Probing dark matter with star clusters

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Declaration

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Part of this work have appeared or will appear in the following publications:

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Abstract

Star clusters are collisional and dark matter (DM) free stellar systems, where their evolution is ruled by two-body interactions and the galactic potential. Using direct summation $N$-body simulations, I study how the observational properties of star clusters can be used to: (i) distinguish between DM free and DM dominated objects. From observations, the nature of several faint stellar systems in the Milky Way halo is not clear, therefore, I quantify the contribution of star clusters to the faint stellar systems population. (ii) Probe the underlying DM density of their host galaxy. I apply a new method to the recently discovered Eridanus II ultra-faint dwarf galaxy that hosts a star cluster in its centre. I find that a cored DM density profile naturally reproduces the observed properties of Eridanus II’s star cluster. (iii) Infer their progenitor properties if they are accreted star clusters, such as Crater. From its properties I find that Crater is likely to be tidally stripped from a dwarf galaxy, and it must have formed extended and with a low concentration. Throughout this thesis, the comparison of simulations and data took into consideration observational biases and uncertainties. I show that the initial conditions of star clusters can heavily influence its present-day properties, and that the stellar evolution prescriptions can also impact the final star cluster properties, such as the neutron stars natal kick distribution. I conclude, through a series of test cases, that $N$-body simulations can be used to reproduce the observed properties of star clusters, and these can ultimately probe their host galaxy DM distribution.
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Chapter 1

Introduction

Understanding the Universe has always been a fixation for humanity, since their existence people have observed the sky pondering on their nature. Despite our effort in the last few thousands of years, we are still far from a complete picture of the Universe. To catch the meaning of where we are now and why, most scientists have focused their studies on the evolution of galaxies, especially our Galaxy - the Milky Way. In the last three decades, photometric and spectroscopic surveys (e.g. HIPPARCOS, 2MASS, SDSS, SEGUE, and APOGEE) have had a major impact in our understanding of galaxy formation, unveiling structures before unseen. Theoretical and numerical (with the development of computers) models have developed in the attempt to reproduce these observational constraints. For instance, rotation curves of galaxies show a flattening in the external region, requiring an additional massive component, commonly referred to as dark matter, which appears to behave as a collisionless fluid that contributes in mass (therefore interactive gravitationally) but not observable. Nowadays, it has been estimated that $\sim 27\%$ of the Universe is made of dark matter (Planck Collaboration et al., 2014a).

One of the most successful models is the $\Lambda$ Cold Dark Matter ($\Lambda$CDM) model. On large scales this model agrees with the growth of large structure in the Universe, on smaller scales there are two problems, the missing satellites and cusp-core problems. The first one is a missing number of objects expected to be observed in the Milky Way, while the second one is a discrepancy between predicted cuspy density profiles (like NFW-profile, Navarro et al., 1996b) and observed core density profiles. These ‘small’ scale problems could tell us more about the nature of dark matter or about physics phenomena that we did not consider. In this thesis, I focused my studies in the understanding of dark matter in the ‘small’ scale, using the properties of globular clusters.

1.1 Globular clusters

The Milky Way hosts $\sim 160$ globular clusters (Harris, 1996, 2010 edition), which are self-gravitating systems orbiting around their host galaxy. Often defined as the ‘building blocks’ of galaxies, they are a perfect laboratory to study several astrophysical problems. For example, they have been used to study the structure of the Milky Way (Shapley, 1918); stellar evolution (Eddington, 1926); the stellar initial mass function (Chabrier, 2003a); and, the formation and evolution of the Milky Way disc (Friel, 1995; Freeman and Bland-Hawthorn, 2002). Moreover, globular clusters and their streams have been
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Figure 1.1: Globular cluster M 9 (NGC 6333). Image from the NASA/ESA Hubble Space Telescope.

used as tracers of the Milky Way potential (Koposov et al., 2010; Bonaca et al., 2014), and additional constraints on the potential are expected in the next years, thanks to the ongoing astrometric mission Gaia (Gaia Collaboration et al., 2016).

In several galaxies like the Milky Way, the globular clusters population can be divided in two subsystems: the majority of globular clusters are metal-poor with metallicities $[\text{Fe}/\text{H}] < -0.8$ which sits in the halo; whereas the metal-rich population with $[\text{Fe}/\text{H}] > -0.8$ sits in the bulge and the disc (Zinn, 1985). This leads to different formation scenarios of the Milky Way. For example, the metal-poor globular clusters in the halo might be from satellite galaxies which have been accreted by the Milky Way, while the metal-rich globular clusters formed in situ (Côté et al., 1998). In Chapter 5, I performed a suite of numerical simulations to study the evolution of one of the halo globular clusters, Laevens I/Crater (Laevens et al., 2014; Belokurov et al., 2014), which is very likely to have an extragalactic origin (Weisz et al., 2016).

Milky Way globular clusters, like M 9 in Fig. 1.1, typically have $10^4 - 10^6$ stars, with luminosities $M_V$ (absolute $V$-band magnitude) from $-4$ to $-10$, an observed size between $\sim 1$ pc and $\sim 20$ pc, high central concentration (high central density $\rho \simeq 10^4 M_\odot \text{pc}^{-3}$), and they are $8 - 13$ Gyr old. However, recently, a family of faint objects that resemble extended globular clusters and compact dwarf galaxies, have been brought to light. In Chapter 2, I present an extensive study of these objects to shed light on their nature.

Due to the high density in the centre of globular clusters, these systems can be considered as one of the most rich laboratories of exotic stars. In globular clusters, it is possible to find millisecond pulsars, cataclysmic variables, binary X-ray sources, blue stragglers,
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which are all thought to be the result of dynamical interactions in dense stellar systems. Moreover, the core of these stellar systems could host one of the merger of two massive stellar black hole, discovered thanks to the emission of gravitational waves (Abbott et al., 2016b). Despite the remarkable amount of observables, the dark remnants populations (white dwarfs, neutron stars, black holes) are still unknown. In Chapters 2 and 3, I studied how some of the properties of the dark remnants could influence the evolution of star clusters.

Globular clusters have been studied intensively with photometric and spectroscopic observations, which have provided their structural and kinematics properties. With these data, projected surface density and velocity dispersion profiles can be constructed. During the analysis of these data, sometimes spherical symmetry is assumed, however some of these stellar systems have a non-zero ellipticity parameter ($e$). Chen and Chen (2010) showed that the majority of globular clusters have $e$ between 0.05 and 0.25, with some exceptional case like NGC 6401 with $e = 0.41 \pm 0.08$. This observed ellipticity is mainly due to internal rotation, pressure anisotropy and external tides, however it is still unknown which one is the dominant effect. Even if it is a challenging task, some degree of anisotropy has been observed also in the velocity space, with a moderate internal rotation (Fabricius et al., 2014). With data from the Gaia satellite, it will be possible to extend these measurements to many other globular clusters and study in more detail their possible internal rotation.

Since their discovery, globular clusters are well known to be unique objects because they have a single stellar population. Therefore, the general idea of their formation is one molecular cloud that collapses gravitationally and forms all the stars. However, recently, photometric and spectroscopic observations have shown that globular clusters do not have a single stellar population. In a colour-magnitude diagram, the stars in globular clusters show different evolutionary sequences (not visible with previous telescopes), this means that the stars have different chemical composition. These observations show that stars in all the globulars have a spread in abundances only for the light elements. The stars in clusters are being selected as pristine and polluted stars, because it may be that these have been ‘polluted’ by massive stars. There are various mechanisms on how the polluted stars are formed, however, each scenario has its flaws (see e.g. Bastian et al., 2015). Therefore, the origin of multiple populations in globular clusters can be considered as one of the main unresolved problem in the star cluster community (see the review by Gratton et al., 2012).

Globular clusters play an important role also in studying dwarf galaxies. Due to a large variety of galaxies, often a lack of observables make the estimation of dark matter not possible. Most faint galaxies, for example, have no remaining gas. However, if one or more star clusters are present, it is still possible to estimate the amount of dark matter in the galaxy. Goerdt et al. (2006) and Sánchez-Salcedo et al. (2006) showed that the survival of globular clusters in dwarf galaxies can be used to probe their dark matter density. In Chapter 4, I used not only the survivability argument but also the structural properties of an observed star cluster in a dwarf galaxy to probe its inner dark matter density profile.

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1The terminology ‘star clusters’ is often used to refer to globular clusters and open clusters, the second one are usually much younger than globular clusters, more metal-rich and they have less than a few thousands stars. In this thesis, I assume that there is no difference between the terms ‘star clusters’ and ‘globular clusters’
Globular clusters have been studied not only observationally, many numerical simulations have been performed in order to investigate their stellar dynamics. These objects are the perfect example to study collisional dynamics, and they can be considered as the physical representation of the \( N \)-body problem. In the following Sections, I will describe the \( N \)-body problem and its mathematical formulation, and the star cluster evolution from a theoretical point of view.

# 1.2 \( N \)-body problem

## 1.2.1 History of the \( N \)-body problem in a nutshell

The gravitational \( N \)-body problem consists in studying, with a mathematical model, the motion of \( N \) particles (point mass particles) interacting through only mutual gravitational attraction, which is expressed by the Newton’s law (Newton, 1687). Already in 1710, Johann Bernoulli provided a complete solution for the two-body problem (Bernoulli, 1710), however for a couple of centuries no one was able to find a solution for \( N \geq 3 \). The \( N \)-body problem has always been considered very important and between 1885 and 1886, a prize in honour of King Oscar II of Sweden and Norway was established, in occasion of the King’s 60th birthday, 21st January 1889. There were four problems to be solved, and one formulated by Karl Weierstrass was (the english version of the problem is taken by Diacu, 1996):

> Given a system of arbitrarily many mass points that attract each other according Newton’s laws, under the assumption that no two points ever collide, try to find a representation of the coordinates of each point as a series in a variable that is some known function of time and for all of whose values the series converges uniformly. […] In the event that the problem remains unsolved at the close of the contest, the prize may also be awarded for a work in which some other problem of Mechanics is treate as indicated and solved completely.

Unfortunately, none of the 12 papers submitted solved the \( N \)-body problem.

In 1887, the German mathematician Ernst Heinrich Bruns showed that: “the \( n \)-body problem has no integrals-algebraic with respect to the time, the position and the velocity coordinates-except the 10 known ones” (Bruns, 1887). Therefore, having ten known integrals of motion, the system of \( 6N \) equations is reduced to \( 6N - 10 \) variables. In 1913, Karl Sundman found the solution of the three-body problem (Sundman, 1913), with a series solution in power of \( t^{1/3} \) uniformly convergent for all real values of \( t \). However, his solution is not applicable if the system collapses producing a three-body collision.

In 1941, the first simulation of a self-gravitating \( N \)-body system was carried out by Holmberg (Holmberg, 1941). Even if computers and computational facilities did not exist, he found a strategy to calculate reciprocal interactions by replacing gravitation by light. He used light bulbs, which represented the point-mass particles, and measured the total light along two different axes by a combination of a photocell and a galvanometer. Since the light obeys an inverse square law, just like gravity, the data collected by Holmberg provided an estimate of the gravitational field and the forces on the individual objects could be evaluated. With this experiment he tried to study the interaction between two massive objects, like galaxies, represented by two circular groups of lamps, each set with
1.2. $N$-BODIES PROBLEM

Figure 1.2: Figure 4 from Holmberg (1941). This figure shows the tidal deformation resulting from the mutual interaction of two galaxies, sampled using 37 light bulbs each, which, in the left panel are assumed in clockwise rotation while in the right panel their rotation is anti-clockwise. This is considered the first simulation of a $N$-body system.

a diameter of 80 cm and each composed by 37 elements (see Fig. 1.2). Holmberg spent weeks in order to set-up and perform this 74-body simulation and the time evolution was quite short. For more efficient simulations, we have to wait for the 1960s, when the first digital computers were introduced. Thanks to this new technologies, in 1963, numerical simulations with $25 \leq N \leq 100$ were performed by Sverre Aarseth (Aarseth, 1963), who can be considered the ‘father’ and the main pioneer of the gravitational $N$-body simulations.

While the numerical simulations were becoming more and more important, in 1991, Quidong Wang, a Chinese student, published the solution (in terms of power-series) of the $N$-body problem (Wang, 1991). Unfortunately, his solution is not practically relevant, because it has a slow convergence and one should sum a very large number of contributions to have a sufficiently accurate solution of the motion of the particles. Probably, this explains why his theoretical approach is not widely known and why the main way to solve the $N$-body problem is numerical.

Nowadays, with the development of integration methods and new technologies, such as the Graphics Processing Units (GPUs), we are able to study the evolution of stellar systems up to $N \sim 10^6$ particles.

1.2.2 The equations of motion

To introduce this problem from a mathematical point of view, we can start with a discrete distribution of $N$ point-mass particles. Assuming that the particles interact only via gravitational interactions, the gravitational force between the $i$-th and $j$-th particle, according
1.2. N-BODY PROBLEM

to Newton’s law, is

$$ F_{ij} = G \frac{m_j m_i}{r_{ij}^3} (\mathbf{r}_j - \mathbf{r}_i) , \quad i \neq j $$  \hspace{1cm} (1.2.1)

where $\mathbf{r}_j$ and $\mathbf{r}_i$ are the positions of the $i$-th and $j$-th particle with coordinates $(x_i, y_i, z_i)$ and $(x_j, y_j, z_j)$, respectively, in a reference frame with origin in $O = (x_0, y_0, z_0)$. $m_i$ and $m_j$ are the mass of the $i$-th and $j$-th particle, respectively, while $G$ is the gravitational constant. It is worth noting the symmetry of the gravitational force (third principle of dynamics):

$$ F_{ij} = -F_{ji} . $$  \hspace{1cm} (1.2.2)

Then, the force on the $i$-th particle will be the sum of all the forces due to the other $N - 1$ particles:

$$ F_i = \sum_{j=1, j \neq i}^{N} F_{ij} = \sum_{j=1, j \neq i}^{N} G \frac{m_j m_i}{r_{ij}^3} (\mathbf{r}_j - \mathbf{r}_i) $$  \hspace{1cm} (1.2.3)

where

$$ r_{ij} = |\mathbf{r}_j - \mathbf{r}_i| = \sqrt{(x_j - x_i)^2 + (y_j - y_i)^2 + (z_j - z_i)^2} $$  \hspace{1cm} (1.2.4)

is the modulus of the distance between the $i$-th and $j$-th particle.

Now we can write the equations of motion for the $i$-th particle as a system of $3N$ second-order differential equations:

$$ \begin{cases} m_i \ddot{\mathbf{r}}_i = \sum_{j=1, j \neq i}^{N} G \frac{m_j m_i}{r_{ij}^3} (\mathbf{r}_j - \mathbf{r}_i) , \quad j \neq i , \quad i = 1, 2, ..., N \\ \dot{\mathbf{r}}_i(0) = \mathbf{v}_i,0 \\ \mathbf{r}_i(0) = \mathbf{r}_i,0 \end{cases} $$  \hspace{1cm} (1.2.5)

We can define the potential function$^2$:

$$ U (\mathbf{r}_1, \mathbf{r}_2, ..., \mathbf{r}_N) = \frac{1}{2} \sum_{i=1}^{N} \sum_{j=1, j \neq i}^{N} G \frac{m_i m_j}{r_{ij}} $$  \hspace{1cm} (1.2.6)

such that $\nabla_k U = F_k$, where $\nabla_k \equiv \left( \frac{\partial}{\partial x_k}, \frac{\partial}{\partial y_k}, \frac{\partial}{\partial z_k} \right)$ for a generic $k$-th particle.

With the potential function $U$, we can rewrite the equations of motion 1.2.5 in a system of $6N$ first-order scalar equations.

$$ \begin{cases} \dot{\mathbf{r}}_i = \mathbf{v}_i \\ \dot{\mathbf{v}}_i = \frac{1}{m_i} \frac{\partial U}{\partial \mathbf{r}_i} \\ \mathbf{r}_i (0) = \mathbf{r}_i,0 \\ \mathbf{v}_i (0) = \mathbf{v}_i,0 \end{cases} $$  \hspace{1cm} (1.2.7)

$^2$If the system is not isolated, there will be an additional external force

$^3$\( U = -\Omega \), where $\Omega$ is the potential energy.
Although the theoretical formulation of the $N$-body problem is very simple, its numerical applications presents several difficulties. Despite the fast development of the supercomputers, many astrophysical challenges are still unsolved. For example we are still far to simulate a typical galaxy, containing $\sim 10^{11}$ stars in reasonable human times. The difficulties on the numerical solution of the $N$-body problem are mainly due to the double-divergence of the gravitational potential, known in particle physics as ultra-violet and infra-red divergence. The ultra-violet divergence occurs when there is a singularity in the potential for very close encounter ($r_{ij} \to 0$), while the infra-red divergence is caused by the never vanishing interactions among the point-mass particles. This double divergence leads to two consequences:

- close encounters ($r_{ij} \to 0$) generate an infinite force between interacting stars ($F_{ij} \to \infty$);
- in a $N$-body system the resulting force on each particle requires the summation over $N - 1$ pair-wise contributions, therefore we have an $O(N^2)$ computational complexity, which can be overwhelming when $N$ is large ($N > 10^6$).

The integrals of motion

In an $N$-body system an integral of motion is any function $I$ of the phase-space coordinates of the $N$ particles $(x, v)$, which is constant along the motion

$$I[x(t_1), v(t_1)] = I[x(t_2), v(t_2)] \Rightarrow \frac{dI[x(t), v(t)]}{dt} = 0. \quad (1.2.8)$$

As shown by Lagrange (1772) the constants of motion do not depend on the number of particles in the system. For an $N$-body system 10 integrals of motion can be found: the total energy conservation, total angular momentum conservation, and position and velocity of the centre of mass (see e.g. Boccaletti and Pucacco, 2003, and references therein). In $N$-body simulations, the conservation of the total energy is often used to verify the quality of the simulation itself.

1.2.3 Time scales of an $N$-body system

To describe an $N$-body system there are two important time scales: the crossing time ($t_{cr}$) and the relaxation time ($t_{rel}$).

$t_{cr}$ (or dynamical time) is the time that a particle takes to cross the system:

$$t_{cr} = \frac{R}{v} = \frac{R^3}{\sqrt{GmN}} \quad (1.2.9)$$

where $R$ is the typical size of the system, $N$ is the number of particles in the system, and $v$ and $m$ are the typical velocity and mass of a particle, respectively; assuming that the system is in a stationary state (virialized).

$t_{rel}$ is the time over which, as a result of collisions between particles, a stellar system completely loses memory of its initial state. Before a system relaxes a particle will pass through the system a number of time $n_{rel}$, therefore
1.3 MODELLING STAR CLUSTERS

\[ t_{rel} = n_{rel} t_{cr} \approx \frac{1}{8 \ln \Lambda} N t_{cr} \]  
\[ (1.2.10) \]

where \( \ln \Lambda \) is the Coulomb logarithm, see Binney and Tremaine (2008). The relaxation time can also be related to physical properties of star clusters, for example the so called ‘half-mass relaxation time’ (Spitzer, 1987):

\[ t_{rh} = 0.138 \frac{\sqrt{N r_{hm}^3}}{\sqrt{G m} \ln \Lambda} \]  
\[ (1.2.11) \]

where \( r_{hm} \) is the half-mass radius of the cluster, and \( \Lambda = \gamma N \). \( \gamma \) is a constant and can have a value between 0.02 and 0.4 depending on the stellar system (Giersz and Heggie, 1994, 1996). \( t_{rh} \) has been computed in the following chapters to evaluate the dynamical evolution of the star clusters studied.

1.3 Modelling star clusters

The evolution of star clusters can be studied adding a collisional term to the collisionless Boltzmann equation or Vlasov equation.

\[ \frac{\partial f}{\partial t} + v \frac{\partial f}{\partial x} - \frac{\partial f}{\partial v} \frac{\partial \phi}{\partial x} = \Gamma[f] \]  
\[ (1.3.1) \]

where \( f \) is the distribution function (probability to find a star in a particular position of the phase space) of the stellar system and \( \Gamma[f] \) represent the effect of two-body interactions. \( \Gamma[f] \) cannot be computed analytically, therefore several numerical methods have been used, such as Fokker-Plack method and Monte Carlo method. Direct \( N \)-body simulations integrate the orbits of each star directly, rather that solving equation 1.3.1.

In this thesis, I mainly used \textsc{Nbody6} (Aarseth, 2003; Nitadori and Aarseth, 2012), which is a direct summation \( N \)-body code. \textsc{Nbody6} is based on a fourth-order Hermite integrator (Makino and Aarseth, 1992) with the Ahmad-Cohen neighbour scheme (Ahmad and Cohen, 1973) and force calculations that are made using Graphics Processing Units (GPUs). Moreover, it contains stellar evolutions prescriptions for single and binary stars for different metallicities (Hurley et al., 2000, 2002). Thanks to the high number of users and mainly Sverre Aarseth, \textsc{Nbody6} has become a code capable to study different astrophysical mechanisms. Other codes do not have the same flexibility because usually some assumptions have to be satisfied (depending on the method used), however they can evolve a star cluster in a day for Monte Carlo codes (e.g. \textsc{MoCCA} code Giersz et al., 2013) and even in few seconds for Scaling Models, such as \textsc{Emacss} (Alexander et al., 2014). While simulations with direct summation \( N \)-body codes could take up to few years, depending on the number of the particles. The Monte Carlo and Scaling methods can be considered less ‘accurate’ than direct summation \( N \)-body codes, however, their fast evolution make them perfect for population and initial conditions studies. Indeed, in this thesis, we used the \textsc{Emacss} code to find the initial conditions of our star clusters, such as the initial size and mass of the clusters.
1.3. MODELLING STAR CLUSTERS

1.3.1 Star cluster models

To simulate a star cluster, first of all we need to find appropriate initial conditions. In 1911, Plummer studied star clusters using the Schuster model (Schuster, 1883), nowadays known as Plummer model (Plummer, 1911), because at that time this model was able to reproduce the observed properties (surface brightness) of clusters. The Plummer-Schuster model is a polytrope of index \( n = 5 \), that has an analytical expression and can be considered as the first model used to study star clusters. However, deeper observations showed that the Plummer model was not a good fit for all star clusters. In 1966, King found a family of models (King, 1966) that could be a better fit for the observed surface density profile. However, star clusters where the King model cannot well reproduce the observed data have been found (e.g. Carballo-Bello et al., 2012), therefore a more generic family of models is needed (see Gieles and Zocchi, 2015, and references therein). Gieles and Zocchi (2015) present a generic expression for the lowered isothermal models (e.g. King models) that can well reproduce the surface density profile of globular clusters.\(^4\) Moreover, they released a publicly available python code, called LIMEPY, that I used in this thesis to fit the number density profile obtained from the \( N \)-body simulations (see Chapters 2, 4, and 5). Plummer and King models has been used not only to fit the observational data, but also as initial conditions of numerical simulations for evolutionary studies of star clusters. In the following paragraphs I will give a brief description on the Plummer and King models, see Binney and Tremaine (2008) for further details.

**Plummer model**

The Plummer model has the following analytic potential (easy to implement in numerical simulations):

\[
\phi(r) = -\frac{GM}{\sqrt{r^2 + a^2}} = -\frac{GM}{a} \left(1 + \frac{r^2}{a^2}\right)^{-\frac{1}{2}}
\]  

(1.3.2)

where \( G \) is the gravitational constant, \( M \) is the total mass of the system, and \( a \) is the scale radius. To obtain the density, we have to solve the Poisson’s equation

\[
\nabla^2 \phi = -4\pi G \rho \tag{1.3.3}
\]

which in the case of spherically symmetric systems can be written in spherical coordinates as

\[
\frac{1}{r^2} \frac{d}{dr} \left( r^2 \frac{d \psi}{dr} \right) = -4\pi G \rho(r) \tag{1.3.4}
\]

where

\[
\rho(r) = \frac{3M}{4\pi a^3} \left(1 + \frac{r^2}{a^2}\right)^{-\frac{5}{2}}
\]

(1.3.5)

is the mass density distribution of the system. Other properties of the Plummer model are summarised in Heggie and Hut (2003).

\(^4\)However, the lowered isothermal profiles cannot reproduce the flattening observed in the external parts of globular clusters (e.g. Drukier et al., 1998; Küpper et al., 2010; Claydon et al., 2017)
1.3. MODELLING STAR CLUSTERS

King models

Other models that can well reproduce the surface density profile of globular clusters are the ‘lowered isothermal models’ to which the King’s models belong (King, 1966). These models resemble the isothermal distribution at small distances from the centre of the mass, while the density profile falls to zero more rapidly at large distances ensuring a finite total mass for the entire system.

The Poisson’s equation for such models it is usually written introducing new dimensionless variables $\tilde{\rho} \equiv \rho/\rho_0$ and $\tilde{r} \equiv r/r_0$, where $\rho_0$ is defined as the central density and $r_0 \equiv \sqrt{9\sigma^2/4\pi G \rho_0}$ as the King radius. These variables were introduced, for the first time, to describe the singular isothermal sphere whose density profile is singular at $r = 0$ while the trend of $\tilde{\rho}(\tilde{r})$ is well defined at the origin. Using the new variables, we can write the Poisson’s equation as

$$\frac{1}{\tilde{r}^2} \frac{d}{d\tilde{r}} \left( \tilde{r}^2 \frac{d\psi}{d\tilde{r}} \right) = -9\sigma^2 \tilde{\rho}. \quad (1.3.6)$$

where $\sigma$ is the velocity dispersion and $\psi \equiv -\phi + \phi_0$ is the relative potential.

For the King models we introduce another parameter $W$ defined as

$$W = \frac{\psi}{\sigma^2} \quad (1.3.7)$$

also known as ‘dimensionless potential’. We can rewrite the Poisson’s equation in terms of $W$

$$\frac{d^2 W}{d\tilde{r}^2} + \frac{2}{\tilde{r}} \frac{dW}{d\tilde{r}} + 9\tilde{\rho}(W, W_0) = 0 \quad (1.3.8)$$

where

$$\tilde{\rho}(W, W_0) = \frac{e^W \text{erf} \left( \sqrt{W} \right) - \sqrt{\frac{4}{\pi}} W \left( 1 + \frac{2}{3} W \right)}{e^{W_0} \text{erf} \left( \sqrt{W_0} \right) - \sqrt{\frac{4}{\pi}} W_0 \left( 1 + \frac{2}{3} W_0 \right)}. \quad (1.3.9)$$

The solution of equation (1.3.8) can be obtained numerically, knowing the initial conditions for $W$ and $\tilde{W} = dW/d\tilde{r}$. Specifically, the value of $W(\tilde{r} = 0) = W_0$ represents the depth of the central potential well, and it is needed to require that its first derivative with respect to $\tilde{r}$, calculated in the centre ($\tilde{r} = 0$), is null. The latter condition corresponds to have a null force at the centre of the system, which is in agreement with its spherical symmetry and corresponds to have a finite (null) mass at $\tilde{r} = 0$. While, as mentioned previously, at large distances the density profile falls to zero, specifically at $\tilde{r} = r_t$, where $r_t$ is the tidal radius of the King model. In general, the bigger the value of the central potential well ($W_0$) the greater the tidal radius ($r_t$) will be.

Initial conditions

We used the Plummer and King models as initial conditions for our simulations. Their respective distribution function can be used to find the initial position and velocity of particles in a cluster (see Binney and Tremaine, 2008, for further details). In NBODY6 it is possible to generate a cluster with a Plummer model using a built-in routine, while for the King models we used the McLUSTER code (Küpper et al., 2011), see Section 3.2.
To generate a star cluster using Plummer profile we need to assign its initial mass \(M_{cl}\) and its initial scale radius \((a)\). Sometimes, it is convenient to assume an initial half-mass radius \(r_{hm}\) and for a Plummer model \(r_{hm} \simeq 1.305a\) (see Table 8.1 at pag. 73 in Heggie and Hut, 2003, for more properties of the Plummer model). For the King models, we have to assume an initial \(r_{hm}, W_0\) and \(M_{cl}\). In general, for higher values of \(W_0\) the model will have higher values of \(r_t\) but also smaller values of \(r_0\) (see fig. 8.3 at pag. 72 in Heggie and Hut, 2003), resulting in more concentrated models. Indeed, the concentration parameter \(c\) for the King models is defined as \(c = \log(r_t/r_0)\).

It is interesting to study not only how the size of a cluster can change its evolution but more importantly its initial density, therefore we can use the density within \(r_{hm}\) as initial condition, see in Section 2.3.1. The initial density can be also varied based on the filling factor. This means that the size of the cluster can be modified based on its Jacobi radius \((r_J)\). I estimated \(r_J\) using eq. (7) in King (1962):

\[
r_J = \left( \frac{GM_{cl}}{\Omega^2 - \frac{d^2\phi}{dR^2}} \right)^{\frac{1}{3}} \tag{1.3.10}
\]

where \(\Omega\) is the angular velocity of the cluster, and \(R_G\) its Galactocentric distance. \(r_J\) is defined as the radius at which the relative acceleration on a star, due to the potential of the cluster and the potential of the host galaxy, is null. It can also be defined as the distance from the centre of the cluster to the Lagrangian points L1 and L2, discovered by Leonhard Euler few years before Lagrange (Euler, 1765).

One of the main difference between King models and Plummer models is that the first one have a finite density, indeed the density will be null at the tidal radius \((r_t)\); while the Plummer model have non-finite density profile, see equation (1.3.5). For this reason, in the King profiles we can fill the Roche-volume assuming that \(r_t = r_J\), while in the Plummer profile we can only assume a ratio between \(r_{hm}\) and \(r_J\), therefore it is possible that few stars can be initially outside \(r_J\).

It is important to note that \(r_J\) depends on the potential of the host galaxy, therefore clusters embedded in different galaxy models will have different \(r_J\), and thus a different evolution (e.g. Tanikawa and Fukushige, 2010).

### Star cluster evolution

The formation of globular clusters is still unknown, the basic idea is that a massive molecular cloud collapses gravitationally forming stars, which expelled the possible remaining gas. Therefore, we can idealise the initial instants of a star clusters as a group of stars (no gas) in virial equilibrium. Initially, due to a spectrum of masses, the most massive stars will lose their kinetic energy interacting with low mass stars, thus the massive ones will sink to the centre while the low mass stars will move to the halo. This process is known as mass segregation, while the energy exchanges brings the system towards energy equipartition, even if total energy equipartition never occurs (Merritt, 1981; Miocchi, 2006). This phenomenon was first studied by Spitzer (1969) and it is also known as ‘Spitzer instability’. While mass segregation occurs, stellar evolution starts (at around 5 Myr), and as shown by Gieles et al. (2010) the stellar evolution phase let the cluster expands at most twice as its initial size. At the same time, mass segregation is still important (e.g. Heggie,
1.3. MODELLING STAR CLUSTERS

because when the black holes form (if they are not expelled out of the cluster due to their natal kick velocity, still matter of debate, see Section 2.4.7 and Chapter 3) they sink rapidly to the centre of the cluster. This will bring to a first core collapse of only black holes (Breen and Heggie, 2013a). Self-gravitating systems, such as star clusters, can be interpreted as systems with negative heat capacity, therefore when the cluster lose stars the core became “hotter” and shrinks towards core collapse (for further details Antonov, 1962; Heggie and Hut, 2003).

