UNIVERSITY OF SURREY

THEORETICAL AND EXPERIMENTAL MODELLING
TECHNIQUES FOR COMPOSITE TUBES UNDER
IMPACT AND VIBRATIONAL LOADING

BY

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A Thesis submitted for the degree of
Doctor of Philosophy in the Faculty
of Engineering of the University of
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January 1990
TO MY FAMILY
AND ANNE
The objective of this thesis is to investigate the impact behaviour of Glass/PES, Glass/Carbon/PES and Carbon/PES composite tubes by performing analytical and experimental analyses. The aim is to obtain information that will serve as a basis for dealing with the design and testing of these composite tubes.

A literature review is presented. This includes the process of manufacture of fibre reinforced composite tubes, impact studies of these materials, test standards, the measurements of elastic material properties and fracture toughness.

The process which is employed to manufacture the composite tubes in this investigation is the film stacking hot press moulding technique. The film stacking method has been developed for fabricating continuous fibre reinforced composite tubes and is discussed in detail. Determining the mechanical properties in the tube form of the above composites is considerably more difficult than for isotropic materials. Several mechanical tests are described, together with the application of the laminate theory to determine the orthotropic properties of the composite materials.

The basic concepts of fracture mechanics for the linear elastic regime are explained. Fracture mechanics parameters (stress intensity factor and strain energy release rate) for the composite are determined by analytical and experimental methods. A reasonable agreement is obtained and discrepancies between the analytical and the experimental techniques are attributed to the orthotropic properties of the composite.

The ACQUIRE software package is used to detect the first three natural frequencies of the glass fibre reinforced PES composite tubes in a free-free condition. The finite element technique is used to determine vibrational frequencies and the dynamic response of these composite tubes. A review of test methods illustrates the importance of developing a test technique for determining the impact force - time and acceleration - time responses of the glass fibre reinforced PES composite tubes. Also two methods to calculate the damping factor of the composite material are described.
ACKNOWLEDGEMENT

I am indebted to many friends and academic staff who have helped, knowingly or unknowingly, in the production of this work. I was extremely fortunate in having the opportunity to discuss some of my ideas with them during my research. I gratefully acknowledge helpful discussions with Professor Leonard Hollaway, M.Sc. (Eng), Ph.D. (Lond), CEng, MInstICE, FICE and Mr Micheal Gunn, MA, Dip.Compl.Sci. (Camb), CEng, MICE, both of whom have contributed constructive suggestions as well as useful comments upon the completion of this work.

I wish to thank Mr. Eric Dawson, Mr. Tony Thorne, Mr. Ian Rankin and Mr. Micheal Napier-Ford for their help during the experiments.

I owe a special debt of gratitude to Anne Duncan for her unfailing support, encouragement and her great help during the task of writing this work.

It is a pleasure to record my indebtedness to Mr Harry Wickens who despite his own heavy commitments, assisted in preparing my graphs for this work.

UNIVERSITY OF SURREY

January 1990
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Composite materials have been developed for use in both building and aerospace structures where high strength and stiffness are required. Glass fibre reinforced polyethersulphone (PES) composites have mechanical properties which may make them a suitable choice for many structural applications. They are stiff and have a high strength which provides assurance of structural integrity during service life.

To explore the potential of these materials as structural elements, an investigation into their performance under impact is undertaken. This investigation examines test techniques and gives an explanation of test approaches; nondestructive test techniques are required to assess mechanical properties relevant to impact.

A study was made of the mechanical properties and impact performance of three types of composite materials, these are:
(a) glass fibre reinforced polyethersulphone (PES) composite tubes,
(b) carbon/glass fibre reinforced polyethersulphone composite tubes,
(c) carbon fibre reinforced polyethersulphone (PES) composite tubes. Comparisons were made between their experimental and theoretical data. This research and analysis of the instrumented impact tests served to provide materials data on these composites. Also the present impact studies led to the application of fracture mechanics as one method of characterising these composite materials.

This investigation details the procedures used to develop testing methods for composite materials. The objective of these studies was to define new methods of assessing the impact behaviour of these materials.

The most common of all tests on composite tubes is the instrumented impact test; this is partly because it is an easy test to perform, and partly because many, though not all, of the desirable characteristics of composites are qualitatively related to its
stiffness; it is also because of the importance of the strength of composites in service.

The instrumented impact tests can be roughly classified as strength tests which allow repeated testing of the same specimen and thus make possible a study of the variation in properties with time.

There is no universally accepted standard test method or technique available for impact studies of composites. Different methods and techniques have been used since the commencement of this project. Because many of these test techniques were carried out in the laboratory, it was necessary to have a knowledge of the influence of these various methods on the impact behaviour of the composite. However the test conditions on the specimens were fixed by the test method adopted. It did not follow that the conditions in a real impact would match those of the instrumented impact test. Hence information obtained from an impact test relate to the composite under the conditions imposed by that test. Because of this, a series of impact tests with different techniques and approaches were carried out carefully.

The results of investigations on composite materials demonstrates that the techniques using the Falling Weight Impact Tester and the Impact Hammer have potential for being able to detect any structural deformation but they are unsuitable for detecting any possible damage or the size of damage area.

The test techniques which have been employed on the composite tubes to establish the effects of impact are discussed in chapter 2.

The dynamic behaviour of the composite tubes were investigated by using finite element methods. The dynamic response of composite tubes are complex, consequently a study of the dynamic behaviour of composites by considering the fundamentals of vibrations of simple systems was undertaken. A guide to the complexity of this dynamic
system was the number of degrees of freedom possessed by it. This meant that calculating the eigenvalues of the system presented difficulties. Therefore for experimental purposes the simplest system with one degree of freedom was believed to represent the tested specimen.

The main aim of this investigation was to establish a method of characterising these composite materials.

The following steps were investigated during this project from an experimental and a theoretical analysis point of view:

a) to establish test techniques,
b) to investigate impact resistance of composite tubes under falling weight (this was to monitor load peak value and strains in specimen),
c) to calculate the damping factor of these materials by an approximate method,
d) to investigate rubber hardness of various impact surfaces,
e) to obtain the natural frequencies of these composite tubes in a free-free condition,
f) to vary fibre make-up which involves glass fibre reinforced polyethersulphone, carbon/glass fibre reinforced polyethersulphone and carbon fibre reinforced polyethersulphone.
g) to undertake analytical parameter studies of
   (i) isotropic fibre arrays,
   (ii) orthotropic fibre arrays,
   when finite element frequency extraction analyses are conducted.
h) to analyse the notch sensitivity of composite materials using compact tension specimen in linear elastic fracture mechanics.
CHAPTER 2

LITERATURE REVIEW

2.1 INTRODUCTION

The objective of this chapter is to review previous work, test standards, papers on impact studies of composites materials, and the process of manufacture of composite tubes. This includes the measurement of elastic or dynamic material properties and fracture toughness.

This chapter will also deal with the properties of laminated composite materials, that is, with the properties of a single layer. The behaviour of a single linear elastic isotropic or orthotropic layer will be considered in detail. As a result, the properties of the laminate will be used in the composite tube analysis.

2.2 PROCESS OF MANUFACTURE OF COMPOSITE TUBES

Composite tubular components have been used for a variety of structural applications. Their geometric simplicity, good structural properties and ease of assembly have caused them to be widely used in aerospace and space structures. For example, reference 1 shows the design and manufacturing of composite tubes for a communications spacecraft where a variety of structural parameters such as thermal stability, low coefficient of expansion and ease of manufacturing, are required.

It is not surprising, therefore, that composite tubes are a desirable material form which can be used as a basic structural shape from which lightweight, efficient structures or components can be constructed. They can be manufactured by several different methods for a broad range of end uses.
Four methods exist for fabricating continuous fibre reinforced tubes. They are prepreg roll wrapping, braiding, filament winding, and pultrusion [Ref 2]. There is a common perception that filament winding is either a natural way or even the only way to produce composite tubes. While filament winding certainly has its place, prepreg roll wrapping offers numerous advantages.

First by using prepreg, parts can be fabricated from either customer-specified materials (i.e. materials already qualified, characterized, and understood by the customer) or manufacturer-specified material. There are a great number of very well characterized prepreg systems available such as the film stacking hot press moulding technique. Roll wrapping also allows for placement of 0 degree material. Filament winding cannot do better than 5 to 10 degrees while braiding is typically limited to + 20 degrees. Pultrusion is generally limited to zero degree orientations although recent off-axis weaves are being tried with mixed results.

Filament winding is clearly the preferred manufacturing method when fabricating a large quantity of very large parts, such as the MX missile, due to its high material placement efficiency per manhour. However, for small parts, particularly the thin wall parts common to aerospace applications, the roll-wrapped prepreg method is used. Part sizes of roll-wrapped tubes range from 0.125" to 16" up to 18 feet length. Wall thickness range from 0.010" to 1". Fibre orientation angles range from 0 to 90 degrees. Basically, the prepreg roll wrapping provides a very flexible production technique for producing a variety of high performance composite tube sizes and orientations.

The fibre reinforced PES composite tubes under investigation here are fabricated by means of the film stacking hot press moulding technique [Ref 42] which is extensively described in section 3.3.1 of Chapter 3.
Impact testing

2.3 **IMPACT TESTING**

In fibre reinforced polymers, the fibres provide the main strength and stiffness, and the polymer acts as an adhesive to transfer the load between the discrete fibres. The polymer also protects the fibre from mechanical damage and prevents ingress of moisture or other solvents into the centre of the composite. Fibres also develop the toughness of the material and thus the material is less susceptible to the propagation of cracks. Hence the impact strength of most polymers is greatly enhanced by the presence of fibres.

The impact strength characteristics of most polymeric materials are readily available, but the data available for fibre reinforced polymeric materials is limited.

The impact test is supposed to measure the toughness and resistance to fracture of a material under impact loading. It is measured by the energy required to break a standard specimen under specified conditions. The field of impact testing is very complex, due mainly to the fact that there are a large number of different types of impact tests available, and the results obtained from one set of tests may be completely different from those obtained from another series of tests on an identical material. This is due mainly to the fact that the different types of test use different shaped samples, are fractured under different types of stress distribution and under different impact loading.

The impact studies of composite materials have been investigated by several authors by extending the established methods for metals to composite materials mainly laminated ones.

The international conference on 'IMPACT TESTING AND PERFORMANCE OF POLYMERIC MATERIALS' 2-3 September 1985 was held at the University
of Surrey. This conference proceeding consists of various papers mostly describing the calculation of the fracture toughness and stress energy release rate ($K_{IC}$ and $G_{IC}$ values).

The objective of the conference was to gather information on, and promote discussion about, the impact testing techniques, analysis, materials evaluation and design of plastics components.

A paper published by Plati and Williams [Ref 3] shows the approach (as used in Refs 4 and 5) which was applied to determine the fracture parameters (fracture toughness, $K_{IC}$ and strain energy release rate, $G_{IC}$) for a range of polymers in Charpy and Izod impact tests. This method is Linear Elastic Fracture Mechanics (LEFM) which has been widely used by many other investigators, e.g., references 6 through 9.

More recently published work [Refs 10-12] shows that when the LEFM theory [Ref 13] is applied to the instrumented falling weight impact (IFWI) techniques then it is possible to measure fracture parameters in thermoplastics, such as short glass fibre reinforced Nylon and carbon fibre reinforced Poly ether-ether Ketone (PEEK) composites. Most test methods record a force versus time curve response during impact test and then operate on this function in order to analyse and interpret toughness behaviour. Three aims were investigated and successfully obtained:

(a) The interpretation of the force versus time curve recorded during impact test and associate features on this curve with observations of the deforming specimen as recorded by high-speed photographic techniques.

(b) A detailed statistical analysis of the impact data of thermoplastics.

(c) The application of IFWI to a number of material science problems, such as the influence of matrix type on the material toughness.
2.4 MEASUREMENT OF ELASTIC PROPERTIES

Because thermoplastics are anisotropic it is more difficult to characterize their mechanical properties than is the case with isotropic materials. However, the deformational characteristics of some anisotropic plastics can be defined relatively simply by treating them as orthotropic materials. This approach has been adopted here with fibre reinforced PES composite tubes whose properties are needed for reliable analytical work.

The mechanical properties of fibre reinforced composites are in many cases known to be anisotropic. Their anisotropy has, however, still received little attention. This applies in particular to reinforced thermoplastics and especially to the characterization of their anisotropic properties. In order to measure the mechanical properties of the composite tubes under investigation, several textbooks have been studied and also numerous research articles. In particular, references 14 through 23 have been most informative and are recommended for additional reading.

From laminate theory [Ref 21] the following approaches, to measure the elastic constants (mechanical properties), were gathered:

Approach 1, for a homogeneous isotropic material in one dimensional stress state, the Hooke's law relationship is

\[ \sigma = E \varepsilon \]  \hspace{1cm} (2.1)

The proportionality constant, \( E \), is the modulus of elasticity. For the homogeneous isotropic material, two elastic constants, \( E \), and the Poisson ratio, \( \nu \), must be determined experimentally in order to specify the stress-strain relationships for a two or three dimensional stress state. For example, step 1 shows a plane stress state where all stresses are acting together. Two independent material elastic constants appear in all the
Measurement of elastic properties

The third elastic constant, shear modulus, \( G \), is a function of the other two elastic constants, \( E \) and \( v \). Therefore, for isotropic materials, two independent elastic constants are needed to be measured experimentally.

Approach 2, The Hooke's law relationships for orthotropic materials in two or three dimensional stress states are more involved than the relationships for isotropic materials. For orthotropic materials, there are four independent elastic constants: The modulus of elasticity in the \( x \) and \( y \) directions (see step 2), \( E_x \) and \( E_y \); the shear modulus, \( G_{xy} \); the major Poisson's ratio, \( \nu_{xy} \). The fifth elastic constant, \( \nu_{yx} \), is a function of the other constants, and it may be determined from the reciprocity relation:

\[
\nu_{yx} E_x = \nu_{xy} E_y \tag{2.2}
\]

Therefore, the orthotropic material necessitates the determination of two more elastic constants than is necessary for isotropic materials.
FULLY ELASTIC DESIGN [After Ref 24]

Step 1: Isotropic

![Diagram showing isotropic stress and strain](image)

For \( \sigma_x \) acting alone, \( \varepsilon_x = \frac{1}{E} \sigma_x \) \( \varepsilon_y = -v \varepsilon_x \) by definition of \( E \) and \( v \).

For \( \sigma_y \) acting alone, \( \varepsilon_y = \frac{1}{E} \sigma_y \) \( \varepsilon_x = -v \varepsilon_y \) by definition of \( E \) and \( v \).

For \( \tau_{xy} \) acting alone, \( \tau_{xy} = \frac{1}{G} \tau_{xy} \) by definition of \( G \).

For \( \sigma_x, \sigma_y \) and \( \tau_{xy} \) acting together, the component strains may be superposed:

\[
\begin{align*}
\varepsilon_x &= \frac{1}{E} \sigma_x - \frac{v}{E} \sigma_y \\
\varepsilon_y &= \frac{1}{E} \sigma_y - \frac{v}{E} \sigma_x \\
\tau_{xy} &= \frac{1}{G} \tau_{xy}
\end{align*}
\]

In matrix notation

\[
\begin{bmatrix}
\varepsilon_x \\
\varepsilon_y \\
\tau_{xy}
\end{bmatrix}
= \begin{bmatrix}
\frac{1}{E} & -\frac{v}{E} \\
-\frac{v}{E} & \frac{1}{E}
\end{bmatrix}
\begin{bmatrix}
\sigma_x \\
\sigma_y
\end{bmatrix}
\]

or \([\varepsilon] = [S][\sigma]\)

and

\[
\begin{bmatrix}
\sigma_x \\
\sigma_y \\
\tau_{xy}
\end{bmatrix}
= \begin{bmatrix}
\frac{E}{1-v^2} & \frac{vE}{1-v^2} & 0 \\
\frac{vE}{1-v^2} & \frac{E}{1-v^2} & 0 \\
0 & 0 & \frac{1}{G}
\end{bmatrix}
\begin{bmatrix}
\varepsilon_x \\
\varepsilon_y \\
\tau_{xy}
\end{bmatrix}
\]

or \([\sigma] = [Q][\varepsilon]\)

Note that \([Q] = [S]^{-1}\).
FULLY ELASTIC DESIGN [After Ref 24]

Step 2: Orthotropic

For \( \sigma_x \) acting alone, \( \varepsilon_x = \frac{1}{E_x} \sigma_x \) \( \varepsilon_y = -v_{xy} \varepsilon_x \)

by definition of \( E_x \) and \( v_{xy} \).

For \( \sigma_y \) acting alone, \( \varepsilon_y = \frac{1}{E_y} \sigma_y \) \( \varepsilon_x = -v_{yx} \varepsilon_y \)

by definition of \( E_y \) and \( v_{yx} \).

For \( \tau_{xy} \) acting alone, \( \gamma_{xy} = \frac{1}{G_{xy}} \tau_{xy} \)

by definition of \( G_{xy} \).

For \( \sigma_x, \sigma_y \), and \( \tau_{xy} \) acting together, the component strains may be superposed:

\[
\begin{align*}
\varepsilon_x &= \frac{1}{E_x} \sigma_x - \frac{v_{yx}}{E_y} \sigma_y \\
\varepsilon_y &= \frac{1}{E_y} \sigma_y - \frac{v_{xy}}{E_x} \sigma_x \\
\gamma_{xy} &= \frac{1}{G_{xy}} \tau_{xy}
\end{align*}
\]

\[
\begin{align*}
\sigma_x &= \frac{E_x}{M} \varepsilon_x + \frac{v_{yx} E_x}{M} \varepsilon_y \\
\sigma_y &= \frac{E_y}{M} \varepsilon_y + \frac{v_{xy} E_y}{M} \varepsilon_x \\
\tau_{xy} &= G_{xy} \gamma_{xy}
\end{align*}
\]

where \( M = 1 - v_{yx} v_{xy} \).

In matrix notation

\[
\begin{bmatrix}
\varepsilon_x \\
\varepsilon_y \\
\gamma_{xy}
\end{bmatrix} =
\begin{bmatrix}
\frac{1}{E_x} & \frac{v_{yx}}{E_y} \\
-\frac{v_{xy}}{E_x} & \frac{1}{E_y} \\
0 & \frac{1}{G_{xy}}
\end{bmatrix}
\begin{bmatrix}
\sigma_x \\
\sigma_y \\
\tau_{xy}
\end{bmatrix}
\text{ or } [\varepsilon] = [S_{AT}] \cdot [\sigma]
\]

and

\[
\begin{bmatrix}
\sigma_x \\
\sigma_y \\
\tau_{xy}
\end{bmatrix} =
\begin{bmatrix}
\frac{E_x}{M} & \frac{v_{yx} E_x}{M} \\
\frac{v_{xy} E_y}{M} & \frac{E_y}{M} \\
\frac{G_{xy}}{M}
\end{bmatrix}
\begin{bmatrix}
\varepsilon_x \\
\varepsilon_y \\
\gamma_{xy}
\end{bmatrix}
\text{ or } [\sigma] = [Q_{AT}] \cdot [\varepsilon]
\]

Note that \([Q_{AT}] = [S_{AT}]^{-1}\).
Sample preparation and test methods for composite characterization are not fully developed or standardized. Test data depend on the test method, and specimen design. In the absence of standard test methods, the data reported by individual researchers cannot be used for accurate analysis. This leads to investigation of test techniques i.e. characterization testing of composites. In addition, because of their non-isotropic and inhomogeneous nature, testing of composites is more extensive than that of metals and is still evolving. Although only two elastic constants are needed to characterize a metal, it takes twenty one independent constants to completely characterise an anisotropic composite. Differences between testing of composites and metals is given in table 2.1 [Ref 25].

**TABLE 2.1: METALS VERSUS COMPOSITES - TESTING**

<table>
<thead>
<tr>
<th>Metals</th>
<th>Composites</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Homogeneous, Isotropic</td>
<td>• Inhomogeneous, Non-Isotropic</td>
</tr>
<tr>
<td>• Specimen Preparation, Test Methods Fully</td>
<td>• Specimen Preparation, Test Methods Not</td>
</tr>
<tr>
<td>Developed and Standardized</td>
<td>Fully Developed or Standardized</td>
</tr>
<tr>
<td>• $\alpha$, $\rho$, $\kappa$, Same in all</td>
<td>• $\alpha$, $\rho$, $\kappa$, Depend on</td>
</tr>
<tr>
<td>Directions</td>
<td>Direction</td>
</tr>
<tr>
<td>• Minimal Testing Needed</td>
<td>• Extensive Testing Needed</td>
</tr>
<tr>
<td>• Isotropic Materials Need Only 2 Out of 3</td>
<td>• Anisotropic - 21 Constants</td>
</tr>
<tr>
<td>Elastic Constants for Complete Characterization</td>
<td>• Monoclinic - 13 Constants</td>
</tr>
<tr>
<td>• $E$ - Young's Modulus</td>
<td>• Orthotropic - 9 Constants, Typical</td>
</tr>
<tr>
<td>• $G$ - Shear Modulus</td>
<td>Composite</td>
</tr>
<tr>
<td>• $\mu$ - Poisson's Ratio</td>
<td>• Orthotropic - 4 Constants (Plane Stress</td>
</tr>
<tr>
<td>• $E = 2G/1 + \mu$</td>
<td>Assumption)</td>
</tr>
<tr>
<td></td>
<td>• Transversely - 5 Constants,</td>
</tr>
<tr>
<td></td>
<td>Isotropic</td>
</tr>
<tr>
<td></td>
<td>Typical Composite</td>
</tr>
</tbody>
</table>

$\alpha = \text{Coefficient of Thermal Expansion}$  \hspace{1cm}  $\rho = \text{Electrical Conductivity}$  \hspace{1cm}  $\kappa = \text{Thermal Conductivity}$

Munjal [Refs 26 and 27] reviews the present status of test methods for characterization of fibre reinforced composites. Reference 26 summarizes test methods available for tension, compression and shear. It discusses the advantages and disadvantages of each test method and recommends which methods are suitable. In some cases, it compares the test data obtained from different test methods or using different specimen designs.
The technical report 85099 published (Nov. 1986) by Royal Aerospace Establishment (RAE) describes test methods suitable for the measurements of the engineering properties of fibre reinforced composites, e.g. resin/matrix composites reinforced with orientated continuous fibres. Specimen configuration and testing procedures are detailed and the applicability of the tests to the different types of fibre reinforced composites (e.g. unidirectional or multi-directional) is discussed.

The above document recommends that a report for any test performed should include:

(a) A description of the test method and type of specimen used, together with its critical dimensions, type of end fittings and adhesives employed.
(b) Fibre and resin type.
(c) The data of the test.
(d) Any observations made during the test that may be relevant to the results obtained or any deviations from the recommendations e.g. incremental instead of continuous loading.

It must be noted that the report indicates, in certain cases some dimensional variations from the defined standards can be permitted without significantly affecting the validity of the test results. Nevertheless, the standard specimens should be employed whenever possible.

The report 85099 recommends two test methods for the measurements of mechanical properties of composite laminates:
(1) The longitudinal tensile test for unidirectional laminates,
(2) The tensile test for multidirectional laminates.

The second test technique was considered to be relevant to the tensile tests carried out in this project and is described as follows;
2.4.1 TENSILE TEST FOR MULTIDIRECTIONAL LAMINATES

This specimen is used to determine the tensile strength and modulus of multidirectional laminates (unnotched). The test may be used for the laminate which is axially orthotropic to obviate induced bending.

In the angled plies, no individual fibres should run under the tags at both ends of the specimens. The free length is chosen so that a non-axial fibre can run across the full width of the specimen and be at least half its specimen width short of the end tags at each end, i.e. minimum $L = W(1+1/\tan \Theta)$.

For testing under dry ambient conditions soft aluminium alloy or glass fibre reinforced plastics (GRP) end tags are suitable. For moist or hot conditions GRP end tags are recommended. The tags are attached using an adhesive suitable for the test environment.

