THE ACOUSTICS OF BRASS WIND-INSTRUMENTS

by

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A Thesis submitted in fulfilment of the requirements
for the degree of Doctor of Philosophy

1979

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This thesis examines the generation of musical sounds in brass wind-instruments. The acoustic behaviour of the instrument is first studied using the trombone and trumpet as examples. The acoustic input impedance and pressure transfer function are measured using a computer controlled, stepped sine wave technique, and these parameters are related to sound propagation in the instrument. Special attention is given to the effect of the mouthpiece. The relationship between the two measured parameters is discussed, which suggests a method of predicting some of the features of the pressure transfer function from the input impedance and an assumed radiation impedance at the bell. The input and transfer responses of an instrument to an impulse of velocity or pressure in the mouthpiece are derived from the frequency-domain data and these results correlated with the instrument's geometry.

After discussion of the parameters of the instrument and player which are of importance in their interaction, particularly the behaviour of the lips when forced by the mouthpiece pressure, the literature on regeneration in wind-instruments and the speech source is reviewed. This leads to the development of a theory predicting the conditions under which a blown note will be sustained, and the harmonic structure of such a note. The theory is tested by measuring the intonation and internal pressure waveforms for a wide variety of notes blown on the trombone. The internal impedance of the pressure source from the lungs is deduced from measurements of alternating pressures in the mouth and mouthpiece and shown to have important consequences. A separate experiment investigates some of the parameters of the lip dynamics by simultaneous measurement of the pressure and velocity in the throat of a mouthpiece while various notes are being played.
'So, too, is there an art of music: but this art should no more blind us to the existence of its corresponding science than do the arts of surgery, photography, or stage illumination, obscure our recognition of the fundamental sciences upon which those arts are founded.'

John Redford (1937)
ACKNOWLEDGEMENTS

I would like to thank Dr. J.M. Bowsher for his help and advice during the course of this study. It is also with pleasure that I acknowledge the support and many helpful suggestions of my fellow research students, particularly the musical insight provided by Mr. P.S. Watkinson and Mr. J.C. Goodwin. The staff of the departmental workshops have been helpful throughout, though particular thanks goes to Mr. E.A. Worpe for the construction of various pieces of electronics. Many hours of patient work have been contributed by the departmental computing staff; the computing experience of Mr. R.A. Bacon has also been invaluable. Finally my thanks to Miss S. Deane for typing the thesis, and to the S.R.C. for funding the research.
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CHAPTER 1

INTRODUCTION

Musical acoustics is an old and diffuse field of study and historically the motivation behind it has been mostly academic. The problems have been so complicated that quantitative scientific theories have been of limited use to musical instrument makers. In many cases the basic mechanism of oscillation in an instrument may be quite well understood, but musically important phenomena may be second or third order effects from a scientific point of view. So the design and manufacture of instruments has been a matter for craftsmen with a wealth of empirical and artistic knowledge, built up over many generations.

Nevertheless, the challenge has been such that many eminent scientists have taken a keen interest in the fundamental physics. The sound production mechanism in wind instruments, for example, has been studied by Helmholtz (1895), A.G. Webster (1919b) and Bouasse (1929). At the moment this problem is more than academic since, with the introduction of mass production techniques the craftsmen's skill becomes alienated and a technological expertise must be sought.

This thesis is specifically concerned with sound production in brass wind-instruments; in other words, instruments which have a cup mouthpiece and are excited into oscillation by the lips of a player. There is a large range of such instruments, which generally consist of a detachable mouthpiece, a section of fairly cylindrical tubing, and a rapidly flaring bell to radiate the sound produced, though there are considerable differences in the relative dimensions of these components. The overall size of brass instruments ranges from that of the piccolo trumpet to that of the contrabass tuba, an increase in total length by a factor of about eight. Most of the work described in this thesis...
has been concerned with two instruments, a trombone and a trumpet. Details of these instruments and their mouthpieces are given in Appendix C.

The trombone in particular has been studied extensively because it is pitched in the middle of the range for the brass instrument family, and a considerable wealth of experience has been accumulated within the research group about this instrument. The history and construction of the trombone are admirably treated by Bate (1972) but for clarity the various parts of the instrument and its mouthpiece are indicated in Fig.1.1.

The influence of the player on sound production in brass instruments is much stronger than other musical instruments because the players' lips play an integral part in the regeneration. Since players differ so much in their physical characteristics and technique of playing, no single instrument is universally suitable for them all. There has been considerable effort directed to understanding the acoustics of the instrument, and some of the important parameters are now well understood, but there is very little scientific literature on the coupling of the instrument to the players' lips, let alone the considerable differences amongst players. It is well known amongst brass players that a good player can largely compensate for the shortcomings of an individual instrument, and the 'sound' he makes is, to some extent, independent of the instrument. It has been one of the objects of this work to attempt to identify the parameters which the player can change to control the regeneration to suit his style of playing. The problem turns out to be not so much identifying what the player can do, as adequately describing the changes he can make so that their influence may be deduced.

We start, however, with a discussion of the acoustics of the instrument alone, in Chapter 2, since an understanding of this is essential before proceeding to consider the interaction between the
Figure 1: Construction of a trombone and mouthpiece.
instrument and the player. This interaction is then discussed in Chapter 3 with reference to the literature in musical acoustics and speech synthesis, and a simplified theory developed for oscillation in brass instruments. This is tested by measurements of actual blown notes in Chapter 4 where some of the properties of the lips and pressure source from the lungs are deduced from measurements of the pressures in the mouth and mouthpiece and the velocity in the mouthpiece for various blown notes.
CHAPTER 2
ACOUSTIC BEHAVIOUR OF BRASS INSTRUMENTS

2.1 Introduction

The purpose of this chapter is to describe the various ways of representing the acoustic response of a brass wind-instrument. This will lay the groundwork for a discussion of the interaction between the player and the instrument in Chapter 3.

The acoustic perturbations in the bulk properties of the air in an instrument are considered to be linear. This is not strictly true for two reasons:

Firstly, the high sound pressure levels in the instrument (which may reach 170 dB in the mouthpiece for loud notes) will give rise to harmonics because of the non-linear relationship between the pressure and volume of the air column. This is discussed at length by Backus and Hundley (1971) who conclude that although this will give a certain amount of harmonic generation it will be far less than the harmonic generation from other sources.

Secondly, the stream of air through the most narrow part of the instrument (the throat of the mouthpiece) becomes turbulent if the velocity exceeds a certain limit. This gives rise to a resistance at the throat which is dependent on the flow, as discussed in section 2.5. However this resistance is small compared to the real part of the acoustic impedance.

These deviations from linearity may play some role, but are not a dominant part of the acoustics at normal levels of playing and are neglected in this study.

The wave is generally plane in the cylindrical part of the instrument, the cutoff frequency of the first higher mode being about
10 kHz (Morse and Ingard, 1968). The wavefront is more complicated at the bends and in the flaring section of the bell, being neither plane (as it would be for a cylinder), or truly spherical (as it would be for a cone). The changing rate of flare partly reflects a wave travelling along the instrument and the point at which this occurs, together with the magnitude of the effect, are functions of frequency and correspond to the phase and amplitude of the reflection coefficient (Jansson and Benade, 1974).

This gives rise to the standing waves in the instrument, and all the resonance phenomenon which are of interest.

2.2 Measures of 'resonance'

There have been a number of methods used to measure the way in which the standing wave changes with frequency.

In the past experimenters have generally measured a series of discrete 'resonant' frequencies by various methods and compared these to the frequencies of blown notes. Typical of such 'resonance' measurements are those of J.C. Webster (1947) or J.G. Woodward (1941) who artificially excited the instrument at the mouthpiece, and measured the variation in pressure outside the bell. The results were uncalibrated and unsuitable for obtaining any information about the 'strength' of each resonance, only frequency information is obtained.

The frequencies measured are dependent on the experimental method used and no set of such frequencies can be taken as a unique measure of intonation when the instrument is played. Even so, the information obtained did enable a better understanding of how the mouthpiece and bell combine to shift the 'resonant' frequencies into the approximate harmonic structure required for playing.
2.3 A two port representation

By considering observable quantities (pressure and volume velocity) of the acoustic wave in the instrument at two planes, a two port representation of the instrument may be used (Fig. 2.1). The implication is that if a suitable coordinate system is used, the propagation is one-dimensional, so the pressure is constant across the surface and the volume velocity is determined over the entire surface. In this thesis the two planes of measurement are defined to be:

1. A plane half way along the throat of the mouthpiece, and
2. A hemisphere of radius equal to that of the bell over the end of the bell.

Plane 1 is discussed later in section 2.4.1. Plane 2 has been previously suggested, as it results from the assumption that purely spherical waves propagate from the end of the horn (Benade and Jansson, 1974). The ratio of the pressure to volume velocity at plane 2 must be a radiation impedance, and a reasonable estimate of this may be obtained by taking the radiation impedance of half a pulsating sphere in an infinite baffle. This is well known (Crandall, 1927) and Fig. 2.2 shows the real and imaginary parts of such an impedance for a hemisphere of radius equal to that of a trombone bell. The input impedance and pressure transfer function defined in Fig. 2.1 are quantities which must be determined experimentally. The remainder of this chapter describes how this has been done and discusses the relationship between the measured parameters.

2.4 Measurement of input impedance

2.4.1 Introduction

The input impedance at some particular frequency is defined as the complex ratio of pressure to volume velocity (units are
$Z_i(f) = \frac{P_i(f)}{U_i(f)}$

$T(f) = \frac{P_o(f)}{P_i(f)}$

$Z_L(f) = \frac{P_o(f)}{U_o(f)}$

Fig. 2.1 The brass instrument as a two port network
Fig. 2.2 The real and imaginary parts of the assumed radiation impedance.
Kg m⁻⁴ S⁻¹ or (S.I.) acoustic ohms). The concept was introduced into
the study of acoustics by A.G. Webster (1919) and has since been widely
used in the formulation of numerous problems, for instance, the study
of acoustic filters by Stewart (1922).

The impedance is probably the single most important parameter
in characterising an instrument, playing as it does a crucial role in
the interaction between instrument and player. It is also a convenient
quantity for laboratory measurement since the acoustics of the room in
which measurements are made have little effect on the impedance over
most of the frequency range of interest. This is further discussed
in section 2.4.6.

The plane in which impedance is measured has to be carefully
defined and ambiguity here has cast doubt on a number of previous studies
(Pratt, 1978). The impedance required is that of the instrument and
mouthpiece, taking into account the volume of mouthpiece cup displaced
by the player's lips. An immediate difficulty here, is that the
player's lips move during each cycle of playing as they are forced into
motion by the pressure in the mouthpiece; this makes the concept of a
single impedance 'seen' by the player rather dubious. Some attempt is
made to resolve this point in Chapter 3. The plane of measurement for
the input impedance was chosen to be half way along the throat of the
mouthpiece for a number of reasons:

1. The acoustic particle velocity is high at this point,
   making it easy to use an anemometer;

2. Although the backbore and half the throat of the mouthpiece
   are still measured together with the instrument, the
   mouthpiece used does not have a strong effect on the
   impedance measured, so the impedance curve characterises
the instrument more than some combination of 
instrument and mouthpiece.

3. The effect of any mouthpiece on the impedance can be 
calculated by using a lumped element model, so that for 
one impedance measurement of an instrument, the effect 
of various mouthpieces can be rapidly computed (see 
section 2.4.8).

2.4.2 Method of Impedance Measurement

The various methods which have been used to measure 
aoustic impedance of wind-instruments, have been extensively reviewed 
by Benade (1973) and Pratt (1978). The method employed here is a 
development of that of Pratt (1978) for measuring the impedance of 
trombones, the original system having been redesigned to give 
measurements more quickly over an extended frequency range. The 
significance of the speed of measurement is that, since the positions 
of the peaks are temperature dependent, the ambient temperature should 
not change significantly during an experiment if the relative positions 
of the impedance peaks are to be accurately measured. The time scale 
for an experiment has been brought down from four and a half hours to 
about half an hour. The frequency range has been extended from 768 Hz 
to 1024 Hz at 1 Hz resolution, and the resolution may now be reduced to 
2 Hz, giving a range of 2048 Hz, enabling instruments of higher pitch 
than the trombone to be measured.

The method of Pratt has been published previously (Pratt et al, 
1977, presented in Appendix A), and so we shall concentrate on the 
improvements made by the author. Basically, the method consists of 
measuring the pressure with a probe microphone (B&K Type 4170) and the
velocity with a hot wire anemometer (DISA constant temperature anemometer type 55D05 and type 55P11 probe), the whole operation, including frequency stepping, being controlled by a mini computer (Data General, Nova 2). The instrument is excited by a loudspeaker (Cetec type 8840) housed in a chipboard box, which feeds the instrument via a brass connecting horn. The block diagram of the apparatus is shown in Fig. 2.3.

Three important new pieces of apparatus have been added:

1. A programmable attenuator (designed and built by Mr. Eric Worpe) which is controlled by a B.C.D. output from the computer can introduce attenuation, for a 1V input signal, of 0 to 59.9 dB in steps of 0.1 dB, and covers the frequency range d.c. to 50 kHz. This replaces the analogue method of regulating the level of the velocity signal which needed to have a long time constant to avoid instability, and introduced a large 'settling time' before readings could be taken.

2. A frequency divider circuit (designed by the author and Mr. Eric Worpe and built by Mr. Worpe). This accepts an input from the programmable oscillator (Adret "Codasyn 201") at 480 times the required frequency and synthesises a triangular wave by generating a rising and falling staircase waveform from the input (Fig. 2.4). This waveform passes through a series of nonlinear networks which shape the waveform to approximate a sine wave (Fig. 2.5). The input frequency is also divided by 20 to form the clock pulse output. The result is a sine wave output at \( f_H \) (say) and an output at \( 24 \times f_H \) which is used to trigger the sampling circuits. Previously the waveforms were sampled at 25 kHz irrespective of driving frequency which led to considerable redundancies in the experimental program, also at higher frequencies the waveform was not sampled at exactly an integer number of points per cycle, leading to sampling errors. The range of this
Fig. 2.3 Block diagram of the apparatus for impedance measurements
Triangular to sinewave converter.

Figure 2.5

E.A.W.
Physics Dept.
1978.
device is limited by the speed of the counter to a maximum output frequency of 2.5 kHz, and the distortion of the output is limited by design compromises in the nonlinear networks which shape the sine wave. The level of distortion is, however, very low, typically 0.7% total harmonic distortion.

3. A 29 bit B.C.D. interface. Because the input on the frequency divider now requires 480 times the excitation frequency, the range of the interface available (14 bits) was not sufficient to cover the range now required (1 MHz) at a resolution of 10 Hz. The author decided to build an interface which would cover the entire range of the programmable oscillator (0 - 1,999,999.9 Hz in steps of 0.1 Hz) to cover any future use, and this required a B.C.D. output of 29 bits. Previously the output had been sent from the computer in binary form and converted to B.C.D. by hardware, but the amount of hardware required to do this for 29 bits is prohibitive. Each output channel on the computer has a capacity of 15 bits and it was decided to convert the output frequency to B.C.D. with a program in the computer and feed 15 bits out through each of 2 channels (channels 2 and 3). This requires the use of a buffer to store the output of one channel until the other has been accessed. A circuit was developed by the author and is represented in Fig.2.6.

These innovations were combined in a program to measure impedance (EZRUN), a flowchart of which is shown in Fig.2.7. A listing of the program is given in Appendix B, and a block diagram of the apparatus in Fig.2.3. The various features which make this system considerably quicker than the one used previously are:

i) Because there is always an integer number of samples per cycle of the pressure and velocity waveforms, the level of the fundamental determined in the program is characterised sufficiently accurately by
I.C. 1 - 12 type 7475 quad. latch
I.C. 13 type 7407 hex. buffer

All outputs from I.C.13 have 470Ω 'pull-up' resistors.
Each I.C. supplied with 6 v from external supply
decoupled with 0.01 μF at pins

Fig.2.6 30 Bit Buffer
Read in: Number of cycles (n)
Velocity reference (UR)
Anemometer constants
Calibration filename

Set frequency to 10 Hz and start attenuation

Take 24 x n samples of each channel
Linearise velocity and take fundamental
Compute extra attenuation to put U = UR
Change attenuation

Is U in range

Increment frequency
Compute fundamental of pressure
Compute Z and write to V.D.U.

Is frequency = 1024

Store data into file

Stop

Fig. 2.7 Flowchart of computer program EZRUN
fewer cycles. Depending on the accuracy required the operator can set the number of cycles sampled at each frequency at the start of the program. Provided the instrument being measured does not have particularly sharp resonances, sampling over 2 cycles is sufficient, which means that a complete run takes approximately 30 minutes.

ii) The use of a feedback loop which changes in discrete steps and allows the system to settle before correcting, overcomes many of the stability problems encountered with an analogue feedback system. Care must be taken, however, to ensure that sampling does not take place before the levels have stabilised. This is particularly important for very sharp impedance peaks, where the transient produced by a change in frequency may take several hundred milliseconds to decay.

The program still maintains the level of the velocity signal about a 'control' level set in the program. This is because of the poor dynamic response of the anemometer under these conditions; the lower level is set by noise and the upper level is limited when the alternating component of the velocity becomes equal to the steady flow through the instrument. The velocity signal then appears to 'rectify' since the anemometer is unable to sense the direction of flow, which leads to considerable errors.

Since the velocity is kept constant, the dynamic range of the measured impedance is equal to that of the pressure signal. The dynamic range of the microphone and amplifier is about 60 dB, and this is not degraded by the A-D converters (14 bits, corresponding to a range of 84 dB).

2.4.3 Calibration of microphone

At the start of the program to measure impedance, a calibration file is read in for the magnitude and phase corrections which
need to be made to compensate for the response of the microphone (discussed in this section) and the boundary layer correction (discussed in section 2.4.4).

The probe microphone (B&K type 4170) was chosen for its exceptionally high and constant input impedance (2G ohms from 20 Hz to 8 kHz) and had previously been calibrated at a facility of the National Gas Turbine Establishment, giving calibration data at intervals of 10 Hz.

By using the program to measure the pressure transfer function (discussed in Section 2.6) and a specially built cavity, it was possible to generate a calibration file in the laboratory. The cavity built was similar to those discussed by Delany (1969) and to that built by Moore (1975) which was used at the N.G.T.E. It was constructed from a VITAVOX S.3 pressure unit with a cylindrical insert, sealed at both ends by 'O' rings, and a flat plastic plate screwed to the top which had holes for a \(\frac{1}{2}\)" 'standard' microphone and the probe microphone. Uniformity of pressure in the cavity was the main concern in construction, so the volume of the cavity was made larger than that of Moore's, whose main design criterion was to achieve high pressure levels. The cavity was, in fact, about 27 mm deep by 13 mm radius, giving a volume of 14 cc. The pressure across the top of the cavity was measured with the plate in various positions and found to be uniform to within ± 0.1 dB and ± 2 deg. of phase up to a frequency of 2.5 kHz. A probe microphone calibration file is shown in Fig.2.8, and was found to be very similar to those obtained at the N.G.T.E.

A separate experiment using an electrostatic actuator, showed that the amplitude response of the microphone used as a standard (B&K type 4134) and its power supply (B&K type 2801) was essentially flat in the frequency range used (Fig.2.9). The small phase error at low frequencies is accounted for with the probe microphone response in the
Fig. 2.8  Calibration file for probe microphone
Fig. 2.9 Response of B & K type 4134 microphone, electrostatically actuated
2.4.4 Boundary layer correction

The acoustic velocity must fall to zero at the walls of a tube, so there exists a 'boundary' layer near the wall where the velocity is different from that on axis. The theoretical form of this boundary layer was first derived by Stokes (1845) and is conveniently presented in Crandall (1927). The form of the velocity profile is shown to be:

\[
V(r) = V_0 \left[1 - \frac{J_0(Kr)}{J_0(Ka)}\right]
\]

where \( V_0 \) = velocity at centre

\( r \) = distance from wall

\( a \) = radius

\( K = \sqrt{\frac{\rho \omega}{\mu}} \)

\( \rho \) = density

\( \mu \) = coefficient of viscosity

so that the mean velocity, \( \frac{V(r)}{V_0} \), divided by the velocity at the centre is:

\[
\frac{V(r)}{V_0} = \left[1 - 2 \frac{J_1(Ka)}{J_0(Ka)}\right]
\]

This is used to calculate the corrections to the measured velocity (on axis) to obtain the mean velocity, and calculate the acoustic impedance. The boundary layer is only appreciable at low frequencies (approx. 1 mm at 10 Hz) where the mean velocity may be only 25% of that measured and -0.25 rad. out of phase with it. A correction file for the velocity profile is calculated by a program using the above equation, written by Mr. P.S. Watkinson, and is shown in Fig.2.10, for a throat diameter of 4 mm.

2.4.5 Calibration of the anemometer

The static calibration of the hot wire anemometer was carried out with the calibrated air supply described by Pratt and
Fig.2.10 Calibration file for velocity profile correction
Bowsher (1978). The amplitude and phase response of the hot wire anemometer is by far the most difficult to determine. The change of resistance with flow for a constant temperature anemometer, is uniform up to several hundred kilohertz, according to the accepted sources (Nielson and Rasmussen, 1966). The frequency response is determined, in a rather complicated way, by the response of the feedback amplifier within the anemometer, since the feedback mechanism is essentially non-linear. Nielson and Rasmussen (1966) have presented graphs of the frequency response of a similar anemometer, indicating that the response approximates to a simple first order system, with cut-off frequency dependent on mean flow across the wire. There is a simple way of measuring this cut-off frequency by impulse testing the anemometer under working conditions ('DISA', 1967). This method, under normal flow conditions, gave a cut-off frequency of 15 kHz. Assuming a simple first order system and extrapolating back, the amplitude and phase response will be < - 0.1% and < 0.1 rad. at 2 kHz. This assumes a steady flow across the wire of some 4 ms\(^{-1}\). At lower velocities there is a distinct tendency for the phase of the measured impedance to drift up with frequency indicating a much reduced cut-off frequency. Figure 2.11, for example, shows the phase of the impedance measured for a straight tube at a mean velocity of 0.26 ms\(^{-1}\), apart from the noise, a clear drift of phase is noticed (the response should be symmetrical about the zero of phase). This graph is further discussed in section 2.4.6.

In conclusion, no corrections were made for the anemometer, but it was always ensured that the steady velocity of an impedance measurement was above 4 ms\(^{-1}\).
Fig. 2.11  Measured impedance of a straight tube
2.4.6 Results

The acoustic impedances of a large number of systems have been measured, and two musical examples are given in Fig. 2.12 and 2.13, those of a trombone measured to $1 \, kH_3$ and trumpet to $2 \, kH_3$, sampling each reading over four cycles. (Details of the particular instruments and mouthpieces used are given in Appendix C.) With frequency scaling the two results show significant similarities, the peaks lie in about the same positions and each show a similar reduction of maximum amplitude at higher frequencies. The latter effect is because of the 'cut-off' frequency of the instrument bell. In crude terms, the wave is completely reflected by the bell at low frequencies which gives large standing wave ratios and so large impedance peaks, above some higher frequency (called the 'cut-off' frequency), the wave is transmitted through the bell (Jansson and Benade, 1974) and the impedance approximates to the characteristic impedance of the cylindrical part of the instrument. It can be seen that the cut-off frequency for a trumpet is about twice that of a trombone. The actual height of each impedance peak varies slightly from instrument to instrument, as does the precise frequency. The frequency and height of each peak may be calculated with a precision much greater than that of the resolution of the measurements if some assumptions are made about the shape of the peak and a curve fitted to many points in that region. Appendix D gives a proof that an isolated impedance peak in a straight tube may be closely modelled as a Lorentzian resonance curve and it is found that impedance peaks in real instruments also approximate such a form. The appendix also describes a computer program developed to interpolate between the measured points and obtain precise figures for the frequency and amplitude of each maxima.

