D} \approx 0 \quad (5.4) \]

As far as the turbulence structure is concerned, this self-preserving structure does not usually develop until about sixty jet nozzle diameters downstream from the exit although, as already mentioned, the mean velocity profiles appear to become similar almost immediately after the disappearance of the potential core.

The theoretical models for predicting the shape of the mean velocity profiles are fully discussed in chapter IV. The solution of Reichardt's inductive theory was applied to predict the mean velocity profiles downstream of a uniform jet exhausting into a stationary atmosphere. The results which are shown in Figure 18 and 19 appeared to be in fair agreement with the measured values in the outer part of the flow. A good empirical fit to these profiles (Figure 4) was given for both plane jet and axi-symmetric circular jet (Bradbury (1967)) which is in the form of:

\[ \frac{\tilde{u}_{x,r}}{\tilde{u}_{x,0}} = f(\gamma) = \exp \left[ -0.6749 \gamma^2 (1 + 0.0269 \gamma^4) \right] \quad (5.5) \]

where

\[ \gamma = r / \delta_{0.5} \]
The self preservation possibility of an axi-symmetric jet issuing into still air and experimental data has led to a simple and useful expression for predicting the centre-line velocity decay over a limited region of the flow (Bradbury (1967)).

It was shown that net momentum flux from the boundary layers on the jet inside walls are small, therefore

\[
\frac{\bar{U}_{x,0}}{U_{0,0}} = 6.82 \ (x/D)^{-1}
\]

(5.6)

for

\[ x/D > 8 \]

In order to predict the spread of the jet or jet radii without the application of numerous traverses across the jet, the momentum integral equation may be applied, hence,

\[
\frac{T}{\rho} = 2 \pi \int_0^\infty r^2 \bar{U}_{x,r}^2 \, dr
\]

(5.7)

where \( T \) is the total thrust of the jet.

The contribution from the turbulence term \( (\sqrt{\sigma^2}) \), which is about five percent, to the momentum integral is neglected.

Substituting the value of \( \bar{U}_{x,r} \) from the expression of Equation (5.5) into the Equation (5.7) leads to the following equation.

\[
\frac{T}{\rho} = 2 \pi \left( \delta^2 \int_0^{0.5} \bar{U}_{x,0}^2 \int_0^\infty \gamma f^2 d\gamma \right) = 2.16 \left( \delta \bar{U}_{x,0} \right)^2
\]

(5.8)

Since, from the numerical integration

\[
\int_0^\infty \gamma f^2 d\gamma = 0.341
\]

(5.9)

The thrust \( T \) may be determined from the isentropic values

\[
T = 2 \pi \rho \frac{D^2}{4} \bar{U}_{0,0}^2
\]

(5.10)
and after substitution of Equation (5.6) into Equation (5.8) leads to an expression which indicates the ratio of spread or the values of jet radii \( \delta_{0.5} \) downstream from the exit in the fully turbulent region

\[
\frac{\delta_{0.5}}{D} = 0.089 \frac{r}{D} \quad (5.11)
\]

It is, however, in the non-equilibrium region which consists of up to twenty jet diameters downstream from the nozzle in which the flow characteristics change from those of quasi-plane mixing layer at the edge of the nozzle to these of an "asymptotic jet" where the noise of circular jet is generated. The near field therefore offers a better opportunity for study of noise generation by turbulence.

5.2.1. The mean velocity

The centre-line mean velocity decay measured downstream of the axi-symmetric jet with a "clean nozzle" at a Reynolds number of \( 5 \times 10^5 \) is shown in Figure 41, in which the ratio of the mean velocity on the centre-line divided on the jet nozzle centre-point mean velocity \( (\bar{U}_{x,0}/\bar{U}_{0,0}) \) is plotted against the distance from the nozzle divided by nozzle diameter \((x/D)\). A reasonably satisfactory collaboration of the data from several traverses made along the centre-line was achieved.

These results are compared with those of Bradbury (1972) measured on quite a different jet rig with a considerably smaller scale circular axi-symmetric uniform jet nozzle of one inch diameter with a Reynolds number of \( 0.5 \times 10^5 \) and also the results of Sami (1967) (see Figure 41). These results are in very good agreement with one another and demonstrate that the potential core extends over about four jet diameters. As a matter of interest, the subsequent centre-line jet mean velocity decay was
reasonably well fitted by the empirical expression \((x/D)^{-1}\) of Equation (5.5) for values of \(x/D > 8\). (see Bradbury 1972).

A set of mean velocity profiles measured at successive positions downstream (Sami (1967)) are reproduced in non-dimensional form in Figures 18 and 19. These are compared with the theoretical results obtained by applying Reichardt's hypothesis.

The measurements of static pressure in the jet flow were carried out by a static tube and the static pressure coefficient

\[
(- \Delta P_{st} / \frac{1}{2} \int_{x,0} U^2)
\]

obtained from the results of several longitudinal traverses along the centre-line and several lateral traverses at sections with a distance of \(x/D\) ratio of two, four, eight, twelve and sixteen. The variation of static pressure coefficient along the centre-line of the axi-symmetric jet shown in Figure 42 and the distributions of static pressure (in mm water) along the radial direction at various stations of \(x/D\) values of five and fifteen downstream from the jet nozzle are shown in Figure 43.

Referring to the discussion of the effect of turbulence on the static pressure measurements in Chapter III, the high turbulence levels and the wide range of flow directions encountered in the present tests causes, of course, substantial errors in the static pressure deduced from static tube readings and also the results shown in Figure 42 and 43, but these results at least
can demonstrate the approximate distribution of static pressure in the jet flow. However, it is probably more accurate to assume a uniform static pressure in the jet flow which is equivalent to ambient pressure rather than to make use of static pressure readings in the measurement of the mean velocities.

The measurements of static pressures, even though the measured and actual static pressure may be quite different, could provide a simple mean of checking how a flow approaches self preservation.

The mean velocity direction is pre-dominantly axial. As a check on this the flow angle measurements were carried out, which are shown in Figure 74 and 75. These diagrams show clearly that the flow angles are comparatively small. The flow directions and related angles are discussed in more details later in this chapter.

5.2.2. Turbulent Intensity

Sufficient work has been done on the turbulent intensity in the potential core region as well as the mixing region of an axisymmetric jet such as Laurance (1956), Ko & Davis (1971) and others. This warrants not to repeating this work.

As a pattern indication, to aid comparison of turbulent intensity of various nozzle configurations and initial flow characteristics at the orifice, it was attempted in the present investigation to show only the results for the variation of the axial velocity fluctuation along the centre-line of the uniform jet.

The v-component of the velocity fluctuation at various points in the jet flow is not equal to the u-component turbulent intensity and may differ by as much as seventeen percent (Laurance (1956)). The maximum value of the u-component turbulent intensity in the radial direction occurs near the position of maximum mean velocity gradient, whereas the v-component maxima occurs near the inner
edge of the flow. The variation of the u-component turbulent intensity level \( \sqrt{\frac{u'^2}{\bar{U}}} \) along the centre-line axis of the axi-symmetric circular "clean nozzle" jet, which is plotted against the distance from the exit divided by the jet nozzle diameter \( (x/D) \), is shown in Figure 44. The maximum rate of increase of the u-component intensity occurs in the region of the maximum mean velocity gradient. The intensity measurements were made also at values of the non-dimensional radius \( (r/D) \) distributed across the mixing zone of several positions downstream of the jet nozzle. A typical result of these measurements at four jet-diameters downstream from the jet nozzle is shown in Figure 45. The turbulent intensity in percent of local mean velocity is presented in Figure 44 and 45.

Some of the intensities measured were large compared with local mean flow \( (U) \), see Figure 44, therefore, errors will be introduced because of the interaction of the components of the turbulence; that is, the effect of \( u, v \) and \( w \) on \( U \) and on each other. No corrections have been applied to the results shown in Figure 44.
5.2.3. The effects of Reynolds' number and the jet nozzle boundary layer thickness emerging from a uniform jet with low turbulence "clean nozzle".

The general mixing is initiated when the stream separates from the solid boundary of the jet and a boundary layer has been developed upstream of the separation point. Thus, at its origin, the shear layer has a non-zero thickness and the velocity profile of the initial boundary layer represents an important boundary condition placed on the subsequent development of the layer. The flow in the early part of the mixing layer is dominated by the transition from velocity profiles of a boundary layer type to those corresponding to fully developed mixing further downstream. The thickness of the initial boundary layer determines the scale of succeeding velocity profiles.

Figure 46 shows the decay of the centre-line mean velocity for jet velocities of 18.80, 70 and 120 ft./sec. corresponding to jet Reynolds numbers based on jet diameters of $1.05 \times 10^5$, $3.85 \times 10^5$ and $6.61 \times 10^5$ respectively. The results for the two higher velocities are not significantly different from one another, but a clear difference can be seen in the results obtained at 18 ft./sec. This difference is also shown in the turbulence intensity measurements on the centre-line of the jet. Figure 47. As might be expected, the higher rate of decay of the 18 ft./sec. jet is accompanied by a higher turbulent intensity on the jet centre-line. The discovery of an apparent influence of jet Reynolds number is surprising because the Reynolds number is very large by most model standards and it had been accepted that provided the shear flows were fully turbulent, then the effects of Reynolds number would have been very small and confined only to the small scale elements of the flow. It seems, therefore, much more likely that the observed effect is due to the increased boundary layer thickness which occurs in the 18 ft./sec. test (see Figure 38). It may be argued that this is a Reynolds number effect, but it is, in fact, an extra
parameter capable of independent control. Some support for this can be obtained from a test reported by Bradbury (1972) on a 1" diameter jet with a jet velocity of 100 ft./sec. and a boundary layer displacement thickness, $\delta^*$, of about 0.007\". The Reynolds number of this test was $0.5 \times 10^5$ which is lower than any of the present tests and yet the results shown in Figure 46 agree very closely with the present tests at 70 ft./sec. and 120 ft./sec. The tentative conclusion from the present tests is, therefore, that it does not appear to be any measureable Reynolds number effects over the range considered, nor is it likely that there will be any influence at higher Reynolds numbers. On the other hand, it does appear that the boundary layer thickness on the jet nozzle does begin to influence the decay rate if the ratio of displacement thickness to jet nozzle diameter exceeds about 0.01.

5.2.4. **Auto-correlation, Cross-correlation and power spectra density functions in the jet flow with a clean uniform nozzle.**

A number of tests were carried out to study the vortex rings produced in a jet flow exhausting from a uniform clean nozzle. Although this was considered as a peripheral investigation, which will be reported in more detail in Chapter VII, it is of great importance to evaluate the flow characteristics of a jet exhausting from a uniform clean nozzle. Therefore, a number of correlation measurements were made with two hot wire anemometers to study the flow characteristics and also to find out whether the vortex motion were rings or spirals.

5.2.5. **Direction of flow in an axi-symmetric circular jet with uniform "clean nozzle"**

The angle measurements were taken in the flow of the uniform jet by means of the five-hole yaw tube mentioned in Chapter III. The
definition of measured yaw and pitch angles, \( \alpha \) and \( \beta \), are illustrated in Figure 74.

In order to compare flow angle characteristics of flow in converging jets with those of an axi-symmetric uniform jet exhausting from a "clean nozzle" the variation of the two yaw and pitch angles, \( \alpha \) and \( \beta \), were measured along and parallel to the Y-axis on the jet exit and at successive positions 2, 4, 8 and 12 jet diameters away from jet nozzle. These measurements are illustrated in Figures 74 and 75. Taking these illustrations into deeper consideration, the following points can easily be recognized. First, the values of both \( \alpha \) and \( \beta \) were found to be zero at the jet exit of the axi-symmetric uniform "clean nozzle" which indicated that the velocities at the jet nozzle plane are parallel to the jet central-axis.

Second, pitch angle, \( \beta \), along and parallel to the y-axis of the cartesian coordinates was, as anticipated, nearly zero. The angle measured for \( \beta \) further away from the centre-line as shown in Figure 75 is small. Third, the three regions of core, transition and fully turbulent can easily be recognized in Figure 74.

Fourth, for distances greater than 8 jet diameters downstream in the fully turbulent flow region the rate of increase of the yaw angle, \( \alpha \), with respect to distance from the nozzle, as was expected, remains constant, which illustrates again that the mean velocity profiles in the fully turbulent region are closely geometrically similar.

5.2.6. Effects of the level and the scale of the initial turbulence at the jet nozzle

Another possible influence on the decay rate of a jet flow could be the initial turbulence level existing in the plane of the jet
nozzle. The scale of this turbulence might also effect the velocity decay downstream. In order to find out how effective these two factors are in influencing the flow of the jet downstream, a higher turbulence level was artificially simulated in the jet nozzle plane. It is well known that screens are used in the flow either for the production or for the reduction of turbulence which is influenced by the screen geometry. The geometry of the screen, in general, may be varied by changing either the relative dimensions of the pattern elements or the pattern itself. The most common pattern is the square-mesh lattice formed of straight bars or wires evenly spaced in both directions in single or double planes. The woven-wire screens commonly used in industry are intermediate between single and biplane types. Another simple pattern in frequent use is represented by perforated plates.

If the screen is imagined to produce a series of jets which coalesce gradually in the downstream direction, see Figure 49, conditions at three sections may be compared (see Figure 48).

i) A cross-section of the free stream well ahead of the screen.

ii) The cross-section at which the jets issuing from the holes are fully contracted but still essentially undiffused.

iii) A cross-section of the free stream well beyond the screen. Between sections 1 and 2 it may be assumed that potential flow exists, while between section 2 and 3 conditions similar to those at any abrupt expansion may be assumed to prevail.

Brian and Peterson (1951) carried out an investigation of flow through lattice-type screens and perforated plates and showed that the u-component turbulent intensity for large values of
fitted the decay law (Equation (5.12)) derived by Frankiel (1948) (see Figure 49).

The fact that the data from the lattices of various \((M/b)\) ratios and perforated plates coincide for large values of \((x/b)\) is significant, indicating that the influence of the screen geometry on the intensity extends only a relatively short distance downstream. The maximum intensity of turbulence along the centre-line of the jet is reached within 2 to 3 mesh lengths from the screen.

In the present investigation a grid was therefore designed to produce a turbulence level of around 5.6 percent of the jet exit velocity which had a longitudinal and lateral scale of 0.75 and 1.5 inches respectively, in the jet nozzle plane. This turbulence grid was built from rectangular cross sectional bars (0.25 inches width) which were evenly spaced in both directions and soldered as a biplane type in the form of a square mesh of one inch side. This grid was mounted at a distance of 1 7/8 jet diameters upstream of the jet nozzle normal to the central axis of the jet between the contraction duct and the parallel part of the jet nozzle.

The \(u\)-component of the turbulent intensity behind the grid was evaluated by traversing the hot-wire along the radial direction, in the parallel part of the jet nozzle, at various positions downstream from the grid. The results, as shown in Figure 48, indicate the influence of the bars upto about \(x/b = 12\) or \(x/M = 3\). Downstream from this formation region the turbulence, as shown also by Taylor (1935), approaches the isentropic condition, that is, the intensity of turbulence becomes the same in each of the three directions \((u, v, w)\). The results of the \(u\)-component turbulent intensity for large values of \(x/b\) on the central axis fit, as shown in Figure 49, the decay law derived by Frankiel (1948).
\[
\frac{\sqrt{\frac{-u^2}{U}}}{U} = 1.12 (\frac{x}{b})^{-5/7}
\]  

This equation is valid for various screens of different \((\frac{M}{b})\) ratio where \(b\) is the width of the bars and \(M\) is the size of the mesh.

The bar width Reynold's number in this test was of the order of 
\[\text{Re} = \frac{U \cdot b}{\nu} = 1.2 \times 10^4\]  
which is larger than the critical Reynolds number of the Frankiel's decay law.

