Filter Bank based Multicarrier Systems for Future Wireless Networks

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Submitted for the Degree of
Doctor of Philosophy
from the
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March 2018

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Abstract

Filter bank based multicarrier (FBMC) systems are one of the promising waveform candidates to satisfy the requirements of future wireless networks. FBMC employs prototype filters with lower side lobe and faster spectral decay, which enables it to have the advantages of reduced out-of-band energy and theoretically higher spectral efficiency (SE) compared to conventional multicarrier scheme i.e., orthogonal frequency division multiplexing (OFDM). These systems also have the ability to facilitate aggregation of non-adjacent bands to acquire higher bandwidths for data transmission. They also support asynchronous transmissions to reduce signaling overhead to meet the ever increasing demand of high data rate transmission in future wireless networks. In this work, the primary research objective is to address some of the critical challenges in FBMC systems to make it viable for practical applications. To this end, the following contributions are provided in this thesis.

First of all, despite numerous advantages, FBMC systems suffers from long filter tails which may reduce the SE of the system. Filter output truncation (FOT) can reduce this overhead by discarding the filter tails but may also destroy the orthogonality in FBMC system. As a result, the signal to interference ratio (SIR) can be significantly degraded. To address this problem, we first presented a theoretical analysis on the effect of FOT in a multiple input multiple output (MIMO) FBMC system, when assuming that transmitter and receiver have the same number of antennas. We derive the matrix model of MIMO-FBMC system which is subsequently used to analyze the impact of finite filter length and FOT on the system performance. The analysis reveals that FOT can avoid the overhead in time domain but also introduces extra interference in the received symbols. To combat the interference terms, we then propose a compensation algorithm that considers odd and even overlapping factors as two separate cases, where the signals are interfered by the truncation in different ways. A general form of the compensation algorithm is then proposed to compensate all the symbols in a MIMO-FBMC block to improve the SIR values of each symbol for better detection at the receiver.

Secondly, transmission of quadrature modulated symbols using FBMC systems has been an issue due to the self-interference between the transmitted symbols both in the time and frequency domain (so-called intrinsic interference). To address this issue, we propose a novel low complexity interference-free FBMC system with QAM modulation (FBMC/QAM) using filter deconvolution. The proposed method is based on inversion of the prototype filters which completely removes the intrinsic interference at the receiver and allows the use of quadrature modulated signaling. The interference terms in FBMC/QAM with and without the proposed system are analyzed and compared in terms of mean square error (MSE). It is shown with theoretical and simulation results that the proposed method cancels the intrinsic interference and improves the output signal to interference plus noise ratio (SINR) at the expense of slight enhancement of residual interferences caused by multipath channel. The complexity of the proposed system is also analyzed along with performance evaluation in an asynchronous multi-service scenario. It is shown that the proposed FBMC/QAM system with filter deconvolution outperforms the conventional OFDM system.

Finally, subcarrier index modulation (SIM) a.k.a., index modulation (IM) has recently emerged as a promising concept for spectrum and energy-efficient next generation wireless communications systems due to the excellent trade-offs they offer among error performance, complexity, and SE. Although IM is well studied for OFDM, FBMC with index modulation (FBMC-IM) has not been thoroughly investigated. To address this topic, we shed light on the potential and
implementation of IM technique for FBMC system. We first derived a mathematical matrix model of FBMC-IM system (FBMC/QAM-IM) along with the derivation of interference terms at the receiver due to channel distortions and the intrinsic behavior of the transceiver model. We have analytically shown that the interference power in FBMC/QAM-IM is smaller compared to that of conventional FBMC/QAM system as some subcarriers are inactive in FBMC/QAM-IM system. We then evaluated the performance of FBMC/QAM-IM in term of MSE, SIR and output SINR. The results show that combining IM with FBMC/QAM can improve the system performance since the inactive subcarriers are not contributing to the overall interference in the system. Based on the interference analysis, we then proposed an improved log-likelihood ratio (LLR) detection scheme for FBMC/QAM-IM system. At the end, BER performance of FBMC/QAM system with and without IM is presented and it can be seen that since the power from inactive subcarriers is reallocated to the active subcarriers in FBMC/QAM-IM, the system shows improved performance compared to conventional FBMC/QAM system.

**Key words:** FBMC, filter output truncation, MIMO, intrinsic interference, filter deconvolution, index modulation, interference analysis

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Acknowledgements

In the name of ALLAH, the most beneficent, the most merciful. I would like to thank my supervisors Prof. Pei Xiao and Prof. Muhammad Ali Imran for their guidance and support at every level. I really appreciate their always-welcoming attitude. I would also like to thank Dr. Lei Zhang for his guidance and technical discussions. His positive feedbacks have always helped me to carry out my research in the right direction. I would also like to thank my colleagues, Hafiz Atta Ul Mustafa, Adnan Akbar, Arsalan Saeed, Junaid Mir, Hassan Malik, Ghulam Ahmad, Syed Sameed Husain, Ahmed Zoha and Haris Pervaiz for their support. Moreover, I would like to acknowledge the unconditional love and support that my wife, my parents, my siblings and rest of my family have shown me during the course of my PhD. Last but not the least, I would like to gratefully acknowledge the support provided by all the staff members at Institute for Communication Systems, home of 5G Innovation Centre.
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# Nomenclature

## Acronyms

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<tr>
<td>ACI</td>
<td>Adjacent Channel Interference</td>
</tr>
<tr>
<td>AFB</td>
<td>Analysis Filter Bank</td>
</tr>
<tr>
<td>BEP</td>
<td>Bit Error Probability</td>
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<tr>
<td>BER</td>
<td>Bit Error Rate</td>
</tr>
<tr>
<td>BPSK</td>
<td>Binary Phase Shift Keying</td>
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<tr>
<td>CDMA</td>
<td>Code Division Multiple Access</td>
</tr>
<tr>
<td>CFO</td>
<td>Carrier Frequency Offset</td>
</tr>
<tr>
<td>CIR</td>
<td>Channel Impulse Response</td>
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<tr>
<td>CMT</td>
<td>Cosine Modulated Multitone</td>
</tr>
<tr>
<td>CP</td>
<td>Cyclic Prefix</td>
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<tr>
<td>CSI</td>
<td>Channel State Information</td>
</tr>
<tr>
<td>DFE</td>
<td>Decision Feedback Equalizer</td>
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<tr>
<td>DFT</td>
<td>Discrete Fourier Transform</td>
</tr>
<tr>
<td>EGF</td>
<td>Extended Gaussian Function</td>
</tr>
<tr>
<td>eMBB</td>
<td>Enhanced Mobile Broadband</td>
</tr>
<tr>
<td>FB-OFDM</td>
<td>Filter Bank OFDM</td>
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<tr>
<td>FBMC</td>
<td>Filter Bank Multicarrier</td>
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<tr>
<td>FMT</td>
<td>Filtered Multitone</td>
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<td>Abbreviation</td>
<td>Description</td>
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<tr>
<td>FOT</td>
<td>Filter Output Truncation</td>
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<tr>
<td>FSE</td>
<td>Frequency Spreading Equalization</td>
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<tr>
<td>GF</td>
<td>Gaussian Function</td>
</tr>
<tr>
<td>IAI</td>
<td>Inter Antenna Interference</td>
</tr>
<tr>
<td>IBI</td>
<td>Inter Block Interference</td>
</tr>
<tr>
<td>ICI</td>
<td>Inter Carrier Interference</td>
</tr>
<tr>
<td>IDFT</td>
<td>Inverse Discrete Fourier Transform</td>
</tr>
<tr>
<td>IIEC</td>
<td>Intrinsic Interference Estimation and Cancellation</td>
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<tr>
<td>IM</td>
<td>Index Modulation</td>
</tr>
<tr>
<td>IoT</td>
<td>Internet of Things</td>
</tr>
<tr>
<td>IOTA</td>
<td>Isotropic Orthogonal Transform Algorithm</td>
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<tr>
<td>ISI</td>
<td>Inter Symbol Interference</td>
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<tr>
<td>LDS</td>
<td>Low Density Signature</td>
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<tr>
<td>LLR</td>
<td>Log Likelihood Ratio</td>
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<tr>
<td>LTE</td>
<td>Long Term Evolution</td>
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<tr>
<td>MBB</td>
<td>Mobile Broad Band</td>
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<tr>
<td>MCM</td>
<td>Multicarrier Modulation</td>
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<tr>
<td>MIMO</td>
<td>Multiple Input Multiple Output</td>
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<tr>
<td>ML</td>
<td>Maximum Likelihood</td>
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<tr>
<td>MLSE</td>
<td>Maximum Likelihood Sequence Estimation</td>
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<tr>
<td>MMSE</td>
<td>Minimum Mean Square Error</td>
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<tr>
<td>MSE</td>
<td>Mean Square Error</td>
</tr>
<tr>
<td>MUSA</td>
<td>Multi User Shared Access</td>
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<tr>
<td>NOMA</td>
<td>Non Orthogonal Multiple Access</td>
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<tr>
<td>OFDM</td>
<td>Orthogonal Frequency Division Multiplexing</td>
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<td>OFDM-IM</td>
<td>OFDM with Index Modulation</td>
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<tr>
<td>Abbreviation</td>
<td>Description</td>
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<tr>
<td>OoBR</td>
<td>Out of Band Radiation</td>
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<tr>
<td>OQAM</td>
<td>Offset Quadrature Amplitude Modulation</td>
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<tr>
<td>PAM</td>
<td>Pulse Amplitude Modulation</td>
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<tr>
<td>PHYDYAS</td>
<td>Physical Layer for Dynamic Spectrum Access</td>
</tr>
<tr>
<td>QAM</td>
<td>Quadrature Amplitude Modulation</td>
</tr>
<tr>
<td>QPSK</td>
<td>Quadrature Phase Shift Keying</td>
</tr>
<tr>
<td>SCMA</td>
<td>Sparse Code Multiple Access</td>
</tr>
<tr>
<td>SD</td>
<td>Spatial Diversity</td>
</tr>
<tr>
<td>SE</td>
<td>Spectral Efficiency</td>
</tr>
<tr>
<td>SFB</td>
<td>Synthesis Filter Bank</td>
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<tr>
<td>SIC</td>
<td>Successive Interference Cancellation</td>
</tr>
<tr>
<td>SIM</td>
<td>Subcarrier Index Modulation</td>
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<tr>
<td>SINR</td>
<td>Signal to Interference plus Noise Ratio</td>
</tr>
<tr>
<td>SIR</td>
<td>Signal to Interference Ratio</td>
</tr>
<tr>
<td>SISO</td>
<td>Single Input Single Output</td>
</tr>
<tr>
<td>SLNR</td>
<td>Signal to Leakage plus Noise Ratio</td>
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<tr>
<td>SM</td>
<td>Spatial Multiplexing</td>
</tr>
<tr>
<td>SMT</td>
<td>Staggered Multitone</td>
</tr>
<tr>
<td>SNR</td>
<td>Signal to Noise Ratio</td>
</tr>
<tr>
<td>STBC</td>
<td>Space Time Block Coding</td>
</tr>
<tr>
<td>URC</td>
<td>Ultra Reliable Communication</td>
</tr>
<tr>
<td>URLLC</td>
<td>Ultra Reliable Low Latency Communication</td>
</tr>
<tr>
<td>V2X</td>
<td>Vehicle to Everything</td>
</tr>
<tr>
<td>WiMax</td>
<td>Worldwide Interoperability for Microwave Access</td>
</tr>
<tr>
<td>WL</td>
<td>Widely Linear</td>
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<tr>
<td>WLP</td>
<td>Widely Linear Processing</td>
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<td>ZF</td>
<td>Zero Forcing</td>
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Introduction

The demand of higher data rates have been increasing drastically over the past several years. One way to fulfil this requirement is to use signals with wider bandwidth but wideband signals are subject to frequency selective fading from the multipath channels. In such cases single carrier systems are not well suited. For wideband signals, multicarrier modulation (MCM) is the most prominent technique that can overcome the fading effect by dividing the wide band signal into several narrow band signals that can handle frequency selective fading very effectively [1].

The MCM scheme used so far in existing systems, such as WiFi based on the IEEE 802.11 standard, WiMax (Worldwide Interoperability for Microwave Access) based on the IEEE 802.16 standard, Long Term Evolution (LTE), LTE-advanced etc. are OFDM based [2,3] and a large body of the literature focuses on the use of OFDM based multicarrier systems for practical applications. The large popularity of OFDM mainly comes from a number of attractive features such as its robustness to multipath fading effects, its high SE due to the closely spaced orthogonal subcarriers and its ability to avoid both intersymbol interference (ISI) by using sufficient guard time and intercarrier interference (ICI) by appending a cyclic prefix (CP) in the guard interval. Additionally, If the length of CP is more than the maximum channel delay spread, the system can also elegantly equalize a frequency selective channel with a single complex coefficient per subcarrier. However, despite its several advantages, OFDM suffers from susceptibility to carrier frequency offset (CFO) resulting in ICI and the use of CP not only reduces the effective throughput of the transmission but also increases transmit power. More importantly, OFDM suffers from significant spectral leakage due to the use of rectangular pulse that has poor frequency localization and thus require large guard bands to protect nearby channels which also reduces the SE of the system. This presents a major source of problem that limits the applicability of OFDM in some present and future communication systems [4]. The
aforementioned shortcomings limit the utilization of OFDM as a suitable waveform for future wireless networks and motivated researchers to look for alternative solutions and propose enhanced physical layers for future wireless networks.

### 1.1 Scope and Objectives

FBMC system is one of the promising candidate waveforms to satisfy the requirements of future wireless networks. Due to numerous advantages, it has emerged as a promising alternative to conventional OFDM scheme. FBMC system utilizes a specially designed prototype filter which is well localized both in time and frequency that enables it to have higher SE and better spectral containment\(^\text{[5,6]}\). FBMC also has the ability to facilitate aggregation of non-adjacent bands to acquire higher bandwidths for data transmission along with asynchronous transmissions to reduce signaling overhead to meet the ever increasing demand of high data rate transmission in future wireless networks. There are mainly three FBMC techniques that are available in literature: filtered multitone (FMT)\(^\text{[7,8]}\), cosine modulated multitone (CMT)\(^\text{[9]}\) and staggered multitone (SMT)\(^\text{[10]}\) which is also known as OFDM with offset quadrature amplitude modulation (OFDM/OQAM) or Saltzberg’s method. Saltzberg showed, in\(^\text{[11]}\), that by introducing a shift of half the symbol period between the real and imaginary components of a complex quadrature amplitude modulation (QAM) symbols, it is possible to achieve a baud-rate spacing between adjacent subcarrier channels and still recover the information symbols free of ISI and ICI. Thus, each subcarrier is modulated with an offset QAM (OQAM) and the orthogonality conditions are considered only in the real field\(^\text{[12]}\). This is because according to Balian-Low theorem\(^\text{[13–15]}\), there is no way to utilize a well-localized prototype filter in both time and frequency, along with maintaining orthogonality and also transmitting at Nyquist rate. However, relaxing the orthogonality condition can guarantee the other two factors. In this case, the data is carried only by the real (or imaginary) components of the signal while the imaginary (or real) parts appear as interference terms at the receiver side. The strict synchronization requirements in conventional OFDM based systems are also much relaxed for FBMC system. This facilitates low complexity implementation of multi-user access in uplink transmissions for FBMC systems\(^\text{[16,17]}\). Due to these advantages, FBMC is considered as a key area of research for the past several years and one of the most promising waveform candidate for future wireless networks\(^\text{[18,19]}\).

Despite various advantages over conventional OFDM systems, there are also some open challenges in FBMC that needs attention to make it viable for practical applications. In this work, the primary research objective is to address some of the critical challenges in FBMC systems.
to make it a strong waveform candidate for future wireless networks. The first challenge is related to the SE of the FBMC system. Although, FBMC has higher SE compared to conventional OFDM system due to the use of well localized prototype filter that ensures ISI and ICI are avoided without the use of CP. However, FBMC systems suffers from long filter tails which may reduce the SE of the system. These long tails results from the fact that transmit filtering affect the localization of FBMC system in time domain. This reduces the actual efficiency of the system due to the filter transients when passing the transmit signal through the polyphase filter. The transmission efficiency $\eta$ can be dropped by the following proportion

$$\eta = \frac{M}{M + K - 1} \quad (1.1)$$

where $M$ is the number of symbols per transmission block and $K$ is the length of each prototype filter. Although, this overhead can be negligible for long transmission blocks. However, this overhead can be significant when the transmitted data is divided into shorter blocks. Hence, the transmission might become challenging in the applications with short messages such as machine-type communications [20][21]. Filter output truncation (FOT) can reduce this overhead by discarding the filter tails at the transmitter but may also destroy the orthogonality in FBMC system. As a result, the SIR can be significantly degraded.

The second challenge is related to the transmission of quadrature modulated symbols in FBMC systems. Since orthogonality in FBMC system only holds in the real field, the received symbols are contaminated with pure imaginary self-interference terms. To satisfy the orthogonality requirements, the FBMC system has to transmit a real-valued symbol every half symbol duration, resulting in the so-called FBMC/OQAM system [22]. The issue of self-interference between the transmitted symbols both in the time and frequency domain (so-called intrinsic interference) is inherent in FBMC/OQAM system. This intrinsic self-interference not only refrain the transmission of quadrature modulated symbols but also result in many other issues in FBMC/OQAM transceivers. Those issues include channel estimation process to be not as straightforward as OFDM systems also the FBMC/OQAM system can not perform properly with just a single-tap equalization in highly dispersive channels [23]. Furthermore, applications such as MIMO, ML detection and the Alamouti space-time block coding (STBC) are not directly applicable to FBMC/OQAM systems. A novel multicarrier waveform called filter bank OFDM (FB-OFDM) is proposed by ZTE in [24]. This waveform is a combination of OFDM and filter bank and adopts transmission of QAM symbols unlike FBMC/OQAM which utilizes real-valued symbols to modulate the subcarriers. This makes it easier to combine FB-OFDM with techniques like MIMO. However, the problem of intrinsic interference still exists in FB-OFDM. Thus, this system is categorized as a non-orthogonal system, and the challenge here is to mitigate the intrinsic interference to make the system quasi-orthogonal while maintaining
per-subcarrier filtering.

Finally, Subcarrier index modulation (SIM) has recently emerged as a promising concept for spectrum and energy-efficient next generation wireless communications systems due to the excellent trade offs they offer among error performance, complexity, and SE [25]. Although IM is well studied for OFDM, FBMC with index modulation has not been thoroughly investigated. The objective is to investigate the potential of combining IM technique with QAM based FBMC systems to meet the demands for future wireless networks.

1.2 Contributions

The main contributions in this work are related to addressing some of the key challenges in conventional FBMC systems. That includes improving SE of FBMC system by avoiding extra overhead introduced by transmit filtering, achieving quadrature modulated symbol transmission in FBMC system by canceling the intrinsic interference inherent in the system and investigating the potential of combining FBMC system with an emerging 5G modulation technique to meet the demands of future wireless networks. The main contributions of this thesis are summarized as follows:

In the first contribution, we targeted SE improvement in MIMO-FBMC system. Since, FBMC systems suffers from long filter tails due to transmit filtering, this may reduce the SE of the system. To avoid this extra overhead we introduced FOT in MIMO-FBMC system. Although, FOT can reduce this overhead but may also destroy the orthogonality in the system. As a result, the SIR in MIMO-FBMC system can be significantly degraded. To address this challenge, the main work in this contribution is summarized as follows:

- We first derive a compact matrix model of MIMO-FBMC system (assuming the transmitter and receiver have the same number of antennas) which lays the ground for the subsequent in depth analysis of the effect of FOT on the detection performance in terms of the SIR and BER.

- Based on the matrix model, we then analyze the impact of finite filter length and different types of FOT on the system performance. Through simulation results, it is shown numerically that FOT can overcome the overhead in time domain but also introduces extra interference in the received symbols and significantly degrade the SIR of the symbols at the edges.

- Thirdly, based on the observations made in the aforementioned numerical analysis, we then propose a compensation algorithm to combat the interference terms. The proposed
algorithm considers odd and even overlapping factors as two separate cases, where the signals are interfered by the truncation in different ways. The general form of the compensation algorithm is then proposed to compensate all the symbols in a MIMO-FBMC block to improve the SIR values of each symbol for better detection at the receiver. The advantage of the algorithm is that it requires no overhead but can still achieve a similar performance compared to the case with no FOT.

In the second contribution, we focused on the transmission of quadrature modulated symbols using FBMC systems i.e., QAM based FBMC systems. So far it has been an issue due to the self-interference between the transmitted symbols both in the time and frequency domain (so-called intrinsic interference). To address this issue, we propose a novel low complexity interference-free FBMC system with QAM modulation (FBMC/QAM) using filter deconvolution. The proposed method is based on inversion of the prototype filters which completely removes the intrinsic interference at the receiver and allows the use of quadrature modulated signaling. The main work in this contribution is summarized as follows:

- A matrix model of the QAM based FBMC system is presented in the presence of additive noise and multipath channel. The interference terms at the receiver due to channel distortions and the intrinsic behavior of the transceiver model are also derived.

- An inverse filter matrix based on prototype filters is then introduced at the receiver to cancel the effects of intrinsic interference in the FBMC/QAM system. It has been shown analytically that the introduction of inverse filter completely removes the intrinsic interference in FBMC/QAM system.

- The interference terms including the ones introduced by the multipath channel are analyzed in terms of MSE with and without the inverse filter. It is also shown that the interference cancellation process significantly improves the system output SINR.

- Complexity analysis of the FBMC/QAM system with and without the inverse filter is also presented. It is shown that the receiver complexity in both cases have the same upper bounds.

- The performance of the proposed FBMC/QAM system is then evaluated for an asynchronous multi-service scenario and the results are compared with conventional OFDM system.

In the third contribution, the target is to investigate the potential of combining QAM based FBMC system with an emerging 5G modulation technique i.e., index modulation to meet the
demands of future wireless networks. In this regard, FBMC/QAM with index modulation system based on ML and LLR detectors are introduced as an improved transmission technique. The main work in this contribution is summarized as follows:

- A mathematical matrix model of the index modulation based FBMC/QAM system is presented along with the derivation of interference terms at the receiver due to channel distortions and the intrinsic behavior of the transceiver model.

- The interference terms including the ones introduced by the multipath channel are analyzed in terms of MSE with and without IM. It is analytically shown that the interference power in FBMC/QAM-IM is smaller comparing to that of the conventional FBMC/QAM system as some subcarriers are inactive.

- The performance of FBMC/QAM-IM is evaluated by comparing the SIR and output SINR with that of the conventional FBMC/QAM system.

- An improved LLR detector is then proposed based on the proposed interference model. It is shown with simulations that the proposed detector provide improved BER performance in FBMC/QAM-IM system compared to the conventional FBMC/QAM system.

1.3 Publications

The research carried out during the course of this PhD results into the following publications.

Journals


The rest of this thesis is organized as follows:

Chapter 2 provides the literature review focused on FBMC systems. We start with a discussion on the main differences between OFDM and FBMC system. We then introduced the principle of FBMC/OQAM modulation technique along with the prototype filters use in this scheme. We then discuss the combination of FBMC system with technique like MIMO and the related work in this particular area. Linear and widely linear processing (WLP) techniques for MIMO-FBMC systems are then introduced in this chapter. At the end, we discuss some
1.4. Thesis Outline

relevant work related to the SE in FBMC systems, quadrature modulated symbol transmission and combination of IM with FBMC systems. In Chapter 3, we first derived a compact matrix model of MIMO-FBMC system which lays the ground for the subsequent in depth analysis on the impact of finite filter length and different types of FOT on the system performance. It is shown analytically that FOT can overcome the overhead in time domain but also introduces extra interference in the received symbols and significantly degrade the SIR of the symbols at the edges. A compensation algorithm is then proposed to compensate all the symbols in a MIMO-FBMC block to improve the SIR values of each symbol for better detection at the receiver. The work done in this chapter has been published in [26]. In Chapter 4, we start with the derivation of a matrix model of the QAM based FBMC system in the presence of additive noise and multipath channel. The interference terms at the receiver due to channel distortions and the intrinsic behavior of the transceiver model are then derived. To cancel the effect of intrinsic interference in FBMC/QAM system, an inverse filter matrix based on prototype filters is then introduced at the receiver. It is shown analytically that the introduction of inverse filter completely removes the intrinsic interference and significantly improves the system output SINR. A complexity analysis of the FBMC/QAM system with and without the inverse filter is then provided. It is shown that the receiver complexity in both cases have the same upper bounds. The end of the chapter discusses the performance of the proposed system in an asynchronous multi-service scenario and the performance is compared with conventional OFDM system. The work done in this chapter has been submitted for publication in IEEE Transactions on Wireless Communications and its preprint is published in [27]. Chapter 5 begins with a mathematical model of the index modulation based QAM-FBMC system along with the derivation of interference terms at the receiver due to channel distortions and the intrinsic behavior of the transceiver model. The interference terms including the ones introduced by the multipath channel are analyzed in terms of MSE with and without IM. It is analytically shown that the interference power in FBMC/QAM-IM is smaller comparing to that of the conventional FBMC/QAM system as some subcarriers are inactive. We then investigate the performance of FBMC/QAM-IM in term of SIR and output SINR and the results are compared with that of the conventional FBMC/QAM system. At the end of the chapter, we proposed an improved LLR detector based on the interference analysis and the BER result show that FBMC/QAM-IM outperforms its conventional counterpart. The work done in this chapter has been submitted for publication in IEEE Access and its preprint is published in [28]. Chapter 6 provides the summary of the thesis, contributions and concluding remarks. It also summarizes the major findings of this thesis and highlights some potential research areas that could be explored as an extension to this work.
Chapter 2

State of the Art

In this chapter, we discuss the background and the state of the art techniques that serve as a basis for the research presented in this PhD thesis. We first discuss the fundamental difference between FBMC and conventional OFDM system along with a comparison between the two techniques in Section 2.1. We then introduce the FBMC system model in Section 2.2 along with an overview of the prototype filters used in this scheme that makes it efficient than conventional OFDM based systems in Section 2.2.1. The possibility of combining MIMO with FBMC/OQAM and related work is discussed in Section 2.3. We then discuss some work related to transmission of quadrature modulated symbols and application of IM in FBMC system in Sections 2.4 and 2.5 respectively. At the end some works related to linear and widely linear processing in MIMO-FBMC/OQAM are highlighted in Section 2.6 and 2.7.

2.1 OFDM vs. FBMC

FBMC (also known as FBMC/OQAM) is an OFDM based scheme that uses offset QAM for modulating each sub-carrier and utilizes a specially designed prototype filter that is well localized both in time and frequency domain. The prototype filter provides FBMC the ability to achieve better SE and the robustness against time and frequency dispersive channels without the use of CP unlike conventional CP based OFDM (CP-OFDM) system [5]. In OFDM, the baseband discrete signal can be written as [29, pp. 24]

\[
x[i] = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} \sum_{m=-\infty}^{\infty} s_{m,n} g[i-mN] e^{j \frac{2\pi}{N} mi}
\]

(2.1)

where, \( n \) denotes the subcarrier index, the data transmitted on the \( n^{th} \) subcarrier of the \( m^{th} \) OFDM symbol is expressed as \( s_{m,n} \) which is a quadrature modulated symbol, \( N \) represent
the total number of subcarriers in a OFDM symbol, the factor \( \frac{1}{\sqrt{N}} \) is introduced for power normalization, and \( g \) is the rectangular window function that separates the sub-channels, with its time domain coefficients defined as

\[
g[i] = \begin{cases} 
\frac{1}{\sqrt{T}} & \text{if } |i| \leq \frac{T}{2} \\
0 & \text{if } |i| > \frac{T}{2} 
\end{cases} \tag{2.2}
\]

where \( T = \frac{1}{\Delta f} = NT_s \) is the OFDM symbol duration, \( T_s \) is the sampling interval and \( \Delta f \) is the subcarrier spacing. To eliminate the ISI and ICI, a CP of length \( L_{cp} \) is added to the OFDM symbol whose length is equal or greater than the channel delay spread. Although the use of CP ensures ISI and ICI free transmission, however, the signal to noise ratio (SNR) is reduced by a factor \( \alpha = \frac{N}{N + L_{cp}} \). Contrary to OFDM, each subcarrier in a FBMC system is modulated with a real-valued symbol to satisfy the orthogonality requirement. To maintain the same data rate as OFDM system without CP, the FBMC system transmit a real symbol every half symbol duration i.e. \( T/2 \), resulting in so called FBMC/OQAM system \[22\]. This is performed at the transmitter side where each complex data symbol \( s_{m,n} \) defined in (2.1), is separated into real/in-phase \( (s_{m,n}^I) \) and imaginary/quadrature phase \( (s_{m,n}^Q) \) components. If \( T \) represents complex OFDM symbol duration with no CP, then \( \tau_0 = \frac{T}{2} \) represent the symbol duration of the real FBMC/OQAM symbol. However, the subcarrier spacing \( \nu_0 \) in FBMC/OQAM is same as OFDM i.e., \( \nu_0 = \Delta f \). Thus for FBMC/OQAM system we have \( \tau_0 \nu_0 = \frac{1}{2} \) which means that the density of the subcarriers in time frequency plan is twice greater in FBMC/OQAM than the conventional OFDM where \( T \Delta f = 1 \). The information carried by one complex-valued OFDM symbol with duration \( T \) is now carried by two real-valued symbols in FBMC/OQAM system each with duration \( \frac{T}{2} \). Hence, the SE of FBMC/OQAM is same as that of conventional OFDM with no CP. The symbol distribution for FBMC/OQAM is shown in Fig. 2.1

![OFDM and FBMC/OQAM symbol distribution](image-url)
The discrete baseband FBMC/OQAM transmit signal can be expressed as \[29\text{, pp. 32}\]

\[
x[i] = \sum_{n=0}^{N-1} \sum_{m=-\infty}^{\infty} a_{m,n} g[i - mN/2] e^{j\frac{2\pi}{N} n(i-D/2)} e^{j\phi_{m,n}}
\] (2.3)

where \(a_{m,n}\) is the real-valued symbol which is either the real or the imaginary part of the input QAM symbol i.e., \(a_{m,n} \in \{s_{m,n}^{I}, s_{m,n}^{Q}\}\). While \(g[i]\) represent the well localized prototype filter. These filters will be introduced in Section 2.2.1 in detail. Also \(D/2\) is the delay term that depends on the length of the prototype filter i.e., \(D = KN - 1\). The phase term \(\phi_{m,n}\) is to ensure that the phase shift of \(\pm \pi/2\) is between adjacent pulse amplitude modulation (PAM) symbols along the time and frequency axis and is given as \(\phi_{m,n} = \frac{\pi}{2}(m + n)\). The details on FBMC/OQAM principle will be discussed in Section 2.2 in detail. Due to the use of these specially designed prototype filters, the ISI and ICI are avoided without the use of CP. This enables FBMC to achieve higher SE compared to conventional OFDM system. An alternate approach for implementing FBMC/OQAM system is to shifts the prototype filter instead of offsetting the QAM symbols, this approach extends the real and imaginary parts of the symbols into the whole symbol duration. This alternate method is equivalent to the traditional FBMC/OQAM implementation and the performance of the two approaches are exactly the same \[22\]. The advantage of this approach is that it avoids the staggered processing of mapping complex QAM symbols into QAM symbols \[23\]. The key differences between OFDM and FBMC systems can be explained using a top level block diagram as follows

---

**Figure 2.2: Key differences between OFDM and FBMC Systems**

It can be seen that OFDM require a cyclic prefix (CP) in its guard interval to combat ISI and ICI terms and also windowing operation is required to suppress the high side lobes in frequency domain. Whereas, FBMC can avoid ICI and ISI terms without the use of CP due to the use of per subcarrier filtering (prototype filtering) as shown in Fig. 2.2. The use of well localized prototype filters enables FBMC/OQAM system to have lower out of band radiation (OoBR) compared to conventional OFDM system. To observe the spectral containment of OFDM and FBMC system, transmission can be carried out on fragmented spectrum to examine the amount of spectral leakage from the transmission sub band into non transmission sub
2.1. OFDM vs. FBMC

band. To observe the spectral leakage, we considered a fragmented spectrum consisting of 2048 subcarriers. Where transmission is done on two sub-bands of 512 subcarriers i.e., sub-carrier 257-768 and sub-carrier 1281-1792. Whereas no transmission is done on the remaining subcarriers. The transmit spectrum of both MCM schemes are shown in Fig. 2.3. It can be observed that OFDM has high level of interference due to higher side lobes in frequency domain as compared to FBMC. If we assume a small guard band between the transmission and non-transmission sub-bands for isolation, the level of interference can be reduced significantly. Fig. 2.4 shows the reduction in the interference level by having different size of guard bands between the transmission and non-transmission sub-bands. It can be observed that with a guard...
2.1. **OFDM vs. FBMC**

Band of just 1% of the total bandwidth (BW) i.e., 2048 subcarriers, the interference level is significantly reduced in case of FBMC while a slight improvement is observed in OFDM. Hence, the low OoBR in FBMC system makes it a potential candidate waveform for future wireless applications that have strict adjacent channel interference (ACI) requirements. Despite several advantages, FBMC is more computationally complex than conventional OFDM. The computational complexity of any multicarrier scheme can be evaluated by calculating the number of real multiplications involved in computing a length-$N$ complex-valued output sequence. In OFDM, the computational complexity comes from the IFFT and FFT operations and its complexity can be written as \cite[pp. 23]{30}

$$C_{OFDM} = 2 \times [N \log_2 N - 3N + 4]$$ (2.4)

The complexity of FBMC can be evaluated by calculating the real multiplications involved in the synthesis filter bank (SFB) at the transmitter and analysis filter bank (AFB) at the receiver. From \cite[pp. 23]{30}, the complexity of the FBMC system can be written as follows

$$C_{FBMC} = 4 \times [2N + N \log_2 N - 3N + 4 + 2KN]$$ (2.5)

It can be seen from Fig. 2.5 that the FBMC is several times more complex than conventional OFDM system. It can also be noticed that the complexity of FBMC depends slightly on the overlapping factor $K$ of the prototype filters. It is worth mentioning that the average bit en-

![Figure 2.5: Computational Complexity of OFDM and FBMC Systems](image)

energy in OFDM is reduced when a CP is introduced in OFDM symbols. The average bit energy reduces with the increase in CP length, resulting in the SNR reduction. To carry out a fair comparison between OFDM and FBMC/OQAM schemes, we have to consider this SNR reduction factor in OFDM. However, FBMC/OQAM does not incur such SNR reduction as it doesn’t
2.1. **OFDM vs. FBMC**

require a CP. However, FBMC/OQAM suffer from long filter tails which may reduce the SNR of the system. These long tails results from the fact that transmit filtering affect the localization

![Retained Tail](image1.png) ![Discarded Tails](image2.png)

\[
\text{Retained Tail} \quad \text{Discarded Tails}
\]

Figure 2.6: Transmit filter output in FBMC/OQAM (K=6)

of FBMC/OQAM system in time domain. This also reduces the efficiency of the system due to the filter transients when passing the input signal through the polyphase filter. In this case the output of the filter will have \((K - 1)N\) more samples than the input due to the linear convolution operation as shown in Fig. 2.6, which means that to keep the orthogonality (minimum interference from other subcarriers and symbols), we need to transmit these tails along with the actual information. The overhead of the system will now be \((K - 1)/M\), which can be very high with large filter overlapping factor \(K\) or small block size \(M\). As a solution, the front and end elements of block could be cut out to improve the SE. A recent study has considered improving the SE in the FBMC system by tackling the over head (tails) caused by the filter operation. The authors in [31] have introduced non data symbols (virtual symbols) before and after each FBMC data packet for shortening the ramp-up and ramp-down periods. It has been discussed in [32] that at least \(N\) extra samples (i.e. 1 FBMC symbol) should be kept to save the performance. However, if we discard all the filter tails without any further compensation algorithm, this will lead to the performance degradation in terms of signal to interference ratio. Therefore, there is a trade-off between the performance and the SE. We will look into this performance trade-off and possible compensation algorithm in detail in Chapter 3. The overhead in OFDM and FBMC systems due to the use of CP and filter tails respectively can lead to a
decrease in SE of the system. The general form of SE can be defined as

\[
\eta = \delta(\Lambda)\alpha\beta
\]  

(2.6)

where \(\delta(\Lambda) = \frac{1}{|\text{det}(L)|}\) is the lattice density i.e., the density of the subcarriers in time-frequency (T-F) plane and \(\text{det}(L) = TF\) defines the determinant of the lattice geometry \(L\). In (2.6), the lattice density of OFDM without CP and FBMC is 1, the variable \(\alpha\) is the SNR reduction factor in OFDM, where as \(\beta\) is the SNR reduction factor in FBMC system. The values of \(\alpha\) and \(\beta\) for OFDM and FBMC systems are tabulated in Table 2.1 where \(T\) is the length of OFDM symbol without CP, \(L_{cp}\) is the length of cyclic prefix, \(M\) is the number of symbols per FBMC block (Note that each block can be considered as a subframe in a LTE frame structure which has 14 symbols per subframe. In this case each FBMC block can be considered as a subframe of \(M\) symbols per block) and \(K\) is the length of the overlapping factor of the prototype filter. Assuming the length of CP as normal mode in the LTE standard i.e., approx 7% of the OFDM symbol duration \((T)\) and the overlapping factor is assumed to be \(K = 5\). The spectral efficiency of OFDM and FBMC, with and without CP and tail cutting is shown in Fig. 2.7. It can be seen

\[
\text{Table 2.1: Spectral Efficiency Reduction Factors}
\]

<table>
<thead>
<tr>
<th></th>
<th>(\alpha)</th>
<th>(\beta)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>OFDM without CP</strong></td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td><strong>OFDM with CP</strong></td>
<td>(\frac{T}{T+L_{cp}})</td>
<td>1</td>
</tr>
<tr>
<td><strong>FBMC/OQAM without Tunc.</strong></td>
<td>1</td>
<td>(\frac{M}{M+K-1})</td>
</tr>
<tr>
<td><strong>FBMC/OQAM with Tunc.</strong></td>
<td>1</td>
<td>(\frac{M}{M+1})</td>
</tr>
</tbody>
</table>

Figure 2.7: Spectral efficiency of OFDM and FBMC Systems
that the SE of OFDM system without CP is maximum since there is no overhead in the system. If a CP is added to OFDM symbols we can see that the SE of the system reduces to around 93%. If we compare the results with FBMC system, we can see that without tail truncation, the SE of the system is very low for short block size. However, if we discard all the filter tails except one, we can see that the efficiency of the FBMC system surpasses the CP based OFDM system when $M \geq 14$. The SE of the FBMC system can be further improved if we are able to discard all the filter tails. However, discarding all tails can lead to performance degradation since orthogonality of the system will be affected.

