CHARACTERISTICS OF NON SINUSOIDAL
MICROWAVE NEGATIVE RESISTANCE OSCILLATORS

by

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of Doctor of Philosophy.

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To my parents for their continual support during the past three years of research.
The primary aim of the work described in this thesis has been to examine the general characteristics of microwave semiconductor oscillators with non sinusoidal waveforms and, in particular, to study the conditions for stability of such oscillators. Single and multiresonant circuits are considered experimentally with the L.S.A. mode being the main point of interest. It is clear from the results that the dependence of the diode power and conductance upon the fundamental voltage amplitude for a multiresonant circuit is quite different compared with a pure sinusoidal oscillator in a single resonant circuit. Similar results are obtained using the describing function technique which proves to be very useful in the analysis and synthesis of circuits for optimum properties of the system.

The study on stability allowing for the presence of harmonic components has led to a generalization of the Kurokawa conditions. The new criteria are more restrictive than the condition of stability for pure sinusoidal oscillators. The describing function technique provides the necessary data to plot the device line and circuit admittance loci and a graphical interpretation of the stability conditions at each frequency component is presented.
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LIST OF PRINCIPAL SYMBOLS

\( B_n \) circuit susceptance at the \( n^{th} \) harmonic

\( C_d \) diode capacitance

\( \frac{dv}{dE} \) differential mobility

\( E_A \) field constant in the analytical \( v - E \) characteristic of GaAs

\( E_B \) d.c. bias electric field

\( E_{th} \) threshold electric field value

\( f \) frequency

\( F(s) \) transfer function

\( G \) average diode conductance

\( G_d \) LSA diode conductance

\( G_L \) load conductance

\( G_o \) low field conductance

\( I_e \) injected signal

\( I_n \) current amplitude at the \( n^{th} \) harmonic

\( I_{th} \) threshold current

\( \ell \) active layer length

\( L(j\omega k) \) transfer function of the linear part of the system

\( L_L(s) \) transfer function of the linear part of the system with neither zeros nor poles at the origin

\( n \) free carrier density

\( n_I \) peak to valley current ratio

\( N_{k\omega} \) multiple input sinusoidal describing function

\( P_i \) poles

\( Q \) quality factor of a resonant circuit
q  electronic charge
R_d  d.c. device resistance
T  total oscillation period
T_{ij}  transmission I-V matrix coefficients
t_p  subthreshold portion of the oscillation period
v  average carrier velocity
v_s  high field carrier saturation velocity
V_A  voltage constant in the analytical I - V characteristic of GaAs
V_B  bias voltage
V_n  voltage amplitude at the n^{th} harmonic
V_{th}  threshold voltage
W  iris width
w(t, \xi)  weighting function
Y_0  characteristic admittance
-\bar{Y}(V)  diode admittance
-\bar{Y}(V_q)  operating point of the device
Y(\omega)  circuit admittance
z_i  zeros
Z_o  characteristic impedance
\epsilon  permittivity of GaAs
\lambda  wavelength
\mu_p  positive differential mobility
\mu_n  negative differential mobility
\mu_o  low field mobility
\psi  angle of intersection between device line and circuit admittance locus
\tau_d  time constant determined by the circuit inductance and the load conductance
\( \tau_p \) positive dielectric relaxation time constant

\( \tau_n \) negative dielectric relaxation time constant

\( \omega \) angular frequency

\( \omega_r \) resonant frequency of the parallel circuit
CHAPTER I

INTRODUCTION

The discovery of the "Transferred Electron Effect" by J.B. Gunn was the starting point of the microwave semiconductor oscillator technology with all its potential applications to communications and radar. Within the last few years microwave semiconductors for signal generation and amplification have divided out in six different technological directions: T.E.O., Impatts, Trapatts, Baritts, Bipolar Transistors and F.E.T's.

During the last decade substantial efforts have been directed in the development of practical oscillators and amplifiers incorporating Gunn diodes as the active element. The semiconductor materials generally used for fabricating microwave devices are silicon for avalanche junction diodes, gallium arsenide for transferred electron devices and more recently indium phosphide also for Gunn effect. Of the three, silicon technology is by far the most advanced because process control is superior, devices are reproducible and several fabrication techniques are available. However GaAs technology has developed quite fast during the last few years. Unlike Si, GaAs permits true semi insulating substrates to be grown for devices that operate at microwave frequencies.

Several modes of oscillation are possible depending upon the internal field distribution within the sample. A GaAs T.E. device may operate in any of the three possible modes: the Gunn or transit time mode, the quenched domain mode, or the limited space charge accumulation (L.S.A.) mo-
The most interesting mode of oscillation is the L.S.A. because the most power can be developed without the limitations inherent in the other oscillation modes. Continuous wave GaAs devices can produce output powers as high as 2 W in X band and can operate at frequencies of almost 100 GHz. For pulsed GaAs it is possible to obtain output powers of the order of 6 kW at relatively low duty cycles from L.S.A. mode devices in L band, dropping to about 2 kW in C band (1).

Device-circuit interaction in relaxation oscillators is not fully understood and no satisfactory explanation has been found for some phenomena observed when working with this type of oscillator. The aim of this thesis is to give a better understanding of negative resistance oscillators which have non sinusoidal voltage waveforms and derives the modifications brought about by the harmonic frequency components, in particular when the stability of the operating point is concerned. The use of describing functions, a technique in control theory not widely used due to the analytical complexity, provides an elegant approach to this problem. For this purpose the circuit and active device are studied independently although they are related by the operating conditions.

This thesis may be grouped in four main sections. The first section consisting of Chapters II, III and IV gives an introduction to the transferred electron effect, a study of the operating conditions and experimental observation of the different waveforms for the current and voltage of the L.S.A. relaxation oscillator. No attempt was made to give a detailed theoretical background on the transferred electron effect as this can be found in several books specialized on this topic.

The second section has two main objectives: the detailed analysis of the describing function technique and feedback model of the system used in the theoretical study of the device circuit interaction and to provide
a better understanding of the usefulness of this technique and its degree of accuracy. For this purpose the describing function technique is applied to a non linear device represented by an I-V cubic characteristic and the results are compared with those obtained using more conventional techniques such as the perturbation method. These two objectives are covered by Chapters V and VI respectively.

In the third section, formed by Chapters VII and VIII, experimental and theoretical results for both single and multiresonant circuits are presented and the synthesis of different lumped element circuits in order to obtain possible stable operating points is shown to be feasible within certain limitations. The fourth and final section, Chapter IX, deals with the discussion of the results and some possible applications of the analytical method used in this thesis are outlined.

Finally, Appendix I gives a comprehensive description of the iterative method used in the computer calculations and a full listing of the program in Fortram IV is also included.
A review of the transferred electron effect and different modes of operation is presented, emphasizing in particular the L.S.A. mode and the relaxation oscillator operation.

II - 1 Introduction to the Transferred Electron Effect.

The possibility of obtaining a bulk semiconductor microwave negative resistance was first predicted theoretically by Ridley and Watkins (2) and Hilsum (3) and although several attempts were made to obtain a negative resistance without the intervention of a p-n union or similar interface, the first experimental evidence of such behaviour was found by Gunn (4) in 1963.

The mechanism to be described has been observed in other materials such as InP, InAs, etc, however at present GaAs is the only material used commercially to make devices, and is the material used throughout all our experimental and theoretical work. Therefore the discussion will be restricted to n type GaAs. The \( \varepsilon - k \) diagram for GaAs is shown in Figure (2.1) and is such that there are satellite valleys in the \( <100> \) and \( <111> \) directions that are above the bottom of the central conduction band valley. Recent experimental and theoretical studies by Vinson et al (5) on the influence of hydrostatic pressure and uniaxial stress on transferred electron effects in GaAs show that a good agreement was found by assuming
Figure (2.1) shows the band structure of GaAs at T = 300 °K. The conduction and valence bands are depicted with specific energy levels and effective masses.

- The conduction band at point $\Gamma$ has $m^* = 0.067 m_0$.
- The valence band at point $\Gamma$ has $\epsilon' = 1.43\text{ eV}$.
- The conduction band at point $X$ has $m^* = 0.85 m_0$.
- The difference in energy between $\Gamma$ and $X$ is $\Delta \epsilon = 0.38\text{ eV}$.
that the L valleys are below the X valleys and have a weaker coupling to the \( \Gamma \) minima. The important point to note about this band structure is that GaAs has an upper valley in the conduction band which has an effective mass that is bigger than that of the lower valleys, as indicate by the curvature - effective mass relation:

\[
m^* = \frac{\hbar^2}{\frac{2}{\frac{d\epsilon}{dk}}} \tag{2.1}
\]

The low field mobility of actual material ranges from 6000 to 9000 \( \text{cm}^2 / \text{v.sec.} \). At a field near 3 \( \text{kv.cm}^{-1} \), the hot electrons (600 \( ^\circ \)K) are scattered into the satellite valleys by polar optical phonons. As the field is increased, the number of electrons in the satellite valleys increases until the number of electrons scattered into these valleys equals the number of electrons back to the central valley by the relaxation process. However Frolich (6) demonstrated that in a single conduction valley there is a maximum value of energy loss by electron-optical phonon interaction and the mean electron energy will start to increase above the value of the lattice for fields above a critical value.

As the electron temperature is raised, electrons with energies equal to the sub-band minimum will transfer because of the higher density of states available in the satellite valleys and the device will present a negative differential mobility because of the decrease of the carrier mobility with increasing electric field. The critical electric field at which \( \frac{dv}{dE} \) changes sign is referred as the threshold field \( E_{th} \) and is determined by the gap between the bottoms of the two valleys and the value of rate of transfer of electrons. It is important that the energy di-
ference between valence band edge and the lowest point of the conduction band is significantly larger than the energy difference between the conduction band minimum or impact ionization will result.

It could be thought that if there is a negative resistivity then there must be a negative resistance. The incorrectness of this argument was first demonstrated theoretically. Ridley suggested that a high field region spontaneously formed to keep the entropy a minimum while Shockley suggested that the field becomes high towards the anode. Both were partially correct, but it was Gunn who observed current fluctuations in bulk GaAs when biased with a constant electric field between 2 - 4 kV/cm and his work gave a better understanding of the nature of a negative resistivity. Following experiments, particularly those by Gunn and Hutson verified that the "Gunn effect mechanism" was that correctly predicted by Ridley and Watkins model.

Once the effect was established the efforts were directed at determining the exact shape of the v-E characteristic. One of the most complete studies of this characteristic has been made by Butcher and Fawcett. The study is based on the Boltzman collision equation together with the detailed scattering probabilities determined from considerations of the band structure for GaAs and the appropriate quantum mechanical formula.

Ruch and Kino experimentally determined the v-E characteristic by injecting electrons into insulating GaAs using an electron beam and observing the drift current as a function of applied field. These results were in good agreement with the theoretical approach by Butcher and Fawcett. Gunn and Elliott tried using very short voltage pulses, in the order or 30 picoseconds in length. It was suggested that this should give a very short time for the field to redistribute itself.
FIG. (2.2) VELOCITY - FIELD CHARACTERISTIC OF GaAs at 300° K
The results of Gunn and Elliott in particular suggested a very low value of the negative differential mobility compared to the theoretical values of Butcher and Fawcett.

Figure (2.2) shows the typical v-E characteristic for GaAs which exhibits approximately an ohmic law field behaviour up to a threshold field $E_{th}$. Above this value electrons will transfer to the upper valleys resulting in a decrease of the average carrier velocity for $E > E_{th}$. For very high fields, above $E = 25 \text{ kV} \cdot \text{cm}^{-1}$ further scattering mechanisms saturate the carrier velocity to a value of $\approx 10^7 \text{ cm} \cdot \text{sec}^{-1}$.

II - 2 Domain and Accumulation Layer Modes

A region of high electric field or domain as it is called is formed due to the change of charge distribution from its original state. A local imperfection in the semiconductor where the electric field is slightly higher will nucleate an accumulation of charge followed by a depletion region due to the difference of velocities.

A domain is usually formed near the cathode because this region often has the greatest non uniformity owing to crystal damage occurring in the process of making the contacts. The domains drift at a velocity of approximately the saturation velocity, due to the high field region until they reach the anode. As the domain enters the anode, the absorbed domain potential is redistributed into the field outside the domain. Gunn observed that the frequency of current oscillations was inversely related to the length of the GaAs sample. Both Heeks and Gunn showed that the velocity of the domain decreases slightly with increasing domain voltage. The theoretical study of Ridley was confirmed by Gunn by further experimentation using a fine capacitive probe, revealing the
changes in voltage along the sample. These experiments established that if the product $n \cdot \lambda$, where $n$ is the free carrier density and $\lambda$ the active layer length, is sufficiently high i.e. $n \cdot \lambda > 10^{11}$ cm$^{-2}$ the domains reached a stable amplitude.

Domain formation leads to a drop in the device current and the electric field outside the domain is below threshold value due to the potential absorbed by the domain. When the carriers' velocity in the high field region equals that in the rest of the sample, the domain continues to travel towards the anode but ceases to grow and the amplitude remains stable. The current raises again to threshold as the domain leaves at the anode and a new domain is formed once more at the cathode. Thus the period of current oscillations is given by

\[ T = \frac{\lambda}{v_s} \]  

(2.2)

where $v_s = 10^7$ cm.sec$^{-1}$ carrier saturation velocity

$T =$ Transit time.

This mechanism is the basic principle of the so called "Domain Transit Time Mode". Although experiments show that the nucleation point of travelling domains occurs near the cathode contact, the exact explanation of cathode nucleation is not clear. Carroll suggests that energy transport and diffusion may play a part in the formation of the domain near the cathode.

When a constant voltage is applied to a sample in which $E = 0$ and $n = n_0$ at each end, Poisson's and charge continuity equations predict an excess of carriers at the cathode. This excess of charge is called an accumulation layer. As the field increases most of the layer separates from the cathode and moves along the sample. The electric field is not the
same at both sides of the layer, being lower than threshold between the cathode and the layer and higher than threshold between the layer and the anode. In general the velocity of the layer is not constant and the layer grows until it reaches the anode and a new accumulation layer is formed.

Unlike the domain, the accumulation layer is not stable in an inhomogeneous medium. The accumulation layer will become a dipolar domain if sufficient time is allowed. This was suggested by Kroemer (17), McCumber and Chynoweth (18) in their theoretical attempts to simulate stable oscillating solutions with GaAs crystals when biased above $E_{th}$ and assuming the doping profile was uniform. This type of oscillation was not observed experimentally because any spatial doping fluctuation or non uniformity would give place to oscillations and the system would be unstable. McCumber and Chynoweth demonstrated by simulation that with a doping variation as small as one part in $10^5$ over a length of 1 μm a depletion layer will form ahead of the accumulation layer and a dipole domain might be generated.

A complete analysis and discussion of the different Gunn modes of operation may be found in some specialized books such as those by Carroll (16), Hobson (19) and Chaffin (20) among others and therefore we will restrict this study to the Limited Space Charge Accumulation (L.S.A.) Mode which is the operation mode selected in this thesis.

II - 3 Limited Space Charge Accumulation Mode . ( L.S.A.)

A new mode of operation of bulk negative resistance devices was predicted by Copeland (21) in computer simulations of a bulk nega-
the resistance device operated in series with a parallel tuned circuit and load conductance as shown in Figure (2.3).

For GaAs diodes the LSA mode has been observed only when the bias voltage is more than twice the threshold voltage for Gunn oscillation. In this mode of operation the frequency of oscillation is determined by the circuit and is higher than the frequency of oscillation of the transit time domain mode. The output power and efficiency are higher than when the same device is operating in the transit time mode. When the diode is oscillating in the LSA mode, the electric field across the diode varies from below threshold value to a value more than twice \( E_t \), so quickly that the space charge distribution due to a high field domain does not have time to form. During the portion of the cycle when the electric field falls below threshold, the relaxation constant is positive and this ensures the redistribution of the accumulation layer near the contact which started building up during the rest of the cycle when the relaxation constant was negative.

The solution of the non linear differential equations describing the system provides the true current and voltage relationship of the oscillator. For steady state oscillation the parallel load conductance presented to the diode must equal the absolute value of the negative resistance of the diode. Figure (2.4) shows the \( v-E \) characteristic with the assumed sinusoidal voltage swing across the device terminals. LSA operation is achieved by the proper design and matching of both the diode and the circuit. For the inhibition of any domain growth we must assume that the doping uniformity is good.

The dynamic \( I-V \) characteristic is to a first approximation a direct scaling of the \( v-E \) characteristic, because of the low growth of the accumulation layer and short distance of propagation, with com-
SINUSOIDAL LSA OSCILLATOR CIRCUIT USED BY COPELAND (21)

FIG. (2.3)

SPACE CHARGE DECAY

SPACE CHARGE GROWTH

SCHEMATIC DIAGRAM OF LSA MODE OPERATION

FIG. (2.4)
plete space charge decay during each cycle. The device will behave as a broad band negative resistance and high values for the efficiency could be achieved. The I-V characteristic of an LSA diode leads to a complex solution of the equations defining the system, due to the non-linear relation between the current and the voltage. Several authors, Bott and Hilsum (22) and Ikoma and Yanai (23) consider a piecewise linear approximation for the v-E characteristic, while Hines (24) uses a digital computer feeding in a large number of points from the curve. Although the first approach has advantages for numerical calculations, boundary conditions should be carefully imposed at the discontinuities. The analytical method used in this thesis and based in control theory allows us to use the I-V characteristic used by Thim (25) and others, although the solution, including Fourier analysis, requires the use of a computer.

The analytical expression for the v-E characteristic is:

\[ v(t) = \frac{\mu_0 E(t) + v_s \left[ \frac{E(t)}{E_A} \right]^{1/4}}{1 + \left[ \frac{E(t)}{E_A} \right]^{1/4}} \]  

(2.3)

where \( \mu_0 \) and \( v_s \) represent the low field mobility and high field carrier saturation velocity respectively. \( E_A \) is a constant field which fixes the threshold field value.

In a first order theory we can consider that there is no space charge accumulation over a complete cycle, covering both positive and negative differential mobilities. This condition is given by

\[ \frac{1}{T} \int_0^T \frac{dv}{dE} \, dt > 0 \]  

(2.4)
where $T = \frac{1}{f}$, $f$ being the oscillation frequency. This is a necessary condition but not sufficient. To allow operation in LSA, the space charge growth during the time spent in the region of negative differential mobility must be small enough so that the electric field inside the sample does not experience too much distortion. Bott and Hilsum allow for this, making arbitrary assumptions about the amount of growth that can be permitted during the time spent in the region of negative mobility.

Dividing the period into two sections, the integral of equation (2.4) can be written as

$$\int_{0}^{T_p} \mu_p \, dt + \int_{T_p}^{T} \mu_n \, dt > 0 \quad (2.5)$$

where we have assumed constant values for the average differential mobilities $\mu_p$ (positive) and $\mu_n$ (negative) during the intervals $0 \rightarrow t_p$ and $t_p \rightarrow T$ respectively. The charge conservation and Poisson's equation can be combined to first order in $\sigma$ to give

$$\frac{\partial n}{\partial t} = -n \mu \frac{\partial E}{\partial t} \quad (2.6)$$

$$\frac{\partial E}{\partial x} = (n - n_0) \frac{q}{\epsilon} = \frac{\sigma}{\varepsilon} \quad (2.7)$$

$$\frac{\partial \sigma}{\partial t} = -n_0 \frac{\mu q}{\varepsilon} \sigma \quad (2.8)$$

The general solution of equation (2.8) is the normal dielectric relaxation equation,

$$\sigma = \sigma_0 e^{-\frac{t}{\tau}} \quad (2.9)$$
where

\[ \tau = \frac{\varepsilon}{n_0 \mu q} \quad (2.10) \]

So, the relative charge growth and decay as a function of time will be given by

\[ \sigma_t = \sigma_o e^{-\frac{n_0 e}{\varepsilon} \frac{dE}{dE} t} \quad (2.11) \]

We see that charge accumulation occurs during the time \( t_p \rightarrow T \) when the electric field is above threshold and the mobility is negative \( (\mu_n) \) and therefore \( \tau \) is negative. If the electric field must be uniform along the sample, that is the accumulation charge formed is very small \( \tau_n \) must be large with respect to the period \( T \),

\[ \frac{1}{\tau} < \left| \frac{e}{n e \mu_n} \right| \quad (2.12) \]

Any accumulation charge formed during \( \tau_n \) is redistributed during the interval \( 0 \rightarrow t_p \), when the relaxation constant is positive \( (\mu_p > 0) \),

\[ \tau_p = \frac{\varepsilon}{n e \mu_p} \quad (2.13) \]

\( \tau_p \) must be shorter than the period to ensure that any charge accumulated is dissipated

\[ \frac{1}{\tau} > \frac{e}{n e \mu_p} \quad (2.14) \]
Equations (2.12) and (2.14) lead to the conclusion that for LSA operation of a good quality n-GaAs diode at room temperature, the ratio of doping to frequency should be within the range

\[ 2.10^{3} \leq \frac{n}{f} \leq 2.10^{5} \text{ sec.cm}^{-2} \quad (2.15) \]

Experimental observation by Copeland\(^{(26)}\) and Kennedy and Eastman\(^{(27)}\) showed a good agreement with this range of values. The theoretical upper frequency limit on LSA operation is related to the electron intervalley scattering time which might be significant in comparison with the period of oscillation.

The calculations presented by Copeland assumed that \( v(E) \) is independent of the speed of variation of \( E \). This will not be true for n-GaAs at frequencies where the period is only several times longer than the intervalley scattering time. This time is considered to be in the region \( 10^{-12} \) to \( 2.10^{-12} \) sec. Rees\(^{(28)}\) has shown that this will put an effective limit to the frequency of LSA at approximately 40 GHz.

Since LSA is not possible for small amplitudes of rf electric field \( E \), large signal calculations must be used to study the behaviour of a diode starting in an unexcited circuit. Copeland\(^{(26)}\) found his experimental LSA oscillator began in transit time mode which switched after several cycles to LSA operation at the circuit resonant frequency. To start LSA mode, the parallel load conductance must be lower than is desirable for maximum efficiency. The initial oscillations are due to some sort of domain and in long samples there will be a high probability of avalanche breakdown before the space charge accumulation is controlled and LSA field oscillations are established. Once the device has started to oscillate and the signal has been reflected in the
different tuning elements of the circuit, the load presented to the diode changes, due to the impedance transformation with the distance along the waveguide. The impedance presented to the diode is related to the time that has passed before the voltage amplitude across the diode falls below threshold voltage.

The idea is to allow the peak rf amplitude to build up quickly so that the oscillation will quickly swing below threshold to prevent domain formation. Spiwack\(^29\) and Kennedy\(^30\) developed several circuit configurations to quick start LSA oscillations. One of the most practical solutions is to connect the circuit inductance in series with the device and the bias. This results in a non sinusoidal waveform relaxation oscillator which has an extremely short starting time and the presence of harmonics in the rf voltage leads to higher values of efficiency compared with those for a pure sinusoidal rf voltage across the diode as suggested by Copeland.

II - 4 LSA Relaxation Oscillator Operation

The term relaxation oscillation is generally applied to an oscillator with a waveform displaying an asymptotic behaviour over a considerable portion of the cycle followed by a discontinuous jump to a new value. This aperiodic portion of the cycle is a direct function of some relaxation time. This type of oscillation is important for successful LSA operation because there is almost no build up time from the start of the oscillation until the amplitude reaches the proper value. In this way the time when space charge occurs is minimized. A natural consequence of the multiharmonic waveform is that the oscillation fundamental frequency is no longer equal to the circuit reso-
nant frequency $f_o$

$$f_o = \frac{1}{2\pi} \frac{1}{\sqrt{L/C}}$$

(2.16)

as shown by Grozskowski$^{(31)}$ and van der Pol$^{(32)}$. A relaxation waveform contains harmonics which have a firm influence in the efficiency as it will be shown later.

A simple equivalent circuit model of an LSA relaxation oscillator with typical values for X band is shown in Figure(2.5). The bias voltage $V_B$ is chosen so that the diode is biased into the negative slope of the I-V characteristic, that is

$$V_B = N \cdot E_{th} \cdot \lambda$$

(2.17)

where the constant $N$ is in the range $1 < N < 10$. The bias supply is fed through the series inductance to the LSA diode and the circuit load conductance $G_L$ which are connected in parallel. The diode element is represented by the parallel combination of the self capacitance $C_d$ and the non linear conductance $G_d$. Figure (2.6) shows the variation of $G_d$ as a function of voltage $V$. When the bias is applied the voltage across the diode initially at zero rises almost exponentially towards $V_B$ with a time constant $\tau_d$ determined by the circuit inductance and the load conductance

$$\tau_d = L \left( G_L + G_d \right)$$

(2.18)

Below threshold $G_d$ is positive and almost constant. However as $V(t)$ approaches $V_{th}$, $G_d$ falls to zero and becomes negative for $V(t) > V_{th}$.
A SIMPLE LUMPED ELEMENT EQUIVALENT CIRCUIT OF AN LSA OSCILLATOR

FIG. (2.5)

ANALYTICAL DIFFERENTIAL CONDUCTANCE - VOLTAGE FUNCTION OF THE LSA DIODE

FIG. (2.6)
Consequently as the voltage equals $V_{th}$ the current through $G_d$ drops sharply. The supply current can not change very quickly because of the series inductance and so the capacitive current increases causing the voltage across the device to rise rapidly. For high voltages $|G_d|$ will fall below $G_L$ and $V(t)$ will reach a maximum value and then decay towards $V_B$. The process now reverses as decreasing $V(t)$ causes an increase of $G_d'$. As before the series inductance restricts the rapid change of supply current and so $V(t)$ is forced down quickly by the capacitive current which flows now in the opposite direction.

