THE OPTICAL MODEL FOR PROTON ELASTIC SCATTERING
FROM LIGHT NUCLEI AND THE SPIN-ORBIT FORCE

A thesis submitted to the
Faculty of Mathematical and Physical Sciences
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Master of Philosophy

by

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We present the optical model analysis of proton elastic scattering on light nuclei, with improved spin-orbit force, in the energy range 50 to 160 MeV.

The particular potentials are determined by varying independently all the parameters of the phenomenological optical potential for p + $^{12}$C elastic scattering and good description is obtained for this target nucleus in the energy range considered. Then, the average potential is deduced. When this average potential is used, without parameter adjustment, to describe p + $^{16}$O elastic scattering a comparable fit to the data is obtained. The dependence on mass number comes only through the half-way radius. Furthermore, when this potential is applied to other neighbouring nuclei, ranging from $^9$Be to $^{14}$N, similar acceptable fits to the above data are also obtained. Hence, the validity of our average potential is not restricted to the target nuclei having only alpha-cluster structure.

The spin-orbit potential is calculated microscopically using the impulse approximation and the first-order multiple scattering theory. This potential is then given finer adjustments to optimize the fit to the polarisation data. This results in the spin-orbit potential having different parameters for their real and imaginary parts. The six-parameter model is more in accord with the microscopic findings and it gives improved description to the experimental data.
To

My father and mother, and

Rio Hirowati
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CHAPTER 1

INTRODUCTION

It is well known now that the atomic nuclei are composed of Z protons and N neutrons, the total number of these nucleons being the mass (atomic) number $A = N + Z$. Ever since its very existence, the presence of nucleus was revealed and its constituents were studied by collision experiments. The suitable probes to study the nucleus have been found in the nuclear and sub-nuclear particles themselves such as neutrons, protons and pions as well as other nuclei. Along this line, the theoretical interpretations of the processes involved are provided by the mathematical theory of scattering and reaction processes on the basis of quantum mechanical idea of wave-particle duality. The wave motion of the projectiles is used for "looking" at the nuclei.

Nucleons incident on a target nucleus interact in many ways. By observing the outgoing particles these interactions can be classified into two main categories: scattering and reaction. The scattering process describes the situation where the outgoing particles are identical with the incident ones; whereas a reaction takes place when the outgoing particles are different, implying the exchange of particles between the projectile and the target. The latter is sometimes called rearrangement collision. Scattering can be further divided into elastic and inelastic. In elastic scattering the outgoing nucleon retains all its energy except for the recoil energy of the target, while in inelastic scattering, some of the energy is "lost" (transferred to the target nucleus) in promoting the target nucleus into one of its excited states.

When a nucleon undergoes elastic scattering, the internal energy of the target is unchanged but the projectile nucleon is scattered out
of the incident beam in a manner which depends on the interaction
between the projectile and the target. The manner of the interaction
is represented mathematically by a potential. Our purpose is to study
the elastic scattering of protons from light nuclei and hence the
potential interaction between them.

The potential is taken to be complex, in analogy with optics,
and hence the nucleus is regarded as a medium of complex refractive
index. There have been many analyses of the experimental data
(differential cross-sections and polarisations) of protons elastically
scattered from many nuclei at many energies in this so-called optical
model of interaction. These analyses are of two main types: the
phenomenological and microscopic approaches. To some extent, the two
types of analyses are complementary to each other in that the microscopic
analyses can give the overall form of the potential and the phenomenological
analyses can make the fine adjustment to fit the data as precisely as
possible. This is, in fact, partly followed in this thesis in
determining the spin-orbit potential.

The present work is an attempt to obtain an average optical
potential phenomenologically and then starting from the microscopic
form of the spin-orbit potential in the framework of Kerman et al(5) a
six-parameter model of the average spin-orbit potential is later derived.

The existence of average potentials shows that the proton elastic
scattering is rather insensitive to the details of nuclear structure.
Further, the average potential is expected to vary smoothly with incident
energies and target mass. This average potential has the advantage
that it facilitates the description of the wave functions in the vicinity
of the nucleus and enables them to be calculated even when no elastic
scattering data are available. This knowledge of proton wavefunctions
is essential for the accurate calculation of nuclear reaction cross-sections
by the distorted wave theories such as Born approximation (DWBA) and impulse approximation (DWIA) in the \((p,2p)\) and \((p,d)\) reactions or in the coupled-channel analysis. Beside that, an accurate determination of the spin-orbit potential will ensure the correct description of polarised protons.

Beyond this utilitarian aspect of the potential, there are reasons to believe that useful information about nuclear forces and nuclear structure can be obtained from the systematic analysis of nucleon elastic scattering\(^{(6)}\).

Also it is an intrinsic interest in the phenomenological optical model itself to determine the systematic energy and target mass dependences which are relatively free from ambiguities in parameterization such as the volume integrals of the various nuclear potentials and their root-mean-square radii. Hence, the potential obtained on a microscopic level, starting from the nucleon-nucleon interaction and incorporating various many-body effects could be compared with the phenomenological results\(^{(7)}\). This development enhances our understanding of the elastic scattering process through the optical model potential.

We will analyse the proton scattering from light nuclei in the intermediate energy range. By intermediate energy we mean from 50 MeV up to 160 MeV. The analyses will concentrate mainly on \(^{12}\text{C}\) target. Since this is a relatively very light nucleus, we expect the fits not to be as satisfactory as the heavier targets.

For heavy nuclei at intermediate energy range, Seth\(^{(8)}\) and recently Nadasen et al\(^{(9)}\) have shown that the optical model gives a satisfactory description of the elastic scattering of nucleons. The model has not enjoyed equal success in its application to light nuclei\(^{(10)}\). The lack of consistency in the optical potential parameters for both light and heavy nuclei might reveal the dependence of optical model parameters
on the nucleon distribution in these nuclei. However, for light nuclei the individual data at a particular energy and target could be reproduced but the resulting parameters of the particular potential fluctuates wildly with energy and mass number.

There are reasons to explain this behaviour. First, even at a fairly high excitation the level density is low for light nuclei and hence the nuclear-structure effects which the optical model cannot describe are not sufficiently averaged out. Second, it may not be appropriate to replace the nucleus with a potential having a simple radial form (for example, a Woods-Saxon form which is often used in the case of heavy nuclei). Inherent in a replacement of this kind is the basic assumption that the nucleus can be regarded as a continuous distribution of nuclear matter to the incident particle. There are so few nucleons in light nuclei that this approximation may not be valid.

However, the parameters obtained by fitting the individual data set in the case of \( p + ^{12}\text{C} \) elastic scattering are then being systematised. An average potential is extracted from the particular potentials obtained for each energy.

This average potential is expected to work as well for the case of \( ^{16}\text{O} \) due to the similarity in structure in terms of spin zero and alpha cluster structure. Carbon-12 can be treated as a three alpha and oxygen-16 as a four alpha cluster structure, respectively. In this average potential, the dependence on mass number comes only through the half-way radii.

The sensitivity of our average potential is tested in the case of \((p,2p)\) reaction to see how well our potential is able to account for the distortion effect in the outgoing channels.

In most of our results we compare with those obtained by Seth\(^{(8)}\). Other analyses involving \( ^{12}\text{C} \) and \( ^{16}\text{O} \) at lower energies include the works
done by Watson et al\textsuperscript{(11)} and van Oers and Cameron\textsuperscript{(12)}. These works cover the energy range between 10-50 MeV, but so far there is no analysis being done in the energy range we are considering using the target nucleus as light as \textsuperscript{12}C. In a way this is a gap we are trying to fill in.

In most of the phenomenological analyses at energies lower than 50 MeV, generally the form-factors were found to be independent of energy and the energy dependence comes in through the behaviour of the dynamical parameters\textsuperscript{(13,14,12,15,16)}. Such a parameter choice has been shown to give satisfactory results whenever the energy range is restricted, at most a few tens of MeV. However, there is no reason a priori to expect a common geometry to work for the whole energy region we are considering.

Precise determination of the spin-orbit part of the potential is important particularly in the applications where the protons involved in the reaction being considered are polarised. This is done by obtaining the overall form of this potential component using the impulse approximation and first-order multiple scattering series of Kerman et al\textsuperscript{(5)}. Using this potential as the starting values the fine adjustment is done phenomenologically against the data to obtain the final form of the spin-orbit potential.

In the analysis to be described, our main problem is the scarcity of the experimental data available such as the differential cross-section, polarisation and the total reaction cross-section data. Even if the data are available they do not extend up to back angle region. The lack of polarisation data limits our ability to determine the spin-orbit force to a higher precision. The missing total reaction cross-section data are obtained by interpolation from the neighbouring values.

The outline of this thesis is as follows. In Chapter 2 a background idea of the nuclear optical model is presented together with
the latest development in this field. This will provide the basic idea of the subject of this thesis. Chapter 3 is devoted to the derivation of the mathematical formulation used in the study of the phenomenological optical potential calculation as well as the choice of the form of the potentials used. In Chapter 4 the particular potentials obtained by independent fits by varying all the potential parameters is described. This leads to the derivation of the average potential in Chapter 5, where the extension to $^{16}_0$ is done. The overall form of the spin-orbit potential obtained microscopically by means of impulse approximation is presented in Chapter 6. Then, starting from the results obtained in this chapter the precise form of the six-parameter average spin orbit potential is derived in Chapter 7. Finally, the conclusions obtained from our work are given in Chapter 8.
2.1 Introduction

One of the most basic interactions in nuclear physics is that between a nucleon and a nucleus. By knowing this interaction one can calculate the physical quantities of the elastic scattering such as the differential cross-sections, polarisations and other quantities and also the distortions of the nucleon waves by the nucleus which are essential for the understanding of the reactions that can take place. Furthermore, it is the usual starting point of the more complicated interactions involving composite projectiles and the nucleus. However, the interaction of a nucleon with a nucleus itself is a complicated one. This arises not only from the many-body problem determined by all the possible interactions between the nucleons present under the influence of the neighbouring nucleons in the nucleus but also the individual structure of the nucleus under consideration.

Essentially the nucleon incident on a nucleus may either be elastically scattered or it may cause a variety of different reactions. To account for this phenomenon the many-body nature of the interaction is represented by a one-body potential. The replacement of this nature was done by Bethe\(^{17}\) using a purely real potential. But the model does not give the predicted physical quantities which are in accord with the experimental values. The inability of such a real potential to account for the non-elastic quantities was then superseded by the compound nucleus model of Bohr\(^{18}\). But Bohr model was in turn shown to be inadequate to account for the experimental results at higher energies. These two rather extreme ideas were then compromised by allowing the one-body potential to be complex where an imaginary part was added to a
real part. Such a complex potential is called the optical potential; a remniscent of the scattering of light by an optical medium. This idea was applied in a semiclassical way by Fernbach et al.(19). In fact, the use of a complex potential in describing the proton scattering was first used by Ostrofsky et al.(20) as early as 1936. But a full quantum mechanical calculation using a complex potential was later done by Le Levier and Saxon(21) for the scattering of 17 MeV protons by aluminium, and gave a good account of the rather imprecise data available at that time. This is the first time the term optical potential was being used.

The finding of Barschall(22) and his colleagues(23) on the strong systematic behaviour of the total cross-section and the differential cross-section of low-energy neutron scattering with the energy and target mass has led Feshbach et al.(24) to analyse these data using the methods of Le Levier and Saxon. They were able to reproduce its overall features. This suggests that the scattering process is determined by the nuclear matter in bulk and the nucleus as a blob of it whose properties are determined by its size. The smooth variation of the physical quantities is attributed to the gradually changing nuclear size. The individual structure of the nuclei enters as a second order effect and causes the observed fluctuations around the systematic behaviour. In the above-quoted works the details of the fluctuations due to the effects of the resonances were neglected. But this can be taken into account by the use of Hauser and Feshbach theory(25).

However, the results of experiments with the relatively poor energy resolution were reproduced by the optical model potential for the low-energy measurements and give the average or gross structure of the differential cross-sections(24). At higher energies the fine structures of the resonances are smoothed out and become giant resonances which can
easily be accounted for by the use of the optical potential.

Since then, and almost thirty years have passed since it was first suggested, the optical potential has developed into a model of considerable power and generality. A wide range of precise scattering data has become available and they can be accounted for by the model to a high degree of precision, and the corresponding wavefunctions are extensively used to extract information on nuclear structure from measurement of nuclear interaction cross-sections. As the precision of the data increased the model was refined to account not only for the differential and reaction cross-sections but also for the polarisations to a high degree of accuracy.

This chapter concerns the nuclear optical model for nucleon, which is the subject of this thesis. The purpose is to supply the background material for the subject. Discussion on the optical model for composite projectiles are found in the articles by Hodgson \(^{(27,28)}\) and by Barrett and Jackson \(^{(29)}\). First, the concept of the model is presented, followed by the discussion on various approaches which exist in determining the optical potentials and the relations between them. Later, the utility of the potentials found in various approaches is discussed.

### 2.2 Concept and Definition

The optical potential is defined as a potential that represents the interaction between a nucleon (or group of nucleons) and a nucleus. When such a potential is inserted into the Schroedinger equation

\[
[-\frac{\hbar^2}{2\mu} \nabla^2 + V(r)]\psi(r) = E\psi(r)
\]  

(2.1)

it gives the differential cross-section and polarisation for elastic scattering, the reaction cross-section and some other less important
observable quantities. In equation (2.1) the optical potential is represented by \( V(r) \), \( \mu \) is the reduced mass, \( E \) is the total energy of the system and \( \psi(r) \) is the total wavefunction of the system described by the time-independent Schroedinger equation (2.1).

Hence, the many-body nature of the nucleon-nucleus interaction is replaced by a one-body potential, \( V(r) \), between the nucleon and the nucleus, which we call optical potential. The name has been borrowed from the term in optics\(^{(21)}\) for the analogy between the interaction of a light wave by a medium of complex refractive index and in the case of the nuclear interaction, the one-body potential is allowed to be complex

\[
V(r) = U_R(r) + i W_I(r)
\]  

For this reason some authors prefer the phrase "complex potential" to "optical". As the result of the potential being complex the time-independent Schroedinger equation (2.1) is no longer hermitian. Hence, constructing the divergence of the probability current density from equations (2.1) and (2.2), we have

\[
\nabla \cdot \mathbf{j} = \frac{2}{\hbar} W_I(r) |\psi(r)|^2
\]  

Thus, in the presence of a complex potential, the current is no longer divergenceless. That the classical continuity equation in the case of a steady state has been attained implies that provided \( W_I(r) \) is negative, the imaginary part of the complex potential has the effect of absorbing flux from the incident beam. Hence, the intensity in the elastic channel has been reduced.

The essential idea of the optical model is that a nucleon incident on a nucleus may be elastically scattered or it may cause a variety of different reactions. If the incident particle is represented...
by a wave, then in classical language it may be scattered or it may be absorbed. This is analogous to the refraction and absorption of a light wave by an optical medium with complex refractive index. In nuclear physics, the nucleus is represented by a complex potential in which the real part has the effect of refracting the incident nucleons and the imaginary part as absorbing them. The absorption takes account of all inelastic scattering and reaction processes which removes the flux from the elastic channel.

The conservation of particle flux links all the reaction channels together, so that any abnormal behaviour in one channel, a resonance or threshold for example, produces corresponding changes in all the others. So elastic scattering depends essentially on both parts of the potential. As the imaginary part determines the absorption, where all the non-elastic processes are lumped together and are treated as a process that removes particles from the incident beam, the real part also affects it by refracting particles towards or away from the region of greatest absorption.

As the consequence of the approximation made the scattering process is essentially determined by the nuclear matter in bulk. The smooth variation of the quantity is due to gradually changing nuclear size, apart from certain small structure effects. This individual structure of the nuclei enters as a second-order effect and causes the observed fluctuations around the systematic behaviour. Hence, the term "optical" applies most accurately at high energies where the real non-elastic processes dominate the absorption, and there is no need to make an energy average. Then only it is always possible to describe exactly the passage of a nucleon through a nuclear medium in terms of an index of refraction, \( n \), such that
\[ n^2 = 1 - \frac{[U_R(r) + iW_I(r)]}{E} \]  \hspace{1cm} (2.4) 

Evidently, the detailed theory of optical potential is a very difficult many-body problem, but substantial progress has been made, for instance making use of the nuclear-matter theory due to Brueckner \(^{(30)}\). However, the formal theory of nuclear reactions developed by Feshbach \(^{(31,32)}\) enables the contributions to the optical potential to be identified and provides a useful framework for a theoretical definition of the potential. This theory is considered below.

By considering only the inelastic scattering channel as open and neglecting the other channels involving transfer reaction and others, we write the total wavefunction \(^{(32,34)}\) as

\[ \psi(r,\xi) = \sum_n \psi_n(r) \phi_n(\xi) \]  \hspace{1cm} (2.5)

where \(\psi_n(r)\) and \(\phi_n(\xi)\) are the wavefunctions for the incident particle and the corresponding target states, respectively. Here, the antisymmetrisation requirement has been neglected. Hence, \(\psi_0(r)\) satisfies \(^{(28)}\)

\[ (T(r) + V(r) - E)\psi_0(r) = 0. \]  \hspace{1cm} (2.6)

where \(v(r)\) is the optical potential.

Defining the projection operator \(P = \langle \phi_0 | \phi_0 \rangle\) which projects on to the ground-state part of the target wavefunction and \(Q = 1 - P\), we have \(^{(34)}\)

\[ [E - T - (\phi_0 | V | \phi_0) - (\phi_0 | VQ \frac{1}{E - Q H Q} QV | \phi_0 \rangle)\psi_0(r) = 0 \]  \hspace{1cm} (2.7)

Comparison with (2.6) gives an exact formal expression for the optical potential.
\[ \begin{align*}
\psi(\tau) &= (\phi_0 | V | \phi_0) + (\phi_0 | VQ \frac{1}{E-Q H Q} \psi | \phi_0) \\
&= (\phi_0 | V | \phi_0) + (\phi_0 | VQ \frac{1}{E-Q H Q} \psi | \phi_0) \quad (2.8)
\end{align*} \]

Furthermore, the second term can be written in the form, using the principle value formula in the limit \( \epsilon \to 0 \),

\[ \frac{1}{E-Q H Q} = \frac{P}{E-Q H Q} - i\pi\delta(E-Q H Q) \quad (2.9) \]

Hence, the first two terms of the optical potential contribute to the real part and the last term to the imaginary part. The first term represents direct transition from the incident to outgoing channel, while the second term involves the energy non-conserving virtual transitions to energetically-unreachable intermediate states followed by a return to the initial state. But the third one involves the energy conserving actual transition to energetically-reachable states which consists of non-elastic events that have the effect of removing flux from the elastic channel. The presence of the propagator causes the second and the last terms to be non-local. It means that the optical potential can be written in the form\(^4\)

\[ <r' | V | r> = V(r) \delta(r-r') + K(r, r') \quad (2.10) \]

so that the Schrödinger equation becomes

\[ [E - T - V(r)]\psi(r) = \int K(r, r')\psi(r')dr' \quad (2.11) \]

The formal theory also shows that the wavefunctions in all the open channels satisfy a set of coupled differential equations, one for each channel. This set is too large to solve in practice, so very often it is severely truncated, and the effect of the channels not considered is taken into account by the complex interaction potential. The most extreme truncation, that which retains only the equation of the wave-function in the shape elastic channel, gives the usual optical potential. A less
severe truncation retains one or more equations for wavefunctions in non-elastic channels; this enables both the elastic and non-elastic cross-sections to be calculated simultaneously provided a model is available for the channels considered and for the corresponding interaction potentials. This approach is important when the coupling between the elastic and non-elastic channels chosen is strong. The coupling to the inelastic scattering channels with the excitation of collective states has often been treated this way, for instance it was done by Buck\(^{(35)}\) and later was extended by Tamura\(^{(33)}\). The coupling to some of the rearrangement collision processes such as the pick-up channel has been done by Mackintosh and Kobos\(^{(36)}\) and also by Comfort and Karp\(^{(37)}\). The contributions from both the inelastic scattering and pick-up channels have been considered by Coulter and Satchler\(^{(38)}\).

Hence, the optical potential is not without limitations. Since the model ignores most of the detailed features of the nuclear structures, any features that depend on the properties of more complicated states differing from one nucleus to the next cannot be accounted for, apart from a few refinements that can be made.

Another condition is that the reaction should excite many states in the compound nucleus, usually fulfilled by medium and heavy nuclei and at high energies.

Of course it also works well if the elastic channel is not strongly coupled to any of the non-elastic channels, for in this case the interaction can significantly perturb the elastic channel.

Mainly, there are two ways of determining the optical potential. One is by phenomenological methods, in that the parameters of an assumed form of the potentials are found by optimising the fit to the experimental
data. The other is by microscopic methods, where the potential is obtained from more fundamental basis and is derived from the known properties of the nucleus and the nucleon-nucleon interaction. This is indicated schematically below:

The phenomenological potential obtained by induction from the nucleon-nucleus scattering and reaction data suffers from ambiguities where it is found that several potentials fit the same data equally well. It is thought that one of these is the 'physical' potential. This physical potential can be identified by deduction from the knowledge of nucleon-nucleon interaction and the known properties of the nucleus. Subsequently, the overall shape of the potential obtained qualitatively by this microscopic method is then given finer adjustments in fitting the experimental data to obtain the phenomenological potential quantitatively.

Hence, the two methods are, in fact, complementary to each other.

2.3 Phenomenological optical potentials

Since this thesis is concerned mainly with this aspect of the optical potential detailed discussion of it is given at the appropriate places throughout this work. However, we mention briefly the components of this potential here which are generally in use.

The need to have the overall (average) potential has been achieved at the expense of the quality of fits to the data. This deviation
from normality can either be understood physically and be treated by inclusion of the physical effects, using coupled-channel method for instance, or can still be treated within the optical model formalism. The latter can be done by the inclusion of a series of small terms to the optical potential which are expressible as a simple analytical function of mass number and energy.

It is an established notion that the optical potential requires the spin-orbit component in order to reproduce the polarisation data besides the fundamental components of central term for the differential cross-section data\(^{(40)}\). This is discussed in detail in the next chapter.

Lane\(^{(41)}\) has shown that the real part of the potential depends on the isospin term which gives rise to the symmetry component. The potential is of the form

\[ V_I(r) = \frac{4V_1}{A} T \cdot f(r) \]  

(2.12)

The existence of this symmetry term has been obtained by Perey\(^{(13)}\), Perey et al\(^{(42)}\), Sinha and Edwards\(^{(43)}\) and Hodgson\(^{(44)}\). So far there is no evidence for its existence in the imaginary part of the potential\(^{(45)}\) due to the lack of knowledge of its form factor.

Another quite sensible possibility is the dependence on the spin of the target nucleus. One of the forms of this spin-spin term is

\[ V_{SS}(r) = V_{SS} f_{SS}(r) \frac{I \cdot \sigma}{I} \]  

(2.13)

But so far no evidence of its presence has yet been obtained\(^{(46)}\) though Nagamine et al\(^{(47)}\) found it to be very small.

Another possibility is to include spin-spin term of the form

\[ V_T(r) = V_T f_T(r) \frac{\{3(I \cdot \hat{r})(\sigma \cdot \hat{r}) - I \cdot \sigma\}}{2 I} \]  

(2.14)
Again its value is very small or zero\textsuperscript{(48,49)}.

There are also other small terms due to the electromagnetic effects due to the interaction of the magnetic moment of the incident particle with the electrostatic field of the nucleus, represented by

$$V_{MM}(r) = \frac{\mu - a}{2MC} \frac{1}{r} \frac{dV_C}{dr}(r) \frac{\hbar}{\sigma}$$

(2.15)

where $\mu$ is the magnetic moment of the incident nucleon and $a = \frac{1}{2}$ for protons and $a = 0$ for neutrons. $V_C$ is the electrostatic Coulomb potential. In most circumstances this term is included into the spin-orbit term of the potential due to similarity in its form.

Other small term is due to the interaction of the induced electric dipole moment of the incident nucleon with the Coulomb field of the target nucleus, written in the form

$$V_{DM}(r) = -\frac{\alpha E^2}{2}$$

(2.16)

where $\alpha$ is the electric polarisability and $E$ is the electric field.

This potential is significant only outside the nucleus where it becomes

$$V_{DM}(r) = -\frac{\alpha z^2 e^2}{2r^4}$$

(2.17)

However, the effect of this potential is so small that they can readily be absorbed into the real part of the potential.

2.4 Microscopic optical potentials

A microscopic calculation of the optical model potential for nucleon is of both fundamental and practical interest. It provides the theoretical understanding of the nuclear structure model and serves to test the underlying many-body theory and the nucleon-nucleon
interaction. It also helps in removing the ambiguities of the phenomenological optical potential. Microscopic calculation is not the main concern of this thesis but it is worth mentioning some of the methods employed and its success.

Most of the realistic calculations of the microscopic optical model potential based on a variety of approaches: multiple-scattering, Green's function and Hartree-Fock or Brueckner-Hartree-Fock theories, and they involve either "effective" (weak) or "realistic" (strong) nucleon-nucleon interactions.

The use of the realistic nucleon-nucleon interaction is usually restricted to higher energy region. It is used together with the multiple scattering expansion of the optical potential which was pioneered by Kerman, McManus and Thaler (5) where impulse approximation is valid. This theory was later extended by Feshbach et al (50) to an energy as high as 1 GeV. Similar work has been undertaken recently by Chaumeaux et al (51). A quite similar approach based on the multiple scattering expansion employing a third-order Bethe-Goldstone reaction matrix to calculate the volume integral of the real optical potential in infinite nuclear matter at intermediate energies (> 200 MeV) has been done by Ray and Coker (52). An approach combining Green's function theory with the Brueckner's low-density expansion has been successfully employed by Jeukenne et al (7) and by Brieva and Rook (53). Their works indicate that it is now possible to calculate optical model potentials that give fits to the data that are comparable to those found phenomenologically.

The effective nucleon-nucleon interactions are being used for constructing the optical potentials at lower energies (< 100 MeV). Greenlees et al (54) pioneered the "reformulated" optical model potential which is, in fact, the folding model potential for the real part and
using an adjusted effective nucleon-nucleon interaction. Later Thomas et al.\textsuperscript{(55)} improved upon this approach by including the exchange term. At very low energies the optical model potential has been calculated using the self-consistent Hartree-Fock approximation which is the lower energy counterpart of the nuclear matter calculations. Works along these lines have been done by Dover and Van Giai\textsuperscript{(56)} and Vinh Mau\textsuperscript{(57)} and collaborators.

2.5 Conclusions

Besides the interest in calculating the optical potential on its own, to get consistent description of scattering process for instance, it can also be used to calculate the distorted waves in nuclear reaction calculations. Also from the knowledge of nucleon-nucleus potential one can extend it to obtain the composite projectile-nucleus scattering, for instance in deriving the alpha-nucleus optical potential from the nucleon-alpha interaction\textsuperscript{(58)}.

However, one can conclude now that the optical model can account for the scattering from a vast range of energy and target nucleus provided proper account is taken of the physical effects involved. Hence, phenomenologically much work remains to be done to improve the calculation of small terms. Microscopically, the techniques for calculating the potential have now been established.
CHAPTER 3

THE PHENOMENOLOGICAL OPTICAL MODEL OF ELASTIC SCATTERING

3.1 Introduction

The elastic scattering of a projectile, whether a nucleon or a composite particle, on a target nucleus can be described by a one-body potential and is called the phenomenological optical potential. The scattering of nucleons by nuclei is the most appropriate because it is for these interactions that we have the best grounds for expecting the model to be valid, and here are to be found the most extensive experimental data. In fact, this is the main aim of our study.