2.2 FBMC/OQAM Principle

The main idea in FBMC/OQAM system is to transmit real (offset QAM) symbols instead of conventional complex (QAM) symbols as in OFDM. We assume that each FBMC block contains $M$ FBMC symbols and each symbol has $N$ subcarriers in frequency domain, then in total $MN$ modulated QAM symbols can be transmitted by one FBMC block. At the transmitter side each complex data symbol $s_{m,n}$, for $m = 0, 1, \ldots, M - 1$ and $n = 0, 1, \ldots, N - 1$, is divided into its in-phase (real part) $s_{m,n}^I$ and quadrature (imaginary part) $s_{m,n}^Q$ components. These components of complex QAM symbol can be viewed as the elements of a pair of PAM sequences that are then transmitted with a time offset $\tau_0 = T/2$. Let us denote the transmitted PAM symbol as $a_{m,n} \in \{s_{m,n}^I, s_{m,n}^Q\}$, at the $n^{th}$ subcarrier and $mT/2$ time instant. Also let us define $g[i]$ as the prototype filter with length $KN$, where $K$ is the overlapping factor of the prototype filter and $N$ is the total number of subcarriers. The discrete time base band transmitted signal in FBMC/OQAM is given in (2.3). The expression can be written in a simpler form as

$$x[i] = \sum_{n=0}^{N-1} \sum_m a_{m,n} g_{m,n}[i] \quad (2.7)$$

where $g_{m,n}[i] = g[i - mN/2] e^{j \frac{2\pi}{N} (i-D)/2} e^{j \phi_{m,n}}$ in which $e^{j \phi_{m,n}} = e^{j \frac{\pi}{2} (m+n)} = j^{m+n}$.

Assuming a noiseless and distortion free channel, the demodulated symbol over the $n^{th}$ subcarrier and the $m^{th}$ instance can be determined by utilizing the inner product of $x[i]$ and $g_{m,n}[i]$ i.e.,

$$r_{m,n} = \langle x, g_{m,n} \rangle = \sum_{i=-\infty}^{\infty} x[i] g^*_{m,n}[i]$$

$$= \sum_{n'=0}^{N-1} \sum_{m'=-\infty}^{\infty} a_{m',n'} \sum_{i=-\infty}^{\infty} g_{m',n'}[i] g^*_{m,n}[i] \quad (2.8)$$
To determine the transmultiplexer impulse response of the system, we can assume that only a unit impulse is transmitted at subcarrier \( n_0 \) and at time instant \( m_0 \). The impulse response of the transmultiplexer can be written as

\[
r_{m,n} = \sum_{i=-\infty}^{\infty} g_{m_0,n_0}[i] g^*_{m,n}[i]
\]

\[
= \sum_{i=-\infty}^{\infty} g[i] g[i - \Delta m] \frac{N}{2} e^{j\frac{2\pi}{N} \Delta n + \gamma} e^{j\pi (\Delta n + n_0) \Delta m} e^{-j\pi (\Delta n + \Delta m)}
\] (2.9)

where \( \Delta n = n - n_0 \) and \( \Delta m = m - m_0 \). It can be seen that the impulse response of the transmultiplexer strongly depends on the prototype filter \( g[i] \). Since, the orthogonality condition in FBMC/OQAM is restricted to the real field only, the prototype filter must be designed to satisfy the following condition i.e.,

\[
\Re\left\{ \sum_{i=-\infty}^{\infty} g_{m',n'}[i] g^*_{m,n}[i] \right\} = \delta_{m,m'} \delta_{n,n'}
\] (2.10)

where \( \delta_{m,m'} = \delta_{n,n'} = 1 \) if \( \{m, n\} = \{m', n'\} \) and \( \delta_{m,m'} = \delta_{n,n'} = 0 \) if \( \{m, n\} \neq \{m', n'\} \).

Applying the orthogonality condition given in (2.10) in (2.8), we obtain

\[
r_{m,n} = a_{m,n} + \sum_{(m',n') \neq (m,n)} a_{m',n'} \sum_{i=-\infty}^{\infty} g_{m',n'}[i] g^*_{m,n}[i]
\] (2.11)

The second term in (2.11) i.e., \( I_{m,n} \) represents the pure imaginary intrinsic interference that is inherent in FBMC/OQAM system i.e., \( I_{m,n} = ju_{m,n} \) with \( u_{m,n} \) being a real-valued term. Since the transmitted symbols are real-valued, we can recover the symbols by retrieving the real part of the received demodulated signal as follows

\[
\hat{a}_{m,n} = \Re\{r_{m,n}\} = a_{m,n}
\] (2.12)

In case of FBMC/OQAM transmission over a realistic channel, the orthogonality property given in (2.10) will be lost since the successive transmitted FBMC/OQAM symbols will overlap due to the channel multipath effect. For simplicity, we assume a time invariant channel and that the number of subcarriers \( N \) is sufficiently large, and the symbol is sufficiently long, so that the channel can be considered as flat fading at each subcarrier. After passing through the multipath channel, the baseband version of the received signal can be written as

\[
r[i] = \sum_{l=0}^{L_h-1} h[l] x[i - l] + \gamma[i]
\] (2.13)
where $\gamma[i]$ is the Gaussian noise with zero mean and variance $\sigma^2$.

Substituting (2.7) into (2.13) yields

$$r[i] = \sum_{n=0}^{N-1} \sum_{m=-\infty}^{\infty} a_{m,n} \sum_{l=0}^{L_h-1} h[l] g_{m,n}[i-l] + \gamma[i]$$

$$= \sum_{n=0}^{N-1} \sum_{m=-\infty}^{\infty} a_{m,n} \sum_{l=0}^{L_h-1} h[l] \left\{ g[i-l-mN/2] e^{j \frac{2\pi}{N} n(i-l-D/2)} e^{j \phi_{m,n}} \right\} + \gamma[i]$$

$$= \sum_{n=0}^{N-1} \sum_{m=-\infty}^{\infty} a_{m,n} e^{j \frac{2\pi}{N} n(D/2)} e^{j \phi_{m,n}} \sum_{l=0}^{L_h-1} h[l] \left\{ g[i-l-mN/2] e^{-j \frac{2\pi}{N} nl} \right\} + \gamma[i] \quad (2.14)$$

Since the filter bandwidth is smaller than the channel coherence bandwidth ($KN \gg L_h$), which also mean that the time variations of $g$ is very limited. Therefore, we can assume that $g[i-l-mN/2] \approx g[i-mN/2]$ for $l \in \{0, 1, ..., L_h-1\}$. We can now approximate $\theta$ as

$$\theta \approx g[i-mN/2] h_{m,n} \quad (2.15)$$

where $h_{m,n} = \sum_{l=0}^{L_h-1} h[l] e^{-j \frac{2\pi}{N} nl}$ is the channel frequency response at $n^{th}$ subcarrier and $m^{th}$ time index. We can now simplify (2.14) by substituting the value of $\theta$ from (2.15) as

$$r[i] \approx \sum_{n=0}^{N-1} \sum_{m=-\infty}^{\infty} h_{m,n} a_{m,n} g[i-mN/2] e^{j \frac{2\pi}{N} n(i-D/2)} e^{j \phi_{m,n}} + \gamma[i]$$

$$\approx \sum_{n=0}^{N-1} \sum_{m=-\infty}^{\infty} h_{m,n} a_{m,n} g_{m,n}[i] + \gamma[i] \quad (2.16)$$

The quadrature modulated symbol $r_{m,n}$ can be obtained by demodulating the received signal given in (2.16) at the time frequency position $(m, n)$ as

$$r_{m,n} = \sum_{i=-\infty}^{\infty} r[i] g_{m,n}^*[i]$$

$$= \sum_{n'=0}^{N-1} \sum_{m'=-\infty}^{\infty} h_{m',n'} a_{m',n'} \sum_{i=-\infty}^{\infty} g_{m',n'}[i] g_{m,n}^*[i] + \sum_{i=-\infty}^{\infty} \gamma[i] g_{m,n}^*[i] \quad (2.17)$$

where $\gamma_{m,n}$ is the noise at the output of the demodulator. Now applying the real orthogonality condition given in (2.10) in (2.17), we have

$$r_{m,n} = h_{m,n} a_{m,n} + \sum_{(m',n') \neq (m,n)} h_{m',n'} a_{m',n'} \langle g_{m',n'}, g_{m,n} \rangle + \gamma_{m,n} \quad (2.18)$$
where $\bar{I}_{m,n}$ is the complex interference term since $h_{m',n'}$ are complex channel coefficients. It should be noted that since we have assumed a time invariant channel, we can consider the channel coefficients to be constant within the neighboring symbols i.e., $h_{m',n'} \approx h_{m,n}$. In this case, we can write the complex interference term as

$$I_{m,n} \approx h_{m,n} \sum_{(m',n') \neq (m,n)} a_{m',n'} \langle g_{m',n'}, g_{m,n} \rangle \approx j h_{m,n} u_{m,n} \tag{2.19}$$

As FBMC/OQAM prototype filter is well localized in both time and frequency domain, the intrinsic interference term $I_{m,n}$ only depends on a limited set of time frequency positions around any symbol as shown in Fig. 2.8. It can be seen that each received PAM symbol $a_{m,n}$ is affected by its neighboring PAM symbols due to the overlap of filter response and thus the received signal is contaminated with an interference term which is in quadrature with the transmitted signal $a_{m,n}$ as expressed in (2.19). Substituting (2.19) into (2.18) will give us the simplified demodulated signal as

$$r_{m,n} \approx h_{m,n} (a_{m,n} + ju_{m,n}) + \gamma_{m,n} \tag{2.20}$$

### 2.2.1 Prototype Filters

FBMC/OQAM system utilizes a specially designed well localized prototype filter both in time and frequency domains. Localization in time aims at limiting ISI while localization in frequency limits the ICI. These prototype filters are based on various design criteria such as energy concentration in time domain, rapid decaying in frequency domain, etc. [15]. In this section, we will introduce two widely used prototype filters in FBMC/OQAM systems i.e., PHYDYAS and IOTA filters. The time and frequency domain comparison of PHYDYAS and IOTA filters are given in Fig. 2.9. It can be observed from Fig. 2.9(a) that the matched filter response of both filters i.e., PHYDYAS and IOTA filters satisfies the Nyquist criteria quite well. It can also
2.2. FBMC/OQAM Principle

Figure 2.9: Time and Frequency Domain Comparison of PHYDYAS and IOTA Filters

be seen from the power spectral density (PSD) comparison of both PHYDYAS and IOTA filters given in Fig. 2.9(b) that the response of the filters are well contained in frequency domain. It can also be seen that the magnitude response of both filters at 2nd subcarrier index is less than -60dB which results in insignificant ICI. Hence, the well localization of PHYDYAS and IOTA filters in both time and frequency domain enables them to avoid ISI and ICI in FBMC system without the use of cyclic prefix.

PHYDYAS Filter

This prototype filter was first developed and investigated by Bellanger in [33]. Then it was employed as a prototype filter in the physical layer for dynamic spectrum access and cognitive radio (PHYDYAS) European project [34].

The PHYDYAS filter response contains $2K - 1$ coefficients in the frequency domain, where $K$ is the overlap factor. This factor implies that the transition phase at the output of the receiver has a length of $K - 1$ symbols. For the PHYDYAS prototype filter, the overlapping factor is chosen to be $K = 4$ and the frequency coefficients $G_k$ are given by [33] and expressed as

\[
G_0 = 1 \\
G_1 = 0.971960 \\
G_2 = \frac{1}{\sqrt{2}} \\
G_3 = \sqrt{1 - G_1^2} = 0.235147 \\
G_k = 0 \text{ for } k > 3
\] (2.21)
2.2. FBMC/OQAM Principle

The continuous frequency response \( G(f) \) of the filter is obtained using the frequency coefficients through the interpolation formula for sampled signals and is given as (2.22).

\[
G(f) = \sum_{k=-\infty}^{K-1} G_k \sin \left\{ \frac{\pi(f - \frac{k}{NK})}{NK} \right\} \frac{N}{NK} \sin \left\{ \pi \left( f - \frac{k}{NK} \right) \right\} \tag{2.22}
\]

The impulse response \( g(t) \) of the filter is obtained by the inverse Fourier transform of (2.22) and is given as (2.23)

\[
g(t) = G_o + 2 \sum_{k=1}^{K-1} G_k \cos \left( \frac{2\pi kt}{KT} \right) \tag{2.23}
\]

The transmultiplexer response of the FBMC system using PHYDYAS prototype filter with overlapping factor \( K = 4 \) is shown in Table. 2.2 [34].

<table>
<thead>
<tr>
<th>( m )</th>
<th>( n )</th>
<th>( n+1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( m-3 )</td>
<td>0.043j</td>
<td>-0.043j</td>
</tr>
<tr>
<td>( m-2 )</td>
<td>-0.125j</td>
<td>-0.125j</td>
</tr>
<tr>
<td>( m-1 )</td>
<td>-0.206j</td>
<td>-0.206j</td>
</tr>
<tr>
<td>( m )</td>
<td>0.24j</td>
<td>0.24j</td>
</tr>
<tr>
<td>( m+1 )</td>
<td>0.206j</td>
<td>0.206j</td>
</tr>
<tr>
<td>( m+2 )</td>
<td>-0.125j</td>
<td>-0.125j</td>
</tr>
<tr>
<td>( m+3 )</td>
<td>-0.043j</td>
<td>-0.043j</td>
</tr>
</tbody>
</table>

Table 2.2: Transmultiplexer response of FBMC using PHYDYAS filter

Extended Gaussian Function and IOTA Filter

Extended Gaussian Function (EGF) is derived from the Gaussian function (GF) i.e. \( g_\alpha \) which is defined as (2.24)

\[
g_\alpha(t) = 2\alpha^{\frac{1}{4}} e^{-\pi\alpha t^2} \tag{2.24}
\]

The GF defines the most well-localized pulse shape in time and frequency domain according to the Heisenberg-Gabor uncertainty principle [35]. The EGF derived from GF in time domain is given in [36] as (2.25)

\[
z_{\alpha,v_0,\tau_0}(t) = \frac{1}{2} \sum_{k=0}^{\infty} d_{k,\alpha,v_0} \left[ g_\alpha \left( t + \frac{k}{v_0} \right) + g_\alpha \left( t - \frac{k}{v_0} \right) \right] \sum_{l=0}^{\infty} d_{l,\frac{1}{\alpha},\tau_0} \cos \left( \frac{2\pi l t}{\tau_0} \right) \tag{2.25}
\]

where \( \tau_0v_0 = \frac{1}{2} \), \( 0.528v_0^2 \leq \alpha \leq 7.568v_0^2 \), \( d_{k,\alpha,v_0} \) are real valued coefficients and can be computed via the rules described in [37]. The Fourier transforms of GF and EGF have the same shape as the function itself except for an axis scaling factor and are expressed as (2.26)

\[
\mathcal{F}[g_\alpha(t)] = g_\alpha^\frac{1}{2}(f), \quad \mathcal{F}[z_{\alpha,v_0,\tau_0}(t)] = z_{\frac{1}{\alpha},\tau_0,v_0}(f) \tag{2.26}
\]
Isotropic Orthogonal Transform Algorithm (IOTA) prototype function is considered as a special case of the EGF. The IOTA prototype function is obtained by setting $\alpha = 1$, $\nu_0 = \frac{1}{\sqrt{2}}$, $\tau_0 = \frac{1}{\sqrt{2}}$ in (2.25). This IOTA prototype function is identical to its Fourier transform as described in [36]. Hence, it is well localization in time and frequency domain and is well suited for FBMC/OQAM system. The transmultiplexer response of the FBMC system using IOTA prototype filter is shown in Table 2.3.

Table 2.3: Transmultiplexer response of FBMC using IOTA prototype filter

<table>
<thead>
<tr>
<th></th>
<th>$m-3$</th>
<th>$m-2$</th>
<th>$m-1$</th>
<th>$m$</th>
<th>$m+1$</th>
<th>$m+2$</th>
<th>$m+3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n-1$</td>
<td>-0.0103j</td>
<td>-0.038j</td>
<td>0.228j</td>
<td>0.441j</td>
<td>-0.228j</td>
<td>-0.038j</td>
<td>0.0103j</td>
</tr>
<tr>
<td>$n$</td>
<td>-0.0183j</td>
<td>0</td>
<td>0.441j</td>
<td>1</td>
<td>0.441j</td>
<td>0</td>
<td>-0.0183j</td>
</tr>
<tr>
<td>$n+1$</td>
<td>0.0103j</td>
<td>-0.038j</td>
<td>-0.228j</td>
<td>0.441j</td>
<td>0.228j</td>
<td>-0.038j</td>
<td>-0.0103j</td>
</tr>
</tbody>
</table>

The prototype filters used in FBMC system have to be well localised both in time frequency domain as discussed in section 2.1. A root-raised cosine filter (RRCF) can only be used as a suitable prototype filters for FBMC system if it satisfies this condition. Otherwise, it would lead to higher intrinsic interferences in the system. The two well-known filters discussed in literature for FBMC system are PHYDYAS and IOTA as discussed in section 2.2.1 which are designed to be well localised both in time and frequency domains by relaxing the orthogonality condition (real domain orthogonality constraint). However, recently a pulse function called dual root raised cosine (DRRC) pulse, which is based on RRCF is proposed for FBMC system which can also reach a good balance between time and frequency locality for FBMC system [24].

2.3 FBMC/OQAM with MIMO Systems

MIMO techniques can be divided into two main categories, spatial multiplexing (SM) and spatial diversity (SD). SM increases data rate by transmitting different data streams in parallel and independently through spatial channels with $N_T$ transmit antenna and $N_R$ receive antenna. The gain from SM is the minimum of $N_T$ and $N_R$ [38]. In SD, transmission is carried out using multiple transmit/receive antennas. The antennas are spaced sufficiently apart to reduce spatial correlation. At receiver, the multiple signal copies experience uncorrelated fading in the channel. In this case the probability that all the signal copies fade simultaneously is reduced dramatically with respect to the probability that a single copy experiences a fade. Hence, SD provides an effective mean to combat fading channels. Hence, the reliability of the received
OFDM is the most prominent multicarrier scheme that combines well with MIMO channels due to its unique capability of converting frequency selective channel into narrow frequency flat subchannels. However, since conventional OFDM technique is not spectrally efficient due to high OoBR, FBMC/OQAM technique has been of great interest in the past several years and their combination with MIMO has been an interesting area of research.

In this section, we will discuss the well known MIMO techniques for FBMC/OQAM system that includes SM and STBC i.e., Alamouti scheme. In the MIMO context, $N_t$ transmit antennas are used for transmission and $N_r$ receive antennas are used at the receiver side. In this case, the demodulated signal at the $i^{th}$ receive antenna in a MIMO-FBMC/OQAM system can be expressed as

$$r(i)_{m,n} = \sum_{j=1}^{N_t} h(ij)_{m,n} (a(j)_{m,n} + j u(j)_{m,n}) + \gamma(i)_{m,n}$$

(2.27)

where $a(j)_{m,n}$ is the real-valued symbol transmitted through antenna $j$ at the $n^{th}$ subcarrier and at the $m^{th}$ time instant, $h(ij)_{m,n}$ is the channel coefficient between the transmit and receive antennas, $u(j)_{m,n}$ is the intrinsic interference term and $\gamma(i)_{m,n}$ is the noise component received at the antenna $i$. Since, the MIMO-FBMC/OQAM system has $N_r$ receive antennas, the complete system model can be expressed in a matrix form as

$$\begin{bmatrix}
  r(1)_{m,n} \\
  \vdots \\
  r(N_r)_{m,n}
\end{bmatrix} = \begin{bmatrix}
  h(11)_{m,n} & \cdots & h(1N_t)_{m,n} \\
  \vdots & \ddots & \vdots \\
  h(N_r1)_{m,n} & \cdots & h(N_rN_t)_{m,n}
\end{bmatrix} \begin{bmatrix}
  a(1)_{m,n} + j u(1)_{m,n} \\
  \vdots \\
  a(N_t)_{m,n} + j u(N_t)_{m,n}
\end{bmatrix} + \begin{bmatrix}
  \gamma(1)_{m,n} \\
  \vdots \\
  \gamma(N_r)_{m,n}
\end{bmatrix} \in \mathbb{C}^{N_r \times 1}$$

(2.28)

The matrix form of MIMO-FBMC/OQAM given in (2.28) can be written as

$$r_{m,n} = H_{m,n} (a_{m,n} + j u_{m,n}) + \gamma_{m,n}$$

(2.29)

In SM context, $N_t$ streams are allocated to each transmit antenna to achieve the SM gain in MIMO-FBMC/OQAM system. The transmitted streams are recovered using the $N_r$ receive antennas at the receiver using the linear receive processing techniques discussed in Section 2.6.1. It is important to have perfect channel state information (CSI) available at the receiver to completely recover the transmitted streams. However, the presence of intrinsic interference terms in FBMC/OQAM system makes the channel estimation process very complicated in MIMO-FBMC/OQAM system and it still remains an open challenge.

The SD can be achieved using STBC i.e., Alamouti scheme. The direct application of Alamouti coding to FBMC/OQAM system is not straightforward as in OFDM since Alamouti scheme relies on complex orthogonality whereas FBMC/OQAM only satisfy real orthogonality. Also the presence of intrinsic interference is problematic when combining STBC with FBMC/OQAM.
R. Zakaria et al. proposed an interference estimation and cancellation process in [40] that facilitates the application of STBC to FBMC/OQAM system. However, the proposed scheme works well only for lower order modulation schemes since interference estimation and cancellation technique suffer from error propagation. When higher order modulation is used, the errors occur more often and propagate through the iterations of the successive cancellation. An alternative to this approach was proposed in [41], where STBC are combined with FBMC/OQAM in a block wise manner. This proposed scheme is feasible when the transmultiplexer response of FBMC/OQAM system is conjugate symmetric along the time axis. The Alamouti scheme can utilize this property to create complex conjugate symbols to be transmitted in the second slot, by transmitting the time reversal version of the data block. A conventional Alamouti decoding scheme can now be applied to FBMC/OQAM system. However, zero padding is required between the blocks to avoid inter block interference (IBI). Other research works are based on modifying the FBMC/OQAM system to enable its combination with STBC. The authors in [42] show that Alamouti scheme can be combined with code division multiple access (CDMA) based FBMC/OQAM system. The scheme proposed in [43] show that the error floor problem in FBMC/OQAM can be avoided by appending a CP in front of each block and by grouping consecutive FBMC/OQAM signals. Another modified version of FBMC/OQAM system was proposed by R. Zakaria et al. in [44]. In the proposed scheme, CP is inserted on each subcarrier before the SFB and then ISI is avoided by channel diagonalization. To avoid ICI, adjacent subcarriers transmit complex data in different time-frequency positions. This enables the combination of STBC with FBMC/OQAM in a straightforward manner. The proposed technique can address the issue of intrinsic interference by using a CP, however, it has a poor BER performance as compared to the conventional OFDM systems. Tensubam et al. in [45] presented a study on recent advancements in MIMO-FBMC and suggest that FMT based FBMC systems offer the same flexibility as OFDM in adopting MIMO technology. However, it is spectrally inefficient compared to other variants of FBMC like CMT and SMT as it requires guard bands between the subcarriers. Unlike conventional OFDM system, the received symbols in CMT and SMT based FBMC systems are contaminated with pure imaginary intrinsic interference. This interference proves to be a huge obstacle in combining MIMO techniques with FBMC.

### 2.3.1 FBMC/OQAM with Massive MIMO

Massive MIMO is an emerging technology for future wireless communication systems in which large number of antennas are used at the base station to mitigate the effects of multi-user interference. The combination of massive MIMO and the FBMC system is of great importance to
exploit the benefits of both technologies. Some recent developments in combining FBMC with massive MIMO have been discussed in [46–48]. The authors in [47] have shown that with the increase in the number of antennas, the equivalent channel after combining the signals from different receive antennas becomes smooth and can be considered as flat at subcarrier level. This is because of the self equalization property of FBMC in massive MIMO channels. According to this property, a channel flattening effect takes place as a result of linear combining of the channel gains in different receiver antennas during the equalization procedure. However, the authors in [47] have assumed perfect CSI at the receiver. The channel estimation issues in FBMC, including the pilot contamination problem still need to be addressed for massive MIMO systems. An idea of frequency spreading equalization (FSE) based FBMC system with massive MIMO was proposed in [48]. It is shown that the proposed FSE scheme for FBMC-based massive MIMO systems enables the use of higher modulation techniques to improve the SE of the system.

### 2.4 Complex Symbol Transmission using FBMC System

The increasing demands for higher data rates in mobile communication and 5G application requirements such as Internet of Things (IoT), Gigabit wireless connectivity, and tactile internet present an ultimate challenge to provide a uniform service experience to users [49, 50]. To this end, the new physical layer should provide two important features. First, variably aggregation of non-adjacent bands to acquire higher bandwidths for data transmission [51]. Second, supporting asynchronous transmissions, reducing signaling overhead and handling sporadic traffic generating devices such as IoT devices [52]. Those features necessitate a new waveform which provides very low OoBR, as well as immunity against synchronization errors.

#### 2.4.1 Why not FBMC/OQAM?

As OFDM is unable to satisfy the new physical layer requirements, several waveforms have been introduced as its potential replacement. FBMC/OQAM is one of the promising candidate waveform. It offers higher SE compared to conventional OFDM systems [26]. It can also provide very low OoBR, as well as immunity against synchronization errors, thanks to its per-subcarrier filtering [53]. The main drawback in FBMC/OQAM is that it relaxes the orthogonality condition to real field to utilize a well-localized filter in time and frequency, and maintain transmission at the Nyquist rate as discussed in section 1.1. Consequently, the transmitted real symbols in this system are contaminated with imaginary interference terms (intrinsic interference) at the receiver. The intrinsic interference is the main issue for FBMC/OQAM
transceivers. First of all, in highly dispersive channels, the system will not perform properly with single-tap equalization. Secondly, MIMO applications with ML detection [54], and the Alamouti STBC [55] are not directly applicable to the system. Finally, due to intrinsic interference, channel estimation process in FBMC/OQAM is not as straightforward as in OFDM systems. To facilitate channel estimation, it is necessary for the transceiver to perform further pilot processing or waste part of the transmit resources [56, 57].

### 2.4.2 Why FBMC/QAM?

The idea of transmitting complex (QAM) symbols using FBMC was first introduced by R. Zakaria et al. in [58]. Two types of FBMC/QAM transmission schemes were proposed, called vertical QAM (VQAM) and horizontal QAM (HQAM).

<table>
<thead>
<tr>
<th>Table 2.4: VQAM structure for FBMC/QAM system</th>
</tr>
</thead>
<tbody>
<tr>
<td>( m-3 )</td>
</tr>
<tr>
<td>n-2</td>
</tr>
<tr>
<td>n-1</td>
</tr>
<tr>
<td>n</td>
</tr>
<tr>
<td>n+1</td>
</tr>
<tr>
<td>n+2</td>
</tr>
</tbody>
</table>

In VQAM approach, all subcarriers are used for transmission. The main difference between this approach and conventional OQAM based FBMC system is that FBMC/OQAM scheme transmit a real symbol with duration \( \tau_0 = \frac{T}{2} \) as discussed in Section 2.1 whereas, this approach transmits a complex symbol with duration \( 2\tau_0 \) i.e., with duration \( T \). The VQAM transmission structures is illustrated in Table 2.4. It should be noted that \( n \) is the subcarrier index while \( m \) is the time index. We can see that all subcarriers are being used for transmitting complex symbol i.e., \( s_{m,n} \). In HQAM approach, only odd or even subcarriers are used for complex symbol transmission with symbol duration \( \tau_0 = \frac{T}{2} \). The HQAM transmission structures is illustrated in Table 2.5.