The effect of a large room resonance on the measured impedance
Fig. 2.12 Measured impedance of the trombone
Fig. 2.13 Measured impedance of the trumpet
was checked by measuring a trombone with and without a large board in front. The board was approximately 1m x 1m and placed 60 cm from the end of the bell. Results of the two experiments are presented in Fig.2.14. Here averaging was over only one cycle which affords the opportunity of assessing how well the measured impedance is characterised in this case. The two measured impedances are remarkably similar when the peaks are large, as one would expect since most of the sound wave is reflected by the bell at these frequencies and the pressure standing wave inside has a large standing wave ratio. As cut-off is approached, distinct differences do become apparent, since here the wave is almost fully transmitted by the bell, making conditions outside the bell more important. These deviations are not important from our point of view, since it is principally the region of distinct peaks, below cut-off, which we are interested in.

One system which proved to be very difficult to measure by this method was the humble straight tube; there are a number of reasons for this and most are linked to the fact that the velocity measurements are not being made in a venturi as they are in most cases, particularly when measurements are made in the throat of a mouthpiece. The first problem is that as the steady volume velocity is increased to achieve a reasonable linear particle velocity (4m s\(^{-1}\)), small discontinuities upstream in the air supply introduce turbulence which 'floods' the acoustic velocities being measured. Even worse, if the airstream is only close to turbulence, the acoustic velocity, if large enough, can push the flow to turbulence at certain parts of the cycle only. This means that small linear flows must be used with subsequent phase distortion (see section 2.4.5). The other main problem with straight tubes is that near impedance maxima there exists considerable 'streaming'
Fig. 2.14 Impedance of the trombone measured with (solid line) and without (dashed line) a screen 0.6 m in front of bell.
flow in the tube, so the small linear flow has a component superimposed on it which varies with frequency. This superimposed velocity is not necessarily in the same direction as the acoustic velocity, and since the anemometer can only measure the magnitude of the velocity, errors arise because of the vector addition of these two flows. The impedance of a straight tube is however fairly easy to calculate, and the author's contribution to the paper presented in Appendix A was the comparison of the measured impedance with that predicted by theory. This afforded a check of the absolute calibration of the measurement system. In retrospect the phase drift of the impedance presented in Appendix A is clearly seen.

2.4.7 The mouthpiece model

Provided the instrument behaves linearly and the wavelength is much greater than any of the dimensions of the mouthpiece, the mouthpiece may be represented by a number of lumped elements. This was suggested by Backus (1976). A computer program has been written which implements this numerically by transforming a file of impedance data measured in the throat to take account of the small inertance formed by the throat and the compliance formed by the cup, as illustrated in Fig.2.15. The resistive part of mouthpiece impedance is due to viscosity and heat conduction but at most is only 5% of the reactance and is therefore neglected. The cup volume used is that measured for the mouthpiece with the volume displaced by the lips subtracted as discussed in Appendix C.

The measured impedance of the trombone and trumpet transformed to take account of the mouthpieces detailed in Appendix C are shown in figures 2.16 and 2.17 where the original measured impedance is also
C is the compliance of the cup volume = \( \frac{v}{\rho c^2} \)

\( M \) is the inertance of the throat = \( \frac{\rho l^*}{S^*} \)

where \( \rho \) is the density of air

\( c \) is the velocity of sound in free air

\( v \) is the volume of the cup

\( l^* \) is the effective length of the throat

\( S^* \) is the cross sectional area of the throat

\[
Z = (j\omega C + \frac{1}{j\omega M + Z_1})^{-1} = \frac{j\omega M + Z_1}{1 - \omega^2 M C + j\omega C Z_1}
\]

Fig.2.15 Lumped element model of mouthpiece
Fig. 2.16 Impedance of trombone (dashed line) with mouthpiece (solid line)
Fig. 2.17 Impedance of trumpet (dashed line) with mouthpiece (solid line)
shown (dashed lines) for comparison. The validity of this model for
the mouthpiece was tested by measuring the impedance of an instrument
in front of the mouthpiece and comparing with the previously measured
impedance and mouthpiece model. The impedance in this plane is not
easily measured, not least because the steady velocity at this point
is very low (0.3 ms\(^{-1}\)) leading to the phase errors discussed in
section 2.4.5. However, in the light of the straight tube measurements
of Fig.2.11 a known correction to the phase may be applied which increases
linearly with frequency. The results of such a measurement for a
trombone are shown in Fig.2.18, together with the impedance of Fig.2.12
transformed to account for the throat of the mouthpiece and the entire
cup volume, including the small volume (1.5cc) because of the spacing
washer behind the mouthpiece rim. The agreement between the two graphs
is very good, and could be considered a validation of the measuring
 technique (particularly the phase accuracy) as much as of the mouthpiece
model.

The effect of various mouthpieces on the measured impedance of an
instrument may now be fairly rapidly investigated. For instance,
Fig.2.19 shows the variation of intonation for the measured trombone
as the volume of the cup is varied.

2.4.8 Alternative excitation methods

It was originally hoped that a broad-band excitation
(impulse or white noise) could be used to drive the instrument and
experimental data would be obtained very quickly for subsequent analysis.
This has proved difficult because of the large changes in the energy
transmission through the driving system, composed of loudspeaker driver
and horn coupler, which would give a very poor signal to noise ratio

* These are further discussed in Appendix F
Fig. 2.18 Impedance of trombone and mouthpiece measured directly (solid line) and calculated with mouthpiece model (dashed line)
Fig. 2.19 Intonation diagram for the trombone with three mouthpiece cup volumes
for the velocity signal at some frequencies unless compensated for.
Figure 2.20 shows the ratio of volume velocity to loudspeaker input
voltage for the loudspeaker and coupling horn driving a trombone.
The sharp resonances below 1 kHz are because of the impedance peaks
in the instrument, and the superimposed modulations together with the
resonances over 1 kHz are due to the driving system. It can be seen
that the system is particularly inefficient above 2 kHz, so a signal
applied to the loudspeaker with a broad frequency content will be
severely degraded by the time it arrives at the instrument. It is not
possible to 'pre-distort' this signal since the transmission curve of
Fig. 2.20 is very dependent on the resonances of the individual instrument
being measured. It was thus considered best to use a stepped sine-wave
measuring system where the velocity levels may be controlled and kept
well above noise level; effort was therefore concentrated on refining
the original systems of Pratt, as described in section 2.4.2 and
Appendix A.

2.5 Measurement of the steady flow resistance

By measuring the pressure head necessary to pass known steady
volume velocities down the instrument, the variation of the flow
resistance with magnitude of flow has been investigated for several
instruments. Figure 2.21 presents data for a trombone and a trombone
mouthpiece on its own, and Fig. 2.22 presents data for a trumpet, and
trumpet mouthpiece only. The flow is plotted as a function of the
linear air velocity in the throat of the mouthpiece, and it can be
seen that at a flow of about 2 ms⁻¹ the resistance begins to rise with
increasing flow and eventually becomes approximately proportional to it.
This is to be expected if the throat of the mouthpiece alone is considered
Fig. 2.20 Ratio of volume velocity in the throat to voltage across loudspeaker coil for the driving system and a trombone
Fig. 2.21 Steady flow resistance of trombone mouthpiece with and without trombone
Fig. 2.22  Steady flow resistance of trumpet mouthpiece with and without trumpet
as an orifice (Ingard and Ising, 1967), through which the flow becomes turbulent. The velocity at which turbulence plays a predominant role agrees well with earlier measurements of orifices in plates (Ingard and Ising, 1967). The magnitude of this resistance is only of the order of 0.3 Mf at most for the trombone and by comparison with the real part of the measured acoustic impedance, this is considered to be generally negligible. The flow resistance for the trumpet shows similar trends to that for the trombone but, apart from the magnitudes, systematic differences can be seen. For example the slope of the resistance with flow is not constant with and without the mouthpiece. This may be due to constrictions in other parts of the trumpet bore, though no systematic attempt has been made to investigate this.

2.6.1 Measurement of pressure transfer function

The pressure transfer function at some frequency is defined in Fig.2.1 to be the complex ratio of the pressure outside the instrument (external pressure), measured one bell radius away, on axis, to the pressure measured in the throat of the mouthpiece. It must be measured in anechoic conditions since the external pressure measurement is very susceptible to distortion from room resonances. For this reason a series of measurements were carried out on the balcony of a second floor building with the instrument elevated at an angle of approximately 30° to the horizontal. The experimental set-up is very similar to that used for impedance, except that a ½ inch microphone (B&K type 4134) is used to measure the external pressure. A block diagram of the apparatus is given in Fig.2.23. The measurements were again made with a computer controlled stepped sine-wave technique, using a very similar program to that for the impedance measurement (Appendix B). In this case, however, the level of the external pressure is maintained
Fig. 2.23 Block diagram of the apparatus for Pressure Transfer Function measurements.
at a constant level by the digital feedback loop in an attempt to keep it above the level of ambient noise on the balcony.

2.6.2 Examples of measured transfer function

The pressure transfer function for a trombone was measured to $1 \text{ kHz}$ (Fig.2.24) and for a trumpet to $2 \text{ kHz}$ (Fig.2.25). The results are very noisy for frequencies below about $150 \text{ Hz}$. This is because the very high attenuation presented by the instrument at these frequencies (60-80 dB) makes it difficult to detect the external pressure in the ambient noise level of 70 dB which was inevitably present on the balcony. This is not too important, however, since the major features show up above this frequency, where the signals are well above the noise level. It is interesting to note the great similarity between the two graphs (Fig.2.24 and Fig.2.25), allowing for scaling of frequency. This is mainly because the cut off frequency of the trumpet is about twice that of the trombone. Above this frequency, sound waves are not appreciably reflected by the bell and propagate straight out to the external microphone which gives a uniform transmission amplitude. The magnitude of the transfer function in this region is predicted reasonably accurately by simple spherical spreading; more will be said about this in section 2.7. The only other calibrated data for the pressure transfer function we have seen published (Benade, 1976), gives completely erroneous values in this region. The quoted value of 0 dB at high frequencies is not possible from considerations of energy conservation. Turning our attention to the phase in this region, it can be seen that we have a linearly falling phase with frequency corresponding to a pure delay. The magnitude of this delay calculated from
Fig. 2.24. Measured pressure transfer function for the trombone
Fig. 2.25 Pressure transfer function for the trumpet measured in two positions on the balcony
\[
\tau = -\frac{d\psi}{d\omega} \quad \text{where} \quad \tau = \text{phase delay} \\
\psi = \text{phase of} \ T \\
\omega = \text{angular frequency}
\]
is 8.5 ms corresponding to an equivalent propagation path in free air of 2.80 m which corresponds very well to the geometric length of the trombone (2.70 m). The effects of unwanted reflections from buildings and other objects were tested by measuring the trumpet in two very different positions. The two results are shown both in Fig.2.25 and are so similar that errors from this source may be considered negligible.

2.7 The relationship between the impedance and pressure transfer function

The two port representation allows some comparison of the two measured parameters (impedance and transfer function) provided some assumptions about the nature of the instrument are made. For instance, if all the sound power put into the instrument is radiated from the instrument, we may write:

\[
\text{Power out} = P_0^2 \times \Re\left(\frac{1}{Z_i}\right) = \text{power in} = P_i^2 \times \Re\left(\frac{1}{Z_0}\right)
\]

The symbols are defined in Fig.2.1 and \(\Re\) means "the real part of" which may be solved to give the magnitude of the transfer function in terms of the radiation and input impedance:

\[
|T| = \left|\frac{P_0}{P_i}\right| = \sqrt{\Re\left(\frac{1}{Z_i}\right) / \Re\left(\frac{1}{Z_0}\right)}
\]

Assuming the load impedance to be that of a pulsating hemisphere discussed in section 2.3 and using the measured impedance for a trombone, the magnitude of the transfer function has been calculated using this equation. The results are given in Fig.2.26 together with the measured transfer function magnitude from Fig.2.24. Prediction of the various peaks and dips in the measured curve is seen to be very good, though the drop in
Fig. 2.26 Calculated magnitude of pressure transfer function (solid line) compared to measured value (dashed line).
level at low frequencies is not predicted. This implies that considerable energy is absorbed by the instrument at low frequencies, while very little is absorbed above cut-off. Various interesting points about the transfer function can be inferred from these graphs. Firstly, the real part of reciprocal radiation impedance used in the equation is obviously the radiation 'conductance' and for the assumed source this quantity does not vary with frequency (Bauer, 1944), i.e. the radiated power is constant with frequency if the driving pressure is constant. This implies that the magnitude of the calculated transfer function is proportional to the square root of the input conductance of the instrument.

It is also worth noting that the transfer function will be the same if measured with or without the cup of the mouthpiece. The cup is assumed to act as a lumped compliance and so no pressure difference can be developed across it. This is in line with what has been stated above, since the conductance of a system is not affected by a shunt susceptance. To calculate the pressure transfer function of the instrument and mouthpiece however, the small portion of mouthpiece throat mentioned in section 2.4.7 also has to be taken into account. However, the impedance of this element is so small that it may normally be ignored, and the transfer function with a mouthpiece is practically identical to that measured above.

2.8 Time domain representation of the characteristics of the instrument

The impedance and transfer function are measured as functions of frequency and by using the Fourier transform the data may be transformed into the time domain to bring out aspects of the behaviour which were not previously clear.
The transfer function will be considered first. A computer program has been written using a Fast Fourier Transform Subroutine which accepts a file of real and imaginary data as a function of frequency and extends the data for negative frequencies by assuming the real and imaginary parts are even and odd functions of frequency respectively, to give an entirely real response in time. The file of transfer function data is converted from log to magnitude form, the real and imaginary parts are taken, and a sampling 'window' superimposed in separate programs. The window used was half a Hanning \((\cosine)^2\) window as shown in Fig.2.27. The complete window will be generated in the Fourier transform program when the negative frequency information is added. Although such windows are usually used in the time domain before transformation to frequency, they perform an exactly analogous task here and prevent spurious results from discontinuities at the high frequency end. The resolution in the time domain is related to the upper frequency limit of the data, so a transfer function run was performed on a trombone up to approximately \(2 \text{ kHz}\), giving a resolution of approximately \(\frac{1}{2} \text{ ms}\). The time scale of the time domain response is set by the frequency resolution, which in this case was \(2 \text{ Hz}\), giving a total time response of \(\frac{1}{2} \text{ second}\). This full time response is shown in Fig.2.28 and the first 1/16 second expanded out in Fig.2.29. There are a number of features which are of interest. First of all the response dies out completely within about 50 ms, with no response near the \(\frac{1}{2} \text{ second}\) point. Since the F.F.T. gives an output which is essentially periodic, any response appearing before the zero of time would show up here, and we can say that the function is causal. There is no a priori reason for any data file to be causal, and causality here reflects the complementary nature of the magnitude and phase which has been recorded, i.e. the real and imaginary parts must be related by Hilbert transforms. The structure
Fig. 2.28 Impulse response calculated from the pressure transfer function extending to \( \frac{1}{2} \) second
Fig. 2.29  Impulse response calculated from the pressure transfer function extending to 1/16 second.
of the response in Fig.2.29 is as one would expect, with a large response at 8.2 ms after the wave has first travelled down the instrument and a few further ones indicating multiple reflections.

The physical interpretation of the response is the pressure that would be seen one bell radius from the end of the instrument if one were able to generate a perfect delta function of pressure in the throat and maintain the pressure at zero thereafter, i.e. terminate it with zero impedance.

Turning our attention to the impedance data, we can compute a similar 'impulse' response by taking the transform of the weighted real and imaginary parts of the measured impedance with mouthpiece. This corresponds to the resultant pressure in the cup if a delta function of velocity is introduced, while the instrument is terminated by an infinite impedance. The results for a trombone measured to 2 kHz are shown in Fig.2.30 and the first 1/16 second in Fig.2.31. For comparison the results of the impulse response program previously used by Pratt (1978) are presented up to 1/16 second, Fig.2.32. Here the resolution is 2 ms and was computed without a 'window'. The improvement in Fig.2.31 can be readily appreciated, though the major features are still clear even with the larger resolution.

To understand the shape of this waveform, at least in its gross features, it is best to return to that simple system, the straight tube. Figure 2.33 shows the impedance of an open tube 2.8 m long by 1 cm diameter, computed by a program written by Mr. P.S. Watkinson. As expected, it has impedance peaks at odd multiples of 60 Hz. The calculated 'impulse response' is shown to 1/2 second in Fig.2.34.

Each peak may be regarded as a Lorentzian resonance with its own impulse response, and the impulse response of all the peaks is the sum
Fig. 2.30 Impulse response calculated from the impedance extending to 1/2 second
Fig. 2.31: Impulse response from the impedance extending to 1/16 second
Fig. 2.32 Impulse response calculated by Pratt (1978) from the impedance extending to 1/16 second
Fig. 2.33  Calculated impedance of straight tube 2.8 m long
Fig. 2.34 Impulse response calculated from impedance for tube 2.8 m long
of these. As discussed in Appendix D, the impulse response of a single Lorentzian resonance is a damped Sinusoid of frequency equal to that of the peak and time constant equal to the reciprocal of bandwidth. The whole impulse response will be made up from the addition of damped sine waves with a harmonic relationship between the 'frequencies'. For each 'cycle' of the fundamental, lasting 16 ms, there will be contributions from the higher peaks, which will be significant for the first few cycles. Since the bandwidth is proportional to the square root of frequency, the impulse responses of the higher peaks are highly damped and after several cycles it is predominantly the fundamental which remains. This behaviour can be seen in Fig.2.34 with the first few peaks being sharp in character while the later ones are more rounded.

The reflected pressure waves alternate in sign because the tube is open and has peaks at only odd multiples of 60 Hertz. If it were closed and so had peaks every 120 Hertz, all the reflected pressure pulses in Fig.2.34 would be positive. The 'rounding' of the later reflections is also seen in the measured response of the trombone in Fig.2.31, though this is complicated by having impedance peaks, particularly the first, which are not exactly harmonic.

The response shown in Fig.2.31 does have a significance to the player, since it is more or less the pressure he would experience if he tried suddenly to start a note, obviously premature reflections here could have a very detrimental effect on the transient characteristics of the instrument. Attempts have previously been made to explain the transient characteristics in a straightforward way from the frequency response (Benade, 1969 and 1976) by considering the group delay at the blown frequency. However, the initial wave propagated down an instrument does not have a 'narrow band' frequency characteristic and the group delay concept cannot be applied, 'broad band' impulse responses such as those
discussed here are necessary.

The fine structure of the two 'impulse' responses shown, Figs. 2.29 and 2.31, should be due to internal features in an instrument. The characteristics of an instrument in this domain should be relatively easily correlated with particular structures in the instrument, allowing a more direct design procedure.

The input impulse response may be confused in its detail for the first 16 ms because of multiple reflections. A peak at 4 ms may be caused by two round trips of 2 ms or one of 4 ms for example. To help sort out the reflection paths, two methods are proposed, which have been successful in similar applications (Kemerait and Childers, 1972, Hassab and Boucher, 1975).

1. Autocorrelation
2. Complex Cepstrum

These are computed by taking the F.F.T. in the normal way of:
1. the square of the amplitude with the phase set to zero
2. the log of the amplitude instead of the real part and the phase instead of the imaginary part.

The precautions which normally need to be taken when computing the autocorrelation with the F.F.T. (Randall, 1977) can be dispensed with in this case since the impulse response dies away before half of the total time scale.

The first 16 ms of the autocorrelation and cepstrum, together with the impulse response are plotted in Fig.2.35. No outstanding features show up on these graphs which did not appear in the original impulse response and it is tentatively concluded that multiple reflections do not play a significant role in the first 16 ms. Much of the fine structure of this period does, however, correlate quite well with specific features of the instrument. Under the impulse response in Fig.2.35 is a
The first 30 ms of the impulse response calculated from the impedance, together with its autocorrelation and complex cepstrum. The delays expected from reflections from various parts of the instrument are also shown.
sketch of the trombone used for the tests, with the time axis replaced by 'effective round trip' time, assuming the velocity to be the same as that for sound in free space. In particular, there do seem to be definite reflections from the discontinuities formed by the beginning and end of the bends in the instrument.

Finally, a simple but elegant experiment to directly determine the impulse response is reported which was suggested by Mr. J.C. Goodwin and carried out by the author. An approximation to an impulse of velocity can be created by slapping the palm of the hand down onto the rim of the mouthpiece. The pressure in the mouthpiece has been recorded for such an 'impulse' and is presented in Fig.2.36. The waveform does show a marked similarity to that computed from the impedance in many of its features. The experimental impulse response would not be expected to be exactly the same as the computed one because:

a) The experimental impulse has a finite width as can be seen from Fig.2.33,

b) The termination impedance at the mouthpiece rim is assumed infinite in the computed impulse response but it is, in fact, terminated by the surface impedance of the palm of the hand in the practical case.
Fig. 2.36 Experimental impulse response
CHAPTER 3

REGENERATION IN BRASS WIND INSTRUMENTS

3.1 Introduction

The sound radiated from a wind instrument is derived from the standing wave pattern inside the instrument. This is, in turn, maintained by the release of pressure from the lungs, via the lips, to the atmosphere. There have been a number of possible mechanisms put forward to explain how the transfer of energy occurs between the pressure source from the lungs and the pressure standing waves in instruments, and some of these are briefly reviewed.

1. It has been suggested that the muscles of the lips can move sufficiently rapidly to modulate the air stream by sheer muscle control. See for example the comments of Ellis in his translation of Helmholtz's tome (Helmholtz, 1895, p.98). A similar theory to account for the movement of the vocal cords was put forward in the 1950s. It was finally concluded however that the time response of the muscles was too slow compared to the period of oscillation to account for the motion which had been observed. A useful discussion of this is found in Fant (1960, p.265) and it seems unlikely that muscular action at this level could be a dominant regeneration mechanism in brass instruments.

2. Edge tone production at the throat of the mouthpiece has been postulated by E.G. Richardson (1929) as the main reason for tone generation. This theory now seems improbable because of the critical changes which would have to be made between the lips and the mouthpiece throat in moving from one note to another. Also the theory completely fails to explain why a note can be produced on an instrument without a mouthpiece, such as a straight tube. Nevertheless the theory is still
regarded highly by some authors with a more musical bias, for example Bate (1972).

3. The mechanism which has found most favour in recent years has been that of the pressure controlled valve. The lips are forced into motion by the pressure in the mouthpiece and the opening between the lips modulates the flow into the instrument. This gives rise to the driving pressure in the mouthpiece because of the instrument's impedance. The history of this theory dates back at least to Weber in 1830 and has been well reviewed by Worman (1971) and Pratt (1978).

4. The alternating flow through the lips must produce a time varying Bernoulli pressure in the lip opening which could also influence the motion of the lips. This is the most important mechanism for oscillation of the vocal cords and because of the dimensional similarity between the glottis and the lips, it may well play some role for brass instruments. A good discussion of the voice mechanism is to be found in Flanagan (1972) and a brief review of the literature is presented in section 2.4. The effect of the Bernoulli pressure on the motion of the lips will, at this stage, be disregarded since it is found that the major features of the regeneration may be accounted for by a pressure controlled valve mechanism.

In formulating the problem for the pressure controlled valve, it is helpful to draw a block diagram of the interaction between player and instrument, as shown in Fig.3.1. The 'reed' as indicated in this diagram, may be the cane reed of the clarinet, the lips of a brass player, or, in the most general case, the vocal cords during speech production.

There is however a distinction to be drawn between reeds which close as the pressure from the source increases (e.g. the reed of a clarinet), and reeds which tend to open under these conditions (e.g.
Fig. 3.1  Block diagram of regeneration in brass instruments
the lips of a brass player). Helmholtz (1895, p.95) clearly distinguishes between these two cases by calling the former reeds which 'strike inwards' and the latter reeds which 'strike outwards', which reflects the action of a reed if the driving pressure becomes very large. The difference is illustrated in Fig.3.2, though either sort of reed may be accounted for in the same theory by suitably defining the 'reed dynamics' discussed in section 3.3.2.

Another important difference between the two cases of Fig.3.2 is that whereas the clarinet has a single moving reed, the lips of the brass player are best represented by a 'double reed'. Each of these two reeds may be treated separately as 'single' reeds and the combined effect readily deduced.

There are two important parameters of the reed behaviour which must be adequately defined if any regeneration theory is to be successful in predicting the behaviour of the system. These are:

1. the dynamic response of the reed when forced by the pressure in the mouthpiece,
2. the equation of flow through the orifice formed by the reed opening as a function of opening area and pressure difference across the reed.