After several relaxation time, the black holes escape from the cluster (see Chapter 2), leaving the other more massive stars, the neutron stars, sink to the centre. Once the neutron stars are ejected, the other most massive stars will sink and so on. When the luminous stars sink to the centre, the core collapse will be observable, indeed some globular clusters in the Milky Way show a core collapsed surface density profile (Harris, 1996, 2010 edition). However, the core collapse process can be delayed and/or halted by the presence of primordial or dynamically formed binaries which are source of energy that do not allow the core to shrink indefinitely. When the core collapse is stopped by binaries, the cluster enter in a post-collapse phase, and successively it dissolves. Depending on the initial conditions, a cluster can dissolve before reaching core collapse, see Chapter 3.

Following Hénon studies (Hénon, 1961, 1965), Gieles et al. (2011) show that the evolution of star clusters in a tidal field can be divided in two phases: expansion and evaporation. Initially, if a cluster forms within the inner part of its Roche-volume, the cluster will expand (also due to stellar evolution) until it fills the Roche-volume. While in the evaporation phase, it has filled its Roche-volume and the two-body relaxation will dominate. In this phase the stars in the clusters will escape due to two-body relaxation, the mass loss will bring the tidal field to decrease and thus the size of the cluster will shrink (as shown by Hénon, 1961, a star cluster evolves at a constant mean density once it fills the Roche-volume).

The evolution of a star cluster also depends on the initial mass spectrum, stellar evolution model and tidal boundary assumed (Chernoff and Weinberg, 1990); furthermore, an initial rotation can play a role (Einsel and Spurzem, 1999). Globular clusters are objects which have been studied for a long time, however there are still some important aspects which are not well understood, such as their formation and the observed ‘multiple populations’ (enhancement in the light elements).

In the Proc IAU 246 (2007), Ivan King said:

There is nothing here that has not been seen before, and for that I apologize...

But as abstruse details accumulate, the simple truths are often lost sight of...
1.4 This Thesis

In the previous Sections, I have discussed how the formation and evolution of star clusters can trace the galaxy formation and I have explained in details the evolution of collisional systems. I have included a mathematical approach to the study of $N$-body systems and I have given a brief description of the methods used to study them.

In this thesis, I use direct $N$-body simulations to study the evolution of star clusters and I analyse the results considering the observational biases, making the comparison between models and data possible.

This thesis is organised as follow:

- in Chapter 2, I use $N$-body simulations to study whether the faint and extended objects in the Milky Way halo could be part of the faint star cluster population;

- in Chapter 3, I study how minor changes in the initial conditions of star clusters could drastically change their evolution;

- in Chapter 4, I use a suite of direct $N$-body simulations to find the dark matter profile of a Milky Way dwarf galaxy, using the observed properties of its central star cluster. Due to the properties of the dwarf galaxy, this result could shed light on the nature of dark matter;

- in Chapter 5, I use the properties of an accreted globular cluster in the Milky Way halo, to find the properties of its progenitor;

- finally, in the Conclusions, I summarise the results of this thesis and describe the future perspectives.
1.4. THIS THESIS
The contribution of dissolving star clusters to the population of ultra faint objects in the outer halo of the Milky Way

In the last decade, several ultra faint objects (UFOs, $M_V \gtrsim -3.5$) have been discovered in the outer halo of the Milky Way. For some of these objects it is not clear whether they are star clusters or (ultra-faint) dwarf galaxies. In this work I quantify the contribution of star clusters to the population of UFOs. I extrapolated the mass and Galactocentric radius distribution of the globular clusters using a population model, finding that the Milky Way contains about $3.3^{+7.3}_{-1.6}$ star clusters with $M_V \gtrsim -3.5$ and Galactocentric radius $\gtrsim 20$ kpc. To understand whether dissolving clusters can appear as UFOs, I run a suite of direct $N$-body models, varying the orbit, the Galactic potential, the binary fraction and the black hole (BH) natal kick velocities. In the analyses, I consider observational biases such as: luminosity limit, field stars, and line-of-sight projection. I find that star clusters contribute to both the compact and the extended population of UFOs: clusters without BHs appear compact with radii $\sim 5$ pc, while clusters that retain their BHs after formation have radii $\gtrsim 20$ pc. The properties of the extended clusters are remarkably similar to those of dwarf galaxies: high inferred mass-to-light ratios due to binaries; binary properties mildly affected by dynamical evolution; no observable mass segregation; and flattened stellar mass function. I conclude that the slope of the stellar mass function as a function of Galactocentric radius and the presence/absence of cold streams can discriminate between DM free and DM dominated UFOs.


\(^1\)I reproduced this Chapter following the guidelines of the publisher license, see Appendix A.1 for details.
2.1 Introduction

The Milky Way halo contains numerous satellite stellar systems with a broad range of luminosities. These stellar systems and their composition contain valuable information about the formation of the Milky Way galaxy (e.g. Majewski 1993; Tolstoy et al. 2009; Belokurov 2013). Up to a decade ago, there was a clear separation between dwarf galaxies (DGs) and globular clusters (GCs). In a diagram of absolute $V$-band magnitude ($M_V$) vs. half-light radius ($r_{\text{eff}}$), GCs (blue squares) and DGs (green circles) with bright luminosities ($M_V \lesssim -3.5$) are separated in size (Gilmore et al., 2007). On the one hand, DGs are large ($r_{\text{eff}} \gtrsim 30$ pc), whereas GCs are compact ($r_{\text{eff}} \lesssim 10$ pc). In addition, stars within DGs display a range of metallicities ($-3 \lesssim [\text{Fe/H}] \lesssim -1.5$, fig. 12 in McConnachie 2012) and their kinematics imply a high mass-to-light ratio, $10 \lesssim M/L_V \lesssim 1000$ (fig. 11 in McConnachie 2012), which is usually explained by a non-baryonic dark matter component (Mateo, 1998; Gilmore et al., 2007; Walker, 2013). Except for a few exceptions, such as $\omega$ Cen (Dickens and Woolley, 1967; Freeman and Rodgers, 1975; Butler et al., 1978) and M54 (Sarajedini and Layden, 1995), GCs have no spread in iron abundance ([Fe/H]), however they do display light-element anomalies (Gratton et al., 2004) which are not seen in DGs. Moreover, the internal kinematics of GCs can be explained by a single old stellar populations with a ‘normal’ initial stellar mass function (IMF), without the need for dark matter (McLaughlin and van der Marel, 2005; De Marchi et al., 2010; Shanahan and Gieles, 2015). Therefore, DGs and GCs were considered to be two totally different classes of stellar systems.

Recently, thanks to the Sloan Digital Sky Survey (SDSS, York et al., 2000), the Panoramic Survey Telescope And Rapid Response System (Pan-STARRS, Læven et al., 2015b), and the Dark Energy Survey (DES, Bechtol et al., 2015; Koposov et al., 2015; Drlica-Wagner et al., 2015b), several ultra faint objects (UFOs, $M_V \gtrsim -3.5$) have been discovered in the outer halo of the Milky Way (MW). For some MW satellites it is still debated whether they are GCs or ultra-faint DGs (Willman et al., 2005; Belokurov et al., 2007; Martin et al., 2016). As shown in Fig. 2.1, at lower luminosities (objects marked in red) the two populations (GCs and DGs) overlap at half-light radii of about 20 to 30 pc (hereafter, I refer to UFO with $r_{\text{eff}} \geq 20$ pc as extended ultra faint objects, eUFO).

Systems in the same magnitude range, but with smaller sizes ($r_{\text{eff}} < 20$ pc), such as Koposov 1 & 2 (Koposov et al., 2007), Balbinot 1 (Balbinot et al., 2013), Kim 1 & 2 (Kim and Jerjen, 2015; Kim et al., 2015b), are most likely ordinary star clusters, although no spectroscopic follow-up has been done for any of these objects.

Kinematic data excludes the possibility of large amounts of dark matter in some UFOs (e.g. Segue 3, Fadely et al. 2011), but for others, the stellar velocities alone do not allow a conclusive classification (e.g. Segue 1, Belokurov et al. 2007; Simon et al. 2011). In several cases a spread in [Fe/H] is indicative of an extended star formation history and therefore argues for a galaxy classification (Willman and Strader, 2012). However, a prolonged star formation history within a dark matter halo does not guarantee that the system contains dark matter at the present day. Tidal stripping and mass segregation could remove the dark matter halo and leave a dark matter free remnant stellar population orbiting the MW (Moore, 1996; Mashchenko and Sills, 2005a,b; Baumgardt and Mieske, 2008).
2.1. INTRODUCTION

Figure 2.1: Distribution of MW satellites in the Galactocentric distance-magnitude space (left) and the size-magnitude space (right). GCs are shown as blue squares, DGs are shown as green circles and the faint stellar systems, whose nature has been topic of debate in literature, are in red ($M_V \gtrsim -3.5$). The data on the GCs were taken from Harris (2010), these on the DGs from McConnachie (2012) and the last satellites discovered were included. The recently discovered GCs (blue and red square) are: Segue 3, Muñoz 1, Balbinot 1, Laevens 1/Crater, Laevens 3, Kim 1, Kim 2, Eridanus III, DES 1, Kim 3. While the recently discovered DGs (green and red circle) are: Hydra II, Laevens 2, Pegasus III, Ret II, Eridanus II, Tucana II, Horologium I, Pictoris I, Phoenix II, Draco II, Sagittarius II, Horologium II, Grus II, Tucana III, Columba I, Tucana IV, Reticulum III, Tucana V, Crater 2, Acquarius 2, Pictor II [Fadely et al. (2011); Muñoz et al. (2012a); Balbinot et al. (2013); Laevens et al. (2014); Belokurov et al. (2014); Paust et al. (2014); Kim and Jerjen (2015); Laevens et al. (2015b,a); Martin et al. (2015); Kim et al. (2015b,a); Bechtol et al. (2015); Koposov et al. (2015); Luque et al. (2015); Drlica-Wagner et al. (2015b); Kim et al. (2016); Torrealba et al. (2016a,b); Drlica-Wagner et al. (2016)].
2.2. THE EXPECTED NUMBER AND RADIUS OF FAINT STAR CLUSTER

In the ΛCDM cosmology (Davis et al., 1985; White et al., 1987; Cen et al., 1994; Navarro et al., 1996b; Springel et al., 2006; Read, 2014), the smallest galaxies are believed to have the highest dark matter density and this makes them promising targets for observing dark matter annihilation signals in γ-rays (e.g. Ackermann et al. 2014). Indeed, the Fermi γ-ray satellite is observing several UFOs (Geringer-Sameth et al., 2015; Drlica-Wagner et al., 2015a), such as the ones that were recently discovered in the Dark Energy Survey data (Bechtol et al., 2015; Koposov et al., 2015). There is an advantage of looking at the UFOs as opposed to the Galactic centre, because they contain fewer known γ-ray sources, such as radio pulsars and low-mass X-ray binaries.

Uniquely establishing whether a UFO contains dark matter is challenging, because only a handful of bright stars are available for spectroscopy and membership determination. In addition, it has been proposed that unbound stars escaping from a dark matter free system could enhance the velocity dispersion and mimic the effect of a dark matter halo (Kroupa, 1997). For the UFOs, apart from the kinematic challenge, it is also difficult to determine $M_V$ and $r_{\text{eff}}$, which affects the virial mass estimate because it is proportional to $r_{\text{eff}}$. It is, therefore, not inconceivable that a dark matter free dissolving star cluster appears to have a massive dark matter halo; this was recently proposed for Segue 1 by Dominguez et al. (2016). In this chapter, I do not focus this study on a particular object, but I aim to shed light on how many star clusters are expected to contribute to the UFO population.

This chapter is organised as follows. In Section 2.2, I estimate how many faint star clusters (dark matter free objects with $M_V \gtrsim -3.5$ and in the MW-halo) we can expect based on an extrapolation from nearby and bright GCs. In Section 2.3, I describe the $N$-body simulations to model star clusters. In Section 2.4, I discuss the results I obtained considering observational biases, and a summary of my results is presented in Section 2.5.

2.2 The expected number and radius of faint star cluster

2.2.1 Number of faint star clusters

In this Section I estimate the number of star clusters that are expected to contribute to the luminosity range of the UFOs by extrapolating from the known GC population.

I use analytic functional forms for the initial distributions of star cluster masses and Galactocentric radii, which I then evolve by a simple mass loss prescription to include the effect of dynamical evolution (two-body relaxation) in the MW potential.

I assume a Schechter function (Schechter, 1976) for the clusters initial mass function (hereafter, CIMF; Jordán et al. 2007 for old clusters; and Gieles et al. 2006 and Larsen 2009 for young cluster):

$$\frac{dN}{dM_i} = AM_i^{-\alpha} \exp \left( - \frac{M_i}{M_*} \right).$$

where $M_i$ is the initial mass of star clusters, $M_*$ is the mass where the exponential drop occurs, $A$ is a constant that sets the total mass in clusters and $\alpha$ is the power-law index at low masses $M_i \lesssim M_*$. Because the MW GCs are old and have lost mass as the result of dynamical evolution, I am interested in the evolved mass function (Jordán et al., 2007), which can be expressed...
in the CIMF by using conservation of number (Fall and Zhang, 2001)

\[ f(M, R_G) = \frac{dN}{dM} = \frac{dN}{dM_i} \left| \frac{\partial M_i}{\partial M} \right|, \]  

(2.2.2)

where \( R_G \) is the Galactocentric radius which enters because the mass evolution depends on the orbit. To proceed, I need an expression for \( \partial M_i/\partial M \) that encapsulates the physics of mass loss of GCs. I assume that the dominant mass-loss process is evaporation, which is the result of two-body relaxation in the Galactic tidal field. Baumgardt (2001) showed that for this process, the dissolution time-scale of GCs, \( t_{\text{dis}} \), scales with their two-body relaxation timescale, \( t_{\text{rh}} \), as 

\[ t_{\text{dis}} \propto t_{\text{rh}}^x, \]  

with \( x \approx 3/4 \). The mass-loss rate, \( \dot{M} \), can then be written as 

\[ \dot{M} = M_5(R_G)(M/10^5 M_\odot)^{1-x}, \]  

where \( M_5(R_G) \approx 20 M_\odot \text{Myr}^{-1} (\text{kpc}/R_G) \) is the \( R_G \)-dependent mass-loss rate found in models of a cluster with a mass of \( 10^5 M_\odot \) on a circular orbit in an isothermal Galactic potential (Gieles et al., 2011). From integrating \( \dot{M} \) we can find an expression for \( M(M_i, \dot{M}_5, \text{Age}) \) (Lamers et al., 2005) from which we derive

\[ M_i = (M^x + \Delta x)^{1/x} \]  

(2.2.3)

\[ \frac{\partial M_i}{\partial M} = M^{x-1}(M^x + \Delta x)^{(1-x)/x}, \]  

(2.2.4)

with \( \Delta x = x(1-\epsilon)^{-1}(10^5 M_\odot)^{1-x} \dot{M}_5 \text{Age/Myr} \), where \( \epsilon \) is the eccentricity of the orbit. The \( (1-\epsilon)^{-1} \) term encapsulates the fact that clusters on eccentric orbits lose mass faster (Baumgardt and Makino, 2003; Cai et al., 2016). I adopt \( \epsilon = 0.5 \), which corresponds to the typical eccentricity of isotropic orbit distribution in a singular isothermal sphere (van den Bosch et al., 1999). Combining equations (2.2.2), (2.2.3) and (2.2.4) we find an expression for the evolved clusters mass function (Gieles, 2009)

\[ f(M, R_G) = A \frac{M^{x-1}}{(M^x + \Delta x)^{\alpha + 1/x}} \exp \left( -\frac{(M^x + \Delta x)^{1/x}}{M_\ast} \right). \]  

(2.2.5)

Because I am interested in finding how many faint star cluster (dark matter free objects with \( M_V \gtrsim -3.5 \) and \( R_G \geq 20 \text{kpc} \), hereafter FSC) we expect in the outer halo of the MW, we need to adopt a Galactocentric radius distribution. I decide to use a simple power-law for the initial distribution

\[ g(R_G) = \frac{dN}{dR_G} \bigg|_i = R_G^{2-\beta}, \]  

(2.2.6)

where \( -\beta \) is the index of the number density distribution \( n(R_G) \), because \( g(R_G) = 4\pi R_G^2 n(R_G) \).

The bivariate distribution that we can compare to the data is thus

\[ h(M, R_G) = \frac{d^2N}{dMdR_G} = f(M, R_G) g(R_G), \]  

(2.2.7)

where \( A \) in the function \( f(M, R_G) \) (equation 2.2.5) is a constant that sets the number of clusters after integrating \( h(M, R_G) \) over \( M \) and \( R_G \). This function can now be used to
2.2. THE EXPECTED NUMBER AND RADIUS OF FAINT STAR CLUSTER

do a maximum likelihood fit to find the set of free parameters for which the distribution
\((h(M, R_G)\) in this case) becomes most probable:

\[
\ln \mathcal{L} = \sum_i \ln \ell_i(p_1, p_2, \ldots, p_j)
\]

where \(\ell_i(p_1, p_2, \ldots, p_j)\) is the probability of finding the datum \(i\) given the set of parameters \(p_1, p_2, \ldots, p_j\). In this case:

\[
\ln \mathcal{L} = \sum_{i=1}^{N_{GC}} \ln [h_i(\alpha, \beta, x, M_*)],
\]

where \(h_i = h(M_i, R_{G_i})\) and \(N_{GC}\) is the number of clusters in the sample.

I use the Harris (2010) catalogue of MW globular cluster properties to get \(M\) and \(R_G\) for each cluster and use \(M/L_V = 2\) to convert luminosities to masses. I then use a Monte Carlo Markov Chain (MCMC) method (the affine-invariant ensemble sampler as implemented in the EMCEE code, Foreman-Mackey et al. 2013) to find the parameters: \(\alpha, \beta, x\) and \(M_*\) that give the highest likelihood. I decide to fit equation (2.2.7) to the GCs in the \(M\) range \(3 \times 10^4 < M/M_\odot < 10^7\) and \(R_G\) range \(0.5 < R_G/kpc < 20\), because this is where I believe the catalogue is complete. The number of selected GCs in that range is \(N_{GC} = 115\). In Table 2.1, I show the results of the best fit parameters. In Fig. 2.2 I show the resulting best-fit distribution.

I then use the best fit distribution to estimate the number of low-mass GCs at large \(R_G\), where the Harris catalogue is incomplete. With the known parameters of the \(h(M, R_G)\) distribution, it is possible to estimate the number of faint star clusters \(N_{FSC}\) by integrating the distribution over the range where the known UFOs are found \((20 \leq R_G/kpc \leq 150; 10^2 \leq M/M_\odot \leq 4.3 \times 10^3\)). The lower and upper limit of the mass range correspond to \(M_V \simeq 0\) and \(M_V \simeq -3.5\), respectively, with the adopted \(M/L_V = 2\).

Therefore, the number of faint star clusters is

\[
N_{FSC} = N_{GC} \int_{10^2 M_\odot}^{4.3 \times 10^3 M_\odot} \int_{20 kpc}^{150 kpc} h(M, R_G) \, dR_G \, dM = 3.3^{+7.3}_{-1.6}
\]

where the constant \(A\) in \(h(M, R_G)\) is such that an integration over the range used for the fit results in 1. The quoted value is the median of posterior distribution of \(N_{FSC}\) shown in Fig. 2.3, and the uncertainties correspond to the region containing 68.3% of the points around the median.

In Fig. 2.2 I show that the extrapolation from the fit to the bright GCs agrees with the number of observed cluster with and without the last observed GC candidates (in orange); however based on this I cannot conclude that a fraction of UFOs need to be galaxies.

### 2.2.2 Size estimate of faint stellar systems

The UFOs have sizes up to approximately 100 pc (Fig. 2.1), but the uncertainties can sometimes be extremely large. Muñoz et al. (2012b) show that it can be challenging to estimate the structural parameters of the ultra-faint DGs within 10% of their true value. For star clusters, it is not known whether it is possible that they appear that large. Here I estimate the maximum radius that a star cluster can have, which corresponds to the
2.2. THE EXPECTED NUMBER AND RADIUS OF FAINT STAR CLUSTER

Figure 2.2: In the bottom left plot I show the MW GCs (blue squares) and the GCs candidates discovered in the last three years (orange squares): Laevens 1/Crater, Laevens 3, Kim 1, Kim 2, Eri III, Balbinot 1, DES 1, Kim 3. The area in the black box is where I compute the fit. In the upper left plot, we have the normalized mass function versus the mass of the GCs, while in the bottom right there is the normalized distribution function versus the Galacticentric distance of the GCs. In blue, the histogram for all the GCs, while the best fit (black line) with $\alpha$, $\beta$, $x$ and $M_*$ as parameters calculated with EMCEE, was found selecting the GCs in this region: $0.5 < R_G / \text{kpc} < 20$ and $3 \times 10^4 < M / M_\odot < 10^7$. The results for the parameters are shown in the upper right plot. In the histograms, the error bars (in grey) are estimated using a Poisson error.
Table 2.1: Best fit parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>0.452 ± 0.236</td>
<td></td>
</tr>
<tr>
<td>$\beta$</td>
<td>3.523 ± 0.128</td>
<td></td>
</tr>
<tr>
<td>$x$</td>
<td>0.724 ± 0.090</td>
<td></td>
</tr>
<tr>
<td>$M_*$</td>
<td>4.041 ± 0.964</td>
<td>$10^5 M_\odot$</td>
</tr>
</tbody>
</table>

Figure 2.3: Posterior of the number of faint star clusters ($N_{\text{FSC}}$) marginalised over $\alpha$, $\beta$, $x$ and $M_*$. The inferred $N_{\text{FSC}} = 3.3^{+7.3}_{-1.6}$. 

\[ N_{\text{FSC}} = 3.3^{+7.3}_{-1.6} \]
situation in which the cluster fills the Roche volume. In that case the half-mass radius \( r_{hm} \) depends on the strength of the tidal field along the orbit. As described by Hénon (1961), a star cluster evolving in a tidal field, evolves at a constant mean density once it fills the Roche-volume, which means that the ratio between the \( r_{hm} \) and the Jacobi radius \( r_J \) is constant: \( r_{hm}/r_J \simeq 0.15 \). This fraction is somewhat \( N \)-dependent (Alexander and Gieles, 2012) and can be as large as \( r_{hm}/r_J \simeq 0.4 \) (Giersz and Heggie, 1997) for very small \( N \), i.e. the region of interest. The Jacobi radius is defined in King (1962) as:

\[
 r_J = \left( \frac{GM}{\Omega^2 - \frac{\partial^2 \phi}{\partial R^2}} \right)^{1/3}, \tag{2.2.11}
\]

where \( \Omega \) is the angular velocity of the cluster around the Galaxy centre, \( \phi \) is the potential of the Galaxy and \( G \) is the gravitational constant. Therefore, using equation (2.2.11) and assuming a singular isothermal halo we obtain

\[
 r_J = \left( \frac{GM}{2\Omega^2} \right)^{1/3}, \tag{2.2.12}
\]

which is only valid for circular orbit, thus \( \Omega = V_C/R_G \).

Therefore, using \( r_{hm}/r_J = 0.2 \) for a cluster with \( M = 500 M_\odot \) at \( R_G = 50 \) kpc, I find \( r_{hm} \simeq 7.6 \) pc (with \( r_{eff} \simeq 5.7 \) pc, if we assume that mass follows light).

From this we see that the radii of the compact UFOs are consistent with being tidally limited star clusters. However, star clusters in the end of their life have lost most of their low-mass stars, and will therefore have a smaller \( M \), resulting in smaller a \( r_J \) and hence a smaller \( r_{hm} \). Also, if the cluster is mass segregated, \( r_{eff} \) can be smaller than \( r_{hm} \) in projection (Hurley, 2007). However, observational biases, such as the presence of unbound stars and dark remnants (Peuten et al., 2016) could perhaps inflate \( r_{eff} \) with respect to \( r_{hm} \). In the next section I consider the evolution of \( r_{eff} \) in numerical models, taking all observational biases into account.

### 2.3 Numerical N-body simulations of FSCS

#### 2.3.1 Description of the \( N \)-body simulations

In this section I describe the details of the simulations. In order to simulate the evolution of star clusters in a tidal field, I used \textsc{nbody6tt} (Renaud et al., 2011; Renaud and Gieles, 2015a), which is an adaptation of the widely used direct \( N \)-body code \textsc{nbody6} developed by Nitadori and Aarseth (2012). I use \textsc{nbody6tt} because I want to consider a Galactic potential that is currently not available in \textsc{nbody6}. With \textsc{nbody6tt} it is straightforward to include the tidal field due to an external galactic potential that is a function of position and time. The tidal force is added to the equation of motion of a star in a non-rotating frame by adding the difference in galactic acceleration on the star and the guide centre. The guide centre is a pseudo-particle (initially at the centre of mass of the cluster), and its motion is integrated separately (Aarseth, 2003).

I adapt three different Galactic potentials: a static ‘NFW-potential’ (Navarro et al., 1996b), a ‘growing NFW-potential’ (hereafter \textsc{gNFW}, Buist and Helmi 2014), and a three component potential (Paczynski, 1990).
34 simulations were performed using the static NFW potential:

\[
\phi_{NFW} = -\frac{GM_0}{R_G} \ln \left( 1 + \frac{R_G}{R_0} \right)
\]  

(2.3.1)

where the scale mass \( M_0 \) is chosen to have a maximum circular velocity \( V_C = 210 \text{ km s}^{-1} \) at \( R_C = 30 \text{ kpc} \), and the scale radius \( R_0 \) is 13.9 kpc.

To test the role of the Galactic potential, 8 simulations were performed using the analytical gNFW model of Buist and Helmi (2014):

\[
\phi_{gNFW} = -\frac{GM_s(z)}{R_G} \ln \left( 1 + \frac{R_G}{R_s(z)} \right)
\]  

(2.3.2)

where the scale mass \( (M_s) \) and the scale radius \( (R_s) \) evolve with the redshift \( z \) as:

\[
\begin{align*}
M_s(z) &= M_0 \exp(-0.2z) \\
R_s(z) &= R_0 \exp(-0.1z)
\end{align*}
\]

with \( M_0 = M_s(z = 0) \) and \( R_0 = R_s(z = 0) \) (same values of eq. 5.3.1).

An additional 4 simulations were performed using a three component potential (bulge, disc and halo). I used the analytical model from Paczynski (1990) (hereafter, P90).

**Bulge:**

\[
\phi_b = -\frac{GM_b}{\sqrt{R^2 + (a_b + \sqrt{z^2 + b_b^2})^2}}
\]  

(2.3.3)

where: \( R \) is the Galactocentric distance in the \( x-y \) plane; \( z \) is the Galactocentric distance in the \( z \)-component; \( M_b = 6.15 \times 10^9 \text{ M}_\odot \); \( a_b = 0.0 \text{ kpc} \); and \( b_b = 0.277 \text{ kpc} \).

**Disc:**

\[
\phi_d = -\frac{GM_d}{\sqrt{R^2 + (a_d + \sqrt{z^2 + b_d^2})^2}}
\]  

(2.3.4)

where: \( M_d = 4.47 \times 10^{10} \text{ M}_\odot \), \( a_d = 3.7 \text{ kpc} \), and \( b_d = 0.20 \text{ kpc} \).

**Halo:**

\[
\phi_h = \frac{GM_h}{d} \left[ \frac{1}{2} \ln \left( 1 + \frac{R_G^2}{d^2} \right) + \frac{d}{R_G} \arctan \frac{R_G}{d} \right]
\]  

(2.3.5)

where: \( M_h = 3.38 \times 10^{10} \text{ M}_\odot \) and \( d = 6.0 \text{ kpc} \).

In this chapter, I choose different values of the masses for different components with respect to the ones from Paczynski (1990). This difference is due to a rescaling factor, such that the NFW and P90 galaxy models have the same virial mass. I used the ratio between the virial mass of the NFW model (\( M_{\text{vir,NFW}} = 1.26 \times 10^{12} \text{ M}_\odot \)) and the virial
mass of the original P90 model \( M_{\text{vir,P90}} = 2.29 \times 10^{12} M_\odot \) to rescale the virial mass of the three components in P90. Once the new virial mass of the single components are known it is possible to derive the new \( M_b, M_d \) and \( M_h \). The virial mass is the mass of the galaxy within the virial radius, when the mean density of the galaxy is equal to \( 200 \rho_c \), where \( \rho_c = 3H_0^2/8\pi G \) is the critical density and \( H_0 = 68.0 \text{ km s}^{-1} \text{ Mpc}^{-1} \) is the Hubble constant.

Depending on the orbit and the Galactic potential, stars escape from the cluster as result of two-body relaxation. We therefore need to find the initial \( N \) that results in near dissolution (i.e. a few bound stars left) at an age of 12 Gyr. I used the fast cluster evolution code EMACSS (Alexander et al., 2014) to iteratively find the initial \( N \) that satisfies these constraints. I consider both circular and elliptical orbits for the clusters, with eccentricities of \( \epsilon = 0, 0.25, 0.5 \) and 0.75 and with apogalactic distances of 50 kpc, 100 kpc and 150 kpc. In the P90-potential, the apocentre of the clusters were chosen such that the orbits are not planar. Escapers were not removed from the simulations to allow stars to move from the tidal tails back into the region of the cluster because of compression at apocentre. For the initial conditions of all the clusters I used a Plummer model (Plummer, 1911) with two different initial densities: clusters that are initially Roche-filling (the stars occupy the total tidal volume), with \( r_{hm}/r_J = 0.1 \); and clusters that are initially Roche-underfilling (the stars occupy the central region of the tidal volume), where the density within \( r_{hm} \) is \( \rho_h = 10^4 M_\odot \text{ pc}^{-3} \). The stars in the cluster initially follow a Kroupa IMF (Kroupa, 2001) between \( 0.1 M_\odot \) and \( 100 M_\odot \), and a metallicity of \( Z = 0.0008 \) (corresponding to \([\text{Fe}/H] \approx -1.5\)). Moreover, for 7 simulations, I consider the possibility that BHs do not receive a natal kick when they form; as a consequence, the cluster retains 100% of stellar mass black hole initially. While in the other simulations the BHs receive a natal kick velocity which is the same kick velocity given to the neutron stars.

Furthermore, in some models primordial binaries were included, where the binaries components have the same mass. I used the description by Kroupa (1995), where the eccentricities are in thermal distribution with eigenevolution. The distribution of the semi-major axis is either derived from the period distribution, which is initially an uniform \( \log \)-period distribution or an uniform distribution for the log of the semi-major axis. The properties of the simulations are presented in Table 2.2.