Dryden (1947) showed that in fact, values of the ratio of the lateral scale of turbulence divided by the bar width \((\frac{L_y}{b})\) for screens of different ratio of \((\frac{M}{b})\) seemed to group to a single curve which is illustrated in Figure 49.

From Figure 49 it may therefore be concluded that for every magnitude of turbulent intensity produced behind a grid in the isentropic region there is only one scale of turbulence and one cannot be varied without changing the level of the other.

Whilst taking measurements of the jet exit velocity profile, it was noted that the positioning of the grid normal to the jet centre-line axis effected the jet exit velocity profile. By altering the angle of attack by one or two degrees the shape of the velocity profile at the exit changed dramatically.

The influence of the grid on the decay of the centre-line mean velocity as well as the \(u\)-component of the centre-line turbulent intensity is illustrated in Figure 41 and 42. This clearly shows that turbulence of this magnitude at the jet exit could not have a significant effect on the jet development, on the jet centre-line velocity decay and on the jet centre-line turbulence level. This is not altogether surprising as it might be argued that jet nozzle turbulence is unlikely to have
any influence until it reaches a level comparable to that occurring naturally in the mixing layers at the edge of the potential core. This implies that jet nozzle turbulence is unlikely to have much effect until it reaches a level of about 20 percent of the jet exit mean velocity.

A few measurements of the spectrum and auto-correlation of the u-component turbulence were also carried out on the centre-line axis of the jet at positions 0, 2, 4, 8 and 12 jet diameters downstream whilst the turbulence grid was mounted between contraction duct and the parallel part of the jet nozzle. The results were then compared with those obtained from the "clean nozzle jet". Figure 97a shows a typical diagram of the one dimensional spectrum of the u-component turbulence measured on the centre-line of the jet at a distance of four jet diameters downstream which is plotted against the non-dimensional frequency (f. D/\nu) and compared with the turbulence spectrum of the clean nozzle jet. The spectrum in both cases with low and relatively high initial turbulence, as shown in Figure 97a exhibited a fairly well defined peak between 0.4 to 0.5 non-dimensional frequency (f. D/\nu). This peak as will be discussed later in Chapter VII is associated with a system of vortex rings which is shed by a jet.

5.2.7. The effects of non-uniform velocity profile at the exit of the jet

Attempts to derive a theoretical method of producing the large variations of a jet entrainment flow due to non-uniform jet exit velocity distribution have been fully discussed in Chapter IV. The theoretical prediction was backed by the experimental results of Stephenson (1968) and also by the experiments carried out in the course of present investigation on two "square" velocity profiles at the nozzle with annular to core velocity ratios (U_a/U_c) of 2.0 and 1.6.
The results of the mean velocity profiles at successive positions of \(x/D = 0.063\), \(x/D = 2\), \(x/D = 4\), \(x/D = 8\) and \(x/D = 16\) downstream from the jet nozzle and also the mean velocity decay on the centre-line of the jet flow are presented in Figures 21 and 22. These clearly show the significant effects of a non-uniform jet exit velocity on the spread of the jet flow compared with the results of a jet with uniform exit velocity.
5.3. Effects of disturbances such as tabs placed on the circumference or upstream of the jet nozzle

In the measurement of one-dimensional spectrum of turbulence taken on the centre-line axis of the circular jet, both with and without the turbulence grid, it was realized that in both cases, the spectrum exhibited a fairly well defined peak, as illustrated in Figure 97a, which was measured at a distance of four jet diameters downstream from the jet nozzle. The presence of this peak which is associated with a system of vortex rings shed by the jet was confirmed by various auto-correlation measurements taken in the jet flow. The presence of this vortex ring structure in the potential core region had already been observed in general accordance with the observation of some other experimenters. The results of a number of measurements on these ring structures will be described in Chapter VII. This vortex ring shedding, which is a comparatively well known phenomenon, is a source of some difficulties in the operation of open jet tunnels, and in order to suppress them, numerous small tabs are usually mounted around the perimeter of the open jet nozzle.

Initially it was thought that the vortex rings might play an important part in the initial development of the jet, so some tests were carried out in which various sizes and numbers of tabs were mounted on the jet nozzle perimeter and on the inside surface of the parallel part of the jet nozzle itself.

The results of the initial tests showed an unexpected and a most interesting effect on the decay of the centre-line velocity as well as the corresponding turbulent intensity. Hence, numerous tests were carried out by using precise machined square tabs mounted with a wide variety of tab arrangements on the circumference of the uniform axi-symmetric jet nozzle or inside the parallel part of the jet nozzle. The results are too numerous to include here but table 2 shows a typical set of tests which
was found to be interesting. This table shows the number and the dimensions of the tabs used in each test on the circumference or within the parallel part of the jet nozzle. In this table \((N)\) represents the number of tabs, \((H)\) and \((B)\) are the heights and widths of the tabs used in each test respectively. The results of these tests, which show the effect of the tabs on the decay rate of the centre-line mean velocity and the corresponding turbulent intensity are plotted against the non-dimensionalized distance from the jet nozzle, are illustrated in Figure 50 to 64. Figure 50 shows the results of the effect of square tabs with a side length of \(\frac{1}{16}\)th of the jet diameter, \((\frac{B}{D} = \frac{H}{D} = \frac{1}{16})\), on the centre-line mean velocity decay of the jet. Figure 51 shows the corresponding effect on the \(u\)-component turbulent intensity. These graphs are plotted in non-dimensionalized form. The interesting effect of the tabs, as shown in Figure 50 and 51, is that when eight tabs were mounted around the perimeter of the nozzle, the effect on the jet development was not very large. However, when the number of tabs was reduced to two, the apparent potential core length was reduced to two jet diameters followed by a rapid decay of the centre-line mean velocity. The centre-line turbulent intensities relative to the local mean velocities were also increased. By increasing the area of each tab, the effect of the tabs on the jet flow became larger. Figure 52 shows the result of the tests with two square tabs of various height \((H)\) and width \((B)\) mounted on the circumference of the jet nozzle. In the case of \(H/D = 1/4\) and \(B/D = 1/8\), as is illustrated in Figure 52, the potential core nearly disappeared and the flow becomes fully turbulent at a distance of 0.5 jet diameters.

Figure 53 shows the results of the mean velocity decay rate on the centre-line axis of the axi-symmetric jet when two or four larger tabs were mounted on the perimeter of the jet nozzle. The effects of a single tab configuration and its mounting
position around the perimeter of the jet nozzle on the centre-line mean velocity decay rate has been checked and the results are shown in Figure 54. These are compared with those of no tab (clean nozzle) and two tab configurations. When mounting the single tab at different positions around the perimeter of the jet, it was found that small discrepancies existed in the results. These were thought to be due to the repositioning of the tabs.

From the numerous tests carried out while the turbulence grid was mounted between the contraction duct and the parallel part of the jet nozzle, it can be realized that the initial turbulence at the exit, with tabs on the circumference of the jet nozzle, did not show any significant effect on the mean radial velocity distribution. However, there seemed to be some changes in the turbulence intensity along the centre-line axis.

Attempts were made to evaluate a kind of correlation between the results of various tab configurations. A parameter which showed to be of significance was the area of each tab divided by the number of tabs used in each test. This parameter was normalized by dividing it by the square jet diameter, hence

\[
\frac{K}{D^2} = \frac{B/D \cdot H/D}{N}
\]  

(5.13)

Figure 52 shows the variation of the mean velocity decay rate on the centre-line axis for each group of tests with the same value of parameter (K) but with different tabs configurations. The agreement in the results of the groups of tests with the same parameter (K) were found to be satisfactory. On the other hand this fair agreement in the results might have been coincidental.

Figure 52 indicates also the effects of the variation of the width and the height of the tabs on the centre-line velocity decay downstream from the jet nozzle.
A number of mean velocity traverses were made across the jet at several stations downstream from the jet nozzle with two square tabs with a length of 1/16th of the jet diameter mounted on the jet nozzle perimeter. Figures 55 to 57 show some of the results of these tests. It is clear that the tabs produce a gross distortion of the jet and it is not until values of x/D greater than about 15 are reached that the mean velocity field becomes approximately axi-symmetric. Figures 58 to 60 show the results of turbulent intensity traverses on the jet nozzle plane and in the region near the nozzle at a distance of 1 and 4 jet diameters downstream and it is clear that the distortion of the mean velocity field is also accompanied by a non-axi-symmetric distribution of turbulent velocity. Moreover, as was noted in the previous discussion and referring to Figures 50 and 51, the general level of the turbulent intensities on the centre-line of the jet are markedly increased by the presence of the tabs. These results showed how tabs tend to split the jet almost in two with high velocity cores on either side of the tabs.

Figures 61 and 62 show the results of a number of mean velocity measurements made across the jet flow at several stations downstream from the jet nozzle with two and four tabs configurations of larger area. This illustrates again the jet flow development downstream from a nozzle with tabs at its perimeter.

This effect was confirmed from some traverses at two and four jet diameters downstream of the jet nozzle with the same two square tabs configuration (with a length of 1/16th jet diameter) and the constant velocity contours shown in Figures 63 and 64 were constructed. Detailed traverses were made in only one quadrant of the jet although check measurements showed a high degree of symmetry in the other quadrants.

Attempts were also made to find out the effect of these tabs on the jet flow downstream, when the tabs were placed in the radial
direction of the inner surface of the parallel part of the jet nozzle. The results of these tests are shown in Figure 65 which show the mean velocity ratio decay on the centre-line axis of the jet. Figure 65 illustrates the interesting effect of the location distance of the tabs, \( s \), away from the jet nozzle plane, on the jet development when two tabs of \( H/D = 1/8 \) and \( B/D = 1/16 \) are used. It shows clearly that the effect of tabs on the jet flow becomes small when the location distance, \( s \), increases.

There seemed to be two possible mechanisms by which such gross distortions could occur by mounting two square tabs with the length of \( 1/16 \) of the jet diameter on the perimeter of the jet nozzle, namely:

(i) by the "stirring" action of trailing vortex motions used from the tabs.
(ii) by the simple deflection of the flow over the tabs such as that which might occur in a potential flow jet with circumferential variations in flow angle. This in turn is thought to have the stirring action on the periodic structure of the vortex rings shed from the clean jet nozzle.

In order to check the trailing vortex motion, that is in case (i), it would strictly be necessary to make circulation measurements in the flow behind the tab, but in view of the magnitude of the distortion, it seemed likely that the presence of vortex motion shed from the tabs would have been easily observable with a simple wool tuft. However, no such motions could be detected and it seemed more probable that the distortion arose from the circumferential variations in flow angle produced by the tabs, that is, case (ii). Some evidence for the possibility that variations in flow angle might be responsible for the distortion of the jet can
be found in a paper by G.I. Taylor (1960) on the formation of thin flat sheets of water. It would appear that, in water jets, angular variations around the circumference of a jet do not die away downstream but can lead to gross changes in the cross-sectional shape of the jet. In fact, on reflection, it is difficult to construct any simple physical argument which would lead to a mechanism involving potential flow pressure changes for reducing circumferential flow angle variations in the case of flow with free surfaces.

Obviously, with air into air jets, the turbulent mixing greatly complicates the flow structure and ultimately the jet structure returns to an axi-symmetric self-preserving type of jet, which has been the subject of many investigations. Nevertheless, in the region near the jet nozzle, it seems very likely that flow angle variations can lead to gross distortions in the jet development. Flow angle variation caused by the presence of tabs which was thought to be responsible for the distortion of the jet flow and changes in the cross-sectional shape of the velocity profiles has been investigated. The test of two square tab configurations with a length of 1/16th jet diameter (configuration No. 4 of table 2) was repeated and flow angles $\alpha$ and $\beta$, have been measured along and parallel to the diameter joining the centre of the two tabs at the jet nozzle ($x/D = 0.06$) and further away from the jet nozzle at positions of $x/D = 0.3125$, $x/D = 0.56$ and $x/D = 1$. For the definition of the yaw and pitch angles $\alpha$ and $\beta$ see Figure 10. The results of yaw angle $\alpha$ are shown in Figure 66. This clearly indicates a flow deflection of more than 14 degrees close to the edge of the tabs near the jet nozzle plane when compared with the results of "clean nozzle" jet shown in Figures 74 and 75, where both angles $\alpha$ and $\beta$ were nearly zero all along the radius of the nozzle. The maxima of the yaw angle variation $\alpha$ which is close to the edge of the tabs at the jet nozzle was reduced and moved towards the centre-line.

* Tests in the fully turbulent region beyond about fifteen diameters downstream showed that an axi-symmetric had already re-established itself.*
axis further downstream. Pitch angle, $\beta$, was nearly zero along the diameter joining the centre of the two tabs.

In order to confirm the "Stirring" action of the deflection of the flow over the tabs on the periodic structure of vortex rings shed from the "clean nozzle jet", various one-dimensional spectrum of the $u$-component turbulence were carried out. Typical results of these tests are illustrated in Figure 97a and compared with those of a circular jet flow with a "clean nozzle". The spectrum tests were carried out at successive positions of $(x/D)$ values of 0, 2, 4, 8 and 12 on the centre-line axis of the jet with two-tabs configuration No.4 shown in table 2. Also spectrum measurements were carried out behind the tab itself, that is by $r/D = 0.5$ and $x/D = 0.1$ which did not show any significant effect on the jet flow.

The results, as shown in Figure 97a, indicated clearly that the periodic structure disappeared with the presence of two comparatively small tabs. This argument was backed up and confirmed by auto-correlation measurements taken at successive positions similar to those of spectrum measurement tests. The periodic variation of auto-correlation coefficients which represent a sort of a periodic loop superimposed on the signal (approximate equivalent to the peaks in the spectrum) disappeared with the presence of the two tabs (see Figure 97b).
5.4 Tests on the effect of wedges on the jet development.

As an addition to the tab results, it seemed interesting to check the assumption that the presence of tabs at the perimeter of the jet nozzle causes the jet flow direction to be varied partially at the jet exit, the mechanism by which the rate of mixing as well as the mean velocity decay rate on the centre-line axis so dramatically changed. Therefore, some further tests were carried out in which two wooden wedges were mounted parallel to the jet central axis on opposite sides of the nozzle walls. The end cross-section of the wedges was the same as the square tabs with side length of 1/16th jet diameter. In actual fact, by varying the wedge angle, the flow direction at the wedge area will be changed. Several tests were carried out using a range of wedges with a half angle \( \theta = 1.9, 5, 20, 45, 60 \) and 90. The effect of these wedges on the centre-line mean velocity ratio variation of the jet is shown in Figure 67. It is clear that there is a progressive effect of increasing the wedge angle and it would seem that wedges with an angle of less than 5 degrees have very little effect on the development of the jet.

Figure 68 shows the yaw angle variation of the flow at the nozzle exit along the diameter joining the two tabs. These results show that the effects of the wedges and tabs produce variation in flow characteristics across the entire jet nozzle and the angles are progressively increased by increasing the wedge angle. It should also be mentioned that several checks were made to ensure that the flow angle on the diameter at right angles to the diameter joining the tabs was always zero in the tests with wedges and tabs.

At this stage it is not obvious to what extent the importance of these 'tab' results could be, however, it does suggest that the presence of comparatively small disturbances at the jet nozzle causes partial change in the flow direction at the jet nozzle.
which in turn produces dramatic effects on the subsequent development and spreading of the jet flow.

Two possible applications of the tabs could be as follows:

1 - In the well known problem of the ground impingement of the downwash from any VTOL aircraft which due to the high velocity and temperature efflux from the lift - engines produce operational problems. The main undesirable problems, depending on the type of landing site being used, are hot gas ingestion, ground erosion, high noise level and infra red radiation. These problems can be alleviated at source by mixing the lift engine efflux with a large quantity of ambient air immediately downstream from the jet nozzle. Several types of high velocity mixing nozzles of various shapes are introduced by Higgins (1964) and Szlenkier (1967).