The end tag adhesive shear strength will limit the level of load input and hence limit the amount of $0^\circ$ fibres present in the laminate, e.g. typically for tests under dry ambient conditions a maximum thickness of 1.5mm of carbon fibres can be allowed. Testing without end tags is permissible provided that
suitable end grips are used. Dimensions are as follows:

\[ t = 1.0 \text{ to } 3.0\text{mm depending on laminate configuration}, \]
\[ W = 25.0\pm0.25\text{mm (for } t = 1.0 \text{ to } 2.49\text{mm)}, \]
or \[ W = 30.0\pm0.25\text{mm (for } t = 2.5 \text{ to } 3.0\text{mm)}, \]
\[ L \text{ not less than } W(1+1/\tan \theta) \text{ or } 100\pm1\text{mm whichever is the greater}, \]

Where \( t \) is measured thickness of specimen,
\( W \) is measured width of specimen,
\( L \) is free length and
\( \theta \) is angle between fibres and longitudinal axis.

The edges of specimen must be parallel to within \( \pm0.1\text{mm}. \) The specimen must be flat and end tag faces parallel and aligned to within \( \pm0.05\text{mm}. \) The width may be reduced to a minimum of 20mm provided that \( W/t \) is not less than 10. For a small angle of \( \theta \) (less than 15°) \( L \) may be reduced to \( W/\tan \theta. \)

The specimen must be carefully aligned in the test machine jaws to avoid inducing specimen bending. The tensile load (or strain) should be increased uniformly to cause failure within 30-90 seconds. For modulus determination strain versus load must be recorded.

The tensile strength for a plain specimen is given by:
\[ f_T = \frac{P}{Wt} \quad (2.3) \]
where \( P \) is the failure load. Since the stress versus strain graph may be nonlinear, the tensile modulus is defined as:
\[ E_T = \text{secant modulus at } 0.25\% \text{ axial strain} \quad (2.4) \]
Poisson's ratio = \( \text{- transverse strain/axial strain} \quad (2.5) \)
at 0.25\% axial strain.

For the test to be valid for design data, failure of the plain specimen must occur in the central region. If the damage extends into the end tag region the result will provide only a lower bound strength value.
2.5 FRACTURE TOUGHNESS [Ref 28]

It has often been stated that impact strength is one of the least understood of the mechanical properties of polymers, in spite of its great technological importance. This is partly because impact strength is not as well defined a mechanical property as, for example, modulus of elasticity in that its definition includes a description of how it is measured. This means that the use of non-standard specimen size may causes a serve limitations on the amount of useful information obtained on these materials.

(a) THE DEVELOPMENT OF FRACTURE TESTING STANDARDS

The development of modern day sophisticated impact tests for plastics started when, in December 1965, a British Standards Institution Committee was requested to recommend methods for providing data for plastics, which could be used for serious engineering design rather than merely for quality control. It was hoped by those responsible for the creation of the committee, that it would be possible for the members to agree on methods of determining, presenting and using the mechanical properties of plastics, which would be more satisfactory than relying on the seriously limited information provided by conventional test methods. This committee echoed the widespread realisation that it was not adequate simply to determine a mechanical property under unique conditions, however carefully standardised, but that it was also essential to study the possible variations of mechanical properties due to changes in conditions. Multi-point testing is more useful than single-point testing. For example, by varying the radius and depth of a notch tip for a set of specimens, rather than keeping them constant, multi-point data is obtained.

In January 1972, six years after the conception of committee PLC/36, BS4618: section 1.2 was published, providing a simple formula for the calculation of impact energy. However, since the
time of the publication of this British Standard, very rapid development of impact behaviour has occurred, necessitating the revision of the standard. This enables a more sophisticated fracture mechanics analysis of impact data to be undertaken, and a British Standard Institution Draft for public comment has been released in B.S.I. Document 78/52049 DC.

(b) CHOICE OF A TEST METHOD

Fairly recent advances in impact testing have greatly extended the scope and complexity of such measurements, often making the choice of the most suitable test procedure far from obvious. Figure 2.3 shows some of the test options available.

The wide selection of test procedures is complicated by the choice between two basic sampling procedures (figure 2.3). Thus tests can be carried out on specimens which are:
1. Fabricated solely for the purpose of testing.
2. Machined from a component.
3. The complete component.

The physical parameters of a product often place constraints on the choice of sampling procedure. A plentiful supply of raw material will frequently be required if finished products are to be tested and the product must be of suitable geometry if specimens are to be machined from it. Conversely, the initial stages of a material selection exercise would probably benefit from low material consumption and rapid specimen production time, these being facilitated by the use of laboratory made specimens.

Finished product testing does, however, possess the advantage of exposing fabrication defects, such as the effects of anisotropy related to either polymer, or polymer and reinforcement orientation, which may not be apparent in laboratory produced samples.
It should be decided before suitable test specimens are made, whether conventional impact testing or fracture mechanics methods are to be used. Fracture mechanics analysis should normally be used only for tests resulting in unstable brittle fractures, and the validity of test results subsequently decided.

The preference shown for determination of either $G_C$, the critical strain energy release rate, or $K_C$, the critical stress field intensity factor, should follow from the initial objective of the test. The parameter, $K_C$, is the more useful in design applications, whilst both are valuable in materials selection.

![Figure 2.3 Test options](image-url)
2.5.1 **PLANE STRAIN FRACTURE TOUGHNESS STANDARD TEST**

The test procedure for plane strain fracture toughness testing is standardised [Refs 29-30] by the American Society for Testing and Materials, ASTM. There are a few requirements to be fulfilled to obtain a condition of plane strain at a crack tip. The ASTM standard provides these criteria.

Srawley and Brown [Refs 31-32] have contributed much to establish the standard for $K\_IC$ testing. The recommended specimens are the three point bend specimen, the compact tension specimen, and the C-shaped specimen. The bend specimen and the compact tension specimen are shown in figure 2.4. They are the general purpose specimens. The C-shaped specimen was especially designed for fracture toughness testing of cylinders and thick bars.

![Figure 2.4 Standard specimens](image)

**Figure 2.4 Standard specimens**

a. Bend specimen; b. Compact tension specimen
Fracture toughness

The dimensions of standard specimens should be such that the width \( W \) is twice the thickness. If this leads to impractical specimen dimensions, alternative sizes are allowed. For bend specimens, the thickness \( B \) may be between \( 0.25 \ W \) and \( W \), compact tension specimens may have a thickness between \( B=0.25 \ W \) and \( B=0.5W \).

The \( K \)-expressions [Ref 33, pages 180-181] are for the bend specimen (the symbols are defined in figure 2.4):

\[
K=(PS/BW^2)x \\
x[2.9(a/W)^2-4.6(a/W)^2+21.8(a/W)^2-37.6(a/W)^2+38.7(a/w)^2] \quad (2.6)
\]

and for the compact tension specimen

\[
K=(P/BW^2)x \\
x[29.6(a/W)^2-185.5(a/W)^2+655.7(a/W)^2-1017(a/a)^2+639(a/w)^2] \quad (2.7)
\]

These expressions are valid only in the range \( 0.45 < a/W < 0.55 \), which covers the allowable range of crack sizes in standard specimens.

Srawley [Ref 34] has proposed new wide range stress intensity expressions. These are for the bend specimen:

\[
K=(PS/BW^2)x3(a/W)^2 \times \\
x[1.99-(a/W)(1-a/W)(2.15-3.93a/W+2.7a^2/W^2)]/2(1+2a/W)(1-a/w) \quad (2.8)
\]

and for the compact tension specimen:

\[
K=(P/BW^2)x(2+a/W) \times \\
x[0.886+4.64a/W-13.32(a/W)^2+14.72(a/W)^3-5.6(a/W)^4]/(1-a/W)^2 \quad (2.9)
\]
Equation (2.8) is accurate within 0.5 per cent over the entire range of $a/W$. Equation (2.9) is also accurate within 0.5 per cent, but only in the range $0.2 < a/W < 1$. Therefore, equations (2.8) and (2.9) can be used for standard specimens as an alternative for equations (2.6) and (2.7). Due to their larger range of validity equations (2.8) and (2.9) are preferable, because they are general purpose expressions.

The specimens have to be provided with a fatigue crack. In order to ensure that cracking occurs at the right place, the specimens contain a starter notch.

2.5.1.1 SPECIMEN SIZE REQUIREMENT

The accuracy with which $K_{IC}$ describe the fracture behaviour depends on how well the stress intensity factor characterizes the conditions of stress and strain immediately ahead of the tip of the fatigue precrack, since it is here that unstable crack extension would originate. The relevant dimensions (see figure 2.4) are [Ref 35, page 98]:

1) The crack length, $a$.
2) Specimen thickness, $B$.
3) The remaining uncracked ligament length, $W-a$.

After considerable experimental work the following minimum specimen size requirements to ensure plane strain behaviour were established:

\[ a \geq 2.5 \left( \frac{K_{IC}}{\sigma_{YS}} \right)^2 \]  \hspace{1cm} (2.10)

\[ B \geq 2.5 \left( \frac{K_{IC}}{\sigma_{YS}} \right)^2 \]  \hspace{1cm} (2.11)

\[ W \geq 5.0 \left( \frac{K_{IC}}{\sigma_{YS}} \right)^2 \]  \hspace{1cm} (2.12)

where $\sigma_{YS}$ is the yield strength.
It is important to note that the above specification of a, B and W (and all the other specimen dimensions) requires that the $K_{IC}$ value to be obtained must already be known or at least estimated. There are three ways of sizing test specimens before the required $K_{IC}$ is actually obtained:

1) Overestimate $K_{IC}$ on the basis of experience with similar materials and correlation with other types of notch toughness test, for example the Charpy V-notch test.
2) Use specimens that have as large a thickness as possible.
3) For high strength materials the ratio of $(Oy_s/E)$ can be used according to the following table, which was drawn up by the ASTM [Ref 36].

**TABLE 2.1: ASTM specifications**

<table>
<thead>
<tr>
<th>$\sigma_{ys}/E$</th>
<th>minimum values of a and B (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0050 – 0.0057</td>
<td>75.0</td>
</tr>
<tr>
<td>0.0057 – 0.0062</td>
<td>63.0</td>
</tr>
<tr>
<td>0.0062 – 0.0065</td>
<td>50.0</td>
</tr>
<tr>
<td>0.0065 – 0.0068</td>
<td>44.0</td>
</tr>
<tr>
<td>0.0068 – 0.0071</td>
<td>38.0</td>
</tr>
<tr>
<td>0.0071 – 0.0075</td>
<td>32.0</td>
</tr>
<tr>
<td>0.0075 – 0.0080</td>
<td>25.0</td>
</tr>
<tr>
<td>0.0080 – 0.0085</td>
<td>20.0</td>
</tr>
<tr>
<td>0.0085 – 0.0100</td>
<td>12.5</td>
</tr>
<tr>
<td>$\geq 0.0100$</td>
<td>6.5</td>
</tr>
</tbody>
</table>
The energy that a material can absorb during fracture is an important property for defining an allowable damage state for a safe structure. The energy release rate with crack extension is important with brittle materials, such as short fibre thermosetting composites or very high strength steels, since their failure tends to be catastrophic. A convenient standard specimen geometry for energy release rate measurement is the compact tension specimen (CTS) [Ref 37].

A closed-form analysis is available for the compact tension specimen composed of isotropic material [Refs 33 and 38]. However, this analysis cannot be applied to a brittle orthotropic material. Wetherhold and Park [Ref 39] conducted a study of energy release calculations for the compact tension specimen using brittle orthotropic materials which were short fibre sheet moulding compound (SMC) materials. They examined the effect of local fibre orientation, load distribution and notch geometry on energy release rate by using a computational fracture mechanics approach called Modified Crack Closure Integral (MCCI) [Ref 40]. This was an indirect method whereby the energy release rate was calculated, and the stress intensity factor was inferred from it.

They applied the MCCI method to the compact tension specimen, for mode I crack propagation and calculated the stress intensity from the energy release rate [Ref 33];

\[ K_1 = \left[ G_1 \sqrt{2E_1E_2 / \sqrt{(E_1/E_2)E_1/E_2 + E_1/2G_{12}}} \right]^{1/2} \] (2.13)

In the case of isotropy, the above reduces to

\[ K_1 = \sqrt{GE} \] (2.14)
where \( E_1 \) is the modulus of elasticity in the longitudinal direction,

\( E_2 \) is the modulus of elasticity in the transverse direction,

\( \nu_{12} \) is the Poisson's ratio,

\( G_I \) is the strain energy release rate for mode I,

\( K_I \) is the stress field intensity factor for mode I.
3.1 **INTRODUCTION**

This chapter discusses the properties of the component parts of the composite. Materials data for the component parts of the composite have been obtained from references 41 through 69. The orthotropic elastic properties of the composite are measured by some simple mechanical testing.

The basic mechanical properties of the composite depend on the fibre, the matrix, the fibre/matrix bond strength and fibre orientations.

The first type of fibre/matrix composites which were investigated, experimentally and analytically, were glass fibre reinforced polyethersulphone (PES) composites; these consisted of specimens containing glass fibres dispersed 90% longitudinally and 10% in the orthogonal direction in the plane of the composite layer in the PES.

3.2 **PROPERTIES OF THE MATRIX**

Polyethersulphone is a high temperature performance engineering thermoplastic. At room temperature it behaves as a traditional engineering thermoplastic, it is tough, rigid and strong and possesses outstanding long-term load-bearing properties.

At temperatures up to 250°C these properties are retained. This material has a unique combination of fabricability, strength, toughness, transparency, elevated temperature creep resistance, electrical, flame resistance and low smoke-emitting properties.
3.2.1 THERMAL STABILITY

PES can be exposed to 200°C for a time in excess of 10000 hours without significant loss of strength. It has an Underwriters' Laboratories temperature index of 180°C.

Perhaps the most widely accepted means of establishing the thermal resistance and endurance of a polymer is that used by Underwriters' Laboratories. The UL Method 746 test procedure generates property-degradation curves (property retention versus time) at various aging temperatures. The time, at each test temperature, that reduces a physical property to 50% of its original value is then plotted and a curve is fitted to the data points. This curve is used to predict the property half-life of the material at a given temperature. A UL temperature index is then determined by the inter-relationships of the test material and control material curves.

Test results on the thermal stability of PES suggest that the material will have a Tensile Half Life (time taken for tensile strength measured at room temperature to drop to 50% of original value as a result of thermal degradation) of approximately 20 years at 180°C and 5 years at 200°C.

PES has a low coefficient of thermal expansion and a low mould shrinkage. Consequently, components can be moulded to close tolerances and will not show large changes because of thermal expansion.
3.2.2 DIMENSIONAL STABILITY AT HIGH TEMPERATURES

The changes in dimensions of PES at 200°C are negligible. Being an amorphous material PES shows no post-moulding changes in dimensions at temperatures up to 200°C. Above 220°C it will start to soften and large dimensional changes will occur. However, such dimensional changes require a finite time, and short-term exposure to high temperatures, such as soldering operations, does not result in dimensional changes.

PES will absorb small quantities of moisture from the atmosphere and small dimensional changes can occur. At equilibrium water content for 65% relative humidity, these dimensional changes are of the order of 0.15% whereas in boiling water they are approximately 0.3%. The water can be removed by heating at 150-180°C. This drying procedure is recommended prior to carrying out soldering operations on film, otherwise there is a danger of water vapour bubbles being formed during the soldering. Provided PES is predried in this way no outgassing occurs at elevated temperatures.

3.2.3 MECHANICAL PROPERTIES

At room temperature PES is a tough material with a drop weight impact strength similar to that of polycarbonate. It is, however, sensitive to notches and sharp corners should be avoided in component design. PES maintains its toughness at low temperatures and even at cryogenic temperatures components fail in a ductile manner.

The tensile strength and flexural modulus of PES are less affected by temperature than those of many other thermoplastics. Even at 180°C PES has a tensile strength of 76 MN/m² and a flexural modulus of 7.8 GN/m².
The long-term load-bearing properties of PES, as measured by creep resistance, are outstanding. At room temperature PES has the best creep resistance of any polymer examined in laboratories. At elevated temperatures a substantial proportion of this creep resistance is maintained so that at 150°C PES can be classed as a good load-bearing material.

### 3.2.4 ENVIRONMENTAL RESISTANCE PROPERTIES

PES possesses good resistance to x-rays, beta rays and gamma rays in the range 20-200°C.

PES does not, however, have good resistance to outdoor weathering. It is recommended for outdoor applications only if stabilised either by incorporating carbon black or by coating with a suitable lacquer.

In common with that of most thermoplastics the resistance of PES to chemicals is dependent on stress levels (external and moulded-in) and on temperature. Normally, higher grades are superior to lower grades but the chemical resistance of all grades can be improved by annealing at 200°C.

Glass-filled PES grades are significantly superior to unfilled grades for chemical resistance.
3.2.5 FLAMMABILITY AND SMOKE

Compared to most plastics PES has low flammability; this is achieved without the aid of fire retardant additives. The Underwriters' Laboratory Standard 94 is a well recognised method for evaluating the flammability (more specifically the burning rate) of materials. The best flammability rating a material can obtain in accordance with UL Standard 94 is a UL 94 "V-O" rating. As specimen thickness has a very significant effect on flammability, the UL 94 burning rating is always accompanied by the minimum thickness for which the material warrants the rating. PES in its natural form (Grades 200P and 300P) has been awarded a Limiting Oxygen Index (L.O.I.) of 34-38 and a UL 94 V-O flammability rating at 0.5mm thickness.

Very few unmodified thermoplastic or thermoset materials feature this rating. Although many materials can be modified to achieve this high UL 94 flammability rating or a high Oxygen Index rating (the ASTM D 2863-70 Oxygen Index (O.I.) test which measures the minimum % oxygen required to support flaming combustion.), the additives adversely affect important mechanical properties (e.g. reduced toughness) and generally increase smoke emissions. The polymer does not drip or melt away from the flame source; instead, it forms a charred structure inhibiting further burning. Additionally the smoke and toxic gas evolution during burning is very low as measured by laboratory tests.

3.3 PROCESSING

Despite its excellent performance at high temperatures PES can be processed by conventional means (i.e. Injection moulding, Extrusion, Blow-moulding and Film Stacking Hot Press moulding techniques). No special equipment or modifications to standard equipment are required.
3.3.1 FILM STACKING HOT PRESS MOULDING TECHNIQUES

The film stacking technique for the production of the reinforced thermoplastics has been developed by Phillips [Ref 42] in which a moulding stack is composed from layers of a thermoplastic polymer in film form interleaved with layers of a reinforcing fabric lightly impregnated with the polymer. The stack is processed by press-moulding. The method, which has the advantage of retaining flexibility in the pre-preg, has been used successfully for a number of polymers and reinforcing fabrics, including hybrid fabrics and the high-temperature thermoplastics polysulphone and polyethersulphone. It has been shown that polyethersulphone laminates have better environmental resistance than polysulphone laminates. The process which was employed to manufacture the specimens used in this investigation was the Film Stacking Hot Press Moulding technique.

In the film stacking process fabrics of fibre and PES which were manufactured by ICI, in the form of prepregs are interdispersed with films of PES to obtain the correct volume fraction and to improve the manufactured composites. Basically the layers of film and pre-preg were stacked, with film outermost on top and bottom surfaces. In this way each layer of pre-preg was sandwiched between two films of polymer. Appropriate heat and pressures were then applied to compress and weld the stack into final moulding.

In the hot press process a split metal mould was used to manufacture the tubes; this mould had a 25mm diameter hole drilled down the 1 meter length. The film stacked material was rolled around the mandrel so that at completion of the production a 25mm diameter tube of 1.5mm wall thickness was produced.
The mould was taken up to a temperature of 280°C at which temperature the material softened and was in a state for moulding. The temperature was maintained at this value under a pressure of 10.3-14 N/mm² for 50 minutes after which time the heat was switched off and the mould was allowed to cool to room temperature.

Figure 3.1 shows the alignment of the fibres in the composite tube.

Figure 3.1 Fibre distribution in composite tube
3.4 FIBRE

There are several continuous glass fibre types with different compositions which are used as fibre reinforcement [Ref 45]:

(a) 'A' glass (alkali grade), a soda-lime glass of high alkali content is cheap but has poor physical and electrical properties although its resistance to acid attack is good. It was formerly used in the aircraft industry but is now gradually going out of production.

(b) 'C' glass (chemical resistant glass), a sodalimeborosilicate glass which has good chemical stability in corrosive environments. Therefore it is often used as the matrix component in which are in contact with, or contain, acidic materials.

(c) 'E' glass (electrical grade), a calcium aluminoborosilicate glass of low alkali content has good physical and electrical properties and reasonable weathering ability. Hence it is the most used in the construction industry.

(d) 'S' glass (high strength), a magnesium aluminosilicate mixture, has better strength properties and can be used at higher temperature. It is therefore used in applications where very high tensile strength and thermal stability is required, e.g. in the aircraft/aerospace industry.

(e) 'Z' glass (zirconia glass) has a high resistance to alkali attack and is used as a reinforcing fibre for cements, mortars and concretes.

Table 3.1 [Ref 43] presents the oxide components and their ranges for the four types of glass fibres that have been produced and used in composites. Table 3.2 [Refs 43, 53 through 61] gives the mechanical properties of these fibres.
Table 3.1 Compositional ranges for glass fibers used in composite materials.

<table>
<thead>
<tr>
<th></th>
<th>E-glass range, % (a)</th>
<th>S-glass range, % (b)</th>
<th>C-glass range, % (c)</th>
<th>A-glass range, % (d)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Silicon dioxide</td>
<td>52-56</td>
<td>65</td>
<td>64-68</td>
<td>72</td>
</tr>
<tr>
<td>Aluminum oxide</td>
<td>12-16</td>
<td>25</td>
<td>3-5</td>
<td>1</td>
</tr>
<tr>
<td>Boric oxide</td>
<td>5-10</td>
<td>...</td>
<td>4-6</td>
<td>...</td>
</tr>
<tr>
<td>Sodium oxide and potassium oxide</td>
<td>0-2</td>
<td>...</td>
<td>7-10</td>
<td>14</td>
</tr>
<tr>
<td>Magnesium oxide</td>
<td>0-5</td>
<td>10</td>
<td>2-4</td>
<td>4</td>
</tr>
<tr>
<td>Calcium oxide</td>
<td>16-25</td>
<td>...</td>
<td>11-15</td>
<td>8</td>
</tr>
<tr>
<td>Barium oxide</td>
<td>...</td>
<td>...</td>
<td>0-1</td>
<td>...</td>
</tr>
<tr>
<td>Zinc oxide</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>Titanium dioxide</td>
<td>0-1.5</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>Zirconium oxide</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>Iron oxide</td>
<td>0-0.8</td>
<td>...</td>
<td>0-0.8</td>
<td>...</td>
</tr>
<tr>
<td>Iron</td>
<td>0-1</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
</tbody>
</table>

(a) Refs 47-51, (b) Refs 48-50, (c) Refs 47-52, (d) Ref 44.
Table 3.2 Typical properties for the glass fiber types.