### 2.3.2 Model for the background stars

Typically, observers use simple colour-magnitude cuts to select cluster stars with respect to a fore/background. This method, however, does not completely eliminate the contamination from MW field stars. In order to account for this issue I adopt a synthetic MW stellar population. I used the code TRILEGAL 1.62 (Girardi et al., 2012), which models the MW stellar population for a given region in the sky, I created a map of stars at two positions \((\ell, b) = (158.6^\circ, 56.8^\circ)\); and \((\ell, b) = (260.98^\circ, 70.75^\circ)\). The simulated backgrounds are at the positions of the known UFOs, Koposov 1 (Ko1, Koposov et al. 2007); and Willman 1 (Will1, Willman et al. 2005, 2006, 2011). Ko1 has a small half-light radius \( \sim 3 \) pc, while Will1 has a large half-light radius \( \sim 25 \) pc, which are extremes in size for this class of objects. The goal is to see whether a cluster with a different background star density

\[\text{Note that TRILEGAL do not reproduce some observations (e.g. Sharma et al., 2016), however this does not affect the conclusions in this Chapter.}\]
### Table 2.2: N-body simulation properties

<table>
<thead>
<tr>
<th>Model</th>
<th>$R_{apo}$ [kpc]</th>
<th>$\epsilon$</th>
<th>$N$</th>
<th>$N_{12,\text{Gyr}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>NFW potential</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$50e00H$</td>
<td>50</td>
<td>0.00</td>
<td>4096</td>
<td>240</td>
</tr>
<tr>
<td>$50e25H$</td>
<td>50</td>
<td>0.25</td>
<td>5000</td>
<td>162</td>
</tr>
<tr>
<td>$50e50H$</td>
<td>50</td>
<td>0.50</td>
<td>6000</td>
<td>184</td>
</tr>
<tr>
<td>$50e75H$</td>
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<td>0.75</td>
<td>10000</td>
<td>147</td>
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<tr>
<td>$50e00L$</td>
<td>50</td>
<td>0.00</td>
<td>2048</td>
<td>67</td>
</tr>
<tr>
<td>$50e25L$</td>
<td>50</td>
<td>0.25</td>
<td>3000</td>
<td>44</td>
</tr>
<tr>
<td>$50e50L$</td>
<td>50</td>
<td>0.50</td>
<td>8192</td>
<td>180</td>
</tr>
<tr>
<td>$50e75L$</td>
<td>50</td>
<td>0.75</td>
<td>20000</td>
<td>91</td>
</tr>
<tr>
<td>$100e00H$</td>
<td>100</td>
<td>0.00</td>
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<td>212</td>
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<td>2048</td>
<td>87</td>
</tr>
<tr>
<td>$100e50H$</td>
<td>100</td>
<td>0.50</td>
<td>3000</td>
<td>217</td>
</tr>
<tr>
<td>$100e75H^*$</td>
<td>100</td>
<td>0.75</td>
<td>3000</td>
<td>217</td>
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<tr>
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<td>1024</td>
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<td>0.25</td>
<td>1024</td>
<td>71</td>
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<td>$100e50L$</td>
<td>100</td>
<td>0.50</td>
<td>2048</td>
<td>159</td>
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<tr>
<td>$100e75L^*$</td>
<td>100</td>
<td>0.75</td>
<td>8192</td>
<td>29</td>
</tr>
<tr>
<td>$150e00H$</td>
<td>150</td>
<td>0.00</td>
<td>1500</td>
<td>172</td>
</tr>
<tr>
<td>$150e25H$</td>
<td>150</td>
<td>0.25</td>
<td>2048</td>
<td>246</td>
</tr>
<tr>
<td>$150e50H$</td>
<td>150</td>
<td>0.50</td>
<td>1500</td>
<td>44</td>
</tr>
<tr>
<td>$150e75H$</td>
<td>150</td>
<td>0.75</td>
<td>2048</td>
<td>53</td>
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<td>$150e00L$</td>
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<td>113</td>
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<tr>
<td>$150e25L$</td>
<td>150</td>
<td>0.25</td>
<td>1024</td>
<td>211</td>
</tr>
<tr>
<td>$150e50L$</td>
<td>150</td>
<td>0.50</td>
<td>1500</td>
<td>183</td>
</tr>
<tr>
<td>$150e75L$</td>
<td>150</td>
<td>0.75</td>
<td>2048</td>
<td>82</td>
</tr>
<tr>
<td>$50e50M-B1$</td>
<td>50</td>
<td>0.50</td>
<td>7200</td>
<td>182</td>
</tr>
<tr>
<td>$50e50M-B2$</td>
<td>50</td>
<td>0.50</td>
<td>7200</td>
<td>227</td>
</tr>
<tr>
<td>$50e50M$</td>
<td>50</td>
<td>0.50</td>
<td>6000</td>
<td>193</td>
</tr>
<tr>
<td>$50e50H-BH$</td>
<td>50</td>
<td>0.50</td>
<td>6000</td>
<td>164</td>
</tr>
<tr>
<td>$50e50L-BH^*$</td>
<td>50</td>
<td>0.50</td>
<td>30000</td>
<td>250</td>
</tr>
<tr>
<td>$50e50L-B2-BH^*$</td>
<td>50</td>
<td>0.50</td>
<td>30000</td>
<td>0</td>
</tr>
<tr>
<td>$50e75H-BH$</td>
<td>50</td>
<td>0.75</td>
<td>10000</td>
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</tr>
<tr>
<td>$50e75L-BH^*$</td>
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<tr>
<td>$150e25H-BH$</td>
<td>150</td>
<td>0.25</td>
<td>2048</td>
<td>139</td>
</tr>
<tr>
<td>$150e25L-BH$</td>
<td>150</td>
<td>0.25</td>
<td>1200</td>
<td>176</td>
</tr>
</tbody>
</table>

### Note
- The capital letter in the model label indicates if the model was, as initial condition, underfilling (high density, H) or Roche-filling (low density, L). In column 4 I show the initial number of stars; column 5 are the number of bound stars at 12 Gyr. The models with the letter M are simulations with a different initial density ($\rho_i = 10^3 M_\odot/pc^3$), with B1 and B2 contains $\sim 20\%$ of primordial binaries, but different semi-major axis distributions; and with BH retain 100% of BHs initially. In gNFW the value of $\epsilon$ is the eccentricity at $\sim 12$ Gyr. The star (*) and the † denote models for which $r_{hm}/r_J = 0.09$ and $r_{hm}/r_J = 0.06$ respectively, i.e. slightly denser to avoid a high escape rate on a dynamical time.
2.3. NUMERICAL N-BODY SIMULATIONS OF FSCS

can appear bigger or smaller.
The TRILEGAL sample was created assuming literature values for the reddening (Schlegel et al., 1998). Assuming \( R_V = 3.1 \) (typical for the MW) and a calibration at infinity, we obtain an extinction of \( A_V(\infty) = 0.0418 \) for Wil1 and \( A_V(\infty) = 0.0757 \) for Ko1, which is used by TRILEGAL to simulate extinctions which are normally distributed. The scatter on the extinction is also taken from Schlegel et al. (1998) dust maps.

In order to introduce some noise in the reddening correction I proceed to correct the TRILEGAL sample assuming a single average value of extinction for the full simulated region. This adds uncertainty to the reddening, which is likely to be the case in real observations.

Furthermore, I assume a photometric error curve \( \nu \), with an exponential form, which represent a typical error in mag for each star.

Here the steps to estimate the background number density:

1. Correction for extinction:

\[
\begin{align*}
g' &= g - A_g \\
r' &= r - A_r
\end{align*}
\]

where \( g \) and \( r \) are apparent magnitudes in SDSS filters and \( g' \) and \( r' \) are the extinction corrected equivalents. For Ko1: \( A_g = 0.091 \) and \( A_r = 0.066 \); whereas for Wil1: \( A_g = 0.013 \) and \( A_r = 0.034 \). These values are estimated using Cardelli et al. (1989) and O’Donnell (1994) extinction curve with \( R_V = 3.1 \).

2. Using a photometric error curve:

\[
\nu(m, a, b, c) = a + e^{\frac{m-b}{c}}
\]  

(2.3.6)

where \( m \) is the observed magnitude corrected for the extinction and \( (a, b, c) \) are parameters which depend on the observations; I compute the magnitudes with simulated errors:

\[
\begin{align*}
g'' &= g' + \chi \nu(g', a, b, c) \\
r'' &= r' + \chi \nu(r', a, b, c)
\end{align*}
\]

where \( \chi \) is a random number sampled from a Gaussian distribution with mean 0 and variance 1. For Ko1 I use \( (a, b, c) = (0.005, 22, 1.2) \); whereas for Wil1, \( (a, b, c) = (0.005, 25, 1.2) \). I choose the value of \( b \) to match the limiting magnitude of the observations (Koposov et al. 2007 for Ko1 and Willman et al. 2006 for Wil1).

I use the above procedure for each star, created with TRILEGAL 1.6, in the field of view of 3 degree, centred in the position of Ko1 and Wil1.

Finally, I applied the following colour-magnitude cuts: \( 16 \leq r'' \leq 22 \) and \( g'' - r'' \leq 1.2 \) for Ko1; while \( 22.6 < r'' < 24.8 \) and \( 0.25 < g'' - r'' < 0.65 \) for Wil1; taking into account only the stars that follow these criteria, we can derive the number of stars per arcsec\(^2\).
2.3.3 Maximum likelihood method to fit half-light radii

To estimate the $r_{\text{eff}}$ of the simulated clusters, I used a maximum likelihood fit following the procedure outlined in Martin et al. (2008). Having the position of the stars on the plane of the sky, the maximum likelihood fit can find the set of free parameters for which the observations become most probable.

I choose a likelihood ($\mathcal{L}$) in the following form:

$$\ln \mathcal{L} = \sum \ln (n_P + n_{BG})$$  \hspace{1cm} (2.3.7)

where $n_P$ and $n_{BG}$ are the probabilities of a star belonging to the cluster and background, respectively. I choose $n_P$ to be a 2-D elliptical Plummer profile, given by:

$$n_P = \frac{N_s}{(1 - e) \pi a^2} \left(1 + \frac{d^2}{a^2}\right)^{-2}$$  \hspace{1cm} (2.3.8)

with

$$d^2 = \left[\frac{1}{1 - e} \left(x \cos(\theta) - y \sin(\theta)\right)\right]^2 + \left[x \sin(\theta) + y \cos(\theta)\right]^2$$  \hspace{1cm} (2.3.9)

In the likelihood analysis I choose the following parameters: the scale radius ($a$) which is also the projected half-number radius, the number of stars in the cluster ($N_s$), the ellipticity\(^3\) ($e$) and the position angle ($\theta$); while $x$ and $y$ are the positions of the stars on the $x$-$y$ plane. I can estimate the number of stars in the background $N_{BG}$, fitting on the parameter $N_s$, and, knowing the number of stars in the snapshot $N_{\text{tot}}$ ($N_{\text{tot}} = N_s + N_{BG}$). Therefore, knowing the area of our simulated field of view, I can derive $n_{BG}$, which is considered to be homogeneous across the simulated field-of-views. I use a downhill simplex method (Nelder and Mead, 1965) to find the parameters that maximizes the likelihood. In the following Section I discuss the results of the analysis.

2.4 Results

In this section I present the results from the analysis, discussing the importance of each observational bias. In this way a comparison between $N$-body simulations and observational data can tell us something about the underlying properties of the observed objects.

2.4.1 Example of the evolution of a low-$N$ cluster

To illustrate the evolution of the underlying cluster properties I first present some of the results without considering observational biases. In Fig. 2.4 I show the properties of the 50e50H model (see Table 2.2). The upper panel shows the evolution of the absolute $V$-band magnitude ($M_V$, see Appendix 2.A for more details on how $M_V$ has been computed) and from this it can be seen that already at approximately 4 Gyr the cluster reaches a luminosity of typical UFOs (see Fig. 2.1). From then onwards, until the end of the evolution the total luminosity drops by a factor of $\sim 15$, and the cluster remains in the luminosity

---

\(^3\)The ellipticity is defined as $e = 1 - b_0/a_0$ where $b_0$ and $a_0$ are the semi-minor and semi-major axis of the ellipse, respectively.
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Figure 2.4: Simulation of a star cluster with initial $N = 6000$, apogalactaction at 50 kpc and eccentricity $\epsilon = 0.5$. Top: evolution of the absolute magnitude in the $V$-band of all the observable particles with $r < 300$ pc. Middle: evolution of the number of bound stars (green line). Bottom: evolution of half-number radius. The blue line is the half-number radius of the bound stars; the cyan line is the tidal radius: in the local minima the cluster is in pericentre, while in the local maxima the cluster is in apocentre.

range of UFOs until complete dissolution. From a comparison to the number of bounds stars ($N_{\text{bound}}$, middle panel), $N_{\text{bound}}$ decreases by a factor of $\sim 100$ in this period. The slow decrease in luminosity compared to $N_{\text{bound}}$ is due to mass segregation and the preferential loss of low-mass stars in the late stages of cluster evolution. This means that the previous estimate of the $N_{\text{FSC}}$ in the right mass range is a lower limit (eq. 2.2.10), because the $N_{\text{FSC}}$ in the correct luminosity range is higher. In the lower panel of Fig. 2.4, I show the evolution of $r_{\text{hm}}$ (bottom blue line) and after about 4 Gyr it levels to a value that is consistent with filling the Roche volume (see Section 2.2.2). The top line (cyan) shows the evolution of $r_J$ computed using equation (2.2.11) which decreases due to the loss of cluster mass because of escaping stars.

Because some of the above properties, such as $N_{\text{bound}}$, $r_{\text{hm}}$ and $r_J$ are not observable, I need to include observational biases in the analyses of the $N$-body results before I can make a meaningful comparison with the observations. Therefore, in the next section I analyse the data in a similar way as is done for the observational data, as described in Sections 2.3.2 and 2.3.3.
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2.4.2 The effect of the background on the size measurements

With the procedure explained in Section 2.3.2, I include background stars in the simulations. I derive the number density of the background stars ($n_{BG}$) for two different observed UFOs: Wil1 and Ko1; where $n_{BG}(\text{Wil1}) < n_{BG}(\text{Ko1})$. Then, with the cluster in the centre, I add randomly the background stars, uniformly distributed, in an area with a radius of 400 pc, far beyond the tidal radius of a low-mass cluster. Finally, as described in Section 2.3.3, I compute the best fit Plummer radii, taking only the ‘observable stars’ into account. I consider ‘observable stars’ all the stars with masses greater than 0.5 $M_\odot$ and which are not dark remnants.

In Fig. 2.5, I show the evolution of $r_{eff}$ for the model 50e50H. In the last three Gyr, the cluster can reach a large size ($\gtrsim 10$ pc), but only near apocentre, where the largest size ($\gtrsim 20$ pc) is found for the cluster with a low background. In Fig. 2.6 I show the best fit number density profile for the model 50e50H (see Table 2.2). The flattening in the external region occurs where the number density of the model roughly equals to $n_{BG}$. The simulated cluster is observed along the orbit, near apocentre, where the tails overlap, in projection, with the cluster itself. From this we see that even though $r_{eff}$ is in the correct size range, the Plummer profile does not fit the data properly, because the stars in the tails generate a bump in the number density profile, which is not observed for UFOs (Martin et al., 2008). Whereas, near pericentre, where the tails are elongated, the number density profile is well reproduced by a Plummer model, see Fig. 2.7.
Figure 2.6: Number density profile of the 50e50H model in apocentre along the \( y \)-axis (in this case along the orbit). The green line is the estimated Plummer model using the parameters obtained with the maximum likelihood fit. The Plummer model is not ideal to fit this number density profile because, at \( r \sim 25 \) pc the bump, caused by the projected positions of the stars in the tails which are close to the centre (in a radius of 400 pc), increases the estimation of the size.

Figure 2.7: Number density profile of the 50e50H model in pericentre along the \( y \)-axis (in this case along the orbit). The green line is the estimated Plummer model using the parameters obtained with the maximum likelihood fit.
2.4.3 The effect of the line-of-sight on the size measurements

Although I showed that orbital phase can greatly influence the apparent size of a cluster, an additional fact must be considered, which is the dependence on the line-of-sight. To study different lines-of-sight, I translated the $N$-body coordinates from a non-rotating frame, to a frame in which the cluster is orbiting in the $x$-$y$ plane with positive angular momentum centred on the Galaxy and with the $x$-axis increasing towards the cluster. As shown in Fig. 2.5, along the $y$-axis near apocentre the cluster reaches $r_{\text{eff}} \sim 20$ pc, while when viewed along the $x$-axis and $z$-axis, Fig. 2.8, I do not see any variations in the $r_{\text{eff}}$ measurements linked to the orbital motion. As a consequence, these star clusters appear as eUFO only when observed along the $y$-axis and when they are near apocentre. For all the simulations, the results for the $x$ and $z$ directions are identical, therefore, in the following figures I will show only one of them.

Because in the chosen reference frame, the $y$-axis is not along the orbit in between pericentre and apocentre, I also considered the cluster’s properties along the orbit at those positions, to see whether the projected tails can influence the measured cluster’s size. Near apocentre and pericentre I expect to obtain the same results as when we observe the cluster along the $y$-axis, because in pericentre the two lines-of-sight overlap. For the entire evolution of the cluster, I found that the estimation of the size along the $y$-axis and along the orbit are comparable.

The $y$-axis is the only line-of-sight along which I can observe clusters with a large size, however it is also the least probable one; because these objects are in the halo of the MW ($R_G \gtrsim 20$ pc). Therefore, unless they have their pericentre within the solar circle, it is impossible to observe them along the orbit.

To quantify the probability to observe an eUFO, I estimated the fraction of orbit ($f_t$) in which a cluster appears extended. Therefore, $f_t$ is the ratio between the time when a cluster appears extended and its orbital period. For the simulation 50e50H along the $y$-axis $f_t \sim 0.08$, but if I take into account the fact that along the other lines-of-sight $f_t = 0$ then the probability to observe the cluster is $<1\%$.  

2.4.4 The effect of the initial cluster density on the size measurement

An additional parameter to take into account is the initial cluster density. As shown in Table 2.2, I divide the simulations in high density (H) and low density (L) clusters. The low density clusters fill the Roche-volume initially while the high density clusters are initially Roche-underfilling.

Hénon (1961) showed that a cluster, once it has filled its Roche-volume, evolves with a constant ratio of $r_{\text{hm}}$ over $r_J$ (Section 2.2.2). Therefore, I expect that the evolutions of $r_{\text{eff}}$ of the clusters on the same orbit but with different initial densities are similar in the final stage of evolution.

I find that this is indeed the case for most of the models (Fig. 2.9). However, I find that there is a difference in the evolution of $r_{\text{eff}}$ depending on the initial density for three of our orbits: $R_G = 50$ kpc and $\epsilon = 0.75$; $R_G = 50$ kpc and $\epsilon = 0.50$; $R_G = 100$ kpc and $\epsilon = 0.75$.

In Fig. 2.10 I show an example of the $r_{\text{eff}}$ evolution for two models on the same orbit with different initial densities and it can be seen that $r_{\text{eff}}$ of the low-density cluster always lays above $r_{\text{eff}}$ of the high-density cluster. I interpret this difference as being due to the
slow removal of stars in the early evolution of the clusters with low densities that stay near the cluster and can enhance $r_{\text{eff}}$ at later stage. The high density cluster loses stars in all directions with higher velocity in the initial phases, and these stars are then too far to affect the $r_{\text{eff}}$ measurement. Furthermore, I observe a greater variation of $r_{\text{eff}}$ due to the orbital motion, visible in all the lines-of-sight for these three orbits, in the simulations with a low initial density. Whereas the clusters with a high initial density appear larger only along the $y$-axis. Therefore, to observe an extended cluster along all the lines-of-sight, this has to initially have a low density. For these simulations $f_t$ can be as high as $\sim 0.54$, this estimate changes for different orbits and whether the cluster is close to dissolution.

### 2.4.5 The effect of cluster’s orbit on the size measurement

To understand the relation between the orbit of a cluster and its $r_{\text{eff}}$, we illustrate in Fig. 2.11 the pericentre ($R_p$) and the apocentre ($R_a$) of each orbit considered in this chapter in a static NFW potential (Table 2.2). Therefore, each point represents an orbit. The colours mark whether the size of a cluster can appear larger than 20 pc (in green) or not (in red), due to the variation in the size evolution, as shown in Fig. 2.5.

Surprising, in Fig. 2.11 the three green dots represent the three orbits mentioned in the previous section (where $r_{\text{eff}}$ evolution for different initial density never overlap). I find that these three orbits have their $R_p$ either close or within the scale radius ($R_0$, blue vertical line) of the Galactic potential, where the slope of the NFW density profile changes.
Figure 2.9: Top: orbit of the cluster. Bottom: Evolution of the half-number radius of the 150e25H (dashed red line) and 150e25L (blue line) models along the $y$-axis.

Figure 2.10: Top: orbit of the cluster. Bottom: Evolution of the half-number radius of the 50e50H (dashed red line) and 50e50L (blue line) models along the $z$-axis (same results for the $x$-axis). Both the models have a Wil1-like background.
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![Figure 2.11: Effect of cluster orbits. The green dots show when I observe a cluster larger than 20 pc in any line-of-sight, while the red dots show orbits of clusters that are always smaller than 20 pc. \( R_0 \) is the scale radius of the NFW potential. As shown, the simulations that have their pericentre close or within the scale radius reach a larger size. For initially Roche-filling clusters and close to dissolution, \( f_t \) can be as high as \( \sim 0.19 \), \( \sim 0.45 \) and \( \sim 0.54 \), for the models 50e50L, 50e75L and 100e75L, respectively.]

variation of the Galactic density profile implies a different evaporation mass loss of the cluster during its pericentre passages. In these orbits, the stars that escape at \( R_p \) are easily coming back close to the cluster, so that they can inflate the size measurement, especially when the cluster is initially Roche-filling.

Among the simulations with a pericentre close to \( R_0 \), the simulations 150e75H and 150e75L which have \( R_p = 21.4 \) kpc do not appear larger. Therefore, we assume that all the clusters with \( R_p < 20 \) kpc appear larger. Considering only orbits with pericentre within 20 kpc and the \( f_t \) in the previous Section, the probability to observe an eUFO, that was initially Roche-filling, can be as high as \( \sim 30\% \).

### 2.4.6 The effect of different potentials

To understand the role of the Galactic potential, I run additional simulations using different MW-like potentials; gNFW and P90.

In the previous section, I conclude that the scale radius of the Galactic potential has an important role to discern between star clusters and extended star clusters.

In a static potential, clusters orbiting around a galaxy have their pericentre fixed in time (dynamical friction is negligible and has not been taken into account), while in a growing potential, clusters that have their pericentre within the scale radius at 12 Gyr could have
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their pericentre beyond the scale radius initially.

Results from the simulations show that a gNFW potential (see Section 2.3.1) does not change the evolution of star clusters, because the stars that inflate the size of a cluster in the last few Gyr are the ones that have escaped recently from the cluster. The properties of the simulations are presented in Table 2.2.

Renaud and Gieles (2015b) showed that cluster evolution does not change in a gNFW potential also for satellites that have \( R_a \leq 50 \text{ kpc} \). In this case, I tested clusters with \( R_a \geq 50 \text{ kpc} \), because, as described in Buijst and Helmi (2014) galaxies form inside out (Helmi et al., 2003; Wang et al., 2011). Which means that the mass of the MW is growing in shells by smooth accretion; therefore, the objects in the halo should be more affected by the growth of the DM potential.

After testing a growing halo potential, I studied the evolution of clusters in a potential which includes a bulge, disc and halo component. The disc could influence clusters which have their pericentre close to the Galactic centre. Moreover, with a multi-component potential I can have non planar orbits, increasing the probability to observe a cluster from different lines-of-sight.

To assess the effects of a multi-component potential, I run four simulations using a static potential for the bulge, the disc and the halo, following the analytical model from P90 (see Section 2.3.1). The properties of the simulations are presented in Table 2.2.

Even in this case the evolution of the observed size is similar to the simulations with a NFW potential. Therefore, I conclude that, with these initial conditions, the passage of a cluster through the disc does not enhance the size of the cluster, because the scale parameter of the disc \( (a_d = 3.7 \text{ kpc}) \) is roughly half of the minimum pericentre distance \( (R_p = 7.14 \text{ kpc}) \).

2.4.7 The effect of stellar mass black holes retained in the cluster

There are no observational constraints on black hole (BH) natal kicks, while there are on neutron star natal kicks, mainly thanks to radio pulsars. For this reason, it is not clear whether the BHs natal kick should be similar (Repetto et al., 2012) or smaller than the neutron stars natal kick (Fryer et al., 2012). Likely, with the discovery of new gravitational waves, further constraints will be set on the BHs natal kick velocity (Abbott et al., 2016a).

It has been shown by Merritt et al. (2004); Mackey et al. (2007, 2008); Lützgendorf et al. (2013) and Peuten et al. (2016) that a higher fraction of dark remnants in a cluster can change its observed properties. Moreover, BH candidates have been observed in several GCs (Strader et al., 2012; Chomiuk et al., 2013); as a consequence, I consider the possibility that BHs do not receive a kick when they form and for these models I retain 100% of stellar mass BHs initially. The properties of the simulations are in Table 2.2 and for the Galactic potential I assumed a NFW potential (eq. 5.3.1).

In Fig. 2.12 I show the evolution of \( r_{eff} \) for the model 50e50L-BH that started with a low initial density. The clusters appear extended \( (r_{eff} \gtrsim 20 \text{ pc}) \) for almost the entire evolution (after roughly 9 Gyr), independent of the projection axis (similar results for other lines-of-sight) and orbital phase. Indeed, the projection effect of the tails are not affecting the fitting results as in Fig. 2.6. Therefore, unlike the models that do not retain BHs, these clusters can be observed as eUFO \( (f_t = 1) \).

In Fig. 2.13, the evolution of the fraction of BHs inside the cluster (within the tidal
Figure 2.12: In the lower plot, evolution of the half-number radius of the 50e50L-BH model along the $x$-axis. Half-number radius for a cluster with a Will-like background (blue line) and a Kol-like background (dashed yellow line), 3D half-mass radius (dotted black line). In the upper plot the black line shows the orbit of the cluster.

radius) shows how fast the BH population escape from the cluster. Breen and Heggie (2013a,b) showed that the escape rate of stellar BHs depend on their half-mass relaxation time.

For clusters with high initial density, for example in the simulation 50e50H-BH, because the short initial half-mass relaxation time ($t_{rh,0}$), all the BHs are dynamically ejected in few Gyr; indeed, the results are similar to the simulation 50e50H where only few percent of BHs are retained in the cluster initially. Whereas, the low density clusters, which have a $t_{rh,0}$ of $\sim 2 - 3$ Gyr, as shown in Fig. 2.13 they retain the BHs up to the dissolution of the cluster. These low density clusters do not appear mass segregated (Peuten et al., 2016). The effect of stellar mass BHs retained in low density clusters is remarkable, because these clusters can appear as large as an eUFO for the last Gyr (not only near apocentre) and along all the lines-of-sight. However, in the absence of kinematics, it is challenging to determine whether these objects are DM free or dominated, because they do not appear mass segregated. Regarding their morphology, if we observe them along the $x$-axis (the most probable line-of-sight), we do not see the typical ‘S’ shape of a star cluster, because the Lagrangian points (L1 and L2) overlap with the centre of the cluster.

2.4.8 Mass function

In this section I want to study the mass function (MF) of collisional system with large $t_{rh,0}$ ($\sim 2 - 3$ Gyr), where the BHs are retained.

In Fig. 2.14, I plot the MF of the $N$-body model 50e50L-BH, for all the stars with-
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Figure 2.13: Evolution of the fraction of BHs (normalized to the maximum value) for the models 50e50L-BH (dashed red line) and 50e75L-BH (black line), within the tidal radius of the clusters.

out the dark remnants (blue line) and for the white dwarfs (WDs, orange line). Then I compare the MF of this model with a single stellar population (SSP) model. For the SSP model I assumed a Kroupa IMF (Kroupa, 2001) between $0.1 \, M_\odot$ and $100 \, M_\odot$, and a metallicity of $Z = 0.0008$ (corresponding to $[\text{Fe}/\text{H}] \simeq -1.5$). I evolve the SSP model, up to 11 Gyr, with the single-star evolution (SSE, Hurley et al. 2000) code, which is the same evolutionary tool available in NBODY6. The MF of the SSP model is in dashed cyan line for all the stars except the dark remnants, and in dashed magenta line I plot the WDs. To compare these models I scaled the SSP MF to the $N$-body MF, such that the number of stars in the last bin of the observable stars ($0.79 < m/M_\odot < 0.87$) is the same for SSP and $N$-body model. From this comparison (Fig. 2.14), we can say that in collisional systems, where dynamical interactions between stars are important, the MF is flattened. Because ultra-faint dwarf galaxies appear to have similar MF slopes ($\sim -1.3$ in the range $0.5 - 0.77 \, M_\odot$, Geha et al., 2013), the flattened MF as a result of dynamical evolution cannot be used to discern between extended star clusters and DGs for an individual object. However, for star clusters I do not expect a relation between the MF slope and the metallicity (as found for DGs), but I do expect the slope to be flatter at smaller $R_G$ (e.g. Vesperini and Heggie, 1997). Hence the MF slope might be useful for addressing the nature of UFOs by considering the MF slope as a function of $R_G$ and $[\text{Fe}/\text{H}]$, simultaneously.

As shown in Fig. 2.14, the model 50e50L-BH shows a large fraction of WD. To estimate how many WD are present in the models with respect to the observable stars, I estimate the number of WD ($N_{\text{WD}}$) between the first bin of the WD and the last bin of the observable stars, and the same for the number of observable stars ($N_*$). Therefore, for
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Figure 2.14: Stellar mass function for the model 50e50L-BH and for a stellar population with the same IMF and age (SSP), but that has undergone no dynamical evolution, after 11 Gyr.

\[ 0.52 < m/M_\odot < 0.87, \quad N_{\text{WD}}/N_* = 0.76 \text{ for the SSP model and } N_{\text{WD}}/N_* = 1.12 \text{ for the } N\text{-body model. Goldsbury et al. (2012) and Heyl et al. (2015) show that, in the UV, the young WD are among the brightest stars in the cluster, which means that for FSC the WD population can potentially be observed. However, in a low- } N\text{-system such as a UFO, the number of young WD is small. For example, with HST in the F225W band, for the model 50e50L-BH at 11 Gyr I expect to be able to observe only 4 out of 499 WD.} \]

2.4.9 Velocity dispersion

To establish the dynamical mass of a system I need \( r_{\text{eff}} \) and the velocity dispersion, \( \sigma \). Despite the fact that it is challenging to determine \( \sigma \) for most UFOs, because of their distance and the limited number of bright stars, for some of them the velocity dispersion has been measured. For example, Will has \( \sigma \sim 0 \text{ km s}^{-1} \) within \( r_{\text{eff}} \) (Willman et al., 2011), which is consistent with a star cluster scenario; whereas Segue 1 has \( \sigma \sim 3.7^{+3.1}_{-1.4} \text{ km s}^{-1} \) within \( \sim 3 r_{\text{eff}} \) with a \( V \)-band mass-to-light ratio of \( 3400 M_\odot/L_\odot \) (Simon et al., 2011), which lead to the conclusion that Segue 1 is a dark matter dominated object. I analysed the simulations keeping in mind the observational biases discussed previously in this chapter, and studied whether it is possible to infer a high velocity dispersion in a dark matter free object. We assumed that with a velocity measurement, member stars and background stars can be separated, and I therefore ignore the effect of background stars

\[^4\text{Giant stars show a lower velocity dispersion, } \sigma \sim 2^{+3.1}_{-1.7} \text{ km s}^{-1}.\]
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on the $\sigma$ measurements. Moreover, I studied the kinematics of the FSCs, to see whether the orbital motion of the cluster could leave some features in the velocity dispersion profile like in $r_{\text{eff}}$ along the orbit near apocentre (Fig. 2.5), as this has been the proposed scenario for the high $\sigma$ of dwarf spheroidal galaxies (Kroupa 1997 but see Mateo 1997 and Olszewski 1998). To compute the $\sigma$, I am taking all the observable stars into account (see Section 2.4.2), within $r_{\text{eff}}$, along a line-of-sight, as a function of time.