The tabs, which are simpler and more economical to produce, could perhaps be used in a much simpler manner in an attempt to attenuate the impingement problems of VTOL aircrafts in the hovering manoeuvres.

2 - In the analysis of the noise emitted from a circular axisymmetric jet, Lilley (1958) indicated that the important factors which increase the noise level, apart from the high speed, are the mean shear, the mean square turbulent velocities and the scale of eddies in the turbulence. The reduction of noise was associated with a rapid spreading of the jet immediately downstream of the jet exit. He introduced and tested various devices such as the "toothed" type of nozzle noise suppressors which reduced the mean shear and the length of the potential core which was followed by a rapid fall off in the mean velocity on the centre-line axis of the jet.

Among the various noise reduction devices Westley & Lilley (1952) also tested the noise level for thirteen different teeth configurations which were found to be satisfactory devices in reducing the noise level of the jet.
A serious objection to this method of noise suppressing technique was that the teeth created a drag which cause a loss in the total thrust of the jets (see also Lilley (1958)). The results of Figure 52 shows, however, that by increasing the number of tabs the decay rate of the centre-line mean velocity nears the decay curve of a 'clean nozzle'. The maximum rate of spread occurred with the two tabs configuration. By using two tabs on the circumference of the jet nozzle the problem of the drag might be very much reduced.

There might be the possibility of even more flow deflection behind the two tabs, hence, there could be a more rapid mean velocity decay rate, by setting the tabs inward towards the direction of upstream flow. This needs to be investigated further.
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<th>Tab s Height ($\frac{H}{D}$)</th>
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<td><img src="0x0" alt="Diagram" /></td>
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**Table 2**: Test on a uniform axi-symmetric circular jet with tabs and wedges on the circumference of the nozzle and in the nozzle throat.
<table>
<thead>
<tr>
<th>Group No.</th>
<th>No. of Test</th>
<th>Number of Tabs (N)</th>
<th>Tab s Width (B/D)</th>
<th>Tab s Height (H/D)</th>
<th>Distance from Nozzle plane (S/D)</th>
<th>Parameter K=64B/D.H/D (N)</th>
<th>Wedge Half Angle (\theta)</th>
<th>Position of the Tabs and Wedges</th>
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**Table 2:** Test on a uniform axi-symmetric circular jet with tabs and wedges on the circumference of the nozzle and in the nozzle throat.
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<th>Tab s Width ($b/D$)</th>
<th>Tab s Height ($h/D$)</th>
<th>Distance from Nozzle plane ($S/D$)</th>
<th>Parameter $K = \frac{B/D}{H/D} N$</th>
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<td>90</td>
<td></td>
</tr>
</tbody>
</table>

**Table 2**: Test on a uniform axi-symmetric circular jet with tabs and wedges on the circumference of the nozzle and in the nozzle throat.
5.5 Effects of converging jet.

Following the above discussion on the effect of tabs and wedges on the jet flow development, it has been concluded that any variation in the flow direction upstream from the jet nozzle or at the jet nozzle plane, causes a substantial change in the mean velocity decay rate on the centre-line and jet flow development downstream. Therefore, attempts have been made to investigate the effect of jets with a wedge shaped ring inserted into the nozzle. These rings were intended to produce axi-symmetric converging jets with wedge angles of 10 and 45 degrees. As was expected, a positive static pressure built up on the jet nozzle plane. The results of static pressure tests are given in Figures 69 and 70 which illustrate the distribution of the ratio of static pressure divided by local total head pressure along the radius of the jet nozzle and also along the jet central axis. Figure 69 shows that the static pressure in the middle point of the converging jet nozzle, with a half angle of 45 degrees, is 50% larger than the middle point static pressure of the converging jet with a wedge angle of 10 degrees. Figure 70 shows the static pressure distribution on the centre-line axis of a converging jet nozzle with a wedge angle of 10 degrees which is compared with the results of a uniform axi-symmetric "clean nozzle" jet.

Owing to the build up of non-uniform static pressure across the converging jet nozzle, the distribution of the mean velocity should not, therefore, be uniform at the jet nozzle. Figure 71 shows the velocity profile along the radius of the two mentioned converging nozzles which is compared with the results of the uniform "clean nozzle" jet. It was clear from both the mean velocity and turbulent intensity measurements along the centre-line axis of the jet, as shown in Figures 72 and 73, that these uniform converging jets behaved in substantially the same way as the jet from an ordinary parallel nozzle. The small variation
in the mean velocity decay rate of the converging jet with a wedge angle of 45 degrees from those results of a uniform "clean nozzle" jet is probably due to the presence of non-uniform velocity profile at the converging jet nozzle.

The two dimensional converging jet of fluid has been investigated by Taylor (1960) and also by Dombrowski (1960). In a theoretical study backed by experimental investigation, they showed an ideal case in which a converging sheet of fluid passes through a point where all the fluid is concentrated into a small cross-section and then diverges into a sheet in a plane perpendicular to its original plane.

The axi-symmetric converging jet of air mentioned above could therefore be formed of an infinite number of converging plane jets. Hence, considering the above discussion on a converging plane jet of fluid there might exist a swirl in any axi-symmetric converging jet flow.

In order to investigate the possibility of swirl in the converging jet flow and also due to the importance of the flow angle variation discussed in sections 5.3 and 5.4 a number of tests were also carried out to investigate the flow angle directions, that is, to find out the development of yaw and pitch angles, $\alpha$ and $\beta$, downstream from both converging jets mentioned above and also the axi-symmetric "clean nozzle" jet. The flow angles $\alpha$ and $\beta$ were measured by means of a five-hole yaw tube along and parallel to the Y-axis of the jets. They were carried out at distances of 0, 2, 4, 8 and 12 jet diameters downstream from the jet nozzle plane. The value of pitch angle, $\beta$, should theoretically be equal to zero along and parallel to the Y-axis. The results of these tests are shown in Figures 74 and 75 for an axi-symmetric uniform "clean nozzle" jet, Figures 76 and 77 for an axi-symmetric converging jet nozzle with a half angle of 10 degrees and Figures 78 and 79 for an axi-symmetric converging jet with a wedge angle of 45 degrees. These results illustrate no significant change in the case of yaw angle $\alpha$ but show a small change in pitch angle, $\beta$, for $r/D$ values greater than 0.5.
5.6 Effects of plenum chamber configuration

In an attempt to study the jet induced lift losses of VTOL aircraft Gentry & Margason (1966) used the fuselage of a model aircraft which had the form of a rectangular cross-section (Figure 82) as the plenum chamber of the lifting jets. It appeared that the loads induced on a circular plate mounted on this small rectangular plenum chamber were four to five times larger than those induced on the same size plate mounted on an ideal circular chamber. A survey of the rate of the mean velocity on the centre-line of a single jet configuration issuing from the rectangular plenum chamber showed a much more rapid velocity decay compared with the results of an ideal circular plenum chamber.

In order to try and consolidate the cause of such a rapid decay the rectangular plenum chamber which was used by Gentry & Margason (1966) was exactly reproduced. To reduce the rather high ratio of total pressure in the plenum chamber divided by atmospheric pressure ($P_{\text{tot}}/P_{\text{atm}}$) applied in Gentry & Margason's tests and also the reliability of the flow angle measurements, especially near the edge of the nozzle (by choosing a higher jet diameter to five-hole-yaw-tube diameter ratio) the overall dimensions of the plenum chamber model used by Gentry & Margason were increased by 100% and reproduced in the course of the present investigation.

The measurements were carried out along and parallel to the $x$, $y$ and $z$ axis of the cartesian coordinates, the original of which lied on the centre point of the circular jet nozzle. The mean velocity profile at the jet nozzle plane was measured in both $y$ and $z$ direction. The results are shown in Figure 80 which indicates a non-uniform velocity distribution within the low velocity region at the centre of the jet nozzle. Figure 81 shows the ratio of total head pressure divided by total head pressure in the plenum chamber along the centre-line axis, $x$. The results
are compared with those of Gentry & Margason (1966) for both the original and modified rectangular plenum chamber. The pressure ratio along the x axis obtained from the measurements on the uniform "clean nozzle" jet of the present investigation is also shown in Figure 81 for comparison. There seems to be two possible mechanisms by which such distortion of the jet development could occur, namely i) the non-uniform velocity distribution at the jet nozzle and ii) by the simple deflection of the flow at the jet nozzle such as tabs and wedges. In view of the magnitude of distortions, it might be argued that the level of non-uniformity of the velocity profile at the nozzle shown in Figure 80 could not be the cause of such a rapid decay. However, it seems more probable that the distortion arises from the variation in the flow angle produced by the shape of the plenum chamber.

A number of tests were therefore carried out and flow angles $\alpha$ and $\beta$, were measured by means of a five-hole-yaw-tube parallel to the y-axis at the position $x/D = 0.1$ and also along the z-axis at the jet nozzle plane. The results are shown in Figures 82 and 83 which indicate a rather large variation of flow direction at the jet nozzle, especially at the centre point of the jet nozzle.

Yaw angle, $\alpha$, is more symmetric along the z-axis with a maxima at $z = 0$, whereas in the case of measurements along the y-axis the maxima has been moved about 0.06 jet diameter towards the direction of the air supply.

Pitch angle, $\beta$, oscillates about zero and, except near the edge of the nozzle, is not significant.

A survey of the effect of the plenum chamber pressure ratio on the flow angles, $\alpha$ and $\beta$ shows a significant change in the mean velocity decay along the centre-line axis. Figure 84 illustrates the effect of the pressure build up in the plenum chamber on the flow angles, $\alpha$ and $\beta$ on the centre-line axis at the position $x/D = 0.1$. 
6. **Entrainment into an axi-symmetric jet**

6.1 **Introduction**

A jet or a close cluster of jets exhausting normally from the lower surface at the central area of a wing or the body entrains the air and a surface pressure field well below ambient is thereby generated on the lower surface of the wing or body which invariably gives a significant rise to a loss of lift often accompanied by undesirable angular movements. This phenomenon is of high interest in dealing with the performance and control of direct jet lift VTOL aircraft in the hovering phase of flight. This reduction in the net upward thrust or lift has previously been studied on a number of VTOL aircraft models by the direct measurement method (see Stephenson (1968); Mayson et al (1972) and Gentry & Margason (1966)). Various techniques were also introduced for measuring the rate of entrainment flow into the axi-symmetric jet (see Ricou & Spalding (1961); see also chapter III of the present study).

The static pressure distribution around the jet nozzle on the plane wall was also studied and measured by various investigators (see Bradbury & Woods (1965) and Wyganski (1964)).
6.2 Theoretical investigation of the entrainment into an axi-symmetric turbulent jet. Representation of entrainment by a line distribution of sinks.

G.I. Taylor (1958) established the stream-line pattern for various forced and thermal turbulent jets by replacing the jet with a continuous sink. A similar approach has been made by Wyganski (1964) and Bradbury & Wood (1965) to predict the pressure distribution on the surface of an infinite wall from the middle of which an axi-symmetric circular jet emerged at right angles. Their investigation was also based on the replacement of the jet with a line sink of variable strength.

In the course of the present investigation, attempts were made to expand the above mentioned jet representation approach in order to forecast the detailed pressure distribution on the plane wall and also in the region surrounding the jet flow (entrainment region, see Figure 3) of various rates of spread, that is, with various mean velocity decay rates on the centre-line.

The flow induced by the jet, in otherwise quiescent fluid, may be obtained by representing the effect of the jet by a series of sinks of various strength equivalent to the rate of increase of volume flow in the jet. The sinks were assumed to be located on the jet centre-line, that is, the x-axis. In order to account for the presence of the infinite plane wall, which coincided on the y-z plane, image sinks were located on the negative side of the x-axis. The entrainment flow into an axi-symmetric jet exhausting into a stationary atmosphere can be considered to be in two regions, namely:

i) The potential core region near the nozzle and
ii) the fully-developed region further downstream from the jet nozzle.
i) Sink strength in the potential core region

The mass flow of the jet in the nozzle, $M_0$, is

$$M_0 = \frac{\pi D^2}{4} \int f \overline{U}_{0,0} \quad (6.1)$$

The mass flow in the jet, $M$, is

$$M = 2 \pi f \int_0^{\infty} \overline{U}_{x,r} \cdot r \cdot dr \quad (6.2)$$

or

$$M = \pi r_c^2 f \overline{U}_{0,0} + 2 \pi f \int_{r_c}^{\infty} \overline{U}_{x,r} \cdot r \cdot dr \quad (6.3)$$

Where $r_c$ is the co-ordinate of the outside edge of the potential core region.

The velocity profile in the mixing layer around the potential core is assumed to be similar everywhere, namely:

$$\overline{U}_{x,r} = \overline{U}_{0,0} f(\gamma) \quad (6.4)$$

where $\gamma$ is equal to $(r-r_c)/r_{0.5}$ and $f(\gamma)$ is a universal function of $\gamma$ where $(r_{0.5})$ is the value of $(r-r_c)$ at which $\overline{U}_{x,r} = \overline{U}_{0,0}/2$.

Substituting equation (6.4) into (6.3) and dividing by equation (6.1) gives:

$$\frac{M}{M_0} = 4 \left( \frac{r_c}{D} \right)^2 + 8 \left( \frac{r_{0.5}}{D} \right)^2 \int_0^{\infty} \gamma f(\gamma) d\gamma + 8 \frac{r_c \cdot r_{0.5}}{D^2} \int_0^{\infty} f(\gamma) d\gamma \quad (6.5)$$

In a two-dimensional free mixing layer, the shear layer spreads linearly with distance downstream from the jet nozzle, $x$. There-
fore, we also assume that

$$\frac{r_c}{D/2} = (1 - p \frac{x}{D/2})$$  (6.6)

and

$$\frac{r_{0.5}}{D} = q \frac{x}{D}$$  (6.7)

where both $p$ and $q$ are constants, substituting equations (6.6) and (6.7) into equation (6.5) gives:

$$\frac{M}{M_0} = (1 - p \frac{x}{D/2})^2 + 2q^2 \left( \frac{x}{D/2} \right)^2 \int_0^\infty \gamma f(\gamma) d\gamma + 2q \frac{x}{D/2} (1 - p \frac{x}{D/2}) \int_0^\infty f(\gamma) d\gamma$$  (6.8)

Now values for constants $p$ and $q$ and a form for the function $f(\gamma)$ should be assumed.

For an axi-symmetric jet with uniform velocity distribution it was shown that the assumption of the following expression for the velocity profile was justified.

$$f(\gamma) = \exp(-0.6932 \gamma^2)$$  (6.9)

This expression, as discussed before, is known to be in good agreement with experimental results of an axi-symmetric jet (Figure 4). However, the use of this function and the linear rate of spread of the mixing layer can be shown to be inconsistent with the conservation of momentum. The momentum integral equation is:

$$\frac{\pi D^2}{4} \bar{U}_{0,0}^2 = 2\pi \int_0^\infty \bar{U}_{x,r}^2 \cdot r \cdot dr$$  (6.10)
This condition is satisfied near the nozzle, where \( r_{0.5} \ll D/2 \)

when

\[
\int_{0}^{\infty} f^2 \, d\gamma = 0.775 \quad (6.11)
\]

By contrast, the momentum integral equation at the end of the potential core requires that:

\[
\frac{\rho}{p} = \sqrt{\frac{2}{2} \int_{0}^{\infty} f^2 \, d\gamma} = 0.85 \quad (6.12)
\]

By taking the average value of \( \rho/p = 0.8 \), the calculated entrainment velocities should be sufficiently accurate for present purposes, in spite of the failure to conserve momentum throughout the potential core region.