The effect of the small series inductance is to cause a very rapid change of voltage through the negative portion of $G_d'$. A transient voltage waveform of the circuit shown in Figure (2.5) is shown in Figure (2.7) which was obtained using a numerical technique to solve the non linear differential equation describing the voltage $V$ as a function of time. The details of this analysis are given in Chapter III. This solution is only valid for true LSA operation because the I-V dynamic characteristic is assumed to be a direct scaling of the instantaneous velocity-field characteristic shown in Figure (2.2). The values corresponding to the ratio $\frac{n}{f}$ within the limits for LSA operation are valid in this case. However with non sinusoidal waveform, the range is extended slightly because of the portion of the oscillation period that the device is below threshold is longer than for single frequency operation and the time that the voltage is in the high negative differential mobility region is reduced.

Although Copeland (33) studied the influence of the harmonic components on the efficiency, the best experimental values were considerably lower than those predicted theoretically. Copeland (26) and Thim (25) studied the effect of spatial variations of doping density in the LSA.
\[ V(t) \sim T, \quad G_L = 0.001 \text{ mho}, \quad V_B = 3 V_{th} \]

**FIG. (2.7)**

**CALCULATED SOLUTION OF** \( V(t) \sim T \)** FOR FIGURE (2.5)**
diode. That could give an explanation for the degradation of the efficiency, but did not explain that the best experimental efficiencies were normally obtained with higher values for \( \frac{n}{f} \) and the voltage waveform contained more components than the fundamental. Reynolds et al.\(^{(34)}\) obtained an experimental efficiency around 30\% at L band frequencies with values for \( \frac{n}{f} \) in the range \( 2 - 5 \times 10^5 \text{ sec.cm}^{-3} \). Jeppesen and Jeppsson \(^{(35)}\) have done computing simulations of LSA operation in real time using a distributed element transmission line suggesting that successful LSA operation should occur for doping to frequency ratios in the range \( 1 - 5 \times 10^5 \text{ sec.cm}^{-3} \).

The frequency of operation is in general a function of the bias voltage as this controls the rate of rise of the asymptotic portion of the cycle. In the case of LSA model, the period of oscillation is given by

\[
T = L \cdot G_0 \ln \frac{V_B}{V_B - V_{th}} + \tau \sqrt{L \cdot C} \quad (2.19)
\]

where \( G_0 = \frac{d}{dV} \) for \( V < V_{th} \). This expression for the period was proposed by Jeppesen and Jeppsson \(^{(36)}\). The first part of equation (2.19) involving the inductance and the low field conductance of the device is due to the exponential voltage rise from zero to threshold, approximately given by

\[
V(t) = V_B \left( 1 - e^{-\frac{t}{L \cdot G_0}} \right) \quad (2.20)
\]

The second term of equation (2.19) corresponds to the sinusoidal oscillation period of the parallel LC circuit. Assuming that equation (2.19) is valid for the period one can see the dependence of the frequency upon the bias voltage. A practical realization of the circuit of
Figure (2.5) for operation at microwave frequencies is extremely difficult to achieve as the circuit elements are distributed and are then frequency dependent. However it is possible to approach the design of a single resonant microwave circuit using a simple model of distributed elements as shown in Chapter VII.
CHAPTER III

THEORETICAL LSA RELAXATION OSCILLATOR. OPERATION CONDITIONS

AND STABILITY OF THE OPERATING POINTS

This Chapter presents a study of the operating conditions of a microwave oscillator, defining the device line, admittance locus of the associated circuit and the stability criteria for the operating points. A full analysis of the LSA relaxation oscillator is performed by computer simulation with discussion of the computed solutions as well as the Fourier analysis of the LSA relaxation waveforms.

III - 1 Operating Conditions

The most general configuration of an oscillator is shown in Figure (3.1) where the oscillator is represented by a waveguide cavity with a negative resistance element. The variable short circuit and the coupling probe allow tuning for maximum output power. Independently of the configuration of the system, the schematic diagram of the oscillator may be drawn as in Figure (3.2). The active element terminals determine the reference plane. $Y(\omega)$ is the admittance presented to the diode at the reference plane and $-\bar{Y}(V)$ represents the admittance of the diode and is a function of the voltage amplitude or current amplitude if we consider impedances ($-\bar{Z}(I)$). In general $-\bar{Y}(V)$ depends also upon the frequency and amplitudes of the different harmonics. We can assume at this stage that the variation of $-\bar{Y}(V)$ with $\omega$ is very
SCHEMATIC DIAGRAM OF A MICROWAVE OSCILLATOR

FIG. (3.1)

EQUIVALENT CIRCUIT OF A FREE RUNNING MICROWAVE OSCILLATOR

FIG. (3.2)
slow compared with $Y'(\omega)$ and that the amplitude of the harmonic components are very small due to the filtering action of the resonant cavity. So the voltage across the diode may be written as:

$$v(t) = \text{Re} \left[ V_o e^{j(\omega t + \phi)} \right] \quad (3.1)$$

where $V_o$ is the amplitude and $\phi$ the phase. The current through the diode is given by

$$-Y(V) = -G(V) + jB(V) \quad (3.2)$$

$$i(t) = \text{Re} \left[ -Y(V) \cdot V_o e^{j(\omega t + \phi)} \right]$$

$$= -G(V) V_o \cos(\omega t + \phi) - B(V) V_o \sin(\omega t + \phi) \quad (3.3)$$

If the admittance presented to the diode is $Y(\omega) = G(\omega) + jB(\omega)$ the current through $Y(\omega)$ is given by

$$i'(t) = \text{Re} \left[ Y(\omega) \cdot V_o e^{j(\omega t + \phi)} \right]$$

$$= G(\omega) V_o \cos(\omega t + \phi) - B(\omega) V_o \sin(\omega t + \phi) \quad (3.4)$$

In steady state and with no external voltage applied we have,

$$i(t) = i'(t) \quad (3.5)$$

$$-G(V) V_o \cos(\omega t + \phi) - B(V) V_o \sin(\omega t + \phi) = G(\omega) V_o \cos(\omega t + \phi) - B(\omega) V_o \sin(\omega t + \phi) \quad (3.6)$$

Multiplying equation (3.6) by $\cos(\omega t + \phi)$ and $\sin(\omega t + \phi)$ respectively and integrating over a complete rf cycle and due to the orthogonality of the trigonometric functions, we have

$$\left[ G(\omega) + B(V) \right] V_o = 0 \quad (3.7)$$
If $V_o$ has a finite value, the operating point is defined then by

$$Y(\omega) = -\overline{Y(V)} \quad (3.9)$$

The same conditions would be derived when more than the fundamental component is present in the voltage across the diode. However the numerical analysis is far more complicated. The use of describing functions, to be introduced in Chapter V, simplifies the analysis and it provides a firmer basis to determine the number of harmonics to be considered in the input signal to the diode.

In the present analysis the active admittance seen looking into the cavity from the output port is a function of $\omega$ as well as the rf voltage amplitude and although the linear part is separated from the non linear part, it is very difficult to apply linear network theory to the complete system. So the determination of the reference plane at the device terminals is very important because once the voltage across the device terminals is known, the voltage at the oscillator output port can be easily obtained through a linear transformation of the rest of the circuit.

III - 2 Admittance Locus, Device Line and Operating Points

We have seen that oscillation in steady state is given by the condition of net conductance and susceptance zero. It is possible to represent in the complex plane $Y(\omega)$ and $-\overline{Y(V)}$ as a function of $\omega$ and $V$ respectively as shown in Figure (3.3). The locus of $Y(\omega)$ as a function
DEVICE AND CIRCUIT ADMITTANCE LINES

FIG.(3.3)
of \( \omega \) is called the circuit admittance or admittance locus and the arrow indicates the direction of increasing values of \( \omega \). The scale in the admittance locus represents equal increments in frequency.

The locus of \(-Y(V)\) as a function of \( V \) is called the device line. For a particular value \( V_0 \) of the rf voltage amplitude, the corresponding point \(-Y(V_0)\) is called the operating point of the device. The arrow in the device line indicates the direction of increasing values of \( V \). Also the scale in the device line corresponds to equal increments in the rf voltage amplitude. In steady state the operating point is determined by the intersection of the circuit admittance and the device line. The rf voltage amplitude \( V_0 \) and frequency \( \omega_0 \) can be read directly from the scales in both loci.

When more than the fundamental component is present in the voltage waveform across the diode we have to consider the different device lines and circuit admittances corresponding to all different harmonic components. The various intersections will provide the values of the amplitudes of the harmonic components of the rf signal.

III - 3 Stability of the Solutions

In order to calculate the instantaneous change in current through \( Y(\omega) \), let us consider the time derivative of \( v(t) \), where \( v \) and \( \phi \) are assumed to be slowly varying functions of time. From equation (3.1) we have

\[
\frac{dv}{dt} = -V (\omega + \frac{d\phi}{dt}) \sin(\omega t + \phi) + \frac{dv}{dt} \cos(\omega t + \phi)
\]

Equation (3.10) can be re-written as:
\[ \frac{dV}{dt} = \text{Re} \left[ V \left( j(\omega \frac{d\phi}{dt} + \frac{1}{V} \frac{dV}{dt}) e^{j(\omega t + \phi)} \right) \right] \quad (3.11) \]

The time derivative corresponds to multiplication by \( j\omega \). We can then replace \( j\omega \) by \( j(\omega + \frac{d\phi}{dt} - \frac{j}{V} \frac{dV}{dt}) \) and \( \text{Re} \left[ Y(\omega) V \right] \) will give us the instantaneous current through \( Y(\omega) \), where \( V \) is the expression for \( v(t) \) in the form \( V e^{j(\omega t + \phi)} \). Therefore,

\[
Y(\omega + \frac{d\phi}{dt} - \frac{j}{V} \frac{dV}{dt}) = Y(\omega) + \frac{dY(\omega)}{d\omega} \left( \frac{d\phi}{dt} - \frac{j}{V} \frac{dV}{dt} \right) \\
= G(\omega) + jB(\omega) + \left( \frac{dG}{d\omega} + j \frac{dB}{d\omega} \right) \left( \frac{d\phi}{dt} - \frac{j}{V} \frac{dV}{dt} \right) \quad (3.12)
\]

where we have assumed that:

\[ \frac{d\phi}{dt} << \omega \]

\[ \frac{1}{V} \frac{dV}{dt} << \omega \]

So,

\[
\text{Re} \left[ Y(\omega) V \right] = \left[ G(\omega) + \frac{dG}{d\omega} \frac{d\phi}{dt} + \frac{dB}{d\omega} \frac{1}{V} \frac{dV}{dt} \right] V \cos(\omega t + \phi) + \\
\left[ -B(\omega) + \frac{dG}{d\omega} \frac{1}{V} \frac{dV}{dt} - \frac{dB}{d\omega} \frac{d\phi}{dt} \right] V \sin(\omega t + \phi) \quad (3.13)
\]

The condition of operation is given by equation (3.5) and multiplying by \( \cos(\omega t + \phi) \) and \( \sin(\omega t + \phi) \) respectively and integrating over one period of oscillation we have:

\[
G(\omega) + \frac{dG}{d\omega} \frac{d\phi}{dt} + \frac{dB}{d\omega} \frac{1}{V} \frac{dV}{dt} = -\overline{G}(V) \quad (3.14)
\]

\[
-B(\omega) + \frac{dG}{d\omega} \frac{1}{V} \frac{dV}{dt} - \frac{dB}{d\omega} \frac{d\phi}{dt} = -\overline{B}(V) \quad (3.15)
\]
Equations (3.14) and (3.15) are the basic equations for the amplitude and phase of the oscillating voltage. For a steady state free running oscillation $\frac{dV}{dt} = \frac{d\phi}{dt} = 0$ and equations (3.14) and (3.15) become

\[ G(\omega) + \bar{G}(V) = 0 \quad \text{(3.16)} \]

\[ -B(\omega) + \bar{B}(V) = 0 \quad \text{(3.17)} \]

equivalent to equation (3.9) defining the operating point.

To study the stability of the operating point, let us consider a small variation of the voltage amplitude around its steady state value $V_o$,

\[ V + V_o + \delta V \quad \text{(3.18)} \]

\[ \bar{G}(V) \rightarrow \bar{G}(V_o + \delta V) = \bar{G}(V_o) + \frac{3G}{3V} V_o \delta V \quad \text{(3.19)} \]

\[ \bar{B}(V) \rightarrow \bar{B}(V_o + \delta V) = \bar{B}(V_o) + \frac{3B}{3V} V_o \delta V \quad \text{(3.20)} \]

Equations (3.16) and (3.17) become

\[ G(\omega) + \bar{G}(V_o + \delta V) = G(\omega) + \bar{G}(V_o) + \frac{3G}{3V} V_o \delta V = 0 \quad \text{(3.21)} \]

\[ -B(\omega) + \bar{B}(V_o + \delta V) = -B(\omega) + \bar{B}(V_o) + \frac{3B}{3V} V_o \delta V = 0 \quad \text{(3.22)} \]

Eliminating $\frac{d\phi}{dt}$ from equations (3.14) and (3.15) we obtain:

\[ G \frac{3B}{3\omega} + \bar{G} \frac{3B}{3\omega} - B \frac{3G}{3\omega} + \bar{B} \frac{3G}{3\omega} + \frac{1}{V} \frac{3V}{3t} \left[ \left( \frac{3G}{3\omega} \right)^2 + \left( \frac{3B}{3\omega} \right)^2 \right] \]

\[ = \frac{3B}{3\omega} (G+\bar{G}) + \frac{3G}{3\omega} (\bar{B} - B) + \frac{1}{V} \frac{3V}{3t} |\dot{\phi}(\omega)|^2 \]

\[ = 0 \quad \text{(3.23)} \]

For a small variation in the voltage amplitude, $V_o + \delta V$,,
equation (3.23) determines the differential equation for $V$. Substituting equations (3.21) and (3.22) in equation (3.23) and for a first order approximation we have,

$$\frac{dB}{d\omega} \left( - \frac{dG}{dV} \right) \delta V + \left( - \frac{dF}{dV} \right) \delta V \frac{dG}{d\omega} + \left| \hat{I}(\omega) \right| \frac{2V}{V} \frac{dV}{dt} = 0 \quad (3.24)$$

If the operating point is stable, $V$ decays with time, so in order to satisfy equation (3.24) it is necessary that:

$$- \frac{dB}{d\omega} \frac{dG}{dV} - \frac{dF}{dV} \frac{dG}{d\omega} > 0 \quad (3.25)$$

Equations (3.16) and (3.17) determine the operating point and this is stable if and only if satisfies the condition of equation (3.25). Kurokawa\(^{(37)}\) presents a complete discussion of the behaviour or pure sinusoidal oscillators with multiple resonant circuits and derives the condition of stability considering an injecting signal in the circuit which would be suitable for injection locking. Kenyon\(^{(38)}\) provides a graphical interpretation for equation (3.25) which is easier to apply. Let $\psi$ to be defined as in Figure (3.3). Stable oscillation at point P is only possible if $0 < \psi < \pi$. We will see that when $\psi \rightarrow 0$ the noise in the oscillation will increase considerably.

### III - Analytical Techniques to Obtain the Operating Point

The value of $V_o$ and $\omega_o$ may be obtained either by solving the differential equation of the system or by graphical representation of the device line and admittance locus of the circuit. The device line is not easy to obtain, specially when harmonics are taken into consideration, however more information is available from the point of view of
studying the stability of the operating point, being even possible to predict new operating points. Control theory provides very useful tools to study the stability of the system, but in order to apply it we must consider carefully the presence of non linear elements in the system, such as the active device. Once the system has been quasilinearized the characteristic equation of the system gives the operating point and also data concerning the device line.

Let us consider the simple model of lumped element circuit for an LSA oscillator of Figure (2.5). The diode conductance $G_d$ is assumed to be a direct scaling of the $V$-$E$ characteristic. Taking the analytical approximation for the $V$-$E$ function used previously as equation (2.3) and scaling this to correspond to a device of low field conductance $G_o = 0.2 \text{ mho}$, as used in these experiments, the device $I$-$V$ characteristic is given by

$$I = G_o \frac{V + \mu_o \frac{\ell}{V_s} \left( \frac{V}{V_A} \right)^4}{1 + \left( \frac{V}{V_A} \right)^4}$$

(3.26)

where $\ell$ is the active device length ($=110\mu$m), $\mu_o$ is the low field mobility $(8.3 \times 10^3 \text{ cm}^2\text{Volt}^{-1}\text{sec}^{-1})$, $v_s$ the saturation velocity $(85000 \text{ m.sec}^{-1})$ and $V_A = E_A \cdot \ell$ where $E_A$ is a field constant of $4 \text{ KV.cm}^{-1}$. The $I$-$V$ characteristic and conductance as a function of voltage for these values of the parameters are shown in Figures (3.4) and (3.5) respectively. The threshold voltage for this characteristic is $V_{th} = 30 \text{ V}$.

Kirchhoff's law for the circuit gives the differential equation for the voltage across the diode:
L.S.A. I-V CHARACTERISTIC

\[ I = G_0 \frac{\frac{2v_s}{\mu_0} \left[ \frac{V}{V_A} \right]^4}{V + \frac{2v_s}{\mu_0} \left[ \frac{V}{V_A} \right]^4} \]

\( \mu_0 = 0.83 \text{ m}^2/\text{sec.} \)

\( v_s = 85000 \text{ m/sec.} \)

\( G_0 = 0.2 \text{ mho} \)

\( V_A = E_A \cdot \lambda \) \((E_A = 4 \text{ KV})\)

\( \lambda = 119 \text{ \mu m} \)
Differential Conductance - Voltage Function

of the LSA I - V Characteristic of Fig. (3.4)
Substituting $I = G_d V$ in equation (3.27) we obtain the basic non linear differential equation of the circuit,

$$\frac{d^2 V}{d\theta^2} + \sqrt{\frac{L}{C}} \left( G_L + \frac{dI}{dV} \right) \frac{dV}{d\theta} + V - 1 = 0 \quad (3.28)$$

where $V$ has been normalized to $V_B$ and the variable $\theta$ is related to time $t$ by

$$\theta = \frac{t}{\sqrt{LC}} \quad (3.29)$$

and $\frac{dI}{dV}$ is given by,

$$\frac{dI}{dV} = \frac{G_o}{1 - 3 \left( \frac{V}{V_A} \right)^4 + \frac{4}{\mu_0} \frac{V}{V_A} \left( \frac{V}{V_A} \right)^4} \left[ 1 + \left( \frac{V}{V_A} \right)^4 \right]^2 \quad (3.30)$$

The initial conditions are $\theta = 0, V = 0, \frac{dV}{d\theta} = 0$

The solutions we are interested in are those which will produce self starting stable oscillations when the switch is activated at $\theta = 0$. There are different methods to obtain these solutions. The first method we have tried is the method of isoclines, following the theory of Cunningham [39], to solve non linear equations with solutions in the $(V, \frac{dV}{d\theta})$ plane, which are transformed by integration into the $(V, \theta)$ plane obtaining a solution of the basic equation.

Equation (3.28) can be written as a first order differential equation in the plane $y=V$,

$$y \frac{dy}{dV} + \sqrt{\frac{L}{C}} \left( G_L + \frac{dI}{dV} \right) y + V - 1 = 0 \quad (3.31)$$
where the change of variable \( y = \frac{dv}{d\theta} \) has been used. The solution of this equation in the (\( y, V \)) plane tends to a limit cycle, which corresponds to the required stable solution. We can represent this equation for constant values of \( \frac{dv}{dV} \) if a linear relation between \( V \) and \( y \) over each small increment of \( V \) is assumed. The curve solution is formed by small sections corresponding to the isoclines with their slope. The solution of equation (3.31) is obtained selecting some initial values for \( V \) and \( y \) and joining the different segments of the curve solution in order to obtain a continuous curve. We have constructed the solution curve using the isocline method for the van der Pol equation, to illustrate the method,

\[
\frac{d^2x}{dt^2} - \epsilon (1-x^2) \frac{dx}{dt} + x = 0
\]  

(3.32)

This is the equation for an oscillatory system having variable damping. If the displacement \( x \) is small, the coefficient of \( \frac{dx}{dt} \) is negative and the damping is negative. If the displacement is large, the damping becomes positive. Equation (3.32) describes then quite well the behaviour of an electronic oscillator. The solution curve is shown in Figure (3.6) for \( \epsilon = 5 \) where only the half upper part of the limit cycle is shown. The solution curve of equation (3.32) rapidly tends to the limit cycle which is a characteristic of the relaxation oscillator, that is, the time for oscillations to build up is very short. Because the equation is not linear the isoclines are curves and not simply straight lines. The most interesting property of the solution for the van der Pol equation is that there is a particular closed solution that is ultimately achieved regardless of initial conditions. By relatively simple incremental integration, the solution curve in the time plane of equation (3.31)
ISOCLINE CONSTRUCTION & PHASE - PLANE DIAGRAM FOR VAN DER POL'S EQUATION FOR $\varepsilon = 5$. ONLY THE UPPER HALF PLANE & THE LIMIT CYCLE ARE SHOWN, THE FIGURE BEING SKEW SYMMETRIC IN THE LOWER HALF PLANE

FIG. (3.6)
is obtained. The isocline method is not easy to use and time consuming, but the main difficulty arises when any singularity exists, because the isocline method applies for all values of $V$ and $y$ except those that correspond exactly to singularities. For example the starting conditions of the oscillation for the circuit are $V = 0$ and $y = 0$, and they are not valid as initial conditions with this method because of the singularity at the origin, and further study of the equation is needed.

It is preferable to use a numerical method to solve the non linear differential equation. If we are interested in finding as much information as possible about the non linear system we may need to apply several methods of analysis. The perturbation method and the method of variation of parameters provide different approaches. The first method gives a steady state solution and it applies only after a steady state has been reached and there is a definite amplitude of oscillation, independently of the initial conditions. The type of solutions we are interested in are of the form

$$V = V_0(t) + a V_1(t) + a^2 V_2(t) + \ldots$$  \hspace{1cm} (3.33)

with the appropriate initial conditions. The initial voltage can not be chosen at will but must have a particular value. The perturbation method is used in the analysis of the cubic characteristic and compared with the describing function technique in Chapter VI.

The method of variation of parameters provides valuable information about the build up or decay of the oscillation and is also very useful for the analysis of the circuit properties for the fast build up of the oscillation. A full analysis of these methods is presented by Cunningham (39) and Raven (40).
It was decided to try the Runge-Kutta numerical method for solving the non-linear differential equation, in an effort to reduce the time taken to obtain the solution. Equation (3.31) can be re-written as a system of two first order differential equations which must be satisfied simultaneously,

\[ V' = p = f(V, \theta) \]  \hspace{1cm} (3.34)

\[ p' = 1 - V + \frac{I}{C} \left( G_L + \frac{dL}{dV} \right) p = f(p, V) \] \hspace{1cm} (3.35)

Basically the Runge-Kutta method can be applied to any first order differential equation of the form

\[ \frac{dx}{d\phi} = f(x, \phi) \]  \hspace{1cm} (3.36)

which is solved from the initial values of \( x = x_0 \) and \( \phi = \phi_0 \), on the assumption that between \( \phi = \phi_0 \) and \( \phi = \phi_0 + \Delta \phi \), \( f(x, \phi) \) may be represented by a Taylor series truncated after the term in \( (\Delta \phi)^4 \). The full details of the method may be found in standard texts on numerical analysis, such as Wilkes (41) and Daniel and Moore (42), and so have not been included in this thesis. The Runge-Kutta is particularly adaptable at the initial point because no historical values of the solution are required to compute subsequent values. The computer program for the solution of this equation was tested by solving the van der Pol equation solved before, for different values of \( \varepsilon \) and the results compared with the solutions published by van der Pol (32) using analytical and theoretical methods.