The equations defining both of the properties, together with the pressure source from the lungs and the behaviour of the instrument will be discussed in section 3.2. The ways in which these equations have been used together to construct analytic theories of regeneration are then reviewed in section 3.3. After a brief review of the speech source in section 3.4, regeneration in brass instruments is discussed in section 3.5.
Fig. 3.2  Idealisation of the 'reed' in a brass instrument and the clarinet.
3.2 Parameters defining the interaction

3.2.1 The pressure source

The main function of the lungs is to supply a pressure source to 'power' the instrument. The steady pressures in the mouth for notes blown on various wind instruments have been measured by a number of authors over the years. The results of two of these measurements for the trombone are presented in Fig. 2.3 (Vivona 1968, Wogram 1972) and are in broad agreement with some measurements of the present author's discussed in Chapter 4. The magnitudes are however only about half of those suggested by Barton (1902) for the trombone. No explanation can be offered for this.

The shape of the graph in Fig. 3.3 is typical of a number of other published results for brass wind-instruments though the overall level is higher for the trumpet; see, for example, Henderson (1942) and Bouhuys (1968). The mouth pressure is not, however, a unique and consistent quantity even for a single note and level, since different players use significantly different pressures and even very competent players may change the mouth pressure by 25% from trial to trial (Vivona 1968). This is in contrast to notes played in a clarinet for example where blowing pressures are very consistent and only vary slightly with level (Worman 1971) which allows a unique 'threshold blowing pressure' to be assigned for each note, which is purely a function of the instrument and cane reed (Backus 1963). It is the well defined properties of the cane reed in comparison to the variable and very frequency dependant parameters of a lip reed (as discussed in section 3.2.2) which makes the blowing pressure consistent in one case but so variable in the other.

Most authors have assumed that the pressure in the mouth is sensibly constant, but measurements discussed in Chapter 4 reveal that there is an
Fig. 3.3  Steady pressure in the mouth while blowing various notes on the trombone measured by different authors
alternating component in the mouth pressure which may be 20% of the steady pressure. This is due to the finite impedance presented by the vocal tract and mouth cavities to the flow from the lungs. It is best considered as a Thévenin equivalent impedance, in series with a constant pressure source, as discussed in Appendix E.

3.2.2 The reed dynamics

The movement of the 'reed' when driven by the pressure in the mouthpiece is probably the most difficult parameter of the interaction to specify for brass instruments. The lips should strictly be modelled as a distributed system of masses coupled by muscles whose restoring force would be a non-linear function of extension. However, to a first approximation, the dynamic properties of the lips may be deduced by considering the idealisation of Fig.3.2, where each lip is considered to be a rigid 'valve' constrained to move in only one direction. A number of studies have been reported where the motion of the lips was observed photographically while the instrument was being blown (Martin, 1942, Hadland 1959 and Leno 1971), though only Martin (1942) has solved the optical problems sufficiently well to give calibrated values of lip displacement. Even in this study, however, the pressure in the mouthpiece was not simultaneously recorded, so the all important phase angle between pressure and lip displacement could not be deduced. The major findings of Martin's observations were that

1. the lip motion is almost sinusoidal,
2. the lips just close once every cycle for all notes observed, and
3. the amplitude of vibration is much larger for notes of low pitch than those of higher pitch.

This supports the assumption of a linear model. The equations of motion for such a model are discussed in this section and solved to give:
1. the opening of the reed per unit pressure in the mouthpiece as a function of frequency, and
2. the effective impedance caused by the movement of the reed into the mouthpiece.

This effective impedance reflects the fact that as well as driving a certain volume velocity down the instrument, the pressure in the mouthpiece also forces the reed into motion, causing a certain reed velocity which, over each cycle, constitutes a volume velocity absorbed by the reed. The ratio of the driving pressure to this volume velocity is defined as the 'reed impedance', which is effectively in parallel with the impedance of the instrument. This effect has been known for some years and was discussed by Worman (1971) for a clarinet reed, but tends to be ignored by most authors in connection with brass instruments.

By considering the volume displaced by the moving reed in this way, much of the ambiguity about the 'plane of measurement' (Pratt 1978) may be resolved.

The model used for each lip is indicated in Fig.3.4 where the 'flap' formed by the reed surface is controlled in the y direction by a lumped stiffness (s), resistance (r) and mass (m). Consider a small sinusoidal pressure $P$ of angular frequency $\omega$ acting in the negative y direction (from the mouthpiece cup). This produces a force on the reed of $PA \cos \Lambda$ where $A$ is the area of the 'flap' and $\Lambda$ is the angle it subtends with the plane of the mouthpiece rim. The displacement produced by this force will be $y$ where

$$PA \cos \Lambda = (j\omega r + s - \omega^2 m) y$$

Since the reed is effectively pivoted, the movement opening the reed ($x$) becomes

$$x = -y \tan \Lambda$$
Fig. 3.4  Geometry for the idealisation of the reed movement
Some numerical estimate of $G(\omega)$ may be made if appropriate values for the lumped parameters are assumed. A computer programme has been written to do this and the parameters we have chosen to specify are:

1. the 'effective area' $(= A \cos \Lambda \tan \Lambda)$
2. the 'mass of the reed' $(=m)$
3. the 'natural frequency of the reed' $(= \sqrt{s/m})$
4. the 'Q' of the reed $(= \sqrt{s m}/r)$

For the lips acting as a reed the natural frequency of the system may be varied by changing the effective stiffness, while it is assumed that the mass remains essentially constant. Figure 3.5 presents the results of such a calculation for two 'resonant frequencies', 200 and 400 Hz. The other parameters were chosen to be representative of the lips in a trombone mouthpiece:

- mass $= 10^{-3}$ Kgm
- effective area $= 2 \times 10^{-4}$ m$^2$
- 'Q' $= 0.5$

The value of 'Q' is deduced from a study by Ishizaka et al (1975) who measured the mechanical impedance of forced vibrations of the flesh on various muscular parts of the body (neck and cheek). They found that the measured impedance approximated that expected from a lumped mass-spring-damper system, and whereas the natural frequency was determined by the tension in the muscles, and varied over a range of 30 to 60 Hz, the 'Q' of the lumped model remained appreciably constant at $0.5 \pm 0.03$ for most of the measurements.

The graphs presented in Fig.3.5 show the modulus and phase of $G$ for the two natural frequencies, and it can be seen that the form
Fig. 3.5 The magnitude and phase of $G$ for the lip reed with natural frequencies of 200 Hz (above) and 400 Hz (below)
of the two graphs is similar while the overall magnitudes show considerable differences, as is to be expected. The units of the magnitudes are Kg\(^{-1}\) m\(^2\) s\(^{-2}\) x 10\(^{-7}\) but from the definition of 'displacement per unit pressure' the units are brought into the more familiar form as 0.1 mm/kPa.

Turning our attention to the effective reed impedance (\(Z_r\)), in our notation the definition becomes

\[
Z_r = \frac{2P}{A\dot{y}}
\]

since the volume velocity absorbed by the reed is \(\dot{y}\frac{A}{2}\).

By using the fact that \(\dot{y} = j\omega y\) in the equation

\[
P A \cos \Lambda = (j\omega r + s - \omega^2 m) \dot{y}
\]

we have

\[
j\omega P A \cos \Lambda = (j\omega r + s - \omega^2 m) \dot{y}
\]

\[
\therefore \quad Z_r = \frac{2}{A} \times \frac{(j\omega r + s - \omega^2 m)}{j\omega A \cos \Lambda}
\]

\[
= \frac{2}{A^2 \cos \Lambda} \left( r + j(\omega m - \frac{S}{\omega}) \right)
\]

The numerical value of this has been calculated for the two cases above and the results presented in Fig.3.6. The units now are Kg m\(^{-4}\) s\(^{-1}\) or the familiar S.I. acoustic ohm. The magnitude of the impedance for the lower frequency is less than for the higher as expected because of the greater movement of the lips. It is important to note that here it is of the same order as the impedance of the instrument, measured in Chapter 2, so it may be expected to have a sensible effect. The nature of this effect is discussed in connection with the source impedance in Chapter 4.

It is of interest to draw a comparison between the lips acting
Fig. 3.6 The magnitude and phase of the effective reed impedance \( Z_r \) for the lip reed with natural frequencies of 200 Hz (above) and 400 Hz (below)
as reeds, discussed above, and the cane reed of a clarinet. The equation for the calculation of $G$ is exactly the same for an 'inward beating' reed as that for an 'outward beating' reed except that the sign of the 'effective area' is positive. The equation for the reed impedance remains unchanged however, since here the motion along the axis of the instrument is unaffected by whether the reed beats 'inwardly' or 'outwardly'. For a clarinet reed the natural frequency is generally much higher than the highest note of the instrument, so the reed operates in the 'stiffness controlled' part for the graph. Typical values for a clarinet reed are given by Worman (1971) as:

- Effective area $= 1.5 \times 10^{-4} \text{ m}^2$
- mass $= 3.2 \times 10^{-6} \text{ Kgm}$
- natural frequency $= 3.7 \times 10^3 \text{ Hz}$
- $'Q' = 8.0$

$G$ and $Z_r$ have been calculated for this reed and are presented in Fig.3.7. The order of magnitude of $|G|$ is much the same as before but since the frequencies are so far below resonance, the graph of $|G|$ is very flat, with virtually no phase shift between displacement and driving pressure. The reed impedance is seen to be largely compliant and drops sharply as the frequency rises.

Finally, we consider the action of two lips forming a double reed. If the two lips behave identically, the response would be exactly the same as a single reed of twice the area. However, important differences have been observed between the motion of the two lips by Henderson (1942) and by Martin (1942) indicating the the two lips have significantly different parameters. The total lip opening may be considered to be the sum of the two individual lips so, using suffixes 1 and 2 to distinguish the upper and lower lip, let
Fig. 3.7 The magnitude and phase of both $G$ and $Z_r$ for a clarinet reed
\[
\frac{x_1}{p} = C_1 \quad \text{and} \quad \frac{x_2}{p} = C_2
\]

So the total \( G = \frac{x_1 + x_2}{p} = C_1 + C_2 \)

Also, let
\[
2p \frac{\dot{X}_1}{\dot{Y}_1} = Z_{r1} \quad \text{and} \quad 2p \frac{\dot{X}_2}{\dot{Y}_1} = Z_{r2}
\]

So the total \( Z_r = \frac{2p}{A(\dot{Y}_1 + \dot{Y}_2)} = \frac{Z_{r1}}{A} + \frac{Z_{r2}}{A} \)

i.e. the two separate reed impedances in parallel.

It is not possible to deduce any numerical parameters for the differences between the two lips from any of the studies cited. However, a good illustration of the effect of the two lips combined may be obtained by considering the case where all the parameters are as defined above for the lips, except that the two lips have natural frequencies of 200 and 250 Hz. The results of calculating the effective value of \( G \) and \( Z_r \) for this combination are presented in Fig.3.8. In fact for this low 'Q' value the combination behaves very much as a single reed of twice the area of the individual reeds (shown in Fig.3.8 as dotted lines) and of resonant frequency about 225 Hz.

3.2.3 The flow equation

In the spirit of the approximation for the reed dynamics, the opening of the lips may be considered as a rectangular slit. The impedance of such a slit for laminar flow is given (Crandall 1927) by
Fig. 3.8 The magnitude and phase of the effective values of $G$ and $Z_r$ for the double lip reed. The parameters of the individual reeds are shown dashed.
\[ Z = \frac{12\mu d}{\xi x^3} + j \omega \frac{5\rho d}{6\xi x} \]

where  
\( \mu \) = the coefficient of viscosity for air  
\( \rho \) = the density of air  
\( d \) = the thickness of the slit (along the axis of flow)  
\( \xi \) = the length of the slit  
\( x \) = the height of the slit

However, this equation does not hold when the flow is rapid enough to become turbulent, as noted by Sivian (1935), and discussed at length by Ingard and Ising (1967). For a fully turbulent flow the resistance takes the form

\[ R = \frac{\rho}{Z} \frac{U}{\xi^2 x^2} \]

where \( U \) is the volume velocity through the slit. For the velocities in the orifice of the lip opening, while a note is being blown, the flow is in general turbulent. So the turbulent resistance always dominates the laminar one.

Several points arise as to the behaviour of the flow equations as both the lip opening and flow change with time.

Firstly, it is assumed that the flow is quasi-static, i.e. it does not depend on the speed of the lip movement. Flanagan (1972) considers this condition for flow through the glottis and comes to the conclusion that it is satisfied because of the 'small dimensions and high flow velocities' involved. Rather more formally Wilson and Beavers (1974) state that the flow through a clarinet reed is quasi-static because the Strouhal number is low.

Secondly, the imaginary part of the impedance for laminar flow describes the effects of the inerterance of the air in the slit, and a
complete analysis would have to include this effect. However, to a first approximation, the impedance due to the inertance is much less than the turbulent resistance at the frequencies considered here. Again Flanagan (1972) has considered this point for the glottis and has shown that the time constant determined by the resistance and inertance of such an orifice is always less than 0.2 ms, and so the impedance was considered completely real at speech frequencies. This conclusion is supported by the numerous theoretical studies on woodwinds where the inertance has been taken into account in the initial formulation but later analysis has shown its effects to be negligible (Backus 1963, Nederveen 1969, Wilson and Beavers 1974, and Fletcher 1979b).

In conclusion, the time independent flow equation we may use is given by

\[ U = \sqrt{\frac{2(P_m - P_i)}{\rho}} \times \]

where \( P_m \) is the instantaneous pressure in the mouth, \( P_i \) is the instantaneous pressure in the cup of the instrument mouthpiece and the other symbols have been defined previously.

The flow equation now only depends on the area of the orifice, not on its shape. So the original assumption of a rectangular slit may be relaxed as long as the orifice area is directly proportioned to its opening. This has been shown by Martin (1942) to be a good approximation for the opening area of the lips.

3.2.4 The behaviour of the instrument

We require the relationship between the pressure in the mouthpiece cup and the volume velocity which travels down the instrument. In Chapter 2 it was shown how this may be considered to
be linear and characterised in the time or frequency domains. For the purposes of the later analysis, which assumes a periodic solution, it is obviously the impedance as a function of frequency which should be used.

3.2.5 Summary

The preceding sections have presented equations, to a first approximation, which define each of the 'boxes' in Fig.3.1. The equations governing the behaviour of the instrument and reed dynamics are considered to be linear, and it is the flow equation for the reed orifice which makes the problem non-linear. This is because the flow depends on the reed opening and the pressure difference, both of which vary with time. The next section will be concerned with the attempts which have been made to solve this set of coupled equations.

3.3 Review of regeneration theories

3.3.1 Introduction

Theories of regeneration have tended to concentrate on woodwind instruments, particularly the clarinet, since here the behaviour of the reed can be directly measured and so all the parameters of the interaction are fully defined. The basic theory formed by combining all the equations discussed in section 3.2 will (with minor modifications) hold true for any pressure controlled valve oscillation. It is the assumptions which have to be made in obtaining simultaneous solutions to this set of equations which are very different for, say, the clarinet and trombone and which leads to the observed differences in behaviour. For example, a reasonable approximation for notes of low amplitudes on a woodwind instrument is that the pressure in the
mouth is always much larger than that in the mouthpiece, whereas this
is not a good approximation for brass instruments. Nevertheless the
various published methods of solution will be reviewed since they
form the basis for the theory presented in section 3.5.

Although the history of regeneration for wind instruments dates
back to before Helmholtz, progress has been relatively slow. In recent
years however there has been a resurgence of interest in the subject as
witnessed by two recent review papers published by Fletcher (1979a, 1979b).

3.3.2 Linear theories

The most straightforward formulation was, in essence,
presented in Appendix VII of Helmholtz (1895):

Assuming $P_m \gg P_i$

then $u(t) = \sqrt{\frac{2 P_m}{\rho}} x(t)$ \quad \therefore \quad u(f) = \sqrt{\frac{2 P_m}{\rho}} x(f)$

since $x(f) = G P_i(f)$ and $u(f) = P_i(f)/Z_i$

we have $\frac{1}{Z_i} = \sqrt{\frac{2 P_m}{\rho}} x G$

This we shall call the Helmholtz regeneration condition and is a very
important starting point in our discussion. When considered as a
condition for oscillation in a linear feedback loop, it constrains the
'loop gain' in exactly the same way as the now-famous Nyquist criterion
(Nyquist 1932) of electrical circuit theory.

The magnitude and phase conditions imposed by Helmholtz above will
be considered separately. The magnitude condition is that the impedance
of the instrument must be large so that the term $\frac{1}{Z_i}$ is smaller than the
magnitude of the right hand side of the above. If the phase angle of
$Z_i$ is $\xi$ and the phase angle of $G$ is $\delta$, the phase relationship becomes
\[ \delta = -\xi \]

An illustration of the phase conditions for oscillation of an 'inward beating' reed and an 'outward beating' reed are shown in Fig. 3.9. There \( \delta \) is taken to be the phase angle of \( \xi \) for the clarinet and double lip reed, discussed in section 3.2.2 and \( \xi \) is the phase angle of a single impedance peak of, for instance, the straight tube discussed in section 2. The frequency of oscillation, assuming the magnitude condition is satisfied, is the frequency at which graphs for the two phase angles cross. For the clarinet this is very close to the frequency of the impedance maximum, but since \( \delta \) must be negative, the blowing frequency is always slightly flat compared to the zero crossing of \( \xi \).

The lip reed on the other hand must have an oscillation frequency above the frequency at which \( \xi \) is zero since \( \delta \) must be positive for this case. Since \( |\delta| \) passes through \( \pi/2 \) at what we may call its natural frequency and \( |\xi| \) can never exceed \( \pi/2 \) and have a positive real part, we can also identify the conditions on the natural frequency of the reed. The natural frequency of the reed must be below the oscillation frequency for the outward beating reed and above it for the inward beating reed.

If \( \omega_r \) is the natural frequency of the reed, \( \omega_i \) is the frequency at which \( \xi = 0 \) and \( \omega_0 \) is the oscillation frequency we may represent the conditions as

\[ \omega_0 < \omega_r \text{ and } \omega_i \text{ for an inward beating reed, and} \]
\[ \omega_0 > \omega_r \text{ and } \omega_i \text{ for an outward beating reed.} \]

All of these conditions are implied in the work of Helmholtz (1895).

This basic theory has been extensively developed for the clarinet, in particular by Backus (1963), Nederveen (1969) and Wilson and Beavers (1974). These authors develop a more complete set of equations, in
Fig. 3.9 Demonstrating the Helmholtz phase condition for the inward beating reed (using the clarinet as an example) and the outward beating reed (using the lip reed as an example)
particular for the flow condition, but finally consider only the fundamental components of the oscillating quantities. These are essentially 'linear' theories since the effect of the harmonics in the alternating quantities has not been considered. Taken to its logical conclusion, such theories would predict oscillation at several frequencies which may not be harmonically related and allow the amplitudes of the alternating quantities to increase without limit. These anomalies can only be resolved by explicitly considering the non-linearity of the interaction.

3.3.3 Non linear theories

The consequences of the essential non-linearity in the regeneration were first discussed by Benade and Gans (1968). They point out that the non-linearity in the equation 'locks together' the amplitudes and phases of the various harmonics which must be present. So the mouthpiece pressure, for instance, has a well defined harmonic structure. These effects had been known and studied previously only with particular attention to non-linear electronic oscillators (Van der Pol 1934) and it is only now that the consequences for wind-instruments are beginning to be put on any firm quantitative foundation (Fletcher 1978). The paper by Benade and Gans (1968) laid the way for this but it was not until Worman (1971) studied the problem for a 'clarinet-like system' that any real headway was made.

Worman first expanded the flow equation as a double Taylor series:

\[ U(t) = \sum_{h,i} F_{hi} (x(t))^h (P_m - P_i(t))^i \]

then, assuming a steady state solution, represented the velocity reed displacement and mouthpiece pressure as Fourier series. So that, for example
\[ P_i(t) = \sum_n P_n \cos(n\omega t + \phi_n) \]

where \( \omega \) is the frequency of oscillation, \( P_n \) is the amplitude of the \( n \)th component of the mouthpiece pressure and \( \phi_n \) its phase.

Now let

\[ U(t) = \sum_n P_n / Z_n \cos(n\omega t + \phi_n - \xi_n) \]

where the impedance of the instrument at a frequency of \( n\omega = Z_n e^{j\xi_n} \)

and \( x(t) = \sum_n P_n \cos(n\omega t + \phi_n + \delta_n) \)

where \( G_n \) at a frequency of \( n\omega \) is \( G_n e^{j\delta_n} \). The complete equation now becomes

\[ U(t) = \sum_n P_n / Z_n \cos(n\omega t + \phi_n - \xi_n) = \sum_{h,i} F_{hi} \left[ \sum_n P_n \cos(n\omega t + \phi_n + \delta_n) \right]^h \]

\[ \times \left[ P_m - \sum_n P_n \cos(n\omega t + \phi_n) \right]^i \]

The final expansion is enormous, even though the Taylor series was truncated at \( h = i = 3 \) and only the first few harmonics of the pressure waveform considered. Nevertheless, the expansion was solved by comparing the terms proportioned to \( \cos(n\omega t) \) and \( \sin(n\omega t) \) on either side yielding a set of coupled equations, two for each harmonic. Since Worman knew all the parameters of the instrument and reed he was working with, this set of equations could (in principle) be solved if the frequency of oscillation was assumed to give the amplitude and phase of each harmonic. In practice each solution involved a number of approximations and considerable effort. The results obtained were in reasonable agreement with the results obtained by blowing.

An alternative approach has been used by Schumacher (1978) who reformulated the equations as a non-linear integral equation of the Hammerstein type.
If 

\[ P_i(t) = \sum_n Z_n e^{j\omega t} \]

and 

\[ P_n = \sum_n U_n \]

the amplitude of the nth harmonic of \( U \) is given by 

\[ U_n = \int U(t') p^{-j\omega t'} dt' \]

Representing the flow equation as a function, \( U \), of \( x \) and \( P_i \)

\[ U(t) = U(x(t), P_i(t)) \]

we have 

\[ P_i(t) = \sum_n Z_n \left[ U(x(t'), P_i(t')) e^{j\omega t'} dt' \right] j\omega t \]

which may be put in the form 

\[ P_i(t) = \int K(t-t') U(x(t'), P_i(t')) dt' \]

where the Kernal \( K(t) \) is given by 

\[ K(t) = \sum_n Z_n e^{j\omega t} \]

which is a non-linear equation of the Hammerstein type. Numerical methods have been developed to solve equations of this type which are considerably quicker than the series method of Worman, or at least are more easily programmed into a digital computer.

Schumacher (1978) also mentions the possibility of incorporating the Bernoulli force into the equations by formulating the motion of the reed as another non-linear equation of the Hammerstein type. However Schumacher ominously notes that "The mathematical territory involving strongly coupled non-linear integral equations is rather uncharted, and one should expect to encounter interesting complications"!

Finally, it should be noted that all the methods of solution discussed assume steady-state solutions, in other words, the pressure in the mouthpiece is assumed to be periodic. One consequence of this is that transient solutions cannot be obtained.
3.4 The voice mechanism

A brief review of the literature on the theory of the speech source and the methods of modelling this source is now given. This throws light on the similarities and differences between the action of the lips as a 'pressure controlled valve' and the glottis, and also suggests convenient models for use in brass instruments acoustics.

It is generally agreed that the main force driving the glottis is due to the time-varying Bernoulli pressure forcing the glottis into motion at right angles to the direction of flow. The dominant force on the lips of a brass player, however, is due to the pressure in the mouthpiece forcing the lips into motion in the same direction as that of the flow. The similarities in the dimensions and rates of flow does suggest however that both the glottis and lips are, in part, controlled by a combination of these two pressures. For instance, Benade has suggested that 'the presence of the Bernoulli force enables a brass player to lip a note on both sides of the impedance peaks, instead of being restricted to the higher side as is required for an outward beating pressure controlled reed' (quoted in Worman 1971, p.111).

In a similar vein St. Hilare et al (1971) suggest that the vibration of the lips and vocal cords may depend on the same 'unsteady boundary effect'. St. Hilare et al (1971) also make the very pertinent remark that "The aerodynamics of many of these sources is still not well understood, whereas the acoustic properties of the resonators associated with the various sources have been extensively studied in many cases".

