STRESSES IN THE CROSS SECTION
OF COMPOSITES
REINFORCED BY UNIDIRECTIONAL FIBRES

by

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Summary

Difficulties in obtaining satisfactory interface bonds in photoelastic models of fibre reinforced materials have focussed attention upon the importance of interface bond strength in determining the failure characteristics of a composite material. In order to simulate the complex stress conditions which exist at the interface in both two and three dimensional models, an apparatus was constructed in which cemented joints were tested to failure under predetermined combinations of tensile and shearing stresses.

The strengths of a limited number of adhesives were investigated by making butt joints between the ends of cylindrical test pieces of dural and epoxy resin, these being the materials used to simulate the fibre and matrix in the model material. Initially a high scatter was observed in the results of strength tests and was only reduced to an acceptable level by adopting a chemical cleansing process for the bonding surfaces of the dural test pieces.

The liability of unidirectionally reinforced composites to failure by the propagation of fractures in planes containing the fibre axes has drawn the attention to the study of stress concentrations associated with the diffusion of load around an array of high modulus circular inclusions embedded in an elastic matrix. As a preliminary to the array arrangements, the stresses around inclusion were investigated using both unbonded and bonded interference fit plug configurations.

The elastic stress distribution associated with the differential thermal shrinkage may be shown to be exactly equivalent to that produced by a suitable initial interference between the fibre and the matrix. This equality has been used in fabricating two dimensional photoelastic models of a regular array of circular inclusions. The effect of initial shrinkage, and additional uniaxial and biaxial loads have been studied for a range of plug sizes representing the fibre volume fractions between 22 and 73 per cent.
Tests on cemented joints showed that chemical cleaning was required in order to obtain consistent strength values. An empirical curve defining the fracture envelope for joints subjected to combined tension and shear loading has been obtained.

Tests on idealised two dimensional models indicate that the critical stresses produced in interference loading can be predicted by a simple theory. Additional tensile load in a plane perpendicular to the fibre axis induces interference slip. The actual load which initiates slip is proportional to the initial interference for a given combination of a fibre and matrix material. The slip process is completed with a small increment in load. Subsequently the linear response to additional load is restored. Essentially similar behaviour was observed in models which incorporate bonded and unbonded inclusions.
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<table>
<thead>
<tr>
<th>LIST OF CONTENTS</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>CHAPTER 1. REVIEW OF PREVIOUS WORK AND SCOPE OF PRESENT INVESTIGATION</strong></td>
<td></td>
</tr>
<tr>
<td>1.1. Introduction</td>
<td>9</td>
</tr>
<tr>
<td>1.2. Mechanical Behaviour of Composites</td>
<td></td>
</tr>
<tr>
<td>1.2.1. Theoretical and Experimental Investigations for Unidirectional Systems</td>
<td>10</td>
</tr>
<tr>
<td>1.2.2. Effect of Fibre Alignment</td>
<td>15</td>
</tr>
<tr>
<td>1.2.3. Effect of Inclusion in Solids of Infinite Dimensions</td>
<td>16</td>
</tr>
<tr>
<td>1.2.4. Disturbance caused by Isolated Pin in an Infinite Plate in Uniaxial Tension</td>
<td>17</td>
</tr>
<tr>
<td>1.3. Problems of Deformation Behaviour of Multi-Fibre Array under Transverse Load</td>
<td>20</td>
</tr>
<tr>
<td>1.4. Failure of Composites</td>
<td>24</td>
</tr>
<tr>
<td>1.4.1. Failure Initiation and Crack Propagation</td>
<td>24</td>
</tr>
<tr>
<td>1.4.2. Problem of Interface and Strength of Interface Bonds under Complex Stress System</td>
<td>26</td>
</tr>
<tr>
<td>1.5. Purpose of Present Investigation</td>
<td>32</td>
</tr>
<tr>
<td>1.6. Choice of Constituent Materials for Composite Models</td>
<td>33</td>
</tr>
<tr>
<td><strong>CHAPTER 2. STRENGTH OF INTERFACE BONDS SUBJECTED TO COMPLEX STATES OF STRESS</strong></td>
<td></td>
</tr>
<tr>
<td>2.1. Introduction</td>
<td>38</td>
</tr>
<tr>
<td>2.2. Test Apparatus</td>
<td>40</td>
</tr>
<tr>
<td>2.3. Test Adhesives</td>
<td>41</td>
</tr>
<tr>
<td>Section</td>
<td>Title</td>
</tr>
<tr>
<td>---------</td>
<td>-----------------------------------------------------------------------</td>
</tr>
<tr>
<td>2.4.</td>
<td>Test Specimens</td>
</tr>
<tr>
<td>2.5.</td>
<td>Test Condition at Interface</td>
</tr>
<tr>
<td>2.6.</td>
<td>Preliminary Test Procedure</td>
</tr>
<tr>
<td>2.6.1.</td>
<td>EASTMAN 910 Adhesive</td>
</tr>
<tr>
<td>2.6.2.</td>
<td>EASTMAN 910 (Selected Lot) with W. T. Bean Catalyst</td>
</tr>
<tr>
<td>2.6.3.</td>
<td>Araldite AY 103 with Hardener HY 991</td>
</tr>
<tr>
<td>2.6.4.</td>
<td>Chemical Treatment of Dural Surfaces</td>
</tr>
<tr>
<td>2.7.</td>
<td>Results</td>
</tr>
<tr>
<td>2.8.</td>
<td>Conclusions</td>
</tr>
</tbody>
</table>

**CHAPTER 3. THE DIFFUSION OF UNIDIRECTIONAL TENSION AROUND A CYLINDRICAL HIGH MODULUS INCLUSION EMBEDDED IN AN ELASTIC MATRIX**

<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.1.</td>
<td>Introduction</td>
<td>51</td>
</tr>
<tr>
<td>3.2.</td>
<td>Preparation of Photoelastic Models</td>
<td>52</td>
</tr>
<tr>
<td>3.3.</td>
<td>Test Procedure</td>
<td>54</td>
</tr>
<tr>
<td>3.4.</td>
<td>Method of Analysis</td>
<td>54</td>
</tr>
<tr>
<td>3.5.</td>
<td>Results</td>
<td>55</td>
</tr>
<tr>
<td>3.5.1.</td>
<td>Effect of Initial Interference</td>
<td>55</td>
</tr>
<tr>
<td>3.5.2.</td>
<td>Effect of Additional Uniaxial Load before Slip</td>
<td>56</td>
</tr>
<tr>
<td>3.5.3.</td>
<td>Slip Mechanism</td>
<td>57</td>
</tr>
<tr>
<td>3.5.4.</td>
<td>Effect of Additional Uniaxial Load after Slip</td>
<td>60</td>
</tr>
<tr>
<td>3.5.5.</td>
<td>Plate with Bonded Plug</td>
<td>60</td>
</tr>
<tr>
<td>3.5.6.</td>
<td>Effect of Uniaxial Load Alone</td>
<td>62</td>
</tr>
</tbody>
</table>
### CHAPTER 4. STRESS DISTRIBUTION IN THE MATRIX

**SURROUNDING A REGULAR ARRAY OF INTERFERENCE FIT PLUGS**

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.1. Introduction</td>
<td>65</td>
</tr>
<tr>
<td>4.2. Manufacture and Storing of Models</td>
<td>67</td>
</tr>
<tr>
<td>4.3. Observations of Optical Patterns</td>
<td>70</td>
</tr>
<tr>
<td>4.3.1. Oblique Incidence Technique</td>
<td>70</td>
</tr>
<tr>
<td>4.3.2. Apparatus for Measuring Oblique Incidence Fringe Numbers</td>
<td>71</td>
</tr>
<tr>
<td>4.4. Analysis of Stress Distribution</td>
<td>74</td>
</tr>
<tr>
<td>4.5. Discussion Including Comparison with Theory</td>
<td>78</td>
</tr>
<tr>
<td>4.5.1. Explanation for Introducing Slots in Plates with 0.8 and 0.9 inch Plugs</td>
<td>79</td>
</tr>
<tr>
<td>4.5.2. Application of Lamé Thick Cylinder Formulae under Extreme Boundary Conditions</td>
<td>83</td>
</tr>
<tr>
<td>4.6. Summary of Important Features of an Array of Interference Fit Plugs</td>
<td>84</td>
</tr>
</tbody>
</table>

### CHAPTER 5. STRESSES IN THE CROSS SECTION OF COMPOSITES LOADED PERPENDICULAR TO THE AXIS OF UNIDIRECTIONAL FIBRES

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.1. Slip Phenomenon</td>
<td>89</td>
</tr>
<tr>
<td>5.2. Importance of Load before Slip</td>
<td>89</td>
</tr>
<tr>
<td>5.3. Application of Load</td>
<td>91</td>
</tr>
<tr>
<td>5.3.1. Choice of Load</td>
<td>91</td>
</tr>
<tr>
<td>5.3.2. Checking for Slip</td>
<td>91</td>
</tr>
<tr>
<td>5.3.3. Reducing the Effect of Creep on Observations</td>
<td>93</td>
</tr>
<tr>
<td>5.3.4. Scaling Procedure to Allow for Error in Initial Interference</td>
<td>93</td>
</tr>
<tr>
<td>Section</td>
<td>Title</td>
</tr>
<tr>
<td>---------</td>
<td>-----------------------------------------------------------------------</td>
</tr>
<tr>
<td>5.4.</td>
<td>Average Ligament Stress Related to Constant Ligament Tension</td>
</tr>
<tr>
<td>5.5.</td>
<td>Effect of Introducing Slots on Mean Ligament Stress</td>
</tr>
<tr>
<td>5.6.</td>
<td>Presentation of Experimental Data</td>
</tr>
<tr>
<td>5.6.1.</td>
<td>Nominal Stress in the Unpierced Plate</td>
</tr>
<tr>
<td>5.6.2.</td>
<td>Mean Ligament Tension in the Unfilled Model</td>
</tr>
<tr>
<td>5.6.3.</td>
<td>Mean Ligament Tension for Uniform Strain at Centre Line of the Plugs</td>
</tr>
<tr>
<td>5.6.4.</td>
<td>Average Stress on the Centre Line of the Ligament</td>
</tr>
<tr>
<td>5.7.</td>
<td>Discussion of Results</td>
</tr>
<tr>
<td>5.7.1.</td>
<td>Effect of Uniaxial Load Alone</td>
</tr>
<tr>
<td>5.7.2.</td>
<td>Effect of Biaxial System of Stress</td>
</tr>
<tr>
<td>5.8.</td>
<td>Conclusions</td>
</tr>
<tr>
<td>6.1.</td>
<td>Introduction</td>
</tr>
<tr>
<td>6.2.</td>
<td>Pretreatment of Interfaces and the Failure Curves for Cemented Joints Subjected to Complex Stresses</td>
</tr>
<tr>
<td>6.3.</td>
<td>Stresses in the Cross Section of Composites Reinforced by Unidirectional Fibres</td>
</tr>
<tr>
<td>6.3.1.</td>
<td>Isolated Circular Inclusion in an Infinite Matrix</td>
</tr>
<tr>
<td>6.3.2.</td>
<td>A Regular Array of Interference Fit Plugs</td>
</tr>
</tbody>
</table>

CHAPTER 6. GENERAL DISCUSSION AND SUGGESTIONS FOR FUTURE WORK

<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.1.</td>
<td>Introduction</td>
<td>110</td>
</tr>
<tr>
<td>6.2.</td>
<td>Pretreatment of Interfaces and the Failure Curves for Cemented Joints Subjected to Complex Stresses</td>
<td>112</td>
</tr>
<tr>
<td>6.3.</td>
<td>Stresses in the Cross Section of Composites Reinforced by Unidirectional Fibres</td>
<td>114</td>
</tr>
<tr>
<td>6.3.1.</td>
<td>Isolated Circular Inclusion in an Infinite Matrix</td>
<td>114</td>
</tr>
<tr>
<td>6.3.2.</td>
<td>A Regular Array of Interference Fit Plugs</td>
<td>117</td>
</tr>
</tbody>
</table>

GRAPHICS AND DRAWINGS

<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>123</td>
</tr>
<tr>
<td>References</td>
<td>Page</td>
<td></td>
</tr>
<tr>
<td>---------------</td>
<td>------</td>
<td></td>
</tr>
<tr>
<td>REFERENCES</td>
<td>171</td>
<td></td>
</tr>
<tr>
<td>APPENDIX A</td>
<td>175</td>
<td></td>
</tr>
<tr>
<td>APPENDIX B</td>
<td>177</td>
<td></td>
</tr>
<tr>
<td>APPENDIX C</td>
<td>178</td>
<td></td>
</tr>
<tr>
<td>APPENDIX D</td>
<td>179</td>
<td></td>
</tr>
<tr>
<td>APPENDIX E</td>
<td>185</td>
<td></td>
</tr>
</tbody>
</table>
1. Review of Previous Work and Scope of Present Investigation

1.1. Introduction

The properties of composites have generally been determined empirically by means of tests on macroscopic samples. It follows that many questions concerning the mechanical and chemical interactions occurring at the interface between the reinforcing fibres and the surrounding matrix still remain unanswered. Previous investigations of the effect of fibre shape, size, volume fraction and fibre orientation on the stress distribution in the matrix have emphasised the importance of these parameters. The presence of stress concentrations, especially in a high-strength thin walled structure, could become the cause of catastrophic failure.

In recent years considerable attention has been paid to the study of fibre reinforced composites mainly as a part of search for materials of low specific weight, high strength and stiffness. In order to achieve the most efficient composite it is important to understand the deformation behaviour of the matrix. The widespread use of unidirectional
composites makes the investigation of this configura-
tion especially valuable.

In a typical fibre reinforced plastic, there are three types of interaction stresses affecting the strength of the composite:

(a) Residual stresses due to shrinkage of the plastic around the fibre during the curing process.

(b) Thermal stresses produced by the differential coefficients of expansion when the composite is subjected to changes in temperature.

(c) Stresses produced by virtue of difference in elastic properties of the component materials when the composite is under the effect of externally applied mechanical loading.

1.2. MECHANICAL BEHAVIOUR OF COMPOSITES:

1.2.1. Theoretical and Experimental Investigations for Unidirectional Systems

Although fibre-reinforced materials have been in use, in one form or another, for many years, it is only recently that much attention has been focussed on the determination of stresses and displacements in such composites. All fibre-reinforced materials function by the transfer of load between the adjacent fibres via the intervening matrix. Since this transfer occurs by shear tractions developed at the fibre-matrix
interface, an accurate knowledge of the stresses at this boundary is essential for the efficient design of composite materials.

Continuous fibres are readily analysed by simple mechanics. However, discontinuous fibres, which are far more widely used, are extremely difficult to solve theoretically due to the discontinuities at the fibre ends. In all the analytical procedures developed, the simplicity of the assumed models has led to large discrepancies when compared with experimental results (Cox 1952, Dow 1963, Tyson & Davies 1965, etc.). This is not surprising as the degree of model idealisation needed to allow analysis is often such that discontinuous effects at the ends of the fibre are neglected.

Several authors have outlined the behaviour of unidirectionally reinforced composites which are subjected to uniaxial tension and have attempted to predict the failure mechanism from these theories.

In a composite material it is usually assumed that the load is carried primarily by the fibres, the matrix serving only to locate and protect the fibres and to transfer the load from one fibre to another. Riley and Reddaway (1968) further assumed that:

(a) all the fibres are of same dimensions,
(b) the fibre and matrix do not react chemically and
(c) a perfect bond was obtained at the fibre-matrix interface.
The assumptions made, by Cox (1952) in deriving the first theoretical solution for determining longitudinal and interfacial-shear stress distributions in matrix, are as follows:

(a) The load transfer between the fibre and the matrix, at any point on the interface, will depend upon the actual displacement and the displacement which would occur if the fibre was absent.

(b) The load is only transferred along the length of the fibre and no load is carried by the fibre-tips.

(c) The effect of the intensification of stress at the fibre ends is neglected.

The theory presented by Cox (1952) can be applied to those fibrous materials which derive their strength and stiffness wholly or mainly from thin fibres capable of transmitting high loads along their lengths but offer no great resistance to loads transmitted perpendicular to their length.

Dow (1963) obtained a similar theoretical solution, for both matrix and fibre in the elastic state, but further assumed that:

(a) straight lines before deformation remain straight after deformation and

(b) the fibre bonded to the matrix along its sides only.
Using photoelastic models Tyson and Davies (1965) confirmed:

(a) the development of high shear stresses near the fibre ends

and (b) the load transfer through the fibre tips becomes important when the end gap is in close proximity.

In deriving stresses analytically Rosen (1964) assumed that:

(a) the matrix carried shear stresses only,

(b) the fibre only sustained tensile stress

and (c) there was a perfect bond between the fibre and the matrix at the interface.

Kelly (1964) presented a theory, for a ductile matrix and fibres, by assuming the stress distribution along the fibre matrix interface at failure of the form as shown in figure 1.1. He suggested that the stress concentration, close to the fibre tip, invokes the yielding of matrix and results in a constant shear stress given by the Tresca's yield criteria.

It must be pointed out that the derivations obtained by Kelly are only suitable for idealised hypothetical cases. In actual practice it would be impossible to obtain:

(a) flawless fibres which would fracture simultaneously,

(b) identical length of all the fibres
and (c) orientation of all the fibres into the direction of the applied load.

Allison and Hollaway (1967) showed the magnitude of the error involved when the stress concentration around the fibre tips is ignored, by comparing it with the theoretical values obtained at corresponding points from the Cox's theoretical solution. Considerably higher shear stresses were experimentally obtained in the vicinity of the fibre end and matched with theoretical values approximately after four fibre widths from the tip.

Carrera and McGarry (1968), employing the finite element method to solve the more practical problem of a cylindrical glass fibre embedded in resin matrix showed that Dow's assumption which is 'straight lines before deformation remain straight after deformation' is only valid at a considerable distance from the fibre tip. However, in the vicinity of the fibre tip the straight lines become curvilinear under the influence of external load.

Allison and Hollaway (1967) showed the effect of stress concentration in the region of the fibre tip by considering curvature of the stress trajectories in the diagram which is reproduced in figure 1.2.

A comprehensive list of analytical solutions available has been prepared by Holister and Thomas (1966).
Most of the work which has been carried out on unidirectionally reinforced discontinuous fibres is primarily theoretical. A number of assumptions are made in these investigations thus producing results which are considerably different from the values obtained experimentally.

1.2.2. Effect of Fibre Alignment:

For most continuous fibre composite applications the fibres will be aligned parallel to the load direction. However, many discontinuously reinforced composites will contain randomly orientated fibres, and this effect of fibre orientation has been studied by Sutton (1962). He concluded that the crux of using the full reinforcing potential of fibres lies in the careful orientation, wetting and bonding of fibres in the matrix.

In order to achieve an indication as to the effect of non-axially aligned fibres, Schuster and Scala (1964) prepared some standard tension specimens having the long axis of the whisker aligned perpendicular to the load. These specimens were stressed in tension while being viewed directly through the microscope in plane-polarized light. No significant stress concentrations were observed around the whisker which was orientated in this case at right angles to the direction of the load. When the load was applied parallel to the axis of the fibre in another tension specimens, the stress had a peak
value occurring in the fibre-matrix interface approximately one-half of a whisker diameter from the tip. The authors concluded that whiskers orientated perpendicular to the load direction, although not acting as reinforcing agents, did not cause appreciable stress concentration to occur in the matrix.