Therefore, the velocity dispersion $\sigma_j$ is defined as:

$$\sigma_j^2 = \frac{1}{(N_h - 1)} \sum_{i} (v_{j,i} - \overline{v}_j)^2$$  \hspace{1cm} (2.4.1)

where $j$ is a chosen line-of-sight, $N_h$ is the number of stars within the projected half-number radius, $v_{j,i}$ is the velocity of the $i$-th star in the line-of-sight and $\overline{v}_j$ is the mean line-of-sight velocity; and I estimate the variance for each velocity dispersion as (Pryor and Meylan, 1993)

$$\Delta \sigma_j^2 = \frac{\sigma_j^2}{2N_h}.$$  \hspace{1cm} (2.4.2)

In Fig. 2.15, I show the velocity dispersion along the $x$-axis (red line) and along the $y$-axis (black dots). The other line-of-sight, $z$, is not shown because it has the same trend and values of the $x$-axis. As shown, there are only features due to the orbital motion along the $y$-axis when the cluster is near apocentre. The cause of increase is similar to what I found for the enhancement of $r_{\text{eff}}$ along the $y$-axis, namely an increased number of unbound stars projected within $r_{\text{eff}}$. Nevertheless, as shown in the Fig. 2.15, this rise happens only for a brief moment with respect to the orbital period, near apocentre. Because of this, and the fact that it is impossible to view a system exactly along its orbit when it is in apocentre (if the apocentre distance is further away than the solar radius), this effect is not expected to play an important role in inflating the velocity dispersion, at least not in the cases studied here. Therefore, any observation of the velocity dispersion of a FSC without binaries in the outer halo should find a value that is consistent with the virial mass of the stars and stellar remnants (i.e. a few $100 \text{m s}^{-1}$).

2.4.10 Binaries

Around one-third of the stars in the solar neighbourhood are in binaries or multiple systems (Lada, 2006) and UFOs may have a higher fraction of binaries (e. g. Simon et al. 2011; Martinez et al. 2011). Previous studies show that binaries play an important role in the cluster’s evolution (Heggie and Hut, 2003; Ivanova et al., 2005; Hurley et al., 2007). NBODY6 includes a prescription for both single star and binary star evolution (Hurley et al., 2000, 2002) and allows us to study these effects combined with their dynamical influence on the evolution of the cluster. In this Section I focus the efforts in understanding and quantifying the effect of primordial binaries on the velocity dispersion, performing three simulations with $\sim 20\%$ of primordial binaries (50e50M-B1, 50e50M-B2 and 50e50L-B2-BH).

I report in Table 2.2 the three runs. For 50e50M-B1 and 50e50M-B2 I have the same initial conditions as for the other clusters, except that the initial density is lower with
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respect to the high density simulations. In these two simulations, as for 50e50M, I have \( \rho_h = 10^3 M_\odot \text{pc}^{-3} \) with an apogalacticon of 50 kpc and eccentricity of 0.50.

In Fig. 2.16 I show the evolution of \( r_{\text{eff}} \) for the model with binaries (50e50M-B1) and without (50e50M). The values for \( r_{\text{eff}} \) between the two simulation are similar therefore I can conclude that primordial binaries do not inflate \( r_{\text{eff}} \). Previous studies (Giersz and Heggie, 2011) showed that the evolution of \( r_{\text{hm}} \) is insensitive to the binary fraction.

For the analyses I treat stars in binary systems in the same way as the single stars, i.e. if their luminosity is above the detection limit, I include them in the analyses of \( \sigma \). This means that there is an additional contribution to \( \sigma \) due to the orbital motion of the binary members. As is often done in observations, I apply a \( \sigma \)-clipping technique iteratively, removing all the stars with velocities larger/smaller than \( 3 \sigma \) from the mean, until the value of the \( \sigma \) does converge (Yahil and Vidal, 1977).

In Fig. 2.17 I show the evolution of the velocity dispersion for the 50e50L-B2-BH model; which dissolve after 11 Gyr. The increase in the observed \( \sigma \) due to binaries is small (green line) with respect to the same model without primordial binaries (black line). Towards the end of the cluster evolution I observe an increase in \( \sigma \) associated with the increased number of binary systems, this increase is due to preferential loss of low-mass single stars (Ivanova et al., 2005; Hurley et al., 2007). However, if the \( \sigma \)-clipping technique it is not taken into account (red line in Fig. 2.17), for example for a low number of observable stars, then the velocity dispersion is roughly 1 km s\(^{-1}\). During the evolution of the cluster, the binary properties hardly change because the model has a large \( t_{\text{rh,0}} \) (see Section 2.4.7), while there typically only a few dynamically formed binaries (which have a short orbital period). Because the dynamical velocity dispersion is low (\( \sim 0.1 \text{ km s}^{-1} \)), binaries with orbital velocities of \( \sim 0.5 \text{ km s}^{-1} \) are significantly affecting the inferred ve-
Figure 2.16: Evolution of $r_{\text{eff}}$ with (blue line) and without (dashed red line) primordial binaries along the $x$-axis. In blue $r_{\text{eff}}$ of 50e50M-B1 and in red $r_{\text{eff}}$ of 50e50M. The black line shows the radial orbit of the cluster.

Locality dispersion. For a primary of 0.7 M$_\odot$ and a secondary 0.4 M$_\odot$ this corresponds to a period of $\sim 1000$ yr, making it very challenging to detect these binaries in repeat observations. Because the binary properties do not evolve much, the only way of taking binaries into account would be to make an assumption about the binary properties and include this in the modelling (e.g. Martinez et al., 2011; Cottaar and Hénault-Brunet, 2014).

Assuming that the model 50e50L-B2-BH at 10 Gyr is in dynamical equilibrium, with the formula by Walker et al. (2009) and Wolf et al. (2010) I can estimate its dynamical mass within the half-light radius of the system:

$$M_{1/2} = \frac{4}{G} \sigma^2 r_{\text{eff}}.$$  \hspace{1cm} (2.4.3)

For example, for $\sigma \simeq 1$ km s$^{-1}$ and $r_{\text{eff}} \simeq 30$ pc, $M_{1/2} \simeq 2.8 \times 10^4$ M$_\odot$. From the simulation I can estimate the half-light luminosity $L_V \simeq 700$ L$_\odot$, therefore the $M/L_V \simeq 40$ M$_\odot$/L$_\odot$, which is consistent with a DM-dominated object interpretation. While if I consider the 3$\sigma$-clipping, $\sigma \simeq 0.4$ km s$^{-1}$ then $M_{1/2} \simeq 4.5 \times 10^3$ M$_\odot$ and $M/L_V \simeq 6$ M$_\odot$/L$_\odot$. Because $r_{\text{eff}}$ and $\sigma$ change with time, in the cluster lifetime I have different estimates of the $M/L_V$, which can be as high as 100 M$_\odot$/L$_\odot$.

The estimation can be interpreted as a lower limit, because I do not have background stars that can contaminate the measurements of the velocity dispersion.
2.5 Conclusions

In this study I present $N$-body simulations of the evolution of faint star clusters (FSCs) using \texttt{NBODY6TT}, which is an adaptation of the direct $N$-body code \texttt{NBODY6}. I focus the analysis on the effects of observational biases on the measurements of the properties of ultra faint objects (UFOs, see Fig. 2.1).

To investigate whether UFOs are part of the (ultra-faint) DGs population, the GCs population or both; I first estimate the total number of FSCs based on a simple GC population synthesis model. This model assumes an initial distributions of star cluster masses and Galactocentric distances, which is then evolved by a simple mass loss prescription. I find that the predicted number of clusters ($N_{\text{FSC}} = 3.3^{+7.2}_{-1.6}$) is consistent with the number of observed star cluster candidates, see Fig. 2.2. However, more star clusters could be discovered and because I am using a very simple model, that for example ignores $M/L_V$ variations, I cannot conclude that part of the UFOs are DGs.

Using a series of $N$-body simulations I further study the biases that may arise from observing FSCs. These simulations were projected along different lines-of-sight and the object properties were derived using traditional state-of-the-art methods (Martin et al., 2008). In order to investigate the role of the MW fore/background stars contamination I adopt a \texttt{TRILEGAL}-simulated field population at the position of two known UFOs (Koposov 1 and Willman 1). I find that the projected density of MW background stars can influence the measured size of a UFO in the sense that denser environments will lead to smaller sizes, when the Plummer model is not able to fit the number density profile properly (i.e. near apocentre and along the orbit, see Fig. 2.6).

Besides the effect of the projected density of MW field stars, I notice that the observed
size of a UFO depends on the orbital phase. Objects closer to apocentre tend to appear larger, however this effect is highly dependent on the viewing angle of the simulations, since the apparent larger size is caused by the overlapping of tidal tail stars. I find that the size enhancement is visible when the object is observed along the $y$-axis (the cluster is orbiting on the $x$-$y$ plane with positive angular momentum and the $x$-axis is the line that connect the Galactic centre to the cluster centre), for a small fraction of the orbit ($\sim 8\%$, for the model 50e50H).

The simulations also reveal a clear relation between cluster size and initial density in the sense that initially low-density clusters are more susceptible to size variations due to orbital phase (in all lines-of-sight). This result suggests that extended UFOs (eUFOs) are more likely to be observed if they formed Roche volume-filling. Furthermore, I observe that simulated clusters with pericentre roughly within the potential scale radius ($R_0$) show variations in the size measurements. This appear to be due to the change of the NFW density slope near the scale radius, which ultimately influences the strength of the tidal forces.

From the analysis I conclude that compact UFO satellites (e.g. Koposov 1 and 2) may naturally arise from a population of collisional systems. However, it is very unlikely that star clusters, that do not initially retain stellar mass BHs, contribute to the eUFO population of a MW-like galaxy. The probability of observing an eUFO becomes insignificant if we consider that it has to be viewed along the $y$-axis (the least probable line-of-sight) and near apocentre. Nevertheless, if a cluster forms filling its Roche volume and has its pericentre within the scale radius, the probability to observe it rise up to 30%. It means that among the FSC population, 1 over 3 could be observed extended. I find that these results hold even on multi-component (P90) or growing Galactic potential.

I find that the retention of stellar mass BHs (no natal kicks) radically changes the results. When the cluster is initially Roche-filling, the observed size (as the 3-D half-mass radius) is expanding for its entire lifetime. In particular, after $9\,\text{Gyr}$ the observed size grows above $20\,\text{pc}$, independent of the line-of-sight and the fore/background. Whereas, when the cluster has an initial high density and retain 100% of BHs, the BHs do not change the evolution of the size because they are rapidly ejected due to two-body interaction. This scenario is partially supported by observations of stellar mass BHs in several GCs (Strader et al., 2012; Chomiuk et al., 2013). Moreover, indirect evidence for BH in GCs comes from the large core radii (Merritt et al., 2004; Mackey et al., 2007, 2008) and the absence of mass segregation in NGC6101 (Peuten et al., 2016).

Finally, I study the effect of primordial binaries in the simulations, in particular their effect on the velocity dispersion. If I combine the size and the velocity dispersion measurements from the simulations with stellar mass BHs and primordial binaries, I estimate a $M/L_V \sim 1 - 100$. High $M/L_V$ values ($M/L_V \gtrsim 10$) are observed in DGs (McConnachie, 2012), which are DM dominated objects, but the simulations show that these measurements are not conclusive for a DGs interpretation. I show that the binaries can inflate the velocity dispersion and that in an extended star cluster the properties of the binaries do not change significantly. Therefore, binaries with different initial properties may inflate the velocity dispersion and $M/L_V$ even more. For this reason, because the initial properties of the binary population is unknown we have to rely on assumptions which may not be correct. However, Peñarrubia et al. (2016) show that wide binaries can be used to constrain the central distribution of DM in ultra-faint DGs. In addition to that,
in some of the eUFOs, metallicity spread has been observed, which is indicative of an extended star formation history. This leads to the conclusion that the satellite is either a tidally disrupted DM-free galaxy, or a DG.

DGs have lower or similar metallicities than GCs but they also have a relaxation time longer than the Hubble time, which means that the dynamical evolution due to two-body relaxation is not important. Therefore, they have a mass function (MF) which is not depleted in low-mass stars as a result of dynamical evolution. However, Geha et al. (2013) showed that the MF of DGs becomes flatter with decreasing metallicity, which they attribute to IMF variations. In GCs, mass segregation and evaporation can change the slope of the MF during the evolution (Vesperini and Heggie, 1997; Baumgardt and Makino, 2003), hence I expect GCs to be depleted in low-mass stars near the end of their lives, and to have a flatter MF for smaller Galactocentric distance.

eUFOs are likely to be accreted objects from DGs interacting with the MW, because if they form in a DG, the probability to be initially Roche-filling (low density) is enhanced (e.g. Elmegreen 2008). Therefore, if they form with a low density they have a large initial half-mass relaxation time and only few BHs will be expelled due to dynamical interactions. Moreover, an initially DM-dominated object will be likely to retain a high number of BHs even when natal kicks are taken into account. The BHs will sit in the centre pushing out the low mass particles due to two-body relaxation. Then, when the dissolution of the system occurs with few hundreds of stars left, a faint DM-free object can be observed.

The results to some extent agree with Dominguez et al. (2016) results, where they claim that Segue 1, an eUFO, can be a DM-free object. Unfortunately, it is not trivial to compare the results with their results, because I am using a direct \(N\)-body code, ideal for collisional systems, while they are using a particle-mesh code, which is not ideal to simulate star clusters but less time consuming, as they stated in their conclusion. Therefore, they do not have stars with different masses and binaries stars, which in these cases are fundamental to increase the observed velocity dispersion. However, the simulations are not fine tuned for Segue 1.

In this chapter, I conclude that star clusters contribute to both the compact and the extended population of UFOs. Retaining stellar mass BHs in an initially low density cluster is vital to have extended star cluster. While a high binary fraction can inflate the velocity dispersion measurements significantly, leading to the conclusion that the object has a high \(M/L_V\) ratio. It is possible to say something about the nature of star clusters that appear as an eUFO by considering the kinematics of the (tidal) tails, because in the case of dissolving star clusters these should be cold (few \(100\) m s\(^{-1}\)). Therefore, if the UFO is a star cluster I expect to observe a flatter MF for smaller Galactocentric distance and uncorrelated with metallicity, and dynamically cold tails; while mass segregation and binary properties cannot be used to discern between DM free and DM dominated object.
2.A Absolute magnitude in V-band

To compute the V-band absolute magnitude ($M_V$) of the simulated clusters we applied two methods.

1) Knowing the luminosity ($L$ in $L_\odot$) and the temperature ($T$ in K) of each star (NBODY6 output) is possible to calculate $M_V$.

$$M_V = -2.5 \log \left( \sum_{i=1}^{N} 10^{-0.4 M_{V,i}} \right)$$ (2.A.1)

where $M_{V,i}$ is the absolute magnitude in band V of the $i$-th star and $N$ is the total number of stars.

$$M_{V,i} = M_{V,\odot} - 2.5 \log \left( \frac{L_i}{L_\odot} \right) - BC$$ (2.A.2)

where $M_{V,\odot} = 4.8$ is the absolute magnitude of the Sun and BC is the bolometric correction:

$$BC = 2.324497 + 2.5 \log(g(T_i))$$ (2.A.3)

with

$$g(T_i) = BB(\lambda, T_i) \cdot \Delta\lambda$$ (2.A.4)

where $\Delta\lambda = 88 \times 10^{-9}$ m is the full width at half maximum (FWHM) of V-band filter. While $BB(\lambda, T_i)$ is the normalised black body radiation formula (Planck law):

$$BB(\lambda, T_i) = \frac{1}{BB_{tot}(T_i)} \frac{BB_0(\lambda)}{e^{\frac{hc}{\lambda K_B T_i}} - 1}$$ (2.A.5)

with $BB_0 = \frac{2hc^2}{\lambda^5}$ and $BB_{tot} = \frac{s}{\pi} T_i^4$. Where $c$ is the speed of light, $h$ is the Planck constant, $s$ is the Stefan-Boltzmann constant, $K_B$ is the Boltzmann constant. In this case we use $\lambda = 551$ nm, which is the central wavelength for the V-band filter.

2) We compute the absolute magnitude in band V using the initial mass of the stars in the simulations.

From eq. (2.A.1) we need to compute $M_{V,i}$. Using CMD 2.7, we can use the PARSEC isochrones v1.2S (Bressan et al., 2012; Chen et al., 2014; Tang et al., 2014), where the initial mass ($m_i^{\text{model}}$) of the stars and their absolute magnitude in V-band ($M_{V,i}^{\text{model}}$) are given for a selected time.

At this point, we can generate a function which interpolate these data, therefore, we have the absolute magnitude in V-band as a function of the initial mass, $M_{V,i}^{\text{model}}(m_i^{\text{model}})$. Using the initial mass of the surviving stars at fixed time in the simulations, we can estimate their absolute magnitude in V-band, $M_{V,i}^{\text{model}}(m_{N\text{-body}}^i)$.

In conclusion the two methods are equivalent because we obtain similar results.
Chapter 3

Neutron star natal kicks and the long-term survival of star clusters

I investigate the dynamical evolution of a star cluster in an external tidal field by using $N$-body simulations, with focus on the effects of the presence or absence of neutron star natal velocity kicks. I show that, even if neutron stars typically represent less than 2% of the total bound mass of a star cluster, their primordial kinematic properties may affect the lifetime of the system by up to almost a factor of four. I interpret this result in the light of two known modes of star cluster dissolution, dominated by either early stellar evolution mass loss or two-body relaxation. The competition between these effects shapes the mass loss profile of star clusters, which may either dissolve abruptly (“jumping”), in the pre-core-collapse phase, or gradually (“skiing”), after having reached core collapse.

This Chapter was published in Monthly Notices of the Royal Astronomical Society (MNRAS), see F. Contenta, A. L. Varri and D. C. Heggie, MNRAS, 2015, 449, L100.\footnote{I reproduced this Chapter following the guidelines of the publisher license, see Appendix A.1 for details.}

3.1 Introduction

The rate at which star clusters lose mass has been one of the enduring problems of stellar dynamics. In one of the earliest results, Ambartsumian (1938) already highlighted the role of relaxation. The landmark survey of Chernoff and Weinberg (1990) added a mass spectrum, stellar evolution and a tidal boundary, and also revealed the importance of the initial structure of the star cluster. But several other factors also influence the lifetime of star clusters, including the binary population (e.g. Tanikawa and Fukushige 2009), the form of the Galactic orbit (e.g. Baumgardt and Makino 2003), the form of the Galactic potential and tidal shocking (e.g. Gnedin and Ostriker 1997), and the crossing time scale (Whitehead et al., 2013).

In this Chapter I add one more influence: natal kicks of neutron stars (NS). Though neutron stars may account for less than 2% of the cluster by mass, I find, astonishingly, that the presence or absence of kicks may change the lifetime of a star cluster by almost a factor of four. Though the existence of natal kicks of neutron stars is not in doubt, their distribution and dispersion are difficult to establish (see, for example, Podsiadlowski et al. 2005).
### Table 3.1: N-body simulation properties

<table>
<thead>
<tr>
<th>Model</th>
<th>$N$</th>
<th>$W_0$</th>
<th>$e$</th>
<th>$M_0$ [M$_\odot$]</th>
<th>$r_h$ [pc]</th>
<th>$r_J$ [pc]</th>
<th>$T_{\text{diss}}$ [Myr]</th>
<th>$T_{\text{diss}}^{\text{BM}}$ [Myr]</th>
<th>$T_{\text{cc}}$ [Myr]</th>
<th>$T_{\text{cc}}^{\text{BM}}$ [Myr]</th>
</tr>
</thead>
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<tr>
<td>8kK</td>
<td>8192</td>
<td>5.0</td>
<td>0.0</td>
<td>4497.3</td>
<td>4.53</td>
<td>24.35</td>
<td>2426</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>16kK</td>
<td>16384</td>
<td>5.0</td>
<td>0.0</td>
<td>8990.7</td>
<td>5.73</td>
<td>30.67</td>
<td>2816</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>32kK</td>
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<td>0.0</td>
<td>18419.2</td>
<td>7.23</td>
<td>38.96</td>
<td>3669</td>
<td>-</td>
<td>-</td>
<td>-</td>
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<tr>
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<td>0.0</td>
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<td>48.79</td>
<td>4516</td>
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<td>0.0</td>
<td>71422.0</td>
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<td>61.21</td>
<td>5927</td>
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<td>61.27</td>
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</tr>
<tr>
<td>128kKe</td>
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<td>0.5</td>
<td>71453.0</td>
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<td>29.43</td>
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<tr>
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<td>5.0</td>
<td>0.5</td>
<td>71453.0</td>
<td>5.50</td>
<td>29.43</td>
<td>11254</td>
<td>11675</td>
<td>8952</td>
<td>9332</td>
</tr>
<tr>
<td>128kK7</td>
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<td>61.31</td>
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<td>-</td>
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<td>-</td>
</tr>
<tr>
<td>128kN7</td>
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<td>0.0</td>
<td>71780.9</td>
<td>7.14</td>
<td>61.31</td>
<td>24494</td>
<td>25506</td>
<td>11886</td>
<td>12620</td>
</tr>
</tbody>
</table>

Note. — The capital letter in the model label indicates if the model is characterized by the presence (#K, e.g. 128kK) or the absence (#N, e.g. 128kN) of NS initial kicks. The star (*) denotes a model for which two different numerical realizations have been evolved; the values are the average of those for the two simulations.

In order to isolate the effect of this one factor I consider models from another landmark survey of the evolution of star clusters: that by Baumgardt and Makino (2003, hereafter BM03). As it happens, they imposed no natal kicks on neutron stars, and it was the attempt to reproduce some of their results that led to this discovery. Indeed their principal models, which begin with a King profile with $W_0 = 5$, evolve very differently, both qualitatively and quantitatively, if natal kicks are applied.

The particular models I considered are described in the following Section, while Section 3.3 presents the results in some detail, including some information on core collapse and mass segregation. The final Section summarises the conclusions, and attempts to interpret them in the context of other recent work.

### 3.2 Description of the Runs

I simulate the evolution of a globular cluster as in BM03, but using NBODY6 (Nitadori and Aarseth, 2012). I have performed a survey of simulations in an accelerating, non-rotating frame, using a number of particles between $N = 8192$ and $N = 131072$, a Kroupa IMF (Kroupa, 2001), with the mass of the stars between 0.1 $M_\odot$ and 15 $M_\odot$ (resulting in a theoretical mean mass $\langle m \rangle = 0.547 M_\odot$), and metallicity $Z = 0.001$. Natal kicks, when they were applied, had a Maxwellian distribution with $\sigma = 190$ km s$^{-1}$ (see eq. 3 in Hansen and Phinney, 1997).

In the simulations, the cluster is in a circular orbit, or in an elliptical orbit with ec-
centricity $e = 0.5$, in a logarithmic Galactic potential $\phi = V_G^2 \ln(R_G)$, where $V_G$ is the circular velocity and $R_G$ is the Galactocentric distance. For the majority of the runs I have used a Roche-lobe filling King (1966) model with $W_0 = 5$ as initial condition. Additional simulations have been performed by increasing the initial concentration of the King profile ($W_0 = 7$). The clusters start at a Galactic radius of 8.5 kpc, with an initial velocity of 220 km s$^{-1}$ (in the circular case); in the elliptical case the apogalacticon is at 8.5 kpc and the initial speed there is reduced appropriately, while the size of the cluster is determined by assuming a Roche-lobe filling condition at perigalacticon. The initial conditions for all the simulations have been generated using MCLUSTER (Küpper et al., 2011).

The tidal radius of the cluster was defined as the Jacobi radius

$$r_J = \left(\frac{GM^2}{2V_G^2} \right)^{1/3} R_G^{2/3},$$

where $M$ is the “bound” cluster mass. The quantities $M$ and $r_J$ were determined self-consistently and iteratively by first assuming that all stars are still bound and calculating the tidal radius with this formula. In a second step, I calculated the mass of all stars inside $r_J$ relative to the density centre of all stars, and used it to obtain a new estimate for $r_J$. This method was repeated until convergence. Escapers were not removed from the simulations.

The properties of the simulations are presented in Table 3.1. The significance of the model label is stated in the note to the Table. Column 4 gives the orbital eccentricity; columns 5, 6 and 7 are the initial values of the total bound mass, the half-mass radius and the Jacobi radius, respectively. Column 8 gives the dissolution time, which, following BM03, is defined as the time when 95% of the mass was lost from the cluster, while column 10 gives the core-collapse time. The corresponding quantities from BM03 are reported in columns 9 and 11, respectively. In this analysis, the moment of core collapse $T_{cc}$ has been determined by inspecting the time evolution of the core radius and of the innermost lagrangian radius enclosing 1% of the total mass.

### 3.3 Results

#### 3.3.1 Lifetime and main properties of the models

The main result of this investigation is that the presence or the absence of NS natal velocity kicks can affect significantly the lifetime of star clusters, up to almost a factor of four. This striking result is illustrated by a series of “reference models” (with $W_0 = 5$, $e = 0$, and $N = 128k$, 64k, 32k, 16k, and 8k), with or without NS velocity kicks; the time evolution of the bound mass of these models is presented in Fig. 3.1. The difference in the behaviour of the models with or without NS kicks starts early in their evolution ($M/M_0 \approx 0.8$) and leads to a dramatic contrast in the slope of the graph at the final stages of evolution ($M/M_0 < 0.2$).

An important aspect of the very rapid dissolution of the models with NS kicks is that, in almost all cases, they fail to reach core collapse during their evolution, as opposed to the models without NS kicks, which show signatures of core collapse at a time corresponding to $0.1 < M/M_0 < 0.2$. The only exception is given by model 8kK, which reaches core collapse at the very late stages of evolution, 240 Myr after the formal dissolution time.
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Figure 3.1: Time evolution of the fraction of bound mass (normalized to the initial value) of models with initial concentration $W_0 = 5$, on circular orbits. The models are characterized by different number of particles and by the presence (red lines; right to left: 128kK, 64kK, 32kK, 16kK and 8kK) or the absence (blue lines; right to left: 128kN, 64kN, 32kN, 16kN and 8kN) of NS kicks. The black dots mark when core collapse occurs. The corresponding models studied by BM03, i.e. without NS kicks, are also shown (green dashed lines; right to left: 128k, 64k, 32k, 16k and 8k); this data was retrieved by means of the data extraction tool Dexter.

(see Tab. 3.1, row 1, and the corresponding black dot in Fig. 3.1). The fact that it reaches core collapse at all, while larger models do not, is attributable to its short initial relaxation time.

I have also considered two additional pairs of models, as representative cases of the regime of high initial concentration ($W_0 = 7$; models 128kK7 and 128kN7) and of the evolution of a star cluster on an elliptic orbit ($e = 0.5$; models 128kKe and 128kNe). Here the effects of the presence of NS kicks on the star cluster lifetime are less severe compared to those on the “reference models”, but they are still significant (see Fig. 3.2). Both models 128kK7 and 128kKe reach core collapse, although at a very late stage of evolution. Of the systems without NS kicks, model 128kNe reaches core collapse at a mass comparable to that of “reference models” without NS kicks, while model 128kN7 has the largest mass at $T_{cc}$ of all the models in this survey which reach core collapse; such a result is not surprising, given its initial concentration.

Another useful diagnostic of the differences between models with and without NS kicks is provided by the mean mass of stars in the innermost lagrangian shell, enclosing 1% of the total bound mass of a system. Its time evolution is illustrated in Fig. 3.3, for all models in this survey with $N = 128k$ particles. In almost all cases, the mean mass in the innermost shell initially shows a decrease, which is due to the early evolution and escape of massive stars; as expected, this effect is more pronounced for systems with NS
3.3. RESULTS

![Graph showing time evolution of the fraction of bound mass of models](image)

Figure 3.2: Time evolution of the fraction of bound mass of models with (i) $W_0 = 5$, on elliptic orbits; (ii) $W_0 = 7$, on circular orbits. As in Fig. 3.1, models with NS kicks are denoted by red lines (right to left: 128kK7 and 128kKe), and without NS kicks by blue lines (right to left: 128kN7 and 128kNe). Dashed green lines show the corresponding models (without kicks) from BM03.

kicks. Nonetheless, after only a few Gyr, the value of the central mean mass starts to increase, reflecting the process of mass segregation. For models that reach core collapse, the mean mass in the final stages of evolution falls within the range $1.2 \leq \langle m \rangle \leq 1.4M_\odot$, which indicates the dominance of neutron stars in the central regions of the system. Not surprisingly, the rapidly dissolving model 128kK (Fig. 3.3, red line) shows a final mean mass which is comparable to the initial value.

3.3.2 Detailed comparison with Baumgardt & Makino (2003)

Despite the efforts in reproducing the initial conditions and the numerical set-up described by BM03, we note that there are still non-negligible discrepancies between our models without NS natal kicks and the corresponding ones in their original investigation (see Table 3.1 and Figs. 3.1 and 3.2). I have attempted to identify the main reasons for these discrepancies in the intrinsic differences between the $N$-body codes used to perform the simulations, and in particular slightly different stellar evolution prescriptions.

I performed the simulations by using the GPU version of NBODY6 (Nitadori and Aarseth, 2012), while BM03 used the public GRAPE-6 version of NBODY4 (Aarseth, 1999). The latter treats components of binaries as single stars, without collisions or exchange of mass, and the resulting differences might partially explain the increasing discrepancy after core collapse for the models depicted in Fig. 3.2, because of the increase in the number of binaries at this time. Moreover, BM03 used a prescription for the properties of stellar remnants by Hurley et al. (2000), while in NBODY6 the Eldridge and Tout
Figure 3.3: Evolution of the mean mass of the stars in the innermost langrangian shell, containing 1% of the bound mass, evaluated for all models with N=128k. The vertical arrows mark the moment of core collapse (in the five models which exhibit core collapse).

(2004) recipe is now used. To test this, I carried out a simulation of model 128kN with the Hurley et al. (2000) prescription for stellar remnants, but I obtained a dissolution time of $T_{\text{diss}} = 23.0$ Gyr, which reduces the discrepancy by only about 30% (see data for model 128kN in Table 3.1).

To assess stochastic effects (such as run-to-run variations) I also performed additional simulations of models 128kN and 64kN by evolving different numerical realizations of the same initial conditions, and by evolving the same realization in several independent simulations (as in BM03). Finally, I performed a simulation of model 128kN in which the escapers were progressively removed (as in BM03), but again without any significant difference ($T_{\text{diss}} = 22.9$ Gyr).

None of these effects was able, individually, to account for the observed discrepancy. Therefore, I believe that the small but systematic discrepancy between the models without NS kicks and the corresponding ones in BM03 results from a combination of all the effects mentioned above, and others which I have not studied, including possible differences in the way in which models are virialised and scaled in different codes. As I shall show later (Section 3.4.1) the sensitivity of these runs to small effects is such that apparently trivial differences could have significant effects.
3.4 Discussion

3.4.1 Two modes of star cluster dissolution

I have found that the presence or absence of neutron star kicks, in the models I have studied, can change the lifetime of a star cluster by a large factor. I shall now try to interpret the results in the context of previous studies of star cluster dissolution mechanisms, with the aim of understanding why it is that a process which affects such a small fraction of the mass can have such a dramatic effect.