The treatment of the nucleon-nucleus elastic scattering which is based on the phenomenological optical model is well-known. The interaction is represented by a potential of appropriate form. Provided that the incident energy is high enough (at least greater than 15 MeV) that the compound nucleus formation is not expected to play any role and that no particular reaction process dominates the interaction, the method of calculating the observables is a straightforward one. So, no physical effect has to be added. Otherwise, we have to include the compound nucleus formation effect and the coupling between the elastic and non-elastic channels into account. Our analysis of the experimental data will be for an incoming proton of above 50 MeV and we assume that all the non-elastic processes can be represented by the imaginary part of the potential alone.

This chapter will mainly describe the mathematical formulation and method of calculating the observables by quantum mechanical arguments. This will be achieved by solving the Schrödinger equation which describes the scattering of a projectile by a target nucleus via an assumed potential. Indeed the problem of finding the solutions to this equation
with the appropriate boundary conditions in the presence of a complex potential is, in general, a difficult one. In most parts, numerical techniques have to be invoked. Moreover, the potential contains parameters which have to be varied in a systematic way such that the use of a high speed computer is indispensible.

In the following we will treat the proton-nucleus scattering from a local, complex and spin-dependent optical potential. Non-relativistic quantum mechanics will be used and the formalism will be in the form suitable for computation.\(^{(4, 59, 60, 61, 62)}\)

### 3.2 The Potential

In order to describe the interaction successfully a suitable choice of the potential has to be made. The potential must be able to account for the interaction based on sound physical ground. In the first place, the potential is assumed to be spherically symmetric.

For proton scattering it is necessary to include a Coulomb term in the optical potential to account for the interaction of the incident proton with the charge distribution due to the protons in the target nucleus. It is sufficiently accurate to take this as the potential due to a uniformly charged sphere, the analytical form of which is given by

\[
V_c(r) = \begin{cases} 
\frac{Z_I Z_T e^2}{2R_c} (3 - \frac{r^2}{R_c^2}) & , \ r < R_c \\
\frac{Z_I Z_T e^2}{r} & , \ r \geq R_c 
\end{cases}
\]  

\[ (3.1) \]

where \( R_c \) is the radius of the charged sphere and is given by \( R_c = r_c A^\frac{1}{3} \). \( Z_I e \) and \( Z_T e \) are the charges of the incident proton and the target.
nucleus, respectively. The exact form of this potential is not important as the interaction is insensitive to its radial form\(^{(60)}\).

As the nucleon-nucleon interaction is of short-range and falls off rapidly at large distances, it is reasonable to expect similar behaviour for the case of nucleon-nucleus interaction as well. From the near-incompressibility of nuclear matter and the saturation of nuclear forces, a nucleon inside the nucleus feels only the nucleus in its immediate vicinity. Hence, well inside the nucleus a nucleon is acted on by the equal forces from all directions so that the corresponding potential is uniform near the centre of the nucleus. Of course, it must be attractive to account for the binding of the nuclei and a smoothly varying function of radial distance. One of the functional forms for the real part of the potential which complies with the above requirements is the Saxon-Woods form\(^{(26)}\)

\[
f(r) = \frac{1}{1 + e \left( \frac{r-R}{a} \right)}
\]  

(3.2)

where \(R\) is the half-way radius. Since the overall extent of the potential is expected to be similar to the nucleus itself, it is taken to be \(R = r_0 A^{1/3}\). Parameter \(a\) is the diffuseness which is a measure of the rapidity of the fall-off of the potential. This form will be used in the present case. It is the same form as the Fermi distribution for nuclear charge or matter distribution but the radius is expected to be larger.

Since the imaginary part of the potential describes different processes, it is not obvious that it should have the same radial behaviour as the real part. At low energies the blocking effect of the Pauli principle inhibits nucleon-nucleon collision in the interior of the nucleus. So the form of this potential is expected to be surface-peaked. But this effect decreases with increasing incident energies
such that the collisions spread throughout the nuclear volume. The
general functional form which combines this behaviour is

\[ W(r) = (W_V - W_D) 4a \frac{d}{dr} f(r) \]  

(3.3)

where \( f(r) \) is defined by (3.1). Factor \( 4a \) is chosen to make the
maximum value of the second function unity and \( W_D \) have the dimension of
MeV.

To account for the polarisation of the scattered nucleons it was
found necessary to include a spin-dependent term in the optical potential
For the scattering of a nucleon from a closed-shell nucleus, on general
invariance consideration \(^{64}\), it is expressed in terms of spin-orbit
coupling \( \mathbf{\ell} \cdot \mathbf{\sigma} \), where \( \mathbf{\ell} \) and \( \mathbf{\sigma} \) are the angular momentum and the Pauli spin
operator respectively. As symmetry requires the spin-orbit force to
be zero in the nuclear interior but appreciable only in the surface
region \(^{65}\), it is rather natural to adopt for this potential a mathematical
function suggested by the Thomas form as in the atomic theory

\[ V_{SO}(r) = V_{SO}^*(r) \mathbf{\ell} \cdot \mathbf{\sigma} = (V_{SO} + iW_S) \left( \frac{\hbar}{m_\pi c} \right)^2 \frac{1}{r} \frac{d}{dr} f(r) \mathbf{\ell} \cdot \mathbf{\sigma} \]  

(3.4)

The constant introduced, to give the correct dimensions, is the
\( \pi \)-meson reduced Compton wavelength. It provides a convenient, if
arbitrary, unit of length, and its value is about \( \sqrt{2} \) fm. The operator
\( \mathbf{\sigma} \) is defined by \( \mathbf{\sigma} = 2\mathbf{s} \), where \( \mathbf{s} \) is the spin-operator for the nucleon.
The imaginary part means that the absorption favours one-spin direction
than the other.

The form-factors given above are the ones most commonly used and
known to work quite well. The requirement for the shapes other than
the present ones remains to be seen and investigated. The exhaustive
compendium of the different shapes of the form-factor is given in the
appendix of Hodgson's book \(^{66}\). Clearly, the optical potential we are
considering is a local one. But, evidence from the many-body theory of the optical potential shows that it is non-local\(^{[51,67]}\). The non-local potential can be shown to be mathematically equivalent to a momentum dependence\(^{[62]}\). Hence, the resulting local potential will exhibit an energy dependent character. In this respect, Lipperheide et al\(^{[68]}\) demonstrated that the energy dependence, in general, arises in two ways: partly because the effective interaction is non-local and partly because of an intrinsic energy dependence.

The final potential for proton has the form

\[
V(r) = V_c(r) - U_R f_R(r) - i[W_V g_V(r) + W_D g_D(r)] \\
+ \left(\frac{\hbar}{m_c}\right)^2 (U_{SO} + iW_{SO}) \frac{1}{r} \frac{d}{dr} f_{SO}(r) e^{-qr}
\]

(3.5)

where,

\[
f_R(r) = (1 + \exp\left(\frac{r-R}{a_R}\right))^{-1}, \quad R_R = r_R A^{\frac{1}{3}}
\]

\[
g_V(r) = (1 + \exp\left(\frac{r-R_I}{a_I}\right))^{-1}, \quad R_I = r_I A^{\frac{1}{3}}
\]

\[
g_D(r) = \frac{4 \exp\left(\frac{r-R_I}{a_I}\right)}{\{1 + \exp\left(\frac{r-R_I}{a_I}\right)\}^{-2}}
\]

\[
f_S(r) = (1 + \exp\left(\frac{r-R_S}{a_S}\right))^{-1}, \quad R_S = r_S A^{\frac{1}{3}}
\]

and \(V_c(r)\) is the Coulomb potential as defined by equation (3.1).

As a result, we have seven geometric parameters: the half-way radii, the diffuseness parameters and the Coulomb radius, as well as five dynamic parameters: the potential depths. In practice usually only one of the forms is used, either volume or surface form, for the
imaginary central potential. Then, the number of parameters is reduced to eleven. However, several of the parameters are related to each other somehow. In fact, this is the main concern of this work: to find the optimum suitable parameters of this potential in order to obtain the best reproduction of the experimental quantities such as differential cross-sections, polarisations and total absorption cross-section.

3.3 Formulation

The scattering of a proton (a charged spin - \( \frac{1}{2} \) particle) by a closed-shell nucleus (spin - 0 target) can be described quantum mechanically by the Schroedinger equation

\[
\left[-\frac{\hbar^2}{2\mu} \nabla^2 + V(r)\right] \psi(r, \sigma) = E\psi(r, \sigma)
\]  

(3.6)

where

\[
\mu = \frac{m_i m_T}{m_i + m_T}
\]  

(3.7)

is the reduced mass of the system, \( m_i \) and \( m_T \) being respectively the masses of the incident and the target particles in a.m.u,

\[
E = \frac{m_T}{m_i + m_T} E_{Lab}
\]  

(3.8)

is the energy in the centre-of-mass system, \( E_{Lab} \) being the Laboratory energy of the incident particle in MeV. \( V(r) \) is the optical potential of equation (3.5) which defines the interaction. The centre-of-mass system is used throughout.

The task is to find the solution to the Schroedinger equation (3.6). The total wavefunction consists of the incident and the scattered waves

\[
\psi(r, \sigma) = \psi_{inc,}(r, \sigma) + \psi_{scat.}(r, \sigma)
\]  

(3.9)
\( \psi_{\text{inc}}(r, \sigma) \) is obtained when the nuclear force is absent and corresponds to the scattering of two point charges due to Coulomb potential. We take

\[
\psi_{\text{inc}}(r, \sigma) = \psi_c(r) \chi_{\text{inc}}(\sigma)
\]  

(3.10)

where \( \psi_c(r) \) is the Coulomb function and solution to the Schroedinger equation

\[
\left[ -\frac{\hbar^2}{2\mu} \nabla^2 + \frac{Z_I^2 Z_T e^2}{r} \right] \psi_c(r) = E \psi_c(r)
\]  

(3.11)

and

\[
\chi_{\text{inc}}(\sigma) = a_{\frac{1}{2}} \chi^{\frac{1}{2}} + a_{-\frac{1}{2}} \chi^{-\frac{1}{2}}
\]  

(3.12)

is the incident spin function. \( \chi^{\frac{1}{2}} \) and \( \chi^{-\frac{1}{2}} \) are the normalised spin eigenfunctions of \( s^2 \) and \( s_z \), and \( a_{\frac{1}{2}} \) and \( a_{-\frac{1}{2}} \) are the corresponding amplitudes. We assume that the incident wave comes from the negative Z-direction.

It is well known that \(^{(69)}\) by separating equation (3.11) in the parabolic coordinates we have the solution

\[
\psi_c(r) = e^{-\frac{i}{2} \pi \gamma} \Gamma(1+i\gamma) e^{ikZ} F(-i\gamma;1;ik(r-Z))
\]  

(3.13)

where \( F(-i\gamma;i\gamma;ik(r-Z)) \) is the confluent Hypergeometric function and

\[
\gamma = \frac{\mu Z_I Z_T e^2}{\hbar^2 k}
\]  

(3.14)

is the Coulomb parameter such that

\[
k = \sqrt{\frac{2\mu E}{\hbar^2}}
\]  

(3.15)

is the wave number.
The asymptotic form of the wavefunction, equation (3.13) is

\[ \psi_c(r) \xrightarrow{r \to \infty} e^{i[kZ-\gamma ln(r-Z)]} \left[ 1 - \frac{\gamma^2}{4k(r-Z)} \right] \]

\[ + \frac{1}{r} f_c(\theta) e^{i(kr-\gamma ln 2kr)} \tag{3.14} \]

where

\[ f_c(\theta) = -\frac{\gamma}{2k} \cos \theta \cos ^2 \frac{\theta}{2} e^{-2i\gamma ln \sin \theta + 2i\sigma_0} \tag{3.15} \]

is the Rutherford scattering amplitude and the Coulomb phase shift is given by

\[ \sigma_\ell = \arg \Gamma(\ell + 1 + i\gamma) \tag{3.16} \]

It is important to note that \( \psi_c(r) \) is

\[ \psi_c(r) = \sum_{\ell=0}^{\infty} \left[ 4\pi(2\ell+1) \right]^{\frac{1}{2}} \frac{i\ell \cos ^2 \frac{\theta}{2}}{kr} \int F_{\ell}(\gamma, kr) Y_\ell^0(\theta, \phi) \tag{3.17} \]

where \( F_{\ell}(\gamma, kr) \) is the Regular Coulomb function. Hence, we write equation (3.10) in the partial wave expansion

\[ \psi_{inc}(x, \sigma) = \sum_{\ell=0}^{\infty} \left[ 4\pi(2\ell+1) \right]^{\frac{1}{2}} \frac{i\ell \cos ^2 \frac{\theta}{2}}{kr} \int F_{\ell}(\gamma, kr) Y_\ell^0(\theta, \phi) \left[ a_{\frac{1}{2}} x^{\frac{1}{2}} + a_{-\frac{1}{2}} x^{-\frac{1}{2}} \right] \tag{3.18} \]

Since the spin-angle parts, \( Y_\ell^0(\theta, \phi) \chi^\frac{1}{2} \) and \( Y_\ell^0(\theta, \phi) \chi^{-\frac{1}{2}} \), are the simultaneous eigenfunctions of the operators \( \ell^2, \ell_Z, \sigma^2 \) and \( \sigma_Z \) but not of the operator \( \ell \cdot \sigma \) which appears in the spin-orbit potential, we want to introduce the spin-angle functions which are simultaneous eigenfunctions of \( \ell^2, s^2, j^2 \) and \( j_Z \), and thus \( \ell \cdot \sigma \) where \( \mathbf{j} \) is the total angular momentum

\[ \mathbf{j} = \ell + s \tag{3.19} \]

and

\[ \ell \cdot \sigma = j^2 - \ell^2 - s^2 \tag{3.20} \]
The new spin-angle function is expanded in terms of the old one

\[ Y_{J,\lambda}^{\pm \frac{1}{2}}(\theta, \phi, \sigma) = \sum_{\lambda, \nu} (\lambda, \lambda, \nu | j \pm \frac{1}{2}) Y_{\lambda}^{\nu}(\theta, \phi) \chi_{\nu}^{\lambda} \]  \hspace{1cm} (3.21)

where

\[ Y_{\lambda}^{\nu}(\theta, \phi) = (-1)^{\frac{\lambda + |\lambda|}{2}} \sqrt{\frac{2\pi + 1}{4\pi}} \sqrt{\frac{(2\pi - |\lambda|)!}{(2\pi + |\lambda|)!}} p_{\lambda}^{\nu}(\cos \theta) e^{im\phi} \]  \hspace{1cm} (3.22)

is the normalised spherical harmonics and \( p_{\lambda}^{\nu}(\cos \theta) \) is the associated Legendre polynomial. \( (\lambda \lambda \nu | jm) \) is the Clebsch-Gordon coefficient for the expansion.

Using equation (3.21) the incident wave (3.18) can be written as

\[ \psi_{\text{inc}}(r, \sigma) = (4\pi)^{\frac{1}{2}} \sum_{\lambda=0}^{\infty} i^{\lambda} e^{i\sigma} F_{\lambda}(\gamma, kr) \sqrt{\lambda + 1} \left\{ a_{\lambda}^{\frac{1}{2}} \sum_{\nu} \frac{Y_{\lambda+\frac{1}{2}, \nu}^{\lambda}(\theta, \phi, \sigma)}{kr} \right\} \]

\[ + a_{-\lambda}^{\frac{1}{2}} \sum_{\nu} \frac{Y_{\lambda-\frac{1}{2}, \nu}^{\lambda}(\theta, \phi, \sigma)}{kr} \]

\[ + (4\pi)^{\frac{1}{2}} \sum_{\lambda=0}^{\infty} i^{\lambda} e^{i\sigma} F_{\lambda}(\gamma, kr) \sqrt{\lambda} \left\{ -a_{\lambda}^{\frac{1}{2}} \sum_{\nu} \frac{Y_{\lambda-\frac{1}{2}, \nu}^{\lambda}(\theta, \phi, \sigma)}{kr} \right\} \]

\[ + a_{-\lambda}^{\frac{1}{2}} \sum_{\nu} \frac{Y_{\lambda+\frac{1}{2}, \nu}^{\lambda}(\theta, \phi, \sigma)}{kr} \]  \hspace{1cm} (3.23)

Similarly, the total wavefunction (3.9) can also be written as

\[ \psi(r) = (4\pi)^{\frac{1}{2}} \sum_{\lambda=0}^{\infty} i^{\lambda} e^{i\sigma} \sqrt{\lambda + 1} \psi_{\lambda}(kr) \left\{ a_{\lambda}^{\frac{1}{2}} \sum_{\nu} \frac{Y_{\lambda+\frac{1}{2}, \nu}^{\lambda}(\theta, \phi, \sigma)}{kr} \right\} \]

\[ + a_{-\lambda}^{\frac{1}{2}} \sum_{\nu} \frac{Y_{\lambda-\frac{1}{2}, \nu}^{\lambda}(\theta, \phi, \sigma)}{kr} \]

\[ + (4\pi)^{\frac{1}{2}} \sum_{\lambda=0}^{\infty} i^{\lambda} e^{i\sigma} \sqrt{\lambda} \psi_{\lambda}(kr) \left\{ -a_{\lambda}^{\frac{1}{2}} \sum_{\nu} \frac{Y_{\lambda-\frac{1}{2}, \nu}^{\lambda}(\theta, \phi, \sigma)}{kr} \right\} \]

\[ + a_{-\lambda}^{\frac{1}{2}} \sum_{\nu} \frac{Y_{\lambda+\frac{1}{2}, \nu}^{\lambda}(\theta, \phi, \sigma)}{kr} \]  \hspace{1cm} (3.24)
When equation (3.24) is inserted into the Schrödinger equation (3.6) it separates for each \( \lambda \) value into a pair of equations for the corresponding radial wavefunctions

\[
\frac{d^2}{dr^2} \psi^+_\lambda(kr) + \left( \frac{2\mu}{\hbar^2} \right) [E-V_c(r) + U_f(r) + i(W_V g_V(r) + W_D g_D(r) + 
\lambda^2 \frac{V_{S0}(r)}{r^2})] \psi^+_\lambda(kr) = 0
\]  
(3.25)

\[
\frac{d^2}{dr^2} \psi^-_\lambda(kr) + \left( \frac{2\mu}{\hbar^2} \right) [E-V_c(r) + U_f(r) + i(W_V g_V(r) + W_D g_D(r) + 
\lambda^2 \frac{V_{S0}(r)}{r^2})] \psi^-_\lambda(kr) = 0
\]  
(3.26)

where \( \psi^+_\lambda(kr) \) and \( \psi^-_\lambda(kr) \) are the radial wavefunctions for the two spin orientations, spin up \((j = \lambda + \frac{1}{2})\) and spin down \((j = \lambda - \frac{1}{2})\). Use has been made of the eigenvalues of \( \hat{\mathbf{L}}^2 \), which are \( \lambda \) and \(-\lambda + 1\) respectively.

The radial wavefunctions, \( \psi^+_\lambda(kr) \) and \( \psi^-_\lambda(kr) \) must satisfy the following boundary conditions:

(i) It must vanish at the origin, i.e. \( \psi^+_\lambda(0) = 0 \).

(ii) The asymptotic form of the solution should, in principle, consist of a plane wave plus an outgoing wave.

In practice, beyond the nuclear field the radial wavefunction tends asymptotically to the form

\[
\psi^+_\lambda(kr) = F^\lambda(kr) + i G^\lambda(kr) + \eta^\pm_\lambda [F^\lambda(kr) - i G^\lambda(kr)]
\]  
(3.27)

where,

\[
\eta_\lambda^\pm = e^{2i\delta^\pm_\lambda}
\]  
(3.28)

is the reflection coefficient of the scattered wave and \( \delta^\pm_\lambda \) is the complex phase shift. \( F^\lambda(kr) \) and \( G^\lambda(kr) \) are the regular and the irregular Coulomb functions. They have the asymptotic forms
\[
F_{\ell}(kr) \xrightarrow{kr \to \infty} \sin(kr - \gamma \alpha n 2kr - \frac{\ell \pi}{2} + \sigma_{\ell}) \quad (3.29)
\]

\[
G_{\ell}(kr) \xrightarrow{kr \to \infty} \cos(kr - \gamma \alpha n 2kr - \frac{\ell \pi}{2} + \sigma_{\ell}) \quad (3.30)
\]

Finally, substituting (3.27) into (3.24) and using (3.14) and (3.21), we have

\[
\psi(r, \theta) \xrightarrow{r \to \infty} \{e^{i[kz - \gamma \alpha nk(r-Z)]}(1 - \frac{\gamma^2}{ik(r-Z)}) \chi_{\text{inc}}(\sigma)
\]

+ \frac{e^{i(kr - \gamma \alpha n 2kr)}}{r} A(\theta)[a_{\frac{1}{2}} \chi_{\frac{1}{2}} + a_{-\frac{1}{2}} \chi_{-\frac{1}{2}}]

+ i B(\theta) [a_{\frac{1}{2}} e^{-i\phi} \chi_{\frac{1}{2}} - a_{-\frac{1}{2}} e^{i\phi} \chi_{-\frac{1}{2}}]

\]

where the non spin-flip and spin-flip scattering amplitudes are respectively

\[
A(\theta) = f_c(\theta) + \frac{1}{2ik} \sum_{\ell=0}^{\infty} \{(\ell+1)\eta_{\ell}^+ - 2\eta_{\ell}^- - (2\ell+1)\} e^{2i\sigma_{\ell}} p_{\ell}^1(\cos \theta)
\]

\[
B(\theta) = \frac{1}{2ik} \sum_{\ell=0}^{\infty} \eta_{\ell}^+ - \eta_{\ell}^- e^{2i\sigma_{\ell}} p_{\ell}^1(\cos \theta)
\]

The second term in equation (3.31) is the scattered wave and can be written as

\[
\psi_{\text{scat}}(r, \theta) \xrightarrow{r \to \infty} e^{i(kr - \gamma \alpha n 2kr)} [A(\theta) + B(\theta) \sigma \cdot \hat{n}] \chi_{\text{inc}}(\sigma)
\]

where

\[
\hat{n} \sin \theta = \frac{k_{\text{inc}} \times k_{\text{out}}}{k_{\text{out}}}
\]

\[k_{\text{inc}} \quad \text{and} \quad k_{\text{out}}\] are the momenta of the incident and the scattered waves respectively. Writing
\[ f(\theta) = A(\theta) + B(\theta) \sigma \cdot \hat{n} \]  

(3.36)

The differential cross-section is defined as

\[ \frac{d\sigma}{d\Omega}(\theta) = \langle [f(\theta)\chi_{\text{inc}}(\sigma)]^+, f(\theta) \chi_{\text{inc}}(\sigma) \rangle \]  

(3.37)

and the polarisation vector is defined as

\[ \hat{P}(\theta) = \langle [f(\theta)\chi_{\text{inc}}(\sigma)]^+, \sigma f(\theta) \chi_{\text{inc}}(\sigma) \rangle \]  

(3.38)

Thus, become

\[ \frac{d\sigma}{d\Omega}(\theta) = |A(\theta)|^2 + |B(\theta)|^2 + [A^*(\theta)B(\theta) + A(\theta)B^*(\theta)] \hat{P}_0 \cdot \hat{n} \]  

(3.39)

and

\[ \hat{P}(\theta) = (|A(\theta)|^2 - |B(\theta)|^2) \hat{P}_0 + [A^*(\theta)B(\theta) + A(\theta)B^*(\theta) + 2|B(\theta)|^2 \hat{P}_0 \cdot \hat{n}] \hat{n} \]  

(3.40)

\[ \frac{d\sigma}{d\Omega}(\theta) = |A(\theta)|^2 + |B(\theta)|^2 \]  

(3.42)

and

\[ \hat{P}(\theta) = P(\theta) \hat{n} = \frac{[A^*(\theta)B(\theta) + A(\theta)B^*(\theta)] \hat{n}}{|A(\theta)|^2 + |B(\theta)|^2} \]  

(3.43)

The above is correct for a pure state. For a mixed state with zero polarisation for the incident beam with \( \hat{P}_0 = 0 \). The scattered beam has
When the potential is complex, the current, \( \dot{J} \), is no longer divergenceless, since (4),

\[
\nabla \cdot \dot{J} = \frac{2}{\hbar} W(r) |\psi(r, \sigma)|^2
\]

(3.44)

The imaginary part of the optical potential, \( W \), acts as a source or sink of the particles depending on its sign. If \( W < 0 \), particles are absorbed or removed from the incident beam at a rate proportional to the local probability density. By the use of the continuity equation (72),

the amount of particles absorbed by the nucleus is at a rate

\[
N_{\text{abs}} = \left\{ \frac{\partial}{\partial t} |\psi(r, \sigma)|^2 dV \right\} = \left\{ \nabla \cdot \dot{J} dV \right\}
\]

\[
= - \int_{S} \dot{J} \cdot ds
\]

(3.46)

The total absorption cross-section is obtained as follows

\[
\sigma_A = \frac{N_{\text{abs}}}{N_{\text{inc}}}
\]

(3.47)

where \( N_{\text{abs}} \) is the absorbed flux which is given by (3.46) and \( N_{\text{inc}} \) is the incident flux which has been assumed to be unity. So, from (3.46) and (3.47) we get

\[
\sigma_A = -\frac{\hbar}{2i\mu} \int_{S} \left[ \psi^\dagger(r, \sigma) \frac{\partial}{\partial r} \psi(r, \sigma) - \psi(r, \sigma) \frac{\partial}{\partial r} \psi^\dagger(r, \sigma) \right] \rho^2 \sin \theta d\theta d\phi dr
\]

(3.48)

where the integral is taken over the surface of a sphere outside the nucleus, say \( r_0 \). Carrying out the surface integration...
Substituting the asymptotic for $i|^{kr}$, equation (3.27), in (3.49) and making use of the Wronskian relation

$$\frac{r^2}{2i\mu} \left[ \psi_+^{-*}(kr) \frac{\partial}{\partial r} \psi_-(kr) - \psi_+^-(kr) \frac{\partial}{\partial r} \psi_+(kr) \right]_{r=r_0}$$

we finally have

$$\sigma_A = 4\pi \sum_{\ell=0}^{\infty} (\ell+1) \left\{ r^2 \left[ \frac{-R}{2i\mu} \left[ \frac{\psi_+^+}{kr} (kr) \frac{\partial}{\partial r} \psi_+^-(kr) - \psi_+^-(kr) \frac{\partial}{\partial r} \psi_+^+(kr) \right] \right]_{r=r_0} \right\}$$

(3.49)

Substituting the asymptotic for $\psi_+^-(kr)$, equation (3.27), in (3.49) and making use of the Wronskian relation

$$G_\ell^+(kr) F_\ell^-(kr) - F_\ell^+(kr) G_\ell^-(kr) = 1$$

(3.50)

we finally have

$$\sigma_A = \frac{\pi}{k^2} \sum_{\ell=0}^{\infty} \left\{ (\ell+1)(1 - |\eta_\ell^+|^2) + \ell(1 - |\eta_\ell^-|^2) \right\}$$

(3.51)

In practice, partial waves with $\ell$ greater than some value, $\ell_{\text{max}}$, do not contribute significantly to these summations; so they may be neglected.