From experimental results of Figure 41, it appears that the potential core region disappears at a distance of four jet nozzle diameters downstream from the jet nozzle. Hence, \( q = 0.125 \) and \( p = 0.15625 \). After integrating the following expressions give:

\[
\int_{0}^{\infty} f(\gamma) \, d\gamma = 0.7213 \quad (6.13)
\]
\[
\int_{0}^{\infty} f(\gamma) \, d\gamma = 1.0645 \quad (6.14)
\]

By inserting the value of the above integrals and values of \( q \) and \( p \) in equation (6.8), the ratio of the mass flow in the jet divided by the mass flow of the jet in the nozzle takes the following form:

\[
\frac{M}{M_0} = 1 + 0.1653 \left( \frac{x}{D} \right) + 0.01853 \left( \frac{x}{D} \right)^2
\]

(6.15)

The strength of the line sink distribution to stimulate entrainment in the potential core region is therefore given by

\[
\frac{dM}{dx} = \frac{M_0}{D} \left( 0.1653 + 0.03716 \left( \frac{x}{D} \right) \right)
\]

(6.16)
ii) The sink strength in the fully turbulent region

A similar analysis to that used above for the potential core region may be used in order to obtain an appropriate line sink distribution to simulate entrainment in the fully turbulent region.

In this region the mass flow in the jet, $M$, takes the following form

$$M = 2 \pi \int_0^\infty \bar{U}_{x,r} \cdot r \cdot dr \quad (6.17)$$

It was shown that

$$\bar{U}_{x,r} = \bar{U}_{x,0} f(\gamma) \quad (6.18)$$

substituting Equation (6.18) into Equation (6.17) gives:

$$M = 2 \pi \int_0^\infty r^{2} \bar{U}_{x,0} \int_0^\infty \gamma f(\gamma) d\gamma \quad (6.19)$$

The momentum integral equation in this section of the flow is:

$$\frac{\pi D^2}{4} \bar{U}_{0,0}^2 = 2 \pi \int_0^\infty \bar{U}_{x,r} \cdot r \cdot dr \quad (6.20)$$

The initial mass flow at the jet nozzle is

$$M_0 = \int_0^\infty \bar{U}_{0,0}^2 = \int_0^\infty \frac{\pi D^2}{4} \frac{\bar{U}_{0,0}^2}{\bar{U}_{0,0}^2}$$

(6.21)

By substituting Equation (6.20) into Equation (6.21) gives

$$M_0 = 2 \pi \int_0^\infty \bar{U}_{x,r}^2 \cdot r \cdot dr \quad (6.22)$$

Substituting Equation (6.18) into (6.22) gives

$$M_0 = 2 \pi \int_0^{0.5} r^{2} \frac{\bar{U}_{x,0}^2}{\bar{U}_{0,0}} \int_0^\infty \gamma f^2 d\gamma \quad (6.23)$$
Hence, the ratio of the mass flow in the jet divided by the initial mass flow at the jet nozzle takes the following form:

\[
\frac{M}{M_0} = \frac{\bar{U}_{0,0}}{\bar{U}_{x,0}} \int_0^\infty \gamma f d\gamma \int_0^\infty \gamma^2 f^2 d\gamma
\]  

(6.24)

Integrating the above integrals and inserting their values in the Equation (6.24) gives:

\[
\frac{M}{M_0} = 2 \frac{\bar{U}_{0,0}}{\bar{U}_{x,0}}
\]  

(6.25)

The mean velocity ratio \(\bar{U}_{x,0}/\bar{U}_{0,0}\) along the x axis of the jets, as are shown in Figure 41, could be replotted in the form shown in Figure 85. These graphs are in good agreement with the following expression:

\[
\frac{\bar{U}_{0,0}}{\bar{U}_{x,0}} = \lambda \frac{x}{D} + \frac{x_0}{D}
\]  

(6.26)

where both \(\lambda\) and \(x_0\) are constants. In the case of a well behaved axi-symmetric jets, such as the "clean nozzle jet" of the present investigation (see section 2.2) the two constants \(\lambda\) and \(x_0/D\) are taken from the Figure 85. Hence, Equation (6.26) takes the following form:

\[
\lambda = 0.131 \quad \text{and} \quad x_0/D = 0.234
\]

\[
\frac{\bar{U}_{0,0}}{\bar{U}_{x,0}} = 0.131 \frac{x}{D} + 0.234
\]  

(6.27)

By substituting Equation (6.27) into Equation (6.25) the ratio of mass flow in the transition and fully turbulent region of the jet divided by the mass flow at the jet nozzle takes the following form:
The strength of the line sink distribution simulating entrainment in the transition and fully turbulent region of the jet can be obtained by integrating Equation (6.28) in respect to (x/D):

\[
\frac{dM}{dx} = 0.262 \frac{M_0}{D}
\]  

(6.29)

### iii) The induced velocities

Following the preceding arguments, it was shown that the strength of the line sink distribution simulating entrainment is similar to that shown in Figure 86. In the potential core region (for \(x/D \leq x_c/D\)) the strength of the sink distribution may be represented by

\[
\frac{dM}{dx} = \frac{M_0}{D} \cdot (A + B \frac{x}{D}) \quad (for \frac{x}{D} \leq \frac{x_c}{D})
\]  

(6.30)

where \(A\) and \(B\) are constants (see Equation (6.16)). In the transition and fully turbulent region for \(x/D > x_c/D\) a line sink distribution of constant strength has been evaluated.

\[
\frac{dM}{dx} = C \frac{M_0}{D} \quad (for \frac{x}{D} > \frac{x_c}{D})
\]  

(6.31)

where \(C\) is a constant (see Equation (6.29)).

This sink distribution and its mirror image gives rise to an entrainment velocity, \(V_r\), which is directed radially inwards toward the jet centre-line axis. Figure 87 shows the induced velocity, \(V_r\), which is directed radially towards the jet centre-line axis on any plane perpendicular to the central axis at a distance of say, \(x_1\), away.
from the jet nozzle. \( V_r \) consists of the two components of velocities \( V_1 \) and \( V_2 \), which are directed towards the centres of two spheres of the radius \( R_1 \) and \( R_2 \) which are located on the jet centre-line axis or the line sink, \( x \).

\[
V_r = V_1 + V_2 \tag{6.32}
\]

The flow towards the centre of each sphere is equal to

\[
Q = 4 \pi R^2 \bar{V} \tag{6.33}
\]

The component of velocity is given by

\[
v = \bar{V} \cos \alpha = \bar{V} \frac{r}{R} \tag{6.34}
\]

therefore

\[
dQ_1 = dv_1 \frac{4 \pi R_1^3}{r} = \frac{2 dM}{f} \tag{6.35}
\]

hence,

\[
v_1 = \int_{x_1}^{x} \frac{2dM}{f} \frac{r}{4 \pi R_1^3} = \int_{x_1}^{x} \frac{2dM}{f} \frac{r}{4 \pi \left(r^2 + (x_1 - x)^2\right)^{3/2}} \tag{6.36}
\]

and

\[
v_2 = \int_{x_1}^{\infty} \frac{2dM}{f} \frac{r}{4 \pi \left(r^2 + (x_1 - x)^2\right)^{3/2}} \tag{6.37}
\]

substituting the values of \( v_1 \) and \( v_2 \) into equation (6.32) and inserting the values of \( dM \) from Equations (6.30) and (6.31) the induced velocity \( \bar{V}_r \) takes the following form:
\[
\frac{V_r}{U_{o,o}} = \frac{r D}{8} \int_0^\infty \frac{X}{(r^2 + (x - x_1)^2)^{3/2}} \, dx + C \int_{X_c}^\infty \frac{dx}{(r^2 + (x - x_1)^2)^{3/2}} \quad (6.38)
\]

Equation (6.38) takes the following form after the integrations have been carried out:

\[
\frac{V_r}{U_{o,o}} = \frac{A D}{4r} \left( \frac{x_c - x_1}{\sqrt{r^2 + (x_c - x_1)^2}} + \frac{x_1}{\sqrt{r^2 + x_1^2}} \right) + \\
\frac{B \cdot r}{2} \left( \frac{x_1^2}{r^2 + x_1^2} + \frac{1}{\sqrt{r^2 + x_1^2}} + \frac{x_1(x_c - x_1)}{\sqrt{r^2 r^2 + (x_c - x_1)^2}} - \frac{1}{\sqrt{r^2 + (x_c - x_1)^2}} \right)
\]

\[
\frac{C \cdot D}{4r} \left( 1 - \frac{x_c - x_1}{\sqrt{r^2 + (x_c - x_1)^2}} \right) \quad (6.39)
\]

This expression was computed by a 9810 HP calculator for various ratios of (x/D) and (r/D). The constants A, B and C and \( x_c \) which are different for different jet configurations and vary with the jet nozzle initial characteristics and the condition of flow upstream from the jet nozzle are taken from the graphs of the mean velocity decays on the centre-line of these jet configurations (see Figures 85 and 52). For an axi-symmetric circular "clean nozzle" jet the value of the constants A, B, C and \( x_c \) were given in the Equation (6.16) and (6.29) which read:

\[ A = 0.1653; \quad B = 0.03716; \quad C = 0.131 \quad \text{and} \quad x_c/D = 4 \]

The induced velocities on the planform of the "clean nozzle jet" of the present investigation were computed by applying the Equation (6.39). The induced pressure coefficient \( C_p \) which is defined as
was plotted in Figure 88. This figure illustrates the induced pressure caused by the entrainment flow at various planes perpendicular to the jet centre-line axis, at different positions away from the x-axis.

Figure 89 illustrates again the induced pressure coefficient along the radius of planes perpendicular to the x axis at distances of (x/D) away from the jet nozzle.

The evaluation of the constant of Equation (6.39) was carried out for two more circular jet configurations with two tabs using Figure 89.

i) N = 2; H/D = 1/16; B/D = 1/16

ii) N = 2; H/D = 1/8; B/D = 1/16

The induced pressure coefficient for these two jet configurations was also computed and are shown in Figure 90.

Comparing the results of Figure 90 with the experimental results which will be discussed later, it was realized that they are not in good agreement near the nozzle. However, further away from the jet nozzle they appeared to be in a better agreement. The reason was thought to be the lack of similarity in the mean velocity ratio profiles near the jet nozzle and the build up of a better similarity further downstream from the jet nozzle (see Figure 90).
6.3 Experimental investigation of entrainment into axi-symmetric jet

Following the theoretical evaluation of induced velocities and induced pressure coefficient in the entrainment flow region, attempts were made to measure the entrainment velocities by applying the pulsed wire anemometer technique. Owing to the low velocity and highly turbulent flow in the entrainment region, it might be rather inaccurate to apply the conventional velocity measurement techniques and instrumentation, such as pitot-static tube and hot-wire anemometer, to obtain the induced velocity and corresponding turbulent intensity levels. Therefore, pulsed wire anemometry, which is shown to be appropriate for low velocity and highly turbulent flows (see Bradbury & Castro (1971)), was used in order to measure the statistical quantities of the entrainment flow into the axi-symmetric jet.

The prototype of this instrument which has been designed and developed in this department was controlled "on-line" by the H-P 9810 calculator (see chapter 3.4.1).

Although initially a number of measurements were made in the entrainment flow outside the jet on the planform, the results of these tests were not satisfactory, because of calibration difficulties. These difficulties were overcome after a calibration rig had been designed and developed.

The velocities at the nozzle of the calibration rig were measured by using a gas-flow-meter and considering the velocity distribution profile at the exit \((Q = V.A)\). Subsequently, further measurements of the entrainment flow were carried out. Entrainment flow symmetry on the ground board was first checked. The results which are illustrated in Figure 91 indicate a non-symmetric flow.

During the course of the design and development of the main rig
attempts were made to locate the main rig in a position where the walls and the ceiling of the laboratory had less effect and did not interfere with the entrainment flow, but in the course of the entrainment flow measurements a high sensitivity of the entrainment flow was noted.

In practice the reflection of the jet flow from the wall had some effect on the entrainment flow and this was thought to be the cause of the non-symmetry in the induced flow on the ground board shown in Figure 91.

Several more entrainment flow measurements were carried out along the radial direction on the planes perpendicular to the x-axis at a distance $x/D$ away from the ground board. The results are shown in Figure 92. Figure 93 illustrates the radial distribution of the turbulent intensity measurements of the induced flow at distances $(x/D)$ away from the ground board.

Although the effect of tabs is to produce a gross distortion of the jet cross-section, it seemed advisable to check directly that this resulted in increased entrainment into the jet. The Figure 94 shows the mean entrainment velocities into a two tabs jet configuration of $H/D = B/D = 1/16$, measured with a pulsed-wire anemometer, at a distance of 0.25 of the jet diameter from the ground board. Although this was a rather arbitrary and isolated test, it does demonstrate the increased entrainment that accompanies the insertion of the two tabs into the jet.

Referring to the previous discussion of Chapter III and looking at direct lift loss measurements reported by Gentry & Margason (1966) which led to the development of an empirical correlation expression of Equation (2.1), there should also be a relationship between the rate of velocity decay on the centre-line of a jet and the induced flow velocity which leads to induced loads
on the planform. The reason is that both of these parameters are functions of the amount of the air being drawn into the jet.

Although these measurements were not enough to lead to an empirical correlation expression, they could indicate the presence of a relationship between the rate of velocity decays on the centre-line of the jet and the induced velocities.

Figure 95 shows the radial distribution of turbulent intensities of the entrainment flow into a two tab jet configuration of $H/D = B/D = 1/16$ at a distance of 0.25 jet diameter from the ground board. The results are compared with the radial turbulent intensity distribution of the entrainment flow of an axi-symmetric circular "clean nozzle" jet.
7. Vortex rings

As a peripheral experimental investigation, in addition to the tests carried out on axi-symmetric low turbulent circular jet, it seemed interesting in the course of the present study, to examine the well-known phenomenon of the existence of a nearly regular puff formation in the potential core region of a jet and evaluate their presence in determining the entrainment into the jet. In the many experimental investigations into the structure of the clean nozzle axi-symmetric jets, the effort has been concentrated on documenting the events in the mixing region where the intense mixing occurs which is also accompanied with a high level turbulence. These studies showed that the fluctuating quantities are random and possess a broad band character. However, the more recent works of Davis, Fisher & Baratt (1963), Bradshaw, Ferriss & Johnson (1964) and Lau, Fisher & Fuchs (1972) showed that beneath this randomness in the mixing region there is a possible existence of a nearly regular pattern which will have some coherence over the entire cross-section, that is, all three regions of the potential core. This was found to be a source of some difficulties in the operation of open jet wind tunnels. Crow & Champagne (1971) refer to these as "puffs" but we shall loosely refer to them as "vortex rings".

At first sight, it is tempting to try to characterize the frequency \( f \) of these vortex rings by a single Strouhal number \( \text{St} = \frac{f D}{U_{0,0}} \), where \( D \) is the jet nozzle diameter and \( U_{0,0} \) is the jet nozzle velocity. However, if the development of the mixing layer is to be considered, which we might expect to develop rather like plane mixing layers at least for the first two jet diameters downstream from the nozzle, then it is necessary if these layers are to have a self-preserving type of structure for the Strouhal number \( \frac{f x}{U_{0,0}} \) to remain constant. This latter condition would require the frequency of the vortex rings to vary.
with distance downstream from the nozzle and this could only be achieved by allowing the vortex rings to coalesce in some way. As a reliable piece of evidence one can discern a large scale of vortex rings in the photographs taken by applying various flow visualization techniques. There are now numerous flow visualization films (see Davis & Yule (1975)) demonstrating that this sort of thing can occur and it will result, of course, in a broadening of the spectrum of the velocity fluctuations associated with the motion of the vortex rings. Under these circumstances, the simple notion of a well-defined Strouhal number becomes ambiguous irrespective of the length scale used in its definition. From the work of Bradshaw et al (1964) and Lau (1971), peaks in the spectra of the turbulence in the mixing layers can be observed and it seems likely that these were associated with vortex-ring structures. The frequency of the peaks in their spectra curves are more nearly consistent with a constant value of $f x / U_{0,0}$ but this is only very approximate and the effects both of axi-symmetry and of the time taken to establish self-preservation in a plane mixing layer combine, no doubt, to ensure that simple notions about vortex-ring structures will not find unambiguous support from experimental data.