The dimensions of the fibre in the matrix being significantly small in these models would have made it difficult to assess the amount stress concentrations in the vicinity of the fibre tips.

4.2.3. Effect of Inclusion in Solids of Infinite Dimensions:

For the purpose of calculations it is assumed that the solid surrounding the inclusion is infinite in extent, and subjected to a uniform stress at infinity, besides having the ideal properties of elasticity, isotropy and homogeneity.

From St. Venant's principal the effect of disturbance would be confined to the neighbourhood of the inclusion. Goodier (1933) found theoretically that at a distance of approximately four diameters away from the inclusion the stress distribution was in no case modified by more than about one per cent. Thus the results would be valid for inclusions which are more than:

(a) four diameters from the boundary

and (b) eight diameters between the inclusions.
In general this would be so if the inclusion:

(a) is small in comparison with the overall dimensions of the body and
(b) is very large when compared with the grain size of the crystalline matrix.

Goodier concluded that the intensification of the applied stress due to a small inclusion not near a boundary depends on its geometric form, but not on the absolute size of the inclusion. He pointed out that the effects of non-metallic inclusions in metals, as a measure of fatigue strength are not very definite. He also suggested that there are instances where the endurance limit is only about one-half of, and others where it is somewhat higher than that of unreinforced matrix. He found that the matrix containing only strong inclusions would have smaller stress concentrations than one containing cavities and the stresses gave no indication but weakening effects due to stress concentrations from any kind of inclusion. These results were confirmed in the present investigation.

1.2.4. Disturbance caused by Isolated Pin in an Infinite Plate in Uniaxial Tension:

Jessop, Snell and Holister (1958) demonstrated, by means of photoelastic observations, the benefits derived by the use of an interference fit between the pin-jointed connection. In the case of alternating loads
they showed that an initial interference between the pin and the plate reduces the increment in the maximum shear stress, which increases in its absolute value, on transverse hole boundary. That is, when pulsating loads are applied, the presence of the initial interference will reduce the alternating component of the maximum shear stress in the plate at the expense of an increase in the shear stress at critical points on the hole boundary. For a given pulsating load, the magnitude of the alternating component of the maximum shear stress falls rapidly, with the increase in initial interference, and gradually levels off at higher interference values.

An investigation of the fatigue characteristics of most of the materials shows that the fatigue life is generally more sensitive to changes in alternating component of the stress than to the changes in mean stress level. Low (1958) suggested that fatigue life of joint would be increased by the introduction of an interference between the pins and the joints and further, an optimum degree of interference exists, for which the fatigue life is maximum, was found by Lambert (1961).

An interesting result obtained experimentally by Jessop, Snell and Holister is shown in figure 1.3 by the two widely different but intersecting slopes of the identical phenomenon for different interferences between the pin and the plate. These tension tests were per-
formed for a ratio of pin diameter to plate width of 3/8 and with 0.3, 0.7 and 1.0 per cent interferences.

An examination of this curve indicates that when the ratio of initial interference shear stress to the applied uniaxial tension is greater than 2.5 the increment in maximum shear stress, on the transverse diameter due to load, is proportional to the applied load stress, and becomes a non-linear relation below the value of 2.5 of the ratio of maximum shear stress due to interference alone divided by the nominal mean tensile stress.

The two distinct slopes of the curve in figure 1.3 are as a result of slipping at the interface and the load at which slip would occur is proportional to the amount of the initial interference.

Holland and Stevenson (1933) derived a mathematical theory which involves the use of a digital computer of fairly large core-store or a tedious and lengthy hand treatment, for calculating the stresses in the plate with interference fit pin, and the basic assumption made was that there is no movement between the pin and the plate at the interface. They concluded that the stress distribution would be linear to the applied load.
Problems of Deformation Behaviour of Multi-Fibre Array under Transverse Load:

The stresses in the cross-section perpendicular to the axis of unidirectional fibres become of greater importance when the loads act in a direction normal to the fibres, since in this case the initiation of failure may be governed by stress and strain concentrations in the plane perpendicular to the fibres.

The review of the literature shows that most of the work done in this field is theoretical. Very limited related experimental work has been carried out and primarily consists of photoelastic models used for obtaining shear stresses at the fibre-matrix interface without making a detailed study of separate principal stresses in the matrix.

Daniel and Purelli (1962) investigated experimentally the shrinkage stresses in resins during the curing cycle by embedding a transducer with properly calibrated electrical-resistance strain gages. They also studied (1961) the shrinkage pressure developed in the matrix, by casting plates of Paraplex P-43 around circular glass inclusions and analysed the resulting isochromatic fringe patterns around the inclusions. This investigation indicated that stress field obtained around the inclusion corresponds to thermal stress field produced by a difference in the coefficients of expansion.
of the matrix and the inclusion material. These authors have concentrated on the case of two inclusions and have only presented illustrations of the multi-plug triangular array models without conducting the detailed stress separations in the matrix.

Kies (1962) used a very simple analysis for evaluating average strain magnification in the ligament of the matrix containing unidirectionally reinforced fibres, forming a square array, and subjected to loads transverse to the fibre direction. He also noticed craze cracking in glass-fibre-reinforced plastics. Kies suggested that the average strain in the resin in a direction transverse to the fibres can be approximately 40 per cent and concluded that no resin in ordinary structural use can stand this strain without cracking.

Herrman and Pister developed a three-dimensional analysis for determining the properties of the composite from the elastic properties of the matrix and the fibres which formed a square array of unidirectionally aligned rods. The restrictions placed on this system, which was subjected to loads normal to fibres, were:

(a) a square packing of fibres,
(b) the deformations were infinitesimally small,
(c) the linear elastic behaviour of the constituents

and
(d) the continuity between the filaments and the resin.
Adams and Doner (1967) followed a finite-difference procedure to develop theoretical solutions of the rectangular array of plugs under transverse loading.

Chen and Cheng (1967) obtained theoretical expressions, by means of classical theory of elasticity, for determining the composite elastic constants for fibre reinforced plastics in terms of elastic moduli and geometric parameters of the fibres forming the triangular array reinforcement.

Marloff and Daniel (1969) conducted a three-dimensional photoelastic analysis using stress-freezing technique for determining the stresses in the matrix of a unidirectionally reinforced composite model subjected to matrix shrinkage and normal transverse loading. This model consisted of a square array of polycarbonate rods in cold setting epoxy resin with a fibre volume fraction of 0.5. The effects of shrinkage were separated from those of external loading by analysing two identical models out of which only one was loaded while the other remained unloaded.

In order to obtain the values of the separate stresses at the starting point of the integration Marloff and Daniel derived the analysis in which a basically incorrect assumption was made for the fibres which are under the influence of shrinkage stresses,
that is,
\[ \int_{o}^{a} \sigma_x \, dy = 0 \]

where \( o \) and \( a \) are the midpoint of the ligament and the point at the interface respectively as shown in the reproduced figure 1.4. To obtain separate values of the principal stresses along the axis of symmetry they used the Lame' Maxwell equilibrium equation.

Javornicky' (1970) investigated the effect of shrinkage stresses in an inclusion and the matrix by manufacturing both from epoxy resin and utilising the hot and cold setting processes. He concluded that a careful and properly controlled hardening process of epoxy resins can decisively eliminate the appearance of shrinkage stresses, but a certain birefringence effect cannot be avoided. He also studied the shear stress developed around three adjacent inclusions which were loaded parallel and transverse directions to the line connecting the centres of the inclusions. He found that the maximum shear stresses were at the boundaries of the inclusions in plates under compressive loading and showed that the distance between inclusions did not fundamentally change the character of the internal state of the stress in an isolated array within the matrix of the plate.

Most of the available information for obtaining stresses around inclusions is theoretical. Some
experimental work which has been carried out in this field consists primarily of stress concentrations at the interface. The problem of diffusion of load and detailed separation of principal stresses in the matrix around inclusions, which represent a cross-section of unidirectional fibres under transverse load, has been neglected. This is due to the difficulty of manufacturing an adequate photoelastic model which would simulate the behaviour of a real composite.

1.4. FAILURE OF COMPOSITES:

1.4.1. Failure Initiation and Crack Propagation

The structural behaviour of a composite material is intimately related to the internal stress level and distribution of load between the fibres and the matrix. The field of micromechanics comprises the study of the internal reactions between the constituent parts due to external forces. Knowledge of the internal stresses serves two main purposes;

(a) these stresses could be utilised in the evaluation of average stress (macroscopic) which forms a useful purpose in the design of efficient structures and

(b) forms the basis for the study of failure modes and establishing failure criteria.

It has been observed that composites reinforced by combined longitudinal and transverse fibres, can fail
by the development of cracks which propagate in planes containing the transverse fibre axes. The low fatigue life observed (Owen, Holbecke & Zienkiewicz 1969) in certain polyester matrices reinforced by woven glass cloth can be attributed to this mode of failure. In practical composites, reinforced by unidirectional fibres similar failure initiation can be produced by parasitic strains or stresses associated with the transverse deformations which are almost inevitably imposed in real loading situations. The importance of this mode of failure, which could provide a barrier to the exploitation of the more desirable properties of the fibre reinforced composites, highlights the need for an investigation of the stress concentrations associated with load passing an array of high modulus cylindrical inclusions. Once this information is available it will be still necessary to introduce an appropriate failure criterion when discussing the possibility of a particular form of fracture initiation. Correct interpretation of the complex stress distribution at the interface implies a knowledge of the interface failure criteria and this involves the collection of further empirical data.
Because of the difficulties of obtaining satisfactory interface bonds in photoelastic models of fibre reinforced materials attention is first focussed upon the interface as a potential source of weakness in real composites. The information obtained about the failure initiation at interfaces subjected to combined stresses is not particular to the fibre-reinforced configurations but can be applied generally to all interface situations which involve combined loading.

In a composite which is reinforced by discontinuous fibres, transfer of the load from one fibre to another is accompanied by the development of a complex stress distribution in the matrix, particularly in the vicinity of the fibre-matrix interface. Even in an ideal situation where the fibres are fully bonded to the matrix, the interface will be subjected to widely varying conditions of shear and normal pressure at different locations. Fracture at the interface will be initiated at a point where the combination of interface shear and tensile stress reaches a critical value. It was therefore decided to undertake a detailed study of the nature of the fibre-matrix interface in order to provide information which might lead to an understanding of the load transfer characteristics and could be of assistance in the development of structurally efficient composites.
The conditions governing interface bonds are not well understood on a macroscopic scale. Although some literature is available on intermolecular attraction at an atomic scale this is of little practical value in predicting the strength of a real cemented joints.

The strength of an interface may be determined by a number of different mechanisms. These include chemical, physical and mechanical bonding. In chemical bonding ionic or covalent bonds are formed between the constituent materials of the composite, that is, certain elements in the polymer chain chemically react with the fibre material to form a chemical bond. Sometimes an additional material is introduced, such as an adhesive layer, one part of which reacts with, or is compatible with, the polymeric matrix while the other part reacts with the fibre.

Two macroscopic objects in contact develop attractive forces which originate at a molecular level and can often exert their influence over distances which are quite large when compared with the relevant molecular dimensions. Van der Waals' (1873) was one of the first to explain the properties of real gases by introducing the constant "a" which allows for the attraction between the molecules of the gas and is universally known as the van der Waals' constant of attraction. These forces of attraction obtained by
Keeson, Debye and London (1930) fall off inversely as the seventh power of the distance and clearly such forces will be infinitesimally small in the bonded joint. However, London also concluded that simultaneous interaction of many molecules can be built up as an additive superimposition of single forces between the pairs. The analysis suggests that the attractive forces between two flat plates, in vacuum, can be obtained by integration of the interaction between all the molecules in one plate with all the molecules in the other plate at a distance \( r \), and leads to the following expression for the forces of attraction per unit area of the plate:

\[
F = \frac{A}{6r^3}
\]

where \( A \) is often known as the Hamaker constant. It has been assumed that the distance between the plates is small in comparison to their thickness. Thus the van der Waals' force of attraction over a comparatively long range vary inversely as the cube of the distance between the plates. The literature reveals that no suitable solution exists to obtain the inter-molecular force of attraction between the fibre and the matrix in a composite material.

In mechanical bonding the shrinkage of the resin around the fibre during the curing process is important.
since this provides the possibility of development of significant frictional forces which will resist relative movement at the interface when additional external loads are applied. The mechanical keying between the fibre and matrix resulting from the surface asperities increases the effect of interface friction which would be expected to be dependent on the normal stress, $\sigma_{nn}$.

The nature of interface friction is very complex and not completely understood. However an empirical law which describes the macroscopic behaviour may be applied using coefficients which have been determined experimentally.

Coulomb (1785) on a macroscopic scale found that the friction between two dry surfaces is:

(a) directly proportional to the normal force,

(b) independent of the area of contact

and

(c) depends on the mechanical properties of the material in contact.

Thus expressing mathematically,

$$F = \mu \cdot R$$

where $\mu$ is the coefficient of friction and $R$ is the normal reaction between the two surfaces in contact.
This may be expressed in a general form as:

$$\int_A \sigma_{nt} \cdot dA \leq \mu \int_A \sigma_{nn} \cdot dA \quad \ldots \ldots \ (1.1)$$

where $A$ is the area of contact, and $\sigma_{nt}$ and $\sigma_{nn}$ are the tangential and normal components of the surface traction respectively. The equation (1.1) may be written as,

$$\sigma_{nt} = \mu \cdot \sigma_{nn}$$

Although the coefficient of friction, $\mu$, is treated as constant, in these equations, in practice it may take one of the two values depending upon whether or not there is relative motion between the pair of surfaces. Thus $\mu_s$ and $\mu_k$ could be the coefficients of static and kinetic friction respectively and equation (1.1) is inequality for $\mu_s$ and equality for $\mu_k$. In general,

$$\mu_k < \mu_s$$

and $\mu_k$ is independent of the relative tangential velocity provided this is fairly low.
The success of adhesive bonding of structural elements in aircraft and spacecraft design, as well as in shipbuilding, in building and bridge construction has increased the interest in the mechanics of the adhesive joint. Most common adhesive joint is a lap joint and is simplest to prepare. However, this joint has a number of drawbacks such as the appearance of bending moments due to the sheet axis displacement, and of stress concentrations at the re-entrant corners. These effects prevent the full utilisation of the adhesive shear strength, and makes this type of joint unsuitable as a tool for testing adhesives. The advantage of bevelling the ends of the adherends has been demonstrated by Mylonas and de Bruyne (1951), while Coker and Filon (1957) employed photoelastic techniques to study the stress distribution in somewhat similar models consisting of straps joined by simple and joggled butt-welds. Such studies indicate various ways for improving the performance of lap joints.

A survey of the literature would be incomplete without mentioning the important photoelastic work by Mylonas (1955) on the stress distribution within the glue line. Lerchenthal (1964) investigated photoelastically the shape of the sheet edges which produced as far as possible a uniform shear stress distribution in the direction of the glue line at the interface. It was observed
that the stress concentration at the re-entrant edges of the conventional square-cut edges may reach 3.6 times the value of the average tangential shear stress at the interface, the improved design resulted in an approximately smooth shear stress distribution, which rises not more than 18 per cent above the average value.

Most of the theoretical work on adhesive bonding in the past has been carried out on lap joints with particular attention focussed on the problem of the elastic failure of scarf joints (Lubkin 1956).

1.5. PURPOSE OF PRESENT INVESTIGATION:

Although considerable effort has been expanded in the past in studying the properties of composites which are loaded uniaxially, the case of transverse loading of fibres has been very much neglected.

The purpose of present investigation is:

(1) To study the behaviour of interface bonds subjected to complex states of stress.

(2) To examine, directly by means of two dimensional photoelastic models;

(a) from the initial tests, the stress disturbance produced around an isolated inclusion in an infinite matrix, due to:

(i) the shrinkage of the matrix during the curing process of a typical glass
reinforced plastic,

and (ii) the application of a uniform stress field at infinity,

(b) in the subsequent tests, the distribution in the cross-section of a regular fibre array with different fibre proportions,

(i) representing initial interference condition,

(ii) subjected to two uniaxial loads separately in mutually perpendicular planes, first along the strongest axis and then along the weakest axis,

and (iii) under a biaxial state of stress.

1.6. Choice of Constituent Materials for Composite Models:

Schuster and Scala (1964) used sapphire ($\alpha$ Al$_2$O$_3$) whiskers as the reinforcing agent because of the high modulus of elasticity ($E \approx 60 \times 10^6$ lbf. per sq. in.). These whiskers are of further practical interest since their high strength does not decrease appreciably with temperature even up to the melting point ($3720^\circ F$). These whiskers also provide a good wetting and therefore bonding interface with the birefringent plastic matrix.

Because of the practical difficulty of obtaining a sapphire, large enough, so that it could be used readily for the manufacture of photoelastic models which represent the cross-section of unidirectionally orientated fibres,
and the modular ratio, of these whiskers to epoxy resin, being considerably high for the simulation of a typical resin-glass composite, it was decided to use dural as the fibre material with birefringent epoxy resin Araldite CT 200 as the surrounding matrix.

In a model composite the weakest link is generally the fibre-matrix interface which is inevitably subjected to a complex stress system of normal and shear stresses. In order to obtain an adequate bond strength in the models it was decided to investigate a technique which will provide information for the preparation and pre-treatment of surfaces to be bonded. This is essential for obtaining a consistent bond. Therefore, a number of test specimens were prepared identically and tested to failure under combined tension and torsion.

Figure 2.2 shows the test specimen which will provide a uniform tensile stress at the interface when loaded in pure tension. Due to tubular shape of the test pieces, a practically uniform stress field will be obtained at the bonded surfaces when subjected to pure torsion. This choice of shape of the test specimen eliminates any stress concentration near the edges of the test pieces and also the intensification of stress does not depend upon the tube edges. Both these drawbacks are experienced in glued lap joints.

A structurally rigid testing rig was manufactured in which the specimen could be supported and subjected to
different combinations of tension and torsion. To obtain a direct and permanent record of the experimental tests, a strain-gage transducer was built-in the rig and wired to an X-Y plotter.

The disturbance produced by an inclusion was examined by manufacturing thin two-dimensional model of sufficiently large size so that the stresses become uniform in the vicinity of plate boundary. The models were viewed in polarised light and the isochromatic fringe number and isoclinic angles were measured at various grid points finely engraved on the plates. The stress separations for interference condition and also with uniaxial load were performed using Lame' Maxwell equilibrium equations (Coker and Filon 1957).

A number of plate models, similar to the one described above, were manufactured so as to represent the cross-section of unidirectionally orientated fibres which are free from interface shear along the axis of the fibres. Two sets of triangular arrays of interference-fit pins were made for each plug size so that the models could be loaded separately, first along the strongest axis and then along the weakest axis of ligament.

In order to study the effect of change in percentage of fibre on the stress distributions in the matrix, five different fibre proportions were tested.
To obtain separate values of the principal stresses in the central ligament of the matrix it was necessary to manufacture a large immersion tank in which the model could be rotated accurately under load and would thus provide oblique incidence measurements.