The existence of $\ell_{\text{max}}$ can be shown by means of the semi-classical approximation. If $r_M$ is the range of the potential, the angular momentum of the projectile is $p_b$; where $p$ and $b$ are the momentum and the impact parameter. The scattering will occur only if $b < r_M$. Doing the transformations

$$p_b \to \hbar [\ell(\ell+1)]^{1/2} \quad \text{and} \quad p \to \hbar k$$

we have

$$[\ell_{\text{max}}(\ell_{\text{max}} + 1)]^{1/2} \gg kr_M$$

(3.52)

Thus,

$$\ell_{\text{max}} \approx kr_M - \frac{1}{2}$$

(3.53)
However, Buck et al \(^{(60)}\) found that it is sufficiently accurate to use the empirical relation,

\[ r_M = R_R + 7a_R \]  

(3.54)

and the maximum partial wave contributes to the scattering process is determined by performing the integration with increasing \( \ell \)-values until

\[ |\text{Re} \, \eta_{\ell}^+ - 1| \leq \epsilon \]  

(3.55)

where \( \epsilon \) is a small number (usually it is chosen to be \( 10^{-4} \)). The value of \( \ell_{\text{max}} \) is the one which satisfies the condition (3.55). Physically it has the same meaning as (3.52): above certain value of \( \ell \) partial waves are not affected by the potential.

### 3.4 Method of Solution

The essential, but time-consuming, part of the computation is the numerical integration of the radial wave equations (3.25) and (3.26) from the origin out to a distance where the nuclear field is negligible. The wavefunction obtained by this integration of the internal equation must join smoothly to the external solution given by equation (3.27).

The numerical integration of the radial wave equations (3.25) and (3.26) was performed using the Fox-Goodwin method which is described in Appendix A1.

At the matching radius, \( r_M \), where the nuclear potential is negligibly small and can be determined by the use of the empirical relation (3.54), the internal solution is matched to the external (asymptotic) one by means of the logarithmic derivative. Thus, if
$f^\pm(kr_M)$ are the logarithmic derivatives of the internal wavefunctions at $r_M$

\[
f^\pm(kr_M) = \frac{\frac{F^\pm(kr_M)+i G^\pm(kr_M)+\eta^\pm_{kr} [F^\pm(kr_M)-i G^\pm(kr_M)]}{F^\pm(kr_M)+i G^\pm(kr_M)+\eta^\pm_{kr} [F^\pm(kr_M)-i G^\pm(kr_M)]}}
\]

Then, from equation (3.56) $\eta^\pm_{kr}$ and hence $\delta^\pm_{kr}$ can be determined. In fact, the whole purpose of the calculation is of course to obtain the reflection coefficient, $\eta^\pm_{kr}$, from which all of the observables are determined. For a complex potential we will have complex phase shifts.

The radial Coulomb functions, $F^\pm_{\lambda}(kr)$ and $G^\pm_{\lambda}(kr)$, as have been used in (3.56) are obtained by solving equations (3.25) and (3.26) in the external region where the nuclear field, being a short-range force, is absent. Coulomb potential is a long-range force, falls-off so slowly with distance that the incident and scattered waves are distorted by it even at very large distances. In this external region the radial wave equations (3.25) and (3.26) reduce to

\[
\frac{d^2}{dr^2} \psi_{\lambda}(kr) + \{\frac{2\mu}{R^2} [E-V_c(r)] - \frac{\lambda(\lambda+1)}{r^2}\} \psi_{\lambda}(kr) = 0
\]

which is satisfied by the regular and the irregular Coulomb functions. A method of solving the above equation is described in Appendix A2.

3.5 Comparison with Experimental Data

The differential cross-sections, polarisations and total reaction cross-section can be calculated using the method of preceding sections. In order to compare these calculated quantities with the experimental data, we have to vary the parameters of the optical potential. The goodness-of-fit to the experimental data with various sets of the
optical potential parameters may be judged by calculating the quantity

\[ \chi^2_\sigma = \frac{1}{N} \sum_{\lambda=1}^{N} \left| \frac{\sigma(\theta_\lambda) - \sigma^X(\theta_\lambda)}{\Delta \sigma^X(\theta_\lambda)} \right|^2 \]

where \( N \) is the number of data points, \( \sigma(\theta_\lambda) \) and \( \sigma^X(\theta_\lambda) \) are the calculated and the experimental differential cross-sections respectively, at the angle \( \theta_\lambda \). \( \Delta \sigma^X(\theta_\lambda) \) is the uncertainty associated with the experimental value. The best-fit parameters are obtained by minimising \( \chi^2_\sigma \). The method of which is described in Appendix A3.

Similar quantities, \( \chi^2_p \) and \( \chi^2_R \), are used to compare fits to the polarisation and reaction cross-section data. If these additional data are available, the quantity to be minimised is

\[ \chi^2 = \chi^2_\sigma + \chi^2_p + \chi^2_R \]

These quantities are the measure of the discrepancy between the theoretical values found from the assumed potential and the experimental data.

Note that the asymptotic radial wavefunction (3.26) are not normalised. But the radial wavefunction as obtained from numerical integration of the radial wave equations (3.25) and (3.26) contains an arbitrary normalisation factor. This factor, however, does not affect the cross-sections and the polarisations, since these quantities are obtained from the phase shifts which in turn are obtained from the ratios of the logarithmic derivatives wherein the normalisation factor cancels out.

In the above consideration we have used the Schroedinger equation without the kinematic corrections due to the relativistic effect.

Similar calculations have been done by Seth covering an energy region of 30-180 MeV using target nuclei heavier than \(^{40}\)Ca. Van Oers et al covering...
investigated the real part of the potential for proton scattering from $^{12}$C and heavier in the energy range 0-1 GeV. Calculations with relativistic corrections at 180 MeV proton on $^{12}$C has recently been done by Ingemarsson et al$^{(74)}$. The relativistic correction factors in the energy range 80-180 MeV proton were found by Nadasen et al$^{(9)}$ to be slightly larger than unity, hence it is not significant. However, they have analysed the proton scattering on the target heavier than $^{40}$Ca and made comparison with the nuclear matter calculations.

Furthermore, we do not include the electromagnetic spin-orbit potential$^{(75)}$ due to the interaction between the magnetic moment of the incident nucleon and the Coulomb field of the target nucleus, since its effect was shown by Batty$^{(76)}$ to be negligible in the energy range we are considering.

The optical model calculations were performed using the 12-parameter automatic search code JIB3 due to F.G. Perey. This code has been modified and the present version is called JIB4. The program performs calculations of differential cross-sections, polarisations, total reaction cross-section and (if the projectile is uncharged) total cross-section for elastic scattering of nucleon on a nucleus whose spin is ignored. It then compares the calculated values to the experimental data by $\chi^2$ values. The method employed is as described earlier in this chapter. The optical potential used is as in equation (3.5). The input consists of the experimental data, initial values of the potential parameters and the options in doing the calculations such as the numerical integration step-lengths, maximum number of guesses allowed and other specifications.

It can search up to 12 parameters varied simultaneously in such a way that the chi-squared deviation between experimental and calculated quantities is minimised. An option is also available to perform a
one-shot calculation in which no search will be done. There is no option for incorporating the kinematic corrections to the Schroedinger equation.

The output consists of the potential parameters, the cross-sections, polarisations, reflection coefficients, phase shifts, chi-squared values and the printing of the central potentials.

This programme was run on the Rutherford Computer IBM 360/195. It requires about 210K words of core for compilation and about 98K words for execution.
4.1 Introduction

Since the success of Le Levier and Saxon\(^{(21)}\), many calculations using the complex optical potential were carried out and able to give the over-all features of the interactions. This suggests that the properties of the nuclear matter in bulk are of significant influence in describing the elastic scattering rather than the individual structure of the nuclei which comes in as a second-order effect. For instance, the work of Feshbach\(^{(24)}\) on neutron scattering was able to account for the gross features of the interaction which vary smoothly with the mass number and incident energy, even when the optical potential used was of the simplest form. Since then, the elastic scattering of nucleons and other projectiles have been studied over a wide energy range and on various nuclei.

Usually proton-nucleus scattering data were analysed for target nuclei heavier than \(^{40}\)Ca. This has been done by Seth\(^{(8)}\), Nadasen et al\(^{(9)}\), Van Oers\(^{(77)}\), Roos and Wall\(^{(78)}\), Horowitz\(^{(79)}\), Kwiatkowski and Wall\(^{(80)}\) and other authors. The lack of optical model analysis for proton scattering on \(^{16}\)O and \(^{12}\)C and other lighter targets can be attributed to the doubtful validity of optical model for light nuclei. As for the heavy and medium weight nuclei they more nearly approach the limit of uniform nuclear matter. Their energy levels are much closer together, so that isolated resonances do not affect the interaction as they do for light nuclei. Moreover, for the very light nuclei, \(^{4}\)He for example, exchange reactions can take place quite frequently. This poses considerable problems for scattering at large angles. However,
Leung and Sherif\(^{81}\) were able to fit proton + \(^4\)He scattering data fairly successfully.

This chapter concerns the use of phenomenological optical model in analysing elastic scattering data for proton on \(^{12}\)C at intermediate energies. Here intermediate energies mean the energy range between about 50 MeV to 160 MeV kinetic energies of the projectile. First, the experimental data on proton + \(^{12}\)C elastic scattering will be presented. Then, the preliminary results of the analysis will be reported, together with the check on the consistency of the various data used in the analysis.

4.2 Experimental Data

The experimental data set for proton - \(^{12}\)C elastic scattering is given in Table 4.1. Some of the data do not include polarisation data and only two of the data, at 156 MeV and 140 MeV, include total reaction cross-section data. Total cross-section data are not available in any of the data because for a charged projectile its values are infinite and hence not being measured experimentally. Most of the data are limited to forward angles only (≤ 90 degrees), except for 145 MeV data and 49 MeV data. Data at 49 MeV have been taken from three different publications.

The data set of Table 4.1 shows that only two of them have the total reaction cross-section data. Those without these data are supplied from the values obtained by the interpolation in Figure 4.1. Data points in Figure 4.1 were obtained from the works of Comparat\(^{82}\), Jarvis\(^{83}\) and Renberg et al\(^{84}\).

Note that the unit for the differential cross-section, \(\sigma(\theta)\), is in millibarns per steradian (mb/sr), for the polarisation, \(P(\theta)\), is
There are several experimental data at energies which are quite close to one another. These are at 156, 145, 144, 143 and 140 MeV laboratory energies of the projectile. They have been plotted on the same graph papers in order to check the consistency between them as shown in Figures 4.2 - 4.4. The trends of the data points are observed to be of similar behaviour in the same angular range. Hence, they are consistent to each other.

4.3 **Analysis**

There are practically three stages in this analysis, according to the energy range of the proton. The first was the analysis of 156 MeV data down to 140 MeV. Then, we continue with the 100 MeV data down to 75 MeV. Later, the 49 MeV data were analysed. In the analyses, the methods of previous chapters were fully used.

Generally, the procedure we adopted is as follows. In order to compare the experimental data with the optical model calculation the parameters of the assumed form of the optical potential have to be varied until calculated values resemble the experimental quantities. This is judged numerically by the value of $\chi^2$. Hence the problem is reduced to the minimization of this quantity. The fitting procedure was started with a physically reasonable potential, usually the parameters from previous analyses. This was done in order to avoid the excessive numerical work and the danger of obtaining the false minima for $\chi^2$ values in the parameter space such that the resulting potentials will be unphysical. The parameters were varied in a systematic way. First, each of the
parameters was varied individually until a minimum value of $\chi^2$ was obtained while the other parameters remain constant. When a minimum had been found in this subspace the parameter corresponding to it was kept fixed. In turn, another parameter, which was kept fixed before, was varied until a minimum $\chi^2$ was again obtained while other parameters remain constant. This procedure was repeated until all the parameters had been varied individually and choosing the parameters to be kept fixed at a corresponding minimum $\chi^2$ values. Then, the number of parameters to be varied was increased by one and the above procedure was again repeated, until all the 12 parameters were varied simultaneously.

The above process of independent fit at each energy for a particular nucleus to obtain the particular potentials is not free from the so-called parameter ambiguities\(^{(66,27)}\). The fact that the potential would not be determined uniquely arises from these ambiguities. As a result, there are very many potentials that give comparable fits to the experimental data. The multi-dimensional surface of $\chi^2$ as a function of the potential parameters shows many local minima in which it is often difficult to identify the one with physical significance. Apart from this reason we encounter other ambiguities, whether continuous or discrete, such as due to the normalisation error of the experimental data.

Besides the ambiguities discussed above, the scarcity of data points and the limited angular range of the data also contribute quite a significant proportion in the parameters determination.

In the following the preliminary results of the unrestricted-parameter searches for the fit to each of the data set will be presented. Elaborate analyses for independent fits of the data and constrained-parameter searches whereby some of the parameters are kept fixed so that an average potential is obtained will be discussed in the next Chapter.
The data have typical features of the intermediate energy region. The differential cross-sections fall smoothly with increasing scattering angle and very little oscillatory behaviour. The polarisations data have marked oscillations.

We have used the parameters found to give the best fit to these data by Comparat\(^{82}\) as the starting values. What we were doing here was to improve the fit to these data resulting with a new set of parameters. Following the above procedure, allowing the parameters of the potential to vary systematically, we hope to get an improved fit.

First, the search was done on the parameters in the central potential only leaving the spin-orbit part unchanged as was found by Comparat\(^{82}\). The imaginary part of the central potential used was volume form as has been well known at this energy, except for very light nuclei such as \(^{6}\)Li which prefers surface absorption\(^{29}\). Then, we let the spin-orbit part equal to zero and the above step was repeated. The results of both calculations were then compared. Surprisingly, the former gave worse fit than the later either visually or \(\chi^2\) values. This contradicts earlier analyses at this energy and those of Comparat. This happens because we did the searches by varying too many parameters at the same time and allowed the \(\chi^2\) values to wander wildly on the parameter surface and consequently hit one of the valleys which was not the true minimum point. This can be judged by the unphysical values of the parameters obtained in the first case. For instance, the optimum \(\chi^2\) gives \(a_I > r_I\).

To change the tactics, we did the searches by fitting only the differential cross-section data alone together with the total reaction cross-section datum excluding the polarisation data. This was done for the two cases above and the resulting parameters are shown in Table 4.2.
of potentials I and II respectively. The corresponding fits to the experimental data are shown in Figure 4.5. It was found that the potential with spin-orbit components gives better fit with tremendously small $\chi^2$ value compared with the one without the spin-orbit component. Searching for the best-fit parameters without spin-orbit component produces oscillatory behaviour at larger angles between 30°-60° in the calculated differential cross-sections.

So far we have been using Comparat's parameters for the spin-orbit part. By varying these parameters as well we get improved fit at the second minimum of the differential cross-section data with still smaller $\chi^2$ value. The new parameters are shown in Table 4.2 of potential III and the plot to the fit in Figure 4.6.

We now include polarisation data in the search procedure and repeat the above steps. The results of the best fit are shown in Figure 4.7 and the potential parameters in Table 4.2 of potentials IV and V. The fits to both differential cross-section and polarisation data are poor, either with or without spin-orbit force being included in the optical potential. The calculated differential cross-sections give oscillatory behaviour beyond 30° for both cases. The discrepancy between the best fits beyond this angle caused by the compensation to achieve fit to the polarisation data. However, the calculated polarisation is lower than the experimental data particularly at small angles.

In order to get the best fit to the polarisation data so that the spin-orbit parameters could be uniquely determined, we searched the parameters by fitting the polarisation data alone. Good fit to these data was obtained and is shown in Figure 4.9. The parameters are as in Table 4.2 of potential VI.

Starting from the parameters of potential VI we search again for the best-fit parameters by fitting the differential cross-section,
polarisation and total reaction cross-section data simultaneously. Following the previous procedure, varying the potential parameters systematically until we achieve the best fit to the data, we obtained the final results as shown in Figures 4.10 and 4.11. The corresponding parameters are in Table 4.3. The fits to both differential cross-section and polarisation data are both reasonably good but the calculated value of the total reaction cross-section is a bit lower than the experimental one.

4.3.2 145 MeV

The data consist of differential cross-sections only and were obtained from Emmerson et al\(^{(85)}\). The total reaction cross-section datum was obtained by interpolation from Figure 4.1.

Starting from the best-fit parameters obtained for 156 MeV data, the 145 MeV data were fitted for the best optimum parameters, following the previous procedure. First, fitting was done without the spin-orbit component, then with the spin-orbit force included.

Good agreement with the data is obtained for the best fit with the spin-orbit force as can be seen from Figure 4.12. The corresponding parameters are in Table 4.3. Hence, we accept that the fit is reasonable.

4.3.3 144 MeV

The data consist of differential cross-section only and are confined to a rather small forward angle region. As usual, the total reaction cross-section was obtained by interpolation.

From previous analyses at higher energies it has been found that the spin-orbit force is necessary in analysing data at this energy range. Henceforth, we would include this force in our analyses since this is
more appropriate than without the spin-orbit force.

The 144 MeV proton scattering on $^{12}$C has been analysed based on this spirit - using spin-orbit force case only. The best fit differential cross-sections are shown in Figure 4.13 and the corresponding parameters in Table 4.3. The differential cross-sections have been correctly reproduced.

4.3.4 143 MeV

The data consist of differential cross-section and polarisation data, but are confined to a quite small forward scattering angle $2^\circ$-$12^\circ$. Consequently, the differential cross-section data points are confined to the large magnitude region. Total reaction cross-section has been obtained by interpolation in Figure 4.1.

Starting from the parameters at 144 MeV data search was done only on the differential cross-section data alone. This step was taken following the previous experience with the 156 MeV data before. The fit to the differential cross-section data was reasonable but as expected has a rather large $\chi^2$ value. Then, starting with the parameters obtained at this step the search was continued by fitting the differential cross-section and polarisation data as well as the total reaction cross-section datum simultaneously. The results of the best fits to these data are rather reasonable as shown in Figures 4.14 and 4.15. The relevant parameters are shown in Table 4.3.

4.3.5 140 MeV

The data consist of the differential cross-sections, polarisations and total reaction cross-sections and are obtained from the work of Jarvis$^{(83)}$. 
Again, both differential cross-section and polarisation data are restricted to small angles.

The search was started using the parameters obtained for 143 MeV data. By experience from 156 and 143 MeV data, the best-fit parameters were obtained by fitting the differential cross-section data alone with the total reaction cross-section datum. Later on the polarisation data being included and the procedure was repeated. The final results of the fitting are shown in Figures 4.16 and 4.17 and the best-fit parameters for this energy are presented in Table 4.3. The fits to both data are reasonably good for the data points available.

4.3.6 100 MeV

The experimental data consist of differential cross-section only. The total reaction cross-section datum has been obtained by interpolation in Figure 4.1.

Starting from the parameters obtained for 140 MeV data the best-fit parameters were obtained by the usual method described above. The result is presented in Figure 4.18 and the best-fit parameters in Table 4.3.

So far we have been using the volume form of the central imaginary part of the optical potential. This form is reasonable for high kinetic energy of the projectile. This is compatible with the Pauli exclusion principle. However, at lower energies the surface-peaked form is needed. So, we would like to determine the form of the central imaginary part preferred at this energy. We used the mixture of volume and surface forms in the fitting procedure until we achieve the best-fit parameters. But, the depth of the surface form of the central imaginary potential obtained was negligibly small and the rest of the parameters including the $\chi^2$ value and the calculated total reaction cross-section were
hardly different from the best-fit using the volume form only. Hence, we conclude that at 100 MeV the optical potential prefers the volume form of the imaginary central part.

4.3.7 96 MeV

The data consist of differential cross-section only. As usual, the total reaction cross-section datum has been obtained by interpolation. At this energy we have used three forms of the central imaginary part: the volume, the mixture of volume and surface and the surface-peaked. Among the three forms the best fit was obtained for the mixture of volume and surface-peaked one. The fit to the differential cross-section data can be seen in Figure 4.19 and the relevant parameters in Table 4.3.

4.3.8 75 MeV

The data consist of differential cross-sections and polarisation. The total reaction cross-section datum was obtained by interpolation in Figure 4.1. Again by the previous procedures, we obtain the best fit optical potential parameters with a mixture of volume and surface form for the imaginary part of the central potential. The plots of the fit are shown in Figures 4.20 and 4.21. The corresponding parameters are in Table 4.3.

4.3.9 49 MeV

The data have been obtained from three separate works. The differential cross-section data are from Fannon et al.(86) and the
polarisation data from Craig et al\(^{(87)}\) and Clarke\(^{(88)}\). Clarke\(^{(88)}\) then extended the differential cross-section measurements into large backward angles.

Analysing these data for the best-fit parameters showed that the surface-peaked form of the imaginary central potential is preferred. Furthermore, the spin-orbit component was found to be real. The same conclusion was also obtained by Fannon et al\(^{(86)}\). The plots to the best-fit are as shown in Figures 4.22 and 4.23. The corresponding parameters are in Table 4.3.

The optical model potential parameters in the energy range 100-156 MeV have been plotted against the centre-of-mass energy in order to study the trend of these parameters with the energy. It is found that the real and imaginary strengths of the central potential vary in a haphazard manner. However, due to the limited energy range considered the real central strength seems to decrease with energy and levels off beyond 145 MeV region. The imaginary part can be taken to exhibit a fluctuation about a constant value. The strength of the spin-orbit part, radius parameters and diffuseness of each component seems to be fairly constant with energy. The behaviour of each of these parameters can be observed in Figures 4.24 to 4.26. The trends seem to be compatible with results at lower energy regions\(^{(89,14,12)}\) and some intermediate energy regions\(^{(81,9,90)}\).

Greenlees et al\(^{(54)}\) have shown that the well-defined quantities involved in the analysis of proton elastic scattering data are the volume integrals and the mean-square radii of the real central potential. The volume integral is defined as

\[
J = \frac{1}{A} \int_{0}^{\infty} V(r) dr 
\]  

(4.1)
which is rather more independent of the radial form and radius chosen for the potential. The mean-square radius is defined as

$$<r^2> = \frac{\int_0^\infty V(r)r^2 \, dr}{\int_0^\infty V(r) \, dr} \quad (4.2)$$

Both of these quantities have been calculated and presented in Table 4.4 for energy range 140-156 MeV. The variations of these quantities are demonstrated in Figures 4.27 to 4.31. No consistent trend could be observed besides the irregularities in the magnitudes as a function of energy. However, the real volume integrals have generally lower values than the published results\(^{(73,91)}\) but fairly in agreement with the work of Ingemarsson et al.\(^{(74)}\)

We have also calculated the imaginary volume integrals for the central part; the average value of which agrees with Hodgson\(^{(92)}\).

The root-mean-square radii of the real central part of the potential have average value about 3.0 fm, which is in agreement with Ingemarsson\(^{(24)}\). The root-mean-square radii of the spin-orbit part are generally lower than the central part as expected\(^{(54)}\).

4.4 Conclusions

The investigation carried out above shows that the optical model gives a fair description of the elastic scattering of protons from \(^{12}\text{C}\), because of its ability to reproduce the elastic scattering data provided the potential parameters are systematically optimised to fit the data. But a unique potential is impossible to derive because of the ambiguities in the potential parameters.
In the above analysis the total reaction cross-section datum has been included because this is an important quantity obtained experimentally which represents the non-elastic processes. This is very valuable in determining the values of the imaginary part of the potential parameters which will be very significant if this potential is used to generate distorted waves in the nuclear reaction calculations.

From our experience in doing the independent fit on all the parameters of the optical potential that the fit to the data can be improved substantially if the polarisation data were deleted. By fitting the polarisation data, if it is available, together with the differential cross-section will cause the steps to become more laborious and troublesome but the fit obtained will not be as good as when the differential cross-section data alone being fitted. The similar problem was encountered by Van Oers in analysing \( p + { }^{40}\text{Ca} \) in the intermediate energy range.