In this section we consider an oscillator entirely controlled by the time-varying Bernoulli pressure. In the general case such an oscillation is very difficult to model analytically, but simplified theories have been presented by Ishizaka et al (1968a) and Gupta et al (1973).
Both of these authors start by modelling the glottis as a single mass-spring-damper system which must now be constrained to move at right angles to the flow. Under these conditions they find that oscillation is only possible if the input reactance of the vocal tract is positive. It is significant that this is exactly the condition imposed on an inward beating reed in section 3.3.2. If the flow equation through the reed opening is entirely governed by Bernoulli's equation, it follows that the Bernoulli pressure acting on the reed tip will be proportional to the pressure difference across the reed. Put explicitly, if the particle velocity ($V$) is given by

$$V(t) = \frac{2(P_m - P_i(t))}{\rho} \ell$$

(from section 3.2.3)

then the force ($F$) on the reed due to the Bernoulli pressure is

$$F(t) = -\frac{\rho}{2} V^2(t) A_2$$

where $A_2$ is the effective area over which the Bernoulli force acts.

$$\therefore F(t) = -A_2 (P_m - P_i(t))$$

So a small alternating pressure downstream of the 'reed' would result in a force on the reed directly proportioned to this pressure, exactly as for an 'inward beating' reed. We have effectively assumed a large steady flow through the 'reed' with a small alternating component superimposed, which may be unrealistic for the glottis in some situations. The equations which result from this analysis are, however, just the same as those which lead to the result for an inward beating reed (section 3.3.2) that the system can only oscillate into an inertive load. This view of the glottis, as a pressure controlled reed, is interesting but possibly not significant since the condition it imposes on the impedance of the vocal tract is unrealistic. Ishizaka et al (1968b) for example have
shown that in reality the voice source will oscillate when loaded by either an inertive or compliant load reactance.

The problem was resolved by both Ishizaka et al (1968a) and Gupta et al (1973) by modelling the glottis as a system of two coupled masses along the direction of flow. It should be emphasised that this 'two mass model' is not the same as the assumption in section 3.2.3 of a 'double reed'. The model of the double reed presented in section 3.2.2 is arranged at right angles to the direction of flow so the openings of the two lips just add together to produce an 'effective' lip opening. The total variation of opening area with time remains sinusoidal if the reed is driven by a sinusoidal pressure, so the system remains linear. A 'two mass model' implies that the glottis has a finite thickness along the direction of the flow so the air must flow through the gap formed by the first mass, then through the gap formed by the second. The 'effective area', as seen by the flow, of the two mass model is not a linear function of driving force, since even if this force is sinusoidal the 'effective area', which is dominated by the opening which is the smaller of the two, will in general not be sinusoidal. If there is any phase difference between the motion of the two masses, for example, the 'effective area' is determined by the displacement of one mass during one part of the period and the displacement of the other mass for the remainder. A two mass model of this sort will oscillate over a much wider and more realistic range of load reactances. The justification for such 'distributed' models of the glottis has been considered in detail by Titze (1975, 1976) who has studied the effects of the differing mechanical properties of the various physiological parts of the vocal cords.

Apart from the analytic theories there has been considerable development of numerical, iterative, methods facilitated by digital
computers. The original pioneering work was carried out by Flanagan and Langraf (1968). In the model developed by these authors, the vocal cords are considered as a single mass, controlled by a linear stiffness and damper, forced by the Bernoulli pressure into motion at right angles to the direction of flow. The vocal tract is represented by a series of ten 'T' filters modelling the variation of an area with length, and the whole system is fed by a constant pressure source. When implemented on a computer, the velocity through the glottis for some 'rest' opening is first calculated, and from this the Bernoulli pressure and hence the incremental displacement of the glottis is calculated. This changes the area of the glottis opening and the program proceeds by calculating the velocity for the next iteration. Each increment of velocity 'propagates' down the ten 'T' sections representing the vocal tract and the resultant sound pressure which radiates from the mouth may be deduced.

The results from this first study were very encouraging, the model not only oscillated but gave realistic waveforms for the glottal opening and flow for a variety of frequencies and levels. A number of later papers reported how the model was developed and refined, in particular, a 'two mass model' was introduced by Ishizaka and Flanagan (1972) in an excellent paper which also elucidated many other aspects of the interaction. The two-mass model was introduced because "the one-mass model was congenitally incapable of sustained oscillation for a capacitive input load of the vocal tract" (Ishizaka and Flanagan 1972), so this model agrees with the analytical theories presented previously.

A similar, but more detailed numerical model, has been presented by Titze (1973,1974) whose careful consideration of the physiological parameters of the vocal cords led him to use a system of sixteen coupled elements to represent each side of the vocal cords. Results from this
model show very good agreement with the observed motion of the various parts of the glottis and has enabled the study of phonation under medically pathological as well as 'normal' conditions.

The possibility of using a model similar to that of Flanagan and Langraf (1968) to represent the regeneration in brass wind instruments has been put forward by Pyle (1969), though no results have been reported. Such a model could have considerable advantages; many effects which are very difficult to consider analytically may be incorporated fairly easily into such a model, for example, the effect of the Bernoulli pressure as well as that in the mouthpiece, or the effect of a two mass model. It is also in the nature of such a model that transient phenomena are readily studied. Starting transients could be investigated by starting the iterations with the model 'at rest' but with a variety of initial conditions, or changes in the different parameters may be made once the oscillation has started to simulate slurs, for example.

At our present state of knowledge however, such an ambitious project was thought not to be justified since it was felt that not enough was known about the fundamentals of the regeneration. In particular, we are still a long way from a complete understanding of the motion of the lips. With hindsight it can be seen that in some instances the mechanics of the model for the vocal cords have been so exclusively studied as to be almost a substitute for studying the physics of the real interaction. One of the aims of the present thesis has been to begin an investigation of the range and characteristics of notes blown by players on brass instruments (Chapter 4). In doing so we hope to test the validity of the assumptions about the regeneration which have been made so far, and attempt to highlight the parameters which are of importance in modelling the regeneration.
3.5 An analytic theory for brass instruments

For notes of low pitch on brass instruments, the waveform of the pressure in the mouthpiece has a very characteristic, non-sinusoidal shape. This is thought to be due to the large amplitudes of lip vibration, such that over a large proportion of the period of vibration, the lip opening is so great that the pressure in the mouthpiece rises to be more or less the same as that in the mouth. When the lips do come together, the flow into the mouthpiece is restricted and a downward 'spike' of pressure results, whose width is small compared to the period. This has been deduced qualitatively by Backus and Hundley (1971), though the analysis which leads to their numerical results is not strictly valid.

For instance, a unique resistance is assigned to the orifice formed by the lip opening as a function of opening area, but not as a function of flow through the reed, though both are shown to be of importance in section 3.2.3. In this section a simplified theoretical treatment of the regeneration in brass instruments is presented which predicts the harmonic structure of this characteristic waveform, as well as the transition to the more sinusoidal waveform at higher frequencies.

The conditions for the existence of any such oscillations (regeneration conditions) are also predicted.

The formulation is similar to the series expansion of Worman (1971). However, the form of the expansions must be very carefully controlled if the equations are not to become as large and unmanageable as his. The equation is first set up using the relationships discussed in section 3.2, with the additional condition that the 'source' and 'reed' impedances are incorporated into a total effective impedance \( Z \) as defined in Appendix E.

We start with the flow equation

\[
U(t) = \frac{\sqrt{2(P_s - P(t))}}{\rho} \times x_1(t)
\]
where $U$ is the volume velocity

$P_r$ is the alternating pressure discussed in Appendix E

$P_s$ is the steady pressure in the mouth

$x_1$ is the total opening of the lips, and the other symbols have been defined elsewhere.

Note that the steady pressure in the mouthpiece may, if desired, by incorporated in $P_s$ though in practice it is only about 1% of the steady pressure in the mouth, and may be ignored.

Expanding the square root to the first two terms of a Taylor expansion gives

$$\sqrt{P_s - P(t)} = \sqrt{P_s} \left(1 - \frac{P(t)}{2P_s}\right)$$

This is not formally valid since $P(t)$ may be of the same order of magnitude as $P_s$, but is a much better approximation than, say, that of Helmholtz (section 3.3.2) who effectively considers only the first term of the equation above.

Now let $x_1(t) = x_0 + x(t)$

where $x_0$ is the average lip opening and let

$$C = \sqrt{\frac{2P_s}{\rho}}$$

so the flow equation becomes

$$U(t) = C(1 - \frac{P(t)}{2P_s})(x_0 + x(t))$$

$$\Rightarrow C\left[x_0 + x(t) - \frac{x_0}{2P_s} P(t) - \frac{P(t)}{2P_s} x(t)\right]$$

The first term in the R.H.S. of this equation represents the steady flow.

The term \(C x(t)\) is the same as that considered by Helmholtz. The term \(\frac{x_0}{2P_s} P(t)\) is a linear term introduced because the pressure difference across the reed has been considered in the flow equation,
and the final term is the only non-linear one, which has now been isolated, so that its effects may be evaluated.

Assuming a periodic solution we may expand \( P(t) \) as a Fourier series

\[
P(t) = \sum P_n \cos(n\omega t + \phi_n), \text{ say.}
\]

The equations for the impedance and reed dynamics now become (as in section 3.2.3)

\[
\frac{p_n}{U_n} = Z_n e^{j\xi_n}, \quad \frac{x_n}{p_n} = G_n e^{j\delta_n}
\]

One final assumption, which greatly simplifies the algebra, is that the displacement of the reed is approximately sinusoidal, as observed by Martin (1942). So we may write

\[
x(t) = G_1 P_1 \cos(\omega t + \phi_1 + \delta_1)
\]

and the complete equation becomes

\[
U(t) = U_0 + \sum P_n \cos(n\omega t + \phi_n - \xi_n)
\]

\[
= C [x_0 + G_1 P_1 \cos(\omega t + \phi_1 + \delta_1)
- \sum \frac{x_0}{2} \sum P_n \cos(n\omega t + \phi_n)
- \frac{G_1 P_1}{2} (\sum P_n \cos(n\omega t + \phi_n)) (\cos(\omega t + \phi_1 + \delta_1))]
\]

The non-linear term

\[
(\sum P_n \cos(n\omega t + \phi_n)) (\cos(\omega t + \phi_1 + \delta_1))
\]

may be expanded to give
\[ P_{1/2} (\cos(-\delta_1) + \cos(2\omega t + 2\phi_1 + \delta_1)) \]

\[ + P_{2/2} (\cos(\omega t + \phi_2 - \phi_1 - \delta_1) + \cos(3\omega t + \phi_1 + \phi_2 + \delta_1)) \]

\[ + P_{3/2} (\cos(2\omega t + \phi_3 - \phi_1 - \delta_1) + \cos(4\omega t + \phi_1 + \phi_3 + \delta_1)) \]

\[ + P_{4/2} (\cos(3\omega t + \phi_4 - \phi_1 - \delta_1) + \cos(5\omega t + \phi_1 + \phi_4 + \delta_1)) \]

The terms on either side for the various harmonics are now compared to give

for \( n = 0 \)
\[ U_0 = C[x_0 - \frac{G_1 P_1^2}{4 P_s} \cos \delta_1] \]

for \( n = 1 \)
\[ \frac{P_1}{Z_1} \cos(\omega t + \phi_1 - \xi_1) = C[G_1 P_1 \cos(\omega t + \phi_1 + \delta_1) \]
\[ - \frac{x_0 P_1}{2 P_s} \cos(\omega t + \phi_1) - \frac{G_1 P_1 P_2}{4 P_s} \cos(\omega t + \phi_2 - \phi_1 - \delta_1)] \]

for \( n = 2 \)
\[ \frac{P_2}{Z_2} \cos(2\omega t + \phi_2 - \xi_2) = C[- \frac{x_0 P_2}{2 P_s} \cos(2\omega t + \phi_2) \]
\[ - \frac{G_1 P_1}{4 P_s} (P_1 \cos(2\omega t + 2\phi_1 - \delta_1) + P_3 \cos(2\omega t + \phi_3 - \phi_1 - \delta_1)) \]

for \( n = 3 \)
\[ \frac{P_3}{Z_3} \cos(3\omega t + \phi_3 - \xi_3) = C[- \frac{x_0 P_3}{2 P_s} \cos(3\omega t + \phi_3) \]
\[ - \frac{G_1 P_1}{4 P_s} (P_2 \cos(3\omega t + \phi_2 + \phi_1 + \delta_1) + P_4 \cos(3\omega t + \phi_4 - \phi_1 - \delta_1))] \]

Since we can define the zero of time arbitrarily, we set \( \phi_1 = 0 \).

Also, since each of the coupled equations above is linear, we may revert
to complex notation.
If \( P_2 < 4 P_s \) we have for \( n = 1 \)

\[
\frac{1}{Z_1} = C(G_1 - \frac{x_0}{2 P_s}) \quad \text{where} \quad Z_1 = G_1 e^{j\xi_1}
\]

and \( G_1 = G_1 e^{j\delta_1} \)

The \( n = 0 \) equation may be put in the form

\[
x_0 = \frac{U_0}{C} + \frac{P_s^2 G_1}{4 P_s} \cos \delta_1
\]

and substituted into the \( n = 1 \) equation to give

\[
P_1 = P_s \left[ \frac{8}{C G_1 \cos \delta_1} (C G_1 - \frac{1}{Z_1} - \frac{1}{2 R_0}) \right]^\frac{1}{2}
\]

Because the term inside the square root must be positive for realistic solutions, we see that the regeneration condition has become

\[
C G_1 \geq \frac{1}{Z_1} + \frac{1}{2 R_0}
\]

This is similar to the Helmholtz condition discussed in section 3.3.2, with the additional term \( \frac{1}{2 R_0} \). Indeed, if we define yet another impedance as the parallel combination of \( Z_1 \) with \( 2 R_0 \), the new condition becomes identical to that of Helmholtz. In general terms this means that our discussion of the conditions for oscillation for an outward beating reed (in section 3.3.2) is still valid. Specifically the reactance of \( Z_1 \) must still be negative for regeneration. The magnitude of \( C G_1 \) may have to be increased to overcome the effect of the \( \frac{1}{2 R_0} \) term on the R.H.S. but this is easily accomplished by raising the blowing pressure.

Turning our attention to the equations for the higher harmonics, we see that all the equations for \( n \geq 2 \) may be represented in the complex form

\[
\frac{P_n}{x_n} = C \left[ -\frac{x_0}{2 P_s} P_n - \frac{P_1}{4 P_s} (G_1 P_{n-1} + G_1^* P_{n-1}) \right]
\]

where \( P_n = P_n e^{j\phi_n} \) and \( G_1^* \) is the complex conjugate of \( G_1 \).
At this point we have to start making assumptions which will affect the form of the harmonic structure, and the most important assumption to make is about the relative magnitudes of $Z_1$ and $2R_0$.

We first consider the case where

$$Z_1 \text{ and } Z_n \gg 2R_0$$

The regeneration condition now becomes

$$\frac{C}{2} \frac{G_1}{R_0} > \frac{1}{2}, \quad \therefore \quad \frac{C}{2} \frac{G_1}{Z_1} \gg \frac{1}{Z_1}$$

and the $n = 1$ equation, which may be written as

$$\frac{C}{2} \frac{x_0}{P_s} = C \frac{G_1}{Z_1} - \frac{1}{Z_1} \quad \text{becomes} \quad \frac{C}{2} \frac{x_0}{P_s} \approx C \frac{G_1}{Z_1}$$

Substituting this back into the general expression for the $n$th harmonic and assuming that $P_{n-1} \gg P_{n+1}$, we obtain

$$\frac{P_n}{Z_n} = C \left[ - \frac{G_1}{4} \frac{P_1}{P_s} \frac{G_1}{P_{n-1}} \right]$$

Since $C \frac{G_1}{Z_n} \gg \frac{1}{Z_n}$, we are left with

$$P_n = - \frac{P_1}{4} \frac{P_{n-1}}{P_s}$$

So, adding up the contributions from all the harmonics

$$P(t) = \sum_{n=1}^{\infty} \left[ - \frac{P_1}{4} \frac{P_{n-1}}{P_s} \right]^{n-1} P_1 \cos(nut)$$

The assumption that $Z_1$ and $Z_n \gg R_0$ means, in effect, that the 'average resistance' of the lip opening is small compared to the instrument's impedance. In other words, the lip opening is large over most of the cycle, which we have associated with notes of low pitch on brass.
instruments. The harmonic relationship is at least of the right form for these low notes since if we take it to its limit by setting

$$\frac{p_1}{4p_s} = 1, \quad \text{we have}$$

$$P(t) = \sum_{n} (-1)^n \frac{p_1}{n} \cos(n \omega t)$$

producing a negative going impulse every cycle. A more detailed comparison with measured waveforms is given in Chapter 4.

As notes of higher pitch are blown the magnitude of $G_1$ will decrease (section 3.2.2), so a smaller lip movement is produced for a given alternating mouthpiece pressure and a larger steady mouth pressure is needed to maintain the oscillation. Since the lip opening is smaller for these notes $R_0$ will increase and become comparable with $Z_1$ and $Z_n$. For very high notes the flow resistance of the orifice is always larger than the impedance of the instrument and the flow becomes proportional to the opening area of the lips. Since this is sinusoidal, the flow and hence the mouthpiece pressure also become much more sinusoidal than before.

In conclusion then, we have developed a simplified theory for the regeneration in brass instruments by making a number of assumptions about the interaction which we believe to be reasonable. The regeneration condition predicted by this theory is similar to that of Helmholtz, in particular we know that the imaginary part of the combined source and instrument impedance (Appendix E) must be negative at the frequency of oscillation. The relative magnitudes of the "average" resistance of the lip opening ($R_0$) and the impedance of the instrument play a large part in determining the harmonic structure of the note produced. For notes of low pitch, where the lip displacement is large, the mouthpiece
pressure has the characteristic harmonic structure presented above, which is independent of the impedance of the instrument provided it is high at all the harmonic frequencies \((Z_n \gg 2 R_0)\). If this condition is not satisfied, for notes of higher pitch for example, the levels of the harmonics will be lower than in the above case. So the pressure waveform in the mouthpiece becomes more sinusoidal. Also, the harmonic structure will become dependant on the magnitude of the impedance at the harmonic frequencies.
4.1 Introduction

A study of the range and characteristics of the notes which can be blown on brass instruments is described, which tests the theory of regeneration which has been put forward and suggests areas in which the theory may be inadequate. Since this thesis has been concerned with the basic regeneration rather than specifically musical effects, it has been best to study the parameters which have the most direct bearing on the interaction between player and instrument. For instance, the pressure waveform in the mouthpiece cup is of more use in evaluating the details of the interaction than, say, the pressure waveform outside the instrument. This does mean however, that our results are not comparable with a number of other studies where the radiated sound was examined (e.g. Luce and Clark 1967, Risset and Mathews 1969 and Beauchamp 1975). In principle all the pressure waveforms measured in the mouthpiece could be transformed by the pressure transfer function to give the pressure outside the instrument. This was not considered worthwhile at this stage especially since the effects of the directionality of the instrument and the room reverberation would be difficult to model.

The major obstacle to conducting a comprehensive study of the characteristics of notes on brass instruments is the large number of variables which can influence the note. The variability of the steady pressure in the mouth (Vivona 1968) has already been discussed in Section 3 and this reflects differences in blowing technique between players, and for one player on different occasions. The reason for this variability is that the player is able to control the interaction in a wide variety of ways. In particular the 'reeds' formed by the
brass player's lips are completely under his control in contrast to, say, the woodwinds. More subtle effects such as the finite impedance of the pressure source from the lungs also play an important role. It is useful at this stage to list the main parameters which will have an influence on the interaction, though no such list can be regarded as comprehensive.

1. **The instrument being played**

   This includes the differences between types of instrument (e.g. the trombone or the trumpet) and also the differences between individual instruments and their mouthpieces. This study has concentrated mainly on one type of instrument, the trombone, whose pitch is approximately in the middle of the range of brass instruments. To illustrate particular points, however, mention has been made of notes blown on both the trumpet and an isolated trombone mouthpiece. One instrument of either type (trombone and trumpet) has been used throughout this study and details of these together with their mouthpieces are given in Appendix C. All the measurements in Chapter 2 and the notes studied in this chapter have been obtained with the instruments in the same condition; all the tuning slides were pushed fully in, the trombone slide was locked in the first position and the valves of the trumpet were left 'open'.

2. **The note played**

   The range of musical notes generally played in first position on the trombone used (a tenor "Bb" instrument) is from Bb2 (approximately 116 Hz) to about C5 (approximately 523 Hz). (The musical notation in this thesis follows the American standard, "A.S.A.T.", 1960). The impedance peaks of the instrument are roughly harmonic and this range of notes corresponds approximately to the frequencies of the second to ninth impedance peaks. The frequency of the first peak is much lower
than would be expected from a strict harmonic series (approximately 40 Hz rather than 60 Hz) and the lowest possible note on a trombone (the 'pedal' note) is of a different quality from the others and is best discussed separately (see section 4.3).

3. Intonation

For each of the musical notes a player can 'lip' the exact frequency over a considerable range by changing his embouchure. The range of intonation is discussed for several players in Section 4.2.

4. Dynamic level

A good player can change the level of most notes by about 35 dB, though this depends on the training and physical conditions of the player.

5. Different players

There are large musical differences between notes produced by different players. This partly reflects the experience and skill of the player and partly what a musician would call his 'style'. The differences in the range of intonation for several players is discussed in section 4.2. Differences in tone quality which may be musically significant are relatively small compared to the major characteristics observed for all competent players, especially when measured in the mouthpiece. Most of the notes studied in sections 4.3, 4.4 and 4.5 were blown with the generous co-operation of one player (Mr. P.S. Watkinson). These notes were, moreover, chosen to be of particular scientific interest and demanded no great musical interpretation, so the characteristics of the notes we have studied may be regarded as typical for all competent players.

6. Embouchure of player

Even for a note blown at one frequency and level, the player can change the tone quality of a note by adjusting his embouchure. While
this is of musical importance the physical differences which result in
the mouthpiece pressure, for example, are generally relatively small.
The interest in such changes in the present study has been in the
parameters which the player can change to bring this about, as much as
the detailed effects of these changes.

7. Time dependence

It has been pointed out in Chapter 3 that all the theoretical
predictions are only valid for the steady-state part of a note, so
that the waveform changes very little from cycle to cycle. This
condition is difficult to obtain with a real player, which may be
because of small muscular changes made involuntarily by the player
or it may be connected with a more fundamental property of the
interaction, as postulated for the violin by McIntyre and Woodhouse(1978).
With a good player, however, a fair approximation to a steady state can
be achieved and it is felt that the small deviations are not of
fundamental importance in the interaction. Such a steady state sound
is, however, not very musically interesting; it is the very changes
in the waveform with time which we cannot predict that make a note
'musical'. The starting transient in particular has been shown to be
of crucial importance in recognising the sound of various wind-instruments
(Clark et al 1963). For example, a note synthesised from a sawtooth
waveform will be identified as that of a plucked string instrument or a
bowed one depending on its initial rise-time (Cutting et al 1976).
However, very little is known about the transient behaviour of the
regeneration loop in wind instruments, except that the harmonic structure
in the transient cannot in general be considered a scaled-down version
of the steady state (Richardson 1954). In this chapter only the steady
state portions of the notes are considered.
4.2 Intonation

A great deal of effort has been spent in the past trying to measure a set of unique frequencies which correspond to those 'typically' blown by players. Wogram (1972) for instance measured the intonation of notes blown by five professional players with repetitions over a period of several days. The players were instructed to play the note at a pitch which 'required least effort' though no instructions seem to have been given as to dynamic level. The range of intonation for all the players was about 30 cents for each note (1 cent = \( \frac{1}{100} \) of a tempered semitone, 30 cents corresponds to about 1.7% variation) and Wogram took the average of these frequencies as the 'genuine pitch'. Webster (1949) conducted a study comparing the intonation differences between players for a number of instruments. Some of these instruments were specifically designed not to play "in tune". He used five players who were asked to play a range of notes in random order, but again with no instruction as to level. The range of intonation found between players and trials was typically 40 cents, though for notes of low pitch on some instruments the range was greater than 120 cents. The differences between the players and the intonation of the instruments was found to be significant, in particular the overall average pitch of some players was some 20 cents higher than for others. The conclusions he drew were that experienced players did more to correct for the inadequacies of the instrument than the inexperienced players, who followed the dictates of the instrument. Finally he states that his experimental conditions "were not sufficient to eliminate experience in the form of oral and/or lip set from the results of these studies".