It can be seen that the transmission of complex symbol i.e., \( s_{m,n} \) is done for either odd or even subcarriers. In both schemes, the overlapping between symbols is reduced, while the symbol rate remains the same as conventional FBMC/OQAM system. The symbol distribution for VQAM and HQAM structures in FBMC/QAM system is shown in Fig. 2.10. In general, the idea behind FBMC with QAM modulation is to reach a quasi-orthogonal signal while maintaining per-subcarrier filtering. There are two types of this system in the literature. Type I
2.4. Complex Symbol Transmission using FBMC System

Table 2.5: HQAM structure for FBMC/QAM system

<table>
<thead>
<tr>
<th></th>
<th>m-3</th>
<th>m-2</th>
<th>m-1</th>
<th>m</th>
<th>m+1</th>
<th>m+2</th>
<th>m+3</th>
</tr>
</thead>
<tbody>
<tr>
<td>n-2</td>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>n-1</td>
<td>(s_{m-3,n-1})</td>
<td>(s_{m-2,n-1})</td>
<td>(s_{m-1,n-1})</td>
<td>(s_{m,n-1})</td>
<td>(s_{m+1,n-1})</td>
<td>(s_{m+2,n-1})</td>
<td>(s_{m+3,n-1})</td>
</tr>
<tr>
<td>n</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
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</tr>
<tr>
<td>n+1</td>
<td>(s_{m-3,n+1})</td>
<td>(s_{m-2,n+1})</td>
<td>(s_{m-1,n+1})</td>
<td>(s_{m,n+1})</td>
<td>(s_{m+1,n+1})</td>
<td>(s_{m+2,n+1})</td>
<td>(s_{m+3,n+1})</td>
</tr>
<tr>
<td>n+2</td>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Figure 2.10: FBMC/QAM (VQAM & HQAM) and OFDM symbol distribution

which was introduced in [59] and also discussed in [60, 61], uses two different prototype filters for odd and even subcarriers to mitigate intrinsic interference. One of the proposed filters in [59] suffers from very poor OoBR which was enhanced in [62] and [63]. However, due to poor orthogonality condition (i.e., real domain orthogonality constraint), transmission with modulation orders higher than 16-QAM was not possible. Type II of FBMC/QAM introduced in [62] uses an optimized prototype filter for all subcarriers i.e., VQAM scheme. The advantage of Type II is that the OoBR rapidly decays to the desired level within one subcarrier spacing, which is an imposed constraint on the cut-off frequency of the prototype filter stated in [64]. This method is also known as FB-OFDM in [65]. Nevertheless, the filter design in this type of system is quite critical in order to achieve an acceptable level of orthogonality, while keeping the Nyquist property in the time domain.
2.5 Index Modulation with FBMC Systems

Index modulation technique, which utilizes an innovative way of conveying information compared to conventional communication systems, has emerged as a promising candidate scheme for 5G wireless networks [66]. In this scheme, information is not only conveyed by the $M$-ary signal constellation, but also by the indexes of the subcarriers that are activated according to the incoming bit stream. The two main promising applications of IM is in multiple antenna systems like MIMO and multicarrier schemes like OFDM. In MIMO, IM can be applied on the transmit antennas of the MIMO system to convey additional information. This application of IM to MIMO systems is termed as SM [67]. On the other hand application of IM with a multicarrier scheme like OFDM considers subcarrier indexes as a source of conveying additional information [68]. IM based OFDM (OFDM-IM) system can also be considered as a frequency domain extension of the SM concept. Recent studies have shown that OFDM-IM can offer appealing advantages over classical OFDM like, improved BER performance and energy efficiency since the power of inactive subcarriers can either be reallocated to active subcarriers to improve the BER performance and can also be saved to improve the energy efficiency of the system [69]. This has enabled IM to emerge as a promising spectral and energy efficient modulation schemes for future wireless networks and is also being proposed as a strong waveform candidate for future wireless networks [70]. The performance improvement in OFDM-IM system comes from the fact that the system utilizes the indexes of the active subcarriers to convey additional information bits. At the transmitter side of the OFDM-IM system, the subcarriers in each symbol are divided in to sub-groups comprising of active and inactive subcarriers. The indexes of these active subcarriers are used as a source of additional information which may leads to improvement in the transmission efficiency of the system. At the receiver side, the active subcarrier indexes can either be detected by a ML detector or a low-complexity LLR detector [71]. A generalization of OFDM-IM have also been proposed in recent years to achieve higher spectral efficiencies [72] and improved system performance using interleaved OFDM-IM system [73]. A direct combination of OFDM-IM with MIMO transmission techniques, called as MIMO-OFDM-IM, is proposed in [74,75]. It is shown that the proposed MIMO-OFDM-IM system can achieve a linear increase in SE of the system. Despite several advantages, conventional OFDM based systems have various short comings as discussed in Chapter [1] including poor OoBR and strict synchronization requirements.

In recent years, FBMC/OQAM has emerged as a promising candidate waveform for future wireless networks due to its profound advantages over conventional OFDM as discussed in Section [2.1]. However, conventional FBMC/OQAM suffers from pure imaginary intrinsic interference caused by neighboring symbols. Some initial work on the application of IM with
FBMC/OQAM (FBMC/OQAM-IM) has been investigated and it is shown that FBMC/OQAM-IM has improved performance compared to conventional FBMC/OQAM system. However, when the IM scheme is applied to FBMC/OQAM system, the intrinsic interferences are partly eliminated and the remaining interferences still affect the BER performance [76–78]. Although IM is well studied for OFDM based systems and some literature on FBMC with OQAM modulation is available, FBMC/QAM with index modulation (FBMC/QAM-IM) has not been thoroughly investigated.

2.6 Linear Processing Techniques

Linear signal processing techniques are useful in estimating the transmit symbols at the receiver and is frequently applied to wireless communications where channel effects can introduce ISI and ICI that significantly degrade the performance of the system. Such linear processing technique includes equalization for uplink transmission and precoding for downlink transmission.

2.6.1 Linear Receive Processing

For uplink transmission, receive processing or equalization is used to overcome the effects of the channel. The block diagram of conventional receiver processing is shown in Fig. 2.11.

In a broad sense, equalization defines any signal processing technique at the receiver to alleviate the interference problem caused by multipath channel. Equalization techniques can be classified as linear and nonlinear. The linear equalization methods are based on Zero Forcing (ZF) and Minimum Mean Square Error (MMSE), while the nonlinear equalization method includes Decision Feedback equalizer (DFE) and the maximum likelihood sequence estimation (MLSE). DFE is the most common nonlinear technique whereas MLSE is the optimal nonlinear equalization technique but its complexity grows exponentially with the length of the channel, it is therefore impractical for most channels of interest. In this section, we will only focus on the linear receive processing techniques i.e., ZF and MMSE. These linear equalization schemes
2.6. Linear Processing Techniques

process the incoming signal to suppress the effects of the multipath channel. From Fig. 2.11, we can see that the equalization process estimates the transmitted symbol \( \hat{s} \) from the received signal \( r \) as follows [79]

\[
\hat{s} = W r
\]  

(2.30)

**Linear ZF Equalization**

The simplest equalization technique is the ZF. This equalization technique makes use of the CSI and applies the inverse of the channel frequency response (\( H \)) to the received signal to restore the signal at the receiver. Equalizer designed using ZF technique is represented as the Moore-Penrose pseudoinverse of the channel matrix in [80] and is expressed as

\[
W_{zf} = (H H^H)^{-1} H^H
\]  

(2.31)

where \( H \) is the frequency domain channel coefficients in diagonal matrix form and is given as \( H = \text{diag}[H_0, H_1, ..., H_{N-1}] \in \mathbb{C}^{N \times N} \). The \( n \)-th diagonal element can be represented as \( H_n = \sum_{l=0}^{L-1} h_l e^{-j \frac{2\pi nl}{N}} \), where \( 0 \leq n \leq N \) is the number of subcarriers, \( h_l \) denotes the channel coefficient of the \( l \)-th tap in the time domain and \( L \) is the length of the multipath channel. Although, ZF equalizer completely removes the effects of multipath channel (if the perfect CSI is available at the receiver). However, at low SNR values, ZF degrades the performance of the system by enhancing the noise in the received signal [80].

**Linear MMSE Equalization**

MMSE equalization technique can mitigate the noise enhancement problem by taking into account the variance of the noise in the system. Equalizer designed using MMSE technique is given as [80]

\[
W_{mmse} = R_s H^H (HR_s H^H + N_o I)^{-1}
\]  

(2.32)

where \( R_s = \mathbb{E}\{ss^H\} = \delta^2 I \) is the autocorrelation matrix of transmitted signal vector \( s \).

Unlike ZF equalizer, MMSE output contains estimation bias error which is a characteristic of MMSE equalizers as it only minimizes the mean square error but does not completely eliminate the interference caused by the multipath channel. Since, MMSE equalization takes into account the noise variance \( (N_o) \) along with the CSI, this method performs better than ZF at low SNRs. However, at high SNR values, ZF and MMSE exhibit similar performances.
2.6.2 Linear Transmit Processing

In the downlink transmission, the base station with multiple antennas transmits to multiple mobile users. The mobile terminals are usually restricted to simple algorithms with low computational complexity. Each user has access only to its received signal and cannot perform decoding jointly with the other users. Thus receiver processing is impractical for downlink transmission and the system must resort to transmit processing or precoding. If the CSI is available at the transmitter, precoding is used for downlink transmission to combat the channel effects and to enable low complexity in the receiver terminals. The most well known linear precoder are ZF and MMSE based [81]. The block diagram of a linear transmit processing is depicted in Fig. 2.12.

![Linear Transmit Processing](image)

**Figure 2.12: Linear Transmit Processing**

**Linear ZF Precoder**

The most simplest precoding technique is ZF based. This technique require CSI at the transmitter, which is a feasible assumption when a feedback channel is present or when the transmitter and receiver operate in time division duplex mode so that the channel transfer function is identical for both uplink and downlink. A precoder designed using ZF technique is represented in [82] as

\[
P_{zf} = \beta \tilde{P}_{zf} = \beta H^H (HH^H)^{-1}
\]  

(2.33)

where \( P_T \) is the transmit power and \( \beta = \sqrt{\frac{P_T}{\text{Tr}(P_{zf} P_{zf}^H)}} \) is an automatic gain control factor, chosen to meet the transmit power constraint for the precoder.

**Linear MMSE Precoder**

Linear precoder designed using MMSE technique can achieve improved performance than linear ZF precoder since they take into account the noise variance as well as the CSI. The linear
2.6. Linear Processing Techniques

MMSE precoder is represented in [82] as

\[
P_{mmse} = \beta \tilde{P}_{mmse} = \beta H^H (HH^H + \frac{N_o}{R_s} \mathbf{I})^{-1}
\]

where \( R_s = \mathbb{E}\{ss^H\} = \delta^2 \mathbf{I} \) is the autocorrelation matrix of transmitted signal vector \( s \). \( N_o \) is the noise variance and \( \beta = \sqrt{\frac{P_T}{\text{Tr}(\tilde{P}_{mmse} \tilde{P}_{mmse}^H)}} \) is the automatic gain control factor as described in (2.33).

2.6.3 Linear Processing Techniques for FBMC/OQAM System

In FBMC/OQAM systems, the frequency selective transmission channel causes ISI, ICI and using multiple antennas creates inter antenna interference (IAI). To mitigate these impairments several processing techniques have been proposed in literature. In this section, we will highlight some of these receiver and transmit processing techniques for the FBMC system.

Receiver Processing Techniques

Equalizing the channel is a challenging issue in MIMO systems, this is because of the distortions caused by the highly frequency selective channels. Ikhlef et al. in [83] assumed SM and proposed successive interference cancellation (SIC) and ordered SIC (OSIC) techniques to extract the transmitted information in MIMO-FBMC/OQAM system. The results are further improved with the use of a two stage OSIC (TS-OSIC) technique where the first stage provides an initial estimate on the transmitted symbol and in the second stage the rough estimates are used to mitigate the effect of ICI from all the transmit antennas and then OSIC is again applied to the ICI removed symbols in order to combat the remaining ISI and IAI effects. The proposed MMSE equalizer design outperforms the classical one-tap equalizer in case of moderate and high frequency selective channels. Ndo et al. in [84] have proposed a new equalization technique for the FBMC system where the imaginary interference terms which are normally discarded at the receiver by taking the real part of the received symbol are exploited to have improved performance of the equalizer. The authors in [84] have shown strong correlation between the real and the imaginary terms of the received symbols and have utilized this correlation to design a new equalizer for the FBMC system. The impact of doubly dispersive channel on a SISO-FBMC system with both ZF and MMSE based one-tap equalization schemes is analyzed in [23]. It is proposed that a complex multi-tap equalization may be required as the performance of the FBMC system is severely limited by strong doubly dispersive channel impact. The authors in [85] have investigated the performance degeneration of OFDM and FBMC systems in doubly-selective channels using a closed-form bit error probability (BEP)
expression. It is shown that FBMC performs better than CP-OFDM in highly time-varying channels due to the use of well localized prototype filter. Inaki Estella et al. in [86] provided a comparison between multi-stream MIMO based OFDM and FBMC systems and revealed that OFDM achieves a lower energy-efficiency than the FBMC. However, unlike OFDM, the use of multiple streams increases interference in FBMC which require more complex equalization techniques. Ana I. Perez-Neira et al. have presented a detailed and comprehensive overview of various challenges in MIMO-FBMC systems and their known solutions in [87].

 transmit Processing Techniques

Precoders have played a key role in improving the performance of FBMC systems. If the CSI is available at the transmitter, precoding can be used to cancel the effects of the channel at the transmitter that allows a much simpler receiver design. One application of precoding is to minimize the effect of a deep fade in one subcarrier on the overall BER of the system. Chang et al. in [88] have presented such a precoded SISO-FBMC system without CSI at the transmitter. The authors have derived an analytical expression for the BER performance of the precoded FBMC system. The proposed system is limited by the assumption of perfect equalization at the receiver whereas in reality imperfect equalization leads to residual ISI and ICI that significantly degrade the system performance. The authors in [89] have analysed the effect of multitap subchannel equalization on error performance of precoded FBMC systems transmitting through highly frequency selective channel. The authors have introduced geometric interpolation based subchannel equalization and the results suggest that subchannel equalizers with more than three taps do not bring improvement in the system performance. Hence, new equalization techniques are needed to compensate the residual ISI. Chang et al. in [90] have analysed the effect of ISI from imperfect equalization and derived an analytical expression for the BER performance of precoded FBMC system. The results suggest that for highly selective channels, one-tap equalization will not be sufficient for FBMC systems as they don’t employ CP like in OFDM. It is therefore required to use multi tap equalization to reduce the effect of ISI for highly frequency selective channels. Jayasinghe et al. in [91] have analyzed the effect of intrinsic interference in the FBMC system and proposed a precoder based on signal to leakage plus noise ratio (SLNR) at the transmitter side to overcome its effects on the FBMC system. It is shown that the proposed precoder design at the transmitter outperforms the equalization based FBMC and OFDM systems. Caus et al. in [92] design a precoding and decoding technique for MIMO-FBMC system to enable multi-user transmission in frequency selective channels. It is achieved by concatenating two precoders, one to cancel the channel interference and the other is jointly designed with the receive filter in the second stage so that MSE is minimized.
The proposed solution is able to deal with the frequency selective channels only if channel frequency response is flat at subcarrier level also it assumes that the number of receive antennas does not exceed the number of transmit antennas. The authors in [93] have presented a single-tap precoder and decoder design for multiuser MIMO-FBMC system for frequency selective channels by optimizing the MSE formula under ZF and MMSE design criteria. Mestre et al. in [94] have proposed a novel architecture for MIMO-FBMC system by exploiting the structure of the analysis and synthesis filter banks using approximation of an ideal frequency-selective precoder and linear receiver. Soysa et al. in [95] have evaluated the performance of precoding and receiver processing techniques for multiple access MIMO-FBMC system for an extended ICI/ISI scenario in uplink and downlink transmissions. There has also been investigations on the performance of MIMO-FBMC system in frequency selective channels. Various precoding and equalization techniques are proposed to achieve robustness against channel frequency selectivity and to improve the SE in MIMO-FBMC system [96].

2.7 Widely Linear Processing

Classical linear signal processing for signal reconstruction, demodulation or parameter estimation is frequently applied to radio communications that employs circular (or proper) signals e.g. MPSK and MQAM etc. For such carrier-modulated transmissions, the received signal is transformed back to baseband before further processing, which leads to a complex valued symbols. However, if the signal comes from real or offset constellations such as BPSK and OQAM, the signal is called improper. As mentioned in [97–102], the proper signals have their second order statistics completely described by their auto-covariance i.e, for a complex random process $s$ with zero mean, the auto-covariance is expressed by $E[ss^H]$. However, for improper signals, the auto-covariance by itself is insufficient to describe its second order statistics, since the pseudo-autocorrelation i.e., $E[ss^T]$ is non-zero. Hence, for improper signals, the linear processing techniques does not take into account all the second order statistics of the received signal and therefore the estimation at the receiver is highly sub-optimal.

![Widely Linear Receiver Diagram](image-url)

Figure 2.13: Widely Linear Estimator
2.7. Widely Linear Processing

The structure of a widely linear (WL) estimator is given in Fig. 2.13. The receiver makes use of the received data $r$ and its conjugate version $r^*$ to estimate the transmitted symbol $s$.

As shown in Fig. 2.13, the output of the WL receiver can be written as

$$\hat{s} = W_1 r + W_2 r^*$$

(2.35)

where $W_1$ and $W_2$ are the two widely linear MMSE receive filters and are represented as [97]

$$\begin{bmatrix} W_1 \\ W_2 \end{bmatrix} = \begin{bmatrix} R_{rr} & R_{rr}^* \\ R_{rr}^* & R_{rr}^* \end{bmatrix}^{-1} \begin{bmatrix} r_s \\ r_v \end{bmatrix}$$

(2.36)

and $R_{rr} = HR_{ss}H^H + N_o I$ is the autocorrelation matrix and $R_{rr}^* = HR_{ss}^*H^T$ is the pseudo-autocorrelation matrix of the received signal vector $r$, also $r_s = R_{ss}H^H$ and $r_v = R_{ss}^*H^T$, where $R_{ss} = E\{ss^H\}$ and $R_{ss}^* = E\{ss^T\}$ is the variance of the transmitted signal vector $s$. It should be noted that $R_{ss} = R_{ss}^* = P_s^2 I$, since $s$ is a real-valued signal vector and $P_s$ is a diagonal power allocation matrix.

2.7.1 Intrinsic Interference Estimation and Cancellation

An intrinsic interference estimation and cancellation (IIEC) methods were proposed in [103]. The first method estimates the intrinsic interference by taking the imaginary part of the linear MMSE equalizer as

$$\hat{I}_{intri} = j\{W_{mmse}r\}$$

(2.37)

The second method estimates the intrinsic interference by re-modulating the demodulated bits using OQAM and then by using the impulse response of the prototype filter, the intrinsic interference is estimated at the receiver as

$$\hat{I}_{intri,m,n} = \sum_{i=m-3}^{m+3} \sum_{j=n-1}^{n+1} c_{i,j} \hat{s}_{j,i} \quad i \neq m \text{ and } j \neq n$$

(2.38)

where the coefficients $c_{i,j}$ represent the filter impulse response coefficients determined by the SFB and AFB and is given in Table 2.2 or 2.3. Both the methods do not provide exact estimate of the interference as the output of the linear MMSE equalizer suffer from estimation bias error. Also these methods require perfect CSI at the receiver to cancel the intrinsic interference from the received symbols i.e.,

$$r = H(s + jI_{intri}) + n - \tilde{H}_{j\epsilon}\hat{I}_{intri} = H(s + j\epsilon) + n$$

(2.39)

where $jI_{intri}$ is the actual intrinsic interference that is contaminating the transmitted symbols $s$, while $j\hat{I}_{intri}$ is the estimated intrinsic interference at the receiver and $j\epsilon$ is the residual.
interference due to the residual error from the linear MMSE equalizer. Secondly, $\tilde{H} = H$ only if perfect CSI is available at the receiver, otherwise it will also contribute to the magnitude of the $j\epsilon$.

### 2.7.2 Widely Linear Processing for MIMO-FBMC system

As we know that FBMC/OQAM employs improper signals, it is possible to exploit the benefits of WLP for FBMC system as well. The WLP technique takes advantage of the impropriety of the real-valued signals, by processing the signal together with its conjugate version to obtain a more precise estimate at the receiver [97]. Hence, significant improvement can be achieved using WLP techniques as compared to conventional linear processing techniques for FBMC/OQAM system. However, intrinsic interference that contaminates the real transmitted symbols in FBMC/OQAM not only represents a major obstacle in combining FBMC with techniques like MIMO but also refrains the exploitation of WLP technique for FBMC/OQAM system. Cheng et al. in [103] proposed a two-step receiver, combining linear and widely linear processing for MIMO-FBMC systems. In the first step a linear MMSE receiver is used to estimate the intrinsic interference terms either by directly taking the imaginary part of receiver output or using the estimates from the already detected adjacent symbols. This estimated intrinsic interference is then cancelled from the received symbols. In the second step a widely linear MMSE receiver provides a more precise estimate of the transmitted symbols due to the cancellation of the intrinsic interference. It is shown in [103] that the proposed two step receiver outperforms the linear MMSE based receiver.
Spectrum Efficient MIMO-FBMC System using Filter Output Truncation

Due to the use of an appropriately designed pulse shaping prototype filter, FBMC system can achieve low out of band emissions and is also robust to the channel and synchronization errors. However, it comes at a cost of long filter tails which may reduce the SE significantly when the block size is small. This may also lead to an issue in dynamic TDD scenarios, where the fast switching causes the long tail to be a considerable part of the transmission duration i.e., overhead increases while effective throughput decreases. Filter output truncation can reduce the overhead by discarding the filter tails but may also significantly destroy the orthogonality of FBMC system, by introducing ICI and ISI terms in the received signal. As a result, the SIR is degraded. In addition, the presence of intrinsic interference terms in FBMC also proves to be an obstacle in combining MIMO technique with FBMC. In this chapter, we present a theoretical analysis on the effect of FOT in an MIMO-FBMC system. First, we derive the matrix model of MIMO-FBMC system which is subsequently used to analyze the impact of finite filter length and FOT on the system performance. The analysis reveals that FOT can avoid the overhead in time domain but also introduces extra interference in the received symbols. To combat the interference terms, we then propose a compensation algorithm that considers odd and even overlapping factors as two separate cases, where the signals are interfered by the truncation in different ways. The general form of the compensation algorithm can compensate all the symbols in a MIMO-FBMC block and can improve the SIR values of each symbol for better detection at the receiver. It is also shown that the proposed algorithm requires no overhead and can still achieve a comparable BER performance to the case with no filter truncation.
3.1 MIMO-FBMC System

The block diagram for both transmitter and receiver of a MIMO-FBMC is shown in Fig. 3.1 where real and imaginary branches i.e. I and Q branches are processed simultaneously and independently. In our analysis of MIMO-FBMC system, we assumed $N_t$ transmit antennas are used to transmit multiple FBMC signals simultaneously which are received by $N_r$ received antennas, where $N_t \leq N_r$. Vectors and matrices are denoted by lowercase and uppercase bold letters. $\{\cdot\}^H$ and $\{\cdot\}^T$ represent conjugate transpose (hermitian conjugate) and transpose operations. $\mathcal{F}$ and $\mathcal{F}^H$ denote the normalized N point DFT and IDFT matrices. $A \otimes B$ represents kronecker product of $A$ and $B$. $\Re(A)$ and $\Im(A)$ are the real and imaginary part of scalar/vector/matrix $A$. $\mathbf{I}_N$ represents an identity matrix for dimension $N \times N$. $A \ast B$ represents the linear convolution of $A$ and $B$. $a \uparrow l$ represents $l$ sample delayed version of the vector $a$ with zero padding at the front. We use $\{\cdot\}$ or $\{\cdot\}$ over any variable to represent the real and imaginary part of that scalar/vector/matrix respectively. The MIMO-FBMC system model follows a block based processing approach where each block contains $M$ FBMC symbols with each symbol containing $N$ subcarriers in frequency domain. Therefore, we can represent each MIMO-FBMC block as $\mathbf{S} = [s_0, s_1, \ldots, s_{M-1}] \in \mathbb{C}^{N_t \times M}$ where $s_m = [s_{m,0}, s_{m,1}, \ldots, s_{m,N_t-1}]^T \in \mathbb{C}^{N_t \times 1}$. The transmitted signal on the $n^{th}$ subcarrier in a

![Figure 3.1: Blocks diagrams for MIMO-FBMC transmitter and receiver in matrix operation form](image)

MIMO system is an $N_t \times 1$ vector, i.e., $\mathbf{s}_{m,n} = [s_{m,n,1}, s_{m,n,2}, \ldots, s_{m,n,N_t}]^T \in \mathbb{C}^{N_t \times 1}$. Each $s_{m,n,j}$ represents a complex signal on $n^{th}$ subcarrier of $m^{th}$ FBMC symbol transmitted by $j^{th}$ transmitting antenna. Hence, $MNN_t$ QAM symbols are transmitted in one FBMC block. Note that a precoding scheme such as ZF can be applied at the transmitter side when the CSI is avail-
3.1. MIMO-FBMC System

able. In such cases the performance of a system can be further enhanced. However, the focus of
this chapter is to analyze the performance of MIMO-FBMC system with finite filter length and
FOT. Therefore, the analysis presented in Section 3.4 is based on unitary precoding matrix but
is easily extendable to the precoding case as well. Moreover, the power of modulated symbols
$s_{m,n,j}$ is represented as $\delta^2$ i.e. $\mathbb{E}\{\|s_{m,n,j}\|^2\} = \delta^2$. The real and imaginary parts of $s_m$ are
represented as $\bar{s}_m$ and $\tilde{s}_m$ respectively.

3.1.1 MIMO Channel Impulse Response

We assume the system operates over a slowly-varying fading channel i.e. quasi-static fading
channel. In such a scenario, it is plausible to assume that the duration of each transmitted
data block is smaller than the coherence time of the channel, therefore, the random fading
coefficients stay constant over the duration of each block \cite{104}. In this case, we define the
multipath channel as a $l$-tap channel impulse response (CIR) matrix with the $l^{th}$-tap power
being $\rho_l^2$. It is also assumed that the average power remains constant during transmission of
whole block. The CIR matrix $H$ is defined as

$$H = \begin{bmatrix} H_0, H_1, \cdots, H_{L-1} \end{bmatrix}^T$$

$$= \begin{bmatrix} \rho_0 Z_0, \rho_1 Z_1, \cdots, \rho_{L-1} Z_{L-1} \end{bmatrix}^T$$

(3.1)

where $H_l$ defines the $l^{th}$ matrix valued CIR coefficient of the channel between all the antennas
and is represented as

$$H_l = \rho_l Z_l = \rho_l \begin{bmatrix} z_{11}(l) & \cdots & z_{1N_t}(l) \\ \vdots & \ddots & \vdots \\ z_{N_r,1}(l) & \cdots & z_{N_r,N_t}(l) \end{bmatrix} \in \mathbb{C}^{N_r \times N_t}$$

(3.2)

The random variable $z_{ij}(l)$ with complex Gaussian distribution as $\mathbb{C}N(0, 1)$ represents the mul-
tipath fading factor for $l^{th}$ tap of the quasi-static rayleigh fading channel between $j^{th}$ transmit
antenna and $i^{th}$ receive antenna. Note that we consider co-located transmit and receive anten-
as to simplify our analysis. However, if we consider either transmit or receive antennas to
be geographically separated, the analysis can easily be extended by considering the common
coefficient $\rho_l$ to be different among the antennas.

3.1.2 Prototype Filters / Filter Matrices

Ideally, an infinite filter length ($K = \infty$) is required to provide the best performance. However,
a finite filter length (e.g. overlapping factor $K = 4 \sim 6$) is used in practice in a FBMC
system to achieve comparable system performance. To generalize the derivation, the filter overlapping factor is taken as \( K \), therefore, \( KN \) is the total length of the prototype filter i.e. \( \bar{\mathbf{w}} = [\bar{w}_0, \bar{w}_1, \cdots, \bar{w}_{K-1}] = [\tilde{w}_0, \tilde{w}_1, \cdots, \tilde{w}_{KN-1}] \in \mathbb{R}^{1 \times KN} \). The \( J \) branch filter matrix \( \mathbf{P}_{\text{orig}} \in \mathbb{R}^{(K+M-1) \times MN} \) can be expressed as

\[
\mathbf{P}_{\text{orig}} = \begin{bmatrix}
\mathbf{P}_{i_F} \\
\mathbf{\bar{W}}_t \\
\mathbf{\bar{W}}_{t+1} \\
\vdots \\
\mathbf{\bar{W}}_{K-1} \\
\end{bmatrix}
\]

(3.3)

where, \( \mathbf{\bar{W}}_k = \text{diag}(\tilde{\mathbf{w}}_k) \in \mathbb{R}^{N \times N} \) for \( k = 0, 1, 2, \cdots, K - 1 \) and \( \mathbf{\bar{W}}_k = [\tilde{w}_{kN}, \tilde{w}_{kN+1}, \cdots, \tilde{w}_{kN+N-1}] \in \mathbb{R}^{1 \times N} \) while \( t = \left\lfloor \frac{K}{2} \right\rfloor \). The value of \( t \) defines the truncated matrix \( \mathbf{P} \) as shown in (3.3). It should be noted that \( \mathbf{P}_{i_F} \) represent the first \( i_FN \) rows and \( \mathbf{P}_{i_R} \) represent the last \( i_RN \) rows of \( \mathbf{P}_{\text{orig}} \) such that \( i_F + i_R \leq K - 1 \). The prototype filter matrix for the \( \Omega \) branch is defined in the same manner. The only difference is that the \( \Omega \) branch filter is a shifted version of the \( J \) branch filter i.e., \( \mathbf{\tilde{w}} = [\mathbf{\tilde{w}}_0, \mathbf{\tilde{w}}_1, \cdots, \mathbf{\tilde{w}}_{K-1}] = [\tilde{w}_0, \tilde{w}_1, \cdots, \tilde{w}_{KN-1}] = [\tilde{w}_{K/2}, \tilde{w}_{K/2+1}, \cdots, \tilde{w}_{KN-1}, \tilde{w}_0, \tilde{w}_1, \cdots, \tilde{w}_{N/2-1}] \in \mathbb{R}^{1 \times KN} \). Shifting prototype filter in the \( \Omega \) branch instead of offsetting the QAM symbols makes the overall design simpler. Similarly, the \( \Omega \) branch truncated filter matrix \( \mathbf{\tilde{P}} \) is defined in the same manner as described for the \( J \) branch with \( \mathbf{\tilde{W}}_k = \text{diag}(\tilde{\mathbf{w}}_k) \in \mathbb{R}^{N \times N} \) for \( k = 0, 1, 2, \cdots, K - 1 \) and \( \tilde{\mathbf{w}}_k = [\tilde{\tilde{w}}_{kN}, \tilde{\tilde{w}}_{kN+1}, \cdots, \tilde{\tilde{w}}_{kN+N-1}] \in \mathbb{R}^{1 \times N} \).

### 3.2 MIMO-FBMC Matrix Model

In this section, the proposed MIMO-FBMC matrix model is derived by extending our previous work on a SISO-FBMC system \[23\]. It is worth mentioning that the derivation of MIMO-
3.2. MIMO-FBMC Matrix Model

FBMC matrix model is not a simple SISO to MIMO mapping. Signal definition, transmit processing, channel modelling, as well as receive processing including channel equalization has to be redefined. The derived MIMO-FBMC model also incorporates FOT as well as the proposed compensation algorithm at the receiver.

3.2.1 Transmit Processing

We will only focus on the real branch in detail since the imaginary branch will follow the same procedure.

Real Branch Processing

According to Fig. 3.1, the signal $\bar{s}_m$ is first multiplied by a phase shifter matrix $\bar{\Phi}_m$ symbol by symbol i.e.,

$$
\bar{a}_m = (\bar{\Phi}_m \otimes I_{N_t}) \bar{s}_m = \bar{\Phi}_{k,m} \bar{s}_m \in \mathbb{C}^{N N_t \times 1}
$$

(3.4)

where $\bar{\Phi}_m$ is a diagonal matrix i.e. $\bar{\Phi}_m = \text{diag}[e^{-j\pi(0+2m)/2}, e^{-j\pi(1+2m)/2}, \cdots, e^{-j\pi(N-1+2m)/2}] \in \mathbb{C}^{N \times N}$. Note that $\bar{\Phi}_{k,m}$ represents the kronecker product $\bar{\Phi}_m \otimes I_{N_t}$ that yields a matrix of size $N N_t \times N N_t$.

Real Branch IDFT Processing

Signal after the phase shifter matrix will pass through an $N$ point IDFT (inverse discrete Fourier transform) block $F^H$ i.e.

$$
\bar{b}_m = (F^H \otimes I_{N_t}) \bar{a}_m = F^H_k \bar{a}_m \in \mathbb{C}^{N N_t \times 1}
$$

(3.5)

where $F^H_k = F^H \otimes I_{N_t}$. Signal after the IDFT block can be represented as $\bar{b} = [\bar{b}_0; \bar{b}_1; \cdots; \bar{b}_{M-1}] = [F^H_k \bar{a}_0; F^H_k \bar{a}_1; \cdots; F^H_k \bar{a}_{M-1}] \in \mathbb{C}^{M N N_t \times 1}$. Here IDFT is a block wise operation since each modulated subcarrier is a column vector of size $N_t \times 1$ and $F^H$ is a generalized $N$ point IDFT matrix.
3.2. MIMO-FBMC Matrix Model

Real Branch Prototype Filter

The signal is then passed through a prototype filter in I and Q branches independently. In general, prototype filters are linearly convolved with the input signal. In order to represent a complete system in matrix form we have defined a prototype filter matrix $\bar{P}_{\text{orig}}$ in a manner that when this filter matrix is multiplied by vector $\bar{b}$; the multiplication of matrices is equivalent to the required linear convolution process. The output of the I branch filter can be written as

$$\bar{o} = \bar{P}_{k,\text{orig}} \bar{b}$$

(3.6)

where $\bar{P}_{k,\text{orig}} = \bar{P}_{\text{orig}} \otimes I_{N_t}$. Note that the output of the real branch filter $\bar{o}$ has $(K - 1)NN_t$ more samples than the input due to the linear convolution process. Hence, to keep the orthogonality (minimum interference from other subcarriers and symbols), all of these samples have to be transmitted to the receiver side. However, the transmission efficiency $\eta$ will drop by the proportion of

$$\eta = \frac{M}{K + M - 1}$$

(3.7)

It can be seen from (3.7), that transmission efficiency $\eta$ is high only if $M$ is large. Another way to achieve higher $\eta$ is to truncate $\bar{P}_{\text{orig}}$ to improve the spectral efficiency. However, truncation may lead to interferences in the system that can significantly degrade the system performance. Without any compensation, the maximum allowable cut off symbols would be $K - 2$ so as to keep the signals detectable [32]. However, with compensation we can completely discard all the $K - 1$ symbols while still keeping the signals detectable. The truncation should take place at the first $i_F$ and the last $i_R$ rows of $\bar{P}_{\text{orig}}$ such that $i_F + i_R \leq K - 1$ as shown in (3.3), where $\bar{P}_{i_F}$ is first $i_F N$ rows and $\bar{P}_{i_R}$ is the last $i_R N$ rows of $\bar{P}_{\text{orig}}$. The middle part of $\bar{P}_{\text{orig}}$ i.e. $\bar{P}$ which is the truncated filter matrix will be used at transmitter side to improve the spectral efficiency of the system. The performance loss due to the truncation of $\bar{P}_{\text{orig}}$ will be compensated at the receiver side and is discussed later in Section 3.4. The output of real branch truncated filter can be written as

$$\bar{o} = (\bar{P} \otimes I_{N_t}) \bar{b}$$

$$= \bar{P}_k \bar{b} \in \mathbb{C}^{MNN_t \times 1}$$

(3.8)
3.2. MIMO-FBMC Matrix Model

**Imaginary Branch Processing Including Prototype Filtering**

Similar process is followed for the $\mathcal{Q}$ branch i.e. the signal $\tilde{s}_m$ is first multiplied by a phase shifter matrix $\tilde{\Phi}_m = j \tilde{\Phi}_m$ symbol by symbol i.e.,

$$\tilde{a}_m = (\tilde{\Phi}_m \otimes I_{N_t}) \tilde{s}_m = \tilde{\Phi}_{k,m} \tilde{s}_m \in \mathbb{C}^{N_t \times 1} \tag{3.9}$$

After the phase shifter matrix, the signal will pass through an $N$ point IDFT block $\mathcal{F}^H$ as

$$\tilde{b}_m = (\mathcal{F}^H \otimes I_{N_t}) \tilde{a}_m = \mathcal{F}^H_k \tilde{a}_m \in \mathbb{C}^{N_t \times 1} \tag{3.10}$$

The signal after IDFT block can be represented as $\tilde{b} = [\tilde{b}_0; \tilde{b}_1; \cdots; \tilde{b}_{M-1}] = [\mathcal{F}^H_k \tilde{a}_0; \mathcal{F}^H_k \tilde{a}_1; \cdots; \mathcal{F}^H_k \tilde{a}_{M-1}] \in \mathbb{C}^{NN_t \times 1}$. Likewise, the following matrix multiplication of truncated filter matrix $\tilde{P}$ and the signal vector $\tilde{b}$ represents the linear convolution of the imaginary branch prototype filter and the imaginary branch input signal.

$$\tilde{o} = (\tilde{P} \otimes I_{N_t}) \tilde{b} = \tilde{P}_k \tilde{b} \in \mathbb{C}^{MNN_t \times 1} \tag{3.11}$$

### 3.2.2 Passing through the Channel

The real and imaginary branch signals $\bar{o}$ and $\tilde{o}$ after the prototype filtering are added together and is then passed through the channel $H$. The received signal is now represented as

$$r = H \ast (\bar{o} + \tilde{o}) + n \tag{3.12}$$

where $n = [n_1, n_2, \cdots, n_{N_t}]^T \in \mathbb{C}^{MNN_t \times 1}$ is a Gaussian noise vector with each element having zero mean and variance $\sigma^2$. To represent the convolution process given in (3.12) as matrix multiplication, we define the $l^{th}$ tap multipath fading factor $Z_l$ of the MIMO channel as a block diagonal matrix by taking the kronecker product of an identity matrix $I_M$ with $Z_l$ as

$$Z_{l,blk} = I_M \otimes Z_{k,l} \tag{3.13}$$

where $Z_{k,l} = I_N \otimes Z_l \in \mathbb{C}^{NN_t \times NN_t}$. The block diagonal matrix $Z_{l,blk}$ has $Z_l$ as its diagonal sub matrices. The definition of $Z_{l,blk}$ implies that each FBMC symbol in a block experiences the same channel i.e. $Z_l$. Hence, we can write (3.12) as

$$r = \sum_{l=0}^{L-1} \rho_l Z_{l,blk}(\bar{o}^{\downarrow N_t l} + \tilde{o}^{\downarrow N_t l}) + n \tag{3.14}$$
where \( \bar{o}^{\downarrow N_l} \), \( \bar{\bar{o}}^{\downarrow N_l} \) represents \( N_l \) samples delayed version of \( o \) and \( \bar{o} \) with zero padding in the front, i.e. \( \bar{o}^{\downarrow N_l} = [0_{N_l \times 1}; \bar{o}_{q,N_l}] \) and \( \bar{\bar{o}}^{\downarrow N_l} = [0_{N_l \times 1}; \bar{o}_{q,N_l}] \) respectively. Note that \( \bar{o}_{q,N_l} \) and \( \bar{\bar{o}}_{q,N_l} \) represents the first \((K+M-1)N_{N_t} - N_l\) elements of \( o \) and \( \bar{o} \) respectively.