<table>
<thead>
<tr>
<th>Material</th>
<th>Density, bulk annealed, g/cm³</th>
<th>at -190 °C (-310 °F)</th>
<th>at 23 °C (72 °F)</th>
<th>at 371 °C (700 °F)</th>
<th>at 538 °C (1000 °F)</th>
<th>Young's modulus of elasticity at 538 °C (1000 °F)</th>
<th>10⁶ psi</th>
<th>Elongation, %</th>
</tr>
</thead>
<tbody>
<tr>
<td>E-glass</td>
<td>2.62</td>
<td>5310</td>
<td>770</td>
<td>3445</td>
<td>500</td>
<td>2620</td>
<td>380</td>
<td>1725</td>
</tr>
<tr>
<td>S-glass</td>
<td>2.50</td>
<td>8275</td>
<td>1200</td>
<td>4585</td>
<td>665</td>
<td>4445</td>
<td>645</td>
<td>2415</td>
</tr>
<tr>
<td>C-glass</td>
<td>2.56</td>
<td>5380</td>
<td>780</td>
<td>3310</td>
<td>480</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Material</th>
<th>ln H₂O 10 h</th>
<th>16% HCl 24 h</th>
<th>16% H₂SO₄ 24 h</th>
<th>1% Na₂CO₃ 24 h</th>
<th>10% NaOH 168 h</th>
<th>Relative permittivity at 1 MHz</th>
<th>Dissipation factor at 1 MHz</th>
</tr>
</thead>
<tbody>
<tr>
<td>E-glass</td>
<td>0.7</td>
<td>0.9</td>
<td>42</td>
<td>43</td>
<td>2.1</td>
<td>6.6</td>
<td>0.0025</td>
</tr>
<tr>
<td>S-glass</td>
<td>0.5</td>
<td>0.7</td>
<td>3.8</td>
<td>5.1</td>
<td>2.0</td>
<td>5.3</td>
<td>0.0034</td>
</tr>
<tr>
<td>C-glass</td>
<td>1.1</td>
<td>2.9</td>
<td>4.1</td>
<td>7.5</td>
<td>24</td>
<td>6.9</td>
<td>0.0025</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Material</th>
<th>Volume resistivity, Ω⋅m</th>
<th>Surface resistivity, Ω</th>
<th>Dielectric strength</th>
<th>Viscosity softening point</th>
<th>Viscosity annealing point</th>
<th>Viscosity strain point</th>
<th>Thermal expansion</th>
<th>High temp. specific heat</th>
<th>Refractive index, bulk annealed</th>
</tr>
</thead>
<tbody>
<tr>
<td>E-glass</td>
<td>0.402 × 10¹⁵</td>
<td>0.42 × 10¹⁶</td>
<td>103</td>
<td>262</td>
<td>846</td>
<td>1555</td>
<td>657</td>
<td>1215</td>
<td>615</td>
</tr>
<tr>
<td>S-glass</td>
<td>0.905 × 10¹³</td>
<td>0.886 × 10¹³</td>
<td>130</td>
<td>330</td>
<td>970</td>
<td>1778</td>
<td>810</td>
<td>1490</td>
<td>760</td>
</tr>
<tr>
<td>C-glass</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
</tbody>
</table>

(a) From -30 °C (-20 °F) to 250 °C (480 °F)
The fibres used in most composite materials are brittle, in that they are elastic to failure with no significant plasticity. Figure 3.2 shows some stress-strain curves for these fibres.

**Fig. 3.2 Stress-strain curves of fibres used in composites**
Abstracted from reference 62.

In terms of mechanical properties, E-glass has higher tensile strength and elastic modulus than does C-glass. It also retains these properties in a wide range of temperatures better than those of C-glass as shown in Table 3.2. The specific heat versus temperature curves for E-glass and C-glass are shown in figures 3.3 and 3.4 respectively. The shape of these curves is typical for the glasses in table 3.1, except for their widely varying glass transition temperature [Refs 63-69].

**Fig.3.3 Specific heat of E-glass**
Abstracted from reference 43

**Fig.3.4 Specific heat of C-glass**
Abstracted from reference 43

The 'E' glass used in this project was mainly continuous strand in the longitudinal direction of the tube with a small proportion of randomly orientated fibre to resist any hoop stress set up in the tubes.
3.5 MEASUREMENT OF ORTHOTROPIC PROPERTIES

In addition to the glass fibre/PES composite discussed earlier glass fibre/carbon fibre/PES and carbon fibre/PES composites were tested. The following tests were designed so that the properties could be interpreted in the framework of orthotropic elastic behaviour.

a) COMPRESSION TESTS,
b) TENSION TESTS,
a) OIL PRESSURE TESTS,
d) TORSION TESTS.

Mechanical properties, therefore, are defined in terms of five constants \( (E_L, E_H, \sqrt{LH}, \sqrt{HL}, G_{LH}) \) which are referred to the orthotropic axis corresponding to the directions of the longitudinal fibres \( L \) and transverse fibres \( H \) (Hoop direction), as shown in Figure 3.5 [Refs 16,17].

![Figure 3.5 Axes defined orthotropic in composites](image)
### 3.6 DENSITY CALCULATION

Density was one of the material properties which was required for future analytical work. A simple method was used (described in the next paragraph) to determine the density of glass fibre reinforced PES composite material. This method was also used for the composite material of hybrid glass fibre/carbon fibre/PES or for the carbon fibre/PES composite material.

A one meter long tube was cut into 10 small samples of approximately 90mm length, these were then weighed and their volumes were measured by the water displacement method. Table 3.3 shows the experimental data obtained.

<table>
<thead>
<tr>
<th>Sample No</th>
<th>Length(mm)</th>
<th>Mass(gram)</th>
<th>Volume(cm$^3$)</th>
<th>DENSITY (kg/m$^3$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>89.75</td>
<td>16.17</td>
<td>10.4</td>
<td>1554.81</td>
</tr>
<tr>
<td>2</td>
<td>89.85</td>
<td>16.27</td>
<td>10.35</td>
<td>1571.98</td>
</tr>
<tr>
<td>3</td>
<td>89.85</td>
<td>16.23</td>
<td>9.2</td>
<td>1764.13</td>
</tr>
<tr>
<td>4</td>
<td>89.75</td>
<td>16.27</td>
<td>10.6</td>
<td>1527.70</td>
</tr>
<tr>
<td>5</td>
<td>89.75</td>
<td>16.18</td>
<td>9.8</td>
<td>1651.02</td>
</tr>
<tr>
<td>6</td>
<td>89.80</td>
<td>16.21</td>
<td>9.5</td>
<td></td>
</tr>
<tr>
<td>(6)</td>
<td></td>
<td></td>
<td>9.6</td>
<td>1688.54 (</td>
</tr>
<tr>
<td>7</td>
<td>89.85</td>
<td>16.21</td>
<td>10.1</td>
<td>1604.95</td>
</tr>
<tr>
<td>8</td>
<td>89.80</td>
<td>16.27</td>
<td>12.0</td>
<td></td>
</tr>
<tr>
<td>(8)</td>
<td></td>
<td></td>
<td>11.2</td>
<td>1452.68 (</td>
</tr>
<tr>
<td>9</td>
<td>89.85</td>
<td>16.19</td>
<td>10.6</td>
<td>1527.36</td>
</tr>
<tr>
<td>10</td>
<td>89.85</td>
<td>16.23</td>
<td>10.5</td>
<td>1547.71</td>
</tr>
</tbody>
</table>

Average value of $\rho = 1589.09$ kg/m$^3$
3.7 Mechanical Tests

The manufacturing cost of these materials was expensive, consequently, only a few composite tubes were tested. Six glass fibre reinforced PES specimens marked as A, B, C, D, E, F (shown in figure 3.6) of approximately 500mm length were tested in compression and tension. Four 8mm strain gauges of gauge factor 2.11+1% were bonded to the middle of the specimens such that two gauges were longitudinal and two were circumferential.

3.7.1 COMPRESSION TESTS

Compression specimens (A, B, C, D, E) were prepared by cutting the ends square to the longitudinal axis. The specimen ends were then encapsulated into a tight fitting steel plug and a metal end cap; this procedure protected the edges of the specimens and prevented local splitting. A ball bearing was placed in series with each end cap and the platens of the Instron testing machine, to ensure that a uniaxial load application through a pinned joint.

![Figure 3.6 Diagram of tested specimens](image-url)
3.7.1.1 Results

Two mechanical properties were determined from these compression tests: $\nu_{LH}$ the longitudinal Poisson's ratio and $E_L$ the longitudinal modulus of elasticity. In order to calculate the value of $\nu_{LH}$, the value of $E_L$ was divided by $E_{ps}$ (pseudo modulus).

$$\nu_{LH} = \text{hoop strain/long strain}$$

$$\nu_L = \left(\text{long stress/long strain}\right)/\left(\text{long stress/hoop strain}\right)$$

$$\nu_{LH} = \frac{E_L}{E_{ps}}$$

A typical Load versus Strain relationship for the five specimens together with a table of their results are given in figures 3.7 to 3.9 and table 3.4.

Each specimen was tested twice and their results do demonstrate linearity which means the tests were well within the elastic region of these composite tubes. The maximum compressive load exerted on these tubes was 1014 Kgf.

To establish whether the positioning of the longitudinal and transverse strain gauges, relation to the middle of the tube, had any effect on the results, two investigations were undertaken; one was conducted with the longitudinal gauge exactly at the centre and the transverse gauge off the mid point and another test was undertaken with the positions reversed. It is shown, in figures 3.8 and 3.9 that the relative positions of the gauges had no effect on their readings. It can also be seen from the results in table 3.4 that the mechanical properties have some spread in their values.
Figure 3.7 Axial load-axial strain for glass/PES.
Compression Tests

Figure 3.8 Axial load-axial strain for glass/PES Composite

Test on Glass-PES Composite

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value 1</th>
<th>Value 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Composites Elastic Modulus</td>
<td>95.8 GN/m²</td>
<td>170.6 GN/m²</td>
</tr>
<tr>
<td>Standard Error of Estimate of Load</td>
<td>1.46 kg</td>
<td>4.10 kg</td>
</tr>
<tr>
<td>Linear Correlation Coefficient</td>
<td>-0.99999</td>
<td>0.99989</td>
</tr>
<tr>
<td>Maximum Load</td>
<td>903 kg</td>
<td>0.20957</td>
</tr>
<tr>
<td>Poisson's Ratio</td>
<td>0.20957</td>
<td></td>
</tr>
</tbody>
</table>

File Name CMM1

Figure 3.8 Axial load-axial strain for glass/PES
Compression Tests

Figure 3.9 Axial load-axial strain for glass/PES Composite

Test on Glass-PES Composite

The Composites Elastic Modulus is 35.8 GPa
The Standard Error of Estimate of Load is 1.20 kg
The Linear Correlation Coefficient is -0.99999
The Maximum Load is 924 kg
The Poisson's Ratio is 0.21225

The Composite's PS Modulus is 168.5 GPa
The Standard Error of Estimate of Load is 3.91 kg
The Linear Correlation Coefficient is 0.99990

File Name: CMM2

Figure 3.9 Axial load-axial strain for glass/PES
3.7.2 **Tension tests**

Six specimens were tested in tension and the preparation for their test were as follows. Aluminium tubes of 75mm lengths were crimped and bonded onto the end of composite tubes. These aluminium ends were then sandwiched between split steel blocks, these latter enabled the specimens to be set in the jaws of the Instron testing machine. A typical load versus strain shown in figure 3.11 and the summary of the tensile test results is given in table 3.4 which indicate a good correlation with compression test results.

![Figure 3.10 Specimen set up in Instron machine for Tension test](image-url)
Test on Glass-PES Composite

- The Composites Elastic Modulus is 30.1 GN/m²
- The Composite's PS Modulus is 129.0 GN/m²
- The Standard Error of Estimate of Load is 1.27 kg
- The Standard Error of Estimate of Load is 3.53 kg
- The Linear Correlation Coefficient is 0.99998
- The Linear Correlation Coefficient is -0.99984
- The Maximum Load is 639 kg
- The Poisson's Ratio is 0.23337

Figure 3.11 Axial load-axial strain for glass/PES
### TABLE 3.4 SUMMARY OF THE RESULTS

<table>
<thead>
<tr>
<th>SAMPLE</th>
<th>COMPRESSION TEST</th>
<th>TENSION TEST</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>V (LH)</td>
<td>E (GN/m)</td>
</tr>
<tr>
<td>A A1</td>
<td>0.234</td>
<td>29.1</td>
</tr>
<tr>
<td>A2</td>
<td>0.236</td>
<td>29.7</td>
</tr>
<tr>
<td>A3</td>
<td>0.239</td>
<td>29.7</td>
</tr>
<tr>
<td>A4</td>
<td>0.236</td>
<td>29.2</td>
</tr>
<tr>
<td>B B1</td>
<td>0.244</td>
<td>36.6</td>
</tr>
<tr>
<td>B2</td>
<td>0.248</td>
<td>36.6</td>
</tr>
<tr>
<td>C C1</td>
<td>0.210</td>
<td>35.8</td>
</tr>
<tr>
<td>C2</td>
<td>0.212</td>
<td>35.8</td>
</tr>
<tr>
<td>D D1</td>
<td>0.240</td>
<td>39.2</td>
</tr>
<tr>
<td>D2</td>
<td>0.250</td>
<td>40.1</td>
</tr>
<tr>
<td>E E1</td>
<td>0.251</td>
<td>30.1</td>
</tr>
<tr>
<td>E2</td>
<td>0.250</td>
<td>30.1</td>
</tr>
<tr>
<td>F</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average</td>
<td>0.2375</td>
<td>33.5</td>
</tr>
</tbody>
</table>

**AVERAGE**

\[
\begin{align*}
V &= 0.23255 \\
E &= 33.45 \text{ GN/m} \\
E &= 144.545 \text{ GN/m}
\end{align*}
\]
3.7.3 OIL PRESSURE TEST

A Budenberg dead-weight pressure tester (figure 3.12) was adopted to set up pressure inside four half metre glass fibre/PES composite tubes. It was found that this tester would sufficiently reach the pressure required. The composite tubes were prepared for oil pressure testing as follows. Two aluminium tubes, 75mm long with 27mm thread section, were bonded to each end of the tubes. One end was blocked off with a steel cap and at the other end of the tube a drilled steel cap was fitted to allow entry of the hydraulic mineral oil (tellus oil). Four strain gauges were bonded to the middle of the tubes, two longitudinal and two in the hoop direction. Using an adapter the tube was fixed to the inlet tube of the dead-weight pressure tester.

Figure 3.12 Budenberg Dead-Weight Tester
3.7.3.1 BACKGROUND THEORY

A half metre glass reinforced PES hollow cylinder of external diameter 25mm and thickness 1.8mm was subjected to an internal oil pressure as a first step towards the characterization of the mechanical properties of these composites. The internal pressure in the tube sets up hoop stresses, $\sigma_l$, and longitudinal stresses, $\sigma_2$, which can be calculated with reference to the free body diagrams shown in figures 3.13 and 3.14 [Ref 14].

![Fig. 3.13 Free body diagram for thin-walled cylinder](image)

Resolving forces in the direction A shown in figure 3.13 gives

$$2 \sigma_1 t \ell = P \ell d$$

i.e. $\sigma_1 = P d/2 t$ \hspace{1cm} (3.1)

![Fig. 3.14 Free body diagram for thin-walled cylinder](image)

Resolving forces in the longitudinal direction

$$\sigma_2 \Pi d t = P \Pi d/4$$

$$\sigma_2 = P d/4 t$$ \hspace{1cm} (3.2)

Thus $\sigma_2 = \sigma_1 / 2$. *  

* The stresses given by equations (3.1) and (3.2) will be valid at the cross sections remote from the ends by St. Venant's principle.
3.7.3.2 Oil Pressure Results

Four specimens were internally pressurised and, from the tests undertaken, values of $\sqrt{\frac{E}{H}}$ and $E_H$ have been calculated. The first step towards the calculation of these two elastic constants was to consider the strain-stress relations for the orthotropic material:

$$\varepsilon_L = \sigma_L' / E_L - (\sqrt{\frac{E}{H}} / \sigma_H')$$

$$\varepsilon_H = - (\sqrt{\frac{E}{L}} / \sigma_L') + \sigma_H' / E_H$$

For a given pressure, longitudinal stress $\sigma_L'$, hoop stress $\sigma_H'$ and the corresponding strain values are known. In addition, the two elastic constants $\sqrt{\frac{E}{L}}$ and $E_L$ have already been obtained from the tension and compression tests. This just left solving these above equations to determine the Poisson's ratio $\sqrt{\frac{E}{H}}$ and modulus of elasticity $E_H$. For simplicity typical data obtained from one specimen is shown in table 3.5 which shows some preliminarily measurements such as specimen dimensions, hoop stresses, and longitudinal stresses. The experimental results are summarised in table 3.6.

Because of end effects (one end of specimen was blocked) the stresses near the end of the tube were not accurately given by the equations 3.1 and 3.2 and this influenced the strain values in both the hoop and the longitudinal directions. This lead to a very large value of $E_H$ and some error in $\sqrt{\frac{E}{H}}$ values. This error could have been minimised by a longer specimen, however, the effect would still have persisted but it would have been slightly smaller. This error can be seen in the table 3.6 and this is why $E_H$ value is not very reliable by this method.
### TABLE 3.5: Sample dimensions, hoop and long. stresses

<table>
<thead>
<tr>
<th>Sample no</th>
<th>Length</th>
<th>Strain gauge length</th>
<th>Resistance</th>
<th>Strain gauge factor</th>
<th>Temp. comp. for</th>
<th>Thermal output</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>500mm</td>
<td>8mm</td>
<td>119.8 Ω</td>
<td>2.11±1%</td>
<td>STEEL</td>
<td>±2 microstrain/°C</td>
</tr>
</tbody>
</table>

**Thread section**

- **Glass fibre/PES**
- **Specimen**

**EXTERNAL DIAMETER**

<table>
<thead>
<tr>
<th>Diameter (mm)</th>
<th>One end of sample no 1</th>
<th>The other end of sample no 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>24.00</td>
<td>24.00</td>
<td>24.05</td>
</tr>
<tr>
<td>24.05</td>
<td>24.05</td>
<td>24.05</td>
</tr>
<tr>
<td>24.00</td>
<td>24.00</td>
<td>24.05</td>
</tr>
<tr>
<td>24.00</td>
<td>24.05</td>
<td>24.05</td>
</tr>
</tbody>
</table>

**AVERAGE VALUE d = 24.005mm**

**INTERNAL DIAMETER**

<table>
<thead>
<tr>
<th>Diameter (mm)</th>
<th>One end of sample no 1</th>
<th>The other end of sample no 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>21.24</td>
<td>21.24</td>
<td>21.20</td>
</tr>
<tr>
<td>21.24</td>
<td>21.24</td>
<td>21.20</td>
</tr>
<tr>
<td>21.24</td>
<td>21.24</td>
<td>21.20</td>
</tr>
<tr>
<td>21.24</td>
<td>21.24</td>
<td>21.20</td>
</tr>
</tbody>
</table>

**AVERAGE VALUE d = 21.245mm**

**THICKNESS**

- t = 1.02mm

**CROSS-SECTIONAL AREA**

- = 132mm²

---

1 Bar = 14.5 1bf/in² = 14.5 x 6894.76 N/m²

1 Bar = 99.97402 N/m² = 100 kN/m²

1 Bar = 0.09997402 N/mm² = 0.1 N/mm²

<table>
<thead>
<tr>
<th>Pressure P (N/mm²)</th>
<th>Q₁² = Pd²/4t (N/mm²)</th>
<th>Q₂² = Pd/4t = Q₁²/2 (N/mm²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>5Bar</td>
<td>0.05</td>
<td>0.05</td>
</tr>
<tr>
<td>10Bar</td>
<td>0.10</td>
<td>0.10</td>
</tr>
<tr>
<td>15Bar</td>
<td>0.15</td>
<td>0.15</td>
</tr>
<tr>
<td>20Bar</td>
<td>0.20</td>
<td>0.20</td>
</tr>
<tr>
<td>25Bar</td>
<td>0.25</td>
<td>0.25</td>
</tr>
<tr>
<td>32Bar</td>
<td>0.32</td>
<td>0.32</td>
</tr>
<tr>
<td>36Bar</td>
<td>0.36</td>
<td>0.36</td>
</tr>
<tr>
<td>40Bar</td>
<td>0.40</td>
<td>0.40</td>
</tr>
<tr>
<td>45Bar</td>
<td>0.45</td>
<td>0.45</td>
</tr>
<tr>
<td>50Bar</td>
<td>0.50</td>
<td>0.50</td>
</tr>
<tr>
<td>56Bar</td>
<td>0.56</td>
<td>0.56</td>
</tr>
<tr>
<td>60Bar</td>
<td>0.60</td>
<td>0.60</td>
</tr>
<tr>
<td>66Bar</td>
<td>0.66</td>
<td>0.66</td>
</tr>
<tr>
<td>72Bar</td>
<td>0.72</td>
<td>0.72</td>
</tr>
<tr>
<td>78Bar</td>
<td>0.78</td>
<td>0.78</td>
</tr>
<tr>
<td>84Bar</td>
<td>0.84</td>
<td>0.84</td>
</tr>
<tr>
<td>90Bar</td>
<td>0.90</td>
<td>0.90</td>
</tr>
<tr>
<td>96Bar</td>
<td>0.96</td>
<td>0.96</td>
</tr>
<tr>
<td>102Bar</td>
<td>1.02</td>
<td>1.02</td>
</tr>
</tbody>
</table>

**SPECIMEN FAILED**
TABLE 3.6: SUMMARY OF OIL PRESSURE TEST RESULTS

<table>
<thead>
<tr>
<th>INTERNAL OIL PRESSURE TEST</th>
<th>( V )</th>
<th>( E_2 ) (GN/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>FIRST SPECIMEN FIRST TEST</td>
<td>0.031</td>
<td>7.913</td>
</tr>
<tr>
<td>FIRST SPECIMEN SECOND TEST</td>
<td>0.033</td>
<td>7.968</td>
</tr>
<tr>
<td>FIRST SPECIMEN THIRD TEST</td>
<td>0.025</td>
<td>7.963</td>
</tr>
<tr>
<td>Average 1st sample data</td>
<td>0.030</td>
<td>7.948</td>
</tr>
<tr>
<td>SECOND SPECIMEN FIRST TEST</td>
<td>0.027</td>
<td>7.605</td>
</tr>
<tr>
<td>SECOND SPECIMEN SECOND TEST</td>
<td>0.016</td>
<td>7.703</td>
</tr>
<tr>
<td>SECOND SPECIMEN THIRD TEST</td>
<td>* 0.016</td>
<td>7.276</td>
</tr>
<tr>
<td>SECOND SPECIMEN FAILURE TEST</td>
<td>Failed at 45 Bars</td>
<td></td>
</tr>
<tr>
<td>Average 2nd sample data</td>
<td>0.020</td>
<td>7.528</td>
</tr>
<tr>
<td>THIRD SPECIMEN FIRST TEST</td>
<td>* 0.050</td>
<td>8.851</td>
</tr>
<tr>
<td>This sample failed near center close to 4 strain gauges.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average 3rd sample data</td>
<td>0.050</td>
<td>8.851</td>
</tr>
<tr>
<td>FOURTH SPECIMEN FIRST TEST</td>
<td>0.041</td>
<td>9.272</td>
</tr>
<tr>
<td>FOURTH SPECIMEN SECOND TEST</td>
<td>0.078</td>
<td>9.581</td>
</tr>
<tr>
<td>FOURTH SPECIMEN THIRD TEST</td>
<td>0.066</td>
<td>9.071</td>
</tr>
<tr>
<td>Average 4th sample data</td>
<td>0.062</td>
<td>9.308</td>
</tr>
<tr>
<td>AVERAGE ALL ABOVE RESULTS =</td>
<td>0.040</td>
<td>8.409</td>
</tr>
</tbody>
</table>
Values of $\sqrt{\nu_{\text{LH}}}$, $\sqrt{\nu_{\text{HL}}}$, $E_{\text{L}}$ and $E_{\text{H}}$ have now been determined from the Budenberg pressure tester and from the mechanical tests. By applying the Maxwell's Reciprocal theorem, which equates the ratio of tensile moduli to that of Poisson's ratios we have:

\[
\frac{E_{\text{L}}}{E_{\text{H}}} = \frac{\sqrt{\nu_{\text{LH}}}}{\sqrt{\nu_{\text{HL}}}}
\]

\[
33.5 \quad \text{LHS} = \frac{3.98}{8.41} = 0.233
\]

\[
5.82 \quad \text{RHS} = \frac{5.82}{0.040} = 0.233
\]

3.7.4 Tests on flat coupon samples

As this theorem was not satisfied it was decided to calculate the mechanical properties of the composite material by using flat coupon samples.

