COMPUTER AIDED PROCESSING
OF GEODESIC STRUCTURAL FORMS

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TO MY MOTHER, WIFE, CHILDREN, BROTHERS, SISTERS
AND ALL MY FRIENDS
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ABSTRACT

Geodesic domes constitute an important family of braced domes offering high degree of regularity and evenness in stress distribution. Data preparation and handling of graphics for geodesic forms are difficult and time consuming tasks and are the stages of analysis where mistakes are most commonly made. In the past, this problem was solved by using already prepared tables of data. However, such a style of presentation is not very convenient.

The present work is concerned with finding a way to prepare data and create configurations for different types of geodesic configurations. Formex algebra and its programming language Formian provide good tools for this purpose. The profile of these configurations can be a circle, a supercircle, an ellipse, a superellipse, a parabola, a hyperbola or a conical section. The floor cross section of these configurations can be circular, supercircular, elliptical or superelliptical. Combinations of different profiles and floor cross sections will result in a wide range of configurations. The generator volume can be a tetrahedron, a cube, a truncated octahedron, an octahedron or an icosahedron.

A new approach of creating geodesic configurations without using generator volumes is introduced and discussed. In this context, the idea of geometric potential function minimisation is used to obtain regular configurations from a simple starting distribution of nodes on spherical and elliptical surfaces.
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CHAPTER ONE

INTRODUCTION

Architect and engineers have been excited about the possibilities of space structures for the past thirty years. They offer opportunities for variation in plan form and building profile, large uninterrupted spans, excellent distribution of loads, optimum utilisation of materials and prefabrication and mass production of easily transportable components. Now that the computers are readily available to handle the complex calculations, there should be little impediment to the widespread use of these interesting structures.

In a computer aided analysis and design involving complex configurations, data preparation and handling of graphics are difficult and time consuming tasks. In addition, data preparation is the stage of analysis that is very much prone to mistakes. The concept of formex algebra together with its computer language Formian provide a basis for solution of these problems, Refs 1 and 2.

Domes and shells have an outstanding role in modern construction. They can be employed for various complexes of structures used for various social programs: sports, exhibitions, etc. Domes and shells are employed not only for covering large spans for major buildings but also for spanning small auxiliary buildings and structures intending for catering people: restaurants, cafes, trade pavilions, etc.

Geodesic domes constitute an important family of domes. The aesthetic aspects, the evenness of stress distribution in these domes and their ease of fabrication and erection
make them ideal for many large constructions. A major problem for the designers in this field relates to data generation. In the past, this problem was solved by preparing tables of data which required a tremendous amount of effort. Dimensional characteristics of several varieties of geodesic domes in the form of tables of trigonometrical parameters were given in a number of papers published in the period of the last 30 years. However, such form of presentation is not very convenient for practical use, Ref 3.

Spreading of computers and their availability make it easy to use automated data generation in the field of geodesics. Data includes topological properties, geometrical properties, load and support information and material properties.

It is the objective of the present work to find a way to prepare data and create configurations for different types of geodesic forms.

Although geodesic subdivision is a good way to obtain a regular spherical dome, it has some drawbacks such as the difficulty in truncating the dome at some intended support level without disturbing its regularity and also the restrictions imposed on the designer by being forced to use some of the already known configurations with little chance to introduce new ideas. A new approach to achieve regularity on domic surfaces without being confined to the traditional ways of subdivision was introduced by J W Butterworth, Ref 4. The idea is to place any number of nodes on a certain predetermined surface or portion of a surface and then allow these nodes to move on the surface by the effect of repulsion forces between them until equilibrium is attained. Some of these nodes can be fixed at certain positions such as the supports or entrances. The problem reduces to obtaining a minimum of a function
called the geometric potential function.

This study is concerned with developing a method for geometric potential function optimisation and use it to obtain spherical or ellipsoidal geodesic configurations by placing a number of points on the required surface or part of a surface. Geometric potential function minimisation method was used also to obtain configurations with more regular shapes than the classical geodesic forms.

The material in this study is organised as follows:

Chapter 2 gives a comprehensive review of the history of geodesic domes, their development, generation of geodesic configurations from regular polyhedra (Platonic polyhedra) and semi-regular polyhedra (Archimedean polyhedra), various methods of subdivisions, advantages and disadvantages of various types of geodesic domes.

Chapter 3 gives an introduction to the basis of formex algebra and its programming language Formian. It covers the fundamentals of formex algebra, formex graphics, formex functions, primitive constituents of Formian, functions and function designators in Formian, information transfer statements, Formian graphics, schemes and induction statements.

In chapter 4 the idea of formex algebra and its programming language Formian are used to develop an easy and reliable way for preparing data and creating configurations for various types of geodesic domes. Five principal polyhedra were used as generator volumes. These polyhedra are tetrahedron, octahedron, truncated octahedron, cube and icosahedron. The profile of the geodesic configurations covered in this chapter can be a circle, a supercircle, an ellipse, a superellipse, a parabola, a hyperbola or a conical section. The cross section of these configurations
can be circular, supercircular, elliptical or superelliptical. Various combinations of profile and floor cross section can be obtained. Combining profiles and cross section of different elliptical shapes will result in a configuration of varying radii. The term elliptical is used throughout this work in a general sense to include circular. The idea of schemes and subroutines were used and by just changing few parameters in the schemes a wide range of geodesic configuration was obtained.

Chapter 5 introduces the idea of geometric potential function minimisation to obtain regular configurations from a simple starting distribution of nodes on certain surfaces. The 'Direct Search Method of optimisation' by Hooke and Jeeves, Ref 5, was used and proved to suit the problem. Different numbers of nodes were placed on constrained domic surfaces with different starting values for the coordinates and different constraint conditions. Five different configurations were studied. These configurations are: 16-node configuration, 21-node configuration, 29-node configuration, 31-node configuration and 46-node configuration. The surface on which these nodes were supposed to lie could be spherical or elliptical.

Finally, chapter 6 states the main conclusions reached in this work. It, also, includes some suggestions and recommendations for future work.
CHAPTER TWO

GEODESIC DOMES

2.1 INTRODUCTION

Domes are of special interest to engineers and architects as they enclose a maximum amount of space with a minimum surface and have proved to be very economic in the consumption of construction materials. Domes are also exceptionally suitable for covering sports stadia, assembly halls, exhibition centres, swimming pools and industrial buildings in which large unobstructed areas are essential and where minimum interference from internal supports is required, Ref 6.

A dome is a typical example of a synclastic surface in which the curvature of any point is of the same sign in all directions. The synclastic surfaces are also called surfaces of positive Gaussian curvature and are not developable, that is, a domic surface cannot be flattened into a plane without stretching or shrinking it. This property is one of the reasons why, in practice, domes cannot be built from members of the same length. Most domes built in practice have a surface which can be generated by the rotation of a plane curve about a vertical line. The rotating curve is called its meridian and the horizontal sections are known as the parallels. Any curve can be used as a meridian; a circle gives rise to a sphere, an ellipse gives rise to an ellipsoid of revolution and a parabola to a rotational paraboloid. Theoretically it is also possible to produce domic surfaces generated by translation. This happens when a plane curve slides on another plane curve, usually at right angles to it and at the same time remaining parallel to itself. Since any combination of curves may be used, a large variety of
surfaces are obtainable by translation, but in civil engineering practice very few skeleton domes have been built having a translational surface.

The development of domes has been closely associated with the development of available materials. In the past, domes were built in stone; brickwork gradually replaced the stone masonry. Timber was the principal roofing material used in the Middle Ages. The introduction of iron opened up an exciting new era for the structural engineer and the architect because of its relatively high strength and comparatively light weight. The introduction of steel, with its greatly improved properties of high strength has a fundamental influence in the development of various types of braced dome and their use for large spans. Reinforced concrete shell domes have been also used for covering large spans, but the drawbacks of this type of construction are the requirement of elaborate and very expensive formwork and that they are slow in construction and often not really economic in their final cost. Nowadays the advantages of steel, aluminum alloys and also glass fibre reinforced plastics were appreciated and are fully used in metal-braced domes.

Domes can be subdivided into four main groups:

(a) frame or skeleton-type single-layer domes;
(b) truss-type domes and double-layer domes;
(c) stressed skin type domes (in which the covering is connected to the bracing members and forms an integral part of the main load carrying structural system);
(d) formed surface type domes (in which thin steel, aluminum or glass reinforced plastics sheets are bent and interconnected along their edges to form the main skeleton system of the dome).

The main types of braced dome frequently used in practice are:
(a) ribbed domes
(b) Schwedler domes
(c) grid domes
(d) parallel-lamella domes
(e) geodesic domes, Ref 6.

2.2 GEODESIC DOMES

A geodesic line on a sphere is the shortest distance between two points on surface the of sphere. Spherical great circles are geodesics. A great circle is a circle on a sphere whose centre is common to that sphere. Any circle which is not a great circle is a small circle.

Richard Buckminster Fuller, the inventor of geodesic domes, has made a phenomenal impact on architects and engineers since 1954. Nature, said Buckminster Fuller, always builds the most economic structures. He claimed that geodesic domes based on mathematical principles embodying force distributions similar to those found in atoms, molecules and crystals will be the lightest, strongest and cheapest construction ever made. The term 'geodesic' given by Buckminster Fuller is related to division of earth into great circles which are the geodesic lines of the sphere.

Geodesic domes consist of a network of framing members which make a more or less uniform pattern over the whole surface. It could be built with curved members which could lie along geodesic curves and thus be a portion of a true surface of a circle or an ellipse or a paraboloid, but it is usually built as a polyhedron with straight members which form the chords of geodesic arcs.

If one attempts to cover a sphere, with such a network, certain basic principles must be observed. Since the triangle is the simplest polygon and also the only one which is rigid in itself, the network will usually consist
of triangles. These form larger configurations, depending on how many triangles meet at a point or vertex. If six equilateral triangles meet on a plane surface, they form a regular hexagon. This is impossible on a sphere because the sum of the angles must be less than $360^\circ$ around the vertex. On the sphere, therefore, all the members cannot be of the same length and the hexagons formed cannot be regular. Even if the pattern is made up of irregular hexagons, no matter how distorted, it is impossible to cover a complete sphere with them. A minimum of 12 pentagons must be introduced.

The basic possibilities and limitations of this type of framework are given by studying all the regular and semi-regular polyhedra and their duals.

2.3 GENERATION OF GEODESIC DOMES

The generation of geodesic domes is based either on regular (Platonic) polyhedra or semi-regular (Archimedean) polyhedra.

2.3.1 THE PLATONIC POLYHEDRA

These polyhedra were not discovered by Plato, but they have been so named because of the study he and his followers made of them. There are five and only five Platonic polyhedra. Each polyhedron is convex and has an equal number of similar regular, convex faces. Platonic polyhedra can have a sphere inscribed within them touching each face in its centre, called the insphere, or can have a sphere circumscribed about them, passing through each vertex, called the circumsphere, a third sphere touching the midpoint of each edge is called the intersphere (or midsphere). These points of tangency or vertices are the only regular systems of points which are equidistant from each other on the surface of a sphere. It can be seen that
all pairs of adjacent faces of a particular polyhedron meet at a constant angle. That angle, called dihedral angle, is measured on the inside of the polyhedron and is always less than 180°. The fact that the dihedral angle is constant is an important characteristic of a regular polyhedron.

The five Platonic polyhedra are:

(a) Tetrahedron This can be described as a solid having 4 triangular faces, 4 apexes and 6 edges or simply: 
F = 4; 3, apexes 4 and edges 6.
(b) Cube 
F = 6; 4, apexes 8 and edges 12.
(c) Octahedron 
F = 8; 3, apexes 6 and edges 12.
(d) Dodecahedron 
F = 12; 5, apexes 20 and edges 30.
(e) Icosahedron 
F = 20; 3, apexes 12 and edges 30.

These Platonic polyhedra are shown in Fig 2.3.1.

2.3.1.1 DUALITY AND THE PLATONIC POLYHEDRA

The octahedron and the cube have the same number of edges, 12, and each polyhedron has the same number of faces as the other has vertices. A suitably-sized model of each of them can be arranged about a common intersphere so that their edges cross at right angles and touch that sphere at those 12 common points, as shown in Fig 2.3.2a. The vertices of each polyhedron lie exactly outside the face centres of the other polyhedron. A similar arrangement can be made with a suitable-sized icosahedron and dodecahedron, since each of those polyhedra has 30 edges and the same number of faces as the other has vertices, Fig 2.3.2b. The remaining Platonic polyhedron, the tetrahedron, has the same number of faces as it has vertices, so two tetrahedra of equal size can be arranged in a similar way to the other pairs of polyhedra, Fig 2.3.2c. The 8 vertices of the two tetrahedra define the vertices of a regular cube. This phenomenon is known as duality, and each polyhedron is
Projections of the Platonic Polyhedra

<table>
<thead>
<tr>
<th>Tetrahedron</th>
<th>Octahedron</th>
<th>Cube</th>
<th>Icosahedron</th>
<th>Dodecahedron</th>
</tr>
</thead>
<tbody>
<tr>
<td>Face View</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Vertex View</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Edge View</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Data for the Platonic Polyhedra

<table>
<thead>
<tr>
<th></th>
<th>Tetrahedron</th>
<th>Octahedron</th>
<th>Cube</th>
<th>Icosahedron</th>
<th>Pentagonal Dodecahedron</th>
</tr>
</thead>
<tbody>
<tr>
<td>Faces</td>
<td>4 triangles</td>
<td>8 triangles</td>
<td>6 squares</td>
<td>20 triangles</td>
<td>12 pentagons</td>
</tr>
<tr>
<td>Vertices</td>
<td>4</td>
<td>6</td>
<td>8</td>
<td>12</td>
<td>20</td>
</tr>
<tr>
<td>Edges</td>
<td>6</td>
<td>12</td>
<td>12</td>
<td>30</td>
<td>30</td>
</tr>
<tr>
<td>Radius of Circumsphere</td>
<td>-612.4</td>
<td>-707.1</td>
<td>-866.0</td>
<td>-951.1</td>
<td>14013</td>
</tr>
<tr>
<td>Radius of Intersphere</td>
<td>-353.6</td>
<td>-5000</td>
<td>-707.1</td>
<td>-609.0</td>
<td>13090</td>
</tr>
<tr>
<td>Radius of Insphere</td>
<td>-204.1</td>
<td>-408.2</td>
<td>-5000</td>
<td>-755.8</td>
<td>11135</td>
</tr>
<tr>
<td>Dihedral Angle</td>
<td>70°32'</td>
<td>105°28'</td>
<td>90°0'</td>
<td>138°11'</td>
<td>116°34'</td>
</tr>
</tbody>
</table>

Radii of circumspheres, interspheres and inspheres are given (to four decimal places) in terms of the edge lengths of the polyhedra. Dihedral angles are given to the nearest minute.

Fig 2.3.1 Platonic polyhedra.
the dual of the other polyhedron that shares the same intersphere in this way. An important characteristic of a regular polyhedron is that the dual of a regular polyhedron is a regular polyhedron, Refs 7 and 8.

2.3.2 THE ARCHIMEDEAN POLYHEDRA

The Archimedean polyhedra can be produced from Platonic polyhedra by relaxing the condition that all the faces meeting at each vertex should be identical regular polygons. There are fifteen Archimedean polyhedra, with similar arrangements of regular, convex polygon of two or more different kinds about each vertex of a polyhedron. All edges are of the same length and every face is a regular polygon, but all the faces are not identical. The vertices are all congruent (identical) but not regular (the angles between pairs of edges are not all the same). These polyhedra can have a sphere circumscribed about them passing through each vertex.

The classification of the Archimedean solids is as follows:

(a) Truncated tetrahedron  \( F = 4;3+4;6\), apexes 12 and edges 18

(b) Truncated cube  \( F = 8;3+6;8\), apexes 24 and edges 36

(c) Truncated octahedron  \( F = 6;4+8;6\), apexes 24 and edges 36

(d) Truncated dodecahedron  \( F = 20;8+12;10\), apexes 60 and edges 90

(e) Truncated icosahedron  \( F = 12;5+20;6\), apexes 60 and edges 90.

(f) Semi-regular prism  \( F = 2;n+n;4\), apexes 2n and edges 3n.

(g) Rhombicuboctahedron  \( F = 8;3+18;4\), apexes 24 and edges 48.

(h) Semi-regular prismoid  \( F = 2;n+2n;3\), apexes 2n and edges 4n.
Fig 2.3.2a
octahedron and cube

Fig 2.3.2b
icosahedron and dodecahedron

Fig 2.3.2c
two tetrahedra

Fig 2.3.3

Fig 2.3.4

Fig 2.3.5

Fig 2.3.5
(i) Cuboctahedron $F = 8; 3+8; 4$, apexes 12 and edges 24
(j) Icosidodecahedron $F = 20; 3+12; 5$, apexes 30 and edges 60.
(k) Snub-cube $F = 6; 4+32; 3$, apexes 24 and edges 60.
(l) Snub-dodecahedron $F = 12; 5+80; 3$, apexes 60 and edges 150.
(m) Rhombicosidodecahedron $F = 20; 3+12; 5+30; 4$, apexes 60 and edges 120.
(n) Truncated cuboctahedron $F = 12; 4+8; 6+6; 8$, apexes 48 and edges 72.
(o) Truncated icosidodecahedron $F = 30; 4+20; 6+12; 10$, apexes 120 and edges 180.

The vertices of the Archimedean duals do not fall on a sphere, but a sphere tangent to every face at its centre can be inscribed with each dual. An important characteristic of the Archimedean polyhedra is that any Archimedean polyhedron can be circumscribed by a regular tetrahedron so that four of its faces lie on the faces of that tetrahedron, Refs 6, 2 and 3.

2.3.3 THE GOLDEN PROPORTION AND THE PLATONIC POLYHEDRA

Consider the series of numbers

\[(1, 1, 2, 3, 5, 8, 13, 21, \ldots)\].

This series can be produced by adding the last two members in the series to give the next one in the series. The next member in the sequence given would be 34 and the subsequent one would be 55. A property of this series is that, if two adjacent numbers are expressed as a ratio (such as 8:13, 13:21, 21:34), the larger the numbers are, the closer the ratio comes to a value of...
or \((\sqrt{5}+1)/2\). This proportion is known as the Golden Proportion. The Golden Proportion is the basis for many interesting relationships. For example, if a line is divided into two parts so that one part is 1.618 times longer than the other part, Fig 2.3.3, it will be found that the overall length of the two parts is 1.618 times the length of the longer part, that is,

\[ a:b = 1:1.618 = b:(a+b). \]

A rectangle whose longer sides are 1.618 times the lengths of its shorter sides is called a Golden Rectangle.

Isosceles triangles whose internal angles are 36°, 72°, 72° or 108°, 36°, 36° have edges whose lengths are in Golden Proportion. Since the diagonal of a regular pentagon divides off 108°, 36°, 36° triangle, the diagonal is 1.618 times the edge length of that pentagon.

A regular decagon can be constructed inside the circle whose radius is 1.618 times the edge length of that decagon, Fig 2.3.4. The ten outside edges of a pentagram (a five-pointed star produced by extending the edges of a regular pentagon) are 1.618 times the length of the edges of the original pentagon, Fig 2.3.5.

Though the preceding examples are two-dimensional, there are some important three-dimensional relationships involving the Golden Proportion and the Platonic polyhedra. Fig 2.3.6 shows a Golden Rectangle arranged inside an icosahedron, the shorter edges following opposite edges of the icosahedron and the longer edges forming diagonals to pentagonal circuits of edges. Three such Golden Rectangle, intersecting at right angles can be constructed inside the icosahedron as in Fig 2.3.7. Since the dodecahedron is the
dual of the icosahedron, similar arrangements of Golden Rectangles can be made within it. In this case the corners of the rectangles touch the midpoints of faces of the dodecahedron instead of the vertices, as was the case with the icosahedron. Fig 2.3.8 shows three Golden Rectangles, intersecting at right angles, whose twelve corners touch the midpoints of the twelve faces of the dodecahedron. A diagonal can be drawn across each pentagonal face of a dodecahedron to define the twelve edges of a regular cube. Clearly, the edges of the cube will be 1.618 times the length of the edges of the dodecahedron.

If each edge of a regular octahedron is divided in the Golden Proportion as in Fig 2.3.9a and those points are joined a regular icosahedron is described as in Fig 2.3.9b. It can be noted that the cube and the dodecahedron share common vertices and that their duals, the octahedron and the icosahedron shares common facial planes Ref 8.

2.3.4 SOME PROPERTIES OF THE POLYHEDRA

(a) The sum of the face angles of a polyhedra = 360*V-120 where V is the number of vertices. The discovery of this relationship has been attributed to Rene' Descartes and it holds true for all polyhedra.

(b) Euler's formula states that in any convex polyhedron the number of faces (F), the number of vertices (V) and the number of edges (E) are related by:

\[ F + V - 2 = E \]

2.4 METHODS OF SUBDIVISION

The polyhedron used as a basis for the geodesic dome is called the principal polyhedron or the generator volume. There are many ways to divide the faces of a principal polyhedron to obtain a suitable face size. All faces must
be triangulated before being divided and then all vertices are pushed outward till they are a common distance from the centre (for sphere) or a harmoniously varying distance (for ellipsoid and other shapes). The aim of the subdivision process is to obtain a reasonably symmetrical spacing, so that the triangles created will be nearly equilateral. The number in which the polyhedron edge is divided is called the frequency of subdivision. Many methods of subdivision exist, some of these methods are discussed below.

2.4.1 CLASS I (ALTERNATE) BREAKDOWN

In this breakdown process the grid lines run roughly parallel to the edges of the polyhedron face. There are a number of different techniques to specify the cross points. One has to choose between similarity of triangle size, or similarity of triangle shape.

(i) Class I Method 1
In this method the flat triangle face edges are divided equally, then the division points are projected outward to obtain equal radii for spherical configuration or the required radii in case of other configurations. This method results in unequal chords, and hence the sighting angles from the centre to the chord are unequal.

(ii) Class I Method 2
This method simply spaces the $\theta$ coordinates equally down the edge, making all the edge elements equal.

Fig 2.4.1 shows the two methods of breakdown on an octahedron face for a 3 frequency breakdown.

It is clear from the methods above that by letting the triangle sizes vary more we can keep their shapes more
Fig 2.4.1 Class I subdivision.
nearly constant. Method 1 is the choice for class I breakdown.

2.4.2 CLASS II (TRICON) BREAKDOWN

The grid lines run roughly perpendicular to the edges of the polyhedron face, Fig 2.4.2. It may be noticed that this class does not really breakdown the polyhedron face from which it derives its frequency count but a smaller and nonequilateral triangle which consists of two polyhedron symmetry triangles back to back. It arose as a method of incorporating the great circles which coincide with the face triangle medians. Triangles which are obtained by drawing the three medians of the face of a polyhedron are called symmetry triangles. Class II triangle is obtained by drawing the medians of the two adjacent faces. If we continue to subdivide the entire polyhedron and then eliminate polyhedron edges themselves, we have a 2 frequency class II breakdown made up entirely of triangles like ADE. Only even frequency subdivision is possible and for a certain frequency on a polyhedron only half the frequency is used in the subdivision of the class II triangle. The nickname tricon came from the fact that the diamond ADEC made up of the triangles is one face of spherical rhombic tricontahedron.

There are two mostly used methods for this class

(i) Class II Method 1

This method in effect divides the diamond median exactly as in class I method 1 to yield equal parts on flat papers but unequal chords when the vertices are pushed out to make equal spherical radii. We then concentrate on one half of the diamond, the class II triangle, and run perpendiculars from medians outward to the sides. The triangle's bottom edge is now subdivided in the way its median was, and then
Fig 2.4.2 Class II subdivision.
lines join these edges to complete the grid. This breakdown is a little more economical in components than class I.

(ii) Class II Method 3

This method permits an absolute minimum member length inventory, 4 members for 4 frequency, 10 for 10 frequency, 12 for 12 frequency and so on. Its economy is paid for by accepting less symmetrical breakdown triangles. The principle of this method is shown in Fig 2.4.3. The horizontal members cross the class II triangle’s median at equally spaced intervals, and the bottom edge point divisions are directly below the corresponding side edge divisions (broken lines are at 90° angles with the bottom edge). It now turns out that each side edge will propagate itself by zigzagging all the way down the main triangle, and each horizontal division will recur as a set of parallel reflections. The figure shows members of eight lengths for 8 frequency. The chord factors shows asymmetries that result. Comparing the saving in different member lengths with other methods we find that, this method requires 12 lengths for 12 frequency whereas a class II method 1 of the same frequency requires 32 and a class I method 1 requires 40. In case of varying radius as for ellipsoids, method 3 has no advantage over method 1.

2.4.3 CLASS III

In this class of subdivision the grid lines run skew to the edges of the polyhedron face. It seems that it is very difficult to determine the coordinates of this grid, but in fact it is not so because it can be obtained from those coordinates of class I subdivision that share common points of intersections. Joseph D. Clinton, Ref 9, shows how this could be obtained.
METHOD 3 BREAKDOWN OF
CLASS II TRIANGLE, 8°.

SYMMETRIES
(LEFT HALF)

CHORD FACTORS

a 0.170287
b 0.199457
c 0.167003
d 0.195678
e 0.161026
f 0.184914
g 0.153316
h 0.168672

Values decrease
toward bottom
left (and, by
symmetry, toward
bottom right).

Fig 2.4.3 Class II subdivision, Method 3.
2.4.3.1 LOWEST COMMON FREQUENCY

Joseph D Clinton, Ref 9, shows that if the class I coordinates are known then the class II and class III geometries of the same method can be found. Using the notion \([p,q']_{b,c}\), Fig 2.4.4, we can classify the complete family of spherical tessellations useful for structural applications as follows:

Class I subscript \(b\) is an integer and \(c\) is always zero or \(c\) is an integer and \(b\) is always zero.

Class II subscript \(b\) is any integer and \(c\) is always equal to \(b\).

Class III subscript \(b\) and \(c\) can be any integer as long as \(b\) is not equal to \(c\) or either \(b\) or \(c\) is not equal to zero.

And for any class, \(p\) = face shape (3 triangular)
\(q\) = number of faces at the vertex of the polyhedron (5 icosahedron)
\(b\) and \(c\) are counters
where \(b + c = NF\) (frequency).

If the lowest common frequency, LCF, is defined as the frequency of a \([p,q']_{b,0}\) polyhedron that shares common coordinates with the \([p,q']_{b,c}\) polyhedron, then LCF can be found as

\[\text{LCF} = b^2 + b*c + c^2\]

where \(b*c\) is from the \([p,q']_{b,c}\) polyhedron and \(\text{LCF} = b\) of the \([p,q']_{b,0}\) polyhedron. If \(b = c\), that is, class II, then LCF can be found from the formula

\[\text{LCF} = 2(3*b^2) / NF\]

and since \(NF = b + c = 2*b\) then
\[\text{LCF} = 3*b.\]

Fig 2.4.5, shows the lowest common frequencies for some class III configurations.
Fig 2.4.4 Notion $[p, q']_{b,c}$
Fig 2.4.5 Lowest common frequencies for some configurations.
2.5 DEVELOPMENT IN SUBDIVISION METHODS

2.5.1 THE METHOD OF SECONDARY GRID DEFORMATION

The mapping properties of the base plane to the corresponding spherical surface in the central and orthogonal projection have been used in the construction of this method, Ref 10. In the central projection equilateral triangular on the plane face of a regular Platonic polyhedra is mapped into a spherical grid of different sizes of triangular fields. The way used to determine the picture of a triangular grid on the plane face of the polyhedron determines the shapes and sizes of the spherical triangles after the grid projection from the sphere centre. This method tries to find a deformed triangular grid on the plane face of regular polyhedron used as a principal polyhedron which gives a better size and shape of the spherical triangular grid after its projection.

The icosahedron is used to introduce the concept of the method. The angle included between two adjacent vertices and the centre of the sphere circumscribed about this basic polyhedron is $w = 63° 26'$, Fig 2.5.1. In the first procedure step of determining the deformed triangular grid, it is necessary to divide the arc of the great circle of the sphere into $n$ equal segments. Fig 2.5.1b presents the icosahedron face view with the section of the sphere with the plane passing through the vertices A and B and the centre of the sphere Ok. The segments, focused in the point Ok, run from the points of the equal division of AB arc. The intersection points of these segments with the edge AB determine new arrangement appropriate for the principles of the central projection. Since the triangle ABC is equilateral we put this new arrangement on its two remaining edges, marking them with appropriate numbers. The points of the same ordinal number are connected with
Fig 2.5.1 Secondary grid deformation.
each base of the triangle ABC by the parallel segments. Then it is necessary to determine the centres of gravity of the triangular fields included between the appropriate sets of segments. These centres of gravity determine the node position of the secondary triangular grid which is shown in Fig 2.5.1c. Deformation of this plane grid appropriate for the central projection, Fig 2.5.1d, makes it possible to assume that after its projection from the centre of the polyhedron we will get a spherical grid of a small degree of size difference of its triangular fields. This method can be used to calculate the lengths of the spherical grid edges spaced over different forms of triangular planes of the base.

The same procedure can be conducted making use of the properties of the orthogonal projection onto the plane. Fig 2.5.2 shows the deformation scheme of the grid on the plane face of a regular icosahedron. Due to the properties of the orthogonal projection, a spherical equilateral triangle determined by this projection, has a smaller surface than the equilateral triangle determined by means of a central projection over the same base shape. In this case it is necessary to determine the position of circle centre Ok1 and its radius, the circle being the common part of the sphere and the plane perpendicular to the plane of the base comprising the edge, that is, BC, Fig 2.5.2a. The angle included between the vertices B and C and the circle centre Ok1 is $\phi = 66^\circ 58'$. The points of the arc BC division into n equal segments determine a new arrangement of the triangular sides of the base ABC in the orthogonal projection. Localization of the nodes of the deformed grid consists of, as previously stated, determining the gravity centres of the appropriate triangular fields, Fig 2.5.2b. The broken curves inside the triangle ABC are directed with their convexities to the opposite vertex. The degree of such a deformation depends on the distance of the base.
Fig 2.5.2 Secondary grid deformation using orthogonal projection onto the plane
plane to its corresponding part of the sphere. It will be greater for the planes situated near the centre of the sphere.

2.5.1.1 COMPARISON OF GRIDS ON BASIS OF ICOSAHEDRON

Fig 2.5.3 shows the dimension relations of the icosahedron obtained by the secondary grid deformation method using the principles of the central projection and the orthogonal projection onto the plane. Also the chord factors of the icosahedral geodesic dome using the conventional class I method of subdivision is shown in Fig 2.5.3c. The frequency of the subdivision is five. The comparison is made on basis of number of edge groups of approximately equal lengths, Es, and the quotient of the longest and shortest edges of the polyhedron. For both grids obtained from the central projection using secondary grid deformation by the central projection or orthogonal projection onto a plane, Es equals the frequency in which the face is subdivided, that is 5. In the conventional class I method 9 different edge lengths is obtained.

The value of the quotient for the central projection is:

\[ N = \frac{0.2455487a}{0.2101608a} = 1.1683 \]

The value of the quotient \( N_1 \) of the triangular spherical grid generated by the orthogonal projection onto the plane, is:

\[ N_1 = \frac{0.2113452a}{0.1887286a} = 1.1198 \]

The value of the quotient \( N_2 \) of the triangular spherical grid generated by class I method is:

\[ N_2 = \frac{0.248794a}{0.188448a} = 1.3202 \]

It is clear that both methods of secondary grid deformation
Fig 2.5.3 Dimension relations using different projection methods.
have the advantage over class I method and yield a more regular spherical grid.

**2.5.2 OPTIMIZATION OF SPHERICAL NETWORK**

In this process a triangular network is constructed on the sphere so that the number of different edge-lengths is a minimum. A combinatorial approach is used to find minimum number of different triangles and different nodes. For very large number of triangles in the network a new method of subdivision is introduced by T Tarnai, Ref 11. A network that satisfies the above three conditions can be termed "optimal". This method is based on the skew, regular triangular tessellation on the icosahedron. Again using the notion \([3,5^*]_{b,c}\) described earlier, the pair of integers \(b,c\) generating the tessellation determine the triangulation number \(T\),

\[
T = b^2 + b*c + c^2
\]

which gives the number of triangles lying on a face of the icosahedron. By using this type of subdivision to obtain a geodesic spherical polyhedron, the number of edges \((E)\), faces \((F)\) and vertices \((V)\) can be expressed in terms of \(T\) as follows:

\[
E = 30*T, \quad F = 20*T \quad V = 10*T + 2
\]

This type of triangular tessellation gives the topological base for Tarnai investigations.

**2.5.2.1 COMBINATORIAL WAY OF ANALYSIS**

This method makes use of the rotational symmetries of the icosahedron in order to decrease the number of different construction elements; that is, 5-fold symmetry with respect to the vertices of the icosahedron, 3-fold symmetry
with respect to the centres of the faces of the icosahedron and 2-fold symmetry with respect to the midpoints of the edges of the icosahedron. It is sufficient to investigate only one third part of a face of the icosahedron. Further simplification is obtained when \( b = 0 \) or \( c = 0 \) or \( b = c \) and only one sixth part of a face of icosahedron needs be investigated due to the presence of plane of symmetry, Fig 2.5.4.

Consider the network on one third (sixth) part of a face of the spherical icosahedron. Let \( n \) denote the maximum number of different edge-lengths of the network, considering the rotational (and mirror) symmetries of the icosahedron. The minimisation can be carried out by moving the nodal points. Since the nodal points can move on the spherical surface they, in general, have two degrees of freedom. However, if the network has a plane of symmetry and the nodal points lies on the plane of symmetry then it, in general, has one degree of freedom. A nodal point which is identical to a vertex, a face centre, an edge midpoint of the icosahedron has no degree of freedom. Let \( f \) denote the sum of the degrees of freedom of the nodal points on one third (sixth) part of a face of the icosahedron. The \( f \) degrees of freedom make possible to prescribe \( f \) conditions in order that \( f \) edges have lengths equal to number of certain other edges. So the minimum number of different edge-lengths, \( m \), in a network of a given topology, in general, is

\[
m = n - f.
\]

However it can occur that the length equality is realized for more than \( f \) edges. Thus \( m \) is an upper bound of the minimum number of different edge-lengths; that is, number of different edges of the polyhedron, \( e_d \), is less than or equal to \( m \). Now, we have a network of a given topology in which the edges are in \( n \) different positions and each edge has one of \( m \) different lengths. The question is: how many
different network with $m$ different edge-lengths do exist? An approximation of this problem is to seek the number of ways such that one or more of the $n$ positions correspond to one different lengths. This generates a great number of different networks. According to combinatorial analysis this number is the Stirling number of the second kind, since the number of ways of putting $n$ different things into $m$ like cells, with no cells empty, is $S(n,m)$, the Stirling number of the second kind. The Stirling numbers $S(n,m)$, however, give only upper bound of the maximum number of geometrically possible networks, $N$, number of different network with given number of different edges is less or equal to $S(n,m)$, because actually not all the networks defined by combinatorial way exist. But, knowing the geometrical conditions we can sharpen this bound. For instance, in case $(b + c)$ is greater or equal to 3 the networks have to be excluded at which the triangles joining in the vertices of the icosahedron are equilateral, and so $N$ is less or equal to $[S(n,m) - S(n-1,m)]$. After determining the geometrical characteristics of all the possible networks we can chose the network with minimum number of different faces and vertices.

The minimum number of different edge-lengths determined by the degrees of freedom of the nodal points and the number of different network constructed by these edge-lengths are presented in Table 2.1 for some first values of $b,c$. 
Table 2.1

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<th>T</th>
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<th>m</th>
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</tr>
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<td>10</td>
<td>4</td>
<td>6</td>
<td>19936</td>
</tr>
</tbody>
</table>

2.5.2.2 NETWORK WITH ONE EDGE-LENGTH

Only one network of icosahedral symmetry can be constructed with one edge-length in the system [3,5\(^{11.0}\) which is the icosahedron itself.

2.5.2.3 NETWORK WITH TWO DIFFERENT EDGE-LENGTHS

Networks of icosahedral symmetry with two different edge
Fig 2.5.4 Combinatorial way of analysis.

Fig 2.5.5 Network with two different edge-lengths.
lengths can be constructed in systems \([3,5^*]_{1,1}, [3,5^*]_{2,0}\) and \([3,5^*]_{2,1}\). Due to Table 2.1 the system \([3,5^*]_{2,1}\) gives the maximum triangulation number. In this system four kinds of network can be constructed. These are shown in Fig 2.5.5 where the edge-lengths (chord factors) are as follows:

(a) \(a_1 = 0.4028362\) \(a_2 = 0.4638569\)
(b) \(a_1 = 0.4124138\) \(a_2 = 0.4744020\)
(c) \(a_1 = 0.4209745\) \(a_2 = 0.4837981\)
(d) \(a_1 = 0.4301484\) \(a_2 = 0.4938359\)

It is easy to see that the number of different faces in variants (a) and (d) is less by one than that in variants (b) and (c). Thus variants (a) and (d) are optimal. If the further requirement is that the maximum edge-length be a minimum, then variant (a) is the best. But, if quotient \(NN\) is required to be a minimum, then variant (d) is the best.

2.5.2.4 A NEW METHOD OF SUBDIVISION (TARNAI METHOD)

It is apparent from Table 2.1 that with an increase of \(b\) and \(c\) the number of geometrically possible networks \(N\), in general rapidly increases. The combinatorial technique becomes difficult because of the great number of the variants. A method of subdivision was suggested by Tarnai. This method of subdivision exists for each \(b\) equal to or greater than 2 and for any fixed \(e_d\), in general, results in triangulation number \(T\) greater than that given by the other methods known so far. This method is developed in system \([3,5^*]_{b,1}\) and uses the method suggested by Tarnai and discovered independently by Pavlov, Ref 11.