Unidirectional fibre-reinforced composites can considered as the basic element from which composite structures can be constructed. The basic properties of this material can be determined by subjecting unidirectional reinforced composite to axial shear and transverse loadings.

A considerable work has been done in the past on axial and shear systems of loads, and the purpose of this study is to concentrate on the stress distribution produced by subjecting the matrix to a uniaxial load perpendicular to the axis of the reinforcement.

The composite material is assumed to consist of a triangular array of unidirectional orientated elastic filaments in an infinite elastic matrix, as shown in figure 4.1. By assuming a regular packing arrangement, a fundamental or repeating unit, for analysis, can be isolated as indicated by the dashed line in figure 4.1.

When more than one stress is known to be operating at a point in a body, it is convenient and valid under Hooke's law to calculate the stress from each source independently and to add the stress of like
type and direction at a particular point. This principal is used to obtain the results for a biaxial system of loads, by performing a tensor addition of the results of two plates with identical size of plugs.
CHAPTER 2

Strength of Interface Bonds
Subjected to Complex States of Stress

2.1. Introduction

This study was undertaken as a part of an investigation of the mechanical behaviour of fibre reinforced materials. In a composite containing discontinuous fibres, a fully bonded interface is generally subjected to complex stress conditions consisting of combined shear and normal pressure during the process of load transfer from one fibre to another through the matrix.

The interface is usually the weakest link in composites exposed to widely varying conditions, such as elevated temperature and humidity, resulting in bond failure at a point where the combination of shear and tensile load reaches a critical value. It was therefore decided to investigate a procedure for obtaining consistently high bond strength under varying combination of predetermined tensile and shear stresses.

A convenient method for examining the details of the stress distribution at the interface of fibre-matrix configuration is provided by the photoelastic technique. Although the method involves a model simulation of the real interface situation it has the advantage of permitting
detailed stress distributions to be obtained for both two and three dimensional fibre configurations. In the analysis, of the latter the stress freezing technique is used and the model is sliced for examination of the optical patterns.

It was decided to establish the stress conditions under which failure could be initiated by conducting a series of tests in which a typical interface was subjected to a known combination of uniform axial and tangential stresses. In the present investigation two types of tests are proposed:

(a) Tests to be carried out at room temperature on two dimensional models in which the fibre is simulated by a dural insert embedded in an epoxy resin matrix. The stresses associated with frictional bonds will be studied but it is also of interest to investigate the conditions under which adhesive bond failure occur, since this will simulate the behaviour of certain real interfaces.

To relate the mode of failure observed in the photoelastic models to the properties of the constituent materials it is necessary to establish the failure criteria for the particular combination of adhesive and adherend. The analysis of the ideal fully bonded configuration also requires that the selected adhesive should be sufficiently strong to accommodate the stresses developed under the loads required to produce satisfactory optical fringe patterns in the model.
(b) In developing three dimensional fibre-matrix models it is necessary to provide an ideal fully bonded configuration which can withstand the effect of externally applied loads together with the differential thermal strains imposed during stress freezing cycle without the introduction of embarrassingly large thermal strains in the model.

In a particular fibre model the normal and shear stresses will vary from point to point along the interface, and it follows that slip will be initiated at a critical point and will then spread over the interface, or a crack will propagate into the matrix.

2.2. Test Apparatus

An apparatus has been designed for testing cemented joints under selected combinations of tensile and shear stresses. The principle of the equipment is illustrated in figure 2.1. Strain gauge bridges were employed as load transducers and the output was recorded on an XY-plotter thus providing a permanent record of the maximum load attained during the test. The plotter is capable of recording from 50 to 0.5 milli-volt per inch with an accuracy of 0.2 per cent of the full scale reading.
The following is the list of adhesives which were tested:

(a) EASTMAN 910;
(b) EASTMAN 910 (Selected Lot) with W. T. Bean 910 Catalyst;
(c) Araldite AY 103 with Hardener HY 991;
and (d) BR-610-2C.

EASTMAN 910 was chosen as one of the binding materials since it has sufficiently high strength also sets very quickly under low pressure which assists in the preparation of number of test specimens in a short time.

The literature supplied by Welwyn Electric suggested that the strength of ordinary EASTMAN 910 is variable and advised that only pre-selected lots which have been subjected to prior tests should be used if consistently high strengths are desired. They also advocate the use of a propriety catalyst which incorporates agents to clean and etch the adherend surface to a limited extent.

From the preliminary results of tests with EASTMAN 910, as outlined in the latter part of this chapter, it will be seen that a considerable inconsistency is evident, and this is partially attributed to the short setting time which made the preparation of the specimens difficult. A number of specimens were prepared using
the Araldite AY 103 and Hardener HY 991 suggested by the Technical Office of CIBA (A.R.L.) Limited. This cold setting adhesive has the low viscosity necessary for assembling three dimensional composite models.

Welwyn Electric Limited also suggested BR-610-2C which retains its strength at high temperature and is therefore of special interest for three dimensional models which are to be subjected to a temperature rise of 120°C during the stress freezing cycle. This adhesive has a slow reaction rate and low viscosity, thus it reduces the possibility of voids or bubbles being formed in the bonded joints. Subsequently it was learned that this adhesive embodied solvent which would make it unsuitable for the type of joints envisaged for three dimensional models.

2.4. Test Specimens

It was planned to manufacture and test five nominally identical test pieces under similar conditions so that the scatter in the results could be ascertained. The details of the test pieces are shown in figure 2.2. These consist of an Araldite CT 200 tube bonded between two dural end pieces. When the test piece is subjected to either torque or axial load the stress condition obtained at the cemented joint are almost uniform.

2.5. Test Condition at Interface

The normal and shear stress directions at the
interface, and the detail of the shape of specimen is shown in figure 2.2. The variation of stress over the glued surface is:

(a) Axial stress,

$$\sigma_{zz} = \frac{W}{\pi d t}$$

and is uniform;

(b) Maximum tangential shear stress at A,

$$\sigma_{z \phi_{\text{max}}} = \frac{16 T D_0}{\pi (D_0^4 - D_i^4)}$$

Minimum tangential shear stress at B,

$$\sigma_{z \phi_{\text{min}}} = \frac{16 T D_i}{\pi (D_0^4 - D_i^4)}$$

Variation of shear stress,

$$\delta \sigma_{z \phi} = \frac{\sigma_{z \phi_{\text{max}}} - \sigma_{z \phi_{\text{min}}}}{\sigma_{z \phi_{\text{max}}} + \sigma_{z \phi_{\text{min}}}}$$

$$= \pm 0.14$$

thus the maximum and minimum values of shear stress developed on the interface vary by 14 per cent from the mean value. This variation in the shear stress is considered to be acceptable since a substantial variation may be obtained in the strength of bonded joint by virtue of other factors which cannot be simply controlled.
2.6. PRELIMINARY TEST PROCEDURE:

2.6.1. EASTMAN 910 Adhesive

The surface of the test pieces was prepared by lapping in a circular motion on Grade '0', Grit No: 220 emery cloth and finally degreased with trichlorethylene. To avoid any formation of oxide layer on dural surfaces, a thin layer of EASTMAN 910 was spread on the surfaces to be bonded as soon as the degreaser evaporated.

The correct alignment of three test piece components was ensured by means of a central rod, and a pressure of 2/3 lbf. per square inch was applied over the glued surfaces. Although great care had been taken to impregnate the aligning jig with Releasil 7 silicon release agent, some difficulty was experienced in removing the jig spindle following the setting of the joint. In some cases it was necessary to apply substantial axial load which could have caused initial damage to the joint and this is thought to be responsible for the wide scatter in the results recorded in the following table:

<table>
<thead>
<tr>
<th>Test No</th>
<th>T (lb. in)</th>
<th>$\sigma_{\phi z}$ (lb. per sq. in)</th>
<th>W (lb)</th>
<th>$\sigma_{zz}$ (lb. per sq. in)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>108</td>
<td>316</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>0</td>
<td>296</td>
<td>865</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>0</td>
<td>27</td>
<td>79</td>
</tr>
<tr>
<td>4</td>
<td>374</td>
<td>2490</td>
<td>75</td>
<td>220</td>
</tr>
<tr>
<td>5</td>
<td>143</td>
<td>952</td>
<td>27</td>
<td>79</td>
</tr>
</tbody>
</table>
It was decided to undertake further tests using these materials and also an improved technique for surface preparation, which is described below:

(a) The dural surfaces were degreased with acetone. Care was taken not to allow the solvent to come in contact with epoxy resin.

(b) The surfaces were allowed to dry. Evaporation of acetone would lower the temperature of the surfaces which subsequently should be allowed to attain the room temperature.

(c) Silicon carbide paper of 400 grit was dipped in metal conditioner and the end piece surfaces were lapped, any grit residue being removed with a clean tissue. Metal conditioner is a mild etching agent with additives which deoxidize, clean and thus promote adhesion.

(d) Neutralizer was applied to the surfaces with a cotton swab and cleaned with one stroke of the tissue.

The surfaces were ready for the application of W. T. Bean 910 Catalyst which was applied sparingly and allowed to dry for approximately one minute. Finally, EASTMAN 910 (Selected Lot) was applied and the specimens were prepared with the aligning jig shown in figure 2.3.

A considerable scatter was observed in the results of these tests which showed little improvement over the previous tabulated results of ordinary EASTMAN 910.
Mr. Hickson (1969) suggested that dural surfaces could be cleaned by eroding with alumina powder as an alternative to lapping with silicon carbide paper which would leave scratches instead of a uniformly pitted surface.

2.6.3. Araldite AY 103 with Hardener HY 991

The test surfaces of the end pieces were eroded at the Central Electricity Research Laboratory using an air containing 27 micron particles of alumina powder. The araldite test piece was degreased with trichlorethylene and a specimen was assembled using Araldite AY 103 with Hardener HY 991. A low strength was obtained on test and subsequent examination showed that the adhesive bond failed at the dural surface. It was thought that the surface produced by the eroding operation was insufficiently rough and a second test piece was manufactured by a similar technique but with dural surfaces additionally roughened with Grade 'O' Grit No: 220 emery cloth. A negligible improvement in shear strength was obtained.

2.6.4. Chemical Treatment of Dural Surfaces

The dural surfaces were etched with a solution of sulphuric and chromic acid prepared as follows:

In 1000 ml. glass beaker 124 gm. of the sulphuric acid of specific gravity 1.82 was slowly added to 317 gm. of clean cold water. During mixing the solution was continuously stirred and care was taken to ensure that there was no excessive rise in the temperature of the solution.
Finally 23 gm. of dry chromic acid was added to the solution which was stirred continuously until this was completely dissolved.

The dural surfaces to be glued were immersed in this acidic solution at 60°C to 65°C for 30 minutes. These were then washed with clean cold water, followed by clean hot water, and finally dried in a blast of hot air. This procedure conforms to the Ministry of Technology Specification D. T. D. 915 B., which specifies that the temperature of hot water and hot air must not exceed 65°C if optimum strength is desired.

A specimen was manufactured with its test surfaces of dural pieces chemically pretreated. The araldite test piece was degreased in the usual way and the specimen was glued with AY 103/HY 991. An excellent bond strength was obtained when subjected to pure torque, and was approximately equal to 5900 lb. per square in. which is the shear of Araldite CT 200.

2.7. RESULTS:

A number of specimens were prepared, with glued surfaces of the dural pieces chemically pretreated while the central Araldite test piece was degreased with trichlorethylene, and using EASTMAN 910 (Selected Lot) along with W. T. Bean Catalyst. All the specimens were cured at room temperature, but one was tested after 24 hours while the others were tested after two weeks.
The results of these tests, which were in pure torsion, are shown in the following table:

<table>
<thead>
<tr>
<th>Test No:</th>
<th>Curing Time</th>
<th>Torque lb. in.</th>
<th>$\sigma_{\phi z}$ lb. per sq. in.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>24 hours</td>
<td>584</td>
<td>4350</td>
</tr>
<tr>
<td>2</td>
<td>2 weeks</td>
<td>687</td>
<td>5120</td>
</tr>
<tr>
<td>3</td>
<td>2 weeks</td>
<td>692</td>
<td>5160</td>
</tr>
<tr>
<td>4</td>
<td>2 weeks</td>
<td>720</td>
<td>5370</td>
</tr>
<tr>
<td>5</td>
<td>2 weeks</td>
<td>694</td>
<td>5170</td>
</tr>
</tbody>
</table>

Another set of test specimens were prepared using the above surface treatment procedure and the adhesive, but the curing period for the subsequent test specimens was three days. These specimens were tested in pure tension. A series of further tests were carried out, using similar specimens, under a predetermined tension which was combined with torsion to produce failure of the bonded butt joint. A fracture curve which is a part of an ellipse, with its major and minor axes chosen from mean experimental values of shear and tensile stresses respectively, is found to fit the experimental values as shown in figure 2.4.

Similar tests were repeated for joints cemented with Araldite AY 103/HY 991. Besides curing these
specimens for a day at room temperature, they were also subjected to a low temperature gradient curing cycle with a maximum increase in temperature of 120°C. Figure 2.5 shows the corresponding plot of elliptical failure locus and the experimental values for the test specimens glued with Araldite adhesive and subjected to combined tension and torsion.

The amount of scatter between failure ellipse and the experimental values, is within the limits of the experimental accuracy for these tests, since a considerable variation in bond strength can be obtained due to other factors which cannot be simply controlled.

2.8. CONCLUSIONS:

Pretreatment of glued surfaces of dural pieces by chemically etching with the solution of sulphuric acid and chromic acid is important if consistent bonds are to be obtained. Eroding of the dural surfaces by blasting with alumina powder, and lapping with a fine silicon carbide paper and emery cloth was not found to be efficient.

The increase in strength of EASTMAN 910 from to a period of few days is consistent with the results obtained by CIBA (A.R.L.) Limited.

In order to obtain a consistently high bond strength with adhesives EASTMAN 910 and Araldite AY 103
with Hardener HY 991, it was necessary to apply a pressure of two-third pound per square inch on glued surfaces. This produced a bond thickness of approximately one-half thousandth of an inch.
CHAPTER 3

The Diffusion of Unidirectional Tension

Around a Cylindrical High Modulus Inclusion

Embedded in an Elastic Matrix

3.1. Introduction

As a preliminary to a study of stresses induced in the matrix surrounding inclusion arrangements representing a wide range of fibre volume fractions, it was decided to examine the disturbance produced by an isolated cylindrical inclusion embedded in a matrix which was subjected to a uniform uniaxial tensile stress field. The investigation was undertaken using two dimensional photoelastic models, which were intended to represent a transverse section located at a sufficiently large distance from the ends of the fibre for the variation in stress along the fibre axis to have become negligible.

It was felt that the initial stresses, induced during fabrication of a composite, and the quality of the bond obtained at the interface could both be of importance in determining the subsequent behaviour under externally applied loads, and provision was made to simulate both of these effects in the present tests.
3.2. Preparation of Photoelastic Models

Both models were manufactured from Araldite CT 200 sheet which had been cast in vertical moulds comprising two polished dural plates separated by 1/8 inch thick spacers. Initial curing was obtained by maintaining the poured mould at 120°C. for 12 hours, when the temperature was reduced at 5°C. per hour to ambient. To complete curing and reduce the initial casting stress to less than 0.2 fringe per inch, the cast plates were de-moulded, laid upon horizontal, paper covered, glass sheets and subjected to an annealing cycle. The surface finish of the cast plates made by this procedure was adequate for optical measurements at normal incidence and it was only necessary to machine the profile as shown in figure 3.1. The outer edges were milled using a high speed steel cutter and the central hole was bored using a single point tool. Both these techniques allow the local temperature of the workpiece to be maintained at a sufficiently low level to avoid the introduction of machining stresses. The dural plugs were machined oversize to provide an interference of 0.4 per cent which represented the differential thermal contraction obtained during cooling of a typical thermosetting resin-glass fibre composite. Assembly was effected after heating the plate and cooling the plug sufficiently to obtain a clearance by virtue of the
differential expansion. The interference then developed as the temperature of the assembly returned to ambient conditions. In the case of the first model an interference fit alone was obtained. However, in the second model it was decided to assess the effect of interface adhesion. The pre-treatment of the adhering surfaces was carried out by degreasing with carbon tetrachloride and applying W.T. Bean 910 Catalyst which was allowed to dry. Finally, a thin layer of EASTMAN 910 (Special Lot) was spread over the surfaces and the model was assembled.

The plates were stiffened around the points at which external loads were applied through a whipple tree shown in figure 3.1. With this arrangement a uniform uniaxial tension, with a variation less than $\pm 0.6$ percent, was obtained in the plates at approximately four inches from the end. Finely engraved grid stations with a suitably diminished mesh size in the immediate vicinity of the plug were inscribed on the plates along both cartesian axes.

The measured physical properties of the Araldite CT 200 plates at ambient temperature were:

(a) Young's modulus of elasticity, $E = 0.45 \times 10^6$ lbf. per sq. inch;

(b) Poisson's ratio, $\nu = 0.38$

and (c) Material fringe value, $f = 52.5$ lbf. per sq. in. per fringe per inch thickness.
3.3. Test Procedure

The models were loaded in small increments with observations being taken at each step. It was found that, at a particular load, slip commenced at the points A shown in figure 3.2. Changes in the isochromatic fringe pattern due to slip were completed with a very small increase in load. On plotting the fringe order against load for various points on the engraved grid it was found that there were two distinct elastic slopes with a transition at the point of slip. Figure 3.3 illustrates the effect of slip on the isochromatic pattern at typical points 0.1 inch from the interface along both the x and y axes.

3.4. Method of Analysis

The separation of principal stresses along the axes of symmetry was undertaken for selected loads using the Lame' Maxwell equation which may be written for each of the Cartesian axes as:

\[
\frac{\partial P_x}{\partial x} = - \frac{P_x - P_y}{\rho_y}
\]

and

\[
\frac{\partial P_y}{\partial y} = - \frac{P_y - P_x}{\rho_x}
\]

where \( P \) is the principal stress in lbf. per square inch.
and the suffices \( x \) and \( y \) refer to the appropriate Cartesian axis;

\[ \rho = \frac{\delta \rho}{\delta \theta} \]

the radius of curvature of the stress trajectory in inches, with

\[ \delta \theta = \Theta_1 - \Theta_2 \]

the inclination between the principal directions at two points equally spaced about the axes of symmetry and at a preselected distance \( \delta s \) apart. The sign convention adopted for the Lame\'s Maxwell equation is shown in figure 3.4.

3.5. RESULTS:

3.5.1. Effect of Initial Interference

The value of initial interference has been chosen to provide a simulation of the differential thermal stresses which are developed during cooling of a thermal setting reinforced resin.

As would be expected similar stress distributions were obtained for interference alone, in the plates containing either unbonded or bonded plugs. In each case it was found that the stresses along both the axes satisfy the axisymmetric plane stress solution as shown in figure 3.5, thereby confirming that the plate boundaries are effectively at infinity in so far as the disturbance due to the plug is concerned.

The stress difference is also an inverse function of
the square of the distance from the plug centre and this feature of the analytical solution is confirmed implicitly by the experimental results shown in figure 3.5.