Finally, the parameters obtained from the analysis above do not show smooth variation with energy except the geometry shows a fairly constant value in the energy range considered. The volume integrals and mean-square radii also do not show consistent variation with the incident projectile energy. Hence, a more systematic method has to be devised, fixing some of the parameters at certain values for example, in order to extract the average potential which we hope to vary smoothly with the energies of the incident particle.
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<td>$P(\theta)$</td>
<td>16.41-174.44</td>
<td>Craig et al (1966)(86) and Clarke (1976)(96)</td>
</tr>
</tbody>
</table>
Table 4.2: Optical Model Potential Parameters for 156 MeV p + $^{12}$C scattering

The parameters of potentials I - VI correspond to figures 4.5 - 4.9
Parameters in the brackets are held constant at Comparat's values

<table>
<thead>
<tr>
<th>Potentials</th>
<th>$U_R$</th>
<th>$r_R$</th>
<th>$a_R$</th>
<th>$W_V$</th>
<th>$r_I$</th>
<th>$a_I$</th>
<th>$U_{S0}$</th>
<th>$W_{S0}$</th>
<th>$r_S$</th>
<th>$a_S$</th>
<th>$r_C$</th>
<th>$\chi^2/N$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Comparat$^{(29)}$</td>
<td>12.84</td>
<td>1.4</td>
<td>0.515</td>
<td>25.40</td>
<td>0.805</td>
<td>0.71</td>
<td>1.84</td>
<td>-2.29</td>
<td>0.915</td>
<td>0.453</td>
<td>1.415</td>
<td>-</td>
</tr>
<tr>
<td>I</td>
<td>14.80</td>
<td>1.31</td>
<td>0.608</td>
<td>17.70</td>
<td>0.890</td>
<td>0.77</td>
<td>(1.84)</td>
<td>(-2.29)(0.915)</td>
<td>(0.453)</td>
<td>(1.415)</td>
<td>9.31</td>
<td></td>
</tr>
<tr>
<td>II</td>
<td>23.25</td>
<td>1.06</td>
<td>0.534</td>
<td>2.50</td>
<td>2.232</td>
<td>0.17</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>(1.415)</td>
<td>488.5</td>
</tr>
<tr>
<td>III</td>
<td>14.35</td>
<td>1.32</td>
<td>0.605</td>
<td>20.27</td>
<td>0.814</td>
<td>0.78</td>
<td>2.58</td>
<td>-0.92</td>
<td>0.917</td>
<td>0.429</td>
<td>1.6</td>
<td>6.09</td>
</tr>
<tr>
<td>IV</td>
<td>11.20</td>
<td>1.47</td>
<td>0.374</td>
<td>17.18</td>
<td>0.670</td>
<td>1.06</td>
<td>2.95</td>
<td>-2.15</td>
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<td>0.537</td>
<td>1.3</td>
<td>145.5</td>
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<tr>
<td>V</td>
<td>11.73</td>
<td>1.48</td>
<td>0.450</td>
<td>22.77</td>
<td>0.492</td>
<td>0.98</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>1.3</td>
<td>457.8</td>
</tr>
<tr>
<td>VI</td>
<td>10.43</td>
<td>1.46</td>
<td>0.483</td>
<td>27.73</td>
<td>0.783</td>
<td>0.67</td>
<td>1.82</td>
<td>-2.81</td>
<td>0.92</td>
<td>0.516</td>
<td>1.415</td>
<td>7.27</td>
</tr>
<tr>
<td>Parameter</td>
<td>Value 1</td>
<td>Value 2</td>
<td>Value 3</td>
<td>Value 4</td>
<td>Value 5</td>
<td>Value 6</td>
<td>Value 7</td>
<td>Value 8</td>
<td>Value 9</td>
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<td></td>
</tr>
<tr>
<td>$E_L$ (MeV)</td>
<td>156</td>
<td>145</td>
<td>144</td>
<td>143</td>
<td>140</td>
<td>100</td>
<td>96</td>
<td>75</td>
<td>49</td>
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<tr>
<td>$E_{cm}$ (MeV)</td>
<td>143.9</td>
<td>133.8</td>
<td>132.8</td>
<td>131.9</td>
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<td>92.2</td>
<td>88.64</td>
<td>69.24</td>
<td>45.2</td>
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<tr>
<td>$U_R$ (MeV)</td>
<td>11.62</td>
<td>8.09</td>
<td>14.96</td>
<td>19.02</td>
<td>37.26</td>
<td>22.78</td>
<td>22.24</td>
<td>19.60</td>
<td>31.31</td>
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<tr>
<td>$r_R$ (fm)</td>
<td>1.454</td>
<td>1.526</td>
<td>1.437</td>
<td>1.261</td>
<td>0.703</td>
<td>1.287</td>
<td>1.352</td>
<td>1.427</td>
<td>1.276</td>
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<tr>
<td>$a_R$ (fm)</td>
<td>0.554</td>
<td>0.408</td>
<td>0.253</td>
<td>0.299</td>
<td>0.894</td>
<td>0.588</td>
<td>0.601</td>
<td>0.204</td>
<td>0.648</td>
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<tr>
<td>$W_V$ (MeV)</td>
<td>22.03</td>
<td>10.40</td>
<td>16.85</td>
<td>14.50</td>
<td>22.65</td>
<td>21.22</td>
<td>0.99</td>
<td>2.24</td>
<td>0</td>
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<tr>
<td>$W_D$ (MeV)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>7.55</td>
<td>6.62</td>
<td>5.98</td>
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<tr>
<td>$r_I$ (fm)</td>
<td>0.782</td>
<td>1.159</td>
<td>0.909</td>
<td>1.346</td>
<td>0.840</td>
<td>2.185</td>
<td>0.811</td>
<td>0.825</td>
<td>0.890</td>
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<tr>
<td>$a_I$ (fm)</td>
<td>0.783</td>
<td>0.532</td>
<td>0.861</td>
<td>0.122</td>
<td>0.737</td>
<td>0.61</td>
<td>0.750</td>
<td>0.702</td>
<td>0.586</td>
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<tr>
<td>$U_{SO}$ (MeV)</td>
<td>2.06</td>
<td>2.81</td>
<td>2.63</td>
<td>2.50</td>
<td>2.72</td>
<td>3.26</td>
<td>0.81</td>
<td>2.79</td>
<td>2.79</td>
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<tr>
<td>$W_{SO}$ (MeV)</td>
<td>-2.27</td>
<td>-1.21</td>
<td>-0.60</td>
<td>-0.07</td>
<td>-0.59</td>
<td>-0.010</td>
<td>-5.48</td>
<td>-1.71</td>
<td>0</td>
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<tr>
<td>$r_S$ (fm)</td>
<td>0.948</td>
<td>0.930</td>
<td>1.004</td>
<td>1.206</td>
<td>1.149</td>
<td>0.919</td>
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<td>0.872</td>
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<td>$a_S$ (fm)</td>
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<td>0.811</td>
<td>0.820</td>
<td>0.744</td>
<td>0.472</td>
<td>0.404</td>
<td>0.540</td>
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<tr>
<td>$r_C$ (fm)</td>
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<td>1.537</td>
<td>1.537</td>
<td>1.645</td>
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<td>1.515</td>
<td>1.516</td>
<td>1.516</td>
<td>1.516</td>
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<td></td>
</tr>
<tr>
<td>$\chi^2/N$</td>
<td>25.0</td>
<td>1.68</td>
<td>3.37</td>
<td>741.6</td>
<td>80.98</td>
<td>1.73</td>
<td>112.5</td>
<td>11.88</td>
<td>28.61</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>$c_0^th$ (mb)</td>
<td>196.5</td>
<td>157.4</td>
<td>243.0</td>
<td>207.3</td>
<td>222.9</td>
<td>245.6</td>
<td>239.5</td>
<td>256.4</td>
<td>243.3</td>
<td></td>
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<tr>
<td>$c_0^{exp}$ (mb)</td>
<td>220.0</td>
<td>226.0</td>
<td>226.0</td>
<td>224.0</td>
<td>245.0</td>
<td>232.5</td>
<td>218.7</td>
<td>342.0</td>
<td></td>
<td></td>
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</table>
Table 4.4: Volume integrals and root-mean-square radii for the optical model potentials of \( p + ^{12}C \) in the energy range 140 - 156 MeV obtained by an independent search on the parameters at each energy

<table>
<thead>
<tr>
<th></th>
<th>Central Potential</th>
<th>Spin-orbit Potential</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Real Part</td>
<td>Imaginary Part</td>
</tr>
<tr>
<td></td>
<td>( J_R (\text{MeV-fm}^3) )</td>
<td>( &lt;r_R^2&gt;^{\frac{3}{2}} (\text{fm}) )</td>
</tr>
<tr>
<td>156</td>
<td>181.66</td>
<td>3.28</td>
</tr>
<tr>
<td>145</td>
<td>150.40</td>
<td>3.10</td>
</tr>
<tr>
<td>144</td>
<td>187.82</td>
<td>2.72</td>
</tr>
<tr>
<td>143</td>
<td>168.45</td>
<td>2.50</td>
</tr>
<tr>
<td>140</td>
<td>215.99</td>
<td>3.43</td>
</tr>
</tbody>
</table>
FIGURE 4.1
Plot of the experimental reaction cross-sections for p - $^{12}$C elastic scattering against the laboratory energies. Data obtained from references (29), (33) and (45).

FIGURES 4.2, 4.3 AND 4.4
Plots of the various experimental data in order to check the consistency between them. Curves drawn through the data points are guides to the eye only.

Figure 4.2 The differential cross-section data for p - $^{12}$C elastic scattering at 156 MeV and 145 MeV.

Figure 4.3 The differential cross-section data for p - $^{12}$C elastic scattering at 144 MeV, 143 MeV and 140 MeV as well as part of the 156 MeV.

Figure 4.4 The polarisation data for p - $^{12}$C elastic scattering at 140 MeV and 143 MeV as well as part of the 156 MeV.

FIGURES 4.5 TO 4.9
Plots of several best fits to the differential cross-section and polarisation data for proton scattering on $^{12}$C at 156 MeV.

Figure 4.5 Fit to the 156 MeV differential cross-section data for p + $^{12}$C scattering by varying the central part of the potential parameters. Spin-orbit parameters used are from Comparat$^{(29)}$. Solid line is the best fit with spin-orbit force and broken line is the best fit without spin-orbit component. These correspond to potentials I and II of Table 4.2.
Figure 4.6  Best fit to 156 MeV differential cross-section data for p + \(^{12}\)C scattering by varying all the parameters, central as well as spin-orbit parts. This corresponds to potential III of Table 4.2.

Figure 4.7  Plots to the differential cross-sections obtained from best fits on differential cross-section and polarisation data simultaneously. Solid line is obtained from the best fit with spin-orbit component and broken line obtained without the spin-orbit component. These correspond to potential IV and V of Table 4.2.

Figure 4.8  (same as Fig.4.7) Plot to the polarisations obtained from the best fits on both differential cross-section and polarisation data simultaneously.

Figure 4.9  Fit to the polarisation data at 156 MeV. The curve obtained by fitting the polarisation data alone, without the differential cross-section data. This corresponds to potential VI of Table 4.2.

FIGURES 4.10 AND 4.11

Best fits to the differential cross-section and polarisation data at 156 MeV for p + \(^{12}\)C scattering obtained by simultaneous fitting of both data and total reaction cross-section datum.

Figure 4.10  Best fit to the differential cross-section data. Parameters are listed in Table 4.3.

Figure 4.11  Best fit to the polarisation data. Parameters are listed in Table 4.3.
FIGURE 4.12
Best fit to the differential cross-section data at 145 MeV for $p + ^{12}C$ scattering. Solid line is the best fit with spin-orbit component and broken line is the best fit without spin-orbit force.

FIGURE 4.13
Best fit to the differential cross-section data at 144 MeV for $p + ^{12}C$ scattering. Fit obtained by including spin-orbit force.

FIGURES 4.14 AND 4.15
Best fits to the differential cross-section and polarisation data at 143 MeV for $p + ^{12}C$ scattering obtained by simultaneous fitting of both data and the total reaction cross-section datum. The spin-orbit force is included in the search.

Figure 4.14 Best fit to the differential cross-section data. Relevant parameters are listed in Table 4.3.

Figure 4.15 Best fit to the polarisation data. Relevant parameters are listed in Table 4.3.

FIGURES 4.16 AND 4.17
Best fits to the differential cross-section and polarisation data at 140 MeV for $p + ^{12}C$ scattering obtained by simultaneous fitting of both data and the total reaction cross-section datum. The spin-orbit component is included in the search.

Figure 4.16 Best fit to the differential cross-section data. Parameters are listed in Table 4.3.

Figure 4.17 Best fit to the polarisation data. The parameters are listed in Table 4.3.
FIGURE 4.18
Best fit to the differential cross-section data at 100 MeV for p + $^{12}$C scattering with the spin-orbit component included.

FIGURE 4.19
Best fit to the differential cross-section data at 96 MeV for p + $^{12}$C scattering with the spin-orbit component included.

FIGURES 4.20 AND 4.21
Plots of the best fit to the experimental data at 75 MeV for p + $^{12}$C elastic scattering.

Figure 4.20 The differential cross-sections.
Figure 4.21 The polarisations.

FIGURES 4.22 AND 4.23
Plots of the best fit to the experimental data at 49 MeV for p + $^{12}$C elastic scattering.

Figure 4.22 The differential cross-sections.
Figure 4.23 The polarisations.

FIGURES 4.24 TO 4.26
Variation of the optical model potential parameters obtained by independent search on all parameters with the centre-of-mass energies of the incident proton. The parameters are listed in Table 4.3.

Figure 4.24a Potential strengths for the real and imaginary parts of central potential.
Figure 4.24b The potential strengths for the real and imaginary part of the spin-orbit potential.
Figure 4.25  Radial parameters of the real and imaginary parts of the central potential together with the spin-orbit part.

Figure 4.26  The diffuseness of the real and imaginary central part together with the spin-orbit part.

**FIGURES 4.27 TO 4.31**

The volume integrals and mean square radii of the optical model potential in the energy range 140-156 MeV. The quantities are plotted against the centre-of-mass energies of the incident proton. The numerical values are in Table 4.4.

Figure 4.27  The volume integrals of the real and imaginary parts of the central potential.

Figure 4.28  The volume integrals of the real and imaginary parts of the spin-orbit potential.

Figure 4.29  The mean-square radii for the real and imaginary parts of the central potential.

Figure 4.30  The mean-square radii of the spin-orbit potential.

In this case $<r_{SR}^2> = <r_{SI}^2>$.

Figure 4.31  The rms values of the optical model potential.
Fig. 4.2
Fig. 4.4
Fig. 4.7

\( p + ^{12}C \) 156 MeV

\[ \frac{\theta}{\text{sr}} \]

- with spin
- no spin

\( \theta (\text{c.m}) \)

\( 0 \rightarrow 80 \)
Fig. 4.8
$p + ^{12}C, 156 \text{ MeV}$
$p + ^{12}\text{C}$ 156 MeV

$P(\theta)$ vs $\theta$ (c.m.)
Fig. 4.12

$p + ^{12}\text{C} \quad 145 \text{ MeV}$

---

- Solid line: with spin
- Dashed line: no spin
Fig. 4.13

$\sigma (\theta)$

$\frac{b}{sr}$

$\theta$ (c.m.)

$p + ^{12}C$ 144 MeV
$p + ^{12}\text{C} \quad 143 \text{ MeV}$

Fig. 4.14
$p + ^{12}C$ 143 MeV

$P(\theta)$

$\theta (c.m.)$

Fig. 4.15
$\rho + ^{12}C \ 140 \text{ MeV}$

$\sigma(\theta)$

$b/sr$

$\theta (\text{c.m.})$

Fig. 4.16
Fig. 4.19

\[ p + ^{12}C \quad 96 \text{ MeV} \]
Fig. 4.20
$p + ^{12}\text{C} \quad 75 \text{ MeV}$

$\rho(\theta)$ vs $\theta (\text{c.m.})$

Fig. 4.21
$p + ^{12}\text{C} \quad 49\text{ MeV}$

Fig. 4.22
Figure 4.28

- Spin-orbit part

- $J_{SR} (\text{real})$

- $J_{SI} (\text{imaginary})$

- Volume Integrals $(\text{MeV} \cdot \text{fm}^3)$

- $E_{cm} (\text{MeV})$
CHAPTER 5

THE AVERAGE POTENTIAL FOR LIGHT NUCLEI AT INTERMEDIATE ENERGIES

5.1 Introduction

The essential feature of the optical model is that nuclei are regarded as blobs of nuclear matter whose properties are determined by their sizes (or equivalently by the number of nucleons comprising them, since nuclear matter is almost incompressible), apart from small deviations due to structure effects. The projectile coming with different kinetic energies will feel the potential, due to the target nucleus, with varying magnitudes. Hence, the resulting potential is expected to show consistent variation with kinetic energy of the projectile and the mass number of the target nucleus. This is the spirit of the optical model - to determine the average properties of nucleon scattering. The potential obtained from this generalisation is called the average (overall) potential.

In the last chapter, the optical potentials for each nucleus at each energy have been determined giving the particular potentials. The potentials are able to reproduce the angular distributions fairly well, but the model parameters fluctuated wildly as a function of energy. The analyses at lower energies and heavier nuclei show that common geometry can be used with the energy dependent dynamical parameters. In this chapter, we will try to find the average energy dependent optical potential for p + 12C scattering. Later, a generalisation to include 16O and light nuclei in general will be considered. We will then compare our potential with that of Seth (8) which is for the nuclei with mass number greater than 40. Soon after we finished this project a new analysis similar to Seth was completed by Nadasen et al. (9) using new data and relativistic kinematic calculations.
Besides the generality of the overall potential obtained it has also the advantage of facilitating the description of the wavefunctions in the vicinity of the nucleus, and enables them to be calculated even though no elastic scattering data are available. This is useful in the studies of nuclear reactions, for example (p,2p) and (p,d) reactions. Of course, the optical-model potential for exit channels are also required.

5.2 Average Potential for p + ^{12}C Elastic Scattering

5.2.1 Procedure I

In the previous chapter (Chapter 4), there are several parameters of the potential which fluctuate around a common value, no matter how random the fluctuations are. The parameters such as \( r_R, a_R, U_{S0}, W_{S0}, r_{S0} \) and \( a_{S0} \) show this behaviour. So it is reasonable to take the average values of these parameters as the common parameter in this energy range. However, instead of doing this averaging we fix them at their values for 156 MeV data. This is adequate for the values at this energy are comparable to average values and in addition these data have been carefully fitted to give the best fit. This can be seen in Figures 4.24b to 4.26 of Chapter 4.

So, keeping \( r_R, a_R, U_{S0}, W_{S0}, r_S \) and \( a_S \) constant at 156 MeV values an unconstrained search to get the best-fit values to the remaining parameters was done for energies 156 MeV down to 75 MeV. By unconstrained search is meant the search by not fitting the total reaction cross-section datum. The final values of the parameters are presented in Table 5.1.

At 100 MeV and 96 MeV a mixture of volume plus surface form of the central imaginary potential has been used. Whereas the volume form is definitely preferred by the 100 MeV data, a mixture of both is favoured by 96 MeV data. The 75 MeV data seem to prefer volume form. However, the rest
of the data at higher energies need volume form of the central imaginary potential.

As can be seen in Figures 5.1 and 5.2, the real potential strength shows a linearly decreasing function of energy but the imaginary strength does not show smooth variation with energy. The imaginary volume integrals and the total reaction cross-sections vary in the same fashion as the imaginary strength, $W_V$. This is not surprising since they represent the non-elastic processes in the interactions.

The real central strength, $U_R$, has been fitted using the formula

$$U_R = 30.76 \left(1.0 - 0.0046 \frac{E}{E_{CM}}\right)$$

but there are no suitable formulae that can be fitted to the imaginary central parameters.

5.2.2 Procedure II

Now the parameters $r_I$ and $a_I$ are held constant at 156 MeV values, hoping that the strength of the imaginary potential will show a smooth variation with energy as does the real part. We continue the unconstrained search by varying only $U_R$ and $W_V$ (and the quantity $W_V + W_D$ at lower energies). Still the $W_V$ does not show a smooth variation with energy but behaves in a similar manner as $\sigma_R$ as it was previously. The $U_R$ varies quite consistently with energy as before. This can be observed in Figure 5.3.

5.2.3 Procedure III

Undoubtedly, we cannot use the same technique for the imaginary part as the real part in getting the smooth variation of the strength
with energy. This is only to be expected, since they involve different processes. The imaginary potential takes account of all non-elastic processes which usually include many different types of reactions. Hence, the treatment is obviously not similar to the real potential. One of the possible ways to treat the imaginary part is to fit the data by including the total reaction cross-section datum as an additional point. This will hopefully give an indication of the behaviour of the imaginary part. This is called constrained optimisation.

Following Procedure I, fixing all the parameters at 156 MeV values except the central imaginary part - \( U_R, W_V \) (or \( W_V + W_D \) and \( W_D \)), \( r_I \) and \( a_I \) - we do the fitting by constrained optimisation. The results are presented in Table 5.2.

The real potential depth, \( U_R \), as before, shows a linearly decreasing function of energy as shown in Figure 5.4. The points have been approximated (by fitting) by the function

\[
U_R (E_{CM}) = 24.134 (1.0 - 0.00359 E_{CM})
\]  
(5.2)

The imaginary potential depth has been plotted in Figure 5.5, though the values are still fluctuating but the general trend can be observed. Its values increase from 75 MeV up to 143 MeV and begin to decrease beyond 144 MeV. However, no general formula can be fitted to the points.

The diffuseness, \( a_I \), shows a fairly linear variation with energy as shown in Figure 5.6. It has been fitted by a functional form

\[
a_I (E_{CM}) = 0.612 - 1.62 \times 10^{-5} E_{CM} \text{ fm.}
\]  
(5.3)

The radial parameter, \( r_I \), shows almost a constant value except at 75 MeV. This constant value is taken to be

\[
r_I = 0.932 \text{ fm.}
\]  
(5.4)
The value is now regarded as the average parameter value for the imaginary central potential.

We also calculated the volume integrals and the root-mean-square radii for each energy. These are plotted in Figure 5.6a. The fits to the reaction cross-sections are fairly good. The volume integrals vary in the same manner as the calculated reaction cross-sections. However, the root-mean-square radii are insensitive to the variation with energy.

5.2.4 Procedure IV

The search is again repeated with the parameters $r_R$, $a_R$, $U_{S0}$, $W_{S0}$, $r_S$ and $a_S$ are held constant at 156 MeV values and $U_R$ according to the formula of equation (5.2), $a_I$ according to the equation (5.3) and $r_I$ at the value of 0.932 fm (equation 5.4). This time only the strength of the imaginary part is varied, either volume part, surface part of a mixture of the two, the results of which are presented in Table 5.3. Below 100 MeV the three kinds of imaginary potential have been used. The 100 MeV data seem not to prefer any of the potential forms. The 96 MeV data have slight preference for the mixture of surface-plus-volume form rather than the volume form, whereas the 75 MeV data prefer the surface form. The total depth of the imaginary central part, $W = W_V + W_D$ has been plotted as a function of centre-of-mass energy in Figure 5.7, where the 100 MeV and 96 MeV data are both chosen to have the mixture of surface-plus-volume form and the 75 MeV data to have only the surface form. The criteria used are the smallness of $\chi^2/N$ values and the closeness of the calculated total reaction cross-sections to that of the experimental values. Since the volume form gives comparably lower values of $\chi^2/N$ as well, the values of $W_V$ have also been plotted on the same graph in Figure 5.7.
The imaginary central potential has two kinds, a mixture of volume plus surface and volume forms, of which the first kind can be represented by the function

\[
W(E_{CM}) = 10.0 + 0.14 E_{CM} - \frac{2 \times 10^6}{(E_{CM})^3} \text{ MeV}
\] (5.5)

with

\[
W = W_V + W_D
\] (5.6)

and the second kind by a constant value

\[
W_V = 27.05 \text{ MeV}
\] (5.7)

with

\[
W_D = 0.0
\]

These are the two forms of the imaginary central potential depth suitable for p + 12C elastic scattering for 75 - 156 MeV proton kinetic energies.

To check the points at lower energies in Figure 5.7, the 49 MeV data were used. Repeating the above procedure IV for these data we obtained the results as presented in Table 5.3 for the 49 MeV data. It prefers the surface or the mixture of surface with small volume forms. But, the total reaction cross-sections are generally lower than the experimental value in this case. However, the values of the potential depth are in agreement with the trend at lower energy values. But, when we use the volume form alone, starting with the value according to equation (5.7) and doing the fitting by varying its value alone, we have better agreement in the total reaction cross-section with the experimental value, with \( W_V = 25.0 \text{ MeV} \) and larger \( \chi^2/N \). The comparison in the fits using only surface and volume forms respectively of the imaginary central potential are shown in Figures 5.8 and 5.9 for the differential cross-
sections and polarisations. Both of the calculated distributions
tend to shift to the left (forward angles) in changing from the small
surface form to a large volume form. The fits to the differential
cross-section data are rather inferior for the large volume form than
the surface one. The fits to the polarisation data are poor for both
forms.

Hence we found that below 100 MeV the forms of the imaginary
central potential depth are complicated. The form is either volume
only or a mixture of surface-plus-volume. So we have to determine
the proportion of $W_V$ and $W_D$ in the general expression of equation (5.5). To do this we plotted the quantity $W_V$ as a function of $W$ for the energies below 100 MeV in Figure 5.10. The points have been fitted with the expression

$$W_V = 1.297W - 7.538 \quad (5.8)$$

Note that the values of $W_V$ are illogical to be negative in the
above expression, because this will cause the absorptive potential to
act as a source rather than as a sink. Clearly, from Figure 5.7 our
formulae (5.5), (5.6) and (5.8) should not be used for very low energies.
The results obtained by Seth are also given in Figure 5.7 for comparison.

5.3 The Average Potential

Below we collect together the formulae for the average potential
we have obtained in analysing the $p + ^{12}C$ elastic scattering data from
49 - 156 MeV, (the strengths are in MeV and the size parameters are
in fermi):
Potential A

\[ U_R = 24.13 \left(1.0 - 0.00359 \frac{E}{E_{CM}}\right) \]

\[ r_R = 1.454 \]

\[ a_R = 0.554 \]

\[ W_V = 27.05 \text{ with } W_D = 0.0 \]

\[ r_I = 0.932 \]

\[ a_I = 0.612 \]

\[ U_{S0} = 2.064 \]

\[ W_{S0} = -2.274 \]

\[ r_S = 0.948 \]

\[ a_S = 0.492 \]

\[ r_C = 1.516 \]

Potential B

It is the same as potential A for the real central and the spin-orbit part as well as the geometrical parameters for the imaginary central part, except for the imaginary central depths which are

\[ W_V = 10.0 + 0.14 \frac{E}{E_{CM}} \]

\[ W_D = 0.0 \]

\[ W = 10.0 + 0.14 \frac{E}{E_{CM}} - \frac{2 \times 10^6}{E_{CM}^3} \]

where

\[ W_V = 1.297W - 7.538 \geq 0, \text{ otherwise } 0 \]

and

\[ W = W_V + W_D \]

The values of \( U_R, W_V \) and \( W_D \) which result from our formulae are presented in Table 5.4, together with the values of \( \chi^2/N \) for the corresponding calculations. The increase in \( \chi^2/N \) ranges between a
factor of 1.5 and 9, but reasonable agreement with the experimental value of $\sigma_R$ is maintained in most cases, except for the potential B at 49 MeV. In this respect, our potentials show better agreement with experiment than that of Seth(8).

The plots to some of the fits for the average and best-fit potentials together with that of Seth are shown in Figures 5.11 to 5.17. We have used average potential B in those figures. They show that our average potential is much better than that of Seth. This is expected as Seth's potential is for target nucleus heavier than $^{40}$Ca.

Of course, the average potential does not yield as good an agreement between the calculated and experimental distributions as the independent best-fit potential, especially around the nuclear-Coulomb interference region. By lowering the values of the central imaginary potential strength this discrepancy can be rectified as can be seen in Figures 5.18 to 5.21. Hence, the main difference between the two potentials is almost entirely due to the choice of the strength of the absorptive potential. However, our values of the absorptive average potential produces quite satisfactory results for the reaction cross-section. For the Seth's potential, the difference in these regions is enhanced.

The effects of potentials A and B are compared in Figures 5.22 to 5.27. Potential A gives marginally better agreement with the data down to 96 MeV in the nuclear-Coulomb interference region. At 75 MeV the agreement is less good beyond 35°. At 49 MeV, the fit to the differential cross-section is slightly less satisfactory at almost all angles (but much better than Seth's) and the result for the reaction cross-section is substantially better, as can be seen in Table 5.4.

The result we get for $r_I$ is somewhat unconventional because the value is smaller than its real counterpart $r_R$. But this is the
behaviour we obtained by our independent fits at most energies, particularly at 156 MeV and 75 MeV where the data are most extensive. This is supported by the analysis of Comparat et al.\(^\text{(97)}\) of the data at 156 MeV on nuclei with \(A = 12, 27, 40, 48, 56, 62, 89\) and 90; only for heavier nuclei do they find \(rR < rI\). Although at 75 MeV, Rolland et al.\(^\text{(98)}\) find \(rR < rI\) for \(^{12}\text{C}\) but their value of \(\chi^2/N\) is a factor of 2.0 greater than our independent fit and their value of \(\sigma_{R}^{\text{th}}\) is \(\sim 70\%\) too large. For \(A = 40 - 209\) Rolland et al find \(rR = rI\). At 49 MeV, Fannon et al.\(^\text{(86)}\) found fits with pure volume absorption or pure surface absorption with \(rR > rI\) and mixed volume and surface absorption with \(rR \approx rI\).

The energy variation of the real part of the optical potential is partly due to the energy dependence of the nucleon-nucleon potential and partly due to the replacement of the non-local part of the optical potential by an equivalent local potential. This energy dependence is manifested in the real strength of our average potential which is a linearly decreasing function of energy. This can be ascribed to a dependence of the potential on the kinetic energy of the nucleon inside the nucleus. For if \(U_R\) is dependent on the kinetic energy, \(T\) of the incident nucleon inside the nucleus, so that

\[
U_R = V_0 - \alpha T \tag{5.9}
\]

where \(\alpha\) and \(V_0\) are constants. For nucleon of energy \(E\),

\[
E = T - U_R + V_C \tag{5.10}
\]

where \(V_C\) is the Coulomb potential inside the nucleus (for proton). Eliminating \(T\), we get

\[
U_R = \frac{1}{1 + \alpha} [V_0 + \alpha V_C] - \frac{\alpha}{1 + \alpha} E \tag{5.11}
\]
which is of the same form obtained from our analysis, and that of
Van Oers and other analyses at lower energies such as by Becchetti and
Greenlees (14), Menet et al (99) and Perey (89).

However, in terms of the basic nucleon-nucleon interaction point
of view the decrease of $U_R$ with energies of the projectile is in
accordance with the decrease of the total elastic cross-sections of the
of the nucleon-nucleon system with the increase of the incident energy.