In the present experiments, a few measurements of spectra were made at various positions downstream on the centre-line of the jet. Unfortunately, owing to limitations on time, these are less comprehensive than one would wish but they do add something to our general store of information about the flow in the initial region of a jet.

Figures 96 and 97a show a complete u-component spectrum on the centre-line of a low turbulent axi-symmetric circular "clean nozzle" jet at x/D = 0, 2 and 4 respectively. The measurements were made with a standard B & K one third octave spectrum analyser with a filter range from 2 Hz to 20 kHz.
The results show a peak in the spectra (except for x/D = 0) although clearly the structures associated with the peak are not anything like as well defined as the very narrow band vortex shedding that occurs, say, downstream of a circular cylinder. The results in Figures 96 and 97a seem to exhibit a peak at a nearly constant Strouhal number \( \frac{fD}{U_0 \sqrt{2}} \) of about 0.45, which agree closely with the observations of Lau (1971). This near constantcy of the Strouhal number \( \frac{fD}{U_0 \sqrt{2}} \) on the flow centre-line seems to contradict the observations of Bradshaw et al. (1964) and Lau (1971) in the mixing regions. However, velocity fluctuations in the region of the peak frequency on the flow centre-line are essentially potential-flow fluctuations and they arise from the integrated contributions from the vortex rings along the length of the potential core. Since, owing to coalescence, these increase in strength as the distance from the nozzle increases, it is quite feasible that the centre-line peaks may be at some average frequency towards the end of the potential core.

It is interesting to note that Crow & Champagne (1971) obtained a Strouhal number \( \frac{fD}{U_0 \sqrt{2}} \) of about 0.3 on the basis of the frequency of smoke "puffs" in some flow visualization studies. It seems likely that the "puffs" counted in such observations would be the largest coherent structures which could be observed before they were absorbed into fully turbulent flow downstream of the potential core. Thus, one might expect this Strouhal number to be even more closely associated with the occurrence of the ring structures at the end of the potential core and this might account for the somewhat lower value of the Strouhal number from such observations compared with the present centre-line spectrum measurements.

By increasing the level of initial turbulence, that is, by placing a turbulence grid at a distance of about 17/8th of the jet diameter upstream from the jet nozzle there will be, as shown in Figure 97a, a minor change in the peak area. However, as expected the amount of energy of turbulence at lower frequencies
is increased. In this case also the spectra clearly exhibits a peak in the vicinity of Strouhal number of 0.4.

Encouraged by the available evidence, and as a step toward building up a further picture of this field and an understanding of the origin a number of sets of auto, cross and spatial correlation (for the definition, refer to Appendix I) measurements were carried out in the potential core, turbulent shear and entrainment region of the axi-symmetric jet.

These measurements were made with a Hewlett Packard 3721 A correlator and the signals of two hot-wire anemometers. In this unit 131072 samples were taken for each run and were averaged continuously after each multiplication of the components.

In turbulence, like any random "noise-like" signals, only a very small shift is sufficient to destroy the similarity, and the similarity never recurs. The auto correlation function is therefore a sharp impulse which decays from the central maximum to very low values at large time shifts. Intuitively, the width of the impulse can be seen to depend on the mean zero-crossing rate of the turbulence waveform, that is, on the band width of the signal (the higher the zero-crossing rate, the smaller the time shift required to destroy similarity). Therefore, the auto correlation function of any signal, random or periodic, depends not on the actual waveform, but only on its frequency content.

The shape of the auto correlation function depends on the shape of the spectrum. Most spectra "roll off" more gradually, and the corresponding auto correlation functions decay more rapidly. (The sharper the "roll off" of the spectrum, the greater the tendency for oscillatory tails in the correlation graph). It is well known that correlations without loops are equivalent to spectra without peaks which correspond to decaying velocity,
whereas peaked spectra represents an auto correlation coefficient which falls into a sort of periodic loop (Bradshaw et al. (1964)). This is clearly illustrated in Figures 97a and 97b which obviously imply some sort of wave motion in a preferred frequency range but with no immediate indication of the number of wavelengths which the notion extends.

Figure 97b shows also the auto correlation measurements on the centre-line axis of the flow from a jet nozzle with two tabs of $B/D = H/D = 16$ at a distance of $x/D = 2$. The results are compared with the auto correlation measurement on the flow from a clean jet nozzle.

Owing to the large number of these auto, cross and spatial correlation tests, it is not intended to include all of the graphs in this report. However, those which are believed to be of some interest to jet flow investigators are shown in Figures 97b to 110.

Figure 98 to 100 show the auto correlation measurements on the flow from a 'clean nozzle' jet at distances of 2, 4 and 6 jet diameters downstream from the jet nozzle. The hot-wire anemometer was mounted in the jet flow at various positions away from the centre-line. The following points are highlighted from these illustrations:

a- There is no oscillatory tail at the jet nozzle
b- The strength of these oscillatory tails increase as the distance from the nozzle increases which is due to coalescence of the vortex rings.
c- They vanish at the end of the potential core.
d- They disappear in the region of the mixing layers.

Figure 101 to 104 illustrate the cross-correlation measurements on the flow from a clean jet nozzle at distances of 0.1, 2, 4 and 8 jet diameters away from the jet nozzle plane. Two hot-wire
anemometers were applied in the flow. One was fixed on the centre-line axis and correlation between them was measured as the other one was positioned at various distances away from centre-line. Two more cross-correlation measurements are reported in the flow from a clean nozzle jet at \( x/D = 2 \) and \( x/D = 4 \). Here the first hot-wire anemometer was mounted in the flow at the fixed radius of \( r/D = 0.25 \) and \( r/D = 0.5 \) from the jet centre-line and correlations were made between this and a second hot-wire anemometer which was placed at various radial positions away from the first one. The results of these correlations are shown in Figure 105 and 106.

The next measurements reported in the flow from a "clean jet nozzle" are spatial correlation measurements at distances of 2, 4, 6 and 12 jet diameters away from the jet nozzle plane. The results are illustrated in Figure 107.

The final measurements to be reported in the series of correlation measurements on the flow from a "clean jet nozzle" are some cross and spatial correlation measurements at \( x/D = 2 \). Two hot-wire anemometers were mounted in the flow at a fixed radius from the jet centre-line and the correlation between them was measured as one wire was traversed circumferentially away from the other, fixed wire. Two tests were carried out, at radii of \( r/D = 0.25 \) and \( r/D = 0.5 \). Thus, in the former the wires were in a mainly potential flow region, whereas in the latter they were near the centre of the fully turbulent mixing layers. The results of these measurements are shown in Figures 108, 109 and 110. Figure 108 illustrates clearly the presence of a structure which is coherent and is similar around the entire circumference of the jet in the potential core at \( r/D = 0.25 \). But since it contains only a small portion of the total turbulent energy, the extent of the correlation in the mixing layers at \( r/D = 0.5 \) is fixed mainly by the scale of the general turbulent motion, which is much less than that of the ring structures (see Figure 109). Figure 110
illustrates the spatial correlation function of the results shown in Figures 108 and 109.

The presence of highly coherent structures in the initial region of a jet has been of some interest to those concerned with jet noise, both theoretically and experimentally, and the only remaining observation from the present tests is that the vortex-ring structure can apparently be suppressed by the insertion of small tabs. Figures 97a and 97b show u-component spectrum and auto correlation functions on the jet centre-line at $x/D = 4$ from a clean nozzle jet and a jet with two tabs. Although the lower frequency content of the spectrum is increased, the peak in the spectrum and also the oscillatory tail in the correlation is entirely eliminated. Since the jet development is so profoundly affected by only two tabs, it is possible that the sound produced by such a jet might be very different from that produced by a jet from a clean nozzle.
8. Concluding remarks

This thesis has described the results of an investigation into a number of factors which it was thought might play an important role in determining the entrainment into an axi-symmetric jet. Over the range of parameters tested, it was found that the effects of nozzle turbulence level, nozzle wall boundary layer thickness and nozzle convergence had little effect on the jet development. However, when small rectangular tabs were mounted on the circumference of the nozzle, it was found that these resulted in gross disturbances to the jet development. The effect was greatest when only two tabs were mounted and diminished as additional tabs were placed on the circumference so that when eight equi-spaced tabs were in position, the jet development was again very similar to that from the plain nozzle. Further experiments showed that the effect was due to circumferential variations in flow angle induced by the tabs. The possibility that this was the cause of various anomalous results obtained on V/STOL model tests was further investigated by measurements on a model from which such anomalous results had already been observed. Measurements with pitch and yaw tubes showed the existence of flow angle variations and it is therefore highly likely that these are responsible for discrepancies between V/STOL model test results.

A few further tests were undertaken to study the structure of the vortex rings shed from the nozzle. Correlation measurements revealed that these structures were indeed rings rather than spirals and the frequency of shedding was found to correspond closely to the Strouhal number obtained previously by other investigators. It was also found that the introduction of two tabs eliminated the vortex ring structures.
The main conclusion to arise from the present work is that in V/STOL model tests, it is clearly not adequate in establishing satisfactory jet flow to ensure a uniform velocity profile at the nozzle or nozzles. A more important criterion would seem to be that there should be no circumferential variations in flow angle and this should be examined more closely in future model tests. In addition, the very large changes in jet development that result from the introduction of tabs suggests that they may be useful in influencing jet noise and it would be extremely interesting to examine this possibility.
Appendix I

Auto-, Cross and Spatial Correlation Functions; Power Spectra Density Functions, Longitudinal and Lateral Scale of Turbulence.

i) Auto-correlation Function

The auto-correlation function for a random signal characterizes the degree of relationship between the waveform and a time shifted version of itself.

The auto-correlation function of a wave form \( x(t) \) is defined as

\[
R_{xx}(\tau) = \lim_{T \to \infty} \frac{1}{T} \int_{0}^{T} x(t) \cdot x(t + \tau) \, dt
\]

That is, the waveform \( x(t) \) is multiplied ordinate by ordinate by a delayed version of itself \( x(t + \tau) \) and the product is averaged over the observation time \( T \), in other words:

\[
R_{xx}(\tau) = \frac{x(t) \cdot x(t + \tau)}{2}
\]

The value of \( x(t + \tau) \) in a random signal must depend on the value of \( x(t) \) and the preceding instant of time. Thus if \( \tau \) is small, then \( x(t + \tau) \) differs little from \( x(t) \) and the ratio \( R_{xx}(\tau)/R_{xx}(0) \) approaches unity. With the increase of \( \tau \) the relation between the quantities \( x(t) \) and \( x(t + \tau) \) weakens and these quantities become independent of each other, and the function \( R_{xx}(\tau) \) tends to zero. By increasing \( \tau \) the similarity never recurs. The value of the auto-correlation function cannot exceed its initial value \( R_{xx}(0) \) for all \( \tau \). The auto-correlation function of any signal, random or periodic, depends not on the actual wave form, but only on its frequency content. The correlation function \( R(\tau) \) is an even function of delay time \( \tau \), that is

\[
R(\tau) = R(-\tau)
\]
Figures 98 to 100 show some typical auto-correlation measurements as a function of delay time in milliseconds.

The periodic loop of an auto-correlation coefficient may also be seen in the spectra density function in the form of a peak. The correlation function of a sine wave

\[ x(t) = a \sin (\omega t + \phi) \]  \hspace{1cm} (4)

is:

\[ R (\tau) \approx \frac{a^2}{2} \cos (\omega t) \]  \hspace{1cm} (5)

Therefore the function \( R(\tau) \) has the same period as \( x(t) \) but in contrast is an even function and does not depend on the phase \( \phi \). Hence, if the signal \( x(t) \) is a random function which includes a superimposed periodic component, then the correlation function will also have a periodic component having the same period. The frequency of this periodic component could be counted from the illustration of an auto-correlation function which should be the same frequency in which the peak occurs in the case of spectra density function. This discussion was handled in more detail in chapter seven.

The initial value of the auto-correlation function \( R_{xx}(0) \) which is equivalent to the root mean square of the signal was checked with the RMS meter and found to be in good agreement. Therefore the initial value of the auto-correlation function \( R_{xx}(0) \) has always a positive value, that is,

\[ R_{xx}(0) = \bar{u}^2 = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{+T} u^2(t) \, dt. \]  \hspace{1cm} (6)
ii) Cross-correlation Function

Two-wire correlation or cross-correlation function deals with two random signals \( x(t) \) and \( y(t) \), therefore it describes the dependence of the values of one signal to the values of the other.

The cross-correlation function of two waveforms \( (x(t) \) and \( y(t) \) is defined, as is done by auto-correlation, by taking the products of the values of \( x(t) \) at time \( t \) and the values of \( y(t) \) at the time \( (t + \tau) \) and averaging the products over the observation time \( (T) \), that is,

\[
R_{xy} (\tau) = \lim_{T \to \infty} \frac{1}{T} \int_{0}^{T} x(t) \cdot y(t + \tau) \, dt \quad (7)
\]

or

\[
R_{xy} (\tau) = \frac{x(t) \cdot y(t + \tau)}{ \int_{0}^{T} x(t) \cdot y(t + \tau) \, dt } \quad (8)
\]

This is always a real-valued function which may be either positive or negative. The cross-correlation function, in contrast to the auto-correlation function, is not an even function.

The variation of cross-correlation coefficient with respect to delay time in milliseconds has been illustrated in Figures 101 to 106 for various lateral traversing across the flow of a "clean nozzle" jet.

iii) Spatial Cross-correlation Function

Special cross-correlation function \( R(r) \) characterizes the similarity between two signals in which their sources are lying a distance \( r \) apart, in other words,
It is more convenient to work with a normalized spatial cross-correlation, that is,

\[ R(r) = \frac{x (\tau = 0) \cdot y (\tau = 0)}{\sqrt{x^2} \cdot \sqrt{y^2}} \]  

Two hot-wire anemometers and a correlator were used to evaluate the lateral and longitudinal cross-correlations by having one wire fixed and by displacing the other either laterally or longitudinally. Each time the cross and auto-correlation functions were evaluated and spatial cross-correlation was determined in the following manner.

\[ -u_x^2 = R_x (\tau) \quad \text{by} \quad \tau = 0 \]

\[ -u_y^2 = R_y (\tau) \quad \text{by} \quad \tau = 0 \]  

\[ u_x \cdot u_y = R_{xy} (\tau) \quad \text{by} \quad \tau = 0 \]

\[ R_{xy} (r) = \frac{R_{xy} (0)}{\sqrt{R_x (0) \cdot R_y (0)}} \]

Some typical measured and evaluated spatial cross-correlation functions are shown in Figure 107 and 110 which indicate the variation of cross-correlation coefficient, with distance of wire separation which in turn were normalized with respect to the jet diameter, D.
iv) **Power Spectra Density Function**

The power spectra density function for a random signal characterizes the general frequency composition of the signal in the form of a spectral density of its mean square value.