The numerical solutions of the set of equations for different values of the bias voltage and load conductance were obtained for initial conditions \( \frac{dV}{d\theta} = 0 \), \( V = 0 \), \( \theta = 0 \), increasing steps of \( \theta \) until the
FIG. (3.7)

COMPUTER SOLUTION OF EQUATION (3.28)

$V(t) = T, \quad V_B = 3V_{th}$

$G = 0.0 \, \text{mhos}$
the period and the amplitude of the oscillation were stable. In all
the calculations the values of \( L, C \) and \( G \) were kept constant being
1.6 nH, 0.1 pf and 0.2 mho respectively. Figure (3.7) shows the vol-
tage waveform across the diode for a bias voltage of \( V_B = 3 \text{ V} \thinspace \text{th} \) and
a load conductance of \( G_L = 0.0 \text{ mho} \). The typical asymmetric waveform
indicates the presence of higher harmonics, most significantly the
second harmonic and as a result the fundamental frequency of \( f_0 = 6.46 \text{ GHz} \) is lower than the resonant frequency of the parallel circuit
(\( \approx 12.33 \text{ GHz} \)). Once a solution was obtained and the voltage waveform
Fourier analyzed, the period of oscillation was compared with the
theoretical value from the expression given by Grozskowski \( ^{31} \);

\[
\sum_n n |V_n|^2 B_n = 0 \tag{3.37}
\]

Where \( n \) represents the number of harmonics. This equation can be
formulated to obtain the period \( T \) as a function of the parameters of
the circuit and the amplitude of the harmonic components,

\[
T = \frac{1}{f} = \sqrt{\frac{4\pi^2 LC \sum_{n=0}^{n=j} n^2 |V_n|^2}{\sum_{n=0}^{n=j} |V_n|^2}} \tag{3.38}
\]

where \( j \to \infty \). Generally \( j \) was limited to a value equal to 10. However
despite this low value the period calculated using equation (3.38)
was found to agree to within \( \approx 3\% \) of the period obtained from the
computed voltage waveform.
As discussed in Chapter II (II-4) increasing the bias voltage increases the oscillation frequency by reducing the portion of the cycle that the voltage is below threshold as shown in the waveforms of Figures (3.8) and (3.9) for $V_B = 5 \ \text{V}_\text{th}$ and $V_B = 8 \ \text{V}_\text{th}$ respectively. Jeppesen and Jeppsson (36) derive an expression for the period-bias dependence

$$T = L. G_o \ln \left( \frac{V_B}{V_B - V_{\text{th}}} \right) + \pi \sqrt{LC} = t_1 + t_2 \quad (3.39)$$

which was quoted before as equation (2.19). $t_1$ represents the time that the device voltage is below threshold, while $t_2$ corresponds to the time that the device is in the high field region, where the device is effectively a constant current source as shown in Figure (3.10). In the derivation of equation (3.39) the time the device is biased in the high negative conductance region is assumed to be very short in comparison with the rest of the cycle and therefore negligible. This assumption is valid for values of $G_L/G_o \ll 1$ and for moderately high values of the bias voltage, and under these conditions the expression provides a reasonable qualitative assessment of the bias tuning. However, for lower values of $V_B$, the rate of voltage transit through the region of high negative differential conductance is not insignificant in comparison with the rest of the cycle. To illustrate this point we have calculated the period of oscillation for different values of $V_B$ keeping the value of $G_L$ constant and equal to 0.005. $G_o$ and compared the results with the theoretical values obtained from equation (3.39). The graphical comparison is shown in Figure (3.11).
$V(t) - T, \ V_B = 5, \ V_{th}, \ G_L = 0.0 \ mho$

FIG. (3.8)

COMPUTER SOLUTION OF EQUATION (3.28)
$V(t) - T, \quad V_B = 8 \cdot V_{th} \quad G_L = 0.0 \text{ mho}$.

COMPUTER SOLUTION OF EQUATION (3.28)

FIG.(3.9)
GaAs CHARACTERISTIC, $V < V_{th}$, DEVICE IS REPRESENTED BY $R_0$

FIG.(3.10a)

GaAs CHARACTERISTIC, $V > V_{th}$, DEVICE IS REPRESENTED BY CURRENT SOURCE.

FIG.(3.10b)
COMPARISON OF THE PERIOD BIAS DEPENDENCE GIVEN BY EQUATION (3.38) WITH THAT OBTAINED BY COMPUTER SOLUTIONS OF EQUATION (3.28)

FIG.(3.11)
Increasing the value of $G_L$ decreases the oscillation period as shown by the voltage waveforms in Figures (3.12) and (3.13) for $V_B = 3V_{th}$ and $G_L = 0.005$ mho and 0.01 mho respectively. It can be seen from these figures that increasing $G_L$ has the effect of reducing the maximum oscillator amplitude on either side of $V_B$ as one would expect. The change in the period is due primarily to the shorter sub-threshold portion of the cycle, the voltage as a function of time in this region being

$$V(t) = V_B \left( 1 - e^{-\frac{t}{L(G_L + G_o)}} \right)$$

(3.40)

This is because the device voltage no longer falls to zero but to a value $V_{th}/x$ where $x > 1$. The first term of equation (3.39) $t_1$ represents the time taken for the current $I_{th} = G_o V_{th}$ to be established in the inductance $L$. To account for the effects of $G_L$ it would be better to replace $t_1$ by

$$t_1' = L(G_o + G_L) \ln \frac{V_B}{V_{th}} \left( \frac{x}{V_B - \frac{(x-1)}{x} V_{th}} \right)$$

(3.41)

where $x = f(G_L)$. Clearly the increase in the inductive time constant from $L_{G_o} \to L(G_o + G_L)$ has an insignificant counteractive effect on the oscillation period in comparison with the strong dependence of $V_{th}/x$ upon $G_L$. When the oscillator was loaded with the maximum permissible value of $G_L$ it was noted that $x \to 2$ for consistent stable oscillations. For this condition and assuming that $V_B/V_{th} >> 1$, $t_1'$ is given by

$$t_1' = L(G_o + G_L) \left( \frac{V_{th}}{2V_B - V_{th}} \right)$$

(3.42)

Christensson et al\(^{(43)}\) propose a similar expression for the period...
$V(t) - T, \quad V_B = 3V_{th} \quad G_L = 0.005 \text{ mho}$

Computer solution of Equation (3.28)

**FIG. (3.12)**
\[ V(t) = T, \quad V_B = 3 \cdot V_{th} \quad G_L = 0.01 \text{ mho} \]

**COMPUTER SOLUTION OF EQUATION (3.28)**

**FIG.(3.13)**
of oscillation. However they consider that the current does not fall below \( I_{\text{th}}/2 \) because of space charge accumulation during the part of the cycle above threshold of the period. This leads to the same expression for \( t_1^* \) as given by equation (3.42). We must also note that \( \frac{dV}{dt} \) in the high negative differential conductance region is reduced with increasing values of \( G_L \), and so the diode exhibits a large negative differential conductance over a greater portion of each cycle. It must be stressed again that the space charge accumulation effects have not been included in this analysis. For the I-V characteristic the average conductance is given by

\[
\bar{G} = \frac{1}{T} \int_0^T \frac{dI}{dV} \, dt \quad (3.43)
\]

and the basic condition for space charge control is that \( \bar{G} > 0 \).

In order to check that the waveform satisfies this condition for space charge control, the differential conductance as a function of time was calculated and shown in Figure (3.14) for \( V_B = 3 \, V_{\text{th}} \) and \( G_L = 0.001 \, \text{mho} \). Since \( \frac{dV}{dE} \) is proportional to \( \frac{dI}{dV} \) we see that the inequality \( \bar{G} > 0 \) is satisfied without being necessary to evaluate the integral. Also the LSA criteria that the voltage remains in the high negative differential mobility region for a small portion of each cycle appears to be satisfied.

III - 6 Physical Interpretation of the Computed Solutions.

The study of a highly non sinusoidal waveform oscillator circuit is appreciably different from that of oscillators where the harmonic content in the output waveform is low and the waveform is practically sinusoidal. Tan and Foulds (44) establish a set of equa-
\[
\frac{dI}{dv} = T, \quad V_B = 3V_{th}, \quad G_L = 0.001 \text{ mho}
\]

\[
\frac{dI}{dv} \sim T \text{ calculated from the solution of equation (3.28)}
\]

'FIG.(3.14)
tions which may be used to obtain important characteristics of the steady state solution, such as frequency and the harmonic voltage amplitudes, assuming that there is no hysteresis in the I-V characteristic and the voltage and current are stable and periodic.

For the circuit of Figure(3.2) Tan and Foulds show that if the voltage and current at the terminals of the non linear device are represented by

\[ V = \sum_{m=-\infty}^{\infty} V_m e^{j\omega t} \quad (3.44) \]

\[ I = \sum_{m=-\infty}^{\infty} I_m e^{j\omega t} \quad (3.45) \]

where \( V_m = V_{m}^{*} \) and \( I_m = I_{m}^{*} \) and \( V_m \) and \( I_m \) are the complex amplitudes of the mth harmonic component, then by forming the summation \( \sum_{m=-\infty}^{\infty} V_m I_m^{*} (jm)^{n} \) and \( \sum_{m=-\infty}^{\infty} V_m^{*} I_m (jm)^{n} \) one obtains the following expressions relating the device current and voltage with the harmonic current and voltage amplitudes in the circuit:

\[ \sum_{m=-\infty}^{\infty} (jm)^{n} V_m I_m^{*} = \sum_{m=0}^{\infty} (jm)^{n} (V_m I_m^{*} + (-1)^{n} V_m^{*} I_m) = S_{nl} \quad (3.46a) \]

\[ S_{nl} = \frac{(-1)^{n+1}}{2\pi} \int_{0}^{2\pi} \frac{\partial V}{\partial I} \frac{\partial I}{\partial \omega t} \frac{\partial^{n-1} I}{\partial (\omega t)^{n-1}} d(\omega t) \quad (3.46b) \]

where \( n \) is an integer. Similarly

\[ \sum_{m=-\infty}^{\infty} (jm)^{n} V_m^{*} I_m = \sum_{m=0}^{\infty} (jm)^{n} (V_m^{*} I_m + (-1)^{n} V_m I_m^{*}) = S_{n2} \quad (3.47a) \]

\[ S_{n2} = \frac{(-1)^{n+1}}{2\pi} \int_{0}^{2\pi} \frac{\partial I}{\partial V} \frac{\partial V}{\partial \omega t} \frac{\partial^{n-1} I}{\partial (\omega t)^{n-1}} d(\omega t) \quad (3.47b) \]
If \( V \) is written as
\[
V = V_0 + V_1 \cos(\omega t + \theta_1) + V_2 \cos(2\omega t + \theta_2) + \ldots \text{ etc}
\]
then \(|V_m|^2\) becomes \(|V_m|^2/4\) in equations (3.46) and (3.47). Using this form of voltage and current, the first three equations in the set (3.47) have particular physical significance in the interpretation of the behaviour of the oscillator circuit. For \( n = 0 \) equation (3.47) may be re-written,

\[
\sum_{m=1}^{\infty} |V_m|^2 \frac{G_m}{2} = \frac{1}{2\pi} \int_0^{2\pi} I V d(\omega t)
\]

where the expressions \( I = V_m (G_m + jB_m) \), \( I^* = V_m^* (G_m - jB_m) \), \( V_m = l_m (R_m + jX_m) \) and \( V_m^* = I_m^* (R_m - jX_m) \) have been used. Equation (3.48) represents the conservation of energy applied to the circuit over a period of one cycle. For \( n = 1 \) equation (3.47) reduces to the Groszkowski relation,

\[
\sum_{m=1}^{\infty} |V_m|^2 \frac{B_m}{2} = \frac{1}{2\pi} \int_0^{2\pi} I dV
\]

For any non-linear device with a loopless current-voltage characteristic equation (3.49) is zero, then

\[
\sum_{m=0}^{\infty} m |V_m|^2 \frac{B_m}{2} = 0
\]

This equation states that the sum of \( m \) times reactive powers of all harmonics (the fundamental included) on the terminals of a loopless negative conductance device must equal zero.

For a sinusoidal oscillator this condition is satisfied by the zero susceptance resonance condition which need not necessarily apply at each harmonic frequency if the waveform is non-sinusoidal.

For \( m = 2 \) we have:
It is difficult to envisage how the LSA oscillator circuit functions at all, when the differential conductance of the device remains positive for such a large portion of each cycle and has a large positive time average value, as shown in Figure (3.14). Equation (3.51) gives a better understanding of this problem, however in order to obtain some confidence in the equation let it be applied at first to a simple sinusoidal oscillator. Then assuming \( V = V_1 \cos(\omega t) \) equation (3.51) can be re-written as

\[
\frac{V_1^2}{2} \frac{G_1}{2} = \frac{-1}{2\pi} \int_0^{2\pi} \frac{\partial I}{\partial V} V_1^2 \sin^2(\omega t) \, d(\omega t) \quad (3.52)
\]

\[
G_1 = \frac{-1}{2\pi} \left[ \int_0^{2\pi} \frac{\partial I}{\partial V} \cos(2\omega t) \, d(\omega t) - \int_0^{2\pi} \frac{\partial I}{\partial V} \cos(\omega t) \, d(\omega t) \right] \quad (3.53).
\]

where the expression : \( \sin^2(\omega t) = \frac{1}{2} - \frac{1}{2} \cos(2\omega t) \) has been used. Assuming \( \frac{\partial I}{\partial V} \) is constant and negative = \(-G_d\) then \( G_1 = G_d \), which is the condition that was derived earlier for the simple sinusoidal case. The frequency is given by equation (3.50), which for a single harmonic frequency component reduces to:

\[
\left| V_1 \right|^2 \frac{B_1}{2} = 0 \quad \text{i.e.} \quad B_1 = 0 \quad \text{as before.}
\]

Generally equation (3.51) may be considered as the rf power generation, since considering the sign of the evaluated integral from the right hand side of the equation, this indicates whether the circuit will generate or absorb rf power.
CALCULATED FROM THE SOLUTION OF EQUATION (3.28) FOR

\[ \frac{dI}{dv} \times \left( \frac{1}{V_B} \frac{dV}{dT} \right)^2 - T, \quad V_B = 3V_{th} \quad G_L = 0.001 \text{ mho} \]

FIG. (3.15)
In order for $|V_m|^2$ to be large, the integral of equation (3.51) must be negative and large. Figure (3.15) shows the plot of $\frac{dI}{dV}$ and $(\frac{dV}{dt})^2$ for $V_B = 3V_{th}$ and $G_L = 0.001$ mho. Considering the product $\frac{dI}{dV} (\frac{dV}{dt})^2$, it is seen from the voltage waveform (see Figure (2.7)) that $(\frac{dV}{dt})^2$ will be maximum when $\frac{dI}{dV}$ is negative and a maximum, also $(\frac{dV}{dt})^2$ is small in the subthreshold region in which $\frac{dI}{dV}$ is positive and large. Therefore the integral of equation (3.51) will be negative, which is confirmed by calculation from the computed solutions.

It is clear then that the harmonic components in a non sinusoidal oscillator impose more restrictive conditions than for the pure sinusoidal case and a great deal of care must be taken in interpreting the behaviour of non sinusoidal oscillators and parallels with sinusoidal operation should be avoided.

III - 7 Fourier Analysis of the LSA Waveforms

The Fourier analysis of the voltage and current waveforms was performed in order to determine the relative amplitudes of the different harmonics. This is necessary if the describing function technique is going to be used in the analysis of the oscillator circuit.

The computer analysis was done for different values of $V_B$ and $G_L$, keeping the values of $L$ and $C$ constant, being 1.6 nH and 0.1 pf respectively. The relative amplitudes up to the 5th harmonic for the voltage and current waveforms corresponding to $V_B = 5V_{th}$ and $G_L = 0.001$ mho are shown in Figure (3.16). It is observed that the voltage swing at the fundamental frequency exceeds the d.c. vol-
tage considerably, which is characteristic of this mode. The second harmonic has a high amplitude (≈52% of the fundamental) and is the predominant harmonic, while the third harmonic is of the order of 10% of the fundamental.

For a high efficiency it is important that the d.c. components are small, the fundamental components are large and the phases between the current and voltages at the different harmonics are such that maximum power is generated at the fundamental and minimum powers at higher harmonics. The phase angles are shown in Figure (3.17). We see that although the second harmonic voltage and current amplitudes are significant, their phase difference is nearly 270°, resulting in low generated power and low efficiency. For the third harmonic although the phase difference is small, the voltage and current amplitudes are also small, ensuring then a very low efficiency. A thorough study of the effect of the harmonics over the tuning range and efficiency is presented by Bosh and Engelmann(45) and because we are not particularly interested in the circuit properties for high efficiency but the interaction between the circuit and the active device, further considerations on the efficiency have not been included in this thesis.

The Fourier series reconstitution of the voltage waveform limited to terms up to and including the fourth harmonic frequency is shown in Figure (3.18). Incorporation of higher harmonics into the voltage waveform changes the frequency, the second harmonic being particularly important. For the values of the parameters mentioned before, the oscillation frequency is 9.05 GHz. This value agrees with the value obtained using equation (3.38) within 1.5%, where j was limited to 10.
**FIG. (3.16a) FOURIER VOLTAGE COMPONENTS**

Harmonic Component

**FIG. (3.16b) CURRENT COMPONENTS**

Harmonic Component

\[
V_B = 5V_{th} \quad G_L = 0.001 \text{ mho}
\]

**FIG. (3.17) V - I PHASE ANGLES**
$V_B = 5 \cdot V_{th}$

$G_L = 0.001 \text{ mho}$

**LSA FOURIER ANALYSIS**

**FIG.(3.18)**
CHAPTER IV

EXPERIMENTAL OBSERVATION OF LSA WAVEFORMS

In this Chapter the device material characteristics and experimental circuit are treated in detail. The experimental observation of the different waveforms provides a substantial basis for the analytical method described in the following chapters and used to derive the operating conditions.

IV - 1 Device Material Characteristics and Mounting Techniques

The GaAs used in this work was purchased from Cayuga Associates\textsuperscript{\dag} as an already contacted slice. The material had been grown by liquid phase epitaxy onto a single crystal wafer of heavily doped GaAs as described by Christensson et al\textsuperscript{[43]}. Figure 4.1 shows the different layers in the slice. The \( n^+ \) Sn doped "buffer layer" is grown onto the substrate to prevent diffusion of impurities from the substrate layer into the active region. The active layer grown above has a free carrier density of \( 1.77 \times 10^{15} \text{ cm}^{-3} \), a thickness of 119 \( \mu \text{m} \) and low field mobility of \( 8.3 \times 10^3 \text{ cm}^2 \text{ Volt}^{-1} \text{ sec}^{-1} \). Onto this an \( n^+ \) Te-doped layer acts as a contacting surface. An Au-Ge eutectic is evaporated onto the complete slice and alloyed into the material. The Ge diffuses faster than the Au resulting in an ohmic contact to the active layer. A final Au layer is evaporated on top for mechanical connection purposes.

\textsuperscript{\dag} Cayuga Associates, Inc., Ithaca, N.Y., U.S.A.
DIODE STRUCTURE

FIG. (4.1)

SCHEMATIC VIEW OF THE S4 ENCAPSULATION SHOWING THE DIODE MOUNTING

FIG. (4.2)
The frequency of operation of a relaxation LSA GaAs oscillator is dependent upon both the low field resistance and the self capacitance. A typical chip area of \(1 \times 10^{-3} \text{ cm}^2\) was used which resulted in a low field resistance of about 5 \(\Omega\) and a capacitance of 0.08 pf. When biased at a typical operating point of \(2 - 4\) times the threshold field the d.c. device resistance lies between \(20 - 40 \Omega\), which results in a reasonable match to the bias pulse unit.

The GaAs was cut using a continuous loop stainless steel saw with an alumina-glycerol slurry as the cutting agent. For handling convenience the chips were mounting into S4 varactor packages as shown in Figure 4.2 using indium solder. The top connection was made with a thin gold strap which was welded to one side of the package and soldered to the other. In order to keep the parasitic lead inductance within appropriate values a wire gold strap was used (\(\approx 300 \mu\text{m}\)), approximately the same dimension as the side of the chip.

For a value of the oscillation frequency of \(f = 4.166 \text{ GHz}\), the ratio \(\frac{n}{f}\) gives a value of \(4.25 \times 10^5 \text{ cm}^{-3}\text{sec}\) which is within the limits proposed for LSA operation: \(10^5 < \frac{n}{f} < 5 \times 10^5 \text{ cm}^{-3}\text{sec}\). The peak to valley current ratio \(n\) is adjusted to the particular value of \(\mu_0\), the low field mobility. For the particular sample used in these experiments this value is 1.81 as shown in Figure 4.3.

If there is any accumulation charge it will take some time to be dissipated, once the voltage has decreased below threshold voltage. According to Thim (46) this time is in the order of tens of picoseconds during which the conduction current is limited to the value of saturation current. The net effect is that the I-V characteristic presents an hysteresis cycle as shown in Figure 4.4. To account for this
d.c. $I - V$ CHARACTERISTICS OF TWO LSA DIODES

FIG. (4.3)
I - V CHARACTERISTIC. CHRISTENSSON MODEL

I - V CHARACTERISTIC. JEFFSSON & JEPESEN MODEL

FIG. (4.4)
we have considered Christensson's model and the period of oscillation is given by,

\[ T = 2\pi \sqrt{LC} + \frac{L}{R_0} \frac{(n-1)V_{th}}{nV_B - V_{th}} \]  

\[ T = \tau_1 + \tau_2 \]  

\[ \tau_1 = 2\pi \sqrt{LC} \]  

\[ \tau_2 = \frac{L}{R_0} \frac{(n-1)V_{th}}{nV_B - V_{th}} \]  

As mentioned before, Jeppesen and Jeppsson consider no formation of accumulation charge, and the I-V characteristic presents no hysteresis cycle. In order to make this study more general we have used Christensson's model, because any accumulation charge may be observed from the voltage waveform obtained experimentally as suggested by Camp. This model is applicable to any Gunn diode independently of the internal configuration of the electric field. Kino shows that this model is applicable even to a device with domains fully formed.

IV - 2 Experimental Circuit

The experimental design consist of a thick rectangular X band iris circuit as shown in Figure (4.5). The diode is mounted inside the iris as shown in Figure (4.6) and a double ridge section is used in order to reduce the mismatch between the diode and the rest of the circuit. This circuit was developed originally at Cornell University and has been

† Electrical Engineering Department, Cornell University, Ithaca, New York 14850, U.S.A.
analyzed in detail by Jeppsson and Jeppesen\(^{(35)}\) and Lidgey\(^{(49)}\). The bias is applied across the series connection of the iris and encapsulated diode, the wave-trap being incorporated to prevent microwave power from coupling into the bias line. It is possible then to have a multiharmonic waveform due to the low Q value of the iris. This is also convenient for a fast build up of the oscillations when \(V>V_{th}\).

For operation in this frequency range it is extremely difficult to obtain a single element non-distributed inductance as shown in Figure (2.5). However if the circuit is designed to allow independent tuning at each of the oscillator harmonics so that the inductive susceptance can be adjusted to be equal at each frequency component, then the circuit will be equivalent to the lumped model of Figure (2.5) used for computer simulations.

The fundamental frequency is below cut-off frequency of the waveguide (\(\approx 6.66\) GHz), and the field components for this frequency are confined to the iris and unaffected by the tuning elements in the waveguide. Higher harmonics propagating into the waveguide are tuned by the variable short circuit and the variable depth probe. It is only practically possible to attempt control of the load at the fundamental, second harmonic and perhaps the third harmonic frequency.

The experimental waveforms were observed on the Hewlett-Packard HP 141A. This sampling scope has a 12.4 GHz capability or the third harmonic of most signals measured. The sampling unit HP 1430A provides the necessary rf trigger signal.
PACKAGE DIODE

(a)

DOBLE STEP λ/4 RIDGE
WAVEGUIDE TRANSFORMER

(b)

X BAND WAVEGUIDE

X BAND IRIS CIRCUIT  (a) FRONT VIEW
(b) SIDE VIEW

SHORT CIRCUIT  VARIABLE DEPTH AND POSITION
PLUNGER  PROBE

IRIS MOUNT  X BAND WAVEGUIDE

COMPLETE L.S.A. OSCILLATOR CIRCUIT

FIG.(4.6)
IV - 3 Bias Circuit

Although efficiencies in excess of 20% may be achieved experimentally from an LSA oscillator, the high power levels involved cause a large amount of heat to be generated in the device and some cooling system is necessary in order to avoid thermally induced non uniformities and therefore a degradation of the characteristics of the active device with increasing temperature. Excessive heating results in loss of space charge control and eventually the device breaks down through impact ionization. These conditions restrict the LSA mode to pulsed applications as the heat involved from CW LSA mode is too high to be removed satisfactorily with conventional techniques.

The pulse unit used in this work is shown in Figure (4.7). The pulse is formed by discharging a transmission line of length $l_c$ which has been charged to a voltage $V_c$. The line of length $l_s$ between $l_c$ and the diode serves to separate the primary pulse from any possible reflection which will occur if the impedance of the diode is different than $Z_o$ (50 Ω), the characteristic impedance of both lines $l_c$ and $l_s$.

The energy stored in $l_c$ is,

$$\varepsilon_{\text{stored}} = \frac{1}{2} C l_c V_o^2 \quad (4.5)$$

where $C$ is the capacitance per unit length of line. Assuming that the line $l_s > 2.1 l_c$, then all the stored energy will pass in a pulse into the line $l_s$. The voltage of the pulse is given by

$$V_{\text{pulse}} = \frac{Z_o}{Z_o + Z_o} V_o = \frac{V_o}{2} \quad (4.6)$$

and the current:
SCHEMATIC DIAGRAM OF THE PULSE UNIT

FIG. (4.7)

EQUIVALENT CIRCUIT OF THE PRIMARY PULSE

FIG. (4.8)
The energy contained in the pulse is

\[ \varepsilon_{\text{pulse}} = \frac{V_0^2}{4Z_0} t \]  

where \( t \) is the pulse length. Applying the conservation of energy, \( t \) may be found.

\[ \varepsilon_{\text{pulse}} = \varepsilon_{\text{stored}} \]

\[ \frac{1}{2} C V_0^2 = \frac{1}{4Z_0} t \]  

\[ Z_0 = \sqrt{\frac{L}{C}} \]  

\[ t = \frac{2.1 C}{L C} \]  

where \( v_c \) is the velocity of a TEM wave in the line.