The new method is a generalization of the subdivision in system \([3,5^*]_{2,1}\), shown by Fig 2.5.5. Its principle is that, in system \([3,5^*]_{b,1}\), the edges of the icosahedron can always be covered by strips consisting of triangles, Fig 2.5.6.
If the nodal points on the boundary of a strip, apart from the vertices of the icosahedron, are fitted to two small circles whose planes are parallel then the method suggested by Tarnai in Ref 11, can be applied in the great triangles bordered by the strips. In order to minimize the number of different faces, only two kinds of edge lengths are used for a strip in analogy to those in Fig 2.5.5. Thus, four variants of the strips are defined, Fig 2.5.7. It is seen that variant (d) of the strips can only be constructed of one kind of triangles and its replicas. The other three contain two kinds of triangles. For \( b = 4 \) a part of the network with strips due to Fig 2.5.7d is shown in Fig 2.5.8 where the edge-lengths (chord factors) are as follows:

\[
\begin{align*}
a_1 &= 0.2485854 \\
a_2 &= 0.2899630 \\
a_3 &= 0.2779841 \\
a_4 &= 0.2657787
\end{align*}
\]

and \( e_d = 4, f_d = 4, v_d = 5 \).

The new method of subdivision with strips according to Fig 2.5.7d seems to result in optimal networks. For these networks, in case \( b \) greater than or equal to 2, the following expressions are valid:

\[
\begin{align*}
T &= b^2 + b + 1, \\
e_d &= b, \\
f_d &= (4b - 4)/3, \text{ if the remainder of } b \text{ divided by } 3 \text{ is } 1, \\
f_d &= (4b - 2)/3, \text{ if the remainder of } b \text{ divided by } 3 \text{ is } 2, \\
f_d &= (4b - 3)/3, \text{ if } b \text{ is divisible by } 3, \\
v_d &= (4b - 1)/3, \text{ if the remainder of } b \text{ divided by } 3 \text{ is } 1, \\
v_d &= (4b - 2)/3, \text{ if the remainder of } b \text{ divided by } 3 \text{ is } 2, \\
v_d &= (4b - 3)/3, \text{ if } b \text{ is divisible by } 3.
\end{align*}
\]

Here two elements of the same size, which can be carried into congruence only by mirroring, are considered different.
Fig 2.5.6 The principle of the new method - the strips.

Fig 2.5.7 Four ways of forming the strips.

Fig 2.5.8 New subdivision for $b = 4$ with strips (d).
2.5.3 GEOMETRIC POTENTIAL FUNCTION MINIMISATION

Although geodesic subdivision is believed to be a good way to obtain a regular spherical dome, it has some drawbacks such as the difficulty in truncating the dome at intended support level without disturbing its regularity and also the restriction imposed on the designer by being forced to use some of the already known configurations with little chance to introduce new ideas. A new approach to achieve regularity on domic surfaces without being confined to the traditional ways of subdivision was introduced by J W Butterworth, Ref 4. The idea is to place any number of nodes on a certain predetermined surface or portion of a surface and then allow these nodes to move on the surface by the effect of repulsion forces between them until equilibrium is attained. Some of these nodes can be fixed at certain positions such as the supports or entrances. The problem reduces to obtaining a minimum of a function called the geometric potential function. The details of this method of subdivision is discussed in chapter 5.

2.6 CHOICE OF SUBDIVISION CLASS

Each of the above classes has some advantages and disadvantages. Class I keeps component variations within narrower bounds but does so at the cost of employing a larger inventory of different member lengths which depend on the frequency and increase rapidly. It permits odd as well as even frequencies, class I icosahedra have uninterrupted equators at all even frequencies and at all frequencies numerous undulant lesser circles which can be straighten out by a certain method and use as truncation planes. Class II yields even frequencies only. Used with icosahedron (though not with the octahedron or tetrahedron) it introduces some truncation problems, since any equator breaks through some of the subdividing triangles. This means figuring some special members to close the gaps. The
lengths of the several components vary over a wider range than class one represents. On the other hand, class II has economic advantages. It yields somewhat larger triangles for the same frequency than class I, some what fewer components, and somewhat fewer different components but the differences between them are greater. In method 3 the shrinkage of the list of different components can be spectacular, and it has special advantages for diamond pattern and space frame applications.

2.7 CHOICE OF SUBDIVISION FREQUENCY

The smaller the polyhedron face we are subdividing, the lower the frequency we shall need for acceptable sphericity and acceptable member length. High frequencies not only use a great many members, they use also a great many different member lengths, a fact that is less apparent when high frequency subdivides an icosahedron face than when we are dealing with an octahedron or, worse still a tetrahedron. In general, to minimize component inventory, the designer can use the lowest frequency he can. Very high frequencies bring each vertex so near to the flatness as to be dangerous unless thickness is imparted to the dome by trussing.

2.8 COMPARISON OF PLATONIC POLYHEDRA AS PRINCIPAL POLYHEDRA

It is usually an advantage if a dome has a few different types of face or edge as possible. Configurations derived from the icosahedron and the dodecahedron tend to have fewer types of component for a given number of faces or edges. This is because the icosahedron and the triangulated dodecahedron starts off with more faces than the other Platonic polyhedra, so they generate more facets when they are subdivided. This is an important reason for using those two as principal polyhedra.
For any polyhedra to be convex there must be some vertices where fewer than six triangles meet and these vertices are defined by the original vertices of the tetrahedron, the octahedron and the icosahedron or by the face centre of the cube and the dodecahedron. The difference between the longest and the shortest edge lengths always tend to be greater for polyhedra derived from the tetrahedron than those derived from the cube and the octahedron. Those differences are in turn, greater than those between the edge lengths of configurations derived from the icosahedron and the dodecahedron. Since it is preferable that the edges of a dome be approximately the same length and hence its faces approximately the same size, this is another reason for basing a dome on geodesic polyhedron derived from the icosahedron or the dodecahedron. With an octahedron, right angles occur at the main vertices. Curves are well distributed with the icosahedron; this is not the case with the octahedron, where some vertical areas are flat and therefore structurally weak. Sometimes other considerations may be more important such as that cubes could be closed packed and that icosahedron and dodecahedron could not. When domes are to be joined, it is often easier to use configurations derived from the cube or the octahedron than to use configurations derived from the icosahedron or the dodecahedron. The main advantage of using an octahedron is that in this system one obtains perpendicular symmetry planes. Thus whatever the breakdown frequency, there will be a natural equator and vertical dividing lines, which provide a good opportunity for the junction within rectangular volumes. In order to obtain similar lengths for the elements, a fifth order frequency is required for an octahedron and a third order frequency for an icosahedron. With the exception of configurations derived from a tetrahedron, all configurations generated from a Platonic polyhedron by 'Tricon' method can be also derived from a different Platonic polyhedron by the 'alternate' method.
2.9 ARCHIMEDEAN POLYHEDRA AS PRINCIPAL POLYHEDRA

There are situations where a configuration derived from a principal polyhedron, other than the five Platonic polyhedra, may be required. Any of the Archimedean polyhedra can be used as a principal polyhedron. There are two Archimedean polyhedra which are particularly important as principal polyhedra, these are the truncated octahedron and the truncated tetrahedron. It is known that the truncated octahedron is the only Archimedean polyhedron which close packs by itself. Each face of the truncated octahedron can be triangulated and triangles subdivided to whatever frequency is required. Clusters of domes packing together like soap bubbles can be derived from such configurations. Configurations derived from the truncated tetrahedron using a suitable frequency cannot be derived from any one of the Platonic polyhedra in the normal way, and their importance is that they have many of the symmetries of the tetrahedron without having all of its disadvantages. Geodesic polyhedra derived from the tetrahedron always have four vertices where only three edges meet and there is a great difference between such a configuration’s longest and shortest edges. The truncated tetrahedron, however has twelve vertices, each of which defines a vertex on the geodesic polyhedron where five edges meet. This means that such configurations have less variations in edge length than similarly sized configurations derived from the tetrahedron.

2.10 KINDS OF FACES ON GEODESIC DOMES

Many kinds of faces other than triangular ones can be used on geodesic domes. For example, each face of a regular cube can be divided into a set of squares and the lines and edges projected on the circumscribing sphere. Such a configuration has quadrilateral faces which are not necessarily flat, Fig 2.10.1. Three important facial
Fig 2.10.1 Geodesic dome with square faces derived from the cube.

Fig 2.10.2 Three different triangular facial pattern.
patterns can be derived from geodesic domes with triangular faces. Each of configurations in Fig 2.10.2 represents a face of a principal polyhedron subdivided to six frequencies by the 'alternate' method, over which one of three pattern has been sketched. Parts of some of the faces are located on adjacent faces of the principal polyhedron as indicated by dotted lines. The first pattern consists of hexagons and triangles and can be superimposed on any polyhedron whose face have been triangulated to a multiple of two frequencies. The polygons formed about the vertices or face centres of the principal polyhedron depend upon the polyhedron being used. The second pattern consists of hexagons and can be superimposed on any polyhedron whose face has been triangulated to a multiple of three frequencies. Pentagons, squares or triangles are formed at some of the vertices, depending on the principal polyhedron. The third pattern consists of diamond shape and can be superimposed on any polyhedron whose faces have subdivided to a multiple of three frequencies. All the faces of that type of configuration are diamond shaped.

Unless a face of a polyhedron is triangular, it may not define a single plane. A plane cutting through a sphere will always define a circle on the surface of the sphere, so, if a face of that configuration is to be flat and all its vertices touch the circumscribing sphere, all of its vertices must touch a common small circle of that sphere.

2.11 SPACE FRAME GEODESIC DOMES

Very large domes keep the lengths of their members within bounds using very large frequencies. Very large frequencies engender member lengths that are very much alike. But zero difference in length is the formula for flatness, so any vertex on such large domes is getting nearly flat. Little stretch here and there (elasticity of
materials; inaccuracy of construction; play of joints) would permit a vertex to pop inward under load. At how low a frequency this problem becomes important depends on materials and tolerance. Also, in such a case, especially when the structure is under the action of unsymmetrical loading, such as snow loads, the dome may fail, not on the account of high stresses exceeding the maximum strength of the material but on the account of insufficient stability of the compression members. This phenomenon was fully appreciated only recently, when the introduction of new light materials, such as aluminum, resulted in a great reduction in the dead weight of the structure and led to the use of slender members prone to elastic instability. This problem is solved by imparting a third dimension to the frame. There are several ways of doing this elegantly and without redundant weight. All of them are reducible to a pair of frames, outer and inner, trussed together.

2.11.1 DIAMOND PATTERN

In this pattern the skin is made integral with the trussing system. This is the principle of Fuller's patent on Laminar Geodesic Domes. It is based on the fact that the same set of points can be joined in two different ways to combine, in effect two different breakdown systems. Fig 2.11.1a shows the principle: instead of joining points A, B, C, D to make two triangle ABC and BCD, the connection BC is omitted and instead A is joined to D. The AD connection follows a 2 frequency Class II breakdown, the others a 3 frequency Class I with a few members omitted. It could all be done with members, though hub details would get complex; Fuller solution was to insert a creased diamond as shown. By building the entire dome in this way we can eliminate the membered frame entirely and allow the joined edges of the diamonds and the creases down the centre of the diamond to take its place. Such a dome has a single circumsphere,
passing through all vertices but two different inspheres, the 3 frequency and 2 frequency; the difference of the radii of the latter is the virtual thickness of the dome. By using one member along the short diagonal one can obtain a hexagonal shape on the surface of the dome when joining six of the diamonds to form a space frame dome. Many tests on panel configurations were done by D.L.Richter; Ref 6, chapter 18, to obtain the best fabrication geometry for optimum structural properties. It was found that by simply varying the number, placement and depth of the creases in a 3.6 m long aluminum alloy sheet 2.0 mm thick, the strength could be varied from 29.4 to 53.0 KN; Fig 2.11.1b.

2.11.2 TRUSSED FRAMES

The other principal method of lending stiffness to a dome of high frequencies (low local curvature) is to use an outer frame and an inner frame and truss them together (double layer domes). To design such a dome, two sets of members are used, using the same chord factors but two different radii. To save weight numerous members of the triangulated surface are omitted, leaving a pattern of hexagons which are then stabilized tensionally the tension cables replacing the omitted members. The outer and the inner shells may or may not be of identical geometry, Ref 7.

2.12 GEODESIC DOMES AND FORMEX ALGEBRA AND FORMIAN

Formex algebra and its computer language Formian provide a good tool for preparing data and creating configurations for complicated types of space structure. The various types of geodesic subdivisions can be easily created using these elegant tools. A wide range of geodesic configurations can be created using the idea of subroutines and schemes introduced in chapter 3 of this work. The idea
Fig 2.11.1a Diamond pattern principles.

- e = diamond edge
- l = long diagonal
- s = short diagonal

Fig 2.11.1b Typical ultimate compression strength values for formed aluminum diamond panels
of formex algebra and Formian and its application on this field of space structures are discussed in chapter 3 and 4, respectively.

A designer using the ideas of formex algebra and Formian will not be confined to use spherical or elliptical geodesic domes of which already prepared data exist. The range of geodesic configurations can include paraboloid, hyperboloid, cones and other shapes which can be imagined. Also the range of the generator volumes can be easily widened to include all Platonic polyhedra and the Archimedean polyhedra as the idea introduced in this work make it easy to shift from one type of polyhedra to another by just changing few parameters. In addition, handling of graphics of these configurations and submission of data to any computer analysis program are made easier and more reliable.
CHAPTER THREE
FORMEX ALGEBRA AND FORMIAN

3.1 INTRODUCTION

Formex algebra is a mathematical system that provides a basis for dealing with data preparation and computer graphics with ease and elegance. Formex algebra allows networks of all kinds to be formulated conveniently even for complex configurations. The universal nature of the formex approach allows one to work with the same set of conceptual tools in all data generation problems, eliminating the need for employment of an assortment of software schemes.

Formian is a preprocessing language that allows problems of data generation to be handled easily. Formian uses formex algebra as its mathematical basis. The origins of Formian as well as formex algebra date back to the seventies and various versions of the language have been in use since that time, Refs 1 and 2.

This chapter gives an introduction to the basis of formex algebra and its programming language Formian.

3.2 FUNDAMENTALS OF FORMEX ALGEBRA

3.2.1 GENERAL DEFINITION OF A FORMEX

A formex is an arrangement of numbers that may be used to represent a part or the whole of a configuration, where the term configuration is used to mean a collection of physical and/or abstract objects. For instance, if R1 denotes the Y-shaped configuration shown in Fig 3.1, then
Fig 3.1
Rl may be represented by

\([\{[4,2;4,4],[4,4;2,5],[4,4;6,5]\}]\)

The above construct is an example of a formex (plural formices), where the nodes are represented by \((4,2), (4,4), (2,5)\) and \((6,5)\) relative to the coordinate system of the type shown. The coordinate system shown in Fig 3.1 is referred to as 'normat', and the point of intersection of the families of 'normat lines' is known as 'normat points'.

The primary components of a formex are enclosed in square brackets and are referred to as 'cantles'. The above formex consists of three cantles and is said to be of 'order' three. Each cantle, in turn, consists of a number of 'signets', that are separated by semicolons, each signet consists of 'uniples' which are separated by commas. The number of uniples in a signet is referred to as the 'grade' of the signet. The number of signets in a cantle is referred to as the 'plexitude' of the cantle. For instance, each of the cantles of the above formex consists of two signets and is said to be of plexitude two, and each signet consists of two uniples and is said to be of grade two. The serial position of a cantle in a formex is referred to as the 'orderate' of that cantle. For instance, the orderate of

\([4,4;6,5]\) and \([4,2;4,4]\)

with respect to the above formex are 3 and 1, respectively.

Graphically, a signet represents a point relative to the normat shown, and each cantle represents a primary component of a configuration. For instance the 1st, 2nd and 3rd cantles of the above formex represent the segments indicated by 1, 2 and 3 in Fig 3.1, respectively. What the little circles and the line segments of Fig 3.1 actually
represent depends on the context. These may, for instance, represent the joints and members of a part of a structure.

When a formex is of the first order it is written without the enclosing curly brackets.

The formex given above is an example of an integer formex because all the numbers in it are integers. An example of a 'floatal formex' is

\[
\{(1,1,2;3,4.5,6), [2,2.4,1; -3.5, -2.9,0]\}.
\]

Among the set of all formices there is one that does not have any cantle, this special formex is denoted by {} and is referred to as the 'empty formex'. The order of the empty formex is equal to zero, but its grade is considered to be arbitrary.

### 3.2.2 HOMOGENEOUS FORMICES AND INGOTS

If a formex has cantles all of which are of the same plexitude, then the formex is said to be 'homogeneous' and is said to 'non-homogeneous' otherwise. A homogenous formex that contains cantles of the \( m \) th plexitude is referred to as a homogeneous formex of the \( m \) th plexitude and may also be referred to as an '\( m \)-plex' formex. For instance,

\[
\{(2,3;2,2), [1,5;6,2],[4,6;3,1]\}
\]

and \[
\{(2,3,4), [5,8,9]\}\]

are homogeneous formices of second plexitude and first plexitude, respectively. A homogeneous formex of the first plexitude like the second example above is referred to as an 'ingot'. The formex of the second example above is an ingot of the second order and third grade, and
is an ingot of sixth order and first grade.

3.2.3 EQUALITY OF FORMICES

Two formices are said to be 'equal' if they are identical.

Thus

\[ [i,j;k,l;p,q] = [4,6;7,8;2,1] \]

implies that \( i=4, \) \( j=6, \) \( k=7, \) \( l=8, \) \( p=2 \) and \( q=1. \)

3.2.4 VARIANTS OF FORMICES

Two formices are said to be 'variants' of each other provided that every cantle in one may be obtained by a rearrangement of the positions of the signets of the cantles of the other. The relationship of equality is regarded as a special case of the relationship of being variants. For instance, if

\[
\begin{align*}
F_1 &= \{ [80,85;60,62;43,33], [90,91], [30,31;72,50] \} \\
F_2 &= \{ [60,62;80,85;43,33], [90,91], [72,50;30,31] \}
\end{align*}
\]

and

\[
\begin{align*}
F_3 &= \{ [58,57;48,49;30,34], [20,21], [22,22;43,59] \}.
\end{align*}
\]

Then, \( F_1 \) and \( F_2 \) are variants of each other, but \( F_3 \) is not a variant of \( F_1 \) or \( F_2 \).

A formex that contains cantles that are variants of each other is said to be 'prolate' and it is said to be 'nonprolate' otherwise. For instance,

\[
\{ [1,4,2], [2,5,3], [3,6,4] \} \]
and
\[
\{ [4,2;6,4], [1,2;3,4], [8,6], [5,3;7,5;9,7] \}
\]
are nonprolate, but
\[
\{ [1,4,2], [2,5,3], [1,4,2], [3,6,4] \}
\]
is a prolate formex since its first and third cantles are equal and being equal is, of course, a special case of being variants. Also
\[
\{ [4,2;6,4], [1,2;3,4], [6,4;4,2], [8,6] \},
\{ [4,2;6,4], [3,4;1,2], [5,3;7,5;9,7] \}
\]
is a prolate formex since its first, third and fifth cantles are variants of each other and, in addition, its second and sixth cantles are variants of each other.

A uniple, a signet or a cantle is nonprolate and so is the empty formex.

3.2.5 SEQUATIONS OF FORMICES

Two formices are said to be sequations of each other provided that one may be obtained from the other by a rearrangement of the position of its cantles. Two equal formices are considered to be sequations of each other. For example, if
\[
E_1 = \{ [2,3;4,5], [7,8], [50,30;35,10] \}
\]
and \[E_2 = \{ [7,8], [50,30;35,10], [2,3;4,5] \}\]

then \(E_1\) and \(E_2\) are sequations of each other.
3.2.6 COMPOSITION OF FORMICES

If \( F_1 \) and \( F_2 \) are two formices of the same grade, then the 'composition' of \( F_1 \) and \( F_2 \) is defined as a formex \( F \) that consists of all the cantles of \( F_1 \) appearing in the same order as in \( F_1 \) followed by all the cantles of \( F_2 \), appearing in the same order as in \( F_2 \), and the relation between \( F, F_1 \) and \( F_2 \) is written as

\[
F = F_1 \# F_2
\]

where the \( \# \) is referred to as the 'duplus symbol' and is read as duplus.

The term 'composition' is used to refer to both the 'process' of composing two or more formices and the 'formex' that is the result of the process.

Formex composition has the following basic properties:

If \( E, F \) and \( G \) are formices of the same grade, then

1. in general, formex composition is not commutative, that is,
   \[
   E \# F \neq F \# E
   \]
2. formex composition is associative, that is,
   \[
   E \# (F \# G) = (E \# F) \# G
   \]
3. for any formex \( H \)
   \[
   H \# \{\} = \{\} \# H = H
   \]
4. \( E \# F \) and \( F \# E \) are sequations of each other.

3.2.7 LIBRA NOTATION

Serial composition of formices is achieved by using the construct \( \text{lib}(i=m,n) \) which is called a 'libra operator'. Let \( F_i \), be a formex which is given in terms of an integer \( i \), the effect of the 'libra operation'
$\text{lib}(i=m,n) | F_i$ is as follows:

if $m < n$ then

$\text{lib}(i=m,n) | F_i = F_m \# F_{m+1} \# \ldots \# F_{n-1} \# F_n$

and if $m = n$, then

$\text{lib}(i=m,n) | F_i = F_m$

and if $m > n$, then

$\text{lib}(i=m,n) | F_i = F_m \# F_{m-1} \# \ldots \# F_{n+1} \# F_n$

where the integer $i$ is referred to as 'libra variable'.

Libra composition may contain a sequence of libra operators as written below:

$\text{lib}(i_1=m_1,n_1) | \text{lib}(m_2,n_2) | \ldots | \text{lib}(m_r,n_r) | F$

if $r = 1$ then the libra composition is said to be 'simple',
if $r > 1$ then it is said to be 'nested' and is referred to as '$r$-nested' libra composition.

3.3 FORMEX GRAPHICS

3.3.1 Formex Plots

A formex may be graphically represented through a rule which is referred to as 'retrobasis' and the resulting configuration is called a formex 'plot'. A geometric configuration may also be represented by a formex through a rule which is referred to as a 'probasis'. A formex plot is constructed from two parts which are referred to as 'tenons' and 'fronds'. A tenon is a part of a plot that represents a signet, and a frond is a part of a plot that
represents a cantle. Every tenon is drawn relative to a 'pivot'.

For example, if

\[ F = \{[2,1;2,2], [2,2;2,3], [1,3;2,3], [2,3;3,3], [3,2;3,3], [3,3;3,4], [2,2;3,2], [3,2;4,2]\} \]

Then the plot of \( F \) is shown in Fig 3.3.1. In this figure, each tenon is represented by a little circle and each frond is represented by two little circles and a straight line with an arrow head.

3.3.2 Retrobases

A 'retrobasis' is a set of rules which is a combination of a 'retronorm' and a collection of 'retrocords' and through which a given formex may be plotted.

3.3.3 Retrocords

An aspect of the shape of a tenon or a frond is defined by a retrocord. For instance, the retrocords of the plot of \( F \) which is shown in Fig 3.3.1 are as follows:

1. A tenon is drawn as a little circle whose centre is a pivot.
2. A frond is drawn as a straight line with an arrowhead which connects two tenons and the direction of the arrow shows the order of the signets in the cantle.

This example of retrocords is merely one possibility, tenons and fronds can be drawn in an infinite variety of ways.

3.3.4 Retronorms

A 'Retronorm' is a set of rules through which the positions
Graphical Retronorms

Fig 3.3.2.
of the pivots are determined. There are three types of retronorms as follows:

(1) 'Formal retronorm', which is defined through mathematical formulae and/or descriptive statements in a natural language.
(2) 'Graphical retronorms', which is defined in terms of graphical construction. 'Normat' is another name for a graphical retronorm.
(3) 'Tabular retronorm', which is defined in terms of a table of values.

Each of these types of retronorm for the formex plot which is shown in Fig 3.3.1 is given below:

(1) \( x = U_1 \)
\( y = U_2 \)

where \( U_1 \) and \( U_2 \) are the uniples of a typical signet of \( F \).

(2) Fig 3.3.2.
(3) Table 3.1.

### Table 3.1

<table>
<thead>
<tr>
<th>( U_1 )</th>
<th>( U_2 )</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( x )</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>( y )</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>( x )</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>( y )</td>
<td>1</td>
<td>3</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>( x )</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>( y )</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>( x )</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>( y )</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
</tbody>
</table>

### 3.3.5 Standard Retronorms

In relation to commonly used coordinate systems, six categories of retronorms may be defined as follows:

(1) A 'unifect retronorm', which relates to a one
dimensional Cartesian coordinate system.

(2) A 'bifect retronorm', which relates to a two
dimensional Cartesian coordinate system.

(3) A 'trifect retronorm', which relates to a three
dimensional Cartesian coordinate system.

(4) A 'polar retronorm' which relates to a polar
coordinate system.

(5) A 'cylindrical retronorm', which relates to a
cylindrical coordinate system.

(6) A 'spherical retronorm', which relates to a
spherical coordinates system.

Moreover, three special cases of these retronorms are
classified as 'standard retronorms' and are referred to as
'basiant retronorms', 'pariant retronorms' and 'metriant
retronorms'.

### 3.3.6 Basiant Retronorms

The family of basiant retronorms is given in Table 3.2.

#### Table 3.2

<table>
<thead>
<tr>
<th>NAME</th>
<th>COORDINATE EQUATIONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Basiunifect</td>
<td>X=b1U1</td>
</tr>
<tr>
<td>Basibifect</td>
<td>X=b1U1, Y=b2U2</td>
</tr>
<tr>
<td>Basitrifect</td>
<td>X=b1U1, Y=b2U2, Z=b3U3</td>
</tr>
<tr>
<td>Basipolar</td>
<td>r=b1U1, s=b2U2</td>
</tr>
<tr>
<td>Basicylindrical</td>
<td>r=b1U1, s=b2U2, z=b3U3</td>
</tr>
<tr>
<td>Basispherical</td>
<td>r=b1U1, s=b2U2, t=b3U3</td>
</tr>
</tbody>
</table>
where \( b_1, b_2 \) and \( b_3 \) are coefficients which are referred to as 'basifactors' and are classified into two groups. One type of basifactors which is associated with linear coordinates \( x, y, z \) or \( r \) is referred to as a 'linear basifactor' and should be given in terms of a unit of length. The other type of basifactor which is associated with angular coordinates \( s \) or \( t \) is referred to as 'angular basifactor' and should be given in terms of a unit of angle. Examples are shown in Figs 3.3.3 to 3.3.8.

Furthermore, for a basipolar or basicylindrical retronorms, if an angular basifactor \( b_2 \) is given as

\[
b_2 = \frac{2\pi}{N}
\]

then the retronorm is referred to as an 'N-sect' retronorm. For instance, the retronorms in the examples of Figs 3.3.6 and 3.3.7 may be referred to as 12-sect retronorms. Similarly, for basispherical retronorm, if angular basifactors \( b_2 \) and \( b_3 \) are given as

\[
b_2 = \frac{2\pi}{N} \\
b_3 = \frac{2\pi}{M}
\]

then the retronorm is referred to as an 'N-M-sect' retronorm. For instance, the retronorm in the example of Fig 3.3.8 may be referred to as a 18-9-sect retronorm.

### 3.3.7 Pariant Retronorms

The family of 'ariant retronorms' is given in Table 3.3. Pariant retronorms are obtained from basiant retronorms whose every linear basifactor is equal to one unit length. Thus pariant retronorms are special cases of basiant retronorms.
Basiunifect (b1=2)

Fig 3.3.3

Basibifect (b1=15, b2=10)

Fig 3.3.4
Basitrifect (b1=1, b2=3, b3=2)

Fig 3.3.5

Basipolar (b1=10, b2=30)

Fig 3.3.6
Basicylindrical \((b_1=4, b_2=30, b_3=5)\)

Fig 3.3.7

Basispherical \((b_1=1, b_2=18, b_3=9)\)

Fig 3.3.8
Table 3.3

<table>
<thead>
<tr>
<th>NAME</th>
<th>COORDINATE EQUATIONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pariunifect</td>
<td>X=U1</td>
</tr>
<tr>
<td>Paribifect</td>
<td>X=U1, Y=U2</td>
</tr>
<tr>
<td>Paritrifect</td>
<td>X=U1, Y=U2, Z=U3</td>
</tr>
<tr>
<td>Paripolar</td>
<td>r=U1, s=b2U2</td>
</tr>
<tr>
<td>Paricylindrical</td>
<td>r=U1, s=b2U2, z=U3</td>
</tr>
<tr>
<td>Parispherical</td>
<td>r=U1, s=b2U2, t=b3U3</td>
</tr>
</tbody>
</table>

3.3.8 Metriant Retronorms

The family of 'metriant retronorms' is given in table 3.4.
Table 3.4

<table>
<thead>
<tr>
<th>NAME</th>
<th>COORDINATE EQUATIONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Metriuniform</td>
<td>X=b1 met(U1,m1)</td>
</tr>
<tr>
<td>Mettribifect</td>
<td>X=b1 met(U1,m1) Y=b2 met(U2,m2)</td>
</tr>
<tr>
<td>Metritrifect</td>
<td>X=b1 met(U1,m1) Y=b2 met(U2,m2) Z=b3 met(U3,m3)</td>
</tr>
<tr>
<td>Metripolar</td>
<td>r=b1 met(U1,m1) s=b2 met(U2,m2)</td>
</tr>
<tr>
<td>Metricylindrical</td>
<td>r=b1 met(U1,m1) s=b2 met(U2,m2) z=b3 met(U3,m3)</td>
</tr>
<tr>
<td>Metrispherical</td>
<td>r=b1 met(U1,m1) s=b2 met(U2,m2) t=b3 met(u3,m3)</td>
</tr>
</tbody>
</table>

where 'met(U,m)' is a function which is referred to as a 'metrifactor' and is described as follows:

if m = 1 or U = 0 then,

met(U,m) = U

and if m != 1 and U != 0 then,

if U > 0 then,

met(U,m) = (1-m^U) / (1-m) = (1+m+m^2+m^3+...+m^{U-1})

and if U < 0 then,

met(U,m) = (1-m^U) / (1-m) = (-1-m-m^2-...-m^{U-1})

where U is a non-zero positive coefficient and is referred to a 'metrifactor'. Examples are shown in Figs 3.3.9 and 3.3.10.
Metriunifect ($b_1=10$, $m_1=2$)

Fig 3.3.9 a

Metribifect ($b_1=3$, $m_1=2$, $b_2=1$, $m_2=3$)

Fig 3.3.9 b
Metripolar ($b_1=1$, $m_1=2$, $b_2=10$, $m_2=2$)

Fig 3.3.10
### 3.4 FORMEX FUNCTIONS

In scalar algebra the relation
\[ y = f(x) \]
is used to represent a relation between \( x \) and \( y \), where \( x \) and \( y \) are independent and dependent variables and the term \( f \) is referred to as a 'function' and symbolizes the rule by which \( y \) is obtained from \( x \).

In an analogous manner, formices may assume the roles of dependent and independent variables. Thus if a rule is established by which from a given formex \( E \) another formex \( G \) is obtained, then this rule may be represented by a symbol, say \( \Phi \), and a relation such as

\[ G = \Phi|E \]

may be used to express \( G \) in terms of \( E \). The symbol \( | \) is referred to as the 'rallus symbol' and is read as 'rallus' or 'of'. The construct \( \Phi|E \) is called a 'function designator' with \( E \) being its 'argument'. Example of a function is the translation function

\[ G = \text{Tran}(h,q)|E \]

where \( h \) and \( q \) are known as 'canonic parameters'.

Formex algebra has three families of functions; 'transflection functions', 'introflection functions' and 'node numbering functions'.

#### 3.4.1 TRANSFLECTION FUNCTIONS

Transflection functions can be divided into three groups of functions which are referred to as 'cardinal', 'tendial' and 'provial' functions, and each group consists of various
functions.

3.4.2 CARDINAL FUNCTIONS

Cardinal functions consist of eight basic functions which are referred to as 'translation', 'rindle', 'reflection', 'lambda', 'vertition', 'rosette', 'projection' and 'dilatation' functions.

3.4.2.1 Translation Functions

A translation function which is denoted by

\[ \text{tran}(h, q) \]

is used to translate a formex plot parallel to the \( U_h \) axis by \( q \) units. Where

\( h \) and \( q \) are the canonic parameters, and \( h \) is a non-zero positive integer less than or equal to the grade of the independent variable, and \( q \) is any number.

For example, the formices

\[ E=\{[1,1;2,3],[2,3;3,1],[3,1;1,1]\} \]
\[ T_1=\text{tran}(1,5)|E \]
and
\[ T_2=\text{tran}(2,4)|E \]

will give rise to the plots shown in Fig 3.4.1.

3.4.2.2 Rindle Functions

A rindle function which is denoted by

\[ \text{rin}(h, s, p) \]

is a combination of the concepts of libra composition and
Fig 3.4.1
translation function and can be represented by

\[ \text{lib}(v=0,s-1)|\text{tran}(h,pv) \]

where the canonic parameters \( h, s \) and \( p \) are defined as follows:

- \( h \): direction of replication
- \( s \): number of replications, called 'spread'
- \( p \): number of units of translation at each step, called 'pace'.

For example, the formices

\[ E=\{[1,1;2,3],[2,3;3,2],[3,1;1,1]\} \]
\[ R_1=\text{rin}(1,4,2)|E \]

and

\[ R_2=\text{rin}(2,3,3)|E \]

will give rise to the formex plots shown in Fig 3.4.2.

### 3.4.2.3 Reflection Functions

A reflection function which is denoted by

\[ \text{ref}(h,q) \]

produces a mirror image of a formex plot with respect to a plane which is normal to the \( U_h \) axis and intersect it at a point for which \( U_h=q \).

For example, the formices

\[ E=\{[1,1;2,3],[2,3;3,1],[3,1;1,1]\} \]
\[ R_{F1}=\text{ref}(1,9/2)|E \]

and

\[ R_{F2}=\text{ref}(2,4)|E \]
R2 consists of all three vertical plots.

R1 consists of all four plots.

Fig 3.4.2
will give rise to the formex plots shown in Fig 3.4.3.

3.4.2.4 Lambda Functions

A lambda function which is denoted by

\[ \text{lam}(h, q) \]

involves the combination of reflection function and composition. Thus,

\[ \text{lam}(h, q) | E \]

means

\[ \text{ref}(h, q) | E \neq E. \]

For example, if

\[ F = [1, 1; 2, 3] \]

and if

\[ L = \text{lam}(1, 2) | F \]

then the plot of \( L \) is shown in Fig 3.4.4.

3.4.2.5 Vertition Functions

A vertition function which is denoted by

\[ \text{ver}(h_1, h_2, q_1, q_2) \]

produces a formex plot rotated through 90° from the plot of original formex. The point of the rotation is given by \( U_{h_1} = q_1 \), \( U_{h_2} = q_2 \) and the plane of rotation is given by the values of \( h_1 \) and \( h_2 \).
Fig 3.4.3 effects of a reflection function
Fig 3.4.4 effects of a lambda function

L consists of the two legs
For example, if

\[ E = \{ [3,1; 4,3], [5,1; 4,3], [5,1; 3,1] \} \]
\[ V_1 = \text{ver}(1,2,5,4) | E \]

and

\[ V_2 = \text{ver}(2,1,4,5) | E \]

then the plots of \( E \), \( V_1 \) and \( V_2 \) are shown in Fig 3.4.5.

3.4.2.6 Rosette Functions

A rosette function which is denoted by

\[ \text{ros}(h_1, h_2, q_1, q_2) \]

produces a combination of a formex and its vertition at 90, 180 and 270 degrees. Thus it means

\[ \text{LIB}(i=0,3) | \text{ver}(h_1, h_2, q_1, q_2)^i. \]

For example, if

\[ E = \{ [3,1; 4,3], [5,1; 4,3], [5,1; 3,1] \} \]

and

\[ R = \text{ros}(1,2,4,4) | E \]

then, the plots of \( E \) and \( R \) are shown in Fig 3.4.6.

3.4.2.7 Projection Functions

A projection function which is denoted by

\[ \text{proj}(h,q) \]

produces the projection of a formex plot onto a plane that is perpendicular to the \( U_h \) axis and intersects it at the
Fig 3.4.5 effects of a vertition function
Fig 3.4.6 effects of a rosette function
point for which \( U_h = q \). For example, if

\[
E = \{[1,1;2,3],[2,3;3,1]\}
\]

\[
P_1 = \text{proj}(1,5) | E
\]

and

\[
P_2 = \text{proj}(2,4) | E
\]

then the plots of \( E, P_1 \) and \( P_2 \) are shown in Fig 3.4.7.

3.4.2.8 Dilatation Functions

A dilatation function which is denoted by

\[
dil(h,q)
\]

produces an elongation or contraction of a formex plot by a factor of \( q \) in a direction parallel to the \( U_h \) axis. For example, if

\[
E = \{[2,1;2,2],[2,2;2,3],[1,3;2,3],[2,3;3,3],
\]

\[
[3,2;3,3],[3,3;3,4],[2,2;3,2],[3,2;4,2]\}.
\]

\[
D_1 = dil(1,4) | E
\]

and

\[
D_2 = dil(2,4,4) | E
\]

then the plots of \( E, D_1 \) and \( D_2 \) are shown in Fig 3.4.8.

3.4.3 TENDIAL FUNCTIONS

Tendial functions are combination of cardinal functions of the same type which may act in two or three directions. There are twenty four tendial functions which are divided into four groups. The first group of tendial function is referred to as 'tendid functions'. A tendid function consists of a cardinal function acting in the first direction preceded by a cardinal function of the same type acting in the second direction. The suffix 'id' is added
Fig 3.4.7 'effects of a projection function
Fig 3.4.8 effects of dilatation function and rindle function
to the corresponding cardinal function for the identifier of a tendid function. The definitions of the tendid functions are shown in Table 3.4.1.