The mean square value of a random function $x(t)$ in a frequency range between $f$ and $f + \Delta f$ may be obtained by filtering the signal with a band pass filter having sharp cut-off characteristics, and computing the average of the squared value of the output. After a long observation of time $T$, the average of the squared value approaches an exact mean square value

$$\bar{\dot{x}}^2(f, f + \Delta f) = \lim_{T \to \infty} \frac{1}{T} \int_{0}^{T} \dot{x}^2(t, f, \Delta f) \, dt \quad (12)$$

where $x(t, f, \Delta f)$ is a portion of the random function $x(t)$ which is in the frequency range from $f$ to $f + \Delta f$. By approaching $\Delta f$ to zero, the power spectra density function $G_x(f)$ can be defined as

$$G_x(f) = \lim_{\Delta f \to 0} \frac{\bar{\dot{x}}^2(f, f + \Delta f)}{\Delta f} \quad (13)$$

$$G_x(f) = \lim_{\Delta f \to 0} \lim_{T \to \infty} \frac{1}{\Delta f} \frac{1}{T} \int_{0}^{T} x^2(t, f, \Delta f) \, dt$$

$G_x(f)$ is therefore a real positive value

$$G_x(f) \Delta f = \bar{\dot{x}}^2(f, f + \Delta f) \quad (14)$$

where

$$\int_{0}^{\infty} G_x(f) \, df = \bar{\dot{x}}^2 \quad (15)$$
The relative energy spectrum is fully discussed by Dryden (1943) and Taylor (1938) which is defined as:

\[ F(f) = \frac{Gx(f)}{\pi^2} \]  

(16)

where

\[ \int_0^\infty F(f) \, df = 1 \]  

(17)

An important property of the power spectra density function lies in its relationship to the auto-correlation function. According to the proof given by Rayleigh and Taylor

\[ R_{xx}(\tau) = \int_0^\infty F(f) \cos(2\pi f \tau) \, df \]  

(18)

or

\[ R_{xx} = \int_0^\infty F(f) \cos(2\pi f \frac{x}{U}) \, df \]  

(19)

or

\[ F(f) = 4 \int_0^\infty R_x(\tau) \cos(2\pi f \tau) \, d\tau \]  

(20)

\[ F(f) = \frac{4}{U_1} \int_0^\infty R_y(\tau) \cos(2\pi f \frac{x}{U}) \, dx \]  

(21)

In other words, the correlation coefficient \( R_{xx} \) and \( \frac{U F(f)}{\sqrt{8\pi}} \) are fourier transforms. If either is measured then the other can be computed.

An Audio Frequency B & K spectrometer was used to analyse the signal from the hot-wire anemomometer and to determine the power
spectra density function, \( F(f) \), which is given by:

\[
F(f) = \frac{1}{B} \frac{\varepsilon_b^2}{\varepsilon_{tot}^2}
\]  

(22)

where \( \varepsilon_b \) and \( \varepsilon_{tot} \) are fluctuating components of hot wire voltage of the band of frequencies and of the total spectrum of the frequencies respectively and \( B \) is the band-pass width of the filters in cycle per second (cps). The resulting display of the spectra density function with respect to frequency shows the content of the signal in each individual frequency band. This method of analysis, as discussed in Chapter seven, shows the frequency bands in which most of the energy is concentrated and also the spectrum can be used to obtain the average eddy size and the length scale of turbulence, \( L_x \).

In order to obtain an accurate value of the energy spectrum at the lower frequencies, the output voltage from the analyser was measured by means of a Time Domain Analyser in which the output was integrated over a time period (sometimes upto 300 seconds). Figures 96 and 97 are logarithmic plots of one dimensional energy spectrum functions, \( F(f) \frac{U}{B} \), of \( u \)-component velocities of the axial velocity fluctuations against the Strouhal number of \( (f \frac{D}{U}) \) measured on the centre-line axis at distances of zero, two, and four jet diameters from the jet exit. These spectra density functions are reasonably fitted by a \((-5/3)\) power law as was predicted by the Lokmogoroff theory (see Gibson (1963)). The change of spectra with respect to distance from the nozzle is primarily a shift of energy to lower frequencies as \( x/D \) increases.

v) **Longitudinal and Lateral Scale of Turbulence**

The definition of lateral cross-correlation \( R_y \) between the values of the \( u \)-component at two points separated by the distance \( y \) in
the direction of the y-coordinate has been given in Equation (10). The curve of $R_y$ plotted against $y$ represents, from the statistical point of view, the distribution of a $u$ along the y axis. If $R_y$ falls to zero and remains zero, a length $L_y$ may be defined as

$$L_y = \int_0^\infty R_y \, dy$$  \hfill (23)

The length $L_y$ can be considered as the average size of the eddies and is considered as the scale of turbulence.

Corresponding longitudinal scale of turbulence $L_x$ may be defined as

$$L_x = \int_0^\infty R_x \, dx$$  \hfill (24)

$R_x$ is the correlation between the values of the component $u$ at two points separated by distance $x$ in the direction of the $x$ coordinate. Lateral and longitudinal correlation $R_y$ and $R_x$ are tied together by a differential equation which was proposed by Von Karman (1937) which deduces to

$$L_y = \frac{1}{2} L_x$$  \hfill (25)

Taylor (1935) indicated that the curvature of the $R_y$ curve at $y = 0$ is a measure of $\left( \frac{\partial u}{\partial y} \right)^2$ which may be shown as

$$\left( \frac{\partial u}{\partial y} \right)^2 = 2 u^2 \lim_{y \to 0} \left( \frac{1 - R_y}{y^2} \right)$$  \hfill (26)

Therefore a length $\chi$ could be defined such as
\[
\frac{1}{\lambda^2} = \lim_{y \to 0} \frac{1 - R_y}{y^2}
\]  \hspace{1cm} (27)

\( \lambda \) may be recorded roughly as a measure of the diameters of the smallest eddies which are responsible for the dissipation of energy or could be considered as the radius of curvature of the \( R_y \) curve at \( y = 0 \), or if a parabola is drawn tangent to the \( R_y \) curve at \( y = 0 \). This parabola cuts the axis at point \( y = \lambda \).

The scale of turbulence can also be evaluated by means of a spectra density function. From Equation (21) it is obvious that the spectra density function for zero frequency, \( F(0) \), is equal to:

\[
F(0) = \frac{4}{U_{1oc}} \int_{0}^{\infty} R_x dx \cos(0)
\]  \hspace{1cm} (28)

and after considering the definition of longitudinal scale of turbulence (equation (24)) takes the form

\[
F(0) = \frac{4}{U_{1oc}} \cdot L_x
\]  \hspace{1cm} (29)

or

\[
L_x = \frac{U_{1oc} F(0)}{4}
\]  \hspace{1cm} (30)

Several schemes have been suggested to evaluate the zero frequency spectra density function, \( F(0) \). The method used by Laurance (1956) was applied in the present study which was simply to find the best curve through which the experimental data of power spectra density function with respect to frequency passed and estimate the maximum value of \( F(f) \). The results of this method proved to be satisfactory compared with the two-wire method and the integration of the area under spatial cross correlation function. The values of \( L \) are
plotted in Figure 49, which shows the variation of lateral scale with the distance from the jet nozzle. A straight line was fitted to the data evaluated.
Appendix II

Calculation of the continuous mean values, least square fit method and probability density histogram

The sample mean value and mean square value are given by

\[ \bar{U} = \frac{1}{N} \sum_{i=1}^{N} U_i \]

\[ \bar{U}^2 = \frac{1}{N} \sum_{i=1}^{N} U_i^2 \]

where the \( U_i \) is the instantaneous data values.

In order to determine the sample mean value and mean square value continuously, after each sample has been taken, the following expressions were applied by the calculator programme.

\[ \bar{U}_{\text{cont}} = \frac{n}{n+1} \bar{U}_{n-1} + \frac{1}{n+1} U_n \text{ for } n = 1, 2, 3, \ldots N \]

\[ \bar{U}_{\text{cont}}^2 = \frac{n}{n+1} \bar{U}_{n-1}^2 + \frac{1}{n+1} \bar{U}_n^2 \text{ for } n = 1, 2, 3, \ldots N \]

where \( U_{\text{cont}} \) is the mean value of continuous data after each sample has been taken. In the case of the calculation of the velocity and turbulence intensity averaged continuously by the method described in 3.4.1. and 3.4.2., they were displayed after each sample had been taken. The calculation of the turbulent intensity after each sample is given by:

\[ \bar{u}^2 = (\bar{U}_{\text{cont}}^2 - \bar{U}_{\text{cont}}^2)^{1/2} \]

To determine the intercept and slope values for the equation

\[ Y = A + BU \]
by applying the least square fit method in the programme of the calculator, the following steps should be carried out after each point of the plot has been evaluated.

\[ Y = A + BU \]

\[
B = \frac{\sum_{i=1}^{n} (U_i - \bar{U}) (Y_i - \bar{Y})}{\sum_{i=1}^{n} (U_i - \bar{U})^2}
\]

or,

\[
B = \frac{\sum_{i=1}^{n} U_i Y_i - \sum_{i=1}^{n} U_i \bar{Y}}{\sum_{i=1}^{n} (U_i - \bar{U})}
\]

\[
A = \bar{Y} - B \bar{U}
\]

where

\[
\bar{U} = \frac{\sum_{i=1}^{n} U_i}{n}
\]

\[
\bar{Y} = \frac{\sum_{i=1}^{n} Y_i}{n}
\]
Appendix III

Selection of the number of samples

The sample size should be selected on the basis of prescribed accuracies in a measurement. Estimates for this selection are fully described by Mandel (1964). For normal velocity distribution the estimate of sample size \( N \), is according to the described size of the velocity confidence interval, \( L \) and the probability of the mean velocity and turbulent intensity of the samples being in the intervals \( \pm \Delta U \) and \( (\Delta u^2)^{1/2} \) respectively.

\[
L = 2\Delta U = 2t \sqrt{\frac{\bar{u}^2}{N}}
\]

\[
L = 2 \Delta (\bar{u}^2)^{1/2} = 2t \sqrt{\frac{\bar{u}^2}{2N}}
\]

where \( t \) is a parameter called the student's 't' distribution function which has values 1.96 for 95% probability and 1.645 for 90% probability of the measured mean velocity and turbulent intensity to be within \( L \) and \( 2\Delta (\bar{u}^2)^{1/2} \) respectively.

As an example, if the turbulent intensity of a flow relative to a reference velocity is 25%, and if it is intended to measure the mean velocity to within \( U = \pm 5% \) of the reference velocity with 95% probability, it would be necessary to take 94 samples.

i.e.

\[
N = \left( \frac{t \sqrt{\bar{u}^2}}{\Delta U} \right)^2 = 94
\]

Under these circumstances there is 95 percent probability that the estimate of intensity would be according to the equation

\[
L = 2 \Delta (\bar{u}^2)^{1/2} = 2t \sqrt{\frac{\bar{u}^2}{2N}}
\]

within \( \Delta (\bar{u}^2)^{1/2} = 14.3 \) percent of true intensity.
Appendix IV

NUMERICAL INTEGRATION

Numerical integration is often known as quadrature. The Gaussian quadrature procedure, which is applied in the theoretical part of the present investigation, to evaluate the definite integral of Equation (4.15), is in many ways better than other methods; it is accurate and requires fewer computational steps. However, a study of this procedure demands a complete understanding of the two types of special polynomials, namely, Lagrange polynomials and Legendre polynomials. The principle properties of the two polynomials will be therefore briefly discussed here.

The Lagrange polynomials:

We assume that for a function $f(x)$ which is continuously differentiable $(n+1)$ times, $(n+1)$ points $(x_1, y_1); (x_2, y_2); (x_n, y_n)$ are known. $f(x)$ may be replaced by a polynomial $P(x)$ of degree $n$

$$P(x) = a_0 + a_1 x + \ldots + a_n x^n \quad (1)$$

We now define the Lagrange polynomial $L_k(x)$ of degree $n$ as

$$L_k(x_i) = 0 \quad \text{if} \quad i \neq k. \quad (2)$$

$$L_k(x_i) = 1 \quad \text{if} \quad i = k$$

where $x_i \ (i=0;1;\ldots; n)$ are the given $n+1$ distinct arguments. Therefore, we can now write $P(x)$ in the form

$$P_n(x) = \sum_{k=0}^{n} L_k(x) y_k \quad (3)$$
where

\[ L_k(x) = \frac{(x-x_0)(x-x_1)(x-x_2) \ldots \ldots \ldots (x-x_n)}{(x_k-x_0)(x_k-x_1)(x_k-x_2) \ldots \ldots \ldots (x_k-x_n)} \quad (4) \]

or

\[ L_k(x) = \prod_{i=0}^{n} \frac{(x-x_i)}{(x_k-x_i)} \prod_{i \neq k} (x_k-x_i) \quad (5) \]

where \( p_i \) notation has the meaning

\[ \prod_{k=1}^{n} = x_1 \cdot x_2 \cdot x_3 \ldots \ldots \ldots \ldots \ldots x_n \quad (6) \]

\( L_k(x) \) can also take the form

\[ L_k(x) = \frac{G_k(x)}{x-x_k} \quad (7) \]

where,

\[ G_k(x) = \frac{F_{n+1}(x)}{x-x_k} \quad (8) \]

and

\[ F_{n+1}(x) = (x-x_0)(x-x_1) \ldots \ldots (x-x_n) \quad (9) \]

The Lagrangian polynomial of degree \( n \) can be constructed by

i) setting up \( F_{n+1}(x) \)

ii) finding \( F_{n+1}(x) \) and then computing \( F_{n+1}(x_k) \) for \( k = 0; 1; 2; \ldots \ldots \); \( n \)

iii) using Eq. (8), finding \( G_k(x) \) for \( x=x_0; x_1; x_2; \ldots \ldots ; x_n \)

and finally
iv) evaluating $L_k(x)$ by applying Eq. (7). After substitution of Eq. (5) into Eq. (3), we obtain;

$$P_n(k) = \sum_{k=0}^{n} \prod_{i=0, i\neq k}^{n} \frac{(x - x_i)}{(x_k - x_i)}$$

(10)

This equation is known as the Lagrange interpolation formula for unequally spaced data.

**Legendre polynomials:**

In general, a set of function $\phi_0(x); \phi_1(x); \phi_2(x); \ldots; \phi_m(x)$ is known as orthogonal in an interval $a \leq x \leq b$ if

$$\int_a^b W(x) \phi_m(x) \phi_n(x) \, dx = 0 \quad m \neq n$$

(10a)

where the weighting function $W(x)$ is non-negative in the given interval $(a,b)$. When all the orthogonal functions $\phi_m(x)$ are polynomials they are known as orthogonal polynomials.

A particular example of such orthogonal polynomials is the set of Legendre polynomials. The first five and the nth are:

$$P_0(x) = 1$$

$$P_1(x) = x$$

$$P_2(x) = \frac{1}{2} (3x^2 - 1)$$

(11)

$$P_3(x) = \frac{1}{2} (5x^3 - 3x)$$

$$P_4(x) = \frac{1}{8} (35x^4 - 30x^2 + 3)$$
The orthogonality and normalization relation, with the weighting function equal to unity, are

\[ \int_{-1}^{+1} P_n(x) \cdot P_m(x) \cdot dx = \begin{cases} 0 & \text{if } n \neq m \\ \frac{2}{2n+1} & \text{if } n = m \end{cases} \] (12)

All the roots of each \( P_n(x) = 0 \) are real and distinct, and are between -1 and +1.

**Gaussian quadrature:**

Gaussian integration method approximate the definite integral

\[ \int_{-1}^{+1} f(x) \cdot dx \]

by the expression

\[ \int_{-1}^{+1} f(x) \cdot dx = W_0 f(x_0) + W_1 f(x_1) + \ldots + W_n f(x_n) \] (14)

i.e.

\[ \int_{-1}^{+1} f(x) \cdot dx = \sum_{k=0}^{n} W_k f(x_n) \] (14)
Where \( W_0; W_1; \ldots; W_n \) are the weighting coefficients and \( x_0; x_1; \ldots; x_n \) are the associated points.

It will be shown later that by a simple change of variables, the procedure can be extended to limits of integration other than \((-1, +1)\).

It can be proved that the associated points are equal to the values of the roots of a Legendre Polynomial \( P_{n+1}(n) = 0 \). There are \( n + 1 \) distinct real roots in the interval \((-1, +1)\). For example, for \( n=2 \) the roots of

\[
P_3(x) = \frac{1}{2} (5x^2 - 3x) = 0
\]

are:

\[
0; \ -\sqrt{3}/5 \ \text{and} \ +\sqrt{3}/5
\]

Therefore in the remainder of this Appendix the subscripted \( x_n \) will be used to indicate the roots of Legendre polynomial.