The equivalent circuit of the bias system is shown in Figure (4.8) including the diode voltage monitoring \( T \). The voltage across the diode is given by

\[ V_d = \frac{R_d}{50 + R_d} V_0 \]  

where \( R_d \) is the d.c. resistance of the LSA device. The threshold field for GaAs is approximately 3 KV cm\(^{-1}\) and for a material of uniform doping density 100 \( \mu \)m width, this corresponds to a voltage of 30 volts. To achieve this value assuming that \( R_d \) is constant and equal to 5Ω the low field value, \( V_0 \) must be 360 volts for a charge line of 50 ohms characteristic
impedance. If $V_0$ is increased just above this value the diode resistance will increase to a new value $R'_d$ because of the negative differential resistance region of the material. Typically $R'_d$ has a value of 30 ohms when the device is operating as an LSA oscillator and so the voltage across the device will jump to a value of $\approx 98$ volts, given by the previous equation, which is an effective bias in excess of $3V_{th}$. Consequently a 600 volt power supply is required to track the diode bias voltage over the range 0 to $8V_{th}$.

A desirable characteristic of the pulse unit is that it should produce sharp edge rectangular pulses. The switch used was a mercury wetted relay which has a switching time of less than 1 nsec and is capable of handling charge voltages of up to 800 volts. In this work we have used a 250 nsec pulse length.

To prevent microwave power from coupling into the bias line an anodized aluminium slug of approximately $\lambda/4$ length at the fundamental oscillation frequency was incorporated in the bias port. This anodized section has extremely low characteristic impedance and is effectively a short circuit to the fundamental frequency. To prevent second harmonic coupling into the bias we used a value somewhat less than $\lambda/4$.

### IV - 4 Voltage and Current Waveforms of the LSA Oscillator.

The rf signal propagating along the waveguide and observed in the output port of the system is shown in Figure (4.9). The frequency of oscillation for the second harmonic is 8.3 GHz and the output power at this frequency is 4.5 W. The total current was measured across a 1 $\Omega$ disk resistor in series with the device as shown in Figure (4.11b).
R.F. SIGNAL. SECOND HARMONIC

FIG.(4.9)

CONDUCTION AND DISPLACEMENT CURRENTS

FIG.(4.10)

L.S.A. TOTAL CURRENT \( f = 4.166 \text{ GHz} \)

FIG.(4.11a)
CURRENT (amps)

Bias
Diode

1 Ω
Resistor
Contacts

Iris

Output

ARRANGEMENT OF CAPACITIVE PROBE

FIG.(4.12a)

ARRANGEMENT OF RESISTIVE PROBE

FIG.(4.11b)

f = 4.166 GHz
DISPLACEMENT CURRENT

V = V_{th}

FIG.(4.12b)

VOLTAGE WAVEFORM OBTAINED BY INTEGRATION OF THE DISPLACEMENT CURRENT OF FIG.(4.12b)

FIG.(4.13)
The total current is the sum of the conduction current and displacement current through the capacitor $C_d$ as indicated in Figure (4.10). The signal proportional to $\frac{dV}{dt}$ and therefore the displacement current is measured by a capacitive probe\(^{(51),(52)}\) placed inside the iris near the diode, adjusting the depth to obtain the best possible waveform. This arrangement is shown in Figure (4.12a).

The voltage waveform is obtained by integrating the displacement current of Figure (4.12b) and setting the minimum of that waveform equal to zero. The average of the voltage waveform is then set equal to the bias voltage $(3V_{th})$. Using Christensson's model we have,

$$T = \frac{1}{f} = 0.24 \times 10^{-9} \text{ sec.}$$

and the values of $\tau_2$ and $\tau_1$ from the experimental waveforms of the displacement current and voltage are 0.08 nsec and 0.16 nsec respectively. The values of $L$ and $C$ may be obtained from equations (4.3) and (4.4) for $n_f = 1.8$, $V_B = 99 \text{ V}$, and $R_o = 4.5 \Omega$,

$$L = 1.98 \text{ nH}$$
$$C = 0.327 \text{ pf}$$

The total capacitance includes the capacitance of the S4 package used in these experiments. An appropriate equivalent circuit of the S4 package is shown in Figure (4.14a). At the fundamental frequency the impedance of this equivalent circuit may be considered as shown in Figure (4.14b). The diode capacitance independently of the package is

$$C_d = 0.327 - 0.22 = 0.107 \text{ pf}.$$ 

The theoretical value calculated using the expression $C_d = \varepsilon \frac{A}{d}$ where $A$ is the area and $d$ the thickness of the active device gives a value of 0.08 pf which gives a good agreement with the experimen-
FIG. (4.14a) EQUIVALENT CIRCUIT OF S4 PACKAGE BASED ON
OWENS’ MODEL

FIG. (4.14b) SIMPLE EQUIVALENT CIRCUIT OF S4 PACKAGE
The maximum value of the displacement current is given by the maximum value of the slope of the voltage waveform.

Current and voltage waveforms were also observed for a second sample in a multiresonant circuit with independent tuning facilities for the second harmonic. This multiresonant circuit is described in Chapter VIII. For the second sample the threshold voltage is 28 volts and the bias $V_B = 3.57 V_{th}$. The peak to valley current ratio is near 1.7 and the ratio $\frac{n}{f}$ has a value of $4.69 \times 10^5 \, \text{cm}^{-3} \cdot \text{sec}$ the fundamental frequency of oscillation being $f_o = 3.77 \times 10^9 \, \text{Hz}$. The total current is measured using a disk resistor of 1 $\Omega$ inside the iris and the waveform is shown in Figure (4.15). Due to the disturbance introduced by the capacitive and resistive probes it was not possible to observe the displacement current and voltage waveforms for the same tuning conditions as for the total current.

The displacement current is shown in Figure (4.16) for a bias voltage $V_B = 4.29 V_{th}$ and the fundamental frequency is $f_o = 4.08 \, \text{GHz}$ giving a value for $\frac{n}{f}$ equal to $4.33 \times 10^5 \, \text{cm}^{-3} \cdot \text{sec}$. The value of the ratio $\frac{n}{f}$ has some influence on the shape of the waveform as shown by Camp (47) using samples with a peak to valley ratio in the range 1.5+2 and values of $\frac{n}{f}$ in the range $1.8 \div 3.2 \times 10^5 \, \text{cm}^{-3} \cdot \text{sec}$. However for this second sample the ratio $\frac{n}{f}$ has a similar value to the first one and the displacement current shows the same variation.

Far more interesting is the voltage waveform, because it was possible to observe the influence of the second harmonic tuning on the voltage waveform. A resistive probe was fitted inside the iris and the zero level determined on the sampling scope. Figure (4.17a) corresponds to high output power at the fundamental and minimum at
FIG.(4.15) TOTAL CURRENT

FIG.(4.16) DISPLACEMENT CURRENT

Horizontal: 0.1 nsec/div
Vertical: 3.15 A/div
f = 3.77 GHz

Horizontal: 0.1 nsec/div
Vertical: 20mV/div
f = 4.08 GHz
VOLTAGE WAVEFORMS

FIG. (4.17a)

FIG. (4.17b)

FIG. (4.17c)
the second harmonic. The voltage waveform changes appreciably as the output power at the second harmonic is increased as shown in Figures (4.17b) and (4.17c). These waveforms and their admittance loci are treated in detail in Chapter VIII.

IV - 5 Some considerations on the voltage and current waveforms

From the experimental observations we note that the oscillation is definitely a relaxation oscillation and therefore space charge accumulation control is achieved. The distance travelled by the accumulation layer during the time \( T = \frac{1}{f} \) (\( f \) being the fundamental frequency) must be very short compared with the active length of the device in order to keep the electric field distribution inside the sample uniform.

On the other hand if a fully formed domain exists, the electric field within the domain will increase considerably if overlength samples are used and cessation of all oscillation and eventual breakdown will occur. In order to study this possibility, let us consider that a fully formed domain exists. The total voltage across the sample is given by

\[
V = E_l \ell + V_d
\]

(4.13)

where \( \ell \) is the active length of the sample and \( d \) is the distance over which the depletion of charge occurs, and \( V_d \) is the excess domain voltage given by

\[
V_d = \frac{1}{2} e n_o \frac{d^2}{\varepsilon}
\]

(4.14)
If the total voltage across the sample is

\[ V = N E_{th} \]  \hspace{1cm} (4.15)\]

and assuming \( E_1 = \frac{1}{2} E_{th} \)

\[ N E_{th} \ell = \frac{1}{2} E_{th} \ell + \frac{1}{2} e n_o \frac{d^2}{\varepsilon} \]  \hspace{1cm} (4.16)\]

\[ d = \sqrt{\frac{E_{th} (2N-1) \varepsilon \ell}{e n_o}} \]  \hspace{1cm} (4.17)\]

Equation (4.13) may be written as

\[ N E_{th} \ell = \frac{1}{2} E_{th} \ell + d (E_p - \frac{3}{2} E_{th}) \frac{1}{2} \]  \hspace{1cm} (4.18)\]

Substituting equation (4.17) in equation (4.18) the peak electric field may be calculated

\[ E_p = \left[ \sqrt{\frac{e n_o \ell (2N-1)}{\varepsilon E_{th}}} + \frac{1}{2} \right] E_{th} \]  \hspace{1cm} (4.19)\]

For the value of the parameters corresponding to the voltage waveforms \( E_p \) has a value of \( \approx 79 E_{th} \). The breakdown field for GaAs is of the order of \( 50 E_{th} \) and therefore if a fully formed domain exists within the sample breakdown will occur and any oscillation will cease.
The theory of describing functions permits the study of the active element and associated circuit in terms of a feedback model similar to that used in control theory. This makes the study of the operating conditions and stability of the solutions easier.

This Chapter is involved with the definition, physical meaning, and practical application of the describing functions. For this, an introduction to quasilinearization techniques and quasilinear approximators is presented. The solution of the characteristic equation of the feedback model provides the operating point of oscillation, with all the information concerning the frequency of oscillation, voltage amplitudes, and phase angles for the different harmonics.

Particularly interesting is the study of the stability of the operating points. The study presented in this Chapter is more general than the original study by Kurokawa (37) for the pure sinusoidal oscillator.

V - 1 Quasilinear Systems

Linear control theory is now very well understood and provides very powerful tools to study different topics of interest in a system. It is then very interesting if we can apply this theory to a system which contains some non-linear as well as linear components,
providing the necessary approximations with minimum error are used. Using a feedback model the non linear and linear parts of the system are separated and individually studied although they are related one to another by the conditions of operation of the system.

A linear system is represented by the equation

$$\sum_{i=0}^{n} a_i \frac{d^i y}{dt^i} = \sum_{j=0}^{m} b_j \frac{d^j x}{dt^j}$$

(5.1)

where x and y represent the input and output signals. In the frequency domain, using Laplace transform (s=jω), equation (5.1) may be written as

$$Y(s) = F(s). X(s)$$

(5.2)

where F(s) is the transfer function of the system. It is possible to solve equation (5.1) in the time domain but there is no criterion to determine if the solution corresponds to a steady state; it is preferable to design a mathematical model where it is possible to impose some conditions without changing the physical significance of the real model. For the simple case of a passive resistance, shown in Figure (5.1) we have

$$v(t) = i(t). R$$

$$V(s) = I(s) . R$$

(5.3)

and F(s) = R = constant. The transfer function which represents R in the block diagram is a constant for any input signal. There is a single valued application between a linear element and its transfer function.

When the element presents a different behaviour for different
SINGLE LINEAR ELEMENT AND EQUIVALENT BLOCK DIAGRAM USING LAPLACE TRANSFORM

FIG.(5.1)

NON LINEAR CHARACTERISTIC WITH TWO DIFFERENT INPUT SIGNALS

FIG.(5.2)
input signals it is not possible to apply the same theory because now the element has different transfer functions for different signals, that is, we are dealing with non linear elements. Control theory is very well treated in several books (53),(54),(55) and therefore a more detailed study is not included in this work, although it is convenient to analyze the approximation of non linear systems to linear systems in order to apply the numerous techniques available in control theory.

Figure (5.2) shows the I-V characteristic of an LSA diode with two different voltage signals superimposed on the bias point $V_B$. If the I-V characteristic was a straight line, there would be no restriction upon the input amplitude or form of the input signal, the output signal being $I(s) = K.V(s)$ where $K = G$ is the transfer function. Due to the non linear I-V characteristic, the device will have different responses for signals $S_1$ and $S_2$ as shown in Figure (5.3).

The problem of studying a non linear system may be simplified by replacing each non linear operator by an approximating linear operation and therefore study the resulting linear system. The linearization of non linear operators can only be justified for small excursions of the variables around the nominal operating values as indicated in Figure (5.4). If the response to an input signal modifies the variables within a range which exceeds the limits of a reasonable linear approximation the system must be re-linearized around the new different operating points. So it is desirable to avoid the small signal restriction and keep at the same time the advantages of a linear approximation.
DIFFERENT OUTPUT SIGNALS CORRESPONDING TO INPUT SIGNALS IN FIG. (5.2)

FIG. (5.3)

ILLUSTRATION OF TRUE LINEARIZATION

FIG. (5.4)
The approximation of a non-linear operation by a linear one is called quasilinearization. This approximation depends on some properties of the input signal. A complete linearized model exhibits only linear behaviour, whereas a quasilinearized model exhibits the basic characteristics of non-linear behaviour, that is, dependence of performance upon signal amplitude. Quasilinearization has a very substantial advantage over true linearization, in that there is no limit to the range of signal magnitudes which can be accommodated.

Quasilinearization is possible for specific forms of input signals, that is, we consider a waveform at the input of the non-linear operator, evaluate the output signal and approximate the result by a linear operation. As it is shown in next section, once the system has been quasilinearized, a feedback model of the system proves to be very useful in determining the input signal at the non linearity.

The basic model in control theory is shown in Figure (5.5) and we can define it as a system comprising one or more feedback loops which compare the controlled output signal x with the command signal e, the difference c (c= e-x) is used to drive the output signal x in correspondence with signal e. In general one can design, except for the non-linear element itself, the feedback loop with the condition that no nonlinearity must be included in it. However the presence of a feedback signal makes it more difficult to determine the waveform input signal to the non-linear element because the feedback signal is modified by the transfer functions in the feedback loop. A practical
BASIC FEEDBACK MODEL IN CONTROL THEORY

FIG.(5.5)
solution to avoid the calculation of the input signal is by assuming its waveform. In this assumption some previous knowledge of the waveform expected across the device is necessary.

The quasilinear approximating functions which describe approximately the transfer characteristics of the non-linearity are called "Describing Functions". The fundamental limitation on the use of describing functions to represent the behaviour of a system is that the true input signal to the non-linear element must approximate the input signal used in the calculation of the describing functions (D.F.), that is the form of the signal at the input to the nonlinearity must be guessed in advance. The principal advantage of the D.F. theory is not that it permits the approximate calculation of the response of a given system to a given input or class of inputs, this can always be done by computer simulation, but it is a valuable tool to the design of non linear systems. This is shown in Chapters VII and VIII, in the synthesis of a single and multiresonant circuit models with similar behaviour to the experimental ones, although they are based on lumped elements instead of distributed elements.

Once we have introduced the D.F. and discussed the advantages and limitations in the use of this technique, the calculation of these functions follows. Let us consider the input signal \( x(t) \) to be formed by the sum of a number of signals \( x_i(t) \) as shown in Figure (5.6). The most general form of linear approximator for the non linearity is a parallel set of linear operators, one to treat each component of the input. If each input component is considered to be steady state and if we restrict our attention to invariant non linearities, the linear operators which form the quasilinear approximators can be taken as
GENERAL LINEAR APPROXIMATOR FOR A NON-LINEAR OPERATOR

Fig. (5.6)
invariant at the outset.

In Figure (5.6), in the time domain

\[ y_a(t) = \int_0^\infty w_1(\xi) x_1(t-\xi) \, d\xi = \text{Convolution Integral} \quad (5.4) \]

where \( w_1(t,\xi) \) is the weighting function and the transfer function for the approximator is given by

\[ \mathcal{L}(w(t,\xi)) = N \quad (5.5) \]

where \( \mathcal{L} \) is the Laplace transform. Once the form for the quasilinear approximator has been chosen it must be decided what is the basis to make the approximation, that is, what criterion to use in choosing the \( w_1 \) functions. The criterion used in this study is the minimum mean square error. The filters in the linear approximator are designed to minimize the mean square difference between the true output and the output of the approximator.

This criterion is universally applicable to all forms of input signal. Another reason to choose this criterion is that it is possible to follow the error analytically and the development based on this criterion is very similar to other optimum linear theories based upon the same criterion. The concept of forming a quasilinear approximator to a non linear operator following the criterion of minimize the mean square error was presented first by Booton \( (56) \). He considered only a single input component and took the approximator to be a static gain. Sommerville and Atherton \( (57) \) took the approximator to be a parallel set of static gains. However in the present analysis we take the approximator to have the most general
form: a parallel set of dynamic linear operators. From Figure (5.6) the error in the approximation is given by

\[ e(t) = y_a(t) - y(t) \]  \hspace{1cm} (5.6)

and its mean square value is

\[ \overline{e(t)^2} = y_a(t)^2 - 2 y_a(t) y(t) + y(t)^2 \]  \hspace{1cm} (5.7)

where

\[ y_a(t) = \sum_{i=1}^{n} \int_{0}^{\infty} w_i(\xi) x_i(t-\xi) \, d\xi \]  \hspace{1cm} (5.8)

\[ y_a(t)^2 = \sum_{i=1}^{n} \sum_{j=1}^{n} \int_{0}^{\infty} \int_{0}^{\infty} \int_{0}^{\infty} w_i(\xi_1) w_j(\xi_2) x_i(t-\xi_1) x_j(t-\xi_2) \, d\xi_1 \, d\xi_2 \]  \hspace{1cm} (5.9)

\[ y_a(t)y(t) = \sum_{i=1}^{n} \int_{0}^{\infty} v_i(\xi) y(t) x_i(t-\xi) \, d\xi \]  \hspace{1cm} (5.10)

Substituting equations (5.9) and (5.10) in equation (5.7) the linear approximators \( w_i \) of the form shown in Figure (5.6) which minimize the mean square error may be obtained.

Gelb and van der Velde\(^{(58)}\) treat the problem of determining the optimum linear approximator to a non-linearity being driven by an input of specified form as a statistical problem. This has the advantage that if there are any random processes in the input components, then statistical techniques are essential. West\(^{(59)}\) and Douce\(^{(60)}\) among several other authors give a detailed analysis of this problem.
Figure (5.7) shows the essential parts of a negative resistance oscillator circuit. The non linear negative resistance is coupled to the load through some passive network and the circuit contains the facility for injecting some external signal. In the final steady state condition the circuit contains currents and voltages not only at the fundamental frequency but also at the harmonic frequencies. The coupling circuit is characterized by its current-voltage transmission matrix

\[
\begin{pmatrix}
V_1 \\
I_1
\end{pmatrix}
= 
\begin{pmatrix}
T_{11} & T_{12} \\
T_{21} & T_{22}
\end{pmatrix}
\begin{pmatrix}
V_2 \\
I_2
\end{pmatrix}
\]

(5.11)

where \(V_1, I_1\) are the rf voltage and current of frequency \(\omega_k\) at the input of the coupling circuit and \(V_2, I_2\) are the corresponding values at the output. The \(T_{ij}\) parameters are then frequency dependent. At this stage the active element is regarded as a voltage dependent negative conductance, assuming that the time domain current variation around a certain bias point is a single valued function of the time domain voltage variation around the same point, that is, we are going to consider no hysteresis cycle in the I-V characteristic of the active device. We will discuss this condition in more extent when considering LSA multiresonant circuits.

Since only stationary cases are going to be treated the complex currents and voltages \(I_{dk}, V_{dk}, I_{ek}, V_{ek}\), i.e., the Fourier
ACTIVE ELEMENT WITH LINEAR COUPLING CIRCUIT, LOAD $G_L$ AND INJECTED SIGNAL $I_e$

FIG.(5.7a)

REPRESENTATION OF LINEAR COUPLING CIRCUIT. $T$ IS THE VOLTAGE CURRENT TRANSMISSION MATRIX DEFINED BY (5.11)

FIG.(5.7b)

FEEDBACK MODEL OF THE CIRCUIT IN FIG.(5.7a)

FIG.(5.8)
components of frequency $\omega_k$ of the stationary currents and voltages $i_d(t), v_d(t), i_e(t)$ and $v_e(t)$ may be used. The index $k$ is used to number the many different frequencies appearing in the non linear system.

Following the straightforward circuit analysis of Gustafsson et al (61) in Figure (5.7b) we have

\begin{align*}
G_L V_{ek} &= I_{ek} - I_{lk} \quad (5.12) \\
V_{ek} &= T_{11} V_{dk} + T_{12} I_{dk} \quad (5.13) \\
I_{lk} &= T_{21} V_{dk} + T_{22} I_{dk} \quad (5.14)
\end{align*}

Substituting equation (5.13) and (5.14) in equation (5.12) the $k$th voltage component of frequency $\omega_k$ across the diode can be expressed as:

\begin{equation}
V_{dk} = \frac{I_{ek} - I_{dk}(G_L T_{12} + T_{22})}{G_L T_{11} + T_{21}} \quad (5.15)
\end{equation}

This equation must be satisfied by each harmonic component of the total voltage and current. It is very useful to develop a feedback model which represents equation (5.15) because this model is the common type used in control theory being very easy to apply all the techniques available in linear control theory and also because the practical application of the describing function technique is based on the approximation of the waveform assumed at the input of the non linearity to the true input signal. A feedback model will determine whether such an approximation is correct. The feedback model which is a mathematical re-formulation of the problem is shown.
in Figure (5.8) where the non linear element is represented by the transfer function \( N_k \). It is necessary then to define this transfer function \( N_k \) and find its physical meaning. For this the describing function is used.

It was shown before that a system with \( n \) input signals at the non linear operator is equivalent to \( n \) quasilinear systems. Each of them treats one component of the \( n \) signals. To each of these quasilinear systems control theory can be applied. In Figure (5.8) the voltages and currents related to the active element can be written in time domain as

\[
v_d(t) = \sum_{k=0}^{\infty} |V_{dk}| \cos(\omega_k t + \theta_k)
\]

\[
i_d(t) = \sum_{k=0}^{\infty} |I_{dk}| \cos(\omega_k t + \phi_k)
\]

Considering the \( k \)th component, the non linear operator may be replaced by the transfer function

\[
N_k = \frac{|I_{dk}|}{|V_{dk}|} e^{j(\phi_k - \theta_k)}
\]

where \( N_k \) is the effective transfer function of the non linear device at the \( k \)th harmonic frequency. These \( N_k \) are the "Multiple Input Sinusoidal Describing Functions" and correspond to the non linear admittance of the active device at the appropriate harmonic frequency. In general \( N_k \) is a function of \( V_{dk} \), \( \theta_k \) and \( \phi_k \). \( N_k \) would also depend upon \( \omega_k \) if the I-V characteristic contains terms of the form \( \left( \frac{d^n v}{dt^n} \right)^m \). In steady state the characteristic equation for the fundamental and all the harmonics is given by,
\[ I_r = I_e - I_{dk} \left( G_L T_{12} + T_{22} \right) \]  
\[ (5.19) \]

\[ V_{dk} = \frac{I_r}{G_L T_{11} + T_{21}} \]  
\[ (5.20) \]

Substituting equation (5.19) in equation (5.20) and using the relation \( I_{dk} = N_k V_{dk} \), we have

\[ V_{dk} \frac{G_L T_{12} + T_{22}}{G_L T_{11} + T_{21}} + 1 = I_e \]  
\[ (5.21) \]

In the steady state (free running) the characteristic equation can be calculated for the fundamental and all the harmonics. This gives an infinite set of equations of the type:

\[ \frac{G_L T_{12} + T_{22}}{G_L T_{11} + T_{21}} N_k + 1 = 0 \]  
\[ (5.22) \]

\[ N_k = -\frac{G_L T_{11} + T_{21}}{G_L T_{12} + T_{22}} = -\frac{1}{L(j\omega_k)} \]  
\[ (5.23) \]

We note that the feedback model allows the linear and non linear parts of the system to be separated. The left hand side of equation (5.23) depends only upon the active element while the right hand side of this equation depends only upon the linear part of the system \( L(j\omega_k) \) and is a function only of \( \omega_k \). We have a set of non linear equations and the aim of the analysis is to derive the values of the D.F. \( N_k \) obtaining the voltage amplitudes \( V_{dk} \) for the different harmonics, the phase angles and the frequency of oscillation.

As the \( N_k \)'s are functions of the voltage components, when they
are known it is possible to plot the device line. One must note that each equation represents a steady state condition and the solution must be checked in order to study the stability of the system.

The D.F. technique is applied on the assumption that the least significant harmonics generated by the non linear characteristic are sufficiently filtered and attenuated by the linear part of the system formed by frequency responsive elements. In Figure (5.8) for steady state condition, if the input signal of frequency $\omega_k$ at the non linearity is

$$V_{dk} = V_{do} e^{j(\omega_k t + \gamma)} \quad (5.24)$$

the output from the non linear element, which is the output signal of the system $I_o$, is given by

$$I_o = I_{dk} = V_{dk} N_k (V_{dk}) = V_{do} N_k (V_{dk}) e^{j(\omega_k t + \gamma)}$$

$$= I_{do} e^{j(\omega_k t + \gamma)} \quad (5.25)$$

where $V_{do} N_k (V_{dk}) = I_{do}$

The input signal $V_{dk}$ can be written in terms of the output signal as

$$V_{dk} = I_{dk} L_2(j\omega_k) L_1(j\omega_k) \quad (5.26)$$

where

$$L_1(j\omega_k) = \frac{1}{G_{11} T_{11} + T_{21}} \quad (5.27)$$

$$L_2(j\omega_k) = G_L T_{12} + T_{22} \quad (5.28)$$
The transfer functions \( L_1(j\omega_k) \) and \( L_2(j\omega_k) \) are complex and can be written as

\[
L_1(j\omega_k) = G_1(\omega_k) e^{j\alpha_1} \\
L_2(j\omega_k) = G_2(\omega_k) e^{j\alpha_2}
\] (5.29) (5.30)

where \( G_1 \) and \( G_2 \) represent real values of the gains of \( L_1 \) and \( L_2 \). So equation (5.26) becomes

\[
V_{dk} = V_{do} e^{j(\omega_k t + \gamma)} = I_{do} e^{j(\omega_k t + \gamma)} G_1(\omega_k) G_2(\omega_k) e^{j(\alpha_1 + \alpha_2)}
\]

\[
l = N_k(V_{dk}) G_1(\omega_k) G_2(\omega_k) e^{j(\alpha_1 + \alpha_2)}
\] (5.31)

So any signal of frequency \( \omega_k \) is attenuated by the linear part of the system by \( \frac{1}{G_1 G_2} \) and its phase modified by \( j(\alpha_1 + \alpha_2) \).