Table 3.4.1 Tendid Functions

<table>
<thead>
<tr>
<th>Function</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>tranid(q1,q2)</td>
<td>tran(2,q2)</td>
</tr>
<tr>
<td>(translation tendid)</td>
<td></td>
</tr>
<tr>
<td>rinid(s1,s2,p1,p2)</td>
<td>rin(2,s2,p2)</td>
</tr>
<tr>
<td>(rindle tendid)</td>
<td></td>
</tr>
<tr>
<td>refid(q1,q2)</td>
<td>ref(2,q2)</td>
</tr>
<tr>
<td>(reflection tendid)</td>
<td></td>
</tr>
<tr>
<td>lamid(q1,q2)</td>
<td>lam(2,q2)</td>
</tr>
<tr>
<td>(lambda tendid)</td>
<td></td>
</tr>
<tr>
<td>projid(q1,q2)</td>
<td>proj(2,q2)</td>
</tr>
<tr>
<td>(projection tendid)</td>
<td></td>
</tr>
<tr>
<td>dilid(q1,q2)</td>
<td>dil(2,q2)</td>
</tr>
<tr>
<td>(dilatation tendid)</td>
<td></td>
</tr>
</tbody>
</table>

For example, to illustrate the rindle tendid function, if

E=\{[1,1;2,3],[2,3;3,1],[3,1;1,1]\}

and

R=rinid(5,3,2,3)|E

then the plots of E and R are shown in Fig 3.4.9.

The second and third groups of tendial functions are referred to as 'tendis' and 'tendit' functions which are similar to the tendid functions but relate to actions in the cardinal direction 1-3 and 2-3, respectively. Moreover, suffixes 'is' and 'it' are used for tendis and tendit functions, respectively.

The fourth group of tendial function is referred to as 'tendix functions' and relates to actions in the cardinal directions 1-2-3. Suffix 'ix' is used for tendix
Fig 3.4.9 effects of a rindle tendid function
functions. The definitions of tendix functions are shown in Table 3.4.2.

Table 3.4.2 Tendix Functions

<table>
<thead>
<tr>
<th>Function</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>tranix(q1,q2,q3)</td>
<td>tran(3,q3)</td>
</tr>
<tr>
<td>(translation tendix)</td>
<td></td>
</tr>
<tr>
<td>rinix(s1,s2,s3,p1,p2,p3)</td>
<td>rin(3,s3,p3)</td>
</tr>
<tr>
<td>(rindle tendix)</td>
<td></td>
</tr>
<tr>
<td>refix(q1,q2,q3)</td>
<td>ref(3,q3)</td>
</tr>
<tr>
<td>(reflection tendix)</td>
<td></td>
</tr>
<tr>
<td>lamix(q1,q2,q3)</td>
<td>lam(3,q3)</td>
</tr>
<tr>
<td>(lambda tendix)</td>
<td></td>
</tr>
<tr>
<td>projix(q1,q2,q3)</td>
<td>proj(3,q3)</td>
</tr>
<tr>
<td>(projection tendix)</td>
<td></td>
</tr>
<tr>
<td>dilix(q1,q2,q3)</td>
<td>dil(3,q3)</td>
</tr>
<tr>
<td>(dilatation tendix)</td>
<td></td>
</tr>
</tbody>
</table>

Tendial functions are not defined for vertition and rosette functions.

3.4.4 PROVIAL FUNCTIONS

'Provial functions' are generalisations of cardinal functions in the sense that their effect may be along any direction or any angle in a given plane.

There are thirty two provial functions which are divided into four groups. The first group of the provial function is referred to as 'proviad functions' and has the suffix 'ad' indicating that the action involves the cardinal direction 1-2.

The second and third groups of provial functions referred to as 'provias' and 'proviat' functions. These groups are
similar to the proviad functions but have suffixes 'as' and 'at' indicating that the actions involve cardinal directions 1-3 and 2-3, respectively.

The fourth group of provial functions is referred to as 'proviax functions' and has suffix 'ax' indicating that the action involves the cardinal directions 1-2-3. The description of proviad and proviax functions are given in Table 3.4.3.

The canonic parameters A1, A2, A3, B1, B2 and B3 in Table 3.4.3 are the coordinates of the end points of a vector AB as shown in Fig 3.4.10 which is referred to as the 'direction vector'.

For example, to illustrate the rindle proviad function if,

\[ E = \{(1,1;3,3), [3,3;5,1], [5,1;1,1]\} \]

and

\[ R = rinad(1,1,4,2,4) | E \]

then, the paribifect plots of E and R are shown in Fig 3.4.11.
for provided functions

Fig 3.4.10
Fig 3.4.11 effects of rindle provided followed by rindle function in the U2 direction
### Table 3.4.3 Proviad and Proviax Functions

<table>
<thead>
<tr>
<th>Function</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>tranad(A1, A2, B1, B2[, q])</td>
<td>Direction of translation is given by vector AB and amount of translation is given by the value of q (or the length of AB, in the absence of q).</td>
</tr>
<tr>
<td>tranax(A1, A2, A3, B1, B2, B3[, q])</td>
<td>(translation proviax)</td>
</tr>
<tr>
<td>rinad(A1, A2, B1, B2, s[, p])</td>
<td>Direction of replication is given by vector AB, number of replications (spread) is given by the value of s and the amount of translation at each step (pace) is given by the value of p (or the length of AB, in the absence of p).</td>
</tr>
<tr>
<td>rinax(A1, A2, A3, B1, B2, B3, s[, p])</td>
<td>(rindle proviad)</td>
</tr>
<tr>
<td>refad(A1, A2, B1, B2)</td>
<td>Direction of replication is given by vector AB, with the plane of reflection being normal to AB at B.</td>
</tr>
<tr>
<td>refax(A1, A2, A3, B1, B2, B3)</td>
<td>(reflection proviad)</td>
</tr>
<tr>
<td>lamad(A1, A2, B1, B2)</td>
<td>Same as for refad and refax.</td>
</tr>
<tr>
<td>lamax(A1, A2, A3, B1, B2, B3)</td>
<td>(lambda proviad)</td>
</tr>
<tr>
<td>verad(A1, A2[, α])</td>
<td>Coordinates of centre of rotation in plane U1-U2 are given by the values of A1 and A2. Amount of rotation is given by the value of α in degrees with the sense of rotation being such that when α=90 then U1 is mapped into U2. Absence of α implies α=90.</td>
</tr>
<tr>
<td>verax(A1, A2, A3, B1, B2, B3, α)</td>
<td>(vertition proviad)</td>
</tr>
<tr>
<td>Function</td>
<td>Description</td>
</tr>
<tr>
<td>----------</td>
<td>-------------</td>
</tr>
</tbody>
</table>
| versax(A₁,A₂,A₃,B₁,B₂,B₃[,α])  
(vertition proviax) | Axis of rotation is given by vector AB and the amount of rotation is given by the value of α in degrees. The sense of rotation is such that when the value of α is positive then the rotation causes a right hand screw to move from A towards B. Absence of α implies α=90. |
| rosad(A₁,A₂[,s,p])  
(rosette proviad) | Coordinate of centre of rotation in plane U₁-U₂ are given by the values of A₁ and A₂. Number of replications (spread) is given by the value of s and the amount of rotation at each step (pace) is given by the value of p in degrees. Sense of rotation is defined as for vertition proviad function (with p instead of α). Absence of s and p implies that s=4 and p=90. |
| projad(A₁,A₂,B₁,B₂)  
(projection proviad) | Direction of projection is given by vector AB, with the plane of projection being normal to AB at B. |
| projax(A₁,A₂,A₃,B₁,B₂,B₃)  
(projection proviax) | |
| dilad(A₁,A₂,B₁,B₂,q)  
(dilatation proviad) | Direction of dilatation is given by vector AB, and factor of dilatation is given by the value of q. |
| dilax(A₁,A₂,A₃,B₁,B₂,B₃,q)  
(dilatation proviax) | |

3.4.5 INTROFLECTION FUNCTIONS

Introflection functions increase the ability of dealing with the problems in a convenient manner for formex formulations. Introflection functions allow formices to be curtailed in various ways and are of particular value when irregular interconnection pattern are being formulated. Two basic classes of introflection functions are known as 'pexum' and 'rendition' functions.
3.4.5.1 PEXUM FUNCTION

In formulating interconnecting patterns, it is often convenient to allow over generation of cantles to occur in the intermediate stages of the formulation. These over generated cantles may be disposed of by using the 'pexum function'. A formex formulation of this nature is known as 'superpansive formulation' and a formex formulation with no over generated cantles is known as a 'prepansive formulation'.

For example, consider the plan view of a flat grid which is shown in Fig 3.4.12 and let it be required to write a formex formulation for grid in terms of the indicated normat.

Then, the formex formulation may be written as

\[ E = \text{rinid}(5,3,1,1) \mid \text{ros}(1,2,3/2,3/2) \mid [1,1; 2,2] \]
\[ F = \text{pex}|E \]

where 'pex' is the pexum function.

Parabifect plots of formices F and E are shown in Fig 3.4.13. As shown in Fig 3.4.13, although the formex E contains more element than the actual configuration, the formex F which represents the actual configuration is obtained by using a pexum function which removes all superfluous elements form E.

The effect of the pexum function is to eliminate any cantle whose signets are the same as those of a preceding cantle.

3.4.5.2 RENDITION FUNCTIONS

Rendition functions consists of six functions which are referred to as 'nexum', 'conexum', 'luxum', 'coluxum', 'pactum' and 'copactum' functions. These functions
Fig 3.4.12

Fig 3.4.13
represent rules for curtailing a formex in the following manners.

Let E and F be two formices of the same grade.

**Nexum Function**

\( \text{Nex}(F) \mid E \) is obtained by deleting every cantle of E that includes one or more signets that not in F.
Where ‘nex(F)’ is referred to as a ‘nexum function’.

**Conexum Functions**

\( \text{Con}(F) \mid E \) is obtained by deleting every cantle of E that consists of signets all of which are in F.
Where ‘con(F)’ is referred to as a ‘conexum function’.

**Luxum Function**

\( \text{Lux}(F) \mid E \) is obtained by deleting every cantle of E that includes one or more signets that are in F.
Where ‘lux(F)’ is referred to as a ‘luxum function’.

**Coluxum function**

\( \text{Col}(F) \mid E \) is obtained by deleting every cantle of E that consists of signets all of which are not in F.
Where ‘col(F)’ is referred to as a ‘coluxum function’.

**Pactum Function**

\( \text{Pac}(F) \mid E \) is obtained by deleting every cantle of E that is not a variant of a cantle in F.
Where ‘pac(F)’ is referred to as a ‘pactum function’.
**Copactum Function**

Cop(F) | E is obtained by deleting every cantle of E that is a variant of a cantle of F. Where 'cop(F)' is referred to as a 'copatum function'. For example, let formices E and F be as follows:

\[
E = \text{rinid}(6,3,2,2) | \text{lamid}(2,2) | [2,1;1,2]
\]

and

\[
F = \text{rin}(1,5,2) | \{[2,3;2,5], [2,3;3,4], [3,4;2,5]\},
\]

then if

\[
G_1 = \text{lux}(F) | E \\
G_2 = \text{col}(F) | E \\
G_3 = \text{nex}(F) | E \\
G_4 = \text{con}(F) | E \\
G_5 = \text{pac}(F) | E
\]

and

\[
G_6 = \text{cop}(F) | E
\]

then the plots of the eight formices are shown in Fig 3.4.14.

**3.4.6 NODE NUMBERING FUNCTIONS**

When a structural system is to be analyzed by a digital computer it would be necessary to prepare a complete description of the system providing information about the interconnection pattern, geometric particulars, material properties, external loads and support conditions.

Generally for the description of the interconnection pattern of a structural system, one may identify the nodal points of the system by a sequence of natural numbers and specify the interconnection pattern in terms of these numbers. Node numbers may also be used to describe the position of the external loads and the supports.

A group of node numbering functions and the concept of
Fig 3.4.14
catena may be used to describe the disposition of the elements, loads or supports into a form relative to an implied sequence of node numbers. These functions are known as 'dictum', 'redictum', 'seviation' and 'novation' functions.

Concept of a Catena

Consider a formex \( \mathbf{E} \) and let \( \mathbf{T} \) be an ingot of the same grade as \( \mathbf{E} \). The ingot \( \mathbf{T} \) is said to be a 'catena' of \( \mathbf{E} \) provided that for every chosen signet \( S \) of \( \mathbf{E} \) there is at least one signet in \( \mathbf{T} \) that is equal to \( S \). Moreover, if an ingot \( \mathbf{T} \) which is a catena of \( \mathbf{E} \) is nonprolate and every signet of \( \mathbf{T} \) is contained in \( \mathbf{E} \), then \( \mathbf{T} \) is said to be an 'exclusive catena' of \( \mathbf{E} \) and is said to be an 'inclusive catena' of \( \mathbf{E} \), otherwise.

For instance, if

\[
\mathbf{E} = \{ [3,7; 4,6], [5,5; 4,6; 5,5], [3,7; 5,5] \}
\]

\[
\mathbf{T}_1 = \{ [4,6], [3,7], [5,5] \}
\]

and

\[
\mathbf{T}_2 = \{ [3,7], [2,8], [5,5], [3,7], [4,6] \}
\]

then \( \mathbf{T}_1 \) is an exclusive catena of \( \mathbf{E} \) and \( \mathbf{T} \) is an inclusive catena of \( \mathbf{E} \).

Dictum and Redictum Functions

Consider a formex \( \mathbf{E} \) and let \( \mathbf{N} \) be a catena of \( \mathbf{E} \). Let a formex \( \mathbf{G} \) be obtained from \( \mathbf{E} \) by replacing every signet \( S \) of \( \mathbf{E} \) by the orderate, with respect to \( \mathbf{N} \), of the first occurrence of \( S \) in \( \mathbf{N} \).

The relation between \( \mathbf{G} \) and \( \mathbf{N} \) is written as

\[
\mathbf{G} = \text{dic}(\mathbf{N})|\mathbf{E}
\]

where 'dic(\( \mathbf{N} \))' is referred to as a 'dictum function' and
the ingot $N$ is referred to the a 'numerant'.
For example, if

$$E=\{[1,1;1,3],[2,2;1,3;3,3],[3,1],[1,1;2,2]\}$$

and

$$N=\{[2,2],[3,3],[4,4],[3,1],[2,4],[1,3],[2,2],[1,1]\}$$

where the numbers in <> are the orderate of the signets of $N$, then

$$\text{dic}(N)\mid E=\{[8;6],[1;6;2],4,[8;1]\}.$$ 

To illustrate the application of a dictum function, consider the flat grid whose plan view is shown in Fig 3.4.15. The formex $E$ which represents the interconnection pattern of the grid and the ingot which represents the nodal points of the grid are written below in terms of the indicated normat.

$$E=\text{rinid}(3,4,2,1)\mid [1,1;3,1]\#\text{rinid}(4,3,2,1)\mid [1,1;1,2]$$

and

$$N=\text{rinid}(4,4,2,1)\mid [1,1].$$

The orderate of every signet of $N$ is written near the node represented by the signet in Fig 3.4.15.

If a formex $G$ is given as

$$G=\text{dic}(N)\mid E$$

then the formex $G$ is found to be of the form

$$G=\{[1;2],[2;3],[3;4],[5;6],\ldots,[11;15],[12;16]\}$$

and the formex $G$ represent the interconnection pattern of the grid in terms of the node numbering scheme of Fig 3.4.15.
Fig 3.4.15

Fig 3.4.16
A dictum function may also be used to express other type of structural data, for example supports or load conditions, in terms of joint numbers.

Every dictum function has an inverse which is referred to as a 'redictum function' and denoted by

\[ \text{red}(N). \]

The definition of a redictum function is as follows:
Let \( E \) be a formex and let an ingot \( N \) be a catena of \( E \).

If
\[ G = \text{dic}(N) | E \]
then
\[ E = \text{dic}^{-1}(N) | G = \text{red}(N) | G. \]

A redictum function has the advantage in relation to saving the storage space in a computer.

Seviation Function

Let \( E \) be a formex of the first grade and let

\[ [U_1; U_2; \ldots ; U_n] \]

be a typical catena of \( E \). Let the differences between all possible pairs of uniples

\[ U_1, \ U_2, \ldots , U_n \]

be compared and the greatest of these differences be denoted by \( \delta \), where if \( n=1 \) then \( \delta \) is considered to be equal to zero.

Finally let \( \delta 's \) for all the cantles of \( E \) be compared and the greatest of these be denoted by \( \Delta \). The integer \( \Delta \) is
referred to as the 'seviation' of \( E \) and the relation between \( E \) and \( \Delta \) is written as
\[ \Delta = \text{sev}|E. \]

For example, if
\[ E = \{ [20; 54], [4; 12; 44], 15, [17; 42] \} \]

then \( \delta \)'s for the first, second, third and fourth cantles of \( E \) are 34, 40, 0 and 25, respectively, and 40 which is the greatest number of these will be \( \Delta \) of \( E \).

Thus
\[ \Delta = \text{sev}|E = 40. \]

The practical significance of the seviation function may be illustrated using the grid which is shown in Fig 3.4.15.

As found before,
\[ G = \text{d}ic(N)|E \]
\[ = \{ [1; 2], [2; 3], [3; 4], \ldots, [11; 15], [12; 16] \}. \]

Now it will be found that
\[ \Delta = \text{sev}|G = 4. \]

This number is the greatest difference between the terminal node numbers for an element in the configuration considered. This number is a measure of the band width of the stiffness matrix of the configuration if stiffness method is used to analyze this structure. Thus, the seviation function can be used to provide band width information in automated structural analysis processes.
Novation function

consider the braced dome shown in Fig 3.4.16 which is represented in terms of a 20-40-sect parispherical normat. The interconnection pattern of this dome may be written as

\[ D = \text{rinit}(10,3,2,2) | \text{rosat}(1,3) | [12,0,3;12,1,4]. \]

If the dictum function is used to transform \( D \) into a formex that describes the interconnection pattern of the dome in terms of a node numbering system, a problem would be encountered. This problem is that both

\[ [12,0,7] \]

and

\[ [12,20,7] \]

represent the same nodal point \( x \) which is shown in Fig 3.4.16.

As a result it is impossible to associate a unique number with this node through a dictum function. This implies that the double represented node will be treated as disconnected. A similar problem arises for all nodes lying on line shown as \((0)\) and \((20)\).

Thus, if \( D \) was to serve as data describing the interconnection pattern of the dome for structural analysis program, then \( D \) has to be modified to take into account the disconnected nodes correctly.

A 'novation function' can be used to solve this problem. Consider a formex \( E \) and let \( F \) be a 2-plex formex of the same grade as \( E \). Let the first cantle of \( F \) be represented by

\[ [S_1;S_2]. \]
Let $E$ be modified by replacing every signet of it which is equal to $S_1$ by $S_2$ and this process be repeated for all the cantles of $F$ proceeding in the natural order. Let the resulting formex be denoted by $G$. The relation between $E$ and $G$ is written as

$$G = \text{nov}(F) \mid E$$

where 'nov($F$)' is referred to as a novation function.

For example, if

$$E = \{[2,3;4,2;6,3],[1,2;5,6],[5,7]\}$$
and

$$F = \{[2,3;1,1],[6,3;1,1],[1,1;3,2],[5,6;1,2]\}$$
then

$$G = \text{nov}(F) \mid E = \{[3,2;4,2;3,2],[1,2;1,2],[5;7]\}$$

The practical application of a novation function is illustrated by using the above mentioned nodal problem of the dome described in Fig 3.4.16. The formex formulation of the dome is described by $D$. Now, if

$$F = \text{rin}(3,3,2) \mid [12,20,3;12,0,3]$$
$$= \{[12,20,3;12,0,3],[12,20,5;12,0,5],[12,20,7;12,0,7]\}$$

then

$$\text{nov}(F) \mid D$$

is a formex representing the interconnection pattern of the dome without any problem regarding discontinuities at the node.

3.4.7 MISCELLANEOUS FORMEX FUNCTIONS

A number of different types of formex functions are introduced in the preceding sections. These functions
provide a wealth of conceptual aids for configuration processing. The objective of this section is to introduce further formex functions that can, at times, enhance the capabilities in dealing with formex formulations. However, even with these additional functions, the possibilities are not completely exhausted and, whenever the situation demands it, one may define other formex functions that are of use in particular classes of problems or are of universal applicability.

RELECTION FUNCTIONS

Consider a formex E and let there be a condition, denoted by P, such that every cantle of E either satisfies or does not satisfy P in an unambiguous manner. That is, P with respect to a cantle of E is either true or false. Let a formex G be obtained from E by examining the cantles of E proceeding in the natural order, and deleting every cantle for which the condition P is false. The rule by which E is transformed into G is symbolized in terms of a function, this function is denoted by

\[ \text{rel}(P) \]

and is referred to as a 'relection function'. The formex G is referred to as the relection of E with respect to P and the relation between E and G is written as

\[ G = \text{rel}(P) | E \]

A condition of the type used in conjunction with relection functions is referred to as a 'perdicant'. In general a perdicant is defined as a Boolean function which has one or more formices as arguments.

Table 3.4.4 contains the description of an assortment of
formex functions. The canonic parameters of some of the functions in this table are monadic and dyadic perdicants. A brief description of monadic perdicants is given above in relation to the relection function. A 'dyadic perdicant' is a condition for comparing two formices or two parts of a formex.

The constructs

\[ U(i, j) \text{ and } W(i, j) \]

may be used for comparing two cantles of a formex. Both of these constructs represent the \( j \)th uniple of the \( i \)th signet of a cantle, but \( U(i, j) \) refers to a preceding cantle and \( W(i, j) \) refers to a following cantle. For instance, consider the formex constant

\[
[6,7;5,3],[5,5;6,7],[7,5;8,6],
[8,6;7,5],[6,4;6,5]\
\]

and let it be required to order the cantles in such a way that the first uniples of the cantles are in the ascending order. This condition may be specified by

\[ U(1,1) < W(1,1) \]

The rearrangement of the cantles in accordance with this condition will give rise to

\[
[5,5;6,7],[6,7;5,3],[6,4;6,5],
[7,5;8,6],[8,6;7,5]\
\]

The above condition is an example of a dyadic perdicant. When the parts of the formices to be compared are signets then the constructs

\[ U(i, j) \text{ and } W(i, j) \]
may be written as

\[ U(j) \text{ and } W(j) \]

respectively.

<table>
<thead>
<tr>
<th>FUNCTION</th>
<th>DESCRIPTION</th>
</tr>
</thead>
<tbody>
<tr>
<td>ras(p) (rapported sequation)</td>
<td>Rapported sequation function transforms a formex into a sequation of it (that is, a formex obtained by a rearrangement of its cantles) with the rearrangement of the cantles controlled by p. For example, if E is a formex expression whose value is {[4,3;1,2],[2,1;3,4],[3,2;2,3]} then the value of ( \text{ras}(U(1),W(1))E ) is {[2,1;3,4],[3,2;2,3],[4,3;1,2]}</td>
</tr>
<tr>
<td>rav(p) (rapported variant)</td>
<td>Rapported variant function transforms a formex into a variant of it (that is, a formex obtained by a rearrangement of the signets of its cantles) with the rearrangement of the signets of the cantles being controlled by p. For example, if E is a formex expression whose value is {[4,3;1,2],[2,1;3,4],[3,2;2,3]} then the value of ( \text{rav}(U(1)&lt;W(1))E ) is {[1,2;3,4],[2,1;3,4],[2,3;3,2]}</td>
</tr>
<tr>
<td>vin(b1,b2) (vinculum)</td>
<td>Vinculum functions transforms a given formex F into a formex every cantle of which consists of a pair of signets of F. Every such cantle {U11,U12,...,U1N;U21,U22,...,U2n} must satisfy the condition that the 'metrum' of the cantle which is defined by ( \sqrt{(U21-U11)^2+(U22-U12)^2+...+(U2n-U1n)^2} ) is within the bounds given by the values of b1 and b2. For instance, if E is a formex expression whose value is {[1,1],[1,2],[2,1],[2,2]} then the value of ( \text{vin}(1.4,1.5)E ) is {[1,1;2,1],[1,2;2,1]}</td>
</tr>
<tr>
<td>Function</td>
<td>Description</td>
</tr>
<tr>
<td>----------</td>
<td>-------------</td>
</tr>
<tr>
<td>pan(d, n)</td>
<td>Pansion function transforms a formex F into a formex which is obtained by inserting the value of n as a uniple in every signet of F at the position indicated by the value of d. For example, if E is a formex expression whose value is ${(4,1,3,2), [2,2], [2,3]}$ then the value of $\text{pan}(2,5)E$ is ${(4,5,1,3,5,2), [2,5,2], [2,5,3]}$.</td>
</tr>
<tr>
<td>dep(d)</td>
<td>Depansion function transforms a given formex F into a formex which is obtained by removing the uniple whose position is specified by the value of d from every signet of F. For instance, if E is a formex expression whose value is ${(4,3,2,1), [5,4,3,2; 6,5,4,3]}$ then the value of $\text{dep}(3)E$ is ${(4,3,1), [5,4,2; 6,5,3]}$.</td>
</tr>
<tr>
<td>med</td>
<td>Medulla function transforms a formex F into an ingot that consists of all the distinct signets of F. For instance, if E is a formex expression whose value is ${(2,1; 3,2), [3,2; 4,3; 2,1], [4,3]}$ then the value of $\text{med}E$ is ${(2,1), [3,2], [4,3]}$.</td>
</tr>
<tr>
<td>ram(p)</td>
<td>Rapported medulla function is equivalent to a composite function that involves the medulla and the rapported sequation functions. To elaborate, $\text{ram}(p)$ is equivalent to $\text{ras}(p)\text{med}$. For example, if E is a formex expression whose value is ${(2,1; 3,2), [3,2; 4,3; 2,1], [4,3]}$ then the value of $\text{ram}(U(1)&gt;W(1))E$ is ${(4,3), [3,2], [2,1]}$.</td>
</tr>
</tbody>
</table>

The types of values of the designators for the functions of Table 3.4.4 are given in Table 3.4.5.
Table 3.4.5 Types of Values of Miscellaneous Formex Functions Designators

<table>
<thead>
<tr>
<th>FUNCTION DESIGNATOR</th>
<th>TYPE OF VALUE</th>
</tr>
</thead>
<tbody>
<tr>
<td>ras(p)</td>
<td>E</td>
</tr>
<tr>
<td>rav(p)</td>
<td>E</td>
</tr>
<tr>
<td>vin(b1,b2)</td>
<td>The same as type of value of E</td>
</tr>
<tr>
<td>pan(d,n)</td>
<td>E</td>
</tr>
<tr>
<td>dep(p)</td>
<td>E</td>
</tr>
<tr>
<td>med</td>
<td>E</td>
</tr>
<tr>
<td>ram(P)</td>
<td>E</td>
</tr>
</tbody>
</table>

3.5 FORMIAN: THE PROGRAMMING LANGUAGE OF FORMEX ALGEBRA

3.5.1 INTRODUCTION

Formian is a preprocessing language that may be employed to generate information about various aspect of a structural system from element connectivity to loading and support arrangements. The information generated may be used for graphic visualisation of the structural system or may be submitted as input data to an analysis package. Formian is mathematically based on the concept of formex algebra. In the following sections various elements of Formian are described. These elements include, primitive constituents, assignment statements, operations and expressions, formex functions, numeric functions, Formian graphics, information transfer, organisational statements and schemes and induction statements.
3.5.2 PRIMITIVE CONSTITUENTS

Characters

Characters are the main building blocks of the syntactic constructs of Formian. A character is a digit or a letter or a symbol or a layout character. A digit is any one of the ten decimal digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. A letter is any one of the fifty two uppercase and lowercase letters of the English alphabet. A symbol is any distinctive printable mark that is not a letter or a digit, for example, +, -, >, &, %, @. A layout character is either a space character or a new line character.

Keywords

A keyword is any one of the following twelve sequences of letters which are used for initiation of various activities.

KEEP  TAKE  PRINT  GIVE  DRAW  SHOW
SUBMIT  ERASE  USE  RECALL  EXIT  STOP.

A keyword may be given in uppercase or lowercase letters.

Identifiers

An identifier is a sequence of letters (no distinction is made between uppercase and lowercase) and digits whose first character is a letter. An identifier may not be the same as a keyword and may not have more than eight characters.

For example,

FLOOR  B  fg77  doMat  Yo34p

are identifiers.
Identifiers are used for naming variables or functions.

3.5.3 CONSTANTS

Formian has the following five types of constants which represent integer numbers, noninteger real numbers, integer formices, noninteger formices and character strings.

Integer Constants

An integer constant is a sequence of one or more digits which may be preceded by a plus or minus sign. For example,

5, -2345, +76, 983201

are integer constants.

Floatal Constants

A floatal constant is one of the following forms:

(1) an integer constant followed by a decimal point followed by a sequence of one or more digits.
(2) a floatal constant of the first form followed by the letter E followed by an integer constant.
(3) an integer constant followed by the letter E followed by an integer constant.

For example,

0.987, 6.23E3, 678.2E-5, 53E4, 5E8

are floatal constants.

The term 'numeric constant' is used to refer to an integer constant or a floatal constant.
Formex Constants

A formex constant is a structured sequence of numeric constants, commas, semicolons, square brackets and curly brackets. An 'integer formex constant' is a formex constant whose uniples are all integer constants. A 'floatal formex constant' is a formex constant which contains one or more floatal constants. For example,

\[
[[3,2;43],[-8,5;45],[3,5;2,8]]
\]

is an integer formex constant and

\[
[[3.5,5;4,5.3],[23,4,5,9.45]]
\]

is a floatal formex constant.

String Constants

A string constant is of the form 'C' where ' is the quote symbol and C is a sequence of characters which is referred to as the value of a string constant. For example,

'We are now typing a string constant'

is a string constant.

If a string constant has a value which is a null character sequence then the string character is denoted by '' and is referred to as the empty string constant.

3.5.4 ASSIGNMENT STATEMENTS AND VARIABLES

A variable is an identifier which has been assigned a value. There are five types of variables, namely, integer variables, floatal variables, integer formex variables,
floatal formex variables and string variables.

The general form of an assignment statement is written as

\[ \text{identifier} = \text{expression} \]

where the symbol '=' is referred to as the assignment symbol.

An 'expression' is structured sequence of constants, variables operators, .........., etc as will be described latter. The effect of an assignment may be conveniently explained in terms of constants as follows:

The assignment statement

\[ N=657 \]

has the effect of creating a variable \( N \) which will represent the integer 657. This variable is said to an integer variable since the expression is an integer constant.

If the above assignment statement is followed by the assignment statement

\[ N=5.238 \]

then it has the effect of discarding the old value of \( N \) which is 657 and assigning a new value which is 5.238 to it. The identifier \( N \) will then become a floatal variable at this point and continue to be so until it is affected by another assignment statement.

Furthermore, if

\[ E=\{[5,4;6,2],[3,7;1,2]\} \]
\[ F=\{[3.5,4.5;1,2],[3,2;4,6.4]\} \]

and
then the first assignment statement creates an integer formex variable E, the second assignment statement creates a floatal formex variable F and the third one creates a string variable FL. In addition, a formex formation is a construct similar to a formex constant in which one or more uniples are given as numeric expressions. For example, the right hand side of the assignment statement

\[
G = [i+4,5; 7,j]
\]

is a formex formation and if i = 2 and j = 6 then the variable G will represent the formex

\[
[6,5;7,6]
\]

3.5.5 OPERATIONS AND EXPRESSIONS

An operation is a rule for production of a value from one or more operands an is represented by an operator. For example the operation of producing the sum of two numeric entities may be denoted by a construct such as

\[
7 + 4
\]

where 7 and 4 are referred to as the 'operands' and the symbol + is referred to as the 'operator'.

Formian has seven operations which are listed in table 3.5.1.
Table 3.5.1 Formian Operations

<table>
<thead>
<tr>
<th>OPERATION</th>
<th>OPERATOR</th>
<th>NUMBER OF OPERATIONS</th>
<th>TYPE OF RESULTING VALUE</th>
</tr>
</thead>
<tbody>
<tr>
<td>addition</td>
<td>+ (plus)</td>
<td>2</td>
<td>If both operands have integer values then the operations yields an integer value, otherwise the operation yields a floatal value.</td>
</tr>
<tr>
<td>subtraction</td>
<td>- (minus)</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>multiplication</td>
<td>* (sidus)</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>division</td>
<td>/(solidus)</td>
<td>2</td>
<td>Operation yields a floatal value.</td>
</tr>
<tr>
<td>exponentiation</td>
<td>^ (tantis)</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>sign prefixion</td>
<td>+ (plus)</td>
<td>1</td>
<td>operation yields a value of the same type as that of the operand.</td>
</tr>
<tr>
<td></td>
<td>or - (minus)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>formex composition</td>
<td># (duplus)</td>
<td>2</td>
<td>If both operands have integer formices then the operation yields an integer formex, otherwise the operation yields a floatal formex.</td>
</tr>
</tbody>
</table>

The first six operations in Table 3.5.1 are for numeric operands and are referred to as numeric operations. The seventh operation is referred to as composition and applies to formex operands.

In Formian two types of expressions are used, which are 'numeric expressions' and 'formex expressions'. A numeric expression is a mathematically meaningful combination of numeric constants, numeric variables, numeric functions designators, numeric operators and parentheses. A formex expression is a meaningful combination of formex variables, formex functions designators, duplus symbols, parentheses and formex formations.
The order of performance of various processes for evaluation of an expression are shown in Table 3.5.2.

Table 3.5.2 Precedence Order

<table>
<thead>
<tr>
<th>PROCESS</th>
<th>PRECEDENCE</th>
</tr>
</thead>
<tbody>
<tr>
<td>sign prefixion</td>
<td>Highest Precedence (executed first)</td>
</tr>
<tr>
<td>evaluation of function designator</td>
<td>Decreasing Degree of Precedence</td>
</tr>
<tr>
<td>exponentiation</td>
<td>Lowest precedence (executed last)</td>
</tr>
<tr>
<td>multiplication and division</td>
<td></td>
</tr>
<tr>
<td>addition and subtraction</td>
<td></td>
</tr>
<tr>
<td>formex composition</td>
<td></td>
</tr>
</tbody>
</table>

Parentheses may be used to alter the order of operator precedence and operations are executed from left to write for operators of equal precedence.

3.5.6 FUNCTIONS AND FUNCTIONS DESIGNATORS

Functions are defined in two different ways. One type of functions are referred to as 'standard functions' whose definitions are incorporated into the Formian Interpreter and are available to all users. The other type of functions which are referred to as 'supplementary functions' may be defined by adding program segment to the Formian Interpreter and creating special versions of Formian.

Numerical Functions

A function that give rise to a numeric value is referred to as a 'numeric function' and there are fourteen standard numeric functions in Formian which are listed in Table 3.5.3.
<table>
<thead>
<tr>
<th>FUNCTION</th>
<th>VALUE OF FUNCTION DESIGNATOR WITH ARGUMENT X BEING A NUMERIC EXPRESSION</th>
<th>TYPE OF VALUE</th>
</tr>
</thead>
<tbody>
<tr>
<td>ric (rounded integer conversion)</td>
<td>The value of ric</td>
<td>X is the integer number nearest to the value of X, e.g. the value of ric</td>
</tr>
<tr>
<td>tic (truncated integer conversion)</td>
<td>The value of tic</td>
<td>X is the integer number obtained by truncating the fractional part of the value of X, e.g. the value of tic</td>
</tr>
<tr>
<td>sign</td>
<td>The value of sign</td>
<td>X is the integer number 1, 0 or -1 depending on the value of X being positive, zero or negative, respectively, e.g. the value of sign</td>
</tr>
<tr>
<td>abs (absolute value)</td>
<td>The value of abs</td>
<td>X is the absolute value of X, e.g. the value of abs</td>
</tr>
<tr>
<td>floc (floatal conversion)</td>
<td>The value of floc</td>
<td>X is the value of X in the floating point form.</td>
</tr>
<tr>
<td>sqrt (square root)</td>
<td>The value of the sqrt</td>
<td>X is the square root of X, e.g. the value of sqrt</td>
</tr>
<tr>
<td>sin (sine)</td>
<td>The value of sin</td>
<td>X is the sine of the value of X, regarded as an angle in degrees, e.g. the value of sin</td>
</tr>
<tr>
<td>cos (cosine)</td>
<td>The value of cos</td>
<td>X is the cosine of the value of X, regarded as an angle in degrees, e.g. the value of cos</td>
</tr>
<tr>
<td>tan (tangent)</td>
<td>The value of tan</td>
<td>X is the tangent of the value of X, regarded as an angle in degrees, e.g. the value of tan</td>
</tr>
<tr>
<td>Function</td>
<td>Description</td>
<td>Type</td>
</tr>
<tr>
<td>-----------</td>
<td>-----------------------------------------------------------------------------</td>
<td>-------</td>
</tr>
<tr>
<td>asin (arcsine)</td>
<td>The value of asin(x) is the angle in degrees (in the range of -90 to 90 degrees) whose sine is equal to the value of (x) (the value of (x) must be in the range -1 to 1), e.g. the value of asin(1) is 90.0.</td>
<td>Floatal</td>
</tr>
<tr>
<td>acos (arccosine)</td>
<td>The value of acos(x) is the angle in degrees (in the range of 0 to 180 degrees) whose cosine is equal to the value of (x) (the value of (x) must be in the range -1 to 1), e.g. the value of acos(0.5) is 60.0.</td>
<td>Floatal</td>
</tr>
<tr>
<td>atan (arctangent)</td>
<td>The value of atan(x) is the angle in degrees (in the range -90 to 90 degrees) whose tangent is equal to the value of (x), e.g. the value of atan(-1) is -45.0.</td>
<td>Floatal</td>
</tr>
<tr>
<td>ln (natural logarithm)</td>
<td>The value of ln(x) is the natural logarithm of the value of (x) (the value of (x) must be positive), e.g. the value of ln(1) is 0.0.</td>
<td>Floatal</td>
</tr>
<tr>
<td>exp (exponential)</td>
<td>The value of exp(x) is (e^v) where (e) is the base of the natural logarithm and (v) is the value of (x), e.g. the value of exp(0) is 1.0.</td>
<td>Floatal</td>
</tr>
</tbody>
</table>

Formex functions

Formex functions in Formian conform to their respective definitions in formex algebra.