I consider initially tidally filling, multi-mass models with stellar evolution. Over the years, several numerical investigations have shown that the dissolution time is strongly affected by two factors: the initial relaxation time and the initial concentration (represented by the King parameter $W_0$). In particular Chernoff and Weinberg (1990) showed that, for a Salpeter-like IMF, their models with $W_0 = 1$ or 3 dissolved quickly, in less than a Gyr, and without core collapse, while models with $W_0 = 7$ all entered core collapse, after about 10 Gyr or longer. Clusters with $W_0 = 3$ and a steeper IMF (and hence a longer time scale for mass loss by stellar evolution) could enter core collapse before dissolution, provided that relaxation was fast enough. Thus there is a tension between the time scales of stellar evolution and relaxation, which plays out differently depending on the concentration.

Recently Whitehead et al. (2013) noted that the clusters which dissolve by the effects of stellar evolution lose their mass in a qualitatively different way from those dominated by relaxation.

The former, as they approach dissolution, lose the last fraction of their mass (which may be substantial) extremely rapidly, whereas the latter lose mass at a rate which is steady, and sometimes even declining. Whitehead et al. also noted that the dividing line between the two modes of dissolution is quite sharp. For that reason it would not be surprising if a very small effect, such as the loss or retention of NS, were to place a cluster in one mode of dissolution or the other.

The two kinds of behaviour described by Whitehead et al. (2013) are plainly visible in several previous studies of star cluster evolution, such as Takahashi and Portegies Zwart (2000), and they are visible in Fig. 3.1 of the present Chapter, where all models with kicks end their evolution by losing mass precipitately (except for the case $N = 8k$), whereas the others lose mass at a more moderate rate. I refer to these two cases as “jumping” and “skiing”, respectively. Fig. 3.1 also illustrates the point made by Chernoff and Weinberg (1990), i.e. the two modes of dissolution are characterised by the presence or absence of core collapse before dissolution. Indeed I see that the clusters with and without natal kicks (except for the case $N = 8k$) lie on either side of the divide between the two modes.

In order to visualise the transition between skiing and jumping models, it has been particularly instructive for us to take on the point of view first suggested by Weinberg (1993), and to explore the evolution of the models in the plane defined by the concentration (parameterized by $c = \log(r_J/r_c)$, where $r_c$ is the core radius) and the mass which remains bound to the system. In this representation, a system which experiences exclusively stel-

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2This unconventional terminology was coined by Simon Portegies Zwart in conversation with one of us (DCH) several years ago. It vividly conveys the difference between skiing down a gentle slope and jumping off a cliff. I note that Whitehead et al. (2013) have conflated the two terms, with a different semantics.
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Figure 3.4: The plot shows the total mass remaining in the cluster as a function of concentration, for the models depicted in Fig. 3.1 with (red lines) and without (blue lines) NS kicks. The black dots show the moment of core collapse, the black horizontal dashed line at $M/M_0 = 0.05$ marks the formal dissolution condition, and the black vertical dashed line denotes “Weinberg’s cliff” (see Fig. 3.3 in Weinberg, 1993).

Lar evolution effects would gradually lose mass, while reducing its concentration due to the progressive expansion, giving rise to a track moving down in the plane and to the left (see Fig. 3.3 in Weinberg, 1993). The tracks qualitatively resemble those of some of the models, as shown in Fig. 3.4. These are four of the models with kicks, shown in red; and one of these (128kK) is also shown in Fig. 3.5.

In Weinberg’s treatment, dealing with the slow evolution of spherical equilibrium models, the tracks end when equilibrium is no longer possible; the tracks end at points along a curve, which is shown as a dashed near-vertical curve in these figures. $N$-body models can cross this curve, but then lose mass on a dynamical time scale, explaining the jumping profile of the corresponding curves in Figs. 3.1 and 3.2. Though its precise position may differ slightly when the simplifying assumptions of Weinberg’s models are relaxed, I refer to this curve as “Weinberg’s cliff”.

In Weinberg’s models, mass-loss is driven by stellar evolution only, and his results should be applicable when this process dominates. When the effects of two-body relaxation are dominant, one of the natural consequences is the progressive increase of the central concentration, leading to core collapse. This results in a track oriented to the right-hand-side of the plane, behaviour which can be immediately recognised in the remaining models in Fig. 3.4 and 3.5. It should not come as a surprise now that all long-lived, “skiding” models show signatures of core collapse, in contrast with short-lived, “jumping” models.

These figures strongly suggest the existence of trajectories in which the two processes, stellar evolution and relaxation, are in a delicate balance overall, even though stellar evo-
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Evidently, the models that I have studied lie close to the separatrix dividing jumping models (which are dominated by stellar evolution, lose mass rapidly at the end of their lives, and do not reach core collapse) and skiing models (which are dominated by two-body relaxation, lose mass gently towards the end of their lives, and reach core collapse). If neutron stars are given no natal kick, as in model 128kN, or the models of Baumgardt and Makino (2003), the trend to mass segregation and core collapse is accentuated, and the model moves across the separatrix into the domain of relaxation-dominated evolution. But I warn the reader against interpreting this as a general rule. Kicks were applied to both model 8kK and model 16kK (the innermost pair of red lines in Fig. 3.4), and they lie on opposite sides of the separatrix. The 8kK model, because of the low particle number and consequently smaller relaxation time, is sufficiently dominated by relaxation to lie in the skiing regime.

These considerations do not immediately explain, however, why the lifetime should be so different as a factor of nearly four. But the example of models 32kK and 8kN, which lose mass in almost the same way until core collapse in the latter model (Fig. 3.1), shows that the effects of skiing and jumping lead to different lifetimes. Though the difference is only a factor 1.13 in this case, it seems plausible that the effect could be much bigger if the event which determines the mode of dissolution occurred very early in the lifetime of
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a model, e.g. the ejection of neutron stars. Furthermore, because the models lie so close to the separatrix between the two modes, it would not be surprising if very minor systematic differences in the initial conditions were to lead to significant systematic differences in the lifetime, as discussed in Section 3.3.2.

While this Chapter has focused on kicks by neutron stars, the lesson to learn is that apparently minor changes can have very large effects, especially for clusters close to the transition between different modes of dissolution. Other factors which should be taken into account include the presence and properties of primordial binaries, variations in the high-mass end of the IMF, and the degree of primordial mass segregation, which influences both the importance of mass loss by stellar evolution and the role of remnants, not only NS but also stellar-mass black holes. The importance of these factors depends on the location of the dividing line between the two modes of dissolution that I have discussed, which can be assessed only by means of appropriate numerical experiments.

3.4.2 Conclusions

I have presented evidence, based on $N$-body simulations of the evolution of initially tidally filling King models with stellar evolution, that the presence or absence of NS natal velocity kicks can play a crucial role in the long-term survival of model star clusters. In particular I show that some of the basic models in the landmark study of Baumgardt and Makino (2003) are especially sensitive to this effect, which can change their lifetime by almost a factor of four. I explain this finding by showing that the models lie close to a dividing line between (i) models which are dominated by the effects of mass-loss from stellar evolution, and whose evolution ends with a steepening rate of mass loss, and (ii) models whose dynamical evolution is dominated by two-body relaxation, which reach core collapse before dissolving, and do so with a gently decreasing rate of mass loss.
Chapter 4

Probing dark matter with star clusters: a dark matter core in the ultra-faint dwarf Eridanus II

I present a new technique to probe the central dark matter (DM) density profile of galaxies that harnesses both the survival and observed properties of star clusters. As a first application, I apply this method to the ‘ultra-faint’ dwarf Eridanus II (Eri II) that has a lone star cluster ∼ 45 pc from its centre. Using a grid of collisional N-body simulations, incorporating the effects of stellar evolution, external tides and dynamical friction, I show that a DM core for Eri II naturally reproduces the size, radial light profile and projected position of its star cluster. By contrast, a dense cusped galaxy requires the cluster to lie implausibly far from the centre of Eri II (> 1 kpc), with a high inclination orbit that must be observed at a particular orbital phase. These results imply that either a cold DM cusp was ‘heated up’ at the centre of Eri II by bursty star formation, or we are seeing the first unambiguous evidence for physics beyond cold DM.


4.1 Introduction

The Λ ‘Cold Dark Matter’ (ΛCDM) model gives a remarkable match to the growth of structure on large scales in the Universe (e.g. Tegmark and Zaldarriaga, 2002; Planck Collaboration et al., 2014b). Yet on smaller scales, inside galaxy groups and galaxies, there have been long-standing tensions (e.g. Klypin et al., 1999; Moore et al., 1999). Key amongst these is the ‘cusp-core’ problem. Pure dark matter (DM) simulations of structure formation in a ΛCDM cosmology predict that galaxies should reside within dense central DM cusps with density $\rho \propto r^{-1}$ (e.g. Dubinski and Carlberg, 1991; Navarro et al., 1996b) whereas observations of the rotation curves of dwarf galaxies have long favoured constant density cores (e.g. Flores and Primack, 1994; Moore, 1994; Read et al., 2017). This may owe to physics beyond CDM, for example self-interacting DM (SIDM; e.g. Spergel and Steinhardt 2000; Kaplinghat et al. 2016), wave-like DM (e.g. Schive et al., 2014)
or ultra-light axions (e.g. González-Morales et al., 2017). However, all of these small-scale tensions with ΛCDM arise when comparing models devoid of ‘baryons’ (stars and gas) with real galaxies in the Universe. There is mounting evidence that bursty star formation during galaxy formation can ‘heat-up’ DM, transforming a DM cusp to a core (e.g. Navarro et al. 1996a; Read and Gilmore 2005; Pontzen and Governato 2012, 2014; Pontzen et al. 2015). The latest simulations, that reach a mass and spatial resolution sufficient to resolve the multiphase interstellar medium, find that DM cores, of approximately the half stellar mass radius in size ($R_{1/2}$), form slowly over a Hubble time (Mashchenko et al., 2008; Governato et al., 2010; Pontzen and Governato, 2012; Teyssier et al., 2013; Madau et al., 2014; Di Cintio et al., 2014; Oñorbe et al., 2015; Tollet et al., 2016; Read et al., 2016; Munshi et al., 2017).

While the above simulations agree on the size and formation timescale of DM cores, there remains some disagreement over the DM halo mass at which cusp-core transformations become inefficient, $M_{200} = M_{\text{pristine}}$. As pointed out by Peñarrubia et al. (2012), for a fixed $M_{200}$ if too few stars form then there will no longer be enough integrated supernova energy to unbind the DM cusp. Depending on the numerical scheme employed, $M_{\text{pristine}}$ has been reported to be as high as $M_{\text{pristine}} \sim 10^{10} \, M_\odot$ (e.g. Chan et al., 2015; Tollet et al., 2016) and as low $M_{\text{pristine}} \sim 10^8 \, M_\odot$ (R16), where the spread owes primarily to different star formation efficiencies in low mass halos (see Section 4.2.2 and Figure 4.1).

The above motivates measuring the central DM density of the very faintest galaxies in the Universe. With little star formation, these may be expected to retain their ‘pristine’ DM cusps (e.g. R16). There is no shortage of such faint dwarf galaxies orbiting the Milky Way, Andromeda and nearby systems (e.g. Belokurov et al., 2007; Collins et al., 2014; Bechtol et al., 2015; Sand et al., 2015). However, most of these are devoid of gas and so the kinematics of their stars must be used to probe their DM halos. This is challenging because of a strong degeneracy between their DM density profiles and the orbit distribution of their stars (e.g. Merrifield and Kent, 1990; Evans et al., 2009; Read and Steger, 2017). For the brighter Milky Way dwarfs, this degeneracy can be broken by using metallicity or colour to split the stars into distinct components with different scale lengths (e.g. Battaglia et al. 2008; Walker and Peñarrubia 2011 and Agnello and Evans 2012, but see Breddels and Helmi 2013 and Richardson and Fairbairn 2014). However, for the fainter dwarfs there are too few stars to obtain strong constraints (Read and Steger, 2017).

An alternative method for probing the central density of dwarf galaxies was proposed by Hernandez and Gilmore (1998), Goerdt et al. (2006) and Sánchez-Salcedo et al. (2006). They showed that the globular clusters (GCs) in the dwarf spheroidal galaxy Fornax would rapidly sink to the centre by dynamical friction if Fornax has a steep DM cusp. By contrast, in a constant density core, dynamical friction is suppressed (Read et al., 2006a; Inoue, 2009, 2011; Petts et al., 2015, 2016), allowing Fornax’s GCs to survive through to the present day. This ‘timing argument’ was refined by Cole et al. (2012) who used 2800 $N$-body simulations of Fornax’s GC system to show that a core is favoured over a cusp, in excellent agreement with split-population modelling of Fornax’s stars (e.g. Walker and Peñarrubia, 2011). (Such survival arguments were extended by Peñarrubia et al. (2009) to the GCs associated with the Sagittarius dwarf.) While it is likely that Fornax has a DM core,}{\footnote{$M_{200}$ is the virial mass. For satellite galaxies, I define this pre-infall.}}
core, its stellar mass ($M_\star \sim 4 \times 10^7 \, M_\odot$; de Boer et al. 2012) is large enough for bursty star formation to drive complete cusp-core transformations (Peñarrubia et al. 2012; R16). Thus, Fornax’s core yields inconclusive constraints on the nature of DM.

In this Chapter, I develop a new method for probing the central DM density of dwarf galaxies that harnesses both the survival and present-day properties of star clusters. Star clusters are dense stellar systems that slowly expand due to two-body relaxation (Hénon, 1965; Gieles et al., 2010). In a tidal field, high-energy stars are pushed over the cluster’s tidal boundary, slowing down the expansion. Eventually, the cluster’s half stellar mass radius becomes a constant fraction of the tidal radius and, from that moment on, the cluster evolves at a constant density set by the tidal field (Hénon, 1961; Gieles et al., 2011). Thus, the observed surface density of low-mass GCs (i.e. those that have undergone sufficient relaxation) can be used as probes of the host galaxy’s tidal field and, therefore, its density distribution (Innanen et al., 1983). This allows us to probe the DM distribution in any dwarf galaxy with low-mass star clusters, including those with a much lower stellar mass than Fornax. This is the key idea that I exploit in this work\(^2\).

To model star clusters sinking in the potential of a host dwarf galaxy, I make use of the semi-analytic dynamical friction model from Petts et al. (2016) (hereafter P16), implemented in the direct-summation code \textsc{nbbody6} (Aarseth, 2003). This allows us to model the survival of star clusters, similarly to Cole et al. (2012), but with a complete \textit{N}-body model of the star cluster itself, including two-body effects, binary formation and evolution and stellar evolution. By comparing a large grid of such models with observational data, we are able to constrain the DM density of dwarf galaxies that host low mass GCs, independently of timing arguments or stellar kinematic measurements.

As a first application, I apply this method to the ultra-faint dwarf galaxy Eridanus II (Eri II) that was recently discovered by the Dark Energy Survey (DES; Bechtol et al. 2015; Koposov et al. 2015). Eri II is situated 366 kpc from the Sun, at the edge of the MW, with $M_V = -7.1$, a half-light radius of $R_{1/2} = 2.31''$, and an ellipticity of 0.48. Eri II appears to show an extended star formation history, but follow-up observations are needed to confirm this. Koposov et al. (2015) and Crnojević et al. (2016) found that Eri II has a lone star cluster at a projected distance $\sim 45$ pc from Eri II’s centre, with $M_V = -3.5$ and a half-light radius of 13 pc (see Crnojević et al., 2016, Table 1). Compared to the MW’s star clusters (Harris, 1996, 2010 edition), Eri II’s star cluster appears faint and extended, contributing $\sim 4\%$ of Eri II’s total luminosity.

This Chapter is organised as follows. In Section 4.2, I describe the method for probing the central DM density of dwarf galaxies using star clusters, and I motivate the priors for modelling Eri II. In Section 4.3, I present the main findings. In Section 4.4, I discuss the implications of the results for galaxy formation and the nature of DM. Finally, in Section 4.5, I present the conclusions.

\(^2\)Note that the cluster’s stellar kinematics are also affected by tides, making them additional probes of the properties of the galactic tidal field (e.g. Küpper et al., 2010; Claydon et al., 2017).
4.2 Method

4.2.1 A new method for measuring the inner DM density of dwarf galaxies

I model the evolution of star clusters orbiting within a host dwarf galaxy using NBODY6DF (P16). This is a publicly available adaptation of NBODY6, which is a fourth-order Hermite integrator with an Ahmad and Cohen (1973) neighbour scheme (Makino and Aarseth, 1992; Aarseth, 1999, 2003), and force calculations that are accelerated by Graphics Processing Units (GPUs, Nitadori and Aarseth, 2012). NBODY6 contains metallicity dependent prescriptions for the evolution of individual stars and binary stars (Hurley et al., 2000, 2002) that I use in the simulations here.

In NBODY6DF, I model the host dwarf galaxy as a static, analytic potential. Dynamical friction is then applied to star cluster members using the semi-analytic model described in P16 (see also Petts et al., 2015). The P16 model has been extensively tested against full N-body simulations of dynamical friction in both cored and cusped background potentials, giving an excellent description of the orbital decay in both cases. In particular, it is able to reproduce the ‘core-stalling’ behaviour, whereby dynamical friction is suppressed inside constant density cores (Goerdt et al. 2006; Read et al. 2006a; Inoue 2009, 2011 and see P16 for further details).

I set up a grid of 200 NBODY6DF simulations, varying the density profile (cusped or cored) and the initial orbit and properties of the star cluster. Comparing this grid with observations, I determine the most likely mass distribution for Eri II, and the initial properties of its star cluster. (Eri II is dominated at all radii by its dark matter halo (see Section 4.2.2) and so its total mass distribution directly provides us with its DM density profile.) In addition, I run a further 26 simulations to determine how the results depend on the mass, concentration and inner logarithmic slope of Eri II’s dark matter halo (Section 4.3.4). I also test whether Eri II’s cluster could form and survive in the very centre of Eri II (Section 4.3.5).

4.2.2 The DM halo of Eri II

I model the DM halo of Eri II using the coreNFW profile from R16. This is described by a mass $M_{200}$ and concentration parameter $c_{200}$, identical to those used for the cusped Navarro-Frenk-White (NFW) profile (Navarro et al., 1996b). However, it allows also for a central DM core. By default, this has a size set by the projected half light radius of the stars $R_{1/2}$, which for Eri II is $R_{1/2} = 0.28$ kpc (Crnojević et al., 2016). The power-law slope of the core is set by $n$, where $n = 1$ produces a flat dark matter core, while $n = 0$ returns the fully cusped NFW profile\(^5\).

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\(^3\)http://github.com/JamesAPetts/NBODY6df.

\(^4\)The latest estimates of $R_{1/2}$ are slightly lower than the value I have assumed here, $R_{1/2} = 246 \pm 13$ pc (Denija Crnojević, private communication), though within the 2σ uncertainties. However, as I show in Section 4.3.4, we are not very sensitive to $M_{200}$, $c_{200}$ or the DM core size. As such, the newer value for $R_{1/2}$ will not affect the results.

\(^5\)In R16, $n$ was parameterised by the total star formation time. However, since this is poorly determine for Eri II, I consider here just a range of values for $n$. I discuss this further in Section 4.4.
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Figure 4.1: The stellar mass-halo mass relation of isolated dwarf galaxies. The symbols correspond to the observational data and the squares to the results from \( N \)-body simulations. The blue solid lines show the results from abundance matching in \( \Lambda \)CDM using the SDSS field stellar mass function (where the lines are dashed, the results are extrapolated). Eri II is marked by the red star. It appears to be consistent with a ‘failed’ Leo T, inhabiting a similar DM halo but having its star formation shut down earlier, lowering its \( M_* \) for the same pre-infall \( M_{200} \).

Eri II’s stellar population may be expected to be like other similar stellar mass dwarf galaxies (e.g. Bootes I and Ursa Major I; see McConnachie 2012) which have predominantly old-age stars. Santana et al. (2013) showed that the apparent intermediate-age population in these galaxies is likely due to the presence of blue straggler stars. However, they could not rule out the presence of an intermediate-age population of up to 3 Gyr old. Recent HST data (propID 14234; subject of a future publication), has confirmed that Eri II’s stellar population is similar to those studied by Santana et al. (2013), hence favoring an older population. In this work, I choose to be conservative and assume that the cluster is older than 5 Gyr.

To obtain an estimate of the (pre-infall) halo mass, \( M_{200} \), for Eri II I use the recent measurement of its mass within the 3D half light radius \( M_{1/2} = 1.2^{+0.4}_{-0.3} \times 10^7 \, M_\odot \), derived from stellar kinematics by Li et al. (2017). I turn this into an \( M_{200} \) by fitting an NFW profile to \( M_{1/2} \) using the \( M_{200} - c_{200} \) relation from Macciò et al. (2007), finding \( M_{200} = 4.7^{+6.9}_{-2.6} \times 10^8 \, M_\odot \).
To test if the above value for $M_{200}$ is reasonable, in Fig. 4.1 I compare the stellar mass ($M_\ast \approx 8 \times 10^4 M_\odot$; Bechtol et al. 2015) and $M_{200}$ for Eri II with measurements for other nearby dwarfs; Eri II is marked by the red star. The purple circles show the $M_\ast - M_{200}$ relation for isolated gas rich dwarfs from Read et al. (2017). The dark cyan triangle shows a measurement for the Carina dwarf spheroidal galaxy from Ural et al. (2015). The light cyan diamond shows an estimate for the isolated dIrr Leo T from Read et al. (2017). The blue solid and dashed lines show the $M_\ast - M_{200}$ relation derived by Read et al. (2017) from abundance matching in $\Lambda$CDM using the SDSS field stellar mass function (the dashed lines show where this is extrapolated). The remaining data points show the latest results from a range of simulations of isolated dwarfs taken from the literature: R16 (magenta); Wang et al. (2015) (green); Chan et al. (2015) (yellow); Wheeler et al. (2015) (blue); and Fitts et al. (2017) (pink). As can be seen, there is a clear discrepancy between most simulations and the data below $M_{200} \sim 10^{10} M_\odot$ that remains to be understood. However, this plot demonstrates that the derived $M_{200}$ for Eri II is in good agreement with estimates for other galaxies of a similar stellar mass. Eri II is consistent with a ‘failed’ Leo T, inhabiting a similar DM halo but having its star formation shut down earlier, lowering its $M_\ast$ for the same pre-infall $M_{200}$. This is further evidenced by the lack of detected HI gas, or recent star formation, in Eri II (Crnojević et al., 2016).

In Fig. 4.2, I show the cumulative mass profiles (left panel) and DM density profiles (right panel) for Eri II that I assume in the fiducial grid of simulations (I explore different halo masses, concentrations and logarithmic cusp slopes in Section 4.3.4). The grey lines show the cusped model, the green lines show the cored model. The middle of the three lines shows $M_{200} = 5 \times 10^8 M_\odot$ that I assume from here on. The top and bottom lines show the upper and lower boundary of $M_{200}$ estimated from the kinematic measurements (Li et al., 2017). The projected half light radius of the stars, $R_{1/2}$, is marked by the vertical blue line. On the right panel, the measurement of $M_{1/2}$ for Eri II from Li et al. (2017) is marked by the red data point. As can be seen, due to the $M_{200} - c_{200}$ relation, changing $M_{200}$ produces only a small effect on the DM density within $R_{1/2}$. Thus, this method will not be very sensitive to $M_{200}$ (I will verify this expectation in Section 4.3.4). However, cusped and cored models look very different within $R_{1/2}$ and this is what I aim to probe in this Chapter.

Finally, the latest version of NBODY6DF only supports a background DM density profile modelled by Dehnen spheres (Dehnen, 1993):

$$\rho(r) = \frac{M_0(3-\gamma)}{4\pi r_0^3} \left( \frac{r}{r_0} \right)^{-\gamma} \left( 1 + \frac{r}{r_0} \right)^{-\gamma-4},$$

where $M_0$ and $r_0$ are the mass and scale length, respectively, and $-\gamma$ is the logarithmic slope of the inner density profile.

Thus, to obtain DM profiles suitable for NBODY6DF, I fit the above Dehnen profile to a coreNFW density profiles. These fits are shown by the dashed lines in the right panel of Fig. 4.2. As can be seen, inside $R_{1/2}$ (the region of interest), these fits are excellent. The best-fit parameters for the cored ($\gamma = 0$, which corresponds to $n = 0.9$ for the coreNFW profile) and cusped ($\gamma = 1$, which corresponds to $n = 0$) models were: $M_0 = 4.79 \times 10^8 M_\odot$ and $r_0 = 0.877$ kpc and $M_0 = 2.94 \times 10^8 M_\odot$ and $r_0 = 1.078$ kpc, respectively.
Figure 4.2: DM halo models for Eri II, chosen as described in Section 4.2.2. The left panel shows the cumulative mass profiles; the right panel the logarithmic density profiles. The mass within the projected stellar half light radius $M_{1/2}$ is marked by the red data point with error bars on the left panel (taken from Li et al. 2017). The dashed black and green lines show the cusped and cored models explored in this work, respectively. These are the best-fit Dehnen profile models to the dwarf galaxy models marked by the solid black and green lines, respectively. For the fiducial grid of simulations, I assume Eri II inhabits DM halo with a virial mass of $M_{200} = 5 \times 10^8 M_\odot$. Models with a halo mass at the 68% upper and lower bound of Eri-II’s stellar kinematics are marked by the upper and lower gray and green lines. For the core profiles, I used the same halo mass as the cusp profiles with $n = 0.9$ (corresponding to a Dehnen model with $\gamma = 0$; dashed green line). The vertical blue line on both panels marks the projected half light radius of Eri II, $R_{1/2}$. I explore the effect of varying $M_{200}$, $\sigma_{200}$ and the inner logarithmic cusp slope of Eri II’s DM halo in Section 4.3.4.
4.2. METHOD

4.2.3 Eri II’s star cluster

I model the initial conditions of Eri II’s star cluster as a Plummer sphere (Plummer, 1911) with a Kroupa IMF (Kroupa, 2001), sampling stars with masses between $0.1 \, M_\odot$ and $100 \, M_\odot$ and assuming a metallicity of $Z = 0.0008$ (corresponding to $[\text{Fe}/\text{H}] \simeq -1.5$). I assumed a range of initial masses $M_{\text{cl},0}$ and half-mass radii $r_{\text{hm},0}$ for the cluster to explore how its initial properties impact its final state.

4.2.4 Exploring parameter space

To explore the parameter space, I ran 200 simulations, 100 for each galaxy model (core and cusp). I varied $r_{\text{hm},0}$, $M_{\text{cl},0}$ and the initial galactocentric distance ($R_{\text{g},0}$) of the cluster. I allowed the cluster to have a $r_{\text{hm},0}$ of 1, 5, 10, 15 and 20 pc; a $M_{\text{cl},0}$ of approximately 13,000, 19,000, 25,000 and 32,000 $M_\odot$; and $R_{\text{g},0}$ of 0.14, 0.28, 0.56, 1.12 and 2.8 kpc. The $r_{\text{hm},0}$ range is based on what is found for young massive clusters (Portegies Zwart et al., 2010). The minimum $M_{\text{cl},0}$ is chosen such that after stellar mass loss the mass is always above the mass of the cluster. The maximum mass was chosen such that less than 20% of all stars in the entire galaxy originated from the star cluster, which is a reasonable upper limit (Larsen et al., 2012, 2014).

For all clusters I adopted circular orbits. This favours the survival of clusters in cusped profiles because eccentric orbits reach closer to the centre of the galaxy where clusters are less likely to survive. (The assumption that the host dwarf galaxy has a spherical potential similarly favours a cusped profile because in triaxial models there are no circular orbits and only the more damaging radial orbits are allowed.)

Contenta et al. 2017b show how the observations of faint star clusters – like Eri II’s star cluster – can be affected by primordial binaries and the retention fraction of black holes, together with observational biases. In the simulations I did not vary these aspects, nor did I vary the initial density profile of the clusters. However, I expect that these aspects affect the evolution of the clusters in the cusped and cored models in the same way.

Eri II’s star cluster is observed at a projected distance $D_{\text{cl}} = 45$ pc from the centre of Eri II. Thus, I also need to take into account the probability for the cluster to be observed at that radius in the total likelihood. I estimate the probability $P(D_{\text{g}} < D_{\text{cl}}^\text{cl} | R_{\text{g}})$ to observe a cluster (on a circular orbit) within $D_{\text{cl}}^\text{cl}$ for a given $R_{\text{g}}$, assuming a random inclination of the orbital plane with respect to the observer. To compute $P(D_{\text{g}} < D_{\text{cl}}^\text{cl} | R_{\text{g}})$, firstly I estimate the angle $\varphi(i, R_{\text{g}})$, which defines the angle in which the cluster is observed to be within $D_{\text{cl}}^\text{cl}$ during 1/4 of an orbit (see Fig. 4.3), where $i \in [0, \pi/2]$ is the angle between the pole of the orbit and the line of sight. For circular orbits, the angle $\varphi(i, R_{\text{g}})$ is given by

$$\varphi(i, R_{\text{g}}) = \begin{cases} \pi/2 - \arcsin \left( \frac{\sqrt{1 - D_{\text{cl}}^\text{cl}/R_{\text{g}}}}{\sin i} \right), & R_{\text{g}} > D_{\text{cl}}^\text{cl}, \\ \pi/2, & R_{\text{g}} < D_{\text{cl}}^\text{cl}. \end{cases}$$

Secondly, I integrate $\varphi(i, R_{\text{g}})$ with respect to $\cos i$, because for random inclinations of the orbital plane, $\cos i$ is uniformly distributed. I then divide by a normalization angle, $\pi/2$, because $\varphi(i, R_{\text{g}})$ considers only 1/4 of an orbit, see Fig. 4.3 and I obtain
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Figure 4.3: Schematic representation of two projected orbits with different inclinations of their orbital plane $i$ (solid black lines). Eri II’s cluster is observed to be 45 pc from the centre of Eri II in projection (dashed red line). I consider that a cluster can be observed as Eri II’s star cluster if its orbit is within 45 pc (solid red line). For a given $i$, a larger distance from the centre results in a smaller $\varphi(i, R_g)$ (see equation 4.2.2). Therefore, clusters that orbit in the outskirts of Eri II are unlikely to be observed near the centre.

\[ P(D_g < D_g^{cl} | R_g) = \frac{2}{\pi} \int_0^1 \varphi(i, R_g) \, \mathrm{d} \cos i. \]  

(4.2.3)

By definition, $0 \leq P(D_g < D_g^{cl} | R_g) \leq 1$.