In most cases in control theory satisfactory results are obtained considering the input signal at the nonlinear element as

\[
v_d = V_o \cos(\omega t + \theta)
\] (5.32)

This is true when all the harmonics are filtered by the linear part of the system. In general it is necessary to study the filtering action in order to determine the number of harmonics which are significant at the input of the active device.

\section*{V - 4 Stability of the Solutions}

The Fourier analysis of the voltage waveform shows that the second harmonic has the biggest influence on the waveform and frequency of operation which is considerably lower than the resonant frequency of the circuit associated with the diode. Multi-
circuits provide independent tuning facilities for the fundamental and second harmonic and negligible power was measured at the third harmonic. The amplitude of the third and fourth harmonics are nearly 10% and 6% respectively and considering the low pass filtering action of the linear part of the system it is reasonable to represent the voltage waveform across the device by the expression

\[ v(t) = V_0 + V_1 \cos(\omega t) + V_2 \cos(2 \omega t + \theta) \]  

(5.33)

The consideration of this waveform across the device is analyzed for every coupling circuit to prove its suitability. At this stage it is convenient to express the current and voltage relative to a chosen d.c. operating point \( V_0 \) and \( I_0 \). For each particular circuit the d.c. terms must be studied independently because d.c. terms are present not only due to the bias voltage but also from possible rectification by the I-V characteristic of the active device. Therefore

\[ v(t) = V_1 \cos(\omega t) + V_2 \cos(2 \omega t + \theta) \]  

(5.34)

\[ i(t) = I_1 \cos(\omega t + \delta) + I_2 \cos(2 \omega t + \gamma) \]  

(5.35)

The set of equations, given by (5.23), to be satisfied is

\[ N\omega L(j\omega) + 1 = 0 \]  

(5.36)

\[ N_{2\omega} L(j2\omega) + 1 = 0 \]  

(5.37)

Since both \( N_\omega \) and \( N_{2\omega} \) are in general complex, equations (5.36) and (5.37) give rise to four equations. The solution of these equations determines the steady state operating point \( V_{10}, V_{20}, \omega_0 \) and \( \theta \) and leads to the values of the diode admittance at the
fundamental and second harmonic frequencies.

Once a solution has been obtained for the set of equations (5.36) and (5.37) we are interested in knowing whether the operating point is stable or not. For a small change in the voltage amplitude \( \Delta V \), the operating point is stable if \( \Delta V \) decays with time, or in other words, the damping factor \( \sigma \) is greater than zero \( (e^{-\sigma t} > 0) \)

As we have used D.F. theory to linearize the system it is convenient to follow the discussion of Loeb\(^{(62)}\) upon stability although the original method is due to Cohen\(^{(63)}\). The same results have been obtained by Popov\(^{(64)}\) using geometrical considerations. The present study extends the stability criterion for multiharmonic waveforms, whereas only the simplest case of pure sinusoidal waveform was studied by Cohen, Loeb and Popov.

It is easily shown that the term \( \frac{1}{L(j\omega)} \) of equation (5.23) is the admittance presented by the passive circuit across the diode. Therefore,

\[
\frac{1}{L(j\omega)} = Y(j\omega) = G(\omega) + j B(\omega) \quad (5.38)
\]

For the operating point, we have from equations (5.36) and (5.37)

\[
Y(j\omega_0) + N(2\chi V_{10}\cdot V_{20}\cdot \theta_0) = 0 \quad (5.39)
\]

\[
Y(j2\omega_0) + N(2\omega V_{10}\cdot V_{20}\cdot \theta_0) = 0 \quad (5.40)
\]

Remembering that \( N_\omega \) corresponds to the diode admittance and has both real and imaginary parts we see that the fundamental equations determining the conditions for steady state oscillations can be expressed as
\[ G_{\text{diode}} + G_{\text{circuit}} = 0 \]  \hspace{1cm} (5.41)

\[ B_{\text{diode}} + B_{\text{circuit}} = 0 \]  \hspace{1cm} (5.42)

Separating the real and imaginary parts

\[ p_1(V_{1o}, V_{2o}, \theta, \omega) + j q_1(V_{1o}, V_{2o}, \theta, \omega) = 0 \]  \hspace{1cm} (5.43)

\[ p_2(V_{1o}, V_{2o}, \theta, \omega) + j q_2(V_{1o}, V_{2o}, \theta, \omega) = 0 \]  \hspace{1cm} (5.44)

where

\[ p_k = \text{Re} \left[ Y_c(k\omega) + N_k \right] \]  \hspace{1cm} (5.45)

\[ q_k = \text{Im} \left[ Y_c(k\omega) + N_k \right] \]  \hspace{1cm} (5.46)

For a small perturbation about the operating point let

\[ V_1 + V_{1o} + \delta V_1 \]  \hspace{1cm} (5.47)

\[ V_2 + V_{2o} + \delta V_2 \]  \hspace{1cm} (5.48)

\[ \theta + \theta_o + \delta \theta \]  \hspace{1cm} (5.49)

\[ \omega + \omega_o + \delta \omega + j \delta \sigma \]  \hspace{1cm} (5.50)

If the operation is to remain stable we have

\[ p_1(V_{1o} + \delta V_1, V_{2o} + \delta V_2, \theta_o + \delta \theta, \omega_o + \delta \omega + j \delta \sigma) + j q_1(V_{1o} + \delta V_1, V_{2o} + \delta V_2, \theta_o + \delta \theta, \omega_o + \delta \omega + j \delta \sigma) = 0 \]  \hspace{1cm} (5.51)

and

\[ p_2(V_{1o} + \delta V_1, V_{2o} + \delta V_2, \theta_o + \delta \theta, \omega_o + \delta \omega + j \delta \sigma) + j q_2(V_{1o} + \delta V_1, V_{2o} + \delta V_2, \theta_o + \delta \theta, \omega_o + \delta \omega + j \delta \sigma) = 0 \]  \hspace{1cm} (5.52)
Since \( \delta V_1, \delta V_2, \delta \theta, \delta \omega \) and \( \delta c \) are small quantities we can carry out a first order expansion about the operating point using a Taylor expansion. This gives

\[
p_1(V_{10}, V_{20}, \theta_0, \omega_0) + \frac{\partial p_1}{\partial V_1} \delta V_1 + \frac{\partial p_1}{\partial V_2} \delta V_2 + \frac{\partial p_1}{\partial \theta} \delta \theta + \frac{\partial p_1}{\partial \omega} \delta \omega +
\]

\[
\frac{\partial q_1}{\partial \omega} \delta \omega + j \frac{\partial q_1}{\partial \sigma} \delta \sigma = 0
\]

Therefore

\[
\frac{\partial p_1}{\partial V_1} \delta V_1 + \frac{\partial p_1}{\partial V_2} \delta V_2 + \frac{\partial p_1}{\partial \theta} \delta \theta + \frac{\partial p_1}{\partial \omega} \delta \omega = 0
\]  

(5.54)

\[
\frac{\partial q_1}{\partial V_1} \delta V_1 + \frac{\partial q_1}{\partial V_2} \delta V_2 + \frac{\partial q_1}{\partial \theta} \delta \theta + \frac{\partial q_1}{\partial \omega} \delta \omega = 0
\]  

(5.55)

Eliminating the \( \delta \omega \) terms leads to

\[
\left[ \frac{\partial p_1}{\partial V_1} \delta V_1 + \frac{\partial p_1}{\partial V_2} \delta V_2 + \frac{\partial p_1}{\partial \theta} \delta \theta \right] \frac{\partial q_1}{\partial \omega} - \left[ \frac{\partial q_1}{\partial \omega} \right]^2 \delta \sigma -
\]

\[
\left[ \frac{\partial q_1}{\partial V_1} \delta V_1 + \frac{\partial q_1}{\partial V_2} \delta V_2 + \frac{\partial q_1}{\partial \theta} \delta \theta \right] \frac{\partial p_1}{\partial \omega} - \left[ \frac{\partial p_1}{\partial \omega} \right]^2 \delta \sigma = 0
\]  

(5.56)

i.e.

\[
\left[ \frac{\partial p_1}{\partial V_1} \delta V_1 + \frac{\partial p_1}{\partial V_2} \delta V_2 + \frac{\partial p_1}{\partial \theta} \delta \theta \right] \frac{\partial q_1}{\partial \omega} -
\]

\[
\left[ \frac{\partial q_1}{\partial V_1} \delta V_1 + \frac{\partial q_1}{\partial V_2} \delta V_2 + \frac{\partial q_1}{\partial \theta} \delta \theta \right] \frac{\partial p_1}{\partial \omega} = \left[ \frac{\partial p_1}{\partial \omega} \right]^2 + \left[ \frac{\partial q_1}{\partial \omega} \right]^2 \delta \sigma
\]  

(5.57)
and an identical equation with \( p_1 \) and \( q_1 \) replaced by \( p_2 \) and \( q_2 \) respectively. For the perturbed condition to decay back to the original stable operation \( \delta \theta \) must be positive. Recognising the bracketed terms of the left hand side of equation (5.57) as the total change in \( p_1 \) and \( q_1 \) brought about by changes in \( V_{10}, V_{20} \) and \( \theta_0 \), \( \Delta p_1 (V_{10}, V_{20}, \theta_0) \) and \( \Delta q_1 (V_{10}, V_{20}, \theta_0) \) say, leads to the equations

\[
\frac{\Delta p_1 (V_{10}, V_{20}, \theta_0)}{\Delta q_1 (V_{10}, V_{20}, \theta_0)} - \frac{\partial p_1}{\partial \omega} \frac{\Delta \omega}{\Delta q_1} > 0 \quad (5.58)
\]

and

\[
\frac{\Delta p_2 (V_{10}, V_{20}, \theta_0)}{\Delta q_2 (V_{10}, V_{20}, \theta_0)} - \frac{\partial p_2}{\partial \omega} \frac{\Delta \omega}{\Delta q_2} > 0 \quad (5.59)
\]

Equation (5.58) is equivalent to Kurokawa's condition for stability of a single frequency oscillator presented in section III-3, and we see that for stability of a non sinusoidal oscillator the corresponding conditions must be satisfied at both the fundamental frequency and at each of the harmonic frequencies. Thus Kenyon's graphical interpretation of the stability condition expressed in terms of the angle between the load line and the device line must also be satisfied at the fundamental and harmonic frequencies, as indicated in Figure (5.9).
SINGLE RESONANT CIRCUIT (THEORETICAL)
PARALLEL L C G CIRCUIT

\[ v = V_1 \cos(\omega t) + V_2 \cos(2\omega t + \phi) \]

\[ L = 4.77 \text{ nH} \]
\[ C = 0.37 \text{ pF} \]

\[ 0 < \phi < 180 \]

STABILITY CONDITIONS FOR THE
FUNDAMENTAL AND SECOND HARMONIC

FIG.(5.9)
The cubic characteristic has been thoroughly studied using different numerical methods, including the describing function technique for pure sinusoidal oscillations \((58),(59)\). In this Chapter we present a detailed analysis of the cubic characteristic for a multiharmonic voltage waveform across the active device using the describing function technique and the results are compared with those obtained using the familiar perturbation method.

In particular we emphasize the advantage of the D.F. technique where no restriction is imposed upon the accuracy as compared with the perturbation method. The analysis of the stability and input waveform for a possible d.c. term in the input signal to the non-linear element is also discussed. Finally the inclusion of the quadratic term of the voltage in the I-V characteristic leads to some interesting modes of oscillation with a strong dependence upon the load conductance.

VI - 1  The Cubic Characteristic

The instantaneous current and voltage of the non-linear voltage controlled negative resistance are related by the curve plotted in Figure (6.1). The analytical expression of this curve,
CUBIC CHARACTERISTIC  $i = ay + by^3$

FIG. (6.1)
with odd order symmetry is given by

\[ i(t) = a v(t) + b v(t)^3 \quad (6.1) \]

For a small signal unstable element the negative conductance condition implies that \( a < 0 \) and since the amount of power available is limited: \( b > 0 \). Devices with a symmetrical characteristic with respect to the bias point \( y(x) = -y(-x) \) do not exhibit a d.c. term in the output. However for non symmetric characteristics the presence of a d.c. term leads to a more complex signal at the input of the non linear element. Such complexity may be avoided by shifting the characteristic to a point where a sinusoidal input signal results in an unbiased output signal. Because we want to study the oscillation conditions and stability in the most general form, we have included the d.c. term in the voltage across the diode for the different circuits. This requires a separate study of the characteristic equation for \( \omega = 0 \) in order to ensure the existence of a limit cycle for the d.c term.

For an input signal \( v = V_0 + V_1 \cos(\omega t) \) the characteristic equations for \( \omega = 0 \) and \( \omega = \omega_0 \) are:

\[
\begin{align*}
N_{0} (V_0, V_1) L(j \omega) & = -1 \quad (6.2) \\
V_0 N_{0} (V_0, V_1) L(j 0) & = -V_0 \quad (6.3)
\end{align*}
\]

Equation (6.3) is immediately satisfied if \( V_0 = 0 \). However this solution has very little interest. For \( V_0 \neq 0 \) we have

\[
N_{0} (V_0, V_1) L(j 0) = -1 \quad (6.4)
\]

where \( L(j 0) \) represents the transfer function of the linear part of the system for \( \omega = 0 \). Equations (6.4) and (6.2) define the limit cycle.
Any periodic output signal of an asymmetrical non linearity with limit cycle contains harmonics. Therefore the condition of low pass filter for the linear part of the system is absolutely necessary in order for the D.F. to have full significance.

Once the non linear element has been characterized by its describing function \( N_0 \) corresponding to \( \omega = 0 \) it is necessary to study the behaviour of the linear part of the system for this same signal. For this purpose the transfer function \( L(s) \), where \( s = j \omega \), which represents the linear part of the system may be written as

\[
L(s) = \frac{s^n(s+z_1)(s+z_2) \ldots (s+z_i)}{s^m(s+p_1)(s+p_2) \ldots (s+p_j)} \quad (6.5)
\]

\[
L(s) = \frac{s^n}{s^m} L_1(s) \quad (6.6)
\]

where \( n = \) number of zeros at the origin

\( m = \) number of poles at the origin

and \( L_1(s) \) has neither zeros nor poles at the origin. Depending on the values of \( n \) and \( m \), the conditions to satisfy equation (6.4) are:

<table>
<thead>
<tr>
<th>( L(s) )</th>
<th>Condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>( s^n L_1(s) )</td>
<td>( n &gt; 1 )</td>
</tr>
<tr>
<td>( L_1(s) )</td>
<td>( N_0(V_0, V_i) L_1(j \omega) = -1 )</td>
</tr>
<tr>
<td>( L_1(s) )</td>
<td>( m &gt; 1 )</td>
</tr>
</tbody>
</table>

TABLE I
It is necessary to study the stability conditions for any particular system and the conditions imposed on the d.c. term \( V_0 \) must be checked carefully.

**VI - 2 Parallel L C G\(_L\) Circuit**

Let us consider a non-linear device with an I-V characteristic defined by equation (6.1) which is coupled to the load conductance \( G_L \) through a parallel L C circuit as shown originally in Figure (2.5) and which has been re-drawn again for convenience in Figure (6.2). The transfer function of the linear part of the circuit is given by

\[
L(j\omega) = \frac{1}{G_L + j(\omega C - \frac{1}{\omega L})} \quad (6.7)
\]

\[
L(s) = \frac{s.L}{s^2L.C. + s.L.G_L + 1} \quad (6.8)
\]

The I-V characteristic is symmetric with respect to the bias point and the stability condition for this coupling circuit establishes, from TABLE I, that \( V_0 = 0 \). Due to the absence of any quadratic term which may produce any second harmonic component, the voltage waveform across the diode is considered to be

\[
v(t) = V_1 \cos(\omega t) + V_3 \cos(3\omega t + \theta) \quad (6.9)
\]

The consideration of the third harmonic in the waveform expands the pure sinusoidal analysis for the cubic characteristic presented by Gustafsson et al (61).

Substituting equation (6.9) into equation (6.1) and expand-
Parallel Circuit with Cubic Characteristic

FIG. (6.2)

Feedback Model for the System of FIG. (6.2)

FIG. (6.3)
ding up to the third harmonic we obtain:

\[ i(t) = (-aV_1 + \frac{3}{2} V_1^2 b + \frac{3}{2} V_1 V_3 b \cos(\omega t) + \frac{3}{4} V_1^2 V_3 b \cos(\omega t + \theta) + \]

\[ (-aV_3 + \frac{3}{2} V_1 V_3 b + \frac{3}{2} V_3^2 b \cos(3\omega t + \theta) + \frac{1}{4} V_1^2 \cos(3\omega t) \]

\[ (6.10) \]

The describing functions for the fundamental and third harmonic are given by

\[ N_\omega = \frac{I_1}{V_1 e^{j(\phi_i - \phi_V)}} = -a + \frac{3}{2} V_1^2 b + \frac{3}{2} V_1 V_3 b e^{j\theta} \]

\[ (6.11) \]

\[ N_{3\omega} = \frac{I_3}{V_3 e^{j(\phi_i - \phi_V)}} = -a + \frac{3}{2} V_1^2 b + \frac{3}{2} V_3^2 b + \frac{1}{4} \frac{V_1}{V_3} b e^{-j\theta} \]

\[ (6.12) \]

\( N_\omega \) and \( N_{3\omega} \) can be written as

\[ N_\omega = D_1 + jD_2 \]  

\[ (6.13) \]

\[ N_{3\omega} = D_3 + jD_4 \]  

\[ (6.14) \]

where

\[ D_1 = -a + \frac{3}{2} V_1^2 b + \frac{3}{2} V_1 V_3 b \cos(\theta) \]

\[ (6.15) \]

\[ D_2 = \frac{3}{2} V_1 V_3 b \sin(\theta) \]

\[ (6.16) \]

\[ D_3 = -a + \frac{3}{2} V_1^2 b + \frac{3}{2} V_3^2 b + \frac{1}{4} \frac{V_1}{V_3} b \cos(\theta) \]

\[ (6.17) \]

\[ D_4 = -\frac{1}{4} \frac{V_3}{V_1} b \sin(\theta) \]

\[ (6.18) \]
The feedback model for the system of Figure (6.2) is shown in Figure (6.3) where the active element is represented by \( N_k \), the describing function or dynamic admittance of the active element at frequency \( \omega_K \). The characteristic equations for \( \omega \) and \( 3\omega \) are

\[
N \frac{1}{\omega G_L + j(\omega C - \frac{1}{\omega L})} + 1 = 0 \quad (6.19)
\]

\[
N \frac{1}{3\omega G_L + j(3\omega C - \frac{1}{3\omega L})} + 1 = 0 \quad (6.20)
\]

Separating the real and imaginary parts

\[
D_1 + G_L = 0
\]

\[
D_2 + \omega C - \frac{1}{\omega L} = 0
\]

\[
D_3 + G_L = 0
\]

\[
D_4 + 3\omega C - \frac{1}{3\omega L} = 0
\]

(6.21)

The simultaneous solution of this set of non-linear equations (6.21) defines the characteristics of the steady state oscillation. The solution provides the values of the different voltage amplitudes \( V_k \), frequency of operation and phase angle. A detailed explanation of the iterative method used in solving this set of equations is given in Appendix I.

To check the accuracy of this technique we have solved the non-linear differential equation which represents the system of Figure (6.2). Applying Kirchhoff's law and after simplification the equation in terms of voltage \( v(t) \) is
\[ \text{L.C.} \frac{d^2v}{dt^2} - L(a - G_L - 3b v^2) \frac{dv}{dt} + v = 0 \quad (6.22) \]

Equation (6.22) can be written as

\[ \frac{d^2v}{dt^2} - \alpha (1 - \beta v^2) \frac{dv}{dt} \omega_r + v \omega_r^2 = 0 \quad (6.23) \]

where

\[ \alpha = \frac{a'}{\omega_r} \]

\[ a' = a - G_L \]

\[ \beta = \frac{3b}{a'} \]

\[ \omega_r^2 = \frac{1}{LC} \quad (6.24) \]

Equation (6.23) is essentially van der Pol's equation except for the presence of the \( \beta \) and \( \omega_r \) terms, but the parameter \( \alpha \) is entirely equivalent to parameter \( \varepsilon \) of van der Pol's equation in section III-4. We shall use the perturbation method to the second order of the small parameter \( \alpha \) given by Cunningham (39) and Clenshaw (65).

A solution is sought of the form

\[ v(t) = E \left[ \cos(\omega t) + \alpha(c_2 \sin(\omega t) + c_4 \sin(3\omega t)) + \alpha^2(c_1 \cos(\omega t) + c_3 \cos(3\omega t) + c_5 \cos(5\omega t)) + \ldots \right] \quad (6.25) \]

According to Clenshaw, the boundary conditions related to equation (6.25) are

\[ v = E \left( 1 + \frac{\alpha^2}{192} \right) \text{ at } t = 0 \quad (6.26) \]
\[
\frac{dv}{dt} = 0 \quad \text{at } t = 0 \quad (6.27)
\]

Following the analysis presented by Tan and Foulds\(^{144}\) the solution to the second order of \(a\) is

\[
v(t) = 2 \sqrt{\frac{a'}{3b}} \left[ \cos(\omega t) + a\left(\frac{-1}{16} \sin(\omega t) - \frac{1}{8} \sin(3\omega t)\right) + \frac{a^2}{16} \left(\frac{-1}{16} \cos(\omega t) + \frac{3}{32} \cos(3\omega t) - \frac{5}{192} \cos(5\omega t)\right) + \ldots \right]
\]

(6.28)

\[
\omega = \omega_0 \left(1 - \frac{a^2}{16}\right) \quad (6.29)
\]

The slight differences in the \(a^2\) term with Cunningham's solution are due to his assumption of the amplitude of the oscillation being simply \(E\) and independent of the higher order terms. For a second order approximation the frequency of oscillation is very near to the proper resonant frequency. At the resonant frequency the \(L C G\) parallel circuit has the admittance of a pure conductance \(G_L\). This positive conductance appears in parallel with the negative conductance element \(-a\). The effect of this conductance \(G_L\) is primarily to modify the equation for the I-V characteristic to

\[
i(t) = (-a + G_L) v(t) + b v(t)^3 \quad (6.30)
\]

It is clear then that \(E = 2(a'/b)^{\frac{1}{2}}\) will be zero or imaginary if \(a'\) is zero or negative respectively. So a condition for oscillation is: \(G_L < |a|\). Three different values of \(a\) have been considered and compared with the solutions obtained from the describing function technique. These results are presented in TABLE II. In all
<table>
<thead>
<tr>
<th>a</th>
<th>$\omega \times 10^9$</th>
<th>$\theta$</th>
<th>$V_1$</th>
<th>$V_2$</th>
<th>$\alpha$</th>
<th>$E$</th>
<th>$a'$</th>
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<tr>
<td>†</td>
<td>-0.0016</td>
<td>79.009</td>
<td>92.14</td>
<td>0.9429</td>
<td>0.0120</td>
<td></td>
<td></td>
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<tr>
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<td>89.0</td>
<td>0.9429</td>
<td>0.0119</td>
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<td>0.9428</td>
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<td>1.635</td>
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<td></td>
<td></td>
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<td>85.0</td>
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<td>0.3036</td>
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<td>2.850</td>
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<td>2.8284</td>
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</table>

**TABLE II**

Comparison of computed solutions of equations (6.19) and (6.20) using describing functions (†) and solutions of equation (6.22) using the perturbation method (*)
the calculations the values of \( L, C, b \) and \( G_L \) were kept constant to 1.6 nH, 0.1 pf, 0.0012 amp/Volt\(^3\) and 0.0008 mho respectively. The resonant frequency of the circuit is \( \omega_0 = 79.056 \times 10^9 \) rad/sec. We observe that for low values of \( \alpha = 0.1 \), there is an excellent agreement in the results. This is the case when the contribution of the higher order is very small as the relative amplitude of the harmonics decreases as \( \alpha \) decreases. Also, the frequency with only a second order correction agrees perfectly well with the value obtained from the describing function as shown in Figure (6.4). However, as the value of \( \alpha \) increases the departure from the describing function calculations starts increasing. This is due to the fact that we are neglecting the contribution of the harmonics in the higher order terms of \( \alpha \), in particular in the frequency where the second order correction is only true for small values of \( \alpha \) as shown in Figures (6.4) and (6.5).