3.5.7 INFORMATION TRANSFER STATEMENTS

The statements which are referred to as 'information transfer statements' and which are described below are used for the transfer of information between different storage and input-output media. A computing system on which Formian is implemented is assumed to include a terminal, a working memory, a repository and some input-output channel as described below.
The terminal is a device through which Formian instructions are inputted and from which system messages and other items of information are outputted. In addition, the terminal has an associated graphics output medium. This may be a region of the screen of the terminal or a separate graphics screen.

The working memory is a medium for storage of values of variables during a Formian session. Storage of information in the working memory is on a temporary basis. At the commencement of a Formian session the working memory is empty. As variables are created and processed, their values are stored in the working memory but at the end of the session all these values are irrecoverably lost.

The repository is a medium for storage of information on a permanent basis. The storage capacity of the repository is normally much greater than that of the working memory but before one can perform any operation on the values in the repository they must be transferred to the working memory.

A printing channel indicates a particular way of textual output on a printing medium. A printing channel is specified by a number. There may be more than one printing channel but, at any given moment, one of them will be the current printing channel. The manner in which a channel is made current will be described latter.

There are also one or more graphical input channels and one or more graphical output channels and, at any given moment, there will be one current graphical input channel and one current graphical output channel.

Information transfer is effected using KEEP, TAKE, PRINT, GIVE, DRAW, SHOW and SUBMIT statements.
Keep Statement

A keep statement is a construct of the form

\[ \text{keep } V_1, V_2, \ldots, V_n \]

where 'keep' is a keyword and \( V_1, V_2, \ldots, V_n \) are variables.
A keep statement is used to store items such as formices and strings permanently, so that they can be used later or kept for future reference. A variable that is kept is referred to as a 'covariable'.

Take Statement

A take statement is a construct of the form

\[ \text{take } C_1, C_2, \ldots, C_n \]

where 'take' is a keyword and \( C_1, C_2, \ldots, C_n \) are covariables.
The retrieval of items stored in the repository is achieved through take statements.

Print Statement

A print statement is a construct of the form

\[ \text{print } V_1, V_2, \ldots, V_n \]

where 'print' is a keyword and \( V_1, V_2, \ldots, V_n \) are variables.
A print statement is used to print the values of the variables on the medium indicated by the current printing channel. Also symbol '>' which is referred to as the trude symbol may be used for a pagethrow (newpage). The comma between a trude symbol and a variable may be omitted.
Give Statement

A give statement is a construct of the form

\[
give \, V_1, \, V_2, \, \ldots, \, V_n
\]

where 'give' is a keyword and \( V_1, \, V_2, \, \ldots, \, V_n \) are variables.

The effect of a give statement is the same as that of the print statement except that the output will appear on the screen of the terminal.

Draw Statement

A draw statement is a construct of the form

\[
draw \, V_1, \, V_2, \, \ldots, \, V_n
\]

where 'draw' is a keyword and \( V_1, \, V_2, \, \ldots, \, V_n \) are formex variables and/or string variables.

A draw statement is used to produce graphical representations of formices together with textual material to appear on the medium indicated by the current graphical output channel.

Similar to the case of print statement, the trude symbol > may be used whose effect is, for example,

1. on graphical screen, to clear the screen
2. on a device that issues sheets of paper, to issue a sheet
3. on a device whose output medium is a roll of paper to shift the plotting area by a certain length along the roll.
Show Statement

A show statement is a construct of the form

\[
\text{show } V_1, V_2, \ldots, V_n
\]

where 'show' is a keyword and \( V_1, V_2, \ldots, V_n \) are formex variables and/or string variables. The effect of a show statement is the same as that of a draw statement except that the output will appear on the graphics medium associated with the terminal.

Submit Statement

A submit statement is a construct of the form

\[
\text{submit } P_1, P_2, \ldots, P_n
\]

where 'submit' is a keyword and each of the entities \( P_1, P_2, \ldots, P_n \) is a data structure called a 'plenix'. The role of a submit statement is to transform formices and other entities into files that may be used as input data for various application programs and packages.

3.5.8 ORGANISATIONAL STATEMENTS

There are five Formian statements that are used for organisational and house keeping purposes and are described as follows:

Erase Statement

An erase statement is a construct of the form

\[
\text{erase } A_1, A_2, \ldots, A_n
\]

where 'erase' is a keyword and \( A_1, A_2, \ldots, A_n \) are
variables or covariables enclosed in parenthesis. The erase statement will cause the listed variables and covariables together with their values to be erased from the working memory and repository, respectively. For example, the execution of the statement

```
erase MAM, DELTA, (DOME)
```

will result in the variables MAM and DELTA and the covariable DOME together with their values to be erased.

Use Statement

A use statement is a construct of the form

```
use A1, A2, ......., An
```

where 'use' is a keyword and each one of the entities A1, A2, ......., An are referred to as a use-item. A use statement is used for definition of standard retronorms, view specifiers as will be described later, and many other specifications as described below:

(1) The use-item for definition of input-output channel is

```
ch(i)
```

where 'ch' stands for channel and 'i' is an identification number of a channel. For example, table 3.5.4 shows a possible set of channels and their identification numbers.
Table 3.5.4

<table>
<thead>
<tr>
<th>CHANNEL NUMBER</th>
<th>DESCRIPTION OF CHANNEL</th>
<th>TYPE OF CHANNEL</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Dot Matrix Printer</td>
<td>Printing channel</td>
</tr>
<tr>
<td>2</td>
<td>Laser printer</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Daisy Wheel Printer</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Mouse</td>
<td>Graphical input channel</td>
</tr>
<tr>
<td>5</td>
<td>Graphics Tablet</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>Light Pen</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>Colour Graphics screen</td>
<td>Graphical output channel</td>
</tr>
<tr>
<td>8</td>
<td>Laser plotter</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>Pen plotter</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>Thermal Plotter</td>
<td></td>
</tr>
</tbody>
</table>

(2) The use-item for definition of line width for tenons and fronds is

\[ lw(r) \]

where 'lw' stands for line width and \( r \) is line width in millimetres.

(3) The use-item for definition for tenon style is

\[ ts(n,d) \]

where 'ts' stands for tenon style and \( n \) is an identification number of the type of symbol as listed below and \( d \) is the dimension of the symbol in millimetres.

\[
\begin{align*}
  n=1 & \quad \circ \\
  n=2 & \quad \circ \\
  n=3 & \quad \bullet \\
  n=4 & \quad \bigcirc \\
  n=5 & \quad \text{empty circle} \\
  n=6 & \quad \square \\
  n=7 & \quad \square \\
  n=8 & \quad \blacksquare \\
  n=9 & \quad \blacksquare \\
  n=10 & \quad \text{empty square}.
\end{align*}
\]
(4) The use-item for definition of the line style is 
\text{ls}(p)
where 'ls' stands for line style and p is an
identification number of the type of line as
listed below.
\begin{align*}
p=1 & \quad \text{full line} \\
p=2 & \quad \text{dashed line} \\
p=3 & \quad \text{dotted line} \\
p=4 & \quad \text{axis line}
\end{align*}

(5) The use-item for definition of frond style is 
\text{fs}(n)
where 'fs' stands for frond style and n is 1, 2 or
3 which is described below:

If the frond represent a cantle with t signets and
the points representing these signets are \(P_1, P_2, \\
\ldots, P_n\), then
\begin{align*}
n=1 & \quad \text{The frond consists of t tenons and t-1} \\
& \quad \text{line segments } P_1 - P_2, P_2 - P_3, \ldots, P_{t-1} - P_t. \\
n=2 & \quad \text{The frond consists of t tenons and t} \\
& \quad \text{line segments } P_1 - P_2, P_2 - P_3, \ldots, P_{t-1} - P_t, P_t - P_1. \\
n=3 & \quad \text{The constitution of the frond is the} \\
& \quad \text{same as for } n=2 \text{ but the area inside of} \\
& \quad \text{the frond is filled with colour.}
\end{align*}

(6) The use-item for definition of the fount of

textual material on graphical output is
\text{tf}(t, h, w)
where 'tf' stands for text fount, t is the
identification number of the typeface of the
characters, h gives the height of each character
in millimetres and w gives the width of each
character in millimetres.

(7) The use-item for definition of position and orientation of text in the picture plane is

\[ \text{tg}(p,q,\alpha) \]

where 'tg' stands for text guide, \( p \) and \( q \) are the coordinates of the position of the bottom left corner of the first character and \( \alpha \) is the angle in degrees which determines the orientation of the text, as shown in Fig 3.5.1.

(8) The use-item for definition of colour effect is

\[ c(n,h) \]

where 'c' stands for colour, \( n \) is an identification number of the item to be coloured as defined below and \( h \) is an identification number of the hue.

- \( n=1 \) line
- \( n=2 \) tenon
- \( n=3 \) infill
- \( n=4 \) text
- \( n=5 \) background

(9) The use-item for definition of pen selection for a pen plotter is

\[ \text{pen}(n) \]

where \( n \) is the number specifying one of the pens of a pen plotter.

Recall Statement

A recall statement of a construct of the form recall V
Fig 3.5.1
where 'recall' is a keyword and V is a string variable. The recall statement enables editing of the contents of string variable V specified in the statement. Absence of V in this statement effects the display of the statement which has been executed last.

**Exit Statement**

An exit statement is a construct of the form

```plaintext
exit
```

where 'exit' is a keyword. The effect of the statement is to leave the Formian session temporarily and go to the operating system of the computer. When the required activities within the operating system are performed, the user may return to Formian where the working environment will be found to be unchanged.

**Stop Statement**

A stop statement is a construct of the form

```plaintext
stop
```

where 'stop' is a keyword. This statement terminates the Formian session and returns the user to the operating system of the computer.

**3.5.9 FORMIAN GRAPHICS**

The concepts of the formian graphics enable the creation of a two dimensional view of a three dimensional plot on graphical output media by using use-items described below.

A plot which is to be viewed is referred to as the 'object'
and the coordinate system relative to which the plot is produced is referred to as ‘the object coordinate system’. The space in which the plot is imagined to be situated is referred to as the ‘object space’. See Fig 3.5.2.

**View Point and View Centre**

The point from which the object is to be viewed is referred to as the ‘view point’. The point which is directly viewed is referred to as the ‘view centre’ and the line that passes through the view point and the view centre is referred to as the ‘view line’. A plane which is normal to the view line at the view centre is referred to the ‘trace plane’. See Fig 3.5.3.

The view point and the view centre may be specified by use-item of the form

\[
\text{vp}(x,y,z) \quad \text{and} \quad \text{vc}(x,y,z)
\]

respectively. Where ‘vp’ stands for view point, ‘vc’ stands for view centre and x, y and z are the coordinates of the view point or the view centre relative to the object coordinate system.

**View Type**

The view type defines the type of projection of the object on the trace plane. The view type may be specified by a use-item of the form

\[
\text{vt}(n)
\]

where ‘vt’ stands for view type and n is the identification number of the type of the view.
Fig. 3.5.2

Fig 3.5.3
The illustration of the perspective projections and parallel projections are shown in Figs 3.5.2 and 3.5.3, respectively.

**View Frame**

The image of a trace called picture lies in the plane which is referred to as 'picture plane', and its coordinate system is referred to as the 'device coordinate system'. The view frame is a rectangular frame in the picture plane, restricting the region for graphic production to the area enclosed by the frame. The view frame may be specified by a use-item of the form

\[ \text{vf}(p_1, q_1, p_2, q_2) \]

where 'vf' stands for view frame and \( p_1, q_1, p_2 \) and \( q_2 \) are coordinates of the corners \( A_1 \) and \( A_2 \) of the view frame relative to the device coordinate system. See Fig 3.5.4.

**View Rise**

The view rise defines the orientation of the trace by specifying a vector in any direction in the object coordinate system where the direction of the vector in the picture plane is parallel to the vertical axis. The view rise may be specified by a use-item of the form

\[ \text{vr}(x_1, y_1, z_1, x_2, y_2, z_2) \]

where 'vr' stands for view rise and \( x_1, y_1, z_1, x_2, y_2 \) and \( z_2 \) are coordinates of the end points of the vector relative to the object coordinate system. See Fig 3.5.5.
Fig 3.5.4

Fig 3.5.5
View Helm

In cases when the starting point of the view rise is coincident with the view centre, it is possible to specify the view point, the view centre and the view rise at the same time by the 'view helm'. The view helm may be specified by a use-item of the form

\[
vh(x_1, y_1, z_1, x_2, y_2, z_2, x_3, y_3, z_3)
\]

where 'vh' stands for view helm and \(x_1, y_1, z_1, x_2, y_2, z_2\) and \(x_3, y_3, z_3\) are coordinates of the view point, the view centre and the arrowhead end of the view rise relative to the object coordinate system, respectively. See Fig 3.5.6.

View Base and View Nave

The position of the picture may be controlled by using the 'view base' and 'view nave'. The view base is a point in the object space and the view nave is a point in the picture plane. The picture will be positioned such that the image of the view base in the picture plane will coincide with the view nave. The view base and the view nave may be specified by use-items of the form

\[
v_b(x, y, z)
\]

and

\[
v_n(p, q)
\]

respectively. Where 'vb' stands for view base and \(x, y\) and \(z\) are coordinates of the view base relative to the object coordinate system. Also, 'vn' stands for view nave and \(p\) and \(q\) are coordinates of the view nave relative to the device coordinate system.
View Scale

The view scale controls the size of the picture and may be specified by a use-item of the form

\[ \text{vs}(r) \]

where 'vs' stands for view scale and \( r \) is a numeric expression whose value controls the size of the picture.

View Gauge

The view gauge is another concept for controlling the size of the picture and may be specified by a use-item of the form

\[ \text{vg}(x_1, y_1, z_1, x_2, y_2, z_2, r) \]

where 'vg' stands for view gauge, \( x_1, y_1, z_1 \) and \( x_2, y_2, z_2 \) are coordinates, relative to the object coordinate system, of the two end points of a line segment in the object space. The size of the picture is chosen such that the length of the image of the line segment in the picture plane is \( r \) millimetres.

View Zone

The view zone also controls the size and the position of the picture by zooming effects. A rectangular solid which is referred to as the 'view zone' is defined and the position and the size of the picture is chosen such that the image of the view zone in the picture plane fits the view frame. The view zone may be specified by a use-item of the form

\[ \text{vz}(x_1, y_1, z_1, x_2, y_2, z_2) \]

where 'vz' stands for view zone, \( x_1, y_1, z_1 \) and \( x_2, y_2, z_2 \)
are coordinates, relative to the object coordinate system, of two diagonally opposite vertices of the view zone.

View Mode

There are three types of modes controlling the position and the size of the picture as below:

Nave Mode: which defines the position and the size of the picture by the view base, view nave and view scale or view gauge.

Range Mode: which allows automatic positioning and scaling of the picture by fitting the 'trace range' in the view frame. Where the trace range is a rectangle in the trace plane where the sides of the rectangle are either parallel or perpendicular to the image of the view rise in the trace plane and where the whole of the trace is inside the rectangle with all four sides of the rectangle just touching the trace.

Zone Mode: which defines the position and the size of the picture by the view zone.

The mode of picture control may be specified by a use-item of the form

\[ \text{vm}(n) \]

where 'vm' stands for view mode and n is 1, 2 or 3 which specifies nave mode, range mode and zone mode, respectively.
Fig 3.5.6

Fig 3.5.7
3.5.10 SCHEMES AND INDUCTION STATEMENTS

In formex algebra it is possible to formulate in terms of parameters. A formulation of this kind is known as 'generic formulation'. Consider the configuration shown in Fig 3.5.7, a formex formulation for the configuration relative to the indicated normat may be written as

\[ G_1 = \text{pex|rinid}(12,8,1,1) \mid (\text{ros}(1,2,3/2,3/2)\mid [1,1;2,1]\# [1,12,2]). \]

This formulation can be written by using the concept of the generic formulation as

\[ G_2 = \text{pex|rinid}(M,N,1,1) \mid (\text{ros}(1,2,3/2,3/2)\mid [1,1;2,1],[1,1;2,2]). \]

When if \( M=12 \) and \( N=8 \) then \( G_2 \) represent \( G_1 \).

A convenient way of employing this formulation in Formian is to include it in an assignment statement of the form

\[ \text{GRID} = ': M,N: \text{G} = \text{pex|rinid}(M,N,1,1) \mid (\text{ros}(1,2,3/2,3/2)\mid [1,1;2,1]\# [1,1,2,2]). \]

The effect of this statement is to create a string variable \( \text{GRID} \) whose value is the generic formulation preceded by a list of parameters enclosed in colons. The part that lies between the quotation marks is referred to as a 'scheme'. A scheme consists of a heading followed by a body. The heading consists of a list of identifiers that are referred to as 'nominal parameters' and are separated by commas and are enclosed in colons. The body consists of a sequence of Formian statements that may incorporate the identifiers listed in the heading. In the above example,
is the heading and M, N are nominal parameters, and

\[ G = \text{pex|rinid}(M,N,1,1) | \]
\[ \text{ros}(1,2,3/2,3/2) | [1,1;2,1]#[1,1;2,2] ) \]

is the body of the scheme.

A string variable whose value is a scheme is referred to as a 'scheme variable'. Thus the above variable GRID is a scheme variable.

The body of the scheme in the above example may be executed through a statement of the form

\[ \text{GRID}(12,8) \]

which is referred to as an 'induction statement'. The statement causes the body of the scheme to be executed with M assuming the value 12 and N assuming the value 8. The result of the execution of the scheme is a formex variable G that represents the configuration of Fig 3.5.7.

In general, an induction statement consists of a scheme variable followed by a list of expressions referred to as 'induction parameters' enclosed in parentheses. When executing a scheme, the nominal parameters are assigned the values of the induction parameters in the given order. A scheme need not include any nominal parameters, for instance,

\[ \text{GRID2=}ireccion:= \]
\[ G = \text{pex|rinid}(12,8,1,1) | \]
\[ \text{ros}(1,2,3/2,3/2) | [1,1;2,1]#[1,1;2,2] ) \]

is a valid scheme statement.
The above scheme may be executed through the induction statement

GRID2

which will represent the configuration of Fig 3.5.7.

As another example, consider the configuration of Fig 3.5.8 which represents the plan view of a double layer grid. The top layer members are drawn in solids lines, the bottom layer members are drawn in dashed lines and the web members are drawn in dotted lines. Let it be required to write a formex formulation for the interconnection pattern in terms of the normat shown. The top layer elements parallel to the U1 axis may be represented by

\[ F_1 = \text{rinid}(10,8,2,2) | [1,1,2;3,1,2], \]

the top layer elements parallel to the U2 axis may be represented by

\[ F_2 = \text{rinid}(11,7,2,2) | [1,1,2;1,3,2], \]

the bottom layer elements parallel to the U1 axis may be represented by

\[ F_3 = \text{rinid}(9,7,2,2) | [2,2,1;4,2,1], \]

the bottom layer elements parallel to the U2 axis may be represented by

\[ F_4 = \text{rinid}(10,6,2,2) | [2,2,1;2,4,1], \]

the web members may be represented by

\[ F_5 = \text{rinid}(10,7,2,2) | \text{rosi}(2,2) | [1,1,2;2,2,1], \]

and the whole configuration may be represented by
Fig 3.5.8 double layer grid.
F = lib(i=1,5) | Fi.

The above formations can be written in a form of a scheme as

\[
\text{GRID1} = '[: :
\begin{align*}
F1 & = \text{rinid}(10,8,2,2) | [1,1,2;3,1,2] \\
F2 & = \text{rinid}(11,7,2,2) | [1,1,2;1,3,2] \\
F3 & = \text{rinid}(9,7,2,2) | [2,2,1;4,2,1] \\
F4 & = \text{rinid}(10,6,2,2) | [2,2,1;2,4,1] \\
F5 & = \text{rinid}(10,7,2,2) \ldots \\
\text{ros}(1,2,2,2) & | [1,1,2;2,2,1] \\
F & = \text{lib(i=1,5) | Fi }'
\end{align*}
\]

If it is required to write a formex formulation for the configuration shown in Fig 3.5.9 then using luxum function in the above scheme will give rise to the configuration required. The scheme will be

\[
\text{GRID2} = '[: :
\begin{align*}
F1 & = \text{rinid}(10,8,2,2) | [1,1,2;3,1,2] \\
F2 & = \text{rinid}(11,7,2,2) | [1,1,2;1,3,2] \\
F3 & = \text{rinid}(9,7,2,2) | [2,2,1;4,2,1] \\
F4 & = \text{rinid}(10,6,2,2) | [2,2,1;2,4,1] \\
F5 & = \text{rinid}(10,7,2,2) \ldots \\
\text{ros}(1,2,2,2) & | [1,1,2;2,2,1] \\
F & = \text{lib(i=1,5) | Fi } \\
F6 & = \text{rinid}(4,4,4,4) | [4,4,1] \\
F7 & = \text{lux(F6) | F }'
\end{align*}
\]

where F7 represents the interconnection pattern required.

Fig 3.5.10 shows the same configuration as in Fig 3.5.9 viewed from a different view point.

It is possible to save the scheme and the variables created by the scheme by using the keep statement. To keep the
scheme the statement should be of the form

    keep GRID

in this case a string covariable GRID is created. But to keep the formex covariable \( G \) the keep statement should be

    keep G

which will create a formex covariable \( G \) whose value is that of the variable \( G \) created through the last induction statement.

For example, if an induction statement such as

    GRID(16,10)

is followed by a keep statement like

    keep G

then a formex covariable \( G \) whose value is

\[
\text{pex|rinid}(16,10,1,1) | \\
(\text{ros}(1,2,3/2,3/2)[1,1;2,1]\#[1,1;2,2])
\]

will be created.
CHAPTER FOUR

FORMEX FORMULATION OF GEODESIC CONFIGURATIONS

4.1 INTRODUCTION

In a computer aided analysis and design involving complex configurations, data preparation and handling of graphics are the most difficult and time consuming tasks. In addition, data preparation is the stage of analysis that is very much prone to mistakes. The concept of formex algebra together with its computer language Formian provide a basis for solution of these problems, Refs 1 and 2. They allow networks of all kinds to be formulated conveniently and they are valuable tools in dealing with complex configurations.

Geodesic domes constitute an important family of braced domes, for its high degree of regularity and evenness in stress distribution in its members. However, data preparation and drawing of this family of domes, without suitable conceptual tools are extremely difficult.

It is the objective of the present work to find a way to prepare data and create configuration for different types of geodesic domes.

Formex algebra and its programming language Formian are used to develop an easy and reliable way for preparing data for the analysis program as well as drawing the configurations.

The set of the topological properties and geometric
particulars of some geodesic configurations are presented in this work. These configurations are based on five principal polyhedra, namely tetrahedron, octahedron, truncated octahedron, cube and icosahedron. Three methods of subdivision are used. These methods of subdivisions are class I, class II and class III.

Sections 4.2 through 4.8 give a detailed description of the derivation of geodesic configurations based on the icosahedron as a generator volume and the subdivided configuration is projected on some circumscribing surfaces. These surfaces may be spherical, superspherical, elliptical, superelliptical, paraboloidal, hyperbolic and conical. Class I method of subdivision is used.

Sections 4.9 and 4.10 show a general approach for formulation to generate data and create configurations for geodesic configurations based on five generator volumes using class I method of subdivision. These generator volumes are tetrahedron, octahedron, truncated octahedron, cube and icosahedron. The subdivided generator volumes are projected on the circumscribing surfaces mentioned in the previous paragraph.

Section 4.11 shows the derivation of some geodesic configurations using class II and class III methods of subdivision. Finally section 4.12 gives some examples which show how data generated from formex formulation can be organised and utilized in structural analysis programs.

4.2 GEODESIC ICOSAHEDRAL FORMULATIONS

The icosahedron is the best of the Platonic polyhedra to create acceptable geodesic domes with low frequencies and reasonable member lengths. The concept of a scheme in Formian is used to generate a formulation for geodesic domes having the icosahedron as generator volume.
The idea of J S Sanchez Alvarez, Ref 12, which formulates a spherical dome from an icosahedral generator volume was developed further in this work and this section shows a method for projecting the geodesic subdivision of the basic polyhedron faces onto some circumscribing surface using different parameters. These surfaces are spherical, elliptical, paraboloidal, hyperbolic and conical.

4.2.1 GENERAL SCHEME OF FORMULATION

The configuration of Fig 4.2.1 is used to illustrate the method of the formex formulation for geodesic forms.

The symmetries of the regular icosahedron help very much in creating the formex formulation required. The whole system is represented through symmetric transformations of a 'basic sub-structure' for which topological and geometric particulars are specified, a basic sub-structure is defined as the structure which can be used in formex formulation processes to develop the final required configuration.

The 20 equal faces of the icosahedron are conveniently grouped in four regions, namely: North-Pole, North-Belt, South-Belt and South-Pole as it is indicated in Figs 4.2.1 and 4.2.2. Furthermore, Fig 4.2.2 shows the triangulated faces of the icosahedron mapped on the plane U1-U2. Also the sub-structure labelled as part-1 is identified as 'the basic symmetry part' of the structure, whereas parts 2 to 20 are interpreted as translations and reflections of the basic symmetry part.

If a formex F1 is taken to represent topological properties, either disposition of nodes or interconnection patterns, within the basic sub-structure, that is, part-1, then the corresponding topological properties of the remaining sub-structures, that is, parts 2 to 20, can be represented as functions of the formex F1. Thus, the
Fig 4.2.1 Geodesic configuration showing basic symmetry part in thick line
North-Pole, parts 1 to 5, may be formulated as

\[ NP = \text{LIB}(k=0,4) | \text{TRAN}(2,k*2\times NF) | F1 \]

where NF is the frequency. \(2\times NF\) is the measure of the basic sub-structure along direction \(U2\) in the plot of Fig 4.2.2.

The North-Belt, parts 6 to 10 may be represented as a reflection of the formex representing the North-Pole as

\[ NB = \text{REF}(1,3\times NF) | NP \]

where \(3\times NF\) is the measure of the basic part along direction \(U1\).

The South-Belt, parts 11 to 15, may be represented as a translation of the formex representing the North-Pole as

\[ SB = \text{TRANID}(3\times NF,NF) | NP. \]

The South-Pole, parts 16 to 20, may also be represented as a translation, this time of the formex representing the North-Belt as

\[ SP = \text{TRANID}(3\times NF,NF) | NB. \]

Finally the whole configuration may be given as the composition of the formices representing the North and South regions as

\[ F = NP#NB#SB#SP. \]

**4.2.2 THE BASIC SUB-STRUCTURE**

The symbol \(F1\) in the above formulation is used in a general sense to represent the topological properties of the basic
symmetry part. Thus any kind of disposition of nodes or interconnection pattern can be assigned to the basic part via $F_1$ to obtain different topological arrangements for the whole system. The set of basic nodes, that is, the nodes in the basic part may be given as

$$G_1 = \text{LIB}(i=0, NF) | \text{LIB}(j=0, i) | \text{PROJID}(3i, NF-i+2j) | [0, 0, 0]$$

where $G_1$ replaces $F_1$ in the general formulation and the signet $[0, 0, 0]$ acts as a 'seed' for the nodes.

The interconnection pattern of the basic part can be now obtained in terms of $G_1$ as

$$F_1 = \text{VIN}(2, \sqrt{10}) | G_1$$

where the vinculum function generates all the two-plex cantles whose 'metrum' is between 2 and $\sqrt{10}$ and whose signets are all in $G_1$. In Fig 4.2.2, a line segment represents a cantle and the metrum of this cantle is simply the length of the line segment.

4.2.3 GEOMETRIC PARTICULARS

The geometric definition of the configuration given in Fig 4.2.1 is obtained by numerically specifying the mapping of the previously treated topological properties on a circumscribing surface.

The three-dimensional Cartesian coordinate system in Fig 4.2.1 is taken as the reference frame for the present coordinate specification and its origin coincides with the centre of the chosen surface which is, in this case, a sphere. Thus the coordinates $x$, $y$, $z$ of a point whose signet is $(U_1, U_2, U_3)$ are given as
\[ x = L*q1 \]  \hspace{1cm} (4.2.1)  
\[ y = L*q2 \]  \hspace{1cm} (4.2.2)  
\[ z = L*q3 \]  \hspace{1cm} (4.2.3)  

where \( q_1, q_2, q_3 \) are the coordinates of a point \( Q \) at the surface of the icosahedron and \( L \) is a positive factor which projects the point onto the sphere and is given as

\[ L = \sqrt{\frac{r + U3*t}{q_1^2 + q_2^2 + q_3^2}} \]  \hspace{1cm} (4.2.4)  

with \( (q_1^2 + q_2^2 + q_3^2) > 0 \) because no point at the surface of the icosahedron coincides with the origin of the reference system. Here, \( r \) is the radius of the desired geodesic configuration, \( t \) is the structural height which adds to \( r \) and \( U3 \), the third uniple in a signet, is used as a 'layer' indicator.

The point \( Q(q_1, q_2, q_3) \) is obtained from the symmetry operations on a corresponding point \( P(p_1, p_2, p_3) \) in the basic sub-structure. The number assigned to each sub-structure in Fig 4.2.2 determines the particular transformation for each part. The symmetry operations may be summarized as follows:

\[ Q = R_z(R_y P) \]  \hspace{1cm} (4.2.5)  

where \( R_y \) and \( R_z \) denote non-commutative rotations of the point \( P \) around the \( Y \) and \( Z \) axes, respectively. The corresponding transformations matrices are

\[
R_y = \begin{bmatrix}
\cos T & 0 & -\sin T \\
0 & 1 & 0 \\
\sin T & 0 & \cos T
\end{bmatrix}
\]

with \( T = T_1 \) for parts 1 to 5

or \( T = T_1 + 2*Tr \) for parts 6 to 10

or \( T = 180 - T_1 - 2*Tr \) for parts 11 to 15

or \( = 180 - T_1 \) for parts 16 to 20.
Fig 4.2.2
Also, \( T_r = \arctan(NF \sqrt{3/3*S}) \)
\( T_1 = \arctan(2*NF \sqrt{3/3*S}) \)
and \( S = NF \sqrt{5 + 6 \cos 72^\circ}/3 \), being geometric constant of the icosahedron expressed in terms of the frequency \( NF \), as illustrated in Fig 4.2.3.

In a similar way
\[
R_z = \begin{bmatrix}
\cos G & -\sin G & 0 \\
\sin G & \cos G & 0 \\
0 & 0 & 1
\end{bmatrix}
\]
with \( G = (n-1)*72^\circ \) for parts 1 to 10
or \( = (n-1)*72^\circ + 36^\circ \) for parts 11 to 20.

The point \( P(p_1, p_2, p_3) \) is then obtained as follows

\[
p_1 = \sqrt{3} \cdot (U_1 - NF \cdot n_1)/3
\]
\[
p_2 = U_2 - (m_1 \cdot NF + m_2 \cdot 2 \cdot NF)
\]
\[
p_3 = S
\]

where \( NF \) is the frequency of subdivision of the geodesic configuration, with the following values for \( n_1, m_1, m_2 \) and \( S \)

for the North Pole
\( n_1 = 2 \)
\( m_1 = 1 \)
\( m_2 = 0,1,2,3,4 \) for parts 1,2,3,4,5, respectively

for the North Belt
\( n_1 = 4 \)
\( m_1 = 1 \)
\( m_2 = 0,1,2,3,4 \) for parts 6,7,8,9,10, respectively

for the South Belt
\( n_1 = 5 \)
\( m_1 = 2 \)
\( m_2 = 0,1,2,3,4 \) for parts 11,12,13,14,15, respectively
Fig 4.2.3
for the South Pole
n1 = 7
m1 = 2
m2 = 0,1,2,3,4 for parts 16,17,18,19,20 respectively

and S = NF*sqrt((5 + 6*cos 72°)/3), being the radius of the insphere of a regular icosahedron with a side length of 2*NF. The insphere touches the mid-point of every face in the icosahedron, as sketched in Fig 4.2.3.

The above specifications for the point P may be interpreted in the intrinsic plot as the translations which makes the mid-point of every symmetry part to coincide with the origin of the x-y coordinate system. In plan view, the x and y axes coincide in turn with the U1 and U2 axes of the intrinsic plot, respectively. In addition, the substructures are reduced by a factor of sqrt(3)/3 along direction U1 to obtain the equilateral triangular faces of the icosahedron and the former are placed at a distance p3 above the plane x-y.

After the coordinates x, y, z of a typical point on a spherical surface are obtained as described above then spherical coordinate system can be used to project the icosahedron’s subdivided faces onto any circumscribing surface whose equation is known. The spherical coordinate system is shown in Fig 4.2.4. The relations between spherical coordinate and Cartesian coordinate systems are shown in the following section.

4.2.4 THE RELATIONS BETWEEN SPHERICAL AND CARTESIAN COORDINATES SYSTEMS

A point in space can be located in a spherical coordinate system by its φ, θ and r coordinates and the relations between these coordinates and the corresponding Cartesian coordinates of the same point can be written as
for x>0, y>0 and z>0 then
\[ \phi = \arctan \left( \frac{y}{x} \right) \]
\[ \theta = \arctan \left( \sqrt{x^2 + y^2} / z \right) \]
\[ r = \sqrt{x^2 + y^2 + z^2} \]

for x>0 and y<0 then
\[ r = \sqrt{x^2 + y^2 + z^2} \]
and for z>0
\[ \phi = 2\pi - \arctan \left( \frac{y}{\text{abs}(x)} \right) \]
\[ \theta = \arctan \left( \sqrt{x^2 + y^2} / z \right) \]
and for z<0
\[ \phi = \arctan \left( \frac{y}{x} \right) \]
\[ \theta = \pi - \arctan \left( \left( \frac{x^2 + y^2}{\text{abs}(z)} \right) \right) \]

for x<0 and y>0 then
\[ r = \sqrt{x^2 + y^2 + z^2} \]
\[ \phi = \pi - \arctan \left( \frac{y}{\text{abs}(x)} \right) \]
and for z>0
\[ \theta = \arctan \left( \sqrt{x^2 + y^2} / z \right) \]
and for z<0
\[ \theta = \pi - \arctan \left( \sqrt{x^2 + y^2} / \text{abs}(z) \right) \]

for x<0 and y<0 then
\[ r = \sqrt{x^2 + y^2 + z^2} \]
\[ \phi = \pi + \arctan \left( \frac{y}{x} \right) \]
and for z>0
\[ \theta = \arctan \left( \sqrt{x^2 + y^2} / z \right) \]
and for z<0
\[ \theta = \pi - \arctan \left( \sqrt{x^2 + y^2} / \text{abs}(z) \right) \].

For the case where both x and y are equal to zero then
\[ \phi = \pi/2 \text{ (in fact } \phi \text{ can assume any value)} \]
and for z>0
\[ r = z \]
\[ \theta = 0 \]
and for z<0
Fig 4.2.4 Spherical Coordinates System
Transformation from spherical coordinates to Cartesian coordinates

To transform from spherical coordinates to Cartesian coordinates the relations are given by

\[ x = r \sin \theta \cos \phi \]  \hspace{1cm} (4.2.6)  
\[ y = r \sin \theta \sin \phi \]  \hspace{1cm} (4.2.7)  
\[ z = r \cos \theta \]  \hspace{1cm} (4.2.8)

4.3 SUPPLEMENTARY RETRONORMS AND SCHEMES

Supplementary retronorms were used as the standard retronorms available on the current Formian interpreter could not meet the requirements of these formulations. A number of subroutines were written to define the required supplementary retronorms. In these subroutines relevant equations were used to define the coordinates of the points on various configurations. The concept of a scheme is used to generate formulations in terms of some nominal parameters.

4.4 THE SUBDIVIDED ICOSAHEDRON FACES

The subdivided icosahedron face or part of the icosahedron can be obtained by directly using the \((q_1, q_2, q_3)\) coordinates of a point \(Q\) at the surface of the icosahedron without projecting it on any circumscribing surface, that is, \(L = 1.0\). In this case, the \((x, y, z)\) of a point whose signet is \((U_1, U_2, U_3)\) will be

\[ x = q_1 \]  \hspace{1cm} (4.4.1)  
\[ y = q_2 \]  \hspace{1cm} (4.4.2)
The remaining operations on a point in the basic substructure are the same as discussed before.

The only parameters required for these configurations are, NF, the frequency of subdivision of the icosahedron's face edge and the radius R from the centre to the vertices of the icosahedron.

The scheme used to generate such configurations is as follows:

:NF,R:
G1=LIB(I=0,NF)|LIB(J=0,1)|PROJID(3*I,NF-I+2*J)|[0,0,0]
F1=VIN(2,10**0.5)|G1
NP=LIB(K=0,4)|TRAN(2,K*2*NF)|F1
NB=REF(1,3*NF)|NP
SB=TRANID(3*NF,NF)|NP
SP=TRANID(3*NF,NF)|NB
F=NP#NB#SB#SP
USE MINE(NF,R)
DRAW F

In the scheme shown above:

- G1 represents the set of basic nodes, that is, the nodes in the basic symmetry part.
- F1 represents the topological properties of the basic symmetry part, that is, the connectivity of the members.
- NP denotes the North-Pole.
- NB denotes the North-Belt.
- SB denotes the South-Belt.
- SP denotes the South-Pole.

The subroutine MINE called by this scheme defines a supplementary retronorm for this configuration. Different
interconnection patterns or disposition of nodes can be assigned to the basic part via F1 to obtain different topological arrangements for the whole or part of the icosahedron.