To compare the $N$-body simulations with the observational data, I assumed a stellar mass-to-light ratio of $M/L_V = 2$, appropriate for old, metal-poor stellar populations (e.g. McLaughlin and van der Marel, 2005), obtaining $M_{\text{ob}} = 4.3 \times 10^3 M_\odot$. I multiply the observed half-light radius ($r_{\text{hl}}$) by 4/3 to correct for projection effects (Spitzer, 1987), to get an estimate for the 3D half-mass radius $r_{\text{hm}} = 17.3 \text{pc}^6$ of Eri II’s star cluster. To find the model that best fit the observational data, I maximise the likelihood for the fitting parameters ($r_{\text{hm,0}}$, $M_{\text{cl,0}}$, and $R_{g,0}$). The log-likelihood function is:

\[ \ln L = -\frac{(r_{\text{hm,ob}} - r_{\text{hm}})^2}{2\sigma_r^2} - \frac{\log^2 (M_{\text{ob}}/M_{\text{cl}})}{2\sigma_{\log(M)}^2} + \ln P(D_g < D_g^{cl} | R_g), \]

(4.2.4)

where $\sigma_r = 1.33 \text{ pc}$ and $\sigma_{\log(M)} = 0.24^7$ are the uncertainties derived from the observation (Crnojević et al., 2016). The last term in the equation above is given by equation (4.2.3) and acts as a prior to the likelihood given that it penalises models that are less likely to be observed simply due to geometrical constraints.

By computing $r_{\text{hm}}$, $M_{\text{cl}}$ (defined as the sum of the mass of all stars within the tidal radius of the cluster) and $R_g$, I calculate the likelihood (equation 4.2.4) for each output time of the simulation.

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6 Assuming that light traces mass, which is not necessarily true if the cluster is mass segregated.

7 $\sigma_r$ is the uncertainties on the 3D half-mass radius; while $\sigma_{\log(M)}$ is estimated assuming $M/L_V = 2$. 

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4.3. RESULTS

4.2.5 Estimation of the number density profile

To study the structural properties of the clusters in the $N$-body simulations, I used a maximum likelihood fit following the procedure described in Martin et al. (2008). I model the stellar distribution of the clusters using 2D elliptical Plummer and spherical, single-component, King models (King, 1966). I fit the models with a Monte Carlo Markov Chain (MCMC) method (EMCEE code, Foreman-Mackey et al., 2013) to optimize the following parameters: projected half-number radius, ellipticity, position angle, and surface density background for the Plummer models; and half-number radius, central dimensionless potential and background surface density for the King models.

4.3 Results

4.3.1 Cusp vs. core

The main result of this investigation is that the presence of a cored DM profile in Eri II allows a star cluster to not only survive in the centre of the galaxy, but also to expand up to $r_{hm} \simeq 17$ pc (i.e. $r_{hl} \simeq 13$ pc), in excellent agreement with observations of Eri II’s lone star cluster. By contrast, a cusped DM profile gives a poorer fit overall and requires special conditions that have to be satisfied. In Fig. 4.4, I show a schematic representation of a simulated cluster in a cusped galaxy (on the left) and a cored galaxy (on the right) at times when they best reproduce the observations (shown in the middle). As can be seen, in the cored galaxy (right), the star cluster (green) stalls at a radius $\sim 45$ pc from Eri II’s centre (see the zoomed image in the blue circle that shows its orbital decay and stalling in red). In this case, no special inclination or time are required to reproduce Eri II’s star cluster. Notice also that the cluster appears visibly extended, similarly to Eri II’s cluster, and that it shows little to no tidal tails, as expected for a cluster orbiting in a constant density core (e.g. Petts et al., 2016). By contrast, in the cusped case (left), the cluster must orbit much farther ($R_g > 1$ kpc) from Eri II’s centre in order to survive. Now its orbit will only be close enough to Eri II in projection when the red circle lies inside the two solid yellow lines. This happens when the cluster orbits with a high inclination ($i > 87.43^\circ$) of the orbital plane, and is in a particular orbital phase (that occurs for $< 3\%$ of the total orbit time). In the cusped galaxy, the cluster is denser than in the cored case and less consistent with the data for Eri II’s cluster. There are also now two visible tidal tails, as expected for a cluster orbiting in a cusped background.

In Fig. 4.5, I show the maximum likelihood of the models as a function of $R_{g,0}$, varying all the other parameters ($M_{cl}$, $r_{hm}$, and $t$), without (left) and with (middle) $P(D_g < D_g^{cl} | R_g)$ in the likelihood; and as a function of time, $t$ (varying all the other parameters and including $P(D_g < D_g^{cl} | R_g)$, right). The shaded grey and green regions show the 68% confidence intervals for the parameters $R_{g,0}$ and $t$ for the cusped (grey) and cored (green) galaxy, respectively. (Assuming that the Wilks’ theorem is valid, I used the likelihood ratio to estimate the confidence intervals (Wilks, 1938); I do not allow the reported confidence interval to be smaller than the distance between two data points.)

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8I used the LIMEPY code (Gieles and Zocchi, 2015) to compute the projected density profiles of King models (https://github.com/mgieles/limepy).
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Figure 4.4: Schematic representation of the cluster orbiting in a cusped Eri II (on the left) and a cored Eri II (on the right) at times when they best reproduce the observations (Crnojević et al. 2016; shown in the middle). On the top right, I show a zoom-in of the simulation in the cored galaxy after 6.3 Gyr. The effect of dynamical friction and stalling can be seen in the red lines that depict the cluster’s orbit. In the cusped case, the cluster can only survive in the outskirts of Eri II, while in the cored case the cluster can survive in the central region where it is much more likely to match the observations.

Figure 4.5: Comparison of the maximum likelihood of the cored and cusped $N$-body models. The left and middle panels show the maximum likelihood as a function of the initial value of $R_g$ ($R_{g,0}$) without (left) and with (middle) $P(D_g < D_g^{\text{cl}}|R_g)$. The right panel shows the maximum likelihood as a function of time, $t$, also with $P(D_g < D_g^{\text{cl}}|R_g)$. The black solid and green dashed lines are for cusp and core models, respectively. The shaded areas show the 68% confidence intervals for $R_{g,0}$ and $t$ for the cored (in green) and cusped (in black) models. The dotted vertical lines show the best-fit values for $R_{g,0}$ and $t$. Notice that the cored models have a higher likelihood than the cusped models, especially once the probability of observing the cluster at its current projected distance, $P(D_g < D_g^{\text{cl}}|R_g)$, is included.
For the clusters in the cored galaxy, the likelihood is bimodal because it is possible to fit the data if the cluster is either in the inner or outer region of the galaxy (see the left panel). For $R_{g,0} = 0.56 \text{kpc}$, the tidal radius of the cluster is close to its minimum and there it is more difficult to increase the cluster’s $r_{hl}$ up to the observed $13 \text{pc}$. This leads to the dip in the likelihood at this point. However, including the probability $P(D_g < D_{cl}^{g} | R_g)$ of observing the cluster at the right position (middle panel) breaks this bimodality, favouring the orbits near the centre with lower $R_{g,0}$.

For the clusters evolving in the cusped galaxy, no star cluster can survive in the inner galaxy $R_{g,0} < R_{1/2}$ for more than $5 \text{Gyr}$ and the likelihood is, therefore, zero for all clusters in that region of parameter space chosen in this study. Considering more massive clusters initially does not necessarily lead to a higher probability of survival, because of the increased importance of dynamical friction. Clusters that orbit outside the scale radius ($R_{g,0} \gtrsim 1 \text{kpc}$) have comparable likelihoods in the cusped and cored models (see left and middle panels) because they are similar by construction at large radii (compare the green and black dashed lines in the right panel of Fig. 4.2). (A measurement of the 3D position of the cluster in Eri II would allow us to completely rule out cusped models. However, to measure a $1 \text{kpc}$ offset from Eri II, an accuracy of 0.006 mag is needed. Even with RR Lyrae, it is only possible at present to reach an accuracy of 0.05 mag.)

The right panel of Fig. 4.5 shows the maximum likelihood at different times. The black line is for clusters in a cusped galaxy, for which the best fits are the models between $6.5$ and $8 \text{Gyr}$ old. The green dashed line is for the cored DM profile, for which the best fits are all models with $t \gtrsim 7 \text{Gyr}$. In the cored galaxy, the best fit models are those that survive in the inner part of the galaxy where they can easily expand up to $\sim 17 \text{pc}$ and survive for $14 \text{Gyr}$. For the cusped galaxy, it is possible to reproduce the observed properties of Eri II star cluster for only a small amount of time and for a small range of $M_{cl,0}$ and $r_{hm,0}$. To reproduce Eri II’s star cluster in a cusped galaxy, it must therefore have an age of $6.5 - 8 \text{Gyr}$. This provides another testable prediction that could fully rule out cusped models.

Finally, even if we accept a high inclination of the orbital plane and the required orbital phase for the cusped case, its star clusters give a poorer fit to the observations than the cored case, because the clusters are not able to expand enough to match the data. I discuss this in more detail, next.

### 4.3.2 Best-fit star cluster models

The best-fit model in the cusped galaxy has $M_{cl,0} \sim 3.2 \times 10^4 \text{M}_\odot$, $r_{hm,0} = 5 \text{pc}$, and $R_{g,0} = 1.12 \text{kpc}$. The best-fit model in the cored galaxy has $M_{cl,0} \sim 1.9 \times 10^4 \text{M}_\odot$, $r_{hm,0} = 10 \text{pc}$, and $R_{g,0} = 0.14 \text{kpc}$. Fig. 4.6 shows the evolution of $M_{cl}$, $r_{hm}$, $R_g$ and $P(D_g < D_{cl}^{g} | R_g)$ for these two models. The red shaded areas show the 68% confidence intervals of the data. (There is only a lower limit for the age of the cluster, because stars younger than $5 \text{Gyr}$ have not been observed; see Section 4.1.) In Fig. 4.6, I show that the properties of the star cluster in a cored DM profile reproduce the properties of the observed star cluster for all times $\gtrsim 5 \text{Gyr}$, while in the cusped case the cluster has to be observed at a specific time. Notice, however, that the cusped model is always at tension with the data, with its size, $r_{hm}$, never quite reaching high enough to match Eri II’s star cluster.
As discussed in Section 4.1, I expect the density of the star cluster to reach an equilibrium due to relaxation-driven expansion and the tidal pruning of high-energy escaper stars. In the left panel of Fig. 4.6, it is possible to see this process happening between 5 and 9 Gyr for the cusped model. Over this period, the cluster evolves at an approximately constant $r_{hm}/r_J$ (Hénon, 1961), where $r_J$ is the ‘Jacobi’ or tidal radius. As a result, the cluster shrinks as $r_{hm} \propto M^{1/3}$ while it loses mass, and it only has a large $r_{hm}$ for a limited time (few Gyr). The cluster in the cored galaxy also expands, but $\gtrsim 5$ Gyr the star cluster evolves at roughly constant $M_{cl}$ and constant $r_{hm}$. This is because the escape rate is very small in compressive tides, and the cluster evolves towards a near isothermal equilibrium configuration, in which the cluster is in virial equilibrium with the tides (Yoon et al. 2011; Bianchini et al. 2015 Webb et al. 2017). This implies that it is more likely to find a cluster in this phase, because it can be in this quasi-equilibrium configuration for a long time ($\gtrsim 10$ Gyr). The asymptotic value of $r_{hm}$ of the cluster in the cored galaxy is in excellent agreement with the data for Eri II’s star cluster (red shaded region). In the $N$-body model the cluster density within $r_{hm}$ evolves to approximately the same value as the (uniform) DM density, hence the cluster density is literally probing the DM density.

4.3.3 Predicted cluster number density profiles

Fig. 4.7 shows the stellar number density profiles of the best-fit models in the cusped (left) and cored (right) case. Crnojević et al. (2016) find the structural parameters of Eri II’s star cluster by fitting a Sersic profile to its surface brightness as measured from integrated photometry. It is possible to similarly derive a surface brightness profile from the $N$-body simulations, however it proved challenging to directly compare the models to the data. Analysing the image from Crnojević et al. (2016), I found that the result is very sensitive
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2.0
1.5
1.0
0.5
0.0 0.5 1.0
log Radius (arcmin)
10^{-4}
10^{-3}
10^{-2}
10^{-1}
10^{0}
10^{1}
N / arcsec^2
Best model in cusped galaxy
Plummer \ r_{pl} = 0.12' (12.51 \pm 0.56 - 0.59 pc)
King \ r_{k} = 0.11' (11.83 \pm 0.39 pc), \ c = 0.83 \pm 0.07

Figure 4.7: (a): Number density profile of the best cusp model at 7.5 Gyr. (b): Number density profile of the best core model at 12 Gyr. The solid red and dashed blue lines indicate the best fit Plummer and King profiles, respectively. The number density of background star is estimated using the number density profile of Eri II (Bechtol et al., 2015). On the right, the number of background stars is higher because the cluster sits in the inner part of Eri II.

to the number of bins used, the subtraction of background sources and which bright stars are masked – all of which can change the result of the fitting. Therefore, for the analysis I used a different approach in which the data from the N-body models are not binned. I used only bright stars (massive stars) that are observable. In this case, I chose only stars that are more massive than 0.75 M_⊙. Furthermore, I included the background stars using the number density profile of Eri II reported in (Bechtol et al., 2015), assuming that the stars are uniformly distributed in the simulated field of view.

As can be seen in Fig. 4.7, a star cluster that evolves in a DM cusp has a different density profile than a star cluster that evolves in a DM core. In the cored galaxy, clusters have a lower concentration parameter, \ c = 0.55 \pm 0.16, compared to the cluster in the cusped galaxy (c = 0.83 \pm 0.07). Here c \equiv \log(r_t/r_0), where r_t is the truncation radius and r_0 is the King/core radius. This means that the core of the cluster is larger (for a given r_{hl}) if it evolves in a DM core. From deeper imaging, it may be possible to derive the projected density profile of the cluster, allowing for a better comparison with the N-body simulations.

4.3.4 The effect of varying the mass, concentration and central logarithmic cusp slope of Eri II’s DM halo

In Section 4.2.2, I showed that for dark matter halos that lie on the \ M_{200} - c_{200} relation, large changes in \ M_{200} produce only a small change in their inner dark matter density. Here, I test whether it is possible to detect such small changes to obtain a constraint on \ M_{200} from the survival and properties of Eri II’s star cluster alone. To test this, I performed an additional ten simulations varying \ M_{200} over the range [1.4 \times 10^8, 10^{11}] M_\odot. (The upper limit of this range is already ruled out by the stellar kinematic measurements for Eri II (Li

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This mass limit was derived from the observational limit reported in Crnojević et al. (2016) using PARSEC’s isochrones v1.2S (Bressan et al., 2012)
et al., 2017), but serves to test the sensitivity to \( M_{200} \).

As anticipated in Section 4.2.2, I found that the results are not sensitive to even large changes in \( M_{200} \) and \( c_{200} \) so long as the Eri II DM halo has a central DM core. By selecting an appropriate \( M_{\text{cl},0} \), \( r_{\text{hm},0} \) and \( R_{g,0} \), I was able in all cases to reproduce the observations within 1\( \sigma \), similarly to the right panel of Fig. 4.6.

I then tested the sensitivity to the inner logarithmic DM density slope at a fixed DM halo mass of \( M_{200} = 10^9 \, M_\odot \), with a concentration set by the \( M_{200} - c_{200} \) relation from Dutton and Macciò (2014). I explored two coreNFW models with \( n = 0.7 \) and \( n = 0.5 \), corresponding to Dehnen models (see equation 4.2.1) with \( (\gamma, M_0, r_0) = (0.25, 8.11 \times 10^8 \, M_\odot, 1.363 \, \text{kpc}) \) and \( (\gamma, M_0, r_0) = (0.5, 9.55 \times 10^8 \, M_\odot, 1.715 \, \text{kpc}) \), respectively. I found that, with an inner density slope of \( \gamma = 0.25 \), I was still able to find star clusters that survive for longer than 5 Gyr and reproduce the observations, in good agreement with recent results from Amorisco (2017). For the galaxy with \( \gamma = 0.5 \), only one cluster (with \( R_{g,0} = 0.14 \, \text{kpc}, r_{\text{hm},0} = 5 \, \text{pc} \) and \( M_{\text{cl},0} \sim 32,000 \, M_\odot \)) survived for more than 5 Gyr. However, this cluster gave a poor match to Eri II’s star cluster since its \( r_{\text{hm}} \) expanded up to 12 pc and then shrank as the cluster’s orbit decayed to the centre of the galaxy. A larger inner density profile slope means a smaller tidal radius for the star cluster and, thus – all other parameters being equal – a shorter dissolution time.

### 4.3.5 A nuclear star cluster in Eri II?

Eri II’s star cluster is offset from the photometric centre of Eri II by \( \sim 45 \, \text{pc} \) (Crnojević et al., 2016). However, given the uncertainties on the photometric centre of Eri II, I consider here the possibility that Eri II’s star cluster is in fact a nuclear star cluster, defining the centre of the galaxy. A star cluster at the centre of a galactic potential will expand due to two-body relaxation, but without the limiting tidal field. Thus, in a cusped galaxy, a cluster that is tidally destroyed close to the centre of the galaxy (for example, the clusters in the cusped models with \( R_{g,0} = 0.14 \, \text{kpc} \); see Section 4.3.2) may still survive if placed at \( R_{g,0} = 0.0 \, \text{kpc} \). As pointed out by Amorisco (2017), this could provide a route to Eri II having a DM cusp without destroying its low density star cluster. To test this, I run six additional simulations of a star cluster with \( M_{\text{cl},0} \sim 25,000 \, M_\odot \) and \( r_{\text{hm},0} \sim 1.5, 10 \) pc, set up to lie at the centre of the cored and cusped Eri II DM halos described in Section 4.2.2.

In Fig. 4.8, I show the evolution of \( r_{\text{hl}} \) for these simulations in a cored (green lines) and a cusped (black lines) DM halo. In the cusped case, for \( r_{\text{hm},0} = 10 \, \text{pc} \), \( r_{\text{hl}} \) does not expand because the cluster is mainly dark matter dominated and so the cluster stars trace the underlying dark matter potential. If Eri II’s star cluster formed in the centre of Eri II, it must have formed with a size similar to that observed today, but with almost double its current mass.\(^{11}\) In Fig. 4.9, I show the velocity dispersion \( \sigma_v \), estimated for observable stars within \( r_{\text{hl}} \), for the simulations with \( r_{\text{hm},0} = 10 \, \text{pc} \) in a cusped (black lines) and cored (green lines) galaxy. In the N-body simulations, both \( M_{\text{cl}} \) and the mass of the galaxy (\( M_g \)) within \( r_{\text{hl}} \) are known. Thus, using the ‘Jeans estimator’ formula from Walker et al. (2009), I can estimate the \( \sigma_v \) due to \( M_{\text{cl}} (\sigma_v (M_{\text{cl}}); \text{dashed lines}) \), and due to the combined mass of the cluster and the galaxy \( (\sigma_v (M_{\text{cl}} + M_g); \text{solid lines}) \). From this, I conclude that

\(^{10}\)This is derived by multiplying \( r_{\text{hm}} \) of the observable stars (defined as being only those stars more massive than 0.75 \( M_\odot \) and excluding any dark remnants) by 3/4 to correct for projection effects.

\(^{11}\)A cluster in the centre of a cusped galaxy loses mass mainly due to stellar evolution.
4.4. DISCUSSION

if Eri II has a DM cusp and hosts a nuclear star cluster, then its star cluster will be DM dominated, with a velocity dispersion\(^{12}\) \(\sigma_v > 2.5 \text{ km s}^{-1}\). By contrast, if Eri II has a DM core, its star cluster will have a much lower dispersion of \(\sigma_v < 1.0 \text{ km s}^{-1}\). These results are in good agreement with Amorisco (2017).

4.4 Discussion

The key result is that a DM core is favoured over a cusp in the ultra-faint dwarf galaxy Eri II. In models with a DM cusp, Eri II’s star cluster is rapidly destroyed by tides, whereas in cored models the star cluster survives for more than a Hubble time, naturally reaching an asymptotic size and mass consistent with observations. I found that this occurs for logarithmic DM cusp slopes steeper than \(\gamma = 0.25\), where \(\gamma\) is the central exponent in the Dehnen profile (equation 4.2.1), independently of even large changes in the assumed DM halo mass or concentration. The only hope for retaining a DM cusp in Eri II is if its star cluster lies at the very centre of the cusp. I found that such a model can work, but is disfavoured by the observed offset between Eri II’s photometric light peak and the projected position of its star cluster. Such a model could be completely ruled out if the velocity dispersion of Eri II’s star cluster is observed to be \(\sigma_v < 1.0 \text{ km s}^{-1}\). These results are in excellent agreement with a recent study by Amorisco (2017).

The mass model for Eri II has a DM core size set by the projected half light radius of the stars \(R_{1/2} \sim 0.28 \text{ kpc}\) (see Fig. 4.2). However, the data only require that there is a

\(^{12}\)Note that none of the clusters in the cusped simulations have a \(r_{hl}\) large enough to be consistent with observations (see Fig. 4.8). However, in a DM cusp, the enclosed mass goes as \(M \propto r^2\) and so \(\sigma_v^2 \propto r\). Thus, increasing the size of the cluster will increase \(\sigma_v\), which is why these results provide a lower bound on the dispersion.
dark matter core where we see Eri II’s star cluster today, at a projected distance of 45 pc from the photometric centre of Eri II. Dynamical friction stalling occurs when the tidal radius of the star cluster approximately matches its galactocentric distance (Read et al., 2006a; Goerdt et al., 2010; Petts et al., 2015, 2016). From this, I can derive a minimum DM core size for Eri II of \( r_c > 45 \) pc. In this section, I explore what such a dark matter core means for galaxy formation and the nature of DM.

### 4.4.1 Dark matter heating

The size and density of the DM core I find in Eri II is in excellent agreement with predictions from the ‘DM heating’ model of R16. However, in the R16 model such a complete DM core would require several Gyr of star formation that may be inconsistent with Eri II’s stellar population (see Section 4.1). However, core formation can be made more efficient if it occurs at high redshift when Eri II’s DM halo was less massive (Madau et al., 2014), or if it owes primarily to angular momentum transfer from cold gas clumps sinking by dynamical friction to the centre of the dwarf (El-Zant et al., 2001; Nipoti and Binney, 2015). Given these complications, following Peñarrubia et al. (2012), R16 and Read et al. (2017), I focus here on the energy required to unbind Eri II’s dark matter cusp:

\[
\frac{\Delta E}{\Delta W} = \frac{M_\star}{\langle m_\star \rangle} \Delta W \xi_{DM} \tag{4.4.1}
\]

where \( \Delta E \) is the total integrated supernova energy, \( \Delta W \) is the energy required to unbind the dark matter cusp, \( M_\star \) is the stellar mass, \( \langle m_\star \rangle = 0.83 \) is the mean stellar mass,
4.4. DISCUSSION

Figure 4.10: The energy required to unbind Eri II’s DM cusp as a function of the DM core size, \( r_c \). I assume for this plot a Chabrier IMF (Chabrier, 2003b) and a coupling efficiency between supernovae and dark matter of \( \epsilon_{\text{DM}} = 0.25\% \) (R16). The horizontal dashed line marks \( \Delta E/\Delta W = 1 \). Above this line, there is enough integrated supernova energy to unbind the cusp; below there is insufficient energy. The vertical green line marks the minimum core size, \( r_{c,\text{min}} = 45 \text{ pc} \) set by the current projected position of Eri II’s star cluster. The vertical blue line marks the core size assumed in this work, \( r_c = R_{1/2} = 0.28 \text{ kpc} \). The black and orange shaded regions assume a DM halo mass of \( M_{200} = 5 \times 10^8 M_\odot \) with a stellar mass \( M_* = 8.3^{+5.1}_{-4.2} \times 10^4 M_\odot \) (Bechtol et al., 2015, assuming three times their uncertainties), and a concentration parameter of \( c_{200} = 23 \) and \( c_{200} = 14 \), respectively. These are the upper and lower 68% confidence intervals of \( c_{200} \) in \( \Lambda \text{CDM} \) (Dutton and Macciò, 2014).

\( \xi = 0.00978 \) is the fraction of mass in stars that go supernova (i.e. those with mass \( m_* > 8 M_\odot \)), and \( \epsilon_{\text{DM}} = 0.0025 \) is the coupling efficiency of the SNe energy to the dark matter. I assume a Chabrier initial stellar mass function over the stellar mass range \( 0.1 < m_* / M_\odot < 100 \) (Chabrier, 2003b). I assume a coreNFW profile when calculating \( \Delta W \) and I take \( \epsilon_{\text{DM}} \) from the simulations in R16. As such, the results are only useful in assessing, at an order-of-magnitude level, whether there is sufficient supernova energy in Eri II’s stellar population to form its apparent central DM core (e.g. Maxwell et al., 2015).

In Fig. 4.10, I plot \( \Delta E/\Delta W \) as a function of the DM core size \( r_c \). The horizontal dashed line marks \( \Delta E/\Delta W = 1 \). Above this line, there is enough integrated supernova energy to unbind the cusp; below there is insufficient energy. The vertical green line marks the minimum core size, \( r_{c,\text{min}} = 45 \text{ pc} \) set by the current projected position of Eri II’s star cluster. The vertical blue line marks the core size assumed in this work, \( r_c = R_{1/2} = 0.28 \text{ kpc} \). The black and orange shaded regions assume a DM halo mass of
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$M_{200} = 5 \times 10^8 M_\odot$ with a stellar mass $M_* = 8.3^{+5.1}_{-4.2} \times 10^4 M_\odot$ (Bechtol et al., 2015, assuming three times their uncertainties), and a concentration parameter of $c_{200} = 23$ and $c_{200} = 14$, respectively. These are the upper and lower 68% confidence intervals of $c_{200}$ in $\Lambda$CDM (Dutton and Macciò, 2014). (Note that the R16 simulations assume the upper envelope of this $c_{200}$ range and so make core formation maximally difficult.)

As can be seen in Fig. 4.10, there is plenty of energy to produce the minimum core size $r_c = 45$ pc independently of the assumed $c_{200}$ or $M_*$. However, for the assumed DM halo mass and supernova energy coupling efficiency that I assume here, a low $c_{200}$ and high $M_*$ for Eri II are required to produce a core as large as $r_c = R_{1/2} = 0.28$ kpc.

While the R16 models are able to produce a DM core in Eri II, most other simulations in the literature to date do not find DM cores in halos below $M_{200} \sim 7.5 \times 10^9 M_\odot$ (e.g. Oñorbe et al. 2015; Chan et al. 2015; Tollet et al. 2016, but see Madau et al. 2014). It is possible to understand the origin of this discrepancy from Fig. 4.1. As can be seen, the R16 simulations (magenta squares) produce a similar total stellar mass to the Chan et al. (2015) (yellow squares) and Wang et al. (2015) (green squares) simulations but in halos an order of magnitude lower in mass. (Note that the Wang et al. (2015) simulations are the same as those discussed in Tollet et al. (2016).) This is why cusp-core transformations in the R16 simulations are energetically feasible. The simulations that find no cores below $M_{200} \sim 7.5 \times 10^9 M_\odot$ form almost no stars below this mass scale and so do not have enough integrated supernova energy to unbind the dark matter cusp (see also the discussion in Read et al. 2017). Understanding this apparent discrepancy between the data and simulations in Fig. 4.1, and the differences between numerical models, remains an open and important problem.

If Eri II is found to have a purely old stellar population, with too few supernovae to provide the energy required to unbind its cusp, then we may be forced to move to models beyond $\Lambda$CDM. I consider some of these, next.

4.4.2 Beyond $\Lambda$CDM

The small scale puzzles in $\Lambda$CDM (see Section 4.1) have motivated the community to consider alternative models. Some of these can solve the cusp-core problem without recourse to baryonic ‘DM heating’. The most popular of these to date is Self-Interacting Dark Matter (SIDM; Spergel and Steinhardt 2000). The latest SIDM models have a velocity-dependent interaction cross section $\sigma/m(v/v_m)$, where $\sigma$ is the dark matter interaction cross section, $m$ is the mass of the dark matter particle and $v_m$ is a velocity scale (e.g. Kaplinghat et al., 2016; Schneider et al., 2017). This is required for the models to be consistent with constraints from weak lensing on galaxy cluster scales that favour $\sigma/m < 0.5\text{cm}^2/\text{g}$ (e.g. Harvey et al., 2015), while maintaining a much higher $\sigma/m \sim 2 - 3 \text{cm}^2/\text{g}$ required to produce large DM cores in nearby gas rich dwarf galaxies (e.g. Kaplinghat et al., 2016).

Since I am studying the low-mass dwarf, I consider a simple velocity independent SIDM model. Following Schneider et al. (2017), the velocity independent interaction cross section can be written as:

$$\frac{\sigma}{m} = \frac{\sqrt{\pi} \Gamma}{4\rho_{\text{NFW}}(r_c)\sigma_s(r_c)}$$  (4.4.2)
where $r_c$ is the coreNFW DM core size (R16), $\rho_{NFW}$ is the initial DM density at $r_c$, $\Gamma = 0.4 \text{ Gyr}^{-1}$ is the SIDM interaction rate (taken from coreNFW fits to numerical simulations in SIDM; Schneider et al. 2017) and:

$$
\sigma_v(r_c)^2 = \frac{G}{\rho_{NFW}} \int_{r_c}^{\infty} \frac{M_{NFW}(r')\rho_{NFW}(r')}{r'^2} dr'
$$

(4.4.3)

is the velocity dispersion of the DM at $r_c$ (assumed in this model to be isotropic).

Using $r_c > 45 \text{ pc}$, I find $\sigma/m > 0.24 \text{ cm}^2/\text{g}$ which is consistent with all known SIDM constraints to date.

Another model that could explain Eri II is ultra-light axions (e.g. González-Morales et al. (2017) and references therein). Assuming an ultra-light axion mass of $m_a \sim 10^{-22} \text{ eV}$, I obtain for the Eri II halo model, $r_c \sim 1.8 \text{ kpc}$ which is consistent with the minimum core size $r_c > 45 \text{ pc}$.

### 4.4.3 Implications of the initial cluster properties

The initial mass of the star cluster in the cored galaxy ($M_{cl,0} \simeq 1.9 \times 10^4 \text{ M}_\odot$) suggests that this star cluster resembled a young massive cluster, similar to those in the disc of the Milky Way (e.g. Arches, Westerlund 1, NGC 3603). The initial radius ($r_{hm,0} \simeq 10 \text{ pc}$) is relatively large compared to these low redshift analogues, which have typical radii of a few pc (Portegies Zwart et al., 2010). However, the model with $r_{hm,0} \simeq 5 \text{ pc}$ expands up to $r_{hm} \sim 15 \text{ pc}$ and fits the data reasonably well. The models did not include primordial mass segregation in the cluster initial conditions. If I had assumed that the massive stars formed more towards the centre of the cluster, it would have expanded more as a result of stellar mass loss (e.g. Zonoozi et al., 2017), allowing for more compact initial conditions. The present day mass of stars and stellar remnants that once belonged to the cluster is $\sim 10^{4} \text{ M}_\odot$, or $\sim 12\%$ of the total stellar mass in Eri II. Such a high cluster formation efficiency has been reported in other dwarf galaxies with a single star cluster (Larsen et al., 2014).