The imaginary part of the average potential $B$ shows energy
dependence as well besides changes in the form of the potential at
different incident energies. Its form changes from all volume for
the incident kinetic energies of the projectile greater than 100 MeV
to all surface for the incident energies 49 MeV downwards. In between
these energies a mixture of the surface-plus-volume form is required.
Furthermore, the magnitude of potential depth decreases when the incident
energy becomes lower. These effects come from the reduction of the
number of reaction channels and the operation of the Pauli exclusion
principle (100). At higher energies the incoming nucleon has enough
energy to penetrate deep into the nuclear volume and excite some of the
nucleon in the nucleus above the Fermi level. As the projectile energy
decreases it interacts only with the nucleons on the surface of the
nucleus where they are not far below the Fermi surface such that the
Pauli principle is being relaxed here. This reason together with the
increase of nucleon-nucleon cross-section with decreasing energy
contributes to the localisation of absorption at the nuclear surface at
lower energies and the absorption throughout the nuclear volume at high
energies (101).

The conclusions derived above are in agreement with those at
lower energies. However, this feature is probably too simple, in
particular some energy-dependence of the radial parameters is to be
expected.
Since the spin-orbit part of the average potential has been chosen to be the value at 156 MeV so good fits are not expected for the polarisation data at lower energies, particularly at 49 MeV. There are indications from other studies\(^{(102)}\) that below 150 MeV projectile energies the shape of the imaginary part changes with energy and peaking outside the real part. Hence, different forms of the spin-orbit potential for the real and imaginary parts are expected in order to give satisfactory fits to the polarisation data. This problem will be investigated in the next two chapters of this thesis.

5.4 The Average Potential for \(p + ^{16}O\) Elastic Scattering

The differential cross-section data for elastic proton scattering from \(^{16}O\) are available at 156 MeV\(^{(103)}\), 142 MeV\(^{(104)}\), 100 MeV\(^{(105)}\) and 65 MeV\(^{(106)}\). We have used the parameters of potential B to calculate the differential cross-sections for comparison with these data. We do not make any changes to these parameters. As can be seen in Figures 5.28 to 5.31, our potential gives quite acceptable agreement with the data. Moreover, the actual values of our parameters at 156 MeV are in close agreement with those obtained by Duhamel who fitted that data at that energy very accurately, as can be seen from Table 5.5. He also found \(r_R > r_I\).

The ability to get satisfactory reproduction of the \(p + ^{16}O\) elastic scattering data using \(p + ^{12}C\) elastic scattering data is not very surprising. Both of them are doubly magic nucleus and having closed shell and subshell for the last nucleons. Hence, the dependence on the nuclear symmetry parameter, \(\frac{N - Z}{A}\), and the target spin term are absent. The dependence comes in through the half-radius parameters on \(A\) only, that is \(R_{\frac{1}{2}} = r_o A^{\frac{1}{3}}\).
5.5 Sensitivity of the optical potential

One of the uses of the average optical potential is for the nuclear reaction calculations. Abdul-Jalil and Jackson\(^{(107)}\) used our potential in the \(^{12}(p,2p)\) reaction calculation for the exit channel. They have used the di-proton model with symmetric coplanar geometry which is particularly sensitive to the effects of distortion because there are two outgoing protons and the energy of the outgoing protons varies with angle.

The result for incident energy of \(E_0 = 160\) MeV for knock-out of proton from \(1P_{\frac{3}{2}}\) level in \(^{12}\)C with separation energy 15.96 MeV are reproduced here in Figure 5.32. Our potential give a much better result, particularly for the left-peak to valley ratio, than that of Seth. Hence, there is a marked difference between the curves obtained using optical potentials with different energy dependence.

5.6 Comparison of the quantities with other studies

We have calculated the rms radii and the volume integrals of our average potential and these are presented in Table 5.5. These are compared with the quantities obtained by Seth. It can be seen that the different parameters do not combine to give similar moments.

However, the first-order optical potential for \(^{12}\)C can be calculated using impulse approximation\(^{(5)}\). Kerman et al\(^{(5)}\) calculated the real volume integral using Gammel-Thaler phase shifts and obtained

\[
J_R \sim 382.0 - 1.03 E_L
\]

with rms radii \(<r^2_R>^{\frac{1}{2}} = <r^2_I>^{\frac{1}{2}} = 3.04\) fm at 90 MeV and 156 MeV.
McDonald and Hull\textsuperscript{(108)} did similar calculations but made use of the Yale phase shifts over the entire energy range 90-200 MeV which gave the volume integral for the real part

\[ J_R \propto 430.0 - 1.5 E_L \]

with \( <r_R>^2 = 3.0 \) fm and \( <r_T>^2 = 3.36 \) fm at 156 MeV, while more recent calculations by Johnson and Martin\textsuperscript{(109,110)} using Watson's multiple scattering theory including second-order potential give

\[ J_R = 392.0 - 0.92 E_L \]

All these calculations are in satisfactory agreement with our result given in Table 5.6, and indicate that the real part of our potential has some physical significance which can be justified by reference to theory. However, due to the complex nature of the imaginary part we do not expect the first-order theory to give an accurate representation.

Due to the many types of reaction involved in the consideration of the imaginary part of the potential a consistent variation of its volume integral with incident energy shows complicated structure as indicated in Figure 5.33. The reasons have been explained earlier in conjunction with the imaginary part of the potential of set B.

In Table 5.7 we compare our real potential depth with those obtained in phenomenological analysis of Becchetti and Greenlees and from various theoretical calculations. But the radial parameters are different in each case and only Slanina\textsuperscript{(111)} has done calculation for a nucleus as light as \( ^{12}\text{C} \). There has been indication that energy dependence may decrease with mass number\textsuperscript{(111)} and hence, the energy dependence potential derived from our work is rather satisfactory. Furthermore, our formula gives the change in sign of the real potential at an energy in reasonable agreement with phenomenological analyses.
spanning the higher energy range \(76,112\).

5.7 Conclusions

In this chapter, we have derived an energy-dependent for proton scattering for light nuclei applicable in the energy range about 50-200 MeV. It is a physically reasonable potential and consequently gives much better description of proton scattering from light nuclei than potentials which are derived by fitting data for much heavier targets.

Our form-factors are energy independent which are of the same behaviour as those in the lower energies, but they are expected to be energy dependent from the theoretical considerations. This indication shows up in our values of \(r_I\) and \(a_I\) in this analysis, but only very slight. To establish this and to improve our potential more complete experimental measurements of elastic scattering are required in this energy range for several light nuclei.

Furthermore, our choice of the spin-orbit potential needs further improvement as we have difficulty in fitting the polarisation data. This will be discussed in the next chapter.
Table 5.1: Unconstrained optimization of parameters $U_R$, $W_S$, $r_I$ and $a_I$. All other parameters are held constant at 156 MeV values: $r_R = 1.454$ fm, $a_R = 0.554$ fm, $U_{S0} = 2.064$ MeV, $W_{S0} = -2.274$ MeV, 
$r_S = 0.948$ fm, $r_C = 1.516$ fm

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Table 5.2: Constraint optimization of $U$, $W_V$ (or $W_0 + W_D$ and $W_0$), $r_1$ and $a_1$. All other parameters are

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<td>0.0</td>
<td>1.083</td>
<td>0.849</td>
<td>0.715</td>
<td>1.091</td>
<td>0.849</td>
<td>0.715</td>
<td>1.091</td>
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<td>1.083</td>
</tr>
<tr>
<td>5</td>
<td>15.93</td>
<td>23.816</td>
<td>23.0.0</td>
<td>0.0</td>
<td>1.091</td>
<td>0.0</td>
<td>1.083</td>
<td>0.849</td>
<td>0.715</td>
<td>1.091</td>
<td>0.849</td>
<td>0.715</td>
<td>1.091</td>
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<td>1.083</td>
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<tr>
<td>2.5</td>
<td>12.74</td>
<td>23.816</td>
<td>23.0.0</td>
<td>0.0</td>
<td>1.091</td>
<td>0.0</td>
<td>1.083</td>
<td>0.849</td>
<td>0.715</td>
<td>1.091</td>
<td>0.849</td>
<td>0.715</td>
<td>1.091</td>
<td>0.0</td>
<td>1.083</td>
</tr>
</tbody>
</table>

At 156 MeV values: $r_s = 0.948$ fm and $r_C = 1.516$ fm.
### Table 5.3: Constrained optimization by varying the imaginary central potential depths alone. Other parameters are fixed at 156 MeV values and according to the derived formulae. $U_R = 24.134 (1.0 - 0.0035 \frac{E_{CM}}{E_{CM}})$, $r_R = 1.454$ fm, $a_R = 0.554$ fm, $r_I = 0.932$ fm, $a_I = 0.612 - 0.000016E_{CM}$ MeV = 0.61 fm, $W_{S0} = -2.274$ MeV, $U_{S0} = 2.064$ MeV, $r_S = 0.948$ fm, $a_S = 0.492$ fm.

<table>
<thead>
<tr>
<th>$E_L$ (MeV)</th>
<th>$W_V$ (MeV)</th>
<th>$W_D$ (MeV)</th>
<th>$\sigma^\text{th}_R$ (mb)</th>
<th>$\sigma^\text{exp}_R$ (mb)</th>
<th>$\chi^2/N$</th>
</tr>
</thead>
<tbody>
<tr>
<td>156</td>
<td>29.31</td>
<td>0.0</td>
<td>233.8</td>
<td>220.0</td>
<td>380.0</td>
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<tr>
<td>145</td>
<td>24.58</td>
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<td>213.4</td>
<td>226.0</td>
<td>9.12</td>
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<td>144</td>
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<td>0.0</td>
<td>227.7</td>
<td>226.0</td>
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<td>143</td>
<td>30.59</td>
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<td>248.4</td>
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<td>140</td>
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<td>214.9</td>
<td>224.0</td>
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<td></td>
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<tr>
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<td>218.7</td>
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<td>49</td>
<td>3.15</td>
<td>0.0</td>
<td>53.7</td>
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<td>106.70</td>
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<td>78.4</td>
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<td>93.18</td>
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</table>
Table 5.4: Values of potential strengths, reaction cross-section and $\chi^2/N$ obtained with our average potentials A and B. Values of $\chi^2/N$ and reaction cross-section obtained with Seth's potential

<table>
<thead>
<tr>
<th>$E_L$(MeV)</th>
<th>156</th>
<th>145</th>
<th>144</th>
<th>143</th>
<th>140</th>
<th>100</th>
<th>96</th>
<th>75</th>
<th>49</th>
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</thead>
<tbody>
<tr>
<td>U_R(MeV)</td>
<td>11.66</td>
<td>12.54</td>
<td>12.62</td>
<td>12.70</td>
<td>12.94</td>
<td>16.16</td>
<td>16.46</td>
<td>18.14</td>
<td>20.21</td>
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<tr>
<td>W_V(MeV)</td>
<td>27.05</td>
<td>27.05</td>
<td>27.05</td>
<td>27.05</td>
<td>27.05</td>
<td>27.05</td>
<td>27.05</td>
<td>27.05</td>
<td>27.05</td>
</tr>
<tr>
<td>$\chi^2/N$</td>
<td>154.8</td>
<td>9.6</td>
<td>20.4</td>
<td>1931.0</td>
<td>1393.0</td>
<td>10.62</td>
<td>229.6</td>
<td>50.91</td>
<td>355.8</td>
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<tr>
<td>$\sigma_{R}^{th}$(mb)</td>
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<td>228.9</td>
<td>229.5</td>
<td>230.1</td>
<td>232.0</td>
<td>264.7</td>
<td>270.9</td>
<td>296.0</td>
<td>352.1</td>
</tr>
<tr>
<td>$\sigma_{R}^{exp}$(mb)</td>
<td>220.0</td>
<td>226.0</td>
<td>226.0</td>
<td>226.0</td>
<td>224.0</td>
<td>245.0</td>
<td>232.5</td>
<td>218.7</td>
<td>342.0</td>
</tr>
<tr>
<td>W_D(MeV)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1.75</td>
<td>3.49</td>
<td>1.48</td>
</tr>
<tr>
<td>$\chi^2/N$</td>
<td>216.7</td>
<td>10.1</td>
<td>12.7</td>
<td>1855.0</td>
<td>1249.0</td>
<td>13.8</td>
<td>762.5</td>
<td>57.2</td>
<td>183.3</td>
</tr>
<tr>
<td>$\sigma_{R}^{th}$(mb)</td>
<td>235.6</td>
<td>233.6</td>
<td>233.4</td>
<td>233.4</td>
<td>232.6</td>
<td>219.1</td>
<td>243.1</td>
<td>243.8</td>
<td>33.0</td>
</tr>
</tbody>
</table>

Seth

| $\chi^2/N$ | 1680.0 | 68.83 | 3265.0 | 20,580.0 | 609,900.0 | 237.2 | 686.1 | 305.4 | 727.4 |
| $\sigma_{R}^{th}$(mb) | 371.0 | 378.5 | 382.7 | 384.9 | 391.4 | 484.7 | 494.5 | 554.9 | 606.7 |
Table 5.5: Parameters for proton scattering from $^{160}$ at 156 MeV
and calculated values of the reaction cross-section

<table>
<thead>
<tr>
<th>Parameter</th>
<th>This work</th>
<th>Duhamel (1975)</th>
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<tbody>
<tr>
<td>$U_R$ (MeV)</td>
<td>11.74</td>
<td>11.57</td>
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<tr>
<td>$a_R$ (fm)</td>
<td>0.554</td>
<td>0.588</td>
</tr>
<tr>
<td>$r_R$ (fm)</td>
<td>1.454</td>
<td>1.46</td>
</tr>
<tr>
<td>$W_V$ (MeV)</td>
<td>29.9</td>
<td>12.61</td>
</tr>
<tr>
<td>$W_D$ (MeV)</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$a_I$ (fm)</td>
<td>0.612</td>
<td>0.728</td>
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<tr>
<td>$r_I$ (fm)</td>
<td>0.932</td>
<td>1.249</td>
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<td>$U_{SO}$ (MeV)</td>
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<td>3.0</td>
</tr>
<tr>
<td>$W_{SO}$ (MeV)</td>
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<td>-2.0</td>
</tr>
<tr>
<td>$a_S$ (fm)</td>
<td>0.492</td>
<td>0.5</td>
</tr>
<tr>
<td>$r_S$ (fm)</td>
<td>0.948</td>
<td>1.0</td>
</tr>
<tr>
<td>$\sigma_R$ (mb)</td>
<td>277</td>
<td>306</td>
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</tbody>
</table>
Table 5.6: Comparison of volume integrals and r.m.s. radii for proton scattering from $^{12}\text{C}$.

<table>
<thead>
<tr>
<th></th>
<th>Seth (1969)</th>
<th>This work</th>
</tr>
</thead>
<tbody>
<tr>
<td>$&lt;\vec{r}^2&gt;^2_R (\text{fm})$</td>
<td>2.36</td>
<td>3.28</td>
</tr>
<tr>
<td>$&lt;\vec{r}^2&gt;^2_I (\text{fm})$</td>
<td>3.46 - 0.0026$E_L$</td>
<td>2.82</td>
</tr>
<tr>
<td>$J_R (\text{MeV fm}^3)$</td>
<td>501.1 exp(-0.01$E_L$)</td>
<td>377.1 - 1.22$E_L$</td>
</tr>
</tbody>
</table>

5.7: Energy dependence of the depth of the real potential

<table>
<thead>
<tr>
<th></th>
<th>Energy range (MeV)</th>
<th>Formula for $U_R (\text{MeV})$</th>
<th>$E_L (U_R=0)$ (MeV)</th>
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</thead>
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<tr>
<td>Becchetti &amp; Greenlees (1969)</td>
<td>10-40</td>
<td>$54(1-0.0059E_L)$</td>
<td>170</td>
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<tr>
<td>Jeukenne et al (1969)</td>
<td>20-150</td>
<td>$56(1-0.0053E_L)$</td>
<td>190</td>
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<tr>
<td>Kidwai &amp; Rook (1971)</td>
<td>&lt; 50</td>
<td>$63(1-0.0044E_L)$</td>
<td>225</td>
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<tr>
<td>Rook (1974)</td>
<td>0-100</td>
<td>$68(1-0.0035E_L)$</td>
<td>285</td>
</tr>
<tr>
<td>Slanina (1969)</td>
<td>10-80</td>
<td>$58(1-0.0036E_L)$</td>
<td>280</td>
</tr>
<tr>
<td>This work</td>
<td>50-160</td>
<td>$24(1-0.0036E_L)$</td>
<td>310</td>
</tr>
</tbody>
</table>
The unconstrained optimisation

Figure 5.1: (for procedure I) The plots of the real central potential depth against the centre-of-mass incident energy. The dots, obtained from the analyses, have been fitted by equation (5.1) which is shown by the solid line.

Figure 5.2: (for procedure I) The plots of the imaginary central potential depth, volume integral and calculated total reaction cross-section as a function of the centre-of-mass incident energy. The lines joining the points serve only as guides to the eyes.

Figure 5.3: (for procedure II) The plots of the calculated total reaction cross-section, central part of the imaginary and real potential depths. The lines joining the points serve only as guides to the eyes.

The constrained optimisation

Figure 5.4: (for procedure III) The plots of the real central potential depth against the centre-of-mass incident energy. The results of the analyses are shown as dots which have been fitted by using equation (5.2) and as shown by the solid line. The formula obtained by Seth for the real potential depth is shown for comparison.

Figure 5.5: (for procedure III) The plots of the imaginary central depth against the centre-of-mass incident energy. The line joining the points serves only as a guide to the eyes.

Figure 5.6: (for procedure III) The plots of the geometrical parameters for the imaginary central part \( \{r_1 \text{ and } a_1 \} \) against the centre-of-mass incident energy. The imaginary central radial parameter, shown as full circle, has been averaged into a
constant value of 0.9317 fm and its diffuseness has been fitted with a solid curve giving equation (5.3).

**Figure 5.6a:** (for procedure III) The plots of the root-mean-square radius of the imaginary central part, volume integral for the imaginary central part and the calculated total reaction cross-section. The experimental total reaction cross-sections are shown as open circles. The lines joining the points are only for the guides to the eyes.

**Figure 5.7:** (for procedure IV) The plots of the imaginary central potential depth against the centre-of-mass incident energy. The open triangles were obtained by using the volume form only. The dots were obtained by either volume form, surface form or a mixture of the two, as explained in the text. The dots have been fitted by equation (5.5) which is shown by the solid line. The volume form of the potential depth has been averaged by a constant value of equation (5.7) and is shown as the chain line. The formula obtained by Seth for this potential depth is shown as the dotted line.

**FIGURES 5.8 AND 5.9**
The effect of using either volume or surface form of the imaginary central potential at 49 MeV for p + $^{12}$C scattering. The solid line is the result of the calculation using the surface form and the dotted line using the volume form. The other parameters are at the average values.

**Figure 5.8:** The differential cross-sections.

**Figure 5.9:** The polarisations.

**Figure 5.10:** The plots of $W_V$ against $W = W_V + W_D$ which have been fitted by equation (5.8) and are shown as a solid line passing through these points.
The fits to the differential cross-section and the polarisation data by the average potential (shown as the solid line). In this case, the potential set B has been used.

The results obtained by using the best-fit potentials and that of using Seth's potential are shown together as chain and dotted lines, respectively.

The effects of lowering the values of the imaginary central potential depth. All other parameters are at the average values.

For 156 MeV, the value of $W_y$ has been lowered from the average value to 19.634 MeV resulting in the lowering of the $\chi^2/N$ values from 216.7 to 74.6. The value of $\sigma_R^{\text{th}}$ has also been lowered to 175.8 mb.

For 75 MeV, the value of the mixture of surface plus volume form of the imaginary potential depths have been lowered from the average values to

$W_y = 8.161$ MeV

and $W_D = 2.979$ MeV

with the value of $\chi^2/N$ being lowered to 6.73 and $\sigma_R^{\text{th}} = 219.7$ mb, in better agreement with the experimental value.

These show the sensitivity of the nuclear-Coulomb interference region to the imaginary central potentials.

The effects of potentials set A and B on the distributions.
The differential cross-sections for elastic scattering for $p + ^{16}O$ calculated using $p + ^{12}C$ average potential parameters at 156 MeV, 142 MeV, 100 MeV and 65 MeV.

Figure 5.32: The cross-section for knock-out for a $1P_2$ proton in the $(p,2p)$ reaction on $^{12}C$ (reproduced from Abdul-Jalil I. and Jackson D.F. to be published in J. Phys. G.).

Figure 5.33: The plot of the volume integral as a function of Laboratory incident energy for the average potential set B.
$U_R = 30.76 \left( 1.0 - 0.0046 E_{cm} \right)$

Fig. 5.1
$U_R = 24.134 (1.0 - 0.035q_{cm})$

$U_R = 53.7 \exp(-0.006q_{cm})$

Fig. 5.4
Fig. 5.6a
$p + ^{12}C \quad 4.9 \text{ MeV}$

- $W_p = 1.63 \text{ MeV}, W_v = 0.0$
- $W_v = 2.50 \text{ MeV}, W_p = 0.0$

Fig. 5.8
\[ p + ^{12}\text{C} \rightarrow ^{4}\text{He} + ^{8}\text{Be}, \quad \theta_{p} = 1.63 \text{MeV}, \quad W_{v} = 0.0 \]

\[ W_{p} = 25.0 \text{MeV}, \quad W_{v} = 0.0 \]
$W_v = 1.297 W - 7.538$

$W = W_v + W_p \ (\text{MeV})$

Fig. 5.10
Fig. 5.11
Fig. 5.12
Fig. 5.13

\[ \sigma(\theta) \] (mb/sr)

\[ \theta (c.m.) \]

- Average potential
- Best-fit potential
- Seth's potential

\( p + ^{12}C \) 143 MeV
Effect of $W_\nu$.

Fig. 5.18
Figure 5.19

$P + ^{12}C$ 156 MeV

Effect of $W_v$
$p + ^{12}C \quad 75 \text{ MeV}$

Effect of $W = W_v + W_d$

Fig. 5.20
$P + ^{12}$C, 75 MeV

Effect of $W = W_v + W_p$

Fig. 5.2.1
\( p + ^{12}C \) 100 MeV

- Potential A
- Potential B

\[ \theta \] (degrees) vs. \( \frac{d\sigma}{d\Omega} \) (b/sr)

Fig. 5.22
\( p + ^{12}C \) : 96 MeV

---

- Potential A
- Potential B

**Fig. 5.23**
Fig. 5.24
$p + ^{12}\text{C} \quad 49\text{ MeV}$

---

**Potential A**

**Potential B**

---

**Fig. 5.26**

**$\theta$(c.m.)**
$p + ^{16}O; \quad 156 \text{ MeV}$

---

Optical model calculation
by potential B

---

Fig. 5.28
$p + ^{16}O \ 142 \ MeV$

- Optical Model

Calculation by potential

Fig. 5.29
Fig. 5.30

$^3_7\text{He} + ^{16}_8\text{O}$ 100 MeV

Optical Model calculation by potential $\beta$
p + $^{16}$O  65 MeV

Optical model calculation by potential B
Volume integrals for the imaginary central potential B.

$J_I$ (MeV fm$^2$)

$E_{Lab}$ (MeV)

Fig. 5.33
MICROSCOPIC SPIN-ORBIT POTENTIAL

6.1 Introduction

The spin-orbit part of the optical potential is a necessary component to be included in the calculations if the polarisation of the scattered nucleon is to be accounted for. So far the form-factor of both the imaginary and the real parts of this potential has been constrained to be equal. Recent theoretical work by Brieva and Rook\(^{102}\) has indicated that they have different shapes and energy dependence. To investigate this part of the optical potential we will use the impulse approximation in the framework of Kerman, McManus and Thaler\(^{(5)}\). This method makes use of the nuclear form-factor and the nucleon-nucleon interaction. This will strengthen the physical basis of the potential and hence the ambiguities between the potentials obtained phenomenologically can be resolved. By doing the microscopic approach of this nature, at least the qualitative feature of this potential can be observed. The potential can then be compared with those obtained from the phenomenological fitting. However, the microscopic spin-orbit part of the potential will be calculated to the first-order only whereby the two-particle and higher correlations are neglected.

This chapter will present the derivation of the nucleon-nucleus elastic scattering optical potential, particularly the spin-orbit part and the reduction of this formalism into the form suitable for computation. The resulting spin-orbit potentials are then used in the optical model calculations to obtain the differential cross-section and the polarisation quantities which will be compared with those obtained previously and the experimental data.
6.2 Formal Theory

This formalism is due to Kerman, McManus and Thaler\(^{(5)}\) which describes the elastic scattering of a nucleon from a target nucleus in terms of scattering from nucleons in the target. This is for high energy projectile of above 100 MeV.

The Schrödinger equation of the system is,

\[(H_T + K + V)|\psi\rangle = E|\psi\rangle\]  \hspace{1cm} (6.1)

where,

- \(H_T\) is the Hamiltonian of the target nucleus,
- \(K\) is the kinetic energy operator of the incident nucleon,
- \(V = \sum_{i=1}^{N} v_i\) is the interaction potential between the incident nucleon and the target nucleus which is the sum of the two-body interactions between the incident particle and the nucleon \(i\) of the target,
- \(E\) is the total energy of the system, and
- \(N\) is the number of nucleons in the target nucleus.

The target nucleus is described by,

\[H_T|n\rangle = \xi_n|n\rangle\]  \hspace{1cm} (6.2)

with

\[\langle \tau_1, \tau_2, \ldots, \tau_N|n\rangle = \phi_n(\tau_1, \tau_2, \ldots, \tau_N)\]  \hspace{1cm} (6.3)

is a completely antisymmetrized eigenstate of the target nucleus including its centre of mass wavefunction in state \(n\), which includes the bound states as well as the continuum states in which one or more particles are unbound. And

- \(\xi_0 = 0\), for the ground state of the nucleus,
- \(\xi_n\), with \(n \geq 1\), is the excitation energy of the particular target excited state and includes the nuclear recoil energy.
The projectile is described by,

\[ \hat{K}|k> = K|k> \]  

(6.4)

with,

\[<r|k> = (2\pi)^{-\frac{3}{2}} \frac{1}{2} e^{-\frac{i}{2} \vec{k} \cdot \vec{r} - i \vec{k} \cdot \vec{r}_0} \]  

(6.5)

as the wavefunction of the incident nucleon which is the plane wave describing the free particle.

For an unperturbed system,

\[(H_T + K)|k,n> = E_n|k,n> \quad (6.6)\]

with,

\[|k,n> = |k>|n> \]

and,

\[E_n = K + \epsilon_n \]

The approximation made now is to neglect the antisymmetrisation between the incident particle and the target nucleus. Moreover, the compound states of the system cannot play any role for energies above about 100 MeV as the lifetime of the compound state is of the order of the transit time across the nucleus.

The Lipmann-Schwinger equation of the Schroedinger equation (6.1) is,

\[|\psi> = |k,0> + \frac{1}{E - H_T - K + i\epsilon} V|\psi> \]  

(6.7)

with the transition probability,

\[T = <k,n|V|\psi> \quad (6.8)\]

Introducing the Möller wave-matrix, \(\Omega\), such that

\[|\psi> = \Omega|k,0> \]  

(6.9)

equation (6.8) becomes,
\[ T = \langle k', n | V \Omega | k, 0 \rangle \]  \hspace{1cm} (6.10)

Hence, the scattering matrix is,

\[ T = V \Omega \]  \hspace{1cm} (6.11)

Then we have,

\[ T = V + V \frac{1}{E - H_T - K + i\epsilon} T \]  \hspace{1cm} (6.12)

As a consequence of using the antisymmetrised target wavefunctions, the two-body potential is the same for all the target nucleons and hence independent of the label, i.e.

\[ \sum_{i=1}^{N} V_i = N V \]  \hspace{1cm} (6.13)

Then, equation (6.12) becomes,

\[ T = NV(1 + \frac{1}{\alpha} T) \]  \hspace{1cm} (6.14)

where

\[ \frac{1}{\alpha} = \frac{\hat{a}}{E - H_T - K + i\epsilon} \]  \hspace{1cm} (6.15)

and \( \hat{a} \), in turn, is a projection operator for completely antisymmetrical nuclear states, \( |n> \).