The distributions of the principal stresses along the x axis which are typical of all radial lines are shown in figure 3.6(a). Values of stresses have been expressed in terms of the interface pressure developed under the interference load. The hoop tension at the interface is almost equal to the interface pressure. The values of both stresses become negligibly small at a radius equal to three pin diameters. These results provide additional confirmation that the model is an adequate representation for the case of a fibre surrounded by an extensive matrix.

3.5.2. Effect of Additional Uniaxial Load before Slip

For the plate with an unbonded plug increments in uniaxial load below the value of 310 lbf. at which slip occurs, give rise to linear changes in fringe number and in the values of the separate stresses at points along both the x and y axes. Typical distributions for a load of 300 lbf. are shown in figure 3.6(b). On the x axis the radial stress is virtually unchanged by the addition of the uniaxial load. The increment in $P_y$ is very small, being only 5 per cent of the value which would have been obtained for a plate containing an unfilled hole. On the y axis, $P_y$ increases uniformly
by an amount equal to the applied tension, and the distribution of $P_x$ is not changed significantly.

3.5.3. Slip Mechanism

With further increase in uniaxial load the linear increase in isochromatic fringe value continued, until slip commenced at 310 lbf. The onset of slip was indicated by the non-linear behaviour illustrated in figure 3.3. A further increase in the applied load of about 15 per cent restored the linear variation in the isochromatic fringe number. However, the slopes of the two curves relating to selected points on both axes were markedly different from the values observed before slip, indicating that the reinforcing characteristics of the plug had changed significantly. Figure 3.2(b) shows the isochromatic pattern immediately before slip occurred, and figure 3.2(c) shows the corresponding pattern after the addition of sufficient load to terminate the non-linear behaviour. As a result of the slip, the isochromatic fringe value at A (shown in figure 3.2) increases by 15 per cent. On the removal of the load it was observed that the slip was not completely a reversible process and a comparison of figures 3.2(a) and 3.2(d) show that the residual isochromatic pattern is somewhat different to that originally obtained prior to loading. The pattern is still symmetrical about $x$ and $y$ axes, but the isochromatics have an elliptical form which is associated with the presence of
residual interface shear stresses.

The stress concentration factor before slip may be defined in the following manner:

Before slip, \( K_b = \frac{\sigma_{\text{max}} - \sigma_{\text{max } i}}{\sigma_{\text{nom}}} \)

where \( \sigma_{\text{max}} \) = the radial or hoop stress due to a uniaxial load plus initial interference,

\( \sigma_{\text{max } i} \) = the stress due to initial interference

and \( \sigma_{\text{nom}} \) = the mean axial tension applied to the plate.

A similar procedure is adopted to obtain the stress concentration factor after slip except that arbitrary values of the stresses at slip are taken instead of the values relating to the initial interference which have been used in the above expression.

After slip, \( K_s = \frac{\sigma_{\text{max } s} - \sigma_{\text{max } s}}{\sigma_{\text{nom } s} - \sigma_{\text{nom } s}} \)

where \( \sigma_{\text{max } s} \) and \( \sigma_{\text{nom } s} \) are the maximum and nominal stresses just after slip respectively. The stress concentration factors before and after slip of the unbonded and bonded plugs are shown in Table 3.1.
### TABLE 3.1

**STRESS CONCENTRATION FACTORS FOR UNBONDED AND BONDED PLUGS**

<table>
<thead>
<tr>
<th>Axis</th>
<th>Stress</th>
<th>Description of Model</th>
<th>Location</th>
<th>$K_b$</th>
<th>$K_s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>X</td>
<td>Hoop</td>
<td>Unbonded plug</td>
<td>B</td>
<td>0.175</td>
<td>0.983</td>
</tr>
<tr>
<td>X</td>
<td>Hoop</td>
<td>Bonded plug</td>
<td>B</td>
<td>0.185</td>
<td>0.938</td>
</tr>
<tr>
<td>Y</td>
<td>Hoop</td>
<td>Unbonded plug</td>
<td>C</td>
<td>-0.125</td>
<td>-0.233</td>
</tr>
<tr>
<td>Y</td>
<td>Hoop</td>
<td>Bonded plug</td>
<td>C</td>
<td>-0.125</td>
<td>-0.150</td>
</tr>
<tr>
<td>X</td>
<td>Radial</td>
<td>Unbonded plug</td>
<td>B</td>
<td>-0.175</td>
<td>-0.816</td>
</tr>
<tr>
<td>X</td>
<td>Radial</td>
<td>Bonded plug</td>
<td>B</td>
<td>-0.250</td>
<td>-1.230</td>
</tr>
<tr>
<td>Y</td>
<td>Radial</td>
<td>Unbonded plug</td>
<td>C</td>
<td>1.200</td>
<td>0.183</td>
</tr>
<tr>
<td>Y</td>
<td>Radial</td>
<td>Bonded plug</td>
<td>C</td>
<td>1.200</td>
<td>0.375</td>
</tr>
</tbody>
</table>
3.5.4. Effect of Additional Uniaxial Load After Slip

An increase in uniaxial load to 600 lbf. produced substantial changes in stress on the x axis, as shown in figure 3.6(c). The stress concentration factor due to hoop tension at the interface is 33 per cent of the value that would be obtained for an unfilled hole. On the y axis, the hoop stress at the interface is reduced slightly. Figure 3.7(a) shows that the rate of increase of the interface pressure \( P_x \) at B, which is 4.5 times as high as was the case prior to slip. The interface pressure at C continues to decrease and becomes zero under the influence of a uniaxial load of 800 lbf.

3.5.5. Plate with Bonded Plug

With interference alone, the radial and hoop stresses at the interface are approximately 5 per cent higher than the values obtained for an unbonded plug, although both models were carefully manufactured to have the same initial interference. The increase observed in the case of the bonded plug is undoubtedly due to the additional thickness of the layer of EASTMAN 910 adhesive.

Again the superimposition of increments in uniaxial tension on the interference load produced a linear response in the isochromatic fringe pattern shown in figure 3.8. A comparison of figures 3.6(b) and 3.6(d) shows that the stress distributions for interference plus
uniaxial load were very similar in the cases of both the unbonded and bonded plugs.

The slip commenced at a uniaxial load of 540 lbf. which is 1.67 times the value obtained for an unbonded plug. An addition of 7.5 per cent in the uniaxial load restored the linear response of the isochromatic fringe number.

The stress distribution shown in figure 3.6(e) which was obtained for an additional uniaxial load of 600 lbf. just after completion of slipping, shows that although a local fracture of the interface bond has occurred at A, the bond at B and C has not been broken. The interface pressure at C has become a small tension under the influence of the large external load.

For an unbonded plug, mentioned above, the isochromatic fringe number at A increased by 15 per cent when slip occurred. In the case of bonded plug, substantially higher stresses were induced around the plug before slip occurs, and a much higher change is observed in the isochromatic fringe number during the slip process. In the present test the stress difference at A increased to 2.25 times the value before slip.

From an independent investigation carried out to study the failure characteristics for the adhesive EASTMAN 910 under combined tension and shear, it was found that the ultimate tensile strength of the bond between dural and Araldite is 900 lbf. per square inch
as shown in figure 2.4.

An extrapolation of the curve giving the variation in interface pressure on the y axis indicates that the interface bond would reach the measured tensile strength with the addition of a uniaxial load of 2200 lbf. It was apparent that the maximum stresses in the Araldite matrix would be substantially less than the strength of material under these conditions, and it follows that debonding of the plug would occur, along arcs which are symmetrically disposed about the y axis, before failure is initiated in the matrix.

3.5.6. Effect of Uniaxial Load Alone:

In order to study the effect of additional uniaxial load upon the interaction between the high modulus plug and the surrounding matrix it was necessary to subtract the stress distribution associated with initial interference from the combined results. It should be remembered, however, that until slip commences the effect of the interference will be to provide a complete bond at the interface, and it follows that similar results should be obtained for both the unbonded and bonded plugs for loads below the value at which slip is initiated. A comparison of the distributions shown in figures 3.9 and 3.10, confirms this assertion within the limits of the experimental accuracy.
In interpreting the results of the present tests it is instructive to examine the deformation behaviour of an extensive plate subjected to uniaxial tension and pierced by an unfilled circular hole. Under the influence of the applied load the boundary of the circular hole becomes elliptical. Hoop stresses of $3\times$ and $-1\times$ times the nominal applied tension are induced at the boundary on the axes which are respectively perpendicular and parallel to the direction of the applied load.

When the circular hole is filled by a high modulus inclusion the maximum values of the hoop stresses are substantially reduced. Under the influence of load alone the transverse diameter of the unfilled hole would tend to be reduced. This deformation is resisted by the inclusion, with a consequent increase in the interface pressure at B.

The resistance to tangential deformation provided by the rigid plug will combine with the increased pressure at B to reduce the hoop tension at the interface on the transverse axis. This effect is confirmed by the behaviour illustrated in figure 3.9.

On the y axis, in the vicinity of C, there is a local increase in the value of $P_y$. This is attributed to the stiffness of the plug which limits deformation in the y direction with a consequent local increase in the radial strain in the matrix.
As a result of the increase in tensile stress at C, a transverse hoop stress of approximately 0.3 to 0.4 times the value of \( P_y \) and of the same sign would be expected to develop at the interface, as the transverse strain developed due to the increase in \( P_y \) is largely restrained by the material of the plate around C in the x direction.

Considerations of the irreversible nature of the slip process leads to the conclusion that a valid measure of the increment in stress induced by the addition of load after slip is not provided by subtracting the results for the interference plus the uniaxial load at slip from the gross values, since this calculation will take no account of the redistribution of stress which accompanies slip. Although the values obtained in the model will have no general significance, being functions of the initial interference and the particular slip mechanism which is operating, they will provide an interesting indication of the plug behaviour under moderately large loads and this aspect of the tests will form the subject for the next chapter.
CHAPTER 4

STRESS DISTRIBUTION IN THE MATRIX

SURROUNDING A REGULAR ARRAY OF INTERFERENCE FIT PLUGS

4.1. Introduction:

A composite reinforced by unidirectional fibres may be idealised simply by a regular array of parallel cylindrical inclusions which have their axes located at the vertices of an equilateral triangular grid as is shown in figure 4.1. Two-dimensional photoelastic models, which are intended to represent typical cross-sections of the composite, have been manufactured by fitting dural plugs into a series of identical holes bored in a thin sheet of Araldite CT 200 epoxy resin. The model materials were chosen to simulate the modular ratio of a typical resin-glass composite. A regular array of plugs fitted in thin plates in this fashion is intended to represent a transverse section, located at a sufficiently large distance from the ends of the fibres for the variation in shear stress along the fibre axis to have become negligible.

Although, in the real composite it would be expected that major loads would be applied in the direction of the fibre axes, inevitably in engineering applications
transverse stresses will also be developed. In many cases it is this biaxial state of stress in the transverse plane which initiates failure in a composite. For the photoelastic models envisaged here it is simpler to analyse the behaviour when uniaxial loads are applied in mutually perpendicular directions during separate tests and subsequently to obtain the results for the biaxial condition by invoking the principal of super-position. For this reason it was decided to test pairs of models which embodied similar inclusion arrays. In the first case the model was arranged so that uniaxial load could be applied in the direction of the major axis of the inclusion array whilst in the second case the load was aligned with the minor axis.

The differential thermal contraction which occurs during the manufacture of a typical thermosetting composite causes the matrix to shrink on to the fibre and thus induces a residual stress distribution. In the case of composites, reinforced by parallel fibres, the resulting stress distribution at sections which are not in the immediate vicinity of the ends of the fibres will be independent of the co-ordinate position along the fibre length. This loading condition has been simulated in the photoelastic model by placing oversize pins in position after heating the plates sufficiently and subsequently cooling them to ambient temperature.
A pair of plates for each plug size provides a check for the results obtained under interference conditions and these are discussed in the later part of this chapter.

Five fibre proportions were examined by testing models with different pin diameters. These involved a variation in fibre content from 22 to 73 per cent.

4.2. Manufacture and Storing of Models:

An Araldite CT 200 plate, 1/8 inch in thickness and 20 inches long, was utilised for the manufacture of the models. A typical pair of models is shown in figure 4.2. An interfibre distance of one inch was maintained for all the models which were fitted plugs with varying diameters between 0.5 and 0.9 inch in 0.1 inch steps.

A given interference would be expected to:

(a) produce a general stress level and hence fewer fringes with increased plug diameter, as has been observed for an isolated plug and

(b) cause an increase in the ligament stress as the ligament width decreases.

It was expected therefore that approximately the same fringe number would be obtained in all the models if the differential between the hole and pin sizes was maintained constant.
To obtain a suitable fringe pattern, an interference 0.001 inch was chosen on the basis of the original tests on single pins. Later the results were scaled by suitable factors to give stress values which would relate to the manufacture of real composite materials.

An excessive increase of temperature of the sheet during machining would induce the machining stresses in the models. In order to avoid this:

(a) a single point diamond tool was used and
(b) very small finishing cuts were made during the final stages of machining the models.

The plates and the plugs were cleaned with carbon-tetrachloride and a generous coating of Releasil 7 was applied to the holes in the plate. The grease assists in minimising the effect of local asperities and ensures a regular isochromatic fringe pattern close to the interface.

The ratio of the coefficients of expansion of Araldite and dural is approximately three at room temperature and increases rapidly with increased temperature. The plates were heated to 70°C. and the plugs, being at ambient temperature, were then placed in the bored holes. An excessive increase in temperature was avoided as this could lead to the introduction of spurious creep effects which, for Araldite, are appreciably higher at elevated temperature. The models assembled
by this technique result in the development of:

(a) the interference pressure, with the inter­
face being free from shear stress along the
plate thickness

and (b) an interference isochromatic fring pattern
when observed in a standard polariscope.

The absorption of moisture by Araldite produces
local swelling and an associated optical effect. For
this reason the plates were stored in a hot cupboard
at a temperature of 70°C, for at least 24 hours before
attempting to make observations of the optical patterns.
The plugs were fitted immediately before making the
measurements.

When the plates were left inside the hot
cupboard, with the plugs in, for a period of three
months it was found that a residual isochromatic fringe
pattern developed around the interference fit plugs.
This is due to creep effect which increases at elevated
temperature. In the case of the 4.0 inch plate with
0.7 inch plugs the residual pattern, on removing the
plugs, had a maximum value of 15 per cent of the fringe
number due to interference alone. This observed spurious
fringe number would be such as to introduce unacceptable
errors in the subsequent analysis and confirmed that the
plates could only be stored if the plugs were removed.
4.3. Observations of Optical Patterns:

In order to obtain separate values of the principal stresses in the central ligament, using integration technique, it would be necessary to carry the summation, numerically, of the increment in stresses along a line, starting either from the region of uniform stress or a free boundary, through regions involving large stress gradients with a possibility of introducing unacceptable errors. To avoid this difficulty it was decided to employ the oblique incidence technique in separating the principal stresses at a series of grid points inscribed along the midline of the central ligament.

4.3.1. Oblique Incidence Technique:

In addition to measuring the fringe numbers at normal incidence, oblique incidence readings were taken after rotating the loaded model in the polariscope so that the plane of wavefront no longer coincided with the plane of the model as is illustrated in figure 4.3.

When making the experimental observations it was necessary:

(a) to avoid the effects of refraction by immersing the model in a fluid which has the same refractive index as Araldite GT 200

and

(b) to ensure that the model is rotated through accurately predetermined angles about an axis which
remains in the plane of the wavefront.

Because of the size of the model it was necessary, therefore, to manufacture a special large tank with provision for rotating the model about a fixed axis parallel to the stress free glass through which the light enters and leaves the tank.

4.3.2. Apparatus for Measuring Oblique Incidence Fringe Numbers:

A large immersion tank was manufactured, together with a loading frame, with the help of which normal and oblique incidence measurements at $45^\circ$ to the incident ray were obtained, for both interference alone and interference plus an additional uniaxial tension up to 800 lbf on the model as shown in figure 4. For convenience, the points of observation were arranged to be close to the axis of rotation so that they would remain within the depth of focus of the microscope objective during the observations at oblique incidence.

The model support holding the loading frame was mounted on a standard optical bench to which it was clamped firmly (shown in figure 4.5), making sure that the axis of rotation of the loading frame and the optical glass ports lay in the plane of the wavefront of the observed wave. This was achieved by placing a mirror on to the glass port and adjusting the tank, by means of knurled screws shown in figure 4.5, until the incident ray is reflected back into the aperture in the
diaphragm of the condensing light system.

A review of the literature shows that the proportion of Aroclor included in the immersion fluid used by different investigators (Allison, Nurse and Ward) varied up to 3 per cent due to the variation in refractive indices of commercially available Aroclor 1242 and liquid paraffin. To obtain an accurate analysis of the separate principal stresses it was decided to match the refractive index of the mixture of commercial Aroclor 1242 and liquid paraffin with Araldite CT 200 used in these tests. The analysis used in the preparation of immersion fluid is described in Appendix A.

After filling the tank with this fluid the loading frame, along the model was lowered into position in the immersion tank where it was held by the model support as shown in figure 4.4.

The oblique incidence technique was used to separate the principal stresses at points on the axes of symmetry of the inclusions. In this case the procedure was particularly convenient as a prior knowledge of the stress directions made possible the provision of suitable means for rotating the model about the required axes. Although the most accurate oblique incidence analysis is obtained by making observations of the fringe numbers for rotations about the two mutually perpendicular axes it was decided to arrange to do this simply for the loaded
plate model. It was decided, therefore, to accept the possibility of slightly reduced accuracy and only make observations for positive and negative rotations about the single chosen axis. Hence considerable care was taken to ensure that the initial orientation of the model in the plane of the wavefront was correct and a system of drilled dowel holes was employed in obtaining accurate rotations of the model.

The advantage of this procedure, over the integration techniques, was that the principal stresses at any point were only dependent upon the normal and oblique incidence measurements made at that point. Any error resulting from the analysis at previous points, along the line, did not affect the accuracy of the stress separation at the point under consideration.

The observations were made, through the microscope, of the isochromatic fringe numbers at various grid points on the model and also corresponding isoclinic angles for some of the obstructed points by the plugs where it was impossible to make oblique incidence observation as shown in figure 4.3.

The Senarmont technique was adopted in making the observations of fractional fringe orders which were measured to an accuracy of ± 0.01 fringes. This method is described briefly in Appendix B.

The details of the analysis adopted in the separation of principal stresses along the Cartesian axes
using normal and oblique incidence readings is given in Appendix C. In some regions, shown in figure 4.3, where oblique incidence measurements could not be obtained, the separation of principal stresses was performed using the Lame’ Maxwell integration. Initial values of the separate stresses were obtained, using the oblique incidence technique, in the regions unobstructed by the plugs.