From (3.8) and (3.11) we can write \( \bar{o}^{\downarrow N_l} = \bar{p}_k^{\downarrow N_l} \bar{b} \) and \( \bar{\bar{o}}^{\downarrow N_l} = \bar{p}_k^{\downarrow N_l} \bar{b} \), where \( \bar{p}_k^{\downarrow N_l} = [0_{N_l \times MN_N}; \bar{p}_{k,q}] \) and \( \bar{\bar{p}}_k^{\downarrow N_l} = [0_{N_l \times MN_N}; \bar{\bar{p}}_{k,q}] \). Here \( \bar{p}_{k,q} \) and \( \bar{\bar{p}}_{k,q} \) are the first \((K+M-1)N_{N_t} - N_l\) rows of \( \bar{p}_k \) and \( \bar{\bar{p}}_k \) respectively. Eq (3.14) can thus be reformed as

\[
r = \sum_{l=0}^{L-1} \rho_l Z_{l,blk}(\bar{p}_k^{\downarrow N_l} \bar{b} + \bar{\bar{p}}_k^{\downarrow N_l} \bar{b}) + n \tag{3.15}\]

The above equation indicates that the truncated filter matrix \( \bar{p}_k \) and \( \bar{\bar{p}}_k \) are distorted because of the channel multipath effect and are represented as \( \bar{p}_k^{\downarrow N_l} \) and \( \bar{\bar{p}}_k^{\downarrow N_l} \) respectively.

To represent (3.15) in a point-wise multiplication form in frequency domain, we apply the circular convolution property by first introducing a block diagonal exchange matrix \( X_{N_l} \in \mathbb{R}^{MN_{N_t} \times MN_{N_t}} \) as

\[
X_{N_l} = \begin{bmatrix}
X_{sub,N_l} & 0 & \cdots & 0 \\
0 & X_{sub,N_l} & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & X_{sub,N_l}
\end{bmatrix}
\tag{3.16}
\]

with

\[
X_{sub,N_l} = \begin{bmatrix}
0_{N_l \times (NN_{N_t} - N_l)} & I_{N_l \times N_l} \\
I_{(NN_{N_t} - N_l) \times (NN_{N_t} - N_l)} & 0_{(NN_{N_t} - N_l) \times N_l}
\end{bmatrix}
\tag{3.17}
\]

As \( X_{N_l}^T X_{N_l} = I \), we have

\[
\bar{o}^{\downarrow N_l} = \bar{p}_k^{\downarrow N_l} \bar{b} = \bar{p}_k^{\downarrow N_l} X_{N_l}^T \bar{b} = \bar{p}_k^{\downarrow N_l} e^{\downarrow N_l} \bar{b} \tag{3.18}
\]

The matrix \( X_{N_l}^T \) and \( X_{N_l} \) are used to exchange the locations of elements of \( \bar{p}_k^{\downarrow N_l} \) and \( \bar{\bar{p}}_k^{\downarrow N_l} \) respectively, such that \( \bar{p}_k^{\downarrow N_l} = \bar{p}_k^{\downarrow N_l} X_{N_l}^T \) and \( \bar{\bar{p}}_k^{\downarrow N_l} = X_{N_l} \bar{b} \). By multiplying the matrix \( X_{N_l} \) with \( \bar{b} \), the last \( N_l \) symbols of its each sub-vector \( \bar{b}_m \) will be moved to the front, i.e.,

\[
\bar{b}_m^{\downarrow N_l} = [b_{m,N_{N_t} - N_l}, \ldots, b_{m,N_{N_t} - 1}, b_{m,0}, \ldots, b_{m,N_{N_t} - N_l - 1}]^T \in \mathbb{C}^{N_{N_t} \times 1} \tag{3.19}
\]

Likewise,

\[
\bar{\bar{b}}_e^{\downarrow N_l} = [\bar{b}_e^{\downarrow N_l}, \bar{b}_e^{\downarrow N_l}, \ldots, \bar{b}_e^{\downarrow N_l}] \in \mathbb{C}^{M_{N_{N_t}} \times 1} \tag{3.20}
\]
The effect is similar when multiplying $X^T_{N_t l}$ with $\bar{P}^{\downarrow N_t l}_{k}$. $X^T_{N_t l}$ only changes the elements location in $\bar{P}^{\downarrow N_t l}_{k}$. Similarly, we can write $\bar{o}^{\downarrow N_t l}_{k}$ as

$$\bar{o}^{\downarrow N_t l}_{k} = \bar{P}^{\downarrow N_t l}_{k} \bar{b} = \bar{P}^{\downarrow N_t l}_{k} X^T_{N_t l} X_{N_t l} \bar{b} = \bar{P}^{\downarrow N_t l}_{k,e} b^{\downarrow N_t l}_{e}$$

(3.21)

Substituting (3.18) and (3.21) into (3.15) yields

$$r \approx L^{-1} \sum_{l=0}^{L-1} \rho_l Z_{l,blk} (\bar{P}^{\downarrow N_t l}_{k,e} b^{\downarrow N_t l}_{e} + \bar{P}^{\downarrow N_t l}_{k,e} \bar{b}^{\downarrow N_t l}_{e}) + n$$

(3.22)

It can be demonstrated that the non zero elements of $\bar{P}^{\downarrow N_t l}_{k,e}$ and $\bar{P}_k$ are very close \( [23] \) i.e., the nonzero elements of $\bar{P}^{\downarrow N_t l}_{k,e}$ are only delayed by $N_t l$ elements as compared to the elements in $\bar{P}_k$. If the non-zero $i^{th}$ row and $k^{th}$ column element of $\bar{P}_k$ is $w_n$, then the element of $\bar{P}^{\downarrow N_t l}_{k,e}$ at the same location will be $w_{n+N_t l}$. Since the length of prototype filter is $KN$ as defined in section 3.1.2, the coefficient $w_n$ and $w_{n+N_t l}$ are the adjacent elements of the prototype filter and are close in value to each other (as $N \gg L$) and therefore the difference between $w_n$ and $w_{n+N_t l}$ is very small. Eq (3.22) can thus be written as

$$r \approx L^{-1} \sum_{l=0}^{L-1} \rho_l Z_{l,blk} (\bar{P}^{\downarrow N_t l}_{k,e} b^{\downarrow N_t l}_{e} + \bar{P}^{\downarrow N_t l}_{k,e} \bar{b}^{\downarrow N_t l}_{e}) + n$$

(3.23)

### 3.2.3 Receive Processing

On the receiver side, the signal $r$ is received by $N_r$ received antennas and is fed to the real and imaginary branches of the receiver as shown in Fig. 3.1 for independent processing.

**Real Branch Processing**

Following the similar approach, we will focus on the real branch processing and the imaginary branch processing follows the same procedure. In the real branch, signals from $N_r$ received antennas are passed through the real branch received filters leading to the following output

$$\bar{x} \approx \bar{P}^H_k r$$

$$\approx \bar{P}^H_k \bar{P}_k \sum_{l=0}^{L-1} \rho_l Z_{l,blk} \bar{b}^{\downarrow N_t l}_{e} + \bar{P}^H_k \bar{P}_k \sum_{l=0}^{L-1} \rho_l Z_{l,blk} \bar{b}^{\downarrow N_t l}_{e} + \bar{P}^H_k n$$

(3.24)

It should be noted that matrix multiplication i.e., $Z_{l,blk} \bar{P}_k$ in (3.23) should satisfy the commutative property i.e., $Z_{l,blk} \bar{P}_k = \bar{P}_k Z_{l,blk}$ for (3.24) to hold true. This property holds in our case since $Z_{l,blk}$ is a block diagonal matrix and $\bar{P}_k$ is generated by kronecker of $\bar{P}$ with an identity
matrix. We can easily verify it using the "Mixed Product Property" of the kronecker product. i.e., \((A \otimes B) \times (C \otimes D) = AC \otimes BD\). As we know that, \(Z_{l,blk} = I_M \otimes Z_{k,l} = I_M \otimes (I_N \otimes Z_l) = I_{MN} \otimes Z_l\) and \(\bar{P}_k = \bar{P} \otimes I_{N_t}\). So, \(Z_{l,blk} \bar{P}_k = (I_{MN} \otimes Z_l) \times (\bar{P} \otimes I_{N_t}) = I_{MN} \bar{P} \otimes Z_l I_{N_t}\) and likewise, \(\bar{P}_k Z_{l,blk} = (\bar{P} \otimes I_{N_t}) \times (I_{MN} \otimes Z_l) = \bar{P} I_{MN} \otimes I_{N_t} Z_l\). As \(I_{MN}\) and \(I_{N_t}\) are identity matrices therefore, \(\bar{P} I_{MN} = I_{MN} \bar{P}\) and \(Z_l I_{N_t} = I_{N_t} Z_l\). Therefore, we can confirm that \(Z_{l,blk} \bar{P}_k = \bar{P}_k Z_{l,blk}\) is a valid assumption in our case. However, the "Mixed Product Property" i.e., \((A \otimes B) \times (C \otimes D) = AC \otimes BD\) only holds if \(AC\) and \(BD\) are possible. Therefore, in our system model, the number of transmit and receiver antenna should be the same i.e., \(N_t = N_r\) (or a SISO case should be considered). It is therefore a limitation of the proposed system model. In (3.24), the Autocorrelation and cross-correlation matrices of \(\bar{P}_k\) and \(\bar{P}_k\) are defined as \(\bar{G}_k = \bar{P}_k H \bar{P}_k^H\), \(\bar{G}_k = \bar{P}_k H \bar{P}_k^H\) \(\bar{G}_k = \bar{P}_k H \bar{P}_k\) and \(\bar{G}_k = \bar{P}_k H \bar{P}_k\). \(\bar{G}_k\), \(\bar{G}_k\), \(\bar{G}_k\) and \(\bar{G}_k\) are \(I_{MNN_r} \times MNN_t\). The above equation (3.24) can now be written as

\[
\bar{x} \approx \bar{G}_k \sum_{l=0}^{L-1} \rho_l Z_{l,blk} \bar{G}_k I_{NN_l} + \bar{G}_k \sum_{l=0}^{L-1} \rho_l Z_{l,blk} \bar{G}_k I_{NN_l} + \bar{P}_k H \bar{n}
\]

(3.25)

**Real Branch DFT Processing and Phase Shifting**

The signal vector at the output of the real branch filter matrix i.e. \(\bar{P}_k^H\) is represented as \(\bar{x} = [x_0, x_1, \ldots, x_{MNN_r-1}]^T \in \mathbb{C}^{MNN_r \times 1}\) and is then passed through a serial to parallel conversion to split the vector into \(M\) segments, each of which has \(NN_r\) elements to perform \(N\)-point DFT and phase shifting process. The \(m\)th segment of the vector \(\bar{x}\) is represented as \(\bar{x}_m = [\bar{x}_{mNN_r}, \bar{x}_{mNN_r+1}, \ldots, \bar{x}_{mNN_r+NN_r-1}]^T \in \mathbb{C}^{NN_r \times 1}\) for \(m = 0, 1, \ldots, M - 1\). The signal is now represented as \(\bar{x} = [\bar{x}_0, \bar{x}_1, \ldots, \bar{x}_{M-1}] \in \mathbb{C}^{NN_r \times M}\) where \(\bar{x}_m = [\bar{x}_{m0}, \bar{x}_{m1}, \ldots, \bar{x}_{mn-1}]^T \in \mathbb{C}^{NN_r \times 1}\) in which \(\bar{x}_{m,n} = [\bar{x}_{m,n,0}, \bar{x}_{m,n,1}, \ldots, \bar{x}_{m,n,NN_r}]^T \in \mathbb{C}^{NN_r \times 1}\). Each \(\bar{x}_{m,n,i}\) represents the real signal on \(n\)th modulated subcarrier for \(m\)th FBMC symbol received by \(i\)th receiving antenna. Using equation (3.25), we can write signal vector \(\bar{x}_m\) as

\[
\bar{x}_m \approx \sum_{i=0}^{M-1} \bar{G}_{k,m,i} \sum_{l=0}^{L-1} \rho_l Z_{k,l,m,i} + \bar{G}_{k,m,i} \sum_{l=0}^{L-1} \rho_l Z_{k,l,m,i} + \bar{P}_k H \bar{n}
\]

(3.26)

where \(\bar{G}_{k,m,i}\) and \(\bar{G}_{k,m,i}\) are the \(m\)th row and \(i\)th column sub-matrices of \(\bar{G}_k\) and \(\bar{G}_k\) respectively. The signal after DFT and phase shifting is represented as

\[
\bar{y}_m \approx \bar{\Phi}_k H \bar{I}_k \bar{x}_m
\]

(3.27)
3.2. MIMO-FBMC Matrix Model

where \( \overline{\Phi}_{k,m}^H = \overline{\Phi}_{m}^H \otimes I_{N_r} \) and \( F_k = \mathcal{F} \otimes I_{N_r} \in \mathbb{C}^{N N_r \times N N_r} \). Hence, \( \overline{y}_m \in \mathbb{C}^{N N_r \times 1} \) can now be simplified by substituting (3.26) into (3.27) as follows before the channel equalization.

\[
\overline{y}_m \approx \overline{\Phi}_{k,m}^H F_k \sum_{i=0}^{M-1} \overline{G}_{k,m,i} L-1 \sum_{l=0}^{L-1} \rho_l Z_{k,l} \overline{b}_{e,i}^{L N_l} + \overline{\Phi}_{k,m}^H F_k \sum_{i=0}^{M-1} \overline{G}_{k,m,i} L-1 \sum_{l=0}^{L-1} \rho_l Z_{k,l} \overline{b}_{e,i}^{L N_l} + \overline{\Phi}_{k,m}^H F_k \overline{u}_{R,m} \tag{3.28}
\]

In (3.28), the third term is the noise processed by the prototype filter, DFT and the phase shifter. The term \( \overline{u}_{I,m} \) is the interference caused by the imaginary part of the signal \( (\overline{s}_m) \).

Whereas, the first term \( \overline{u}_{R,m} \) contains the actual desired symbol \( (\overline{s}_m) \). In \( \overline{u}_{R,m} \), we can write

\[
\sum_{l=0}^{L-1} \rho_l Z_{k,l} \overline{b}_{e,i}^{L N_l} = \mathbf{H}_{c_{ir}} \overline{b}_i. \]

The matrix \( \mathbf{H}_{c_{ir}} \) is an \( N N_r \times N N_t \) block circulant matrix. In general, an \( N N_r \times N N_t \) block circulant matrix is fully defined by its first \( N N_r \times N N_t \) block matrices \( [105 \text{ pp. 38}] \). In our case, \( \mathbf{H}_{c_{ir}} \) is determined by \( [\mathbf{H}_0, \mathbf{H}_1, \cdots, \mathbf{H}_{L-1}, \mathbf{0}_{(N-1)N_r \times N_t}]^T \in \mathbb{C}^{N N_r \times N N_t} \)

\[
\overline{u}_{R,m} = \overline{\Phi}_{k,m}^H F_k \sum_{i=0}^{M-1} \overline{G}_{k,m,i} \mathbf{H}_{c_{ir}} \overline{b}_i \tag{3.29}
\]

where \( F_k^H F_k = \mathbf{I} \). Then we can use the circular convolution property as follows (pp.129-130) \([106]\).

\[
\overline{u}_{R,m} = \overline{\Phi}_{k,m}^H F_k \sum_{i=0}^{M-1} \overline{G}_{k,m,i} F_k^H \mathbf{H}_{c_{ir}} C \mathcal{F}_k \overline{b}_i \tag{3.30}
\]

where \( C \) is the frequency domain channel coefficients in block diagonal matrix form and is given as \( C = \text{blkdiag}[\mathbf{C}_0, \mathbf{C}_1, \cdots, \mathbf{C}_{N-1}] \in \mathbb{C}^{N N_r \times N N_r} \). The \( n^{th} \) block diagonal element in the frequency response of the MIMO channel can be represented as \( \mathbf{C}_n = \sum_{l=0}^{L-1} \mathbf{H}_{i} e^{-j \frac{2\pi}{N} n l} \in \mathbb{C}^{N_r \times N_t} \). \( \mathcal{F}_k \overline{b}_i \) denotes the DFT processing of \( \overline{b}_i \) and according to (3.5) and (3.4), we have \( \mathcal{F}_k \overline{b}_i = \overline{a}_i = \overline{\Phi}_{k,i} \overline{s}_i \), substituting it into (3.30) leads to

\[
\overline{u}_{R,m} = \overline{\Phi}_{k,m}^H F_k \sum_{i=0}^{M-1} \overline{G}_{k,m,i} F_k^H \overline{\Phi}_{k,i} \overline{s}_i \tag{3.31}
\]

The order of \( C \) and \( \overline{\Phi}_{k,i} \) are exchangeable since both are diagonal, we can thus obtain the following expression

\[
\overline{u}_{R,m} = \sum_{i=0}^{M-1} \overline{\Phi}_{k,m}^H F_k \overline{G}_{k,m,i} F_k^H \overline{\Phi}_{k,i} \overline{s}_i \tag{3.32}
\]
Similarly using the same method we can derive the expression for $\bar{u}_{I,m}$ as

$$\bar{u}_{I,m} = \sum_{i=0}^{M-1} \Phi_{k,m}^H \tilde{\mathcal{F}}_k \tilde{G}_{k,m,i} \Phi_{k,i} \bar{C} \bar{s}_i$$ (3.33)

Substituting (3.32) and (3.33) into (3.28) yields

$$\bar{y}_m \approx \Phi_{k,m}^H \mathcal{F}_k \sum_{i=0}^{M-1} \tilde{G}_{k,m,i} \Phi_{k,i} \bar{C} \bar{s}_i + \Phi_{k,m}^H \mathcal{F}_k \sum_{i=0}^{M-1} \tilde{G}_{k,m,i} \Phi_{k,i} \bar{C} \bar{s}_i$$

$$+ \Phi_{k,m}^H \mathcal{F}_k \bar{P}_{k,m} \bar{n}$$ (3.34)

We can further reduce (3.34) as

$$\bar{y}_m \approx \Phi_{k,m}^H \mathcal{F}_k \tilde{G}_{k,m,m} \Phi_{k,m} \bar{C} \bar{s}_m + \sum_{i=0, i \neq m}^{M-1} \Phi_{k,m}^H \mathcal{F}_k \tilde{G}_{k,m,i} \Phi_{k,i} \bar{C} \bar{s}_i$$

$$+ \Phi_{k,m}^H \mathcal{F}_k \bar{P}_{k,m} \bar{n}$$ (3.35)

**Channel Equalization**

We represent one tap channel equalizer as a block diagonal matrix $E$ and is applied to the real branch signal $\bar{y}_m$ as

$$\bar{u}_m \approx E \bar{y}_m$$ (3.36)

Substituting (3.35) into (3.36) we get the equalized signal $\bar{u}_m$ as

$$\bar{u}_m \approx \mathcal{E} \left( \Phi_{k,m}^H \mathcal{F}_k \tilde{G}_{k,m,m} \Phi_{k,m} \bar{C} \bar{s}_m + \sum_{i=0, i \neq m}^{M-1} \Phi_{k,m}^H \mathcal{F}_k \tilde{G}_{k,m,i} \Phi_{k,i} \bar{C} \bar{s}_i \right) + \mathcal{E} \Phi_{k,m}^H \mathcal{F}_k \bar{P}_{k,m} \bar{n}$$ (3.37)

Eq (3.37) can be written as

$$\bar{u}_m \approx \mathcal{E} \left( \Phi_{k,m}^H \mathcal{F}_k \tilde{G}_{k,m,m} \Phi_{k,m} \bar{C} \bar{s}_m + \sum_{i=0, i \neq m}^{M-1} \Phi_{k,m}^H \mathcal{F}_k \tilde{G}_{k,m,i} \Phi_{k,i} \bar{C} \bar{s}_i \right) + \mathcal{E} \Phi_{k,m}^H \mathcal{F}_k \bar{P}_{k,m} \bar{n}$$ (3.38)

where $\mathcal{E}$ can be either ZF or MMSE based linear channel equalizer

$$\mathcal{E} = C^H \left( CC^H + \nu^2 / \delta^2 I \right)^{-1}$$ (3.39)
where \( \nu = 0 \) for ZF while \( \nu = 1 \) is for MMSE case. With a simple ZF equalization i.e. \( \mathbf{E} = (\mathbf{C})^{-1} \), we can write (3.38) as

\[
\bar{\mathbf{u}}_m \approx \bar{\Phi}_k^H \mathcal{T}_k \bar{G}_{k,m,m} \mathcal{T}_k \bar{\Phi}_k \bar{s}_m + \sum_{i=0, i \neq m}^{M-1} \bar{\Phi}_k^H \mathcal{T}_k \bar{G}_{k,m,i} \mathcal{T}_k \bar{\Phi}_k \bar{s}_i + E \bar{\Phi}_k^H \mathcal{T}_k \bar{\mathbf{P}}_k \mathbf{n} \]

(3.40)

The autocorrelation and cross correlation matrices used in (3.40) are defined using the non-truncated filter matrices as \( \bar{\mathbf{G}}_k = \bar{\mathbf{P}}_k^H \mathbf{P}_k^\text{orig} \) and \( \bar{\mathbf{G}}_k = \mathbf{P}_k^H \mathbf{P}_k^\text{orig} \) respectively. According to the orthogonality of FBMC with infinite filter length and no truncation [23], \( \bar{\mathbf{Q}}_{k,m,i} \) and \( \bar{\bar{\mathbf{Q}}}_{k,m,i} \) defined in (3.40) have the following property:

\[
\bar{\mathbf{Q}}_{k,m,i} = \begin{cases} 
\mathbf{I} + j \Re\{\bar{\mathbf{Q}}_{k,m,i}\} & \text{for } i = m \\
 j \Im\{\bar{\mathbf{Q}}_{k,m,i}\} & \text{for } i \neq m 
\end{cases}
\]

(3.41)

The property given in (3.41) suggest that the matrix \( \bar{\mathbf{Q}}_{k,m,i} \in \mathbb{C}^{NN_t \times NN_t} \) has a particular structure when \( i = m \) and \( i \neq m \). When \( i = m \), the diagonal entries in \( \bar{\mathbf{Q}}_{k,m,i} \) are unity (real value) like in an identity matrix while the off diagonal entries are all imaginary values. Whereas, when \( i \neq m \), the diagonal entries are zero and off diagonal entries are all imaginary values. So if we take the real part of (3.41), the property is then simplified as

\[
\Re\{\bar{\mathbf{Q}}_{k,m,i}\} = \begin{cases} 
\mathbf{I} & \text{for } i = m \\
 0 & \text{for } i \neq m 
\end{cases}
\]

(3.42)

The simplified property satisfies the real orthogonality condition and the received signal will be free from intrinsic interference. Now using the property given in (3.41), we can write (3.40) as

\[
\bar{\mathbf{u}}_m \approx \bar{s}_m + j \left[ \sum_{i=0}^{M-1} \Re\{\bar{\mathbf{Q}}_{k,m,i}\} \bar{s}_i + \sum_{i=0}^{M-1} \Im\{\bar{\mathbf{Q}}_{k,m,i}\} \bar{s}_i \right] + \bar{\mathbf{u}}_{\text{noise},m}
\]

(3.43)

The \( \bar{\mathbf{u}}_{\text{intri},m} \) term is the pure imaginary intrinsic interference that is inherent in the FBMC system. It can be avoided by taking the real part of (3.43) which is also satisfied by the simplified property given in (3.42). Hence, we can write (3.43) as

\[
\Re\{\bar{\mathbf{u}}_m\} \approx \bar{s}_m + \Re\{\bar{\mathbf{u}}_{\text{noise},m}\}
\]

(3.44)
Eq (3.44) shows that with infinite filter length and no FOT, the actual transmitted symbol in real branch i.e., $\bar{s}_m$ can be recovered without any intrinsic interference. The term $\Re(\bar{u}_{\text{noise},m})$ is the real part of the processed noise.

The equalized imaginary branch symbol, $\tilde{u}_m$ can be written as follows using the same approach as adopted for the real branch.

$$\tilde{u}_m \approx \tilde{\Phi}_{k,m}^H \mathcal{F}_k \tilde{G}_{k,m,m} \mathcal{F}_k^H \tilde{\Phi}_{k,m} \tilde{s}_m + \sum_{i=0, i \neq m}^{M-1} \tilde{\Phi}_{k,m}^H \mathcal{F}_k \tilde{G}_{k,m,i} \mathcal{F}_k^H \tilde{\Phi}_{k,i} \tilde{s}_i + \tilde{\Phi}_{k,m}^H \mathcal{F}_k \tilde{P}_{k,m} \mathcal{F}_k^H \tilde{n}$$

Using the property of infinite filter length given in (3.46), we can write (3.45) as

$$\tilde{u}_m \approx j \tilde{s}_m + \sum_{i=0}^{M-1} \Re\{\tilde{Q}_{k,m,i}\} \tilde{s}_i + \tilde{u}_{\text{noise},m}$$

Taking the imaginary part of (3.47), we have

$$\Im\{\tilde{u}_m\} \approx \tilde{s}_m + \Im\{\tilde{u}_{\text{noise},m}\}$$

### 3.2.4 Combining Real and Imaginary Branches

From (3.44) and (3.47), we can write the estimation of the transmitted symbol $s_m$ as follows

$$\hat{s}_m \approx \Re\{\tilde{u}_m\} + j \Im\{\tilde{u}_m\}$$

### 3.3 Widely Linear Receiver for MIMO-FBMC System

As we have already discussed in Section 2.6 that WLP can provide more precise estimate of the transmitted signals (improper or non-circular signals) compared to linear processing techniques. As we know that FBMC/OQAM employs improper signals, it is possible to exploit
the benefits of WLP for FBMC system as well. However, intrinsic interference that contaminates the real transmitted symbols in FBMC/OQAM is not only the main obstacle in combining FBMC with techniques like MIMO but also it refrains the exploitation of WLP technique for FBMC/OQAM system. We have seen some methods of canceling the intrinsic interference in Section 2.7.1. However, the proposed methods suffer from residual errors due to imprecise estimation of the intrinsic interference. The method I defined in (2.37) suffers from estimation bias error while method II defined in (2.38) just rely on a few neighboring symbols to estimate the intrinsic interference. This method also introduces delay in the system since it follows a shifting window approach to estimate intrinsic interference at all the time frequency positions in a FBMC/OQAM block.

Using the proposed MIMO-FBMC matrix model, we can estimate the intrinsic interference for both real and imaginary branches using already detected symbols. The precise estimation of the intrinsic interference can be achieved by combining the response of SFB and AFB as defined in (3.41), where the pure imaginary components contribute to the intrinsic interference in the FBMC/OQAM system. We can estimate the pure imaginary intrinsic interference in the real branch signal as follows

\[
\hat{u}_{\text{intri},m} = j \left[ \sum_{i=0}^{M-1} \Im\{\tilde{Q}_{k,m,i}\} \tilde{s}_i + \sum_{i=0}^{M-1} \Im\{\tilde{\tilde{Q}}_{k,m,i}\} \tilde{s}_i \right] \quad (3.49)
\]

Assuming perfect CSI is available at the receiver we can cancel this intrinsic interference from the real branch received signal vector given in (3.35) as follows

\[
\tilde{y}'_m = \tilde{y}_m - H\hat{u}_{\text{intri},m} \quad (3.50)
\]

A widely linear equalizer can now be applied to have a more precise estimate of the transmitted signal as shown in Fig. 3.2.

\[
\hat{s}_m = W_1\tilde{y}'_m + W_2\tilde{y}'_m^{\text{fs}} \quad (3.51)
\]
The widely linear filters i.e., $W_1$ and $W_2$ in (3.51) are defined in (2.36).

Similarly, the intrinsic interference can be canceled from the imaginary branch received signal vector and a widely linear equalizer can be subsequently used to obtain a more precise estimate of the transmitted signal. The improved performance of WLP in MIMO-FBMC system can be seen from the BER results given in Fig. 3.3. It can be seen that widely linear equalizer significantly improves the performance of MIMO-FBMC/OQAM system compared to conventional linear equalizer. The improvement comes from the fact that WLP process the received signal together with its conjugate version to obtain a more precise estimate at the receiver by exploiting the impropriety of the MIMO-FBMC/OQAM signals. This result in improved BER performance compared to conventional linear receivers. However, since the main difference between the two receivers is that WL receiver first utilize the conventional linear MMSE receiver for intrinsic interference estimation and cancellation and then a WL equalizer is subsequently used for better estimation at the receiver. As WL stage is also designed using MMSE approach, this result in two equalizers i.e., $W_1$ and $W_2$ to process the incoming signal and its conjugate version as shown in (3.51). Hence, we can consider the complexity of a WL-MMSE receiver to be three times the complexity of a conventional MMSE receiver.

### 3.4 Finite Filter Length and Filter Output Truncation Analysis

This section presents the impact of finite filter length and FOT on the system performance. We will first derive the interferences caused by finite filter length and the FOT and then FOT...
analysis will be presented in detail. We will focus on the real branch in detail since imaginary branch will follow the same procedure.

3.4.1 Finite Filter Length ($K \neq \infty$) with FOT

As it is impractical to use infinite filter length from implementation point of view, we now consider the practical case where we consider a finite filter length ($K \neq \infty$) with FOT. In this case the autocorrelation and cross correlation matrices given in (3.40) are now defined using the truncated matrices define in (3.3) i.e. $\tilde{G}_k = \tilde{P}_k^H \tilde{P}_k$ and $\tilde{G}_k = \tilde{P}_k^H \tilde{P}_k$ respectively. In this case, (3.41) will now be modified as

$$
\mathbb{R}\{\tilde{Q}_{k,m,i}\} = \begin{cases} I + \mathbb{R}\{\Delta \tilde{Q}_{k,m,m}\} & \text{for } i = m \\ \mathbb{R}\{\Delta \tilde{Q}_{k,m,i}\} & \text{for } i \neq m \end{cases}
$$

$$
\mathbb{R}\{\tilde{Q}_{k,m,i}\} = \mathbb{R}\{\Delta \tilde{Q}_{k,m,i}\} \text{ for } i = 0, \cdots, M-1
$$

(3.52)

where $\Delta \tilde{Q}_{k,m,i} = \tilde{F}_{k,m}^H \tilde{G}_{k,m,i} \tilde{F}_{k,i}^H$ and $\Delta \tilde{Q}_{k,m,i} = \tilde{F}_{k,m}^H \tilde{G}_{k,m,i} \tilde{F}_{k,i}^H$ in which $\Delta \tilde{G}_{k,m,i}$ and $\Delta \tilde{G}_{k,m,i}$ are the error matrices due to the finite filter length and truncating effect. Hence, Eq (3.44) will now be modified as

$$
\mathbb{R}\{\tilde{u}_m\} = \tilde{s}_m + \sum_{i=0}^{M-1} \mathbb{R}\{\Delta \tilde{Q}_{k,m,i}\} \tilde{s}_i + \sum_{i=0}^{M-1} \mathbb{R}\{\Delta \tilde{Q}_{k,m,i}\} \tilde{s}_i + \mathbb{R}\{\tilde{u}_{noise,m}\}
$$

(3.53)

The variance of elements in the error matrices not only depends on the filter length $K$ and the truncation number $i_F$ and $i_R$, but more importantly on the odd or even value of $K$. The truncation causes the filter correlation matrices to be unsaturated at both the edges i.e., the symbols at the start and at the end of the block will experience truncation effect while the truncation causes the filter correlation matrix to be saturated in the middle part. Hence, the symbols in the middle of the filtered MIMO-FBMC block are least effected. This can be confirmed from [32], where we have demonstrated that with finite filter length ($K = 6$), the filter output contains $K-1$ symbols and that these extra tails at the edges of the FBMC block have small average energy compared to the middle part of the block.