Defining a scattering operator, \( \tau \), which describes the interaction of the nucleon with one of the target nucleons by

\[ \tau = V + V \frac{1}{\alpha} \tau \]  \hspace{1cm} (6.16)

then from equations (6.14) and (6.15) we have,

\[ T' = U^{(0)}(1 + \frac{1}{\alpha} T') \]  \hspace{1cm} (6.17)

where

\[ T' = \left( \frac{N-1}{N} \right) T \]  \hspace{1cm} (6.18)

and
Define the projection operators,

\[ P_0 = |0><0| \] (6.20)

which projects onto the ground state of the nucleus, and

\[ Q_0 = \sum_{n \neq 0} |n><n| \] (6.21)

which projects off the nuclear ground state, such that

\[ P_0 + Q_0 = 1 \] (6.22)

Then, using (6.20) and (6.21) we can write equation (6.17) as

\[ T' = U(1 + i - P_0 T') \] (6.23)

where

\[ U = \frac{1}{1 - U^{(0)} - \frac{1}{\alpha} Q_0} \] (6.24)

and is called the potential matrix which can also be written in the form,

\[ U = U^{(0)} (1 + \frac{1}{\alpha} Q_0 U) \] (6.25)

So, for elastic scattering, equation (6.23) is

\[ <0|T|0> = <0|U|0> + <0|U|0> \frac{1}{\alpha_0} <0|T|0> \] (6.26)

which is the scattering from a single-particle potential \( <0|U|0> \), the optical potential and,

\[ \frac{1}{\alpha_0} = \frac{a}{E - K + i\epsilon} \] (6.27)

Writing the diagonal and non-diagonal parts of \( U^{(0)} \) as \( U_D^{(0)} \) and \( U_N^{(0)} \), equation (6.25) can be rewritten in the form,

\[ U = w(U^{(0)} + U_N^{(0)} \frac{1}{\alpha} Q_0 U) \] (6.28)

where,
The propagator in the second term of equation (6.28) can be written as

\[ w = \frac{1}{1 - \frac{U_0^{(0)}}{Q_0}} \]

(6.29)

and is the propagator, which describes the propagation of a nucleon in an average nuclear potential or equivalently as the free propagation via \( \alpha \) followed by the multiple scattering due to \( w \).

Hence, the optical potential for an elastic scattering is

\[ <0|U|0> = <0|U^{(0)}|0> + \sum_{n\neq 0} <0|U|n> \frac{1}{<n|\alpha|n>-<n|U^{(0)}|n>} <n|U^{(0)}|0> + \ldots \]

(6.31)

The second term and higher, in equation (6.31), describes the virtual nuclear excitation in intermediate states which contain the two body and higher nuclear correlation functions. But the first term \( U^{(0)} \) contains all the corrections due to multiple elastic scattering.

Recently, a slightly different approach to the above formulation has been done by Amiet\(^{(113)}\) in terms of scattering amplitudes instead of Möller operators. His approach is less involved and can easily be extended to inelastic scattering.

6.3 The Approximations

There are basically two approximations to be used: the multiple scattering approximation and impulse approximation. Both of the approximations are considered reasonable for higher energies.

The multiple scattering approximation consists of neglecting the second and higher terms in equation (6.30). The only term left is the first-order potential,
In the impulse approximation, the scattering operator, \( \tau \) is replaced by the free two-nucleon operator,

\[
t = \nu + \nu \frac{1}{E - K_1 - K_0 + ie} t
\]

(6.32)

where \( K_1 \) and \( K_0 \) are the kinetic energy operators of the two colliding nucleons. The impulse approximation is,

\[
\tau \approx t
\]

(6.33)

As a result of these two approximations, the first-order potential becomes

\[
\langle 0 | U | 0 \rangle \approx (N-1) \langle 0 | t | 0 \rangle
\]

(6.34)
Equation (6.34) can be separated out into the problem of nuclear structure and the nucleon-nucleon scattering. The actual expression for the first-order optical potential is,

\[ <0, k^- | U | k, 0> = (N-1) <0, k^- | t | k, 0> \]  

(6.35)

where \( k \) and \( k^- \) are the initial and first momentum of the projectile respectively. Neglecting the Fermi motion of the target nucleus, equation (6.35) reduces to

\[ <0, k^- | U | k, 0> = (N-1) <0 | \sum_{i=1}^{N} \frac{1}{N} e^{-i\mathbf{q} \cdot \mathbf{r}_i} | k^- | t | k> \]  

(6.36)

\[ = (N-1) \int \mathcal{D} \mathbf{r} e^{-i\mathbf{q} \cdot \mathbf{r}} \rho(\mathbf{r}) <k^- | t | k> \]  

(6.37)

where we have defined the one-particle density distribution as

\[ \rho(\mathbf{r}) = <0 | \sum_{i=1}^{N} \delta(\mathbf{r} - \mathbf{r}_i) | 0> \]  

(6.38a)

with the momentum transfer \( \mathbf{q} = k^- - k \) and,

\[ F(q) = \int \mathcal{D} \mathbf{r} e^{-i\mathbf{q} \cdot \mathbf{r}} \rho(\mathbf{r}) \]  

(6.38b)

is the form-factor of the nucleus which is just the Fourier transform of the nucleon density distribution, and \( <k^- | t | k> \) is the free two-nucleon scattering matrix fixed by the nucleon-nucleus system. Hence it requires the off-energy-shell values. But, the free two-nucleon scattering matrix,
the relation between \( |k^\perp| \) and \( |k| \) is fixed by the two-particle energy-momentum conservation laws and changes with the coordinate system so that it is on the energy shell. In relating the free nucleon-nucleon case to nucleon-nucleus case, it is quite correct at small forward angles scattering. This quantity is related to the free two-nucleon scattering amplitude by

\[
M(q) = -\frac{(2\pi)^2}{2\hbar^2} <k^-|t|^k>
\]

where it is related to the phase-shift, \( \delta_\perp \) for the nucleon-nucleon elastic scattering\(^{(114)}\) by

\[
M(q) = \frac{2\pi}{4\hbar} \sum_{\lambda} (2\lambda+1) (e^{2i\delta_\perp}-1) P_{\lambda}(\cos \theta)
\]

where \( \theta \) is the scattering angle.

The phase-shifts, \( \delta_\perp \) are obtained by analysing the experimental data on nucleon-nucleon scattering.

Finally, the optical potential is of the form,

\[
<0,k^-|U|^k,0> = -\frac{2\hbar^2(N-1)}{(2\pi)^2 m} M(q) F(q)
\]

which has been expressed in terms of the known quantities. To evaluate (6.41) we need the knowledge of the nuclear form-factor, \( F(q) \) and the nucleon-nucleon scattering amplitude, \( M(q) \).

6.4 Free two-nucleon scattering

The scattering wavefunction for two nucleon scattering is\(^{(114)}\)

\[
\psi^{(n)}(r,\sigma) = e^{ikr} \chi^{(n)}(\sigma) + f^{(n)}(\theta,\phi) \frac{e^{ikr}}{r}
\]

where the spin-scattering amplitude can be written as
\[ f^{(n)}(\theta, \phi) = M(k, \theta, \phi) \chi^{(n)}(\sigma) \quad (6.43) \]

Assuming the invariance under space rotations and reflection and time reversal, the scattering amplitude can be expressed in a general form

\[ M = A + B(\sigma^{(1)} \cdot \hat{n})(\sigma^{(2)} \cdot \hat{n}) + C(\sigma^{(1)} \cdot \hat{n} + \sigma^{(2)} \cdot \hat{n}) + E(\sigma^{(1)} \cdot \hat{q})(\sigma^{(2)} \cdot \hat{q}) + F(\sigma^{(1)} \cdot \hat{p})(\sigma^{(2)} \cdot \hat{p}) \quad (6.44) \]

For the charge independence in isospin space it becomes

\[ M = M_0 P_0 + M_1 P_1 \quad (6.45) \]

where \( P_0 \) and \( P_1 \) are the projection operators for states with total isospin 0 and 1, respectively. Each \( M_0 \) and \( M_1 \) are of the form (6.44).

\( \hat{n}, \hat{q} \) and \( \hat{p} \) are the unit vectors orthogonal to each other with \( \hat{n} \) perpendicular to the scattering plane.

For triplet state in isospin state,

\[ M(pp \rightarrow pp) = M(nn \rightarrow nn) = M_1 \quad (6.46) \]

but the p-n scattering system may be either in a singlet or triplet state, so that

\[ M(pn \rightarrow pn) = \frac{1}{2}(M_1 + M_0) \quad (6.47) \]

and

\[ P_0 = \frac{1}{4}(1 - \tau^{(1)} \cdot \tau^{(2)}) \quad (6.48) \]

\[ P_1 = \frac{1}{4}(3 + \tau^{(1)} \cdot \tau^{(2)}) \quad (6.49) \]

The scattering coefficient can be written as the function of isotopic spin, for example,
\[ A = \frac{1}{2}(3A_1 + A_0) + \frac{1}{2}(A_1 - A_0) \tau^{(1)} \tau^{(2)} \]

\[ = \frac{1}{2}(A_{pp} + A_{pn}) + \frac{1}{2}(A_{pp} - A_{pn}) \tau^{(1)} \tau^{(2)} \quad (6.50) \]

where \( A_1 \) and \( A_0 \) are the coefficients of triplet and single isotopic spin respectively. \( A_{pn} \) and \( A_{pp} \) are for the p-n and p-p scattering.

All these quantities are expressed in terms of the two-body centre of mass and they are related by

\[ q = k_0 - k' \quad (6.51) \]

\[ \frac{k}{k_0} = \frac{2N}{N + 1} \quad (6.52) \]

where \( k_0 \) and \( k' \) are the initial and final momenta of the nucleon in the two-nucleon system and \( \theta_0 \) is its scattering angle which is related to the nucleon-nucleus system by

\[ k_0 \sin \theta_0/2 = k \sin \theta/2 \quad (6.53a) \]

From equations (6.51) and (6.53a) the nucleon-nucleus scattering angle can be expressed as

\[ \theta = 2 \sin^{-1} \left[ \left( \frac{N+1}{2N} \right) \sin \frac{\theta_0}{2} \right] \quad (6.53b) \]

6.5 The nuclear form-factors

The form-factor, \( F(q) \) is the Fourier transform of the nucleon density distribution in the nucleus defined by equations (6.38b) and (6.38a) respectively. If the normalised, fully antisymmetrised wavefunction for the nuclear ground state, \( |0> \) is a single Slater determinant
of a single-particle wave function, $\psi_\mu(r_i)$

$$|0> = (A!)^{-\frac{1}{2}} \det|\psi_\mu(r_i)|$$

(6.54)

where $\mu$ is the single-particle state for a particle at $r_i$, then equation (6.38a) becomes

$$\rho(r) = \sum_\mu |\langle r|\mu\rangle|^2 = \sum_\mu |\psi_\mu(r)|^2$$

(6.55)

The states $|\mu>$ must be determined in a realistic way, usually by generating them in a Saxon-Woods potential well with a spin-orbit component. The parameters of this potential are fixed by requiring that the single-particle binding energies are fitted to the experimental separation energies in nuclear reaction studies, such as $(p,2p)$ and $(p,d)$, and the charge density should fit the elastic electron scattering. Such calculations have been done by Elton and Swift\(^{(116)}\). However, Jackson and Murugesu\(^{(117)}\) have shown that in the case of $^{12}$C and $^{16}$O, the form-factors generated by the harmonic oscillator potential well and that of Saxon-Woods are in agreement to each other up to $Q \sim 2.5 \text{ fm}^{-1}$.

The harmonic oscillator wavefunctions are convenient to handle analytically. The algebra of these functions are dealt with in many books including that of von Buttler\(^{(118)}\). We choose these functions to generate the form-factors for $^{12}$C and $^{16}$O. In this case, the spin-orbit force and the difference between neutron and proton potential wells are ignored.

The required wavefunctions are:

$$\psi_{\frac{1}{2}}(r_i) = 2\left(\frac{1}{\pi a^6}\right)^{\frac{1}{4}} e^{2a^2/\rho} y_0(\theta, \phi) \chi_\pm$$
\[ 1P_{\frac{3}{2}} \text{ State} \]

\[ \psi_{\frac{3}{2}}(r) = 2 \left( \frac{4}{\pi a^2} \right)^{\frac{1}{4}} r e^{2a^2} \left\{ \begin{array}{l}
\sqrt{\frac{r^2}{3}} Y_{1}^{0}(\theta, \phi) X_{+} + \sqrt{\frac{r^2}{3}} Y_{1}^{1}(\theta, \phi) X_{-}, \quad m_J = \frac{3}{2} \\
\sqrt{\frac{r^2}{3}} Y_{1}^{1}(\theta, \phi) X_{+} + \sqrt{\frac{r^2}{3}} Y_{1}^{0}(\theta, \phi) X_{-}, \quad m_J = \frac{1}{2} \\
Y_{1}^{-1}(\theta, \phi) X_{-}, \quad m_J = -\frac{1}{2} \\
\end{array} \right. \]

\[ 1P_{\frac{1}{2}} \text{ State} \]

\[ \psi_{\frac{1}{2}}(r) = 2 \left( \frac{4}{\pi a^2} \right)^{\frac{1}{4}} r e^{2a^2} \left\{ \begin{array}{l}
\sqrt{\frac{r^2}{3}} Y_{1}^{0}(\theta, \phi) X_{+} - \sqrt{\frac{r^2}{3}} Y_{1}^{1}(\theta, \phi) X_{-}, \quad m_J = \frac{3}{2} \\
\sqrt{\frac{r^2}{3}} Y_{1}^{-1}(\theta, \phi) X_{+} - \sqrt{\frac{r^2}{3}} Y_{1}^{0}(\theta, \phi) X_{-}, \quad m_J = \frac{1}{2} \\
\end{array} \right. \]

For \(^{12}\text{C}\) of configuration \((1S_{\frac{1}{2}})^{2} (1S_{\frac{1}{2}})^{2} (1P_{\frac{3}{2}})^{4} (1P_{\frac{3}{2}})^{4}\), the nucleon density distribution is,

\[ \varphi(r) = \frac{1}{12} \sum_{\mu=1}^{12} |<\mu|r>|^2 \]

\[ = \frac{1}{12} \left\{ 4 |\psi_{\frac{1}{2}}(r)|^2 + 8 |\psi_{\frac{3}{2}}(r)|^2 \right\} \]

\[ = \frac{1}{3} \left( \frac{1}{\pi a^2} \right)^{\frac{3}{2}} \left( 1 + \frac{4}{3} \frac{r^2}{a^2} \right) e^{a^2} \]

\[ (6.56) \]

The form-factors are obtained by substituting equation (6.56) or (6.55) into (6.36b) and making use of the partial waves expansion

\[ e^{-i\mathbf{q}\cdot\mathbf{r}} = \sum_{\ell} i^{-\ell}(2\ell+1)j_{\ell}(qr)P_{\ell}(\cos \theta), \]

also remembering,
\[ P_0(\cos \theta) = 1 \quad \text{and} \quad j_0(qr) = \frac{\sin qr}{qr} \]

so,

\[
F(q) = 2\pi \int_0^\infty \int_0^{2\pi} \sum_{\ell} i^{-\ell}(2\ell+1) j_\ell(qr) P_\ell(\cos \theta) \rho(r) \sin \theta \ d\theta \ r^2 \ dr.
\]

\[
= 2\pi \int_0^\infty dr \ r^2 \sum_{\ell} i^{-\ell}(2\ell+1) j_\ell(qr) \rho(r) \ d\cos \theta \ P_\ell(\cos \theta) P_0(\cos \theta).
\]

\[
= 2\pi \int_0^\infty dr \ r^2 \sum_{\ell} i^{-\ell}(2\ell+1) j_\ell(qr) \rho(r) \frac{2}{2\ell+1} \delta_{\ell0}.
\]

\[
= 4\pi \int_0^\infty dr \ r^2 j_0(qr) \rho(r)
\]

\[
= 4\pi \int_0^\infty dr \ r^2 \frac{\sin qr}{qr} \rho(r)
\]

(6.57)

Hence, the analytical forms of the form-factor are:

For \(^{12}\text{C}\), \(F(q) = \frac{1}{3}(3 - \frac{q^2a^2}{3}) \exp\left(-\frac{q^2a^2}{4}\right)\) \hspace{1cm} (6.58)

For \(^{16}\text{O}\), \(F(q) = \frac{1}{4}(4 - \frac{q^2a^2}{2}) \exp\left(-\frac{q^2a^2}{4}\right)\) \hspace{1cm} (6.59)

The values of the length parameter are obtained by fitting the elastic electron scattering data. In the case of \(^{12}\text{C}\), a good description of the data on electron scattering over a range of momentum of up to \(q \approx 2.5 \text{ fm}^{-1}\) using oscillator wavefunctions with a length parameter \(a = 1.64 \text{ fm}\). For \(^{16}\text{O}\) it is possible to use harmonic oscillator wavefunctions to describe elastic electron scattering for \(q < 2.5 \text{ fm}^{-1}\) with a length parameter \(a = 1.76 \text{ fm}\).
6.6 Tractable form of the optical potential

The optical potential obtained earlier has to be reduced to a form suitable for computation. This is necessary because the analytical technique is no longer possible at this stage.

The factorisation of equation (6.35) into nuclear structure and nucleon-nucleon scattering parts is done only after the matrix elements of the spin and isospin operator have been evaluated, that is equation (6.34) is averaged over spin and isospin of the nucleons in the nucleus. For nuclei with total spin 0 and isospin 0, \(^{12}\text{C}\) and \(^{16}\text{O}\) for instance, the optical potential becomes

\[
<0,k^-|U|k,0> = -\frac{2\hbar^2(N-1)}{(2\pi)^2 m} [\bar{A}(\mathcal{Q}) + \bar{C}(\mathcal{Q}) \sigma \cdot \hat{n}] F(\mathcal{Q}) \quad (6.60)
\]

where

\[
\bar{A}(\mathcal{Q}) = \frac{1}{2} (A_{pp} + A_{pn})
\]

and

\[
\bar{C}(\mathcal{Q}) = \frac{1}{2} (C_{pp} + C_{pn})
\]

such that \(A_{pp}, A_{pn}, C_{pp}\) and \(C_{pn}\) are the relevant coefficients for p-p and p-n scattering.

We can write

\[
\bar{C}(\mathcal{Q}) \sigma \cdot \hat{n} = \frac{\bar{C}(\mathcal{Q})}{k^2 \sin \theta} \sigma \cdot k \times k^-
\]

\[
= \frac{\bar{C}(\mathcal{Q})}{k^2 \sin \theta} \sigma \cdot k \times k^-
\]

\[
(6.61)
\]

Hence, the first-order optical model potential for nucleon elastic scattering on even-even nucleus in momentum space is

\[
<0,k^-|U|k,0> = V(\mathcal{Q})
\]

\[
= -\frac{2\hbar^2(N-1)}{(2\pi)^2 m} [\bar{A}(\mathcal{Q}) + \bar{C}(\mathcal{Q}) \sigma \cdot k \times k^-] F(\mathcal{Q}) \quad (6.62)
\]
This is a purely nuclear potential without the Coulomb component for proton scattering. To the first order, it gives the same effect as the nuclear part of equation (3.5) (Chapter 3). Writing the nuclear part of equation (3.5) in momentum space

\[ V(q) = \langle k' | V(r) | k \rangle = \int \, d\mathbf{r} \, e^{-i\mathbf{q} \cdot \mathbf{r}} \, V(q) \cdot \]

\[ = V_{\text{cen}}(q) + \int \, d\mathbf{r} \, e^{-i\mathbf{q} \cdot \mathbf{r}} \, V_{S0}(r) \cdot \mathbf{r} \times \mathbf{k} \cdot \mathbf{r} \cdot (6.63) \]

where \( V_{\text{cen}}(q) \) is the nuclear component of the central part and the second term on the right of equation (6.63) is the spin-orbit part which is of the form

\[ V_{S0}(r) = \left( \frac{\hbar}{m_c} \right)^2 \frac{1}{r} \int \frac{d\mathbf{r}}{r} \, V_{S0}(r) \cdot (6.64) \]

Equation (6.63) can be reduced to (4)

\[ V(q) = V_{\text{cen}}(q) - \left( \frac{\hbar}{m_c} \right)^2 \, V_{S0}(q) \cdot \mathbf{r} \times \mathbf{k} \cdot (6.65) \]

Then, comparing the coefficients of equations (6.62) and (6.65) we have

\[ V_{\text{cen}}(q) = - \frac{2\hbar^2 (N-1)}{(2\pi)^2 m} \hat{A}(q) \cdot F(q) \cdot (6.66) \]

and

\[ \left( \frac{\hbar}{m_c} \right)^2 \, V_{S0}(q) = - \frac{i2\hbar^2 (N-1)}{(2\pi)^2 m} \hat{C}(q) \cdot F(q) \cdot (6.67) \]

Writing \( V_{S0}(q) \) in configuration space,

\[ V_{S0}(r) = \int \, dq \, e^{-i\mathbf{q} \cdot \mathbf{r}} \, V_{S0}(q) \]

and following the same procedure as before,

\[ V_{S0}(r) = 4\pi \int_{0}^{\infty} dq \, q^2 \, j_0(qr) \, V_{S0}(q) \cdot (6.68) \]
Both of these equations have been evaluated numerically. The two-body scattering coefficients, $\hat{C}(q)$, are plotted in figures 6.1 to 6.8. These quantities have been calculated from the on-shell two-body nucleon-nucleon scattering using the Gammel-Thaler potential. The phase-shifts obtained from fitting the data have been reduced to the scattering coefficients $\tilde{C}(q)$ which are available in the paper by Kerman et al.\(^5\).

As can be seen from Figures 6.1, 6.2, 6.5 and 6.6 the scattering coefficients $\tilde{C}(q)$ for 156 MeV cover the momentum transfer up to the value $q \approx 2.75$ fm\(^{-1}\), which is equivalent to $\theta \approx 71.62^\circ$. At 90 MeV, the scattering coefficients cover up to $q = 2.1$ fm\(^{-1}\), which is equivalent to $\theta = 76.3^\circ$. Hence, due to the inadequacy of the ranges covered by these coefficients and the reliability of $F(q)$ which is only up to $q = 2.5$ fm\(^{-1}\) ($\theta = 64.2^\circ$), the integration procedures in equations (6.68) and (6.69) are not adequate. In fact, to complete the integrations to cover the entire q-space we need the above quantities, $\tilde{C}(q)$ and $F(q)$, to cover reliably up to $q \approx 4.7$ fm\(^{-1}\) for 156 MeV and $q \approx 3.14$ fm\(^{-1}\) for 90 MeV. However, in doing these types of calculations Ingemarsson et al.\(^74\) approximated the values of $\tilde{C}(q)$ beyond a certain region by an analytical function of Saxon-Woods type. In our case we did not include this approximation for the q values beyond the region given by the nucleon-nucleon scattering data and therefore, we would not expect these potentials to yield agreement with the data beyond 70°.

The form-factors and the spin-orbit potentials are as shown in Figures 6.9 to 6.16. The form-factors seem to have either Saxon-Woods shape or Gaussian shape. To make the comparison with the phenomenological results which so far we have been using the Saxon-Woods parametrisation;
this shape has been assumed and being fitted to the form-factors. The results of these fittings are shown as dotted lines and considered to be reasonably good. The Saxon-Woods parameters so obtained are listed in Table 6.1. However, the resulting fit to the potential appears to be less good. This may indicate that the Saxon-Woods form is not the appropriate shape favoured by the impulse.

One notable feature of these potentials is that the imaginary part is extremely shallower than the real part and they have signs similar to the phenomenological results obtained previously. However, one thing in which it differs from the conventional phenomenological optical potential is that the real part has different form-factors from the imaginary part besides the huge difference in its depths.

These potentials have been inserted into the Schroedinger equation of Chapter 3 with the central potential obtained earlier from the phenomenological analysis. We have calculated the differential cross-sections and polarisations at 156 MeV and 90 MeV, both using the average central potential (denoted by AVKMT) and the best-fit central potential (denoted by BFKMT). The comparison of these calculations and with that of the experimental data are shown in Figures 6.17 to 6.20. The calculations at 90 MeV have been compared with the data at 75 MeV due to the availability of polarisation data at this energy. But the polarisation results agree quite nicely with the experimental results at small angles and give the general shape in agreement with the data. However, at both energies the differential cross-sections calculated using AVKMT and BFKMT potentials are generally higher than the experimental results at all angles.

6.7 Conclusions

From the studies in this chapter on the first-order optical
potential obtained using impulse approximation we can conclude that the Saxon-Woods parametrisation of the spin-orbit force may not be adequate as at the microscopic level the surface-peak form fails to show up.

Furthermore, different form-factors, and probably entirely different shapes, are needed for the real and imaginary parts of the spin-orbit potential as indicated by the impulse consideration. This idea will be tested in the phenomenological analysis in the next chapter. Hopefully, better fit to the polarisation data would be achieved.
Table 6.1: Spin-orbit parameters obtained by fitting Saxon-Woods form to the potential obtained from impulse approximation

<table>
<thead>
<tr>
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<th>$p + ^{12}\text{C}$</th>
<th></th>
<th>$p + ^{16}\text{O}$</th>
</tr>
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<tr>
<td></td>
<td>156 MeV  90 MeV</td>
<td>156 MeV</td>
<td>90 MeV</td>
</tr>
<tr>
<td>$r_{SR}$ (fm)</td>
<td>0.926  0.906</td>
<td>0.845</td>
<td>0.825</td>
</tr>
<tr>
<td>$a_{SR}$ (fm)</td>
<td>0.606  0.622</td>
<td>0.601</td>
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<td>$U_{S0}$ (MeV)</td>
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<td>8.11</td>
<td>9.12</td>
</tr>
<tr>
<td>$r_{SI}$ (fm)</td>
<td>0.939  1.026</td>
<td>0.845</td>
<td>0.952</td>
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<tr>
<td>$a_{SI}$ (fm)</td>
<td>0.695  0.729</td>
<td>0.699</td>
<td>0.724</td>
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<td>$W_{S0}$ (MeV)</td>
<td>-1.25 -1.38</td>
<td>-1.64</td>
<td>-1.82</td>
</tr>
</tbody>
</table>
FIGURES 6.1 AND 6.2
Real and imaginary parts of the two-body scattering coefficients for the spin-orbit part at 156 MeV with $^{12}$C target.

FIGURES 6.3 AND 6.4
Same as Figures 6.1 and 6.2 but at 90 MeV with $^{12}$C target.

FIGURES 6.5 AND 6.6
Same as Figures 6.1 and 6.2 but for $^{16}$O target.

FIGURES 6.7 AND 6.8
Same as Figures 6.3 and 6.4 but for $^{16}$O target.

FIGURES 6.9 TO 6.16
The spin-orbit form-factors and potentials. The solid lines are the potentials obtained from impulse approximation and the dotted lines as the results of fitting the potential by Saxon-Woods form.

FIGURES 6.17 TO 6.20
Comparisons of fits to the differential cross-section and polarisation data using various spin-orbit potentials: average central part with the average 4-parameter spin-orbit force (AV4), Best-fit potential (BF4), average central with KMT spin-orbit part (BFKMT).