Different parameters were used to run the scheme and different configurations were obtained. Combinations of the four separate regions can be used to obtain different configurations. For example

Fig 4.4.1 shows a South-Belt level truncation of a 6 frequency icosahedron.

Fig 4.4.2 is a North-Pole truncation of a 10 frequency icosahedron.

4.5 VARIOUS GEODESIC CONFIGURATIONS FORMULATIONS

Once the spherical coordinates for a typical point in space are obtained then extending the vector that joins this point to the reference centre will locate new points with the same $\phi$ and $\theta$ coordinates as the original point but with different $r$ coordinates. This new point can be chosen to lie on any surface that can be defined by an equation using spherical coordinate system.

Various types of geodesic forms obtained by projecting the subdivided generator volume onto some circumscribing surfaces are discussed below.

4.5.1 GENERALISED ELLIPSOIDS

The most commonly used type of geodesic domes other than spherical domes is the elliptical domes, that is, domes having ellipses as their profiles.

The general equation of ellipsoids family can be written as
where $x$, $y$ and $z$ are ordinary Cartesian coordinates, $a$, $b$ and $c$ denote the radii of the generalized ellipsoid in the $x$, $y$ and $z$ Cartesian directions, respectively, and $u$, $v$ and $w$ define the form of the surface. By varying the above values an array of symmetrical shapes can be obtained. We may start in two dimensions by letting $z = 0$. Then if $a$ and $b$ are both equal to 1.0 and $u$ and $v$ are both equal to 2.0, we get

$$x^2 + y^2 = 1.0 \quad (4.5.2)$$

which describes a circle centred at $(0,0)$.

To get an ellipse we may let $a = 1.0$ and take up the variation in $b$ and obtain

$$x^2 + (y/b)^2 = 1.0 \quad (4.5.3)$$

Obviously $b$ determines how the radius varies as we travel round the curve. It is, in fact, the ratio of the long and short axes and it may be given the name, $E$, for expansion, Fig 4.5.1. If this curve describes the profile of a dome with equatorial truncation and if the radius of the dome at the ground level is $r_1$, then its radius at the zenith is $E*r_1$. If the $E>1.0$ the dome will be tall and if $E<1.0$ the dome will be flattened. Thus the dome headroom can be increased or diminished without altering the area of the circular floor.

Using spherical coordinates and substituting for the values of

$$x = r*sin \theta$$

and

$$y = r*cos \theta$$
in the equation of the ellipse shown below

\[ x^2 + \left(\frac{y}{E}\right)^2 = 1.0 \]  
(4.5.4)

we will have

\[ (r \sin \theta)^2 + \left(\frac{r \cos \theta}{E}\right)^2 = 1.0 \]

and rearranging we obtain

\[ r = \sqrt{\frac{E^2}{E^2 \sin^2 \theta + \cos^2 \theta}} \]  
(4.5.5)

**4.5.2 SUPERSPHERES AND SUPERELLIPSOIDS**

If the exponents \( u \) and \( v \) in equation 4.5.1 are assigned any value greater than two, then a supercircle or a superellipse is obtained depending on the value of \( E \), that is, if \( E \) is equal to 1.0, the curve is a supercircle, and if \( E \) has some other value, then the curve is a superellipse.

The supercircle radius formula is given by

\[ r = \left(\frac{1}{\sin^n \theta + \cos^n \theta}\right)^{1/n} \]  
(4.5.6)

where \( n \) is the value of the exponents \( u \) and \( v \).

The superellipse radius formula is given by

\[ r = \left(\frac{E^n}{E^n \sin^n \theta + \cos^n \theta}\right)^{1/n}. \]  
(4.5.7)

Fig 4.5.2 shows a supercircle of exponent 2.5 compared with the normal circle, it is clear that the supercircle has shoulders. Fig 4.5.3 shows a superellipse of exponent 2.5 which is less tapered than a normal ellipse of the same expansion.
A dome with either profile retains more of its headroom further from the centre than does the nonsuper version. A supercircle profile results in a supersphere and a superellipse profile results in a superellipsoid.

4.5.3 VARYING TWO RADII

The ellipsoids discussed before have been ellipsoids of revolution which means that one cross-section is circular, and such domes, unless they are truncated obliquely will have either a circular floor plan and special profile, or special floor plan and semi-circular profile. The structure’s natural sphericity is expanded (or contracted) in only one direction. If we specify two separate expansions, $E_1$ and $E_2$, one for floor and the other for profile, then we obtain a dome of varying two radii. So we could have a supercircle profile to get headroom and superelliptical floor plan, to get a walking space.

Fig 4.5.4 shows how the two radii vary. Let a floor plane passing through the centre has radius $r_1$ which varies with the $\phi$ angle. This radius is a function of the expansion of the floor ellipse, $E_1$, and of the $\phi$ angle of the vertex in question, Fig 4.5.4. The equation of $r_1$ is given by

$$r_1 = \left(\frac{E_1n_1}{(\cos^n\phi + E_1n_1\sin^n\phi)}\right)^{1/n_1} \tag{4.5.8}$$

where $n_1$ is the exponent of the floor equation.

The radius $r_2$, which is the radius from the centre of the system to vertex $(\phi, \theta)$, can be expressed as follows

$$r_2 = \left(\frac{r_1n_2^2E_2n_2}{(E_2n_2^2\sin^n\theta + r_1n_2^2\cos^n\theta)}\right)^{1/n_2} \tag{4.5.9}$$

where $n_2$ is the exponent of the profile equation.

When the two exponents are identical, that is, $n_1 = n_2 = n,$
Fig 4.1 S. supercircle exponent = 2.5

Fig 4.5.1

Fig 4.5.2

Fig 4.5.3
the radius $r_2$ is given by

$$r_2 = \frac{((E_1E_2)^n}{((E_2\sin \theta)^n * (\cos^n \phi + (E_1\sin \phi)^n(E_1\cos \theta)^n)^{1/n}} \quad (4.5.10)$$

Equation 4.5.10 is a general equation for the radius $r_2$ of any ellipsoid of revolution. Assigning different values for expansions $E_1$, $E_2$ and exponent $n$ will result in one of the expressions for radius $r$ of the spheres, superspheres, ellipsoids and superellipsoids shown above. For example, if $E_1 = E_2 = 1.0$ and let $\phi = 0.0$ to obtain a two dimensional equation, then equation 4.5.10 will reduce to

$$r_2 = \frac{1}{(\sin^n \theta + \cos^n \theta)^{1/n}}$$

which is the expression of the radius of a circle if $n = 2$ and supercircle if $n > 0$.

If $E_1 = 1.0$, that is, a circular floor, and $E_2 = E$, that is, an elliptical profile, then $r_2$ is given by

$$r_2 = \frac{E^n}{(E^n\sin^n \theta + \cos^n \theta)^{1/n}}.$$ 

which is the same as equation 4.5.7 for ellipse and superellipse.

Since the distance $r$, that is, the distance from the origin of the system to any point on the domic surface, is obtained and the $\phi$ and $\theta$ values are known then the position of this point in space is uniquely defined.

The next step is to transform these spherical coordinates to Cartesian coordinates to simplify the drawing process. Again the relations between spherical coordinate system and Cartesian coordinate system are invoked, this time to transform from spherical coordinates to Cartesian coordinates.
4.5.4 FORMIAN APPLICATIONS

A subroutine MINE was written to define a supplementary retronorm for this type of configurations. In this subroutine, equation 4.5.10 is used to define the various radii of any ellipsoidal configuration. The term ellipsoidal configuration is used in the sense that it includes the sets of spherical, superspherical, ordinary ellipsoids and superellipsoids. The scheme used to create the configuration is

: NF, R, E1, E2, EN:
G1 = LIB(i=0,NF) | LIB(j=0,i) | PROJID(3*i,NF-i+2*j) | [0,0,0]
F1 = VIN(2,10**0.5) | G1
NP = LIB(k=0,4) | TRAN(2,k*2*NF) | F1
NB = REF(1,3*NF) | NP
SB = TRANID(3*NF,NF) | NP
SP = TRANID(3*NF,NF) | NB
F = NP#NB#SB#SP
USE MINE(NF,R,E1,E2,EN)
DRAW F.

Where

NF = frequency of the subdivision of the basic sub-structure edge.
R = radius of the dome along the x-axis at the equator level, that is, at $\theta = 90^\circ$.
E1 = expansion of the floor radius.
E2 = expansion of the profile radius.
EN = an exponent which defines the shape of the profile of the dome and it equals to n in equation 4.5.10.

In the scheme shown above

G1 represents the set of basic nodes, that is, the
nodes in the basic symmetry part. 

$F_1$ represents the topological properties of the basic symmetry part, that is, the connectivity of the members.

NP denotes the North-Pole. 
NB denotes the North-Belt. 
SB denotes the South-Belt. 
SP denotes the South-Pole.

Different parameters were used to run the scheme and different configurations were obtained. Combinations of the four separate regions can be used to obtain different configurations. Examples of some configurations obtained are

(1) Configuration 1

This configuration is spherical geodesic dome. The values of the parameters used to run the above scheme are

- $NF = 4$
- $r = 1.0$
- $E_1 = 1.0$
- $E_2 = 1.0$
- $EN = 2.0$

Fig 4.5.5 shows an isometric view of this configuration.

(2) Configuration 2

Fig 4.5.6a and Fig 4.5.6b show a dome of elliptical profile and circular floor with the following parameter values

- $NF = 6$
- $r = 1.0$
- $E_1 = 1.5$
- $E_2 = 1.0$
- $EN = 2.0$
(3) Configuration 3

Fig 4.5.6c and Fig 4.5.6d show a dome of elliptical profile and elliptical floor with

\[ \text{NF} = 6 \]
\[ r = 1.0 \]
\[ E1 = 1.5 \]
\[ E2 = 1.5 \]
\[ EN = 2.0 \]

(4) Configuration 4

Fig 4.5.7 shows a cap truncation of an elliptical dome with

\[ \text{NF} = 6 \]
\[ r = 1.0 \]
\[ E1 = 1.0 \]
\[ E2 = 2.0 \]
\[ EN = 2.0 \]

(5) Configuration 5

Fig 4.5.8 shows a South-Belt level truncation dome with

\[ \text{NF} = 4 \]
\[ r = 1.0 \]
\[ E1 = 1.0 \]
\[ E2 = 1.5 \]
\[ EN = 2.0 \]

(6) Configuration 6

In this configuration the parameter \( EN \) is assigned the value of 4.0, and the other parameters have the following values
NF = 6
r = 1.0
E1 = 1.2
E2 = 1.5.

Both the profile and the floor of this configuration are superellipses. The expansions of the superellipses are different. Fig 4.5.9 shows the top view, isometric view and side view of this superellipsoidal geodesic configuration.

(7) Configuration 7

Fig 4.5.10 shows a North-Pole icosahedral superspherical dome with
NF = 6
r = 1.0
E1 = 1.0
E2 = 1.0
EN = 4.0

4.6 PARABOLOIDS

A paraboloid is a volume of revolution of a parabola about an axis of symmetry. Let us consider the equation

\[ y = ax^2 + bx + c \quad \text{a} \neq 0.0 \quad (4.6.1) \]

which is an equation of a parabola, the three coefficients a, b and c determine the position of the vertex and the width of the parabola.

The x coordinate of the vertex of the parabola is \(-b/(2a)\) which is obtained by equating to zero the first derivative of the function y with respect to x for the maximum value of function y. The y coordinate can be found by substituting for the value of x in equation 4.6.1 above.
Fig 4.5.7

Side view

Top view
Side view

Top view

Fig 4.5.8
Fig 4.5.9
If the vertex of the parabola is required to lie on the y-axis, that is,

\[ x = 0.0 \]

then

\[ -\frac{b}{2a} = 0.0 \]

which implies that \( b = 0.0 \), hence the equation of the parabola reduces to

\[ y = ax^2 + c. \quad (4.6.2) \]

The parabola opens upward if \( a > 0.0 \) and it opens downward if \( a < 0.0 \).

When \( y = 0.0 \) then

\[ x^2 = -\frac{c}{a}. \]

If \( c > 0.0 \) and \( a > 0.0 \) then the parabola will not cross the x-axis at all.

If \( c > 0.0 \) and \( a < 0.0 \) then the parabola will intercept the x-axis at

\[ x = \sqrt{-\frac{c}{a}}. \]

The vertex of the parabola is at \((0, c)\). Fig 4.6.1a shows the sketch of a parabola which opens downward.

If \( a \) is taken to be equal to \(-1.0\) then the equation of the parabola is

\[ y = -x^2 + c. \quad (4.6.3) \]

Using spherical coordinates system and substituting for the
Fig 4.5.10
Fig 4.6.1a A parabola opening downward

Fig 4.6.1b A paraboloid

Antennas shaped like pieces of paraboloids of revolution. (a) Radio telescopes use the same principles as optical telescopes. (b) A "rectangular-cut" radar reflector. (c) Horn antenna in a microwave radio link.

Fig 4.6.1c
values of $x$ and $y$ in terms of $\theta$, $\phi$ and $r$ in equation 4.6.2, we obtain

$$(a \sin^2 \theta) r^2 - (\cos \theta) r + c = 0.0$$

solving this quadratic equation for $r$, we get

$$r = (\cos \theta - \sqrt{(\cos^2 \theta - 
4 \times a \times (\sin^2 \theta) \times c) / (2 \times a \times \sin^2 \theta)}). \quad (4.6.4)$$

Equation 4.6.4 above can be used to obtain the distance $r$ for all values of the angle $\theta$ if the cross-section of the paraboloid is circular. Paraboloid having elliptic cross-section, need the introduction of the effect of the horizontal position of the vertex, that is, the angle $\phi$, in the equation of the distance $r$ to any vertex on its surface. The radius $r_1$ of the ellipse is a function of the angle $\phi$ and the expansion of the ellipse, $E$, Fig 4.6.2. The radius of ellipse $r_1$ can be expressed as

$$r_1 = \sqrt{(E^2 \times R^2) / (E^2 \times \cos^2 \phi + \sin^2 \phi)}). \quad (4.6.5)$$

where $R$ is the radius of the parabola along the $x$-axis and is given by

$$R = \sqrt{-c/a}.$$ 

The effect of the change in the width of the parabola can be introduced into the equation of the parabola in the coefficient of the $x^2$ term. If the variable coefficient of $x^2$ is taken as $a_1$, then

$$r_1 = \sqrt{-c/a_1}$$

and

$$a_1 = (-c/r_1^2).$$
The distance $r$ from the centre of the elliptic paraboloid to any vertex on its surface is given by

$$r = (\cos \theta - \sqrt{\cos^2 \theta - 4a_1^2 (\sin^2 \theta)c}) / (2a_1\sin^2 \theta)$$

Equation 4.6.6 above is the general equation for the distance $r$ for both circular and elliptic paraboloid. When the expansion $E = 1.0$, that is, circular paraboloid, $a_1 = a$ and equations 4.6.4 and 4.6.6 are identical.

Fig 4.6.1b shows an elliptical paraboloid of radii $a$, $b$ and $c$ in the three Cartesian axis. Fig 4.6.1c shows some uses of parts of paraboloidal shapes.

4.6.1 FORMIAN APPLICATION

A supplementary retronorm for this type of configurations was written and called through the following scheme.

: NF, R, E, C:
G1=LIB(i=0,NF)|LIB(j=0,i)|PROJID(3*i,NF-i+2*j)|[0,0,0]
F1=VIN(2,10**0.5)|G1
NP=LIB(k=0,4)|TRAN(2,k*2*NF)|F1
NB=REF(1,3*NF)|NP
SB=TRANID(3*NF,NF)|NP
F=NP#NB#SB
USE MINE(NF,R,E,C)
DRAW F.

Where

NF = frequency of the subdivision of the basic sub-structure edge.
R = radius of the dome along the x-axis at the equator level, that is, at $\theta = 90^\circ$.
E = expansion of the floor radius.
$E = \frac{AB}{CD}$

Fig 4.6.2

$x = r_1 \cdot c \cdot \sin \phi$

$y = r_1 \cdot \sin \phi$
The height of the parabola along the z-axis.

and where

\( G_1 \) represents the set of basic nodes, that is, the nodes in the basic symmetry part.

\( F_1 \) represents the topological properties of the basic symmetry part, that is, the connectivity of the members.

\( \text{NP} \) denotes the North-Pole.

\( \text{NB} \) denotes the North-Belt.

\( \text{SB} \) denotes the South-Belt.

Different parameters were used to run the scheme and different configurations were obtained.

Fig 4.6.3 shows a geodesic configuration having a paraboloid shape with the following parameter values

\[ \begin{align*}
\text{NF} & = 9 \\
R & = 4.0 \\
E & = 1.0 \\
c & = 12.0
\end{align*} \]

The cross-section of this configuration is circular and its interconnection pattern is triangular.

The configuration shown in Fig 4.6.4 also has a paraboloid shape with the following parameter values

\[ \begin{align*}
\text{NF} & = 25 \\
R & = 5.0 \\
E & = 1.0 \\
c & = 12.0
\end{align*} \]

The cross-section of the above configuration is circular and its interconnection pattern is triangular.

The configuration shown in Fig 4.6.5 is an elliptical paraboloidal geodesic configuration with
The configuration shown in Fig 4.6.6 is an elliptical paraboloidal geodesic configuration with

\[ \begin{align*}
\text{NF} &= 6 \\
R &= 2.0 \\
E &= 1.6 \\
c &= 4.0
\end{align*} \]

The configuration shown in Fig 4.6.7 is a circular paraboloidal geodesic configuration with

\[ \begin{align*}
\text{NF} &= 8 \\
R &= 5.0 \\
E &= 1.5 \\
c &= 12.0
\end{align*} \]

4.7 HYPERBOLOIDS

The equation of a hyperboloid is given by

\[ (x/a)^2 + (y/b)^2 - (z/c)^2 = 1.0 \quad (4.7.1) \]

where \( x, y \) and \( z \) are ordinary Cartesian coordinates, \( a, b \) and \( c \) denotes the radii of the hyperboloid in the \( x, y \) and \( z \) Cartesian directions, respectively. This hyperboloid is symmetric with respect to each of the coordinates planes. The sections cut out by the coordinate planes are

\[ \begin{align*}
x &= 0.0 & \text{the hyperbola} & (y/b)^2 - (z/c)^2 = 1.0, \quad (4.7.2) \\
y &= 0.0 & \text{the hyperbola} & (x/a)^2 - (z/c)^2 = 1.0, \quad (4.7.3) \\
z &= 0.0 & \text{the ellipse} & (x/a)^2 + (y/b)^2 = 1.0. \quad (4.7.4)
\end{align*} \]

The plane \( z = z_1 \) cuts the surface in an ellipse with centre on the \( z \)-axis and vertices on the hyperbolas in equation
Fig 4.6.3
Fig. 4.6.4
Fig 4.6.5
4.7.2 and equation 4.7.3 above. In the special case where a = b, the surface is a hyperboloid of revolution and equation 4.7.4 will be an equation of a circle, Figs 4.7.1a and 4.7.1b.

If a = b, then, using spherical coordinates and substituting for the values of x and y in any of the hyperbola equations above, we obtain

\[ \left(\frac{r \sin \theta}{a}\right)^2 - \left(\frac{r \cos \theta}{c}\right)^2 = 1.0, \]

from which

\[ r = \sqrt{\frac{(c^2 a^2)}{(c^2 \sin \theta^2) - (a^2 \cos \theta)^2}}, \quad (4.7.5) \]

where r is the distance from the centre 0 to any point on the hyperboloid surface.

The value of the coefficient c determines the height required for the circular cross-section of the hyperboloid to change its radius from 1 to \(\sqrt{2}\) and hence it determines the steepness of the hyperboloid. For any value of z, the radius of the circular cross-section of the hyperboloid is given by

\[ a_1 = a \sqrt{1 + \left(\frac{z}{c}\right)^2}. \quad (4.7.6) \]

Points that lie on the flat circular cross-section at the top of the hyperboloid at any height z, have distance r from the centre 0 which is expressed as

\[ r = \frac{z}{\cos \theta} \quad (4.7.7) \]

for all values of angle \(\theta\) which is less than or equal to \(\theta_1\), where

\[ \theta_1 = \arctan \left(\frac{\sqrt{1 + \left(\frac{z}{c}\right)^2}}{z}\right). \quad (4.7.8) \]
Fig 4.6.6

Top view

Side view
An elliptic hyperboloid can be obtained by assigning different values to a and b. Let $E = b/a$, expansion of the ellipse, then the radius $r_1$ to any point on the ellipse is given by

$$r_1 = \sqrt{\frac{(Ea)^2}{(E\sin \phi)^2 + \cos^2 \phi}}. \quad (4.7.9)$$

Inserting this value of $r_1$ instead of $a$ in equation 4.7.9 of the hyperbola will give

$$r = \sqrt{\frac{(c^2 r_1)^2}{(c \sin \theta)^2 - (r_1 \cos \theta)^2}}. \quad (4.7.10)$$

For the flat elliptic cross-section at the top of the hyperboloid, the distance $r$ to any point on it is given by

$$r = \frac{z}{\cos \theta} \quad (4.9.11)$$

for all values of $\theta$ which is less than or equal to $\theta_1$, where

$$\theta_1 = \arctan \left( \frac{r_1 \sqrt{1 + (z/c)^2}}{z} \right). \quad (4.9.12)$$

### 4.7.1 FORMIAN APPLICATION

A supplementary retronorm, MINE, for this type of configurations was written and called through the following scheme

: NF, R, E, C, Z:

G1 = LIB(i=0,NF) | LIB(j=0,i) | PROJID(3*i,NF-i+2*j) | [0,0,0]
F1 = VIN(2,10**0.5) | G1
NP = LIB(k=0,4) | TRAN(2,k*2*NF) | F1
NB = REF(1,3*NF) | NP
SB = TRANID(3*NF,NF) | NP
F = NP#NB#SB
USE MINE(NF,R,E,C,Z)
DRAW F.
Fig 4.7.1a A hyperbola

Fig 4.7.1b A hyperboloid
Where

\[ \text{NF} = \text{frequency of the subdivision of the basic sub-structure edge.} \]
\[ R = \text{radius of the hyperboloid along the x-axis at the equator level, that is, at } \theta = 90^\circ. \]
\[ E = \text{expansion of the floor radius.} \]
\[ c = \text{the height of the hyperbola along the z-axis required for the radius } r \text{ of the hyperboloid along the x-axis to change its value from 1 to } \sqrt{2}. \]
\[ z = \text{height of the hyperboloid required.} \]

and where

\[ G_1 \text{ represents the set of basic nodes, that is, the nodes in the basic symmetry part.} \]
\[ F_1 \text{ represents the topological properties of the basic symmetry part, that is, the connectivity of the members.} \]
\[ \text{NP denotes the North-Pole.} \]
\[ \text{NB denotes the North-Belt.} \]
\[ \text{SB denotes the South-Belt.} \]

Different parameters were used to run the scheme and different configurations were obtained.

Fig 4.7.2 shows a hyperboloid geodesic configuration having a circular cross-section with the following values of the parameters

\[ \text{NF} = 8 \]
\[ R = 1.0 \]
\[ E = 1.0 \]
\[ c = 3.0 \]
\[ z = 9.0 \]

The interconnection pattern on the surface of the configuration is triangular.
Fig 4.7.3 shows a hyperboloid geodesic configuration having the following values of parameters:

- \( NF = 10 \)
- \( R = 1.0 \)
- \( E = 1.0 \)
- \( c = 1.0 \)
- \( z = 4.0 \)

The cross-section of this configuration is circular and its interconnection pattern is triangular.

Fig 4.7.4 shows a hyperboloid geodesic configuration having a circular cross-section with the following values of parameters:

- \( NF = 7 \)
- \( R = 2.5 \)
- \( E = 1.0 \)
- \( c = 4.0 \)
- \( z = 4.0 \)

### 4.8 CONES

A cone is obtained by rotating a straight inclined line a complete revolution about an axis. Fig 4.8.1 may be taken to represent a cross-section of a cone of base radius \( a \) and height \( h \). The equation of the right side line \( P-P1 \) may be written as:

\[
y = -(h/a)x + h. \tag{4.8.1}
\]

In terms of spherical coordinates, the distance \( r \) may be written as:

\[
r = h/(\cos \theta + (h/a)\sin \theta) \tag{4.8.2}
\]

where

- \( h \) = the height of the cone, and
Fig 4.7.2

Top view

Side view

Fig 4.7.2
\[ a = \text{radius of the cone base.} \]

Furthermore, if point \( P_1 \) traces an ellipse as it travels around the \( y \)-axis, the resulting surface will be an elliptic cone.

The radius \( r_l \) of the base ellipse can be expressed in terms of the radius \( a \) along the \( x \)-axis at the equator level, the \( \phi \) coordinate and the expansion of the ellipse, \( E \), as follows

\[ r_l = \sqrt{(E^2 a^2) / (E^2 \sin^2 \phi + \cos^2 \phi)}. \quad (4.8.3) \]

The distance \( r \), from the centre of the coordinate system to a point on the surface of the cone is given by

\[ r = h / (\cos \theta + (h/r_l) \sin \theta) \quad (4.8.4) \]

### 4.8.1 FORMIAN APPLICATION

A supplementary retronorm, MINE, for this type of configurations was written and called through the following scheme

: NF, a, E, h:
\[ G_1 = \text{LIB}(i=0, \text{NF}) | \text{LIB}(j=0, i) | \text{PROJID}(3*i, \text{NF} - i + 2*j) | [0, 0, 0] \]
\[ F_1 = \text{VIN}(2, 10**0.5) | G_1 \]
\[ N_P = \text{LIB}(k=0, 4) | \text{TRAN}(2, k*2*\text{NF}) | F_1 \]
\[ N_B = \text{REF}(1, 3*\text{NF}) | N_P \]
\[ S_B = \text{TRANID}(3*\text{NF}, \text{NF}) | N_P \]
\[ F = N_P \# N_B \# S_B \]
\[ \text{USE} \ MINE(\text{NF}, a, E, h) \]
\[ \text{DRAW} \ F. \]

Where

\[ \text{NF} = \text{frequency of the subdivision of the basic} \]
Top view

Side view

Isometric view

Fig 4.7.4
Fig 4.8.1
sub-structure edge.

\[ a = \text{radius of the dome along the x-axis at the equator level, that is, at } \theta = 90^\circ. \]

\[ E = \text{expansion of the floor radius.} \]

\[ h = \text{the height of the cone} \]

and where

\[ G_1 \] represents the set of basic nodes, that is, the nodes in the basic symmetry part.

\[ F_1 \] represents the topological properties of the basic symmetry part, that is, the connectivity of the members.

\[ \text{NP denotes the North-Pole.} \]

\[ \text{NB denotes the North-Belt.} \]

\[ \text{SB denotes the South-Belt.} \]

Different parameters were used to run the scheme and different configurations were obtained.

Fig 4.8.2 shows a conical geodesic configuration having the following parameter values

\[ \text{NF} = 7 \]
\[ a = 1.0 \]
\[ E = 1.0 \]
\[ h = 6.0 \]

The cross-section of this configuration is circular and its interconnection pattern is triangular.

Fig 4.8.3 shows a South-Belt level truncation of a conical geodesic configuration having the following parameter values

\[ \text{NF} = 7 \]
\[ a = 1.0 \]
\[ E = 1.0 \]
\[ h = 6.0. \]
4.9 A GENERAL APPROACH FOR GEODESIC FORMULATION

As the previous discussed methods of formulation are only concerned with configurations generated from one kind of polyhedron, a general approach is developed in this work to tackle the problem of data generation for configurations generated from five principal polyhedra. These are tetrahedron, octahedron, truncated octahedron, cube and icosahedron.

Spherical coordinate system is again used to facilitate the process of formulations. Each $\phi$, $\theta$ pair of the 'basic triangle' of the five polyhedra can easily be generated from a set of three integers whose sum is always the frequency of the breakdown, Ref 7. The basic triangle is the face of the polyhedron, or part of the face of the polyhedron, as in the case of the cube. If the three integers are taken as $x$, $y$ and $z$ then the $\phi$ and $\theta$ for each of the polyhedron basic triangle are as follows

**Tetrahedron**

\[
\phi = \arctan \left( \frac{y_1}{x_1} \right) \quad (4.9.1)
\]

\[
\theta = \arctan \left( \frac{\sqrt{x_1^2 + y_1^2}}{z_1} \right) \quad (4.9.2)
\]

where

\[
x_1 = \sqrt{3} \times x
\]

\[
y_1 = 2y - x
\]

and \[
z_1 = \frac{3z - x - y}{\sqrt{2}}
\]

**Octahedron**

\[
\phi = \arctan \left( \frac{y}{x} \right) \quad (4.9.3)
\]

\[
\theta = \arctan \left( \frac{\sqrt{x^2 + y^2}}{z} \right) \quad (4.9.4)
\]
Fig 4.8.2

Top view

Side view

Fig 4.8.2
Fig 4.8.3

Top view

Side view

Fig 4.8.3
Truncated octahedron

\[ \phi = \arctan \left( \frac{y}{x} \right) \]  
\[ \theta = \arctan \left( \frac{\sqrt{x^2+y^2}}{z} \right) \]

where if

\[ NTC = \frac{NF}{3} \]

and \[ z = NF - x - y \] then

if \((x+y)\) is less than or equal to \(NTC\) then \(z = 2 \times NTC\)

and if \((x+z)\) is less than or equal to \(NTC\) then \(y = 2 \times NTC\)

and if \((y+z)\) is less than or equal to \(NTC\) then \(x = 2 \times NTC\)

else values of \(x\), \(y\) and \(z\) are unchanged.

Cube

\[ \phi = \arctan \left( \frac{y}{x} \right) \]
\[ \theta = \arctan \left( \frac{\sqrt{x^2+y^2}}{z} \right) \]

where \(z = \frac{NF}{\sqrt{2}}\)

Icosahedron

\[ \phi = \arctan \left( \frac{y^2}{x^2} \right) \]
\[ \theta = \arctan \left( \frac{\sqrt{x^2+y^2}}{z^2} \right) \]

where

\[ x^2 = x \times \sin 72^\circ \]
\[ y^2 = y + x \times \cos 72^\circ \]

and \(z^2 = 0.5 \times NF + \frac{z}{T}\)

where \(T\) is the Golden Proportion \((T = 1.6180)\)
4.9.1 DERIVATION OF THE COORDINATES

The reason why the spherical coordinates shown above are generated from triads of small integers is shown in this section.

The derivations of the spherical coordinates for the tetrahedron, octahedron, truncated octahedron, cube and the icosahedron are shown in details.

Octahedron Coordinates

Fig 4.9.1 shows one face ABC of an octahedron with NF = 3 outlined upon it. The breakdown is projected onto the horizontal plane OBC, every point on the projection lying directly beneath its original. Fig 4.9.2 shows the projection plane by itself. It can be seen that this projection is a 45° right-angled triangle, OB and OC coincide with the Cartesian x and y axes. Thus all the points on the projected breakdown have simple integer x, y coordinates and so do the points on the octahedron face directly above them (p is at 1,1).

Fig 4.9.3, a side view, shows how the 45° slope of the octahedron face translates integer distances (0-p1) from the origin into integer verticals (p1-p). Thus the z coordinates of the breakdown vertices are also integer.

Fig 4.9.4 shows that the coordinates φ and θ of point p are

\[ \phi = \arctan \frac{y}{x} \]
\[ \theta = \arctan \left( \sqrt{x^2 + y^2} / z \right) \]

Truncated Octahedron coordinates

Fig 4.9.5 shows that to get a frequency NF on a truncated
octahedron one must start with an octahedron of $3^{*}NF$ frequency. The octahedron face edges are divided into thirds and the top corner is folded back until it is parallel to the projection plane and hence it will become a quarter of the square face. Its $x$ and $y$ values remain as they were, but the fact that it is parallel with the projection plane makes its $z$ retain everywhere the value of $z$ along the line $MN$, that is, $2^{*}NF$. Triangle 2 is a portion of the octahedron face unchanged from its former orientation so its points can be evaluated as if they were in a $3^{*}NF$ octahedron subdivision.

Cube coordinates

Consider Fig 4.9.6 which shows the upper face of a cube. One quarter of this face, ABC, is subdivided into smaller triangular net. The whole face of the cube consists of four parts like ABC part rotated around the z-axis. The Cartesian coordinates are as shown while the z-axis is perpendicular to the plane of the paper. The centre of the Cartesian coordinate system coincides with the centre of the cube. The $x$ and $y$ axes are parallel to the diagonals of the upper face of the cube. The subdivided part ABC lies between the $x$ and $y$ axes planes at a distance equal to half the cube edge length from the origin along the z-axis. Since the face of the cube is parallel to the horizontal plane $z = 0$, the projection of the breakdown on this plane is identical to the subdivided face and projections of the edges AB and AC coincide with the $x$-axis and $y$-axis respectively. Thus all points on the projected breakdown have simple integer $x,y$ coordinates and so do all points on the cube face directly above them.

Fig 4.9.7 shows a cross-section of the cube passing through point p and point A. This figure shows that the $z$ coordinates of any point on this face is constant and equals to $L/2$, where $L$ is the edge length of the cube.
Referring to Fig 4.9.6, If the length of half the diagonal of the cube face is taken to be equal to \( NF \), which is the frequency of subdivision of the two equal edges of the triangle ABC, then the edge length of the cube will be

\[
L = NF \times \sqrt{2}.
\]

And the z coordinate is

\[
z = L/2 = NF/\sqrt{2}.
\]

From the above discussion we see that the spherical coordinates \( \phi \) and \( \theta \) can be written as

\[
\phi = \arctan \left( \frac{y}{x} \right)
\]

\[
\theta = \arctan \left( \frac{\sqrt{x^2 + y^2}}{z} \right)
\]

where \( z = NF/\sqrt{2} \).

Icosahedron Coordinates

Fig 4.9.8 shows the projection of an icosahedron face \( O_1A_1C_1 \) onto the plane where the Cartesian axes \( OB \) and \( OA \) are located. The 72° angle at the icosahedron apex makes the projection line skew. Fig 4.9.9 shows the projection \( OAC \) nestled between equal axes \( AO \) and \( OB \). The point \( p \) is midway along \( AC \); this special point is chosen for simplicity, but any point on the icosahedron face would do. \( Q \) is similarly located at the midpoint of \( AB \). Its coordinates are \( x,y \) and we want to find the coordinates \( x_2,y_2 \) of \( p \).

If \( OC = OB = 1 \) then

\[
DC = \sin 72^\circ;
\]

\[
\frac{DC}{OB} = \sin 72^\circ / 1
\]

\[
x_2 = x \times \sin 72^\circ
\]

similarly
Fig 4.9.8

Fig 4.9.9

Fig 4.9.10
\[ y_2 = y + pQ \]
and \[ pQ/x_2 = \tan 18^\circ \]
then \[ pQ = (x \cdot \sin 72^\circ) \cdot \tan 18^\circ = x \cdot \cos 72^\circ \]
thus \[ y_2 = y + x \cdot \cos 72^\circ \]

For \( z_2 \), see Fig 4.9.10, which is a rear wall of Fig 4.9.8 with the same lettering. \( O_iA_i \) is the icosahedron's edge, \( O_iO \) is the \( z \) axis with \( O \) at the centre of the polyhedron. From the properties of the icosahedron we have:

In triangle \( O_iA_iL \)
\[ \tan A_i = 1/T \]
where \( T \) is the Golden Proportion (\( T = 1.618 \)).
and in the rectangle \( LOAA_i \)
\[ OA = 2 \cdot LO. \]

If the icosahedron edge is divided into \( N_F \) equal parts and by dropping perpendiculars \( L A_i \) and \( O A \) will be divided likewise. we have

\[ N_F = x + y + z, \]
and since \( O A \) is the \( y \)-axis, then
\[ x = 0 \]
\[ z = N_F - y \]

The \( z_2 \) consists of two parts, the part in the triangle \( O_iA_iL \) and the part in the rectangle \( LOAA_i \). The former is

\[ (f - y)/ \tan A_i, \]
that is, \( z/T \). The latter is equivalent to \( LO \) which is half \( OA \), but \( OA = N_F \). Hence

\[ z_2 = N_F/2 + z/T \]
Tetrahedron coordinates

Fig 4.9.11 shows a tetrahedron face projected onto the plane that contains the Cartesian axes OB, OA. The projection of the tetrahedron face is AOC. We follow the same procedure as in the derivation of the coordinates of the icosahedron. The point P is midway along AC and Q is similarly located at the midpoint of AB. The coordinates of Q are x,y, and we want to find the coordinates x1,y1 of P. If OC = OB =1, then

DC = cos 30°

and

DC/OB = (cos 30°)/1

and

x1/DC = x/OB
x1 = x*cos 30° = x*sqrt(3)/2.

Similarly, y1 = y - PQ
PQ/x1 = tan 30°
PQ = (tan 30°)*x1.

Substituting for x1 by x*sqrt(3)/2 we get

y1 = y - x/2.

For z1, see Fig 4.9.12 which is the projection of the tetrahedron onto the y-z plane. From the properties of the tetrahedron

angle O01A1 = arctan(1/sqrt(2))
angle O1A1L = arctan(sqrt(2))
angle LOA1 = arctan(2*sqrt(2)).

The z1 coordinate consists of two parts, the constant part which is equivalent to OL, and the variable part.
Fig 4.9.11a

Fig 4.9.11b

Fig 4.9.12
The constant part OL = - NF*cot LOA1. The variable part is equal to z*tan O1A1L.

\[ z_1 = z*\sqrt{2} - \frac{x+y+z}{2*\sqrt{2}} \]

rearranging

\[ z_1 = \frac{3z - x - y}{2*\sqrt{2}} \]

The spherical coordinates of any point on the surface of a tetrahedron are given by

\[ \phi = \arctan \left( \frac{y_1}{x_1} \right) \]
\[ \theta = \arctan \left( \frac{\sqrt{x_1^2+y_1^2}}{z_1} \right) \]

4.9.2 THE METHOD OF NUMBERING

The sets of the three integers for each node are obtained by following a certain way of numbering. The first integer, x, is obtained by numbering the first drawing as shown, Fig 4.9.13. Rotating the pattern 60° clockwise will result in the second integer y, and another rotation 60° clockwise will give rise to the third integer z. Each vertex has a unique three integers the sum of which is always equal to the frequency. When Cartesian coordinate system is used then the three integers can be represented as shown in Fig 4.9.14.