One can see the advantage of using D.F. analysis because when the third harmonic is considered in the waveform across the diode, this will be the total content of third harmonic in the waveform, in the sense that there is no restrictions or further possible signals across the device which will contribute to the third harmonic. However, the D.F. technique is more tedious when expansion of the current through the device in terms of the different harmonics is not straightforward as in the case of the cubic characteristic, and generally requires the use of a computer.

In order to expand the analysis related to the possible d.c. terms that might be present in the output signal, we have considered the cubic characteristic including the \( v(t)^2 \) term. The
Describing Function and Perturbation Method $\alpha = 0.1012$

Perturbation Method values $\alpha = 0.3036$

VOLTAGE WAVEFORM FOR THE $i=\text{av} + \text{cv}^3$ CHARACTERISTIC

FIG. (6.4)
VOLTAGE WAVEFORM FOR THE $i = -av + cv^3$ CHARACTERISTIC

FIG.(6.5)
new characteristic is given by

\[ i(t) = -a v(t) + b v(t)^2 + c v(t)^3 \]  \hspace{1cm} (6.31)

The \( b v(t)^2 \) term generates a d.c term in the output signal, which will be present in the input signal through the feedback loop. If for the I-V characteristic we are considering \( b > c > 0 \), the voltage across the diode is given by:

\[ v(t) = V_o + V_1 \cos(\omega t) + V_2 \cos(2\omega t + \theta) \]  \hspace{1cm} (6.32)

The describing functions for the d.c, fundamental and second harmonic frequencies are given by

\[ N_0 = -a + b V_0 + \frac{1}{2} \frac{V_1^2 b}{V_0} + \frac{V_2^2 b}{2 V_0} + c V_0^2 + \frac{3}{2} c V_1^2 + \frac{3}{2} c V_2^2 \cos(\theta) \]  \hspace{1cm} (6.33)

\[ N_\omega = -a + \frac{3}{2} c V_2^2 + \frac{3}{4} c V_1^2 + b V_2 e^{j\theta} \]  \hspace{1cm} (6.34)

\[ N_{2\omega} = -a + \frac{3}{2} c V_1^2 + \frac{3}{4} c V_2^2 + \frac{1}{2} V_2^2 e^{-j\theta} \]  \hspace{1cm} (6.35)

In absence of a d.c. signal at the input, the output presents a d.c. term given by

\[ d.c_{out} = \frac{1}{2} b V_1^2 + \frac{1}{2} b V_2^2 + \frac{3}{4} c V_1^2 V_2 \cos(\theta) \]  \hspace{1cm} (6.36)

The condition of stability for the d.c. term can be easily shown using the "Incremental Input Describing Function". For this purpose a perturbation about zero of the input bias is assumed and the associated perturbation about the \( d.c_{out} \) of the output bias is analyzed.
The incremental input describing function (I.D.F.) is defined as

\[ N_{inc}(V_1, V_2, V_0 = 0) = \lim_{\Delta V_0 \to 0} \frac{N_0(V_1, V_2, \Delta V_0)}{\Delta V_0} - d.c. \]  

(6.37)

Equation (6.37) becomes:

\[ N_{inc}(V_1, V_2, V_0 = 0) = \lim_{\Delta V_0 \to 0} N_0(V_1, V_2, \Delta V_0) - (\frac{bV_1^2}{2\Delta V_0} + \frac{bV_2^2}{2\Delta V_0}) + \]

\[ \frac{3cV_1^2V_2\cos(\theta)}{4\Delta V_0} \]

\[ \frac{c(\Delta V_0)^2 + \frac{3}{2}cV_1^2 + \frac{3}{2}cV_2^2 + \frac{3cV_1^2V_2\cos(\theta)}{4\Delta V_0} - \frac{bV_1^2}{2\Delta V_0} - \frac{bV_2^2}{2\Delta V_0}}{4\Delta V_0} = \]

\[ \frac{3cV_1^2V_2\cos(\theta)}{4\Delta V_0} = -a + \frac{3}{2}cV_1^2 + \frac{3}{2}cV_2^2 = K = \text{constant} \]  

(6.38)

So we are interested in the stability of the system of Figure (6.6) where the active element is represented by the constant factor K.

The open loop transfer function of this linear system is given by:

\[ F(s) = \frac{Ks}{s^2L.C. + sL.G_L + 1} \]  

(6.39)

which corresponds to a type zero system and stable as indicated by the zero-pole plane.

The oscillating voltage waveform is shown in Figure (6.7) for two different values of the load conductance \(G_L\). For the lowest value of the conductance the voltage presents almost a relaxation waveform with similar behaviour to an LSA diode in the parallel L.C.G circuit. For higher values of the load conductance the ampli-
EQUIVALENT FEEDBACK MODEL FOR D.C. SIGNAL

FIG. (6.6)

$$F_1 = \frac{s}{s^2LC + sLG_L + 1}$$

VOLTAGE WAVEFORM FOR THE $i = -av + bv^2 + cv^3$ CHARACTERISTIC
IN A PARALLEL L C G CIRCUIT

FIG. (6.7)
tudes for the fundamental and second harmonic decreases, decreasing the ratio $V_1/V_2$ and the phase angle $\theta$ increases and the waveform tends to be more sinusoidal. This same behaviour will be discussed in more detail with LSA devices.
CHAPTER VII

CHARACTERIZATION OF AN LSA DIODE OPERATING IN A SINGLE RESONANT CIRCUIT

A single resonant circuit with an active device operating in LSA mode is studied both theoretically, using multiple input describing functions, and experimentally by means of a network analyzer. The multiple input describing function technique provides valuable information about the device line which characterizes the active element, and therefore it is possible to study the stability of the different operating points. The good agreement obtained between the theoretical and experimental results shows the describing function technique as a potential tool in the design and synthesis of single resonant circuits.

VII - 1 LSA Computer Simulations Using Describing Functions

The I-V characteristic for the limited charge accumulation mode of operation was introduced in section III-4, and for the parameters of the diodes used the analytical expression for this characteristic is given by

\[ I = G_o \frac{V + A V^h}{1 + B V^h} \]  \hspace{1cm} (7.1)

where \( G_o = 0.2 \text{ mho} \), \( A = 3.77 \times 10^{-6} \text{ Volt}^{-3} \) and \( B = 3.61 \times 10^{-7} \text{ Volt}^{-4} \).
The threshold voltage for this characteristic has a value of \( V_{th} = 35 \) V. The oscillation waveform will be basically an inductive relaxation below \( V_{th} \) and damped sine wave above threshold.

The first circuit we have considered is a single resonant parallel circuit, originally studied in Chapter III and shown in Figure (7.1). The current is a single valued function of the voltage and is given by:

\[
I = - I_0 + \frac{V + NV_{th} + A(V + NV_{th})^4}{1 + B(V + NV_{th})^4} G_0
\]  

(7.2)

where \( I_0 = I(V)|_{V = NV_{th}} \). Equation (7.2) represents the current variation around \( I_0 \) corresponding to the voltage variation around \( V = NV_{th} \), where \( N > 1 \). In this analysis we have assumed that there is no hysteresis in the I-V characteristic and that it is also time invariant. This is reasonable for the case of a LSA relaxation oscillator in which there is no significant accumulation of charge during any part of the rf cycle. Although experimentally we have used distributed circuit elements, the consideration of lumped circuit elements as in Figure (7.1) provides very useful information about the device - circuit interaction and will also help in the comparison with the transmission line circuit analysis.

For a specific voltage waveform across the diode the expansion of the current in terms of the different harmonics is not straightforward as in the case of the cubic characteristic, and therefore the calculation of the describing functions will require the use of a computer. The Fourier analysis of the voltage waveform in section V-7 shows that the consideration of the fundamental and
PARALLEL L C G CIRCUIT WITH LSA DIODE

**FIG. (7.1)**

\[ f = \frac{1}{T} \text{ (GHz)} \]

**FIG. (7.2)**

Frequency bias dependence obtained by computer solutions of equations (7.4) and (7.5) using describing function technique.
second harmonic provides a reasonable representation of the voltage waveform across the diode. Therefore the voltage is given by

\[ v(t) = V_1 \cos(\omega t) + V_2 \cos(2\omega t + \phi) \quad (7.3) \]

where the condition \( V_0 = 0 \) is imposed by the linear part of the system as shown in section VI-1, if a limit cycle is to be reached. For the different circuits described in this analysis, there is no rectification, i.e. the d.c. voltage remains at the bias voltage \( V_B \).

If we represent \( N_\omega \) and \( N_{2\omega} \) by their real and imaginary parts the fundamental equations (5.36) and (5.37) expressed in terms of the device and circuit admittances are written very easily for the simple parallel circuit and take the form

\[ \text{Re} |N_\omega| + G_L = 0 \quad (7.4a) \]

\[ \text{Re} |N_{2\omega}| + G_L = 0 \quad (7.5a) \]

\[ \text{Im} |N_\omega| + \omega C - \frac{1}{\omega L} = 0 \quad (7.4b) \]

\[ \text{Im} |N_{2\omega}| + 2\omega C - \frac{1}{2\omega L} = 0 \quad (7.5b) \]

where \( N_k \) are functions of \( V_1, V_2 \) and \( \phi \) but they are not dependent upon \( \omega \) as no terms of the form \( \frac{dR_N}{dt} \) exist in the I-V characteristic.

Steady state oscillations exist to these equations only if the diode is not too heavily loaded. A typical result of frequency versus bias voltage is shown in Figure (7.2) and demonstrates the general behaviour. For X band operation with values for \( L \) and \( C \) of 1.6 nH and 0.1 pf respectively the resulting waveform for \( G_L = 0.0046 \) mho is shown in Figure (7.3). A full description of
RF VOLTAGE WAVEFORM

\[ V = V_1 \cos(\omega t) + V_2 \cos(2\omega t + \theta) \]

\[ V_B = 3V_{th} \]

\[ g_L = 0.0046 \, \text{S}^{-1} \]

\[ f = 10.56 \, \text{GHz} \]

FIG.(7.3). PREDICTED VOLTAGE WAVEFORM ASSUMING THE FUNDAMENTAL AND SECOND HARMONIC
the computer program including the Fourier analysis of the conduction current and the several subroutines used to derived the various currents is presented in Appendix I. Figures (7.4a) and (7.4b) show the displacement and conduction currents respectively for \( g_L = 0.0046 \) mho, while the total current is shown in Figure (7.5). Campanelli (66) has pointed out that the effect of changing \( R_L \) from constant with frequency to varying with frequency, was to change the current waveform. This change in the current waveform may have a considerable effect on the efficiency. However the voltage waveform is predominantly determined by the value of \( \frac{n}{f} \). In the computer simulations fixed values for \( R_L \) and low values of \( \frac{n}{f} \) have been used, and predictions for the waveforms were only possible with constant values for \( R_L \).

This results in different shaped current waveforms in comparison with the experimental ones presented in section (IV-4) for different values of \( \frac{n}{f} \) and load conductance. The D.P. technique would be then very useful for the analysis of efficiency from the point of view of the current waveform and load conditions.

VII - 2 Stability of the Computed Solutions

Figures (7.6) and (7.7) show the variation of the circuit admittance with frequency for the fundamental and second harmonic frequencies respectively. We observe that the circuit behaves inductively for the fundamental and capacitively for the second harmonic. This behaviour is a general characteristic for all the circuits both theoretical and experimental we have considered and it is discussed in detail in Chapter VIII. The diode admittance locus or device line was plotted by obtaining different
$V_B = 3 V_{th}$

$G_L = 0.0046 \, \Omega^{-1}$

$f = 10.56 \, \text{GHz}$

DISPLACEMENT CURRENT

FIG.(7.4a)
\[ V_B = \frac{3}{V_{th}} \]
\[ q_L = 0.0016 \ \Omega^{-1} \]
\[ f = 10.56 \ \text{GHz} \]

**CONDUCTION CURRENT**

**FIG. (7 kb)**

![Graph showing conduction current over time](attachment:graph.png)
$V_B = 3 V_{th}$

$G_L = 0.0046 \, \Omega^{-1}$

$f = 10.56 \, \text{GHz}$

TOTAL CURRENT

FIG. (7,5)
COMPUTED OPERATING POINTS AND STABILITY CONDITION AT THE FUNDAMENTAL FREQUENCY FOR THE CIRCUIT OF FIG.(7.1)
SECOND HARMONIC FREQUENCY FOR THE CIRCUIT OF FIG. (7.1)

COMPUTED OPERATING POINTS AND STABILITY CONDITION FOR THE

\[ 0 < \psi < \pi \]

\[ \phi \]

\[ \omega \]

\[ \theta \]

\[ \alpha \]

\[ \beta \]

\[ \gamma \]

\[ \delta \]

\[ \epsilon \]

\[ \zeta \]

\[ \eta \]

\[ \theta \]

\[ \vartheta \]

\[ \kappa \]

\[ \lambda \]

\[ \mu \]

\[ \nu \]

\[ \xi \]

\[ \omicron \]

\[ \pi \]

\[ \rho \]

\[ \sigma \]

\[ \tau \]

\[ \upsilon \]

\[ \phi \]

\[ \chi \]

\[ \psi \]

\[ \omega \]

\[ \varepsilon \]

\[ \zeta \]

\[ \eta \]

\[ \theta \]

\[ \vartheta \]

\[ \kappa \]

\[ \lambda \]

\[ \mu \]

\[ \nu \]

\[ \xi \]

\[ \omicron \]

\[ \pi \]

\[ \rho \]

\[ \sigma \]

\[ \tau \]

\[ \upsilon \]

\[ \phi \]

\[ \chi \]

\[ \psi \]

\[ \omega \]

\[ \varepsilon \]

\[ \zeta \]

\[ \eta \]

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\[ \phi \]

\[ \chi \]

\[ \psi \]

\[ \omega \]

\[ \varepsilon \]

\[ \zeta \]

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\[ \theta \]

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\[ \psi \]

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\[ \varepsilon \]

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\[ \nu \]

\[ \xi \]

\[ \omicron \]

\[ \pi \]

\[ \rho \]

\[ \sigma \]

\[ \tau \]

\[ \upsilon \]

\[ \phi \]

\[ \chi \]

\[ \psi \]
operating points as the load conductance was changed and it was checked that the diode would continue to oscillate in the transitions from one point to the next. Both operating points $P_{0i}$ and $P_{1i}$ satisfy Kenyon's stability criterion as $0 < \psi < \pi$. A more formal approach than the graphical interpretation of Kenyon to the study of the stability of the operating points would be by assuming small variations around $P_{0i}$ and $P_{1i}$ and using the incremental input describing function as shown in section VI-5. For a fixed value of $G_L$, the admittance locus has a single intersection with the device line and the selection of the operating point if this is stable is primarily determined by the load conductance and the voltage amplitude across the device. For a more complex circuit the admittance locus presents one or more loops at both the fundamental and second harmonic frequencies and therefore several operating points are possible and the selection of a stable solution requires a detailed study of the device - circuit interaction as presented in Chapter VIII. However very useful information concerning the stability of the solution may be gained by the analysis of the single intersection case considered in this section.

The rf voltage amplitude is expected to be small just after the bias supply is turned on, as the diode presents a very high conductance. Under this condition the steady state oscillation condition may be expressed as

$$|\bar{Y}(0) - Y(\omega)| V = 0$$

where $\bar{Y}(0)$ is the diode admittance for small signal. The voltage $V$ is not zero whether due to the perturbation introduced when the bias source was switched on or due to noise, and because $Y(\omega)$ is not equal to $\bar{Y}(0)$ as shown in Figure (7.9) for any real value of $\omega$. 
in order to satisfy equation (7.6) \( \omega \) must be complex i.e.,

\[
\omega = \omega' - j\alpha 
\]  

(7.7)

The time factor corresponding to \( \omega \) is \( e^{j\omega t} \cdot e^{\alpha t} \), consequently a positive value of \( \alpha \) indicates that the amplitude will increase with time. For the parallel circuit of Figure (7.1) we have

\[
Y(\omega) = G_L + j\left(\omega C - \frac{1}{\omega L}\right) 
\]  

(7.8)

where \( G_L \) includes the load conductance and possible losses in the cavity. In the proximity of resonance:

\[
\omega = \omega_r + \Delta\omega 
\]  

(7.9)

\[
Y(\omega) = j\left((\omega_r + \Delta\omega) C - \frac{1}{(\omega_r + \Delta\omega) L}\right) + G_L 
\]

\[
= 2j\Delta\omega C + G_L 
\]

(7.10)

where \( \omega_r = \frac{1}{\sqrt{LC}} \) and the expansion

\[
\frac{1}{1 + \frac{\Delta\omega}{\omega_r}} = 1 - \frac{\Delta\omega}{\omega_r} + \left(\frac{\Delta\omega}{\omega_r}\right)^2 - \ldots
\]

to the first order of \( \Delta\omega \) has been used. Equation (7.10) can be written as

\[
2j\Delta\omega C + G_L = 2j(\omega' - \omega_r) C + G_L = 2j(\omega' - j\alpha - \omega_r) C + G_L 
\]

\[
= 2j(\omega' - \omega_r) C + 2\alpha C + G_L 
\]

(7.11)

The values of the susceptance and conductance corresponding to equa-
tion (7.11) are indicated in Figure (7.9). The admittance locus \( Y(\omega) \) is a line parallel to the imaginary axis and the \( \alpha = \text{constant} \) loci are lines parallel to the admittance locus, while \( \omega' = \text{constant} \) are lines perpendicular to \( Y(\omega) \). The right hand side of the admittance locus corresponds to positive values of \( \alpha \) while the left hand side is for negative values of \( \alpha \). To reach the steady state operation \( \alpha \) must be positive for the circuit under consideration. That means that the device-circuit interaction makes the voltage amplitude increase in value, shifting the operating point along the device line towards the imaginary axis until it meets the admittance locus. Figure (7.9) shows that not only the amplitude changes but also the frequency, as the operating point moves, because the value of \( \omega' \) changes along the device line. When the operating point reaches the intersection of both the admittance locus and the device line we have

\[
Y(V_0) - Y(\omega_0') = 0 \tag{7.12}
\]

which is equivalent to

\[
2 \begin{smallmatrix} C \quad (\omega_0 - \omega_r) \end{smallmatrix} + B_{\text{diode}} = 0 \quad (7.13a)
\]

\[
G_L + G_{\text{diode}} = 0 \quad (7.13b)
\]

\( V_0 \) and \( \omega_0 \) are the voltage amplitude and frequency of oscillation of the operating point for steady state condition. If the voltage amplitude experiences a further increase, the device-circuit interaction will make it decrease to its original value, because for values of \( V > V_0 \), \( \alpha \) is negative. A more general discussion independently of the circuit configuration is given by Slater\textsuperscript{[67]} and Kurokawa\textsuperscript{[68]}.
EXPLAINING THE ONSET OF THE OSCILLATION IN A SINGLE TUNED OSCILLATOR

FIG. (7.9)
VII - 3 Efficiency and Load Conductance

The normalized resistance \( -R_{\text{rf}}/R_o \) as a function of rf voltage amplitude for the fundamental and second harmonic frequencies is shown in Figure (7.10) and Figure (7.11) respectively and these confirm that the voltage amplitude increases for decreasing values of the load conductance as shown in section III-5. If the load resistance is larger the rf voltage amplitude will increase until the diode's negative resistance has the same absolute value. An advantage of using \( R_o \) is that its value for a particular diode can be measured with an ohmmeter.

The bias voltage and the parameters \( L \) and \( C \) were kept constant for all operating points at \( V_B = 3 V_{\text{th}}, L = 1.6 \text{ nH and } C = 0.1 \text{ pf.} \) A maximum theoretical value for the efficiency of 25.8% was obtained for the fundamental, the frequency being 11.08 GHz and the load conductance \( G_L = 9 \times 10^{-3} \text{ mho.} \) It happens that the GaAs I-V characteristic is sufficiently non linear to allow the device to match a variety of loads and retain the same waveshape. However for high values of the load conductance, that is \( G_L > 0.01 \text{ mho,} \) the second harmonic amplitude and the phase angle increase rapidly, and the voltage waveform loses its relaxation shape as shown in Figure (7.12).

Due to the high value of the conductance, the efficiency increases but the device is not operating in LSA as the average conductance during one complete rf cycle is negative (\( \approx -0.0212 \text{ mho) } \). If a domain is formed, due to operation in some other mode, breakdown may occur if overlength samples are used. We have not observed this behaviour experimentally because as it is shown later if the device is heavily loaded any oscillation will stop.
EFFICIENCY AND AVERAGE RF RESISTANCE/LOW FIELD D.C. RESISTANCE AS A FUNCTION OF RF FUNDAMENTAL VOLTAGE AMPLITUDE

FIG.(7.10)
V = 3 Vth
L = 1.6 \times 10^{-9} H
C = 0.1 \times 10^{-12} F
G_L = 0.014 \, \Omega^{-1}
G = -0.0212
f = 9.412 \, GHz

Voltage waveform for a high value of the load conductance. Fig. (7.12)
VII - 4 Considerations of the Waveform at the Input of the Non Linear Active Device

We have considered for the describing function calculation an input waveform for the voltage across the diode described by equation (7.3). A relaxation waveform cannot be represented by a single sinusoidal signal. For that reason multiple input describing functions have been used. It was mentioned in section V-5 that the linear part of the system must provide enough low pass filtering to the signal at the output of the non linear element. The transfer function of the linear part of the system of Figure (7.1) is given by

\[ L(j\omega) = \frac{1}{G_L + j(\omega C - \frac{1}{\omega L})} = K e^{-j\phi} \]  

(7.14)

where

\[ K = \sqrt{\frac{1}{G_L^2 + (\omega C - \frac{1}{\omega L})^2}} \]  

(7.15)

\[ \phi = \arctan \frac{\omega C - \frac{1}{\omega L}}{G_L} \]  

(7.16)

In the frequency domain the response to an input signal of the type \( x = A \cos(\omega t) \) is given by

\[ y = K.A.\cos(\omega t - \phi) \]  

(7.17)

The gain as a function of frequency for the parallel L C G circuit is shown in Figure (7.13) and bearing in mind the relative amplitudes of the harmonics from the Fourier analysis of the voltage waveform, we conclude that the assumed input waveform of equation (7.3) is a
$G_L = 0.0046 \text{ mho}$

$V_B = 3 V_{th}$

$L = 1.6 \times 10^{-9} \text{ H}$

$C = 0.1 \times 10^{-12} \text{ f.}$

$$K = \sqrt{\frac{1}{G_L^2 + (\omega C - 1/\omega L)^2}}$$

**Fig. (7.13)**

GAIN $K$ AS A FUNCTION OF FREQUENCY $\omega$ FOR THE PARALLEL $L C G_L$ CIRCUIT
good approximation to the real input signal to the non linear element. We have also tried input waveforms considering the second and third harmonics to check if our assumption was correct. It was found that the inclusion of the third harmonic in the waveform has no significant effect on the original values for the fundamental and second harmonic amplitudes, and the frequency changes slightly ($\pm 2.7\%$) as was expected. However the number of non linear equations and unknown variables increases from four to six, and that represents a considerable increase in computer time because making a judicious trial solution is quite difficult with so many variables involved, and also due to the accuracy of the solution of the set of equations we are interested in. Because of the number of programs for different values of the parameters, it was decided not to include the third harmonic.

If initially we look for a sinusoidal voltage solution i.e., $v = V_1 \cos(\omega t)$ and use only equation (7.4), one obtains the results shown in Figure (7.1) which are effectively the same as Hines (24). The LSA mode requires that there is no build up of charge from one cycle to the next. The essential criterion for this can be expressed by the condition: $\bar{G} > 0$ where $\bar{G}$ is the time average diode conductance defined by equation (3.43). Figure (7.1) indicates therefore that if the output is purely sinusoidal the device will begin to work in the LSA mode at the maximum power point and that subsequent increases in rf voltage are associated with a smaller diode conductance and a decrease in power. Efficiency however is maximized at a device voltage slightly beyond this point. This occurs in this range because the d.c. current falls as the drive level is increased.

Using a more realistic input signal $v = V_1 \cos(\omega t) + V_2 \cos(\omega t + \theta)$
FIG. (7.14). CHARACTERISTICS OF A SINGLE PARALLEL CIRCUIT OSCILLATOR
ASSUMING A PURE SINUSOIDAL VOLTAGE WAVEFORM
SINGLE RESONANT CIRCUIT

\[ V = V_1 \cos(\omega t) + V_2 \cos(2\omega t + \theta) \]

\[ \bar{G} = \frac{G}{V_C/I_C} \]

\[ I_1/I_C \]

\[ -\eta_1 \]

\[ \bar{G} \]

\[ \bar{P}/I_C \cdot V_C \]

FIG.(7.15). CHARACTERISTICS OF A SINGLE PARALLEL CIRCUIT OSCILLATOR ASSUMING A NON SINUSOIDAL VOLTAGE WAVEFORM
and using equations (7.4) and (7.5) leads to the results shown in Figure (7.15). Comparing Figures (7.14) and (7.15) shows that including the second harmonic component allows a considerable greater fundamental voltage swing. Noting the point at which \( G = 0 \) implies that the device could work in the LSA mode on the low voltage side of the peak power point as well as the high voltage side as restricted by Figure (7.14).

The frequency of operation for Figures (7.14) and (7.15) is in S band and the values of \( L \) and \( C \) are 4.777 nH and 0.37 pf respectively, the bias voltage being \( V_B = 3. V_{th} \).