The author's approach to this problem has been to try to determine the range of intonations obtainable by a number of players at certain specified dynamic levels. The intonation was determined by a lissajous
figures method. The pressure in the throat of the mouthpiece was measured with a probe microphone whose output was amplified and fed to the 'Y' channel of an oscilloscope. The 'X' channel was driven by a stable sinusoidal oscillator (Adret "coDa'syn 201") whose output frequency was varied until the resulting lissajous figures were (relatively) stable. The accuracy of this method is primarily determined by how constant the play can maintain the pitch of the note (the oscillator was accurate to 1 part in 10^9). The frequency could be estimated to about $1 \frac{H}{z}$ at low frequencies and about $2 \frac{H}{z}$ above 350 Hz. The first experiment conducted was to test the range of possible frequencies which could be blown for each note from Bb 2 to C5 at a constant S.P.L. in the mouthpiece of 154 dB (r.m.s.). Each of the players in this experiment (P.S.W., J.M.B. and J.C.G.) were tested at least twice over a period of several weeks. The results for the three players are shown in Figs.4.1, 4.2 and 4.3 where the 'normalised frequency' is plotted against the number of the impedance peak (n). Distinct differences between players and trials show up on these graphs with one player (J.C.G.) showing a considerably wider range of intonation than the other two, which will be discussed below.

A second experiment was then conducted in which the players were instructed to play as quietly as possible, irrespective of pitch. The intonation of notes blown under these conditions for two players (P.S.W. and J.M.B.) are shown in Fig.4.4 where it can be seen that, apart from the bottom Bb, the results are quite consistent. The minimum S.P.L. at which these notes could be sustained was also recorded and found to be consistent for each note and both players at 144 dB ± 2 dB. The third player (J.C.G.) who had the widest range in the first experiment, was able to lip the intonation at these low levels almost as much as at 154 dB, and was also able to consistently play the notes at a level 3 dB
Fig. 4.1 Limits of intonation for notes played at an S.P.L. of 154 dB (r.m.s.) on the trombone by P.S.W.
Fig. 4.2 Limits of intonation for notes played at an S.P.L. of 154 dB (r.m.s.) on the trombone by J.M.B.
Fig. 4.3 Limits of intonation for notes played at an S.P.L. of 154 dB (r.m.s.) on the trombone by J.C.G.
Fig. 4.4 Intonation for notes played as quietly as possible on the trombone by P.S.W. and J.M.B. (also the relative intonation of the equitempered scale)
below that of the other two players. The difference between the intonation of the two players in Fig.4.4 seems to be consistent with the average differences in pitch found by Webster (1949) but the trends in the intonation diagram are relatively independent of player or trial and so it is postulated that these are a property of the individual instrument. The frequencies of the equitemperal scale are also shown in Fig.4.4 for comparison.

The theory presented in section 3.5 predicts that the frequencies of the blown notes should all be slightly greater than those of the impedance maxima, or strictly the zero crossing of the phase of the impedance. The intonation measured above cannot be directly compared to the frequencies of impedance maxima measured in section 2 because the temperature of the instrument was different in the two cases. All the measurements on the intonation, as well as in the rest of this chapter, were taken with the instrument fully 'warmed up', i.e. the player blew the instrument for several minutes before any measurements were taken. The temperature distribution along a trombone under these conditions has been measured by Wogram (1972) who finds that the temperature falls almost exponentially from about 33°C at the mouthpiece, to 26°C half way down the instrument, to 24°C (room temperature in his case) at the bell. The warmest portion of the instrument is thus the cylindrical part and so it is reasonable to assume a simple correction for the changing velocity of sound with temperature in this region. The mean temperature of Wogram's distribution was 27°C and since the temperature in the laboratory for the impedance measurement was 22°C, a correction of + 0.6% has been applied to the frequencies of the impedance maxima measured in section 2. There will be small tuning differences due to changes in the ambient temperature during the blown trials, but the correction for this is less than 1 cent/°C (Young 1946) and may be ignored.
The corrected frequencies at which the phase of the impedance of the trombone and mouthpiece (Fig.2.18) is zero are plotted in Fig.4.5 and these are seen to predict the frequencies of Fig.4.4 quite well above the fourth maximum.

As mentioned above, the theoretical prediction of the blown frequency of the impedance of the instrument on its own is slightly sharp of the impedance maxima for each peak. The actual frequencies measured in this section, even for soft blowing (Fig.4.4) where we would expect the theory to hold reasonably well, may in fact fall flat of the frequencies predicted (Fig.4.5). We have however not considered the effect of the 'reed' impedance or the 'source' impedance discussed in Appendix E, and these are expected to have a small, but important effect on the intonation. The reed impedance appears in parallel with the impedance of the instrument and since the reed must operate above its natural frequency, the reactive part of its impedance must be positive (Fig.3.6). It will effectively appear as in inertance shunting the instrument's impedance and will thus raise the resonant frequency. The source impedance however, is thought to be dominated by the compliance of the mouth cavities and appears in series with the impedance of the instrument and reed to form the 'total' impedance of Appendix E. This will lower the resonant frequency of the whole system. So although the intonation may be primarily determined by the instrument, both of these effects will slightly shift the resonant frequency and could give rise to the effects observed in Fig.4.4.

Another effect which may be of importance for notes of low pitch is the influence of the impedance at higher harmonic frequencies, for instance to explain the sharpening of the frequency of note F3 found in Fig.4.4. It is possible that the non-sinusoidal nature of the waveform even for low levels of this note makes the regeneration condition,
Fig. 4.5  Intonation of the measured impedance and sum function of the trombone
involving only the fundamental frequency discussed in section 3, inaccurate and so the influence of the impedance at harmonic frequencies must be taken into consideration. This has been suggested by Wogram (1972) who postulated that it is the intonation of the "sum-function" which is important. The sum-function is defined to be

\[ S F(f) = \frac{1}{n} \sum_{n} \Re Z_i(nf) \]

This has been computed for the instrument used here and is presented in Fig.4.6. The intonation of the main peaks of this diagram are also given in Fig.4.5 and it is found that the shape of the intonation diagram of Fig.4.4 is predicted accurately by this even for low pitches. Since the impedance peaks are small above 1 kHz the intonation of the sum-function is much the same as for the normal impedance peaks above 500 Hz.

4.3 The alternating mouthpiece pressure

The alternating pressure in the mouthpiece of an instrument was measured using a probe microphone in the same position as that used for the impedance measurement, i.e. half way along the mouthpiece throat. It is assumed that the pressure measured in this plane is the same as that in the mouthpiece cup. This is a reasonable approximation below about 2 kHz since the pressure dropped by the small portion of mouthpiece throat between the microphone and the cup will be small, as can be confirmed by comparing the series impedance of this portion of throat with the impedance of the rest of the instrument. The probe microphone used in the impedance measurements (B & K 4170) was found to be unsuitable for measurements of blown notes since its small orifice soon became blocked under playing conditions. The probe microphone used was a B & K probe attachment (type UA 0040) coupled to a ½" microphone (B & K type 4134). The probe had a length of 29 mm and an internal diameter of 1.5 mm.
Fig. 4.6  Calculated sum function for the impedance of the trombone
The microphone and amplifier (B & K type 2121) used were calibrated using the same technique as for the probe microphone used in the impedance measurements (section 2.4.3) and the results of such a calibration experiment are presented in Fig.4.7. There is purposely an overall phase-shift of $\pi$ in the phase calibration to account for the inversion of the pressure waveform by the microphone.

The pressure in the mouthpiece for the steady state portion of about fifty notes has been studied to examine the change of waveform with level and intonation. The waveforms were recorded by using a suite of programs written for the Nova 2 computer by the departmental computing staff, which effectively turns it into a digital recorder, capable of recording a twenty second note on two channels at a sampling rate of 25 kHz. Representative cycles for all the notes studied were edited out and stored for later use.

As a preliminary example, the mouthpiece pressure for four notes (Bb2, Bb3, F4 and Bb4) are presented in Fig.4.8 which were measured at the same r.m.s. level. The transition from the characteristic low-frequency waveform to the more sinusoidal waveform at higher frequencies can be clearly seen. The waveforms as functions of time are only of limited use however when we wish to compare them to those suggested by the theory, since it is the harmonic structure which is predicted. A pitch synchronous analysis program was therefore used to analyse the waveforms into their Fourier components. This technique seems to have been used originally by Mathews et al (1961) for the analysis of speech, and the program used here is similar to his in many respects. The program used had been written by Mr. R.A. Bacon and has certain advantages over other such programs. For instance, the period is determined by examining the time at which the waveform falls through some pre-determined level, which proved to be a more reliable indicator.
Fig. 4.7 Calibration file for the short probe microphone
Fig. 4.8 Alternating mouthpiece pressure for the notes Bb2, Bb3, F4 and Bb4 played on the trombone.
of the period than the peak detection used by Risset (1966) for example (one of Mathew's co-workers). After determining the limits of each period the sampled points are analysed to determine the Fourier expansion of the waveform. The form of this expansion is important (particularly the definition of the phase angle), since the results are to be compared with theoretical predictions. The form used was

\[ f(t) = \sum_{n} a_n \cos(n\omega t) + b_n \sin(n\omega t) \]

\[ = \sum_{n} p_n \cos(n\omega t + \phi_n) \]

where \( p_n^2 = a_n^2 + b_n^2 \) and \( \phi_n = -\tan^{-1} \frac{b_n}{a_n} \)

with due care being taken to add \( \pi \) to \( \phi_n \) if \( b_n \) was negative. The program was checked by analysing a square wave from a waveform generator, the frequency of which was purposely chosen to be a 'worse case' where the period was not exactly an integer number of sample points.

The waveform to be analysed and the resulting analysis are presented in Fig.4.9 and are in good agreement with the computed Fourier series for a square wave

\[ f(t) \propto \sum_{n} \frac{1}{n} \cos(n\omega t + (n-1) \frac{\pi}{2}), \text{ n odd} \]

A pitch synchronous analysis of the notes presented in Fig.4.8 was carried out and the amplitudes of the harmonics for the various notes presented in Fig.4.10. As expected the lower notes are much richer in harmonics than the higher ones, and for these notes the graph of \( \log P_n \) against \( n \) approximates a straight line supporting the theoretical harmonic structure derived in section 3.5 that

\[ |p_n| = K|p_{n-1}| \]

where \( K \) is a constant less than unity for each note.
Fig. 4.9 Pitch synchronous analysis giving the amplitudes and phases of the Fourier series for a square wave.
The phase relationship predicted by the theory is that

$$\phi_n = 0 \text{ (n odd)} \text{ or } \pi \text{ (n even)}$$

and Fig.4.11 presents a graph of the deviation of $\phi_n$ from this relationship for the notes analysed above. The phases have been corrected for the microphone response shown in Fig.4.7. It can be seen that the agreement is fairly good for notes of low pitch, but deviations from the simple relationship occur at higher frequencies. At very high frequencies the magnitudes of the harmonics are so low that no reliable estimation of the phase can be obtained.

The change of the harmonic structure of these notes with level and intonation may be investigated by analysing each cycle of a prolonged note in which (hopefully), only one parameter is varied by the player. Provided the change is slow, each cycle will only be slightly different from the last and 'steady-state' conditions will be maintained.

The pitch synchronous analysis for a note of pitch Bb2 with slowly changing frequency was computed, for example, and the change of 'frequency' (really the reciprocal of period) with 'time' (really the number of the cycle) is shown for 800 cycles in Fig.4.12 where it is seen that the frequency falls from about 120 Hz to about 90 Hz. This is from the frequency at which oscillation is predicted according to the theory, into a region where other effects must be of importance to maintain the oscillation. The change in the waveforms with frequency is well defined for this 'slur' and the recorded waveform at various frequencies are presented in Fig.4.13. Figure 4.14 shows the resulting amplitudes and phases of the first four harmonics for this note, in which the amplitude and phase start off in the well defined structure discussed above, but slowly change until the magnitude of the second
Fig. 4.11 Phase angle (taking account of sign changes) for the Fourier coefficients of the notes Bb2, Bb3 and F4 played on the trombone.
Fig. 4.12 Change of reciprocal period ('frequency') against the number of periods ('time') for a slur, nominally Bb2 played on the trombone.
Fig. 4.13: Waveforms of the mouthpiece pressure for various frequencies about the note Bb₂ played on the trombone.
Fig. 4.14 Amplitudes and phases of the first four Fourier coefficients (f, 2f, 3f and 4f) for the slur about Bb2 played on the trombone.
and fourth harmonic begin to rise. At these frequencies the oscillation has begun to be 'supported' by higher peaks of the impedance. The second harmonic for instance falls on the third impedance maxima (at about 180 Hz) and the fourth impedance maximum falls on the sixth impedance maximum (at about 360 Hz). This is a very non-linear 'regime' of oscillation, effectively involving sub-harmonic generation and has been termed 'privileged' by Bouasse (Benade and Gans 1968). Good players can play a number of these 'privileged' notes in addition to the 'normal' musical ones and such notes will show up as smaller peaks on the sum function diagram of Fig.4.6. The 'pedal' note is the most easily obtained of this class of note and represents an extreme case of oscillation maintained by impedance peaks at harmonic frequencies. The frequency of the pedal note is about 60 Hz and here the impedance of the instrument is low, but the impedance at all the harmonic frequencies is high and all of these peaks combine to maintain the oscillation.

A slur for this pedal note has been recorded and the variation of 'frequency' with 'time' is presented in Fig.4.15 for 256 cycles of the waveform. Typical waveforms at various frequencies are shown in Fig.4.16 and the pitch synchronous analysis for the first four harmonics shown in Fig.4.17. The waveforms and harmonic structure can be seen to retain many of the features found for the note Bb2 played at about 120 Hz. Because of the assumptions made in section 3.5 (in particular the assumption of a sinusoidal lip motion), the theory developed could not predict the existence of these 'privileged' notes.

The assumptions we have made do, however, lead to a theory which predicts the harmonic structure of 'normal' notes quite well. For example, the pitch synchronous analysis of the note Bb2 at constant frequency (118 Hz) and slowly increasing level is given in Fig.4.18.
Fig. 4.15 The change of reciprocal frequency with number of periods for a slur about the pedal note played on the trombone.
Fig. 4.16 Waveforms of the mouthpiece pressure for various frequencies about the pedal played on the trombone.
Fig. 4.17 Amplitudes and phases of the first four Fourier coefficients ($f, 2f, 3f$ and $4f$) for the slur about the pedal played on the trombone.
Fig. 4.18 Amplitudes and phases of the first four Fourier coefficients (f, 2f, 3f, and 4f) for the crescendo played at Bb2 on the trombone.
Here the amplitudes of the harmonics change proportionately to each other and the phases remain fairly constant. The waveforms of the mouthpiece pressure for three representative levels are shown in Fig.4.19 and the graph of log \( (P_n) \) against \( n \) for these notes shown in Fig.4.20. The change of the harmonic structure with level of Fig.4.20 is predicted quite well by the formula derived in section 3.5, i.e. that

\[
P(t) = \sum_n (-K)^n P_1 \cos n\omega t
\]

with \( K \) and \( P_1 \) increasing with level. The deviations from the theoretical straight line are the greatest for the note of high amplitude and this is probably because the assumption

\[
|P_{n+1}| \ll |P_{n-1}|
\]

made in section 3.5 becomes less and less valid as the overall amplitude is increased.

A similar change of harmonic structure with level is found for other notes of low pitch. Figure 4.21, for example, shows the results of a pitch synchronous analysis for the note F3 at constant frequency (174 Hz) with gradually increasing level, and the form of this is very like Fig.4.17. As the pitch of a note is raised, however, the amplitudes of the higher harmonics fall off and the waveform of the mouthpiece pressure becomes more sinusoidal, as can be seen in Fig.4.8. At very high pitches the higher harmonics can barely be detected. Figure 4.22, for example, shows the results of a pitch synchronous analysis of a crescendo of the note Bb4 at constant frequency (464 Hz). Only the first two harmonics are shown in this diagram, however, since the amplitude of the higher harmonics are more than 30 dB below the fundamental, even at high amplitudes.

Apart from these changes of waveform as the pitch and level of
Fig. 4.19 Waveforms of the mouthpiece pressure for various levels of the note Bb2 played on the trombone.
Fig. 4.20 Amplitudes of the Fourier coefficients for various levels of the note B♭2 played on the trombone.
Fig. 4.21 Amplitudes and phases for the first four Fourier coefficients (f, 2f, 3f, and 4f) for a crescendo played at F3 on the trombone.
Fig. 4.22 Amplitudes and phases for the first two Fourier coefficients (f and 2f) for a crescendo played at Bb4 on the trombone.
the note are changed, the player is able to change the tone of a note, to a certain extent by adjusting his embouchure. This is illustrated in Fig. 2.23 where the mouthpiece pressure has been recorded for the same note (Bb2) and level (approximately 160 dB r.m.s.) blown by the same player who used these embouchure adjustments to play notes which he judged to be 'bright', 'normal' and 'dark'. These changes in waveform appear similar to those which would be produced if the overall amplitude was changed (Fig. 4.19) and the harmonic structures of the notes all conform to the pattern predicted for these notes, i.e. the plot of log Pn vs. n (Fig. 2.24) are straight lines to a good approximation with changing slope. This change in waveform we have previously associated with the increase in lip movement as a note is played at higher amplitudes, and we may suggest that this change in lip movement is brought about here, not by the change in level but the change in embouchure. The most likely explanation for this change is that by adjusting the position of his lips, the player can make changes in the effective mass of the moving parts of them. As the note is played more 'darkly' for example, the player puckers his lips. This would increase their effective mass and reduce the amplitude of lip vibration for a given driving pressure, and so account quite well for the observed changes.

4.4 The alternating pressure in the mouth

Because of the source impedance presented by the vocal tract and mouth cavities, the pressure in the mouth will not be constant while a note is being blown. The influence of this source impedance on the regeneration is discussed in Appendix E where it is suggested that it is the magnitude of the Thévenin equivalent source impedance (Z_s), compared to that of the instrument in parallel with the reed impedance (Z_i//Z_r) which is important. Since the flow through both of these impedances
Fig. 4.23 Waveforms of the mouthpiece pressure for three notes played on the trombone with the player changing his embouchure to produce various tone qualities.
Fig. 4.24 Amplitudes of the Fourier coefficients for three notes of different tone quality played on the trombone.
is the same we may obtain some estimate of their ratio by observing the relative alternating pressures in the mouth and mouthpiece:

\[
\frac{Z_s}{Z_i/Z_f} = -\frac{P_m}{P_i}
\]

(from the diagram E.1).

It is not in general possible to determine the value of \(Z_s\) absolutely since \(Z_r\) is not observable. For example, notes of low pitch produce large movements of the lips and so \(Z_r\) is small and may form an appreciable part of the parallel combination of \(Z_i\) and \(Z_f\) (section 3.2.2). The source impedance expected from the trachea and mouth cavities can be estimated by considering them as a tube of changing cross sectional area, and there is a considerable body of literature on this subject because of its importance in speech transmission. Fant (1960), for example, presents graphs of the variation of cross sectional area with length for the vocal tract while various vowels were being uttered, which were deduced from X-ray photographs. It is clear that the source impedance will be strongly dependent on the positions of the tongue and soft palate since these have a marked effect on the effective volume of the mouth cavity.

The input impedance at the mouth has, in fact, been measured by various authors in rather different contexts. Maslen and Rowlands (1966), for example, measured the impedance up to 100 Hz in connection with a study on the stability of aircrew breathing equipment. Gupta et al (1973) measured the input impedance at the mouth to test a simplified model of the acoustic response of the trachea. The results of Gupta et al extend up to 400 Hz but were conducted with only one position of the mouth cavity (the subjects were asked to prepare to pronounce the letter 'P'). He found that the impedance had a maximum value of some
2.5 MΩ at about 200 Hz and was predicted reasonably accurately by a model consisting of a volume of 40 cc for the mouth cavity connected to an open tube representing the vocal tract.

In the present study, the pressure in the mouth (-Pm) has been measured together with the pressure in the mouthpiece of the instrument (P�) by using a pair of matched probe microphones. The probe microphones used were constructed from the B & K kit (type UA 0040) and were similar to the ones in section 4.3, except that extensions to the 29 mm of the probe kit were constructed from 20 mm of small bore plastic tubing to make it easier for a player to keep one in the corner of his mouth. Some care was taken to ensure that the sensitivities and frequency response of the two probe microphones were as close as possible. The B & K probe kit can be used with or without a small plastic tightening gasket between the probe body and the ½" microphone used ("Brue! and Kjaer" 1972). Normally this gasket is used to block the pressure equalisation vent in the microphone (type 4134 in this case) but it also has the effect of changing the volume of the air gap between the probe body and the microphone diaphragm. The measurements in section 4.3 were made with a probe microphone without this gasket, but it was found that if the frequency responses of two such probe microphones were compared, large differences in the responses occurred. This was thought to be due to the slightly different air gaps left between the probe microphone body and the diaphragm for the two probe bodies used. Indeed, if the responses were compared with gaskets in each of the probes, so the air gaps were identical, the differences in response were much smaller. Since the gaskets seal the pressure release vent, it was felt that the microphones might incur damage from the steady pressure in the mouth, so two small grooves were carved on the 'lip' of each of the gaskets allowing an airflow down the probe tube, via the gasket, to the
pressure release vent at the back of the diagram. The relative frequency responses of two such microphones, with plastic extension tubes, and amplified by B & K type 2111 spectrometers (set to 'linear') was measured by using the pressure cavity described in section 2.4.3. The results are presented in Fig.4.25 and show a maximum difference in the response of some 1.6 dB and 0.2 radians below 2 kHz. The outputs from the two spectrometers were then fed into the A-D converters of the Nova 2 computer and simultaneously sampled at a rate of 25 kHz. This arrangement was used to record the alternating pressure in the mouth by placing the plastic tubing of one probe microphone through the corner of the player's lips, and the alternating pressure in the mouthpiece by coupling the other probe through a hole bored in the cup of a trombone mouthpiece.

It was found that the player could change the magnitude of the alternating pressure in the mouth over a small range, for notes blown at one level and frequency, by changing the position of his tongue and so make adjustments to the size of the mouth cavities. The alternating pressure amplitude was fairly constant, however, for what the player judged to be 'normal' playing and this pressure increased in proportion to the pressure in the mouthpiece as the note was played louder. The two recorded pressures for a number of notes played on the trombone are given in Figs. 4.26, 4.27, 4.28 and 4.29, corresponding to notes of frequency 116 Hz, 232 Hz, 350 Hz and 464 Hz. The magnitudes of the fundamental components of these two pressures were computed using the pitch synchronous analysis program and the ratio of the magnitudes of the fundamental pressure in the mouth to that in the mouthpiece was found to vary between 5% and 20%. This implies that for some notes the source impedance is an appreciable part of 'total' impedance of Appendix E, and could have some effect while 'lipping' a note flat.
Fig. 4.25 Relative frequency response of the two probe microphones
Fig. 4.26  Alternating pressure in the mouth (above) and mouthpiece (below) for the note Bb2 (116 Hz) played on the tromb
Fig. 4.27 Alternating pressure in the mouth (above) and mouthpiece (below) for the note Bb3 (232 Hz) played on the trombone.
Fig. 4.28 Alternating pressure in the mouth (above) and mouthpiece (below) for the note F4 (350 Hz) played on the trombone.
Fig. 4.29 Alternating pressure in the mouth (above) and mouthpiece (below) for the note Bb4 (464 Hz) played on the trombone.
for instance. This was investigated by recording the two pressures for the note Bb2 while the note was lipped flat (to about 100 Hz) (Fig. 4.30). We find that the fundamental component of the pressure in the mouth does increase slightly but also there is a marked increase in the amplitude of the second harmonic of the mouth pressure as the note gets into the privileged regime discussed above.