Figure 6.17: The differential cross-section at 156 MeV for AVKMT, BFKMT and AV4.

Figure 6.18: The polarisation at 156 MeV for AVKMT, BFKMT and AV4.

Figure 6.19: The differential cross-section at 75 MeV for AVKMT, BFKMT and AV4.
Figure 6.20: The polarisation at 75 MeV for AVKMT, BFKMT and AV4.

Note that in calculating the KMT spin-orbit potentials in Figures 6.19 and 6.20, the parameters used are at 90 MeV and being compared to the data at 75 MeV.
Real part of the two-body scattering amplitude for $^{12}$C, at 156 MeV.

Fig. 6.1
Imaginary part of the two-body scattering amplitude for $^{12}\text{C}$ at 156 MeV

Fig. 6.2
Real part of the two-body scattering amplitude for $^{12}C$, at 90 MeV.

Fig. 6.3
Imaginary part of the two-body scattering amplitude for $^{12}$C at 90 MeV

$\Im |T_{C}^{1,2}|^2$ vs. $q$ (fm$^{-1}$)

Fig. 6.4
Real part of the two-body scattering amplitude for $^{16}$O at 156 MeV.
Imaginary part of the two-body scattering amplitude for $^{16}O$ at 156 MeV

Fig. 6.6

$C_t(q) \times 10^m$
Imaginary part of the two-body scattering amplitude for $^{16}O$ at 90 MeV

Fig. 6.7

$\tilde{C}_R(q) \times 10^{-2}$ (fm)
Imaginary part of the two-body scattering amplitude for $^{16}O$ at 90 MeV

Fig. 6.8
N + $^{12}$C at 156 MeV

--- microscopic

--- W-S fits

Fig. 6.9
\[ V_{so}(r) \text{ (eV)} \]

\[ N + ^{12}C \text{ at 156 MeV} \]

\[ V_{so}'(r) \text{ (eV)} \]

- --- microscopic
- --- W-S fits

\[ r (fm) \]

Fig. 6.10
Fig. 6.11
$V_{so}(r)$

$N + ^{12}C$ at 90 MeV

$r (fm)$

$eV_{so}'(r)$
eV

---

- microscropic

- W-$\gamma$ fits

Fig. 6.12
$N + ^{16}O$ at 156 MeV

- microscopically

- W-S fit

Fig. 6.13
Fig. 6.15

\[ N + ^{16}\text{O} \text{ at } 90 \text{ MeV} \]

\[ \text{microscopic} \]

\[ \text{W-S fits} \]
Fig. 6.17

\[ p + ^{12}\text{C} \rightarrow 156 \text{ MeV} \]

- BFKMT
- AVKMT
- AV4
Fig. 6.18
$p + ^{12}C$ 75 MeV

- BFKMT
- AVKMT
- AV4

Fig. 6.19
CHAPTER 7

THE SIX-PARAMETER MODEL OF SPIN-ORBIT FORCE

7.1 Introduction

In the previous chapters, that is chapters 4 and 5, we have used the four-parameter model of the spin-orbit potential where the forms of the real and the imaginary parts of the potential are constrained to be equal. This model has been used in most of the works on the phenomenological calculations of this type until now. But, so far, most of the works concern the lower energy region in which the spin-orbit potential is real and hence, the need to have different shape for the imaginary part does not arise.

However, recent theoretical works by Brieva and Rook\textsuperscript{(102)} based on a nuclear matter approach and by Ingemarrson et. al\textsuperscript{(74)} at 180 MeV and also our work in the last chapter using impulse approximation require that the spin-orbit potential should be complex and that the imaginary part has a different shape from that of the real part. Moreover, the impulse approximation does not favour the use of the Saxon-Woods form for the imaginary component of this potential.

Our purpose now is to allow the real and the imaginary parts of the spin-orbit potential to have different form-factors and the parameters of these potential components will then be optimized to fit the experimental angular distribution, particularly the polarisation data. Hopefully this modification will give rise to a better fit to the polarisation data and consequently the differential cross-sections as well. This modification is essentially the course of development followed by the central part of the potential in its early history. The change is physically justifiable since each of the components,
real and imaginary, represents different physical processes.

Furthermore, the need to have precise spin-orbit parameters which fit the polarisation data well is important in the studies of nuclear reaction calculations involving the polarised protons. By making the real and imaginary parts to have different form-factors we hope to be able to get much improved set of the spin-orbit parameters.

This chapter will describe the analysis of the data at 156, 75 and 49 MeV which have the polarisation data using the six-parameter model of the spin-orbit force. A general formula for this potential model will be derived. We will also discuss the effects and implications of this model in relation to our earlier studies and the studies done by other workers.

7.2 The six-parameter model

In this model the spin-orbit potential of equations (3.4) in chapter 3 is replaced by

$$V_{S0}(r) = \left(\frac{\hbar}{m_c}\right)^2 \frac{1}{r} \frac{d}{dr} \left[ V_{S0}(r) + i W_{S0}(r) \right] \tag{7.1}$$

where, each of the components, real and imaginary parts, has the Saxon-Woods form-factor,

$$V_{S0}(r) = U_{S0} \left\{ 1 + \exp \left[ \frac{R - R_{SR}}{a_{SR}} \right] \right\}^{-1}$$

$$W_{S0}(r) = W_{S0} \left\{ 1 + \exp \left[ \frac{R - R_{SI}}{a_{SI}} \right] \right\}^{-1}$$

The radial parameters $R_{SR}$ and $R_{SI}$, and the diffuseness parameters, $a_{SR}$ and $a_{SI}$, have the usual definitions, and they are no longer constrained to be equal to each other. In other words, the real
and imaginary parts of the spin-orbit potential have now different form-
factors, contrary to the conventional definition.

We are now in a position to vary these 6 parameters of the spin-
orbit potential, \( U_{S0}, W_{S0}, R_{SR}, a_{SR}, R_{SI}, \) and \( a_{SI}, \) so as to achieve better fits to the data which have the polarisation data in addition to the differential cross-section data. As explained earlier, this part of the potential is responsible for the reproduction of the polarisation data and this modification is intended to facilitate the improvement in the fit to the polarisation data and consequently the differential cross-section data.

The central part of the potential will be kept fixed at the previous values, the best-fit values and the average values. Only the six parameters of the spin-orbit potential will be varied to fit the experimental data for each of the central potential. We start with the average central potential followed by the best fit potential. In each of these cases, the starting parameters for the spin-orbit potential are the values obtained from impulse approximation of chapter 6 and at lower energies those of phenomenological values.

7.2.1 The average central potential

In this section we will describe the optical potential fit obtained by varying the six-parameter spin-orbit part while keeping the central part at the average values obtained previously.

i) 156 MeV

Starting with the values for the spin-orbit potential obtained from the impulse approximation of the last chapter, the spin-orbit parameters have been searched by fitting both the polarisation as well as the differential cross-section data. The procedure followed is exactly the same as the one used for the central part in chapter 4.
The results for the best-fit parameters obtained by this independent fit to the six spin-orbit parameters are presented in table 7.1 and it is denoted by $AV_6$.

The fit to the polarisation data is shown in figure 7.1 by the dotted line. It fits the data nicely for larger angles, from about $30^\circ$ to $45^\circ$. The agreement between the calculated values and the experimental data at small angles are not good. The former is lower than the latter.

The fit to the differential cross-section data is shown in figure 7.2. It is observed that by optimizing the spin-orbit parameters the fit at mid angular region has been improved. The improvement in the fit at this region, the second dip in the differential cross-section, has been noted earlier by other people using the four-parameter spin-orbit force, for instance Snow et. al, Fernbach et. al, and recently by Ingemarsson et. al and Nadasen et. al. This phenomena is largely due to the influence of the spin-orbit potential on oscillations in the differential cross-section. The fit to these data at small angles is not good due to the deficiencies in the imaginary central potential, which have been discussed earlier.

ii) 75 MeV

Following the same procedure as above, we have the best parameter for the independent fit for the six-parameter spin-orbit potential. There is much improvement to the quality of fit compared to the one obtained by the use of average four-parameter potential obtained earlier. The parameters are listed in table 7.1 and denoted by $AV_6$. The fit to the polarisation data is presented in figure 7.3 by the dotted line. The fit is good at small angles but slightly lower at $45^\circ$. 
iii) 49 MeV

Again following the above procedure, we obtained the best independent fit parameters for the spin-orbit potential, starting with the phenomenological four-parameter average values of chapter 5. The resulting values are presented in table 7.1 with the name AV6. The fits to the polarisation data are shown in figure 7.4.

From the above analyses it is found that the six-parameter model gives extremely small values of the diffuseness parameters for the real part of the spin-orbit potential compared with the imaginary part of the four parameter model at higher energies. The $\chi^2/N$ values obtained using this model are generally much lower than the average potential obtained previously. Hence, the contributions of this part of the potential to the quality of fit, are quite substantial.

In order to investigate further, we are now using the best-fit potential for the central part and repeating the procedure of the previous section.

7.2.2 The best-fit central potential

This section describes the optical model fit to the differential cross-section and polarisation data for $p + {}^{12}\text{C}$ by optimizing the six parameters but fixing the central part of the potential at their best-fit values which we have obtained earlier in chapter 4.

i) 156 MeV

By keeping the central part of the optical potential at its best-fit values, the spin-orbit parameters have been optimized starting with the values obtained by impulse approximation of chapter 6. The best-fit parameters at this energy are presented in table 7.1 and denoted by BF6. The fits to the polarisation data are shown in figure 7 by the solid line.
We notice that the fit at small angles is excellent for the polarisation data but at larger angles deteriorate slightly, but still reproducing the second minimum as before. The good quality of fit at small angles is ascribed to the values of the central potential which has been discussed earlier in relation to the Coulomb-nuclear interference region.

Again we obtained here the value of $a_{SR}$ which is relatively small, about 0.2 fm. To check that this is the value corresponding to the minimum $\chi^2/N$ value, without any other minimum valleys, the plots of $\chi^2/N$ against $a_{SR}$ for several values of the real potential depth, $U_{S0}$ keeping $r_{SR}$ at its best value are presented. This can be observed in figure 7.5. It is clear that the value at 0.254 fm is the best value corresponding to the true minimum valley. The conventional value at larger magnitude is not justified because it will correspond to the larger values of $\chi^2/N$. Hence, the $a_{SR}$ value has been unambiguously determined.

ii) 75 MeV

Again these data give extremely small value of $a_{SR}$ as can be seen in table 7.1 denoted by the name BF6. The fit to the polarisation data is shown in figure 7.3 by the solid line. The fit to these data is good at almost all angles, but on the whole the quality of fit is only slightly better than the best independent fit using four-parameter model.

iii) 49 MeV

Following the same procedure as above, but starting with the parameters at phenomenological values we obtain the resulting parameters in table 7.1 with the name BF6. The quality of fit is slightly better than the four-parameter model. The fit to the polarisation data is shown in figure 7.4.
One thing we notice is that the value of $a_{SI}$ has been pushed to a very small value of less than 0.1 fm. To check the uniqueness of this value we have plotted the $\chi^2/N$ value against $a_{SI}$ at this energy for various values of $W_{S0}$. This is presented in figure 7.6. It is observed that the minimum $\chi^2/N$ values can be obtained for $a_{SI} < 0.1$ fm or $a_{SI} > 1.5$ fm. Furthermore, the curves do not show definite trends of minimum valleys for each value of $W_{S0}$. We consider this $a_{SI}$ value for the best-fit as unphysical, because either we get an extremely sharply peaked potential or very broad one. Hence, this parameter, $a_{SI}$ can not be uniquely determined.

We have also plotted the $\chi^2/N$ against $W_{S0}$ for the value of $a_{SI}$ reasonably chosen at 0.6 fm, which is the average value at higher energies. We found that for a minimum $\chi^2/N$, $W_{S0}$ tends to zero, as can be seen in figure 7.7. By plotting the values of $\chi^2/N$ against $r_{SI}$ for a fairly small value of $W_{S0}$ and again using a 'reasonable' value of $a_{SI}$ we obtained curve in figure 7.8. It is observed that the minimum $\chi^2/N$ value at $r_{SI} = 0.6$ fm is a false one and that the 'true' minimum is situated somewhere beyond 1.5 fm which we consider as unphysical. Hence, we conclude that the $W_{S0}(r)$ is not well-determined at this energy.

Figure 7.9 shows the effect of fitting the polarisation data at 49 MeV by using the real spin-orbit component only. It can be seen that the fit is of comparable quality to that with the couplex spin-orbit potential. In the case of average central potential being used, the fit is better.

Hence, at 49 MeV the imaginary component of the spin-orbit force is not important. It can be taken to be zero or an extremely small value. However, the real part is uniquely determined and hence, the need for the spin-orbit potential at this energy is justified.
This can be seen from figures 7.10 to 7.12 for the plots of $\chi^2/N$ against its real part of the spin-orbit potential parameters.

To see the forms of the real and imaginary parts of the spin-orbit potential, we have plotted the potential in figures 7.13 and 7.14 for the 156 MeV and 75 MeV. It is clear that they have different form-factors and consequently different shapes. The real part is a sharply-peaked potential and the imaginary part does not show this surface-peaked appearance.

7.3 Average six-parameter spin-orbit potential

The parameters of the six-parameter spin-orbit potential corresponding to BF6 have been plotted in figures 7.15 to 7.17. The energy-dependence of the size parameter is slight. However, we have fitted the points and obtained the energy dependence of the average potential which is given below.

The real part of the spin-orbit potential can be reproduced quite well by

\[
\begin{align*}
U_{S0} &= 3.81 - 0.019 E_L \text{ MeV} \\
r_{SR} &= 0.583 + 0.00274 E_L \text{ fm} \\
a_{SR} &= 0.225 \text{ fm}
\end{align*}
\]

(7.2)

This potential will yield the sharply-peaked potential as shown in figure 7.22. This representation predicts that $U_{S0}$ goes to or through zero at $E_L \approx 200$ MeV, although we have no justification for assuming that this linear dependence holds outside the energy region we have studied.

The imaginary part of the spin-orbit potential can be fitted by the formulae:
\[ W_{S0} = -4.0 + 10.85 \exp(-0.0205 E_L) \text{ MeV} \]
\[ r_{SI} = 1.83 - 0.00645 E_L \text{ fm} \]
\[ a_{SI} = 0.594 + 0.000616 \text{ fm} \]  
(7.3)

The fits to the points obtained from the analyses are shown as the dotted line in figures 7.16 and 7.17. The fits to the differential cross-section and polarisation data using these formulae and the average formulae of set B are shown in figures 7.18 to 7.21. The fits are reasonably good.

In figure 7.22 we have plotted the real spin-orbit potentials of Average S06 at 50, 100 and 150 MeV together with the real central potential normalised to -4.5 MeV at \( r = 0 \). It is observed that the real spin-orbit potential is peaked well inside \( R_R \) (the half-way radius of the central real potential).

However, the imaginary spin-orbit potential always has a volume form. Hence, the total effects of the real and imaginary components of the optical potential on the scattering process are different as will be discussed later.

Table 7.2 shows the volume integrals, root-mean-square radii and products of the volume integral and diffuseness parameter for the real and imaginary spin-orbit potentials. It is observed that the three quantities \( J_{SR}, J_{SR}a_{SR} \) and \( \langle r_{SR}^{2.5} \rangle \) for the real spin-orbit BF6 potential show constant values as a function of energy. Whereas the imaginary part of BF6 the values \( \langle r_{SI}^{2.5} \rangle \) are constant, except at 49 MeV where we have indicated earlier that the parameters are not well-defined. The values of \( J_{SI} \) and \( J_{SI}a_{SI} \) increase in strength as the energy increases.

For the AV6 potential, the magnitude of \( J_{SI} \) and \( J_{SI}a_{SI} \) show
similar trend as usual (increasing numerically as the energy increases). The values of $<r_{SI}^2>^{\frac{1}{2}}$ can be taken to be constant with energy. $<r_{SR}^2>^{\frac{1}{2}}$ can be regarded as increasing slowly with energy. For the case of $J_{SR}$, it seems to decrease with energy. The magnitudes of $J_{SR}$ do not show any trend at all.

Since the values of the above quantities are for the three incident energies only, the conclusions regarding their trends can not be taken to be definite. If more values are available, the trend might change as a result.

In the case of average S06 potential, trends of the quantities discussed above are as shown in figures 7.23 and 7.24.

7.4 The effect of a spin-orbit potential

The nuclear potential affecting the $\ell^{th}$ partial wave can be written in the form

$$\text{Re} \; V_\ell^+(r) = - U_R f(r) + \left( \frac{k}{m c} \right)^2 \frac{d}{dr} \left[ \frac{d}{dr} - \frac{1}{r} \right] V_{S0}(r) \left\{ - (\ell+1) \right\}$$

$$\text{IM} \; V_\ell^+(r) = - \left[ W_{V_S^+}(r) + W_{D_D}(r) \right] + \left( \frac{k}{m c} \right)^2 \frac{d}{dr} \left[ \frac{d}{dr} - \frac{1}{r} \right] W_{S0}(r) \left\{ - (\ell+1) \right\}$$

where the superscripts $\pm$ correspond to $j = \ell \pm \frac{1}{2}$, respectively. Since, we have established that,

$$\frac{1}{r} \frac{d}{dr} V_{S0}(r) < 0$$

$$\frac{1}{r} \frac{d}{dr} W_{S0}(r) > 0$$

Hence, we may write
The effect of the above potentials in terms of Fermi model is well-known. For \( j=\frac{\ell+1}{2} \), the \( \text{Re} V^+_{\ell}(r) \) becomes more attractive in the surface region and extends to a larger radius \( R^+ > R \) which consequently increases the real phase shift, \( \delta^+_{\ell} \). The imaginary spin-orbit part weakens the magnitude of \( \text{Im} V^+_{\ell}(r) \) in the surface region, thus raising the reflection coefficient \( |\eta^+_{\ell}| \).

For the effects are in the opposite direction. These effects of \( U_{S0} \) on \( \delta^+_{\ell} \) and \( W_{S0} \) on \( |\eta^+_{\ell}| \) are almost completely decoupled: \( U_{S0} \) almost exclusively causes the splitting of real phase shifts without affecting \( |\eta^+_{\ell}| \), while \( W_{S0} \) alone is responsible for the splitting of \( |\eta^+_{\ell}| \) without affecting the real phase shifts.

So, in the Fermi model we have \( \delta^+_{\ell} > \delta^-_{\ell} \) and expect \( \text{Re} \eta^+_{\ell} < \text{Re} \eta^-_{\ell} \) and that \( L^+ > L^- \), where \( L^\pm \) are the critical angular momenta for the transition from minimum to maximum values of \( \text{Re} \eta^\pm_{\ell} \). The essential assumption of this model is that the spin-orbit potential is surface-peaked and the peaks are close to half-way radius \( R \) of the real central potential.

But, in our case the real spin-orbit potential is peaked a long way inside \( R \) and the imaginary spin-orbit part has a volume behaviour and hence, even for a relatively small values of \( \ell \) which are important in such a light nucleus, \( \text{Re} V^\pm_{\ell}(r) \) and \( \text{Im} V^\pm_{\ell}(r) \) are dramatically different from each other and from the central parts. Hence, the usual Fermi model of the phenomenon of polarisation can not be used to elucidate our results. This can be seen from figure 7.25 where our spin-orbit potentials (BF6 and BF4) show that \( \text{Re} \eta^+_{\ell} > \text{Re} \eta^-_{\ell} \) and \( L^+ < L^- \).
in contradiction to expectations from the Fermi model. The main change
between the values obtained with potentials BF4 and BF6 is the enhanced
difference between \( \eta_{\pm}^{+} \) and \( \eta_{\pm}^{-} \) with \( |L^+ - L^-| \) increasing from one
unit to two.

In figure 7.26 we observe that at 49 MeV \( |\eta_{\pm}^{+}| = |\eta_{\pm}^{-}| \) over a range
of values of \( \ell \) due to the small imaginary spin-orbit term, but the
difference is increasing as the imaginary spin-orbit component is
increased (at higher energies). The effect is greater for BF6 potential
than BF4 at higher energies. But, \( \delta^-_\ell > \delta^+_\ell \) due to the real term, as can
be seen in figure 7.25.

The behaviour of the reflection coefficients can be understood in
terms of the behaviour of the classical turning point. The turning
point \( r_0 \) for the \( \ell \)th partial wave is defined through the relation,

\[
Re V_{\ell,\text{tot}}^\pm(r_0) = V_c(r_0) + Re V_{\ell}^\pm(r_0) + \frac{\hbar^2}{2\mu} \frac{\ell(\ell+1)}{r_0^2} \quad \text{(7.6)}
\]

for various values of \( \ell \). At 50 MeV, the differences in \( \text{Re } V_{\ell,\text{tot}}^{+}(r) \)
and \( \text{Re } V_{\ell,\text{tot}}^{-}(r) \) appear outside the classical turning point only for
\( \ell < 4 \) and the differences occur where \( \text{Im } V_{\ell,\text{tot}}(r) \) is largest as can be
seen in figure 7.28. At 156 MeV, the differences between \( \text{Re } V_{\ell,\text{tot}}^{+}(r) \)
and \( \text{Re } V_{\ell,\text{tot}}^{-}(r) \) appear outside the classical turning point for \( \ell \leq 8 \)
as shown in figure 7.28. Except for \( \ell=1 \), the difference between
\( \text{Im } V_{\ell,\text{tot}}^{+}(r) \) and \( \text{Im } V_{\ell,\text{tot}}^{-}(r) \) is more significant then the difference
between \( \text{Re } V_{\ell,\text{tot}}^{+}(r) \) and \( \text{Re } V_{\ell,\text{tot}}^{-}(r) \) both in magnitude and in extent
in configuration space.
7.5 Conclusions

From our analysis it has been found that the need for different shapes and different energy dependence of the real and imaginary part of the spin-orbit potential is justified. The imaginary spin-orbit potential has a volume form with the parameters having rather conventional values. The real spin orbit potential is surface-peaked and peaks quite a long way inside the real central potential.

In our six-parameter spin-orbit potential model, the real diffuseness $a_{SR}$ has a value of consistently of the order of $0.2 \text{ fm}$ and the imaginary diffuseness $a_{SI}$ has a value quite similar to the central part, which is slightly larger than the conventional values. On the other hand, in the four-parameter model the diffuseness $a_S$ is of the order of $0.2 \text{ fm}$ at lower energies, where the imaginary spin-orbit part is not important, and about $0.4$ to $0.5 \text{ fm}$ at higher energies. The values obtained by other workers are also similar, as can be seen in table 7.3, where the four-parameter model parameters for the spin-orbit potential obtained by us and by Ingemarsson et. al and Fannon et. al are shown. Hence, we conclude that the values of $a_{SR}$ and $a_{SI}$ are different and that at higher energies, the value of $a_S$ represents a compromise between the required values of $a_{SR}$ and $a_{SI}$.

We have also found that the imaginary spin-orbit potential is not important, it is small or zero, in the energy region $E_L \sim 49 \text{ MeV}$, particularly if the central potential is optimized. In fact, Kobos and Mackintosh (119) found that the imaginary spin-orbit potential for $p + ^{16}\text{O}$ changes sign between 45 and 50 Mev in a direction which is consistent with our results.

Our real spin-orbit potential strength diminishes more rapidly with energy than the real central potential one, as can be seen from
equations (7.2) and (5.2). The trend of $r_{SR}$ is increasing and $r_{SI}$ is decreasing with energy which is consistent with the prediction of the impulse approximation of the previous chapter, but the behaviour of other parameters is not. In any case, the calculations using impulse approximation is a useful means of predicting the starting values for $r_{SR}$, $r_{SI}$ and $a_{SI}$ in the higher energy region where we may expect this approximation to have reasonable validity.
Table 7.1: Values obtained in fits to proton scattering from $^{12}\text{C}$ with a six-parameter spin-orbit potential. Parameters underlined were not included in the search. The last column refers to the corresponding 4-parameter fit.

<table>
<thead>
<tr>
<th>Potential</th>
<th>$E_L$ (MeV)</th>
<th>$U_{S0}$ (MeV)</th>
<th>$r_{SR}$ (fm)</th>
<th>$\alpha_{SR}$ (fm)</th>
<th>$W_{S0}$ (MeV)</th>
<th>$r_{SI}$ (fm)</th>
<th>$\alpha_{SI}$ (fm)</th>
<th>$\chi^2/N$</th>
<th>$\chi^2/N(4)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>BF6</td>
<td>156</td>
<td>0.86</td>
<td>1.077</td>
<td>0.254</td>
<td>-3.55</td>
<td>0.872</td>
<td>0.690</td>
<td>13.46</td>
<td>25.0</td>
</tr>
<tr>
<td></td>
<td>75</td>
<td>2.31</td>
<td>0.801</td>
<td>0.180</td>
<td>-1.75</td>
<td>1.035</td>
<td>0.640</td>
<td>10.38</td>
<td>11.88</td>
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<tr>
<td></td>
<td>49</td>
<td>2.88</td>
<td>0.721</td>
<td>0.225</td>
<td>-0.38</td>
<td>0.629</td>
<td>0.098</td>
<td>27.8</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>-0.1</td>
<td>1.0</td>
<td>0.6</td>
<td>28.6</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0</td>
<td>-</td>
<td>-</td>
<td>28.8</td>
<td>28.6</td>
</tr>
<tr>
<td>AV6</td>
<td>156</td>
<td>0.56</td>
<td>1.057</td>
<td>0.111</td>
<td>-3.06</td>
<td>0.825</td>
<td>0.665</td>
<td>97.37</td>
<td>216.7</td>
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<td></td>
<td>75</td>
<td>1.69</td>
<td>0.583</td>
<td>0.293</td>
<td>-0.78</td>
<td>1.468</td>
<td>0.618</td>
<td>8.75</td>
<td>57.2</td>
</tr>
<tr>
<td></td>
<td>49</td>
<td>2.37</td>
<td>0.766</td>
<td>0.133</td>
<td>-1.10</td>
<td>1.050</td>
<td>0.730</td>
<td>75.58</td>
<td>183.3</td>
</tr>
</tbody>
</table>
Table 7.2: The volume integrals, root-mean-square radii and products of volume integral and diffuseness for potentials AV6 and BF6. Note that these are the spin-orbit parts of the optical potential.

<table>
<thead>
<tr>
<th>$E_L$ (MeV)</th>
<th>$J_{SR}$ (MeV-fm$^3$)</th>
<th>$J_{SR}^a_{SR}$ (MeV-fm$^4$)</th>
<th>&lt;r$^2_{SR}$&gt;$(\text{fm})$</th>
<th>$J_{SI}$ (MeV-fm$^3$)</th>
<th>$J_{SI}^a_{SI}$ (MeV-fm$^4$)</th>
<th>&lt;r$^2_{SI}$&gt;$(\text{fm})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>156</td>
<td>4.744</td>
<td>1.205</td>
<td>2.13</td>
<td>-20.517</td>
<td>-14.157</td>
<td>2.94</td>
</tr>
<tr>
<td>BF6 75</td>
<td>5.197</td>
<td>0.935</td>
<td>1.57</td>
<td>-13.314</td>
<td>-8.521</td>
<td>2.97</td>
</tr>
<tr>
<td>49</td>
<td>5.102</td>
<td>1.148</td>
<td>1.52</td>
<td>-0.395</td>
<td>-0.0387</td>
<td>1.17</td>
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<tr>
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<td>0.299</td>
<td>1.92</td>
<td>-15.289</td>
<td>-10.167</td>
<td>2.82</td>
</tr>
<tr>
<td>AV6 75</td>
<td>1.971</td>
<td>0.577</td>
<td>1.49</td>
<td>-13.135</td>
<td>-8.118</td>
<td>3.45</td>
</tr>
<tr>
<td>49</td>
<td>4.499</td>
<td>0.598</td>
<td>1.44</td>
<td>-9.711</td>
<td>-7.089</td>
<td>3.24</td>
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</table>
Table 7.3: Values obtained in best fits to proton scattering from $^{12}$C with a 4-parameter spin-orbit potential (BF4).