### 4.4.4 Comparison with other work in the literature

Concurrent with this work, Amorisco (2017) have recently modelled Eri II’s central star cluster sinking under dynamical friction, finding that it cannot survive long in a cusped potential, in good agreement with the results presented here (They also report similar results for the star cluster in Andromeda XXV.) The key difference between these studies is that I model the internal structure of Eri II’s star cluster, accounting for two-body relaxation and stellar evolution, for the first time. Amorisco (2017) used a collisionless $N$-body code which cannot capture two-body relaxation effects that drive the expansion of the cluster over time\(^{13}\). The key advantage of modelling the collisional effects in Eri II’s star cluster is that its final mass and size then depend on the local tidal field and, therefore, on the

\(^{13}\) Amorisco (2017) argue that collisional effects can be ignored in extended and faint star clusters. However, this depends on the assumed initial conditions for the cluster. The initial half-mass relaxation time (Spitzer, 1987) of the best-fit cluster model is $\sim 2.2 \text{ Gyr}$, computed assuming $\ln \Lambda = \ln(0.02 N)$ (Giersz and Heggie, 1996). Since the cluster is older than this, two-body relaxation was important during the evolution of this cluster.
mass distribution at the centre of Eri II. This gives us an additional probe of the central
dark matter density profile in Eri II that goes beyond a survival argument.

4.5 Conclusions

I have presented a new method for probing the central dark matter density in dwarf galax-
ies using star clusters. Low mass star clusters orbiting in the tidal field of a larger host
galaxy are expected to reach an equilibrium size due to relaxation-driven expansion and
the tidal pruning of high-energy escaper stars. I have used the \textsc{NBody6DF} collisional
\textsc{N}-body code, which includes stellar evolution and dynamical friction, to show that this is
indeed the case. As a first application, I have applied this method to the recently discov-
ered ultra-faint dwarf, Eri II. This has a lone star cluster that lies some $\sim 45$ pc from its
centre in projection. Using a suite of 226 full \textsc{N}-body simulations, I showed that models
with a central dark matter core (with an inner logarithmic density slope of $\gamma < 0.25$) are
favoured over those with a dark matter cusp. A DM core naturally reproduces the size,
radial light profile and projected position of Eri II’s star cluster. By contrast, dense cusped
galaxy models require the cluster to lie implausibly far from the centre of Eri II ($> 1$ kpc),
with a high inclination orbit ($i > 87.43^\circ$) that must be observed at a special orbital phase
($< 3$ per cent of the orbital period).

The models make several clear predictions that can be tested with deeper observations. If Eri II is cored, then:

- the cluster can have any age older than $\sim 7$ Gyr (as compared to a narrow age range
  of $6.5 - 8$ Gyr in the cusped case);
- there are no tidal tails associated with the cluster;
- the cluster has a low concentration ($c \sim 0.5$ as compared to $c \sim 0.8$ in the cusped
  case).

I also considered the possibility that Eri II’s star cluster lies at the very centre of a DM
cusp, allowing it to survive tidal disruption. This is already disfavoured by the observed
offset between Eri II’s photometric light peak and the projected position of its star cluster.
However, such a model could be completely ruled out if the velocity dispersion of Eri II’s
star cluster is found to be $\sigma_v < 1.0$ km s$^{-1}$.

I have shown that extended faint star clusters can survive at the centres of dwarf galax-
ies with DM cores. Such faint star clusters could be liberated from their host dwarf
galaxy by Galactic tides that act more efficiently on cored dwarfs (e.g. Read et al., 2006b;
Peñarrubia et al., 2010), providing an explanation for some of the recently discovered
ultra-faint objects found in the Milky Way (for a discussion see Contenta et al., 2017b).

The presence of a DM core in the ultra-faint dwarf galaxy Eri II implies that either its
CDM cusp was ‘heated up’ by bursty star formation, or we are seeing the first unambigu-
ous evidence for physics beyond CDM.
4.5. CONCLUSIONS
Chapter 5

The peculiar properties of the distant halo star cluster
LDû 11/Laevens 1/Crater, nature or nurture?

Most halo globular clusters formed in satellite galaxies and were accreted onto the Milky Way. In this study I investigate whether there are cluster properties that are inherited from the evolution in their previous hosts, which can be used as complementary tool in unravelling the accretion history of the Milky Way. I performed a suite of direct N-body simulations of star clusters, to find the initial properties of the recently discovered outer halo cluster LDû 11/Laevens 1/Crater. Claimed to be an extended faint young accreted object, it is a perfect candidate to test whether globular clusters can be used to infer their progenitor properties and the Milky Way accretion history. I explored three evolutionary scenarios for the origin of this cluster: an in-situ formed star cluster on a circular orbit, a tidally stripped former member of one of the Milky Way satellites and the surviving nucleus of a now disrupted dwarf galaxy. Considering the observational biases, I found that if it was an in-situ cluster or a cluster that was stripped from a dwarf galaxy, it must have formed extended and with a low concentration, making the present-day properties something that needs to be understood from the formation of star clusters. If the cluster is the remnant of a disrupted dwarf galaxy, it was tidally heated by the disruption of the dwarf and the disc crossing providing a possible explanation for its large radius. However, the tidal shocks make the cluster significantly more centrally concentrated than observed, therefore it is unlikely that the cluster is the remnant of a dwarf galaxy. Combined with its relatively young age and low metallicity, it is possible to conclude that LDû 11/Laevens 1/Crater was stripped from a dwarf galaxy that may still be orbiting the Milky Way. The previous host of LDû 11/Laevens 1/Crater can be found with measurements of its proper motion and that of the outer satellite galaxies. Therefore, star clusters with similar properties of LDû 11/Laevens 1/Crater can be used to shed a light on their accretion history.

This Chapter will be submitted for publication in Monthly Notices of the Royal Astronomical Society (MNRAS), and will be published as Contenta et al., (in prep).
5.1. Introduction

In January 2014 a faint stellar system was discovered in the outskirts of the Milky Way (MW) by the amateur astronomer Pascal Le Dû (Acker and Le Dû, 2014), who called the object LDû 11. A few months later this object got the interest of the scientific community after two independent follow-up studies\(^1\) by Laevens et al. (2014) and Belokurov et al. (2014), using the Panoramic Survey Telescope and Rapid Response System 1 (Pan-STARRS1) Survey and the ESO VST ATLAS survey, respectively. Laevens et al. (2014) claimed to have observed the most distant globular cluster, Laevens 1; while Belokurov et al. (2014) called the object Crater and suggested that it is a faint dwarf galaxy, because of two blue-loop stars which these authors interpreted as a signature of a younger stellar population, which would imply a dark matter dominated system with an extended star formation history (Willman and Strader, 2012). Laevens et al. (2014) found an effective radius of \(r_{\text{eff}} \approx 20\,\text{pc}\) and a luminosity of \(M_V = -4.3\), while Belokurov et al. (2014) found \(r_{\text{eff}} \approx 30\,\text{pc}\) and a luminosity of \(M_V = -5.5\). They both inferred a heliocentric distance \(>140\,\text{kpc}\), and further observations were needed to establish its nature.

Bonifacio et al. (2015) estimated the line-of-sight velocity and metallicity of two giants ([Fe/H] = −1.73 and −1.67), using X-Shooter at the Very Large Telescope (VLT). From this, they estimated a velocity dispersion \(\sigma > 3.7\,\text{km\,s}^{-1}\), and with deep photometry they claimed that the stars to the blue of the turn-off are a younger population of stars, suggesting that the system was a dwarf galaxy.

Spectroscopic follow-up with DEIMOS at Keck observatory showed that there is no spread in metallicity in their sample of 14 stars (Kirby et al., 2015). These authors found that the blue stars that lead Belokurov et al. (2014) to suggest that there is an age spread, were in fact not members of LDû 11/Laevens 1/Crater. The velocity dispersion measured by Kirby et al. (2015) is \(\sigma < 3.9\,\text{km\,s}^{-1}\). From the absence of metallicity spread, small \(r_{\text{eff}}\), low mass-to-light ratio and the fact that it does not agree with the luminosity-metallicity relation for dwarf galaxies, they conclude that Crater\(^2\) is likely to be a globular cluster.

From stellar population modelling of Hubble Space Telescope (HST) photometry, Weisz et al. (2016, hereafter W16) inferred a present-day mass of \(9.9 \times 10^8\,\text{M}_\odot\), an age of 7.5 Gyr, \(r_{\text{eff}} = 19.4\,\text{pc}\), a luminosity of \(M_V = -5.3\), and a distance of \(D = 145\,\text{kpc}\). From King (1966) model fits to the number density profile, a low concentration was found (i.e. a large core radius relative to \(r_{\text{eff}}\)). They supported the conclusion that Crater is an accreted globular cluster, and based on the relatively young age and low metallicity ([Fe/H] \(\sim -1.66\)), they argued that the progenitor must have been a small dwarf galaxy, less massive than WLM.

Finally, Crater was observed using the Multi-Unit Spectroscopic Explorer (MUSE) at the VLT by Voggel et al. (2016). They observed 26 members and found that Crater has a systemic line-of-sight velocity (\(v_{\text{sys}}\)) of 148 km s\(^{-1}\) and a velocity dispersion (\(\sigma_v\)) of \(2.04^{+2.19}_{-1.06}\) km s\(^{-1}\). From these measurements, they inferred a mass-to-light ratio of \(8.52^{+28.0}_{-6.5}\,\text{M}_\odot/\text{L}_\odot\), again leading to the conclusion that Crater is a globular cluster.

All these measurements (see Table 5.1), using different instruments were essential to establish the nature of Crater and its properties. Its origin, however, is still unknown.

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\(^1\)Unaware of Acker and Le Dû (2014) paper, they claimed that it was a discovery.

\(^2\)In this paper we will use the most used name in the literature.
5.2. Numerical \( N \)-body simulations of Crater

Cluster shares similar distances with Leo V, Leo IV and Crater 2, and similar line-of-sight velocities in the Galactic standard rest frame (GSR) with Leo IV and Leo II (Belokurov et al., 2014; Torrealba et al., 2016a; Voggel et al., 2016; Collins et al., 2017; Caldwell et al., 2017). This means that any of these objects could be the progenitor host of Crater, however, no streams have been observed, neither from the cluster, nor from these dwarfs. The cluster could have also been stripped from one of the Magellanic clouds during their interaction with the MW. Indeed, there is another star cluster in the Small Magellanic Cloud (SMC) with a similar age and metallicity (Lindsay 38, Glatt et al., 2008, 2009). The SMC hosts another star cluster, Lindsay 1, which has a similar \( r_{\text{eff}} \) as Crater, but it has a much higher metallicity ([Fe/H] = −1.14, Glatt et al., 2008, 2009).

Alternatively, Crater could be the remnant of a dwarf galaxy, that was disrupted during its interaction with the MW (Peñarrubia et al., 2010; Errani et al., 2017). The progenitor dwarf galaxy could be comparable to Eridanus II (Bechtol et al., 2015; Koposov et al., 2015; Crnojević et al., 2016), where a loose cluster, such as Crater, can survive in the central region of the dwarf galaxy; if its inner dark matter profile is cored, rather than cusped like a Navarro et al. (1996b) profile (Hernandez and Gilmore, 1998; Goerdt et al., 2006; Sánchez-Salcedo et al., 2006; Amorisco, 2017; Contenza et al., 2017a).

In this study, we perform a suite of \( N \)-body simulations of star clusters evolving in galactic potentials, to estimate the initial properties of the cluster, with the aim of finding which of the proposed scenario (tidally stripped or remnant) is more likely.

This Chapter is organized as follows: in Section 5.2, I describe the \( N \)-body simulations with the initial conditions, and the method used to analyse the simulation data. In Section 5.3, I describe the results from the \( N \)-body simulations analysed in observational space. In Section 5.4, I discuss the implications of the results on the nature of possible progenitors. Finally, in Section 5.5, I present the conclusions.

5.2 Numerical \( N \)-body simulations of Crater

In this Section, I describe the details of the simulations that were performed in this study. In order to simulate the evolution of star clusters in a tidal field, I used \textsc{nbody6tt} (Reinaud and Gieles, 2015a), which is an adaptation of the widely used, Graphics Processing Unit (GPU)-enabled direct \( N \)-body code \textsc{nbody6.gpu} developed by Nitadori and Aarseth (2012). With \textsc{nbody6tt} it is straightforward to include the tidal field due to an external galactic potential that is a function of position and time. The tidal force is added to the equation of motion of a star in an inertial frame by adding the difference in galactic acceleration on the star and the guide centre. The guide centre is a pseudo-particle (initially at the centre of mass of the cluster), and its motion in the galaxy is integrated separately (Aarseth, 2003). Finally, it contains metallicity dependent prescriptions for the evolution of individual stars and binary stars (Hurley et al., 2000, 2002).

The stars in the cluster initially follow a Kroupa IMF (Kroupa, 2001), which is between 0.1 \( M_\odot \) and 100 \( M_\odot \), and a metallicity of \( Z = 0.0005 \) (corresponding to [Fe/H] ≃ −1.65). For the initial density profile of our clusters I used both Plummer model (Plummer, 1911) and King models (King, 1966). The King models allow us to change the concentration of the cluster, which can affect how our simulations match the observed cluster.

Escapers were not removed from the simulations to allow stars to move from the tidal tails back into the cluster during the expansion of the Roche lobe at apocentre.
### Table 5.1: Properties of LDû 11/Laevens 1/Crater.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Decl. (J2000)</td>
<td>−10:52:37.1</td>
<td></td>
</tr>
<tr>
<td>$D$</td>
<td>145 ± 3</td>
<td>kpc</td>
</tr>
<tr>
<td>$M_V$</td>
<td>−5.3 ± 0.1</td>
<td></td>
</tr>
<tr>
<td>$M$</td>
<td>$9.9^{+0.1}_{-0.05} \times 10^3$</td>
<td>$M_\odot$</td>
</tr>
<tr>
<td>Age</td>
<td>7.5 ± 0.4</td>
<td>Gyr</td>
</tr>
<tr>
<td>$r_{\text{eff}}$ (Plummer)</td>
<td>0.46 ± 0.01</td>
<td>arcmin</td>
</tr>
<tr>
<td>$r_{\text{eff}}$ (King)</td>
<td>0.42 ± 0.01</td>
<td>arcmin</td>
</tr>
<tr>
<td>$r_{\text{eff}}$ (King)</td>
<td>0.46 ± 0.01</td>
<td>arcmin</td>
</tr>
<tr>
<td>$r_{\text{eff}}$ (King)</td>
<td>19.4 ± 0.4</td>
<td>pc</td>
</tr>
<tr>
<td>$r_{\text{King}}$</td>
<td>0.39 ± 0.02</td>
<td>arcmin</td>
</tr>
<tr>
<td>$c = \log(r_t/r_{\text{King}})$</td>
<td>0.76 ± 0.05</td>
<td></td>
</tr>
<tr>
<td>$1 - b_0/a_0$</td>
<td>&lt; 0.055 (90%)</td>
<td></td>
</tr>
</tbody>
</table>

Note. — In the first section the data comes from W16: positions, distance ($D$), luminosity ($M_V$), mass ($M$), age, observed size using the Plummer and King models, King radius ($r_{\text{King}}$, size of the core of the King model), concentration parameter, and ellipticity of the cluster. In the second section the data come from Voggel et al. (2016): line-of-sight velocity, velocity dispersion, and velocity dispersion of the stars within 35 arcsec from the centre of the cluster.
5.2. Maximum likelihood method to fit half-light radii

To analyse the $N$-body simulations in a similar way as the observations, I use a maximum likelihood fit following the procedure outlined in Martin et al. (2008), to find the structural properties of the cluster (e.g. $r_{\text{eff}}$). Having the position of the stars on the plane of the sky, the maximum likelihood fit can find the set of free parameters for which the observations become most probable.

I choose a likelihood ($L$) in the following form:

$$\ln L = \sum \ln (n_P + n_{\text{BG}}),$$

(5.2.1)

where $n_P$ and $n_{\text{BG}}$ are the probabilities of a star belonging to the cluster and background, respectively. I choose $n_P$ to be a 2-D elliptical Plummer model (Plummer, 1911), given by:

$$n_P = \frac{N_*}{(1 - e) \pi a^2} \left( 1 + \frac{d^2}{a^2} \right)^{-2},$$

(5.2.2)

with $d$ the elliptical radius:

$$d^2 = \left[ \frac{1}{1 - e} (x \cos(\theta) - y \sin(\theta)) \right]^2 + [x \sin(\theta) + y \cos(\theta)]^2.$$

(5.2.3)

For $n_{\text{BG}}$, I use the values derived by W16 for the King and Plummer best models to generate an homogeneous field of fore/background stars, because in $N$-body simulations there are not fore/background stars.

In this likelihood analysis I choose the following parameters: the scale radius ($a$) which is also the projected half-number radius$^3$, the number of stars in the cluster ($N_*$), the ellipticity$^4$ ($e$) and the position angle ($\theta$); while $x$ and $y$ are the positions of the stars in the field of view. I can estimate the number of stars in the background $N_{\text{BG}}$, fitting on the parameter $N_*$, and knowing the number of stars in our snapshot $N_{\text{tot}}$ ($N_{\text{tot}} = N_* + N_{\text{BG}}$). Therefore, knowing the area of our simulated field of view, I can derive $n_{\text{BG}}$. I use a Monte Carlo Markov Chain (MCMC) method (the affine-invariant ensemble sampler as implemented in the EMCEE code, Foreman-Mackey et al. 2013) to find the parameters that give the highest likelihood.

The same method was used to find the best King model, where instead of $n_P$ in equation (5.2.1), I used the King number density profile. I used the public available LIMEPY python module$^5$ (Gieles and Zocchi, 2015). With this tool I can find the following parameters: the projected half-number radius$^3$ ($r_{\text{eff}}$), the number of stars in the cluster ($N_*$), and the dimensionless central potential ($W_0$) that give the highest likelihood. (I assumed zero ellipticity for the King models.)

However, using a MCMC chain for each time step of an $N$-body simulation is computationally expensive, therefore, to study the evolution of the concentration ($c$), I used a downhill simplex method (Nelder and Mead, 1965), which finds the King model parameters that maximizes the likelihood. (To estimate the evolution of $c$, I keep the same

$^3$Assuming that light trace mass, it is also the half-light radius, which is not correct if the cluster is mass segregated.

$^4$the ellipticity is defined as $e = 1 - b_0/a_0$ where $b_0$ and $a_0$ are the semi-minor and semi-major axis of the ellipse, respectively

$^5$https://github.com/mgieles/limepy
5.3. POSSIBLE SCENARIOS AND RESULTS

observational limit, F606W < 27, used in the HST observations (W16), therefore at every snapshot of the simulation I will have a different minimum mass observed.) For clarity, I smoothed the N-body results, because I are more interested in the evolution of the concentration than in the actual value at each time step. In the following Section I discuss the results of the analysis.

5.2.2 Galactocentric position and radial velocity

To set up the N-body simulations, first of all, I need to find the Galactocentric position and radial velocity of Crater with respect to the Galactic centre. From the heliocentric position and the line-of-sight velocity (see Tabel 5.1), using the public available ASTROPY module (Astropy Collaboration et al., 2013), I estimate the Galactocentric coordinates of the cluster to be \((X_{Cra1}, Y_{Cra1}, Z_{Cra1}) = (0.16, -96.97, 107.50)\) kpc, with \(R_{G_{Cra1}} = 145 \pm 3\) the Galactocentric distance, and \(v_r = -1.13 \pm 1.12\) km s\(^{-1}\) its line-of-sight velocity in the GSR. I assume that \(v_r\) is equal to the radial velocity with respect to the Galactic centre \((V_{R\odot})\), because the angle between \(\vec{V}_{R\odot}\) and \(\vec{v}_r\) is very small (\(\sim 8/145\)). I assumed that the Sun is at \((X_{\odot}, Y_{\odot}, Z_{\odot}) = (-8, 0, 0)\) kpc with a circular velocity of 220 km s\(^{-1}\) and that the uncertainty in \(R_{G_{Cra1}}\) follows a Gaussian distribution. If \(V_{R\odot}\) is close to zero then the cluster is either close to apocentre or pericentre, or it is on a circular orbit. I studied three different scenarios:

1. MW1: the cluster is on a circular orbit in a MW-like potential (Section 5.3.1);

2. MW2: the cluster evolves in the centre of a cored dwarf galaxy which get disrupted when it crosses the disc of the MW. Then, the remaining nucleus, the cluster, evolves in a MW-like potential, reaching the observed position.

3. SMC1: the cluster has been tidally stripped from a dwarf galaxy, e.g. the SMC. To study this, I simulated the cluster on a circular orbit around an SMC-like galaxy (Section 5.3.3). In this case, I am omitting the transition phase from the dwarf to the MW, and the evolution of the cluster in the MW (after the cluster is tidally stripped from its host).

In the next Section I describe the galactic potential for these three scenarios with the respective results from the simulations.

5.3 Possible scenarios and Results

5.3.1 In-situ formation in the Milky Way (MW1)

I perform 18 direct N-body simulations of a cluster on a circular orbit in a MW-like galaxy (see Table 5.2), assumed to be an NFW potential (Navarro et al., 1996b):

\[
\phi_{\text{NFW}}(R_g) = -\frac{G M_{\text{NFW}}}{R_g} \ln \left(1 + \frac{R_g}{\alpha_{\text{NFW}}}\right)
\]
5.3. POSSIBLE SCENARIOS AND RESULTS

Table 5.2: $N$-body simulation properties

<table>
<thead>
<tr>
<th>Model</th>
<th>$W_0$ (10^3 M$_\odot$)</th>
<th>$M$ (10^3 M$_\odot$)</th>
<th>$r_{hm}$ [pc]</th>
<th>$c^a$</th>
<th>$M$ (10^3 M$_\odot$)</th>
<th>$r_{eff}$ [pc]</th>
<th>$c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>MW1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>P4</td>
<td>-</td>
<td>17.0</td>
<td>4.4</td>
<td>1.17</td>
<td>9.5</td>
<td>5.5 ± 0.2</td>
<td>1.75 ± 0.05</td>
</tr>
<tr>
<td>P4BH</td>
<td>-</td>
<td>17.0</td>
<td>4.4</td>
<td>1.17</td>
<td>9.5</td>
<td>9.3 ± 0.3</td>
<td>1.23 ± 0.05</td>
</tr>
<tr>
<td>P8</td>
<td>-</td>
<td>17.0</td>
<td>8.8</td>
<td>1.13</td>
<td>9.5</td>
<td>10.8 ± 0.4</td>
<td>1.34 ± 0.05</td>
</tr>
<tr>
<td>P8BH</td>
<td>-</td>
<td>17.0</td>
<td>8.8</td>
<td>1.13</td>
<td>9.6</td>
<td>15.9 ± 0.5</td>
<td>1.08 ± 0.06</td>
</tr>
<tr>
<td>P10</td>
<td>-</td>
<td>17.0</td>
<td>10.3</td>
<td>1.11</td>
<td>9.4</td>
<td>13.0 ± 0.5</td>
<td>1.27 ± 0.06</td>
</tr>
<tr>
<td>P10BH</td>
<td>-</td>
<td>17.0</td>
<td>10.3</td>
<td>1.11</td>
<td>9.7</td>
<td>18.0 ± 0.6</td>
<td>1.05 ± 0.06</td>
</tr>
<tr>
<td>P10-B</td>
<td>-</td>
<td>16.8</td>
<td>10.3</td>
<td>1.13</td>
<td>9.2</td>
<td>13.1 ± 0.5</td>
<td>1.42 ± 0.06</td>
</tr>
<tr>
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<td>-</td>
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<td>1.13</td>
<td>9.4</td>
<td>19.9 ± 0.7</td>
<td>1.06 ± 0.06</td>
</tr>
<tr>
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<td>-</td>
<td>17.0</td>
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<td>9.7</td>
<td>17.8 ± 0.7</td>
<td>1.24 ± 0.06</td>
</tr>
<tr>
<td>P13-B</td>
<td>-</td>
<td>16.8</td>
<td>13.5</td>
<td>1.13</td>
<td>9.1</td>
<td>17.8 ± 0.6</td>
<td>1.23 ± 0.06</td>
</tr>
<tr>
<td>2K10BH</td>
<td>2</td>
<td>18.5</td>
<td>10.3</td>
<td>0.50</td>
<td>10.3</td>
<td>18.2 ± 0.5</td>
<td>0.86 ± 0.07</td>
</tr>
<tr>
<td>3K10BH</td>
<td>3</td>
<td>18.1</td>
<td>10.3</td>
<td>0.67</td>
<td>10.2</td>
<td>17.6 ± 0.5</td>
<td>0.88 ± 0.06</td>
</tr>
<tr>
<td>4K10BH</td>
<td>4</td>
<td>18.4</td>
<td>10.3</td>
<td>0.84</td>
<td>10.2</td>
<td>20.8 ± 0.6</td>
<td>0.97 ± 0.06</td>
</tr>
<tr>
<td>7K10BH</td>
<td>7</td>
<td>18.0</td>
<td>10.3</td>
<td>1.53</td>
<td>10.0</td>
<td>20.3 ± 0.7</td>
<td>1.16 ± 0.06</td>
</tr>
<tr>
<td>2K13</td>
<td>2</td>
<td>18.2</td>
<td>13.5</td>
<td>0.50</td>
<td>9.9</td>
<td>17.7 ± 0.5</td>
<td>0.86 ± 0.07</td>
</tr>
<tr>
<td>3K13</td>
<td>3</td>
<td>18.2</td>
<td>13.5</td>
<td>0.67</td>
<td>9.9</td>
<td>17.8 ± 0.5</td>
<td>0.90 ± 0.06</td>
</tr>
<tr>
<td>4K13</td>
<td>4</td>
<td>17.3</td>
<td>13.5</td>
<td>0.84</td>
<td>9.9</td>
<td>18.3 ± 0.5</td>
<td>0.90 ± 0.06</td>
</tr>
</tbody>
</table>

SMC1

| P13-6rg | -                       | 17.4                 | 13.5        | 1.11 | 7.4                  | 12.6 ± 0.3    | 0.77 ± 0.06 |
| 3K10BH-6rg | 3                   | 18.1                 | 10.3        | 0.67 | 7.9                  | 12.0 ± 0.2    | 0.81 ± 0.04 |
| 2K13-6rg | 2                     | 18.2                 | 13.5        | 0.50 | 6.8                  | 15.3 ± 0.3    | 0.55 ± 0.13 |
| 3K13-6rg | 3                     | 18.2                 | 13.5        | 0.67 | 6.4                  | 14.8 ± 0.3    | 0.74 ± 0.06 |
| 3K13-10rg| 3                     | 18.2                 | 13.5        | 0.67 | 8.7                  | 16.2 ± 0.3    | 0.78 ± 0.05 |
| 3K13-13rg| 3                     | 18.2                 | 13.5        | 0.67 | 8.8                  | 17.4 ± 0.3    | 0.88 ± 0.04 |

Note. — The first capital letter in the model label indicates if I used as initial condition a Plummer (P) or a King (K) model. BH denotes a simulation with 100% of BH’s retention; and -B when the model contains primordial binaries. (a) $W_0$ and $c$ are not parameters of the Plummer model, however I estimate $c$ at $t = 0$ Gyr for comparison, fitting a Plummer model with a King model.
where $G$ is the gravitational constant, $M_{\text{NFW}} = 6.6 \times 10^{11} M_\odot$ is the scale mass, and $a_{\text{NFW}} = 13.9 \, \text{kpc}$ is the scale radius. These were derived by the Aquarius simulations (Springel et al., 2008), with $M_{\text{NFW}}$ chosen to have a maximum circular velocity of $210 \, \text{km s}^{-1}$ at $30 \, \text{kpc}$. I choose this potential because I will simulate the cluster on a circular orbit at a Galactocentric distance $R_{G_{\text{Cra1}}} = 145 \, \text{kpc}$, therefore only the total enclosed mass and the local shape of the dark matter halo are important.

To find the initial $N$ of the simulated star clusters (see Table 5.2) that results in the same observed properties of Crater at 7.5 Gyr (see Table 5.1), I used the fast cluster evolution code EMACSS (Alexander et al., 2014).

**Evolution of the size**

Collisional systems, such as star clusters, have relaxation times that are shorter than their ages, and this makes the finding of their initial condition challenging. In a few relaxation time most of the initial conditions are wiped out and it is possible to arrive at similar final conditions by starting with different initial conditions. However, in this respect, Crater is a peculiar object, because if the simulated cluster is initially too small, despite its short relaxation time, it does not have enough time to expand to the observed size. As studied by Hurley and Mackey (2010), some extended clusters must have formed extended. Here I find the same result, in Fig. 5.1 I show the evolution of the size of the simulations P13 (cyan line), P10BH (orange line), and P4 (black line), see Table 5.2. The simulation P10BH expands faster because 100% of its stellar mass black holes (BHs) were retained initially (no supernovae kicks). A cluster that has 100% of its BHs retained initially expands faster because the BHs sink to the centre making the dynamical interactions between the stars stronger, leading to more energetic particles and a faster expansion of the cluster. However, in clusters with initial small $r_{\text{hm}}$, the BHs retained in a cluster will not affect the evolution of the size because they will rapidly escape. As shown by Breen and Heggie (2013a,b), the escape rate of stellar BHs depends on the half-mass relaxation time of the cluster (higher densities means smaller half-mass relaxation time).

Therefore, Crater needs to form extended, with an initial 3D half-mass radius ($r_{\text{hm}}$) of about 10.3 pc if all the stellar-mass black holes (BHs) are retained, or 13.5 pc if the BHs are ejected after supernova kicks.

I performed a suite of $N$-body simulations with the purpose of finding the initial conditions that could best fit the observed number density profile (see fig. 4 in W16). In Fig. 5.2, I show the number density profile with the best fit Plummer and King models estimated with the method described in Section 5.2.1, using only the observable stars$^6$. The results shown in Fig. 5.2 are consistent with the observed values and better agreement can be achieved with a small increase of the initial size of the cluster. However, the aim is to find whether this study can tell us more about the progenitor of Crater, instead of finding the perfect initial conditions, which still depends on many assumptions, such as the MW-like potential and the orbit (circular in this case).

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$^6$F606W < 27, observational limit of W16, which can be converted in the minimum mass observed, $m = 0.64 \, M_\odot$, using PARSEC’s isochrones v1.2S (Bressan et al., 2012)
5.3. POSSIBLE SCENARIOS AND RESULTS

Figure 5.1: MW1 - Evolution of $r_{\text{eff}}$ of the models: P13 (cyan line), P10BH (orange line), and P4 (black line). This plot illustrates that not all initial radii expand toward the observed $r_{\text{eff}}$ of Crater. To be extended, at an age of 7.5 Gyr, a Crater-like cluster must have formed relatively extended.

Figure 5.2: MW1 - Number density profile of the model P13. The red and blue lines are the best fit Plummer and King profiles, respectively.
5.3. POSSIBLE SCENARIOS AND RESULTS

Evolution of the concentration

The observed concentration parameter is $c = 0.76$, while the $N$-body models are more concentrated ($c = 1.24$, for the model P13, Fig 5.2, Table 5.2), i.e. they have a smaller core. The stellar mass BHs and/or primordial binaries stars can increase the size of the core (Merritt et al., 2004; Mackey et al., 2007, 2008; Peuten et al., 2016), however, in this case, I do not see a major improvement. With 100% BHs retention, I obtain $c = 1.04$, for the model P13BH (see Table 5.2). Changing the initial parameters, I find that if the cluster initially has a low concentration, then it will better fit the observations. In Fig. 5.3, I show the number density profile of the model 2K13, which agrees with the observations within $1\sigma$.