In an early abstract, Hall (1955) described a series of experiments in which he observed the change of the shape of the mouth cavities during brass instrument playing. He says that this has a major influence on the tone quality produced by various players. This effect has been ignored in subsequent investigations until the present one. We are now in a position to see how these changes can have some effect on the note produced, by changing the source impedance. If, for instance, the source impedance is large at some harmonic frequency of the note, the pressure dropped by the source impedance will be relatively large and so reduce the mouthpiece pressure and hence the radiated pressure at this frequency.

A study of the alternating pressure in the mouth also throws light on some otherwise puzzling phenomena. For instance, a player is able to 'buzz' a note on a mouthpiece alone at almost any pitch, and this is frequently done by players for training and practice. The input impedance of a mouthpiece on its own is dominated by the Helmholtz resonance of the volume of the mouthpiece cup and inertance of the backbore. The impedance of the backbore of a trombone mouthpiece has been measured by the method described in section 2.4 and is shown as the dashed line in Fig. 4.31. The solid line in this figure is the impedance computed for the whole mouthpiece by assuming the cup to be a lumped compliance as described in section 2.4.7. The total impedance has a peak at the Helmholtz resonance frequency, and is inertive at
Fig. 4.30 Alternating pressure in the mouth (above) and mouthpiece (below) for the note Bb2 lipped flat to 100 Hz on the trombone
Fig. 4.31 Measured impedance of the backbore of a trombone mouthpiece (dashed line) and the computed impedance of the whole mouthpiece (solid line)
frequencies below this and compliant at higher frequencies. The resonant frequency and 'Q' of this resonance are in good agreement with measurements made by Dr. J.M. Bowsher of the 'popping' characteristics of this mouthpiece. The technique used for these measurements was to examine the frequency and decay rate of the oscillation created by sharply striking the rim of the mouthpiece onto the palm of the hand. The frequency of ringing is termed the 'popping' frequency and the effective 'Q' can be deduced from the decay rate.

If the mechanism of a player 'buzzing' on a mouthpiece is the same as that when he blows an instrument, i.e. a pressure controlled valve, we would not expect him to be able to buzz the mouthpiece below this popping frequency. This is because the reactance below resonance is positive and we have seen in section 3.5 that the reactance of an instrument must be negative for sustained oscillation. The magnitude of this reactance is small, however, and it was thought that the impedance from the mouth could play some part in controlling the oscillation. This was tested by measuring the alternating pressures in the mouth and mouthpiece cup using the technique described above for a number of notes buzzed on the mouthpiece. We can now see why the pressure has been recorded directly in the cup of the mouthpiece, rather than in the more convenient plane of the throat, as in section 4.3. When measuring the pressure in the mouthpiece while blowing an instrument, the impedance of the small portion of throat between the measuring plane and the mouthpiece cup was very small in comparison to the impedance of the rest of the instrument, so the pressure in the measuring plane and that in the cup were substantially the same. If there is no instrument on the end of the mouthpiece, however, this portion of the throat forms a significant proportion of the total inertance of the system and pressures measured in this plane were found to have significant phase errors compared to
those measured in the cup.

A player is able to 'buzz' notes on a trombone mouthpiece up to a frequency of about 600 Hz, though he reports a distinct 'change of feel' when going from notes below about 400 Hz to notes above this frequency. All these frequencies are below the measured 'popping' frequency of the mouthpiece and 400 Hz is the frequency at which the impedance of the mouthpiece begins to rise markedly (Fig. 4.31). The alternating pressures in the mouth and mouthpiece for notes buzzed by a player (P.S.W.) on a trombone mouthpiece were recorded for notes at frequencies of approximately 120, 240, 350, 470 and 590 Hz and are presented in Figs. 4.32, 4.33, 4.34, 4.35 and 4.36. The mouthpiece pressure for notes of low pitch are very non-sinusoidal, as it was for notes played on the whole instrument, though the waveforms do not display the same characteristic form as those discussed in section 4.3. These waveforms do seem to be due to the large amplitude of lip vibrations for the notes of low pitch, however, since as the pitch is raised and the amplitude of lip vibration becomes less, the waveforms become much more sinusoidal. The amplitudes of the fundamental components of these two pressures were again determined using the pitch synchronous analysis program. It was found that for the first three notes the fundamental amplitudes of the pressure in the mouth and mouthpiece are very similar (within 1.5 dB of each other), and were also nearly out of phase with each other. Since the impedance of the mouthpiece is largely inertive at these low frequencies, we can conclude that the source impedance is of the same order of magnitude as this and also largely compliant. This means that the imaginary part of the total impedance discussed in Appendix E is no longer predominantly positive and may, in fact, fulfil the condition imposed on it in section 3.3 for oscillation with an outward beating reed: that it be small and negative. This would explain
Fig. 4.33 Alternating pressure in the mouth (above) and mouthpiece (below) for a note of frequency 240 Hz buzzed on the trombone mouthpiece.
Fig. 4.34 Alternating pressure in the mouth (above) and mouthpiece (below) for a note of frequency 350 Hz buzzed on the trombone mouthpiece.
Fig. 4.35 Alternating pressure in the mouth (above) and mouthpiece (below) for a note of frequency 470 Hz buzzed on the trombone mouthpiece.
Fig. 4.36  Alternating pressure in the mouth (above) and mouthpiece (below) for a note of frequency 590 Hz buzzed on the trombone mouthpiece.
the oscillation below 400 Hz in terms of the theory for regeneration in a pressure-controlled valve which has been presented above, but the situation above 400 Hz is rather different. Here the fundamental of the mouth pressure is only about 30% of that in the mouthpiece and cannot play a dominant role in determining the right conditions for regeneration. Indeed, the mouth pressure in this region can be changed by the player moving his tongue until the ratio of fundamental pressure is far less than 30% while still maintaining the oscillation. An example of this effect is given in Fig.4.37 which is for the same frequency as in Fig.4.36 (590 Hz) but different tongue position. A different mechanism of regeneration must be present for these notes, and this may be supplied by the Bernoulli pressure. The magnitudes of the alternating pressures for notes buzzed on the mouthpiece are far less than for notes blown on the trombone and so the driving force on the lips will be proportionately less, which may be important at high frequencies where less movement of the lips is caused by unit pressure in the mouthpiece (section 3.2.2). The magnitude of the alternating flow, however, is still quite large and may give rise to the Bernoulli pressure necessary to maintain the oscillation. This is only a tentative proposal, however, since measurements on the lip motion and the flow through the lips are necessary before any definite conclusions can be drawn.

4.5 Relationship between the pressure and flow in the mouthpiece

The alternating pressure in the mouthpiece is accompanied by an alternating flow through the lips into the cup, determined by the impedance of the instrument and the lips at the particular frequency considered. This alternating velocity modulates the steady flow through the lips which is due to the steady pressure in the mouth, and because of the non-linear nature of the regeneration, the ratio
Fig. 4.37 Alternating pressure in the mouth (above) and mouthpiece (below) for a note of frequency 590 Hz buzzed on the trombone mouthpiece with a different tongue position.
of this steady mouth pressure to the average flow, is of major importance in determining the resultant pressure waveforms. In Chapter 3, this quantity \( R_0 \) was called the 'average resistance' of the lip opening and it is the magnitude of this quantity in relation to the impedance of the instrument and lips which determine whether the pressure waveform will have the characteristic low frequency form discussed in section 4.3 or will have the more sinusoidal form as have associated with higher frequencies.

The use of the hot-wire anemometer to determine the acoustic velocity in the throat of the mouthpiece while the instrument was excited artificially, was discussed in section 2.4, and here we use this device to measure the flow in the same position while the instrument is actually being blown. Early results from experiments where the velocity in the throat of the mouthpiece was measured with the anemometer and the alternating pressure measured with a probe microphone, have been reported verbally (Elliott 1978, 1978b, 1979). These studies brought to light a number of facts which were not expected and were very important in the early stages of formulating the work presented in this thesis. As an example, Fig.4.38 presents the measured mouthpiece pressure and particle velocity for a range of played notes spanning two octaves, on both the trumpet and trombone. These notes were recorded for an approximately constant r.m.s. pressure in the mouthpiece which corresponded to a playing level of about m.f.

The similarity between the magnitudes and waveforms of notes played an octave apart on the two instruments is striking. In Chapter 2 the measured impedance in the throat was seen to be similar for the two instruments, allowing for frequency scaling, and we can now see that this similarity is reflected in the measured waveforms. The two instruments should not, however, be considered as identical in operation since the
Fig. 4.38 Measured pressure and velocity in the throat of the mouthpiece for notes Bb2, Bb3, F4, Bb4 played on the trombone and notes Bb3, Bb4, F5, Bb5 played on the trumpet.
impedances computed for the two instruments in front of the mouthpiece show marked differences (Figs. 2.16 and 2.17). This is because the cup volume and throat size of the trumpet are proportionally much smaller than that found in the trombone, the effects of which are discussed in Appendix F.

One final point to be made in connection with Fig. 4.38, is that we can estimate the complex impedance in the throat by computing the fundamental components of the pressure and velocity waveforms with the pitch synchronous analysis program. These results are in reasonable agreement with those measured at the same frequency in Chapter 2, which provides a convenient check on the calibration of the transducers used in this section.

In a separate series of experiments, the steady pressure in the mouth and the average flow in the throat have been measured to estimate the parameter $R_0$. The steady pressure in the mouth has been measured by using a water manometer connected to a length of small bore plastic tubing which was placed in the corner of the player's mouth. The velocity was measured simultaneously in the throat with the hot wire anemometer and recorded by the computer for subsequent linearisation and averaging. A number of notes were recorded in this way on the trombone and trumpet, in this case the level of the fundamental pressure in the mouthpiece was kept constant for all notes (at an S.P.L. of 154 dB), for reasons which will become apparent later on. The computed value of $R_0$ for these notes is presented in Fig. 4.39 with different frequency axis for measurements on the trombone and trumpet. As expected, the value of $R_0$ increases with frequency from a value which is below the measured impedance for the instruments to one which is of the same order of magnitude or higher than the measured impedances. This supports the prediction in Chapter 3 that it is the relative magnitude of $R_0$ which
Fig. 4.39 'Average' resistance of the lip opening for a number of notes played on the trombone and trumpet.
is the important factor in determining the transition from low to high frequency waveforms for the pressure in the mouthpiece. Using a different technique Bouhuys (1965, 1968) has also measured the average pressure in the mouth and flow down the instrument for notes on various wind instruments, and used it to deduce the "equivalent d.c. resistance" of the opening between the player's lips. His results for the trombone (Bouhuys 1968) are in good agreement with ours. He also measured the variation of $R_0$ with dynamic level and found it to be approximately constant for any one note, i.e. the average flow increases in proportion to the mouth pressure.

The equation for the average flow derived in section 3.5 was

$$U_0 = C x_0 + \frac{C G_1 p^2}{4 \rho} \cos \delta_1, \quad C = \sqrt{\frac{2}{\rho}} \frac{p_s}{\xi}$$

If we assume that $4 p_s >> p_1$ (a reasonable assumption for the notes studied here), this equation reduces to that used in section 3.2.3 for the average flow through a slit of opening $x_0$. We have determined $U_0$ and $P_s$ in the experiment above, so if we estimate the value of $\xi$ (taken here as half the diameter of the mouthpiece rim), we can determine the variation of the average opening of the lips with the frequency of the note

$$x_0 = \frac{U_0}{\xi} \sqrt{\frac{\rho}{2 \rho_s}}$$

A graph of this variation is presented in Fig.4.40 and it can be seen that the average opening of the lips for notes on the trombone are of the same order of magnitude as those for notes played on the trumpet. The magnitudes and variation with frequency of these average openings are also similar to those directly observed by Martin (1942) though no accurate comparison can be made since the S.P.L. in the mouthpiece for Martin's measurements is not reported. We can make use of another of
Fig. 4.40  Computed value of 'average' lip opening for a number of notes played on the trombone and trumpet.
the observations of Martin, however, to estimate the magnitude of the
effective parameters of the lips which had to be assumed at the
beginning of Chapter 3. Martin found that for all the notes he studied,
the lip motion was sinusoidal and the lips just came together once
every cycle, so we can see that $x_1 = x_0$. The equation for the motion
of the lips was given in section 3.2.2 as

$$\frac{x_1}{p_1} = \frac{A^\prime}{(j\omega r + s - \omega^2 m)}$$

where $A^\prime$ is the effective area of the lips,
$x_1$ is the amplitude of lip vibration
and $p_1$ is the pressure in the mouthpiece.

We assumed that the effective mass ($m$) and 'Q' ($\sqrt{\frac{m}{r}}$) of the
system remained constant while the stiffness ($s$) was changed to alter
the natural frequency of the lips. At the natural frequency ($\omega_r$) of
the lips this equation reduces to

$$\left|\frac{x_1}{p_1}\right| = \frac{A^\prime Q}{\omega_r^2 m}$$

and since the 'Q' is low (the assumed value was 0.5) this equation is
also valid for frequencies near the natural frequency. We can now
obtain some estimate of the effective mass of the lips by using the
computed value of $x_0$ ($= x_1$), assumed values of $A^\prime$ and $Q$, the angular
frequency of the note as an estimate of $\omega_r$, and the known, constant,
pressure level of the fundamental

$$m = \frac{A^\prime Q}{\omega_r^2} \left|\frac{p_1}{x_1}\right|$$
The mass calculated from this equation for the various notes above is plotted against frequency in Fig.4.41. It is obvious from this graph that the working assumption made in section 3, that the effective mass of the lips was constant is not realistic. This does not invalidate any of the theoretical results, however, since this assumption was not carried over into the theory presented in section 3.5. With this rather crude experiment, it is not possible to independently investigate the variation of all the reed parameters with frequency and the computed change of mass as the pitch rises may also reflect the fact that the magnitude of some of the other parameters are not constant with frequency. The effective width of the lip opening, for instance, will decrease as a smaller proportion of the lips is forced into motion at higher frequencies as has been observed by Martin (1942). We can, however, suggest a modified model of the lip dynamics which would account for the observed behaviour, by assuming that the natural frequency of the lips is changed by adjustment of both the tension and the effective mass of the lips. A form which would fit our results quite well would be that $s \propto \omega_r$ and $m \propto \frac{1}{\omega_r}$. Such a model has in fact been used in the numerical model of the glottis of Flanagan and Langraff (1968) though no quantitative justification has been given, to quote these authors: "It appears more realistic to introduce a cord-tension parameter as a combined stiffening and lightening of the moving system".

The purpose of this chapter, then, has been to observe directly some of the parameters which are of importance in the interaction between player and instrument. These observations have been used to assess the theory for the regeneration which was presented in Chapter 3 and identify areas in which this simplified theory may be inadequate. The steady state portions of most of the notes studied have been in good agreement with that predicted by the theory presented, with significant deviations
Fig. 4.41: Computed value of the effective mass of the lips for a number of notes played on the trombone and trumpet.
occurring for 'privileged' notes on the trombone and notes of high pitch buzzed on the trombone mouthpiece. The calibrated results we have obtained have also been useful in estimating the magnitude of some of the effects which have not been adequately discussed in other studies, such as the impedance of the pressure source from the mouth, and in deducing some of the basic parameters of the moving lips (such as the effective mass) which were not previously available in the published literature.
CHAPTER 5

SUMMARY AND CONCLUSIONS

The main purpose of this study has been to examine the significance of the acoustics of the instrument in the interaction with a player to produce musical blown notes. The instrument acoustics have been shown to be only one determining factor in the overall regeneration, while the player is able to control the regeneration by a number of different and sometimes rather subtle means.

Nevertheless, a clear understanding of the significant parameters of the instrument is of major importance. By considering the behaviour of the instrument in the context of an essentially linear, one dimensional, model (the two port network) we have been able to interpret various measurements made on the instrument in a relatively simple way. The two parameters chosen for the study were the input impedance at the throat of the mouthpiece and the pressure transfer function between this point and a point one bell radius from the end of the instrument. Both of these parameters have been directly measured with a computer controlled, stepped-sine wave technique, with some care being taken over the calibration of the results (particularly for the phase data). One great advantage of measuring the impedance in the throat of the mouthpiece is that the effect of the mouthpiece used in the instrument is minimised, and by using a lumped model for the cup volume and throat, the impedance of the instrument with a wide variety of mouthpieces can be subsequently computed. The impedance as 'seen' by the player is made up of the impedance of the instrument and mouthpiece (taking due account of the reduction in cup volume by the volume of the player's protruding lips), in parallel with an effective impedance arising from the motion of the player's lips. This effective impedance of the lips will change as
different notes are played, so the impedance 'seen' by the player is, to some extent, controlled by him. It is certainly not a single, unique, parameter for all players as is strongly implied in most discussions.

The 'impulse response' for both input and transfer characteristics has been computed by transforming the data from the frequency to the time domain. The results are compared to the delays expected by propagation to and from various parts of the instrument and many of the features of the impulse response correlate well with these. The impulse response at the instrument mouthpiece is particularly rich in detail and is thought to be of importance to players during the initial transient portion of a note. A premature reflection here could interfere with the build-up of oscillation of the lips, spoiling the 'attack'. It is in the nature of the Fourier transform that all frequency information is considered in predicting the response at any one time and so the impulse response calculated here is essentially 'broad band'. The suggestion put forward by Benade (1969, 1976) that it is the group velocity which must be considered in predicting the transient behaviour is not correct since, during the initial transient, it is not a 'narrow band' signal which is propagated in the instrument. The main criterion for objectively judging the 'quality' of an instrument has, in the past, been the impedance as a function of frequency, though this has had limited success (Pratt and Bowsher 1979). It may be worth studying the impulse response of a wide variety of instruments to decide if this provides a more meaningful discriminator.

We now turn our attention to the 'steady state' portion of a note, i.e. the quasi-periodic portion after the initial transient. Here the interaction between player and instrument gives rise to a number of complexities. Some of these are due to the fact that the player can
control, relatively independently, many of the parameters discussed above which control the regeneration, and he does so, at times, quite unconsciously. The influence may not be direct, for example a change in embouchure will change both the action of the lips as a valve and their effective impedance. The source impedance formed by the mouth cavities and vocal tract is another important parameter which can be controlled by the player, to force a note over a range of intonation, for example. It is the combination of these effects, all controlled by the player, which make the variety of notes which can be played on a brass instrument so large, and dependent on the experience of the individual player. Although this makes for a rich variety of musical interpretation, it makes repeatable laboratory studies very difficult!

The non-linearity of the equation of flow through the orifice formed by the lip opening gives rise to a characteristic pressure waveform in the mouthpiece when a note of low pitch is played near an impedance maximum. It can also cause a 'regime' of oscillation to be set up where the impedance at the fundamental frequency is low, but the impedances at higher harmonic frequencies are high, these are called 'privileged' notes.

The parameter which is the most difficult to specify in the regeneration loop is the dynamic response of the lips. It has been postulated that the lips act as separate valves controlled by lumped mass-spring-damper systems and many effects can be explained on this basis, but the validity of the approach can only be checked by direct observation. The excellent photographic study by Martin (1942) lends some support to this theory but no numerical data can be extracted from his observations since the magnitude and phase of the pressure in the mouthpiece was not recorded. Although other studies have been carried out since Martin's (Hadland 1959, Leno 1971) none has been as carefully conducted as his, and a clear gap in our understanding is very apparent.
Some of the gross features of the lip dynamics have been deduced from the measurement we have made of the pressure and velocity in the mouthpiece while notes were being played at various pitches and levels.

The most promising method of modelling the complexities of the interaction seems to be by using a digital model similar to that used successfully to model the vocal cords by Flanagan and Landgraf (1968) or Titze (1973, 1974). The model holds many attractions since the influence of the Bernoulli pressure or a non-linear model of the lip movement for example could be incorporated fairly easily, whereas these effects are very difficult to incorporate in an analytical theory. The main obstacle to constructing such a model has been the lack of an adequate description of its component parts. If incorrect assumptions are made in the initial program, then the model would not be sufficiently realistic and the results obtained would be at best unhelpful and at worst misleading. It is hoped that the results of the present study will provide some guidance as to the parameters necessary to build such a model.
The Measurement of the Acoustic Impedance of Brass Instruments

by R. L. Pratt, S. J. Elliott and J. M. Bowsher
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Dedicated to Dr. R. W. B. Stephens on the occasion of his 75th birthday

Summary

An apparatus is described which is largely free of the limitations of earlier methods of measuring the input impedance of brass instruments. The concept of acoustic impedance is discussed and those earlier methods are briefly reviewed and are shown to suffer from the following disadvantages: (i) absolute calibration is difficult to achieve, (ii) the plane of measurement is not readily movable, (iii) the effect of steady air-flows cannot be studied, (iv) phase information is not easily extracted, (v) only sine wave test signals may be used, and (vi) measurements cannot be made when the instrument is being played. The new apparatus measures both the acoustic pressure and the particle velocity simultaneously using a horn coupled probe microphone and a hot wire anemometer and is controlled by an on-line mini-computer. The calibration procedures of both pressure and velocity transducers are described in detail in view of the fact that hot wire anemometers, in particular, are not commonly used in musical acoustics. The method used to make a swept sine wave measurement of the input impedance is described and the corrections necessary to achieve an accurate result discussed. The results obtained from measurements on a straight cylindrical tube are compared with theoretical predictions developed by the authors from many sources and show good agreement. Results of measurements on trombones are illustrated and an indication of the way that computer processing may aid analysis and interpretation is given.

Messung der akustischen Impedanz von Blechblasinstrumenten

Zusammenfassung


Mesure de l'impédance acoustique des cuivres

Sommaire

On décrit un appareil exempt des faiblesses constatées dans les procédés antérieurement connus, dans leur application aux cuivres, pour la mesure de leur impédance d'entrée. Après une discussion de la notion d'impédance acoustique, on passe brièvement en revue les méthodes connues, et l'on montre qu'elles présentent les inconvénients suivants: 1° difficulté d'un étalonnage absolu; 2° difficulté de déplacer le plan où se fait la mesure; 3° impossibilité d'étudier les effets d'un écoulement constant d'air; 4° difficulté d'obtenir des données sur l'angle; 5° seuls les signaux sinusoidaux sont utilisables pour l'essai; 6° impossibilité de procéder à la mesure pendant qu'on joue de l'instrument.

Le nouvel appareil proposé mesure simultanément la pression acoustique et la vitesse des particules, en utilisant une sonde microphonique reliée à un pavillon et un anémomètre à fil chaud; il est commandé par un mini-ordinateur. Les anémomètres à fil chaud n'étant pas d'utili-
sation courante en acoustique musicale, on décrit en détail les procédés d'étalonnage des traducteurs de pression et de vitesse. Nous décrivons la méthode que nous employons pour mesurer l'impédance d'entrée au moyen d'un signal sonore à variation continue, et nous discutons les corrections nécessaires pour arriver à des résultats exacts. On compare les prévisions théoriques provenant de diverses sources avec les résultats de cette méthode pour un tube cylindrique droit : l'accord est satisfaisant. On illustre par des figures les résultats obtenus sur des trombones, et l'on indique la façon dont l'informatique pourrait contribuer à leur analyse et à leur interprétation.

1. Introduction

The term “impedance” was introduced by Heaviside [1] towards the end of the last century in connection with the analysis of electrical circuits. The concept was extended by Webster [2] who introduced acoustic impedance in a study of the theory of horns and the phonograph. Webster also studied the behaviour of sound production in wind instruments and in the previous year had delivered a paper to the American Physical Society entitled “A mechanically blown wind instrument” [3]. The principle described in that paper is of a valve, controlled by a spring under tension, which could admit a puff of air into the instrument. The pressure wave generated is reflected by the bell and arrives back at the valve at the appropriate phase to maintain oscillation. This model is adopted by Benade in his book “The fundamentals of musical acoustics” [4] to explain the function of the lips when playing brass instruments.