In an endeavour to improve the above ratios as well as using the actual fabricated composite tube, a sample of the latter was cut along the longitudinal direction, heated and a pressure was applied to it to form the sample into a flat plate of dimension 71mm long, 25.4mm wide and 1.53 thick. This flat sample had values of $E_{\text{H}}=6.3$ GN/m² and $\sqrt{\nu_{\text{HL}}}=0.033$. The two values of $E_{\text{H}}$ and $\sqrt{\nu_{\text{HL}}}$ clearly do not agree.

![Figure 3.15 Short flat sample set-up](image)
Test on Glass-PES Composite

The Composites Elastic Modulus is 6.3 GPa
The Composite's PS Module is 195.1 GPa
The Standard Error of Estimate of Load is 7.68 kg
The Standard Error of Estimate of Load is 3.92 kg
The Linear Correlation Coefficient is 0.99134
The Linear Correlation Coefficient is -0.99775
The Maximum Load is 171 kg
The Poisson's Ratio is 0.03219

Figure 3.16 Axial load-strain for short flat sample
Three long and four short strips with dimensions as given in table 3.7 were prepared for testing. As E values in tension and compression, in both the hoop and longitudinal directions, were considered to be the same, only tension tests were carried out on these long and short samples.

**Table 3.7 Samples size**

<table>
<thead>
<tr>
<th></th>
<th>t thickness</th>
<th>w width</th>
<th>L Length</th>
</tr>
</thead>
<tbody>
<tr>
<td>Long samples</td>
<td>1 t=1.54mm</td>
<td>w=25.80mm</td>
<td>L=302 mm</td>
</tr>
<tr>
<td></td>
<td>2 t=1.54mm</td>
<td>w=25.82mm</td>
<td>L=302 mm</td>
</tr>
<tr>
<td></td>
<td>3 t=1.78mm</td>
<td>w=25.34mm</td>
<td>L=285 mm</td>
</tr>
<tr>
<td>Short samples</td>
<td>1 t=1.91mm</td>
<td>w=25.45mm</td>
<td>L=71.3 mm</td>
</tr>
<tr>
<td></td>
<td>2 t=1.91mm</td>
<td>w=25.45mm</td>
<td>L=71.3 mm</td>
</tr>
<tr>
<td></td>
<td>3 t=1.745mm</td>
<td>w=25.44mm</td>
<td>L=72.5 mm</td>
</tr>
<tr>
<td></td>
<td>4 t=1.55mm</td>
<td>w=25.2 mm</td>
<td>L=71.0 mm</td>
</tr>
</tbody>
</table>

Four 8mm strain gauges were bonded to the middle of each sample, two in the longitudinal direction, two in the hoop direction. A selection of the experimental results obtained from these samples, are shown in figures 3.17 to 3.21.

MOMTV1 Trial Sample Figure 3.16
MOMTV2 Short Sample no. 1 Figure 3.17
MOMTV4 Short Sample no. 3 Figure 3.18
MOMTV7 Short Sample no. 3 Figure 3.19
MOMTV8 Short Sample no. 4 Figure 3.20

Sample no. 1, reference MOMTV2 failed at 100 kg load about one end just outside the Instron supports.

Figure 3.18 (reference MOMTV4) shows sample no. 3 produce high value of $E_{ps} = 738.3$ consequently very low value of $V_{HL} = 0.0088$. 
Test on Glass-PES Composite

The Composites Elastic Modulus is 5.9 GPa
The Standard Error of Estimate of Load is 2.70 kg
The Linear Correlation Coefficient is 0.9545
The Maximum Load is 99 kg

The Composite's PS Modulus is 150.2 GPa
The Standard Error of Estimate of Load is 2.24 kg
The Linear Correlation Coefficient is -0.99687
The Poisson's Ratio is 0.03928

Figure 3.17 Axial load-strain for short flat sample
Test on Glass-PES Composite

- The Composites Elastic Modulus is 6.5 GN/m²
- The Composite's PS Modulus is 738.3 GN/m²
- The Standard Error of Estimate of Load is 1.89 kg
- The Standard Error of Estimate of Load is 15.67 kg
- The Linear Correlation Coefficient is 0.99700
- The Linear Correlation Coefficient is -0.76719
- The Maximum Load is 84 kg
- The Poisson's Ratio is 0.00879

File Name MGMT4

Figure 3.18 Axial load-strain for short flat sample
Oil Pressure Test

![Test on Glass-PES Composite](image)

- Axial Load (kg)
- Axial Strain (microstrain)
- Transverse Strain Gauges
- Longitudinal Strain Gauges

**Figure 3.19 Axial load-strain for short flat sample**
### Test on Glass-PES Composite

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Composites Elastic Modulus</td>
<td>6.9 GPa</td>
</tr>
<tr>
<td>Composite’s PS Modulus</td>
<td>170.5 GPa</td>
</tr>
<tr>
<td>Standard Error of Estimate of Load</td>
<td>2.77 kg</td>
</tr>
<tr>
<td>Standard Error of Estimate of Load</td>
<td>1.54 kg</td>
</tr>
<tr>
<td>Linear Correlation Coefficient</td>
<td>0.99530</td>
</tr>
<tr>
<td>Linear Correlation Coefficient</td>
<td>-0.99855</td>
</tr>
<tr>
<td>Maximum Load</td>
<td>100 kg</td>
</tr>
<tr>
<td>Poisson’s Ratio</td>
<td>0.04029</td>
</tr>
</tbody>
</table>

**Figure 3.20** Axial load-strain for short flat sample
Because of the apparently unreliable value of $E_p$, the short sample no. 3 was tested three more times, still producing unreasonable data. For instance at one point there was no lateral strain recording and at another a piece of the sample broke off because of a crack near one end of the sample. Figure 3.19 (reference MOMTV7) is the result of a short sample clamped near the four strain gauges at one end. Sample no. 4 (figure 3.20, reference MOMTV8) was taken to replace sample no. 3 for tension test.

### 3.7.4.1 Long and Short samples Results

**Three Long samples**

<table>
<thead>
<tr>
<th>$E_L$ (GN/m²)</th>
<th>$\gamma_{LH}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>27.6</td>
<td>0.197</td>
</tr>
<tr>
<td>28.2</td>
<td>0.200</td>
</tr>
<tr>
<td>25.2</td>
<td>0.190</td>
</tr>
</tbody>
</table>

Average values 27.0 0.196

**Three Short samples**

<table>
<thead>
<tr>
<th>$E_H$ (GN/m²)</th>
<th>$\gamma_{HL}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.9</td>
<td>0.039</td>
</tr>
<tr>
<td>7.4</td>
<td>0.042</td>
</tr>
<tr>
<td>6.9</td>
<td>0.040</td>
</tr>
</tbody>
</table>

Average values 6.73 0.040

Maxwell's Reciprocal theorem

$E_L/E_H = 4.01$

$\gamma_{LH}/\gamma_{HL} = 4.9$
The mechanical properties of the flat coupon samples satisfied Maxwell's Reciprocal theorem better than the composite tube properties. The flat samples properties therefore will be used for analytical work.

3.7.5 TORSION TEST

Two glass fibre/PES specimens of identical length 375mm were tested by means of a 'TECQUIPMENT' torsion testing machine. The test specimens were fitted with two machined steel end blocks to allow them to be mounted into the testing machine. Each end consisted of a steel cylindrical block A, at one side and a pin of hexagonal cross-section B to be fixed on to the testing machine at the opposite end; Figure 3.21 shows the arrangement. The end blocks were machined to allow block A to be fixed to specimens by means of screws. Epoxy resin was used to bond the specimen to the end blocks. During testing, the applied torsional moment and the corresponding deflections, $x_1$, and $x_2$, (of two marked positions on the two rods) were recorded. The two deflections were measured by means of dial gauges in a direction perpendicular to the plane containing the rod and the axis of the specimen.

3.7.5.1 THE TORSION TEST RESULTS

In the first test the intention was to fail the sample in torsion, unfortunately dial gauges could not record any deflections after a torque value of 10Nm. Therefore, a second torsion test with a different lever arm $(y_1, y_2)$ was performed. This time sample no. 1 at 14Nm torque value did not take any more load which indicated that the sample had failed. From these two torsion tests an average value of $G_{LH} = 3.15$ GN/m² was obtained.
Sample no. 2 was prepared for more torsion tests. In the first test the sample was twisted up to a torque value of 10Nm. Twenty minutes was allowed to carry out the second test which was done up to 10Nm again. From this test a value of \( G_{\text{LH}} = 2.82 \text{ GN/m}^2 \) was calculated.

In the third test, sample no. 2 at torque value of 17Nm continued to extend without taking any further load.

It was then decided that the fourth test should fail sample no. 2. This sample took torque of 18Nm without dropping off any load. For this reason, the fifth and final test, approximately three hours later, was carried out.

This time sample no. 2 reached a torque value of 21Nm and test had to be stopped since the dial gauges reached their limit of recording deflections.

Analysing the last three tests results of sample no. 2, it was clearly established that after the 10 or 11th points on the graph of Torque vs \( J*\Theta/L \) the rest of experimental points lied on a curve i.e. they illustrate non-linearity. For a better and reliable result only the first 10 points on each of three tests graphs were considered to be used for calculation of \( G_{\text{LH}} \) value.

The value of the shear modulus (\( G_{\text{LH}} \)) was obtained from the displacements \( x_1 \) and \( x_2 \), the applied torque, and the geometrical dimensions of the tested specimens.

The value of the shear modulus (\( G_{\text{LH}} \)) was determined by averaging the results of all the tests and this is summarised in table 3.8. A typical data, fifth test on sample no. 2 is selected to illustrate the torsion tests results.
Figure 3.21 The torsion test arrangement

L = Gauge Length
FIGURE 3.22 Polar moment x Twist angle x longitudinal length versus Torque of Glass Fibre/PES
TABLE 3.8: TORSION TEST RESULTS

<table>
<thead>
<tr>
<th>TORQUE VALUE (Nm)</th>
<th>SHEAR MODULUS (GN/m²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>3.2626581</td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>3.1476593</td>
</tr>
<tr>
<td>13</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>2.8223060</td>
</tr>
</tbody>
</table>

First specimen

Second specimen

It seemed that some nonlinearity appeared in the experimental results because of this only linear part of the next three tests (graphs) was used to determine G value (i.e. 10 points on each of the graphs).

| 10                | 2.8662157             |
| 18                | 2.8873000             |
| 21                | 2.8870053             |
|                   | 2.9788574             |

Average of the above results, Shear Modulus, G = 2.98 GN/m²

LH

E =27.0 GN/m²

L

ν =0.196

E =2.6 GN/m²

LH

PES

E =6.73 GN/m²

H

ν =0.040

ν =0.42

PES
3.8 Mechanical tests on carbon/glass/PES composite tubes

3.8.1 Tension tests on coupon samples

At this point of the research, the second type of material (carbon fibre/glass fibre/PES) composite tubes were tested in a similar manner to the glass fibre/PES composite tubes. Three long and three short flat samples of carbon fibre/glass fibre/PES were prepared for tension tests. Since there were two types of fibre running in the longitudinal direction, it was decided to carry out a trial test on sample no. 1 as follows;

![Diagram of strain gauges position on sample no. 1](image)

**Figure 3.23 Strain gauges position on sample no. 1**

In the longitudinal direction (figure 3.23), there were two 5mm strain gauges; one gauge was positioned on a carbon fibre and another one was positioned on a glass fibre. However, in the transverse direction there was only one 8mm strain gauge which layed across both the glass and carbon fibres. Both type of gauges had an identical gauge factor of 2.11±1%.

Tension tests on the three long samples started by considering the strain gauges on the carbon fibre first. The results were summarized in table 3.9.

This was followed by three tests on the strain gauges on the glass fibre. For no apparent reason the first test gave a
different pseudo modulus value of 170.3 GN/m. From the experimental data (table 3.9), it was concluded that it might be right to have one gauge positioned longitudinally regardless of where this gauge should be (i.e. either on a glass or on a carbon fibre) and another one in the transverse direction.

A long flat sample (no. 2) was therefore prepared and two tension tests were carried out on it. Both these tests produced high values of pseudo modulus and consequently low values of Poisson's ratios. This is in comparison with the pseudo modulus values from the sample no. 1 results.

Two more tension tests were carried out on sample no. 3. It was concluded from both samples no. 2 and 3 results that there might be an error in the strain gauge reading in the transverse direction. For this very reason, it was decided to remove the transverse strain gauge on samples no. 2 and 3. The buckled surfaces of both samples were sanded and some PES resin was sprayed on these surfaces and then two new 8mm strain gauges were bonded on to these flat (prepared) surfaces of both samples no. 2 and 3.

Two more samples were prepared for testing (samples no. 4 and 5) and two tension tests were carried out on each sample. Notice that sample no. 4 was turned through 180° laterally in order to produce a better data as shown in figure 3.24 (C/GMOMT11). Data obtained using these long samples did, however, gave lower values of $E_L, E_{ps}$ consequently a lower value of $V_{LH}$. This was a good indication that preparing the samples (as described above) did improve the results.
Test on Carbon/Glass-PES Composite

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elastic Modulus</td>
<td>36.6 GPa</td>
</tr>
<tr>
<td>Composite's PS Modulus</td>
<td>186.6 GPa</td>
</tr>
<tr>
<td>Standard Error of Estimate of Load</td>
<td>0.89 kg</td>
</tr>
<tr>
<td>Standard Error of Estimate of Load</td>
<td>3.42 kg</td>
</tr>
<tr>
<td>Linear Correlation Coefficient</td>
<td>0.99997</td>
</tr>
<tr>
<td>Linear Correlation Coefficient</td>
<td>-0.99954</td>
</tr>
<tr>
<td>Maximum Load</td>
<td>401 kg</td>
</tr>
<tr>
<td>Poisson's Ratio</td>
<td>0.19627</td>
</tr>
</tbody>
</table>

Figure 3.24 Axial load-strain for long flat sample
<table>
<thead>
<tr>
<th>Sample no</th>
<th>Tension test</th>
<th>Sample size width (mm)</th>
<th>Thickness (mm)</th>
<th>E (GN/m)</th>
<th>2</th>
<th>V</th>
<th>LH</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>First test</td>
<td>25.0</td>
<td>2.2</td>
<td>1.9</td>
<td>38.7</td>
<td>0.216</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>Second test</td>
<td>25.4</td>
<td>1.9</td>
<td>1.7</td>
<td>38.9</td>
<td>0.220</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
<td>288 mm</td>
<td>30.8</td>
<td>2.2</td>
<td>2.0</td>
<td>50.2</td>
<td>0.220</td>
</tr>
<tr>
<td>2</td>
<td>First test</td>
<td>25.7</td>
<td>1.6</td>
<td>1.9</td>
<td>40.0</td>
<td>0.158</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Second test</td>
<td>25.4</td>
<td>1.9</td>
<td>1.95</td>
<td>40.4</td>
<td>0.152</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>287.85 mm</td>
<td>39.9</td>
<td>2.2</td>
<td>39.9</td>
<td>0.229</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>First test</td>
<td>24.8</td>
<td>1.6</td>
<td>1.7</td>
<td>43.5</td>
<td>0.199</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Second test</td>
<td>24.6</td>
<td>1.6</td>
<td>1.6</td>
<td>44.2</td>
<td>0.200</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>287.8 mm</td>
<td>43.8</td>
<td>2.2</td>
<td>43.8</td>
<td>0.200</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>First test</td>
<td>25.0</td>
<td>1.66</td>
<td></td>
<td>37.2</td>
<td>0.195</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Second test</td>
<td></td>
<td></td>
<td></td>
<td>36.6</td>
<td>0.196</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>First test</td>
<td>25.7</td>
<td>1.7</td>
<td></td>
<td>31.7</td>
<td>0.153</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>Second test</td>
<td></td>
<td></td>
<td></td>
<td>31.5</td>
<td>0.153</td>
<td></td>
</tr>
<tr>
<td><strong>Average all above results</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>36.54</td>
<td>0.192</td>
<td></td>
</tr>
</tbody>
</table>
3.8.2 Tension tests on short carbon/glass/PES samples

To determine mechanical properties $E_H$ and $\sqrt{L}$ of carbon fibre/glass fibre/PES composite tubes, three short flat specimens, each 72 mm long, 26 mm wide were tested in tension. A number of 2 mm strain gauges were stuck on these specimens approximately midway along the length, one in the longitudinal and another in the transverse direction (Note: strain gauges were mounted on both side of the specimen) as shown in figure 3.25;

![Figure 3.25 Position of strain gauges on a short sample](image)

Each specimen was tested twice in the Instron Machine with a loading speed of 0.05 cm/minute. The longitudinal strain gauge on sample no. 3 measured tension followed by compression behaviour. The positive slope of the stress versus strain for this particular gauge was high and consequently, more tests were carried out on specimen no. 3.

Initially the tensile test was stopped at approximately 71 kg as the integrity of one of the longitudinal strain gauges was suspect due to a drop in tensile strain (figure 3.28). Subsequent inspection showed the gauge to be intact and this was confirmed as the modulus calculated from the average of the two longitudinal strain gauges gave a comparable value to that obtained from the previous specimens. The specimen was again tested in tension (figure 3.29) and again the tensile elastic modulus ($E_H$) calculated from the average of the two gauges was in good agreement, however, one of the gauges still showed a sudden change from a tensile to compressive strain.
On all three specimens lateral strain measurement proved extremely difficult due to positioning of strain gauges on either carbon or glass fibres. These gauges seemed to reflect local fibre type behaviour and not the overall composite behaviour.

Figure 3.30 was tested at low load.
Figure 3.31 was tested at high load.
Both above tests were carried out to show the strain gauge producing this uncharacteristic behaviour.

During the manufacturing of these tubes (in the Film Stacking Hot Press Moulding Technique [Ref 42]) small areas of the composite material tend to be squeezed at the joint of the split steel mould as shown in the diagram below. The fibres at this point will undoubtedly be pinched and when the composite tube is opened out to enable test specimens to be formed from the tube two distinct ripples appear in the transverse fibres.

![Diagram](diagram.png)

**Figure 3.26 Manufacturer's moulding arrangement**

Three tests were carried out using the Instron testing machine. The jaws of the machine were positioned over the two ripples to minimise their effect. However, the results showed that a possible slip occurred.
Test on Carbon/Glass-PES Composite

The Composite Elastic Modulus is 6.0 GPa
The Composite's PS Modulus is 300.1 GPa
The Standard Error of Estimate of Load is 1.34 kg
The Standard Error of Estimate of Load is 5.25 kg
The Linear Correlation Coefficient is 0.99748
The Linear Correlation Coefficient is -0.98298
File Name C/SMQMT18

The Maximum Load is 71 kg
The Poisson's Ratio is 0.01971

Test on Carbon/Glass-PES Composite

The Composite Elastic Modulus is 6.0 GPa
The Composite's PS Modulus is 300.1 GPa
The Standard Error of Estimate of Load is 1.34 kg
The Standard Error of Estimate of Load is 5.25 kg
The Linear Correlation Coefficient is 0.99748
The Linear Correlation Coefficient is -0.98298
File Name C/SMQMT18

The Maximum Load is 100 kg
The Poisson's Ratio is 0.01679

Figure 3.28 Axial load-strain for short flat sample

Figure 3.29 Axial load-strain for short flat sample
Test on Carbon/Glass-PES Composite

The Composite Elastic Modulus is 7.2 GPa
The Composite's PSM Modulus is 393.0 GPa
The Standard Error of Estimate of Load is .87 kg
The Standard Error of Estimate of Load is 4.26 kg
The Linear Correlation Coefficient is 0.99719
The Linear Correlation Coefficient is -0.99101
The Maximum Load is 41 kg
The Poison's Ratio is 0.01963

File Name C/GM0HT20

Figure 3.30 Axial load-strain for short flat sample

Test on Carbon/Glass-PES Composite

The Composite Elastic Modulus is 6.4 GPa
The Composite's PSM Modulus is 769.3 GPa
The Standard Error of Estimate of Load is 1.32 kg
The Standard Error of Estimate of Load is 6.87 kg
The Linear Correlation Coefficient is 0.99917
The Linear Correlation Coefficient is -0.97711
The Maximum Load is 180 kg
The Poison's Ratio is 0.00003

File Name C/GM0HT21

Figure 3.31 Axial load-strain for short flat sample
**TABLE 3.10: SUMMARY OF TEST RESULTS ON SHORT SAMPLES**

<table>
<thead>
<tr>
<th>Sample no.</th>
<th>Tension test</th>
<th>Sample size</th>
<th>E (GN/m)</th>
<th>2 V HL</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>width (mm)</td>
<td>thickness (mm)</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>First test</td>
<td>25.2</td>
<td>1.725</td>
<td>6.1</td>
</tr>
<tr>
<td>1</td>
<td>Second test</td>
<td>Sample cracked</td>
<td>5.3</td>
<td>0.003</td>
</tr>
<tr>
<td>2</td>
<td>First test</td>
<td>25.6</td>
<td>1.45</td>
<td>6.0</td>
</tr>
<tr>
<td>2</td>
<td>Second test</td>
<td></td>
<td>6.2</td>
<td>0.051</td>
</tr>
<tr>
<td>3</td>
<td>First test</td>
<td>25.5</td>
<td>1.55</td>
<td>6.0</td>
</tr>
<tr>
<td>3</td>
<td>Second test</td>
<td></td>
<td>6.0</td>
<td>0.017</td>
</tr>
<tr>
<td>3</td>
<td>Third test</td>
<td></td>
<td></td>
<td>7.2</td>
</tr>
<tr>
<td>3</td>
<td>Fourth test</td>
<td>No readings from</td>
<td>6.4</td>
<td>0.008</td>
</tr>
<tr>
<td>3</td>
<td>Fifth test</td>
<td>two gauges.</td>
<td></td>
<td>5.2</td>
</tr>
<tr>
<td>3</td>
<td>Sixth test</td>
<td></td>
<td>6.4</td>
<td>0.014</td>
</tr>
<tr>
<td>3</td>
<td>Seventh test</td>
<td></td>
<td>6.0</td>
<td>0.017</td>
</tr>
<tr>
<td>3</td>
<td>Eighth test</td>
<td></td>
<td>6.0</td>
<td>0.016</td>
</tr>
</tbody>
</table>

**AVERAGE ALL ABOVE RESULTS**

|          | 6.21 | 0.023 |
3.8.3 TORSION TESTS ON Carbon/Glass/PES

Three samples were cut out in 330mm length from a one metre length of carbon fibre/glass fibre/PES composite tube. Their dimensions are:

Table 3.11: Samples size

<table>
<thead>
<tr>
<th>Sample</th>
<th>External Diameter (φ)</th>
<th>Internal Diameter (d)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>24.95</td>
<td>21.225</td>
</tr>
<tr>
<td>2</td>
<td>24.95</td>
<td>21.1</td>
</tr>
<tr>
<td>3</td>
<td>25.00</td>
<td>21.0</td>
</tr>
</tbody>
</table>

Each sample was tested in torsion up to a torque value of 10Nm three times and their results is shown in table 3.12.

First torsion test on the third sample indicated a very unusual low value of shear modulus which was considered as unreliable data. However, second torsion test gave more reasonable shear modulus value and two more torsion tests were carried out on the third sample.