4.9.3 BASIC SUB-STRUCTURE FORMULATION

The face of the basic polyhedron is taken as the basic sub-structure for the formex formulations of the whole structure for the tetrahedron, octahedron, truncated octahedron and the icosahedron. One quarter of the cube face is taken as the basic sub-structure.
Fig 4.9.13 Integers numbering.
Fig 4.9.14 Cartesian representation of \((X,Y,Z)\) integers.
The formex representing the triangulated sub-structure can be written as

$$E_1 = \text{LIB}(i=0, NF-1) \text{RIN}(1, NF-i, 1) \text{TRAN}(2, I) \mid E$$

where

$$E = \{[0, 0, 1; 1, 0, 1], [1, 0, 1; 0, 1, 1], [0, 1, 1; 0, 0, 1]\}$$

and \(NF\) is the frequency of the subdivision of the basic triangle.

\(E_1\) is taken to represent the topological properties of the elements. The set of the nodes of the basic sub-structure may be given as

$$F_1 = \text{LIB}(i=0, NF) \text{RIN}(1, NF-i+1, 1) \text{TRAN}(2, I) \mid [0, 0, 1]$$

where \([0, 0, 1]\) is the seed of the set of nodes.

The symbol \(E_1\) in the above formulation is used in a general sense to represent the topological properties of the basic symmetry part. Thus any kind of disposition of nodes or interconnection pattern can be assigned to the basic part via \(E_1\) to obtain different topological arrangements for the whole system.

4.9.4 GEOMETRIC PARTICULARS

The geometric definition of the configuration required can be obtained by numerically specifying the mapping of the previously given topological properties onto the required circumscribing surface. For a point having a signet \([U_1, U_2, U_3]\) the geometric particulars for each of the five generator volumes are as follows
For tetrahedron

\[ x_1 = \sqrt{3} \times U_1 \quad (4.9.11) \]
\[ y_1 = 2 \times U_2 - U_1 \quad (4.9.12) \]
\[ z_1 = \frac{3 \times (N - \text{abs}(U_1) - \text{abs}(U_2)) - U_1 - U_2}{\sqrt{2}} \quad (4.9.13) \]

For octahedron

\[ x = U_1 \quad (4.9.14) \]
\[ y = U_2 \quad (4.9.15) \]
\[ z = N - \text{abs}(U_1) - \text{abs}(U_2) \quad (4.9.16) \]

For truncated octahedron

If \( NTC = \frac{N}{3} \)
and \( z = N - \text{abs}(U_1) - \text{abs}(U_2) \)
then if \( (\text{abs}(U_1) + \text{abs}(U_2)) \) is less than or equal to \( NTC \) then

\[ x = U_1 \quad (4.9.17) \]
\[ y = U_2 \quad (4.9.18) \]
\[ z = 2 \times NTC. \quad (4.9.19) \]

And if \( (\text{abs}(U_1) + z) \) is less than or equal to \( NTC \) then

\[ x = U_1 \quad (4.9.20) \]
\[ y = \frac{2 \times NTC \times U_2}{\text{abs}(U_2)} \quad (4.9.21) \]
\[ z = N - \text{abs}(U_1) - \text{abs}(U_2). \quad (4.9.22) \]

And if \( (\text{abs}(U_2) + z) \) is than or equal to \( NTC \) then

\[ x = \frac{2 \times NTC \times U_1}{\text{abs}(U_1)} \quad (4.9.23) \]
\[ y = U_2 \quad (4.9.24) \]
\[ z = N - \text{abs}(U_1) - \text{abs}(U_2). \quad (4.9.25) \]

Else

\[ x = U_1 \quad (4.9.26) \]
\[ y = U_2 \quad (4.9.27) \]
\[ z = N - \text{abs}(U_1) - \text{abs}(U_2) \quad (4.9.28) \]
For cube

\[ x = U_1 \]  
\[ y = U_2 \]  
\[ z = NF/\sqrt{2} \]

For icosahedron

\[ x_2 = U_1 \sin 72^\circ \]  
\[ y_2 = U_2 + U_1 \cos 72^\circ \]  
\[ z_2 = 0.5NF + (NF-\text{abs}(U_1)-\text{abs}(U_2))/T \]

where \( T \) is the Golden Proportion defined in chapter 2. The relevant equations of each of the basic polyhedra are used to obtain \( \phi, \theta \) and \( r \).

4.10 FORMIAN APPLICATIONS

The concept of a scheme is again used to produce generic formulations of different kinds for geodesic configurations. Subdivided polyhedra, ellipsoids, superellipsoids, paraboloids, hyperboloids and cones having one of the five polyhedra discussed above as their generator volume are shown in this section. The relevant equations of the profile required together with the equations for the geometric particulars of the generator volume are used to obtain the required configuration. A supplementary retronorm for each of the configurations was written. The scheme and the supplementary retronorm contain some parameters which are required to define the configuration concerned. Some schemes and the produced configurations on different circumscribing surfaces are shown below.

4.10.1 SUBDIVIDED POLYHEDRA

The relevant equations that defines the \((x, y, z)\)
coordinates of any point on the subdivided polyhedra were used in the supplementary retronorm for each type of polyhedra. The parameters used are, NT, an identifier that defines the type of the polyhedron and NF, the frequency of subdivision of the polyhedron's edge.

The scheme that generates such configurations is

\[ NT, NF: \]
\[ E = \{[0,0,1;1,0,1],[1,0,1;0,1,1],[0,1,1;0,0,1]\} \]
\[ BE1 = LIB(I=0,NF-1)|RIN(1,NF-1,1)|TRAN(2,I)|E \]
\[ F1 = LIB(I=0,NF)|RIN(1,NF-I+1,1)|TRAN(2,I)|[0,0,1] \]
\[ FF = RINID(NF,NF,3,3)|\{[0,0,1],[1,1,1],[2,2,1]\} \]
\[ FA = LUX(FF)|BE1 \]
\[ FG = RINID(NF/2,NF/2,2,2)|[0,0,1] \]
\[ FB = LUX(FG)|BE1 \]
\[ DIA = [1,0,1;0,1,1] \]
\[ DIAG = RINID(NF,NF,1,1)|dia \]
\[ DIAG1 = COP(DIAG)|BE1 \]
\[ DIAG2 = RINID(NF,NF,2,2)|[1,0,1;0,1,1] \]
\[ DIAG3 = COP(DIAG2)|BE1 \]
\[ USE MINE(NT,NF) \]

where

\[ NT \] = type of the generator volume and has the following values
1 for tetrahedron
2 for cube
3 for octahedron
4 for truncated octahedron
5 for icosahedron

\[ NF \] = frequency of the subdivision of the basic sub-structure edge

In the scheme shown above

BE1 is a formex that represents the topological
properties of the basic triangle, that is, the connectivity of the members,

F1 is a formex that represents the set of nodes in the basic triangle,

FA is a formex that represents the hexagonal configuration connectivity on the basic triangle,

and each of the formices FB, DIAG1 and DIAG3 represents different connectivity on the basic triangle.

The formices that represent the final configurations depend on the type of the polyhedron used, that is, on NT. For instance; if NT = 1, that is, a tetrahedron, then the formex

\[ TET1 = \text{ROSAX}(0,0,0,0,0,1,3,120) | \text{BE1} \]

represents three faces of a tetrahedron having triangular connectivity.

The formex

\[ TET2 = \text{ROSAX}(0,0,0,0,0,1,3,120) | \text{FA} \]

represents a tetrahedron having a hexagonal connectivity.

For NT = 2 , that is, a cube, then the formex

\[ CUBE1 = \text{LAM}(2,0) | \text{LAM}(1,0) | \text{BE1} \]

represents one face of the cube having a triangular connectivity.

The formex

\[ CUBE2 = \text{ROSAX}(0,0,0,0,1,1,4,90) | \text{CUBE1} \]

\[ \text{ROSAX}(0,0,0,1,1,0,4,90) | \text{CUBE1} \]
represents a cube having triangular connectivity.

For NT = 3, that is, an octahedron, the formex

\[
\text{OCT1} = \text{LAM}(2,0) | \text{LAM}(1,0) | \text{DIAG1}
\]

represents four subdivided faces of the octahedron.

The formex

\[
\text{OCT2} = \text{LAM}(3,0) | \text{LAM}(2,0) | \text{LAM}(1,0) | \text{BE1}
\]

represents a subdivided complete octahedron.

For NT = 4, that is, a truncated octahedron, the formices that represent different configurations on the complete truncated octahedron or part of it can be obtained in the same manner as for the octahedron.

For NT = 5, that is, an icosahedron, the formex

\[
\text{NP} = \text{ROSAX}(0,0,0,0,0,1,5,72) | \text{BE1}
\]

represents a cap truncation of an icosahedron and it includes five faces, that is, the North-Pole.

The formex that represents the North-Belt is of the form

\[
\text{NB} = \text{ROSAX}(0,0,0,0,0,1,5,72) | \text{NB1}
\]

where

\[
\text{NB1} = \text{ROSAX}(0,0,0,0,NF,0.5\times NF,1,72) | \text{BE1}.
\]

The South-Belt is given by the formex

\[
\text{SB} = \text{ROSAX}(0,0,0,0,0,1,5,72) | \text{SB1}
\]
where

$$SB_1 = ROSAX(0,0,0,0,NF,0.5*NF,1,72)|NB_1.$$  

The South-Pole is given by the formex

$$SP = ROSAX(0,0,0,0,0,1,5,72)|SP_1$$

where

$$SP_1 = ROSAX(0,0,0,0,0,1,1,36)|ref(3,0)|BE_1.$$  

The complete configuration is represented by

$$CONFIG = NP \# NB \# SB \# SP.$$  

Some of the configurations obtained by running the above scheme are shown below.

Fig 4.10.1 is a triangulated configuration on a tetrahedron of 16 frequency.

Fig 4.10.2 is a hexagonal configuration on a tetrahedron of 16 frequency.

Fig 4.10.3 is a triangulated configuration on a truncated octahedron of 6 frequency.
Fig 4.10.1 Tetrahedron (triangular configuration)

Fig 4.10.2 Tetrahedron (hexagonal configuration)

Fig 4.10.3 Truncated octahedron

Fig 4.10.4 Cube

Fig 4.10.5 Octahedron (triangular configuration)

Fig 4.10.6 Octahedron (hexagonal configuration)
Fig 4.10.4 is a triangulated configuration on a cube of 6 frequency.

Fig 4.10.5 is a triangulated configuration on an octahedron of 6 frequency.

Fig 4.10.6 is a hexagonal configuration on an octahedron of 6 frequency.

Fig 4.10.7a is a basic substructure of a truncated octahedron of 12 frequency and Fig 4.10.7b is the complete triangulated octahedron of 12 frequency.

Fig 4.10.8 is a North-Pole of an icosahedron having a triangulated configuration of 20 frequency.

4.10.2 ELLIPSOIDS AND SUPERELLIPSOIDS

Equations 4.5.1 through 4.5.10, shown in section 4.5 are used here again in the supplementary retronorm that defines the family of ellipsoids and superellipsoids geodesic configurations. The term ellipsoids and superellipsoids include spheres and superspheres.

The scheme that generates such configurations is

: NF,R,E1,E2,NT,EN:
E={ [0,0,1;1,0,1],[1,0,1;0,1,1],[0,1,1;0,0,1] }
BE1=LIB(I=0,NF-1)|RIN(1,NF-I,1)|TRAN(2,I)|E
F1=LIB(I=0,NF)|RIN(1,NF-I+1,1)|TRAN(2,I)|[0,0,1]
FF=RINID(NF,NF,3,3)|{ [0,0,1],[1,1,1],[2,2,1] }
FA=LUX(FF)|BE1
FG=RINID(NF/2,NF/2,2,2)|[0,0,1]
FB=LUX(FG)|BE1
DIA=[1,0,1;0,1,1]
DIAG=RINID(NF,NF,1,1)|DIA
DIAG1=COP(DIAG)|BE1
Fig 4.10.7a
Basic substructure

Fig 4.10.7b Truncated octahedron

Fig 4.10.8
\text{DIAG2=RINID}(\text{NF}, \text{NF}, 2, 2) | [1, 0, 1; 0, 1, 1]
\text{DIAG3=COP}(\text{DIAG2}) | \text{BE1}
\text{USE MINE}(\text{NF}, \text{R}, \text{E1}, \text{E2}, \text{NT}, \text{EN})

where

\begin{align*}
\text{NF} & = \text{frequency of the subdivision of the basic sub-structure edge.} \\
\text{R} & = \text{radius of the dome along the x-axis at the equator level, that is, at } \theta = 90^\circ. \\
\text{E1} & = \text{expansion of the floor radius.} \\
\text{E2} & = \text{expansion of the profile radius.} \\
\text{NT} & = \text{type of the generator volume and has the following values}
\begin{align*}
1 & \text{ for tetrahedron} \\
2 & \text{ for cube} \\
3 & \text{ for octahedron} \\
4 & \text{ for truncated octahedron} \\
5 & \text{ for icosahedron}
\end{align*}
\text{EN} & = \text{an exponent which defines the shape of the profile of the dome and it equals to } n \text{ in equation 4.5.10.}
\end{align*}

In the scheme shown above

\begin{align*}
\text{BE1} & \text{ is a formex that represents the topological properties of the basic triangle, that is, the connectivity of the members,} \\
\text{F1} & \text{ is a formex that represents the sets of nodes in the basic triangle,} \\
\text{FA} & \text{ is a formex that represents the hexagonal configuration connectivity on the basic triangle,}
\end{align*}

and each of the formices FB, DIAG1 and DIAG3 represents different connectivity on the basic triangle.
The formices that represent the final configurations depend on the type of generator volume used, that is, on $NT$. For instance; if $NT = 1$, that is, a tetrahedron generator volume, then the formex

$$TET1 = ROSAX(0,0,0,0,0,1,3,120)\mid BE1$$

represents three faces of a tetrahedral geodesic configuration having triangular connectivity.

The formex

$$TET2 = ROSAX(0,0,0,0,0,1,3,120)\mid FA$$

represents a tetrahedral geodesic configuration having a hexagonal connectivity.

For $NT = 2$, that is, a cube generator volume, then the formex

$$CUBE1 = LAM(2,0)\mid LAM(1,0)\mid BE1$$

represents a geodesic configuration on one face of the cube having a triangular connectivity.

The formex

$$CUBE2 = ROSAX(0,0,0,0,1,1,4,90)\mid CUBE1 \#$$
$$ROSAX(0,0,0,1,1,0,4,90)\mid CUBE1$$

represents a geodesic configuration on the six faces of a spherical cubic dome having a triangular connectivity.

For $NT = 3$, that is, an octahedron generator volume, then the formex

$$OCT1 = LAM(2,0)\mid LAM(1,0)\mid DIAG1$$
represents a geodesic configuration on four faces of the octahedron.

The formex

\[ \text{OCT}_2 = \text{LAM}(3,0) | \text{LAM}(2,0) | \text{LAM}(1,0) | \text{BE}_1 \]

represents a geodesic configuration on the complete octahedron.

For \( NT = 4 \), that is, a truncated octahedron, the formices that represent different configurations on the complete generator volume surface or part of it can be obtained in the same manner as for the octahedron.

For \( NT = 5 \), that is, an icosahedron generator volume, the formex

\[ \text{NP} = \text{ROSAX}(0,0,0,0,0,1,5,72) | \text{BE}_1 \]

represents a cap truncation of a geodesic configuration on an icosahedron and it includes five faces, that is, the North-Pole. This formex is valid for all values of \( E_1 \) and \( E_2 \).

The formex that represents the North-Belt for spherical geodesic configurations is of the form

\[ \text{NB} = \text{ROSAX}(0,0,0,0,0,1,5,72) | \text{NB}_1 \]

where

\[ \text{NB}_1 = \text{ROSAX}(0,0,0,0,\text{NF},0.5\times\text{NF},1,72) | \text{BE}_1. \]

The South-Belt is given by the formex

\[ \text{SB} = \text{ROSAX}(0,0,0,0,0,1,5,72) | \text{SB}_1 \]
where

\[ SB1 = \text{ROSAX}(0,0,0,0,NF,0.5*NF,1,72)|\text{NB1}. \]

The South-Pole is given by the formex

\[ SP = \text{ROSAX}(0,0,0,0,0,1,5,72)|\text{SP1} \]

where

\[ SP1 = \text{ROSAX}(0,0,0,0,0,1,1,36)|\text{ref}(3,0)|\text{BE1}. \]

The complete spherical geodesic configuration is represented by

\[ \text{CONFIG} = \text{NP} \# \text{NB} \# \text{SB} \# \text{SP}. \]

Some of the configurations obtained by running the above scheme are shown below.

Fig 4.10.9 shows a tetrahedral spherical geodesic configuration having the following values of parameters

\[ \begin{align*}
NF &= 15 \\
R &= 1.0 \\
E1 &= 1.0 \\
E2 &= 1.0 \\
EN &= 2.0
\end{align*} \]

Fig 4.10.10 shows a tetrahedral spherical geodesic configurations having parameters as follows

\[ \begin{align*}
NF &= 20 \\
R &= 1.0 \\
E1 &= 1.0 \\
E2 &= 1.0 \\
EN &= 2.0.
\end{align*} \]

The configuration has a hexagonal interconnection pattern.
Fig 4.10.9

Top view

Side view
Fig 4.10.10

Side view

Top view

Fig 4.10.10
Fig 4.10.11 shows a tetrahedral elliptic geodesic configuration having the following values of parameters

\[
\begin{align*}
NF &= 16 \\
R &= 1.0 \\
E1 &= 1.0 \\
E2 &= 1.4 \\
EN &= 2.0
\end{align*}
\]

The floor section of this configuration is circular and its profile is elliptical.

Fig 4.10.12 shows a cubic spherical geodesic configuration having the following values of the parameters

\[
\begin{align*}
NF &= 8 \\
R &= 1.0 \\
E1 &= 1.0 \\
E2 &= 1.0 \\
EN &= 2.0
\end{align*}
\]

Fig 4.10.13 shows a cubic hexagonal spherical geodesic configuration having the following values of parameters

\[
\begin{align*}
NF &= 8 \\
R &= 1.0 \\
E1 &= 1.0 \\
E2 &= 1.0 \\
EN &= 2.0
\end{align*}
\]

Fig 4.10.14 shows a hexagonal geodesic configuration having the cube as its generator volume and projected on a surface of a supercircular floor and a superelliptical profile with the following values of parameters

\[
\begin{align*}
NF &= 12 \\
R &= 1.0 \\
E1 &= 1.0 \\
E2 &= 1.8 \\
EN &= 6.0
\end{align*}
\]
Fig 4.10.11
Fig 4.10.15 shows a configuration of the same parameters as the configuration in Fig 4.10.14 but with a hexagonal interconnection pattern.

Fig 4.10.16 shows isometric views for Figs 4.10.14 and 4.10.15.

Figs 4.10.17 and 4.10.18 show a triangular and hexagonal interconnection pattern on a cubic elliptical geodesic configuration having the following parameters:

\[
\begin{align*}
NF &= 13 \\
R &= 1.0 \\
E_1 &= 1.5 \\
E_2 &= 2.0 \\
EN &= 2.0
\end{align*}
\]

Fig 4.10.19 is a spherical geodesic configuration based on a truncated octahedron as a generator volume of frequency NF=12.

Fig 4.10.20 is a superspherical geodesic configuration based on a truncated octahedron as a generator volume of NF=10, E1=E2=1.0, EN=6 and R=1.0.

Fig 4.10.21 is an elliptical geodesic configuration based on an octahedron as a generator volume and have NF=9, R=1.0, E1=1.0, E2=1/1.5 and EN= 2.0.

Fig 4.10.22 is a geodesic configuration of the same parameters as those for the configuration in Fig 4.10.21 but with a hexagonal interconnection pattern.

Fig 4.10.23 is an elliptical octahedral geodesic configuration with NF=8, R=1.0, E1=1.0, E2=4.0 and EN=2.0.

Fig 4.10.24 is an elliptical octahedral geodesic configuration with NF=12, R=1.0, E1=1.0, E2=3.0 and EN=2.0.
Fig 4.10.16a Isometric view for Fig 4.10.15

Fig 4.10.16b Isometric view for Fig 4.10.14
Fig 4.10.19

Fig 4.10.20
Fig 4.10.21

Side view

Top view
Fig 4.10.22

Side view

Top view
Fig 4.10.25 is an elliptical octahedral geodesic configuration of hexagonal and triangular interconnection patterns with \( NF=12, R=1.0, E_1=1.0, E_2=1.5 \) and \( EN=2.0 \).

Fig 4.10.26 is a superspherical octahedral geodesic configuration with \( NF=15, R=1.0, E_1=1.0, E_2=1.0 \) and \( EN=3.0 \).

Fig 4.10.27 is a superelliptical octahedral geodesic configuration with \( NF=15, R=1.0, E_1=1.2, E_2=2.0 \) and \( EN=4.0 \). Both the cross section and the profile of the configuration are elliptical.

Fig 4.10.28 is a North-Pole of an icosahedral spherical configuration having \( NF=13, R=1.0, E_1=E_2=1.0 \), and \( En=2.0 \).

Fig 4.10.29 and Fig 4.10.30 show top views of two icosahedral geodesic configuration of two different interconnection patterns.

Fig 4.10.31 shows the basic substructure and the side view and the top view of an elliptical geodesic configuration with \( NF=15, R=1.0, E_1=1.0, E_2=1.5 \) and \( EN=2.0 \).

Fig 4.10.32 shows a configuration of the same parameters as the configuration in Fig 4.10.31 but with hexagonal interconnection pattern.

4.10.3 PARABOLOIDS

The configurations having paraboloidal shapes are obtained using equations 4.6.1 through 4.6.6 shown in section 4.6.

The scheme for this type of geodesic configurations is

: \( NF, R, E, NT, CA: \)
Fig 4.10.25

Side view

Top view
Isometric view  Top view

Side view

Fig 4.10.26
Fig 4.10.27
Top view

Side view

Fig 4.10.28
Fig 4.10.29 Top view

Fig 4.10.30 Top view
Fig 4.10.31

Basic substructure

Side view

Top view
Fig 4.10.32

Side view

Top view
El={[0,0,1;1,0,1],[1,0,1;0,1,1],[0,1,1;0,0,1]}
BE1=LIB(I=0,NF-1)|RIN(1,NF-I,1)|TRAN(2,I)|El
Fl=LIB(I=0,NF)|RIN(1,NF-I+1,1)|TRAN(2,I)|[0,0,1]
FF=RINID(NF,NF,3,3)|{[0,0,1],[1,1,1],[2,2,1]}
FA=LUX(FF)|BE1
FG=RINID(NF/2,NF/2,2,2)|[0,0,1]
FB=LUX(FG)|BE1
DIA=[1,0,1;0,1,1]
DIAG=RINID(NF,NF,1,1)|DIA
DIAG1=COP(DIAG)|BE1
DIAG2=RINID(NF,NF,2,2)|[1,0,1;0,1,1]
DIAG3=COP(DIAG2)|BE1
USE MINE(NF,R,E,NT,CA)

Where

\[
\begin{align*}
NF & = \text{frequency of the subdivision of the basic sub-structure edge.} \\
R & = \text{radius of the dome along the x-axis at the equator level, that is, at } \theta = 90^\circ. \\
E & = \text{expansion of the floor radius.} \\
NT & = \text{indicates type of the generator volume and has the following values} \\
& \quad 1 \text{ for tetrahedron} \\
& \quad 2 \text{ for cube} \\
& \quad 3 \text{ for octahedron} \\
& \quad 4 \text{ for truncated octahedron} \\
& \quad 5 \text{ for icosahedron} \\
CA & = \text{the height of the parabola along the z-axis.}
\end{align*}
\]

In the scheme shown above

BE1 is a formex that represents the topological properties of the basic triangle, that is, the connectivity of the members,

Fl is a formex that represents the sets of nodes in the basic triangle,
FA is a formex that represents the hexagonal configuration connectivity on the basic triangle,

and each of the formices FB, DIAG1 and DIAG3 represents different connectivity on the basic triangle.

The formices that represent the final configurations depend on the type of generator volume used, that is, on NT. For instance, if NT = 1, that is, a tetrahedron generator volume, then the formex

\[ \text{TET1} = \text{ROSAX}(0,0,0,0,0,0,0,1,3,120) \mid \text{BE1} \]

represents three faces of a tetrahedral geodesic configuration having triangular connectivity.

The formex

\[ \text{TET2} = \text{ROSAX}(0,0,0,0,0,0,0,1,3,120) \mid \text{FA} \]

represents a tetrahedral geodesic configuration having hexagonal connectivity.

For NT = 2, that is, a cube generator volume, then the formex

\[ \text{CUBE1} = \text{LAM}(2,0) \mid \text{LAM}(1,0) \mid \text{BE1} \]

represents a geodesic configuration on one face of the cube having triangular connectivity.

For NT = 3, that is, an octahedron generator volume, then the formex

\[ \text{OCT1} = \text{LAM}(2,0) \mid \text{LAM}(1,0) \mid \text{DIAG1} \]
represents a geodesic configuration on four faces of the octahedron.

For \( NT = 4 \), that is, a truncated octahedron, the formices that represents different configurations on the complete generator volume surface or part of it can be obtained in the same manner as for the octahedron.

For \( NT = 5 \), that is, an icosahedron generator volume, the formex

\[
NP = \text{ROSAX}(0, 0, 0, 0, 0, 1, 5, 72) \mid \text{BE1}
\]

represents a cap truncation of a geodesic configuration on an icosahedron and it includes five faces, that is, the North-Pole.

Some of the configurations obtained by running the above scheme are shown below.

Fig 4.10.33 is a tetrahedral paraboloidal geodesic configuration with \( NF=20 \), \( R=1.0 \), \( E=1.0 \) and \( CA=4.0 \).

Figs 4.10.34 and 4.10.35 shows a tetrahedral paraboloidal geodesic configuration with triangular and hexagonal interconnection pattern, respectively, and the frequency of subdivision is 22.

Fig 4.10.36 is an octahedral paraboloidal geodesic configuration with \( NF=10 \), \( R=3.0 \), \( E=1.0 \) and \( CA=3.0 \).

Fig 4.10.37 shows an isometric view and top view of an octahedral paraboloidal geodesic configuration with \( NF=12 \), \( R=2.0 \), \( E=1.0 \) and \( CA=4.0 \).

Fig 4.10.38 is an elliptical octahedral paraboloidal geodesic configuration with \( NF=16 \), \( R=1.0 \), \( E=1.6 \) and \( CA=4.0 \).
Fig 4.10.33

Side view

Top view
Top view

Side view

Fig 4.10.34
Top view

Side view

Fig 4.10.35
Side view

Top view

Fig 4.10.36
Fig 4.10.37

Isometric view

Top view

Fig 4.10.37
Fig 4.10.38

Side view

Top view
The interconnection pattern of this configuration is triangular.

Fig 4.10.39 shows an elliptical truncated octahedral paraboloidal geodesic configuration with NF=16, R=1.0, E=1.5 and CA=6.0. The interconnection pattern of this configuration is hexagonal.

Fig 4.10.40 is an elliptical truncated octahedral paraboloidal geodesic configuration with NF=15, R=4.0, E=2.0 and CA=4.0. The interconnection pattern of this configuration is triangular.

Fig 4.10.41 shows a cubic paraboloidal geodesic configuration with NF=13, R=2.0, E=1.0 and CA=2.0.

Fig 4.10.42 shows a cubic paraboloidal geodesic configuration with hexagonal interconnection pattern with NF=10, R=2.0, E=1.0, and CA=4.0.

Figs 4.10.43 and 4.10.44 show a cap truncation of an icosahedral paraboloidal geodesic configuration with triangular and hexagonal interconnection pattern, respectively. The configurations have NF=14, R=4.0, E=1.0 and CA=5.0.

Fig 4.10.45 shows an icosahedral paraboloidal geodesic configuration of hexagonal interconnection pattern with NF=25.

4.10.4 HYPERBOLOIDS

The hyperboloid’s equations in section 4.7 are used in the supplementary retronorm that generates this type of geodesic configurations.
Side view

Top view

Fig 4.10.39
Fig 4.10.41

Side view

Isometric view
Side view

Top view

Fig 4.10.42
Fig 4.10.44

Side view

Top view
Fig 4.10.45
The scheme that generates the configurations is:

\[
\begin{align*}
E_1 &= \{[0,0,1;1,0,0,1], [1,0,1;0,1,1], [0,1,1;0,0,1] \} \\
BE_1 &= \text{LIB}(I=0, NF-1) | \text{RIN}(1, NF-1, 1) | \text{TRAN}(2, I) | E_1 \\
F_1 &= \text{LIB}(I=0, NF) | \text{RIN}(1, NF-I+1, 1) | \text{TRAN}(2, I) | [0,0,1] \\
F_1 &= \text{RINID}(NF,NF,3,3) | \{[0,0,1], [1,1,1], [2,2,1] \} \\
F_A &= \text{LUX}(F_F) | BE_1F_G = \text{RINID}(NF/2,NF/2,2,2) | [0,0,1] \\
F_B &= \text{LUX}(F_G) | BE_1 \\
DIA &= [1,0,1;0,1,1] \\
\text{DIAG} &= \text{RINID}(NF,NF,1,1) | \text{DIA} \\
\text{DIAG}_1 &= \text{COP}(\text{DIAG}) | \text{BE}_1 \\
\text{DIAG}_2 &= \text{RINID}(NF,NF,2,2) | [1,0,1;0,1,1] \\
\text{DIAG}_3 &= \text{COP}(\text{DIAG}_2) | \text{BE}_1 \\
\text{USE MINE}(NF,R,E,NT,C,Z)
\end{align*}
\]

Where

- **NF** = frequency of the subdivision of the basic sub-structure edge.
- **R** = radius of the dome along the x-axis at the equator level, that is, at \( \theta = 90^\circ \).
- **E** = expansion of the floor radius.
- **NT** = an integer indicates type of the generator volume and has the following values:
  1 for tetrahedron
  2 for cube
  3 for octahedron
  4 for truncated octahedron
  5 for icosahedron
- **C** = the height of the hyperbola along the z-axis required for the radius \( r \) of the hyperboloid to change its radius from 1 to \( \sqrt{2} \).

In the scheme shown above, BE1 is a formex that represents the topological
properties of the basic triangle, that is, the connectivity of the members,
F₁ is a formex that represents the sets of nodes in the basic triangle,
FA is a formex that represents the hexagonal configuration connectivity on the basic triangle,

and each of the formices FB, DIAG₁ and DIAG₃ represents different connectivity on the basic triangle.

The formices that represent the final configurations depend on the type of generator volume used, that is, on NT. For instance; if NT = 1, that is, a tetrahedron generator volume, then the formex

\[ \text{TET₁} = \text{ROSAX}(0,0,0,0,0,1,3,120)\mid \text{BE₁} \]

represents three faces of a tetrahedral geodesic configuration having triangular connectivity.

The formex

\[ \text{TET₂} = \text{ROSAX}(0,0,0,0,0,1,3,120)\mid \text{FA} \]

represents a tetrahedral geodesic configuration having a hexagonal connectivity.

For NT = 2, that is, a cube generator volume, then the formex

\[ \text{CUBE₁} = \text{LAM}(2,0)\mid \text{LAM}(1,0)\mid \text{BE₁} \]

represents a geodesic configuration on one face of the cube having a triangular connectivity.

For NT = 3, that is, an octahedron generator volume, then
the formex

\[ \text{OCT1} = \text{LAM}(2,0)|\text{LAM}(1,0)|\text{DIAG1} \]

represents a geodesic configuration on four faces of the octahedron.

For NT = 4, that is, a truncated octahedron, the formices that represent different configurations on the complete generator volume surface or part of it can be obtained in the same manner as for the octahedron.

For NT = 5, that is, an icosahedron generator volume, the formex

\[ \text{NP} = \text{ROSAX}(0,0,0,0,0,0,1,5,72)|\text{BE1} \]

represents a cap truncation of a geodesic configuration on an icosahedron and it includes five faces, that is, the North-Pole.

Some of the configurations obtained by running the above scheme are shown below.

Figs 4.10.46 and 4.10.47 shows a tetrahedral hyperboloidal geodesic configuration with triangular and hexagonal interconnection pattern, respectively, with NF=20, E=1.0, R=1.0, C=4.0 and Z=4.0.

Fig 4.10.48 shows an elliptical cubic hyperboloidal geodesic configuration with triangular interconnection pattern with NF=18, E=1.5, R=1.0, C=4.0 and Z=6.0.

Fig 4.10.49 shows an octahedral hyperboloidal geodesic configuration with hexagonal interconnection pattern with NF=16, E=1.0, R=1.0, C=2.0 and Z=2.0.
Fig 4.10.48a Top view

Fig 4.10.48b Front view

Fig 4.10.48c Side view
Fig 4.10.49a Top view
Fig 4.10.49b Isometric view
Fig 4.10.49c Side view
Fig 4.10.50 shows an elliptical octahedral hyperboloidal geodesic configuration with triangular interconnection pattern with \( NF=12, \ E=1.20, \ R=1.0, \ C=4.0 \) and \( Z=6.0 \).

Fig 4.10.51 shows an icosahedral hyperboloidal geodesic configuration with \( NF=13, \ E=1.0, \ R=1.0, \ C=3.0 \) and \( Z=3.0 \).

Fig 4.10.52 shows an icosahedral hyperboloidal geodesic configuration with \( NF=15, \ E=1.0, \ R=1.0, \ C=2.0 \) and \( Z=3.0 \).

Fig 4.10.53 shows an icosahedral hyperboloidal geodesic configuration with \( NF=18, \ E=1.0, \ R=1.0, \ C=2.0 \) and \( Z=5.0 \).

4.10.5 CONES

Conical configurations are obtained from equations 4.8.1 through 4.8.4 shown in section 4.8.

The relevant scheme is

\[
E_1 = \{[0,0,1;1,0,1],[0,1,0;1,0,1],[0,1,1;0,0,1]\}
\]

\[
B_{E1} = \text{LIB}(I=0,NF-1) | \text{RIN}(1,NF-I,1) | \text{TRAN}(2,I) | E_1
\]

\[
F_1 = \text{LIB}(I=0,NF) | \text{RIN}(1,NF-I+1,1) | \text{TRAN}(2,I) | [0,0,1]
\]

\[
F_F = \text{RINID}(NF,NF,3,3) | \{[0,0,1],[1,1,1],[2,2,1]\}
\]

\[
F_A = \text{LUX}(F_F) | B_{E1}
\]

\[
F_G = \text{RINID}(NF/2,NF/2,2,2) | [0,0,1]
\]

\[
F_B = \text{LUX}(F_G) | B_{E1}
\]

\[
D_{IA} = [1,0,1;0,1,1]
\]

\[
D_{IA} = \text{RINID}(NF,NF,1,1) | D_{IA}
\]

\[
D_{IA} = \text{COP}(D_{IA}) | B_{E1}
\]

\[
D_{IA} = \text{RINID}(NF,NF,2,2) | [1,0,1;0,1,1]
\]

\[
D_{IA} = \text{COP}(D_{IA}) | B_{E1}
\]

\[
\text{USE MINE}(NF,R,E,NF,H)
\]

where

\[
NF = \text{frequency of the subdivision of the basic}
\]
Fig 4.10.50c Side view  
Fig 4.10.50b Front view  

Fig 4.10.50a Top view
Fig 4.10.51
Fig 4.10.52
the sub-structure edge.

\begin{align*}
R &= \text{radius of the dome along the x-axis at the equator level, that is, at } \theta = 90^\circ. \\
E &= \text{expansion of the floor radius.} \\
NT &= \text{type of the generator volume and has the following values} \\
&\quad 1 \text{ for tetrahedron} \\
&\quad 2 \text{ for cube} \\
&\quad 3 \text{ for octahedron} \\
&\quad 4 \text{ for truncated octahedron} \\
&\quad 5 \text{ for icosahedron} \\
H &= \text{the height of the cone}
\end{align*}

In the scheme shown above

\begin{align*}
\text{BE1} \text{ is a formex that represents the topological properties of the basic triangle, that is, the connectivity of the members,} \\
\text{F1} \text{ is a formex that represents the sets of nodes in the basic triangle,} \\
\text{FA} \text{ is a formex that represents the hexagonal configuration connectivity on the basic triangle,} \\
\text{and each of the formices FB, DIAG1 and DIAG3 represents different connectivity on the basic triangle.}
\end{align*}

The formices that represent the final configurations depend on the type of generator volume used, that is, on NT. For instance; if NT = 1, that is, a tetrahedron generator volume, then the formex

\begin{align*}
\text{TET1} &= \text{ROSAX}(0,0,0,0,0,1,3,120)|\text{BE1}
\end{align*}

represents three faces of a tetrahedral geodesic configuration having triangular connectivity.

The formex
represents a tetrahedral geodesic configuration having hexagonal connectivity.

For \( NT = 2 \), that is, a cube generator volume, then the formex

\[ \text{CUBE1} = \text{LAM}(2,0) | \text{LAM}(1,0) | \text{BE1} \]

represents a geodesic configuration on one face of the cube having triangular connectivity.

For \( NT = 3 \), that is, an octahedron generator volume, then the formex

\[ \text{OCT1} = \text{LAM}(2,0) | \text{LAM}(1,0) | \text{DIAG1} \]

represents a geodesic configuration on four faces of the octahedron.

For \( NT = 4 \), that is, a truncated octahedron, the formices that represents different configurations on the complete generator volume surface or part of it can be obtained in the same manner as for the octahedron.