The values given for 156 MeV were used at all energies in the average potential (AV4)

<table>
<thead>
<tr>
<th>$E_L$ (MeV)</th>
<th>$U_{SO}$ (MeV)</th>
<th>$W_{SO}$ (fm)</th>
<th>$r_{SO}$ (fm)</th>
<th>$a_S$ (fm)</th>
<th>$\chi^2/N$</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>185</td>
<td>3.63</td>
<td>-3.09</td>
<td>0.870</td>
<td>0.463</td>
<td>3.82</td>
<td>Ingemarsson et al(74)</td>
</tr>
<tr>
<td>156</td>
<td>2.06</td>
<td>-2.27</td>
<td>0.948</td>
<td>0.492</td>
<td>25.0</td>
<td>This work</td>
</tr>
<tr>
<td>75</td>
<td>2.79</td>
<td>-1.71</td>
<td>0.872</td>
<td>0.540</td>
<td>11.88</td>
<td>&quot;</td>
</tr>
<tr>
<td>49</td>
<td>2.79</td>
<td>0</td>
<td>0.716</td>
<td>0.222</td>
<td>28.6</td>
<td>&quot;</td>
</tr>
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<td>49</td>
<td>4.83</td>
<td>0</td>
<td>0.67</td>
<td>0.21</td>
<td>~22</td>
<td>Fannon et al(86)</td>
</tr>
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**Figure 7.1:** The best independent fit to the polarisation data for $p + ^{12}\text{C}$ elastic scattering at 156 MeV using the six-parameter spin-orbit potential. The dotted line was obtained by keeping the central part at the average values and has been denoted by $AV_6$. The solid line was obtained by using the best-fit parameters obtained previously and has been denoted by $BF_6$.

**Figure 7.2:** The fit to the differential cross-section data for $p + ^{12}\text{C}$ at 156 MeV. The dotted line was obtained by independent fit to the differential cross-section and polarisation data using six-parameter spin-orbit potential but keeping the central part at the average values ($AV_6$). The solid line is the result using the average four-parameter optical potential obtained previously ($AV_4$) and is given for comparison to show the improvement in the second dip of the differential cross-section.

**Figure 7.3:** The best independent fit to the polarisation data for $p + ^{12}\text{C}$ elastic scattering at 75 MeV using the six-parameter spin-orbit potential. The dotted line was obtained using the average central potential and optimising the spin-orbit parameters. The solid line was obtained using the best-fit central potential ($BF_6$).

**Figure 7.4:** The best independent fit to the polarisation data for $p + ^{12}\text{C}$ at 49 MeV using the six-parameter spin-orbit potential. The dotted line was obtained using the average central potential and searching for best values of the spin-orbit parameters ($AV_6$). The solid line was obtained using the best-fit central potential ($BF_6$).
Figure 7.5: The plots of $\chi^2/N$ values against $a_{SR}$ for various values of $U_{S0}$ at 156 MeV. The value of $r_{SR}$ is held at its best value. The minimum $\chi^2/N$ corresponds to the best value of $U_{S0}$.

Figure 7.6: The plots of $\chi^2/N$ values against $a_{SI}$ for various values of $W_{S0}$ at 59 MeV. The value of $r_{SI}$ is being held at its best value of 0.629 fm.

Figure 7.7: The plot of $\chi^2/N$ values as a function of $W_{S0}$ at 49 MeV. The value of $r_{SI}$, and $a_{SI}$ is at the average values at higher energies which we consider as reasonable.

Figure 7.8: The plot of $\chi^2/N$ values against $r_{SI}$ for various values of $r_{SI}$ at 49 MeV. We take the value of $W_{S0}$ as small (according to figure 7.7) and $a_{SI}$ as 'reasonable'. Consequently, the minimum at $r_{SI} = 0.6$ is a false one.

Figure 7.9: The fits to the polarisation data at 49 MeV, using BF6 (solid line) and AV6 (dotted line) by putting the imaginary component, $W_{S0}(r)$ equal to zero.

FIGURES 7.10 TO 7.12

The plots of various $\chi^2/N$ values against the real parameters of the spin-orbit potential at 49 MeV to show their uniqueness.

Figure 7.13: The plots of six-parameter spin-orbit potential at 156 MeV for AV6 and BF6 potentials.

Figure 7.14: The plots of six-parameter spin-orbit potential at 75 MeV for AV6 and BF6 potentials.
FIGURES 7.15 TO 7.17

The plots of the six-parameters spin-orbit potential parameters for the best independent fits to both the differential cross-section and polarisation data. The central potentials are fixed at their best-fit values obtained in Chapter 4.

The dotted lines are the fits to these points using the average formulae of Section 7.3.

FIGURES 7.18 TO 7.21

The fits to the experimental data using the average formulae of Section 7.3 and the average formulae of Chapter 5 for the central part. The combination of these formula is assigned the name Average S06.

FIGURE 7.22

The behaviour of real part of the average S06 potential with energy, compared with the real central potential.

FIGURE 7.23

The behaviour of the real quantities of average S06 potential - $J_{SR}$, $a_{SR}$, and $<r_{SR}^{2}>^\frac{1}{2}$ - with energy.

FIGURE 7.24

The behaviour of the imaginary quantities of average S06 potential - $J_{SI}$, $a_{SI}$ and $<r_{SI}^{2}>^\frac{1}{2}$ - with energy.

FIGURE 7.25

The reflection coefficients as a function of the partial waves at 156 MeV for BF4 and BF6 potentials.

FIGURE 7.26

The modulus of the reflection coefficients as a function of the partial waves at 49 MeV, 75 MeV and 156 MeV for BF4 and BF6 potentials.
FIGURES 7.27 AND 7.28

The plots of the total potentials of equation (7.6) versus the partial waves at 50 MeV and 156 MeV for the average S06 potential and the average central potential.
Fig. 7.2

\[ \sigma(\theta) \quad (\text{mb/} \text{sr}) \]

\[ p + ^{12}\text{C} \quad 156 \text{ MeV} \]

- Dashed line: AV 6
- Solid line: AV 4
Fig. 7.3

$P + ^{12}C, 75\, \text{MeV}$

$\rho(\Theta)$

$\Theta (\text{c.m.})$
Figure 7.4

$p + ^{12}\text{C}, 4.9\text{MeV.}$
$156 \text{ MeV}$

($r_{sp} = 1.077 \text{ fm}$.)

$\chi^2/N$ vs. $a_{SR} (\text{fm})$

$U_{so} = 1.2$

$U_{so} = 1.0$

$U_{so} = 0.864$

$U_{so} = 0.6$

Fig. 7.5
Figure 7.7

$49 \text{ MeV}$

$r_{51} = 1.0 \text{ fm}$

$a_{31} = 0.6 \text{ fm}$
$W_{so} = 0.2 \text{ fm.}$

$a_{sz} = 0.6 \text{ fm.}$

Fig. 7.8

$r_{sz} (\text{fm.})$
$p + ^{12}C \quad 49 \text{ MeV}$

$\gamma_{5R} = 0.721 \text{ fm}$

$U_{50} = 2.884 \text{ MeV}$

Fig. 7.10
$p + ^{12}\text{C} \quad 49 \text{ MeV}$

$r_{SR} = 0.721 \text{ fm}$

$q_{SR} = 0.225 \text{ MeV}$

Fig. 7.11
$p + ^{12}C \quad 49 \text{ MeV}$

$a_{SR} = 0.225 \text{ fm}$

$U_{SO} = 2.884 \text{ MeV}$

Figure 7.12
\[ p + ^{12}\text{C} \rightarrow 156 \text{ MeV} \]

- BF6
- AV6

**Fig. 7.13**
\[ p + ^{12}\text{C} \quad 75 \text{ MeV} \]

![Graph showing \( r'(r) \) and \( V'(r) \) with curves for BF6 and AVG at various r (fm) values.](image)

**Fig. 7.14**
\( U_{so} = 3.81 \left( 1.0 - 0.005 E_L \right) \)

**Fig. 7.16**

\( U_{so} \) (MeV)

\( E_L \) (MeV)
$p + ^{12}C$ 156 MeV

---

$\sigma(\theta)$ (mb/sr)

---

$\theta$ (c.m.)

---

Fig. 7.18
Fig. 7.19
$\rho + ^{12}C \ 49 \text{ MeV}$

---

**Fig. 7.20a**

Average S06.

---

$\theta \text{(c.m.)}$

---

$R_g$
Fig. 7.21
Fig. 7.21a
Fig. 7.22
Real part of Average 506
Fig. 7.24
$E_L = 50 \text{ MeV}$

- $j = l + \frac{1}{2}$
- $j = l - \frac{1}{2}$

Figure 7.27
$E_L = 156\,\text{MeV}$

$\ell = \ell + \frac{1}{2}$

$\ell = \ell - \frac{1}{2}$

Re $V_{\ell,\text{tot}}^\pm$

Im $V_{\ell,\text{tot}}^\pm$

$E_{\text{cm}}$

Fig. 7.28
SUMMARY AND CONCLUSIONS

We have presented a detailed analysis of the optical model potential for elastic scattering of proton on $^{12}$C and $^{16}$O. The central potential that we have obtained is simpler and able to give a fair description of the differential cross-section data for proton scattering on light nuclei in the energy range 50 to 160 MeV. The six-parameter model of our spin-orbit potential has different shapes for the real and imaginary components with the volume form for the imaginary part and the surface-peaked form for the real part. However at around 49 MeV the imaginary component of this spin-orbit potential is negligibly small or can be taken to be absent. In general, the polarisation data can be adequately reproduced by our spin-orbit potential.

By what we have obtained, it is clear that the phenomenological optical model can sufficiently describe the elastic scattering of proton on light nuclei at intermediate energy region, at least for the closed-shell nuclei such as $^{12}$C and $^{16}$O. We have obtained the averaged potential which depends on energy and apart from the mass dependence of the radial parameters the form-factors for the central potential are independent of energy. Hence, the dependence on energy arises only from the central potential depths. Similarly, the spin-orbit potential is dependent on the incident projectile energy but in this case the form-factors are no longer independent of energy, in addition to dependence of the radial parameters on the target mass numbers.

We conclude that our potential can confidently be used to generate the distorted waves in nuclear reaction calculations involving proton and light target nuclei in the energy range we have considered.
Apart from this, the intrinsic interest in the scattering of protons on light nuclei itself can facilitate the comparison of our work with the more fundamental theory of the optical potential starting from the nucleon-nucleon interaction. For instance, most of the optical model calculations involve heavier target nuclei and hence the finer details of the potential for these nuclei have been adequately explored. So far, comparison of the microscopic calculations with the phenomenological results as light as $^{12}\text{C}$ has been constantly avoided, usually the $^{40}\text{Ca}$ or sometimes $^{16}\text{O}$ is considered light enough. Hence, our results on light nuclei appeal to such comparison so that more physical insight into the optical potential of nucleon on target nuclei, which involve different mechanism from the heavier targets due to its structure, will be achieved.

The average potential with its six-parameter spin-orbit part has been further tested by comparing its predictions with data on proton scattering from $^9\text{Be}$, $^{11}\text{B}$ and $^{14}\text{N}$. For $^9\text{Be}$, agreement with the available differential cross-sections and the total reaction cross-sections in the energy range 100 - 160 MeV is good. For $^{11}\text{B}$, agreement with the differential cross-section at 100 MeV is quite good but at 155 MeV there are discrepancies in both differential cross-section and polarisation in the angular range 35° - 55°. For $^{14}\text{N}$, agreement with the new differential cross-section data from Indiana at 144 MeV is excellent, agreement with the differential cross-sections at 100 and 155 MeV is not so good but the polarisation at 155 MeV is satisfactory. At 49 MeV agreement with the polarisation is very good up to 60° and is in accord with the general trend of the data all the way out to 160°. The differential cross-section is systematically underestimated but the shape is satisfactory out to 130°.

Hence, our average potential is able to successfully describe the
scattering of protons from light nuclei, as light as $^9$Be. So, the
ability to reproduce the $^{16}$O data using the average potential obtained
from the analysis of $^{12}$C data cannot be ascribed to their alpha cluster
structures$^{(121)}$, as our average potential works as well for non-alpha
structure nuclei such as $^9$Be, $^{11}$B and $^{14}$N as has been described above.
However, the possible contributions from the alpha cluster structure to
the $p + ^{12}$C elastic scattering has been done semi-classically by Bonetti
et al$^{(122)}$.

The interpretation of our average potential in terms of alpha
particle structure of $^{12}$C and $^{16}$O is quite appealing in view of the fact
that the results of analysis on $^{12}$C give striking agreement to the fits
for $^{16}$O and the real spin-orbit parameters $a^R_{SR}$ is of the order of 0.2 fm
which is similar to the $^4$He case$^{(81)}$. However, by testing our potential
on non-alpha like target disproves this connection.

Apart from the possible alpha cluster contribution to the proton
scattering on light nuclei$^{(122)}$, the contribution from back coupling of
the deuteron pick-up channel$^{(36)}$ is quite substantial in view of the work
at lower energy. This deuteron pick-up effect has been shown by
Mackintosh and Cordero$^{(123)}$ and Mackintosh and Kobos$^{(124)}$ to be represented
phenomenologically by an $\lambda$-dependent complex potential. But at higher
energies Confort and Karp$^{(37)}$ have shown that the back couplings from
$(p,d)$ channels have had an enormous impact on the imaginary component of
the optical potential. However, the effects of all back coupling decrease
with energy. Hence, we believe that by including these effects, apart
from the exchange effect, can further improve the fits to the experimental
data. Furthermore, the nuclear structure effect can be incorporated into
the expression for the average potential using the isospin term, $T_1$, of
Lane$^{(125)}$ which is called nuclear symmetry term in order to fit the proton
scattering on non-closed shell nuclei. By including this term as well
as the term due to the Coulomb correction in our potential can further
generalise the validity of our potential.

Our phenomenological optical model analysis is a simple one in
the sense that all the non-elastic channels are lumped together in the
imaginary part of the potential. So the imaginary part of the potential
is rich with the information concerning the reactions involved in the
interaction processes as well as those of nuclear structure. It is
not surprising that the difference in the fits to the differential
cross-section data using our average and particular potentials around
the nuclear-Coulomb interference region almost entirely due to the
imaginary central potential.

The radial parameters, $r_R$ and $r_I$, of our central potential have
rather unconventional values with $r_R > r_I$. But this choice of parameter
values is clearly indicated by our independent fits at most energies.

However, we have two forms of the imaginary central potential.
One of them is the constant volume form at all incident energies and
the other is more compatible with the Pauli exclusion effect, which is
increasing in depth with the incident energies and changing shapes from
volume form at higher energies and surface-peaked form at lower energies.

The calculation of the spin-orbit potential using the method of
impulse approximation gives quite a useful starting value for the
form-factors in the higher energy region where we expect this approximation
to have reasonable validity. However, the fits to the data using the
potential calculated from the impulse approximation and the first-order
multiple scattering theory of Kerman et al.\(^{(5)}\) do not produce as good
as the fit obtained from our average potential. However, calculations
using the Kerman et al.\(^{(5)}\) approach gives no support for the use of the
Thomas form of the spin-orbit part. The same conclusion in this respect
has also been arrived at by Ingemarsson et al at 180 MeV\(^{(74)}\).

Indeed, good fit to the polarisation data entails the use of the precise form of the spin-orbit potential. The use of our six-parameter model for the spin-orbit potential has improved the fits resulting in different shapes for its components. As a by-product, the improved fit to the second minimum in the mid-angular region is achieved as a result. This sensitivity of the mid-angular region to the spin-orbit potential has been noted by Ingemarsson et al\(^{(74)}\) and Nadeson et al\(^{(9)}\) recently. However, there are indications\(^{(126)}\) that the imaginary spin-orbit potential changes sign at the energies around 50 MeV and becomes positive at lower energies. This is in agreement with our finding at around 50 MeV data.

In conclusion, we would like to remark that the proton scattering is of fundamental importance in nuclear physics. Clear understanding of this interaction is necessary because upon it the firm foundation of nuclear physics is built. Any uncertainties in the understanding of elastic scattering must propagate into all applications of this and more complex reactions, such as the extraction of nuclear structure information and heavy-ions interactions. It is a pity, therefore, that the study of the proton elastic scattering is being hampered by the lack of precision data at sufficient energy, angular and mass ranges both for cross-section and polarisation, than by any other factors. By having more precise data we hope further progress in this aspect could be achieved.
APPENDIX

A1: Fox-Goodwin Formula

The Fox-Goodwin formula (127) is

\[
\left(1 - \frac{h^2}{12} f_1\right)y_1 = \left(2 + \frac{5h^2}{6} f_0\right)y_0 - \left(1 - \frac{h^2}{12} f_{-1}\right)y_{-1} + \Delta \tag{A1.1}
\]

where

\[
\Delta = \left(\frac{1}{240} - \frac{6^6}{15120} \frac{13}{6^8} - \ldots\right)y_0 \tag{A1.2}
\]

is the error. Writing the radial Schroedinger equations (3.25) and (3.26) in the form

\[
\frac{d^2}{dr^2} y(r_i) = f(r_i)y(r_i) \tag{A1.3}
\]

at \(i\)th point. For convenience we write \(y(r_i) = y_i\). \(h\) is the step-length.

The algorithm to solve (A1.3) is as follows. Given the function at \((i-2)\)th and \((i-1)\)th steps, we obtain

\[
\xi_i = x_i y_i \tag{A1.4}
\]

where

\[
x_i = 1 - \frac{h^2}{12} f_i
\]

Hence, Fox-Goodwin formula becomes

\[
\xi_i = (2 + \frac{5h^2}{6} f_{i-1})y_{i-1} - \xi_{i-2} \tag{A1.5}
\]

since

\[
y_{i-1} = \frac{\xi_{i-1}}{x_i}
\]

Finally we have
Equation (A1.6) is used iteratively starting from the origin outward to a point beyond the nuclear field. This method is the most suitable because of the absence of the first derivative and one of the most rapid ones.

A2: Coulomb functions and Phase Shifts

Coulomb phase shifts are conveniently calculated from the recurrence relation

$$\sigma_\lambda (\gamma) = \sigma_{\lambda+1} (\gamma) - \tan^{-1} \left( \frac{\gamma}{\lambda+1} \right)$$  \hspace{1cm} (A2.1)

and the asymptotic expansion (66)

$$\sigma_\lambda (\gamma) = \alpha [\lambda+\frac{1}{2}] + \gamma [\log \beta - 1] + \frac{1}{\beta} \left\{ \frac{-\sin \alpha}{12} + \frac{\sin 3\alpha}{360} \beta^2 + \frac{\sin 5\alpha}{1260} \beta^4 + \frac{\sin 7\alpha}{1680} \beta^6 + \frac{\sin 9\alpha}{1188} \beta^8 + \ldots \right\}$$  \hspace{1cm} (A2.2)

where

$$\alpha = \tan^{-1} \left( \frac{\gamma}{\lambda+1} \right), \text{ and } \beta = \left\{ \gamma^2 + (\lambda+1)^2 \right\}^{\frac{3}{2}}$$

The regular Coulomb function was calculated by Miller's Method. This method set $F_{\lambda+1}(kr) = 0$, $\alpha F_{\lambda}(kr) = \epsilon$ where $\lambda$ is an integer $> \lambda_{\text{max}}$, $\alpha$ is a constant to be determined and $\epsilon$ is a very small number. Using the recurrence relation

$$\frac{\left[ \gamma^2 + (\lambda+1)^2 \right]^{\frac{3}{2}}}{\lambda+1} F_{\lambda+1}(kr) + \frac{\left[ \gamma^2 + \lambda^2 \right]^{\frac{3}{2}}}{\lambda} F_{\lambda-1}(kr)$$

$$= (2\lambda+1) \left[ \frac{\gamma}{\lambda(\lambda+1)} + \frac{1}{kr} \right] F_{\lambda}(kr)$$  \hspace{1cm} (A2.3)
All the $\alpha F_{k}^{(kr)}$'s down to $\alpha F_{0}^{(kr)}$ can be calculated. The constant $\alpha$ was found from the Wronskian

\[ F_{0}^{(kr)}G_{0}^{(kr)} - F_{0}^{(kr)}G_{0}^{(kr)} = 1 \]  

(A2.4)

where $F_{0}^{(kr)}$ was found from the recurrence relation

\[ \frac{\gamma}{k+1} F_{k}^{(kr)} - F_{k}^{(kr)} = \left[ \frac{\gamma^{2} + (k+1)^{2}}{k+1} \right]^{\frac{1}{2}} F_{k+1}^{(kr)} \]  

(A2.5)

and $G_{0}^{(kr)}$ and $G_{0}^{(kr)}$ were calculated using the expression

\[ G_{0}^{(kr)} = s \cos(-\gamma \ln 2kr + kr + \sigma_{0}) - t \sin(-\gamma \ln 2kr + kr + \sigma_{0}) \]

\[ G_{0}^{(kr)} = s \cos(-\gamma \ln 2kr + kr + \sigma_{0}) - t \sin(-\gamma \ln 2kr + kr + \sigma_{0}) \]

(A2.6)

where

\[ s = \sum_{n=0}^{25} s_{n}, \quad t = \sum_{n=0}^{25} t_{n} \]

\[ S = \sum_{n=0}^{25} S_{n}, \quad T = \sum_{n=0}^{25} T_{n} \]

The number of terms in the sum was found to be adequate, and the individual terms were calculated from the recurrence relation.
\[ S_{n+1} = \frac{(2n+1)y}{2k(n+1)} s_n - \frac{y^2-n(n+1)}{2k(n+1)} t_n \]
\[ t_{n+1} = \frac{(2n+1)y}{2k(n+1)} t_n + \frac{y^2-n(n+1)}{2k(n+1)} s_n \]
\[ S_{n+1} = \frac{(2n+1)y}{2k(n+1)} S_n - \frac{y^2-n(n+1)}{2k(n+1)} T_n - \frac{S_n+1}{kr} \]
\[ T_{n+1} = \frac{(2n+1)y}{2k(n+1)} T_n + \frac{y^2-n(n+1)}{2k(n+1)} S_n - \frac{t_{n+1}}{kr} \]

(A2.7)

with the initial values

\[ s_0 = 1, \quad s_1 = \frac{y}{2kr}; \quad S_0 = 0, \quad S_1 = \frac{y^3-y^2}{2k^2r^2} - \frac{y^2}{2kr}; \]
\[ t_0 = 0, \quad t_1 = \frac{y^2}{2kr}; \quad T_0 = 1 - \frac{y^2}{kr}, \quad T_1 = \frac{y^2}{k^2r^2} + \frac{y}{2kr}. \]

The summations satisfy the relation

\[ sT - St = 1 \]  \hspace{1cm} (A2.8)

As \( r \) increases the convergence of these series improves, and accurate results are obtained for \( r \) substantially greater than \( r_M \).

A3: Chi-squared Minimization

The sets of optical model parameters which optimize the fits to experimental data were obtained by minimizing \( \chi^2 \) function defined by equations (3.58) and (3.59). Minimization process is achieved by the method of steepest descent. It consists of three phases: exploration, descent and termination.
Let the theoretical angular distributions be functions of $n$ parameters $p_1, p_2, p_3, \ldots, p_n$ represented by a vector $f$ in the $n$-dimensional parameter space. In the $(n+1)$-dimensional space of $\chi^2$ and $f$ the function

$$\chi^2 = F(f)$$

(A3.1)

generates a hypersurface. Equation (A3.1) is essentially the same as equation (3.58). Our objective is to locate a point $f$ at which $F$ has a minimum.

We approximate the theoretical angular distribution by the first-order Taylor expansion

$$\sigma_\lambda(f) = \sigma_\lambda(f_0) + \sum_{i=1}^{n} \frac{\partial \sigma_\lambda}{\partial p_i} f_0^0 p_i + \ldots$$

(A3.2)

where

$$f = f_0 + \delta f$$

(A3.3)

At the minimum point, this relation is satisfied

$$\left. \frac{\partial^2 F(f)}{\partial p_i} \right|_{f_0} = 0, \ (i=1, 2, \ldots, n)$$

(A3.4)

that is

$$\left. \frac{\partial^2 F(f)}{\partial p_i} \right|_{f_0} + \sum_{j=1}^{n} \frac{\partial^2 \sigma_\lambda}{\partial p_i \partial p_j} f_0 \delta p_j = 0$$

(A3.5)

Again we have approximated the function $F(f)$ by $\bar{F}(f)$ using the first-order Taylor series.

Since,

$$\left. \frac{\partial^2 F(f)}{\partial p_i} \right|_{f_0} = 2 \frac{\sum_{\lambda=1}^{N} \sigma_\lambda(f) - \sigma^x_\lambda}{(\Delta \sigma^x_\lambda)^2} \left. \frac{\partial \sigma_\lambda}{\partial p_i} \right|_{f_0}$$

(A3.6)
and
\[ \frac{\partial^2 F(f)}{\partial p_j \partial p_i} \big|_{f^0} = \frac{2}{N} \sum_{\lambda=1}^{N} \frac{1}{(\Delta \sigma^X)^2} \frac{\partial \sigma^X(f)}{\partial p_j} \bigg|_{f^0} \delta p_k + \cdots - \delta^X \]

Substituting equations (A3.7) and (A3.6) into (A3.5) and using (A3.2) we have

\[ \begin{align*}
\frac{2}{N} \sum_{\lambda=1}^{N} \frac{1}{(\Delta \sigma^X)^2} & \left[ \sigma^X(f^0) + \sum_{k=1}^{n} \frac{\partial \sigma^X(f)}{\partial p_k} \right] \bigg|_{f^0} \delta p_k + \cdots - \sigma^X \\
+ \frac{2}{N} \sum_{\lambda=1}^{N} \frac{1}{(\Delta \sigma^X)^2} \sum_{j=1}^{n} \frac{\partial \sigma^X(f)}{\partial p_j} \bigg|_{f^0} \delta p_j = 0
\end{align*} \]
(A3.8)

After rearranging, we have

\[ \sum_{\lambda=1}^{N} \frac{1}{(\Delta \sigma^X)^2} \left[ \sigma^X f^0 - \sigma^X(f) \right] \frac{\partial \sigma^X(f)}{\partial p_j} \bigg|_{f^0} \delta p_j = 0 \]
(A3.9)

There are n simultaneous equations of the form (A3.9). Solving this system of linear equations for \( \delta f \) we obtain the predicted minimum point at \( f = f^0 + \delta f \). Due to the approximations used, the method is valid for \( f^0 \) near the minimum point.

We write (A3.9) in matrix form

\[ \sum_{j=1}^{n} A_{ij} \Delta_j = B_{i\lambda}, (i=1, 2, \ldots, n) \]  
(A3.10)

where

\[ A_{ij} = \sum_{\lambda=1}^{N} a_{i\lambda} b_{j \lambda} \quad \text{and} \quad B_{i\lambda} = \sum_{\lambda=1}^{N} c_{i\lambda} d_{\lambda} \]

which can easily be solved for \( \Delta_j \) by the matrix inversion method.
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