TABLE 3.12: TORSION TEST RESULTS

<table>
<thead>
<tr>
<th>TORQUE VALUE (Nm)</th>
<th>SHEAR MODULUS (GN/m²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>2.37</td>
</tr>
<tr>
<td></td>
<td>2.40</td>
</tr>
<tr>
<td></td>
<td>2.40</td>
</tr>
<tr>
<td>First specimen</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>2.39</td>
</tr>
<tr>
<td></td>
<td>2.40</td>
</tr>
<tr>
<td>second specimen</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>1.98</td>
</tr>
<tr>
<td></td>
<td>2.16</td>
</tr>
<tr>
<td></td>
<td>2.12</td>
</tr>
<tr>
<td>Third specimen</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>2.20</td>
</tr>
</tbody>
</table>

Average of the above results, Shear Modulus, $G_{LH} = 2.284$GN/m²

Typical data is shown in figure 3.32.
FIGURE 3.32: Polar moment [Twist angle/Longitudinal length] versus Torque of Glass Fibre/PES
The conclusion is that the mechanical properties of carbon fibre/glass fibre reinforced PES composite tubes are as follows:

**TABLE 3.13: Summary of the results**

Shear Modulus, $G = 2.284 \text{ GN/m}^2$

<table>
<thead>
<tr>
<th></th>
<th>$\gamma$</th>
<th>$E$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$LH$</td>
<td>0.192</td>
<td>38.5</td>
</tr>
<tr>
<td>$L$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$E$</td>
<td>0.023</td>
<td>6.21</td>
</tr>
<tr>
<td>$H$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

3.9 **TORSION TESTS ON Carbon fibre/PES**

Two types of carbon fibre reinforced PES composite tubes were tested in torsion. First type had 90% carbon fibres in the longitudinal direction and 10% carbon fibres in the transverse direction. Three samples of this type were chosen but only two were tested in torsion up to a torque value of 10Nm and then this torque value was decreased. Their results is summarised in table 3.14. Second type had $+10/-10/+10/-10\%$ carbon fibres distributed in the matrix PES and each samples of this type was tested in torsion up to a torque value of 10Nm three times and their results is shown in table 3.15.

First torsion test on the sample A indicated a very unusual (low) value of shear modulus which was considered as unreliable data. However, the second torsion test gave more reasonable shear modulus value and two more torsion tests were carried out on this sample.
### TABLE 3.14: TORSION TEST RESULTS

<table>
<thead>
<tr>
<th>Torque Value (Nm)</th>
<th>Length (mm)</th>
<th>Shear Modulus (GN/m²)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>First specimen</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td></td>
<td>2.05</td>
</tr>
<tr>
<td>10</td>
<td></td>
<td>2.07</td>
</tr>
<tr>
<td>10</td>
<td></td>
<td>2.04</td>
</tr>
<tr>
<td>2.05</td>
<td></td>
<td>2.04</td>
</tr>
<tr>
<td>2.07</td>
<td></td>
<td>2.06</td>
</tr>
<tr>
<td>2.14</td>
<td></td>
<td>2.06</td>
</tr>
<tr>
<td>2.00</td>
<td>308</td>
<td>2.08</td>
</tr>
<tr>
<td>2.01</td>
<td></td>
<td>2.01</td>
</tr>
<tr>
<td>2.07</td>
<td>307</td>
<td>2.07</td>
</tr>
<tr>
<td>2.01</td>
<td></td>
<td>2.01</td>
</tr>
<tr>
<td>2.07</td>
<td></td>
<td>2.01</td>
</tr>
<tr>
<td>2.01</td>
<td></td>
<td>2.01</td>
</tr>
<tr>
<td>2.14</td>
<td>308.5</td>
<td>2.24</td>
</tr>
<tr>
<td>2.22</td>
<td></td>
<td>2.22</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average of the above results, Shear Modulus, G = 2.081GN/m²</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### TABLE 3.15: TORSION TEST RESULTS

<table>
<thead>
<tr>
<th>Torque Value (Nm)</th>
<th>Length (mm)</th>
<th>Shear Modulus (GN/m²)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sample A</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>258.45</td>
<td>3.10</td>
</tr>
<tr>
<td>10</td>
<td></td>
<td>3.21</td>
</tr>
<tr>
<td>10</td>
<td></td>
<td>3.25</td>
</tr>
<tr>
<td>10</td>
<td></td>
<td>3.29</td>
</tr>
<tr>
<td>Sample B</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>322.0</td>
<td>3.43</td>
</tr>
<tr>
<td>10</td>
<td></td>
<td>3.52</td>
</tr>
<tr>
<td>10</td>
<td></td>
<td>3.52</td>
</tr>
<tr>
<td>Sample C</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>261.35</td>
<td>3.51</td>
</tr>
<tr>
<td>10</td>
<td></td>
<td>3.65</td>
</tr>
<tr>
<td>10</td>
<td></td>
<td>3.68</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average of the above results, Shear Modulus, G = 3.416GN/m²</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

LH
3.10 Summary and conclusions

Chapter 3 has dealt with the properties of the component parts as well as the elastic properties of the fibre reinforced composite. The relationships between the elastic constants of an orthotropic material are first assumed and the elastic moduli of fibre reinforced composites from the mechanical testing are then measured. The elastic properties of laminates are used since they satisfied Maxwell's Reciprocal theorem. The mechanical properties of all fibre reinforced composites under investigation are summarised in table 3.16.

**TABLE 3.16: SUMMARY OF PROPERTIES OF COMPOSITES**

<table>
<thead>
<tr>
<th>COMPOSITE TYPE</th>
<th>E (GN/m²)</th>
<th>E H(GN/m²)</th>
<th>V LH</th>
<th>V HL</th>
<th>G LH(GN/m²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>glass/PES</td>
<td>27.0</td>
<td>6.73</td>
<td>0.196</td>
<td>0.040</td>
<td>2.98</td>
</tr>
<tr>
<td>carbon/glass/PES</td>
<td>38.5</td>
<td>6.21</td>
<td>0.192</td>
<td>0.023</td>
<td>2.284</td>
</tr>
<tr>
<td>carbon/PES type 1</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>2.081</td>
</tr>
<tr>
<td>carbon/PES type 2</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>3.416</td>
</tr>
</tbody>
</table>

Note that type 1 carbon fibre reinforced composites has 90% fibres in the longitudinal direction and 10% fibres in the transverse direction. Type 2 has +10/-10/+10/-10% fibres distributed in the matrix PES.
Impact testing is widely used to give a better understanding of the properties of a material because impact loads are often the most severe that the material will experience. The conventional impact tests (e.g., Izod, Charpy) used for metals are not appropriate for the composite studied here because the failure mode (in which the matrix fractures but the fibres do not) prevents the normal measurements being made. Hence this chapter contains separate studies of the fracture toughness of the composite (Part A) and the composite's response to dynamic loads (Part B).
4.1 INTRODUCTION

The aim of part A of chapter 4 is to discuss the experimental and numerical techniques employed for the solution of practical fracture problems in polymers and also to present the use of computer software for undertaking the finite element method of analysis.

4.2 BACKGROUND THEORY

In this part a brief summary is provided of the basic concepts and definitions employed in the theory of fracture mechanics [Refs 70 through 98].

4.2.1 Griffith's Relationship

The basic theory on which fracture mechanics is founded emanates from the work of Griffith [Ref 70]. This involves the calculation of the fracture strength of a brittle solid (glass) which contained a sharp crack. The model, Fig. 4.1, is that of a through-thickness crack of length 2a in an infinite body, lying normal to a uniform applied tensile stress, \( \sigma'_{\text{app}} \). Plane-strain conditions are assumed (i.e. a condition of zero strain in the direction orthogonal to both the crack length and that of the applied stress).

![Figure 4.1 Model](image_url)

**FIGURE 4.1 MODEL**
An energy balance, assuming linear elastic behaviour, then gives for the fracture stress, $\sigma'_F$:

$$\sigma'_F = \left( \frac{2E \gamma}{\Gamma (1-\nu^2) a} \right)^{\frac{1}{2}} \quad (4.1)$$

where $E$ is Modulus of Elasticity
$\gamma$ is Poisson's Ratio
$2 \gamma$ is the work of fracture ($\gamma$ is often taken as the surface energy)

This expression therefore provides a relationship between fracture stress and crack length if the material's work of fracture ($2 \gamma$) is known.

4.2.2 Orowan/Irwin Relationship

The above relationship was later modified by Orowan and Irwin [Ref 70] to take account of the occurrence of plastic flow at the crack tip before the onset of crack extension. Then, using elastic relationships for the body as a whole, which can be justified only if the size of the plastic zone is very small, equation (4.1) becomes:

$$\sigma'_F = \left( \frac{E G_{Ic}}{\Gamma (1-\nu^2) a} \right)^{\frac{1}{2}} \quad (4.2)$$

where $G_{Ic}$ is the material's plane strain (opening mode I) fracture toughness

In plane-stress deformation for a very thin sheet, the $(1-\nu^2)$ factor is missing from the denominators of equations 4.1 and 4.2 and the material's toughness is then written as $G_C$.

Usually, equation 4.2, or its plane-stress version, is used
to calculate the maximum size of defect that can be tolerated under a given design stress. This process can, however, be reversed to estimate the maximum stress that can be applied to a component which contains a crack of known length.

A crack which is present in a loaded body can be deformed in different ways. Irwin [Ref 72] observed that there are three independent kinematic movements of the upper and lower crack surfaces with respect to each other, as shown in figure 4.2, and these are categorised as:

![Fundamental modes of fracture](image)

**Figure 4.2** Fundamental modes of fracture: (a) Opening Mode I; (b) Shearing Mode II; (c) Tearing Mode III.

Opening mode I: in which the two crack surfaces are pulled apart in the y-direction, but where the deformation are symmetric about the x-z and x-y planes.

Shearing mode II: in which the two crack surfaces slide over each other in the x-direction, but where the deformations are symmetric about the x-y plane and skew symmetric about the x-z plane.

Tearing mode III: in which the two crack surfaces slide over each other in the z-direction, but where the deformations are skew-symmetric about the x-y and x-z planes.
4.3 FRACTURE TOUGHNESS

The fracture toughness $K$ is a material property characterizing the crack resistance which is measured in terms of stress intensity expressed in units of $(\text{stress}) \times (\text{length})^2$.

To determine a $K$ value, a notched specimen of suitable dimensions is increasingly loaded until a crack occurs and extends abruptly. The ratio of $K_I$ to the applied load is a function of specimen design and dimensions which is evaluated by stress analysis. This $K$ value is a function of temperature and strain rate.

The parameter governing tensile fracture can be stated as a critical stress intensity, either $K_C$ (plane stress) or $K_{IC}$ (plane strain). The value of $K$ depends on specimen thickness and constraint. The limiting value of $K_I$ for maximum constraint (plane strain) is $K_{IC}$ (plane strain fracture toughness).

4.4 EXPERIMENTAL DETAILS

The recommended procedure for plane strain toughness in the ASTM designation E399-94 [Refs 75, 76 and 77] using reduced 'compact tension specimen' (CTS) are used in this work as shown below;

\[ \text{Figure 4.3 Geometry of compact tension specimen} \]
The reduced size is chosen to eliminate the instability-related tear fracture mode III; this latter can be minimised by the use of antibuckling plates restraining the specimen lateral movement. Two kinds of machine-notch geometry are used, these are:
(a) L specimen (Θ=90°),
(b) T specimen (Θ=0°),
where Θ denotes the angle between the longitudinal direction of fibres and notch as shown in Figure 4.4.

![Figure 4.4 Specimen notch geometry](image)

For these thin specimens (1.6mm on average), it was found impossible to maintain the planar shape of the sample when the load was high i.e. instability took place as shown in figure 4.4. When such instability develops, the crack and final fracture mode become tear-like. Such an instability was eliminated by decreasing the notch-crack distance 'a' so that mode I fracture was accomplished instead of mode III in the case of L-specimens. Specifically modified metal side support plates were employed also to eliminate mode III tear fracture.

![Figure 4.5 The mode of instability](image)
A glass fibre reinforced PES composite tube was cut through the centre and then two 22cm halves were made of two long halves as shown in figure 4.6;

![Figure 4.6 Composite section sizes for flattening method](image)

In order to obtain reliable test results ten samples were tested as recommended in the fracture testing procedures. A "MOORE" Hand Operated Press with Temperature Control was set to 300°C prior to placing the samples in it. Also the 22cm samples were placed in an oven at 150°C temperature before placing them onto the two steel plates covered by PTFE (service temperature=260°C). At 300°C temperature, these samples were placed on the steel plates which were already heated to the required temperature and a small pressure was exerted through the top and bottom platens of the hot Press, onto these steel plates for one hour. The pressure was gradually increased every 20 minutes until 30 kg/cm² (approximately 420 lb/in² ) pressure was achieved. The samples were kept at this pressure for about one hour. After this time the pressure was reduced and the specimens were cooled down to room temperature. This flattening method produced samples of 1.6mm thickness.

Selecting the best flattened samples, 20 small reduced size specimens [as used in Ref 78] were made.
The dimensions of the 20 samples were 34mm in length, 26mm wide and 1.6mm thick and each had a milled 60° x 2.4mm notch width and a notch depth of 10mm. The fracture tests on the first 10 'T' samples were carried out using the JJ instrument machine (figure 4.7). The first two samples failed suddenly and their results were therefore ignored. However, the remaining samples were failed in a conventional failure pattern through their 10% fibre direction. Their results is shown in figure 4.8.
Figure 4.8 Load - Deflection graphs
The second 10 'L' samples were also fractured in the JJ instrument machine. Each sample was failed through the matrix with a few fibres resisting the tensile load. During the test, each sample buckled when the load was high (i.e. Mode III behaviour was experienced). Three millimeter thick anti-buckling metal plates were therefore used to restrain the samples (figure 4.9). There was some concern, however, about friction between the metal plates and samples during the tests and this friction was considered to be minimised by the use of PTFE inside the metal plates. Apart from sample no 6 which failed at the edge of the V-notch, all the other samples failed at the tip of the V-notch. A typical result of load against deflection is shown in figure 4.10.

Figure 4.9 Anti-buckling plates dimensions
Samples from the second glass fibre reinforced PES composite tube are referred to according to the numbering system shown in figure 4.11; in this case the composite tube was cut to these sizes in order to be fitted to the Hand operated Hot Press platens. From each piece nos. 3, 8, 9 and 10, five small samples were made and in total 20 samples were made for fracture tests (figure 4.12).

A 45° cutter with a width 1/8" was used to create the required V-notch in these samples and this changed the recommended sample size as shown in figure 4.13.
The test samples were 34mm in length, 26mm in width and 1.6mm thick with a milled 45° x 4mm notch and notch depth 10mm.

The ten samples were fractured by using the JJ instrument machine and the first five samples were failed through the weakest plane (i.e. 10% fibre direction) and at the bottom of the notch. The other five samples were pre-cracked at the tip of the notch to encourage failure through 90% fibres direction. Only samples nos. 8 and 10 exhibited this required failure pattern and actually failed through the strongest plane (90% fibres direction). Their results were used to evaluate the validity of the K-value only. The load-deflection graph obtained for the 10 samples is shown in figure 4.14.
Figure 4.14  Load - Deflection graphs
4.5 TENSION TESTS ON FLAT LONG STRIPS

Four long strips were made out of the flattened pieces shown in figure 4.15 and subsequently four 8mm strain gauges with a gauge factor of 2.09±1% were bonded to the middle of the strips, two in the longitudinal direction, two in the transverse direction. Their dimensions are summarised in table 4.1.

<table>
<thead>
<tr>
<th>Sample no.</th>
<th>Width</th>
<th>Thickness</th>
<th>Length</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>12.7mm</td>
<td>1.72mm</td>
<td>190mm</td>
</tr>
<tr>
<td>2</td>
<td>12.95mm</td>
<td>1.64mm</td>
<td>190mm</td>
</tr>
<tr>
<td>4</td>
<td>13.26mm</td>
<td>1.56mm</td>
<td>190mm</td>
</tr>
<tr>
<td>6</td>
<td>13.17mm</td>
<td>1.70mm</td>
<td>189mm</td>
</tr>
</tbody>
</table>

Figure 4.15 Flattened samples
Two aluminium plates were bonded to each side of the sample and at both ends; this prevented any stress concentration being developed in the sample when placed in the jaws of the Instron testing machine; a tensile load was applied. Having carried out three tensile tests on the first sample, it was found that these long strips gave modulus value of 32 GN/m² compared to modulus value of 27 GN/m² in chapter 3. Similarly five tests were carried out on the second sample and this time the loading speed rate was changed from 0.05 to 0.1 cm/minute. Both sample nos. 4 and 6 were twice tested in tension. A typical load versus strain relationship is show in figure 4.16.

**Test on Glass-PES Composite**

- The Composite Elastic Modulus is 32.6 GN/m²
- The Standard Error of Estimate of Load is 1.82 kg
- The Linear Correlation Coefficient is 0.99983

*Figure 4.16 Axial load-strain graph*
4.6 The experimental results

To establish whether a valid $K$ was obtained from the fracture tests it was first necessary to calculate the following minimum specimen size requirements [Ref 35] which ensured nominal plane strain behaviour.

$$a > 2.5 \left( \frac{K_{IC}}{\sigma_{ys}} \right)^2$$...............................................................................(4.3)

$$B > 2.5 \left( \frac{K_{IC}}{\sigma_{ys}} \right)^2$$...............................................................................(4.4)

$$W > 4.0 \left( \frac{K_{IC}}{\sigma_{ys}} \right)^2$$...............................................................................(4.5)

where $K_{IC}$ was the plane strain fracture toughness and $\sigma_{ys}$ was the material yield strength. It must also be noted that the relevant dimensions were:
1) The crack length, $a$.
2) Specimen thickness, $B$.
3) Specimen width, $W$.

It should also be noted that to ensure cracking occurred correctly, the specimens were initially cracked at the tip of the notch.

For the 'L' specimens; at peak load value, from theory [Ref 35];

$$K_{I} = \left( \frac{P}{B \sqrt{W}} \right) f(a/w)$$...............................................................................(4.6)

Where $P$ is the peak load and

$$f(a/w) = \frac{(2+a/w) \left( 0.886 + 4.64(a/w) - 13.32(a/w)^2 + 14.72(a/w)^3 - 4.6(a/w)^4 \right)}{(1-a/w)^{3/2}}$$

substituting values for sample no. 8 from figure 4.14, the load-displacement curve, into the equation 4.6 gives;

$$K_{I} = 9.55734 \text{ MN/m}^{3/2}$$

Subsequently this leads to the $B$ values in table 4.2.
Experimental results

TABLE 4.2: The required thickness

<table>
<thead>
<tr>
<th>Sample Number</th>
<th>$\mathcal{E}$ (N/m²)</th>
<th>E (GN/m²)</th>
<th>$\sigma_{ys}$ (N/m²)</th>
<th>B (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10567x10^-6</td>
<td>30.5</td>
<td>3.22x10+8</td>
<td>2.2</td>
</tr>
<tr>
<td>2</td>
<td>14583x10^-6</td>
<td>31.9</td>
<td>4.65x10+8</td>
<td>1.1</td>
</tr>
<tr>
<td>4</td>
<td>15357x10^-6</td>
<td>32.6</td>
<td>5.01x10+8</td>
<td>0.9</td>
</tr>
<tr>
<td>6</td>
<td>10971x10^-6</td>
<td>34.9</td>
<td>3.94x10+8</td>
<td>1.5</td>
</tr>
<tr>
<td><strong>AVERAGE</strong></td>
<td>12870x10^-6</td>
<td>32.73</td>
<td>4.20x10+8</td>
<td>1.4</td>
</tr>
</tbody>
</table>

From theory $B \geq 1.4$mm while the experimental value of $B$ was 1.6mm. Therefore, the $K_1$ value obtained experimentally was valid for the sample no. 8 which failed through the 90% fibre direction.

Similarly for the 'T' specimens; substituting peak load values from figure 4.8, equation 4.6 gives table 4.3.

TABLE 4.3: The required thickness

<table>
<thead>
<tr>
<th>Sample Number</th>
<th>P (N)</th>
<th>K (N/m²)</th>
<th>$\sigma_{ys}$ (N/m²)</th>
<th>B (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>132.0</td>
<td>2.25x10+6</td>
<td>3.51x10+8</td>
<td>0.103</td>
</tr>
<tr>
<td>4</td>
<td>132.0</td>
<td>2.25x10+6</td>
<td>3.51x10+8</td>
<td>0.103</td>
</tr>
<tr>
<td>5</td>
<td>128.0</td>
<td>2.18x10+6</td>
<td>3.51x10+8</td>
<td>0.097</td>
</tr>
<tr>
<td>6</td>
<td>120.0</td>
<td>2.05x10+6</td>
<td>3.51x10+8</td>
<td>0.085</td>
</tr>
<tr>
<td>7</td>
<td>100.0</td>
<td>1.71x10+6</td>
<td>3.51x10+8</td>
<td>0.059</td>
</tr>
<tr>
<td>8</td>
<td>118.0</td>
<td>2.01x10+6</td>
<td>3.51x10+8</td>
<td>0.082</td>
</tr>
<tr>
<td>9</td>
<td>114.0</td>
<td>1.95x10+6</td>
<td>3.51x10+8</td>
<td>0.077</td>
</tr>
<tr>
<td>10</td>
<td>96.0</td>
<td>1.64x10+6</td>
<td>3.51x10+8</td>
<td>0.054</td>
</tr>
</tbody>
</table>

**Average**

$K = 2$ MN/m²

From the above results, it is clear that the $K$ value in the 10% fibre direction is consistent with the specimen size and material yield strength according to the equations (4.3), (4.4) and (4.5). This value as well as the test is therefore valid even though the specimen size was not the ASTM standard value.
4.7 STRESS INTENSITY FACTOR EVALUATION BY FINITE ELEMENT

The aim of this section is to use a standard computer program [Ref 72] for stress intensity factor evaluation in the analysis of linear elastic fracture problems. Basically, this section applies two methods out of four methods which were incorporated in the computer software program to the compact tension specimen whose geometry is shown in figure 4.3. This type of analysis has been the subject of an experimental investigation by several other researchers with a specimen of different geometry [Refs 81 and 82].

The finite element mesh for one half of the specimen about the centre line; used in the analysis is shown in figure 4.17 and contains 54 elements and 194 nodal points. It can been seen that a relatively fine element subdivision was taken around the crack tip and that the pin loading was assumed to be a single concentrated nodal load and a solution was obtained for the two methods listed below. The input data for (a) and (b) are provided in appendices A and B.

(a) Virtual crack extension approach

In this method the crack was advanced by pertubating the nodal points in the crack tip zone in the direction of crack advance [Ref 72]. In particular, nodes 17, 19, 21, 68, 70 and 72 were displaced by 0.001mm in the x-direction. Stress intensity factor $K$ was directly calculated and had a value of $K \approx 1.3$ MN/m$^{\frac{3}{2}}$; this differed from the experimental value of $K \approx 2$ MN/m$^{\frac{3}{2}}$.

(b) J-integral method

In this approach the J-integral [Ref 72] was evaluated along a contour path surrounding the crack tip. The path employed in solution passed through the lines $b = \frac{a}{2}$ of elements 6, 23, 38, 39, 40, 41, 28 and 11 and is shown as the broken line in figure 4.17. The stress intensity factor was again obtained and had a value of $K \approx 1.0$ MN/m$^{\frac{3}{2}}$. 
The computations were repeated for several different integration contours corresponding to paths through different elements; giving a total of nine contour paths. Table 4.4 shows the comparison of the experimental and theoretical values of $K_1$ and $G_1$.

**TABLE 4.4: The results**

<table>
<thead>
<tr>
<th>Experimental values</th>
<th>Theoretical values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K$ (MN/m$^{1/2}$)</td>
<td>$K$ (MN/m$^{1/2}$)</td>
</tr>
<tr>
<td>$G$ (MN/m)</td>
<td>$G$ (MN/m)</td>
</tr>
<tr>
<td>2</td>
<td>1.4E-04</td>
</tr>
<tr>
<td>1.3</td>
<td>0.6E-04</td>
</tr>
</tbody>
</table>

4.8 **LUSAS finite element analysis**

In order to ensure that the program of reference 72 had been used correctly, the finite element package, LUSAS, was used to determine crack stress intensity factor. It allowed the analysis of linear elastic static fracture of the compact tension specimen using plate elements, QPM8 and crack tip elements, QPK8; and provided a stress and displacement distribution for each element. By taking the displacement, calculated for a particular element, a value of $K_1$ was determined by the energy balance approach.