For \( NT = 5 \), that is, an icosahedron generator volume, the formex

\[ \text{NP} = \text{ROSAX}(0,0,0,0,0,1,5,72) | \text{BE1} \]

represents a cap truncation of a geodesic configuration on an icosahedron and it includes five faces, that is, the North- Pole.

Some of the configurations obtained by running the above scheme are shown below.

Fig 4.10.54 shows a tetrahedral conical geodesic
Top view

Fig 4.10.54

Side view
configuration with NF=15, R=1.0, E=1.0 and H=4.0.

Fig 4.10.55 shows a cubic conical geodesic configuration with NF=13, R=1.0, E=1.0 and H=2.0.

Figs 4.10.56 and 4.10.57 are octahedral conical geodesic configuration with NF=20, E=1.0, R=1.0 and H=4.0. The interconnection pattern is triangular and hexagonal, respectively.

Fig 4.10.58 is an elliptical octahedral conical geodesic configuration with NF=8, E=1.5, R=1.0 and H=8.0.

Fig 4.10.59 is a conical geodesic configuration based on the truncated octahedron as a generator volume with NF=12, E=1.0, R=1.0 and H=4.0.

Fig 4.10.60 is a conical geodesic configuration based on the truncated octahedron as a generator volume with NF=18, E=1.0, R=1.0 and H=8.0.

Figs 4.10.61 and 4.10.62 are icosahedral conical geodesic configuration with triangular and hexagonal interconnection pattern, respectively. The parameters of these configuration are NF=12, E=1.0, R=1.0 and H=8.0.

Fig 4.10.63 a South-Belt level truncation of an icosahedral conical geodesic configuration having NF=7, E=1.0, R=1.0 and H=5.0.

4.11 CLASS II AND CLASS III METHODS OF SUBDIVISION

Geodesic configurations generated using class II and class III methods of subdivision can be obtained using the method of the lowest common frequency, LCF, discussed in chapter 22 section 2.4.3. The same notation used in section 2.4.3
Fig 4.10.55

Top view

Side view
Top view

Side view

Fig 4.10.58
Fig 4.10.60

Top view

Side view
Top view

Side view

Fig 4.10.62
Top view

Side view

Fig 4.10.63
is used here again.

If the coordinates of class III, that is, \([p,q']_{b,c}\) polyhedron, are sought, the lowest common frequency of a \([p,q']_{b,0}\) polyhedron, that is, class I that shares common coordinates may be found by applying the formula

\[
LCF = b^2 + b*c + c^2
\]  
(4.11.1)

where \(b\) and \(c\) are from the \([p,q']_{b,c}\) polyhedron and

\[
LCF = b \text{ of the } [p,q']_{b,0} \text{ polyhedron}.
\]

In case of class II method, the lowest common frequency, LCF, is given by

\[
LCF = 2(b^2 + b*c + c^2)/NF
\]

where \(NF\) is the frequency of subdivision of the sub-structure edge and it equals \((b + c)\), and since \(b = c\), then

\[
LCF = 3*b.
\]  
(4.11.2)

The coordinates of the nodes of the basic sub-structure for any of the five principal polyhedra considered earlier, that is, tetrahedron, truncated octahedron, octahedron, cube and icosahedron can be obtained from the coordinates of class I method of subdivision. These sub-structures can be projected on any circumscribing surfaces as shown in the previous sections. Some examples of class II and class III methods of subdivision are shown below. Some schemes that generate some geodesic configurations based on class II and class III methods of subdivision are shown below. The subroutines MINE called through these schemes are the same as those subroutine derived in section 4.10 for class I method of subdivision.
4.11.1 CLASS II

4.11.1.1 SUBDIVIDED POLYHEDRA

:NT, NF, NF1:

\[ E = \{ [0,0,1;1,0,0], [0,1,0;0,1,1], [0,1,1;1,0,0] \} \]

\[ \text{BEl} = \text{LIB}(I=0, NF-1) | \text{RIN}(1, NF-I, 1) | \text{TRAN}(2, I) | E \]

\[ \text{Q1} = \text{LIB}(I=0, NF1/2) | \text{RIN}(1, 0.5*NF1-I+1, 3) | \text{TRAN}(2, 3*I) | [0,0,1] \]

\[ \text{Q2} = \text{LIB}(I=0, NF1/2-1) | \text{RIN}(1, 0.5*NF1-I, 3) | \text{TRAN}(2, 3*I) | [1,1,1] \]

\[ \text{Q3} = \text{LIB}(I=0, NF1/2-2) | \text{RIN}(1, 0.5*NF1-I-1, 3) | \text{TRAN}(2, 3*I) | [2,2,1] \]

\[ Q = \text{Q1} \# \text{Q2} \# \text{Q3} \]

USE MINE(NT, NF)

\[ \text{QA} = \text{MED} | Q \]

\[ \text{QB} = \text{ROSAX}(0,0,0,0,1,N1,DG) | \text{QA} \]

\[ \text{QC} = \text{VIN}(b1,b2) | \text{QB} \]

USE VN(200,200)

USE VS(50)

DRAW QC

where

\[ \text{NT} \quad \text{= type of the generator volume and has the following values} \]

1 for tetrahedron
2 for cube
3 for octahedron
4 for truncated octahedron
5 for icosahedron

\[ \text{NF} \quad \text{= frequency of the subdivision of the basic sub-structure edge for class I method of subdivision} \]

\[ \text{NF1} \quad \text{= frequency of subdivision of the basic sub-structure using class II method and it is equal} \]
In the scheme shown above

**BE1** is a formex that represents the topological properties of the basic sub-structure, that is, the connectivity of the members,

**Q** is a formex that represents the set of nodes in the basic sub-structure,

**QB** is a formex that represents the set of nodes of the configuration obtained by rotating the basic sub-structure a given number of rotations,

**N1** an integer that defines the number of the basic triangles meeting at the apex of the generator volume

- \( N_1 = 3 \) for tetrahedron
- \( N_1 = 4 \) for octahedron, truncated octahedron and cube
- \( N_1 = 5 \) for icosahedron,

**DG** is the amount of rotation at each step (pace) and it equals \( 360/N_1 \) degrees,

**QC** is a formex that represents the connectivity of the members,

**b1** and **b2** are numeric expressions that define the bounds of the metrum of each cantle of the formex **QA**.

### 4.11.1.2 GENERALISED ELLIPSOIDS

: **NF, E1, E2, NT, EN, NF1:**

\[
E = \{ [0,0,1;1,0,1], [1,0,1;0,1,1], [0,1,1;0,0,1] \}
\]

**BE1** = \( \text{LIB}(I=0,NF-1) | \text{RIN}(1,NF-I,1) | \text{TRAN}(2,I) | E \)

**F1** = \( \text{LIB}(I=0,NF) | \text{RIN}(2,NF-I+1,1) | \text{TRAN}(1,I) | [0,0,1] \)

**Q1** = \( \text{LIB}(I=0,NF1/2) | \text{RIN}(1,0.5\ast NF1-I+1,3) | \text{TRAN}(2,3*1) | [0,0,1] \)

**Q2** = \( \text{LIB}(I=0,NF1/2-1) | \text{RIN}(1,0.5\ast NF1-I,3) | \text{TRAN}(2,3*1) | [1,1,1] \)
Q3=LIB(I=0,NF1/2-2) | RIN(1,0.5*NF1-I-1,3) | TRAN(2,3*I) |
[2,2,1]
Q=Q1#Q2#Q3

USE MINE(NF,E1,E2,NT,EN)

QA=MED|Q
QB=ROSAX(0,0,0,0,0,1,N1,DG)|QA
QC=VIN(b1,b2)|QB
USE VN(200,200)
USE VS(50)
DRAW QC

The parameters used in the scheme above are as follows:

NF = frequency of the subdivision of the basic sub-structure edge.
E1 = expansion of the floor radius.
E2 = expansion of the profile radius.
NT = type of the generator volume and has the following values:

1 for tetrahedron
2 for cube
3 for octahedron
4 for truncated octahedron
5 for icosahedron

EN = an exponent which defines the shape of the profile of the dome and it equals to n in equation 4.5.10.

In the scheme shown above the formices and the parameters used have the same definition as those in section 4.11.1.1 above.

4.11.1.3 PARABOLOIDS

: NF,R,EX,NT,CA,NF1:
E={[0,0,1;1,0,1],[1,0,1;0,1,1],[0,1,1;0,0,1]}
BE1=LIB(I=0,NF-1) | RIN(1,NF-I,1) | TRAN(2,I) | E
F1=LIB(I=0,NF) | RIN(2,NF-I+1,1) | TRAN(1,I) | [0,0,1]

Q1=LIB(I=0,NF1/2) | RIN(1,0.5*NF1-I+1,3) | TRAN(2,3*I) | [0,0,1]
Q2=LIB(I=0,NF1/2-1) | RIN(1,0.5*NF1-I,3) | TRAN(2,3*I) | [1,1,1]
Q3=LIB(I=0,NF1/2-2) | RIN(1,0.5*NF1-I-1,3) | TRAN(2,3*I) | [2,2,1]
Q=Q1#Q2#Q3

USE MINE(NF,R,EX,NT,CA)

QA=MEDIQ
QB=ROSAX(0,0,0,0,0,1,N1,DG)|QA
QC=VIN(b1,b2)|QB
USE VN(200,200)
USE VS(50)
DRAW QC

The parameters NF,R,EX,NT and CA are the same as those used in section 4.10.3.

Other parameters and formices are as defined in section 4.11.1.1.

4.11.2 CLASS III

4.11.2.1 SUBDIVIDED POLYHEDRA

:NT,NF,NF1,b,c:

E={ [0,0,1;1,0,1], [1,0,1;1,0,1], [0,1,1;0,0,1] }
BE1=LIB(I=0,NF-1) | RIN(1,NF-I,1) | TRAN(2,I) | E

R0=LIB(I=-1,NF1) | TRANID(b*I,C*I) | { [b,c,1;2*b,2*c,1], [2*b,2*c,1;b-c,b+2*c,1], [b-c,b+2*c,1;b,c,1] }
R01=LIB(I=0,b) | TRANID(0-c*I,NF1*I) | R0
R02=LIB(I=0,c) | TRANID(C*I,0-NF1*I) | R0
\[ R = R_0 \# R_01 \# R_02 \]

\texttt{USE MINE(NT,NF)}

\texttt{H1 = NEX(BE1) \mid R}
\texttt{H12 = MED\mid H1}
\texttt{H13 = ROSAX(0, 0, 0, 0, 1, N1, DG) \mid H12}
\texttt{NP = VIN(b1, b2) \mid H13}

\texttt{USE VN(200, 200)}
\texttt{USE VS(150)}
\texttt{USE LS(2)}
\texttt{DRAW BE1}
\texttt{USE LS(1)}
\texttt{DRAW NP}

\text{where}

\[ NF_1 = \text{frequency of subdivision of the basic sub-structure using class III method and it is equal to LCF.} \]

\text{All other parameters are as defined in section 4.11.1.1.}

\text{In the scheme shown above}

\begin{itemize}
  \item \texttt{BE1} is a formex that represents the topological properties of the basic sub-structure, that is, the connectivity of the members,
  \item \texttt{H12} is a formex that represents the set of nodes in the basic sub-structure,
  \item \texttt{H13} is a formex that represents the set of nodes of the configuration obtained by rotating the basic sub-structure a given number of rotations,
  \item \texttt{NP} is a formex that represents the connectivity of the of the members.
\end{itemize}
4.11.2.2 GENERALISED ELLIPSOIDS

: NF, E1, E2, NT, EN, NF1, b, c:

E = {[0, 0, 1; 1, 0, 1], [1, 0, 1; 0, 1, 1], [0, 1, 1; 0, 0, 1]}
BE1 = LIB (I = 0, NF - 1) | RIN(1, NF - 1, 1) | TRAN(2, I) | E

RO = LIB (I = -1, NF1) | TRANID(b*I, c*I) | {[b, c, 1; 2*b, 2*c, 1], ...
[2*b, 2*c, 1; b-c, b+2*c, 1], [b-c, b+2*c, 1; b, c, 1]}
R01 = LIB (I = 0, b) | TRANID(0-c*I, NF1*I) | RO
R02 = LIB (I = 0, c) | TRANID(c*I, 0-NF1*I) | RO
R = R0#R01#R02

USE MINE (NF, E1, E2, NT, EN)

H1 = NEX(BE1) | R
H12 = MED | H1
H13 = ROSAX(0, 0, 0, 0, 1, N1, DG) | H12
NP = VIN(b1, b2) | H13
USE VN(200, 200)
USE VS(150)
USE LS(2)
DRAW BE1
USE LS(1)
DRAW NP

The parameters used in the scheme above are as follows

NF = frequency of the subdivision of the basic sub-structure edge.
E1 = expansion of the floor radius.
E2 = expansion of the profile radius.
NT = type of the generator volume and has the following values
   1 for tetrahedron
   2 for cube
   3 for octahedron
4 for truncated octahedron
5 for icosahedron

EN = an exponent which defines the shape of the profile of the dome and it equals to n in equation 4.5.10.

In the scheme shown above

BE1 is a formex that represents the topological properties of the basic triangle, that is, the connectivity of the members.

4.11.2.3 PARABOLOIDS

: NF, R, EX, NT, CA, NF1, b, c:
E=[[0,0,1;1,0,1],[1,0,1;0,1,1],[0,1,1;0,0,1]]
BE1=LIB(I=0,NF-1)|RIN(1,NF-I,1)|TRAN(2,I)|E

R0=LIB(I=-1,NF1)|TRANID(b*I,c*I)|{[b,c,1;2*b,2*c,1],...
[2*b,2*c,1; b-c, b+2*c, 1], [b-c, b+2*c, 1; b, c, 1]}
R01=LIB(I=0,b)|TRANID(0-c*I,NF1*I)|R0
R02=LIB(I=0,c)|TRANID(c*I,0-NF1*I)|R0
R=R0#R01#R02

USE MINE(NF,R,EX,NT,CA)

H1=NEX(BE1)|R
H12=MED|H1
H13=ROSAX(0,0,0,0,0,1,N1,DG)|H12
NP=VIN(b1,b2)|H13
DRAW NP

4.11.3 EXAMPLES OF CLASS II AND CLASS III METHODS OF SUBDIVISION

Some of the configurations obtained using class II and class III methods of subdivision are shown below.
Fig 4.11.1 shows an icosahedron cap subdivided using class II method of subdivision with frequency $NF=10$, that is, $[3,5^\ast]_{5,5}$.

Fig 4.11.2 shows an icosahedron cap subdivided using class II method of subdivision with frequency $NF=8$, that is, $[3,5^\ast]_{4,4}$.

Fig 4.11.3 is a South-Belt level truncation of a class II icosahedron with frequency $NF=8$, that is, $[3,5^\ast]_{4,4}$.

Fig 4.11.4 is a South-Belt level truncation of a class III icosahedron with frequency $NF=6$, that is, $[3,5^\ast]_{4,2}$.

Fig 4.11.5 is a spherical class III icosahedral geodesic configuration with frequency $NF=6$, that is, $[3,5^\ast]_{4,2}$.

Fig 4.11.6 is an upper half of class III paraboloidal octahedral geodesic configuration with frequency $NF=6$, that is, $[3,4^\ast]_{4,2}$.

Fig 4.11.7 is a class III paraboloidal icosahedral geodesic configuration with frequency $NF=6$, that is, $[3,5^\ast]_{4,2}$.

4.12 ORGANISATION OF DATA

Let us consider the dome shown in Fig 4.12.1. This dome is a spherical dome based on North-pole truncation of the geodesic subdivision of an icosahedron, using the frequency of 5. The radius of the complete dome is 50 metre. The structural framework is composed of 200 circular tubular members and 76 joints. These members consist of two types according to their cross-sections properties. The outer lower members between supports are of type 1 and the remaining members are of type 2. There are also two types of supports. Those supports which are completely constrained are of type S1 and others which are vertically
Fig 4.12.1

Top view

Side view

Fig 4.12.1
constrained only are of type S2. One loading condition is
to be analysed. This loading condition is a combination of
two types of loads. One type of loads, \( L_1 \), represents the
loads on the support joints, and the other type of loads, 
\( L_2 \), represents the loads on the remaining joints of the
structure.

Let it be assumed that this structure is to be analysed
using a computer program which is based on the standard
stiffness method. The information required as input data
for the analysis program includes information about the
interconnection pattern, geometric particulars, material
properties, external loads and support conditions. These
information can be easily generated and transformed to the
analysis program with the aid of formex algebra and its
programming language Formian. It is convenient to include
the whole of the information for a particular case in a
single scheme. This scheme may be divided into several
parts according to the different information required to be
generated.

The scheme for the structure under consideration is shown
below

\[
\text{DOME} = ': NF, R, E1, E2, EN:
\]

\[
\text{G1} = \text{LIB}(I=0, NF) | \text{LIB}(J=0, I).
\]

\[
\text{PROJID}(3*I, \text{NF-I}+2*J) | [0, 0, 0]
\]

\[
\text{F1} = \text{VIN}(2, 10**0.5) | \text{G1}
\]

\[
\text{NP} = \text{LIB}(K=0, 4) | \text{TRAN}(2, K*2*\text{NF}) | \text{F1}
\]

\[
\text{MB1} = \text{LIB}(I=0, \text{NF}-1) | [3*\text{NF}, 2*I, 0; 3*\text{NF}, 2*I+2, 0]
\]

\[
\text{MEMB1} = \text{LIB}(K=0, 4) | \text{TRAN}(2, K*2*\text{NF}) | \text{MB1}
\]

\[
\text{MEMB2} = \text{CON(MEMB1)} | \text{NP}
\]

\[
\text{K1} = \text{LIB}(I=1, 4) | [0, (2*I+1)*\text{NF}, 0; 0, \text{NF}, 0]
\]
K2 = LIB(I=0, NF) | [3*I, (9*NF+I), 0; 3*I, (NF-I), 0]
K3 = LIB(I=0, NF) | [3*I, (3*NF-I), 0; 3*I, (NF+I), 0]
K4 = LIB(I=0, NF) | [3*I, (5*NF-I), 0; 3*I, (3*NF+I), 0]
K5 = LIB(I=0, NF) | [3*I, (7*NF-I), 0; 3*I, (5*NF+I), 0]
K6 = LIB(I=0, NF) | [3*I, (9*NF-I), 0; 3*I, (7*NF+I), 0]

K = K1#K2#K3#K4#K5#K6
GD1 = NOV(K) | NP
GD = PEX|GD1
GD2 = MED|GD
T = LIB(I=0, 76) | [I]
D1 = DIC(GD2) | GD

S = [3*NF, 0, 0]
S1 = LIB(K=0,4) | TRAN(2, K*2*NF) | S
SS = LIB(I=0, NF) | [3*NF, 2*I, 0]
S2 = LIB(K=0, 4) | TRAN(2, K*2*NF) | SS

L1 = S1#S2
L2 = LUX(L1) | GD2

USE MINE(NF, R, E1, E2, EN)
USE PEN(2, 10)
USE VN(200, 200)
USE VS(100)
DRAW NP
DRAWN GD2, T'

Where

NF = frequency of the subdivision of the basic sub-structure edge which equals 5.
R = radius of the dome along the x-axis at the equator level, that is, at \( \theta = 90^\circ \) and it
equals 50 m.

\[ E_1 = \text{expansion of the floor radius and it equals 1.} \]

\[ E_2 = \text{expansion of the profile radius and it equals 1}. \]

\[ E_N = \text{an exponent which defines the shape of the profile of the dome and it equals 2}. \]

This scheme consists of 6 parts. Part 1 of the scheme consists of 7 statements in which type 1 and type 2 elements connectivity information is generated. The formex variable that represents the connectivity of type 1 elements is identified as MEMB1. The formex variable MEMB2 represents the connectivity of type 2 elements of the structure.

In part 2, information about node numbering to be adopted in the analysis program is generated. The formex variable GD2 is the numerant of the structure. The result of this node numbering pattern is shown in Fig 4.12.2.

Information about support nodes is generated in part 3, where the formex variable S1 represents all the support nodes of type 1 and the formex variable S2 represents the support nodes of type 2.

Part 4 of the scheme generates information about loaded nodes, where the formex variable L1 represents the loaded nodes with type 1 loads and the formex L2 represents the nodes loaded with type 2 loads.

The USE statement in part 5 specifies the supplementary retronorm to be used to obtain the actual node coordinates from the corresponding normat coordinates. The subroutine MINE in this scheme is the same subroutine defined in section 4.5.3.

The last part of the scheme, that is, part 6, consists of
Fig 4.12.2
5 statements which have the effect of producing a plot of the formices required using a suitable scale and a convenient viewpoint.

The induction statement

\[
\text{DOME}(5,50,1,1,2)
\]

will generate all information defined in the body of the scheme above.

The procedure discussed above can be used to produce information for any kind of ellipsoids by just entering the relevant values for the nominal parameters in the induction statement. The produced information can be submitted to any computer analysis program or can be kept for future reference.

As another example, let us consider the octahedral paraboloidal geodesic dome shown in Fig 4.12.3. This dome is obtained by projecting the subdivided upper half of an octahedron on a circumscribing paraboloid. The frequency of the subdivision of the octahedron edge is 8. The 'alternate' method of subdivision is used and the interconnection pattern of the structure is hexagonal with squares at the vertices of the generator volume. The structure consists of 132 tubular members and 96 joints. The radius of the dome is 30 metres and its height to the apex is 20 metres. All members are of the same type. The supports of the structure are of two types. Supports which are completely constrained are of type 1 and the supports which are only vertically constrained are of type 2. One half only of the structure was subjected to loading of one type concentrated on the nodes of that half including the supports nodes. The analysis program to be used and the information required to be generated and submitted are the same as those explained in the previous example.
Top view

Side view

Fig 4.12.3
The scheme below will generate the required information for the example under consideration.

\[\text{DOME}_1 = \': \text{NF}, \text{R}, \text{E}, \text{NT}, \text{CA}:\]

\[\text{E}_1 = \{[0, 0, 1; 1, 0, 1], [1, 0, 1; 0, 1, 1], [0, 1, 1; 0, 0, 1]\}\]

\[\text{ME}_1 = \text{LIB}(I=0, \text{NF}-1) | \text{RIN}(1, \text{NF}-I, 1) | \text{TRAN}(2, I) | \text{E}_1\]

\[\text{FF} = \text{RINID}(\text{NF}, \text{NF}, 3, 3) \{[0, 0, 1], [1, 1, 1], [2, 2, 1]\}\]

\[\text{ME}_2 = \text{LUX}(\text{FF}) | \text{ME}_1\]

\[\text{DOM} = \text{ROSID}(0, 0) | \text{ME}_2\]

\[\text{F}_1 = \text{LIB}(I=0, \text{NF}) | \text{LAMID}(0, 0) | \text{RIN}(1, \text{NF}-I+1, 1) | \text{TRAN}(2, I) | [0, 0, 1]\]

\[\text{FF}_1 = \text{ROSID}(0, 0) | \text{FF}\]

\[\text{FN} = \text{PEX} | \text{LUX}(\text{FF}_1) | \text{F}_1\]

\[\text{T} = \text{LIB}(I=0, 148) | [I]\]

\[\text{ZN}_1 = \text{LIB}(I=0, \text{NF}) | \text{TRANID}(-I, I) | [\text{NF}, 0, 1]\]

\[\text{ZN}_2 = \text{PEX} | \text{ROSID}(0, 0) | \text{ZN}_1\]

\[\text{H}_1 = \text{MED} | \text{FA}_1\]

\[\text{HGF} = \text{COL}(\text{ZN}_2) | \text{H}_1\]

\[\text{S}_1 = \text{ROSID}(0, 0) | [\text{NF}, 0, 1]\]

\[\text{S}_2 = \text{LUX}(\text{S}_1) | \text{HGF}\]

\[\text{F}_2 = \text{LIB}(I=0, \text{NF}) | \text{RIN}(1, \text{NF}-I+1, 1) | \text{TRAN}(2, I) | [0, 0, 1; 0, 0, 1]\]

\[\text{FK} = \text{PEX} | \text{LUX}(\text{FF}) | \text{F}_2\]

\[\text{L} = \text{LAM}(1, 0) | \text{FK}\]

\[\text{USE MINE}(\text{NF}, \text{R}, \text{E}, \text{NT}, \text{CA})\]
USE VN(200,200)
USE VS(50)
USE VP(0,0,100)
DRAWN DOM

where

\[
\begin{align*}
NF & = \text{frequency of the subdivision of the basic sub-structure edge and it equals 8.} \\
R & = \text{radius of the dome along the x-axis at the equator level, that is, at } \theta = 90^\circ \text{ and it equals 30 m.} \\
E & = \text{expansion of the floor radius and it equals 1.} \\
NT & = \text{indicates type of the generator volume and has the value of 3 for octahedron.} \\
CA & = \text{the height of the parabola along the z-axis and it equals 20 m.}
\end{align*}
\]

Part 1 of the scheme generates information about elements connectivity. The formex variable DOM represents this connectivity.

In part 2, information about node numbering to be adopted in the computer analysis program is generated. The formex FN is the numerant of the structure. The result of this node numbering pattern is shown in Fig 4.12.4.

In part 3, information about support nodes is generated. The formex S1 represents all the support nodes of type 1 and the formex S2 represents the support nodes of type 2.

In part 4, information about loaded nodes is generated. The formex variable L represents the loaded nodes.

The USE statement in part 5 specifies the supplementary
Fig 4.12.4
retronorm to be used to obtain the actual node coordinates from the corresponding normat coordinates. The subroutine MINE in this scheme is the same subroutine defined in section 4.10.3.

The last part of the scheme, that is, part 6, consists of 5 statements which have the effect of producing a plot of the formices required using a suitable scale and a convenient view point.

Information required can be obtained by using the induction statement

\[ \text{DOME1(8,30,1,3,20).} \]
CHAPTER FIVE

GEOMETRIC POTENTIAL FUNCTION
MINIMISATION

5.1 INTRODUCTION

Although geodesic subdivision is a good way to obtain a regular spherical dome, it has some drawbacks such as the difficulty in truncating the dome at intended support level without disturbing its regularity and also the restriction imposed on the designer by being forced to use some of the already known configurations with little chance to introduce new ideas. A new approach to achieve regularity on domic surfaces without being confined to the traditional ways of subdivision was introduced by J W Butterworth, Ref 4. The idea is to place any number of points on a certain predetermined surface or portion of a surface and then allow these points to move on the surface by the effect of repulsion forces between them until equilibrium is attained. Some of these points can be fixed at certain positions such as the supports or entrances. The problem reduces to obtaining a minimum of a function called the geometric potential function.

This study is concerned with developing a method for geometric potential function optimisation and use it to obtain spherical or ellipsoidal geodesic configurations by placing any number of points on the required surface or part of a surface. Geometric potential function minimisation method was used also to obtain configurations with more regular shapes than those obtained from directly
projecting the network of the subdivided polyhedron on a circumscribed ellipsoid.

5.2 ELLIPSOIDS AND GENERALISED ELLIPSOIDS

In addition to the drawbacks of geodesic subdivision on spherical surfaces mentioned in section 5.1, the spherical form also has limitations, such as excessive volume to plan area ratio in the case of hemispherical truncations and low edge headroom in the case of cap truncations. These limitations necessitate turning to nonspherical surfaces such as ellipsoids and generalised ellipsoids.

The general equation of ellipsoids family can be written as

\[(x/a)^u + (y/b)^v + (z/c)^w - 1 = 0\]  \hspace{1cm} (5.1)

where \(a\), \(b\) and \(c\) denote the radii of the generalised ellipsoid in the \(X\), \(Y\) and \(Z\) Cartesian directions, respectively, and \(u\), \(v\) and \(w\) define the form of the surface. For example, \(u = v = w = 2\) defines an ellipsoid, and if in addition \(a = b = c\), a sphere results.

If the geodesic subdivisions of the generalised ellipsoids are accomplished by projecting from a subdivided polyhedron onto a circumscribing ellipsoid, the regularity will not be the same as in the case of a sphere. Many modifications have been proposed to obtain a reasonable regularity but no general and satisfactory solution appears to have been found. In a flat normal ellipsoidal geodesic dome the short members are found around the zenith where they are subjected to low loads, whereas the longest members with widely separated vertices are found around the sides where the load is being transmitted to the foundations. Hugh Kenner, Ref 7, suggested to modify the coordinates of the
subdivided polyhedron faces before projecting the points on the circumscribing surface. He suggested, when using spherical coordinate system, to replace θ by θ₁ obtained from

$$θ₁ = \text{atan} \left( \frac{(\tan θ)}{E} \right) \quad (5.2)$$

where $E =$ ratio of the radius at the zenith to the radius at the ground level and is called the 'Expansion'. This correction is called the E Correction.

Another modified θ value which gives a better configuration can be obtained from

$$θ₁ = \text{atan} \left( \frac{(\tan θ)}{E^{0.5}} \right) \quad (5.3)$$

Which is called the Root E Correction.

The new approach of the geometric potential seems to provide a feasible way of creating regular configurations from a simple starting point especially on the generalised ellipsoid surfaces.

5.3 THE GEOMETRIC POTENTIAL FUNCTION

Let there be a set of $n$ points numbered from 1 to $n$ with associated weightings of $m_1$ to $m_n$ and constrained to lie on a surface $S$.

Let $d_{ij}$ denote the length of chord joining points $i$ and $j$. Assume that forces of mutual repulsion act between these points defined by certain law.

The geometric potential function $G$, is defined as

$$G = Σ m_i * m_j * (d_{ij})^{-q} \quad (5.4)$$
for all values of \( i \) and \( j \) with \( i \) not equal to \( j \) and where \( q \) is a positive integer defining the power law of the repulsion between points.

The value of \( G \) is taken as a measure of the regularity of the point distribution and the set of point coordinates which minimise \( G \) will be the optimum for the given weightings and power law.

### 5.4 MINIMISING GEOMETRIC POTENTIAL FUNCTIONS

The distance between two points in space using the spherical coordinate system \((\phi, \theta, R)\) is given by

\[
d_{ij} = [R_i^2 + R_j^2 - 2R_iR_j(\cos\theta_i \cos\theta_j + \\
\cos(\phi_i - \phi_j) \sin\theta_i \sin\theta_j)]^{1/2} \tag{5.5}
\]

When an aspherical surface is used, it is possible to introduce some simplification into the geometric potential function in equation 5.4. In the case of spherical surface \( R_i = R_j = R \), a constant, then

\[
d_{ij} = \sqrt{2} \times R \times [1 - \cos\theta_i \cos\theta_j - \\
\cos(\phi_i - \phi_j) \sin\theta_i \sin\theta_j]^{1/2} \tag{5.6}
\]

The geometric potential may be written as

\[
G = \frac{q}{2} \sum \left[ m_i m_j (1 - \cos\theta_i \cos\theta_j - \\
- \cos(\phi_i - \phi_j) \sin\theta_i \sin\theta_j)^{q/2} \right] \tag{5.7}
\]

For ellipsoidal surfaces

\[
R_i = \left( \frac{E^2}{E^2 \sin^2 \theta + \cos^2 \theta} \right)^{1/2} \tag{5.8}
\]

Where \( R_i \) is the radius of any point on the surface.
The values of the relevant radii for each pair of points is calculated using equation 5.8 above and substituted for in equation 5.5 to obtain the distance between these points, that is, \( d_{ij} \), then equation 5.4 is used to obtain the value of the geometric potential function, \( G \).

5.5 OPTIMISATION TECHNIQUES

Since \( G \) is a nonlinear function of the node coordinates then its minimisation involves a nonlinear optimisation problem. The philosophy underlying the nonlinear optimisation programs is that no one strategy can be relied on to be the best for any given problem. Although some methods more commonly do better than others, it is not possible to predict in advance which is the best one to use. Several methods must be tried to solve the same problem and if more than one converge to the same solution this will improve credibility of the solution, Ref 13.

5.6 CONSTRAINED AND UNCONSTRAINED PROBLEMS

The optimisation problem can be treated either as constrained or unconstrained. If the points are free to take up positions anywhere on the spherical or ellipsoidal surface the problem is an unconstrained optimisation in which the variables are \((\phi, \theta)\) coordinates of each node. If the configuration is generated directly on the portion of the sphere or the ellipsoid that is required for the structure the problem is a constrained optimisation. The two types of optimisation problems can be solved by different techniques which give the required solution.

Some applications have been tried on parts of spherical and ellipsoidal surfaces using different optimisation strategies and penalty functions. Of all these strategies the best one that suits our problem was found to be the Direct Search Method by Hooke and Jeeves, Ref 5.
5.7 THE DIRECT SEARCH METHOD

The method of R. Hooke and T. A. Jeeves, that is, 'Direct Search Solution of Numerical and Statistical Problems' is used to solve the problem

\[ U = U(x_1, x_2, \ldots, x_n) = \text{minimum} \]

subject to

\[ W_i(x_1, x_2, \ldots, x_n) = 0 \quad i = 1, p \]
\[ V_j(x_1, x_2, \ldots, x_n) \geq 0 \quad j = 1, M \]

The phrase 'direct search' is used to describe sequential examination of trial solutions involving comparison of each trial solution with 'best' obtained up to that time together with a strategy for determining (as a function of earlier results) what the next trial solution will be.

The application of direct search to a problem requires a space of points \( P \) which represent possible solutions, together with a means of saying that \( P_1 \) is a 'better' solution than \( P_2 \) (written \( P_1 \preceq P_2 \)) for any two points in the space. There is presumably a single point \( P^* \), the solution, with the property \( P^* \preceq P \) for all \( P \neq P^* \).

In these terms, the basic form of direct search is as follows. A point \( B_0 \) is arbitrarily selected to be the first 'base point'. A second point, \( P_1 \), is chosen and compared with \( B_0 \). If \( P_1 \preceq P_0 \), \( P_1 \) becomes the second base point, \( B_1 \); if not, \( B_1 \) is the same as \( B_0 \). This process continues, each new point being compared with the current base point. The 'strategy' for selecting new trial points is determined by a set of 'states' which provide the memory. The number of states is finite. There is an arbitrary initial state \( S_0 \), and a final state which stops the search. The other states represent various conditions which arise as a function of the results of the trials made. The strategy itself comprises the choice of \( B_0 \) and \( S_0 \), the rules of transition
between states, and the rules for selecting trial points as a function of the current state and the base point.

Suppose, for example, that the problem is to minimize a function

\[ f(x_1, x_2, \ldots, x_n). \]

A solution point \( P_i \) is a vector \((x_{i1}, x_{i2}, \ldots, x_{in})\), and we say that \( P_i \preceq P_j \) if and only if

\[ f(x_{i1}, x_{i2}, \ldots, x_{in}) < f(x_{j1}, x_{j2}, \ldots, x_{jn}). \]

The base point \( B_r \), then, is simply that point, from among \( B_0, P_1, P_2, \ldots, P_n \) which has produced (apart from ties) the smallest value of

\[ f(x_1, x_2, \ldots, x_n). \]

The next trial point, \( P_{r+1} \) is determined (relative to \( B_r \)) by the present state \( S_r \). It is convenient to think of a trial at \( P_{r+1} \) as a 'move' or 'step' from the base point \( B_r \). The move is a 'success' if \( P_{r+1} \subseteq B_r \) and it is a 'failure' otherwise. Roughly speaking, the states make up part of the logic, influencing moves to be proposed in the same general direction (assuming that direction is meaningful in the solution space) as those which recently succeeded; they suggest new directions if recent moves have failed; and finally, they decide when no further progress can be made. The fact that no further progress can be made does not always indicate that the solution has been found. Thus direct search may fail.

5.7.1 A Formal Definition of Direct Search

A more precise definition of the basic form of direct search is given below.
There is a space $\Sigma$ of points $P$. There is a comparative relation $C$ on the points of $\Sigma$ satisfying the transitive relation $(P \subset Q, Q \subset R) \Rightarrow P \subset R$. There is an extremal point $P^*$ of $\Sigma$ with the property $P' \subset P$ for any other $P$ in $\Sigma$.

The point $P^*$ represents the solution to a problem, other points of $\Sigma$ representing possible solutions.

A direct search procedure makes sequential comparisons, using $C$ the relation, to determine:

1. A set of 'trial points' $P_r$. (Here and below, $r = 1, 2, \ldots, N$ where $N$ is determined below.)
2. An initial 'base point' $B_0$ and a set of base points $B_r$, where

$$B_r = P_s \text{ for some } s \leq r.$$  

3. A set of integers, called the 'states' of the procedure, including an initial state $S_0$, the state $S_r$ being associated with the base point $B_r$.
4. A 'stop rule' for terminating the procedure, that is for determining $N$.
5. An 'approximate solution', or an approximation to $P^*$, which is $B_N$.

The 'strategy' for the determination of these sets is defined by the following rules:

a. The initial base point $B_0$ and the initial state $S_0$ ($\neq 0$) are arbitrary.

b. The remaining trial points are $P_r = h(B_{r-1}, S_{r-1})$, where $h$ is a function onto $\Sigma$, and $S_{r-1} \neq 0$.

c. If $P_r \subset B_{r-1}$ then $B_r = P_r$ and $S_r = f(S_{r-1})$. Otherwise, $B_r = B_{r-1}$ and $S_r = g(S_{r-1})$.

d. When the first time $S_i = 0$, the procedure stops; that is, $N = i$. 
5.7.2 Pattern Search - A Specific Kind of Strategy

Pattern search is a direct search routine for minimising a function \( S(\phi) \) of several variables \( \phi = (\phi_1, \phi_2, \ldots, \phi_k) \). The argument \( \phi \) is varied until the minimum of \( S(\phi) \) is obtained. The pattern search routine determines the sequence of values for \( \phi \); an independent routine computes the values of \( S(\phi) \).