4.4. ANALYSIS OF STRESS DISTRIBUTION:

The very small difference between the diameter of the plug and corresponding hole bore which is required to produce an acceptable optical pattern implies that small changes in plug size will involve substantial changes in the overall stress levels. If standard tool-makers’ tolerances were adopted in manufacturing models, it was found that a consistent optical pattern could be obtained in individual models as shown in figure 4.6. It was not, however, possible to maintain sufficiently close control over the effective magnitude of the interference in different models without resorting to extremely specialised machining techniques. Therefore, it was decided to adopt the usual instrument-makers’ tolerance and to eliminate the effect of random differences, in interference which occur between separate models, by suitably non-dimensionalising the experimental results.
The isochromatic fringe numbers at any point in a model, must be linearly related to the magnitude of the initial interference. If the observed inconsistency in the values measured in different models is only due to the error in the initial interference, it follows that normalisation of the measured stresses by the appropriate values of the stress difference, at centre point of the ligament, should provide regular curves.

The hoop and radial stresses at points \( A_1 \) and \( B_1 \), when expressed as a fraction of the difference in principal stresses at \( A \), versus plug diameter were found to be regular curves as shown in figure 4.7. The ratio of hoop strain at \( A_1 \) and \( B_1 \) to that at \( A \), was also a smooth curve when plotted against pin diameter as shown in figure 4.8. This procedure confirms that the apparent inconsistencies between the results of different plug arrangements are entirely associated with variations in the relative interference and cannot be attributed to experimental errors. However this does not eliminate the possibility of a consistent error for any one plate.

The hoop strain distribution around the inclusion interface is characterised by six maxima and six minima by virtue of the geometric symmetry of the surrounding inclusions. Assuming that the strain distribution can be expressed by a Fourier series, it follows that the mean value of the maximum at \( A_1 \) and the minimum at \( B_1 \) will
provide a good approximation to the overall mean strain which is related directly to the measured diametral interference.

In order to study the effect of variation in plug diameter on the principal stresses, the following normalisation procedure was adopted:

After determining the separate values of the hoop, $\sigma_{\phi m}$ and radial, $\sigma_{rr m}$ stresses at points $A_1$ and $B_1$ the mean hoop strain in the matrix was calculated:

$$\overline{\varepsilon}_m = \frac{1}{E_m} \left[ \frac{\sigma_{\phi m A_1} + \sigma_{\phi m B_1}}{2} - \nu_m \frac{\sigma_{rr m A_1} + \sigma_{rr m B_1}}{2} \right]$$

$$... (4.1)$$

where $E_m$ and $\nu_m$ are the Young's modulus and Poisson's ratio of the matrix.

It follows that the mean pressure applied to the plug,

$$\overline{p} = \frac{\sigma_{rr m A_1} + \sigma_{rr m B_1}}{2}$$

in which case the mean hoop strain is given by

$$\overline{\varepsilon}_p = \left( \frac{1 - \nu}{E_p} \right) \left( \overline{\sigma}_{rr m A_1} + \overline{\sigma}_{rr m B_1} \right)$$

$$... (4.2)$$
Therefore, the proportional interference,

\[ \frac{U}{r} = \bar{\varepsilon}_m - \bar{\varepsilon}_p \]  

Substituting \( \bar{\varepsilon}_m \) and \( \bar{\varepsilon}_p \) from equations (4.1) and (4.2) respectively, the experimental proportional interference was calculated from the measured values as follows:

\[ \frac{U}{r} = \left( \frac{\sigma_{\phi m A_1} + \sigma_{\phi m B_1}}{2E_m} \right) - \left( \sigma_{rr m A_1} + \sigma_{rr B_1} \right) \left( \frac{\nu_m}{2E_m} + \frac{1 - \nu_p}{E_p} \right) \]

where \( U \) is the radial displacement at a radius, \( r \) of the interface and the suffices \( m \) and \( p \) denote the matrix and the plug respectively.

The stress values, corresponding to a proportional interference of 0.4 per cent, were then obtained by multiplying the experimentally determined stresses by \( \frac{0.004}{U/r} \).

The resulting values thus represented the stresses obtained as a result of the differential thermal contraction which would occur during the cooling of a typical thermo-setting resin-glass fibre composite. The normalised values of the stresses at \( A_1 \) and \( B_1 \) are shown in figure 4.9.
The hoop tension at A, which is shown in figure 4.10 for both plates, increases rapidly in a linear manner with the increase in pin size. The radial stress at A also increases linearly, with the exception of 4.0 inch plate in which case the radial stress at A appears to reach a maximum at a plug size of 0.8 inch and then decreases to zero for the plate containing 0.9 inch plugs.

If the interference is standardised at 0.4 per cent for all the plug sizes, the hoop and radial stresses at A would be expected to increase with the increase in pin diameter. This is due to the decrease in ligament width as a result of which point A is virtually closer to the plug-matrix interface at $A_1$ or $B_1$ where the stresses would be maximum. Figure 4.11 and 4.12 illustrate the individual stress distributions for 4.0 and 6.2 inch plates respectively, and show that the average value of the ligament stress also increases with plug size.

In the case of 6.2 inch plate with 0.9 inch plugs the ligament width $B_1B_1$ (shown in figure 4.10) is 0.83 inch which is sufficient to resist the radial force developed, due to interference-fit plugs, in the X-direction without a substantial change in radial stress. Whereas, for the 4.0 inch plate, a large tensile strain is developed in the Y-direction by the vertical components of the radial pressure on 0.1 thin ligaments separating 0.9 inch interference-fit plugs. In the X-direction, the associated
transverse strain which is mainly prevented by the rigid plugs causes a local reduction in the interface pressure developed at the neck of the ligament.

4.5.1. **Explanation for Introducing Slots in Plates with 0.8 and 0.9 inch Plugs:**

At point A in the 6.2 inch plate, the radial stress, $P_x$ has a higher value than at the corresponding points in 4.0 inch plate.

The obvious difference between the models is represented by:

(a) the restraint on the elastic deformation of the plate in the lateral direction provided by the portion of the unpierced plate beyond the end of the plug array (figure 4.13)

and

(b) the local deformation of the matrix at point C (shown in figure 4.13) is restricted, in the direction parallel to the free boundary, by the unpierced length of the plate which prevents it from representing a typical section of the composite array. This effect is predominant in 6.2 inch plate because of the arrangement of the plate boundary.

It was thought that these effects could be reduced by slotting the plate beyond the hole array. Since this would at least provide a more symmetrical array arrangement.
In the case of models with 0.8 and 0.9 inch plugs, due to the large increase in the percentage of the matrix, when traversing across the last row of plugs towards the solid length of the plate, the movement of these plugs is restricted before the introduction of slots. For example, the percentage of Araldite in the unpierced plate is 3.7 times more than in an array containing 0.9 inch plugs fitted on the vertices of an equilateral triangle of one-inch sides.

With the increase in plug diameter both the above factors become sufficiently large so as to restrict the behaviour of the plate as a typical sample of an array of composite material. The discrepancy between the interface pressures observed at the corresponding points A and A׳ in the original models suggested that even the central ligament might not be behaving in the same way as an infinite array. It was therefore decided to cut slots in plates with 0.8 and 0.9 inch plugs along the unpierced plate close to the plugs as shown in figure 4.13.

The effect of introducing slots is shown, in figure 4.10, by the decrease in radial pressure at A in the 6.2 inch plate to the same value as given by the linear portion of the curve for 4.0 inch plate. In figure 4.13 it would be observed that a small release of local restraint at point C, by cutting slots, has a predominant effect on the radial pressure at A for 6.2 inch plate with larger plugs.
by virtue of a large decrease in ligament width $A_1A_1$ as compared with $B_1B_1$ for 4.0 inch plate.

At points $A_1$ and $B_1$, the variation of radial stress, when expressed as the ratio of stress which acting alone would produce a strain in the direction of the radial stress, versus the plug diameter is shown in figure 4.14. This radial stress, at $A_1$ for 4.0 inch plate in the $X$-direction and at $B_1$ for 6.2 inch plate in the $Y$-direction, falls rapidly with the increase in plug diameter, whereas the same stress, in 6.2 inch plate $A_1$ and 4.0 inch plate at $B_1$, approximates to a linear relation. In the case of larger plug sizes there is a considerable reduction in the matrix which is unable to resist the radial pressure, thus resulting in the deformation of the matrix in the direction of the free boundary as shown in figure 4.15.

Within the limits of accuracy of the experimental stress separations it was found that the normalised values of hoop stress at points $A_1$ and $B_1$ change linearly with plug diameter for both model arrangement as shown in figure 4.9. Further, the hoop stresses $P_{x_{B_1}}$ and $P_{y_{A_1}}$ have the same value for corresponding plug sizes on both plates. Similarly, the interface pressures at $A_1$ and $B_1$ in the 4.0 inch plate coincide with the values at $B_1$ in the 6.2 inch plate. It is surprising to find that the interface pressure at $A_1$ in the 6.2 inch plate is significantly higher than the other values, since it could be
expected that similar results should be obtained at corresponding points if these models really represent infinite plug arrays.

It should be noted here that the symmetry of the plug array would suggest that dissimilar values should be observed at points $A_1$ and $B_1$ with the stress distribution around the interface reaching a maximum at one of these points and a minimum at the other. It would be expected that the smaller plugs would act in isolation and that the difference between the values at $A_1$ and $B_1$ would vanish as the stress distribution around the interface becomes more uniform. The consistent equality of the stresses at $A_1$ and $B_1$ with increasing plug size implies that the interface stress values due to interference alone are independent of circumferential position and suggests that these might be predicted by a relatively simple theory.

In general, slotting the plates appears to have little effect and does not offer an acceptable explanation for the discrepancy between the interface pressures observed on the 4.0 and 6.2 inch plates (figure 4.9). In considering the original experimental results it is significant that the stress differences observed at $A_1$ and $B_1$ are consistent between the 6.2 and 4.0 inch plates for all plug sizes.

In making oblique incidence measurements the 6.2 inches wide plate is rotated about the Y-axis whilst the 4.0 plate is rotated about the x axis. Inevitably the oblique
incidence method implies an averaging process which will introduce different types of error in the stress separations for the two models. This effect is likely to be greater in the case of 6.2 inch plates in which a large percentage of the ligament $A_1A_1$ is obstructed by the plugs during oblique incidence measurements and it is possible that the consistent discrepancy observed in the values of $P_x$ at locations $A$ and $A_1$ can be attributed to this cause. Unfortun- tunately in the time available it was not possible to pursue a detailed quantitative analysis of this effect since this would have involved a substantial increase in the experimental programme at a time when it was not possible to introduce additional work.

4.5.2. Application of Lame' Thick Cylinder Formulae under Extreme Boundary Conditions:

The simplest analysis of the stresses associated with initial interference involves in approximating the geometry of the typical cell (shown in figure 4.16) to a pair of concentric cylinders and assuming that the deformation and stresses are of the form given by the Lame' solution for a thick walled cylinder. In the case of matrix reinforced by a regular array of interference-fit plugs, the radius of the equivalent cylinder was calculated by equating its area with that of a typical hexagonal cell as shown in figure 4.16.
It is not possible to specify the boundary condition on the outer surface of the idealised cylinder. Therefore, it was decided to examine the problem under the extreme conditions in which:

(a) the boundary is unloaded and
(b) the boundary is undeformed.

The interface pressure and the hoop tension in the matrix were both calculated for free and fixed outer boundary with 0.4 per cent interference at the interface with the pin. Table 4.1 shows the interface stresses for various plug sizes and a detailed procedure for obtaining these values has been outlined in the Appendix D.

On plotting the interface pressure and hoop stress for both free and fixed outer boundary condition, it is clear that the experimental values fit closely to the free outer boundary condition, as shown in figure 4.17, and it may be concluded that the fall in radial pressure with increasing plug size observed experimentally is a significant effect.

4.6. SUMMARY OF IMPORTANT FEATURES OF AN ARRAY OF INTERFERENCE FIT PLUGS:

(a) The principal stresses were obtained along the mid-lines of a typical element using a combination
<table>
<thead>
<tr>
<th>Plug Size (in.)</th>
<th>Free</th>
<th>Fixed</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\sigma_{rr_1}$</td>
<td>$\sigma_{\phi\phi_1}$</td>
</tr>
<tr>
<td>0.5</td>
<td>900</td>
<td>1435</td>
</tr>
<tr>
<td>0.6</td>
<td>755</td>
<td>1490</td>
</tr>
<tr>
<td>0.7</td>
<td>620</td>
<td>1610</td>
</tr>
<tr>
<td>0.8</td>
<td>430</td>
<td>1625</td>
</tr>
<tr>
<td>0.9</td>
<td>260</td>
<td>1695</td>
</tr>
</tbody>
</table>

**TABLE 4.1**

Interface Pressure and Hoop Tension in the Matrix using Axisymmetric Solution for Free and Fixed Outer Boundary

Loading Condition: Interference of 0.4 per cent
of the oblique incidence and the Lamé\textsuperscript{1} Maxwell techniques. The stress distributions have been obtained for models which represent fibre volume fractions from 22 to 73 per cent.

(b) Experimental values of the hoop and radial stresses at $A_1$ and $B_1$ fit closely with the approximate analytical solution obtained from the free boundary condition in Lamé\textsuperscript{1} thick cylinder analysis, except for the largest plug sizes. The equality between the corresponding interface stresses observed at $A_1$ and $B_1$ implies that the stress distribution around the interface is effectively axisymmetric.

(c) Interface pressures at $A_1$ and $B_1$, when expressed as the ratio of stress which when acting alone would produce a strain in the direction of the radial stress, versus plug diameter, although had a smooth curve also showed a drop in radial stress in the direction of the free boundary for larger plug sizes, that is 0.8 and 0.9 inch. With the increase in plug diameter the volume of the matrix falls rapidly and thus is unable to sustain the force developed by the radial pressure resulting in a local deformation near the plugs in the direction of the free boundary and hence a reduced radial pressure.

(d) The hoop stress at $A$ increases rapidly in a linear fashion with the increase in plug diameter. Due to the decrease in ligament width the point $A$ becomes
closer to the interface where the stresses have the
highest value.

(e) A simple axisymmetric solution which assumes
that the outer boundary of an idealised cylindrical shell
is unloaded, will predict the interface stresses in the
matrix.
CHAPTER 5

Stresses in the Cross-section of Composites

Loaded Perpendicular to the Axis of Unidirectional Fibres

The models which were analysed under interference alone were tested again under uniaxial load. Observations were made of the optical patterns due to the stress values associated with the additional load obtained by subtraction of the original interference stresses. By testing pairs of corresponding models results were obtained from inclusion arrays in which the load direction is aligned first with the major axis and then with the minor axis of the inclusion array. The stress values obtained for separate uniaxial loads were combined to provide information about the biaxial loading condition.

The model was loaded by whipple trees attached one to each end of the plate which was reinforced at the loading points. By this arrangement a uniform additional load was imposed on the unreinforced matrix. The variation in the nominal axial stress on a transverse section, DE (figure 4.2) is less than ± 1 per cent across the full width of the plate. Observations made on this section were employed in deriving the applied stress directly in terms of the optical sensitivity, of the photo-elastic material Araldite CT 200, which is obtained from the usual
expression,

\[ P - Q = \frac{Nf}{t} \]

where \( P \) and \( Q \) are the principal stresses in lbf. per sq. inch, and in this case \( Q = 0; \)

\[ N = \text{the isochromatic fringe number}, \]
\[ t = \text{the thickness of the plate in inches}, \]
\[ f = \text{the material fringe value in lbf. per sq. in. per fringe per inch thickness}. \]

5.1. SLIP PHENOMENON:

In the case of an isolated pin in an infinite plate the isochromatic fringe number due to interference plus load changes linearly with the application of additional uniaxial load until interface slip occurs. Slip appears to be initiated at the point \( C \) (figure 4.2) where the shear stress is maximum. The onset of slip results in a sudden change in fringe number in the vicinity of the interface at \( C \). Similar behaviour was observed on loading the multi-pin models.

5.2. IMPORTANCE OF LOAD BEFORE SLIP:

Slip is a non-linear process which may involve significant changes in the stress distributions in a particular plate. It follows, that the principal of superposition
only applies prior to the initiation of interface slip. and that the results for interference plus load must be obtained before slip if valid results are to be derived for the biaxial stress condition. This condition limited the maximum uniaxial load which could be applied to the model inclusion arrays.

The local interface shear at which slip occurs is proportional to the amount of initial interference. As has already been explained a consistent value of interference was not obtained for the individual plate models. It was also difficult to obtain a sufficiently accurate assessment of the value of initial interference due to the unavailability of instruments which would measure diameters, both internal and external, with an accuracy of better than \( \pm 0.0001 \) inch. At this degree of precision it also seemed likely that the surface finish and the surface contamination would reduce the accuracy of the measurements of initial interference.

From observations of the isochromatic fringe pattern it is possible to correlate the values of interference in different models. However, the additional interface shear induced by the uniaxial load will be a complex function of the array geometry and the material properties. Therefore, it follows that an evaluation of the initial interface pressures will be of little help in predicting the load at which slip will occur.
5.3. APPLICATION OF LOAD:

5.3.1. Choice of Load

The nominal load for the model, was chosen so as to produce a constant ligament stress, $\sigma_m$ in all the plates which were assumed to be unfilled. In order to obtain the values of the nominal load for 4.0 inch plate with intermediate size of plugs, that is 0.7 inch, a load which would cause slip was first determined. Subsequently, the plugs were refitted and a slightly reduced value of load was applied to this model.

Due to interference alone, the random variation of the isochromatic fringe number, at corresponding points in different models, has been explained previously. Also the load at slip is proportional to the initial interference. Therefore, the nominal load was proportionately reduced, for plates which had a low value of the fringe number due to initial interference, so as to avoid the initiation of slip. The actual load applied, to all the plates, during the experiment and the corresponding values of the nominal stresses in the unpierced section of the plate are shown in Table 5.1.

5.3.2. Checking for Slip

Before taking the measurements, an initial load unload cycle was performed to confirm that no slipping would occur at the test load. In case of slip, the plugs were removed by heating the model to 70°C. and refitted.
<table>
<thead>
<tr>
<th>Plate Width w inches</th>
<th>4.0</th>
<th>6.2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plug Diameter d inch</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.5</td>
<td>0.6</td>
<td>0.7</td>
</tr>
<tr>
<td>Applied W Load lbf.</td>
<td>76</td>
<td>93</td>
</tr>
<tr>
<td>Nominal Stress lbf./sq. in.</td>
<td>144</td>
<td>177</td>
</tr>
<tr>
<td>Load Direction</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Uns. = Unslotted  
S. = Slotted

**TABLE 5.1**

Loads Applied to the Models and the Corresponding Nominal Stresses

\[
\sigma_{\text{nom}} = \frac{W}{(w - nd) \cdot t}
\]
This procedure was repeated at a reduced load until a satisfactory cycle was obtained at which the slip would not occur.

5.3.3. Reducing the Effect of Creep on Observations

Sufficient time was allowed for the initial creep rate to diminish to a negligible value that significant errors will not be introduced by virtue of changes in isochromatic patterns throughout the time during which measurements were performed. In Araldite CT 200 creep rate is proportional to the applied stress and it follows that the effect of creep will not cause a redistribution of the stress but can be effectively represented by the decrease in overall stress distribution.

5.3.4. Scaling Procedure to Allow for Error in Initial Interference

Separate values of the principal stresses were calculated along both the Cartesian axes in the central ligament, due to interference plus load, by using the Lamé, Maxwell and oblique incidence techniques which have been described previously. A typical stress distribution, when expressed as the ratio of the nominal tension in the unfilled plate, along the Cartesian axes in the central ligament for 0.7 inch plugs in 4.0 and 6.2 inch plates under the influence of interference plus additional load is shown in figure 5.1.