The test piece and loading conditions were symmetric, only half the specimen needed to be considered. The finite element discretisation is shown in figure 4.18 consisting of 54 eight-noded plane membrane elements, two of which are the crack tip elements with the midside nodes situated at the quarter point to reproduce the crack tip singularity. First analysis was conducted with an initial crack length 10mm and the second analysis, figure 4.19, was undertaken after repositioning the crack tip by 0.001mm. Figure 4.20 shows the crack tip elements with the crack tip position at node 17. The change in energy for this node was used to give the value of $K_1$ which is shown in table 4.5.
Figure 4.18
FINITE ELEMENT IDEALISATION OF COMPACT TENSION SPECIMEN
FINITE ELEMENT IDEALISATION OF CRACKED COMPACT TENSION SAMPLE
Figure 4.20
COMPACT TENSION SAMPLE SHOWING CRACK TIP AT NODE 17=8.001mm
4.9 SUMMARY AND CONCLUSIONS

Fracture toughness depends upon many parameters and it will often be difficult to find a toughness value for a material in a particular application [Ref 98].

This section has shown how the principles of fracture toughness testing was applied to the glass fibre reinforced PES composite materials. The two consistent values of toughness, $K$ and the fracture strain energy release rate, $G$ were obtained experimentally and compared with the finite element method results.

Basically, it was concluded that

1. Linear elastic analysis with a coarse mesh was considered to be appropriate for the stress intensity factor evaluation of composite materials.

2. The virtual crack extension approach was found to give reasonably accurate results compared with the experimental values (table 4.4).

3. It must also be appreciated that this type of computer software program is set up to model crack propagation in isotropic materials whereas the composite tube has orthotropic properties.

4. The fracture parameters from LUSAS were slightly closer to the experimental values than those obtained using the computer program from reference 72 (table 4.5).
4.10 **INTRODUCTION**

This part of chapter describes the dynamic test techniques used on the specimens and highlights the limitations and shortcomings of the test methods. It also considers natural frequency measurements using a software package.

Two types of dynamic finite element analysis were performed. The first was eigenvalue extraction to determine the natural frequencies and mode shapes of the system. The second type was a step-by-step analysis in which the response of the system to a transient load was modelled. Different element types and material properties were employed and the analytical results were compared with those from the dynamic tests. The finite element package LUSAS-A (the advanced version) was used for the eigenvalue problem and dynamic analysis [Refs 99 through 122].

4.11 **WAVEFORM PROCESSING AND GRAPHICS SOFTWARE**

This section describes the use of the ACQUIRE software package which was used to detect natural frequencies of these composite tubes.

ACQUIRE is a modular software package fully compatible with the Datalab 1200 recorder for use with HP 200 Series computers. It includes facilities for the acquisition, display and processing of analogue data, graphics plot generation and the creation of permanent files. In addition to a selection of time and frequency domain processing functions, ACQUIRE enables users to readily create their own measurement routines through its sequence generation function.
In the dynamic (impact) tests a span length of 500mm was chosen for the composite tube. Other span lengths showed varying, but always greater evidence of interference from the supports in the acceleration-time graphs that were obtained. Theoretically the natural frequency of a beam which is clamped between supports is the same as the natural frequency of a beam of the same length in the free-free condition. For this reason a tube of length of 500mm was chosen for the natural frequency measurements.

Figure 4.21 shows plots of the theoretical mode shapes for the first five modes of a free-free beam. From the experimental point of view a knowledge of these mode shapes is important for at least two reasons. Firstly, it shows the best choice of location for accelerometer in order to detect the required frequencies. Secondly, it allows the supports to be located near to the nodal positions and thus minimise effects due to springs stiffness or damping.

![Theoretical mode shapes for a free-free beam](image-url)

Figure 4.21 Theoretical mode shapes for a free-free beam
Two types of hammer tip were used, steel and plastic, and the following experimental procedures were common to both.

1. The springs were accurately positioned at 112mm from either end of the beam.
2. The accelerometer was placed at 145mm from one end of the beam in x-direction.
3. The sampling rate was set to an interval of 200 μsec.
4. The beam was impacted at 150mm from the opposite end.

The following notations are used in this section;

RFFT LIN MAG A real time domain waveform is transformed into its frequency spectrum using the Fast Fourier Transform.
TF LIN MAG Transfer Function Linear Magnitude.
RFFT LOG MAG Real FFT with Logarithmic output.

(a) Plastic tip

The composite beam was impacted in a free-free condition and five responses were recorded which are listed below;
The impact force - time response figure 4.22(a).
The acceleration - time response figure 4.22(b).
The RFFT LIN MAG of the beam figure 4.23(a).
The TF LIN MAG of the beam figure 4.23(b).
The RFFT LOG MAG of the input signal of the beam figure 4.24.

The first and second natural frequencies of the beam were accurately determined whereas the third excited frequency seemed inaccurate because the amplitude of the signal was smaller than the stated accuracy range of the equipment. This can be seen from close examination of the RFFT LOG MAG of input plot (figure 4.24).
Figure 4.22  
(a) Force - Time graph  
(b) Acceleration - Time graph
Impact hammer at 150mm (plastic tip)
4096 points
Accelerometer at 145mm from one free end
Accelerometer in X-direction
Sampling interval 0.2msec

Figure 4.23 (a) RFFT Lin Mag of glass/PES sample in a free-free condition. (b) TF Lin Mag of glass/PES sample in a free-free condition.
RFFT Log Mag for impact hammer (plastic tip)
4096 points
Accelerometer at 145mm from one free end
Impact hammer at 150mm from opposite free end
Accelerometer in X-direction
Sampling interval 0.2msec
Mean removed rectangular window

Figure 4.24 RFFT Log Mag of input glass/PES sample in a free-free condition
(b) Steel tip

Similarly five responses were obtained and the first two natural frequencies were accurately found and again the amplitude of the signal corresponding to the third frequency was smaller than the stated accuracy range of the equipment.

Figure 4.25 (a) Force-time graph. (b) Acceleration-time graph.
Waveform recording techniques

Impact hammer at 150mm (steel tip)
4096 points
Accelerometer at 145mm from one free end
Accelerometer in X-direction
Sampling interval 0.2msec

Figure 4.26 (a) RFFT Lin. Mag. of glass/Pes sample in a free-free Condition. (b) TF Lin. Mag. of glass/Pes sample in a free-free condition.
RFFT Log Mag for impact hammer (steel tip)
4096 points
Accelerometer at 145mm from one free end
Impact hammer at 150mm from the opposite free end
Accelerometer in the X-direction
Sampling interval 0.2msec
Mean removed rectangular window

Figure 4.27 RFFT Log Mag of input glass/PES sample in a free-free condition
4.12 Comment on the frequency results

The results from the frequency tests are shown in table 4.6. As can be seen, the frequency remained the same, regardless of changing the impact point or impact hammer tip.

![Diagram showing impact and accelerometer positions]

Figure 4.28 Showing the impact and accelerometer positions

### TABLE 4.6: SUMMARY OF THE FREQUENCY RESULTS

<table>
<thead>
<tr>
<th>Hammer tips type</th>
<th>Accelerometer position from one end of the beam in mm</th>
<th>Impact point at the other end of the beam in mm</th>
<th>Frequency (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PLASTIC</td>
<td>145</td>
<td>150</td>
<td>475</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1161</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>2078</td>
</tr>
<tr>
<td>PLASTIC</td>
<td>150</td>
<td>150</td>
<td>474</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1161</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>2080</td>
</tr>
<tr>
<td>STEEL</td>
<td>145</td>
<td>150</td>
<td>475</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1161</td>
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<tr>
<td></td>
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<td></td>
<td>2080</td>
</tr>
<tr>
<td>STEEL</td>
<td>150</td>
<td>150</td>
<td>474</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1177</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>2100</td>
</tr>
</tbody>
</table>
The equations of motion are:

\[ M \ddot{d} + C \dot{d} + Kd = P(t) \quad (4.7) \]

where
- **M** mass matrix
- **C** damping matrix
- **K** stiffness matrix
- **P(t)** applied nodal load vector at time *t*
- **\ddot{d}** acceleration vector
- **\dot{d}** velocity vector
- **d** displacement vector

There are two distinct reasons for calculating the natural frequencies (eigenvalues). The first is to assist in the general description of the dynamic properties of the system which would be useful in design. The lowest natural frequencies are of interest here. The second reason is to furnish an estimate of the largest time step that is permitted in the dynamic step-by-step analysis. This requires a knowledge of the largest natural frequency of the finite element model [Refs 101 and 102].

In practice we do not need to calculate the largest natural frequency of the finite element model because an important theorem proposed by Irons [Ref 100] shows that the highest system eigenvalue will always be less than the highest eigenvalue of any individual element.

The solution methods used in the eigenproblem, are based on some fundamental facts. Considering the effectiveness of the solution procedures, none of the methods is completely efficient, but, the solution technique to be used, should be selected according to the specific problem to be solved. Indeed, given a specific eigenproblem, the choice of the most appropriate method depends on the characteristics of **K** (the
stiffness matrix) and M (the mass matrix) and the number of
eigenpairs required. However, for this project the subspace
iteration method was used since in finite element analysis the
smallest eigenvalues were specifically required. This required
the solution of a large eigenproblem i.e. all the eigenvalues and
eigenvectors were required. Nevertheless, the subspace iteration
method has been extended to calculate very effectively all
eigenvalues and can also be employed in calculating the largest
eigenvalue and corresponding eigenvector.

4.14 SUBSPACE ITERATION METHOD

The subspace iteration method [Ref 99] consists of the following
three steps:
1- To Establish q starting iteration vectors, q>p where p is
the number of eigenvalues and vectors to be calculated.

2- To use simultaneous inverse iteration on the q vectors and
Ritz analysis to extract the best eigenvalue and eigenvector
approximations from the q iteration vectors.

3- To use the sturm sequence check to verify that the required
eigenvalues and corresponding eigenvectors have been
calculated, after iteration convergence.

Altogether, the subspace iteration method is largely based on
simultaneous vector iteration, sturm sequence information, and
Rayleigh-Ritz analysis.

The basic objective in the subspace iteration method is to satisfy

$$K \delta = M \delta X$$

where

\begin{align*}
K & \quad \text{stiffness matrix} \\
\delta & \quad = [\delta_1, \ldots, \delta_p] \\
M & \quad \text{mass matrix} \\
X & \quad = \text{diag}(x_i) \\
x & \quad \text{is the eigenvalue}
\end{align*}
In addition to the relation in (4.8), the eigenvectors also satisfy the orthogonality conditions:

\[ \mathbf{\phi}^T K \mathbf{\phi} = \lambda \quad (4.9) \]

and also

\[ \mathbf{\phi}^T M \mathbf{\phi} = I \quad (4.10) \]

where \( I \) is a unit matrix of order \( p \) because \( \mathbf{\phi} \) stores only \( p \) eigenvectors.

An important aspect in subspace iteration is the convergence of the method. Experience has shown that with the starting iteration vector \( q = \text{min}[2p, p+8] \), of the order of ten iterations are needed to calculate the largest eigenvalue \( \lambda \) to about six digit precision, with the smaller eigenvalues being predicted more accurately.

Most of this section describes how this precision is obtained.

The technique of inverse iteration is used to calculate an eigenvector, and at the same time the corresponding eigenvalue can also be evaluated. Inverse iteration is employed in the subspace iteration method.

In the following, the basic equations used in inverse iteration and a more effective form of the technique will be discussed in detail. In the solution a starting iteration vector \( v \) is assumed and then in each iteration step \( k=1,2, \ldots \), the
following are evaluated:

\[ \begin{align*}
  K v_{k+1} &= M v_k \quad (4.11) \\
  v_{k+1} &= \frac{v_k}{\sqrt{v_k^T M v_k}} \quad (4.12)
\end{align*} \]

where provided that \( v \) is not \( M \)-orthogonal to \( \delta_1 \), meaning that \( v^T M \delta_1 \neq 0 \), so it is true that

\[ v_{k+1} \to \delta_1 \quad \text{as} \quad k \to \infty \]

The basic step in the iteration is the solution of the equations in (4.11) in which a vector \( v_{k+1} \) is evaluated with a direction closer to an eigenvector than the previous iteration vector \( v_k \). The calculation in (4.12) merely assures that the \( M \)-weighted length of the new iteration vector \( v_{k+1} \) is unity; i.e., \( v_{k+1} \) satisfies the mass orthogonality relation

\[ v_{k+1}^T M v_{k+1} = 1 \quad (4.13) \]

substituting for \( v_{k+1} \) from (4.12) into (4.13), it is found that (4.13) is indeed satisfied.
The relations in (4.11) and (4.12) state the basic inverse iteration algorithm. However, in actual computer implementation it is more effective to iterate as follows. Assuming that $y_1 = M \hat{v}_1$, we evaluate

for $k=1,2, \ldots$,

$$K \hat{v}_{k+1} = y_k \quad (4.14)$$

$$y_{k+1} = M \hat{v}_{k+1} \quad (4.15)$$

$$\rho(\hat{v}_{k+1}) = \frac{\hat{v}_{k+1}^T y_k}{\hat{v}_{k+1}^T \hat{v}_{k+1}} \quad (4.16)$$

$$y_{k+1} = (\hat{v}_{k+1}^T \hat{v}_{k+1})^{-1/2} \hat{v}_{k+1}^T y_k \quad (4.17)$$

where, provided that $y_1^T \hat{v}_1 \neq 0$,

$y_{k+1} \rightarrow M \hat{v}_1$ and

$\rho(\hat{v}_{k+1}) \rightarrow x_1$

as $k \rightarrow \infty$
It should be noted that in the steps (4.14) to (4.17) the calculation of the matrix product $M \mathbf{v}$ in (4.11) is dispensed with by iterating on $y$. But the value of $y_k$ is evaluated in either procedure; i.e., $y_{k+1}$ must be calculated in (4.12) and is evaluated in (4.15). Using the second iteration procedure, an approximation to the eigenvalue $\lambda_1$ given by the Rayleigh quotient $\rho_{k+1}(\mathbf{v})$ is obtained in (4.16). It is this approximation to $\lambda_1$ which is conveniently used to determine convergence in the iteration. Denoting the current approximation to $\lambda_1$ by $\lambda_{1}^{(k+1)}$

[i.e., $\lambda_{1}^{(k+1)} = \rho(\mathbf{v}_{k+1})$],

convergence exists when

$$\frac{|x_{1}^{(k+1)} - x_{1}^{(k)}|}{x_{1}^{(k+1)}} < \text{tol} ; i = 1, \ldots, p \quad (4.18)$$

where tol should be $10^{-2s}$ when the eigenvalues shall be accurate to about 2s digits. For example, if iteration is carried out until all $p$ ratios in (4.18) are smaller than $10^{-6}$, it is very likely that $x_1$ has been approximated to about six-digit accuracy, and the smaller eigenvalues have been evaluated more accurately. Since the eigenvalue approximations are calculated using a Rayleigh quotient, the eigenvector approximations are accurate to only about $s$ (or more) digits. It should be noted that the iteration is performed with $q$ vectors, $q<p$, but convergence is measured only on the approximations obtained for the $p$ smallest eigenvalues.

Let $l$ be the last iteration, then

$$x_{1} = \rho_{k+1}(\mathbf{v}) \quad (4.19)$$
and

\[
\delta_1 = \frac{\mathbf{v}_{l+1}}{\mathbf{T} \mathbf{v}_{l+1}}^{\frac{1}{2}}
\]  

(4.20)

Another important aspect when using the subspace iteration technique is that of verifying that in fact the required eigenvalues and vectors have been calculated since the relations in (4.8), (4.9) and (4.10) are satisfied by any eigenpairs. This verification is the third important phase of the subspace iteration method (see section 4.14).

A conservative estimate for a region in which the exact eigenvalues of the problem \(K \delta = x M \delta\) lies is given by

\[
0.99x_1^{(l+1)} < x_1^{(l+1)} < 1.01x_1^{(l+1)}
\]  

(4.21)

where only the smallest eigenvalues that converged to a tolerance of \(10^{-2}\) should be included. The relation in (4.21) can be used to establish bounds on all exact eigenvalues, and hence a realistic Strum sequence check can be applied.
4.15 **FINITE ELEMENT MODELS**

The art of finite element analysis lies in the development of a suitable idealisation of a structure to provide the required results.

The mesh of elements must not be too fine to make the preparation of data, computer time and interpretation of results difficult, but must not be too coarse to make the accuracy of the results unacceptable. To develop a suitable idealisation, one must have some knowledge of the likely distribution of stresses in the structure, and the fineness of mesh required to provide results of acceptable accuracy at required points in the structure. It is not always possible to estimate the likely stress distribution in a structure before an analysis and it may be necessary to run a pilot coarse mesh idealisation in the first instance.

The mesh can then be refined or modified in the areas of interest, to obtain acceptable results.

There are many different types of elements available in the LUSAS element library for modelling a structure. All elements in LUSAS pass a stringent test called the patch test. This means that as a mesh of elements is made finer, the results are guaranteed to converge to the correct solution. Nearly all elements in LUSAS are numerically integrated.

Elements which were considered to be suitable for the present analysis, are shown figures 4.29 and 4.30.
Figure 4.29 THREE-DIMENSIONAL SOLID ELEMENTS

Figure 4.30 SEMILOOF THIN SHELL ELEMENTS
4.15.1 **HX16 SOLID ELEMENTS**

![Figure 4.31 16 noded thick shell element](image)

Hexahedral isoparametric solid elements with 16 nodes are capable of modelling curved boundaries (figure 4.31). These elements are numerically integrated. Nodes are numbered according to the right-hand screw rule in the local z-direction, i.e. numbering in an anticlockwise direction. These three-dimensional solid elements possess one nodal degree of freedom in each direction, i.e. three nodal degrees of freedom in the x, y and z-directions.

When considering composite specimens it is important to remember that they are highly anisotropic so that properties measured in any one direction may not necessarily be the same as those in other directions. The variation in the properties with direction posed the problem of adequate definition in the finite element model.

In LUSAS it is possible to define orthotropic properties in the directions of local axes. This procedure, however, is very tedious for the composite tube considered here and so a simplified approach was adopted in which the following properties were assumed relative to the global coordinate axes:

- Modulus of elasticity in x-direction, $E_x = 6.73 \, \text{GN/m}^2$,
- Modulus of elasticity in y-direction, $E_y = 27.0 \, \text{GN/m}^2$, 
4.15.2 QSL8 SEMILOOF ELEMENTS

A quadrilateral shell element with arbitrary geometry is shown in figure 4.32. This element can accommodate curved shell geometry with varying thicknesses and anisotropic materials may also be specified. The element formulation takes account of both membrane (in-plane) and flexural (out-of-plane) deformations and, as required by thin shell theory, shearing deformations are excluded. The semiloof element possess one nodal degree of freedom in each direction (i.e. three nodal degrees of freedom in the x, y, and z-directions) at corner nodes plus five nodal degrees of freedom (3 displacements and 2 rotations) at midside nodes. These rotations refer to the loof rotations about the edge of the element at the loof points. The +ve direction of the loof rotations is defined by the right-hand screw rule applied to a vector running in the direction of the lower to higher numbered corner node numbers along an edge. The loof points are located at \( \frac{1}{\sqrt{3}} \) of the distance from a midside node to a corner node.

As before the orthotropic properties were specified and the following values were taken from Chapter 3.

- Modulus of elasticity in x-direction, \( E_x = 6.73 \) GN/m²,
- Modulus of elasticity in y-direction, \( E_y = 27.0 \) GN/m²,
- Shear modulus, \( G_{xy} = 2.98 \) GN/m²,
Modulus of elasticity in z-direction, $E_z = 6.73$ GN/m$^2$,
Shear modulus, $G_{xy} = 2.98$ GN/m$^2$,
Shear modulus, $G_{yz} = 0.92$ GN/m$^2$,
Shear modulus, $G_{xz} = 0.92$ GN/m$^2$,
Poisson's ratio, $\nu_{xy} = 0.196$,
Poisson's ratio, $\nu_{yz} = 0.42$,
Poisson's ratio, $\nu_{xz} = 0.040$,
Mass density, $\rho = 1589.09$ kg/m$^3$,
Hysteretic damping factor, $\mu = 0$.

These properties can be compared with those measured in Chapter 3 (table 3.8). The basis for the simplified properties is that the longitudinal modulus ($E_z$) is most important in determining the behaviour of the composite tube.

The theory implemented in the LUSAS is as follows: the element formulations are based on the standard isoparametric approach. This particular arrangement of nodes means that the element is suitable for modelling the bending and in plane stresses in a thick shell. The numerical integration rules employed are 2x2x2 points for HX16. HX16 elements pass the patch test.

HX16 SOLID ELEMENTS were chosen to be the first suitable idealisation of modelling the PES specimens in this investigation. First of all, for this analysis a quarter of PES tube was considered, i.e. simplicity was considered to reduced unnecessary calculations and computer time.

The pilot coarse mesh (16 elements total) was run and then the mesh was modified to a very fine mesh (100 elements total) which satisfied the tolerances of the chosen elements, in order to obtain acceptable results.
Poisson's ratio in x-direction, \( \nu_{xy} = 0.196 \),
Angle of orthotropy relative to reference axis, \( \Theta = 0^\circ \),
Mass density, \( \rho = 1589.09 \text{ kg/m}^3 \),
Hysteretic damping factor, \( \mu = 0 \).

In contrast to the solid elements, properties for the semiloof elements are always specified relative to the local axes of the element. Therefore, there was no difficulty in using the full range of material properties measured in Chapter 3.

The element formulations are based on an isoparametric approach with constraints to invoke the Kirchhoff hypothesis for thin shells. The variation of stresses within the elements can be regarded as linear. These QSL8 elements are numerically integrated with a 5 point rule and they all pass the patch test for convergence.

After having worked with solid elements, QSL8 SEMILOOF ELEMENTS were chosen for the second analysis. The 4 elements pilot coarse mesh was used initially and then the mesh was modified to a 16 elements mesh which was satisfactory.

4.16 FREQUENCY RESULTS

For analytical purposes only the first three frequencies were summarised in table 4.7 even though ten of them were demanded in each LUSAS input data. The purpose of the eigenvalue extraction method was to establish the most suitable element to model the glass fibre reinforced composite tubes. A number of elements were used for calculation of natural frequencies but only the ones put in table 4.7 showed reasonable correlation between their results and that of the RFFT plots (see table 4.6). It must be mentioned that boundary conditions and
material properties were common problems to most of these elements. Therefore, the best obvious approach to establish the most suitable element was to consider three distinct factors.

(a) An element whose first three natural frequencies were the closest to the experimental ones.
(b) A mesh which used less computer time.
(c) An element which was previously used in impact studies of polymers.

Experimental frequencies for the 500mm glass fibre reinforced PES composite tube are found to be as follow; the first frequency is equal to 475 Hz, the second frequency is equal to 1161 Hz and the third frequency is considered to be approximately 2080 Hz. Mode shapes from analysis and theory for beams suggest that the first and second modes are as shown in figures 4.33 and 4.35. The third mode is suggested to be either figure 4.37 from analysis or figure 4.39 from theory.

One hundred solid elements, a quarter of the tube with isotropic properties gave 474 Hz as the first frequency, missed the second one and also gave 2366 Hz as the third frequency which could be either figure 4.37 or figure 4.39 (this value was 12% higher than the experimental one). With orthotropic properties where modulus of elasticity in x-direction was considered to be the same as the z-direction i.e. Ex equal to Ez, all frequencies were found to be inaccurate.

Four hundred solid elements modelling the whole tube with isotropic properties gave 475 Hz as the first frequency (which was in a good agreement with the experimental one), 1202 Hz as the second frequency (which was within 4% of the experimental value) and 2307 Hz as the third frequency which was considered to be either figure 4.37 or figure 4.39. Here again with the orthotropic properties the frequency values were found to be
inaccurate. It was concluded that the thick shell solid elements would only produced accurate results when they possess isotropic properties. This result seems surprising as the data of Chapter 3 shows that composite tube has anisotropic properties. On the other hand, it should be remembered that the anisotropic properties assumed for the solid elements are based on a simplified approach.