The successive values of \( \phi \) can be interpreted as points in a \( K \)-dimensional space. The procedure of going from a given point to the following point is called a move. A move is termed a success if the value of \( S(\phi) \) decreases; otherwise it is a failure. The pattern search routine makes two types of move. The first type of move is an exploratory move designed to acquire knowledge concerning the behaviour of the function \( S(\phi) \). This knowledge is inferred entirely from the success or failure of the exploratory moves without regard to any quantitative appraisal of the functional values. The rudimentary information of success or failure is utilised by combining it into a 'pattern' which indicates a probable direction for a successful move. The exploratory moves form a (vector) basis for the argument space. For simplicity, the exploratory moves are here taken to be simple, that is, at each move only the value of a single coordinate is changed. The second type of move is a pattern move designed to utilize the information acquired in the exploratory moves, and accomplish the actual minimisation of the function by moving in the direction of the established 'pattern'. As set up each pattern move is followed by a sequence of exploratory moves which continually revise the pattern. The point from which a pattern move is made is designated a base point, and the direct search procedure may be conceived as fundamentally proceeding from base point to base point.
The pattern move from a given base point duplicates the combined move from the previous base point. That is, all coordinates are changed by an amount equal to the difference between the present base point and the previous base point. The intuitive basis for this type of move is the presumption that whatever constituted a successful set of moves in the past is likely again to prove successful. The result of the pattern move may either be a success or a failure.

Following a successful pattern move it is reasonable to conduct a series of exploratory moves and attempt to further improve the result. Each exploratory move is carried out as follows: A single coordinate of the point is varied to see whether a successful can be made by either increasing or decreasing this coordinate by a prescribed step size. If a success is obtained the altered value of the coordinate is retained; otherwise the original value is restored. Such exploratory moves are made for each coordinate, and a final point reached becomes a new base point. In this way the successful pattern move is further improved and only the information as to whether the exploratory moves succeed or fail is used.

If the pattern move fails, the simplest way of continuing the search would be to begin over again from the base point with a series of exploratory moves, and thus establish an entirely new pattern. Typically a pattern once established will, through continuous modification, grow until the length of the pattern move is 10 to 100 times the basic step size. When the pattern move then fails, it is generally not possible to make any further significant progress in a direction similar to that established by the pattern, and no simple modification of the pattern permits a good new direction to be selected.

The direct search procedure outlined above has been termed
pattern search because it is based on the determination of a ‘pattern’ of simple moves that will give a successful direction in which to move. From the above description it can be realized that the major reduction in \( S(\phi) \) is produced by the pattern move. Some reduction is made by the exploratory moves, but their primary function is to supply information for the improvement of the pattern move.

The method of terminating the search is shown below. For any given value of the step size, the search procedure will reach an impasse when the pattern move having failed to determine a new base point, all the exploratory moves from the base point fail. To further continue the search, it is necessary to reduce the prescribed step size. The amount of reduction should be enough to permit a new pattern to be established. However, too large a reduction in step size will result in slowing down the search. The total search time has not to be overly sensitive to the amount of reduction. The final termination of the search is made when the step size is sufficiently small to insure that the optimum has been closely approximated. In any case, the step size must be kept above a practical limit imposed by means of computation.

An exact description of a pattern search routine, which has been applied, is given in Fig 5.7.1 (from Ref 5).

5.7.3 Examples

some examples of function minimisation are given in this section.

Example 1

As a first example of using direct search pattern, 9 points were placed on a plane square of 10 units side. Five of these points were fixed and the remaining four were free to
Chart 1. Descriptive flow diagram for pattern search

Fig 5.7.1a

Chart 2. Detailed flow diagram for pattern search

For notation see Table B.

Starting Conditions: Initially, the variables $\phi$ and $K$ as well as $\Delta$, $\rho$, and $\delta$ are assigned values; and a routine to compute $S(\phi)$ is supplied.
CHART 3. Descriptive flow diagram for exploratory moves (Program $E$).
The routine shown is carried out for each coordinate separately.

Fig 5.7.1c

(a) The program $E$ is:

(b) The program $E_k$ is:

Note: The loop implicit in (a) may be carried out explicitly (for $k \geq 1$) by the routine:

CHART 4. Detailed flow diagram for exploratory moves

Fig 5.7.1d
TABLE A

Variables and Their Value Interpretations for Charts 2 and 4

The variables $\psi$, $\theta$, and $\phi$ are points in a $K$-dimensional space; the rest of the variables are unidimensional.

- $\theta$: the previous base point
- $\psi$: the current base point
- $\phi$: the base point resulting from the current move
- $S(\psi)$: the functional value at the base point
- $S(\phi)$: the functional value for this move
- $S$: the functional value before this move (usually, the smallest value so far attained by the set of exploratory moves)
- $\Delta$: current step size
- $\delta$: "minimum" step size
- $\rho$: reduction factor for step size, $\rho < 1$
- $w_k$: one of the coordinate values for $\phi$, $k = 1, 2, \ldots, K$
- $K$: number of coordinates for the points

Fig 5.7.1e

TABLE B

Notation for Detailed Flow Diagrams

- $x \rightarrow y$: means the value of the variable $x$ is to become the new value of the variable $y$.
- $\text{P}$: stands for the question "Is the statement $p$ true?".
- $E: S. \phi$: indicates that a program $E$ (See Charts 3 and 4) is to be carried out which will affect the values of the variables $S$ and $\phi$.
- calculate $S(\phi)$: indicates that the value of $S(\phi)$ is to be calculated by an independent program.

Fig 5.7.1f
move on the surface prescribed by this square. Four of the fixed points were at the corners of the square and the fifth point was placed at the intersection of the diagonals of the square. Cartesian coordinate system was used. The coordinates of the five fixed points were 
\((0,0),(10,0),(0,10),(10,10)\) and \((5,5)\). The other four points were placed at \((5,2),(5,4),(5,6)\) and \((5,8)\) coordinates. All points were given the weighting of 1 and the value of the power law of repulsion between points was taken as 4.

The geometric potential function, \(G\), which was defined in section 5.3 was used as a measure of regularity of the points' distribution with the prescribed surface. The length of the chord joining points \(i\) and \(j\) is given by

\[d_{ij} = \left[ (x_i - x_j)^2 + (y_i - y_j)^2 \right]^{1/2} .\]

Substituting for \(d_{ij}\) in the equation of the geometric potential \(G\) and using \(m_i = m_j = 1.0\)

\[G = \sum [(x_i - x_j)^2 + (y_i - y_j)^2]^{-q/2} .\]

The minimisation of this function as a constrained optimisation resulted in the distribution of points shown in Fig 5.7.2.
Fig 5.7.2

Fig 5.7.3
Example 2

As a second example, thirteen points were placed on a similar square as in the first example above. Five points were fixed and had the same coordinates as those of the fixed points of the above example. Four points were placed along the y-axis at intervals of 2 units with the coordinates (0,2), (0,4), (0,6), (0,8). The remaining four points were placed along the line whose equation was x = 10, with coordinates (10,2), (10,4), (10,6), (10,8). The equation of the geometric potential function, G, and the expression of chord length between point i and j shown in example 1 were used. The value of the power law q was taken as 4. The optimisation process carried out using direct search method resulted in points distribution shown in Fig 5.7.3.

Example 3

As a further example, consider the rectangle shown in Fig 5.7.4. The dimensions of this rectangle are 15 by 10 units. Seventeen points were placed as shown in the figure. Points 1, 2, 3, 4 and 17 were fixed and the remaining 12 points were free to move on a surface constrained with the prescribed rectangle. The result of the optimisation process using a power law with q = 4 is shown in Fig 5.7.5. Table 5.7.1 shows the starting coordinates and the resulting coordinates of the optimisation process.
Fig 5.7.4.

Fig 5.7.5
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<th>Resulting coordinate</th>
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<td>y</td>
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<tr>
<td>17</td>
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<td>5</td>
</tr>
</tbody>
</table>

Table 5.7.1
5.8 LOCAL MINIMA

The use of different starting points for the optimisation procedure may help to overcome the problem of obtaining a local minimum in the minimisation process.

5.9 EFFECTS OF POWER LAW AND NODE WEIGHTING

The effect of a high power law is to force node spacing to be more uniform. Increasing the points weighting in a certain area will give greater node spacing in that area. This is useful in minimising the geometric potential function on ellipsoidal surface subdivisions projected from Platonic polyhedra. Variation in points spacing could also be required for structural or architectural reasons.

5.10 APPLICATIONS AND RESULTS

A computer program for optimisation using 'Direct Search Method' by R. Hookes and T.A. Jeeves introduced in section 5.7 was used to find a set of point distribution that give minimum value of G. Different values for power law and weighting were used. Different ellipsoidal surfaces were used as surfaces for the optimisation.

5.10.1 Optimisation Using Certain Numbers of Points

Trials were made by placing different numbers of points at random on spherical or ellipsoidal surfaces and then allowing the optimisation procedure to alter their coordinates until a minimum value of G was obtained. With equal weightings and a power law with q = 2, low numbers of points generated the classical regular polyhedra and with higher numbers of points the geodesic patterns were obtained, Ref 4, for example:
Nodes | Resulting minimum G configuration
---|---
4  | tetrahedron
6  | octahedron
12 | icosahedron
32 | 2-frequency tricon geodesic
42 | 2-frequency alternate geodesic.

Many examples have been considered using some predetermined numbers of points on a chosen part of ellipsoidal surfaces. Some of these examples are given in this section. The configurations obtained are plotted on polar coordinate system with θ varying linearly from the centre and the pattern of interconnection was subsequently added by hand.

**Example 1: A 16-node configuration**

A total of sixteen points were placed on the part of an ellipsoidal surface that lies between θ = 0° and θ = 90°. Six of these points were fixed at 60° intervals around the ring having θ = 90°. Six of the remaining 10 points were spaced at 60° intervals at θ = 60°. Three points were spaced at 120° intervals at θ = 30°. The last point was fixed at the apex. The fixed points had zero degree of freedom and the remaining points were free to move on the surface constrained between θ = 0° and θ = 90°.

Different values of expansion E, power law q and weighting m were used in the geometric potential function optimisation process. The resulting configurations are shown in Figs 5.10.1a to 5.10.3f. Figure numbers together with the relevant values of E, q and m are given below.

Fig 5.10.1a (E=1.0, starting configuration)
Fig 5.10.1b (E=1.0, q=4, m=1.0)
Fig 5.10.1c (E=1.0, q=80, m=1.0)
Fig 5.10.1d (E=1.0, q=2, m=2 for fixed ring m=1 otherwise)
Fig 5.10.1e (E=1.0, q=4, m=2 for fixed ring m=1 otherwise)
Fig 5.10.1f (E=1.0, q=8, m=2 for fixed m=1 for free)
Fig 5.10.1g (E=1.0, q=80, m=2 for fixed m=1 for free)
Fig 5.10.1h (E=1.0, q=80, m=2 for fixed ring m=1 otherwise)

Fig 5.10.2a (E=1.5, starting configuration)
Fig 5.10.2b (E=1.5, q=8, m=1.0)
Fig 5.10.2c (E=1.5, q=20, m=1.0)
Fig 5.10.2d (E=1.5, q=50, m=1.0)
Fig 5.10.2e (E=1.5, q=10, m=6 for fixed ring m=1 otherwise)
Fig 5.10.2f (E=1.5, q=10, m=6 for fixed m=1 for free)
Fig 5.10.2g (E=1.5, q=80, m=2 for fixed ring and m=1 otherwise)
Fig 5.10.2h (E=1.5, q=80, m=6 for fixed ring and m=1 otherwise)
Fig 5.10.2i (E=1.5, q=80, m=6 for fixed m=1 for free)
Fig 5.10.2j (E=1.5, q=80, m=10 for fixed ring and m=1 otherwise)

Fig 5.10.3a (E=1/1.5, starting configuration)
Fig 5.10.3b (E=1/1.5, q=2, m=1.0)
Fig 5.10.3c (E=1/1.5, q=6, m=1.0)
Fig 5.10.3d (E=1/1.5, q=40, m=1.0)
Fig 5.10.3e (E=1/1.5, q=80, m=1.0)
Fig 5.10.3f (E=1/1.5, q=10, m=6 for fixed and m=1 for free)

In all the above cases it is noted that cyclic symmetry of order 3 appears and that the configurations obtained with high values of power law are more uniform than others.

Example 2: A 21-node configuration

A total of 21 points were placed on the part of an ellipsoidal surface that lies between \( \theta = 0^\circ \) and \( \theta = 90^\circ \). Eight points were fixed at 45\(^\circ\) intervals around the ring \( \theta = 90^\circ \). Another 8 points were placed at 45\(^\circ\) intervals at
and 4 points were spaced at 90° intervals at 
θ = 30°. The last point was fixed at the apex. The fixed
points had zero degree of freedom and the remaining points
are free to move on the surface constrained between θ = 0°
and θ = 90°.

Different values of expansion E, power q and weighting m
were used in the geometric potential function optimisation
process. The resulting configurations are shown in Figs
5.10.4a to 5.10.6b. Figure numbers together with the
relevant values of E, q and m are given below.

Fig 5.10.4a (E=1.0, starting configuration)
Fig 5.10.4b (E=1.0, q=80, m=1.0)

Fig 5.10.5a (E=1.5, starting configuration)
Fig 5.10.5b (E=1.5, q=4, m=1.0)
Fig 5.10.5c (E=1.5, q=80, m=1.0)
Fig 5.10.6a (E=0.5, starting configuration)
Fig 5.10.6b (E=0.5, q=40, m=1.0)

In all of these configurations a cyclic symmetry of order
4 appears.

Example 3: A 29-mode configuration

Twenty nine points were placed on the same surface defined
in examples 1 and 2 above. Sixteen points were fixed at
22.5° intervals at the ring θ = 90°. Eight points were
spaced at 45° at the ring θ = 60° and they were free to
move on the defined surface. Four points were spaced at
90° intervals at the ring θ = 30° and they were also free
to move on the defined surface. The last point was fixed
at the apex.

Different values of expansion E, power q and weighting m
were used in the geometric potential function optimisation
Fig 5.10.5c
process. The resulting configurations are shown in Figs 5.10.7a to 5.10.9c. Figure numbers together with the relevant values of $E$, $q$ and $m$ are given below.

Fig 5.10.7a ($E=1.0$, starting configuration)
Fig 5.10.7b ($E=1.0$, $q=6$, $m=1$ for fixed and $m=2$ for free)
Fig 5.10.7c ($E=1.0$, $q=6$, $m=1.0$)
Fig 5.10.7d ($E=1.0$, $q=4$, $m=2$ for fixed and $m=1$ for free)
Fig 5.10.8a ($E=1.5$, $q=4$, $m=1.0$)
Fig 5.10.8b ($E=1.5$, $q=6$, $m=1.0$)
Fig 5.10.8c ($E=1.5$, $q=6$, $m=2$ for fixed and $m=1$ for free)
Fig 5.10.9a ($E=0.75$, $q=4$, $m=1.0$)
Fig 5.10.9b ($E=0.75$, $q=2$, $m=1$ for fixed and $m=1$ for free)
Fig 5.10.9c ($E=0.75$, $q=4$, $m=1$ for fixed ring and $m=2$ otherwise)

In these configurations also cyclic symmetry of order 4 appears.

Example 4: A 31-node configuration

Thirty one nodes were placed on the ellipsoidal surface defined in the previous examples. Two cases were considered. Case 1 is the configuration obtained when 12 nodes were fixed around the ring $\theta = 90^\circ$. Case 2 is the configuration obtained when 15 nodes were fixed around the ring $\theta = 90^\circ$.

Case 1

In this case 12 nodes were fixed around the $\theta = 90^\circ$ at $30^\circ$ intervals. Twelve nodes were spaced at $30^\circ$ intervals at $\theta = 60^\circ$ and they were free to move on the prescribed surface. Six nodes were spaced at $60^\circ$ intervals at $\theta = 30^\circ$ and they were also free to move. The last node was fixed at the apex.
Fig 5.10.7a

Fig 5.10.7b
Fig 5.10.8a
Fig 5.10.9a

Fig 5.10.9b
Fig 5.10.9c
Different values of expansion $E$, power $q$ and weighting $m$ were used in the geometric potential function optimisation process. The resulting configurations are shown in Figs 5.10.10a to 5.10.12b. Figure numbers together with the relevant values of $E$, $q$ and $m$ are given below.

Fig 5.10.10a ($E=1.0$, $q=4$, $m=1$ for fixed and $m=2$ for free)
Fig 5.10.10b ($E=1.0$, $q=80$, $m=1.0$)
Fig 5.10.10c ($E=1.0$, $q=6$, $m=1.0$)
Fig 5.10.10d ($E=1.0$, $q=4$, $m=2$ for fixed and $m=1$ for free)

Fig 5.10.11a ($E=1.5$, $q=10$, $m=1.0$)
Fig 5.10.11b ($E=1.5$, $q=80$, $m=1.0$), different starting point was used. The nodes are initially distributed as follows: twelve nodes were fixed around the ring $\theta = 90^\circ$ at $30^\circ$ intervals. Twelve nodes were placed free to move at $\theta = 75^\circ$ at $30^\circ$ intervals. Six nodes were placed free to move at $\theta = 45^\circ$ at $60^\circ$ intervals. The last node was fixed at the apex.

Fig 5.10.12a ($E=1/1.5$, $q=6$, $m=1.0$)
Fig 5.10.12b ($E=1/1.5$, $q=80$, $m=1.0$)

In these configurations a cyclic symmetry of order 6 appears.

Case 2

In this case 15 nodes were fixed around the $\theta = 90^\circ$ at $24^\circ$ intervals. Ten nodes were spaced at $36^\circ$ intervals at $\theta = 60^\circ$ and they were free to move on the prescribed surface. Five nodes were spaced at $72^\circ$ intervals at $\theta = 30^\circ$ and they were also free to move. The last node was fixed at the apex.

Different values of expansion $E$, power $q$ and weighting $m$ were used in the function optimisation process.
Fig 5.10.10a

Fig 5.10.10b
Fig 5.10.10c

Fig 5.10.10d
Fig 5.10.11a

Fig 5.10.11b
The resulting configurations are shown in Figs 5.10.13 to 5.10.17. Figure numbers together with the relevant values of E, q and m are given below.

Fig 5.10.13 (E=1.0, q=60, m=1.0)
Fig 5.10.14 (E=1/1.25, q=40, m=1.0)
Fig 5.10.15a (E=1.25, q=4, m=1 for fixed and m=4 for free)
Fig 5.10.15b (E=1.25, q=1, m=1 for fixed and m=2 for free)
Fig 5.10.16 (E=1.0, q=60, m=1.0)
Fig 5.10.17 (E=1.5, q=4, m=1 for fixed and m=2 for free)

In these configurations the cyclic symmetry is of order 5 except for Fig 5.10.17 where a 3 order symmetry appears with 20 different member lengths.

Example 4: A 46-node configuration

In this example 15 nodes were fixed around the θ = 90° at 24° intervals on a similar surface as defined in the previous examples. Fifteen nodes were spaced at 24° intervals at θ = 60° and they were free to move on the prescribed surface. Ten nodes were spaced at 36° intervals at θ = 45° and they were also free to move. Five nodes were spaced at 72° intervals at θ = 30° and they were also free to move on the prescribed surface. The last node was fixed at the apex.

Different values of expansion E, power q and weighting m were used in the geometric potential function optimisation process. The resulting configurations are shown in Figs 5.10.18a to 5.10.18d. Figure numbers together with the relevant values of E, q and m are given below.

Fig 5.10.18a (E=1.0, q=4, m=1.0)
Fig 5.10.18b (E=1.0, q=4, m=1 for fixed and m=2 for free)
Fig 5.10.18c (E=1.0, q=4, m=2 for fixed and m=1 for free)
Fig 5.10.18d (E=1.0, q=4, m=2 for fixed and m=4 for free)

The cyclic symmetry is of order 5.

5.10.2 Optimisation Using Geodesic Subdivision on Ellipsoidal Surfaces

The spherical coordinates (ϕ, θ) of classical geodesic subdivisions on some polyhedra faces using the 'alternate' method of subdivision were used as starting values for optimisation of point distribution on ellipsoidal surfaces of different expansion, E.

The symmetry was utilized to carry the optimisation on one face or more (in the case of icosahedron) to cover values of θ less than or equal to 90°. This minimises time and effort for both human and machine.

Many cases have been tried using different values for expansion, E, and power q. Some of the results are given in this section.

5.10.2.1 Generation of Configuration Coordinates

Obtaining the starting values of the coordinates and their submission to the optimisation program may itself seem to be a problem, but as mentioned and shown in the previous chapters formex algebra and Formian provide a convenient tool. Any configuration can be created on the generator volume as explained in chapter 4 and the required data may be transferred to the relevant computer program. The process of automated data generation makes it easier to optimise node distribution in many configurations having different interconnection patterns by just changing the parameters in the generating scheme in Formian.
5.10.2.2 Presentation of Results

The values of the coordinates $\phi$ and $\theta$ for each node in the configuration are shown in tables. These tables show three different sets of coordinates. The first set of coordinates are the values obtained using classical geodesic subdivision as starting values, the second set contains the values obtained using geometric potential function optimisation technique and the last set of coordinates are the values of $\theta$ obtained using root E correction, remembering that the value of $\phi$ coordinates for the root E correction method are the same as the corresponding values in the starting value sets. These values are tabulated together for the sake of comparison.

Chord factors for configurations are also tabulated in a similar manner.

5.10.2.3 EXAMPLES

Example 1

A 2-frequency icosahedron face and part of the adjacent face that crosses the line of $\theta = 90^\circ$ (which are considered as a basic sub-structure) are used as a surface for a constrained optimisation process. Fig 5.10.19 shows the configuration to be optimised and the basic sub-structure used for optimisation. The results are shown in Tables 5.1 to 5.4.

Table 5.1 shows the spherical coordinates ($\phi$, $\theta$) of points obtained from classical subdivision method, the geometric potential function minimisation method and root E-correction method for expansion $E = 1.5$ and $q = 6$.

Table 5.2 shows the spherical coordinates ($\phi$, $\theta$) of points obtained using the above three mentioned methods for expan-
Fig 5.10.19 Top view

Basic sub-structure
### Table 5.1 (Two Frequency Icosahedron, $E=1.5$ and $Q=6.0$)

<table>
<thead>
<tr>
<th>NODE NO.</th>
<th>STARTING VALUES</th>
<th>GEOMETRIC POTENTIAL</th>
<th>ROOT $E$ CORRECTION</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\phi$</td>
<td>$\theta$</td>
<td>$\phi$</td>
</tr>
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<td>31.7174743</td>
<td>72.00000</td>
</tr>
<tr>
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<td>63.4349488</td>
<td>0.000000</td>
</tr>
<tr>
<td>2,1</td>
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<td>58.2825255</td>
<td>36.00000</td>
</tr>
<tr>
<td>2,2</td>
<td>72.00000</td>
<td>63.4349488</td>
<td>72.00000</td>
</tr>
<tr>
<td>3,1</td>
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</tr>
<tr>
<td>3,2</td>
<td>54.00000</td>
<td>90.000000000</td>
<td>54.00000</td>
</tr>
</tbody>
</table>

### Table 5.2 (Two Frequency Icosahedron, $E=1/1.5$, $Q=6.0$)

<table>
<thead>
<tr>
<th>NODE NO.</th>
<th>STARTING VALUES</th>
<th>GEOMETRIC POTENTIAL</th>
<th>ROOT $E$ CORRECTION</th>
</tr>
</thead>
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<tr>
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<td>$\theta$</td>
<td>$\phi$</td>
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<td>3,2</td>
<td>54.00000</td>
<td>90.000000000</td>
<td>54.00000</td>
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</table>
Table 5.3 (Chord Factors For a Two Frequency Icosahedron, E=1.5 and Q=6.0)

<table>
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<tr>
<th>MEMBER</th>
<th>STARTING VALUES</th>
<th>GEOMETRIC POTENTIAL</th>
<th>ROOT E CORRECTION</th>
</tr>
</thead>
<tbody>
<tr>
<td>0,0/1,0</td>
<td>0.7887837</td>
<td>0.6768710</td>
<td>0.6757682</td>
</tr>
<tr>
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<td>0.7992141</td>
<td>0.7104180</td>
<td>0.7094949</td>
</tr>
<tr>
<td>1,0/2,0</td>
<td>0.6809909</td>
<td>0.7489052</td>
<td>0.7068145</td>
</tr>
<tr>
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<td>0.7611889</td>
<td>0.6751937</td>
<td>0.7484096</td>
</tr>
<tr>
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<td>0.5874070</td>
<td>0.6077963</td>
<td>0.5732769</td>
</tr>
<tr>
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<td>0.6086171</td>
<td>0.6451866</td>
</tr>
<tr>
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</tr>
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</table>

Table 5.4 (Chord factors for a Two frequency Icosahedron, E=1/1.5 and Q=6.0)

<table>
<thead>
<tr>
<th>MEMBER</th>
<th>STARTING VALUES</th>
<th>GEOMETRIC POTENTIAL</th>
<th>ROOT E CORRECTION</th>
</tr>
</thead>
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<tr>
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<td>0.5296085</td>
</tr>
<tr>
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<td>0.4716218</td>
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</tr>
<tr>
<td>1,0/2,1</td>
<td>0.5074592</td>
<td>0.5379871</td>
<td>0.5428410</td>
</tr>
<tr>
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<td>0.5413932</td>
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</tr>
<tr>
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<td>0.5275506</td>
<td>0.4322156</td>
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</tr>
<tr>
<td>2,1/3,1</td>
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<td>0.5201625</td>
<td>0.5301040</td>
</tr>
<tr>
<td>3,1/3,2</td>
<td>0.6180339</td>
<td>0.6180339</td>
<td>0.6180339</td>
</tr>
</tbody>
</table>
sion \( E = 1/1.5 \) and \( q = 6 \).

Table 5.3 shows the chord factors obtained using the three methods for expansion \( E = 1.5 \) and \( q = 6 \) and Table 5.4 shows the chord factors obtained using the three methods for \( E = 1/1.5 \) and \( q = 6 \).

**Example 2**

A 4-frequency icosahedron face and part of the adjacent face that crosses the line of \( \theta = 90^\circ \) are used as a surface for a constrained optimisation process. The coordinates \((\phi, \theta)\) of the points that lie on apices of the icosahedral generator volumes were fixed. In addition, the \( \phi \)-coordinates of all point that lie on the polyhedron's edges were fixed whereas their \( \theta \)-coordinates were free to move and to participate in the optimisation process. Fig 5.10.20 shows the basic sub-structure used for optimisation. The results are shown in Tables 5.5 to 5.9.

Table 5.5 shows the spherical coordinates \((\phi, \theta)\) of points obtained from classical subdivision method, the geometric potential function minimisation method and root \( E \)-correction method for expansion \( E = 1.5 \) and \( q = 6 \).

Table 5.6 shows the spherical coordinates \((\phi, \theta)\) of points obtained using the above three mentioned methods for expansion \( E = 1.0 \) and \( q = 6 \).

Table 5.7 shows the spherical coordinates \((\phi, \theta)\) of points obtained using the above three mentioned methods for expansion \( E = 1/1.5 \) and \( q = 6 \).

Table 5.8 shows the chord factors obtained using the three methods for expansion \( E = 1.5 \) and \( q = 6 \) and Table 5.9 shows the chord factors obtained using the three methods for \( E = 1/1.5 \) and \( q = 6 \).
Fig 5.10.20 Basic sub-structure
<table>
<thead>
<tr>
<th>NODE NO.</th>
<th>STARTING VALUES</th>
<th>GEOMETRIC POTENTIAL</th>
<th>ROOT E CORRECTION VALUES OF θ</th>
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<td>72.000000 13.167302</td>
<td>11.961334</td>
</tr>
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<td>0.000000 26.767479</td>
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<tr>
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<td>26.776545</td>
</tr>
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Table 5.7 (Four Frequency icosahedron, E=1/1.5 Q=6.0)

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Table 5.8 (Chord factors for four Frequency Icosahedron, \( E=1.5 \) and \( Q=6.0 \))

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Table 5.9 (Chord Factor for Four Frequency Icosahedron, 
\(E=1/1.5, \ Q=6.0\))

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Example 3

A 4-frequency octahedron face is used as a basic sub-structure to optimise the configuration shown in Fig 5.10.21. The basic sub-structure is shown in Fig 5.10.22. The results of the optimisation process is shown in Tables 5.10 to 5.15.

Table 5.10 shows the spherical coordinates ($\phi$, $\theta$) of points obtained from classical subdivision method, the geometric potential function minimisation method and root $E$-correction method for expansion $E = 1.0$ and $q = 6$.

Table 5.11 shows the spherical coordinates ($\phi$, $\theta$) of points obtained using the above three mentioned methods for expansion $E = 1/1.5$ and $q = 6$.

Table 5.12 shows the spherical coordinates ($\phi$, $\theta$) of points obtained using the above three mentioned methods for expansion $E = 1.5$ and $q = 6$.

Table 5.13 shows the chord factors obtained using the three methods for expansion $E = 1.0$ and $q = 6$, Table 5.14 shows the chord factors obtained using the three methods for $E = 1/1.5$ and $q = 6$ and Table 5.15 shows the chord factors obtained using the three methods for $E = 1.5$ and $q = 6$.

5.11 DISCUSSION

The results of the above examples show that there are many factors which affect the minimisation process and its results. These factors are

1. The nature of the surface on which the points are supposed to lie.
2. The number of points assumed.
Fig 5.10.21

Side view

Fig 5.10.22 Basic sub-structure

Top view
Table 5.10 (Four frequency octahedron, \( E=1.0, \; Q=6.0 \))

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Table 5.11 (Four Frequency octahedron, $E=1/1.5$, $Q=6.0$)

<table>
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Table 5.12 (Four frequency octahedron, E=1.5, Q=6.0)

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Table 5.13 (Chord Factor for Four Frequency Octahedron, \( E=1.0, Q=6.0 \))

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<th>GEOMETRIC POTENTIAL</th>
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</thead>
<tbody>
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<td>0.3898622</td>
</tr>
<tr>
<td>1,0/1,1</td>
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<td>0.5407719</td>
</tr>
<tr>
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<td>0.3904991</td>
</tr>
<tr>
<td>1,0/2,1</td>
<td>0.438871</td>
<td>0.4724512</td>
</tr>
<tr>
<td>2,0/2,1</td>
<td>0.517638</td>
<td>0.5211371</td>
</tr>
<tr>
<td>2,0/3,0</td>
<td>0.459506</td>
<td>0.3904991</td>
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<tr>
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<td>0.5211371</td>
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<tr>
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<td>0.4724512</td>
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Table 5.14 (Chord Factors for Four Frequency Octahedron, E=1/1.5, Q=6.)

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Table 5.15 (Chord Factors for Four Frequency Octahedron, $E=1.5$, $Q=6.0$)

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<th>CORRECTION</th>
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<tr>
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</table>
(3) The initial starting point.
(4) The weightings of the points.
(5) The power law of the repulsion forces between points.

The effects of each of the above factors are shown below:

(1) The Nature of The Surface on Which The Points Lie

The surfaces used for minimisation process includes spherical surfaces, ellipsoidal surfaces having different expansions. The results show that the distributions of points tend to be uniform and other factors shown above have the same effect in all cases of ellipsoidal surfaces. Some trials have been taken to minimise the geometric potential functions on paraboloidal surfaces but the results obtained are not good which means that either the optimisation method used is not suitable for this kind of problem or some of the factors discussed below need to be changed in order to arrive to a feasible solution.

(2) The Number of Points Used for Minimisation

A feasible number of points which can give a suitable distribution on the assumed surface can affect the final result or the existence of the solution itself. The number of points which can be distributed in a way of cyclic symmetry will result in saving the execution time of the program and will result in an acceptable distribution and hence a sensible configuration.

(3) The Initial Starting Point

Choosing an initial starting point can direct the minimisation process in a certain direction and can help very much in arriving to the required solution. In some cases using an infeasible initial starting point will
increase the program execution time or will result in an unacceptable solution or no solution at all.

(4) The Weightings of The Points

Increasing the weightings of some points will result in greater node spacings in the area related to these points. Hence the density of node distribution can be controlled by varying the weightings of some points at the intended area. This effect can be seen clearly in the previous examples especially when ellipsoidal surfaces are used. Increasing the weightings of the fixed points around the ring having \( \theta = 90^\circ \) resulted in lengthening the members connected to these fixed points. On the other hand, increasing the weightings of the free points resulted in shortening the members connected to the fixed points. If the weightings of all points are assigned the same value, the result will be the same no matter what the value of the weighting is.

(5) The Power Law of Repulsion Between Points

The contribution of one point to the geometric potential function depends on its distance from all other points. If the power \( q \) is high then more distant nodes will have very little effect and the major contributions to the geometric potential function come from the interaction of a node with its nearest neighbours only. The more local action of a high power \( q \) is thus to force node spacing to be uniform, Ref 4. What high value of power \( q \) to be used in order to obtain a uniform point distribution depends on other factors, mainly on the nature of the surface used and the number of points. Examples shown in this chapter show that for some cases a uniform distribution can be obtained using power \( q \) as low as 6 whereas some others need a power \( q \) as high as 80.

Comparison of the results of examples shown in section
5.10.2 can be made on the basis of:

(1) Regularity of the configuration that results.
(2) The ratio between the maximum and minimum length of members.
(3) The position of the longest and shortest set of members in the structure.
(4) The total length of members.

(1) Regularity of The Configuration

For all the configurations studied in this work it is apparent that the classical method of projecting a subdivided face of a regular polyhedron onto a circumscribing surface fails to yield good regularity when ellipsoidal surfaces are used. On the other hand, the geometric potential function minimisation method yields, almost regular configurations in all cases.

(2) Ratio Between Maximum and Minimum Length of Members

The geometric potential function minimisation method is superior in this respect as it gives the lowest ratio of maximum to minimum length of members in all cases compared with the values obtained using classical methods of subdivision. The range of member lengths is small which helps to simplify the process of manufacturing and transportation a structure, Table 5.16.

(3) Position of Longest and Shortest Members

For all shallow ellipsoidal configurations (that is, E < 1.0), obtained by projecting the classical subdivision of a regular polyhedron face onto a circumscribing elliptical surface, the shortest members will concentrate near the zenith and the longest members are found to be near the foundations. For tall ellipsoidal configurations (that is,
### Table 5.16 (Ratio of Longest/Shortest member)

<table>
<thead>
<tr>
<th>CONFIGURATION</th>
<th>STARTING VALUES</th>
<th>GEOMETRIC POTENTIAL</th>
<th>ROOT E CORRECTION</th>
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<tr>
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<td>1.355</td>
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<tr>
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<td>1.644</td>
<td>1.514</td>
<td>1.55</td>
</tr>
<tr>
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<tr>
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</tr>
<tr>
<td>E=1/1.5</td>
<td>2.314</td>
<td>1.80</td>
<td>1.915</td>
</tr>
</tbody>
</table>
E > 1.0), the situation is reversed and the shortest members concentrate near the foundation whereas the longest members are found to be near the zenith.

The geometric potential function minimisation method tends to distribute the members regularly.

(4) Total Length of Members

It turns out that the regularity of configuration is accompanied by an increase in the total length of members in the structure, but this increase is found to be very small. However, this increase in the total member length does not imply an increase in the amount of material, because the regularity obtained will yield a more even stress distribution which will result in smaller member sizes.

5.12 OBSERVATIONS AND COMMENTS

It is observed that using the geometric potential function minimisation method on even spherical geodesic surfaces obtained using conventional methods yields a better configuration.

The potential function minimisation method tends to divide the angles subtended by the edges of the substructure with the centre equally, which is not the case in the classical method.

Geometric potential function minimisation method presents an ingenious method to revolutionise the world of geodesic and braced domes, taking advantage of the fast spreading of digital computers.
CHAPTER SIX

CONCLUSIONS

A new approach for formex formulation of geodesic configurations using different methods of subdivision has been presented. This approach uses formex algebra and its computer programming language Formian. It has been shown by various examples that by use of this approach previously difficult and tedious data preparation and generation of geodesic configuration can be solved in a simple manner. This new approach also opens unlimited opportunities to create and prepare data for structures which were extremely difficult to be generated in the past. This approach also provides a useful tool to study various alternatives by just changing few parameters at the stage of running the Formian program and hence saves time and effort.

The generator volumes used for these configurations can be a tetrahedron, a cube, a truncated octahedron, an octahedron or an icosahedron. Class I, class II and class III methods of subdivisions have been studied and many examples have been presented. The profile of these configurations can be a circle, a supercircle, an ellipse, a superellipse, a parabola, a hyperbola or a conical section. The floor cross section of these configuration can be circular, supercircular, elliptical or superelliptical. Many combinations of profiles and floor cross sections have been tried and the resulting configurations are shown.

The new approach of generating geodesic configurations using the method of geometric potential function minimisation on spherical and elliptical surfaces has been presented and studied. Some examples of regular
configurations having number of nodes not obtainable from the classical methods of subdivision have been shown. The effects of many factors on the minimisation process have been studied and examples have been presented. The main factors that affect the minimisation process are found to be:

(1) The nature of the surface on which the points lie.
(2) The number of points used for minimisation.
(3) The initial starting point.
(4) The weightings of the points.
(5) The power law of the repulsion forces between points.

This new approach opens a new era in the world of geodesic configurations and gives designers more freedom in choosing their designs. The configurations obtained using this method have no truncation problems as they can be generated directly on the required surface.

The approach using formex algebra and Formian to create geodesic configurations can be extended to include the rest of the Platonic polyhedra and Archimedean polyhedra as generator volumes. The circumscribing surfaces can be extended to include many other shapes which have not been covered in this work.

The geometric potential function minimisation approach can be extended to optimise the distribution of any number of nodes, with a feasible solution, on other possible surfaces not covered in this work to obtain new regular configurations. This approach can also be used to obtain more regular geodesic configurations created on paraboloidal, hyperboloidal and conical surfaces using the classical subdivision coordinates of nodes as starting values for the optimisation process.
REFERENCES