A mathematical treatment of the interaction of the horn impedance with the time varying impedance function of the lips was given by Bouasse in 1929 [5]. He noted that the interaction of the horn with the player's vibrating lips created a non-linear flow of air into the instrument, and hence deduced that for stable oscillation horns should possess natural frequencies that correspond closely to the harmonics of the note being played ([4], p. 395). More recently Smith and Daniel have developed a perturbation theory which may be used to alter the frequency of a single resonance by a specified amount. The use of a temperament system in the musical scale means that the positions of the maxima are “... a compromise between their intonation when used as fundamentals, and their reinforcement of the harmonics of other notes” [6].

An example is quoted where the tenth impedance maximum on a particular trumpet was moved in frequency to overcome a deficiency of second harmonic when the note D₅ was played, since this note involves the co-operation of the fifth and tenth maxima. The nature of this compromise also has implications for the “responsiveness” or feel of the instrument. It was decided for the purposes of this study to measure the impedance curves of a range of trombones of varying quality, and to relate the detailed nature of the curves to the subjective impressions of professional musicians. This paper will describe a new method for the measurement of acoustic impedance.

Before proceeding, acoustic impedance will be defined since modern practice differs from Webster's original definition. The acoustic impedance \( Z \) at a given section is the complex quotient of the acoustic pressure \( P \) divided by the volume velocity \( U \), namely:

\[
Z = \frac{P}{U} \text{ kg m}^{-4} \text{s}^{-1} \text{ or acoustic Ohm (S.I.)}.
\]

Since \( Z \) is complex, eq. (1) may be re-written as,

\[
Z = |Z| e^{i\varphi}
\]

or

\[
Z = R + jX.
\]

This corresponds, for example, with the usage of Kinsler and Frey [7].

Normally it is the quantity \(|Z|\) to which musical acousticians refer as acoustic or “input” impedance. The measurement of \( \varphi \), the phase angle, is usually neglected.

2. Existing impedance measuring systems

The measurement of the acoustic impedance of brass instruments has received attention from several workers, all employing a basically similar technique. A review of these variations has already been given by Benade [8]. A system devised by Kent and co-workers at Conn, Co., the musical instrument manufacturers, and later used by Benade is shown in Fig. 1. The output of a variable frequency sine wave oscillator (V.F.O.) is fed via a controlled attenuator and power amplifier (P.A.) to a horn loudspeaker driver unit. This unit is

![Fig. 1. Impedance measuring system devised by Kent.](image-url)
coupled via a glass-fibre filled tube to a capillary and finally to the mouthpiece and the instrument under test. A microphone placed at the loudspeaker end of the capillary (control microphone) is incorporated in a feed-back loop which maintains a constant pressure amplitude at that point. Since the impedance of the capillary is high, this in effect produces a high impedance, constant volume velocity source at the mouthpiece. A pressure response microphone placed in the plane of the mouthpiece rim records the pressure while the oscillator is swept over the frequency range of interest and its output is thus directly proportional to the acoustic impedance.

Merhaut [9] and Coltman [10] using the same basic technique have described variations in the method used to provide a constant volume velocity source. Merhaut places a thin nylon diaphragm, driven by a loudspeaker, before the mouthpiece cup. The diaphragm is coated with aluminium and forms one electrode of a condenser microphone, the other electrode being held rigid. The motion of the diaphragm produces a signal proportional to the diaphragm displacement. The signal is then differentiated to produce a signal proportional to the diaphragm velocity. This forms the error signal in a feed-back circuit providing a constant volume velocity source. Coltman’s method of deriving a constant volume velocity source involves the mounting of a secondary coil (suspended between magnets) on the piston connected to the driver coil. This secondary coil thus provides a signal proportional to the motion of the loudspeaker, and is used as the error signal in a feed-back circuit.

All the systems just described suffer from a number of drawbacks: Firstly, the technique used to achieve a constant volume velocity discourages absolute calibration since the velocity signal is derived indirectly. Backus, using a system similar to that devised by Kent has only recently published calibrated impedance data [11]. He measured the impedance of a specially devised annular capillary and recorded the calibrated values for both control and response microphones. Secondly, it is not generally possible to move the plane of measurement from the mouthpiece cup rim to other parts of the instrument, for example the mouthpiece throat, without some modification to the experimental apparatus. It is clearly desirable to be able to measure the acoustic impedance of the instrument alone, in addition to the more complex case of mouthpiece and instrument combined. The advantage of measuring acoustic impedance at the throat of the mouthpiece is that the impedance maxima obtained correlate well with playing frequencies [12], [13], [14], and the influence of the cup volume is removed. Thirdly, all the above systems neglect the effect produced on the magnitude of the impedance maxima by a steady airflow through the instrument (provided normally by the player) although this can be shown to be significant [15]. Fourthly, extracting phase information is a matter of considerable difficulty when using one of the above methods. Finally, it is not possible to collect pressure and velocity data when the instrument is blown by a player.

More recently Wogram in his Ph.D. thesis [16] has shown how the methods described may be extended to give absolute data for acoustic impedance. He did this by comparing his results for an instrument with those for a 55 m length of tubing whose impedance is calculable, and whose phase response is assumed to be zero. This assumes that the value for the impedance of a length of tube may be calculated accurately, and that the tube impedance comprises only a real part. This method is not entirely satisfactory as it does not permit an independent, experimental check on such calculations and assumptions to be made.

Finally it should be noted that Backus and Hundley [17] have successfully used the impedance tube method [18], which is generally employed for determining the acoustic impedance of absorbent materials. Although this method yields calibrated data for the acoustic impedance, as well as the phase, it is very laborious to perform, especially if a resolution of better than 1 Hz is required. Such a method is not therefore suitable, where comparisons between many instruments are to be made.

3. A new approach

A brass instrument may be characterized by the 2-port network shown in Fig. 2. The system has a transfer function \( \frac{P_0}{P_1} \) and input impedance \( \frac{P_1}{U_1} \). Note that a brass instrument only becomes non-linear when coupled to a player’s lips. Under normal playing conditions non-linearity in the air may be ignored [17]. The input impedance, \( Z(f) \), may be measured by applying one of several excitation functions, \( U_1(f) \), and measuring the response, \( P_1(f) \). Four popular excitation functions are:

(i) step function,
(ii) impulse function,
(iii) random noise,
(iv) sine wave.

Since the techniques described above are unable to measure the volume velocity directly, only the
sine wave excitation method is possible. The volume velocity is not measurable and is merely held constant, hence only an uncalibrated signal proportional to $Z(f)$ is obtainable, unless one of the two techniques described above is employed [11], [16]. A method which makes possible the use of any of the four excitation methods, and which is largely free from the five limitations listed earlier, has been developed at Surrey University. The work has been presented verbally at conferences during 1976 [19], [20], [21]. The most important single feature of this system is that the velocity is measured directly using a hot wire anemometry system, allowing calibrated data for the acoustic impedance (both modulus and phase) to be obtained. In addition the system is controlled by a digital computer, instead of coupled analogue devices. The computer used is a Data General Nova 2, equipped with a 2.5 M byte moving head disc, a 150 kbyte floppy disk, a visual display unit (VDU), a line printer, and 2 channel analogue to digital (A/D) and digital to analogue (D/A) converters. These converters have 14 bit resolution and can sample continuously at a rate of 25000 samples/s on each channel. A Binary Coded Decimal (BCD) output register is also available.

The principal point at which impedance is measured has been moved to the throat of the mouthpiece, although measurements in the plane of the mouthpiece cup rim may also be taken. The change enables the impedance of the instrument (without the mouthpiece cup volume) to be measured, although it should be noted that the mouthpiece backbore is still present and has an influence on the impedance. Measurements without the mouthpiece being present are possible but would not be close to actual playing conditions. Several mouthpieces have been modified to allow a Bruel and Kjaer Probe Microphone (type 4170) and a DISA hot-wire anemometer probe (type 55 P 11) to be inserted in the throat (Fig. 3). The probe microphone is fitted flush with the surface of the mouthpiece and the hot-wire probe is placed in the centre of the throat (Fig. 4). The two probes are mounted in the same plane and their signals are sampled simultaneously by the A/D converters. The impedance at that plane is then calculated by the computer.

### 4. Calibration procedures

Before discussing operation of the experimental technique devised to measure the impedance, the calibration procedures for both the probe microphone and the hot-wire probe will be described, since the accuracy of the measurements is naturally dependent on reliable calibration curves, and the use of hot-wire anemometry is not widespread in musical acoustic measurements. A calibration curve for the probe microphone is supplied by the manufacturers, but does not include the phase response. However a facility exists at the National Gas Turbine Establishment for calibrating probe microphones, and with their generous help and co-operation, the microphone used in the present work has been calibrated at three overall sound pressure levels; 120, 140 and 150 dB. The microphone is horn coupled and the results show that it does not suffer the disadvantage of the conventional cylindrically coupled units, whose calibration
changes appreciably with level, due to changes in boundary layer losses in the probe tube. The probe microphone under test and a ½ inch Bruel and Kjaer condenser microphone (type 4135) are placed in a uniform sound field and excited by broad band noise. The signals from each device are compared, and the difference between them expressed as a correction in both amplitude (expressed in dB) and phase (expressed in degrees). The amplitude and phase response of the ½ inch microphone is assumed to be flat over the frequency range of interest (see section 6). The system was used to provide a calibration curve to cover the range 10 Hz to 5 kHz, and the resolution was 10 Hz.

The hot-wire anemometry system also requires calibration, and a special apparatus was constructed as follows: Compressed air at a line pressure of 750 kPa is fed to a regulating system. This system comprises three basic stages; filtration, regulation and heating. A two-stage filter removes all particles down to a size of 0.01 μm diameter. This precaution is necessary to avoid damage to the 5 μm diameter hot-wire probes. The air is then regulated by a pressure release valve set to 600 kPa which provides a constant pressure head to a two stage regulator which reduces the pressure to 270 kPa. A needle valve (18 turn) allows a precise airflow setting to be made, and the optional heater completes the system. All that remains is for the airstream from this regulated supply to be accurately calibrated, which task is performed using a pitot-tube [22]. The arrangement used is shown in Fig. 5, and makes use of a small wind tunnel whose final section is a trombone mouthpiece. The pitot-tube is positioned in the mouthpiece throat, in line with the airflow. The differential pressure between the pitot-tube and a tube in the throat normal to the airflow is measured with an inclined tube manometer with 1/100 mm of water resolution. For each turn of the needle valve the pressure is recorded, and hence the velocity may be calculated. Calibration is carried out over a range of velocities in the mouthpiece throat of 1 m s⁻¹ to 16 m s⁻¹. For comparison, velocities during playing are typically 5 m s⁻¹ of steady airflow and 1 m s⁻¹ of alternating flow for a middle register mezzoforte note. Having calibrated the regulating system the pitot-tube is removed, and a hot-wire probe inserted. The signal from the hot-wire anemometer may be expressed thus [23].

\[ V^2 = A + BU^p \]  

(4)

where

- \( V \) is the output voltage from the hot-wire anemometer,
- \( U \) is the air velocity,
- \( A, B \) and \( p \) are constants.

The original theoretical work of King [24] indicated that \( p = \frac{1}{2} \), but in practice the optimum value for \( p \) depends on several practical factors and may be considerably different. The value for \( p \) is optimized during calibration, and is found to lie in the range 0.35 < \( p < 0.45 \). Hardware linearizers may be used to assist the calibration procedure, but they are unnecessary in this case since the computer may be used to linearize the velocity signal. This is done by feeding the velocity signal to one channel of the A/D converters, and then 1000 samples are taken and averaged for each turn of the needle valve. Since the velocity characteristics of the needle valve are known, the constants \( A \) and \( B \) may be computed for a given value of \( p \). \( p \) is initially taken as 0.35 and is incremented in steps of 0.01 until the error of fit in eq. (4) is a minimum. The corresponding values of \( A, B, \) and \( p \) are then recorded for subsequent use.

5. The experiment

Having discussed the calibration technique for both the pressure and velocity probes in some detail, the experimental procedure used to determine acoustic impedance will now be described. A diagram of the system is shown in Fig. 6. The
The cross-sectional area of the mouthpiece throat is also entered (typical diameters fall in the range 5.80 mm to 7.35 mm). The computer then sets an initial frequency on a programmable frequency oscillator (P.F.O.). The oscillator used in an Adret Codasyn 201, whose frequency may be selected by a BCD coded input. The sine wave output from the oscillator is amplified (Quad 50) and fed to a loudspeaker (Gauss 40) which in turn is coupled to the instrument under test. Although this method does not require a constant volume velocity source, in practice some degree of control is used to ensure that the signal from the hot-wire anemometer is adequate at all times. This is achieved by smoothing the velocity signal, and feeding the result into the amplitude modulation (a.m.) input of the oscillator. The R.M.S. value of the velocity waveform is thus kept to within approximately 15% of any set value. The A/D converters are then instructed to acquire simultaneously one cycle of both the pressure and velocity waveforms. Using the calibration constants for the hot-wire probe, the particle velocity may be calculated. This is transformed into the volume velocity by multiplying by the cross-sectional area of the mouthpiece throat. A small error arises due to boundary layer effects, this point is discussed more fully in section 7.

Owing to the unusual load that a brass instrument puts on a conventional loudspeaker, a certain amount of distortion appears in both pressure and velocity waveforms near the resonance frequencies of the instrument. This may be filtered out digitally by expanding the sampled waveform as a Fourier Series and extracting only the fundamental. In addition a phase value for the fundamental component may also be extracted. By subtracting the velocity phase value from the pressure phase value the phase of velocity with respect to pressure may be obtained. This value is finally corrected automatically taking into account the phase response of the probe microphone mentioned previously. The pressure signal may now be calibrated. The probe microphone is coupled to a Power Supply (Brüel and Kjaer type 2801) and an Impulse Precision Sound Level Meter (type 2204) provides additional amplification. This arrangement has a combined sensitivity of approximately 120 Pa/V and hence the pressure signal may be converted to Pascals, and then to decibels. The pressure (now in dB) is then corrected using the calibration curves supplied by the National Gas Turbine Establishment and the R.M.S. value calculated. The fully calibrated values for the pressure, velocity and phase are printed on the VDU and held in a temporary store. The process of sampling the waveforms and comput-
ing the values is repeated 11 times, and the 12 results are averaged to improve the accuracy of the data stored in a disc file. The frequency is incremented by 1 Hz and the process is repeated until the upper frequency limit (typically 768 Hz) is reached.

6. Determination of the input impedance of trombones

The detailed analysis of the input impedance of trombones is a matter for other papers (to be published), but it is convenient here to illustrate the capabilities of the apparatus and to indicate the very considerable advantages of having a computer-based system. The input impedances of two medium bore trombones (the bore of a trombone is conventionally taken to be the inside diameter of the inner tube of the slide section) of bores approximately 12.7 mm are shown in Fig. 8. The magnitude, $|Z|$, of the impedance is shown in the upper part of the Figure and the phase angle, $\phi$, of the impedance in the lower part. The data stored in the disc file are plotted by an X-Y plotter driven by the D/A converters, and it is apparent that any combination of results may be plotted on the same piece of paper for immediate visual comparison. The examples of trombones chosen for Fig. 5 are of a very highly respected make and of the cheapest readily available on the British market.

A more subtle advantage of the equipment is that the data may be transformed before being plotted; in Fig. 9 the input impedance of one of the trombones shown in Fig. 8 is plotted showing the real and imaginary components separately. Since the real component of the impedance is the one more immediately associated with the transfer of energy through the instrument (see below), this form of display is often more helpful in visualizing the interaction of player and instrument. The data may also be corrected without having to re-run the experiment. For example, it was discovered some time after calibration that the calibration of the probe microphone performed by the National Gas Turbine Establishment was in error in phase by approximately 0.14 radian, due to an incorrect value being assigned to the phase response of the reference microphone; all that had to be done was to change one line in the plotting program used by the computer.
The input impedance of a brass cylinder of bore 10.9 mm, and length 1.42 m is shown in Fig. 10, which may be compared with Fig. 8. The data from this experiment are discussed more fully in section 8.

The analysis of the input impedance by eye is quite demanding and a program which sorts out the maxima and minima and then prints out the values of the maxima and minima, together with the frequencies at which they occur, and the Q's of the maxima is of very great value in making assessments of several instruments — or of the same instrument in different slide positions. The Q's are calculated by examining the real part of the input impedance at the frequency of a peak and at 5 Hz above and below the peak. It is, perhaps, pertinent to point out that the most generally accepted definition of Q implies that it is incorrect to follow most authors in musical acoustics and calculate Q from the magnitude of the impedance. For this reason our values will be found to differ from those previously reported. An example of the print-out from this program is given in Fig. 11. The example corresponds to the instrument shown in Fig. 9.

In studying the interaction of player and instrument, the energy transfer is, as already mentioned, a very useful guide. In particular, the playing frequencies may be predicted quite closely. In his thesis, Wogram [16] used the concept of a "sum function" which he calculated by determining the real part of the input impedance and then summing the input resistance values at harmonics of a fundamental frequency (some normalisation is necessary but need not be considered here). As the fundamental frequency is swept through the usable range of the instrument, the sum function is found to have peaks at frequencies corresponding to the played frequencies and at the "privileged" frequencies mentioned by Bouasse [5]. Hence the conclusion may be drawn that a player tends to excite the instrument at frequencies for which maximum energy transfer to the room will ensue. Wogram simply considered all harmonics equally and did not appear to take into consideration the work of Benade and Worman [26] who showed that at low dynamic levels the spectrum of the sound inside the instrument has relatively few harmonics, but as the playing dynamic increased the nth harmonic increased as the nth power of the increase in fundamental. It is a simple matter to extend Wogram's sum function to take Worman's result into account and in a later paper we shall discuss how the variation of intonation with playing dynamic may be calculated from the input impedance data.

7. Measurement accuracy

Errors introduced during the measurement of acoustic impedance may be separated into two groups, calibration errors and system errors. The calibration values for the probe microphone supplied by the National Gas Turbine Establishment have an amplitude error of ±0.5 dB and a phase error of ±1 degree. The inclined tube manometer had a typical error of 0.25%, and the repeatability when testing the regulated air supply was typically ±1% over an 18 month period. The voltage data from the hot-wire anemometer fitted equation (4) with negligible error (Pearson correlation coefficient > 0.99999).

The repeatability for |Z| is typically ±5% and for Q is typically ±2%, although these figures could probably be improved if a frequency step of 0.1 Hz were used. This is because the Q's of the resonances...
are very high. The positions of the maxima very rarely moved by more than 1 Hz on successive trials. The temperature of the room was held at 21 °C ± 1 °C. Probably the most important source of error (and certainly the hardest to estimate) is the non-uniform velocity profile across the mouthpiece throat. This profile may be measured by traversing the hot-wire across the section and observing the change in flow that occurs for frequencies from D.C. to 800 Hz. Thus the correct volume velocity may be obtained by multiplying the on-axis particle velocity by an “effective” cross-sectional area obtained from the velocity profile data. This effective area is naturally a function of frequency.

8. The theoretical prediction of impedance

One object of the apparatus was to obtain calibrated values for the impedance, and to verify that this had been achieved a system of known impedance was measured.

The theoretical derivation of the input impedance of any brass instrument is not possible at present, so the test impedance must be a simpler system, and a straight cylindrical tube, whose properties are (in principle) well understood, was chosen. The theoretical input impedance of a cylindrical conduit may be derived from analysis of a transmission line analogy, assuming that only the dominant (plane) mode propagates at the frequencies used [27]. The input impedance of a length $l$ of transmission line with distributed loss, terminated by an impedance $Z_i$, is given by [28]:

$$Z_l = Z_0 \tanh \Gamma l + Z_0 \tanh^{-1} \left( \frac{Z_0}{Z_L} \right)$$

where

- $Z_0$ is the characteristic impedance, which in general is complex, and
- $\Gamma$ is the propagation constant $= \alpha + j \beta$,
- $\alpha$ is the attenuation coefficient, and
- $\beta$ is the wave number.

Using the ‘impedance analogy’ so that electrical impedance is directly analogous to acoustic impedance, as defined above, this expression may be applied to a cylindrical tube of length $l$. Expressions for the distributed parameters of such a line (series impedance and shunt admittance) are given by Daniels [29], and these may be used to obtain values of $Z_0$ and $\Gamma$.

In the practical case of a tube about 1.4 m long and 10 mm internal diameter considerable simplification of eq. (5) may be obtained. To begin with, at the frequencies used, the ‘large’ tube approximation is valid [30], i.e. the parameters:

$$a \sqrt{\frac{\omega \rho}{\eta}} \quad \text{and} \quad \alpha \sqrt{\frac{\omega \rho C_p}{K}}$$

are both much larger than unity, where

- $a$ is the internal radius of the tube,
- $\omega$ is the angular frequency,
- $\rho$ is the density of the air,
- $\eta$ is the coefficient of shear viscosity of air,
- $C_p$ is the specific heat of air at constant pressure,
- $K$ is the thermal conductivity of air.

This allows the following assumptions to be made:

1) that the velocity of sound in the tube ($v$) is equal to that in free air ($c$),
2) that $Z_0$ is a real constant of value $\frac{\rho c}{\pi a^2}$,
3) that $\beta = \frac{\omega}{v}$,
4) that $\alpha = \frac{\omega}{v} (g + h)$,

where

$$g = \frac{1}{a} \sqrt{\frac{\eta}{2 \omega \rho}}$$

$$h = \gamma - 1 \sqrt{\frac{K}{2 \omega \rho C_p}}$$

and

$\gamma$ is the ratio of the principal specific heats of air.

The free air attenuation (proportional to the square of frequency) and the attenuation (proportional to frequency) reported by Fay [31] and others, are negligible.

The value of the load impedance (due to radiation) in this case is approximately given by [32]:

$$Z_l = \frac{\rho c}{\pi a^2} \left[ \left( \frac{\beta a}{4} \right)^2 + j 0.61 \beta a \right] = Z_0 (R_1 + j X_1), \text{ say.}$$

The magnitude of this is less than 6% of $Z_0$ below a frequency of 1 kHz. Therefore

$$\tanh (R_1 + j X_1) \approx R_1 + j X_1$$

and from eq. (5) the input impedance becomes,

$$Z_l = Z_0 \tanh (\alpha l + j \beta l + R_1 + j X_1).$$

If $\Lambda = \alpha l + R_1$ and

$$l' = l + \frac{X_1}{R} = l + 0.61 a,$$
then

\[ Z_i = Z_q \tanh (\Delta + j \beta V) = \frac{\tanh \Delta + j \tan \beta V}{1 + j \tanh \Delta \tan \beta V}. \]

The magnitude of \( \alpha l \) is also small (less than 0.3 for frequencies below 1 kHz) but dominates the radiation resistance, so

\[ \tanh \Delta \ll \Delta \ll \alpha l, \text{ and} \]

\[ Z_i = Z_q \alpha l + j \tan \beta V \frac{1 + j \alpha l \tan \beta V}{|Z| e^{j\varphi}}. \]

This is considerably simpler to manipulate than the exact form of eq. (5). A simplified form of this equation is used by Jansson and Benade [33].

Impedance maxima occur for frequencies at which \( \beta V = (n + 1/2) \pi \) where \( |Z| = Z_q(\alpha l) \) and \( \varphi = 0 \). Similarly impedance minima occur for frequencies at which \( \beta V = n \pi \) where \( |Z| = Z_q\alpha l \) and \( \varphi = 0 \).

The turning points of the phase of the input impedance occur when

\[ \beta V = (n + 1/2) \pi \text{ or } (n + 3/2) \pi, \]

so

\[ \varphi = \pm \tan^{-1}(1/2\alpha l) \text{ and } |Z| \ll Z_q. \]

In Figs 12 and 13 maxima and minima of magnitude and phase are presented, measured for a straight tube of length 1.42 m and internal radius 5.45 mm at a temperature of 21 °C. The results have been corrected for non uniform flow (see section 7).

In fact at low frequencies second order terms are not negligible in the equations, so

\[ v = c(1 - g - h) \text{ and} \]

\[ Z_0 = \frac{\rho c a}{\pi a^2 (1 + g - h)}. \]

This modifies the predicted values of the lowest frequency points, for instance the frequency of the first maximum is lowered by about 4% and its magnitude decreased by about 3%.

This was conveniently shown by Benade [30] although his work has been shown to be in error for 'small' tubes by Backus [34] and the present authors have found inconsistencies in the derivation for 'large' tubes. The theory forms a special case of the very general work of Weston [35], another student of Dr. R. W. B. Stephens.