Sixteen semiloof elements, a quarter of the tube with isotropic properties gave 476 Hz as the first frequency, missed the second one and gave 2407 Hz as the third frequency which was again considered to be either figure 4.37 or figure 4.39. The mesh with orthotropic properties gave 442 Hz (within 7% of the experimental value) as the first frequency, missed the second one and 1934 Hz as the third frequency which was closer to the experimental value (within 7%) than the isotropic result. Now this mesh was refined using sixty four elements and the analyses were repeated, both isotropic and orthotropic results were smaller than the sixteen elements mesh results. This was expected from the finite element analysis since a more flexible structure was considered.

Finally, sixty four semiloof elements, the whole tube with isotropic properties produced 476 Hz as the first frequency, 1272 Hz as the second frequency and 2407 Hz as the third frequency which was 16% higher than the experimental value and was considered to be either figure 4.37 or figure 4.39. It must be noted here that this 2407 Hz frequency value was produced as the second frequency by the sixteen elements mesh for the quarter of the tube. The sixty four elements mesh with the orthotropic properties gave smaller values as shown in figures 4.33 to 4.40.

Semiloof elements were found to be the required type and their first three mode shapes are illustrated in figures 4.33, 4.34, 4.35, 4.36, 4.37, 4.38, 4.39, and 4.40.
### TABLE 4.7: SUMMARY OF THE FINITE ELEMENT FREQUENCY RESULTS

<table>
<thead>
<tr>
<th>Number of elements</th>
<th>Specimen span (mm)</th>
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Figure 4.33
1st mode shape of clamped tube

Figure 4.34
1st mode shape of clamped tube
Mode shapes

2nd mode shape of clamped tube

Figure 4.35

2nd mode shape of clamped tube

Figure 4.36
Figure 4.37
3rd mode shape of clamped tube

Figure 4.38
3rd mode shape of clamped tube
Figure 4.39
4th mode shape of clamped tube

Figure 4.40
4th mode shape of clamped tube
4.17 TEST TECHNIQUES

Nondestructive tests were conducted on the glass fibre reinforced PES composite specimens to determine the impact force - time and acceleration - time relationships. This section describes the procedures and problems associated with the impact tests.

In attempting to develop a satisfactory test procedure, several different factors were examined:

1) For tests a) and b) below the type of rubber tip that is used to cushion the impact on the beam. Also in tests a) and b) different heights of drop were considered.

2) The span length of the clamped beam.

3) Method of load application: three different types of test were considered

(a) The Falling Weight Impact Test,
(b) The Swinging Hammer Test,
(c) The Impact Hammer Test.

4.17.1 RUBBER TIP HARDNESS

The reason for using rubber tips on the falling weight and hammer was to impose an impact force which contained a frequency band of low order values. This allows the force transmitted to the specimen to be accurately measured and represented in subsequent analysis. Five different hardness tips were investigated (see Appendix F).
A relatively stiff tip leads to a force characteristic similar to A whereas a more flexible tip gives a force characteristic like B.

To enable the stiffness of the end tips of the falling weight or the hammer to be empirically calibrated, a penetration into the material of a metal hemisphere of diameter 19mm was measured against the penetration load, applied by a back-loader odometer. Figure 4.42 shows the results obtained for the five different tips used.

To investigate the effect of the tip shape, the stiffness of a flat headed black rubber 1 tip was compared with a dome headed black rubber 2 tip (as shown in figure 4.43)

From the results of these tests and some use of different tips in impact tests the black rubber tip 1 was selected for use.
### Impact Surfaces

**THE ODOMETER TEST RESULT FOR FIVE IMPACT SURFACES**

**FIGURE 4.42 Deflection - Load graph**

<table>
<thead>
<tr>
<th>Load (kg)</th>
<th>Penetration (mm)</th>
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</thead>
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<td>.289560E -1</td>
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</table>

- **Green Rubber**
- **Black Rubber (1)**
- **Black Rubber (2)**
- **Pink Rubber**
- **Nylon Rubber**
**2 Black Rubbers**

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<thead>
<tr>
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**THE ODOMETER TEST RESULT FOR TWO IMPACT SURFACES**

**FIGURE 4.43 Deflection - Load graph**
4.17.2 **SPAN LENGTH**

After comparing the experimental results for spans in the range 100mm to 900mm it was clear that a span of 500mm should be used for the subsequent tests. Span lengths shorter than 500mm were too stiff and exhibited only small vibrations. Span lengths greater than 500mm exhibited a discontinuity in the applied impact force - time graph which was attributed to the effect of the supports or the system response. An example of this behaviour for a 900mm span is shown in figure 4.44.

No matter whether the falling weight or impact hammer test techniques was used for the two different boundary conditions which were the fixed end support and the free-free support, in the force versus time response data a discontinuity occurred at the peak impact force.

This type of discontinuity in the force - time graph was regarded as unacceptable. This was mainly because of the problems associated with representing this data response in subsequent analytical work.
Figure 4.44 Force - Time graph

Figure 4.45 Acceleration - Time graph
4.17.3 The Falling Weight Impact Equipment

The falling weight test apparatus was used to deliver impacts of different potential energy by varying the drop height. The falling weight consisted of a cylinder of perspex of diameter 18mm and height 45mm. Onto the lower end of this cylinder was bonded a cylindrical piece of aluminium, the same diameter as the perspex and on to this was seated a hardened mild steel ball bearing of diameter 12.6mm. The falling weight was contained within a thin walled plastic tube into which a longitudinal slot was cut to reduce air resistance effects. The tube was set up vertically above the test specimen and over the point to be impacted as shown in Figure 4.46.

Rubber tips of various stiffnesses were mounted on to the test specimen in series with a perspex saddle which encircled the tubes.

Figure 4.46 The Falling Weight Impact apparatus
Strain gauges or accelerometers were positioned under the specimen as shown in figures 4.47 and 4.48.

a) Falling weight impact Tester with a strain gauge.

![Diagram of load direction, force transducer, specimen, and strain gauge.]

Specimen span was 100mm.

**Figure 4.47 Longitudinal section**

b) Falling weight impact tester with an accelerometer.

![Diagram of load direction, force transducer, specimen, and accelerometer.]

Specimen span was 200mm.

**Figure 4.48 Longitudinal section**

The results of this method are shown in figures 4.50, 4.51 and 4.52.
Another falling weight configuration (figure 4.49) was used to monitor the deformational behaviour of the composite tube during impact tests; this method did not involve a knowledge of the impact force being applied to the structure. It was desirable for this test to reduce the mass in the middle of the composite tube to a minimum; consequently the force transducer was omitted.

These two test techniques whose longitudinal cross-section is shown below, exhibit distinct advantages, but no one technique demonstrated the capability of completely characterizing these composite tubes.

c) Falling Weight Impact Tester with only an accelerometer.

Specimen span was 600mm.

Figure 4.49 Longitudinal section
Figure 4.50 Force - Time graph
Falling Weight: 23.31 gr
Drop Height: 455 mm
Impact Surface: Black Rubber
Specimen Span: 200 mm

Figure 4.51 Acceleration - Time graph
Falling Weight: 23.31 gr
Drop Height: 455 mm
Impact Surface: Black Rubber
Specimen Span: 200 mm
Figure 4.52 Strain - Time graph

- Falling weight: 23.31 grams
- Drop height: 455mm
- Impact surface: Blackrubber
- Specimen Span: 200mm
4.17.4 The Swinging Hammer Test

The swinging Hammer method was used to determine the true impact force which was experienced by the specimen. By introducing this technique it was hoped that an improvement in the force-time and acceleration-time graphs could be achieved. The apparatus consisted of a mild steel half meter long lever arm with an adjustable head. This lever arm was hinged to a heavy support to allow it to swing and to impact the specimen. The adjustable head was designed to move along the lever arm creating smaller or larger impact energy. As shown in Figure 4.53 a force transducer was mounted on to either the adjustable head or the perspex saddle.

Figure 4.53 Swinging Hammer apparatus
Swinging Hammer with different test arrangements.

a) Spherical metal tip on hammer,
   Black rubber + Force transducer on specimen,
   Specimen span was 200mm.

   ![Figure 4.54 Transverse section]

b) Spherical metal tip +. Force transducer on hammer,
   Black rubber on specimen,
   Specimen span was 600mm.

   ![Figure 4.55 Transverse section]

One particular problem associated with this method was that the force-time response data gave more than one impact force due to a rebound effect.

This method led on to using the hand-held impact hammer.

Figures 4.56 and 4.57 show the results of the swinging hammer method.
4.17.5 The Impact Hammer

A Bruel & Kjaer impact hammer (type 8202) with a force transducer in which an impact tip was mounted was used. In this case the tip is provided with the hammer. When the member is excited by the hammer, energy is transferred to the member in a very short period of time giving a typical input force signal as shown in figure 4.61. The shape of this force signal depends upon the type of the hammer tip, mass of the hammer and the dynamic characteristic of the system under investigation. As the frequency bandwidth of the force spectrum is determined by the length of the signal, these characteristics will determine the cut off frequency of the excitation signal. The stiffer the hammer tip the shorter will be the signal and the wider will be the frequency span.

The force transducer built into the hammer measures the input force and an accelerometer mounted on the system measures the response. The advantages of using the impact hammer are:
1. No fixtures are required as was the case for the falling weight and swinging hammer tests.
2. It may be used in restricted spaces where the falling weight and the swinging hammer could not be used.

The disadvantages of the impact hammer include:
1. It has a very high crest factor which may drive the system beyond its region of linear response. The method is therefore not suitable for non-linear systems.
2. Since there is little energy input to the system it has poor signal to noise characteristics.
3. Skill is needed from the operator in order to achieve repeatable signals and to avoid multiple impacts.

Four different test methods using the Impact Hammer, were investigated in the laboratory and these are discussed in the
following steps numbered a, b, c and d.

a) Impact Hammer Tester with a rubber tip.

Specimen span varied from 400mm to 800mm.

Figure 4.58 Longitudinal section

Figure 4.58 shows the first test arrangement. It illustrates the use of a perspex boss which is fixed to the specimen by a threaded rod. An accelerometer is fitted to the other end of the rod. Experimental results obtained using this method are shown for two of the specimen of spans of 500mm and 600mm; figures figures 4.61 and 4.63 represent the impact force versus time response of the spans respectively, and figures 4.62 and 4.64 represent the corresponding output acceleration versus time response.

b) Impact Hammer Tester with a split thread.

Specimen span varied from 400mm to 800mm.

Figure 4.59 Longitudinal section
It was not certain whether the use of a continuous thread had any effect on the experimental results. To clarify this, the following test set-up was prepared using a split thread as shown in the figure 4.59. Figures 4.65 and 4.67 are plots of the impact force versus time response of the system and figures 4.66 and 4.68 are the respective results of the acceleration output versus time response. On comparing the relevant graphs it can be seen that they are essentially the same input for the amplitude of the impact force and acceleration output. So the conclusion was to use continuous thread for further tests.
Figure 4.61 Force - Time graph

Figure 4.62 Acceleration - Time graph
Figure 4.63 Force - Time Graph

Impact Hammer
Impact Surface Rubber Tip
Specimen Span 600mm

Figure 4.64 Acceleration - Time Graph

Impact Hammer
Impact Surface Rubber Tip
Specimen Span 600mm
Figure 4.65 Force - Time graph

Figure 4.66 Acceleration - Time graph
Figure 4.67 Force - Time graph

Impact Hammer
Impact Surface Rubber Tip
Specimen Span 600mm

Figure 4.68 Acceleration - Time graph

Impact Hammer
Impact Surface Rubber Tip
Specimen Span 600mm
c) Impact Hammer Tester with a strain gauge.

![Specimen with strain gauge](image1)

Specimen span varied from 300mm to 800mm.

**Figure 4.69 Longitudinal section**

This step illustrates the idea of no extra mass but simply a strain gauge bonded to the mid-point of the specimen.

d) Impact Hammer Tester with a Perspex Boss.

![Perspex boss with strain gauge](image2)

Specimen span varied from 300mm to 800mm.

**Figure 4.70 Longitudinal section**

Figure 4.71 shows the use of a perspex boss held in position by a thread insert and a steel strain gauge bonded to the mid-point of the specimen; this test set-up is shown in figure 4.70. The purpose of this arrangement was to investigate any change in the experimental results obtained using this test technique compared with step c). Again no change was detected and the results are shown in figures 4.72, 4.73, 4.74 and 4.75 for specimen span of 600mm. Figures 4.72 and 4.74 represent the force-time graphs and figures 4.73 and 4.75 represent the strain-time graphs.
Figure 4.71 Test specimen with split thread using Impact Hammer test arrangement
Figure 4.72 Force - Time graph

Impact Hammer
Impact Surface Rubber Tip
Specimen Span 600mm

Figure 4.73 Strain - Time graph

Impact Hammer
Impact Surface Rubber Tip
Specimen Span 600mm
Figure 4.74 Force - Time graph

Figure 4.75 Strain - Time graph
4.17.6 DISCUSSION OF IMPACT TEST RESULTS

These techniques were used to investigate the flexural impact vibration characteristics of glass fibre reinforced PES composite tubes for spans of 100mm to 900mm when the beams were encastre. The height of the falling weight was varied between 300mm to 455mm. Spans less than 500mm were impractical due to the very stiff nature of the system.

Some of the methods employed exposed shortcomings, these included inaccuracies such as double impact of the hammer the effect of the support fixities which gave discontinuities in the output graph.

There were several difficulties associated with different test techniques as can be clearly seen in the graphs obtained from different test set-ups. For example, see figures 4.50, 4.63, 4.67, 4.72 and 4.74 and also consider the first peak response in these figures. In the falling weight method figure 4.50 a discontinuity occurred after the peak impact force whereas this phenomenon occurred before the peak impact force in the impact hammer method. Also this discontinuity increased in magnitude as the specimen span increased in the impact hammer test technique. In addition to this, for the falling weight method an oscillation continued in the force-time graph.

From independent tests, it was shown that these discontinuities were the result of the end fixity of the specimens. In the light of this the tubes were clamped as tightly as practical and the discontinuities were then reduced to a minimum. These latter did not affect the maximum value of the load given by the maximum peak. The acceleration versus time response of the system is shown in figures 4.51, 4.62, 4.64, 4.66 and 4.68. A knowledge of the
natural frequency of the system is needed in the evaluation of the damping factor. It was difficult to determine the natural frequency of the system since these experimental results represented the superimposed higher order frequencies which were excited during the impact methods. To be able to determine the natural frequencies of the system it would have been necessary to obtain a transfer function of the input and output. At the time the impact tests were conducted the FFT software package was not available. Later measurements of natural frequency of the composite tube have been reported in section 4.11.

During the impact, which was of very short duration, extremely large forces act momentarily on the specimen at the points of impact and produce dynamic vibration in the specimen. It was necessary to find the vibrational behaviour of the specimen after impact. In dealing with this problem, it was regarded as justified to neglect, during impact, the mass of the accelerometer and the perspex saddle.

Considering the effect of damping which decreases the amplitude of the acceleration in the acceleration versus time graph, the most useful observation was the maximum amplitude of the acceleration in the first $1/2$ cycle (during the first millisecond) of the vibration. The accelerometer also provided a record from which it was possible to determine the damping factor of the specimen as well as providing data which can be used in the analysis of the deformational behaviour of the specimen at any time.

Further investigation into the test techniques showed that the results obtained using the impact hammer were sufficiently reliable and accurate so that they could be used for the analytical analyses such as determining the specimen damping factor (using the acceleration versus time response
of the system, see section 4.19 and figure 4.63). The impact load varied at each impact as it is impossible to control the energy input with hammers; this problem is not present with the falling weight impact tester.

The experimental results on the glass fibre reinforced PES are shown in figures 4.50 to 4.75. These figures illustrate examples of force-time, acceleration-time and strain-time curves for these composite tubes.

These results describe impact events for the various test techniques which were carried out in the laboratory. The voltmeter was used to record all output readings and the calibration was 10 mvolts per N.

The individual data is described as follows:

1. Force versus time curve obtained by a force transducer showing impact force trace when the specimen was impacted.

2. Acceleration versus time curve obtained by an accelerometer positioned in the mid-point of the specimen expressing modes of vibrations.

3. Strain versus time curve obtained by a strain gauge bonded to the mid-point of the specimen illustrating deformational behaviour.

From the study of the experimental results of different test arrangements and specimen span lengths, it was concluded that the impact hammer test technique would certainly provide adequate data for the subsequent analytical analyses.
4.18 IMPACT DAMPING FACTOR

Damping has been difficult to evaluate. It was appreciated that damping during testing had a considerable effect on the response of the system.

An approximate damping factor was determined experimentally by a number of methods. For example, the specimens, supported on different spans, were impacted and the acceleration at their mid-points was recorded. The decay of the amplitudes of the acceleration shown in figure 4.77 gave an indication of the percentage of critical damping of the specimen. Critical damping is dependent upon the geometry and the frequency of oscillation. Damping in the Lusas finite element system is introduced as a proportion of the mass and the stiffness of the structure, thus taking account of changes in damping due to geometry. The percentage of critical damping is related to these factors and proportions of both the mass and stiffness matrices were chosen by LUSAS for damping calculations.

The following shows an approximate method [Ref 120] of calculating the specimen damping factor, but for simplicity, this method was only carried out for a specimen span of 600mm with fixed end supports.

First of all, two specimen pieces were weighed:

<table>
<thead>
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<th>LENGTH (mm)</th>
<th>MASS (gram)</th>
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<td>99.88</td>
<td>18.38</td>
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<tr>
<td><strong>Average</strong></td>
<td><strong>99.87</strong></td>
</tr>
<tr>
<td></td>
<td><strong>18.35</strong></td>
</tr>
</tbody>
</table>

Assume a single degree of freedom system.

The equations of motion are:

\[ M \ddot{d} + C_d + K_d = P(t) \] (4.22)
where $M$ mass matrix
$C$ damping matrix
$K$ stiffness matrix
$P(t)$ applied nodal load vector at time $t$
$\ddot{d}$ acceleration vector
$\dot{d}$ velocity vector
$d$ displacement vector

In a damped vibration system, the solution of the above equation can be written as:

$$d = A \exp(-\frac{\nu}{\omega_n} t) \sin(\omega t + \phi) \quad (4.23)$$

where $\nu$ damping ratio = $C/C_c$

i.e. ratio of the actual damping constant to the critical damping ratio.

$$C_c = 2m \omega_n$$

$\omega_n$ circular natural frequency

$\phi$ phase angle

The equation (4.23) is plotted in figure 4.76; the displacement represented by the solid line, lies between the two envelope curves $\pm A \exp(-\frac{\nu}{\omega_n} t)$. It should be noted that these envelope curves will be of the same magnitude for velocity and acceleration versus time graphs. Therefore, acceleration = $\exp -\frac{\nu}{\omega_n} \text{displacement}$

$$\log_n \text{acc.} = -\frac{\nu}{\omega_n} \text{displacement}$$

which is a straight line with slope($\frac{\nu}{\omega_n}$).

This approximate numerical method covers the determination of the damping factor ($C$) of these glass fibre reinforced PES composite tubes by test techniques which were described in the section 4.17. The details of the various specimen and test configurations are discussed in the section 4.17.6.
Impact Damping Factor

\[ A \exp\left( -\gamma \omega_n t \right) \]

\[ -A \exp\left( -\gamma \omega_n t \right) \]

Figure 4.76 Free damped vibration system
Figure 4.77 Acceleration - Time graph
Damping Factor

![Damping Factor Graph]

**DAMPING FACTOR ANALYSIS** For specimen span = 600mm

**FIGURE 4.78** ln acceleration-Time graph
### SPECIMEN SPAN (mm) | MASS (gram)
---|---
100 | 18.37388605
200 | 36.74777210
300 | 55.12165815
400 | 73.49554420
500 | 91.86943025
600 | 110.2433163
700 | 128.6172024
800 | 146.9910884

A segment of this acceleration versus time graph (figure 4.77) was chosen for calculation of $K$ where $K = \frac{\gamma}{\omega_n}$. All the peak values that lie in this segment were noted (23 points) from which log acceleration versus time was plotted as shown in figure 4.78. Then the following procedures were followed to obtain the specimen damping factor i.e. $C$ value.

From figure 4.78: $K = 3.7/14 \times 10^{-2} = 26.43$

$T = 4 \times 10^{-3}$ seconds $\Rightarrow f = 1/T = 250$ Hz

$\omega_n = 2\pi f = 1570.80$ rad/second

But $K = \frac{\gamma}{\omega_n} \Rightarrow \gamma = \frac{K}{\omega_n} = 0.0168$

$C = 2mw = 346.34$ kg rad/s

Since $\gamma = \frac{C}{\omega_n} \Rightarrow C = \gamma \omega_n$

Hence for a specimen span 600mm, $C = 5.827$ kg/s.

It must be noted that this high value of damping factor includes the system damping (not just material damping) since the composite tube was fully clamped. Because of this, the system damping included the effect of the supports.
4.19 National Physical Laboratory Dynamic Tests

4.19.1 Dynamic Mechanical properties [Ref 114]

The dynamic properties may be specified by two basic quantities: the dynamic storage modulus (E' or G'), which provides a measure of the effective stiffness of the material and is proportional to the peak energy stored and recovered during each cycle of deformation, and, the loss factor (or damping factor, tan $\delta$) which is proportional to the ratio of net energy dissipated per cycle as heat to the peak stored energy.

The dynamic behaviour of a material is strongly dependent on frequency and temperature and provides valuable information on molecular motional frequencies, activation energies, and transition temperatures.

4.19.2 Audiofrequency Resonance Methods [Ref 115]

To enable a theoretical analysis to be undertaken it was necessary to determine dynamic mechanical properties of composite tubes. The two desired quantities: the dynamic storage modulus (E') and the loss factor (or damping factor, tan $\delta$) of two flat specimens (Glass fibre/PES and Carbon fibre/PES) were determined by means of resonance techniques. In this work the flexural vibration was employed.

The dynamic tests were made using free-free vibrational modes. For the flexural modes the specimen was suspended by two nylon loops (illustrated in Figure 4.79) which were individually positioned by hand at the calculated nodal points. Bring steel strips were bonded to the specimen in the positions shown in Figure 4.80. Sinusoidal forces were applied to one end of the specimen by means of an electromagnetic transducer fed by a variable frequency.
oscillator, the resulting alternating magnetic field produced a force upon the magnetic strip and so excited vibrations in the specimen. Detection of the vibrations was made by a proximitor probe, which faced another conducting strip, and yielded an output voltage proportional to the vibration amplitude.

In the case of flexural resonance only one transducer and one detector probe was necessary, and they were located beneath the specimen at the two respective ends.

The audiofrequency test method was undertaken as described in the following steps:

1. The length, width and thickness of the specimen were measured accurately, for example the thickness was measured in several positions with a sensitive micrometer and average value of thickness was taken.

2. The specimen was weighed.

3. The nodal positions of the specimen were calculated and the specimen was positioned on the nylon supports.

4. The nth resonance frequency \(f_n\) was found by increasing the frequency until a maximum output was obtained.

To enable a determination of loss factors to be undertaken the width of the resonance peak \(f_{u,n} - f_{l,n}\) where the \(f_{u,n}\) - \(f_{l,n}\) are frequencies above and below \(f\) at which the vibration amplitude is \(1/\sqrt{2}\) of the peak amplitude (3 dB lower) (shown in Figure 4.81).

Knowing \(f_{u,n}\) and \(f_{l,n}\), the dynamic storage moduli \(E'\), and loss factor \(\tan \delta\), were calculated using a computer package programme which contained the necessary formulae.