3.4.2 Filter Output Truncation (FOT) Analysis

To analyze the impact of these factors on the filter output truncation, we consider the following cases. We first consider the even value of filter length ($K = 6$), which will introduce $K-1$ tails i.e. 5 extra symbols at the output of the transmit filter (IOTA prototype filter). We have considered $N = 64$ i.e., FFT size and $M = 8$ i.e. symbols per block at the input of the filter.
3.4. Finite Filter Length and Filter Output Truncation Analysis

Note that this value of \( M \) is considered just as an example and does not affect the outcomes of the analysis.

1. **Use it all**: No cut at all \( (i_F = 0, i_R = 0) \), i.e., input 8 symbols and output 13 symbols.

2. **One symbol (front and end)**: Cut 2 at the front and 1 at the end \( (i_F = 2, i_R = 1) \), i.e., input 8 symbols and output 10 symbols.

3. **One symbol (front)**: Cut 2 at the front and 2 at the end \( (i_F = 2, i_R = 2) \), i.e., input 8 symbols and output 9 symbols.

4. **One symbol (end)**: Cut 3 at the front and 1 at the end \( (i_F = 3, i_R = 1) \), i.e., input 8 symbols and output 9 symbols.

5. **The same length**: Cut the front 3 and last 2 symbols \( (i_F = 3, i_R = 2) \), to keep the number of symbols the same i.e. input 8 symbols and output 8 symbols.

Fig. 3.4 shows the desired signal and interference powers for real and imaginary branches in case of finite filter length \( (K = 6) \) with different FOT scenarios. The observations drawn from Fig. 3.4 regarding the aforementioned cases are discussed as follows

- **Use it all** case i.e. no truncation can achieve very good performance for both real or imaginary branches. As in this case, the second and third terms in (3.53) will not exist and therefore the desired symbols are free from interference terms.

- **One symbol (front and end)** case can achieve similar performance as in use it all case, only marginal difference is at the edge symbols. This is because the one symbol at the front has significant energy as compared to the other tails [32]. Leaving this symbol at the front will significantly reduce the interference level and the effect of cutting other two symbols at the front and one at the end has much less affect on the neighboring symbols as can be seen from Fig. 3.4(c).

- **One symbol (front)** case introduces interference at the last symbols i.e. \( m = 7, 8 \) compared to the one symbol (front and end) case. This loss is tolerable as the signal power loss and the increase in the interference level for \( m = 7 \) and \( m = 8 \) are insignificant as can be seen from Fig. 3.4(c). These losses are acceptable as we are avoiding an extra symbol overhead compared to the one symbol (front and end) case. This performance loss at the last symbols is due to the truncation at the end of the filter that introduces interference in the last two symbols.
3.4. Finite Filter Length and Filter Output Truncation Analysis

Figure 3.4: Signal and interference power with output SIR in real and imaginary branches ($K = 6$)
• However, One symbol (end) case does not work as the signal power for \( m = 1 \) is reduced and the interference level has increased significantly which are both unacceptable. It is because in this case we are truncating the front part of the filter that discards all the symbols at the front of the block and introduces significant interference in the neighboring symbols. Hence, leaving one symbol at the end is not a good strategy.

• In the same length case, the desired signal power and interference power for the symbols at the edges \((m = 1, 2, 7, 8)\) are affected significantly. This is because the extra symbols at the start and the end of the block are truncated that affects their neighboring symbols. In this case, the second and third terms in (3.53) will exist and as a result, the detected symbols will be affected by these interference terms.

The output signal to interference ratio (SIR) for real and imaginary branches is illustrated in Fig. 3.4(e) and Fig. 3.4(f) respectively, where we can see that with a finite filter length \((K = 6)\), the best SIR can be obtained with use it all case; however, the overhead is quite high in this case. While the same length case can completely remove the overhead but significantly reduces the SIR of the symbols at the edges. A good balance is to adopt the one symbol (front) case for even \( K \) which offers an acceptable trade-off between the overhead and the performance.

However, the observations are totally reversed when we consider the odd number of filter length e.g. \((K = 5)\). In this case the last symbol in the imaginary branch is significantly affected by the FOT as can be seen from Fig. 3.5. The one symbol (end) case is now more effective in case of odd filter length as it provides better SIR compared to the other cases as can be seen from Fig. 3.5(e) and Fig. 3.5(f).

Since the target branch and symbol are totally different for odd and even \( K \); in the next section, we will focus on the even \( K \) only for proposing the compensation algorithm. The compensation algorithm for the odd \( K \) can be derived using the same approach.

3.5 Proposed Compensation Algorithm

Although adding one symbol (front) case can provide acceptable performance \((\text{SIR}>20\text{dB})\). However, this approach is valid only when the block size \( M \) is large. For instance when \( M = 20 \), the total overhead is only 5% and this percentage further drops when \( M \) goes to larger value \([32] \). However, considering different traffic models and also the latency of the data, the solution that one symbol (front) case may cause significant overhead e.g. with moderate \( M = 5 \), the total overhead is 20%, which is very inefficient.

In order to overcome this inefficiency for moderate \( M \), we propose a compensation approach
3.5. Proposed Compensation Algorithm

Figure 3.5: Signal and interference power with output SIR in real and imaginary branches ($K = 5$)
which allows complete removal of the overhead caused by the filtering operation. Note that when \( K \) is even, if we consider the same length case only the first symbol on the \( I \) branch has unacceptable level of SIR whereas the corresponding symbol on the \( Q \) branch has sufficient SIR level (20dB) as can be see from Fig. 3.4(e) and Fig. 3.4(f) respectively. While in the odd \( K \) case, the situation is opposite (only the last symbol on the \( Q \) branch has unacceptable level of SIR) as can be seen from Fig. 3.5(f). With this observation, we can state that all of the other symbols (both real and imaginary, except the first real symbol for even \( K \) or last imaginary symbol for odd \( K \)) can be easily detected. Considering the even \( K \) case with the assumption that the channel is known and that we only need to compensate the first symbol in the real branch to have sufficient SIR value to detect all the symbols. According to (3.53), the first \( I \) branch symbol can be written as
\[
\Re \{ \bar{u}_0 \} = \bar{s}_0 + \sum_{i=0}^{M-1} \Re \{ \bar{\Phi}^H H k,0 \Delta \bar{G}_k,0,i \bar{H} H \Phi k,i,0 \} \bar{s}_i + \sum_{i=0}^{M-1} \Re \{ \bar{\Phi}^H H k,0 \Delta \bar{G}_k,0,i \bar{H} H \Phi k,i,1 \} \bar{s}_i 
\]
\[
= \bar{s}_0 + \sum_{i=0}^{M-1} \Re \{ \bar{\Phi}^H H k,0 \Delta \bar{G}_k,0,i \bar{H} H \Phi k,i,0 \} \bar{s}_i + \sum_{i=1}^{M-1} \Re \{ \bar{\Phi}^H H k,0 \Delta \bar{G}_k,0,i \bar{H} H \Phi k,i,1 \} \bar{s}_i 
\]
\[
+ \sum_{i=0}^{M-1} \Re \{ \bar{\Phi}^H H k,0 \Delta \bar{G}_k,0,i \bar{H} H \Phi k,i,1 \} \bar{s}_i 
\]
(3.54)
The first term in (3.54) is the desired signal, the second term is the ICI and the third and fourth terms are the ISI caused by the \( I \) and \( Q \) branches respectively. For simplicity, we omit the noise term. In order to improve the SIR of the first symbol in the \( I \) branch, we need to compensate the ICI and the ISI terms at the receiver. For this, we need to find the compensation matrices i.e. \( \Delta \bar{G}_k,0,i \) and \( \Delta \bar{G}_k,0,i \) in (3.54). Note that the \( \Delta \bar{G}_k,0,i \) and \( \Delta \bar{G}_k,0,i \) are caused by the FOT which brings significant SIR reduction for some symbols. To derive the matrices, we define the perfect autocorrelation matrices \( \bar{G}_k,orig \) and \( \bar{G}_k,orig \) as follows
\[
\bar{G}_k,orig = \bar{P}_k,orig \bar{P}_k,orig = \begin{bmatrix} \bar{P}_k,iF \bar{P}_k,RF \end{bmatrix} \begin{bmatrix} \bar{P}_k,iF \\ \bar{P}_k,RF \end{bmatrix} 
\]
\[
= \bar{P}_k,iF \bar{P}_k,iF + \bar{P}_k,RF \bar{P}_k,RF 
\]
(3.55)
Similarly,
\[
\bar{G}_k,orig = \bar{P}_k,iF \bar{P}_k,iF + \bar{P}_k,RF \bar{P}_k,RF 
\]
(3.56)
We can write the compensation matrices \( \Delta \bar{G}_k \) and \( \Delta \bar{G}_k \) using the perfect autocorrelation matrices (\( \bar{G}_k,orig \) and \( \bar{G}_k,orig \)) and the truncated autocorrelation matrices (\( \bar{G}_k = \bar{P}^H H \bar{P}_k \)) as:
\[
\Delta \bar{G}_k = \bar{G}_k,k,orig - \bar{G}_k = \bar{P}_k,iF \bar{P}_k,iF + \bar{P}_k,RF \bar{P}_k,RF 
\]
(3.57)
\[ \Delta \tilde{G}_k = \tilde{G}_{k,orig} - \tilde{G}_k = \bar{P}^H_{k_i} \tilde{P}_{k,i,R} + \bar{P}^H_{k_i} \tilde{P}_{k,i,R} \]  

(3.58)

Now for even \( K \) case, we propose the following compensation algorithm to determine \( \Delta \tilde{G}_{k,0,i} \) and \( \Delta \tilde{G}_{k,0,i} \) in (3.54) for compensating ISI in the first real symbol. Using (3.57) and (3.58), we can determine \( \Delta \tilde{G}_{0,0,i} = \Delta \tilde{G}_{0,0,i} \otimes I_{N_r} \) and \( \Delta \tilde{G}_{k,0,i} = \Delta \tilde{G}_{k,0,i} \otimes I_{N_r} \) for \( i = 0 \cdots M - 1 \) using (3.3) as

\[
\begin{align*}
\Delta \tilde{G}_{0,0} &= W_0^H W_0 + W_1^H W_1 + W_2^H W_2 \\
\Delta \tilde{G}_{0,1} &= W_1^H W_0 + W_2^H W_1 \\
\Delta \tilde{G}_{0,2} &= W_2^H W_0 \\
\Delta \tilde{G}_{0,j} &= 0 \quad \text{for} \quad 3 \leq j \leq M - 1 
\end{align*}
\]

(3.59)

and

\[
\begin{align*}
\Delta \tilde{G}_{0,0} &= W_0^H W_0 + W_1^H W_1 + W_2^H W_2 \\
\Delta \tilde{G}_{0,1} &= W_1^H W_0 + W_2^H W_1 \\
\Delta \tilde{G}_{0,2} &= W_2^H W_0 \\
\Delta \tilde{G}_{0,j} &= 0 \quad \text{for} \quad 3 \leq j \leq M - 1 
\end{align*}
\]

(3.60)

### 3.5.1 Compensating the Real Branch Signal

The real branch signal is affected by ISI and ICI terms as shown in (3.54). The proposed algorithm can compensate these two interferences as follows

#### Compensating the ISI

The third and fourth terms in (3.54) are the ISI terms caused by the \( J \) and \( Q \) branch symbols. Using (3.59) and (3.60), we can compensate these ISI terms at the receiver side as

\[
\begin{align*}
\bar{u}_{0,comp} = & \Re \{ \bar{u}_0 \} - \sum_{i=1}^{M-1} \Re \{ \Phi^H_{k,0} F_k \Delta \tilde{G}_{k,0,i} \tilde{F}^{H}_k \tilde{\Phi}_{k,i} \} \bar{s}_i - \sum_{i=0}^{M-1} \Re \{ \Phi^H_{k,0} F_k \Delta \tilde{G}_{k,0,0} \tilde{F}^{H}_k \tilde{\Phi}_{k,0} \} \bar{s}_i \\
= & \bar{s}_0 + \Re \{ \Phi^H_{k,0} F_k \Delta \tilde{G}_{k,0,0} \tilde{F}^{H}_k \tilde{\Phi}_{k,0} \} \bar{s}_0 \\
= & \Re \{ I + \Re \{ \Phi^H_{k,0} F_k \Delta \tilde{G}_{k,0,0} \tilde{F}^{H}_k \tilde{\Phi}_{k,0} \} \} \bar{s}_0 \\
= & \Re \{ I + \Phi^H_{k,0} F_k \Delta \tilde{G}_{k,0,0} \tilde{F}^{H}_k \tilde{\Phi}_{k,0} \} \bar{s}_0 
\end{align*}
\]

(3.61)
### Compensating the ICI

To compensate the ICI in (3.61), we need to determine the term $\bar{\Phi}^H_{k,0}J_k \Delta \bar{G}_{k,0,0} J^H_k \bar{\Phi}_{k,0}$. Since, $\bar{\Phi}_{k,0}$ and $J_k$ are already known, we can evaluate the value of $\Delta \bar{G}_{k,0,0}$ from (3.59). Hence, we can compensate this term by using a ZF (or if we consider the noise term in (3.54) we can use MMSE) equalization at the receiver to estimate $\bar{s}_0$ with relatively higher SIR as

$$\hat{s}_0 = (\mathbb{R}[\mathbb{I} + \bar{\Phi}^H_{k,0} J_k \Delta \bar{G}_{k,0,0} J^H_k \bar{\Phi}_{k,0}])^{-1} \bar{u}_{0,\text{comp}}$$

(3.62)

It can be seen from (3.62) that both ICI and ISI can be compensated at the receiver side. The proposed compensation algorithm can provide the same SIR for the first real symbol as in the use it all case by compensating the effect of FOT as can be seen from Fig. 3.6(e). Further, we can derive a generalized expression of (3.62) which can be used to further improve the SIR of other symbols as well by compensating their ICI and ISI terms. The generalized expression of (3.61) can be derived as

$$\bar{u}_{m,\text{comp}} = \mathbb{R}\{\bar{u}_m\} - \sum_{i=0,i \neq m}^{M-1} \mathbb{R}\{\bar{\Phi}^H_{k,m} J_k \Delta \bar{G}_{k,m,i} J^H_k \bar{\Phi}_{k,i}\} \bar{s}_i - \sum_{i=0}^{M-1} \mathbb{R}\{\bar{\Phi}^H_{k,m} J_k \Delta \bar{G}_{k,m,m} J^H_k \bar{\Phi}_{k,m}\} \bar{s}_m$$

$$= \bar{s}_m + \mathbb{R}\{\bar{\Phi}^H_{k,m} J_k \Delta \bar{G}_{k,m,m} J^H_k \bar{\Phi}_{k,m}\} \bar{s}_m$$

$$= \mathbb{R}[\mathbb{I} + \bar{\Phi}^H_{k,m} J_k \Delta \bar{G}_{k,m,m} J^H_k \bar{\Phi}_{k,m}] \bar{s}_m$$

(3.63)

Similarly, (3.62) can be generalized as

$$\hat{s}_m = (\mathbb{R}[\mathbb{I} + \bar{\Phi}^H_{k,m} J_k \Delta \bar{G}_{k,m,m} J^H_k \bar{\Phi}_{k,m}])^{-1} \bar{u}_{m,\text{comp}}$$

(3.64)

In (3.63), it is worth mentioning that the term $\sum_{i=0,i \neq m}^{M-1} \mathbb{R}\{\bar{\Phi}^H_{k,m} J_k \Delta \bar{G}_{k,m,i} J^H_k \bar{\Phi}_{k,i}\} \bar{s}_i$ should be treated carefully for $i = 0$, since only accurate $\hat{s}_0$ will bring accurate compensation to other symbols, otherwise errors will be introduced, which implies that $\hat{s}_0$ should be always compensated first.

#### 3.5.2 Compensating the Imaginary Branch Signal

We can now compensate the $\Omega$ branch as well using the same approach used for the $I$ branch; however, since all the symbols already have good initial SIR, it will be easier to compensate them in this branch compared to the $I$ branch. The compensation approach for the $\Omega$ branch is as follows

$$\hat{s}_m = (\mathbb{R}[\mathbb{I} + \bar{\Phi}^H_{k,m} J_k \Delta \bar{G}_{k,m,m} J^H_k \bar{\Phi}_{k,m}])^{-1} \bar{u}_{m,\text{comp}}$$

(3.65)
where,
\[
\tilde{u}_{m,\text{comp}} = \Im\{\tilde{u}_m\} - \sum_{i=0, i \neq m}^{M-1} \Im\{\tilde{\Phi}^H_{k,m} \Delta \tilde{G}_{k,m,i} \tilde{\Phi}_{k,i}\} \tilde{s}_i - \sum_{i=0}^{M-1} \Im\{\tilde{\Phi}^H_{k,m} \Delta \bar{G}_{k,m,i} \bar{\Phi}_{k,i}\} \bar{s}_i
\]  
(3.66)

It can be seen from (3.65) that both ICI and ISI can also be compensated for the imaginary branch at the receiver side. The SIR of the symbols at the edges are significantly improved with the use of the proposed compensation algorithm by compensating the effect of FOT as can be seen from Fig. 3.6(f).

### 3.6 Simulation Results

In this section, we present a set of simulation results to demonstrate the effectiveness of the proposed compensation algorithm in the *the same length* case. For simulations, the selected parameters for the MIMO-FBMC system includes the IOTA prototype filter with overlapping factor \( K = 6 \). The number of transmit and receive antennas are \( N_t = N_r = 2 \). The FFT size is considered as \( N = 64 \) and the block size is considered as \( M = 8 \). The desired signal is modulated by QPSK with normalized power and input SNR is controlled by the noise power. The LTE channel model considered in our simulation is the extended pedestrian A model (EPA) \[108\]. For the equalization, the MMSE based equalizer is selected as it is more generic. As we have concluded in Section 3.5 that our main concern is the first real branch symbol which has a very low SIR value of around 2dB. The proposed compensation algorithm given in (3.62) significantly improves the SIR of the first real symbol i.e. the signal power increase from -5.1dB to 0dB while the interference level drops from -5dB to -48dB. Hence, increasing the SIR of the first real symbol from 2dB to 48dB as can be seen from Fig. 3.6. The proposed general form of the compensation algorithm compensates all the symbols in a block. However, it is important that the first symbol in the real branch i.e., \( \bar{s}_0 \) is decoded first since it will be used to estimate the ISI and ICI effect on the neighboring symbols. In our proposed system we are concerned with the SIR value of each symbol and since we have compensated the interference in the first real symbol and consequently the SIR has also improved we have assumed that the symbol \( \bar{s}_0 \) will be perfectly detected at the receiver. Since the compensate all case is compensating the interference terms in all the neighboring symbols, the SIR performance has improved compared to use it all case as can be seen from Fig. 3.6. Note that we do not need to compensate the imaginary branch as it already has sufficient SIR values for detecting all the symbols at the receiver as discussed in Section 3.5. However, the term acceptable SIR value is strongly dependent on the modulation order as higher modulation order require...
3.6. Simulation Results

Figure 3.6: Signal and interference power with output SIR using compensation algorithm ($K = 6$)
high SIR values for achieving a specific required performance. The proposed general form of
the compensation algorithm significantly improves the SIR of all the symbols in the real and
imaginary branches as can be see from Fig. 3.6. Compensating all the symbols can help in
improving the probability of detection at the receiver. The coded results (convolutional code
with code rate 1/2) for the BER performance of various FOT schemes in MIMO-FBMC system
with and without compensation algorithm are presented in Fig. 3.7.

Figure 3.7: BER performance of OFDM and FBMC system with and without compensation

It can be observed that the system performance in case of use it all and one symbol (front)
has similar BER performance but the latter required only one extra tail compared to the for-
mer, which required $K - 1$ extra tails. The same length case requires no extra tail but has a
relatively poor BER performance. We have used conventional MIMO-OFDM as a baseline
scheme to show the advantage of the proposed algorithm over such conventional multicarrier
schemes. For a fair comparison between MIMO-FBMC and MIMO-OFDM systems, the SNR
loss, due to the cyclic prefix (overhead) in OFDM, must be considered. For this reason, we
have calculated the noise power for both systems as discussed in [32]. The comparison shows
the significance of the proposed algorithm especially for higher modulation schemes. It can
be seen from Fig. 3.7 that for low order modulation schemes like QPSK, MIMO-FBMC sys-
tem without compensation can still perform better than conventional MIMO-OFDM but if we
increase the modulation schemes to higher order like 16QAM or 64QAM, the performance of
MIMO-FBMC system without compensation becomes poorer than MIMO-OFDM due to self-
interference caused by FOT. In such cases, use of the proposed compensation algorithm is very
significant as it not only provides better performance but also improves the SE of the system.
The spectral efficiency of the system has been simulated using Shannon equation [104] which
3.6. Simulation Results

gives an upper bound of the capacity that the system can achieve i.e. maximum error free transmission rate. Note that the capacity is measured using only the simulation and is not the exact representation of achievable capacity. The objective is to provide an idea regarding the SE gain that can be achieved using the FOT and the compensation algorithm at the receiver. The SE expression with respect to the block size \( M \) used in the simulation is given as follows

\[
SE = \min\{N_T, N_R\} \times \frac{M}{M + \alpha} \left\{ \frac{1}{M} \sum_{i=1}^{M} \log_2 (1 + SINR_i) \right\}
\]

(3.67)

where \( \alpha \) represents the overhead in each case i.e. \( K - 1 \) for use it all, 1 for one symbol (front) and 0 for compensate all. The SE in each case is given in Fig. 3.8. It can be observed from Fig. 3.8(a) that the SE is independent of \( M \) for the compensate all case whereas it is dependent for the use it all and one symbol (front) cases as they have one and \( K - 1 \) tails respectively with each block. It can also be seen from Fig. 3.8(b) that the SE gain obtained using the compensate all case reduces with the increase in \( M \) for both use it all and one symbol (front) cases. Hence, the compensation algorithm is best suited for applications that has a frame structure based on moderate \( M \).

The SE results for a range of SNR \((E_b/N_o)\) values are also shown in Fig. 3.9. With a fixed block size, the SE of the system increases as input SNR \((E_b/N_o)\) increases as shown in Fig. 3.9(a). It can be observed that SE performance of the compensate all case is better than one symbol (front) and use it all cases, as these FOT schemes require certain overhead to achieve improved BER performance. However, the proposed compensation algorithm provides similar BER performance by compensating the effects of FOT in the same length case without introducing any overhead. This enables compensate all case to have a certain SE gain compared to other FOT schemes as can be seen from Fig. 3.9(b).
3.7 Conclusion

The impact of finite filter length and different types of FOT has been theoretically analyzed in a MIMO-FBMC system. The analysis is based on a compact matrix model of a MIMO-FBMC system, which was then used for investigating the effects of FOT on the detection performance in terms of the SIR and BER. The analysis showed that although FOT can avoid overhead but it also destroys the orthogonality in the FBMC system thus introducing interferences. However, only real part of the first symbol or the imaginary part of the last symbol are affected by the aforementioned interferences. A general form of compensation algorithm based on our analysis has been designed to compensate the symbols in a MIMO-FBMC block to improve the SIR of each symbol. The advantage of this algorithm is that it improves the SE of the system as it requires no overhead and at the same time can still achieve similar performance compared to the case without FOT. However, the SE gain tends to reduce with the increasing $M$ as the overhead tends to decrease with increase in frame size. The proposed analytical framework developed in this chapter provide useful insights into the effect of finite filter length and FOT on the system performance and the proposed compensation algorithm can enable MIMO-FBMC system to achieve higher SE compared to its conventional counterpart.
Chapter 4

Quadrature Modulated Symbol Transmissions in FBMC Systems using Inverse Filter

Transmission of quadrature modulated symbols using filter bank multicarrier systems has been an issue due to the self-interference between the transmitted symbols both in the time and frequency domain (so-called intrinsic interference). In this chapter, we propose a novel low-complexity interference-free FBMC system with QAM modulation (FBMC/QAM) using filter deconvolution. The proposed method is based on inversion of the prototype filters which completely removes the intrinsic interference at the receiver and allows the use of quadrature modulated signaling. The interference terms in FBMC/QAM with and without the proposed system are analyzed and compared in terms of MSE. It is shown with analytical and simulation results that the proposed method cancels the intrinsic interference and improves the output SINR at the expense of slight enhancement of noise and residual interferences caused by multipath channel. The complexity of the proposed system is also analyzed along with performance evaluation in an asynchronous multi-service scenario. It is shown that the proposed FBMC/QAM system with filter deconvolution outperforms the conventional OFDM system.

4.1 FBMC/QAM System

As we have discussed in Section 2.4.2, that the idea behind FBMC with QAM modulation is to reach a quasi-orthogonal signal while maintaining per-subcarrier filtering. Therefore, the
FBMC/QAM system is still affected by the intrinsic interference. To mitigate this interference in FBMC/QAM system, a remedial system is required which is known as *inverse system* in the general context of linear systems theory \[109\]. The inverse system, is cascaded with the multicarrier filtering, and thus yields a replica of the transmitted symbols without interference terms, after channel equalization. Since the inverse system counteracts the effect of multicarrier filtering, the process is called *deconvolution*. In this process, the transmitted symbols are separated from the filtering characteristics of the system. We propose this novel interference-free FBMC/QAM system based on inversion of the prototype filters. The advantage of this system is that it can retain the positive features of FBMC and OFDM at the same time e.g. the channel estimation and equalization can be performed in a straightforward way as in OFDM together with other advantages that can be achieved in FBMC systems, such as low OoBR and robustness to synchronization errors.

### 4.1.1 System Model

In this section, we define the FBMC/QAM system in matrix form which will be subsequently used to propose an inverse system based on prototype filters to cancel the effect of intrinsic interference. The system model is divided into transmit processing, multipath channel and receive processing blocks as follows

**Transmit Processing**

The FBMC/QAM system follows a block based processing approach where each block contains $M$ FBMC/QAM symbols and each symbol has $N$ subcarriers in the frequency domain i.e. each block is represented as $S = [s_0, s_1, \cdots, s_{M-1}] \in \mathbb{C}^{N \times M}$ where $s_m = [s_{m,0}, s_{m,1}, \cdots, s_{m,N-1}]^T \in \mathbb{C}^{N \times 1}$. Hence, the total number of QAM symbols transmitted in one FBMC/QAM block is $MN$. Furthermore, the power of the modulated symbol $s_{m,n}$ is represented as $\delta^2$ i.e., $\mathcal{E}\{\|s_{m,n}\|^2\} = \delta^2$. The block diagram for both transmitter and receiver of FBMC/QAM is shown in Fig 4.1. In which the signal $s_m$ is first passed through an $N$ point IDFT processor and the output can be expressed as

$$b = [b_0; b_1; \cdots; b_{M-1}] = [\mathcal{F}^H s_0; \mathcal{F}^H s_1; \cdots; \mathcal{F}^H s_{M-1}] \in \mathbb{C}^{MN \times 1}.$$ (4.1)
4.1. FBMC/QAM System

Prototype Filter / Filter Matrix

The signal then passes through a prototype filter \( w \). It has been reported that a well-designed prototype filter with moderate length (e.g., overlapping factor \( K = 4 \sim 6 \)) incurs negligible interference \([20]\). To generalize our derivation, let us suppose the filter overlapping factor is \( K \), so the total length of the prototype filter is \( KN \) i.e., \( w = [w_0, w_1, \ldots, w_{K-1}] = [w_0, w_0, \ldots, w_{KN-1}] \in \mathbb{R}^{1 \times KN} \). In general, the prototype filters are linearly convolved with the input signal but to represent the complete system in matrix form we have to present the filtering process in matrix form as well. The multiplication of the filter matrix with the input vector is equivalent to the required linear convolution process. The prototype filter matrix \( P \in \mathbb{R}^{(K+M-1)N \times MN} \) is therefore defined as

\[
P = \begin{pmatrix}
W_0 & 0 & 0 & \cdots & 0 \\
W_1 & W_0 & 0 & \cdots & 0 \\
\vdots & W_1 & W_0 & \cdots & 0 \\
W_{K-1} & \vdots & W_1 & \cdots & 0 \\
0 & W_{K-1} & \vdots & \cdots & W_0 \\
0 & 0 & W_{K-1} & \cdots & W_1 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & 0 & \cdots & W_{K-1}
\end{pmatrix}, \tag{4.2}
\]

where \( W_k = \text{diag}(w_k) \in \mathbb{R}^{N \times N} \) for \( k = 0, 1, 2, \ldots, K - 1 \) and \( w_k = [w_{kN}, w_{kN+1}, \ldots, w_{kN+N-1}] \in \mathbb{R}^{1 \times N} \). The output of the filter matrix \( P \) is formed as

\[
o = Pb \in \mathbb{C}^{(K+M-1)N \times 1}. \tag{4.3}
\]

The output of the filter i.e., \( o \) has \( (K-1)N \) more samples due to the linear convolution process.
Channel Impulse Response

We assume the system operates over a slowly-varying fading channel i.e., quasi-static fading channel. In such a scenario, we can assume that the duration of each of the transmitted data block is smaller than the coherence time of the channel, therefore the random fading coefficients stay constant over the duration of each block [104]. In this case, we define the multipath channel as a $L$-tap CIR with the $l^{th}$-tap power being $\rho_l^2$. It is also assumed that the average power remains constant during the transmission of the whole block. Let us define the CIR $h$ as

$$h = [h_0, h_1, \cdots, h_{L-1}]^T = [\rho_0 z_0, \rho_1 z_1, \cdots, \rho_{L-1} z_{L-1}]^T,$$  

(4.4)

where $h_l$ defines the $l^{th}$ tap in the time domain CIR and the complex random variable $z_l$ with complex Gaussian distribution as $\mathbb{C}N(0, 1)$ represents the multipath fading factor of the $l^{th}$ tap of the quasi-static rayleigh fading channel.

Passing through the Channel

The signal $o$ after the prototype filtering is then passed through the channel $h$. The received signal is now represented as

$$r = h \ast o + n,$$  

(4.5)

where $n$ is Gaussian noise with each element having zero mean and variance $\sigma^2$. To represent the convolution process given in (4.5) as matrix multiplication, we first define the $l^{th}$ tap multipath fading factor $z_l$ in a diagonal matrix form as follows

$$Z_l = z_l \times I_{(K+M-1)N \times (K+M-1)N}.$$  

(4.6)

The definition of $Z_l$ implies that each FBMC/QAM symbol in a block experiences the same channel. With all these definitions we can reform (4.5) as

$$r = \sum_{l=0}^{L-1} \rho_l Z_l o_l + o_{IBI} + n,$$  

(4.7)

where $o_{IBI} = \sum_{l=0}^{L-1} \rho_l Z_l r_{B,l}$ is the IBI caused by channel multipath effect with $r_{B,l} = [r_{p,l}; 0_{(M+K-1)N \times l}]$ and $r_{p,l} \in \mathbb{C}^{l \times 1}$ is the interfering signal from the previous FBMC/QAM block. When guard time is longer than the channel duration, we have $r_{B,l} = 0$ and consequently, $o_{IBI} = 0$. In (4.7), $o_l$ represents $l$-sample delayed version of $o$ with zero padding in the front and is represented as $o_l = [0_{l \times 1}; o_{q,l}]$. Where $o_{q,l}$ represents the first $(K + M -$
4.1. FBMC/QAM System

1) $N - l$ elements of $o$. From (4.3) we can write $o^{ll} = P^{ll}b$, where $P^{ll} = [0_{l \times MN}; p_{q,l}]$. Here $P_{q,l}$ is the first $(K + M - 1)N - l$ rows of $P$. We can thus reform (4.7) as follows

$$r = \sum_{l=0}^{L-1} p_l Z_l P^{ll} b + o_{IBI} + n. \quad (4.8)$$

Equation (4.8) indicates that as a result of channel multipath effect, the original $P$ is replaced by distorted filter matrix $P^{ll}$. In order to demonstrate the relationship of the distortion and the multipath effect on the FBMC/QAM system, we first introduce a block diagonal exchanging matrix $X_l \in \mathbb{R}^{MN \times MN}$ as follows

$$X_l = \begin{bmatrix}
X_{sub,l} & 0 & \cdots & 0 \\
0 & X_{sub,l} & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & X_{sub,l}
\end{bmatrix} \in \mathbb{R}^{MN \times MN}, \quad (4.9)$$

with

$$X_{sub,l} = \begin{bmatrix}
0_{l \times (N-l)} & I_{l \times l} \\
I_{(N-l) \times (N-l)} & 0_{(N-l) \times l}
\end{bmatrix} \in \mathbb{R}^{N \times N}. \quad (4.10)$$

As $X_l^T X_l = I$, we have

$$o^{ll} = P^{ll}b = P^{ll} X_l^T X_l b = P_e^{ll} b_e^{ll}. \quad (4.11)$$

The matrix $X_l^T$ and $X_l$ are used to exchange the locations of elements of $P^{ll}$ and $b$ respectively, such that $P_e^{ll} = P^{ll} X_l^T$ and $b_e^{ll} = X_l b$. By multiplying the matrix $X_l$ with $b$, the last $l$ symbols of its each sub-vector $b_m$ will be moved to the front, i.e.

$$b_e^{ll} = [b_{m,N-l}, b_{m,N-l-1}, b_{m,0}, \cdots, b_{m,N-l-1}]^T. \quad (4.12)$$

Likewise,

$$\tilde{b}_e^{ll} = [b_{e,0}; b_{e,1}; \cdots; b_{e,M-1}] \in \mathbb{C}^{MN \times 1}. \quad (4.13)$$

The effect is similar when multiplying $X_l^T$ with $P^{ll}$. $X_l^T$ only changes the elements location in $P^{ll}$. Substituting (4.11) into (4.8) yields

$$r = \sum_{l=0}^{L-1} p_l Z_l P^{ll} b_e^{ll} + o_{IBI} + n. \quad (4.14)$$

It can be observed that the non zero elements of $P_e^{ll}$ and $P$ are very close i.e. the nonzero elements of $P_e^{ll}$ are only delayed by $l$ elements as compared to the elements in $P$. If the nonzero $i^{th}$ row and $k^{th}$ column element of $P$ is $w_n$, then the element of $P_e^{ll}$ at the same location...
will be \( w_{n+l} \). Since \( N \gg L \), the difference between \( w_n \) and \( w_{n+l} \) is very small as the adjacent elements of the prototype filter are close to each other. In order to show the interference caused by the multipath on the filter distortion, we define \( P_{\text{e}}^{\downarrow l} \) as follows

\[
P_{\text{e}}^{\downarrow l} = P + \Delta P^{\downarrow l}.\tag{4.15}
\]

Eq (4.14) can thus be written as

\[
r = \sum_{l=0}^{L-1} \rho_l Z_l P_{\text{e}}^{\downarrow l} + o_{fd} + o_{IBI} + n,
\]

where \( o_{fd} = \sum_{l=0}^{L-1} \rho_l Z_l \Delta P^{\downarrow l} b_{\text{e}}^{\downarrow l} \) is the interference caused by the filter distortion due to channel multipath effect.