To obtain stresses due to the effect of additional load alone, the stresses by virtue of interference pattern
were subtracted from the corresponding values of the stresses due to interference plus load at each of the grid points for all the models. Assuming elastic conditions, the principal stresses were subsequently increased, for plates subjected to a reduced load, to a value which would be obtained with the nominal load. Thus, principal stresses were obtained in the central ligament of all ten plates for the conditions: (a) load alone and (b) constant ligament tension assuming that the holes in the plate to be unfilled.

5.4. Average Ligament Stress Related to Constant Ligament Tension

The nett ligament load was calculated by integrating the principal stress due to load alone along the ligament $A_1A_1$ for 4.0 inch plate and $B_1B_1$ for 6.2 inch plate as shown in figure . . The average ligament stress was derived by dividing the load so obtained by the minimum cross-sectional area of the ligament.

The variation of average ligament tension, with plug diameter is shown in figure 5.2. It can be seen that experimentally determined value of corresponds closely to the nominal value which had been used in determining the magnitude of the load to be applied during the experimental investigation. From this it is concluded that the portion of applied load carried by the plugs is almost negligible.
5.5. **Effect of Introducing Slots on Mean Ligament Stress**

In the case of 6.2 inch plates, with larger plug sizes, the ligament load has a higher value than shown by the regular curves in figures 5.2 and 5.3. On slotting these plates the load becomes closer to the curves due to the change in stiffness associated with the release of the lateral constraint which had been developed by virtue of the extension of the matrix beyond the plug array. Hence, by introducing slots in the plates with larger sized plugs the behaviour of the model becomes a more realistic simulation of an infinite array of plugs in a composite material.

5.6. **Presentation of Experimental Data**

In order to present the experimental stresses in a non-dimensional form so that these values can be compared to the behaviour of different arrays, it is necessary to choose an arbitrary definition of the nominal stress. Although the form of the loading applied to the model is very simple, this results in a complicated interaction between the plugs and the matrix. The complexity of the load diffusion process through the inclusion array makes it very difficult to assess the proportion of load carried by the inclusions and the ligaments in the matrix. It is clear that any simple definition will idealise the true
behaviour and may result in a misleading presentation of the experimental data. For this reason it is necessary to consider the implication of using a number of different definitions and subsequently to assess which one is most appropriate. Four definitions have been examined.

5.6.1. **Nominal Stress in the Unpierced Plate**, $\sigma_{\text{nom}}$:

The nominal stress in the undrilled plate is defined as:

$$\sigma_{\text{nom}} = \frac{W}{Ag}$$

where $W$ is the uniaxial load on the plate having an overall width $b$ and thickness $t$ such that,

$$Ag = bt.$$  

It is important to note that this choice of nominal stress is arbitrary and assumes a uniform tension in the unpierced matrix, that is, the interference fit plugs carry load proportional to the cross-section of the pins.

5.6.2. **Mean Ligament Tension in the Unfilled Model**, $\sigma_m$:

Another possible definition involves calculating the mean tension in the ligaments with the inclusions removed. This proposal obviously takes account of changes in ligament geometry, but does not allow for the load diffusion effects.
Although at first sight this definition does not seem to be reasonable, in practice the plugs carry a small proportion of the applied load. This effect has been observed in case of an isolated pin and also for an array of plugs loaded at infinity (figure 5.2).

As has been explained previously this definition was used in calculating the experimental value of the model loads to be applied during testing. The results of these experiments indicate that this procedure is not entirely satisfactory in deriving load values which will consistently prevent the onset of slipping.

The load on the model was calculated by assuming that typical ligaments carry equal loads. Therefore, total load,

\[ W = \sum \sigma_m t l_w \]

\[ = \sigma_m t \sum l_w \]

where \( \sigma_m = \) the constant ligament tension,

\( t = \) the plate thickness,

and \( l_w = \) the ligament width

\[ = (D - d) \] for typical ligaments; \( D \) and \( d \) represent the plug spacing and diameter respectively.

Hence,

\[ W = \sigma_m t (w - nd) \]

where \( w = \) the width of the plate

and \( n = \) the number of plugs on the centre line perpendicular to the direction of the applied load.
This would be an acceptable definition of nominal stress if the effective flexibility of the inclusions in the transverse direction was small when compared to the matrix mean ligament stiffness.

Although the additional load imposed, in this case, was proportional to the ligament width the load shared by the plugs was not accounted for, that is, the model was assumed to be an unfilled plate. Also, for 6.2 inch plates with larger plug sizes, it is improper to take the ligament width as $B_1 B_1$ (figure 5.2) when calculating the applied load.

5.6.3. Mean Ligament Tension for Uniform Strain at the Centre Line of the Plugs, $\bar{\sigma}_m$:

A further definition may be obtained by assuming a constant value for the uniaxial strain across a transverse section passing through the centres of a series of inclusions, that is:

$$\frac{\bar{\sigma}_m}{E_m} = \frac{\sigma_f}{E_f}$$

where $\bar{\sigma}_m$ and $\sigma_f$ represent the stresses in the matrix and the fibre respectively, and $E_m$ and $E_f$ are the corresponding moduli of elasticity. It must be observed that the Poisson's effect of the composite materials has been neglected.
The total load applied to the plate is:

\[ W_T = \bar{\sigma}_m (w - nd) t + \sigma_T n d t \]

and the nominal stress in the unpierced plate is:

\[ \sigma_{\text{nom}} = \frac{W_T}{A_g} \]

and \( A_g = wt \)

where \( w \) = the width of the plate,

\( t \) = the plate thickness,

\( n \) = the number of plugs at the centre line perpendicular to the applied load,

\( d \) = the diameter of plugs,

and \( A_g \) = the gross area of undrilled plate.

Therefore, the nominal stress can be written as:

\[ \sigma_{\text{nom}} = \bar{\sigma}_m (1 - \frac{nd}{w}) t + \bar{\sigma}_m \frac{E_f \cdot nd}{E_m \cdot w} t \]

or \( \bar{\sigma}_m = \frac{\sigma_{\text{nom}}}{1 + \frac{nd}{w} \left( \frac{E_f}{E_m} - 1 \right)} \)

This does not give a realistic description of the stress distributions (figure 5.4) and is thus likely to provide an inadequate definition for larger size of plugs, owing to the restrictive assumptions which implies in the formulation that proportion of load carried by inclusions
and matrix are related to the elastic moduli of the component materials.

5.6.4. Average Stress on the Centre Line of the Ligament, $\bar{\sigma}$:

The average stress on ligament $A_1A_1$ in the 4.0 inch plate is defined as:

$$\bar{\sigma} = \frac{2 \int_{A_1} P_y \, dx}{(D - d)}$$

and the corresponding expression for the 6.2 inch plate is:

$$\bar{\sigma} = \frac{2 \int_{A_1} B_1 \, P_x \, dy}{(D - d)}$$

where $P_y$ and $P_x$ are the stresses due to additional load in the $y$ and $x$ directions respectively, and $(D - d)$ represents the corresponding ligament width $B_1A_1$ and $A_1A_1$ for 6.2 and 4.0 inch plates as shown in the figure 5.2.

This procedure has the advantage over the previous methods in that the values of the average ligament tension are calculated from the stresses determined experimentally, thus eliminating errors due to load difference and calibration of the optical sensitivity of the material.

The variation of nominal tension in the unpierced plate and the mean ligament stress when plotted against plug diameter, in figure 5.4, show that both these stresses decrease with increasing pin diameter. Also these stresses have a lower value for the 4.0 inch plate, as a result of
smaller ligament width $A_1A_1$ in the direction perpendicular to the load. In figure 5.5, it would be observed that the stress difference, when expressed as the ratio of nominal stress and mean ligament tension for uniform strain, at points $A_1$ and $B_1$ increases with plug diameter, the increase being greater in the case of the 4.0 inch plate with larger plugs. On examining the difference of principal stresses at points $A_1$ and $B_1$ it was noticed that these do not increase appreciably with pin diameter as shown in figure 5.6. Hence the value of the difference of stresses would be expected to increase more rapidly when expressed as the ratio of the factors $\sigma_{\text{nom}}$ and $\bar{\sigma}_m$ which decrease with increasing plug diameter. It is thus possible to account for the rapid increase in the stress differences when expressed as the ratio of $\sigma_{\text{nom}}$ and $\bar{\sigma}_m$. The increase is larger for 4.0 inch plates with larger plug sizes, and emphasises the inadequacy of the nominal stress in the unpierced plate and the mean ligament stress definitions.

The variation of average ligament tension, when expressed as the ratio of mean ligament stress is shown in figure 5.7. The values of this relationship are far greater than unity and increase with pin diameter, thus confirming that the proportion of the load carried by the ligament is much larger than would be calculated by assuming a uniform strain at the centre line of the ligament perpendicular to the load. That is, the assumption
that the load shared by the fibre and matrix is in proportion of their moduli of elasticity is not valid in this case.

5.7. DISCUSSION OF RESULTS:

5.7.1. Effect of Uniaxial Load Alone

Due to load alone the hoop stresses $P_y A_1$ and $P_x B_1$ for the load in Y and X directions respectively, remain constant with plug diameter as shown in figure 5.8(a). Because of the higher stiffness of the plug the deformation of the matrix, on the transverse axis at the interface, is restricted in the direction of the load, thus preventing the expected increase in hoop stress with increasing pin diameter.

In figure 5.8(b) the hoop tension, at $B_1$ and $A_1$ for 4.0 and 6.2 inch plates respectively, increase with plug diameter, except in the case of 6.2 inch plate with larger plug sizes, its value gradually decreases and becomes compressive for 0.9 inch plugs. Due to an excessive decrease in matrix width $A_1 A_1$ for 0.9 inch pins of 6.2 inch plate in the direction of the load that the stress geometry changes, that is, Poisson's effect of the additional load which produces a compressive stress in the y direction becomes larger than the stress developed in the same direction by rigid plugs which are expected to remain circular under the imposition of a uniaxial load on the plate.

The lateral contraction of the plate, under load is largely prevented by the rigid plugs at $A_1$ and $B_1$ for
4.0 and 6.2 inch plates respectively, thus producing a compressive radial force which increases with the increase in percentage of the rigid fibres as shown in figure 5.9(a).

On the pin diameter parallel to load direction, the radial stress has a tensile value as is shown in figure 5.9(b). Because of the higher stiffness of the plugs the longitudinal strain in the matrix close to points B1 and A1, is largely prevented by the rigid plugs, for 4.0 and 6.2 inch plates respectively, thus developing a tensile stress at these points.

There is little effect of introducing slots in plates with smaller ligament widths and the variation in stresses is attributed to the experimental errors which would be of larger magnitude for these plates due to the difficulty of making measurements as explained previously. The maximum change in stresses is approximately 15 per cent. This is in the case of 6.2 inch plate containing 0.9 inch inclusions.

Figure 5.10 shows the variation of hoop stress, when expressed as the ratio of nominal stress in the unpierced plate $\sigma_n$, mean ligament stress $\bar{\sigma}_m$ and the average ligament tension $\bar{\sigma}$, at points A1 and B1 with the increase of plug diameter. It is seen that there is a larger increase in the values of hoop stress with plug diameter when presented as the fraction of $\sigma_n$ and $\bar{\sigma}_m$, whereas the hoop tension when non-dimensionalised with $\bar{\sigma}$
becomes closer to the experimental values given in the figures 5.8 and 5.9. These figures also show the variation of principal stresses when expressed as the ratio of mean ligament tension in the unfilled model $\bar{\sigma}_m$, which is constant for all the plates and was used to calculate the nominal load applied to the models as explained previously.

It would be observed in figure 5.4 that the values of $\bar{\sigma}_m$ and $\bar{\sigma}_n$ decrease with the increase in pin diameter. Therefore, on dividing by these nominal tensions, the experimental stresses would give higher values for larger plug sizes. Whereas in figure 5.2, it can be observed that $\bar{\sigma}$ has a fairly constant value for all the plug arrays and thus the slope of experimental stress curves would not change appreciably with plug diameter even when the experimental values are expressed in terms of $\bar{\sigma}$.

Also the radial stress, at $A_1$ and $B_1$ when expressed as a ratio of $\bar{\sigma}_n$ and $\bar{\sigma}_m$, showed a rapid increase with plug diameter, but had values closer to the actual radial stress when represented as a fraction of $\bar{\sigma}$ as shown in figure 5.11.

5.7.2. Effect of Biaxial System of Stress

A tensor addition of the principal stresses, obtained in the two plates for a given plug size loaded separately in $x$ and $y$ directions, produced stresses which would be obtained in a similar plate under a biaxial state of stress. It must be emphasised that these values of
principal stresses were obtained prior to the initiation of interface slip.

The variation of hoop and radial stresses at points A₁ and B₁, with plug diameter, is shown in figure 5.12. It would be observed that the stress in the y direction at A₁, \( P_{yA_1} \), decreases rapidly after the ratio of the plug diameter to its spacing has reached 0.8 and attains a small compressive value for 0.9 inch plugs. In figure 5.8, it would be observed that \( P_{yA_1} \) for loads in x and y directions separately, when superimposed would result in a small compressive stress at A₁ in the y direction.

In order to obtain the stresses in a plane transverse to the axis of the fibres of a real composite, with a modular ratio identical to the one used in this analysis, and subjected to a biaxial stress system perpendicular to the axis of the fibres, the following technique would be adopted:

The stresses acting along the two Cartesian axes are:

\[
\sigma_x = \frac{W_x}{A_x}
\]

and

\[
\sigma_y = \frac{W_y}{A_y}
\]

where \( W_x \) and \( W_y \) are the external loads acting on the
areas $A_x$ and $A_y$ away from the region where the stresses are required.

As explained earlier the most suitable definition of the nominal stress is the average ligament tension,

$$\overline{\sigma} = \frac{\int P_x \, dy}{(D - d)}$$

and hence this is used to express the stresses which are to be evaluated. At any point in the matrix, the components of stresses $x$ and $y$ in the $x$ and $y$ directions respectively are given by:

$$\frac{\sigma_x}{\overline{\sigma}} = K_x^x \left( \frac{\overline{\sigma}}{\sigma_x} \right) + K_y^x \left( \frac{\overline{\sigma}}{\sigma_y} \right)$$

$$\frac{\sigma_y}{\overline{\sigma}} = K_x^y \left( \frac{\overline{\sigma}}{\sigma_x} \right) + K_y^y \left( \frac{\overline{\sigma}}{\sigma_y} \right)$$

where

$$K_x^x = \frac{\sigma_x}{\overline{\sigma}} \quad \text{and} \quad K_y^y = \frac{\sigma_y}{\overline{\sigma}}$$

The suffices $x$ and $y$ represent the experimental values due to load in $x$ and $y$ directions respectively. The values of $K_x^x$ and $K_y^y$ for both the directions of loading and for different fibre volume fractions can be obtained from the curves in figures 10(c) and 11(c). Thus by calculating the values of nominal stresses $\overline{\sigma}_x$ and $\overline{\sigma}_y$ at any point in
a composite in which the proportion of the fibres lies approximately between 20 and 75 per cent, the principal stresses $\sigma_x$ and $\sigma_y$ can be obtained in terms of the average ligament tension in the matrix.

It must be remembered that these results are only applicable to triangular array of fibres aligned uniaxially and the section is chosen away from the ends of the fibres where the interfacial shear stress has become uniform.

5.8. **CONCLUSIONS:**

The variation of load carried by the ligaments, due to additional load imposed in the $x$ and $y$ directions versus plug diameter, is shown in figure 5.3. It would be observed that the ligament load, which was calculated by integrating the ligament stress due load alone in the direction of the applied load, was a smooth curve. It follows that a reasonably correct choice of the procedure had been made for calculating the applied load for various models.

In an idealised plate model with unfilled holes the decrease in load, for a constant ligament stress, would be linearly proportional to the ligament width, whereas in figure 5.3 the slope of the load curve decreases with the increase in plug diameter. This divergence from the straight line behaviour is the result of a larger proportion of the load being shared by the pins for smaller ligament widths.
The average ligament stress, shown in figure 5.2, has a constant value and is equal to the mean ligament tension for an unfilled hole array. Hence the average ligament tension, shown in Table 5.2, on the ligament in a cross-section of unidirectional fibres subjected to transverse loading in the x and y directions separately, can be calculated by simply assuming the holes to be unfilled and obtaining the mean ligament tension in the direction of the load. There is a limitation to this procedure when the volume of the fibres exceeds 60 per cent. The average ligament tension, in this case, gradually falls below the nominal ligament stress due to a greater percentage of the applied load now being sustained by the interference fit plugs.
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<td>0.7</td>
<td>0.8</td>
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<tr>
<td>Plate</td>
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<td>Uns.</td>
<td>S.</td>
<td>Unslotted</td>
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<tr>
<td>Average Ligament Tension 1bf./sq. in.</td>
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<td>414</td>
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</tbody>
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\[
\bar{\sigma} = \frac{2 \int A_y Py \, dx}{(D - d)}
\]

\[
\bar{\sigma} = \frac{2 \int A_x Px \, dy}{(D - d)}
\]

Uns. = Unslotted  
S. = Slotted

**TABLE 5.2**

Average Ligament Tension in the Matrix
CHAPTER 6

General Discussion

and Suggestions for Future Work

6.1. INTRODUCTION:

The failure of composites is generally precipitated by cracks initiated in the vicinity of fibre breaks or discontinuities. Stress concentrations around these fibre tips have been analysed in the past by several investigators using both theoretical (Cox 1952 and Dow 1963) and experimental (Schuster and Scala 1964, Allison and Hollaway 1967) techniques.

Most of the work, on the deformation behaviour of multi-fibre arrays under transverse loading, is theoretical and primarily consists of evaluating the average strain magnification in the ligament (Kies 1962) and calculating the elastic constants of the composite in terms of the elastic and geometric parameters of the fibre array which forms the reinforcement.

Some of the experimental work carried out in this field involves the determined atom of the shrinkage stresses around two (Daniel and Durelli 1961) or three inclusions (Javornicky 1970) using two dimension photoelastic models. Stress freezing and the slicing technique were adopted
in the case of three dimensional models by Marloff and Daniel (1969) during the photoelastic of the principal stresses in matrix of a unidirectionally reinforced composite model subjected to matrix shrinkage and external loading transverse to the axis of the fibres. To obtain the separate values of the principal stresses at the starting point of the Lame' Maxwell integration they, authors made a basically incorrect assumption by equating the total load in the matrix to zero which will be erroneous for fibres under the influence of shrinkage stresses.

In practical composites reinforced by combination of longitudinal and transverse fibres the development of cracks which propagate in planes perpendicular to the axes of the main reinforcement may result in premature failure of the material. Typical examples of this behaviour have been observed by Owen, Holbecke and Zienkiewicz (1969) and McGarry (1966) in composites reinforced by indirectional fibres. Failure initiation may be attribution to small parasitic stresses which are variably present in real loading situations. The importance of this point emphasises the need for an investigation of the type presented here.

The additional study of adhesion conditions at the interface although undoubtedly failing to model the
prototype conditions exactly provides some valuable data about the stress conditions at the interface.