NOTE: For this flexural resonance \(E'\) and \(\tan \delta\) were measured at room temperature in modes 1, 2 and 3.
TABLE 4.8: Glass fibre/PES Specimen Specification

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<tr>
<th>Thickness (mm)</th>
<th>Width (mm)</th>
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</thead>
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<td>1.549</td>
<td>1.516</td>
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</table>

Specimen mass, \( M = 4.33 \) grams
(plus added mass) \( m + M = 4.36 \) grams
metal end mass, \( m = 0.03 \) grams
mass length (metal ends length) = 4mm
Specimen length, \( \ell = 164 \) mm
Poisson Ratio, \( \nu = 0.3 \) (guess value)

$$\text{string position}$$

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<tr>
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<th>( 1-2x )</th>
<th>( x )</th>
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</tbody>
</table>

Mode | Temp | Freq | \( E' \) | \( \tan \delta \) | \( E'' \) |
<table>
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<td>1184</td>
<td>21.80</td>
<td>1.521E-3</td>
<td>3.316E-2</td>
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</table>

The formula used to calculate damping factor is
$$\tan \delta = \frac{\tan \delta}{\text{f}^2} - \frac{\tan \delta}{\text{f}^2_n} = \frac{\Delta \text{f}^2}{\text{f}^2}$$

whereas previously damping factor calculated as follows
$$\lambda = 2 \pi \nu = \pi \tan \delta$$  \( \nu = \tan \delta \)

The damping is low but is comparable with other materials. Because of air damping and structural damping, the damping factor obtained experimentally is an effective damping factor (equal to 1.521E-03).
**TABLE 4.9: Carbon fibre/PES Specimen Specification**

<table>
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Specimen mass, \( M = 3.59 \) grams  
(plus added mass) \( m+M = 3.63 \) grams  
metal end mass, \( m = 0.04 \) grams  
mass length (metal ends length) = 4mm  
Specimen length, \( l = 164 \)mm  
Poisson Ratio, \( \nu = 0.3 \) (guess value)

<table>
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<th>n</th>
<th>x</th>
<th>1-2x</th>
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<td>132.9</td>
<td>58.3</td>
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</tbody>
</table>

Mode | Temp | Freq | \( E' \) | \( \tan \delta \) | \( E'' \) |
<table>
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</tbody>
</table>

The damping is low but comparable with other materials, and the value calculated is an effective damping factor (equal to 1.055E-3).
Figure 4.79 Support methods used in free-free vibration for (a) flexural resonance

Figure 4.80 Positions of spring steel strips on sample for (a) flexural resonance

Figure 4.81 Positions of $f_n, f_{n-1}$ and $f_{u,n}$ on peak
4.20 THE FINITE ELEMENT ANALYSIS OF IMPACT TESTING

Using the finite element package LUSAS the following results were obtained for the specimen span in constrained boundary conditions. Dynamic analysis using data from figure 4.61 was carried out to determine the deflection of the composite tube specimen.

Initially using SOLID elements gave poor data consequently 16 SEMILOOF elements were chosen to be the most suitable elements to model the 500mm composite tube specimen.

For this analysis a quarter of the composite tube was considered, because symmetry was considered to reduced unnecessary calculations and computer time. The accuracy of the finite element method was increased by halving the initial time step (dt = 0.00025 seconds).

The composite tube deflection from 15 load cases in the dynamic analysis is shown in figure 4.83.

Figure 4.82 The proposed finite element mesh
Initially, it was required for a good accuracy to choose a time step of 1/5 to 1/10 of largest natural frequency of the proposed mesh in the step-by-step dynamic analysis. This time step value was soon changed with the new modified version of LUSAS (version 86.07 May 1986) as explained below.

Within LUSAS two completely general procedures have been implemented and these cover most of the conventional procedures, such as Central difference, Newmark Beta, ..., etc. These procedures were:
(a) The three-point integration scheme.
(b) The four-point integration scheme.

It was recommended for conditionally stable integration scheme such as the three-point one, to use a time step given by

$$dt = \frac{2}{w_{\text{max}}}$$

where $w_{\text{max}}$ was the maximum eigenvalue of the system. However, an unconditionally stable algorithm should use a $dt/T$ smaller than about 0.01 where $T$ was the fundamental period of the system.

In this project, the three-point integration scheme (Newmark's method) with a series of different time steps was used.

It seemed impossible to determine the highest frequency of the proposed mesh by applying Iron's theorem. It was therefore, decided to use the experimental value of $dt$ (five milliseconds) which was halved four times in order to increase sensitivity of the LUSAS output. However, a time increment, $dt$ of 0.25 milliseconds was finally chosen for the dynamic analysis (figure 4.83).
Figure 4.83 15 load cases deflections
Semiloof elements had the distinct advantage that modelling of the whole composite tube was not necessary (only a quarter of it was considered for the step-by-step dynamic analysis) and accurate results were obtained by the use of relatively coarse mesh (only sixteen elements were used in the analysis).

The following figures illustrate the mid node responses of 500mm glass fibre reinforced PES composite tube which were obtained from the finite element computer outputs using the step-by-step dynamic analysis. The displacement versus time response data is given in figure 4.84 and the acceleration versus time response data is given in figure 4.85. As the accuracy of the finite element step-by-step dynamic analysis is time dependent, a time step of 0.0000625 seconds was used which was a quarter of the time interval separating experimental measurements of the acceleration.

The response of the tube was monitored for about 60 milliseconds but only a segment of the experimental force versus time graph which is presented in figure 4.61 was used in the step-by-step dynamic analysis to produce the displacement and acceleration versus time responses. This segment had 11 milliseconds of the time response and impact forces after 5.5 milliseconds were considered to be of zero magnitude. Comparing the finite element acceleration versus time response (for this segment) presented in figure 4.85 with the equivalent segment of the experimental acceleration versus time response, it was found the finite element results indicated acceleration values of 96g whereas the experimental results showed acceleration values of 60g for the first peak in the first cycle in both responses. This discrepancy may have caused by the fact that the sixteen semiloof elements mesh have lower frequency values. However, both responses are approximately in a good
agreement with each other for the 11 milliseconds segment.

The computer input data program which was used for the eigenvalue extraction analysis, is given in appendix D and its output is given in the table 4.7. In addition the computer input data program which was used for the step-by-step dynamic analysis, is given in appendix F and its nodal displacements output is shown in figure 4.83.
FIGURE 4.84 Displacement-Time graph

16 SEMILOOF ELEMENTS
DYNAMIC CONTROL 3
WITH ALL ROTATIONS

ACCELERATION-TIME RESPONSE

For time step = 0.00000625s
and specimen span = 500 mm
FIGURE 4.85 Acceleration-Time graph
5 DISCUSSION

Composites have many advantages when used as structural components, including high specific strength and stiffness, resistance to cracks by impact loading and good corrosion resistance. Thus, the use of composites should provide more durable structures compared with those manufactured from conventional materials. For example, in the aircraft industry, the constant demand for light weight efficient structures has led structural engineers to use materials, such as fibre reinforced composites.

The aim of this investigation was to characterize these composites by their mechanical properties. This was achieved by carrying out nondestructive test techniques under tensile, compressive and torsional loading.

The experimental part of the investigation started with the mechanical tests to ascertain the properties of the composite tubes which received attention in the second part of Chapter 3. The number of flat specimens used to obtain these mechanical properties was considered to be sufficient. It was also shown that the mechanical properties which were obtained using cylindrical specimens did not satisfy Maxwell's theorem, consequently they were not used for any analytical work. From the experimental results, it seemed that this discrepancy may have been caused by some fundamental difference in the way load is carried by fibres in a composite tube compared to a flat specimen. Alternatively the assumption of orthotropic properties may not be adequate.

Chapter 4 consists of two parts, namely part A which presents the plane strain fracture toughness testing and part B which shows the dynamic tests and analysis.

Engineering fracture mechanics can deliver the methodology to compensate the inadequacies of conventional design concepts. The
conventional design criteria are based on tensile strength, yield strength and buckling stress. These criteria are adequate for many engineering structures, but they are insufficient when there is the likelihood of cracks. Now, after approximately three decades of development, fracture mechanics has become a useful tool in design with high strength materials. Chapter 4 part A has a brief introductory summary of the concepts of engineering fracture mechanics. The rest of the Chapter (part A) deals with the determination of the fracture intensity factor, \( K \) for the composite materials under investigation in a plane strain condition. In principle, knowledge of the stress intensity factor, \( K \), for a crack in a particular structural element enables prediction of crack growth and fracture. In part A of Chapter 4 fracture analysis is used to obtain approximate values of the elastic parameters \( K \) and \( G \) (the strain energy release rate). All analyses are based on Linear Elastic Fracture Mechanics (LEFM) which is the case when the stress is low with respect to the yield stress for brittle materials. It must also be appreciated that the stress intensity factor, \( K \) cannot be measured directly in an experiment, but it can be found through the relations between \( K \) and a measurable quantity, such as the strain energy release rate, \( G \). This approach was used in the finite element analyses (both the LUSAS fracture analysis and the computer programme of reference 72) to obtain the required stress intensity factor. Unfortunately both the computer programme of reference 72 and LUSAS are designed to be used for homogeneous and isotropic types of material in the linear elastic fracture analysis. This surely underestimated the true values of stress intensity factor, \( K \), and the energy release rate, \( G \) for the glass fibre reinforced PES composite tubes in both analyses.

In order to improve the \( K \)-value in the finite element analyses, two different approaches were investigated. The first approach was the effect of element size on the accuracy of the finite element prediction of the stress intensity factor, \( K \). In order to do this, the mesh was refined around the crack region. Unfortunately the
K-value resulting from this refined mesh remained the same as before [Ref 90]. The second approach was the way failure load was applied to the mesh. Again this was done by applying the load through either a single node (node 138) or a number of nodes (nodes 136, 138 and 86 as shown in figure 4.17). This approach also did not improve the K-value [Ref 90]. Nevertheless, all information gathered so far seemed to emphasis the fact that these K and G values were calculated as accurately as possible for the proposed mesh.

As mentioned already, it was suspected that the stress intensity factor, K for mode I fracture may have been overestimated experimentally since it was discovered that the metal side support plates could not always prevent shear fracture, because, when the shear stress was high, the metal plates themselves could not maintain their planar shape. It was therefore concluded that these plates are only effective under a low load acting on the compact tension specimen.

With respect to the dynamic testing, the experimental tests and results (from Chapter 3) provided sufficient data for the finite element analyses which successfully modelled the mode shapes (as shown in figures 4.33 through to 4.40) and the deformational patterns (as shown in figure 4.83). The first four mode shapes were plotted for the front elevation as well as the plan view with a magnification factor of 1 by selecting 64 thin shell semiloof elements. The deformational patterns were also plotted by choosing 16 thin shell semiloof elements and taking their nodal displacement results from the LUSAS output. Then these displacement values were multiplied by a factor of 100 in order to draw an exaggerated deformational pattern. This procedure was carried out by the plotting facility associated with LUSAS (MYSTRO) which also produced the mode shapes.

Frequently in finite element analysis, in order to reduce arithmetic and computer time, data is presented for only a half or a segment of the structure. This means use of symmetry in any proposed finite
element mesh. To save computing time and data storage, it was decided to consider only a quarter of the composite tube when finding its natural frequencies. But as it can be seen from table 4.7, the use of symmetry led to missing a frequency in the quarter of the tube analysis which is present in the analysis of the whole tube. Basically, symmetry ignored the asymmetric bending mode and the squeezing mode shown in figures 4.37 and 4.39 respectively. Therefore, for the frequency analysis the 64 thin shell semiloof finite element mesh was used. However, it must be stated here that the use of symmetry did not create any difficulty for the step-by-step dynamic analysis since the applied loading was symmetric and did not excite the asymmetric modes.

Vibrations of a simple system (i.e. a glass fibre reinforced PES composite tube in a free-free condition) was undertaken to give natural frequencies of the system. These values were then compared with the LUSAS finite element analysis results (the eigenvalue extraction method) in order to establish the most suitable element to model the composite tube in an impact situation. The element which gave a good correlation between their results and that of the experimental values, was chosen to create the required mesh. This mesh was then used to study dynamic behaviour of glass fibre reinforced PES composite tubes by considering its deformational patterns (figure 4.83) under dynamic loads.

It was the objective of this investigation to establish experimental and analytical techniques for the assessment of glass fibre reinforced PES composite tubes when they are subjected to dynamic loads.

In this investigation a number of test techniques were undertaken but only two were felt worthy of further investigations, these are:

(1) Falling Weight Impact Tester

The span lengths of the test specimen were varied between
100 mm and 900 mm and the falling weight was a constant value of 23.31 grams for every span lengths investigated.

(2) Impact Hammer Test

The specimens were subjected to impact under a hammer action.

The above two test techniques provided data from the plots of Force versus Time for impact force and the Acceleration versus Time for deformational behaviour.

A selection of the experimental results which were obtained from different test arrangements and specimen span lengths was presented in part B of Chapter 4 to illustrate the vibration characteristics of glass fibre reinforced PES composite tubes. The best test technique was found to be the impact hammer which provided a set of reasonable results (force versus time and acceleration versus time graphs) for comparison with the finite element static, frequency extraction and step-by-step dynamic analyses.

In addition to this, the experimental results obtained using the impact hammer test technique were found to be sufficiently accurate to be used for determining the composite material damping factor (by the use of the acceleration versus time response of the system).

The damping factor of any material can be approximately obtained by a number of methods such as the method described in the case 2 of the free vibration section of reference 120 or the audiofrequency resonance method as explained in section 3 of reference 115. An approximate material damping factor was determined experimentally by both methods. But neither of the two values of damping factor were used in the analytical investigations here since there existed a number of inaccuracies in determining these values. For example, in the first method, it was found difficult to measure the true value of system time period, $T$, from the acceleration-time graph since this
response data represented all vibrational frequencies superimposed upon one another. Consequently, the natural frequency of the system was affected by the time period value, T. It must be also added here that the high value of damping factor measured by this method included the system damping (not just material damping) since the composite tube was fully clamped. Because of this, the system damping included the effect of the supports. In the second method, the fundamental natural frequency of the composite was guessed and the equipment was set to this value. This may have introduced error in determining the damping factor value. However, the material damping factor value, \( \mu \), did not have any significant effect on both the frequency extraction and the step-by-step dynamic analyses. On the contrary, the LUSAS user manual suggested that a damping factor of zero value be used, if necessary, to obtain any results.

The study of dynamic behaviour of glass fibre reinforced PES composite tubes was undertaken by using the step-by-step finite element dynamic analysis. Semiloof elements were found to be the most suitable elements to model the composite tubes. They had the distinct advantage that modelling of only a quarter of the composite tube was considered for the step-by-step dynamic analysis (with only sixteen elements in the mesh). The acceleration response of the mid node of the mesh was approximately in a good agreement with the experimental acceleration response.

A series of dynamic analyses with different time steps were carried out to increase the accuracy in the LUSAS output of nodal displacements. This finally produced the plot shown in figure 4.83. It must be noted that the nodal displacements were given for only 15 load cases because of limited computer data storage capacity. However, for these types of analyses, the given storage allocation was considered to be sufficient to draw a general trend for the deformational behaviour of glass fibre reinforced PES composite tubes under investigation here.
6 CONCLUSIONS

A structural material is required to be strong and stiff enough to carry loads without being deformed significantly by them. At the same time it should be easily shaped and joined to other components so as to form an engineering structure. It should be resistant to a range of corrosive environments and it should provide all these facilities as inexpensively as possible. For a considerable period of time metals provided the best compromise in meeting these requirements but, over the last few years, they have been supplanted for many applications by fibre reinforced composite materials. There are now many types of composite materials in commercial use that show particular advantages in certain applications.

Polyethersulphone (PES) resin has been characterized and developed as a matrix for fibre reinforced composites. This resin offers toughness as well as chemical resistance. A successful processing technique has been developed to form tubes using the film stacking technique. Previous thermoplastic composite development has led to a film stacking technique that has proven to be a simple as well as successful method. Characterization of resin properties including mechanical properties, chemical and environmental resistance have been performed and documented by Imperial Chemical Industries, limited (ICI). This resin is sold under the trade name 'victrex' PES by ICI.

In general, polyethersulphone appears to have good potential as a matrix resin for composites. It offers superior mechanical properties such as toughness especially where it is of primary value.

The fibre reinforced PES composite tubes are suitable as structural elements because they possess a unique combination of strength and toughness which are the two main factors required for structural applications.
In general, composites more than other materials require a comprehensive set of data, because of their complex composition, than other materials to characterise them adequately for design purposes and more detailed information about the conditions under which they are going to be used if they are to be successfully designed. Mainly there is a concerned need for further effort to develop or establish methods which would use the available data (such as mechanical properties) to predict, with a reasonably high degree of confidence, the behaviour of composite components in service.

Generally speaking, fibre reinforced composite materials have anisotropic elastic characteristics (i.e. properties). The idealised composite structure is orthotropic, having three mutually perpendicular planes of symmetry. The properties of the fibre reinforced PES composite tubes under investigation here are interpreted as orthotropic behaviour. It is concluded from the experimental tests that the mechanical properties which are obtained using flat coupon specimens, do satisfy Maxwell's Reciprocal theorem. This indicates that the assumption of orthotropic properties is adequate for using in the analytical analyses. This can be seen from the results of the finite element analyses such as dynamic analysis when they are approximately in good agreement with the experimental results.

The three different types of composite materials; glass fibre/PES, carbon/glass fibre/PES and carbon fibre/PES composite tubes are successfully characterised by their mechanical properties. This is achieved by using laminate theory to ascertain their mechanical properties. In addition to this, the assumption of orthotropic properties is adequate for these composite tubes when their flat coupon specimens are used for the mechanical testing instead of cylindrical specimens.
Strength and toughness are primary requirements in structural materials. The toughness of metals is achieved by a quite different process from that used in the design of fibre reinforced composites.

Fibre reinforced composites offer certain advantages over metals. This is because their failure mechanism is different from metal when they resist failure by the growth of a crack from a region of damage.

To study crack growth and fracture of these fibre reinforced composite materials, the fracture toughness testing is applied to them. Generally, to characterise these composite materials by their fracture mechanics parameters (i.e. fracture toughness and strain energy release rate), it is required to carry out standard fracture mechanics tests such as Charpy or Izod tests. Unfortunately, there are no accepted standard test method available for impact studies of composites. Therefore, the recommended procedure for plane strain toughness in the ASTM using reduced 'compact tension specimen' are used for the glass fibre reinforced PES composite materials.

The discrepancy between the experimental values of toughness and strain energy release rate and the analytical ones, is suspected to be from a few factors. For example, the proposed test technique over-estimates the stress intensity factor and the energy release rate for mode I because of the test arrangement. Use of homogeneous and isotropic properties in both the computer program and LUSAS package surely underestimate the true values of fracture mechanics parameters.

It is therefore concluded that reasonable values of fracture mechanics parameters will be ascertained when both a computer program and a standard test technique are designed for these composite materials with their orthotropic properties since the principal aim of the computer program and the test technique is to determine the stress intensity factor and the strain energy release rate.
Nowadays, vibration studies are more often required in the engineering practice, being decisive for the design of structural components. There are three main types of vibration measurement namely:

a) the measurement of vibration levels, i.e. of the system output, in order to compare them with standards or base-line data.

b) the analytical method for a system response to a given excitation (it is required to measure the input, which most often is an excitation force or torque).

c) the class of measurements which use known excitation and resulting response data. The problem is to find a mathematical model of the system. Usually, a single force is used and measured, in addition to the response, so that it is possible to derive the response characteristics of the system or component under test.

Type c (above) vibration measurement was investigated in this study and the objective was to perform:
(a) an identification of natural frequencies and mode shapes,

(b) measurements of specific dynamic properties of the system. The one of interest in this study was the damping factor.

(c) a finite element model of the analysed structure,

(d) a check of analytical natural frequencies against measured data and to undertake further analyses.

The three natural frequencies of the composite tube in a free-free condition is accurately obtained and this successfully led to the establishment of the most suitable elements which are thin shell semiloof elements, for modelling the test specimens in the analytical work.
SUGGESTION FOR FUTURE WORKS

1. Comparison of the analytical results produced by LUSAS with those from another finite element package e.g. ABAQUS. ABAQUS uses different shell and solid finite elements to LUSAS as well as employing more flexible and general ways of specifying orthotropic properties. This would serve as a valuable independent check of the accuracy of the analyses presented in this thesis.

2. Representation of material and structural damping in the analytical work (this would involve more experimental measurements of damping).

3. A study of the influence of the boundary conditions (free-free/clamped supports and symmetry) on the tubes behaviour from the experimental and analytical points of view.

4. Undertaking comparative results on glass/carbon/PES carbon/PES

4. Investigating notch sensitivity of a tube member impacted by a falling weight. It must be mentioned here that preliminary study was conducted on notch sensitivity of the composite tube, however it was difficult to make progress because there is no established interpretation of this test.

5. Manufacturing an element of a skeletal system and impacting one member when the system is under known boundary conditions.
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### INPUT DATA

Stress intensity factor evaluation for the compact tension specimen figure 4.17 using the virtual crack extension approach

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Stress intensity factor evaluation for the compact tension specimen figure 4.17 using the J-integral method

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Appendix C

LUSAS FRACTURE MECHANICS INPUT DATA COMPUTER PROGRAM

SYSTEM
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UNITS N mm KG
OPTION 18 25 44 91
QPM8 ELEMENT TOPOLOGY

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12 23 24 25 46 76 75 74 45
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Appendix C

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QPK8 GEOMETRIC PROPERTIES
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MATERIAL PROPERTIES
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SUPPORT NODES
1 17 1 F R
33 0 0 R F
51 0 0 R F
88 0 0 R F

LOAD CASE TITLE Y-dirn. load
CONCENTRATED LOAD
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ELEMENT OUTPUT
1 54 1 1 1

PLOT FILE
END
FREQUENCY ANALYSIS INPUT DATA COMPUTER PROGRAM

SYSTEM
NLPZ=350000
EXIT

PROBLEM TITLE NATURAL FREQUENCY ANALYSIS OF CANTILEVERED TUBE
UNITS N M KG
OPTION 2 18 74
QSL8 ELEMENT TOPOLOGY
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INC 1 2 2 2 1 2 2 2 1 4
INC 4 4 14 14 14 14 14 14 14 4

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SUPPORT NODES
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10 9 1 R R R R R
15 52 14 R F F F R
14 43 14 R F F R R
23 51 14 R F F R R
23 51 14 R F F R R
57 0 0 R R F R R
58 64 1 F R F R R
65 0 0 R R F R R

LOAD CASE TITLE Z-dirn. load
CONCENTRATED LOAD
1  9 1 0.0 0.0 0.0

EIGENVALUE CONTROL
CONSTANTS 10
PLOT FILE
END
ODEOMETER TEST PROCEDURES

The test method was as follows:
(1) the odometer was balanced,
(2) the deflection dial gauge was set to a reference point,
(3) loading process was started with 100 gram load; since lever arm ratio was 11 to 1 therefore the actual load on the rubber was 1100 grams or 1.10 kg,
(4) one minute was allowed and then the amount of penetration was recorded,
(5) the above procedures were repeated for 200, 300, 400 and 500 gram loads which was equivalent to 2.2, 3.3, 4.4, and 5.50 kg total load respectively.

Figure 2.6 Impact Surface set-up in an Odometer
THE STEP-BY-STEP DYNAMIC ANALYSIS INPUT DATA COMPUTER PROGRAM

SYSTEM
NLPZ=350000
EXIT
PROBLEM TITLE COMPOSITE
UNITS N M KG
OPTION 18
QSL8 ELEMENT TOPOLOGY
FIRST 1 1 2 3 11 17 16 15 10
INC 1 2 2 2 1 2 2 2 1 4
INC 4 14 14 14 14 14 14 14 14
NODE COORDINATES

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**QSL8 GEOMETRIC PROPERTIES**

1 16 1 0.0015 0.0015 0.0015 0.0015 0.0015 0.0015 0.0015 0.0015

**MATERIAL PROPERTIES ORTHOTROPIC**

1 16 1 6.73E+9 27E+9 2.98E+9 0.196 0 1589.09 80E-6 14E-6 0 0

**SUPPORT NODES**

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**CONCENTRATED LOAD**

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**ELEMENT OUTPUT**

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**NODE OUTPUT**

1 65 1 1

**DYNAMIC CONTROL 3**

**INCREMENTATION 0.00025**

**SUPPORT NODES**

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1 9 1 R R R R R R
10 52 14 R F F R R
15 43 14 R F F R R
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23 51 14 R F F R R
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58 64 1 F R F R R
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