**Receive Processing**

The received signal \( r \) is first passed through the receive filter bank, represented by the matrix \( P^H \). The output of the receive filter bank becomes

\[
x = P^H r,
\]

\[
x = G \sum_{l=0}^{L-1} \rho_l Z_l b_{\text{e}}^{\downarrow l} + P^H (o_{fd} + o_{IBI} + n).
\]

where \( G = P^H P \in \mathbb{R}^{MN \times MN} \) is the autocorrelation matrix and has a structure as shown in (4.18) with each element being a diagonal sub-matrix of size \( N \times N \).

\[
G = \begin{bmatrix}
\sum_{i=0}^{K-1} W_i W_i & \sum_{i=1}^{K-1} W_i W_{i-1} & \cdots & W_{K-1} W_0 & 0 & \cdots & 0 \\
\sum_{i=1}^{K-1} W_{i-1} W_i & \sum_{i=0}^{K-1} W_i W_i & \cdots & W_{i-K+2} W_{K-1} W_0 & \vdots & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\
W_0 W_{K-1} & \sum_{i=0}^{K-1} W_{i-K+2} W_i & \cdots & \sum_{i=0}^{K-1} W_i W_i & \sum_{i=1}^{K-1} W_i W_{i-1} & \cdots & W_{K-1} W_0 \\
0 & W_0 W_{K-1} & \cdots & \sum_{i=1}^{K-1} W_i W_i & \sum_{i=0}^{K-1} W_i W_i & \cdots & \sum_{i=K-2}^{K-1} W_i W_{i-K+2} \\
\vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & \cdots & W_0 W_{K-1} & \sum_{i=K-2}^{K-1} W_i W_i & \cdots & \sum_{i=0}^{K-1} W_i W_i
\end{bmatrix}
\]

(4.18)

We will now analyze the FBMC/QAM system performance with and without an inverse filter at the receiver.
4.1.2 Case 1: FBMC/QAM without inverse filter

The FBMC/QAM system is affected by intrinsic interference introduced by the transmit and receive filters. These interference terms can significantly limit the system performance. In this sub-section, we will derive these interference terms to analyze their impact on the system performance.

DFT Processing of the filtered signal

The signal vector at the output of the receive filter matrix, i.e., $P^H$ is represented as $x = [x_0, x_1, \cdots, x_{MN-1}]^T \in \mathbb{C}^{MN \times 1}$ and is then passed through a serial to parallel converter to split the vector into $M$ segments each of which has $N$ elements to perform $N$-point DFT. The $m^{th}$ segment of the vector $x$ is represented as $x_m = [x_m^N, x_m^{N+1}, \cdots, x_m^{N+L-1}]^T \in \mathbb{C}^{N \times 1}$ for $m \in 0, 1, \cdots, M-1$. The signal is now represented as $x = [x_0, x_1, \cdots, x_{M-1}] \in \mathbb{C}^{N \times M}$ where $x_m = [x_m, x_{m+1}, \cdots, x_{m+N-1}]^T \in \mathbb{C}^{N \times 1}$.

The signal vector after DFT is represented as follows

$$y_m = G x_m \in \mathbb{C}^{N \times 1},$$

$$= \mathcal{F} \sum_{i=0}^{M-1} G_{m,i} \sum_{l=0}^{L-1} p_l z_l b_{e,i}^l + \mathcal{F} P^H_m (o_{fd} + o_{IB} + n), \quad (4.19)$$

where $G_{m,i}$ is the $m^{th}$ row and $i^{th}$ column sub-matrices of $G$. We can show that channel circular convolution property holds in (4.19) and that the channel coefficients and the transmitted signal $s_i$ for $i = 0, 1, \cdots, M$ can be written as point-wise multiplication form in the frequency domain. We can write $\sum_{l=0}^{L-1} p_l z_l b_{e,i}^l = H_{cir} b_i$ in (4.19), where the matrix $H_{cir} = [h_0^l, h_1^l, \cdots, h_{L-1}^l]$ being a $N \times N$ circulant matrix. In general, an $N \times N$ circulant matrix is fully defined by its first $N \times 1$ vector. In our case, $H_{cir}$ is determined by $[h_0, h_1, \cdots, h_{L-1}, 0_{(N-L) \times 1}]^T \in \mathbb{C}^{N \times 1}$ i.e.,

$$H_{cir} = \begin{bmatrix}
  h_0 & 0 & \cdots & 0 & h_{L-1} & \cdots & h_1 \\
  \vdots & h_0 & \ddots & \vdots & \ddots & \vdots & \vdots \\
  h_{L-2} & \vdots & \ddots & \ddots & \ddots & \ddots & \vdots \\
  h_{L-1} & h_{L-2} & \cdots & h_0 & 0 & \cdots & \vdots \\
  0 & h_{L-1} & \cdots & h_0 & 0 & \cdots & \vdots \\
  \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\
  0 & \cdots & h_{L-1} & h_{L-2} & \cdots & h_0 & 0
\end{bmatrix}, \quad (4.20)$$
Also by introducing $\mathcal{F}^H \mathcal{F} = \mathbf{I}$ in (4.19), we obtain

\[
\mathbf{y}_m = \mathcal{F} \sum_{i=0}^{M-1} \mathbf{G}_{m,i} \mathcal{F}^H \mathcal{F}^H \mathcal{F} \mathbf{b}_i \mathcal{F}^H \mathcal{F} \mathbf{b}_i + \mathcal{F} \mathbf{p}_m^H (\mathbf{o}_{fd} + \mathbf{o}_{IBI} + \mathbf{n}).
\]

Using the circular convolution property (pp.129-130) \[106\], we can write $\mathcal{F}^H \mathcal{F} = \mathbf{C}$, where $\mathbf{C}$ is the frequency domain channel coefficients in diagonal matrix form and is given as $\mathbf{C} = \text{diag}[\mathbf{C}_0, \mathbf{C}_1, \cdots, \mathbf{C}_{N-1}] \in \mathbb{C}^{N \times N}$. The $n^{th}$ block diagonal element in the frequency response of the channel can be represented as $\mathbf{C}_n = \sum_{l=0}^{L-1} h_l e^{-j \frac{2\pi}{N} nl}$, $0 \leq n \leq N$. Also $\mathcal{F} \mathbf{b}_i$ denotes the DFT processing of $\mathbf{b}_i$ and according to (4.1), we have $\mathcal{F} \mathbf{b}_i = \mathbf{s}_i$, by substituting it into (4.21) we get

\[
\mathbf{y}_m = \sum_{i=0}^{M-1} Q_{m,i} \mathbf{c}_i + \mathcal{F} \mathbf{p}_m^H (\mathbf{o}_{fd} + \mathbf{o}_{IBI} + \mathbf{n}),
\]

where $Q_{m,i} = \mathcal{F} \mathbf{G}_{m,i} \mathcal{F}^H$ has the following property

\[
Q_{m,i} = \begin{cases} 
\mathbf{I} + \Delta Q_{m,m} & \text{for } i = m \\
\Delta Q_{m,i} & \text{for } i \neq m
\end{cases}
\]  \hspace{1cm} (4.23)

Note that $\Delta Q \in \mathbb{C}^{MN \times MN}$ denotes the interference coefficient matrix that determines the magnitude of intrinsic interference in the received signal block. Using (4.23), we can write (4.22) as follows

\[
\mathbf{y}_m = (\mathbf{I} + \Delta Q_{mm}) \mathbf{c}_m + \sum_{i=0,i\neq m}^{M-1} \Delta Q_{m,i} \mathbf{c}_i + \mathcal{F} \mathbf{p}_m^H (\mathbf{o}_{fd} + \mathbf{o}_{IBI} + \mathbf{n}),
\]

\[
= \mathbf{c}_m + \Delta Q_{mm} \mathbf{c}_m + \sum_{i=0,i\neq m}^{M-1} \Delta Q_{m,i} \mathbf{c}_i + \mathcal{F} \mathbf{p}_m^H (\mathbf{o}_{fd} + \mathbf{o}_{IBI} + \mathbf{n}).
\]

### Channel Equalization

We represent one tap channel equalizer as a diagonal matrix $\mathbf{E}$ and is applied to the signal $\mathbf{y}_m$ as follows

\[
\hat{\mathbf{s}}_m = \mathbf{E} \mathbf{y}_m,
\]

\[
= \mathbf{E} \mathbf{c}_m + \mathbf{E} \Delta Q_{mm} \mathbf{c}_m + \mathbf{E} \sum_{i=0,i\neq m}^{M-1} \Delta Q_{m,i} \mathbf{c}_i + \mathbf{E} \mathcal{F} \mathbf{p}_m^H (\mathbf{o}_{fd} + \mathbf{o}_{IBI} + \mathbf{n}),
\]  \hspace{1cm} (4.25)

Let us now assume $\mathbf{E}$ to be either ZF or MMSE i.e. the two most popular linear channel equalizers.

\[
\mathbf{E} = \mathbf{C}^H (\mathbf{C} \mathbf{C}^H + \nu \sigma^2 / \delta^2 \mathbf{I})^{-1},
\]  \hspace{1cm} (4.26)
where \( \nu = 0 \) for ZF while \( \nu = 1 \) is for MMSE. We can now write (4.25) as

\[
\hat{s}_m = \beta s_m + \mathbb{E}Q_{mm}C_m + \sum_{i=0,i\neq m}^{M-1} \mathbb{E}Q_{m,i}C_i
\]

\[+ \mathbb{E}JP^H_m(o_{fd} + o_{IBI} + n), \tag{4.27}\]

where \( \beta = \mathbb{E}C \) is a diagonal matrix with its \( n^{th} \) diagonal element being defined as

\[
\beta_n = \frac{|E_n|^2}{|E_n|^2 + \nu\sigma^2/\delta^2}, \tag{4.28}\]

The estimated signal \( \hat{s}_m \) can now be expressed as follows

\[
\hat{s}_m = \underbrace{s_m}_{\text{Desired Signal}} + (I - \beta)s_m + \underbrace{\mathbb{E}Q_{mm}C_m}_{\text{MMSE Estimation Bias}} + \sum_{i=0,i\neq m}^{M-1} \underbrace{\mathbb{E}Q_{m,i}C_i}_{\text{ICI}}
\]

\[+ \underbrace{\mathbb{E}JP_m^H o_{fd}}_{\text{Filter Distortion by Multipath}} + \underbrace{\mathbb{E}JP_m^H o_{IBI}}_{\text{IBI by Multipath}} + \underbrace{\mathbb{E}JP_m^H n}_{\text{Noise}}. \tag{4.29}\]

Note that the estimation bias error \((I - \beta)\) is an effect of compromising the interference and noise of the MMSE equalizer. However, \((I - \beta) = 0\) when the ZF receiver is used.

### 4.1.3 Case 2: FBMC/QAM with inverse filter

We can see from (4.29) that the transmitted signal \( s_m \) is accompanied by ICI and ISI (intrinsic interference) terms along with interferences caused by the multipath channel and the noise. We can overcome the intrinsic interference by introducing an inverse filter matrix \( R \) at the receiver as shown in Fig. 4.2. Let the inverse filter matrix be defined as the inverse of the autocorrelation matrix \( G \) defined in (4.17) i.e. \( R = G^{-1} \in \mathbb{R}^{MN \times MN} \). Since the autocorrelation matrix \( G \) is a band diagonal matrix, the inverse of the band diagonal matrix will result in a sparse matrix that consists of diagonal sub-matrices as shown in Fig. 4.3. The sparse structure of the inverse filter can lead to a low complex deconvolution process at the receiver. It should be noted that the
inverse filter matrix is a symmetric matrix i.e. \( R = R^T \) and that the entries of the off diagonal sub-matrices are symmetric with respect to the main diagonal (as shown with different colours). It can also be noted that each off diagonal sub-matrix in \( R \) has negligible middle \( N/2 \) diagonal elements represented as dotted sections in Fig. 4.3. An arbitrary number of elements in the range of \([0, N/2]\) can be replaced here by zero to reduce the complexity. Let the elements of the dotted section considered in the complexity analysis be defined as \( \eta \) i.e., if \( \eta = 0 \), all \( N/2 \) diagonal elements are considered, \( \eta = 0.5 \) represent \( N/4 \) diagonal elements in the range of \([3N/8, 5N/8]\) are considered, whereas \( \eta = 1 \) means none of the middle \( N/2 \) diagonal elements are considered in the complexity analysis given in Sec. 4.3.

Using (4.18), the output of the inverse filter matrix \( R \) at the receiver side can be written as follows:

\[
v_m = \sum_{i=0}^{M-1} U_{m,i} \sum_{l=0}^{L-1} \rho_l z_l b_{c,i}^{kl} + R_m P^H_m (o_{fd} + o_{IBI} + n),
\]  

(4.30)
where $\mathbf{R}_m \in \mathbb{C}^{N \times MN}$ is the $m^{th}$ row of sub-matrices of matrix $\mathbf{R}$, while $\mathbf{U}_{m,i} = \sum_{j=0}^{M-1} \mathbf{R}_{m,j} \mathbf{G}_{j,i} \in \mathbb{R}^{N \times N}$ and has the following property

$$\mathbf{U}_{m,i} = \begin{cases} \mathbf{I}_{N \times N} & \text{for } i = m, \\ \mathbf{0} & \text{for } i \neq m \end{cases}.$$  \hfill (4.31)

Eq. (4.30) can now be written as follows

$$\mathbf{v}_m = \sum_{l=0}^{L-1} \rho_l \mathbf{z}_l \mathbf{b}^H_{e,m} + \mathbf{R}_m \mathbf{P}_m^H (\mathbf{o}_{fd} + \mathbf{o}_{IBI} + \mathbf{n}). \hfill (4.32)$$

**DFT Processing of the filtered signal**

The signal after DFT processing is now represented as follows

$$\mathbf{y}_m = \mathbf{C}_s \mathbf{s}_m + \mathcal{F} \mathbf{R}_m \mathbf{P}_m^H (\mathbf{o}_{fd} + \mathbf{o}_{IBI} + \mathbf{n}). \hfill (4.33)$$

where $\mathbf{C}_s = \mathcal{F} \sum_{l=0}^{L-1} \rho_l \mathbf{z}_l \mathbf{b}^H_{e,m}$.

**Channel Equalization**

The estimated symbol $\hat{s}_m$ after equalization can be expressed as follows

$$\hat{s}_m = E\mathbf{y}_m,$$

$$= \beta \mathbf{s}_m + E\mathcal{F} \mathbf{R}_m \mathbf{P}_m^H (\mathbf{o}_{fd} + \mathbf{o}_{IBI} + \mathbf{n}), \hfill (4.34)$$

where $\beta = \mathbf{E} \mathbf{C}$ is a diagonal matrix with its $n^{th}$ diagonal element represented as (4.28). The estimated signal $\hat{s}_m$ can now be expressed as follows

$$\hat{s}_m = \underline{\underline{\text{Desired Signal}}} + \underline{\underline{\text{MMSE Estimation Bias}}} + \underline{\underline{\text{Filter Distortion by Multipath}}} + \underline{\underline{\text{IBI by Multipath}}} + \underline{\underline{\text{Noise}}}.$$  \hfill (4.35)

As we can see from (4.35) that the transmitted signal $\mathbf{s}_m$ is free from ICI and ISI terms as compared to the case with no inverse filter. However, the use of inverse filter matrix $\mathbf{R}$ enhances the interferences caused by the multipath channel and noise as shown in (4.35). Therefore, in what follows, we will investigate the interference and noise power to analyze the usefulness of inverse filter matrix at the receiver.
4.2 Interference Analysis

Although the use of inverse filter matrix removes ISI and ICI, however, the interference caused by the multipath channel and noise is enhanced due to the use of inverse filter matrix $R$. Therefore we need to investigate the interference and noise power enhancement due to the inverse filter at the receiver. This provides deep insights and useful guidelines for receiver design in the FBMC/QAM system.

4.2.1 Interference / noise power in case of no inverse filter

As we can see from (4.29) that in case of no inverse filter, the estimated symbol is accompanied with MMSE estimation bias, interference terms like ICI, ISI, filter distortion and IBI due to multipath channel and noise i.e.,

$$\hat{s}_m = s_m + \psi_{\text{resd,m}} + \psi_{\text{ICI,m}} + \psi_{\text{ISI,m}} + \psi_{\text{fd,m}} + \psi_{\text{IBI,m}} + \psi_{\text{noise,m}}.$$  \hspace{1cm} (4.36)

The MSE of the $n$-th modulation symbol estimation in the $m$-th FBMC/QAM symbol can be derived as

$$\gamma_{\text{tot},m,n} = \mathbb{E}||\hat{s}_{m,n} - s_{m,n}||^2$$

$$= \mathbb{E} \left[ ||\psi_{\text{resd,m}}||_n^2 + ||\psi_{\text{ICI,m}}||_n^2 + ||\psi_{\text{ISI,m}}||_n^2 + ||\psi_{\text{fd,m}}||_n^2 + ||\psi_{\text{IBI,m}}||_n^2 + ||\psi_{\text{noise,m}}||_n^2 \right].$$  \hspace{1cm} (4.37)

MSE of signal estimation bias

The desired signal estimation bias is caused by the MMSE receiver since it minimizes the MSE between the transmitted and received signal. This leads to residual interference in the estimated signal. From (4.37) and (4.29), we can write the variance of the signal estimation bias as

$$\gamma_{\text{resd,m,n}} = \mathbb{E}||\psi_{\text{resd,m}}||_n^2 = \mathbb{E}\left\{ ||(I - \beta)s_m||_n^2 \right\}$$

$$= \delta^2 (I - \beta_n)^2;$$  \hspace{1cm} (4.38)
4.2. Interference Analysis

As $\mathbb{E}\{\|s_{m,n}\|^2\} = \delta^2$ and according to (4.28), $\beta_n = \frac{|C_n|^2}{|C_n|^2 + \nu \sigma^2 / \delta^2}$. Substituting $\beta_n$ in (4.38) yields

$$\gamma_{\text{resd},m,n} = \delta^2 (I - \beta_n)^2 = \delta^2 (I - 2\beta_n + \beta_n^2),$$

$$\gamma_{\text{resd},m,n} = \delta^2 \left[ I - \frac{2|C_n|^2}{|C_n|^2 + \nu \sigma^2 / \delta^2} + \frac{|C_n|^4}{(|C_n|^2 + \nu \sigma^2 / \delta^2)^2} \right],$$

$$\gamma_{\text{resd},m,n} = \delta^2 \left[ \frac{\nu^2 \sigma^4}{(|C_n|^2 + \nu \sigma^2)^2} \right].$$

(4.39)

Apparently, when the ZF receiver is adopted, $\gamma_{\text{resd},m,n} = 0$ since $\nu = 0$. However, the ZF equalization leads to noise enhancement unlike MMSE receivers.

MSE of ICI

We can write the variance of the ICI from (4.37) and (4.29) as

$$\gamma_{ICI,m} = \mathbb{E}\|\psi_{ICI,m}\|^2 = \mathbb{E}\|\mathbf{E}\Delta Q_{mm} \mathbf{Cs}_m\|^2,$$

$$\gamma_{ICI,m} = \mathbb{E}[\mathbf{E}\Delta Q_{mm} \mathbf{Cs}_m \mathbf{C}^H \Delta Q_{m,m}^H \mathbf{E}^H],$$

(4.40)

As $\mathbb{E}\{\|s_{m,n}\|^2\} = \delta^2$ and from (4.23), we know that $\Delta Q_{m,m} = \mathbf{Q}_{m,m} - \mathbf{I}$ for $i = m$, we can thus reformulate the above equation as

$$\gamma_{ICI,m} = \delta^2 \left[ \mathbf{E}(\mathbf{Q}_{m,m} - \mathbf{I}) \mathbf{C} \mathbf{C}^H (\mathbf{Q}_{m,m} - \mathbf{I})^H \mathbf{E}^H \right],$$

$$\gamma_{ICI,m} = \delta^2 \left[ \mathbf{E} \mathbf{Q}_{m,m} \mathbf{C} \mathbf{C}^H \mathbf{Q}_{m,m}^H \mathbf{E}^H - \mathbf{E} \mathbf{Q}_{m,m} \mathbf{C} \mathbf{C}^H \mathbf{E}^H \right. $$

$$\left. - \mathbf{E} \mathbf{C} \mathbf{C}^H \mathbf{Q}_{m,m}^H \mathbf{E}^H + \mathbf{E} \mathbf{C} \mathbf{C}^H \mathbf{E}^H \right],$$

(4.41)

Taking the $n^{th}$ diagonal element of (4.41), we have

$$\gamma_{ICI,m,n} = \delta^2 \| \mathbf{E} \mathbf{Q}_{m,m} \mathbf{C} \mathbf{C}^H \mathbf{Q}_{m,m}^H \mathbf{E}^H \|_n,$$

$$\gamma_{ICI,m,n} = \delta^2 \| \mathbf{E} \mathbf{Q}_{m,m} \mathbf{C} \mathbf{C}^H \mathbf{Q}_{m,m}^H \mathbf{E}^H \|_n - \delta^2 \| \mathbf{E} \mathbf{Q}_{m,m} \mathbf{C} \mathbf{C}^H \mathbf{E}^H \|_n,$$

$$\gamma_{ICI,m,n} = \delta^2 \| \mathbf{E} \mathbf{Q}_{m,m} \mathbf{C} \mathbf{C}^H \mathbf{Q}_{m,m}^H \mathbf{E}^H \|_n + \delta^2 \| \mathbf{E} \mathbf{C} \mathbf{C}^H \mathbf{E}^H \|_n,$$

$$\gamma_{ICI,m,n} = \delta^2 \| \mathbf{E} \mathbf{Q}_{m,m} \mathbf{C} \mathbf{C}^H \mathbf{Q}_{m,m}^H \mathbf{E}^H \|_n - \delta^2 \| \mathbf{E} \mathbf{Q}_{m,m} \mathbf{C} \mathbf{C}^H \mathbf{E}^H \|_n,$$

$$\gamma_{ICI,m,n} = \delta^2 \| \mathbf{E} \mathbf{Q}_{m,m} \mathbf{C} \mathbf{C}^H \mathbf{Q}_{m,m}^H \mathbf{E}^H \|_n,$$

(4.42)

where $\alpha_{ICI,n} = \| \mathbf{Q}_{m,m} \mathbf{Q}_{m,m}^H - \mathbf{Q}_{m,m} - \mathbf{Q}_{m,m}^H + I_{N \times N}\|_n.$
4.2. Interference Analysis

MSE of ISI

We can write the variance of the ISI from (4.37) and (4.29) as
\[
\gamma_{ISI,m} = \mathbb{E} \| \psi_{ISI,m} \|^2 = \mathbb{E} \left\| \sum_{i=0, i \neq m}^{M-1} E \Delta Q_{m,i} C s_i \right\|^2,
\]
\[
= \mathbb{E} \left[ \sum_{i=0, i \neq m}^{M-1} E \Delta Q_{m,i} C s_i s_i^H C^H \Delta Q_{m,i}^H E^H \right],
\] (4.43)

As \(\mathbb{E} \{\| s_{m,n} \|^2 \} = \delta^2\) and from (4.23), we know that \(\Delta Q_{m,i} = Q_{m,i}\) for \(i \neq m\), we can thus write the above equation as
\[
\gamma_{ISI,m} = \delta^2 \sum_{i=0, i \neq m}^{M-1} E Q_{m,i} C C^H Q_{m,i}^H E^H,
\] (4.44)

Taking the \(n^{th}\) diagonal element of (4.44), we obtain
\[
\gamma_{ISI,m,n} = \delta^2 \left\| \sum_{i=0, i \neq m}^{M-1} E Q_{m,i} C C^H Q_{m,i}^H E^H \right\|_n,
\]
\[
= \delta^2 |E_n|^2 |C_n|^2 \alpha_{ISI,n}.
\] (4.45)

where \(\alpha_{ISI,n} = \left\| \sum_{i=0, i \neq m}^{M-1} Q_{m,i} Q_{m,i}^H \right\|_n\).

MSE of Filter Distortion due to multipath channel

We can write the variance of the interference caused by filter distortion due to multipath channel from (4.37) and (4.29) as
\[
\gamma_{fd,m} = \mathbb{E} \| \psi_{fd,m} \|^2 = \mathbb{E} \| E F P_m^H o_{fd} \|^2,
\]
\[
= \mathbb{E} \left[ E F P_m^H o_{fd} o_{fd}^H P_m m F^H E^H \right],
\]
\[
= E F P_m^H E [o_{fd} o_{fd}^H] P_m m F^H E^H,
\]
\[
= E F P_m^H \alpha_{fd} P_m m F^H E^H,
\] (4.46)

Using (4.16), we can determine \(\alpha_{fd} = \mathbb{E} [o_{fd} o_{fd}^H]\) as follows
\[
\alpha_{fd} = \mathbb{E} \left[ \left\{ \sum_{l=0}^{L-1} \rho_l Z_l \Delta P^{ij}_l b^{ij}_l \right\} \left\{ \sum_{l=0}^{L-1} \rho_l Z_l \Delta P^{ij}_l b^{ij}_l \right\}^H \right],
\]
\[
= \sum_{l=0}^{L-1} \rho_l^2 \mathbb{E} [Z_l \Delta P^{ij}_l b^{ij}_l b^{ij}_l^H \Delta P^{ij}_l H Z_l^H],
\] (4.47)
4.2. Interference Analysis

From (4.4) and (4.6), \( \mathbb{E}\{Z_l Z_l^H\} = 1 \) since \( z_l \in \mathbb{C}N(0, 1) \) also we know that \( \mathbb{E}\{b_l^H b_l^H\} = \delta^2 \), consequently

\[
\alpha_{fd} = \delta^2 \sum_{l=0}^{L-1} \rho_l^2 \text{Tr}\{\Delta P_l \Delta P_l^H\},
\]

\[
= \delta^2 \sum_{l=0}^{L-1} \rho_l^2 T_l^l,
\]

(4.48)

where \( T_l^l = \text{Tr}[\Delta P_l \Delta P_l^H] \). Since \( T_l^l \) is a scalar value, therefore \( \alpha_{fd} \) is also a scalar value.

Now substituting (4.48) into (4.46), yields

\[
\gamma_{fd,m} = \alpha_{fd} \mathbb{E}[\mathcal{F}_m P_m \mathcal{F}_m^H \mathcal{F}_m],
\]

(4.49)

By taking the \( n \)th diagonal element of \( \gamma_{fd,m} \), we have

\[
\gamma_{fd,m,n} = \alpha_{fd} \mathbb{E}[\mathcal{F}_m P_m \mathcal{F}_m^H \mathcal{F}_m]^n,
\]

\[
= \alpha_{fd} |E_n|^2.
\]

(4.50)

where \( \|\mathcal{F}_m P_m \mathcal{F}_m^H\|_n = \|\mathbb{I}_{N \times N}\|_n \).

MSE of IBI

Let us consider the case when we have inter-block interference due to the lack of guard time. We can write the variance of the interference caused by IBI from (4.37) and (4.29) as

\[
\gamma_{IBI,m} = \mathbb{E}\|\psi_{IBI,m}\|^2 = \mathbb{E}\|\mathcal{F}_m P_m \mathcal{F}_m^H \mathcal{F}_m\|^2,
\]

\[
= \mathbb{E}[\mathcal{F}_m P_m \mathcal{F}_m^H \mathcal{F}_m \mathcal{F}_m^H \mathcal{F}_m]
\]

\[
= \mathbb{E}[\mathcal{F}_m P_m \alpha_{IBI} P_m \mathcal{F}_m^H \mathcal{F}_m],
\]

(4.51)

where \( \alpha_{IBI} = \mathbb{E}[\mathcal{F}_m P_m \mathcal{F}_m^H \mathcal{F}_m] \), now using (4.7), we can determine \( \alpha_{IBI} \) as

\[
\alpha_{IBI} = \mathbb{E} \left[ \left\{ \sum_{l=0}^{L-1} \rho_l Z_l y_{B,l} \right\} \left\{ \sum_{l=0}^{L-1} \rho_l Z_l y_{B,l} \right\}^H \right],
\]

\[
= \mathbb{E} \left[ \sum_{l=0}^{L-1} \rho_l^2 Z_l \mathbb{E}\{y_{B,l} y_{B,l}^H\} Z_l^H \right],
\]

(4.52)

Since \( Z_l \) has a complex Gaussian distribution i.e. \( \mathbb{C}N(0, 1) \) and also \( Z_l \) and \( y_{B,l} \) are uncorrelated, we can write the above equation as follows

\[
\alpha_{IBI} = \sum_{l=0}^{L-1} \rho_l^2 \mathbb{E}\{y_{B,l} y_{B,l}^H\},
\]

(4.53)
4.2. Interference Analysis

$\mathcal{E}\{y_B,l y_B^H\}$ is dependent on the signal type of the last block, where we assume it is also occupied by an FBMC symbol with the same power, then we have

$$
\mathcal{E}\{y_B,l y_B^H\} = \mathcal{E}\|P(l) b_{last}\|^2 = \text{Tr}\left[ P(l) \mathcal{E}\{b_{last} b_{last}^H\} P^H\right],
$$

$$
= \delta^2 \text{Tr}\left[ P(l) P^H\right] = \delta^2 \text{Tr}\left[ P_{corr}\right],
$$

where $P(l) = P_{(last-l)}:0_{(M+K-1)N-\times M N}$ in which $P_{(last-l)}$ contains the last $l$-th rows of $P$ also $b_{last}$ is the symbol (after IDFT) in the last block and that $\mathcal{E}\{b_{last} b_{last}^H\} = \delta^2 I$. Substituting (4.54) in (4.53), we obtain

$$
\alpha_{IBI} = \delta^2 \sum_{l=0}^{L-1} \rho_l^2 P_{corr},
$$

(4.55)

Since $P_{corr}$ is a scalar value, therefore $\alpha_{IBI}$ is also a scalar value. Substituting it into (4.51), yields

$$
\gamma_{IBI,m} = \alpha_{IBI} \text{Tr}\left[ P^H m P m^H E^H\right],
$$

(4.56)

By taking the $n$th diagonal element of $\gamma_{IBI,m}$, we derive the MSE due to IBI as

$$
\gamma_{IBI,m,n} = \alpha_{IBI} \|E^H P m^H P m F H E H\|_n
$$

$$
= \alpha_{IBI} \|E_n\|^2.
$$

(4.57)

where $\|E^H P m^H P m F H E H\|_n = \|I_{N \times N}\|_n$. If we further notice that the elements of $P_{(last-l)}$ are very small and contains the last $l$ rows of matrix $W_{K-1}$. Therefore, $P_{corr}$ will be a diagonal matrix with first $l$-th diagonal elements being the square of the last $l$-th elements of filter $w$ i.e., $w_{K,N-1}, w_{K,N-1+1}, \ldots, w_{K,N-1}$. We can therefore represent $P_{corr}$ as follows

$$
P_{corr} = \text{diag}[w_{K,N-1}, w_{K,N-1+1}, \ldots, w_{K,N-1}, 0_{(K+M-1)N-l}] 
$$

(4.58)

Using (4.58), we can have the following approximation

$$
\gamma_{IBI,m,n} \approx \delta^2 |E_n|^2 \sum_{l=0}^{L-1} \rho_l^2 \sum_{k=0}^{l-1} w_{K,N-1-k}^2.
$$

(4.59)

MSE of Noise

We can write the variance of the noise from (4.37) and (4.29) as

$$
\gamma_{noise,m} = \mathcal{E}\|\psi_{noise,m}\|^2 = \mathcal{E}\|E^H P m\|^2,
$$

$$
= \mathcal{E}[E^H P m n^H P m^H E^H],
$$

$$
= \sigma^2 E^H P m P m^H E^H,
$$

(4.60)
4.2. Interference Analysis

where $\mathcal{E}\{nn^H\} = \mathcal{E}\|n\|^2 = \sigma^2$ since $n$ is Gaussian noise with each element having zero mean and variance $\sigma^2$. Taking the $n^{th}$ diagonal element of (4.60), we have

$$\gamma_{\text{noise},m,n} = \sigma^2\|\mathcal{F}\mathbf{P}_m^H\mathbf{P}_m\mathcal{F}^H\|n = \sigma^2|E_n|^2.$$  \hspace{1cm} (4.61)

where $\|\mathcal{F}\mathbf{P}_m^H\mathbf{P}_m\mathcal{F}^H\|n = \|I_{N \times N}\|n$.