Equation (14) must involve no approximation if the integrand \( f(x) \) is a polynomial of degree \( 2n+1 \) or less. By the definition of Lagrange polynomial we have

\[
h_n(x) = \sum_{k=0}^{n} h(x_k) L_k(x) \tag{15}
\]

Where \( h(x_k) \) is a constant

hence

\[
\int_{-1}^{+1} h_n(x) \, dx = \sum_{k=0}^{n} h(x_k) \int_{-1}^{+1} L_k(x) \, dx \tag{16}
\]
By comparing equation (16) with equation (14), the following equation can be obtained;

\[ w_k = \int_{-1}^{+1} L_k(x) \, dx \quad (17) \]

where \( k = 0; 1; 2; \ldots; n \) the weighting coefficient \( w_k \) will usually be calculated in terms of Legendre polynomials \( P_n(x) \) from equation (7), it can be obtained:

\[ L_k(x) = \frac{1}{P_{n+1}(x_k)} \frac{P_{n+1}(x)}{x - x_k} \quad (18) \]

hence

\[ w_k = \frac{1}{P_{n+1}(x_k)} \int_{-1}^{+1} \frac{P_{n+1}(x) \, dx}{x - x_k} \]

The weighting coefficients \( w_k \) and associated points \( x_k \) are tabulated below against the degree \( n \) (Kuo, S. S. (1965)).

<table>
<thead>
<tr>
<th>( n )</th>
<th>Weighting coefficient ( w_k )</th>
<th>Associated points ( x_k )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>8/9</td>
<td>0.774 596 669</td>
</tr>
<tr>
<td></td>
<td>5/9</td>
<td>± 0.774 596 669</td>
</tr>
<tr>
<td>3</td>
<td>0.652 145 154</td>
<td>± 0.339 981 043</td>
</tr>
<tr>
<td></td>
<td>0.347 854 845</td>
<td>± 0.861 136 311</td>
</tr>
<tr>
<td>4</td>
<td>0.568 888 888</td>
<td>0.0</td>
</tr>
<tr>
<td></td>
<td>0.478 628 670</td>
<td>± 0.538 469 310</td>
</tr>
<tr>
<td></td>
<td>0.236 926 885</td>
<td>± 0.906 179 845</td>
</tr>
<tr>
<td>5</td>
<td>0.467 913 934</td>
<td>± 0.238 619 186</td>
</tr>
<tr>
<td></td>
<td>0.360 761 573</td>
<td>± 0.661 209 386</td>
</tr>
<tr>
<td></td>
<td>0.171 324 492</td>
<td>± 0.932 469 514</td>
</tr>
</tbody>
</table>
When the lower and upper limits of integration are A and B, respectively, a relation can be obtained that holds for the limits of integration, -1 to +1.

Hence, the following substitution is to be made:

$$x = \frac{(B - A) t + (B + A)}{2}$$

Therefore, to evaluate the integration of $f(x) \, dx$ between the limits $A$ and $B$ by using 6 point Gaussian quadrature we have

$$\int_{A}^{B} f(x) \, dx = \frac{B - A}{2} \sum_{i=1}^{6} W_i \, f\left(\frac{(B - A) t_i + (B + A)}{2}\right)$$

Gaussian quadrature formula can also be applied for computing an N-dimensional integral of the form

$$\int_{-1}^{+1} \int_{-1}^{+1} \cdots \int_{-1}^{+1} f(x_1; x_2; \ldots; x_n) \, dx_1 \, dx_2 \ldots dx_n$$

If the quadrature formula for functions of single variable is according to Eq. (14), then it is not difficult to calculate the N-dimensional integral which takes the form,

$$\int_{-1}^{+1} \int_{-1}^{+1} \cdots \int_{-1}^{+1} f(x_1, x_2, \ldots, x_n) \, dx_1 . dx_2 . dx_3 \ldots dx_n = \sum_{i=1}^{n} \sum_{i=1}^{n} \cdots \sum_{i=1}^{n} W_i \, W_j \ldots W_k \cdot f(x_i) \cdot f(x_j) \ldots f(x_k)$$
The Gaussian quadrature formulae are fully described by Stroud and Secrest (1966).

The double integrations indicated in Eqs. 4.14 - 4.16 were performed in a data processing computer by the use of the library programme written in algol language (see next pages).

Nine data cards were provided to be read in as the weighting coefficient (w) and the roots of legendre polynomials (in the programme shown as U).
12/04/73  COMPILED BY XALE MK. 5C

LIST'(LP, 45)
SENDTO'(ED, MZGKCOMP, AXX)
WORK'(ED, MZGKWORK)

DUMPON'(ED)

ICONCERN:  **** INPUT: FROM DOC SOURCE ****

DUMP'(ED, MZGK_DUMP)
PROGRAM'(MZGK)
COMPACT DATA
TRACE1  2
BEGIN!

'INTEGER' N, M, I, L, K, TI:

BLOCK  1
'REAL' PI, INT, O, F05, F1, F2, F3, Z16, Z17, Z18, NA, DELR;
SELECTINPUT(S);
SELECTOUTPUT(U);
N := 2;
M := 61;
T1 := 20;
BEGIN'

'ARRAY' U[1, 10], P[10], M1, M2, M3, M5, M10, M20, M50, M100,

BLOCK  2
V05, V1, V2, V3, V5, V10, V20, V50, V100, W05, W1, W2, W3, W5, W10, W20, W50, W100,
INT1, INT2, H05, H10, H50, H100, V05, V01, V02, V03, V05, V010, V020, V050, V0100,
INT1, INT2, H05, H10, H50, H100, V05, V01, V02, V03, V05, V010, V020, V050, V0100,
VR, Integer, array S[T; N];

'REAL' PROCEDURE: MULTIINT(N, LOW, UPP, FUNEV, S, M, U, W)

BLOCK  3

'VALUE' N, M;

'INTEGER' N, M;
'ARRAY' U, W

'INTEGER' 'ARRAY' S;
'REAL' 'PROCEDURE' LOW, UPP, FUNEV;

ICONCERN: COMPUTES AN N-DIMENSIONAL INTEGRAL USING AN M-POINT
GAUSSIAN QUADRATURE,
LOW(X, Y) IS USED TO EVALUATE THE J-TH LOWER BOUND
UPP(X, Y) IS USED TO EVALUATE THE J-TH UPPER BOUND
FUNEV(X, Y) IS USED TO EVALUATE THE FUNCTION
THE ARRAYS ARE DIMENSIONED
BEGIN!
 ARRAY A,H,G,X[1:N],D,R[1:N+1];
 INTEGER! ARRAY K,H[1:N];
 REAL E,F,AA,DD,XU;
 INTEGER I,J;
 I=1;
 R[N+1]=D[N+1]=1.0;
 SETUP;
 FOR J=1 STEP 1 UNTIL N DO;
 BEGIN!
 AA=A[J] = LOW(N,X[J]);
 B[J]=UPP(N,X[J]);
 E=S[J];
 DD=D[J]=D[J]-E;
 C[J]=AA + 0.5*DD;
 X[J]=C[J] + 0.5*DD*XU;
 R[J]=0.0;
 H[J]=K[J]=1;
 END! OF SETUP;
 J=1+N;
 SUM;
 F=SUM(N,X[J]);
 R[J]=R[J]*C[J]*F[1]*C[J] + R[J];
 IF K[J]<N THEN;
 GO TO LABK;
 IF H[J]<S[J] THEN;
 GO TO LABH;
 J=J+1;
 IF J=0 THEN;
 GO TO EXIT;
 GO TO SUM;
 LABH;
 H[J]=H[J]+1;
 C[J]=(H[J] - 0.5)*D[J] + A[J];
 K[J]=0;
 LABK;
 K[J]=K[J]+1;
 INITIAL;
 K[J]=0.5*C[J]*D[J] + C[J];
 IF J=N THEN;
 GO TO SUM;
**MULTINT = R[I,J] * D[I] * O, 5**

**END OF MULTIPLE INTEGRATING PROCEDURE**

**REAL** PROCEDURE **LOW** (N, X, J)

**BLOCK 4**

- **VALUE** N, N, J
- **INTEGER** N, J
- **ARRAY** X

**BEGIN**

- **IF** J = 1 **THEN** LOW := LIT[1] **ELSE** LOW := 0
- **END OF LOW**

**REAL** PROCEDURE **UPP** (N, X, J)

**BLOCK 5**

- **VALUE** N, N, J
- **INTEGER** N, J
- **ARRAY** X

**BEGIN**

- **IF** J = 1 **THEN** UPP := LIT[1] **ELSE** UPP := 2 * PI
- **END OF UPP**

**REAL** PROCEDURE **FUNEV** (N, X, J)

**BLOCK 6**

- **VALUE** N, N, J
- **INTEGER** N, J
- **ARRAY** X

**BEGIN**

- **Z16 := 0.005625 * 16, 000**
- **IF** J = 1 **THEN** FUNEV := ((X[1]) / (PI * Z16)) * (EXP((-(PI * (Z16)))) + 2 * (PI) / 2)
- **ELSE**

**FUNEV** := (EXP((2 * (PI) * (X[1]) * COS(X[2]) / (Z16))))

**END OF FUNEV**

**DELRI := 0.025:**

- **FOR** I := 1 **STEP** 1 **UNTIL** M **DO**

**BEGIN**

- **V[I] := READ**
- **S[I] := READ**
- **N[I] := READ**
- **L[I] := READ**
- **K[I] := READ**

**FOR** I := 1 **STEP** 1 **UNTIL** T **DO**

**BEGIN**

- **Q[I] := 0**
- **FOR** K := 1 **STEP** 1 **UNTIL** L **DO**

**BEGIN**

- **Q := Q + INTEG[I] * VM[I]**
- **PRINT** (P[K], 1, 2)
- **PRINT** (Q, 2.0)
- **NEWLINE(0)**
- **END**

**END**

**END**

**END**
<table>
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<tr>
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<th>Title</th>
<th>Publication</th>
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<td>&quot;Measurements of Intensity and Scale of Wind Tunnel Turbulence.&quot;</td>
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FIGURE 2A: THE SIDE VIEW OF THE MAIN JET RIG.
**FIGURE 3:** SCHEMATIC REPRESENTATION OF THE JET FLOW.

**FIGURE 4:** MEAN VELOCITY PROFILES IN THE FULLY TURBULENT FLOW REGION.
Figure 6: The decay of the jet centre-line mean velocity from Gentry & Margason (1966).
FIGURE 7: THE EFFECT OF PRESENCE OF ACOUSTIC WAVES IN THE JET EXIT ON THE DECAY OF THE JET CENTRE-LINE MEAN VELOCITY.
Figure 8: The effect of geometrical form of the jet nozzle on the decay of the jet centre-line velocity. Higgins (1964).

Figure 9: The decay of the jet centre-line mean velocity.
FIGURE 10: CALIBRATION OF FIVE HOLES PITCH AND YAW TUBE.
FIGURE 11: INTERFACE, USE OF DESK CALCULATOR FOR "ON-LINE" PULSED-WIRE MEASUREMENTS.
Figure 12. Low velocity pulsed - wire calibration

Reciprocal of the flight time in microseconds.

(-wire) \( 1/T = 0.000369U + 0.000144 \)

(+ wire) \( 1/T = 0.000219U + 0.000047 \)

Mean velocity in feet per second.

0 1 2 3 4

0 1 2 3 4
FIGURE 13: INTERFACE, USE OF DESK CALCULATOR FOR "ON - LINE" HOT-WIRE MEASUREMENTS.
FIGURE 14: HOT-WIRE CALIBRATION, USE OF DESK CALCULATOR FOR "ON-LINE" HOT-WIRE CALIBRATION MEASUREMENTS.

\[ v^2 = 9.69 + 2.649 u^{0.45} \]
FIGURE 15: USE OF DESK CALCULATOR FOR "ON-LINE" HOT-WIRE MEASUREMENTS
THE DECAY OF CENTRE-LINE MEAN VELOCITY OF A "CLEAN NOZZLE JET".
FIGURE 16: TURBULENT INTENSITY MEASUREMENTS ON THE CENTRE-LINE OF A "CLEAN NOZZLE" JET.
USE OF DESK CALCULATOR FOR "ON-LINE" HOT-WIRE MEASUREMENTS.
Figure 18: Mean velocity profiles in an axi-symmetric circular "clean nozzle" jet.
Figure 19: Mean velocity profiles in an axi-symmetric circular "clean nozzle" jet.

SAMI (1967).

Reichardt's theory.
FIGURE 20: THE DECAY OF THE JET CENTRE-LINE MEAN VELOCITY FROM A TRIANGULAR AND SQUARE JET NOZZLE.
FIGURE 21: THE MEAN SQUARE VELOCITY PROFILES OF AN AXI-SYMMETRIC CIRCULAR JET WITH "SQUARE" VELOCITY PROFILE AT THE EXIT.
FIGURE 22: THE MEAN SQUARE VELOCITY PROFILES DOWNSTREAM OF AN AXI-SYMMETRIC JET WITH "SQUARE" VELOCITY PROFILE AT

- REICHAHR'S THEORY
- EXPERIMENTAL RESULTS

\[ \frac{r}{D_a} \]

\[ x/D = 4 \]
\[ 0.0 \quad 0.1 \quad 0.2 \quad 0.3 \quad 0.4 \]

\[ x/D = 8 \]
\[ 0.0 \quad 0.1 \quad 0.2 \quad 0.3 \]

\[ x/D = 12 \]
\[ 0.0 \quad 0.1 \quad 0.2 \]

\[ x/D = 16 \]
\[ 0.0 \quad 0.1 \]

\[ \left( \frac{u^2}{x,r/\bar{u}_a} \right) \]
FIGURE 23: THE MEAN SQUARE VELOCITY PROFILES DOWNSTREAM OF AN AXI SYMMETRIC JET WITH "SQUARE" VELOCITY PROFILE AT THE EXIT.

- REICHARDT'S THEORY
- MAYSON et al. (1972)
  - $U_a = 831$ ft./sec.
  - $U_c = 761$ ft./sec.

---

$x/D = 0$

$x/D = 1.5/1.57$

$x/D = 3/1.57$
FIGURE 24: MEAN SQUARED VELOCITY PROFILES DOWNSTREAM OF AN AXI SYMMETRIC JET WITH "SQUARE" VELOCITY PROFILE AT THE EXIT.
FIGURE 25: THEORETICAL EVALUATION (REICHARDT'S THEORY) OF THE SQUARED VELOCITY PROFILES DOWNSTREAM OF A MULTI JET NOZZLE CONFIGURATION.
FIGURE 26: THEORETICAL EVALUATION (REICHARDT'S THEORY) OF THE SQUARED VELOCITY PROFILES DOWNSTREAM FROM A MULTI JET NOZZLE CONFIGURATION.
Figure 27: The decay of the jet centre-line mean velocity from a segment nozzle.