VII - 5 LSA Single Resonant Circuit. Experimental Results

The oscillator used in the first series of experiments is the simple coaxial circuit shown in Figure (7.16). This is very similar to the circuit described by Wasse. The essential series inductance in this case is variable and is determined by the position of the movable short circuit plunger. This inductance also determines the oscillation frequency, the coupling to the load and hence the output power. The general behaviour of the circuit is shown in Figure (7.17) in terms of the power and frequency variation with the number of turns of the movable short circuit for two different values of the bias voltage \( V_B = 2.65 \, V_{th} \) and \( V_B = 2.5 \, V_{th} \). The frequency decreases when the distance of the short circuit from the diode increases, thereby increasing the inductance.

Experimental measurements were carried out as follows. A fixed bias voltage was chosen so that the diode oscillated steadily for as wide a range of the short circuit plunger as possible. At each setting of the short circuit the fundamental frequency was measured.
FIG. 7.6a, 7.6b

Simple oscillator circuit (after Wasse et al. [69])

Modification for measuring the admittance presented to the oscillating diode
CHARACTERISTICS OF THE EXPERIMENTAL CIRCUIT OF FIG. (7.16)

FIG. (7.17)
together with the power at both fundamental and harmonic frequencies using suitable low and high pass filters. The bias choke system was then dismantled and the unpackaged chip removed, without disturbing the rest of the circuit. With the bias circuit replaced by the coaxial measuring line as indicated in Figure (7.16b) the admittance was measured by a Hewlett-Packard network analyzer at the reference plane originally occupied by the chip over a suitable range of frequencies centered on the fundamental and second harmonic frequencies. The chip and bias network were then replaced, a new short circuit position chosen and the measuring procedure repeated.

For all the settings of the short circuit the output power at the second harmonic remains practically constant and one can assume that the plunger has no influence upon the second harmonic, although there should be an appreciable variation in the voltage amplitude for the second harmonic as the load conductance changes appreciably.

It was also observed that for high values of the bias voltage the diode only oscillates for short distances of the plunger from the diode. This is due to the lower values of the negative conductance of the diode with high values of bias. A maximum power of 18.2 W rf was obtained with this single resonant circuit at \( V_B = 2.5 \ V_{th} \), the frequency being 2.817 GHz and the efficiency 4.95 %. Figure (7.18) shows the different values of efficiency corresponding to different bias voltages.

A typical set of admittance curves for different positions of the short circuit is shown in Figure (7.19). Indicating on each curve the appropriate frequency of oscillation determines the corresponding circuit admittance presented to the chip. The admittance of
FIG. (7.18). \( \eta \sim \frac{V_B}{V_{th}} \) FOR THE EXPERIMENTAL CIRCUIT OF FIG. (7.16)
SINGLE RESONANT CIRCUIT

A = 1 TURN
B = 4 TURNS
C = 5 TURNS

$A_f = 4.08 \text{ GHz}$
$B_f = 3.08 \text{ GHz}$
$C_f = 2.817 \text{ GHz}$

FIG. (7.19)
CIRCUIT ADMITTANCE AS A FUNCTION OF
FREQUENCY AND POSITION OF MOVABLE SHORT
CIRCUIT
the circuit is independent of the bias voltage but the operating point will change as $V_B$ varies. We note that when the plunger is far from the diode, the conductance presented to the diode is very high, hence the high output power. The value of the load conductance limits the number of turns of the plunger, confirming the theoretical results shown in section VII-3 which indicated the cessation of any oscillation for high values of the conductance presented to the diode.

This immediately identifies the device line as well as enabling one to relate output power, frequency and other characteristics as a function of the load admittance presented to the chip. For convenience the device line is replotted in Figure (7.20) and confirms the stability condition described by Kenyon in terms of the angle of intersection of the device line and the circuit line. The variation of output power, diode conductance and fundamental voltage is shown in Figure (7.21).

Over a frequency range of more than an octave the circuit admittance lines shown in Figure (7.19) lie approximately along constant conductance circles, i.e. the circuit does behave like a parallel L C G circuit. The effective capacitance arises from the evanescent modes set up at the T junction of the inner conductors and also at the step at the entrance to the short circuit stub line. The effective values of L and C have been estimated from the measured susceptance variation with frequency. With the chip in place this circuit capacitance is augmented by the chip capacitance (≈ 0.1 pf).

Solving equations (7.4) and (7.5) using the estimated values of L and C and G, leads to the predicted device line shown in Figure (7.20) and the theoretical power variation shown in Figure (7.21).
MEASURED AND PREDICTED DEVICE LINES FOR THE DIODE
IN THE PARALLEL CIRCUIT

FIG.(7.20)
Fig. (7.21). Diode conductance and output power as a function of the fundamental voltage component.
The variation of frequency with load conductance is shown in Figure (7.22). This corresponds to the decrease of frequency as the separation of the short circuit from the diode increases, increasing therefore the load conductance as shown in Figure (7.17).

The agreement between the theoretical and experimental results of Figures (7.20), (7.21) and (7.22) are very satisfactory bearing in mind the complexity of the T junction of the experimental model, which for high frequencies deviates substantially from the theoretical L C C parallel model. We note that the output power decreases for increasing values of the voltage amplitude and therefore the device works only on the high voltage side of the peak power point shown in Figure (7.14) as suggested by Hines by computer simulations using a pure sinusoidal voltage drive signal. However the presence of the second harmonic in the voltage waveform across the diode is more significant in multi-resonant circuits, particularly when tuning facilities are available for the different harmonics as is presented in next chapter.
FIG. (7.22). FREQUENCY VARIATION WITH LOAD CONDUCTANCE
CHAPTER VIII

CHARACTERIZATION OF AN LSA DIODE OPERATING IN A MULTIPLE RESONANT CIRCUIT

This Chapter presents the detailed analysis of a double resonant circuit similar to the one studied by Kenyon for pure sinusoidal waveforms. The multiple input describing function technique is used to deal with non sinusoidal waveforms and the results are compared with the experimental ones obtained using a modified version of the Wasse's original circuit.

The study is further extended to multiple resonant circuits. A substantially different behaviour compared with the single resonant case is observed experimentally, where one or more loops are present in the locus admittance of the circuit at both the fundamental and second harmonic. The model used in the theoretical analysis shows a similar behaviour to the experimental multiresonant circuit. Special attention is paid to the analysis of the effects of the second harmonic on the behaviour of the fundamental.

VIII - 1 Double Resonant Circuit. Theoretical Analysis.

In this section we study a double resonant circuit which is similar to the circuit studied by Kenyon (38). However we emphasize the usefulness of the describing function technique to obtain the different operating points and in particular the selection of the operating point for multiharmonic waveforms.
The addition of a series resonant circuit to the original parallel circuit as shown in Figure (8.1) makes possible the study of some characteristics that are only associated with multiresonant circuits. The resonant frequency is the same for both circuits and if we define the Q factor for each circuit and the parameter $b$ as

$$Q_1 = \frac{\omega_0 C_1}{G_L} \quad \text{Parallel circuit} \quad (8.1)$$

$$Q_2 = \frac{\omega_0 L_2 G_L}{C_1} \quad \text{Series circuit} \quad (8.2)$$

$$b = B \cdot G_L \quad (8.3)$$

then

$$\frac{\partial b}{\partial \omega} \bigg|_{\omega = \omega_0} = \frac{\partial}{\partial \omega} \left[ \left( \frac{\omega C_1}{\omega L_1} - \frac{1}{\omega L_1} \right) - \frac{\omega L_2}{\omega L_2 - \frac{1}{\omega C_2}} \right]$$

$$= \frac{2C_1}{G_L} - 2L_2 G_L = \frac{2}{\omega} (Q_1 - Q_2) \quad (8.4)$$

We see that the susceptance of the series circuit tends to compensate that of the original parallel circuit in the proximity of resonance ($\omega_0$), and the combination of the two has a broader bandwidth than either single circuit.

The effective Q at $\omega_0$ is the difference $Q_1 - Q_2$ which may be made as low as we desire. The transmission matrix of the coupling circuit is shown in Figure (8.2) and is given by

$$\begin{pmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{pmatrix} = \begin{pmatrix} 1 & j(\omega L_2 - \frac{1}{\omega C_2}) \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ j(\omega C_1 - \frac{1}{\omega L_1}) & 1 \end{pmatrix}$$
SYNTHESIS OF THE DOUBLE TUNED CIRCUIT

FIG. (8.1)

COUPLING CIRCUIT

FIG. (8.2)
\[
\begin{pmatrix}
\frac{\omega^2 L_1 C_2 - 1 + 2\omega^2 L_1 C_1 - (\omega^2 C_1 L_2)^2}{\omega^2 L_1 C_2} & j(\omega L_2 - \frac{1}{\omega C_2}) \\
\frac{j(\omega C_1 - \frac{1}{\omega L_1})}{1} & 1
\end{pmatrix}
\]
(8.5)

The characteristic equations for frequencies \(\omega\) and \(2\omega\) are

\[
N_\omega \frac{G_L \cdot j(\omega L_2 - \frac{1}{\omega C_2})}{\omega^2 L_1 C_2 - 1 + 2\omega^2 L_1 C_1 - (\omega^2 L_2 C_1)^2} + j(\omega C_1 - \frac{1}{\omega L_1}) = 0
\]
(8.6)

\[
N_{2\omega} \frac{G_L \cdot j(2\omega L_2 - \frac{1}{2\omega C_2})}{4\omega^2 L_1 C_2 - 1 + 8\omega^2 L_1 C_1 - (4\omega^2 L_2 C_1)^2} + j(2\omega C_1 - \frac{1}{2\omega L_1}) = 0
\]
(8.7)

The transfer function of the linear part of the system is given by

\[
L(s) = \frac{s(\frac{s^2 L_1 L_2 C_2 G_L}{s^4 L_1 L_2 C_1 C_2 + s^3 L_1 C_1 C_2 + s^2 (L_1 C_1 G_L + L_2 C_2 G_L + L_1 G_L) + s C_2 G_L})}{s^4 L_1 L_2 C_1 C_2 + s^3 L_1 C_1 C_2 + s^2 (L_1 C_1 G_L + L_2 C_2 G_L + L_1 G_L) + s C_2 G_L}
\]
(8.8)

where \(s = j\omega\)

Equation (8.8) corresponds to a zero order type system and therefore the condition \(V_0 = 0\) is a stable one.

The set of equations which describe the steady state solution is
We shall consider three cases corresponding to the relative values of $Q_1$ and $Q_2$.

a) $Q_1 > Q_2$

Figures (8.3) and (8.4) show the circuit admittance variation with frequency and the device lines for the fundamental and second harmonic frequencies respectively. By comparison with the single resonant case the frequency points are relatively crowded at the center of the locus and the conductance changes considerably in this region. From the stability point of view we see that the operating point is stable as $0 < \psi < \pi$. However for this circuit as the device is connected the oscillation generated, will vary continuously in frequency and amplitude and there are no unstable points. The combination of both circuits with similar values for $Q$ may lead to a very low effective $Q$ and therefore the locking range increases considerably.
DOUBLE TUNED CIRCUIT BEHAVIOUR AND OPERATING POINT
POINT AT THE FUNDAMENTAL FREQUENCY. CASE $Q_1 > Q_2$

FIG. (8.3)
Double tuned circuit behavior and operating point at the fundamental frequency, Case $Q_1 = Q_2$.

$\omega_0 = 60.86 \times 10^9$ rad/sec.

$C_1 = C_2 = 0.00006 \, \text{F}$.

$\omega_0 \approx 2 \pi f$.

FIG. 8.5
b) $Q_1 = Q_2$

With this condition a cusp appears in the admittance locus as shown in Figure (8.5). A less linear variation than for the former case is observed for $N(V)$ and therefore this representation is only valid for small variations around the operating point. Very noisy oscillations will take place when the operating point is near the cusp. When $Q_1$ is slightly greater than $Q_2$ the device line may be parallel to the admittance locus ($\psi = 0$) over a considerable band. Figure (8.6) shows the admittance locus for the second harmonic. The operating points at both frequencies are stable.

c) $Q_2 > Q_1$

For this condition a loop appears in the admittance locus $Y(\omega)$. The slope of the device line is very small and we note that three different operating points (0, A, B) are possible, corresponding to the three intersections with the admittance locus. The device line has the slope indicated in Figure (8.7) and therefore the region between $D'$ and $B'$ is unstable because for any intersection $0 > \psi > -\pi$. Tuning the circuit by decreasing the value of $L_2$, changes the circuit admittance locus and assuming that the device line is unchanged, the oscillation frequency will change from 0 to $D'$ at which it jumps to $A'$. Increasing the value of $L_2$ the oscillation frequency moves down to $A$ and then jumps further down to 0. Therefore there exists two regions $B'A'$ and $0D'$ in which oscillation is entirely stable. If a large conductance $G_L'$ is added in parallel all oscillation ceases as shown in Figure (8.8). Figure (8.9) shows the admittance locus and device line for the second harmonic. We note that there is no loop at this frequency and therefore the variation of susceptance is very pronounced.
FIG. (8.7). DOUBLE TUNED CIRCUIT BEHAVIOUR AND OPERATING POINTS AT THE FUNDAMENTAL FREQUENCY

CASE $Q_2 > Q_1$

$G_L = 0.009 \Omega^{-1}$

$\omega_0 = 58.694 \times 10^9 \text{ rad/sec}$

$\omega_A = 96.87 \times 10^9 \text{ rad/sec}$
DOUBLE RESONANT CIRCUIT

SECOND HARMONIC OPERATING POINTS

\[ V = V_1 \cos(\omega t) + V_2 \cos(2\omega t + \theta) \]

FIG. (8.6) \( Q_1 = Q_2 \)
FIG. (8.4) \( Q_1 > Q_2 \)
FIG. (8.9) \( Q_1 < Q_2 \)

\( 0 < \psi < 180 \)
VIII - 2 Selection of the Operating Point

It was introduced in Chapter VII how the operating point is selected for the case of a single resonant circuit and in particular for the L C G parallel circuit. However when the admittance locus presents one or more loops the stability of the different operating points play an important role in the final steady state oscillation. When the bias is applied to the active element the oscillation starts growing according to the increase of the total voltage amplitude across the device terminals. This process corresponds to the shifting of the operating point along the device line from the right \(( G_{\text{diode}} \gg V << )\) towards the imaginary axis \(( G_{\text{diode}} << V >> )\). In general the final operating point is the intersection of the device line and admittance locus which is stable and which lies furtherest on the right and therefore the first point to be met during the setting up of the oscillation. For the case of a loop in the admittance locus, the loci \(\alpha = \text{constant}\) is shown in Figure (8.10), where \(\alpha\) is the imaginary part of the complex frequency \(\omega' = \omega - j\alpha\) introduced in section VII-2. The diode admittance for small signal \(\bar{Y}(0)\) corresponds now to two values of \(\alpha\), one associated with the dotted line and the other with the continuous line at the right.

If we consider that a small voltage exists with \(\alpha < 0\), the frequency will be determined by the position of \(\bar{Y}(0)\) along the device line. the amplitude will decrease exponentially with time and eventually will be zero after some time.

If the small voltage amplitude is associated with \(\alpha > 0\), the amplitude will increase with time and the operating point will move to the left, towards point A. At intersection B, \(\alpha\) is positive, the amplitude increases and the operating point will be selected either at point A or 0
EXPLAINING THE ONSET OF OSCILLATION AND STABILITY OF POSSIBLE OPERATING POINTS IN A MULTIPLE TUNED OSCILLATOR

FIG. (8.10)
depending upon the stability of the corresponding points for the different harmonics. If more than one stable operating point is possible, as shown in Figure (8.10), the computer simulations performed with this double resonant circuit showed that operation will occur at the stable intersection which lies furthest to the right.

The operating point thus created is stable because if the amplitude increases from its steady state value, the point will move to the left where \( \alpha < 0 \), and the amplitude will decay back to its original value. If the amplitude decreases from its steady state value the operating point appears on the \( \alpha \) positive side and the amplitude will increase again. The operating point \( B \) is unstable because if the amplitude decreases the operating point appears in the \( \alpha \) negative region and the amplitude will decrease further with time. When the operating point moves in the \( \alpha > 0 \) region the amplitude will continue increasing in value moving towards point \( A \).

VIII - 3 Double Resonant Circuit. Experimental Results.

Figure (8.11) shows the original simple circuit modified by a double coaxial tuner. The experimental technique is similar to that described earlier. For a particular bias voltage the circuit is tuned to give either maximum or some intermediate power output at the fundamental frequency. In general we found that the first stub selects the oscillation state, being very critical from the point of view of stability and the second stub modifies the output power, that is, modifies the matching conditions. No form of tuning at the second harmonic is available for this circuit and therefore no particular conditions were impo-
SIMPLE OSCILLATOR CIRCUIT WITH ADDITIONAL DOUBLE STUB TUNER

FIG.(8.11)
CIRCUIT ADMITTANCE AND OPERATING POINTS FOR THE DOUBLE RESONANT CIRCUIT

$F_A = 3.25 \text{ GHz}$
$F_C = 3.24 \text{ GHz}$

GRAPH DATA REF. 7910
sed on the output power at the harmonic frequency.

A typical admittance locus for the fundamental frequency is shown in Figure (8.12). The arrows indicate the operating points for the different loci. A maximum output power of 29 W rf was obtained at the fundamental frequency for \( V_B = 3.23 \ V \text{th} \) (\( V \text{th} = 28 \ V \)) and the efficiency was 6.51\%. The inner curve A corresponds to the admittance locus for maximum output power while the outer curve C corresponds to the locus for minimum power. The admittance locus in the same frequency range presented to the diode's reference plane for a non oscillation state corresponds to curve D in Figure (8.13) and we note that the admittance locus is almost a constant conductance. No oscillation is possible due to the high values of the susceptance presented to the diode at the possible operating frequencies (3.15 - 3.25 GHz), in comparison with the values of the susceptance for the case of oscillation, where the susceptance is about the minimum possible value for each particular locus. The admittance locus variation from an oscillation state to a non oscillation state is shown in Figure (8.14).

The stability conditions at the fundamental frequency are shown to a large scale in Figure (8.15). All the operating points satisfy the stability criterion and are therefore stable.

The admittance locus for the second harmonic for the same operating conditions is shown in Figure (8.16). The inner curve corresponds to curve A for the fundamental and the operating point corresponds to a frequency of 6.5 GHz with an output power of 5W rf. The outer curve corresponds to curve C for the fundamental, the frequency being 6.46 GHz and the output power 4.6 W rf. Experimentally a very small variation in power at the second harmonic frequency was observed. Figure (8.17) corres-
TRANSITION FROM OSCILLATION TO NON OSCILLATION STATE

FIG.(8.14)
FIG. (8.15) DEVICE LINE, ADMITTANCE LOCUS AND STABILITY CONDITIONS FOR THE DOUBLE RESONANT CIRCUIT.
\( f_A = 6.5 \text{ GHz} \)

\( \text{POWER}_A = 5 \text{ W} \)

\( f_B = 6.48 \text{ GHz} \)

\( \text{POWER}_B = 4.6 \text{ W} \)

**FIG. (8.16)**

DOUBLE RESONANT CIRCUIT. SECOND HARMONIC ADMITTANCE LOCUS
ponds to the admittance locus for the non oscillation condition and for the possible range of oscillation frequency the values of the susceptibility and conductance are similar to those for an oscillation condition. However as mentioned in Chapter V the oscillation conditions for a non sinusoidal oscillator are more restrictive than for the simple sinusoidal one, in the sense that they must be satisfied by each frequency component of the signal.

The general characteristics of the circuit are shown in Figure (8.18) in terms of the variation of output power, conductance and voltage amplitude.

All the diagrams show immediately that the circuit behaves inductively at the fundamental and capacitively at the second harmonic, and maximum output power corresponds to a relatively high value of the conductance at the fundamental while the conductance has a lower value for each particular admittance locus at the second harmonic.

VIII - 4 Fundamental Frequency Spectrum of the Oscillator

For the spectral analysis of the pulsed oscillator in a double resonant circuit a Hewlett Packard spectrum analyzer was used to monitor the frequency of the oscillator and the relative change in harmonic power as the circuit elements were adjusted. Figure (8.19) shows the typical \( \frac{\sin x}{x} \) frequency spectrum of the fundamental oscillation at 3.23 GHz pulsed for 100 nsec at a repetition rate of 333 Hz. The ordinate scale is proportional to \( \log_{10} \) of the power. The output power at the fundamental was 10 W rf for \( V_B = 3.4 V \) \( \text{th} \) \( \times \) th \( \text{th} = 28 V \). Figure (8.19) shows the envelope of the display which was obtained from the analyzer output terminal connected via a peak detector to an X - Y recorder. The high side
DIODE CONDUCTANCE AND OUTPUT POWER AS A FUNCTION OF THE FUNDAMENTAL VOLTAGE COMPONENT

FIG.(8.18)
$f_0 = 3.23 \text{ GHz}$

PULSE LENGTH $\approx 100$ nsec

PRF $= 333 \text{ Hz}$
FIG. (8.20) FUNDAMENTAL FREQUENCY SPECTRUM OF THE LSA OSCILLATOR

(a) Horizontal: 10 MHz/div. Vertical: 10 dB/div

(b) Horizontal: 3 MHz/div. Vertical: 10 dB/div.
lobe level was due to the presence of some frequency modulation and the
asymmetry was a result of a small amount of residual amplitude modulation.
Figure (8.20) shows the spectrum for \( y = 10 \text{ MHz/cm} \) and \( x = 10 \text{ dB/cm} \).

For a single resonant circuit the shape of the spectrum worsened
considerably although it improved by locking the oscillator to an injected
signal of 20 dB down in power. Figure (8.19) corresponds to the spectrum
after tuning the circuit with the double stub.

VIII - 5  Multiresonant Circuits. Computer Simulations

We have seen from several of the figures that the multiple
tuned circuits exhibit resonant loops at both the fundamental and second
harmonic frequencies. The first multiresonant circuit we have considered
gives a good understanding of the selection of the operating point and
locking range, depending upon the values of \( Q_1 \) and \( Q_2 \), but is not suita­
ble for an adequate study of the second harmonic.

In the oscillator circuit, physical structures near the device
such as the short circuit plunger and the waveguide post of the device
for the bias supply will create large reflections which rotate slowly
with frequency \( \rho \rho e^{-2j\beta l} \) and \( l \) is small \) in the Smith Chart repre­
sentation. However the slide screw tuner, double stub, etc \( .. \) which are
located at a distance from the diode gives a reflection which rotates
quickly with frequency. The quick rotation is the result of the large
electrical distance from the device to the reflection point. The longer
the distance the quicker the phase rotation of the reflection. To syn­
thesize this general behaviour it is necessary to consider a fairly
complicated lumped circuit such as that shown in Figure (8.21a). For
this circuit the load conductance is different for the fundamental and second harmonic. The problem as before is to determine the operating point for steady state oscillation. The analysis follows the same pattern as in section VII-1 for the simple circuit. The transmission matrix of the coupling circuit is obtained as follows.

In the frequency domain, for Figure (8.21b) and Figure (8.21c)

\[ G_2 V_{lk} = I_{ek} - I_{lk} \]  \hspace{1cm} (8.13)

\[ V_{lk} = t_{11} V_{dk} + t_{12} I_{dlk} \]  \hspace{1cm} (8.14)

\[ I_{lk} = t_{21} V_{dk} + t_{22} I_{dlk} \]  \hspace{1cm} (8.15)

\[ I_{ek} = V_{dk} (t_{21} + t_{11} G_2) + I_{dlk} (t_{22} + t_{12} G_2) \]  \hspace{1cm} (8.16)

\[ G_1 V_{2k} = -I_{2k} \]  \hspace{1cm} (8.17)

\[ V_{2k} = T_{11} V_{dk} + T_{12} I_{d2k} \]  \hspace{1cm} (8.18)

\[ I_{2k} = T_{21} V_{dk} + T_{22} I_{d2k} \]  \hspace{1cm} (8.19)

\[ I_{d2k} = -V_{dk} \frac{G_1 T_{11} + T_{21}}{T_{12} G_2 + T_{22}} \]  \hspace{1cm} (8.20)

The total current through the diode is given by

\[ I_{dk} = I_{dlk} + I_{d2k} \]  \hspace{1cm} (8.21)

From equations (8.16) and (8.21) we have

\[ V_{dk} = \frac{I_{ek} - (t_{22} + t_{12} G_2) I_{dk}}{t_{21} + t_{11} G_2} + \frac{I_{d2k} (t_{22} + t_{12} G_2)}{t_{21} + t_{11} G_2} \]  \hspace{1cm} (8.22)

Substituting equation (8.20) in equation (8.22) we have:
FIG. (8.21a) SYNTHESIS OF THE INDEPENDENTLY TUNED CIRCUIT

\[ F_1 = \frac{1}{(1+A.B)(t_{21} + t_{11} G_2)} \]
\[ F_2 = t_{22} + t_{12} G_2 \]

FIG. (8.22) FEEDBACK LOOP
This leads to the feedback model which is a mathematical reformulation of equation (8.23). This model is shown in Figure (8.22).

The characteristic equation of the system is given by

\[ N_k L(j\omega) + 1 = 0 \]  

(8.25)

or in terms of admittances:

\[ N_k + Y_c(j\omega) = 0 \]  

(8.26)

where \( N_k \) is the appropriate describing function or diode admittance and \( Y_c \) the circuit admittance at the frequency \( \omega_k \). For an input signal \( v = V_1 \cos(\omega t) + V_2 \cos(2\omega t + \theta) \), equation (8.25) leads to a set of four non-linear equations which when satisfied simultaneously determine the operating point \((V_{1o}, V_{2o}, \omega_o, \theta_o)\).