### 4.2.2 Interference / noise power in case of inverse filter

As can be seen from (4.35) that with the inverse filter, the estimated symbol is accompanied with MMSE estimation bias, filter distortion and IBI due to channel multipath effect and noise i.e.,

$$\hat{s}_m = s_m + \psi_{\text{resd},m} + \psi_{\text{fd},m} + \psi_{\text{IBI},m} + \psi_{\text{noise},m}.$$  \hspace{1cm} (4.62)

Similar to (4.37), we can write the MSE of the $n$-th modulation symbol estimation in the $m$-th FBMC/QAM symbol as follows

$$\gamma_{\text{tot},m,n} = \mathcal{E}\|\hat{s}_{m,n} - s_{m,n}\|^2$$

$$= \mathcal{E}\left[\|\psi_{\text{resd},m}\|_{n}^2 + \|\psi_{\text{fd},m}\|_{n}^2 + \|\psi_{\text{IBI},m}\|_{n}^2 + \|\psi_{\text{noise},m}\|_{n}^2 \right].$$  \hspace{1cm} (4.63)

**MSE of signal estimation bias**

This residual interference caused by the MMSE equalizer is same as (4.39) since it is independent from the effect of inverse filter matrix $\mathbf{R}$. Hence, the interference power of the MMSE estimation biased is

$$\gamma_{\text{resd},m,n} = \mathcal{E}\|\psi_{\text{resd},m}\|_{n}^2 = \delta^2\left[\frac{\nu^2\sigma^4}{(\delta^2|C_n|^2 + \nu^2)^2}\right].$$  \hspace{1cm} (4.64)
MSE of Filter Distortion due to multipath channel

From (4.63) and (4.35), the variance of interference caused by filter distortion due to channel multipath effect in case of inverse filter is as

\[
\gamma_{fd,m} = E\|\psi_{fd,m}\|^2 = E\|E^H R_m P_m^H o_{fd}\|^2,
\]

\[
= E\{E^H R_m P_m^H o_{fd} o_{fd}^H P_m R_m^H \sigma^H \sigma^H E^H\},
\]

\[
= E^H R_m P_m^H E^f o_{fd} o_{fd}^H P_m R_m^H \sigma^H \sigma^H E^H,
\]

\[
= E^H R_m P_m^H \alpha_{fd} P_m R_m^H \sigma^H \sigma^H E^H,
\] (4.65)

From (4.48), we know

\[
\alpha_{fd} = E[ o_{fd} o_{fd}^H] = \delta^2 \sum_{l=0}^{L-1} \rho_l^2 T_{l,l}^H,
\] (4.66)

where \(T_{l,l}^H = \text{Tr}[\Delta P_l^H \Delta P_l^H]\). Since \(T_{l,l}^H\) is a scalar value, therefore \(\alpha_{fd}\) is also a scalar value.

Now substituting (4.66) into (4.65), yields

\[
\gamma_{fd,m} = \alpha_{fd} E^H R_m P_m^H P_m R_m^H \sigma^H \sigma^H E^H,
\] (4.67)

By taking the \(n^{th}\) diagonal element of \(\gamma_{fd,m}\), we obtain

\[
\gamma_{fd,m,n} = \alpha_{fd} E\|E^H R_m P_m^H P_m R_m^H \sigma^H \sigma^H E^H\|_n,
\]

\[
= \alpha_{fd} |E_n| \| \zeta_{m,n} \|_n^2,
\] (4.68)

where \(\|E^H R_m P_m^H P_m R_m^H \sigma^H \sigma^H E^H\|_n = \zeta_{m,n} \| I_{N \times N} \|_n\).

MSE of IBI

Let us consider the case when we have inter-block interference due to the lack of guard time.

From (4.63) and (4.35), we can write the variance of the interference caused by IBI in case of inverse filter as

\[
\gamma_{IBI,m} = E\|\psi_{IBI,m}\|^2 = E\|E^H R_m P_m^H o_{IBI}\|^2,
\]

\[
= E\{E^H R_m P_m^H o_{IBI} o_{IBI}^H P_m R_m^H \sigma^H \sigma^H E^H\},
\]

\[
= E^H R_m P_m^H E^f o_{IBI} o_{IBI}^H P_m R_m^H \sigma^H \sigma^H E^H,
\]

\[
= E^H R_m P_m^H \alpha_{IBI} P_m R_m^H \sigma^H \sigma^H E^H,
\] (4.69)

From (4.55), we already know

\[
\alpha_{IBI} = \delta^2 \sum_{l=0}^{L-1} \rho_l^2 P_{corr}^{(l)},
\] (4.70)
Since $P_{corr}^{(l)}$ is a scalar value, therefore $\alpha_{IBI}$ is also a scalar value. Substituting (4.70) into (4.69), yields

$$
\gamma_{IBI,m} = \alpha_{IBI} E F R_m P_m^H P_m R_m^H E^H \gamma_{IBI,m,n} = \alpha_{IBI} |E_n|^2 \zeta_{m,n},
$$

(4.71)

By taking the $n^{th}$ diagonal element of $\gamma_{IBI,m}$, we derive the MSE of IBI as

$$
\gamma_{IBI,m,n} = \alpha_{IBI} E F R_m P_m^H P_m R_m^H E^H \parallel_n = \alpha_{IBI} |E_n|^2 \zeta_{m,n}.
$$

(4.72)

where $\parallel F R_m P_m^H P_m R_m^H E^H \parallel_n = \zeta_{m,n} I_{N \times N}$. 

MSE of Noise

From (4.63) and (4.35), the variance of noise in case of inverse filter as

$$
\gamma_{noise,m} = E \| \psi_{noise,m} \|^2 = E \| E F R_m P_m^H n \|^2 = E [E F R_m P_m^H n n^H P_m R_m^H E^H],
$$

(4.73)

As $E \{ n n^H \} = E \| n \|^2 = \sigma^2$ since $n$ is Gaussian noise with each element having zero mean and variance $\sigma^2$. Taking the $n^{th}$ diagonal element of (4.73), we obtain

$$
\gamma_{noise,m,n} = \sigma^2 \| E F R_m P_m^H P_m R_m^H E^H \parallel_n = \sigma^2 |E_n|^2 \zeta_{m,n}.
$$

(4.74)

where $\| E F R_m P_m^H P_m R_m^H E^H \parallel_n = \zeta_{m,n} I_{N \times N}$. 

Note that the $\zeta_{m,n}$ is the noise / interference enhancement factor which is introduced when we
use an inverse filter matrix $\mathbf{R}$ at the receiver. To show the enhancement factor per symbol of the FBMC/QAM block, we have considered, $N = 64$ i.e., the FFT size to define the FFT matrix $\mathcal{F}$, the block size is considered as $M = 14$, the prototype filter matrix $\mathbf{P}$ is defined using IOTA prototype filter. The noise/interference enhancement factor per subcarrier per symbol ($\zeta_{m,n}$ subcarrier/symbol) is presented in Fig. 4.4. It can be seen that the noise enhancement factor is constant for every subcarrier in each symbol and its effect is maximum for the symbols in the middle of the FBMC/QAM data block as shown in Fig. 4.4(a). However, the impact is not significant since the average enhancement factor in a block is 1.32 as can be seen from Fig. 4.4(b).

4.3 Complexity Analysis

In this section we have presented the complexity analysis of the FBMC/QAM system with and without inverse filter at the receiver. The objective is to determine if there is a significant increase in the complexity of the system with the introduction of the inverse filter matrix at the receiver. Typically the complexity of a system is measured by the number of floating point operations (FLOPS), we however, only focus on the number of real multiplications in our complexity analysis. Since we have already presented the FBMC/QAM transmitter and receiver processes in matrix multiplication form in Section 4.1, we have adopted the naive matrix multiplication algorithm [110] to perform the complexity analysis.

4.3.1 Complexity analysis in case of no inverse filter

To determine the complexity of the FBMC/QAM system without inverse filter, we have to look at the structure of the system as shown in Fig. 4.1.

Transmitter Complexity

Since our model follows a block based processing approach, the input to the system is a vector of size $MN$. Each QAM symbol in the FBMC/QAM transmitter requires an $N$-point IDFT operation. The most efficient FFT algorithm i.e. split-radix requires $N \log_2 N - 3N + 4$ real multiplications [111, 112]. The complex vector $\mathbf{b}$ at the output of the IDFT processor is then processed through the transmit filter matrix $\mathbf{P}$. Since, the structure of $\mathbf{P}$ is sparse as shown in (4.2), the number of real multiplications involved in filtering operation is determined as $\sum_{k=1}^{MN} P_k b_k = 2MNK$ per block. Where $P_k$ is the number of nonzero elements in the $k^{th}$
4.3. Complexity Analysis

The total number of real multiplications involved in the transmitter per FBMC/QAM block is

\[ C_{\text{Tx}} = MN \log_2 N + (2K - 3)MN + 4M \]  

(4.75)

**Receiver Complexity**

It can be seen from Fig. 4.1, the transmitted signal \( o \), after passing through the channel, is received by the receiver as a complex vector \( r \) and is processed by the receiver filter. Using the *naive matrix multiplication algorithm*, the number of real multiplications involved in this stage is \( 2MNK \) per block. After serial to parallel conversion, each symbol is processed by a \( N \) point DFT operation resulting in \( N \log_2 N - 3N + 4 \) real multiplications for processing one symbol. The quadrature modulated symbols after the DFT processing are then equalized using \( E \) as defined in (4.26). The equalization process requires \( 4MN \) real multiplications to estimate one transmitted FBMC/QAM block. Hence, the total number of real multiplications per FBMC/QAM block in the case of no inverse filter (NIF) at the receiver is

\[ C_{\text{Rx}}^{\text{NIF}} = MN \log_2 N + (2K + 1)MN + 4M \]  

(4.76)

![Figure 4.5: Complexity comparison of FBMC/QAM with and without inverse filter for \( K=5 \) and \( M=14 \)]
4.4. Simulation Results

4.3.2 Complexity analysis in case of inverse filter

The transmitter complexity in this case is the same as (4.75) since inverse filter only increases the complexity of the receiver. The total number of real multiplications per FBMC/QAM block in case of inverse filter ($IF$) at the receiver is

$$C_{IF \text{Rx}} = C_{\text{NIF Rx}} + C_R. \quad (4.77)$$

where $C_R = 2MN - \eta N(M - 1)$ is the number of additional real multiplications per symbol introduced by the inverse filter and depends on the value of $\eta$ as defined in Sec. 4.1.3. It can be seen from (4.77) that the complexity of the receiver with inverse filter ($C_{IF \text{Rx}}$) depends on the block size ($M$). Hence, the additional complexity will be higher for large block size. The complexity in terms of real multiplications per symbol (assuming $M = 14$) with and without the inverse filter is presented in Fig. 4.5. It is worth mentioning that the worst case or the upper bound of the receiver complexity i.e., Big–$\mathcal{O}$ is $O(N \log_2 N)$ for both cases and can be determined by dropping the lower order terms and the constant multipliers in (4.76) and (4.77).

4.4 Simulation Results

In this section we present the simulation results for MSE and output SINR of the FBMC/QAM system with and without inverse filter along with the BER performance in case of synchronous and asynchronous multi-service scenarios. The simulation parameters includes, $N = 64$ i.e., the FFT size to define the FFT matrix $\mathcal{F}$, the block size is considered as $M = 14$, the prototype filter matrix $P$ is defined using IOTA prototype filter with overlapping factor $K = 6$. The desired signal is modulated by 16QAM and 64QAM (for BER analysis) with normalized power and input SNR is controlled by the noise power. The received signal is equalized using MMSE equalizer and the LTE channel model considered in our simulation is the extended pedestrian A model (EPA) [108].

4.4.1 MSE and output SINR

The performance of FBMC/QAM system without inverse filter matrix at the receiver can be observed from the simulation results given in Fig. 4.6. The overall MSE performance of FBMC/QAM system without inverse filter matrix is shown in Fig. 4.6(a). It can be observed

\footnote{Although matrix inversion has a general complexity of $O(N^3)$ which can significantly increase the complexity of the receiver. However, since the coefficients in the autocorrelation matrix $G$ are constant, we can calculate the inverse filter matrix $R = G^{-1} \in \mathbb{R}^{MN \times MN}$ \textit{off-line}.}
that the system becomes interference limited at an input SNR of around 30dB and the MSE is around -11.2dB. It can also be confirmed that the derived model of the interference in the system completely matches with the simulation results. It can also be observed from the results in Fig. 4.6(b) that due to the presence of ISI and ICI terms, the output SINR of the system is limited to around 11.2dB. This shows that the presence of these interference terms make the system highly interference limited. The individual MSE sources like noise, residue from the equalization, IBI, ISI, ICI and filter distortion due to multipath effect are shown in Fig. 4.7. It can be seen that without inverse filter matrix the contribution of ICI and ISI is quite significant i.e. around -16.5dB and -13dB respectively. The interference caused by the multipath effect includes filter distortion and IBI which contributes around -32.5dB and -45dB respectively in the overall interference in the FBMC/QAM system. The performance of FBMC/QAM system with inverse filter at the receiver can be observed from the simulation results given in Fig. 4.8.
The overall MSE performance of FBMC/QAM system with inverse filter matrix is shown in Fig. 4.8(a). It can be observed that the system becomes interference limited at an input SNR of around 50dB and the MSE is around -31dB, which is significantly lower than the case with no inverse filter matrix. It can also be confirmed that the derived model of the interference in the system completely matches with the simulation results. It can also be observed from the results in Fig. 4.8(b) that with very insignificant ISI and ICI terms, the output SINR of the system is now limited to around 31dB. This shows that the use of inverse filter matrix significantly improves the FBMC/QAM system performance. The individual MSE sources like noise, residue from the equalization, IBI, ISI, ICI and filter distortion due to multipath effect are shown in Fig. 4.9. It can be seen that the inverse filter matrix completely removes the intrinsic interference in the received signal i.e., ISI and ICI terms. The residual interferences caused by the multipath effect includes filter distortion and IBI which now contributes around -32dB.
Figure 4.10: Performance comparison of FBMC/QAM with and without Inverse Filter

and -43.5dB respectively in the overall interference in the FBMC/QAM system (A slightly higher interference is due to the enhancement factor $\zeta_{m,n}$ introduced by the use of inverse filter matrix). The performance comparison in term of MSE and input SNR vs. output SINR in a FBMC/QAM system with and without the inverse filter matrix are shown in Fig. 4.10. It can be seen in Fig. 4.10(a) that the composite MSE performance is significantly improved with the use of inverse filter matrix. This improvement is also evident in Fig. 4.10(b) where the output SINR with respect to input SNR is shown and it can be seen that the use of inverse filter matrix significantly improves the system performance. This performance improvement in terms of MSE and output SINR is achieved due to the cancellation of the intrinsic interference i.e., ICI and ISI terms in the conventional FBMC/QAM system as shown in section 4.1.3. We have also shown that without the inverse filter, the interference contribution from these ICI and

Figure 4.11: Noise & Interference Enhancement
ISI terms are very significant i.e., -16.5dB and -13dB respectively which makes the system interference limited as shown in Fig. 4.7. Since the use of inverse filter cancels this intrinsic interference, the MSE and output SINR performance are improved significantly as shown in Fig. 4.10. However, it is worth mentioning that the use of inverse filter is a suitable option for medium to high SNR cases only since at low SNR values, the system is noise limited rather than interference limited. As we have shown that the use of inverse filter can enhance the noise power in section 4.2.2, this may lead to more harm than good at low SNR values. It is therefore highlighted in chapter 6 as a part of future work to propose a proper filter design to counteract the noise enhancement problem in our proposed system and to make the system suitable for low SNR cases. The noise and interference ($\gamma_{IBI}$ and $\gamma_{fd}$) enhancement due to the use of inverse filter (IF) is illustrated in Fig. 4.11. It can be seen that the enhancement is very small compared to the no inverse filter case (NIF). The interference enhancement is therefore negligible in comparison to the level of performance improvement achieved with the use of inverse filter.

4.4.2 BER Performance

The coded results (convolutional code with code rate 1/2 and generator polynomials defined as [133, 171]) for the BER performance of FBMC/QAM system with and without inverse filter are presented for synchronous and asynchronous multi-user transmissions. We have considered a multi-user (multi-service) transmission scenario since next generation wireless systems are expected to provide a flexible framework for heterogeneous services. In such a case, services like mobile broadband (MBB), IoT, ultra reliable communication (URC) may coexist in adjacent sub-bands. To evaluate the performance of FBMC/QAM system with multi-service transmission, we segregate the whole bandwidth into three consecutive sub-bands, each for different user (services). The BER performance of the FBMC/QAM system with and without inverse filtering for synchronous segregated spectrum is shown in Fig. 4.12. We have used conventional OFDM as a baseline scheme to compare the performance of the FBMC/QAM system with and without inverse filter. It should be noted that we have considered no guard bands between adjacent sub-bands in our simulations. Although OFDM system is not a spectral efficient scheme for such applications due to higher side lobes in frequency domain and therefore require guard bands in practical systems. We have considered same parameters for OFDM and FBMC/QAM with and without inverse filter to evaluate the performance in a multi-service scenario. For a fair comparison between the two systems, the SNR loss due to the CP (overhead) in OFDM, must be considered. For this reason, we have calculated the noise power for both systems as discussed in [32]. It can be seen from Fig. 4.12 that without inverse filter
(FBMC/QAM-NIF), the FBMC/QAM system has poor performance compared to conventional OFDM system due to the presence of intrinsic interference. Since, FBMC/QAM with inverse filter (FBMC/QAM-IF) can cancel the intrinsic interference at the receiver, the system can provide comparable performance to the conventional OFDM system as shown in Fig. 4.12.

Figure 4.12: BER performance of OFDM and FBMC/QAM system with synchronous sub-bands

However, it is likely that the adjacent sub-bands in a multi-service transmission are out of sync since simple IoT devices in future wireless networks may only have coarse synchronization. Thus it is very desirable for the system to be robust against asynchronism between adjacent sub-bands. To evaluate the performance of FBMC/QAM under asynchronous sub-bands, we have considered the timing offset between two adjacent sub-band transmissions to be 50% of the symbol interval as shown in Fig. 4.13.

Figure 4.13: Asynchronous user streams

Since the second sub-band transmission suffers interference from the first and third sub-band transmissions, it is appropriate to investigate the BER performance of the second sub-band user. In case of multi-service asynchronous transmission, the BER performance of FBMC/QAM with and without inverse filter is shown in Fig. 4.14. It can be seen that in case of asynchronous
multi-service transmission, FBMC/QAM with inverse filter significantly outperforms the conventional OFDM system. It is due to the good frequency domain localization of the prototype filter in FBMC/QAM system which means less interference leakage from adjacent sub-bands. Since, OFDM suffers from higher side lobes in frequency domain it will have more interference from the neighboring subbands as discussed in [24]. We can also observe the impact of neglecting the diagonal elements in matrix $R$ on the system BER performance. It can be seen from Fig. 4.12 and Fig. 4.14 that for $\eta=0$ the BER performance is better than $\eta=1$ since we are using all the diagonal elements in the inverse filter matrix. However, the complexity of system is lower for $\eta=1$ as neglecting middle $N/2$ elements in the off-diagonal sub-matrices of $R$ leads to less additional real multiplications. It is therefore a trade-off between the complexity and the system BER performance i.e., a higher value of $\eta$ leads to lower complexity as well as relatively poor BER performance. Whereas a lower value of $\eta$ leads to higher complexity as well as relatively better BER performance. In the light of all the results, the improved performance of FBMC/QAM compared to conventional OFDM systems along with its inherent good out-of-band leakage performance, robustness to asynchronous multi-service transmissions and ability to have flexible scheduling on subcarrier level makes it a suitable candidate for next generation wireless applications, especially for massive machine type communications.

Figure 4.14: BER performance of OFDM and FBMC/QAM system with asynchronous sub-bands
4.5 Conclusion

We have proposed a novel low-complexity interference-free FBMC/QAM system based on matrix inversion of the prototype filters that mitigates the intrinsic interference in a FBMC/QAM system. The proposed system enables the use of quadrature modulated symbol transmission, while maintaining per subcarrier based filtering. The proposed system is based on a compact matrix model of the FBMC/QAM system, which also laid the ground for an in-depth analysis of the interferences affecting the system when operating in a multipath environment. The interference terms due to channel distortions and the intrinsic behavior of the transceiver model have been derived in detail and analyzed in terms of MSE with and without the inverse filter. It was shown through the analytical and simulation results that inverse filtering significantly reduces the interference in FBMC/QAM system at the expense of slight enhancement in IBI, interference due to filter distortion caused by multipath channel and noise. The complexity analysis of the system with and without the inverse filter is also provided which shows that complexity in both cases have the same upper bounds. The performance of the system is then evaluated for synchronous and asynchronous multi-service scenarios. Simulation result shows that FBMC/QAM with inverse filter can provide comparable performance to the conventional OFDM system in case of synchronous multi-service transmission while it outperforms OFDM in the asynchronous case. The improved performance of the proposed FBMC/QAM system makes it highly suitable for next generation wireless applications, especially for massive machine type communications.
Index Modulation based Filter Bank Multicarrier (FBMC/QAM) System

Subcarrier index modulation has recently emerged as a promising concept for spectrum and energy-efficient next generation wireless communications systems since it strikes a good balance among error performance, complexity, and SE. IM technique when applied to a multicarrier waveforms yields the ability to convey the information not only by $M$-ary signal constellations as in conventional multicarrier systems, but also by the indexes of the subcarriers, which are activated according to the incoming bit stream. Although IM is well studied for OFDM based systems, FBMC with index modulation has not been thoroughly investigated. In this chapter, we shed light on the potential and implementation of IM technique for QAM based FBMC system. We start with a mathematical model of the IM based QAM-FBMC system (FBMC/QAM-IM) along with the derivation of interference terms at the receiver due to channel distortions and noise. The interference terms including the ones introduced by the multipath channel are analyzed in terms of MSE and output SINR. It is shown with analytical and simulation results that the interference power in FBMC/QAM-IM is smaller comparing to that of the conventional FBMC/QAM system as some of the subcarriers are inactive. The performance of FBMC/QAM with IM is investigated by comparing the SIR and output SINR with that of the conventional FBMC/QAM system along with the BER performance which shows that the FBMC/QAM-IM is a promising transmission technique for future wireless networks.
5.1 FBMC/QAM System with Index Modulation

To overcome the shortcomings of OQAM based FBMC systems, we have proposed an improved QAM based FBMC system in Chapter 4. We have shown that an inverse filter can be used at the receiver to cancel the intrinsic interference in the FBMC/QAM system. However, the use of inverse filter enhances the noise and interferences caused by multipath channel as discussed in Section 4.2.2. To reduce the effect of enhanced residual interferences, IM can be a suitable option. As some of the subcarriers in IM based MCM systems are inactive, this gives us an opportunity to combine IM with FBMC/QAM to reduce the impact of residual interference in this transmission scheme. In this chapter, we introduce IM concept to the FBMC/QAM system proposed in Chapter 4. It is shown analytically and by simulation that the interference power is smaller compared to that of the conventional FBMC/QAM system as some subcarriers carry nothing but zeros. We further investigate the advantages of FBMC/QAM with index modulation by comparing the SIR with that of the conventional FBMC/QAM system. The analytical and simulation results clearly indicates the superiority of the FBMC/QAM-IM system in terms of SIR, output SINR and BER performance over the conventional FBMC/QAM system.

5.1.1 System Model

In this section, we define the index modulation based FBMC/QAM system in matrix form which will be subsequently used to analyze the interference reduction due to the introduction of index modulation with conventional FBMC/QAM system. The system model of FBMC/QAM-IM is developed as an extension to the FBMC/QAM model presented in Section 4.2. We will focus on the index modulation block in detail while the FBMC/QAM model will be briefly discussed.

Index Modulation

The block diagram of the FBMC/QAM-IM system is illustrated in Fig. 5.1. Since, index modulation utilizes indexes of certain subcarriers according to the incoming bit stream to convey information, we define the subcarriers bearing M-ary signal constellation symbols as active subcarriers and the rest as inactive. The proposed model of FBMC/QAM-IM follows a block based processing approach where each block contains M symbols and each symbol has N subcarriers in the frequency domain. Let us consider that transmitted information of each block has T bits. These information bits are then divided into g groups each containing p bits which are then mapped to an FBMC/QAM subblock of length n, where \( n = \frac{N}{g} \). Each group of p
5.1. FBMC/QAM System with Index Modulation

bits can be split into two parts. The first $p_1$ bits are used for index selection in which $k$ out of $n$ available subcarrier are activated according to a predefined look up table [71]. The remaining $p_2$ bits are mapped on to the $M$-ary signal constellation to define the data symbols that will modulate the active subcarriers. The number of bits $p_1$ carried by the indexes of the active subcarriers are defined as

$$ p_1 = \lfloor \log_2 C_n^k \rfloor, \quad (5.1) $$

where $C_n^k$ denotes the binomial coefficient. In other words, selected indexes can have $c = 2^{p_1}$ possible realizations. The number of bits carried by the $M$-ary constellation symbols are defined as

$$ p_2 = k \log_2 M, \quad (5.2) $$

The total number of bits that are transmitted by one FBMC/QAM-IM block can be defined as

$$ T = (p_1 + p_2)g = (\lfloor \log_2 C_n^k \rfloor + k \log_2 M)g \quad (5.3) $$

In other words, in FBMC/QAM-IM system, the information is conveyed by both the indexes of the active subcarriers and also the $M$-ary constellation symbols modulating these active subcarriers. Also we can infer that, as we are not using all of the available subcarriers for the data transmission, the loss in transmission efficiency is compensated by transmitting additional bits in the spatial domain of the FBMC/QAM block. From Fig. 5.1 it can be seen that for each subblock $\beta$, the first $p_1$ bits are used for index selection i.e., $k$ out of $n$ available indexes are selected as

$$ i_{\beta} = [i_{\beta,1}, ..., i_{\beta,k}] \quad (5.4) $$
where $i_{\beta, \gamma} \in [1, ..., n]$, $\beta = 1, ..., g$ and $\gamma = 1, ..., k$. To modulate these active subcarriers, the remaining $p_2$ bits are mapped on the $M$-ary signal constellation to define the transmitted data symbols as

$$a_{\beta} = [a_{\beta,1}, ..., a_{\beta,k}]$$

where $a_{\beta}$ is the symbol vector transmitted on the active sub-carriers in each sub-block. Such that $a_{\beta, \gamma} \in S$ for $\beta = 1, ..., g$ and $\gamma = 1, ..., k$, where $S$ is the set of all possible complex symbols from the $M$-ary constellation. Using $i_\beta$ and $a_{\beta}$ from (5.4) and (5.5), the FBMC/QAM block creator generates all of the subblocks $A_{m,\beta} \in \mathbb{C}^{n \times 1}$ for $\beta = 1, ..., g$ and form FBMC/QAM-IM symbol as $s_m = [s_{m,0}, s_{m,1}, ..., s_{m,N-1}]^T = [A_{m,1}, A_{m,2}, ..., A_{m,g}]^T \in \{0, a_{\beta, \gamma}\}$. The generated FBMC/QAM-IM block can be expressed as $S = [s_0, s_1, ..., s_{M-1}] \in \mathbb{C}^{N \times M}$. Furthermore, to have unit average power, we reallocate the power from inactive subcarriers to the active subcarriers in a subgroup. The average power of the modulated symbol $s_{m,n}$ can be represented as $\mathbb{E}\{\|s_{m,n}\|^2\} = \delta^2$. It should be noted that when the number of active subcarriers $k$ is equal to the number of carriers in a subgroup i.e., $k = n$, the FBMC/QAM-IM system will become a conventional FBMC/QAM system. Hence, the FBMC/QAM system can be viewed as a special case of the FBMC/QAM-IM system.

It should be noted that by properly choosing $n$ and $k$, the spectral efficiency of FBMC/QAM-IM system can be improved. The spectral efficiency $\eta_{FBMC/QAM-IM}$ can be calculated as follows

$$\eta_{FBMC/QAM-IM} = \frac{M^N}{N} \left( \frac{\log_2 C_n^k + k \log_2 M}{NM} \right) \text{ bits/sec/Hz} \hspace{1cm} (5.6)$$

As we already mentioned that FBMC/QAM-IM system reduces to FBMC/QAM system in case of $k = n$, the spectral efficiency of FBMC/QAM can now be derived using (5.6) as

$$\eta_{FBMC/QAM} = \log_2 M \hspace{1cm} \text{bits/sec/Hz} \hspace{1cm} (5.7)$$

The spectral efficiency gain of FBMC/QAM-IM over conventional FBMC/QAM system can be expressed as

$$\eta_{gain} = \frac{\eta_{FBMC/QAM-IM}}{\eta_{FBMC/QAM}} = \frac{M^N}{NM} \left( \frac{\log_2 C_n^k + k \log_2 M}{\log_2 M} \right) = \frac{\log_2 C_n^k}{n \log_2 M} + \frac{k}{n} \hspace{1cm} (5.8)$$

We can see from (5.8), that by properly selecting $n$, $k$ and $M$-ary modulation, we can achieve spectral efficiency gain $\eta_{gain} > 1$. For example, when $n = 4$, $k = 3$ and $M = 2$ i.e., BPSK modulation, we can calculate $\eta_{gain} = 1.25$, which indicates that the spectral efficiency of FBMC/QAM-IM has exceeded that of conventional FBMC/QAM.


5.1. FBMC/QAM System with Index Modulation

Transmit Processing

It can be seen from Fig. 5.1 that after index modulation, the signal $s_m$ is passed through the conventional FBMC/QAM transmitter i.e., $N$ point IDFT processor, parallel to serial conversion and the transmit filter matrix $P$. The structure of $P$ is already defined in (4.2). The output of the transmit filter matrix can be expressed as

$$o = Pb \in \mathbb{C}^{(K+M-1) \times 1}, \quad (5.9)$$

where $b$ is the signal vector processed symbol by symbol through the IDFT block i.e., $F^H$. The signal vector can be represented as $b = [b_0; b_1; \cdots; b_{M-1}] = [F^H s_0; F^H s_1; \cdots; F^H s_{M-1}] \in \mathbb{C}^{MN \times 1}$. It should be noted that the prototype filter matrix $P$ is designed in a manner that when it is multiplied by vector $b$, the multiplication of matrices is equivalent to the required linear convolution process. As a result the output of the transmit filter will have $(K-1)N$ more samples than the signal vector $b$ as can be seen from (5.9).

Passing through the Channel

We assume the system operates over a slowly-varying fading channel i.e., quasi-static fading channel. In such a scenario, we can assume that the duration of each of the transmitted data block is smaller than the coherence time of the channel, therefore the random fading coefficients stay constant over the duration of each block [104]. In this case, we define the multipath channel as a $L$-tap channel impulse response (CIR) with the $l^{th}$-tap power being $\rho_l^2$. It is also assumed that the average power remains constant during the transmission of the whole block. Let us define the CIR $h$ as

$$h = [h_0, h_1, \cdots, h_{L-1}]^T = [\rho_0 z_0, \rho_1 z_1, \cdots, \rho_{L-1} z_{L-1}]^T, \quad (5.10)$$

where $h_l$ denotes the channel coefficient of the $l^{th}$ tap in the time domain and the complex random variable $z_l$ with complex Gaussian distribution as $\mathbb{C}N(0, 1)$ represents the multipath fading factor of the $l^{th}$ tap of the quasi-static rayleigh fading channel.

The signal vector $o$ is then passed through the multipath channel $h$ as discussed in Section 4.1.1. The received signal after passing through the channel can be represented as

$$r = \sum_{l=0}^{L-1} \rho_l Z_l P b_e^l + o_{fd} + o_{IBI} + n, \quad (5.11)$$

The first term in (5.11) represents the linear convolution process between the channel and the transmitted signal where $Z_l$ implies that each FBMC symbol in a block experiences the same
5.1. FBMC/QAM System with Index Modulation

channel i.e., \( Z_l = z_l \times I_{(K+M-1)N} \times (K+M-1)N \) and \( b^l_j = X_l b \). Note that the matrix \( X_l = I_M \otimes X_{\text{sub},l} \) is a block diagonal exchange matrix where \( X_{\text{sub},l} = [0_{l \times (N-l)}, I_l, 0_{(N-l) \times l}] \) and is used to exchange the locations of elements of \( b \). Furthermore, \( o_{fd} = \sum_{l=0}^{L-1} \rho_l Z_l \Delta P^l b^l e^l \) is the interference caused by the filter distortion due to channel multipath effect, \( o_{\text{IBI}} = \sum_{l=0}^{L-1} \rho_l Z_l r_{B,l} \) is the inter-block interference (IBI) caused by multipath channel with \( r_{B,l} = [r_{p,l}^l; 0_{(M+K-l)N-l \times 1}] \) and \( r_{p,l}^l \in \mathbb{C}^{l \times 1} \) is the interfering signal from the previous FBMC/QAM block and \( n \) is a Gaussian noise vector with each element having zero mean and variance \( \sigma^2 \).

Figure 5.2: Block diagram of FBMC/QAM-IM Receiver

**Receive Processing**

The receiver process of FBMC/QAM-IM is shown in Fig. 5.2. At the receiver, the received signal \( r \) is first passed through the receive filter \( P^H \), which serve as a matched filter in FBMC/QAM receiver. The signal after matched filtering is then passed though the inverse filter \( R \) to cancel the intrinsic interferences in the received signal block. The signal after the inverse filtering is then processed symbol by symbol through the DFT processor i.e., \( \mathcal{F} \). The resulting frequency domain signal \( y_m \) is then equalized using ZF or MMSE equalizer \( E \) to counter the effects of multipath channel. The complete receive processing is already derived in detail in Section 4.2.2. Following the same receiver processing steps, the equalized symbol \( \hat{s}_m \) can now be represented as follows

\[
\hat{s}_m = s_m + (\mathbf{I} - \beta) s_m + E \mathcal{F} R_m P_m^H o_{fd} + E \mathcal{F} R_m P_m^H o_{\text{IBI}} + E \mathcal{F} R_m P_m^H n \quad (5.12)
\]

As we can see from (5.12) that the transmitted signal \( s_m \) is free from ICI and ISI terms due to the use of inverse filter matrix \( R \). However, the received symbols are affected by interferences...
caused by MMSE estimation bias, IBI and filter distortion due to multipath channel. Since, we know that some of the sub-carriers in FBMC/QAM-IM system are in-active and therefore they will not contribute to the overall interference in the system. In what follows, we will evaluate the interference and noise power in FBMC/QAM and FBMC/QAM-IM systems to estimate the performance improvement in term of MSE and output SINR due to the introduction of index modulation.