Acknowledgements

The work on musical acoustics at the University of Surrey is supported by a Research Grant from the Science Research Council.

(Received April 1st, 1977.)

References


A number of programs have been written in FORTRAN for use on the NOVA 2 computer and they divide conveniently into four classes:

1. Programs which control experimental runs,
2. Programs which present the data files,
3. Programs which manipulate the data files,
4. Programs which calculate parameters from formulae used in the thesis.

We present a list of the programs used with brief descriptions of their function. Listings are then given for examples of each type of program (which are marked by asterisks in the list below).

**EZRUN** controls the experiment to measure impedance up to 1024 Hz in steps of 1 Hz.

**EZZRUN** controls the experiment to measure impedance up to 2048 Hz in steps of 2 Hz.

**ETRUN** controls the experiment to measure the pressure transfer function up to 1024 Hz in steps of 1 Hz.

**ETTRUN** controls the experiment to measure the pressure transfer function up to 2048 Hz in steps of 2 Hz.

**ESING** controls an experiment to determine the impedance at single frequencies.

**EKAL** samples the anemometer output voltage for sixteen steady velocities and calculates the steady-flow anemometer calibration.

**ETDRAW** plots the magnitude and phase of a data file on an oscilloscope or X-Y plotter.

**EDDRAW** plots a data file as a dashed line onto the X-Y plotter.

**EIMPDRAW** plots the file of the impulse response on an oscilloscope or X-Y plotter.
SQPDRAW (written by Mr. D. Munro). See Appendix D.

EFIND locates the frequency at which the impedance is nearest to a maximum or minimum.

ERITZ* calculates the real and imaginary parts of a data file.

ELOGZ calculates the log of the amplitude of a data file.

EINVZ calculates the reciprocal of a data file.

ESQZ squares the magnitude of a data file and sets the phase to zero.

EWEIGHTZ multiplies the magnitude of a data file by a Hanning window.

EMPIECE calculates the effect of a lumped mouthpiece model on an impedance data file.

ESUM calculates the "sum function" of a data file.

EIMP* calculates the impulse response of a data file by computing the inverse F.F.T.

PPHASE (written by Mr. P.S. Watkinson) corrects a data file for a linear phase error with frequency.

ECALADD adds together a number of calibration files to form the one used in a run program.

EZAP calculates the series or parallel combination of two data files.

EZRAD calculates the radiation impedance of a hemisphere in an infinite baffle.

ETFCAL calculates the magnitude of the pressure transfer function from the input and radiation impedances.

EBOUND (written by Mr. P.S. Watkinson) calculates the amplitude and phase correction necessary for the boundary layer.

EGCAL calculates the displacement per unit pressure for the reed model.

EZRCAL calculates the effective "reed impedance".

PIMPLE (written by Mr. P.S. Watkinson) calculates the theoretical input impedance of a straight tube.
DOUBLE PRECISION F
DIMENSION CALM(1024),CALPH(1024),IDATA(500),VP(250)
DIMENSION Z(1024),PH(1024),UD(1024),UK(1024),NAME(10)
DIMENSION U(1024),IAT(1),MANE(10)

TYPE*PROGRAM TO MEASURE ACOUSTIC IMPEDANCE*
TYPE*FROM 10HZ TO 1024HZ IN STEPS OF 1HZ*
ACCEPT *NUMBER OF CYCLES = *,N
ACCEPT *VELOCITY REFERENCE (CC/S) = *,UR
ACCEPT *UPPER VELOCITY LIMIT (CC/S) = *,URL
ACCEPT *LOWER VELOCITY LIMIT (CC/S) = *,URL
ACCEPT *START ATTENUATION (DB) = *,DB

TYPE *ANEMOMETER CONSTANTS ; *
ACCEPT "DIA. OF THROAT (MM) = *,D
ACCEPT *MICROPHONE CALIBRATION (PA/V) = *,CALP

TYPE *FILENAME OF CALIBRATION CURVE ? *
READ(11,1)MANE(1)

1 FORMAT(S20)
  CALL FOPEN(2,MANE)
  TYPE *FILENAME OF RUN ? *
  READ(11,2)NAME(1)

2 FORMAT(S20)
  CALL FOPEN(1,NAME)
  CALL RDBLK(2,0,CALM,8,IER)
  CALL RDBLK(2,8,CALPH,8,IER)
P=3.1415926
  IAT(1)=DB*X10.0
  CALL GIV(7,7,IAT,1,0)
  CALL FREAK(48000.0)
  PAUSE
  DO 10 L=10,1024
  IS=N*X48
  10 CALL GET(2,IDATA(1),IS)
  IF=N*X24
  UT=0.0
  DO 20 I=1,IP
  K=I*2
  J=K-1
  VP(I)=IDATA(J)/1638.4
  VU=IDATA(K)/1638.4
  UI(I)=(A+B*VU*VU)**(1.0/P)
  UT=UT+UI(I)
  20 CONTINUE
  CALL FUNDN(24,N,U1,URMS,UPH)
  UMS=URMS*P1*D*D/4.0E6
  UCCS=UMS*1.0E6
  AER=UCCS/UR
  DBADD=20.0*ALO610(AER)
  DB=DB+DBADD
  IF(DB.LT.0.0)DB=0.0
  IF(DB.GT.59.9)DB=59.9
  IAT(1)=DB*X10.0
  CALL GIV(7,7,IAT,1,0)
  IF(UCCS.LT.URL.OR.UCCS.GT.URU)60 TO 16
  F=DBLE((L+1)*4800.0)
  CALL FREAK(F)
  UDC=UT/IP
CALL FUNDN(24, N, VP, VPRMS, VPPH)
P1=VPRMS*CALP
UD(L)=UDC
UK(L)=UMS
Z(L)=(P1/UD(L))*CALM(L)
PH(L)=UPH-VPPH+CALPH(L)+PI

30 IF(PH(L).GT.PI) PH(L)=PH(L)-2.0*PI
IF(PH(L).LT.-PI) PH(L)=PH(L)+2.0*PI
IF(PH(L).GT.PI.OR.PH(L).LT.-PI) GO TO 30
ZMEG=Z(L)/1.0E6
WRITE(10,40) L, ZMEG, PH(L), P1, UCCS

40 FORMAT( " " , I4, " Hz Z=" , F5.1, " MEGOHMS PH=" ,
        X F5.2, " RAD PRESS.=" , F5.2, " PA VEL=" , F5.2, " CC/S" )
WRITE(10,50) UDC

50 FORMAT( " MEAN LINEAR VELOCITY = " , F6.3, " M/S" )
10 CONTINUE
CALL WRBLK(1,0,Z,8,IER)
CALL WRBLK(1,8,PH,8,IER)
CALL WRBLK(1,16,UD,8,IER)
CALL WRBLK(1,24,U,8,IER)
CALL FSTAT(1,3,IER)
CALL FCLOS(1)
CALL FCLOS(2)
CALL GIV(7,7,500,1,0)
CALL FREAK(48000.0)
GO TO 100

16 IDELY=N*1000
CALL FDELY(IDELY)
GO TO 15

100 STOP
END
DIMENSION T(1024), P(1024), IT(1024), IP(1024), NAME(20)

10 TYPE*FILE TO BE DRAWN?
READ(11,1)NAME(1)
1 FORMAT(20)
CALL FOPEN(1, NAME)
CALL FTCLEAR
CALL FTPENUP
ACCEPT 'IS FIRST OR SECOND 16 BLOCKS IN THE FILE TO
* BE PLOTTED? TYPE 1 OR 2 ', IB
IA=(IB-1)*16
ACCEPT "RANGE OF UPPER GRAPH?", RDB
ACCEPT "RANGE OF LOWER GRAPH?"
* (IF 0 ENTERED RANGE = PI)*, RPH
IF(RPH.EQ.0.0) RPH=3.1415926
ACCEPT "UPPER GRAPH AXIS TO BE AT
* BOTTOM , MIDDLE OR TOP (0, 1/2) ", IPOSS
ITPOS=390+195*IPOSS
CALL RDDBLK(1, IA, T, 8, IER)
CALL RDDBLK(1, IA+8, P, 8, IER)
DO 90 I=10,1024
RT=ITPOS+T(I)*195.0/RDB
IT(I)=RT
RP=195.0+P(I)*195.0/RPH
IF(I)=RP
90 CONTINUE
CALL FTDISP
CALL FTPENUP
ACCEPT ' AXES PLOTTED? YES(1) OR NO(0) ', KP
IF(KP.EQ.0) GO TO 11
CALL FTPLOT(0,0,1)
CALL FTPLOT(0,780,0)
CALL FTPLOT(1023, ITPOS, 1)
CALL FTPLOT(0, ITPOS, 0)
CALL FTPLOT(1023, 195, 1)
CALL FTPLOT(0, 195, 0)
CALL FTPLOT(0, 0, 1)
CALL FTPLOT(15, 0, 0)
CALL FTPLOT(0, 390, 1)
CALL FTPLOT(15, 390, 0)
CALL FTPLOT(0, 585, 1)
CALL FTPLOT(15, 585, 0)
CALL FTPLOT(0, 780, 1)
CALL FTPLOT(15, 780, 0)
IX=0
IY=0
DO 51 M=1,10
IX=M*100
CALL FTPLOT(IX, 210, 1)
CALL FTPLOT(IX, 195, 0)
51 CONTINUE
DO 52 N=1,10
IX=N*100
CALL FTPLOT(IX, ITPOS+15, 1)
CALL FTPLOT(IX, ITPOS, 0)
52 CONTINUE
CALL FTPENUP
PAUSE
11 CALL FTAWAY
CALL FCLOS(1)
GO TO 10
STOP
END
DIMENSION Z(1024), P(1024), RZ(1024), AIZ(1024)

TYPE *PROGRAM TO CONVERT A FILE OF*
TYPE * MAGNITUDE AND PHASE DATA*
TYPE * TO A FILE OF REAL AND IMAGINARY DATA*
TYPE *FILE TO BE CONVERTED ?*
READ (11,1) NAME(1)

1 FORMAT(S20)
CALL FOPEN(1,NAME)
TYPE *NEW FILENAME ?*
READ (11,1) MANE(1)
CALL FOPEN(2,MANE)
CALL RDBLK(1,0,Z,8,IER)
CALL RDBLK(1,8,P,8,IER)
DO 5 I=10,1024
RZ(I)=Z(I)*COS(P(I))
AIZ(I)=Z(I)*SIN(P(I))
5 CONTINUE
CALL WRBLK(2,0,RZ,8,IER)
CALL WRBLK(2,8,AIZ,8,IER)
CALL FCLOS(1)
CALL FCLOS(2)
STOP
END
DIMENSION ARRAY(4098), NAME(10), MANE(10)

TYPE "PROGRAM TO COMPUTE INVERSE F.T. OF FILE"

READ(11,1) NAME(1)

1 FORMAT(S20)

CALL FOPEN(1, NAME)
CALL RDQLK(1,0, ARRAY(1), 8, IER)
CALL RDQLK(1,8, ARRAY(1025), 8, IER)

JFLAG=1

DO 20 I=1,10

ARRAY(1024+I)=0.0

20 CONTINUE

DO 30 I=1,9

ARRAY(I)=ARRAY(10)*I/10

30 CONTINUE

DO 40 I=1,4097

ARRAY(4097-2*I)=ARRAY(I)

ARRAY(4098-2*I)=-ARRAY(1024+I)

40 CONTINUE

DO 50 J=2,2048,2

ARRAY(J+1)=ARRAY(4097-J)

ARRAY(J+2)=-ARRAY(4098-J)

50 CONTINUE

ARRAY(1)=0.0

ARRAY(2)=0.0

ARRAY(2050)=0.0

TYPE "START F.F.T."

CALL NDFT4CARRAY, 2048, JFLAG

IF(JFLAG .NE. 0) STOP ERROR IN F.F.T.

TYPE "END F.F.T."

DO 60 I=1,2048

J=I*2

IF (ARRAY(J).GT.1.0E-6) TYPE "ARRAY(*,J,) = " , ARRAY(J)

ARRAY(I+1)=ARRAY(2*I+1)

60 CONTINUE

TYPE "NEW FILENAME?"

READ(11,1) MANE(1)

CALL FOPEN(2, MANE)

CALL WRQLK(2,0, ARRAY(1), 16, IER)

CALL FCLOS(1)

CALL FCLOS(2)

STOP

END
APPENDIX C

DESCRIPTION OF INSTRUMENTS AND MOUTHPIECES USED

The trombone used was a Boosey and Hawkes 'Imperial' model, Serial NO.591802. In very new condition, the instrument had hardly been used at all.

The trumpet used was an Olds 'recording' model, Serial No.330753. This was about 15 years old and had received considerable use.

One mouthpiece was used with each instrument and the dimensions of these are listed below. Also listed are the measured volumes which the lips displace in the cups of the two mouthpieces. This was measured by attaching the mouthpiece to a length of perspex tubing which was stopped at the other end. The tube is half filled with water, the mouthpiece rim placed on a firmly held piece of rubber and the tube inverted. The water now shows the level of a volume including the mouthpiece cup and backbore. This is recorded and the system again inverted. Next a player places his lips over the rim and forms an embouchure as normal as can be expected. The tube is inverted and the difference in levels noted to give the cup volume displaced by the lips. This was repeated several times for several different embouchures, and the results were found to be very repeatable. There seems to be little change in the volume displaced as the player changes embouchure from a low to high register, but this probably dependent on the individual player.

<table>
<thead>
<tr>
<th>Mouthpiece type</th>
<th>Total cup volume (cc)</th>
<th>Diameter of throat (mm)</th>
<th>Length of throat from cup to measuring plane (mm)</th>
<th>Measured volume displaced by the lips (cc)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trombone</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dennis Wick</td>
<td>7.2 ±0.1</td>
<td>6.7 ±0.1</td>
<td>11.5 ±0.5</td>
<td>2.5 ±0.5</td>
</tr>
<tr>
<td>&quot;9BS&quot;</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Trumpet</td>
<td>1.2 ±0.1</td>
<td>4.2 ±0.1</td>
<td>11.0 ±0.5</td>
<td>0.3 ±0.2</td>
</tr>
<tr>
<td>Vincent Bach</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>&quot;1C&quot;</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
APPENDIX D

ON THE BEHAVIOUR OF THE IMPEDANCE NEAR RESONANCE

For a straight tube (Appendix A)

\[ Z_i = Z_0 \frac{A + j \tan(\beta \ell)}{1 + j \frac{A}{\delta} \tan(\beta \ell)} \]

where the symbols are defined in Appendix A.

At an impedance maximum \( \omega = \omega_i \), say

So \( \beta \ell = \frac{\omega_i \ell}{c} = n\pi + \frac{\pi}{2} \)

Near the resonance let \( \omega = \omega_i + \omega' \)

So \( \beta \ell = \frac{(\omega_i + \omega')\ell}{c} = n\pi + \frac{\pi}{2} + \delta \), say

where \( \delta = \frac{\omega'\ell}{c} = (\omega - \omega_i) \frac{\delta}{\ell} \)

So \( \tan(\beta \ell) = \tan\left(\frac{\pi}{2} + \delta\right) = -\cot \delta = -\frac{1}{\delta} \)

If \( \delta \ll 1 \)

\[ \frac{Z_i}{Z_0} = \frac{\Delta + \frac{j}{\delta}}{1 - j \frac{\Delta}{\delta}} = \frac{\delta \Delta - j}{\delta + j \Delta} \]

\[ = \frac{1}{\Delta + j \delta} \text{ if } \Delta \delta \ll 1 \]

This is a Lorentzian resonance curve with the following useful properties:

1. It can be shown that the Argand diagram of such a peak is a circle of radius \( \frac{1}{2\Delta} \) and centre \( (0, \frac{1}{2\Delta}) \).

2. The angle between the centre and any point can be shown to be \( \theta \), where:

\[ \tan \theta = -\frac{2 \Delta \delta}{\Delta^2 + \delta^2} \]
It has been found that the Argand diagram of the impedance peaks for the measured instruments is also circular to a good approximation. By modelling each peak as a Lorentzian resonance the frequency as a function of angle round the circle may be fitted to an equation similar to that above and the parameters \( \omega_1 \) and \( \Delta \) may be accurately determined. A computer program (SQPDRAW) has been written by Mr. D. Munro to derive these parameters from the Argand diagram and so determine the precise location and magnitude of each peak. An example of the Argand diagram and distribution of frequency with angle produced by this program for an impedance peak on the trombone is given in Fig.D.1. This method of locating the precise frequency of each peak has been published in an appendix to a recent paper (Pratt and Bowsher, 1979). For a single resonance peak

\[
Z_i = \frac{Z_0}{\Delta + j\delta} = \frac{Z_0}{\frac{c/\ell}{\alpha c + j(\omega - \omega_1)}},
\]

if \( \Delta = \alpha \ell \) (as shown in Appendix A). The impulse response can be calculated by taking the Fourier transform of this, which gives a response proportional to

\[
\exp(-\alpha t) \exp(-j \omega t) \quad (\text{Pippard, 1978})
\]

which is a damped sinusoid of period \( = 2\pi/\omega_1 \) and time constant \( = 1/\alpha c \).

Assuming that the energy in the system is proportional to the square of the response, the loss of energy per cycle must be

\[
\exp\left(-\frac{4\pi \alpha c}{W_i}\right) = 1 - \frac{4\pi \alpha c}{W_i},
\]

provided \( \frac{2\pi}{W_i} \ll \frac{1}{\alpha c} \)

And we are now in a position to calculate the 'Q' of the peak directly from the definition

\[
\frac{\text{Energy stored}}{\text{Energy dissipated per cycle}} \times 2\pi
\]
Fig.D.1  The left hand part of the figure shows the polygon defined by the original data points and the circle fitted by the interpolation procedure. The upper right part shows the original and fitted curves relating frequency to angle around the circle. The lower right part shows the distance (multiplied by 20) between the two curves in the upper right part.
If the energy stored is unity the energy lost per cycle must be
\[
\frac{4\pi ac}{\omega_i}
\]
\[
'Q' = \frac{2\pi \omega_i}{4\pi ac} = \frac{\omega_i}{2ac}
\]
This implies that the bandwidth of the resonance is 2ac, and we find that if \( \delta = \pm ac \)
\[
\frac{Z_i}{Z_0} = \frac{1}{ac (1 \pm j)}
\]
while at resonance \( \omega = \omega_i \) and
\[
\frac{Z_i}{Z_0} = \frac{1}{ac}
\]
So the bandwidth also corresponds to the 'half power points' in the normal definition of 'Q', which is used in SQPDRAW to obtain the value of the 'Q' for each impedance peak.
APPENDIX E

THE EFFECTS OF THE SOURCE AND REED IMPEDANCES

By allowing some flexibility in the definition of the alternating pressure used in the regeneration theory, the effects of both the 'source' impedance (section 3.2.1) and the 'reed' impedance (section 3.2.2) may be accounted for with the impedance of the instrument. Consider the 'circuit diagram' of Fig.E.1, the volume velocity from the constant pressure source ($P_s$) flows via the source impedance ($Z_s$), through the orifice formed by the lip opening (represented as a time-varying resistance, $R$). The impedance presented at this junction is the parallel combination of the impedance of the instrument ($Z_i$) and the reed impedance ($Z_r$). Let the alternating pressure across the source impedance by $P_m(t)$ and that across the instrument and reed impedance by $P_i(t)$. The flow equation now becomes

$$U(t) = \sqrt{\frac{2(P_s - (P_i(t) - P_m(t)))}{e}} \times x(t)$$

At any one frequency the reed will be driven by the alternating pressure difference across it so

$$x = G \left( P_i - P_m \right)$$

where $G$ is unchanged. Also we have

$$P_{im} = -U \frac{Z_s}{Z_i} \quad \text{and} \quad P_i = U \frac{Z_i}{Z_i + Z_r}$$

so

$$U = \frac{(P_i - P_m)/(Z_s + Z_i/Z_r)}{x(t)}$$

where $Z_i/Z_r$ represents $Z_i$ and $Z_r$ in parallel. This set of equations is exactly the same as those discussed in section 3.2 provided,
Fig. E.1  Diagram indicating the flow path from the lungs via opening formed by the player's lips into the instrument
1. the mouth pressure is considered constant at its average value,

2. instead of just considering the alternating pressure in the mouthpiece, the total alternating pressure is considered:

\[ P(t) = P_i(t) - P_m(t), \text{ say.} \]

3. the impedance used in the regeneration theory is the combination of all three impedances discussed above, i.e.

\[ Z = Z_s + \frac{Z_i}{Z_r}. \]

The limiting case considered by most authors assumes \( Z_s \ll Z_i \) and \( Z_r \gg Z_i \). So \( Z \Rightarrow Z_i \) and \( P(t) \Rightarrow P_i(t) \).
APPENDIX F

EFFECT OF THE MOUTHPIECE ON THE MAGNITUDE OF THE IMPEDANCE PEAKS OF THE INSTRUMENT

If the mouthpiece can be represented by a lumped compliance \( C \) in parallel with the series combination of a lumped inertance \( M \) and the impedance of the instrument \( Z_i \) (as in Fig. 2.15) the input impedance is given by

\[
Z = \frac{j\omega M + Z_i}{1 - \omega^2 MC + j\omega C Z_i}
\]

Suppose that the impedance of the instrument is equal to the characteristic impedance at the 'popping' frequency, i.e.

\[
Z_i = Z_0 \text{ at } \omega = \omega_p = \frac{1}{\sqrt{MC}}
\]

then the input impedance will be approximately equal to its real part at this frequency

\[
Z = \frac{M}{C Z_0} \text{ provided } \frac{\sqrt{M}}{C} \gg Z_0
\]

Let this = \( N Z_0 \) where \( N = \frac{M}{C Z_0^2} \)

There will be a slight change in the Helmholtz resonance frequency because of \( Z_0 \), but the magnitude of \( Z \) will remain the same as that deduced above, provided

\[
N \gg 1
\]

The instrument will have a maximum and minimum of impedance at frequencies near to \( \omega = \omega_p \) and near to these frequencies the reactance of the
instrument will combine with those of the mouthpiece to give other resonances. So the frequencies of the impedance peaks of the instrument will be shifted, but it is the magnitude of these peaks that we are concerned with here. Near a minimum of the instrument's impedance \( Z_i = \Delta Z_0 \) (Appendix A)

\[
Z = \frac{M}{C \Delta Z_0} = N \frac{Z_0}{\Delta}
\]

Also, near a maximum of the instrument's impedance \( Z_i = \frac{Z_0}{\Delta} \) (Appendix A)

\[
Z = \frac{M \Delta}{C Z_0} = N \Delta Z_0
\]

so an impedance minimum is converted to a maximum by the parallel resonance of the mouthpiece and an impedance maximum is converted to a minimum. Moreover the magnitudes of the impedances in this region are all higher than those for the instrument alone by a factor of \( N \).

This amplification of all the impedances can be thought of as an increase in the characteristic impedance from \( Z_0 \) to \( N Z_0 \). Amplification of the impedance by the mouthpiece in this way has been discussed by Backus (1976) and Benade (1976), both authors using the trumpet as an example. The implication (particularly in Benade 1976) is that this is an effect applicable to all brass instruments and so it was surprising to find that the impedance of the trombone and mouthpiece (Fig.2.16) does not show the pronounced peaks of the trumpet and mouthpiece (Fig.2.17). It has been shown that the relative sizes of the cup and throat reactances at the popping frequency, compared to the characteristic impedance of the instrument determines the magnitude of this amplification and we have expressed this in the numerical term

\[
N = \frac{M}{C Z_0^2}
\]
If the magnitudes of the inertance, compliance and characteristic impedance are worked out and used to determine $N$ for the trumpet, its value is found to be about 7, whereas the value of $N$ from the parameters of a trombone is only about 0.6. So we can deduce that the proportionally larger cup volume and backbore of the trombone lead to changes in the magnitude of instrument impedance which are far less than for the trumpet. Indeed, since $N$ is so small in the case of the trombone the analysis above is invalid, and the effect of the mouthpiece on the instrument's impedance can no longer be predicted by the simple formulae of Benade (1976).
Note that J.A.S.A. refers to the Journal of the Acoustical Society of America

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