To understand how $c$ evolves in time, I analysed all the snapshots of our $N$-body simulations, see Fig. 5.4 and 5.5. I find that the observed $c$ increases with time for clusters where BHs natal kicks are present, while the evolution of $c$ changes when 100% BHs are retained in the clusters. In the latter case, $c$ increases for clusters with a low concentration, reaching a plateau after $\sim 2$ Gyr, whereas it decreases for concentrated clusters. However, in all the studied cases, at around 7.5 Gyr, $c$ seems to be always higher than $\sim 0.8$ (it is still consistent within $1\sigma$ with the observed value). From this, I conclude that to achieve an observed low concentration value, the cluster has to form with a low concentration.

5.3.2 Nuclear cluster of a tidally disrupted dwarf galaxy in the MW (MW2)

In this scenario, the evolution of the cluster can be divided in two parts:

1. the cluster evolves in the centre of a cored dwarf galaxy;
5.3. POSSIBLE SCENARIOS AND RESULTS

Figure 5.4: MW1 - Evolution of $c$ for simulations with BHs natal kicks. The black data point is derived from the observations (see Table 5.1).

Figure 5.5: MW1 - Evolution of $c$ for simulations with 100% BHs retention. The black data point is derived from the observations (see Table 5.1).
2. the cluster evolves in the MW, after the dwarf galaxy gets disrupted by tidal interaction with the MW.

For the first part, I used the simulations performed by Contenta et al. (2017a). In their paper, they ran 100 simulations of star clusters in a cored dwarf galaxy, because they wanted to reproduce the properties of the central star cluster in Eridanus II. Therefore, I can use some of these simulations that could represent the ‘initial’ properties of Crater. As shown in their paper, different dark matter halo masses and central densities do not change the evolution of the cluster, what it is important is the central slope of the dark matter density profile. Therefore, I assumed that the first part of the evolution of the cluster is in a cored dwarf galaxy with an inner density radial slope of 0 (see Contenta et al., 2017a, for further details).

In the centre of a cored dwarf galaxy, the cluster experience a compressive tidal field. In this configuration while the mass loss of the cluster is limited, the energy of the stars is enhanced. This brings the cluster to expand reaching an equilibrium density where the size is roughly constant and the core continues its slow expansion. Bianchini et al. (2015) show that when a cluster switches from compressive to isolation, the cluster will expand due to its supervirialised state. However, they also show that the expansion will be not as large as a cluster that spends its lifetime in isolation. A following study by Webb et al. (2017) show that a cluster that switches from strong compressive tides (centre of a cored dwarf galaxy) to an extensive tidal field (like the MW halo), suffers a drastic expansion and mass loss. This process could explain extended objects like Crater.

For the second part of the cluster evolution (when the cluster is in the MW), I need to find an orbit that reaches the observed position (see Table 5.1). An orbit can be defined by three quantities: \( V_R \), the tangential velocity \( (V_t) \), and an angle \( (\theta_G) \), which determines the orbital plane. In the MW2 scenario, we can find an orbit making two assumptions:

1. the orbit is in the \( Y-Z \) plane, which sets \( \theta_G \), and ends at the current position of Crater: \( (Y_{Cra1}, Z_{Cra1}) = (-96.97, 107.50) \) kpc;

2. the cored dwarf galaxy gets disrupted when it crosses a region where the MW density \( (\rho_G) \) is higher than the central density of the dwarf galaxy \( (10^8 \text{ M}_\odot \text{ kpc}^{-3}) \), which sets the lower limit of \( \rho_G \) (Peñarrubia et al., 2010). Contenta et al. (2017a) show that the cluster evolves towards an equilibrium density within its \( r_{hm} (\rho_h) \) that is a factor of \( \sim 2 \) higher than its host \( (\rho_h/\rho_G \sim 2) \). For \( \rho_G > 2 \times 10^8 \text{ M}_\odot \text{ kpc}^{-3} \) the cluster would also get disrupted, therefore I use this as the upper limit for \( \rho_G \).

Once I have defined the MW potential, I can estimate \( V_t \) using the above limits for \( \rho_G \). I adopt a three-component potential, based on the ‘MWPotential2014’ described by Bovy (2015).

The three components are:

- **Bulge** (Hernquist, 1990):

\[
\phi_H = -\frac{GM_H}{R_G + a_H} \tag{5.3.2}
\]

\(^7\)For simplicity, I use an analytic potential, while Bovy (2015) adopts a power-law density profile with an exponential cut-off.
where $M = 0.5 \times 10^{10} M_\odot$ is the mass of the bulge and $a_H = 0.6 \text{kpc}$ is the scale radius.

- Disc (Miyamoto and Nagai, 1975):
  \[
  \phi_d = -\frac{GM_d}{\sqrt{R^2 + \left(a_d + \sqrt{Z^2 + b_d^2}\right)^2}} \tag{5.3.3}
  \]
  where $R$ is the Galactocentric distance in the $X$-$Y$ plane and $Z$ is the Galactocentric distance in the $Z$-component; $M_d = 6.8 \times 10^{10} M_\odot$ is the mass of the disc, $a_d = 3.0 \text{kpc}$ and $b_d = 0.28 \text{kpc}$ are the scale radius and scale height, respectively.

- Halo (NFW): see equation (5.3.1), with $M_{\text{NFW}} = 4.3 \times 10^{11} M_\odot$ and $a_{\text{NFW}} = 16.0 \text{kpc}$.

I used a three-component potential because the disc diminishes the survivability of cored dwarf galaxies (e.g. Errani et al., 2017), and I chose the 'MWPotential2014' because it was derived using several observational constraints (for further details see Bovy, 2015).

With the three component potential and the two assumptions made, I find that the dwarf galaxy must cross the disc between $6$ and $8 \text{kpc}$ (and for symmetry between $-6 \text{kpc}$ and $-8 \text{kpc}$) from the centre. Assuming that the $R_G^{\text{Cra1}}$ and $V_{R_G}$ have a Gaussian distribution with $R_G^{\text{Cra1}} = 145.1 \pm 3 \text{kpc}$ and $V_{R_G} = -1.13 \pm 1.12 \text{km s}^{-1}$, we have a ‘family’ of orbits that crosses the disc at the chosen distances. To compute the orbit of the cluster, I used the orbit integrator in NBODY6TT (Renaud and Gieles, 2015a), using the analytical potential above, i.e. the same as used for the evolution of our $N$-body system (see Section 5.2).

I am also assuming that the MW potential is fixed, however Renaud and Gieles (2015c) showed that this has a negligible effect on the evolution of the cluster. In addition, the cluster evolves in this potential for less than $\sim 4 \text{Gyr}$ (for a redshift $< 0.5$), when only the halo of the MW is growing.

The events that will affect the cluster are the sudden change of the potential and the tidal shock due to the disc crossing. For this reason, I have chosen an orbit that crosses the disc twice within $\sim 8 \text{Gyr}$ (i.e. the age of the cluster). I know that the dwarf galaxy has been accreted less than $8 \text{Gyr}$ ago (W16), however it is unknown when its disruption has taken place. Therefore, I studied the two cases shown in Fig. 5.6:

1. the cluster goes directly to the observed position after the disruption of its host (red line). In this case, the cluster evolves for $t_0 + t_1 = 6316 \text{Myr}$ in the dwarf galaxy and $t_1 \approx 1184 \text{Myr}$ in the MW\(^8\), with $(Y_0, Z_0) = (-6.186, 0.023) \text{kpc}$ and $(V_{Y_0}, V_{Z_0}) = (89.343, 462.447) \text{km s}^{-1}$ the initial positions and velocities\(^9\) of the cluster in the MW, respectively;

\(^8\) $t_0 + t_1 \approx 7.5 \text{Gyr}$, age of the cluster.

\(^9\) I converted $V_{R_G}$, $V_t$ and $\theta_G$ to Cartesian coordinates.
5.3. POSSIBLE SCENARIOS AND RESULTS

2. the cluster crosses the disc one more time before it reaches the observed position (blue line), with \( t_0 = 3950 \text{ Myr} \), \( t_1 \simeq 3550 \text{ Myr} \), \((Y_0, Z_0) = (7.752, 0.001) \text{ kpc} \) and \((V_{Y0}, V_{Z0}) = (241.197, -376.160) \text{ km s}^{-1}\).

For the second part of the MW2 scenario (cluster in the MW), I ran 6 simulations using as initial conditions a cluster at \( t_0 \) in Contenta et al. (2017a) simulations.

**Evolution of the size**

I present the evolution of a cluster as if it was the remnant of a cored dwarf galaxy. I ran 6 simulations where the cluster starts in the three-component potential of the MW and it is affected by one or two disk shocks, in addition to the sudden change of the potential from the dwarf galaxy to the MW. In Fig. 5.7, I show with a dashed green line the evolution of \( r_{\text{eff}} \) for a cluster in a cored dwarf galaxy (models P10-2sh and P10-1sh, see Table 5.3). I then evolve the same cluster in a MW-like potential, with two different orbits, where the cluster crosses once (solid red line) and twice (solid blue line) the disc (recall Fig. 5.6). Every time that the cluster crosses the disc, its size rapidly expands, because, due to the disc shock, the energy of the stars in the outer region of the cluster is enhanced (Kundic and Ostriker, 1995). Therefore, the stars in the outer region become unbound, however, once the cluster has crossed the disc, it regains the unbound stars, due to the rapidly increase of the tidal radius (due to the high eccentricity of the orbit).

In this scenario, in the MW, the cluster is initially Roche overfilling, because the tidal radius changes from 50 and 49 pc (when the cluster is in the central region of the dwarf galaxy) to 28 and 20 pc (when the cluster is in the disc of the MW) for P10-2sh and P10-1sh models, respectively (see Table 5.3). However, what determine the expansion of the cluster is its density when the dwarf galaxy gets tidally stripped, as shown by Gieles and Renaud (2016), due to tidal heating: low density clusters expand while high density clusters contract. As shown in Fig. 5.7, \( r_{\text{eff}} \) of the cluster at the final position is \( \sim 25 \text{ pc} \), therefore to have a smaller radius, consistent with the observations (Table 5.1), a different initial density is needed.

**Evolution of the concentration**

In Fig. 5.8, I show the evolution of \( c \) in compressive tides, such as the central region of a cored dwarf galaxy, where the observed \( c \) decreases with time. As mentioned above, in compressive tides, the mass loss of the cluster is limited by the tides, with a tidal radius which is roughly constant. Moreover, the cluster reaches an equilibrium density where the size is roughly constant and the core is expanding, explaining the decrease of \( c \). However, when the cluster evolves in a MW-like potential the disc shock drastically enhances the observed \( c \) of the cluster. The disc shock removes the outer layer of stars and when the cluster re-adjusts to the tidal field it ends with a higher concentration. With different initial conditions, it is possible to find a final size comparable to the observed value (see Table 5.1), but low values of \( c \) cannot be achieved (as shown in Fig 5.8).

### 5.3.3 Tidal stripping from an SMC-like galaxy (SMC1)

In this scenario, the cluster is tidally stripped from a MW satellite. To test this, I ran 6 simulations of clusters on a circular orbit in a SMC-like galaxy at different galactocentric
Figure 5.6: MW2 - Orbit of the cluster in the \(Y-Z\) plane, in the scenario MW2. The thick red line shows the orbit of the cluster that has only recently lost its host, while the thin blue line shows the orbit of the cluster that crosses the disc twice. Isodensity contour \((\rho_G = 10^8 \, M_\odot \, \text{kpc}^{-3})\) of the three-component galaxy model in the \(Y-Z\) plane are shown with a thin black line.
5.3. POSSIBLE SCENARIOS AND RESULTS

Figure 5.7: MW2 - Evolution of the $r_{\text{eff}}$ of the remnant of a dwarf galaxy. Dashed green line represents the evolution of $r_{\text{eff}}$ of a cluster in the centre of a cored dwarf galaxy. The blue (P10-2sh) and red (P10-1sh) lines are the evolution of $r_{\text{eff}}$ of clusters in the MW-like potential, affected by two and one disc shocks, respectively. The blue and red lines do not match initially with the green line because the fitting process gives numerically different results which are consistent within 1σ, and I also smoothed the data for clarity, see Section 5.2.1.

Figure 5.8: MW2 - Evolution of $c$ in the MW2 scenario. Dashed green line represents the evolution of $c$ of a cluster in centre of a cored dwarf galaxy. The blue (P10-2sh) and red (P10-1sh) lines are the evolution of $c$ for clusters in a MW-like potential, affected by two and one disc shocks, respectively. The black data point is derived from the observations (see Table 5.1).
5.3. POSSIBLE SCENARIOS AND RESULTS

As discussed in Section 5.1, Crater has similar structural properties of Lindsay 1 and metallicity as Lindsay 38 (Glatt et al., 2008, 2009). These clusters are 6.27 and 13.28 kpc away from the centre of the SMC, therefore I chose to simulate a Crater-like object on a circular orbit at 6.27, 10, and 13.28 kpc. For the properties of the potential I adopt the model used by Besla et al. (2012), which is a Hernquist potential, see equation (5.3.2), where $M_H = 2.0 \times 10^{10} M_\odot$ is the mass of the galaxy and $a_H = 7.3$ kpc is the scale radius.

In both the MW1 and the SMC1 scenarios, the cluster is in the outskirts of its host galaxy and it can expand because it is initially Roche underfilling (the stars are in the inner part of the Roche volume). I do not know whether clusters are on circular orbits in the SMC, but I want to test how a different potential influences the observed properties of the cluster, which I vary by changing $R_G$ of the orbit. The evolution of star clusters in different potentials is well studied in the literature (Baumgardt and Makino, 2003; Tanikawa and Fukushige, 2010; Miholics et al., 2014; Bianchini et al., 2015; Miholics et al., 2016; Claydon et al., 2017; Webb et al., 2017), however how this affects the observed properties, in particular $r_{\text{eff}}$ and $c$, has not been studied in much detail (but see Sippel et al., 2012).

Miholics et al. (2014, 2016) studied the evolution of accreted star clusters in the MW. They show that when a cluster switches from a dwarf galaxy to a MW-like potential, its $r_{\text{hm}}$ rapidly adjusts to the new tidal field, therefore they conclude that $r_{\text{hm}}$ alone cannot be used to distinguish between accreted and in-situ star clusters. Even if I am omitting the phase when the cluster is tidally stripped by the dwarf and get accreted by the MW, from Miholics et al. (2014, 2016) results, it is possible to conclude that the simulations in the MW1 scenario could be considered as the last part of the evolution of a cluster that has been tidally stripped from a dwarf galaxy. However, how the tidal stripping event affects the observational properties of the cluster is not clear.

**Evolution of the size**

As mentioned in Section 5.3.3, the evolution of a cluster depends on the galactic potential. For different galactic potential (MW1 and SMC1) and different $R_G$ (6.27, 10 and 13.28 kpc), I ‘observe’ a different $r_{\text{eff}}$ at $t = 7.5$ Gyr. For large $R_G$ the galactic tidal field is weaker and $r_{\text{eff}}$ is larger. In this case, better fits for $r_{\text{eff}}$ can be obtain changing the initial $r_{\text{hm}}$ and mass of the cluster, but, I stress that the new initial conditions will depends on the galactic potential and the galactocentric distance assumed. In this scenario, due to stronger tidal field, the clusters lose more mass respect to the clusters in MW1 scenario. However, this does not not affect the observed size, see Table 5.2.

**Evolution of the concentration**

In this scenario I found a different evolution of the observed $c$, see Fig. 5.9. I show that lower values of $c$ can be easily achieved in a dwarf galaxy, due to a stronger tidal field, indeed $c$ increases for larger $R_G$, where the tidal field is weaker. However, from these $N$-body simulations, it is not possible to determine which was the host of Crater. Taking into account all the simulations in MW1 and SMC1, I can only conclude that the observed $c$ does vary with the choice of potential and $R_G$. Therefore, making a prediction on the former host galaxy using the observed properties of the cluster is challenging.
5.4. Discussion

Table 5.3: $N$-body simulation properties of the MW2 scenario.

<table>
<thead>
<tr>
<th>Model</th>
<th>$t = 0$ Gyr</th>
<th>$t = 3.9$ Gyr</th>
<th>$t = 6.3$ Gyr</th>
<th>$t = 7.5$ Gyr</th>
</tr>
</thead>
<tbody>
<tr>
<td>P5-2sh</td>
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<td>5.0 &gt; 1</td>
<td>13.1</td>
<td>11.5</td>
</tr>
<tr>
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<td>25.5</td>
<td>5.0 &gt; 1</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>P10-1sh</td>
<td>25.5</td>
<td>10.0 &gt; 1</td>
<td>12.3</td>
<td>14.8</td>
</tr>
<tr>
<td>P10-1sh</td>
<td>25.5</td>
<td>10.0 &gt; 1</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>P15-1sh</td>
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<td>15.0 &gt; 1</td>
<td>13.1</td>
<td>19.1</td>
</tr>
<tr>
<td>P15-1sh</td>
<td>31.6</td>
<td>15.0 &gt; 1</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Note. — The capital letter P indicates that the cluster was initially a Plummer model. -1sh and -2sh indicate whether the cluster crosses the disc once or twice, respectively. In the table I include also the initial conditions of the clusters in the three-component potential, just before they are affected by the disc shock.

In the SMC1 scenario, I am assuming that the tidal stripping event from the dwarf to the MW will not affect the cluster, which may be not correct. Therefore, additional simulations are needed to understand how the stripping event could affect the observed $c$. Bianchini et al. (2015) show that when the cluster switches from a compressive tides to isolation, an impulsive approximation for the change of the potential respect to an adiabatic approximation does not affect the final evolution of the cluster (see their fig. 3). In this case, during the stripping event the cluster is going from an extensive tidal field (dwarf galaxy) to another extensive tidal field (MW), therefore the change of the potential should less violently affect the evolution of the cluster and an impulsive approximation for the stripping event could be used.

5.4 Discussion

In this Chapter, I am interested in understanding if the observed properties of clusters, such as Crater, are due to specific initial conditions, or to its evolution in a specific environment. As already mentioned by previous papers (e.g., W16), Crater is likely to be an accreted star cluster. W16 proposed that Crater could be a stripped cluster from the interaction between the Magellanic system and the MW. (Considering that the SMC hosts Lindsay 38 (Glatt et al., 2008, 2011), which has a similar age and metallicity as Crater.) Whereas, from the age-metallicity relation of the LMC, SMC and WLM, derived by Leaman et al. (2013), the Magellanic clouds are too massive to be the former host of Crater, while a Carina/Leo I-like galaxy is a more likely progenitor. Furthermore, Crater can be associated with: Crater 2, Leo V, Leo IV, and Leo II, due to similar distances and line-of-sight velocities (Belokurov et al., 2014; Torrealba et al., 2016a; Voggel et al., 2016; Collins et al., 2017; Caldwell et al., 2017). Therefore, Crater might be a stripped object from one of these satellites, however, without any proper motion measurements and no streams detected, it is not possible to make a definite conclusion.

These simulations show that if the cluster was tidally stripped the cluster was initially an extended and low concentrated object. However, I do not take into account the stripping event. Miholics et al. (2014, 2016) studied how the cluster reacts to the change of different extensive tides. They consider two cases: a cluster on a circular orbit in a dwarf galaxy that falls in the MW, and a similar set up where the dwarf galaxy evaporates with time, reaching its dissolution after 3 Gyr. They show that the cluster will adjust rapidly to the
last tidal field it experiences. Therefore, if the cluster is in a dwarf galaxy and gets accreted it will continue its evolution as if it was an in-situ star cluster. In this case, I want to study a similar effect, where the cluster gets tidally stripped, and the dwarf galaxy is still a MW satellite (it does not evaporate). More importantly, I want to study how this process affects the observational properties of the cluster and to understand this, additional simulations will be performed.

Alternatively, Crater could be the remnant of a Milky Way satellite. For the cluster to survive in the centre of its host, the galaxy needs to be cored (Hernandez and Gilmore, 1998; Goerdt et al., 2006; Sánchez-Salcedo et al., 2006). For the cored dwarf galaxy to get tidally disrupted, it must cross a high density region of the MW (e.g. the disc), but at the same time, the cluster must survive. I assumed that the dwarf galaxy will be disrupted when it crosses regions where the MW density is between the central density of the dwarf galaxy ($\sim 10^8 \text{M}_\odot \text{kpc}^{-3}$) and the minimum density for a star cluster ($\sim 2 \times 10^8 \text{M}_\odot \text{kpc}^{-3}$). I used the simulations from Contenta et al. (2017a) for the evolution of the cluster in the cored dwarf galaxy and NBODY6TT for the evolution of the cluster in a three-component potential (see Section 5.3.2). A similar study was done by Bianchini et al. (2015) and Webb et al. (2017). In Bianchini et al. (2015) they studied the evolution of a star cluster (with equal mass particle) that evolves in compressive tides up to 8 Gyr, when they switch off the tidal field, leaving the cluster in isolation for $\sim 2.5$ Gyr. They show that there is no difference between an adiabatic or impulsive change of the tidal field (from compressive to no tides), and more importantly that the cluster does not expand more than it would have if it was in isolation initially. A following study by Webb et al. (2017) investigated the evolution of a cluster (with Kroupa IMF and including stellar
5.5. CONCLUSIONS

evolution) that evolves 300 Myr in a compressive tidal field and other 700 Myr in an extensive tidal field (using a MW-like potential with different Galactocentric distances). They show that if a cluster evolves in a compressive tidal field initially, its evolution in a MW potential will be greatly affected. Due to the large expansion and mass loss that the star cluster suffers, this mechanism could explain the observed extended faint objects in the MW halo. Here, the star cluster falls into the dwarf galaxy, embedded in a compressive tidal field for few Gyr, and then I switched to a MW-like potential, where the cluster suffers also a disc shock initially. In this scenario, I obtain similar results to Webb et al. (2017), where the cluster can expand if the cluster has a low density when it crosses the disc. However, it is not trivial to compare these simulations, because in this case the cluster has settled in a compressive tidal field for few Gyr, while, in Webb et al. (2017), the cluster evolves in compressive tides for only 300 Myr after formation, therefore its observational properties should be highly affected by the tidal change.

5.5 Conclusions

In this Chapter, I have used the \textsc{nbbody6tt} collisional \textit{N}-body code to study the evolution of accreted globular clusters in different galactic potentials, with and without BHs natal kicks, and on circular and eccentric orbits. I analysed the \textit{N}-body simulations in observational space, using the same detection limit as the HST observations and the same models (King and Plummer models, W16).

There are two possible evolutionary scenarios for an accreted star cluster: either it has been stripped from one of the MW satellites or it is the nuclear remnant of a cored dwarf galaxy that has been tidally disrupted.

In the first scenario, the direct \textit{N}-body simulations show that in a dwarf galaxy-like potential, where the probability to find initially low density clusters is enhanced (e.g. Elmegreen, 2008), a cluster with initially large radius ($r_{hm} > 10$ pc) and low concentration is needed to observe a Crater object. I also notice that retaining 100% of BHs initially, does not change the main results. However, the evolution of a Crater-like globular cluster depends on the galactic potential and the orbit of the cluster itself. Therefore, if the cluster has been stripped from a dwarf galaxy, with only \textit{N}-body simulations, it is not possible to conclude from which dwarf galaxy the cluster might have come from.

For the second scenario, where Crater could be the remnant of a cored dwarf galaxy, I found that the cluster expands every time it crosses the disc (see Fig. 5.7), making the observed size easy to match even for clusters with an initial small size. However, every time the cluster crosses the disc, $c$ drastically increases (see Fig. 5.8), making this scenario very unlikely. On the other hand, this could be the formation scenario for some of the ultra-faint objects observed in the halo of the Milky Way (Contenta et al., 2017b, and reference therein). To confirm this, their proper motion needs to be consistent with orbits that crossed the central region of the Milky Way.
Chapter 6

Conclusions

6.1 Summary of the Thesis

In this thesis, I show that globular clusters can be used to differentiate between dark matter dominated and dark matter free objects, and they can probe the underlying dark matter potential.

In Chapter 2, I used direct \( N \)-body simulations to study whether the ultra-faint objects in the Milky Way (MW) could be part of the faint star cluster population. From the observations, it is often not clear where the boundary between extended faint star clusters and compact ultra-faint dwarf galaxies lies. With a simple model is possible to explain the observed faint star clusters (see Section 2.2.1), but many uncertainties and assumptions can play a role in classifying an object. Usually, the inferred mass-to-light ratio (\( M/L \)) is used to find whether the object is dark matter dominated (\( M/L > 10 M_\odot/L_\odot \)) or free (\( M/L < 10 M_\odot/L_\odot \)). With \( N \)-body simulations, I show that a low density cluster that retains 100% of its black holes, expands up to a size comparable to a dwarf galaxy, and the assumed binaries population can enhance the inferred line-of-sight velocity dispersion. Therefore, these two factors together could enhance the \( M/L \) of dark matter free objects.

Stellar evolution of low metallicity stars is far from understood, and natal kicks of dark remnant are still unknown (matter of debate, see Spera et al., 2015, and references therein), however these assumptions in \( N \)-body simulations are often underestimated. In Chapter 3, I show that a minor change, like the kick velocity of the neutron stars, could be extremely important for the cluster evolution. I found that even if the neutron stars represent less than 2% of the total mass of the cluster, the presence of natal kicks affects the lifetime of the cluster by up to almost a factor of four. Therefore, minor changes early in the lifetime of the cluster could have very large effects on its evolution.

Motivated by the recent discovery of a star cluster in the centre of a low mass dwarf galaxy, Eridanus II (Bechtol et al., 2015; Koposov et al., 2015; Crnojević et al., 2016), I studied the structural properties of this star cluster, see Chapter 4, to probe the dark matter profile of its host. Thanks to the public available code, \textsc{nbody6df} (Petts et al., 2015, 2016), I was able to simulate the cluster in a dwarf galaxy, taking into account the dynamical friction. Using a similar analysis discussed in Chapter 2, I could compare the observational data with the simulations. From my study, I found that the observed cluster can survive in the centre of Eridanus II only if the dwarf galaxy has a cored dark
manner density profile. The core/cusp problem\(^1\) was initially solved with a continuous star
formation that can ‘heat-up’ the dark matter particles, transforming the dark matter cusp in
a core. However, in cosmological hydrodynamical simulations of galaxies, the formation
of a core does not occur for low mass dwarf galaxies (Peñarrubia et al., 2012), such as
Eridanus II, unless a reionisation model is not included (Read et al., 2016). Furthermore,
if follow up observations show that Eridanus II has a short star formation history, then
there will be no Cold Dark Matter (CDM) model (up-to-date) able to explain the presence
of a dark matter core. Then, this would be the first evidence for physics beyond CDM.

Finally, in Chapter 5, I studied a specific MW halo globular cluster, Laevens 1/Crater
(Laevens et al., 2014; Belokurov et al., 2014), which is claimed to be a young accreted
globular cluster (Weisz et al., 2016). I use direct \(N\)-body simulations to find the initial
conditions and to study the evolution of this cluster. Loose clusters have a long relaxation
time, therefore, Crater-like star clusters can be used to derive their formation properties. I
found that Crater can be an accreted object from a MW satellite, but to have the properties
we now observe, it must form extended and with a low concentration (with a large core
radius). Whereas, it is unlikely that Crater is the nucleus of a disrupted dwarf galaxy. To
disrupt its progenitor, an orbit that crosses the disc is needed, and every time that the clus-
ter crosses the disc, its concentration increases by almost a factor of two. Unfortunately,
with only \(N\)-body simulations it is not possible to find Crater’s progenitor, because the
evolution of the observed properties of the cluster depends on the assumed galactic poten-
tial and orbit. I concluded that, with follow up observations, the cluster’s proper motion
should indicate its progenitor.

6.2 Future perspectives

In this thesis, I focused my studies on the extended star clusters, in the MW halo and in
one of its satellites (Eridanus II). Thanks to the Sloan Digital Sky Survey (SDSS, York
et al., 2000) and the Dark Energy Survey (DES, Bechtol et al., 2015; Koposov et al., 2015;
Drlica-Wagner et al., 2015b), it was possible to find most of the halo stellar systems, and
many more will come with the advent of new telescopes, such as the Large Synoptic
Survey Telescope (LSST; LSST Science Collaboration et al., 2009). New MW satellites
and streams will be used to investigate the accretion history and the formation of the MW.

The ongoing Gaia mission (Gaia Collaboration et al., 2016) will make a major contri-
bution to our understanding of the dynamics of stars in the MW, in particular in the Solar
neighbourhood and nearby open and globular clusters. Gaia is measuring the position,
parallax and proper motion of more than a billion of stars, while the radial velocity will
be provided only for the brighter stars.

All these data from observations, together with numerical simulations, are the perfect
combination to find the nature and the properties of the halo of the MW and its satel-
lites. However, the comparison between data and models is not trivial. In my studies,
I showed how \(N\)-body models differ from the observed properties. Therefore, further
studies should be done analysing the extended stellar systems individually. With a suite
of \(N\)-body simulations it is possible to find constraints on the initial conditions of an ob-
ject, if its current structural properties and orbit are known. More interestingly, numerical

\(^1\)Discrepancy between predicted cuspy density profiles and observed core density profiles.
simulations with both collisional and collisionless codes could be used to find the nature of the MW halo objects (e.g. Willman 1 and Segue 1).

I showed that extended globular clusters, such as Crater, require to be initially extended and with a low concentration and that it is unlikely for this cluster to be the remnant of a cored dwarf galaxy. However, accreted objects could be affected by the transition phase between different potentials. In Chapter 5, I only explore the case when the cluster switch from a dwarf galaxy to a MW potential (impulsive approximation). This approximation does influence the evolution of the cluster and its observed properties, therefore an adiabatic approximation (see Bianchini et al., 2015) should be tested. Further studies on the accreted objects and how they are affected by the accretion events would help to unravel the evolution of these globular clusters and their initial conditions, shedding a light on how globular clusters form. Moreover, the HSTPROMO collaboration has recently observed the Crater-Leo group, and the proper motion of Leo IV, Leo V and Crater 2 will soon be available, and it will be possible to understand if these objects are on the same orbit and if they were the previous host of Crater.

There are few dwarf galaxies that host a globular cluster in the centre: Eridanus II (see Chapter 4), Pegasus dwarf Irregular, Andromeda I, and Andromeda XXV. With different methods (see also Amorisco, 2017), globular clusters can be used to probe the dark matter density profile of their host. The inferred dark matter density profiles from HI rotation curves are often matter of debates, due to the many biases that are involved in the analysis and in the observations (e.g. Oman et al., 2017). Whereas, studying the structural properties of these central globular clusters, with the aim of reproducing the observed properties, it is possible to distinguish between cusp and core dark matter profiles. These central globular clusters are also the only probe for dark matter density profile when the host galaxy (e.g. Eridanus II) is devoid of gas and other dynamical proxies (e.g. Read and Steger, 2017). Furthermore, the absence of globular clusters, in similar dwarf galaxies, could shed light on the nature of dark matter and in solving the cusp/core problem.
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BIBLIOGRAPHY


Appendix A

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A.1 Publications

The papers Contenta et al. (2015, 2017a,b) were published and submitted to Monthly Notices of the Royal Astronomical Society (MNRAS). These papers were published under the Oxford Journals Licence to publish (see http://www.oxfordjournals.org/ for details). Under this licence, as an author (first and third) respectively I have the right to include these papers in full or in part in this thesis (see https://academic.oup.com/journals/pages/authors/authors_faqs/online_licensing#three for details).

A.2 Figures

A.2.1 NASA/ESA Figure

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A.2.2 Figure 4 of Holmberg 1941

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