5.2 Interference and Noise Power Estimation

It can be seen from (5.12), that the transmitted symbol vector $s_m$ is accompanied with interference terms caused by the multipath channel and noise. In this section we estimate the interference and noise power in the FBMC/QAM-IM system. First, let us introduce a diagonal matrix $D_m \in \mathbb{R}^{N \times N}$ such that the $n^{th}$ diagonal element of $D_m$ is defined as

$$D_{m,n} = \begin{cases} 1 & \text{if } s_{m,n} \in a_{\beta,\gamma} \\ 0 & \text{if } s_{m,n} = 0 \end{cases}$$

(5.13)

The diagonal matrix $D_m$ represent the active and inactive indexes of the transmitted symbol vector $s_m$. It should be noted that introducing $D_m$ will have no effect on the system model since $D_m s_m = s_m$. The purpose of defining $D_m$ is to evaluate the impact of inactive subcarriers on the system performance. Since some of the subcarriers in FBMC/QAM-IM are inactive, therefore they will not contribute to the overall interference in the system. Furthermore, it should be noted that $D_m$ will be an identity matrix in case of conventional FBMC/QAM system since all of the subcarriers are active i.e., $n = k$ and contribute to the overall interference in the system. The interference and noise power calculation follows the same procedure as discussed in Section 4.3. It can be seen from (5.12) that the estimated symbol is accompanied with MMSE estimation bias, interference terms like filter distortion and IBI due to multipath channel and noise i.e.,

$$\hat{s}_m = s_m + \psi_{\text{resd},m} + \psi_{\text{fd},m} + \psi_{\text{IBI},m} + \psi_{\text{noise},m}$$

(5.14)

The MSE of the $n$-th modulation symbol estimation in the $m$-th FBMC/QAM-IM symbol can be derived as

$$\gamma_{\text{tot},m,n} = \mathbb{E}||\hat{s}_{m,n} - s_{m,n}||^2$$

$$= \mathbb{E}[||\psi_{\text{resd},m}||^2 + ||\psi_{\text{fd},m}||^2 + ||\psi_{\text{IBI},m}||^2 + ||\psi_{\text{noise},m}||^2]$$

(5.15)
5.2. Interference and Noise Power Estimation

5.2.1 MSE of signal estimation bias

The desired signal estimation bias is caused by the MMSE receiver since it minimizes the MSE between the transmitted and received signal. This leads to residual interference in the estimated signal. From (5.15) and (5.12), we can write the MSE of the signal estimation bias as

\[
\gamma_{\text{resd},m,n} = E\| \psi_{\text{resd},m} \|^2_n = E\{ \| (I - \beta)s_m \|^2_n \}
\]

\[
= E\{ \| (I - \beta)D_m s_m \|^2_n \}
\]

\[
= \delta^2 \| (I - \beta)D_m \|^2_n
\]

\[
= \delta^2 D_m,n (I - \beta_n)^2
\]

(5.16)

where \( E\{s_m s_m^H\} = \delta^2 I \) and similar to (4.39), substituting \( \beta_n \) i.e., \( \beta_n = \frac{|C_n|^2}{|C_n|^2 + \nu \sigma^2} \) into (5.16) yields

\[
\gamma_{\text{resd},m,n} = \delta^2 D_m,n \left[ \frac{\nu^2 \sigma^4}{(\delta^2 |C_n|^2 + \nu \sigma^2)^2} \right].
\]

(5.17)

Apparently, when the ZF receiver is adopted, \( \gamma_{\text{resd},m,n} = 0 \) since \( \nu = 0 \). However, the ZF equalization leads to noise enhancement unlike MMSE receivers.

5.2.2 MSE of filter distortion due to multipath channel

We can write the MSE of the interference caused by filter distortion due to multipath channel from (4.65) as follows

\[
\gamma_{\text{fd},m} = E^H \text{FR}_m \text{D}_m^H \alpha_{\text{fd}} \text{P}_m \text{R}_m^H \text{E}_m^H,
\]

(5.18)

Using (5.11), we can determine \( \alpha_{\text{fd}} = E[\alpha_{\text{fd}}^H] \) as follows

\[
\alpha_{\text{fd}} = E \left[ \left\{ \sum_{l=0}^{L-1} \rho_l \text{Z}_l \text{D}_m \text{P}^l \text{b}_e^l \text{H} \} \left\{ \sum_{l=0}^{L-1} \rho_l \text{Z}_l \text{D}_m \text{P}^l \text{b}_e^l \text{H} \}^H \right\} \right],
\]

\[
= \sum_{l=0}^{L-1} \rho_l^2 E[\text{Z}_l \text{D}_m \text{P}^l \text{b}_e^l \text{H} \Delta \text{P}^l \text{H} \text{Z}_l^H]
\]

\[
= \sum_{l=0}^{L-1} \rho_l^2 E[\text{Z}_l \text{D}_m \text{P}^l \text{b}_e^l \text{H} \Delta \text{P}^l \text{H} \text{Z}_l^H]
\]

(5.19)

From (5.10), we know that \( E\{\text{Z}_l \text{Z}_l^H\} = 1 \) since \( z_l \in \mathbb{C}N(0, 1) \) also \( E\{s_m s_m^H\} = \delta^2 I \) and \( \| \text{X}_l^H \text{D}_m \text{D}_m^H \text{X}_l^H \|_n = \frac{k}{\nu} \| I_{N \times N} \|_n \), consequently

\[
\alpha_{\text{fd}} = \frac{k}{n} \delta^2 \sum_{l=0}^{L-1} \rho_l^2 \text{Tr}\{\Delta \text{P}^l \Delta \text{P}^l \text{H}\},
\]

\[
= \frac{k}{n} \delta^2 \sum_{l=0}^{L-1} \rho_l^2 T^{l},
\]

(5.20)
5.2. Interference and Noise Power Estimation

where $T_{\downarrow l} = \text{Tr}[\Delta P_{\downarrow l} \Delta P_{\downarrow l}^H]$. Since $T_{\downarrow l}$ is a scalar, $\alpha_{fd}$ is also a scalar. Now substituting (5.20) into (5.18), yields

$$\gamma_{fd,m} = \alpha_{fd} \mathbb{E}[\mathcal{F} R_m P_m^H P_m R_m^H \mathcal{F} E],$$

(5.21)

By taking the $n^{th}$ diagonal element of $\gamma_{fd,m}$, we obtain

$$\gamma_{fd,m,n} = \alpha_{fd} ||E FR_m P_m^H P_m R_m^H F^H||_n,$$

(5.22)

where $||F R_m P_m^H P_m R_m^H F^H||_n = \zeta_m,n ||I_{N \times N}||_n$.

Comparing (5.22) with (4.68), we can see that the variance of filter distortion in FBMC/QAM-IM system is $k_n$ times the variance of filter distortion in FBMC/QAM system. Hence, we can say that the inactive subcarriers do not contribute to the total interference in the FBMC/QAM-IM system since $k_n < 1$ in FBMC/QAM-IM system.

5.2.3 MSE of IBI

We can write the MSE of the interference caused by IBI from (4.69) as follows

$$\gamma_{IBI,m} = \mathbb{E}[\mathcal{F} R_m P_m^H \alpha_{IBI} P_m R_m^H \mathcal{F} E],$$

(5.23)

where $\alpha_{IBI} = \mathbb{E}[\omega_{IBI} \omega_{IBI}^H]$, now using (5.11), we can determine $\alpha_{IBI}$ as

$$\alpha_{IBI} = \mathbb{E}\left[\left\{ \sum_{l=0}^{L-1} \rho_l Z_l Y_{B,l} \right\} \left\{ \sum_{l=0}^{L-1} \rho_l Z_l Y_{B,l} \right\}^H \right],$$

(5.24)

$$\mathbb{E}\{Y_{B,l} Y_{B,l}^H\}$$

is dependent on the signal type of the last block, where we assume it is also occupied by an FBMC symbol with the same power, then we have

$$\mathbb{E}\{Y_{B,l} Y_{B,l}^H\} = \mathbb{E}[||P_{(l)} b_{last}||^2] = \text{Tr}[P_{(l)} \mathbb{E}[b_{last} b_{last}^H] P_{(l)}^H],$$

$$= \text{Tr}[P_{(l)} \mathbb{E}[D_{last} s_{last} s_{last}^H D_{last}^H \mathcal{F}] P_{(l)}^H],$$

$$= \frac{k}{n} \delta^2 \text{Tr}[P_{(l)} P_{(l)}^H] = \frac{k}{n} \delta^2 \text{Tr}[P_{(l)}^{\text{corr}}],$$

(5.25)
where \( P^{(l)} = [P_{(last-l)}; b_{last}; 0_{(N-K-1)N \times MN}] \) in which \( P_{(last-l)} \) contains the last \( l \)-th rows of \( P \) also \( b_{last} \) is the symbol (after IDFT) in the last block and \( \mathcal{E}\{s_{last}s_{last}^H\} = \delta^2 I \) and \( \|\mathcal{H}^H D_{last} D_{last}^H \|= kN \). Substituting (5.26) into (5.25), we obtain

\[
\alpha_{IBI} = \frac{k}{n} \delta^2 \sum_{l=0}^{L-1} \rho_l^2 P_{(l)}^{corr},
\]

(5.27)

Since \( P_{(l)}^{corr} \) is a scalar, \( \alpha_{IBI} \) is also a scalar. Substituting (5.27) into (5.23), yields

\[
\gamma_{IBI,m} = \alpha_{IBI} \mathcal{E}\{\mathcal{F}R_m P_m^H R_m^H \mathcal{F} E^H\},
\]

(5.28)

By taking the \( n \)th diagonal element of \( \gamma_{IBI,m} \), we derive the MSE of IBI as

\[
\gamma_{IBI,m,n} = \alpha_{IBI} \|\mathcal{E}\{\mathcal{F}R_m P_m^H R_m^H \mathcal{F} E^H\}\|_n,
\]

(5.29)

where \( \|\mathcal{F}R_m P_m^H R_m^H \mathcal{F} E^H\|_n = \zeta_{m,n} \|I_{N \times N}\|_n \). It should be noted that if we consider a sufficient guard interval between the data blocks then we can safely assume the inter-block interference to be negligible i.e., \( \gamma_{IBI,m,n} = 0 \). Also by comparing (5.29) with (4.72), we can see that the variance of IBI in FBMC/QAM-IM system is also \( \frac{k}{n} \) times the variance of IBI in FBMC/QAM system.

### 5.2.4 MSE of Noise

We can write the MSE of the noise from (4.73) as follows

\[
\gamma_{noise,m} = \sigma^2 \mathcal{E}\{\mathcal{F}R_m P_m^H R_m^H \mathcal{F} E^H\}
\]

(5.30)

where \( \mathcal{E}\{nn^H\} = \mathcal{E}\|n\|^2 = \sigma^2 \) since \( n \) is Gaussian noise with each element having zero mean and variance \( \sigma^2 \). Taking the \( n \)th diagonal element of (5.30), we have

\[
\gamma_{noise,m,n} = \sigma^2 \|\mathcal{E}\{\mathcal{F}R_m P_m^H R_m^H \mathcal{F} E^H\}\|_n,
\]

(5.31)

where \( \|\mathcal{F}R_m P_m^H R_m^H \mathcal{F} E^H\|_n = \zeta_{m,n} \|I_{N \times N}\|_n \). Note that the \( \zeta_{m,n} \) is the noise / interference enhancement factor which is introduced when we use an inverse filter matrix at the receiver. Also by comparing (5.31) with (4.74), we can see that the variance of noise in FBMC/QAM-IM system is the same as the variance of noise in FBMC/QAM system. Hence, the performance improvement in FBMC/QAM-IM system comes from the reduction in the variance of interferences due to the use of index modulation.
5.3 Improved Receiver for FBMC/QAM-IM

The conventional receivers in FBMC/QAM system are used for detecting the $M$-ary symbols to extract the transmitted information. However, the FBMC/QAM-IM receiver needs to detect the indexes of the active sub-carriers and also the corresponding information ($M$-ary) symbol transmitted on those active subcarriers. To detect the active sub-carrier indexes and the $M$-ary symbols on the active subcarriers, the received symbol vector $\hat{s}_m$ is divided into $g$ groups by the detection group creator block i.e., $\hat{s}_m = [B_{m,1}, B_{m,2}, ..., B_{m,g}]^T \in \mathbb{C}^{N \times 1}$ as shown in Fig. 5.2. Each sub-block $B_{m,g} \in \mathbb{C}^{n \times 1}$ can now be detected by an optimum ML detector. The output of the detector is then used to extract the information embedded in the indexes of the active subcarriers as well as the constellation symbols transmitted on those active subcarriers. However, the optimal ML detector suffers from high complexity. In the following section, using the interference and noise power analysis presented in Section 5.2, we propose a low complexity detector based on the LLR approach.

5.3.1 Maximum likelihood (ML) Detector

The ML detector is an optimum detector that considers all possible sub-block realizations by searching for all possible sub-carrier index combinations and signal constellation points to make a joint decision on the active indexes and the constellation symbols for each sub-block by minimizing the following metric

$$\hat{A}_{m,\beta} = \arg\min_{A_{m,g} \in \Gamma} \|B_{m,\beta} - A_{m,\beta}\|^2$$

(5.32)

Thus, the ML detector chooses the sub-block $A_{m,g} \in \Gamma$, where $\Gamma$ is the set of all the possible sub-blocks, that yields the smallest distance with the received sub-block $B_{m,g}$ to estimate the transmitted sub-block. From the estimated sub-block $\hat{A}_{m,\beta}$, the index bits and $M$-ary symbol bits can then be decoded using the index decoder and $M$-ary demodulator as shown in Fig. 5.2. Although the ML detector can provide optimal performance but its complexity is unaffordable i.e., $\sim \mathcal{O}(2^pM^k)$. Therefore, to reduce the complexity of the ML detector, we propose an improved LLR detector based on the interference and noise power analysis provided in Section 5.2. The complexity of a LLR detector is $\sim \mathcal{O}(M)$ which makes it less complex than ML detector [77].

5.3.2 Log-likelihood Ratio (LLR) Detector

In this section we proposed an improved detector for FBMC/QAM-IM based on the interference analysis given in Section 5.2. A general LLR detector provides the logarithm of the ratio
of a posteriori probabilities of the frequency domain symbols by considering the fact that their values can either be zero or non-zero depending upon the sub-carrier being active or inactive. To determine the status of any subcarrier being active or inactive, we can use the following ratio

\[ \lambda_{m,n} = \log_e \frac{\sum_{\chi=1}^{M} P(s_{m,n} = a_\chi | \hat{s}_{m,n})}{P(s_{m,n} = 0 | \hat{s}_{m,n})} \]  

(5.33)

where \( a_\chi \in S \) and \( \lambda_{m,n} \) is the LLR value of the \( n^{th} \) subcarrier of \( m^{th} \) symbol in a FBMC/QAM-IM block. It should be noted that a larger value of \( \lambda_{m,n} \) means it is more probable that the \( n^{th} \) subcarrier under consideration was selected by the index selection block at the transmitter or in other words the subcarrier was active. The LLR expression given in (5.33) can be simplified by applying Bayes’ formula as follows

\[ \lambda_{m,n} = \log_e \frac{\sum_{\chi=1}^{M} P(s_{m,n} = a_\chi | \hat{s}_{m,n}) P(s_{m,n} = a_\chi)}{P(s_{m,n} = 0 | \hat{s}_{m,n}) P(s_{m,n} = 0)} \]  

(5.34)

As we already know that

\[ \sum_{\chi=1}^{M} P(s_{m,n} = a_\chi) = \frac{k}{n} \]  

(5.35)

and,

\[ P(s_{m,n} = 0) = \frac{n-k}{n} \]  

(5.36)

Using (5.35) and (5.36), we can update (5.34) as

\[ \lambda_{m,n} = \log_e \frac{k \sum_{\chi=1}^{M} P(s_{m,n} = a_\chi)}{(n-k)P(s_{m,n} = 0)} \]  

(5.37)

Eq. (5.37) can be further simplified as

\[ \lambda_{m,n} = \log_e (k) - \log_e (n-k) - \log_e \frac{\sum_{\chi=1}^{M} P(s_{m,n} = a_\chi)}{P(s_{m,n} = 0)} \]  

(5.38)

According to (5.12), the equalized symbol vector can be modeled as

\[ \hat{s}_m = s_m + \psi_{tot,m} \]  

(5.39)

where \( \psi_{tot,m} = \psi_{resd,m} + \psi_{fd,m} + \psi_{IBI,m} + \psi_{noise,m} \) is the sum of interference terms and the processed noise in the FBMC/QAM-IM system. It should be noted that the noise term
\( \psi_{\text{noise},m} \) is independent of all other terms and interference; the IBI contribution \( \psi_{\text{IBI},m} \) is also independent of all other terms since the interference comes from the previous FBMC/QAM-IM block. However, the MMSE estimation bias error \( \psi_{\text{resd},m} \) and filter distortion due to multipath channel \( \psi_{\text{fd},m} \) are correlated since they both depend on the desired signal \( s_m \). A ZF equalizer can be used to avoid the MMSE estimation bias error and to have all the remaining interference terms and noise independent with each other. We can now write the third term in (5.38) as follows

\[
\log_e \sum_{\chi=1}^{M} P(\hat{s}_{m,n} | s_{m,n} = a_\chi) = \log_e \sum_{\chi=1}^{M} \frac{1}{\gamma_{\text{tot},m,n}} \exp \left( \frac{-|\hat{s}_{m,n} - a_\chi|^2}{\gamma_{\text{tot},m,n}} \right)
\]

Substituting (5.40) into (5.38) yields,

\[
\lambda_{m,n} = \log_e (k) - \log_e (n - k) + \frac{|\hat{s}_{m,n}|^2}{\gamma_{\text{tot},m,n}} + \log_e \left\{ \sum_{\chi=1}^{M} \exp \left( \frac{-|\hat{s}_{m,n} - a_\chi|^2}{\gamma_{\text{tot},m,n}} \right) \right\}
\]

where \( \gamma_{\text{tot},m,n} \) is the total noise plus interference power of the \( n \)th subcarrier of the \( m \)th symbol in a FBMC/QAM-IM block and can be calculated using (5.15). It can be seen from the block diagram of our proposed LLR based detector in Fig. 5.3 that after calculating the \( n \) LLR values of each FBMC/QAM-IM symbol, the LLR values are used for index decoding and \( M \)-ary demodulation. The \( n \) LLR values are first used to create sub-groups of the \( m \)th FBMC/QAM-IM symbol, the \( k \) subcarriers in each sub-group which have maximum LLR value are assumed to be active. After detection of active sub-carrier indexes in each sub-group, the information is passed to the index decoder which provides hard detection of the index selecting bits based
on the indexes of the active sub-carriers in each sub-group. The M-ary symbols transmitted on each active sub-carriers is demodulated by the M-ary demodulator in a conventional manner to estimate the remaining transmitted bits. The bit combiner block then combines bits from all the sub-groups to generate the transmitted bits vector \( \hat{T} \) as shown in Fig. 5.2.

5.4 Simulation Results

In this section we present the simulation results for MSE, output SINR and SIR in FBMC/QAM system with and without index modulation along with the BER performance comparison of index modulation based FBMC/QAM system and conventional FBMC/QAM system. The simulation parameters used in the following simulation results are same as the parameters defined in section 4.4. However, for the BER results, the desired signal is modulated by QPSK modulation with normalized power. The size of each sub-group is considered as \( n = 4 \) and the number of active sub-carriers per sub-group is considered as \( k = 3 \) to have same transmission rate in FBMC/QAM system with and without IM.

5.4.1 MSE and output SINR

Since we know that not all of the sub-carriers in FBMC/QAM-IM system are active i.e., \( n > k \) unlike conventional FBMC/QAM system where \( n = k \). In this case the interference power will be smaller than the conventional FBMC/QAM system. For our analysis, we consider \( n = 4 \) and \( k = 3 \) FBMC/QAM-IM system with QPSK as the M-ary signal constellation. The individual interference terms like noise, residue from the MMSE equalization, IBI, ISI, ICI and filter distortion due to multipath channel in the proposed FBMC/QAM and FBMC/QAM-IM systems are derived in Section 4.2.2 and 5.2 and the results are presented in Fig. 5.4(a) and Fig. 5.4(b) respectively. The results show the power of each interference component that is affecting the multicarrier system. It can be seen that the contribution of ICI and ISI (intrinsic interference) is quite insignificant with the use of inverse filter at the receiver i.e., ISI is around -320dB and ICI cannot be even displayed on the same scale. However, the system is still affected by residue from the MMSE equalization, IBI and filter distortion due to multipath channel. Since some of the sub-carriers in IM based FBMC/QAM system are in-active, they will not contribute to these residual interferences. As a result, the interference level would be smaller compared to conventional FBMC/QAM system. The performance in terms of total MSE and output SINR in a FBMC/QAM system with and without index modulation is presented in Fig. 5.5. It can be seen from Fig. 5.5(a) that the MSE in FBMC/QAM system is improved with the
5.4 Simulation Results

Figure 5.4: Interference components in FBMC/QAM and FBMC/QAM-IM (4,3,QPSK)

use of IM. The improvement gain depends on the selection of $n$ and $k$ values. In this case we have considered $n = 4$ and $k = 3$ which result in a gain around $10 \log_{10} \left( \frac{n}{k} \right)$, i.e., $\sim 1.25$ dB. The improvement in MSE performance can be enhanced by a higher $\frac{n}{k}$ ratio. The output SINR of the system also improves with the use of IM as can be seen from the Fig. 5.5(b). It can also be confirmed that the interference terms in the system model give in (5.12) completely matches with the simulation results, which verifies the accuracy of the derived analytical model.

Figure 5.5: Performance comparison of FBMC/QAM and FBMC/QAM-IM (4,3,QPSK)

5.4.2 SIR Performance

The output SIR of FBMC/QAM system with and without the IM is presented in Fig. 5.6. As we have discussed earlier that since some of the subcarriers are inactive in FBMC/QAM-IM sub-
5.4. Simulation Results

block. Their power can either be saved to improve the energy efficiency of the system or it can be reallocated to the active subcarriers in a subgroup to improve the system BER performance. In our case we have considered the later option and distribute the power of inactive subcarriers to the active subcarriers. This results in the total average power of the FBMC/QAM-IM symbol to be unit as can be seen from Fig. 5.6(a). The interference power of each symbol

Figure 5.6: Output SIR performance comparison of FBMC/QAM and FBMC/QAM-IM (4,3,QPSK) with inverse filter in FBMC/QAM system with and without IM is shown in Fig. 5.6(b). It can be seen that the interference power in FBMC/QAM-IM system has been reduced by $10 \log_{10}(\frac{n}{k}) \sim 1.25 dB$ compared to conventional FBMC/QAM system. We can also see that the interference power is affecting the middle symbols more than the symbols at the edges of the FBMC/QAM block. The main reasons for this behavior is the intrinsic interference in FBMC/QAM system. As we know that symbols in FBMC/QAM system overlap each other both in time and frequency domain due to per subcarrier filtering. So it is obvious that the symbols at the edges of the block will experience less interference from the neighboring symbols compared to the symbols
Secondly, as we have already established that the use of inverse filter enhances the residue interferences in the FBMC/QAM system as discussed in Section 4.2.2 and we have also shown in Fig. 4.4 that the interference enhancement factor is constant for every subcarrier in a symbol. However, it affects the middle symbols more than the symbols at the edges. The output SIR of FBMC/QAM system with and without IM is shown in Fig. 5.6(c). It can be seen that the use of IM with FBMC/QAM improves the SIR of the system by reducing the variance of the interferences existing in the conventional FBMC/QAM system.

5.4.3 BER Performance

The results for the BER performance of FBMC/QAM system with and without IM are presented in Fig. 5.7. For FBMC/QAM-IM system, we have selected \((n,k)\) as \((4,3)\), which means that the SE of FBMC/QAM-IM is the same as the conventional FBMC/QAM system. It can be seen that FBMC/QAM-IM (with inverse filter) has better performance compared to conventional FBMC/QAM system (with inverse filter) i.e., FBMC-IF, FBMC-IM-ML-IF and FBMC-IM-LLR-IF results in Fig. 5.7.

![Figure 5.7: BER performance of FBMC/QAM system with and without IM](image)

The use of inverse filter cancels the intrinsic interference in FBMC/QAM system making it quasi-orthogonal as discussed in section 4.1.3. Since, index modulation in FBMC/QAM reduces the effect of residual interference at the receiver and also the power from inactive subcarriers are reallocated to the active carriers, the system can provide improved BER performance compared to its conventional counterpart. It can also be seen that the proposed LLR detector exhibit same performance as ML detector but with much lower complexity. The similar
performance of LLR detector and ML detector is also exhibited in [77]. We can also explain this similar performance as well since the ML detector is comparing the received sub-group of subcarriers with all the possible sub-groups which leads to a complexity of order $O(2^p \cdot M^k)$ as discussed in section [5.3.1]. When ML detector is comparing the sub-groups instead of each subcarrier, it is actually performing the joint index detection and $M$-ary detection. However, in LLR detector each subcarrier is first checked to be active or inactive in a subgroup and then $M$-ary detection is applied on the active sub-carriers leading to a complexity of order $O(M)$ as discussed in section [5.3.1]. Since LLR is also based on likelihood function as ML detector, their performance will be similar in case of hard detection. The only advantage is reduced complexity in case of LLR compared to ML detection whereas the performance will be similar. It can also be seen from Fig. [5.7] that without the use of inverse filter, the FBMC/QAM-IM system has poor performance compared to conventional FBMC/QAM system i.e., FBMC-NIF, FBMC-IM-ML-NIF and FBMC-IM-LLR-NIF results in Fig. [5.7]. It is because the FBMC/QAM-IM system relies on ML or LLR detectors which basically needs to detect the active indexes and then the corresponding $M$-ary symbols on the active subcarriers (joint detection in case of ML) to recover the transmitted information. Since with no inverse filter the system becomes non-orthogonal and a poor index decoding not only result in erroneous index bit detection but also lead to the erroneous $M$-ary symbol detection, resulting in poor BER performance as can be seen from the results in Fig. [5.7]. Hence, IM is not suitable for non-orthogonal schemes since the interference itself can result in erroneous index detection and subsequently the $M$-ary detection will be erroneous as well. This is why IM has been used for schemes like OFDM and FBMC/OQAM in literature which are both orthogonal. Our objective was to first utilize an inverse filter to cancel the intrinsic interference in FBMC/QAM system making it almost orthogonal and then combine IM with the proposed system to exploit the benefits of both the schemes. In the light of all the results, the improved performance of our proposed FBMC/QAM with IM compared to conventional FBMC/QAM systems makes it a suitable candidate for next generation wireless applications.

5.5 Conclusion

We have evaluated the performance of IM based QAM-FBMC system to highlight the potential of combining an emerging 5G modulation technique with our proposed FBMC/QAM system in Chapter [4]. We first derived a mathematical model of the IM based QAM-FBMC system along with the derivation of interference terms at the receiver due to channel distortions and the intrinsic behavior of the transceiver model. We have shown that the interference power in
FBMC/QAM-IM is smaller compared to that of conventional FBMC/QAM system as some subcarriers are inactive in IM based FBMC/QAM system. We then evaluated the performance of FBMC/QAM-IM in term of MSE and SINR and the results are compared with that of the conventional FBMC/QAM system. The results show that combining IM with FBMC/QAM can improve the system performance since the inactive subcarriers do not contribute to the overall interference in the system. The SIR performance of a FBMC/QAM block with and without IM is also presented. The results show the effect of interference on each FBMC/QAM symbol in a block. It can seen that the interference is higher for symbols in the middle of the block. At the end, BER performance of FBMC/QAM system with and without IM is presented and it can be seen that FBMC/QAM-IM has improved performance compared to conventional FBMC/QAM since the power from inactive subcarriers are reallocated to the active subcarriers.
Conclusion and Future Work

FBMC system has emerged as a promising alternative waveform candidate to replace conventional OFDM scheme to satisfy the requirements of future wireless networks. This is because FBMC offers the possibility of shaping subcarrier signals with waveform that is well localized both in time and frequency domains. This results into reduced OoBR due to steep side lobe decay, allowing a flexible spectrum usage and offering an increased resilience against time and frequency offsets, compared to conventional OFDM systems making it very suitable for asynchronous multi-service scenarios.

Despite various advantages, some key challenges in FBMC systems have been identified that needed attention to make it viable for future wireless applications. In this thesis we have addressed some of these critical challenges to make FBMC system a potential candidate waveform for future wireless networks. In what follows, the summary of the thesis is presented along with a general conclusion drawn from this research. At the end, some potential future directions along with some possible extensions to the current work are highlighted as future work.

6.1 Summary and Conclusion

This thesis has mainly focused on addressing some of the key challenges in FBMC systems to make it a potential waveform candidate for future wireless networks. We first provided a comprehensive literature review focused on FBMC systems. We start with a discussion on the main differences between OFDM and FBMC system along with the introduction on the principle of FBMC/OQAM modulation technique and the prototype filters used in this scheme. A discussion on the combination of FBMC system with technique like MIMO is presented along
with some related work in this particular area. Linear and widely linear processing techniques for MIMO-FBMC systems are then introduced. At the end, we discussed some relevant work related to the SE in FBMC systems, quadrature modulated symbol transmission and combination of IM with FBMC systems.

In Chapter 3, we first derived a compact mathematical matrix model of MIMO-FBMC system which laid the ground for the subsequent in depth analysis on the impact of finite filter length and different types of FOT on the system performance. It is shown numerically that FOT can overcome the overhead in time domain but also introduces extra interference in the received symbols and significantly degrade the SIR of the symbols at the edges. A compensation algorithm is then proposed to compensate all the symbols in a MIMO-FBMC block to improve the SIR values of each symbol for better detection at the receiver. The proposed model also identified the self-interference components (also known as intrinsic interference) in MIMO-FBMC system which serve as a barrier in combining FBMC with technique like MIMO and WLP. An intrinsic interference estimation and cancellation model was proposed based on the MIMO-FBMC matrix model to exploit the benefits of WLP. A two step receiver based on linear and widely linear processing is utilized for MIMO-FBMC system such that the output of the linear MMSE receiver is first used to estimate and cancel the intrinsic interference and a widely linear MMSE equalizer is then used to get improved estimation of the transmitted symbols.

In Chapter 4, we investigated the possibility of quadrature modulated symbol transmission in FBMC system. We started with the derivation of a matrix model of the QAM based FBMC system in the presence of additive noise and multipath channel. The interference terms at the receiver due to channel distortions and the intrinsic behavior of the transceiver model are also derived. Based on the proposed mathematical model of FBMC/QAM system, we proposed an inverse filter at the receiver to cancel the effects of self interference in FBMC/QAM block. It is shown with theoretical analysis that the introduction of inverse filter completely removes the intrinsic interference and significantly improves the system output SINR. A complexity analysis of the FBMC/QAM system with and without the inverse filter is also provided. It is shown that the receiver complexity in both cases have the same upper bounds. We then evaluated the performance of the proposed system in an asynchronous multi-service scenario and the performance is compared with conventional OFDM system. The FBMC/QAM system with the proposed inverse filter represented a viable solution for the 5G mMTC use case.

In chapter 5, we evaluated the performance of IM based QAM-FBMC system to highlight the potential of combining an emerging 5G modulation technique with our proposed FBMC/QAM system. We first provided a mathematical model of the IM based QAM-FBMC system along with the derivation of interference terms at the receiver due to channel distortions and the intrinsic behavior of the transceiver model. It is analytically shown that the interference power
in FBMC/QAM-IM is smaller compared to that of the conventional FBMC/QAM system as some subcarriers are inactive in FBMC/QAM-IM system. We then evaluated the performance of FBMC/QAM-IM in term of MSE, SIR and output SINR. The results are then compared with that of conventional FBMC/QAM system. It can be seen that combining IM with FBMC/QAM can further improve the system performance since the inactive subcarriers are not contributing to the overall interference in the system. The BER performance of FBMC/QAM-IM is also shown to be better than FBMC/QAM since the power from inactive subcarriers can be reallocated to the active subcarriers. On the other hand, the power from the inactive subcarriers can also be completely suppressed to achieve better power efficiency.

Currently the waveform choice for 5G enhanced mobile broad band (eMBB) and ultra reliable low latency communication (URLLC) is sub-band filtered OFDM [113]. However, the waveform for 5G massive machine type communication (mMTC) has not been finalized and we believe FBMC/QAM can be a promising candidate also companies like ZTE and Huawei are showing interest in this waveform as a potential multicarrier scheme for mMTC.

6.2 Future Work

This thesis has addressed some of the key challenges associated with filter bank based multicarrier systems. However, we have localized some areas of research that are not fully explored. In this context, we briefly highlight some potential research areas in this section that could be studied as an extension to the current research.

1. **FOT analysis for generic MIMO-FBMC system:** In chapter 3, we have proposed a compact matrix model of MIMO-FBMC system which was subsequently used for the in depth analysis of the effect of FOT on the detection performance. However, the proposed matrix model is limited to the case when transmitter and receiver have same number of antennas. The analysis of chapter 3 could be extended to the generic MIMO-FBMC scenario, i.e. when transmitter and receiver have different numbers of antennas.

2. **Noise enhancement in FBMC/QAM system:** In chapter 4, we have shown that the effects of intrinsic interference can be removed using an inverse filter at the receiver side. Although the proposed system significantly improves the output SINR of the system, this method suffers from noise enhancement. This noise enhancement might be a significant problem at low SNR values. Hence, a proper filter design is required to counteract the noise enhancement problem and to make the system suitable for low SNR cases.
3. **FBMC/QAM system in high mobility scenario:** It would be interesting to evaluate the performance of FBMC/QAM system in high mobility scenarios e.g., vehicle-to-everything (V2X) applications.

4. **Window optimization for FBMC/QAM systems:** For FBMC/QAM system, it is possible to use windowing instead of per subcarrier filtering to enhance the orthogonality of the system. It could be interesting to investigate a joint design of window functions to reduce OoBR and ISI in FBMC/QAM system.

5. **FBMC/QAM with NOMA:** Non-orthogonal multiple access (NOMA) is an emerging technology for future cellular systems in order to accommodate more users via non-orthogonal resource allocation. The combination of FBMC/QAM system with NOMA schemes such as low density signature (LDS) and sparse code multiple access (SCMA) could be an interesting area of research. In this regard, ZTE has shown interest in combining our proposed FBMC/QAM system with their state of the art multiuser shared access (MUSA) technique.

6. **Soft detector for FBMC/QAM-IM system:** In chapter 5, we propose an improved LLR detector based on our interference analysis. It was shown that the proposed LLR detector shows similar performance as optimal ML detector. However, the proposed detector provides hard detection of the index bits and therefore cannot work well with coding since the index bits have no soft information to achieve a significant coding gain. In order to exploit channel coding gains in FBMC/QAM-IM system, soft information from the $M$-ary symbols as well as the index bits are required. In order to obtain soft index bits, an exact LLR detector is required. However, the design of LLR detector for IM based multicarrier system can be complex since higher modulation schemes and the combination of active/inactive subcarriers could result in large number of possible metric calculations to obtain soft information. It would be interesting to look at a low complexity LLR soft detector for FBMC/QAM-IM system as a part of future work.
Bibliography


[34] M Bellanger et al., “FBMC physical layer: a primer”.