DISTANCE FROM JET NOZZLE EQUIVALENT CIRCULAR NOZZLE DIAMETER
FIGURE 28: THEORETICAL EVALUATION OF VELOCITY PROFILES AT TWO JET NOZZLE DIAMETERS AWAY FROM A CIRCULAR JET WITH "SQUARE" VELOCITY PROFILES (Ua/Uc) AT THE JET EXIT.
FIGURE 29: THEORETICAL EVALUATION OF VELOCITY PROFILES DOWNSTREAM FROM
A JET WITH "SQUARE" VELOCITY PROFILE AT THE EXIT.
FIGURE 30: THEORETICAL EVALUATION OF VELOCITY PROFILES DOWNSTREAM FROM A JET WITH "SQUARE" VELOCITY DISTRIBUTION PROFILE AT THE EXIT.
FIGURE 30: THEORETICAL EVALUATION OF VELOCITY PROFILES DOWNSTREAM FROM A JET WITH "SQUARE" VELOCITY DISTRIBUTION PROFILE AT THE EXIT.
FIGURE 31:
THE MASS FLOW RATIO OF CIRCULAR JET WITH "SQUARE" VELOCITY DISTRIBUTION PROFILE AT THE EXIT.
FIGURE 31a: THE MASS FLOW RATIO AT \( (x/D_a) = 10 \).
FIGURE 32: THE NON-DIMENSIONAL RATE OF ENTRAINMENT INTO THE JET WITH "SQUARE" VELOCITY DISTRIBUTION AT THE EXIT.
FIGURE 33: PRESENTATION OF ENTRAINMENT INTO AN AXI-SYMMETRIC UNIFORM JET BY A DISTRIBUTION OF SINKS.

\[ C = \frac{\partial \rho}{\partial x} \]

- REICHARDT'S THEORY
- HILL (1972)
- BRADBURY & WOOD (1964)
FIGURE 34: THE ENTRAINMENT INTO THE JETS OF NON-UNIFORM "SQUARE" VELOCITY DISTRIBUTION PROFILE AT THE EXIT.
FIGURE 35: THE ENTRAINMENT VELOCITY INTO AN AXI-SYMMETRIC UNIFORM JET.
FIGURE 36: THE ENTRAINMENT VELOCITY INTO A RING AND A CORE JET.
FIG. 37 THE DECAY OF THE JET CENTRE-LINE VELOCITY FROM VARIOUS EXPERIMENTERS.
\[
\frac{\bar{U}_{0,r}}{\bar{U}_{0,0}} = (1 - \frac{U_{0,r}}{U_{0,0}}) \, dr
\]

- \(U_{0,0} = 120 \, \text{ft/sec.}; \, \delta^* = 1.716 \, \text{mm}; \, \delta^*/D = 0.0084\)
  - \(\text{Re} = 6.61 \times 10^5\)
- \(U_{0,0} = 66 \, \text{ft/sec.}; \, \delta^* = 1.574 \, \text{mm}; \, \delta^*/D = 0.0077\)
  - \(\text{Re} = 3.14 \times 10^5\)
- \(U_{0,0} = 21.5 \, \text{ft/sec.}; \, \delta^* = 1.855 \, \text{mm}; \, \delta^*/D = 0.0091\)
  - \(\text{Re} = 1.18 \times 10^5\)
- \(U_{0,0} = 11.6 \, \text{ft/sec.}; \, \delta^* = 3.147 \, \text{mm}; \, \delta^*/D = 0.0155\)
  - \(\text{Re} = 6.4 \times 10^4\)

**FIGURE 38: THE EFFECT OF REYNOLDS NUMBER ON THE JET NOZZLE MEAN VELOCITY PROFILE.**
Figure 39: The effect of Reynolds number on the turbulence intensity at the jet nozzle.
Figure 41: The decay of the jet centre-line mean velocity from an axi-symmetric "clean nozzle" jet.
FIGURE 42: THE STATIC PRESSURE ALONG THE CENTRE-LINE AXIS OF AN AXI-SYMMETRIC "CLEAN NOZZLE" JET.
FIGURE 43: THE STATIC PRESSURE DISTRIBUTION.
FIGURE 45: THE TURBULENT INTENSITY ALONG THE RADIAL DIRECTION OF AN AXI-SYMMETRIC CIRCULAR JET AT \( x/D = 4 \).
FIGURE 46  THE INFLUENCE OF BOUNDARY LAYER THICKNESS ON VELOCITY DECAY ON THE CENTRE-LINE OF AN AXI-SYMMETRIC CIRCULAR "CLEAN NOZZLE JET."
FIGURE 47: THE INFLUENCE OF BOUNDARY LAYER THICKNESS ON THE CENTRE-LINE TURBULENT INTENSITY OF A "CLEAN NOZZL
FIGURE 48: RADIAL DISTRIBUTION OF TURBULENCE LEVEL BEHIND THE SCREEN IN THE PARALLEL PART OF THE JET NOZZLE.
FIGURE 49: INTENSITY AND LATERAL SCALE OF TURBULENCE PRODUCED BY A LATTICE TYPE BIPLANE SCREEN.

\[ \frac{\sqrt{u^2}}{u_{10c}} = 1.12 (x/b)^{-5/7} \]

Frankel (1947)

- - - M/b=2.
- M/b=4.x- M/b=8. - - M/b=1.5

Bin (1951).

\[ \Delta \] M/b=4 PRESENT MEASUREMENTS.
- - M/b=\( \frac{1}{2} \) 1/8

\[ \bigcirc \] M/b= 1 / 1/4

\[ \odot \] M/b= 1 1/3 / 1/3

\[ \bullet \] M/b= 12/3

\[ \bigotimes \] M/b= 4/1

Corrsin (1951)
FIGURE 50: THE EFFECT OF TABS ON THE DECAY OF THE JET CENTRE-LINE MEAN VELOCITY.
FIGURE 51: THE EFFECT OF TABS ON THE JET CENTRE-LINE TURBULENT INTENSITY.
FIGURE 52: EFFECTS OF TABS ON THE CENTRE-LINE MEAN VELOCITY DECAY
FIGURE 53: EFFECTS OF TABS ON THE JET CENTRE-LINE MEAN VELOCITY DECAY.
FIGURE 55: MEAN RADIAL VELOCITY DISTRIBUTIONS PROFILE OF A JET WITH TWO TABS.
FIGURE 56: RADIAL DISTRIBUTION PROFILE OF THE MEAN VELOCITY OF A JET WITH TWO TABS.
FIGURE 57: RADIAL DISTRIBUTION OF THE MEAN VELOCITY OF A JET WITH TWO TABS.
FIGURE 58-59 RADIAL DISTRIBUTION OF TURBULENT INTENSITY DOWNSTREAM OF A JET WITH TWO TABS. B/D = H/D = 1/16.
TURBULENT INTENSITY (\%)

\[
\frac{\left<(u^2)\right>^\frac{1}{2}}{\bar{u}_{10c}}
\]

PARALLEL TO Y-AXIS
PARALLEL TO Z-AXIS

FIGURE 60: RADIAL TURBULENCE LEVEL OF A JET WITH TWO TABS OF B/D=H/D=1/16
Figure 61: Radial distribution of the mean velocity profile downstream of a jet with two tabs of H/D = 1/8 and B/D = 1/16.
FIGURE 62: RADIAL VELOCITY DISTRIBUTION PROFILE OF A JET WITH 4 TABS OF \( H/D = 3/8 \) AND \( B/D = 1/8 \).
FIGURE 63: CONSTANT VELOCITY CONTOURS DOWNSTREAM OF A JET WITH TWO TABS.
FIGURE 64: CONSTANT VELOCITY CONTOURS DOWNSTREAM OF A JET WITH ONE TAB AT X/D = 4.
FIGURE 65: EFFECTS OF TABS ON THE JET CENTRE-LINE MEAN VELOCITY DECAY.
Figure 66: Flow angle variation of a jet with two tabs.
FIGURE 67: THE EFFECT OF WEDGE ANGLE ON THE DECAY OF THE JET CENTRE-LINE VELOCITY.
FIGURE 68: THE EFFECT OF WEDGE ANGLE ON THE FLOW-ANGLE VARIATION ACROSS THE JET NOZZLE EXIT.
FIGURE 69: STATIC PRESSURE DISTRIBUTION ACROSS THE CONVERGING JET NOZZLE EXIT
FIGURE 70: THE STATIC PRESSURE DISTRIBUTION ALONG THE CENTRE-LINE OF A CONVERGING JET.

- CONVERGING JET
- WEDGE ANGLE = 10°
- AXI-SYMMETRIC "CLEAN NOZZLE"

\[ \frac{P_{st}}{P_{tot}} \] vs \( (x/D) \) (DISTANCE FROM NOZZLE / JET DIAMETER)
FIGURE 71: THE EFFECT OF CONVERGING JET NOZZLE ON THE VELOCITY DISTRIBUTION PROFILE ACROSS JET NOZZLE.
FIGURE 72: THE EFFECT OF JET NOZZLE CONVERGENCE ON THE DECAY OF THE JET CENTRE-LINE VELOCITY.
FIGURE 73: THE EFFECT OF NOZZLE CONVERGENCE ON THE JET CENTRE-LINE TURBULENT INTENSITY.
**FIGURE 74:** THE FLOW ANGLE (YAW ANGLE) DOWNSTREAM OF AN AXI-SYMMETRIC "CLEAN NOZZLE" JET.
FIGURE 75: THE FLOW ANGLE (PITCH ANGLE) DOWNSTREAM OF AN AXI-SYMMETRIC "CLEAN NOZZLE" JET. SYMBOLS AS IN FIGURE 74.
FIGURE 76: THE FLOW ANGLE (YAW ANGLE) DOWNSTREAM OF A CONVERGING JET NOZZLE.
FIGURE 77: THE FLOW ANGLE (PITCH ANGLE) DOWNSTREAM OF A CONVERGING JET NOZZLE.
Figure 78: The flow angle (yaw angle) downstream from a converging jet nozzle.
FIGURE 79: THE FLOW ANGLE (PITCH ANGLE) DOWNSTREAM FROM A CONVERGING JET NOZZLE.
FIGURE 80: THE MEAN VELOCITY PROFILE ACROSS THE JET NOZZLE OF A JET OF THE GENTRY & MARGASON TYPE.
FIGURE 81: MEAN VELOCITY DISTRIBUTION ON THE CENTRE-LINE OF A JET OF THE GENTRY & MARGASON TYPE.

- PLENUM CHAMBER PRESSURE TAPPING = 17.5 mm WATER.
- PLENUM CHAMBER PRESSURE TAPPING = 33 mm WATER.
- ORIGINAL RECTANGULAR PLENUM CHAMBER OF GENTRY & MARGASON.
- MODIFIED RECTANGULAR PLENUM CHAMBER OF GENTRY & MARGASON.
- CYLINDRICAL PLENUM CHAMBER AND "CLEAN NOZZLE".

PLAEMUM CHAMBER PRESSURE
TOTAL HEAD PRESSURE ON THE CENTRE-LINE
FIGURE 82: FLOW ANGLES IN A JET NOZZLE OF THE GENTRY & MARGASON TYPE.
FIGURE 83: FLOW ANGLES IN A JET NOZZLE OF THE GENTRY & MARGASON TYPE AT (x/D) = 0.1

- YAW ANGLE ($\alpha^\circ$)
- PITCH ANGLE ($\beta^\circ$)

DISTANCE FROM JET CENTRE-LINE AX
JET NOZZLE DIAMETER
FIGURE 84: THE EFFECT OF PRESSURE BUILD UP IN THE PLENUM CHAMBER OF THE GENTRY & MARGASON TYPE ON THE FLOW ANGLES IN THE JET NOZZLE AT (x/D) = 0.1.
FIGURE 85: THE RECIPROCAL OF CENTRE-LINE MEAN VELOCITY DISTRIBUTION.
Figure 86: Presentation of entrainment into an axi-symmetric jet by line distribution of sinks.

Figure 87: Theoretical approach to the calculation of induced velocities into a circular jet.
FIGURE 88: THEORETICAL EVALUATION OF ENTRAINMENT FLOW INTO AN AXI-SYMMETRIC JET.

DISTANCE FROM THE GROUND B.

JET NOZZLE DIAMETER

$C_p \times 10^{-4}$

PRESSURE COEFFICIENT

$r/D$
FIGURE 89: THEORETICAL EVALUATION OF ENTRAINMENT VELOCITY INTO AN AXI-SYMMETRIC CIRCULAR JET.

DISTANCE FROM CENTRE-L1

JET NOZZLE DIAMETER
FIGURE 90: THEORETICAL EVALUATION OF INDUCED VELOCITIES INTO CIRCULAR JETS WITH TWO TABS. \((x/D = 0.25)\)
Figure 91: Axi-symmetry check of the induced flow into the jet on the ground board at $x/D = 0.25$. 

Induced Flow Pressure Coefficient $C_p = \frac{V_p^2}{U_{1oc}^2}$

- $X$-direction
- $(-X)$-direction
- $(-Y)$-direction
- Turbulence intensity

Distance from jet centre-line: jet nozzle diameter.
FIGURE 92: THE ENTRAINMENT FLOW INTO THE JET NEAR THE GROUND BOARD.
FIGURE 93: The turbulent intensity of the entrainment flow into the jet near the ground board.
FIGURE 94: The effect of tabs on the entrainment into the jet. (x/D = 0.25).
FIGURE 95: THE EFFECT OF TABS ON THE TURBULENT INTENSITY OF THE ENTRAINMENT FLOW INTO THE JET AT $x/D = 0.25$. 
FIGURE 96: TURBULENCE SPECTRUM IN AN AXI-SYMMETRIC "CLEAN NOZZLE JET"
FIGURE 97b: THE EFFECTS OF TABS ON THE AUTOCORRELATION FUNCTION OF THE TURBULENCE ON THE JET CENTR-LINE AT x/D
FIGURE 98: AUTOCORRELATION FUNCTIONS ON THE CENTRE-LINE OF AN AXI-SYMMETRIC "CLEAN NOZZLE" JET AT $x/D = 2$. 

$r/D = 0$

$r/D = 1/8$

$r/D = 1/4$

$r/D = 1/2$

$r/D = 3/4$
Figure 100: The autocorrelation functions on the centre-line of an axi-symmetric "clean nozzle" circular jet at $x/D = 6$. 
FIGURE 101: CROSS-CORRELATION FUNCTIONS IN THE FLOW OF AN AXI-SYMMETRIC "CLEAN NOZZLE" CIRCULAR JET AT x/D = 0.1.
<table>
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<th>$y/D$</th>
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<tbody>
<tr>
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<td><img src="image1" alt="Graph" /></td>
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<tr>
<td>$1/4$</td>
<td><img src="image2" alt="Graph" /></td>
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<td>$1/2$</td>
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<td>$1$</td>
<td><img src="image5" alt="Graph" /></td>
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FIGURE 102: CROSSCORRELATION FUNCTIONS IN A FLOW OF AN AXI-SYMMETRIC JET OF CIRCULAR "CLEAN" NOZZLE AT $x/D = 2$. 33.3 millisecond.
FIGURE 103: THE CROSSCORRELATION FUNCTIONS IN A FLOW OF AN AXI-SYMMETRIC "CLEAN NOZZLE" JET AT $x/D = 4$. 
FIGURE 104: THE CROSSCORRELATION FUNCTIONS IN A FLOW OF AN AXI-SYMMETRIC CIRCULAR "CLEAN NOZZLE" JET AT x/D = 6.
FIGURE 105: THE CROSSCORRELATION FUNCTIONS IN A FLOW OF AN AXI-SYMMETRIC CIRCULAR "CLEAN NOZZLE" JET AT $x/D = 2$. 

$y/D = 1/8$

$y/D = 1/4$

$y/D = 3/8$

$y/D = 5/8$

$y/D = 17/16$

3.33 millsec.
FIGURE 106: THE CROSSCORRELATION FUNCTION IN A FLOW OF AN AXI-SYMMETRIC CIRCULAR "CLEAN NOZZLE" JET AT x/D = 4.
FIGURE 107: SPATIAL CROSSCORRELATION FUNCTIONS IN A FLOW OF AN AXI-SYMMETRIC CIRCULAR "CLEAN NOZZLE" JET.
FIGURE 108: CIRCUMFERENTIAL CORRELATIONS IN A "CLEAN NOZZLE" JET AT x/D=2.
FIGURE 109: CIRCUMFERENTIAL CORRELATIONS IN A "CLEAN NOZZLE" JET AT x/D = 2.
FIGURE 110: CIRCUMFERENTIAL CORRELATIONS IN THE AXI-SYMMETRIC CIRCULAR "CLEAN NOZZLE" JET AT \( x/D = 2 \).