6.2. Pretreatment of Interfaces and the Failure Curves for Cemented Joints Subjected to Complex Stresses:

Owing to the difficulty of obtaining an adequate bond strength at the interface of the photoelastic fibre reinforced composite models, it was decided to study the bond strength of the cemented joints under complex states of stress. A fully bonded interface is generally subjected to complex stress conditions of combined shear and normal pressure during the process of load transfer from one fibre to another through the matrix.

The interface is usually the weakest link in composites which are loaded in conditions of elevated temperature and humidity resulting in initiation of fracture at a point where the combination of interface shear and tensile stress reaches a critical value. In order to simulate the complex conditions which exist at the interface in both two dimensional models an apparatus was constructed in which cemented joints could be tested to failure under predetermined combinations of tensile and shearing stresses.

Some of the factors such as chemical, physical and mechanical binding which affect strength of the inter-
face bond discussed in detail in section 1.4.2 and in Appendix E.

The strength of a limited number of adhesives has been investigated by making butt joints between the ends of cylindrical test pieces of dural and epoxy resin, these being the materials used to simulate the fibre and matrix in the model composite. Initially a very high scatter was observed in the results of strength tests in which the conventional workshop techniques were adopted for cleaning and degreasing the mating surfaces of dural cemented interface. Erosion of the surfaces by an air blast loaded with alumina powder did little to improve the consistency of the bond and it was found necessary to chemically cleanse the dural surface, with a solution of sulphuric and chromic acid at a controlled temperature between 60° and 65°C, in order to obtain a satisfactory joint. This procedure of chemical pretreatment of dural surfaces is described in some detail in section 2.6.4.

The results of the tests carried out on specimens prepared by chemical cleaning of dural surfaces showed that the failure curve for EASTMAN 910 (Selected Lot), which has a sufficiently high strength and is particularly suitable for the quick assembly of two dimensional photoelastic models at room temperature, is a part of the ellipse (figure 2.4).

A similar failure curve was obtained for Araldite
AY103/HY991 (figure 2.5) which is a cold setting adhesive of low viscosity necessary for the assembling of three dimensional composite models. Besides curing these specimens at room temperature for 24 hours, they were also subjected to a low temperature gradient stress freezing cycle with a maximum temperature rise of 120°C.

It would be expected that the pattern of failure under combined loading will apply to the prototype system although the stress levels at which failure is initiated might well be somewhat different.

6.3. STRESSES IN THE CROSS SECTION OF COMPOSITES REINFORCED BY UNIDIRECTIONAL FIBRES:

6.3.1. Isolated Circular Inclusion in an Infinite Matrix:

As a preliminary to the analysis of stresses around fibre arrays, the stresses in the matrix surrounding an isolated circular inclusion were examined. The photoelastic models employed in these tests consisted of an interference fit plug fitted in an epoxy resin plate which was sufficiently wide for the disturbance associated with the plug to attenuate completely before the edge of the plate was reached. Provision was made for the plate to be subjected to uniaxial tension and the behaviour of the model was studied as the magnitude of the unidirect-
ional load was increased past the point at which slip occurred at the interface. A second model which was incorporated with an interference fit plug bonded into an epoxy resin plate and tested in the similar manner.

The analysis of the optical patterns for interference alone, interference plus load and interference plus load after slipping were made for both the unbonded and bonded plug models. The effect of uniaxial load alone was obtained by subtracting the stress distribution due to initial interference from the combine results before slip. It was confirmed experimentally that prior to the initiation of slip the effect of interference was to provide a complete bond at the interface since similar results were obtained for both unbonded and bonded plugs.

The variation in uniaxial tension was less than \( \pm 1 \) per cent, at approximately four inches from the points where the external load was applied, and this was achieved by means of whipple trees attached one to each end of the plate. Further the change in values of both the principal stresses become negligibly small at a radius equal to three pin diameters. These results confirm that the model is an adequate representation for the case of a fibre surrounded by an extensive matrix loaded at infinity.

At the interface the hoop and radial stresses, when expressed as the ratio of interface pressure, are approximately equal to unity for both unbonded and bon-
ded plugs. In both cases the stress distribution around the interference fit plugs approximates closely to axisymmetric plane stress solution, thereby providing further evidence that the plate boundary is effectively at infinity in so far as the disturbance due to the plug is concerned.

On the application of uniaxial load, in the y direction, (a) the stress Py, increases sensibly uniformly across the plate and (b) the hoop stress at the interface on the x axis, increases by a value considerably less than the nominal stress. The radial on this axis is virtually unchanged by the addition of load. On the y axis, the increase in Py is equal to the value of the applied tension while the distribution of Px does not change appreciably. It follows effectively that the matrix behaves as a homogeneous plate.

With the increase of additional load the isochromatic fringe number varied linearly on both the axes until slip commenced at approximately 45° to the direction of the applied load. A small further increase in load for both unbonded and bonded plugs restored the linear variation in the isochromatic fringe value. However, the slopes of the curves after slip, for both the axes, were considerably different from those observed before slip, thus indicating that the reinforcing characteristics of the plug had changed significantly. On the removal of the
load it was observed that the isochromatic pattern was no longer axisymmetric.

In the case of bonded plug the slip initiated at a uniaxial load which was 1.67 times the value obtained for an unbonded plug. For the unbonded plug, the increase in shear stress, at a point 45° to the direction of the load, was only 15 per cent when slip occurred. Whereas, in the case of bonded plug the increase, in critical stress produced by slipping at the corresponding point, was 2.2 times the value immediately before slip.

6.3.2. A Regular Array of Interference Fit Plugs:

The cross section of a composite reinforced by unidirectional fibres, has been idealised by a regular array of parallel cylindrical interference fit plugs located at the vertices of an equilateral triangular grid. These models represent a transverse section situated at sufficiently large distance from the ends of the fibres for the variation in shear stress along the fibre axis to have become negligible. Five different fibre volume fractions were tested by fitting a range of dural plugs into suitable arrays of identical holes bored in thin epoxy resin plate models.

The shrinkage stresses, which are developed during the fabrication of a typical thermosetting composite, were simulated by fitting oversize pins. This method of
assembly provides a correct simulation of the shrinkage process achieved in real composites if it is assumed that the elastic modulus of the prototype matrix remains constant as the composite cools to room temperature. A method has been proposed for eliminating the effect of small variations in interference which were obtained experimentally when normal machining tolerances were used in manufacturing the photoelastic models. It has been found possible to express the observed stresses as normalised values which do not involve errors from this source.

The effect of mechanical creep in the matrix was avoided by fitting the plugs immediately before making the measurements and the plates were stored in a hot cupboard during the period prior to testing. It was thus possible to prevent the absorption of moisture which would induce a spurious optical pattern at the edges of the models.

Stress separations in the central ligament were undertaken using the oblique incidence technique. It was not possible to utilise the most accurate procedure which involves making rotations about both the principal directions. Since the separations were required along axes of symmetry which implies either a maximum or a minimum in one of the principal stresses, the application of oblique incidence technique can give rise to consistent error in the values of the separate stresses. There seems
no reason to believe that this has produced an immoderately large error in the current experiment, since consistent results have been obtained for models which involve substantial changes in stress distribution and different forms of applied load.

The effect of introducing slots, in plates with larger plug sizes, was negligible thus confirming that the models represented an infinite triangular array of unidirectional orientated fibres.

For the corresponding pair of plates, the hoop and radial stresses at A increase linearly for all but the largest plug sizes, whereas both these stresses have a sensibly uniform value at A₁ and B₁.

Using Lamé's analysis the hoop and radial stresses, in the matrix at the interface, were calculated for both free and fixed outer boundaries and with 0.4 per cent interference at the fibre-matrix interface. It was observed that the experimental values, of both hoop tension and radial pressure, fit closely to the unloaded outer boundary condition. Therefore it is possible to obtain these critical stresses simply by considering an isolated plug model of the form described above.
By extrapolating the curve of interface pressure (figure 4.9) to a very low fibre volume fraction, corresponding to the volume fraction of the isolated pin, it was found that the interface pressure asymptotes to a value of 1270 lbf. per square inch which is the theoretical figure for a single plug fitted with 0.4 per cent interference.

With the application of uniaxial load the isochromatic fringe pattern varies linearly until interface slip commences at a critical load which is proportional to the amount of interference between the plug diameter and the size of the bore in the matrix. To analyse the elastic behaviour of an array under biaxial load, it is only possible to apply the principle of superposition before the initiation of slip.

Due to uniaxial load alone the hoop stress, on the transverse diameter of the plug, remains constant. This is the result of higher stiffness of the plugs which restricts the movement of the matrix on the transverse axis at the interface, thereby presenting the expected increase in hoop stress at the interface increases linearly for most of the plug sizes. This is due to the rigid plugs preventing the deformation of the matrix along the longitudinal axis.

The lateral contraction of the plate under load, on the transverse axis is largely prevented by the rigid
plugs, thus a compressive stress is developed which increases with an increase in fibre volume fraction. On the diameter parallel to the direction of load, the radial stress has a tensile value. Because of the higher stiffness of the plugs the longitudinal strain in the matrix is prevented by the rigid plugs and hence a tensile stress is developed at these points.

A biaxial stress system, obtained by a tensor addition of the principal stresses in the two plates loaded separately in x and y directions, showed that both the hoop and radial stresses had a primarily tensile value for various plug sizes. The magnitude of the radial stress at the interface along the direction of uniaxial load has a higher value than at the same point under biaxial state of stress. But the hoop stress at the above point has a higher value when compared with the values at the corresponding point under uniaxial loads.

At the interface on the transverse diameter of the plugs the hoop stress, in general, has a higher value, under biaxial stress system, when compared to the values of stresses developed at the corresponding points in plates under uniaxial loads. Whereas the radial stress at the point under consideration, becomes tensile under biaxial state of stress and has a smaller value than in the plates loaded separately in x and y directions.
6.4.  SUGGESTIONS FOR FUTURE WORK:

(a) The effect of various combinations of biaxial stress could be studied using the results reported in this present investigation.

(b) Test the adequacy of an approximate theoretical treatment, such as finite element analysis, by making comparison with photoelastic results and hence obtain solution for fibre-matrix combination with different modular ratios.

(c) Develop three dimensional solution by both analytical and experimental techniques.
\[ \sigma_{f,av} = \left(1 - \frac{L}{2L}\right) \sigma_{f,\text{max}} \]

Fig. 1.1 Schematic representation of an aligned discontinuous fibre composite subjected to an axial stress.
Fig. 1.2. Isoclins and stress trajectories around an isolated rounded ended fibre subjected to uniaxial tension
O PUSH FIT
□ LOW INTERFERENCE.
+ HIGH INTERFERENCE

Fig. 1.3. Increment in maximum shear stress versus maximum shear stress due to interference alone.

\[ \frac{\delta S_m}{\bar{\sigma}} = 0 \]

\( S_m \) = maximum shear stress, on the transverse hole boundary A, due to initial interference

\( \delta S_m \) = increment in \( S_m \) due to increase pulsating load

\( \bar{\sigma} \) = nominal mean tensile stress
Fig. 1.4. Coordinate system for graphical integration (Marloff and Daniel)
FIG. 2.2. TEST SPECIMEN WITH DIRECTION OF STRESSES
FIG. 2.3. ALIGNING JIG

ARALDITE BUSH FOR ALIGNING ARALDITE PIECE SECTION DURING BONDING

SPINDLE

BUSHES INTEGRAL WITH END PIECES
Adhesive: EASTMAN 910

Fig. 2-4 Failure locus for a cemented joint subjected to combined bending and shear
Adhesive: Araldite AY 103/HY 991

Fig. 2-5. Failure locus for a cemented joint subjected to combined bending and shear
Fig. 3-1. The model and the arrangement of loading.
Fig. 3.2. Light field isochromatic fringe patterns around an isolated plug.
Modular ratio of plug/matrix = 20.
Fig. 3-3. Variation of fringe number with uniaxial load at points E and F.
Fig.3-4. Sign convention for Lame' Maxwell equation
Fig. 3.5: Interference stresses satisfying axisymmetric plane stress condition
Fig. 3-7. Variation of interface stresses with load
Fig. 38. Variation of fringe number with uniaxial load at points E and F
Fig. 3-9. Stress distributions on X axis for an additional load of 300 lbf. alone.
Fig. 3-10. Stress distributions on Y axis for an additional load of 300 lbf. alone
Fig. 4-1. Arrangement of fibre array representing the cross section of a typical composite.
Fig. 4.3. Restriction of measurement area on rotation of the model
Plug size: 0.5 inch
Plate width: 4.0 inches
Plug size: 0.5 inch
Plate width: 6.2 inches
Plug size: 0.6 inch
Plate width: 4.0 inches
Plug size: 0.7 inch;  Plate width: 6.2 inches

Figure 4.6. Consistent Isochromatic Fringe Pattern in Individual Models
Plug size: 0.7 inch
Plate width: 4.0 inches
Plug size: 0.9 inch
Plate width: 6.2 inches
Fig. 4.7. Variation of hoop and radial stresses at A₁ and B₁ with plug diameter. Loading condition: Interference alone.
Figure 4.8. Hoop strains at points A₁ and B₁.

**Loading condition:** Interference alone.
Fig. 4.9. Variation of hoop and radial stresses at $A_1$ and $B_1$ with plug diameter. Loading condition: Interference of 0.4%.
Fig. 4.10. Variation of hoop and radial stresses with plug diameter at point A

Loading condition: Interference alone

nominal interface pressure = 1270 lbf./sq. in.
Figure 4.12: Hoop and Radial Stress Distributions in the Ligament Plate width: 6.2 inches.
Fig. 4.13. Location of slots in plates of larger plug sizes.
Fig. 4.14. Variation of interface pressure at $A_1$ and $B_1$ with plug diameter

Loading condition: Interference of 0.4 per cent
Fig. 4.15. Deformation of matrix towards the free boundary for largest plug size
Fig. 4.16. A typical hexagonal cell fitted around each pin.
Fig. 4.17. Lame axisymmetric solution for extreme boundary condition along with a set of experimental points.

Loading condition: Interference of 0.4%
Fig. 5.1. Hoop and radial stress distributions in the central ligament for 0.7 inch plugs
Loading condition: Interference plus load
Fig. 5.2. Variation of average ligament tension with plug diameter.

\[ d = \text{plug diameter} \]
\[ V_f = \text{fibre volume fraction} \]

\[ \bar{\sigma} = \text{average ligament tension} = \frac{2 \int_{A} A f P_y \, dy}{(D-d)} \text{ for 4.0 plate} \]

\[ \sigma_m = \text{mean ligament tension} = \frac{W}{(w-nd)t} \]
unslotted slotted

X 4.0 inch plate
+
+ 6.2 inch plate

uniaxial load in:
X direction
Y direction

plug diameter d in.

ligament load due to force in:
X-direction, $W_{\text{lig}x} = 2t \int_{A}^{B} P_x \, dy$
Y-direction, $W_{\text{lig}y} = 2t \int_{A}^{B} P_y \, dx$

Fig. 5.3. Additional load imposed on ligaments by forces in X and Y directions separately
uniform strain at \( \Psi \) for mean ligament tension

\[ \sigma_m = \frac{\sigma_{nom}}{1 - \frac{nd}{W}(\frac{E_f}{E_t} - 1)} \]

- \( \sigma_m \): mean ligament tension
- \( \sigma_{nom} \): nominal stress
- \( n \): number of plugs
- \( d \): plug diameter
- \( W \): width of plate
- \( A_g \): area of unpierced section of the plate

\[ \sigma_{nom} = \frac{W}{A_g} = \sigma_m (1 - \frac{nd}{W}) \]

Fig. 5.4. Variation of mean ligament tension and nominal stress with plug diameter
Fig. 5-5. Variation of stress difference at points $A_1$ at $B_1$ with plug diameter

Loading condition: uniaxial load alone
Fig. 5.6. Variation of principal stress difference at $A_1$ at $B_1$ with plug diameter

Loading condition: uniaxial load alone
Fig. 5.7. Variation of average ligament tension, expressed as the ratio of mean ligament stress, with pin diameter. Loading condition: uniaxial load alone.
Load in:

\begin{align*}
\sigma_m &= \frac{W}{t \leq l_w} = \text{mean ligament tension in the unfilled model}
\end{align*}

Fig. 5.8. Variation of hoop tension at points A_1 and B_1 with plug diameter. Loading condition: uniaxial load alone.
Fig. 5.9. Variation of radial stress at points A₁ and B₁ with plug diameter
Loading condition: uniaxial, load alone
\[ \frac{P_{XB1}}{\sigma} = [K_x]_{B1} \quad \frac{P_{YA1}}{\sigma} = [K_y]_{A1} \]
Fig. 5.10. Variation of hoop stress at points $A_1$ and $B_1$ with plug diameter. Loading condition: uniaxial load alone.
The diagrams illustrate the relationship between radial stress and plug diameter for two different conditions. The equations shown are:

1. For the condition where \( P_{XB_1} \):
   \[
   \frac{P_{XB_1}}{\sigma} = \left[ K_Y \right]_{B_1}^y
   \]

2. For the condition where \( P_{XA_1} \):
   \[
   \frac{P_{XA_1}}{\sigma} = \left[ K_X \right]_{A_1}^x
   \]

The graphs show the variation of radial stress with plug diameter in inches (d inch) for different values of the pressure ratio and the constant coefficients.
radial stress

\[ \frac{P_{x_{A1}}}{\sigma_m} \quad \frac{P_{y_{B1}}}{\sigma_m} \]

plug diameter \( d \) inch

radial stress

\[ \frac{P_{x_{A1}}}{\sigma_m} \quad \frac{P_{y_{B1}}}{\sigma_m} \]

plug diameter \( d \) inch
Fig. 5.11. Variation of radial stress at points A₁ and B₁ with plug diameter.

Loading condition: uniaxial load alone.
Fig. 5.12. Variation of principal stresses at A₁ and B₁ with plug diameter

Loading condition: Equal biaxial loads
(without interference)
REFERENCES

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The immersion tank was filled to approximately three quarters the depth of viewing window with a mixture of Aroclon and liquid paraffin in the ratio 4 : 1. The shift in passage of a collimated pencil beam of light observed through the immersion fluid when a rectangular block of Araldite, $\frac{1}{2}$ inch in thickness was traversed across the beam which made an angle $\theta$ with one of the block faces as shown in figure below. The light beam shifted towards the right thus indicating that the refractive index, $\mu_1$ of immersion fluid was greater than the refractive index, $\mu_2$ of Araldite as shown by the following analysis:

For \[ \frac{\mu_1}{\mu_2} > 1.0 \quad \text{and} \quad \frac{\mu_1}{\mu_2} < 1.0 \]

Shift $= CB = S$

\[ C_1B_1 = S_1 \]

\[ S = AB \sin (r-\theta) \]

\[ S_1 = AB \sin (\theta-\theta_1) \]

\[ S = \frac{t}{\cos r} x \sin (r-\theta) \quad 4.1 \]

\[ S_1 = \frac{t}{\cos r_1} x \sin (\theta-\theta_1) \quad 4.2 \]

where $r$ and $\theta$ are the angle of incidence and refraction respectively.
Taking the absolute value of $r-\theta$ in equations 4.1 and 4.2, we obtain,

$$\text{Shift} = \frac{t}{\cos r} \times \sin |r-\theta| \quad \ldots \ldots 4.3$$

Also

$$\frac{\sin r}{\sin \theta} = \frac{\mu_1}{\mu_2}$$

or

$$r = \sin^{-1} \left( \sin \theta \times \frac{\mu_1}{\mu_2} \right)$$

Substituting the value of $r$ in eq. 4.3

$$\text{Shift} = \frac{t \times \sin \left| \sin^{-1} \left( \sin \theta \times \frac{\mu_1}{\mu_2} \right) - \theta \right|}{\cos \left[ \sin^{-1} \left( \sin \theta \times \frac{\mu_1}{\mu_2} \right) \right]}$$

Small quantities of liquid paraffin were added to the immersion fluid and a decrease in shift was observed each time. This process was continued until the shift vanished altogether. In case the shift had increased with the addition of liquid paraffin, the procedures would have been carried out by adding Aroclon.
APPENDIX B

Having determined the integral part of the fringe numbers by inspection, the fractional part, at a general point M, was measured as follows:

(a) Crossed polariser and analyser were rotated until an isoclinic passes through point M.

