Impact of removing nodes on the controllability of complex networks

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1 Introduction

Complexity theory has been used to study a wide range of systems in biology and nature but also business and socio-technical systems, e.g., see [1]. The ultimate objective is to develop the capability of steering a complex system towards a desired outcome. Recent developments in network controllability [2] concerning the reworking of the problem of finding *minimal control configurations* allow the use of the polynomial time Hopcroft-Karp algorithm instead of exponential time solutions. Subsequent approaches build on this result to determine the precise control nodes, or drivers, in each minimal control configuration [3], [4]. A browser-based analytical tool, *CCTool*[1], for identifying such drivers automatically in a complex network has been developed in [5].

One key characteristic of a complex system is that it continuously evolves, e.g., due to dynamic changes in the roles, states and behaviours of the entities involved. This means that in addition to determining driver nodes it is appropriate to consider an evolving topology of the underlying complex network, and investigate the effect of removing nodes (and edges) on the corresponding minimal control configurations. The work presented here focuses on arriving at a classification of the nodes based on the effect their removal has on controllability of the network.

2 Results

Methodology. We consider three categories in terms of cardinality of the maximum matching, C_{MM}: a node is *delete-redundant*, iff C_{MM} is unchanged; *delete-ordinary*, iff C_{MM} is reduced by one; and, *delete-critical* iff C_{MM} is reduced by more than one.

We applied the following adaptation of the algorithm found in [6], [2], [5] for classifying nodes based on the effect their removal has on controllability of the network.

1. Find one maximum matching of the graph $G(V,E)$ by applying Hopcroft-Karp.
   - Convert the graph into a bipartite graph: $G_b\ (V_+, V_-, E)$ where V_+ is the out-set, and V_- the in-set
   - Run the Hopcroft-Karp algorithm on the bigraph G_b to find a maximum matching (denote as MMG) and denote the size of MMG as CG

[1] Available at: http://cctool.herokuapp.com

2. Obtain sets of matched nodes and S-matched nodes of G with respect to MMG
   - A set of matched nodes in V_- is one of matched nodes of G (denote as M_-)
   - A set of matched nodes in V_+ is one of S-matched nodes of G (denote as M_+)
   - Denote a set of matched or S-matched nodes of G as M ($M = M_+ \cup M_-$)
   - Nodes that are not contained in the set M are not S-redundant and are therefore delete-redundant
3. Pick one node in M and identify the category of the node
   - Pick one node n in M and denote the node in V_+ as n_+, and in V_- as n_-
   - Create a subgraph S_b by removing all edges incident with node n_+ and n_-
   - Create a matching MS by removing matched edges incident with nodes n_+, n_-
   - Find a maximum matching of S_b and denote the size as CS
     - Find an augmenting path with regards to MS in subgraph S_b
     - If there is an augmenting path, use it to augment MS
     - Once more, find an augmenting path and augment MS
     - Augmented matching MS is a maximum one of S_b
   - Identify the category of the node n
     - If CS = GS then n is delete redundant
     - If CS = $GS - 1$ then n is delete ordinary
     - If CS = GS - 2 then n is delete critical
4. Repeat step 3 until all nodes in M are identified by using MMG and G_b

where a matched node is the end point of a matched link; an S-matched node is the start point of a matched link; an S-redundant node is a node that is always a matched node or an S-matched node, in all maximum matchings; and, a node is a delete redundant node if and only if the node is not an S-redundant node.

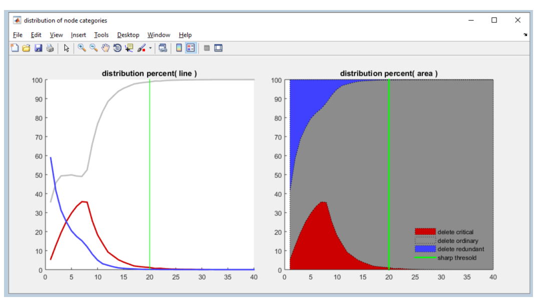

Fig. 1. Distribution of node categories ($N = 1000$)

Summary. Fig.1 shows the distribution of the different categories of nodes considered. If the edge probability increases, all nodes tend to be delete-ordinary, while delete-redundant nodes disappear gradually. When the edge probability exceeds a threshold, the fraction of delete-critical nodes, which was increasing, starts to rapidly decrease.

Fig.2 shows the initial state of the network (top left) and the effect of removing different categories of nodes. Our experiments showed that when 3 delete-critical

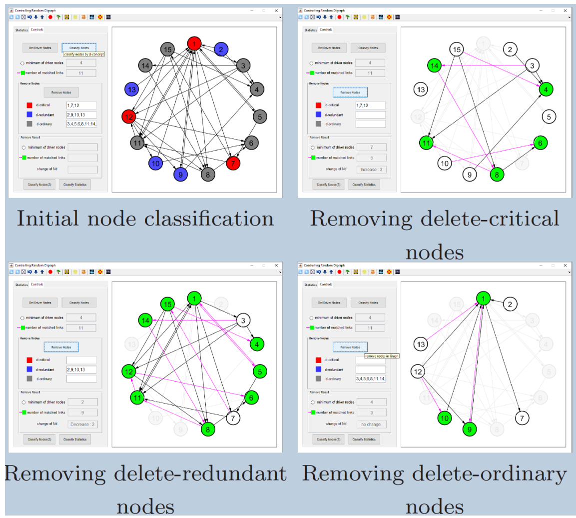

Fig. 2. Node classification based on impact on network controllability

nodes are removed the number of driver nodes increases by 3. When 4 delete-redundant nodes are removed the number of driver nodes increases by 2 while removing 8 delete-ordinary nodes results in no change in the number of driver nodes.

In terms of robustness of the node categories devised, we note the following results.

When some nodes are removed from the network, all node classifications are stable with delete-redundant nodes being the most unstable. When the edge probability increases, the delete-ordinary nodes are the most stable. When 10% of the nodes are removed from the network, then over 70% of the nodes' categories are not changed. Also, the delete-ordinary category is the most stable one.

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