Although Figure (8.21a) contains a large number of variable discrete components it is not easy to synthesize the observed admittance-frequency relation of the experimental distributed circuit. The parameters
MULTIPLE RESONANT CIRCUIT

FIG.(8.23b)
PREDICTED BEHAVIOUR FOR THE MULTIRESONANT CIRCUIT
PREDICTED DIODE CONDUCTANCE AND OUTPUT POWER AS A FUNCTION OF THE FUNDAMENTAL VOLTAGE COMPONENT

FIG.(8.24)
have therefore been chosen to give the essential resonant behaviour at the fundamental and second harmonic frequencies. Figure (8.23) shows the computed results for this circuit. The three loci correspond to three different values of $G_1$ and $G_2$ which cause significant changes in the diode loading at the fundamental frequency. The device line and the corresponding characteristics of operation are very similar to those found experimentally. The largest output occurs at the highest conductance and both the power and the diode conductance decrease as the rf fundamental voltage decreases, as shown in Figure (8.24). The operating point A is not quite the maximum possible power output but it is clearly nearly the maximum since the device line is not far off from being tangential to the admittance locus.

**VIII - 6  Experimental Analysis of a Multiresonant Circuit with an Active Device Operating in LSA Mode**

Figure (8.25) shows a more complicated experimental circuit in which the diode is mounted in a waveguide iris and the fundamental output power is taken out via a ridged waveguide. The coaxial output line contains a double stub coaxial tuner which controls the admittance at the fundamental. The other two tuning elements, the movable short circuit at the back of the diode (in reduced height X band waveguide) and the tuning slug in the ridged waveguide section affect only the second harmonic tuning so that tuning at the fundamental and second harmonic are reasonably independent. This slug is shown in Figure (8.26) and is designed to present a high VSWR at the second harmonic and a match at the fundamental. The second harmonic tuning is then localized in the vicinity of the diode. The tuning slug is basically a two port
LSA OSCILLATOR CIRCUIT WITH INDEPENDENT TUNING FOR THE FUNDAMENTAL

AND THE SECOND HARMONIC.

FIG. (8.25)
WAVEGUIDE RIDGE

$1 = \lambda/4$ AT THE FUNDAMENTAL FREQUENCY

IMPEDANCE AT AA AT THE FUNDAMENTAL FREQUENCY

$$Z_{AA} = Z_0 \quad \text{IF} \quad Z_1^2 = Z_0 Z_2$$

IMPEDANCE AT AA AT THE SECOND HARMONIC FREQUENCY

$$Z_{AA} = \frac{Z_2^2}{Z_0}$$

$$|\rho| = \frac{(Z_2^2 - Z_0^2)(Z_2^2 + Z_0^2)^{-1}}{}$$

SCHEMATIC DIAGRAM OF THE SECOND HARMONIC TUNING SLUG AND
EQUIVALENT CIRCUIT OF THE TUNING SLUG

FIG.(8.26)
network and by choosing the ports at appropriate planes any two port network can act as a simple transformer. The turns ratio of this equivalent transformer is related to the input VSWR presented by the device when terminated in a matched load. It follows therefore that if there is another mismatch at the second harmonic frequency further down the waveguide the tuning slug can in fact partially match this out.

The experimental technique is as follows. For a particular bias voltage the circuit is tuned to give minimum power out at the second harmonic frequency and either maximum or some intermediate power out at the fundamental frequency or alternatively maximum power out at the second harmonic frequency etc. At each setting the frequency and output powers are measured. Without changing the tuning controls the diode is removed and the admittance at the original plane of the diode measured on the network analyzer. The precise operating point on each \( Y(\omega) \) versus \( \omega \) is located by knowing the corresponding frequency of oscillation.

For this circuit the chip is mounted in S4 package and therefore one must consider the admittance transformation through the package as presented in section IV-4. All the results to be presented have already been transformed through S4 package.

Typical results for conditions of minimum output power at the second harmonic frequency are given in Figure (8.27) which corresponds to \( V_B = 5.17 \ V_{th} \). At the fundamental frequency the admittance is inductive and maximum power occurs at the largest conductance, and power decreases as the conductance decreases. At the second harmonic frequency the admittance is capacitive and the conductance the minimum value of the locus. Figure (8.28) shows the condition at the fundamental frequency
to a larger scale. The stability conditions are satisfied but are only
just satisfied at the point of maximum power. The output pulse and the
bias voltage pulse, each showed significant flicker at this setting.
For this setting of maximum output power the oscillation does become
noisy. Transitions between different operating points were observed to
be perfectly stable.

A particularly interesting feature of these results is that
the diode conductance and the power output at the fundamental frequency
each increase as the rf fundamental voltage amplitude increases. Figures
(8.29) and (8.30) show that the same behaviour is seen at other bias
voltages although the actual operating point is of course different. It
could be thought from Figure (8.30) that the output power would increa­
se indefinitely with voltage amplitude. However one must remember that
the maximum output power will correspond to the value of the voltage
amplitude when the device line is tangential to the admittance locus and
is stable.

Figure (8.31) shows the admittance locus for \( V_B = 3.92 \ V_{th} \).
We observe that the fourth intersecting point \( P_4 \) of the device line and
the admittance locus is unstable and experimentally no oscillation was
observed, but breakdown in the output and bias pulse. It could be thought
that the diode would oscillate at intersection \( P_0 \) as this is a stable
one. However the reason it does not oscillate at all is because at this
intersection the oscillation conditions are not satisfied, because the
frequency is too low for the applied bias.

Very different conditions are obtained when the circuit is
tuned to give maximum output power at the second harmonic frequency as
shown in Figures (8.32) and (8.34) for \( V_B = 5.17 \ V_{th} \) and \( V_B = 3.92 \ V_{th} \).
MULTIPLE RESONANT CIRCUIT

--- MAX. RF POWER
--- INTERMEDIATE RF POWER
--- MINIMUM RF POWER

\( f_A = 4.82 \text{ GHz} \)
\( f_B = 4.82 \text{ GHz} \)
\( f_C = 4.80 \text{ GHz} \)

FIG.(8.27)
CIRCUIT ADMITTANCE AND OPERATING POINTS
FOR THE INDEPENDENTLY TUNED CIRCUIT
DEVICE AND CIRCUIT ADMITTANCE LINES AT THE FUNDAMENTAL FREQUENCY FOR MINIMUM OUTPUT AT THE SECOND HARMONIC

FIG.(8.28)
MULTIPLE RESONANT CIRCUIT

\[ \begin{align*}
    O V_B &= 5.17 \times V_{th} \\
    \times V_B &= 3.92 \times V_{th} \\
    f_A &= 4.82 \text{ GHz} \\
    f_B &= 4.42 \text{ GHz}
\end{align*} \]

FIG.(8.29)

CIRCUIT ADMITTANCE AND OPERATING POINTS
CORRESPONDING TO DIFFERENT BIAS VOLTAGES
AND MINIMUM OUTPUT AT THE SECOND HARMONIC
$V_B = 3.92 \, V_{th}$

△ MAX. RF POWER ( $f=4.42 \, \text{GHz}, \, P=16.5 \, \text{W}$)

◇ INTERMEDIATE RF POWER ( $f=4.4 \, \text{GHz}, \, P=8.5 \, \text{W}$)

▼ MINIMUM RF POWER ( $f=4.435 \, \text{GHz}, \, P=3.6 \, \text{W}$)

▼ NO OSCILLATION

CIRCUIT ADMITTANCE AND OPERATING POINTS FOR THE INDEPENDENTLY TUNED CIRCUIT

FIG.(8.31)
respectively. The output power at the fundamental and second harmonic are 3.4 W and 3.8 W for $V_B = 5.17 V_{th}$ and 3.3 W and 3.6 W for $V_B = 3.92 V_{th}$. There is no resonant behaviour at the fundamental and therefore only the single admittance value is plotted. We also note that the locus admittance for the non oscillation condition corresponds to high values of conductance. While tuning from the operating point A to the new operating point B the diode ceases to oscillate. The corresponding admittance locus shown in Figure (8.33) indicates that the load conductance becomes too large for the diode to accommodate. The general performance characteristics of the multiple-tuned oscillator are given in Figures (8.35) and (8.36).

In order to study the influence of the harmonics on the waveform, the voltage waveform was observed experimentally for different settings of the tuning elements. A resistive probe was fitted inside the iris and the zero level determined on the sampling scope. The displacement current picked up by the probe was minimized by an appropriate value of the resistor. Three different voltage waveforms are presented. Figure (8.37a) corresponds to high output at the fundamental and minimum at the second harmonic. The admittance locus for this setting is shown in Figure (8.38), curve A. The conductance at the fundamental is quite high although for that particular locus higher values could be reached. The second harmonic presents a minimum value of the conductance within the locus, and it is clear that the probe inside the iris modified the susceptance for the second harmonic, because it is near the iris where the tuning for the second harmonic is localized.

The voltage waveform changes appreciably as the output power at the second harmonic is increased, Figure (8.37b) and Figure (8.37c) corresponding to curves B and C for intermediate and high power at the
MULTIPLE RESONANT CIRCUIT
SECOND HARMONIC:
— MAX. RF POWER
— MIN. RF POWER
— NO OSCILLATION

\[ f_{A'} = 10.20 \text{ GHz} \]
\[ f_{B'} = 10.20 \text{ GHz} \]
\[ V_B = 5.17 \times V_{th} \]

FIG. (8.32)
CIRCUIT ADMITTANCE AND OPERATING POINTS
FOR CIRCUIT TUNED TO GIVE MAXIMUM OUTPUT
AT THE SECOND HARMONIC FREQUENCY
MULTIPLE RESONANT CIRCUIT.
SECOND HARMONIC

FIG. (8.33). DEVICE AND CIRCUIT ADMITTANCE LINES FOR MAXIMUM OUTPUT AT THE SECOND HARMONIC
MULTIPLE RESONANT CIRCUIT

SECOND HARMONIC:

○ MAX. RF POWER
× MIN. RF POWER
□ NO OSCILLATION

\[ V_B = 3.92 \text{ V}_{\text{th}} \]
\[ f_A = 4.635 \text{ GHz} \]
\[ f_{B'} = 9.3 \text{ GHz} \]
\[ P_{A'} = 3.6 \text{ W} \]
\[ P_{B'} = 3 \text{ W} \]
\[ f_1 = 9.16 \text{ GHz} \]
\[ f_2 = 9.32 \text{ GHz} \]
\[ f_3 = 9.42 \text{ GHz} \]

FIG.(8.34)

CIRCUIT ADMITTANCE AND OPERATING POINTS
FOR CIRCUIT TUNED TO GIVE MINIMUM OUTPUT
AT THE FUNDAMENTAL
EFFICIENCY, FREQUENCY AND MAXIMUM POWER OUTPUT AT THE FUNDAMENTAL AND SECOND HARMONIC FREQUENCIES AS A FUNCTION OF $\frac{V_B}{V_{th}}$

FIG.(8.35)
\[ \frac{P_1}{P_0} \sim \frac{V_B}{V_{th}} \text{ for the independently tuned circuit} \]

FIG.(8.36)
second harmonic respectively. The waveform tends to be more sinusoidal as the fundamental voltage amplitude decreases, increasing the ratio $\frac{V_2}{V_1}$.

The conductance at the second harmonic increases for increasing values of the power, while the corresponding conductance at the fundamental decreases. Again a low value of the susceptance at the fundamental combined with a high conductance ensures high output power at this frequency. Transitions from different states of oscillation are stable as can readily be seen from the intersections of the device line and the admittance loci.

VIII - 7 Discussion of the Results

Perhaps the most obvious result from both experiment and analysis shown by all the circuits is that the admittance across the diode is inductive at the fundamental frequency but capacitive at the second harmonic frequency. This is a direct consequence of the restrictions which negative resistance oscillators must satisfy. Grozskowski[^31] and van der Pol[^32] and more recently Quine[^70] have shown that provided the I-V characteristic of any element shows no hysteresis the harmonic voltage components must satisfy the relation

$$\sum_{n=0}^{n} n |v_n|^2 b_n = 0$$

(8.27)

where $b_n$ is the circuit susceptance across the diode at the nth harmonic frequency. Tan and Foulds[^hl] have shown that this is only one of a set of equations which must be satisfied. For a voltage waveform in which only the fundamental and second harmonic voltage components are significant we must have
FIG. (8.37a). VOLTAGE WAVEFORM CORRESPONDING TO FIGURE (4.17a)

$V_B = 4.285 \, V_{th}$
$V_{th} = 28 \, V$
$V_{pp} = 493.272V$
$f = 4.715 \, GHz$
$P_{out} = 11 \, W$
$P_{2 \, arm.} = 2.4 \, W$

FIG. (8.37b). VOLTAGE WAVEFORM CORRESPONDING TO FIGURE (4.17b)

$V_{pp} = 543.86V$
$f = 4.725 \, GHz$
$P_\omega = 7.5 \, W$
$P_{2\omega} = 3.2 \, W$

FIG. (8.37c). VOLTAGE WAVEFORM CORRESPONDING TO FIGURE (4.17c)

$V_{pp} = 581.81$
$f = 4.715 \, GHz$
$P_\omega = 5 \, W$
$P_{2\omega} = 3.8 \, W$
$f_A = 4.715 \text{ GHz}$

$\bar{f}_B = 4.725 \text{ GHz}$

**FIG. (8.38a)**

CIRCUIT ADMITTANCE AND OPERATING POINTS
CORRESPONDING TO FIGURES (4.17a) and (4.17b)
$f_c = 4.715 \text{ GHz}$

**FIG. (8.38b)**

CIRCUIT ADMITTANCE AND OPERATING POINTS
CORRESPONDING TO FIGURE (4.17c)
Thus \(B_1\) and \(B_2\) must have opposite signs and for the LSA mode it is known that the admittance at the fundamental must be inductive. Equation (8.27) applies to all circuits, not merely a simple parallel LC circuit and therefore provided \(B_1\) and \(B_2\) are not zero the equation relates the relative harmonics and fundamental voltages components, i.e.

\[
\frac{v_1}{v_2} = \frac{2B_2}{B_1}
\]  

(8.29)

Substituting for \(B_1\) and \(B_2\) from the computed results shown on Figure (8.23) for the independently tuned circuit gives

\[
\frac{v_1}{v_2} = 2.7
\]

The actual computed values give

\[
\frac{v_1}{v_2} = 2.67
\]

This gives a comforting check on the computing. The experimental values for the fundamental and second harmonic components do not give such good agreement because of the existence of higher harmonics in the voltage waveform, and although the voltage amplitudes for higher frequencies are small as the Fourier analysis showed, the susceptances presented to the diode at these frequencies have a significant contribution in equation (8.27).

We note also that maximum power out at the fundamental occurs at conditions of relatively high conductance at the fundamental while the susceptance is about the minimum possible value for the particular locus. For this setting the conductance at the second harmonic is a mini-
mum. Correspondingly high conductance and small susceptance ensures high output power at the second harmonic. In almost all the cases the output power at the second harmonic was not very large, however the presence of this harmonic in the waveform has a significant effect on the fundamental as well as making the stability conditions more restrictive.
CHAPTER IX

CONCLUSIONS

It has been shown that the technique of describing functions, a technique well known in non linear control theory, can be applied very successfully to give a good description of non linear negative resistance oscillators in which the resultant voltage waveform is non sinusoidal. The describing function provides an accurate load at the harmonics and leads to realistic and accurate waveforms. These are not the sinusoidal voltage waveforms assumed in some earlier GaAs oscillator theories which were the result of assuming a zero load impedance at all frequencies other than the fundamental. Furthermore with this wide range model, the frequency can be free to move and still see a realistic load, as it does in the case of bias tuning of frequency.

This technique is further enhanced by using the simple I-V characteristic of the device discussed in Chapter II. Although computing time depends upon the number of harmonics considered in the waveform it is shorter compared with earlier simulations, Kennedy (71) because it is not necessary to compute the electric field configuration within the device at each instant of time. Also no waveform has been excluded by imposing a linear ramp below the threshold voltage and no restriction has to be put on the linear coupling circuit. This is in contrast to conventional studies of non linear oscillators (e.g. van der Pol oscillations) which require a simple form of the non-linearity and the linear circuit. Furthermore no conditions were imposed upon the relative phase of the harmonics and the susceptance at different frequencies.
It is obvious from the preceding chapters that as soon as the input to the non linear element has been defined a unique describing function exists which will not change even if the linear network is altered. In this way it is fearly easy to obtain general conclusions about the influence of the linear part on the behaviour of the total circuit. It is possible then to synthesize the linear network giving the total circuit optimal properties.

The theoretical model shows that the usual condition that the diode admittance be the negative of the circuit admittance must hold not only at the fundamental but also at each significant harmonic frequency. The analysis has also shown that the Kurokawa condition for stability at the operating point must also be satisfied at each frequency. These combined conditions are more restrictive than for the single frequency sinusoidal oscillator since the diode admittance at the various frequencies are inter-related by the I-V characteristic. It is now clear that different LSA oscillators can work over different regimes of the relation between the rf voltage and the output power and diode conductance.

Experimentally it was found that for the simple parallel circuit the power and diode conductance always decreased as the rf voltage increased. It is shown theoretically that this is an essential requirement if the voltage waveform is purely sinusoidal, and that it is the likely operation even if there is a second harmonic voltage component. On the other hand experiments showed that the multiple tuned circuit always oscillated such that the output power and diode conductance increased up to some critical value as the fundamental component of the rf voltage increased. This agrees exactly with the theoretically pre-
dicted results.

Although we have studied an active non linear device the technique presented could be applied to passive non linear elements. We realize the disadvantage of this technique, i.e. solving a complex set of non linear equations simultaneously, but we believe that developing a feedback model for non linear microwave circuits and applying describing function techniques to the resulting model can be a useful approach to the analysis of the stability of non sinusoidal microwave oscillators.
After careful analysis of the different iterative methods available, the one developed by Powell\(^{(72),(73)}\) was chosen for the computer calculations throughout this thesis. This iterative method is similar to the conjugate gradient method originally developed by Hestenes and Stiefel\(^{(74)}\). It has second order convergence and near a stationary value it converges more quickly than the steepest descents method. Although each iteration is quite long because the method is applicable to a general function, the rate of convergence is comparable to that of the more powerful of the gradient methods such as the one by Martin and Tee\(^{(75)}\).

However, a deficiency of this method is that the user has to make some decisions carefully. In particular, the parameter DSTEP must be so small that for \(|\delta|<\text{DSTEP}, F_k(x+\delta) ; k = 1,2,3,\ldots n\), is nearly a linear function of \(\delta\), but it must not be so small that the differences are dominated by computer rounding errors. Also, the user's scaling of the unknowns \((x_1,x_2,\ldots x_n)\) must be such that it is sensible to apply the usual euclidean definitions of vector scalar products and orthogonality.

This iterative method also takes some time to program and although it will converge from a poor approximation to the stationary value, computer time may be saved if the user makes a judicious approximation of the solution. In this respect, the information obtained from the experimental results is very helpful in providing a good approximation to the solution.
The computer program is written in FORTRAN IV and the master program makes use of EO4FBF and FOURIER main subroutines. Subroutine EO4FBF is substantially the Harwell subroutine VA05A and is based on Powell's iterative method. EO4FBF subroutine calls the subroutines IN and MON. Subroutine FN calculates the values of the functions F(n) for any point x_1, x_2, ... x_n and the subroutine MON is used for monitoring the output information.

Subroutine FOURIER provides the Fourier analysis of the current waveform. It calls the subroutine F4INTSMF which performs the numerical integration by Simpson's rule, with difference correction and self adjusting step length. The coefficients of the Fourier series are obtained by calling the subroutines FUNB, FUNC and FUNS corresponding to a_0, a_n and b_n respectively. Figure 1 shows the schematic structure of the master program and different subroutines while Figure 2 shows the general flowchart. Finally a complete listing of a computer program corresponding to a multiresonant circuit is included. The program is for use with ICL 1905 computer system.
MAIN PROGRAM AND SUBROUTINES. FORTRAN IV
THIS PROGRAM CALCULATES THE STEADY STATE OPERATING POINT
OF AN L.S.A OSCILLATOR IN A MULTIRESONANT CIRCUIT. THE USER SETS
AN INITIAL SOLUTION X(1), X(2), X(3) AND X(4) CORRESPONDING TO
W, TH, V1 AND V2 OF THE VOLTAGE WAVEFORM V=V1*COS(WT)+V2*COS(2WT+TH)
ACROSS THE DEVICE AND THE PUKEEL ITERATIVE METHOD PROVIDES THE
SIMULTANEOUS SOLUTION FOR F(1), F(2), F(3) AND F(4) WITHIN AN ERROR
FIXED BY THE USER (EPS).
SUBROUTINE FOURIER CALCULATES THE DIFFERENT COMPONENTS OF THE
CONDUCTION CURRENT.
L1, L2, L3, G1, G2, G3, SIGD AND SIGU ARE THE COMPONENTS OF THE MULTI
RESONANT CIRCUIT WHICH MAY BE CHANGED FOR DIFFERENT OPERATING
POINTS.
The I-V CHARACTERISTIC OF THE DIODE IS GIVEN BY:
I=K1*(V+K3*V**4)/(1+K4*V**4)
K3=(VS*L)/(MO*VA**4)
K4 = 1/(VA**4)
K1 = LOW FIELD CONDUCTANCE
VS=SATURATION VELOCITY (85000 M/SEG)
MO=LOW FIELD MOBILITY (0.83 M*M/SEG)
L = ACTIVE LENGTH OF THE SAMPLE (50E-6 M)
VA=E*A* L WHERE E*A IS A CONSTANT FIELD (4 KV/CM)
THE BIAS CURRENT AND VOLTAGE ARE GIVEN BY I0 AND K4.
MULTIRESONANT CIRCUIT

    13 -- C3 --
    |    |    |
    | 1  | 1  | 1
    |    |    |    
    L2 -- C2--

1  | 1  | 1
2  | 1  | 1
3  | 1  | 1
4  | 1  | 1

MASTER SEBASTIAN
DIMENSION X(4), F(4), W(300)
COMMON/B/C/D/E
COMMON MN, V1, V2, TH
EXTERNAL FN, MON
X(1)=26.8
X(2)=-4.91
X(3)=148.2
X(4)=56.5
M=4
EPS=1E-7
EPS1=1E-4
DMAX=100
E=2+2*M*N+N+2*M*5+N
F=1
M=300
IFAIL=0
CALL EOSFR(M,N,X,F,S,EPS,DMAX,W,N,MON,IP,IX,IFAIL)
W=4*X(1)+1.09
V1=X(3)
V2=X(4)
WRITE(3,99) W, X(2), V1, V2
WRITE(2,99) IFAIL
WRITE(3,200) M, (C(I), I=1, 2), (D(I), I=1, 2)
99 FORMAT(///1X,SHW, E14.6,SH, MTHETA=E14.6,SH, NV1=E14.6, E14.6)
200 FORMAT(///1X,SHW, E14.6,SH, IFAIL=I11)
S1=STOP
END
SUBROUTINE FN(M,N,X,F)
DIMENSION X(M),F(N)
DIMENSION B(10),C(10),D(10),E(10)
COMMON/B,C,D,E
COMMON NN,V1,V2,TH
PI=3.141592654
TMXX(2)=PI/180.

VWXX(I)=1.0

V1=VX(I)
V2=VX(I)

REAL L1,L2,L3,L4
REAL I1,II,12

L1=2.6E-9
L2=4E-8
L3=4E-9

G1=2.34;E-13
G2=3E-14
G3=6.5E-14

SIGU=0.007
SIGD=0.0082

I1=SURT((C(1))**2+(D(1))**2)
I2=SURT((C(2))**2+(D(2))**2)

GAM=ACOS(C(1)/I1)
DEL=ACOS(C(2)/I2)

TH=X(2)*PI/180.
WWW=X(1)*1.E9

V1=VX(I)
V2=VX(I)

CALL FOURIER

REAL L1,L2,L3,L4
REAL I1,II,12

L1=2.6E-9
L2=4E-8
L3=4E-9

G1=2.34;E-13
G2=3E-14
G3=6.5E-14

SIGU=0.007
SIGD=0.0082

I1=SURT((C(1))**2+(D(1))**2)
I2=SURT((C(2))**2+(D(2))**2)

GAM=ACOS(C(1)/I1)
DEL=ACOS(C(2)/I2)

TH=X(2)*PI/180.
WWW=X(1)*1.E9

V1=VX(I)
V2=VX(I)

CALL FOURIER

FUNCTION FN(X)

RETURN
END
FUNCTION FUNS(X)
COMMON NN,V1,V2,TH
V=V1*COS(X)*V2*COS(2*X*TH)  COMMON NN,V1,V2,TH
FUNS=FNA(V)*COS(NN*X)   COMMON NN,V1,V2,TH
RETURN
END

END OF SEGMENT, LENGTH 68, NAME FNA

FUNCTION FNA(V)
REAL K1,K2,K3,K4
REAL IO
I0=2.4147
K1=0.2
K2=3.77E-6
K3=3.61E-7
K4=115
FNA=-I0*K1*(V+K4+K2*(V+K4)**4)/(1+K3*(V+K4)**4)
RETURN
END

END OF SEGMENT, LENGTH 68, NAME FNA

FINISH

END OF COMPILATION - NO ERRORS

DOCUMENT MONITOR , NORM(MPSF) : LP00 ON 22/07/76 AT 17.28
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