(b) Rotated the crossed polariser and analyser through a further angle of 45°.

(c) Analyser quarter wave plate was inserted with its axes parallel to the polarising directions, to retain the dark field.

(d) Polariser and analyser were uncoupled, and the Senarmont scale was set to zero.

(e) The analyser was rotated until the next integral fringe, below the value at M, moved to span the point. If the rotation required to bring the nth fringe to M was equivalent to a reading r on the Senarmont scale then;

total retardation at M = n + r fringes.
APPENDIX C

The separation of principal stresses along the Cartesian axes using normal and oblique incidence readings may be written as:

From normal measurements,

\[ P_x - P_y = \frac{N_1 f}{t} \quad \ldots 4.4 \]

Rotation of the model by \( \theta \) about \( x \)-axis gives,

\[ P_x - P_y \cos^2 \theta = \frac{N_2 f}{t} x \cos \theta \quad \ldots 4.5 \]

where \( P \) is the principal stress in lbf. per sq.inch and the suffices \( x \) and \( y \) refer to the appropriate cartesian axes.

\( N \) is the isochromatic fringe number with suffices 1 and 2 corresponding to the normal and oblique measurements respectively.

Material fringe value, \( f = 52.5 \) lbf. per sq.in.

per fringe

per inch thickness;

and \( t \) is the thickness of the model in inches.

From equations 4.4 and 4.5 we obtain

\[ P_x = \frac{f \cos \theta}{t \sin^2 \theta} \left\{ N_2 - N_1 \cos \theta \right\} \]

and

\[ P_y = P_x - \frac{N_1 f}{t} \]
APPENDIX D

Lame's thick cylinder formulae are:

\[ \sigma_{rr} = A - \frac{B}{r^2} \quad \ldots \quad 4.6 \]

\[ \sigma_{\phi\phi} = A + \frac{B}{r^2} \quad \ldots \quad 4.7 \]

where \( \sigma_{rr} \) and \( \sigma_{\phi\phi} \) are the radial and hoop stresses respectively in the matrix at any radius; \( r \) and, \( A \) and \( B \) are the constants.

For the Matrix

With Free Outer Boundary condition

\[ \sigma_{rr} = 0 \quad \text{at} \quad r = r_o \]

and \[ \sigma_{rr} = -p \quad \text{at} \quad r = r_i \]

where \( p \) is the pressure at interface.

Substituting these values of \( \sigma_{rr} \) in equation 4.6 and eliminating constants \( A \) and \( B \) from equations 4.6 and 4.7 we obtain;

\[ \sigma_{rr} = \frac{p r_i^2}{r_o^2 - r_i^2} \left( 1 - \frac{r_o^2}{r^2} \right) \quad \ldots \quad 4.8 \]
and
\[ \sigma_{\phi\phi} = \frac{p r_i^2}{r_o^2 - r_o^2} \left( 1 + \frac{r_o^2}{r^2} \right) \] 

Area of the hexagon and also of the superimposed circle = \( 6 \times \frac{0.5}{\sqrt{3}} \times 0.5 = 0.866 \text{ sq. in.} \)

Therefore, the radius of the outer boundary, \( r_o = 0.525 \text{ in.} \)

Relative interference at the fibre-matrix interface

\[ \frac{U}{r_i} = \frac{U_m - (-U_p)}{r_i} \]

where \( U \) is the radial displacement and suffixes \( m \) and \( p \) denote the matrix and the plug respectively.

If \( \varepsilon \) represents the hoop strain at the interface

then
\[ \varepsilon_m = \frac{2\pi(r_i + U_m) - 2\pi r_i}{2r_i} \]

\[ \varepsilon_m = \frac{U_m}{r_i} \]

Similarly \( \varepsilon_p = \frac{-U_p}{r_i} \)
Usually, \( r_i \) is far greater than \( U \), therefore \( r_i \) is taken as the nominal radius of the plug and the bore before assembly.

Also

\[
\epsilon_m = \frac{\sigma_{\Phi m}}{E_m} - \nu \frac{\sigma_{rrm}}{E_m} \quad \ldots \ldots \quad 4.13
\]

and

\[
\epsilon_p = \frac{\sigma_{\Phi p}}{E_p} - \nu \frac{\sigma_{rrp}}{E_p} \quad \ldots \ldots \quad 4.14
\]

For the plug, \( \sigma_{rrp} \) and \( \sigma_{\Phi p} \) to be finite when \( r = 0 \) in equations 4.6 and 4.7, the value of constant \( B \) should be zero;
therefore

\[
\sigma_{rrp} = \sigma_{\Phi p} = -p \quad \ldots \ldots \quad 4.15
\]

Also

\[
\sigma_{rrm} = -p \quad \ldots \ldots \quad 4.16
\]

Substituting \( \sigma_{\Phi m} \) from equation 4.9 and \( \sigma_{rrp} \), \( \sigma_{\Phi p} \) and \( \sigma_{rrm} \) from equations 4.15 and 4.16 in equation 4.13 and 4.14, we obtain

\[
\epsilon_m = \frac{p}{E_m} \left[ \frac{r_i^2}{r_o^2 - r_i^2} \left( 1 + \frac{r_o^2}{r_i^2} \right) + \nu \right] \quad \ldots \ldots \quad 4.17
\]

and

\[
\epsilon_p = \frac{-p}{E_p} \left( 1 - \nu \right) \quad \ldots \ldots \quad 4.18
\]

From equations 4.11, 4.12, 4.17 and 4.18 the percentage interference for a free outer boundary condition may be
written as

\[
\frac{U}{r_i} = p \left[ \frac{1}{E_m} \left( \frac{r_o^2 + r_i^2}{r_o^2 - r_i^2} \right) + \frac{1}{E_p} (1 - \nu_p) \right] \quad \ldots \quad 4.19
\]

For free outer boundary condition, the values of interface pressure, \( p \) were calculated for different plug sizes and the corresponding hoop stresses were subsequently obtained from the equation 4.9.

Fixed Outer Boundary Condition:

For the matrix;

At the outer boundary, \( r_o \)

\[ \epsilon_{\phi\phi} = 0 \]

or \( \sigma_{\phi\phi} - \nu \sigma_{rr} = 0 \) \quad \ldots \quad 4.20

Substituting the values of \( \sigma_{rr} \) and \( \sigma_{\phi\phi} \) from equations 4.6, 4.7 in equation 4.20, we obtain

\[
A = -\frac{B}{r_o^2} \left( \frac{1 + \nu_m}{1 - \nu_m} \right) \quad \ldots \quad 4.21
\]

At the inner boundary, \( r_i \)

substituting \( A \) from equation 4.21 in equation 4.6

\[
\sigma_{rr_i} = -p = -\frac{B}{r_o^2} \left( \frac{1 + \nu_m}{1 - \nu_m} \right) - \frac{B}{r_i^2}
\]
or \[ B = \frac{p_i r_i^2 r_o^2 (1 - \nu^m)}{r_i^2 (1 + \nu^m) + r_o^2 (1 - \nu^m)} \] \[ \cdots \cdots \cdots 4.22 \]

and \[ A = \frac{-p_i r_i^2 (1 + \nu^m)}{r_i^2 (1 + \nu^m) + r_o^2 (1 - \nu^m)} \] \[ \cdots \cdots \cdots 4.23 \]

From equations 4.22 and 4.23, substituting A and B in equations 4.6 and 4.7, we obtain

\[ \sigma_{\Phi m} = \frac{p_i r_i^2}{r_i^2 (1 + \nu^m) + r_o^2 (1 - \nu^m)} \left[ -(1 + \nu^m) + \frac{r_o^2}{r_i^2} (1 - \nu^m) \right] \] \[ \cdots \cdots \cdots 4.24 \]

and

\[ \sigma_{rrm} = \frac{p_i r_i^2}{r_i^2 (1 + \nu^m) + r_o^2 (1 - \nu^m)} \left[ -(1 + \nu^m) - \frac{r_o^2}{r_i^2} (1 - \nu^m) \right] \]

At \( r = r_i \), \( \sigma_{rrm} = -p \) \[ \cdots \cdots \cdots 4.25 \]

For the plug, the radial and hoop stresses are given by the equation 4.15 of the free outer boundary condition.

From equation 4.10, the relative interference is

\[ \frac{U}{r_i} = \left( \frac{\sigma_{\Phi m}}{E_m} - \nu_m \frac{\sigma_{rrm}}{E_m} \right) + \left( \frac{\sigma_{\Phi p}}{E_p} - \nu_p \frac{\sigma_{rrm}}{E_p} \right) \] \[ \cdots \cdots \cdots 4.26 \]

Substituting the values of hoop and radial stresses, from equations 4.24, 4.25 and 4.15, in equation 4.26 the values
of interface pressures, $p_i$ for fixed outer boundary condition were calculated for various plug sizes. Substituting these values of $p_i$ in equation 4.24, the corresponding hoop stresses were obtained at the interface.

The values of Poisson's ratio $\nu$, Young's modulus of elasticity $E$, and the relative interference $U/r_i$ were taken as

- $\nu_m = 0.38$
- $E_m = 0.45 \times 10^6$ lbf. per sq. in.
- $\nu_p = 0.31$
- $E_p = 11 \times 10^6$ lbf. per sq. in.
- and $\frac{U}{r_i} = 0.004$

Using these values the hoop and radial stresses in the matrix at the interface were obtained for both free and fixed outer conditions and are shown in Table 4.1.
ADHESION:

Since early man first noted that blood caused his hair to stick together very tenaciously, the science of adhesion has made a considerable advance. Sir Isaac Newton wrote in his book on Opticks, two and one-half centuries ago: "There are agents in nature which able to make the particles of joints stick together by very strong attractions and it is the business of experimental philosophy to find them out." Newton was far ahead of his time, and it is only within the past few years that a real attempt has been made in understanding the forces operating in adhesion.

In early 1950's, the days of advent of fibre reinforced plastics, glass fibre was dominantly used in fabric form with polyester resin as matrix. The glass fibre has low modulus of elasticity, and the development of new higher modulus fibres such as boron, graphite, silicon carbide, and beryllium which have strength similar to that of glass fibres and densities as low as or lower than glass. The chemical fibres offer a greater degree of freedom with regard to fibre diameter and length. Newly developed high modulus resins, such as the cyclo-aliphatic epoxies, and high temperature
resistant resins, such as polybenzimidazole and polyimide resins give another degree of freedom for selecting the appropriate type of resin and fibre for a given application.

In making selection of new or more versatile materials it must be recognized that the fibre and matrix materials must function in an integrated manner in a composite to resist externally applied loads, with the load being transferred from fibre to fibre through the matrix and through the fibre-matrix interface. Therefore, the matrix must provide a sufficiently high interfacial cohesive shear strength and must have a sufficiently high interfacial shear strength, either through chemical or mechanical adhesion or a combination of the two. The shear strengths must be adequate under the anticipated surface conditions, such as elevated temperature and high humidity, both of which reduce the interfacial shear strength at a faster rate than the matrix cohesive shear strength. It is therefore necessary to investigate the nature of fibre-matrix interface to provide scientific information which will lead to improved load transfer characteristics and thereby result in more structurally efficient composites.

There are two principal theories on how load transfer between fibres and the matrix is achieved:

(1) The mechanical bonding, friction or shrink
fit theory.

(2) Chemical coupling between the constituents.

According to the first theory the resin shrinks around fibre during polymerization or cure of the resin, and provides sufficient force to give frictional resistance to the movement of the fibre through the cured resin.

In a loaded composite the tangential forces, developed at the interface, are very complex and not completely understood. Among the effects that must be considered are the deformation of the surface, material interchange, and chemical and thermal changes. Therefore empirical "laws" are used with coefficients determined experimentally.

L. da Vinci (1452-1519), G. Amontous (1699) and C.A. Coulomb (1785) observed that in many situations "friction produces double the effort if the weight be doubled" and "friction made by same weight will be of equal resistance at the beginning of the movement, although the contact may be of different breadths or lengths".

The second factor is the possible chemical coupling between the constituents. The elements of this theory are that certain functional groups in the polymer chemically react with the fibre surface to form a chemical bond, or a separate chemical coupling agent is used, such as adhesive materials, one part of which reacts with the fibre surface
and another part of which reacts with, or is compatible with the polymeric matrix.

In adhesion, the attractive forces, between two macroscopic objects, originate at a molecular level can often exert their influence over distance which are quite large when compared with the atomic dimensions.

Van der Waals' (1873) was one of the first in the field of intermolecular attraction to explain the properties of real gases with his equation:

\[( p + \frac{a}{V^2} ) \cdot ( V - b ) = n R T \]

where \( p \) is the pressure of the gas,

\( V \) its volume,

\( n \) the number of moles,

\( R \) is the gas constant, and

\( T \) is the absolute temperature.

The constant "a" was introduced to allow for the attraction between the molecules of the gas and is universally known as the van der Waals' constant of attraction.

The constant "b" allows for the finite volume of the gas molecules, a factor which becomes important at high pressure. The constants a and b can readily be determined if the critical pressure, volume and temperature of the gas are known.

Considering the potential energy curve in the
following figure, for the interaction of two molecules. The expression for "a" may be written as:

\[ a = \frac{1}{2} \int_{X}^{\infty} U(r) \cdot dv \]

where \( U(r) \) is the potential energy at distance \( r \), and \( dv \) is the volume element over which the integration is performed.

Energy and Force of interaction between two molecules.
The distance \( x \) represents the diameter of the molecule which is responsible for the magnitude of the co-volume term that is, it is the distance at which a strong repulsion between the molecules is first experienced.

The force of attraction \( F \) can be written as:

\[
F = -\frac{d[U(r)\,]}{dr}
\]

The molecular forces which determined the van der Waals' attraction was not clear until 1930 when it was found that the following three different effects contributed to the overall van der Waals' attraction.

(1) Interaction of permanent dipole moments; by Keesom.
(2) The polarising action of one molecule on another; by Debye.
(3) With apolar atoms on attraction which originated in a quantum mechanical effect; by London.

The Keeson effect:

Keeson pointed out that the attractive forces, between two molecules with permanent dipole movements, are maximum when the dipoles are aligned end-to-end \((\delta^+ \text{ to } \delta^-)\), this effect is usually known as "orientation effect".

The mutual potential energy of attraction between two molecules of dipole moments \( \mu_1 \) and \( \mu_2 \), is
given by

\[ U_{\text{Keeson}} = - \frac{2 \mu_1 \mu_2}{3 kT \cdot r^6} \]

where \( k \) is the Boltzmann constant, and 
\( T \) is the absolute temperature.

The Debye effect:

It was observed by Debye that the attractive forces did not reduce appreciably at high temperatures. Therefore, he concluded that an additional attractive effect existed on account of polarising action of one molecule on another molecule, and is also known as "induction effect".

If \( \alpha_1 \) and \( \alpha_2 \) are the polarisability of the two molecules, the mutual attractive potential energy of the molecules is:

\[ U_{\text{Debye}} = - \frac{\alpha_1 \mu_2^2 - \alpha_2 \mu_1^2}{r^6} \]

This could not explain the attraction between the molecules in inert gases. From the point of view of wave mechanics such molecules were spherically symmetrical and on time-average basis did not possess a dipole or quadrupole moment.

The London effect:

In 1930 London pointed out that various electronic
configurations would exist if one considered an instantaneous picture of the molecules, instead of time-average effect. Thus for a short time intervals, the molecules would possess dipoles, which would act on the neighbouring molecules and induce dipoles in them, resulting attraction between molecules.

By quantum mechanical treatment of the problem, London found that the mutual attractive potential energy was given by:

\[
U_{\text{London}} = -\frac{3\hbar}{2} \frac{\nu_1 \nu_2}{(\nu_1 + \nu_2)} \frac{\alpha_1 \alpha_2}{r^6}
\]

where \( \hbar \) is Planck's constant,
\( \nu_1 \) and \( \nu_2 \) the characteristic frequencies,
\( \alpha_1 \) and \( \alpha_2 \) the polarisabilities of the two molecules.

The London forces are also known as "dispersion forces" since the characteristic frequency \( \nu \) is directly related to the dispersion formula.

The quantum energy, \( \frac{2\hbar}{\nu_1 + \nu_2} \frac{\nu_1 \nu_2}{(\nu_1 + \nu_2)} \) (\( \approx \hbar \nu_0 \) when \( \nu_1 = \nu_2 = \nu_0 \)), was taken by London as being approximately equal to the ionisation potential of the molecule.

One other conclusion reached by London was that "the simultaneous interaction of many molecules can simply be built up as an additive superimposition of single forces between pairs".
The attractive forces between two plates:

The van der Waals' forces of attraction obtained by the three contributions fall off inversely as the sixth power of the distance and therefore are essentially short range forces. However, Kallman and Willstätter suggested that summation of the London forces might explain the long range attractive forces responsible for a number of colloidal phenomena.

Summation of the London forces for the molecules in one flat plate and evaluation of their interaction with similar arrays of molecules in another flat plate, in vacuum, at a distance \( r \) leads to the following expression for the potential energy of attraction per square centimeter of the plate,

\[
U_A = -\frac{\pi^2 q^2 \lambda}{12} \left\{ \frac{1}{r^2} + \frac{1}{(r + 2 \delta)^2} + \frac{2}{(r + \delta)^2} \right\}
\]

where \( \delta \) is the thickness of the plate,

\( q \) the number of atoms per cm\(^3\) of the plates, and

\( \lambda \) equals \( \frac{3}{4} h \nu_0 \kappa^2 \)

The quantity \( \pi^2 q^2 \lambda \) is a constant for a given material and is usually given the symbol \( A \) and often referred to as Hamaker constant.

If the distance between the plates is small compared with their thickness, we obtain

\[
U_A = -\frac{\pi^2 q^2 \lambda}{12 r^2}
\]
Thus the mutual potential energy of attraction now varies inversely as the square of the distance between the plates. Under these conditions the London forces can exert their influence over a comparatively long range. The force of attraction per unit area between the plates is

\[ F = \frac{d U_A}{dr} = \frac{\pi^2 q^2 \lambda}{6r^3} \]